Estimation of Transmission Losses in a Changing Electric Power Industry

by

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B.S., Electrical Engineering
University of Puerto Rico, 1994

Submitted to the Department of Electrical Engineering and Computer Science
in Partial Fulfillment of the Requirements for the Degree of

Master of Science in Electrical Engineering and Computer Science

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1996

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Abstract

This thesis is concerned with the basic problem of computing how much power is lost in transferring across an electric power network, as the power is injected into particular nodes of the network that represent points of power supply and taken out at other nodes, that represent points of power consumption. This topic is revisited because of an increased tendency for economic transfers that go beyond the transfer levels at which transmission loss is relatively small, together with the questions of compensation for transmission losses in a deregulated/competitive power industry, including possible compensation of transmission losses at the end-user level, i.e., at the level of their cause. The question posed in this thesis is if the localized response of system voltages and angles to input changes could be used to: (i) locally estimate transmission losses caused by an end user, keeping in mind availability of real-time information networks, and, (ii) compensate for these losses by injecting the power corresponding to this estimated transmission loss, and not be dependent on the cost for this service dictated by someone else.

Thesis Supervisor: Marija Ilić
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Chapter 1

Introduction

This thesis is concerned with what may appear to be a well-understood and extensively researched topic in the electric power engineering. The basic problem is the one of computing how much power is lost in transferring across an electric power network, as the power is injected into particular nodes of the network that represent points of power supply and taken out at other nodes, that represent points of power consumption. This problem is not unique to the electric power networks. The problem has been of much interest in all other electric networks, for various reasons. In smaller (than electric power networks) scale electric networks the losses of the network are of interest either because of thermal constraints on the hardware, or because of a need to design circuits whose efficiency, as measured in terms of voltage and/or current sources needed to meet the performance specifications of the circuit, is high. The well-known maximum power transfer theorem [5, 4] provides an analytic bound on how much is the most power that could be transferred from a source to a demand point. In the simplest case of a single source connected to a single load, maximum power that can be delivered to the load is half of total sent; the other half amounts to the resistive losses of the network. Obviously, at the point of maximum power transfer the loss is very high compared to the ratio of the power delivered, i.e., the efficiency is only 50%. For this reason, network design is such that the system operates relatively far from the conditions defined by the maximum power transfer.

To consider the problem of transmission losses for electric power networks in the context of more general electric networks, electronic circuits in particular, we recognize that a power network has the basic function to deliver specified power to the points of demand; the objectives of typical electronic circuits are not defined in terms of power delivered. It is for this reason that the problem of transmission losses becomes more of a dominant one in the electric power networks, than in many others. A power network design is subject to two obviously conflicting objectives, of (i) delivering prespecified power, and (ii) minimizing transmission losses in the network as this is being done. It follows from the maximum power transfer theorem that the more power is transferred between two nodes of a power network, the higher transmission loss is. As a result, electric power networks are typically designed so that for nominal (expected) system demand, the total transmission loss is not higher than 5% of the total power injected into the system. This, furthermore, implies that the system operating conditions are quite far from the maximum power the network could transfer.

Power industry has well-developed tools for computing total transmission loss as a function of all power injected into the network. The formulae range from approximate formulae that only require knowing power into the network, and do not require solving the network [6], through the completely accurate formulae that require full-blown load flow computations [6]. These are briefly summarized in this thesis.

The main reasons for re-visiting the topic of transmission losses at this time are at least threefold:

1. Increased tendency for economic transfers that go beyond the transfer levels at which transmission loss is relatively small.

2. Question of compensation for transmission losses in a deregulated/competitive power industry.

3. Question concerning possible compensation of transmission losses at the end-user level, i.e., at the level of their cause.

More specifically, the Federal Energy Regulatory Commission (FERC) in the United States has over the past couple of years been particularly concerned with the problems of systems services traditionally provided by utilities in the process of meeting their obligation to serve customers. These services are referred to in several FERC-originated documents as ancillary services [18], implying that they are in some sense auxiliary to the primary supply/demand power market that has been becoming increasingly competitive. Transmission loss compensation is considered to be one of such ancillary services, among several others.

These services are necessary to balance generation and demand in an interconnected electric power network in response to system input changes and various uncertainties. Transmission loss is a consequence of activities in the primary supply/demand market, and, as such, it is not known ahead.

In most of the strictly engineering-based literature, an a priori assumption is made that all ancillary services are provided at the interconnected system level in a somewhat coordinated manner [10]. The same assumption is made in the FERC documentation [18]. The so-called pro forma tarrifs recommended by FERC are intended to compensate utilities for providing power associated with transmission loss, and for all other interconnected operations services. The tarrifs are, unfortunately, insensitive to the locational and temporal differences. As such, they are approximate and do not provide incentives to their users (participants in the primary supply/demand) for minimizing the need for such services. In particular, the pro forma tarrifs cannot differentiate effects of two economic transactions[1] on the operating conditions of the interconnected system, as long as the power quantities traded are the same. Consequently, two transactions that inject the same amount of power into the system, are required to pay utilities for transmission losses caused on the interconnected system the same amount of money, independently form the actual losses caused by the two transactions. We observe this without an intent to be critical of these tarrifs. As a matter of fact, the real reason that a simple, approximate tarrif of this type is being proposed, comes from recognizing the generic complexity involved in computing and allocating losses associated with individual transactions in a potentially very active primary market. It has been proven recently that, strictly speaking, power flows through transmission lines of an interconnected network depend on all system inputs, and cannot be uniquely allocated to individual transactions [30]. This is not a surprising result, given the inherent non-linearity of the governing equations, known as load flow equations [23], when expressed in terms of power. There always exists an interaction component in line power flows caused by each specific system input, that is non-zero only when other system inputs are present.

Aside from concerns regarding computational complexities, this non-linear character of the problem points out into an entire new set of questions that must be addressed prior to attempting to claim accuracy and equity of tarrifs for transmission loss compensation. The

[1] By an economic transaction one means a contractual agreement between two parties on the interconnected system to exchange power.

most important issue is the dependence of these tarrifs on the type of primary market (industry structure) adopted. In a strictly bilateral, competitive market the cost of transmission loss created on the system by each bilateral transaction depends on the order in which the transactions are implemented. In a fully coordinated market management in which an independent system operator simultaneously coordinates the specific supply/demand requests and manages/charges for ancillary services associated with these requests, one could envision periodic computations of costs caused by all transactions present on the system[2]. In the most general, multilateral primary supply/demand market in which both bilateral transactions and coordinated supply/demand management through an independent system operator, take place at arbitrary times, the allocation of transmission losses and their dependence on the other activities throughout the system is even more complex.

This dependence of ancillary services on the specifics of the primary market is not well understood at present. Depending on the motivation and the level of general understanding of the problem, one encounters at least three distinct schools of thought when it comes to dealing with the cost of ancillary services, including transmission losses.

1. Traditional providers of interconnected operations services in a regulated industry assume that these services can only be provided in a coordinated way from the level of an independent system operator.

   Much effort is put into developing computational methods for allocating cost of transmission losses and other services in response to the primary market in a poolco-like environment [30, 31, 32]. Often only a *bundled* cost of accommodating a specific supply/request through an ISO is seen, without an explicit unbundling of the price/cost of transaction from the cost of system services. It is well known that this bundled cost is computed as a short-run marginal cost (SRMC) at each node and it reflects the cost of providing net power input into the system[3] and cost compensation for transmission losses and other types of ancillary services. This SRMC is referred to as a spot price, and it generally varies as the net system inputs at various locations in the system vary. The poolco-based primary market, in principle, makes the allocation

[2]This coordinated management very closely resembles the operations of present power pools, and is often referred to as a poolco [9]

[3]When the net nodal power into the system is positive, a market player connected to this node supplies power to the system; when it is negative, this player is seen as a power demand to the rest of the system. In this sense, there is no qualitative difference between a supply and demand.

of ancillary services fairly straightforward since it does not require unbundling of the cost of primary market activities from the cost of ancillary services. In particular, assuming full information about the system, in such structure the need for explicit computation of losses is un-necessary.

2. Proponents of strictly bilateral markets base their fundamental thinking on parallels with other competitive industries. Opportunities for profit making are viewed by each specific market player in dealing directly with other market players. In such environment cost of all ancillary services is an un-necessary burden that market player would like to minimize as much as possible.

   This primary market structure creates to the providers of ancillary services a challenge to develop cost allocation methods for ancillary services provided to the bilateral market players. This is in concept new thinking to the traditional power engineers whose software methods rest on seeing the system as one.

3. Finally, a hybrid industry structure that allows for mix of bilateral and coordinated primary market players, combines complexities of the previous two structures.

These non-traditional industry structures raise at least three new conceptual questions. These are:

1. Do all ancillary services have to be provided in a coordinated way by an independent system operator, or the end users, i.e., the market participants could estimate the power mismatch caused at their own level and compensate by adding more (or less) into the system at their location than trading?

2. If the ancillary services are provided to the market players in a coordinated way, how could one at the ISO level *unbundle* primary level activities from the ancillary services, and, furthermore, how can one allocate these costs to the specific market participants in a fair and rigorous way?

3. If it is shown to be possible to decentralize compensation of ancillary services to each individual end user level, how can this be done?

1.1 Approximation of Transmission Losses

It ought to be clear, based on the above, that an absolute accuracy and equity in providing for transmission loss is basically not possible. This is consistent with one of the conclusions in Joskow's recent article concerned with the power industry in transition; the article clearly communicates author's assessment that the cost of system services at the end of transition never got computed and allocated exactly [14]. This is at first somewhat bothersome conclusion to the power engineering community used to challenges of numerical accuracy. It leads to the new question if it is possible to develop simple compensation methods for ancillary services, whose accuracy could be estimated and somehow tested[4].

This question is the main topic of this thesis. It is restricted to the context of transmission loss sub-problem. The idea is to re-visit some of the fundamental understanding of electric power network responses to system input changes that has been used in the past for development of computationally effective algorithms of various types. Specifically, the so-called localized response property of power networks to system input changes is recalled in the context of the problem of interest in this thesis. This property basically states that under certain mild assumptions the effect of power input change at the node i on voltage changes throughout the system is most seen at the nodes (electrically) closest to the node i, and it decreases in a concentric relaxation-like manner away from the input change. This property has been studied in the past both for qualitative understanding of power network characteristics, as in [16, 13, 11], as well as for the development of simplified computational algorithms as in [26, 29, 28]. The question posed in this thesis is if this localized response of system voltages to input changes could be used to

1. Locally estimate transmission losses caused by an end user, keeping in mind availability of real-time information networks required by FERC [17, 19].

2. Compensate for these losses by injecting the power corresponding to this estimated transmission loss, and not be dependent on the cost for this service dictated by someone else.

An important observation here is that these approximate formulae should be independent from the activities on the rest of the system, except through the information exchange

[4] It is important to observe that the pro forma approximate tarrifs proposed by FERC do not have this feature.

provided in real time by a real-time information network of some sort. It is expected that such information network would at least regularly provide information about the status/parameters of the network, and the system-wide state (voltages) as they are computed at the ISO level. No information about the specific transactions and cost data is needed.

In order for the formulae of this type to exist, it is necessary to make the case that the interaction component in flows is small relative to the main flows caused by each transaction separately. While this claim was made in [30, 31, 32], it is re-visited here by further recognizing that generation input patterns are evolving into smaller changes distributed throughout the system, rather than large generation inputs only at a limited number of locations in the system. The premise made here is that the theoretical problem of computing the effect of many distributed small changes into the system is somewhat simpler than the problem of computing impact of large changes at selected number of allocations[5]. This is because the impact of specific changes is more localized, and, also because linearized models are more likely to be valid. While the interaction component in the power line flows could be non-negligible for large deviations in specific system inputs, the dispersed effects of smaller deviations at many locations are likely to be less interacting with each other. It is with this in mind that the derivations in this thesis are conceived.

1.2 Thesis Organization

In Chapter 2, the basic governing equations of electric power networks in steady-state, i.e., the load flow equations are briefly reviewed. These equations define constraints on real and reactive power inputs into the network, by stating that the power injected into the system at each node must equal the power transferred by the network. Next, the decoupling assumption under which real power load flow equations are separable from the reactive power load flow equations is defined and conditions under which this assumption is valid are stated. It is recalled that under the real/reactive power decoupling assumption, the real power load flow problem can be interpreted as a problem of a DC nonlinear resistive network. Similarly, it is reviewed how is the decoupled reactive power/voltage problem interpreted as a problem of a DC nonlinear resistive network.

A localized response property, essential for the methods described in this thesis, is

[5] In both cases the same total demand is met.

stated next for the decoupled nonlinear real power load flow problem. This result is a direct consequence of interpreting the problem as a nonlinear resistive network with nondecreasing resistors, that is known to have the localized response property. Loosely speaking, the localized response property in the context of the decoupled real power load flow problem means that the largest voltage phase angle at the nodes directly connected to the location i, at which an increment in real power ΔP_i takes place, is never smaller than the largest voltage phase angle change at nodes one tier away from bus i, and so on.

This property is not sufficient, however, for the changes in phase angle differences across the transmission lines to have the same property, i.e. to be decreasing away from the cause of their change. For this to hold true, it is sufficient to have a relatively meshed network, as seen from the location i; starting with a relatively small number of lines directly connected to the location i, the number of lines across the cutsets away from the location i increases. It is intuitively clear that for a transmission network whose reactances are uniform, the changes in real power line flows decrease in proportion to the increase of number of lines across the cutsets away from node i.

This qualitative property is important for the real power loss estimation methods proposed in this thesis. To introduce the problem of transmission losses basic formulae for transmission loss calculation in power networks are briefly reviewed. Present state-of-the-art is briefly reviewed for computing transmission losses.

Next, in Chapter 3 a closed form formula for computing voltage phase angle changes created by the end user located at bus i is introduced. This formula only requires knowledge of the power increase ΔP_i at the location i and the network parameters of the entire system. The algorithm is based on the linearized, decoupled real power load flow equations. Numerical methods similar in concept could be derived without making the linearizating assumption. The results of this algorithm, combined with the updated nominal voltage phase angles of the entire network are used as the starting information for estimating real power transmission loss created by the end user at bus i when injecting ΔP_i into the system. First, a formula for computing real power losses is described that reflects the interaction of nodal power increments in the interconnected system. This formula cannot be used by each end user independently from the others. It is next proposed that for the most typical power networks the line flow changes ΔP_{ij} decrease in absolute value away from the location where power is injected into the system. Given this property, it is possible to claim that the voltage phase angle differences across the transmission lines also decrease in proportion with the line reactances. This further leads to the conjecture that the effects of power input changes resulting from economic transactions are separable to a significant degree, unless the transactions are very close electrically. This conjecture is formally derived.

In Chapter 4, the problem of estimating reactive power losses in an interconnected power network is studied. First, the decoupled reactive power-voltage (QV) load flow equations are briefly reviewed. These form the governing equations of direct interest. Next, the state of-the-art results concerned with a non-linear network interpretation of the QV load flow problem are summarized. It is concluded that such an interpretation is possible. However, the resulting non-linear network, because of the presence of shunt capacitors on a primarily inductive network, is analogous to a non-linear DC resistive network, not all of whose resistors are monotonically increasing. A qualitative implication of this situation is that it is not possible to state unconditionally that a change in reactive power injection ΔQ_i into bus i leads to uniform decrease in voltage changes ΔV_i [13]. This can only be proven when the shunt capacitors are not present.

This obstacle could be overcome in the context of the functions of an end user in a competitive environment by decomposing the problem of reactive power loss compensation into[6]

1. The shunt reactive power loss component, measurable directly in terms of local power factor compensation.

2. The reactive power losses created in the planar transmission grid that interconnects all nodes.

An underlying modeling assumption here is that the reactive power inputs into the nodes are represented as ideal reactive power injections into the grid[7]. The total reactive power injection could be thought of as consisting of the portion flowing from the node to the ground, and the portion flowing into the planar transmission network. It is proposed here that the shunt reactive losses be directly estimated and compensated by each end user; this is trivial to do. The method proposed in this thesis introduces an approach to estimate the second component. It is proven in this thesis that the voltage changes ΔV_i away from

[6]The same idea was recognized recently in [1].
[7]This includes flows through the capacitors.

the reactive power injection into the planar portion at node i of the grid decrease uniformly away from this location. A closed form solution for estimating the voltage deviations in the entire network caused by the specific end user is derived by using only the information about the local injection into the grid ΔQ_i and the network parameters of the entire grid. Next, an approximate formula for reactive power losses in response to reactive power changes at several locations in this system is derived. This formula requires knowledge about nominal voltages, that is assumed to be provided by a real time information network of some sort [17, 19]. It is proposed that for system input changes that are not very close electrically, reactive power loss can be estimated and compensated individually by each end user.

In Chapter 5, numerical results on the standard IEEE 39 bus system are described in support of theoretical propositions made in this thesis. It is concluded that an acceptable accuracy is achievable.

In Chapter 6, possible ways of using the proposed method for real time loss compensation by the end users themselves, instead of paying for loss compensation at the interconnected system level are described. It is pointed out that this approach is not intended to be exclusive. End users not interested in compensating for transmission loss themselves could continue to pay for the ancillary services according to agreed upon tarrifs.

In Chapter 7, the main intent in this thesis has been recaptured. It is suggested that, at least in concept, it is possible to allow the end users to provide for transmission loss themselves by injecting an additional amount of power at the location where the injection for primary market is carried. The estimates are based on the recognition of several characteristics associated with power flow propagation in electric power networks. Much of this knowledge is hidden in the highly technical literature, and is just beginning to play an important role for providing and developing systems service tarrifs under competition. The validity of the approach is demonstrated on the IEEE 39 bus system. Several immediate open questions are stated in the concluding remarks of this thesis.

Chapter 2

Theoretical Background

In this chapter, the basic governing equations of electric power networks in steady-state, i.e., the load flow equations are briefly reviewed. These equations define constraints on real and reactive power inputs into the network, by stating that the power injected into the system at each node must equal the power transferred by the network. Next, the decoupling assumption under which real power load flow equations are separable from the reactive power load flow equations is defined and conditions under which this assumption is valid are stated. It is recalled that under the real/reactive power decoupling assumption, the real power load flow problem can be interpreted as a problem of a DC nonlinear resistive network. Similarly, it is reviewed how is the decoupled reactive power/voltage problem interpreted as a problem of a DC nonlinear resistive network.

A localized response property, essential for the methods described in this thesis, is stated next for the decoupled nonlinear real power load flow problem. This result is a direct consequence of interpreting the problem as a nonlinear resistive network with nondecreasing resistors, that is known to have the localized response property. Loosely speaking, the localized response property in the context of the decoupled real power load flow problem means that the smallest voltage phase angle at the nodes directly connected to the location i, at which an increment in real power ΔP_i takes place, is always larger than the largest voltage phase angle change at nodes one tier away from bus i, and so on.

This property is not sufficient, however, for the changes in phase angle differences across the transmission lines to have the same property, i.e. to be decreasing away from the cause of their change. For this to hold true, it is sufficient to have a relatively meshed network, as seen from the location i; starting with a relatively small number of lines directly connected to the location i, the number of lines across the cutsets away from the location i increases. It is intuitively clear that for a transmission network whose reactances are uniform, the changes in real power line flows decrease in proportion to the increase of number of lines across the cutsets away from node i.

This qualitative property is important for the real power loss estimation methods proposed in this thesis. To introduce the problem of transmission losses basic formulae for transmission loss calculation in power networks are briefly reviewed. Present state-of-the-art is briefly reviewed for computing transmission losses.

2.1 Load Flow Equations

In order to present the exact formulation of real power losses in a transmission network it is necessary to review the load flow problem. The load flow equations form a consistent set of algebraic equations for computing voltage magnitudes and angles throughout the system for a given network topology and parameters, and the specified generated and required power throughout the system. They basically represent power flow balance equations resulting from the familiar nodal equations in electric circuits [3].

Certain assumptions made about the steady state models of different system components which lead to the simplification of the general form of steady state equations to the specific form recognized as the load flow equations are reviewed first. The load flow formulation assumes that both exciter and governor controls are ideal, so that they maintain voltages at the generator buses at any reference constant value, and the mechanical power output at the specified value. For this reason generators are often referred to in load flow computations as the "PV" buses, since their steady state specifications are parametric in terms of real power P and voltage magnitude V. The same way for load buses, loads are characterized as "PQ" buses with voltages magnitude and angles unknown. A sufficient number of equations for defining a load flow problem consists of $2n$ real and reactive power balances at all n load buses and the k real power balances at generator buses. They are of the form

$$P_{Li} = \sum_{j \in K_i} \left(\frac{V_i^2}{Z_{ij}} \cos \zeta_{ij} - \frac{V_i V_j}{Z_{ij}} \cos(\delta_i - \delta_j - \zeta_{ij})\right) \tag{2.1}$$

$$Q_{Li} = \sum_{j \in K_i} (\frac{V_i^2}{Z_{ij}} \sin\zeta_{ij} - \frac{V_i V_j}{Z_{ij}} \sin(\delta_i - \delta_j - \zeta_{ij})) \tag{2.2}$$

for $i = 1, ..., n$.

$$P_{Gi} = \sum_{j \in K_i} (\frac{V_i^2}{Z_{ij}} \cos\zeta_{ij} - \frac{V_i V_j}{Z_{ij}} \cos(\delta_i - \delta_j - \zeta_{ij})) \tag{2.3}$$

for $i = n+1, ..., n+k$. The notation P_{Li}, Q_{Li} and P_{Gi}, stands for real and reactive power injections into load buses and real power into generator buses, respectively. $V_i \angle \delta_i$ stands for nodal voltage magnitude and phase angle at each bus i in the system. $Z_{ij} \angle \zeta_{ij}$ is the complex-valued impedance of a transmission line connecting nodes i and j.

Two explanations for distinguishing among all generator busses and the "slack" bus are, first: an engineering argument that it is not possible to specify real power inputs to all busses (loads and generators), prior to knowing transmission system losses indicates the necessity not to specify real power at least at one of the generators before load flow calculations are performed and the losses are known. In practice, it is common to designate one of the largest generators as the "slack" bus to assure it has sufficient real power output to compensate for the transmission losses. Second, it is needed for mathematical reasons; it is straightforward to show that the real power balance equation of type (2.3) is linearly dependent on real power equations at all other generators. It is, therefore, necessary to assume phase angle of the "slack" generator known and not directly use equation (2.3) to compute it. The slack bus is modeled as an ideal voltage source whose magnitude and phase angle are fixed.

2.2 Real/Reactive Power Decoupling Assumption

Two simpler formulations of the load flow problem can be found under operating conditions of the network. These formulations are based on the real and reactive power decoupling. The basic idea on which this decoupling assumption is found can be best illustrated on the simplest case of a two bus example as the one shown on Figure 2-1.

Computing sensitivity of real power P_j delivered to the load with respect to change in phase angle difference $(\delta_i - \delta_j)$, one obtains,

$$\frac{\partial P_j}{\partial(\delta_i - \delta_j)} = \frac{V_i V_j}{z_{ij}} \sin(\delta_i - \delta_j - \zeta_{ij}) \tag{2.4}$$

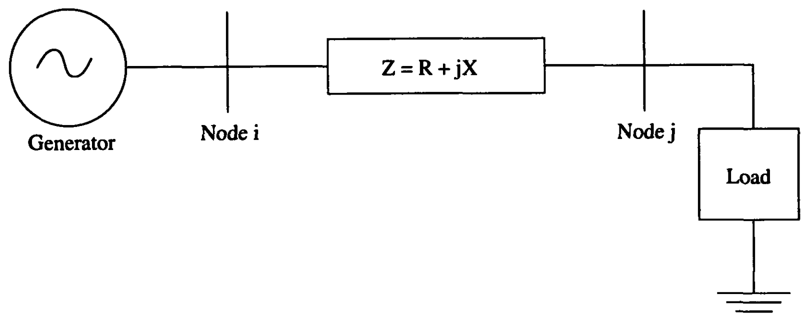

Figure 2-1: Simple 2 Bus System

Similarly, sensitivity of real power P_j with respect to change in voltage magnitude V_j is

$$\frac{\partial P_j}{\partial V_j} = \frac{V_i}{z_{ij}} \cos(\delta_i - \delta_j - \zeta_{ij}) \tag{2.5}$$

For highly inductive lines $\zeta_{ij} = \frac{\pi}{2} rad$, and voltage magnitudes close to 1 p.u., it follows that,

$$\left| \frac{\partial P_j}{\partial(\delta_i - \delta_j)} \right| \gg \left| \frac{\partial P_j}{\partial V_j} \right| \tag{2.6}$$

The same way for the reactive power, we have

$$\left| \frac{\partial Q_j}{\partial V_j} \right| \gg \left| \frac{\partial Q_j}{\partial(\delta_i - \delta_j)} \right| \tag{2.7}$$

Taking this into account, one can separate the load flow problem into sub-problems: one for analyzing the effect of real power and nodal voltage phase angles ($P - \delta$ problem), and the other for studying dependence of reactive power and voltage magnitudes ($Q - V$ problem). Since we are more interested in estimating real power losses we place more emphasis on the $(P - \delta)$ formulation.

2.3 Linearized $P - \delta$ Problem

The decoupled real power-phase angle problem is defined as the problem of computing phase angle differences $(\delta_i - \delta_j)$ for specified real power demand P_i at loads $i = 1, ..., n$ and real power generation at buses $i = (n+1), ..., (n+k)$, and the specified angle at the slack bus δ_0.

Suppose that the resistive part of transmission lines is negligible relative to its inductive part. Let $A \in R^{N \times \ell}$,$(N = n + k)$ be the reduced incidence matrix of the system obtained by deleting all shunts and the ground node. The slack bus is taken as reference. Let $[y]$ be the diagonal matrix with elements $B_{ki} | k \neq i, k, i = 0, ..., N$ and define $J_p \in R^{N \times N}$ as

$$J_p = A[y]A^T \tag{2.8}$$

It is simple to show that the linearized decoupled load flow equations take on the form

$$P_k(\delta) = \sum_{i=0}^{N} B_{ki}(\delta_k - \delta_i) \tag{2.9}$$

for $k = 1, ..., N$. Under the assumptions that the network is connected, J_p is non-singular and a compact linearized matrix representation of the $P - \delta$ problem takes on a form

$$\underset{\sim}{P}(\underset{\sim}{\delta}) = J_p \underset{\sim}{\delta} \tag{2.10}$$

This formulation, also known as the DC load flow, is equivalent to the nodal equations representing a linear resistive network whose branch parameters are B_{ik}, and in which currents and voltages are replaced by real power flows and phase angles, respectively. Note that this formulation assumes linearization of nonlinear constituent relations, which is only meaningful for small phase angle differences. For large changes in phase angles problems may arise in certain ranges of operating conditions. This leads to the need to study a decoupled $P - \delta$ problem as a non-linear resistive network problem.

2.4 Nonlinear $P - \delta$ Problem

Employing the commonly used fact in network theory that each network which consists of devices whose constituent $v-i$ relations are nonlinear monotone functions defined in the first quadrant has a behavior of a linear resistive network [3], a nonlinear network formulation for the real power-phase angle problem was introduced in [21, 20]. The formulation is similar to the linearized problem formulation given above, except for the constituent relations defining line power flows in terms of phase angle differences being nonlinear

$$p_{ik} = V_i V_k B_{ik} \sin(\delta_i - \delta_k) \tag{2.11}$$

Since this constituent relation is not monotone for the entire range of phase angle differences, limits on phase angle changes $\Delta\delta_{ij}$ around a nominal operating point δ_{ij} need to be defined first for which the real power-phase angle problem can be interpreted as a nonlinear resistive problem of the certain range of operating conditions.

From the decoupled real power equations, the change in real power input at each node i can be expressed as

$$\Delta P_i = V_i \sum_{j \in K_i} V_j B_{ij} (\sin(\delta_{ij} + \Delta\delta_{ij}) - \sin\delta_{ij}), i = 1, ..., (n+k) \tag{2.12}$$

or,

$$\Delta P_i = \sum_{j \in K_i} c_{ij} h_{ij}(\Delta\delta_{ij}) \tag{2.13}$$

where, $h_{ij}(\Delta\delta_{ij}) = \sin(\delta_{ij} + \Delta\delta_{ij})$ and K_i are nodes directly connected to node i, excluding i. It can be seen that $h_{ij}(\Delta\delta_{ij})$ is monotonically increasing in the following region of interest: Assuming $\Delta\delta_{ij} \leq r_{ij}$ where r_{ij} is prespecified, the function defined before will be monotone increasing as long as

$$\frac{dh_{ij}(\Delta\delta_{ij})}{d\Delta\delta_{ij}} = \cos(\delta_{ij} + \Delta\delta_{ij}) \geq 0 \tag{2.14}$$

or,

$$-\frac{\pi}{2} \leq (\delta_{ij} + \Delta\delta_{ij}) \leq \frac{\pi}{2} \tag{2.15}$$

Next, create a linear system of equations of the form

$$\Delta P_i = \sum_{j \in K_i} \Delta\delta_{ij} g_{ij} c_{ij} \tag{2.16}$$

where $g_{ij} = g_{ji}$. If we examine solutions of (2.12) for which (2.15) holds, then it is only necessary to consider g_{ij} in (2.16) which are bounded by

$$\frac{\sin(\delta_{ij}+r_{ij})-\sin\delta_{ij}}{-r_{ij}} \leq g_{ij} \leq \frac{\sin(\delta_{ij}-r_{ij})-\sin\delta_{ij}}{r_{ij}} \tag{2.17}$$

Every solution to (2.12) with bounded $\Delta\delta_{ij}$then corresponds to one solution of (2.16) with some set of coefficients bounded by (2.17). The set of non-linear equations (2.12) has a non-linear resistive network interpretation, since it is of the form

$$\Delta \underline{P} = H_P \Delta \underline{\delta} \tag{2.18}$$

The off-diagonal elements are

$$h_{ij} = c_{ij} g_{ij} \tag{2.19}$$

and the diagonal elements

$$h_{ij} = -\sum_{j\in K_i, i\neq j} c_{ij} g_{ij} \tag{2.20}$$

The matrix H_P can be thought of as a conductance matrix of a resistive network with the same topology as the underlying power network.

2.5 Localized Response Property

It is known, that the change in voltage phase angles decreases monotonically as the electrical distance from the triggering event increases [12, 11]. This is what is known as the localized response property. In order to study this property, it is convenient to introduce another bus enumeration dependent on the bus where the triggering event (change in power input, ΔP_i) occurred [29]. We partition buses into tiers as in Figure 2-2. Let us call the bus where the change in power input ΔP_i occurs, tier 1. The buses directly connected to it are tier 2. The buses directly connected to tier 2 are tier 3 and so on. Now let us partition vectors $\Delta \underline{P}$ and $\Delta \underline{\delta}$ into subvectors $\Delta \underline{P_k}$ and $\Delta \underline{\delta_k}$ corresponding to all bus voltage angles and power injections in buses in tier k.

Now, for a $P-\delta$ network partitioned into tiers, the localized response property establishes that for a single change in power injection in bus 1, ΔP_1,

$$\|\Delta \underline{\delta_1}\|_\infty \geq \|\Delta \underline{\delta_2}\|_\infty \dots \geq \|\Delta \underline{\delta_N}\|_\infty \tag{2.21}$$

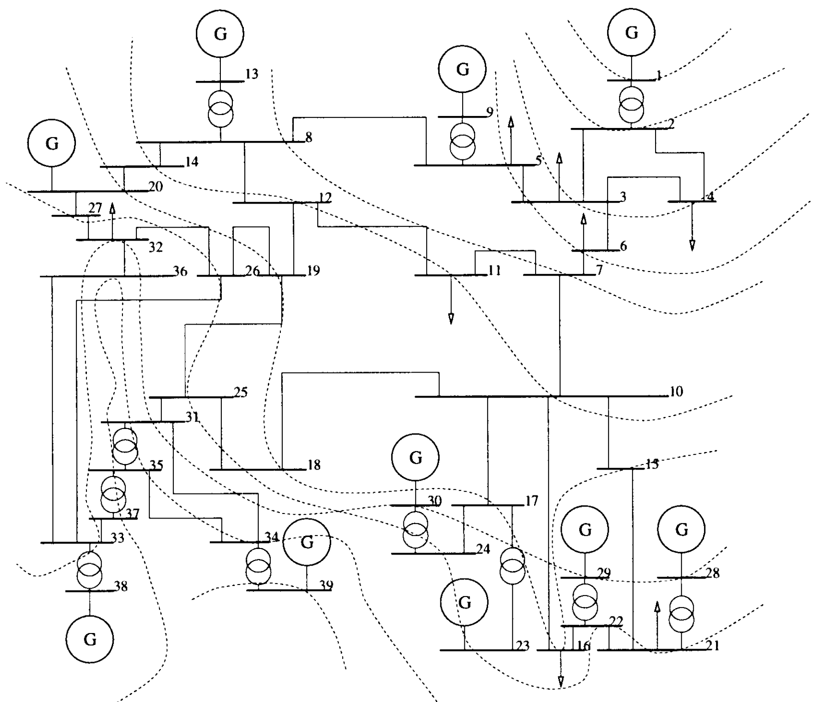

Figure 2-2: Tier-based Enumerated Network

where $\|x\|_\infty$ denotes the sup norm of x and $\Delta\delta_{\underline{k}}$ is the vector of phase angles changes in tier k. In other words, it states that the largest change in phase angle in tier 1 is not smaller that the largest change in phase angle in tier 2, and so on.

Proof: Note that it is enough to show that,

$$\|\Delta\delta_{\underline{k-1}}\|_\infty \geq \|\Delta\delta_{\underline{k}}\|_\infty \tag{2.22}$$

Note also that by our proceeding results it is enough to show the result for the linear $P-\delta$ formulation defined in equation (2.10).

Lemma: In the case of the linear problem it is enough to show the previous equation for $k = N$, where N is the number of tiers.

Proof: For $k < N$, one can by star-mesh transformation eliminate the nodes in tier $(k+1)$ through tier N without affecting the solutions in tiers 1 through k. Also the reduction will only introduce new lines between busses in tier k and thus not affect the tier structure of the network.

Proof of the Localized Response Property (linear case): Since we have ordered the network tier-wise, we have the following block tri-diagonal system of equations after the tier based bus re-enumeration

$$\begin{bmatrix} B_{11} & B_{12} & 0 & 0 & \cdot & 0 \\ B_{21} & B_{22} & B_{23} & 0 & \cdot & 0 \\ 0 & B_{32} & B_{33} & B_{34} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & B_{(N-1)(N-2)} & B_{(N-1)(N-1)} & B_{(N-1)N} \\ 0 & 0 & 0 & \cdot & B_{N(N-1)} & B_{NN} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\underline{\delta}_2 \\ \Delta\underline{\delta}_3 \\ \cdot \\ \Delta\underline{\delta}_{N-1} \\ \Delta\underline{\delta}_N \end{bmatrix} = \begin{bmatrix} \Delta P_1 \\ 0 \\ 0 \\ \cdot \\ 0 \\ 0 \end{bmatrix} \tag{2.23}$$

From above we can see that,

$$B_{N(N-1)}\Delta\underline{\delta}_{N-1} + B_{NN}\Delta\underline{\delta}_N = 0 \tag{2.24}$$

$$\Delta\underline{\delta}_N = (-B_{NN}^{-1}B_{N(N-1)})\Delta\underline{\delta}_{N-1} \tag{2.25}$$

Hence,

$$\|\Delta\underline{\delta}_N\|_\infty \| - B_{NN}^{-1}B_{N(N-1)}\|_\infty \|\Delta\underline{\delta}_{N-1}\|_\infty \tag{2.26}$$

where,

$$\| - B_{NN}^{-1}B_{N(N-1)}\|_\infty = \max_i \sum_j \|(-B_{NN}^{-1}B_{N(N-1)_{ij}})\| := v \tag{2.27}$$

Denote by $\underline{1}$ the vector all of whose components are 1. From elementary properties fo the admittance matrix it follows that every element in the diagonal of B_{NN} is non-negative and every element of $B_{N(N-1)}$ is non-positive. Hence,

$$v = \max_i \sum_j (-B_{NN}^{-1}B_{N(N-1)_{ij}}) = \max_i (-B_{NN}^{-1}B_{N(N-1)}\underline{1})_i \tag{2.28}$$

Since the sum of entries in each row is greater than or equal to zero

$$B_{NN}\underline{1} + B_{N(N-1)}\underline{1} \geq 0 \tag{2.29}$$

where $\geq$ means a component-wise inequality. Multiplying the above equation on both sides by B_{NN}^{-1},

$$\underline{1} + B_{NN}^{-1}B_{N(N-1)}\underline{1} \geq 0 \tag{2.30}$$

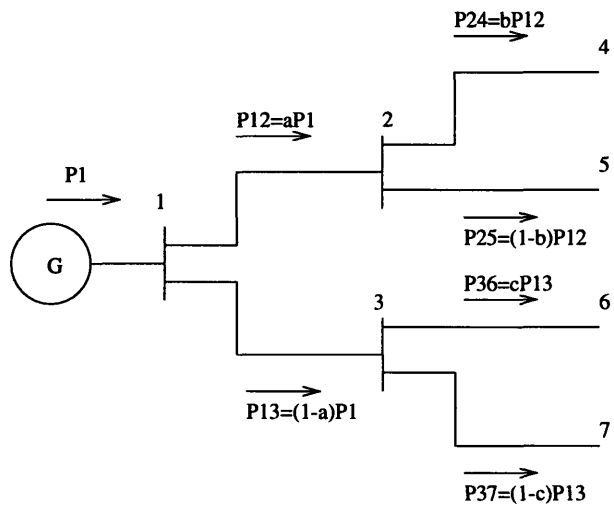

Figure 2-3: Relatively Meshed Network

or

$$-B_{NN}^{-1}B_{N(N-1)}\underline{1} \leq \underline{1} i.e.v \leq 1 \tag{2.31}$$

which proves the property.

2.5.1 Localized Response of Phase Angle Differences

This property, however, is not sufficient for the changes in phase angle differences across the transmission lines to have the same property, to be decreasing away from the cause of their change. Let us introduce an arbitrary relatively meshed network as in Figure 2-3, that branches out from a single node, that we are going to call node 1. From the DC load flow formulation we can see that linearity holds and we have,

$$P_{ij} + \Delta P_{ij} = B_{ij}(\delta_i + \Delta\delta_i - (\delta_j + \Delta\delta_j)) \tag{2.32}$$

Thus,

$$\Delta P_{ij} = B_{ij}(\Delta\delta_i - \Delta\delta_j) = B_{ij}\Delta\delta_{ij} \tag{2.33}$$

Since, the DC load flow formulation assumes lossless lines by an argument analogous to the current dividers of circuit theory, we can see that the change in power injection dies out from node 1. Assuming uniform (or at least the same order of magnitude) line parameters, we can see that if the change in power injection dies out, so does the change in angle differences,

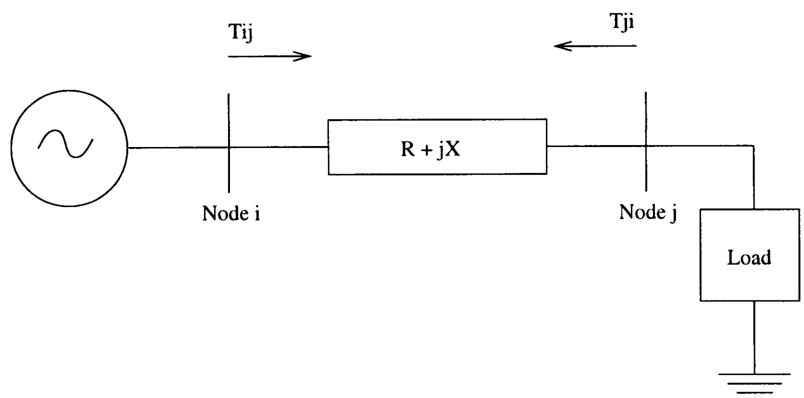

Figure 2-4: Simple 2 Bus System

making valid our assumption that if the changes in power injection occur 'far away' from each other, transmission losses can be approximated in a localized way. From Figure 2-3 we can see that the power P_1 decreases away from node 1, being a, b and c the ratios of input impedances seen from nodes 1, 2 and 3, respectively. These ratios will always be smaller than 1, thus making our assumption valid.

2.6 Real Power Losses

From the simplest power system of a single generator and a single load connected through a transmission line as in Figure 2-4, real power T_{ij} going out of node i, is given by

$$T_{ij} = \Re\{V_i e^{j\delta_i} I^*\} \tag{2.34}$$

where I^* is the complex conjugate of the current and is given by,

$$I^* = \left(\frac{V_i e^{j\delta_i} - V_j e^{j\delta_j}}{R_{ij} + jX_{ij}}\right)^* \tag{2.35}$$

Recalling that $\frac{1}{R_{ij}+jX_{ij}} = G_{ij} - jB_{ij}$ and applying the conjugate to the expression in parenthesis, we have that,

$$T_{ij} = \Re\left\{V_i e^{j\delta_i}\left(\frac{V_i e^{j\delta_i} - V_j e^{j\delta_j}}{R_{ij}+jX_{ij}}\right)^*\right\} \tag{2.36}$$

or,

$$T_{ij} = \Re\{V_i e^{j\delta_i}(V_i e^{-j\delta_i} - V_j e^{-j\delta_j})(G_{ij} + jB_{ij})\} \tag{2.37}$$

$$T_{ij} = \Re\{(V_i^2 - V_i V_j e^{j(\delta_i - \delta_j)})(G_{ij} + jB_{ij})\} \tag{2.38}$$

This takes the following form.

$$T_{ij} = V_i^2 G_{ij} - V_i V_j (G_{ij}\cos(\delta_i - \delta_j) + B_{ij}\sin(\delta_i - \delta_j)) \tag{2.39}$$

Similarly, the real power flowing from node j to node i takes on the form,

$$T_{ji} = V_j^2 G_{ij} - V_i V_j (G_{ij}\cos(\delta_i - \delta_j) - B_{ij}\sin(\delta_i - \delta_j)) \tag{2.40}$$

Then, the losses in line $i - j$ are given as the sum of both power flows as,

$$P_{Lij} = T_{ij} + T_{ji} \tag{2.41}$$

$$\begin{aligned} &= G_{ij}(V_i^2 - 2V_i V_j \cos(\delta_i - \delta_j) + V_j^2) \\ &\approx G_{ij}\left[V_i^2 - 2V_i V_j\left[1 - \frac{(\delta_i - \delta_j)^2}{2}\right] + V_j^2\right] \\ &= G_{ij}((V_i - V_j)^2 + V_i V_j(\delta_i - \delta_j)^2) \end{aligned}$$

If all voltages are at nominal values, it is $V_i = V_j = 1.0 p.u.$ then,

$$P_{Lij} \approx G_{ij}(\delta_i - \delta_j)^2 \tag{2.42}$$

or in matrix form,

$$P_L \approx [\underline{\delta_{ij}}]^T [B_{ij}] [\underline{\delta_{ij}}] \tag{2.43}$$

where δ_{ij} is the vector of angle differences and is given by,

$$\delta_{ij} = \delta_i - \delta_j \tag{2.44}$$

$$\underline{\Gamma} = M\underline{\delta} \tag{2.45}$$

$\underline{\delta}$ being the vector of node angles, and M a reduce incidence matrix whose elements are 0, 1 and -1.

Chapter 3

Estimation of Real Power Losses

In this chapter a closed formula for computing voltage phase angle changes created by the end user located at bus i is introduced. This formula only requires knowledge of the power increase ΔP_i at the location i and the network parameters of the entire system. The algorithm is based on the linearized, decoupled real power load flow equations. Numerical methods similar in concept could be derived without making the linearizating assumption. The results of this algorithm, combined with the updated nominal voltage phase angles of the entire network are used as the starting information for estimating real power transmission loss created by the end user at bus i when injecting ΔP_i into the system. First, a formula for computing real power losses is described that reflects the interaction of nodal power increments in the interconnected system. This formula cannot be used by each end user independently from the others. It is next proposed that for the most typical power networks the line flow changes ΔP_{ij} decrease in absolute value away from the location where power is injected into the system. Given this property, it is possible to claim that the voltage phase angle differences across the transmission lines also decrease in proportion with the line reactances. This further leads to the conjecture that the effects of power input changes resulting from economic transactions are separable to significant degree, unless the transactions are very close electrically. This conjecture is formally derived.

3.1 A Localized Method for Computing Changes in Voltage Phase Angles

Starting from the last two rows in equation (2.23) in Chapter 2 one obtains,

$$B_{(N-1)(N-2)}\Delta\delta_{N-2} + B_{(N-1)(N-1)}\Delta\delta_{N-1} + B_{(N-1)N}\Delta\delta_{N} = 0$$

$$B_{(N-1)(N-2)}\Delta\delta_{N-2} + B_{(N-1)(N-1)}\Delta\delta_{N-1} - B_{(N-1)N}B_{NN}^{-1}B_{N(N-1)}\Delta\delta_{N-1} = 0$$

$$B_{(N-1)(N-2)}\Delta\delta_{N-2} + (B_{(N-1)(N-1)} - B_{(N-1)N}B_{NN}^{-1}B_{N(N-1)})\Delta\delta_{N-1} = 0$$

This leads to a recursive relation,

$$\Delta\delta_{N-1} = (-\tilde{B}_{(N-1)(N-1)}^{-1}B_{(N-1)(N-2)})\Delta\delta_{N-2}$$

where,

$$\tilde{B}_{(N-1)(N-1)} = B_{(N-1)(N-1)} - B_{(N-1)N}B_{NN}^{-1}B_{N(N-1)}$$

By solving equation (2.23) backwards, $\Delta\delta_1$ is found as a function only of ΔP_1 and the system matrix, B, as,

$$\Delta\delta_1 = (B_{11} - B_{12}\tilde{B}_{22}^{-1}B_{21})\Delta P_1 \tag{3.1}$$

Using this expression and the expressions for all changes in voltage angles, one can find by forward substituting the changes introduced by the power injection at bus 1 as,

$$\Delta\delta_j = (-\tilde{B}_{jj}^{-1}B_{j(j-1)})\Delta\delta_{j-1} \tag{3.2}$$

for $j = 2, ..., N$ where

$$\tilde{B}_{kk} = B_{kk} - B_{k(k+1)}\tilde{B}_{(k+1)(k+1)}B_{(k+1)k} \tag{3.3}$$

for $k = 2, ..., (N-1)$ and $\tilde{B}_{NN} = B_{NN}$.

It is now important to note that this formulation will have a solution as long as the B matrix is nonsingular. This suggests the use of a slack bus, similar to the one in the load flow formulation.

This result is potentially useful for distributed decision-making in a deregulated energy market. What this result implies is that given the change in power injection in a single bus, nominal operating conditions on the network known, it is possible to calculate phase angle changes for all nodes. This angle changes at the same time can be used to compute line losses locally.

3.2 An Approximate Method for Computing Real Power Transmission Losses

Given nominal operating conditions of the network are known (in a deregulated power industry this is going to be given by an On-Line Same-Time Information System (OASIS)), voltage angle changes $\Delta\delta_i$, introduced in the network by the power injection ΔP_i in bus i can be approximated using this forward-backward substitution algorithm. Next, one can approximate total real power losses in the network as follows,

$$P_L \approx [\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}]^T[B_{ij}][\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}] \tag{3.4}$$

or line by line,

$$P_{\underline{L}ij} \approx [[\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}]][B_{ij}][[\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}]] \tag{3.5}$$

where, B_{ij} is the matrix whose diagonal is the line conductances, $[\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}]$ is the vector of phase angle changes in a line, and $[[\underline{\delta_{ij}} + \Delta\underline{\delta_{ij}}]]$ is a diagonal matrix whose elements are defined as phase angle differences across each line.

$$\delta_{ij} = \delta_i - \delta_j \tag{3.6}$$

$$\underline{\delta_{ij}} = M[\delta_i ... \delta_N]^T \tag{3.7}$$

where $i, j = 1, 2, ..., N$ and M is an $\ell \times N$ matrix of zeros, ones and minus ones as defined in [5], where ℓ is the number of lines and N is the number of nodes. The elements of this matrix are defined by the following rule,

$$m_{ik} = \left\{ \begin{array}{c} 1 \\ -1 \\ 0 \end{array} \right\} \tag{3.8}$$

if line k leaves, enters or is not incident with node i, respectively.

Let us introduce a vector $\underline{\Gamma}$, such that $\underline{\Gamma} = M\underline{\delta}$. Following the tier-wise enumeration, then M takes on the following form,

$$\Gamma = \begin{bmatrix} 1 & -1 & 0 & . & . & . & . & & & \\ 1 & 0 & -1 & 0 & . & . & . & . & & \\ 0 & 1 & 0 & -1 & 0 & . & . & & & \\ . & 0 & 1 & 0 & -1 & 0 & . & . & & \\ & 1 & 0 & . & 0 & -1 & 0 & . & . & \\ & & 1 & 0 & . & . & -1 & 0 & . & \\ & & & & 0 & 1 & 0 & -1 & 0 & . \\ & & & & & 0 & 1 & -1 & 0 & . \\ & & & & & & & . & . & \\ & & & & & & & & . & . \end{bmatrix} \begin{bmatrix} \delta_1 \\ \underline{\delta_2} \\ . \\ . \\ . \\ . \\ . \\ . \\ \delta_{(\underline{N}-1)} \\ \delta_{\underline{N}} \end{bmatrix} \tag{3.9}$$

where the vector composed of δ_1 to $\delta_{\underline{N}}$, is the vector of node angles in tiers 1 to N. The vector of angle differences Γ is divided in subvectors Γ^I to $\Gamma^{(N-1)}$ according to tiers, for example Γ^I is a subvector of angle differences between busses in tier 1 and tier 2, plus intra-tier connections, that is busses in tier 2 connected between each other. The same way, Γ^{II} is a subvector of angle differences between busses in tier 2 and tier 3, plus intra tier connections in tier 3. It is easy to see that the matrix M is rank deficient. In order to make this compatible with the rest of the formulation, it is needed to eliminate the column which corresponds to the slack bus.

Now, consider various transactions occurring simultaneously. The vector $\Delta\delta_{ij}$ reflects the effect of all transactions on phase angle deviations from nominal conditions. In a competitive industry, this information is not going to be available, so it is imperative to find a way to estimate losses locally, or as a function only of the change in power injection at only one bus. To do this, let us first recall the formula for losses given in equation (2.42),

$$P_L \approx G_{ij}(\delta_{ij} + \Delta\delta_{ij})^2 \tag{3.10}$$

Since $\Delta\delta_{ij}$ is estimated using linearized $P-\delta$ model formulation, superposition holds, i.e.,

$$\Delta\delta_{ij} = \Delta\delta_{ij,1} + \Delta\delta_{ij,2} + ... + \Delta\delta_{ij,k} \tag{3.11}$$

Here $\Delta\delta_{ij,k}$ is the vector of change in angle differences due to a change in injection at bus k. Expanding equation (3.10) we have,

$$P_L \approx G_{ij}(\delta_{ij}^2 + 2\delta_{ij}\Delta\delta_{ij} + \Delta\delta_{ij}^2) \tag{3.12}$$

The terms $2\delta_{ij}\Delta\delta_{ij}$ and $\Delta\delta_{ij}^2$ are now

$$2\delta_{ij}\Delta\delta_{ij} = 2\delta_{ij}(\Delta\delta_{ij,1} + \Delta\delta_{ij,2} + ... + \Delta\delta_{ij,k}) \tag{3.13}$$

$$\Delta\delta_{ij}^2 = \Delta\delta_{ij,1}^2 + \Delta\delta_{ij,2}^2 + ... + \Delta\delta_{ij,k}^2 + 2\Delta\delta_{ij,1}\Delta\delta_{ij,2} + 2\Delta\delta_{ij,1}\Delta\delta_{ij,k} + ... \tag{3.14}$$

Now, recalling the localized response property and asumming the change in power injections are electrically distant, we can approximate $\Delta\delta_{ij}^2$ as,

$$\Delta\delta_{ij}^2 = \Delta\delta_{ij,1}^2 + \Delta\delta_{ij,2}^2 + ... + \Delta\delta_{ij,3}^2 \tag{3.15}$$

For example, let us assume that only one bilateral transaction is occurring between nodes 1 and k and that they are separate enough that the localized response property holds. It can be seen that the term $2\Delta\delta_{ij,1}\Delta\delta_{ij,k}$ will be close to zero, because the elements of $\Delta\delta_{ij,1}$ that are significant will be multiplied by the elements of $\Delta\delta_{ij,k}$ that are close to zero. The same is true the other way around. Now we can approximate the power losses as,

$$P_L \approx G_{ij}[\delta_{ij}^2 + 2\delta_{ij}\Delta\delta_{ij,1} + 2\delta_{ij}\Delta\delta_{ij,2} + ... + 2\delta_{ij}\Delta\delta_{ij,k} + \Delta\delta_{ij,1}^2 + \Delta\delta_{ij,2}^2 + ... + \Delta\delta_{ij,k}^2] \tag{3.16}$$

$$\begin{aligned} P_L \approx G_{ij}\delta_{ij}^2 + G_{ij}(2\delta_{ij}\Delta\delta_{ij,1} + \Delta\delta_{ij,1}^2) + G_{ij}(2\delta_{ij}\Delta\delta_{ij,2} + \Delta\delta_{ij,2}^2) + ... \\ + G_{ij}(2\delta_{ij}\Delta\delta_{ij,k} + \Delta\delta_{ij,k}^2) \end{aligned} \tag{3.17}$$

$$P_L \approx P_L(\delta_{ij}) + \Delta P_L(\Delta\delta_{ij,1}) + \Delta P_L(\Delta\delta_{ij,2}) + ... + \Delta P_L(\Delta\delta_{ij,k}) \tag{3.18}$$

where $P_L(\delta_{ij})$ are the base case losses to be provided by the OASIS [19] and $\Delta P_L(\Delta\delta_{ij,k})$ is a change in transmission losses due to change in power injection at bus k.

3.3 Algorithm for Estimating Real Power Losses

In this section, we describe the steps involved in estimating real power losses on transmission lines, given that nominal operating conditions of the network are provided by the OASIS. We describe, how this estimation will be done by each individual player in a de-regulated energy market.

Step 1:

The first step in estimating real power transmission losses is to re-enumerate the network using the tier based re-enumeration. At the same time, it is necessary to define directions of power flows, in order to construct the M matrix. Special attention should be placed in order to make this matrix correspond to the nominal conditions provided by the OASIS. At this time, the B matrix can be constructed and sub-divided into sub-matrices $B_{11}, B_{12}, B_{21}, B_{22}, ..., B_{N,N-1}, B_{N,N}$.

Step 2:

After all data is available, changes in node angles caused by this player can be computed by the same player and independently of the others. This is done using the forward/backward substitution introduced in section 3.1, i.e.,

$$\Delta\delta_1 = (B_{11} - B_{12}\tilde{B}_{22}^{-1}B_{21})\Delta P_1 \tag{3.19}$$

$$\Delta\underline{\delta_j} = (-\tilde{B}_{jj}^{-1}B_{j(j-1)})\Delta\underline{\delta_{j-1}} \tag{3.20}$$

for $j = 2, ..., N$ where

$$\tilde{B}_{kk} = B_{kk} - B_{k(k+1)}\tilde{B}_{(k+1)(k+1)}B_{(k+1)k} \tag{3.21}$$

for $k = 2, ..., (N-1)$ and $\tilde{B}_{NN} = B_{NN}$ After change in node angles are known, one can compute change in angle differences as,

$$\Delta\underline{\delta_{ij}} = M\Delta\underline{\delta} \tag{3.22}$$

Step 3:

The next step in estimating real power transmission losses is to compute line by line change in real power losses introduced by a transaction using the formulation described as,

$$\Delta P_L(\Delta\delta_{ij,1}) = G_{ij}(2\delta_{ij}\Delta\delta_{ij,1} + \Delta\delta_{ij,1}^2) \tag{3.23}$$

where $\Delta\delta_{ij,1}$ is defined, in the same way as before, as the vector of change in angle differences due to a change in power injection at node 1.

Step 4:

Now it is appropriate to mention, how would each individual player act in order to compensate for the losses introduced by each transaction. The idea is that every individual player compute the losses introduced by its transaction and then compensate for it. Let us analyze how would each player act in a bi-lateral transaction. Let us say there is a load X that wants to buy 100 MW from Generator Y. What will actually happen is that load X will estimate the losses that a 100 MW change at its node will introduce. Let us call this losses ΔP_X. Then the load will have to make a contract to buy $(100 + \Delta P_X)$MW. From the generator side, Generator Y will have then to compute how much losses this increase of $(100 + \Delta P_X)$MW will create. Let's call this ΔP_Y. In effect, Generator Y will have to increase its generation by $(100 + \Delta P_X + \Delta P_Y)$MW, in order to compensate for its losses.

Chapter 4

Reactive Power-Voltage Magnitude Problem

In this chapter, the problem of estimating reactive power losses in an interconnected power network is studied. First, the decoupled reactive power-voltage (QV) load flow equations are briefly reviewed. These form the governing equations of direct interest. Next, the state of-the-art results concerned with a non-linear network interpretation of the QV load flow problem are summarized. It is concluded that such an interpretation is possible. However, the resulting non-linear network, because of the presence of shunt capacitors on a primarily inductive network, is analogous to a non-linear DC resistive network, not all of whose resistors are monotonically increasing. A qualitative implication of this situation is that it is not possible to state unconditionally that a change in reactive power injection ΔQ_i into bus i leads to uniform decrease in voltage changes ΔV_i [13]. This can only be proven when the shunt capacitors are not present.

This obstacle could be overcome in the context of the functions of an end user in a competitive environment by decomposing the problem of reactive power loss compensation into[1]

1. The shunt reactive power loss component, measurable directly in terms of local power factor compensation.

2. The reactive power losses created in the planar transmission grid that interconnects

[1]The same idea was recognized recently in [1].

all nodes.

An underlying modeling assumption here is that the reactive power inputs into the nodes are represented as ideal reactive power injections into the grid[2]. The total reactive power injection could be thought of as consisting of the portion flowing from the node to the ground, and the portion flowing into the planar transmission network. It is proposed here that the shunt reactive losses be directly estimated and compensated by each end user; this is trivial to do. The method proposed in this thesis introduces an approach to estimate the second component. It is proven in this thesis that the voltage changes ΔV_i away from the reactive power injection into the planar portion at node i of the grid decrease uniformly away from this location. A closed form solution for estimating the voltage deviations in the entire network caused by the specific end user is derived by using only the information about the local injection into the grid ΔQ_i and the network parameters of the entire grid. Next, an approximate formula for reactive power losses in response to reactive power changes at several locations in this system is derived. This formula requires knowledge about nominal voltages, that is assumed to be provided by a real time information network of some sort [17, 19]. It is proposed that for system input changes that are not very close electrically, reactive power loss can be estimated and compensated individually by each end user.

4.1 Reactive Power - Voltage Magnitude Problem

Under the real-reactive power decoupling assumption the reactive power-voltage magnitude problem is defined as the problem of determining load voltage magnitudes for specified load reactive power demand; generator voltages are assumed fixed, as long as generator reserves are available. Under the decoupling assumption all phase angles are known parameters. The mathematical formulation of the reactive power-voltage problem follows directly from the load flow equations defining reactive power balance at all loads,

$$V_i^2\left((\sum_{k\neq i} B_{ik}) - b_i\right) - V_i \sum V_k B_{ik} \sin(\delta_{ik} - \zeta_{ki}) - Q_i = 0, i = 1, ..., n \tag{4.1}$$

[2]This includes flows through the capacitors.

Typical inequality constraints directly relevant for voltage are

$$Q_{gi}^{min} < Q_{gi} < Q_{gi}^{max}, i = n+1, ..., n+k \tag{4.2}$$

$$V_i^{min} < V_i < V_i^{max}, i = 1, ..., n+k \tag{4.3}$$

Reactive power steady state problems are directly related to the operating conditions under which operationally acceptable voltage solution does not exist within these limits, or the solution becomes very sensitive to small variations in system inputs and parameters.

The main theoretical difficulty in formulating the reactive power - voltage magnitude problem arises because loads are modeled as constant power models. It is simple to show that in the case when loads are modeled as either constant impedance or constant current devices, the reactive power-voltage problem degenerates into a linear problem of the form

$$A\underline{x} = \underline{c} \tag{4.4}$$

where matrix A and input vector $\underline{c}$ are defined in the following way [25]:

- (i) In the case of assumed constant impedance load models as

$$Q_i = B_{Di} V_i^2, B_{Di} = const., i = 1, ..., n \tag{4.5}$$

equation (4.1) degenerates into

$$\left(\sum_{j=1}^{n} B_{ik} - b_i + B_{Di}\right) E_i - \sum_{j=1}^{n} B_{ij} E_j \cos \delta_{ij} = \sum_{j=n+1}^{n+k} B_{ij} E_j \cos \delta_{ij} \tag{4.6}$$

The matrix representation of these equations is of the form (4.4) where

$$A = B + diag(B_{Di}) \tag{4.7}$$

$$B = \begin{bmatrix} (B_{11} - b_1) & -B_{12} & ... & -B_{1n} \\ -B_{21} & (B_{22} - b_2) & ... & -B_{2n} \\ ... & ... & ... & ... \\ -B_{n1} & ... & ... & (B_{nn} - b_n) \end{bmatrix} \tag{4.8}$$

and

$$\underset{\sim}{c}^T = [c_1, ..., c_n] \tag{4.9}$$

with

$$c_i = \sum_{j=n+1}^{n+k} B_{ij} V_j \cos \delta_{ij} \tag{4.10}$$

and the unknown vector $\underset{\sim}{x}$ being the vector of load voltage

$$\underset{\sim}{x}^T = [V_1, ..., V_i] \tag{4.11}$$

Clearly, as long as network and load parameters are such that matrix A remains nonsingular, the problem of voltage solution nonexistence is avoided. The only remaining question is if the voltage solution is within the pre-specified operating limits defined previously and it it is achievable within the specified generation limits.

- (ii) Similarly, in the case of constant current load models of the form

$$Q_{Di} = \Im\{\hat{S}_{Di}\} = E_i I_i \sin \phi_i \tag{4.12}$$

with ϕ_i being the load power factor angle, current I_i - constant (under the constant current load model assumption) and the power factor angle ϕ_i = constant (under the real-reactive power decoupling assumption). The reactive power load flow equations take on the form of linear equations in unknown voltages

$$V = [V_1, ..., V_n]^T \tag{4.13}$$

with the system matrix equal B defined in (8-8), and the known vector $\underset{\sim}{c}$

$$\underset{\sim}{c}^T = [I_i \sin \phi_1, ..., I_n \sin \phi_n]^T \tag{4.14}$$

These particular cases of reactive power-voltage problem formulation under simplifying load modeling assumptions show an obvious problem in interpreting any theoretical results to the reactive power-voltage problem and its dependence on load models assumed. Any deviation from the assumed load modes could lead to different solutions. The hope is that the system would operate in regions with low sensitivity of voltage solution to the choice of load model. Otherwise, there would be too much risk involved in employing theoretical results for estimating regions within which a robust, technically acceptable voltage solution exists. The most frequent modeling assumption of constant reactive power loads leads to a genuine nonlinear problem, whose solution may not exist, or it may be non-unique.

4.2 The Q-V Problem as a Nonlinear Resistive Network Problem

The reactive power-voltage (Q-V) problem can be formulated as a non-linear network problem in two slightly different ways[3]:

- As a non-linear resistive network with independent sources
- As a linear resistive network with dependent sources

We place more emphasis on the first formulation, since it is the one we use to estimate reactive power losses. Recall that the real power-phase angle problem is inherently a problem of non-linear resistive networks with independent sources. It is less obvious how can the reactive power-voltage problem be interpreted as a network problem than it was with the real power-phase angle problem formulation. To show this, start with the decoupled reactive power-voltage load flow equations whose form under the assumption of negligible transmission line resistances becomes,

$$Q_i = -V_i \sum_{k \in K_i} V_k c_{ik} \tag{4.15}$$

where K_i are the busses directly connected to bus i (including i) and

$$c_{ik} = c_{ki} = B_{ik} \cos \delta_{ik} \tag{4.16}$$

and

$$c_{ii} = B_{ii} - b_i = \sum_{k \in K_i} B_{ik} \tag{4.17}$$

To clarify sign convention in (4.15) notice that for inductive transmission lines $B_{ik} < 0$ and capacitive shunts $b_i > 0$ where b_i is a shunt connection at bus i. Since the E_i are voltage

[3]This section is a summary of the formulation in [24]

magnitudes and as such are always positive, write

$$V_i = e^{x_i} \tag{4.18}$$

to form

$$Q_i = \sum_{k \in K_i} e^{(x_i + x_k)} c_{ik} \tag{4.19}$$

or,

$$Q_i = -\sum_{k \in K_i} f(x_i + x_k) c_{ik} \tag{4.20}$$

Similarly as in the real power-phase angle studies, we are concerned with perturbations around a nominal steady state, and therefore we form

$$\Delta Q_i = -\sum_{k \in K_i} [f(x_i + x_k + \Delta x_i + \Delta x_k) - f(x_i + x_k)] \tag{4.21}$$

or,

$$\Delta Q_i = -\sum_{k \in K_i} h_{ik}(\Delta x_i + \Delta x_k) c_{ik} \tag{4.22}$$

where the function h_{ik} is defined as

$$h_{ik} = e^{x_i + x_k} \cdot (e^{\Delta x_i + \Delta x_k} - 1) \tag{4.23}$$

It can be seen that $h_{ik}(\Delta x_i + \Delta x_k)$ is monotone increasing and restricted to the first and third quadrants in $(\Delta x_i + \Delta x_k)$. We create a linear system of equations of the form

$$\Delta Q_i = -\sum_{k \in K_i} (\Delta x_i + \Delta x_k) c_{ik} g_{ik} \tag{4.24}$$

where $g_{ik} = g_{ki}$. If we examine solutions of the previous equation for which $|\Delta x_j| < r$ for $j = 1, ..., n$ then it is only necessary to consider g_{ik} which are bounded by

$$V_i V_k \frac{(e^{-2r} - 1)}{-2r} \leq g_{ik} \leq V_i V_k \frac{(e^{2r} - 1)}{2r} \tag{4.25}$$

Every solution (4.22) with bounded Δx_i then corresponds to a solution of (4.24) with some set of coefficients bounded by inequalities (4.25). We show next that the set of equations (4.22) has a resistive network interpretation. To put the linear system (4.24) in more recognizable form, rewrite it as

$$-\Delta \underline{Q} = H_Q \Delta \underline{x} \tag{4.26}$$

where

$$h_{ik} = c_{ik} g_{ik} = g_{ik} B_{ik} \cos \delta_{ik} \tag{4.27}$$

and

$$h_{ii} = -\sum_{k \in K_i, k \neq i} c_{ik} g_{ik} + 2 c_{ii} g_{ii} \tag{4.28}$$

The conventions involved in (4.15) produce non-positive off-diagonal entries in the admittance matrix for the power system. If we associate $-\Delta \underline{Q}$ with a set of currents and $\Delta \underline{x}$ with the set of node voltages, then the matrix can be thought of as a conductance matrix of a resistive network with the same topology as the power system. Nodes i and k in the resistive network are connected with positive conductance $-g_{ik} B_{ik} \cos \delta_{ik}$. The conductance to ground at node i is given by

$$h_{is} = 2 g_{ii} c_{ii} + 2 \sum_{k \in K_i, k \neq i} c_{ik} g_{ik} \tag{4.29}$$

$$h_{is} = 2 g_{ii} \left[-b_i - \sum_{k \in K_i, k \neq i} B_{ik} \right] + 2 \sum_{k \in K_i, k \neq i} g_{ik} B_{ik} \cos \delta_{ik} \tag{4.30}$$

To verify that the conductance h_{ii} are typically negative, consider perturbations small enough so that

$$g_{ik} \cong V_i V_k \tag{4.31}$$

In this case

$$h_{is} \cong -2 Q_i \tag{4.32}$$

The typical load bus (PQ bus) in the power system has $Q_i > 0$ so that for small disturbances the equivalent resistive network is composed of positive resistors to ground at the nodes corresponding to load busses. The DC load flow assumptions of small angles δ_{ik} and voltage magnitudes near unity give, for small perturbations

$$h_{is} \cong -2 B_i \tag{4.33}$$

Again, the typical PQ bus has a capacitive connection to ground from the transmission line models and possible shunt capacitors which with our sign convention is a negative value of h_{is}. Estimates fo the size of perturbations r, for which the shunt connections remain negative can also be obtained. If we assume

$$\left[-b_i - \sum_{k\in K_i, k\neq i} B_{ik}\right] > 0 \tag{4.34}$$

which is typical at PQ busses the h_{is} is maximized if g_{ii} takes on the maximum value possible while each g_{ik} is the minimum. For example, for the voltage reduction case,

$$h_{is} = Q_{is} + \frac{1 - e^{2r}}{2r} Q_{i2} \tag{4.35}$$

where

$$Q_{i2} = 2V_i^2 \left[b_i - \sum_{k\in K_i} B_{ik}\right] \tag{4.36}$$

and

$$Q_{i2} = 2 \sum_{k\in K_i, k\neq i} V_i V_k B_{ik} \cos\delta_{ik} \tag{4.37}$$

Since Q_{is} is available from the diagonal entry of the Jacobian of the load flow solution and

$$Q_i = -\frac{(Q_{is} + Q_{i2})}{2} \tag{4.38}$$

the range of r for which (4.35) is negative can be computed directly.

An alternate, less combinatorial approach is presented next. The approach is based on formulating the Q-V problem as a linear resistive network problem with dependent sources.

4.3 Reactive Power-Voltage Problem as a Linear Resistive Network with Dependent Sources

In order to clarify differences between the problem formulation of the $Q - V$ nonlinear problem as a nonlinear resistive network problem with independent sources [13] summarized above, on one side, and the formulation given in [27, 24], which is presented in this section, on the other side, the following should be recognized:

A general nonlinear resistive network problem can be thought of as an algebraic problem fo n coupled equations in n unknowns of the form

$$g_1(x_1, ..., x_n) = c_1 \tag{4.39}$$

$$g_2(x_1, ..., x_n) = c_2 \tag{4.40}$$

$$g_n(x_1, ..., x_n) = c_n \tag{4.41}$$

where each function g_i depends on all unknown variables. While conditions under which a solution to this general problem exists are not known, by recognizing a linear resistive network structure of the class of the problem of interest here, it becomes possible to solve this problem in terms of bounds on changes within which unique solution guaranteed to exist.

It has been shown in [24] that the Q-V problem can be formulated as a particular subclass of the general algebraic problem defined previously of the form

$$A\underline{x} + \underline{f}(\underline{x}) = \underline{c} \tag{4.42}$$

This problem and conditions for its solution existence have been studied extensively in the area of nonlinear monotone networks [25]. Here each nonlinear function f_i is a function of one variable x_i only. In [27, 24] nonlinear reactive power-voltage problem was formulated as a problem of type (4.42).

4.4 Forward-Backward Algorithm for Estimating Changes in Nodal Voltage Magnitudes

Starting from equation (4.26) and rearranging it in a tier wise manner, we have,

$$\begin{bmatrix} H_{11} & H_{12} & 0 & 0 & \cdot & 0 \\ H_{21} & H_{22} & H_{23} & 0 & \cdot & 0 \\ 0 & H_{32} & H_{33} & H_{34} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & H_{(N-1)(N-2)} & H_{(N-1)(N-1)} & H_{(N-1)N} \\ 0 & 0 & 0 & \cdot & H_{N(N-1)} & H_{NN} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta \underline{x}_2 \\ \Delta \underline{x}_3 \\ \cdot \\ \Delta x_{\underline{N}-1} \\ \Delta \underline{x}_N \end{bmatrix} = \begin{bmatrix} -\Delta Q_1 \\ 0 \\ 0 \\ \cdot \\ 0 \\ 0 \end{bmatrix} \tag{4.43}$$

From the last two rows of this equation we have,

$$H_{(N-1)(N-2)}\Delta x_{\underline{N}-2} + H_{(N-1)(N-1)}\Delta x_{\underline{N}-1} + H_{(N-1)N}\Delta \underline{x}_N = 0$$

$$H_{(N-1)(N-2)}\Delta x_{\underline{N}-2} + H_{(N-1)(N-1)}\Delta x_{\underline{N}-1} - H_{(N-1)N}H_{NN}^{-1}H_{N(N-1)}\Delta x_{\underline{N}-1} = 0$$

$$H_{(N-1)(N-2)}\Delta x_{\underline{N}-2} + (H_{(N-1)(N-1)} - H_{(N-1)N}H_{NN}^{-1}H_{N(N-1)})\Delta x_{\underline{N}-1} = 0$$

This leads to a recursive relation,

$$\Delta x_{\underline{N}-1} = (-\tilde{H}_{(N-1)(N-1)}^{-1}H_{(N-1)(N-2)})\Delta x_{\underline{N}-2}$$

where,

$$\tilde{H}_{(N-1)(N-1)} = H_{(N-1)(N-1)} - H_{(N-1)N}H_{NN}^{-1}H_{N(N-1)}$$

By solving equation (4.43) backwards, Δx_1 is found as a function only of ΔQ_1 and the system matrix, H, as,

$$\Delta x_1 = (H_{11} - H_{12}\tilde{H}_{22}^{-1}H_{21})(-\Delta Q_1) \tag{4.44}$$

Using this expression and the expressions for all changes in x, one can find by forward substituting the changes introduced by the reactive power injection at bus 1 as,

$$\Delta \underline{x}_j = (-\tilde{H}_{jj}^{-1}H_{j(j-1)})\Delta x_{\underline{j}-1} \tag{4.45}$$

for $j = 2, ..., N$ where

$$\tilde{H}_{kk} = H_{kk} - H_{k(k+1)}\tilde{H}_{(k+1)(k+1)}H_{(k+1)k} \tag{4.46}$$

for $k = 2, ..., (N-1)$ and $\tilde{H}_{NN} = H_{NN}$.

It is now important to note that this formulation will have a solution as long as the H matrix is nonsingular. This is true as long as the our assumption that the PV buses have regulated voltage magnitudes, thus enabling us to get rid of the line and columns corresponding to them.

After the vector $\Delta\underline{x}$ is computed, voltage magnitudes at all nodes can be computed as follows,

$$\Delta V_i = \frac{dV_i}{dx_i}\Delta x_i$$

$$\Delta V_i = V_i \Delta x_i \tag{4.47}$$

This result is potentially useful for distributed decision-making in a deregulated energy maket. What this result implies is that given the change in reactive power injection in a single bus, nominal operating conditions on the network known, it is possible to calculate voltage magnitude changes for all nodes. This voltage magnitude changes at the same time can be used to compute reactive line losses locally.

4.5 Reactive Power Losses and Their Estimation

Following the same approach used for the real power losses formulation, from the simplest system of a single generator and a single load connected through a transmission line as in Figure 4-1, reactive power Q_{ij} going out of node i is given by

$$Q_{ij} = \Im\{V_i e^{j\delta_i} I^*\} \tag{4.48}$$

or,

$$Q_{ij} = \Im\{V_i e^{j\delta_i}(V_i e^{-j\delta_i} - V_j e^{-j\delta_j})(G_{ij} + jB_{ij})\} \tag{4.49}$$

$$Q_{ij} = \Im\{(V_i^2 - V_i V_j e^{j(\delta_i - \delta_j)})(G_{ij} + jB_{ij})\} \tag{4.50}$$

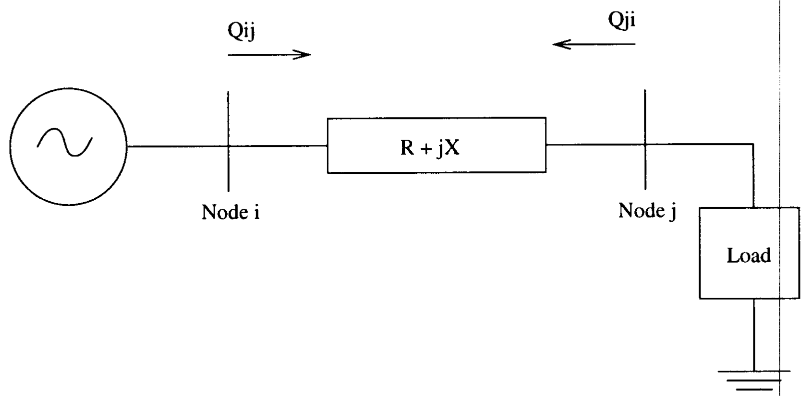

Figure 4-1: Simple 2 Bus System

This results in,

$$Q_{ij} = V_i^2 B_{ij} - V_i V_j (B_{ij} \cos\delta_{ij} + G_{ij} \sin\delta_{ij}) \tag{4.51}$$

Similarly, the reactive power Q_{ji} flowing from node j to node i

$$Q_{ji} = V_j^2 B_{ij} - V_i V_j (B_{ij} \cos\delta_{ij} - G_{ij} \sin\delta_{ij}) \tag{4.52}$$

where $\delta_{ij} = \delta_i - \delta_j$. Then the reactive losses in line $i - j$ are given as the sum of both reactive power flows as,

$$Q_{Lij} = B_{ij}(V_i^2 - 2V_i V_j \cos\delta_{ij} + V_j^2) \tag{4.53}$$

$$\approx B_{ij}\left[V_i^2 - 2V_iV_j\left[1 - \frac{(\delta_{ij})^2}{2}\right] + V_j^2\right]$$

$$= B_{ij}((V_i - V_j)^2 + V_iV_j(\delta_{ij})^2) \tag{4.54}$$

Unlike in the real power losses formulation, we can not say that $\delta_{ij} = 0$. It is easily shown by comparing the results with the exact load flow analysis that this approximation would introduce unacceptable inaccuracy in computing Q_{Lij}. However, recalling the decoupling assumption, we can say that angles do not change much because of a change in reactive power injection.

Provided that the OASIS is going to have a base case for the network and system parameters, changes in voltage magnitudes at all busses due to a single change in reactive power, can be computed using the $Q-V$ formulation as a non-linear resistive network with independent sources previously described.

$$-\Delta \underline{Q} = H_Q \Delta \underline{x} \tag{4.55}$$

and

$$V_i = e^{x_i} \tag{4.56}$$

From this formulation we can see that, if we assume that voltages at PV busses are regulated and kept at nominal conditions (i.e. 1p.u.), the corresponding value of x is going to be zero, thus enabling us to get rid of all the columns and lines of matrix H_Q corresponding to PV busses and taking them as reference. In that way, we also assure that this matrix is non-singular. After re-enumeration of the busses using the tier-based enumeration introduced earlier, the matrix H_Q has the topology of the B matrix of the real power-phase angle problem. For the matrix H_Q to correspond to a nonlinear resistive network with positive resistances only, we have to assume that no shunts are present in the network. Therefore, we are modeling capacitors at nominal conditions as known reactive power injections. It is, furthermore assumed that the end users compensate locally to have a unity power factor at each bus. With this in mind, the proof for a localized response on the Q_V problem is analogous to the proof for the real power-phase angle in section 2.5.

Now, let us consider how to compute changes in reactive power losses due to a change in reactive injections in a deregulated energy market. First, let's begin with the reactive losses formulation,

$$Q_{Lij} \approx B_{ij}((V_i + \Delta V_i - (V_j + \Delta V_j))^2 + (V_i + \Delta V_i)(V_j + \Delta V_j)(\delta_{ij})^2) \tag{4.57}$$

Remember that ΔV_i and ΔV_j are actually the sum of all the changes due to changes in injections at different busses,

$$\Delta V_i = \Delta V_{i,1} + \Delta V_{i,2} + ... + \Delta V_{i,k} \tag{4.58}$$

$$\Delta V_j = \Delta V_{j,1} + \Delta V_{j,2} + ... + \Delta V_{j,k} \tag{4.59}$$

For simplification purposes let us introduce $V_{ij} = V_i - V_j$ and $\Delta V_{ij} = \Delta V_i - \Delta V_j$.

It can be seen that the first term of equation (4.57) is similar to the estimation of real power losses, thus by the same argument we can approximate it as,

$$(V_{ij} + \Delta V_{ij})^2 \approx V_{ij}^2 + (2V_{ij}\Delta V_{ij,1} + \Delta V_{ij,1}^2) + (2V_{ij}\Delta V_{ij,2} + \Delta V_{ij,2}^2) + ...$$

$$+ (2V_{ij}\Delta V_{ij,k} + \Delta V_{ij,k}^2) \tag{4.60}$$

Now, let us investigate how the second term can be approximated in a localized way. First,

$$(V_i + \Delta V_i)(V_j + \Delta V_j) = V_iV_j + V_i\Delta V_j + V_j\Delta V_i + \Delta V_i\Delta V_j \tag{4.61}$$

The first three terms do not present any problems to a localized formulation, as we can see,

$$V_i\Delta V_j = V_i(\Delta V_{j,1} + \Delta V_{j,2} + ... + \Delta V_{j,k})$$

$$V_j\Delta V_i = V_j(\Delta V_{i,1} + \Delta V_{i,2} + ... + \Delta V_{i,k})$$

However, on the fourth term interactions among changes in voltage magnitudes due to different changes in injection, prevent us from having a localized formulation.

$$\Delta V_i\Delta V_j = (\Delta V_{i,1} + \Delta V_{i,2} + ... + \Delta V_{i,k})(\Delta V_{j,1} + \Delta V_{j,2} + ... + \Delta V_{j,k})$$

It is now that we recall the localized response property, to say that

$$\Delta V_i\Delta V_j \approx \Delta V_{i,1}\Delta V_{j,1} + \Delta V_{i,2}\Delta V_{j,2} + ... + \Delta V_{i,k}\Delta V_{j,k} \tag{4.62}$$

since we are assuming that the changes in reactive power injection are occurring with enough separation for the localized response property to be valid. It is to say that, for example, the change in voltage magnitude in bus 1 due to change in injection at bus 1 is much larger than the change in voltage magnitude in bus 1 due to change in injection at bus k, which is "far" from bus 1.

In this way we can summarize the estimation of reactive power losses due to a change in reactive power injection as,

$$Q_L \approx Q_L(V_i, V_j) + \Delta Q_L(\Delta V_{i,1}, \Delta V_{j,1}) + \Delta Q_L(\Delta V_{i,2}, \Delta V_{j,2}) + ... + \Delta Q_L(\Delta V_{i,k}, \Delta V_{j,k}) \quad (4.63)$$

where $Q_L(V_i, V_j)$ stands for the nominal operating conditions provided by the OASIS and the change in reactive power losses throughout the grid created by injecting ΔQ_k into the network is given by

$$\Delta Q_L(\Delta V_{i,k}, \Delta V_{j,k}) \approx B_{ij}[(2V_{ij}\Delta V_{ij,k} + \Delta V_{ij,k}^2) + \delta_{ij}^2(V_i \Delta V_{j,k} + V_j \Delta V_{i,k} + \Delta V_{i,k} \Delta V_{j,k})] \quad (4.64)$$

Chapter 5

Numerical Examples

In this chapter, numerical results on the standard IEEE 39 bus system are described in support of theoretical propositions made in this thesis. It is concluded that an acceptable accuracy is achievable.

5.1 Localized Response Property Revisited

The localized response property establishes that the changes in voltage phase angles decrease monotonically as the electrical distance from the triggering event increases [12, 11]. Now recall the tier based enumeration and this property can be written as,

$$\|\Delta\underline{\delta_1}\|_\infty \geq \|\Delta\underline{\delta_2}\|_\infty ... \geq \|\Delta\underline{\delta_N}\|_\infty \tag{5.1}$$

where $\|x\|_\infty$ denotes the sup norm of x and $\Delta\underline{\delta_k}$ is the vector of phase angles changes in tier k. In other words, it states that the maximum change in phase angle in tier 1 is not less that the maximum change in phase angle in tier 2.

Let us consider the 39 buses network shown in the figure 5-1, with nominal conditions given in tables 5.1, 5.2 and 5.3 (in real world this would be provided by the OASIS). Table 5.1 shows values of node voltages, phase angles and real and reactive powers generated and demanded at each node. Table 5.2 shows values of real and reactive power flows through the network with its respective losses. Line parameters are given as in Table 5.3.

Now, assume that there is a change in real power injected at bus 1 of 50% of the nominal conditions. It is $\Delta P_1 = 4.15$p.u. This will create a change in the phase angles, $\Delta\underline{\delta_i}$ as shown

Tier	Node	Bus Type	V	δ	P_{gen}	Q_{gen}	P_{load}	Q_{load}
1	1	PV	1.02650	8.41892	8.29999	2.04813	0.00000	0.00000
2	2	PQ	1.02163	1.24098	-0.00001	0.99999	2.83500	1.26900
3	3	PQ	1.00522	-5.40954	0.00006	-0.00004	1.39000	0.17000
	4	PQ	1.01084	-1.64399	0.00000	0.00000	2.06000	0.27600
4	5	PQ	1.03432	-4.34744	0.00147	0.00622	2.24000	0.47200
	6	PQ	0.99370	-7.54943	0.00003	-0.00005	2.81000	0.75500
	7	PQ	0.99869	-7.34344	0.00000	0.00000	0.00000	0.00000
5	8	PQ	1.02624	-5.86400	-0.00152	-0.00658	0.00000	0.00000
	9	PV	1.02780	2.55651	5.39991	1.01253	0.00000	0.00000
	10	PQ	1.00342	-6.17106	0.00000	0.00000	3.29400	0.32300
	11	PQ	0.99694	-8.33359	-0.00001	-0.00002	1.58000	0.30000
6	12	PQ	1.00018	-8.74858	-0.00013	0.00007	3.22000	0.02400
	13	PV	1.04750	-3.39111	2.50004	2.73385	0.00000	0.00000
	14	PQ	1.02677	-8.92140	-0.00003	0.00003	0.00000	0.00000
	15	PQ	1.00972	-6.04671	-0.00001	0.00000	3.08600	-0.92200
	16	PQ	1.00799	-3.65616	0.00000	0.00000	2.74000	1.15000
7	17	PQ	1.03827	-1.44251	0.00001	-0.00008	0.00000	0.00000
	18	PQ	0.98574	-7.49823	-0.00001	0.00000	3.20000	1.53000
	19	PQ	0.97745	-9.71276	-0.00004	0.00001	5.00000	1.84000
	20	PV	1.03000	-10.76364	9.99998	3.65052	11.04000	2.50000
	21	PQ	1.02925	0.73030	0.00000	0.00000	2.47500	0.84600
	22	PQ	1.03510	0.93252	0.00000	-0.00001	0.00000	0.00000
	23	PV	0.99720	3.80171	6.32000	1.85971	0.00000	0.00000
8	24	PQ	0.98455	-2.89404	-0.00001	0.00001	6.80000	1.03000
	25	PQ	0.98589	-6.71044	0.00001	-0.00001	0.00000	0.00000
	26	PQ	0.98499	-9.55915	0.00000	-0.00001	0.00000	0.00000
	27	PQ	1.00687	-11.14197	-0.00002	0.00000	0.00000	0.00000
	28	PV	1.06350	8.68635	5.60000	1.62360	0.00000	0.00000
	29	PV	1.04930	5.96491	6.50000	3.18047	0.00000	0.00000
9	30	PV	1.01230	2.31130	5.08000	2.02686	0.00000	0.00000
	31	PQ	0.99587	-4.04835	0.00001	0.00000	0.00000	0.00000
	32	PQ	0.97311	-11.67785	-0.00003	0.00001	5.22000	1.76000
	33	PQ	0.98697	-9.03396	-0.00004	-0.00001	0.00000	0.00000
	34	PQ	1.00128	-2.98922	0.00001	0.00005	0.00000	0.00000
	35	PQ	0.99676	-3.49542	0.00000	0.00002	0.08500	0.88000
10	36	PQ	0.97446	-11.21005	-0.00001	0.00001	2.33800	0.84000
	37	PQ	0.98758	-8.15208	0.00000	0.00000	0.00000	0.00000
	38	SB	0.98200	0.00000	5.78112	2.83664	0.09200	0.04600
11	39	PV	0.98310	5.13449	6.50000	2.78804	0.00000	0.00000

Table 5.1: Nominal Node Operating Conditions for IEEE 39 Bus Network

Tier	Line	from-to	P_{send}	Q_{send}	P_{rec}	Q_{rec}	P_{loss}	Q_{loss}
I	1	1-2	8.29999	2.04826	-8.24450	-0.96623	0.05549	1.08202
II	2	2-3	1.92154	0.20362	-1.90115	0.01996	0.02039	0.22358
	3	2-4	3.48795	0.49362	-3.47130	-0.31409	0.01665	0.17953
	4	3-4	-1.40292	0.05438	1.41131	0.03809	0.00839	0.09246
III	5	3-5	-0.67916	-0.83270	0.68281	0.86960	0.00366	0.03691
	6	3-6	2.59329	0.58836	-2.58349	-0.48549	0.00980	0.10287
IV	7	6-7	-0.22648	-0.26952	0.22664	0.27169	0.00016	0.00217
	8	5-8	2.46142	-0.98916	-2.41537	1.04572	0.04604	0.05657
	9	5-9	-5.38277	-0.35052	5.39991	1.01347	0.01715	0.66295
V	10	7-10	-2.32927	-0.32401	2.33315	0.37336	0.00388	0.04935
	11	7-11	2.10263	0.05232	-2.09952	-0.01595	0.00310	0.03637
	12	8-12	3.55435	1.55136	-3.53579	-1.33572	0.01857	0.21564
	13	8-13	-2.50004	-2.50656	2.50004	2.73286	0.00000	0.22630
	14	8-14	1.35955	-0.09248	-1.35338	0.16495	0.00617	0.07247
	15	11-12	0.51951	-0.28405	-0.51913	0.28874	0.00039	0.00469
VI	16	10-15	-0.42605	-1.04958	0.42643	1.05710	0.00038	0.00752
	17	10-16	-3.29176	-0.07227	3.30038	0.21762	0.00861	0.14536
	18	10-17	-4.50611	-1.24157	4.54082	1.66467	0.03472	0.42311
	19	10-18	2.59677	1.66706	-2.58825	-1.57816	0.00851	0.08890
	20	12-19	0.83478	1.02299	-0.83251	-0.98587	0.00227	0.03712
	21	14-20	1.35335	-0.16495	-1.35158	0.20902	0.00176	0.04408
VII	22	15-21	-3.51244	-0.13510	3.53910	0.55925	0.02666	0.42415
	23	16-22	-6.04038	-1.36763	6.07058	1.89614	0.03020	0.52851
	24	21-22	-0.42912	-0.59925	0.42943	0.60417	0.00031	0.00492
	25	17-23	-6.28945	-1.23988	6.32000	1.85964	0.03055	0.61976
	26	17-24	1.74864	-0.42481	-1.74628	0.47139	0.00236	0.04658
	27	18-25	-0.61176	0.04815	0.61245	-0.03974	0.00070	0.00841
	28	19-26	-0.23671	-0.56069	0.23702	0.56565	0.00031	0.00496
	29	20-27	0.31156	0.94122	-0.31063	-0.91805	0.00093	0.02316
	30	19-25	-3.93082	-0.29345	3.94383	0.50324	0.01301	0.20979
VIII	31	21-28	-5.58497	-0.80601	5.60000	1.62357	0.01503	0.81756
	32	22-29	-6.50000	-2.50031	6.50000	3.18042	0.00000	0.68011
	33	24-30	-5.05373	-1.50138	5.08000	2.02684	0.02627	0.52546
	34	25-31	-4.55627	-0.46349	4.57569	0.68144	0.01942	0.21795
	35	26-32	3.22622	0.87258	-3.21701	-0.74363	0.00921	0.12895
	36	26-33	-3.46324	-0.46803	3.46575	0.50076	0.00252	0.03273
	37	27-32	0.31062	0.91805	-0.30849	-0.88442	0.00213	0.03363
IX	38	31-34	-4.36137	-0.80608	4.36930	0.89136	0.00793	0.08529
	39	31-35	-0.21432	0.12463	0.21442	-0.12194	0.00010	0.00270
	40	34-35	2.13070	0.86148	-2.12860	-0.83882	0.00211	0.02265
	41	32-36	-1.69453	-0.13195	1.69575	0.14598	0.00122	0.01403
	42	33-37	-1.82133	0.09579	1.82372	-0.06779	0.00239	0.02800
	43	33-36	4.04465	1.15305	-4.03375	-0.98599	0.01090	0.16706
	44	33-38	-5.68912	3.18174	5.68912	2.79058	0.00000	5.97232
	45	35-37	1.82918	0.08076	-1.82372	0.06779	0.00546	0.14855
X	46	34-39	-6.50000	-1.75285	6.50000	2.78800	0.00000	1.03515

Table 5.2: Nominal Line Operating Conditions for IEEE 39 Bus Network

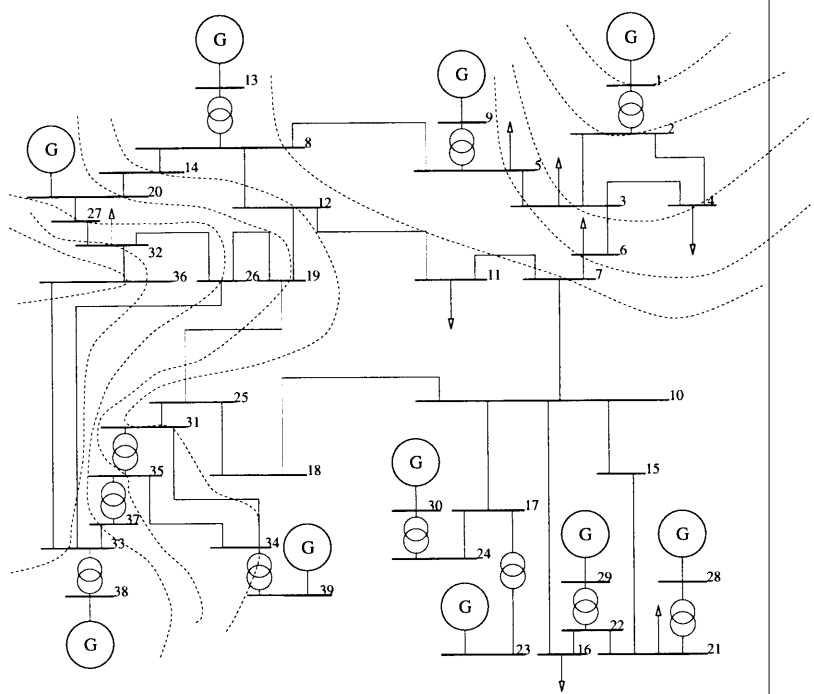

Figure 5-1: IEEE 39 Bus Network

Line	R	X	Line	R	X
1	.0008	.0156	24	.0006	.0096
2	.0057	.0625	25	.0007	.0142
3	.0014	.0151	26	.0007	.0138
4	.0043	.0474	27	.0018	.0217
5	.0032	.0323	28	.0008	.0128
6	.0014	.0147	29	.0010	.0250
7	.0013	.0173	30	.0008	.0129
8	.0070	.0086	31	.0005	.0272
9	.0006	.0232	32	.0000	.0143
10	.0007	.0089	33	.0009	.0180
11	.0007	.0082	34	.0009	.0101
12	.0013	.0151	35	.0008	.0112
13	.0000	.0181	36	.0002	.0026
14	.0035	.0411	37	.0023	.0363
15	.0011	.0133	38	.0004	.0043
16	.0003	.0059	39	.0016	.0435
17	.0008	.0135	40	.0004	.0043
18	.0016	.0195	41	.0004	.0046
19	.0009	.0094	42	.0007	.0082
20	.0013	.0213	43	.0006	.0092
21	.0010	.0250	44	.0000	.0250
22	.0022	.0350	45	.0016	.0435
23	.0008	.0140	46	.0000	.0200

Table 5.3: Line Parameters

in Table 5.4. The change in node angles can be computed using the forward/backward substitution method described in section 3.1 as,

$$\Delta\delta_1 = (B_{11} - B_{12}\tilde{B}_{22}^{-1}B_{21})\Delta P_1 \tag{5.2}$$

$$\Delta\underline{\delta}_j = (-\tilde{B}_{jj}^{-1}B_{j(j-1)})\Delta\delta_{\underline{j}-1} \tag{5.3}$$

for $j = 2, ..., N$ where

$$\tilde{B}_{kk} = B_{kk} - B_{k(k+1)}\tilde{B}_{(k+1)(k+1)}B_{(k+1)k} \tag{5.4}$$

for $k = 2, ..., (N-1)$ and $\tilde{B}_{NN} = B_{NN}$. In Table 5.4 we compare the nodal phase angle changes obtained using the forward/backward substitution method to actual values computed using load flow simulations.

From these results, it can be seen that the localized response property holds, i.e., that the largest change in tier i is always larger than the largest change in tier $(i+1)$, for all $i = 1, ..., N$, where N is the number of tiers, and that the DC load flow (forward/backward) formulation used to compute phase angle changes is very accurate. Note that we are introducing a fairly large change in real power injection (50% of the nominal value) and change in phase angles are actually larger than the nominal values, and still we can still be predicted with high accuracy using the forward/backward method.

5.2 Numerical Accuracy of the Proposed Method for Estimating Real Power Losses

For estimating real power losses at the bus where the input change occurs, and independently from other system input changes, the localized spread of phase angles away from the input change is not sufficient. Since the transmssion loss is a function fo phase angle differences, $\Delta\delta_{ij}$, it is important that they also become smaller away from the system input change causing them. The changes in $\Delta\delta_{ij}$ are easily computed from $\Delta\delta$ as $\Delta\underline{\delta}_{ij} = M\Delta\underline{\delta}$. It is convenient to show these values, so that it can be seen that our assumption that the angle differences decrease away from the node where the change in injection occured is valid (Recall our argument analogous to the current dividers of circuit theory [5]). It was shown

Tier	Node	$\Delta\delta$(DC)	$\Delta\delta$(LF)
1	1	27.6875	27.6519
2	2	23.9684	23.8180
3	3	16.4761	15.9800
	4	22.1578	21.8902
4	5	13.1835	12.6063
	6	14.4465	13.9557
	7	12.0659	11.7235
5	8	11.7401	11.5103
	9	13.1835	12.6315
	10	11.3496	11.0688
	11	11.5965	11.2805
6	12	10.8355	10.5932
	13	11.7401	11.5375
	14	10.0230	9.8274
	15	11.3496	11.0686
	16	11.3496	11.0930
7	17	11.3496	11.0944
	18	10.5907	10.3487
	19	8.3490	8.2246
	20	8.9844	8.8551
	21	11.3496	11.1210
	22	11.3496	11.1220
	23	11.3496	11.1009
8	24	11.3496	11.0848
	25	8.8428	8.7215
	26	6.3645	6.3557
	27	7.9458	7.8979
	28	11.3496	11.1409
	29	11.3496	11.1359
9	30	11.3496	11.0884
	31	8.4166	8.3450
	32	6.4341	6.4206
	33	5.9444	5.9654
	34	8.2652	8.2122
	35	8.1138	8.0687
10	36	6.2705	6.2642
	37	6.2902	6.2996
11	39	8.2652	8.2499

Table 5.4: Changes in Node Angles

earlier that the changes in real power losses can be estimated locally for a change in real power injection at node 1 as,

$$\Delta P_L(\Delta\delta_{ij,1}) \approx G_{ij}(2\delta_{ij}\Delta\delta_{ij,1} + \Delta\delta_{ij,1}^2) \tag{5.5}$$

These results are shown on Table 5.5.

From the results shown in Table 5.5, we can conclude two things: first, that the accuracy of the localized method for computing real power transmission losses is high, and second, that losses are, to certain extent, localized. These two conclusions help us prove the validity of our assumptions.

It is important to recognize that for a single power input change around nominal conditions, both power imbalance and the transmission loss is compensated for solely by the slack bus. This should be kept in mind since in the actual operation of a power system this function is distributed among several buses. The implications of theis modeling factor on cost for transmission loss in a deregulated industry should be carefully treated.

5.3 Real Power Losses Caused by Bilateral Transactions

Consider next a scenario of a bilateral transaction in a deregulated electric power industry, instead of a single bus change in system input. A bilateral transaction represents a simultaneous increase of system input of X MW at location i, and decrease of X MW at location j. Bus i is referred to as a supplier, and bus j as a buyer in the primary supply/demand market. It is particularly important to develop means of effective loss estimate cause by these transactions since their impact on the system-wide operation is unbunbled from the profit-driven power trade. This is unlike a simple injection scenario in which system-wide effects are managed in a bundled way with the power imbalances caused by intentional, profit-driven changes in system input.

A premise in this thesis is that the bilateral market participants could estimate locally their impact on the system and have a choice to compensate for them by modifying their original trades slightly. The following numerical setup illustrates typical accuracy of this approach.

Let us consider a change in demand at node 36 of $\Delta P_{36} = -4.15$p.u. Note the negative sign, implying an increase in demand. The same way as before, we compute changes in

Tier	Line	δ_{ij}	$\Delta\delta_{ij,1}$	$\Delta P_L(\Delta\delta_{ij,1})$	ΔP_L(LF)
I	1	7.1779	3.7191	0.0671	0.0742
II	2	6.6505	7.4924	0.0687	0.0702
	3	2.8850	1.8107	0.0255	0.0270
	4	-3.7656	-5.6817	0.0434	0.0437
III	5	-1.0621	3.2926	0.0036	0.0084
	6	2.1399	2.0296	0.0250	0.0230
IV	7	-0.2060	2.3805	0.0062	0.0058
	8	1.5166	1.4433	0.1120	0.0896
	9	-6.9039	0	0	0.0003
V	10	-1.1724	0.7164	-0.0031	-0.0024
	11	0.9902	0.4694	0.0036	0.0032
	12	2.8846	0.9047	0.0104	0.0104
	13	-2.4729	0.0000	0	0
	14	3.0574	1.7171	0.0084	0.0085
	15	0.4150	0.7610	0.0023	0.0019
VI	16	-0.1243	0	0	0.0001
	17	-2.5149	0	0	0.0002
	18	-4.7285	0	0	0.0015
	19	1.3272	0.7589	0.0080	0.0072
	20	0.9642	2.4865	0.0095	0.0080
	21	1.8422	1.0386	0.0024	0.0024
VII	22	-6.7770	0.0000	0.0000	0.0004
	23	-4.5887	0.0000	0.0000	0.0007
	24	-0.2022	0.0000	0.0000	0.0000
	25	-5.2442	0	0	0.0005
	26	1.4515	0	0	0.0001
	27	-0.7878	1.7479	0.0003	0.0001
	28	-0.1536	1.9845	0.0049	0.0044
	29	0.3783	1.0386	0.0009	0.0009
	30	-3.0023	-0.4938	0.0047	0.0044
VIII	31	-7.9560	0	0	0.0002
	32	-5.0324	0.0000	0	0
	33	-5.2053	0	0	0.0003
	34	-2.6621	0.4262	-0.0056	-0.0048
	35	2.1187	-0.0696	-0.0006	-0.0006
	36	-0.5252	0.4201	-0.0024	-0.0022
	37	0.5359	1.5117	0.0021	0.0022
IX	38	-1.0591	0.1514	-0.0019	-0.0016
	39	-0.5529	0.3028	-0.0001	-0.0001
	40	0.5062	0.1514	0.0012	0.0011
	41	-0.4678	0.1636	-0.0007	-0.0007
	42	-0.8819	-0.3458	0.0023	0.0021
	43	2.1761	-0.3261	-0.0028	-0.0026
	44	-9.0340	5.9444	0	0
	45	4.6567	1.8236	0.0052	0.0048
X	46	-8.1237	0	0	0
Total	–	–	–	.4006	.3929

Table 5.5: Changes in Losses Due to a Change in Injection at Node 1

nodal phase angles using the forward/backward substitution method. It is important to note here, that the enumeration for this computation will change. Recall the tier-based bus enumeration, and in this case bus 36 will be bus 1, and busses directly connected to bus 36, will form the second tier, and so on. To make it easier to understand, we will keep the same enumeration throughout all our examples, except where we explicitly establish the contrary. With the changes in node angles known, we can compute changes in angle differences throughout the network and similarly to the first case, changes in real power losses due to a change in demand at node 36. In Table 5.6, we show the line by line values of changes in angle differences due to changes in power at node 1 and 36, $\Delta\delta_{ij,1}$ and $\Delta\delta_{ij,36}$, together with changes in real power losses, $\Delta P_{L,1}$ and $\Delta P_{L,36}$

From these results, it can be observed how accurate the method is for estimating real power losses locally, that is how real power transmission losses can be estimated knowing only the change in power injection at a given node and nominal or base case conditions of the network. It is important to note (from Table 5.6) that in lines where the changes in angle difference and power losses due to the change in power at node 1 are significant (lines close to node 1), the changes in angle difference and power losses due to the change in power at node 36 are small (far from node 36), and vice versa. It can be seen how this method can be used even for computing real power transmission losses of a bilateral transaction provided that the two busses involved in it are separate "enough" for our assumptions to be valid.

More work is needed to develop notions of electrical distances below which the impact of transmssion loss is such that all injections should compensate for transmission loss uniformly. In other words, because of very close electrical distances among these system inputs, the interaction component is so large relative to the component caused by individual transactions that for all practical purposes these must be treated as one transaction. More work is needed to develop knowledge of a minimum electrical distance below which this is true.

In the context of the 39 bus system, the following scenario illustrate such need. Let us introduce a change in real power demand at node 4, instead of node 36, of $\Delta P_4 = -4.15$p.u. It can be seen from Figure 5-1 that both nodes are close together, so it can be expected that our approximation will be poor, due to the fact that the interactions will be significant and can not be neglected. This is documented on table 5.7. It can be seen that because the interaction component is dominant, losses are very inaccurate.

Line	$\Delta\delta_{ij,1}$	$\Delta P_L(\Delta\delta_{ij,1})$	$\Delta\delta_{ij,36}$	$\Delta P_L(\Delta\delta_{ij,36})$	$\sum_{k=1,36} \Delta P_L(\Delta\delta_{ij,k})$	ΔP_L (LF)
1	3.7191	0.0671	0	0	0.0671	0.0743
2	7.4924	0.0687	0	0	0.0687	0.0704
3	1.8107	0.0255	0	0	0.0255	0.0271
4	-5.6817	0.0434	0	0	0.0434	0.0438
5	3.2926	0.0036	0.0343	-0.0001	0.0035	0.0087
6	2.0296	0.0250	-0.0156	-0.0001	0.0249	0.0227
7	2.3805	0.0062	-0.0183	0.0000	0.0062	0.0056
8	1.4433	0.1120	0.0150	0.0008	0.1128	0.0904
9	0	0	0.0000	0.0000	0.0000	0.0003
10	0.7164	-0.0031	-0.0166	0.0001	-0.0030	-0.0023
11	0.4694	0.0036	0.0066	0.0000	0.0037	0.0033
12	0.9047	0.0104	-0.0659	-0.0006	0.0098	0.0100
13	0.0000	0	0	0	0	0
14	1.7171	0.0084	0.2228	0.0009	0.0093	0.0101
15	0.7610	0.0023	0.0107	0.0000	0.0023	0.0019
16	0	0	0.0000	0.0000	0.0000	0.0001
17	0	0	0.0000	0.0000	0.0000	0.0002
18	0	0	0	0	0	0.0019
19	0.7589	0.0080	-0.0176	-0.0001	0.0078	0.0073
20	2.4865	0.0095	-0.0754	-0.0001	0.0094	0.0079
21	1.0386	0.0024	0.1348	0.0003	0.0026	0.0029
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0006
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0009
24	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
25	0	0	0.0000	0.0000	0.0000	0.0006
26	0	0	0.0000	0.0000	0.0000	0.0001
27	1.7479	0.0003	-0.0405	0.0001	0.0004	0.0000
28	1.9845	0.0049	-0.0288	0.0000	0.0049	0.0039
29	1.0386	0.0009	0.1348	0.0001	0.0010	0.0015
30	-0.4938	0.0047	-0.0167	0.0001	0.0048	0.0049
31	0	0	0	0	0	0.0002
32	0.0000	0	0.0000	0	0	0
33	0	0	0	0	0	0.0004
34	0.4262	-0.0056	-0.0320	0.0005	-0.0051	-0.0041
35	-0.0696	-0.0006	0.8586	0.0085	0.0079	0.0082
36	0.4201	-0.0024	-0.2053	0.0023	-0.0001	-0.0013
37	1.5117	0.0021	0.1962	0.0001	0.0022	0.0036
38	0.1514	-0.0019	-0.0114	0.0002	-0.0018	-0.0014
39	0.3028	-0.0001	-0.0227	0.0000	-0.0001	-0.0001
40	0.1514	0.0012	-0.0114	-0.0001	0.0011	0.0011
41	0.1636	-0.0007	0.3785	-0.0012	-0.0019	-0.0012
42	-0.3458	0.0023	0.0259	-0.0001	0.0022	0.0019
43	-0.3261	-0.0028	1.4424	0.0180	0.0151	0.0146
44	5.9444	0	-5.9444	0	0	0
45	1.8236	0.0052	-0.1368	-0.0003	0.0049	0.0044
46	0	0	0	0	0	0
Total	–	0.4006	–	0.0290	0.4296	0.4254

Table 5.6: Changes in Losses Due to Changes in Injection at Nodes 1 and 36

Line	$\Delta\delta_{ij,1}$	$\Delta P_L(\Delta\delta_{ij,1})$	$\Delta\delta_{ij,4}$	$\Delta P_L(\Delta\delta_{ij,4})$	$\sum_{k=1,4}\Delta P_L(\Delta\delta_{ij,k})$	ΔP_L (LF)
1	3.7191	0.0671	0	0	0.0671	0.0725
2	7.4924	0.0687	-5.6817	-0.0191	0.0496	0.0116
3	1.8107	0.0255	1.3731	0.0182	0.0436	0.0547
4	-5.6817	0.0434	7.0548	-0.0019	0.0415	-0.0051
5	3.2926	0.0036	-3.2926	0.0165	0.0201	0.0017
6	2.0296	0.0250	-2.0296	-0.0089	0.0161	-0.0006
7	2.3805	0.0062	-2.3805	0.0087	0.0149	0.0003
8	1.4433	0.1120	-1.4433	-0.0398	0.0723	-0.0003
9	0	0	0.0000	0.0000	0.0000	0.0002
10	0.7164	-0.0031	-0.7164	0.0059	0.0027	0.0003
11	0.4694	0.0036	-0.4694	-0.0022	0.0014	-0.0001
12	0.9047	0.0104	-0.9047	-0.0076	0.0028	-0.0002
13	0.0000	0	0	0	0	0
14	1.7171	0.0084	-1.7171	-0.0047	0.0037	-0.0002
15	0.7610	0.0023	-0.7610	-0.0001	0.0022	0.0000
16	0	0	0	0	0	0.0000
17	0	0	0.0000	0.0000	0.0000	0.0000
18	0	0	0.0000	0.0000	0.0000	0.0004
19	0.7589	0.0080	-0.7589	-0.0044	0.0035	-0.0002
20	2.4865	0.0095	-2.4865	0.0012	0.0108	-0.0002
21	1.0386	0.0024	-1.0386	-0.0013	0.0010	0.0000
22	0.0000	0.0000	0.0000	0.0000	0.0000	0.0001
23	0.0000	0.0000	0.0000	0.0000	0.0000	0.0002
24	0.0000	0.0000	0.0000	0.0000	0.0000	0
25	0	0	0.0000	0.0000	0.0000	0.0001
26	0	0	0.0000	0.0000	0.0000	0.0000
27	1.7479	0.0003	-1.7479	0.0067	0.0071	0.0001
28	1.9845	0.0049	-1.9845	0.0067	0.0117	0.0001
29	1.0386	0.0009	-1.0386	0.0001	0.0010	0.0000
30	-0.4938	0.0047	0.4938	-0.0040	0.0007	-0.0001
31	0	0	0	0	0	0.0000
32	0.0000	0	0.0000	0	0	0
33	0	0	0	0	0	0.0001
34	0.4262	-0.0056	-0.4262	0.0065	0.0010	0.0003
35	-0.0696	-0.0006	0.0696	0.0006	0.0000	0.0000
36	0.4201	-0.0024	-0.4201	0.0055	0.0032	0.0001
37	1.5117	0.0021	-1.5117	0.0004	0.0024	0.0000
38	0.1514	-0.0019	-0.1514	0.0022	0.0003	0.0001
39	0.3028	-0.0001	-0.3028	0.0001	0.0000	0
40	0.1514	0.0012	-0.1514	-0.0009	0.0003	0.0000
41	0.1636	-0.0007	-0.1636	0.0010	0.0003	0.0000
42	-0.3458	0.0023	0.3458	-0.0015	0.0008	-0.0001
43	-0.3261	-0.0028	0.3261	0.0033	0.0005	0.0001
44	5.9444	0	-5.9444	0	0	0
45	1.8236	0.0052	-1.8236	-0.0035	0.0017	-0.0001
46	0	0	0	0	0	0
Total	–	0.4006	–	-0.0163	0.3843	0.1358

Table 5.7: Changes in Losses Due to Changes in Injection at Nodes 1 and 4

Note that changes in real power transmission losses due to changes in injection at nodes 1 and 4 are significant in the same lines, meaning this, that there is a strong interaction between both changes in injections, making our assumptions invalid. This kind of bilateral transactions can be approximated if the changes in injection is small compared to the nominal conditions. Remember that in our example the change in injection is 50% of the nominal conditions. Further explanations and simulations of this case are not given in this thesis, because that would be a complete different problem that does not have to do with our premise that losses can be estimated in a localized way. The problem of estimating effects of many electrically close yet small input changes is not the topic of this thesis. This is the problem related to the operating/pricing principles for distributed generation that may emerge as relevant in the near future.

5.4 Numerical Accuracy of the Proposed Method for Estimating Reactive Power Losses

In this section we present an example on how reactive power losses caused on the transmission grid, excluding shunt losses can be computed on a localized way. Again we use the same IEEE 39 busses network to show our results. However there is a small variation on the nominal conditions on the system. In order to compute changes in nodal voltage magnitudes due to changes in reactive power injections using the $Q - V$ formulation of a non-linear resistive network with independent sources given in equation (4.26) [12, 13], we have to define voltages at all PV busses as regulated to 1 p.u[1]. This will allow us to get rid of all the lines and colums of the matrix H_Q when we use the transformation,

$$V_i = e^{x_i} \tag{5.6}$$

since for these PV buses, x_i will be zero and we can take them as a reference for voltage. Not only the nominal voltage will be 1 p.u., but it is also regulated, meaning, that there will not be any change in its magnitude. This implies that there will not be any changes in x_i consequently, making our formulation valid not only for the nominal case, but also for the estimation of changes in node voltage magnitudes. The nominal conditions for this case

[1]This is not essential but it is done here for simplicity

Node	V	δ	P_{gen}	Q_{gen}	P_{load}	Q_{load}
1	1.00000	10.10225	8.29999	2.32815	0.00000	0.00000
2	0.96551	2.50690	-0.00001	1.00000	2.83500	1.26900
3	0.94275	-4.95302	0.00007	-0.00005	1.39000	0.17000
4	0.95258	-0.72291	0.00000	0.00000	2.06000	0.27600
5	0.97426	-3.78188	0.00271	0.00507	2.24000	0.47200
6	0.92819	-7.35888	0.00004	-0.00004	2.81000	0.75500
7	0.93100	-7.08144	0.00000	0.00001	0.00000	0.00000
8	0.96361	-5.40700	-0.00280	-0.00542	0.00000	0.00000
9	1.00000	3.55935	5.39992	1.31311	0.00000	0.00000
10	0.93379	-5.71570	0.00000	0.00003	3.29400	0.32300
11	0.93007	-8.21395	0.00000	-0.00001	1.58000	0.30000
12	0.93514	-8.67977	-0.00012	0.00009	3.22000	0.02400
13	1.00000	-2.71541	2.50004	2.07025	0.00000	0.00000
14	0.98417	-8.94619	-0.00002	0.00005	0.00000	0.00000
15	0.93987	-5.57025	0.00001	0.00001	3.08600	-0.92200
16	0.93572	-2.80293	0.00000	0.00004	2.74000	1.15000
17	0.96917	-0.26335	-0.00001	0.00000	0.00000	0.00000
18	0.91703	-7.24756	0.00002	0.00004	3.20000	1.53000
19	0.91280	-9.74550	0.00002	0.00009	5.00000	1.84000
20	1.00000	-10.97680	9.99998	4.62494	11.04000	2.50000
21	0.95800	2.27200	-0.00001	0.00003	2.47500	0.84600
22	0.96256	2.51556	-0.00002	0.00001	0.00000	0.00000
23	1.00000	4.96071	6.32000	2.14283	0.00000	0.00000
24	0.96945	-1.73820	0.00000	0.00001	6.80000	1.03000
25	0.92250	-6.33569	-0.00002	0.00003	0.00000	0.00000
26	0.92161	-9.51997	-0.00001	-0.00001	0.00000	0.00000
27	0.96439	-11.37768	-0.00003	0.00000	0.00000	0.00000
28	1.00000	11.36363	5.59999	1.88346	0.00000	0.00000
29	1.00000	8.05700	6.49999	2.93291	0.00000	0.00000
30	1.00000	3.58456	5.08000	1.67551	0.00000	0.00000
31	0.93645	-3.32899	-0.00001	0.00001	0.00000	0.00000
32	0.91248	-11.94044	0.00003	0.00008	5.22000	1.76000
33	0.92583	-8.92328	0.00002	-0.00005	0.00000	0.00000
34	0.94310	-2.13965	-0.00003	0.00000	0.00000	0.00000
35	0.93706	-2.70535	-0.00001	0.00005	0.08500	0.88000
36	0.91340	-11.40442	0.00001	0.00003	2.33800	0.84000
37	0.92753	-7.92932	-0.00001	0.00001	0.00000	0.00000
38	1.00000	0.00000	5.83626	3.46119	0.09200	0.04600
39	1.00000	5.78341	6.49998	3.29520	0.00000	0.00000

Table 5.8: Nominal Node Operating Conditions for the $Q - V$ Problem

are given in Tables 5.8 and 5.9.

As in the estimation of real power transmission losses, let us introduce next a change of 50% at bus 2 of the amount $\Delta Q_2 = .6345$p.u. Note the positive sign, in order to make it consistent with the earlier.

$$-\Delta \underline{Q} = H_Q \Delta \underline{x} \tag{5.7}$$

This formulation already accounts for the sign, since we are assuming that changes can only occur at PQ busses. With this formulation we compute changes in x at every node, which correspond to changes in node voltages with the following relationship,

$$\Delta V_i = V_i \Delta x_i \tag{5.8}$$

Line	from-to	P_{send}	Q_{send}	P_{rec}	Q_{rec}	P_{loss}	Q_{loss}
1	1-2	8.3000	2.3042	-8.2405	-1.1551	0.0595	1.1491
2	2-3	1.9182	0.2959	-1.8952	-0.0454	0.0231	0.2506
3	2-4	3.4873	0.5987	-3.4685	-0.3962	0.0188	0.2025
4	3-4	-1.3990	-0.0166	1.4085	0.1207	0.0095	0.1041
5	3-5	-0.6652	-0.8477	0.6694	0.8899	0.0042	0.0422
6	3-6	2.5695	0.7407	-2.5582	-0.6225	0.0113	0.1182
7	6-7	-0.2517	-0.1313	0.2519	0.1329	0.0001	0.0016
8	5-8	2.4747	-0.7639	-2.4253	0.8246	0.0495	0.0607
9	5-9	-5.3814	-0.5915	5.3999	1.3022	0.0185	0.7107
10	7-10	-2.3345	-0.0804	2.3389	0.1364	0.0044	0.0560
11	7-11	2.0827	-0.0516	-2.0791	0.0927	0.0035	0.0411
12	8-12	3.5454	1.6058	-3.5242	-1.3597	0.0212	0.2461
13	8-13	-2.5000	-1.8765	2.5000	2.0669	0	0.1904
14	8-14	1.3771	-0.5545	-1.3688	0.6519	0.0083	0.0974
15	11-12	0.4991	-0.3937	-0.4986	0.3999	0.0005	0.0062
16	10-15	-0.4254	-0.9402	0.4258	0.9474	0.0004	0.0072
17	10-16	-3.2804	0.1446	3.2903	0.0221	0.0099	0.1667
18	10-17	-4.5013	-1.1082	4.5408	1.5870	0.0395	0.4787
19	10-18	2.5742	1.4504	-2.5652	-1.3563	0.0090	0.0941
20	12-19	0.8027	0.9386	-0.8004	-0.9014	0.0023	0.0372
21	14-20	1.3688	-0.6528	-1.3664	0.7122	0.0024	0.0593
22	15-21	-3.5118	-0.0232	3.5425	0.5073	0.0307	0.4841
23	16-22	-6.0303	-1.1658	6.0648	1.7667	0.0345	0.6008
24	21-22	-0.4350	-0.4270	0.4352	0.4309	0.0002	0.0039
25	17-23	-6.2888	-1.5031	6.3200	2.1329	0.0312	0.6298
26	17-24	1.7480	-0.0858	-1.7457	0.1308	0.0023	0.0450
27	18-25	-0.6348	-0.1735	0.6357	0.1847	0.0009	0.0112
28	19-26	-0.2968	-0.6092	0.2973	0.6163	0.0004	0.0071
29	20-27	0.3264	1.4123	-0.3243	-1.3597	0.0021	0.0525
30	19-25	-3.9028	-0.3278	3.9175	0.5648	0.0147	0.2371
31	21-28	-5.5825	-0.9211	5.6000	1.8587	0.0175	0.9376
32	22-29	-6.5000	-2.1953	6.5000	2.9191	0	0.7238
33	24-30	-5.0542	-1.1543	5.0800	1.6672	0.0257	0.5128
34	25-31	-4.5532	-0.7491	4.5758	1.0015	0.0225	0.2524
35	26-32	3.2131	0.5880	-3.2031	-0.4475	0.0100	0.1406
36	26-33	-3.5104	-1.2079	3.5136	1.2501	0.0032	0.0422
37	27-32	0.3243	1.3597	-0.3194	-1.2834	0.0048	0.0763
38	31-34	-4.3561	-0.9985	4.3652	1.0964	0.0091	0.0979
39	31-35	-0.2197	-0.0039	0.2198	0.0063	0.0001	0.0024
40	34-35	2.1348	1.1361	-2.1322	-1.1078	0.0026	0.0283
41	32-36	-1.6975	-0.0270	1.6989	0.0429	0.0014	0.0159
42	33-37	-1.8185	-0.0209	1.8212	0.0526	0.0027	0.0316
43	33-36	4.0491	1.0717	-4.0369	-0.8836	0.0123	0.1882
44	33-38	-5.7443	-2.2707	5.7443	3.3737	0	1.1030
45	35-37	1.8273	0.2199	-1.8212	-0.0527	0.0062	0.1671
46	34-39	-6.5000	-2.2117	6.5000	3.2637	0	1.0520

Table 5.9: Nominal Line Operating Conditions for the $Q - V$ Problem

Tier	Node	Bus Type	V	Δx	ΔV(F/B)	ΔV(LF)
1	1	PV	1.0000	0	0	0
2	2	PQ	0.9655	-0.0078	-0.0075	-0.0083
3	3	PQ	0.9427	-0.0038	-0.0031	-0.0037
	4	PQ	0.9526	-0.0068	-0.0064	-0.0073
4	5	PQ	0.9743	-0.0010	-0.0009	-0.0010
	6	PQ	0.9282	-0.0023	-0.0021	-0.0026
5	7	PQ	0.9310	-0.0010	-0.0010	-0.0012
	8	PQ	0.9636	-0.0005	-0.0005	-0.0007
	9	PV	1.0000	0	0	0
6	10	PQ	0.9338	-0.0006	-0.0006	-0.0007
	11	PQ	0.9301	-0.0009	-0.0008	-0.0011
	12	PQ	0.9351	-0.0006	-0.0006	-0.0008
	13	PV	1.0000	0	0	0
	14	PQ	0.9842	-0.0002	-0.0002	-0.0002
7	15	PQ	0.9399	-0.0005	-0.0005	-0.0007
	16	PQ	0.9357	-0.0004	-0.0004	-0.0005
	17	PQ	0.9692	-0.0002	-0.0002	-0.0002
	18	PQ	0.9170	-0.0005	-0.0005	-0.0006
	19	PQ	0.9128	-0.0004	-0.0003	-0.0005
	20	PV	1.0000	0	0	0
8	21	PQ	0.9580	-0.0002	-0.0002	-0.0003
	22	PQ	0.9626	-0.0002	-0.0002	-0.0003
	23	PV	1.0000	0	0	0
	24	PQ	0.9695	-0.0001	-0.0001	-0.0001
	25	PQ	0.9225	-0.0003	-0.0003	-0.0004
	26	PQ	0.9216	-0.0002	-0.0002	-0.0003
	27	PQ	0.9644	-0.0001	-0.0001	-0.0001
9	28	PV	1.0000	0	0	0
	29	PV	1.0000	0	0	0
	30	PV	1.0000	0	0	0
	31	PQ	0.9365	-0.0002	-0.0002	-0.0003
	32	PQ	0.9125	-0.0002	-0.0002	-0.0003
	33	PQ	0.9258	-0.0002	-0.0002	-0.0003
10	34	PQ	0.9431	-0.0002	-0.0002	-0.0003
	35	PQ	0.9371	-0.0002	-0.0002	-0.0003
	36	PQ	0.9134	-0.0002	-0.0002	-0.0003
	37	PQ	0.9275	-0.0002	-0.0002	-0.0003
	38	SB	1.0000	0	0	0
11	39	PV	1.0000	0	0	0

Table 5.10: Changes in Nodal Voltage Magnitudes

The results of the forward/backward substitution method for our example are given in Table 5.10. From these results we can see high accuracy of the method for computing locally changes in voltage magnitudes. It is important to note that these changes in voltage magnitude are also localized, as was expected from the localized response property under our assumptions, neglecting PV busses.

The next step, then, is to compute changes in reactive power losses in each line by using our formulation given in equation (4.64). This relationship is given by,

$$\Delta Q_L(\Delta V_{i,k}, \Delta V_{j,k}) \approx B_{ij}[(2V_{ij}\Delta V_{ij,k} + \Delta V_{ij,k}^2) + \delta_{ij}^2(V_i\Delta V_{j,k} + V_j\Delta V_{i,k} + \Delta V_{i,k}\Delta V_{j,k})] \quad (5.9)$$

Line	V_{ij}	$\Delta V_{ij,2}$	$\Delta Q_L(\Delta V_{i,2}, \Delta V_{j,2})$	ΔQ_{loss}(LF)
1	0.0345	0.0075	0.0280	0.0316
2	0.0228	-0.0044	-0.0055	-0.0060
3	0.0129	-0.0011	-0.0046	-0.0048
4	-0.0098	0.0033	-0.0021	-0.0024
5	-0.0315	-0.0022	0.0043	0.0053
6	0.0146	-0.0010	-0.0025	-0.0028
7	-0.0028	-0.0011	0.0004	0.0005
8	0.0106	-0.0004	-0.0007	-0.0006
9	-0.0257	-0.0009	0.0014	0.0016
10	-0.0028	-0.0004	0.0002	0.0002
11	0.0009	-0.0001	-0.0001	-0.0001
12	0.0285	0.0001	0.0001	0.0001
13	-0.0364	-0.0005	0.0020	0.0027
14	-0.0206	-0.0003	0.0002	0.0003
15	-0.0051	-0.0002	0.0002	0.0002
16	-0.0061	0.0000	0.0001	0.0001
17	-0.0019	-0.0002	-0.0001	-0.0002
18	-0.0354	-0.0004	0.0010	0.0013
19	0.0168	-0.0001	-0.0004	-0.0003
20	0.0223	-0.0003	-0.0006	-0.0007
21	-0.0158	-0.0002	0.0002	0.0003
22	-0.0181	-0.0003	-0.0001	-0.0001
23	-0.0268	-0.0002	0.0003	0.0004
24	-0.0046	0.0000	0.0000	0.0000
25	-0.0308	-0.0002	-0.0007	0.0009
26	-0.0003	-0.0001	0.0000	0.0000
27	-0.0055	-0.0002	0.0001	0.0001
28	-0.0088	-0.0001	0.0002	0.0002
29	0.0356	0.0001	0.0002	0.0003
30	-0.0097	0.0000	-0.0001	-0.0002
31	-0.0420	-0.0002	0.0005	0.0006
32	-0.0374	-0.0002	0.0009	0.0012
33	-0.0305	-0.0001	0.0003	0.0004
34	-0.0140	-0.0001	0.0001	0.0001
35	0.0091	0.0000	-0.0001	-0.0001
36	-0.0042	0.0000	0.0000	0.0000
37	0.0519	0.0001	0.0003	0.0005
38	-0.0067	0.0000	0.0001	0.0001
39	-0.0006	0.0000	0.0000	0.0000
40	0.0060	0.0000	0.0000	0.0000
41	-0.0009	0.0000	0.0000	0.0000
42	-0.0017	0.0000	0.0000	0.0000
43	0.0124	0.0000	-0.0001	-0.0001
44	-0.0742	-0.0002	0.0009	0.0014
45	0.0095	0.0000	-0.0001	-0.0001
46	-0.0569	-0.0002	0.0009	0.0012
Total	–	–	0.0266	0.0334

Table 5.11: Reactive Losses Changes Due to Change in Injection at Node 2

Chapter 6

Implications on Pricing for Transmission Losses in a Changing Industry

The theoretical and numerical results developed in the previous chapters of this thesis are motivated by the recognition that the quest for entirely exact recovery of costs of interconnected operations services in a changing industry forms a very complex challenge. It is possibly unrealistic to attempt this task prior to evaluating tradeoff between the cost borne by the need for additional real-time monitoring of individual market players, and the inaccuracies in pricing for these services in some approximate ways that do not require extensive technical developments.

Typical power systems are highly non-linear, high order systems. Because of their non-linear character it is hard to separate effects of individual market players on system changes independently from other market players. At least in concept, everything depends on everything else, and this makes pricing for system-wide support in response to the primary supply/demand market activities a problem uniquely different than the problems in many other industries.

Expressed in a different way, this implies that incremental calculations of individual market players do not sum up to the system-wide effects of all market players present on the system. This, in turn, implies that the order in which economic transactions take place also makes a difference.

It is this complexity that has led FERC to recently propose approximate costs for ancillary services. The pro forma tarrifs are of this type and are based on previous estimates of average costs associated with the ancillary services in the regulated industry.

In this thesis an attempt is made to re-visit one particular ancillary service, i.e., compensation of transmission losses. The question is asked if, based on certain fundamental characteristics of electric power networks, one could develop an approximate, simple technique for estimating transmission losses caused in the grid by each individual market player, and independently from the others.

The results indicate that this actually is possible since most of the large electric power networks show a localized response to a system input change; an incremental change in real power input of ΔP_i into node i leads to the largest changes in voltage phase angles $\Delta\delta_j$ at nodes directly connected to the location where the change takes place, and it decreases in a tier-wise manner away from bus i. Similar property is shown to hold in terms of the effects of a reactive power input increment ΔQ_i at bus i on changes in nodal voltage magnitudes[1]

The localized response property is used in this thesis to develop a forward-backward substitution algorithm for computing changes in voltage angles caused by the market player i who is injecting power ΔP_i. Only knowledge of transmission grid parameters, such as line reactances is assumed.

However, since the real power losses are a function of voltage phase angle differences (and not the actual angles), for transmission losses to be somewhat localized relative to bus i where the power input is made, one must ask the question concerning the qualitative response of phase angle differences. It is demonstrated in this thesis that relatively meshed networks, such as typical EHV transmission networks effectively act as a power flow divider as one goes away from the location of the power injected. A highly meshed power network has the characteristic that changes in phase angle differences further away from the point of power injection decrease in magnitude.

It is shown next how the real power losses caused by several market players, i.e., by several injections into the power grid could be approximated by each individual market player, independently of the others.

[1]More caution is needed in the reactive power case, by modeling shunt devices as equivalent reactive power sources, which can always be managed locally. Only the effect of reactive power input on reactive power propagation throughout the grid shows the localized response property.

Only the knowledge of the system-wide nominal voltages and network parameters is assumed. In the context of present deregulation debate in the United States this knowledge is expected to be made available in real time [19].

The estimates of transmission losses made by each individual market player could be used by themselves to inject power $(\Delta P_i + \Delta P_{loss,i})$ into the system, and , effectively cancel out power loss created on the entire network.

This, further, implies that the market players who make an option to compensate for transmission losses themselves would not be obliged to pay for the transmission loss ancillary service.

This concept is simple to implement and attractive since it gives a competitive option to the market players to compensate for their effects on the system.

It is also important to note that this solution avoids the assymetry caused by the presence of a slack bus when compensating for transmission losses. The losses are compensated for in a distributed way, rather than by the slack bus.

A word of caution is needed here, particularly as related to the real power losses. Namely, if the market players are not sufficiently far apart from each other, measured in terms of electrical distances, the interaction effects could become significant. Further work is needed to quantify a relation between the electrical distance between two buses and the error made in estimating real power losses in the manner proposed in this thesis. The result should lead to certain guidelines as to which market players must be viewed as a single entity when pricing for ancillary services.

The issue is less pronounced when compensating for reactive power losses. The reactive power losses are a function of actual nodal voltages, and not their differences. The localized response property guarantees tier-wise decrease in the voltage magnitude changes away from the reactive power input causing them.

The method suggested in this thesis implies that individual market players compensate for the reactive power losses created themselves by injecting additional reactive power. This is qualitatively different than having to design tarrifs for reactive power losses. This is fortunate, since the economic value of anything related to reactive power and voltage is somewhat elusive and hard to justify[2].

[2]Recall that only heat curves for generators are in terms of real power generated, and not reactive.

Chapter 7

Conclusions

This thesis is concerned with what may appear to be a well-understood and extensively researched topic in the electric power engineering. The basic problem is the one of computing how much power is lost in transferring across an electric power network, as the power is injected into particular nodes of the network that represent points of power supply and taken out at other nodes, that represent points of power consumption.

The main reasons for re-visiting the topic of transmission losses at this time were at least threefold:

1. Increased tendency for economic transfers that go beyond the transfer levels at which transmission loss is relatively small.

2. Question of compensation for transmission losses in a deregulated/competitive power industry.

3. Question concerning possible compensation of transmission losses at the end-user level, i.e., at the level of their cause.

The question posed in this thesis was if the localized response of system voltages to input changes could be used to

1. Locally estimate transmission losses caused by an end user, keeping in mind availability of real-time information networks required by FERC [17, 19].

2. Compensate for these losses by injecting the power corresponding to this estimated transmission loss, and not be dependent on the cost for this service dictated by someone else.

In Chapter 2, the basic governing equations of electric power networks in steady-state, i.e., the load flow equations were briefly reviewed. These equations define constraints on real and reactive power inputs into the network, by stating that the power injected into the system at each node must equal the power transferred by the network. Next, the decoupling assumption under which real power load flow equations are separable from the reactive power load flow equations was defined and conditions under which this assumption is valid were stated. It was recalled that under the real/reactive power decoupling assumption, the real power load flow problem can be interpreted as a problem of a DC nonlinear resistive network. Similarly, it was reviewed how is the decoupled reactive power/voltage problem interpreted as a problem of a DC nonlinear resistive network.

A localized response property, essential for the methods described in this thesis, was stated next for the decoupled nonlinear real power load flow problem. This result is a direct consequence of interpreting the problem as a nonlinear resistive network with nondecreasing resistors, that is known to have the localized response property. Loosely speaking, the localized response property in the context of the decoupled real power load flow problem means that the largest voltage phase angle change at the nodes directly connected to the location i, at which an increment in real power ΔP_i takes place, is never smaller than the largest voltage phase angle change at nodes one tier away from bus i, and so on.

This property is not sufficient, however, for the changes in phase angle differences across the transmission lines to have the same property, i.e. to be decreasing away from the cause of their change. For this to hold true, it is sufficient to have a relatively meshed network, as seen from the location i; starting with a relatively small number of lines directly connected to the location i, the number of lines across the cutsets away from the location i increases. It is intuitively clear that for a transmission network whose reactances are uniform, the changes in real power line flows decrease in proportion to the increase of number of lines across the cutsets away from node i.

This qualitative property is important for the real power loss estimation methods proposed in this thesis. To introduce the problem of transmission losses basic formulae for transmission loss calculation in power networks were briefly reviewed. Present state-of-the-art was briefly reviewed for computing transmission losses.

Next, in Chapter 3 a closed form formula for computing voltage phase angle changes created by the end user located at bus i was introduced. This formula only requires knowledge of the power increase ΔP_i at the location i and the network parameters of the entire system. The algorithm is based on the linearized, decoupled real power load flow equations. Numerical methods similar in concept could be derived without making the linearizating assumption. The results of this algorithm, combined with the updated nominal voltage phase angles of the entire network were used as the starting information for estimating real power transmission loss created by the end user at bus i when injecting ΔP_i into the system. First, a formula for computing real power losses was described that reflects the interaction of nodal power increments in the interconnected system. This formula cannot be used by each end user independently from the others. It was next proposed that for the most typical power networks the line flow changes ΔP_{ij} decrease in absolute value away from the location where power is injected into the system. Given this property, it is possible to claim that the voltage phase angle differences across the transmission lines also decrease in proportion with the line reactances. This further leads to the conjecture that the effects of power input changes resulting from economic transactions are separable to a significant degree, unless the transactions are very close electrically. This conjecture was formally derived.

In Chapter 4, the problem of estimating reactive power losses in an interconnected power network was studied. First, the decoupled reactive power-voltage (QV) load flow equations were briefly reviewed. These form the governing equations of direct interest. Next, the state of-the-art results concerned with a non-linear network interpretation of the QV load flow problem were summarized. It was concluded that such an interpretation is possible. However, the resulting non-linear network, because of the presence of shunt capacitors on a primarily inductive network, is analogous to a non-linear DC resistive network, not all of whose resistors are monotonically increasing. A qualitative implication of this situation is that it is not possible to state unconditionally that a change in reactive power injection ΔQ_i into bus i leads to uniform decrease in voltage changes ΔV_i [13]. This can only be proven when the shunt capacitors are not present.

This obstacle could be overcome in the context of the functions of an end user in a competitive environment by decomposing the problem of reactive power loss compensation into

1. The shunt reactive power loss component, measurable directly in terms of local power factor compensation.

2. The reactive power losses created in the planar transmission grid that interconnects all nodes.

An underlying modeling assumption here is that the reactive power inputs into the nodes are represented as ideal reactive power injections into the grid[1]. The total reactive power injection could be thought of as consisting of the portion flowing from the node to the ground, and the portion flowing into the planar transmission network. It was proposed here that the shunt reactive losses be directly estimated and compensated by each end user; this is trivial to do. The method proposed in this thesis introduced an approach to estimate the second component. It was proven in this thesis that the voltage changes ΔV_i away from the reactive power injection into the planar portion at node i of the grid decrease uniformly away from this location. A closed form solution for estimating the voltage deviations in the entire network caused by the specific end user was derived by using only the information about the local injection into the grid ΔQ_i and the network parameters of the entire grid. Next, an approximate formula for reactive power losses in response to reactive power changes at several locations in this system was derived. This formula requires knowledge about nominal voltages, that is assumed to be provided by a real time information network of some sort [17, 19]. It was proposed that for system input changes that are not very close electrically, reactive power loss can be estimated and compensated individually by each end user.

In Chapter 5, numerical results on the standard IEEE 39 bus system were described in support of theoretical propositions made in this thesis. It is concluded that an acceptable accuracy is achievable.

In Chapter 6, possible ways of using the proposed method for real time loss compensation by the end users themselves, instead of paying for loss compensation at the interconnected system level were described. It was pointed out that this approach is not intended to be exclusive. End users not interested in compensating for transmission loss themselves could continue to pay for the ancillary services according to agreed upon tarrifs.

[1]This includes flows through the capacitors.

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