A Hybrid Approach of Data-driven and Physics-based Methods for Estimation and Prediction of Fatigue Crack Growth

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ABSTRACT

Lamb-wave-based nondestructive testing and evaluation (NDT/E) methods have drawn much attention due to their potential to inspect plate-like structures in a variety of industrial applications. To estimate and/or predict fatigue crack growth, many research efforts have been made to develop data-driven or physics-based methods. Data-driven methods show high predictive capability without the need for physical domain knowledge; however, fewer data can lead to overfitting in the results. On the other hand, physics-based methods can provide reliable results without the need for measured data; however, small amounts of physical information can worsen their predictive capability. In real applications, both the measurable data and the physical information of systems may be considerably limited; it is thus challenging to estimate and/or predict the crack length using either the data-driven or physics-based method alone. To make use of the advantages and minimize the disadvantages of each method, the work outlined in this paper aims to develop a hybrid approach that combines the data-driven and the physics-based methods for estimation and prediction of fatigue crack growth with and without Lamb wave signals. First, with Lamb wave signals, a data-driven method based on signal processing and the random forest model can be used estimate crack lengths. Second, in the absence of Lamb wave signals, a physics-based method based on an ensemble prognostics approach and Walker's equation can be used to predict crack lengths with the help of the previously estimated crack lengths. To demonstrate the validity of the proposed approach, a case study is presented using datasets provided in the 2019 PHM Conference Data Challenge by the PHM Society. The case study confirms that the proposed method shows high accuracy; the RMSEs for specimens T7 and T8 are calculated as 0.2021 and 0.551, respectively. A penalty score is calculated as 7.63; this result led to a 2nd place finish in the Data Challenge. To the best of the authors' knowledge, this is the first attempt to propose a hybrid approach for estimation and prediction of fatigue crack growth.

1. INTRODUCTION

Nondestructive testing and evaluation (NDT/E) methods have attracted a great deal of attention due to their ability to inspect machines, vehicles, and structures (Büyüköztürk & Taşdemir, 2012). A number of different NDT/E methods have been developed over several decades. Among the various techniques available, Lamb waves, which are guided elastic waves that propagate along thin, plate-like structures, have provided a convenient method for prompt and

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continuous inspection (Cawley & Alleyne, 1996; Yashiro, Takatsubo, & Toyama, 2007). In particular, the Lamb-wave-based NDT/E method has been employed for inspecting riveted structures. When transducers are permanently attached to these structures, this method has the advantages of being able to continuously monitor large areas with a small number of transducers (Konstantinidis, Wilcox, & Drinkwater, 2007; Wilcox, 1998). This method has been used to detect and locate fatigue cracks in structures that are caused by repeated cyclic loading.

Many research efforts have been made to develop data-driven or physics-based methods for estimation and prediction of fatigue crack growth. As data-driven methods, Mohanty *et al.* presented principal component analysis to extract features from Lamb wave signals and proposed the Bayesian-based Gaussian process approach to predict fatigue crack growth (Mohanty, Chattopadhyay, Peralta, & Das, 2011). Wang *et al.* demonstrated three different machine learning algorithms (i.e., extreme learning machine, radial basis function network, and a genetic algorithm optimized back propagation network) for fatigue crack growth models (Wang, Zhang, Sun, & Zhang, 2017). As physics-based methods, Maslouhi explained an approach by which Lamb wave signals could be incorporated into the empirical model (i.e., Nasgro model) of fatigue crack growth (Maslouhi, 2011). He *et al.* presented Lamb-wave-based crack length quantification using finite element simulations (He, Ran, Liu, Yang, & Guan, 2017). To quantify crack length, a response surface with damage-sensitive features (normalized amplitudes and phase change) was developed using finite element simulations.

The data-driven method shows high predictive capability without the need for physical domain knowledge; however, fewer data (e.g., Lamb wave signals) can lead to overfitting in the results. On the other hand, the physics-based method can provide reliable results without the need for measured data; however, a small amount of physical information (e.g., geometric dimensions and material properties of specimens) can worsen the predictive capability. In real applications, both the measurable data and physical information of systems are considerably limited; it is thus challenging to estimate and/or predict the crack length using either the data-driven or physics-based method alone. To make use of the advantages of each method, while minimizing the disadvantages, the research outlined in this paper aims to develop a hybrid approach that combines both data-driven and physics-based methods for estimation and prediction of fatigue crack growth with and without Lamb wave signals.

First, when the Lamb wave signals are given, the data-driven method is considered, with signal pre-processing and the use of a random forest model. A set of features is extracted from the pre-processed signals. Then, a random forest model is used to estimate crack lengths via optimal feature selection and grid-search-based hyper-parameter optimization. Next, compared with the data-driven method case, there could be two different situations: one is under the same loading condition and the other is under a different loading condition. Therefore, different physics-based approaches are used to predict the crack lengths without the Lamb wave signals. Using the assumption that similar fatigue crack growth patterns occur under homogeneous loading conditions, an ensemble prognostics approach with simplified particle-filter-based weight updating is used to predict the crack lengths. In contrast, when the loading conditions are different, it is difficult to use the information from the training specimens because the crack propagation patterns would be different. Therefore, it is necessary to predict crack lengths for the next cycles, only using the previously estimated crack lengths for the corresponding test specimen. In this study, Walker's equation-model-based approach is chosen and Monte Carlo methods are used to predict the remaining crack lengths for the case of different loading conditions. To the best of the authors' knowledge, this is the first attempt to propose a hybrid approach for estimation and prediction of fatigue crack growth.

To demonstrate the validity of the proposed approach, a case study is presented using datasets provided in the 2019 PHM Conference Data Challenge by the PHM Society. The ultimate goal of the Data Challenge problem was to estimate crack lengths of a few loading cycles using the given Lamb wave signals under constant loading conditions and, further, to perform crack prediction without the signals for two validation specimens (T7 and T8) under different loading conditions (i.e., constant and variable loading conditions for specimens T7 and T8, respectively).

The remainder of this paper is organized as follows. Section 2 provides a brief review of the problem description outlined for the 2019 PHM Conference Data Challenge. Section 3 demonstrates the proposed hybrid approach that combines data-driven and physics-based methods. Validation of the proposed method is covered in Section 4. Finally, the conclusions of this paper are provided in Section 5.

2. PROBLEM DEFINITION

2.1. System Description

Figure 1 shows the system description outlined for the Data Challenge. Piezoelectric sensors (i.e., an actuator and a receiver) were placed on the aluminum lab joint specimens to test the growth of fatigue cracks. The distance between the actuator and the receiver was 161 mm. The actuator generated a tone burst signal of a few cycles, and a Lamb wave propagated along the path. The receiver measured the propagating Lamb wave signals. If a crack formed along the path, local geometry deformation (caused by the crack) would make the received signals different, as compared to the signals that propagate in the absence of a crack.

Training Set												Test Set	
T1		T2		T3		T4		T5		T6		T7	T8
Cycle	**Crack (mm)**	**Cycle**	**Crack (mm)**	**Cycle**	**Crack (mm)**	**Cycle**	**Crack (mm)**	**Cycle**	**Crack (mm)**	**Cycle**	**Crack (mm)**	**Cycle**	**Cycle**
50000	0	50000	0	14000	0	55900	0	42000	0	55000	0	36001	40000
60000	2.18	70033	3.25	50000	0	60200	1.61	46000	0	60078	0.82	40167	50000
62500	2.76	72000	4.95	57038	2.57	65001	2.17	51000	2.7	68091	2.36	44054	70000
65500	3.51			60035	4.02	67054	2.74	56000	3.64	69018	3.36	47022	74883
69025	4.51			62017	4.72	70016	3.13			72516	4.65	49026	76931
70026	4.90			64019	5.49	71130	4.06			73211	5.08	51030	89237
70766	7.46			65029	5.9	73210	4.96					53019	92315
				66012	6.52	75045	7.24					55031	96475
				66510	6.93								98492
													100774

Table 1. Given datasets (crack length and cycle) for specimens T1-T8

2.2. Data Description

In the Data Challenge, specimens from the training datasets are labeled from T1 to T6; those from the validation datasets are labeled T7 and T8. Training datasets from all specimens consist of (1) crack lengths and (2) Lamb wave signals measured by actuators and receivers for (3) the corresponding cycles. Lamb wave signals were measured two times in the time domain. For example, the signals for specimen T1 are presented in Figure 2. For the validation datasets of specimens T7 and T8, Lamb wave signals for some cycles were not provided. The signals for T7 are only given for cycles 36001, 40167, 44054, and 47022; the signals for T8 include only cycles 40000, 50000, 70000, 74883, and 76931. Table 1 summarizes the given cycles for specimens T1-T8 and crack lengths for specimens T1-T6.

Note that specimens T1-T7 were tested under constant loading, while the remaining specimen (T8) was tested under variable loading. Figures 3 (a) and (b) depict the constant and variable loading conditions, respectively. The minimum and maximum values of the sinusoidal stress-constant loading in specimens T1-T7 are 4.77 MPa and 100.21 MPa, respectively. On the other hand, in the fatigue loading spectra under variable loading conditions, the initial 500 cycles have a maximum value of 90 MPa and a minimum value of 4.77 MPa. The final 500 cycles have a maximum value of 100.21 MPa and a minimum value of 4.77 MPa.

2.3. Scoring Process

The main objective of the Data Challenge was to minimize the discrepancies between the estimated crack lengths and the true crack lengths for specimens T7 and T8; the true crack lengths were not provided to participants during the Data Challenge. For scoring, the 2019 Data Challenge included three main penalty functions: (1) a time penalty function; (2) an asymmetric penalty function; and (3) a monotonicity penalty function. First, the time penalty function was defined as

$$T(i) = 2 + 10x_i \tag{1}$$

where x_i stands for the true crack length. This penalty function penalized prediction error at the end of life more than at the initial stages of crack growth. Second, the asymmetric penalty function was defined as

$$\begin{aligned} A(i) &= \exp\left(\frac{|\tilde{x}_i - x_i|}{0.5}\right) - 1,\ if\,(\tilde{x}_i - x_i) \geq 0 \\ A(i) &= \exp\left(\frac{|\tilde{x}_i - x_i|}{0.2}\right) - 1,\ if\,(\tilde{x}_i - x_i) \geq 0 \end{aligned} \tag{2}$$

where $\tilde{x}_i$ stands for the estimated crack length. This penalty function penalized underestimation of crack length more than overestimation, since underestimation has dire consequences. Lastly, the monotonicity penalty function was defined as

$$\begin{aligned} M(i) &= 1 + 10 \times (|\tilde{x}_i - \tilde{x}_{i-1}|),\ if\,(\tilde{x}_i - \tilde{x}_{i-1}) < 0 \\ M(i) &= 1,\ if\,(\tilde{x}_i - \tilde{x}_{i-1}) \geq 0 \end{aligned} \tag{3}$$

This penalty function penalized when the estimated crack lengths did not follow the physics of the monotonic trend of the crack growth problem. The penalty score $S(i)$ for any cycle was defined as the multiplication of the three penalty functions. The overall penalty score S_{sum} was calculated by a cumulative summation, specifically

$$\begin{aligned} S(i) &= T(i) \cdot A(i) \cdot M(i) \\ S_{\text{sum}} &= \sum_{i=1}^{N} S(i) \end{aligned} \tag{4}$$

Thus, a perfect cumulative penalty score was 0, and would occur when the predicted crack lengths were exactly equal to the true crack lengths.

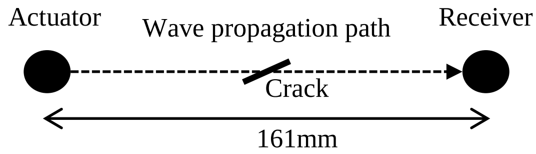


Figure 1. Description of the system examined to test the fatigue crack growth

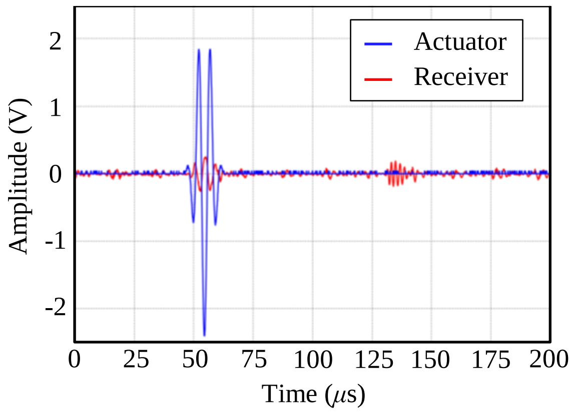
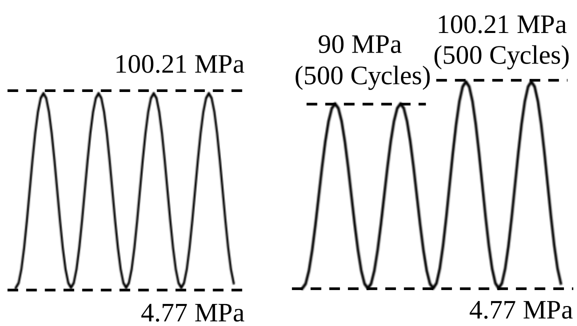


Figure 2. Lamb wave signals measured by the actuator and receiver for specimen T1



(a) (b)

Figure 3. Loading conditions in the experiment: (a) constant and (b) variable loading

3. METHOD

3.1. Flowchart of the Proposed Hybrid Approach

Figure 5 presents the overall flowchart used for the proposed hybrid approach that combines data-driven and physics-based methods to estimate and predict fatigue crack growth with and without Lamb wave signals. When Lamb wave signals are measured, the proposed data-driven method enables estimation of crack lengths. The flowchart of the data-driven method consists of five steps, including: (Step 1) pre-processing of Lamb wave signals, such as via band-pass filter and phase alignment; (Step 2) feature extraction based on physical interpretation; (Step 3) development of a random forest model; (Step 4) K-fold validation for hyper-parameter optimization and optimal feature selection; and (Step 5) crack length estimation. Details of each step are provided in Section 3.2.

Using the crack length data estimated by the data-driven method, the proposed physics-based method enables prediction of crack lengths without Lamb wave signals. The first step is pre-processing of previously estimated crack length data to normalize the crack cycles where the crack occurs. After this step, compared with the data-driven method case, there could be two different situations: one is under the same loading condition and the other is under a different loading condition. Assuming that the homogeneous loading condition leads to similar fatigue crack growth patterns, an ensemble prognostics approach with simplified weight updating based on the particle filter is used. The ensemble prognostics approach consists of two steps, including: generating a probability density function (PDF) for each particle and calculating the weights from each PDF. For the case of a different loading condition, Walker's equation is used. Walker's equation approach consists of two steps, including: linear regression of crack lengths for the first several cycles and model constant estimation. Finally, the crack lengths for the remaining cycles can be predicted. Details of the two physics-based models are provided in Section 3.3.

3.2. Data-driven Method with L*a*mb Wave Signals

The field of Prognostics and Health Management (PHM), utilizes several data-driven methods, including signal processing, machine learning, and deep learning (Benkedjouh, Medjaher, Zerhouni, & Rechak, 2013; Ha et al., 2016; Oh, Jung, Jeon, & Youn, 2017). Even though deep-learning-based PHM techniques have the advantage of autonomous feature extraction, a significant amount of data is required to successfully perform the required tasks (e.g., classification and regression). In the Data Challenge, however, there are some issues to consider related to deep learning, in particular: (1) the small number of specimens in the training datasets, (2) the irregular trends of the cycles, and

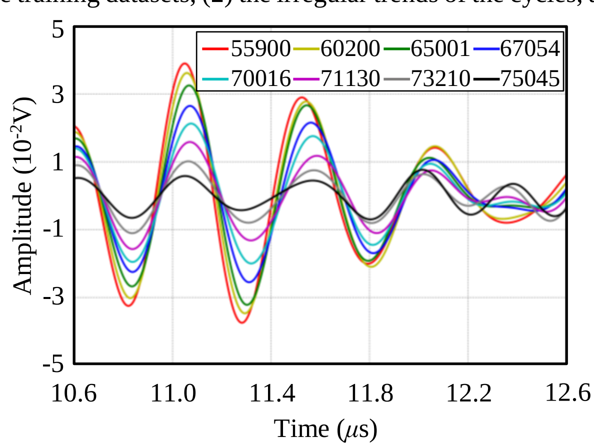


Figure 4. Pre-processed Lamb wave signals in specimen T4

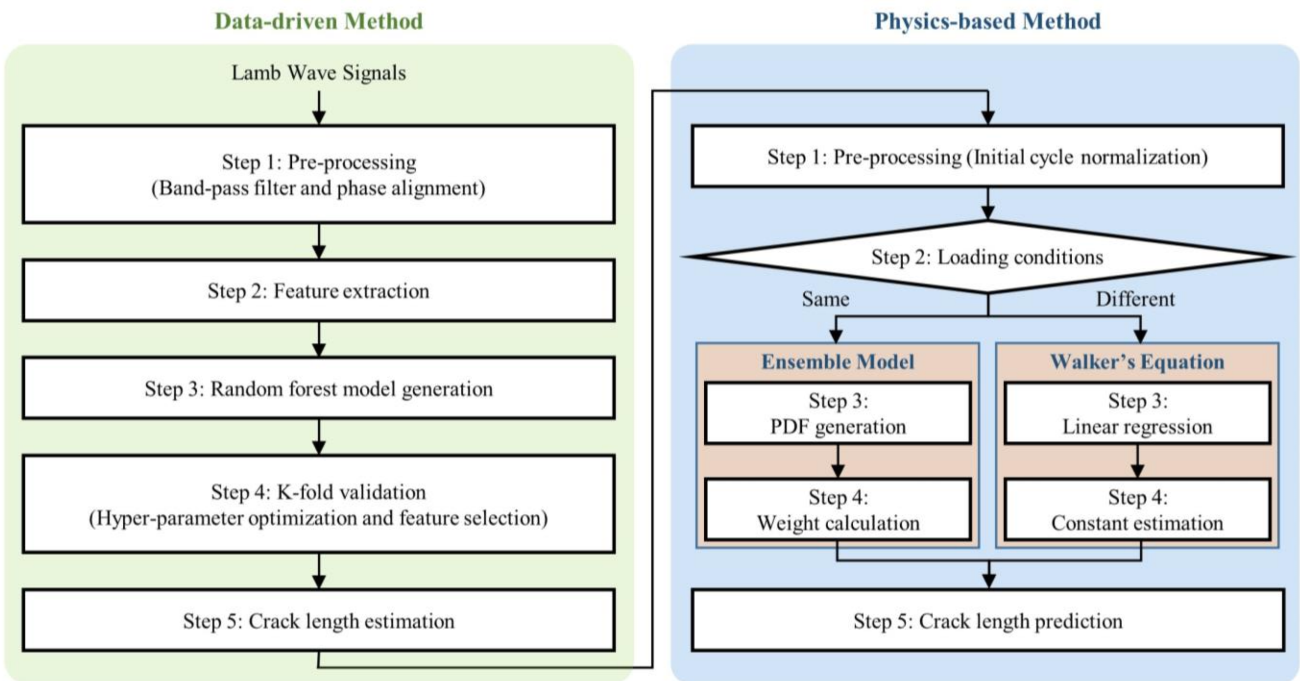


Figure 5. A flowchart of the proposed hybrid approach for estimating and predicting the fatigue crack growth

(3) the specimen-dependent data characteristics. These issues make it difficult to apply deep learning to develop a generalized predictive model in this case. Thus, we proposed a data-driven method based on signal processing and machine learning. First, pre-processing was needed, as described in Section 3.2.1. Then, several features were extracted, as outlined in 3.2.2. Using the features, a random forest model and *k*-fold validation are presented in Section 3.2.3 and 3.2.4., respectively.

3.2.1. Pre-processing

It is worth noting that there are two key points in the given raw data. First, the raw signals are contaminated by various sources, such as environmental noise, boundary reflections, complex Lamb wave propagation, and local geometry deformation. Second, since the piezoelectric sensor pairs (actuator and receiver) are different for each specimen, the different distances between the sensor pairs result in phase differences between the specimens.

To solve these problems, two pre-processing techniques were considered: (1) band-pass filter and (2) phase alignment. First, band-pass filter was used to remove the noise. The frequency of the noise ranged from 150 kHz to 350 kHz. When applying the band-pass filter to raw signals, which were measured two times, the two denoised signals were found to be very similar. Therefore, only the first measured signals were used for the training datasets. Second, based on the maximum value of the actuator signals, the phases of the actuator signals were aligned across both specimens. This phase alignment was applied to the signals measured by all actuators and receivers for all cycles. The signal processing technique reduced both noise and uncertainty. Furthermore, the separation between the signals was noticeable in a certain time range; the principal S0 mode of the Lamb wave was observed in this range. For example, Figure 4 depicts pre-processed Lamb wave signals of specimen T4 in the time domain of interest. Several features were extracted in this range; details of these features are covered in following subsection.

3.2.2. Feature Extraction

Feature extraction is a basic step in which factors that reflect the characteristics of the signals are obtained (Jeon, Jung, Youn, Kim, & Bae, 2015). Based on physical interpretation of crack length effects on the received Lamb wave signals, three important assumptions were considered. First, the energy of the received signals decreased as the crack size increased; this is due to a partial reflection of the signal at the interface of the crack (Staszewski, Lee, & Traynor, 2007). Therefore, the amplitude change might be a property of importance when estimating the crack length. Features associated with the Lamb wave energy loss were extracted, including (1) maximum amplitude, (2) maximum energy, and (3) dynamic time warp residual energy. Second, the phase change takes place due to scattering (i.e., reflection and transmission) at the crack location and the crack-length-dependent traveling distance (He et al., 2017). Features associated with the phase change were extracted, including (1) cross-correlation time lag, (2) point time delay, and (3) dynamic time warp distance. Next, the correlation between specimens with and without a crack decreased as the crack length increased; this is because discontinuities at the crack location result in distortion of the shape of the transmitted Lamb wave (Le Clézio, Castaings, & Hosten, 2002). Therefore, the correlation coefficient was extracted for the loss of the similarity properties. Finally, since fatigue crack growth is a sequential process, only the previously estimated crack length was additionally taken into account. Considering the different ranges of the features, they were standardized with a standard normal distribution.

We thoroughly investigated the trends of all extracted features for all cycles in specimens T1-T6. Since the distributions of features in the T5 specimen were significantly different from the others, the T5 specimen was considered an outlier. Therefore, the T5 specimen was excluded from the following data-driven model.

3.2.3. Random Forest Model

This subsection describes an ensemble-based model, a so-called random forest model. As compared to a single model, the ensemble-based model has the merit of reducing the possibility of overestimation. In an ensemble model, multiple models are trained to solve the same problem and combined for the purpose of getting better results. The random forest model has been widely used. The random forest model constructs multiple decision trees for the training datasets and yields the mean value of the crack lengths estimated in individual trees for the validation datasets. By averaging the results obtained in the various models, it guarantees a high generalization performance.

In this study, the random forest model uses the n_{tree} bootstrap sample data of source samples to build a variety of n_{tree} decision tree models with randomness. The random forest model is robust against overfitting, as compared to artificial neural networks or support vector machines. The random forest model was implemented by scikit-learn packages in Python (Pedregosa et al., 2011). It should be noted that there are two important hyper-parameters required to achieve high performance in estimating the fatigue crack growth: (1) maximum depth and (2) number of trees.

3.2.4. *K*-Fold Validation

Since neither the T7 nor the T8 specimen is included in the training specimens (T1-T6), any proposed model should offer good performance when applied to general specimens. Therefore, it is of great importance to minimize performance metrics (loss function) through optimal feature selection and hyper-parameter optimization. Keeping this purpose in mind, a *k*-fold cross validation technique was used with a randomly selected set of features from specimens T1-T6 (except T5).

As shown in Figure 6, the training datasets were partitioned into five sub-datasets that correspond to each specimen in the *k*-fold cross validation. Of the five sub-datasets, a single dataset was regarded as the test data; the remaining four sub-datasets were considered training datasets. The *k*-fold cross-validation was thus performed five times. The performance metric *PM* was defined as

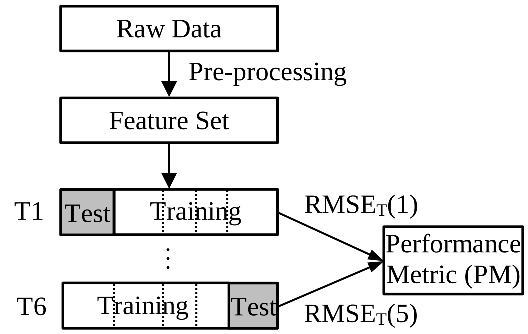


Figure 6. A schematic diagram of the *k*-fold cross validation

$$PM = \frac{1}{5}\sum_{i=1}^{5} RMSE_T\left(i\right) \tag{5}$$

where $RMSE_T(i)$ stands for the root mean square error (RMSE) between the predictive crack lengths and the true crack lengths for all cycles of the i^{th} specimen (here, the 5^{th} specimen is T6).

For a certain set of features, the grid-search-based hyper-parameter optimization was executed to prevent an overfitting problem. The objective function was to minimize the performance metric *PM*. The search spaces of maximum depth and number of trees, which are discrete variables, were set as n_{depth}={1,2,3} and n_{trees}={5,10,15,20,25}, respectively. The grid search optimization provided not only the performance metric *PM* but also a set of optimal hyper-parameters for a certain set of features.

Next, it is desirable to select the best features; this is because highly correlated features can worsen the predictive performance (Yu & Liu, 2003). Thus, we randomly selected a set of features and repetitively performed the hyper-parameter optimization. By comparing the calculated performance metric *PM*, the five optimal features were selected as (1) maximum amplitude, (2) maximum energy, (3) phase delay, (4) correlation coefficient, and (5) previous crack length information. This set of features includes all properties mentioned in Section 3.2.2.

It should be noted that the random forest model with optimal feature selection and hyper-parameters is a stochastic model. This model leads to inherent predictive uncertainties, even though the same input features are given. Therefore, 20 independent models were ensembled. Finally, the ensemble model estimated the fatigue crack length with high regression performance when Lamb wave signals were given.

3.3. Physics-based Method without Lamb Wave Signals

When Lamb wave signals are not given, the regression model must be developed using only the estimated crack length and

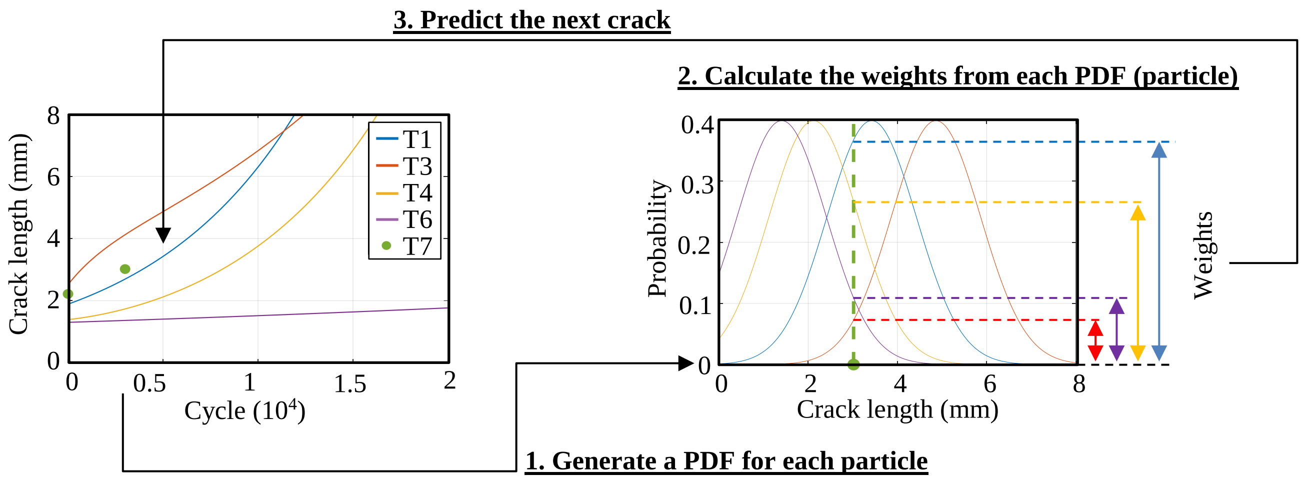


Figure 7. An example of simplified particle-filter-based weight calculation for prognostics

cycles. However, due to the different loading conditions of specimens T7 and T8, two different approaches were considered. For the T7 data set, an ensemble prognostics method with simplified particle-filter-based weight updating was used, as the loading condition was identical to that of the training specimens (T1-T6). In contrast, Walker's method based on the equation model was used to estimate the crack length for specimen T8, since the loading condition was different. For both approaches, pre-processing occurred first, as described in Section 3.3.1. The details of each approach are described in Sections 3.3.2 and 3.3.3, respectively.

3.3.1. Pre-processing

The first step for both approaches was to normalize the validation datasets of specimens T7 and T8. Referring to the studies of the effects of a sinusoidal, stress-constant amplitude on fatigue crack lengths (Rolfe & Barsom, 1977), it should be emphasized that the initial crack length corresponds to the zero cycle in the field of fatigue crack initiation and propagation. In Section 3.2, the data-driven method is used to estimate the initial nonzero crack length at a certain cycle, denoted as N_{initial}. Assuming that the crack initiates at cycle N_{initial}, N_{initial} (44054 in specimen T7 and 70000 in specimen T8) is subtracted from the cycles in Table 1, while the cycles that do not have the crack are excluded.

3.3.2. Ensemble Prognostics Approach

An ensemble-based prognostics approach was used to estimate the crack length of specimen T7. Since the loading condition of T7 is identical to that of specimens T1-T6, we assumed that the trend of crack propagation would be similar. Thus, using a weighted sum of the models for T1 through T6, the crack length of specimen T7 was estimated. The prognostics approach utilized the following two steps.

First, exponential models of T1 through T6 were established. The sum of two exponentials were used as the model, as

$$a_N = A_0 \exp\left(A_1 N\right) + B_0 \exp\left(B_1 N\right) \qquad (6)$$

where a_N is the crack length, N is the number of fatigue cycles, and A_0, A_1, B_0, and B_1 are the parameters of the model. These two exponentials can represent more complex fatigue crack propagation trends than a single exponential model. Specifically, the model outlined above fitted T1-T6 data sets for the latter cycles, where the penalty weights were high. Thus, these two exponentials were used as the ensemble models. Note that specimens T2 and T5 were not used, as the data sets did not have enough data points for the regression.

Typical ensemble prognostics derives the weight from the difference between the true value and the values from each model found in the prior step. It should be noted that fatigue crack growth is a nonlinear engineering problem. A Kalman filter method, one of the most widely used techniques in the field of prognosis modeling, has the ability to analytically find model parameters for linear systems. It is well known that high accuracy can be guaranteed only for linear systems. Due to the nonlinear trends of fatigue crack growth with respect to the number of cycles, we used the simplified particle filter approach. Since this approach is a simulation-based prediction technique that is based on trial and error, it is appropriate for – and often used for – non-linear systems.

Figure 7 presents the process of the simplified particle-filter-based ensemble prognostics approach. First, the exponential model (Eq. (6)) of each specimen (T1, T3, T4, and T6) can be used to interpolate the crack length for an arbitrary cycle. Through the exponential models, the crack lengths of the specimens for prior cycles of specimen T7, in which Lamb wave signals are given, can be estimated. Next, the Gaussian distribution PDFs are generated, as shown in Figure 7; the mean of each PDF is equal to the estimated crack length for each specimen at the certain cycle. It should be noted that this suggested approach limits the maximum weight, as a Gaussian distribution has a maximum value at the mean; if there is no limitation of maximum weight with only a few models, the weight could quickly converge to one of the models and it may lead to a large error at the end. The weight of each specimen can be calculated from the value of the generated PDFs at the crack length of specimen T7. Finally, the summation of crack lengths at the next cycle, multiplied by the weights for each specimen, provides the predicted crack length.

The advantages of this proposed method are as follows: (1) all specimens can be considered through the limited weight, rather than requiring that the predicted crack lengths follow the results of a specific specimen with similar tendencies, and (2) the crack length can be predicted considering the uncertainties from the generated PDFs, which can be found from a small number of specimens.

3.3.3. Walker's Equation Model

Many empirical models have been developed for characterizing fatigue crack growth rate curve in the field of fracture mechanics (Bannantine, Comer, & Handrock, 1990). The most common models are the Paris equation, the Walker's equation, and the NASGRO equation. It should be noted the Paris equation does not account for the stress ratio since it's only for zero minimum stress of loading conditions. On the other hands, the Walker's equation and the NASGRO equation are generalized Paris equation to take stress ratio effects into account. However, since the NASGRO equation has more model parameters than the Walker equation, it can lead to serious uncertainties if the data is not enough to estimate model parameters. Therefore, in this study, the Walker's equation is under consideration.

Using normalized datasets of specimen T8, three estimated crack lengths are valid for predicting the remaining five crack lengths. To apply Walker's equation models, several model parameters (or coefficients) should be determined to allow an accurate model to be obtained. However, three datasets are not enough to acquire parameters of an accurate model, as the variance of the parameters is too large. To predict the crack lengths for cycles 89237 and 92315, a simple linear regression model was thus used. Thus, five crack lengths were used to build the Walker's equation model.

Denoting the initial crack length as a_i, the fatigue crack length a_N after N cycles can be numerically obtained from

$$a_N = a_i + \sum_{j=1}^{N} \left(\frac{\Delta a}{\Delta N} \right)_j \tag{7}$$

Under the assumption that the variable loading condition can be equivalent to the repeated constant loading, Walker's equation model provides that the increment in crack length Δa_j for one cycle (ΔN=1) can be expressed as

i	N_i	$S_{\max}$ (MPa)	$S_{\min}$ (MPa)	R
1	500	91	4.77	0.053
2	500	100.21	4.77	0.0476
3	1	100.21	4.77	0.0476

Table 2. Information on variable loading conditions for Walker's equation model

$$\Delta a_j = C_0 \left(K_{\max} \left(1-R\right)^{\gamma} \right)_j^m \tag{8}$$

where C_0 is a constant and m is the slope on the log-log plot; γ stands for a constant of the material; R, which is associated with the loading conditions, indicates the ratio of minimum nominal stress $S_{\min}$ to maximum nominal stress $S_{\max}$; and $K_{\max}$ indicates the maximum stress intensity factor K_j, which is defined as

$$K_j = FS\sqrt{\pi a_j} \tag{9}$$

Here, the stress intensity factor K_j is a function of geometric parameter F, nominal stress S, and crack length a_j. If the aluminum plate is large enough, F can be regarded as one. If the repeating history contains N_{B} cycles, the increase in crack length Δa_B during one repetition is obtained by

$$\Delta a_B = \sum_{j=1}^{N_B} C_0 \left(K_{\max} \left(1-R\right)^{\gamma} \right)_j^m \tag{10}$$

The average crack growth per cycle during one repetition of the variable loading history can thus be expressed as

$$\left(\frac{\Delta a}{\Delta N} \right)_{\mathrm{avg}} = \frac{1}{N_{\mathrm{B}}} \sum_{j=1}^{N_B} C_0 \left(K_{\max} \left(1-R\right)^{\gamma} \right)_j^m \tag{11}$$

By substituting Eq. (11) into Eq. (7), the final crack length a_f for a certain cycle can be obtained. Table 2 summarizes some information about the variable loading conditions, such as the number of cycles N_i, the minimum nominal stress $S_{\min}$ to maximum nominal stress $S_{\max}$, and ratio R.

It should be noted that the constants C_0, γ, and m could not be determined because they depend on the material properties of the specimen. In addition, we assumed that there are uncertainties σ_{FC} in the first cycle, where a nonzero crack length is estimated, because no information was given about the exact cycle at which the crack initiates. In summary, there are four uncertain parameters.

In the study outlined in this paper, generic algorithm-based optimization was considered for parameter estimation. The optimization problem can be formulated as

$$\text{Minimize } RMSE_{WE}\left(\mathbf{d}\right) = \frac{1}{5}\sum_{i=1}^{5} i^2 \left(y_{WE}\left(\mathbf{d}\right) - y \right)^2$$
$$\mathbf{d} = \{C_0, \gamma, m, \sigma_{FC}\} \tag{12}$$
$$\text{Subject to } \mathbf{d}_{\mathbf{L}} \le \mathbf{d} \le \mathbf{d}_{\mathbf{U}}$$

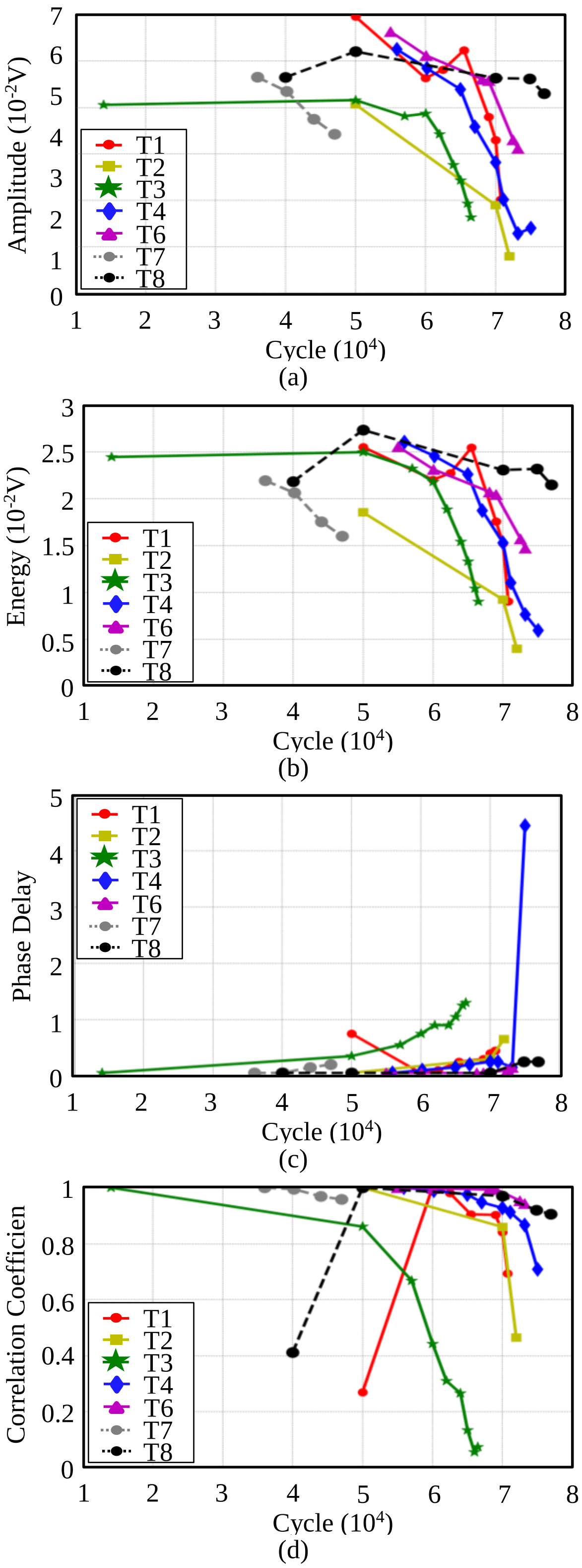


Figure 8. Trends of selected optimal features with respect to cycles in the data-driven method: (a) maximum amplitude; (b) maximum energy; (c) phase delay; and (d) correlation coefficient

where a set of four parameters is denoted as $\mathbf{d}$; $y_{WE}(\mathbf{d})$ is the predicted crack length from Walker's equation model; y stands for the estimated crack length from the random forest model and the linear regression model for the first five cycles 70000, 74883, 76931, 89237 and 92315, and; $\mathbf{d}_L$ and $\mathbf{d}_U$ are the lower and upper bounds of the parameters, respectively. Here, the objective function $RMSE_{WE}$ is defined as RMSE with more weights on the cracks in the latter cycles. The weighting is used to prevent the predicted crack length from being overestimated.

Depending on the initial design variables (parameters of the Walker's equation model), the optimization problem has various solutions. Therefore, the sets of estimated parameters provided various curves of crack length versus cycle. Taking these uncertainties into account, Walker's equation models were generated by Monte Carlo methods. To examine the trends of the generated models, we gradually increased the number of models. It was observed that the average of the models converged to a certain regression model as the number of models approached 100. Considering the computing cost, an average of 100 generated models was used to predict the remaining crack lengths of specimen T8.

4. VALIDATION OF THE PROPOSED METHOD

This section describes the validation of the proposed method. Recall that the *k*-fold cross validation provided an optimal set of five features: (1) maximum amplitude, (2) maximum energy, (3) phase delay, (4) correlation coefficient, and (5) previous crack length, as outlined in Section 3.2.4. Figure 8 presents the trends of optimal features in the training specimens (T1-T6) and the validation specimens (T7 and T8) with respect to cycles when Lamb wave signals are given. Figures 8 (a) and (b) depict the trends of maximum amplitude and energy. As the cycle increases, both maximum amplitude and energy tend to decrease, due to the impedance mismatch (or discontinuities) at the crack location. Figure 8 (c) presents the trend of the phase delay. The larger the crack size, the greater the phase delay, as the Lamb wave travels a longer distance due to local geometry deformation. Figure 8 (d) illustrates the trend of the correlation coefficient. The correlation coefficients between the specimens without and with cracks tends to decrease as the crack length increases; this is due to the distorted shapes of the transmitted Lamb wave at the interface of the crack. The crack lengths that are estimated through use of the data-driven method are listed in Table 3.

Figures 9 (a) and (b) present the predicted crack lengths obtained from the ensemble prognostics approach (Section 3.3.2) and from Walker's equation model (Section 3.3.3), respectively. Cross points of orange and sky-blue colors indicate the true crack lengths provided by the PHM Society after closing the Data Challenge. Dotted lines of gray and black colors indicate predicted crack length for specimens T7 and T8, respectively. It should be noted that the physics-

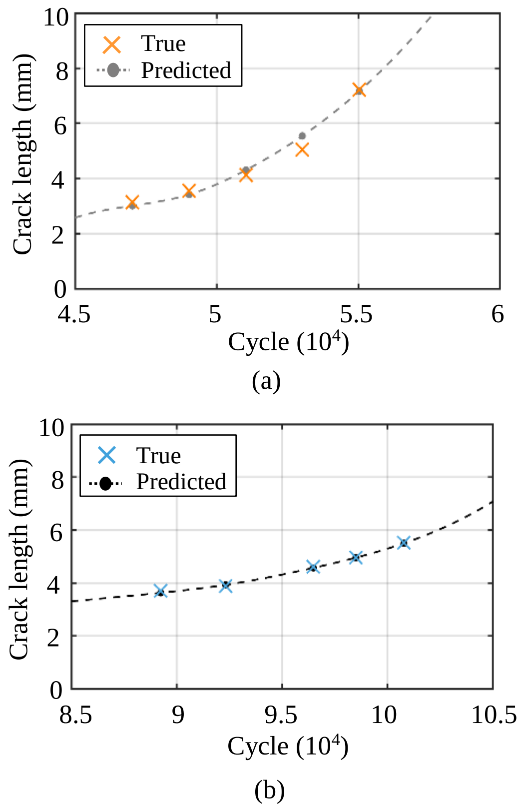


Figure 9. Predicted fatigue crack length using physics-based methods: (a) specimen T7 and (b) specimen T8

T7			
Method	**Cycle**	**True**	**Prediction**
Data-driven	36001	0	0
	40167	0	0
	44054	2.07	2.175
	47022	3.14	3.017
Physics-based	49026	3.56	3.423
	51030	4.13	4.310
	53019	5.05	5.547
	55031	7.22	7.170
T8			
Data-driven	40000	0	0
	50000	0	0
	70000	0	1.722
	74883	1.94	2.291
	76931	2.5	2.565
Physics-based	89237	3.71	3.630
	92315	3.88	3.930
	96475	4.61	4.571
	98492	4.96	4.956
	100774	5.52	5.500

Table 3. Results of the proposed method

based method can predict the crack length for an arbitrary cycle. The predicted crack lengths derived from the physics-based method for the cycles given in Table 1 are summarized in Table 3. As a result, the RMSEs for specimens T7 and T8 were calculated as 0.2021 and 0.551, respectively. The penalty score in the Data Challenge was calculated to be 7.63.

5. CONCLUSION

This paper proposed a hybrid approach that combines data-driven and physics-based methods to estimate and predict the fatigue crack growth of an aluminum lap joint specimen with and without Lamb wave signals. First, a data-driven method based on signal processing and machine learning was used to estimate the crack lengths for a few cycles, for which Lamb wave signals were given. Band-pass filter and phase alignment were used to de-noise the raw Lamb wave signals. Next, a random forest model was used to estimate crack lengths through optimal feature selection and grid-search-based hyper-parameter optimization. Second, a physics-based method was used to predict the remaining crack lengths without the use of Lamb wave signals. Due to different loading conditions, two approaches were considered: (1) an ensemble prognostics approach that simplified particle-filter-based weight updating for use under the same loading conditions, and (2) Walker's equation models with Monte Carlo methods for use under different loading conditions. To demonstrate the validity of the proposed approach, a case study was presented using the datasets provided in the 2019 PHM Conference Data Challenge by the PHM Society. The case study confirmed that the hybrid approach showed high accuracy; the RMSEs for specimens T7 and T8 were calculated as 0.2021 and 0.551, respectively. A penalty score was calculated as 7.63; this resulted in a 2nd place finish in the competition. It can be thus concluded that the proposed method overcomes the overfitting characteristics of either data-driven or physics-based methods that are caused by the lack of data or physical information, respectively. To the best of the authors' knowledge, this is the first attempt to propose a hybrid approach for estimation and prediction of fatigue crack growth with and without Lamb wave signals. The proposed hybrid model can be potentially incorporated into the maintenance and management systems. If the model is used in an embedded system or a cloud system, crack lengths can be estimated and predicted in real-time, as long as data acquisition is possible in real-time. Moreover, this approach is applicable not only to specimens, but also to large engineering systems such as rotors, bearings, and wind turbines. However, there are some limitations of commercialization such as dependence on hardware performance, manual procedures in signal processing, and difficulties in automated model improvement even under the big data.

This study has focused on estimating and predicting a deterministic value of the crack length at a certain cycle. The predictive results will have uncertainties that arise in the process of determining the random forest model, simplified particle filter, and empirical constant estimation in Walker's equation. In future work, statistical distributions and confidence intervals of the predicted crack lengths will be investigated using uncertainty propagation analysis. In addition, it should be noted that this study was performed with a limited number of training and test samples. In future work, by generating virtual data (e.g., Lamb wave signals) through developing a multiphysics finite element model with the help of statistical model calibration, deep-learning-based fatigue crack growth prediction will be studied.

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REFERENCES

Bannantine, J., Comer, J., & Handrock, J. (1990). Fundamentals of metal fatigue analysis. *Research supported by the University of Illinois. Englewood Cliffs, NJ, Prentice Hall, 1990, 286*.

Benkedjouh, T., Medjaher, K., Zerhouni, N., & Rechak, S. (2013). Remaining useful life estimation based on nonlinear feature reduction and support vector regression. *Engineering Applications of Artificial Intelligence, 26*(7), 1751-1760.

Büyüköztürk, O., & Taşdemir, M. A. (2012). *Nondestructive Testing of Materials and Structures*: Springer Science & Business Media.

Cawley, P., & Alleyne, D. (1996). The use of Lamb waves for the long range inspection of large structures. *Ultrasonics, 34*(2-5), 287-290.

Ha, J. M., Youn, B. D., Oh, H., Han, B., Jung, Y., & Park, J. (2016). Autocorrelation-based time synchronous averaging for condition monitoring of planetary gearboxes in wind turbines. *Mechanical Systems and Signal Processing, 70*, 161-175.

He, J., Ran, Y., Liu, B., Yang, J., & Guan, X. (2017). A fatigue crack size evaluation method based on lamb wave simulation and limited experimental data. *Sensors, 17*(9), 2097.

Jeon, B. C., Jung, J. H., Youn, B. D., Kim, Y.-W., & Bae, Y.-C. (2015). Datum unit optimization for robustness of a journal bearing diagnosis system. *international journal of precision engineering and manufacturing, 16*(11), 2411-2425.

Konstantinidis, G., Wilcox, P. D., & Drinkwater, B. W. (2007). An investigation into the temperature stability of a guided wave structural health monitoring system using permanently attached sensors. *IEEE Sensors Journal, 7*(5), 905-912.

Le Clézio, E., Castaings, M., & Hosten, B. (2002). The interaction of the S0 Lamb mode with vertical cracks in an aluminium plate. *Ultrasonics, 40*(1-8), 187-192.

Maslouhi, A. (2011). Fatigue crack growth monitoring in aluminum using acoustic emission and acousto‐ultrasonic methods. *Structural control and health monitoring, 18*(7), 790-806.

Mohanty, S., Chattopadhyay, A., Peralta, P., & Das, S. (2011). Bayesian statistic based multivariate Gaussian process approach for offline/online fatigue crack growth prediction. *Experimental mechanics, 51*(6), 833-843.

Oh, H., Jung, J. H., Jeon, B. C., & Youn, B. D. (2017). Scalable and unsupervised feature engineering using vibration-imaging and deep learning for rotor system diagnosis. *IEEE Transactions on Industrial Electronics, 65*(4), 3539-3549.

Pedregosa, F., Varoquaux, G., Gramfort, A., Michel, V., Thirion, B., Grisel, O., . . . Dubourg, V. (2011). Scikit-learn: Machine learning in Python. *Journal of machine learning research, 12*(Oct), 2825-2830.

Rolfe, S. T., & Barsom, J. M. (1977). *Fracture and fatigue control in structures: Applications of fracture mechanics*: ASTM International.

Staszewski, W., Lee, B., & Traynor, R. (2007). Fatigue crack detection in metallic structures with Lamb waves and 3D laser vibrometry. *Measurement science and technology, 18*(3), 727.

Wang, H., Zhang, W., Sun, F., & Zhang, W. (2017). A comparison study of machine learning based algorithms for fatigue crack growth calculation. *Materials, 10*(5), 543.

Wilcox, P. D. (1998). *Lamb wave inspection of large structures using permanently attached transducers.* University of London.

Yashiro, S., Takatsubo, J., & Toyama, N. (2007). An NDT technique for composite structures using visualized Lamb-wave propagation. *Composites Science and Technology, 67*(15-16), 3202-3208.

Yu, L., & Liu, H. (2003). *Feature selection for high-dimensional data: A fast correlation-based filter solution.* Paper presented at the Proceedings of the 20th international conference on machine learning (ICML-03).

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