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Research Article

Improved Knowledge Measures for q-Rung Orthopair Fuzzy Sets

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ABSTRACT

The q-rung orthopair fuzzy set (qROFS) defined by Yager is a generalization of Atanassov intuitionistic fuzzy set (IFS) and Pythagorean fuzzy sets (PyFSs). In this paper, we define the knowledge measure for qROFS by using the cosine inverse function. The information precision and information content are two facets of knowledge measure. Both facets of knowledge measure are considered. The properties of knowledge measure and their graphical explanations are discussed. An application of the knowledge measure in multi-attribute group decision-making (MAGDM) problem under the confidence level approach is given. A numerical example of the selection of renewable energy sources is discussed.

1. INTRODUCTION

The membership function is employed to represent the information in the fuzzy sets theory [1]. Real-world hesitations can be handled impressively by fuzzy set theory. Atanassov explicated intuitionistic fuzzy set (IFS) as a generalization of the fuzzy set theory [2]. The information in IFS is portrayed in the form of membership (favor) and nonmembership (against) functions. The membership and nonmembership degrees allocate the values from the unit interval [0,1] with the constraint that their sum is less than or equal to one that is if we represent the associate (membership) and nonassociate (nonmembership) functions by ξ and ν, respectively, than $0 \leq \xi + \nu \leq 1$. This constraint identifies a range of ξ and ν. The triangular region in Figure 1 represents the set of order pairs available for membership and nonmembership grades.

Yager defined Pythagorean fuzzy set (PyFS) as a generalization of IFSs and extends the range of membership and nonmembership functions that is decision-makers express their judgments more freely than IFSs [3,4]. The membership ξ and nonmembership functions ν satisfies the following condition $0 \leq \xi^2 + \nu^2 \leq 1$. The circular region in Figure 1 represents the set of order pairs available for membership and nonmembership grades for PyFSs. For more about PyFSs, we refer to [5–8].

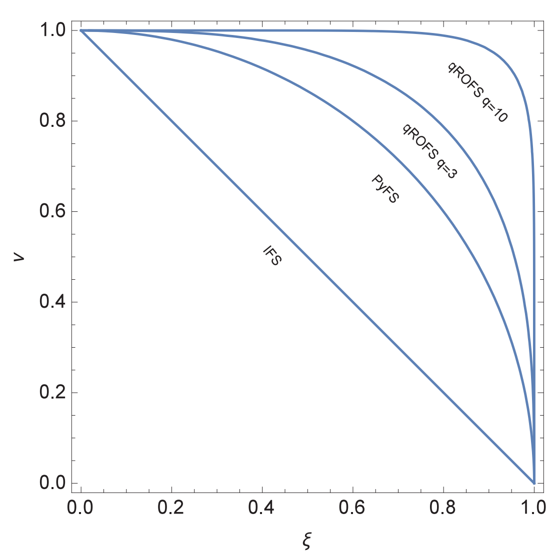

Figure 1 Graphical representation of IFS, PyFS and qROFSs (q = 3,10).

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Yager introduced the concept of q-rung orthopair fuzzy set (qROFS) as a generalization of both IFSs and PyFSs [9,10]. The condition for the membership ξ and nonmembership ν functions is $0 \leq \xi^q + \nu^q \leq 1$ where $q \geq 1$. In Figure 1, the set of order pairs available for membership and nonmembership grades for qROFSs are displaced.

Figure 1 differentiate the set of order pairs available for membership and nonmembership grades for IFSs, PyFSS and qROFSs (or margin for decision-makers to make their judgments). According to Ali [11], the range of the membership functions in IFS has an area 0.5 square unit in Figure 1. For PyFS the area increases to 0.77854 square unit which is 57% more than from the IFS case. For qROFS (q=3), the area increases to 0.8832, which is approximately 13% more than the PyFS case. The range of the membership function covers up to 99% region of the unit square $[0,1] \times [0,1]$ for q=10 (see Figure 1). So for real-life applications, the value of q up to 10 is more suitable although q is the real number.

The notion of neutrosophic sets is quite different from qROFSs. In neutrosophic sets $\xi + \mathrm{i} + \nu \leq 3$ (where ξ, i and ν are membership, indeterminacy and nonmembership grades respectively). There is a paradoxical situation when ξ=i=ν=1, that is we cannot draw conclusion for any element for which we have value (1,1,1), same happen for the values in its neighborhood. That is there are infinite many paradoxical values when we consider neutrosophic sets. But such situations do not arise in qROFSs. Also, the neutrosophic set [12] has three membership functions but the uncertainty index cannot be dealt with in the neutrosophic set, and also there does not exist any specific method to obtain an indeterminacy degree.

While dealing with real-life situations qROFSs provide more flexible ways to define membership and non-membership grades. That is why nowadays many researchers are showing keen interest in qROFSs. Hussain *et al.* [13] combined the qROFS with soft sets and defined their aggregation operators. The generalized Maclaurin symmetric mean operators, the exponential aggregation operators, the aggregation operators, the interaction Hamy mean operators, the Dombi power partitioned Heronian mean operators, the power Maclaurin symmetric mean operators, the Heronian mean operators and the power aggregation operators for qROFSs were expounded in [14–20], respectively. The multiple heterogeneous relationships between membership functions and criterion were explored by Yang *et al.* [21]. Verma [22] explained order-α divergence and entropy measures for qROFSs and applied to multi-attribute group decision-making (MAGDM) problems. The enterprise resource planning systems selection problem was solved by Hamy mean operators for qROFSs by Wang *et al.* [23]. The multiplicative consistency of preference relation of qROFSs was analyzed by Zhang *et al.* [24]. Du discussed the correlation coefficient of qROFSs both on bounded and unbounded continuous universes [25]. For more about decision-making and qROFSs, we refer to [26–33].

De Luca and Termini [34] introduced the axioms for fuzzy entropy which measures the fuzziness for fuzzy sets. Entropy has been a rich area of interest for many researchers. Entropy for IFSs was expounded by Bustince and Burillo [35]. Later Szmidt and Kacprzyk [36] defined a ratio based entropy by using the Hamming distance. The information about an intuitionistic fuzzy value (IFV) is conveying properly by this entropy that is how much intuitionistic fuzzy is an IFV? The distance, similarity, entropy and inclusion measures for qROFSs were expounded by Peng and Liu [37].

The dissimilarity and similarity measures is a rich area for scholars. Keeping in mind the applications of this area many researchers work on dissimilarity and similarity measures. The applications of this area was discussed in medical diagnosis, data mining, pattern recognition, decision-making, clustering and in image processing. The qROF multi-parametric similarity measure and combinative distance-based assessment were used to assess the classroom teaching quality [38]. The dissimilarity, similarity, entropy and inclusion measures for qROFSs were defined by Peng and Liu [37]. The similarity measures of qROFSs based on cosine and cotangent functions were expounded by Wang *et al.* [39]. The Minkowski dissimilarity measures (Hamming, Euclidean and Chebyshev distances) for qROFSs were discussed in [40]. The TOPSIS method based on improved cosine similarity measures was interpreted in [41]. A new dissimilarity measure was defined with a nice interpretation. The TOPSIS and ELECTRE were described based on this novel dissimilarity measure for qROFSs in [42].

Knowledge is basically related to the information considered in a particular useful context. The amount of information and the amount of knowledge are closely linked. From practical point of view, the transformation of information into the knowledge is very important, like in decision-making. Generally it is thought that knowledge measure is the dual of the entropy [43]. This approach is also adopted in [44,45]. In this regard the point of view given by Szmidt *et al.* [46] is bit different and they consider that a knowledge measure must depend on both hesitancy index and the entropy. Therefore while defining a knowledge measure in IFSs, both uncertainty index and entropy must be taken into account [46]. So a knowledge measure for IFSs is given in [46], which is based on both uncertainty index and entropy. It was expected that for a constant value of uncertainty index the knowledge measure defined in [46] must behave dually to entropy. Actually this does not happen for the above mentioned knowledge measure. Guo [47] and Guo and Xu [48] think that the entropy and knowledge are two distinct measures therefore these should be dealt independently. An axiomatic definition of knowledge measures is given in [47,48]. Knowledge measure in [48] is based on information clarity and information content. Singh *et al.* [49] proposed the knowledge, entropy and inclusion measures for fuzzy sets and it's application in image processing. For more literature on knowledge measure, refer to [50,51].

In the first approach, the axiomatic framework of knowledge measures for qROFSs was defined [52]. The previous approach has considered the hesitancy index only. The higher value of knowledge measure is attached to qROFS with lower hesitancy indices. In other words, the approach considers the information content that is the maximum information content that results in the maximum knowledge measure. But the approach not considered the information clarity or information precision. So, the proposed approach contemplate the both aspect of knowledge measure that is the information content and information precision.

Since the qROFS is a generalization of both IFSs and PyFSs. Thus the existing models to quantify the knowledge from the intuitionistic fuzzy information are not suitable for Pythagorean and q-rung orthopair fuzzy environment. Therefore, we need to define a new generalized knowledge measure for q-rung orthopair fuzzy environment that quantifies the information from qROFSs.

Motivated by the Yager approach, we propose a method to quantify the knowledge associated with qROFS. The knowledge associated with q-rung orthopair fuzzy values (qROFVs) increases for assured and precise information and decreases when the ambiguity and uncertainty factor increases. The knowledge measure also depend both on information precision and information content. Also, the confidence level approach towards MAGDM problems is essential in the information fusion step. But there does not exist any study about how to find the confidence level. We have formulated a procedure to find the confidence level in the information fusion step.

To measure the amount of knowledge from information provided in the form of qROFS, a novel axiomatic framework is proposed in this paper that consider the information precision and information content. The main contributions of our work are:

- A knowledge measure for qROFS based on inverse cosine function is defined.
- The axiomatic characterization results are obtained for the proposed knowledge measure.
- Properties of knowledge measure with their graphical representation are discussed.
- A method to solve MAGDM problems under confidence level is proposed.

The proposed knowledge measure differentiates among the two clearly different situations: (i) the membership and nonmembership functions both have same values (i.e., we have equal number of arguments in favor and against). (ii) We have no information at all (i.e., the membership and nonmembership functions both have values zero).

The remaining part of the paper is designed as follows: Section 2 consists of the basic definitions related to the IFS, PyFS and qROFS. An axiomatic definition of knowledge measure and its related properties with graphical explanations are investigated in Sections 3–5. The applications of the proposed knowledge measure in MAGDM problems under confidence level approach are explored in Section 6. The summary, limitations and future directions are debated in Section 8.

2. PRELIMINARIES

In this segment, the definitions of IFS, PyFS and qROFS and their properties are mentioned. Let $X=\{t_1,t_2,\ldots,t_n\}$ represents the universal set throughout the paper which is discrete, finite and non-void discourse set. The membership and nonmembership functions and unit interval $[0,1]$ are represented as ξ_R, ν_R and Δ, respectively, throughout the paper.

Definition 1. An IFS R on a universal set X is defined as

$$R=\{(\xi_R(t_i),\nu_R(t_i))\,|\,t_i\in X\},$$

where $\xi_R : X\to\Delta$ and $\nu_R : X\to\Delta$ with constraint $\xi_R(t_i)+\nu_R(t_i)\le 1$. The quantity $\pi_R(t_i)=1-(\xi_R(t_i)+\nu_R(t_i))$ is called the hesitancy degree of the element $t_i\in X$.

Definition 2. [3,4] A PyFS R on a universal set X is defined as

$$R=\{(\xi_R(t_i),\nu_R(t_i))\,|\,t_i\in X\},$$

where $\xi_R : X\to\Delta$ and $\nu_R : X\to\Delta$ with constraint $\xi_R^2(t_i)+\nu_R^2(t_i)\le 1$. The quantity $\pi_R(t_i)=\sqrt{1-(\xi_R^2(t_i)+\nu_R^2(t_i))}$ is called the hesitancy degree of the element $t_i\in X$.

Definition 3. [9,10] A qROFS R on a universal set X is defined as

$$R=\{(\xi_R(t_i),\nu_R(t_i))\,|\,t_i\in X\},$$

where $\xi_R : X\to\Delta$ and $\nu_R : X\to\Delta$ with constraint $\xi_R^q(t_i)+\nu_R^q(t_i)\le 1$. The quantity $\pi_R(t_i)=\left(1-(\xi_R^q(t_i)+\nu_R^q(t_i))\right)^{\frac{1}{q}}$ is called the hesitancy degree of the element $t_i\in X$. For a fixed element $t_i\in X$, the qROFS $(\xi_R(t_i),\nu_R(t_i))$ is called the q-rung orthopair fuzzy value (qROFV). For simplicity, we write (ξ_{R_i},ν_{R_i}) a qROFV instead of $(\xi_R(t_i),\nu_R(t_i))$.

For any two qROFVs $b_1=(\xi_{b_1},\nu_{b_1})$ and $b_2=(\xi_{b_2},\nu_{b_2})$, the basic laws are defined as follows:

1. $b_1^c=(\nu_{b_1},\xi_{b_1})$
2. $b_1\vee b_2=(\max\{\xi_{b_1},\xi_{b_2}\},\min\{\nu_{b_1},\nu_{b_2}\})$
3. $b_1\wedge b_2=(\min\{\xi_{b_1},\xi_{b_2}\},\max\{\nu_{b_1},\nu_{b_2}\})$

3. KNOWLEDGE MEASURE

Knowledge is basically related to the information considered in a particular useful context. The amount of information and the amount of knowledge are closely linked. From practical point of view, the transformation of information into the knowledge is very important, like in decision-making. In this part, we define the knowledge measure for qROFS by using the cosine inverse function. The information precision and information content are two facets of knowledge measure. Both facets of knowledge measure are consider in the given method.

In the following, we provide the axiomatic definition of knowledge measure for qROFSs.

Definition 4. The knowledge measure I^q of a qROFS in X is a function from a qROFS to a unit interval, that is, $I^q:\text{qROFS}\to[0,1]$, which satisfies the following properties:

K1. $I^q(R)=1$, iff R is a crisp set.

K2. $I^q(R)=0$, iff $\pi_R(t_i)=1,\ \forall t_i\in X$.

K3. $I^q(R)\ge I^q(S)$, iff the higher information content of the elements of R with greater information precision in comparison with S, that is,

$$\left(\xi_R^q(t_j)+\nu_R^q(t_j)\right)\ge\left(\xi_S^q(t_j)+\nu_S^q(t_j)\right)\ \textit{and}$$
$$\left|\xi_R^q(t_j)-\nu_R^q(t_j)\right|\ge\left|\xi_S^q(t_j)-\nu_S^q(t_j)\right|,\ \forall\, t_j\in X.$$

K4. $I^q(R)=I^q(R^c)$, where R^c represents the complement of R.

Now, we define the knowledge measure for qROFS by using cosine inverse function. The knowledge is measured by considering membership, nonmembership and hesitancy degrees.

Definition 5. Let R be a qROFS in X. Then, the knowledge measure of R is defined and expressed as

$$\mathrm{I}_c^q(\mathrm{R}) = \left(\frac{1}{2n} \sum_{j=1}^{n} \left(|\xi_R^q(t_j) - \nu_R^q(t_j)| + \frac{2}{\pi} \cos^{-1}\left[\pi_R^q(t_j)\right] \right) \right)^{\frac{1}{q}} \tag{1}$$

Since $\pi_R(t_j) = \left(1 - \left(\xi_R^q(t_j) + \nu_R^q(t_j)\right)\right)^{\frac{1}{q}}$, $\mathrm{I}_c^q(\mathrm{R})$ can be expressed as

$$\mathrm{I}_c^q(\mathrm{R}) = \left(\frac{1}{2n} \sum_{j=1}^{n} \left(|\xi_R^q(t_j) - \nu_R^q(t_j)| + \frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t_j) + \nu_R^q(t_j)\right)\right] \right) \right)^{\frac{1}{q}} \tag{2}$$

If $X = \{t\}$, then the knowledge measure for a qROFS R is represented as

$$\mathrm{I}_c^q(\mathrm{R}) = \left(\frac{1}{2} \left(|\xi_R^q(t) - \nu_R^q(t)| + \frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] \right) \right)^{1/q} \tag{3}$$

Remark 1. It is important to note that the knowledge measure presented in Equation (3) has two parts, that is, $|\xi_R^q(t) - \nu_R^q(t)|$ and $\frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]$. Since the knowledge measure has two facets that is information content and information precision. The first part $\left(|\xi_R^q(t) - \nu_R^q(t)|\right)$ represents the information precision, while the second part $\left(\frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]\right)$ indicates the information content.

Since there is maximum opacity at $\xi(t) = \nu(t)$ and no information precision. So, the first part becomes zero for this case and the knowledge is measured by the second part that is information contents based knowledge is measured.

The information content is maximum at $\left(\xi_R^q(t) + \nu_R^q(t)\right) = 1$. There are infinite many combinations of membership and nonmembership degrees for which $\left(\xi_R^q(t) + \nu_R^q(t)\right) = 1$. The second part remains maximum one for this case that is constant. Therefore, the knowledge is measured based on the first part.

The graphical representation of both parts displaced in Figures 2 and 3. It can easily be seen from Figure 2 that the first part have value one two times for crisp cases and remains zero for $\xi(t) = \nu(t)$. While the second part displaced in Figure 3 and touches zero at (0,0). The graph remains maximum one for $\left(\xi_R^q(t) + \nu_R^q(t)\right) = 1$.

The following axiomatic properties are satisfied for the proposed knowledge measure I_c^q (Definition 5).

Theorem 1. *If* R *and* S *be two qROFSs in* $X = \{t\}$, *then knowledge measure* I_c^q *satisfies the following axiomatic properties:*

(K1). $\mathrm{I}_c^q(\mathrm{R}) = 1$, *iff* R *is a crisp set.*

(K2). $\mathrm{I}_c^q(\mathrm{R}) = 0$, *iff* $\pi_R(t) = 1$.

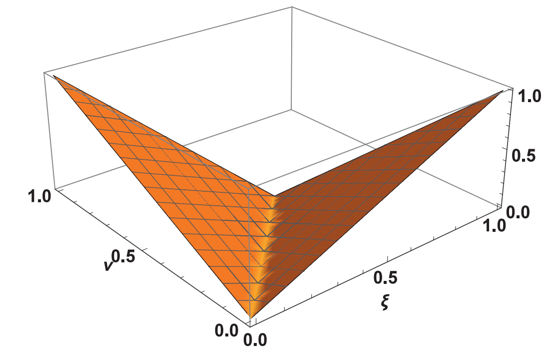

Figure 2 First part of Equation (3).

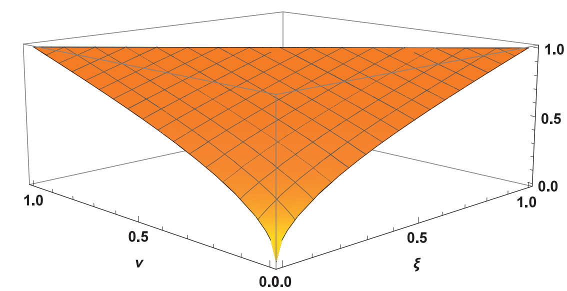

Figure 3 Second part of Equation (3).

(K3). $\mathrm{I}_c^q(\mathrm{R}) \geq \mathrm{I}_c^q(\mathrm{S})$, *iff the higher information content of the elements of* R *with greater information precision in comparison with* S, *that is,*

$$\left(\xi_R^q(t) + \nu_R^q(t)\right) \geq \left(\xi_S^q(t) + \nu_S^q(t)\right) \text{ and}$$
$$|\xi_R^q(t) - \nu_R^q(t)| \geq |\xi_S^q(t) - \nu_S^q(t)|, \ t \in X.$$

(K4). $\mathrm{I}_c^q(\mathrm{R}) = \mathrm{I}_c^q(\mathrm{R}^c)$.

Proof. Proof. (K1). Since $\pi_R = \left(1 - \xi_R^q - \nu_R^q\right)^{\frac{1}{q}}$, we have

$$\begin{aligned} & \mathrm{I}_c^q(\mathrm{R}) = 1 \\ \Leftrightarrow\ & \frac{1}{2}\left(|\xi_R^q(t) - \nu_R^q(t)| + \frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] \right) = 1 \\ \Leftrightarrow\ & \left(|\xi_R^q(t) - \nu_R^q(t)| + \frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] \right) = 2. \end{aligned} \tag{4}$$

The left-hand side of the Equation (4) has two parts, that is, $|\xi_R^q(t) - \nu_R^q(t)|$ and $\frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]$. The both parts have the following boundaries:

$$0 \leq |\xi_R^q(t) - \nu_R^q(t)| \leq 1 \text{ and}$$
$$0 \leq \frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] \leq 1$$

Thus Equation (4) holds only when both parts have value one, that is,

$$\begin{aligned} & |\xi_R^q(t) - \nu_R^q(t)| = 1 \text{ and} \\ & \frac{2}{\pi} \cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] = 1. \end{aligned} \tag{5}$$

Now, $|\xi_R^q(t) - \nu_R^q(t)| = 1$ only when $\xi_R(t) = 1$ and $\nu_R(t) = 0$ or $\xi_R(t) = 0$ and $\nu_R(t) = 1$. The second part $\frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]$ is one when $\xi_R^q(t) + \nu_R^q(t) = 1$. This can also be obtained for $\xi_R(t) = 1$ and $\nu_R(t)$=0 or $\xi_R(t) = 0$ and $\nu_R(t) = 1$. Simultaneously, the both parts of Equation (5) obtained only when $\xi_R(t) = 1$ and $\nu_R(t) = 0$ or $\xi_R(t) = 0$ and $\nu_R(t) = 1$.

Therefore, the necessary and sufficient condition for Equation (4) to hold is $\xi_R(t)$=1 and $\nu_R(t)$=0 or $\xi_R(t)$=0 and $\nu_R(t)$=1.

Hence it holds only for crisp sets.

(K2). From Equation (2), we have

$$\begin{aligned} & I_c^q(R) = 0 \\ \Leftrightarrow & \tfrac{1}{2}\Big(|\xi_R^q(t) - \nu_R^q(t)| + \\ & \tfrac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]\Big) = 0 \\ \Leftrightarrow & \Big(|\xi_R^q(t) - \nu_R^q(t)| + \\ & \tfrac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]\Big) = 0. \end{aligned} \tag{6}$$

The left-hand side of the Equation (6) has two parts, that is, $|\xi_R^q(t) - \nu_R^q(t)|$ and $\frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]$. Thus Equation (6) holds only when

$$\begin{aligned} & |\xi_R^q(t) - \nu_R^q(t)| = 0 \ \textit{and} \\ & \tfrac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right] = 0. \end{aligned} \tag{7}$$

Now, $|\xi_R^q(t) - \nu_R^q(t)| = 0$ when $\xi_R(t) = \nu_R(t)$ and this includes $\xi_R(t) = \nu_R(t) = 0$. The second part $\frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_R^q(t) + \nu_R^q(t)\right)\right]$ is zero only when $\xi_R^q(t) + \nu_R^q(t) = 0$. This can only be obtained for $\xi_R(t) = \nu_R(t) = 0$. Simultaneously, the both parts of Equation (7) obtained only when $\xi_R(t) = \nu_R(t) = 0$.

Therefore, the necessary and sufficient condition for Equation (6) to hold is $\xi_R(t) = \nu_R(t) = 0$. Thus $I_c^q(R) = 0$, iff $\pi_R = 1$.

(K3). To prove the axiom K3, let $u = |\xi_R^q(t) - \nu_R^q(t)|$ and $v = \left(\xi_R^q(t) + \nu_R^q(t)\right)$, where $0 \le u \le v \le 1$. Equation (3) can take the from as

$$I_c^q(R) = \left(\frac{1}{2}\left(u + \frac{2}{\pi}\cos^{-1}[1 - v]\right)\right)^{1/q} \tag{8}$$

To complete the proof, we need to show the function in Equation (8) monotonically increasing with respect to u and v.

$$\frac{\partial I_c^q(R)}{\partial u} = \frac{2^{-1/q}\left(u + \frac{2}{\pi}\cos^{-1}[1 - v]\right)^{\frac{1}{q}-1}}{q} > 0, \tag{9}$$

$$\frac{\partial I_c^q(R)}{\partial v} = \frac{2^{1-\frac{1}{q}}\left(u + \frac{2}{\pi}\cos^{-1}[1 - v]\right)^{\frac{1}{q}-1}}{\pi q\sqrt{1 - (1 - v)^2}} > 0 \tag{10}$$

So, the Equations (9) and (10) show the required monotonicity.

(K4). $I_c^q(R) = I_c^q(R^c)$ is obvious from Equation (2).

4. PROPERTIES OF KNOWLEDGE MEASURE I_c^q

This part consist of properties and analysis of the knowledge measure I_c^q. The graphical explanations are given to support the analysis.

P1: Figure 4 shows the geometric interpretation of knowledge measure I_c^q for qROFSs. For each qROFS, there exists a point from region ADE. By analyzing Figure 4, we have noticed following properties:

1. Point A represents the most fuzzy qROFS $(0, 0, 1)(\xi_R(t) = \nu_R(t) = 0)$ and the value of I_c^q is zero. It means, we have no information.
2. While points B and C represent the crisp sets and the value of I_c^q is one for both points. From Figure 4, we observed that the graph approaches value 1 two times for crisp sets (i.e., $\xi_R(t) = 1$ *or* $\nu_R(t) = 1$). It means, we are sure about data or have complete information.
3. The graph remains symmetric around line AD, which shows the complement case that is the values remains same for complement of the qROFSs.

P2: The proposed knowledge measure I_c^q differentiates among the two clearly different situations:

(i) The membership $(\xi_R(t))$ and nonmembership $(\nu_R(t))$ functions have same values (we have equal number of arguments in favor and against, i.e., $\xi_R(t) = \nu_R(t)$).

(ii) We have no information at all (the membership and nonmembership functions have values zero, i.e., $\xi_R(t) = \nu_R(t) = 0$).

In fact, the information measure should be different for the most fuzzy qROFS R = (x,0,0) ($\xi_R(t) = \nu_R(t) = 0$) and the case when $\xi_R(t) = \nu_R(t) \neq 0$. The Equation (3) can be rewritten as

$$I_c^q(R) = \left(\frac{\cos^{-1}\left[1 - 2\xi_R(t)^q\right]}{\pi}\right)^{\frac{1}{q}} \tag{11}$$

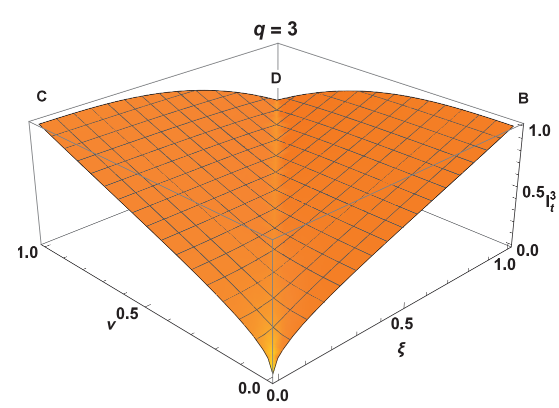

Figure 4 Graphical representations of knowledge measure I_c^3.

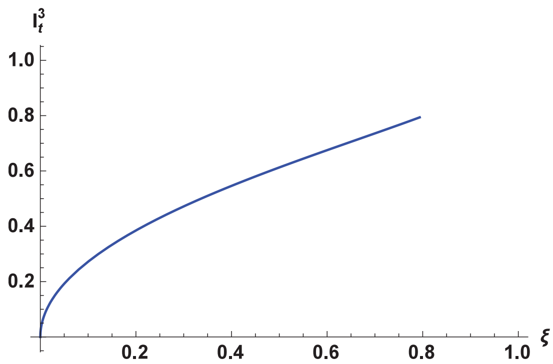

Figure 5 Knowledge measure when $\xi_R(t)=\nu_R(t)$.

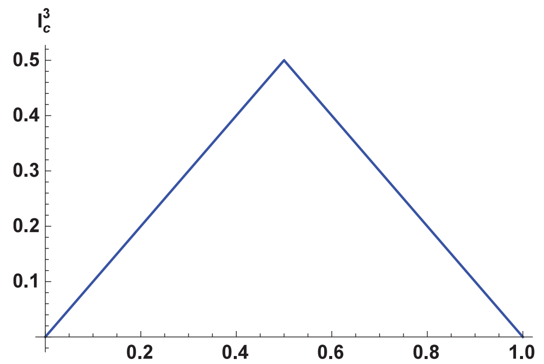

Figure 6 Knowledge measure as De Luka and Termini entropy.

Equation (11) shows that there is no information precision for $\xi(t)=\nu(t)$ and the first part becomes zero for this case. Thus the knowledge is measured by the second part that is information contents based knowledge is measured.

Figure 5 shows the geometric interpretation of knowledge measure I_c^3 for qROFSs when $\xi_R(t)=\nu_R(t)$ and I_c^3 is a strictly increasing function with respect to membership degree. It means the knowledge measure I_c^3 is consistent with the case when $\xi_R(t)=\nu_R(t)$.

P3: Now we consider the case when $\pi_R(t)=0$. Since $\pi_R(t)=\left(1-\xi_{R_j}^q-\nu_{R_j}^q\right)^{\frac{1}{q}}$, therefore, $\nu_{R_j}^q=1-\xi_{R_j}^q$. Then Equation (3) can be rewritten as

$$
\begin{aligned}
I_c^q(R)&=\left(\frac{1}{2}\left(|2\xi_R^q(t)-1|+\frac{2}{\pi}\cos^{-1}[0]\right)\right)^{1/q}\\
I_c^q(R)&=\left(\frac{1}{2}\left(|2\xi_R^q(t)-1|+1\right)\right)^{1/q}
\end{aligned}
\tag{12}
$$

Since the information content is maximum at $\pi_R(t)=0$, therefore, the second part is maximum, that is, one. Also, it is important to note that there are infinite combinations of $\xi_R(t)$ and $\nu_R(t)$ for which $\pi_R(t)=0$. Graphically view of that points can be seen in Figure 1, where the graph start from (1,0) and end at (0,1). Since the knowledge is maximum at crisp points, therefore, the knowledge got highest value at end points and minimum at the middle point, that is, $\xi_R(t)=\nu_R(t)$. So, when move from (1,0) to $\xi_R(t)=\nu_R(t)$, the knowledge decreases, while increases from $\xi_R(t)=\nu_R(t)$ to (0,1).

P4: The proposed knowledge measure I_c^1 is consistent with entropy measure of De Luca and Termini[34]. Since every element belonging to the fuzzy set R can be written as a point $=(\xi_R(t),\nu_R(t))$, where $\nu_R(t)=1-\xi_R(t)$. Therefore, I_c^1 can be expressed as

$$
I_c^1(R)=\left(\frac{1}{2}\left(|2\xi_R(t)-1|+1\right)\right)
$$

As a dual measure, the entropy measure of fuzzy set R is given by

$$
\begin{aligned}
E(R)&=1-\left(\frac{1}{2}\left(|2\xi_R(t)-1|+1\right)\right)\\
&=\frac{1}{2}\left(1-|2\xi_R(t)-1|\right)
\end{aligned}
\tag{13}
$$

The geometric interpretation of E(R) is given in Figure 6. From Figure 6, we can easily see that E(R) has value zero only for crisp cases. E(R) has maximum value at (x,0.5,0.5). At last, E(R) is greater than E(S) where R is any "sharpened" version of S, that is any fuzzy set such that $E(S)\geq E(R)$ if $\xi_R(t)\geq 0.5$ and $E(S)\leq E(R)$ if $\xi_R(t)\leq 0.5$. Hence all the properties for De Luca and Termini entropy are satisfied for E(R).

P5: Next, we have seen that when $\pi(t)$=c (constant), then I_c^q reaches a minimum for $\xi(t)=\nu(t)$. Since $\pi(t)=(1-(\xi^q(t)+\nu^q(t)))^{\frac{1}{q}}$ and $(\xi^q(t)+\nu^q(t))=1-\pi^q(t)=1-c^q$, therefore, $\nu^q(t)=1-c^q-\xi^q(t)$. Then Equation (2) takes the form

$$
\begin{aligned}
I_c^q(R)&=\left(\frac{1}{2}\left(|\xi_R^q(t)-\nu_R^q(t)|+\right.\right.\\
&\left.\left.\frac{2}{\pi}\cos^{-1}\left[1-\left(\xi_R^q(t)+\nu_R^q(t)\right]\right)\right)^{1/q}\\
&=\left(\frac{1}{2}\left(|2\xi_R^q(t)+c^q-1|+\frac{2}{\pi}\cos^{-1}[c^q]\right)\right)^{1/q}
\end{aligned}
\tag{14}
$$

Now, the part $|2\xi_R^q(t)+c^q-1|$ in Equation (14) is responsible for the minimal point. The minimal point exists at $2\xi_R^q(t)+c^q-1=0$, that is $\xi_R(t)=((1-c^q)/2)^{1/q}$.

Definition 6. To make analysis more clear and understandable, for each fixed value of π=c (constant), we define a relation $\sim_{I_R^q}$ such that

$$
\alpha\sim_{I_R^q}\beta\Leftrightarrow\pi(\alpha)=\pi(\beta).
\tag{15}
$$

Remark 2. For each fixed value of π, we have an infinite number of combinations for qROFVs. But the value of knowledge measure is not necessarily the same for two qROFVs having the same value of π. For example, (0.4,0.2) and (0.3,0.3) have same hesitancy index π=0.4. But the values of knowledge measure are 0.46901 and 0.36901, respectively. Now all such points having the same hesitancy index constitute an equivalence class. We observe the behavior of knowledge measures in particular equivalence classes.

Theorem 2. *The relation $\sim_{I_R^q}$ define in Equation (15) is an equivalence relation.*

Proof. Straightforward □

From Theorem 2, it is clear that the quotient set is defined with the help of $\sim_{I_R^q}$ and ® (set of all qROFVs). That is,

$$
K=®/\sim_{I_R^q}=\{[\alpha]_{\sim_{I_R^q}}:\alpha\in ®\}.
\tag{16}
$$

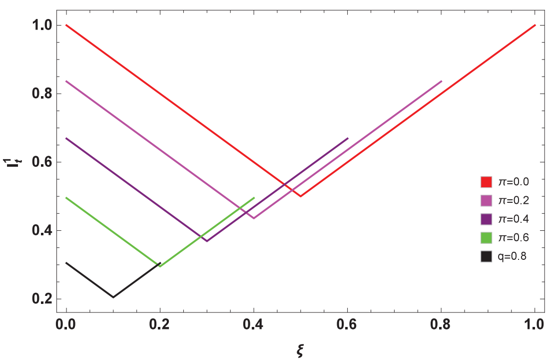

Figure 7 The shape of some classes generated by relation $\sim_{I_R^1}$ (q=1).

It is important to note that for each class $[\alpha]_{\sim_{I_R^q}} \in K$, the value of hesitancy index π is constant (fixed) for all qROFVs $\alpha \in ®$. The graphical view of some classes in K obtained by the relation $\sim_{I_R^1}$ is given in Figure 7.

It is necessary to observe the behavior of the classes in K. We have seen from Figure 7 that the minimal point of classes exists at $\xi = \nu$. Since the maximum uncertainty/ambiguity is attached when the membership degree is equal to the nonmembership degree. Also, knowledge is related to information. Therefore, it is intuitive to obtain the minimum value of knowledge measure when membership degree is equal to the non-membership degree in a class.

5. ENTROPY INDEPENDENT KNOWLEDGE MEASURE

In this section, the analytical and numerical proofs are provided to prop the entropy independent knowledge measure. Entropy for qROFSs was defined by Peng and Liu [37] and discussed twelve different models that fulfilled the required properties for entropy measure. To discuss the relation between knowledge and entropy measures for qROFSs, let us recall the axioms for entropy measure presented in [37].

Definition 7. The entropy measure E^q of a qROFS in X is a function from a qROFS to a unit interval, that is, E^q:qROFS $\rightarrow [0,1]$, which satisfies the following properties:

E1. $E^q(R) = 0$, iff R is a crisp set.

E2. $E^q(R) = 1$, iff $\xi_R(t_i) = \nu_R(t_i)$, $\forall t_i \in X$.

E3. $E^q(R) \leq E^q(S)$, iff R is less fuzzy than S, that is, $\xi_R(t_j) \leq \xi_S(t_j) \leq \nu_S(t_j) \leq \nu_R(t_j)$ or $\nu_R(t_j) \leq \nu_S(t_j) \leq \xi_S(t_j) \leq \xi_R(t_j)$.

E4. $E^q(R) = E^q(R^c)$, where R^c represents the complement of R.

Axioms E1 and E4 in Definition 5 are closely related to the axioms K1 and K4 in Definition 3. The Axioms E2 and K2 are significantly different from each other, like if we consider the knowledge measure as a dual of entropy then it attains value one whenever $\xi_R(t_i)=\nu_R(t_i)$. It means entropy or its dual can't differentiate among them. But I_c^q has ability to differentiate between them, that is, the information content plays an important role to measure the knowledge when $\xi_R(t_i)=\nu_R(t_i)$.

Now, the conditions in axiom E3 implies that $|\xi_R^q(t_j) - \nu_R^q(t_j)| \geq |\xi_S^q(t_j) - \nu_S^q(t_j)|$. Whenever, $\left(\xi_R^q(t_j) + \nu_R^q(t_j)\right) \geq \left(\xi_S^q(t_j) + \nu_S^q(t_j)\right)$ then both measures behave as a dual (Theorem 3). For example, if we consider the two single element qROFSs R and S defined as R=(0.3,0.6) and S=(0.4,0.4). Then $\xi_R(t_j) \leq \xi_S(t_j) \leq \nu_S(t_j) \leq \nu_R(t_j)$ with $\left(\xi_R^q(t_j) + \nu_R^q(t_j)\right) \geq \left(\xi_S^q(t_j) + \nu_S^q(t_j)\right)$. The results are $E^1(R) = 0.571429 \leq E^1(S) = 1$ and $I_c^1(R)$=0.618116 $\geq I_c^1(S)$=0.435906. Thus the qROFS with less entropy carries higher value of knowledge measure.

On the other hand if $|\xi_R^q(t_j) - \nu_R^q(t_j)| \geq |\xi_S^q(t_j) - \nu_S^q(t_j)|$ with $\left(\xi_R^q(t_j) + \nu_R^q(t_j)\right) \leq \left(\xi_S^q(t_j) + \nu_S^q(t_j)\right)$ then it is not necessary that a qROFS with less entropy carries a higher value of knowledge measure. For example, if we consider the two single element qROFSs R and S defined as R=(0,0.1) and S=(0.1,0.1). Then $\xi_R(t_j) \leq \xi_S(t_j) \leq \nu_S(t_j) \leq \nu_R(t_j)$ with $\left(\xi_R^q(t_j) + \nu_R^q(t_j)\right) \leq \left(\xi_S^q(t_j) + \nu_S^q(t_j)\right)$. The results of entropy and knowledge measures are $E^1(R) = 0.9 \leq E^1(S) = 1$ and $I_c^1(R) = 0.193566 \leq I_c^1(S) = 0.204833$, respectively. This means it is possible that a qROFS with less entropy carries less knowledge measure. Thus the proposed knowledge measure is independent of entropy measure.

6. APPLICATION IN MCGDM PROBLEMS

This section is devoted to the application of knowledge measure in MCGDM problems. The confidence q-rung orthopair fuzzy Einstein weighted averaging (CqREWA) operator, decision-making process, CRITIC method are discussed in this section. At the end, a step-wise procedure for MCGDM problems is discussed.

6.1. q-Rung Orthopair Fuzzy Einstein-Weighted Averaging Operator Under Confidence Level

Besides these achievements under the generalizations of a fuzzy environment, few existing approaches consolidate the familiarity degree in the information fusion step. The specialist in an MCDM problem assesses the alternatives based on the mentioned criteria only, that is, the familiarity (called confidence levels) of the specialist with the evaluation objects is not incorporated in most of the existing studies. So, it is a must to include the familiarity of an expert in the original information. Xia *et al.* [53] formulated the induced aggregation operators for fuzzy and hesitant fuzzy sets. The aggregation operators under the confidence level for IFSs were considered by Yu [54]. Garg [55] expounded the confidence levels based on Pythagorean fuzzy aggregation operators. The qROF aggregation operators under confidence level were proposed by Joshi and Gegov [56].

Yu [54] and Rahman *et al.* [57] discussed the confidence intuitionistic fuzzy Einstein weighted averaging operators and generalized confidence intuitionistic fuzzy Einstein hybrid averaging/geometric operators, respectively. They discussed the closure property with respect to their environments. So, we will extend their approach to qROFSs and define CqREWA operator. Since, the operator CqREWA is a natural extension of them, so it definitely closed with respect to qROFSs. The details of the proposed aggregation operator and geometric and hybrid aggregation operators will be discuss in another paper.

Table 1 A linguistic ratings and their corresponding.

Linguistic Variables	Corr. qROFVs	Linguistic Variables	Corr. qROFVs
EH	(0.9, 0.1)	L	(0.4, 0.6)
VH	(0.8, 0.2)	VL	(0.25, 0.75)
H	(0.7, 0.3)	EL	(0.1, 0.9)
F	(0.5, 0.5)		

Definition 8. Let $\rho_i = (\xi_i, \nu_i) 1 \le i \le m$, are the qROFVs with λ_i as the confidence level of ρ_i with $0 \le \lambda_i \le 1$. If ϖ_i, $1 \le i \le m$ are the weights of the qROFVs with $\varpi_j \ge 0$ and $\sum_{i=1}^{m} \varpi_i = 1$. Then the CqREWA operator under confidence level is define as follows:

$$
\begin{aligned}
&\mathrm{CqREWA}_{\varpi}\left(\left(\rho_1, \lambda_1\right), \left(\rho_2, \lambda_2\right), \ldots, \left(\rho_m, \lambda_m\right)\right) \\
&= \oplus_{i=1}^{m} \varpi_i \left(\lambda_i \rho_i\right) \\
&= \left(\left[\frac{\prod_{i=1}^{m} \left(1+\xi_i^q\right)^{\lambda_i \varpi_i} - \prod_{i=1}^{m} \left(1-\xi_i^q\right)^{\lambda_i \varpi_i}}{\prod_{i=1}^{m} \left(1+\xi_i^q\right)^{\lambda_i \varpi_i} + \prod_{i=1}^{m} \left(1-\xi_i^q\right)^{\lambda_i \varpi_i}} \right]^{1/q}, \right. \\
&\qquad \left. \frac{(2)^{1/q} \prod_{i=1}^{m} \left(\nu_i\right)^{\lambda_i \varpi_i}}{\left[\prod_{i=1}^{m} \left(2-\nu_i^q\right)^{\lambda_i \varpi_i} + \prod_{i=1}^{m} \left(\nu_i^q\right)^{\lambda_i \varpi_i}\right]^{1/q}} \right)
\end{aligned} \tag{17}
$$

6.2. Decision-Making Process

The aim of decision making process is to select the favorite alternative on the basis of the criteria defined by the experts. In decision-making process, let $X=\{t_1, t_2, \ldots, t_m\}$ be the m alternatives which are assessed against n attributes (criteria) represented as $E=\{e_1, e_2, \ldots, e_n\}$. Each alternative t_i evaluated with respect to each criteria e_j and the evaluated values are saves in the form of linguistic variables. Then linguistic information converted into qROF information by seven point scale given in Table 1. The qROF decision matrix $M=[\rho_{ij}]_{m \times n}$, where ρ_{ij} represents the evaluation of i^{th} alternative against j^{th} criteria. The qROF decision matrix $M=[\rho_{ij}]_{m \times n}$ can be represented as follows:

$$
\begin{aligned}
M = \left[\rho_{ij}\right]_{m \times n} &= \begin{pmatrix} \rho_{11} & \rho_{12} & \cdots & \rho_{1n} \\ \rho_{21} & \rho_{22} & \cdots & \rho_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} & \rho_{m2} & \cdots & \rho_{mn} \end{pmatrix} \\
&= \begin{pmatrix} \left(\xi_{11}, \nu_{11}\right) & \left(\xi_{12}, \nu_{12}\right) & \cdots & \left(\xi_{1n}, \nu_{1n}\right) \\ \left(\xi_{21}, \nu_{21}\right) & \left(\xi_{22}, \nu_{22}\right) & \cdots & \left(\xi_{2n}, \nu_{2n}\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\xi_{m1}, \nu_{m1}\right) & \left(\xi_{m2}, \nu_{m2}\right) & \cdots & \left(\xi_{mn}, \nu_{mn}\right) \end{pmatrix}
\end{aligned} \tag{18}
$$

6.3. CRITIC Method

Diakoulaki *et al.* [58] have proposed the CRITIC (The Criteria Importance Through Intercriteria Correlation) method that uses correlation analysis to detect contrasts between criteria and standard deviation, which quantifies the contrast intensity of the corresponding criterion [59]. The method developed is based on the analytical investigation of the evaluation matrix for extracting all information contained in the evaluation criteria [59]. The CRITIC method reflects the inner information of data transmission and the approximation of subjective weight to some extent. Thus Yalcin and Unlu [60] used CRITIC method to obtain weights for IPO performance analysis, Tus and Adali [61] used for attendance software assessment, and Peng *et al.* [62] used to obtain weights for 5G industry evaluation.

The weights of criteria are formulated by CRITIC method by following steps:

Step 1: Calculate the knowledge measure $A=[a_{ij}]_{m \times n}$ of each qROFV $\rho_{ij}=(\xi_{ij}, \nu_{ij})$ by Equation (19).

$$
\begin{aligned}
a_{ij} = \Bigg(&\frac{1}{2} \Big(|\xi_{ij}^q - \nu_{ij}^q| + \\
&\frac{2}{\pi} \cos^{-1} \left[1 - \left(\xi_{ij}^q + \nu_{ij}^q \right) \right] \Big) \Bigg)^{1/q}.
\end{aligned} \tag{19}
$$

Step 2: Normalize the matrix A by using Equation (20) and represented as $A'=[a'_{ij}]_{m \times n}$.

$$
a'_{ij} = \frac{a_{ij} - \min_i a_{ij}}{\max_i a_{ij} - \min_i a_{ij}} \tag{20}
$$

Step 3: The standard deviations for each criterion are calculated by Equation (21).

$$
\Theta_j = \sqrt{\frac{\sum_{i=1}^{m} \left(a'_{ij} - \hat{a}_j\right)^2}{m}}, \tag{21}
$$

where $\hat{a}_j = \frac{\sum_{i=1}^{m} a'_{ij}}{m}$.

Step 4: Correlation coefficients between each criterion are calculated by Equation (22) which constitute the correlation coefficient matrix $B=[b_{ij}]_{n \times n}$.

$$
b_{jk} = \frac{\sum_{i=1}^{m} (a'_{ij} - \hat{a}_j) \sum_{i=1}^{m} (a'_{ik} - \hat{a}_k)}{\sqrt{\sum_{i=1}^{m} (a'_{ij} - \hat{a}_j)^2 \sum_{i=1}^{m} (a'_{ik} - \hat{a}_k)^2}}. \tag{22}
$$

Step 5: The quantity of information of each criterion are computed using standard deviation and correlation coefficient as follows:

$$
\Upsilon_j = \Theta_j \sum_{k=1}^{n} (1 - b_{jk}). \tag{23}
$$

Step 6 The weights of criteria are based on quantity of information and determine by Equation (24) as follows:

$$
\varpi_j = \frac{\Upsilon_j}{\sum_{j=1}^{n} \Upsilon_j}. \tag{24}
$$

6.4. Proposed MCGDM Method

To understand the whole procedure of the MCGDM method, we divided into 6 steps.

Let $D = \{d_1, d_2, \ldots, d_r\}$ be the group of experts/decision makers with weights τ_ℓ, $1 \leq \ell \leq r$. Individual expert evaluates each alternative against criteria and give his preferences in the form of qROFV which composed the qROF decision matrix $M^\ell = [\rho_{ij}^\ell]_{m\times n} = (\xi_{ij}^\ell, \nu_{ij}^\ell)_{m\times n}$. The confidence level λ for each qROFV is calculated by using knowledge measure I_c^q. The qROF decision matrix M^ℓ is converted to $N^\ell = ((\xi_{ij}^\ell, \nu_{ij}^\ell), \lambda_{ij}^\ell)_{m\times n}$ by including the confidence level values λ_{ij}^ℓ for each qROFV $(\xi_{ij}^\ell, \nu_{ij}^\ell)$.

Step 2: Normalize the qROF decision matrix N^ℓ for each expert according to the benefit and cost criteria as follows:

$$\rho_{ij}^\ell = \begin{cases} (\xi_{ij}^\ell, \nu_{ij}^\ell), & \textit{for benefit criteria}, \\ (\nu_{ij}^\ell, \xi_{ij}^\ell), & \textit{for cost criteria}. \end{cases} \tag{25}$$

Since the knowledge measure I_c^q is symmetric, therefore, the value of confidence level for each qROFV remains unchanged.

Step 3: The information from qROF decision matrices N^ℓ, $1 \leq \ell \leq r$ is aggregated by using CqREWA operator (Equation (17)) and constitutes a single qROF decision matrix $N = [\sigma_{ij}]_{m\times n} = (\xi_{ij}, \nu_{ij})_{m\times n}$, where σ_{ij} are calculated as follows:

$$\begin{aligned} \sigma_{ij} &= \oplus_{\ell=1}^r \tau_\ell \left(\lambda_{ij}^\ell \rho_{ij}^\ell\right) = \\ &\left(\left[\frac{\prod_{\ell=1}^r \left(1 + \left(\xi_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell} - \prod_{\ell=1}^r \left(1 - \left(\xi_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell}}{\prod_{\ell=1}^r \left(1 + \left(\xi_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell} + \prod_{\ell=1}^r \left(1 - \left(\xi_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell}} \right]^{1/q} \right. \\ &\left. , \frac{(2)^{1/q} \prod_{\ell=1}^r \left(\nu_{ij}^\ell\right)^{\lambda_{ij}^\ell \tau_\ell}}{\left[\prod_{\ell=1}^r \left(2 - \left(\nu_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell} + \prod_{\ell=1}^r \left(\left(\nu_{ij}^\ell\right)^q\right)^{\lambda_{ij}^\ell \tau_\ell} \right]^{1/q}} \right) \\ &= (\xi_{ij}, \nu_{ij}). \end{aligned} \tag{26}$$

Step 4: Weights (ϖ_j, $1 \leq j \leq n$, $\varpi_j \geq 0$ and $\sum_{j=1}^n \varpi_j = 1$) are calculated by using CRITIC method as described in Section 6.3. CRITIC method use information aggregated in qROF decision matrix N.

Step 5: Again, the CqREWA operator is use to aggregate the information from qROF decision matrix N with weights obtained in previous step and qROFVs χ_i, $1 \leq i \leq m$ are obtain as follows:

$$\begin{aligned} \chi_i &= \oplus_{j=1}^n \left(\varpi_j \sigma_{ij}\right) = \\ &\left(\left[\frac{\prod_{j=1}^n \left(1 + \xi_{ij}^q\right)^{\varpi_j} - \prod_{j=1}^n \left(1 - \xi_{ij}^q\right)^{\varpi_j}}{\prod_{j=1}^n \left(1 + \xi_{ij}^q\right)^{\varpi_j} + \prod_{j=1}^n \left(1 - \xi_{ij}^q\right)^{\varpi_j}} \right]^{1/q}, \right. \\ &\left. \frac{(2)^{1/q} \prod_{j=1}^n \left(\nu_{ij}\right)^{\varpi_j}}{\left[\prod_{j=1}^n \left(2 - \nu_{ij}^q\right)^{\varpi_j} + \prod_{j=1}^n \left(\nu_{ij}^q\right)^{\varpi_j} \right]^{1/q}} \right) \end{aligned} \tag{27}$$

Step 6: Rank the qROFVs χ_i, $1 \leq i \leq m$ and optimal value $Opt(\chi_i)$ corresponds to the optimal alternative $Opt(t_i)$. To rank the alternatives t_i, the graphical method for ordering the qROFVs based on the uncertainty index and entropy is used. The graphical method for ranking the qROFVs was proposed by Khan *et al.* [63], where we have seen that the most of the already proposed ranking methods are not locally orthodox criterion.

6.5. Renewable Energy Source Selection by Proposed Method

The dependence on imported sources are very much reduced and the security of supply are provided by renewable energy (RE). RE addressed our energy needs by replacing foreign energy imports with reliable and clean home-grown electricity. Also, RE added bonus of fantastic local economic opportunities. By using RE instead of fossil fuels, we would significantly decrease the current levels of greenhouse gas emissions, and this would have positive environmental impact for our entire planet. RE is not all about environment as it can also give strong boost to our economy in form of new jobs. RE often referred to as clean energy, comes from natural sources or processes that are constantly replenished. For example, sunlight or wind keep shining and blowing, even if their availability depends on time and weather. Many studies have made to analyze and prioritize the RE sources for a region. These studies were based on some suitable MCGDM methods. Interested reader can find the relevant studies in [64,65]. We discuss here a general method to select and prioritize the RE source against the prescribed criteria.

Let $X = \{t_1, t_2, t_3, t_4, t_5\} = \{$ wind energy, biomass energy, tidal energy, solar energy, hydro-power$\}$ represents five renewable energy sources (alternatives) and the set E represents the criteria, where $E = \{e_1, e_2, e_3, \ldots, e_{12}\} = \{$ water pollution, need for waste disposal, air pollutant emissions, land requirement, economic risk, security, sustainable energy, land disruption, durability, adaptability to energy policy, cost, feasibility$\}$. The qROFV is given to each alternative t_i, $1 \leq i \leq 5$ on the basis of each criterion e_j, $1 \leq j \leq 12$. Let $D = \{d_1, d_2, d_3\}$ be the group of three experts/ decision-makers with weight vector $\Upsilon = \{\Upsilon_1, \Upsilon_2, \Upsilon_3\}^T = \{0.3, 0.3, 0.4\}^T$. To understand well, we follow the procedure discussed above.

Step 1 & 2: Each individual decision maker d_ℓ makes his/her assessment of alternatives against criteria and give their preferences in the form of linguistic variable. The information in the form of linguistic variables are presented in Tables 2–4. Table 1 is used to convert information from linguistic to qROFVs and obtained qROF decision matrices $M^\ell = [\rho_{ij}^\ell]_{5\times12} = (\xi_{ij}^\ell, \nu_{ij}^\ell)_{5\times12}$. The confidence level λ for each qROFV is calculated by using knowledge measure I_c^3(q=3). The qROF decision matrix M^ℓ is converted into $N^\ell = ((\xi_{ij}^\ell, \nu_{ij}^\ell), \lambda_{ij}^\ell)$ by including the confidence level values and presented in Tables 5–7. We supposed that the matrices obtained are normalized, that is each criterion deals as benefit criteria. The confidence level λ_{ij}^ℓ for each qROFV is calculated by using knowledge measure I_c^3 for q=3 as follows:

$$\lambda_{ij}^\ell = I_c^3\left(\rho_{ij}^\ell\right) = \left(\frac{1}{2}\left(\mid \xi_{\rho_{ij}^\ell}^3 - \nu_{\rho_{ij}^\ell}^3 + \frac{2}{\pi}\cos^{-1}\left[1 - \left(\xi_{\rho_{ij}^\ell}^3 + \nu_{\rho_{ij}^\ell}^3\right)\right]\right)\right)^{1/3}$$

Table 2 Performance comparison with state-of-the-art methods. The number in red indicates the best result and the number in blue indicates the second best result.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	VH	F	H	EH	F	EH	VH	F	H	EL	F	EH
t_2	L	F	VH	H	EH	H	L	F	VH	H	EH	H
t_3	EH	L	F	VH	VL	VH	EH	L	F	VH	VL	VH
t_4	H	F	VH	L	H	F	H	F	VH	L	H	F
t_5	F	H	VH	VL	VH	F	F	H	VH	VL	VH	VL

Table 3 Associated linguistic information by d_2 expert.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	F	F	H	H	F	EH	VH	F	H	L	F	EH
t_2	EL	F	VH	H	H	H	L	F	VH	H	F	H
t_3	H	L	F	H	VL	VH	EH	L	F	H	VL	H
t_4	H	H	H	L	H	EH	H	F	VH	VL	H	F
t_5	H	L	VH	L	VH	F	F	H	VH	L	VH	VL

For example,

$$\lambda_{11}^1 = I_c^3\left(\rho_{11}^1\right)$$
$$= \left(\frac{1}{2}\left(|\xi^3_{\rho^1_{11}} - \nu^3_{\rho^1_{11}}| + \frac{2}{\pi}\cos^{-1}\left[1-\left(\xi^3_{\rho^1_{11}} + \nu^3_{\rho^1_{11}}\right)\right]\right)\right)^{1/3}$$
$$= \left(\frac{1}{2}\left(|0.8^3 - 0.2^3| + \frac{2}{\pi}\cos^{-1}\left[1-\left(0.8^3+0.2^3\right)\right]\right)\right)^{1/3}$$
$$= 0.82.$$

Similarly, we calculate the confidence levels of all qROFVs.

Step 3: To aggregate the information from each qROF decision matrix N^{ℓ}, (ℓ=1,2,3), CqREWA operator defined in Equation (26) is used. A single qROF decision matrix $N=[\sigma_{ij}]_{5\times12}=(\xi_{ij},\nu_{ij})_{5\times12}$ is obtain and display in Table 8.

Step 4: Information in Table 8 is used to calculate weights by CRITIC method described in Section 6.3 and the weights are as follows:

$$\varpi_1 = 0.0693, \varpi_2 = 0.0676, \varpi_3 = 0.1127,$$

$$\varpi_4 = 0.0753, \varpi_5 = 0.0900, \varpi_6 = 0.0688,$$

$$\varpi_7 = 0.0652, \varpi_8 = 0.0695, \varpi_9 = 0.1370,$$

$$\varpi_{10} = 0.0817, \varpi_{11} = 0.0957, \varpi_{12} = 0.0672.$$

Step 5: Again, the CqREWA operator define in Equation (27) is use to aggregate the information from qROF decision matrix N with

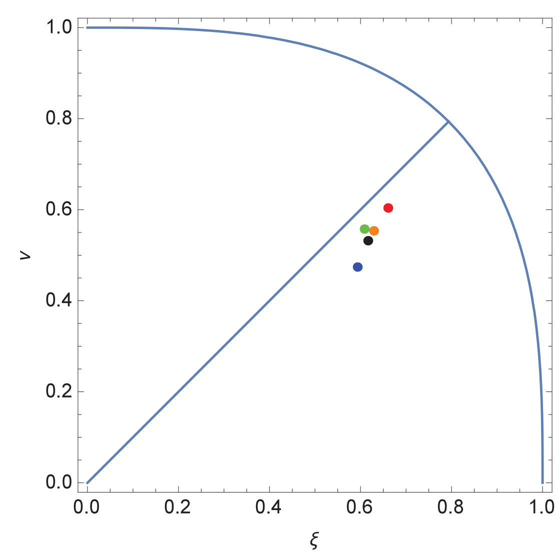

Figure 8 Position of the associated qROFVs

weights obtained in previous step and qROFVs χ_i, $1 \leq i \leq m$ are obtain as follows:

$$\chi_1 = (0.6165, 0.5319), \chi_2 = (0.6604, 0.6038),$$

$$\chi_3 = (0.5933, 0.4740), \chi_4 = (0.6085, 0.5575),$$

$$\chi_5 = (0.6292, 0.5536).$$

Step 6: We used the graphical ranking method to rank the qROFVs. Figure 6 shows that all qROFVs are below the equal line and thus rank based on the hesitancy index. The qROFVs with less hesitancy index ranked highest. The hesitancy degrees of qROFVs are as follows:

$$\pi(\chi_1) = 0.8505,\ \pi(\chi_2) = 0.7893,\ \pi(\chi_3) = 0.881,$$

Table 4 Associated linguistic information by d_2 expert.

	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	VH	F	F	EH	F	H	VH	L	H	EL	F	H
t_2	H	F	H	H	EH	H	L	F	F	H	EH	H
t_3	H	F	F	VH	L	H	EH	L	F	VH	VL	VH
t_4	VH	F	H	L	H	F	H	F	VH	F	H	F
t_5	H	H	VH	L	VH	F	F	H	VH	VL	L	L

Table 5 Converted qROFVs information for d_1 expert (N^1).

	e_1	e_2	e_3	e_4	e_5	e_6
t_1	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)	((0.5, 0.5), 0.61)	((0.9, 0.1), 0.92)
t_2	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)	((0.7, 0.3), 0.76)
t_3	((0.9, 0.1), 0.92)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)
t_4	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)
t_5	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)
	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.1, 0.9), 0.92)	((0.5, 0.5), 0.61)	((0.9, 0.1), 0.92)
t_2	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)	((0.7, 0.3), 0.76)
t_3	((0.9, 0.1), 0.92)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)
t_4	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)
t_5	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)

Table 6 Converted qROFVs information for d_2 expert (N^2).

	e_1	e_2	e_3	e_4	e_5	e_6
t_1	((0.5, 0.5), 0.61)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.9, 0.1), 0.92)
t_2	((0.1, 0.9), 0.92)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)
t_3	((0.7, 0.3), 0.76)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)
t_4	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)
t_5	((0.7, 0.3), 0.76)	((0.4, 0.6), 0.68)	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)
	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.9, 0.1), 0.92)
t_2	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)
t_3	((0.9, 0.1), 0.92)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.25, 0.75), 0.80)	((0.7, 0.3), 0.76)
t_4	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)
t_5	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)

$$\pi(\chi_4) = 0.8441,\ \pi(\chi_5) = 0.8346$$

$$\pi(\chi_2) < \pi(\chi_5) < \pi(\chi_4) < \pi(\chi_1) < \pi(\chi_3)$$

$$t_2 \succ t_5 \succ t_4 \succ t_1 \succ t_3$$

Remark 3. It can be easily observed from Table 9 that optimal alternative remain same for all values of parameter q. However, there is a slightly change in the developed ranking for $q = 1$. Table 9 contains the hesitancy values for aggregated qROFVs.

7. COMPARISON ANALYSIS

This paper provides us the improved knowledge measure for qROFSs. Khan *el al.* [52] was first discussed the knowledge measures for qROFSs. According to them, the knowledge measure decreases with higher values of the hesitancy index, that is, the maximum information content leads to maximum knowledge. But they have not considered the information clarity facets of knowledge measure. But, as we discussed in Section 3, our proposed knowledge measure considers both perspectives of knowledge measure, that is, information content and information clarity. Hence the proposed approach is better than the previous one.

The entropy for qROFSs was discussed by Peng and Liu [37] and Verma [22]. The maximum entropy is obtained when the membership degree is equal to the nonmembership degree, that is, $\xi = v$. So, in these complex situations, the entropy alone can't handle the situation. Thus it is necessary and significant work to develop an independent technique with robust properties to take measurements of the amount of knowledge in the context of qROFSs to distinguish between them. From Remark 1, we have seen that when the membership degree is equal to the nonmembership

Table 7 Converted qROFVs information for d_3 expert (N^3).

	e_1	e_2	e_3	e_4	e_5	e_6
t_1	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.5, 0.5), 0.61)	((0.9, 0.1), 0.92)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)
t_2	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)	((0.7, 0.3), 0.76)
t_3	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)
t_4	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)
t_5	((0.7, 0.3), 0.76)	((0.7, 0.3), 0.76)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)
	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	((0.8, 0.2), 0.84)	((0.4, 0.6), 0.68)	((0.7, 0.3), 0.76)	((0.1, 0.9), 0.92)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)
t_2	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.9, 0.1), 0.92)	((0.7, 0.3), 0.76)
t_3	((0.9, 0.1), 0.92)	((0.4, 0.6), 0.68)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.8, 0.2), 0.84)
t_4	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)	((0.8, 0.2), 0.84)	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.5, 0.5), 0.61)
t_5	((0.5, 0.5), 0.61)	((0.7, 0.3), 0.76)	((0.8, 0.2), 0.84)	((0.25, 0.75), 0.80)	((0.4, 0.6), 0.68)	((0.4, 0.6), 0.68)

Table 8 Aggregated information (Matrix N).

	e_1	e_2	e_3	e_4	e_5	e_6
t_1	(0.70, 0.36)	(0.43, 0.68)	(0.58, 0.51)	(0.84, 0.18)	(0.43, 0.68)	(0.82, 0.20)
t_2	(0.50, 0.64)	(0.43, 0.68)	(0.72, 0.32)	(0.64, 0.42)	(0.84, 0.18)	(0.64, 0.42)
t_3	(0.75, 0.29)	(0.38, 0.71)	(0.43, 0.68)	(0.73, 0.31)	(0.29, 0.77)	(0.72, 0.32)
t_4	(0.69, 0.36)	(0.51, 0.59)	(0.68, 0.37)	(0.35, 0.73)	(0.64, 0.42)	(0.38, 0.70)
t_5	(0.59, 0.49)	(0.64, 0.43)	(0.74, 0.3)	(0.28, 0.78)	(0.74, 0.3)	(0.38, 0.72)
	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}
t_1	(0.76, 0.27)	(0.40, 0.70)	(0.64, 0.42)	(0.24, 0.85)	(0.43, 0.68)	(0.82, 0.20)
t_2	(0.35, 0.73)	(0.43, 0.68)	(0.67, 0.40)	(0.64, 0.42)	(0.82, 0.21)	(0.64, 0.42)
t_3	(0.88, 0.12)	(0.35, 0.73)	(0.43, 0.68)	(0.73, 0.31)	(0.23, 0.80)	(0.73, 0.31)
t_4	(0.64, 0.42)	(0.43, 0.68)	(0.76, 0.27)	(0.36, 0.73)	(0.64, 0.42)	(0.43, 0.68)
t_5	(0.49, 0.63)	(0.60, 0.47)	(0.76, 0.27)	(0.23, 0.81)	(0.63, 0.45)	(0.37, 0.71)

Table 9 Sensitivity analysis.

q	$\pi(\chi_1)$	$\pi(\chi_2)$	$\pi(\chi_3)$	$\pi(\chi_4)$	$\pi(\chi_5)$	Developed Ranking
1	0.10617	0.01938	0.14110	0.14645	0.10162	$t_2 \succ t_5 \succ t_1 \succ t_3 \succ t_4$
2	0.65516	0.54507	0.70729	0.64501	0.61932	$t_2 \succ t_5 \succ t_4 \succ t_1 \succ t_3$
3	0.85052	0.78930	0.88100	0.84412	0.83461	$t_2 \succ t_5 \succ t_4 \succ t_1 \succ t_3$
4	0.92558	0.88906	0.94338	0.92204	0.91676	$t_2 \succ t_5 \succ t_4 \succ t_1 \succ t_3$
5	0.95887	0.93592	0.96909	0.95738	0.95383	$t_2 \succ t_5 \succ t_4 \succ t_1 \succ t_3$

degree, the information clarity becomes zero and the knowledge is measured from an information content perspective. The higher value of knowledge measure is attached to higher value information content. Hence our proposed knowledge measure is appropriate for such situations.

8. CONCLUSION

In this paper, the knowledge measure for qROFSs is discussed with respect to information clarity and information content. Both facets of knowledge are used to quantify the knowledge from associated qROFSs. The knowledge measure is monotonically nonincreasing with respect to hesitancy index. The maximum and minimum values of knowledge measure are attached at crisp and (0,0) cases, that is, the knowledge is maximum at sure information and minimum when there is not information. For constant values of hesitancy index (say c), the equivalence classes are generated and each class has a minimum at $\xi=\nu=(\frac{1-c^q}{2})^{1/q}$. It has been shown that the knowledge measure is entropy independent. At the end, a MCGDM method has established based on CqREWA operator under confidence level, CRITIC method and knowledge measure. The proposed approach is effective only when the value of qis less than or equal to 10. When the value of qincreases from 10the knowledge measures approaches to zero from both the ξ-axis and ν-axis. That is the knowledge measure approaches zero around the neighborhood of the point (0,0). In the future, we will apply the proposed method for ranking and image processing, and so on.

CONFLICT OF INTEREST

The authors declare that they have no conflict of interest.

AUTHORS' CONTRIBUTIONS

All authors contribute equally for this study.

COMPLIANCE WITH ETHICAL STANDARDS

This article does not contain any studies with human participants or animals performed by any of the authors.

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