



GREEN SUPPLIER SELECTION BASED ON CODAS METHOD IN PROBABILISTIC UNCERTAIN LINGUISTIC ENVIRONMENT

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Abstract. Probabilistic uncertain linguistic sets (PULTSs) have widely been used in MADM or MAGDM. The CODAS method, which is a novel MADM or MAGDM tool, aims to acquire the optimal choice which have the largest Euclidean & Hamming distances from the NIS. This paper designs the probabilistic uncertain linguistic CODAS (PUL-CODAS) method with sine entropy weight. Finally, a numerical example for green supplier selection is given and the obtained results are compared with some existing models.

Keywords: MAGDM, PULTSs, CODAS method, green supplier, selection.

JEL Classification: C43, C61, D81.

Introduction

The CODAS method was firstly designed by Keshavarz Ghorabae (2016). It is a novel and useful model used to solve MADM problems with aid of deriving the Euclidean distance and Hamming distances to select the best alternative. Ghorabae, Amiri, Zavadskas, Hooshmand, and Antuchevičienė (2018) defined the fuzzy CODAS method to select suppliers. Panchal et al. (2017) applied fuzzy CODAS to tackle the maintenance decision issue. (Badi et al., 2018) employed CODAS method to select the optimal desalination plant location in Libya. Yeni and Ozelik (2019) defined the CODAS method for MAGDM under IVIFSs. Peng and Li (2019) designed the hesitant fuzzy soft CODAS method. Karasan, Bolturk, and Kahraman (2019) proposed neutrosophic CODAS method. Pamucar, Badi, Sanja, and Obradovic (2018) introduced linguistic neutrosophic CODAS method.

Due to certain complexity, experts couldn't depict their preferences through real numbers (Liao & Xu, 2014a, 2014b, 2014c), thus with help of other mathematical qualitative tool (Beg et al., 2019; Lu & Wei, 2019; Wang, 2019; Wu et al., 2019a, 2019b). For example, the DMs

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could employ the linguistic terms to depict satisficing degree of a car (Herrera & Martinez, 2000b). In order to give qualitative assessment, Herrera and Martinez (2000a) designed the 2TLTs for calculating along with words. Sohaib, Naderpour, Hussain, and Martinez (2019) defined 2-tuple linguistic TOPSIS for MAGDM issues. Furthermore, Rodriguez, Martinez, and Herrera (2012) designed the HFLTss which depicts some possible linguistic values. Wei (2019a) defined the GDSM under HFLTss. Liao, Xu, and Zeng (2015) developed VIKOR model under HFLTss. Liao, Yang, and Xu (2018b) gave the ELECTRE II model with HFLTss and gave two ELECTRE II model based on the score-deviation and positive and negative ideal.

Furthermore, Pang, Wang, and Xu (2016) defined the PLTs. Recently, PLTs have become hot issues for HFLTss (Liao et al., 2018a; Wei et al., 2018) and HFSs (Xia & Xu, 2011). Bai, Zhang, Qian, and Wu (2017) formed a comparison model to tackle PLTs. Gou and Xu (2016) designed some basic operations of PLTs. Liao, Jiang, Xu, Xu, and Herrera (2017) defined linear programming model to cope with MADM under PLTs. Lin, Chen, Liao, and Xu (2019) defined the ELECTRE II model under PLTs. Feng, Liu, and Wei (2019) built the PL-QUALIFLEX. Liao, Jiang, Lev, and Fujitac (2019) researched the PL-ELECTRE III model. Bai, Zhang, Shen, Huang, and Fan (2018) built the PLTs in MAGDM under uncertainty. Jin, Wang, and Xu (2019) gave uncertain PLTs in GDM. Kobina, Liang, and He (2017) defined the power operators under PLTs based on power operators (Wei, 2019b; Yager, 2001). Cheng, Gu, and Xu (2018) studied the GDM under PLTs setting. Liang, Kobina, and Quan (2018) defined the GRA algorithms for PL-MAGDM under geometric BM (Wang et al., 2018; Wei et al., 2019). Xie, Xu, and Ren (2019) studied the incomplete hybrid probabilistic linguistic problem. Lu, Wei, Wu, and Wei (2019) proposed TOPSIS algorithm to solve the PL-MAGDM.

In certain situations, some DMs may depict their preferences through ULTs (Xu, 2004). Inspired by PLTs (Pang et al., 2016) and ULTs (Xu, 2004), Lin, Xu, Zhai, and Yao (2018) defined probabilistic ULTs (PULTs). Xie, Ren, Xu, and Wang (2018) depicted some preference relation under PULTs and designed the distance and similarity. But there are no recent existing literatures to use CODAS method to solve PUL-MAGDM. Therefore, it is very necessary to investigate such issue. The other remaining section of such paper is given. Section 1 reviews the definition of PULTs. In Section 2, the CODAS method is defined for PUL-MAGDM along with sine entropy weight. In Section 3, a detailed example is developed and some comparative analysis is given. This paper finishes with conclusions in last Section.

1. Preliminaries

In such section, some basic mathematical definitions are simply reviewed.

Definition 1 (Gou et al., 2017). Let $L = \{l_\alpha | \alpha = -\theta, \dots, -2, -1, 0, 1, 2, \dots, \theta\}$ be an LTS, the l_α could depict the corresponding information with β which is defined by using g :

$$g: [l_{-\theta}, l_\theta] \rightarrow [0, 1], \quad g(l_\alpha) = \frac{\alpha + \theta}{2\theta} = \beta, \quad (1)$$

β could also depicts the equivalent assessing information for l_α which is defined with g^{-1} :

$$g^{-1}:[0,1] \rightarrow [l_{-\theta}, l_{\theta}], \quad g^{-1}(\beta) = l_{(2\beta-1)\theta} = l_{\alpha}, \quad (2)$$

Definition 2 (Pang et al., 2016). Given an LTS $L = \{l_j | j = -\theta, \dots, -2, -1, 0, 1, 2, \dots, \theta\}$, the PLTS is simply defined:

$$L(p) = \left\{ l^{(\phi)}(p^{(\phi)}) \left| l^{(\phi)} \in L, p^{(\phi)} \geq 0, \phi = 1, 2, \dots, \#L(p), \sum_{\phi=1}^{\#L(p)} p^{(\phi)} \leq 1 \right. \right\}, \quad (3)$$

where $l^{(\phi)}(p^{(\phi)})$ is the ϕ th $l^{(\phi)}$ along with corresponding probability values $(p^{(\phi)})$, and $\#L(p)$ denotes the number of $L(p)$. The $l^{(\phi)}$ in $L(p)$ are listed with ascending order.

Furthermore, Lin et al. (2018) defined the PULTSs based on ULTSs (Xu, 2004) and PLTSs (Pang et al., 2016).

Definition 3 (Lin et al., 2018). The PULTS is defined:

$$PULTS(p) = \left\{ [L^{\phi}, U^{\phi}](p^{\phi}) \left| p^{\phi} \geq 0, \phi = 1, 2, \dots, \#PULTS(p), \sum_{\phi=1}^{\#PULTS(p)} p^{\phi} \leq 1 \right. \right\}, \quad (4)$$

where $[L^{\phi}, U^{\phi}](p^{\phi})$ expresses the uncertain linguistic values $[L^{\phi}, U^{\phi}]$ with probability values p^{ϕ} , L^{ϕ} , U^{ϕ} are LTSs, $L^{\phi} \leq U^{\phi}$, and $\#PULT(p)$ is the length of $PULTS(p)$.

Definition 4 (Lin et al., 2018). Let $PULTS_1(p) = \left\{ [L_1^{\phi}, U_1^{\phi}](p_1^{\phi}) \left| \phi = 1, 2, \dots, \#PULTS_1(p) \right. \right\}$ and $PULTS_2(p) = \left\{ [L_2^{\phi}, U_2^{\phi}](p_2^{\phi}) \left| \phi = 1, 2, \dots, \#PULTS_2(p) \right. \right\}$ be PULTSs, and the $\#PULTS_1(p)$ and $\#PULTS_2(p)$ are called as length of $PULTS_1(p)$ and $PULTS_2(p)$. If $\#PULTS_1(p) > \#PULTS_2(p)$, then $\#PULTS_1(p) - \#PULTS_2(p)$ ULTSs are added to $PULTS_2(p)$. The added ULTSs are the smallest ULTSs in $PULTS_2(p)$ and the corresponding probabilities values of newly added ULTSs are zero.

Definition 5 (Lin et al., 2018). Let $PULTS(p) = \left\{ [L^{\phi}, U^{\phi}](p^{\phi}) \left| \phi = 1, 2, \dots, \#PULTS(p) \right. \right\}$, the defined expected values $EV(PULTS(p))$ and corresponding deviation degree $DD(PULTS(p))$ is proposed:

$$EV(PULTS(p)) = \frac{\sum_{\phi=1}^{\#PULTS(p)} \left(\frac{g(L^{\phi})p^{\phi} + g(U^{\phi})p^{\phi}}{2} \right)}{\sum_{\phi=1}^{\#PULTS(p)} p^{\phi}}, \quad (5)$$

$$DD(PULTS(p)) = \frac{\sqrt{\sum_{\phi=1}^{\#PULTS(p)} \left(\frac{g(L^{\phi})p^{\phi} + g(U^{\phi})p^{\phi}}{2} - E(PULTS(p)) \right)^2}}{\sum_{\phi=1}^{\#PULTS(p)} p^{\phi}}, \quad (6)$$

Definition 6. Let $PULTS_1(p) = \left\{ \left[L_1^\phi, U_1^\phi \right] \left(p_1^\phi \right) \mid \phi = 1, 2, \dots, \#PULTS_1(p) \right\}$ and $PULTS_2(p) = \left\{ \left[L_2^\phi, U_2^\phi \right] \left(p_2^\phi \right) \mid \phi = 1, 2, \dots, \#PULTS_2(p) \right\}$, along with $\#PULTS_1(p) = \#PULTS_2(p) = \#PULTS(p)$, then the Euclidean distance $ED(PULTS_1(p), PULTS_2(p))$ and the Hamming distance $HD(PULTS_1(p), PULTS_2(p))$ is listed:

$$ED(PULTS_1(p), PULTS_2(p)) = \sqrt{\sum_{\phi=1}^{\#PULTS(p)} \left(\left| g(L_1^\phi) p^\phi - g(L_2^\phi) p^\phi \right|^2 + \left| g(U_1^\phi) p^\phi - g(U_2^\phi) p^\phi \right|^2 \right)} / 2\#PULTS(p); \quad (7)$$

$$HD(PULTS_1(p), PULTS_2(p)) = \frac{\sum_{\phi=1}^{\#PULTS(p)} \left(\left| g(L_1^\phi) p^\phi - g(L_2^\phi) p^\phi \right| + \left| g(U_1^\phi) p^\phi - g(U_2^\phi) p^\phi \right| \right)}{2\#PULTS(p)}. \quad (8)$$

Then, the sine entropy of PULTS is defined to get unknown attribute weights in MAGDM issue based on the idea of simplified Neutrosophic sine entropy (Cui & Ye, 2018).

Definition 7. Let $PULTS(p) = \{PULTS_1(p), PULTS_2(p), \dots, PULTS_n(p)\}$ be the PULTS, where $PULTS_j(p) = \left\{ \left[L_j^\phi, U_j^\phi \right] \left(p_j^\phi \right) \mid \phi = 1, 2, \dots, \#PULTS_j(p) \right\}$ is the j -th probabilistic uncertain linguistic number (PULN), $j = 1, 2, \dots, n$. Then the probabilistic uncertain linguistic sine entropy measure (PULSEM) is designed:

$$PULSEM(PULTS(p)) = \frac{1}{2n\#PULTS(p)} \sum_{j=1}^n \left(\sum_{\phi=1}^{\#PULTS(p)} \sin \left(g(L_j^\phi) p_j^\phi \pi \right) + \sin \left(g(U_j^\phi) p_j^\phi \pi \right) \right). \quad (9)$$

2. CODAS method for PUL-MAGDM issue

In such part, the PUL-CODAS model for MAGDM is designed. $A = \{A_1, A_2, \dots, A_m\}$ is named a group of given alternatives, $G = \{G_1, G_2, \dots, G_n\}$ is called a group of given attributes along with weight $w = (w_1, w_2, \dots, w_n)$, where $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$ and $E = \{E_1, E_2, \dots, E_q\}$ is named a set of experts. Suppose that G_j is evaluated by E_k for A_i and is expressed as uncertain linguistic variables $[L_{ij}^k, U_{ij}^k]$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, $k = 1, 2, \dots, q$.

Then, the CODAS model is devised to deal with PUL-MAGDM issues. The calculating steps are given soon afterwards and the flowchart is given in Figure 1.

Step 1. Convert cost attribute into beneficial attribute. If cost attribute value is $[L_{ij}^k, U_{ij}^k]$, then the corresponding beneficial attribute value is $[-U_{ij}^k, -L_{ij}^k]$.

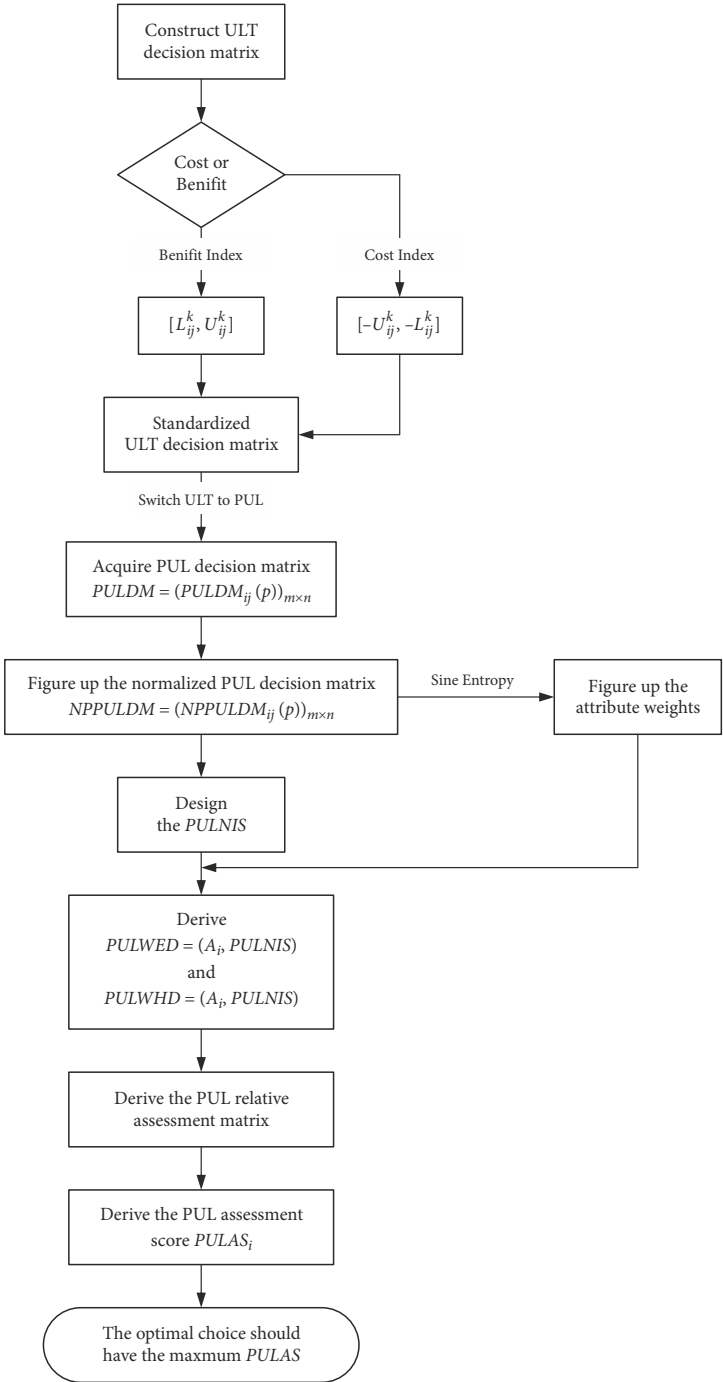


Figure 1. The flowchart of the CODAS method for the PUL-MAGDM

Step 2. Switch the $[L_{ij}^k, U_{ij}^k]$ into PUL matrix $PULDM = (PULDM_{ij}(p))_{m \times n}$, $PULDM_{ij}(p) = \{[L_{ij}^\phi, U_{ij}^\phi](p_{ij}^\phi) \mid \phi = 1, 2, \dots, \#PULDM_{ij}(p)\}$.

Step 3. Figure up the normalized PUL matrix $NPULDM = (NPULDM_{ij}(p))_{m \times n}$.

Step 4. Figure up the attributes weight by sine entropy.

Since the uncertainty of one attribute increases, the attribute weight should decrease correspondingly. Thus, we may figure up unknown weights of each attribute based on the sine entropy measure formula Eq. (9). Firstly, the probabilistic uncertain linguistic sine entropy measure (PULSEM) of $NPULDM_{ij}(p)$ are designed as follows:

$$PULSEM(NPULDM_{ij}(p)) = \frac{1}{2m\#NPULDM_{ij}(p)} \sum_{i=1}^m \left(\sum_{\phi=1}^{\#PULDM_{ij}(p)} \sin(g(L_{ij}^\phi)p_{ij}^\phi\pi) + \sin(g(U_{ij}^\phi)p_{ij}^\phi\pi) \right), \quad (10)$$

Then, the attribute weights is:

$$w_j = \frac{1 - PULSEM(NPULDM_j(p))}{\sum_{j=1}^n (1 - PULSEM(NPULDM_j(p)))}, \quad j = 1, 2, \dots, n. \quad (11)$$

Step 5. Design the PULNIS:

$$PULNIS = (PULNIS_1, PULNIS_2, \dots, PULNIS_n), \quad (12)$$

$$PULNIS_j = \{[L_j^\phi, U_j^\phi](p_j^{(\phi)}) \mid \phi = 1, 2, \dots, \#NPULDM_{ij}(p)\}, \quad (13)$$

$$E(PULNIS_j) = \left\{ \min_i E(NPULDM_{ij}(p)) \right\}, \quad (14)$$

Step 6. Derive the probabilistic uncertain linguistic weighted Euclidean distance $PULWED(A_i, PULNIS)(i = 1, 2, \dots, m)$ and Hamming distance $PULWHD(A_i, PULNIS)(i = 1, 2, \dots, m)$:

$$PULWED(A_i, PULNIS) = \sum_{j=1}^n w_j ED(NPULDM_{ij}(p), PULNIS_j), \quad (15)$$

$$PULWHD(A_i, PULNIS) = \sum_{j=1}^n w_j HD(NPULDM_{ij}(p), PULNIS_j), \quad (16)$$

$$ED(NPULDM_{ij}(p), PULNIS_j) = \sqrt{\frac{\sum_{\phi=1}^{\#NPULDM_{ij}(p)} \left(\left| g(L_{ij}^\phi)p_{ij}^\phi - g(L_j^{(\phi)})p_j^{(\phi)} \right|^2 + \left| g(U_{ij}^\phi)p_{ij}^\phi - g(U_j^{(\phi)})p_j^{(\phi)} \right|^2 \right)}{2\#NPULDM_{ij}(p)}}, \quad (17)$$

$$HD\left(NPULDM_{ij}(p), PULNIS_j\right) = \frac{\sum_{\phi=1}^{\#NPULDM_{ij}(p)} \left(\left| g\left(L_{ij}^{\phi}\right) p_{ij}^{\phi} - g\left(L_j^{\phi}\right) p_j^{(\phi)} \right| + \left| g\left(U_{ij}^{\phi}\right) p_{ij}^{\phi} - g\left(U_j^{\phi}\right) p_j^{(\phi)} \right| \right)}{2\#NPULDM_{ij}(p)}, \quad (18)$$

Step 7. Derive the PUL relative assessment matrix (PULRAM):

$$PULRAM = [PULRAM_{ik}]_{m \times m}, \quad (19)$$

$$PULRAM_{ik} = \left(PULWED(A_i, PULNIS) - PULWED(A_k, PULNIS) \right) + \left(f\left(PULWED(A_i, PULNIS) - PULWED(A_k, PULNIS)\right) \times \left(PULWHD(A_i, PULNIS) - PULWHD(A_k, PULNIS)\right) \right), \quad (20)$$

where $k \in \{1, 2, \dots, m\}$ and the threshold formula is given by Eq. (21):

$$f(x) = \begin{cases} 1 & \text{if } |x| \geq \tau \\ 0 & \text{if } |x| < \tau \end{cases}, \quad (21)$$

where the threshold parameter τ is between 0.01 and 0.05. In this paper, $\tau = 0.02$ are always used to compute (Lin et al., 2018).

Step 8. Derive the PUL assessment score $PULAS_i (i = 1, 2, \dots, m)$ by Eq. (22).

$$PULAS_i = \sum_{k=1}^m PULRAM_{ik}. \quad (22)$$

Step 9. Sort the alternatives with $PULAS_i$, the optimal choice should have maximum value.

3. A numerical example and comparative analysis

3.1. A numerical example

In today's world of resource shortage and increasingly serious environmental pollution, facing the strict environmental protection system, how to make the coordinated development of supply chain and environment while pursuing economic benefits will become an important means and decisive factor for enterprises to succeed in market competition (Tavana et al., 2017; Tong, 2017; Wang et al., 2017). Green supply chain management, as a new management mode of core enterprises under sustainable development, has been widely recognized and valued by the academic and business circles (Wei et al., 2020a, 2020b, 2020c, 2020d). Green supplier management is the important part of green supply chain management through the coordination and cooperation with suppliers to achieve cost reduction to reduce resource consumption to improve the environment, and can make the enterprise faster response to market demand, improve the core competitiveness, establish corporate social image. Supplier selection is also an important link along with implementation of green supplier management. Green supplier selection is a very common decision issue (Wang et al., 2020; Wei et al., 2020;

2020d; Zavadskas et al., 2019; Zhang et al., 2020). Thus, in such section, an example about green supplier selection is given to proof the defined method. There are some green suppliers $GS_i (i = 1, 2, 3, 4, 5)$ for experts to select according to four assessing attributes: ① Q_1 is the environmental competencies; ② Q_2 is transportation cost of suppliers; ③ Q_3 is environmental improvement quality; ④ Q_4 is financial conditions of suppliers. All these four attributes are adapted from Lei, Wei, Gao, Wu, and Wei (2020). The Q_2 is cost index and other indices are beneficial. These potential green suppliers $GS_i (i = 1, 2, 3, 4, 5)$ could be assessed with employing the LTSs

$$S = \{s_{-3} = \text{extremely poor}(EP), s_{-2} = \text{very poor}(VP), s_{-1} = \text{poor}(P), s_0 = \text{medium}(M), s_1 = \text{good}(G), s_2 = \text{very good}(VG), s_3 = \text{extremely good}(EG)\}$$

by five DMs according to these given attributes, as given in the Tables 1–5.

Table 1. The ULTSs through DM_1

Alternatives	Q_1	Q_2	Q_3	Q_4
GS_1	[M, G]	[M, G]	[G, VG]	[P, G]
GS_2	[M, G]	[G, VG]	[M, G]	[VG, EG]
GS_3	[VP, P]	[P, M]	[G, VG]	[P, M]
GS_4	[G, VG]	[M, G]	[G, EG]	[VG, EG]
GS_5	[M, G]	[P, M]	[M, G]	[P, M]

Table 2. The ULTSs through DM_2

Alternatives	Q_1	Q_2	Q_3	Q_4
GS_1	[M, G]	[G, VG]	[VG, EG]	[P, G]
GS_2	[G, VG]	[P, M]	[G, VG]	[G, VG]
GS_3	[M, G]	[P, M]	[VG, EG]	[P, M]
GS_4	[VG, EG]	[M, G]	[VP, P]	[G, VG]
GS_5	[P, M]	[VP, P]	[M, VG]	[M, VG]

Table 3. The ULTSs through DM_3

Alternatives	Q_1	Q_2	Q_3	Q_4
GS_1	[M, VG]	[M, G]	[VG, EG]	[M, G]
GS_2	[M, G]	[G, VG]	[P, M]	[VG, EG]
GS_3	[M, G]	[M, G]	[VG, EG]	[G, VG]
GS_4	[VG, EG]	[G, VG]	[G, EG]	[VG, EG]
GS_5	[G, VG]	[VP, P]	[M, VG]	[M, VG]

Table 4. The ULTSs through DM₄

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[G, VG]	[VG, EG]	[P, G]
GS ₂	[G, VG]	[M, G]	[VG, EG]	[VG, EG]
GS ₃	[VP, P]	[M, G]	[VG, EG]	[P, M]
GS ₄	[VG, EG]	[M, G]	[G, EG]	[VG, EG]
GS ₅	[P, M]	[VP, P]	[VG, EG]	[M, VG]

Table 5. The ULTSs through DM₅

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[VG, EG]	[M, G]	[VP, P]
GS ₂	[P, G]	[G, VG]	[VG, EG]	[G, VG]
GS ₃	[G, VG]	[P, M]	[G, VG]	[EP, VP]
GS ₄	[M, G]	[M, VG]	[G, VG]	[VG, EG]
GS ₅	[M, G]	[P, M]	[M, VG]	[G, VG]

Then, we employ the PUL-CODAS model designed to choose the optimal green supplier.

Step 1. Convert cost index Q₂ into beneficial index (See Tables 6–10). For example, in Table 1, the ULTS [M, G] is given for alternative A₁ under G₂ by the first DM, the converted beneficial attribute value is [P, M].

Table 6. The ULTSs through DM₁

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[P, M]	[G, VG]	[P, G]
GS ₂	[M, G]	[VP, P]	[M, G]	[VG, EG]
GS ₃	[VP, P]	[M, G]	[G, VG]	[P, M]
GS ₄	[G, VG]	[P, M]	[G, EG]	[VG, EG]
GS ₅	[M, G]	[M, G]	[M, G]	[P, M]

Table 7. The ULTSs through DM₂

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[VP, P]	[VG, EG]	[P, G]
GS ₂	[G, VG]	[M, G]	[G, VG]	[G, VG]
GS ₃	[M, G]	[M, G]	[VG, EG]	[P, M]
GS ₄	[VG, EG]	[P, M]	[VP, P]	[G, VG]
GS ₅	[P, M]	[G, VG]	[M, VG]	[M, VG]

Table 8. The ULTSs through DM₃

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, VG]	[P, M]	[VG, EG]	[M, G]
GS ₂	[M, G]	[VP, P]	[P, M]	[VG, EG]
GS ₃	[M, G]	[P, M]	[VG, EG]	[G, VG]
GS ₄	[VG, EG]	[VP, P]	[G, EG]	[VG, EG]
GS ₅	[G, VG]	[G, VG]	[M, VG]	[M, VG]

Table 9. The ULTSs through DM₄

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[VP, P]	[VG, EG]	[P, G]
GS ₂	[G, VG]	[P, M]	[VG, EG]	[VG, EG]
GS ₃	[VP, P]	[P, M]	[VG, EG]	[P, M]
GS ₄	[VG, EG]	[P, M]	[G, EG]	[VG, EG]
GS ₅	[P, M]	[G, VG]	[VG, EG]	[M, VG]

Table 10. The ULTSs through DM₅

Alternatives	Q ₁	Q ₂	Q ₃	Q ₄
GS ₁	[M, G]	[EP, VP]	[M, G]	[VP, P]
GS ₂	[P, G]	[VP, P]	[VG, EG]	[G, VG]
GS ₃	[G, VG]	[M, G]	[G, VG]	[EP, VP]
GS ₄	[M, G]	[VP, M]	[G, VG]	[VG, EG]
GS ₅	[M, G]	[M, G]	[M, VG]	[G, VG]

Step 2. Convert the ULTSs into PULTSs (See Table 11).

Table 11. Decision matrix with PULTSs

Alternatives	Q ₁	Q ₂
GS ₁	$\left\{ \left\langle [l_0, l_1] 0.8 \right\rangle, \left\langle [l_0, l_2] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_0, l_1] 0.4 \right\rangle, \left\langle [l_1, l_2] 0.4 \right\rangle, \left\langle [l_2, l_3] 0.2 \right\rangle \right\}$
GS ₂	$\left\{ \left\langle [l_{-1}, l_1] 0.2 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle, \left\langle [l_1, l_2] 0.4 \right\rangle \right\}$	$\left\{ \left\langle [l_{-1}, l_0] 0.2 \right\rangle, \left\langle [l_0, l_1] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.6 \right\rangle \right\}$
GS ₃	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.4 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_{-1}, l_0] 0.6 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle \right\}$
GS ₄	$\left\{ \left\langle [l_0, l_1] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_0, l_1] 0.6 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \left\langle [l_0, l_2] 0.2 \right\rangle \right\}$
GS ₅	$\left\{ \left\langle [l_{-1}, l_0] 0.4 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.6 \right\rangle, \left\langle [l_{-1}, l_0] 0.4 \right\rangle \right\}$

End of Table 11

Alternatives	Q_3	Q_4
GS_1	$\left\{ \left\langle [l_0, l_1] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.2 \right\rangle, \left\langle [l_0, l_1] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_{-1}, l_1] 0.6 \right\rangle \right\}$
GS_2	$\left\{ \left\langle [l_{-1}, l_0] 0.2 \right\rangle, \left\langle [l_0, l_1] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_1, l_2] 0.4 \right\rangle, \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$
GS_3	$\left\{ \left\langle [l_1, l_2] 0.4 \right\rangle, \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_{-3}, l_{-2}] 0.2 \right\rangle, \left\langle [l_{-1}, l_0] 0.6 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.2 \right\rangle \right\}$
GS_4	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_1, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_1, l_2] 0.0 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.8 \right\rangle \right\}$
GS_5	$\left\{ \left\langle [l_0, l_2] 0.6 \right\rangle, \left\langle [l_0, l_1] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_{-1}, l_0] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_0, l_2] 0.6 \right\rangle \right\}$

Step 3. Compute the normalized PULTSs (Table 12).

Table 12. Normalized PULTSs

Alternatives	Q_1	Q_2
GS_1	$\left\{ \left\langle [l_0, l_1] 0 \right\rangle, \left\langle [l_0, l_1] 0.9 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.1 \right\rangle \right\}$	$\left\{ \left\langle [l_0, l_1] 0.4 \right\rangle, \left\langle [l_1, l_2] 0.4 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.2 \right\rangle \right\}$
GS_2	$\left\{ \left\langle [l_{-1}, l_0] 0.1 \right\rangle, \left\langle [l_0, l_1] 0.5 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.4 \right\rangle \right\}$	$\left\{ \left\langle [l_{-1}, l_0] 0.2 \right\rangle, \left\langle [l_0, l_1] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.6 \right\rangle \right\}$
GS_3	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.4 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_{-1}, l_0] 0 \right\rangle, \left\langle [l_{-1}, l_0] 0.6 \right\rangle, \right. \\ \left. \left\langle [l_0, l_1] 0.4 \right\rangle \right\}$
GS_4	$\left\{ \left\langle [l_0, l_1] 0.2 \right\rangle, \left\langle [l_1, l_2] 0.2 \right\rangle, \right. \\ \left. \left\langle [l_2, l_3] 0.6 \right\rangle \right\}$	$\left\{ \left\langle [l_0, l_1] 0 \right\rangle, \left\langle [l_0, l_1] 0.7 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.3 \right\rangle \right\}$
GS_5	$\left\{ \left\langle [l_{-1}, l_0] 0.4 \right\rangle, \left\langle [l_0, l_1] 0.4 \right\rangle, \right. \\ \left. \left\langle [l_1, l_2] 0.2 \right\rangle \right\}$	$\left\{ \left\langle [l_{-2}, l_{-1}] 0.6 \right\rangle, \left\langle [l_{-2}, l_{-1}] 0.6 \right\rangle, \right. \\ \left. \left\langle [l_{-1}, l_0] 0.4 \right\rangle \right\}$

End of Table 12

Alternatives	Q ₃	Q ₄
GS ₁	$\left\{ \left\langle \left[l_0, l_1 \right] 0.2 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.6 \right\rangle \right\}$	$\left\{ \left\langle \left[l_{-2}, l_{-1} \right] 0.2 \right\rangle, \left\langle \left[l_{-1}, l_0 \right] 0.3 \right\rangle, \right. \\ \left. \left\langle \left[l_0, l_1 \right] 0.5 \right\rangle \right\}$
GS ₂	$\left\{ \left\langle \left[l_{-1}, l_0 \right] 0.2 \right\rangle, \left\langle \left[l_0, l_1 \right] 0.2 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.6 \right\rangle \right\}$	$\left\{ \left\langle \left[l_1, l_2 \right] 0 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.4 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.6 \right\rangle \right\}$
GS ₃	$\left\{ \left\langle \left[l_1, l_2 \right] 0 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.4 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.6 \right\rangle \right\}$	$\left\{ \left\langle \left[l_{-3}, l_{-2} \right] 0.2 \right\rangle, \left\langle \left[l_{-1}, l_0 \right] 0.6 \right\rangle, \right. \\ \left. \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle \right\}$
GS ₄	$\left\{ \left\langle \left[l_{-2}, l_{-1} \right] 0.2 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.5 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.3 \right\rangle \right\}$	$\left\{ \left\langle \left[l_1, l_2 \right] 0.0 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.8 \right\rangle \right\}$
GS ₅	$\left\{ \left\langle \left[l_0, l_1 \right] 0.5 \right\rangle, \left\langle \left[l_1, l_2 \right] 0.3 \right\rangle, \right. \\ \left. \left\langle \left[l_2, l_3 \right] 0.2 \right\rangle \right\}$	$\left\{ \left\langle \left[l_{-1}, l_0 \right] 0.2 \right\rangle, \left\langle \left[l_0, l_1 \right] 0.3 \right\rangle, \right. \\ \left. \left\langle \left[l_1, l_2 \right] 0.5 \right\rangle \right\}$

Step 4. Derive the attributes weight from Eq. (10)–(11), the attributes weight is given in Table 13.

Table 13. The attributes weight

Weight	w ₁	w ₂	w ₃	w ₄
w	0.2535	0.2621	0.2110	0.2734

Step 5. Obtain the PULNIS (Table 14).

Table 14. PULNIS

	Q ₁	Q ₂
PULNIS	$\left\{ \left\langle \left[l_{-2}, l_{-1} \right] 0.4 \right\rangle, \left\langle \left[l_0, l_1 \right] 0.4 \right\rangle, \right. \\ \left. \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle \right\}$	$\left\{ \left\langle \left[l_{-2}, l_{-1} \right] 0.0 \right\rangle, \left\langle \left[l_{-2}, l_{-1} \right] 0.6 \right\rangle, \right. \\ \left. \left\langle \left[l_{-1}, l_0 \right] 0.4 \right\rangle \right\}$
	Q ₃	Q ₄
PULNIS	$\left\{ \left\langle \left[l_{-1}, l_0 \right] 0.2 \right\rangle, \left\langle \left[l_0, l_1 \right] 0.6 \right\rangle, \right. \\ \left. \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle \right\}$	$\left\{ \left\langle \left[l_{-3}, l_{-2} \right] 0.2 \right\rangle, \left\langle \left[l_{-1}, l_0 \right] 0.6 \right\rangle, \right. \\ \left. \left\langle \left[l_1, l_2 \right] 0.2 \right\rangle \right\}$

Step 6. Compute the $PULWED(GS_i, PULNIS)$ and $PULWHD(GS_i, PULNIS)$ ($i=1,2,3,4,5$) through Eqs. (15)–(18) (See Table 15).

Table 15. $PULWED(GS_i, PULNIS)$ and $PULWHD(GS_i, PULNIS)$

Alternatives	$PULWED(GS_i, PULNIS)$	$PULWHD(GS_i, PULNIS)$
GS_1	0.1828	0.1463
GS_2	0.2016	0.1446
GS_3	0.1153	0.0521
GS_4	0.2323	0.1466
GS_5	0.1017	0.0648

Step 7. Compute the $PULRAM = [PULRAM_{ik}]_{5 \times 5}$ through Eqs. (19)–(21) (Table 16).

Table 16. $PULRAM = [PULRAM_{ik}]_{5 \times 5}$

	GS_1	GS_2	GS_3	GS_4	GS_5
GS_1	0.0000	−0.0188	0.1617	−0.0498	0.1626
GS_2	0.0188	0.0000	0.1788	−0.0327	0.1797
GS_3	−0.1617	−0.1788	0.0000	−0.2115	0.0136
GS_4	0.0498	0.0327	0.2115	0.0000	0.2123
GS_5	−0.1626	−0.1797	−0.0136	−0.2123	0.0000

Step 8. Calculate the $PULAS_i$ ($i=1,2,3,4,5$) by Eq. (22) (See Table 17).

Table 17. PULAS decision matrix

Alternatives	GS_1	GS_2	GS_3	GS_4	GS_5
PULAS	0.2556	0.3446	−0.5383	0.5062	−0.5682

Step 9. According to $PULAS_i$ ($i=1,2,3,4,5$), the order is $GS_4 > GS_2 > GS_1 > GS_3 > GS_5$. That's to say, GS_4 is the optimal alternative.

3.2. Comparative analysis

Then, the PUL-CODAS is compared with ULWA operator (Xu, 2004), PUL-TOPSIS (Lin et al., 2018) and PULWA (Lin et al., 2018).

3.2.1. Compared with ULWA

Firstly, we deal with such example by using the ULWA (Xu, 2004) with same weight to aggregate these ULTSs into a group matrix (See Table 18).

Table 18. Group uncertain linguistic matrix

Alternatives	Q_1	Q_2	Q_3	Q_4
GS_1	$[l_{0.0}, l_{1.2}]$	$[l_{0.8}, l_{1.8}]$	$[l_{1.4}, l_{2.4}]$	$[l_{1.1}, l_{0.6}]$
GS_2	$[l_{0.2}, l_{1.4}]$	$[l_{0.4}, l_{1.4}]$	$[l_1, l_2]$	$[l_{1.6}, l_{2.4}]$
GS_3	$[l_{-0.6}, l_{0.4}]$	$[l_{-0.6}, l_{0.4}]$	$[l_{1.6}, l_{2.4}]$	$[l_{-1}, l_0]$
GS_4	$[l_{1.4}, l_{2.4}]$	$[l_{0.2}, l_{1.4}]$	$[l_{0.4}, l_2]$	$[l_{1.8}, l_{2.8}]$
GS_5	$[l_{-0.2}, l_{0.8}]$	$[l_{-1.6}, l_{-0.6}]$	$[l_{0.4}, l_2]$	$[l_0, l_{1.6}]$

The weight is: $w_1 = 0.2535, w_2 = 0.2621, w_3 = 0.2110, w_4 = 0.2734$, then the total value is derived by employing ULWA (Xu, 2004).

$$Z_1(w) = [s_{0.2318}, s_{1.4465}], Z_2(w) = [s_{0.8040}, s_{1.800}], Z_3(w) = [s_{-0.2451}, s_{0.7127}], \\ Z_4(w) = [s_{0.9838}, s_{2.1628}], Z_5(w) = [s_{-0.3857}, s_{0.9049}].$$

Then, the score of five potential alternative are derived through Definition 9 (Lin et al., 2018):

$$E(Z_1(w)) = s_{0.8391}, E(Z_2(w)) = s_{1.3020}, E(Z_3(w)) = s_{0.2338}, \\ E(Z_4(w)) = s_{1.5733}, E(Z_5(w)) = s_{0.2596}.$$

Furthermore, the order is derived: $GS_4 > GS_2 > GS_1 > GS_5 > GS_3$. Thus, we could obtain the optimal green supplier GS_4 .

3.2.2. Compared with PUL-TOPSIS

Then, the PUL-CODAS is compared with the PUL-TOPSIS model (Lin et al., 2018), then the derived result is obtained (Table 19). Thus, the best green supplier is GS_4 .

Table 19. The calculating results through PUL-TOPSIS

TOPSIS	Calculating results
The distances from PULPIS	$d_1^+ = 0.5423, d_2^+ = 0.4089, d_3^+ = 0.6594, d_4^+ = 0.1651, d_5^+ = 0.7746$
The distances from PULNIS	$d_1^- = 0.6250, d_2^- = 0.7830, d_3^- = 0.4272, d_4^- = 0.8823, d_5^- = 0.4599$
Closeness coefficients	$CI_1 = -2.5773, CI_2 = -1.5899, CI_3 = -3.5110, CI_4 = 0.0000, CI_5 = -4.1714$
Ordering	$GS_4 > GS_2 > GS_1 > GS_3 > GS_5$

3.2.3. Compared with PULWA

Finally, the PUL-CODAS is compared with PULWA (Lin et al., 2018), the attributes weight is: $w_1 = 0.2535, w_2 = 0.2621, w_3 = 0.2110, w_4 = 0.2734$, then the total value is derived through PULWA operator.

$$\begin{aligned}
Z_1(w) &= \left\{ \left[s_{-0.1094}, s_{0.0924} \right], \left[s_{0.0650}, s_{0.5222} \right], \left[s_{0.3834}, s_{0.7245} \right] \right\}, \\
Z_2(w) &= \left\{ \left[s_{-0.1200}, s_{0.0000} \right], \left[s_{0.1094}, s_{0.4401} \right], \left[s_{0.8399}, s_{1.3893} \right] \right\}, \\
Z_3(w) &= \left\{ \left[s_{-0.3668}, s_{-0.2107} \right], \left[s_{-0.2369}, s_{0.2702} \right], \left[s_{0.3586}, s_{0.6954} \right] \right\}, \\
Z_4(w) &= \left\{ \left[s_{-0.0844}, s_{0.0085} \right], \left[s_{0.2109}, s_{0.6053} \right], \left[s_{0.9468}, s_{1.4595} \right] \right\}, \\
Z_5(w) &= \left\{ \left[s_{-0.1561}, s_{0.1055} \right], \left[s_{-0.2513}, s_{0.1527} \right], \left[s_{0.1669}, s_{0.5014} \right] \right\}.
\end{aligned}$$

Then, the score of these five alternatives are derived through Definition 9 (Lin, et al., 2018):

$$\begin{aligned}
E(Z_1(w)) &= s_{0.1865}, E(Z_2(w)) = s_{0.2954}, E(Z_3(w)) = s_{0.0566}, \\
E(Z_4(w)) &= s_{0.3496}, E(Z_5(w)) = s_{0.0577}.
\end{aligned}$$

The order is: $GS_4 > GS_2 > GS_1 > GS_5 > GS_3$ and the optimal green supplier is GS_4 .

Conclusions

In such paper, we developed the CODAS model for MAGDM based on PULTSs and sine entropy weight. Firstly, the Euclidean and Hamming distance under PULTSs are introduced. Then, the CODAS method is proposed for PUL-MAGDM and its main merit is that it highlights Euclidean and Hamming distance from PULNIS. Finally, an example analysis about green supplier selection is utilized to show the defined algorithms and some detailed comparative analysis are used to elucidate the effectiveness in practical decision making.

However, there still remains some unfinished work to be done. Since the computational process of the PULTSs is complicated, we need to further investigate the operations of PULTSs. Except that, the consensus analysis between different groups should be take into account. In future, we are also going to carry out researches on these two aspects and devote to apply the designed methods to other fields, such as pattern recognition, industrial engineering, E-commerce, and so on. At the same time, the corresponding application of the designed algorithms under PULTSs are studied through some other uncertain MADM and uncertain settings and the basic concept of PULTSs could be employed to expand some other fuzzy settings with help of their corresponding probability.

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APPENDIX

Abbreviations

CODAS	– Combinative distance-based assessment;
DMs	– Decision makers;
MAGDM	– Multiple attribute group decision making;
NIS	– Negative-ideal solution;
PUL	– Probabilistic uncertain linguistic;
PULTSs	– Probabilistic uncertain linguistic sets;
ULTSs	– Uncertain linguistic terms sets;
PUL-MAGDM	– Probabilistic uncertain linguistic MAGDM;
PULNIS	– Probabilistic uncertain linguistic negative ideal solution;
IVIFSs	– Interval-valued intuitionistic fuzzy sets;
2TLTSs	– 2-tuple linguistic term sets;
PL-MAGDM	– Probabilistic linguistic MAGDM;
GDSM	– Generalized dice similarity measures;
HFLTSSs	– Hesitant fuzzy linguistic term sets.