AN EXTENDED YIELD CURVE MODEL FOR BOND OPTION PRICING USING A JUMP/GARCH-M FORWARD RATE PROCESS

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ABSTRACT

Purpose of this paper is to develop a pricing model for bond options with long term to expiration using results from time series analysis on interest rate movement.

In this paper the yield curve model for bond option pricing (Ho and Lee, 1986) is extended in three aspects.

1 Forward short-term rate structure is used instead of zero-coupon bond price structure.
2 Interest rate movements were modeled at three different points on the forward short-term rate curve.
3 ARMA process with Jump/GARCH-M disturbance is used to model the movements of forward short-term rates.

We performed time series analysis on innovation in forward short-term structure $\{I_t\}$, defined as $I_t=R_{j,t}-R_{j+1,t-1}$, where $R_{j,t}$ is j period forward short-term rate observed at time t.

Our findings on innovation series are as follows.

1 For shorter j, $\{I_t\}$ has positive autocorrelation. Risk premium have serial autocorrelation and/or market is inefficient.
2 For longer j, $\{I_t\}$ is negatively autocorrelated. This is consistent with mean reversion in interest rate movement.
3 The series $\{I_t\}$ is heteroskedastic. For shorter j, jump style heteroskedasticity is dominant, while for longer j, autoregressive conditional heteroskedasticity (ARCH) is dominant.
4 Time varying risk premium (GARCH-M effect) are detected.

Although we worked on data from Japanese bond market, our methodology can be applied to other markets as well.

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1. Introduction

1.1 Purpose of this paper

Immediate purpose of this paper is to develop a pricing model for bond option with long period to its expiration. We also expect that we develop better understanding on movements of term structure of interest rates and that our model will be applicable to interest rate contingent claims in general.

Our methodology in this paper is to develop a extended version of yield curve model for bond option pricing using technique of financial time series analysis. Here we briefly list the major sources of our key ideas. The base of our model is the yield curve model by Ho and Lee[1], which is flexible enough to be accommodated with theories and findings on interest rate financial time series. From the field of

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1. Ho, Thomas S. Y., and Sang-bin Lee, 1986a. "Term Structure Movement and Pricing Interest Rate Contingent Claims." The Journal of Finance, Vol.41 No.5, pp.1011-1029.

   Ho, Thomas S. Y., and Sang-bin Lee, 1986b. "Term Structure Movements and Interest Rates Contingent Claims Pricing." Working paper series, number 375. Salomon Brothers Center for the Study of Financial Institutions, Graduate School of Business Administration New York University.

   Ho, Thomas S. Y., 1990a Strategic Fixed Income Investment. Dow Jones Irwin, Chapter 11.

empirical study on interest rate movement Fama and Bliss (1987)[2] provided us useful discussion on mean reversion and time varying risk premium. In the field of financial time series analysis we obtained the concept of time varying conditional variance from Taylor model[3] and ARCH models[4], and we also learned from discussions on alternatives of random walk hypothesis in financial time series in Lo and MacKinlay (1988)[5].

1.2 Structure of this paper

We start this paper with an overview of relevant theories and methodologies of option pricing, interest rate movement,

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2. Fama, Eugene F. and Robert R. Bliss 1987 "The Information in Long-Maturity Forward Rates" The American Economic Review Vol.77 No.4, pp.680-692

3. Taylor, Stephen 1986 Modeling Financial Time Series, John Wiley & Sons Ltd.

4. Many versions of autoregressive conditional heteroskedasticity models are developed. Bollerslev, Chou and Kroner (1990) provides exclusive survey of those models.

   Bollerslev, Tim, Ray Y. Chou, and Kenneth F. Kroner 990 "ARCH Modeling in Finance: A Review of The Theory and Empirical Evidence," Working Paper No.97 November, 1990, Kellogg Graduate School of Management, Northwestern University

5. Lo, Andrew W. and A. Craig MacKinlay 1988 "Stock Market Prices Do Not Follow Random Walks: Evidence from a Simple Specification test," The Review of Financial Studies Vol.1 No.1, pp.41-66

and financial time series analysis in section 2. In section 3, we describe observed characteristics of forward short-term interest rate movement. Our findings are serially correlated prediction error of pure expectation hypothesis, mean reversion, and autoregressive conditional heteroskedasticity. In section 4, we build ARMA model (autoregressive moving average model) with jump and GARCH-M (generalized autoregressive conditional heteroskedasticity in mean) disturbance and perform maximum likelihood estimation of its parameters. Estimated model is used to extended yield curve model in section 5. We conclude in section 6.

2. Theories and Models

2.1 Pricing Models for Interest Rate Options

Our direct motivation is to fill the needs for better method in pricing callable corporate bonds. A callable corporate bond can be valued as the portfolio of a non-callable corporate bond and a call option or series of call options. Typical structure of a call provision embedded in a callable bond is;

(a) European call for the first half of the life of the host bond, and American call thereafter with call price declining according to a schedule over time, or

(b) Series of European calls expiring in sequence on each coupon payment date during the latter half of the life of the host bond, having call price declining sequentially to the par value.

Dyer and Jacob (1989)[6] reported that three categories of

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6. Dyer, L. J., and D.P. Jacob 1989 "A Practitioner's Guide to Fixed-Income Option Models," The Journal of International Securities Markets, Spring 1989, IFR

interest rate option pricing models are used in practice. They are:

(1) Black-Scholes models, which assume log-normally distributed bond price,

(2) binomial models, which assume log-normal distribution of yield to maturity, and

(3) yield curve models, which model dynamics of the yield curve.

Dyer and Jacob 1989 reasonably commented that the former two categories involve inappropriate assumptions (constant discount rate and constant volatility) and uncomfortable results (arbitrage opportunities and negative interest rates), while yield curve models can be set up in some consistent way. Furthermore, it is reported that although the three categories are in reasonable agreement for short-term options, yield curve model is the clearly preferred method for long term options.

Hull (1989)[7], and Hull and White (1990)[8] provides useful

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7. Hull, John, 1989, Options, Futures, and Other Derivative Securities, Prentice Hall, Chapter 10.

overview of methods for interest rate derivative securities. According to Hull and White (1990), yield curve model initiated by Ho and Lee (1986) is unique in that their model is consistent with any shape of currently observed term structure. Two other major models developed by, Cox, Ingersoll, and Ross (1985)[9] and by Vasicek (1977)[10] were also extended to be consistent with given shape of term structure by Hull and White (1990).

The information contained in the shape of the yield curve observed at time t implies a structure of forward rates. The forward rate structure at time t can be interpreted as the sum of expected spot rates and risk premium. Therefore, models that are consistent with particular shape of yield curve observed in the market at time t, are consistent with expectation by market participants about the future interest rates as of time t. This is one of the common advantages of the yield curve models that are consistent with yield curve at t=0, while models of older generation examined in Dobson,

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8. Hull, John, and Alan White 1990 "Pricing Interest-Rate-Derivative-Securities." The Review of Financial Studies Vol.3 No.4, pp.573-592

9. Cox, J. C., J. E. Ingersoll, and S. A. Ross 1985 "A Theory of the Term Structure of Interest Rates," Econometrica, Vol. 53 No. 2, pp. 385-467

10. Vasicek, O. A., 1977, "An Equilibrium Characterization of the Term Structure," Journal of Financial Economics, Vol.5 pp. 177-188

Sutch, and Vanderford (1976)[11] are not based on expectation incorporated in forward rates prevailing in the market.

Another major advantage of yield curve models is that it allows us to concentrate on modeling process of change in interest rates, while other models those deal price change of bonds require us to trace mixed effects of seasoning of bonds and movement of interest rates. Since we attempt to perform time series analysis of rate movement, free from contamination by seasoning effects, this property is important.

The original version of the yield curve model by Ho and Lee (1986a) was a single factor model of yield curve. The yield curve was to be transformed under path dependent condition and arbitrage-free condition in some stochastic way by a perturbation function, which is a function of term and independent of time and state. As noted in the Ho and Lee (1986a) and developed somehow in Ho and Lee (1986b), their model can easily be extended to have time varying parameters,

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11. Dobson, Sutch, and Vanderford (1976) provides a list of models for expectation on interest rate. Most of them are some kind of ARIMA model based on time series of the past and present short term rates, and some others are ARIMA model for the transformed same series. Clearly, these models do not count for unique information included in the currently prevailing forward rates, which are implied in the currently prevailing yield curve.

    Dobson, Steven W., Richard C. Sutch, and David E. Vanderford 1976 "An Evaluation of Alternative Empirical Models of The Term Structure of Interest Rates," The Journal of Finance Vol.31 No.4, pp.1035-1065

state dependent form and/or multiple factors.

This flexibility allows an extension of the yield curve model using the idea such as "key rate durations" in Ho (1990b)[12], which suggests to expand the single factor method of modelling term structure movement to multi factor method. Key rate duration is an idea to explicitly model interest rates movements at multiple points along the yield curve, rather than to look only at short-term rate.

Heath, Jarrow, and Morton (1990)[13] is an attempt to extend Ho and Lee model in the three ways;

(1) Extension from single factor model to two factor model (it allows more general N factor model),

(2) Generalization from discrete time model to continuous time model, and

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12. Ho, Thomas S. Y., 1990b, Key Rate Durations: A Measure of Interest Rate Risks Exposure, Working Paper Series S-90-17, Salomon Brothers Center for Study of Financial Institutions, Leonard N. Stern School of Business, New York University

13. Heath, David, Robert A. Jarrow, and Andrew Morton 1990 "Contingent Claim Valuation With a Random Evolution of Interest Rates," Review of Futures Markets, Vol.9 No.1, pp.54-82, Chicago Board of Trade

    Heath, Jarrow, and Morton (1990) contains two comments from Hull, John of Toronto University and Habeeb, Gregory G. of PaineWeber Inc., and some discussions.

(3) Use of forward rates instead of price of zero-coupon bonds, i.e. yields.

Regarding the appropriate direction to which yield curve models are to be extended, Hull, John commented to Heath, Jarrow, and Morton (1990) that it is possible to model either of bond price, forward rate, or short-term rate, and also that short-term based model do not need significant arbitrage-free condition except non-negativity of short-term rate, while other two types needs careful treatment to avoid arbitrage.

2.2 Interest Rate Movement Theories

ARMA models, or distributed lag models, have been popular method for modeling interest rate movements. A catalogue of empirical models on interest rate movement is presented by Dobson, Sutch, and Vanderford (1976). All of the models discussed were single factor time series models, which can be seen as some kind of ARIMA model. Such models had been presented as candidates for good linear estimator of interest rate movements. However, it is not clear why interest rate movement follow such process that has serial autocorrelations and whether such serial correlation implies violation of martingale. If there exists predictability, which can be represented using ARMA models, in interest rate movement on

average, why such opportunities are faded out through arbitrage ?

For survey of interest rate theories, the following two literatures provided good starting points. Wood and Wood (1985)[14] provided useful overview of development in expectation theory of interest rates, starting from traditional expectation theories to modern expectation theories by Cox, Ingersoll, and Ross (1981)[15]. Melino (1986)[16] also provides thorough overview of development both in theoretical literatures and empirical studies on term structure of interest rates.

After survey of literatures, our major concern on modeling interest rate movement were summarized as the following points;

(a) market inefficiency (in the sense of martingale),

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14. Wood, John H., and Norma L. Wood 1985 Financial Markets, Harcourt Brace Jovanovich, San Diego, California, Chapter 19

15. Cox, John C., Jonathan E. Ingersoll, and Stephen A. Ross 1981 "A Reexamination of Traditional Hypothesis about the Term Structure of Interest Rates," The Journal of Finance Vol.36 No.4 pp..769-799

16. Melino, Angelo, 1986, The Term Structure of Interest Rates: Evidence and Theory, NBER Working Paper Series No.1828, National Bureau of Economic Research

(b) mean reversion (autoregressive rate movement),

(c) risk premium (constant term premium, or time varying risk premium, etc.), and

(d) source of observed excess kurtosis and predictability of conditional variance (ARCH effects, jump effects, etc.).

For discussion on martingale, some aspects of mean reversion, and risk premium, we found helpful discussions in the literatures on interest rate movements. For discussion on some other aspects of mean reversion and excess kurtosis (fat tailed shape) of distribution, we found relevant discussions in several literatures on financial time series other than interest rate movements.

2.2.1 Martingale, risk premium, and serial correlation

Martingale in forward short-term interest rates is studied theoretically and empirically by Roll (1970)[17]

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17. Roll, Richard 1970, The Behavior of Interest Rates, An Application of The Efficient Market Model to U.S. Treasury Bills, Basic Books, Inc., Publishers, New York, London

following model of Samuelson (1965)[18]. Roll (1970) stated that:

> Denoting;
>
> $R_{j,t}$ as j-1 year forward one year rate observed at time t,
>
> $L_{j,t}$ as risk premium included in $R_{j,t}$, and
>
> B_t as knowledge available as of time t,
>
> and defining
>
> $X_{j,t}$ as $R_{j,t}-L_{j,t}$,
>
> the sequence $\{X_{j,t}\}$ follows pure martingale, or equivalently,
>
> $$E_{t-1}(\check{R}_{j,t}-\bar{L}_{j,t} \mid B_{t-1}) = R_{j+1,t-1}-L_{j+1,t-1},$$
>
> where $\check{R}_{j,t}$ and $\acute{L}_{j,t}$ at time t-1 are random variabls.

This provides definition of martingale for process of risk premium adjusted forward short-term interest rates, which Hull, John (1990) suggested to use as the easiest-to-handle building block for extended yield curve models[19]. The observable variable $\{I_{j,t}\}$ defined as $I_{j,t}=R_{j,t}-R_{j+1,t-1}$ here is the sum of the following two components.

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18. Samuelson, Paul A. 1965 "Proof That Property Anticipated Prices Fluctuate Randomly," Industrial Management Review Vol.6 No.2, pp..41-49

19. Hull, John (1990) comment to Heath, Jarrow, and Morton (1990), Review of Futures Markets, Vol.9 No.1, pp..77-78, Chicago Board of Trade

(1) Unexpected innovation in forward short term rate, $X_{j,t}-X_{j+1,t-1}$.

(2) Difference between $L_{j,t}$ and $L_{j+1,t-1}$.

Since risk premium $L_{j,t}$ is unobservable, we have to identify functional forms of $L_{j,t}$ to be tested. This is necessary, when we attempt to answer the questions,

(1) if $\{X_{j,t}\}$ follows martingale, and/or

(2) if there are some evidence for certain type of risk premium.

Cox, Ingersoll, and Ross (1985) model assumes term premium increasing with maturity, while Fama and Bliss (1987) pointed out that behavior of expected returns is inconsistent with simple term structure models in which expected returns always increase with maturity, and suggested time varying risk premium. We consider the following two types of functional representation of risk premium.

(1) L_j as a monotonously increasing function of j and independent on t. This corresponds to the

original idea of liquidity premium by Hicks (1946)[20]. In this case, series of innovation in forward rate, $\{I_{j,t}\}$ defined as $I_{j,t}=R_{j,t}-R_{j+1,t-1}$, follows random walk with negative drift $C=L_j-L_{j+1}<0$. From martingale hypothesis,

$$E_{t-1}[X_{j,t}] = X_{j,t-1},$$

and by definition,

$$E_{t-1}[X_{j,t}]+L_{j,t} = E_{t-1}[R_{j,t}], \text{ and}$$

$$X_{j+1,t-1}+L_{j+1,t-1} = R_{j+1,t-1}.$$

Then,

$$E_{t-1}[R_{j,t}]-R_{j+1,t-1}$$
$$= \{E_{t-1}[X_{j,t}]-X_{j,t-1}\}+\{L_{j,t}-L_{j+1,t-1}\}$$
$$= L_j-L_{j+1} = C < 0.$$

(2) The second method is the use of ARCH-M model by Engle, Lilien, and Robins (1987)[21], which revealed that expectation hypothesis holds under the ARCH-M style time varying risk premium and that the result is robust. Time varying risk premium also provides explanation for the rejection of expectation hypothesis by recent studies where constant term premium hypothesis is wrongly applied

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20. Hichs, John (1946) Value and Capital, 2nd ed., Oxford Press, London

21. Engle, Robert F., David M. Lilien, and Russell P. Robins 1987 "Estimating Time Varying Risk Premia in the Term Structure: The ARCH-M Model," Econometrica Vol.55 No.2, pp.391-407

for volatile periods.

The risk premium at time t, $L_{j,t}$, is defined as a linear function of $\sqrt{H_t}$, where H_t is conditional variance of R_j at time t. Specifically, we use jump/GARCH process as $\{H_t\}$, and define $L_{j,t}=P\sqrt{H_t}$, where P is a positive constant. This is a extended version of the GARCH-M model by Bollerslev, Engle, and Wooldridge (1988)[22] to include jump components. In this case, time series of innovation in the forward rate $\{I_{j,t}\}$, defined as $I_{j,t}=R_{j,t}-R_{j+1,t-1}$, follows a process as follows.

$$\begin{aligned} I_{j,t} &= [R_{j,t}-R_{j+1,t-1}]+P[\sqrt{H_{j,t}}-\sqrt{H_{j+1,t-1}}] \\ &= X_t + P\Delta[\sqrt{H_t}] + \varepsilon_t, \end{aligned}$$

while denoting D as difference operator and assuming change of $H_{j,t}$ due to small change in j is negligible. Then, it is clear that test for serial correlation of $\{I_t\}$ is insufficient as the test of martingale property of $\{X_t\}$. We need to estimate $\{H_t\}$ process and adjust for serial correlation caused by it. We need to test $I_t-P\Delta[\sqrt{H_t}]$ for serial correlation in order to make inference if $E[X_t]=0$, i.e. whether the rate movement is efficient in the

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22. Bollerslev, Tim, Robert F. Engle, and Jeffrey M. Wooldridge 1988 "A Capital Asset Pricing Model with Time-varying Covariances," Journal of Political Economy Vol.96, No.1

sense of martingale.

2.2.2 Mean reversion

Although the idea of market efficiency leads to the concept that risk premium adjusted forward short-term rate process $\{R_t\}$ should follow pure martingale, there exists another important point of view for interest rate movement. The idea of mean reversion in the interest rate movement process leads to stationary ARMA style process rather than random walk suggested by martingale concept. Importance of mean reversion is examined by Hogan and Breidbart (1990)[23]. Hogan and Breidbart compared a yield curve model without mean reversion against another yield curve model with mean reversion features, and concluded that different handling of mean reversion results large price difference in long-term options, while price difference is smaller for shorter-term options.

Theoretical and empirical study on forward interest rate by Fama and Bliss (1987) presented a mean reverting model.

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23. Hogan, Michael and Seth Breidbart 1990, "The Long-Term Behavior of Interest Rates And Options Pricing," The Journal of International Securities Markets, Spring 1990, pp..49-56

They used AR(1) time series to show that mean reversion is a general property of stationary ARMA processes, and that mean reversion counts for half of the forward rate volatility in the long run. Suppose a time series of forward one year rate observed at time t follows the AR(1) process,

$$R_t = C + \theta R_{t-1} + \varepsilon_t, \quad |\theta| < 1$$

where R_t is forward one period rate at time t,

$1-\theta$ is a parameter for the speed of reversion,

$C/(1-\theta)$ is the unconditional mean, say μ, or the target of the reversion,

ε_t is disturbance at time t, and

the inequality imposed on θ is condition for stationarity.

The following equivalent expression,

$$E_{t-1}[R_t] = \theta R_{t-1} + (1-\theta)\mu,$$

presents that $E[R_t]$ is the point which divides the vertex of $[R_{t-1},\mu]$ with the proportion of $(1-\theta):\theta$. Another expression,

$$E_{t-1}[R_t - R_{t-1}] = -(1-\theta)(\mu - R_{t-1}),$$

shows that expected one period change in R_t on condition R_{t-1} has a size proportional to the distance between R_{t-1} and μ and that the change has direction from R_{t-1} to μ. As a general property of stationary ARMA model, when s gets larger;

(1) Conditional expectation of $E[R_{t+s}|R_t]$ quickly approaches unconditional mean of $E[R_t]$, and

(2) Covariance between R_t and R_{t+s}, $COV(R_t,R_{t+s})$, converges to zero.

Therefore, conditional expectation and conditional variance of rate change between time t and time t+s can be written as follows.

(1) $$E[\ R_{t+s}-R_t \mid R_t\ ] = E[R_{t+s}|R_t] - R_t$$

When s get larger, this approaches to

$$\mu - R_t,$$

where μ is the unconditional mean $E[F_t]$.

(2) $$VAR[\ R_{t+s}-R_t \mid R_t\ ]$$

$$= VAR[R_{t+s}|R_t] + VAR[R_t] + 2COV[R_{t+s},R_t]$$

When s gets larger,

$VAR[R_{t+s}]$ approaches $VAR[R_t]$,

$COV[R_{t+s},R_t]$ approaches 0, and

$VAR[\ R_{t+s}-R_t \mid R_t\ ]$ approaches $2VAR[R_t]$.

Therefore, the proportional contribution of mean reversion to the entire variance,

$VAR[R_{t+s}] / VAR[R_{t+s}-R_t]$ approaches 50% from the lower side.

There are several bond option pricing models, which applied the idea of mean reversion. Jamshidian (1989)[24] used a mean reverting stochastic process, Ornstein-Uhlenbeck process, for the pricing of option on zero coupon bonds and derived closed form solution. Cox, Ingersoll, and Ross (1985) also applied the mean reversion hypothesis. From the point of view of empirical study, Fama and Bliss (1987) provided supporting evidence for mean reversion hypothesis.

2.3 Models for other financial time series

Outside the studies on the term structure of interest rates, we could find relevant literatures for our purpose.

Mean reversion had long been discussed for movement of term structure of interest rate. However, recently mean reversion is reported and discussed in the stock markets and foreign exchange markets as well with discussion on

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24. Jamshidian, F., 1989 "An Exact Bond Option Formula" The Journal of Finance Vol.44 No.1 pp.205-209

statistical techniques for detection and modeling.

Heteroskedasticity is reported for stock market data, foreign exchange market data, commodity market data, and almost all other kinds of financial time series data. Techniques to analyze such heteroskedasticity have also been developed and ready to be used in the field of interest rate study.

2.3.1 Other aspects of mean reversion

Lo and Mackinlay (1988), Poterba and Summers (1988)[25], and Fama and French (1988)[26] provides discussion on mean reverting properties of financial time series. Although those discussions are based on data of stock returns, methodological points are easily applied to other financial time series. Among these, Poterba and Summers (1988) reported that;

(1) stock return over short horizon tend to show positive autocorrelation, while those over longer horizon tend to show negative autocorrelation, and

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25. Poterba, James M., and Lawrence H. Summers 1988 "Mean Reversion in Stock Prices Evidence and Implications," Journal of Financial Economics Vol.22 pp.27-59

26. Fama, Eugene F., and French R. Kenneth 1988 "Permanent and Temporary Components of Stock Prices," Journal of Political Economy Vol.96 No.2 pp.246-273

that

(2) these observations on serial correlation are so subtle that random walk hypothesis cannot be rejected in many cases at conventional size of statistical tests.

Poterba and Summers (1988) used sum of a random walk and a stationary mean reverting process as their model and also reported that standard statistical software packages fail to estimate an ARMA(1,1) plus a random walk model from a data set generated by Monte Carlo simulation, when random walk components contributed 75% or more of the entire variance.

Variance ratio test is examined and suggested to detect serial correlation from both homoskedastic or heteroskedastic data sets by Lo and MacKinlay (1988), Lo and MacKinlay (1989)[27], and Poterba and Summers (1988).

2.3.2 Excess Kurtosis in distribution

Financial time series tend to have leptokurtic distribution rather than to have normal distribution. Early

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27. Lo, Andrew W., and A. Craig MacKinlay 1989 "The Size and Power of the Variance Ratio Test in Finite Samples," Journal of Econometrics Vol.40 pp.203-238

study on this point can be found in Davies, Speeding, and Watson (1980), where skewness and kurtosis is analyzed for ARMA process with non-normal residuals[28]. This can result either from some generating process with leptokurtic distribution, like student's t-distribution and logistic distribution, or from heteroskedastic behavior of conditionally normally distributed generating processes of financial time series. Further, as is pointed out in Mandelbrot (1963)[29] large change in speculative price series do not distributed uniformly, rather they are somewhat clustered. This suggests some serial dependence among conditional variances, i.e. heteroskedasticity with serial dependence.

These observations and ideas combined, lead to a group of models called autoregressive conditional heteroskedasticity (ARCH), where series of conditional variance is assumed to follow ARMA processes. The original ARCH model, where conditional variance was assumed to follow MA(4) process with

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28. Davies, Neville, Trevor Speeding, and William Watson 1980 "Autoregressive Moving Average Processes with Non-Normal Residuals," Journal of Time Series Analysis, Vol.1 No.2, pp.103-109

29. Mandelbrot, Benoit. 1963 "Forecasts of Future Prices, Unbiased Markets, and 'Martingale' Models" The Journal of Business Vol.36, pp.394-419

specific lag pattern, was developed by Engle (1982)[30]. This model was analyzed by Milhøj (1985)[31], Weiss (1986)[32] and also extended to ARMA model with ARCH disturbance by Weiss (1984)[33]. The model followed a quite natural course of development and was extended by Bollerslev (1986)[34] to have conditional variance time series that follows ARMA process (generalized autoregressive conditional skedasticity, GARCH). GARCH process is discussed by Engle and Bollerslev (1986)[35], and by Bollerslev (1988)[36]. In ARMA process with ARCH disturbances by Weiss (1984), ARCH model was applied to residuals from usual ARMA method. However, from the point of view of risk-return trade off, the estimated magnitude of

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30. Engle, Robert F. 1982 "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of U.K. Inflation," Econometrica Vol.50 No.4, pp.987-1008

31. Milhøj, Anders 1985 "The Moment Structure of ARCH Processes," Scandinavian Journal of Statistics Vol.12 pp.281-292

32. Weiss, A.A. 1986 "Asymptotic theory for ARCH models: Estimation and testing," Econometric Theory Vol.2 pp.107-131

33. Weiss, Andrew A. 1984 "ARMA models with ARCH errors," Journal of Time Series Analysis Vol.5 No.2 pp.129-143

34. Bollerslev, Tim 1986 "Generalized Autoregressive Conditional Heteroskedasticity," Journal of Econometrics Vol.31, pp. 307-327

35. Engle, R.F., and Tim Bollerslev 1986 "Modeling the persistence of conditional variances," Econometric Review Vol.5 pp.1-50

36. Bollerslev, Tim 1988 "On the Correlation Structure for the Generalized Autoregressive Conditional Heteroskedastic Process," Journal of Time Series Analysis Vol.9 No.2 pp.121-131

conditional variance should be reflected in the estimation of conditional expectation, i.e. conditional expectation and conditional variance must be estimated simultaneously, not one after another. Then, ARCH-M model was developed for time varying risk premium by Engle, Lilien, and Robins (1987). This course was followed by GARCH(1,1)-M model for testing CAPM in Bollerslev, Engle, and Wooldridge (1988), and by Factor-ARCH model in Engle, Ng, and Rothchild (1990)[37]. Nelson (1990)[38] studied continuous time version of ARCH models. Bollerslev, Chou, and Kroner (1990) provides exclusive survey of ARCH models, and Akgiray (1989)[39] provides comprehensive introduction of several types of ARCH models with applications.

While ARCH type models provide explanation for some portion of excess kurtosis reported in many financial time series, residuals from ARCH models still tend to have excess kurtosis. There should be different source of heteroskedasticity. Here jump processes, which are not serially correlated, are introduced as a candidate for the

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37. Engle, Robert F., Ng, Victor K., and Rothchild, Michael 1990 "Asset Pricing with a Factor-ARCH Covariance Structure" Journal of Econometrics Vol.45, pp. 213-237

38. Nelson, Daniel B. "ARCH Models as Diffusion Approximations," Journal of Econometrics Vol.45, pp.7-38

39. Akgiray, Vendat 1989 "Conditional Heteroskedasticity in Time Series of Stock Returns: Evidence and Forecasts," Journal of Business Vol.62 No.1, pp.55-80

model of remaining excess kurtosis. This model was studied for stock returns and foreign exchange markets by Jorion (1989)[40]. A model for interest rates which includes both jump and diffusion components are also developed in Ahn and Thompson (1988)[41].

2.4 Summary

We analyze the time series of $\{I_{j,t}\}$, innovation in forward rate structure, defined as $I_{j,t}=R_{j,t}-R_{j+1,t-1}$. We concentrate on detecting and estimating the following features of the $\{I_t\}$ process at different length of j, since these are relevant for bond option pricing.

(1) Market efficiency in the sense of martingale.

(2) Evidence for constant term premium hypothesis or for time varying risk premium (jump/GARCH-M).

(3) Evidence for identifying source of excess kurtosis (jump or GARCH).

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40. Jorion, Philippe 1989 "On Jump Precesses in the Foreign Exchange and Stock Markets," The Review of Financial Studies Vol.1 No.4, pp.427-445

41. Ahn, Chang Mo, and Howard E. Thompson 1988 "Jump-Diffusion Processes and the Term Structure of Interest Rates," The Journal of Finance Vol.43 No.1 pp.155-174

(4) Evidence for mean reversion.

3. Time Series Analysis on Innovation in Forward Short-Term Rates

3.1 Description of Data and Notation

In this section we explain what the data used are, how we transformed them, and what the significant properties of the time series are.

Data used here are three daily time series of Japanese bond yield. The three series are "NIKKEI bond indexes" for "short-term bonds," "medium term bonds," and "long term bonds." Description of the indexes are as follows.

NIKKEI Bond Index

[Publisher]

The Japan Economic Journal (Nihon Keizai Shimbun)

[Distinction of terms]

Short-term, medium-term, or long-term bonds indexes are the average of yield to maturity (YTM) of each group of bonds which fall in the same maturity class defined as follows.

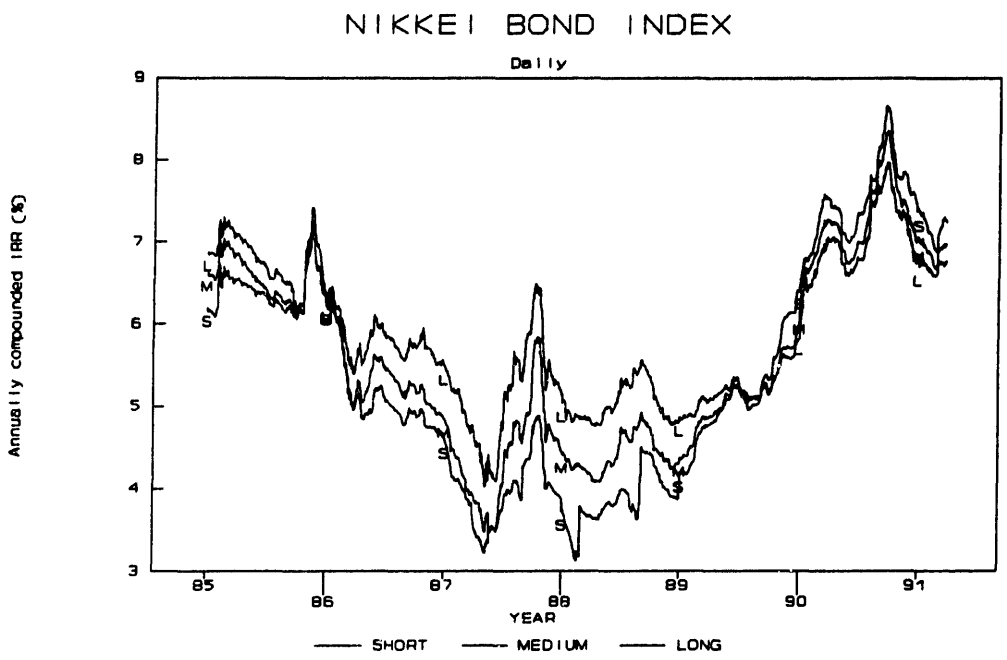

Fig. 3.1.

Short-term : less than 3 years,

Medium-term : 3 years to 7 years, or

Long-term : 7 years to 10 years,

respectively.

[Type of bonds included]

Government bonds, government guaranteed bonds, municipal bonds, bank debentures, corporate bonds, and Samurai bonds.

[Method of calculation]

Yield to maturity (YTM) is calculated for each bond using internal rate of return (IRR) method based on 6 month of compounding period length. This is because

Japanese bonds have semiannual payments. Annual YTM is obtained by just doubling the 6 month period based IRR. To obtain index arithmetic average is calculated for bonds within each maturity classification.

[Source of data]

Mean of the bid-ask quotations is used for the YTM calculation. The quotations are taken from "bench-mark bond quotations, daily" published by the Japan Securities Dealers Association.

The bench-mark quotations are arithmetic average for each of bid and ask quotations reported by market makers who are members of the association. Quoted bid and ask prices at 9 A.M. are reported.

[Notes]

NIKKEI Bond Index is not adjusted for early redemptions and durations. Bonds are classified to each term based only on the remaining period to final maturity. When bonds are called, such bonds are removed from calculation.

The index is subject to change in membership of the "bench-mark quotation". The membership is changed by the association considering maturity change caused by seasoning, announcement of early redemption, and trading

volume.

A major problem of using the NIKKEI bond index for our study is the difficulty in deriving implied forward rate structure. Since the indexes are neither representing spot rate curve, nor are par-yield curve, there is no straight-forward way to derive forward rate structure from these data.

To derive accurate implied forward rates, detailed information on each bonds included in the index is required. Since such detailed information was not available, we prepared rough estimator of duration for each class of maturity and assumed that the rate of index of each maturity class represents the spot rate, or IRR of discount bonds, having such time horizon that equal to the duration of the maturity class. Such duration changes depending on coupon rates of bonds included within each class and also on market discount rate. However, for simplicity, we assumed that coupon rates of all bonds were 5.5% par annum throughout the period, and discount rate were also constant st 6% for the purpose of duration derivation.

This rough adjustment shows that short, medium, and long term bond index correspond to discount rate for 1.86 years, 5.0 years and 6.93 years, respectively.

Table 3.1

TERM LENGTH ADJUSTMENT USING DURATION

	Short	Medium	Long
Duration (Years)	1.86	5.00	6.93
Accumulated (Days)	679	1823	2528
Incremental (Days)	679	1145	705

For future study we strongly urge to use spot rate data from the market, which is currently under joint development by Industrial Bank of Japan and J.P.Morgan.

For this paper, we assume piecewise constant forward rate structure. Assuming the figures for short-term, medium-term, and long-term indexes are representing discount rates for 1.86 years, 5.00 years, and 6.93 years, we can derive forward rate structure which is piecewise constant for each interval of [year 0 to year 1.86], (year 1.86 to year 5.00], and (year 5.00 to year 6.93]. Figure 3.2 presents the movement of the three forward rates. When compared with the original NIKKEI Bond Index presented in the figure 3.1, the effect of the transformation from yield curve into forward rate curve is evident. Short term forward rates show quite similar movement with short term yield, long term forward rate turned out to stay within relatively stable range, and medium term forward

rate is in-between. This is understandable because yield is geometric average of forward rates, or logarithm of yield is arithmetic average of logarithm of forward rates.

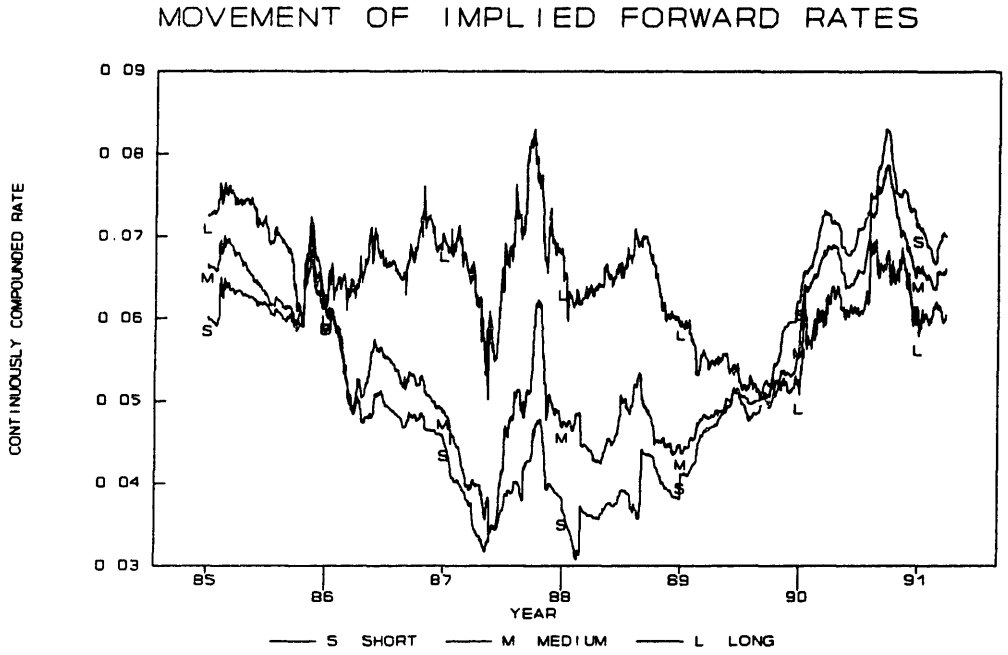

Fig. 3.2

For convenience of the following discussions, we take log of the rates and use the following notation.

(1) We basically follow the notation used by Roll (1970). $R_{j,t}$ represents j period forward short-term rate (one year rate starting year j-1 and ending year j) observed at time t. Since we only have three different j,

$$j = 1.86 \text{ years},$$

5.00 years, or

6.93 years,

we also use the word "short", "medium", or "long" term forward rate" or $R_{short,t}$, $R_{medium,t}$, or $R_{long,t}$ equivalently.

(2) We also analyze innovation in the forward rate series $\{I_{j,t}\}$. This is not equal to the first order difference of $\{R_{j,t}\}$. Beside the innovation caused by information that newly arrived to the markets, there are the following two sources of change in $\{R_{j,t}\}$.

(a) Change in $\{R_{j,t}\}$ caused by time-varying risk premium.

(b) "Expected" change in $\{R_{j,t}\}$ which is already built-in to the slope of forward rate structure observed at time t.

By the word "innovation," we will mean the portion of change in $\{R_{j,t}\}$ that is not expected. If j and t are measured on the same scale, unexpected innovation can be defined as $I_t=R_{j,t}-R_{j+1,t-1}$ as in Roll (1970). In our data set, j is only three step with

interval of 1.86 years, 3.14 years, and 1.93 years, while t is measured by day. Then, we pro-rated the logarithm of yield according to the length of j.

$$I_{j,t} = R_{j,t} - \{R_{j,t}D_j + (d)(R_{j^+,t-1}-R_{j,t})\}/D_j,$$

where $j=\{short,medium\}$, and $j^+=\{medium, long\}$ respectively,

$R_{j,t}$ is logarithm of forward rate for period j observed at time t,

D_j is number of days included within the period j, and

d is number of days between time t and t-1, i.e. d=3 if t is Monday and d=1 for week days.

For the long term forward rate series, we defined I_t just as the first order difference, since no information on further longer term is available from the market data.

3.2 Comparison of Forward Rate Process $\{r_t\}$ and Innovation Process $\{I_t\}$

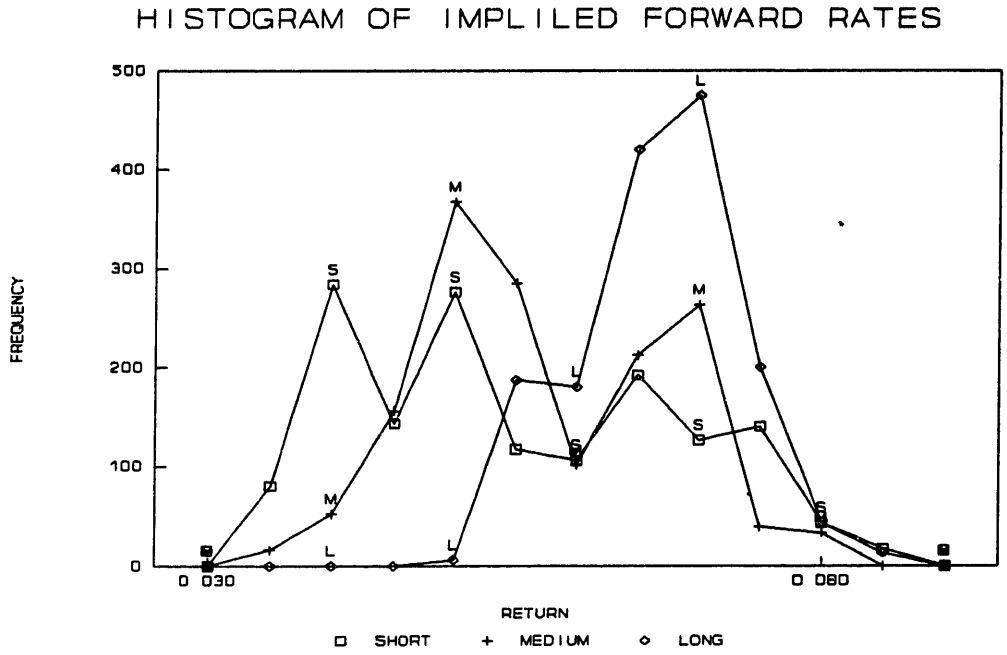

Fig. 3.3

Figures 3.3 is the histogram of forward rates during the 6 year and 3 month period between January 1985 to March 1991, which shows that $\{R_t\}$ is concentrated within relatively narrow range for longer j, while $\{R_t\}$ is distributed over wider range for shorter j.

How the different distribution of $\{R_t\}$ for different j can be explained ? Do $\{R_{j,t}\}$ have different shape of distribution for different j ? To answer this question, we prepared histogram of the $\{R_t\}$ for each j, year by year for the 6 sub-periods which correspond to the calendar years.

Figures 3.4, 3.5, and 3.6 show that the most of the difference was caused by the shift of conditional distribution rather than the different shape of distribution itself. For shorter j, histogram of $\{R_t\}$ shift around a lot over time, while histogram is almost still throughout the six sub-periods for longer j.

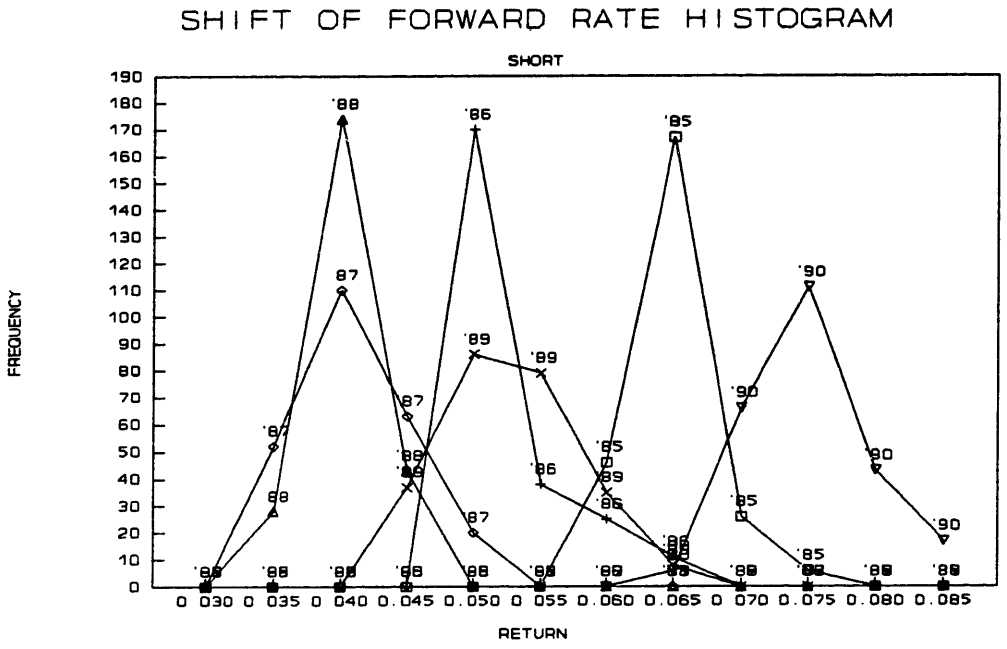

Fig. 3.4

This relative persistence in $\{R_{j,t}\}$ for longer j seems to be consistent with the idea of mean reversion. Because risk adjusted forward rate is estimator for spot rate in the future, persistence in $\{R_{j,t}\}$ with longer j implies persistence in expectation on spot rate in the distant future.

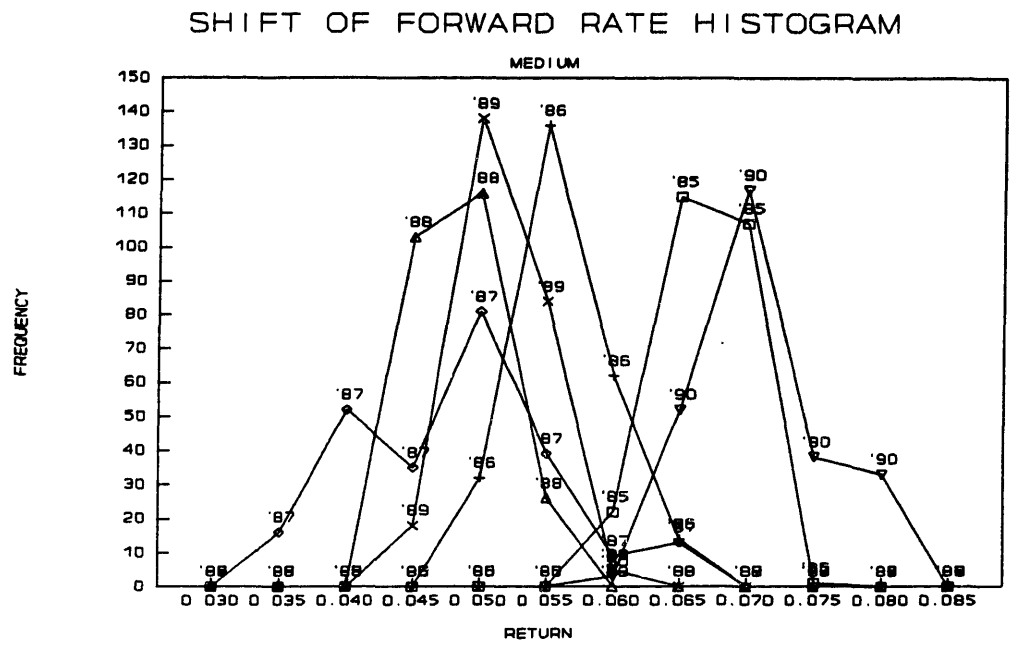
SHIFT OF FORWARD RATE HISTOGRAM
MEDIUM
FREQUENCY
RETURN
0 030 0 035 0.040 0.045 0 050 0 055 0.060 0.065 0.070 0.075 0.080 0.085

Fig. 3.5

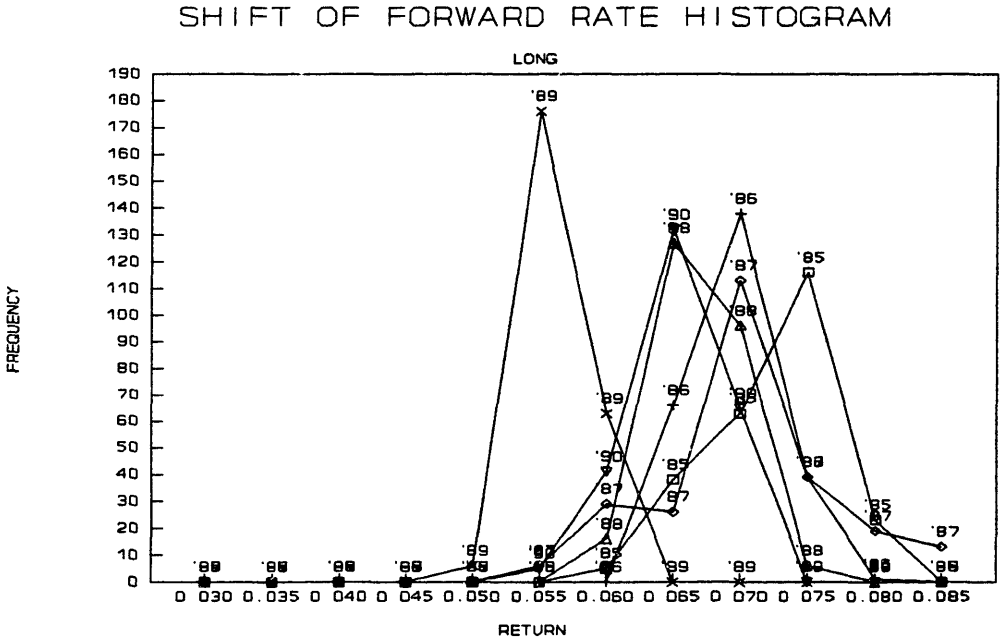
SHIFT OF FORWARD RATE HISTOGRAM
LONG
FREQUENCY
RETURN
0 030 0.035 0 040 0 045 0.050 0.055 0.060 0 065 0 070 0 075 0.080 0.085

Fig. 3.6

The figure 3.3 also shows that for longer j the mode of histogram of $\{R_{j,t}\}$ lays in higher class. This fact can be interpreted in the following two ways.

(1) Term premium incorporated in $\{R_{j,t}\}$ increase with the length of j. This leads to the idea of constant liquidity premium.

(2) Time varying risk premium as some increasing function of uncertainty in $\{R_{j,t}\}$. As we see later, data show that variance of innovation, $VAR[I_{j,t}]$, is greater for longer j.

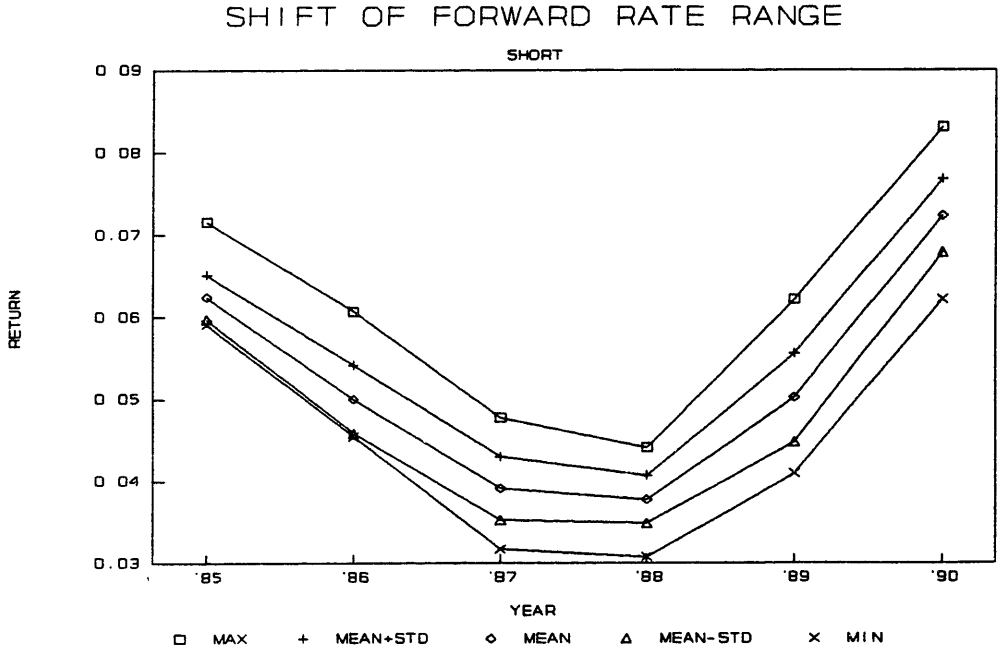

Fig. 3.7

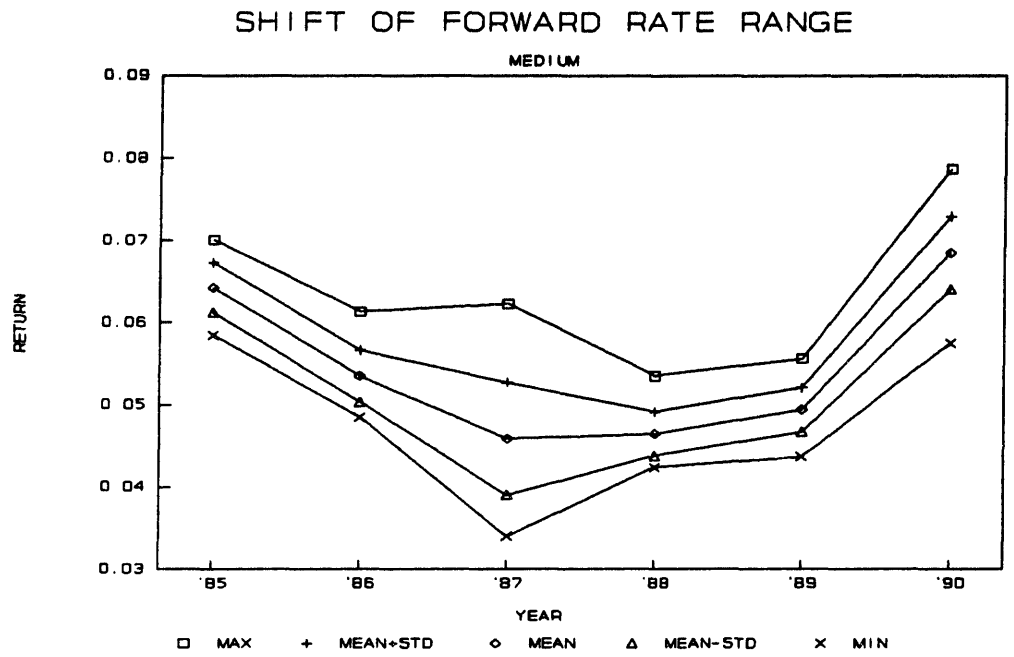
SHIFT OF FORWARD RATE RANGE
MEDIUM
RETURN
0.09
0.08
0.07
0.06
0 05
0 04
0.03
'85
'86
'87
'88
'89
'90
YEAR
MAX
MEAN+STD
MEAN
MEAN-STD
MIN

Fig. 3.8

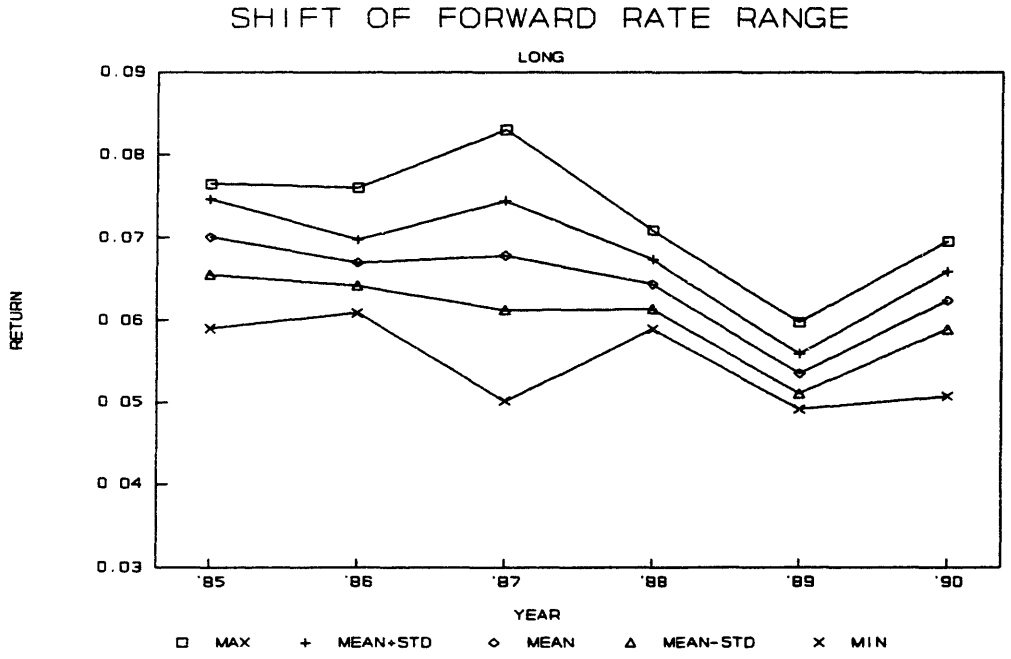
SHIFT OF FORWARD RATE RANGE
LONG
RETURN
0.09
0.08
0.07
0 06
0 05
0 04
0.03
'85
'86
'87
'88
'89
'90
YEAR
MAX
MEAN+STD
MEAN
MEAN-STD
MIN

Fig. 3.9

Figure 3.7 to 3.9 also present year-to-year shift in range of forward rate distribution. These figures confirm that most of the variance of $\{R_{j,t}\}$ for short j is caused by shift in distribution over time.

Although $\{R_{j,t}\}$ with shorter j have larger unconditional variance over the entire period (January 1985 to March 1991), this relationship reverse when unconditional variance is calculated for smaller sub-periods.

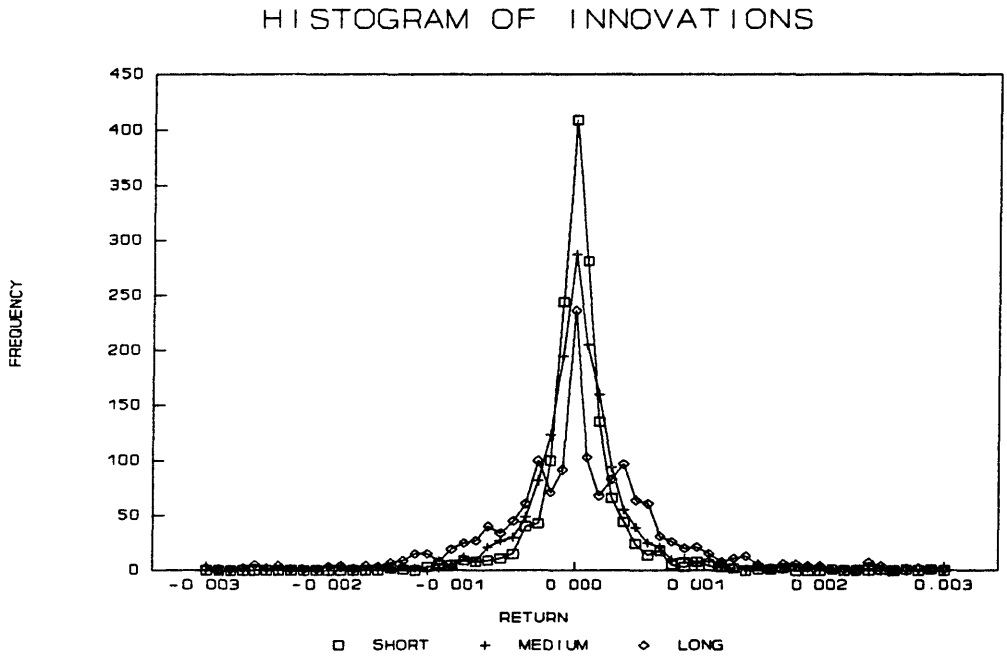

Fig. 3.10

This point becomes more clear when we examine $\{I_{j,t}\}$. Figure 3.10 compares histograms of $\{I_{j,t}\}$ throughout the 6 year and 3 month period for different j. Histograms are quite similar for all j, which is quite different from the

histograms of $\{R_{j,t}\}$.

Figure 3.11 through 3.13 shows year to year shift in histogram of $\{I_{j,t}\}$ for each j. All three distributions are fairly stable throughout the entire period. This is quite different from the behavior of $\{R_{j,t}\}$. This implies that great portion of the shift in $\{R_{j,t}\}$ distribution is expected changes and that such expectation is implied in the forward rate structure observed in the market.

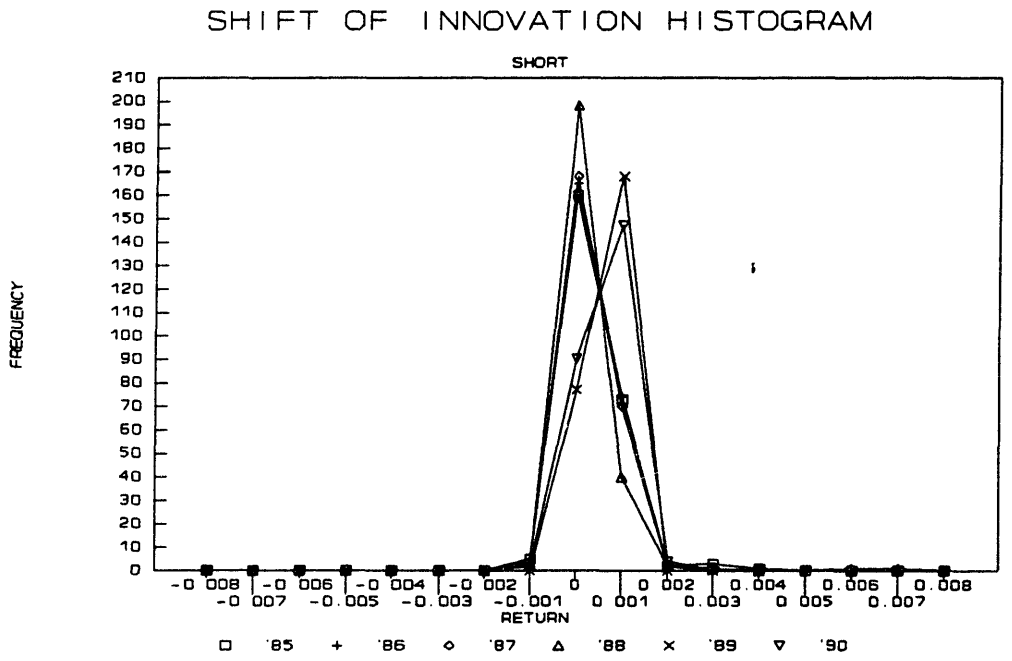

Fig. 3.11

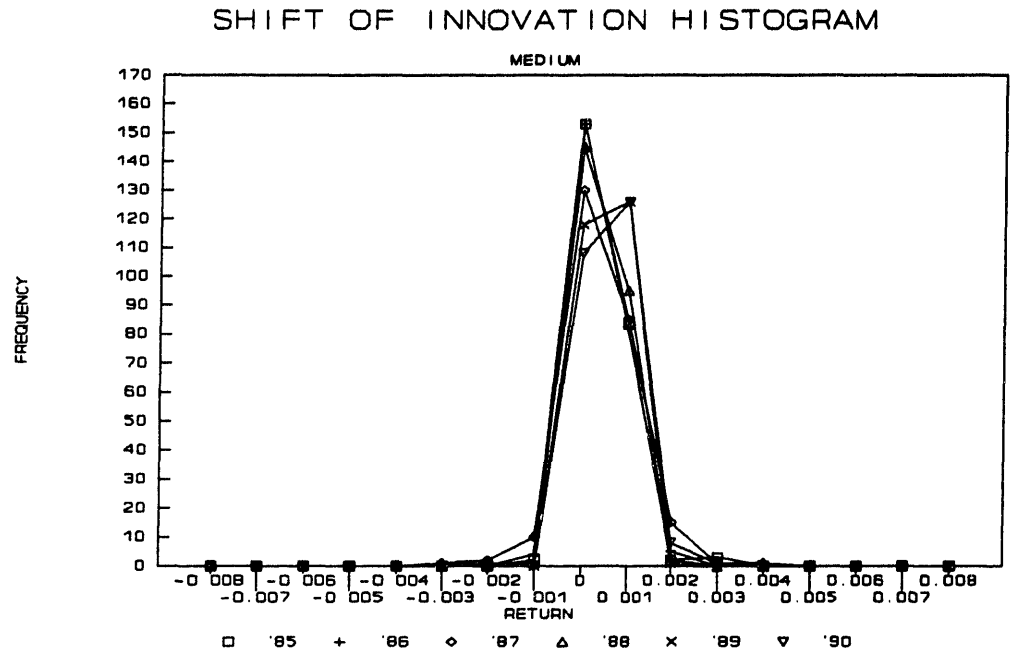

Fig. 3.12

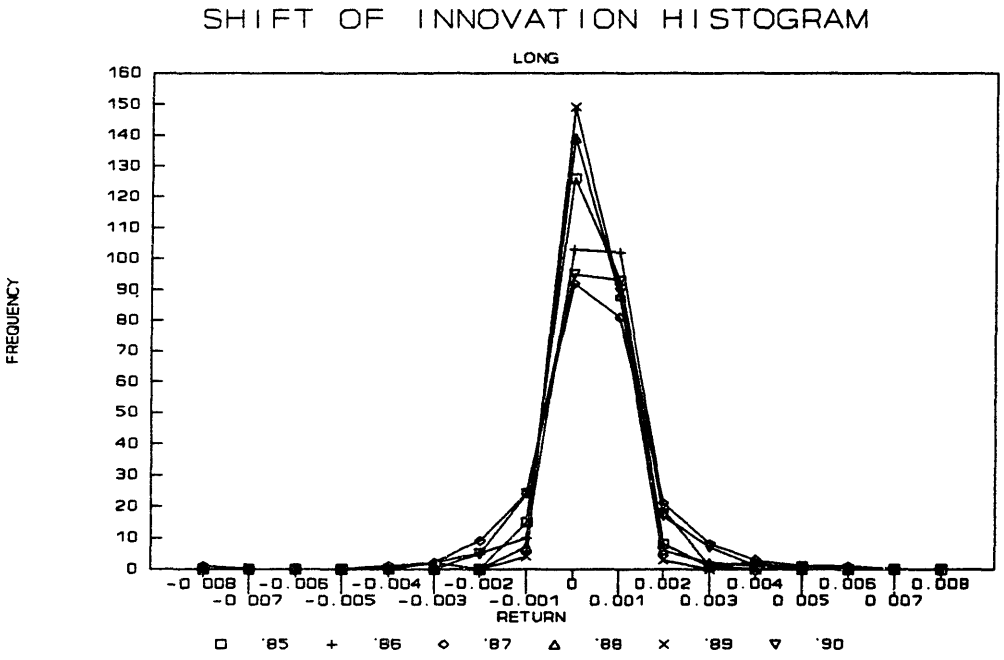

Fig. 3.13

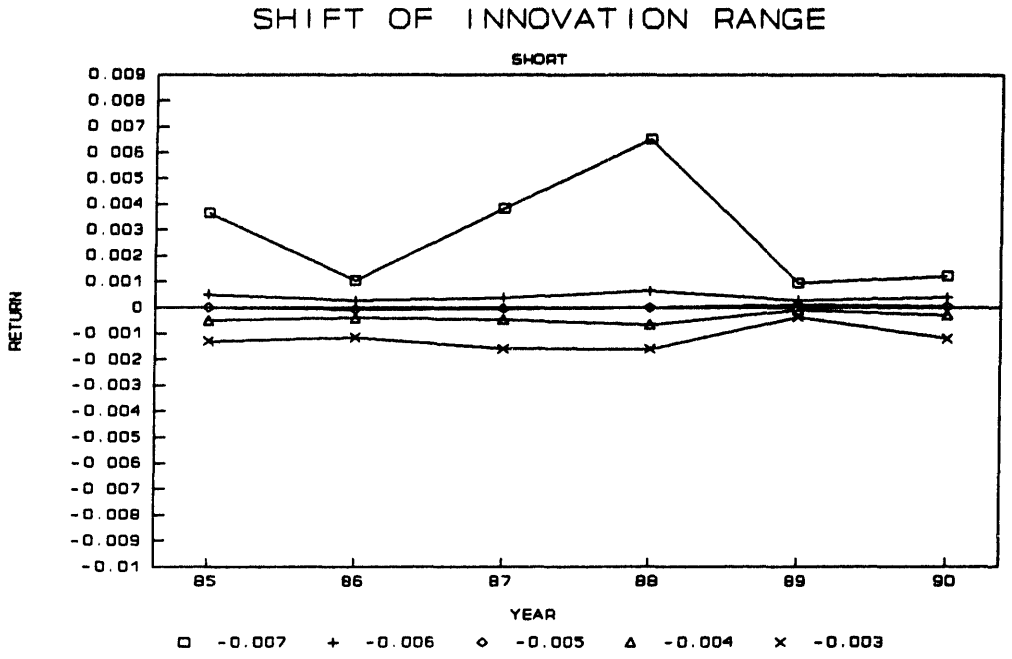

Fig. 3.14

Figures 3.14 to 3.16 present year to year shift in range of mean plus/minus 1 standard deviation, maximum, and minimum of $\{I_t\}$ distribution. Figure 3.17 presents the comparison of the mean plus/minus 1 standard deviation range for the three different j. When we see annual mean, they are almost zero throughout the entire period. Based on t-statistics, they are not significantly different from zero at each year. When we observe annual standard deviation, it is apparent that;

(1) Annual variance of $\{I_{j,t}\}$ is usually greater for longer j, and that

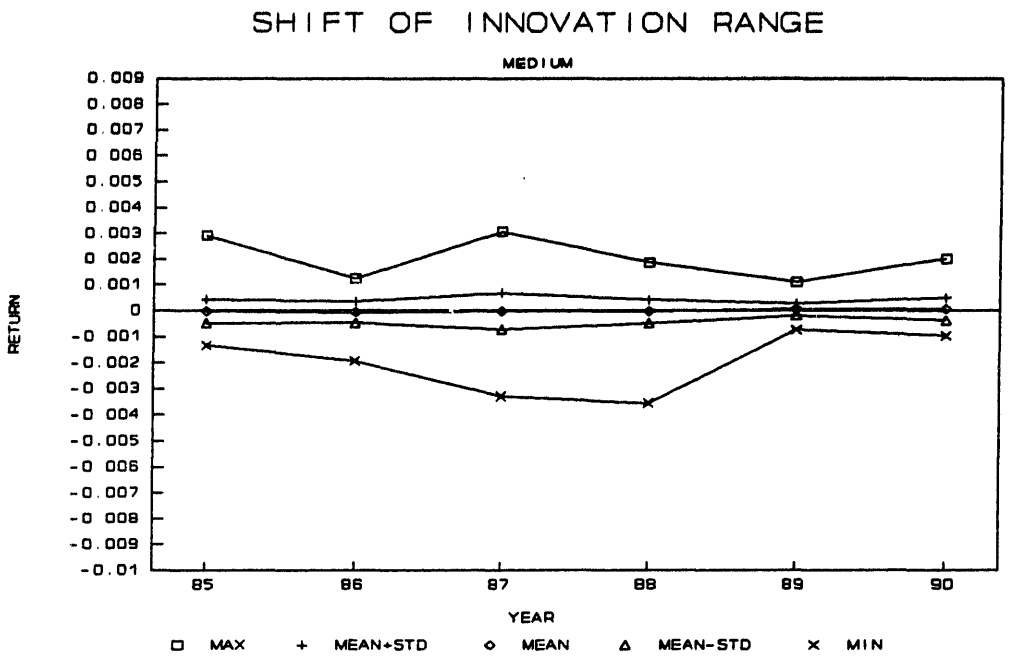

Fig. 3.15

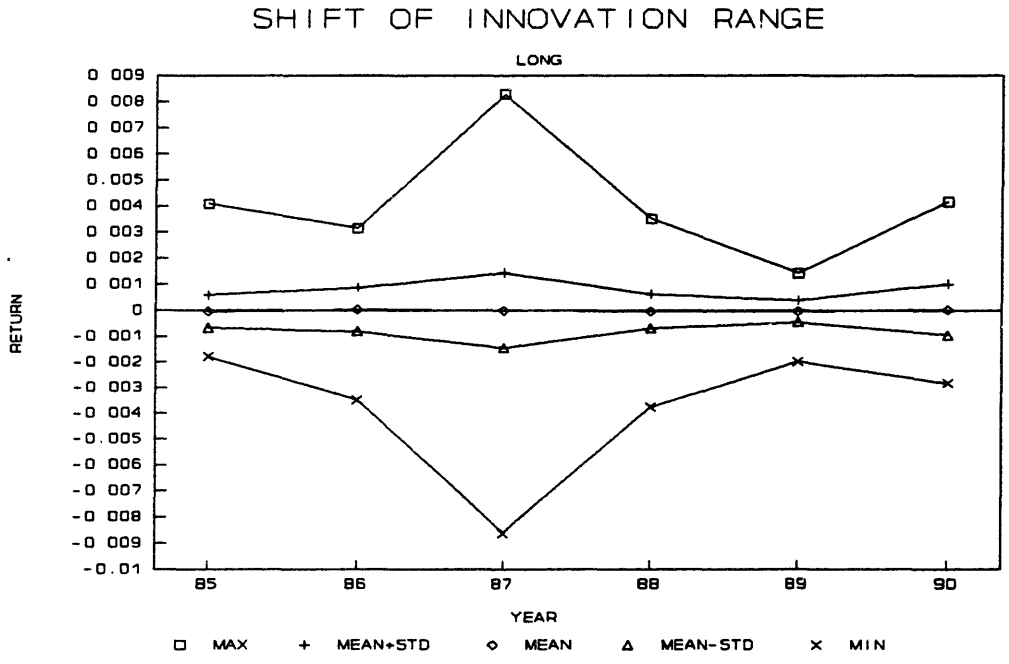

Fig. 3.16

(2) Annual variance of $\{I_{j,t}\}$ for different j, seems to keep stable relationship roughly proportional to j.

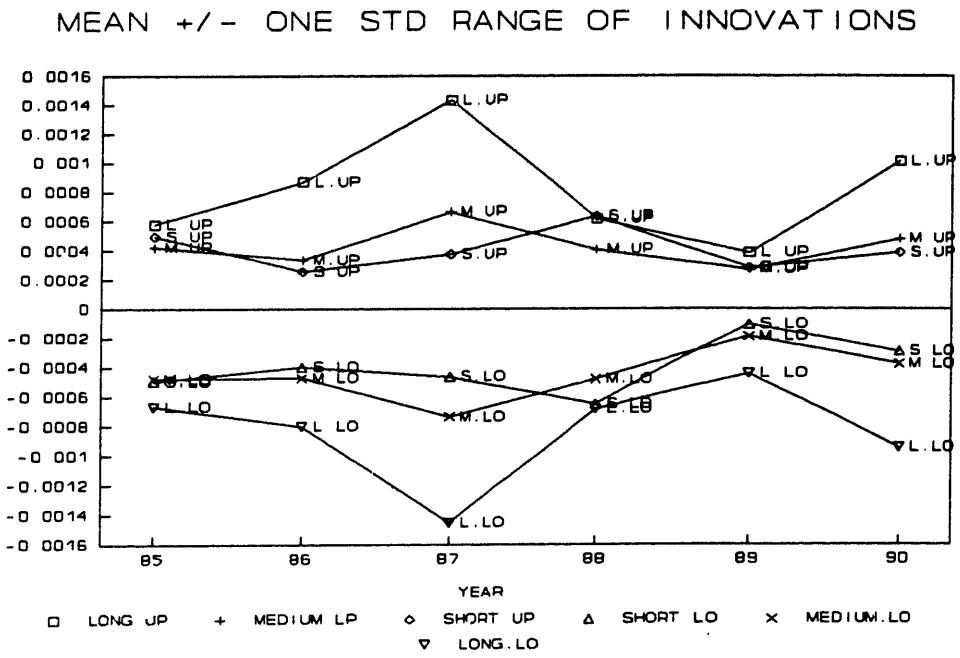

Fig. 3.17

When compared to the forward rate series of $\{R_t\}$, the innovation series $\{I_{j,t}\}$ are fairly stable both in the level and shape of distribution over the 6 years. Also, as figure 3.10 shows, all three distributions are tightly concentrated around zero, and have quite similar sharp peaked shape. It is apparent that modeling innovation precesses $\{I_{j,t}\}$ is much easier than modeling forward short-term rates $\{R_{j,t}\}$.

3.3 Mean Reversion, Risk Premium, Heteroskedasticity and Market Efficiency of $\{I_{j,t}\}$

In the following two subsections we examine characteristics of $\{I_{j,t}\}$ process. Figures 3.18 to 3.20 presents $\{I_{j,t}\}$ for different j. Summary statistics of $\{I_{j,t}\}$ are presented in table 3.2 through table 3.4 for j=short, medium or long, and data set observed at different frequency of daily, weekly and monthly. For most series average is not significantly distant from zero. T-statistics are insignificant at the usual 5% level. No evidence to support constant term premium hypothesis is found, which expects $\{I_{j,t}\}$ has bias and tendency to take negative value.

Figures 3.18 to 3.20 show subtle differences among the behavior of daily $\{I_{j,t}\}$ for each j. These figures suggest that the $\{I_{long,t}\}$ has negative autocorrelation and is oscillating around zero, the unconditional mean, while $\{I_{j,t}\}$ (j=short, medium) have positive autocorrelation and are meandering around zero. This is consistent with the ideas that $\{R_{long,t}\}$ is relatively stable and mean reverting tendency, and that $\{R_{j,t}\}$ (j=short or medium) shift a lot over time and have relatively strong positive autocorrelation, implying that market is somewhat inefficient in the sense that it takes several days to absorb new information.

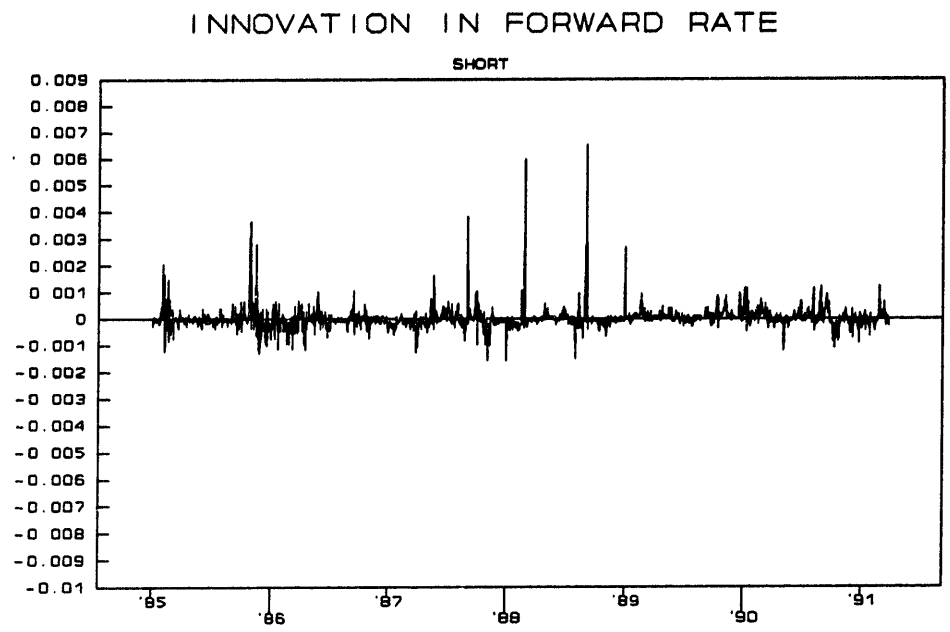

Fig. 3.18

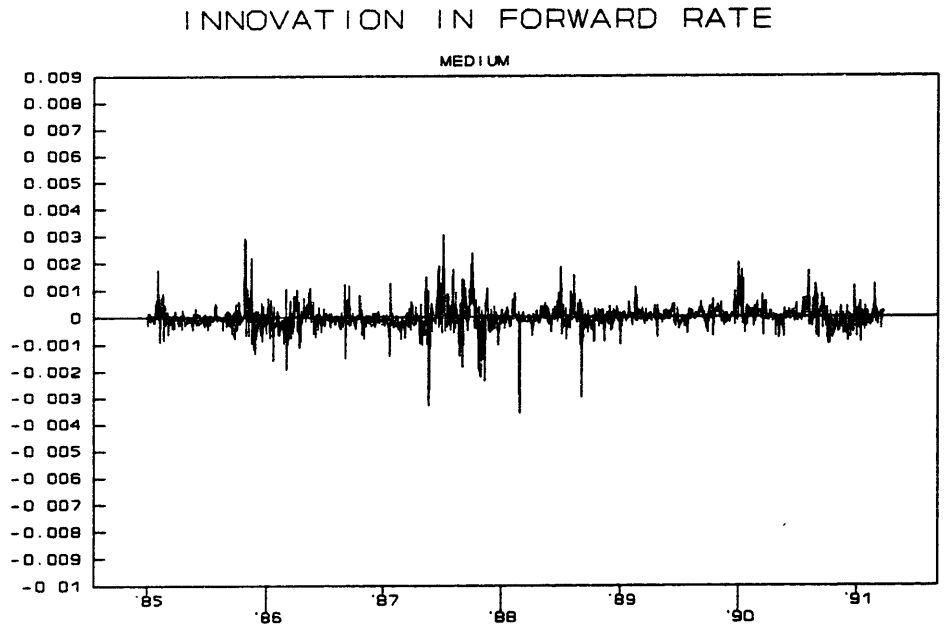

Fig. 3.19

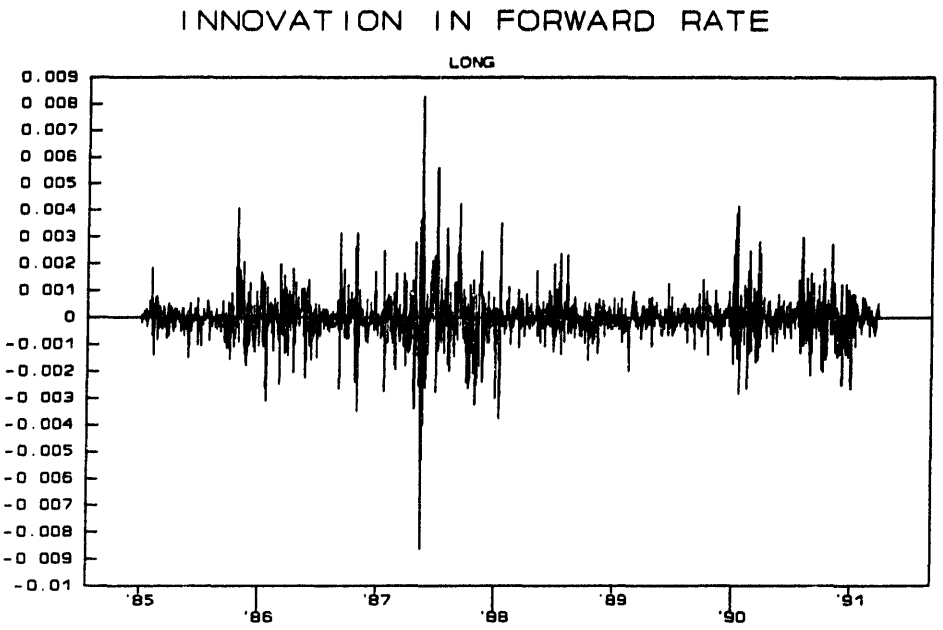

Fig. 3.20

These findings about the behavior of conditional mean are confirmed by examining autocorrelation functions (ACFs) and partial autocorrelation functions (PACFs). Figures 3.21 to 3.38 are correlograms of the three innovation series using ACFs and PACFs. ACF and PACF of the series of absolute value and squared value of innovation are also presented.

For testing the significance of autocorrelation function (ACF) of k-th order, ρ_k, asymptotic distribution of $\rho_k \sim N(0, VAR(\rho_k))$ is used. Bartlett's formula, $VAR(\rho_k)=1/N$ can be used. However, when we remind that financial time series tend to have larger magnitude of autocorrelation at lower order, it might be more useful to use the following

cummuratively adjusted Bartlett's formula

$$VAR(\rho_k) = \frac{1+2\sum_{i=1}^{k-1}\rho_i^2}{N}$$

to test marginal significance of ρ_k, when k increases.

Moreover, in cases where ARCH effects are expected, it is important to notice that Bartlett's formula tend to provide too small variance, resulting too frequent rejection of the null hypothesis that $\rho_k=0$. For ARCH(1) effect, Diebold (1986) provided adjustment for Bartlett's formula[42]. Taylor (1984) also pointed out that empirical study for varicus kind of financial time series resulted variance for autocorrelation coefficient being 2.5/n, 1.6/n, and 1.3/n for commodities, foreign exchange, and stocks respectively[43].

Since we expect more general form of GARCH effect than Diebold's ARCH(1), Diebold's adjustment for Bartlett's formula is not directly available for us. We used the accumulation adjusted version of Bartlett's formula for our correlogram

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42. Diebold, Francis X. 1986 "Testing for Serial Correlation in The Presence of ARCH'" American Statistical Association 1986 Proceedings of the Business and Economic Statistics Section, pp.323-328

43. Taylor, Stephen J. 1984 "Estimating the Variances of Autocorrelations Calculated from Financial Time Series," Applied Statistics Vol.33 No.3, pp.300-308

analysis as a benchmark of VAR(ρ_k), while keeping it in mind that the criteria is biased. Then, our 5% significance level is,

$$\pm 1.96\sqrt{\frac{1+2\sum_{i=1}^{k-1}\rho_i^2}{N}}$$

We use correlogram analysis for the purpose of model identification and have chance to further examine significance of parameters in the stage of model diagnosis. Then, we might take the risk of choosing too deep order of autocorrelation at this stage.

For the similar test of partial autocorrelation function (PACF)s of k-th order, ϕ_k, is much easier. We can just apply the Bartlett's formula and use VAR(ϕ_k)=1/N for all k. Therefore, usual 5% level criteria is $\pm 1.96/\sqrt{N}$.

Excess kurtosis is the most noticeable characteristics of these data. Magnitude of excess kurtosis seems to depend on two factors.

(1) Excess kurtosis of $\{I_{j,t}\}$ is greater for shorter j.

(2) For each length of j, excess kurtosis of $\{I_{j,t}\}$ depends on the frequency of the data (i.e. daily,

weekly, or monthly). For daily data set excess kurtosis is extremely large for all j. However, with the observation frequency declines, excess kurtosis also diminish quickly for all j.

Excess kurtosis can be explained in the following two ways.

(1) Generating process of time series is conditionally Gaussian, however, time varying conditional variance causes spurious excess kurtosis.

(2) Generating process itself is fat-tailed. Student's t-distribution, logistic distribution, Palate distribution, etc. had proposed.

For our data set, heteroskedasticity seems to be suitable. Since for all j, excess kurtosis is huge for daily and weekly data, while it is insignificant at 5% level for the monthly data, such change in variance can be seen as almost averaged out for observation period longer than a month. Clustering tendency of observation with large absolute value of $I_{j,t}$ also suggests heteroskedasticity rather than fat-tailed generating processes. Such heteroskedasticity seems to be larger for shorter j, since excess kurtosis of $\{I_{j,t}\}$ is greater for shorter j.

Table 3.2 Summary statistics of short term innovation $\{I_{short}\}$

	$\{I_{short}\}$-daily	$\{I_{short}\}$-weekly	$\{I_{short}\}$-monthly
N	1523	318	74
AVG	0.00000118	-0.00000544	0.00000996
STD.	0.00042632	0.00116631	0.00298994
T-RATIO	0.003 I	-0.005 I	0.003 I
SKEWNESS	5.50906869	-1.39278524	-0.1610953
T-RATIO	87.771 S	-10.140 S	-0.566 I
KURTOSIS	74.2145408	9.89154718	2.90245451
T-RATIO	567.300 S	25.086 S	-0.171 I
D.W.	1.65564963 M	1.41679815 S	0.66072620 S
RUNS	0.00000000 S	0.00000000 S	0.00390000 S
ABS.D.W.	1.23506006 S	1.05405689 S	0.68051083 S
ABS.RUNS	0.00000000 S	0.00010000 S	0.16830000 I
SQ.D.W.	1.92963064 I	1.78096992 I	1.39414056 S
SQ.RUNS	0.48720000 I	0.47510000 I	0.41560000 I

Note: S, M, and I mean statistically significant, marginal, or insignificant at 5% level.
ABS., and SQ. mean series of absolute and squared values respectively.

Table 3.3 Summary statistics of medium term innovation $\{I_{medium}\}$

	$\{I_{medium}\}$-daily	$\{I_{medium}\}$-weekly	$\{I_{medium}\}$-monthly
N	1523	318	74
AVG	-0.00001222	0.00006071	0.00027045
STD.	0.00046167	0.00125236	0.00340022
T-RATIO	-0.026 I	0.048 I	0.080 I
SKEWNESS	0.07414016	-0.65827238	-0.73884445
T-RATIO	1.181 I	-4.792 S	-2.595 S
KURTOSIS	14.47122413	7.87524352	3.76369005
T-RATIO	91.381 S	17.746 S	1.341 I
D.W.	1.68783012 M	1.24494833 S	1.73092678 I
RUNS	0.00000000 S	0.00020000 S	0.00080000 S
ABS.D.W.	0.94505203 S	0.75174014 S	0.51265854 S
ABS.RUNS	0.00000000 S	0.00200000 S	0.09020000 I
SQ.D.W.	1.54590632 S	1.23183166 S	1.02249835 S
SQ.RUNS	0.00000000 S	0.02040000 S	0.34240000 I

Note: S, M, and I mean statistically significant, marginal, or insignificant at 5% level.
ABS., and SQ. mean series of absolute and squared values respectively.

Table 3.4 Summary statistics of long term innovation $\{I_{long}\}$

	$\{I_{long}\}$-daily		$\{I_{long}\}$-weekly		$\{I_{long}\}$-monthly	
N	1523		318		74	
AVG	-0.00000799		0.00004050		0.00000000	
STD.	0.00087769		0.00177209		0.00341096	
T-RATIO	-0.009	I	0.023	I	0.000	I
SKEWNESS	0.30163894		-0.65601082		-0.49501800	
T-RATIO	4.806	S	-4.776	S	-1.738	I
KURTOSIS	18.22614754		6.52875150		3.52191631	
T-RATIO	121.293	S	12.845	S	0.916	I
D.W.	2.20385784	I	2.06052583	I	1.96353238	I
RUNS	0.30140000	I	0.24170000	I	0.24210000	I
ABS.D.W.	0.82495765	S	0.80947049	S	0.80309896	S
ABS.RUNS	0.00000000	S	0.00000000	S	0.11170000	I
SQ.D.W.	1.12504864	S	1.50424043	S	1.41276741	S
SQ.RUNS	0.00000000	S	0.00010000	S	0.48390000	I

Note: S, M, and I mean statistically significant, marginal, or insignificant at 5% level.
ABS., and SQ. mean series of absolute and squared values respectively.

Then, the next question of practical interest is, whether conditional variance VAR[$I_{j,t}|t-1$] performs significantly better than unconditional variance VAR[$I_{j,t}$]. Denoting the process of conditional variance of $\{I_{j,t}\}$ as $\{H_{j,t}\}$, the question is if $\{H_{j,t}\}$ is independent process or not. Even if $\{I_{j,t}\}$ or risk premium adjusted $\{I_{j,t}\}$ is uncorrelated, $\{H_{j,t}\}$ can be serially correlated or, more generally, serially dependent.

Such serial correlation of $\{H_{j,t}\}$ can be detected through examining correlogram of $\{|I_{j,t}|\}$ or $\{(I_{j,t})^2\}$ presented in the figures 3.21 through 3.38. Other tests for serial correlation of time series are also available. Results of Durbin-Watson statistics test and runs check for $\{|I_{j,t}|\}$ and $\{(I_{j,t})^2\}$ are contained in tables 3.5 through 3.7.

Results of these analysis on correlation structure of $\{I_{j,t}\}$, $\{|I_{j,t}|\}$, and $\{(I_{j,t})^2\}$ can be summarized as follows.

(1) $\{I_{j,t}\}$ for j=short or medium have positive significant autocorrelation up to about 5 days.

(2) Correlograms show that evidence for GARCH style of heteroskedasticity is stronger for longer j, where ACF or PACF for $\{|I_{j,t}|\}$ and/or $\{(I_{j,t})^2\}$ exceed the magnitude of those for $\{I_{j,t}\}$ of corresponding order.

For shorter j, auto correlation of $\{|I_{j,t}|\}$ and/or $\{(I_{j,t})^2\}$ are almost equal to those of $\{I_{j,t}\}$. Hence, such autocorrelation of $\{|I_{j,t}|\}$ and/or $\{(I_{j,t})^2\}$ do not imply anything but autocorrelation of $\{I_{j,t}\}$.

(3) Although greater excess kurtosis is observed for shorter j, autocorrelation of $\{|I_{j,t}|\}$ and $\{(I_{j,t})^2\}$ are smaller for shorter j. Then some source of heteroskedasticity other than GARCH type should be considered. Provable explanation might be jump processes, which do not cause autocorrelation in $\{|I_{j,t}|\}$ or $\{(I_{j,t})^2\}$ processes, while generating heteroskedasticity.

$I_{j,t}$ and $I_{j,t+s}$ for j=short and medium have strong positive correlation for small s. Since $I_{j,t}=[R_{j,t}-R_{j+1,t-1}]+[L_{j,t}-L_{j+1,t-1}]$, serial correlation in $\{I_{j,t}\}$ can be explained in the following two ways.

(1) $L_{j,t}-L_{j+1,t-1}$ is positively autocorrelated. This is not consistent with constant term premium hypothesis. However, some kind of time varying risk premium, e.g. GARCH-M, might be consistent with this explanation.

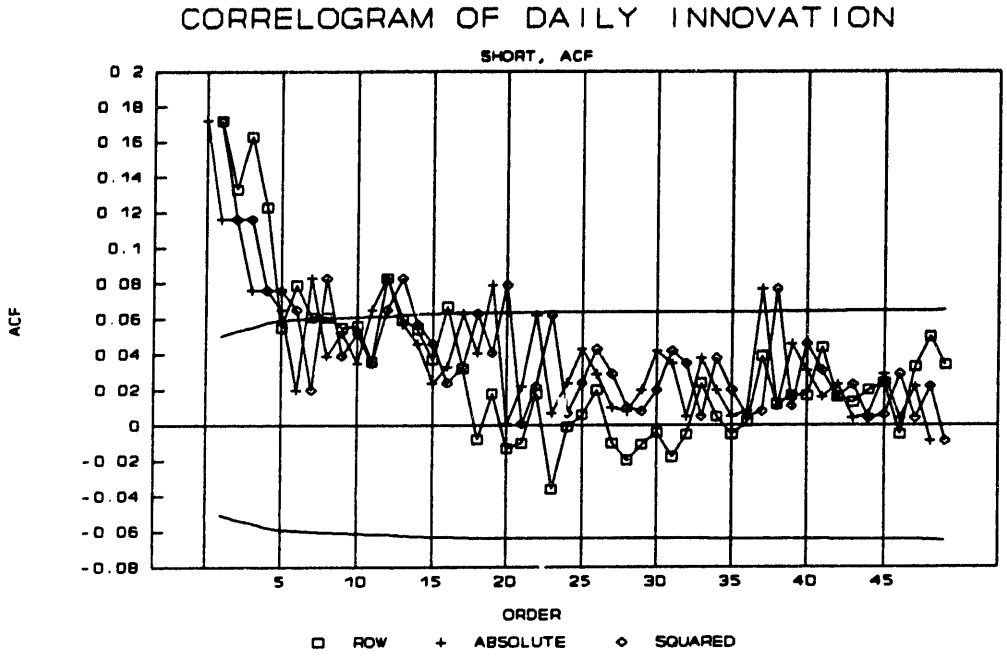

Fig. 3.21

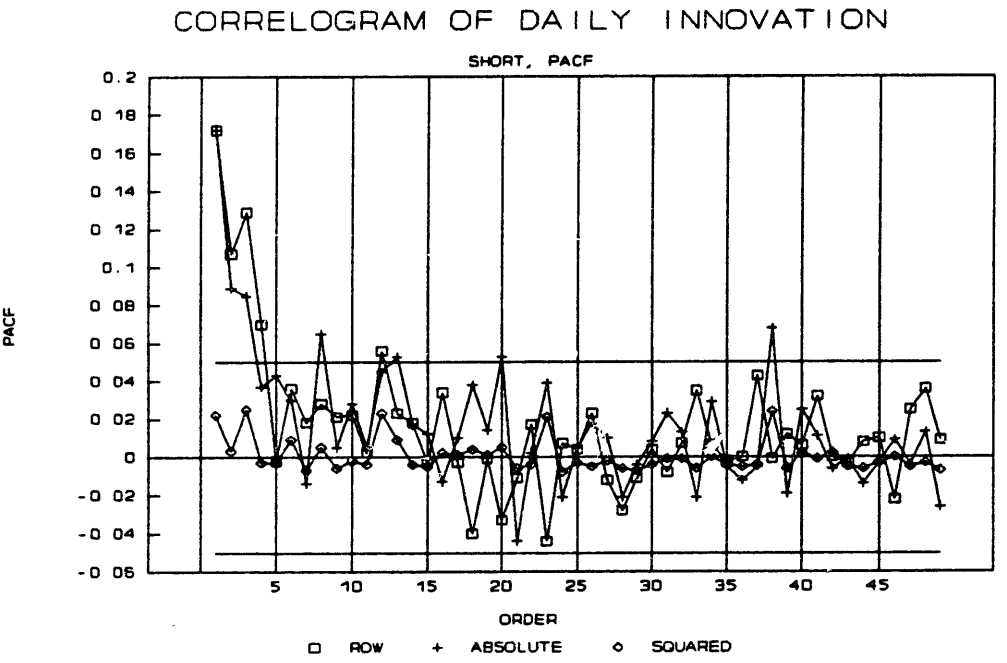

Fig. 3.22

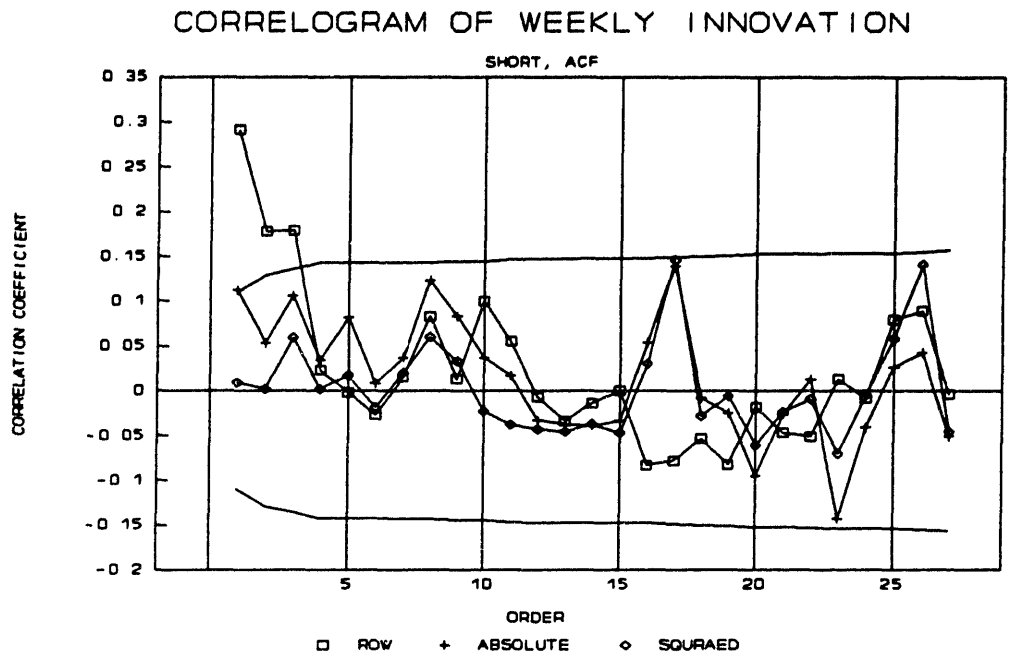

Fig.3.23

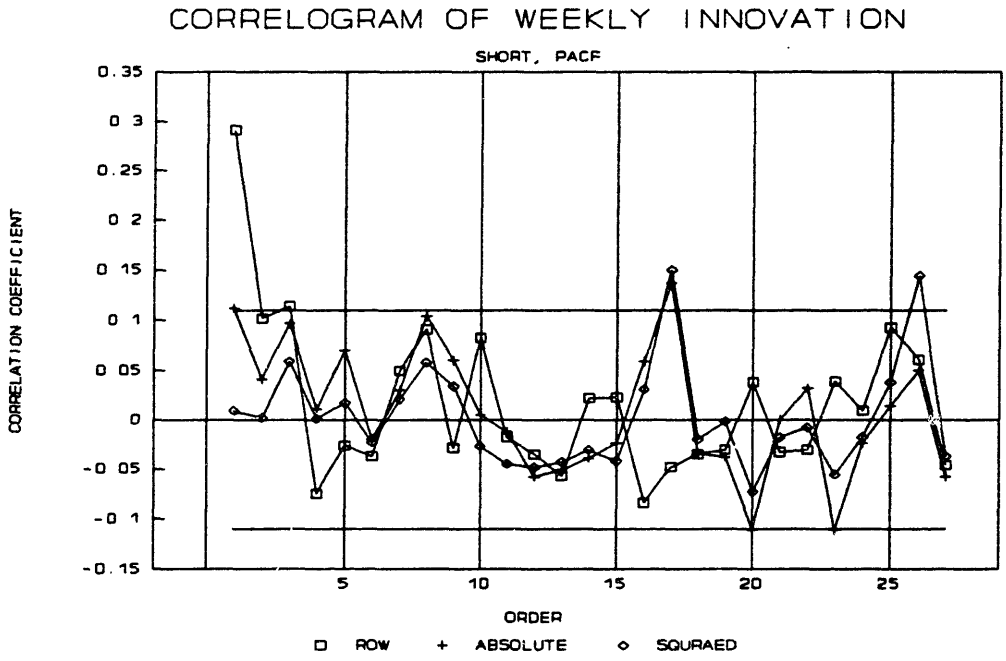

Fig.3.24

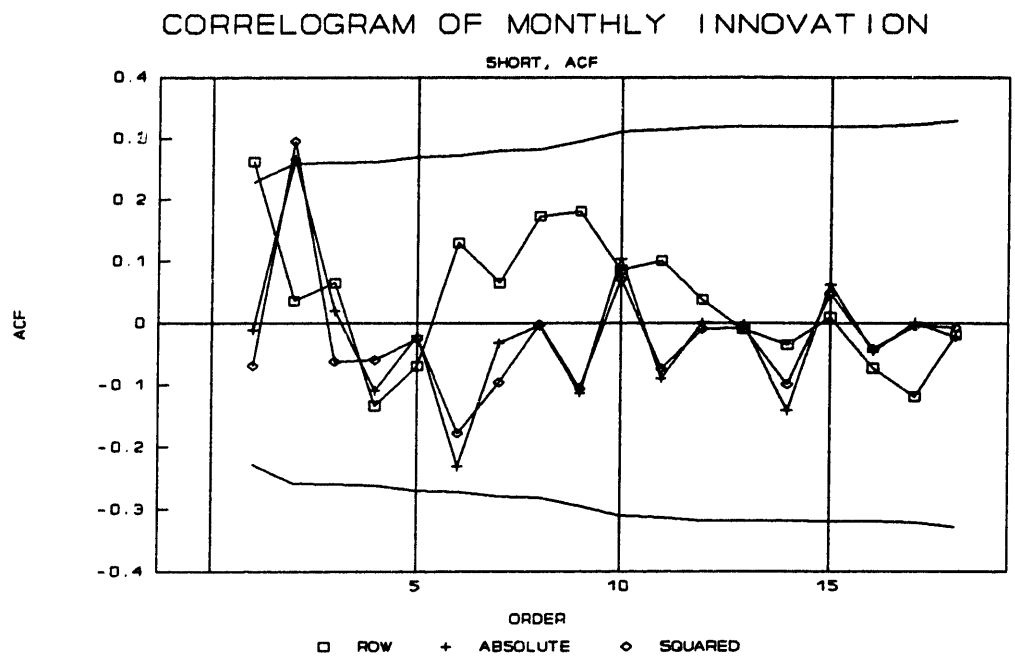

Fig.3.25

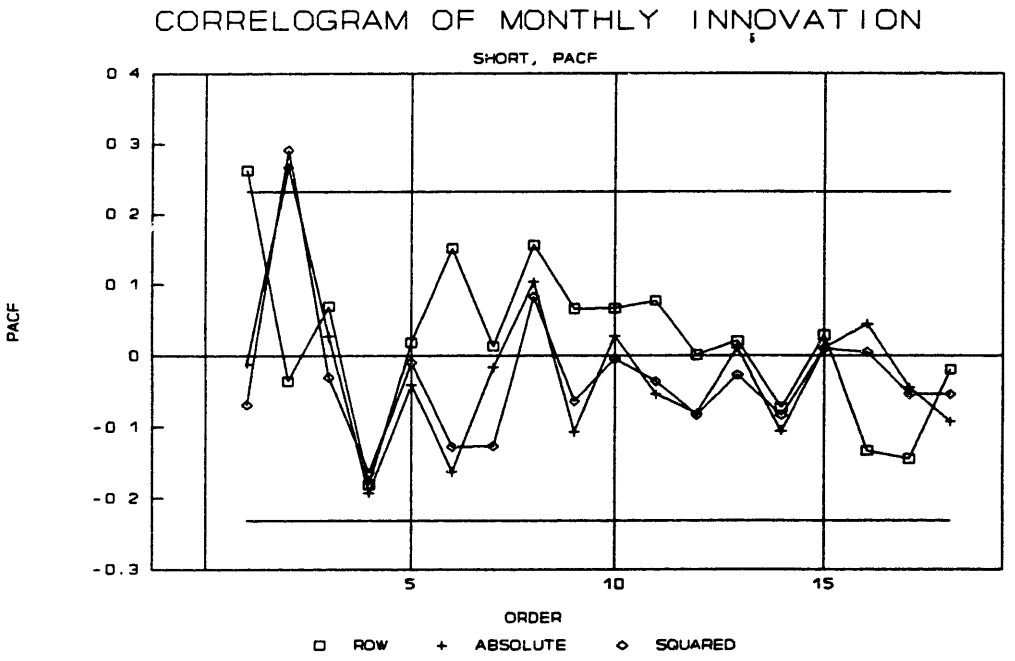

Fig.3.26

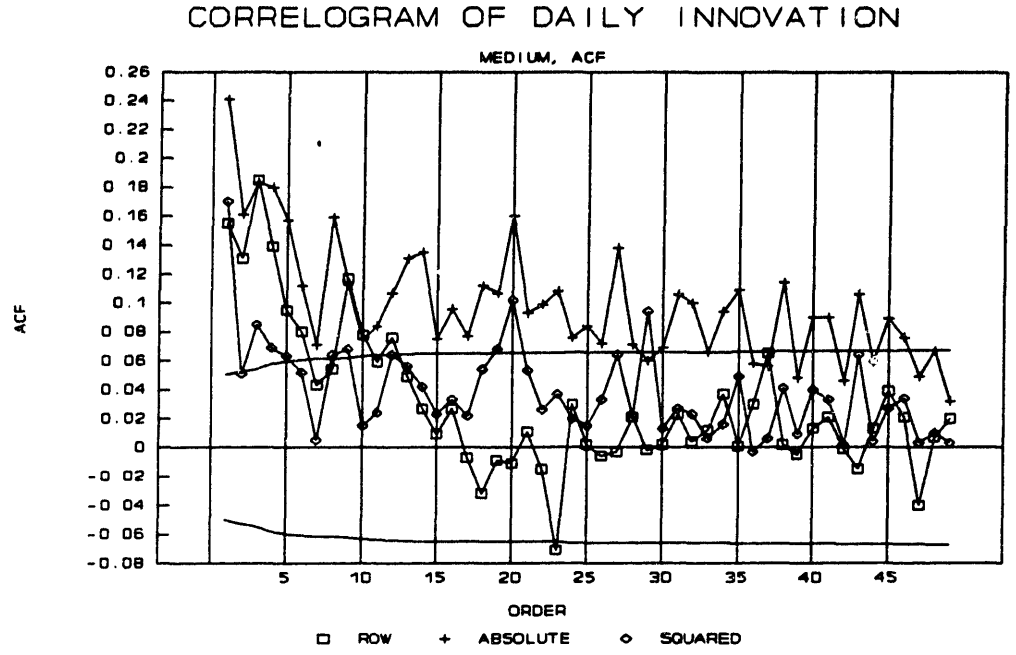

Fig. 3.27

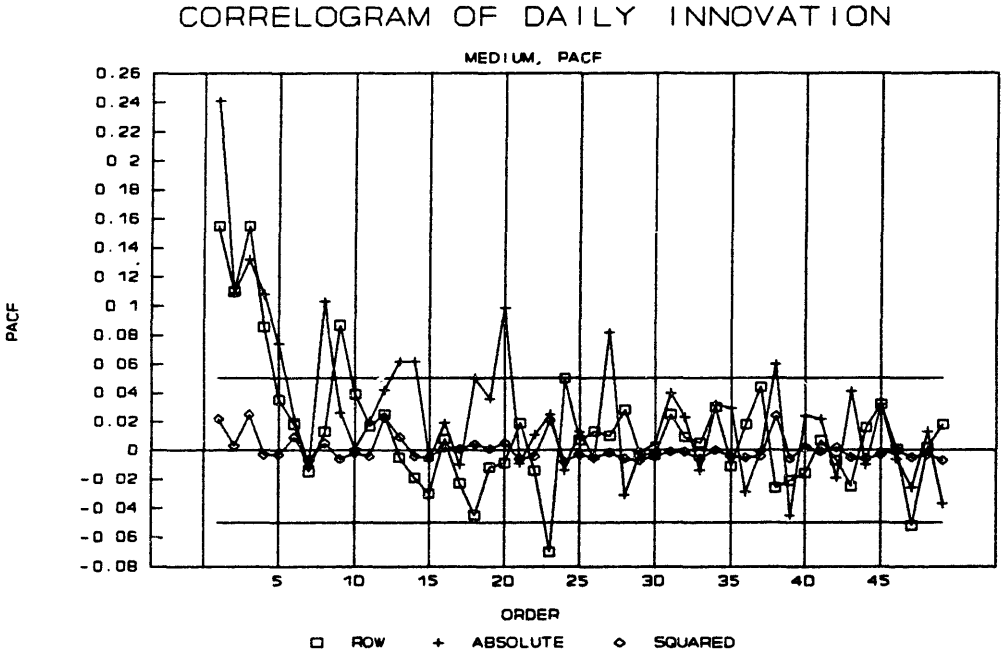

Fig. 3.28

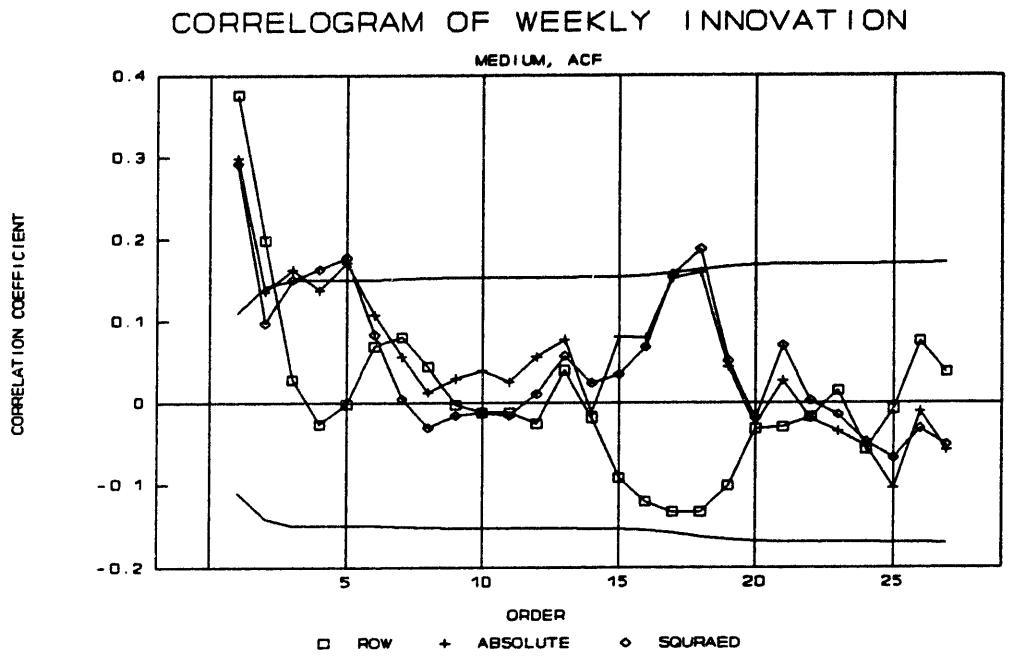

Fig.3.29

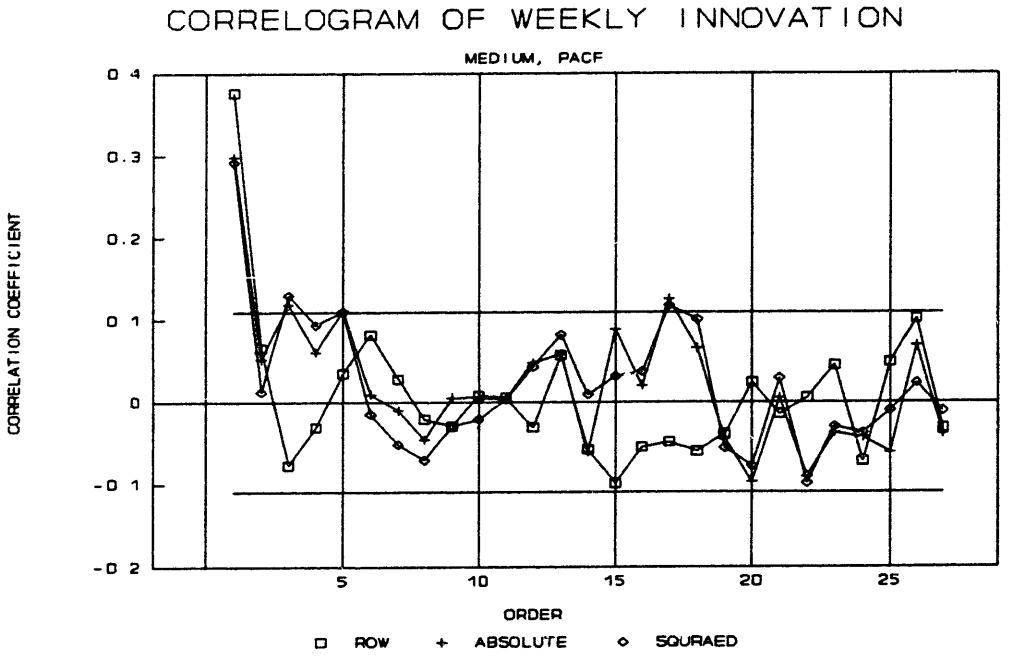

Fig.3.30

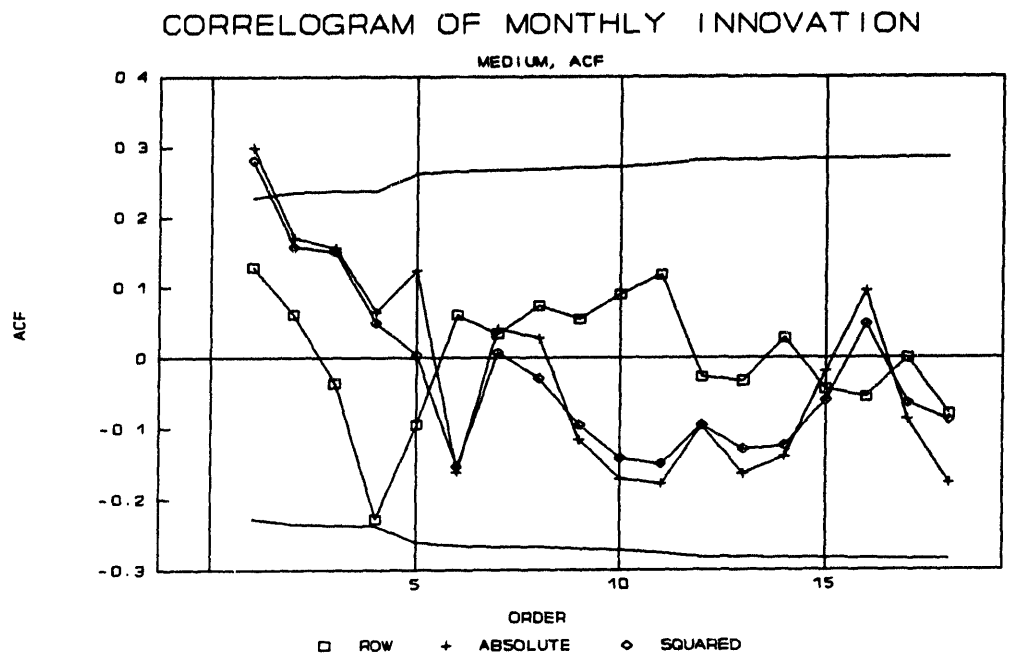

Fig.3.31

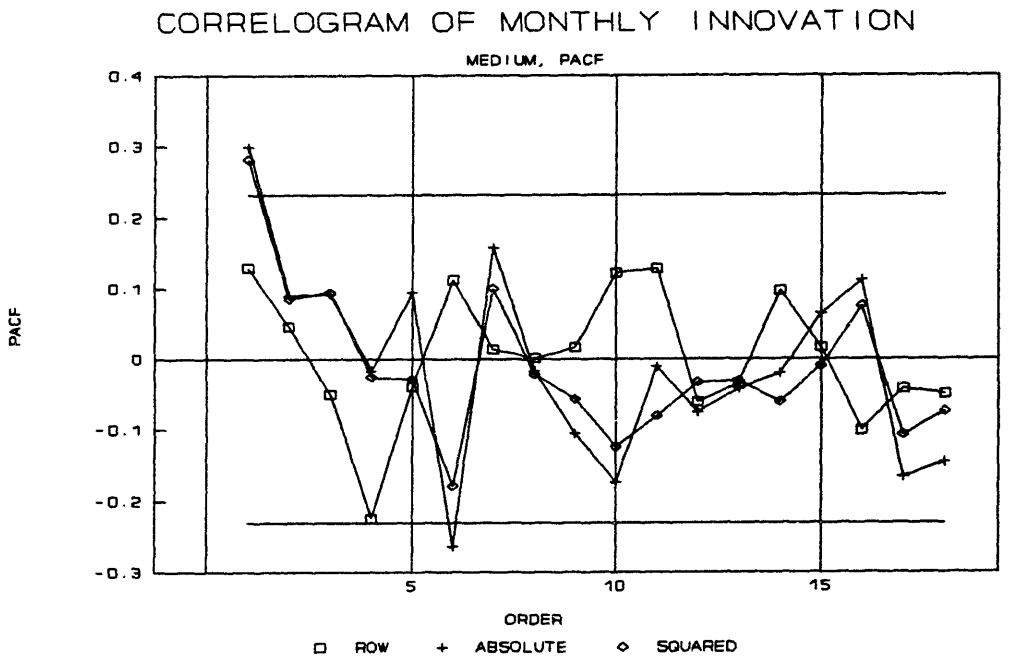

Fig.3.32

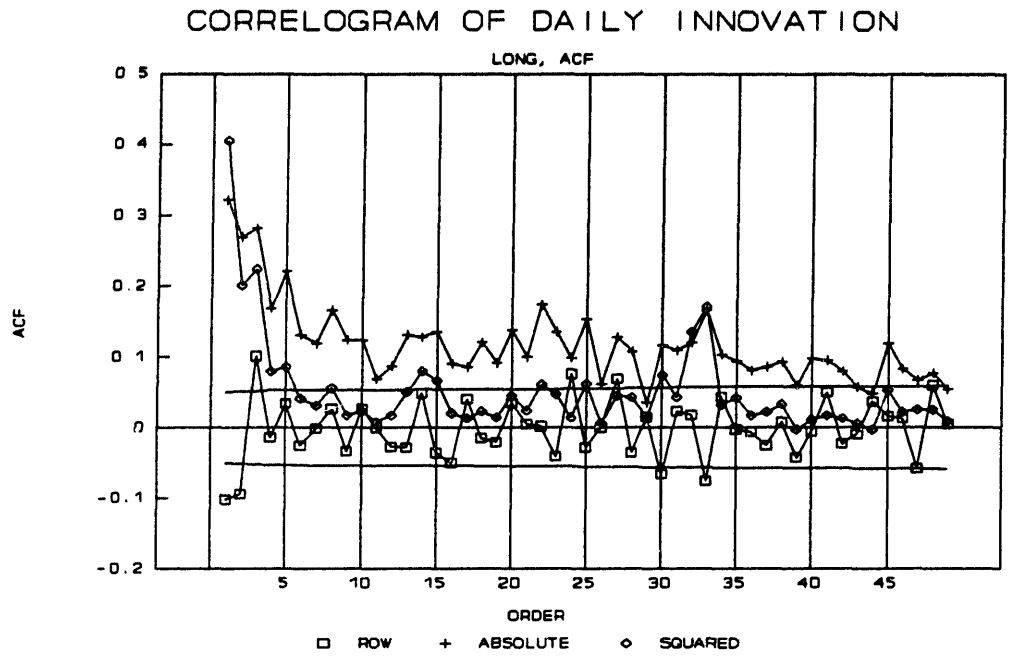

Fig. 3.33

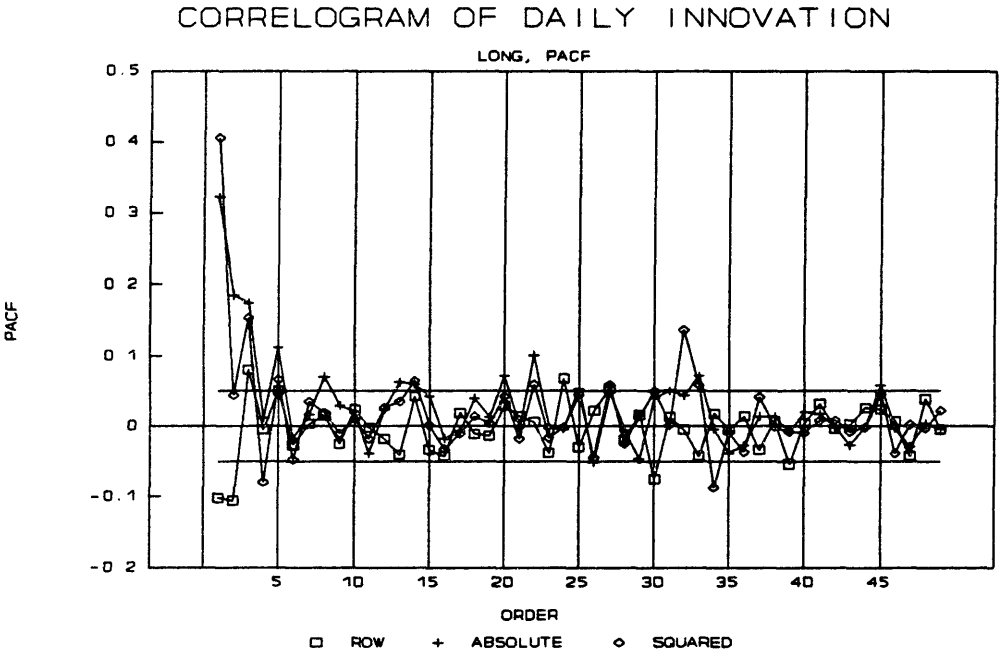

Fig. 3.34

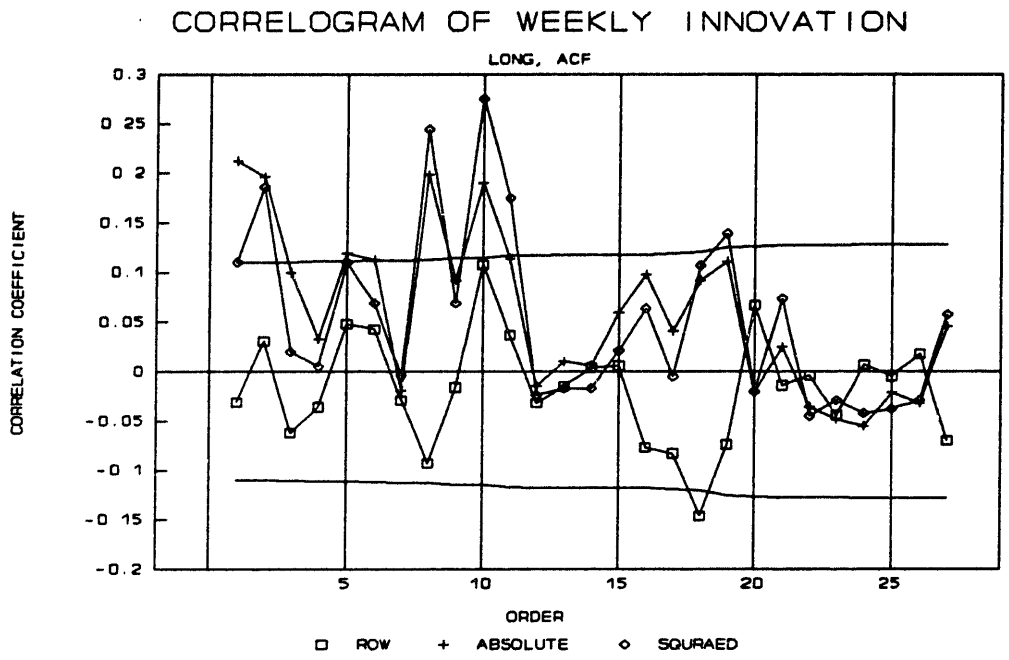

Fig.3.35

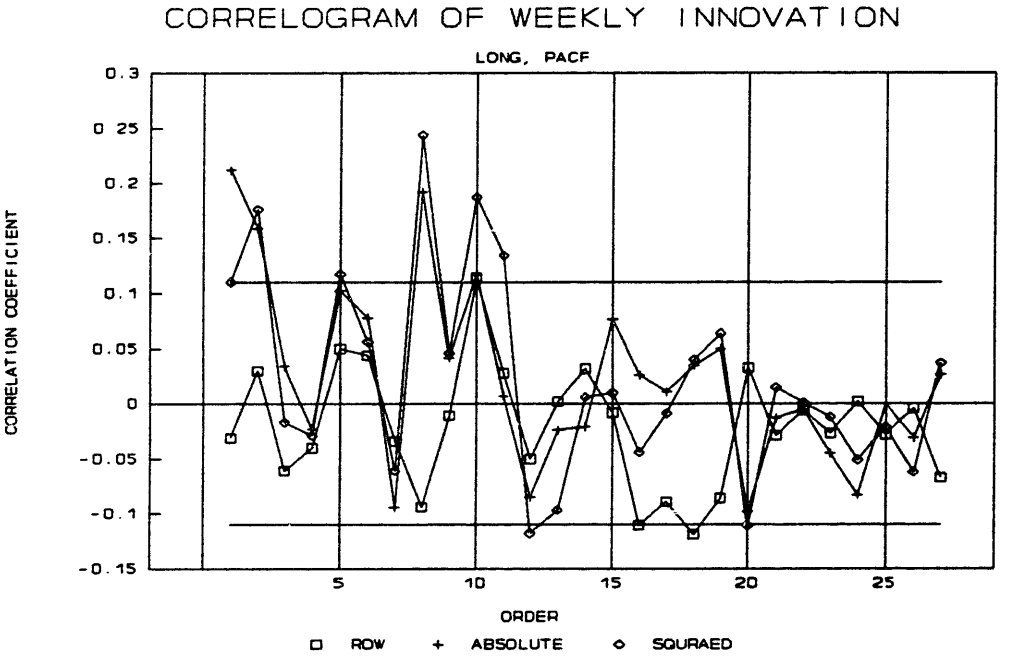

Fig.3.36

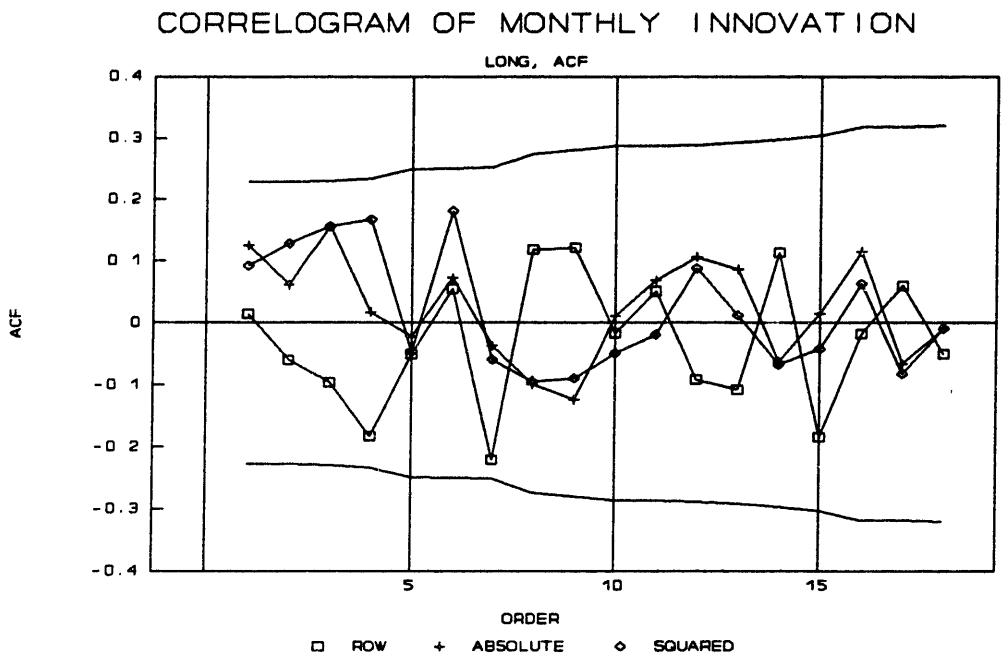

Fig.3.37

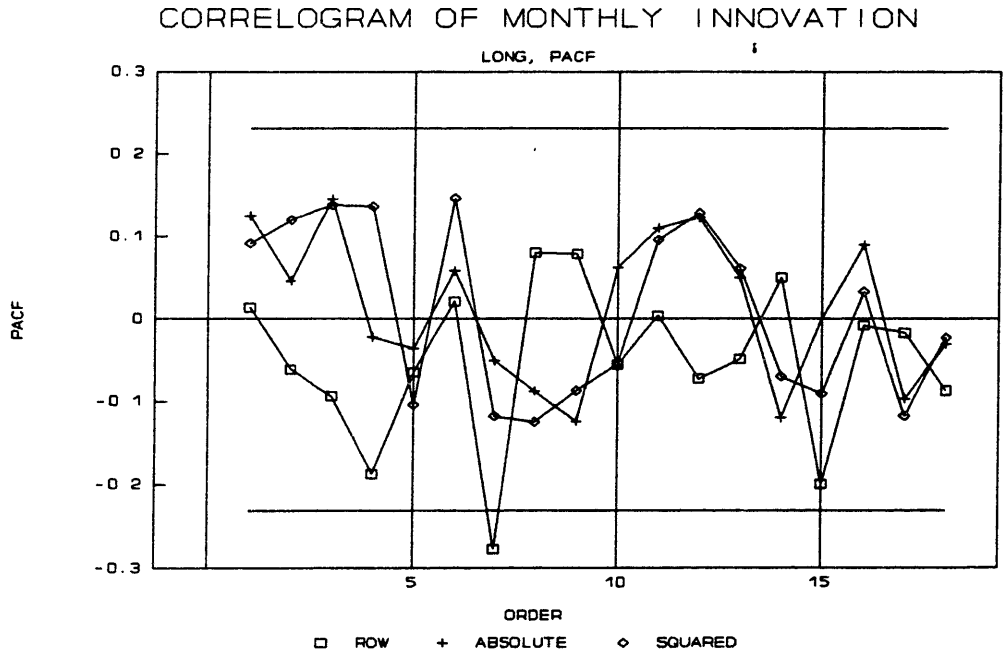

Fig.3.38

(2) $R_{j,t}-R_{j+1,t-1}$ is positively autocorrelated. This implies violation of martingale, and therefore, suggests bond market is inefficient in reflecting new information into price or, in this case, bond yields.

There is another possible source of serial correlation in $R_{j,t}-R_{j+1,t-1}$ other than the two sources discussed above. Since some of the bonds included in NIKKEI Bond Index are not actively traded, there might be seeming autocorrelation caused by non-trading, which is similar to that for small stocks. However, since quotations are revised at least once a day, it is difficult to explain the observed positive autocorrelation being significant at the order of over of 4 days.

We attempted to identify ARMA order for the $\{I_{j,t}\}$ process over the 1523 daily observations from the 6 year and 3 month period from January 1985 to March 1991, using Akaike Information Criteria (AIC) and Shwarts Beysian Information Criteria (SBIC). AIC is known to be inconsistent and to have bias to suggest greater order, while SBIC is known to be consistent and to have opposite bias for small data set. Summary of these tests are in table 3.5 and all attempted

models (ARMA(p,q), $p+q \leq 2$) are reported in tables 3.6 through 3.8.

Table 3.5 ARMA model selection using information criteria

Term	SHORT	MEDIUM	LONG
AIC	ARMA(1,1)	ARMA(1,1)	ARMA(2,0)
SBIC	ARMA(1,1)	ARMA(1,1)	ARMA(2,0)

ARMA(1,1) was selected for $\{I_{short,t}\}$ and $\{I_{medium,t}\}$, and AR(2) was selected for $\{I_{long,t}\}$. We need to have some intuition on how these ARMA processes will behave when applied to long run forecast. For this purpose we calculated variance ratio using estimated parameters for these ARMA precesses.

Variance ratio for q periods, VR(q), is calculated as,

$$VR(q) = 1 + \frac{2}{q}\sum_{i=1}^{q-1}(q-1)\rho_i + (0).$$

K-th order autocorrelation coefficient of ARMA process, $\rho(k)$, can be calculated as a linear combination of lower order

correlation coefficients[44]. For AR(2) with parameters ϕ_1, ϕ_2, using the relationship of

$\rho_i=\phi_1\rho_{i-1}+\phi_2\rho_{i-2}$, $\rho_0=1$, and $\rho_i=\rho_{-i}$,

$$\rho_1 = \phi_1\rho_0+\phi_2\rho_{-1}$$
$$= \phi_1+\phi_2\rho_1$$

then,

$\rho_1 = \phi_1/(1-\phi_2)$ for i=1,

$\rho_2 = \phi_1^2/(1-\phi_2)+\phi_2$ for i=2, and

$\rho_i = \phi_1\rho_{i-1}+\phi_2\rho_{i-2}$ for $i\geq 3$.

For ARMA(1,1) with parameters ϕ and θ,

$\rho_1 = (1+\phi\theta)(\phi+\theta)/(1+2\phi\theta+\phi^2)$,

$\rho_i = \phi\rho_{i-1}$ for $i\geq 2$.

Numerical results are presented in the figure 3.39. Variance ratios for all j, show convergence. For j=long, convergence is relatively quick and the target of convergence is less than 1, while for j=short and medium convergence is slower and target values are far greater than 1.

Finally we develop some view on the behavior of $\{R_{j,t}\}$ based on the above analysis on $\{I_{j,t}\}$. We need to remind the

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44. Anderson, Oliver D. 1984 "Mapping the Parameter Domain onto the Autocorrelation Range for ARMA(p,q) Models, $p+q\leq 2$," Time Series Analysis: Theory and Practice 5, pp.303-314, North-Holland

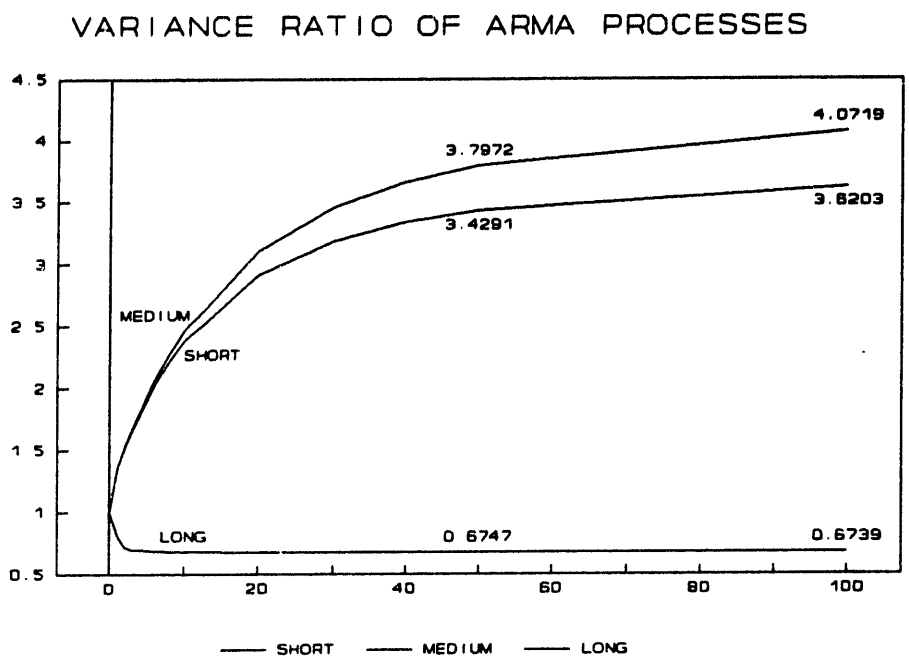

Fig. 3.39

relation between $\{I_{j,t}\}$ and $\{R_{j,t}\}$, that

(1) Source of change in $\{R_{j,t}\}$ is not only $\{I_{t,j}\}$, but also the expected rate movements, which are incorporated in the forward rate structure implied in the market rates, and that

(2) Expected portion of rate movement seems to have greater magnitude compared to $\{I_{j,t}\}$.

Since we have $\{I_{j,t}\}$ process for three different length of j, and we also prepared ARMA estimators, which are linear best estimators, for each $\{I_{j,t}\}$, we can forecast $\{R_{j,t}\}$ using these ARMA estimators of $\{I_{j,t}\}$. Since forecast on $\{R_{j,t+s}\}$, s>0, is

affected by $\{I_{j^*,t}\}$, where j* is any of {short, medium, long} that is longer than j, $\{R_{j,t}\}$ process can be understood as a kind of vector autoregressive (VAR) process. From the above results we expect a VAR process which has the following characteristics.

(1) $R_{long,t}$ and $R_{long,t+s}$ are negatively autocorrelated. The magnitude of negative autocorrelation reaches stability in about 5 days around the level of 0.57. When compared with random walk having equivalent magnitude of daily variance, $\{R_{long,t}\}$ process is persistent on its level, which suggests existence of mean reversion.

(2) $R_{medium,t}$ and $R_{medium,t+s}$ are positively autocorrelated for small value of s. In the shorter time horizon, the variance ratio of $\{R_{medium,t}\}$ converges to about 4 within half a year. This positive autocorrelation is inherent in the $\{I_{medium,t}\}$ process. However, in much longer time horizon, effect of negative autocorrelation inherent in the $\{I_{long,t+s}\}$ process will overwhelm the positive autocorrelation through VAR process.

(3) $R_{short,t}$ and $R_{short,t+s}$ are strongly positively correlated for small value of s. This is caused by strong

positive autocorrelation of $\{I_{short,t}\}$. As it was the case in $\{R_{medium,t}\}$, this positive autocorrelation in $\{R_{short,t}\}$ will be overwhelmed in the very long time horizon by the negative autocorrelation of $\{I_{long,t}\}$, which penetrates into $\{R_{short,t}\}$ through VAR process. However, VAR effect from $\{I_{medium,t}\}$ having stronger positive autocorrelation than $\{I_{short,t}\}$ might work to the opposite direction, and over the middle length of time horizon, $\{R_{short,t}\}$ might show complicated behavior.

However, these discussions on mean reversion and market inefficiency are subject to further examination about the characteristics of the process of risk premium $\{L_{j,t}\}$. ARMA models examined for $\{I_{j,t}\}$ process above are not adjusted for ARCH in mean (ARCH-M) effects, which might cause spurious autocorrelation in $\{I_{j,t}\}$ through autocorrelation in risk premium $\{L_{j,t}\}$. If $\{H_{j,t}\}$ follows process of ARMA type, i.e. processes with serial correlation, $\{L_{j,t}\}$ might also follow some process with serial correlation. If ARCH-M effects could be identified and removed, serial correlation in $\{|I_{j,t}|\}$ and $\{(I_{j,t})^2\}$ would be somewhat weakened. On the other hand, for the long term innovation series, which has negative autocorrelation before adjustment for $\{L_{j,t}\}$, it is difficult to guess what the ARMA process after removing ARCH-M effect is.

Table 3.6

ESTIMATED ARMA(p,q) MODEL FOR $\{I_{short,t}\}$ PROCESSES FOR p+q≤2

SHORT	CONSTANT	AR(1)	AR(2)	MA(1)	MA(2)	AIC	SBIC
ARMA(0,1)	0.000001			-0.14		-15.543	-15.539
T-RATIO	0.10			-5.75			
ARMA(0,2)	0.000001			-0.13	-0.08	-15.548	-15.541
T-RATIO	0.10			-5.46	-3.16		
ARMA(1,0)	0.000000	0.172				-15.548	-15.544
T-RATIO	0.09	6.81					
ARMA(1,1)	0.000000	0.872		0.75		-15.578	-15.571
T-RATIO	0.04	24.11		15.29		MIN	MIN
ARMA(2,0)	0.000000	0.153	0.106			-15.557	-15.550
T-RATIO	0.09	6.03	0.03				

Table 3.7

ESTIMATED ARMA(p,q) MODEL FOR $\{I_{medium,t}\}$ PROCESSES FOR $p+q \leq 2$

MEDIUM	CONSTANT	AR(1)	AR(2)	MA(1)	MA(2)	AIC	SBIC
ARMA(0,1)	-0.00001			-0.13		-15.379	-15.375
T-RATIO	-0.92			-5.19			
ARMA(0,2)	-0.00001			-0.12	-0.07	-15.384	-15.377
T-RATIO	-0.88			-4.76	-3.04		
ARMA(1,0)	-0.00001	0.155				-15.383	-15.379
T-RATIO	-0.89	6.13					
ARMA(1,1)	-0.00000	0.892		0.77		-15.422	-15.415
T-RATIO	-0.57	28.14		17.52		MIN	MIN
ARMA(2,0)	-0.00000	0.138	0.109			-15.393	-15.386
T-RATIO	-0.49	5.43	4.31				

Table 3.8

ESTIMATED ARMA(p,q) MODEL FOR $\{I_{long,t}\}$ PROCESSES FOR p+q≤2

LONG	CONSTANT	AR(1)	AR(2)	MA(1)	MA(2)	AIC	SBIC
ARMA(0,1)	-0.00000			0.122		-14.086	-14.083
T-RATIO	-0.40			4.80			
ARMA(0,2)	-83.10000			0.096	0.078	-14.09	-14.083
T-RATIO	-0.45			3.78	3.05		
ARMA(1,0)	-0.00000	-0.10				-14.084	-14.08
T-RATIO	-0.39	-4.00					
ARMA(1,1)	-0.00001	-0.81		-0.83		-14.072	-14.065
T-RATIO	-0.35	-4.88		-5.33			
ARMA(2,0)	-0.00000	-0.11	-0.10			-14.093	-14.086
T-RATIO	-0.44	-4.42	-4.15			MIN	MIN

3.5 Correlation Among Forward Rates With Different Terms

Stability of correlation among innovations of different terms are examined. All combinations of correlation between two of the three innovation series are calculated for each year (see figure 3.40), and each quarter (see figure 3.41). The results show that medium-long correlation and short-long correlation are slowly moving within the positive area, and that short-medium correlation had negative coefficient in one year (1988), when it was highly unstable, while for the other 5 years the coefficient stays positive.

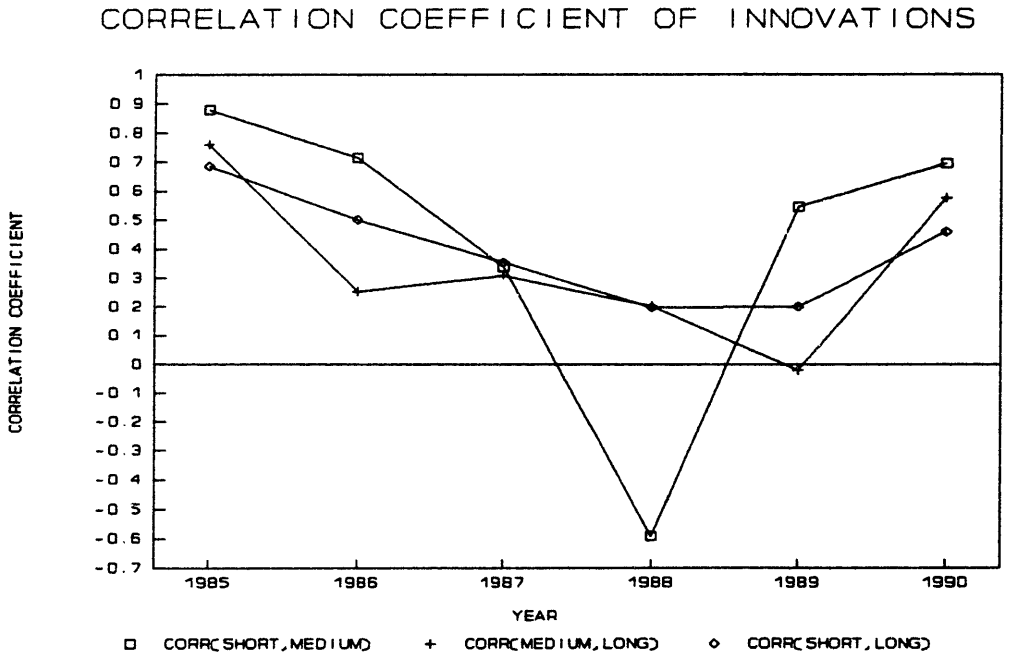

Fig. 3.40

When we examine figure 3.41, the negative correlation of

short-medium are observed in Q1 1988, Q3,1988, and Q4 1988. On the other hand, in other quarters during the period of late 1987 and early 1989, short-medium correlation took large positive value. Furthermore, figure 3.42 presents that correlation coefficient, $CORR(I_{short,t}, I_{medium,t})$, has absolutely clear single peak, while it also has long left tail. These facts suggest existence of some stable correlation between $\{I_{short,t}\}$ and $\{I_{medium,t}\}$ over some sufficiently long period.

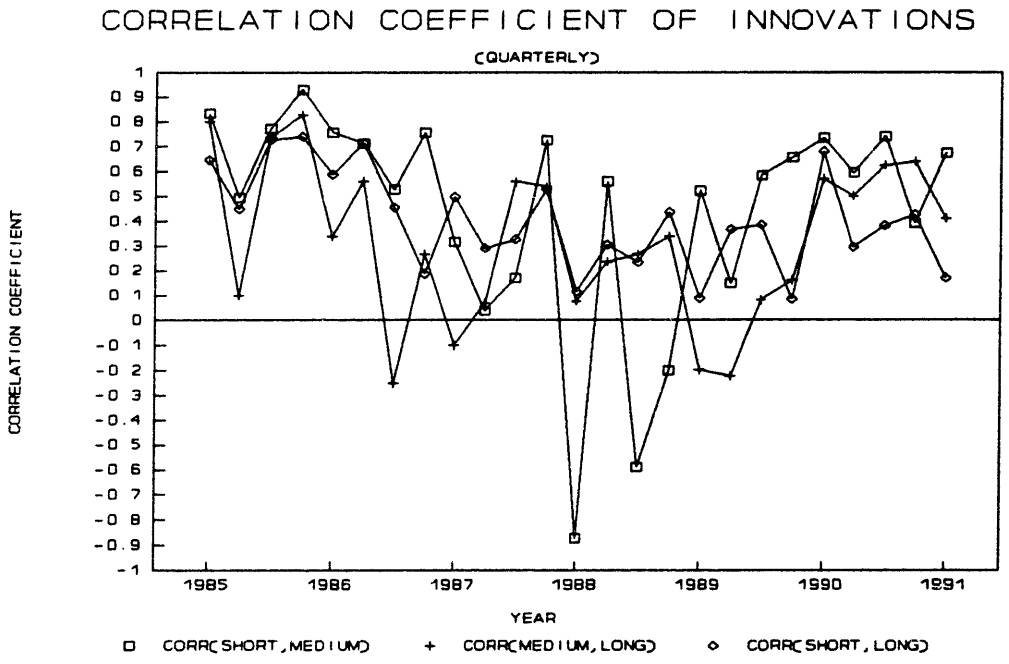

Fig. 3.41

Figures 3.40 and 3.41 suggests that correlation among the innovation series of different terms are changing slowly within certain ranges, and that such correlations are roughly predictable in the long run. This long run stability is important, when we attempt to construct our extended yield

curve model for bond options with long period to expiration.

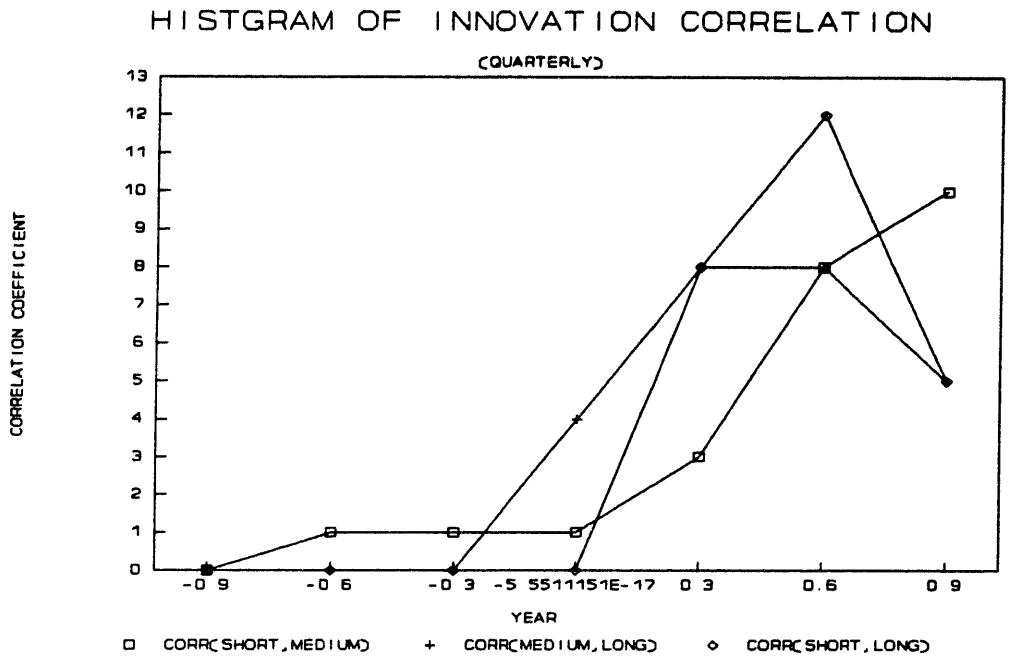

Fig. 3.42

Section 4. ARMA Model with Jump/GARCH-M effect

In this section we propose a time series model for $\{I_{j,t}\}$ process, and discuss on estimation methodology. We are interested in modeling characteristics of $\{I_{j,t}\}$ process movements, especially taking care of the following points.

(1) Serial correlation in the risk premium adjusted innovation series, which is related to market inefficiency and/or mean reversion of interest rate movement. We will attempt to estimate ARMA model to capture this feature. ARMA(1,1) or lower order is expected for $\{I_{j,t}\}$, j=short, medium. AR(2) or lower order is expected for $\{I_{j,t}\}$, j=long.

(2) Heteroskedasticity in GARCH form and jump form. We consider heteroskedasticity as combined GARCH(1,1) process and Poisson jump process. We expect for shorter j, jump component might have larger relative contribution, while GARCH effect will dominant component of change in variance of $\{I_{j,t}\}$ for longer j.

(3) Time varying risk premium, especially in the form of jump/GARCH-M. Since our data are already in

logarithm, we will consider multiplicative form of risk premium $L_{j,t}$,

$$L_{j,t}=P\sqrt{H_{j,t}},$$

where $H_{j,t}$ is conditional variance of risk premium adjusted $I_{j,t}$ process, and C is positive real constant.

It is convenient to reconfirm our notation, before proceeding further.

Forward short term rate $\{R_{j,t}\}$ is decomposed into two components

$$R_{j,t}=X_{j,t}+L_{j,t},$$

where, $L_{j,t}$ is time varying risk premium and $X_{j,t}$ is estimator for j period future spot rate at time t.

Correspondingly Innovation in forward rate $\{I_{j,t}\}$ is decomposed as,

$$I_{j,t}=\Delta X_{j,t}+\Delta L_{j,t}.$$

Risk premium is assumed to be expressed as,

$$\Delta L_{j,t}=L_{j,t}-L_{j,t-1}=P\Delta\sqrt{H_{j,t}}.$$

Then, following Jordin (1988), our model can be specified as follows.

$$I_t - P\Delta\sqrt{H_t} = C + \sum_{i=1}^{p} \phi_i (I_{t-i} - P\Delta\sqrt{H_{t-i}}) + \sum_{i=1}^{q} \theta_i \varepsilon_{t-i} + \varepsilon_t,$$

$$\varepsilon_t = \sqrt{H_t} Z_t + \sum_{i=1}^{n_t} Y_i, \quad Z_t \sim N(0,1),$$

$$H_t = C_H + \sum_{i=1}^{p_H} \phi_{Hi} H_{t-i} + \sum_{i=1}^{q_H} \theta_{Hi} \varepsilon_{t-i}^2,$$

where C, ϕ_i $(i=1,\cdots,p)$, θ_i $(i=1,\cdots,q)$ are parameters for ARMA(p,q) process,

$\{H_t\}$ is conditional variance in GARCH form,

C_H, ϕ_{Hi} $(i=1,\cdots,p_H)$, θ_{Hi} $(i=1,\cdots,q_H)$ are parameters for GARCH(p_H,q_H) process,

n_t follows Poisson distribution with parameter λ,

$Y \sim N(\theta_J, \delta^2)$ is random variable for size of jump,

P is coefficient for the term of time varying risk premium, and

$$\Delta\sqrt{H_t} = \sqrt{H_t} - \sqrt{H_{t-1}}.$$

For maximum likelihood estimation of this model, likelihood function is given in the following formula.

$$l=-T\lambda-\frac{T}{2}\ln(2\pi)+\sum_{t=1}^{T}\ln\left[\sum_{j_c=0}^{\infty}\frac{\lambda^{j_c}}{j_c!}\frac{1}{\sqrt{H_t+\delta^2 j_c}}\exp\left(\frac{-(I_t-P\Delta\sqrt{H_t}-\mu_t-\theta_J j_c)^2}{2(H_t+\delta^2 j_c)}\right.\right.$$

$$\mu_t=C+\sum_{i=1}^{p}\phi_i(I_{t-i}-P\Delta\sqrt{H_{t-i}})+\sum_{i=1}^{q}\theta_i e_{t-i}$$

$$H_t=C_H+\sum_{i=1}^{p_H}\phi_{Hi}H_{t-i}+\sum_{i=1}^{q_H}\theta_{Hi}e_{t-i}^2,$$

where T is the number of observation,

λ is a parameter representing density of Poisson jump,

θ_J and δ size of Poisson jump, $Y\sim N(\theta_J,\delta^2)$,

j_c is count for number of jump in a unit period, which can be neglected but for small j_c for its amall provability, and

Parameters in the formula of H_t and μ_t are the same as those in the model definition.

Estimation of these complicated time series using maximum

likelihood method is not easy. Judge, etc. (1985)[45] contains overview of numerical optimization algorithms. Harvey (1989)[46] gave some practical comments on this problem. According to Harvey (1989) the following methods are suggested.

(1) Scoring method, or Gauss-Newton method, can be used. While they are asymptotically efficient and guaranteed to converge, they take time. An algorithm by Berndt, Hall, Hall, and Hausman (1974)[47] (BHHH algorithm) is mostly used example of this category.

(2) EM algorithm by Watson and Engle (1983)[48] can also be used. This is also slow.

(3) As numerical optimization using computers, FORTRAN

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45. Judge, George G. 1985 Appendix B, Numerical Optimization Methods, The Theory and Practice of Econometrics, 2nd Edition, pp.951-979, Wiley, New York

46. Harvey, Andrew C. 1989 Forecasting, Structural Time Series Models and the Kalman Filter, Cambridge University Press, London

47. Berndt, E.R., B. H. Hall, R. E. Hall, and J. A. Hausman 1974 "Estimation and Inference in Nonlinear Structural Models," Annals of Economic and Social Measurement, Vol.3 pp.653-665

48. Watson, M.W., and R. F. Engle 1983 "Alternative Algorithm for the Estimation of Dynamic Factor, MIMIC and Varying Coefficient Regression," Journal of Econometrics, Vol.23 pp.385-400

subroutine E04JBF of NAG library is said to work well in practice[49].

(4) As a method for parameter estimation in the frequency domain using fast Fourier transformation (FFT) is recommended. Although this method is efficient and speedy, sometimes results from this method are different from those from time domain estimation. In such cases estimator obtained from frequency domain procedure should not be used.

Although we attempted maximum likelihood estimation by BHHH algorithm implemented in FORTRAN 77 using IBM 4381 hardware, it did not performed well. Severe computational difficulties were encountered. Even using the double precision of the environment, execution errors of underflow were unavoidable.

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49. In Mark 12 of NAG library E04JBF is announced to be suspended but not to be withdrawn before Mark 14. E04UCF will be the alternative.

Section 5. Taylor-M Model: An Easier Alternative

In this section we use Taylor model as an alternative method for modeling financial time series with time varying variance. We attempt to extend Taylor model to include time varying risk premium. Taylor in mean (Taylor-M) model is proposed. This extension is led by the identical idea that extended ARCH model into ARCH-M model.

Taylor (1986) contains exclusive introduction and development of many versions of Taylor model, while variety of functional form of $\{V_t\}$ process and estimation method for it were proposed in many papers[50].

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50. Taylor, Stephen J. and Kingsman, Brian G. 1978 "Non-stationarity in Sugar Prices," Journal of Operations Research Society, Vol.29 No.10, pp.971-980

    Taylor, Stephen J., and Kingsman, Brian G. 1979 "An Analysis of the Variance and Distribution of Commodity Price Changes," Australian Journal of Management, 1979, pp. 135-149

    Taylor, Stephen J. 1980 "Conjectured Models for Trends in Financial Prices, Tests and Forecasts," Journal of Royal Statistical Society, Series A Vol.143 (1980) Part 3, pp.338-362

    Taylor, Stephen J. 1982 "Tests of the Random Walk Hypothesis Against a Price-Trend Hypothesis," Journal of Financial and Quantitative Analysis, Vol.17 No.1, pp.37-61

    Taylor, Stephen J. 1982 "Financial Returns by the Product of Two Stochastic Processes-A Study of Daily Sugar Prices, 1961-79" Time Series Analysis: Theory and Practice 1, North-Holland

The most basic idea of Taylor model is to decompose given financial time series $\{X_t\}$ into $\{U_t\}$ process and $\{V_t\}$ process,

$$(X_t-\mu) = V_t \times (U_t-\mu)$$

where, μ is unconditional mean of X_t,
V_t represents conditional standard deviation at time t, and
U_t has unit variance.

Analysis on $\{V_t\}$ process leads to understanding on behavior of time varying variance, and analysis on serial correlation in $\{U_t\}$ process leads to examination of market efficiency in the sense of random walk.

Since $\{V_t\}$ process is unobservable, functional form of $\{V_t\}$ process must be specified before parameters are estimated. Taylor (1986) proposed many variation for $\{V_t\}$ process. Taylor (1983)[51] applied his framework to develop a trading rule which exploits market inefficiency.

We pick up the simplest version with the least number of

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51. Taylor, Stephen J. 1983 "Trading Rules for Investors in Apparently Inefficient Futures Markets," Futures Markets: modelling, managing and monitoring futures trading, pp.165-198, Basil Blackwell

parameters, where $\{V_t\}$ follows exponentially weighted moving average (EWMA) process, and then extend it to Taylor-M model.

$$I_t-\mu = V_tU_t$$
$$V_t = M^*_t/0.798$$
$$M^*_t = \theta M_{t-1}+(1-\theta)M^*_{t-1}$$
$$M_t = |I_{t-1}-\mu-\Delta(L_{t-1})|$$
$$L_t = P(V_t)$$

where, U_t is heteroskedasticity-rescaled stochastic process with $VAR(U_t)=1$,

V_t is unobservable process of conditional standard diviation of $\{I_t-\mu\}$ process,

0.798 is the ratio, $E[|X|]/\sigma_X$, for $X\sim N(0,1)$,

M^*_t is estimator for M_t based only on information as of time t-1,

θ is a parameter for speed of adjustment of M^*_t process,

μ is unconditional mean of I_t,

P is a positive real coefficient for risk premium,

L_t is risk premium, and

$\Delta(L_t)=L_t-L_{t-1}$.

As the initial value of M^* we used average of $|I_t-\mu|$ for $1\le t\le 20$.

$$M^*_{20} = E[|I_t-\mu|], \qquad 1 \le t \le 20$$

Then we estimated the two parameters of θ and P using $21 \le t$. We selected the value of parameters that minimize the following sum of squared error.

$$SSE = \Sigma(M^*_t-M_t)^2$$

Figure 5.1 through 5.6 presents sensitivity of the sum of the squared error (SSE) against each of the parameters estimated. Results for estimation of Theta and P for different j are presented in table 5.1.

Table 5.1 Summary of Taylor-M model

PARAMETER	SHORT	MEDIUM	LONG
P	1.4	0.87	1.5
θ	0.07	0.08	0.13

Estimated θs in the Taylor-M model are greater for longer j. This result seems to be insensitive for existence of risk premium, when compared with the θs for the usual Taylor model

of 0.065, 0.08, and 0.15 for short, medium, and long j respectively. Estimated values for risk premium parameter of P are interesting. Figure 5.7 presents the magnitude of risk premium which is the products of estimated P_j and $\{V_{j,t}\}$ for each j. Risk premium for the longest term rate is by far the largest, while risk premium for short and medium term are almost equivalent magnitude and reverse their order from time to time.

We may think that discount rates for longer j may contain larger risk premium, because longer duration causes greater magnitude of price risk for bond holders. Then, greater value of $L_{j,t}$ for long j is understandable, while it needs different explanation that $L_{j,t}$ for short and medium j are almost indifferent. Our hypothetical explanation is that short-term rate movements might contain some additional sources of risk different from that contained in the longer term rates. One of possible source of such risk might be jump component of rate movement, which are not predictable from the past and present rate movements.

We attempted another version of Taylor-M model, using AR(1), instead of EWMA, for $\{V_t\}$ process. AR(1) process was constructed so that M^*_t reverts to unconditional mean of M_t. However, Taylor-M with AR(1) could not outperform that with EWMA in terms of SSE minimization.

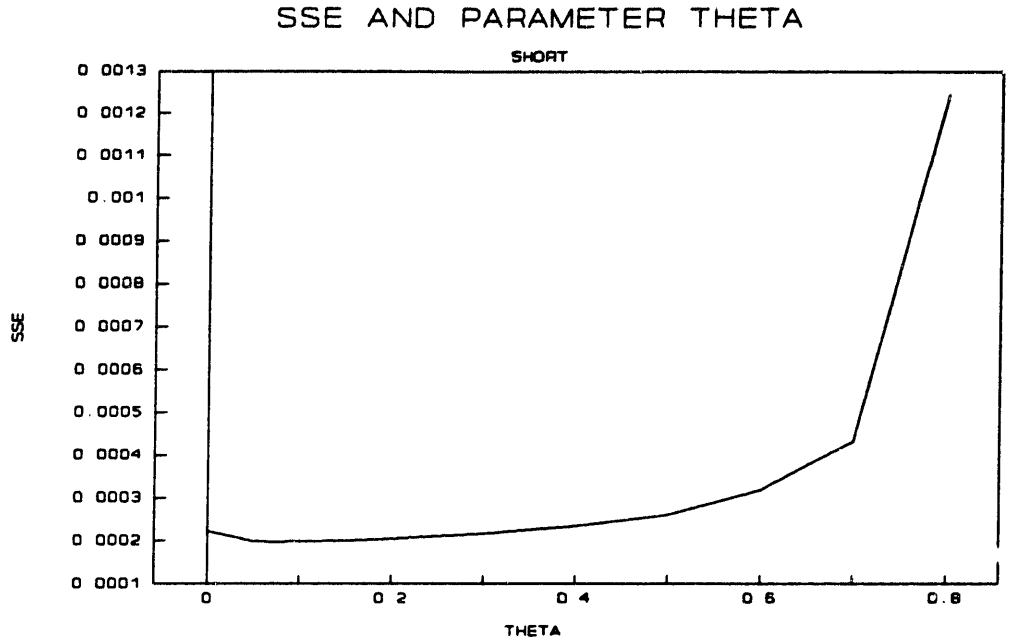
SSE AND PARAMETER THETA
SHORT
0 0013
0 0012
0 0011
0.001
0 0009
0 0008
0 0007
0 0006
0.0005
0 0004
0 0003
0 0002
0 0001
SSE
0
0 2
0 4
0 6
0.8
THETA

Fig.5.1

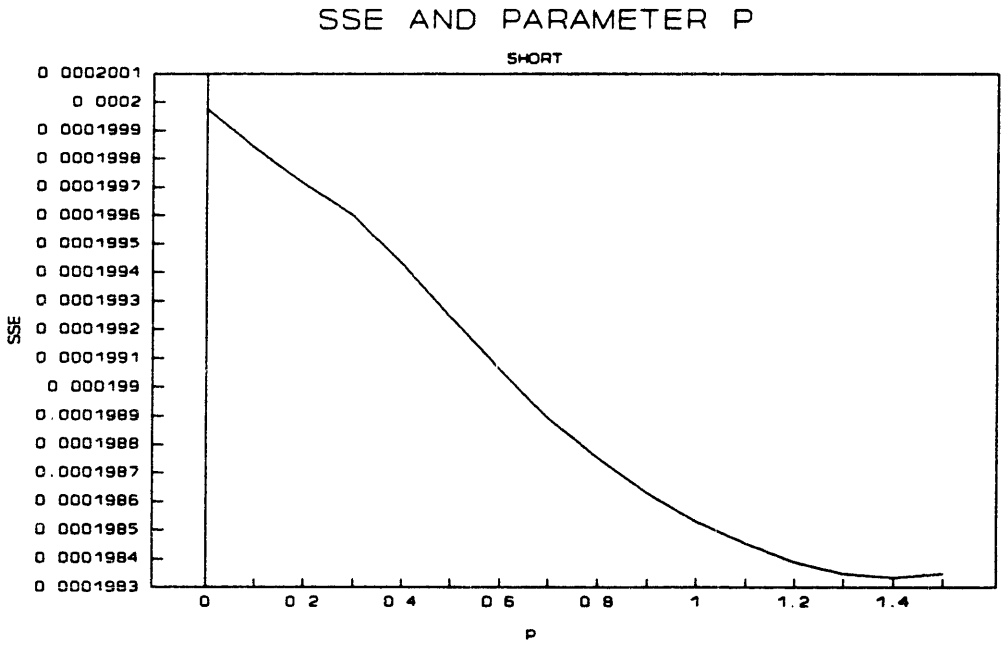
SSE AND PARAMETER P
SHORT
0 0002001
0 0002
0 0001999
0 0001998
0 0001997
0 0001996
0 0001995
0 0001994
0 0001993
0 0001992
0 0001991
0 000199
0.0001989
0 0001988
0.0001987
0 0001986
0 0001985
0 0001984
0 0001983
SSE
0
0 2
0 4
0 6
0 8
1
1.2
1.4
P

Fig.5.2

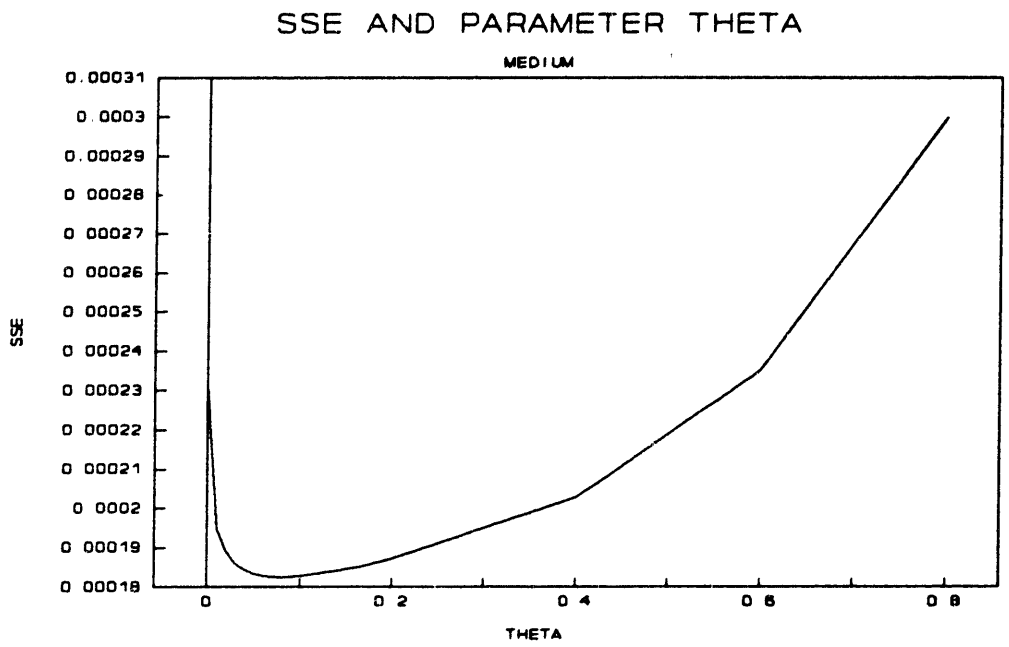
SSE AND PARAMETER THETA
MEDIUM
0.00031
0.0003
0.00029
0 00028
0 00027
0 00026
0 00025
0 00024
0 00023
0 00022
0 00021
0 0002
0 00019
0 00018
SSE
0
0 2
0 4
0 6
0 8
THETA

Fig.5.3

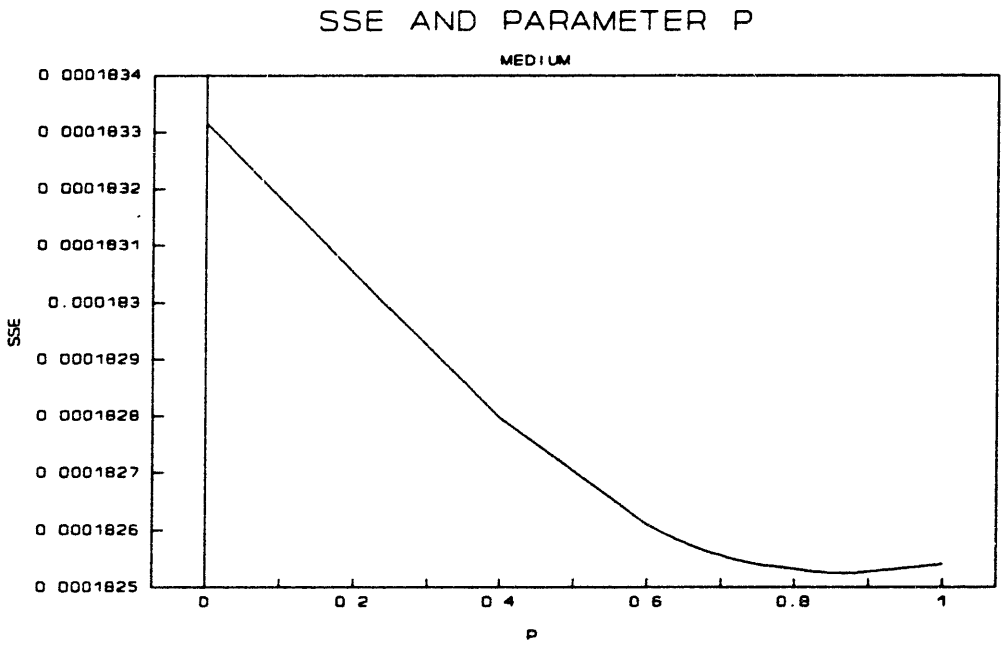
SSE AND PARAMETER P
MEDIUM
0 0001834
0 0001833
0 0001832
0 0001831
0.000183
0 0001829
0 0001828
0 0001827
0 0001826
0 0001825
SSE
0
0 2
0 4
0 6
0.8
1
P

Fig.5.4

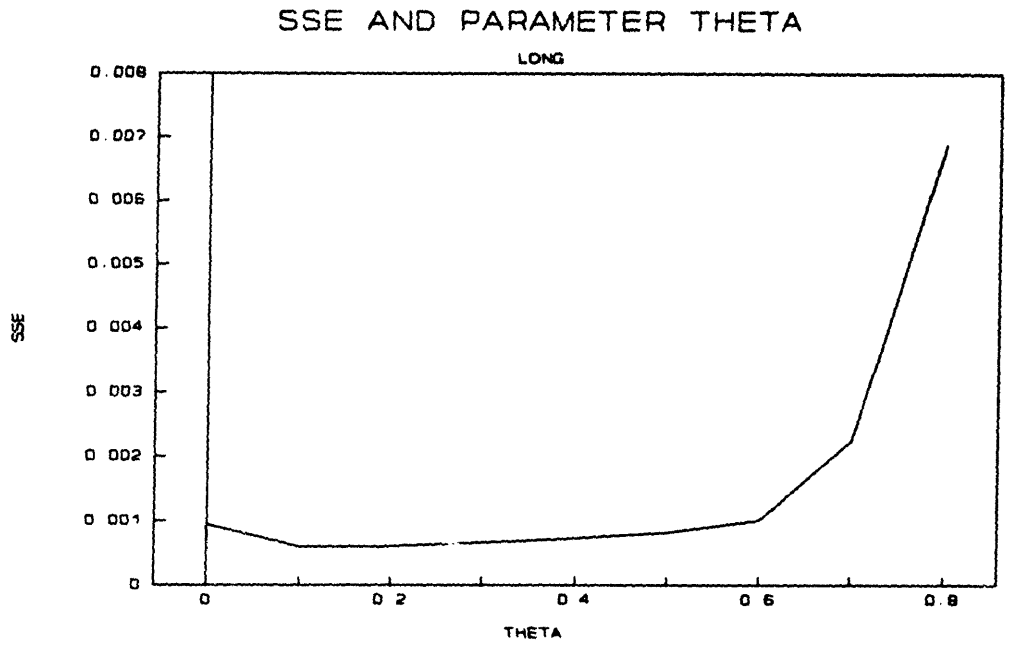
SSE AND PARAMETER THETA
LONG
0.008
0.007
0 006
0.005
0 004
0 003
0 002
0 001
0
SSE
0
0 2
0 4
0 6
0.8
THETA

Fig.5.5

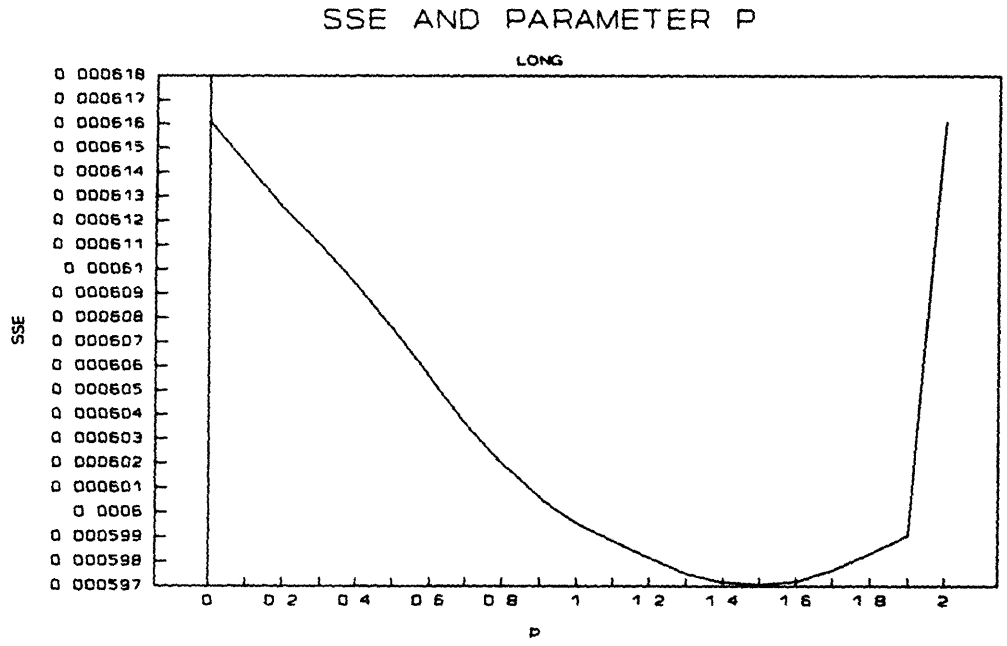
SSE AND PARAMETER P
LONG
0 000618
0 000617
0 000616
0 000615
0 000614
0 000613
0 000612
0 000611
0 00061
0 000609
0 000608
0 000607
0 000606
0 000605
0 000604
0 000603
0 000602
0 000601
0 0006
0 000599
0 000598
0 000597
SSE
0
0 2
0 4
0 6
0 8
1
1 2
1 4
1 6
1 8
2
P

Fig.5.6

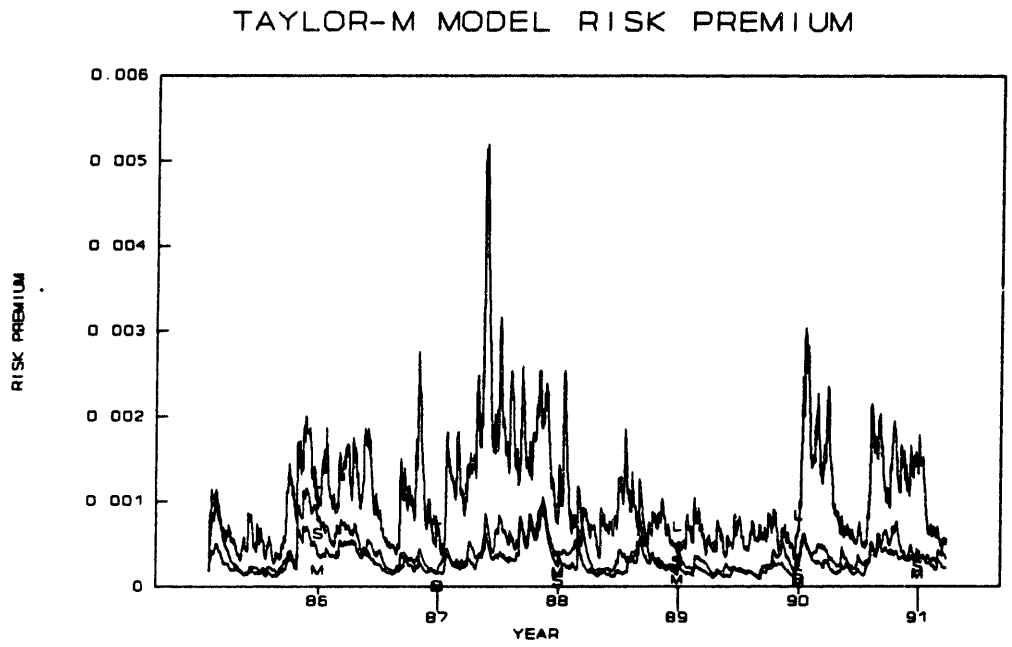

Fig.5.7

Comparing summary statistics of $\{U_t\}$ processes from the usual Taylor model and from Taylor-M model (see table 5.2 and 5.3), we find that risk premium contributed a significant portion of the excess kurtosis. This suggests an understanding that excess kurtosis of $\{I_{j,t}\}$ process is not only caused by time varying variance, but also time varying risk premium. Table 5.4 presents result of regression of $I_{j,t}$ against change in risk premium, i.e. first order difference of $\{L_{j,t}\}$. Coefficients were, of course, highly significant. The magnitude of R-squared, being 30% to 40%, implies that roughly one third of change in forward rate $\{R_{j,t}\}$ is attributed to change in risk premium.

Table 5.2 Summary statistics of Taylor-M {Ut} process

	SHORT	MEDIUM	LONG
N	1502	1502	1502
AVG	-0.04515103	0.06031681	0.01463858
STD	0.13365335	0.09924514	0.16998269
T-RATIO	-0.338 I	0.608 I	0.086 I
SKEWNESS	1.29922663	-0.31057040	1.35513798
T-RATIO	20.556 S	-4.914 S	21.441 S
KURTOSIS	11.24210842	10.47036042	5.39873405
T-RATIO	65.203 S	59.098 S	18.976 S
D.W.	1.35912665 S	97798220 S	1.87045222 I
RUNS	0.0000 S	0.0000 S	0.8238 I

Note: S, M, and I in the table mean Significant, Marginal, or Insignificant at 5% level respectively.

Table 5.3 Summary statistics of $\{U_t\}$ process from usual Taylor model

	U_s	U_m	U_l
N	1513	1523	1513
AVG	0.04403108	0.00743489	-0.01231278
T-RATIO	0.03 I	0.01 I	-0.01 I
SKEWNESS	5.12674677	0.53190358	0.21198975
T-RATIO	81.41 S	8.45 S	3.37 S
KURTOSIS	66.14266157	21.52328210	9.09983082
T-RATIO	501.35 S	147.07 S	48.43 S
D.W.	1.62242739 S	1.68840758 M	2.07543024 I

Note: S, M, and I in the table mean Significant, Marginal, or Insignificant at 5% level respectively.

Table 5.4 Analysis on the magnitude of Taylor-M effects

REGRESSEE	REGRESSOR	CONSTANT	COEFFICIENT	R-SQUARE	D.W.
I_s	$D[V_s]$	0.000002	0.00010826	30.5%	1.05
		0.19	25.72		
I_m	$D[V_m]$	-0.00001	0.00015047	32.0%	1.06
		-1.23	26.64		
I_l	$D[V_l]$	-0.00000	0.00030225	39.0%	1.73
		-0.44	31.06		

Figures 5.8 to 5.13 presents plot and correlogram of $\{I_{j,t}\}$ series after eliminating risk premium. For short and medium j, $\{I_{j,t}\}$ show strong and long lasting positive autocorrelation. Shape of correlograms suggest moving average, MA(q), processes. On the other hand, $\{I_{j,t}\}$ with long j have almost no significant correlation at 5% level.

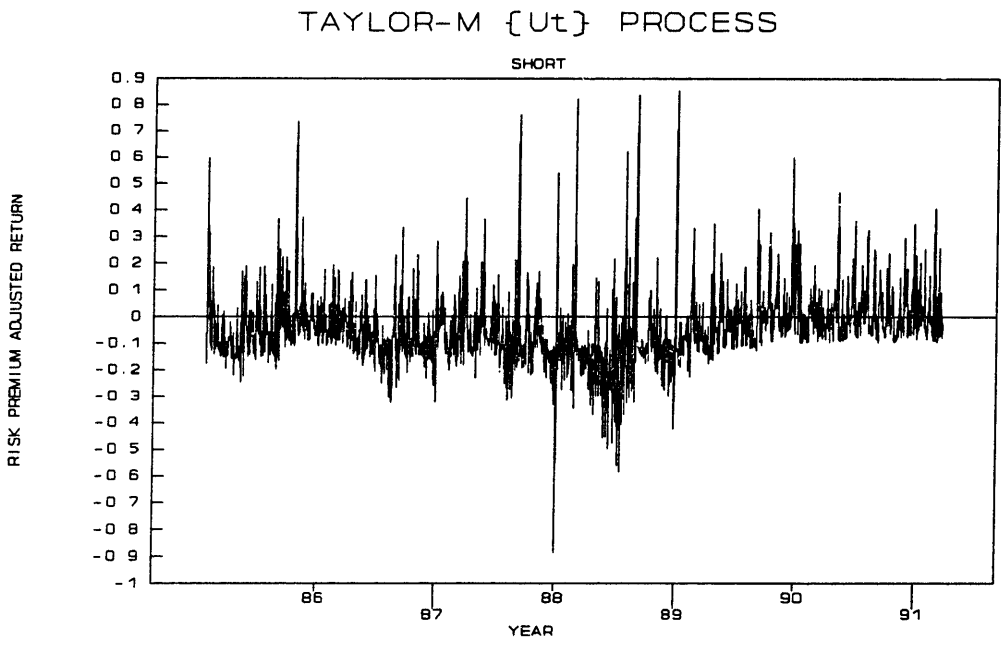
TAYLOR-M {Ut} PROCESS
SHORT
RISK PREMIUM ADJUSTED RETURN
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
-0.8
-0.9
-1
86
87
88
89
90
91
YEAR

Fig.5.8

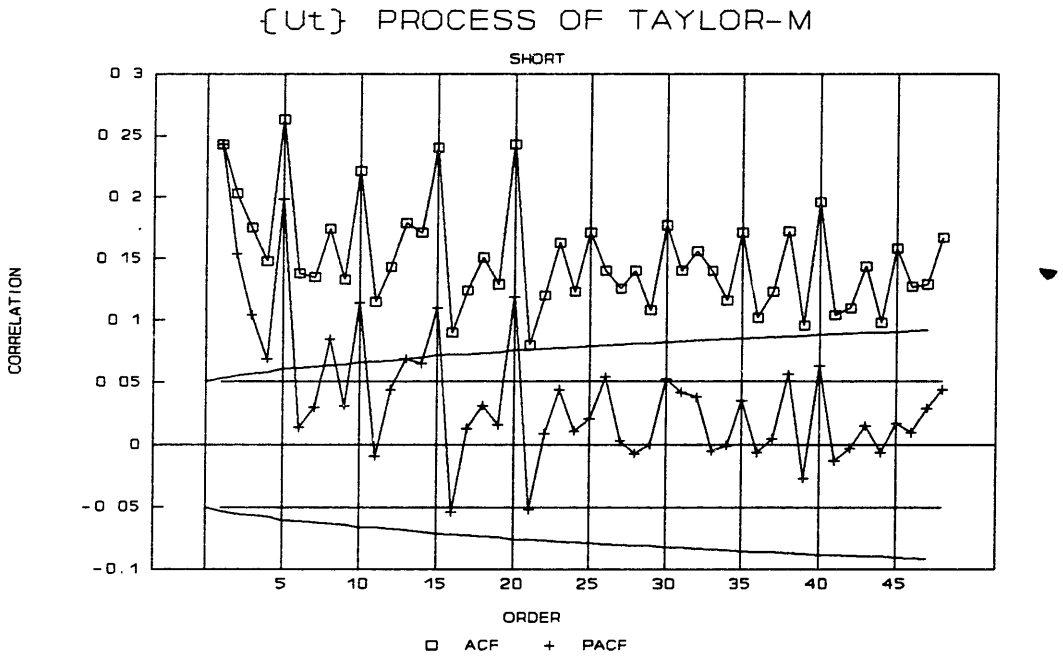
{Ut} PROCESS OF TAYLOR-M
SHORT
CORRELATION
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
-0.1
5
10
15
20
25
30
35
40
45
ORDER
ACF
PACF

Fig.5.9

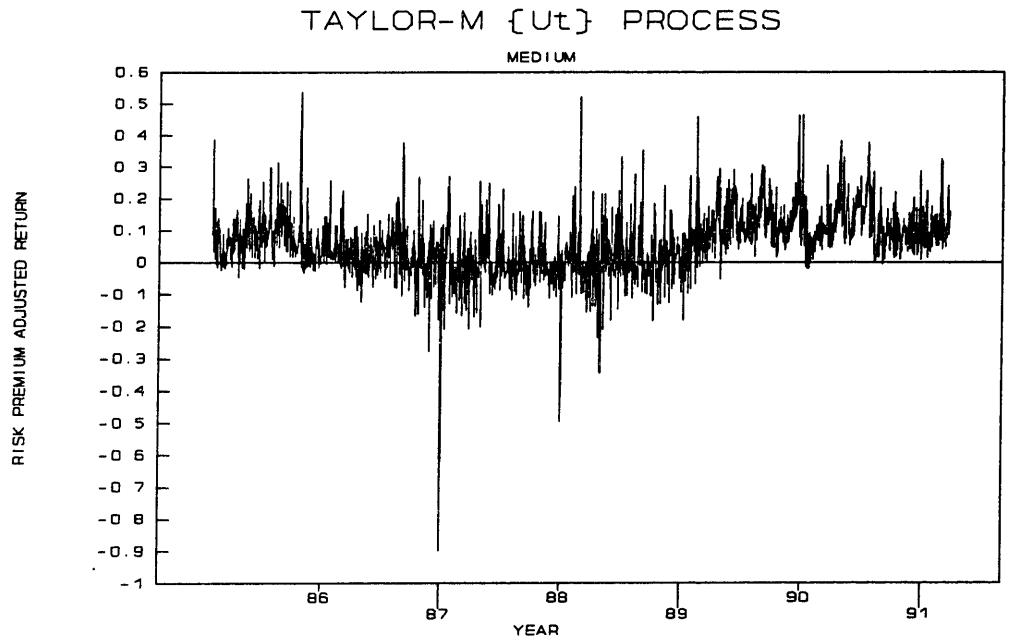

Fig.5.10

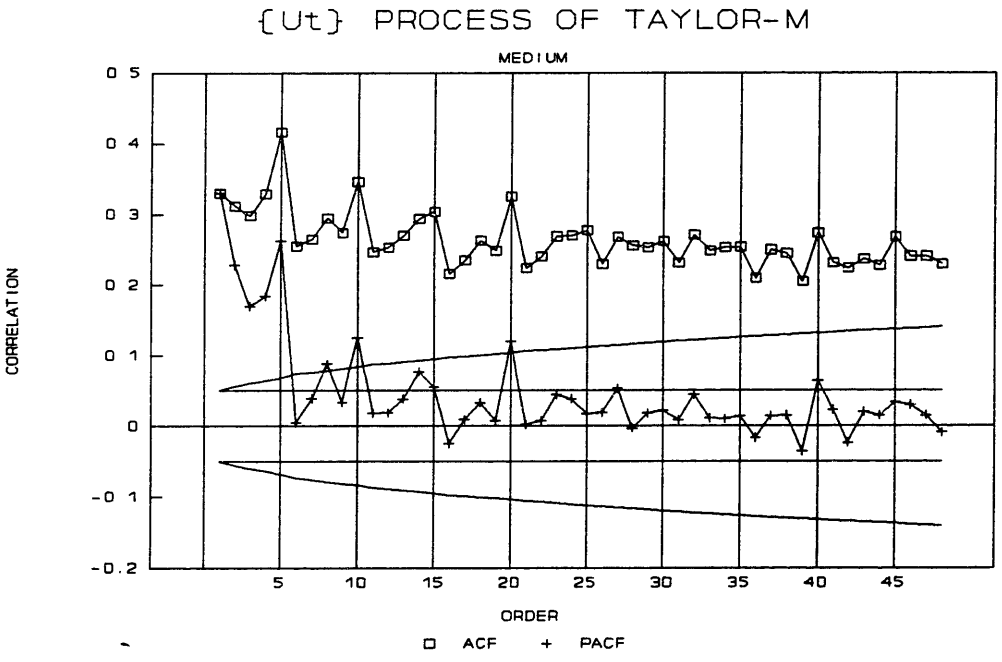

Fig.5.11

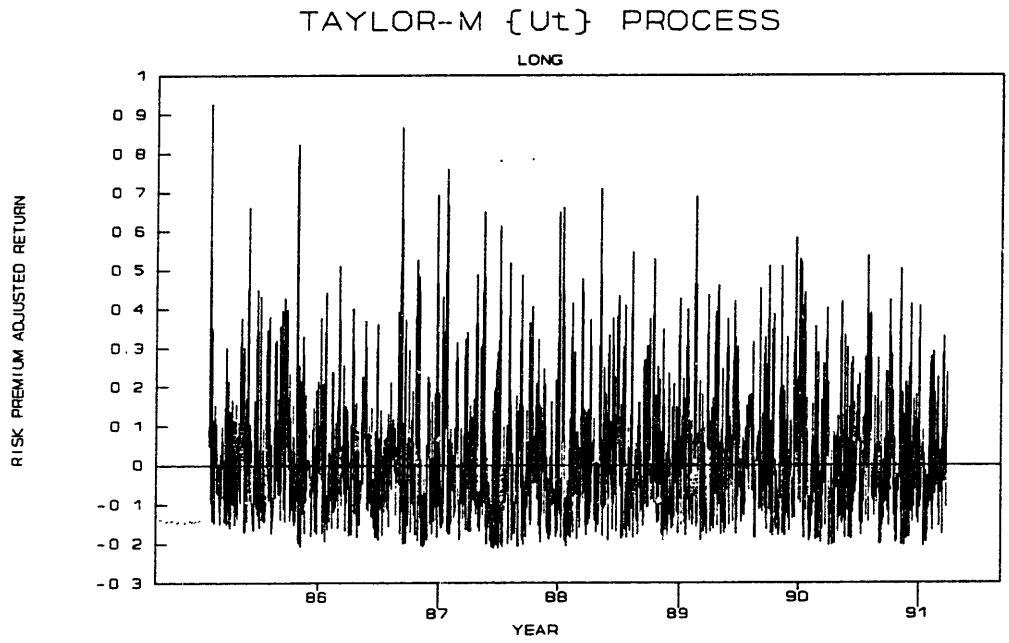

Fig.5.12

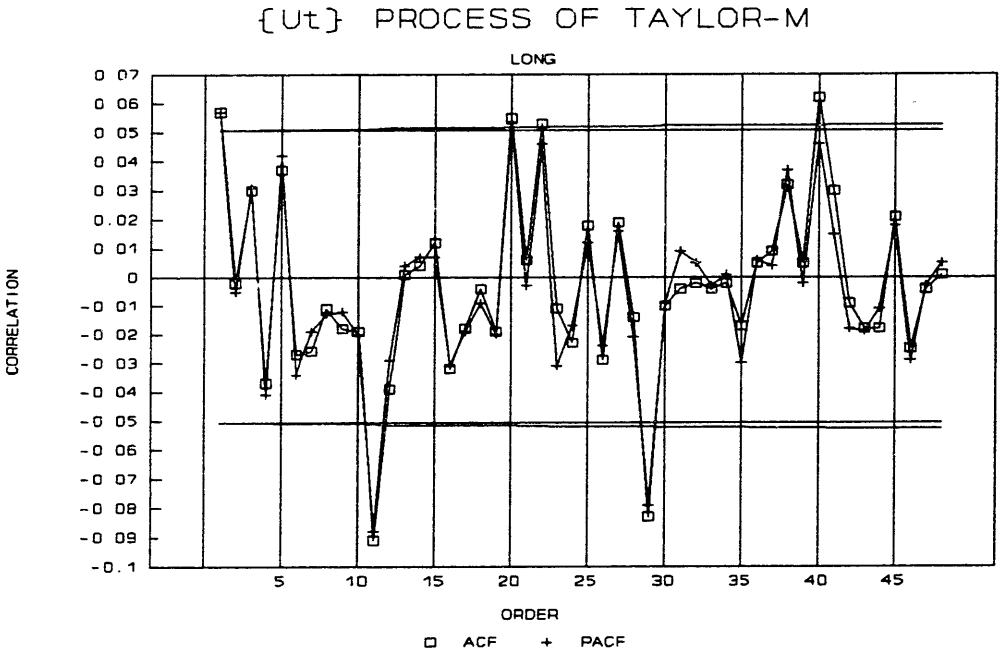

Fig.5.13

With our Taylor-M model, so far, it is difficult to analyze if mean reversion effect exists. This is mostly due to the functional form of $\{V_t\}$ process. EWMA process is, unlike ARMA processes, non-stationary, and its long term forecast is the very state where the process currently is, while long term forecast of stationary ARMA processes converges to their unconditional mean. In reality, large magnitude of interest rate volatility, caused by some shock, will be expected to gradually die out. It is not likely, as EWMA implies, that a volatility shock changes the level of expected volatility permanently.

Therefore, we should increase number of parameters and use some stationary process for specification of $\{V_t\}$, especially for the use of long run forecasting. From the same reason, the long lasting serial correlation presented in figures 5.9 and 5.11, should not be taken as shown. It is unlikely that martingale is violated over such a long time horizon.

After all, with only two parameters, our Taylor-M model performed well, except for the failure in capturing long run effects of mean reversion.

Section 6. Extended Yield Curve Model

In this section we discuss about construction of an extended version of yield curve model, using our findings on behavior of $\{R_{j,t}\}$ and $\{I_{j,t}\}$. We will provide some mathematical way for pricing discount bonds, while we propose Monte Carlo method for coupon bonds or more general interest rate contingent claims.

Using the ARCH model with jump/GARCH-M effects discussed in section 4, we can forecast $\{I_{j,t}\}$, i.e. we can have $E[I_{j,t+s}|t]$ and $VAR[I_{j,t+s}|t]$ for $s\geq 0$. As the common property of stationary ARMA process, $E[I_{j,t+s}|t]$ converges to zero as s gets larger. While, $VAR[I_{j,t+s}|t]$ do not converge as s gets larger, variance ratio of $I_{j,t+s}$ converges to certain level above or below 1, depending on whether $\{I_{j,t}\}$ is positively or negatively autocorrelated for each j.

Using these forecast on $I_{j,t+s}$, $s\geq 0$ and structure of $R_{j,t}$ at time t, we can derive conditional expectation and conditional variance of spot discount factor for j period zero coupon bonds at time t, say $S_{j,t}$, which usually defined as the products of annual yield over the time horizon. In our case, since data sets are in logarithm, $S_{j,t}$ is just the sum of logarithm of annualized yields. If the bond market is

efficient in the sense that the series of risk adjusted forward short-term rate follows martingale,

$$E[S_{j,t+s}|t] = \sum_{j=1}^{s} (R_{j,t} - L_{j,t}) ,$$

and

$$VAR[S_{j,t+s}|t] = \sum_{j=1}^{s} VAR[R_{j,t} - L_{j,t}] + 2\sum_{i<j} COV[R_{j,t} - L_{j,t}, R_{i,t} - L_{i,t}]$$

$$= \sum_{j=1}^{s} VAR[I_{j,t} - \Delta L_{j,t}] + 2\sum_{i<j} COV[I_{j,t} - \Delta L_{j,t}, I_{i,t} - \Delta L_{i,t}] ,$$

where $CORR(I_{j,t}, I_{i,t})$, $i \neq j$ are stable and positive over period longer than a year. Using these formula, expectation and variance of price of zero coupon bonds, which is $1/S_{j,t}$, can be obtained under the assumption the $\{I_{j,t} - \Delta L_{j,t}\}$ has no autocorrelation. There is need for adjustment using Jensen's inequality,

$$E[\frac{1}{X}] \geq \frac{1}{E[X]} .$$

The piecewise constant structure of forward rates used in this paper, makes it fairly easy to calculate these conditional expectation and conditional variance.

There remain two points to be discussed regarding the violation of martingale property of innovation in risk adjusted forward rates, $\{I_{j,t}-\Delta L_{j,t}\}$. Positive and negative autocorrelation of the series can be interpreted in the following way. Positive autocorrelation of the adjusted innovation series may be caused by delay of market reflecting new information into interest rates. Market is "efficient" with delay of about a week or so, taking that length of period to fully reflect new information into interest rates. We think this effect exists in our data for short and medium j. Negative autocorrelation can be interpreted either as the result of overreaction of market participants or the result of mean reversion of risk premium adjusted forward rate series, $\{R_{j,t}-L_{j,t}\}$. For our data set from Japanese bond market, it is difficult to tell either of the overreaction hypothesis and/or the mean reversion hypothesis is there. Since long term bonds are traded by dealers of institutional investors with heavy volume and extremely short investment horizon of less than a day, overreaction hypothesis might explain why negative autocorrelation emerges only for long j. However, mean reversion of $\{R_{j,t}-L_{j,t}\}$ is also plausible, when we remind stable distribution of $\{R_{j,t}\}$ over time. To properly account for these serial correlation in the risk adjusted innovation, we need vector autoregressive (VAR) model.

To price coupon bonds or other interest rate contingent

claims in mathematical way, highly complicated, non-linear calculations are needed. Cox and Ross (1976)[52] provided option pricing model using alternative distributions other than lognormal distribution. Non centered chi-squared distribution, etc. are discussed. However, for practical use, we think it is better to develop a Monte Carlo simulation, which can be applied for pricing of any interest rate options under estimated parameter set for movement of $\{R_{j,t}\}$ and $\{I_{j,t}\}$. When we develop such Monte Carlo simulation system, we should care about the following points.

(1) The parameter for density of Poisson jump, λ, should be common for all j, because jump in $\{I_{j,t}\}$ processes with different j are caused by the same information which arrives at time t. Parameters for magnitude of such jumps may differ for each j.

(2) Correlation between $\{I_{j,t}\}$ for different j, must be taken into account. Since these are positive over long time horizon, failure to account for these positive correlations leads to underestimation of conditional variance of $S_{j,t}$. This underestimation in volatility of interest rates causes underpricing of interest rate contingent claims.

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52. Cox, John C. and Stephen A. Ross 1976 "The Valuation of Options for Alternative Stochastic Processes," Journal of Economics Vol.3, pp.145-166

Section 7. Conclusion

In this paper we attempted to capture (1) market inefficiency as violation of martingale in the series of risk premium adjusted forward rate $\{R_{j,t}-L_{j,t}\}$, (2) mean reversion, (3) time varying risk premium as linear function of conditional standard deviation, and (4) conditional heteroskedasticity specified either as GARCH and/or jump process. Then we tried to construct an extended version of yield curve model for bond option pricing.

We found supporting evidences for the following points.

(1) Market is slow in fully reflecting new information into interest rates. For short j, forward short-term rates tend to shift in the same direction for up to 5 business days, i.e. a week.

(2) For long j, forward short-term rate process is persistent, when compared to random walk. Day to day shift in forward rates with long j are negatively autocorrelated, implying mean reversion and/or overreaction.

(3) Time varying risk premium incorporated in forward

short-term rates are significant, contributing about one third of variance of forward rates.

(4) Excess kurtosis of $\{I_{j,t}\}$ is significant for all j. Although excess kurtosis is greater for shorter j, autocerrelation of $\{|I_{j,t}-\Delta L_{j,t}|\}$ and $\{(I_{j,t}-\Delta L_{j,t})^2\}$ processes are greater for longer j. This implies that heteroskedasticity associated with longer j is explained relatively well by GARCH process, while that associated with shorter j needs other source of change in variance. Jump process might be a possible explanation.

Although results from maximum likelihood estimation are not available for this paper yet, Lagrange multiplier tests and t-tests using such estimators will allow further discussion on interest rate movement and development of model. Use of VAR (vector autoregressive) model should also be considered.

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