# SPREAD SPECTRUM FOR INDOOR WIRELESS <br> DATA COMMUNICATION 

by
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#### Abstract

This thesis investigates the merits of direct sequence spread spectrum multiple access for wireless local area networks. The primary obstacles are multipath fading and multiaccess interference. These effect both bit error rate and code acquisition performance.

Since code acquisition is not a severe problem for matched filter receivers, conditions under which matched filtering can be implemented are determined. These conditions are the result of current technological limitations.

It is shown that if matched filtering can not be done then system-wide synchronization is required to facilitate rapid code acquisition. Under these circumstances spread spectrum may not be an attractive multiple access protocol.

Two methods are demonstrated to determine system performance in terms of bit error rate and multipath outage. One method is easy to compute and uses a Gaussian interference approximation. The other method, which must be computed numerically by computer, uses a more realistic model but generates similar multipath outage results.

Performance is found to compare favorably with several other simple multiple access protocols including fixed assignment FDMA and TDMA. However, it is shown that spread spectrum can not compete with more sophisticated protocols such as CSMA/CD or BRAM that allow cooperation between terminals.


Thesis Supervisors: Jerome B. Wiesner, Institute Professor Emeritus
Robert S. Kennedy, Professor of Electrical Engineering

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## CONTENTS

Chapter 1 Introduction ..... 5
Chapter 2 Channel and Signal Model ..... 9
Chapter 3 Receiver Technology ..... 13
Chapter 4 Code Acquisition ..... 18
Chapter 5 System Performance ..... 31
Appendix 5A ..... 49
Chapter 6 Other Multi-access Protocols ..... 52
Chapter 7 Conclusions ..... 72
Glossary of Notation ..... 75
References ..... 78

## Chapter 1

## Introduction

Recent years have seen a blossoming in the amount of computer hardware. Personal computers are now commonplace in the home. It is increasingly common to find a terminal or PC on every desk not only in research labs but in business offices as well. At the same time advances in technology and design have led to reductions in size, weight, and cost of equipment. It is possible to imagine in the not-too-distant future a battery-powered, notebook-sized terminal with capabilities similar to that of current CRT terminals.

To get full use out of all this hardware, communications ability must be provided for host connection, file transfer, electronic mail, database access, etc. Currently the bulk of this local communication is carried over cables. Each piece of equipment has a cable attaching it to the network. As a result, installing or moving a piece of equipment is expensive and slow. Furthermore, lack of conduit space is a problem in many buildings. To accomodate truly portable terminals an entirely different approach must be taken. We would like to avoid cables altogether and communicate via radio frequency links.

Since we have chosen to use unguided radio communications, users will encounter interference of various types. Signals travelling over different paths will arrive at the receiver with different amplitudes, phases, and delays causing what is known as multipath interference. Signals intended for other receivers cause multi-access interference. Finally there may be other systems in the vicinity generating what we will call cosite interference.

The ability of spread spectrum signalling to reject interference is well-known so it is natural to consider spread spectrum radio transceivers for wireless local area networks. Another advantage of spread spectrum is that its low spectral density may allow it to
coexist with other systems in the same frequency band (see, for example, [1]). A special reason for considering spread spectrum at this time is the regulatory environment. The Federal Communications Commission (FCC) recently opened up three bands for spread spectrum use $[2],[3]$. Since obtaining FCC approval can be a large part of a product development project, these new regulations are likely to have a great impact on radio technology [4]. This thesis discusses the use of spread spectrum radio for local digital communication.

Some related work has been presented in the literature. There are several commercially available digital radio networks. The Motorola product is typical [5]. It provides communication between a large number of terminals over a metropolitan area by way of strategically located base stations which are connected by land lines. It does not use spread spectrum; multi-access is achieved by contention.

Many applications of spread spectrum radio have been studied. There is a vast literature on military uses where the emphasis is on antijam and anti-intercept capability. Much of this material can be found in [6]. There has been much discussion of using spread spectrum for land mobile radio though to my knowledge it has not been implemented anywhere [1], [7]-[11]. Several researchers have considered the suitability of spread spectrum for indoor cordless telephones [12], [13]. Finally a handful of people have reported on spread spectrum for indoor wireless data communication. [14] and [15] were primarily concerned with using spread spectrum to combat multipath. [16]-[18] reported on the performance of star-configured spread spectrum systems both with and without antenna diversity.

This paper assumes the following goals:

1) We want an easily reconfigurable network to accomodate portable users as well as frequent entrances and withdrawals from the network. To accomplish this communicators will use RF transceivers.
2) We want to handle communications for terminals and small computers (large computers are generally sufficiently stationary to justify cabling). This traffic is bursty and has a low data rate. For most purposes 1200 baud is sufficient.
3) A network which serves a large office building or lab might encounter several hundred active users (and perhaps many more subscribers who are inactive). On the other hand a network which serves a house or a single room may need to serve only ten or twenty users at most. It will be seen that technology may dictate different strategies for these two environments.
4) The network must be able to deal with concurrent communication between various independent pairs of users. Direct sequence spread spectrum modulation will be used to allow operation in the presence of multi-access interference.
5) For simplicity and reliability we wish to avoid centralized control. This is particularly important in a small network where the incremental cost of the controller may be prohibitive. We also want to avoid system-wide synchronization. For one thing asynchronous operation is intrinsically simpler. More importantly, differences in propagation times make perfect synchronization physically impossible.
6) Finally as in any system we want a solution which is inexpensive and reliable.

This paper will discuss how well the goals above can be fulfilled by a fully connected radio network utilizing direct sequence spread spectrum and differential phase shift keying (DPSK). Each receiver is assigned a spread spectrum code. For simplicity, it is assumed that more than one transmitter never attempts to communicate with a single receiver simultaneously. The spectral spreading provides multi-access capability as well as rejecting interference from other systems. It should be pointed out that systems of sufficient bandwidth to effectively combat multipath will not be considered.

Chapter 2 describes the pertinent characteristics of the channel and signal models. The most important phenomena to be included in the model are multipath fading and
multi-access interference. Chapter 3 describes the features of various technologies available to implement the receiver. It will be shown that charge coupled devices (CCD's) can be used to build matched filter receivers with sufficient process gain to accomodate small to medium-sized systems. At the current state of technology large systems must use correlation receivers which can be built using digital logic. Chapter 4 explains methodologies for the crucial process of acquiring code synchronization. Synchronization is not difficult for matched filter receivers, but the only way a correlation receiver can achieve rapid acquisition is by restricting the initial timing uncertainty. The most obvious way to do this is to use system-wide synchronization. Under these circumstances spead spectrum is not attractive. Chapter 5 gives analytical and numerical results on system performance. The primary measure of performance is the probability that a desired bit error rate will be achieved. It is shown that some form of diversity is required and antenna diversity is recommended. Indeed even with several antennas it may sometimes be necessary to adjust an antenna to improve a link. Chapter 6 compares this system to several other possible multi-access protocols. Spread spectrum is simple and requires no cabling. It can have a performance advantage over other simple protocols such as ALOHA and fixed assignment FDMA and TDMA. It can not, however, provide the performance of more sophisticated protocols such as BRAM or cable-based protocols such as CSMA/CD. Finally Chapter 7 gives my conclusions.

## Chapter 2

## Channel and Signal Model

This chapter describes the relevant features of the indoor RF channel and the received spread spectrum signal and ways of modeling these features.

The signal attenuates as it propagates to greater distances from the transmitter. Generally a power law model is used [19]:

$$
\begin{equation*}
s(\mathrm{~dB})=-m \log d, \tag{2.1}
\end{equation*}
$$

where $s$ is the signal strength in dB , and $d$ is the distance from the transmitter to the receiver. The constant $m$ depends on many factors including the floorplan and materials used in the construction of the building. Measured values of $m$ range from 1.2 in an open hangar which apparently acted as a waveguide up to 6.5 in a cinder block laboratory with steel partitioning.

Some materials actually absorb certain frequencies causing attenuation, This phenomenon is well-known in long-distance microwave propagation which suffers serious attenuation during rainfall because of absorption by water. Such affects can be significant even over the small distances encountered in indoor communications. It has been reported [20] that signals at 60 GHz , which is absorbed by oxygen, will hardly propagate outside a single room. One would expect such effects to obey an exponential attenuation law rather than a power law. Nonetheless the issue has received little attention in the literature and the tendency has been to assume that a power law is sufficient to model the attenuation. It may well be that the fading on the channel is sufficiently complicated to mask any weaknesses in the attenuation model. It is also possible that these effects are incorporated into the shadowing which is discussed next.

A signal propagating over a single path from the transmitter to the receiver is attenuated by the walls and floors through which it passes. This causes fading correlated over entire rooms since the signal arriving anywhere in the room has encountered approximately the same walls and floors. Attenuation is multiplicative so the logarithm of the attenuation is additive. Using central limit theorem arguments one is led to Gaussian statistics for the logarithm of the attenuation. Thus the signal level averaged over the area of a room is modeled as having lognormal fading statistics with mean given by the power law. The standard deviation is typically less than $10 \mathrm{~dB}[21],[22]$. The sum of lognormal random variables is itself well-modeled by a lognormal variable so this argument is not adversely affected by the existence of several paths.

Due to reflections there are generally multiple paths between the transmitter and the receiver. The signal components from the various paths add noncoherently causing fading which is correlated over distances on the order of a quarter wavelength [23]. If the antennas are stationary (within a quarter wavelength) the link is stable and a constant fade is experienced.

Multipath fading has been modeled in two distinct ways in the literature. If one intends to resolve the various paths and use them as a basis for diversity one needs statistics for the individual paths [24]-[27]. Alternatively one can try to characterize the gross properties of the received signal. The quadrature components of the signal are sums of components from various paths. Central limit arguments lead to Gaussian quadrature components which in turn leads to a Rayleigh distributed signal amplitude (or equivalently an exponentially distributed power). If there is a strong direct path signal, a Rician distribution may be more appropriate. Signal phase is generally modeled as uniformly distributed.

One other multipath effect that requires discussion is the delay spread (or the coherence bandwidth which is proportional to its reciprocal). Measurements have shown
that this varies widely from building to building. RMS delay spread can range from as low as $30 \mathrm{nsec}[15]$ to as high as 250 nsec [28]. Spread depends on the structure of the building. This leads to coherence bandwidths on the order of $1-10 \mathrm{MHz}$. If the system bandwidth exceeds the coherence bandwidth, the fading will be frequency-selective.

Motion of the transmitter, receiver, or transmission medium causes a Doppler shift. In a multipath environment different components may also have different shifts leading to Doppler spread. Because of the slow speeds normally encountered in an indoor environment, Doppler bandwidth is unlikely to exceed 100 Hz for a carrier around $1 \mathrm{GHz}[7]$.

The channel described above is quite complicated. To account for all features it is necessary to use complex computer simulations, so I will make some simplifying assumptions to allow for tractable analysis. I assume a carrier frequency small enough that vibration and unintentional antenna movement will not effect link stability but large enough that antenna diversity can be conveniently implemented on a small terminal. This dictates a carrier frequency in the range 500 MHz to 10 GHz . I also assume that a frequency can be chosen such that absorption loss is insignificant. I assume stationary terminals and ignore Doppler shifts caused by moving reflectors (such as crowds in the hall). Finally I consider only Rayleigh fading which I assume to be constant and due to a single resolved path. This presupposes that the system bandwidth is less than several megahertz (the coherence bandwidth).

The received signal is modeled as

$$
\begin{equation*}
s(t)=n(t)+\sqrt{2 P} \sum_{k=0}^{K-1} A_{k} a_{k}\left(t-t_{k}\right) b_{k}\left(t-t_{k}\right) \cos \left(\omega_{c} t+\varphi_{k}\right) \tag{2.2}
\end{equation*}
$$

where $P$ is the mean signal power, $K$ is the number of active users, $A_{k}$ is the amplitude of the signal intended for the $k$ th receiver normalized by $P, a_{k}(t)$ is the code waveform,
$b_{k}(t)$ is the data waveform, $t_{k}$ is the delay of the $k$ th signal due to propagation and asynchronism, $\omega_{c}$ is the carrier frequency common to all signals, $\varphi_{k}$ is the phase of the $k$ th signal, and $n(t)$ is an additive white Gaussian noise process with spectral density $N_{0} / 2$. $A_{k}$, for $k=0, \ldots, K-1$, are independent identically distributed Rayleigh random variables with probability density

$$
\begin{equation*}
f(A)=2 A e^{-A^{2}}, \quad A \geq 0 . \tag{2.3}
\end{equation*}
$$

The code waveform is defined by

$$
\begin{equation*}
a_{k}(t)=a_{j}^{(k)}, \quad j T_{c} \leq t<(j+1) T_{c} \tag{2.4}
\end{equation*}
$$

where $a_{j}^{(k)}$ is 1 or -1 for each $j$ and forms an infinite sequence with period $N_{c}$. Thus the code waveform is a sum of contiguous rectangular pulses called chips of duration $T_{c}$. Similarly

$$
\begin{equation*}
b_{k}(t)=b_{l}^{(k)}, \quad l T \leq t<(l+1) T \tag{2.5}
\end{equation*}
$$

where $T$ is the bit time. I will assume that $T=N_{c} T_{c}$, although it is not alpays necessary and in some cases may be inadvisable [29]. Due to the difficulty of establishing absolute phase over a fading channel, the data is differentially encoded. Signal phases are independent and uniformly distributed between 0 and $2 \pi$. Delays are independent and uniformly distributed between 0 and $T_{u}$. In an asynchronous system $T_{u}=T$. In a synchronous system $T_{u}$ represents imperfect synchronization due to different propagation delays and timing error.

## Chapter 3

## Receiver Technology

This chapter describes several technologies applicable to spread spectrum receivers. By seeing what available technology is capable of, we will get some guidance as to what direction to proceed. We will be especially interested in ways of implementing filters matched to the spread spectrum signal as this allows for a particularly simple receiver. Such filters require large time bandwidth products and complicated impulse responses. Currently three technologies compete for this application: surface acoustic wave (SAW) devices, digital devices, and charge coupled devices (CCD's).

In all three cases the filter is implemented as a delay line with weighted taps connected to a summing bus. In a SAW device an acoustic wave is launched at one end of the device. Transducers are placed along the length of the device and serve as taps. The typical propagation velocity for the wave is $3 \times 10^{3} \mathrm{~m} / \mathrm{sec}[30]$. Thus bandwidth is determined by the density of lines that can be printed on the substrate. The device is generally made of quartz or lithium niobate and is manufactured using the same techniques and equipment used for making semiconductor chips. 200 MHz bandwidths are easily achievable using inexpensive production techniques. With state of the art equipment bandwidths of several gigahertz can be attained [31]. Since we anticipate chip rates in the several megahertz range, SAW devices certainly have sufficient bandwidth. Indeed there may be enough bandwidth to perform filtering at the carrier frequency.

Unfortunately SAW devices are in other ways totally inadequate for our application. The impulse response of the filter must be at least one bit time long. At 1200 baud we need an $833 \mu \mathrm{sec}$ impulse response resulting in a device 2.5 meters long. It is possible to use folded geometries but this makes it difficult to tap the delay line. Thus
the required device is much too bulky. In any case it is very difficult and expensive to fabricate devices even one tenth this length.

Digital technology is more promising and is rapidly improving. To perform correlation the waveform must be sampled and read in to the processor which must then perform a multiplication and an accumulation. These operations must be performed at least once each chip time and require several clock cycles depending on the processor architecture. We therefore need a processor whose clock rate is several times the chip rate. The popular TMS32020 signal processing chip from Texas Instruments runs at 5 MHz (a newer version is expected to run at 10 MHz ) and requires four instruction cycles for each chip [32]. Thus the new version will be able to perform correlation on signals with a 2.5 MHz chip rate which is precisely in the range of interest. It is certainly possible to produce more specialized hardware which can correlate somewhat faster.

A correlator must be synchronized to the signal. In the next chapter we will see that this is a problem in our environment. Thus we would like to do convolution in order to implement a matched filter. We now need to perform $N_{c}$ multiplies and accumulates during each chip time for a total of $R N_{c}^{2}$ operations each second where $R$ is the baud rate. As an example the TMS32020 operating at 10 MHz and using pipelining to multiply and accumulate in a single clock cycle can achieve a maximum process gain $\left(N_{c}\right)$ of 91 at 1200 baud. This is not enough gain to support more than a handful of users. While special purpose hardware could no doubt increase the achievable process gain to several hundred, a large system (100 or more users) requires a process gain of at least 1000. At 1200 baud this demands a processor clock rate of 1.2 GHz which is well beyond current digital technology. VLSI parallel processing chips may someday relieve this situation, but, for the time being, it is wise to seek a different solution.

Charge coupled devices are made up of cells which store charge and a mechanism for transferring charge from one cell to another. If we create a long line of these cells and add voltage to charge conversion at the input, charge to voltage conversion at the output, and a clock to regulate the transfer of charge, we have an analog delay line. Since CCD's can be fabricated in silicon, all of this can be put on a single chip.

Indeed much more can be put on the chip. To build what is called an analog binary correlator we add a digital shift register. Each stage of the register and each cell of the delay line is tapped. The taps are fed into a multiplier. The multiplier may be a fairly simple device containing only a few transistors which, according to the value from the shift register, switches a current proportional to the charge on the delay line onto the appropriate bus [33]-[35]. The currents, which sum on the bus, are fed into a summing amplifier. If the shift register contains only positive and negative unit values then this is just a differential amplifier. To use this device the chip sequence is fed into the shift register and the received waveform (processed into the baseband) is fed into the delay line. A full correlation is performed at each clock cycle. The result is a discrete time matched filter.

As an example of what CCD's can do I give some of the specifications of an analog binary correlator currently being fabricated [36]. It is a dual device with two 1024-stage delay lines. These can be used for in-phase and quadrature channels. It comes in a small 68 -pin package, dissipates 1 W , and uses a single 5 V power supply. It has been tested at room temperature for clock rates up to 47.5 MHz and down to 10 kHz (total time delay and thus minimum clock rate is limited by thermal leakage [37]) on both the delay line and the shift register (allowing rapid programmability). The shift register holds 3 -bit weights. Zero weights allow the length to be changed. The dynamic range is 8 or 9 bits. The device has been produced with 80 to $90 \%$ yield and is expected ultimately to cost under $\$ 100$.

All of these features are attractive except for the length. To compensate for synchronization error it is desirable to sample at least twice per chip, in which case this device can accomodate a process gain of 512 at most. This is enough to be of interest for small systems with less than 50 users but is inadequate for larger systems. While it may be possible to make devices somewhat longer, length is limited by charge transfer inefficiency (CTI), the attenuation of the signal at each transfer of charge. While the CTI of this device has been improved greatly, it is not anticipated that the device will be lengthened. Naturally a longer device would also be physically larger and consequently more expensive.

We see that all three devices have important limitations. SAW devices are limited to a total delay of about $100 \mu \mathrm{sec}$. This can not be improved without building an unreasonably large device. Thus SAW devices are not well adapted to data rates below 10 kHz . Digital correlators are limited to chip rates of several megahertz. This is sufficient for our application. On the other hand, for digital matched filters, the product of the chip rate (in chips per second) and the process gain is limited to several million which is adequate only for very small systems. CCD's are limited to process gains of around 1000. This is not a fundamental limit but no dramatic improvements are expected soon. If we do not need process gains in excess of 1000, CCD matched filtering is the way to go. Otherwise we must use a correlator.

A word about frequency hopping is in order here. None of the technologies discussed are well-suited to frequency hopping receivers. SAW devices are again unsuitable because they cannot be made with sufficiently long impulse responses. Both the digital devices and the charge coupled devices described above operate at baseband. This requires removing the frequency hopping. To do this one can use a frequency synthesizer synchronized to the signal to mix down to a common frequency or one can use a filter bank with a bandpass filter centered at each hopping frequency. Both of these methods
are difficult. By contrast direct sequence chips differ only in amplitude so the signal can be brought to baseband by a mixer with fixed frequency or by a single bandpass filter. Thus we have technologies available to implement a correlation receiver (or, for sufficiently small systems, a matched filter receiver) for direct sequence systems. Frequency hopping systems are harder to implement. This is the primary reason why this thesis focuses on direct sequence techniques.

## Chapter 4

## Code Acquisition

This chapter describes how to synchronize the receiver to the desired transmitter. It is assumed that each receiver is assigned a unique code. Any transmitter wishing to send a message modulates the signal using the appropriate code for the intended receiver. The receiver must acquire code timing (which we have previously assumed is equivalent to bit timing) in order to demodulate the data. This is made especially difficult by the bursty nature of the data. Acquisition must be done rapidly at the beginning of each message so as not to lose part of the message.

Conceptually, the simplest way to acquire the signal is by use of a matched filter. Indeed a matched filter receiver passively responds to the signal and does not necessarily need an acquisition circuit. A matched filter has an impulse response which is a time-reversed version of the desired signal possibly scaled in amplitude. The impulse response is given explicitly by

$$
\begin{equation*}
h(t)=a_{k}(T-t) \cos \left(\omega_{I F}(T-t)\right), \quad 0 \leq t \leq T \tag{4.1}
\end{equation*}
$$

where $\omega_{I F}$ is the intermediate frequency and, as in chapter $2, a_{k}(t)$ is the spread spectrum waveform, and $T$ is the bit time. The output of the filter is given by the familiar convolution integral

$$
\begin{equation*}
y(t)=\int_{0}^{T} h(\tau) s(t-\tau) d \tau \tag{4.2}
\end{equation*}
$$

where $s(t)$ is the signal as in equation (2.2) except that it too has been translated to the intermediate frequency. I will assume, for the sake of simplicity, perfect phase tracking for coherent detection.

If we sample the output at time $T$ (recall that both the digital and the CCD filters of chapter 3 naturally produce sampled outputs) we obtain, following the notation of [38],

$$
\begin{equation*}
y=\eta+\sqrt{P / 2} T\left[A_{0} b_{1}^{(0)}+\sum_{k=1}^{K-1} A_{k} I_{k, 0}\left(b_{1}^{(k)}, b_{2}^{(k)}, t_{k}, \varphi_{k}\right)\right] \tag{4.3}
\end{equation*}
$$

where $\eta$ has a Gaussian distribution with zero mean and variance $N_{0} T / 4$ and is due to the additive white Gaussian noise. Recall from chapter 2 that $.4_{k}, b_{l}^{(k)}, t_{k}$, and $\varphi_{k}$ denote, respectively, the amplitude, $l$ th data bit, delay and phase of the $k$ th signal. The first term inside the brackets is the signal term. The sum is the interference term. The function $I_{k, 0}$ represents the multi-access interference due to the $k$ th user. We note for use in the next chapter that the interference is given explicitly by

$$
\begin{equation*}
I_{k, 0}\left(b_{1}^{(k)}, b_{2}^{(k)}, t_{k}, \varphi_{k}\right)=\frac{\cos \varphi_{k}}{T}\left[b_{1}^{(k)} R_{k, 0}\left(t_{k}\right)+b_{2}^{(k)} \hat{R}_{k, 0}\left(t_{k}\right)\right] \tag{4.4}
\end{equation*}
$$

The functions $R_{k, 0}$ and $\hat{R}_{k, 0}$ are partial correlation functions of the spread spectrum waveform and are defined by

$$
\begin{align*}
& R_{k, 0}(t) \equiv C_{k, 0}\left(l-N_{c}\right) \hat{R}_{q}\left(t-\zeta T_{c}\right)+C_{k, 0}\left(l+1-N_{c}\right) R_{q}\left(t-l T_{c}\right)  \tag{4.5}\\
& \hat{R}_{k, 0}(t) \equiv C_{k, 0}(l) \hat{R}_{q}\left(t-l T_{c}\right)+C_{k, 0}(l+1) R_{q}\left(t-l T_{c}\right)
\end{align*}
$$

where $l$ is the integer part of $t_{k} / T_{c}, C_{k, 0}$ is the aperiodic crosscorrelation function of the chip sequence [39]

$$
C_{k, 0}(l) \equiv \begin{cases}\sum_{j=0}^{N_{c}-1-l} a_{j}^{(k)} a_{j+l}^{(0)}, & \text { if } 0 \leq l \leq N_{c}-1  \tag{4.6}\\ \sum_{j=0}^{N_{c}-1+l} a_{j-l}^{(k)} a_{j}^{(0)}, & \text { if } 1-N_{c} \leq l<0\end{cases}
$$

and $R_{q}$ and $\hat{R}_{q}$ are the partial autocorrelation functions for the chip waveform, $q(t)$

$$
\begin{align*}
\hat{R}_{q}(u) & \equiv \int_{u}^{T_{c}} q(t) q(t-u) d t  \tag{4.7}\\
R_{q}(u) & \equiv \int_{0}^{u} \cdot q(t) q\left(T_{c}+t-u\right) d t=\hat{R}_{q}\left(T_{c}-u\right)
\end{align*}
$$



Figure 4.1. Detection of signal in additive Gaussian noise

Generally we will use rectangular chips giving

$$
\left.\begin{array}{l}
\hat{R}_{q}(u)=T_{c}-u  \tag{4.8}\\
R_{q}(u)=u
\end{array}\right\} \quad 0 \leq u \leq T_{c}
$$

If we assume that the combined interference and noise is approximately Gaussian with zero mean and variance $V$ then the sample is Gaussian with variance $V$ and mean $s \equiv A_{k} \sqrt{P / 2} T$. We establish a threshold $h=\epsilon s$ and declare signal presence if the threshold is exceeded. Defining the signal to noise ratio as $s^{2} / V$, the probability of missing the signal (see figure 4.1) is

$$
\begin{equation*}
P_{m}=\operatorname{Prob}(y<h \mid \text { signal present })=Q\left(\frac{s-h}{\sqrt{V}}\right)=Q((1-\epsilon) \sqrt{S N R}) \tag{4.9}
\end{equation*}
$$

where the $Q$ function is given by

$$
\begin{equation*}
Q(z)=\int_{z}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-x^{2} / 2} d x \tag{4.10}
\end{equation*}
$$

If the filter output is sampled at some other time, $t$, the expression for the sample looks very much like (4.3) except the signal term is replaced by a self interference term
which has the same form as (4.4) with $k=0$. Generally codes are chosen so that this autocorrelation term is near zero as long as $|t-T|>T_{c}$. If we assume that this term is dominated by the noise and the other interference terms then, as above, the sample will be Gaussian with variance $V$ but the mean will now be zero (see figure 4.1). Proceeding as before the probability of false alarm (falsely detecting a signal when none is present) is

$$
\begin{equation*}
P_{f a}=\operatorname{Prob}(y \geq h \mid \text { signal absent })=Q(\epsilon \sqrt{S N R}) \tag{4.11}
\end{equation*}
$$

Figure 4.2 piots $P_{f a}$ against $P_{m}$ parameterized by $S N R$.
Although matched filtering is conceptually simple, we saw in the previous chapter that building the necessary filter can be technologically difficult. Another choice is to build a correlation receiver which actively multiplies the incoming signal by a locally generated copy of the desired signal. The output of the multiplier is integrated. The integrator is sampled and subsequently zeroed each bit time. The output is exactly equivalent io the output of a matched filter sampled at time $T+t_{d}$ where $t_{d}$ is the delay between the received signal and the local reference. As mentioned before, a matched filter produces a spike of width $2 T_{c}$ in response to the signal. The acquisition system must make $\left|t_{d}\right|<T_{c}$ in order to take advantage of this correlation peak. A separate tracking system, which will not be discussed further, generally takes over to bring $t_{d}$ near zero to maximize the signal strength.

The acquisition problem has been much studied and numerous techniques have been proposed (for a summary see [29]). For example one simple method is to transmit an unmodulated reference with the signal. This is impossible in a multi-access environment since if the references share the same bandwidth there is no way of separating them at the receiver.

Another method, known as sequential estimation, relies on codes which are generated by shift registers with feedback. There are many classes of such codes [39]. The receiver attempts to demodulate correctly enough consecutive chips to fill a local shift register. This too is inappropriate to a multi-access environment where the signal is likely to be many dB below the interference at the receiver input making the error rate very close to .5 .

Another approach is to use a separate channel for synchronization. Prior to each data message a synchronization message is sent to indicate exactly when (relative to the synchronization message) the data message will arrive. While this may be useful in some situations there is at least one very important case where the value of this scheme is questionable. In many networks a large percentage of the traffic consists of data being sent from ASCII terminals to their host computers. Frequently each character is sent as a separate message. :In this case the synchronization messages would not look much different from the data messages in terms of length and arrival rate. Thus it would be foolish to have two different systems (with the accompanying additional hardware complexity) to do essentially similar tasks. Whichever system works better should probably be used for both channels.

Probably the most common acquisition method is to use what is called a sliding correlator. A preamble is added to the front of each message by the transmitter to allow for synchronization. The preamble consists of the spread spectrum waveform without any data modulation. It lasts long enough to acquire synchronization with high probability. A local reference is generated and correlated for $T_{e}$ seconds with the incoming signal. $T_{e}$ is calk the examination time or dwell time. If the result exceeds a threshold then the local reference is assumed to be synchronized. Otherwise the local reference is time-shifted $T_{s}$ seconds and correlation is restarted. This process
is repeated until the threshold is crossed at which time control is handed over to the tracking circuit.

How long must the preamble be? The statistics of the acquisition time depend on the initial timing uncertainty (denoted $T_{u}$ ), the time shift $\left(T_{s}\right)$, the dwell time ( $T_{e}$ ), the probability of missing synchronization $\left(P_{m}\right)$, and the probability of false alarm ( $P_{f a}$ ). $P_{f a}$ and $P_{m}$ depend in turn upon the threshold and the signal statistics as well as $T_{e}$. An estimate of the mean acquisition time is given by [40]

$$
\begin{equation*}
T_{\mathrm{acq}}=\frac{T_{u} T_{e}}{2 T_{s}\left(1-P_{f a}\right)} \frac{1+P_{m}}{1-P_{m}} \tag{4.12}
\end{equation*}
$$

Without prior timing information $T_{u}=T$, the bit time. In order to avoid missing the correlation peak altogether we must have $T_{s}$ be no larger than the chip time, $T_{c}$, so $T_{u} / T_{s}$ is at least $N_{c}$. In order to get small $P_{f a}$ and $P_{m}$ we must have $T_{e}$ on the order of a bit time. If $N_{c}<1000$ we can use CCD's to build matched filters and avoid correlation, so we see that the preamble is likely to be at least 500 bits long. This is unacceptable.

To improve the situation it is necessary to drastically reduce either $T_{e}$ or $T_{u}$. While there are several methods for reducing the average dwell time, the only way to effect the huge reduction (hundredfold in order to make the synchronization overhead comparable to the data load in the short message case) is to have many correlators working in parallel examining different time shifts. This is likely to be expensive and to use a lot of space.

The remaining option is to reduce the initial timing uncertainty. This can be done in a number of ways. We may consider two level systems in which an idle member of a set of one or more correlators is triggered by a threshold crossing in a filter matched to a short section of the code thereby reducing the uncertainty region to only those cells . which cross the threshold [41]. The problem here is that the partial code has many
fewer chips than the full code and so can reject less interference. Thus the number of users will be limited by the length of this matched filter and not by the process gain. In this case we may as well use shorter codes for which we can build matched filters. For similar reasons we do not wish to use component code schemes where long codes are generated by adding shorter codes with different periods which can be acquired separately. I hasten to add that these methods may be useful if, for other reasons, it is desirable to have codes which are much longer than bits.

In a more promising approach, the transmitter sends a full preamble as described above only at the beginning of a session. The terminals then use stable clocks to remain synchronized between messages. Subsequent messages require only enough preamble to account for the clock drift and, if one of the terminals has moved, change in propagation time. To reduce the preamble to a few bits the clocks must remain synchronized within several microseconds.

If the network traffic is not described well in terms of sessions between pairs of transmitters and receivers then the entire network must be synchronized. There are essentially two ways of doing this. The users may periodically exchange messages so as to maintain synchronization at all times. The me sages may have to be quite lengthy or have a special form to allow new users to acquire.

Alternatively each terminal may synchronize to a common timing signal. This signal may come from a master clock node or it may be an external signal such as the synchronization signal from a public television broadcast or navigation satellite. In the former case a spread spectrum signal can be used. Each receiver can periodically synchronize to this signal when it is not receiving any other message. In the latter case extra hardware is most likely needed to receive the signal but no master clock transmitter is required. In either case, assuming negligible local clock instability, the timing uncertainty has two components. The message must propagate from the transmitter
to the receiver over an unknown range and there is a difference in the local clocks at the transmitter and receiver due to the propagation delay of the clock signal. Thus the timing uncertainty has been reduced to at most $2 T_{p}$, the maximum round trip propagation delay. If no two terminals are more than 300 meters apart, as would be the case in most buildings, the maximum round trip propagation delay would be $2 \mu \mathrm{sec}$.

In order to determine exactly how long the preamble must be it is necessary to compute the probability of acquisition. I make four assumptions. First I assume that the signal delay is uniformly distributed over the uncertainty region. Second I assume that, once acquired, synchronization is not lost. Third, in case of a false alarm the clock is not shifted but no additional processing delay is incurred. Finally, I assume that there is exactly one cell where the signal is and all other cells are statistically identical. There are $N_{u} \equiv T_{u} / T_{s}^{\prime}$ cells in all. If the preamble is of length $L T_{e}$ so that $L$ cells can be examined, then the probability of acquisition is given by

$$
\begin{align*}
P_{\mathrm{acq}}= & \frac{1-P_{m}}{N_{u}} \\
& +\sum_{i=2}^{N_{u}} \frac{1-P_{m}}{N_{u}} \sum_{k=i}^{L}\binom{k-2}{i-2}\left(1-P_{f a}\right)^{i-1} P_{f a}^{k-i} \\
& +\sum_{i=2}^{N_{u}} \frac{1-P_{m}}{N_{u}} \sum_{j=2}^{1+\left\lfloor\frac{L-i}{N_{u}}\right\rfloor} P_{m}^{j-1} \sum_{k=l_{\mathrm{min}}}^{L}\binom{k-j-1}{l_{\min }-j-1}\left(1-P_{f a}\right)^{l_{\mathrm{win}}-j} P_{f a}^{k-l_{\mathrm{min}}} \tag{4.13}
\end{align*}
$$

where $l_{\min }=(j-1) N_{u}+i$. The cell in which the signal is present is indexed by $i$. The number of times the $i$ th cell is searched is indexed by $j$. The total number of cells searched is indexed by $k$. The combinatorial term gives the number of ways of dividing $k-l_{\min }$ false alarms into $l_{\min }-j$ strings each terminated by a correct rejection of the cell.

Given a particular signal to noise ratio we can make a tradeoff between $P_{f a}$ and $P_{m}$ by choice of the threshold. In figure 4.2 $P_{f a}$ is plotted against $P_{m}$ parameterized by signal to noise ratio assuming coherent detection and Gaussian noise. Thus we can optimize probability of acquisition over operating point. Given a specified allowable probability of failure to acquire, $\hat{P}_{f}$, we can then determine the required preamble length. The length was found not to be very sensitive to operating point so optimization was done by choosing the best of only three or four operating points. The results are plotted in figures 4.3 and 4.4 for signal to noise ratios of 4 and 10 respectively. Such low $S N R$ 's are of interest because if there are $n$ examination cells per chip ( $n=T_{c} / T_{s}$ ) and the correlation peak is triangular with width $2 T_{c}$ and height $S N R_{\max }$ then the worst case (see figure 4.5 which depicts the case where $n=1$ ) $S N R$ is

$$
\begin{equation*}
S N R=\frac{2 n-1}{2 n} S N R_{\max } \tag{4.14}
\end{equation*}
$$

Thus if $n=1$ half the signal power may be lost to the acquisition system.
If $T_{e}=T$ then the $y$-axes of figures 4.3 and 4.4 are in terms of bit times. If we expect a significant part of the network traffic to consist of short messages (such as single 8-bit ASCII characters) we would want to limit preamble length to 10 or 20 bits. From the figures it is apparent that to do wnis and have an acceptable probability of failure it is necessary to limit the uncertainty region to a small number of cells (and consequently a small number of chips). Since the uncertainty due to propagation delays is on the order of a microsecond this is a reason for limiting the chip rate to several megahertz. I caution that this argument applies only to a system using a single sliding correlator. A matched filter, in effect, searches all the cells in a single bit time.

The implications for correlation receivers are dire. An asynchronous system using correlation receivers will devote most of its bandwidth to code acquisition and is inefficient to the point of ridiculousness. To overcome this the system must be synchronized


Figure 4.2. Coherent receiver operating curves with Gaussian noise


Figure 4.3. Required acquisition preamble length for $S N R \doteq 4$


Figure 4.4. Required acquisition preamble length for $S N R=10$


Figure 4.5. Sampling of the autocorrelation function
to within a few chip times. For data rates and network diameters of interest this limits chip rates to several megahertz. The resulting process gain is in the low thousands. This is not much gain over what can be attained with CCD matched filters while the cost of synchronization is high. Furthermore, as we will see in chapter 6, there are far more efficient protocols which rely on synchronization. If a spread spectrum system must also be synchronized much of the system's advantage in simplicity is lost.

## Chapter 5

## System Performance

This chapter discusses system performance measured primarily in terms of two related parameters: multipath outage and maximum supportable number of simultaneous users. While the meaning of the latter is obvious the former requires some explanation. As mentioned in chapter 2, multiple paths between transmitter and receiver cause fading which will be modeled as having a Rayleigh distribution. A minimum bit error rate (BER) is specified. If terminals are placed randomly, some links will not achieve the required BER due to fading. The probability that a link between two randomly chosen sites is nonoperational is called multipath outage. It may also be interpreted as the percentage of volume in which an antenna will not pick up a satisfactory signal from a given transmitter. I will also use the word reliability to refer to the complement of multipath outage: the percentage of volume in which a satisfactory signal can be received.

Multipath outage is determined in two ways. First, by making some simplifying assumptions, analytic results can be found. Secondly, to verify that the assumptions do not significantly reduce the accuracy of the results, a numerical method can be used with the aid of a digital computer. The primary assumptions that must be made for the analytic approach concern the codes used and the nature of the interference. The spread spectrum codes are assumed to be random. In other words the chips are independent and take on the values $\pm 1$ with equal probability. The other assumption is that the multi-access interference, $I$, can be modeled as a Gaussian process. This is based on central limit theorem arguments. We are interested in systems with many users. The interference terms from these users are independent (because of the first
assumption) and add as in equation (4.3) so this assumption follows from the central limit theorem.

At the receiver the signal is correlated with the receiver code. Interference is caused by nonzero crosscorrelation of the receiver code and the unwanted signals. We assume that this interference is Gaussian. Appendix 5A shows that the interference has zero mean and variance given by

$$
\begin{equation*}
E\left[I^{2}\right]=\sum_{k=1}^{K-1} \frac{E_{b_{k}}}{3 N_{c}} \tag{5.1}
\end{equation*}
$$

where $E_{b_{k}}$ is the bit energy of the $k$ th signal. The mean interference power averaged over the Rayleigh fading statistics is

$$
\begin{equation*}
\frac{\bar{E}_{b}}{3 N_{c} / N_{I}}, \quad \bar{E}_{b} \equiv E_{\text {fade }}\left[E_{b_{k}}\right], \quad N_{I} \equiv K-1 \tag{5.2}
\end{equation*}
$$

where $\bar{E}_{b}$ is is the average bit energy and $N_{I}$ is the number of interferers.
Let $C$ be the ratio of (5.1) and (5.2).
;

$$
\begin{equation*}
C=\sum_{k=1}^{K-1} \frac{E_{b_{k}}}{N_{I} \bar{E}_{b}} \tag{5.3}
\end{equation*}
$$

$C$ is a measure of excess interference. $C$ is a sum of squares of Rayleigh random variables. From any elementary probability text we can find that the square of a Rayleigh random variable has an exponential distribution and the sum of exponentials has a gamma distribution. The probability distribution function for $C$ is given by

$$
\begin{equation*}
F(C)=1-\sum_{j=0}^{N_{I}-1} \frac{\left(C N_{I}\right)^{j} e^{-C N_{I}}}{j!} \tag{5.4}
\end{equation*}
$$

If the number of interferers exceeds 20 then $C>2$ with probability less than $10^{-4}$ :

Any communications textbook (e.g., [42]) gives the BER of DPSK in additive Gaussian noise

$$
\begin{equation*}
P_{E}=\frac{1}{2} \exp \left(-E_{b_{11}} / 2 V\right) \tag{5.5}
\end{equation*}
$$

where $V$ is the variance of the noise. In this case this variance is given by

$$
\begin{equation*}
V=\frac{N_{0}}{2}+\frac{C N_{I} \bar{E}_{b}}{3 N_{c}} \tag{5.6}
\end{equation*}
$$

In the interference dominated case (5.5) simplifies to

$$
\begin{equation*}
P_{E}=\frac{1}{2} \exp \left(-\frac{3 A_{0}^{2} N_{c}}{2 C N_{I}}\right), \quad \frac{\bar{E}_{b}}{N_{0}} \gg \frac{3 N_{c}}{2 C N_{I}} \tag{5.7}
\end{equation*}
$$

If we specify a probability of error $\hat{P}_{E}$, we can solve for acceptable fade

$$
\begin{equation*}
A_{0}^{2} \geq \frac{2 C}{3 N_{c} / N_{I}} \ln \frac{1}{2 \hat{P}_{E}} . \tag{5.8}
\end{equation*}
$$

The density function for the squared fade is exponential:

$$
\begin{equation*}
\operatorname{Prob}\left[A_{0}^{2} \geq z\right]=e^{-z} \tag{5.9}
\end{equation*}
$$

Thus reliability is given by

$$
\begin{equation*}
R=\left(2 \hat{P}_{E}\right)^{\frac{2 C}{3 N_{c} / N_{I}}} . \tag{5.10}
\end{equation*}
$$

Figures 5.1 and 5.2 give reliability as a function of $\hat{P}_{E}$ and $N_{c} / N_{I}$ for excess noise factors of $C=1$ and 2 respectively. $N_{c} / N_{I}$ can be interpreted as a system bandwidth expansion factor. Figure 5.3 shows the required $\bar{E}_{b} / N_{0}$ in the absence of interference and the required $N_{c} / C N_{I}$ in the absence of thermal noise as a function of required reliability and $\hat{P}_{E}$. Table 5.1 below shows the required $\bar{E}_{b} / N_{0}$ in dB as a function of required reliability, $\hat{P}_{E}$, and $C$. No entry is given if the reliability can not be achieved under the given conditions. It can be seen that in most cases either the requirements

| Table 5.1. Required $\bar{E}_{b} / N_{0}$ in dB |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $R$ | . 9800 | . 9500 | . 9000 | . 8000 | . 6837 |
| C | $\hat{P}_{E}$ | $N_{c} / N_{I}$ |  |  |  |  |  |
| 1 | $10^{-2}$ | 10 | - | - | - | - | 15.2 |
|  |  | , 20 | - | - | - | 16.3 | 11.9 |
|  |  | 40 | - | - | 19.9 | 13.9 | 10.9 |
|  |  | 80 | - | 23.2 | 17.3 | 13.1 | 10.5 |
|  | $10^{-4}$ | 10 | - | - | - | - | - |
|  |  | 20 | - | - | - | - | 19.5 |
|  |  | 40 | - | - | - | 20.2 | 15.5 |
|  |  | 80 | - | - | 23.9 | 17.5 | 14.4 |
|  | $10^{-6}$ | 10 | - | - | - | - | - |
|  |  | 20 | - | - | - | - | - |
|  |  | 40 | - | - | - | 34.7 | 19.1 |
|  |  | 80 | - | - | - | 20.6 | 16.9 |
| 2 | $10^{-2}$ | 10 | - | - | - | - | - |
|  |  | 20 | - | - | - | - | 15.2 |
|  |  | 40 | - | - | - | 16.3 | 11.9 |
|  |  | 80 | - | - | 19.9 | 13.9 | 10.9 |
|  | $10^{-4}$ | 10 | - | - | - | - | - |
|  |  | 20 | - | - | - | - | - |
|  |  | 40 | - | - | - | - | 19.5 |
|  |  | 80 | - | - | - | 20.2 | 15.5 |
|  | $10^{-6}$ | 10 | - | - | - | - | - |
|  |  | 20 | - | - | - | - | - |
|  |  | 40 | - | - | - | - | - |
|  |  | 80 | - | - | - | 34.7 | 19.1 |

on $\bar{E}_{b} / N_{0}$ are modest or they are entirely unachievable. In other words the system exhibits a threshold effect with respect to reliability.

We conclude from all this data that it is not possible to attain very high reliabilities. Diversity methods to counteract this problem will be discussed later. If we can satisfy ourselves with reliability of .6 or lower we will be able to achieve low error rates $\left(10^{-6}\right)$ with bandwidth expansion factors $\left(N_{c} / N_{I}\right)$ of 20 to 40 as long as the excess interference does not exceed 2.

It is now time to dispense with the Gaussian approximation and the assumption of


Figure 5.1. Link reliability as a function of bandwidth expansion for excess interference $C=1$


Figure 5.2. Link reliability as a function of bandwidth expansion for excess interference $C=2$


Figure 5.3. System requirements as a function of reliability
random codes. Without these assumptions analysis becomes much more difficult and results must be obtained numerically with the help of a computer. In this section Gold codes are used. Gold codes are formed from maximal length shift register sequences (msequences). M-sequences can be generated using shift registers by appropriate choice of feedback. An $n$-stage shift register produces a sequence of length $2^{n}-1$. Gold codes are formed by the modulo 2 addition (exclusive or) of two distinct m-sequences of equal length. There are some other restrictions on how the two sequences are chosen. For details on both m-sequences and Gold sequences see [39]. Since the addition can be performed with arbitrary time shift between the two m-sequences, $2^{n}+1$ Gold sequences are formed including the two m-sequences.

The generation of Gold sequences requires only two $n$-stage tapped shift registers and some exclusive or circuits for the addition and the feedback. To generate random codes the entire sequence must be stored. Consequently, Gold codes are much more practical and are commonly used today. They also have very desirable autocorrelation properties so synchronization can be acquired. Their crosscorrelation properties allow interference to be kept low.

In the following paragraphs I will extend the characteristic function method of [38] to include Rayleigh fading. We define the total interference, $I$, as

$$
\begin{equation*}
I=\sum_{k=1}^{K-1} A_{k} I_{k, 0}\left(b_{1}^{(k)}, b_{2}^{(k)}, t_{k}, \varphi_{k}\right) \tag{5.11}
\end{equation*}
$$

where the functions $I_{k, 0}$ are defined in (4.4). The probability of error at the distinguished receiver is

$$
\begin{align*}
P_{E} & =\frac{1}{2} \operatorname{Prob}\left[y \leq 0 \mid b_{1}^{(0)}=1\right]+\frac{1}{2} \operatorname{Prob}\left[y \geq 0 \mid b_{1}^{(0)}=-1\right] \\
& =\frac{1}{2} \operatorname{Prob}\left[\eta^{\prime}+I \leq-A_{0}\right]+\frac{1}{2} \operatorname{Prob}\left[\eta^{\prime}+I \geq A_{0}\right]  \tag{5.12}\\
& =\frac{1}{2}-\frac{1}{2} \operatorname{Prob}\left[-A_{0} \leq \eta^{\prime}+I \leq A_{0}\right],
\end{align*}
$$

where $\eta^{\prime} \equiv\left(P T^{2} / 2\right)^{-1 / 2} \eta$ is the normalized thermal noise with variance $N_{0} / 2 \bar{E}_{b}$ where $\bar{E}_{b}=P T$ is the bit energy averaged over the fading.

The characteristic function of $\eta^{\prime}$ is

$$
\begin{equation*}
\phi_{2}(u)=E\left[e^{j u \eta^{\prime}}\right]=\exp \left[-\left(N_{0} / 4 \bar{E}_{b}\right) u^{2}\right] \tag{5.13}
\end{equation*}
$$

Similarly, we let $\phi_{1}(u)$ and $\phi(u)$ denote the characteristic functions of $I$ and $\eta^{\prime}+I$ respectively. These functions are all real-valued and symmetric and hence we can use the inverse transform to write

$$
\begin{equation*}
P_{E}=\frac{1}{2}-\frac{1}{\pi} \int_{0}^{A_{u}} \int_{0}^{\infty} \phi(u) \cos (u x) d u d x \tag{5.14}
\end{equation*}
$$

Performing the outer integration and using the fact,

$$
\begin{equation*}
\phi(u)=\phi_{2}(u)-\phi_{2}(u)\left[1-\phi_{1}(u)\right] \tag{5.15}
\end{equation*}
$$

gives

$$
\begin{equation*}
P_{E}=\frac{1}{2}-\frac{1}{\pi} \int_{0}^{\infty} \cdot \frac{\sin \left(u A_{0}\right)}{u} \phi_{2}(u) d u+\frac{1}{\pi} \int_{0}^{\infty} \frac{\sin \left(u A_{0}\right)}{u} \phi_{2}(u)\left[1-\phi_{1}(u)\right] d u \tag{5.16}
\end{equation*}
$$

Solving the first integral gives

$$
\begin{equation*}
P_{E}=Q\left(A_{0} \sqrt{2 \bar{E}_{b} / N_{0}}\right)+\frac{1}{\pi} \int_{0}^{\infty} \frac{\sin \left(u A_{0}\right)}{u} \phi_{2}(u)\left[1-\phi_{1}(u)\right] d u \tag{5.17}
\end{equation*}
$$

The multi-access interference is a sum of independent terms so its characteristic function can be written as a product

$$
\begin{align*}
& \phi_{1}(u)=\prod_{k=1}^{K-1}\left\{\frac{1}{8 \pi T} \sum_{b_{1}, b_{2}}\right. \int_{0}^{2 \pi} \int_{0}^{T} \int_{0}^{\infty} \\
&\left.2 A_{k} \exp \left[-A_{k}^{2}+\jmath u A_{k} I_{k, 0}\left(b_{1}^{(k)}, b_{2}^{(k)}, t_{k}, \varphi_{k}\right)\right] d A_{k} d t_{k} d \varphi_{k}\right\} \tag{5.18}
\end{align*}
$$

Some algebraic handiwork reduces this to

$$
\begin{equation*}
\phi_{1}(u)=\prod_{k=1}^{K-1}\left\{\frac{1}{2 N_{c}} \sum_{l=0}^{N_{c}-1} \int_{0}^{\infty} 2 A_{k} e^{-A_{k}^{2}}\left[f\left(u A_{k} ; l, \theta_{k, 0}\right)+f\left(u A_{k} ; l, \hat{\theta}_{k, 0}\right)\right] d A_{k}\right\} \tag{5.19}
\end{equation*}
$$

where we define the function $f(u ; l, g)$ by

$$
\begin{equation*}
f(u ; l, g)=\frac{2}{\pi T_{c}} \int_{0}^{\pi / 2} \int_{0}^{T_{c}} \cos \left\{u\left[g(l+1) R_{q}(t)+g(l) \hat{R}_{q}(t)\right] \frac{\cos \varphi}{T}\right\} d t d \varphi \tag{5.20}
\end{equation*}
$$

and $\theta$ and $\hat{\theta}$ are periodic and odd crosscorrelation functions given by

$$
\begin{align*}
& \theta_{k, 0}(l)=C_{k, 0}(l)+C_{k, 0}\left(l-N_{c}\right), \\
& \hat{\theta}_{k, 0}(t)=C_{k, 0}(l)-C_{k, 0}\left(l-N_{c}\right) . \tag{5.21}
\end{align*}
$$

Assuming rectangular chips we use (4.8) to compute the inner integral

$$
\begin{align*}
f(u ; l, g) & =\frac{2}{\pi} \int_{0}^{\pi / 2} \operatorname{sinc}\left\{u[g(l+1)-g(l)] \frac{\cos \varphi}{2 \pi N_{c}}\right\} \cos \left\{u[g(l+1)+g(l)] \frac{\cos \varphi}{2 N_{c}}\right\} d \varphi \\
\operatorname{sinc}(x) & \equiv \frac{\sin \pi x}{\pi x} \tag{5.22}
\end{align*}
$$

It is also possible to compute the integral found in (5.19) by using (5.22). Using the trigonometric identities

$$
\begin{equation*}
\sin (a x) \sin (b x)=\frac{1}{2}[\sin (a+b) x+\sin (a-b) x], \quad \sin (-x)=-\sin (x) \tag{5.23}
\end{equation*}
$$

and reversing the order of integration gives

$$
\begin{align*}
h(u ; l, g) \equiv & \int_{0}^{\infty} 2 A e^{-A^{2}} f(u A ; l, g) d A \\
= & \frac{4 N_{c}}{\pi u[g(l+1)-g(l)]} . \\
& \int_{0}^{\pi / 2} \int_{0}^{\infty} \frac{e^{-A^{2}}}{\cos \varphi}\left[\sin \left(\frac{A u g(l+1) \cos \varphi}{N_{c}}\right)-\sin \left(\frac{A u g(l) \cos \varphi}{N_{c}}\right)\right] d A d \varphi . \tag{5.24}
\end{align*}
$$

Using (equation (7.4.7) from [43])

$$
\begin{equation*}
\int_{0}^{\infty} e^{c^{2} t^{2}} \sin 2 x t d t=\frac{1}{c} e^{-x^{2} / c^{2}} \int_{0}^{x / c} e^{t^{2}} d t=\frac{1}{c} D i(x / c) \tag{5.25}
\end{equation*}
$$

where $D i(x)$ is Dawson's integral, we write

$$
\begin{equation*}
h(u ; l, g)=\frac{4 N_{c}}{\pi} \int_{0}^{\pi / 2} \frac{D i\left(\frac{u g(l+1) \cos \varphi}{2 N_{c}}\right)-D i\left(\frac{u g(l) \cos \varphi}{2 N_{r}}\right)}{u[g(l+1)-g(l)] \cos \varphi} d \varphi . \tag{5.26}
\end{equation*}
$$

This allows us to write the characteristic function as

$$
\begin{equation*}
\phi_{1}(u)=\prod_{k=1}^{K-1}\left\{\frac{1}{2 N_{c}} \sum_{l=0}^{N_{c}-1}\left[h\left(u ; l, \theta_{k, 0}\right)+h\left(u ; l, \hat{\theta}_{k, 0}\right)\right]\right\} . \tag{5.27}
\end{equation*}
$$

To evaluate this we need to compute Dawson's integral by [43]

$$
\begin{equation*}
D i(x)=-D i(-x) \equiv e^{-x^{2}} \int_{0}^{x} e^{t^{2}} d t=\sum_{n=0}^{\infty} \frac{(-2)^{n} x^{2 n+1}}{\prod_{i=0}^{n} 2 i+1} \tag{5.28}
\end{equation*}
$$

This can be evaluated to any desired accuracy by truncating the sum. We will also encounter several indeterminate forms:

$$
\begin{align*}
& \lim \frac{D i\left(\frac{u g(l+1) \cos \varphi}{2 N_{r}}\right)-D i\left(\frac{u g(l) \cos \varphi}{2 N_{c}}\right)}{u[g(l+1)-g(l)] \cos \varphi}= \\
& \begin{cases}\frac{1}{2 N_{c}}, & u \cos \varphi \rightarrow 0, \\
\frac{1}{2 N_{c}}\left[1-\frac{u g(l) \cos \varphi}{N_{c}} D i\left(\frac{u g(l) \cos \varphi}{N_{r}}\right)\right], & g(l+1) \rightarrow g(l) .\end{cases} \tag{5.29}
\end{align*}
$$

Probability of error is now determined by numerically computing the integral in (5.17) which in turn requires numerical evaluation of (5.27) using (5.28) and (5.29). The integral is truncated at $L \pi$ and evaluated by the composite Simpson's rule. Since

$$
\begin{equation*}
\left|\frac{\sin u A}{u}\right| \leq \frac{1}{L \pi}, \quad u \geq L \pi \tag{5.30}
\end{equation*}
$$

the truncation error is bounded by

$$
\begin{align*}
\frac{1}{\pi} \int_{L \pi}^{\infty} \frac{\sin (u A)}{u} \phi_{2}(u)\left[1-\phi_{1}(u)\right] d u & \leq \frac{2}{L \pi^{2}} \int_{L \pi}^{\infty} \phi_{2}(u) d u \\
& =\sqrt{\frac{16 \bar{E}_{b}}{L^{2} \pi^{3} N_{0}}} Q\left(\frac{L \pi}{\sqrt{2 \bar{E}_{b} / N_{0}}}\right) \tag{5.31}
\end{align*}
$$

Thus the truncation error can also be made arbitrarily small by choice of $L$. The computational load grows linearly with the product of code length and number of interferers. Most of the work is involved in the evaluation of (5.27) which does not depend on $\bar{E}_{b} / N_{0}$ so the computation of error rate at different noise levels can be done at little expense. It should be noted from (5.31), however, that the number of points on the $u$-axis at which (5.27) must be evaluated depends very much on $\bar{E}_{b} / N_{0}$. I was unable to obtain results for $\bar{E}_{b} / N_{0}$ in excess of 17 dB .

The algorithm described above was implemented on a VAX 11/785 computer in PASCAL. Figures $5.4-7$ show some of the data obtained. In all cases randomly selected 255-chip Gold codes were used. Curves marked as theoretical come from an analysis like that described earlier in the chapter. These curves show that the analytic approach, which is much easier, is adequate in most cases. Figure 5.4 shows the effect of fading of the desired signal on probability of error for various noise levels and interference from 11 other users. Figure 5.5 shows the dramatic effect of increasing the number of interferers. If the desired signal is faded 1 dB and $\bar{E}_{b} / N_{0}$ is 17 dB , the probability of error increases by an order of magnitude each time the number of users is doubled. Figure 5.6 shows the same effect in a different way as well as demonstrating a noise floor when the thermal noise level is high. One would also find that if the interference level were high compared to the thermal noise, an error rate floor would be reached where reducing the noise would have no effect. If we specify an allowable probability
of error we can determine from the data what fading level is allowable and hence the probability of achieving this goal. This is depicted in Figure 5.7 with $\bar{E}_{b} / N_{0}=17$ dB. The horizontal axis in this plot ir $N_{I} / N_{c}$ which can be interpreted as bandwidth efficiency.

Figure 5.7 demonstrates that unless we are willing to live with low bandwidth efficiency, low reliability, and high BER it is necessary to somehow counteract fading. The easiest way to do this is to use antenna diversity. Recall from chapter 2 that two antennas situated more than a quarter wavelength apart will receive signals with independent fades. Thus if the carrier frequency is chosen appropriately antennas placed at opposite corners of the terminal will receive independently faded signals. The receiver chooses the stronger signal (A more sophisticated receiver would form a weighted sur :.) Ghereby changing the effective fade distribution. If $\mathcal{R}_{1}$ is the reliability achieved with a single antenna then $R_{D}$, the reliability achieved with $D$ antennas, is given by

$$
\begin{equation*}
R_{D}=1-\left(1-R_{1}\right)^{D} \tag{5.32}
\end{equation*}
$$

Thus diversity greatly increases reliability. For example, if $R_{1}$ is only .6837 (the last column in Table 5.1), an additional antenna will increase the reliability to 9 .

Is diversity sufficient to allow us to establish a fully connected network? To determine this we divide space up in to cells such that fading in different cells is independent. Suppose we have a network of $N_{s}$ terminals (subscribers) each of which has $D$ receiver antennas giving a reliability $R_{D}$ as in (5.32). Further suppose that we will site the terminals in sequence so as to establish an adequate link to all the previously fixed stations. The procedure is to search up to $J$ cells in combinations of $D$ until a good combination is found. No single cell will be tried more than once so at most $J / D$ combinations will be tried. Terminal $n+1$ must establish links with $n$ other terminals.


Figure 5.4. Effect of signal fading on $P_{E}$


Figure 5.5. Effect of fading and interference on $P_{E}$


Figure 5.6. Effect of interference on $P_{E}$


Figure 5.7. Effect of bandwidth efficiency on reliability

A particular combination of cells will succeed with probability $1-R_{D}^{n}$. This may be retried $J / D$ times giving the terminal a probability of success, $P_{n+1}$

$$
\begin{equation*}
P_{n+1}=1-\left(1-R_{D}^{n}\right)^{J / D} . \tag{5.33}
\end{equation*}
$$

The probability that the entire system can be connected is thus

$$
\begin{equation*}
P_{\text {sys }}=\prod_{n=2}^{N_{n}} P_{n}=\prod_{n=1}^{N_{n}-1} 1-\left(1-R_{D}^{n}\right)^{J / D} \tag{5.34}
\end{equation*}
$$

In the case $D=1$, let us suppose that we determine what signal to interference ratio ( $S I R \equiv N_{c} E_{b_{0}} / C N_{I} \bar{E}_{b}$ ) will give adequate performance. We accept a cell if the signal received from each transmitter has bit energy at least $\gamma \bar{E}_{b}$. How should $\gamma$ be chosen? The received energies will have a truncated exponential distribution with mean $\bar{E}_{b}(\gamma+1)$. In the worst case the desired signal will have eriergy $\gamma \bar{E}_{b}$ and the interference will have energy $C N_{I} \bar{E}_{b}(\gamma+1)$. After despreading we have

$$
\begin{equation*}
S I R \geq \frac{3 N_{c} \gamma}{C N_{I}(\gamma+1)} \tag{5.35}
\end{equation*}
$$

In terms of $\gamma$ we have

$$
\begin{equation*}
\gamma=\frac{S I R_{\min } C}{\left(3 N_{c} / N_{I}\right)-S I R_{\min } C} \tag{5.36}
\end{equation*}
$$

The probability that the energy of a signal exceeds the required $\gamma \bar{E}_{b}$, that is the reliability, is just $e^{-\gamma}$. A reasonable tradeoff between bandwidth efficiency and synchronization requirements suggests that $N_{c} / N_{I}$ be in the range 20 to 40 . To allow for a maximum excess interference $(C)$ of 2 and a minimum $S I R$ of 10 dB leads to reliabilities, $R_{1}$, in the range .6 to .8 .

Generously, the number of cells searched is unlikely to exceed 50 before the installer becomes frustrated. Table 5.2 below gives the probability of finding fixed sites for all terminals in order to establish a fully connected network. It is apparent that in a large
network it is simply impossible to maintain a fully connected network. While this is a serious drawback in some situations, in many cases it is easily overcome. In a typical local area network made up of ASCII terminals, personal computers, and larger host computers, it is rare that one station needs to communicate with a large percentage of the other station in any short period. We assumed at the outset that terminals were portable so, if a required link has an unacceptable BER, all we need to do is move the terminal (or, if they are exposed, just the antennas) several inches into the next cell. This is essentially a primitive form of diversity. With error detection or power monitoring, it is possible to have the terminal inform the user that the link needs adjustment.

| Table 5.2. Probability of Full Connection with 50-cell Search |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| $:$ |  | $N_{s}$ | 10 | 20 | 100 | 200 |
| $:$ |  |  |  |  |  |  |
| $:$ |  | $R_{1}$ |  |  |  |  |
|  | 1 | .6 | $4.0 \times 10^{-2}$ | $1.3 \times 10^{-19}$ | 0.0 | 0.0 |
|  |  | .8 | 0.996 | $4.7 \times 10^{-2}$ | 0.0 | 0.0 |
|  | 2 | .6 | 0.988 | $8.5 \times 10^{-2}$ | 0.0 | 0.0 |
|  |  | .8 | 1.000 | 1.000 | $3.3 \times 10^{-10}$ | 0.0 |
|  | 3 | .6 | 1.000 | 0.985 | 0.0 | 0.0 |
|  |  | .8 | 1.000 | 1.000 | 0.999 | 0.558 |

We covered a lot in this chapter, but really the conclusions are rather simple. Using a Gaussian approximation we found that to achieve low error rates without unreasonably low reliability we need a bandwidth expansion factor $\left(N_{c} / N_{I}\right)$ of at least 20. We also found that without antenna diversity it is nearly impossible to achieve reliability near unity. Some complicated and time-consuming numerical work merely served to confirm these observations. We also found that the use of two (or more) antennas improves the situation greatly but still does not eliminate multipath outage. It seems that a certain amount of antenna tweaking comes with the territory.

## Appendix 5A

In this appendix we compute the mean and variance of the interference, $I$. Since the product of the data and the chips has the same statistics as the chips alone we can ignore data modulation. Since all chips have identical statistics we may also assume, without loss of generality, that the time delays $\left(t_{k}\right)$ are all in the range $\left[0, T_{c}\right]$ by considering the delays modulo $T_{c}$. Using rectangular chip pulses, the interference (after correlation) is given by

$$
\begin{align*}
I= & \sum_{k=1}^{K-1} \sum_{i=1}^{N_{c}} \frac{2 \sqrt{E_{b_{k}}}}{T_{c} N_{c}}\left[\int_{0}^{t_{k}} a_{i-1}^{(k)} a_{i}^{(0)} \cos \left(\omega_{I F} t+\varphi_{k}\right) \cos \omega_{I F} t d t\right. \\
& \left.+\int_{t_{k}}^{T_{c}} a_{i}^{(k)} a_{i}^{(0)} \cos \left(\omega_{I F} t+\varphi_{k}\right) \cos \omega_{I F} t d t\right] \\
= & \sum_{k=1}^{K-1} \sum_{i=1}^{N_{c}} \frac{2 \sqrt{E_{b_{k}}} a_{i}^{(0)}}{T_{c} N_{c}}  \tag{5A.1}\\
& \left\{a_{i-1}^{(k)}\left[\cos \varphi_{k}\left(\frac{t_{k}}{2}+\frac{1}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}\right)-\frac{\sin \varphi_{k}}{2 \omega_{I F}} \sin ^{2} \omega_{I F} t_{k}\right)\right] \\
& \left.\left.+a_{i}^{(k)}\left[\cos \varphi_{k}\left(\frac{T_{c}-t_{k}}{2}-\frac{1}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}\right)+\frac{\sin \varphi_{k}}{2 \omega_{I F}} \sin ^{2} \omega_{I F} t_{k}\right)\right]\right\}
\end{align*}
$$

where $E_{b_{k}}=a_{k}^{2} P T$ is the bit energy of the $k$ th signal and, as in previous chapters, $\omega_{I F}$ is the intermediate frequency, $P$ is the mean signal power, and $A_{k}, \varphi_{k}$ and $a_{i}^{(k)}$ are the normalized amplitude, phase, and $i$ th chip of the $k$ th signal. Also recall that $K, N_{c}, T_{c}$, and $T$ are the number of active users, the number of chips per bit, the chip duration, and the bit duration respectively. The chips have zero mean so clearly the mean interference is zero.

Since (5A.1) is a double sum of independent terms we can compute the variance as a sum of variances for each term. The variance of a single term (denoted $\overline{r^{2}}$ ) is found
by squaring and taking the expectation

$$
\begin{align*}
\overline{r^{2}}=E & {\left[\frac { 4 E _ { b _ { k } } } { T _ { c } ^ { 2 } N _ { c } ^ { 2 } } \left\{\cos ^{2} \varphi_{k}\left(\frac{t_{k}^{2}}{4}+\frac{t_{k}}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}+\frac{1}{16 \omega_{I F}^{2}} \sin ^{2} 2 \omega_{I F} t_{k}\right)\right.\right.} \\
& \quad-\frac{\sin 2 \varphi_{k}}{2 \omega_{I F}}\left(\frac{t_{k}}{2}+\frac{1}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}\right) \sin ^{2} \omega_{I F} t_{k}+\frac{\sin ^{2} \varphi_{k}}{4 \omega_{I F}^{2}} \sin ^{4} \omega_{I F} t_{k} \\
& +2 a_{i-1}^{(k)} a_{i}^{(k)}\left(-\frac{\sin ^{2} \varphi_{k}}{4 \omega_{I F}^{2}} \sin ^{4} \omega_{I F} t_{k}\right. \\
& +\cos ^{2} \varphi_{k}\left(\frac{t_{k}\left(T_{c}-t_{k}\right)}{4}+\frac{T_{c}-2 t_{k}}{8 \omega_{I F}} \sin 2 \omega_{I F} t_{k}-\frac{1}{16 \omega_{I F}^{2}} \sin ^{2} 2 \omega_{I F} t_{k}\right) \\
& \left.\quad+\frac{\sin 2 \varphi_{k}}{4 \omega_{I F}} \sin ^{2} \omega_{I F} t_{k}\left(\frac{2 t_{k}-T_{c}}{2}+\frac{1}{2 \omega_{I F}} \sin 2 \omega_{I F} t_{k}\right)\right) \\
& +\cos ^{2} \varphi_{k}\left(\frac{\left(T_{c}-t_{k}\right)^{2}}{4}-\frac{T_{c}-t_{k}}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}+\frac{1}{16 \omega_{I F}^{2}} \sin ^{2} 2 \omega_{I F} t_{k}\right) \\
& \left.\left.+\frac{\sin 2 \varphi_{k}}{2 \omega_{I F}}\left(\frac{T_{c}-t_{k}}{2}-\frac{1}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}\right) \sin ^{2} \omega_{I F}^{\prime} t_{k}+\frac{\sin ^{2} \varphi_{k}}{4 \omega_{I F}^{2}} \sin ^{4} \omega_{I F} t_{k}\right\}\right] \tag{5A.2}
\end{align*}
$$

The phase variable is uniformly distributed on $[0,2 \pi]$. To take the expectation over the phase variable and the chips we use

$$
\begin{align*}
E\left[\cos ^{2} \varphi_{k}\right] & =E\left[\sin ^{2} \varphi_{k}\right]=\frac{1}{2} \\
E\left[\sin 2 \varphi_{k}\right] & =0  \tag{5A.3}\\
E\left[a_{i-1}^{(k)} a_{i}^{(k)}\right] & =0
\end{align*}
$$

Plugging these in gives

$$
\begin{align*}
\overline{r^{2}}=E & {\left[\frac { 2 E _ { b _ { k } } } { T _ { c } ^ { 2 } N _ { c } ^ { 2 } } \left(\frac{t_{k}^{2}}{4}+\frac{\left(T_{c}-t_{k}\right)^{2}}{4}\right.\right.}  \tag{5A.4}\\
& \left.\left.+\frac{2 t_{k}-T_{c}}{4 \omega_{I F}} \sin 2 \omega_{I F} t_{k}+\frac{1}{8 \omega_{I F}^{2}} \sin ^{2} 2 \omega_{I F} t_{k}+\frac{1}{2 \omega_{I F}^{2}} \sin ^{4} \omega_{I F} t_{k}\right)\right]
\end{align*}
$$

Now to take the expectation over the delay variable, which is uniformly distributed over $\left[0, T_{c}\right]$, we use

$$
\begin{align*}
E\left[t_{k}^{2}\right] & =E\left[\left(T_{c}-t_{k}\right)^{2}\right]=\frac{T_{c}^{2}}{3} \\
E\left[\sin ^{2} 2 \omega_{I F} t_{k}\right] & =\frac{1}{2} \\
E\left[\sin ^{4} \omega_{I F} t_{k}\right] & =\frac{3}{8}  \tag{5A.5}\\
E\left[t_{k} \sin 2 \omega_{I F} t_{k}\right] & =-\frac{1}{2 \omega_{I F}} \\
E\left[\sin 2 \omega_{I F} t_{k}\right] & =0
\end{align*}
$$

leading to

$$
\begin{equation*}
\overline{r^{2}}=\frac{E_{b_{k}}}{3 N_{c}^{2}} \tag{5A.6}
\end{equation*}
$$

There are $N_{c}$ terms in the inner sum in (5A.1) so the variance of the interference from the $k$ th user is

$$
\begin{equation*}
E\left[I_{k, 0}^{2}\right]=\frac{E_{b_{k}}}{3 N_{c}} . \tag{5A.7}
\end{equation*}
$$

Summing over the interferers gives the variance of the interference

$$
\begin{equation*}
E\left[I^{2}\right]=\sum_{k=1}^{K-1} \frac{E_{b_{k}}}{3 N_{c}} \tag{5A.8}
\end{equation*}
$$

This is the desired result.

## Chapter 6

## Other Multi-access Protocols

This chapter compares the spread spectrum system discussed in the previous chapters with other network strategies. Many multi-access networks have been proposed and implemented. Some systems are appropriate for wire-based networks while others may be used in a radio environment. Some systems allocate bandwidth to users in a fixed way while others allocate bandwidth randomly through contention or according to demand. There are also other spread spectrum methods available. This chapter briefly discusses the relative merits of a broad range of systems covering examples of all the possibilities mentioned above.

The primary advantages of spread spectrum systems are freedom from cabling and simplicity. I call spread spectrum simple because it requires no cooperation between terminals. Of course I have not really specified a complete protocol. I have not specified any procedure for recovering from (or even detecting) bit errors or for preventing two terminals from transmitting to the same receiver simultaneously. Dealing with these issues may add complexity to spread spectrum. If system-wide synchronization is required to accomodate correlation receivers, then complexity is certainly increased.

While comparisons based on whether a system requires cabling are completely straightforward, simplicity is difficult to quantify. Most of this chapter dwells upon more traditional (and easily quantified) performance measures: bandwidth efficiency and delay. These are used to compare systems in terms of the number of users that can be supported with comparable delay. Spread spectrum does not use bandwidth in the most efficient manner nor does it have dazzling delay performance. As a result, much of this chapter may give a rather negative view of spread spectrum. The point is not
that spread spectrum is bad. The point is that spread spectrum should be considered only if freedom from cabling and simplicity are major priorities.

While copper or fiber-based networks have all the disadvantages mentioned in chapter 1 , it is worthwhile seeing what we lose by giving up cables. Cable-based networks have an intrinsic performance advantage. They can use the cable to control multi-access interference by spatially separating signals. They also eliminate multipath fading. In some cable-based topologies it is also possible to synchronize the network with arbitrary accuracy at all points.

To determine the capabilities of cable-based networks I will use Ethernet (Ethernet is a registered trademark of the Xerox corporation) as an example. Ethernet operates at 10 Mbps . It has a maximum diameter of 2.5 km and can have as many as 1024 nodes [44]. It uses a multi-access protocol called CSMA/CD (carrier sense multiple-access with collision detection). A user wishing to send a message first breaks the message up into packets of fixed maximum length. Before sending a packet the interface unit monitors the cable to determine if it is already in use. If a signal is found the packet is deferred for a later time; otherwise the packet is sent immediately. Due to propagation delay a terminal may fail to detect that the channel is already in use. Hence packet collisions may occur if two nodes begin transmitting within $T_{p}$ seconds of each other where $T_{p}$ is the propagation delay. While sending the packet the receiver continues to monitor the line and compares the received signal to the signal it is transmitting. If they do not match, a collision has occurred and the packet is retried later. Collision detection is not generally possible in a radio network since the locally generated signal usually overwhelms the receiver.

Let us consider the common measures of network performance: bandwidth efficiency (throughput) and mean packet delay. As I said before, these will be used to determine how many users can be supported with comparable delay. In the spread
spectrum system, the bandwidth efficiency is just $K / N_{c}$ where $K$ is the number of simultaneous users and $N_{c}$ is the number of chips in the code sequence which is assumed to be of the same duration as a bit. Since messages are sent as soon as they are ready, the delay is just the packet transmission time (denoted $T_{t}$ ) given by the ratio of the packet length in bits and the baud rate. For the purposes of comparison I will assume a baud rate of 1.2 kbps and a chip rate around 2.5 MHz leading to $N_{c}$ around 2100 which may eventually be within the limits of CCD technology.

Simple approximate formulas exist [45] for the efficiency, $S$, and mean delay, $T_{d}$, of networks of the Ethernet type:

$$
\begin{gather*}
S=\frac{T_{t}}{T_{t}+2 e T_{p}}  \tag{6.1}\\
T_{d}=\frac{2 e T_{p} K}{1-S} \tag{6.2}
\end{gather*}
$$

where $K$ is now the number of users waiting to send packets. If the load is heavy almost all active users will have packets waiting at any time. Figure 6.1 shows the relationship between efficiency and packet length assuming a 2.5 MHz data rate. If long packets are used efficiencies approaching unity can be achieved. Even using the maximum specified Ethernet round trip propagation delay of $45 \mu \mathrm{sec}$ (includes a long detection circuit response time) packets need only be 64 bits long (only 8 ASCII characters) to achieve $10 \%$ efficiency. From the previous chapter we know that this is about the best that can be expected from a spread spectrum system if we desire low error rates. Thus in hisost cases Ethernet is, as expected, much more efficient of bandwidth than spread spectrum.

The spread spectrum packet delay is just $L / 1200$ seconds where $L$ is the packet length in bits. Using the same length packets on an Ethernet system operating at


Figure 6.1. Ethernet bandwidth efficiency at 2.5 Mbps .

### 2.5 MHz results in a delay

$$
\begin{equation*}
T_{d}=\left(4 \times 10^{-7} L+2 e T_{p}^{-}\right) K \tag{6.3}
\end{equation*}
$$

The mean delay is less than the spread spectrum delay if

$$
\begin{equation*}
K<\frac{1}{1200\left(4 \times 10^{-7}+2 e\left(T_{p} / L\right)\right)} \approx \frac{2083}{1+13.6 \times 10^{6}\left(T_{p} / L\right)} \tag{6.4}
\end{equation*}
$$

This bound is plotted in figure 6.2 as a function of packet length with $T_{p}$ as a parameter. It is apparent that, if the load is heavy so that most of the active users have packets waiting, and if the packets are very short, the spread spectrum system may be able to support more users without excessive delay. However even if the packets are only 64 bits and the propagation delay is the maximum allowed an Ethernet system can handle about 200 active users. Thus, in most cases, spread spectrum should only be considered if avoiding wiring is a priority.

Now let us turn to radio networks. Perhaps the simplest strategy to provide a high-capacity (In this context capacity is defined as the greatest obtainable efficiency.) fair, multi-access radio channel is to allocate a fixed amount of bandwidth to each user. This can be done using either static time division multiple access (TDMA) or frequency division multiple access (FDMA). Static refers to the fixed allocation. Dynamic versions of these protocols exist but I do not consider them here.

In FDMA the total bandwidth is divided up into $N_{s}$ subbands where $N_{s}$ is the number of system subscribers. Each subscriber receives messages in its own subband without interference. Clearly if there are messages for all subscribers simultaneously then the efficiency will be $100 \%$. In general (assuming that all messages are intended for different users) the efficiency is $K / N_{s}$, the fraction of subscribers who are active. The bandwidth allocated to inactive users is wasted. Since users do not have to wait


Figure 6.2. Active users supported by 2.5 Mbps Ethernet with delay less' than spread spectrum
to gain access to the channel, the delay is just the packet transmission time. If $R$ is the data rate achievable by using the entire system bandwidth then the delay is given by

$$
\begin{equation*}
T_{d}=\frac{L N_{s}}{R} \tag{6.5}
\end{equation*}
$$

which is exactly the same delay suffered by a spread spectrum system with process gain $N_{c}=N_{s}$. Thus we see that FDMA has high efficiency and low delay if most of the subscribers are active at any given time. If most ( $90 \%$ or more) of the subscribers are inactive a spread spectrum system will provide more efficient channel usage and lower delay. A similar argument applies to any protocol with a fixed allocation.

For TDMA there is an additional snag. A TDMA system must be synchronized. Since perfect synchronization is not possible a guard band must be left between each time slot reducing the bandwidth efficiency and increasing the delay. In all fairness it should be said that many FDMA systems also require guard bands to reduce interference caused by imperfect filtering, oscillator drift, Doppler shifts, and receiver nonlinearities. Furthermore, TDMA does not have a problem when more than one message is sent to the same receiver. Time slots can be allocated to transmitters because, in contrast to FDMA (or spread spectrum), it is not difficult for the receiver to monitor all channels.

The simplest multi-access protocol that does not use fixed allocation is the ALOHA scheme. In its simplest form no synchronization is needed. Any user with a message to send simply sends it using the full system bandwidth. If no other user's transmission overlaps, the message will arrive successfully barring corruption by noise. If a collision occurs the transmitter waits a random amount of time and retransmits. Collisions may be detected by failing to receive an acknowledgement in some specified amount of time. This time must of course be at least equal to the round trip propagation delay, $2 T_{p}$.

If $G$ is the offered load (the total bit rate of the incoming packets divided by the maximum system bit rate) and the packet arrival process is assumed to be Poisson, then the bandwidth efficiency is given by [44]

$$
\begin{equation*}
S=G e^{-2 G} \tag{6.6}
\end{equation*}
$$

If $G=.5$ the capacity of $1 / 2 e=.18$ is reached. $S / G=e^{-2 G}$ can be interpreted as the probability of successful packet transmission. The probability that a packet requires $J$ transmissions before successful reception is given by

$$
\begin{equation*}
\operatorname{Prob}(J=j)=e^{-2 G}\left(1-e^{-2 G}\right)^{j-1}, \quad j=1,2,3 \ldots \tag{6.7}
\end{equation*}
$$

This is a geometric density with mean $e^{2 G}$. Let $H$ be the average time a terminal waits before retrying a transmission normalized by the packet transmission time. We must have $H \gg 1$ to avoid the same packets colliding continuously. Mean delay is then approximately

$$
\begin{equation*}
T_{d} \approx\left[\frac{G}{S}+H\left(\frac{G}{S}-1\right)\right] T_{t} \tag{6!8}
\end{equation*}
$$

At capacity this is approximately $H(e-1) T_{t}$. Thus an ALOHA system with $G<.5$ will have less mean delay than a spread spectrum system with the same system bandwidth as long as

$$
\begin{equation*}
H<\frac{N_{c}}{e-1} \approx .582 N_{c} \tag{6.9}
\end{equation*}
$$

If $N_{c}$ is large, as would be necessary to support many users, (6.9) is not a very stringent requirement. Thus ALOHA compares favorably with spread spectrum in terms of mean delay. On the other hand, the delay for spread spectrum is constant while some packets may have very long delays in an ALOHA system. Also, if packets have varying length, $H$ must be normalized by the maximum packet duration. Consequently
short packets will be delayed nearly as much as long packets. In the spread spectrum system delay is proportional to packet length.

ALOHA is also unstable. As the load is increased the efficiency is reduced by increased probability of collision. A similar statement can be made of spread spectrum systems: as the number of users and thus the load increases the error rate increases. Ultimately the error rate becomes unacceptable. If retransmissions of erroneous packets were requested, the system would also be unstable. Nonetheless, by monitoring error rate, the receiver knows that it has received a corrupted message and may send a negative acknowledgement. ALOHA requires positive acknowledgement. If packets tend to be short, acknowledgements may form a significant part of the load. As long as the packet error rate is less than .5 there is an advantage to using negative acknowledgements.

Now let us consider what is lost by insisting on simplicity. If we are willing to use a more sophisticated protocol, bandwidth can be allocated according to the needs of the user. One such protocol is BRAM (broadcast recognizing access method). In this protocol all subscribers are ordered. Each subscriber is given a time slot in round robin fashion. If a subscriber has a message to transmit it begins transmission in its time slot. All other users sense the transmission and suspend the round robin until it is complete. The time slots must be $2 T_{p}$ in length to allow for sensing the beginning and end of messages. If $q$ is the active fraction of users $\left(K / N_{s}\right)$ then the efficiency is given by

$$
\begin{equation*}
S=\frac{T_{t}}{T_{t}+\frac{2 T_{p}(1-q)}{q}} . \tag{6.10}
\end{equation*}
$$

Note the similarity in form to (6.1). Thus qualitatively the performance is similar to Ethernet performance. However, because there are no collisions and hence no need for collision detection, it can be implemented using radio links as long as the network is
fully connected (i.e., all receivers can detect signals from all transmitters). In terms of the offered load the response is nearly ideal as long as the propagation delay is small compared to the packet length. The efficiency is given by [44]

$$
\begin{equation*}
S=1+G-\sqrt{1+G^{2}} . \tag{6.11}
\end{equation*}
$$

When $G$ is small this is approximately $G$. When $G$ is large the channel saturates and a capacity of 1 is reached.

To determine mean delay we consider two cases. Ender light load a user waiting to send a message must simply wait for its slot. On average it will have to wait $N_{s} / 2$ slots for a mean delay of $N_{s} T_{p}$. This is smaller than the spread spectrum delay as long as $N_{s}<L T / T_{p}$. As an example suppose the network diameter is 600 meters (a large building) so $T_{p}=2 \mu \mathrm{secs}$. Even if the packets are single characters ( 8 bits ) a lightly loaded BRAM system can handle 3300 subscribers with less delay than the spread spectrum system. At the other extreme suppose every subscriber is waiting to send a message. Then the average delay is $N_{s} T_{t} / 2$. This will be less than our systems delay as long as $N_{s}<2 N_{c}$. Of course our system could not handle such a heavy load at all. We conclude our system will outperform BRAM only if $N_{s}$ is very large but the load is very light.

There is a variation on BRAM that is insensitive to the fraction of active users. To determine who accesses the channel next all users with messages begin broadcasting there addresses using binary on-off keying with one bit in each time slot. As soon as a user receives a 1 which it did not send, it determines that there is a user with higher priority and it stops broadcasting. After $\log _{2} N_{s}$ time slots the user with the highest address acquires the channel and sends its message. For fairness this user goes into a dormant state until the channel becomes idle either because no other user wishes to
use the channel or all other active users are also dormant. This approach is better if there are a very large number of subscribers most of whom are inactive.

BRAM requires synchronization. However synchronization is relative to the end of the most recent message so, while stable clocks are required, no external synchronization signal or master clock node is required. A more important objection is that collisions may occur if the signal from some transmitter is too weak to be detected at some points in the network. In this situation two signals may be sent simultaneously and while they will not interfere at either transmitter they may very well interfere at the intended receivers. In the spread spectrum system the same situation has the effect of reducing interference. Of course the two affected terminals can not communicate with each other directly in either system.

Let us summarize what we have learned so far in this chapter. If freedom from cables and simplicity are not important, performance can be improved greatly by using protocols which take advantage of cabling (such as Ethernet) or cooperation between. terminals (such as BRAM). We should consider spread spectrum radio if it is important to avoid wiring and complexity. It is not, however, the only option that should be ! considered. If most of the subscribers are active most of the time, a fixed allocation system is probably better. There are also conditions under which an ALOHA system may be more appropriate.

There are other spread spectrum techniques and I would be remiss if I did not say something about them. The rest of this chapter is devoted to frequency hopping. Frequency hopping systems, rather than reducing interference, try to avoid interference. There are $N_{f}$ carrier frequencies. Signals are sent on one carrier frequency for $T_{c}$ seconds and are then switched to a different carrier. Each receiver has two sequences of frequencies associated with it: one for a mark and the other for a space. Frequently the two sequences are identical except for a constant frequency shift. If, during a chip, no
other transmitter is using either of the possible carriers, then energy will be detected only at the frequency corresponding to the data. If another transmitter is using this frequency as well, no harm is done except in the unlikely event that the two signals exactly cancel. If the other frequency is being used, a hit is said to have occurred and the chip contains no information. Thus chips corrupted by interference are ignored.

To use frequency hopping for multiple access we need to find large sets of frequency hopping patterns with good properties. Essentially what is required of sequence sets is that they have low autocorrelation sidelobes and low crosscorrelation. The sequences we consider are periodic with period $N_{c} T_{c}$. On the interval $\left[0, N_{c} T_{c}\right]$ the $m$ th waveform is given by

$$
\begin{equation*}
s_{m}(t)=\sum_{l=0}^{N_{c}-1} f_{n(m, l)}\left(t-l T_{c}\right) \tag{6.12}
\end{equation*}
$$

where $n(m, l)$ is an integer between 0 and $N_{f}-1$, and $f_{n}(t)$, for $n=0,1, \ldots, N_{f}-1$, is a pulse time-limited to $\left[0, \boldsymbol{T}_{c}\right]$ that satisfies

$$
\int_{0}^{T_{c}} f_{n}(t) f_{p}(t) d t= \begin{cases}1, & \text { if } n=p  \tag{6.13}\\ 0, & \text { otherwise }\end{cases}
$$

The periodic crosscorrelation; $c_{i, j}(t)$, is defined as

$$
\begin{equation*}
c_{i, j}(t)=\int_{0}^{N_{c} T_{c}} s_{i}(r) s_{j}(r-t) d t \tag{6.14}
\end{equation*}
$$

For the special case $i=j$ this is called autocorrelation. For the time being we assume chip synchronous operation. Thus we are only interested in the value of crosscorrelation functions at multiples of $T_{c}$ where the value is just the number of matching chips. We define $c_{\max }$ as the value of $c_{i, j}(t)$ maximized over all such $t$ and $i, j=0, \ldots, M-1$ excluding the cases where $t=0$ and $i=j$.

Given $N_{c}, N_{f}$, and $c_{\max }$ we would like to have upper and lower bounds for the maximum achievable number of sequences, $M$. This will give us some idea how many
users a system can handle with some maximum level of interference. We begin by deriving a simple upper bound. Suppose we have $M$ sequences. Corresponding to each of the $N_{c}$ chip positions in each of the $M$ sequences there is a $k$-chip subsequence which starts at that location. There are thus $M N_{c}$ such subsequences. If any two of these are identical it follows that $c_{\max }$ is greater than or equal to $k$. Consequently if $k=c_{\text {max }}+1$ all $M N_{c}$ subsequences of length $k$ must be different. Since each of the chips is chosen from an alphabet of size $N_{f}$ there are only $N_{f}^{k}$ possible $k$-chip subsequences. We see immediately that

$$
\begin{equation*}
M \leq \frac{N_{f}^{c_{\max }+1}}{N_{c}} \tag{6.15}
\end{equation*}
$$

which is the desired upper bound.
To obtain a lower bound it is necessary to construct a set of sequences with the required properties. What follows is an extension and clarification of work done by Gustave Solomon [46]. Let the alphabet size, $N_{f}$, and the sequence length, $N_{c}$, be such that $N_{f}=p^{r}$ where $p$ is a prime number and $N_{c}$ divides $N_{f}-1$. In most cases it will be possible to find such a pair fairly close to any given design goal. Label each member of the alphabet in any convenient way with a different element of $G F\left(p^{r}\right)$, the Galois field of order $p^{r}$. Let $b$ be a primitive element of $G F\left(p^{r}\right)$. Define $Q \equiv\left(p^{r}-1\right) / N_{c}$ and let $a=b^{Q}$. Note that $a$ has order $N_{c}$. Define the sequences $x_{q}=\left(x_{i}^{(q)}\right)$, for $i=0, \ldots, N_{c}-1$, by

$$
\begin{equation*}
x_{i}^{(q)}=P_{x}\left(b^{q} a^{i}\right), \quad P_{x}(y)=y^{t}+\sum_{k=0}^{t-1} c_{x k} y^{k} \tag{6.16}
\end{equation*}
$$

where $q$ is an integer between 0 and $Q-1$ inclusive. Considering all possible coefficients for the polynomials and choices of $q$ the total number of sequences obtained is

$$
\begin{equation*}
M=N_{f}^{t} Q=\frac{N_{f}^{t}\left(N_{f}-1\right)}{N_{c}} \tag{6.17}
\end{equation*}
$$

We would like to find the crosscorrelation and autocorrelation of these sequences. First note that $P_{x}\left(a^{j} \cdot \vec{a}\right)=P_{x}\left(b^{Q j} \cdot \vec{a}\right)$, where $\vec{a}$ is the vector of powers of $a$, generates the same sequence as $P_{x}$ cyclicly shifted left by $j$ chips. To compute correlation we consider two sequences one of which is shifted by $j$ chips. The correlation is the number of positions in which these two sequences match. We are thus led to consider the roots of the equation

$$
\begin{equation*}
P_{x_{1}}\left(b^{q_{1}} y\right)-P_{x_{2}}\left(a^{j} b^{q_{2}} y\right)=\left(b^{q_{1} t}-b^{\left(j Q+q_{2}\right) t}\right) y^{t}+\sum_{k=0}^{t-1}\left(b^{q_{1} k} c_{x_{1} k}-b^{\left(j Q+q_{2}\right) k} c_{x_{2} k}\right) y^{k}=0 \tag{6.18}
\end{equation*}
$$

If the two polynomials are the same this reduces to

$$
\begin{equation*}
P_{x_{1}}\left(b^{q_{1}} y\right)-P_{x_{1}}\left(a^{j} b^{q_{2}} y\right)=\left(b^{q_{1} t}-b^{\left(j Q+q_{2}\right) t}\right) y^{t}+\sum_{k=0}^{t-1}\left(b^{q_{1} k}-b^{\left(j Q+q_{2}\right) k}\right) c_{x_{1} k} y^{k}=0 . \tag{6.19}
\end{equation*}
$$

Both of these formulas are written as polynomials of degree $t$ so if the coefficients are not identically zero they can have no more than $t$ roots and consequently $t$ is an upper bound on correlation (which may be lower than $t$ if we do not evaluate the polynomial at all roots or if the polynomial has irreducible factors). We would thus have the lower bound

$$
\begin{equation*}
M \geq \frac{N_{f}^{c_{\max }}\left(N_{f}-1\right)}{N_{c}} \tag{6.20}
\end{equation*}
$$

which differs from our upper bound only by the factor $\left(N_{f}-1\right) / N_{f}$. Unfortunately we cannot guarantee in general that the coefficients are not identically zero in (6.18) and (6.19), but there are four important speciai cases for which we can guarantee it.

If $t=1,(6.18)$ reduces to

$$
\begin{equation*}
\left(b^{q_{1}}-b^{j Q+q_{2}}\right) y+c_{x_{1} 0}-c_{x_{2} 0}=0 . \tag{6.21}
\end{equation*}
$$

If the polynomials are different the constant term will be nonzero. Since $b$ is primitive it follows that the linear coefficient is zero if and only if $q_{1}=j Q+q_{2}$. This is only possible if $j=0$ (unshifted sequences) and $q_{1}=q_{2}$ (polynomials evaluated on the same coset of the multiplicative subgroup generated by $a$ ). Since the constant term is nonzero we get in this case an equation with no roots. In other words distinct sequences generated by the same cosets are orthogonal. If the sequences are shifted or different cosets are used the crosscorrelation may be 1 since a linear equation has one root. If $a$ is primitive $y$ takes on all nonzero values including the root so the crosscorrelation is exactly 1.

If the polynomials are the same the equation is zero if and only if $j=0$ and $q_{1}=q_{2}$ which is to say when we are considering the same sequence unshifted in which case we get the obvious result that the peak autocorrelation is $N_{c}$. Otherwise there will be no roots. Thus if two sequences are generated by the: same polynomial and they are either shifted or use different cosets the correlation is zero.

The second special case is when $N_{c}=N_{f}-1$ is prime. In this case $Q=1$ so $q_{1}=q_{2}=0$ so the high order coefficient of the polynomial reduces to

$$
\begin{equation*}
b^{q_{1} t}-b^{\left(j Q+q_{2}\right) t}=1-b^{j t} . \tag{6.22}
\end{equation*}
$$

For cases of interest both $j$ and $t$ are less than $N_{c}$ and so relatively prime to $N_{c}$. Consequently $j t$ is relatively prime to $N_{c}$ and in particular $j t$ does not equal $0\left(\bmod N_{c}\right)$. Since $b$ is primitive we conclude that $b^{j t}$ is not 1 so (6.18) cannot be identically zero unless $j=0$ (unshifted sequences). If $j=0$ of course (6.19) is identically zero, but, if the polynomials are different in (6.18), then this is clearly sufficient to insure that (6.18) is not identically zero. Summarizing, the crosscorrelation between two unshifted sequences is at most $t-1$ while the crosscorrelation (or autocorrelation) between two shifted sequences is at most $t$.

It should be noted that this situation is relatively rare. For $N_{c}=N_{f}-1$ to be prime and $N_{f}$ to be a power of a prime, $N_{f}$ must be either 3 or a power of 2 . There are only seven such allowable values of $N_{f}$ less than 100,000: $2,3,4,8,32,128$, and 8192.

The third special case is when we force $j=0$ (i.e., we consider a bit synchronous system). If $q_{1}$ and $q_{2}$ are distinct in (6.18) the high order term will not vanish. The result is that if we are using different cosets we will get a correlation of at most $t$. If $q_{1}=q_{2}$ but the polynomials are different, the high order term will vanish but the other terms will not leading to a correlation of at most $t-1$. If $N_{c}=N_{f}-1$ then we automatically have $q_{1}=q_{2}=0$ so $c_{\max }=t-1$. This was the case that Solomon primarily considered and he claimed that in this case this set of sequences is of maximal size.

The final special case is when $t$ is relatively prime to $N_{f}-1$. Using arguments similar to those already seen in the second case it can be shown that the maximum 4 crosscorrelation is $t$ unless $j=0$ and $q_{1}=q_{2}$ in which case the maximum is $t-1$.

Let us use the codes generated by the first case mentioned above. The number of these codes is

$$
\begin{equation*}
M=\frac{N_{f}\left(N_{f}-1\right)}{N_{c}} \tag{6.23}
\end{equation*}
$$

and each has length $N_{c}$. Each frequency is used once in exactly $N_{f}-1$ codes and no frequency is repeated in any code. Since, in the asynchronous case, two of the $N_{c}$ chips from an interfering user overlap with any given chip, the probability that a particular interferer will hit a specified chip is

$$
\begin{equation*}
P_{\mathrm{HIT}}^{\prime}=\frac{2}{N_{c}} \frac{N_{f}-2}{M-1} \approx \frac{2}{N_{f}} \tag{6.24}
\end{equation*}
$$

Since there are $N_{I}$ interferers the probability of a specified chip being hit is

$$
\begin{equation*}
P_{\mathrm{HIT}} \approx 1-\left(1-\frac{2}{N_{f}}\right)^{N_{I}} \tag{6.25}
\end{equation*}
$$

If the signal level is high compared to the thermal noise a bit can be correctly demodulated unless all $N_{c}$ chips are hit in which case an erasure occurs. The probability of erasure is approximately

$$
\begin{equation*}
P_{E} \approx\left(1-\left(1-\frac{2}{N_{f}}\right)^{N_{I}}\right)^{N_{c}} \approx e^{-N_{c} e^{-2 N_{I} / N_{f}}}=e^{-N_{c} e^{-2 N_{I} N_{c} / G_{p}}} \tag{6.26}
\end{equation*}
$$

where $G_{p}$ is the process gain, $N_{c} N_{f}$, for the frequency hopping system. This can be optimized over $N_{c}$. The optimum $N_{c}$ is $G_{p} / 2 N_{I}$. The resulting optimized erasure probability is plotted in figure 6.3. It can be seen that low error rates require an inordinate amount of bandwidth.

The situation can be improved if the system operates chip synchronously so that an interferer has only one opportunity to hit a given chip. Since the hopping rate of a frequency hopping system may be much lower than the chip rate of a direct sequence system with the same process gain, it is much easier to synchronize to the required accuracy for chip synchronous operation. The resulting erasure probability is

$$
\begin{equation*}
P_{E} \approx\left(1-\left(1-\frac{1}{N_{f}}\right)^{N_{I}}\right)^{N_{c}} \approx e^{-N_{c} e^{-N_{I} N_{c} / G_{p}}} \tag{6.27}
\end{equation*}
$$

which is optimized at $N_{c}=G_{p} / N_{I}$. Plugging this in gives

$$
\begin{equation*}
P_{E}^{(O P T)}=e^{-G_{p} / N_{I} e} \tag{6.28}
\end{equation*}
$$

This is also plotted on figure 6.3 and the results are much more satisfying.


Figure 6.3. Probability of erasure for frequency hopping with optimum chip rate

Comparing (6.28) to equation (5.14) we see that the direct sequence system will have worse performance in terms of BER if

$$
\begin{equation*}
\frac{3 A_{0}^{2}}{2 C}<e^{-1} \tag{6.29}
\end{equation*}
$$

This occurs with approximately $22 \%$ probability if $C=1$. The conclusion is that the direct sequence system will usually have better performance but, if there is enough system margin to keep the signal above the thermal noise, frequency hopping is insensitive to either signal or interference power levels. Thus, with high signal level, frequency hopping does not have a problem with multipath outage.

How high must the signal level be? In order for the statement to hold the minimum signal level must be high enough that the chip error rate due to thermal noise is much iess than the expected erasure rate. It must be easy to obtain this minimum signal level by antenna placement so that the probability of failing to receive this signal level is also much less than the erasure rate. As an example if $G_{p} / N_{I}$ is set at 20 the erasure rate (using the optimum $N_{c}=20$ ) is $6.4 \times 10^{-4}$. We wish the probability of error in any of the 20 chips to be an order of magnitude below this. This requires a chip error rate of $3.2 \times 10^{-6}$. Chip error rate is given by [42]

$$
\begin{equation*}
P_{C}=\frac{1}{2} e^{-E_{b} / 2 N_{0} N_{c}} \tag{6.30}
\end{equation*}
$$

Thus the minimum $E_{b} / N_{0}$ must be about 27 dB . So that the probability of fading below this level is less than $6.4 \times 10^{-5}$ an extra 42 dB of margin is required for a stupendous $69 \mathrm{~dB} E_{b} / N_{0}$. If double antenna diversity is used the margin is approximately halved leading to a $48 \mathrm{~dB} E_{b} / N_{0}$. Even this may be difficult.

We conclude that if the signal level is very high (perhaps impractically high) then frequency hopping is more robust then direct sequence. This is because it is sensitive only to the number of interferers and not to their power levels. On the other hand, the
average probability of erasure is worse. If the signal levels are lower so that thermal noise can not be ignored, then the analysis above gives no indication of what level of performance can be expected.

## Chapter 7

## Conclusions

This thesis investigated the merits of direct sequence spread spectrum multiple access for wireless local area networks. The primary obstacles are multipath fading and multi-access interference. These effect both bit error rate and code acquisition performance.

Since code acquisition is not a severe problem for matched filter receivers, we considered the technologies available for implementing filters matched to the code waveform. Although several competing technologies exist, currently CCD technology is the most promising for the data rates and process gains of interest. CCD's can attain process gains of about 1000 over a range of chip rates covering several orders of magnitude. The $1-10 \mathrm{MHz}$ chip rates that we contemplate is right in the middle of this range.

For very large systems we need process gains exceeding the capabilities of even CCD technology. In this case correlation receivers must be used and much effort must be put into acquiring code synchronization. Messages must be prefixed with a synchronization preamble. If timing is completely unknown at the receiver the preamble must be so long that the synchronization overhead is likely to dominate the system bandwidth. Thus system-wide synchronizatioii is required to reduce the initial timing uncertainty.

There are two reasons why I would not recommend system-wide synchronization for a spread spectrum system of this type. The first is data rate dependent. In order to keep synchronization overhead low, timing uncertainty must be kept to a small multiple of the chip time. Uncertainty can not be reduced below the propagation time of the network which is typically several microseconds. Thus the chip rate is limited to sperhaps 5 MHz . We are interested in data rates from $1-10 \mathrm{kbps}$ leading to process gains
of $500-5000$ at most. This is little or no gain over what is achievable with matched filters which do not require system synchronization.

The second reason is even more persuasive and does not depend on the desired data rate at all. One of the prime motivations for considering spread spectrum was that it needs very little control. Pairs of users can communicate without regard for other users as long as some maximum load is not exceeded. Yet synchronization is a form of control and if it is necessary for any reason it should be used to increase system capacity by cooperation between system users. In other words synchronization adds considerable complexity to a spread spectrum system making it attractive to consider protocols such as BRAM which rely upon synchronization to improve bandwidth efficiency.

The process gain of 1000 attained by a CCD matched filter is sufficient to accomodate fairly large networks (over 50 terminals) under some conditions. Since the fade level at a particular location does not vary with time good performance can be attained if sites are carefully chosen so that no signal is in a deep fade. This is only practical for very small networks and is inconvenient for portable terminals. A more useful method of counteracting multipath fading is to use antenna diversity. Indeed it would be unwise to build a system without some form of diversity, and antenna diversity is probably the simplest to implement. Even with antenna diversity we found that, in order to communicate with all other system subscribers, it will be necessary to occasionally adjust antenna positions to get out of a deep fade. At carrier frequencies in the low gigahertz range adjustments of only a few inches are sufficient. While this would not pose any problem for a handheld terminal, the need for adjustment should be kept in mind when considering this type of network since it could be a problem for some types of terminals.

There are several topics not covered in this thesis that may be promising for further research. In chapter 2 it was mentioned that there is lognormal fading in addition to
the Rayleigh fading treated here. Lognormal fading is caused by the layout of the building and has a much larger coherence distance so that it is not easily combatted by antenna diversity. It would be worthwhile to determine the impact of this effect and, if necessary, find means to avoid it.

A great deal more work could be done with frequency hopping. In this paper only the interference dominated case was considered, but the powers needed to enter this regime may be unrealistic. An analysis should be done which can account for all the effects of thermal noise.

A couple of interesting topics which were not mentioned at all are uses for error correction coding to improve reliability and mobile terminals which experience timevarying fading. Networks with higher data rates may also be interesting. In this case frequency-selective fading may be encountered. Finally the compatibility of spread spectrum networks with neighboring systems should be considered.

## Glossary of Notation

| SYMBOL | EXPLANATION | PAGE |
| :---: | :---: | :---: |
| $A_{k}$ | Amplitude of the $k$ th signal normalized by mean power | 11 |
| $a$ | Element of $G F\left(N_{f}\right)$ used to generate frequency hopping codes | 64 |
| $\vec{a}$ | Vector, ( $\left.a^{0}, \ldots, a^{N_{f}-1 / Q}\right)$, of powers of $a$ | 64 |
| $a_{k}(t)$ | $k$ th code waveform | 11 |
| $a_{j}^{(k)}$ | $j$ th chip of $k$ th code sequence | 12 |
| $b$ | Primitive element of $G F\left(N_{f}\right)$ | 64 |
| $b_{k}(t)$ | $k$ th data waveform | 11 |
| $b_{l}^{(k)}$ | $l$ th bit of $k$ th data sequence | 12 |
| $C$ | Excess interference factor | 32 |
| $C_{j, k}(l)$ | Aperiodic crosscorrelation function of the $j$ th and $k$ th chip sequences | 19 |
| $c_{j, k}(t)$ | Periodic crosscorrelation of $j$ th and $k$ th code waveforms | 63 |
| $c_{\text {max }}$ | Maximum value of $c_{j, k}(t)$ excluding $j=k$ and $t=0$ | 63 |
| D | Antenna diversity | 43 |
| d | Distance from the transmitter to the receiver. | 9 |
| Di(x) | Dawson's integral: $e^{-x^{2}} \int_{0}^{x} e^{t^{2}} d t$ | 40 |
| $\bar{E}_{b}$ | Mean bit energy averaged over fading | 32 |
| $E_{b_{k}}$ | Bit energy of $k$ th signal | 49 |
| $G$ | Offered network load | 58 |
| $G_{p}$ | Process gain ( $N_{c} N_{f}$ ) for frequency hopping | 68 |
| $h$ | Signal detection threshold | 20 |
| $h(t)$ | Filter impulse response | 18. |
| I - | Total multi-access interference | $38^{\circ}$ |
| $I_{j, k}\left(b_{1}, b_{2}, t, \varphi\right)$ |  |  |
|  | Multi-access interference to the $k$ th user due to the $j$ th user | 19 |
| $j$ | Used to indicate cyclic shift of frequency hopping code | 64. |
| $J$ | $\sqrt{-1}$ | 39 |
| $K$ | Number of active users | 11 |
| $L$ | Packet length in bits | 54 |
| M | Number of frequency hopping codes | 63 |
| $m$ | Gradient of signal power with distance | 9 |
| $N_{c}$ | Number of chips per bit; process gain for direct sequence | 12 |
| $N_{f}$ | Number of available carrier frequencies for frequency hopping | 62 |
| $N_{\text {I }}$ | Number of interferers ( $K-1$ ) | 32 |
| $N_{s}$ | Number of system subscribers | 43 |
| $N_{u}$ | Timing uncertainty in cells ( $T_{u} / T_{s}$ ) | 25 |
| $N_{0} / 2$ | Spectral density of $n(t)$ | 11 |
| $n$ | Number of cells examined per chip ( $T_{c} / T_{s}$ ) | 25 |
| $n(t)$ | Additive white Gaussian noise process | 11. |
| $P$ | Mean signal power - | 11 |


| $P_{E}$ | Probability of error for direct sequence; probability of erasure for frequency hopping | 32 67 |
| :---: | :---: | :---: |
| $\hat{P}_{E}$ | Required probability of error or erasure | 33 |
| $\hat{P}_{f}$ | Probability of failure to acquire | 25 |
| $P_{f a}$ | Probability of false alarm | 20 |
| $P_{m}$ | Probability of missing signal | 20 |
| $Q$ | ( $N_{f}-1$ )/ $N_{c}$; number of cosets of multiplicative subgroup of $a$ in $G F\left(N_{f}\right)$ | 64 |
| $Q(z)$ | $\int_{z}^{\infty} \frac{1}{\sqrt{2 \pi}} e^{-x^{2} / 2} d x$ | 20 |
| $q$ | Used to index cosets of multiplicative subgroup of $a$ in $G F\left(N_{f}\right)$ | 64 |
| $q(t)$ | Chip pulse waveform | 19 |
| $R$ | Data rate, baud rate | 14 |
| R | Reliability | 33 |
| $R_{1}$ | Reliability with one antenna | 43 |
| $R_{D}$ | Reliability with $D$ antennas | 43 |
| $r^{2}$ | See equation (5A.2) | 49 |
| $R_{j, k}(t), \hat{R}_{j, k}(t)$ |  |  |
|  | Partial crosscorrelations between $j$ th and $k$ th code waveforms | 19 |
| $R_{q}(t), \hat{R}_{q}(t)$ |  |  |
|  | Partial autocorrelation functions of the chip pulse waveform | 19 |
| $S$ | Bandwidth efficiency | 54 |
| $s$ | Mean signal amplitude | 20 |
| $s(t)$ | Received signal | 11 |
| $s_{m}(t)$ | $m$ th frequency hopping code waveform | 63 |
| $S I R$ | Signal to interference ratio | 47 |
| SNR | Signal to noise ratio | 20 |
| $T$ | Bit duration | 12 |
| $T_{\text {acq }}$ | Mean acquisition time | 23 |
| $T_{c}$ | Chip duration | 12 |
| $T_{d}$ | Mean packet delay | 54 |
| $T_{e}$ | Examination or dwell time. | 22 |
| $T_{p}$ | Propagation delay | 24 |
| $T_{s}$ | Sliding correlator time shift | 22 |
| $T_{t}$ | Packet transmission time | 53 |
| $T_{u}$ | Initial timing uncertainty | 12 |
| $t$ | Degree of polynomial used to produce frequency hopping codes | 64 |
| $t_{d}$ | Delay between the received signal and local reference | 21 |
| $t_{k}$ | Delay of the $k$ th signal | 11 |
| $V$ | Variance of received signal | 20 |
| $y(t)$ | Filtered signal | 18 |
| $y$ | Sample of $y(t)$ | 18 |
| $\epsilon$ | Detection threshold as a fraction of the signal mean | 20 |
| $\eta$ | Sampled filtered Gaussian noise | 19 |

$\eta^{\prime} \quad$ Normalized thermal noise: $\left(P T^{2} / 2\right)^{-1 / 2} \eta$ ..... 38

$$
\theta_{j, k}(l), \hat{\theta}_{j, k}(l)
$$Periodic and odd crosscorrelations between $j$ th and $k$ th code40sequences

$\varphi_{k} \quad$ Phase of the $k$ th signal ..... 11
$\phi(u) \quad$ Characteristic function of $\eta^{\prime}+I$ ..... 39
$\phi_{1}(u) \quad$ Characteristic function of $I$ ..... 39
$\phi_{2}(u) \quad$ Characteristic function of $\eta^{\prime}$ ..... 39
$\omega_{c}$ Carrier frequency ..... 11
$\omega_{I F} \quad$ Intermediate frequency ..... 18

## References

[1] R. J. Samuel, "The application of spread spectrum modulation to land mobile radio," Proc. of IEE Intl. Conf. on Radio Spectrum Conservation Techniques, 1980, pp. 29-34.
[2] M. J. Marcus, "Recent U. S regulatory decisions on civil uses of spread spectrum," Proc. Globecom, Dec. 1985, pp. 504-506.
[3] D. B. Newman, 'Jr.; "FCC authorizes spread spectrum," IEEE Communications Magazine, vol. 24, July 1986, pp. 46-47.
[4] R. D. Haggarty, E. L. Key, D. R. Kramer, and E. A. Palo, "Spread spectrum communications and signal processing requirements," Intl. Specialist Seminar on Case Studies in Advanced Signal Processing, Sep. 1979, pp. 76-84.
[5] J. Krebs, "R/F linked portable digital communications system system overview," and J. L. Sandvos, "PCX - a portable data terminal," Proc. National Communications Forum, 1984, pp. 402-409.
[6] M. K. Simon, J. K. Omura, R. A. Scholtz, and B. K. Levitt, Spread Spectrum Communications, vols. I-III, Computer Science Press, Rockville, Maryland, 1985.
[7] G. R. Cooper, and R. W. Nettleton, "A spread-spectrum technique for high-capacity mobile communications," IEEE Trans.: Veh. Technol., vol. VT-27, Nov. 1978, pp. 264-275.
[8] D. J. Goodman, P. S. Henry, and V. K. Prabhu, "Frequency-hopped multilevel FSK for mobile radio," BSTJ, vol. 59, Sep. 1980, pp. 1257-1275.
[9] G. Niyonizeye, M. Lecours, and T. H. Huynh, "Mutual interferences in a binary FH-FSK spread spectrum system for mobile radio," IEEE Trans. Veh. Technol., vol. VT-34, Feb. 1985, pp. 28-34.
[10] O. Yue, "Spread spectrum mobile radio, 1977-1982," IEEE Trans. Veh. Technol., vol. VT-32, Feb. 1983, pp. 98-105.
[11] U. Langewellpott, "Autotel: a novel wideband mobile telephone and data system at 900 megahertz," Proc. of 2nd IEE Intl. Conf. on Radio Spectrum Conservation Techniques, 1983, pp. 85-88.
[12] J. E. Padgett, "Analysis of spread spectrum modulation for cordless telephones," Proc. Globecom, Dec. 1985, pp. 507-512.
[13] K. Yamada, K. Daikoku, and H. Usui, "Performance of portable radio telephone using spread spectrum," IEEE Trans. Commun., vol. COM-32, Jul. 1984, pp. 762-768.
[14] P. Freret, R. Eschenbach, D. Crawford, and P. Braisted, "Applications of spread-spectrum radio to wireless terminal communications," Proc. National

Telecommunications Conf., Nov. 1980, pp. 69.7.1-4.
[15] J. H. Winters, and Y. S. Yeh, "On the performance of wideband digital radio transmission within .buildings using diversity," Proc. Globecom, Dec. 1985, pp. 991-996.
[16] M. Kavehrad, "Performance of nondiversity receivers for spread spectrum in indoor wireless communications," AT\&T Tech. J., vol. 64, Jul.-Aug. 1985, pp. 1181-1210.
[17] M. Kavehrad, and P. J. McLane, "Performance of direct sequence spread spectrum for indoor wireless digital communication," Próc. Globecom, Dec. 1985, pp. 974-979.
[18] M. Kavehrad, and P. J. McLane, "Performance of low-complexity channel coding and diversity for spread spectrum in indoor, wireless communications," AT\&T Tech. J., vol. 64, Oct. 1985, pp. 1927-1965.
[19] S. E. Alexander, "Characterising buildings for propagation at 900 MHz ," Electron. Lett., vol. 19, Sep. 1983, p. 860.
[20] R. S. Swain, "Cordless telecommunications in the UK," Proc. National Communications Forum, 1984, pp. 163-168.
[21] S.: E. Alexander, "Radio propagation within buildings at 900 MHz ," Electron. Lett., vol. 18, Oct. 1982, pp. 913-914.
[22] D: C. Cox, "Universal portable radio communications," Proc. National Communications Forum, 1984, pp. 169-174.
[23] M. Kavehrad, personal communication.
[24] G. L. Turin, "Introduction to spread-spectrum antimultipath techniques and their application to urban digital radio," Proc. IEEE, vol. 68, Mar. 1980, pp. 328-353.
[25] U. Charash, "Reception through Nakagimi fading multipath channels with random delays," IEEE Trans. Commun., vol. COM-27, Apr. 1979, pp. 657-670.
[26] H. Hashemi, "Simulation of the urban radio propagation channel," IEEE Trans. Veh. Technol., vol. VT-28, Aug. 1979, pp. 213-225.
[27] H. Suzuki, "A statistical model for urban radio propagation," IEEE Trans. Commun., vol. COM-25, Jul. 1977, pp. 673-680.
[28] D. M. J. Devasirvatham, "Time delay spread measurements of wideband radio signals within a building," Electron. Lett., vol. 20, Nov. 1984, pp. 950-951.
[29] R. C. Dixon, Spread Spectrum Systems, Wiley, New York, 1984.
[30] J. Heighway, and E. G. S. Paige, "Analogue signal processing with SAW devices," Intl. Specialist Seminar on the Impact of New Technologies in Signal Processing, Sep. 1976, pp. 27-43.
[31] D. T. Bell, Jr., J. D. Holmes, and R. V. Ridings, "Application of acoustic surface-wave technology to spread spectrum communications," IEEE Trans. Microwave Theory Tech., vol. MTT-21, Apr. 1973, pp. 263-271.
[32] Texas Instruments Inc., TMS32020 User's Guide, 1985.
[33] P. B. Denyer, and J. Mavor, " 256 point CCD programmable transversal filter," Proc. of 5th Intl. Conf. on Charge-Coupled Devices, 1979, pp. 253-253e.
[34] D. M. Grieco, "The application of charge-coupled devices to spread-spectrum systems," IEEE Trans. Commun., vol. COM-28, Sep. 1980, pp. 1693-1705.
[35] J. R. Tower, D. A. Gandolfo, L. D. Elliott, B. M. McCarthy, F. D. Shallcross, and S. C. Munroe, "A 512-stage analog-binary programmable transversal filter," Proc. of 5th Intl. Conf. on Charge-Coupled Devices, 1979, pp. 247-252.
[36] S. C. Munroe, personal communication.
[37] D. D. Buss, C. R. Hewes, and M. de Wit, "Charge coupled devices for analogue signal processing," Intl. Specialist Seminar on the Impact of New Technologies in Signal Processing, Sep. 1976, pp. 17-26.
[38] E. A. Geraniotis, and M. B. Pursley, "Error probability for direct-sequence spread-spectrum multiple-access communications - part II: approximations," IEEE Trans. Commun., vol. COM-30, May 1982, pp. 985-995.
[39] D. V. Sarwate, and M. B. Pursley, "Crosscorrelation properties of pseudorandom and related sequences," Proc. IEEE, vol. 68, May 1980, pp. 593-619.
[40] S. S. Rappaport, and D. M. Grieco, "Spread-spectrum signal acquisition: methods and technology," IEEE Communications Magazine, vol. 22, Jun. 1984, pp. 6-21.
[41] S. S. Rappaport, and D. L. Schilling, "A two-level coarse code acquisition scheme for spread spectrum radio," IEEE Trans. Commun., vol. COM-28, Sep. 1980, pp. 1734-1742.
[42] J. M. Wozencraft, and I. M. Jacobs, Principles of Communication Engineering, Wiley, New York, 1965.
[43] M. Abramowitz, and I. Stegun, Handbook of Mathematical Functions, National Bureau of Standards, Washington, D. C., 1972.
[44] W. R. Franta, and I. Chlamtac, Local Networks, D. C. Heath, Lexington, Massachusetts, 1981.
[45] A. S. Tanenbaum, Computer Networks, Prentice-Hall, Englewood Cliffs, New Jersey, 1981.
[46] G. Solomon, "Optimal frequency hopping sequences for multiple access," Proc. of the 1973 Symp. on Spread Spectrum Communications, 1973, pp. 33-35.

