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RESEARCH ARTICLE

An efficient alternative approach to solve a transportation problem

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Abstract: Determination of an Initial Feasible Solution (IFS) to a transportation problem plays an important role in obtaining a minimal total transportation cost solution. Better initial feasible solution can result less number of iterations in attaining the minimal total cost solution. Recently, an efficient method denoted by JHM (Juman and Hoque's Method) was proposed to obtain a better initial feasible solution to a transportation problem. In JHM only column penalties are considered. In this paper, a new approach is proposed with row penalties to find an IFS to a transportation problem. The new method is illustrated with a numerical example. A comparative study on a set of benchmark instances shows that the new method provides the same or better initial feasible solution to all the problems except one. Thus, our new method can be considered as an alternative technique of attaining an initial feasible solution to a transportation problem.

Keywords: Transportation Problem; Initial feasible solution; minimal total cost solution.

INTRODUCTION

An essential component of our modern life is the shipping of goods from where they are produced to markets worldwide. Nationally, companies spend billions of dollars annually in transporting goods. The transportation problem (TP) is a special class of network optimization problem that deals with transporting a homogeneous product from multiple sources (e.g., factories) to multiple destinations (e.g., warehouses). The aim of the TP is to find a way of carrying out this transfer of goods at minimum total cost.

The TP was formalized by the French mathematician Gaspard Monge (1781). Major advances were made in the field during World War II by the Russian mathematician and economist Leonid Kantorovich. Tolstoi was the one of the first to study the transportation problem mathematically. An article named "Method of finding of finding the minimal total kilometrage in Cargo transportation planning in space, in the collection of transportation planning"; volume I of the national commissariat of transportation of the Soviet Union was published by Tolstoi (1930). In 1939, Leonid Kantorovich was formulated the transportation problem as linear programming problem and he published a book named "Mathematical methods in the Organization and planning of Production". After few years United States of American mathematician Frank Lauren Hitchcock (1941) has published a paper "The Distribution of a Product from Several Sources to Numerous Localities" where he formulated an algorithm for the transportation problem similar to the primal simplex algorithm. In 1949, Koopmans has presented an independent study not related to Hitchcock's, and called "Optimum utilization of transportation systems. The two contributions which developed by Hitchcock and Koopmans helped in the development of transportation methods which involve a number of plants and a number of destinations. After that Charnes, and Cooper (1954), have published the stepping stone method of explaining linear programming calculations in transportation problems. In 1951, Dantzig applied the concept of Linear Programming in solving the Transportation models. Reinfeld and Vogel (1958) have published a book named Mathematical Programming. In this book, they proposed a method to find the initial basic feasible solution for a transportation problem. Dantzig (1963) has published book named Linear Programming Extensions. Hoffman (1963) has given a necessary and sufficient condition under which a family of O(mn)-time greedy algorithms solve the classical two dimensional transportation problem with m sources and n destinations. Then Hammer (1969), Szwarc (1971), Garfinkel and Rao (1971), Bhatia, Kanti Swaroop and Puri (1976) studied about time minimizing transportation problem. A paper which is named by "Cost operator algorithms for the transportation problem" was published by Sirinivasan, and Thompson in 1977. After few years Shimshak, Kaslik and Barclay (1981) proposed a Modification of Vogel's approximation method through the use of heuristic. Moreover, Many research works have been carried out on the Vogel's approximation method (Juman and Hoque, 2012; Juman and Hoque, 2013a; Juman and Hoque, 2013b; Juman and Hoque, 2014b; Juman and Perera, 2015b). Adlakha and Kowalski (1999) proposed an alternative solution algorithm for certain transportation problems.

In the beginning of the 20th century, Veena Adlakha and Krzysztof Kowalski (2000) introduced an alternative solutions analysis for transportation problems. Also Gen, Choi and Ida (2000) improved genetic algorithm for generalized transportation problem. In the same year Sharma and Sharma have proposed a new dual based procedure for the transportation problem. In 2003, Adlakha and Kowalski introduced a simple heuristic for solving small fixed charge transportation problems. Sharma and Prasad (2003) studied about obtaining a good primal solution to the uncapacitated

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transportation problem. Winston (2004) published a book which named Operation Research; Applications and Algorithms. After few years Imam, Elsharawy, Gomahand Samy (2009) introduced a method to solving transportation problem using object oriented model. Pargar, Javadian and Ganji (2009) formulated a heuristic for obtaining an initial solution for the transportation problem with experimental analysis.

Kulkarni and Datar (2010) published a paper which is named "on solution to modified unbalanced transportation problem". In the same year Pandian and Natarajan obtained a new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems. Sen et al. (2010) published a paper named a study of transportation problem for an essential item of southern part of north eastern region of India as an OR model and use of object oriented programming. Schrenk, Finke and Cung (2011) studied about two classical transportation problems revisited: pure constant fixed charges and the paradox. Pandian and Natarajan (2011) introduced a new method for solving bottleneck-cost transportation problems. Korukoglu and Balli (2011) improved Vogel's approximation method for the transportation problem. Deshmukh (2012) presented a method for solving transportation problem. Ramadan and Ramadan (2012) proposed a hybrid two-stage algorithm for solving transportation problem. Also in 2012, Samuel improved zero point method for the transportation problems. Juman, et al. (2013) studied about a sensitivity analysis and an implementation of the well-known Vogel's approximation method for solving an unbalanced transportation problem. In 2014, Kowalski, Lev, Shen and Tu obtained a fast and simple branching algorithm for solving small scale fixed-charge transportation problem. Juman and Hoque (2014) developed a heuristic solution technique to attain the minimal total cost bounds to the transportation problem with varying demands and supplies. Juman and Hoque (2015a) presented an efficient heuristic to obtain a better initial feasible solution to the transportation problem. They demonstrated a deficiency of recently developed method and they developed the method to obtain a better initial feasible basic solution. In 2016 Speranza discussed the trends in transportation logistics.

The classical transportation problem is concerned with a set of nodes or places called plants ($S_1, S_2, S_3, \dots, S_m$) which

Table 1: Transportation Tableau.

Plants \ Destination	D_1	D_2	D_3		D_{n-1}	D_n	Supply quantity
S_1	x_{11}	x_{12}	x_{13}	…	$x_{1,m-1}$	$x_{1,n}$	s_1
S_2	x_{21}	x_{22}	x_{23}	…	$x_{2,m-1}$	$x_{2,n}$	s_2
S_3	x_{31}	x_{32}	x_{33}	…	$x_{3,m-1}$	$x_{3,n}$	s_3
	.	.	.		.	.	.
	.	.	.		.	.	.
	.	.	.		.	.	.
S_{m-1}	$x_{m-1,1}$	$x_{m-1,2}$	$x_{m-1,3}$	…	$x_{m-1,n-1}$	$x_{m-1,n}$	s_{m-1}
S_m	$x_{n,1}$	$x_{n,2}$	$x_{n,3}$	…	$x_{m,n-1}$	$x_{m,n}$	s_m
Demand quantity	d_1	d_2	d_3	…	d_{n-1}	d_n	

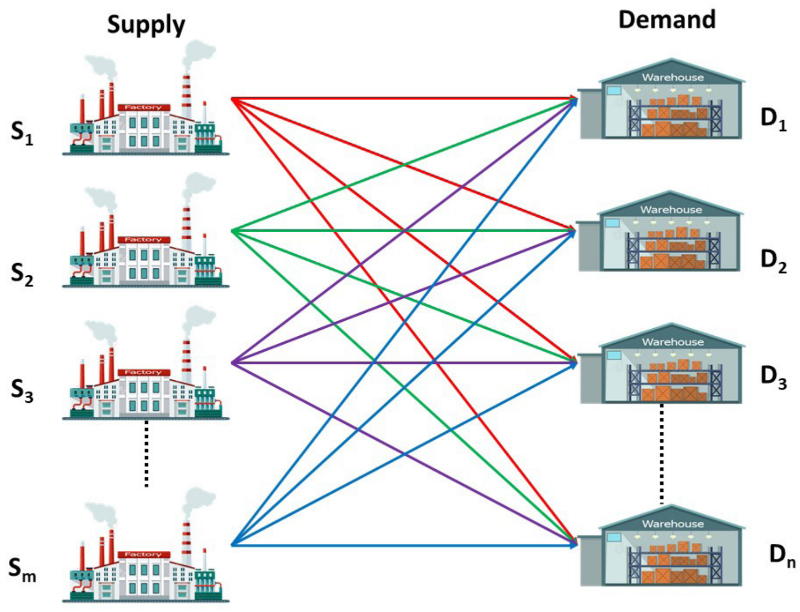

Figure 1: Network representation of general transportation problem

have a commodity available for shipment, and another set of places called destinations ($D_1, D_2, D_3, \ldots, D_n$) which require this commodity. The data consists of availability at each plant ($s_1, s_2, s_3, \ldots, s_m$), the requirement at each destination ($d_1, d_2, d_3, \ldots, d_n$), and the cost of transporting the commodity per unit from each plant to each destination, C_{ij}. The problem is to determine the quantity to be transported from each plant to each destination, x_{ij}, so as to meet the requirements at minimum total shipping cost. The tableau (Table 1) and the network diagram (Figure 1) of the transportation problem are provided.

Mathematical Formulation for a classical Transportation Problem

In this section we further discuss about the mathematical formulation of the transportation problem (TP). The following notations are used in formulating the TP.

<u>Notations</u>

s_i Supply quantity (in units) from i^{th} supply node

d_j The demand in units per unit time

c_{ij} Unit transportation cost from i^{th} supply node to j^{th} demand node

x_{ij} Number of units transported from i^{th} supply node to j^{th} demand node

m Total number of supply nodes (suppliers)

n Total number of demand nodes (buyers)

The basic problem (sometimes called as the general, classical or Hitchcock transportation problem) can be stated mathematically as follows.

$$\left.\begin{aligned} &\text{Min } \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \\ &\text{Subject to } \sum_{j=1}^{n} x_{ij} \le s_i, \quad i=1,2,\ldots,m \\ &\qquad\qquad \sum_{i=1}^{m} x_{ij} \ge d_j, \quad j=1,2,\ldots,n \\ &\qquad\qquad \text{where } x_{ij} \ge 0 \ \forall\, i, j. \end{aligned}\right\} \quad (1)$$

A necessary and sufficient condition for the existence of a feasible solution to the transportation problem is;

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

This means that the total supply is equal to total demand. Then the Transportation problem is called as a Balanced Transportation Problem. If not the problem is called as an Unbalanced Transportation Problem. Although, TP can be solved by the simplex algorithm, the specialized algorithm for this problem is much more efficient due to its special structure.

Methods of solving a Transportation Problem

There are several methods to find the initial feasible solution for a transportation problem. Few of them are

- North – West Corner Rule (2004)
- Minimum Cost Method (2004)
- Vogel's Approximation Method (1958)
- Juman and Hoque Method (2015)

After finding the Initial Feasible Solution (IFS) for a transportation problem using one of the above methods, a minimal total cost solution technique can be applied to attain the Minimum Total Cost Solution (MTCS).

Minimum Total Cost Solution to the Transportation problem

The transportation problem is solved by using transportation algorithm. The transportation algorithm consists with two steps.

Step 1: Determine an initial feasible solution (IFS) for the problem

Step 2: Test for optimality of the IFS by the well-known Stepping Stone method or Modified Distribution method

Objective of this Research

The main objective of this research is to develop an alternative method especially, for the Juman and Hoque (2015)'s Method of finding an Initial Feasible Solution for a balanced transportation problem.

METHODOLOGY

An Alternative Method to find an IFS for a TP

We have here developed a New Method (NM) to obtain an efficient IFS (optimum or very near optimum) to solve the conventional transportation problem. The steps involved in NM in producing the initial feasible solution are described below:

Algorithm for New Method

Step 1: Check whether the problem is balanced or not. If it is unbalanced, then add dummy demander or dummy supplier to make the problem balanced with zero transportation cost.

Step 2: For each row of the transportation matrix, identify the least cost cell. Assign the respective supply quantities there.

Step 3: For assigned allocation in each of columns (without taking into account any crossed column, if exist) check whether the column sum is less than or equal to the respective demand quantity. If so, go to step 10

Step 4: for each of the allocations in an unmet column considering the row containing that allocation determine the difference between the second least and the least unit cost, and identify the smallest of them (in case of tie, identify the smallest with the largest unit cost) Else identify the smallest difference for each of them separately and go to step 5

Step 5: check whether there exist a cell (or cells) in an unmet column not containing the second least unit cost corresponding to the smallest of the difference between the second least and the least unit cost in a row for each of the allocation in an another unmet column. If such a column exists, identify the former unmet column go to step 8

Step 6: pick up any two unmet columns. For each of them, find differences between the 2nd least and the least unit cost of that unmet column.

Let the smallest of the differences for an unmet column corresponds to the least unit cost$c_1 c_1$, and the smallest of the differences for the other unmet column corresponds to the smallest unit cost $e_1 e_1$. Let c_1 c_1, c_2 c_2 and $c_3 c_3$ be the 1st, 2ndand 3rd least unit costs in a row and e_1 e_1, $e_2 e_2$ and $e_3 e_3$ be the 1st, 2nd and 3rd least unit cost in another row.

If (c_3 c_3 - c_1 c_1)> (e_3 e_3 - e_2 e_2), then identify the unmet column containing the least unit cost c_1 c_1. Else identify the other unmet column (containing the least unit cost$e_1 e_1$)

Step 7: Pick up any two unmet column for each of them. If there is no such connection between the two columns, then select the column with minimum difference. And go to step 8

Step 8: Considering the identified unmet column in step 4, 5, 6 or 7 and corresponding to the smallest of the differences between the second least and the least unit cost in a row for each of the allocations in this column, transfer the maximum possible amount of excess demand quantity which can make next column met, from the least unit cost cell to the next least unit cost cell in a row. (If the demand value is smaller than the value of the allocation in the unmet column, transfer the full amount of the allocation)

If there remains more excess demand in the selected column, do the same for the next smallest difference of the second least and least unit costs and continue the transferring process until the selected unmet column become met.

Step 9: Cross of the column that has completely been satisfied by removal of excess demand quantity just made, and go to step 3

Step 10: Stop, and take the current solution as the initial feasible basic solution.

Benefit of New Method over VAM

1. Only row penalties are calculated in New Method whereas both row and column penalties are calculated in VAM.

2. In the former, in each of the iteration row penalties are calculated for the rows with allocations only whereas in this latter row penalties are calculated for all rows.

3. The tie breaking feature is incorporated in New Method also.

Differences between JHM and New Method

1. In JHM, only column penalties are calculated. But in New Method only row penalties are calculated.
2. For the unbalanced transportation problem (i.e. total supply >total demand) New Method needs balancing by adding dummy destination whereas the JHM does not need such balancing.

Similarities of JHM and New Method

1. If total demand > total supply, both JHM and New Method follow the same way of balancing for find an initial solution.
2. Both methods are incorporated with tie breaking feature.

Illustration of a numerical example problem by New Method

Consider the following numerical example problem provided in Table 2.

In this problem, the total supply > total demand. Therefore, it is unbalanced problem. Then we must add dummy destination to balance the problem.

Table 2 : A numerical example problem data.

	D_1	D_2	D_3	Supply
S_1	3	4	6	**100**
S_2	7	3	8	**80**
S_3	6	4	5	**90**
S_4	7	5	2	**120**
Demand	110	110	60	390 / 280

Table 3: The balanced transportation problem with an unmet column D_4.

	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	3	4	6	0 **100**	**100**	3-0=3
S_2	7	3	8	0 **80**	**80**	3-0=3
S_3	6	4	5	0 **90**	**90**	4-0=4
S_4	7	5	2	0 **120**	**120**	2-0=2
Demand	**110**	**110**	**60**	**110**	**390** **390**	
	Met column	Met column	Met column	**Unmet column**		

Table 4: Transportation table with an allocation in S_4 - D_3 of the met column.

	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	3	4	6	0 **100**	**100**	**3-0=3**
S_2	7	3	8	0 **80**	**80**	3-0=3
S_3	6	4	5	0 **90**	**90**	4-0=4
S_4	7	5	2 **60**	0 **60**	**120**	
Demand	**110**	**110**	**60**	**110**		
	Met column	Met column	**Met column**	**Unmet column**		

Table 5: Transportation table with allocations in S_4 - D_3 and S_1 - D_1 of the met columns.

	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	3 **100**	4	6	0	**100**	
S_2	7	3	8	0 **80**	**80**	**3-0=3**
S_3	6	4	5	0 **90**	**90**	4-0=4
S_4	7	5	2 **60**	0 **60**	**120**	
Demand	**110**	**110**	**60**	**110**		
	Met column	Met column	**Met column**	**Unmet column**		

Table 6: Transportation table with an unmet column D_2.

	D_1	D_2	D_3	D_4	Supply	Penalty
S_1	3 **100**	4	6	0	**100**	
S_2	7	3 **80**	8	0	**80**	7-3=4
S_3	6	4 **40**	5	0 **50**	**90**	**6-4=2**
S_4	7	5	2 **60**	0 **60**	**120**	
Demand	**110**	**110**	**60**	**110**		
	Met column	**Unmet column**	**Met column**	**Met column**		

Table 7: IFS obtained from NM– New Method of this paper.

	D_1	D_2	D_3	D_4	Supply
S_1	3 **100**	4	6	0	**100**
S_2	7	3 **80**	8	0	**80**
S_3	6 **10**	4 **30**	5	0 **50**	**90**
S_4	7	5	2 **60**	0 **60**	**120**
Demand	**110**	**110**	**60**	**110**	
	Met column	**Met column**	**Met column**	**Met column**	

Table 8: Performance measure of NM over VAM and JHM for 9 benchmark instances.

Problem chosen from	Initial cost with an IFS by			% decrease in TCIFS by NM over VAM and JHM		Minimal T. cost (by Lingo)	% increase from the minimal total cost by		
	VAM	JHM	NM	VAM	JHM		VAM	JHM	NM
Sirinivasan and Thompson	955	880	880	7.85	0.00	880	8.52	0.00	0.00
Sen *et al*.	2,164,000	2,146,750	2,146,750	0.80	0.00	2,146,750	0.80	0.00	0.00
Deshmukh	779	743	743	4.62	0.00	743	4.85	0.00	0.00
Ramadan and Ramadan	5,600	5,600	5,600	0.00	0.00	5,600	0.00	0.00	0.00
Kulkarni and Datar	880	840	840	4.55	0.00	840	4.76	0.00	0.00
Schrenk *et al*.	59	59	59	0.00	0.00	59	0.00	0.00	0.00
Samuel	28	28	28	0.00	0.00	28	0.00	0.00	0.00
Imam *et al*.	475	460	435	8.42	5.43	435	9.20	5.75	0.00
Adlakha and Kowalski	390	390	390	0.00	0.00	390	0.00	0.00	0.00

Table 9: Performance measure of NM over VAM and JHM for 7 benchmark instances from Juman and Hoque (2015).

Problem No.	Initial cost with an IFS by			% decrease in TCIFS by NM over VAM and JHM		Minimal T. cost (by Lingo)	% increase from the minimal total cost by		
	VAM	JHM	NM	VAM	JHM		VAM	JHM	NM
1	5,125	4,525	4,525	11.71	0.00	4,525	13.26	0.00	0.00
2	3,520	3,460	3,460	1.70	0.00	3,460	1.73	0.00	0.00
3	960	920	920	4.17	0.00	920	4.35	0.00	0.00
4	849	809	809	4.71	0.00	809	4.94	0.00	0.00
5	465	417	465	0.00	11.51	417	11.51	0.00	11.51
6	3,663	3,458	3,458	6.00	0.00	3,458	5.93	0.00	0.00
7	109	109	109	0.00	0.00	109	0.00	0.00	0.00

The cost associated with an IFS obtained by NM is Z = 840, along with X_{11}=100, X_{22}=80, X_{31}=10, X_{32}=30, X_{33}=60. It should be noted that the initial cost obtained by NM for the problem is 840 which is an optimal and **zero** additional iterations are required to get the optimal solution.

Comparative study among VAM, JHM and New Method (NM) of 16 benchmark instances are discussed in next section.

Comparative study

In this section, a comparative study is done for Vogel (1958)'s Approximation Method (VAM), Juman and Hoque (2015)'s Method (JHM) and New Method (NM). Comparison among these three methods is performed using the solutions of 16 numerical example problems chosen from the literature. Data for these 16 instances are provided in Appendix A and Appendix B. The Stepping Stone Method (SSM) is then used to obtain the minimal total cost solution (MTCS) starting from the initial feasible solution (IFS) which is obtaining from VAM, JHM or New Method throughout this section.

The performance measure and the comparison on the iteration numbers of these three methods are provided in **Tables 8-11** and the **Fig. 2-7.**

Comparison on the iteration numbers

Table 10: Comparative study on the iteration number taken by SSM coupled with NM, VAM and JHM for 9 benchmark instances.

Problem Chosen from	Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by		
	VAM	JHM	NM
Srinivasan and Thompson(1977)	1	0	0
Sen *et al*. (2010)	2	0	0
Deshmukh (2012)	1	0	0
Ramadan and Ramadan (2012)	0	0	0
Kulkarni and Datar (2010)	2	0	0
Schrenk*et al*. (2011)	0	0	0
Samuel (2012)	0	0	0
Imam *et al*. (2009)	1	1	0
Adlakha and Kowalski (2009)	0	0	0

Table 11: Comparative study on the iteration number taken by SSM coupled with NM, VAM and JHM for 7 benchmark instances from Juman and Hoque (2015).

Instance No.	Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by		
	VAM	JHM	NM
1	1	0	0
2	2	0	0
3	1	0	0
4	1	0	0
5	1	0	1
6	2	0	0
7	0	0	0

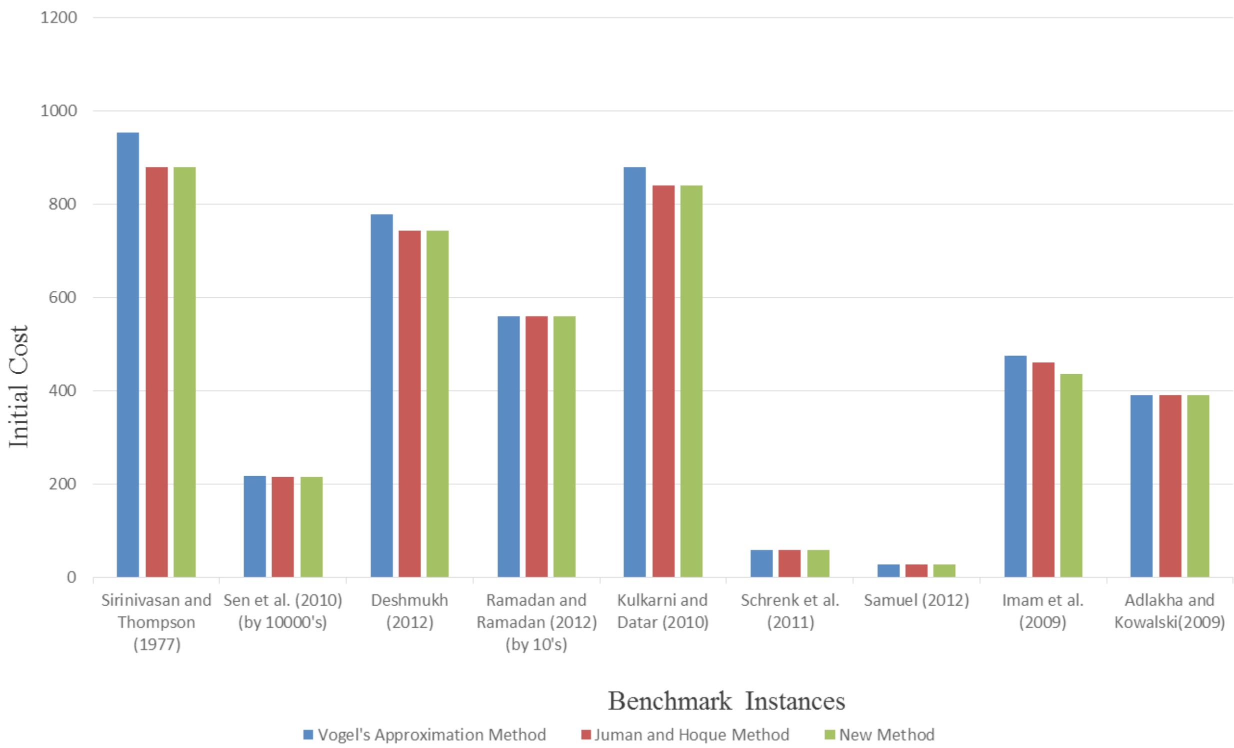

Figure 2: Bar chart of initial cost vs. benchmark instances in case of solving by VAM, JHM and NM.

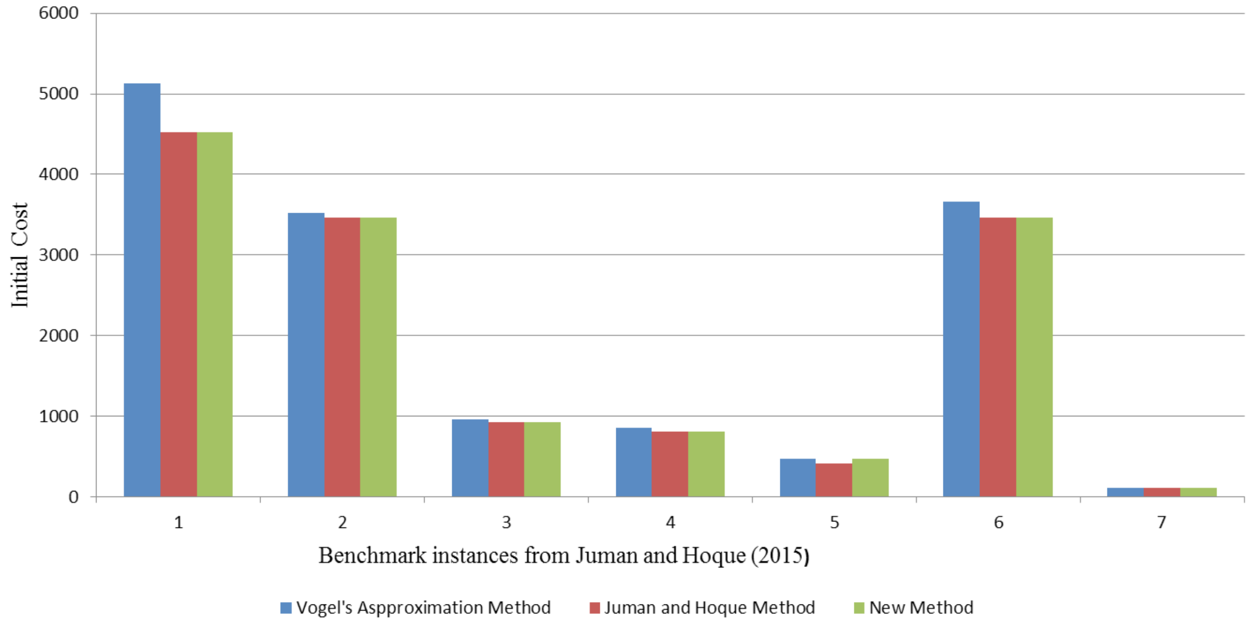

Figure 3: Bar chart of initial cost vs. 7 benchmark instances from Juman and Hoque (2015) in case of solving by VAM, JHM and NM.

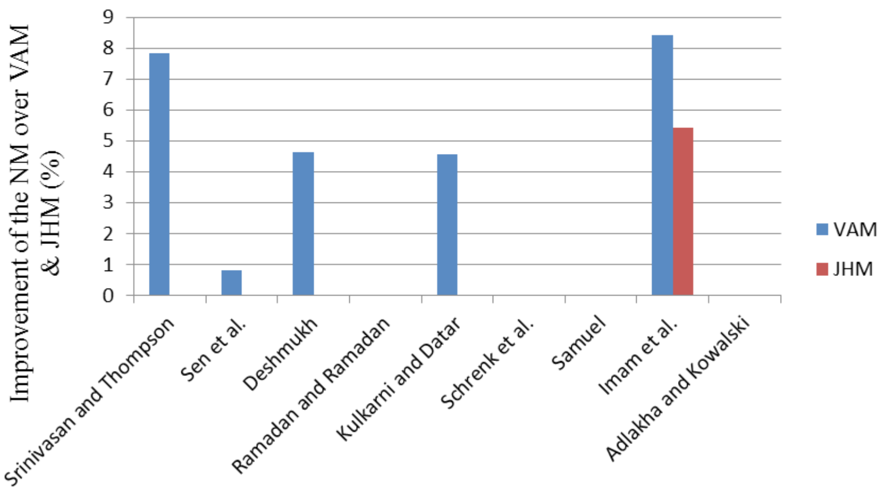

Figure 4: Percentage decreases in total cost for IFS by NM over VAM and JHM for 9 benchmark instances.

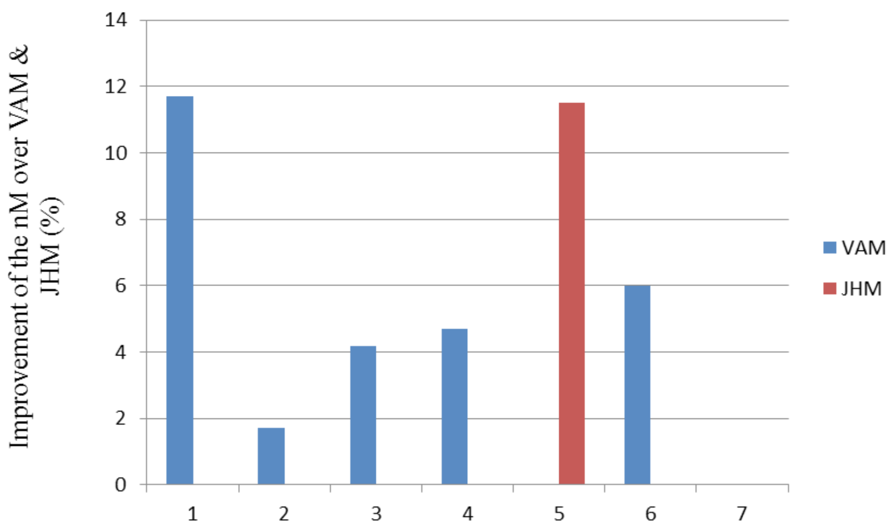

Figure 5: Percentage decreases in total cost for IFS by NM over VAM and JHM 7 benchmark instances from Juman and Hoque (2015).

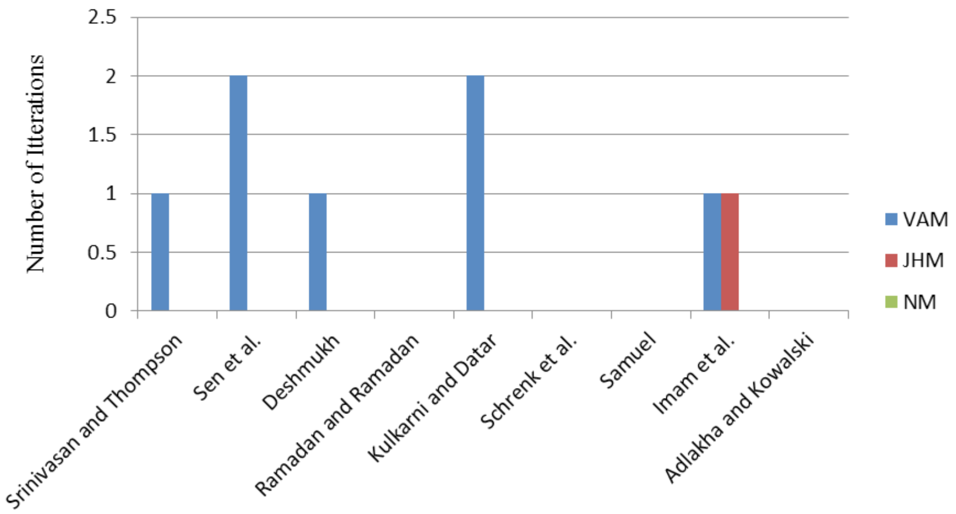

Figure 6: Number of Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by VAM, JHM and NM for 9 benchmark instances.

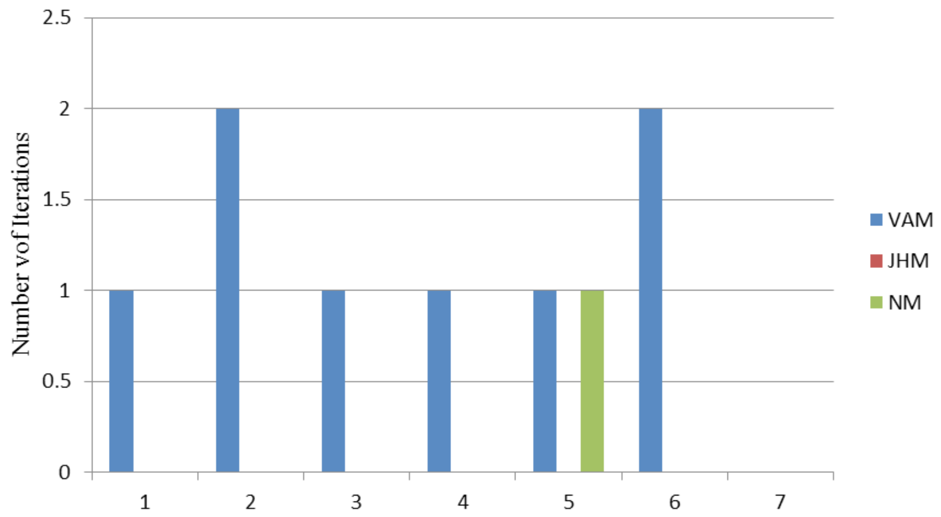

Figure 7: Number of Iterations taken by SSM to obtain MTCS by starting with an IFS obtained by VAM, JHM and NM for 7 benchmark instances from Juman and Hoque (2015).

It can be easily observed from the **Tables 8-11** and the **Figures 2-7** that our new method provides the same or better IFS to all the studied benchmark instances except one. Thus, our new method developed in this paper can be considered as an alternative approach to determine the IFS to a transportation problem.

CONCLUSION

Finding an initial feasible solution is the prime requirement to obtain a minimal total cost solution to a transportation problem. Based on this initial feasible solution of the problem, the number of iterations to get the minimum total cost solution can be changed. If an initial feasible solution becomes close to the minimum total cost, the minimum total cost solution can obtained with lesser number of iterations. There are several methods to obtain IFS. Among them, Vogel's Approximation Method (1958) and JHM Solution algorithm (2015) can be considered as a state of the art initial solution providers available in the literature (refer to Juman and Hoque, 2015). In this research, a new alternative method is developed to attain an efficient initial feasible solution to the transportation problems. In the development of our new method only row penalties containing allocations are incorporated whereas JHM is incorporated only with columns containing allocations. A comparative study shows that the new method gives the minimal total cost solution to 15 out of 16 benchmark instances. For the remaining problem, the number of iterations taken to obtaining the optimal solution by Vogel's Approximation Method (VAM) and new method are the same. However, for this problem JHM provides a better near-optimal cost solution compared to our new method. Similarly, our new method provides a minimum total cost solution to a problem (Imam *et al*., 2009) whereas the VAM and JHM does not. Thus, JHM Solution Algorithm and our new proposed method are two of the best available methods in finding the IFS to the transportation problem.

Future research might be carried out to develop this method for the case of large number of plants and destinations. Throughout this research supply and demand quantities are all fixed. But in real world supply and demand quantities may vary. This research considers linear transportation problems. But the transportation problem can become nonlinear also. In those cases the current method cannot be applied. Thus the new proposed method can be extended to include those variations. We intend to devote ourselves in this direction of future research.

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APPENDIX A

Numerical Example (Srinivasan and Thompson, 1977)

$[C_{ij}]_{3\times4}$= [3 6 3 4; 6 5 11 15; 1 3 10 5]

$[S_i]_{3\times1}$= [80, 90, 55]

$[D_j]_{1\times4}$= [70, 60, 35, 60]

Numerical Example(Sen et al., 2010)

$[C_{ij}]_{5\times4}$ = [60 120 75 180; 58 100 60 165; 62 110 65 170; 65 115 80 175; 70 135 85 195]

$[S_i]_{5\times1}$= [8000, 9200, 6250, 4900, 6100]

$[D_j]_{1\times4}$= [5000, 2000, 10000, 6000]

Numerical Example (Deshmukh, 2012)

$[C_{ij}]_{3\times4}$= [19 30 50 10; 70 30 40 60; 40 8 70 20]

$[S_i]_{3\times1}$= [7, 9, 18]

$[D_j]_{1\times4}$= [40, 8, 7, 14]

Numerical Example (Ramadan and Ramadan, 2012)

$[C_{ij}]_{3\times3}$= [32 40 120; 60 68 104; 200 80 60]

$[S_i]_{3\times1}$= [20, 30, 45]

$[D_j]_{1\times3}$= [30, 35, 30]

Numerical Example (Kulkarni and Datar, 2010)

$[C_{ij}]_{4\times3}$= [3 4 6; 7 3 8; 6 4 5; 7 5 2]

$[S_i]_{4\times1}$= [100, 80, 90, 120]

$[D_j]_{1\times3}$= [110, 110, 60]

Numerical Example (Schrenket al., 2011)

$[C_{ij}]_{3\times4}$= [3 6 1 5; 7 9 2 7; 2 4 2 1]

$[S_i]_{3\times1}$= [6, 6, 6]

$[D_j]_{1\times4}$= [4, 5, 4, 5]

Numerical Example (Samuel, 2012)

$[C_{ij}]_{3\times4}$= [1 2 3 4;4 3 2 0; 0 2 2 1]

$[S_i]_{3\times1}$= [6, 8, 10]

$[D_j]_{1\times4}$= [4, 6, 8, 6]

Numerical Example (Imam et al. , 2009)

$[C_{ij}]_{3\times4}$= [10 2 20 11;12 7 9 20; 4 14 16 18]

$[S_i]_{3\times1}$= 15, 25, 10]

$[D_j]_{1\times4}$= [5, 15, 15, 15]

Numerical Example (Adlakha and Kowalski, 2009)

$[C_{ij}]_{4\times5}$= [2 1 3 2 2; 3 2 1 1 1; 5 4 2 1 3; 7 5 5 3 1]

$[S_i]_{4\times1}$= [20, 70, 30, 60]

$[D_j]_{1\times5}$= [50, 30, 30, 50, 20]