MANAGING THE DISCOVERY LIFE CYCLE OF A FINITE RESOURCE: A CASE STUDY OF U.S. NATURAL GAS

by

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ABSTRACT

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In an economy such as that of the United States whose growth is based on exponentially increasing usage of nonrenewable resources, policy makers need management tools to ensure a long-term supply of these resources at acceptable prices. This thesis presents a System Dynamics simulation model of the factors controlling the discovery life cycle of a nonrenewable resource. The model is fitted to data from the U.S. natural gas industry as a case study.

Classical mineral economic theory dictates that continued use of a finite nonrenewable resource will lead eventually to a price at which consumption of the resource drops effectively to zero. It is important for policy makers to understand how alternative price, technology, and exploration policies will influence the amount and price of the resource available over time. This study indicates that in the case of finite, nonrenewable resources such as the fossil fuels, the normal behavior mode of the system is an initial period of exponential growth in consumption, a period of rising prices where growth in consumption is halted, and finally a decline in consumption.

The exact timing of the occurrence of the end of growth is determined by many factors, including, for instance, the growth rate of potential usage, the initial level of unproven reserves, the shape of the cost of exploration curve, and the occurrence of various policies such as subsidies or ceiling price regulations. It appears that the short-term effectiveness of such policies which aim at postponement of a decline of usage rate is minimal. Price regulation greatly reduces the long-term level of reserves available, however.

A detailed statistical analysis was performed in order to obtain better estimates of the nonlinear relationships in the model. This analysis provides a better fit to historical data, but does not alter the overall behavior modes of the model, or the response of the model to various sensitivity or policy tests.

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CHAPTER I

INTRODUCTION

The United States today depends on the fossil fuels coal, oil, and natural gas for 96 percent of its energy supply.[1] Although the total amount of fossil fuel reserves is not precisely known, it is finite and nonrenewable during time periods of less than millions of years. Coal is still in relative abundance. It has been estimated that the United States' coal reserves have a current ratio of recoverable reserves to production of 2,620.[2] At a projected rate of growth of production of 5 percent per year, however, these reserves would last only a century. It also has been estimated that more than half of the total U.S. supplies of petroleum have been consumed. The U.S. became a net importer of oil in 1948 and has become increasingly dependent on foreign sources despite government limitations of oil imports.[3] It has been suggested that U.S. oil production reached its historical peak near 1970, and that production will decline henceforth until economic depletion.[4]

The natural gas industry seems to be facing the most imminent crisis of all. The United States depends on natural gas for over thirty percent of its energy supply. Although it has been estimated that from 400 to 900

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1) Statistical Abstract of the United States 1970, U.S. Government Printing Office, (1970) P. 506.

2) Mineral Facts and Problems, 1970, [Washington] U.S. Dept. of the Interior, Bureau of Mines, (1970) p. 35.

3) Ibid., p. 168.

4) M. K. Hubbert, "Energy Resources", in NAS-NRG, Resources and Man, W. H. Freeman and Co., San Francisco, (1969), p. 183.

trillion cubic feet of natural gas still remains undiscovered in the U.S.,[5] proven reserves have been falling rapdily since 1967. The discovery rate DR, currently less than the production rate PR, is decreasing, while production of natural gas is rising at almost seven percent per year.[6] The trends in proven reserves, discovery and production of natural gas over the past decade are shown in Table 1.

Year	Proven Reserves (trillion ft^3)	Discoveries plus extensions and revisions (trillion ft^3)	Production (trillion ft^3)	R/P Ratio (Years)
1960	262.3	13.9	13.0	20.2
1961	266.3	17.2	13.4	20.0
1962	272.3	19.5	13.6	20.0
1963	276.2	18.2	14.5	19.2
1964	281.3	20.3	15.3	18.4
1965	286.5	21.3	16.3	17.6
1966	289.3	20.2	17.5	16.5
1967	292.9	21.8	18.4	15.9
1968	287.3	13.7	19.4	14.9
1969	275.2	8.5	20.7	13.3
1970	266.2	12.7	21.7	12.3

TABLE 1: DISCOVERY AND PRODUCTION OF U.S. NATURAL GAS[7]

(1960-1970)

The producers of natural gas cite price regulation as the major cause for decreasing discoveries: "'Frankly, there is no incentive for wildcatting,' says W.W. Keeler, chairman of Phillips Petroleum Co., a major

5) Ibid., p. 188.
6) M.K. Hubbert, "Energy Resources," in W.W. Murdock, Ed., Environment: Resources, Pollution and Society, Sinauer Assoc., Inc., Stamford, Conn. (1971), p. 97.
7) American Gas Association, Gas Facts, 605 Third Ave., N.Y., 10016.

gas producer. "Until there is a break in these FPC regulations,' he adds, 'I don't think we'll spend a lot of money trying to find gas.'"[8] Others outside the industry contend that a decline in the overall supply of gas is beginning to have an impact on rates of discovery.

What are the major factors controlling long-term discovery of fossil fuel and other resources? It is immediately apparent that if a resource is finite and nonrenewable, a trade-off between short and long-term goals is necessary. With a finite resource one can enjoy a high usage rate for a relatively short period of time, depleting resources quickly, or one can sustain a lower usage rate over a longer period. What then are the effects of government policies such as ceiling price regulation or tax incentives on the short and long-term behavior of discovery and usage rates of a resource?

The answer to these questions depends on many factors including, for example, the estimate of existing reserves in the U.S., the cost of exploration, investment in exploration, price of the resource, sales revenue, proven reserves, demand, and usage rate. Rather good data are available on each of these factors, but their interaction over time is not intuitively obvious.[9] Nor are traditional econometric tools fully suitable for making long-term projections over a period where one can expect significant reversals of trends in price, discovery rate, usage rate, etc. However, tools developed at M.I.T. over the past fifteen years in the System Dynamics Group

8) Wall Street Journal, April 12, 1971.

9) see J.W. Forrester, "Counterintuitive Behavior of Social Systems," M.I.T. Technology Review, V. 73, No. 3, (Jan. 1971) for a discussion of this point.

do permit the representation for analysis (through simulation) of complex relationships like those important in determining the rate of natural gas discovery and usage. These tools are described in several texts,[10] and have been used to study the behavior of many other systems.[11]

In this report, a System Dynamics model of discovery of U.S. natural gas is developed as an example of the dynamics of the natural resource discovery process. This model permits one to test, through simulation, the probable effects of alternative policies on the discovery life cycle of the resource.

Chapter II evaluates three existing models of the U.S. natural gas industry in terms of model criteria established by the FPC. Chapter III presents the System Dynamics model of discovery of U.S. natural gas, and describes in detail the functional relationships among variables included in the model. Chapter IV analyzes the applicability of linear statistical inference methods to an attempt to obtain better estimates of model parameters using time series data. The implications of various possible and existing policies on the supply of natural gas are discussed in Chapter V. Chapter VI discusses the major conclusions to be drawn from the work to date, and the potential for further research.

10) J.W. Forrester, Industrial Dynamics, M.I.T. Press, Cambridge, Mass. (1961). J.W. Forrester, Principles of Systems, Wright-Allen Press, Cambridge, Mass., (1968). A.L. Pugh, Dynamo II User's Manual, M.I.T. Press, Cambridge, Mass., (1970).

11) For example, see [Meadows (1970)], [Randers, Meadows (1972)], [Hamilton, et. al. (1969)].

CHAPTER II

EVALUATION OF OTHER MODELS OF THE U.S. NATURAL GAS INDUSTRY

The U.S. natural gas industry provides an excellent example of a socio-economic system where simulation modeling can provide useful information to government policy makers. The gas industry has been regulated by the Federal Power Commission (FPC) since 1954. This one agency must make regulatory decisions affecting over three thousand separate companies and over four billion dollars in annual revenues. In a search for models to aid in policy making, the FPC has compiled a list of fifteen issues critical to future policy decisions (included as Appendix I). The following four points summarize the general model characteristics necessary to address these issues:

1. The model must show the behavior of reserves, discoveries, price, and production over time; thus the model must be dynamic in nature.
2. The model should include the important economic relationships among new reserves, price, demand, revenue, and capital investment.
3. The time horizon for the model is relatively long-term (20 years) - the FPC is interested in estimates of supply for the next two decades to the year 1990.
4. The model should be capable of testing the effects of various possible policies such as regulation, technological improvements or tax treatments on long-term supply.

Many models of various aspects of the U.S. natural gas industry have been developed to date. The models belong to two general categories: static economic models and dynamic econometric models. The purpose of this chapter is to present examples from each of these two model classes and to evaluate the models' usefulness as management tools for long-term policy planning.

Static Economic Models

The term "static economic model" here refers to any analysis using classic supply and demand curves, or schedules of quantities of gas that would be supplied or demanded, ceteris paribus, at various prices. The qualifier ceteris paribus is an important one, since it requires that other factors such as cost of gas, population, and per capita consumption of gas be held constant.

An example of a static economic model of the gas industry is given by Milton Russell in "Producer Regulation for the 1970's."[12] In this analysis, Russell attempts to evaluate the distributive effects of area price regulation as it is currently being practiced by the FPC. Distribution effects arise because the Natural Gas Act states that regulation only applies to interstate sales. Intra-state sales and direct industrial sales by the producer to the ultimate consumer are exempted from regulation. These exempt or non-jurisdictional sales have comprised approximately one-half of total gas sales by volume in recent years.

12) M. Russell, "Producer Regulation for the 1970's," first draft of a paper to be included in an RFF volume edited by Keith Brown, (1971).

The model assumes there is one single supply function for the aggregated U.S. gas industry, indicated by line S in Figure 1. The demand for gas is divided into jurisdictional and non-jurisdictional demand components, BR and AM respectively. If there were no regulation or restrictions on interstate price, the market clearing price P_e would hold in both the jurisdictional and non-jurisdictional markets, with Q_e the total amount sold.

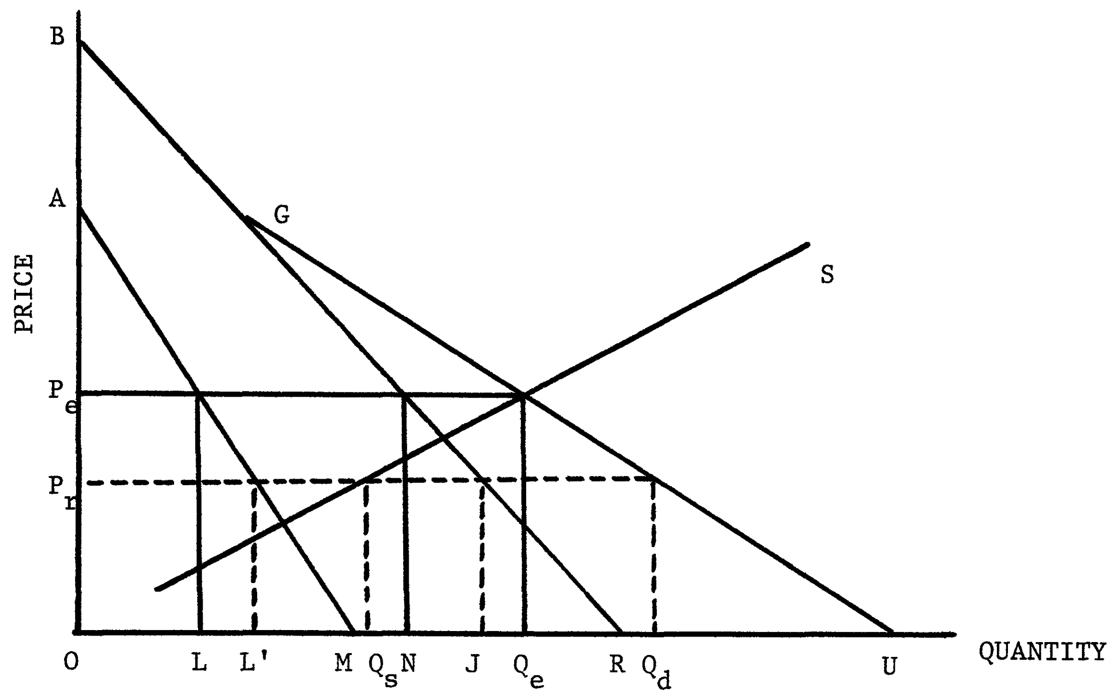

Figure 1: Russell Field Price Model

OL would be sold in the non-jurisdictional market and ON would be sold in the jurisdictional market.

Suppose we impose a ceiling price P_r below the market price P_e on the jurisdictional market for gas. In this case the total amount demanded Q_d

exceeds the total amount supplied Q_s.[13] Russell argues that the same price P_r will exist for both jurisdictional and non-jurisdictional customers, for if the unregulated price were higher, producers would shift sales from regulated to unregulated markets. This would tend to drive the unregulated price down to equal P_r.

Thus, with P_r below P_e, amount OQ_s would be supplied, with OL' going to the unregulated market, and the remaining $L'Q_s$ available for purchase by jurisdictional consumers. As compared with the unregulated market, a ceiling price P_r below P_e expands the amount of gas going to the nonjurisdictional market, and restricts jurisdictional sales causing a shortage for interstate markets.

Russell concludes that regulation has led to a misallocation of resources, and to gas shortages in the regulated markets. Under the ceteris paribus assumptions in his Field Price Model, he finds regulation reduces economic welfare in the short-term by allocational inefficiencies.

These ceteris paribus assumptions limit the usefulness of the model as a tool for long-term management of resources, however. Over the long time period involved, total demand is being forced up exponentially at over six percent per year because of growth in population and in per capita consumption of gas. At the same time, the cost of producing gas is rising (because of the difficulty of finding new reserves of gas), reducing the supply

13) There exists ample evidence that this is the case presently in the U.S. natural gas industry. In a Washington Post article, March 2, 1972, entitled "Gas Firm Restricts Sales in District Area," W.H. Jones states, Washington Gas Light Co. yesterday stopped all sales of natural gas to new customers in the metropolitan area and said it could not increase sales of gas to any present customer. Cutbacks of gas sales have become common throughout the Midwest and Middle Atlantic states in recent months because of severe shortages of natural gas."

of gas at any given price. It is clear, then, that both supply and demand curves will shift substantially over the long time horizon held by the FPC.

The first criterion of the list of model characteristics necessary to address critical FPC issues states that the model must permit analysis of the behavior of the industry over time. In utilizing a static model such as the Field Price Model for long-term analysis, it is possible that violations of the ceteris paribus assumptions could either invalidate the conclusions drawn from the model, or cause a dynamic behavior that completely overshadows the effects predicted by the static model. In analyzing regulation policies in terms of welfare and optimal resource allocation, it seems imperative to take the time dimension into account. Over such a long time frame as twenty years, it is conceivable that short-term welfare benefits or losses may result in long-term effects of the opposite nature (this point will be discussed more fully in Chapter V).

Some static analyses have attempted to include the effects of supply and demand schedules which shift over time. However, when conjectures about dynamic behavior are made verbally rather than modeled explicitly, it is difficult to come to any concrete conclusions about relative magnitudes or even directions of policy effects. If verbal assumptions are included explicitly in a dynamic model, one often finds results which run counter to one's initial intuition. Russell is certainly aware of this danger. He states that "the dynamic effects of gas regulation would appear to reinforce the results of the static model, though this conclusion is based only on casual observation and on behavioral assumptions which are plausible, but which have not been demonstrated."[14]

14) M. Russell, "Special Group Interests in Natural Gas Price Policy," Southern Illinois University, Carbondale, Illinois, (1971) (unpublished paper).

Dynamic Econometric Models

In order to avoid the restrictions of the ceteris paribus assumptions of static analyses, most of the models of the natural gas industry are dynamic in nature. The most common type of dynamic models are econometric models, where a linear relationship is hypothesized between a dependent variable at time t and one or more independent variables at time t or t-1. The normal statistical procedure for estimation of the coefficients which define the relationship between the dependent and the independent variables is Ordinary Least Squares analysis (OLS). In order for OLS to give unbiased estimates of the real-world parameters, the system generating the real-world data should meet several requirements:

1. The real-world system should be in the same form as the model. In the case of OLS, the system should be intrinsically linear. That means that each dependent variable Y_i should be a linear function of the dependent variables X_i:

$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \ldots + \tilde{\varepsilon}_i \quad i = 1, 2, \ldots, n$$

2. The residuals $\tilde{\varepsilon}_i$ (the amount of variance which the equation has not been able to explain) must be random, normally distributed, and uncorrelated with the X_{ij} (independent variables) or Y_i (dependent variables).
3. The X_{ij} must be independent with respect to each other, and must not depend on Y_i-i.e., they must be exogenous.
4. Projections should only be made within the range of observed data. Prediction outside this range is subject to substantial error--the variance of the predicted value increases with the square of the distance from the mean of the observations.

Most economic systems violate one or more of the assumptions on which OLS techniques are based. One must be careful, therefore, that in using the model to address specific issues such as those designated by the FPC, the violations of assumptions do not significantly reduce the utility of the model.

The following two types of dynamic econometric models are both designed to cope to varying degrees with the assumptions necessary for econometric analysis of the U.S. natural gas industry. The first, the FPC staff's econometric model developed by J. Daniel Khazzoom, is a single-equation model estimated using OLS. The second, a simultaneous equations model developed by P. MacAvoy, uses two-stage least squares regression techniques.

Single-Equation Models: The Khazzoom Model

J. Daniel Khazzoom has developed a dynamic econometric model of the U.S. natural gas industry typical of those which use OLS regression techniques. It is employed as the FPC staff's econometric model of natural gas supply in the United States.[15] More complicated single-equation models of the gas industry have been developed by others,[16] but these models are similar in that price is considered exogenous in each, and all report similar results and conclusions. The Khazzoom model is dynamic in nature, and assumes linear single-equation relationships for new discoveries ND and

15) J. Daniel Khazzoom, "FPC Staff's Econometric Model of Natural Gas Supply in the United States," Bell Journal of Economics and Management Science Vol. 2, No. 1 (Spring, 1971).

16) See F.M. Fisher, Supply and Costs in the U.S. Petroleum Industry, Two Econometric Studies, Baltimore, Md., Johns Hopkins Press, (1964) or E.W. Erickson and R.M. Spann, "Supply Response in a Regulated Industry: the Case of Natural Gas," Bell Journal of Economics and Mgt. Science, Vol. 2, No. 1, (Spring, 1971), p. 99.

extensions and revisions XR, which together comprise the supply of new gas. Both are assumed to have a lagged linear dependency on the price of gas, oil, and natural gas liquids, and each includes carryover effects from the preceding year. The relationships are:

$$ND_t = \beta_o + \beta_1 \sum_{t=1}^{2} C_{t-i} \pm \beta_2 \sum_{i=1}^{2} P_{o_{t-i}} + \beta_3 \sum_{t=1}^{2} P_{L_{t-i}} + \beta_4 \sum_{i=1}^{2} ND_{t-i} + \tilde{\epsilon}_t$$

$$XR_t = \beta_o + \beta_1 C_t - \beta_2 P_{o_t} + \beta_3 P_{6_t} + \beta_4 ND_{t-1} + \beta_5 XR_{t-1} + \tilde{\epsilon}_t$$

where ND = new discoveries
XR = extensions and revisions
C = ceiling price of gas
P_o = price of oil
P_L = price of natural gas liquids

This model formulation contains implicitly a number of assumptions. First, the dependency of price is assumed to be linear (a quadratic model is tested, but results are not significantly different from the linear model). Price is assumed exogenous, and is the only exogenous variable affecting supply. With these assumptions, Khazzoom performs an OLS analysis on data from 1961-1968 and 1961-1969, and estimates that discoveries in 1970 will suffer a drop of 20-26 percent (in comparison with 1969 discoveries). In addition, he performs a sensitivity analysis to see how future discoveries respond to various increases in gas prices.

When used to predict supply response over short-term periods, the Khazzoom model performs well, predicting 1969 discoveries with about a 5 percent error. When policy makers such as the FPC become concerned with the long-term, however, it is clear many of the underlying assumptions of the Khazzoom model are invalid. Khazzoom notes that over longer periods of perhaps ten years or more, discoveries may be decreased due to depletion of resources, a factor omitted from his model.[17] Depletion may also affect gas price by increasing the cost of gas. Thus there may exist an interrelationship between price and new discoveries over the long run which would cause price to increase with additional discoveries, violating Khazzoom's assumption that price is exogenous. In addition, it is likely that other factors besides price (such as increasing costs or sales revenue) affect discoveries in the long-term, and that these variables are also interdependent in the long run. Thus a single-equation model such as Khazzoom's is valid for short-term projections, but is not a good tool for evaluating the alternative policies which may be employed in the long-term management of U.S. natural gas reserves.

When a system contains variables which are interdependent as in the case of supply of natural gas in the U.S., the model can generally be put in the form of a simultaneous equation system. An excellent example of this approach is the model developed by P.W. MacAvoy of M.I.T.

Simultaneous-Equations Models: The MacAvoy Model

P.W. MacAvoy of M.I.T. has developed a simultaneous-equations model of the U.S. natural gas industry[18] which recognizes the longer-term interdependencies among parameters. MacAvoy develops equations for price, new discoveries, production, and well drilling activity for various regions in the U.S. The model is fitted to data from 1955-1960 (before regulation), and then used to predict price and discoveries in the period

17) J. D. Khazzoom, Op. Cit. (1971), p. 58.
18) P. W. MacAvoy, "The Regulation-Induced Shortage of Natural Gas," The Journal of Law and Economics, Volume XIV(1), (April, 1971), p. 167.

1961-1967 (during regulation). If the model is realistic and the coefficients obtained from pre-regulation data are correct, then the difference between simulated and actual results can be attributed to regulation effects. MacAvoy concludes from this modeling effort that without regulation, price of natural gas would have more than doubled, and discoveries would have substantially increased during the period 1961-1967.

MacAvoy's econometric model of supply and demand consists of the following four simultaneous equations for new discoveries, wells drilled, production, and price of gas:

$$\log \Delta\hat{R}_{tj} = 0.2765 + \underset{(.0292)}{.5894} \log (\Delta R_{t-1,j}) + \underset{(.0489)}{.4364} \log (\hat{W}_{tj}); R^2 = .9400$$

$$\log \hat{W}_{tj} = 32.0582 + \underset{(0.1206)}{1.0296} \log (\hat{P}_{tj}) + \underset{(0.5656)}{6.7314} \log (fp_t) + \underset{(0.6188)}{0.6872} \log (W_{t-1,j}); R^2 = .9848$$

$$\log \Delta\hat{Q}_{tj} = 44.7285 + \underset{(0.0322)}{1.1091} \log (\Delta\hat{R}_{tj}) + \underset{(0.3514)}{1.1567} \log (i_t) - \underset{(1.9389)}{10.9032} \log (fp_t); R^2 = .9528$$

$$\log \hat{P}_{tj} = 28.8494 + \underset{(0.0055)}{0.1210} \log (\Delta\hat{R}_{tj}) - \underset{(0.0879)}{0.7138} \log (\sum_j \Delta\hat{R}_{tj}) - \underset{(0.0461)}{0.1232} \log (M_j) + \underset{(0.8726)}{6.6067} \log (fp_t) + \underset{(0.1131)}{1.3798} \log (\Delta N\Delta Y); R^2 = .9205$$

where ΔR_{tj} = new reserve in year t, district j

W_{tj} = wells drilled in year t, district j

ΔQ_{tj} = production from new contracts signed in year t, district j

P_{tj} = average price during time t, district j

fp_t = all-fuels price index in year t

i_t = current rate of interest in year t (as a measure of cost of investment in reserves)

M_j = distance from point of production to final markets in district j

$\Delta N \Delta Y$ = (population increase) x (per capita income increase)

Literature on regression techniques[19] warns that estimates of coefficients in a simultaneous equations system obtained through Ordinary Least Squares analysis will tend to be biased (i.e., they will not be equal to the real-world relationships). In order to rid the estimates of the coefficients of bias in his simultaneous equations model, MacAvoy used a procedure known as "Two-Stage Least Squares." He regresses each dependent variable against the exogenous variables, and then uses the predicted values of the dependent variables to estimate coefficients in the "second stage" of the regression (thereby specifying the equations shown above). With this technique, he reports high R^2 coefficients for all four equations when based on data from the East Coast and Mid-West markets. When the model was extended to the West Coast markets, however, some coefficients had wrong signs, and the R^2 values were much lower. He attributes this lack of fit to the monopsony pipeline control of prices in the West in the 1950's, a factor which could not be expected to continue through the 1960's given that a substantial number of pipelines entered the demand side of the market during the later period.

How well does MacAvoy's model address the FPC criteria for models of the gas industry listed earlier? The model is dynamic in nature, explaining

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19) See, for example, J. Johnston, Econometric Methods, McGraw-Hill, N.Y., (1963), p. 233.

the behavior of reserves, discoveries, price, and production over time. The model does include the important economic relationships for the time period on which it is based. In this sense, it is useful for the time period which it analyzes--in this case, 1955-1968. The FPC is interested in a model which explains behavior up through 1990, however. When linear statistical models are used to analyze long-term behavior of a system, several problems may arise.

First, many of the variables can be expected to go out of the range in which estimations were made--in the case of natural gas, costs and production are rising, price is regulated, and discoveries are falling, causing the reserve-production ratio to decrease in recent years to unprecidented levels. It is clear that some powerful forces are building up in the industry, and that one cannot expect trends to continue through 1990 as they have in the past. This implies that the system may well be nonlinear. When one assumes an intrinsically linear form in a nonlinear system and estimates coefficients based on behavior in a limited range of variation of the variables, it is important to remember that the error in prediction increases greatly when using these coefficients to predict behavior outside that variable range. Because of the nonlinearities, behavior may actually be quite different, causing the model to project incorrectly the effects of policies on the gas industry over the long term.

In addition, it is often possible that when one lengthens the time horizon of the model, the primary determinants of behavior may be substantially different. In the very long-term, for instance, costs of exploration for reserves may become a major component of price as gas becomes more scarce. These rising costs also decrease the quantity of gas discovered at a given level of investment, and thus affect discovery rate of new reserves.

Dynamic econometric models predict system behavior well when real-world relationships can be assumed linear in the range of behavior, and the real-world system can not be expected to show long-term behavior which deviates widely from the range in which the estimation was made. Over the long term (through 1990), however, one would expect to see the effects of diminishing returns in investment in exploration for gas--in the long run, discoveries must decrease due to rising costs of exploration, caused by continual depletion of a finite resource. Evidence cited in Appendix II does indicate that diminishing returns due to rising costs were a factor during the 1960's. The MacAvoy model will not reflect diminishing returns--new reserves ΔR_{tj} are assumed to be some constant times the number of wells drilled W_{tj}, while in the long term this constant will actually decrease due to diminishing returns.

In addition, one would expect the number of variables which can be considered exogenous to the system to diminish when the model is used to explain long-term behavior. For instance, fuel prices fp_t, treated as exogenous in the MacAvoy model, will certainly be a function of gas price P_{tj} in the long term. This effect can cause problems of identification in long-term econometric models, which limit the applicability of linear regression techniques to long-term system modeling. This issue will be discussed more fully in Chapter IV.

Summary

In order to avoid simulation modeling as a purely academic exercise, and to increase the number of models actually used to solve practical problems of policy making, models should be built with the user in mind. A powerful potential user of simulation models in government is the Federal Power Commission, which regulates the natural gas industry. To elicit help in determining the effects of regulatory policies on the U.S. natural gas industry, the FPC has provided a list of criteria for prospective models. The criteria listed in full in Appendix I can be summarized as: dynamic capability, long time frame (through 1990), capability of including all the important economic relationships simultaneously, and flexibility in testing of alternative policies.

With these criteria in mind, two types of models of the U.S. natural gas industry were examined. The first, static economic models, were found to be too restrictive because the large number of ceteris paribus assumptions inherent in the models were severely violated over the long term in the case of natural gas supply and demand. Demand is increasing at almost seven percent per year. Costs of finding gas may be expected to rise sharply over the long term, forcing the supply curve to shift. Imports can be expected to rise in the future, and extraction and exploration technologies will improve, also shifting the supply curve.

The second type of models, dynamic econometric models, require fewer ceteris paribus assumptions than static models because they can include the effects of interactions among variables through time. Because of the very long time frame of interest to the FPC, parameters can be expected to vary beyond the range of estimation, making prediction of long-term policy effects tenuous in those cases. In addition, assumptions of linearity made in most econometric models are invalid over the wide variation of variables which can be expected in the long term.

Thus the two types of models (static, dynamic econometric) most commonly used to analyze the U.S. natural gas industry may be useful tools to evaluate short-term effects of policies, but they are not well suited for an extremely long-term framework such as that required by the FPC. What is needed is a modeling technique which enables explicit incorporation of all nonlinearities and feedback interrelationships which significantly influence the system's behavior.

A third methodology, System Dynamics, appears better suited to meet the model criteria outlined by the FPC. The modeling technique is dynamic, and capable of including simultaneously all the important factors controlling long-term supply of a resource. System Dynamics models are especially well suited to long-term analysis because they can easily incorporate nonlinear relationships.

Finally, the most effective use of System Dynamics models is for evaluation of policy effects on long-term behavior of a system. It is impossible to predict the future, but managers can use models to identify alternative outcomes and to seek policies which will increase the probability of realizing the desired goals. The following chapter describes a System Dynamics model of the U.S. natural gas industry, and presents the assumptions and functional relationships which underlie the model.

CHAPTER III

A DYNAMIC MODEL OF DISCOVERY OF U.S. NATURAL GAS

The domestic supply of the fossil fuels seems to be at a turning point, for it is becoming more and more difficult to supply the fuels needed to continue past trends in growth of energy consumption from U.S. sources. The goal of the model presented here is to incorporate the elements and interactions which control the discovery and supply of U.S. natural gas, to determine the nature of this turning point in supply, and to examine the effectiveness of various policies in alleviating the problem.

Parameter values will be derived from data from the natural gas industry in the United States. It is important to recognize, however, that the underlying model structure is representative of other finite, nonrenewable resources. The end use of fossil fuels dictates that the process of recycling be excluded from this model. The impact of recycling on the supply of a mineral resource has been analyzed in a similar model by Randers [Randers (1971)].

Many aspects of the natural gas industry such as seasonal demand fluctuations or pipeline distribution problems certainly deserve attention, but the purpose of this paper is to examine the effects of policies on long-term trends and behavior modes in industry supply. The following assumptions are those which appear most appropriate for that purpose.

Model Assumptions

The model assumes that the natural gas industry is composed of many firms, all producing one undifferentiated product, natural gas. The interdependency between the oil and the gas industry has been ignored for the purposes of this study. Although this assumption certainly affects the specific data produced by the model in the early stages of natural gas discovery, it does not affect overall behavior, or the relative effects of policies. Furthermore, this assumption is becoming more and more valid at present. Directional drilling for either gas or oil is becoming more successful, and over 70 percent of all gas wells are now un-associated with oil.[20]

The model includes those relationships which govern the behavior of the industry as a whole: the separate producer's actions in discovery and production are aggregated together to obtain this industry behavior. The producers of natural gas are taken as those engaged in the discovery of natural gas and the preparation of gas for use. Pipeline distribution companies are considered consumers, not producers. The gas is "sold" at the wellhead, usually under long-term contracts.

The cost of exploration is assumed to be monotonically rising as resources are depleted. Of course the actual cost of exploration contains random fluctuations, and an example of the model's response to random fluctuations in cost, new field size, and consumption is shown in Chapter V. However, data supports the assumption of a long-term relationship between costs and the fraction of total resources remaining to be discovered.

The individual natural gas industry firms also exhibit an eight to fifteen year cycle in the reserve-production ratio caused by capacity

---

20) M.A. Adelman, The Supply and Price of Natural Gas, supp. to Journal of Industrial Economics, Basil Blackwell, Oxford, 1962.

acquisition delays in building pipelines. It is not the concern of this paper to model this effect; however an analysis of this behavior can be found in Meadows [Meadows (1969)].

In the model, demand is assumed to be an endogenous function of product price, with other determinants such as growth in population and per capita consumption causing increased demand at any given price. This rate of growth in potential demand is determined exogenously to the model, and is assumed in the case of natural gas to be 6.57 percent per year.[21]

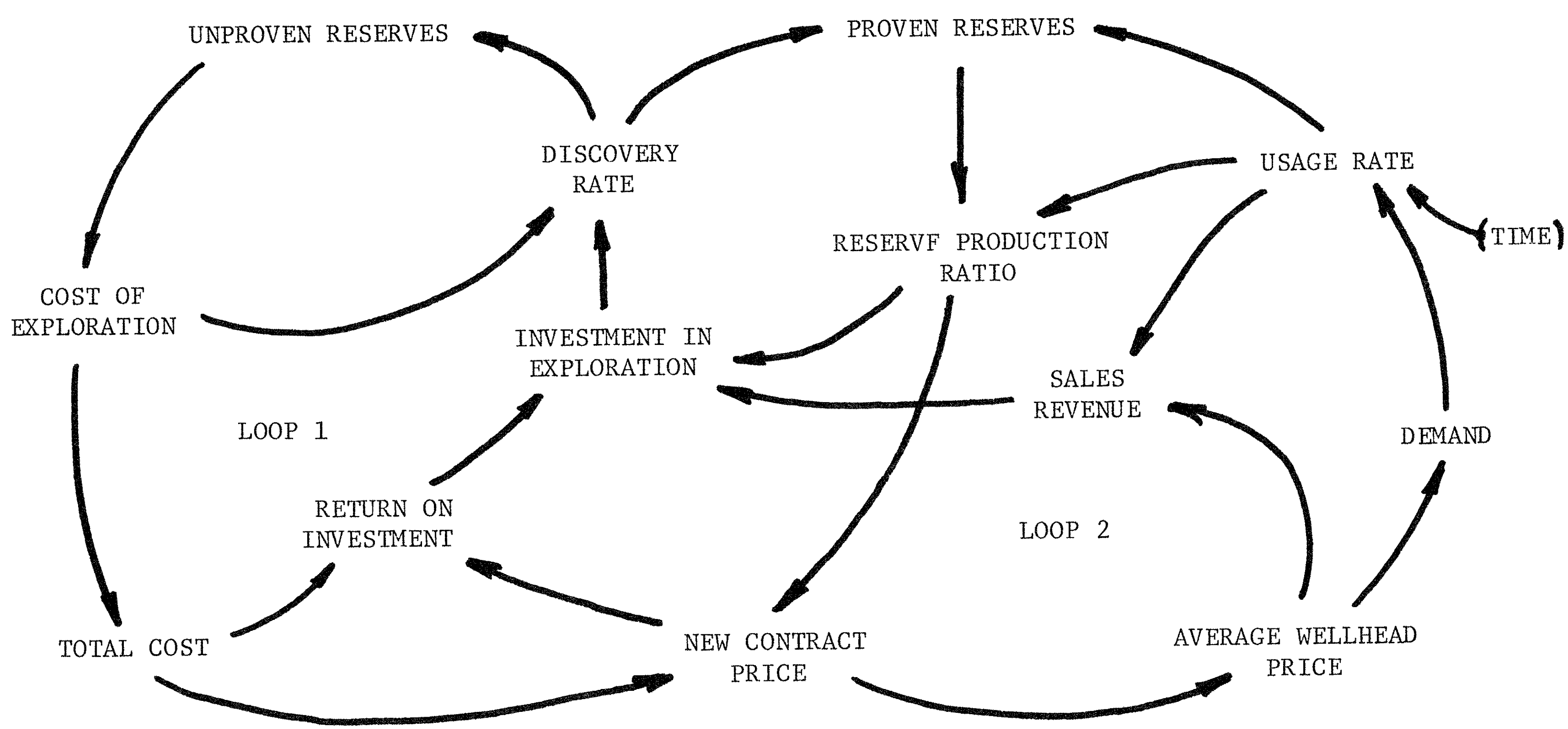
The model does not explicitly include changes in exploration or extraction technology, substitution effects, or the effects of imports in its structure. These effects can, however, be implicitly studied by their influence on existing parameter values in the model, as will be illustrated later for extraction technology and imports. The effects of technology and substitution on resource availability are treated in a separate paper by William W. Behrens of the M.I.T. System Dynamics Group [Behrens, (1971)].

Description of the Model

Figure 2 is a causal loop diagram which illustrates the major feedback loops of the model. The model contains only two major state variables, or levels, corresponding to unproven reserves UPR[22] and proven reserves PR. Loop 1 is a negative feedback loop interrelating the level of unproven reserves, the cost of exploration, and the discovery rate. As unproven reserves are depleted, the cost of exploration rises because less gas is

21) M.K. Hubbert, Op. Cit. (1971), p. 97.
22) Uppex case letters are used to indicate the abbreviation for each parameter which is used in the DYNAMO flow diagram (Figure 3).

FIGURE 2: CAUSAL LOOP DIAGRAM OF THE NATURAL GAS DISCOVERY MODEL



NOTE: The algebraic sign "+" or "-" at the head of each arrow expresses the polarity of influence between the two variables linked by that arrow. For example, as AVERAGE WELLHEAD PRICE increases, SALES REVENUE increases (a "+" relationship), and as AVERAGE WELLHEAD PRICE increases, DEMAND decreases (a "-" relationship).

found per foot of drilling, and because producers must look in less accessible places. This decreases the discovery rate, slowing the depletion of unproven reserves. The rise in cost of exploration also increases the total unit cost of production, which decreases return on investment in exploration, causing a decrease in investment in exploration. A decrease in investment in exploration then decreases discovery rate after a few years delay. The delays included in the system are not indicated in the causal loop diagram. They are critical determinants of system behavior, however, and are included explicitly in the system flow diagram and the model equations.

Loop 2 is the demand loop relating the level of proven reserves, price, demand, and usage rate. An increase in proven reserves increases the reserve-production ratio, which decreases new contract price because of excess coverage.[23] A price decrease results in an increase in demand, increasing usage rate which decreases proven reserves. The increase in usage rate also increases sales revenue, which increases investment in exploration, for it is assumed that the industry allocates its investment in proportion to its revenues. Investment in exploration is decreased when the reserve-production ratio exceeds a desired coverage, because the need for further discoveries is momentarily diminished.

Figures 3 and 4 show the variables in the model as related by a DYNAMO flow diagram with and without the effects of regulation. The following describes the important relationships among the system elements and presents the mathematical equations which express these relationships

23) Coverage is the number of years one can support current production out of current inventories, and is equal to the reserve-production ratio.

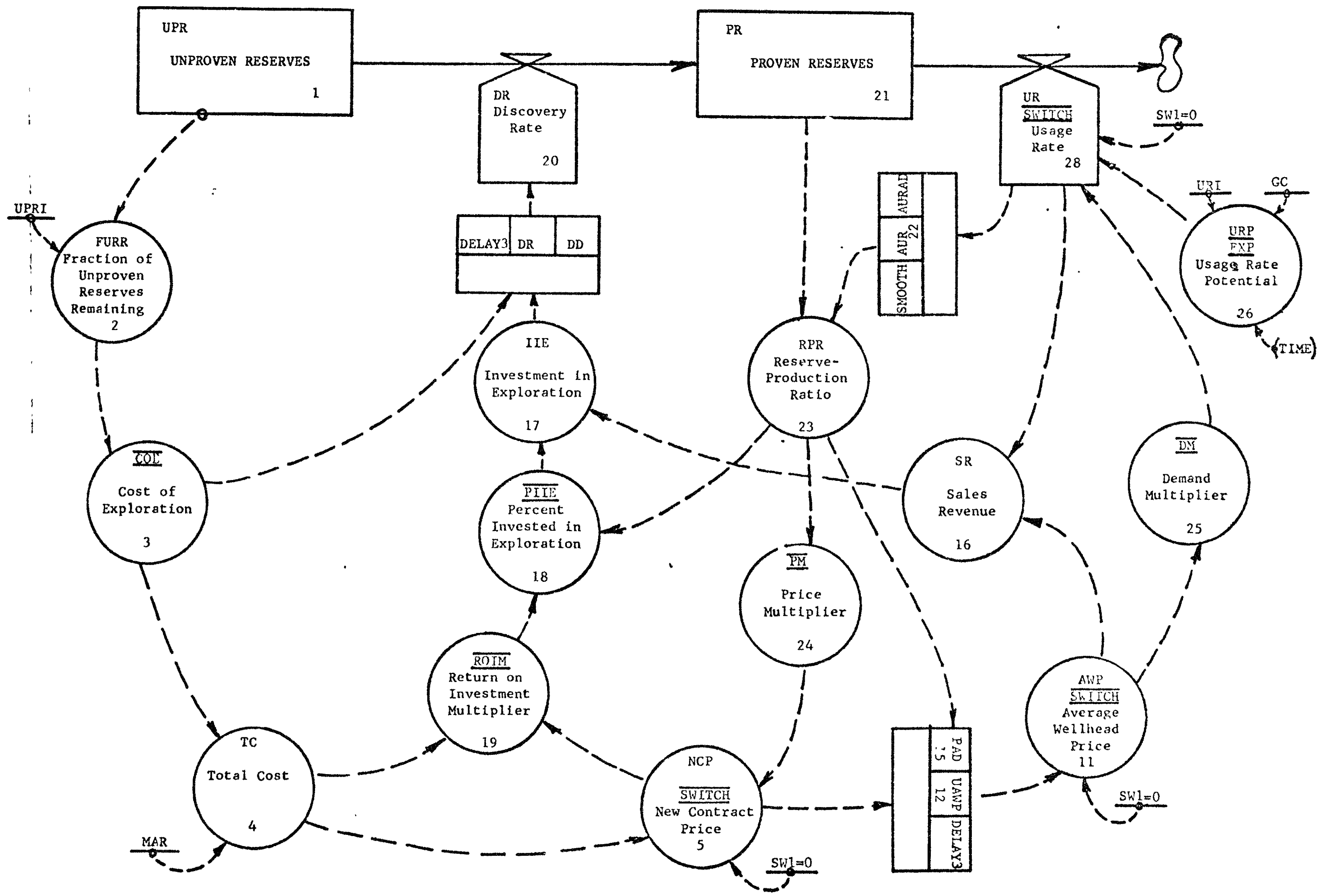


FIGURE 3: DYNAMO FLOW DIAGRAM OF THE U.S. NATURAL GAS
DISCOVERY MODEL: UNREGULATED CASE

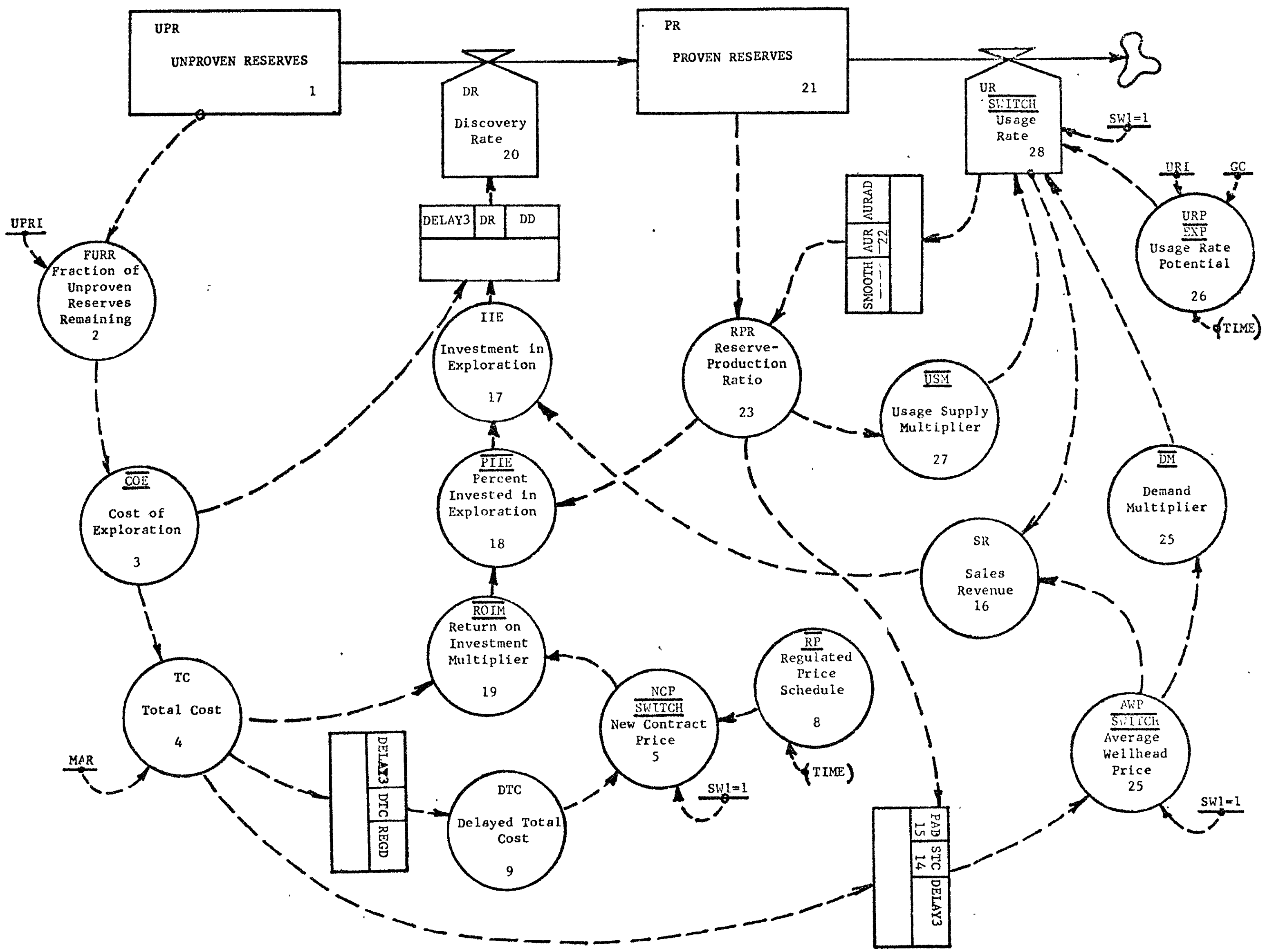


FIGURE 4: DYNAMO FLOW DIAGRAM OF THE U.S. NATURAL GAS DISCOVERY MODEL: REGULATED CASE

in a form suitable for processing by the computer.[24] Numbers at the right indicate the variable's location in the flow diagram.

LOOP 1: DISCOVERY LOOP

Unproven Reserves

```
UPR.K=UPR.J+(DT)(-DR.JK)                                  1, L
UPR=UPRI                                                  1.1, N
UPRI=1.04E15                                              1.2, C
    UPR     - UNPROVEN RESERVES (CUBIC FEET)
    DT      - TIME INCREMENT BETWEEN CALCULATIONS (YEARS)
    DR      - DISCOVERY RATE (CUBIC FEET/YEAR)
    UPRI    - UNPROVEN RESERVES INITIAL (CUBIC FEET)
```

Perhaps the simplest and yet most important concept of the model is the fact that the total amount of unproven reserves UPR in the system is finite. The total extent of the reserves will be unknown initially, existing largely as unproven reserves. Even after those reserves are discovered they may not be economically exploited, but the total quantity is fixed. This implies that unproven reserves begin with some initial value UPRI, and can only decrease over time as unproven reserves are discovered.

There has been much controversy over the estimation of the value of initial unproven reserves. Because of the close relationship between gas and oil finds, most gas estimates are based on the results of existing extensive oil analyses. Estimates of 2,000 trillion cubic feet or more[25] have been made on the basis of Zapp's assumption that "oil to be discovered

24) The conventions of DYNAMO are fully explained in <u>DYNAMO II User's Manual</u>, (Pugh, 1970). Here it is sufficient to note that the equations are primarily algebraic. J, K and L are used in place of the time subscripts t-1, t, t+1 respectively. X.K means the value of X at time K, X.KL means the value of X over the interval from time K to time L.
25) T.A. Hendricks, "Resources of Oil, Gas, and Natural Gas Liquids in the United States and the World," U.S. Geol. Survey Circ. 522, (1965).

per foot of exploratory drilling in any given petroliferous region will remain essentially constant until an areal density of about one exploratory well per two square miles has been achieved."[26] Hubbert shows that in fact discoveries per foot drilled have fallen off exponentially with cumulative footage drilled, and makes an estimate of 1,040 trillion cubic feet on the basis of this hypothesis.[27] Because the Hubbert estimate is derived from the discovery data, his value of 1,040 trillion cubic feet of initial unproven reserves of gas for the U.S. excluding Alaska and Hawaii is used in the model. This value can, however, be easily changed from one simulation run to the next to determine the dynamic implications of alternative possible unproven reserve bases. This is done in Chapter V. One advantage of a simulation model lies in the ease with which one may analyze the implications of changes in underlying assumptions such as variations in reserve estimates.

Fraction of Unproven Reserves Remaining

```
FURR.K=UPR.K/UPRI                                        2, A
    FURR    - FRACTION OF UNPROVEN RESERVES REMAINING
                (DIMENSIONLESS)
    UPR     - UNPROVEN RESERVES (CUBIC FEET)
    UPRI    - UNPROVEN RESERVES INITIAL (CUBIC FEET)
```

As unproven reserves are depleted, the level of unproven reserves at any given time divided by the initial amount of unproven reserves UPRI gives the fraction of unproven reserves remaining to be discovered FURR.

26) M.K. Hubbert, Resources and Man, p. 185. For the original formulation by Zapp, see A.D. Zapp, "Future Petroleum Producing Capacity of the United States," U.S. Geol. Survey Bull., 1962, 1142-H.
27) M.K. Hubbert, Op. Cit. (1969), p. 188.

Cost of Exploration

```
COE.K=TABHL(COET,LOGN(10*FURR.K),-3.5,2.5,.5)*          3, A
  (COEN)*(CNM.K)
COEN=1E-5                                                3.1, C
COET=1.3E4/6E3/2.7E3/1E3/545/245/110/50/22/9.98/         3.2, T
  4.48/2.02/.91
     COE     - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)
     TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
     COET    - COST OF EXPLORATION TABLE
     LOGN    - NATURAL LOGARITHMIC FUNCTION
     FURR    - FRACTION OF UNPROVEN RESERVES REMAINING
                 (DIMENSIONLESS)
     COEN    - COST OF EXPLORATION NORMAL (DOLLARS/CUBIC
                 FOOT)
     CNM     - COST NOISE MULTIPLIER (DIMENSIONLESS)
```

It has been found that for all nonrenewable natural resources, the cost of exploration is a function of reserves remaining--as reserves are depleted, the cost of exploration increases.[28] Initially, when the fraction of unproven reserves remaining is one, the industry will explore for new gas reserves in the most accessible places and exploit the largest fields available, making the cost of exploration relatively low. As most of the larger deposits are discovered, producers must look in less accessible places like the mountains or off-short Louisiana, causing the cost of exploration to rise. In addition, the size of reserves found and the success ratio of wildcat wells drilled decreases, further increasing costs as the fraction of unproven reserves diminishes. Finally, as the fraction of unproven reserves remaining approaches zero, cost of exploration approaches infinity as no more gas can be found at any cost. The graph of cost of exploration COE as a function of fraction of unproven reserves remaining FURR is given in Figure 5. Equations 3 and 3.1 above are simply

28) See for example the discussion of costs in [Ayres, Kneese, (1971)]

a shorthand notation expressing the relationship between cost of exploration COE and fraction of unproven reserves remaining FURR shown in Figure 5, and defined by the table COET multiplied by the normalizing constant COEN.

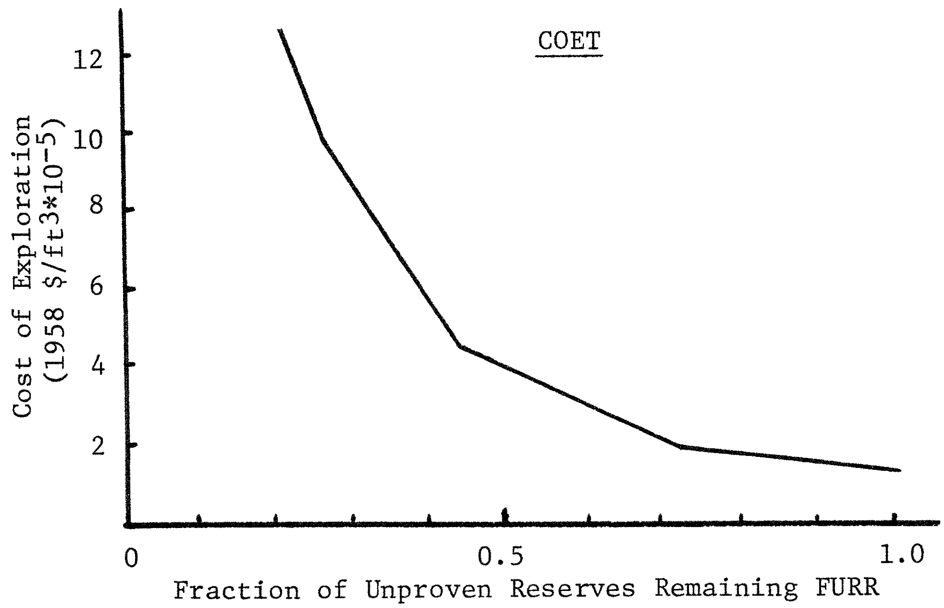


FIGURE 5: COST OF EXPLORATION TABLE

According to a study by M. Adelman,[29] there is little doubt that these costs are rising: "We must be content with the limited but firm conclusion that finding cost has almost certainly been on the increase from the late 1940's to the middle 1960's." In fact we can be more precise.

This long-run relationship has been derived from data given in NAS-NRC's Resources and Man[30] relating rate of discovery of oil per foot drilled as a function of cumulative feet drilled (Figure 6). Hubbert

29) M. A. Adelman, "Trends in Costs of Finding and Developing Oil and Gas in the U.S.," Essays in Petroleum Economics, Proc. 1967 Rocky Mountain Petroleum Economics Institute, Colorado School of Mines, (1967), p. 65.
30) M.K. Hubbert, Op. Cit. (1969), p. 186.

notes that the rate of gas discovered to oil discovered has averaged about 6,000 ft^3/bbl over the past 20 years. The trend towards directional

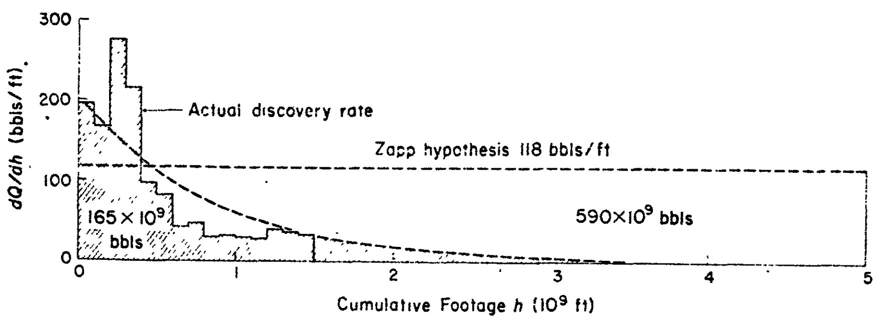


FIGURE 6: OIL/FT DRILLED VS. CUMULATIVE FEET DRILLED

(After Figure 8.19, Resources and Man)

drilling tends to increase this ratio for the future. We assume, however, that the gains are largely offset by recently rising costs of drilling because of the increasing average well depths.[31] With this assumption, one can derive the cost of exploration COE versus fraction of unproven reserves remaining FURR curve (Figure 5) from the above curve.

The values of cost of exploration normal COEN and unproven reserves initial UPRI are set to fit the theoretical curve to the actual data (Appendix II). This data, as noted by Adelman,[32] is extremely difficult to obtain because of the problem of allocation of costs between oil and gas, or between exploration and development. An estimate was made by multiplying total exploration expenditures for oil and gas by the percentage of gas wells, weighted by the cost per well (see Appendix II). It is

31) American Petroleum Institute, Independent Petroleum Association of America, Mid-Continent Oil and Gas Association, "Joint Association Survey of Industry Drilling Costs.", (1959-1967).
32) M.A. Adelman, Op. Cit. (1967), p. 58.

useful to note here that the shape of the cost of exploration curve, and not the actual values on the curve is the important factor in determining system behavior.

Total Cost

```
TC.K=(MAR)(COE.K)                                                  4, A
MAR=3.7                                                            4.1, C
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    MAR     - COST MARGIN (DIMENSIONLESS)
    COE     - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)
```

During the period 1955-1967, exploration costs remained a relatively constant fraction of thirty percent of the total exploration, development, and production costs for the oil and gas industry.[33] Included in the cost margin is a profit percentage over costs of 12 percent to cover normal return on investment. Thus total cost is assumed to be a constant multiple of 3.7 times the cost of exploration for the natural gas industry.

Determination of Price

Two prices, the new contract price NCP and average wellhead price AWP, are significant determinants of behavior in the natural gas industry. The new contract price NCP is a function of total cost TC and the current producer supply situation, measured by the reserve-production ratio RPR. The new contract price is used by producers to determine return on investment in their decision to invest in exploration for new gas reserves.

33) U.S. Energy Study Group, Energy R&D and National Progress, U.S. Gov't. Printing Office, Wash., D.C. (1964), p. 147. Also American Petroleum Institute, et.al., Joint Association Survey (Section II), (1967), p. 5.

Average wellhead price is a delayed function of new contract price, and influences sales revenue and consumption of gas. Both average wellhead price AWP and new contract price NCP are affected when price regulation occurs.

New Contract Price

```
NCP.K=SWITCH(UNCP.K,RNCP.K,SW1)                          5, A
    NCP    - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    UNCP   - UNREGULATED NEW CONTRACT PRICE (DOLLARS/
               CUBIC FOOT)
    RNCP   - REGULATED NEW CONTRACT PRICE (DOLLARS/CUBIC
               FOOT)
    SW1    - REGULATION SWITCH
```

New contract price is represented in the model as a switching function, whose value is unregulated new contract price UNCP without regulation (SW1=0), and regulated new contract price RNCP when simulating a run in which regulation takes place (SW1=1). The regulated new contract price RNCP is assumed to be a function only of total cost, while the unregulated new contract price is a function of both total cost and the relative supply of gas.

When price is a function of relative quantity supplied (through the reserve-production ratio RPR), it is free to seek an equilibrium level depending on quantity supplied and quantity demanded. Price regulation upsets this process, for price is then set on a cost-plus margin basis, and no longer responds to shortages in supply indicated by a low reserve-production ratio RPR.

Unregulated New Contract Price

```
UNCP.K=(TC.K)(PM.K)                                         6, A
    UNCP    - UNREGULATED NEW CONTRACT PRICE (DOLLARS/
                CUBIC FOOT)
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    PM      - PRICE MULTIPLIER (DIMENSIONLESS)
```

The unregulated new contract price UNCP is equal to total cost plus a margin whose size is determined by the price multiplier PM. The price multiplier reflects the producers' new contract price response to relative abundance or scarcity of reserves. If the reserve-production ratio RPR is above that desired by the industry, the margin above cost that producers will charge in establishing new contracts will be relatively small, for the producers wish to avoid the costs of carrying excess inventory, and therefore will sell near cost.

The normal gas contract commits a producer to deliver quantities of gas for an average of 20 years. Thus if the reserve-production ratio is below 20, the producers run the risk of not being able to fulfill a future delivery if they choose to sell additional gas now. The alternative is to tap marginally productive reserves whose costs are higher. In any case a low reserve-production ratio forces the producers to charge a higher price for gas.

Regulated New Contract Price

```
RNCP.K=CLIP(MAX(RP.K,DTC.K),UNCP.K,TIME.K,1955)        7, A
    RNCP    - REGULATED NEW CONTRACT PRICE (DOLLARS/CUBIC
                FOOT)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    MAX     - FUNCTION WHICH CHOOSES THE MAXIMUM OF TWO
                ARGUMENTS
```

RP - REGULATION PRICE SCHEDULE (DOLLARS/CUBIC FOOT)
DTC - DELAYED TOTAL COST (DOLLARS/CUBIC FOOT)
UNCP - UNREGULATED NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
TIME - TIME (YEARS)

As a result of the 1954 Supreme Court decision in Phillips Petroleum Company v. Wisconsin[34] the natural gas industry has been subject to price ceiling regulation by the Federal Power Commission. The price is based on the producer's operating costs. To represent this regulatory policy in the model, new contract price is taken as unregulated up to 1955, a year after the Supreme Court decision. Up to this point new contract prices had been continually rising, for producers could command a higher price due to increasing demand for gas. After the Supreme Court decision, however, new contract prices fell until 1964 and then remained constant at their 1964 level,[35] for regulation imposed ceiling prices which were based on cost plus a 12 percent margin.[36] This effect is formulated as an exogenous time series RP representing the downward drift of new contract prices from 1955-1964, and a constant value thereafter.

The ceiling prices set in 1960 were admittedly above the cost-plus margin. They were rationalized as a provision against uncertainties during the change-over period and an added stimulus to exploratory activity.[37] Ceiling prices therefore remain near their 1960 level until costs rise

34) Phillips Petroleum Co. v. Wisconsin, 347 U.S. 672, (1954).
35) F.P.C., Sales by Producers of Natural Gas to Interstate Pipeline Companies, (1960-1964), Wash. D.C. 20426.
36) R.S. Spritzer, "Changing Elements in the Natural Gas Picture: Implications for the Federal Regulatory Scheme," first draft of a paper for RFF volume edited by Keith Brown (1971), p. 12.
37) Ibid., p. 11.

enough to warrant further increases. Because of the difficulties involved in perceiving cost increases and translating them into new price guidelines, a four year perception delay is assumed between actual rises in total cost and resulting rises in new contract prices.

The simulation of the effects of regulation on new contract price are as follows: new contract price is assumed unregulated until 1955. In 1955 new contract price is determined exogenously by RP, representing a transition period where price drifts downward towards the total cost regulatory guidelines imposed by the FPC. As costs rise above the 1964 new contract price level, it is assumed new contract price NCP rises in accordance with a delayed measure of total cost DTC.

Regulated Price Schedule

```
RP.K=TABHL(TRP,TIME.K,1955,1964,9)*(1E-4)                3, A
TRP=1.95/1.6                                             3.1, T
     RP      - REGULATION PRICE SCHEDULE (DOLLARS/CUBIC
                 FOOT)
     TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
     TRP     - REGULATED PRICE SCHEDULE TABLE
     TIME    - TIME (YEARS)
```

The regulated price schedule RP represents an exogenous time series reflecting the behavior of regulated new contract price RNCP from 1955 through 1969. Real-World data and the regulated price schedule RP are graphed in Figure 7. Data are obtained from F.P.C., Sales by Producers of Natural Gas to Interstate Pipeline Companies.

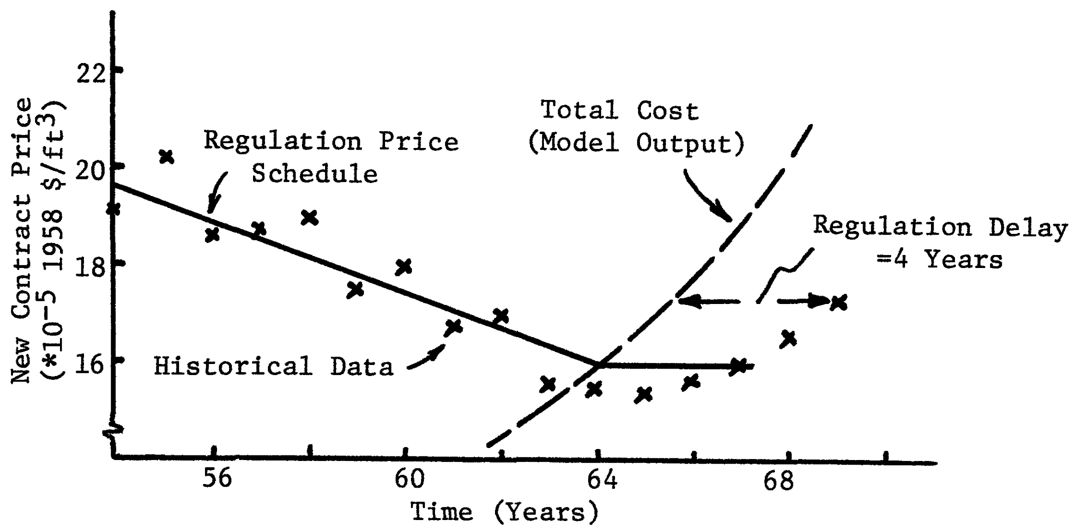


FIGURE 7: NEW CONTRACT PRICE, 1954-1969 (1954 DOLLARS)

Delayed Total Cost

```
DTC.K=DELAY3(TC.K,REGD)                                              9, A
REGD=4                                                               9.1, C
    DTC     - DELAYED TOTAL COST (DOLLARS/CUBIC FOOT)
    DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    REGD    - REGULATION DELAY (YEARS)
```

The delayed total cost function represents the delay in response of the FPC to rises in cost. The inherent difficulties in measuring unit costs in gas makes any rise in cost uncertain. Even after the cost rise is perceived, there are additional delays in structuring new rates. The magnitude of this delay has been approximated by the analysis shown in Figure 7. When fitted to data, the model reproduces the time series behavior of total cost shown in Figure 7. The actual rise in new contract price can be seen to be delayed behind rises in total cost by about four years.

Average Wellhead Price

```
AWP.K=SWITCH(UAWP.K,RAWP.K,SW1)                          10, A
    AWP    - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    UAWP   - UNREGULATED AVERAGE WELLHEAD PRICE
               (DOLLARS/CUBIC FOOT)
    RAWP   - REGULATED AVERAGE WELLHEAD PRICE (DOLLARS/
               CUBIC FOOT)
    SW1    - REGULATION SWITCH
```

The average wellhead price AWP represents the average price per cubic foot for all gas sold in a given year. This value determines sales revenue SR and demand for gas through the demand multiplier DM. Average wellhead price AWP is controlled by a switching function activated by the regulation switch SW1, which determines whether average wellhead price will be regulated (SW1=1 implies AWP=RAWP) or unregulated (SW1=0 implies AWP=UAWP).

Unregulated Average Wellhead Price

```
UAWP.K=DELAY3(NCP.K,PAD.K)                               11, A
    UAWP   - UNREGULATED AVERAGE WELLHEAD PRICE
               (DOLLARS/CUBIC FOOT)
    DELAY3 - TERM DENOTING A LAGGED RELATIONSHIP
    NCP    - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
    PAD    - PRICE AVERAGING DELAY (YEARS)
```

When wellhead price is unregulated, average wellhead price is simply a delayed function of new contract price NCP. This represents the fact that average wellhead price reflects the price of gas sold under all contracts, new and old. A higher new contract price will not significantly affect average wellhead price initially, for it must be weighted by the quantity of gas sold at that price. A continuing trend of rising new contract prices will gradually lead to a rise in average wellhead price after a delay determined by the average turnover time of gas flowing through proven reserves PR.

Regulated Average Wellhead Price

```
RAWP.K=CLIP(MAX(P60,STC.K),UAWP.K,TIME.K,1960)        12, A
P60=1.5E-4                                            12.1, C
    RAWP    - REGULATED AVERAGE WELLHEAD PRICE (DOLLARS/
                CUBIC FOOT)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    MAX     - FUNCTION WHICH CHOOSES THE MAXIMUM OF TWO
                ARGUMENTS
    P60     - CEILING PRICE SET IN 1960 (DOLLARS/CUBIC
                FOOT)
    STC     - SMOOTHED TOTAL COST (DOLLARS/CUBIC FOOT)
    UAWP    - UNREGULATED AVERAGE WELLHEAD PRICE
                (DOLLARS/CUBIC FOOT)
    TIME    - TIME (YEARS)
```

The ceiling price regulation imposed by the FPC also has direct effects on average wellhead price. Regulation of average wellhead prices occurred effectively in 1960, when a ceiling price was finally set for all pre-1960 contracts (as well as for all new contract prices) after years of negotiation.[38] This price was also above the cost-plus-margin guideline, which established a 12 percent margin above average costs.

Thus to approximate the effects of regulation on the average wellhead price AWP, the average wellhead price is assumed equal to the unregulated average wellhead price UAWP until 1960. After 1960, the price is held at its 1960 level, until average total cost (STC) rises above that value. From that point on, it is assumed that the FPC will raise the ceiling average wellhead price AWP in accordance with the cost-plus-margin guideline, so average wellhead price AWP becomes equal to smoothed total cost STC.

38) Ibid., p. 11.

Smoothed Total Cost

```
STC.K=DELAY3(TC.K,PAD.K)                                    13, A
     STC     - SMOOTHED TOTAL COST (DOLLARS/CUBIC FOOT)
     DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
     TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
     PAD     - PRICE AVERAGING DELAY (YEARS)
```

The response of average wellhead price AWP to changes in total cost (when the industry is regulated) or new contract price (when unregulated) is delayed by a smoothing delay PAD to represent the fact that although the discovery cost of an additional unit of gas may be high, its impact on the average wellhead price AWP of proven reserves must be weighted by the fraction of total proven reserves with the higher price. This is represented in the model with a delay function. If total cost were to rise in year ten from five cents/Mcf to ten cents/Mcf, the response of average wellhead price AWP, represented by a third order delay, is shown in Figure 8. This representation is not as precise as one which would keep track of each different cost category of gas and would specify usage and discovery rates for each. However, the error introduced by this simplification has no influence on the conclusions derived from this model. It might, however, be important in shorter-term studies of the natural gas industry.

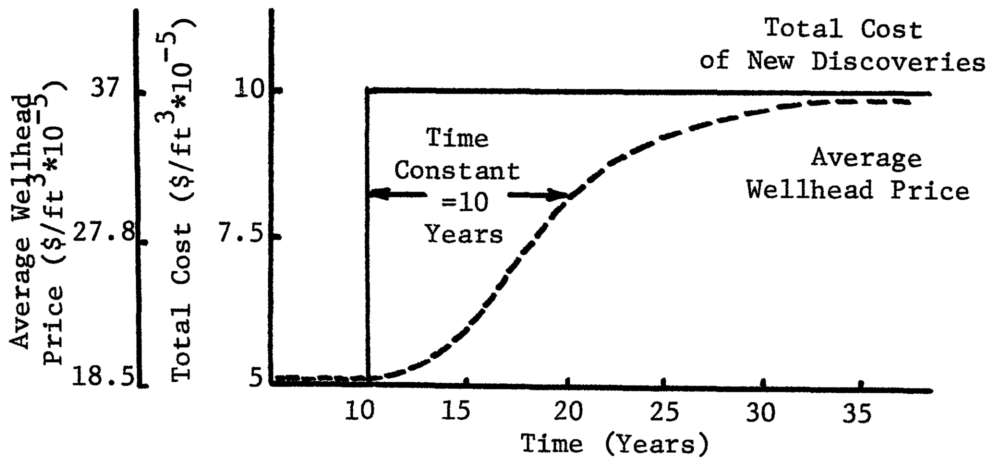


FIGURE 8: EFFECTS ON AVERAGE WELLHEAD PRICE OF STEP RISE IN TOTAL COST

Price Averaging Delay

```
PAD.K=RPR.K                                                  14, A
    PAD     - PRICE AVERAGING DELAY (YEARS)
    RPR     - RESERVE-PRODUCTION RATIO (YEARS)
```

As the cost of exploration rises, the total cost of the new additions to proven reserves also rises. The average wellhead price of these proven reserves does not rise immediately in response to changes in the cost of new reserves, however. In the natural gas industry, reserves are sold to pipeline of other distribution companies under a long-term contract, usually twenty years.[39] Thus the average wellhead price in any given year is associated with revenue from recent contracts negotiated at a higher price because of rising costs, and with revenue from gas discovered as much as twenty years ago, when discovery costs were much lower. Assuming regular cost increases, the cost which affects average wellhead price today is really total cost averaged over a period of PAD.

39) M.A. Adelman, Op. Cit. (1962), p. 37.

The value of the price averaging delay PAD depends on the relative magnitudes of discovery rate DR, level of proven reserves PR, and usage rate UR. The reserve-production ratio RPR gives a measure of the average time gas remains in the proven reserve category. If discovery rate and usage rate were equal, the time delay on cost which would determine the average wellhead price would be the reserve-production ratio RPR divided by two. If discovery rate DR were nearly zero, this time delay would be best represented simply by the value of reserve-production ratio RPR. This difference in delay representation becomes significant in relation to model behavior only when discovery rate DR is near zero, so the price averaging delay PAD is assumed equal to the reserve-production ratio RPR.

Sales Revenue

```
SR.K=(UR.JK)(AWP.K)                                   15, A
     SR       - SALES REVENUE (DOLLARS)
     UR       - USAGE RATE (CUBIC FEET/YEAR)
     AWP      - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)
```

Sales revenue is simply the product of usage rate and price, reflecting the total revenue obtained from the sale of the resource. The industry's ability to invest in exploration for new reserves is assumed to be proportional to its current revenue. Thus growth in sales revenue provides the positive impetus which increases the discovery rate through increased investment in exploration during the growth phase of the industry.

Investment in Exploration

```
IIE.K=(PIIE.K)(SR.K)                                     16, A
    IIE     - INVESTMENT IN EXPLORATION (DOLLARS)
    PIIE    - PERCENT INVESTED IN EXPLORATION (FRACTION)
    SR      - SALES REVENUE (DOLLARS)
```

The total amount invested in exploration per year is defined to be equal to the sales revenue for that year multiplied by the percentage of sales revenue invested in exploration. From historical data it appears that the industry tends to invest about 35 percent of its sales revenue[40] when return on investment is adequate and relative supply (RPR) is near the desired value of 20 years. Deviations dependent on the apparent return on investment and relative coverage or abundance of reserves, are reflected in the function PIIE described below.

Percent Invested in Exploration

```
PIIE.K=TABHL(PIIET,RPR.K/DRPR,.2,2.0,.2)*(ROIM.K)*   17, A
  (INM.K)
PIIET=.5/.5/.5/.48/.39/.24/.12/.05/.01/0             17.1, T
    PIIE    - PERCENT INVESTED IN EXPLORATION (FRACTION)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    PIIET   - PERCENT INVESTED IN EXPLORATION TABLE
    RPR     - RESERVE-PRODUCTION RATIO (YEARS)
    DRPR    - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
    ROIM    - RETURN ON INVESTMENT MULTIPLIER
                (DIMENSIONLESS)
    INM     - INVESTMENT NOISE MULTIPLIER (DIMENSIONLESS)
```

As stated by C. Hawkins, the decision to spend money in drilling for oil or gas is a capital investment decision. ...Capital will be shifted

40) American Petroleum Institute, "Joint Association Survey of Industry Drilling Costs," and American Gas Association, Gas Facts.

away from exploration if there is the expectation of a higher return in other investments; the decision to drill for oil or gas is no different from any other type of investment decision."[41]

As represented in the model the decision to invest in exploration is dependent on two factors: 1) A return on investment multiplier that encourages investment when return on investment is adequate and decreases investment when the return is low. The measure for return on investment is taken as new contract price/average cost. 2) A function of the reserve-production ratio RPR which states that when the RPR exceeds a desired reserve-production ratio (assumed to be 20 years for the gas industry), investment in discovery will be reduced. This is due to the high inventory costs of maintaining a reserve larger than needed to ensure the fulfillment of current contracts. The percent invested in discovery as a function of

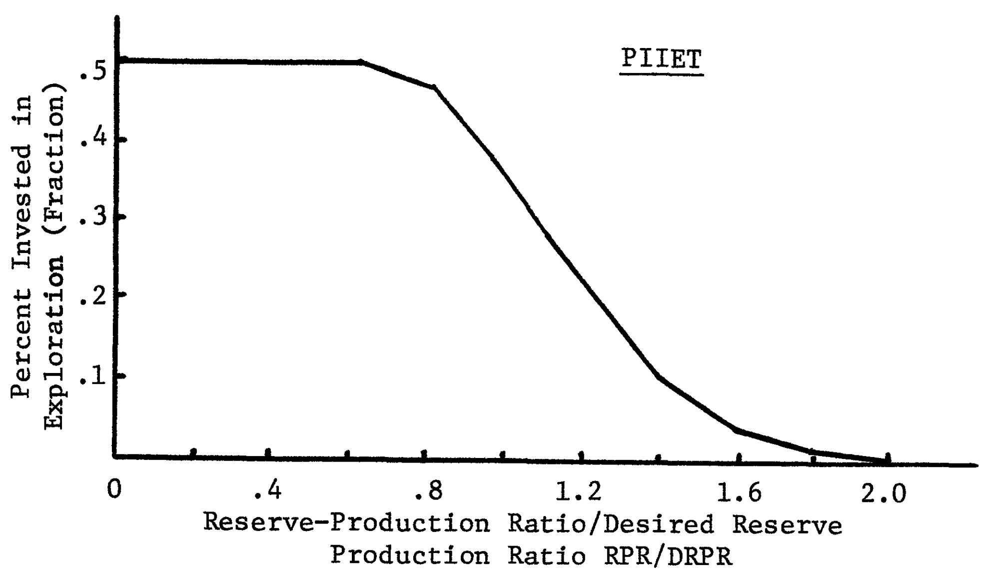


FIGURE 9: PERCENT INVESTED IN EXPLORATION TABLE

41) C.A. Hawkins, "Structure of the Natural Gas Producing Industry," Forthcoming in a RFF Volume ed. by Keith Brown, First Draft, (1971), p. 29.

RPR/DRPR is given in Figure 9. Producers are expected to limit their investment in discovery to 35 percent when return on investment is normal, and the reserve-production ratio RPR is about 20 years, for this has been the normal cost of exploration as a percentage of average revenue. When reserves become higher than desired, producers will reduce the investment in exploration to discourage new discoveries. This happened in 1939, for example, when the percentage invested in discovery fell to about 15 percent[42] at a high reserve-production ratio RPR of 28 years. If the reserve-production ratio falls below the desired ratio, producers are assumed to increase their investment in exploration up to maximum of 50 percent of sales revenue, if return on investment is normal.

The investment noise multiplier INM allows the analyst to test the response of the model to random noise fluctuations. The value of the investment noise multiplier INM is nominally set at 1, and is controlled by the noise switch SWN.

Return on Investment Multiplier

```
ROIM.K=TABHL(ROIMT,NCP.K/TC.K,0,2.2,.2)                      18, A
ROIMT=0/.08/.25/.44/.55/.67/.76/.82/.88/.92/.96/1            18.1, T
     ROIM     - RETURN ON INVESTMENT MULTIPLIER
                  (DIMENSIONLESS)
     TABHL    - TERM DENOTING A TABULAR RELATIONSHIP
     ROIMT    - RETURN ON INVESTMENT MULTIPLIER TABLE
     NCP      - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
     TC       - TOTAL COST (DOLLARS/CUBIC FOOT)
```

42) Estimated from data in U.S. Bureau of the Census, Census of Mineral Industries - 1963, Vol. I, U.S. Government Printing Office, Wash., D.C., (1967), p. 13B-67.

The percentage invested in exploration is also a function of the return on investment. It is assumed that producers' investment response to changes in levels of new contract price/total cost is relatively linear within the normal range of behavior (Figure 10). This has been found to

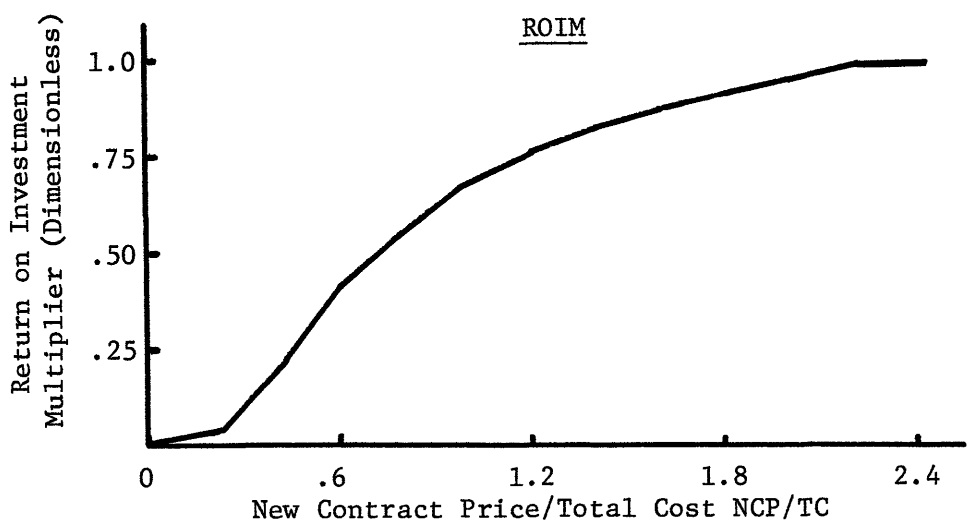


FIGURE 10: RETURN ON INVESTMENT MULTIPLIER TABLE

be the case in the Khazzoom study carried out for the FPC.[43] When the ratio of new contract price/total cost becomes very large, however, investment approaches an upper limit of 50 percent of sales revenue. At smaller values of new contract price/total cost, investment falls rapidly to zero.

The effects of the return on investment multiplier seem to be in evidence today, where costs of exploration are rising faster than the average price. In the late 60's, according to the gas producers,[44]

---

43) J.D. Khazzoom, Op. Cit. (1971), p. 74-80.
44) Wall Street Journal, April 12, 1971.

percent invested in exploration had decreased due to decreased return on investment caused by low prices. Between 1958 and 1968, additions to gas reserves had been averaging 19 trillion cubic feet annually. But in 1968, additions to reserves dropped to less than 14 trillion cubic feet, and in 1969, less than 9 trillion cubic feet. Because of the four to five year delay between investment in exploration and its effects, we should infer that a drop in investment (here measured by drilling expenditures of gas wells) occurred around 1963. This seems to be the case, as shown in Figure 11, for drilling expenditures seem to peak in 1962.

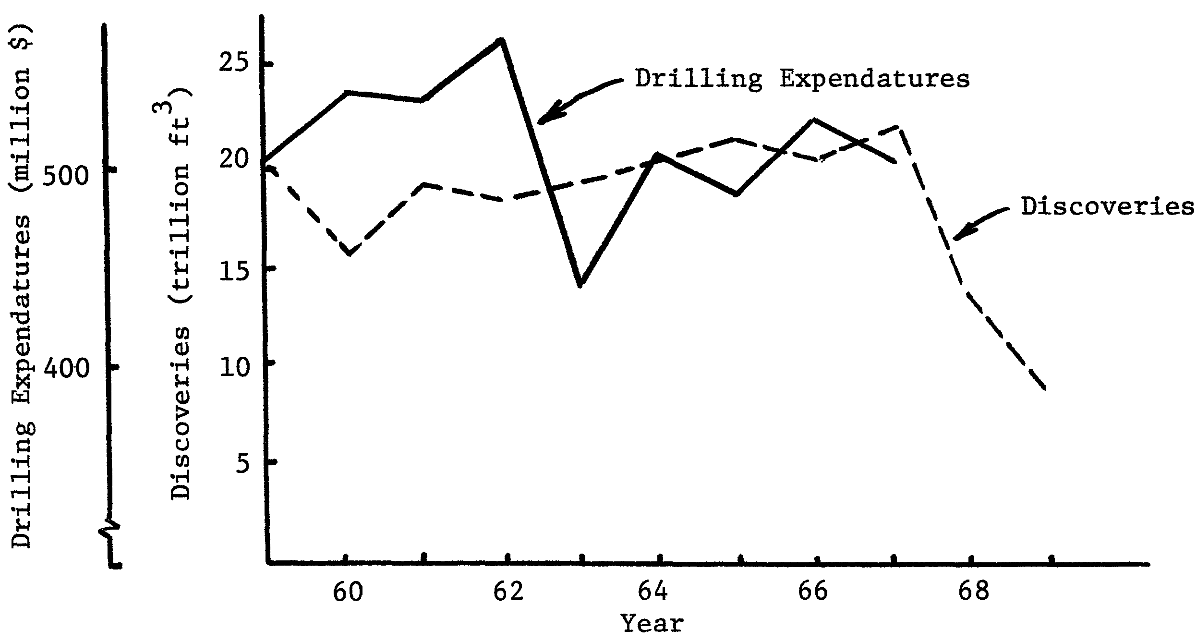


FIGURE 11: DRILLING EXPENDITURES AND DISCOVERIES, 1959-1969[45]

45) American Petroleum Institute, et. al., "Joint Association Survey of Industry Drilling Costs,"(1960-1967),and American Gas Association, <u>Gas Facts</u>,(1960-1969).

Discovery Rate

```
DR.KL=DELAY3(IIE.K/COE.K,DD)                                    19, R
DD=4.5                                                          19.1, C
    DR      - DISCOVERY RATE (CUBIC FEET/YEAR)
    DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
    IIE     - INVESTMENT IN EXPLORATION (DOLLARS)
    COE     - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)
    DD      - DISCOVERY DELAY (YEARS)
```

The outcome of an investment in exploration is generally not known for four to five years due to the time involved in finding a prospective site for drilling, setting up and drilling the wells, and making an accurate estimate of the size of discovery. M.A. Adelman cites, as a rule of thumb, that four to five years development time is required for wells.[46] Using lagged regression analyses, J. Daniel Khazzoom has determined the response of gas discoveries in subsequent years as a function of a one-year impulse rise in the price of gas.[47] The relationship he presents (Figure 12) is incorporated in the model as a third-order delay with a delay time of 4.5 years.

---

46) M.A. Adelman, Supply and Price of Natural Gas, p. 6.
47) J.D. Khazzoom, "FPC Gas Model," Bell Journal of Economics and Management Science, Vol. 2, No. 1, (Spring, 1971), p. 71.

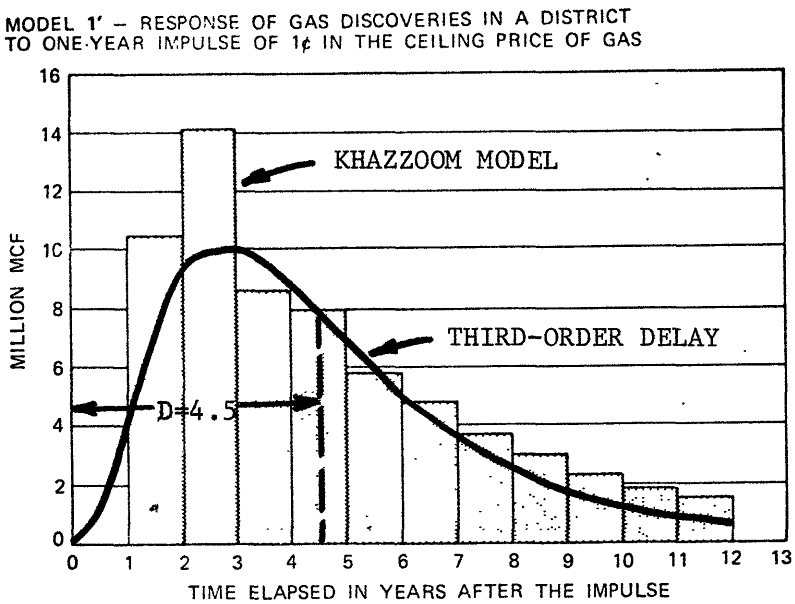


FIGURE 12: KHAZZOOM MODEL RESPONSE OF DISCOVERIES TO A ONE-CENT IMPULSE RISE IN PRICE[48]

In the model, the total amount added to reserves per year, the discovery rate DR, is assumed to be a delayed function of investment in exploration divided by the cost of exploration COE in dollars/ft^3 of gas. The discovery rate thus includes both "new discoveries" and "extensions and revisions" as distinguished by the American Gas Association. The discovery delay DD is assumed to be 4.5 years.

LOOP 2: DEMAND LOOP

The demand loop (Figure 13) is a negative or goal-seeking loop which works to establish an equilibrium between discovery rate and usage rate.

48) <u>Ibid.</u>, p. 71.

The reserve-production ratio RPR is a measure of the relative levels of supply (proven reserves PR) and demand (usage rate UR). Producers work to stabilize this ratio by raising new contract price NCP when reserve-production ratio RPR is lower than the desired reserve-production ratio DRPR (and lowering price when RPR>DRPR) through the price multiplier PM. Consumers

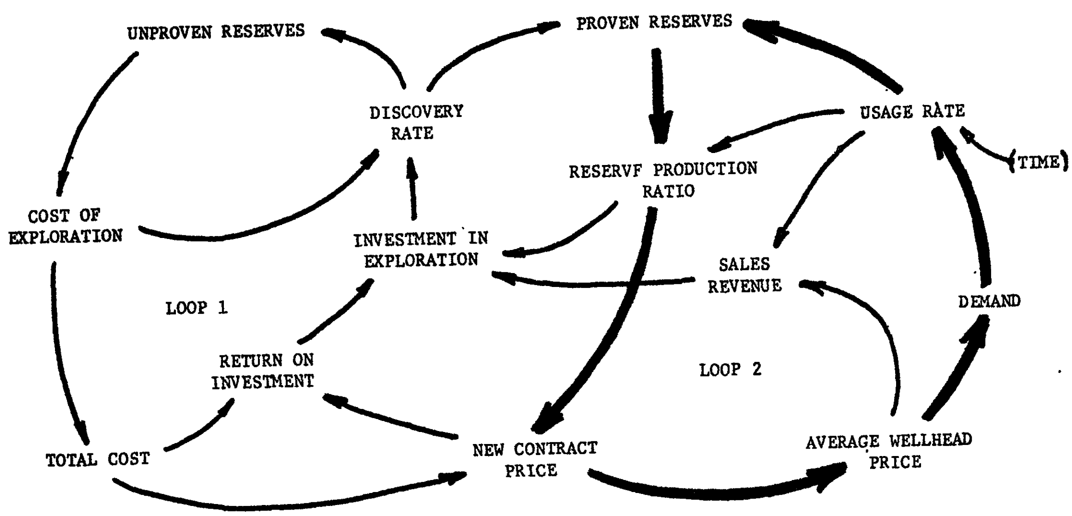


FIGURE 13: DEMAND LOOP

respond through the demand multiliper DM by reducing consumption at higher prices, thus lowering the usage rate. This acts to raise the reserve-production ratio RPR towards the desired level. Note that the level of average wellhead price which equates discovery and usage is constantly being driven upward by rising total cost.

Proven Reserves

```
PR.K=PR.J+(DT)(DR.JK-UR.JK)                                  20, L
PR=PRI                                                       20.1, N
PRI=6.4E12                                                   20.2, C
    PR      - PROVEN RESERVES (CUBIC FEET)
    DT      - TIME INCREMENT BETWEEN CALCULATIONS (YEARS)
    DR      - DISCOVERY RATE (CUBIC FEET/YEAR)
    UR      - USAGE RATE (CUBIC FEET/YEAR)
    PRI     - PROVEN RESERVES INITIAL (CUBIC FEET)
```

Proven reserves PR are increased by the discovery rate DR and decreased by the usage rate UR. Proven reserves is a level which corresponds to the natural gas already discovered by drilling but not yet used. In most cases it remains in its original reservoir in the ground. For this reason it is extremely difficult to obtain an accurate estimate of reserves discovered when a new field is first drilled, and so each year a large fraction of discoveries (in most years over half) result from extensions and revisions of already-discovered fields.

Average Usage Rate

```
AUR.K=SMOOTH(UR.JK,AURAD)                                    21, A
AUR=AURI                                                     21.1, N
AURI=3.2E11                                                  21.2, C
AURAD=1                                                      21.3, C
    AUR     - AVERAGE USAGE RATE (CUBIC FEET/YEAR)
    SMOOTH  - FIRST-ORDER EXPONENTIAL SMOOTHING FUNCTION
    UR      - USAGE RATE (CUBIC FEET/YEAR)
    AURAD   - AVERAGE USAGE RATE ADJUSTMENT DELAY (YEARS)
    AURI    - AVERAGE USAGE RATE INITIAL (CUBIC FEET/
                YEAR)
```

The average usage rate AUR is represented as an exponential average of usage rate UR, with a one-year averaging delay. This tends to smooth out short-term variations in usage, reflecting only the longer-term trends. This corresponds with industry accounting practices of calculating usage rates and levels of reserves year by year.

Reserve-Production Ratio

```
RPR.K=PR.K/AUR.K                                              22, A
DRPR=20                                                       22.1, C
    RPR    - RESERVE-PRODUCTION RATIO (YEARS)
    PR     - PROVEN RESERVES (CUBIC FEET)
    AUR    - AVERAGE USAGE RATE (CUBIC FEET/YEAR)
    DRPR   - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
```

The reserve-production ratio RPR has traditionally been the industry's measure of reserve adequacy. It indicates how long current reserves would supply current consumption. Usage rate will not remain at current levels, however. Consumption has been climbing at about seven percent per year, making the reserve-production ratio a gross overestimate of the current reserves' actual expected life. Figure 14 shows the relationship between the reserve-production ratio and actual time before current reserves would be depleted at various growth rates for total usage. When

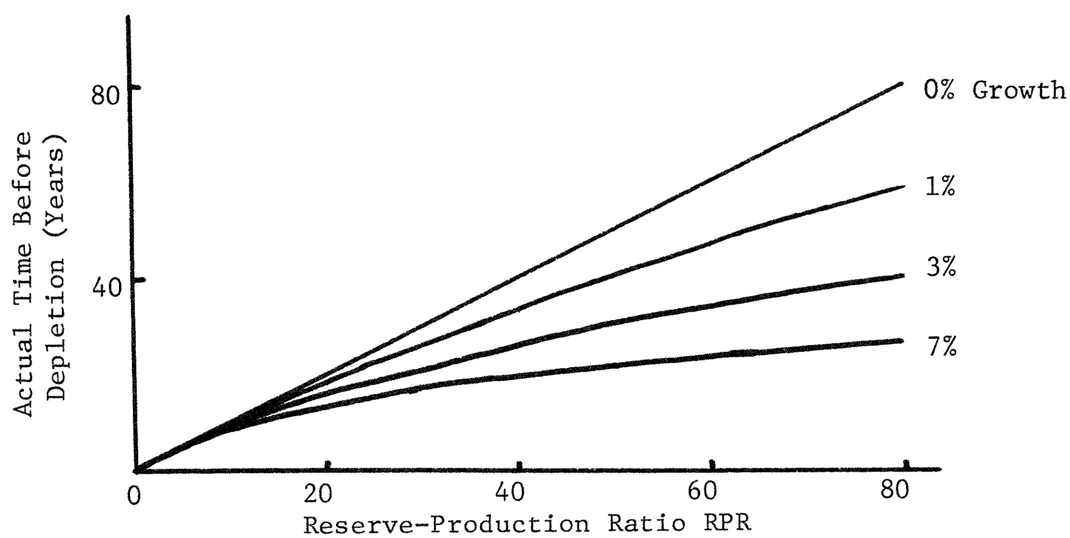


FIGURE 14: ACTUAL TIME BEFORE DEPLETION AT VARIOUS GROWTH RATES VERSUS NOMINAL RESERVE-PRODUCTION RATIO RPR

there is no growth, the relationship is linear, indicating that the time before depletion is equal to the reserve-production ratio. At seven percent growth, a 20 year reserve-production ratio actually promises to last only 12.7 years. The reserve-production ratio continues, however, to be used by producers as a measure of relative abundance of the resource.[49]

Price Multiplier

```
PM.K=TABHL(PMT,RPR.K/DRPR,0,1.8,.3)*(PNM.K)                23, A
PMT=8/5.5/3.75/2.35/1.35/1/1                               23.1, T
     PM      - PRICE MULTIPLIER (DIMENSIONLESS)
     TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
     PMT     - PRICE MULTIPLIER TABLE
     RPR     - RESERVE-PRODUCTION RATIO (YEARS)
     DRPR    - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
     PNM     - PRICE NOISE MULTIPLIER (DIMENSIONLESS)
```

The price multiplier (Figure 15) represents the response of the producers to changes in the reserve-production ratio, and defines the producer supply curve in the unregulated case. Price never drops below the cost-plus margin determined by cost of exploration, for it is assumed producers would never sell below cost. As usage rate or quantity demanded increases, the reserve-production ratio decreases, and the price at which producers are willing to sell rises. This is caused by the nature of the selling process in the industry. Reserves are normally committed to long-term contracts, in which the producer agrees to deliver a prescribed quantity at a given price for as long as 20 years. Because of the contracts, a producer runs a certain risk by selling reserves now that he may need to fulfill contracts in the future. If he does this, and further reduces his

49) Industry practices cited in M.A. Adelman, Op. Cit. (1962), pp. 66-68.

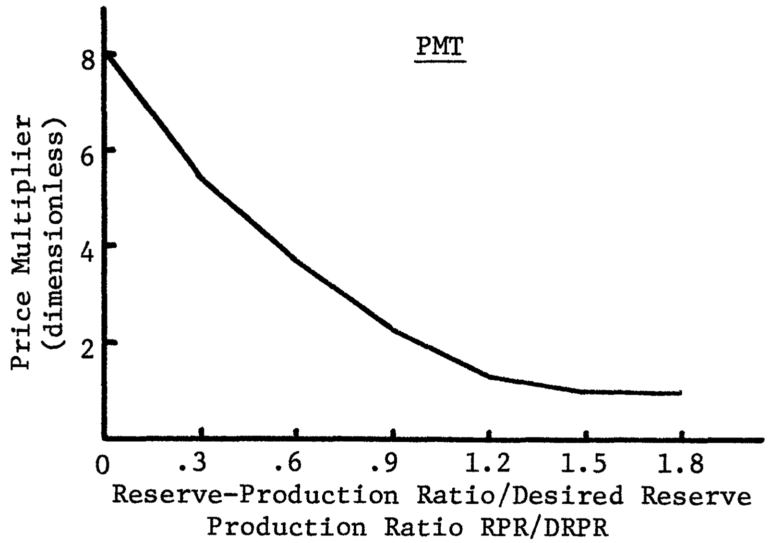


FIGURE 15: PRICE MULTIPLIER TABLE

reserve-production ratio RPR, he must either discover new reserves to replace those sold, or tap more costly known reserves. In either case, the effect is to cause the producer to sell reserves at a higher price when reserve-production ratio RPR is low.

The price multiplier PM acts to define the reaction of producers in setting price to changing conditions to supply and demand. The traditional approach is to attempt to define a static industry supply and demand curve, valid at only one point in time. The System Dynamics approach employed here recognizes the importance of changing economic conditions when modeling long-term behavior, and thus models the dynamic responses of producers and consumers to changing conditions of supply and price. System Dynamics extends the ability of static theory to capture real-world behavior. Producers react to changing conditions of resource availability by charging a higher or lower price. This response affects the quantity demanded (demand curve) and thus the usage rate. A change in usage of course changes the conditions of availability. This is a goal-seeking process which is the dynamic analog of the static concept of equilibrium. In the real world production and consumption are seldom precisely balanced, for the equilibrium price is continually changing as producers' costs change and potential demand rises exponentially. This focus on actual consumption and discovery, rather than on the abstract concepts of supply and demand curves, facilitates the evaluation of actual policies, for actual policies usually have as their objective some change in actual discovery or consumption.

Demand Multiplier

```
DM.K=TABHL(DMT,LOGN(AWP.K*1E5),1,8.5,.5)*(DNM.K)      24, A
DMT=2.1/1.59/1.21/.9/.69/.5/.24/.14/.067/.031/.014/  24.1, T
   5.5E-3/1.5E-3/2E-4/1.7E-5/0
      DM      - DEMAND MULTIPLIER (DIMENSIONLESS)
      TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
      DMT     - DEMAND MULTIPLIER TABLE
      LOGN    - NATURAL LOGARITHMIC FUNCTION
      AWP     - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)
      DNM     - DEMAND NOISE MULTIPLIER (DIMENSIONLESS)
```

The demand multiplier (Figure 16) reflects the consumer response in quantity demanded to changes in price. Data reflecting the demand response to price are only available in a very small range of price variation. Since price will undoubtedly rise in the future due to rising costs, no historical data exists to explain demand behavior at future values of price. However, the model is sensitive only to the assumption that the response of consumer demand is negative with respect to price rises, which is certainly the case. Changes in the individual values on the demand curve do

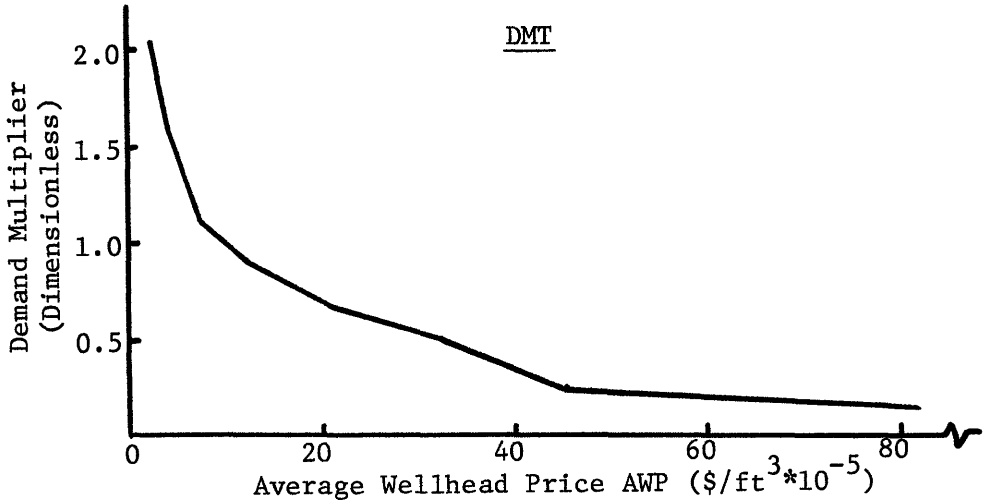


FIGURE 16: DEMAND MULTIPLIER TABLE

not affect the general behavior modes of the model or the various relative effects of the policies.

Usage Rate Potential

```
URP.K=URI*EXP(GC*(TIME.K-1900))                              25, A
GC=.0657                                                     25.1, C
URI=3.2E11                                                   25.2, C
TIME=1900                                                    25.3, N
    URP     - USAGE RATE POTENTIAL (CUBIC FEET/YEAR)
    URI     - USAGE RATE INITIAL (CUBIC FEET/YEAR)
    EXP     - EXPONENTIAL FUNCTION
    GC      - GROWTH CONSTANT (FRACTION)
    TIME    - TIME (YEARS)
```

The Potential Usage Rate represents the rate of use of natural gas each year that consumers would demand, if the price were held constant and the supply were unlimited. Up to the 1960's the natural gas usage rate has exhibited an average exponential growth in demand of 6.57 percent per

50) Equation #25 is DYNAMO notation for $URP(t) = URI*e^{(GC1)(t-1900)}$
51) M.K. Hubbert, Op. Cit. (1971), p. 97.

year, while other market forces such as a higher price or restrictions in supply will eventually limit the growth in actual usage rate. In reality usage is not free to respond to price changes. However, the delay in responding to price changes is so short as to be dynamically unimportant in this study.

Usage Supply Multiplier

```
USM.K=TABHL(USMT,RPR.K/DRPR,0,2,.2)*(UNM.K)               26, A
USMT=0/.12/.7/.86/.95/1/1.02/1.04/1.05/1.06/1.06          26.1, T
     USM     - USAGE SUPPLY MULTIPLIER (DIMENSIONLESS)
     TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
     USMT    - USAGE SUPPLY MULTIPLIER TABLE
     RPR     - RESERVE-PRODUCTION RATIO (YEARS)
     DRPR    - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
     UNM     - USAGE NOISE MULTIPLIER (DIMENSIONLESS)
```

The influence of the usage supply multiplier in the model reflects the tendency of producers to restrict the sale of reserves under regulation when relative supply (reserve-production ratio RPR) is low. For example, the Wall Street Journal (April 12, 1971) reports a waiting list of 17,000 residents of Chicago who have ordered gas but are unable to obtain it. Producers are reluctant to take on new customers when reserve-production ratio RPR is low, and will not sell additional gas to existing customers. The multiplier (Figure 17) is a function of relative coverage of reserves, or the reserve-production ratio RPR divided by the desired reserve-production ratio DRPR. It is assumed that when the reserve-production ratio is near the desired ratio, producers will not restrict supply, selling as much as is demanded at the current price. But as reserve-production ratio RPR drops, suppliers restrict the amount of gas they will sell at the regulated price, for reasons outlined earlier. E.W. Erickson estimated that in 1967,

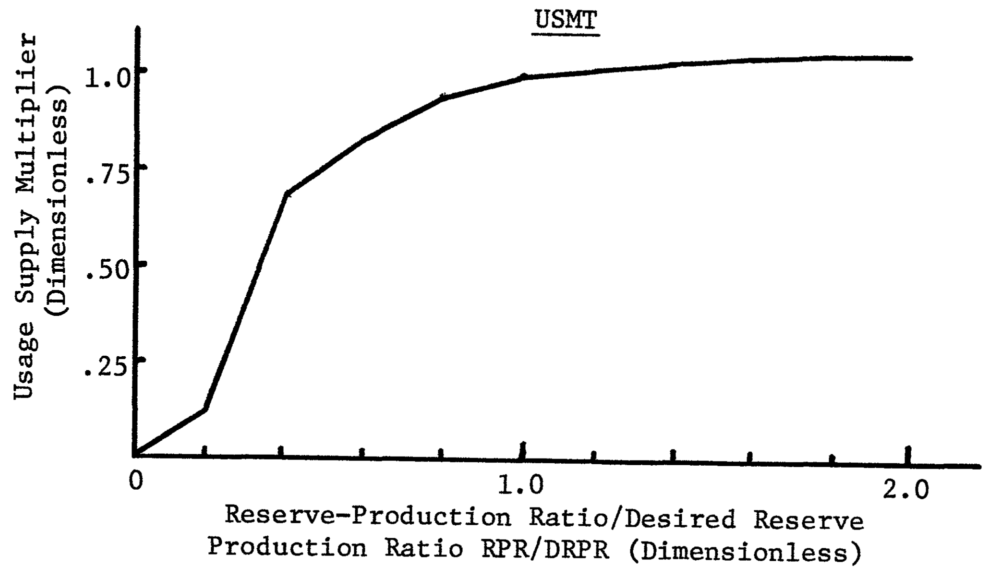


FIGURE 17: USAGE SUPPLY MULTIPLIER TABLE

the production rate was 10 to 25 percent below current demand.[52] This would correspond to the point indicated with an asterisk on Figure 17, since the reserve-production ratio RPR in 1967 was 16 years.[53] The desired reserve-production ratio DRPR represents simply the average producer's goal for his reserve-production ratio RPR. That goal certainly can vary from producer to producer: some are willing to tolerate 18 years, while others feel safer operating at 25 years. But because the standard contract length is 20 years, it is assumed in the model that the desired reserve-production ratio DRPR is 20.

52) E.W. Erickson, "Supply Response in a Regulated Industry: the Case of Natural Gas," Bell Journal of Economics and Management Science, Vol. 2, No. 1, (Spring, 1971), p. 120.
53) American Gas Association, Gas Facts, (1969), p. 11.

Usage Rate

```
UR.KL=SWITCH(UUR.K,RUR.K,SW1)                          27, R
    UR      - USAGE RATE (CUBIC FEET/YEAR)
    SWITCH  - FUNCTION WHOSE VALUE IS SET INITIALLY BY
                ANALYST
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    RUR     - REGULATED USAGE RATE (CUBIC FEET/YEAR)
    SW1     - REGULATION SWITCH
```

The usage rate UR is controlled by a switch function which permits the analyst to control whether response of usage rate to supply is through the usage supply multiplier USM (regulated case, SW1 = 1) or through changes in price P (unregulated case, SW1 = 0). In both the unregulated and regulated cases, producers must respond to changes in supply, represented by the reserve-production ratio RPR. When the industry is not regulated, producers raise price P when reserve-production ratio RPR is low. In the regulated case, producers cannot raise price, so they restrict the amount of gas they offer to the market through the usage supply multiplier USM. The usage rate UR is controlled by the same regulation switch SW1 as the price function.

Unregulated Usage Rate

```
UUR.K=(URP.K)(DM.K)                                    28, A
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    URP     - USAGE RATE POTENTIAL (CUBIC FEET/YEAR)
    DM      - DEMAND MULTIPLIER (DIMENSIONLESS)
```

The unregulated usage rate UUR is equal to the consumer demand for gas at any given price multiplied by an exponentially increasing function, usage rate potential URP, growing at 6.57 percent per year. The demand multiplier DM represents the response of consumers to a change in average wellhead price, ceteris paribus. It is assumed that this curve is constantly being shifted due to growth in population and per capita gas consumption. In the unregulated industry, the unregulated usage rate UUR is equal to the usage rate UR, and producers respond to changes in relative supply (reserve-production ratio RPR) through the price multiplier PM.

Regulated Usage Rate

```
RUR.K=CLIP((USM.K)(UUR.K),UUR.K,TIME.K,1960)            29, A
    RUR     - REGULATED USAGE RATE (CUBIC FEET/YEAR)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    USM     - USAGE SUPPLY MULTIPLIER (DIMENSIONLESS)
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    TIME    - TIME (YEARS)
```

When price is regulated, supply and demand are no longer in equilibrium at the established price. It is no longer true that consumers can buy as much gas as they would demand at the now regulated average wellhead price RAWP. Instead, when the average wellhead price AWP is set to be lower than its equilibrium value, producers will satisfy only a fraction of the potential demand. The fraction actually supplied will depend on their relative coverage, or reserve-production ratio RPR/desired reserve-production ratio DRPR, through the usage supply multiplier USM. In the case of the natural gas industry, regulation had little effect on the average wellhead price AWP until 1960, although the Supreme Court decision took place in 1954. Thus the regulated usage rate UREG is expressed through a switching function, activated in 1960, whose value is equal to unregulated usage rate UUR before 1960, and unregulated usage rate UUR multiplied by the usage supply multiplier USM after 1960.

CHAPTER IV

INCORPORATION OF EMPIRICAL DATA

The preceding chapter has presented a theoretical model of the U.S. natural gas industry derived from a survey of empirical literature and conversations with industry policy makers. The model does not contain elaborate detail, but is an approximation of the real world composed of those relationships necessary and just sufficient to explain the major determinants of long-term discovery and sales. Chapter III gave a verbal description and defense of the assumed system structure, and illustrated the shape of the nonlinear relationships or table functions in the model. In this chapter statistical inference techniques are employed to incorporated empirical data into the model. The first section examines the applicability of statistical techniques to System Dynamics models. The second section describes the parameterization of the natural gas model. The third section examines the behavior of the model compared to real-world time series to provide some measure of model validity.

The Use of Linear Statistical Methods for Parameter Estimation in System Dynamics Models

System Dynamics is a technique of modeling which has been specifically developed for analysis of nonlinear, multi-feedback loop systems too complex for the derivation of general analytical solutions. Most System Dynamics models adopt a long time horizon which generally means that one or more variables will be related nonlinearly over their range of behavior. It is these nonlinear relationships (generally expressed as table functions in

System Dynamics models) which are the most difficult to estimate, for they represent a time-independent theory of a relationship between variables which often must be extracted from time-dependent data. Yet it is the nonlinear hypotheses which are most in need of empirical data, for they are extremely difficult to estimate without some empirical basis. This chapter will therefore focus on the estimation of nonlinear relationships or table functions using linear, statistical techniques. Problems arising due to nonlinearities and feedback-loop structures will be addressed separately.

Nonlinearities

To use linear statistical methods in estimating nonlinear table functions in System Dynamics models, it is first necessary to transform the hypothesized relationships into a linear form. Often it is possible to approximate the nonlinear relationship as linear in a specific range of available data. When this is an unreasonable approximation to make, there exists a number of useful transformations which convert nonlinear relationships into linear form, thus extending the applicability of linear regression techniques.[54] When such a transformation is possible, the relationship transformed is termed intrinsically linear.

Linear regression analysis is therefore applicable only for linear or intrinsically linear systems. When making linear assumptions about nonlinear relationships in System Dynamics models, two problems may occur. First, many nonlinear forms exist which cannot be linearized, and are

54) See, for example, Draper and Smith, Applied Regression Analysis, John Wiley & Sons, New York, (1966), p. 131.

not intrinsically linear. Second, when one makes a linear assumption about a relationship in a certain region of variation of the variables, it is important to realize that the variance of the estimation increases proportionately with the square of the distance from the mean of the data. This implies that extrapolation of the relationship outside the range of behavior can be extremely inaccurate.

Feedback-Loop Structure

The feedback loop structure of System Dynamics models provides their most limiting characteristic in any attempt to employ linear statistical methods for estimation. The existence of a feedback loop implies that two or more variables are interdependent--that, for instance, Y depends on X and X depends on Y. This implies that the system can be represented econometrically by a set of simultaneous equations.

When simultaneity exists, the literature on regression analysis[55] warns that the use of Ordinary Least Squares regression techniques can give biased estimates of the regression coefficients. This means that the relationship between two variables as inferred from time series data by OLS regression is different from the actual time-independent relationship which exists in the real world. Since we are after the real-world relationship and not simply that inferred through time series data, we must be wary of OLS regression results in feedback-loop systems.

What can be done to recover the real-world relationships when normal regression techniques yield biased results? Johnston [Johnston (1963)]

55) See, for example, J. Johnston, Op. Cit. (1963), p. 233.

proposes more sophisticated methods of regression analysis aimed at purging the regression coefficients of bias due to simultaneous equations--Indirect Least Squares and Two-Stage Least Squares (2SLS) are two of the most frequently used techniques. These techniques can only be used in certain equation systems, however. The condition under which any advanced regression can be used to analyze a specific equation is called the identification condition.[56] The conditions of identification relate the number of exogenous and endogenous variables included in an equation. An equation is said to be exactly identified if the number of exogenous variables excluded from the individual equation is equal to the number of endogenous variables included less one. An equation is overidentified if the number of excluded exogenous variables is greater than the number of included endogenous variables less one, and underidentified if the opposite is true. Indirect Least Squares can only be used for exactly identified relations, and Two-Stage Least Squares for either overidentified or exactly identified relationships.

What are the implications of the identification condition in attempting to use statistical methods in System Dynamics models? Roughly, the conditions of identifiability state that the more exogenous variables a system has, the more likely that the system is exactly or over-identified, thus allowing one to use advanced regression techniques. However, the long time horizons of most System Dynamics models tends to reduce the number of variables which can be considered exogenous. It is generally true therefore that many of the relationships in System Dynamics models will be

56) See F.M. Fisher, The Identification Problem in Econometrics, McGraw-Hill Book Co., N.Y., (1966).

underidentified and thus, strictly speaking, not susceptable to the more advanced techniques of recovery from bias due to simultaneous equations.

This last statement must be qualified, however, because the issue of identifiability should not be viewed as a dichotomous situation. To quote F.M. Fisher, "it is quite possible to define a concept of 'near identifiability'".[57] In terms of System Dynamics models, a relationship which is strictly underidentified can be forced to be identified if some endogenous variables are assumed exogenous. For example, most econometricians assume any delayed or lagged endogenous variable an exogenous variable. Fisher warns that this practice is a dangerous one if used as a rule-of-thumb procedure to escape the identification problem, for it can still lead to biased regression coefficients.[58] Yet in many cases it is possible to assume that in the short term, certain variables are exogenous in estimating relationships in System Dynamics models, and to obtain a reasonable estimate from the process. It is important, however, to recognize the assumptions one makes in obtaining those estimates, and their possible consequences on the validity of the regression coefficient.

How is it possible to determine the reasonableness of the assumption that a specific variable or set of variables is exogenous? An important objective is to measure the bias which one introduces in assuming that certain endogenous variables are exogenous. The bias can be approximated by simulation of System Dynamics models. One can hypothesize or measure by regression on time series data the relationship which would occur if the

57) F.M. Fisher, "Simultaneous Equations Estimation - The State of the Art," IDA Economic Papers (July, 1970), IDA, 400 Army-Navy Drive, Arlington, Va. 22202, p. 41.
58) Ibid., p. 76.

variables could be considered exogenous. The appropriate model equations are then altered to reflect this relationship, and the model is simulated to create time series data over the same period as the real-world data. The difference between the coefficients inferred statistically from simulation time series (where no endogenous variables are assumed exogenous) and that from real-world time series gives a measure of the bias due to the assumption that the variables are exogenous.[59]

This method is similar to the econometric technique known as synthetic data analysis, where in place of the System Dynamics model one uses a set of simultaneous equations to estimate the bias.[60] Thus, simulation gives us a method to check the validity of assumptions made when applying linear regression techniques to System Dynamics models. This estimation procedure was applied to the table functions of the natural gas model, and the results are described in the following section.

Parameterization of the Natural Gas Model

The preceding section described a procedure for incorporation of empirical data into System Dynamics table functions. This procedure can be outlined as follows:

1. Test identification of relationship (difference between the number of exogenous variables excluded to the number of endogenous variables included in the equation). If exactly or over-identified, proceed to regression.

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59) This measure is in reality only an approximation of the bias, for it assumes all other relationships and the structure of the simulation model are correct.
60) See, for example, J. Johnston, Op. Cit. (1960), pp. 275-295.

2. If relationship is underidentified, assume some variables exogenous, and proceed to regression.[61]

3. Incorporate regression results in table function coefficients.

4. Test for bias due to simultaneous equations by creating time series data with the model, and performing the same regression on the model data as performed in step 2.

If the results of the regression on synthetic data are the same as those obtained from real-world data, then the assumption that some variables are exogenous did not necessarily add significant bias to the results. If the two regression results are different, then different variables must be assumed exogenous, and the estimating procedure repeated. It may be impossible, however, to obtain an unbiased estimate from real-world data. In that case, one can use trial and error simulation to obtain the table function which best recreates time series data (without, however, obtaining any quantitative measure of confidence in the results). The following section describes the estimation of the six table functions in the model, following the procedure outlined above. The tests for bias were performed by creating time series from the model run with a normally distributed random error added to cost of exploration COE, percent invested in exploration PIIE, price multiplier PM, demand multiplier DM, and usage supply multiplier USM. This test checks the stability of the relationships in the presence of noise in the system, and provides a measure of bias when one compares the actual relationship in the model to the relationship obtained by OLS regression on the synthetic data. The noise-generating

61) See [Fisher (1970)] for a discussion of the choice of exogenous or "instrumental" variables.

function is assumed multiplicative, and takes the form:

X.K = SAMPLE (NORMRN(MEAN, DEV), SMPLI, INVAL)

where SAMPLE is a sample-and-hold function with initial value INVAL, sample interval SMPLI, and value NORMRN. NORMRN generates random numbers normally distributed with mean MEAN and standard deviation DEV.

Cost of Exploration

Real data on the cost of exploration is not generally available, but can be derived from available data on investment in exploration and on gas discovery rate. The following approximate relationship was used to generate the data:

$$COE_t = \frac{IIE_t}{DR_{t+5}}$$

The function is approximate because the lagged five-year delay is really a third-order delay with average delay time equal to 4.5 years. The cost of exploration is assumed in the model to be a function of the fraction of unproven reserves remaining FURR. The initial amount of unproven reserves UPRI is assumed to be 1,040 X 10^{12} ft^3 of gas for the continental U.S., so data on the fraction of unproven reserves can be derived from the formula:

$$FURR_t = \frac{UPRI - (\sum_{t=0}^{t} DR_t)}{UPRI}$$

The derivation of cost of exploration data can be found in Appendix II. The nonlinear relationship for cost of exploration assumed in Chapter III can be approximated as:

$$COE_t = \beta_o' (FURR_t)^{\beta_1} (\tilde{\epsilon}_t')$$

which transforms to the linear relationship:

$$\ln (COE_t) = \beta_o + \beta_1 \ln (FURR_t) + \tilde{\epsilon}_t$$

This equation is exactly identified. This implies that advanced statistical techniques could be used to estimate the relationship, but as a first approximation in each table function, it is useful to test the validity of Ordinary Least Squares. This would involve the assumption that the fraction of unproven reserves remaining is exogenous. The resulting OLS regression gives:

$$\ln\ (COE_t) = \underset{(.96)}{3.9} - \underset{(.52)}{1.6}\ \ln\ (10 * FURR_t)$$ [62]

$$R^2 = 51 \qquad SER = 0.26$$

The results are significant at the 5 percent level ($F^1_q = 9.35$). Both β_o and β_1 are significant at the 1 percent level. Most of the observed variance is due to earlier values of cost of exploration COE, which aren't as reliable as current data because the industry only began accurate reporting of cost in 1959.

When the above function derived from historical data is entered into the model as the cost of exploration table function COET, the average of three regressions on synthetic data created by the model for the same time period is:

$$\ln\ (COE_t) = 3.73 - 1.53\ \ln\ (10 * FURR_t)$$

Details on the synthetic data analyses are given in Appendix III.

Figure 18 gives a graphic comparison of the results of the synthetic data analysis performed on the cost of exploration table COET. The solid line is a linear approximation of the relationship obtained from OLS regression on historical data, and is the relationship assumed in the

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62) In cases where variables range below one, it is best to multiply the data by 10 to avoid the behavior of the log function in that region. This transformation does not affect the coefficients--only the constant term.

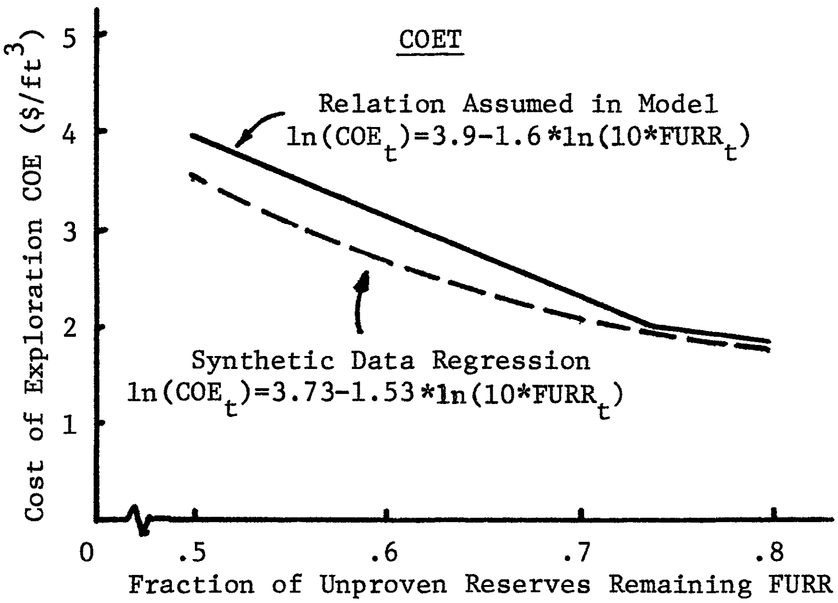

FIGURE 18: RESULTS OF SYNTHETIC DATA ANALYSIS OF THE COST OF EXPLORATION TABLE COET

model over the range of FURR from .5 to .8. The dotted line represents the average of regressions on three sets of synthetic data, the results of which are reported in Appendix III. The synthetic data regressions seem to converge satisfactorily on the model relationship, giving evidence that the assumption that the fraction of unproven reserves remaining FURR is exogenous adds little bias to the relationship.

Price Multiplier

The price multiplier table PMT relates the ratio of new contract price NCP divided by total cost TC to the reserve-production ratio RPR:

$$PM_t = \frac{NCP_t}{TC_t} = f(RPR_t)$$

This relationship is valid only until regulation occurs. After regulation is enforced, the price margin is exogenously determined by the FPC, and is no longer free to be set by producers in response to relative supply. Since no reliable data could be obtained for new contract price NCP before regulation, the analysis was carried out using average wellhead price AWP. When using average wellhead price AWP, the relevant total cost is that which occurred approximately ten years ago, TC_{t-10}.[63] The equation expressing the general form of the relationship hypothesized for the price multiplier PM is:

$$\ln\,(AWP_t/COE_{t-p_{10}}) = \beta_o + \beta_1 \ln\,(RPR_t) + \tilde{\varepsilon}_t$$

This equation is underidentified, for all three variables included in the equation are endogenous, and the system contains only one exogenous variable. In order to perform an OLS regression, cost of exploration COE_{t-10} and reserve-production ratio RPR must be assumed exogenous. No time series data on cost of exploration exist for the 1950's, the period before regulation. Yet a regression can be done using the predicted value of cost of exploration COE from simulated time series. This procedure is similar to Two-Stage Least Squares, where one uses predicted values of endogenous variables to purge the equation of simultaneity bias. The assumption that the reserve-production ratio RPR is exogenous may introduce bias--this can be tested by synthetic data analysis.

The historical data on the price multiplier PM is given in Appendix II. Arguments advanced in Chapter III implied that producers will charge an average wellhead price equal to smoothed total cost (TC_{t-10}) if the reserve-production ratio is large. Thus the price multiplier should

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63) This delay can be approximated by Proven Reserves/(Discovery Rate + Usage Rate). From 1950-1960, PR=24, DR=15, UR=9. Therefore delay = 10 years.

approach 1.0 as the reserve-production ratio rises above the desired level of twenty years. This theory is well supported by the data--Figure 19 shows a plot of ln (AWP_t/COE_{t-10}) versus ln $(RPR_t/10)$.

A regression on historical data gives highly significant results--the regression is significant at the 1 percent level. It can be seen from

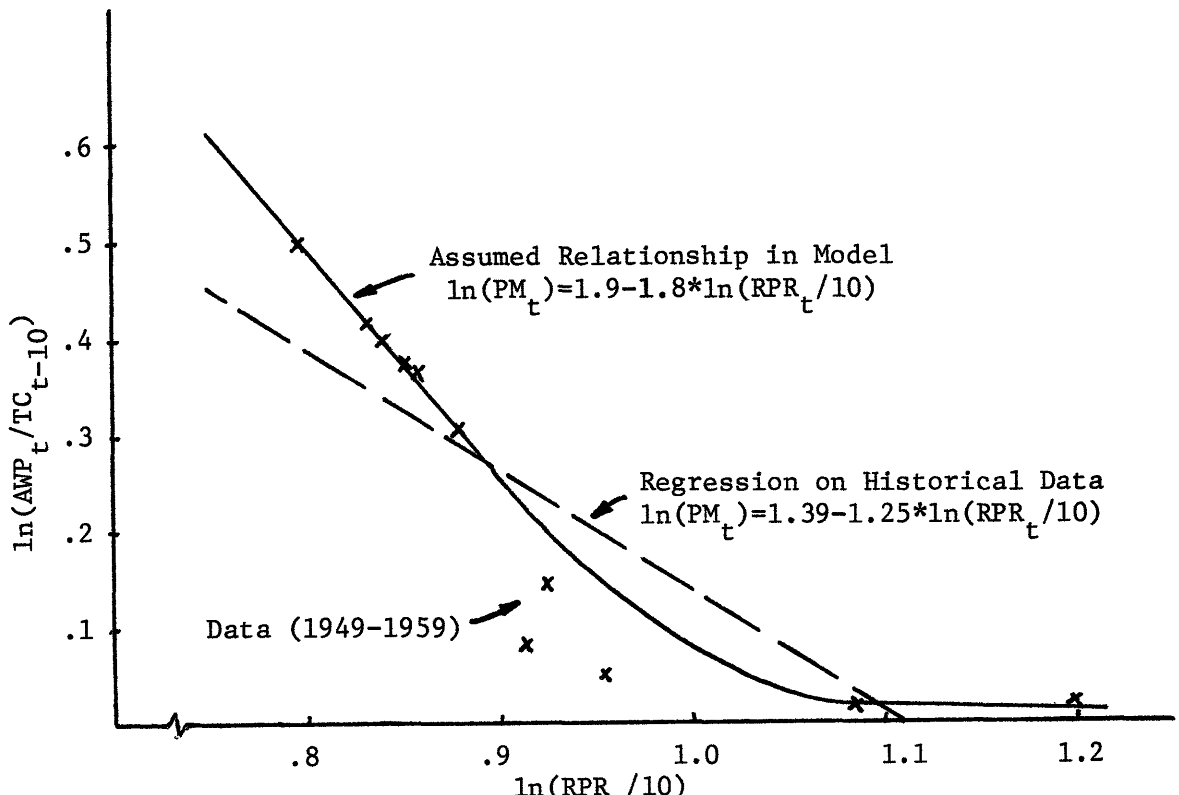


FIGURE 19: DATA AND REGRESSION RESULTS FOR PRICE MULTIPLIER TABLE (log-linear scale)

Figure 19 that the data do not conform well to a long-linear hypothesis over their entire range. A better fit to data is obtained by breaking the data into two parts: above a reserve-production ratio of 24 years [ln (RPR/10) = .9], the model relationship approaches 1.0 asymptotically. Below this point, historical data leads to the relationship:

$$\ln(PM_t) = \underset{(.142)}{1.92} - \underset{(.168)}{1.82} \ln(RPR_t/10)$$

$$R^2 = .97 \qquad SER = .0138$$

The relationship indicated by the solid line in Figure 18 was incorporated in the price multiplier table function PMT.

When the model is used to create synthetic time series data from 1949-1959, reserve-production ratio stays relatively constant around twenty years, while the real-world data decreases from thirty-three to twenty years. This is due to the fact that the model assumes investment in exploration IIE will decrease if reserve-production ratio RPP exceeds the desired value DRPR, while in actuality this was not the case through 1950. Much of the gas found from 1900-1950 was found while looking for oil, and thus the actual investment in exploration for gas was higher than planned, if one allocates investment according to the resource found (gas or oil). The model assumes investment in gas can be made idependently from oil, and so the normal mode of industry behavior is to hold reserve-production ratio RPR constant near twenty years. For reasons stated earlier, this assumption is becoming more valid as firms explore and drill selectively for gas.

This discrepancy makes it difficult to measure the bias due to the assumption that cost of exploration and reserve-production ratio are exogenous. Instead of using data from the period 1959-1969, a later period was chosen where the reserve-production ratio RPR did vary in the simulation run (see Appendix III). Figure 20 shows the results of the synthetic data analysis performed on the price multiplier table PMT. The solid line is the linear approximation of the relationship from Figure 19 contained in the model. The dotted line is the average of three regre-

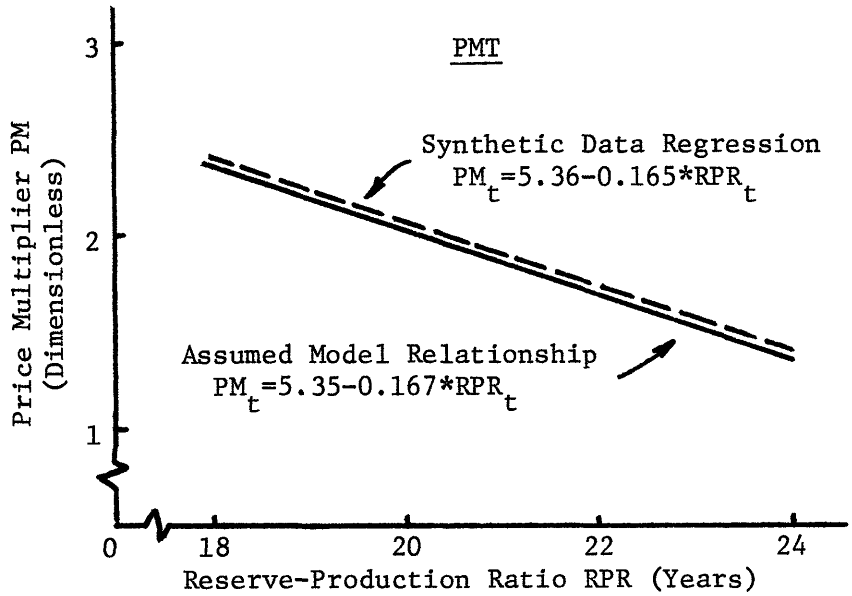


FIGURE 20: RESULTS OF SYNTHETIC DATA

ANALYSIS OF THE PRICE MULTIPLIER TABLE PMT

ssion on synthetic data which clearly converges on the relationship assumed in the model. This implies that the OLS assumptions of exogenous variables did not significantly bias the estimate.

Demand Multiplier

The unregulated usage rate UUR is a function of average wellhead price AWP and an exogenous function of time which represents exponential increases in population and per-capita demand for gas. The regression equation is then:

$$\ln (UUR_t) = \beta_o + \beta_1 \ln (AWP_t) + \beta_2(t) + \tilde{\epsilon}_t$$

This equation is underidentified, for it includes two endogenous variables and no exogenous variables are excluded. The simplest procedure is again Ordinary Least Square--average wellhead price AWP is assumed exogenous.

The regression was performed on 1945-1959 data for average wellhead price AWP and usage rate UR, presented in Appendix II. The regression yielded:

$$\ln(UUR_t) = \underset{(.275)}{2.46} - \underset{(.143)}{(.556)} \ln(AWP_t) + \underset{(.0074)}{(.11)}\, t$$

$$R^2 = .99 \qquad SER = .044$$

This regression yielded highly significant results ($F_2^{12} = 507$). The exponential growth factor is higher than the average with no price effects (11 percent versus 7 percent), because the reaction of consumer demand to a rising price tends to decrease the observed growth in consumption. The

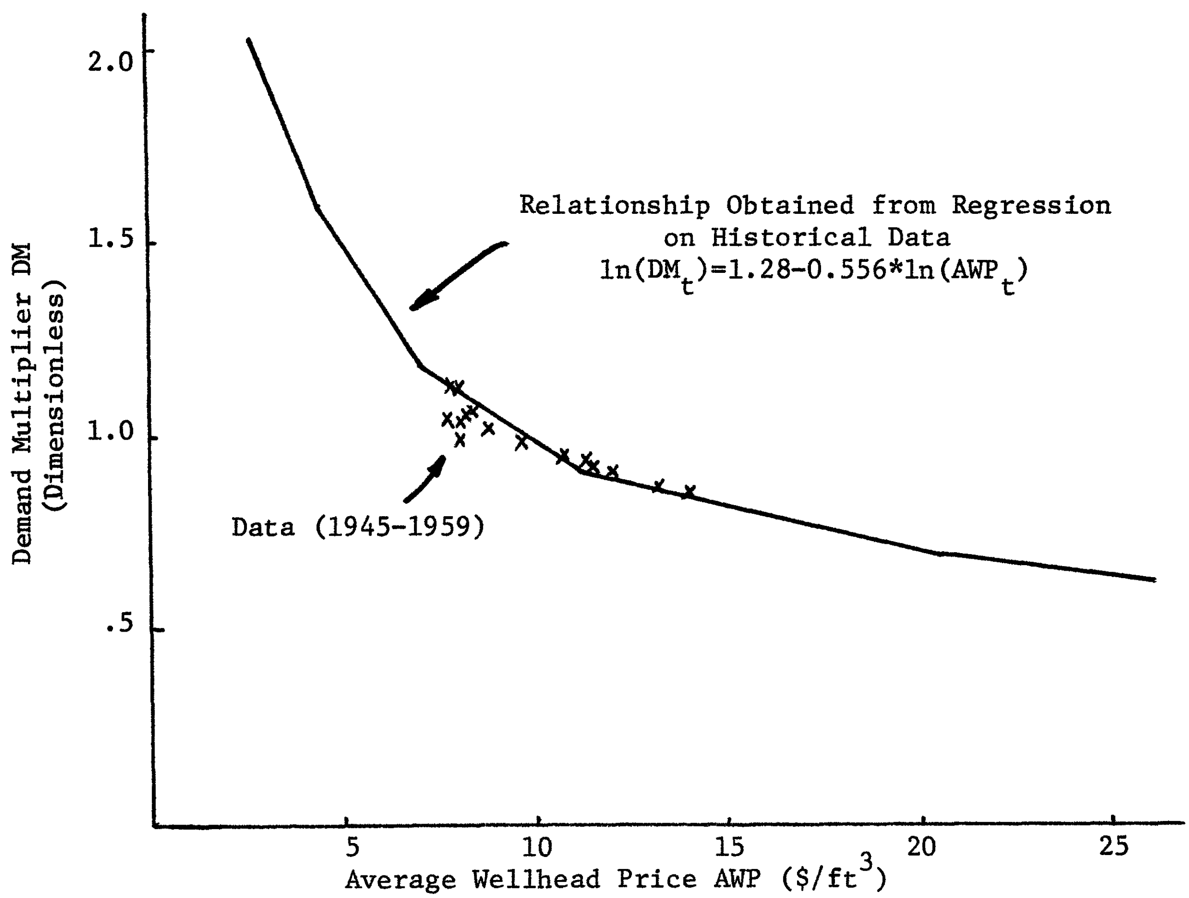


FIGURE 21: DEMAND MULTIPLIER TABLE DMT AND HISTORICAL DATA

price coefficient is negative and relatively inelastic, which is to be expected when prices are so low. The demand multiplier table DMT was obtained from the above relationship by normalizing its value to 1.0 at an average wellhead price AWP of 10¢/Mcf. The relative position of the curve can then be changed by varying the initial value of usage rate URI. The relationship between demand and price resulting from this regression is shown in Figure 21. The data plotted were obtained using an 11 percent growth in potential usage rate. In running the model, however, a 7 percent rate was used in order to be conservative about future growth in usage. The sensitivity of the model to growth in potential usage rate is tested in Chapter V.

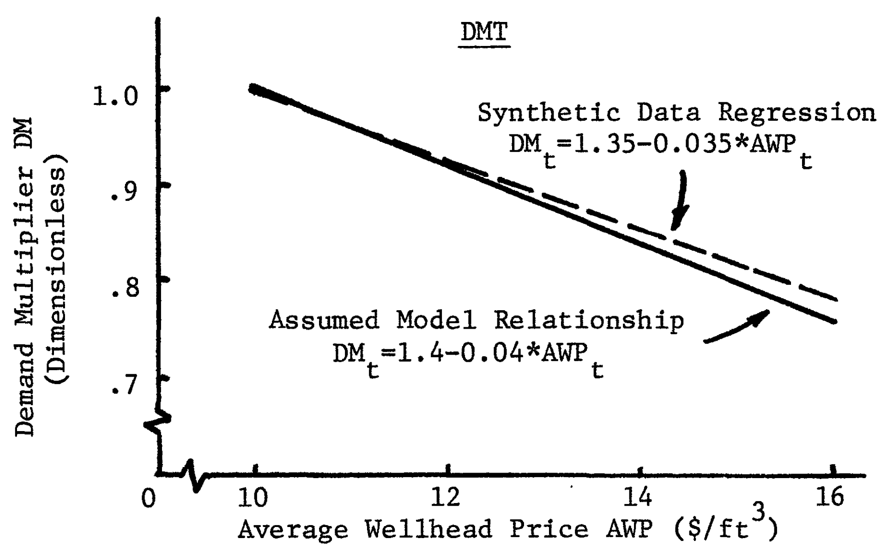


FIGURE 22: RESULTS OF SYNTHETIC DATA ANALYSIS ON THE DEMAND MULTIPLIER TABLE DMT

In order to test for bias, the model was run with stochastic noise elements, and synthetic data from 1945-1959 was obtained. When regressions are run on the synthetic data, the resulting relationship converges on the assumed demand multiplier relationship DMT during that period (Figure 22) indicating little bias due to OLS assumptions.

It is important to note how limited this information acquired from time series data really is. Price of gas is continually rising, and this guarantees that future behavior will be outside the range of available data. The behavior of the demand multiplier at higher prices is a hypothesis about future consumer tastes—it is assumed in the model that demand becomes more elastic as price rises. Although this hypothesis is critical in pinpointing future values of usage rate or proven reserves, the exact determination of the demand multiplier DM does not affect the relative effects of policies on the behavior modes of the system.

Usage Supply Multiplier

The usage supply multiplier USM incorporates into the model the mechanism by which producers ration the dwindling supplies of gas when price is regulated. When there is a free market, producers raise the new contract price of gas in response to shortages (measured by reserve-production ratio RPR). With regulation, producers must respond by limiting the quantity of gas sold. The usage supply multiplier thus restricts usage rate as the reserve-production ratio drops below the desired ratio, twenty years.

Data for the usage supply multiplier are derived in Appendix II. To obtain an estimate of this effect, it was assumed that price is held constant from 1960 - 1970, and potential usage rate rises at 6.6 percent per year. An Ordinary Least Squares Regression can be performed if the reserve-production ratio RPR is assumed exogenous. The results are:

$$\ln(USM_t) = -\ 0.653 + 0.213\ \ln(RPR_t)$$
$$\qquad\qquad (.107) \quad (.038)$$

$$R^2 = .78 \qquad SER = .020$$

The relationship is significant at the 1 percent level ($F_1^9 = 32$). When a synthetic data analysis is performed on the usage supply multiplier table USMT, there appears to be an upward bias on the values of the usage supply multiplier at lower values of the reserve-production ratio RPR (Figure 23; RPR = 12-14 years). In order to correct for this, the model

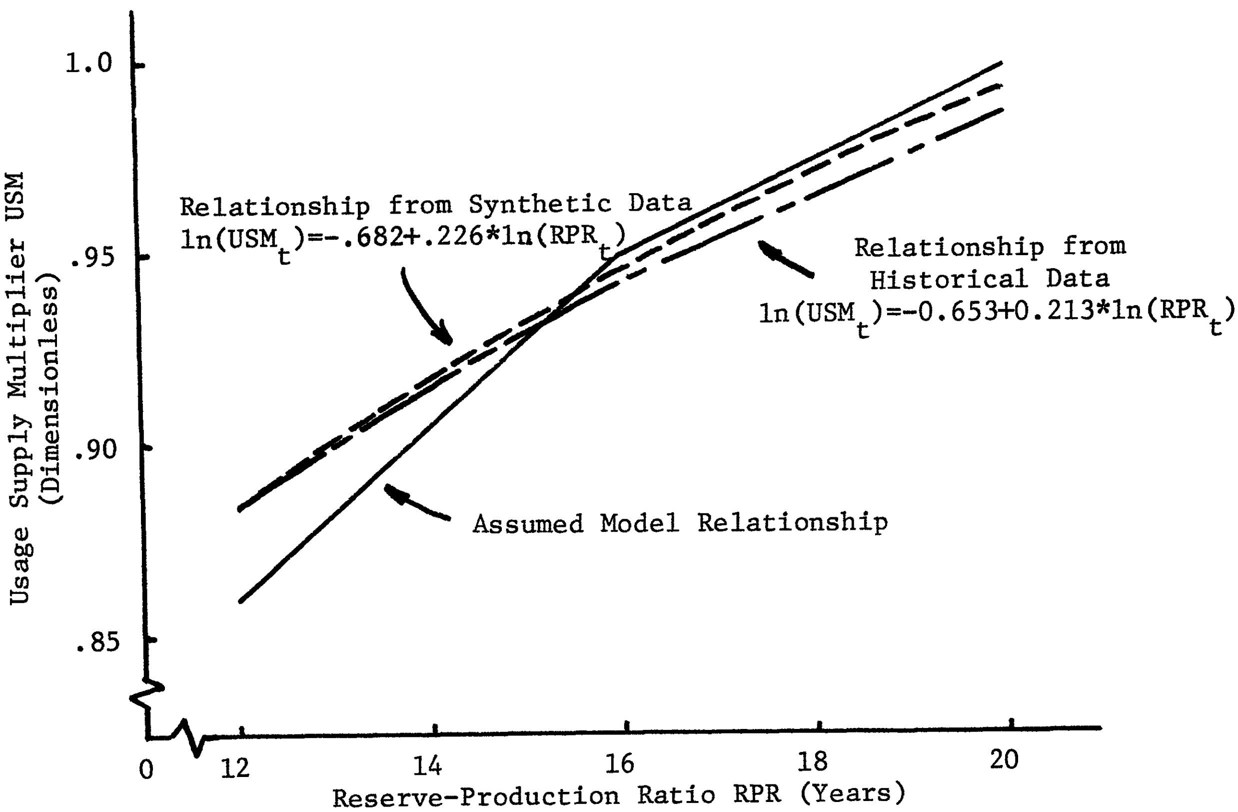


FIGURE 23: RESULTS OF SYNTHETIC DATA ANALYSIS OF THE USAGE SUPPLY MULTIPLIER TABLE USMT

relationship (solid line) was adjusted downward in that region of reserve-production ratio RPR. With the assumed model relationship, regressions on three sets of synthetic data yield the relationship:

$$\ln (USM_t) = - 0.682 + 0.226 \ln (RPR_t)$$

which is very close to that obtained from historical data, indicating the existing bias has been taken into account in the assumed model relationship. Thus the assumed model relationship is a closer approximation to the real-world relationship than that observed from historical data.

Percent Invested in Exploration

The amount of investment in exploration in any given year is defined to be the sales revenue SR for that year multiplied by the percent invested in exploration PIIE. Percent invested in exploration is assumed to be a function of return on investment (measured by the ratio of new contract price to total cost NCP/TC) and relative supply (measured by the reserve-production ratio RPR).

This formulation will recreate time series data only in a period where there is little interaction between exploration in the gas and the oil industries. Carryover effects, chance discoveries of gas when searching for oil, have been small since the 1950's, so the data used for estimation of the percent invested in exploration function PIIE are taken from the period 1955-1967 (see Appendix II).

A regression equation in the form of the hypothesized relationship between percent invested in exploration PIIE, reserve-production ratio RPR, and return on investment (measured by new contract price to total cost NCP/TC) is given by:

$$\ln (PIIE_t) = \beta_o + \beta_1 \ln (RPR_t) + \beta_2 \ln (NCP_t/TC_t) + \varepsilon_t$$

This relationship is underidentified, for it includes four endogenous variables and excludes only one exogenous variable, potential usage rate URP. The first approximation tested was the assumption that reserver-production ratio RPR and new contract price over total cost NCP/TC are exogenous. With that assumption, Ordinary Least Squares is appropriate.

The OLS analysis was performed, and the regression gave results significant at the 5 percent level ($F_2^8 = 5.07$). Further analysis of the regression reveals a number of problems, however. The coefficient of new contract price over total cost NCP/TC is negative and not significantly different from zero, and the coefficient of the reserve-production ratio RPR is positive. The latter result would imply that as reserve-production ratio declines, producers would tend to decrease their investment in exploration, given a constant return on investment. The regression results are as follows:

$$\ln (10 * PIIE_t) = \underset{(1.49)}{-(2.21)} + \underset{(.73)}{(1.14)} \ln (RPR_t) \underset{(.346)}{-(.022)} \ln (10 * NCP_t/TC_t)$$

$$R^2 = .56 \qquad SER = .14$$

An examination of the variance-covariance matrix shows that the two independent variables, reserve-production ratio RPR and new contract price over total cost NCP/TC, are highly correlated (correlation coefficient = .89). This implies that the variables are in fact <u>not</u> independent of each other. This is not surprising, given the circumstances of the period 1955-1967. In that period, both reserve-production ratio RPR and the ratio or new contract price to total cost NCP/TC decrease due to rising usage rate. Usage rate decreases reserve-production ratio directly, and decreases the ratio of new contract price to total cost by increasing sales revenue, which stimulates discoveries, and causes total cost to rise due to rising cost of exploration. Thus one can expect the reserve-production ratio RPR to be correlated with new contract price over total cost NCP/TC, as is shown in Figure 24.

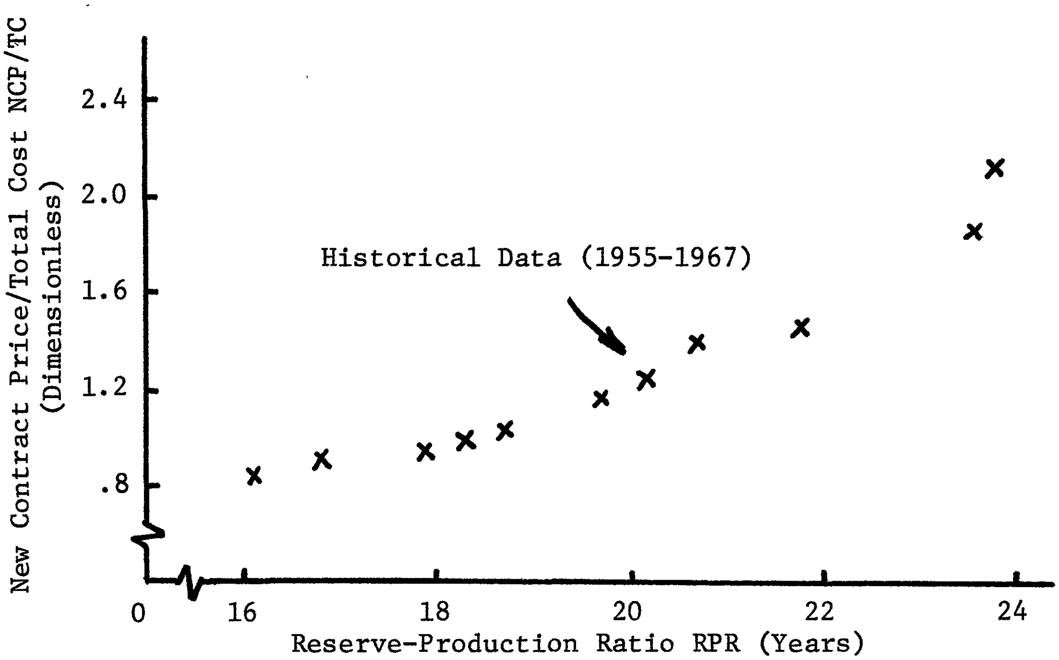


FIGURE 24: CORRELATION OF NEW CONTRACT PRICE/TOTAL COST WITH RESERVE-PRODUCTION RATIO RPR, 1955-1967

A second regression on historical data using two less data points reveals the effects of multicollinearity on OLS regressions. The second regression is still significant, yet both coefficients have changed drastically - the coefficient of reserve-production ratio RPR is now negative and about three times as large, and the coefficient of new contract price to total cost NCP/TC is positive, significant, and two orders of magnitude larger than before. The results of the second regression are as follows:

$$\ln (10 * PIIE_t) = \underset{(2.46)}{5.44} - \underset{(1.35)}{(3.27)} \ln (RPR_t) + \underset{(0.66)}{(2.20)} \ln (10 * NCP_t/TC_t)$$

$$R^2 = .78 \qquad SER = .093$$

This phenomenon is not unusual for regressions whose independent variables are highly collinear - the coefficients become highly unstable, making it impossible to determine the real-world relationship from OLS. This second regression reveals a second problem: the Durbin-Watson statistic is 3.2, indicating autocorrelation, a symptom of bias due to simultaneous equations.

It is clear that Ordinary Least Squares gives unsatisfactory estimates for the relationship determining percent invested in exploration PIIE. Conceivably it may be possible to make other less naive assumptions of exogenous variables, and perform a more sophisticated analysis using Two-Stage Least Squares, or some other higher-order regression technique. Instead of this approach, the relationship was estimated using simulation. The two table functions were hypothesized from the information presented in Chapter III, and then the model was used to create synthetic time series data corresponding to the same period as the real-world data, 1955-1967. By iterative changes in the two table functions, the synthetic data created by the model can be made to fit real-world time series data as shown in Figure 25. The final table functions are defined in Chapter III.

The fit to data relating percent invested in exploration PIIE to new contract price over total cost NCP/TC shown in Figure 25a is quite satisfactory. The fit of data relating percent invested in exploration PIIE to reserve-production ratio RPR seems to be shifted to the left (Figure 25b). This can be explained by the fact that the model does not include carry-over effects between the oil and the gas industry. Those effects caused

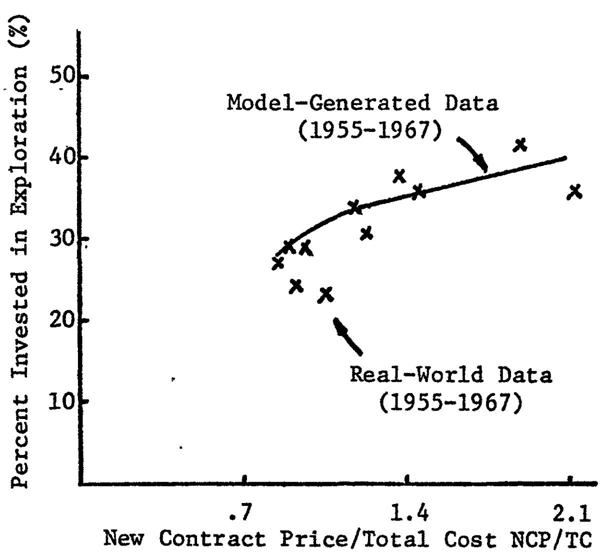


Figure 25a

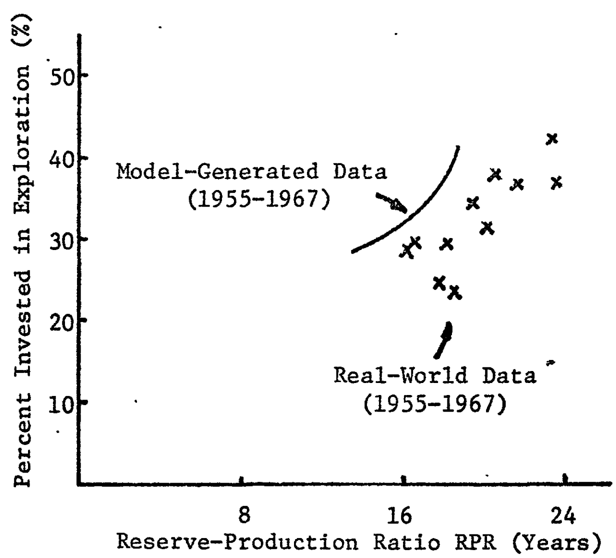


Figure 25b

FIGURE 25: REAL-WORLD AND MODEL-GENERATED DATA RELATING PERCENT INVESTED IN EXPLORATION PIIE, NEW CONTRACT PRICE/TOTAL COST NCP/TC, AND RESERVE-PRODUCTION RATIO RPR

the reserve-production ratio RPR to be high before 1950. Although the model data and real-world data converge at lower values of reserve-production ratio RPR, earlier values of reserve-production ratio RPR from model data are lower than real-world data, for the model tends to stabilize reserve-production ratio RPR at twenty years.

An examination of the results of OLS regressions performed on synthetic data gives further support for the hypothesized relationships. A regression on time series created by the model without noise inputs (data graphed in Figure 25) reveals substantial bias due to the feedback effects of the system -- the regression on synthetic data shows a positive relationship between investment and reserve-production ratio RPR, while the underlying relationship assumed in the model is negative:

$$\ln (10 * PIIE_t) = - \underset{(.223)}{0.791} + \underset{(.131)}{0.558} \ln (RPR_t) + \underset{(.064)}{0.169} \ln (10 * NCP_t/TC_t)$$

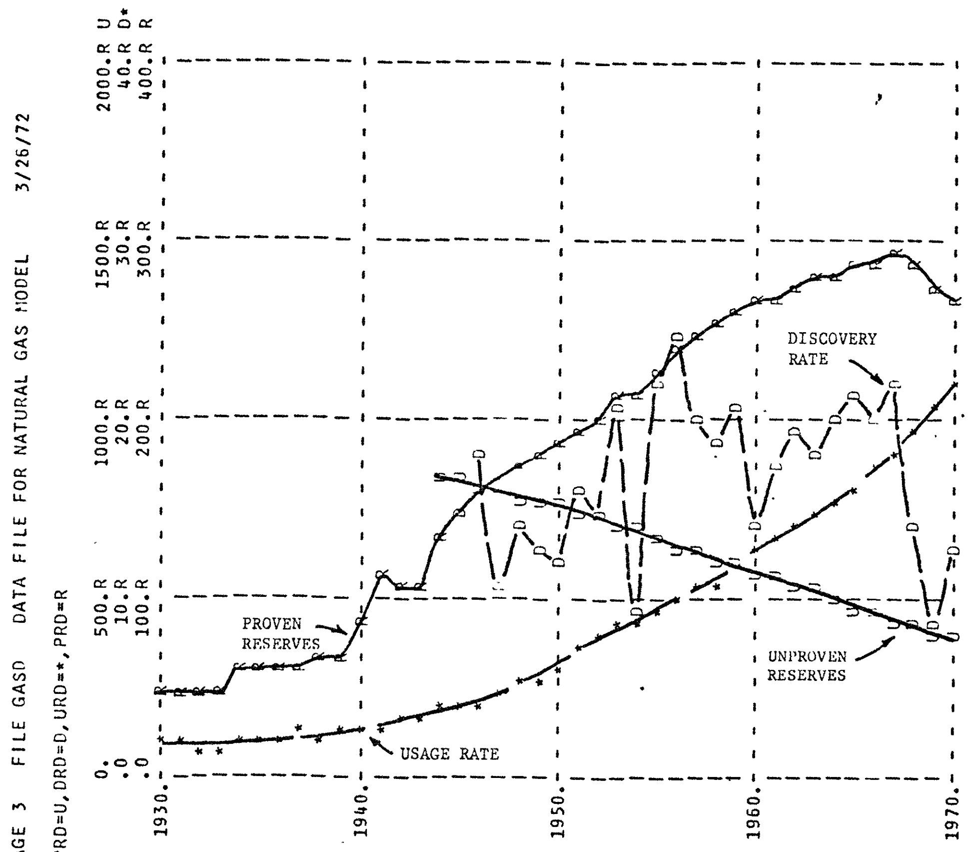
$$R^2 = .99 \qquad SER = .01$$

One cause of this phenomenon is the feedback between percent invested in exploration PIIE and reserve-production ratio RPR--as investment goes down, discoveries decrease, lowering the reserve-production ratio RPR. Another contributor to this effect is the exponentially increasing usage rate UR, which tends to decrease the reserve-production ratio RPR exogenously during this period when investment is also decreasing.

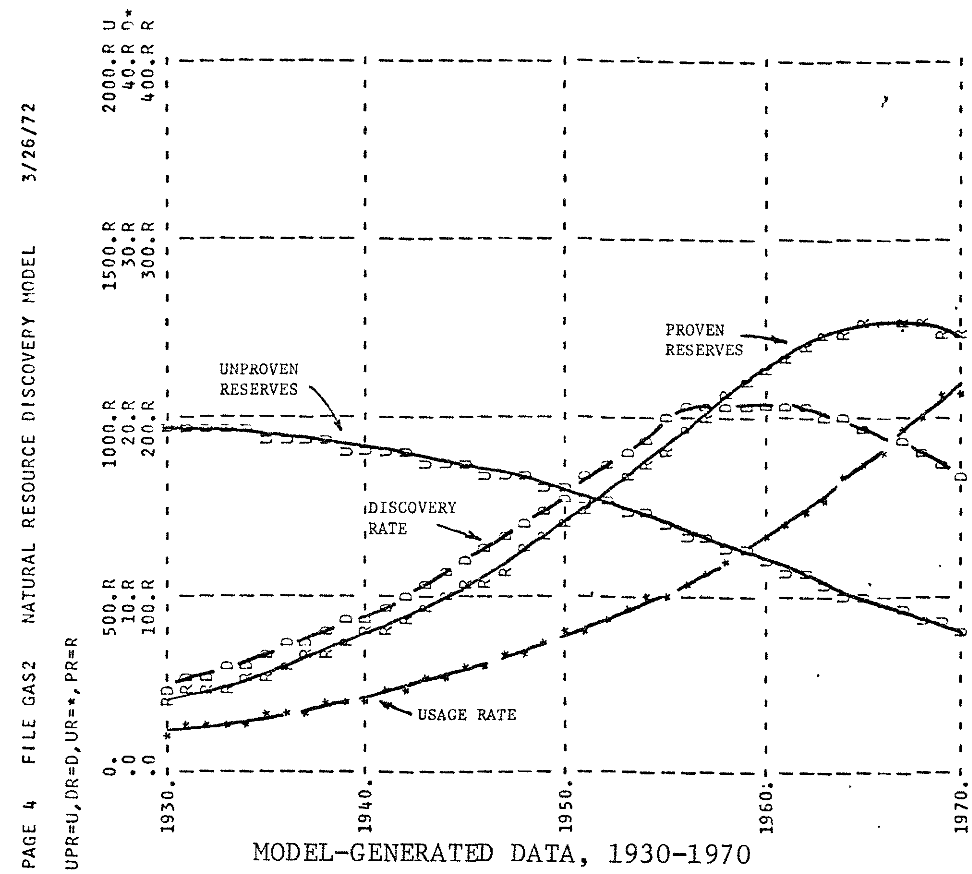
A series of regressions on data created by the model with stochastic components in the five system equations reveals the effects of multicollinearity--the values of the regression coefficients from synthetic data become very unstable, as is the case with real-world data (Appendix III). The correlation coefficient between reserve-production ratio RPR and new contract price over total cost NCP/TC is .88, as compared to the real-world data coefficient of .89. The evidence supplied by simulation analysis thus supports the formulation of the investment function presented in Chapter III.

Comparison of Model Behavior with Historical Data

Figures 26 and 27 compare real-world time series data to model-generated data for the period 1930 to 1970, a period where data are available on most of the model variables. The model shows excellent agreement with time series data for unproven reserves UPR, usage rate UR, and cost of exploration COE. Discovery rate data is extremely noisy, but converges well with model-generated data when smoothed. Discrepancies in proven

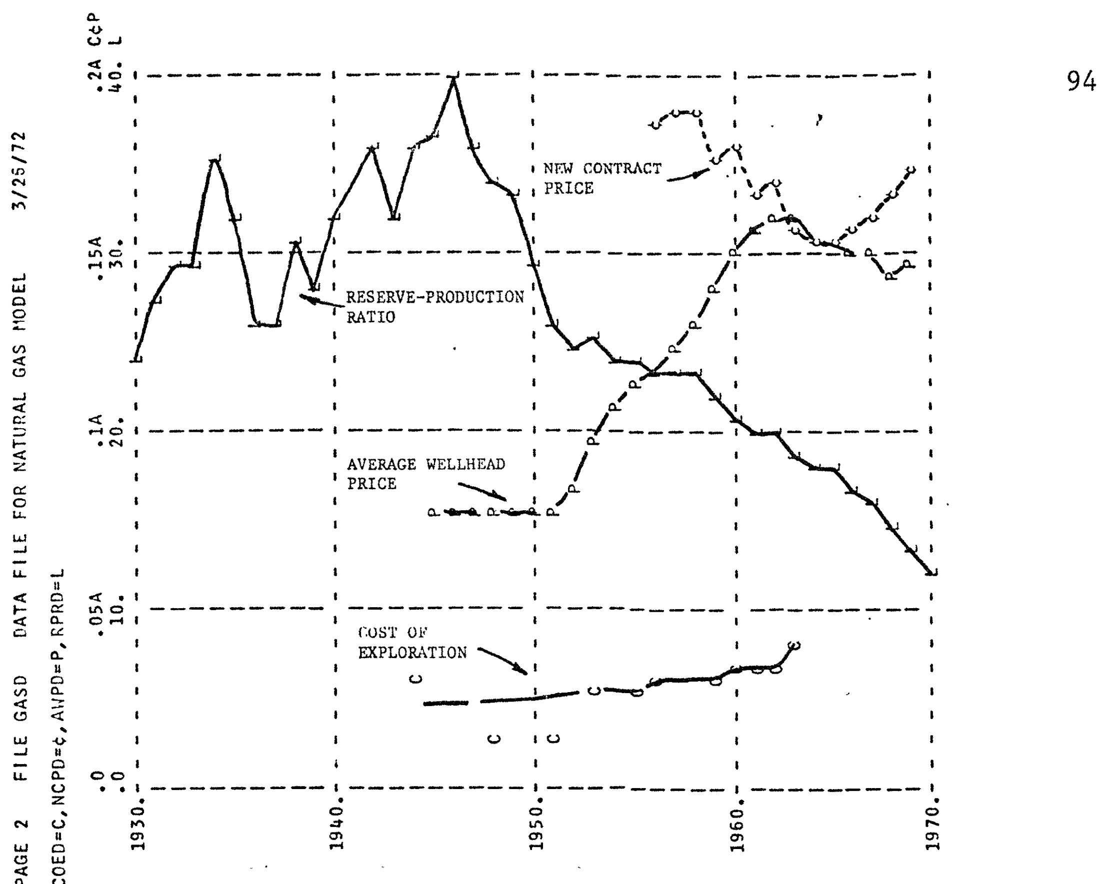


HISTORICAL DATA, 1930-1970

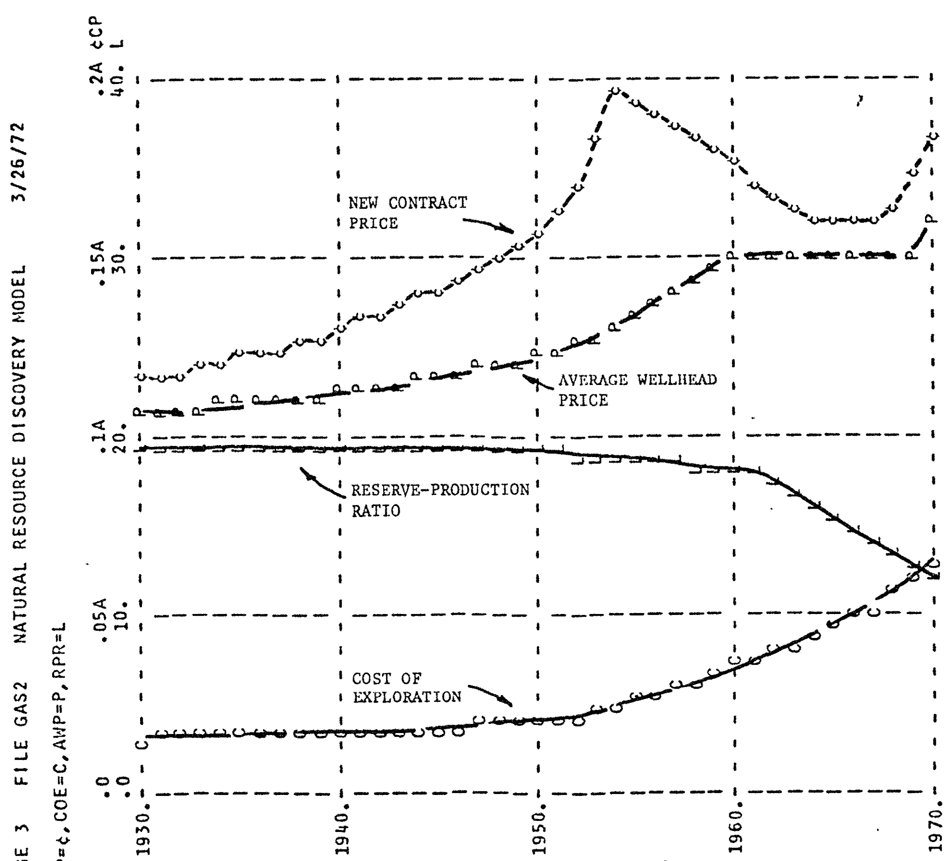


MODEL-GENERATED DATA, 1930-1970

FIGURE 26: REAL-WORLD AND MODEL-GENERATED DATA FOR UNPROVEN RESERVES, DISCOVERY RATE, PROVEN RESERVES, AND USAGE RATE



HISTORICAL DATA, 1930-1970



MODEL-GENERATED DATA, 1930-1970

FIGURE 27: REAL-WORLD AND MODEL-GENERATED DATA FOR NEW CONTRACT PRICE, AVERAGE WELLHEAD PRICE, COST OF EXPLORATION, AND RESERVE-PRODUCTION RATIO

reserves, price, and reserve-production ratio RPR arise again due to carryover effects from oil exploration. As gas was discovered while searching for oil, proven reserves PR were increased beyond the normal values generated by the model. This increases reserve-production ratio RPR in this period, which decreases average wellhead price AWP and new contract price NCP as well.

It can be seen that the model recreates phasing effects very well, however. Discovery rate DR drops below usage rate UR in 1966-1967 in both model and real-world data, causing proven reserves PR to peak in 1967. If one smooths the discovery rate DR data, it appears discovery rate DR peaks around 1960, as the model shows. While demonstrated ability to reproduce past behavior is no guarantee of a model's utility in projecting the impact of future changes in the system it represents, the fidelity of the model's phasing and turning points is a strong indicator of the model's utility.

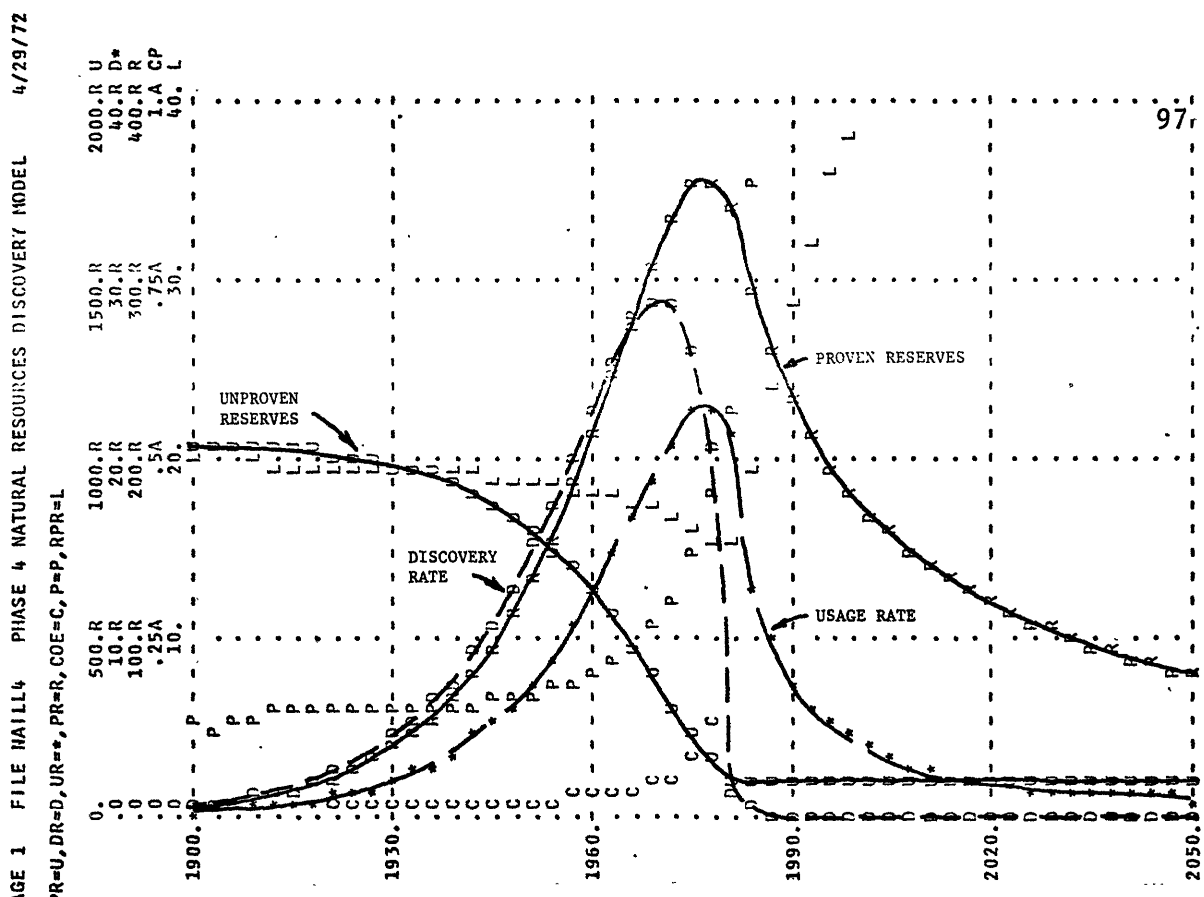
Implications of Statistical Data Analysis for System Dynamics Studies

The model of the U.S. natural gas industry was first constructed and simulated extensively without a formal statistical analysis of the relationships expressed by the six table functions. These relationships were hypothesized to be consistent with an intuitive impression of the industry formed from extensive research of the literature on U.S. natural gas. Because the formal analysis described in this chapter involved an additional amount of effort comparable to the initial formulation of the model, it is appropriate to ask how much was gained through the additional investment in time.

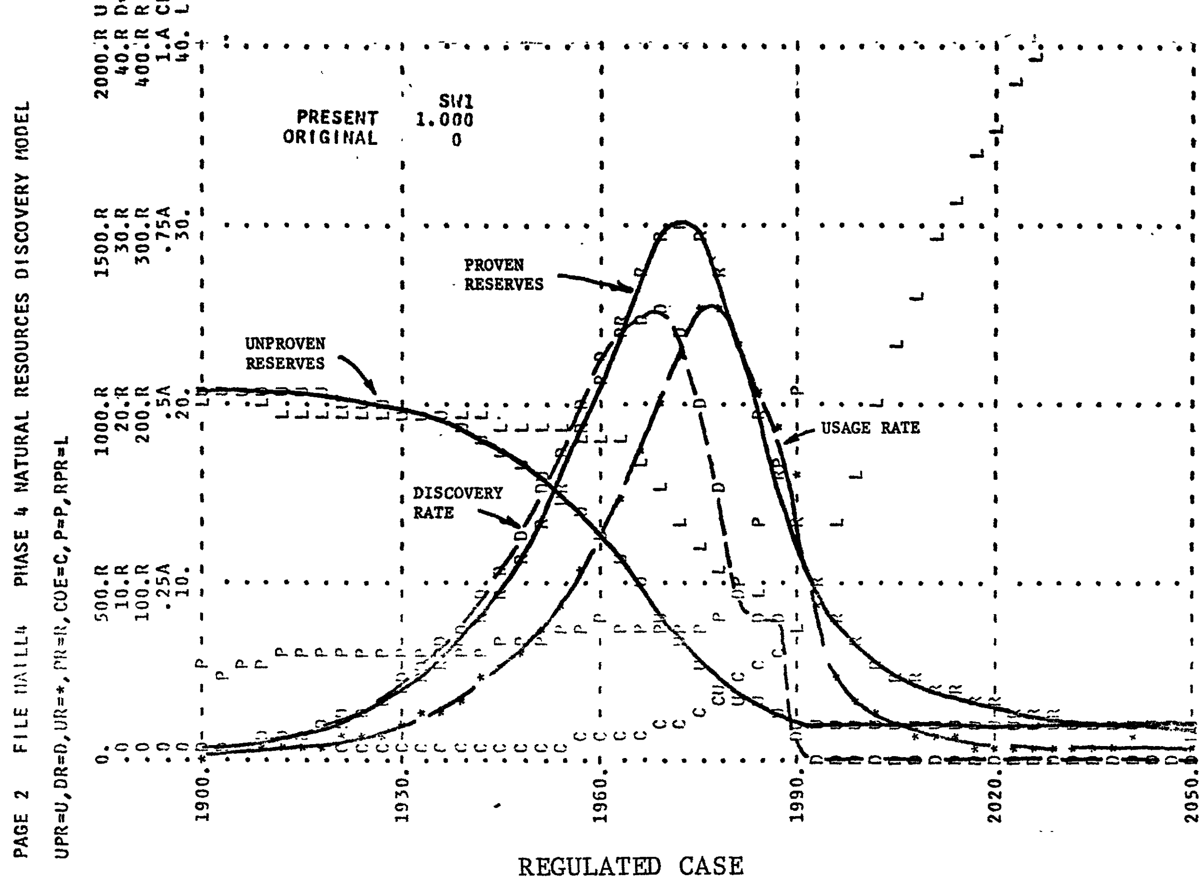
A qualitative answer to this question can be obtained by comparing extended simulations of the model with and without the refined tabular relationships, and both with and without policy changes. Given the role of such a model in policy analysis it is the change in the relative impact of a policy, not changes in the absolute value of a model parameter at some point in the future, which is the measure of increased utility. The model is not appropriately used for point predictions, but rather for the selection of the best policy given some goal. If the relative desirability of two policies were preserved by refining the model, that would constitute a significant increase in model utility due to statistical analysis.

Figure 28 and 29 show the relative effects of regulation on the long-term behavior of the industry before (Figure 28) and after (Figure 29) the preceding analysis of historical data was performed. One can say generally that the relative impact of regulation on the two models is the same. In both cases, regulation holds the price down, causing discovery rate DR to peak earlier, and usage rate UR to rise further due to increased demand. In both cases, proven reserves PR is greatly depressed in the long run with regulation. For a further comparison of policy effects before and after statistical analysis, refer to Chapter V and [Naill (1972)].

Although the overall behavior modes and policy response of the model are essentially unchanged with the additional statistical analysis, it is important to note that the fit of the model to existing historical data is considerably improved by the additional effort. The relative importance of the contributions of a structurally oriented modeling effort or one

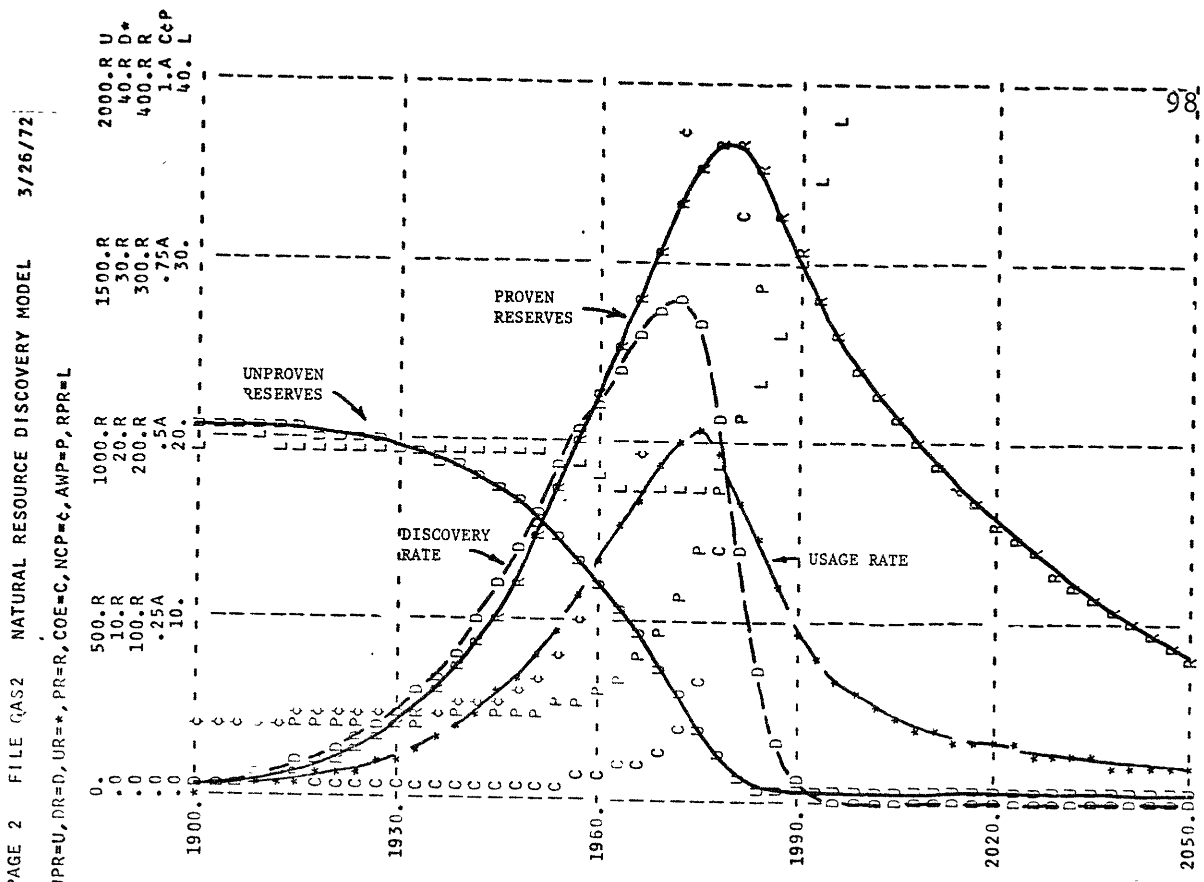


UNREGULATED CASE

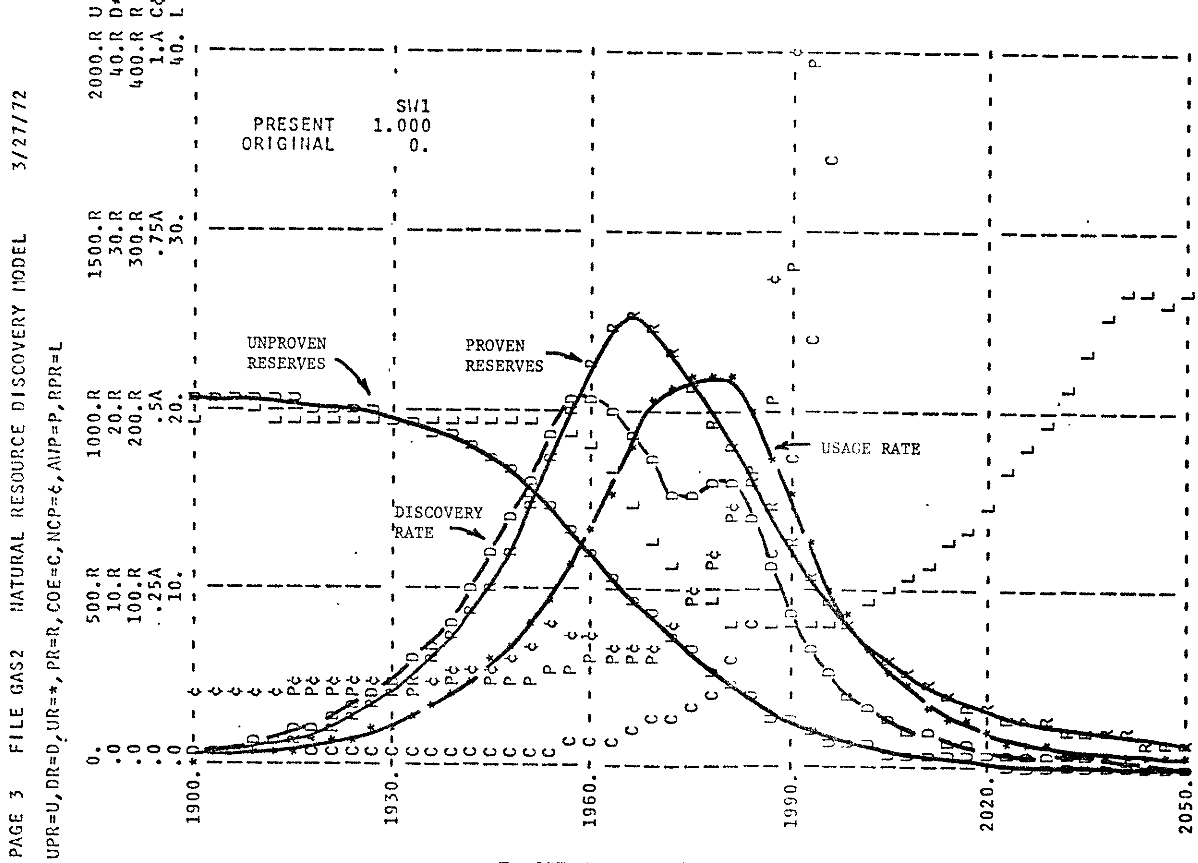


REGULATED CASE

FIGURE 28: BEHAVIOR OF THE MODEL WITH AND WITHOUT REGULATION
<u>BEFORE</u> STATISTICAL ANALYSIS



UNREGULATED CASE



REGULATED CASE

FIGURE 29: BEHAVIOR OF THE MODEL WITH AND WITHOUT REGULATION
<u>AFTER</u> STATISTICAL ANALYSIS

with more emphasis on historical data depends on the purpose of the model. Models are generally used for either the prediction of future values of variables, or as a tool to test the relative effects of various policies.

System Dynamics models are generally employed in the analysis of problems where the available data are incomplete and often inaccurate, and where one is interested in the behavior of the system beyond some turning point or after the introduction of a substantial change. Then point prediction is not a valid use of the model.[64]

It is generally true for systems which contain many negative feedback relationships that the behavior modes of the system are insensitive to the exact shape of the individual table functions, but rather are determined by the overall causal structure and delays in the system. This is fortunate, for one can expect the individual coefficients of a real system to change slowly over time--the general causal structure in a socio-economic system is, however, much more enduring.

Fitting historical data very precisely to an invalid system structure will produce little of use in long-term policy analysis. However, once the structure is determined, detailed analysis of data can serve to verify hypothesized relationships in the system, and to derive better estimates of the nonlinear relationships and individual coefficients in the system.

64) For an example of a System Dynamics-econometric study where this is not the case, see [Weymar (1969)].

CHAPTER V

DESCRIPTION OF MODEL BEHAVIOR AND EVALUATION OF POLICIES

This chapter describes general model behavior and the response of the model to various sensitivity tests and policy changes. The model is first run without any policy changes, representing the behavior of the industry if no regulation had occurred. Next, the effects of regulation on long-term behavior of the industry are evaluated. This represents a baseline run of existing policy, to which several additional possible policies are compared.

General Model Behavior: The Unregulated Industry

Figure 30 shows the behavior of the model of the U.S. natural gas industry under conditions of no regulation. Discovery rate DR and usage rate UR rise freely until 1972, for exploration costs COE are low and rising only very gradually, and proven reserves PR are initially high with respect to usage rate UR. When the fraction of unproven reserves remaining FURR drops to about .4, costs begin to rise more quickly (Figure 3), causing average wellhead price AWP to rise. Because of the delay in the response of average wellhead price AWP to total cost TC, the cost of additional discovery rises faster than price and revenues, forcing discoveries to begin declining in 1975.

The rising average wellhead price AWP slows the exponentially rising usage rate UR through reduced demand (demand multiplier DM, Figure 16), and eventually causes it to drop in 1978. Proven reserves PR reach a peak of about 360 trillion cubic feet in 1978, and begin to decrease as

PAGE 2 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 3/26/72

UPR=U,DR=D,UR=*,PR=R,COE=C,NCP=¢,AWP=P,RPR=L

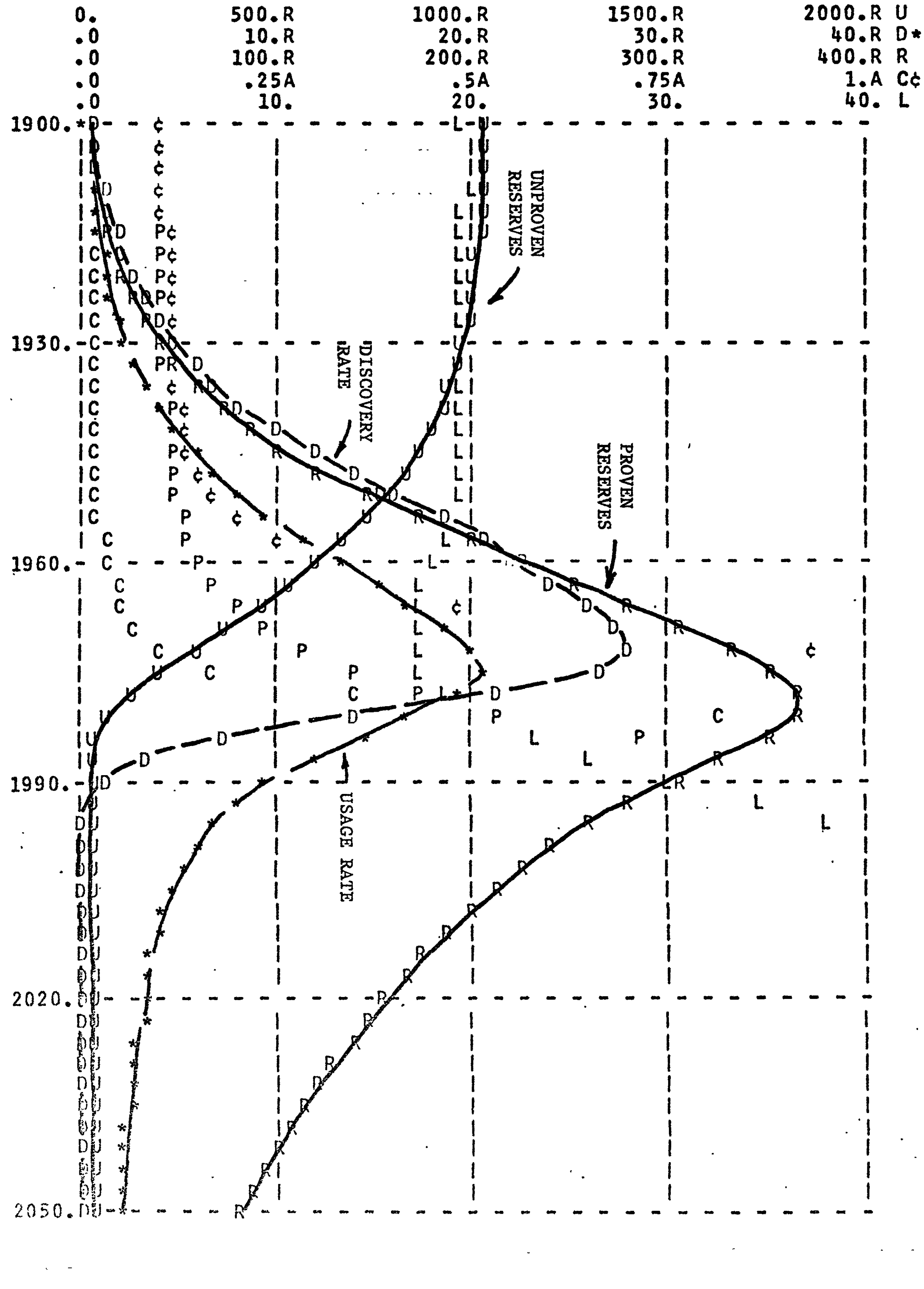


FIGURE 30: UNREGULATED BASELINE RUN

discovery rate DR falls below usage rate UR. As the fraction of unproven reserves remaining FURR drops to about .3, costs are rising so fast that the return on investment on new discoveries becomes unattractive, and the return on investment multiplier ROIM forces the percent invested in exploration PIIE to drop almost to zero in 1990. Discoveries thus also drop almost to zero, causing about one-twentieth of the initial unproven reserves to remain undiscovered, for the cost of this last amount's discovery is so high that it is not worth finding. Concurrently, the reserve-production ratio RPR has begun to rise due to high proven reserves PR and low usage rate UR, causing the total cost smoothing delay to increase, and slow the rise in average wellhead price AWP (reflecting the smoothed total cost of proven reserves). This causes usage rate UR to decay more slowly from 1985 on, and the accumulated proven reserves PR decline slowly over time. By 2050, almost one-tenth of the total initial U.S. supply of gas remains in proven reserves, for its high price has discouraged demand.

It is reasonable to expect that conditions like those after year 1990 would stimulate government intervention in ways that could alter several assumptions underlying the model. Nevertheless, reserves remaining at that point are so low that no significant change in the total life cycle is any longer possible beyond 1990. One also could speculate what role reserve-production ratio RPR would play at the extreme end of the life cycle. Desired reserve-production ratio RPR would probably increase when depletion is imminent.

These possibilities lead one to recognize the difference between using a model to predict what will happen and using it to show the logical outcome of current policies. This study has the second objective.

System Behavior Under Regulation

Figure 31 shows the effects of ceiling-price regulation based on cost plus a margin, as described in Chapter III. This corresponds to the actions taken by the FPC after the 1954 Supreme Court Decision on the Phillips case.[65] The behavior of the model is identical to the unregulated case up to 1954, with both discovery rate DR and usage rate UR exhibiting exponential growth at about 7 percent per year, keeping up with potential demand. After 1954, however, the effects of the Phillips decision cause new contract price NCP to fall, and average wellhead price AWP to remain constant at its 1960 level. The regulated level of new contract price NCP is lower than the gas companies would charge without regulation. Thus investment in discovery is depressed almost immediately through the return on investment multiplier. This causes discovery rate DR to slow its growth and finally peak in 1960 at a level well below the unregulated case. The lower average wellhead price AWP has encouraged usage rate UR to grow faster than in the unregulated case through stimulated demand, causing the reserve-production ratio RPR to fall dramatically during the 1960's.

Because of heavy industry pressure and clearly rising costs of exploration, the ceiling price rises in the mid-1970's, but not to the price level of the unregulated run. This stimulates some additional discovery through 1990 due to higher return on investment, but not enough to increase the reserve-production ratio RPR, which continues to fall due to higher usage rate. Proven reserves, meanwhile, have peaked in 1967 at a lower level than in the unregulated case (about 250 trillion cubic feet

65) R. S. Spritzer, Op. Cit. (1971), p. 10-12.

PAGE 3 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 3/27/72 REG

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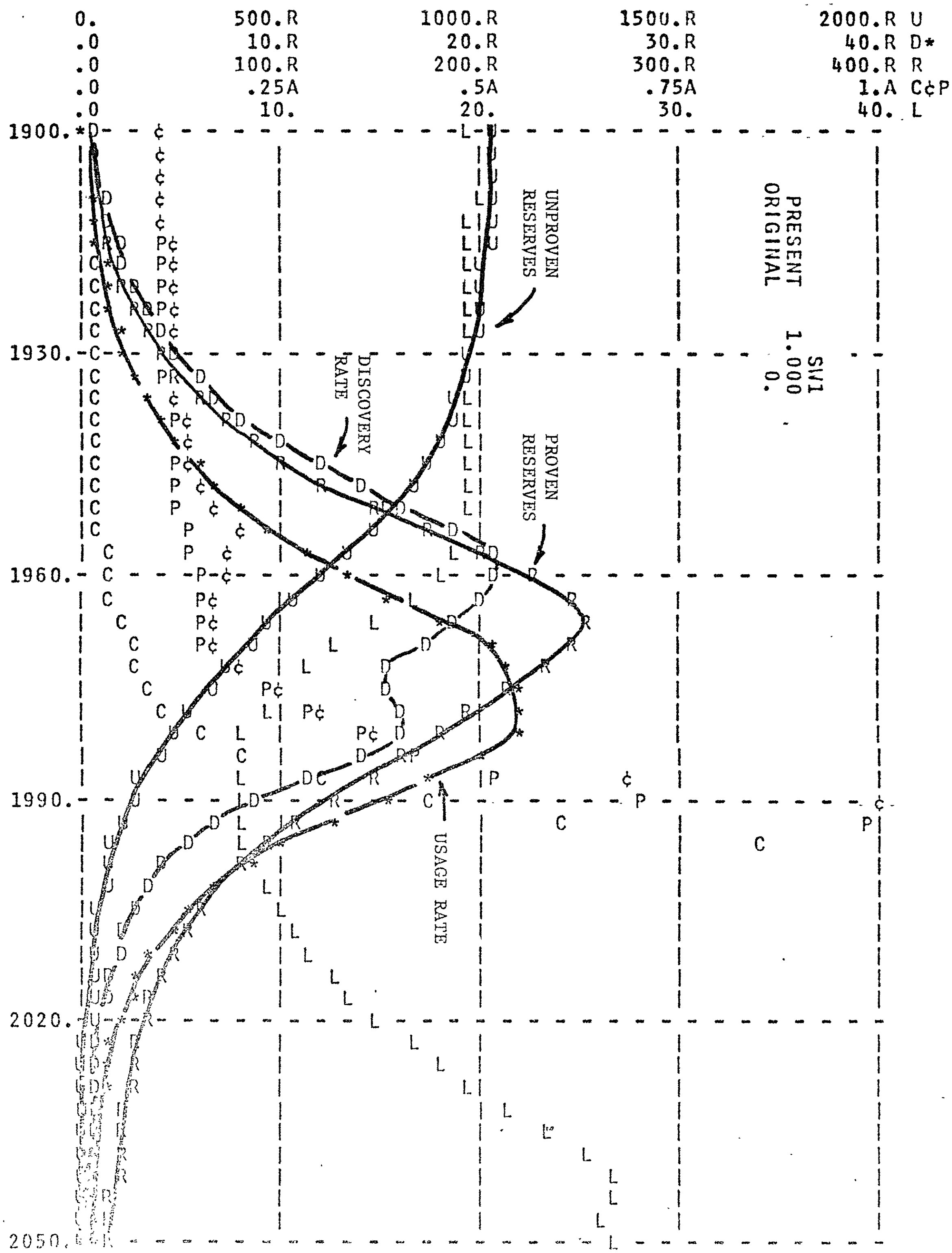

FIGURE 31: RUN WITH CEILING PRICE REGULATION

as compared to 360 trillion cubic feet). Proven reserves also drop faster than in the unregulated case, for usage rate is higher due to the lower average wellhead price. Thus by 2050 proven reserves have fallen to one-fifth of their value in the unregulated run, or about 15 trillion cubic feet.

The overall effect of regulation on the behavior of the model is to transfer usage away from the future, increasing the satisfaction of present needs, while decreasing the reserves available for the future. Under regulation, the industry is left in the year 2000 with few reserves left to discover, and few reserves left to use--the domestic supply of natural gas has essentially been depleted. This is in contrast to the unregulated case, where although the discovery activity is completed earlier (stimulated by higher prices), proven reserves remain high, allowing the usage rate to continue higher after the year 2000. It need hardly be mentioned that the conversion of coal into synthetic gas **reserve** may be a major source of natural gas by the year 2000. It is not the objective of this model to study that influence. However, this model would be an extremely useful starting point in the development of a model to test the impact of different coal conversion strategies.[66]

Figure 32 shows the same run as Figure 31 the regulated case, but with noise multipliers added to the system. The mean standard deviations of the noise multipliers have been estimated from the regressions performed in Chapter IV. The exact form of the noise multipliers are shown

66) A study of the dynamics of coal conversion will be conducted in 1972 by the staff of the Dartmouth Research Program on the Management of Technology and Public Policy.

PAGE 6 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 3/26/72 REGULATED

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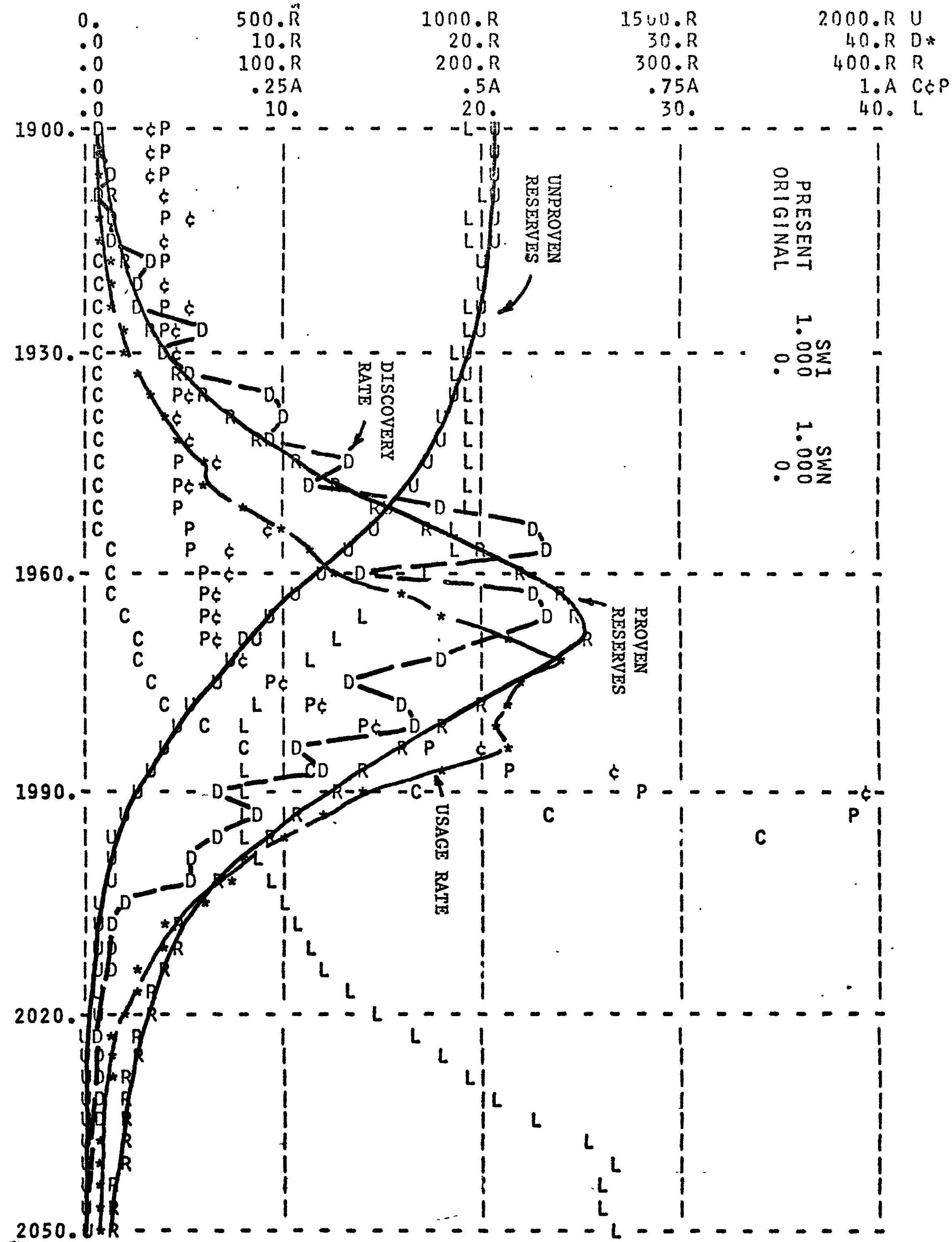


FIGURE 32: REGULATED RUN WITH NOISE ELEMENTS

in Appendix IV, with one exception—the uncertainty in the size of gas reserves discovered has been assumed to affect discovery rate DR directly, and not the cost of exploration as is assumed in the synthetic data analyses. An examination of Figure 32 shows that although the noise inputs have visible influence, the system's overall behavior is hardly affected. In fact, the stock of proven reserves PR acts as a buffer which filters out the short-term variations in discovery and usage. The behavior over time of unproven and proven reserves is almost identical in the two runs.

Effect of an Increase in the Estimate of Unproven Reserves

The simplest, and until recently, the most popular response to the energy crisis is to say that there is no crisis in fuel supply. It is often suggested that the estimate of initial unproven reserves is wrong, and that there are considerably more reserves to be found than the current estimates indicate. Figure 33 shows the behavior of the model under price regulation when the estimate of initial unproven reserves UPRI is doubled to 2,080 trillion cubic feet of gas, corresponding to estimates based on the Zapp hypothesis.[67] Note that the vertical level and rate scales of Figure 32 have changed by a factor of two. The principal effect of doubling initial reserves is to allow the exponential growth of discovery rate DR and usage rate UR to continue for an additional ten years. In this time, discovery rate DR, proven reserves PR, and usage rate UR reach twice their values in Figure 31 where initial unproven reserves UPRI is only 1,040 trillion cubic feet. This run exhibits an important aspect of the

67) M.K. Hubbert, Op. Cit. (1969), p. 188.

PAGE 9 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 3/26/72

UPR=U,DR=D,UR=*,PR=R,COE=C,NCP=¢,AWP=P,RPR=L

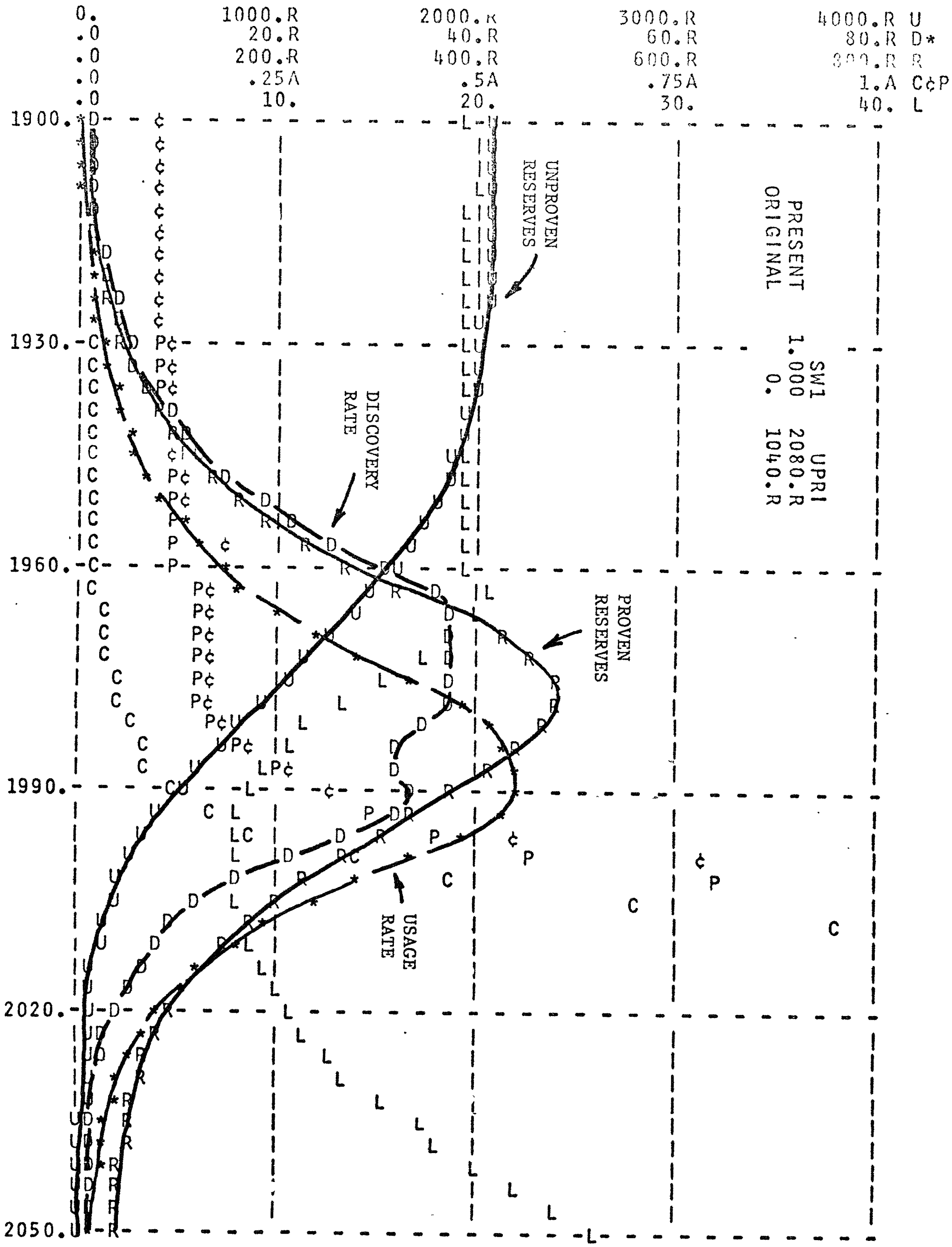


FIGURE 33: REGULATED RUN WITH 2,080 TRILLION CUBIC FEET ESTIMATED INITIAL RESERVES

behavior of exponential growth--the general model behavior is relatively insensitive to changes in the initial values of parameters. With the assumption of exponential growth in usage rate UR, doubling the estimate of reserves would allow only a ten-year postponement of the time when usage is forced to decline.

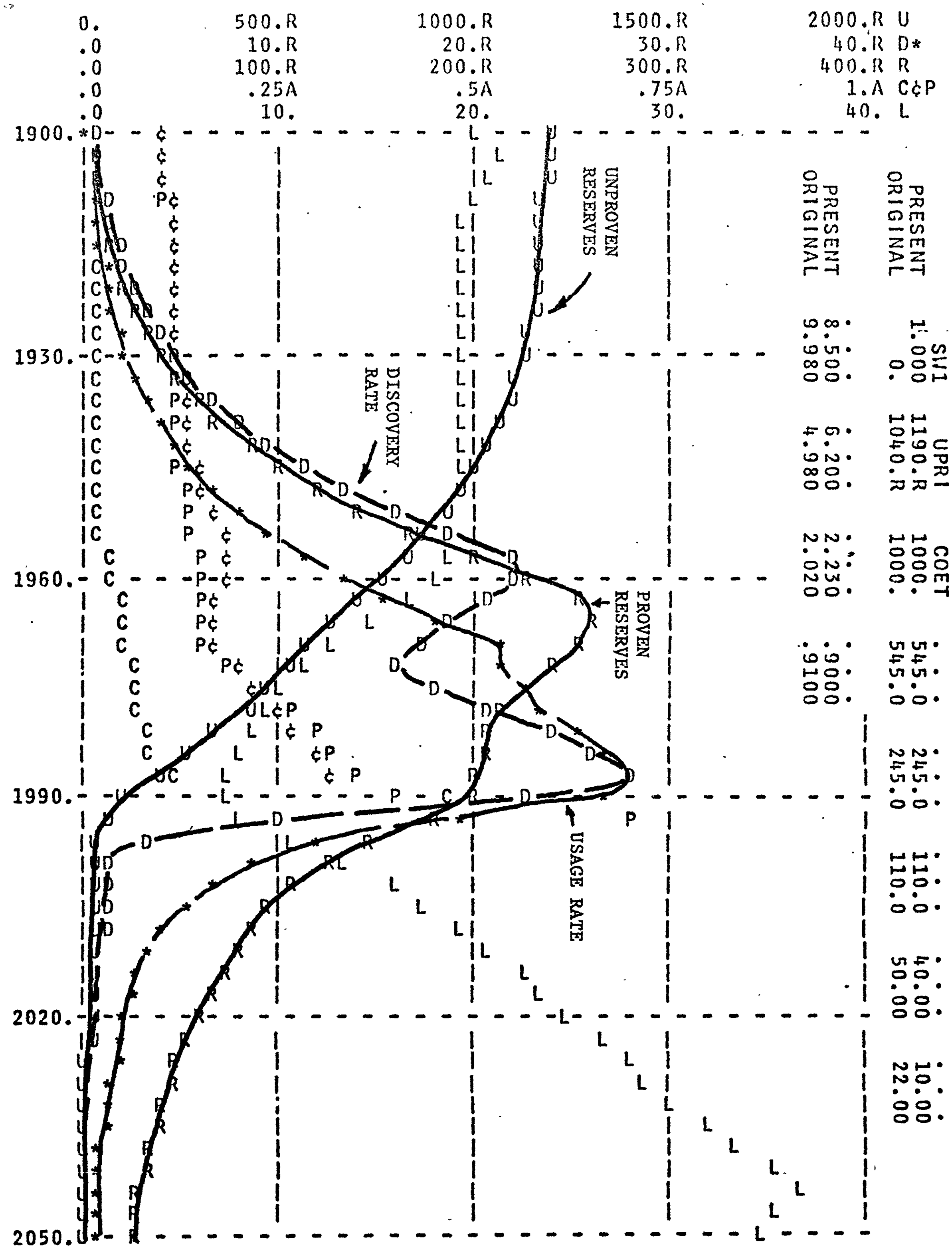
This run is also a rough representation of the behavior of the system in response to a large increase in future imports of natural gas. Expanding the system boundaries to include other countries and permitting large scale imports simply has the effect of increasing the initial value of unproven reserves (and probably shifting their relationship to cost). The actual value of unproven reserves in the case of imports would be determined by the world estimates of reserves and the total amount each country is willing to export to the U.S.

Effects of an Alaska Discovery

A likely future influence on the system is a large discovery of natural gas in Alaska in conjunction with the oil strikes already made there. Hubbert has estimated that as much as 150 trillion cubic feet of gas may be discovered there.[68] Figure 34 shows the effects on behavior of the regulated run if that amount of natural gas were discovered in Alaska around 1975. This discovery can be simulated by changing the cost of exploration curve to reflect a period of lower costs after 1975, and by increasing the initial value of unproven reserves to 1,190 trillion cubic feet. The effects as shown in Figure 34 are a dramatic increase in

68) M.K. Hubbert, Op. Cit. (1969), p. 193.

UPR=U,DR=D,UR=*,PR=R,COE=C,NCP=¢,AWP=P,RPR=L



WITH ALASKA DISCOVERY IN 1975

discoveries up until 1987, allowing usage rate UR to continue climbing six years longer than without the discovery, or until 1987. Thus usage rate UR and discovery rate DR peak at higher values than without the discovery (Figure 31), but the net result is a postponement of the limitations on growth in usage for only a few years.

Effects of a Change in Growth of Potential Demand

Suppose through a large campaign to conserve resources it were possible to decrease the growth rate of potential demand from seven percent to two percent after 1970. Would this significantly extend the period of abundance in supply of natural gas? Figure 35 shows the effects of this policy on the regulated case (Figure 31). Discovery rate DR is virtually unaffected immediately after 1970, for there is already much desire for new reserves due to the low reserve-production ratio RPR, and further increases in potential demand only makes the situation worse. Usage rate UR peaks at the 1970 value and begins to fall, but not as quickly as in the seven percent growth case. The effects of the lower growth rate in potential demand on usage rate UR are partially offset by the higher reserve-production ratio RPR which puts less restrictions on usage rate UR through the usage supply multiplier. The net result is a lower peak in usage rate UR and a slower decline, postponing the depletion of unproven reserves to 10 percent of their initial value by about ten years.

Effects of an Improvement in Exploration Technology

Suppose that in 1972 the U.S. gas industry stepped up its research

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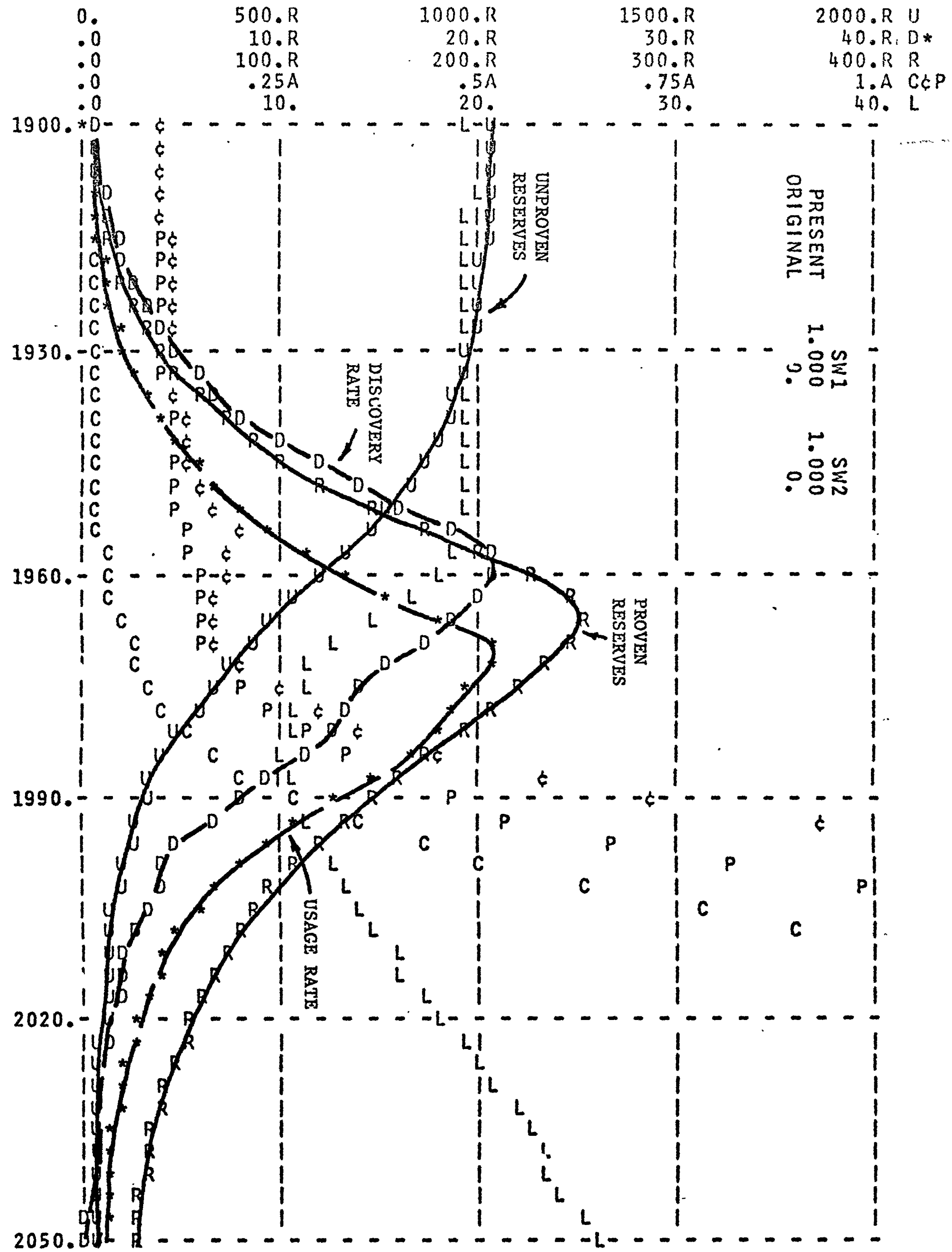

ANGE IN GROWTH IN POTENTIAL
1970 ON THE REGULATED RUN

effort and greatly advanced the technology of exploration and extraction. One example of the result of such an effort might be increased efficiency of extraction through use of underground nuclear explosions. This policy can be simulated by holding the cost of exploration down near the 1972 level until almost all of the unproven reserves have been discovered. Figure 36 shows the behavior of the regulated case when an increase in technology holds down exploration costs for several years beyond 1972. The lower costs cause an increase in discoveries through 1981, for lower costs provide a higher return on investment. Usage rate UR continues to grow until 1982 and then falls abruptly as average wellhead price AWP rises steeply in response to cost increases. Costs rise when the fraction of unproven reserves remaining approaches zero. The overall effect of a large improvement in technology is to increase the severity of the eventual shortage. Usage and discovery rates rise longer with technology but fall very steeply as unproven reserves are nearly depleted.

Government Subsidy of Exploration Costs

To alleviate the supply problems caused by declining discoveries it has been suggested that the government subsidize the gas industry's exploration costs. Figure 37 shows the effects on the regulated run of a 25 percent government subsidy of exploration enacted in 1972. It can be seen that the subsidy accomplishes its objective, but only in the short run. Discoveries increase up until 1978, and then fall, for rising costs of exploration again discourage investment in discovery. Usage rate UR rises for a few years more than without a subsidy (Figure 31), but again falls due to producer restrictions on supply caused by the low reserve-production

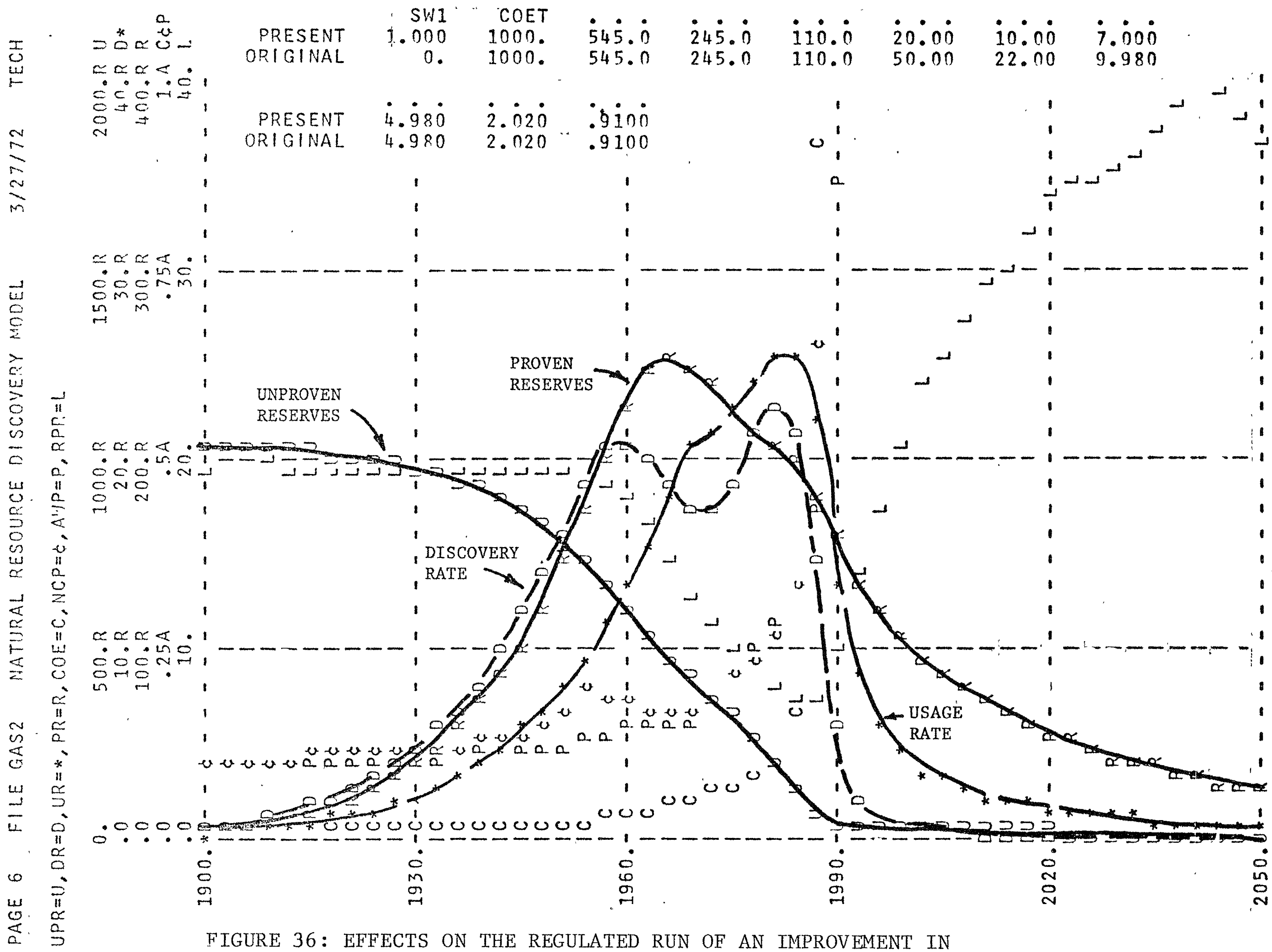

FIGURE 36: EFFECTS ON THE REGULATED RUN OF AN IMPROVEMENT IN EXPLORATION TECHNOLOGY BEGINNING IN 1972

PAGE 3   FILE GAS2   NATURAL RESOURCE DISCOVERY MODEL   3/27/72   SUBSIDY

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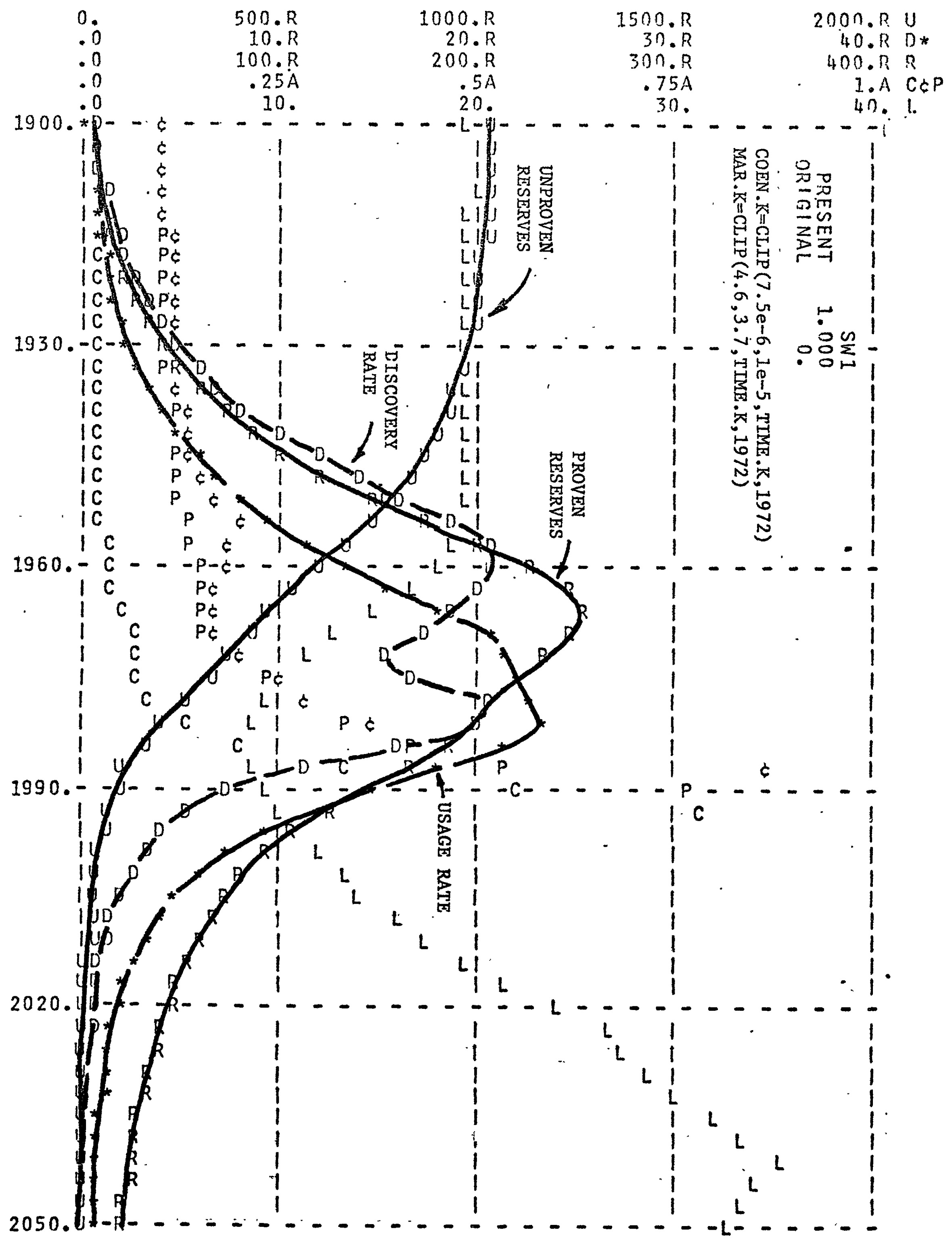

FIGURE 37: REGULATED RUN WITH A 25% SUBSIDY ON EXPLORATION COSTS IN 1972

ratio RPR. The net result of the subsidy on exploration is therefore to provide only a short-run solution to the supply problem. The long-term supply of natural gas remains virtually unchanged.

Effects of De-Regulation in 1975

Figure 38 shows the behavior of the model if the regulated run (Figure 31) were de-regulated in 1972. De-regulation is simultated by functions which return the average wellhead price AWP and usage rate UR to their unregulated values. In order to avoid a discontinuous rise in new contract price NCP, the de-regulated value of new contract price NCP is assumed to be a delayed function of unregulated new contract price UNCP.

The immediate effect of the rise in new contract prices is the stimulation of discoveries through increased return on investment. This is short-lived, however, for as unproven reserves approach zero, the cost of exploration rises rapidly. Usage rate UR rises immediately after 1975, for the increase in discoveries increases the reserve-production ratio RPR, allowing producers to sell more gas. The rapidly rising average wellhead price AWP depresses usage rate UR more severely than in the regulated case (Figure 31) after 1980, causing proven reserves to remain high through 2050. Thus de-regulation of the industry is consistent with long-term supply goals, sacrificing immediate usage for possible future usage.

Summary of Policy Effects

The effects of the various policies and sensitivity tests on the behavior of the system are summarized in Table 2. It is important to remember that in evaluating a policy, it is the relative effects of the

PAGE 3 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 3/28/72 REG,UNREG

UPR=U,DR=D,UR=*,PR=R,COE=C,NCP=¢,AWP=P,RPR=L

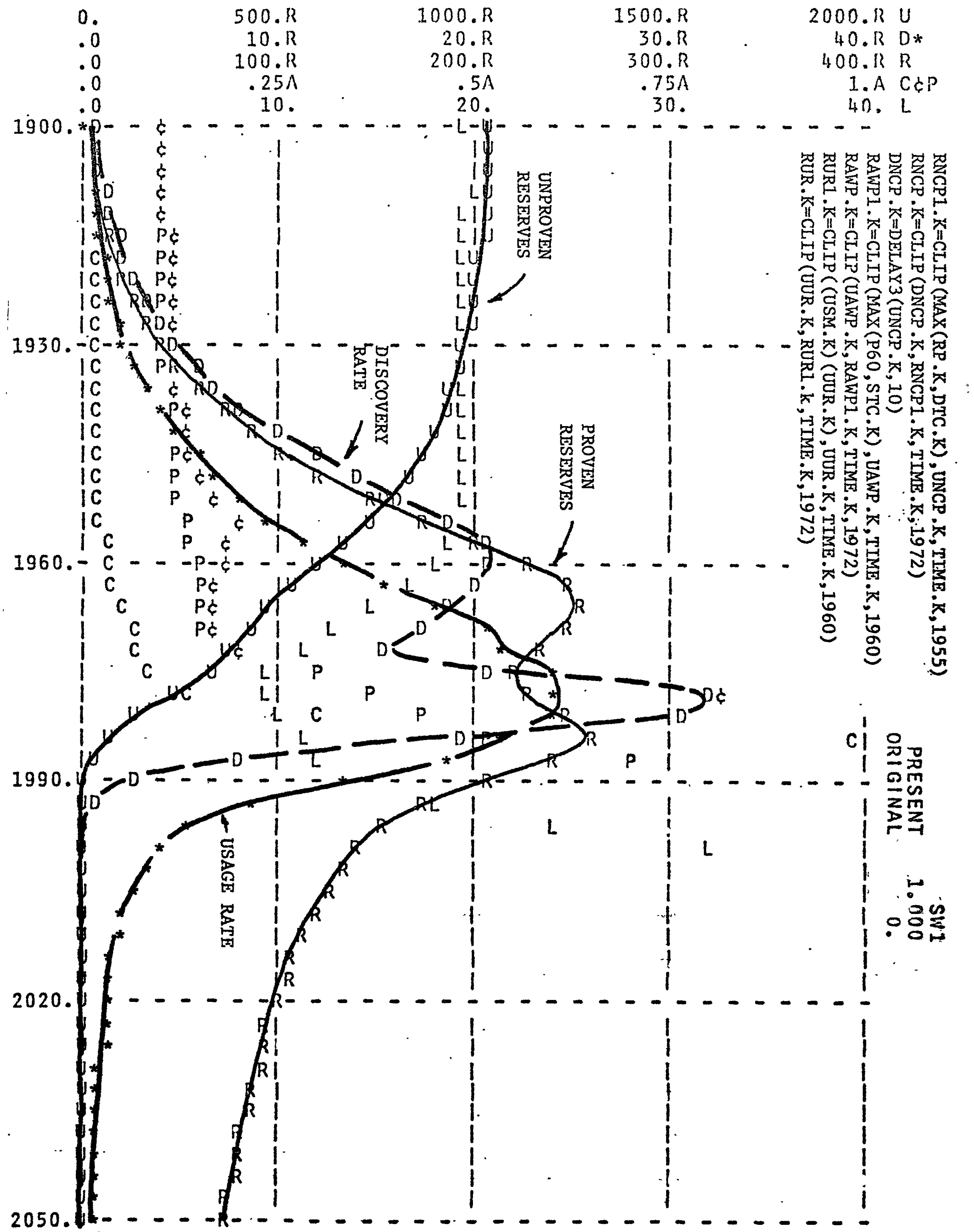

FIGURE 38: EFFECTS ON THE REGULATED RUN IF THE INDUSTRY WERE DE-REGULATED IN 1972

policy compared to a baseline case which has the most validity. Thus, for instance, a valid evaluation of the effects of regulation on discovery rate DR might be that regulation reduces the peak value of discovery rate DR by 25 percent, and forces discovery rate DR to peak twelve years earlier than if no regulation had occurred.

POLICY	EFFECT OF POLICY						
	Discovery Rate		Usage Rate		Proven Reserves		Time when Average Wellhead Price AWP exceeds 50¢/Mcf
	Peak Value (trillion ft^3)	Turning Point	Peak Value (trillion ft^3)	Turning Point	Peak Value (trillion ft^3)	Turning Point	
1) Unregulated	28	1972	21	1975	270	1978	1980
2) Ceiling Price Regulation	21	1960	22	1980	250	1967	1986
3) Double unproven reserves	38	1972	44	1990	450	1977	1996
4) Alaska Discovery	28	1987	28	1987	250	1967	1991
5) 2 percent growth rate in 1970	21	1960	21	1972	250	1967	1991
6) Technology Improvement in 1972	23	1980	26	1983	250	1967	1988
7) 25 percent Subsidy on exploration in 1972	21	1978	23	1981	250	1967	1986
8) De-Regulation in 1972	32	1978	24	1978	260	1984	1983

TABLE 2: EFFECTS OF POLICIES ON SYSTEM BEHAVIOR

CHAPTER VI

CONCLUSIONS AND POTENTIAL FOR FURTHER RESEARCH

As finite, nonrenewable resources continue to be depleted at exponentially increasing rates, industry and government policy makers (such as the FPC) are becoming more aware of the need to adopt a long-term viewpoint in policy planning. In order to test the long-term effects of policies on resource supply, managers need explicit planning tools such as dynamic simulation models. Several existing models of the U.S. natural gas industry have been surveyed and found to be inadequate for long-term testing.

In response to this need for long-term resource planning models, a dynamic model of the discovery process of U.S. natural gas is presented in Chapter III. In order to fully incorporate all the available data in the model, an extensive analysis of the model's nonlinear table functions is included in Chapter IV. It is important to note that although the resulting model fits existing data better after this analysis, the relative effects of various policies on the major behavior modes of the model remain unchanged.

General Conclusions

The most fundamental conclusion to be drawn from the behavior of the model is that in the case of finite, nonrenewable resources such as the fossil fuels, the normal behavior mode is an initial period of unrestricted growth in production, a turning point when growth is slowed, and finally a period during which there is a decline in production. This conclusion has been drawn by others.[69] However, the type of model developed here does offer an unprecedented experimental tool for determining how various technological, physical, economic, and political factors might alter the pattern of growth and decline.

The exact timing of the turning point and the magnitude of production at that point is determined by many factors, including, for instance, the growth rate of potential usage rate UR, the initial level of unproven reserves, and the shape of the exploration cost curve. However, once the structure or interrelationships of these elements has been determined, the behavior of the model and the time of decline is remarkably insensitive to small changes in system parameter values. For instance, an increase by a factor of two in the actual quantity of initial inproven reserves results in a postponement of the turning point in supply by only ten years. A 25 percent subsidy on exploration postpones the turning point for only three years.

If a decline in supply is unavoidable, it is imperative that we think about <u>how</u> we want to allocate our available resources: should we take precautions that some modest but useful amount of the scarce resource be available over the long term, or should we encourage the immediate use of the resource to postpone the decline in usage? It appears from the runs made that the short-term advantages of policies such as regulation or subsidies toward the postponement of decline in usage are minimal. The model supports the claims of the industry spokesmen that depressed price through regulation tends to depress discoveries in the short run. But it is also apparent that the industry was nearing a period of limitations in supply

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69) M.K. Hubbert, Op. Cit. (1969), p. 167.

regardless of regulation. In addition, price regulation carries a large penalty in terms of long-term supply--a low price encourages demand and thus usage, greatly reducing the long-term level of available proven reserves.

The particular schedule of gas consumption most consistent with our national goals depends on many factors outside the gas industry. Probably most important is the achievement of an orderly transition over the next century from primary dependence on fossil fuels to reliance on some renewable or essentially infinite energy resource (eg. solar or fussion power). The model described here is not designed to suggest how quickly gas reserves should be used--that is a political issue--but once society has identified its short and long-term goals, the model does permit one to compare the relative effectiveness of alternative policies in achieving that usage rate UR which will best serve our society's interests.

Potential for Further Research

The economic impact of a future decline in production rate of domestic natural gas depends on the possibilities for substitution of other fuels for gas, and on the amount of services rendered by a unit of gas as it is used to produce final products. Substitution of other fuels for gas is implicit in the model in the relationship of average wellhead price AWP to demand for gas--as price rises, it is assumed that consumers switch to other fuels. A valuable extension of this research would be to represent explicitly the process of substitution by combining the gas model with a model of production and consumption of some other resource, like coal, which is a substitute for natural gas. Then one could study the research and investment policies which would facilitate an orderly shift of demand away from gas and towards a greater reliance on the substitute. An endogenous technology sector like that developed by Behrens [Behrens (1971)] would also be needed in order to represent the delays inherent in developing sufficient technologies of substitution.

The central goal of this project has been to integrate the many geological and economic processes which determine the investment and discovery of domestic natural gas over an extended period. In order to study more completely the factors controlling the flow of a resource from unproven reserves to pollution or solid waste, this model could be extended to explicitly represent technological changes which affect a resource system. Such a project has been proposed in 1972 to the National Science Foundation as a part of Dartmouth College's Research Program on the Management of Technology and Public Policy. It is the goal of the proposed project to work closely with user agencies such as the Department of the Interior, and to develop disaggregated models useful in forming long-term resource management policies.

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APPENDIX I

COMPLETE LIST OF MODELING ISSUES CRITICAL TO THE FPC[70]

FEDERAL POWER COMMISSION

NATIONAL GAS SURVEY

All of the issues to be considered are not necessarily limited to the following list:

1. Estimated proven and potential gas reserves for the next two decades and the magnitude thereof relative to other energy sources to meet the projected gas requirements.

2. Economic basis for additions to reserves.

3. The present and estimated future market requirements for gas, including gas for feed stocks for the next two decades to the year 1990.

4. Supply-demand price relationships.

5. Price structures and interfuel competition.

6. Impact of environmental standards on gas production, transmission, distribution and markets, including the role of gas in the environment.

Environmental legislation can have a great influence on our lives of tomorrow. The continued growth in power consumption is now projected to increase by 450 percent by 1990 which would mean according to some expertises "an intolerable air pollution, radiation hazards and the ruination of our landscape" unless proper fuels are made available and such properly used.

7. Current and projected reserves and production needed to fulfill predicted market requirements.

8. The capital required to meet the projected demands for all three phases of the gas industry. These funds will not be in the magnitude of millions but shall be in billions of dollars.

9. Present and future adequacy and efficiency of all facilities required for production, transmission and distribution of gas.

10. Prospective technological improvements and ample funds for research and development.

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70) FPC private communication.

11. Possible new gas sources to supplement the conventional domestic reserves such as gas from Mexico, Canada, Alaska and the Arctic Islands. Other sources of gas to be considered are liquified natural gas, coal gasification, gas from oil and oil shales and tar sands, and gas from nuclear stimulation including a determination of the anticipated volumes which could be derived from each with present and improved technology.

Investments required to produce gas from these alternate sources, prices to be charged for such, necessary lead time to make these gases available, ownership and location of these sources and where conversion plants are required, the accessibility of such to the markets.

12. Determine from the projected natural gas supply versus projected gas requirements to 1990 the necessary gas facilities to meet the increasing market demands, both national and regional, and the capital requirements associated therewith, including, among other factors, required drilling, adequacy of inventory level of gas reserves, and the capability of pipelines and distributing systems, individually and collectively, as pertinent to supply and market areas, including the magnitude of gas resources and reserves relative to other energy sources.

13. The impact of regulation and other governmental influences on natural gas demand and supply and the possible effect of changes in governmental policies, including such areas of consideration as supply-price relationships; regulatory policy with regard to pipelines' rate of return, depreciation with tax treatment in rate proceedings; environmental pollution abatement; gas imports and exports; mandatory oil import controls; laws and customs controlling international trade and legislative actions in such matters as treaties, conventions and the Jones Act; national security; balance of payments; taxes, including depletion allowance; leasing of lands in the public domain; public safety; research and development; and the conservation practices in being in the respective gas producing states.

14. The long-range potential for cost reduction and possible patterns of development in production, transmission and distribution of natural gas, including, among other factors, economies of scale, increased use of storage, technological developments, a more comprehensive statistics and information system, interconnections, remote control operations, load factor improvement, and improved appliances and equipment.

15. The Survey may indicate upon its completion that new guidelines and revisions to the Natural Gas Act, FPC regulatory policy and possible changes in federal legislation may be in order.

APPENDIX II: HISTORICAL DATA

Cost of Exploration

Year	(1) Cost of Exploration, Gas & Oil (Billions 1958 $)	(2) Gas Wells / Oil & Gas Wells	(3) Cost/Well Gas / Cost/Well Total	(4) Investment in Exploration = 1 X 2 X 3 (Billions 1958 $)	(5) Discoveries, Year + 5 (Trillion ft^3)	(6) Cost of Exploration = 4/5 (¢/Mcf)	Fraction of Unproven Reserves Remaining
1944	1.05	.191	1.8 est.	.36	12.7	2.84	.81
1948	1.25	.115	1.8 est.	.26	20.5	1.27	.75
1951	1.77	.114	1.8 est.	.36	24.7	1.46	.71
1953	2.10	.128	1.8 est.	.48	18.9	2.54	.68
1955	2.13	.103	1.8 est.	.39	13.9	2.81	.65
1956	2.24	.124	1.8 est.	.50	17.2	2.91	.63
1959	1.98	.165	1.88	.61	20.3	3.01	.57
1960	1.99	.197	1.87	.73	21.3	3.44	.55
1961	1.78	.210	1.74	.65	20.2	3.22	.54
1962	2.21	.215	1.66	.75	21.8	3.44	.52
1963	1.73	.187	1.68	.54	13.7	3.94	.50

Sources: American Petroleum Institute, et. al., Joint Association Survey of Industry Drilling Costs, and American Gas Association, Gas Facts.

The cost of exploration of gas in any given year is difficult to determine because of the joint nature of expenditures for gas and oil. For the purposes of this study, however, an estimate of the cost of exploration of gas alone is needed. This cost has been derived above from total exploration cost by allocating gas exploration cost according to the number of gas wells drilled as a percentage of total oil and gas wells drilled. This value is then weighted by the cost/well of gas wells relative to the cost/well of all wells drilled. The resulting product is the year's investment in exploration. The discoveries resulting from this investment are taken from data five years later, because of the approximate five-year lag in the effects of investment. The investment in exploration divided by the resulting discoveries gives an estimate of the year's cost of exploration.

Price Multiplier

Year	(1) Average Wellhead Price (1958 ¢/Mcf)	(2) Cost of Exploration, t-10 (simulated) (1958 ¢/Mcf)	(3) Total Cost = 3.7 X (2) (1958 ¢/Mcf)	(4) Price Multiplier = (1)/(3)	(5) Reserve-Production Ratio RPR (Years)
1949	7.68	2.03	7.51	1.02	33.3
1950	7.77	2.05	7.59	1.02	29.6
1951	8.08	2.07	7.66	1.05	26.0
1952	8.43	2.10	7.77	1.09	24.9
1953	9.87	2.13	7.87	1.25	25.2
1954	10.7	2.15	7.95	1.35	24.2
1955	11.5	2.18	8.06	1.43	23.8
1956	11.8	2.20	8.15	1.45	23.6
1957	12.3	2.24	8.28	1.49	23.1
1958	12.9	2.29	8.46	1.53	23.0
1959	14.2	2.32	8.60	1.65	21.8

Sources: American Gas Association, Gas Facts, and API, et. al., Joint Association Survey of Industry Drilling Costs.

Demand Multiplier

Year	Usage Rate (Trillion ft^3)	Average Wellhead Price (1958 ¢/Mcf)
1945	3.92	7.81
1946	4.03	7.95
1947	4.58	7.71
1948	5.15	7.76
1949	5.42	7.68
1950	6.28	7.77
1951	7.46	8.08
1952	8.01	8.43
1953	8.40	9.87
1954	8.74	10.7
1955	9.41	11.5
1956	10.08	11.8
1957	10.68	12.3
1958	11.03	13.9
1959	12.05	14.2

Source: American Gas Association, *Gas Facts*.

Usage Supply Multiplier

Year	(1) ln (Usage Rate$_t$)	(2) ln (UR$_t$) + .066 (1961-t)	(3) ln (Usage Supply Multiplier) (2) - 2.584	Usage Supply Multiplier
1960	2.542	2.608	.024	1.024
1961	2.584	2.584	0	1.000
1962	2.631	2.565	-.019	.981
1963	2.691	2.559	-.025	.975
1964	2.738	2.540	-.044	.957
1965	2.776	2.512	-.072	.930
1966	2.845	2.515	-.079	.931
1967	2.899	2.503	-.081	.922
1968	2.962	2.500	-.084	.919
1969	3.030	2.502	-.082	.921
1970	3.070	2.476	-.108	.898

Source: American Gas Association, Gas Facts.

Percent Invested in Exploration

Year	(1) New Contract Price (1958 ¢/Mcf)	(2) Total Cost Simulated (1958 ¢/Mcf)	(3) Sales Revenue (AWP X UR) (billions 1958 $)	(4) Investment in Exploration (billions 1958 $)	(5) Percent Invested in Exploration (¢)/(3)	(6) NCP/TC (1)/(2)	(7) RPR (Years)
1955	20.4	9.5	1.08	.39	.36	2.14	23.8
1956	18.7	10.0	1.19	.50	.42	1.85	23.6
1959	17.6	12.2	1.70	.61	.36	1.44	21.8
1960	17.9	13.0	1.93	.73	.38	1.38	20.7
1961	16.8	13.8	2.08	.65	.31	1.22	20.2
1962	17.0	14.6	2.22	.75	.34	1.16	19.7
1963	15.8	15.4	2.35	.54	.23	1.03	18.8
1964	15.5	16.2	2.39	.69	.29	.96	18.2
1965	15.3	17.1	2.47	.60	.24	.90	17.9
1966	15.7	17.9	2.61	.76	.29	.88	16.8
1967	16.0	19.1	2.70	.75	.28	.84	16.1

Sources: FPC, Sales by Producers of Natural Gas to Interstate Pipeline Companies, AGA, Gas Facts, API, et. al., Joint Association Survey of Industry Drilling Costs.

APPENDIX III: SYNTHETIC DATA ANALYSIS

The purpose of the synthetic data analysis performed on the natural gas model is to determine the presence of any significant bias in the coefficients of the table functions obtained from OLS regression. Bias can occur due to the feedback loop structure of the system, which violates the OLS assumption that each independent variable is exogenous.

To create the synthetic data, noise multipliers were added to the table function equations for cost of exploration COE, percent invested in exploration PIIE, price multiplier PM, demand multiplier DM, and usage supply multiplier USM. The noise multiplier equations take the form:

A CNM.K = SAMPLE (NORMRN (1,.286) ,1,1)

A INM.K = SAMPLE (NORMRN (1,.152) ,1,1)

A PNM.K = SAMPLE (NORMRN (1,.107) ,1,1)

A DNM.K = SAMPLE (NORMRN (1,.045) ,1,1)

A UNM.K = SAMPLE (NORMRN (1,.020) ,1,1)

where SAMPLE is a sample-and-hold function with initial value equal to 1.0, sample interval equal to one year, and subsequent values equal to NORMRN. NORMRN generates random numbers normally distributed with mean equal to 1.0, and standard deviation SD derived directly from the regressions on historical data with the formula:

$$SD = \log_e^{-1} (SD_{regression}) - 1$$

This takes into account the logarithmic form of the data regressions.

The regressions on the historical data reported in Appendix II take the form:

$$\ln (COE_t) = \underset{(.96)}{3.9} - \underset{(.52)}{1.6} \ln (10 * FURR_t) \qquad R^2 = .51, \quad SER = .262$$

$$\ln(10 * PIIE_t) = \underset{(1.49)}{-2.21} + \underset{(.73)}{1.14} \ln(RPR_t) - \underset{(.35)}{.022} \ln(10 * NCP_t/TC_t)$$

$R^2 = .56$, SER = .142

$$\ln(AWP_t/COE_{t-10}) = \underset{(.25)}{1.39} - \underset{(.27)}{1.25} \ln(RPR_t/10) \qquad R^2 = .71, \ \text{SER} = .104$$

$$\ln(UUR_t) = \underset{(.28)}{2.46} - \underset{(.14)}{.56} \ln(AWP_t) + \underset{(.0074)}{.11t} \qquad R^2 = .99, \ \text{SER} = .044$$

$$\ln(USM_t) = \underset{(.11)}{-.65} + \underset{(.038)}{.21} \ln(RPR_t) \qquad R^2 = .78, \ \text{SER} = .020$$

To perform the synthetic data analysis, three sets of synthetic data were created by varying the DT slightly in the model, thus changing the random generation function NORMRN so that a new distribution is created. An example of the synthetic data created by the model is given in Table AIII-1.

The regressions on synthetic data were done with data from the same years as the regressions on historical data (Appendix II)[71] for comparison with the historical coefficients in order to estimate bias. The results of the synthetic data repressions are as follows:

Cost of Exploration

(1) $$\ln(COE_t) = \underset{(.76)}{4.37} - \underset{(.41)}{1.88} \ln(10 * FURR_t) \qquad R^2 = .70, \ \text{SER} = .224$$

(2) $$\ln(COE_t) = \underset{(.57}{3.68} - \underset{(.31)}{1.49} \ln(10 * FURR_t) \qquad R^2 = .72, \ \text{SER} = .173$$

72) Except for the price multiplier, PM, for reasons explained in Chapter IV. Synthetic data was used from the period 2006-2018, when RPR varies from 18-24 years, in the case of the price multiplier PM.

PAGE 2 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 4/28/72 SYNTHETIC DATA,DT=.1

	SW1	SWN
PRESENT	1.000	1.000
ORIGINAL	0.	0.

PAGE 3 FILE GAS2 NATURAL RESOURCE DISCOVERY MODEL 4/28/72 SYNTHETIC DATA,DT=.1

TIME	COE	FURR	PIIE	AWP	TC	RPR	UR	PM	DM	USM
E+00	E-03	E+00	E+00	E-03	E+00	E+00	E+12	E+00	E+00	E+00
1945.	.017	.8167	.27914	.114	.00006	19.403	5.894	1.8078	.9557	.9702
1946.	.014	.8042	.36872	.114	.00005	19.496	6.418	1.9380	.9746	.9801
1947.	.013	.7910	.37442	.113	.00005	19.771	6.695	2.0976	.9520	.9838
1948.	.018	.7772	.34650	.114	.00007	19.785	6.937	2.0818	.9238	.9999
1949.	.016	.7629	.50252	.116	.00006	19.522	7.907	1.8639	.9862	.9961
1950.	.029	.7481	.31574	.119	.00011	19.331	7.537	1.9116	.8804	.9905
1951.	.020	.7326	.31129	.119	.00007	19.500	8.184	1.6589	.8952	.9460
1952.	.023	.7163	.43651	.122	.00008	19.258	9.113	2.0164	.9335	.9896
1953.	.018	.6988	.38767	.124	.00007	19.132	9.290	2.2490	.8912	1.0215
1954.	.025	.6803	.35924	.125	.00009	19.281	9.774	2.2247	.8781	.9876
1955.	.033	.6613	.37593	.127	.00012	19.197	10.727	2.2945	.9025	.9799
1956.	.025	.6417	.35285	.129	.00009	19.160	10.340	2.0791	.8147	1.0065
1957.	.026	.6219	.33102	.134	.00010	18.833	11.215	2.2401	.8275	1.0317
1958.	.027	.6015	.38817	.137	.00010	18.753	12.663	2.2132	.8751	.9781
1959.	.025	.5802	.42400	.141	.00009	18.608	13.742	2.2955	.8893	1.0003
1960.	.032	.5575	.37529	.150	.00012	18.135	14.374	2.7733	.8734	.9974
1961.	.041	.5343	.35158	.150	.00015	18.574	13.420	2.2863	.7889	.9655
1962.	.031	.5112	.39723	.150	.00012	18.495	16.089	2.3438	.8498	1.0064
1963.	.033	.4882	.32528	.150	.00012	17.904	15.740	2.1566	.8175	.9585
1964.	.038	.4654	.21504	.150	.00014	17.427	16.960	2.9495	.8090	.9774
1965.	.041	.4431	.31048	.150	.00015	16.669	18.383	2.7985	.8523	.9417
1966.	.060	.4211	.28384	.150	.00022	15.881	18.263	3.2670	.7831	.9535
1967.	.064	.3991	.30984	.150	.00024	15.206	19.651	3.3078	.8158	.9224
1968.	.089	.3776	.12671	.150	.00033	14.703	22.972	3.7302	.8946	.9203
1969.	.082	.3568	.19852	.150	.00030	14.112	22.097	3.2001	.8156	.9098
1970.	.093	.3371	.16032	.150	.00035	13.191	23.490	3.3178	.8399	.8796

FIGURE AIII-1: EXAMPLE OF SYNTHETIC DATA

(3) $\ln (COE_t) = 3.12 - 1.23 \ln (10 * FURR_t)$ $\qquad R^2 = .26, \quad SER = .365$

$\qquad\qquad (1.28) \quad (.69)$

AVE: $\ln (COE_t) = 3.73 - 1.53 \ln (10 * FURR_t)$

<u>Percent Invested in Exploration</u>

(1) $\ln (10 * PIIE_t) = -3.97 + 1.95 \ln (RPR_t) - .115 \ln (10 * NCP_t/TC_t)$

$\qquad\qquad (5.23) \quad (3.08) \qquad\qquad (1.46)$

$R^2 = .56, \quad SER = .142$

(2) $\ln (10 * PIIE_t) = -0.460 - 0.135 \ln (RPR_t) + 0.814 \ln (10 * NCP_t/TC_t)$

$\qquad\qquad (2.87) \quad (1.55) \qquad\qquad (.694)$

$R^2 = .59, \quad SER = .149$

(3) $\ln (10 * PIIE_t) = -2.35 + 1.65 \ln (RPR_t) - 0.412 \ln (10 * NCP_t/TC_t)$

$\qquad\qquad (2.96) \quad (1.67) \qquad\qquad (.733)$

$R^2 = .30, \quad SER = .127$

<u>Price Multiplier</u> (Model Linear in range RPR = 18-24 years):

(1) $PM_t - 5.58 - 0.182 \, RPR_t$ $\qquad R^2 = .85, \quad SER = .176$

$\qquad (.637) \quad (.031)$

(2) $PM_t = 5.67 - 0.178 \, RPR_t$ $\qquad R^2 = .73, \quad SER = .289$

$\qquad (.940) \quad (.0445)$

(3) $PM_t = 4.83 - 0.136 \, RPR_t$ $\qquad R^2 = .62, \quad SER = .259$

$\qquad (.903) \quad (.0435)$

AVE: $PM_t = 5.36 - 0.165 \, RPR_t$

<u>Note</u>: Model Assumes $PM_t = 5.35 - .167 \, RPR_t$ when RPR_t varies from 18-24 years.

<u>Demand Multiplier</u> (Model linear in region AWP = 10 - 20 ¢/Mcf):

(1) $DM_t = \underset{(.138)}{1.33} - \underset{(.0109)}{0.0342}\ AWP_t$ $R^2 = .42$, SER = 0.481

(2) $DM_t = \underset{(.120)}{1.41} - \underset{(.0092)}{0.0400}\ AWP_t$ $R^2 = .58$, SER = 0.0428

(3) $DM_t = \underset{(.119)}{1.29} - \underset{(.0094)}{0.0316}\ AWP_t$ $R^2 = .45$, SER = 0.0391

AVE: $DM_t = 1.35 - .0352\ AWP_t$

<u>Note</u>: Model Assumed $DM_t = 1.4 - .04\ AWP_t$ when AWP_t varied from 10-20 ¢/Mcf when this analysis was done. Since then, a logarithmic table function for Demand Multiplier has been incorporated (see Appendix IV).

<u>Usage Supply Multiplier</u>

(1) $\ln(USM_t) = \underset{(.238)}{-0.631} + \underset{(.0857)}{0.206} \ln(RPR_t)$ $R^2 = .39$, SER = .0336

(2) $\ln(USM_t) = \underset{(.139)}{-0.803} + \underset{(.0496)}{0.272} \ln(RPR_t)$ $R^2 = .77$, SER = .0184

(3) $\ln(USM_t) = \underset{(.154)}{-0.613} + \underset{(.0560)}{0.199} \ln(RPR_t)$ $R^2 = .58$, SER = .0196

AVE: $\ln(USM_t) = -0.682 + 0.226 \ln(RPR_t)$.

Historical Data Regression gives:

$\ln(USM_t) = \underset{(.107)}{-0.653} + \underset{(.038)}{0.213} \ln(RPR_t)$ $R^2 = .78$, SER = .0200

APPENDIX IV: LISTING OF COMPUTER PROGRAM

```
UPR.K=UPR.J+(DT)(-DR.JK)                                1, L
UPR=UPRI                                                1.1, N
UPRI=1.04E15                                            1.2, C
    UPR     - UNPROVEN RESERVES (CUBIC FEET)
    DT      - TIME INCREMENT BETWEEN CALCULATIONS (YEARS)
    DR      - DISCOVERY RATE (CUBIC FEET/YEAR)
    UPRI    - UNPROVEN RESERVES INITIAL (CUBIC FEET)

FURR.K=UPR.K/UPRI                                       2, A
    FURR    - FRACTION OF UNPROVEN RESERVES REMAINING
                (DIMENSIONLESS)
    UPR     - UNPROVEN RESERVES (CUBIC FEET)
    UPRI    - UNPROVEN RESERVES INITIAL (CUBIC FEET)

COE.K=TABHL(COET,LOGN(10*FURR.K),-3.5,2.5,.5)*          3, A
  (COEN)*(CNM.K)
COEN=1E-5                                               3.1, C
COET=1.3E4/6E3/2.7E3/1E3/545/245/110/50/22/9.98/        3.2, T
  4.48/2.02/.91
    COE     - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    COET    - COST OF EXPLORATION TABLE
    LOGN    - NATURAL LOGARITHMIC FUNCTION
    FURR    - FRACTION OF UNPROVEN RESERVES REMAINING
                (DIMENSIONLESS)
    COEN    - COST OF EXPLORATION NORMAL (DOLLARS/CUBIC
                FOOT)
    CNM     - COST NOISE MULTIPLIER (DIMENSIONLESS)

TC.K=(MAR)(COE.K)                                       4, A
MAR=3.7                                                 4.1, C
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    MAR     - COST MARGIN (DIMENSIONLESS)
    COE     - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)

NCP.K=SWITCH(UNCP.K,RNCP.K,SW1)                         5, A
    NCP     - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
    SWITCH  - FUNCTION WHOSE VALUE IS SET INITIALLY BY
                ANALYST
    UNCP    - UNREGULATED NEW CONTRACT PRICE (DOLLARS/
                CUBIC FOOT)
    RNCP    - REGULATED NEW CONTRACT PRICE (DOLLARS/CUBIC
                FOOT)
    SW1     - REGULATION SWITCH

UNCP.K=(TC.K)(PM.K)                                     6, A
    UNCP    - UNREGULATED NEW CONTRACT PRICE (DOLLARS/
                CUBIC FOOT)
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    PM      - PRICE MULTIPLIER (DIMENSIONLESS)
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RNCP.K=CLIP(MAX(RP.K,DTC.K),UNCP.K,TIME.K,1955)        7, A
    RNCP    - REGULATED NEW CONTRACT PRICE (DOLLARS/CUBIC
                FOOT)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    MAX     - FUNCTION WHICH CHOOSES THE MAXIMUM OF TWO
                ARGUMENTS
    RP      - REGULATION PRICE SCHEDULE (DOLLARS/CUBIC
                FOOT)
    DTC     - DELAYED TOTAL COST (DOLLARS/CUBIC FOOT)
    UNCP    - UNREGULATED NEW CONTRACT PRICE (DOLLARS/
                CUBIC FOOT)
    TIME    - TIME (YEARS)

RP.K=TABHL(TRP,TIME.K,1955,1964,9)*(1E-4)              8, A
TRP=1.95/1.6                                           8.1, T
    RP      - REGULATION PRICE SCHEDULE (DOLLARS/CUBIC
                FOOT)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    TRP     - REGULATED PRICE SCHEDULE TABLE
    TIME    - TIME (YEARS)

DTC.K=DELAY3(TC.K,REGD)                                9, A
REGD=4                                                 9.1, C
    DTC     - DELAYED TOTAL COST (DOLLARS/CUBIC FOOT)
    DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    REGD    - REGULATION DELAY (YEARS)

AWP.K=SWITCH(UAWP.K,RAWP.K,SW1)                        10, A
    AWP     - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)
    SWITCH  - FUNCTION WHOSE VALUE IS SET INITIALLY BY
                ANALYST
    UAWP    - UNREGULATED AVERAGE WELLHEAD PRICE
                (DOLLARS/CUBIC FOOT)
    RAWP    - REGULATED AVERAGE WELLHEAD PRICE (DOLLARS/
                CUBIC FOOT)
    SW1     - REGULATION SWITCH

UAWP.K=DELAY3(NCP.K,PAD.K)                             11, A
    UAWP    - UNREGULATED AVERAGE WELLHEAD PRICE
                (DOLLARS/CUBIC FOOT)
    DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
    NCP     - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
    PAD     - PRICE AVERAGING DELAY (YEARS)
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RAWP.K=CLIP(MAX(P60,STC.K),UAWP.K,TIME.K,1960)        12, A
P60=1.5E-4                                            12.1, C
    RAWP    - REGULATED AVERAGE WELLHEAD PRICE (DOLLARS/
                CUBIC FOOT)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    MAX     - FUNCTION WHICH CHOOSES THE MAXIMUM OF TWO
                ARGUMENTS
    P60     - CEILING PRICE SET IN 1960 (DOLLARS/CUBIC
                FOOT)
    STC     - SMOOTHED TOTAL COST (DOLLARS/CUBIC FOOT)
    UAWP    - UNREGULATED AVERAGE WELLHEAD PRICE
                (DOLLARS/CUBIC FOOT)
    TIME    - TIME (YEARS)

STC.K=DELAY3(TC.K,PAD.K)                              13, A
    STC     - SMOOTHED TOTAL COST (DOLLARS/CUBIC FOOT)
    DELAY3  - TERM DENOTING A LAGGED RELATIONSHIP
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
    PAD     - PRICE AVERAGING DELAY (YEARS)

PAD.K=RPR.K                                           14, A
    PAD     - PRICE AVERAGING DELAY (YEARS)
    RPR     - RESERVE-PRODUCTION RATIO (YEARS)

SR.K=(UR.JK)(AWP.K)                                   15, A
    SR      - SALES REVENUE (DOLLARS)
    UR      - USAGE RATE (CUBIC FEET/YEAR)
    AWP     - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)

IIE.K=(PIIE.K)(SR.K)                                  16, A
    IIE     - INVESTMENT IN EXPLORATION (DOLLARS)
    PIIE    - PERCENT INVESTED IN EXPLORATION (FRACTION)
    SR      - SALES REVENUE (DOLLARS)

PIIE.K=TABHL(PIIET,RPR.K/DRPR,.2,2.0,.2)*(ROIM.K)*    17, A
  (INM.K)
PIIET=.5/.5/.5/.48/.39/.24/.12/.05/.01/0              17.1, T
    PIIE    - PERCENT INVESTED IN EXPLORATION (FRACTION)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    PIIET   - PERCENT INVESTED IN EXPLORATION TABLE
    RPR     - RESERVE-PRODUCTION RATIO (YEARS)
    DRPR    - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
    ROIM    - RETURN ON INVESTMENT MULTIPLIER
                (DIMENSIONLESS)
    INM     - INVESTMENT NOISE MULTIPLIER (DIMENSIONLESS)

ROIM.K=TABHL(ROIMT,NCP.K/TC.K,0,2.2,.2)               18, A
ROIMT=0/.08/.25/.44/.55/.67/.76/.82/.88/.92/.96/1     18.1, T
    ROIM    - RETURN ON INVESTMENT MULTIPLIER
                (DIMENSIONLESS)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    ROIMT   - RETURN ON INVESTMENT MULTIPLIER TABLE
    NCP     - NEW CONTRACT PRICE (DOLLARS/CUBIC FOOT)
    TC      - TOTAL COST (DOLLARS/CUBIC FOOT)
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DR.KL=DELAY3(IIE.K/COE.K,DD)                               19, R
DD=4.5                                                     19.1, C
    DR       - DISCOVERY RATE (CUBIC FEET/YEAR)
    DELAY3 - TERM DENOTING A LAGGED RELATIONSHIP
    IIE      - INVESTMENT IN EXPLORATION (DOLLARS)
    COE      - COST OF EXPLORATION (DOLLARS/CUBIC FOOT)
    DD       - DISCOVERY DELAY (YEARS)

PR.K=PR.J+(DT)(DR.JK-UR.JK)                                20, L
PR=PRI                                                     20.1, N
PRI=6.4E12                                                 20.2, C
    PR       - PROVEN RESERVES (CUBIC FEET)
    DT       - TIME INCREMENT BETWEEN CALCULATIONS (YEARS)
    DR       - DISCOVERY RATE (CUBIC FEET/YEAR)
    UR       - USAGE RATE (CUBIC FEET/YEAR)
    PRI      - PROVEN RESERVES INITIAL (CUBIC FEET)

AUR.K=SMOOTH(UR.JK,AURAD)                                  21, A
AUR=AURI                                                   21.1, N
AURI=3.2E11                                                21.2, C
AURAD=1                                                    21.3, C
    AUR      - AVERAGE USAGE RATE (CUBIC FEET/YEAR)
    SMOOTH - FIRST-ORDER EXPONENTIAL SMOOTHING FUNCTION
    UR       - USAGE RATE (CUBIC FEET/YEAR)
    AURAD    - AVERAGE USAGE RATE ADJUSTMENT DELAY (YEARS)
    AURI     - AVERAGE USAGE RATE INITIAL (CUBIC FEET/
                YEAR)

RPR.K=PR.K/AUR.K                                           22, A
DRPR=20                                                    22.1, C
    RPR      - RESERVE-PRODUCTION RATIO (YEARS)
    PR       - PROVEN RESERVES (CUBIC FEET)
    AUR      - AVERAGE USAGE RATE (CUBIC FEET/YEAR)
    DRPR     - DESIRED RESERVE-PRODUCTION RATIO (YEARS)

PM.K=TABHL(PMT,RPR.K/DRPR,0,1.8,.3)*(PNM.K)                23, A
PMT=8/5.5/3.75/2.35/1.35/1/1                               23.1, T
    PM       - PRICE MULTIPLIER (DIMENSIONLESS)
    TABHL    - TERM DENOTING A TABULAR RELATIONSHIP
    PMT      - PRICE MULTIPLIER TABLE
    RPR      - RESERVE-PRODUCTION RATIO (YEARS)
    DRPR     - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
    PNM      - PRICE NOISE MULTIPLIER (DIMENSIONLESS)

DM.K=TABHL(DMT,LOGN(AWP.K*1E5),1,8.5,.5)*(DNM.K)           24, A
DMT=2.1/1.59/1.21/.9/.69/.5/.24/.14/.067/.031/.014/        24.1, T
  5.5E-3/1.5E-3/2E-4/1.7E-5/0
    DM       - DEMAND MULTIPLIER (DIMENSIONLESS)
    TABHL    - TERM DENOTING A TABULAR RELATIONSHIP
    DMT      - DEMAND MULTIPLIER TABLE
    LOGN     - NATURAL LOGARITHMIC FUNCTION
    AWP      - AVERAGE WELLHEAD PRICE (DOLLARS/CUBIC FOOT)
    DNM      - DEMAND NOISE MULTIPLIER (DIMENSIONLESS)
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URP1.K=URI*EXP(GC*(TIME.K-1900))                          25, A
GC=.0657                                                  25.1, C
URI=3.2E11                                                25.2, C
TIME=1900                                                 25.3, N
    URP1    - NORMAL POTENTIAL USAGE RATE (CUBIC FEET/
                YEAR)
    URI     - USAGE RATE INITIAL (CUBIC FEET/YEAR)
    EXP     - EXPONENTIAL FUNCTION
    GC      - GROWTH CONSTANT (FRACTION)
    TIME    - TIME (YEARS)

USM.K=TABHL(USMT,RPR.K/DRPR,0,2,.2)*(UNM.K)               26, A
USMT=0/.12/.7/.86/.95/1/1.02/1.04/1.05/1.06/1.06          26.1, T
    USM     - USAGE SUPPLY MULTIPLIER (DIMENSIONLESS)
    TABHL   - TERM DENOTING A TABULAR RELATIONSHIP
    USMT    - USAGE SUPPLY MULTIPLIER TABLE
    RPR     - RESERVE-PRODUCTION RATIO (YEARS)
    DRPR    - DESIRED RESERVE-PRODUCTION RATIO (YEARS)
    UNM     - USAGE NOISE MULTIPLIER (DIMENSIONLESS)

UR.KL=SWITCH(UUR.K,RUR.K,SW1)                             27, R
    UR      - USAGE RATE (CUBIC FEET/YEAR)
    SWITCH  - FUNCTION WHOSE VALUE IS SET INITIALLY BY
                ANALYST
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    RUR     - REGULATED USAGE RATE (CUBIC FEET/YEAR)
    SW1     - REGULATION SWITCH

UUR.K=(URP.K)(DM.K)                                       28, A
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    URP     - USAGE RATE POTENTIAL (CUBIC FEET/YEAR)
    DM      - DEMAND MULTIPLIER (DIMENSIONLESS)

RUR.K=CLIP((USM.K)(UUR.K),UUR.K,TIME.K,1960)              29, A
    RUR     - REGULATED USAGE RATE (CUBIC FEET/YEAR)
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    USM     - USAGE SUPPLY MULTIPLIER (DIMENSIONLESS)
    UUR     - UNREGULATED USAGE RATE (CUBIC FEET/YEAR)
    TIME    - TIME (YEARS)

PRTPER.K=CLIP(0,PRTMAX.K,BTIME,TIME.K)                    31, A
    PRTPER  - VARIABLE PRINT PERIOD
    CLIP    - FUNCTION WHOSE VALUE CHANGES DURING RUN
    PRTMAX  - VARIABLE VALUE OF PRINT PERIOD
    BTIME   - BEGINNING TIME FOR PRINTOUT
    TIME    - TIME (YEARS)
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PRTMAX.K=CLIP(1,0,ETIME,TIME.K)                                   32, A
BTIME=1945                                                        32.1, C
ETIME=1970                                                        32.2, C
    PRTMAX - VARIABLE VALUE OF PRINT PERIOD
    CLIP   - FUNCTION WHOSE VALUE CHANGES DURING RUN
    ETIME  - END TIME FOR PRINTOUT
    TIME   - TIME (YEARS)
    BTIME  - BEGINNING TIME FOR PRINTOUT

URP2.K=UR2I*EXP(GC2*(TIME.K-1970))                                33, A
UR2I=3.979E13                                                     33.1, C
    URP2   - POTENTIAL USAGE RATE AFTER CHANGE IN GROWTH
               RATE (CUBIC FEET/YEA
    UR2I   - VALUE OF POTENTIAL USAGE RATE IN 1970
               (CUBIC FEET/YEAR)
    EXP    - EXPONENTIAL FUNCTION
    GC2    - NEW GROWTH RATE EFFECTIVE IN 1970
               (FRACTION)
    TIME   - TIME (YEARS)

URP3.K=CLIP(URP2.K,URP1.K,TIME.K,1970)                            34, A
    URP3   - POTENTIAL USAGE RATE WITH CHANGE IN GC IN
               1970 (CUBIC FEET/YEAR)
    CLIP   - FUNCTION WHOSE VALUE CHANGES DURING RUN
    URP2   - POTENTIAL USAGE RATE AFTER CHANGE IN GROWTH
               RATE (CUBIC FEET/YEA
    URP1   - NORMAL POTENTIAL USAGE RATE (CUBIC FEET/
               YEAR)
    TIME   - TIME (YEARS)
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URP.K=SWITCH(URP1.K,URP3.K,SW2)                                 35, A
GC2=.02                                                         35.1, C
SW1=0                                                           35.5, C
SW2=0                                                           35.6, C
SWN=0                                                           35.7, C
    URP    - USAGE RATE POTENTIAL (CUBIC FEET/YEAR)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    URP1   - NORMAL POTENTIAL USAGE RATE (CUBIC FEET/
               YEAR)
    URP3   - POTENTIAL USAGE RATE WITH CHANGE IN GC IN
               1970 (CUBIC FEET/YEAR)
    SW2    - GROWTH RATE SWITCH
    GC2    - NEW GROWTH RATE EFFECTIVE IN 1970
               (FRACTION)
    SW1    - REGULATION SWITCH
    SWN    - NOISE SWITCH

INM.K=SWITCH(1,SAMPLE(NORMRN(1,.15),1,1),SWN)                   37, A
    INM    - INVESTMENT NOISE MULTIPLIER (DIMENSIONLESS)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    SAMPLE - SAMPLE AND HOLD FUNCTION
    NORMRN - RANDOM NOISE FUNCTION
    SWN    - NOISE SWITCH

CNM.K=SWITCH(1,SAMPLE(NORMRN(1,.26),1,1),SWN)                   38, A
    CNM    - COST NOISE MULTIPLIER (DIMENSIONLESS)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    SAMPLE - SAMPLE AND HOLD FUNCTION
    NORMRN - RANDOM NOISE FUNCTION
    SWN    - NOISE SWITCH

UNM.K=SWITCH(1,SAMPLE(NORMRN(1,.02),1,1),SWN)                   39, A
    UNM    - USAGE NOISE MULTIPLIER (DIMENSIONLESS)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    SAMPLE - SAMPLE AND HOLD FUNCTION
    NORMRN - RANDOM NOISE FUNCTION
    SWN    - NOISE SWITCH

PNM.K=SWITCH(1,SAMPLE(NORMRN(1,.11),1,1),SWN)                   40, A
    PNM    - PRICE NOISE MULTIPLIER (DIMENSIONLESS)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    SAMPLE - SAMPLE AND HOLD FUNCTION
    NORMRN - RANDOM NOISE FUNCTION
    SWN    - NOISE SWITCH

DNM.K=SWITCH(1,SAMPLE(NORMRN(1,.045),1,1),SWN)                  41, A
    DNM    - DEMAND NOISE MULTIPLIER (DIMENSIONLESS)
    SWITCH - FUNCTION WHOSE VALUE IS SET INITIALLY BY
               ANALYST
    SAMPLE - SAMPLE AND HOLD FUNCTION
    NORMRN - RANDOM NOISE FUNCTION
    SWN    - NOISE SWITCH
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