THE SOLID STATE STRUCTURE AND PROPERTIES
OF STIFF CHAIN ARAMIDS

by<br>GREGORY CHARLES RUTLEDGE<br>Bachelor of Science in Chemical Engineering University of Virginia (1983)<br>Submitted to the Department of Chemical Engineering in Partial Fulfillment of the Requirements for the Degree of<br>DOCTOR OF PHILOSOPHY<br>at the<br>MASSACHUSETTS INSTITUTE OF TECHNOLOGY<br>(January, 1990)<br>© Massachusetts Institute of Technology, 1990

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Professor Ulrich W. Suter Thesis Supervisor

Accepted by $\qquad$
'The time has come,' the Walrus said, 'To talk of many things:
Of shoes-and ships-and sealing waxOf cabbages-and kings-
And why the sea is boiling hotand whether pigs have wings.'

Lewis Carroll, The Walrus and the Carpenter

# THE SOLID STATE STRUCTURE AND PROPERTIES OF STIFF CHAIN ARAMIDS 

by<br>Gregory Charles Rutledge<br>Submitted to the Department of Chemical Engineering on January 18, 1990 in partial fulfillment of the requirements for the degree of Doctor of Philosophy


#### Abstract

Stiff chain aramids are polymers composed of alternating aromatic ring and amide moieties which, due to the rigidity of the individual components and the parallel propagation of the chain contour, exhibit rodlike behavior in the individual macromolecule. This rodlike behavior leads to the formation of nematic liquid crystal mesophases above a critical concentration of polymer in solution and enables the realization of unusually high degrees of molecular orientation and crystallization in spun fibers. These aramids are characterized by high melting points and good thermal stability up to $550^{\circ} \mathrm{C}$, and the practicality of high orientation of fully extended chains makes their fibers attractive as high strength, high modulus reinforcing materials. This work addresses the topic of the solid state structure and properties of such molecular architectures, with specific emphasis on poly(p-phenylene terephthalamide) (PPTA), at the scale of atomic level organization.


A static atomistic model was developed to simulate the geometry and potential energy characteristics in the ordered solid state of a crystal-forming polymer. This model represents the local atomic structure as an imperfect pseudocrystalline parallelepiped embedded within a larger crystalline matrix. It requires no adjustable parameters. Minimization of potential energy simultaneously with respect to both intramolecular and intermolecular degrees of freedom permits the prediction of detailed atomic structure and cohesive energy and estimation of the Hildebrand solubility parameter. Quantitative prediction and diagrammatic representation of the x-ray scattering pattern from these structures was made. Application of the simulation methods to PPTA yielded a plurality of viable polymorphs having cohesive energies on the order of $39 \mathrm{kcal} /$ mole of repeat units; both energy and (in select cases) geometry predictions were in agreement with available literature information. Detailed analysis of interchain interactions was used to demonstrate the hierarchy of the primary behaviors leading to the formation of hydrogen-bonded sheets and three dimensional structure.

A series of simulated deformations of the minimum energy structures was used to predict the full twenty-one independent elements of the stiffness and compliance tensors. Thermodynamic and statistical mechanical analyses demonstrate that our values, calculated from considerations of potential energy only, overestimate the true elastic constants by $3 \%$ to $27 \%$. Elastic properties of aramid fibers were estimated by symmetrizing the single crystal elastic tensors in the limits of uniform distribution of strain and stress to yield Voigt and Reuss bounds, respectively, for the elastic moduli. For one structure of PPTA the calculated extensional, transverse, and torsional moduli are in the ranges $215-325 \mathrm{GPa}, 5.2-21.1 \mathrm{GPa}$, and 4.1-13.3 GPa, respectively, in good agreement with observed values.

Validation of the model was performed through an experimental program which included the design and construction of a dry-jet wet spinning apparatus suitable for the production of highly oriented crystalline monofilament of $20 \mu \mathrm{~m}$ to $40 \mu \mathrm{~m}$ dry fiber diameter from quantities of polymer less than one gram. Analysis by wide angle $x$-ray scattering of as-spun and annealed fibers confirmed the polymorphic behavior of PPTA and the accuracy of the structure predictions from the simulation.

The effects of atomic scale modification were addressed through the simulation of chlorinated PPTA's. Polymer chains having chlorines substituted in the 2 and 6 positions of the phenylene ring and their regular isomeric forms exhibiting entirely head-to-head, tail-to-tail or entirely head-to-tail linkages of asymmetric units were considered by simulation. Cohesive energies roughly $90 \%$ to $93 \%$ of that for

PPTA were found for these modified polymers. The primary cause for this reduction is the expansion of the atomic lattice. The hierarchy of structure-determining factors and the estimated fiber moduli are little changed from their PPTA counterparts.

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## TABLE OF CONTENTS

TITLE PAGE
Page
ABSTRACT ..... 3
ACKNOWLEDGEMENTS ..... 5
TABLE OF CONTENTS ..... 6
LIST OF FIGURES ..... 10
LIST OF TABLES ..... 13

1. INTRODUCTION ..... 15
1.1 Motivation ..... 15
1.2 Statement of Objectives ..... 17
2. STATE OF THE ART ..... 19
2.1 Aramids ..... 19
2.1.1 Early Development ..... 19
2.1.2 Chain Rigidity and Solution Structure ..... 22
2.1.3 Constitutional Isomerism ..... 26
2.1.3.1 Definition ..... 26
2.1.3.2 Kinetic Model and Homogeneous Phase Amidation ..... 27
2.2 Solid State Structure and Properties ..... 29
2.2.1 General ..... 29
2.2.2 Aramid Solid State ..... 32
2.2.3 Processing ..... 33
2.3 Atomistic Simulation of Dense Polymer Systems ..... 40
2.2.1 General ..... 40
2.2.2 Polymer Glasses ..... 40
2.2.3 Polymer Crystals ..... 41
3. ATOMISTIC SIMULATION OF ORDERED STRUCTURE ..... 44
3.1 General Approach ..... 44
3.2 Structural Representation ..... 44
3.2.1 Chain Generation and Helix Alignment ..... 44
3.2.2 Multichain Packing ..... 47
3.3 Energy Representation ..... 49
3.3.1 Explicit Description ..... 49
3.3.2 Lattice Summation ..... 51
3.3.3 Force Field ..... 53
3.4 Prediction of X-ray Scattering ..... 57
3.4.1 Structural Contributions to Intensity ..... 57
3.4.2 Modifications to the Structure Factor Intensity ..... 60
4. APPLICATION TO POLY(P-PHENYLENE TEREPHTHALAMIDE) ..... 64
4.1 Introduction ..... 64
4.2 Chain Description ..... 65
4.3 Parameterization of the Force Field ..... 65
4.4 Simulation Procedure ..... 69
4.5 PPTA Simulation Results: Single Chain ..... 75
4.6 PPTA Simulation Results: Multichain ..... 75
4.6.1 Selection and Presentation of Results ..... 75
4.6.2 Evaluation of Polymorphs ..... 85
4.6.3 Calculation of X-ray Scattering ..... 90
4.7 Summary ..... 94
5. MECHANICAL PROPERTIES BY SIMULATION ..... 96
5.1 Thermodynamic Analysis ..... 96
5.1.1 Classical Thermodynamics of Deformation ..... 96
5.1.2 Statistical Mechanics of Deformation ..... 99
5.1.3 Estimation of Entropic Contributions ..... 104
5.2 Method of Calculation ..... 107
5.3 Elasticity of Poly(p-phenylene terephthalamide) ..... 108
5.3.1 Modification of the Chain Description ..... 108
5.3.2 Determination of Crystallite Compliance and Stiffness Matrices ..... 110
5.3.3 Isolated Chain Compliance and Comparison to Crystallite Compliance ..... 112
5.3.4 Consideration of a Second Allomorph ..... 114
5.4 Moduli Involving Fiber Symmetry ..... 116
5.4.1 Derivation of Fiber Symmetry Relations ..... 116
5.4.2 Voigt and Reuss Limits for PPTA ..... 119
5.5 Estimation of Resistance to Shear ..... 121
6. FIBER SPINNING AND X-RAY EVALUATION ..... 125
6.1 Objective ..... 125
6.2 Equipment Design ..... 125
6.2.1 Preliminary Considerations ..... 125
6.2.2 Design Specifications ..... 127
6.3 Experimental Procedure ..... 128
6.3.1 Dope Preparation ..... 128
6.3.2 Fiber Formation ..... 131
6.3.3 Heat Treatment ..... 131
6.4 Spinning Trials for PPTA ..... 132
6.5 X-ray Analysis ..... 136
6.5.1 Equipment and Procedures ..... 136
6.5.2 Methods of Data Analysis ..... 138
6.5.3 Evaluation of PPTA Fibers and Comparison to Simulation ..... 139
7. EXTENSION TO ISOMERS BASED ON DICHLORO-SUBSTITUTED MONOMERS ..... 145
7.1 Introduction ..... 145
7.2 Chain and Force Field Modification ..... 146
7.3 Simulation Results: Single Chain ..... 151
7.4 Simulation Results: Multichain ..... 154
7.4.1 Approaches to Identification of Minima ..... 154
7.4.2 Packing of Head-to-Head, Tail-to-Tail Isomers ..... 154
7.4.3 Packing of Head-to-Tail Isomers ..... 161
7.5 Prediction of Fiber Mechanical Properties ..... 172
8. CONCLUSION ..... 175
8.1 Summary ..... 175
8.2 Recommendations for Future Work ..... 177
NOTATION ..... 179
ABBREVIATIONS ..... 184
REFERENCES ..... 185
APPENDICES ..... 192
APPENDIX A: Helix Representation of Chain Conformation ..... 192
APPENDIX B: Summation Representation for a Periodic Structure and ..... 198 Convergence of Terms
APPENDIX C: Calculations using Semi-Empirical Quantum Mechanical Methods ..... 203
APPENDIX D: Atomic Scattering Factor Functions ..... 223
APPENDIX E: Detailed X-ray Scattering Intensities from Simulated Structures ..... 225
and Simulated X-ray Fiber Diffraction Patterns for Chlorine- modified Isomers
APPENDIX F: Details of Apparatus for Fiber Spinning ..... 239
APPENDIX G: FORTRAN Listing of Program for Simulation of Ordered ..... 256 Polymer Structure
APPENDIX H: FORTRAN Listing of Program to Calculate and Plot X-ray ..... 328 Scattering Diagrams (IRIS 4D-GT Version)

## LIST OF FIGURES

Figure Page
$\begin{array}{lll}\text { 1.1: } & \begin{array}{l}\text { Plot of specific strength versus specific modulus for a variety of } \\ \text { fibers [Adams and Eby; 1987]. }\end{array} & 16\end{array}$
2.1: Common aramid polymers. 20
2.2: Critical volume concentration $\mathrm{Vp}^{*}$ versus axial ratio and comparison 24 between threory and experiments for PBA and PPTA [Flory, 1984].
2.3: Schematic representations of ordered phase formations in polymers:
(a) semicrystalline matrix of unoriented crystallites embedded in an amorphous matrix; (b) quasinematic alignment of semicrystal domains; (c) microfibril paracrystal.
2.4: The two reported crystal structures for poly(p-phenylene terephthalamide): (a) Modification I [Northolt, 1974]; (b) Modification II [Haraguchi et al., 1979].
2.5: Effect of heat treament of a few seconds duration on properties of poly(1,4-benzamide) fibers [Kwolek et al., 1977].
3.1: Definition of chain packing parameters.
$\qquad$


a
3.2: Representation of the packing structure as a local domain, described explicitly in terms of mutable atomic positions, embedded within an implicit perfect crystal.
4.1: $\begin{aligned} & \text { A segment of poly(p-phenylene terephthalamide) (PPTA) with all } \\ & \text { torsion angles in their zero positions. }\end{aligned}$
4.2: Rotation energy functions suggested for the phenylene ring connected to the amide nitrogen in benzanilide: (a) $\pi$-bond additivity [Tashiro et al., 1977]; (b) empirical fit [Hummel and Flory, 1980]; (c) PCILO calculation [Lauprêtre and Monnerie, 1978]; (d) AM1 calculation (this work); (e) composite force field (this work).
4.3: Rotation energy functions suggested for the phenylene ring connected to the amide carbon in benzanilide: (a) $\pi$-bond additivity [Tashiro et al., 1977]; (b) empirical fit [Hummel and Flory, 1980]; (c) PCILO calculation [Lauprêtre and Monnerie, 1978]; (d) AM1 calculation (this work); (e) composite force field (this work).

4.4: $\quad$| Potential energy contour for benzanilide as a function of the ring |
| :--- |
| torsions $\phi_{1}$ and $\phi_{3}$. |$\quad 76$

4.5: Energy contours for packing of extended conformations of two
enendently orientable chains of like conformation of PPTA, as a
function of the two setting angles $\omega_{1}$ and $\omega_{2}$.
4.6: Orthographic perspectives of the eight primary structures for PPTA
suggested by simulation. The sets (a) through (h) correspond to the
structures labelled 1 through 8, in order, in Table 4.6. Left: z-axis
perspective; Midde: perspective perpendicular to the ac-plane; Right:
perspective perpendicular to the previous two perspectives.
4.7: $\quad$ Simulated $x$-ray fiber diffraction patterns for the eight primary struc-
tures for PPTA. The sets (a) through (h) correspond to the structures labelled 1 through 8, in order, in Table 4.6.
5.1 Illustration of the mode of least resistance to shear deformation in the simulated PPTA Structure \#3. Shown are the unit cell, the orientation of the [102] crystallographic planes which contain this deformation mode, and the principle axes describing the orientation of deformation within this plane.
6.1: $\quad$ Schematic of the experimental program for polymer dissolution, fiber spinning, and post-spin heat treatment.
6.2: $\quad$ Shear stress at the wall and die swell as functions of shear rate at the wall for the spinning of PPTA from a $10 \mathrm{wt} \%$ solution in $100 \%$ $\mathrm{H}_{2} \mathrm{SO}_{4}$ at $30^{\circ} \mathrm{C}$ using a spinnerette of diameter $60 \mu \mathrm{~m}$ and capillary lengh $100 \mu \mathrm{~m}$.
6.3: Experimental and calculated equatorial intensity distributions for a sample of annealed PPTA fiber. The calculated intensity is a summation of fitted Gaussian distributions which best reproduce the experimental trace.
6.4: Representative $x$-ray fiber patterns for PPTA fibers spun during the course of this work: (a) as-spun fiber produced at a nominal drawdown ratio of 3.2 ; (b) same fiber after annealing for 2 sec at $500^{\circ} \mathrm{C}$.
7.1: Segments of poly(2,6-dichloro-p-phenylene terephthalamide) (2,6-DiClPPTA) with all torsion angles in their zero positions: (a) head-tohead, tail-to-tail isomer; (b) head-to-tail isomer.
7.2: Definition of "atom types" for the dihalo-substituted phenylene ring moiety.
7.3: $\quad$ Phenylene ring rotation energy functions suggested for dichlorosubstituted rings.
7.4: Potential energy contour for N -[2,6-dichlorophenyl]-benzamide as a function of the ring torsions $\phi_{1}$ and $\phi_{3}$.
7.5: Potential energy contour for 2,6 -dichlorobenzanilide as a function of the ring torsions $\phi_{1}$ and $\phi_{3}$.
7.6: Orthographic perspectives of the four primary structures for the

HHTT isomers of $2,6-$ DiCl-PPTA suggested by simulation. The sets (a) through (d) correspond to the structures labelled 1 through 4, in order, in Table 7.3. Left: z-axis perspective; Middle: perspective perpendicular to the bc-plane; Right: perspective perpendicular to the previous two perspectives.
7.7: Orthographic perspectives of the ten primary structures for the HT isomers of $2,6-\mathrm{DiCl}$-PPTA suggested by simulation. The sets (a) through (j) correspond to the structures labelled 1 through 10 , in order, in Tables 7.5a and 7.5b. Left: z-axis perspective; Middle: perspective perpendicular to the bc-plane; Right: perspective perpendicular to the previous two perspectives.
A.1: Definition of the vectors $\mathbf{B}, \mathbf{B}^{\prime}, \mathbf{B}^{\prime \prime}, \mathbf{C}$, and $\mathbf{C}^{\prime}$ and the helical param- ..... 193 eters $\rho$ and $\Theta$ (after Sugeta and Miyazawa [1967]).
A.2: Definition of Euler angles for orientation of a chain about its center ..... 197 of gravity.
B.1: Definition of angles describing the relative orientation of two linear ..... 202 charge distributions (from Maitland et al., 1981)
D.1: Atomic scattering factor functions for hydrogen, carbon, nitrogen, ..... 224 oxygen, and chlorine (in order from bottom to top). Points are recommended mean atomic scattering factors from self-consistent or variational wave functions (from the International Tables for X-Ray Crystallography [1968]); curves are fitted functions of the form given by Equation D. 1 using parameters listed in Table D.1.
E.1: $\quad$ Simulated $x$-ray fiber diffraction patterns for the four primary struc- ..... 235 tures for 2,6-DiCl-PPTA, head-to-head, tail-to-tail isomers. The sets (a) through (h) correspond to the structures labelled 1 through 4 , in order, in Table 7.3.
E.2: $\quad$ Simulated $x$-ray fiber diffraction patterns for the ten primary struc- ..... 236 tures for 2,6-DiCl-PPTA, head-to-tail isomers. The sets (a) through (j) correspond to the structures labelled 1 through 10 , in order, in Table 7.5.
F.1: Front and side views of the extrusion apparatus. ..... 241
F.2: $\quad$ Diagram of the fiber spinning extrusion apparatus. ..... 242
F.3: Diagram of the syringe and plunger design to spin polymer dope. ..... 243
F.4: Diagram of the solution pre-filter for homogenization and loading of ..... 244 the solution to the syringe.
F.5: $\quad$ Schematic of the process control and data acquisition loop for fiber ..... 245 spinning.
F.6: Diagram of the fiber heat treatment process step. ..... 246
F.7: Data output from a sample spinning experiment illustrating motor ..... 247 control at various rates. The spinnerette used in this example was $120 \mu \mathrm{~m}$ in diameter.

## LIST OF TABLES

Table ..... Page
2.1: $\quad$ Properties of Kevlar ${ }^{(8)}$ aramid fiber ..... 21
2.2: Conformational behavior of some aramids ..... 25
2.3: $\quad$ Polymer order versus kinetic parameters under different mixing ..... 28 mechanisms
2.4: $\quad$ Crystallographic parameters of previously reported modifications of ..... 35 PPTA
2.5: $\quad$ Some reported properties of aramid fibers as functions of process ..... 38 parameters
4.1: $\quad$ Fixed bond lengths and Valence Force Field (VFF) constants for ..... 68 variable backbone angles
4.2: $\quad$ 6-12 Nonbonded atomic potential parameters and elementary charges ..... 68 used in the electrostatic potential
4.3: Intrinsic torsional energy function parameters ..... 72
4.4: . Starting point mesh used in multichain search-minimization ..... 74
4.5: $\quad$ Most favored isolated chain conformations ..... 77
4.6: Multichain energy minimization results: structural parameters for eight ..... 80 most probable unit cells
4.7: $\quad$ Chain conformations realized in packed structures ..... 86
4.8: Intermolecular bonding energy: component contributions to chain-chain ..... 88 interaction within a sheet (all values in $\mathrm{kcal} /$ mole of repeat units)
4.9: Intermolecular bonding energy: component contributions to chain-chain ..... 88 interaction between sheets (all values in $\mathrm{kcal} /$ mole of repeat units)
5.1: $\quad$ Thermomechanical properties of PPTA and PBA ..... 105
5.2: $\quad$ Strain dependence of Raman frequencies in PPTA ..... 105
5.3: $\quad$ Comparison of the two chain descriptions used for structure and ..... 109 mechanical property calculations
5.4: Distribution of strain energy among the degrees of freedom of the ..... 113 structure for isolated chain and packed chain simulations
5.5: PPTA fiber elastic constants (GPa): theoretical (Structure \#3) and ..... 120 experimental
5.6: PPTA fiber theoretical elastic constants (GPa): Structures \#4 and \#5 ..... 120
6.1: Equipment design specifications ..... 130
6.2: $\quad$ Summary of fiber spinning conditions ..... 135
6.3: Experimentally determined reflections in PPTA fibers produced in this ..... 142 work and those reported in the literature for Modifications I and II
7.1: 6-12 Nonbonded potential parameters and elementary charges used in ..... 149 the electrostatic potential
7.2: Intrinsic torsional energy function parameters ..... 149
7.3: $\quad$ Multichain energy minimization results: structural parameters for four ..... 155 most probable unit cells of 2,6-dichloro-PPTA: head-to-head, tail-to- tail polymorphs
7.4: Chain-to-chain bonding energies for 2,6-dichloro-PPTA: head-to-head, ..... 160 tail-to-tail polymorphs
7.5: Multichain energy minimization results: structural parameters for ten ..... 162 most probable unit cells of 2,6-dichloro-PPTA: head-to-tail poly- morphs
7.6: Chain-to-chain bonding energies for 2,6-dichloro-PPTA: head-to-tail ..... 170 polymorphs
7.7: 2,6-DiCl-PPTA theoretical fiber elastic constants (GPa): HHTT ..... 174
Structures \#2 and \#3; HT Structures \#1, \#5, and \#6
C.1: Tabulated results of semiempirical quantum mechanics calculations ..... 207
C.2: Partial atomic charges from AM1 calculations: unsubstituted com- ..... 214 pounds
C.3: Tabulated results of semiempirical quantum mechanical calculations: ..... 215 dichloro compounds.
C.4: Partial atomic charges from AM1 calculations: dichloro compounds. ..... 222
D.1: Function parameters for atomic scattering factors. ..... 223
E.1: Summary of $x$-ray reflections in molecular simulation structures of ..... 225 PPTA
E.2: $\quad$ Summary of $x$-ray reflections in molecular simulation structures for ..... 229 2,6-dichloro-PPTA; head-to-head, tail-to-tail polymorphs
E.3: $\quad$ Summary of x-ray reflections in molecular simulation structures for ..... 230 2,6-dichloro-PPTA; parallel head-to-tail polymorphs
E.4: $\quad$ Summary of x-ray reflections in molecular simulation structures for ..... 232 2,6-dichloro-PPTA; antiparallel head-to-tail polymorphs
F.l: Equipment parts list for fiber spinning apparatus ..... 240

## 1. INTRODUCTION

### 1.1 Motivation

The rapidly expanding field of rigid rod polymers represents a commercially significant and scientifically fascinating class of new, high performance materials. Interest in this general class of polymers dates back to the work of Morgan and Kwolek in the sixties and seventies on wholly aromatic polyamides which resulted in patents [Kwolek, 1972; Morgan, 1975] to the Dupont Company and eventual commercialization of the trademark fibers Nomex ${ }^{\circledR}$ and Kevlar ${ }^{\oplus}$. Today, this class has expanded to include, in addition to the original polyamides, a wide variety of new polyamides, polyesters, polyhydrazides, polyoxazoles and polythiazoles, to name a few. Commercially, these materials have attracted interest due to their unusual thermal stability and impressive mechanical properties, which hold promise for a new era of light weight, high strength composites and cords; an indication of the impressive strength-to-weight properties of these polymers, relative to conventional reinforcing materials, is illustrated in Figure 1.1 [Adams and Eby, 1987]. In the scientific arena, the rigid rod polymers have stimulated renewed interest in the theoretical and phenomenological ramifications of long range molecular rigidity to melt and solution behavior, processing, and solid state properties. The ease with which high degrees of molecular orientation may be realized and the impressive mechanical properties achievable with such stiff chains suggest the importance of atomic scale structural character to macroscopic structure and properties of the bulk polymer.

Efforts to develop improved high strength organic fibers have followed several paths. Some research has been directed towards modifications of aramids or development of entirely new chemistries to produce polymers which have a different combination of material properties or which may not be susceptible to some of the performance flaws characteristic of aramids, such as hydrolytic instability or poor compressive strength. Other efforts have sought to facilitate or improve processing methods to yield better performing fibers at reduced costs, either through chemical modification or process design. In our own laboratory research into the placement of substituents on the ring moiety of the diamide monomer and control of polymerization over the range of constitutional isomers which may result has led to the production of more soluble polymers which appear to retain the essential rigidity of the unsubstituted molecules. These new polymers are well-characterized in terms of the locus of substituents along the

Figure 1.1: Plot of specific strength versus specific modulus for a variety of fibers [Adams and Eby, 1987].

length of the chain; as such, they provide appropriate model polymers for exploring the effects of atomic level modifications on bulk properties.

Lacking to date is a complete description of the linkage between atomic scale detail and macroscopic material properties for these and other new, high performance polymers. Such understanding is required before one can proceed to the ideal of designed materials for engineered applications. The concept of perceptive design of new materials is a topic of great current interest. The goal is to understand the source of, and by so doing reliably predict, the properties of new materials in the bulk from knowledge of the constituent structure. In the past, such attempts have been largely empirical and phenomenological in nature, devised to mimic nature rather than provide an atomistic basis for understanding. With the ongoing explosion in computational resources, the application of more exacting and detailed approaches to property prediction than have heretofore been possible may now be attempted. Such methods as Molecular Mechanics (MM) and Molecular Dynamics (MD), and even the computationally demanding calculations of quantum mechanics, have been and are currently being developed for idealized conceptualization of increasingly complex systems, beginning with isolated molecules but including in a few instances polymeric liquids and solids of glassy and crystalline structure. However, no such attempt has previously been made to apply these techniques to the unique problems of conformationally constrained stiff chain macromolecules in their highly ordered solid state.

With the congruence of enhanced computational resources and the current emphasis on the design of new materials, the opportunity arises to further the development of novel computational techniques as well as establish a uniquely detailed basis for understanding the connection between structure and properties of a new class of materials. The insights gained should serve to expand our understanding of the fundamental relations between atomic structure and material behavior as well as augment the palette of computational software available for the development of such information. The results obtained should provide some direction for future synthetic efforts towards the next generation of high performance polymers.

### 1.2 Objectives

The goal of this work is to simulate the solid state structure of a commercially relevant aramid and its family of ring-substituted isomers and correlate calculated predictions with experimental results. For this purpose, we have selected the polymer poly(p-phenylene terephthalamide), referred to hereafter as PPTA, and its di-substituted isomers having chlorines on the aromatic ring of either the diamine or diacid monomer in the ortho positions. This broad scope entails the following specific objectives:

1) Development of a static atomistic model of a polymer chain in a highly ordered (i.e. pseudocrystalline, crystalline) solid state using available a priori geometric and energy information characteristic of the constituent subunits of the polymer, without the introduction of adjustable parameters. The model must be applicable to the specific features of extended chain conformation polymers and encompass all features relevant to the quantitative determination of geometry and potential energy.
2) Use of this model to derive predictions of structure, $x$-ray diffraction, selected thermodynamic properties, and detailed mechanical properties of the solid state of a model stiff chain polymer system and the atomic scale source of these properties.
3) Validation of the model by comparison with experimental data on structure and mechanical behavior. For this purpose, this work encompasses an experimental program to design and build an apparatus for spinning small quantities of fiber of sufficient crystalline perfection for comparison to simulation results. The validation procedure entails wide angle x-ray scatcring (WAXS) analysis of such fibers produced under different processing conditions and direct comparison of simulation results with experimental data at both the atomic and the phenomenological levels.
4) Extension of the simulation process to the analysis of derivatives of the parent polymer for purposes of relating discrete atomic-scale variations to observable macroscopic properties. These derivatives may vary in the type of substitution (e.g. chiorine, methyl, nitro, or methoxy), the location of substitution (e.g. on the diacid phenylene ring or the diamine phenylene ring), and in the order of substitution along the polymer backbone (e.g. "head-to-head, tail-to-tail" polymerized comonomers or "head-to-tail" polymerized comonomers). For this purpose, 2,6-dichloro-substituted derivatives of poly(p-phenylene terephthalamide) are to be studied in detail.

The remainder of this document is organized to address these objectives in order. The state of the art leading up to this work is summarized in Chapter 2. Chapter 3 addresses the development of the computer model and discusses the special considerations required to simulate the oriented solid state. Chapter 4 details the application of the model to the parent polymer, PPTA. Chapter 5 then discusses the simulation procedure for the prediction of mechanical properties, again with application to the parent polymer. Chapter 6 deals with the experimental aspects of fiber spinning, $x$-ray analysis of fibers, and validation of the model. Finally, Chapter 7 deals with the extension of the method to the prediction of properties of the related polymers, with particular attention to the specific effects of atomic substitution and constitutional order.

## 2. STATE OF THE ART

### 2.1 Aramids

2.1.1 Early Development

Aromatic polyamides, or aramids, represent the first, chronologically, and to date commercially foremost of the high performance stiff chain polymers. By definition, aramids are synthetic long chain molecules containing amide moieties wherein at least $85 \%$ of the amide linkages are attached directly to two aromatic rings [Preston, 1987]. They are generally produced via step growth polymerization, or polycondensation, of (1) a monomer consisting of one or more aromatic moicties and having a primary amine at one end and a carboxylic acid or acid chloride at the other, or (2) comonomers comprised of an aromatic diamine on the one hand and an aromatic diacid (or diacid chloride) on the other. The introduction of aramids into the literature resulted from the work of Morgan, Kwolek, and co-workers at Dupont in the early seventies on the self-condensation of p-aminobenzoyl chloride to yield poly(p-benzamide) (PBA) and bipolymer condensation of m - or p -phenylene diamine with a phthaloyl dichloride to yield poly(m-phenylene isophthalamide) (MPIA) or poly(p-phenylene terephthalamide) (PPTA), commercialized under the trade names Nomex ${ }^{\circledR}$ and Kevlar ${ }^{\circledR}$ [Kwolek, 1972; Morgan, 1977; Kwolek et al., 1977; Bair et al., 1977; Schaefgen et al., 1979]; these polymers are shown in Figure 2.1. Of these, Kevlar ${ }^{*}$ has attracted the greatest attention for its high temperature stability and good strength-to-weight properties; a list of its characteristics is given in Table 2.1. Since that time, numerous variations in the area of rigid and semirigid polymers that lead to high performance materials have been reported. These include molecular chains with modifications of the aromatic rings [Kwolek, 1972; Bair et al., 1977; Jadhav et al., 1988], terpolymers with flexible subunits [Lenk et al., 1977; Aoki et al., 1978], azomethines [Morgan et al., 1984], and poly(amide-hydrazides) [Morgan, 1978; Preston and Hofferbert, 1978], to name a few. As a result of their unusual properties, these aramids are finding application in such end uses as structural composites, ropes and cables, protective apparel, and abrasion-resistant moldings. The high performance tensile properties of these aramid fibers are commonly attributed to the unusually stiff extended conformational characteristics of the molecules and the relative ease with which these macromolecular chains may be aligned along the fiber axis during fiber formation. The resulting fibers and yarns are conjectured to bear tensile loads almost entirely along the molecular axis which,

Figure 2.1: $\quad$ Common aramid polymers.


## Table 2.1

Properties of Kevlar ${ }^{\circledR}$ aramid fiber ${ }^{\text {,b }}$

High crystallinity (>95\%)
High orientation
Extended chain structure
High melting point (ZST $640^{\circ} \mathrm{C}$ )
High glass transition temperature $\left(375^{\circ} \mathrm{C}\right)$
Low density ( $1.45 \cdot 1.48 \mathrm{~g} / \mathrm{cm}^{3}$ )
Low creep
High cut resistance
Good flex resistance and textile processability
Outstanding tensile properties (modulus: 180 GPa ; strength: 3.5 GPa )
Low elongation at break ( $2-4 \%$ )
High damping vs. organic fibers
Nonconductive; good diclectric properties
Good abrasion resistance
Moderate compressive strength ( 500 MPa )
${ }^{2}$ from Schaefgen et al., (1979), as per Gentile [1988]
${ }^{\text {b }}$ Dupont Kevlar product literature
being extended, may not elongate significantly without bond scission. Tensile strengths approach the theoretical limits of chemical bond strengths.

### 2.1.2 Chain Rigidity and Solution Structure

$\pi$-bonding electron orbitals above and below the plane of the phenylene ring and the conjugated double bond character of the carbon-nitrogen and carbon-oxygen bonds in the amide link lend structural rigidity to the individual backbone moieties; thus molecular flexibility is possible only through rotation about bonds connecting successive moietics. The barrier to rotation and chain folding about the amide bond is on the order of $20 \mathrm{kcal} / \mathrm{mol}$, with only a rare occurrence of cis conformations [Jorgensen and Swensen, 1985; Ramachandran and Mitra, 1976]. Rotation about the bonds connecting successive phenylene rings and amide groups is also restricted by the energetically advantageous tendency toward electron delocalization realized when these moicties are coplanar, on the one hand, and repulsive overlap of atomic spheres, on the other. Furthermore, such rotation produces little conformational change when the chain extending bonds on the ring are in the para positions. As such, these polymers resemble wormlike chains in solution, with typical persistence lengths on the order of 150 to $250 \AA$ [Arpin and Strazielle, 1977; Zero and Aharoni, 1987; Millaud and Strazielle, 1978].

The lack of conformational flexibility and high degree of chain extension has two important ramifications. First, a relatively limited number of additional conformations are available to the polymer in solution or melt over that available in the solid state; for this reason, the entropies of solution and melting are expected to be unusually small. This contributes to the characteristically high melting points of aramids, on the order of $500+^{\circ} \mathrm{C}$; these polymers typically undergo decomposition concurrent with or prior to melting. As such, they must be processed from lyotropic phases. However, since dissolution is little aided by entropic effects, the solubility of these aramids is largely determined by the enthalpies of dissolution. PPTA, for example, is only soluble in highly aggressive polar solvents, such as concentrated sulfuric acid, chloro or methyl sulfonic acids, or hydrogen fluoride, making processing difficult and expensive.

Second, above a critical concentration the polymer forms a stable anisotropic solution mesophase. In general, such anisotropic mesophases may arise upon cooling of the neat liquid (thermotropic liquid crystals) or upon increasing concentration in a solvent (lyotropic liquid crystals). This ordered phase formation in aramids has been anticipated through the use of lattice models predicting nematic liquid crystal phase formation for monodispserse and polydisperse rods [Flory, 1956; Flory and Ronca, 1979; Flory and Ronca, 1979; Flory, 1984]. In liquid crystals, the molecular chains are already ordered in local domains in the quiescent melt or solution. The nematic mesophase is characterized by coalignment of rods within each domain but with no preferred ordering perpendicular or translationally parallel to the
axis of local orientation; smectic and cholesteric mesophases, which will not be dealt with in detail here, exhibit additional translational ordering into molecular layers and rotational correlation between layers. The formation of the nematic phase results from directionally dependent interactions between molecular chains, which may be either hard (i.e. hard body steric exclusion) or soft (i.e. distance-dependent anisotropic intermolecular energetic interactions) in nature. The former case has been effectively described in terms of the aspect ratio $x$ of the molecule, which may be calculated using the expressions:

$$
\begin{array}{ll}
x & =\left(\mathrm{M}_{\mathrm{w}} / \mathrm{M}_{\mathrm{ru}}\right)\left(\mathrm{l}_{\mathrm{ru}} / \mathrm{d}_{\mathrm{ch}}\right) \\
\mathrm{d}_{\mathrm{ch}} & =\left(\mathrm{M}_{\mathrm{ru}} / \rho \mathrm{N}_{\mathrm{A}} \mathrm{~L}_{\mathrm{u}}\right)^{1 / 2} \tag{2.2}
\end{array}
$$

From this, the critical volume fraction at which a metastable ordered phase forms may be estimated by the equation:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{p}}^{*} \quad \approx(8 / x) /(1-2 / x) ; \quad x>10 \tag{2.3}
\end{equation*}
$$

While this equation was derived for metastable phase formation, its slight tendency towards overestimation has shown it to be a useful predictor for the onset of the stable nematic phase. The aspect ratio $x$ must exceed $x_{\text {crit }}=6.42$ for the formation of an ordered phase; as $x \rightarrow \infty$, the ratio of volume fractions in the two phases (ordered/disordered) at the transition approaches 1.465 . The effects of polydispersity serve to widen the concentration range over which both isotropic and anisotropic phases are stable, and lead to fractionation of species such that higher aspect ratios are distributed preferentially into the anisotropic phase. During processing, imposition of an external field (e.g. flow or magnetic) induces common alignment of domains with minimal entanglement of molecules, which is important to achieving high orientation in conventional fiber-forming processes [Morgan, 1977].

Experimental studies on PBA, PPTA, and some of their chlorinated derivatives in dilute solution confirm the extended conformations of these chains (Table 2.2). Further studies [Papkov et al., 1974; Kwolek et al., 1977; Bair et al., 1977; Panar and Beste, 1977; Balbi et al., 1980; Gentile, 1988] of phase behavior for PBA and chloro-substituted PPTA in dimethyacetamide (DMAc) plus LiCl and PPTA in $\mathrm{H}_{2} \mathrm{SO}_{4}$ demonstrate (1) the tendency among these systems (including the chloro-substituted) to form nematic phases, (2) the reduction in critical polymer concentration for nematic phase formation at higher molecular weights (i.e. higher aspect ratio) in qualitative agreement with the lattice model (Figure 2.2), and (3) the critical point behavior associated with adding LiCl (i.e. increasing the interaction between

Figure 2.2: $\quad$ Critical volume concentration $\mathrm{Vp}^{*}$ versus axial ratio and comparison between threory and experiments for PBA and PPTA [Fiory, 1984].


Table 2.2
Conformational behavior of some aramids

| Polymer | Solvent | Persistence <br> Length ( $\AA$ ) | Mark-Houwink <br> Viscosity Dependence |
| :---: | :---: | :---: | :---: |
| PPTA $^{\text {a,b }}$ | 96\% $\mathrm{H}_{2} \mathrm{SO}_{4}$ | 150-200 | 1.09 |
| PPTA ${ }^{\text {b }}$ | $\mathrm{HSO}_{3} \mathrm{Cl}$ | 400-500 | 1.36 |
| PBA ${ }^{\text {a,b }}$ | $96 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ | 400-600 | 1.5 |
| CI-PPTA ${ }^{\text {b }}$ | $96 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ | 150 | - |
| Cl-PPTA ${ }^{\text {b }}$ | $\mathrm{HSO}_{3} \mathrm{Cl}$ | 400 | - |
| $\mathrm{Cl}-\mathrm{PPTA}{ }^{\mathrm{c}}$ | $96 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ | - | 0.8 |
| PPTA (calculated) ${ }^{\text {d }}$ |  | 420 | - |
| PBA (calculated) ${ }^{\text {d }}$ |  | 410 | - |

${ }^{\text {a }}$ Arpin and Strazielle, 1977
${ }^{\mathrm{b}}$ Millaud and Strazielle, 1978
${ }^{\text {c }}$ Aoki et al., 1978
${ }^{\text {d }}$ Erman et al., 1980
chains) to the solvent/polymer system. Deviations from theoretical behavior have been explained in terms of wormlike flexibility for PPTA at higher molecular weight (higher $x$ in Figure 2.2) in concentrated sulfuric acid, resulting from protonation of the carbonyl oxygens and loss of double bond conjugation [Gardner et al., 1984].

### 2.1.3 <br> Constitutional Isomerism

2.1.3.1

Definition

Recently, the possibility for control of constitutional isomerism in condensation polymers has been applied to the development of new and well-characterized substituted aramids from both theoretical and experimental perspectives [Murano, 1972; Korshak et al.,1975; Pino et al., 1978; Slonim et al., 1980; Steinman et al., 1981; Suter and Pino, 1984; Schmucki et al., 1985; Gentile, 1988|. Constitutional isomerism, as dealt with in this work, refers to polymers which have the same chemical composition and are produced from the same set of monomer reactants, but which differ in the nature or sequence of bonding, or atom connectivity. This is distinct from configurational isomerism and conformational isomerism, which refer to polymers of like constitution which differ either "permanendy" in their sequence of sites of stereoisomerism or "nonpermanently" in the spatial arrangements of the constituent atoms such as might be induced by rotation about an intemal bond. Such constitutional isomerism has long been recognized in vinyl polymers such as polystyrene, where the pendant phenyl rings may be on adjacent carbons (i.e. "head-to-head, tail-to-tail" isomer) or may be separated by an intervening carbon (i.e. "head-to-tail" isomer). Similar isomerism in condensation polymers is likewise possible in those cases where one or more of the monomers is nonsymmetric.

For the simplest case of bicomponent polycondensation, the nonsymmetric monomer may be denoted XabX and the symmetric monomer YccY , which may then be reacted to yield sequences of the type -acca-, -accb-, or bcccb-, with the elimination of XY. Polymerization may then result in a variation of molecular constitution between three possible extremes: 1) strictly altemating head-to-head, tail-to-tail sequences; 2) completely random distribution of head-to-head and head-to-tail sequences; 3) strictly head-to-tail sequences. The degree of order may be quantified using the parameter $s$, defined as [Suter and Pino, 1984]:

$$
\begin{equation*}
s=[a c c b] /([a c c a]+[a c c b]+[b c c b]) \tag{2.4}
\end{equation*}
$$

where [] is the molar concentration of each sequence

By this convention, $s=0$ describes the regular head-to-head, tail-to-tail isomer, $s=1$ describes the regular head-to-tail isomer, and an equal distribution of head-to-head and head-to-tail junctions corresponds to $s=1 / 2$. However, as noted by Gentile [1988], intermediate values of the order parameter provide no information about the distribution, or "blockiness", of the head-to-head and head-to-tail sequences; for this, one requires details of diad and triad statistics, which are currently unavailable.

Of particular interest to this work are the demonstrably large variations in physical properties with variations in structural regularity exhibited by constitutional isomers of polycondensates. Previous experimental work with polyesters, polyureas, and polyamides has demonstrated the anticipated variations in isomeric order and correlated this degree of regularity with changes in physical properties. Order parameters as low as $\mathrm{s}=0.005$ have been achieved with polyamide condensation and as high as $\mathrm{s}=0.89$ with polyurea condensation [Steinman, 1981]. Melting point decreases on the order of $30^{\circ}$ to $40^{\circ} \mathrm{C}$ and solubility increases of an order of magnitude have been reported for several irregular isomers over their more regular head-to-head, tail-to-tail counterparts. The work of Gado [1985], and more recently Gentile [1988], have dealt directly with the considerations of atomic substitution on monomers composing the parent chain of PPTA. These works resulted in the realization of asymmetric constitutional units in the polymer chain and the controlled synthesis of polymers having different sequences of these asymmetric units (i.e. constitutional isomers).

### 2.1.3.2 Kinetic Model and Homogencous Phase Amidation

The development of structural isomerism for homopolymer and bipolymer systems has been modeled under the assumptions of first order, single step, irreversible kinetics in a homogeneous reaction phase [Suter and Pino, 1984; Gentile, 1988]. By this model, structural order (s) at high conversion depends upon: (1) the degree of conversion $X_{n}$; (2) the relative reactivity ( $r$ ) of the two functional groups of the asymmetric monomer XabX; (3) the reactivity induction parameters ( $\mathrm{g}_{\mathrm{a}}$ and $\mathrm{g}_{\mathrm{b}}$ ), indicative of a change in reactivity of the second -cY end of the symmetric monomer upon reaction of the first -cY end with either -aX or -bX , respectively; and (4) the rate and order of addition of each monomer to the reaction mixture; this refers to the possibility for immediate combination of both monomers, or the finite rate addition of one monomer to the other. Different combinations of these variables leads to several possible polymerization cases; these are summarized in Table 2.3, abstracted from Gentile [1988].

By means of controlled addition syntheses, wherein a novel homogeneous phase amidation system was required, ring-substituted derivatives of PPTA were successfully polymerized over a range of intermediate degrees of isomeric order. Ring substitutions were performed exclusively on the diamine monomer. Polymers included poly(2,6-dichloro-p-phenylene terephthalamide), poly(2-nitro-p-phenylene terephthalamide), poly(2-methoxy-p-phenylene terephthalamide), and poly( 2,6 -dimethoxy-p-phenylene terephthalamide). Of these, the first met with the greatest success, with inherent viscosities $\eta_{\text {inh }}$ as high

## Table 2.3

Polymer order versus kinetic parameters under different mixing mechanisms

Case A: $\quad \mathrm{r}=\mathrm{l}, \mathrm{g}_{\mathrm{a}}=\mathrm{g}_{\mathrm{b}}=1$
( $s=1 / 2$ under all mixing conditions)

Mixing mechanisms involving infinitely slow or fast addition of one monomer to the other.
Case B: $\quad r \neq 1, g_{a}=g_{b}=1$
( $0 \leq \mathrm{s} \leq 1 / 2$ )

- YccY slowly added to XabX $\quad s=s(r)$
- XabX slowly added to YccY or infinitely fast mixing of XabX and Ycc Y

$$
s=7 / 2
$$

Case C $\quad \mathrm{r} \neq 1, \mathrm{~g}_{\mathrm{a}}=\mathrm{g}_{\mathrm{b}} \neq 1$
and Case $\mathrm{D}: \quad \mathrm{r} \neq 1, \mathrm{~g}_{\mathrm{a}} \neq \mathrm{g}_{\mathrm{b}} \neq 1$
$\begin{aligned} & \text { - YccY slowly added to XabX } \\ & \text { (same as Case B) }\end{aligned} \quad s=s(r)$

- XabX slowly added to YccY
$s=1 / 2$ (same as Case B)
- Infinitely fast mixing of XabX and $\mathrm{Ycc} Y$

$$
\begin{aligned}
& s=s\left(g_{a}, r\right) \\
& \mathrm{g}_{\mathrm{a}} \rightarrow 0, \mathrm{~s} \rightarrow 1 \\
& \mathrm{~g}_{\mathrm{a}} \rightarrow \infty, \mathrm{~s} \rightarrow 0
\end{aligned}
$$

Mixing mechanisms involving finite rates of addition of one monomer to the other.
Case B: $\quad r \neq 1, g_{a}=g_{b}=1 \quad(0 \leq s \leq 1 / 2)$

- YccY added to XabX
- XabX added to YccY

$$
\begin{aligned}
& s=s(r, \text { feed rate }) \\
& s=1 / 2
\end{aligned}
$$

Case C: $\quad r \neq 1, g_{a}=g_{b} \neq 1$
and Case D: $\quad r \neq 1, g_{a} \neq g_{b} \neq 1$

- YccY added to XabX
$s=s\left(g_{a}, r\right.$, feed rate $)$
- XabX added to YccY

$$
s=s\left(g_{a}, g_{b}, r, \text { feed rate }\right)
$$

- XabX and YccY added to each (slower controls as listed above)
as $5.68 \mathrm{dl} / \mathrm{g}$ (for the $s=1 / 2$ isomer) and order parameters ranging from 0.07 up to 0.60 (inherent viscosities around $0.5 \mathrm{dl} / \mathrm{g}$ were produced at the extremes of order). These dichloro-substituted polymers were further demonstrated to be soluble in N -methylpyrrolidone (NMP) up to $80 \mathrm{mg} / \mathrm{ml}$ ( $s=1 / 2$ isomer); maximum soluble concentrations decreased with increases in isomeric order in the polymer chain. It was further demonstrated that these substituted polymers exhibited nematic phase formation at concentrations comparable to those exhibited by the unsubstituted polymer; in the absence of changes in enthalpic interactions, such behavior confirms the retention of the essential rigidity of the chain in the modified polymers. Thus minute changes in atomic structure, and even more subtle variations in structural order, were shown to have measurable effects on polymer solubility without greatly altering chain flexibility. By process of elimination, one may conjecture that it is changes in enthalpic or energetic interactions between polymer chains and between polymer/solvent combinations that are responsible for the observed variation in macroscopic phase behavior, and that these changes arise due to subtle changes at the atomic level.

In light of these results, it seems clear that the ultimate cohesive behavior of the polymer is a function not only of composition, but of structural order as well. Qualitatively, one could argue simply that larger substituents and a higher degree of substitution serve to force main chain segments apart, disrupting specific interactions associated with the condensation linkages (e.g. hydrogen bonding of amide linkages), and that increasing disorder in the placement of substituents accentuates interferences. This work exploits our ability, by the methods described above, to preselect variations in (1) substituent composition and (2) isomeric order as part of the effort to clarify the relation between molecular level parameters and macroscopic material properties. These polymers have the advantages that they are well characterized and that they show changes in macroscopic behavior as a result of minute variations in atomic structure.

### 2.2 Solid State Structure and Properties

### 2.2.1 General

Because of the close proximity of atomic groups in the solid state, the specifics of molecular structure should play a large role in determining condensed phase behavior. Stereoregularity and tacticity in vinyl polymers are known to be important factors in determining whether these polymers can crystallize and in what form [Alexander, 1969]; regularity of structure is vital to the organization of crystalline order. In most cases, polymers which tend to crystallize do so only partially, resulting in a patchwork of crystallites embedded within an amorphous matrix, such as illustrated in Figure 2.3a, the degree of

Figure 2.3: $\quad$ Schematic representations of ordered phase formations in polymers: (a) semicrystalline matrix of unoriented crystallites embedded in an amorphous matrix; (b) quasinematic alignment of semicrystal domains; (c) microfibril paracrystal.

crystallinity and crystallite orientation being highly dependent upon processing history. In this morphology individual chains weave in and out of both phases. Flexible chain polymers may crystallize by chain folding, giving the common lamellar structures seen in polyethylene and the resulting spherulitic crystallites common in melt crystallization. In oriented (e.g. fiber) form, these flexible semicrystalline polymers may be represented by a two-phase composite model (Figure 2.3b) [Alexander, 1969; Prevorsek, 1982]. In this picture, crystal domains contain chains or chain folded lamellae aligned along the major orientation axis and are in turn separated from one another by discrete amorphous zones; this model is representative of aliphatic polyamides and polyesters, as well as polyethylene terephthalate. Depending upon the molecular rigidity, polymers which do not exhibit folding over long distances may be considered as a single phase "condensed nematic", wherein individual molecules are aligned along a common pseudocrystalline axis, and there is no distinct amorphous phase. This view is illustrated in Figure 2.3c. The solid state structure may also demonstrate translational registry between neighboring repeat units and dimensional order perpendicular to the fiber axis [Moncricff, 1963; Norhtolt and Van Aartsen, 1977]. Ultimately, order in all three dimensions would denote a single crystal or polycrystal solid state.

In conventional crystallography of low molecular weight compounds, one identifies a unit cell, consisting of one or a few molecules, which when replicated periodically in three dimensions in space describes the long range ordered structure of the crystalline solid. By analogy, the common practice for crystal-forming polymers has been to define unit cells which consist of a finite and small number of constitutional repeat units of the polymer chain. However, in polymeric solids one encounters the additional constraints of intramolecular bonding between unit cells, which is not a priori compatible with the periodicity convention assumed by crystallography. In order to satisfy periodicity within a single macromolecule, the individual chain must assume a preferred helical (or degenerate helical, i.e. rodlike or zig-zag) conformation. These helices may then pack into one or more energetically favorable crystal lattice geometries, depending upon the helix itself and the possibility for interactions between helices. Translational periodicity along the unit cell dimension parallel to the chain axis implies commensuration of the helix over a finite section of the chain. The individual crystallites then exhibit dimensional periodicity, which may be classified using conventional crystallographic nomenclature. Multiple crystal forms may result where either two or more individual helix conformations or two or more packing geometries for the same helix are comparably stable.

Conventional crystal nomenclature begins to break down upon detailed consideration of real polymers, where the unit cells are composed of portions of molecular chains and must conform as well to intramolecular bond geometries. In this case one must consider the possibility of nonintegral, or incommensurate, helical structures, determined by preferred bond geometries, which preclude perfect translational periodicities over finite distances. Poly(oxymethylene) and poly(tetrafluoroethylene) are two polymers which have been proposed to have practically incommensurate helical structures in the solid [Carazzolo, 1963; Gramlich, 1976; Weeks et al., 1981]. Saruyama et al. [1985] have proposed periodic
defects (discommensurations) within a regular (commensurate) helix, as an alternative to the incommensurate helix description, in order to explain additional reflections in drawn poly(oxymethylene) and to suggest a means for interconversion between polymorphic forms.

A further categorization of crystalline structure relevant to these polymers has been described by Hosemann and Bagchi [1962] and is designated paracrystallinity. In this description of the atomic lattice, the true atomic positions are displaced from the regular lattice points according to some a priori distribution function. Hosemann postulates that real crystal structures are subject to two kind of distortions. Distortions of the first kind preserve the long range periodicity of the lattice; atomic positions are displaced about the points of the lattice described by the reference unit cell. Such distortions are indistinguishable from thermal vibration distortions. Distortions of the second kind involve fluctuations in the periodicity vectors in relation to the nearest-neighbor lattice points, rather than the ideal lattice points. Thus the absolute deviation of atomic positions from the ideal lattice points defined by the reference unit cell is cumulative and increases as the square of the distance from the reference cell.

### 2.2.2 Aramid Solid State

Previous investigators have reported on the considerable structural complexity of fibrous PPTA [Dobb et al., 1977; Dobb et al., 1977; Panar et al., 1983]. For length scales greater than tens of nanometers, one speaks of a supramolecular structure composed of superposed periodic defect layers ( 35 nm period) and radially-oriented sheet structures with periodic pleats ( 500 nm period), microfibrillar morphology ( 600 nm diameter), and a skin-core differentiation in Kevlar ${ }^{\circledR}$ fibers; recently, Kelvar $149^{\circledR}$ has been reported to exhibit a higher degree of crystallinity and crystallite size and to be devoid of both pleat and microfibrillar macromorphologies as well as of skin-core differentiation [Kwolek et al., 1987]. On the molecular scale, chain ordering has been portrayed using crystalline and paracrystalline descriptions. Correlations of the breadths of meridional x-ray diffraction peaks with the square of reflection order, as predicted by a one-dimensional paracrystal model having distortions of the second kind, and with tensile modulus have been used to suggest a distorted latuice paracrystalline structure in PPTA [Northolt and Van Aartsen, 1977; Barton, 1985; Hindeleh et al., 1984]. In addition to unusual chain rigidity, the aramids experience strong intermolecular cohesive forces in the solid state due to interactions between the polar amide moieties on neighboring chains. This cohesive ability presumably improves the mechanical properties of the bulk polymer transverse to the axis of orientation, for resistance to shear and compressive failure, and leads to high thermal stability; however, this strong cohesion also causes the chains to associate and reduces the enthalpic driving force for mixing, thus contributing to the poor solubility of the polymer. The choice of balance in this trade-off has been previously considered only subjectively and the optimum condition has been unclear.

Other investigators have suggested that PPTA may coexist in several packing modes, or polymorphs [Northolt, 1974; Hancock et al., 1977; Haraguchi et al., 1979; Gardner et al., 1984]. To date, two different but related polymorphs for the pure polymer [Northolt, 1974; Haraguchi et al., 1979], as well as one crystal solvate structure, involving PPTA and sulfuric acid [Gardner et al., 1984], have been isolated and identified through the use of conventional x-ray analytical techniques. In the first, denoted Modification I, the derived unit cell is pseudo-orthorhombic with two chains per unit cell; this structure is shown in Figure 2.4 a . The chain conformation is fully extended with ring rotation angles of $-30^{\circ}$ and $+38^{\circ}$. The chains are rotated about their axes such that the amide planes lie $4^{\circ}$ out of the bc crystallographic facet. The second polymorph, denoted Modification II and shown in Figure 2.4b, was derived as a variation of Modification I. Here, the same chain conformation is maintained, with location of the second chain shifted from [ $z_{2}, 2 / 2$ ] in the ab-facet to $[2 / 2,0]$; however, the resolution of the scattering data in this case was insufficient to determine a unique setting angle. The occurrence of the two polymorphs results from the different environments in which each is coagulated and the ability of the coagulant to interfere with interchain hydrogen bonding during solidification [Haraguchi et al., 1979]. Under certain conditions Modification II may transform to Modification I upon annealing. The unit cell descriptions for each of the pure polymer structures are listed in Table 2.4. The reported structure for the crystal solvate is essentially a swollen version of the pure polymer Modification I, wherein the sulfuric acid molecules coordinate primarily with the amide moieties of the polymer chains in a two-to-one ratio and incorporate into the hydrogen bonding sheets of the bc crystal facet, with the excess solvent nonstoichiometrically included between sheets.

### 2.2.3 Processing

Because of the multiple crystal forms often exhibited by polymers, processing history is a necessary consideration in solid state analysis. For aramids, the process of primary interest is fiber formation, which is best suited to achieving high uniaxial orientation for mechanical reinforcement applications. Fiber spinning operations usually fall into one of three categories:
(1) melt spinning; in this process, neat polymer is extruded as a melt, with solidification induced by cooling of the polymer filament below its melting temperature. Reported spin velocities may be quite high (e.g. $4000 \mathrm{~m} / \mathrm{min}$ ) [Montgomery, 1971] as solidification requires only thermal transfer and not diffusive transfer. Such processing is possible for aliphatic amides and even some stiff chain polyesters. However, as previously stated, the melt temperatures for aramids are typically comparable to the temperatures at which these materials decompose; thus this class of processes is precluded for aramid fiber spinning.

Figure 2.4: The two reported crystal structures for poly(p-phenylene terephthalamide): (a) Modification I [Northolt, 1974]; (b) Modification II [Haraguchi et al., 1979].
(a)

```
PPTA Modification 1
```



PPTA Modification 2
(b)


Table 2.4
Crystallographic parameters of previously reported modifications of PPTA
Modification $\mathrm{I}^{\text {a.d }} \quad$ Modification $\mathrm{II}^{\mathrm{b}, \mathrm{d}} \quad$ Crystal Solvate ${ }^{\mathrm{c}, \mathrm{d}}$

| a ( $\AA$ ) | 7.87 | 8.0 | 16.35 |
| :---: | :---: | :---: | :---: |
| b ( $\AA$ ) | 5.18 | 5.1 | 9.59 |
| c $(\AA)$ | 12.9 | 12.9 | 12.9 |
| $\gamma$ (degrees) | 90 | 90 | 90 |
| chain locations | [ 0,0$] ;[1 / 2,12]$ | [0,0];[12, 0 ] | [0,0];[132, $3_{2}$ ] |
| \# of $\mathrm{H}_{2} \mathrm{SO}_{4}$ | - | - | ca. 9 per cell |
| Space Group | Pn or P 21 n | Pa or $\mathrm{P}_{1} \mathrm{a}$ | - 12 |
| density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.50 | 1.50 | 1.12 |

${ }^{a}$ Northolt, 1974.
${ }^{\text {b }}$ Haraguchi et al., 1979.
${ }^{\text {c }}$ Gardner et al., 1984.
${ }^{\mathrm{d}}$ monoclinic cell type.

(2) dry spinning; in this process, polymer is extruded as a solution in a volatile solvent. Solidification of the extruded filament is induced by volatilization of the solvent species, which is then carried away by an inert gas. Because diffusion and volatilization of a light compound is required, process rates are generally slower (e.g. $1000 \mathrm{~m} / \mathrm{min}$ ) than those in melt spinning. Filament uniformity is affected by the relative rates of diffusion of solvent within the filament and evaporation of solvent at the filament surface. This in turn may be controlled by the temperature and flow rate of the carrier gas. Again, this process is precluded in aramid fiber manufacture due to the insolubility of these polymers in readily volatilized solvents.
(3) wet spinning; this process resembles dry spinning, except that solidification of the extruded filament is induced by diffusive exchange of solvent and nonsolvent species in a coagulating bath. This process typically requires the slowest spin rates (e.g. $100 \mathrm{~m} / \mathrm{min}$ ), depending upon the rates of diffusion of the solvent and nonsolvent species involved and the diameter of the filaments extruded. Best results are generally obtained when polymer concentrations are as high as possible, subject to the limits imposed by polymer solubility and solution viscosity [Knudsen, 1963 (from Montgomery, 1971)]. Coagulation should ideally occur as slowly as possible, to allow for uniform precipitation of the polymer filament and to avoid dissimilar morphologies within the cross section of the fiber. Coagulation rate may usually be reduced either by addition of solvent to the coagulant bath or by reduction of the coagulating temperature [Allcock and Lampe, 1981]. Because a temary system is involved, filament solidification may actually entail separation into a biphasic system, from which the residual solvent/nonsolvent component must subsequently be removed.
(4) gel spinning; this process requires polymer of sufficient molecular weight to form a gel at polymer concentrations as low as $2 \%$ to $8 \%$ by weight. This gel phase is extruded as a filament which is precipitated in a coagulant bath such that up to $98 \%$ of the solvent/nonsolvent phase remains within the fiber. The primary difference lies in the character of the precipitated fiber, which consists of high molecular weight, unoriented, low entanglement density polymer chains. Hot drawing of this filament then results in primary extension and alignment of the unentangled chain segments between crossover points in the gel to yield well-oriented, high strength fibers upon removal of the solvent/nonsolvent phase. Elongation during the drawing process may range from 30 to 100 times the original length, but process rates are generally very slow compared even to wet spinning processes, on the order of $\mathrm{cm} / \mathrm{min}$ [Jaffe, 1987]. Gel spinning essentially seeks to achicve, by means of dilution, the low entanglement densities in flexible polymers that are realized by stiff chain polymers upon microdomain formation in the quiescent liquid crystal phase. Low entanglement density is a prerequisite to achieving high alignment of polymer chains in the solid filament.

Aramids have been processed into both film and fiber form by means of solvent/nonsolvent exchange. For the formation of highly oriented, pseudocrystalline morphologies, the dry-jet wet spinning
process described by Blades [Blades, 1973; Blades, 1975], represents the best means currently available. By this process, the polymer in solution is extruded first through a "dry zone" prior to entry into the coagulation bath. While isotropic spin dopes may be used, the highest degrees of molecular orientation require the use of a liquid crystal solution dope. These domains may be conveniently oriented on a molecular scale prior to solidification [Hancock et al., 1977; White and Fellers, 1978; Valenti et al., 1981; Conio et al. 1987]. Without preorientation in solution, the long relaxation time for polymer motion prevents adequate molecular orientation within the time frame of the spinning process [Chung, 1986]. An extensional flow gradient, which is most pronounced in the dry zone, is realized by collecting the precipitated fiber at a greater linear velocity than that at which the solution exits the spinning die; increasing the extensional flow field improves the degree of orientation along the major axis of the fiber [Kwolek et al., 1977]. In the coagulation bath, a shear gradient is created due to the differential between filament and coagulant flow velocities, which may be reduced by providing for cocurrent coagulant flow in the vicinity of the fiber. Depending on the relative solvency strengths and diffusivities of the solvent and coagulant, a skin layer may form on the fiber, resulting in a differentiation between a fiber "skin", wherein solidification has occurred most rapidly, and a fiber core, where solidification and crystallization proceeds more slowly and to greater perfection. Thus the final product may be affected by changes in many processing factors, such as solution and coagulant temperatures, residence time and relaxation time of the solution/solid mixture in each zone, shear rates at the spin die and in the bath, the magnitude and distribution of extensional stresses along the fiber path, and the evolution of polymer/solven//nonsolvent composition along the spin line. Such processing variables may enable the formation of kinetically-frozen structures, especially important in the morphology of the fiber skin, analogous to the concept of mechanically equilibrated structure in polymer glasses.

Postspin heat treatments near the melt temperature may provide the chains with sufficient mobility in the solid state to "melt out" kinetically frozen crystal forms in favor of the thermodynamically most favorable form. Table 2.5 demonstrates some reported variations in mechanical behavior for PBA and PPTA as functions of spin solution composition, draw ratio during spinning, and annealing. A significant jump in tensile modulus follows upon heat treatment and the average molecular weight increases due to end linking of chains [Kwolek et al., 1977]. The resulting increase in mechanical properties is illustrated for PBA in Figure 2.5. Tenacity and modulus both increase with decreases in orientation angle, a measure of crystallite alignment obtained through x-ray diffraction analysis. Orientation is improved by increased solution anisotropy in the quiescent state or by an increased extensional flow gradient during solidification.

Table 2.5
Some reported properties of aramid fibers as functions of process parameters

| polymer | variable | $\eta_{\mathrm{inh}}$ <br> (dl/g) | tensile strength (GPa) | tensile modulus (GPa) | orientation ${ }^{\text {c }}$ <br> (degrees) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| ${ }^{\text {d P PBA }}$ | iso. soln. ${ }^{\text {a }}$ | 2.1 | 0.56 | 23.4 | 33 |
|  | low aniso. ${ }^{2}$ | 2.1 | 1.09 | 42.4 | 20 |
|  | high aniso. ${ }^{\text {a }}$ | 2.1 | 1.24 | 54.6 | 16 |
| ${ }^{\text {d P P }}$ | free fall ${ }^{\text {b }}$ | 1.48 | 0.41 | 18.0 | 39 |
|  | $1.90{ }^{\text {b }}$ | 1.48 | 0.50 | 30.2 | 37 |
|  | $2.56{ }^{\text {b }}$ | 1.48 | 0.71 | 36.0 | 26 |
|  | $3.83{ }^{\text {b }}$ | 1.48 | 0.89 | 50.2 | 22 |
|  | $5.11{ }^{\text {b }}$ | 1.48 | 1.10 | 60.5 | 22 |
| ${ }^{\text {e PBA }}$ | - | - | 2.11 | 72.1 | - |
| ${ }^{\text {e }}$ PPTA | - | - | 2.53 | 53.6 | - |
| ${ }^{\text {fPBA }}$ | annealed | - | . 5 | 180. | - |
| ${ }^{\text {f }}$ PPTA | annealed | - | - | 139. | - |

${ }^{\text {a }}$ a solution morphology: isotropic or anisotropic fractions
${ }^{\text {b }}$ spin draw ratio: linear velocity of draw wheel over the linear velocity at the die exit
${ }^{c}$ orientation is expressed in degrees as the angle of orientation, which is calculated as the azimuthal peak width at which the intensity of $x$-ray scattering in a single reflection falls to half its peak intensity; the angle is a measure of deviation of the crystallites from perfect alignment, so that smaller angles correspond to more perfect crystallite alignment.
${ }^{d}$ Kwolek et al., 1977, p1390.
${ }^{\text {e }}$ Dobb et al., 1977, p237.
${ }^{f}$ Ii et al., 1986, pl809.

Figure 2.5: Effect of heat treatment of a few seconds duration on properties of poly(1,4-benzamide) fibers [Kwolek et al., 1977].


## 2.3

## Atomistic Simulation of Dense Polymer Systems

2.3.1

General

The methods of Molecular Mechanics and Molecular Dynamics have proven considerably useful in the correlation of macroscopic behavior with atomic level structural detail. Usually, these methods are employed to obtain atomic potential energy functions in systems of experimentally known structure or, conversely, to elucidate the characteristics of a previously identified region of phase space. The methods prove especially useful in those instances where simplifying assumptions, such as independent behavior of single molecules, structural periodicity, or idealized stochastic disorder, are both realistic and quantifiable. Applications to isolated macromolecules are numerous [e.g. Brant and Flory, 1965; Suter, 1979; Brisson and Brisse, 1986; Allegra et al., 1970; Scott and Scheraga, 1966; Ooi et al., 1967; Tashiro et al., 1977]. However, the detailed treatment of atomic level interactions and chain packing of macromolecules at glass and crystal densities are considerably fewer.

### 2.3.2 Polymer Glasses

An extensive analysis of local structure in glasses originated in the work of Theodorou and Suter [1985, 1986]. This model entailed the simulation of a vinyl polymer glass, atactic polypropylene at $-40^{\circ} \mathrm{C}$, as a static structure in detailed mechanical equilibrium. A single chain of degree of polymerization (DP) 76 was generated in a cube having edge lengths of $18.15 \AA$ by a Metropolis Monte Carlo procedure using the Rotational Isomeric State (RIS) scheme with fixed bond lengths and bond angles. The cube filling process employed periodic boundary conditions (i.e. a chain segment which extends through one face of the cube re-enters through the opposite face) and proceeded by means of an algorithm which took into consideration long range interactions in order to avoid large vacancies in the structure. The initial guess so generated was then relaxed to mechanical equilibrium by minimization of the potential energy of the system with respect to the torsional degrees of freedom of the chain. Here it is important to note that the final structure need not satisfy the criteria for thermodynamic equilibrium (i.e. minimum free energy) but instead satisfies the criteria for mechanical equilibrium (i.e. minimum potential energy in a static structure), consistent with the view of a polymer glass as a state of frozen liquid disorder. Nonbonded interatomic interactions were calculated explicitly up to a finite distance ( $\mathrm{R}=$ $2.3 \sigma$, where $\sigma$ refers to the minimum energy separation in the nonbonded interatomic potential function used, and is pairwise atom-dependent). The contribution to the system potential energy from long range interactions was then made by assuming a spatially uniform distribution of all species at large distances (i.e. no long range structural order) and integrating over all species pair interactions from $2.3 \sigma$ to infinity. Macroscopic and microscopic properties of the glass were deduced from averages over fifteen such
minimized structures. Imposed deformations of the cube were then employed successfully to predict the elastic constants to within $15 \%$ of their experimental values.

This model was extended by Ludovice and Suter [1989] to include polar interactions and was applied to polyvinylchloride (PVC) of DP 76 (cube edge $=17.84 \AA$ ), and in a single instance to a chain of DP 200 (cube edge $=24.64 \AA$ ), at $25^{\circ} \mathrm{C}$. Significant here was the introduction of charged atomic species, which was handled through the assignment of partial atomic charges to the individual atoms and the addition of a Coulombic interaction term. A distance-dependent dielectric constant was introduced to reflect the diminution effects due to induced polarization of the intervening medium on field strength experienced by charges separated by larger distances. This approach was shown to reproduce consistent dipole moments and reasonable cohesive energies in glassy PVC. This model of the glassy state has since been further employed to simulate constrained dynamics in PVC [Ludovice, 1989] and glassy polymer structure of atactic polypropylene confined between semi-infinite surfaces [Mansfield and Theodorou, 1989].

### 2.3.3 <br> Polymer Crystals

Modelling efforts aimed at single crystals of flexible polymers tend to fall into two categories: (1) predictions of elastic properties of known crystal forms, or (2) derivation of structural and thermodynamic properties. For elastic constants, the investigator generally assumes that it is the response of potential energy to deformation that dominates elastic behavior in crystals. As early as 1967 Anand applied the concepts of atom-based force fields to predict the elastic constants for a known structure of orthorhombic polyethylene. McCullough and co-workers $[1973,1974]$ developed a matrix technique for representing crystalline assemblies of chains of fixed conformation, with consideration of certain chain defects (e.g. chain kinking or folding). Elastic constants in polyethylene were studied by considering potential energy interactions within a single chain and between selected nearest-neighbor chains in a given orthorhombic assembly. Crystal/crystal transformation pathways were proposed by mapping the potential energies of assemblies of chains as a function of selected packing parameters. However, minimization of the potential energies with respect to either intramolecular or intermolecular parameters was not attempted. Tashiro and co-workers also reported a method for the calculation of the three dimensional elastic moduli of assemblies of chains. This method relied largely upon a priori crystal symmetries to expedite calculations and was applied to polyethylene (PE), poly(vinyl alcohol), and Nylon 6 [Tashiro et al., 1978; Tashiro et al., 1978; Tashiro and Tadokoro, 1981].

Tripathy et al. [1981] dealt with the prediction of crystalline packing of chain molecules by minimizing the intermolecular interaction energies of assemblies of chains of fixed conformation using a row formalism for the single chain which enabled one to calculate interactions between infinitely long
chains. However, this formalism was not amenable to extension to three dimensions. From their study of PE, these authors concluded that, lateral to the chain axis, packing geometry was fixed primarily by interactions with the nearest-neighbor shell of chains, while for packing energy one required the additional inclusion of at least the next-nearest-neighbor shell. A limited investigation of poly(vinylidene fluoride) in a fixed geometry led these authors to conclude that a distance-dependent dielectric term such as that employed by Ludovice for glasses is most suitable for correct geometry predictions. These authors also recognized the problems associated with the identification of all local minima in such densely packed systems.

Recently, Sorensen et al. [1988] have incorporated the simultaneous minimization of packing energy with respect to both intramolecular conformation and intermolecular packing parameters, a feature important to the prediction of structure at solid state densities. These authors went on to deduce elastic constants, using the second derivative matrix of potential energy with respect to the independent variables at the minimum energy configuration, and vibrational dispersion curves, from which the thermodynamic quantities were calculated. Again, application to PE and poly(oxymethylene) showed remarkably good agreement between calculated and experimental values for packing geometry and lattice energy, and reasonable agreement for elastic constants and heat capacities between $50^{\circ} \mathrm{C}$ and $350^{\circ} \mathrm{C}$. Lattice energy differences were proposed to explain the stability of the two polymorphs of poly(oxymethylene).

Despite this considerable attention to simulations of flexible chain aliphatic polymers, there have been no reports of application of these methods to the case of the rigid rod polymers. Flory and coworkers [Hummel and Flory, 1980; Erman et al., 1980] addressed the question of conformational energetics in p-phenylene polyamides and polyesters, with subsequent predictions of chain persistence lengths. Tashiro et al. [1977] also performed single chain calculations on the three relevant aramids, poly(p-benzamide), poly(m-phenylene isophthalamide), and poly(p-phenylene terephthalamide), in order to elucidate conformational energetics and tensile moduli, based on the assumption that intermolecular effects contribute negligibly to axial moduli, with reasonable success. Both groups produced parameterized force fields to describe the conformational behavior of the isolated chain and, in the latter case, its response to extensional deformation. However, in the solid state one encounters an environment that is both conformationally constrained and densely packed. Under such strong opposing forces, one cannot expect to decouple the problems associated with the possibly antagonistic criteria for optimization of chain conformation and long range crystalline regularity. It is especially important to recognize that intermolecular forces may be sufficient to induce changes in chain conformation and, by so doing, influence the intramolecular interactions. Conflicts between conformational intramolecular constraints and crystallographic intermolecular constraints are sufficient to induce the formation of nonrational helices or helix discommensurations such as those proposed by Saruyama et al. [1985]. One may further anticipate that intermolecular interactions between nonuniform helices or helices possessing some form of constitutional disorder would lead to anomalous packing behavior between neighboring chain segments.

It becomes apparent, then, that some attempts have been made to address the simulation of polymeric systems in the limit of isolated chain behavior and, in densely packed systems, at the extremes of crystalline order and liquidlike disorder. However, the emerging materials technologies include polymers which do not fit neatly into either extreme. For example, even in atactic PVC one observes local association of polymer segments and gelation. Ludovice and Suter [1987] have proposed subtle electrostatic interactions suggested by glass simulation results to account for this behavior. The presence of strong intermolecular as well as intramolecular forces and localized or anisotropic assemblages of chains or chain segments may be responsible for many of the desirable properties of the new polymeric materials, but pose particular problems of representation and numerical analysis to the materials investigator. The task remains to establish the quantitative connection between that which can be described and designed, and that which is actually observed in such cases.

## 3. ATOMISTIC SIMULATION OF ORDERED STRUCTURE

### 3.1 General Approach

The static atomistic modelling methodology assumes that the energy of the material in question may be calculated from a set of force interactions, corresponding to a total potential energy characteristic of the structure. It is composed of two fundamental parts: (1) a method for representation of the material as an ensemble of atoms, or atomic groups; (2) a realistic representation of the forces operating between individual members of the ensemble. In the ideal crystal representation, the first is defined completely throughout space by the local coordinates of the conformational repeat unit, in conjunction with transformations required to describe symmetry-related elements within the unit cell and the three periodicity vectors which define the replication of the unit cell in space. This periodicity may also be used to simplify the calculation of interatomic interactions, as will be described later. The approach which follows begins with the determination of the conformation behavior of the single chain, as prescribed by intramolecular interactions. From the single chain an explicit multichain structure is generated which reflects short-range intermolecular interactions in the nearest-neighbor and next-nearestneighbor shells, and invokes periodicity simplifications only to estimate the appropriate long-range compaction forces. The calculations assume that the force interactions may be realistically represented by empirical bond length, bond angle, and bond torsion deformation functions, and additive two-body interactions in the case of nonbonded species or species whose interaction distance is conformationally dependent. Iterations along a decreasing potential energy trajectory are performed using a Quasi-Newton algorithm with simultaneous variation of all intramolecular and intermolecular degrees of freedom.

### 3.2 Structural Representation

3.2.1 Chain Generation and Helix Alignment

The generation of the representative chain involves the specification of the set of three dimensional coordinates locating the atoms or rigid moieties which comprise the chain and entails $3 n-3$ intramolecular
degrees of freedom, $n$ being the number of atoms (or moieties) in the (finite) chain. This set may be specified directly in Cartesian space or generated as a function of internal parameters, such as bond lengths, bond angles, and dihedral angles, as is common in amorphous polymer glass simulations; the latter has been facilitated by the introduction of the generator matrix formalism of Flory [1969]. In the case of polymer glasses, successive atom placements are generated stochastically, so that one can not expect repetition in structure along the chain. One special feature of macromolecular crystals is the coupling between periodicity along the contour of the polymer chain and periodicity along the lattice dimension parallel to the chain propagation. In the case of ideal crystals the higher level order of the crystal implies translational periodicity along the chain and thus the formation of a rational helical conformation (one may consider rodlike and zig-zag conformations to be degeneracies of the general helix description); to ensure periodicity, one employs a feature of the multichain ensemble, a lattice dimension, in describing the structure of the helix and hence the chain conformation. Previous simulations of ideal crystals have taken advantage of the periodicity characteristic of perfect crystals to generate additional transformations whose elements involve intermolecular distances and orientations derived from the dimensions, angles, and symmetries of the crystal latice [McCullough, 1974; Sorensen and Boyd, 1988]. The result is a perfectly periodic ensemble created by replication in three dimensions of the minimum basis set of coordinates belonging to a single repeat unit of the rational helix. If one assumes only the existence of a "conformational repeat unit" (referred to hereafter as CRU), a substructure which is internally indistinguishable from any other along the chain contour in terms of conformation, the chain generation method satisfies the more general condition of helical periodicity. The special case of translational periodicity, or rational helicity, may be superposed by substituting a helix axis translation parameter for one of the internal degrees of freedom. In these cases, the number of intramolecular positional degrees of freedom reduces to $3 \mathrm{~N}_{\mathrm{CRL}}-3$, where $\mathrm{N}_{\mathrm{CRU}}$ is the number of atoms in the CRU. This repeat unit is typically taken to be one chemical repeat, but could just as well be an integer multiple thereof. This number of internal degrees of freedom may be further reduced through the introduction of virtual bonds or other conformational constrains.

In the procedure developed here, we treat the polymer chain as a finite segment of the general coiling backbone polymer. Following Flory [1969], the Cartesian coordinate vector $\mathbf{X}$ for all atoms in a single chain in the frame of reference of the first atom is gencrated:

$$
\begin{align*}
& \mathbf{X}_{\mathrm{i}}^{1}=\mathbf{L}_{1}+\mathbf{T}_{1} \mathbf{L}_{2}+\mathbf{T}_{1} \mathbf{T}_{2} \mathbf{L}_{3}+\ldots+\mathbf{T}_{1} \mathbf{T}_{2 . .} \mathbf{T}_{\mathrm{i} \cdot \mathbf{1}} \mathbf{L}_{\mathrm{i}}  \tag{3.1}\\
& \mathbf{L}_{\mathrm{i}}=\left[\begin{array}{lll}
l_{\mathrm{i}} & 0 & 0
\end{array}\right] \tag{3.2}
\end{align*}
$$

$$
\mathbf{T}_{\mathrm{i}}=\left[\begin{array}{llc}
\cos \left(\vartheta_{\mathrm{i}}\right) & \sin \left(\vartheta_{\mathrm{i}}\right) & 0  \tag{3.3}\\
\sin \left(\vartheta_{\mathrm{i}}\right) \cos \left(\phi_{\mathrm{i}}\right) & -\cos \left(\vartheta_{\mathrm{i}}\right) \cos \left(\phi_{\mathrm{i}}\right) & \sin \left(\phi_{\mathrm{i}}\right) \\
\sin \left(\vartheta_{\mathrm{i}}\right) \sin \left(\phi_{\mathrm{i}}\right) & -\cos \left(\vartheta_{\mathrm{i}}\right) \sin \left(\phi_{\mathrm{i}}\right) & -\cos \left(\phi_{\mathrm{i}}\right)
\end{array}\right]
$$

$l_{\mathrm{i}}, \vartheta_{\mathrm{i}}$, and $\phi_{\mathrm{i}}$ are the bond length, bond angle complement, and dihedral angle locating atom i in the frame of reference of atom $\mathrm{i}-1$. In this convention, $\phi_{\mathrm{i}}=0$ corresponds to the trans dihedral conformation. This may be accomplished using the generator matrix $\mathbf{A}$ :

$$
\begin{align*}
& \begin{array}{ll}
\mathbf{X}_{\mathrm{i}}^{1} & = \\
\mathbf{A}_{[1} & \mathbf{A}_{2}
\end{array} \mathbf{A}_{3} \mathbf{A}_{4} \ldots \mathbf{A}_{\mathrm{i}-1} \mathbf{A}_{\mathrm{i}]}, \quad \mathrm{i}>1  \tag{3.4}\\
& \mathbf{A}_{\mathrm{j}}=\left[\begin{array}{cc}
\mathbf{T}_{\mathrm{j}} & \mathbf{L}_{\mathrm{j}} \\
\mathbf{0} & 1
\end{array}\right]  \tag{3.5}\\
& 4 \times 4 \text { matrix }  \tag{3.6}\\
& \text { with } \quad \mathbf{A}_{[1}=\left[\begin{array}{cc}
\mathbf{T}_{1} & \mathbf{L}_{1}
\end{array}\right]  \tag{3.7}\\
& \text { and } \quad \mathbf{A}_{\mathrm{i}]}=\left[\begin{array}{c}
\mathbf{L}_{\mathrm{i}} \\
1
\end{array}\right]
\end{align*}
$$

In highly ordered polymer structures, it is appropriate to limit the set of unique conformational parameters to those of the CRU. The helical superstructure may be defined in terms of the translation along the helix axis (pitch, $d_{h}$ ), the directional rotation (twist, $\Theta_{h}$ ) between identical points in successive structural units, and the radius $\left(\rho_{h}\right)$, the distance from the axis to a reference point on the chain. The nonuniform helix is considered to be a perturbation of the uniform conformation. In order to capture local perturbations in helical structure, this model allows continuous variation of chain conformation during energy minimization through nonrational as well as rational helix conformations and includes the special cases of zig-zag and extended chain conformations. For the fully extended (i.c. zero helix twist) conformation it is necessary to introduce a switch from true helix representation and alignment to an approximation based on alignment of the major axis of the radius of gyration of the finite chain segment, in order to overcome the singularity in helix radius and consequent alignment problem which occurs in the helix treatment. These calculations are presented in greater detail in Appendix A. Imperfect helix alignment, rigorously interpreted, creates the possibility for steric interference between neighboring chains at points remote from the region explicitly described in the model. In allowing imperfect alignment of neighboring helices within the dominant explicit representation one must ensure that the deviation from alignment be quantifiably small over large distances; that is, either (a) minor, low energy torsional correc-
tions (e.g. discommensurations) spaced at large intervals along the chain are capable of "periodically realigning" the set of chain propagation vectors or (b) the correlation lengths of the "crystallites" are sufficiently small relative to the alignment deviation to preclude any significant remote steric interferences. The first condition may be allowed through minor torsions which produce changes in chain alignment but which are energetically insignificant (e.g. an amide torsion of only $1^{\circ}$ out of plane requires less than $0.05 \mathrm{kcal} / \mathrm{mol}$, but may atter the backbone contour vector by $2^{\circ}$, and can create a local helix twist displacement of approximately $10^{\circ}$ ); the second condition may be deduced from the minimum dimension at which energetically-significant steric overiap occurs.

### 3.2.2 Multichain Packing

The multichain structure is created through successive independent molecular coordinate gencration accompanied by alignment of the primary chain axes. Again, the problem may be computationally simplified by assuming that individual chains are conformationally indistinguishable, analogous to the CRU assumption. In this case structure generation reduces to replication and reorientation of the parent chain conformation at points on a two-dimensional ficld. In a "disordered lattice", each neighboring chain may be located by a set of independent parameters describing orientation and translation of that chain from the "parent" position. In the paracrystalline structure, the vectors joining points in the "net" vary in magnitude and direction according to an a priori statistical probability. One may further limit the degrees of freedom by requiring correlation between the orientation of chains located at regular points in the net. In the perfect crystal, the net simplifies to a regular grid of translationally periodic points with only one or two characteristic orientations describing the setting of the chain at each of the grid points.

In these calculations, we employ the single conformation and periodic grid assumptions. The primitive cell parallelepiped has edge vectors $\mathrm{A}, \mathrm{B}$, and C , and angles $\angle \mathrm{AB}=\zeta, \angle \mathrm{AC}=0$, and $\angle \mathrm{BC}=\tau$. Figure 3.1 illustrates this description of the crystal. We define the translation lengths, $A$ and $B$, and the angles, $\zeta, v$, and $\tau$, that orient the point net with respect to the major chain propagation axis $\mathbf{C}$, chosen to coincide with the z coordinate axis; the $\mathbf{A}$ vector is arbitrarily chosen to lie in the xz -plane of the parent frame of reference. Chains are located at $\mathrm{mA}+\mathrm{nB}, \mathrm{m}$ and n being positive or negative integers. Each of the $\mathrm{N}_{\mathrm{c}}$ independent chains may be oriented about its center of mass through the use of appropriate Euler angles and $\mathrm{N}_{\mathrm{c}}-1$ chains may be translated (relative to the parent chain) along their axes. From this selection of $\mathrm{A}, \mathrm{B}, \tau, v$, and $\zeta$, it follows that, in Cartesian coordinates:

$$
\mathbf{A}=\left[\begin{array}{c}
A \cos \left(90^{\circ}-v\right)  \tag{3.8}\\
0 \\
A \sin \left(90^{\circ}-v\right)
\end{array}\right] \quad \mathbf{B}=\left[\begin{array}{l}
\mathrm{B} \cos \left(90^{\circ}-\tau\right) \cos \zeta^{\prime} \\
\mathrm{B} \cos \left(90^{\circ}-\tau\right) \sin \zeta^{\prime} \\
\mathrm{B} \sin \left(90^{\circ}-\tau\right)
\end{array}\right]
$$

Figure 3.1: Definition of chain packing parameters.

where

$$
\cos \zeta^{\prime}=(\cos \zeta-\cos \tau \cos v) /(\sin \tau \sin v)
$$

The calculation of individual atomic coordinates for each atom i in chain j (which is located at $\mathbf{R}_{\mathrm{j}}=\mathrm{m}_{\mathrm{j}} \mathbf{A}+\mathrm{n}_{\mathrm{j}} \mathbf{B}$ in the frame of the "parent") is accomplished by:

$$
\begin{align*}
& {\left[\begin{array}{c}
\mathbf{X}_{i \mathrm{i}} \\
1
\end{array}\right]=\left[\begin{array}{cc}
\mathbf{W}_{\mathrm{j}} & \mathbf{R}_{\mathrm{j}} \\
\mathbf{0} & 1
\end{array}\right]\left[\begin{array}{c}
\mathbf{X}_{\mathrm{i}, 1} \\
1
\end{array}\right]+\mathrm{f}_{\mathrm{j}} \mathrm{~d}\left[\begin{array}{cc}
\mathbf{W}_{\mathrm{j}} & \mathbf{0} \\
\mathbf{0} & 1
\end{array}\right]\left[\begin{array}{l}
0 \\
0 \\
1 \\
0
\end{array}\right]}  \tag{3.9}\\
& \mathbf{W}_{j}=\left[\begin{array}{lll}
\cos \chi_{j} \cos \psi_{j} \cos \omega_{j} & -\cos \chi_{j} \cos \psi_{j} \sin \omega_{j} & -\cos \chi_{j} \sin \psi_{j} \\
-\sin \chi_{j} \sin \omega_{j} & -\sin \chi_{j} \cos \omega_{j} & \\
\sin \chi_{j} \cos \psi_{j} \cos \omega_{j} & -\sin \chi_{j} \cos \psi_{j} \sin \omega_{j} & -\sin \chi_{j} \sin \psi_{j} \\
+\cos \chi_{j} \sin \omega_{j} & +\cos \chi_{j} \cos \omega_{j} & \\
\sin \psi_{j} \cos \omega_{j} & -\sin \psi_{j} \sin \omega_{j} & \cos \psi_{j}
\end{array}\right] \tag{3.10}
\end{align*}
$$

d is the helix translation per structural repeat length (pitch), $\mathrm{f}_{\mathrm{j}}$ is the translation, expressed as a fraction of the pitch, of chain j along its axis, and $\chi_{\mathrm{j}}, \psi_{\mathrm{j}}$, and $\omega_{\mathrm{j}}$ are the Euler angles for placement of the jth chain (for details, refer to Appendix A: Figure A.2). For the crystalline case, A, B, $\tau, v$, and $\zeta$ are constant, while in the paracrystal these may vary. While the $\mathbf{W}_{j}$ matrix is formulated generally above, in long chain polymers $\Psi_{j}$ is limited to values near $0^{\circ}$ and $180^{\circ}$ due to constraints on helix alignment. Chains which are mirror images of the parent are handled through multiplication of $\mathbf{W}_{\mathrm{j}}$ with an inversion matrix. Under these conditions, helix translation along its axis may be sufficienly considered by $-0.5<\mathrm{f}_{\mathrm{j}}<+0.5$. With the introduction of the regular point net the number of intermolecular degrees of freedom becomes $4 N_{c}+4$, while further simplification via $\chi_{j}=0$ and $\Psi_{j}=0$ yields $2 N_{c}+4$ degrees of freedom.

### 3.3 Energy Representation

### 3.3.1 Explicit Description

In order to calculate the potential energy for a representative crystal form, this method first assumes pairwise additivity of effective intermolecular interactions. The completely general description of the total potential energy for a construction consisting of L polymer chains may be represented as:

$$
\begin{equation*}
\mathrm{E}^{\text {expl }}=\sum_{\substack{\text { all chains } \\ l}} \mathrm{E}_{l}^{\text {intra }}+1 / 2 \sum_{\text {all chains }}^{l} \sum_{\substack{\text { all } \\ l+l^{\prime} \neq l}} \mathrm{E}_{l, l^{\text {inter }}}^{\text {ins }} \tag{3.11}
\end{equation*}
$$

$l$ and $l$ ' refer to each of the $L$ chains in the structure. Under the simplifying assumptions of structural repetition within and between chains, the average potential energy per structural repeat unit for a construction may be calculated explicitly as:

$$
\begin{align*}
E^{\text {unit }} & =E^{\text {expl }} / L N_{r}  \tag{3.12}\\
& =\left(1 / N_{c} N_{r}\right)\left[\quad \sum_{m=1}^{N c} E_{m}^{\text {intra }}+3 / 2 \sum_{m=1}^{N c} \sum_{\substack{l=1 \\
l \neq m}}^{L} E_{m, l^{i n t e r}}^{\text {incer }}\right] \tag{3.13}
\end{align*}
$$

$m$ refers to the each of the $N_{c}$ chains in the structure which are not related by a symmetry operation. The first term represents the total of the intramolecular interactions. For a single chain of helix units, each consisting of k interacting centers, the intramolecular energy may be calculated as:

$$
\begin{equation*}
E^{\text {intra }}=\sum_{i=1}^{k} \sum_{j=1}^{h \cdot k} E(i, j) \tag{3.14}
\end{equation*}
$$

where $E(i, j)$ is the general form describing the interaction between two atoms $i$ and $j$. The second term in Equation (3.13) represents the intermolecular potential energy. For two conformationally identical, parallel, interacting chains involving only distance-dependent interactions, one may take advantage of the nondirectional nature of spherically symmetric interactions to obtain the complete potential energy description from a reduced set of coordinates; this is employed only as an expedient to keep the memory requirements of the program to a minimum. For two interacting chains $m$ and $l$, each of helical repeat units consisting of $k$ interacting centers per helical repeat, the interaction energy may be calculated as (where $i_{m}$ refers to the $i$ th atom of chain $m$ ):
in general:

$$
\begin{equation*}
E^{\text {inter }}=\sum_{i=h \cdot k / 2}^{(h \cdot k / 2)+k} \sum_{j=1}^{h \cdot k} E\left(j_{l}, i_{m}\right) \tag{3.15a}
\end{equation*}
$$

special case for parallel helices:

$$
\begin{equation*}
E_{l, m}^{\text {imer }}=\sum_{i=1}^{k} \sum_{j=1}^{h \cdot k} E\left(j, i_{m}\right)+\sum_{i=k+1}^{h \cdot k} \sum_{j=1}^{k} E\left(j, i_{m}\right) \tag{3.15b}
\end{equation*}
$$

Latice Summation

While the above equations apply generally to multibodied systems, crystalline solids are most often idealized as perfectly periodic structures in three dimensions. This simplification precludes the representation of defects such as dislocations or discommensurations. However, it allows one to expedite the calculation of pairwise interactions through the use of lattice summations in three dimensions. A similar summation was employed by Anand [1967] in arriving at an analytical equation for the cohesive energy of the infinite crystal in the absence of electrostatic forces. The simplification is conceptually identical to a numerical approximation of the Madelung Constant for each new simulation. The translationally periodic infinite solid in three dimensions may be viewed as a construction consisting of unit cells displaced from a "parent" by an integer multiple of its edge vectors, where the only unique pairwise interactions that must be considered are those between the atoms making up one helical repeat unit of the parent chain and those of one helical repeat unit in each of the neighboring independent chains (i.e. those chains not displaced from the parent by a symmetry operation). The distance-dependent interaction may then be computed for an arbitrarily large construction through the use of a lattice summation:

$$
\begin{equation*}
E^{\text {latice }}=\left(N_{\text {CRL }} / N_{i}\right) \sum_{n=1}^{\mathrm{N}_{\mathrm{p}}} \sum_{\mathrm{i}=1}^{\mathrm{Ni}} \sum_{\mathrm{j}=1}^{\mathrm{Ni}_{\mathrm{i}}} \mathrm{~A}_{\mathrm{ij}, \mathrm{n}} \sum_{\mathrm{L}} \mathrm{~V}_{\mathrm{n}}\left(\mathrm{~d}_{\mathrm{ij}}\right) \tag{3.16}
\end{equation*}
$$

where i and j refer to the $\mathrm{N}_{\mathrm{i}}$ interacting atoms that are crystallographically unique (i.e. belonging to the $N_{c}$ independent chains). Thus if $N_{c}=1, N_{i}$ is equal to $k$, the number of atoms comprising one period of the helix. In general, $\mathrm{N}_{\mathrm{i}}=\mathrm{kN}_{\mathrm{c}}$. The premultiplier ensures that $\mathrm{E}^{\text {unit }}$ and $\mathrm{E}^{\text {lanice }}$ are calculated on the same basis. $\mathrm{V}_{\mathrm{n}}$ is the distance-dependant potential function (e.g. $\mathrm{d}_{\mathrm{ij}}{ }^{-1}, \mathrm{~d}_{\mathrm{ij}}{ }^{-6}$ or $\mathrm{d}_{\mathrm{ij}}^{-12}$ ) having multiplier $\mathrm{A}_{\mathrm{ij}, \mathrm{n}}$ for atoms i and $\mathrm{j} ; \sum \mathrm{A}_{\mathrm{ij}, \mathrm{n}} \mathrm{V}_{\mathrm{n}}\left(\mathrm{d}_{\mathrm{i}}\right)$ is the potential energy of interaction between a pair of atoms i and j (summation over n implied). The summation over the distance-dependent function in Equation 3.16, $\sum_{\mathrm{L}} \mathrm{V}_{\mathrm{n}}\left(\mathrm{d}_{\mathrm{ij}}\right)$, is the lattice sum:

$$
\begin{equation*}
\Sigma_{\mathrm{L}} \mathrm{~V}_{\mathrm{n}}\left(\mathrm{~d}_{\mathrm{ij}}\right)=\sum_{\lambda a} \Sigma_{\lambda b} \sum_{\lambda c} V_{\mathrm{n}}\left(\left[d_{\mathrm{ij}, \mathrm{abc}}^{\mathrm{T}} d_{\mathrm{ij}, \mathrm{abc}}\right]^{1 /}\right) \tag{3.17}
\end{equation*}
$$

$$
\begin{equation*}
\mathbf{d}_{\mathrm{i}, \mathrm{bac}} \quad=\quad \mathbf{d}_{\mathrm{j}} \cdot \mathbf{d}_{\mathrm{i}}+\lambda_{2} \mathbf{A}+\lambda_{b} \mathbf{B}+\lambda_{\mathrm{c}} \mathbf{C} \tag{3.18}
\end{equation*}
$$

$\mathbf{d}_{\mathrm{i}}$ is the location vector of atom i. A, B, and C are, again, the parallelepiped edge vectors; the summation coefficients $\lambda_{a}, \lambda_{b}$, and $\lambda_{c}$ are selected large enough to effectively capture all significant interactions. Values of $\lambda_{2}, \lambda_{b}$,and $\lambda_{c}$ up to $20(\approx 100 \AA$ for most organic crystals) have been explored in this work, but in general truncation of $\lambda_{2}, \lambda_{b}, \lambda_{c}$ at values between 4 and 8 (depending upon geometry and potential energy description) was found to be adequate to approximate the energy of the infinite lattice to within $0.1 \mathrm{kcal} / \mathrm{mole}$. This is in agreement with the findings of Tripathy et al. [1981], who concluded that the inclusion of a next-nearest-ncighbor shell in the simulation of polyvinylidene fluoride was required to accurately estimate stabilization energy but that the inclusion of additional shells was unnecessary.

The above equations may be rewritten for helical periodicity as a summation over only the atoms of the helical subunits of each crystallographically unique chain. Then Equation (3.16) holds with $\mathrm{N}_{\mathrm{i}}$ being the number of interacting centers in the helical subunit. The lattice sum becomes:

$$
\begin{align*}
& \Sigma_{\mathrm{L}} \mathrm{~V}_{\mathrm{n}}\left(\mathrm{~d}_{\mathrm{ij}}\right)=\Sigma_{\mathrm{a}} \Sigma_{\mathrm{b}} \Sigma_{\mathrm{s}} \mathrm{~V}_{\mathrm{n}}\left(\left[\mathrm{~d}^{\mathrm{T}, \mathrm{j}, \mathrm{abs}} \mathrm{~d}_{\mathrm{ij}, \mathrm{abs}}\right]^{\mathrm{H}}\right)  \tag{3.19}\\
& \mathbf{d}_{\mathrm{i}, \mathrm{j}, \mathrm{bs}}=\quad \hat{\mathbf{W}}_{\mathrm{s}}\left(\mathbf{d}_{\mathrm{j}}-\mathbf{d}_{\mathrm{ij}}{ }^{\mathrm{A}}\right)+\mathbf{d}_{\mathrm{ij}}{ }^{\mathrm{A}}-\mathbf{d}_{\mathrm{i}}+\lambda_{\mathrm{a}} A+\lambda_{\mathrm{b}} \mathbf{B}+\mathrm{s} \mathbf{F}  \tag{3.20}\\
& \mathbf{F}=\left[\begin{array}{lll}
0 & 0 & d_{h}
\end{array}\right]  \tag{3.21}\\
& \hat{\mathbf{w}}_{\mathrm{s}}=\left[\begin{array}{ccc}
\cos \left(s \Theta_{\mathrm{h}}\right) & -\sin \left(s \Theta_{\mathrm{h}}\right) & 0 \\
\sin \left(\mathrm{~s} \Theta_{\mathrm{h}}\right) & \cos \left(\mathrm{s} \Theta_{\mathrm{h}}\right) & 0 \\
0 & 0 & 1
\end{array}\right] \tag{3.22}
\end{align*}
$$

In this equation, $\mathbf{d}_{\mathrm{ij}}{ }^{\boldsymbol{j}}$ is the vector describing the displacement of a point on the axis of the helix containing atom j from that on the axis of the helix containing atom i . The latter representation, while more computationally demanding than the case of translational periodicity in three dimensions, has the advantage that it can simulate incommensurate helices throughout the latice. However, time trials using code optimized for concurrency on an Alliant FX/80 superminicomputer resulted in roughly an eightfold increase in CPU time for the helical periodicity representation. Under the special condition of helix
commensuration (i.e. $\mathrm{C}=\mathrm{nd}, \mathrm{s}=\mathrm{nc}$, and $\mathrm{n} \Theta_{\mathrm{h}}=\mathrm{m} 360^{\circ}, \mathrm{m}$ and n being integers) Equation 3.20 reduces to the form of Equation 3.18.

In our calculations we have retained the explicit potential energy calculation (Equation 3.13) within a central "core" region consisting of the reference chain segment and its nearest-neighbor shell ( $3 \times 3 \times$ DP6: an array of chain segments threc chains on a side, each segment possessing six monomer units) or in some cases the next-nearest-neighbor shell ( $5 \times 5 \times$ DP10). This region contains the dominant energy contributions which define the geometry of the structure. The explicit calculation allows one to represent local inhomogeneities such as defects in chemical constitution or tacticity or helix discommensuration and accounts for over $90 \%$ of the total calculated energy ( $98 \%$ in the $5 \times 5 \times$ DP10 case). The latice summation calculates the contribution to the system energy of interactions with the surroundings, which ensures an accurate estimate of densification and the total cohesive energy of the condensed phase structure. Thus the total energy to be minimized consists of both an explicit energy per unit contribution within a central parallelepiped and a lattice summation contribution for points outside of this parallelepiped:

$$
\begin{equation*}
E^{t \alpha}=E^{\text {unit }}\left(K \times K \text { array }, N_{r} \text { repeat units }\right)+E^{1 a t i c e}\left(\left|\lambda_{a}\right|\right\rfloor \lambda_{b}\left|>K / 2,\left|\lambda_{c}\right|>N_{r} / 2\right) \tag{3.24}
\end{equation*}
$$

The resulting picture, illustrated in Figure 3.2, may be viewed as a potentially defective pseudocrystalline parallelepiped embedded within a defect-free crystalline universe. In this manner the model attempts to capture both geometry and energy features of pseudocrystalline solid structures.

### 3.3.3 Force Field

The choice of a potential energy force field depends upon the molecular architecture. The potential energy summation must converge as its range is extended over successive interaction shells. Akin to earlier work described in the literature [McCullough, 1974; Tashiro et al., 1977; Pertsin and Kitaigorodsky, 1987], this model uses a modified Valence Force Field (VFF) description for bonded twoand three-atom deformations, and empirical torsional energy functions for bonded four-atom deformations. Nonbonded interactions are represented by a modified 12-6-1 Lennard-Jones/Coulomb energy function:

Figure 3.2: Representation of the packing structure as a local domain, described explicitly in terms of mutable atomic positions, embedded within an implicit perfect crystal.


$$
\begin{equation*}
\mathrm{E}_{\mathrm{ij}} \quad=\mathrm{A}_{\mathrm{ij}, 12} / \mathrm{d}_{\mathrm{i} j}{ }^{12}+\mathrm{A}_{\mathrm{ij}, \mathrm{j}} / \mathrm{d}_{\mathrm{ij}}{ }^{6}+\mathrm{A}_{\mathrm{ij}, 1} /\left(\mathrm{Dd}_{\mathrm{ij}}\right) \tag{3.25}
\end{equation*}
$$

where

$$
\begin{array}{rll}
A_{i j, 12} & =0.5 A_{i j, 6}\left(r_{i}+r_{j}\right)^{6} & \\
A_{i, 6} & =-\left(\varepsilon_{i} \varepsilon_{j}\right)^{1 / 2}\left(r_{i}+r_{j}\right)^{6} & \\
A_{i j, 1} & =q_{i} q_{j} / 4 \pi \varepsilon_{0} & \\
D\left(d_{i j}\right) & =1 & ; d_{i j}<d^{*} \\
& =D_{B} \exp \left(-\kappa / d_{i j}\right) \quad ; d_{i j}>d^{*} \\
\kappa & =\ln \left(D_{B}\right) d^{*} & \tag{3.30}
\end{array}
$$

$i$ and $j$ refer to the atoms separated by the distance $d_{i j} ; \varepsilon$ and $r$ are atom contributions to the van der Waals potential energy well depth and location, respectivcly, and the $q$ are partial atomic charges, which may be derived from data on low molecular weight compounds or estumated using well-established quantum mechanical methods. $\varepsilon_{0}$ refers to the vacuum permittivity. A distance-dependent dielectric constant has been employed, using the approximation of Block and Walker [1973] to reflect the different dielectric environments experienced by short and long range charge interactions. At short distances, the effective dielectric constant is unity; at larger distances, it asymptotically approaches its static bulk value, $D_{B}$. The constant $\kappa$ is chosen to ensure convergence of the two portions of the dielectric function at the crossover distance $d^{*}$. A quintic spline in $D$ between $1.0<d_{i j} / d^{*}<1.1$ is used to join the regions smoothly. A similar attenuation was employed with good results in the modeling of a polar polymer glass [Ludovice, 1989]. It is readily demonstrated that interactions of higher than third order in inverse distance (e.g. the sixth order and twelfth order induction/dispersion terms) are convergent as interaction distances become infinite (Appendix B); such is not necessarily the case of the first order electrostatic term. To ensure convergence of the summation over the electrostatic interactions one requires that the structure be electrically neutral; the electrostatic term then becomes equivalent to a summation of interactions only of dipoles and higher moments, which interact as functions of distance of third order or higher. Secondly, it is necessary in this method that the characteristic crossover distance $\mathrm{d}^{*}$ be constant for all atoms in the structure, in order to ensure that there is no net accumulation of residual electrostatic interaction as an artifact of the boundary between explicit energy calculation and lattice summation.

The representation of electrostatic interactions as partial atomic charges merits further discussion. An alternative to the use of a partial charge representation for polar species and subsequent use of a Coulombic potential would be to represent the polar character of each electrically neutral moiety by its dominant, or lowest nonzero, multipole and then to employ multipole interaction potentials. For all electrically neutral conformational subunits, the zeroth moment (total charge) is, by definition, zero. For
the amide moiety, the first nonzero moment is the dipole moment, and for the phenylene ring it is, by symmetry, the quadrupole moment. All higher nonzero moments depend on the choice of the frame of reference [Gray and Gubbins, 1984] and are of secondary importance. Thus one may employ an electrostatic interaction potential involving dipole-dipole, dipole-quadrupole, and quadrupole-quadrupole interactions, as shown in Equation 3.31.

$$
\begin{aligned}
\mathrm{E}^{\text {elec }}= & {\left[1 / 4 \pi \mathrm{D}\left(\mathrm{~d}_{\mathrm{ij}}\right) \varepsilon_{0}\right] \quad\left[\left(-\mu_{\mathrm{i}} \mu_{\mathrm{j}}^{\prime} / d_{\mathrm{ij}}^{3}\right)\left(2 \cos \theta_{1} \cos \theta_{2}-\sin \theta_{1} \sin \theta_{2} \phi\right)\right.} \\
+ & \left(3 / 2 d_{\mathrm{i}}{ }^{4}\right)\left(\mu_{\mathrm{i}} \Theta_{\mathrm{j}}\left(\cos \theta_{1}\left(3 \cos ^{2} \theta_{2}-1\right)-2 \sin \theta_{1} \sin \theta_{2} \cos \theta_{2} \cos \phi\right\}-\mu_{\mathrm{j}} \Theta_{\mathrm{i}}\left(\cos \theta_{2}\left(3 \cos ^{2} \theta_{1}-1\right)\right.\right. \\
& \left.\left.-2 \sin \theta_{1} \sin \theta_{2} \cos \theta_{1} \cos \phi\right\}\right) \\
+\quad & \left(3 \theta_{\mathrm{i}} \Theta_{j} / 4 d_{\mathrm{ij}}{ }^{5}\right)\left(1-5 \cos ^{2} \theta_{1}-5 \cos ^{2} \theta_{2}+17 \cos ^{2} \theta_{1} \cos ^{2} \theta_{2}+2 \sin ^{2} \theta_{1} \sin ^{2} \theta_{2} \cos ^{2} \phi\right. \\
& \left.\left.-16 \sin \theta_{1} \sin \theta_{2} \cos \theta_{1} \cos \theta_{2} \cos \phi\right)\right]
\end{aligned}
$$

$\theta_{1}, \theta_{2}$, and $\phi$ are angles of orientation (see Appendix B: Figure B. 1 for details). This representation requires the identification and assignment of higher order moments to subunits of the polymer chain. If we may assume that the distances over which interactions occur are large with respect to the separation of the poles of the relevant electrostatic moment, then the field may be approximated by a set of charges located at vectorial positions $\mathbf{r}_{\mathbf{i}}$. One may then define a series of multipole moments [Maitland et al., 1981]:

| zeroth moment: total charge | $q$ | $=\sum q_{i}$ |
| :--- | :--- | :--- |
| first moment: dipole | $\mu$ | $=\sum q_{i} \mathbf{r}_{\mathbf{i}}$ |
| second moment: quadrupole | $\Theta$ | $=\sum q_{i} \mathbf{r}_{\mathbf{i}} \mathbf{r}_{\mathbf{i}}$ |
| third moment: octopole | $\mathbf{0}$ | $=\sum q_{i} \mathbf{r}_{\mathbf{i}} \mathbf{r}_{\mathbf{i}} \mathbf{r}_{\mathbf{i}}$ |

etc.

The use of partial atomic charges has the advantages that (1) the charge may be selected to reproduce any higher order moments for neutral moieties, (2) select atomic substitutions are readily handled through introduction of atomic potential parameters characteristic of the new atoms, and (3) the energy calculation is entirely a function of atomic separation distance, and not orientations of groups of atoms. This last feature facilitates calculation and reduces the simulation time requirements. This is especially crucial to
the implementation of the rapid lattice sum calculation, which assumes that interactions are only distancedependent.

### 3.4 Prediction of X-ray Scattering

### 3.4.1 Structural Contributions to Intensity

Information about the microstructure of matter may be obtained experimentally only by indirect means. The most common approach to materials analysis procecds by deduction of atomic structure from observations of scattered radiation; this is the method of conventional crystallography, whereby one proposes a unit cell which is consistent with, but by no means unique to, the observed scattering of x-rays at wide angles. Our detailed atomistic modelling approach instead predicts stable structures from a priori considerations of potential energy. This process of construction from atomic building blocks may be extended to the a priori prediction of the unique pattern of scattering of $x$-rays to be observed from such energetically stable structures. In this sense, our calculation of theoretical $x$-ray diffraction patterns is conceptually the reverse of the conventional crystallographic method and ensures the additional constraint of stability of structure with respect to potential energy.

The scattering of $x$-ray radiation at wide angles (i.e. at angles greater than $5^{\circ}$ divergent from the incident beam) is the primary source of indirect information concerning atomic structure, where the scattering bodies are roughly 2 to $20 \AA$ apart. Here, the incident radiation interacts with the electron cloud distribution centered around the atomic nuclei. Scattcring, or secondary emission associated with the electrons, takes the form of coherent scattering, which is unmodified in wavelength and phase, and incoherent scattering, which is modified. The latter type, frequently termed Compton scattering, contributes to the diffuse background in x-ray diffraction and is not related to structural features, and for this reason is not considered further in the following discussion. The coherent scattering power $f_{j}$ of an atom j at zero scattering angle $2 \theta$, where all scattering is in phase, increases in proportion to the number of electrons around the atom and decreases with increasing scattering angle.

For two scattering bodies displaced from each other by the vector $\mathbf{r}$, we define the scattering vector $\mathbf{S}$ as:

$$
\begin{equation*}
\mathbf{S} \quad=\left(\mathbf{s}-\mathbf{s}_{o}\right) / \lambda \tag{3.36}
\end{equation*}
$$

where $s_{0}$ is the path of the incident radiation and $s$ is the path of the diffracted radiation, forming an angle of $2 \theta$ with the incident beam path. $\lambda$ is the wavelength of radiation. Thus the phase difference of the diffracted radiation is $2 \pi \mathrm{~S} \cdot \mathrm{r}$ and the magnitude of the scattering vector is given by Equation 3.37 . We may express $f_{j}$, the atomic scattering factor alternatively as $f_{j}(S)$ or $f_{j}(2 \theta)$.

$$
\begin{equation*}
S \quad=\quad|S|=(2 \sin \theta) \lambda \tag{3.37}
\end{equation*}
$$

For a system of N atoms, the intensity of the scattered radiation may be expressed in terms of the intensity $I_{e}$ of $x$-rays scattered by an electron by defining the structure factor $F(S)$ and its complex conjugate $\mathrm{F}^{*}(\mathbf{S})$ as follows:

$$
\begin{align*}
& \mathrm{I}(\mathbf{S})=\mathrm{I}_{\mathrm{e}} \mathrm{~F}(\mathbf{S}) \mathrm{F}^{*}(\mathbf{S})  \tag{3.38}\\
& \mathrm{F}(\mathbf{S})=\sum_{\mathrm{i}}^{\mathbf{N}} \mathrm{f}_{\mathrm{j}}(\mathbf{S}) \exp \left(2 \pi i \mathbf{S} \cdot \mathbf{r}_{\mathfrak{j}}\right) \tag{3.39}
\end{align*}
$$

The nature of the problem then becomes to calculate the structure factor. In crystalline systems, where the entire structure is simply a composite of identical unit cells, one proceeds by describing all the atoms $\mathrm{j}=1$ to N in terms of the unit cell k to which the atom belongs and the atom j within the unit cell which corresponds to i .

$$
\begin{align*}
& \mathrm{I}_{\mathrm{cr}}(\mathbf{S})=\mathrm{I}_{\mathrm{c}} \mathrm{G}(\mathbf{S}) \mathrm{F}_{\mathrm{cr}}(\mathbf{S}) \mathrm{G}^{*}(\mathbf{S}) \mathrm{F}_{\mathrm{cr}}^{*}(\mathbf{S})  \tag{3.40}\\
& \mathrm{G}(\mathbf{S}) \mathrm{F}_{\mathrm{cr}}(\mathbf{S}) \quad=\quad \sum_{\mathrm{k}} \exp \left(2 \pi i \mathbf{S} \cdot \mathbf{R}_{\mathrm{k}}\right) \sum_{\mathrm{j}} \mathrm{f}_{\mathrm{j}}(\mathbf{S}) \exp \left(2 \pi i \mathbf{S} \cdot \mathrm{r}_{\mathrm{j}}\right) \tag{3.41}
\end{align*}
$$

where now $\mathbf{R}_{\mathbf{k}}$ is the placement vector which locates the $\mathbf{k t h}$ unit cell and $\mathbf{r}_{\mathrm{j}}$ is the local placement vector which locates the jth atom within the unit cell. By this decomposition of the spatial description, it is clear that:

$$
\begin{equation*}
r_{i}=R_{k}+r_{j} \tag{3.42}
\end{equation*}
$$

By invoking the familiar reciprocal lattice coordinate system, defined by

$$
\begin{align*}
& \mathbf{a}^{*}=(\mathbf{b} \times \mathbf{c}) / \mathrm{V} \quad \mathbf{b}^{*}=(\mathbf{c} \times \mathbf{a}) / \mathrm{V} \quad \mathbf{c}^{*}=(\mathbf{a} \times \mathbf{b}) / \mathrm{V}  \tag{3.43}\\
& \mathrm{~V}=\mathbf{a} \cdot(\mathbf{b} \times \mathbf{c})=\mathbf{b} \cdot(\mathbf{c} \times \mathbf{a})=\mathbf{c} \cdot(\mathbf{a} \times \mathbf{b}) \tag{3.44}
\end{align*}
$$

and expressing the scattering vector in reciprocal space and the coordinates of the jth atom in the unit cell in term of its fractional (real space) coordinates ( $\mathrm{x}_{\mathrm{j}}, \mathrm{y}_{\mathrm{j}}$, and $\mathrm{z}_{\mathrm{j}}$ ), one can take advantage of the Laue functions for $G(\mathbf{S})$ in the limit of a very large parallelepiped crystal having sidelengths La, Mb, and Nc, to obtain the following expression for the scattered intensity [Tadokoro, 1979]:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{cr}}(\mathrm{hk} l)=\quad \mathrm{I}_{\mathrm{e}}\left(\mathrm{~L}^{2} \mathrm{M}^{2} \mathrm{~N}^{2}\right) \mathrm{F}_{\mathrm{cr}}(\mathrm{hk} l) \mathrm{F}_{\mathrm{cr}}^{*}(\mathrm{hk} l) \tag{3.45}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathbf{S}=h \mathbf{a}^{*}+\mathbf{k} \mathbf{b}^{*}+l \mathbf{c}^{*}  \tag{3.46}\\
& \mathbf{r}_{\mathrm{j}} \quad=\quad \mathrm{x}_{\mathrm{j}} \mathbf{a}+\mathrm{y}_{\mathrm{j}} \mathbf{b}+\mathrm{z}_{\mathrm{j}} \mathbf{c}  \tag{3.47}\\
& \mathrm{~F}_{\mathrm{cr}}(\mathrm{hk} l)= \sum_{\mathrm{j}=1}^{n} f_{\mathrm{j}}(\mathrm{~S}) \exp \left[2 \pi i\left(\mathrm{~h} \mathrm{x}_{\mathrm{j}}+\mathrm{k} y_{j}+l z_{\mathrm{j}}\right)\right]  \tag{3.48}\\
&=\sum_{\mathrm{j}=1}^{n} f_{j}(\mathrm{~S})\left[\cos 2 \pi\left(\mathrm{~h} \mathrm{x}_{\mathrm{j}}+\mathrm{ky} \mathrm{j}_{\mathrm{j}}+l \mathrm{z}_{\mathrm{j}}\right)+i \sin 2 \pi\left(\mathrm{~h} \mathrm{x}_{\mathrm{j}}+\mathrm{ky} \mathrm{y}_{\mathrm{j}}+l \mathrm{z}_{\mathrm{j}}\right)\right]
\end{align*}
$$

The summation in Equation 3.48 is taken over n , the number of atoms in the unit cell. $\mathrm{h}, \mathrm{k}$, and $l$ are indices defined by

$$
\begin{equation*}
\mathbf{S} \cdot \mathbf{a}=\mathrm{h} \quad \mathrm{~S} \cdot \mathbf{b}=\mathrm{k} \quad \mathbf{S} \cdot \mathbf{c}=l \tag{3.49}
\end{equation*}
$$

Using the relation

$$
\begin{align*}
& \text { he relation } \quad \mathrm{F}_{\mathrm{cr}}(\mathrm{hk} l) \mathrm{F}_{\mathrm{cr}}^{*}(\mathrm{hk} l)=\left|\mathrm{F}_{\mathrm{cr}}(\mathrm{hk} l)\right|^{2} \quad \text { we obtain finally } \\
& \mathrm{I}_{\mathrm{cr}}(\mathrm{hk} l)=\quad \mathrm{I}_{\mathrm{e}}\left(\mathrm{~L}^{2} \mathrm{M}^{2} \mathrm{~N}^{2}\right) \quad \sum_{\mathrm{j}=1}^{\mathrm{n}} f_{\mathrm{j}}(\mathrm{~S})\left[\cos ^{2} 2 \pi\left(\mathrm{~h} \mathrm{x}_{\mathrm{j}}+\mathrm{k} y_{j}+l \mathrm{z}_{\mathrm{j}}\right)+\sin ^{2} 2 \pi\left(\mathrm{~h} x_{\mathrm{j}}+\mathrm{ky} y_{\mathrm{j}}+l \mathrm{z}_{\mathrm{j}}\right)\right] \tag{3.50}
\end{align*}
$$

This is the structural contribution to diffraction intensity for a static periodic structure in the limit of large crystallite size. Since the atom positions are fixed on a threc dimensional composite lattice, the diffraction pattern consists of a set of Dirac spikes of magnitude $I_{\mathrm{cr}}(\mathrm{hk} l)$ whose positions are described by the vectors $\mathbf{S}(\mathrm{hk} l)$.

### 3.4.2 Modifications to the Structure Factor Intensity

Thermal Motion. In practice, several experimental factors contribute to modify the observed intensity. At finite temperatures the atoms vibrate about their lattice points. If we presume that these vibrations are isotropic for all atoms, then we may express the temperature factor correction to observed intensity in terms of the mean square displacement $\mu_{\mathrm{s}, \mathrm{i}}{ }^{2}$ of each atom i. This effect is indistinguishable from that caused by isotropic displacements considered in the Hosemann theory of paracrystals of the first kind [Hosemann and Bagchi, 1962]. For simplicity, we may assume a single common $\mu_{\mathrm{s}}{ }^{2}$ for all atoms and carry the temperature correction term out of the summation in Equation 3.50. The premultiplier becomes:

$$
\begin{align*}
\mathbf{M}_{\mathrm{iso}} & =\exp \left(-2 \pi^{2} S^{2} \mu_{\mathrm{s}}^{2}\right)  \tag{3.51}\\
& =\exp \left[-8 \pi^{2} \mu_{\mathrm{s}}^{2}\left(\sin ^{2} \theta / \lambda^{2}\right)\right]
\end{align*}
$$

In reality, due to atomic bonds and directionally-corrclated interactions in the crystal, such motions are quite probably anisotropic. As a first approximation, one would expect the greatest anisotropy to occur between motions parallel to the chain axis and those lateral to it. To reflect this anisotropy, Equation 3.51 may be modified by recasting the dependence upon $(\sin \theta / \lambda)^{2}$ in terms of $\mathbf{h}^{2}, \mathbf{k}^{2}$, and $l^{2}$, the squared indices of the reflection, and introducing two mean square displacements $\mu_{11}{ }^{2}$ and $\mu_{\perp}{ }^{2}$. The anisotropic thermal correction may then be expressed as:

$$
\begin{equation*}
\mathbf{M}_{\mathrm{an}}=\quad \exp \left[-2 \pi^{2}\left[\left(\mathrm{~h}^{2}+\mathrm{k}^{2}\right) \mu_{\perp}^{2}+l^{2} \mu_{11}^{2}\right]\right] \tag{3.52}
\end{equation*}
$$

Absorption. The absorption factor A reflects the reduction in scattered intensity due to absorption of radiation as it travels through the specimen. This factor depends upon specimen size, shape and elemental composition, and the wavelength of $x$-rays used. The absorption correction is therefore addressed in the discussion of experimental x-ray measurement.

Multiplicity. The multiplicity factor J reflects the number of identical reflections which superimpose during the scattering experiment. This depends upon the crystal system and the distribution of crystallite orientations within a polycrystalline sample. For example, for the cubic crystal system and completely random orientation of crystallites, the vectors $\mathbf{a}^{*}, \mathbf{b}^{*}$, and $\mathbf{c}^{*}$ are of equal magnitude and equally distributed in all orientations; thus only the total magnitude of $|\mathbf{h}|+|\mathbf{k}|+|l|$ is significant. The multiplicity in this system of $(\mathrm{h} 00)=(0 \mathrm{~h} 0)=(00 \mathrm{~h})$ is 6 , while the multiplicity of (hhh) is 8 and that of $(\mathrm{hh} 0)=(\mathrm{hOh})=(0 \mathrm{hh})$ is 12 . By including the entire set of hkl indices greater than, equal to, and less than zero in the calculation of the set of intensities $I_{c r}(h k l)$ and summing redundant reflections, multiplicity due to crystal symmetry is implicitly considered. For the distribution of crystallite orientation in fiber samples, we assume cylindrical symmetry about the molecular chain axis, $\mathbf{c}$ in our convention, and uniform distribution of crystallite orientations lateral to this axis. In this case, those reflections which scatter in the equatorial or meridional planes contribute equally to two reflections, resulting in a halving of each reflection intensity prior to summing of redundant reflections, while those which scatter within a quadrant contribute equally to reflections in all four quadrants, resulting in a quartering of each reflection intensity.

Polarization and Lorentz Factors. The $x$-rays scattered by the electrons are plane polarized to a degree that is dependent upon the scattering angle. This correction may be expressed as:

$$
\begin{equation*}
\mathrm{P}(\mathrm{hk} l)=(1+\cos 2 \theta) / 2 \tag{3.53}
\end{equation*}
$$

The Lorentz factor corrects for the relative fraction of time in which a given family of planes exists in a narrow angular range to reflect x-rays and is a function of the inclination of the planes to the axis of rotation of a crystal or the fiber axis in the case of cylindrical symmetry. Tadokoro [1979] suggests the use of the correction:

$$
\begin{equation*}
L(h k l)=\cos \theta /\left[\sin 2 \theta\left(\cos ^{2} \phi-\sin ^{2} \theta\right)^{1 / 2}\right] \quad \text { for all reflecting planes } \tag{3.54}
\end{equation*}
$$

Here, $\phi$ is the angle between the normal to the reflecting planes and the fiber axis. However, de Wolff [1962] suggests that in the case of fiber patterns, more satisfactory results may be obtained using:

$$
\begin{array}{ll}
L(h k l)^{\circ}=\quad\left(\sin ^{2} \theta \cos \theta \sin \phi\right)^{-1} & \text { for general reflections } \\
L(00 I)=\quad\left(t \sin ^{2} \theta \cos \theta\right)^{-1} & \text { for meridional reflections } \tag{3.56}
\end{array}
$$

$t$ is given to a reasonable approximation in terms of the aximuthal angle $\beta_{1 / 2}$, in radians, at which the intensity of the reflection falls to one half of its peak value (at $\beta=0^{\circ}$ ), by

$$
\begin{equation*}
t=0.815 \beta_{1 / 2} \tag{3.57}
\end{equation*}
$$

The final observed intensity, then becomes:

$$
\begin{equation*}
\mathrm{I}_{\mathrm{cbs}}(\mathrm{hk} l)=\quad \mathrm{A} \mathrm{~J}(\mathrm{hk} l) \mathrm{P}(\mathrm{hk} l) \mathrm{L}(\mathrm{hk} l) \mathrm{I}_{\mathrm{cr}}(\mathrm{hk} l) \tag{3.58}
\end{equation*}
$$

In application, the structures generated by our model are highly ordered, pseudocrystalline geometries which deviate only locally from the ideal crystal description. Thus we consider it appropriate to apply the intensity analysis for periodic latices to our structures; this is also consistent with the aforementioned view of our model as a local structure embedded within a perfect crystal matrix. The calculation first assumes a "pseudo unit cell", consisting of the domain of explicitly represented atoms or some substructure thereof. In this work, we have selected a substructure composed of one helix repeat unit or an approximation thereof from each chain type placed independently on the lattice. Then the Cartesian coordinates of all atoms comprising the cell are converted to fractional coordinates in a straightforward manner (see Appendix B). For all combinations of $h, k$, and $l$ having $|h|,|k|$, or $|l|$ less than some preselected value (e.g. 6 to 9 ) we calculate the scatering vector, from which one obtains the scattering angle $2 \theta$ by Equations 3.46 and 3.37 and the azimuthal angle $\beta$ by use of the relation

$$
\begin{equation*}
\cos \beta=(\mathrm{s} \cdot \mathrm{c} /|\mathrm{s}||\mathrm{c}|) \tag{3.59}
\end{equation*}
$$

which for a fiber sample oriented perpendicular to the incident beam, simplifies to

$$
\begin{equation*}
\cos \beta=(\mathbf{S c} /|\mathrm{S}||c|) \tag{3.60}
\end{equation*}
$$

The structure factor square $\mathrm{F}_{\mathrm{ar}}{ }^{2}(\mathrm{hkl})$ is obtained from a summation of Equation 3.50 over all atoms in the pseudo unit cell and the final intensity is corrected for thermal motion, polarization and Lorentz factors, Equation 3.58, and normalized by the most intense observed reflection; in this manner, all factors which moderate equally the intensities of all reflections are eliminated and we obtain a set of relative observable intensities which are directly comparable to experimental x-ray data. The following chapter details the specifics of application of these calculations to the case study of PPTA.

## 4. APPLICATION TO POLY(P-PHENYLENE TEREPHTHALAMIDE)

### 4.1 Introduction

This model was developed primarily with an interest in the particular features of highly extended, stiff chain polymers such as the wholly aromatic polyamides, polyesters, and other conformationally restricted long chain polymers. In the solid state, these materials are characterized by a high degree of molecular alignment. However, as a result of their relative inflexibility, they do not exhibit the conventional crystalline domain/amorphous domain segregation typical of more flexible polymers, as evidenced by the lack of an amorphous halo in x-ray diffraction measurement [Northolt and Van Aartsen, 1977]. Instead, the solid state structures tend to arrange into condensed phase nematic arrays. Aramid fibers specifically exhibit evidence of paracrystalline distortions of the second kind along the chain alignment axis. The modelling approach described above lends itself to the analysis of imperfection along the chain contour and the elucidation of the nature and molecular scale source of multiple orders (i.e. polymorphism).

For purposes of developing and testing the model we directed out attention to the in depth analysis of poly(p-phenylene terephthalamide). This structure possesses several features which recommend it for our purposes. First, PPTA has been discussed extensively in the literature and is the only such rigid rod polyamide for which there exists a sufficient set of experimental data, encompassing crystallographic and macromorphological descriptions, thermo-mechanical behavior, solution behavior, radiation absorption, and response to chemical exposure. Second, as was reviewed in the discussion of constitutional isomerism, a procedure exists to introduce controlled atomic substitutions on the parent chain and to vary the isomeric order of the PPTA derivatives; this is particularly useful in studying in a stepwise manner the sensitivity and range of applicability of the model. Third, PPTA is readily available, in the form of Dupont's $\mathrm{Kevlar}^{\circledR}$ fiber, for additional processing and testing against model predictions.

Even on a stand-alone basis, PPTA possesses several features of particular interest for molecular analysis. 1) This polymer consists entirely of alternating para-linked phenylene rings and amide linkages; each of these rigid moieties is a polyatomic group which acts in a concerted manner. The phenylene moiety may rotate without significant effect on the helical chain conformation, but with possibly major impact on chain packing. 2) The amide moiety is known to form intermolecular hydrogen bonds between
proximate hydrogens and oxygens. 3) There is the possibility for further energetically-favorable interactions due to $\pi$-bond overlap between adjacent phenylene and amide moieties within a chain or between proximate phenylene moieties between chains. These features create a vastly different environment from the neutral, flexible chain aliphatics addressed by other investigators. New problems of both intramolecular description and intermolecular behaviors must be addressed in order to produce a meaningful picture by atomistic modelling.

### 4.2 Chain Description

The polymer chain is described as an ensemble in Cartesian space of its constituent atoms, all of which are expressed explicitly. The repeat unit of the polymer chain is illustrated in Figure 4.1. All atoms in the chain are treated as point centers of force for calculation of interaction energies. We hold all bond lengths and bond angles constant, with the exception of the eight "backbone" angles (labelled $\vartheta_{1}$ through $\vartheta_{8}$ in Figure 4.1); during the course of this work, we discovered that the optimization trajectory was considerably improved when these crucial angles were allowed to change during ring rotation. The phenylene rings are represented as rigid, hexagonal constructions of the ten bonded constituent atoms. The bonding at the trivalent amide carbons and nitrogens in the amide group are fixed in the planar $\mathrm{sp}^{2}$ hybridization configuration. In this way the number of internal degrees of freedom was reduced from eighty-four to fourteen: the eight bond angles and six torsion angles ( $\phi_{1}$ through $\phi_{6}$ in Figure 4.1) of the structural unit.

### 4.3 Parameterization of the Force Field

Where possible, we have drawn upon proven force field paramcterizations for use in this model. The reasons for this are twofold: first, the parametcrizations taken from the literature have already been shown to reproduce structural features and thermodynamic quantities in related low molecular weight models and thus have some degree of credibility prior to their usage here; second, such a selection is philosophically compatible with the idea of the self-consistent force field and transferability of atomic parameters from one system to another, a concept central to the generalization of atomistic modelling methods.

A simple VFF description was employed for the eight bond angles, using values taken from Tashiro et al. [1977]. Table 4.1 gives the important parameters for the VFF description, along with all the bond lengths and bond angles required. Parameterization of the 12-6-1 nonbonded interatomic potential energy

Figure 4.1: A segment of poly(p-phenylene terephthalamide) (PPTA) with all torsion angles in their zero positions.

function for the amide moiety has been taken directly from the work of Lifson and co-workers on the development of a self-consistent force field encompassing low molecular weight alkane, alkene, amide, and carboxylic acid crystals [Lifson and Warshel, 1968; Warshel and Lifson, 1970; Hagler et al., 1974; Lifson et al., 1979]; this force field has been designed to reproduce hydrogen-bonding interactions implicitly. For the carbons and hydrogens of the aromatic rings, we initially attempted to use the values suggested by Lifson and co-workers for aliphatic carbons and hydrogens, as has been done previously by others [Bernstein and Hagler, 1978; Hummel and Flory, 1980]. The van der Waals radii for the aromatic carbons were eventually reduced by ten percent from the aliphatic carbon values suggested by these investigators, in order to improve the torsional energy approximation for ring rotation (see below). We deem this slight correction to be minor in light of the margin of error inherent in such estimates of van der Waals radii.

The partial atomic charges required for the atom-centered electrostatic interactions involving phenylene ring atoms, however, are not generally available. To estimate these, it was necessary to embark upon a series of calculations using various semiempirical quantum mechanics packages for a homologous series of low molecular weight aromatic amides. In each case, molecules in the series were completely optimized with respect to bond lengths, bond angles, and bond torsions, subject to the restriction of planar hexagonal phenylene rings, in accord with the model description. Results for the members of the series were compared for internal consistency of geometry and charge distribution. Where possible, individual members were checked for agreement with available experimental data for ring torsions and nonzero electrostatic moments. The procedures used and results obtained are detailed in Appendix C. The final atomic charges were chosen to reproduce dipole and quadrupole data through assignment of partial charges and to ensure local charge neutrality in the amide $\mathrm{N}-\mathrm{H}$, the amide $\mathrm{C}=\mathrm{O}$, and the phenylene ring. The calculated charges on the atoms of the amide group are comparable to those proposed independently by Hagler et al. [1974] and used in conjunction with their induction/dispersion parameters; the latter were retained in our work for purposes of consistency. Partial atomic charge and nonbonded potential energy parameters for all atoms are tabulated in Table 4.2.

For the C-N amide torsion, we have used the potential energy function of Jorgensen and Swensen [1985], derived from NMR data on N-methylacetamide. The interaction between the adjacent ring and amide moieties prefers coplanarity of the bond planes of these two groups in order to effect the delocalization of electrons in $\pi$-bond orbitals; however, this coplanarity is precluded by steric overlap between the hydrogens on the ring and the pendant atoms $(\mathrm{O}$ and H$)$ on the amide group. Numerous (disparate) estimates for the phenylene-amide correlated torsion energy have been reported, indicating the general lack of consistent quantitative information on this type of distributed interaction. Estimates have originated from semiempirical quantum mechanics calculations [Lauprêtre and Monnerie, 1978], $\pi$-bond group additivity considerations [Tashiro et al., 1977, p413], and empirical fitting to experimental crystal data for low molecular weight analogues [Hummel and Flory, 1980]. We have further explored this

Table 4.1
Fixed bond lengths and Valence Force Field (VFF) constants for variable backbone angles

| Angle ${ }^{\text {a,b }}$ | $\underset{\left(\mathrm{kcal} / \mathrm{deg}^{2}\right)}{\mathrm{k}_{\mathrm{i}}}$ | Bond | ( ${ }_{\text {¢ }}$ ) |
| :---: | :---: | :---: | :---: |
| $\mathrm{CC}_{\mathrm{o}} \mathrm{O}\left(\vartheta_{\vartheta}\right)$ | 0.088 | $\mathrm{C}_{\text {ar }}-\mathrm{C}_{\text {ar }}$ | 1.40 |
| $\mathrm{CC}_{0} \mathrm{~N}\left(\vartheta_{1}, \vartheta_{7}\right)$ | 0.088 | $\mathrm{Cara}_{\text {ar }}-\mathrm{H}^{\text {ar }}$ | 1.10 |
| $\mathrm{C}_{0} \mathrm{NH}\left(\vartheta_{4}\right)$ | 0.050 | $\mathrm{Cax}_{\mathrm{ar}} \mathrm{C}_{\text {am }}$ | 1.50 |
| $\mathrm{C}_{0} \mathrm{NC}\left(\vartheta_{3}, \vartheta_{5}\right)$ | 0.088 | $\mathrm{Cax}^{-\mathrm{N}^{-} \mathrm{N}}$ | 1.41 |
| CNH ( $\hat{6}_{6}$ ) | 0.050 | $\mathrm{C}_{\mathrm{am}}=0$ | 1.24 |
| $\mathrm{NC}_{0} \mathrm{O}\left(\vartheta_{8}\right)$ | 0.088 | $\mathrm{Cam}_{\mathrm{mm}} \mathrm{N}$ | 1.39 |
|  |  | $\mathrm{N}-\mathrm{H}$ | 1.00 |

${ }^{2}$ Angle bending energy $=\mathbf{k}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}}-\vartheta_{0}\right)$ where $\vartheta_{0}=120^{\circ}$ in all cases.
${ }^{\mathrm{b}} \mathrm{C}_{\mathrm{O}}$ is the sp${ }^{2}$-carbonyl carbon atom

Table 4.2
6-12 Nonbonded atomic potential parameters and elementary charges used in the electrostatic potential

| Atom | $\varepsilon$ <br> (kcal/mole) | r <br> $(\AA)$ | $\mathrm{q}^{\mathrm{a}}$ <br> (elementary charge) |
| :--- | :--- | :---: | :---: |
| C (phenylene, 1 and 4) | 0.039 |  |  |
| C (phenylene, 2 and 3) | 0.039 | 1.96 | -0.06 |
| H (phenylene) | 0.038 | 1.37 | -0.12 |
| C (amide) | 0.147 | 2.03 | +0.14 |
| H (amide) | 0.0 | 0.0 | +0.38 |
| N | 0.169 | 1.96 | +0.28 |
| O | 0.232 | 1.60 | -0.28 |
|  |  |  | -0.38 |

${ }^{2}$ The value for the bulk dielectric constant $\mathrm{D}_{\mathrm{B}}$ was 3.5 , and the Block-Walker crossover distance $\mathrm{d}^{*}$ was $3.3 \AA$.
torsion using AM1 semiempirical quantum mechanics calculations, as described in Appendix C. Figure 4.2 shows several energy functions reported in the literature, along with AM1 results for the rotation of the phenylene ring connected to the amide nitrogen in benzanilide. Figure 4.3 shows similar predictions, including AM1 rotation results for the ring connected to the amide carbon in benzanilide. For parameterization purposes, we have employed two-parameter cycloidic torsion energy functions which, in combination with the nonbonded atomic interactions, are consistent with the available torsional energy estimates. These, along with the amide torsional energy function of Jorgensen and Swensen, are reproduced below, with parameters presented in Table 4.3. Because the nonbonded interactions are retained explicitly for interactions between atoms in the ring and the amide group, the torsional potentials describing ring rotation may be attributed to the phenomenon of electron distribution between the two moieties.

$$
\begin{align*}
& E^{\text {tor,amide }} \quad=\quad 1 / 2 V_{a, 1}\left(1-\cos \phi_{2}\right)+1 / 2 V_{a, 2}\left(1-\cos 2 \phi_{2}\right) \quad \text { [same for } \phi_{4} \text { ] }  \tag{4.1}\\
& E^{\text {del.phenylene- } C=0}=\quad 1 / 2 V_{d}(1+\cos (y))  \tag{4.2}\\
& \left.\phi_{5}=1 r_{2}\left(y-m_{d} \sin (y)\right)+\pi \quad \text { [same for }\left(\phi_{5}-\phi_{1}\right)\right] \\
& E^{\text {del.phenylene-N-H }}=\quad 1 / 2 V_{d}(1-\cos (z))  \tag{4.3}\\
& \phi_{6}=1 / 2\left(\mathrm{z}-\mathrm{m}_{\mathrm{d}} \sin (\mathrm{z})\right) \quad \text { [same for }\left(\phi_{6}-\phi_{3}\right) \text { ] }
\end{align*}
$$

### 4.4 Simulation Procedure

The approach to structure optimization plays a major role in the validity of model predictions. Time and computational constraints limit the thoroughness of any such general investigation. Approaches to analysis by simulation may, however, be classified according to the goals they seek to achieve. Where one is interested primarily in elucidation of a known structure or defined region of parameter space, or in comparisons between such predefined domains, it is possible to take advantage of previous (e.g. experimental) information to limit the range of investigation, making it relatively straightforward to identify minima of immediate interest and to establish the nature of the local parameter space. This solution contains no assertions about the global nature of the minima thus identified and may depend strongly upon the choice of a starting point. Where one is concerned with a first principles approach to

Figure 4.2: Rotation energy functions suggested for the phenylene ring connected to the amide nitrogen in benzanilide: (a) $\pi$-bond additivity [Tashiro et al., 1977]; (b) empirical fit [Hummel and Flory, 1980]; (c) PCILO calculation [Lauprêtre and Monnerie, 1978]; (d) AM1 calculation (this work); (e) composite force field (this work).


Figure 4.3: Rotation energy functions suggested for the phenylene ring connected to the amide carbon in benzanilide: (a) $\pi$-bond additivity [Tashiro et al., 1977]; (b) empirical fit [Hummel and Flory, 1980]; (c) PCILO calculation [Lauprêtre and Monnerie, 1978]; (d) AM1 calculation (this work); (e) composite force field (this work).


## Table 4.3

Intrinsic torsional encrgy function parameters ${ }^{\text {a }}$

| Torsion | $\mathrm{V}_{\mathrm{a}, 1} \mathrm{~b}$ | $\mathrm{~V}_{\mathrm{a}, 2}^{\mathrm{b}}$ | $\mathrm{V}_{\mathrm{d}}^{\mathrm{b}}$ | $\mathrm{m}_{\mathrm{d}}$ |
| :--- | :---: | :---: | :---: | :---: |
| amide $\left(\phi_{2}, \phi_{4}\right)$ | 2.8 | 21.2 |  | - |
| diacid ring $\left(\phi_{5}, \phi_{5}-\phi_{1}\right)$ | - | - | 9.0 | 0.2 |
| diamide ring $\left(\phi_{6}, \phi_{6}-\phi_{3}\right)$ | - | - | 13.0 | 0.4 |

${ }^{2}$ see text for details, Equations 4.1-4.3
ball values in $\mathrm{kcal} / \mathrm{mol}$
materials investigation, it is necessary not only to locate minima, but to establish confidence in their global relation. Unfortunately, this may only be assured by investigating the whole of parameter space and identification of all minima therein, a time intensive process whose end cannot be a priori guaranteed. The latter problem necessitates an initial grid scan of parameter space, with a decision criterion established to identify relevant regions of the parameter field for further analysis. This approach, of course, suffers from grid density dependencies and response of the minimization algorithm to the convolution of the multivariate phase space.

Our analysis to idenuify likely structures for polymer packing proceeds according to the following strategy. As an initial approximation, we assume that chain conformation and chain packing considerations may be decoupled and considered independently. This is the most common approach taken in molecular mechanics and $x$-ray structure factor fitting procedures and, as was mentioned previously, is where most previous effors have stopped. As a result of this approximation, we may first employ single chain simulations to identify those conformations which are of lowest energy and are most likely to meet packing density criteria. In addition, we are primarily interested in the highly oriented structures produced during fiber spinning. Such elongational processing steps are known to favor the formation of extended chain conformations prior to vitrification or crystallization and concurrent loss of conformational mobility. Based on this, we exclude conformations of low helical pitch from further consideration.

As a second step, we perform successive grid scans of the multivariate packing parameter space using fixed chain conformations at a predetermined packing density, initially without minimization of energy with respect to intermolecular packing parameters, starting with conformations of smallest helix diameter; we conclude the search with those conformations which are of sufficiently large helix diameter to preclude the realization of reasonable packing energies in the entire parameter space. In some cases, such scans are repeated with minimization of potential energy with respect to packing parameters. The grid density was chosen based on experience with the polymer in question and reflects a subjective tradeoff between computation time and comprehensiveness. Table 4.4 lists the starting point mesh parameters, ranges, and mesh densities employed in PPTA.

In the third step, we drop the approximation of independent intramolecular and intermolecular behaviors and concentrate on those areas of conformation/chain packing space that signal the location of low energy local minima. The relevant domains thus identified are searched by means of nonlinear multivariate potential energy minimizations using an unconstrained Davidon-Fletcher-Powell Quasi-Newton algorithm with "approximate Hessian" estimation [Fletcher, 1972].

## Table 4.4

Starting point mesh used in multichain search-minimization

| Parameter | Range | Step Size |
| :--- | :--- | :--- |
| interchain distance $(\AA)$ | $4.5-9.0$ | 2.25 |
| interaxial angle (degrees) | $30-90$ | 20 |
| chain placement angle (degrees) | $0-360$ | 20 |
| chain translation | $-0.5-+0.5$ | 0.25 |

${ }^{2}$ chain translation is expressed as a fraction of the helix translation distance; values outside of this range are redundant with respect to rotation/translation

In order to reduce the parameter space of relevance for chain packing calculations, we first looked for the preferred conformation behavior of the isolated chain. For this purpose, we consider the PPTA chain as a string of torsion angle triplets, composed of the phenylene-C-N, amide, and C-N-phenylene torsions ( $\phi_{3}, \phi_{2}$ and $\phi_{1}$, respectively, in Figure 4.4), each triplet being identical and altemating in order along the chain due to the symmetry of the monomer units. This simplification arises from the assumption that intramolecular interactions are essentially decoupled across the rigid phenylene moieties. Since the amide torsion exhibits a strong preference for the trans or cis conformation, it suffices to study the conformation energy as a function of $\phi_{1}$ and $\phi_{3}$, with $\phi_{2}$ always assuming a position of minimum energy. Figure 4.4 shows such an energy contour map for benzanilide with the amide torsion optimized about its trans conformation; a similar map with minimization of the amide torsion about the cis conformation exhibits minima near $\left[30^{\circ}, 150^{\circ}\right.$ ] and $\left[150^{\circ}, 30^{\circ}\right]$, but these are energetically less favorable (by $4-5 \mathrm{kcal} /$ mole of repeat units) compared to the trans amide conformation. Of the possible monomer conformations, only those of a highly extended character will pack to densities realized in the solid state for PPTA without unreasonable penalties in intermolecular energy. Based on this, we have concentrated on those conformations having rodlike or crankshaftlike conformations for chain packing studies. Of these, only the all-trans rodlike conformations appear in the final list of lowest energy packed structures, as will be seen in the next section. Table 4.5 lists the internal coordinates of the lowest energy rodlike conformations assumed by the PPTA chain in the current force field representation. This essential rigidity ensures that chain alignment plays a dominant role in packed structure formation.

### 4.6 PPTA Simulation Results: Multichain

### 4.6.1 Selection and Presentation of Results

A screening of the packing energy using the most probable conformations in a general grid assembly and minimizing simultaneously with respect to both intramolecular and intermolecular degrees of freedom indicates considerable correlation between chain orientations in the packed structures. This is demonstrated in Figure 4.5, where the concentration of packing minima along the diagonals of a plot of total potential energy versus chain setuing angles $\omega$ (for a lauice composed of two independent chains initially in the all-trans conformation with successive phenylene rings oppositely rotated; $\omega$ is defined by the rotation of the first amide plane of the chain out of the AC-plane of the parallelepiped, Figure 3.1) is indicative of the predominant formation of sheetlike structures in PPTA; similar maps were found for packing of chains in other initial conformations.

Figure 4.4: Potential energy contour for benzanilide as a function of the ring torsions $\phi_{1}$ and $\phi_{3}$.



Table 4.5
Most favored isolated chain conformations


Figure 4.5: Energy contours for packing of extended conformations of two independently orientable chains of like conformation of PPTA, as a function of the two setting angles $\omega_{1}$ and $\omega_{2}$.


From such maps, those structures having the lowest total potential energy were identified as possible crystal structures. By means of the scan-and-search procedure, a listing of local minima is obtained; the length of this listing depends on the severity of the convolution of the potential energy hypersurface, the number of reasonably advantageous packing geometries available to the material in question, and the comprehensiveness of the scan-and-search employed. If we are only interested in the most probable polymorphs, we may discard as irrelevant those minima whose Boltzman-weighted probabilities fall below a designated significance level. That is, for some cut-off criterion $0<f<1$ :
$\Delta \mathrm{E}_{\mathrm{i}}=\mathrm{E}_{\text {toh allomorph } \mathrm{i}}-\mathrm{E}_{\text {to, global minimum allomorph }}-2 \delta \mathrm{E}$
where $\delta \mathrm{E}$ is the estimated accuracy of the energies calculated
discard all allomorphs i such that: $\exp \left(-\Delta \mathrm{E}_{\mathrm{i}} / \mathrm{RT}\right)<f$

For a $\delta E$ of roughly $1 \mathrm{kcal} / \mathrm{mol}$ and a cut-off criterion $f$ of 0.2 , or $20 \%$ probability of occurrence, at 298 K , the cut-off in energy for a significant local minimum above that of the global minimum corresponds to $2.9 \mathrm{kcal} / \mathrm{mol}$ of repeat units ( $12 \mathrm{cal} / \mathrm{g}$ ). On this basis, we discard those local minima whose energy exceeds this criterion to obtain a final basis set of significant energy-minimized structures.

The final analysis suggests up to eight distinct and viable crystalline polymorphs, all having cohesive energies within a variance of $6 \%$, or a range of $2.4 \mathrm{kcal} / \mathrm{mol}(10 \mathrm{cal} / \mathrm{g})$ between the highest and the lowest energies. These eight structures are demonstrated in the sets of orthographic projections presented in Figure 4.6(a)-(h). In these projections, the leftmost perspective looks down the chain axis, and van der Waals spheres (the dashed lines) have been included to illustrate the density of atomic packing in these structures. The middle perspective is perpendicular to the ac-facet of the structure, and the rightmost perspective is orthogonal to the previous two. In these perspectives, the carbons are black, the amide hydrogens white, the nitrogens lightly shaded, the oxygens more darkly shaded, and the ring hydrogens have been omitted from the second two perspectives for clarity. Quantitative description is given in Table 4.6. It may be noted that in the construction of this model we have intentionally favored a general parallelepiped description for chain packing, involving the vectors $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$, over a presumed unit cell or superposition-of-unit cells description. There are also some minor quantitative differences between the parameters listed in Table 4.6 and those which may be deduced from Figure 4.5, due to refinements of the model and elimination of redundant structures prior to tabulation in Table 4.6. For ease of clarification, we have converted the structural parameters in Table 4.6 from the generalized parallelepiped description to a parameter set more amenable to crystallographic interpretation and have reported parameters in both conventions. The appropriate unit cell descriptions were selected from the infinite

Table 4.6
Multichain energy minimization results: structural parameters for eight most probable unit cells

| Structure ID $\rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Structural Parameters |  |  |  |  |  |  |  |  |
| a (A) | 6.8 | 7.1 | 8.3 | 8.4 | 8.4 | 8.5 | 8.4 | 4.8 |
| b ( $\AA$ ) | 6.2 | 5.9 | 5.0 | 4.9 | 4.9 | 4.6 | 4.9 | 5.0 |
| c (A) | 13.2 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 |
| $\alpha$ (degrees) | 90 | 93 | 90 | 86 | 80 | 79 | 85 | 55 |
| $\beta$ (degrees) | 90 | 92 | 90 | 78 | 89 | 94 | 93 | 95 |
| $\gamma$ (degrees) | 88 | 92 | 92 | 89 | 90 | 93 | 90 | 96 |
| Chain Locations | $[0,0]$ | $[0,0]$ | [0,0] | [0,0] | [0,0] | [0,0] | [0,0] | [0,0] |
| (ab projection) | [ $32,2,2]$ | [122,2\%] | $[2 / 2,2,2]$ | [ 322,26 ] | [22,0] | [ 23,0$]$ | [ 22,0$]$ |  |
| Chain Setuing |  |  |  |  |  |  |  |  |
| Angles (degrees), | 6 | 2 | 18 | 12 | -2 | -9 | 2 | 2 |
| relative to the bc-facet | -174 | 2 | -161 | 13 | -1 | -9 | 1 | - |
| Inter-sheet | 6.2 | 6.6 | 5.9 | 6.6 | 2.4 | 3.1 | 6.6 | 2.7 |
| Translation ( $\AA$ ) |  |  |  |  |  |  |  |  |
| Helix Twist $\Theta_{h}$ (degrees) | 0 | 0 | 4 | 6 | 3 | 0 | 1 | 4 |
| Monomer Phenylene |  |  |  |  |  |  |  |  |
| Ring Rotation (degrees) diacid ring $\phi_{s}$ : | -16 | -30 | -26 | -35 | -43 | 39 | 43 | 34 |
| diamide ring $\phi_{6}$ : | 15 | 21 | 43 | 36 | 40 | 38 | 33 | 29 |
| Cohesive Energy (kcal/mole of repeat units ) | 40.5 | 38.5 | 38.3 | 38.2 | 38.1 | 39.3 | 39.7 | 39.6 |
| Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.42 | 1.45 | 1.45 | 1.49 | 1.50 | 1.57 | 1.47 | 1.56 |
| Parallelepiped |  |  |  |  |  |  |  |  |
| Description: |  |  |  |  |  |  |  |  |
| A (A) | 4.7 | 4.5 | 4.8 | 4.9 | 4.2 | 6.2 | 6.4 | 5.0 |
| B ( $\AA$ ) | 4.6 | 4.7 | 4.9 | 4.8 | 6.4 | 4.4 | 4.2 | 4.8 |
| C ( $\AA$ ) | 13.2 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 |
| $\tau$ (degrees) | 90 | 90 | 90 | 81 | 97 | 98 | 93 | 95 |
| $\checkmark$ (degrees) | 90 | 94 | 90 | 77 | 89 | 88 | 88 | 55 |
| $\zeta$ (degrees) | 85 | 81 | 62 | 61 | 50 | 48 | 50 | 96 |
| $\omega_{1}$ (degrees) | 127 | 128 | 101 | 110 | 92 | 146 | 138 | 92 |
| $\omega_{2}$ (degrees) | -53 | 128 | -79 | 110 | 92 | 146 | 138 | 92 |
| $\mathrm{f}_{2}$ | 0.47 | 0.50 | 0.45 | 0.50 | 0.18 | -0.25 | 0.50 | 0.0 |

Figure 4.6: Orthographic perspectives of the eight primary structures for PPTA suggested by simulation. The sets (a) through (h) correspond to the structures labelled 1 through 8, in order, in Table 4.6. Left: z-axis perspective; Middle: perspective perpendicular to the ac-plane; Right: perspective perpendicular to the previous two perspectives.
(a)

```
PPTA Structure #1
```



## PPTA Structure \#2

(b)


Figure 4.6: Orthographic perspectives of the eight primary structures for PPTA suggested by simulation (continued).
(c)

PPTA Structure \#3


PPTA Structure \#4
(d)


Figure 4.6: Orthographic perspectives of the eight primary structures for PPTA suggested by simulation (continued).
(e)

```
PPTA Structure #5
```



PPTA Structure \#6
(f)


Figure 4.6: Orthographic perspectives of the eight primary strucures for PPTA suggested by simulation (continued).
(g)

```
PPTA Structure #7
```



PPTA Structure \#8
(h)

number of possible descriptions based on a preference for interaxial angles $\alpha=\angle \mathbf{b c}, \beta=\angle \mathbf{a c}$, and $\gamma=\angle \mathbf{a b}$ closest to $90^{\circ}$. It may readily be seen that the crystallographic cell, described by the vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$, obeys the relations: $\mathbf{c} \| \mathbf{C}$ and $\mathbf{a x b} \|_{\mathbf{A} \times \mathbf{B}}$. Similarly, the chain setting angles are reported relative to the bc-facet (the bxc plane), as was done by Northolt for PPTA, rather than relative to the arbitrarily selected $\mathrm{A} \times \mathrm{C}$ plane of the parallelepiped.

### 4.6.2 Evaluation of Polymorphs

From this basis set of lowest energy packing minima, one may draw several conclusions concerning the preferred structure of PPTA:

1) The backbone features of individual chain conformations are in reasonable agreement with those employed by Northolt in his crystal structure analysis. Table 4.7 shows the final values of the important backbone angles and torsions of the chain conformation in the packed geometries. Angles of $10^{\circ} \pm 4^{\circ}$ are formed between the phenylene rings and the chain propagation axis, and torsions about the amide bond vary from $2^{\circ}$ to $7^{\circ}$ and are generally balanced by alternate plus/minus torsions of equal magnitude in consecutive amide moieties. This conformation orients the amide dipoles in all cases in nearly antiparallel fashion as one travels along the chain axis. Phenylene ring positions about the $\mathrm{C} 1-\mathrm{C} 4$ axes relative to the plane of the adjacent amide bond show a preference for out-of-plane rotation (see Table 4.7), as expected from the isolated chain energetics reported in the previous section and from repors in the literature [Northolt, 1977; Tashiro et al., 1977 p413; Hummel and Flory, 1980]. Packing considerations do not appear to preclude either of the two possible directions of ring rotation. However, the chain packing does appear to have a considerable effect on the magnitude of out-of-plane rotation, with values ranging between roughly $15^{\circ}$ and $43^{\circ}$ for either ring.
2) While no hydrogen bonding potential energy function has been explicitly included, nevertheless the geometry and energy character of "hydrogen bonding" between amide groups in neighboring chains is reasonably reproduced. This is in good agreement with the findings of Hagler et al. [1974]. For the final eight structures, the average $0 \cdots \mathrm{H}$ "bond" distance is $2.3 \AA$, with a $\mathrm{N}-\mathrm{H} \cdots \mathrm{O}$ angle of $160^{\circ}$. The interaction energy atributable to the hydrogen bond varies between 1.8 and $3.8 \mathrm{kcal} / \mathrm{mole}$, and is largely due to the strong electrostatic attraction between the hydrogen-oxygen pair. Hagler et al. suggest that their force field for the amide moiety, employed unchanged in this work, yields an optimum hydrogen bonding distance of $2.1 \AA$ and a "bond energy" of $2.4 \mathrm{kcal} / \mathrm{mol}$; in their study of low molecular weight amide crystals, predicted hydrogen bond lengths were slighty lower, ranging from $1.9 \AA$ up to $2.1 \AA$, with a N-H...O angle of $145^{\circ}$ to $175^{\circ}$. Our calculated bond length of $2.3 \AA$ lies intermediate between these values and the $2.9 \AA$ proposed by Ladell and Post [1954] to be typical for amide crystals. The $160^{\circ}$

Table 4.7

## Chain conformations realized in packed structures

| Structure $\rightarrow$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Internal Coordinate |  |  |  |  |  |  |  |  |
| Bond Angles (degrees) |  |  |  |  |  |  |  |  |
| $\mathrm{Ca}_{\mathrm{ar}}-\mathrm{Cam}_{\text {a }} \mathrm{N}$ | 118.6 | 118.9 | 118.9 | 119.3 | 118.3 | 118.6 | 118.7 | 119.0 |
| $\mathrm{Ca}_{\mathrm{ar}}-\mathrm{Cam}_{\mathrm{am}}=0$ | 120.2 | 119.4 | 122.0 | 120.6 | 121.8 | 121.7 | 120.7 | 120.8 |
| $\mathrm{Cam}_{\text {am }}$ - $\mathrm{N}-\mathrm{H}$ | 115.1 | 115.4 | 118.8 | 118.2 | 118.4 | 117.8 | 117.3 | 117.3 |
| $\mathrm{Cam}_{\text {m }}-\mathrm{N}-\mathrm{C}_{2 \mathrm{r}}$ | 129.3 | 128.3 | 123.1 | 124.1 | 123.1 | 124.2 | 124.8 | 126.0 |
| $\mathrm{Car}_{2 \mathrm{r}} \mathrm{N}-\mathrm{H}$ | 115.7 | 116.2 | 118.4 | 118.5 | 117.7 | 117.8 | 117.9 | 116.8 |
| $\mathrm{C}_{\mathrm{ar}} \mathrm{N}^{-} \mathrm{C}_{2 \mathrm{~m}}$ | 129.1 | 128.4 | 122.7 | 123.9 | 125.0 | 124.5 | 124.8 | 125.9 |
| $\mathrm{N}-\mathrm{C}_{\mathrm{am}}=\mathbf{O}$ | 121.3 | 121.7 | 119.2 | 120.4 | 120.7 | 119.6 | 120.6 | 120.4 |
| $\mathrm{N}-\mathrm{C}_{\mathrm{am}} \mathrm{C}_{\mathrm{ar}}$ | 120.4 | 119.4 | 121.9 | 120.2 | 119.0 | 121.4 | 120.6 | 120.4 |
| Torsions (degrees) |  |  |  |  |  |  |  |  |
| $\stackrel{\mathrm{N}}{ }-\mathrm{C}_{2 \mathrm{~m}}-\mathrm{C}_{2 m}-\mathrm{N}$ | 0.3 | 0.5 | -3.0 | -3.3 | -4.1 | -0.4 | -0.6 | -5.7 |
| $\mathrm{C}_{2 \mathrm{~m}}-\mathrm{C}_{\text {am }}-\mathrm{N}-\mathrm{N}$ | 7.3 | 7.3 | 5.6 | 3.8 | 4.4 | 3.1 | 4.1 | 7.8 |
| $\mathrm{C}_{\text {am }} \mathrm{N}-\mathrm{N}-\mathrm{C}_{\text {am }}$ | -0.4 | 0.3 | -0.8 | -1.9 | 0.1 | 0.4 | -0.1 | -1.6 |
| $\mathrm{N}-\mathrm{N}-\mathrm{C}_{2 m}-\mathrm{C}_{2 m}$ | -7.1 | -7.3 | -6.6 | -4.8 | -3.5 | -3.3 | -4.4 | -4.3 |
| Out-of-Plane Ring Rotations (degrees) diacid phenylene diamide phenylene |  |  |  |  |  |  |  |  |
|  | $\begin{array}{r} 164 \\ 15 \end{array}$ | $\begin{array}{r} 151 \\ 19 \end{array}$ | $\begin{array}{r} 154 \\ 43 \end{array}$ | $\begin{array}{r} 146 \\ 37 \end{array}$ | $\begin{array}{r} 138 \\ 40 \end{array}$ | $\begin{aligned} & 37 \\ & 35 \end{aligned}$ | $\begin{aligned} & 43 \\ & 33 \end{aligned}$ | $\begin{aligned} & 34 \\ & 29 \end{aligned}$ |
| Helix Parameters |  |  |  |  |  |  |  |  |
| Pitch ( $\AA$ ) | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 |
| Twist (degrees) | 0 | 1 | 4 | 6 | 3 | 0 | 1 | 4 |

angle is essentially identical to that assumed by Schroeder and Lippincott [1957] in their explicit potential function representation of the $\mathrm{N}-\mathrm{H} \cdots \mathrm{O}$ hydrogen bond.
3) The nature of the chain-chain interactions are consistent across the set of eight structures, as suggested by the orientation correlation maps, and determine the extent of lateral order (perpendicular to the chain axis) seen in the polymorphs. The hydrogen bonding interactions for each chain lie roughly within a plane as a result of the chains' all-trans conformations; this leads to the association of chains by hydrogen bonding into sheets and allows one to distinguish between interchain energies within and between sheets. These energies may be broken down into their van der Waals and electrostatic components for each cell structure. Table 4.8 shows the sources of interactions within a sheel. In each case, the dominant contribution ( $66 \%$ to $90 \%$ of the total) is elcctrostatic. Furthermore, the electrostatic component may be decomposed into amide-amide interactions and phenylene-phenylene interactions as shown, with the remainder being a cross contribution from amide-phenylene interactions; clearly, the electrostatic interaction is predominantly between amide moieties and is quite specific in nature. Thus one finds an interaction of considerable magnitude and moiety-to-moiety specificity between chains within a hydrogen-bonded sheet, which ensures axial register and consistent interchain separation distances. This interaction takes priority in organizing the packing of chains lateral to the axis of chain alignment in the solid state. Table 4.9, on the other hand, illustrates the nonspecific nature of the interchain energetics between chains in neighboring sheets. Of particular interest is the magnitude of the total energy, which, contrary to previous conjecture, is comparable to that between chains within a sheet. However, the source of this interaction energy is evenly split between van der Waals and electrostatic contributions, and the latter in turn is relatively evenly split among the interactions between different moieties. From this one may conclude that while sheet-sheet separation is a high energy process, sheet-sheet slip remains a viable deformation mode, and the lack of preferred register between sheets is largely responsible for the multiplicity of crystal forms witnessed in the model analysis. This tendency towards slip is consistent with compressive failure observations reported in the literature [Knoff, 1987; Morgan et al., 1983].
4) The predicted densities lie between 1.42 and $1.57 \mathrm{~g} / \mathrm{cm}^{3}$; these values result largely from the choice of $12-6-1$ potential energy parameters. While the model does not explicitly consider thermal effects, the force field parameters implemented were derived from experimental data on low molecular weight amide crystals at $25^{\circ} \mathrm{C}$, and therefore incorporate thermal motion effects into the force field parameters themselves. The resulting densities are in good agreement with reported PPTA solid densities of 1.45 to $1.50 \mathrm{~g} / \mathrm{cm}^{3}$.
5) The cohesive energy is defined as the increase in internal energy per mole of a substance upon removal of all intermolecular forces [Van Krevelen, 1976]. This value is readily calculated as the difference between the energy of the packed structure and the energy of the most stable isolated chain

## Table 4.8

Intermolecular bonding energy; component contributions to chain-chain interaction within a sheet (all values in $\mathrm{kcal} /$ mole of repeat units)

| Cell ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total $12-6$ | 2.1 | 2.6 | 2.8 | 2.2 | 2.6 | 1.4 | 2.5 | 0.9 |  |
| Total <br> electrostatic | 5.8 | 5.1 | 5.4 | 6.9 | 7.1 | 8.1 | 7.5 | 8.3 |  |
| Amide-amide <br> electrostatic | 3.8 | 3.3 | 3.5 | 5.7 | 5.9 | 7.4 | 5.7 | 6.8 |  |
| Phenylene-phenylene <br> electrostatic | 0.7 | 1.1 | 1.9 | 1.7 | 1.5 | 1.9 | 2.1 | 2.1 |  |
| Total | 7.9 | 7.7 | 8.2 | 9.1 | 9.7 | 9.5 | 10.0 | 9.2 |  |

Table 4.9
Intermolecular bonding energy; component contributions to chain-chain interaction between sheets (all values in $\mathrm{kcal} /$ mole of repeat units)

| Cell ID | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Total 12-6 | 4.4 | 3.8 | 3.2 | 3.4 | 4.4 | 4.3 | 4.5 | 4.6 |
| Total <br> electrostatic | 2.5 | 2.5 | 4.6 | 2.7 | 3.8 | 2.5 | 4.8 | 3.1 |
| Amide-amide <br> electrostatic | 0.8 | 1.1 | 1.7 | 1.2 | 0.8 | 0.3 | 1.7 | 0.2 |
| Phenylene-phenylene <br> electrostatic | 1.3 | 1.6 | 1.5 | 1.2 | 1.3 | 2.1 | 2.6 | 2.2 |
| Total | 6.9 | 6.3 | 7.8 | 6.1 | 8.2 | 6.8 | 9.3 | 7.7 |

conformation. The Hildebrandt solubility parameter may then be calculated as the square root of the cohesive energy density:

$$
\begin{equation*}
\delta=\left(\mathrm{E}_{\mathrm{coh}} / \mathrm{V}\right)^{1 / 2}=\left(\mathrm{E}_{\mathrm{coh}} \mathrm{p} / \mathrm{M}_{\mathrm{w}}\right) \tag{4.6}
\end{equation*}
$$

In this manner, for the cohesive energy and corresponding solubility parameter for PPTA based on the set of eight packing structures we obtain (per mole of repeat units):

$$
\begin{aligned}
& \mathrm{E}_{\mathrm{coh}}=39.0 \pm 1 \mathrm{kcal} / \mathrm{mol} \\
& 15.3\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}<\delta<16.1\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2} \\
& \text { or } 31.2 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}<\delta<32.8 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}
\end{aligned}
$$

This range is quite reasonable in light of the relative insolubility and high temperature melting behavior of PPTA. While experimental values for solubility parameters of aromatic polyamides are not available, values for some related aliphatic polyamides are 12.7 and $13.6\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}$ (or $25.9 \times 10^{3}$ and $27.7 \times 10^{3}$ $\left[\mathrm{J} / \mathrm{m}^{3}\right]^{1 / 2}$ ) for poly(8-aminocaprylic acid) and poly(hexamethylene adipamide), respectively [Van Krevelen, 1972, p88]; we would expect the aromatic polyamide value to be somewhat higher, reflective of its reduced solubility in all but the most aggressive solvents.

Finally, significant differences between the eight final structures should be noted. All geometries except Structure \#8 contain two chains per unit cell. The chains in Structures \#1 through \#4 all intersect the ab-plane at $[0,0]$ and $[1 / 2,1 / 2]$. Of these, Structures \#1 and \#3 each contain conformations having oppositely rotated phenylene rings and sheet packing such that amide moieties in successive sheets are in register along the c dimension with alternating direction of the amide dipoles; Structures \#2 and \#4 are the analogues, respectively, of \#1 and \#3 with the distinction being the codirection of amide dipoles in successive sheets. The chains in Structures \#5 through \#7 intersect the ab-plane at [0,0] and [ 32,0 ]. Only Structure \#5 has the adjacent rings rotated in the opposite sense as seen in the first four structures; Structures \#6 and \#7 have similarly rotated rings along the chain. The seventh structure is an analogue to the sixth, with the former having improved register between amide moieties in neighboring sheets. The eighth structure contains only one chain per unit cell but with an $\alpha$ angle of $55^{\circ}$ that effectively creates the axial shift between hydrogen-bonded sheets seen in the other seven structures.

Applying the principles described in the previous chapter, we calculated the sets of $x$-ray diffractions anticipated from each of the eight possible structures for PPTA. Atomic scattering factors were calculated using exponential functions of $x=(\sin \theta / \lambda)$ in the range $0 \leq x \leq 1$, as described in Appendix $D$, using tabulated values of the atomic scatering factors readily available in the International Table for X-Ray Crystallography [1968]. By invoking fiber symmetry, the complete scaturing patuern may be predicted from a plot of reflection intensity versus scattering angle $2 \theta$ and azimuthal angle $\beta$ in a single quadrant (e.g. the $0 \leq 2 \theta, 0 \leq \beta \leq 90$ quadrant). The calculated intensities are normalized by the most intense reflection (generally the [200] reflection for PPTA) to yicld values between 0 and $100 \%$; these are tabulated for all reflections greater than $1 \%$ observable intensity in Appendix E, Table E.1. More illustrative, however, is the graphical recreation of the entire fiber diffractogram for immediate visual comparison to experimental films. We have presented patterns of intensity directly in polar form as functions of the scattering angle $2 \theta$ and the azimuthal angle $\beta$, which corresponds to projection of the scattering on a hemisphere with the sample positioned at its origin. For purposes of display, the calculated reflections were assumed to be distributed about their mean positions according to Gaussian probabilities in $2 \theta$ and $\beta$. The breadths of these distributions represent the expected peak broadening due to finite crystallite sizes, the crystal mosaic, and paracrystal displacements of the second kind, which are not available a priori from the model calculations. Additionally, we require an estimate for $\beta_{1 / 2}$ for the Lorentz correction of meridional reflection intensities. The peak broadening distributions were selected of the form:

$$
\begin{equation*}
y \quad=\quad A \exp \left[-\ln 2\left(2\left(x-x_{0}\right)^{2} / \omega_{x}^{2}\right)\right] \tag{4.7}
\end{equation*}
$$

A is the amplitude of the distribution at $\mathrm{x}_{0}$ and $\omega_{\mathrm{x}}$ is the breadth of the distribution at $\mathrm{y}=\mathrm{A} / 2$. Then the intensity at any point of the diagram may be calculated by summation of the contributions from all reflections at that point; the calculated intensity $I_{\text {obs }}$ of each reflection $i$ located at $\left(2 \theta_{i}, \beta_{i}\right)$ equals the integrated volume of the distribution in $2 \theta$ and $\beta$ :

$$
\begin{align*}
& \mathrm{I}_{\mathrm{obs}}(\mathrm{hkl})=\quad \mathrm{A}_{\mathrm{hkl}} \omega_{2 \theta} \omega_{\beta} \pi / 4 \ln 2  \tag{4.8}\\
& I_{\text {cbs }}\left(x_{1}, x_{2}\right)=\sum_{i=1}^{n p} A_{i}\left(\exp \left\{-\ln 2\left[2\left(x_{1}-2 \theta_{i}\right) / \omega_{29}\right]^{2}\right\}\right) \\
& \times\left(\exp \left(-\ln 2\left[2\left(x_{2}-\beta_{j}\right) / \omega_{\beta}\right]^{2}\right]\right) \tag{4.9}
\end{align*}
$$

We require only three parameters not available through the model, the root mean square thermal displacement amplitude and estimates of $\omega_{2 \theta}$ and $\omega_{\beta}$, in order to recreate the entire fiber diffractogram. For our universal isotropic thermal displacement factor we employed the B value used successfully by Boon and Magré [1970] for poly(p-phenylene oxides) and by Northolt [1974] for the experimental analysis of scattering from PPTA:

$$
\text { B } \equiv \quad 8 \pi^{2} \mu_{\mathrm{s}}^{2}=5 \times 10^{-16} \mathrm{~cm}^{2}
$$

corresponding to a root mean square displacement of $0.25 \AA$. For $\omega_{2 \theta}$ and $\omega_{\beta}$ values of 1.5 degrees and 15 degrees, respectively, were found to be appropriate, corresponding to $\beta_{1 / 2}$ equal to 7.5 degrees. At this level of projection, it was not deemed justified to incorporate anisotropy of the thermal motion correction into the pattern simulations.

Due to the nature of the simulation, the calculated structures are inherently triclinic, so that observed reflections are actually groups of reflections of related diffraction planes; the slight displacement of these planes further contributes to the diffuseness of reflections expected in observable pauerns and illustrates the potential complexity of reflections whose composite appears as a single "reflection" on film. The composites of reflections are illustrated in the recreated fiber diffractograms presented in Figures 4.7 (a) through (h), again corresponding to the eight PPTA structures described previously. These simulated fiber diffraction pattems were generated on an IRIS 4D series workstation with GT graphics and a $1280 \times 1024$ pixel display terminal. The grid density ( $2 \theta$ by $\beta$ ) was 500 points by 450 points per quadrant, or steps of 0.1 degrees in $2 \theta$ and 0.2 degrees in $\beta$. Visual resolution was enhanced using a nonproportional gray scale to achieve the greatest visual sensitivity at the lower intensities characteristic of the majority of reflections; a scale is provided in each diffractogram for clarification.

Diagrammatic representations of the calculated x -ray diffraction patterns provide a simple yet effective means for distinguishing between polymorphs at the phenomenological level, both for comparing and contrasting the various results of the model analyses, and for latter comparison to actual experimental data. For example, common to all the final polymorphs are the intense [002], [004], and [006] meridional reflections, characteristic of the extended chain conformations and the approximate $2_{1}$ screw symmetry of chains; inclination of the normal to the ab-facet away from the chain axis results in these reflections appearing slightly off-meridian in a few cases. Only in the fifth, sixth, and eighth structures, where translation between neighboring sheets is 2.5 to $3.0 \AA$, are weak [001] and [005] meridional reflections observed. Most characteristic of the simulated PPTA pauterns is the fingerprint of the equatorial and first layer lines of each structure. Here one can easily distinguish, by means of the

Figure 4.7: $\quad$ Simulated $x$-ray fiber diffraction patterns for the eight primary structures for PPTA. The sets (a) through (h) correspond to the structures labelled 1 through 8, in order, in Table 4.6.
(a)


Figure 4.7: $\quad$ Simulated $x$-ray fiber diffraction patterns for the eight primary structures for PPTA (continued).
(e)
(g)

(h)
presence or absence of [011] or [111] families of reflections (relative to the intense [110] reflection), between structures \#1 and \#3, having oppositely-directed amide dipoles in successive hydrogen-bonded sheets, and structures \#2 and \#4, having codirected dipoles in successive sheets. Structures \#6 and \#7, having face-centered unit cells and phenylene rings that are similarly rotated with respect to the amide bonding planes (and hence with respect to the hydrogen bonded sheets) have characteristically intense second layer reflections (i.e. [112] or [212] families of reflections). Structures \#1 and \#3 bear the hallmarks of the first modification of PPTA, reported by Northolt. As can be seen from a comparison of the unit cell parameters listed in Table 4.6 with those presented carlier in Chapter 2, Table 2.4, Structure \#3 agrees most closely with the reported unit cell dimensions of Modification I. Only Structure \#5 appears to exhibit any resemblance to Modification II reported by Haraguchi et al. (Table 2.4), most notably in the appearance of a [010] reflection and the combination of [210] and [211] sets of reflections, which reproduces the intensity attributed by Haraguchi entircly to the [211] planes in Modification II. However, in this case the agrecment between simulation and experiment is not as satisfying in general. This is due in part to the considerable tilt $\left(12^{\circ}\right)$ of the normal to the ab-facet with respect to the chain axis in the simulated structure, resulting in the spliting of most reflections along the layer lines. However, it should also be noted that Modification II shows evidence of being a kinetically-frozen structure which can convert irreversibly to Modification I upon high temperature annealing. That this modification is not as faithfully predicted by simulation as is the first modification could be due in large part to the specific set of process conditions, namely spinning into an aqueous environment with no subsequent annealing, required to realize Modification II in practice, conditions which are in no way incorporated into the model.

### 4.7 Summary

The methodology presented in the preceding chapter serves to generate from a priori considerations predictions for the atomistic structural detail of highly ordered polymeric solids. In addition to its suitability in representing ideal periodic structures of packed helical molecules, such as have been almost exclusively dealt with in the past, this representation provides a method by which imperfections of either a periodic or local nature may be accurately incorporated into the analysis. This additional capability becomes especially important as interest increases in polymers of a paracrystalline or polycrystalline nature. Application of this procedure to the study of a representative stiff chain polymer, PPTA, reveals that, due to the convolution of the potential energy hypersurface, one may expect a multiplicity of viable candidates for stable geometries; the first point to address, then, becomes the selection of what information one chooses to derive from such an analysis and reduction of the output data to a minimum relevant basis set. Second, in the presence of strong intermolecular packing forces, one encounters competing trends of similar magnitude, one driven by bonded interactions within a chain and the other by non-
bonded interactions of the type commonly attributed to hydrogen bonds between chains. One may prioritize the development of structure according to the magnitude and specificity of interaction energies; for PPTA, the key traits are the chain stiffness arising from the rigidity of the moieties, the coplanar hydrogen bonding between chains arising from the rodlike conformation of each chain and the consequent persistent formation of sheet structures, and the relative variety of packing of sheets which leads to the occurance of distinguishable polymorphs having comparable stability and capable of coexistence at the microscopic level. Lastly, where several polymorphs are equally stable, each may be distinguished and identified by its characteristic $x$-ray diffraction pattem. This enables one to address the comparison between different simulated structures and between simulation and experimental data at the level of the identifying phenomenon (i.e. scatering) itself. Such analysis allows one to group the polymorphs according to intensity and location of reflections and to identify those which most closely agree with experimental observations.

## 5. MECHANICAL PROPERTIES BY SIMULATION

### 5.1 Thermodynamic Analysis

### 5.1.1 Classical Thermodynamics of Deformation

Thermodynamic stability of structure is defined by minimization of the free energy with respect to the degrees of freedom describing the structure; as such, it is composed of both an internal energy contribution (potential and kinctic) and an entropic contribution, as described by Equation 5.1. It is worth emphasizing at this point that so far we have spoken entirely of the relative stability of geometries based on a criterion of minimum potential energy, as is common practice in molecular mechanics methods. The justification for this simplification is several fold. First, we speak only of static structures in which atoms are assumed to behave as stationary centers of force; kinetic energy effects enter only through the selection of the force field parameters themselves, such as the effective van der Waals radii, which are derived from crystal data at finite temperatures. Thus for determination of relative stability of structure, kinetic energy factors are lumped into the potential energy implicitly. Second, we relax our criterion of thermodynamic equilibrium in favor of one of mechanical equilibrium, a condition more suitable to the visualization of dense polymer systems, which is by definition the minimum internal energy with respect to the degrees of freedom describing the structure. It may be mentioned that to the extent that we are simulating crystalline or pseudocrystalline solids at room temperature or lower, we would not anticipate large entropic contributions to stabilization energy. For example, Tadokoro [1979] calculates a total contribution of roughly 1 kcal per mole of $\mathrm{CH}_{2}$ units in crystalline polyethylene between 0 K and 300 K . This translates into $17 \mathrm{kcal} /$ mole in the PPTA case, on an equivalent per gram basis, or less than $50 \%$ of the stabilization potential energy. This estimate should be even smaller for stiff chain polymers such as PPTA. Even so, it is not the absolute magnitude of the entropic contribution, but its difference from structure to structure, that affects the relative stability; this is certain to be smaller yet.

The calculation of elastic compliances and clastic stiffnesses, however, is another mater. Here, we require the second derivatives of the Helmholtz free energy A with respect to deformation stress or strain:

$$
\begin{align*}
& A=U-T S  \tag{5.1}\\
& d A=d U-T d S-S d T  \tag{5.2}\\
&=d U-T d S \quad  \tag{5.3}\\
& \\
& \\
& \text { for isothermal deformations }
\end{align*}
$$

The elements of the fourth order elastic stiffness tensor are defined as:

$$
\begin{equation*}
\left.\left(\partial^{2} \mathrm{~A} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)\right|_{\mathrm{O}}=\mathrm{V}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMNK}} \tag{5.4}
\end{equation*}
$$

where the elements of the Lagrangian (material) strain tensor $\varepsilon_{\mathrm{LM}}$ are defined using:

$$
\begin{equation*}
2 \varepsilon_{\mathrm{LM}}=\partial \mathrm{u}_{\mathrm{L}} / \partial \mathrm{x}_{\mathrm{M}}+\partial \mathrm{u}_{\mathrm{M}} / \partial \mathrm{x}_{\mathrm{L}} \quad+\sum_{\mathrm{l}=1}^{3} \partial \mathrm{u}_{\mathrm{I}} / \partial \mathrm{x}_{\mathrm{L}} \partial \mathrm{u}_{\mathrm{l}} / \partial \mathrm{x}_{\mathrm{M}} \tag{5.5}
\end{equation*}
$$

$\mathbf{u}$ is the displacement vector describing deformation. The subscript LM refers to deformation of the L surface acting in the $M$ direction. For small deformations, we may neglect the terms of second order in displacement.

Consider first an arbitrary elastic solid subjected to an arbirary isothermal small deformation; the internal energy $U$ may be expressed as a Taylor series expansion about the undeformed state. Neglecting terms higher than second order, one writes for the difference in internal energy between the deformed state and the ground state:

$$
\begin{align*}
\mathrm{dU} & =U_{\mathrm{def}}-U_{\mathrm{O}} \\
& =\Sigma_{\mathrm{LM}}\left(\partial \mathrm{U} / \partial \varepsilon_{\mathrm{LM}}\right) \|_{\mathrm{O}} \varepsilon_{\mathrm{LM}}+\not 2 / 2 \Sigma_{\mathrm{LM}} \Sigma_{\mathrm{NK}}\left(\partial^{2} \mathrm{U} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right) l_{\mathrm{O}} \varepsilon_{\mathrm{LN}} \varepsilon_{\mathrm{NK}} \tag{5.6}
\end{align*}
$$

From Weiner [1983], we have the following relations:

$$
\begin{align*}
& \left(\partial \mathrm{U} / \partial \varepsilon_{\mathrm{LM}}\right)\left\|_{\mathrm{O}}=\left(\partial \mathrm{A} / \partial \varepsilon_{\mathrm{LM}}\right)\right\|_{\mathrm{O}}-\mathrm{T}\left(\partial \mathrm{~S} / \partial \varepsilon_{\mathrm{LM}}\right) \|_{\mathrm{O}} \quad \text { (isothermal) }  \tag{5.7a}\\
& \left(\partial \mathrm{A} / \partial \varepsilon_{\mathrm{LM}}\right) \|_{\mathrm{O}}=\mathrm{V}_{\mathrm{O}} \sigma_{\mathrm{LM}} \tag{5.7b}
\end{align*}
$$

$$
\begin{align*}
& \left.\left(\partial S / \partial \varepsilon_{L M}\right)\right)_{o}=\gamma_{L M} C_{\varepsilon}  \tag{5.7c}\\
& \left(\partial^{2} \mathrm{U} / \partial \varepsilon_{L M} \partial \varepsilon_{N K}\right)\left\|_{O}=\left(\partial^{2} A / \partial \varepsilon_{L M} \partial \varepsilon_{i K}\right)\right\|_{O}-T\left(\partial^{2} S / \partial \varepsilon_{L M} \partial \varepsilon_{i K}\right) l_{O}  \tag{5.7d}\\
& \left(\partial^{2} \mathrm{~S} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{YK}}\right) \|_{\mathrm{O}}=-\mathrm{V}_{\mathrm{O}}\left(\partial \sigma_{\mathrm{L} . \mathrm{M}} / \partial \mathrm{T} \partial \varepsilon_{\mathrm{NK}}\right)=-\mathrm{V}_{\mathrm{O}}\left(\partial \mathrm{C}_{\mathrm{LMNK}} / \partial \mathrm{T}\right) \tag{5.7e}
\end{align*}
$$

which in conjunction with Equation 5.4 may be used to rewrite Equation 5.6 as follows [Theodorou and Suter, 1986]:

$$
\begin{equation*}
d U=\Sigma_{L M} V_{O}\left[\sigma_{L M}+\rho_{O} C_{\varepsilon} T \gamma_{L M}\right] \varepsilon_{L M}+2 / 2 \Sigma_{L M} \Sigma_{\mathrm{NK}} V_{\mathrm{O}}\left[C_{\mathrm{LMNK}}-T\left(\partial C_{\mathrm{LMNK}} / \partial T\right)\right] \varepsilon_{\mathrm{LN}} \varepsilon_{\mathrm{NK}} \tag{5.8}
\end{equation*}
$$

For direct application to the model description presented in Chapter 3, we convert this equation to an expression for the change in internal energy on the basis of a mole of polymeric repeat units:

$$
\begin{equation*}
d U=\Sigma_{L M} M_{r u}\left[\sigma_{L M} / \rho_{O}+C_{\varepsilon} T \gamma_{L M}\right] \varepsilon_{L M}+1 / 2 \sum_{L M} \Sigma_{\mathrm{NK}}\left(M_{\pi} / \rho_{O}\right)\left[C_{L M N K}-T\left(\partial C_{L M N K} / \partial T\right)\right] \varepsilon_{L N} \varepsilon_{\mathrm{VK}} \tag{5.9}
\end{equation*}
$$

The first term in Equation 5.9 is the internal residual stress, which by formulation is zero in our simulations where we have allowed for variation of density during minimization. The second term reflects the connection between the elastic stiffnesses and the second derivatives of internal energy with respect to strain. To assess the relative importance of the contributions of internal energy and entropy to the elastic constants, we may proceed to evaluate the second derivative of Helmholtz energy with respect to strain:

$$
\begin{equation*}
\mathrm{V}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMAK}}=\left(\partial^{2} \mathrm{~A} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)\left\|_{\mathrm{O}}=\left(\partial^{2} \mathrm{U} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)\right\|_{\mathrm{O}}-\mathrm{T}\left(\partial^{2} \mathrm{~S} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right) \|_{\mathrm{O}} \tag{5.10}
\end{equation*}
$$

Entropic effects are relatively unimportant if:
or

$$
\begin{equation*}
\left|\mathrm{T}\left(\partial^{2} \mathrm{~S} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)\right|_{\mathrm{o}} /\left(\partial^{2} \mathrm{~A} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right) \|_{\mathrm{O}} \mid \ll 1 \tag{5.11a}
\end{equation*}
$$

$$
\begin{equation*}
\left|\mathrm{T}\left(\partial \mathrm{C}_{\mathrm{LMNK}} / \partial \mathrm{T}\right) / \mathrm{C}_{\mathrm{LMNK}}\right|_{\mathrm{O}}=\left|\partial \ln \mathrm{C}_{\mathrm{LMNK}} / \partial \ln \mathrm{T}\right|_{\mathrm{O}} \ll 1 \tag{5.11b}
\end{equation*}
$$

The subscript O indicates the minimum internal energy (i.e. reference) structure. Evaluation of this inequality requires data for the elastic stiffnesses as functions of temperature at constant strain levels. Because such constant strain data is generally difficult to obtain, this criterion has been re-expressed in terms of constant stress derivatives [Theodorou and Suter, 1986]:
$\left|\left(T / C_{\mathrm{LMNK}}\right)\left[\left(\partial \mathrm{C}_{\mathrm{LMNK}} / \partial \mathrm{T}\right)_{\sigma}-\Sigma_{\mathrm{L}, \mathrm{M}} \Sigma_{\mathrm{NK}} \mathrm{C}_{\mathrm{LMNK}} \alpha_{\mathrm{NK}}\left(\partial \mathrm{C}_{\mathrm{LMNK}} / \partial \sigma_{\mathrm{LM}}\right)_{\mathrm{T}, \sigma \mathrm{LLM})}\right]\right| \ll 1$

For an isotropic solid characterized in the reference (undeformed) state by an isotropic stress distribution, the above criterion simplifies to:

$$
\begin{equation*}
\left|\left(1 / C_{\text {LMNK }}\right)\left[T\left(\partial C_{\text {LMNK }} / \partial T\right)_{p}+\left.\left(\alpha_{\rho} T / K_{T}\right)\left(\partial C_{\text {LVNK }} / \partial P\right)\right|_{T}\right]\right| \ll 1 \tag{5.13}
\end{equation*}
$$

However, such is not the case for the crystal lattice, where elastic response to imposed stress is generally anisotropic; instead, one requires the complete thermal expansion tensor $\alpha$, expressed alternatively by the Grüneisen tensor components $\gamma_{L M}$ :

$$
\begin{equation*}
\alpha_{P Q}=\left(C_{\varepsilon} / v_{O}\right) S_{P Q L M} \gamma_{L M} \tag{5.14}
\end{equation*}
$$

Unfortunately, the complete thermal expansion tensor or the Grüneisen tensor are rarely available for the anisotropic solid. One must usually settle for a few select elements, typically the diagonal elements, of either tensor. This issue will be readdressed in Section 5.1.3.

### 5.1.2 Statistical Mechanics of Deformation

Alternatively, in order to understand the contributions to the macroscopic thermodynamic properties of changes in structural description at the atomic scale, we may recast these state variables in terms of the canonical (ensemble) partition function for an $\mathbf{N}$ body system:

$$
\begin{equation*}
\mathrm{Q}(\mathrm{~N}, \mathrm{~V}, \mathrm{~T}) \quad=\quad \sum_{j} \exp \left(-\beta \mathrm{E}_{j}(\mathrm{~N}, \mathrm{~V})\right) \tag{5.15}
\end{equation*}
$$

where $\beta=1 / \mathrm{kT}$ and the $\mathrm{E}_{\mathrm{j}}$ are the total (free) energies of the states j available to the system. The thermodynamic state variables then become:

$$
\begin{align*}
\mathrm{A} & =-\mathrm{kT} \ln \mathrm{Q}  \tag{5.16}\\
\mathrm{U} & =\mathrm{kT}^{2}(\partial \ln \mathrm{Q} / \partial \mathrm{T})_{\mathrm{N}, \mathrm{~V}}  \tag{5.17}\\
\mathrm{~S} & =\mathrm{kT}(\partial \ln \mathrm{Q} / \partial \mathrm{T})_{\mathrm{N}, \mathrm{~V}}+\mathrm{k} \ln \mathrm{Q} \tag{5.18}
\end{align*}
$$

For a system of distinguishable particles, such as the lattice points in a crystal, the canonical partition function may be decomposed into parts for additional treatment by either classical or quantum mechanical methods:

$$
\begin{equation*}
Q(N, V, T)=q_{\text {crans }} q_{\mathrm{roa}} q_{\mathrm{vib}}=Q_{\text {class }} Q_{q u a n!} \tag{5.19}
\end{equation*}
$$

Now, we choose to treat the crystal lattice in the vicinity of its minimum internal energy configuration as a regular structure of discrete lattice points at which are located the N bodies (or atoms) of the system; each body vibrates about its local lattice point. The total internal energy may then be expressed as a function of an arbitrary set of structure-defining variables $s$ as a Taylor series expansion about the minimum energy configuration $s=s_{0}$ :

$$
\begin{equation*}
U(s)=U\left(s=s_{0}\right)+\sum_{i=1}^{N}\left(\partial U / \partial\left(\Delta s_{i}\right)\right) \|_{0} \Delta s_{i}+1 / 2 \sum_{i=1}^{N} \quad \sum_{j=1}^{N}\left(\partial^{2} U / \partial\left(\Delta s_{i}\right) \partial\left(\Delta s_{j}\right)\right)_{0} \Delta s_{i} \Delta s_{j} \tag{5.20}
\end{equation*}
$$

where the first term, the potential energy at the minimum, is a function only of the soft variables and will be referred to hereafter as $U^{p o t}{ }_{\text {min }}$, and the second term, the kinetic term, is a function of vibratory displacements about this static minimum, due either to oscillations in such hard variables as bond lengths and bond angles or small oscillations in soft variables such as bond torsion or interchain distance, which are further assumed to be independent of $\mathbf{s}_{\mathbf{O}}$.

The positions described by the vector of change $\Delta \mathbf{s}=0$ correspond to the minimum energy configuration, where by the definition of mechanical equilibrium $\left(\partial U / \partial\left(\Delta s_{i}\right)\right)_{0}=0$. The quadratic term represents a set of coupled harmonic oscillators. By next assuming that the crystal behaves as a large polyatomic molecule whose atoms occupy latice points connected by "springs", and introducing normal coordinate analysis, one may decompose the system into a set of independent oscillators, with a maximum of $3 \mathrm{~N}-6=3 \mathrm{~N}$ (for N on the order of $10^{20}$ atoms) normal mode vibration frequencies $v_{j}$, each contributing its own component $\mathrm{q}_{\mathrm{vibj}, \mathrm{j}}$ to the quantized portion of the partition function. Equating $\mathrm{U}(\mathrm{s})$ with the set of energies $\mathrm{E}_{\mathrm{j}}$ available to the system in the vicinity of the lattice minimum and substituting the independent partition functions $q_{\text {vibj, }}$ for the component oscillators in the quadratic term, one obtains:

$$
\begin{equation*}
Q(N, V, T)=\exp \left(-\beta U^{p o t}\right) \quad \prod_{j=1}^{3 N} q_{\text {vib }} \tag{5.21}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathrm{q}_{\text {vib }{ }_{j}}=\exp \left(-\beta h v_{j} / 2\right) /\left(1-\exp \left(-\beta h v_{j}\right)\right) \tag{5.22}
\end{equation*}
$$

For the Helmholz energy and its second derivatives, this yiclds:

$$
\begin{gather*}
\mathrm{A}=\mathrm{U}_{\text {pot }}^{\text {min }}  \tag{5.23}\\
+\mathrm{kT} \sum_{\mathrm{j}=1}^{3 \mathrm{~N}} \ln \left(q_{\mathrm{vibj}}\right)  \tag{5.24}\\
\left(\partial^{2} \mathrm{~A} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)=\left(\partial^{2} U^{\text {pot }}{ }_{\min } / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)+\mathrm{kT} \sum_{\mathrm{j}=1}^{3 \mathrm{~N}}\left(\partial^{2} \ln \left(q_{\mathrm{vib}, j}\right) / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)
\end{gather*}
$$

For the calculation of elastic stiffnesses, our interest lies in the term $\left(\partial^{2} A / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)=\mathrm{VC}_{\mathrm{LMNK}}$. Clearly, if the normal mode frequencies are independent of deformation in the small deformation regime, then the second term on the right hand side drops out and ( $\left.\partial^{2} \mathrm{~A} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{NK}}\right)$ may be replaced by $\left(\partial^{2} U^{\text {Pot }}{ }_{\text {min }} / \partial \varepsilon_{L M} \partial \varepsilon_{\mathrm{NK}}\right)$, suggesting that oscillatory freedom makes at most a minor contribution to elastic response; this is the strict Harmonic Approximation. Thus for evaluation of the significance of entropic contributions to elastic stiffnesses, one requires either normal mode vibrational frequencies for deformed structures as well as undeformed structures, or experimental data for elastic constants versus temperature. Applying the same criterion for significance of entropic contributions as given in Equation 5.11a, one obtains:

$$
\begin{equation*}
k T \quad \sum_{j=1}^{3 N}\left(\partial^{2} \ln \left(q_{\text {vibj }}\right) / \partial \varepsilon_{L M} \partial \varepsilon_{\text {NK }}\right) / V C_{\text {LMNK }} \ll 1 \tag{5.25}
\end{equation*}
$$

Both the numerator and the denominator of this equation are size dependent, the numerator on the number of oscillators in the system and the denominator on the volume of the system. Taking advantage of the equality $(\mathrm{N} / \mathrm{V})_{\text {system }}=\left(\mathrm{N} / \mathrm{V}_{\text {unit cell }}=\mathrm{N}_{\mathrm{O}} / \mathrm{V}_{\mathrm{O}}\right.$ and differentiating Equation 5.22 with respect to the components of strain, one may write:

$$
\begin{equation*}
N_{\mathrm{O}} \mathrm{kT} \quad \sum_{\mathrm{j}=1}^{3 \mathrm{~N}}\left(\partial^{2} \ln \left(\mathfrak{q}_{\mathrm{Vibj},}\right) / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{VK}}\right) / N V_{0} C_{\text {LMNK }} \ll 1 \tag{5.26}
\end{equation*}
$$

or

$$
\begin{align*}
& \left(N_{\mathrm{O}} / \beta N V_{O} C_{L M N K}\right) \sum_{j=1}^{3 N} h \beta\left(\frac{12}{2}+\Lambda\right)\left(\partial^{2} v_{j} / \partial \varepsilon_{L M} \partial \varepsilon_{\mathrm{NK}}\right)-(h \beta)^{2}\left(\Lambda+\Lambda^{2}\right)\left(\partial v_{j} / \partial \varepsilon_{L M}\right)\left(\partial v_{j} / \partial \varepsilon_{\mathrm{NK}}\right) \ll 1 \\
& \Lambda=1 /\left(\exp \left(h \beta v_{j}\right)-1\right) \tag{5.27}
\end{align*}
$$

As will be suggested by the data presented in the next section, if we assume that:
(1) $\partial v_{j} / \partial \varepsilon_{L M}=K_{v}$, a constant for all $j, L M$
and (2) $\quad \partial^{2} v_{\mathrm{j}} / \partial \varepsilon_{\mathrm{LM}} \partial \varepsilon_{\mathrm{iK}}=0$ for all $\mathrm{j}, \mathrm{LM}, \mathrm{NK}$
then for a new criterion we obtain:

$$
\begin{equation*}
-\left(\mathrm{N}_{\mathrm{J}} / \beta \mathrm{NV}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMNK}}\right)\left(\mathrm{h} \beta \mathrm{~K}_{v}\right)^{2} \sum_{\mathrm{j}=1}^{3 \mathrm{~N}}\left(\Lambda+\Lambda^{2}\right) \quad \ll 1 \tag{5.28}
\end{equation*}
$$

For large $N$, we may approximate the summation by introducing a distribution function $g(v)$ for the normal mode frequencies and integrate over all frequencies:

$$
\begin{gather*}
-\left(\mathrm{N}_{\mathrm{O}} / \beta N V_{\mathrm{O}} C_{\mathrm{LMNK}}\right) \quad\left(h \beta K_{v}\right)^{2} \quad \int_{0}^{\infty} \exp (\mathrm{h} \beta v) \mathrm{g}(v) /(\exp (\mathrm{h} \beta v)-1)^{2} d v \quad \ll 1  \tag{5.29}\\
\int_{0}^{\infty} \mathrm{g}(v) d v=3 \mathrm{~N}
\end{gather*}
$$

It only remains to determine the distribution function $\mathrm{g}(\mathrm{v})$, which should be easier than the complete set of individual frequencies. According to McQuarrie [1976], the normal frequencies of a crystal vary from essentially zero to some value of the order of $10^{13}$ cycles per second. Normal frequencies are not due to the vibrations of single atoms, but are concerted harmonic motions of all the atoms; such concerted motion is called a normal mode. Unfortunately, complete distributions are not generally available. Nevertheless, we may proceed by introducing the same approximations for the normal mode vibrations of a crystal as have been employed so successfully in the prediction of low temperature heat capacities, namely the Einstein and Dcbye approximations:

Einstein Approximation: Here it is assumed that each normal mode has the same frequency $\mathrm{v}_{\mathrm{E}}$, i.e. each member of the latuice sees the same environment and acts as an independent oscillator:

$$
\begin{equation*}
\mathrm{g}_{\mathrm{E}}(\mathrm{v})=3 \mathrm{~N} \delta\left(v-v_{\mathrm{E}}\right) \tag{5.30}
\end{equation*}
$$

which, when substituted into Equation 5.29, yields:

$$
\begin{equation*}
-\left(3 N_{\alpha} / \beta V_{O} C_{L N N K}\right)\left(h \beta K_{v}\right)^{2}\left[\exp \left(h \beta v_{\mathrm{E}}\right) /\left(\exp \left(h \beta v_{\mathrm{E}}\right)-1\right)\right] \ll 1 \tag{5.31}
\end{equation*}
$$

Debye Approximation: Here it is reasoned that only low frequencies, up to a characteristic frequency $\mathrm{v}_{\mathrm{D}}$, are important; these long waveiength oscillations are insensitive to the detailed atomic character of the solid and may be calculated by assuming that the crystal is a continuous elastic body. This leads to:

$$
\begin{array}{rlrl}
g_{D}(v) & = & 9 \mathrm{Nv}^{2} / v_{D}^{3} &  \tag{5.32}\\
& =0 \leq v \leq v_{D} \\
& 0 & & v>v_{D}
\end{array}
$$

which, when substituted into Equation 5.29 and making the substitution $x=h \beta v$, leads to:

$$
\begin{equation*}
-\left(3 N_{\mathrm{O}} \mathrm{kT} / \mathrm{V}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMNK}}\right) \mathrm{K}_{\mathrm{v}}{ }^{2} /\left(\mathrm{h} \beta v_{\mathrm{D}}{ }^{3}\right) \cdot \int_{0}^{h \beta v_{\mathrm{D}}} \mathrm{x}^{2} \exp (\mathrm{x}) /(\exp (\mathrm{x})-1) \mathrm{dx} \ll 1 \tag{5.33}
\end{equation*}
$$

which may be integrated numerically for a given $v_{D}$. These criteria for the significance of the entropic contributions to the elastic moduli, expressed in equations 5.12, 5.31, and 5.33, are addressed in the next section with respect to the particular case of the stiff chain polymer.

### 5.1.3 Estimation of Entropic Contributions

Ii et al. [1986, 1986, 1987] in a series of papers reported latice thermal expansion data and some thermomechanical propertics for PPTA and PBA aramid fibers obtained by means of x-ray analysis of the crystal lattice spacings under given axial (c-axis) loadings. Relevant properties which may be deduced from this work are reproduced in Table 5.1. Based on these values, we may calculate the stiffness criterion for $\partial^{2} \mathrm{~A} / \partial \varepsilon_{33}{ }^{2}$, assuming that only the axial stiffness constant $\mathrm{C}_{3333}$ is significantly affected by the introduction of an axial stress $\sigma_{33}$. In the second term of Equation 5.12, one would expect that for such oriented chains $\partial \mathrm{C}_{3333} / \partial \sigma_{33}$ should far exceed all other $\partial \mathrm{C}_{\mathrm{LMNK}} / \partial \sigma_{\mathrm{LM}}$ due to the deformation of internal bond lengths, angles, and torsions required by an imposed stress in this direction, rather than the deformation of weaker intermolecular interactions allowed under stress in the other directions; the possible exception in the aramid case is the $\partial \mathrm{C}_{111} / \partial \sigma_{11}$, which involves deformation of the hydrogen bonds. However, the fact that $C_{3333}$ should also be much greater than the other moduli, and that $\alpha_{11}$ and $\alpha_{22}$ are greater than zero and thus would offset somewhat the negative $\alpha_{33}$ contribution, should ensure that the estimate calculated below is a worst case scenario:

$$
\begin{aligned}
\mid\left(\mathrm{T} / \mathrm{C}_{3333}\right)\left[\partial \mathrm{C}_{3333} /\left.\partial \mathrm{T}\right|_{\sigma}-\mathrm{C}_{3333} \alpha_{33} \partial \mathrm{C}_{3333} /\left.\partial \sigma_{33}\right|_{\mathrm{T}} \mid\right. & =0.27 \text { (PPTA) } \\
& =0.32(\mathrm{PBA})
\end{aligned}
$$

These numbers imply that the true modulus of PPTA is at least $79 \%$ of the value calculated solely from internal energy contributions.

For our consideration of the analysis based on statistical mechanics, we refer to the work of Galiotis et al. [1985], who have investigated the strain dependence of the Raman frequencies of Kelvar $49^{\circ}$ fiber, in the range 1100 to $1700 \mathrm{~cm}^{-1}$, between $0 \%$ and $2 \%$ axial strain $\left(\varepsilon_{33}\right)$. The frequencies and peak shifts of six vibrational modes are reproduced in Table 5.2. From their data, the gradients in frequency appear to be roughly independent of both frequency and strain. Hence we make the simplifications suggested in the previous section, i.e. that the second derivatives of frequency with respect to strain are essentially

Table 5.1

Thermomechanical properties of PPTA and PBA ${ }^{\text {a }}$

|  | PPTA | PBA |
| :---: | :---: | :---: |
| $\alpha_{11}$ | $8.3 \times 10^{-5} \mathrm{~K}^{-1}$ | $7.0 \times 10^{-5} \mathrm{~K}^{-1}$ |
| $\alpha_{22}$ | $4.7 \times 10^{-5} \mathrm{~K}^{-1}$ | $4.1 \times 10^{-5} \mathrm{~K}^{-1}$ |
| $\alpha_{33}$ | $-2.9 \times 10^{-6} \mathrm{~K}^{-1}$ | $-7.7 \times 10^{-5} \mathrm{~K}^{-1}$ |
| $\mathrm{C}_{3333} \approx 1 / \mathrm{S}_{3333} \quad($ at 298 K$)$ | 168 GPa | 188 GPa |
| $\partial \mathrm{C}_{3333} / \partial \mathrm{T}_{633=0.5 \mathrm{GPa}}$ |  |  |
| $\approx \mathrm{S}_{3333}{ }^{-2} \partial \mathrm{~S}_{3333} / \partial \mathrm{T}_{633=0.5} \mathrm{GPa}$ | -0.181 GPa/K | -0.247 GPa/K |
| $\left.\left.\partial \mathrm{C}_{3333} / \partial \sigma_{33}\right\}_{300 K}=-\mathrm{S}_{3333}{ }^{-2} \partial \mathrm{~S}_{3333} / \partial \sigma_{33}\right\}_{300 \mathrm{~K}}$ | 59 | 32 |

${ }^{2}$ from li et al. [1986, 1986, and 1987].

Table 5.2
Strain dependence of Raman frequencies in PPTA ${ }^{a}$

| Frequency $\left(\mathrm{cm}^{-1}\right)$ | $\partial \nu_{j} / \partial \varepsilon_{33}\left(\mathrm{~cm}^{-1} / \%\right)$ | $\Lambda+\Lambda^{2}$ |
| :--- | :--- | :--- |
| 1649.5 | $-2.2 \pm 0.2$ | $3.65 \times 10^{-4}$ |
| 163.5 | $-4 \pm 0.2$ | 4.33 |
| 1519.2 | $-4.2 \pm 0.7$ | 6.82 |
| 1330.9 | $-3.4 \pm 0.3$ | 16.9 |
| 1280.1 | $-3.6 \pm 0.3$ | 21.5 |
| 1183.7 | $-0.5 \pm 0.3$ | 34.3 |

${ }^{2}$ from Galiotis et al. [1985].
zero, and take an average value for $\mathrm{K}_{\mathrm{v}}=-3 \mathrm{~cm}^{-1} / \%$ (i.e. $-300 \mathrm{~cm}^{-1}$ ). At 298 K , we have $\mathrm{h} \beta=0.0048 \mathrm{~cm}$, $\mathrm{V}_{\mathrm{O}}$ is $5.26 \times 10^{-22} \mathrm{~cm}^{3}$ (the unit cell volume), $\mathrm{N}_{\mathrm{O}}$ is taken to be the 56 atoms for PPTA (two repeat units per unit cell) and $\left(\mathrm{kT} / \mathrm{V}_{\mathrm{O}} \mathrm{C}_{3333}\right)=0.00466$.

Estimation of the Einstein and Debye frequencies of such a material is subjective, at best. Bondi [1968] reports Debye temperatures ( $\Theta_{\mathrm{D}}$ ) between $100^{\circ}$ and $400^{\circ} \mathrm{C}$ for monatomic solids, which corresponds to $v_{D}=k \Theta_{D} / h$ between $70 \mathrm{~cm}^{-1}$ and $280 \mathrm{~cm}^{-1}$. Values for glassy polymers are on the order of $120^{\circ} \mathrm{C}$. Unfortunately, there exist no good estimates for Debye or Einstein temperatures for polymeric crystals. However, guided by those systems for which such data is available, we may presume a conservative value of:

$$
\Theta_{D}=100^{\circ} \mathrm{C} ; \quad \Theta_{\mathrm{E}}=0.75 \Theta_{\mathrm{D}}=75^{\circ} \mathrm{C}
$$

which corresponds to:

$$
v_{D}=70 \mathrm{~cm}^{-1} ; \quad v_{\mathrm{E}}=52 \mathrm{~cm}^{-1}
$$

Einstein Approximation:

$$
-\left(3 \mathrm{~N}_{\mathrm{O}} \mathrm{kT} / \mathrm{V}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMNK}}\right)\left(\mathrm{h} \beta \mathrm{~K}_{v}\right)^{2}\left[\exp \left(\mathrm{~h} \beta v_{\mathrm{E}}\right) /\left(\exp \left(\mathrm{h} \beta v_{\mathrm{E}}\right)-1\right)\right]=.066 \ll 1
$$

Debye Approximation:

$$
-\left(3 \mathrm{~N}_{\mathrm{O}} \mathrm{kT} / \mathrm{V}_{\mathrm{O}} \mathrm{C}_{\mathrm{LMNK}}\right) \mathrm{K}_{\mathrm{v}}{ }^{2} /\left(\mathrm{h} \beta \mathrm{v}_{\mathrm{D}}{ }^{3}\right) \quad \int_{0}^{\mathrm{h} \beta v_{\mathrm{D}}} \mathrm{x}^{2} \exp (\mathrm{x}) /(\exp (\mathrm{x})-1) \mathrm{dx}=.024 \ll 1
$$

These values would suggest that entropic and kinetic energy contributions to the elastic moduli in polymeric crystals are actually less than $7 \%$ of the total response of free energy to deformation, a quarter of that determined from thermomechanical data for axial deformation. In the following sections, we will adhere to calculations of the elastic constants from changes in the potential energy for deformed structures, with the realization that these estimates may be $10 \%$ to $30 \%$ too high.

### 5.2 Method of Calculation

In the case of static crystals fully minimized in potential energy with respect to both intramolecular and intermolecular degrees of freedom, the first order coefficients of Equation 5.6 are all zero (i.e. there is no internal residual stress or thermal expansion). To obtain the twenty-one independent $\mathrm{C}_{\mathrm{LNNK}}$ 's one requires twenty-one deformation "experiments"; these are selected as follows:

> 3 uniaxial tensions $\left(\varepsilon_{\mathrm{LL}}\right)$
> 3 simple shears $\left(\varepsilon_{\mathrm{LM}}\right)$
> 3 biaxial tensions $\left(\varepsilon_{\mathrm{LL}}, \varepsilon_{\mathrm{MM}}\right)$
> 3 dual component shears $\left(\varepsilon_{\mathrm{LM}}, \varepsilon_{\mathrm{NK}}\right)$
> 9 combined tension/shear $\left(\varepsilon_{\mathrm{LL}}, \varepsilon_{\mathrm{NK}}\right)$

The first six require the minimum energy configuration plus two deformed configurations ( $\pm \varepsilon$ ), from which an estimate of $\mathrm{C}_{\mathrm{LNMK}}$ may be calculated using a three point formula. The other fifteen cases each require an additional two deformed configurations ( $+\varepsilon_{\mathrm{LM}}+\varepsilon_{\mathrm{NK}}$ and $-\varepsilon_{\mathrm{LM}},-\varepsilon_{\mathrm{NK}}$ ), which in combination with the previous deformed structures yield estimates of $\mathrm{C}_{\text {LMNK }}$ through use of a seven point formula.

3-poin: $\partial^{2} \mathrm{f} / \partial \mathrm{x}^{2}=\quad(\mathrm{f}(1,0)-2 \mathrm{f}(0,0)+\mathrm{f}(-1,0)) /\left(\mathrm{h}_{\mathrm{x}}\right)^{2}$

7-point: $\partial^{2} f \partial x \partial y=\quad-(f(1,0)+f(0,1)+f(-1,0)+f(0,-1)-2 f(0,0)-f(1,1)-f(-1,-1)) / 2 n\left(h_{\mathrm{x}}\right)^{2}$
$h_{x}$ is the step size in $x, h_{y}$ is the step size in $\mathrm{y}, \mathrm{n}=\mathrm{h}_{\mathrm{y}} / \mathrm{h}_{\mathrm{x}}>1$
$\partial^{2} f / \partial \mathrm{x} \partial \mathrm{y}$ provides an estimate of $\mathrm{dU} / \varepsilon_{\mathrm{LM}} \varepsilon_{\mathrm{NK}}$. From Equation 5.9 , in the absence of entropic contributions, the elements of the compliance matrix may be obtained as

$$
\begin{equation*}
C_{\mathrm{LMNK}}=\left(\mathrm{dU} / \varepsilon_{\mathrm{LM}} \varepsilon_{\mathrm{NK}}\right)\left(\rho_{\mathrm{O}} / \mathrm{M}_{\mathrm{Tu}}\right) \tag{5.36}
\end{equation*}
$$

Because of the feature of chain alignment within the model, deformation parallel to the chain propagation direction and lateral to it may not be treated identically. Tensile deformation in the latter case is solely a function of intermolecular packing parameters and may be imposed by appropriately straining the packing geometry. Tensile deformation along the chain direction is an implicit function of intramolecular parameters and may be imposed by applying a forcing function to cause deformation of the
chain axis. On the other hand, simple shear in a plane containing the chain axis may be induced by altering the intermolecular description, while shear in a plane cutting the chain axis is precluded by formulation of the model, which defines the z -axis of the global coordinate system as the alignment direction of the chain axes. In the actual "experiments" we impose engineering strains $\mathrm{e}_{\mathrm{LM}}$ where it is possible to impose deformations of lateral dimensions in the chain direction (i.e. $\dot{\epsilon}_{13}$ and $\epsilon_{23}$ may be nonzero). Then $\varepsilon_{L L}=e_{L L}$ and $\varepsilon_{L M}=\varepsilon_{M L}={ }^{2} / e_{L M}$. With the imposition of strain and reminimization with respect to intramolecular and intermolecular degrees of freedom, the components of the structure may respond in a nonaffine manner, this process is conceptually consistent with a realistic phenomenon of atomic rearrangement in a system involving bonded and nonbonded atoms.

### 5.3 Elasticity of Poly(p-phenylene terephthalamide)

### 5.3.1 Modification of the Chain Description

The model used for PPTA is a modification of the simple chain description used to identify optimal crystal structures. The latuer described the single chain using fixed bond lengths, hexagonal planar phenyl rings, and planar $\mathrm{sp}^{2}$ bond orientations at the amide carbons and nitrogens, requiring only six torsional degrees of freedom and eight valence bond angle degrees of freedom. While this was deemed sufficient internal freedom for the chain to adjust to its preferred packing configuration, this chain description was too restricted to respond realistically to imposed strains acting along the chain axis. The new chain description relaxes the planarity constraint at the amide nitrogens and carbons, resulting in ten torsional degrees of freedom. The rigid phenylene ring was replaced by a planar symmetric ring which may be described using only one bond angle and two bond length degrees of freedom. Finally, the bonds in the chain backbone were allowed to respond to deformation, adding six more bond iengths to the degrees of freedom, for a total of ten bond angle and ten bond length degrees of freedom for the PPTA repeat unit. Valence Force Field potentials [Tashiro et al., 1977] were introduced to describe the potential energy of these deformations, with the equilibrium positions chosen so as to reproduce the original chain upon energy minimization. Table 5.3 shows the correlation between corresponding values for the above mentioned degrees of freedom at the minimum energy cell configuration for PPTA Structure \#3 using the two chain descriptions. The intermolecular degrees of freedom remained unchanged (i.e. two periodicity lengths, three periodicity orientation angles, two setting angles, and one translational displacement). The cohesive energy of this structure calculated using the second chain description is within $0.5 \mathrm{kcal} / \mathrm{mol}$ of that obtained using the simpler description; this lends further confidence to the accuracy and adequacy of the simpler version for structure and cohesive energy calculations. Lastly, it remains to define the coordinate axes of mechanical deformation. It must be noted that the selection of axes in use at this

Table 5.3
Comparison of the two chain descriptions used for structure and mechanical property calculations

|  | Type 1 (22 DOF) | Type 2 (38 DOF) |
| :---: | :---: | :---: |
| Bond Length ( $\AA$ ) |  |  |
| $\mathrm{C}^{\text {- }} \mathrm{Car}, 1$ | 1.50 (fixed) | 1.504, 1.508 |
| $\mathrm{C}_{\mathrm{ar}, 1} \mathrm{C}_{\text {a }} \mathrm{C}_{\mathrm{ar}, 2}$ | 1.40 (fixed) | $1.404,1.402$ |
| $\mathrm{Cax}_{\text {ar }, 2} \mathrm{C}_{\mathrm{ar}, 3}$ | 1.40 (fixed) | 1.400, 1.396 |
| $\mathrm{C}-\mathrm{N}$ | 1.39 (fixed) | 1.385, 1.385 |
| $\mathrm{C}_{\mathrm{ar}, 1}$ - N | 1.41 (fixed) | 1.398, 1.400 |
| Bond Angle (degrees) |  |  |
| $\mathrm{C}_{\mathrm{ar}, 6}-\mathrm{Cax}, 1,1^{-\mathrm{C}_{\mathrm{ar}, 2}}$ | 120 (fixed) | 119.4, 120.3 |
| $\mathrm{C}_{a, 1,1} \mathrm{C}=0$ | 118.9, 118.9 | 118.9, 119.9 |
| $\mathrm{C}_{\mathrm{arar}, 1} \mathrm{l}^{\text {a }}$ C-N | 122.0, 121.9 | 121.8, 122.0 |
| $\mathrm{Carar}, 1^{1} \mathrm{~N}-\mathrm{H}$ | 118.1, 118.4 | 118.1, 118.1 |
| $\mathrm{Carar}_{2}$ - $\mathrm{N}-\mathrm{C}$ | 123.1, 122.7 | 122.9, 122.6 |
| Bond Torsion (degrees) |  |  |
| $\mathrm{Cax}_{\text {ar } 2} \mathrm{C}^{-} \mathrm{Car}, 1 \mathrm{C}-\mathrm{N}$ | -26.0, -153.7 | -25.7, -159.0 |
| $\mathrm{C}_{\text {ar } 2,2} \mathrm{C}_{\text {ara }, 1}-\mathrm{C}=0$ | (dependent) | 19.0, 172.0 |
| $\mathrm{C}_{\text {ara }, 1} \mathrm{C}$ - $-\mathrm{N}-\mathrm{C}_{\text {ar, }}$, | 5.6, -6.6 | 5.8, -6.9 |
| $\mathrm{C}_{\text {ara }, 2} \mathrm{C}_{\text {ar, } 1}$ - $\mathrm{N}-\mathrm{C}$ | -137.3, -43.1 | $-134.6,-44.4$ |
|  | (dependent) | 137.8, -167.8 |
| Intermolecular |  |  |
| A ( $\mathrm{A}^{\text {) }}$ | 4.78 | 4.78 |
| B ( $\AA$ ) | 4.90 | 4.93 |
| $\alpha$ (degrees) | 90.3 | 89.7 |
| $\beta$ (degrees) | 89.9 | 89.5 |
| $\boldsymbol{\gamma}$ (degrees) | 62.1 | 61.5 |
| $\omega_{1}$ (degrees) | 101.4 | 99.5 |
| $\omega_{2}$ (degrees) | -79.5 | -81.4 |
| $\mathrm{f}_{2}$ | 0.45 | 0.45 |
| Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.46 | 1.46 |
| Cohesive Energy (kcal/mol) | 38.3 | 37.8 |

point differ slightly from those used to report crystal structures in Chapter 4. In the mechanical property presentation, as in the model development, the $x_{3}$-axis corresponds to the c -axis of the pseudocrystal, along which the chain axes are aligned; the $\mathrm{x}_{1}-\mathrm{x}_{3}$ plane, i.e. the " 2 " face, contains the ac -facet of the pseudocrystal wherein lie the hydrogen-bonded shects; the $\mathrm{x}_{2}$-axis and the bc-facet are defined by orthogonality and crystal periodicity relationships, respectively. Crystal structures provided in Table 4.6 follow the convention that the hydrogen-bonded sheets lic in the bc-facet; however, this is employed purely for reasons of consistency with the common practice in crystallography of selecting the "a" dimension such that " a " is larger than " b ", as has been done for the structures previously reported in the literature.

### 5.3.2 Determination of Crystallite Compliance and Stiffness Matrices

Initially, the multichain structure was subjected to $\pm 0.4 \%$ tensile strains along the crystallographic c (chain) axis, with all other internal and external degrees of freedom allowed to adjust (i.e. lateral contraction of the structure in addition to chain conformation adjustment). The resulting $\mathrm{E}_{33}$ modulus was 303 GPa ; the resulting densitics of the strained structures deviated from the strain free value by $\pm 0.16 \%$, indicative of a slight expansion (compression) of the latice upon extension (compression).

In a second set of experiments, the full complement of twenty-one deformations was performed, allowing only degrees of freedom internal to the unit cell to relax (i.e. the optionally-strained lattice dimensions $\mathrm{a}, \mathrm{b}, \mathrm{c}$, and the interaxial angles $\alpha, \beta$, and $\gamma$ were fixed). The complete stiffness matrix is presented below in the $6 \times 6$ Voigt format. In this convention, only one subscript is required to identify the appropriate element of stress or strain, and two for the corresponding stiffness and compliance matrices (e.g. $\varepsilon_{\mathrm{L}}=\varepsilon_{\mathrm{LL}}$ for $\mathrm{L}=1,2,3$, and $\varepsilon_{4}=\varepsilon_{23}, \varepsilon_{5}=\varepsilon_{13}, \varepsilon_{6}=\varepsilon_{12}$ ). The magnitudes of imposed strains were selected to be large enough to give numerically significant differences in potential energy (i.e. $\mathrm{e}_{\mathrm{L}}=0.4 \%$ for $\mathrm{L}=3$ and $0.8 \%$ for $\mathrm{L}=1,2 ; \mathrm{e}_{\mathrm{L}}=1.6 \%$ for $\mathrm{L}=4,5,6$ ). The resulting precision in the reported values are on the order of $\pm 5 \%$, or $\pm 1$ to 5 GPa . Deformations imposed by $\mathrm{e}_{33}$ and $\mathrm{e}_{23}$, representing roughly the extremes of elastic response, where checked for various strain levels; the calculated stiffness elements remained consistent up to $3.2 \%$ strain or higher. Removal of the imposed deformation and reminimization of potential energy with respect to the internal coordinate degrees of freedom gives back the original undeformed configuration, ensuring that the strain levels employed are within the domain of elastic response.

The compliance matrix is calculated by inversion of the stiffness matrix:

$$
\begin{equation*}
\mathbf{S}=\mathbf{C}^{-1} \tag{5.37}
\end{equation*}
$$

From the compliance matrix are calculated the Young's moduli $\mathrm{E}_{\mathrm{i}}$, the shear moduli $\mathrm{G}_{\mathrm{i}}$, and the first six Poisson's ratios $\mathrm{v}_{\mathrm{ij}}$ :

$$
\begin{align*}
& E_{\mathrm{i}}=1 / \mathrm{S}_{\mathrm{ii}} \quad ; \mathrm{i}=1,2,3  \tag{5.38a}\\
& \mathrm{G}_{\mathrm{j}}=1 / \mathrm{S}_{\mathrm{ij}} ; \mathrm{i}=4,5,6 \text { and } \mathrm{j}=\mathrm{i}-3  \tag{5.38b}\\
& \mathrm{v}_{\mathrm{ij}}=-\mathrm{S}_{\mathrm{i} j} / \mathrm{S}_{\mathrm{ij}} ; \mathrm{i}, \mathrm{j}=1,2,3 \tag{5.38c}
\end{align*}
$$

For the simulated structure which most closely resembles the thermodynamically stable Modification I reported in the literature, PPTA Structure \#3, the full stiffness and compliance matrices are calculated to be:

$$
\begin{aligned}
& \mathbf{C}=\left[\begin{array}{rrrrrr}
40.5 & 23.3 & 12.9 & 0.1 & 1.2 & 2.4 \\
23.3 & 30.6 & 40.7 & 1.5 & 3.2 & 1.9 \\
12.9 & 40.7 & 360 . & 0.3 & 5.0 & 11.2 \\
0.1 & 1.5 & 0.3 & 5.5 & 3.7 & 2.7 \\
1.2 & 3.2 & 5.0 & 3.7 & 22.0 & 3.2 \\
2.4 & 1.9 & 11.2 & 2.7 & 3.2 & 7.5
\end{array}\right] \quad \mathrm{GPa} \\
& \mathbf{S}=\left[\begin{array}{rrrrrr}
4.85 & -4.17 & 0.35 & 1.80 & 0.21 & -1.76 \\
-4.17 & 7.52 & -0.74 & -2.47 & -0.52 & 1.67 \\
0.35 & -0.74 & 0.37 & 0.50 & 0.02 & -0.66 \\
1.80 & -2.47 & 0.50 & 25.0 & -2.82 & -8.52 \\
0.21 & -0.52 & 0.02 & -2.82 & 5.26 & -1.16 \\
-1.76 & 1.67 & -0.66 & -8.52 & -1.16 & 18.0
\end{array}\right] \times 10^{-2} \mathrm{GPa}^{-1} \\
& \\
& \mathrm{E}_{1}=20.6 \mathrm{GPa} \\
& \mathrm{G}_{1}=4.0 \mathrm{GPa} \\
& v_{12}=0.55 \\
& v_{31}=-0.07
\end{aligned}
$$

The obvious trends to be noted here are that the transverse moduli $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ are roughly $5 \%$ to $8 \%$ of the axial modulus $\mathrm{E}_{3}$. As was presaged in the discussion of interchain total interaction energies within and between hydrogen bonded sheets in Chapter 4, the two transverse moduli are of comparable
magnitude. The shear moduli $\mathrm{G}_{1}$ and $\mathrm{G}_{3}$, corresponding to motions of the hydrogen-bonded sheets of chains parallel and perpendicular, respectively, to the chain axes, are roughly $1 \%$ to $2 \%$ of the axial modulus; the large value calculated for $G_{2}$ is indicative of the expected resistance to shear deformation of the hydrogen-bonded sheet structure itself. The valucs for $\mathrm{v}_{\mathrm{ij}}$ satisfy the inequalities for positive strain energy for a body having orthorhombic symmetry [Ward, 1983], which this structure approximates:

$$
\begin{align*}
& \mathrm{S}_{12}^{2}<\mathrm{S}_{11}^{2}  \tag{5.39}\\
& \mathrm{~S}_{13}^{2}<1 / 2 \mathrm{~S}_{33}\left(\mathrm{~S}_{11}+\mathrm{S}_{12}\right) \tag{5.40}
\end{align*}
$$

### 5.3.3 Isolated Chain Compliance and Comparison to Crystallite Compliance

For comparison to calculations in the literature, the isolated chain was also subjected to fixed axial strains of $\pm 0.4 \%$, which yielded a chain modulus of 195 GPa . By comparison, Tashiro et al. [1977] calculate a value of 182 GPa for the isolated chain modulus (referred to there as the crystallite modulus). Fielding-Russell [1971] calculates a value of 200 GPa for the all-trans planar conformation. The observed tensile modulus falls in the range 120 GPa to 200 GPa [Tashiro et al., 1977; Gaymans et al., 1976; Kwolek et al., 1987; Allen, 1988]. It is significant that whereas the tensile stiffness $\mathrm{C}_{3333}$ calculated for the single chain is in good agreement with previous estimates, that estimated for the packed structure is considerably higher, both with and without inclusion in the optimization parameter list of those variables describing lateral packing (i.e. 303 GPa and 360 GPa , respectively). In order to determine the source of this difference, the component breakdowns of energy contributions in both the single chain and packed chain estimations were computed. For this purpose, the intramolecular contribution was defined as that change in energy attributable to changes in intramolecular variables upon deformation, which includes bond stretching, bond angle bending, torsion angle rotation, intramolecular van der Waals interactions, and intramolecular Coulombic interactions. For the packed struclure, intermolecular interaction contributions were defined as changes upon deformation atributable to intermolecular van der Waals and Coulombic interactions. The contributions to changes in energy upon $\pm 0.4 \%$ strain along the c -axis (the chain axis) is shown in Table 5.4.

A careful study of these values in the case of the isolated chain indicates that the minimum energy conformation represents a balance between interactions tending to promoted chain compression (i.e. bonds and valence angles) and others promoting chain extension (i.e. torsions and nonbonded interactions); thus small deformations are accompanied by trade-offs in these interactions, such that the total change in internal energy is not large. In the packed chain structure, however, it is the intramolecular and

## Table 5.4

Distribution of strain energy among the degrees of freedom of the structure for isolated chain and packed chain simulations

Isolated Chain (intramolecular only)

|  | $\Delta \mathrm{E}_{\text {bond }}$ | $\Delta \mathrm{E}_{\text {angl }}$ | $\Delta \mathrm{E}_{\text {lors }}$ | $\Delta \mathrm{E}_{\mathrm{vdW}}$ | $\Delta \mathrm{E}_{\text {Coul }}$ | $\Delta \mathrm{E}_{\text {intra }}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  |  |
| $+0.4 \%$ | 0.427 | 0.464 | -0.210 | -0.597 | -0.007 | 0.078 |
| $-0.4 \%$ | -0.243 | -0.339 | 0.583 | 0.002 | 0.037 | 0.042 |

## Packed Chains

(Intramolecular)

|  | $\Delta \mathrm{E}_{\text {bond }}$ | $\Delta \mathrm{E}_{\text {angl }}$ | $\Delta \mathrm{E}_{\text {tors }}$ | $\Delta \mathrm{E}_{\mathrm{vdW}}$ | $\Delta \mathrm{E}_{\text {Coul }}$ | $\Delta \mathrm{E}_{\text {intra }}$ |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
|  |  |  |  |  |  | 0.193 |
| $+0.4 \%$ | -0.331 | -0.389 | 0.093 | 0.649 | 0.040 | 0.061 |

(Intermolecular)

|  | $\Delta \mathrm{E}_{\mathrm{vdW}}$ | $\Delta \mathrm{E}_{\text {Coul }}$ | $\Delta \mathrm{E}_{\text {inter }}$ | $\Delta \mathrm{E}_{\text {rotal }}$ |
| :--- | ---: | :--- | :--- | :--- |
| $+0.4 \%$ | 0.030 | -0.105 | -0.074 | 0.118 |
| $-0.4 \%$ | -0.032 | 0.075 | 0.042 | 0.103 |

intermolecular interactions that compensate each other. Due to impinging intermolecular interactions, the chain conformation is no longer the minimum energy conformation represented by the isolated chain, and as a result the trade-off between intramolecular contributions concurrent with axial strain are no longer balanced, but tend to pay a higher penalty for extension than witnessed previously. On the other hand, intermolecular interactions, primarily the Coulombic interactions, actually favor extension, thus offsetting the higher penalty for conformational extension. This trend may be understood as follows: extension leads to a net separation of atom centers, which decreases the cohesive energy contributions due to van der Waals interactions. However, charge intcractions in the packed structure are not randomly oriented, but are so arranged such that attractive interactions predominate. In particular, species of opposite charge are preferentially located closer together. Even an affine extension imposes larger absolute changes in charge separation for those species which are farther apart, i.e. specics of like charge, resulting in a net improvement of the total Coulombic interaction. Nonaffinc deformations such as the one performed here will be even more preferential in distributing charge separation. In the sum, both intramolecular and intermolecular interactions contribute to resist compression, while the advantages gained in intermolecular interaction only partially offset the resistance of chain conformation to extension. The net result is the increase in the stiffness constant $\mathrm{C}_{3333}$. This illustrates not only the importance of intermolecular contributions to resistance to deformation, but the complexity of changes in intramolecular response which accompany the changes in conformation that result from interactions with neighboring chains.

### 5.3.4 <br> Consideration of a Second Allomorph

From the results of the atomistic simulation, several discrete pseudocrystalline polymorphs of comparable energy are obtained. The results presented above correspond to that model structure which most closely resembles the reported most stable crystal structure for PPTA [Northolt, 1974]. However, at least one other crystal allomorph has been isolated by precipitation from acid solution into water under controlled conditions of polymer concentration and postprecipitation heat treatment [Haraguchi, 1979]. Other energetically stable packing geometrics may also exist to lesser extents in the actual polymer fiber. The above procedure was repeated for a second model geometry which more closely approximates the second crystal polymorph reported in the literature, although it should be noted that none of the model structures derived by minimization of total potential energy replicated this reported structure to the extent seen in the first case. Shown below is a summary of the stiffness and compliance matrices for the PPTA Structure \#5. (It should be noted that negative values for elements of the stiffness matrix are physically impossible, but arise here due to the limit of numerical precision involved in performing the computer strain "experiments".)

$$
\begin{aligned}
& \mathbf{C}=\left[\begin{array}{rrrrrr}
54.6 & 18.2 & 0.6 & 17.6 & 9.1 & -4.9 \\
18.2 & 59.9 & 12.0 & 2.0 & 1.9 & -2.9 \\
0.6 & 12.0 & 291 . & 9.0 & 36.3 & 30.3 \\
17.6 & 2.0 & 9.0 & 9.4 & 1.5 & 2.6 \\
9.1 & 1.9 & 36.3 & 1.5 & 20.2 & -2.4 \\
-4.9 & -2.9 & 30.3 & 2.6 & -2.4 & 20.4
\end{array}\right] \mathrm{GPa} \\
& \mathbf{S}=\left[\begin{array}{rrrrrr}
8.72 & -2.02 & 0.79 & -16.8 & -3.61 & 2.38 \\
-2.02 & 2.20 & -0.32 & 3.48 & 1.01 & -0.03 \\
0.79 & -0.32 & 0.69 & -1.58 & -1.54 & -0.85 \\
-16.8 & 3.48 & -1.58 & 43.8 & 6.09 & -6.13 \\
-3.61 & 1.01 & -1.54 & 6.09 & 9.00 & 1.84 \\
2.38 & -0.03 & -0.85 & -6.13 & 1.84 & 7.74
\end{array}\right] \times 10^{-2} \mathrm{GPa}^{-1} \\
& \\
& \mathrm{E}_{1}=11.5 \mathrm{GPa} \\
& \mathrm{G}_{1}=2.3 \mathrm{GPa} \\
& v_{12}=0.92 \\
& v_{31}=-0.09
\end{aligned}
$$

Significant differences to note between these two model structures are the values for extensional and tranverse moduli, which in the second structure would appear to be considerably smaller for axial deformation but larger for transverse deformation. This would be expected for a chain packing in which the conformation of the individual chains is reduced in pitch ( $12.87 \AA$ in the second geometry) relative to the most stable isolated chain conformation ( $13.10 \AA$ in the first geometry) in exchange for more favorable lateral interchain interactions in the packed geometry; in this case, the penalty in energy for extensional deformation of the chain is smaller than that in the previous case, where the chains were closer to a conformational energy minimum, while the penalty in energy for deformation of the lateral interactions is proportionally greater. The range for torsional modulus is roughly the same for both cases.

### 5.4 Moduli Involving Fiber Symmetry

5.4.1 Derivation of Fiber Symmetry Rclations

One may derive estimates for the moduli of the polycrystal mosaic from the complete elastic constant matrices by applying assumptions concerning the crystal packing morphology and the distribution of stress and strain over the packed crystals. For this purpose, we treat the fiber as a polycrystalline structure possessing perfect alignment of the molecular axes along the fiber axis, but random orientation of the crystallites in the planes lateral to the fiber axis. We also presume the absence of any second phase elastic matrix material. We can then calculate the mean elastic constants for the assumed fiber orientation function. If we assume that an imposed strain is distributed uniformly over all elements of the polycrystalline fiber, then we may calculate the fiber stiffness matrix as the cylindrical average of the crystalline stiffness matrix. We invert this matrix to obtain the fiber compliance matrix and obtain the elastic moduli as previously described; this is the volume average of stiffnesses, or Voigt limit, which provides an upper bound to the clastic constants. Altematively, we may assume that stress, rather than strain, is uniformly distributed, resulting in the analogous volume average of compliances, or Reuss limit. These two assumptions, while neither being exactly correct, provide the upper and lower limits to the elastic constants for the cylindrically symmetric polycrystal "composite", within which the true elastic constants must lie [Arridge, 1985].

In general, the mean value of an element of the stiffness matrix, for example, may be expressed analytically as
where $\Phi(\mathrm{X}, \Psi, \Omega)$ is the function describing the distribution of crystallite orientations and the $l_{\mathrm{i}^{\prime} \mathrm{i}}$ are given by Arridge [1985]:

$$
\begin{align*}
& l_{1^{\prime} 1}=\cos \Omega \cos X \cos \Psi-\sin \Omega \sin \Psi  \tag{5.42a}\\
& l_{1^{\prime} 2}=-\cos \Omega \sin \Psi-\sin \Omega \cos X \cos \Psi  \tag{5.42b}\\
& l_{1^{\prime} 3}=\sin X \cos \Psi  \tag{5.42c}\\
& l_{2^{\prime} 1}=\cos \Omega \cos X \sin \Psi+\sin \Omega \cos \Psi  \tag{5.42d}\\
& l_{2^{\prime} 2}=-\sin \Omega \cos X \sin \Psi+\cos \Omega \cos \Psi \tag{5.42e}
\end{align*}
$$

$$
\begin{align*}
& l_{2^{\prime} 3}=\sin X \sin \Psi  \tag{5.42f}\\
& l_{3^{\prime} 1}=-\cos \Omega \sin X  \tag{5.42g}\\
& l_{3^{\prime} 2}=\sin \Omega \sin X  \tag{5.42h}\\
& l_{3^{\prime} 3}=\cos X \tag{5.42i}
\end{align*}
$$

For the case of fiber symmetry, where we assume perfect alignment of the crystal elements along the z -axis and a uniform orientation distribution lateral to this axis, we may express the compliance as a function of crystallite orientation in tensorial form as:

$$
\begin{equation*}
\mathbf{C}_{\Omega}=\mathbf{T}_{\Omega} \mathbf{T}_{\Omega} \mathbf{C} \mathbf{T}_{\Omega}{ }^{\mathrm{T}} \mathbf{T}_{\Omega}{ }^{T} \tag{5.43}
\end{equation*}
$$

where $\mathbf{T}_{\Omega}$ is the rotation matrix:

$$
\mathbf{T}_{\Omega}=\left[\begin{array}{ccc}
\cos \Omega \sin \Omega & 0  \tag{5.44}\\
-\sin \Omega & \cos \Omega & 0 \\
0 & 0 & 1
\end{array}\right]
$$

Given any three conformable matrices, A, B, and C, it can be shown that

$$
\begin{equation*}
\{\mathbf{A B C}\}_{\text {col }}=\left(\mathbf{A} \otimes \mathbf{C}^{\mathrm{T}}\right)\{\mathbf{B}\}_{\text {col }} \tag{5.45}
\end{equation*}
$$

where $\left\}_{\text {col }}\right.$ denotes the column array of the elements of the matrix enclosed thercin, the elements being arranged in "reading" order, i.e. by rows, and $\otimes$ signifies the direct matrix product. Then the above equation for $\mathrm{C}_{\Omega}$ becomes:

$$
\begin{equation*}
\left\{\mathbf{C}_{\Omega}\right\}_{\text {col }}=\left(\mathbf{T}_{\Omega} \otimes \mathbf{T}_{\Omega} \otimes \mathbf{T}_{\Omega} \otimes \mathbf{T}_{\Omega}\right)\{\mathbf{C}\}_{\text {col }} \tag{5.46}
\end{equation*}
$$

or, using the reduced notation where $\mathbf{T}^{\mathrm{xp}}$ indicates the self-direct product of degree p of the matrix $\mathbf{T}$

$$
\begin{equation*}
\left\{C_{\Omega}\right\}=T_{\Omega}{ }^{\times 4}\{C\} \tag{5.47}
\end{equation*}
$$

Then the cylindrical average over $\Omega$ becomes:

$$
\begin{equation*}
\left\langle\left\{C_{\Omega}\right\}\right\rangle=(1 / 2 \pi) \int_{0}^{2 \pi} T_{\Omega 2^{x^{4}}}\{C\} d \Omega=\left\langle T_{\Omega}^{x^{4}}\right\rangle\{C\} \tag{5.48}
\end{equation*}
$$

Upon integration, one obtains 41 nonzero terms in $\left\langle\mathbf{T}_{\Omega}{ }^{\mathbf{x 4}}\right\rangle$, of which 21 are independent. Taking advantage of the symmetries in C , one can derive simple equations for the six independent terms of the cylindrically averaged stiffness matrix $\left\langle\mathrm{C}_{\Omega}\right\rangle$, which may be collected into Voigt form as shown below:

$$
<\mathrm{C}_{\Omega}>=\left[\begin{array}{llllll}
\mathrm{A} & \mathrm{~B} & \mathrm{C} & & &  \tag{5.49}\\
\mathrm{~B} & \mathrm{~A} & \mathrm{C} & & 0 & \\
\mathrm{C} & \mathrm{C} & \mathrm{~F} & & & \\
& 0 & & \mathrm{E} & \mathrm{E} & \\
& & & & & \mathrm{D}
\end{array}\right] \quad \text { Voigt Average }
$$

where

$$
\begin{aligned}
& \mathrm{A}=(1 / 8)\left(3 C_{11}+3 C_{22}+2 C_{12}+4 C_{66}\right) \\
& B=(1 / 8)\left(C_{11}+C_{22}+6 C_{12}-4 C_{66}\right) \\
& C=(1 / 2)\left(C_{13}+C_{23}\right) \\
& D=(1 / 8)\left(C_{11}+C_{22}-2 C_{12}+4 C_{66}\right) \\
& E=(1 / 2)\left(C_{44}+C_{55}\right) \\
& F=C_{33}
\end{aligned}
$$

Similarly, the cylindrically averaged compliance matrix may be expressed as:

$$
\left\langle\mathbf{S}_{\Omega}\right\rangle=\left[\begin{array}{llllll}
A & B & C & & &  \tag{5.50}\\
B & A & C & & 0 & \\
C & C & F & E & & \\
& 0 & & & E & \\
& & & & & D
\end{array}\right] \quad \text { Reuss Average }
$$

$$
\text { where } \quad \begin{aligned}
A & =(1 / 8)\left(3 S_{11}+3 S_{22}+2 S_{12}+S_{66}\right) \\
B & =(1 / 8)\left(S_{11}+S_{22}+6 S_{12}-S_{66}\right) \\
C & =(1 / 2)\left(S_{13}+S_{23}\right) \\
D & =(1 / 2)\left(S_{11}+S_{22}-2 S_{12}+S_{66}\right) \\
E & =(1 / 2)\left(S_{44}+S_{55}\right) \\
F & =S_{33}
\end{aligned}
$$

These equations are in agreement with the more general orientation equations given by Arridge to describe averaged elasticity matrices where actual crystallite alignment functions are experimentally available. Where only the moduli of the fibers are ultimately required, the number of simulated deformation "experiments" is drastically reduced from forty-two to eighteen. However, the reduced experiment set must be used with caution in those cases where one obtains the compliance matrix by inversion of the stiffness matrix. For structures that are clearly triclinic, some of the elements of the single crsytal stiffness matrix which are not required by Equation 5.49 may nevertheless deviate significantly from zero. This is not a problem for calculating the Voigt average elastic properties. However, in cases where these elements are nonzero, inversion of this estimation of the single crystal stiffness matrix introduces error into the estimation of the single crystal compliance matrix and in turn into the estimation of the Reuss average elastic constants. For this reason, the latter may in some cases be artificially high. In the following, we have only reported Reuss average elastic constants in those cases where the complete twenty-one independent element stiffness matrices were determined.

### 5.4.2 Voigt and Reuss Limits for PPTA

Applying this approach to the crystallographic stiffness and compliance matrices given previously, we arrive at upper (Voigt) and lower (Reuss) limits for the fiber elastic constants. In Table 5.5, these constants are listed for PPTA Structure \#3, using the more usual, experimentally-defined terminology. Also shown are the available experimentally-determined moduli for PPTA (Kevlar) fibers, as well as the results of calculations reported by Northolt and van Aartsen using bond deformation models. Values for $\mathrm{G}_{3}$ and $v_{31}$ are not independent, but may be determined easily by the relations:

$$
\begin{align*}
& G_{3}=1 / 2 E_{1} /\left(1+v_{12}\right)  \tag{5.51}\\
& v_{31}=v_{13} E_{1} / E_{3} \tag{5.52}
\end{align*}
$$

Table 5.5
PPTA fiber elastic constants (GPa): theoretical (Structure \#3) and experimental

|  | Voigt | Reuss | Predicted Range ${ }^{\text {h }}$ | Experiment | Northolt ${ }^{\text {a }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Extensional Modulus ( $\mathrm{E}_{3}$ ) | 335 | 272 | 325-215 | 120-200 ${ }^{\text {b,c,c, ec }}$ | 220 |
| Transverse Modulus ( $\mathrm{E}_{1}, \mathrm{E}_{2}$ ) | 21.8 | 6.6 | 21.1-5.2 | - | 16-29f |
| Torsional Modulus ( $\mathrm{G}_{1}, \mathrm{G}_{2}$ ) | 13.7 | 5.2 | 13.3-4.1 | $2^{\text {b }}, 0.95-2.2^{\text {b }}$ | $2.5-5.7^{\text {f }}$ |
| Extensional Poisson's Ratio ( $v_{13}, v_{23}$ ) | 0.46 | 0.54 | - | - | . |
| Transverse Poisson's <br> Ratio ( $v_{12}, v_{21}$ ) | 0.60 | 0.66 | - | - | - |

${ }^{\text {a }}$ Northolt and van Aartsen [1977].
${ }^{\text {b }}$ Allen [1988].
c Tashiro et al. [1977].
${ }^{d}$ Gaymans et al. [1976].
c Kwolek et al. [1987].
${ }^{f}$ based on the assumption that linear hydrogen bonds are the dominant interactions between chains
${ }^{8}$ Knoff [1987].
${ }^{h}$ after correction for entropy contributions, upper bound is $97 \%$ of the Voigt limit and lower bound is $79 \%$ of the Reuss limit.

Table 5.6
PPTA fiber theoretical elastic constants (GPa): Structures \#4 and \#5

|  | Structure \#4 |  | Structure \#5 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Voigt | Predicted Range ${ }^{\text {a }}$ | Voigt | Reuss | Predicted Range ${ }^{2}$ |
| Extensional Modulus ( $\mathrm{E}_{3}$ ) | 369 | 358-292 | 290 | 146 | 281-115 |
| Transverse Modulus ( $\mathrm{E}_{1}, \mathrm{E}_{2}$ ) | 55.1 | 53.4-43.5 | 52.2 | 21.9 | 50.6-17.3 |
| Torsional Modulus ( $\mathrm{G}_{1}, \mathrm{G}_{2}$ ) | 15.8 | 15.4-12.5 | 14.8 | 3.8 | 14.4-3.0 |
| Extensional Poisson's Ratio ( $\nu_{13}, \nu_{23}$ ) | -0.19 | - | 0.08 | -0.34 | - |
| Transverse Poisson's Ratio ( $v_{12}, v_{21}$ ) | 0.14 | - | 0.31 | 0.24 | - |

${ }^{2}$ after correction for entropy contributions, upper bound is $97 \%$ of the Voigt limit and lower bound is $79 \%$ of the Voigt limit for Structure \#4 and $79 \%$ of the Reuss limit for Structure \#5.

The values calculated for the PPTA Structure \#3 exhibit considerable spread between the upper (Voigt) and lower (Reuss) bounds. The values for all moduli tend to be somewhat higher than the experimentally measured values, as expected; the former may be as much as $30 \%$ higher than the true crystal lattice values, due to neglect of entropic effects, while the latter may be expected to be lower due to imperfections in the bulk. Taking these considerations into effect, the calculated lower bounds for $\mathrm{E}_{3}$ and $\mathrm{G}_{1}$ may be as low as 215 GPa and 4.1 GPa , respectively, in excellent agreement with experimental values.

Table 5.6 lists the fiber moduli calculated for two additional predicted structures of PPTA: Structure \#4, which is closely related to Structure \#3 above and differs primarily in the orientation of neighboring sheets, and Structure \#5 previously discussed. It must be noted here that the data for Structure \#4 were calculated using the reduced experimental data set. Based on the Voigt averages for Structure \#4, one observes similar magnitudes for the extensional and torsional moduli of PPTA structures \#3 and \#4 but a significant increase in the transverse modulus, probably due to slightly higher hydrogen-bonding energies in the latter geometry. Structure \#5, on the other hand, exhibits somewhat lower Reuss bounds than Structure \#3 for $E_{3}$ and $G_{1}$ (based on the full stiffness matrix calculation); the implication is that this structure might lead to a slightly more extensible fiber if predominant in the actual final product (e.g. a predominance of Modification II).

### 5.5 Estimation of Resistance to Shear

Returning to the discussion of Section 5.3, one may take advantage of the complete stiffness matrix to look at candidates for orientation of failure. It is not possible to draw rigorous conclusions about yield behavior based only on an estimation of the elastic compliance matrix for a body. However, we may attempt to define that mode of deformation for which the required work is a minimum, or the orientation at which the resistance to shear deformation is least. For this purpose, we proceed analogously to the consideration of yield criteria for isotropic bodies. We first define our criteria for imposed strain and the resulting stress response in terms of the first three tensorial invariants, described by equations 5.53 , 5.54 , and 5.55 , in order to establish a condition of equal deformation. We define a constant state of strain using the first three invariants of the matrix $\alpha$, where $\alpha$ may be strain $\varepsilon$ or stress $\sigma$ :

$$
\begin{array}{lll}
\mathrm{J}_{1}=(1 / 3) \operatorname{Tr}(\alpha) & \begin{array}{l}
\text { (for } \alpha=\sigma \text { this is the negative of hydrostatic } \\
\text { pressure) }
\end{array} \\
\mathrm{J}_{2}=3 \operatorname{Tr}\left(\alpha-(1 / 3) \operatorname{Tr} \alpha \mathrm{I}_{3}\right)^{2}
\end{array}
$$

such that our critical stress criterion may be described as:

$$
\begin{equation*}
r=f\left(\mathrm{~J}_{1}, \mathrm{~J}_{2}, \mathrm{~J}_{3}\right) \tag{5.56}
\end{equation*}
$$

To a first approximation, we assume that the critical mode of shear deformation is independent of the hydrostatic component of stress, as in metals, although this may not be strictly true for polymers. Then, following the approach of von Mises for analysis of yield phenomena, we propose that the critical shear mode does not involve $\mathrm{J}_{3}$ and that stresses in simple tension and compression are equal (i.e. no Bauschinger effect (Ward, 1983 p 340 ]). This leads to a criterion analogous to the von Mises yield criterion; Equation 5.56 simplifies to:

$$
\begin{equation*}
r=f\left(\mathrm{~J}_{2}\right)=\text { constant } \tag{5.57}
\end{equation*}
$$

For a fixed value of $\mathrm{J}_{2}$ of $\varepsilon$ we can then compute $\sigma$ and $\mathrm{J}_{2}$ of $\sigma$ via equations 5.54 and 5.58 :

$$
\begin{equation*}
\sigma \quad=\quad C \varepsilon \tag{5.58}
\end{equation*}
$$

We propose to define for r a "stress resistance", which is independent of the magnitude of $\mathrm{J}_{2}(\varepsilon)$, as the low shear equivalent of the Von Mises Yield Stress (the second invariant of the stress matrix) over the Work Equivalent Shear Strain (the second invariant of the strain matrix):

$$
\begin{equation*}
\mathrm{r}=\left(\mathrm{J}_{2}(\sigma) / \mathrm{J}_{2}(\varepsilon)\right)^{1 / 2} \tag{5.59}
\end{equation*}
$$

For simplicity, we take the condition $\mathrm{J}_{1}(\varepsilon)=\mathrm{J}_{3}(\varepsilon)=0$. The minimum stress resistance, $\mathrm{Y}_{\text {min }}$, may be idenified by selecting an arbitrary strain tensor and operating rotations of this tensor on the elastic stiffness tensor describing the response of the elastic body to determine those orientations at which Y is a minimum. For the calculation of $\mathrm{J}_{2}(\alpha)$ where $\alpha$ is $\varepsilon$ or $\sigma$, we use:

$$
\begin{equation*}
\mathrm{J}_{2}=\left(\alpha_{11}-\alpha_{22}\right)^{2}+\left(\alpha_{22}-\alpha_{33}\right)^{2}+\left(\alpha_{33}-\alpha_{11}\right)^{2}+6\left(\alpha_{12}{ }^{2}+\alpha_{23}{ }^{2}+\alpha_{13}{ }^{2}\right) \tag{5.60}
\end{equation*}
$$

Applying this approach to the stiffness tensor calculated for the PPTA Structure \#3, we obtain a minimum shear resistance $r$ of 6.9 GPA for a deformation having principle axes and corresponding deformation eigenvalues of:
$\left.\begin{array}{llllllll}\mathbf{X} & =+0.4082 \mathrm{~J}_{2}(\varepsilon) & \mathbf{X} & = & {[ } & 0.486 & -0.662 & -0.571\end{array}\right]$
corresponding to the following tensorial strain description:

$$
\varepsilon \quad=\quad \pm \mathrm{J}_{2}(\varepsilon)\left[\begin{array}{rrr}
0.031 & -0.254 & -0.027  \tag{5.61}\\
-0.254 & -0.051 & 0.316 \\
-0.027 & 0.316 & 0.020
\end{array}\right]
$$

One may immediately observe that the $\varepsilon_{12}$ and $\varepsilon_{23}$ elements are predominant, indicative of motions primarily of sheet-sheet slip lateral and parallel, respectively, to the chain axis. This could be anticipated from the nearly equal but low magnitudes of the shear moduli $G_{1}$ and $G_{3}$ relative to that for $G_{2}$ for this structure. The plane of this deformation corresponds closely to deformation in the [102] crystallographic plane; this is illustrated in Figure 5.1, showing the calculated unit cell, the [102] plane, and the principle axes of deformation in this plane. From the minimum energy configuration, the lowest energy deformation process involves a motion of the chain simultaneously longitudinal and lateral to neighboring chains in the next hydrogen-bonded sheet. This is consistent with the view that deformation in PPTA occurs with the retention of the integrity of the hydrogen-bonded sheets.

Figure 5.1 Illustration of the mode of least resistance to shear dcformation in the simulated PPTA Strucure \#3. Shown are the unit cell, the orientation of the [102] crystallographic planes which contain this deformation mode, and the principle axes describing the orientation of deformation within this plane.


### 6.1 Objective

Simulations of real materials are only as good as the validity of information they provide. Any computational development inherently invokes a series of approximations, whose purpose is to make the problem tractable without sacrificing accuracy. In the end, the validity of the model must be checked against a known material system. For this reason, we found it necessary to undertake an experimental program whereby we could produce aramid polymers of the types to be modelled in a highly ordered form for comparison with simulation results. Our objective did not involve an exhaustive research into the complex interrelationships between material and process conditions, but rather exploitation of the best technology available sufficient for the purpose of producing highly oriented and crystalline samples of PPTA and its derivatives. As was addressed in Chapter 2, the method of choice for this purpose is the dry-jet wet-spinning of aramid fiber from anisotropic solutions. This chapter discusses the criteria behind the design of our embodiment of the dry-jet wet-spinning process, followed by a description of the equipment and the process itself. Experimental procedures and general considerations are discussed prior to a description of the fibers produced from solutions of PPTA in sulfuric acid. Finally, the analysis of fibers via $x$-ray diffractometry is discussed.

### 6.2 Equipment Design

### 6.2.1 Preliminary Considerations

Several requirements guided the choice of process design and selection of methods and materials for our application. These are considered below; where possible, the equipment was constructed to maximize versatility without sacrificing on these minimum requirements.

First, the process must be capable of processing small quantities of polymer into representative fiber. This requirement was based on the limit imposed by current synthetic capabilities in our lab to produce
constitutionally modified and constitutionally isomeric polymers of both sufficient molecular weight and sufficient regularity of structure. The polymers reported by Gentile [1988] and referred to in the section on constitutional isomerism by controlled Homogenous Phase Amidation were in all cases isolated in quantities of less than one gram. The maximum batch size of polymer currently achievable is on the order of five grams, the primary limitation being lack of in-house capability to purify the requisite monomers in sufficient quantity to conduct larger scale polymerizations. The importance of this limit on polymer availability (for the modified polymers) is exacerbated by the nature of fiber spinning, which is inherently a continuous or semicontinuous process. Each trial consists of start-up, steady state, and transition phases; fiber generated during start-up or between changes of operating conditions during the spin is not discussed. One would ideally opt for a process involving the largest quantity of polymer possible, in order to reduce the significance of the start-up phase. As a balance, solution quantities on the order of 5 ml , corresponding to roughly one gram of polymer at $10 \mathrm{wt} \%$ in sulfuric acid, were considered to be the appropriate requisite spin dope volume which could be obtained and converted into quality fiber.

Second, the primary feature believed to be responsible for the desirable mechanical properties of aramid fibers and the dominant assumption in the constructions simulated in the model is a high degree of molecular alignment. Two major factors contribute to the development of orientation in aramid fibers. (1) The first factor is the formation of an anisotropic phase prior to filament formation; this may be controlled procedurally, by working at concentrations and temperatures at which liquid crystal phases form. For simplicity, dope preparation may be decoupled from subsequent processing. (2) The second requirement is the capability to control both the spin line speed and the draw-down ratio $D_{R}$, defined as the ratio of the linear velocity of the fiber at the collection wheel to that at the die exit, which produces the extensional flow field that causes co-orientation of the directors of the nematic domains.

Third, the corrosive nature of the polymer/solvent system poses special problems. The use of concentrated acids imposes severe restrictions on the selection of materials of construction, and the complete removal of solvent is critical if the fibers are to be exposed to high temperatures, as residual acid will accelerate degradation of the polymer. The use of small filament diameters reduces the magnitude of concentration gradients of solvent within the filament during coagulation and permits more complete removal of solvent from the fiber core. The smaller diameter also ensures a more homogeneous coagulation process, which should minimize the skin/core morphology differentiation often noted in wetspun fibers. Previous workers typically report the greatest success with spinnerette dies of diameter on the order of $25 \mu \mathrm{~m}$ to $200 \mu \mathrm{~m}$ [Montgomery, 1971; Blades, 1973; Valenti et al., 1981]. The selection of the spinnerette diameter reflects a trade-off between the improved coagulation efficiency possible with small diameter and the ease of handling of larger diameters.

Fourth, the dopes considered are quite viscous. Ideally, one would prefer that the concentration of polymer in the spin dope be as high as possible, in order to promote solution anisotropy and to reduce solvent removal requirements. However, such concentrations lead to very high viscosities in the quiescent solution. Viscosities as high as 4000 poise are reported for PBA ( $\left.\eta_{\text {inh }}=3.9 \mathrm{~g} / \mathrm{dl}\right)$ in hydrofluoric acid at $0^{\circ} \mathrm{C}$ [Schaefgen et al., 1979]. Aoki et al. [1980] report viscosity versus shear rate curves for Kevlar ${ }^{\otimes}$ in $\mathrm{H}_{2} \mathrm{SO}_{4}$ at both $25^{\circ} \mathrm{C}$ and $60^{\circ} \mathrm{C}$, in concentrations up to $12 \%$ polymer by weight; this data shows considerable shear thinning behavior (from 30000 poise at a shear rate of $0.01 \mathrm{sec}^{-1}$ down to 300 poise at $8 \mathrm{sec}^{-1}$ ) of the anisotropic solution ( $10 \mathrm{wt} \%$ solution) at $25^{\circ} \mathrm{C}$, but relatively constant viscosities on the order of 1000 poise or lower at $60^{\circ} \mathrm{C}$. We have assumed an representative value of 1000 poise for design purposes.

Fifth, we require certain postspin treatments; these include multiple washing and neutralization steps and heat treatment of the fiber. Postspin treatments are decoupled from the spin line for ease of operation. Washing and neutralization ensure complete removal of acid and prevent later degradation. Annealing at temperatures above $450^{\circ} \mathrm{C}$ have been shown to produce significant improvements in fiber mechanical properties in aramids. Blades [1973] reports increases in modulus as large as $100 \%$ with heat treatments at $550^{\circ} \mathrm{C}$ for 1 to 6 seconds. Morgan et al. [1983] and Chatzi and Koenig [1987] suggest that the fibers should be elongated by about $0.5 \%$ under such conditions with a tension of $6 \mathrm{~g} /$ denier $(0.8 \mathrm{GPa}$ for PPTA). Kwolek et al. [1977] report increases in modulus for PBA from $450 \mathrm{~g} /$ denier ( 60 GPa ) up to $1000 \mathrm{~g} /$ denier ( 132 GPa ) at $540^{\circ} \mathrm{C}$, and similar increases in tensile strength from $6.5 \mathrm{~g} /$ denier ( 0.86 GPa ) to $17 \mathrm{~g} /$ denier ( 2.3 GPa ). These large increases in mechanical stiffness are indicative of changes in the morphology of the fibers, presumably in the direction of higher orientation of the molecules or increased crystallinity or both. The morphology change could also reflect a transformation from one polymorph characterized by a low tensile modulus, such as that for PPTA Structure \#5 presented in Table 5.6, to another polymorph characterized by a higher tensile modulus, such as that for either PPTA Structure \#3 or Structure \#4. For purposes of comparison with the results generated under the idealized view presumed by the simulation, experimental data from fibers of greatest molecular orientation and highest degree of crystallinity should prove most appropriate.

### 6.2.2 Design Specifications

In light of the these considerations, our process as a minimum requires: firsh, an effective means for forming the highly viscous spin dopes; second, equipment and controls sufficient to spin this dope as a very fine diameter filament under controlled extrusion and wind-up velocity constraints; third, a means for conducting very short duration annealing of the fibers under at least modest tension. The process and requisite apparatus were implemented as follows:
(1) Decoupled dissolution, spinning, and postspin washing, neutralization and heat treatment steps.
(2) Low-shear dope preparation, by means of a special mechanically-driven threaded shaft mixer, to ensure complete polymer dissolution at high concentration without causing shear-induced degradation
(3) Volumetric displacement extruder (syringe and plunger design) for extrusion, rather than a pressure-driven extrusion, in order to ensure accurate control of extrusion velocity regardless of solution inhomogeneities or variations in solution dynamic properties from trial to trial.
(4) Monofilament spinning, to simplify start-up and operation of the spin line
(5) Washing and neutralization of the wet filament while on the collection roller by submersion in agitated baths.
(6) High temperature, short duration heat treatment, by passage of the fiber samples through a furnace at a predetermined velocity to realize the desired residence time, under sufficient tension to maintain the carrier lines taut. This serves to generate high degrees of crystallinity and to mitigate the effects of imperfect process conditions on microstructure.

Figure 6.1 illustrates the entire fiber preparation process, including the prespinning and postspinning treatment steps. During the course of this work, fibers were generated through two different embodiments of this process. However, the fundamental features of process operation and fiber formation did not vary between these two pieces of equipment; the later version included accurate computer control, measurement, and data acquisition of important process variables. Monitored parameters were the extruder and collector drive motor rates, the extruder temperature, and the plunger drive force, from which were calculated extrusion and collection linear velocities, plunger position, and extrusion pressure. The critical components are presented in greater detail in Appendix F. Table 6.1 summarizes the most important design specifications which determine the range of available spin conditions.

### 6.3 Experimental Procedure

### 6.3.1 Dope Preparation

Our PPTA solutions consisted exclusively of Kevlar $29^{\circledR}$, supplied by E.I. DuPont de Nemours and known to consist primarily of PPTA [Penn et al., 1976; Morgan et al., 1983], redissolved in $100 \%$ $\mathrm{H}_{2} \mathrm{SO}_{4}$. The inherent viscosity of the polymer was $6.1 \mathrm{dl} / \mathrm{g}$ in $96 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ (determined using an Ostwald viscometer and a standard concentration of 0.05 g of polymer in 10 ml of solvent) at $25^{\circ} \mathrm{C}$. Assuming that at concentrations of $0.5 \mathrm{~g} / \mathrm{dl} \eta_{\text {inh }}=[\eta]$ and using Mark-Houwink constants of $\mathrm{K}=8 \times 10^{-5} \mathrm{dl} / \mathrm{g}$ and a $=1.09$ [Arpin and Strazielle, 1977], this corresponds to an approximate molecular weight of 30000

Figure 6.1: $\quad$ Schematic of the experimental program for polymer dissolution, fiber spinning, and postspin heat treatment.


## Table 6.1

## Equipment design specifications

| Syringe volume | 6.2 ml |
| :--- | :--- |
| Syringe barrel diameter | 10 mm |
| Spinnerette orifice diameter | $50 \mu \mathrm{~m}$ to $120 \mu \mathrm{~m}$ |
| Spinnerette die profile | 2 mm to 1 mm to orifice diameter |
| Spinnerete temperature | $25^{\circ}$ to $80^{\circ} \mathrm{C}$ |
| Plunger velocity | -42.5 to $+42.5 \mathrm{~mm} / \mathrm{min}$ |
| Wind-up wheel radius | 35 mm |
| Wind-up velocity | -3000 to +3000 rpm |
| Maximum plunger force | $10,000 \mathrm{~N}\left(1.27 \times 10^{8} \mathrm{~Pa}, 1260 \mathrm{~atm}\right)$ |

$\mathrm{g} / \mathrm{mole}(\mathrm{DP}=240)$. The spin dopes were prepared by combining the polymer staple with solvent under an inert atmosphere, heating to $60^{\circ}$ to $80^{\circ} \mathrm{C}$, and thoroughly mixing by slow stirring (i.e. $\leq 60 \mathrm{rpm}$ ) using the threaded mixer. Concentrations from $4.0 \mathrm{wt} \%$ to $17.5 \mathrm{wt} \%$ were made ( $\mathrm{V}_{\mathrm{p}}^{*}$ is about $7 \mathrm{wt} \%$ to $8 \mathrm{wt} \%$ polymer in sulfuric acid at $25^{\circ} \mathrm{C}$ ); for most trials, a solution of $10 \mathrm{wt} \%$ polymer in solvent was used. In some instances, the polymer was first dried at $110^{\circ} \mathrm{C}$ under vacuum for two hours prior to combination with the solvent, which was then allowed to impregnate the fiber staple overnight before heating was initiated. In either case, homogeneous anisotropic dopes could be prepared in thirty minutes to one hour. Change in molecular weight due to shear-induced degradation or thermal decomposition was checked by viscometry and found to be insignificant. Anisotropy was confirmed by stir opalescence and polarized light microscopy.

### 6.3.2 Fiber Formation

Approximately 5 ml of the polymer dope was loaded by spatula into a prefilter, which attached to the open end of the assembled syringe barrel. By means of a hand-operated screw which forced the solution through a stainless steel filter screen, the solution was filtered and homogenized while being loaded to the syringe. The dope was then extruded through the die orifice by means of a motor-driven plunger, with the extruded filament falling vertically from the spinnerette die face upon exit. Spinnerette die diameters from $60 \mu \mathrm{~m}$ up to $120 \mu \mathrm{~m}$ were used, but the $60-\mu \mathrm{m}$ dies were preferred (generally specified to have a 1:1 length to diameter ratio, but in all but a few cases the capillary length could not be confirmed). Operative filament velocities at the orifice ranged from 5 to $80 \mathrm{~m} / \mathrm{min}$, depending upon plunger rate and orifice diameter; $40 \mathrm{~m} / \mathrm{min}$ was typical. The filament passed initially through an air gap of 5 to 10 mm and then through the aqueous coagulation bath at room temperature $\left(24^{\circ} \mathrm{C}\right)$, along a path length on the order of 105 cm , at ambient temperature. Finally, the precipitated polymer filament was collected on the wind-up roller, whose rotation speed was selected relative to the linear velocity of the plunger to induce draw-down of the filament. The collected fiber sample was then washed in water, followed by neutralization overnight of residual acid in a $1 \mathrm{wt} \% \mathrm{NaOH}$ solution bath, followed in turn by a final water wash and drying at $60^{\circ}$ to $80^{\circ} \mathrm{C}$. This constitutes the denoted "as-spun" fiber. Fiber samples for $x$-ray analysis were prepared by impregnating a section of the wound filament with epoxy and then cutting a multifilament sample of length 110 mm directly from the collection spool.

### 6.3.3 Heat Treatment

The samples were optionally tied or glued to long leaders of Kevlar $29^{\circ}$ which were used to thread the annealing line prior to heat-up. The sample was drawn through a high temperature zone under inert atmosphere one or more times to produce the denoted "annealed" fiber. A residence time of two seconds
at $500^{\circ} \mathrm{C}$ was employed, in accordance with recommendations in the literature [Kwolek, 1972; Morgan et al., 1980]. Subsequent annealing passes did not appear to alter the structural characteristics observed by x -ray analysis to any significant extent.

Polarized and unpolarized light microscopy (Nikon Optiphot-pol light microscope with Microflex AFX-II photomicrographic atlachment, magnifications of $40 \mathrm{x}, 200 \mathrm{x}$, and 400 x ) were used to verify the liquid crystal phase formation and to evaluate fiber diameter variations. Magnifications typically used were 40 x for the former and 400 x for the latter.

### 6.4 Spinning Trials for PPTA

The primary factor effecting molecular orientation during the spinning process is the draw-down ratio $D_{R}$, a measure of the extensional flow field acting to orient the nematic domains. We generally quote a nominal draw-down ratio, or spin-stretch factor, based on the known extrusion and wind-up linear velocities. The actual extensional flow field depends upon the true draw-down ratio, which is augmented by the tendency for the extruded filament to swell as it exits the spinnerette; this swelling is caused by the relaxation upon transition from confined to free surface flow of residual stresses imposed upon the extrudate by flow through the die geometry. Die swell is defined as the ratio of the diameter of the relaxed fiber to that of the extrudate exiting the die face. In the range of shear rate $\gamma_{w}$, defined as in Equation 6.1, from 6000 to $17200 \mathrm{~s}^{-1}$, Conio et al. [1987] found die swell in PBA ( $\eta_{\text {inh }}$ of $1.37 \mathrm{dl} / \mathrm{g}$ and $1.85 \mathrm{dl} / \mathrm{g}$ ) to increase from 1 at very low extrusion speeds up to 1.3 at an extrusion speed of $13 \mathrm{~m} / \mathrm{min}$. Jaffe and Jones [1985] report a linear correlation between the logarithms of die swell and shear rate at the wall for PPTA; in the range $\gamma_{w}=25000$ to $90000 \mathrm{~s}^{-1}$, die swell varied from roughly 1.1 up to 1.7 . For a $10 \%$ by weight solution of polymer in $100 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ spun at $30^{\circ} \mathrm{C}$, we determined that wind-up velocity necessary to match exactly the linear velocity of the swollen filament without inducing draw or allowing slack in the spin line. By assuming that the filament does not undergo additional dimensional changes, other than those due to relaxation, before the collection roller, we may calculate the die swell as the square root of the ratio of linear velocities, as indicated in Equation 6.2. With the assumption of incompressible Newtonian flow behavior in the die capillary, we obtain values for the shear rate and shear stress at the wall of the die using Equations 6.1 and 6.3 , respectively:

$$
\begin{array}{ll}
\gamma_{w} & =8 v_{e} / D_{\text {orifice }} \\
\text { die swell } & =\left(v_{e} / v_{w}\right)^{1 / 2} \tag{6.2}
\end{array}
$$

$$
\begin{equation*}
\tau_{w} \quad=\quad D_{\text {orifice }} P / 4 L_{\text {orifice }} \tag{6.3}
\end{equation*}
$$

$P$ refers to the driving pressure for extrusion, assumed to be equal to the force per unit area at the end of the moving plunger. Figure 6.2 shows the variation of die swell and shear stress with shear rate in the range $30000 \mathrm{~s}^{-1}<\gamma_{\mathrm{w}}<185000 \mathrm{~s}^{-1}$ for our apparatus using a spinnerette of diameter $60 \mu \mathrm{~m}$ and capillary length of $100 \mu \mathrm{~m}$ and a $10 \mathrm{wt} \%$ polymer dope. Die swell remains essentially constant over the range of shear rates employed in this work, while shear stress increases almost linearly with shear rate. From the slope of the latter curve, we obtain a coefficient of viscosity of 160 poise for the $10 \mathrm{wt} \%$ dope at $30^{\circ} \mathrm{C}$. The shear thinning behavior under these conditions is considerably less pronounced than that reported by Aoki et al. [1979] for a similar dope at $25^{\circ} \mathrm{C}$. However, the shear rates employed here are considerably higher than those reported by Aoki et al., and the calculated coefficient of viscosity is half the value obtained at their ultimate (highest) shear rates; this would suggest a transition from shear thinning to Newtonian behavior at higher shear rates, consistent with some rheological observations for rigid rod polymers [Larson, 1988]. The die swell is somewhat larger than that observed by other investigators; this is probably due to differences in the extrusion geometry and the imposition and relaxation of applied stresses prior to the extruder exit. Conio et al. suggest that this effect should decrease with increasing polymer concentration and higher extrusion temperatures. In agreement with these observations, using a $17.5 \mathrm{wt} \%$ solution of PPTA in $100 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ spun at $80^{\circ} \mathrm{C}$ through the same orifice (i.e. $60 \mu \mathrm{~m} \times 100$ $\mu \mathrm{m}$ ), we observe a die swell of only 1.3 in the range $37000 \mathrm{~s}^{-1}<\gamma_{\mathrm{w}}<127000 \mathrm{~s}^{-1}$.

Table 6.2 summarizes some results of variations in draw-down ratio. Here we adhere to the use of nominal draw-down ratios and report the estimated die swell, where possible. Unfortunately, we do not have die swell data for fibers spun using the first apparatus; however, based on the similarity of design we expect the phenomenon to be similar to that observed in trials using the second spin apparatus under similar operating conditions. The trends are consistent with those suggested in the literature for the related polymer, polybenzamide [Kwolek et al., 1977]. Specifically, we note that fiber radius decreases with increasing draw-down ratio for as-spun fiber, consistent with dimensional changes normally accompanying extensional flow. Orientation angle, determined as the azimuthal breadth at one half the maximum intensity of the x-ray diffraction peak at $2 \theta=22.5^{\circ}$, decreases with increasing draw ratio for as-spun fiber; this is in line with expectations for alignment of nematic domains upon exposure to linear accelerations after leaving the spinnerette. Our annealing method produces no measurable dimensional change in the fiber, but results in a decreased orientation angle for those fibers initially spun at low draw ratio. At higher draw ratios we observe a limiting orientation angle which is actually lower (i.e. better alignment) for the $10 \mathrm{wt} \%$ polymer dope. At this point, we presume that this discrepancy is due to geometry factors different between the two embodiments of the spin apparatus design. A similar limit in fiber properties is reported by Valenti et al. [1981] for modulus as a function of draw ratio for their work

Figure 6.2: $\quad$ Shear stress at the wall and die swell as functions of shear rate at the wall for the spinning of PPTA from a $10 \mathrm{wt} \%$ solution in $100 \% \mathrm{H}_{2} \mathrm{SO}_{4}$ at $30^{\circ} \mathrm{C}$ using a spinnerette of diameter $60 \mu \mathrm{~m}$ and capillary length $100 \mu \mathrm{~m}$.


Table 6.2
Summary of Selected Fiber Spinning Trials

|  |  |  | Nominal draw ratio | Dry fiber diameter ( $\mu \mathrm{m}$ ) | Orientation Angle (degrees) as spun [annealed] |
| :---: | :---: | :---: | :---: | :---: | :---: |
| TRIAL 1: | Polymer concentration | $10 \mathrm{wt} \%$ | 0.3 | 28 | 66 |
|  | Extrusion temperature | $30^{\circ} \mathrm{C}$ | 0.5 | 23 | 43 |
|  | Extrusion rate | $31 \mathrm{~m} / \mathrm{min}$ |  |  |  |
|  | Spinnerette diameter | . $55 \mu \mathrm{~m}$ |  |  |  |
|  | Die swell |  |  |  |  |
| TRIAL 2: | Polymer concentration | $10 \mathrm{wt} \%$ | 1.3 | 38 | 63 [46] |
|  | Extrusion temperature | $25^{\circ} \mathrm{C}$ | 2.0 | 24 | 34 [28] |
|  | Extrusion rate | [ $36 \mathrm{~m} / \mathrm{min}$ | 2.6 | 21 | 20 [20] |
|  | Spinnerette diameter | : $60 \mu \mathrm{~m}$ | 2.9 | 22 | 25 [20] |
|  | Die swell | : (unknown) | 3.2 | 22 | 20 [22] |
| TRIAL 3: | Polymer concentration | : $17.5 \mathrm{wt} \%$ | 0.9 |  |  |
|  | Extrusion temperature | : $80{ }^{\circ} \mathrm{C}$ | 1.2 | 32 | 32 |
|  | Extrusion rate | : $30 \mathrm{~m} / \mathrm{min}$ | 1.7 | 24 | 30 |
|  | Spinnerette diameter | : $63 \mu \mathrm{~m}$ |  |  |  |
|  | Die swell | : 1.3 |  |  |  |
| TRIAL 4: |  | $: 17.5 \mathrm{wt} \%$ | 1.0 | 31 | 36 |
|  | Extrusion temperature | : $80{ }^{\circ} \mathrm{C}$ | 1.7 | 25 | 33 |
|  | Extrusion rate | : $60 \mathrm{~m} / \mathrm{min}$ | 2.0 | 22 | 35 |
|  | Spinnerette diameter | : $63 \mu \mathrm{~m}$ | 2.5 | 19 | 31 |
|  | Die swell | : $1.3 \mu \mathrm{~m}$ | 3.0 | 18 | 31 |

on $\mathrm{X}-500$, the polyterephthalamide of p -aminobenzhydrazide, with a break occuring near $\mathrm{D}_{\mathrm{R}}=1.5$. The ultimate orientation angle values are comparable to values reported in the literature. Hindeleh et al. [1984] report values of $12.4^{\circ}$ and $18.9^{\circ}$ for commercial Kevlar $49^{\circledR}$ and Kevlar $29^{\circledR}$, respectively, while the original patent of Kwolek [1972] reports values between $11^{\circ}$ and $50^{\circ}$. This suggests that our process produces representative fibers. Lastly, we mention that we observe no evidence of an amorphous halo in the x-ray diffraction scans for the annealed fiber. Even in the as-spun fiber, adequate reproductions of the diffraction traces were achieved with the consideration solely of the apparent peaks. From this we conclude that our fiber contains at most a very small amorphous fraction.

### 6.5 X-ray Analysis

### 6.5.1 Equipment and Procedures

As our initial interest lies in the analysis of microstructure, wide angle $x$-ray scattering is the natural choice for evaluation of the quality of fiber spinning and the concurrence between experimental and simulation results. This technique provides the most accurate and direct information about atomic structure and molecular orientation. Infrared absorption provides information about the vibratory freedom of the component atoms and was used as a second independent check for variations in the local atomic environment and for the identification of specific intermolecular interactions such as hydrogen bonds.

Two different WAXS apparatuses were used to gather data during the course of this work. Both machines were rotating anode diffractometers equipped with four-circle goniometers and scintillation counters and employed $\mathrm{Cu} \mathrm{K} \alpha$ radiation ( $\lambda=1.5418 \AA$ ). The first machine was a Rigaku D 4148 H 212 kW High Brilliance generator operated at 40 kV and 150 mA , with a $0.1^{\circ}$ slit collimator and nickel filtered radiation. The second machine was a Siemens D500 generator with DACO-MP control, operated at 40 kV and 35 mA , with a 1.2 mm pinhole collimator and bent quartz crystal monochromator. Multifilament samples were held together by embedding the filament ends in epoxy. The samples were mounted on a frame suitable for positioning in the incident $x$-ray beam and were manually extended and fixed to the opposite end of the frame with tape; good filament alignment was thus possible in the fiber holder, but no extensional deformation was applied. Typical sample sizes were 2 mm in diameter and 20 mm long but contained considerable void space. Absorptions of individual samples were determined from a ratio of the scattering intensity of the $2 \theta=21.67^{\circ}$ peak of polyethylene with and without the sample placed in front of the counter; the absorption (expressed as $\mu \mathrm{t}$, the linear absorption coefficient $\mu$ in $\mathrm{cm}^{-1}$ times the sample thickness in cm ) was determined using Equation 6.4.

$$
\begin{equation*}
(\mu \mathrm{t})=-\ln \left(\mathrm{I}_{\mathrm{A}} / \mathrm{I}_{\mathrm{O}}\right) \tag{6.4}
\end{equation*}
$$

$\mathrm{I}_{\mathrm{O}}$ refers to the unmodified intensity and $\mathrm{I}_{\mathrm{A}}$ refers to the absorption-modified intensity. Typical values for $(\mu \mathrm{t})$ were generally about 0.2 . At this level of absorption, corrections were deemed unnecessary for our samples in either normal or symmetrical transmission modes [International Tables for X-ray Crystallography, 1968; Tadokoro, 1979].

WAXS analysis of the experimental fiber samples proceeded in two stages. The first was an initial evaluation of crystallinity and molecular order by means of equatorial (perpendicular to the fiber axis) and meridional (parallel to the fiber axis) scans of diffracted intensity versus scattering angle $2 \theta$. This also included a routine scan of diffracted intensity versus azimuthal angle $\beta$ for a major reflection, generally the intense equatorial reflection at $2 \theta=22.5^{\circ}$, for purposes of determining orientation angle and crystallite orientation within the fiber sample. Care was taken in all cases to first align the fibers and identify $\beta=0^{\circ}$, where the equatorial diffractions are maxima. This analysis provided a rapid evaluation of the fiber spinning process and recommended samples for further evaluation. The second stage entailed the generation of the full two-dimensional fiber diffraction pattern for comparison to simulation results. Scans of intensity versus $2 \theta$ were generally in the range of $10^{\circ}$ to $55^{\circ}$ by steps of $0.1^{\circ}$ in $2 \theta$ at the desired $\beta$. Azimuthal scans were performed in steps of $1^{\circ}$ or $2^{\circ}$ in $\beta$ at the desired $2 \theta$. Complete fiber diagrams were recreated from successive radial scans of intensity versus $2 \theta$ at different $\beta$ between $0^{\circ}$ and $90^{\circ}$. The full fiber diagram was generated by mirror replication of the first quadrant in the other three quadrants. Counting times were selected to ensure a signal error level (i.e. the square root of intensity divided by intensity) $<5 \%$.

Infrared measurements were made using a Mattson Cygnus 100 FTIR apparatus. Samples were prepared from cut fiber cast in potassium bromide at roughly $2 \%$ by weight. Measurements were generally made in the range $4000 \mathrm{~cm}^{-1}$ to $400 \mathrm{~cm}^{-1}$.

It is worth noting that neither small angle $x$-ray scattering nor mechanical property measurement were undertaken within the scope of this work. The first provides information primarily about the macrostructure of the polymer solid on the length scale of tens to hundreds of angstroms, which is beyond the scale of the information currently generated by the model. Mechanical property evaluation of fibers, on the other hand, could perform two useful functions: First, it could provide useful information about the elastic properties of fibers for evaluation of fiber integrity and uniformity; it has been suggested [Prof. P. Smith, UCSB, personal communication] that mechanical behavior is more sensitive to small changes in molecular orientation in the limit of high degrees of order than is the magnitude of the measured orientation angle. Second, the obtained elastic moduli would be available for direct comparison to
simulation predictions, in much the same manner as x-ray scattering has been used to check structure predictions. However, such a program has not been attempted here, due to the sensitivity of such results to many other factors not related to atomic microstructure and for which no account has been made in the theoretical analysis. For example, small imperfections in fiber macrostructure, such as internal or surface voids, are to a large extent invisible to WAXS analysis, but may dominate and grossly distort the conclusions of a mechanical property analysis. A necessary component of any rigorous mechanical property evaluation, especially for purposes of comparison to an idealized model, would be the more extensive evaluation of the fiber spinning process than has been attempted here, concentrating on the development of both microstructure and macrostructure as a function of operating variables.
6.5.2 Methods of Data Analysis

The experimental x-ray diffraction data for fiber patterns were checked visually in the same manner as were the predicted patterns from the simulation structures. The only correction applied was the subtraction of background radiation and normalization of intensities prior to construction of the intensity versus ( $2 \theta, \beta$ ) grid for interpolation and display on the IRIS 4D GT system; all other corrections have already been applied in the calculation of the simulated reflection intensities. The same routines were used to create the visual display as werc employed in the cases of the simulated structures; in this manner, comparable fiber diffraction patterns were displayed for both simulated and experimental data sets.

As a further check for experimental accuracy, selected experimental diffraction patterns were analyzed by conventional methods for indexing of significant reflections. This evaluation proceeded by selecting a finite number of Gaussian peaks to fit to the two-dimensional experimental pattern, of the form described by Equations 4.9 and 4.10 used for the representation of reflections generated by the simulated structures. The fitting procedure entailed minimization of the relative deviation, calculated as shown in Equation 6.5; of the complete pattern intensity distribution, as a function of the peak heights $A_{i}$, positions ( $2 \theta_{\mathrm{i}}, \beta_{\mathfrak{i}}$ ) and widths at half intensity $\left(\omega_{2 \theta}, \omega_{\beta}\right)$ for all peaks $\mathrm{i}=1, \mathrm{n}_{\mathrm{p}}$.

$$
\begin{equation*}
R_{D}=\frac{\sum_{i=1}^{n 2 \theta} \sum_{j=1}^{n \beta}\left(I_{\text {exp, }, \mathrm{i} j}-I_{\text {calc, }, \mathrm{i},}\right)^{2}}{\sum_{i=1}^{n 2 \theta} \sum_{j=1}^{n \beta}\left(I_{\text {exp. }, \mathrm{i},}\right)^{2}} \tag{6.5}
\end{equation*}
$$

Allowing all parameters to vary independently describes a set of $5 \mathrm{n}_{\mathrm{p}}$ independent variables; this set was initially restricted by requiring all peaks to have the same set of peak widths, reducing the independent parameter set to $\left(3 n_{p}+2\right)$. The fitting procedure was initiated with different numbers of peaks and different initial peak assignments, until a best fit (i.e. minimum relative deviation $R_{D}$ ) was achieved; the variance was generally less than $10 \%$. Subsequent reoptimization allowing each peak to vary independently in width did not produce significant improvements in fitting. Figure 6.3 illustrates in one dimension (i.e. 2日) the quality of fit possible. From the final set of Gaussian peaks describing the best fit to the experimental pattern, the total intensity attributable to each reflection was calculated using Equation 4.9.
6.5.3

Evaluation of PPTA Fibers and Comparison to Simulation

Figures 6.4 (a) and (b) show reconstructed experimental x-ray diffractograms for our $D_{R}=3.2$ fiber (Trial 2, Table 6.2) before and after annealing, respectively. Results from the quantitative analysis and deconvolution of peaks are reported in Table 6.3, along with a summary of the reflection identifications reported by Northolt [1974] and Haraguchi et al. [1979] for Modifications I and II, respectively. In comparing our results with the two crystal modifications reported in the literature, we note the following:

The as-spun fiber exhibits a poorly resolved diffraction pattern, which is to be expected from a morphology determined primarily by kinetic constraints operative during coagulation. Prior to annealing, the possibility for imperfect crystallinity or multiple coexisting crystalline polymorphs is significantly greater. There is no assurance that such a morphology should consist entirely, or even primarily, of the thermodynamically most stable allomorph; it is not expected, then, that the as-spun fiber diffraction pattern must conform to one predicted by simulation calculations. The most notable feature of the asspun fiber is the appearance of equatorial reflections at $17.9^{\circ}$ and $22.6^{\circ}$ in $2 \theta$, in agreement with Modification II, and of a new equatorial reflection at $27.7^{\circ}$, indicative of an [hk0] reflection not previously reported; Haraguchi et al. report an off-equatorial reflection of medium intensity that they attribute to the [211] diffraction planes. These reflections are in agreement with a structure possessing chains at the $[0,0]$ and $\left[\frac{1}{2}, 0\right]$ locations in the ab-facet of the unit cell, suggestive of the simulation structures \#5 through \#8. The last three of these, \#6, \#7 and \#8, all possess similarly-rotated phenylene rings with respect to the amide bond plane in the chain conformation, with the resulting generation of significant second layer nonmeridional reflections. This leaves PPTA Structure \#5 as that most consistent with the as-spun fiber. This structure also predicts significant reflections atributable to both [210] and [211] sets of diffraction planes; the former, while not noted by Haraguchi et al. in Modification II, is consistent with our as-spun fiber and, in conjunction with the close-lying [211] reflection, reproduces well the intensity previously atributed solely to the [211] planes. However, peak resolution in our as-spun fiber is generally insufficient to justify a more detailed analysis of crystal structure. We do observe that,

Figure 6.3: Experimental and calculated equatorial intensity distributions for a sample of annealed PPTA fiber. The calculated intensity is a summation of fitted Gaussian distributions which best reproduce the experimental trace.


Figure 6.4: Representative x-ray fiber patterns for PPTA fibers spun during the course of this work: (a) as-spun fiber produced at a nominal draw-down ratio of 3.2; (b) same fiber after annealing for 2 sec at $500^{\circ} \mathrm{C}$.

(b)

## Table 6.3

Exprimentally determined reflections in PPTA fibers produced in this work and those reported in the literature for Modifications I and II

| As-spun fiber |  |  | Annealed fiber |  |  | Modification I ${ }^{\text {d }}$ |  |  | Modification $\mathrm{II}^{\text {e }}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I |
|  |  |  | 13.7 | 84 | 3 | 13.7 | m | 7 |  |  |  |
| 17.9 | 0 | 31 |  |  |  |  |  |  | 17.4 | e | s |
|  |  |  | 20.4 | 0 | 81 |  |  |  |  |  |  |
|  |  |  |  |  |  | 21.7 | g |  |  |  |  |
| 22.6 |  | 100 | 22.5 | 0 | 100 | 22.6 | e | 100 | 22.2 | e | vs |
| 27.7 | 0 | 45 |  |  |  |  |  |  |  |  |  |
| 28.48 |  | 13 | 28.0 | 86 | 12 |  |  |  |  |  |  |
|  |  |  | 29.1 |  | 23 | 29.3 | g |  |  | g | m |
|  |  |  | 29.8 |  | 5 |  |  |  |  |  |  |
|  |  |  | 33.4 |  | 5 |  |  |  |  |  |  |
| 39.2 |  | 8 |  |  |  |  |  |  |  |  |  |
| 41.029 |  | 9 | 37.3 |  | 6 | 37.4 | g | 4 |  |  |  |
|  |  |  | 38.4 | 0 | 10 | 38.5 |  |  |  |  |  |
| 43.785 |  | 35 | 43.4 |  | 26 | 42.0 | m | 35 |  | m | m |
|  |  |  | 45.6 |  | 15 |  |  |  |  |  |  |
| 49.0 |  | 8 |  |  |  | 50.0 | g | 4 |  |  |  |
|  |  |  | 49.9 | 0 | 8 | 46.1 | e | 1 |  |  |  |
|  |  |  |  |  |  | 47.2 | g |  |  |  |  |

a $\quad \lambda=1.542 \AA$.
b $\quad e, m$, and $g$ refer to equatorial, meridional, and general reflections, respectively, since azimuthal angles of reflection are not reported.
c the observed d spacing is not reported.
d Northolt [1974].
e Haraguchi et al. [1979].
f corresponds to 3.2 draw ratio fiber from Trial 2 (Table 6.2)
in agreement with the simulation results, the meridional reflections located at $28.4^{\circ}$ and $43.7^{\circ}$, attributable to the [004] and [006] reflections and determined primarily by chain conformation, are indicative of an extended chain conformation.

Upon annealing we note a structural transformation such as that reported by other investigators [Haraguchi et al., 1979; Haraguchi et al., 1979; English, 1986]. The equatorial reflections at $17.9^{\circ}$ and $27.7^{\circ}$ disappear in favor of a strong reflection at $20.5^{\circ}$ and an off-equatorial reflection at $29.1^{\circ}$, characteristic of Modification I. Other strong equatorial reflections occur at $25.5^{\circ}$ and $38.4^{\circ}$. Only the [111] reflection predicted to occur at $21.7^{\circ}$ fails to appear in our patterns. Reflections along the meridian shift to $28.0^{\circ}$ and $43.4^{\circ}$, indicative of a slight extension of the chains during annealing, but remain consistently higher in $2 \theta$ than those reported by Northolt [1974]. Intensities of meridional reflections are also lower, but we see increased intensities of off-meridional reflections such as [106] and [104]. The final dominant structure in the annealed fiber agrees well with Modification I.

The same trend in structure transformation was followed using IR spectroscopy, with particular attention paid to the band shifts from $630 \mathrm{~cm}^{-1}$ to $673 \mathrm{~cm}^{-1}$ and from $720 \mathrm{~cm}^{-1}$ to $730 \mathrm{~cm}^{-1}$, reported by Haraguchi et al. [1979] to signal the transition from Modification II to Modification I. However, our IR data was not sensitive to such a transition upon annealing. Rather, we observe an obvious absorption at $720 \mathrm{~cm}^{-1}$, characteristic of Modification II, in both as-spun and annealed fibers, despite the clear predominance suggested by the x-ray results of Modification I over Modification II in the annealed fibers. The breadth of the absorption band between $600 \mathrm{~cm}^{-1}$ and $690 \mathrm{~cm}^{-1}$ prevented us from drawing any conclusions about a band shift in this range as the result of annealing.

From these results we infer that, prior to annealing, the individual chains are essentially extended and arranged in a reasonably oriented packing array, but that lateral chain packing is not well defined, and two or more such packing modes for the extended chains are probably present. Both of the structures previously reported are partially consistent with the observed diffractogram and could be present in the as-spun fiber. Annealing serves to refine the packing structure in favor of a single dominant packing geometry, as a result of paracrystal-crystal transformation, crystal-crystal transformation, or similar processes.

Of the eight polymorphs suggested by simulation, the PPTA structures identified in Chapter 4 as Structure \#3 and Structure \#4 bear the greatest resemblance to Modification I and hence to our annealed fiber. With the exception of the overlap of the intense [200] and [110] reflections in the simulated patterns for PPTA Structures \#3 and \#4 in Figure 4.7(c) and (d), the agreement in position and intensity of the remaining significant reflections is encouraging. Strucure \#3 most closely approximates the experimental pattern. One notes that in the simulated patern for Structure \#4, the [00l] reflections lie off the meridian; this is due to the deviation in the crystallographic angle $\beta$ from $90^{\circ}$, indicative of a slight
shift between successive sheets of hydrogen-bonded chains. The annealed fiber appears to exhibit characteristics of both polymorphic forms. The reflection "families" (Appendix E, Table E.1) may be compared directly with the integrated intensities for the annealed fiber listed in Table 6.3, deduced from the two-dimensional Gaussian deconvolution procedure. The major reflections are represented in each calculated structure, while the minor reflections could derive from the presence of either or both possible contributors. The weaker meridional reflections predicted by simulation may be traced to the distortion, dictated by local packing constraints, of the 2 screw symmetry of the chain conformation. The unit cells deduced by Northolt [1974] and Tashiro et al. [1977] directly from WAXS data on well-annealed fibers are similar to Structure \#3 but possess higher crystal symmetry.

## 7. EXTENSION TO ISOMERS BASED ON DICHLORO-SUBSTITUTED MONOMERS

### 7.1 Introduction

A primary consideration of model development, and one of the ultimate goals of this work, is the extension of the simulation procedures to the analysis of other similar molecular architectures. At present, the first step in the development of a new model of this type remains the validation of the method using a material of known behavior. This study has been discussed in detail in the previous chapters. However, the real advantage of the method lies in the insight it provides into the connection between chemical structure and material behavior and in its accuracy as a predictive tool and a guide for targeting synthetic efforts. One of the original sources of motivation for this computational development and a running consideration guiding selection of options during development of the model was the more general interest in our group in producing improved, modified aramids which circumvent the weaknesses of the existing list of available stiff chain aramid reinforcement polymers, namely the poor solubility and inferior compressive strength of the $\mathrm{Kevla}{ }^{\circledR}$ family and other rigid rod polymers such as poly(p-phenylene benzobisoxazole) (PBO) and poly(p-phenylene benzobisthiazole) (PBT). The constitutionally isomeric versions of ring-substituted PPTA show promise as reinforcing materials of improved processability, and are especially attractive from the point of view of simulation as materials having controllable atomic-level variations leading to observable macroscopic behavioral changes.

In order to test and demonstrate the extension of the method to new materials, we chose to concentrate on a single representative modification. Variations of interest, based on the previous discussion of constitutional isomerism and the reaction kinetics model, are several: 1) Location of substitution may occur on the phenylene ring of either or both monomers, in order to create the one or two asymmetric monomer units required for control of the reaction kinetics. 2) Substitution may be mono- or di-substitution on one or both carbons, respectively, ortho to the C 1 position on the ring. 3) The substituent considered may be either monatomic in nature, such as a halogen, or polyatomic, such as a methoxy group. 4) One may envision an analysis which entails either extreme of order of isomerism (i.e. entirely head-to-head, tail-to-tail linkages or entirely head-to-tail linkages), the completely randomly structured isomer, or some intermediate degree of regularity. We concentrate here on the analysis of modifications involving dual substitution of chlorines in the 2 and 6 positions of the phenylene ring in the limits of complete isomeric regularity, that is, either head-to-head, tail-to-tail (referred to hereafter as

HHTT) or head-to-tail (referred to hereafter as HT). For purposes of model and force field development we consider dichloro-polyamides based on the polymerization of 2,6-dichloro-p-phenylene diamine with terephthalic acid, referred to hereafter as $2,6-\mathrm{DiCl}-\mathrm{PPTA}$, and the polymerization of 2,6 -dichloroterephthalic acid with p-phenylene diamine, referred to hereafter as PP-2,6-DiCl-TA; these dichloropolyamides will be denoted collectively as Cl-PPTA. For packing calculations, we limit ourselves to the study of the HHTT and HT isomers of $2,6-\mathrm{DiCl}-\mathrm{PPTA}$. One structural repeat unit of each of these two isomers is illustrated in Figure 7.1. Schematically, the symmetric diacid monomer is represented as an open circle and the asymmetric diamine monomer as a filled triangle. Substitution on the diamine monomer was chosen based upon previous successes in the actual synthesis of polymers based on this monomer [Gentile, 1988].

### 7.2 Chain and Force Field Modification

The selection of Cl-PPTA for the study of controlled variation has the advantage that it requircs only minor changes in the chain description and parameterization of the force field and may be considered as a perturbation of the unsubstituted PPTA. In the first instance, it is sufficient to employ the same chain creation and duplication procedure, with the simple removal of pendant hydrogens and replacement by chlorines at appropriate positions along the chain. In practice, this is performed by a delete-and-replace subroutine prior to replication of the parent chain on the lattice. Provisions for generation of packed structures of the random isomer were also implemented by allowing for substitution after chain replication, in randomly selected sequences of head-to-head and head-to-tail orientations, which may differ from chain to chain; however, such calculations will not be discussed within the scope of this work.

Parameterization of the force field requires a redefinition of the set of unique interactions associated with the substituted phenylene ring. In our previous considerations, we treated all the rings as symmetric units. In the modified description, this view is altered to distinguish between a "substituted end" of the ring and an "unsubstituted end" of the ring. In implementation, this requires the definition of a new set of atoms for the phenylene ring, as shown in Figure 7.2: the substituted end consists of the Cl carbon, located between the two chlorines and bonded to the amide moiety, the C2 and C6 carbons, positioned ortho to the Cl carbon and bonded to the two chlorines, and the chlorines themselves; the unsubstituted end consists of the C 4 carbon, located between the two hydrogens and bonded to the opposite amide moiety, the C3 and C5 carbons, positioned meta to the Cl carbon and bonded to the two remaining hydrogens, and the hydrogens, for a total of six atomic species, as opposed to the previous description composed of three atomic species. Secondly, we require a new parameterization of the ring-amide torsional interaction potential to describe the modified behavior of the substituted end. For the unmodified ring and the unsubstituted end of the modified ring, we retain the previous torsion potentials

Figure 7.1: Segments of poly(2,6-dichloro-p-phenylene terephthalamide) (2,6-DiCl-PPTA) with all torsion angles in their zero positions: (a) head-to-head, tail-to-tail isomer; (b) head-to-tail isomer.


Figure 7.2: Definition of "atom types" for the dihalo-substituted phenylene ring moiety.

from the PPTA force field. In this way we have tried to incorporate the predominant changes involved in performing the stated ring substitutions with a minimum of changes to the chain construction or the force field description.

The parameters for the modified force field were generated in exactly the same manner as was previously employed for the parameterization of the ring potential for PPTA. The carbon-carbon and carbon-hydrogen bond lengths and the van der Waals radii and well depth parameters $r_{i}$ and $\varepsilon_{i}$ for carbon and hydrogen were carried over from the PPTA simulation unchanged. The corresponding values for the chlorine atoms were taken from the work of Bernstein and Hagler [1978, 1978] on the crystal structure of the model compound p-(N-chlorobenzylidene)-p-chloroaniline, based on their ab initio quantum mechanical calculations. For partial atomic charges, we again resorted to the procedure of estimation based on AM1 semiempirical quantum mechanical calculations, discussed previously in Chapter 4 and outlined in greater detail in Appendix C. The revised set of atomic nonbonded potential parameters is listed in Table 7.1. The intrinsic torsional energy function was determined for substitution on either the ring of the diamine monomer or the ring of the diacid monomer by appropriate rotations of each ring during AM1 simulations of N -[2,6-dichlorophenyl]-benzamide and 2,6-dichlorobenzanilide, respectively. The variation in potential energy with ring position in each case was best reproduced by the use of a curtate of the form given previously by Equation 4.3. The parameters for these two ring-amide interaction potentials are given in Table 7.2. It is interesting to note that the best fit to the semiempirical AM1 calculations for substitution on the C - N -phenylene torsion requires a significantly increased potential well depth of 32 $\mathrm{kcal} /$ mole and a very steep curtate curvature, whereas an adequate fit in the case of the $\mathrm{N}-\mathrm{C}$-phenylene torsion is obtained with no additional torsion potential; these curves are shown in Figure 7.3 along with the data points from AM1 calculation. This would imply a significant differentiation between $\pi$-bond electron delocalization in the case of the substituted diamine monomer and that in the case of the substituted diacid monomer. However, it would be premature to draw any conclusions concerning electron behavior between moieties based on such a simplified representation. The revised curtates probably reflect a procedural requirement necessitated by the larger hard sphere repulsion of the chlorine atoms in order to emulate the curvature of the rotation potential suggested by AM1, rather than any

## Table 7.1

6-12 Nonbonded atomic potential parameters and elementary charges used in the electrostatic potential
Atom
$\varepsilon$
$(\mathrm{kcal} / \mathrm{mole})$
$(\stackrel{\uparrow}{\AA})$ $\stackrel{\mathrm{q}^{\mathrm{a}}}{\text { (elementary charge) }}$

| C (phenylene, 1) | 0.039 | 1.96 | -0.02 |
| :--- | ---: | ---: | ---: |
| C (phenylene, 4) | 0.039 | 1.96 | -0.06 |
| C (phenylene, 2 and 6) | 0.039 | 1.96 | -0.03 |
| C (phenylene, 3 and 5) | 0.039 | 1.96 | -0.12 |
| H (phenylene) | 0.038 | 1.37 | +0.17 |
| C (amide) | 0.147 | 2.03 | +0.38 |
| H (amide) | 0.0 | 0.0 | +0.28 |
| N | 0.169 | 1.96 | -0.28 |
| O | 0.232 | 1.60 | -0.38 |
| Cl | 0.243 | 1.80 | +0.02 |

${ }^{2}$ The value for the bulk dielectric constant $D_{B}$ was 3.5 , and the Block-Walker crossover distance $\mathrm{d}^{*}$ was $3.3 \AA$.

Table 7.2
Intrinsic torsional energy function parameters ${ }^{\mathbf{a}}$

| Torsion | $V_{d}{ }^{b}$ | $m_{d}$ |
| :--- | ---: | :---: |
| diacid ring $\left(\phi_{5}, \phi_{5}-\phi_{1}\right)$ | 0.0 | 0.0 |
| diamide ring $\left(\phi_{6}, \phi_{6}-\phi_{3}\right)$ | 32.0 | 0.8 |

${ }^{\text {a }}$ see text for details, Equation 4.3
${ }^{\mathrm{b}}$ all values in $\mathrm{kcal} / \mathrm{mol}$

Figure 7.3: Phenylene ring rotation energy functions suggested for dichloro-substituted rings.

Dichlorophenyl Rotational Potentials

significant redistribution of electron probability; the very low barrier at $90^{\circ}$ in the C - N -phenylene case is considered to be of minor significance relative to the general form of the curve. More interesting is the distribution of partial atomic charges among the components of the substituted ring. The pendant chlorines exhibit a very slight positive charge, and appear to effectively counterbalance the tendency for redistribution of electron density to the carbon members of the ring. The pendant hydrogens, on the other hand, retain a significant positive charge, resulting in a notably biased atomic charge distribution at the aromatic C-H bonds. Such assignment of atomic charges accurately reproduces the dipole moment of m-dichlorobenzene (see Appendix C).

### 7.3 Simulation Results: Single Chain

At this stage, we still consider chains of both 2,6-DiCl-PPTA and PP-2,6-DiCl-TA. As in the case of the unsubstituted polymer PPTA, the isolated chain energetics were first considered in terms of torsion angle triplets. Again, one assumes that intermolecular interactions are essentially decoupled across the rigid phenylene rings. With this simplification, the triplets of the isolated chains of the HHTT isomer and the HT isomer become equivalent; one need only consider the two cases of substitution location. Figures 7.4 and 7.5 show the energy contours for N -[2,6-dichloiophenyi]-benzarside and 2,6-dichlorobenzanilide, respectively, as a function of the phenylene-C-N and C-N-phenylene torsions ( $\phi_{3}$ and $\phi_{1}$ ) with the amide torsion $\phi_{2}$ assuming a position of minimum energy in the vicinity of the trans conformation. As before, contours with the amide torsion in the vicinity of the cis conformation all lie 5 $\mathrm{kcal} /$ mole or more above that of the minimum energy conformations. Not surprisingly, the location of the minima and the barrier heights of the unsubstituted rings remain essentially unaltered from the plain benzanilide case. The substituted rings rotate considerably out of the plane of the amide linkage, showing broad minima between 60 and 120 degrees, with very low barriers to rotation within this range. This increased freedom of rotation about the phenylene ring axis should not alter greatly the extensional rigidity of the chain, since it involves rotation about the $\mathrm{C} 1-\mathrm{C} 4$ axis of the ring; this has been confirmed by Gentile [1988] for both random and ordered isomers of Cl-PPTA, which exhibit the onset of a nematic phase at concentrations comparable to those for PPTA of similar molecular weight and consistent with Flory's lattice model for the mesophase transition in rodlike molecules. However, such rotational freedom implies an increase in the entropy of the isolated chains in the solution phase over that for the unsubstituted PPTA.

Figure 7.4: Potential energy contour for N -[2,6-dichlorophenyl]-benzamide as a function of the ring torsions $\phi_{1}$ and $\phi_{3}$.



Figure 7.5: Potential energy contour for 2,6-dichlorobenzanilide as a function of the ring torsions $\phi_{1}$ and $\phi_{3}$.



## 7.4

### 7.4.1 <br> Approaches to Identification of Minima

At this point we focus on the packing of chains of 2,6-DiCl-PPTA. Two approaches recommend themselves to the identification of the set of structures of local minimum energy likely to occur in nature. The first is a repetition of the process described in Section 4.4 for the initial identification of energy minima in PPTA; this involves a multistep procedure of scanning and searching through the multivariate parameter space. However, equally consistent with the concept of a first principles approach to structure elucidation and considerably less time-consuming is a process of minimization which assumes that the modified structures are likely to exhibit packed geometries which are perturbations of the geometries of the unsubstituted polymer. This assumption obviates the lengthy grid scanning phase and recommends a finite set of minimizations within predetermined regions of parameter space. Its major weakness lies in the implication that the modification does not involve any significant new interactions which would stabilize new, unrelated geometrics. In order to determine the comparability of the two methods and in particular the comprehensiveness of the latter, both procedures were followed in the analysis of the HHTT and HT isomers of $2,6-$ DiCl-PPTA.

### 7.4.2 Packing of Head-to-Head, Tail-to-Tail Isomers

CRU Definition: .For the HHTT isomer, two possible representations of the CRU were initially considered. The first and simplest assumes that the dimer constitutes the basic conformational unit, analogous to PPTA. In a head-tail-tail-head sequence, consisting of four monomers defining two oppositely-directed dimers, the set of conformation-determining angles and torsions are identical for both dimer subunits, regardless of their respective orientations; in this description, the number of independent internal degrees of freedom required to describe the chain remains fourteen. The validity of such a simplification was checked by considering a second representation in which the tetramer constitutes the basic conformational unit, consisting of the complete head-tail-tail-head sequence, or two independently conformable dimers, for a total of twenty-eight internal degrees of freedom

## Location and Description of HHTT Energy Minima: Employing the dimer CRU representation and

 following the same procedure as described in Chapter 4 with a cut-off criterion of 0.2 for identification of those polymorphs of interest, the analysis suggests a list of four viable crystal polymorphs for the fully ordered HHTT isomer of 2,6 -DiCl-PPTA. These are presented in Table 7.3 in the same format as was used for the polymorphs of the unsubstituted PPTA; orthographic projections are provided in Figure $7.6(\mathrm{a})$-(d) for ease of visualization. Of these, only Structure \#2 derives from one of the eight originalTable 7.3
Multichain energy minimization results: structural parameters for four most probable unit cells of 2,6-dichloro-PPTA; head-to-head, tail-to-tail polymorphs

| Structure ID $\rightarrow$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Structural |  |  |  |  |
| $\begin{aligned} & \text { Parameters } \\ & \downarrow \end{aligned}$ |  |  |  |  |
| a ( $\AA$ ) | 8.54 | 8.88 | 8.53 | 5.46 |
| b ( $\AA$ ) | 6.11 | 5.79 | 6.15 | 4.58 |
| c ( $\AA$ ) | 25.4 | 26.2 | 26.2 | 26.2 |
| $\alpha$ (degrees) | 107 | 90 | 96 | 107 |
| $\beta$ (degrees) | 87 | 90 | 89 | 88 |
| $\gamma$ (degrees) | 98 | 82 | 97 | 101 |
| Chain Locations | $[0,0]$ | [0,0] | [0,0] | [0,0] |
| (ab projection) | [ 122,12 ] | [ $122,2 / 2]$ | [ 212,26 ] | - |
| Chain Setting Angle (degrees) |  |  |  |  |
| relative to the bc-facet | -20 | 23 | -16 | 22 |
|  | -20 | -157 | -16 | 22 |
| Inter-sheet Translation ( $\AA$ ) | 0.0 | 6.6 | 13.1 | 0.0 |
| Helix Twist $\Theta_{\mathrm{h}}$ (degrees) | 1 | 0 | 1 | 0 |
| Monomer Phenylene |  |  |  |  |
| Ring Rotation (degrees) |  |  |  |  |
| diacid ring $\phi_{5}$ : diamide ring $\phi_{6}$ : | $\begin{array}{r} 5 \\ 61 \end{array}$ | -6 57 | 63 | -6 |
| Cohesive Energy (kcal/mol) | 35.8 | 35.4 | 35.4 | 35.1 |
| Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.63 | 1.53 | 1.69 | 1.69 |
| Parallelepiped Description: |  |  |  |  |
| A ( $\AA$ ) | 4.89 | 5.63 | 4.95 | 5.46 |
| B ( $\AA$ ) | 5.59 | 4.95 | 5.55 | 4.58 |
| $\mathrm{C}(\AA)$ | 25.4 | 26.2 | 26.2 | 26.2 |
| $\tau$ (degrees) | 102 | 90 | 94 | 107 |
| $v$ (degrees) | 98 | 90 | 93 | 88 |
| $\zeta$ (degrees) | 110 | 66 | 108 | 102 |
| $\omega_{1}$ (degrees) | 61 | 107 | 78 | 83 |
| $\omega_{2}$ (degrees) | 61 | -73 | 78 | 83 |
| $\mathrm{f}_{2}$ | 0.0 | 0.50 | 1.0 | 0.0 |

Figure 7.6: Orthographic perspectives of the four primary structures for the HHTT isomers of 2,6-DiCl-PPTA suggested by simulation. The sets (a) through (d) correspond to the structures labelled 1 through 4, in order, in Table 7.3. Left: z-axis perspective; Middle: perspective perpendicular to the bc-plane; Right: perspective perpendicular to the previous two perspectives.


Figure 7.6: Orthographic perspectives of the four primary structures for the HHTT isomers of 2,6-DiCl-PPTA suggested by simulation (continued).
(c)
2. 6-C1-PPTA (HHTT) Structure \#3

(d)


PPTA structures. Significantly, this second structure derives from PPTA Structure \#3, the calculated allomorph which bears the greatest likeness to the experimentally-determined Modification I.

Trends in Structure: The density of the HHTT isomers is on the average roughly $10 \%$ greater than that of PPTA. Accounting for the higher molecular weight of the substituted polymer, this corresponds to an average increase in unit cell volume of $15 \%$ to $20 \%$. One may immediately note that again there is no significant deviation from the fundamental extended conformational behavior of the chain; the helix twist deviates insignificantly from zero degrees. Successive phenylene rings may be either similarly or oppositely rotated with respect to the amide bond plane. However, the trend is for the ring rotation angles to remain within a relatively narrow range of five to six degrees or fifty-seven to sixty-three degrees for the phenylene ring of the diacid monomer or the phenylene ring of the diamine monomer, respectively. This suggests two striking implications for the substituted polymer: First, both rings tend to fall within a very narrow angular range in the packed structure, an unexpected result given the magnitude of the intermolecular forces and the range of low energy angular fluctuation available to the rings suggested by single chain simulations. Second, the unsubstituted ring of the diacid monomer has been forced considerably and consistently away from its minimum energy position for the isolated chain. From these observations, one would conclude that the packing of chains of the HHTT isomer significantly alters and restricts the conformation space of the chain itself; qualitatively, this implies a significanty different environment for both the potential energy and conformational entropy of the chain in the solid state from that in the isolated (e.g. dilute solution) state. Consistent with the PPTA case, the 2,6-DiCl-PPTA HHTT packed chain geometries exhibit a strong correlation between chain seuing angles, with the implication again that the chains associate into sheets held together by hydrogen bonds between chains. Within the unit cell, the predominance of hydrogen bonding interactions ensures the maintenance of translational register between amide moieties and phenylene moieties, respectively. Between chains in successive sheets, the substituted phenylene ring may be in register with another substituted ring of like orientation (HHTT structures \#1 and \#4), with a substituted ring of opposite orientation (HHTT Strucure \#3), or with an unsubstituted ring (HHTT Structure \#2). That is, all three types of register are represented in these four equipotential crystal structures. There does not appear to be any justification for the supposition that the introduction of chlorines on the rings creates or prohibits any specific structural juxtaposition between the modified moieties.

Trends in Potential Energy: The total cohesive energy, expressed on a per dimer basis, is slightly lower than that found for PPTA. Predicted densities range from as low as $1.53 \mathrm{~g} / \mathrm{cm}^{3}$ up to $1.69 \mathrm{~g} / \mathrm{cm}^{3}$. For the set of four HHTT isomer packing geometries, we obtain:

$$
\mathrm{E}_{\mathrm{coh}}=\quad 35.4 \pm 0.5 \mathrm{kcal} / \mathrm{mol}
$$

$$
\text { or } \quad \begin{aligned}
& 13.3\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}<\delta<14.0\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2} \\
& 27.1 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}<\delta<28.5 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}
\end{aligned}
$$

These values are roughly $90 \%$ of the corresponding values predicted for PPTA. Despite the similarities in the nature of interactions between the unsubstituted polymer and the HHTT isomer, the total cohesive forces are measurably smaller in the latter. The implication is that the introduction of ring substituents compels an expansion of the lattice which in turn leads to a weakening of the net cohesive forces. This reduced cohesiveness provides the main argument for the greater solubility of the HHTT isomer over that of the unsubstituted polymer. The new Hildebrand solubility parameter estimates are comparable to those reported for aliphatic polyamides.

These trends are further confirmed by the decomposition of the chain-chain bonding energies (on a per dimer basis) listed in Table 7.4, this time in terms of chains related by crystallographic location; the hydrogen bonded sheets, by earlier convention, lie in the bc-facet in each case. The moiety-specific amide-to-amide electrostatic interaction accounts for $40 \%$ to $65 \%$ of the total interaction energy between neighboring chains within the bc-facel, corresponding to hydrogen bond energies of 1.5 to $3.0 \mathrm{kcal} / \mathrm{mole}$, in perfect accord with the hydrogen bond energies expressed in PPTA. By contrast, interactions between neighboring chains in the ac-facet exhibit very low total interaction energies. The "diagonal" interactions listed in Table 7.4 refer to the largest interchain interaction energies which cross from one sheet to another. With the exception of Structure \#4, intersheet interactions are comparable to intrasheet interactions, with magnitudes of 6.5 to $7.3 \mathrm{kcal} /$ mole. We conclude that the introduction of substitution on the ring appears to alter the magnitude, but not the distribution or moiety-to-moiety specificity, of interaction energy between chains from that seen in the PPTA case.

The Tetramer CRU: The second chain representation, involving four rings in the CRU, was adopted for repetition of the minimizations initiated at the original PPTA structures. The resulting minima realized, on the average, an increase of $1.5 \mathrm{kcal} /$ mole of dimer in cohesive energy. Densities were comparable to those found using the CRU representation with only two rings. The greater freedom allowed to the chain conformation resulted in new twists in the helical structure of the molecule, usually consisting at most of compensating rotations of $\pm 10^{\circ}$ to $15^{\circ}$ about the virtual bonds spanning the phenylene rings of the two diamine monomer components; such compensating rotations result in a net helix twist close to zero degrees, but with every two successive amide dipoles along the chain contour lying in a plane $15^{\circ}$ rotated from the preceding pair. However, this slight variation did not justify the large increase in computational resources required to accommodate the more demanding tetramer calculations. Packing geometry remained essentially unchanged. For this reason, further exploration using this representation was not decmed necessary.

Table 7.4
Chain-to-chain bonding energies for 2,6-dichloro-PPTA; head-to-head, tail-to-tail polymorphs (all values in $\mathrm{kcal} / \mathrm{mole}$ of dimer)

Cell ID
Cell type ${ }^{2}$
ac-facet:
total 12-6
total electrostatic
amide-amide elec.
phenyl-phenyl elec. total
bc-face::

| total 12-6 | 2.0 | 3.1 | 1.8 | 3.4 |
| :--- | :--- | :--- | :--- | :--- |
| toal electrostatic | 4.5 | 4.2 | 4.6 | 6.0 |
| amide-amide elec. | 3.4 | 2.8 | 3.5 | 6.0 |
| phenyl-phenyl elec. | 0.4 | 0.8 | 0.4 | 2.0 |
| total | 6.5 | 7.3 | 6.4 | 9.4 |

diagonal:

| total 12-6 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| total electrostatic | 4.7 | 3.8 | 2.8 | 1.4 |
| amide-amide elec. | 2.8 | 3.2 | 4.3 | 1.6 |
| phenyl-phenyl elec. | 1.0 | 1.8 | 2.7 | 0.7 |
| total | 6.7 | 7.0 | 2.0 | 0.2 |
|  |  |  |  | 3.1 |

${ }^{2}$ Cell type identifies chain intersections with the ab-facet: fc indicates chains at $[0,0]$ and [ $2 / 2,26]$; ec indicates chains at $[0,0]$ and $[2 / 2,0] ; p$ refers to a primitive cell with chains only at the comers. Intrasheet interactions are consistently in the bc-facet. Closest intersheet interactions are diagonal in the fc cell type but are in the ac-facet in the ec cell type; in the p cell type both types of interactions are intersheet in nature.

### 7.4.3

Packing of Head-to-Tail Isomers

Chain Directionality: With the HT isomer, the problems of appropriate chain representation are slightly different from those of the HHTT isomer. In this case, it is again reasonable and sufficient to consider only the dimer CRU, as was suggested in the case of the unsubstituted polymer, since there is no distinction of orientation between dimer units along the contour of the chain. However, the HT isomer possesses directionality as a whole. For this reason, one must now consider, instead of multiple representations of the internal chain description, multiple non-interconvertible intermolecular relationships between chain directions during packing. Specifically, we consider two cases: one where all chains pack with parallel directionality, and one where chains may pack with antiparallel directionality. These two cases will be designated as $\mathrm{HT}(\mathrm{P})$ and $\mathrm{HT}(\mathrm{A})$ variations, respectively. In the latter case, one could as well assume that the two distinguishable chain types (e.g. "up" and "down" chains) have different CRU's, leading again to a doubling of the number of internal degrees of freedom; for simplicity, we have considered only one common CRU for both chain types in all cases. However, we have considered the possibility for the "up" and "down" chain types to be mirror images of one another, effected during simulation by reflecting the parent chain of "up" type through the yz-plane prior to replication and reorientation of a "down" chain.

Location and Definition of HT Energy Minima: Because of the directionality of the HT chain and the consequent escalation in number of variations of packing to be considered, the final list of viable structures is somewhat longer than for the HHTT isomer, even for the same conditions of search and selection. Table 7.5(a) lists the four $\mathrm{HT}(\mathrm{P})$ polymorphs, identified through the scan and search procedure; orthographic projections are presented in Figure 7.7(a)-(d). As in the case of the HHTT isomers, the minimizations were also performed based on initial conditions defined by the original eight PPTA structures. Here, as many as three of the four $\mathrm{HT}(\mathrm{P})$ allomorphs, structures \#1, \#2, and \#3, derive from the original PPTA structures. Furthermore, these three allomorphs derive in particular from PPTA structures \#3, \#4 and \#7; again, the first of these is that which most closely resembles the experimentallydetermined Modification I. This continued recurrence and persistence lends further credence to the accuracy of the PPTA analysis reported in Chapter 4 and hints at the relative importance of this basic geometric unit, in both the substituted and unsubstituted polymers. The HT(P) structures \#2 and \#3 bear the same relation to \#1 as was reported earlier for the corresponding structures in PPTA.

Table 7.5(b) shows the features of the $\mathrm{HT}(\mathrm{A})$ isomers, with corresponding orthographic projections in Figure 7.7(e)-(j). These must be considered in common with the HT(P) versions, since these are all structures based on the same chemistry and could be obtained separately or mixed in the actual bulk solid state. Of the structures listed in Table 7.5(b), \#5 through \#7 contain "up" and "down" chains that are related only by rigid body translation and rotation; \#8 through \#10 contain chains which are reflected through the yz-plane prior to translation and rotation operations. In this set of antiparallel structures, one

Table 7.5a
Multichain energy minimization results: structural parameters for four most probable unit cells of 2,6-dichloro-PPTA: parallel head-to-tail polymorphs

| Structure ID $\rightarrow$ | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Structural |  |  |  |  |
| Parameters |  |  |  |  |
| a ( $\AA$ ) | 8.68 | 8.97 | 8.04 | 4.96 |
| b ( $\AA$ ) | 5.88 | 5.60 | 5.96 | 5.52 |
| c ( $\AA$ ) | 13.1 | 13.1 | 13.1 | 13.0 |
| $\alpha$ (degrees) | 90 | 87 | 87 | 91 |
| $\beta$ (degrees) | 85 | 81 | 82 | 92 |
| $\gamma$ (degrees) | 81 | 80 | 92 | 72 |
| Chain Locations | [0,0] | [0,0] | [0,0] | [0,0] |
| (ab projection) | [1/2, $1 / 2]$ | [12/2,2] | [12/2,0] |  |
| Chain Setting Angle (degrees) relative to the bc-facet | 25 | 24 | 22 | 32 |
|  | -154 | 24 | 20 | 32 |
| Inter-sheet |  |  |  |  |
| Translation ( $\AA$ ) | 6.6 | 6.6 | 6.6 | 0.0 |
| Helix Twist $\Theta_{\mathrm{h}}$ (degrees) | 3 | 2 | 8 | 1 |
| Monomer Phenylene |  |  |  |  |
| Ring Rotation (degrees) |  |  |  |  |
| diacid ring $\phi_{5}$ : | -17 | -23 | 18 | 5 |
| diamide ring $\phi_{6}$ : | 59 | 54 | 60 | 65 |
| Cohesive Energy (kcal/mol) | 36.7 | 35.2 | 36.1 | 34.9 |
| Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.55 | 1.59 | 1.65 | 1.51 |
| Parallelepiped |  |  |  |  |
| Description: |  |  |  |  |
| A (A) | 5.60 | 5.70 | 7.04 | 5.51 |
| B ( $\AA$ ) | 4.85 | 4.88 | 4.02 | 4.96 |
| C ( $\AA$ ) | 13.1 | 13.1 | 13.1 | 13.0 |
| $\tau$ (degrees) | 86 | 84 | 82 | 92 |
| $v$ (degrees) | 86 | 81 | 83 | 91 |
| $\zeta$ (degrees) | 68 | 63 | 58 | 72 |
| $\omega_{1}$ (degrees) | 105 | 107 | 112 | 162 |
| $\omega_{2}$ (degrees) | -76 | 108 | 114 | 162 |
| $\mathrm{f}_{2}$ | 0.50 | 0.50 | 0.50 | 0.0 |

Table 7.5b
Multichain energy minimization results: structural parameters for most probable unit cells of 2,6-dichloro-PPTA: antiparallel head-to-tail polymorphs

| Structure ID $\rightarrow$ | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Structural |  |  |  |  |  |  |
| Parameters |  |  |  |  |  |  |
| a $(\AA)$ | 8.73 | 9.37 | 7.86 | 10.06 | 8.78 | 8.87 |
| b (A) | 5.68 | 5.35 | 5.89 | 4.65 | 5.85 | 6.58 |
| c ( $\AA$ ) | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 |
| $\alpha$ (degrees) | 92 | 98 | 92 | 92 | 90 | 83 |
| $\beta$ (degrees) | 81 | 84 | 86 | 82 | 91 | 75 |
| $\gamma$ (degrees) | 91 | 82 | 90 | 83 | 82 | 115 |
| Chain Locations | $[0,0]$ | [0,0] | $[0,0]$ | $[0,0]$ | $[0,0]$ | $[0,0]$ |
| (ab projection) | [ 3,20$]$ | [ 22,0$]$ | [ 2,0 ] | [ $22,2,2]$ | [ 212,12 ] | [ $\quad 2,0]$ |
| Chain Setting |  |  |  |  |  |  |
| Angles (degrees), relative to the | 21 | 35 | 31 | 18 | 24 | 10 |
| bc-facet | 17 | 74 | 20 | 11 | -20 | 14 |
| Inter-sheet |  |  |  |  |  |  |
| Translation ( $\AA$ ) | 5.3 | 0.4 | 6.4 | 2.9 | 6.7 | 0.5 |
| Helix Twist $\Theta_{h}$ (degrees) | 1 | 4 | 17 | 2 | 1 | 6 |
| Monomer Phenylene |  |  |  |  |  |  |
| Ring Rotation (degrees) |  |  |  |  |  | -29 |
| diamide ring $\phi_{6}$ : | -27 60 | -17 61 | 62 | -35 | 57 | 62 |
| Cohesive Energy (kcal/mol) | 37.1 | 37.1 | 36.0 | 37.3 | 35.8 | 35.3 |
| Density ( $\mathrm{g} / \mathrm{cm}^{3}$ ) | 1.59 | 1.59 | 1.69 | 1.71 | 1.53 | 1.58 |
| Parallelepiped |  |  |  |  |  |  |
| Description: |  |  |  |  |  |  |
| A (A) | 7.13 | 7.57 | 3.92 | 5.81 | 5.60 | 6.19 |
| B ( $\AA$ ) | 4.37 | 4.68 | 7.10 | 5.26 | 4.93 | 4.43 |
| C ( $\AA$ ) | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 | 13.1 |
| $\tau$ (degrees) | 92 | 98 | 95 | 96 | 90 | 83 |
| $v$ (degrees) | 84 | 91 | 92 | 89 | 91 | 69 |
| $\zeta$ (degrees) | 53 | 45 | 56 | 49 | 67 | 74 |
| $\omega_{1}$ (degrees) | 120 | 99 | 104 | 115 | 113 | 168 |
| $\omega_{2}$ (degrees) | 168 | 58 -0.19 | 159 | -72 -0.42 | 108 0.29 | -42 -0.08 |
| $\mathrm{f}_{2}$ | 0.25 | -0.19 | 0.26 | -0.42 | 0.29 | -0.08 |

Figure 7.7: Orthographic perspectives of the ten primary structures for the HT isomers of $2,6-\mathrm{DiCl}-$ PPTA suggested by simulation. The sets (a) through (j) correspond to the structures labelled 1 through 10, in order, in Tables 7.5 a and 7.5 b . Left: z -axis perspective; Middle: perspective perpendicular to the bc-plane; Right: perspective perpendicular to the previous two perspectives.
(a)
2, 6-C1-PPTA (HT) Structure \#1

2. 6-C1-PPTA (HT) Structure \#2.
(b)


Figure 7.7: Orthographic perspectives of the ten primary structures for the HT isomers of 2,6-DiClPPTA suggested by simulation (continued).
(c)
2. 6-C1-PPTA (HT) Structure \#3

(d) 2, 6-C1-PPTA (HT) Structure \#4


Figure 7.7: Orthographic perspectives of the ten primary structures for the HT isomers of 2,6-DiClPPTA suggested by simulation (continued).
(e)

> 2. 6-C1-PPTA (HT) Structure \#5

(f)
2, 6-C1-PPTA (HT) Structure \#6.


Figure 7.7: Orthographic perspectives of the ten primary structures for the HT isomers of 2,6-DiClPPTA suggested by simulation (continued).
(g)

> 2. 6-C1-PPTA (HT) Structure \#7

(h)


Figure 7.7: Orthographic perspectives of the ten primary structures for the HT isomers of $2,6-\mathrm{DiCl}-$ PPTA suggested by simulation (continued).
(i)
2, 6-Cl-PPTA (HT) Structure \#9

(j)

may note that, with the restriction that all chains possess the same conformation, the correlation of chain orientations is significantly altered in order to accommodate steric packing of the large ring moieties. Thus in Structure \#5 the identification of hydrogen-bonded sheets is still possible, but with the amide dipoles in one sheet rotated with respect to those in successive sheets, while in structures \#6 and \#7 the hydrogen bonding between chains in every second (i.e. antiparallel) sheet completely breaks down. It was this observation that led to the suggestion that antiparallel chains based on the same CRU may realize better packing as mirror images; in structures \#8 through \#10 are again realized the common pattern of correlated chain orientation and hydrogen bonded sheet formation. HT(A) Structures \#8 and \#9 bear a marked resemblance to the HT(P) Structure \#2, and in turn to the PPTA Structure \#4, differing primarily in the axial register between shects. HT(A) Structure \#10 exhibits the basic features found also in HT(P) Structure \#4 and PPTA Structure \#7.

Trends in Potential Energy: Considering these ten HT packing geometries in unison, we observe that the cohesive energy lies intermediate between that of the HHTT isomer geometries and that reported for the PPTA geometries:

$$
\mathrm{E}_{\mathrm{coh}}=36.2 \pm 1.3 \mathrm{kcal} / \mathrm{mol}
$$

The best cohesive energies are found in the $\mathrm{HT}(\mathrm{A})$ simulations, although within the margin of accuracy of the energy calculations both parallel and antiparallel packings may be considered feasible. The cohesive energy is roughly $93 \%$ of that for PPTA. Calculated densities range from $1.51 \mathrm{~g} / \mathrm{cm}^{3}$ up to $1.71 \mathrm{~g} / \mathrm{cm}^{3}$. The predicted Hildebrand solubility parameters are:

$$
\begin{aligned}
& 13.1\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}<\delta<14.4\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2} \\
& 26.7 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}<\delta<29.4 \times 10^{3}\left[\mathrm{~J} / \mathrm{m}^{3}\right]^{1 / 2}
\end{aligned}
$$

The decomposition of the interaction energy into its chain-to-chain contributions is presented in Tables 7.6(a) and (b), in the same manner as before, with identifiable "sheets" parallel to the be crystallographic facet. Total energies of interaction within sheets range from 6 to $10 \mathrm{kcal} / \mathrm{mol}$. Interactions between sheets vary from 4.5 to $10 \mathrm{kcal} / \mathrm{mol}$ depending upon the orientation of interaction. For intrasheet interactions, the trends in hydrogen-bond energies and percentage contributions from each set of moietymoiety interactions are the same as in the HHTT cases. Contributions vary considerably for interactions between shects. In both structure and distribution of cohesive forces, the HT isomers exhibit behaviors intermediate between those of PPTA and those of the substituted HHTT counterpart. The HT structures

## Table 7.6a

Chain-to-chain bonding energies for 2,6-dichloro-PPTA: parallel head-to-tail polymorphs (all values in $\mathrm{kcal} /$ mole of dimer).

| Cell ID | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Cell type ${ }^{2}$ ac-facet: | fc | fc | ec | p |
| total 12-6 | 0.3 | 0.2 | 6.1 | 2.9 |
| total electrostatic | 0.0 | 0.0 | 3.4 | 0.0 |
| amide-amide elec. | 0.0 | 0.0 | 2.8 | -0.9 |
| phenyl-phenyl elec. | 0.0 | 0.0 | -0.7 | 0.0 |
| total | 0.3 | 0.2 | 9.5 | 2.9 |
| bc-facet: |  |  |  |  |
| total 12-6 | 3.0 | 3.6 | 3.1 | 3.2 |
| total electrostatic | 4.5 | 4.4 | 4.7 | 4.0 |
| amide-amide elec. | 3.0 | 3.2 | 3.5 | 2.7 |
| phenyl-phenyl elec. | 0.7 | 0.7 | 0.7 | 0.9 |
| total | 7.5 | 8.0 | 7.8 | 7.2 |
| diagonal: |  |  |  |  |
| total 12-6 | 4.0 | 4.0 | 1.4 | 1.8 |
| total electrostatic | 3.5 | 1.6 | -0.8 | 4.5 |
| amide-amide elec. | 1.6 | 1.2 | -0.2 | 3.4 |
| phenyl-phenyl elec. | 0.2 | 0.6 | -0.4 | 0.4 |
| total | 7.5 | 5.6 | 0.6 | 6.3 |

${ }^{\text {a }}$ Cell type identifies chain intersections with the ab-facet: fc indicates chains at $[0,0]$ and [ $12 / 2,2 / 2$ ]; ec indicates chains at $[0,0]$ and $\left[\frac{122,0]}{}\right.$; $p$ refers to a primitive cell with chains only at the corners. Intrasheet interactions are consistently in the bc-facet. Closest intersheet interactions are diagonal in the fc cell type but are in the ac-facet in the ec cell type; in the p cell type both types of interactions are intersheet in nature.

Table 7.6b
Chain-to-chain bonding energies for 2,6-dichloro-PPTA: antiparallel head-to-tail polymorphs (all values in $\mathrm{kcal} /$ mole of dimer)

Cell ID
Cell type ${ }^{\text {a }}$
ac-face:
total 12-6
total electrostatic
amide-amide elec.
phenyl-phenyl elec. total
bc-facet:
total 12-6
total electrostatic amide-amide elec. phenyl-phenyl elec. total
diagonal:
total 12-6
total electrostatic amide-amide elec. phenyl-phenyl elec. total

| 5 | 6 | 7 | 8 | 9 | 10 |
| ---: | :---: | :---: | :---: | :---: | :---: |
| ec | ec | ec | fc | fc | ec |
|  |  |  |  |  |  |
| 5.4 | 4.6 | 3.0 | 0.1 | 2.4 | 4.6 |
| 4.6 | 4.1 | 3.8 | 0.0 | 0.0 | 2.2 |
| 2.7 | 1.4 | 2.8 | 0.0 | 0.0 | -1.0 |
| 1.0 | 1.5 | 0.6 | 0.0 | 0.0 | 1.0 |
| 10.0 | 8.7 | 6.8 | 0.1 | 2.4 | 6.8 |


| 4.1 | 3.8 | 4.3 | 6.5 | 4.4 | 4.4 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 3.7 | 2.6 | 3.9 | 6.8 | 3.0 | 2.4 |
| 0.4 | 0.9 | 0.4 | 0.9 | 0.7 | 0.2 |
| 7.5 | 7.0 | 9.8 | 9.9 | 7.5 | 6.1 |

$-\quad 0.1$

| 0.9 | 0.6 | 0.1 | 3.6 | 3.9 | 2.7 |
| ---: | ---: | ---: | ---: | ---: | ---: |
| -0.8 | -0.2 | 0.2 | 0.9 | 3.1 | 4.9 |
| -0.4 | -0.2 | 0.2 | 0.7 | 1.7 | 2.6 |
| -0.3 | 0.0 | 0.0 | 1.0 | 0.2 | 0.6 |
| 0.1 | 0.4 | 0.3 | 4.5 | 7.0 | 7.6 |

${ }^{2}$ Cell type identifies chain intersections with the ab-facet: fc indicates chains at $[0,0]$ and $[1 / 2,1 / 2]$; ec indicates chains at $[0,0]$ and $[1 / 2,0]$; $p$ refers to a primitive cell with chains only at the comers. Intrasheet interactions are consistently in the bc-facet. Closest intersheet interactions are diagonal in the fc cell type but are in the ac-facet in the ec cell type; in the p cell type both types of interactions are intersheet in nature.
are more clearly perturbations of the unsubstituted polymer, and exhibit a multiplicity of forms. Both parallel and antiparallel arrangements of neighboring chains realize satisfactory cohesive energies. Again, lattice expansion appears to be the primary factor leading to reductions in total cohesive energy relative to the unsubstituted polymer. One may observe that the antiparallel structures are, on the average, roughly $3 / 4 \mathrm{kcal} / \mathrm{mol}$ more stable that their parallel-oriented counterparts; however, the long range total dipole moments of the asymmetric chains appear to have relatively little effect at the local scale, where structure definition and stability are primarily determined. We may assert that, in this case of substitution for PPTA, monatomic substitution on every second ring of the chain is sufficient to force expansion of the lattice, but the low density of substitution and the specific features of ring orientation within the most probable structures, namely the misalignment of ring edges and the greater or lesser isolation of each modified ring from other modified rings, preclude any specific interaction, steric or otherwise, which would alter properties anisotropically.

In the absence of experimental data for direct comparison, and in consideration of the multiplicity of polymorphs for the substiuted isomers, we shall not autempt to discuss at this point the calculated $x$-ray fiber diffraction pauerns for the dichloro-substituted polymers. Such predictions have been made, however, and both tabulated intensities and recreated diffractograms are provided in Appendix E for reference purposes.

### 7.5 Prediction of Fiber Mechanical Properties

By analogy to the study described in Chapter 5 for the prediction of the clastic behavior of PPTA, we may simulate small scale deformation of the dichloro-substituted polymers in order to predict their mechanical behavior. For this purpose, we concentrate on five of the predicted geometries from the total of fourteen viable candidates. These were selected as follows: The first two, HHTT Structure \#2 and HT(P) Structure \#1, are the modified geometries for the two isomers which are analogous to the PPTA Structure \#3 considered in depth in Chapter 5; these were selected for direct comparison to that important PPTA polymorph. The third, HHTT Structure \#3, represents a packing geometry whercin the chains have been shifted to bring chlorine substituents on neighboring chains into closest proximity (see Figure 7.6(c)), where they are likely to have the greatest effect on mechanical properties. This structure was selected for comparison to the HHTT Structure \#2. The last two, HT(A) Structures \#5 and \#6, describe packing geometries analogous to PPTA Structure \#5 and the related Modification II; the two differ primarily in the axial register of the chlorine substituents between neighboring chains, with the substituents in closest proximity in $\mathrm{HT}(\mathrm{A})$ Structure \#6 (see Figure 7.7(f)). A comparison of mechanical behavior for these two geometries should describe the largest difference to be observed in the HT isomers attributable to the placement of the monatomic ring substituents.

The modification of the chain description, the method of calculation, and the magnitudes of imposed deformations, were identical to those employed for PPTA and described in depth in Chapter 5. However, rather than generate the full twenty-one element compliance matrix, we concentrated on those elements necessary to determine the Voigt model fiber elastic constants. These are presented in Table 7.7(a) and 7.7(b) for the HHTT isomers and the HT isomers, respectively. In all five cases, the extensional moduli are of the same order of magnitude, ranging from 230 GPa up to 290 GPa . While this range still exceeds the estimated tensile modulus of the isolated chain, it is somewhat lower than the 290 to 370 GPa range of upper limits anticipated for the PPTA structures, indicative of a slight weakening of the load bearing capacity along the fiber axis. Based on the previous discussion, one may conjecture that this slight reduction is due primarily to changes in intermolecular packing interactions. Transverse moduli in the substituted structures HHTT \#2 and HT(P) \#1 tend to be $50 \%$ to $100 \%$ greater than the corresponding PPTA \#3 transverse modulus. However, the alternate HHTT \#3 allomorph involving proximate chlorines again exhibits a transverse modulus comparable to the PPTA value. A comparison of the HHTT structures \#5 and \#6, which differ primarily in the register between chlorine-modified rings, reveals almost identical sets of elastic constants for these two geometries. In all cases, the torsional moduli are consistently $2 \%$ to $5 \%$ of the extensional modulus; the absolute values of these moduli are comparable to those for PPTA. It appears that the introduction of chlorines as substituents on every other ring of the chain should have only a moderate effect on the mechanical behavior of the resulting fibers. The simulations suggest that the most likely effect is a slight reduction in the extensional modulus, but little or no change in the torsional modulus. The orientation of the dichloro-substituted rings in each structure appears to preclude the operation of the bulky chlorines as barriers to chain slip. As a result, one should expect to observe litule change in shear deformation behavior from that seen previously in PPTA.

## Table 7.7a

2,6-DiCl-PPTA HHTT theoretical fiber elastic constants (GPa): Structures \#2 and \#3

|  | Voigt | $\begin{array}{l}\text { Structure \#2 } \\ \text { Predicted } \\ \text { Range }\end{array}$ |  | Voigt |
| :--- | :--- | :--- | :--- | :--- | \(\left.\begin{array}{c}Structure \#3 <br>

Predicted <br>
Range^{2}\end{array}\right]\)
${ }^{2}$ after correction for entropic contributions.

Table 7.7b
2,6-DiCl-PPTA HT theoretical fiber elastic constants (GPa): Structures \#1, \#5 and \#6

|  | Structure \#1 |  | Structure \#5 |  | Structure \#6 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Voigt | Predicted Range ${ }^{\text {a }}$ | Voigt | Predicted Range ${ }^{\text {a }}$ | Voigt | Predicted Range ${ }^{2}$ |
| Extens. $\operatorname{Mod}\left(\mathrm{E}_{3}\right)$ | 232 | 225-183 | 275 | 268-217 | 252 | 244-199 |
| Transv. Mod ( $\mathrm{E}_{1}, \mathrm{E}_{2}$ ) | 31 | 30-24 | 32 | 31-25 | 31 | 30-24 |
| Tors. Mod ( $\mathrm{G}_{1}, \mathrm{G}_{2}$ ) | 9.4 | 9.1-7.4 | 7.3 | 7.1-5.8 | 8.0 | 7.8-6.3 |
| Extens. Poisson's Ratio ( $v_{13}, v_{23}$ ) | 0.29 |  | 0.08 | - | 0.27 | - |
| Transv. Poisson's Ratio ( $v_{12}, v_{21}$ ) | 0.36 | - | 0.39 | - | 0.49 | - |

a after correction for entropic contributions.

## 8. CONCLUSION

### 8.1 Summary

As was proposed at the outset of this work, we have attempted to take advantage of current computational capabilities and unique chemistries in order to address the issues of structure and property prediction relevant to the growing field of engineered polymeric materials. The materials selected for study were the para-linked aramids based on poly(p-phenylene terephthalamide) oriented in the solid state. The contents of this study may be summarized as follows:
(1) A complete methodology was developed for the geometric representation of a general ordered polymer structure composed of many polymer chains aligned along their helical or pseudohelical axes, the generation of requisite parameters for force field potentials, and exploration of the entire multivariate phase space in order to identify the structures of minimum potential energy in a densely packed solid state. The methods used are readily applicable to other helix-forming polymers through a simple change of repeat unit generation and force field calculation routines. The importance of helix imperfection and long range interatomic interactions have been addressed quantitatively. The method is based on atomicscale considerations and requires no adjustable parameters
(2) Select thermodynamic and phenomenological characteristics, in particular the solid state cohesive energy, Hildebrand solubility parameter, and scatering of x-rays from fiber specimens, were generated deterministically from simulated structures. These predictive methods do not require or presume the availability of experimental data for the material in question, except to the extent that appropriate force field parameterizations may be deduced from data of low molecular weight analogues or other calculational procedures.
(3) The model was validated through application to the study of PPTA, a well-known stiff chain aramid. The results suggest the viability of a plurality of pseudocrystalline forms for PPTA. Predictions of cohesive energy density ( $39 \pm 1 \mathrm{kcal} /$ mole ), Hildebrand solubility parameter ( 15.3 [ $\left.\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}<\delta<16.1$ $\left[\mathrm{cal} / \mathrm{cm}^{3}\right]^{1 / 2}$ ), and x -ray scattering are in accord with expectations based on available data in the literature. Through a detailed analysis of the polymorphic forms, we are able to distinguish, prioritize, and quantify the features which lead to the development of ordered structure, or lack thereof.
(4) The model description was modified to enable deformation "experiments" of select pseudocrystalline polymorphs. The experiments account only for the potential energy contribution to the elastic stiffnesses, which is shown to overestimate moduli by $3 \%$ to $27 \%$. We have calculated the complete twenty-one independent element elastic stiffness and compliance tensors for the general triclinic crystal. We have also predicted the complete set of conventional fiber clastic constants in the Voigt and Reuss limits. Finally, we have proposed a novel approach based on our knowledge of the complete material stiffness tensor for the single crystal to identify the orientation of least resistance to shear.
(5) The design of a fiber spinning apparatus suitable for the production of quality fibers from small (less than one gram) quantities of polymer is presented. The equipment described is suitable for temperaturecontrolled dry-jet wet spinning (or alternatively, of course, simple wet spinning) of polymer solutions in corrosive solvents up to $100^{\circ} \mathrm{C}$, with postspin fiber treatments at temperatures up to $500^{\circ} \mathrm{C}$. Representative fibers of PPTA were generated, along with data concerning die swell, draw ratio, and annealing effects in this process. The resulting fibers were analyzed by IR spectroscopy and WAXS for comparison with the model predictions.
(6) The simulation methodology was extended to the study of the effects of controlled atomic variation on solid state structure, cohesive energy density and solubility parameter, x-ray scattering, and mechanical property evaluation. Atomic substitution and the order of substitution along the chain contour were addressed using the regular head-to-head, tail-to-tail and head-to-tail isomers of the polyterephthalamides of 2,6 -dichlorophenylene diamine. The results confirm the reduced cohesiveness of these modified pseudocrystalline structures, leading to the observed improvement of solubility in common solvents. Mechanical property predictions suggest that such substitutions should play a relatively minor role in the mechanical behavior of these fibers.

We conclude from this work that the methods of atomistic modelling are applicable to the prediction of detailed structure even in cases where the chemical complexity is considerable. The wealth of thermodynamic, phenomenological, and mechanical property information which may be anticipated by this method makes it especially attractive. Current limitations to applicability result primarily from the numerical difficulties associated with the highly convoluted potential energy hypersurface characteristic of the dense solid state and the limited accuracy in predicting small strain moduli.

### 8.2 Recommendations for Future Work

Based on our experience developed during the current work, a number of recommendations for improvements and extensions of the methods and ideas described herein may be proposed.
(1) Improvement of the efficiency of the search procedure for minima. To our knowledge, there is no theory or method currently available to guarantee a priori the identification of a global minimum; one may only identify a set of local minima, under the assumption that by means of a sufficiently comprehensive search procedure the global minimum is contained therein. However, it would be desireable to develop an "intelligent" search algorithm which recognizes the constraints of the independent parameter set and takes advantage of these to step past irrelevant local minima.
(2) The concepts of paracrystallinity are relevant to the study of crystallinity in polymers; it would be feasible to reformulate the structure generation procedures $w$ incorporate distribution functions which describe distortions of the second kind. It may be possible to redescribe the polymorphism presented in this work in terms of a reduced number of structurally significant parameter distributions employing internal coordinates rather than Cartesian coordinates.
(3) It would be of further value to take advantage of the software and minimum potential energy packing geometries generated within this work to explore specific structure transformation processes such as ring flip motion, chain rotation about or translation parallel to the molecular axis, and deviation from perfect co-alignment of molecular axes. Such analyses could produce information about intermediate activated state geometries and energy barriers to transformation of particular interest to spectroscopic analyses.
(4) The method may be applied to more varied chemistries. The simulation procedure is especially valuable as a guide to proposing modified chemistries in that it can describe the structural changes that occur upon the introduction of chemical changes. Here we have only considered the most minor form of structural modification, the introduction of a larger monatomic species at regular points along the chain contour. Further interesting changes would include the study of random substitutions along the chain contour, the substitution of functional moieties which would force the introduction of a new intermolecular interaction, or introduction into the chain backbone of specific moieties which either reduce the stiffness of the main chain or create a fixed alteration of the chain helicity, which would break down the hydrogen bonded sheets and force the formation of a new set of intermolecular interactions. Specific examples, some of which are currently under pursuit, are the study of the effects of fixed chain twist, such as that found in MPIA, the characterization of solid state crosslinking in a system containing fusible rings, and the analysis of dopant retention in a crystalline polymer.
(5) The experimental apparatus and WAXS scattering procedures are readily applicable to the production of fibers of new materials; a program of experimental work would go hand in hand with the theoretical analysis of proposed new materials suggested by Point 4. Of immediate relevance to the results presented here would be the processing of regularly substituted dichloro-PPTA's into fiber for x-ray and structure analysis and comparison to simulation results. The means currently exist to produce the HHTT isomer with an order parameter of $s<0.1$; this synthesis and processing effort is currently in progress and will be presented in a later publication. The dipole moment of the repeat unit of the 2,6-DiCl-PPTA isomer is on the order of 2.8 Debye, based on our AM1 calculations for p -( 2,6 -dichlorophenylene) diformamide. The simulation results presented here suggest the possibility for an electrically active polymer fiber containing parallel arrays of regular HT isomers of 2,6-DiCl-PPTA; Synthetic routes to the HT isomer of this polymer having order parameter $\mathrm{s}>0.9$ are currently under investigation;
(6) A final ambitious recommendation would be a study of the linkage between structure at the atomic scale addressed within this work and structure at the macromorphological scale tenfold or a hundredfold larger than the current simulation scale. Such an undertaking would require the consideration of the extent to which local structure determines long range structure and how kinetic processing parameters influence this determination. The fruits of such a study would include a more accurate predictive method for real mechanical properties and could lend new insight into the formation of macrofeatures in the bulk, such as microfibrils or periodic defect structures.

## NOTATION

| a,b,c | real space continuation vectors for the crystallographic unit cell. | ( $\AA$ ) |
| :---: | :---: | :---: |
| $\mathbf{a}^{*}, \mathbf{b}^{*}, c^{*}$ | reciprocal space continuation vectors for the crystallographic unit cell. | $\left(\AA^{-1}\right)$ |
| A,B,C | latice continuation vectors in the parallelopiped description. | (A) |
| A,B,C | magnitudes of the lattice continuation vectors $\mathbf{A}, \mathbf{B}$, and $\mathbf{C}$. | (A) |
| A | Helmholiz free energy. | (kcal/mol) |
| $\mathrm{A}_{\mathrm{ij}, \mathrm{n}}$ | the generalized potential function multiplier |  |
| $\mathrm{A}_{\mathrm{H}}, \mathrm{B}_{\mathrm{H}}$ | helix rotation matrix and translation vector, as defined by Miyazawa et al., describing the helix conformation. |  |
| $\mathrm{C}_{\text {LMNK }}$ | an element of the material stiffness tensor $\mathbf{C}$. | (GPa) |
| $\mathrm{C}_{\varepsilon}$ | Specific heat of a solid at constant strain. | ( $\mathrm{kcal} / \mathrm{mol} / \mathrm{K}$ ) |
| $\mathrm{d}_{\mathrm{h}}$ | helix pitch, or projected length of one repeat unit on the helix axis. | ( $\AA$ ) |
| $\mathrm{d}_{\mathrm{ch}}$ | diameter of the molecular "cylinder". | ( $\AA$ ) |
| $\mathrm{d}_{\mathrm{ij}}$ | the interatomic distance between i and j | ( A $^{\text {) }}$ |
| $\mathrm{d}^{*}$ | the critical crossover distance for the distance-dependent dielectric constant | ( $\AA$ ) |
| $\mathrm{D}_{\text {B }}$ | the bulk dielectric constant. |  |
| $\mathrm{D}_{\mathrm{R}}$ | the in-line draw-down ratio during fiber spinning, defined as the ratio of the linear velocity of the fiber at collection over the linear velocity at the spinnerette. |  |
| $\mathrm{D}_{\text {crifice }}$ | the diameter of the spinnereute orifice. | ( $\mu \mathrm{m}$ ) |
| $\mathrm{e}_{\text {LM }}$ | an element of the engineering strain tensor e. |  |
| $\mathrm{E}_{\mathrm{i}}$ | elastic modulus. | (GPa) |
| $\mathrm{E}_{\text {coh }}$ | cohesive energy of a multichain geometry. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| Eexpl | the total internal energy calculated in the explicit domain. | (kcal/mol) |
| $E_{l}^{\text {inra }}$ | the intramolecular internal energy of chain $l$. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| $\mathrm{E}_{l, l}{ }^{\text {inter }}$ | the intermolecular internal energy between chains $l$ and $l$. | (kcal/mol) |


| $E^{\text {latice }}$ | the total internal energy calculated in the outer lattice domain, per repeat unit. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| :---: | :---: | :---: |
| $E^{\text {cot }}$ | the total internal energy of a structure, per repeat unit. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| $E^{\text {unit }}$ | the internal energy of the explicit domain, per repeat unit. | $(\mathrm{kcal} / \mathrm{mol})$ |
| $\mathrm{f}_{\mathrm{i}}$ | the fractional translation in the lattice of chain i along its axis. |  |
| $\mathrm{f}_{\mathrm{j}}(\mathrm{S})$ | the atomic scattering factor for atom type $j$, which is a function of the length of the scattering vector $S$. |  |
| F | the helix translation vector, akin to $\mathbf{B}_{\mathbf{H}}$, employed in imposing helical symmetry. | ( $\AA$ ) |
| $\mathrm{F}(\mathbf{S}), \mathrm{F}_{\mathrm{cr}}(\mathbf{S})$ | general and crystallographic x-ray structure factor based on atom positions. |  |
| $\mathrm{g}_{\mathrm{a}}, \mathrm{g}_{\mathrm{b}}$ | induction parameters, in reaction kinetics model for constitutional isomerism. |  |
| $\mathrm{G}_{\mathrm{i}}$ | shear modulus. | (GPa) |
| G(S) | crystallographic x-ray structure factor based on unit cell periodicity. |  |
| G | real space metric tensor, defined by $\mathbf{H}^{\mathrm{T}} \mathbf{H}$. |  |
| h | Planck constant ( $0.662 \times 10^{-33} \mathrm{~J}$-s). | (J•S) |
| h,k,l | Miller indices denoting the sets of crystallographic planes. |  |
| H | coordinate transformation matrix for Cartesian to fractional coordinate systems. |  |
| $\mathrm{I}_{\text {e }}$ | the intensity of $x$-rays scattered by an electron. |  |
| $\mathrm{I}(\mathbf{S}), \mathrm{I}_{\mathrm{cr}}(\mathbf{S})$ | calculated general and crystallographic intensities of scautered x-ray radiation, as functions of the scattering vector $\mathbf{S}$. |  |
| $\mathrm{I}_{\text {obs }}(\mathbf{S})$ | observed $x$-ray intensity. |  |
| $1_{\mathrm{ru}}$ | the projected "length" of the repeat unit. | ( $\AA$ ) |
| $l_{\text {i }}$ | the length of bond $i$. | ( $\AA$ ) |
| $\mathrm{J}_{1}, \mathrm{~J}_{2}, \mathrm{~J}_{3}$ | first, second, and third tensorial invariants. |  |
| k | Boltzmann constant (8.31 J/mole-K). | (J/mole $\cdot \mathrm{K}$ ) |
| L(hkl) | Lorentz Factor for x-ray intensity correction for diffraction from a set of planes hkl. |  |
| $L_{\text {orifice }}$ | the capillary length of the spinnerette orifice. | ( $\mu \mathrm{m}$ ) |
| $\mathrm{M}_{\mathbf{w}}$ | weight average molecular weight. | (g/mol) |


| $\mathrm{M}_{\mathrm{ru}}$ | molecular weight of the repeat unit. | (g/mol) |
| :---: | :---: | :---: |
| $\mathrm{N}_{\text {A }}$ | Avogadro Number $\left(6.022 \times 10^{23}\right)$. | $\left(\mathrm{mol}^{-1}\right)$ |
| $\mathrm{N}_{\mathrm{CRU}}$ | the number of atoms in a conformational repeat unit (CRU). |  |
| $\mathrm{N}_{\mathrm{c}}$ | the number of independent chains on the lattice. |  |
| $\mathrm{N}_{\mathrm{i}}$ | the number of crystallographically unique atoms. |  |
| $\mathrm{N}_{\mathrm{p}}$ | the number of additive components of the total nonbonded potential description. |  |
| $\mathrm{N}_{\mathrm{r}}$ | the number of repeat units in the chain. |  |
| 0 | octopole moment of an atomic moiety. | $\left(\AA^{3}\right)$ |
| P | Pressure. | (Pa) |
| $\mathrm{P}(\mathrm{hk} l)$ | the polarization factor for $x$-ray intensity correction for diffraction from a set of planes hkl. |  |
| $\mathrm{q}_{\mathrm{i}}$ | the partial atomic charge of atom i. |  |
| Q | canonical partition function in statistical mechanics. |  |
| r | relative reactivity of ends of an asymmetric monomer, used in reaction kinetics model for constitutional isomerism. |  |
| $\mathrm{r}_{\mathrm{i}}$ | the van der Waals radius of atom i . | ( $\AA$ ) |
| $\mathrm{R}_{\mathrm{k}}, \mathrm{r}_{\mathrm{i}}$ | location vectors describing unit cell and atomic positions in an arbitrary frame of reference. | ( $\AA$ ) |
| $s$ | the order parameter to describe constitutional isomerism. |  |
| $\mathbf{S}_{\mathbf{O}}, \mathbf{S}$ | the paths of incident and scattered radiation, respectively. |  |
| S | the scattering vector for incident radiation along the path $\mathbf{s}_{\mathbf{o}}$ and radiation scattered along the path s . |  |
| S | entropy. | ( $\mathrm{kcal} / \mathrm{mol} / \mathrm{K}$ ) |
| $S_{\text {LMNK }}$ | an element of the material compliance tensor $\mathbf{S}$. | $\left(\mathrm{GPa}^{-1}\right)$ |
| $\mathrm{T}_{\mathrm{H}}$ | the rotation matrix describing alignment of the helix axis of the polymer chain along the Cartesian z-axis. |  |
| T | temperature. | ( ${ }^{\circ} \mathrm{C}, \mathrm{K}$ ) |
| u | displacement vector of atomic coordinates describing deformation. | ( $\AA$ ) |
| U | internal energy. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| $v_{e}$ | linear extrusion velocity of a filament during spinning. | $(\mathrm{m} / \mathrm{min})$ |


| $\mathrm{v}_{\mathrm{w}}$ | linear collection, or wind-up, velocity of filament during spinning. | (m/min) |
| :---: | :---: | :---: |
| $\mathrm{V}, \mathrm{V}_{\mathrm{o}}$ | volume (eg. unit cell). | $\left(\AA^{3}\right)$ |
| $\mathrm{V}_{\mathrm{n}}$ | the generalized distance dependent potential function. | (kcal/mol) |
| $V_{p}^{*}$ | the critical volume fraction for anisotropic phase formation. |  |
| $\mathbf{W}_{\mathrm{j}}$ | the rotation matrix used to orient the polymer chain j about its center of gravity by means of the appropriate Euler angle rotations $\chi_{j}, \Psi_{j}$, and $\omega_{j}$. |  |
| $\hat{\mathbf{W}}$ | the helix twist matrix employed in imposing helical symmetry in the lattice summation for potential energy. |  |
| $x$ | aspect ratio of a rigid rod molecule. |  |
| $\mathbf{x}$ | the eigenvector matrix for the radius of gyration tensor $\mathbf{H}$ |  |
| $\mathrm{X}_{\mathrm{n}}$ | degree of conversion during polymerizaton. |  |
| X | the vector of Cartesian coordinates for atoms. | ( $\AA$ ) |
| $\alpha, \beta, \gamma$ | real space interaxial angles for the crystallographic unit cell. | (degrees) |
| $\alpha_{P Q}$ | an element of the thermal expansion tensor $\alpha$. | $(\mathrm{K})^{-1}$ |
| $\beta$ | the azimuthal angle of scattered radiation in x-ray scattering. | (degrees) |
| $\gamma_{w}$ | shear rate al the wall of the spinnerette orifice in fiber spinning | $\left(\sec ^{-1}\right)$ |
| $\gamma_{L M}$ | an element of the Grüneisen tensor $\gamma$. |  |
| $\zeta, v, \tau$ | interaxial angles describing lattice continuation in the parallelepiped description. | (degrees) |
| $\delta$ | Hildebrandt solubility parameter. | $\left(\mathrm{cal} / \mathrm{cm}^{3}\right)^{1 / 2}$ |
| $\varepsilon_{\text {i }}$ | the Lennard-Jones potential well depth parameter for atom i. | ( $\mathrm{kcal} / \mathrm{mol}$ ) |
| $\varepsilon_{0}$ | the permittivity of a vacuum. | $\left(\mathrm{kcal} \cdot \AA / \mathrm{C}^{2}\right)^{-1}$ |
| $\varepsilon_{\text {LM }}$ | an element of the material strain tensor $\varepsilon$. |  |
| $\eta_{\text {inh }}$ | inherent viscosity, approximated at 0.5 g polymer per dl. | ( $\mathrm{dl} / \mathrm{g}$ ) |
| $\theta$ | the scattering angle in $x$-ray diffraction, also referred to as half the Bragg angle $2 \theta$. | (degrees) |
| $\vartheta_{i}$ | the complement of the bond angle at atom $\mathrm{i}-1$, used in describing the location of atom i . | (degrees) |
| $\Theta_{\mathrm{h}}$ | helix twist, or the amount of rotation about the helix axis occuring over one repeat unit. | (degrees) |
| $\Theta_{\mathrm{E}}, \Theta_{\mathrm{D}}$ | Einstein and Debye temperatures, defined as $\nu_{E} h / k$ or $\nu_{D} h / k$. | (K) |


| $\theta$ | quadrupole moment of an atomic moiety. | ( $\AA^{2}$ ) |
| :---: | :---: | :---: |
| ${ }^{k_{T}}$ | isothermal compressibility. | ( $\mathrm{kcal} / \mathrm{mol} \cdot \mathrm{m}^{3}$ ) |
| $\lambda$ | wavelength of radiation. | ( $\AA$ ) |
| $\lambda_{2}, \lambda_{b}, \lambda_{c}$ | summation coefficients along the lattice continuation vectors. |  |
| $\lambda_{1}, \lambda_{2}, \lambda_{3}$ | eigenvalues of the radius of gyration tensor $\Xi$ of the finite chain segment. | (A) |
| $\mu$ | elementary dipole moment of an atomic moiety. | ( $\AA$ ) |
| $\mu_{s, i}{ }^{2} \mu_{s}{ }^{2}$ | isotropic mean square displacement of atoms due to thermal vibration, for an atom i and universally for all atoms. | ( $\mathrm{cm}^{2}$ ) |
| $\mu \mathrm{t}$ | sample absorption in an x-ray beam, as the product of the linear absorption coefficient $\mu$ and the sample thickness $t$ |  |
| $v$ | vibrational frequency. | $(\mathrm{cm})^{-1}$ |
| $v_{E}$ | characteristic frequency in the Einstein Approximation. | $(\mathrm{cm})^{-1}$ |
| $v_{\text {D }}$ | characteristic frequency in the Debye Approximation. | $(\mathrm{cm})^{-1}$ |
| $v_{i j}$ | Poisson's ratio. |  |
| $\Xi$ | the radius of gyration tensor for the finite chain segment. | (A) |
| $\rho$ | density. | (g/cm ${ }^{3}$ ) |
| $\rho_{\text {h }}$ | helix radius, defined at an arbitrary point on the helix. | (A) |
| $\sigma_{\text {LM }}$ | an element of the material stress tensor $\sigma$. | (GPa) |
| $\tau_{w}$ | shear stress at the wall of the spinnerete orifice in fiber spinning. | (Pa) |
| $\phi_{i}$ | the torsion angle between atoms i-2 and i-1, used in describing the location of atom i. | (dcgrees) |
| $\chi_{i}, \psi_{i}, \omega_{i}$ | the Euler angles orienting chain i on the latice. | (degrees) |
| $\mathrm{X}, \Psi, \Omega$ | the Euler angles describing the crystallite orientation | (degrees) |

## ABBREVIATIONS

| AM1 | Austin Model 1. AMPAC version available through the Department of <br> Chemistry, Indiana University, Bloomington, Indiana, 47405, QCPE Program |
| :--- | :--- |
| \#506. |  |

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## APPENDIX A: Helix Representation of Chain Conformation

The helix described entirely in terms of internal bond coordinates may be fairly simple or severely convoluted within a single helical repeat distance and there is no constraint on the rationality of the helix. We derive the corresponding helix rotation matrix $\mathbf{A}_{\mathbf{H}}$ and translation vector $\mathbf{B}_{\mathbf{H}}$ of Miyazawa and Sugeta [Miyazawa, 1961; Sugeta and Miyazawa, 1967] from our earlier generator matrices:

$$
\begin{align*}
\mathbf{A}_{\mathbf{H}} & =\mathbf{Y}_{\mathrm{k}} \mathbf{Y}_{\mathrm{j}}^{-1}  \tag{A.1}\\
\mathbf{B}_{\mathbf{H}} & =\mathbf{Y}_{\mathrm{k}}\left[\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right] \quad-\quad \mathbf{Y}_{\mathrm{j}}\left[\begin{array}{l}
0 \\
0 \\
0 \\
1
\end{array}\right] \tag{A.2}
\end{align*}
$$

$\begin{array}{llll}\text { where: } & \mathbf{Y}_{j} & = & \mathbf{A}_{\mid} \mathbf{A}_{2} \ldots \mathbf{A}_{j-1} \\ & \mathbf{Y}_{k} & = & \mathbf{A}_{\mid} \mathbf{A}_{2} \ldots \mathbf{A}_{j-1} \mathbf{A}_{j} \ldots \mathbf{A}_{\mathrm{k}-1} \\ \text { such that: } & \mathbf{X}_{\mathrm{k}}{ }^{1} & = & \mathbf{A}_{\mathbf{H}} \mathbf{X}_{\mathrm{j}}{ }^{1}+\mathbf{B}_{\mathbf{H}} \\ & \mathbf{X}_{j}{ }^{1} & = & \mathbf{A}_{\mathbf{H}}{ }^{\mathrm{T}} \mathbf{X}_{\mathrm{k}}{ }^{1}-\mathbf{B}_{\mathbf{H}}\end{array}$
$j$ and $k$ are the corresponding atoms in two successive conformational repeat units $m$ and $m+1$, and $j$ is greater than or equal to 3 .

From $\mathbf{A}_{\mathbf{H}}$ and $\mathbf{B}_{\mathbf{H}}$ one can calculate the three helix parameters for a uniform, nondegenerate helix and the rotation matrix required to align the helix axis with the z -axis.

$$
\begin{align*}
& \mathrm{C}=\left(\mathbf{A}_{\mathbf{H}}^{\mathrm{T}}-\mathbf{E}_{4}\right) \mathbf{B}_{\mathrm{H}}  \tag{A.7}\\
& \mathrm{C}^{\prime}=\left(\mathbf{E}_{4}-\mathbf{A}_{\mathbf{H}}\right) \mathbf{B}_{\mathrm{H}} \quad\left(\mathbf{E}_{\mathbf{S}} \text { is the identity matrix of order } \mathbf{S}\right)  \tag{A.8}\\
& \mathrm{B}=\left(\mathbf{B}_{\mathbf{H}}^{\mathrm{T}} \mathbf{B}_{\mathbf{H}}\right)^{\text {i/2 }}  \tag{A.9}\\
& \mathrm{C}=\left(\mathbf{C}^{\mathrm{T}} \mathbf{C}\right)^{i / 2} \tag{A.10}
\end{align*}
$$

from which it follows that: (refer to Figure A.1)

Figure A.1: Definition of the vectors $\mathbf{B}, \mathbf{B}^{\prime}, \mathbf{B}{ }^{\prime \prime}, \mathbf{C}$, and $\mathbf{C}^{\prime}$ and the helical parameters $\rho$ and $\Theta$ (after Sugeta and Miyazawa [1967]).


$$
\begin{array}{ll}
\cos \Theta_{h} & =\left(\mathbf{C}^{\mathrm{T}} \mathbf{C}^{\prime}\right) / \mathrm{C}^{2} \\
\rho_{\mathrm{h}} & =1 / 2 \mathrm{C} /\left(1-\cos \Theta_{\mathrm{h}}\right) \\
d_{\mathrm{h}} & =\mathbf{B}_{\mathbf{H}}\left(\mathbf{C}^{\mathrm{T}} \mathbf{C}^{\prime}\right) /\left(\mathrm{C}^{2} \sin \Theta\right) \tag{A.13}
\end{array}
$$

and then:

$$
\begin{equation*}
\mathbf{X}_{i}^{h} \quad=\quad T_{H} X_{i}^{1}+\mathbf{D} \tag{A.14}
\end{equation*}
$$

where

$$
\begin{align*}
\mathbf{D} & =\left[\begin{array}{lll}
\rho_{\mathrm{h}} & 0 & 0
\end{array}\right]  \tag{A.15}\\
\mathbf{T}_{\mathbf{H}} & =\left[\begin{array}{lll}
\mathrm{e}_{1}(\mathrm{x}) & \mathrm{e}_{1}(\mathrm{y}) & \mathrm{e}_{1}(\mathrm{z}) \\
\mathrm{e}_{2}(\mathrm{x}) & \mathrm{e}_{2}(\mathrm{y}) & \mathrm{e}_{2}(\mathrm{z}) \\
\mathrm{e}_{3}(\mathrm{x}) & \mathrm{e}_{3}(\mathrm{y}) & \mathrm{e}_{3}(\mathrm{z})
\end{array}\right]  \tag{A.16}\\
\mathbf{e}_{1} & =\mathbf{C} / \mathrm{C}  \tag{A.17}\\
\mathbf{e}_{2} & =\mathbf{e}_{3}^{\mathrm{T}} \mathbf{e}_{1}  \tag{A.18}\\
\mathbf{e}_{3} & =\left(\mathbf{C}^{\mathrm{T}} \mathbf{C}^{\prime}\right) / C^{2} \sin \Theta_{\mathrm{h}} \tag{A.19}
\end{align*}
$$

For the special case where $\Theta=180^{\circ}$, one must substitute an alternative calculation for the pitch and for a section of the orientation matrix:

$$
\begin{array}{ll}
\mathbf{B}_{\mathbf{H}}^{*} & =\left(\mathbf{A}_{\mathbf{H}}+\mathbf{E}\right) \mathbf{B}_{\mathbf{H}} \\
\mathrm{d}_{\mathrm{h}} & =1 / 2\left|\mathbf{B}_{\mathbf{H}}^{*}\right| \\
\mathbf{e}_{3} & =1 / 2 \mathbf{B}_{\mathbf{H}}^{*} / \mathrm{d}_{\mathrm{h}} \tag{A.22}
\end{array}
$$

However, in applications involving finite chain lengths, this helical coordinate conversion encounters a singularity in radius $\rho_{h}$ for the degenerate straight rod helix ( $\Theta_{h}=0, C=0$ ). This singularity produces a discontinuity in helix alignment, whereby conformations infinitesimally displaced from the straight rod become sections of infinitely large spirals, with the result that alignment is rotated by $90^{\circ}$ in space. To surmount this numerical problem, we have chosen to substitute in this vicinity of parameter space a coordinate transformation which approximates the helix axis with the major axis of the radius of gyration tensor for the finite chain segment, with a transition between the two transformations at a finite value of
$\Theta_{h}$. The radius of gyration tensor for the finite chain segment is calculated using only the coordinates of the "backbone" atoms, i.e. those carbon and nitrogen atoms which lie on the backbone bond (and virtual bond) contour of the chain. For a segment containing $N_{i}$ such atoms, the elements of the radius of gyration tensor are calculated simply by:

$$
\begin{equation*}
\Xi_{j k}=\sum_{i=1}^{N_{i}}\left(e_{j} \cdot r_{i}\right)\left(e_{k} \cdot r_{i}\right) / N_{i}-\sum_{i=1}^{N_{i}}\left(e_{k} \cdot r_{i}\right) / N_{i} \quad \sum_{i=1}^{N_{i}}\left(e_{k} \cdot r_{i}\right) / N_{i} \tag{A.23}
\end{equation*}
$$

Here, $\mathbf{e}_{\mathrm{j}}$ is a coordinate axis unit vector, where j and k take the values 1,2 , and 3 , corresponding to the Cartesian $x-, y$ - and $z$-axes of the real space coordinate system. Diagonalization of $\Xi$ by successive Jacobi transformations yields the eigenvalues $\left(\lambda_{1}, \lambda_{2}\right.$, and $\left.\lambda_{3}\right)$ of this tensor, which correspond to the magnitudes of the principle axes of the chain segment, and the matrix $\mathbf{x}$ of eigenvectors, which correspond to the unit vectors of the principle axes. The principle axes are sorted such that $\lambda_{3}>\lambda_{1}>\lambda_{2}$ to yield the rotation matrix $\mathbf{T}_{\mathbf{H}}=\mathbf{x}^{T}$. The helix is centered and aligned such that its primary axis lies along the Cartesian $z$-axis and the secondary axis lies in the Cartesian xz-plane:

$$
\begin{equation*}
\mathbf{X}_{\mathrm{i}}=\mathbf{T}_{\mathbf{H}}\left(\mathbf{r}_{\mathrm{i}}-\sum_{\mathrm{i}=1}^{\mathbf{N}_{\mathrm{i}}}\left(\mathbf{e} \cdot \mathbf{r}_{\mathrm{i}}\right) / \mathbf{N}_{\mathrm{i}}\right) \tag{A.24}
\end{equation*}
$$

This approximation allows the model to handle all cases of chain conformation in a numerically continuous manner for finite chains segments of sufficient length. However, it may readily be seen that the above transformation of coordinates is dependent upon $\mathrm{N}_{\mathrm{i}}$, or the length of the chain segment; this dependence becomes less sensitive as $N_{i}$ increases for segments which are rodlike in conformation, since $\lambda_{3}$ increases faster than either $\lambda_{1}$ or $\lambda_{2}$. It does not, of course, ensure perfect alignment of the near-rod helices along their true propagation axes except in the limit of extremely large $\mathrm{N}_{\mathrm{i}}$.

Individual chains are characterized by their local Cartesian frames of reference, defined by the helix axis (or the longest principle axis of the radius of gyration) along the $z$-axis, the first amide bond plane (or the second longest principle axis) in the xz-plane, and the $y$-axis chosen to form a right-handed orthonormal coordinate system. The chain may then be oriented in general by application of three rotations, $\chi, \psi$, and $\omega$, about the $z$-axis (counterclockwise), the $y^{\prime}$-axis (clockwise), and the $z$ "-axis (counterclockwise), respectively, as shown in Figure A.2. The necessary rotation matrix $\mathbf{W}$ by this definition is as follows:

$$
\begin{align*}
& \mathbf{W} \quad=\quad \mathbf{W}_{\omega} \quad \mathbf{W}_{\psi} \mathbf{W}_{\chi}  \tag{A.25}\\
& \mathbf{w}_{\chi}=\left[\begin{array}{ccc}
\cos \chi & -\sin \chi & 0 \\
\sin \chi & \cos \chi & 0 \\
0 & 0 & 1
\end{array}\right]  \tag{A.26}\\
& \mathbf{W}_{\psi}=\left[\begin{array}{ccc} 
& \\
\cos \psi & 0 & -\sin \psi \\
0 & 1 & 0 \\
\sin \psi & 0 & \cos \psi
\end{array}\right]  \tag{A.27}\\
& \mathbf{W}_{\omega}=\left[\begin{array}{ccc}
\cos \omega & -\sin \omega & 0 \\
\sin \omega & \cos \omega & 0 \\
0 & 0 & 1
\end{array}\right] \tag{A.28}
\end{align*}
$$

which, when multiplied as shown in Equation A.25, yield the rotation matrix given by Equation 3.10.

Figure A.2: Definition of Euler angles for orientation of a chain about its center of gravity.


## APPENDIX B: Summation Representation of a Periodic Structure and Convergence of Terms

Lattice Summation In general, for a force field of the type described below, with $p$ terms of inverse power $m(p)$ in distance:

$$
\begin{equation*}
E \quad=\quad \sum_{p} A_{p}(1 / d)^{-m(p)} \tag{B.1}
\end{equation*}
$$

one may write the total energy of interaction of one atom $i$ with an entire latice of $N_{j}$ atoms of type $j$ in the form:

$$
\begin{equation*}
E_{i=1, j}=\quad \sum_{j} \sum_{p} A_{p}\left(1 / d_{i=1, j}\right)^{-m(p)} \tag{B.2}
\end{equation*}
$$

In general, for a periodic lattice described in three dimensions by the periodicity vectors $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$, the interatomic distance may be described by the lattice point ( $\mathrm{h}, \mathrm{k}, l$ ) and the vector $\mathbf{d}_{\mathbf{o}}$ locating the j lattice relative to the atom $\mathrm{i}=1$ :

$$
\begin{align*}
\mathbf{d}_{1, j}^{2} & =\left(\mathbf{d}_{1, \mathrm{j}}{ }^{\mathrm{T}} \mathbf{d}_{1, \mathrm{j}}\right)  \tag{B.3}\\
& =\left(\mathrm{ha}+\mathrm{k} \mathbf{b}+l \mathbf{c}+\mathbf{d}_{\mathbf{o}}\right)^{\mathrm{T}}\left(\mathrm{ha}+\mathrm{kb}+l \mathbf{c}+\mathbf{d}_{\mathbf{o}}\right)  \tag{B.4}\\
& =\left(\mathbf{d}_{1, \mathrm{hk} l}^{\mathrm{T}} \mathbf{d}_{1, \mathrm{hk} l}\right) \tag{B.5}
\end{align*}
$$

which yields, in Cartesian coordinates, for the interaction of the lattice centered on atom $j=1$ with the atom $i=1$ :

$$
\begin{equation*}
\mathrm{E}_{1,1}=\sum_{\mathrm{p}} \sum_{\mathrm{h}} \sum_{\mathrm{k}} \sum_{l} \mathrm{~A}_{1,1, \mathrm{p}}\left(\mathbf{d}_{1, \mathrm{hk} l} \mathrm{~T}_{1, \mathrm{hk} l}\right)^{-\mathrm{m}(\mathrm{p}) / 2} \tag{B.6}
\end{equation*}
$$

This may be generalized for a structural unit consisting of $\mathrm{N}_{\mathrm{i}}$ atoms interacting with an entire composite lattice having $\mathrm{N}_{\mathrm{j}}$ atoms in its simplest subunit. The total energy per unit of $\mathrm{N}_{\mathrm{i}}$ atoms is given by Equation B.7; this equation is essentially the lattice summation in the form presented in the text in Equations 3.16, 3.17, and 3.18.

$$
\begin{equation*}
\mathrm{E}_{\text {lattice }}=\quad \sum_{\mathrm{p}} \sum_{\mathrm{i}}^{\mathrm{N}_{\mathrm{i}}} \sum_{\mathrm{j}}^{\mathbf{N}_{\mathrm{j}}}\left[\sum _ { \mathrm { h } } \sum _ { \mathrm { k } } \sum _ { l } \mathrm { A } _ { \mathrm { i } , \mathrm { j } , \mathrm { p } } \left(\mathbf{d}_{1, \mathrm{hk} l \text { of } j} \mathrm{~T}_{\left.\left.\mathbf{d}_{1, \mathrm{hk} l \text { of } j}\right)^{-\mathrm{m}(\mathrm{p}) / 2}\right]}^{]}\right.\right. \tag{B.7}
\end{equation*}
$$

Transformation from Cartesian to lattice coordinate system This may be converted into a lattice-based coordinate system (i.e. "fractional coordinates") by means of the transformation matrix $\mathbf{H}$

$$
\begin{align*}
& \mathbf{X}_{\mathbf{c}}=\mathbf{H} \mathbf{x}_{\mathbf{r}} \quad \text { where } \mathbf{x}_{\mathbf{r}} \text { refers to the lattice coordinates and } \mathbf{X}_{\mathbf{c}} \text { refers }  \tag{B.8}\\
& \text { to the Cartesian coordinates. }
\end{align*}
$$

$$
\mathbf{H}=\left[\begin{array}{lll}
a \sin \beta & b(\cos \gamma-\cos \alpha \cos \beta) / \sin \beta & 0  \tag{B.9}\\
0 & b\left(\sin ^{2} \beta-\cos ^{2} \gamma-\cos ^{2} \alpha+2 \cos \alpha \cos \beta \cos \gamma\right)^{1 / 2} & 0 \\
a \cos \beta & b \cos \alpha & c
\end{array}\right]
$$

$\mathbf{H}$ has been defined with the crystallographic system oriented within the Cartesian system as described in Chapter 3, i.e. $\mathbf{c}$ parallels the Cartesian z-axis, a lies in the xz-plane, and $\mathbf{b}$ completes a right-handed system. Then we may write:

$$
\begin{equation*}
\mathbf{d}_{\mathrm{ij}} \quad=\cdots \mathbf{X}_{\mathrm{cj}}-\mathbf{X}_{\mathrm{c} i}=\mathbf{H}\left(\mathbf{x}_{\mathrm{f}_{\mathrm{j}}}-\mathbf{x}_{\mathrm{fi}}\right) \tag{B.10}
\end{equation*}
$$

In general, the atoms reside at fractional coordinates described by $\mathbf{x}_{\mathrm{f}}=\left[\begin{array}{lll}\mathrm{h} & \mathrm{k} & l\end{array}\right]$ in the lattice-based coordinate system. We redefine the vector $\mathbf{d}_{\mathbf{o}}$ in lattice space as:

$$
\mathbf{d}_{\mathbf{o}}=\mathbf{H} \mathbf{d}_{\mathrm{fO}} ; \quad \mathbf{d}_{\mathrm{fO}}=\left[\begin{array}{ll}
\mathrm{h}^{\prime} & \mathrm{k}^{\prime} l^{\prime} \tag{B.11}
\end{array}\right]
$$

For ease of expression we define the following condensed notation:

$$
\begin{equation*}
\mathbf{x}_{\mathrm{hk} l / \text { of } \mathrm{j}} \equiv\left(\mathbf{x}_{\mathrm{fj}}-\mathbf{x}_{\mathrm{fi}}\right)=\left[\mathrm{h}^{\prime}+\mathrm{h}^{\prime} \mathrm{k}+\mathrm{k}^{\prime} l+l^{\prime}\right] \tag{B.12}
\end{equation*}
$$

The summation may be rewritten as:

$$
\begin{equation*}
E_{\text {latice }}=\quad \sum_{\mathbf{p}} \sum_{i} \sum_{i} \sum_{j}\left[\sum_{h} \sum_{k} \sum_{l} A_{i_{i, j p}}\left(\mathbf{x}_{h k l} \text { of }{ }^{\mathrm{T}} \mathbf{G x}_{\mathrm{hk} l \text { of } j}\right)^{-\mathrm{m}(\mathrm{p}) / 2}\right] \tag{B.13}
\end{equation*}
$$

$\mathbf{G}$ is the real space metric tensor:

$$
\mathbf{G}=\mathbf{H}^{\mathrm{T}} \mathbf{H}=\left[\begin{array}{lll}
a^{2} & a b(\cos \gamma) & a c(\cos \beta)  \tag{B.14}\\
a b(\cos \gamma) & b^{2} & b c(\cos \alpha) \\
a c(\cos \beta) & b c(\cos \alpha) & c^{2}
\end{array}\right]
$$

Convergence of Lattice Terms To demonstrate convergence, we write $\mathrm{E}_{\text {latice }}$ as a summation of $\Delta \mathrm{E}$ 's from successive triclinic "shells" of atoms $\mathrm{N}_{\mathrm{j}}$

$$
\begin{equation*}
\mathrm{E}_{\text {laticc }}=\Delta \mathrm{E}_{\text {sheli } 1}+\Delta \mathrm{E}_{\text {shell } 2}+\Delta \mathrm{E}_{\text {shell } 3}+\ldots+\Delta \mathrm{E}_{\text {shell } n}+\ldots \tag{B.15}
\end{equation*}
$$

We demonstrate convergence of $\mathrm{E}_{\text {latice }}$ as n goes to $\infty$ by scaling arguments. For large $\mathrm{h}, \mathrm{k}, \mathrm{l}:$

$$
\begin{equation*}
\left|\mathrm{x}_{\mathrm{hk} k}\right|^{2} \approx \mathrm{~h}^{2}+\mathrm{k}^{2}+l^{2} \propto \mathrm{n}^{2} \tag{B.16}
\end{equation*}
$$

and the number of terms in the shell goes as $\left(n^{3}-(n-2)^{3}\right) \propto n^{2}$. Therefore, $\lim _{n \rightarrow \infty} E_{\text {shell }} n$ is a " $p$ series" of
order of magnitude $\mathrm{O}\left(\mathrm{n}^{2-\mathrm{m}(\rho)}\right)$, which converges for $\mathrm{m}>3$ and converges conditionally for $\mathrm{m}=3$.

Thus all induction/dispersion terms and interactions of electrostatic moments higher than dipole/dipole and charge/quadrupole are convergent. The last two are slowly divergent (harmonic) series in the limit $n \rightarrow \infty$. The charge/quadrupole term may be discarded in the limit $n \rightarrow \infty$ for systems in which the total charge is zero, since separation of charges becomes insignificant in the limit of infinite interaction distance, and only the total charge is important.

The special case of the dipole/dipole term may be writen as [Maitland et al., 1981]:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{d} / \mathrm{d}}=\quad=-\left(\mu \mu^{\prime} / r^{3}\right)\left(2 \cos \theta_{1} \cos \theta_{2}-\sin \theta_{1} \sin \theta_{2} \cos \phi\right) \tag{B.17}
\end{equation*}
$$

where $\theta_{1}, \theta_{2}$, and $\phi$ are angles of orientation defined as in Figure B.1. In the amorphous limit, one may assume that at very large distances, all orientations of the second dipole with respect to the first are sampled equally. In this case, it is easily shown, by integration of $\theta_{2}$ and $\phi$ over all possible orientations relative to the first dipole, that the total contribution from each set of interacting dipoles becomes zero. Such an approximation is not rigorous in the case of periodic structures; however, in translationally periodic locations of dipoles of fixed local orientation, as one proceeds through successive shells of radius $r$ and thickness $\delta \mathbf{r}$, each shell contains an increasing number of dipoles describing a broader range of (discrete) dipole orientations relative to the first dipole, making such an approximation reasonable, if not rigorous, at large distances.

Figure B.1: Definition of angles describing the relative orientation of two linear charge distributions (from Maitland et al., 1981)

(b)


## APPENDIX C: Semiempirical Quantum Calculations

Central to the success in application of the atomistic modelling methods is the validity and comprehensiveness of the force fields used to describe internal energy and their dependence on conformational and structural variables. Ideally, one would attempt to achieve the greatest degree of validity and comprehensiveness by applying the most fundamental concepts of intcractions between bodies characterized by mass and charge distributions. For this purpose, the theories of quantum mechanics could, in principle, be used to describe the energetics of many body/many electron systems. However, in practice these calculations are far too complex even in relatively small polyatomic molecules for rigorous evaluation. Thus one must strike a balance between rigor and sufficiency. Even $a b$ initio quantum calculations on small molecules already involve (albeit well accepted) simplifications of de rigueur quantum theory. Next are the semiempirical quantum calculations, which employ approximations of the wave functions to simplify calculations with (hopefully) little loss of validity; however, these approximations must be parameterized using data from real molecules, and thus have individual limits of utility, depending upon the nature of the approximations and parameterizations used. For atomistic modelling of large multichain systems such as are involved in this work, time and calculational constraints mandate even further simplification of the set of equations used to describe the system energetics. Most common are the two-body interaction potentials used to describe the energy of interaction as a function of distance of separation, such as the Lennard-Joncs or Buckingham potentials. Other modelling efforts have employed orientation-dependent interactions, such as the "hydrogen bond potential", inductive effects in electrostatics, such as ficld-induced polarizabilities, or higher moment interactions, such as dipole-dipole or quadrupole-quadrupole interactions.

We have attempted to satisfy the criteria of sufficiency and simplicity entircly through the use of atom-centered, distance-dependent, two-body interactions to describe all nonbonded interactions. Where possible, these potentials are parameterized using experimental data from real systems; where such data is unavailable, we resort to predictions based on the more rigorous semiempirical quantum methods mentioned previously. The validity of these results, in turn, are checked against available structural and electrostatic moments of real molecules. For the aramid system, the following approach was undertaken.

A homologous series of amide, aromatic, and aromatic amide compounds representing subunits of poly(p-phenylene terephthalamide), end-capped with hydrogens, were optimized for heat of formation with respect to geometry. The series consisted of:
(1) formamide $\left(\mathrm{CH}_{3} \mathrm{NO}\right)$
(2) benzene $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$
(3) benzamide $\left(\mathrm{C}_{7} \mathrm{H}_{7} \mathrm{NO}\right)$
(4) formanilide $\left(\mathrm{C}_{7} \mathrm{H}_{7} \mathrm{NO}\right)$
(5) terephthalamide $\left(\mathrm{C}_{8} \mathrm{H}_{8} \mathrm{~N}_{2} \mathrm{O}_{2}\right)$
(6) p-phenylene diformamide $\left(\mathrm{C}_{8} \mathrm{H}_{8} \mathrm{~N}_{2} \mathrm{O}_{2}\right)$
(7) benzanilide $\left(\mathrm{C}_{13} \mathrm{H}_{11} \mathrm{NO}\right)$

The heat of formation of each compound was minimized (using the Davidon-Fletcher-Powell Method) with respect to bond lengths, bond angles, and bond torsions, with the restrictions that equivalent bond lengths and bond angles be equal (e.g. benzene) and all phenylene rings retain a hexagonal planar conformation (consistent with the description of the phenylene ring in the simulations). The results are tabulated in Table C.1(a)-(g), showing atomic charge assignment (in units of "partial atomic charge"), dipole moments, heats of formation, and relevant bond torsions. Initial conformations were chosen with trans-planar amide bonds and phenylene rings rotated by $30^{\circ}$ from the plane of the amide bond. In each case, AM1, MNDO, and CNDO (in part) semiempirical quantum calculations were performed and compared. Partial atomic charges for each atom are calculated as the difference between the number of outer shell electrons and the calculated electron density attributed to that atom in the compound. No conformational averaging was involved in any case. The calculated dipoles were compared, where possible, with experimentally measured values. Also noted are the percentages of the dipole moments attributed to hybridization, as these represent that part of the dipole moment not captured by the atomcentered partial charges; a larger calculated percentage hybridization translates into a greater error in using only the stated partial atomic charges to describe the electrostatic moments. Also shown are the final values for the amide and ring torsions.

From this analysis, we note that CNDO tends to predict the smallest charges, and in many cases of different sign from the MNDO and AM1 results, and the largest hybridization percentages. AM1 generally calculates the largest partial atomic charges and the lowest percentage hybridization. CNDO and AM1 both appear to predict reasonably good estimates of dipole moments across the series, but only the AM1 charges comes close to reproducing the experimental quadrupole moment of benzene of $-8.7 \times 10^{-26}$ esu $\mathrm{cm}^{2}$ [Battaglia et al., 1981]. MNDO performs poorest in estimating dipole moments and generally overestimates the torsions of the amide bond and that about the bond connecting the rings and amide moieties; in this respect, it is worth noting that AM1 was designed specifically to correct problems experienced by MNDO in predicting correct bond geometries in aromatic compounds. Based on these observations, we used AM1 exclusively from this point onward to obtain estimates for partial atomic charges.

Partial atomic charges for use in the 12-6-1 energy function in the polymer crystal simulations were determined as follows. (1) We require a minimum number of distinct atom types; in the unsubstituted polymer, this corresponds to seven atomic species: the amide nitrogen, amide oxygen, amide carbon and
amide hydrogen, the aromatic hydrogen, the aromatic carbons in the para position (no distinction is made between those connected to the amide nitrogen and those connected to the amide carbon), and the aromatic carbons in the ortho position. For these, the weighted average of AM1 charges across the homologous series are given in Table C.2, along with the standard deviations of acual values from the mean. In calculating the averages, charges in the larger compounds, which more closely approximate the electronic structure of the polymer, are preferentially weighted: the last three compounds in the series each contribute fully to the average; the first two are weighted by $1 / 3$, and the second two by $2 / 3$. (2) We require total charge neutrality on each of the $\mathrm{N}-\mathrm{H}$ and $\mathrm{C}=\mathrm{O}$ bond groups of the amide moiety (as per Hagler et al., [1974]) and on the phenylene ring. To achieve this, the residual charge for each of these three groups was uniformly redistributed among the elements of the group. This is a subjective correction, but one of only minor significance, given the small magnitude of residual charge in all cases but the $\mathrm{N}-\mathrm{H}$ bond group. The final "adjusted partial atomic charges" are also listed in Table C.2, along with the charges suggested by the self-consistent force field of Hagler et al. [1974] for the atoms of the amide moiety.

The procedure described above has a significant advantage in that it may be readily extended to the parameterization of chains of modified constitution. This is, of course, a critical advantage to this work, which presumes to analyze chlorine-substituted polymers as an extension of the parent polymer poly-(p-phenylene terephthalamide). This procedure, which is tested for PPTA and with results compared to available experimental data, must be applied blindly to the homologous series based on aromatic amides having chlorines substituted in the ortho position on the ring, as experimental data, other than the dipole moment of $m$-dichlorobenzene, is essentially nonexistent. In this case, the homologous series consists of:
(1) m-dichlorobenzene $\left(\mathrm{C}_{6} \mathrm{H}_{4} \mathrm{Cl}_{2}\right)$
(2) 2,6-dichlorobenzamide $\left(\mathrm{C}_{7} \mathrm{H}_{5} \mathrm{NOCl}_{2}\right)$
(3) 2,6-dichloroformanilide $\left(\mathrm{C}_{7} \mathrm{H}_{5} \mathrm{NOCl}_{2}\right)$
(4) 2,6-dichloroterephthalamide $\left(\mathrm{C}_{8} \mathrm{H}_{6} \mathrm{~N}_{2} \mathrm{O}_{2} \mathrm{Cl}_{2}\right)$
(5) p-(2,6-dichloro)phenylene diformamide $\left(\mathrm{C}_{8} \mathrm{H}_{6} \mathrm{~N}_{2} \mathrm{O}_{2} \mathrm{Cl}_{2}\right)$
(6) N -[2,6-dichlorophenyl]-benzamide $\left(\mathrm{C}_{13} \mathrm{H}_{9} \mathrm{NOCl}_{2}\right)$
(7) 2,6-dichlorophenylbenzanilide $\left(\mathrm{C}_{13} \mathrm{H}_{9} \mathrm{NOCl}_{2}\right)$

The set of unique species in the phenylene ring now become: the carbon in the 1 position (between the chlorines), the carbon in the 4 position (between the hydrogens), the carbons in the 2 and 6 positions (bonded to the chlorines), the carbons in the 3 and 5 positions (bonded to the hydrogens), the hydrogens, and the chlorines. As before, the charge and geometry results for each compound in the series are presented in Table C.3(a)-(g); the average, standard deviation, and adjusted partial atomic charges are tabulated in Table C.4.

It is worth noting that the amide charges have changed insignificantly from their values calculated for the unsubstituted compounds, which lends support to the premise that the substituted materials may be
treated as simple modifications of the unsubstituted materials. Secondly, the charges on the 1 and 4 carbons remain very close to zero and the carbon and hydrogn charges of the $\mathrm{C}-\mathrm{H}$ bonds remain close to their original values (within the confidence limit of the averaging method employed); it is the $\mathrm{C}-\mathrm{Cl}$ bond species in the ring that alter, to the extent that now charge is actually more evenly distributed between the two bonded atoms than beforc.

Rotation potentials for the unsubstituted and substituted phenylene rings about the bond connecting to the amide nitrogen or amide carbon were calculated using benzanilide, (2,6-dichlorophenyl)benzamide, and ( 2,6 -dichlorophenyl)formanilidc. In the first case, each ring was driven through a $90^{\circ}$ rotation in increments of $15^{\circ}$, with the opposite ring fixed at $30^{\circ}$ out-of-plane rotation. In the latter cases, only the substituted rings were driven in this manner; it was assumed here, and in the simulation work, that the ring rotation potential was a function only of the connected ring and amide atoms, and not of substitution at the other side of the ring or on the ring across the amide group. These results have been dealt with in greater detail in chapters 4 and 7.

Table C. 1
Tabulated results of semiempirical quantum mechanics calculations


| ATOM |  | CHARGE |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  | CNDO | MNDO | AM1 |
| H1 | -.041 |  |  |
| C2 | +.348 | +.056 | +.119 |
| N3 | -.247 | -.376 | +.257 |
| H4 | +.225 | +.182 | -.448 |
| H5 | +.210 | +.187 | +.219 |
| O6 | -.632 | -.371 | +.223 |
|  |  |  | -.371 |

DIPOLE MOMENT: total (\% hybrid) Exp
3.728
3.361
3.703
3. 37-3. 86
(17.6)
$-44.75$
TORSIONS:
$1-2-3-5: 180^{\circ} 180^{\circ}$
(b)

BENZENE


| ATOM |  | CHARGE |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  | CNDO | MNDO | AM1 |
| C1 |  |  |  |
| C2 | +.008 | -.059 | -.130 |
| C3 | +.008 | -.059 | -.130 |
| C4 | +.008 | -.059 | -.130 |
| C5 | +.008 | -.059 | -.130 |
| C6 | +.008 | -.059 | -.130 |
| H7 | +.008 | -.059 | -.130 |
| H8 | -.008 | +.059 | +.130 |
| H9 | -.008 | +.059 | +.130 |
| H10 | -.008 | +.059 | +.130 |
| H11 | -.008 | +.059 | +.130 |
| H12 | -.008 | +.059 | +.130 |
|  |  | +.059 | +.130 |

DIPOLE MOMENT: total (\% hybrid)
$\begin{array}{lll}0.0 & 0.0 & 0.0\end{array}$
Exp.
0.0

HEAT OF FORMATION: (kcal/mole)
$-47.10$
21.32
22.02


| ATOM | Charge |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | CNDO | MNDO | AM1 |  |
| C1 | +. 018 | -. 034 | -. 108 |  |
| C2 | $+.005$ | -. 071 | -. 140 |  |
| C3 | +. 018 | -. 007 | -. 104 |  |
| C4 | -. 027 | -. 121 | -. 121 |  |
| C5 | +. 024 | -. 017 | -. 077 |  |
| C6 | $+.005$ | -. 069 | -. 138 |  |
| H7 | -. 005 | +. 064 | +. 135 |  |
| H8 | -. 005 | $+.066$ | +. 136 |  |
| H9 | -. 009 | $+.071$ | $+.133$ |  |
| H10 | +. 006 | +. 066 | $+.157$ |  |
| H11 | -. 004 | +. 066 | $+.139$ |  |
| C12 | +. 319 | +. 361 | $+.343$ |  |
| 013 | -. 353 | -. 336 | -. 371 |  |
| N14 | -. 260 | -. 336 | -. 434 |  |
| H15 | +. 117 | +. 146 | $+.220$ |  |
| H16 | +. 128 | +. 152 | +. 231 |  |
| DIPOLE MOMENT: | total (\% hybrid) |  |  | Exp. |
|  | $\begin{aligned} & 3.857 \\ & (58.2) \end{aligned}$ | $\begin{aligned} & 3.411 \\ & (26.8) \end{aligned}$ | $\begin{aligned} & 3.710 \\ & (17.4) \end{aligned}$ | 3.65-3.88 |

HEAT OF FORMATION: (kcal/mole)

$$
-15.69 \quad-16.03
$$

TORSIONS :

```
3-4-12-14 :
13-12-14-15 :
\begin{tabular}{rr}
\(69^{\circ}\) & \(33^{\circ}\) \\
\(-150^{\circ}\) & \(-174^{\circ}\)
\end{tabular}
```

(d)

FORMANILIDE


| ATOM |  | CHARGE |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  | CNDO | MNDO | AM1 |
| C1 | -.014 | -.068 | -.154 |
| C2 | +.024 | -.048 | -.106 |
| C3 | -.036 | -.046 | -.149 |
| C4 | +.141 | +.086 | +.065 |
| C5 | -.044 | -.090 | -.181 |
| C6 | +.023 | -.046 | -.106 |
| H7 | -.003 | +.062 | +.133 |
| H8 | -.004 | +.063 | +.135 |
| H9 | +.012 | +.076 | +.169 |
| H10 | -.002 | +.059 | +.128 |
| H11 | -.004 | +.061 | +.131 |
| N12 | +.204 | -.370 | -.339 |
| H13 | +.342 | +.172 | +.230 |
| C14 | -.291 | +.372 | +.267 |
| O15 | -.042 | -.343 | -.345 |
| H16 |  | +.058 | +.122 |

DIPOLE MOMENT: total (\% hybrid) Exp

| 3.397 | 2.849 | 3.156 |
| :--- | :--- | :--- |
| $(81.4)$ | $(32.2)$ | $(25.0)$ |

$-9.716$
TORSIONS:

| $3-4-12-14$ | $:$ | $60^{\circ}$ | $17^{\circ}$ |
| :--- | ---: | ---: | ---: |
| $13-12-14-15$ | $-168^{\circ}$ | $-176^{\circ}$ |  |

(e)

TEREPHTHALAMIDE


ATOM

|  | MNDO | AM1 |
| :--- | :--- | :--- |
| H1 |  |  |
| N2 | +.160 | +.234 |
| C3 | -.343 | -.432 |
| C4 | +.361 | +.341 |
| C5 | -.094 | -.099 |
| C6 | -.033 | -.089 |
| C7 | -.016 | -.111 |
| C8 | -.095 | -.101 |
| C9 | -.030 | -.089 |
| H10 | -.016 | -.114 |
| H11 | +.077 | +.142 |
| C12 | +.071 | +.159 |
| N13 | +.358 | +.341 |
| H14 | -.337 | -.428 |
| H15 | +.149 | +.221 |
| O16 | +.152 | +.224 |
| H17 | -.334 | -.366 |
| H18 | +.069 | +.159 |
| O19 | +.077 | +.141 |
| H20 | -.332 | -.364 |
|  | +.156 | +.231 |

DIPOLE MOMENT: total (\% hybrid)

| 0.248 | 0.156 |
| :--- | :--- |
| $(25.0)$ | $(92.9)$ |

HEAT OF FORMATION: (kcal/mole)

$$
-52.30 \quad-16.03
$$

TORSIONS:

```
16-3-2-15
2-3-4-5 :
13-12-7-6 :
14-13-12-19:
\begin{tabular}{rr}
\(-153^{\circ}\) & \(-177^{\circ}\) \\
\(-98^{\circ}\) & \(-141^{\circ}\) \\
\(-75^{\circ}\) & \(-35^{\circ}\) \\
\(150^{\circ}\) & \(170^{\circ}\)
\end{tabular}
```



| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  |  |  |
| H1 | MNDO | AM1 |
| C2 | +.058 | +.122 |
| N3 | +.384 | +.266 |
| C4 | -.390 | -.333 |
| C5 | +.097 | +.041 |
| C6 | -.077 | -.156 |
| C7 | -.024 | -.128 |
| C8 | +.072 | +.043 |
| C9 | -.060 | -.156 |
| H10 | -.048 | -.126 |
| H11 | +.074 | +.174 |
| N12 | +.065 | +.134 |
| C13 | -.388 | -.335 |
| O14 | +.379 | +.268 |
| O15 | -.350 | -.350 |
| H16 | -.349 | -.350 |
| H17 | +.182 | +.230 |
| H18 | +.066 | +.133 |
| H19 | +.075 | +.172 |
| H20 | +.178 | +.230 |

DIPOLE MOMENT: total (\% hybrid)

| 0.212 | 0.138 |
| :--- | :--- |
| $(18.9)$ | $(2.9)$ |

HEAT OF FORMATION: (kcal/mole)

$$
\begin{array}{ll}
-38.06 & -41.56
\end{array}
$$

TORSIONS:

```
16-3-2-15 :
-172*
-1770
2-3-4-5 : -1270
13-12-7-6 : -740 -6%
14-13-12-19: 174. 1790
```



| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  | MNDO | AM1 |
|  |  |  |
| C1 | -.056 | -.155 |
| C2 | -.058 | -.107 |
| C3 | -.029 | -.145 |
| C4 | +.070 | +.071 |
| C5 | -.072 | -.181 |
| C6 | -.054 | -.107 |
| H7 | +.062 | +.132 |
| H8 | +.062 | +.135 |
| H9 | +.072 | +.168 |
| H10 | +.059 | +.129 |
| H11 | +.061 | +.130 |
| N12 | -.357 | -.326 |
| H13 | +.171 | +.231 |
| C14 | +.394 | +.351 |
| O15 | -.334 | -.343 |
| C16 | -.118 | -.127 |
| C17 | -.009 | -.105 |
| C18 | -.068 | -.137 |
| C19 | -.035 | -.108 |
| C20 | -.068 | -.137 |
| C21 | -.021 | -.078 |
| H22 | +.069 | +.137 |
| H23 | +.066 | +.138 |
| H24 | +.065 | +.136 |
| H25 | +.065 | +.140 |
| H26 | +.064 | +.155 |

DIPOLE MOMENT: total (\% hybrid) Exp.

| 3.193 | 3.449 | $3.38,3.83$ |
| :--- | :--- | :--- |
| $(27.2)$ | $(22.1)$ |  |


| HEAT OF FORMATION: (kcal/mole) |  |  |
| :--- | ---: | ---: |
| TORSIONS: | 16.19 | 19.66 |
| $3-4-12-14$ |  |  |
| $15-14-12-13:$ | $84^{\circ}$ | $24^{\circ}$ |
| $12-14-16-17:$ | $-164^{\circ}$ | $-172^{\circ}$ |

## Table C. 2

Partial atomic charges from AM1 calculations: unsubstituted compounds

| Atom type: | $\mathrm{H}_{\text {amide }}$ | Namide | $\mathrm{C}_{\text {amide }}$ | $\mathrm{O}_{\text {amidc }}$ | $\mathrm{C}_{\text {ph.para }}$ | $\mathrm{C}_{\text {ph.ortho }}$ | $\mathrm{H}_{\mathrm{ph}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Weighted average charge | 0.226 | -0.378 | 0.309 | -0.356 | -0.065 | -0.124 | 0.144 |
| Standard deviation | 0.005 | 0.051 | 0.040 | 0.011 | 0.085 | 0.028 | 0.015 |
| Residual charge on $\mathrm{N}-\mathrm{H}$ | -0.151 |  |  |  |  |  |  |
| Residual charge on $\mathrm{C}=\mathrm{O}$ | -0.047 |  |  |  |  |  |  |
| Residual charge on phenylene | -0.050 |  |  |  |  |  |  |
| "Adjusted" charge | 0.302 | -0.302 | 0.333 | -0.333 | $-0.060$ | -0.119 | 0.149 |
| HHL charge ${ }^{\text {a }}$ | 0.28 | -0.28 | 0.38 | -0.38 | - | - | - |

[^0]
## Table C. 3

Tabulated results of semiempirical quantum mechanical calculations: dichloro compounds
(a) M-DICHLORO-BENZENE


| ATOM |  | CHARGE |  |
| :--- | :--- | :--- | :--- |
|  |  |  |  |
|  | CNDO | MNDO | AM1 |
| C1 |  |  |  |
| C2 | +.016 | -.020 | -.121 |
| C3 | +.076 | -.000 | -.055 |
| C4 | +.022 | -.033 | -.125 |
| C5 | +.002 | -.052 | -.112 |
| C6 | +.022 | -.033 | -.125 |
| H7 | +.076 | -.000 | -.055 |
| C18 | +.029 | +.094 | +.161 |
| H9 | -.142 | -.096 | -.006 |
| H10 | +.016 | +.081 | +.149 |
| H11 | +.011 | +.076 | +.144 |
| C112 | +.016 | +.081 | +.149 |
|  | -.142 | -.096 | -.006 |

DIPOLE MOMENT: total (\% hybrid) Exp.
1.649
$(0.1)$
1.745
1.222
1.22-1. 68
heAT Of FORMATION: (kcal/mole)
-----
6.63
8.23


| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  | MNDO | AM1 |
| C1 | -.033 | -.100 |
| C2 | -.041 | -.130 |
| C3 | +.045 | -.034 |
| C4 | -.076 | -.108 |
| C5 | +.029 | -.023 |
| C6 | -.039 | -.130 |
| H7 | +.077 | +.146 |
| H8 | +.087 | +.155 |
| C19 | -.075 | +.007 |
| C110 | -.094 | +.017 |
| H11 | +.087 | +.156 |
| C12 | +.366 | +.352 |
| O13 | -.315 | -.342 |
| N14 | -.326 | -.426 |
| H15 | +.149 | +.226 |
| H16 | +.159 | +.234 |

DIPOLE MOMENT: total (\% hybrid)

| 4.266 | 4.315 |
| :--- | :--- |
| $(22.1)$ | $(12.7)$ |

HEAT OF FORMATION: (kcal/mole)

$$
-25.83 \quad-25.15
$$

TORSIONS :

| $3-4-12-14$ | $:$ | $88^{\circ}$ | $77^{\circ}$ |
| :--- | ---: | ---: | ---: |
| $13-12-14-15$ | $-152^{\circ}$ | $177^{\circ}$ |  |

(c)

2,6-DICHLORO-FORMANILIDE


| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  | MNDO | AM1 |
|  |  |  |
| C1 | -.051 | -.119 |
| C2 | -.031 | -.116 |
| C3 | +.022 | -.046 |
| C4 | +.113 | +.061 |
| C5 | -.010 | -.079 |
| C6 | -.027 | -.112 |
| H7 | +.075 | +.146 |
| H8 | +.084 | +.153 |
| C19 | -.080 | +.014 |
| C110 | -.099 | -.018 |
| H11 | +.084 | +.154 |
| N12 | -.376 | -.344 |
| H13 | +.184 | +.236 |
| C14 | +.380 | +.268 |
| O15 | -.335 | -.326 |
| H16 | +.067 | +.128 |

DIPOLE MOMENT: total (\% hybrid)

$$
2.854
$$

2.917
(30.5)
(24.5)

HEAT OF FORMATION: (kcal/mole)
$-20.35$
-19.01
TORSIONS:

| $3-4-12-14$ | $:$ | $88^{\circ}$ | $66^{\circ}$ |
| :--- | :--- | ---: | ---: |
| $13-12-14-15$ | $-163^{\circ}$ | $-163^{\circ}$ |  |



| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  |  |  |
|  | MNDO | AM1 |
| H1 |  |  |
| N2 | +.162 | +.236 |
| C3 | -.334 | -.429 |
| C4 | +.357 | +.342 |
| C5 | -.092 | -.089 |
| C6 | -.004 | -.084 |
| C7 | +.033 | -.043 |
| C8 | -.054 | -.092 |
| C9 | +.015 | -.032 |
| C110 | +.010 | -.109 |
| C111 | -.064 | +.024 |
| C12 | -.082 | +.024 |
| N13 | +.362 | +.348 |
| H14 | -.326 | -.423 |
| H15 | +.152 | +.228 |
| O16 | +.152 | +.226 |
| H17 | -.322 | -.357 |
| H18 | +.089 | +.175 |
| O19 | +.096 | +.158 |
| H20 | -.313 | -.340 |
|  | +.162 | +.235 |

DIPOLE MOMENT: total (\% hybrid)

| 1.531 | 1.988 |
| :--- | :--- |
| $(18.3)$ | $(12.5)$ |

heat of formation: (kcal/mole)

$$
-60.89 \quad-61.70
$$

TORSIONS:

| $16-3-2-15$ | $:$ | $-153^{\circ}$ | $-176^{\circ}$ |
| :--- | :--- | :--- | :--- |
| $2-3-4-5$ | $:$ | $-108^{\circ}$ | $-140^{\circ}$ |
| $13-12-7-6$ | $:$ | $-88^{\circ}$ | $-77^{\circ}$ |
| $14-13-12-19$ | $:$ | $151^{\circ}$ | $-176^{\circ}$ |




## ATOM

## CHARGE

|  | MNDO | AM1 |
| :--- | :--- | :--- |
| C1 | -.054 | -.117 |
| C2 | -.030 | -.119 |
| C3 | +.019 | -.040 |
| C4 | +.124 | +.062 |
| C5 | -.011 | -.075 |
| C6 | -.027 | -.117 |
| H7 | +.075 | +.144 |
| H8 | +.084 | +.153 |
| C19 | -.085 | +.017 |
| C110 | -.100 | -.004 |
| H11 | +.083 | +.151 |
| N12 | -.373 | -.337 |
| H13 | +.188 | +.237 |
| C14 | +.413 | +.352 |
| O15 | -.329 | -.326 |
| C16 | -.121 | -.124 |
| C17 | -.009 | -.100 |
| C18 | -.071 | -.139 |
| C19 | -.033 | -.108 |
| C20 | -.070 | -.138 |
| C21 | -.012 | -.080 |
| H22 | +.071 | +.142 |
| H23 | +.067 | +.137 |
| H24 | +.064 | +.136 |
| H25 | +.067 | +.139 |
| H26 | +.068 | +.154 |

DIPOLE MOMENT: total (\% hybrid)

| 2.807 | 2.946 |
| :--- | :--- |
| $(28.3)$ | $(22.5)$ |

heat of formation: (kcal/mole)

## TORSIONS

| $3-4-12-14$ | $:$ | $83^{\circ}$ | $75^{\circ}$ |
| :--- | :--- | ---: | ---: |
| $15-14-12-13:$ | $-163^{\circ}$ | $-162^{\circ}$ |  |
| $12-14-16-17$ | $77^{\circ}$ | $42^{\circ}$ |  |

(g)


| ATOM | CHARGE |  |
| :--- | :--- | :--- |
|  |  |  |
|  | MNDO | AM1 |
| C1 | -.065 | -.154 |
| C2 | -.050 | -.107 |
| C3 | -.044 | -.146 |
| C4 | +.092 | +.072 |
| C5 | -.081 | -.178 |
| C6 | -.049 | -.107 |
| H7 | +.062 | +.133 |
| H8 | +.062 | +.135 |
| H9 | +.076 | +.169 |
| H10 | +.062 | +.130 |
| H11 | +.060 | +.131 |
| N12 | -.361 | -.317 |
| H13 | +.181 | +.235 |
| C14 | +.409 | +.360 |
| O15 | -.320 | -.317 |
| C16 | -.079 | -.118 |
| C17 | +.040 | -.027 |
| C18 | -.042 | -.130 |
| C19 | -.033 | -.097 |
| C20 | -.041 | -.130 |
| C21 | +.034 | -.029 |
| C122 | -.080 | +.018 |
| H23 | +.086 | +.156 |
| H24 | +.078 | +.148 |
| H25 | +.087 | +.156 |
| C126 | -.086 | +.017 |

DIPOLE MOMENT: total (\% hybrid)

| 4.185 | 4.078 |
| :--- | :--- |
| $(20.1)$ | $(17.8)$ |

HEAT OF FORMATION: (kcal/mole)
$6.64 \quad 10.57$
TORSIONS :
$3-4-12-14:$
$15-14-12-13:$
$12-14-16-17:$

| $58^{\circ}$ | $22^{\circ}$ |
| ---: | ---: |
| $-169^{\circ}$ | $-177^{\circ}$ |
| $87^{\circ}$ | $88^{\circ}$ |

Table C. 4
Partial atomic charges from AM1 calculations: dichloro compounds

| Atom type: | $\mathrm{H}_{\text {amide }}$ | $\mathrm{N}_{\text {amide }}$ | $\mathrm{C}_{\text {amide }}$ | $\mathrm{O}_{\text {amide }}$ | $\mathrm{C}_{1}$ | $\mathrm{C}_{4}$ | $\mathrm{C}_{2.6}$ | $\mathrm{C}_{3.5}$ | $\mathrm{H}_{\mathrm{ph}}$ | $\mathrm{Cl}_{\mathrm{ph}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Average Charge | 0.233 | -0.368 | 0.321 | -0.334 | -0.033 | -0.071 | -0.042 | -0.125 | 0.155 | 0.013 |
| Standard Deviation | 0.005 | 0.045 | 0.041 | 0.012 | 0.082 | 0.075 | 0.018 | 0.021 | 0.029 | 0.028 |
| Residual charge Residual charge Residual charge | on $\mathrm{N}-\mathrm{H}$ on $\mathrm{C}=0$ on pheny | ylene | $\begin{aligned} & -0.102 \\ & -0.136 \\ & -0.014 \end{aligned}$ |  |  |  |  |  |  |  |
| "Adjusted" Charge | 0.301 | -0.301 | 0.328 | -0.328 | -0.023 | -0.061 | -0.031 | -0.115 | 0.165 | 0.023 |

## APPENDIX D: Atomic Scattering Factor Functions

For purposes of calculating the structure factors for the crystal based on the ensemble of atoms and atom coordinates, the atomic scattering factors are required in functional form. Tables of mean atomic scattering values are readily available from the International Tables of X-Ray Crystallography, [Volume III, 1968] page 201. Following Moore [1963], we have used functions of the form:

$$
\begin{align*}
& f_{j}\left(x_{i}\right)=A_{j} \exp \left(-a_{j} x_{i}^{2}\right)+B_{j} \exp \left(-b_{j} x_{i}^{2}\right)+C_{j}  \tag{D.1}\\
& j=H, C, N, O, C l \\
& x_{i}=\sin \left(\theta_{i}\right) \lambda
\end{align*}
$$

These have been fitted to the mean atomic scatering factors derived by self-consistent or variational wave functions in the range $\left(0 \leq \sin \left(\theta_{i}\right) / \lambda \leq 1\right)$ using the recommended weighting function for $\mathrm{CuK} \alpha$ radiation:

$$
\begin{equation*}
w_{i}=\exp \left[-\left(\mathrm{x}_{\mathrm{i}}-1 / 2\right)^{2}\right] \tag{D.2}
\end{equation*}
$$

The resulting function parameters and the corresponding sum of squares of the regression obtained using the RS1 graphics and statistical analysis software package [BBN Research Systems, Cambridge, MA], are presented in Table D.1. The quality of fit is illustrated in Figure D.1.

## Table D. 1

Function parameters for atomic scattering factors

| Atom type | $\mathbf{A}_{\mathbf{i}}$ | $\mathbf{a}_{\mathbf{i}}$ | $\mathbf{B}_{\mathbf{i}}$ | $\mathbf{b}_{\mathbf{i}}$ | $\mathbf{C}_{\mathbf{i}}$ | $\boldsymbol{\varepsilon}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| H | 0.391 | 7.483 | 0.596 | 30.072 | 0.011 | 0.268 |
| C | 1.248 | 2.517 | 3.678 | 23.747 | 1.044 | 0.414 |
| $\mathbf{N}$ | 1.778 | 3.836 | 3.938 | 19.226 | 1.260 | 0.309 |
| O | 2.836 | 4.423 | 3.773 | 17.229 | 1.373 | 0.218 |
| Cl | 8.066 | 0.973 | 7,983 | 22.093 | 0.946 | -- |

Figure D.1: Atomic scattering factor functions for hydrogen, carbon, nitrogen, oxygen, and chlorine (in order from bottom to top). Points are recommended mean atomic scatlering factors from self-consistent or variational wave functions (from the International Tables for XRay Crystallography [1968]); curves are fitted functions of the form given by Equation D. 1 using parameters listed in Table D.1.

## Atomic Scattering Factor Functions



# APPENDIX E: Detailed X-ray Scattering Intensities from Simulated Structures and Simulated X-ray Fiber Diffraction Patterns for Chlorine-modified Isomers 

## Table E.1a

Summary of x-ray reflections in molecular simulation structures of PPTA

|  | Structure \#1 |  |  | Structure \#2 |  |  | Structure \#3 |  |  | Structure \#4 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hikl | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I |
| $\begin{aligned} & 110 \\ & 1 \overline{1} 0 \end{aligned}$ | 19.6 | 0 | 87 |  |  | 72 | 21.0 | 0 | 74 | 21.0 | 0 | 50 |
|  | 19.0 | 0 | 65) | 19.3 | 0 | 80 | 20.5 | 0 | 23) | 21.2 | 0 | 21 |
| 200 | 26.2 | 0 | 100 | 25.2 | 0 | 100 | 21.4 | 0 | 100 | 21.5 | 0 | 100 |
| $\begin{array}{r} 310 \\ 3 \overline{10} 0 \end{array}$ |  | 0 |  |  |  |  | 37.7 | 0 |  | 37.3 | 0 |  |
|  | 42.9 | 0 |  | 40.8 | 0 |  |  | 0 |  | 37.7 | 0 |  |
| 400 | 54.0 | 0 | 1 |  |  |  |  |  |  |  |  |  |
| $\overline{1} 01$ |  |  |  |  |  |  |  |  |  | 13.8 | 29 | 3 |
| $\begin{aligned} & 011 \\ & 0 \overline{1} 1 \end{aligned}$ |  |  |  | 16.9 | 24 |  |  |  |  | 19.0 | 21 | 2 |
|  |  |  |  | 16.2 |  |  |  |  |  |  |  | 4 |
|  | 20.2 |  | $3)$ |  |  |  | 22.1 | 18 |  |  |  |  |
| 111 |  |  | 4 |  |  |  |  |  |  |  |  |  |
| 111 |  |  | 4 |  |  |  |  |  |  |  |  |  |
| 111 |  |  | 3 |  |  |  | 22.1 | 18 | 2 |  |  |  |
| $\stackrel{201}{201}$ | 27.1 |  | 1 |  |  |  | 22.5 | 18 | 2 |  |  |  |
|  |  |  |  |  |  |  | 22.5 |  | 2 |  |  |  |
| 211 |  |  | 2 |  |  | 12 |  |  | 4 |  |  | $2)$ |
| $\frac{211}{}$ |  |  | 3 | 29.8 | 13 | 13 |  | 14 | 5 |  |  | 6 |
| $2 \overline{1} 1$ |  |  | 3 | 30.0 | 13 | 1 |  | 14 | 5 |  | 14 | 4 |
| 211 | 30.4 |  | 2 | 30.4 |  | 9 | 29.2 |  | 4 | 30.3 |  | 7 |
| $\begin{aligned} & 121 \\ & 121 \end{aligned}$ |  |  |  | 34.6 |  |  |  |  |  | 38.0 |  | 1 |
|  |  |  |  | 33.0 |  |  |  |  |  |  |  |  |
| $\overline{3} 01$ |  |  |  |  |  |  |  |  |  | 34.7 |  | 2 |
| 002 | 28.5 | 0 | 1 | 13.5 |  | 5 | 13.5 |  | 5 | 13.8 |  | 5 |
| $\frac{112}{112}$ |  |  |  | 23.4 |  | 1 2 |  |  |  | 24.9 |  | 2 |
|  |  |  |  | 23.5 |  | 2 |  |  |  |  |  |  |
| $\frac{202}{202}$ |  |  |  | 29.2 |  | 2 |  |  |  | 27.9 |  | 2 |
| 022 |  |  |  | 34.2 |  | 2 |  |  |  |  |  |  |
| $\frac{312}{312}$ |  |  |  |  |  |  | 40.2 | 20 | $1)$ | 37.0 |  | 1 |
|  |  |  |  |  |  |  | 40.2 | 20 | 1 |  |  |  |
| $\overline{1} 32$ |  |  |  | 50.5 | 16 | 1 |  |  |  |  |  |  |

Table E.1a (continued)

| $\begin{aligned} & 103 \\ & 103 \end{aligned}$ |  |  |  |  |  |  | $\begin{aligned} & 21.472 \\ & 25.3 \quad 54 \end{aligned}$ | $\left.\begin{array}{l}2 \\ 1\end{array}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} 013 \\ 013 \end{gathered}$ |  |  | 26.151 24.755 | $\left.\begin{array}{l}4 \\ 2\end{array}\right]$ |  |  | 26.750 | 2 |
| $\begin{aligned} & \overline{1} 13 \\ & 113 \end{aligned}$ | 28.346 28.346 | $\left.\begin{array}{l}2 \\ 2\end{array}\right]$ |  |  |  |  |  |  |
| $\begin{array}{r} 203 \\ \mathbf{2} 03 \end{array}$ |  |  |  |  | 29.843 29.744 | $\left.\begin{array}{l}2 \\ 1\end{array}\right)$ |  |  |
| 213 213 |  |  | $\begin{aligned} & 35.835 \\ & 37.6 \quad 33 \end{aligned}$ | $\left.\begin{array}{l} 2 \\ 1 \end{array}\right\}$ |  |  | 31.640 | 2 |
| 303 |  |  |  |  |  |  | 35.135 .8 | 2 |
| 004 | 27.190 | 17 | 27.386 | 25 | 27.290 | 13 | 27.878 | 17 |
| 104 |  |  | 40.043 | 3 |  |  |  |  |
| $\frac{114}{114}$ |  |  | $\begin{aligned} & 33.355 \\ & 32.956 \end{aligned}$ | $\left.\begin{array}{l}2 \\ 1\end{array}\right)$ |  |  | 34.552 | 3 |
| 105 |  |  |  |  |  | 34.5 | $82 \quad 1$ |  |
| 015 |  |  | 38.463 | 2 |  |  |  |  |
| 006 | 41.290 | 17 | 41.486 | 36 | 41.390 | 9 | 42.378 | 17 |
| $\begin{array}{r} 106 \\ 106 \end{array}$ | $\begin{array}{ll} 43.4 & 72 \\ 43.4 & 72 \end{array}$ | 1 1 |  |  | 42.875 42.875 | $\left.\begin{array}{l}4 \\ 4\end{array}\right)$ |  |  |
| $\begin{array}{r} 016 \\ 016 \end{array}$ | $\begin{array}{ll} 43.7 & 71 \\ 43.8 \quad 71 \end{array}$ |  |  |  |  |  |  |  |
| 116 116 |  |  | $\begin{aligned} & 47.761 \\ & 46.464 \end{aligned}$ | 1 2 |  |  | 44.369 | 3.4 |
| $\begin{array}{r} 206 \\ 206 \end{array}$ |  |  | $\begin{aligned} & 50.057 \\ & 48.260 \end{aligned}$ | 1 2 |  |  | 43.772 | 2 |

## Table E.1b

Summary of x-ray reflections in molecular simulation structures of PPTA

|  | Structure \#5 |  |  | Structure \#6 |  |  | Structure \#7 |  |  | Structure \#8 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hkI | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I |
| 100 |  |  |  |  |  |  |  |  |  | 21.9 | 01 | 100 |
| 010 | 18.4 | 0 | 49 | 19.5 | 0 | 47 | 18.1 | 0 | 46 |  |  |  |
| 110 |  |  |  |  |  |  |  |  |  | 29.8 | 0 | 27 |
| 200 | 21.3 | 0 | 100 | 21.1 | 0 | 100 | 21.1 | 0 | 100 |  |  |  |
| 020 |  |  |  |  |  |  |  |  |  | 44.7 | 0 | 1 |
| 210 | 28.2 | 0 0 |  | 29.4 | 0 0 |  |  |  |  | 49.9 | 0 | 1 |
| 210 |  |  |  |  |  |  | 28.1 | 0 | 35 |  |  |  |
| 220 |  |  |  | 44.6 | 0 | 2 | 42.8 | 0 | 1 |  |  |  |
| 400 |  |  |  |  |  |  | 43.3 | 0 | 1 |  |  |  |
| $4 \overline{10}$ |  |  |  | 46.8 | 0 | 1 |  |  |  |  |  |  |
| 001 | 6.9 |  | 9 | 6.9 | 79 | 4 |  |  |  | 8.3 |  | 2 |
| 101 |  |  |  |  |  |  |  |  |  | 20.2 | 20 | 2 |
| 011 |  |  |  | 19.5 |  | 1 |  |  |  |  |  |  |
| 011 |  |  |  |  |  |  |  |  |  |  |  |  |
| $\begin{gathered} 111 \\ 1111 \\ 111 \\ 111 \end{gathered}$ | $\begin{aligned} & 21.5 \\ & 23.4 \end{aligned}$ |  | 2 4 | 21.6 |  | 1 | 21.8 21.4 22.8 | 18 18 17 | $\left.\begin{array}{l}1 \\ 2 \\ 1\end{array}\right]$ |  |  |  |
| $\begin{array}{r}211 \\ 211 \\ \hline 211\end{array}$ | 28.5 29.9 29.9 | 14 13 13 | $\left.\begin{array}{l}6 \\ 3 \\ 5\end{array}\right)$ |  |  |  |  |  |  |  |  |  |
| 002 | 13.8 |  | 5 |  |  |  | 13.6 | 84 | 10 | 16.6 | 54.8 | 2 |
| 102 |  |  |  | 16.9 |  | 1 |  |  |  | 34.5 |  | 2 |
| 012 |  |  |  |  |  |  | 23.6 |  | 3 |  |  |  |
| 112 |  |  |  |  |  | 2 |  |  |  |  | 29.4 | 5 |
| 112 |  |  |  |  |  | 1 |  |  |  |  |  |  |
| 112 |  | 30 | 2 |  |  |  |  |  |  |  |  | 2 |
| 112 |  |  |  |  |  |  |  |  |  |  |  |  |
| $\overline{2} 02$ |  |  |  |  |  |  | 24.6 | 33 | 4 |  |  |  |
| 212 |  |  |  |  |  |  | 31.1 | 26 | 3 ) |  | 18 | $1)$ |
| $2 \overline{1} 2$ |  |  |  |  |  |  | 32.6 |  | 6 | 40.5 | 20 | 1. |
| $\begin{array}{r} 122 \\ 122 \end{array}$ |  |  |  | 41.6 | 19 | 2 |  |  |  | 40.6 | 20 | 1 |

Table E.1b (continued)
$\left.\begin{array}{llllllllllllll}302 & 34.9 & 23 & 1 \\ 302 & 35.3 & 23 & 2\end{array}\right)$

## Table E. 2

Summary of x-ray reflections in molecular simulation structures for 2,6-dichloro-PPTA; head-to-head, tail-to-tail polymorphs

|  | Structure \#1 |  |  | Structure \#2 |  |  | Structure \#3 |  |  | Structure \#4 |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hkl | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I | $2 \theta$ | $\beta$ | I |  |
| 100 |  |  |  |  |  |  |  |  |  | 16.5 | 0 | 14 |  |
| 010 |  |  |  |  |  |  |  |  |  | 20.7 | 0 | 1 |  |
| $\bar{T} 10$ | 17.4 | 0 | $2)$ |  |  |  | 16.9 | 0 | $5)$ | 23.9 | 0 | 2 |  |
| 110 | 6.1 | 0 | 6 | 17.2 | 0 | 9 | 19.0 | 0 | 15) | 29.1 | 0 |  |  |
| $1 \overline{10}$ |  |  |  | 19.6 | 0 | 22 |  |  |  |  |  |  |  |
| 200 | 21.0 | 0 | 3 | 20.2 | 0 | 17 | 21.0 | 0 | 5 |  |  |  |  |
| 020 | 31.0 | 0 | 1 |  |  |  | 29.4 | 0 | 2 |  |  |  |  |
| $\overline{2} 10$ |  |  |  |  |  |  |  |  |  | 35.9 | 0 | 2 |  |
| 310 | 37.3 | 0 | 1 | 32.3 | 0 | 9 | 36.7 | 0 | 3 |  |  |  |  |
| 001 | 3.7 | 72 | 100 | 3.4 | 90 | 100 |  |  |  | 3.6 | 73 | 100 |  |
| $0 \overline{11}$ |  |  |  |  |  |  | 14.6 | 13 | 1 |  |  |  |  |
| 002 | 7.3 | 72 | 37 |  |  |  | 6.8 | 84 | 100 | 7.2 | 73 | 58 |  |
| $\frac{102}{102}$ | $\begin{aligned} & 22.2 \\ & 22.4 \end{aligned}$ | $\begin{aligned} & 18 \\ & 18 \end{aligned}$ | $\left.\begin{array}{l} 1 \\ 1 \end{array}\right)$ |  |  |  |  |  |  |  |  |  |  |
| 112 |  |  |  |  |  |  |  |  |  | 31.5 | 13 | 2 |  |
| $\underline{202}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $\overline{202}$ |  |  |  |  |  |  |  | $18$ | 2) |  |  |  |  |
| 022 |  |  |  |  |  |  | 30.9 | 13 | 2 |  |  |  |  |
| $\overline{2} 12$ |  |  |  | 28.1 | 14 | $3)$ |  |  |  |  |  |  |  |
| 212 |  |  |  | 24.7 | 16 | 2 |  |  |  |  |  |  |  |
| $2 \overline{12}$ |  |  |  | 28.1 | 14 | 3 |  |  |  |  |  |  |  |
| 212 |  |  |  | 24.7 | 16 | 2 |  |  |  |  |  |  |  |
| 003 | 11.0 | 72 | 37 | 10.1 | 90 | 59 |  |  |  | 10.8 | 73 | 32 |  |
| $0 \overline{13}$ |  |  |  |  |  |  | 16.9 | 37 | 2 |  |  |  |  |
| $\overline{103}$ |  |  |  |  |  |  | 14.6 | 44 | 1 | 19.7 | 32 | 1 |  |
| $\frac{113}{113}$ |  |  |  | $\begin{aligned} & 20.0 \\ & 20.1 \end{aligned}$ | $\begin{aligned} & 31 \\ & 30 \end{aligned}$ | $\left.\begin{array}{l}1 \\ 1\end{array}\right)$ |  |  |  |  |  |  |  |
| 004 |  |  |  | 13.5 | 90 | 4 | 13.6 | 84 | 3 | 14.4 | 73 | 3 |  |
| 005 | 18.3 | 72 | 8 | 16.9 | 90 | 19 |  |  |  | 18.0 | 73 | 3 |  |
| $0 \overline{15}$ |  |  |  |  |  |  | 21.3 | 53 | 1 |  |  |  |  |
| 105 |  |  |  |  |  |  | 20.1 | 58 | 1 | 24.4 | 45 | 1 |  |
| 006 |  |  |  |  |  |  | 20.5 | 84 | 3 |  |  |  |  |
| $0 \overline{16}$ |  |  |  |  |  |  |  |  |  | 25.2 | 55 | 2 |  |
| 007 |  |  |  | 23.8 | 90 | 2 |  |  |  |  |  |  | $\therefore$ |
| 008 |  |  |  |  |  |  |  |  |  | 28.9 | 73 | 1 |  |

## Table E. 3

Summary of x-ray reflections in molecular simulation structures for 2,6-dichloro-PPTA;
parallel head-to-tail polymorphs

|  | Structure \#1 |  |  | Structure \#2 |  |  | Structure \#3 |  |  | Structure \#4 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| hkl | $2 \theta$ | $\beta$ | 1 | $2 \theta$ | $\beta$ | I |  | $\beta$ | I | $2 \theta \beta$ | I |
| 010 |  |  |  |  |  |  | 14.9 | 0 | 14 | 16.90 | 3 |
| 100 |  |  |  |  |  |  |  |  |  | 18.80 | 7 |
| 110 |  | 0 | 13 | 17.6 | 0 | 19 |  |  |  | 21.10 |  |
| $1 \overline{10}$ |  | 0 |  |  | 0 |  |  |  |  | 29.20 | 1 |
| 200 | 20.8 | 0 | 25 | 20.2 | 0 | 37 | 22.4 | 0 | 34 |  |  |
| 210 |  |  |  |  |  |  | 27.6 | 0 | 35 | 36.60 | 2 |
| 210 | 27.8 | 0 | 1 |  |  |  |  | 0 | 25 |  |  |
| 220 | 34.5 | 0 | 1 | 35.5 | 0 | 2 |  |  |  |  |  |
| $\begin{array}{r} 3 \overline{10} \\ 310 \end{array}$ | 32.8 | 0 | 9 | $\begin{array}{r} 37.3 \\ 32.1 \end{array}$ | 0 0 | $\left.\begin{array}{r} 2 \\ 15 \end{array}\right)$ |  |  |  |  |  |
| 001 |  |  |  |  |  |  |  |  |  | 6.888 | 100 |
| 101 | 11.9 |  | $4)$ | 11.3 |  | 87 | 12.2 |  | $3)$ |  |  |
| 101 | 12.8 |  | 2 |  |  | 4 |  |  | 3 |  |  |
| $\begin{aligned} & 011 \\ & 011 \end{aligned}$ | 16.6 |  | 1 | $\begin{aligned} & 17.7 \\ & 17.4 \end{aligned}$ |  | $\left.\begin{array}{r} 10 \\ 2 \end{array}\right]$ |  |  |  |  |  |
| 111 | 18.7 |  | $1)$ |  |  |  |  |  | 4 | 22.018 |  |
| 1111 | 18.2 |  | 1. |  |  |  |  |  | 2 2 |  |  |
| 211 | 24.4 |  | 3 |  |  | 6 |  |  |  |  |  |
| 211 | 28.2 |  | 4 | 28.3 |  | 8 |  |  |  |  |  |
| 211 | 25.3 |  | 3 | 25.7 |  | 4 |  |  |  |  |  |
| 211 | 29.1 |  | 4 | 29.5 |  | 6 |  |  |  |  |  |
| 121 |  |  |  |  |  |  | 32.7 |  | 1 |  |  |
| 301 | 31.6 |  | 1 | 30.2 |  | 3 | 33.6 |  | 2 |  |  |
| $3 \overline{1} 1$ |  |  |  |  |  |  | 36.3 |  | 1 |  |  |
| 002 | 13.6 |  | 100 | 13.78 |  | 100 | 13.7 | 82 | 100 | 13.688 | 20 |
| $\begin{gathered} 012 \\ 012 \end{gathered}$ |  |  |  |  |  |  | 19.7 20.9 | $\begin{aligned} & 44 \\ & 41 \end{aligned}$ | 1 5 |  |  |
| 112 |  |  |  |  |  |  |  |  |  |  |  |
| 112 | 22.3 |  | $3)$ | 23.5 |  | 2 |  |  |  |  |  |
| T12 |  |  |  | 25.3 |  | 2 |  |  |  |  |  |
| $1 \overline{12}$ |  |  |  | 24.0 |  |  |  |  |  |  |  |
| $\overline{2} 02$ |  |  |  | 26.2 |  | 3 |  |  |  |  |  |

Table E. 3 (continued)


## Table E.4a

Summary of x-ray reflections in molecular simulation structures for 2,6-dichloro-PPTA; antiparallel head-to-tail polymorphs


## Table E.4b

Summary of x-ray reflections in molecular simulation structures for 2,6-dichloro-PPTA; antiparallel head-to-tail polymorphs


Table E.4b (continued)

| $\begin{array}{r} 212 \\ \frac{2}{2} 12 \end{array}$ | 33.424 | 2 |  |  | 25.832 | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 302 \\ & \frac{3}{3} 02 \end{aligned}$ | $\begin{array}{ll} 31.1 & 26 \\ 29.5 & 28 \end{array}$ | 1 2 |  |  |  |  |  |
| 003 | 20.781 | 11 | 20.389 | 9 |  |  |  |
| 103 | 23.062 | 1 |  |  |  |  |  |
| $\begin{aligned} & 013 \\ & 013 \end{aligned}$ | $\begin{array}{ll} 26.2 & 51 \\ 30.7 & 42 \end{array}$ | 2 1 |  |  |  |  |  |
| $\overline{2} 03$ | 26.550 | 3 |  |  |  |  |  |
| 303 | 35.236 | 3 |  |  |  |  |  |
| $\begin{aligned} & 104 \\ & 104 \end{aligned}$ | $\begin{aligned} & 29.767 \\ & 28.673 \end{aligned}$ | $\left.\begin{array}{l}1 \\ 4\end{array}\right)$ |  |  |  |  |  |
| $\begin{array}{r} 114 \\ 114 \end{array}$ | $\begin{aligned} & 32.458 \\ & 33.056 \end{aligned}$ | 22 |  |  | 30.564 | 1 |  |
| 204 | 32.159 | 2 |  |  |  |  |  |
| 214 | 38.845 | 2 |  |  |  |  |  |
| 005 | 34.881 | 2 |  |  |  |  |  |
| 105 |  |  |  |  | 35.675 | 1 | - |
| 205 | 38.364 | 2 |  |  |  |  |  |
| 006 | 42.081 | 3 | 41.389 | 4 |  |  |  |
| 106 | 43.672 | 2 |  |  | 41.783 | 1 |  |
| 016 | 43.772 .0 | 4 |  |  |  |  |  |
| 116 |  |  |  |  | 42.379 | 1 |  |

Figure E.1: $\quad$ Simulated x-ray fiber diffraction patterns for the four primary structures for 2,6-DiClPPTA, head-to-head, tail-to-tail isomers. The sets (a) through (d) correspond to the structures labelled 1 through 4, in order, in Table 7.3.
(a)
(c)

(b)
(d)

Figure E.2: Simulated $x$-ray fiber diffraction patterns for the ten primary structures for 2,6-DiCl-PPTA, head-to-tail isomers. The sets (a) through (j) correspond to the structures labelled 1 through 10, in order, in Table 7.5.
(a)

(b)
(d)

Figure E.2: $\quad$ Simulated x-ray fiber diffraction patterns for the ten primary structures for $2,6-\mathrm{DiCl}-\mathrm{PPTA}$, head-to-tail isomers (continued).
(e)

(h)

Figure E.2: $\quad$ Simulated x-ray fiber diffraction patterns for the ten primary structures for 2,6-DiCl-PPTA, head-to-tail isomers (continued).
(i)


## APPENDIX F: Description of Apparatus for Fiber Spinning

The basic process flow has been provided in Chapter 6. Included here are the details of the important components of the process. Figure F. 1 shows front and side views of the extrusion apparatus. Figures F. 2 through F. 6 present, in order, the fiber spinning extruder (F.2), the detail of the syringe-andplunger design (F.3), the solution pre-filter, used to homogenize and filter the solution while loading to the syringe for spinning (F.4), the control loop for process control and data acquisition (F.5) and the annealing operation (F.6). The dope was prepared using a teflon "screw" mixer, composed of a shaft 9.5 mm in diameter with a thread 6.4 mm in width and height winding the shaft with a pitch of 38.1 mm , fitted into a round-ended glass tube of roughly 30 mm inside diameter. The coagulation tank consisted simply of a plexiglass trough 91.3 cm long by 15.2 cm wide by 20.2 cm deep, with two freely-rotating, grooved wheels 7.6 cm in diameter located 9.0 cm from either end of the trough, to serve as guides for the coagulating filament. Table F. 1 lists the parts specifications for the major components of the spinning apparatus. Figure F. 7 illustrates the data output from a sample spinning trial, indicating control of temperature and both motor speeds, and monitoring of plunger position and extrusion force required. Finally, a listing of the FORTRAN control program follows.

## Table F. 1

Equipment parts list for fiber spinning apparatus

| Drive Screw | Star Linear-Schlitten \#1285-110-02, <br> Traverse: Ausführung B, 240 mm Hub | 5 mm pitch screw |
| :---: | :---: | :---: |
| Drive Motor | Sanyo "Super R" R 511-T002E11 DC Servo-Motor <br> Sanyo E500200C31 Oplical Shaft Encoder <br> 353:1 Gear Box <br> 4-Quadrant PDT-A03-20 regulator with PS1116 transformer | $-3000 \mathrm{rpm}-+3000 \mathrm{rpm}$ $200 \mathrm{impulses} / \mathrm{rcv}$ |
| Wind-up Motor | Sanyo "Super R" R 301-T011 DC Servo-Motor Litton 820A LDBZ-500-236-12-A Optical Shaft Encoder 4-Quadrant PDT-A01-10 regulator with PS1114 transformer | $-3000 \mathrm{rpm}-+3000 \mathrm{rpm}$ 500 impulses/rev |
| Force Transducer | Hottinger Baldwin Messtechnik C9/10kN with MC3 converter | 0-10kN |
| Heating Elements | VIMO-CNS 200 watt |  |
| Thermocontrol | Pt 100 passive thermal sensing element Amplification to $10 \mathrm{mV} /{ }^{\circ} \mathrm{C}$, accuracy to $\pm 0.1^{\circ} \mathrm{C}$ PID temperature regulator | $0-100^{\circ} \mathrm{C}$ |
| Data Acquisition | MetraByte DAS-20 for IBM PC/XT/AT compatibles |  |

Figure F.1: Front and side views of the extrusion apparatus.


Figure F.2: Diagram of the fiber spinning extrusion apparatus.


Figure F.3: $\quad$ Diagram of the syringe and plunger design to spin polymer dope.


Figure F.4: Diagram of the solution pre-filter for homogenization and loading of the solution to the syringe.


Figure F.5: $\quad$ Schematic of the process control and data acquisition loop for fiber spinning.


| (1) | Optical Shaft Encoder (A signal, B signal, Reference) |
| :---: | :---: |
| (2) | DC Servo-Motor |
| (3) | Tachogencrator |
| (1) | Upper Interrupt Switch for Drive Motor |
| (6) | Lower Interrupt Switch for Drive Motor |
| (6) | Plunger Position Refercace Switch |
| (1) | Plunger Stop Switch |
| (2) | Force |
| (0) | Temperature |
| 48 | Program Control Interrupt Switch |
| 0 | Regulator |
| (9) | MBC DAS-20 Digital Input |
| (3) | MBC DAS-20 Analog Output ( -10 to +10 volts) |
| 0 | MBC DAS-20 Analog Input (differential inputi -10 to +10 volts) |
| - | Control Program |

Figure F.6: Diagram of the fiber heat treatment process step.


Furnace

Figure F.7: Data output from a sample spinning experiment illustrating motor control at various rates. The spinnerette used in this example was $120 \mu$ in diameter.


PROGRAM SPINCON

```
C
C Control program for operation and data acquisition during fiber
C spinning experiments.
Version: December, 1989
    G.C. Rutledge
    Microsoft FORTRAN77 v4.01
    Motor control via DAS-20 modes 3 (A/D), 7 (D/A), 14 (DI) and 23
    (Frequency counter).
    CHARACTER*10 OUTF1
    CHARACTER*10 OUTF2
    INTEGER*2 DAP1(10), DAP2(10), FRP1(10),FRP2(10),DIP (10), ADP (10)
    INTEGER*2 BASE, INTLEV, DMALEV
    INTEGER*2 IERR, DAMODE, FRMODE, DIMODE, INP, IMODE, ADMODE
    INTEGER*2 DAS20, SEGADR, OFFADR, SEGPTR, OFFPTR
    INTEGER*2 TRIG, RCODE
    INTEGER*2 I, J, K, N
    INTEGER*2 JHR, JMIN, JSEC, JHUN, KHR, KMIN, KSEC, KHUN
    INTEGER*4 JCLOCK
    INTEGER*4 CRATE, BUFFER, ALLOC
    INTEGER*4 WORD1,WORD2,MASK1 (16),mask2(16)
    INTEGER*4 BSZ,BSZM1,BSZD
    REAL*8 SRATE, RATE, RLIMIT, PI, Z
    REAL R301(2),R501(2)
C
C
C
C R501 and R301 motor calibration curve parameters (in rpm):
(eg) DAO = R301(1)*RPM + R301(2)
    DATA R501/ 0.6804, -6. /
    DATA R301/ 0.6338, -5. /
    DATA PLDIAM/ 10. /
    DATA DWDIAM/ 35./
    DATA PLIMIT/ 138.5 /
    DATA VISC /500./
    DATA FSLP,FINT / 1846, -20 /
    DATA TSLP,TINT / 100, 0 /
CONSTANTS
    EPS = 1E-5
    PI = 3.1415926535
    SRATE = 5000000.0
    RLIMIT = 76.3
    OFFSET = 0
    BASE = #300
    INTLEV = 2
    DMALEV = 1
    BSZ = 32766
    BSZD = 2*BSZ
C
Input error message and redistribution
GO TO }5
6 0 ~ C O N T I N U E ~
    WRITE(*,*) 'Invalid Input. Please, try again !'
    GO TO (101,201) IERR
    CONTINUE
C Initialize DAS-20 Board using mode 0 and allocate buffer:
    IMODE = 0
    DAP1(1) = BASE
    DAP1(2) = INTLEV
    DAP1(3) = DMALEV
```

```
    RCODE = DAS20(IMODE, DAP1)
    IF (RCODE .NE. 0) WRITE(*, 120) IMODE, RCODE
    CONTINUE
    BUFFER = ALLOC (BSZ)
    IF (BUFFER .EQ. O) WRITE(*,*) 'Cannot Allocate Buffer'
Step 0: Print out experiment configuration and start clock
    WRITE(*,*) 'Insert spinnerette diameter (mm) > '
    READ(*,*) DIEDIAM
    WRITE(*,*) 'Insert capillary length (mm) >'
    READ(*,*) DIELEN
    WRITE(*,*) 'Spinnerette Diameter (mm) : ',DIEDIAM
    WRITE(*,*) 'Capillary Length (mm) : ',DIELEN
    WRITE(*,*) 'Approx. Solution Visc (p) : ',VISC
    WRITE(*,*) 'Draw Wheel Diameter (mm) : ',DWDIAM
    WRITE(*,*) ' '
C
C "Start" clock
C
    CALL GETTIM( KER,KMIN, KSEC,KHUN )
    LHR = KHR
    LMIN = KMIN
    LSEC = KSEC
    LHUN = KHUN
C
C Set modes of operation:
C
C
C
C
C
C
C
c
C Step 1: Raise plunger from current position to Positionschalter
```

```
    OUTF1 = 'TSPIN1.DAT'
    OUTF2 = 'CSPIN1.DAT'
    OREN (UNIT=11,FILE=OUTF1)
    OREN (UNIT=10,FILE=OUTF2)
    890
    CLOSE (UNIT=11)
    CLOSE (UNIT=10)
    DAP2(2) = NINT( R3O1(2) )
    RCODE = DAS20( DAMODE,DAP2 )
    WRITE(*,*) 'Ralsing plunger to zero position ... '
    DRI = -2900.
    DAP1(2) = NINT( R501(1)*DR1 + R501(2) )
    RCODE = DAS2O( DAMODE,DAP1 )
    800 RCODE = DAS2O( DIMODE,DIP )
    IBIT7 = DIP(1)/128
    IF( IBIT7*KOSHTDN.GT.0 ) GO TO }81
    KOSHTDN = MAX( O,IBIT7 )
    GO TO 800
    810 CONTINUE
    POS1 = 0.
    WRITE(*,*) 'Plunger position initialized '
    Step 2: Lower plunger to Load Point position (30 mm)
    DR1 = 0
    DAP1(2) = NINT( R501(2) )
    RCODE = DAS20( DAMODE,DAP1 )
    qAUSE 'Hit RET to lower plunger to load point pos. '
    WRITE(*,*) 'Lowering plunger to Load Point ... '
    DRI = 2500.
    PLOAD = 25.
    DAP1 (2) = NINT( R501(1)*DR1 + R501 (2) )
    CALL GETTIM( KHR,KMIN,KSEC,KHUN )
    RCODE = DAS2O( DAMODE,DAP1 )
    820 CALL GETTIM( JHR,JMIN, JSEC, JHUN )
    JCLOCK = 100*(60*(60*(JHR-KHR) + (JMIN-KMIN)) + (JSEC-KSEC))
        + + (JHUN-KHUN)
            KHR = JHR
            KMIN = JMIN
            KSEC = JSEC
            KHUN = JHUN
            DCLOCK = REAL(JCLOCK)/6000.
            POS1 = POS1 + DR1*DCLOCK*5./353.
            IF( POS1.GE.PLOAD ) GO TO }83
            GO TO 820
    830
            CONTINUE
            DR1 = 0
            DAP1(2) = NINT( R501(2) )
            RCODE = DAS20( DAMODE,DAP1 )
            RCODE = DAS20( FRMODE,FRP1 )
            RCODE = DAS20( FRMODE, FRP2 )
            FREQ1 = REAL( FRP1(3) )*300./REAL( FRP1(1) )
            FREQ2 = REAL( FRP2(3) )*120./REAL( FRP2(1) )
            IF( FREQ1.GT.2 ) WRITE(*,*) 'Motor 1 not stationary'
            IF( FREQ2.GT.2 ) WRITE(*,*) 'Motor 2 not stationary'
            PAUSE 'Load Plunger and hit RETURN to continue >>>'
C
C Step 3: Lower plunger into barrel
            WRITE(*,*) 'Enter approximate quantity of solution loaded (ml) > '
            READ(*,*) QSOLN
            PFULL = 4000.*QSOLN/PI/PLDIAM**2
            WRITE(*,*) 'Lowering plunger to Solution ...',
            DR1 = 2000.
            PSOLN = PLIMIT - PFULL - 2.
            IF(PLIMIT-PSOLN.LE.O.1) WRITE(*,*) 'WARNING: No Solution'
            DAP1(2) = NINT( R501(1)*DR1 + R501 (2) )
            CALL GETTIM( KHR, KMIN, KSEC, KHUN )
            RCODE = DAS2O( DAMODE,DAP1 )
840 CALL GETTIM( JHR, JMIN, JSEC,JHUN )
    JCLOCK = 100*(60*(60*(JHR-KHR) + (JMIN-KMIN)) + (JSEC-KSEC))
                                + (JHUN-KHUN)
```

```
        KHR = JHR
        KMIN = JMIN
        KSEC = JSEC
        KHUN = JHUN
    DCLOCK = REAL(JCLOCK)/6000.
    POS1 = POS1 + DR1*DCLOCK*5./353.
    IF( POS1.GE.PSOLN ) GO TO 850
    GO TO }84
    850 CONTINUE
    DR1 = 0.
    DAP1(2) = NINT( R501(2) )
    RCODE = DAS2O( DAMODE,DAP1 )
    RCODE = DAS20( FRMODE,FRP1 )
    RCODE = DAS20( FRMODE,FRP2 )
    FREQ1 = REAL( FRP1 (3) )*300./REAL(FRP1 (1) )
    FREQ2 = REAL( FRP2(3) )*120./REAL( FRP2(1) )
    IF( FREQI.GT.3 ) WRITE(*, *) 'Motor 1 not stationary'
    IF( FREQ2.GT.3 ) WRITE(*,*) 'Motor 2 not stationary'
    PAUSE 'Plunger in barrel position; hit RETURN to continue >>> '
    NOUT = NOUT + 1
    OUTF1 = 'TSPIN'//CHAR(NOUT + ICHAR('O'))//'.DAT'
    OUTF2 = 'CSRIN'//CHAR(NOUT + ICHAR('O'))//'.DAT'
    OPEN(UNIT=11,FILE=OUTF1)
    OPEN(UNIT=10,FILE=OUTF2)
    Enter loop size
    IERR = 1
    CONTINUE
    WRITE(*,*) 'Enter size of control loop > '
    READ(*, 132, ERR = 60) NLP
    IF( NLP.LT.1 ) GO TO 60
    Load motor speed setpoint
    (max range is -42.5 to +42.5 mm/min;
    maximum is approximated using F=8(pi)uLv
    F in newtons, }u\mathrm{ in poise, L in mm, and v in mm/min
        FMAX = 10000.
        FVMAX = 2.387E7*FMAX*(DIEDIAM/PLDIAM)**2/VISC/DIELEN
C
    400 WRITE (*,41)
    WRITE(*,42) FVMAX/1000.
    FORMAT(1X,'Input linear extrusion rate ie nom fiber velocity')
    FORMAT (1X,' (up to ',f8.2,' m/min) >')
    READ(*,*) FASER
    VRPM1SET = (DIEDIAM/PLDIAM)**2*FASER*1000
    RPM1SET = VRPM1SET*70.6
    WRITE(*,*) 'Input draw wheel linear rate ',
    '(-329 to +329 m/m1n) >'
    READ(*,*) VRPM2SET
    IF (ABS (FASER).GE.EPS) THEN
        DRAWRAT = VRPM2SET/FASER
    ELSE
        DRAWRAT = 9999999
    END IF
    WRITE(*,*) 'Nominal draw ratio: ',DRAWRAT
    RPM2SET = VRPM2SET*1000./DWDIAM/PI
    WRITE(*,*) 'Is this correct? ( }0=n=\mathrm{ no, 1=yes) > '
    READ(*,*) KORREKT
    CALL GETTIM( JHR,JMIN,JSEC,JHUN )
    LCLOCK = 100*(60*(60*(JHR-LHR) + (JMIN-LMIN)) + (JSEC-LSEC))
        + (JHUN-LHUN)
            CUMTIME = REAL (LCLOCK)/100.
            WRITE(10,1) CUMTIME ,faser,vrpm2set,drawrat
            IF(KORREKT.NE.1) GO TO 400
            IF (ABS (FASER).LE.ERS) THEN
            DR1 = 0
            DAP1(2) = NINT( R501(2) )
            RCODE = DAS2O( DAMODE,DAP1 )
            RCODE = DAS20( FRMODE,FRP1 )
```

```
            FREQ1 = REAL( FRP1(3) )*300./REAL( FRP1(1) )
        END IF
        IF (ABS(VRPM2SET).LE.EPS) THEN
            DR2 = 0
            DAP2(2) = NINT( R3O1(2) )
            RCODE = DAS20( DAMODE,DAP2 )
            RCODE = DAS20( FRMODE,FRP2 )
            FREQ2 = REAL( FRP2(3) )*120./REAL( FRP2(1) )
    END IF
    WRITE(*,*) 'Beginning Spin ...,'
C
C Gate interval must be set based on RPM at low RPM:
Motor1 sends 200 1mpulses/cycle; Motor2 sends 500 impulses/cycle
C
    450
C
C Set a maximum speed step for transitions
C
    STPCNT = 50
    IBIT6C = 0
    IBIT7C = 0
C
C BegIn Control Loop >>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>
C
C Compute signal(s) to send to motor(s) (variable gain):
    DELTA1 = RPM1SET - FREQ1*KDIR1
    IF( ABS(DELTA1/(RPM1SET+EPS)).LE.0.025 ) THEN
        GADJ1 = . 05
    ELSE IF( ABS(DELTA1/(RPMISET+EPS)).LE.O.1 ) THEN
        GADJ1 = . 2
    ELSE
        GADJ1 = . 7
    END IF
    STPMAX = REAL( FREQ1/100 + 1 )*STPCNT
    STPM = MIN( STPMAX, ABS (DELTA1) )
    SGN = SIGN( 1.,DELTAI )
    DGAIN1 = STPM/( DELTA1*SGN + EPS )*GADJ1
    DR1 = ( DELTA1*DGAIN1 + DR1 )
    DR1 = MAX( -3000.,MIN( 3000.,DR1) )
    DAP1(2) = NINT( R501(1)*DR1 + R501(2) )
C
    DELTA2 = RPM2SET - FREQ2*KDIR2
    IF( ABS(DELTA2/(RPM2SET+EPS)).LE.0.025 ) THEN
        GADJ2 = . 05
    ELSE IF( ABS(DELTA2/(RPM2SET+EPS)).LE.0.1 ) THEN
        GADJ2 = . 2
    ELSE
        GADJ2 = . }
    END IF
    STPMAX = REAL( FREQ2/100 + 1 )*STPCNT
    STPM = MIN( STPMAX ,ABS (DELTA2) )
    SGN = SIGN( 1.,DELTA2 )
    DGAIN2 = STPM/( DELTA2*SGN + EPS )*GADJ2
    DR2 = ( DELTA2*DGAIN2 + DR2 )
    DR2 = MAX ( -3000.,MIN( 3000.,DR2) )
    DAP2(2) = NINT( R3O1(1)*DR2 + R301(2) )
C
C Get time of iteration, update postions, and send signal(s)
    CALL GETTIM( JHR, JMIN,JSEC,JHUN )
    JCLOCK = 100*(60*(60*(JHR-KHR) + (JMIN-KMIN)) + (JSEC-KSEC))
```

```
    + + (JHUN-KHUN)
    LCLOCK = 100*(60*(60*(JHR-LHR) + (JMIN-LMIN)) + (JSEC-LSEC))
                + (JHUN-LHUN)
    DCLOCK = REAL(JCLOCK)/6000.
        CUMTIME = REAL (LCLOCK)/100.
        POS1 = POS1 + REAL (KDIR1)*FREQ1*DCLOCK*5./353.
    IF( (IP.NE.1).AND.((MAX (REAL (KDIR1)*PLIMIT, 0.)-REAL (KDIR1)*POS1)
    + .LE.O.01) ) GO TO 490
        POS2 = POS2 + FREQ2*DCLOCK
    RCODE = DAS20( DAMODE,DAP1 )
    RCODE = DAS2O( DAMODE,DAP2 )
    KHR = JHR
    KMIN = JMIN
    KSEC = JSEC
    KHUN = JHUN
c
C Read frequency counters
    RCODE = DAS20( FRMODE,FRP1 )
    RCODE = DAS2O( FRMODE,FRP2 )
    FREQ1 = REAL( FRP1 (3) )*FR1CNV
    FREQ2 = REAL( FRP2 (3) )*FR2CNV
Force Transducer (CHO), Temperature (CH1) and Program Switch (CH2)
******* For reasons not understood, it appears to be necessary to read
******* each AI twice in succession to get accurate values
******* possibly a board communicaiton problem
    ADP(1) = 3
    ADP(2) = 0
    RCODE = DAS2O( ADMODE,ADP )
    ADP(1) = 3.
    ADP(2) = 0
    RCODE = DAS2O( ADMODE,ADP )
    IF(RCODE.NE.0) WRITE(*,*) 'RCODE ERROR ',RCODE
    FVOLTS = ADP (1)*10./4096.
    ADP(1) = 4
    ADP (2) = 1
    RCODE = DAS2O( ADMODE,ADP )
    ADP(1) = 4
    ADP(2)=1
    RCODE = DAS20( ADMODE,ADP )
    IF(RCODE.NE.0) WRITE(*,*) 'RCODE ERROR ',RCODE
    TVOLTS = ADP(1)*1./4096.
    ADP (1) =1
    ADP(2) = 2
    RCODE = DAS2O( ADMODE,ADP )
    ADP(1) = 1
    ADP(2) = 2
    RCODE = DAS20( ADMODE, ADP )
    IF(RCODE.NE.O) WRITE(*,*) 'RCODE ERROR ',RCODE
    SVOLTS = ADP (1)*20./4096.
    INTERUPT = NINT(SVOLTS/5.)
C
C Record data
C
    CEXP = CUMTIME
    PEXP = POS1
    R1EXP = REAL (KDIR1)*((FREQ1/70.6)*(PLDIAM/DIEDIAM)**2)
        /1000
    R2EXP = REAL(KDIR2)*FREQ2*DWDIAM*PI/1000
    FEXP = FVOLTS*ESLP + FINT
    TEXP = TVOLTS*TSLP + TINT
    WRITE (11,1) CEXP/60,PEXP,R1EXP,R2EXP,FEXP/100,TEXP/10
    WRITE(*,5) LP,CEXP,PEXP,R1EXP,R2EXP,FEXP,TEXP
C
C Check for software shutdown endswitches open
```

```
RCODE = DAS2O( DIMODE,DIP )
```

RCODE = DAS2O( DIMODE,DIP )
IBIT7 = DIP(1)/128
IBIT7 = DIP(1)/128
NDIP = DIP(1) - IBIT7*128

```
NDIP = DIP(1) - IBIT7*128
```

```
    IBIT6 = NDIP/64
    KNSHTDN = KDIR1*( IBIT6 - IBIT7 )
    IF( KNSHTDN*KOSHTDN.GT.O ) GO TO 490
    KOSHTDN = MAX( O,KNSHTDN )
    IF(LP.EQ.1) KOSHTDN = 0
C
C If motor(s) currently run below critical speed, determine direction of
C rotation (unless motor(s) is essentially idle; ie rpm < 1)
C
    LOW1 = 1
    LOW2 = 1
    IF( (FREQ1.GT.DR1CRT).OR.(FREQ1.LE.2.0) ) LOW1 = 0
    IF( (FREQ2.GT.DR2CRT).OR. (FREQ2.LE.2.0)) LOW2 = 0
    IF( (LOW1.NE.1).AND.(LOW2.NE.1) ) GO TO 1100
        NCNT = 0
        NDXO1 =0
        NDXO2 = 0
        NCYC1 = 0
        NCYC2 = 0
        NERR1 = 0
        NERR2 = 0
        KTLY1 = 0
        KTLY2 = 0
        IDO1 = 0
        IDO2 = 0
        RCODE = DAS2O( DIMODE,DIP )
c
C Check for software shutdown endswitches open
C
    IBIT7 = DIP(1)/128
    NDIP = DIP(1) - IBIT7*128
    IBIT6 = NDIP/64
    KNSHTDN = KDIR1*( IBIT6 - IBIT7 )
    IF( KNSHTDN^KOSHTDN.GT.O ) GO TO 490
    KOSHTDN = MAX( 0,KNSHTDN )
C
C Signal decomposition:
C
    NDIP = NDIP - IBIT6*64
    IBIT5 = NDIP/32
    NDIP = NDIP - IBIT5*32
    IBIT4 = NDIP/16
    NDIP = NDIP - IBIT4*16
    IBIT3 = NDIP/8
    NDIP = NDIP - IBIT3*8
    IBIT2 = NDIP/4
    NDIP = NDIP - IBIT2*4
    IBIT1 = NDIP/2
    IBITO = NDIP - IBIT1*2
    IF( LOW1.NE.1) GO TO }121
            NDXN1 = IBITO
                NCYC1 = NCYC1 + ABS(NDXN1 - NDXO1)
            IDN1 = 2*IBIT1 + IBIT2
            NELEM = 4*IDO1 + IDN1 + 1
            KDR = MASK1 (NELEM)
            NERR1 = NERR1 + KDR/2
                IF( ABS (KDR).EQ.1 ) THEN
                NERR1 = NERR1 + ABS (KDR-KDIR1)/2
                KTLY1 = KTLY1 + KDR
                    KDIR1 = SIGN(1,KTLY1)
            END IF
            IDO1 = IDN1
            NDXOI = NDXN1
            IF (NCYC1.GT.1) LOW1 = 2
1210
            IF( LOW2.NE.1) GO TO 1220
            NDXN2 = IBIT3
            NCYC2 = NCYC2 + ABS(NDXN2 - NDXO2)
            IDN2 = 2*IBIT4 + IBIT5
            NELEM = 4*IDO2 + IDN2 + I
            KDR = MASK2 (NELEM)
            NERR2 = NERR2 + KDR/2
```

```
            IF( ABS(KDR).EQ.1 ) THEN
            NERR2 = NERR2 + ABS (KDR-KDIR2)/2
            KTLY2 = KTLY2 + KDR
            KDIR2 = SIGN(1,KILY2)
            END IF
            IDO2 = IDN2
            NDXO2 = NDXN2
            IF (NCYC2.GT.1) LOW2 = 2
            IF( (LOW1.EQ.1).OR. (LOW2.EQ.1) ) GO TO 1200
            IF( LOW1.EQ.2 ) THEN
                ERROR1 = REAL (NERR1)/REAL (200*NCYC1)
                IF( ERROR1.GT.0.009 ) WRITE(*,*) 'Low speed error 1 = ',
            ERROR1
            END IF
            IF( LOW2.EQ.2 ) THEN
                ERROR2 = REAL (NERR2)/REAL(500*NCYC2)
                IF( ERROR2.GT.0.009 ) WRITE(*,*) 'Low speed error 2 = ',
                ERROR2
            END IF
        GO TO 1010
    CONTINUE
        KDIR1 = SIGN( 1,DR1 )
        KDIR2 = SIGN( 1,DR2 )
        IF(INTERUPT.EQ.1) GO TO 220
        CONTINUE
        WRITE(*,*) ' 2,2,3,4,5 - REINITIALIZE, Reset LOOPSIZE, SPEEDS,',
                            REPEAT, END 
        IERR = 2
        CONTINUE
        READ(*, 132, ERR = 60) INP
        IF ((INP .GT. 5) .OR. (INP .LE. 0)) GOTO 60
        GOTO ( 890,101,400,450,600 ) INP
        CONTINUE
        IF( IBIT6.EQ.I ) WRITE(*,*) 'Endschalter unten geschlossen'
        IF( IBITT.EQ.1 ) WRITE(*,*) 'Positionschalter oben geschlossen'
        CONTINUE
        DAPI(2) = NINT(R501(2) )
        DAP2(2) = NINT( R3O1(2))
        RCODE = DAS20( DAMODE,DAP1 )
        RCODE = DAS20( DAMODE,DAP2 )
        GO TO 220
6 0 0
    ONTINUE
        DAP1(2) = NINT( R501 (2) )
        DAP2(2) = NINT( R3O1(2) )
        RCODE = DAS20( DAMODE,DAP1 )
        RCODE = DAS2O( DAMODE,DAP2 )
        STOP
        FORMAT (' Mode ', I2, ' , Error = ', I4)
        FORMAT (16I7)
        FORMAT(1X,F10.2,2X,F6.2,2X,F6.2,2X,F6.2,2X,F7.1, 2X,F6.1)
        FORMAT (1X,I4,F10.2,2X,F6.2,2X,F6.2,2X,F6.2,2X,F7.1,2X,F6.1)
        END
```


## APPENDIX G: FORTRAN Listing of Program for Simulation of Ordered Polymer Structure

This appendix contains one version of the FORTRAN code used to simulate the polymer solid state, calculate potential energy, and minimize with respect to degrees of freedom. It also contains the subroutines required for imposing deformation. The following subroutines are listed for the main program:

| CMODEL (main) | UNITCEL | RDSTRAIN | EES |
| :--- | :--- | :--- | :--- |
| CELGEN | DENS | ATRNS | VECTORIZ |
| LDVAR | NONBOND | MM | CURTATE |
| LDCHN | SUBSET | MV | CURINV |
| RANDS 1 | ENERGY | VV | MULMAX |
| COORDN | RTRNS | MT | VA10A |
| PHENYL | STRAIN | MA | MC11A |
| HELIX | DATARD | JACOBI |  |
| GYRA | RDWRITE | ELJ |  |

Following the main code is a listing of the major replacement subroutines required for simulating the modified chain description used to generate mechanical property information. Lastly, sample input files required by the program are provided.

C
C


* POLYMER CRYSTAL SIMULATION PROGRAM * *
* *OLIMER CRYSIAL SIMULAIION PROGRAM *
* GREGORY C. RUTLEDGE $\quad$ *
DEPARTMENT OF CHEMICAI ENGINEERING
MASSACHUSETTS INSTITUTE OF TECHNOLOGY *
NOVEMBER 1989 *
* (C) Copywrite 1989: Gregory C. Rutledge, Ulrich w. Suter *
$\star$ *

The Polymer Crystal Simulation Program (PCSP) is designed for the
purpose of conducting general studies into the nature and structure
of linearly connected (macromolecular) systems. The simulation is
based upon a Molecular Mechanics interpretation of atomic level
energetic interactions. In simulating crystalinity, the program
assumes an infinite connectivity along the backbone of the molecular
chain, such that the crystal may be represented by a subset of the
molecular architecture which describes a helical curve in space.
The program then constructs a regular repetition of this represent-
ative unit in three dimensions, subject to variations and con-
constraints specified in the material representation. The internal
energy, both intramolecular and intermolecular, of such a material
representation is then calculated based on appropriately parameter-
ized two-body force interactions between atoms or atomic groups and
other functions appropriate to an energetic description of the
displacement degrees of freedom available to the atoms in the
structure. Minimization of the total (summed) energy with respect
to all degrees of freedom leads then to the prediction of the
energetically most stable local material representation.
The main program serves to control the call and execution of the
Polymer Crystal Simulation. Other subroutines control the genera-
tion of atomic coordinates for the representative (parent) chain
subset and the corresponding helical parameters for the parent chain
conformation (either a true helix or an estimate based on the radius
of gyration tensor of the suset). The parent chain is mapped onto a
regular non-orthogonal lattice of chain positions, subject to
additional translational and/or orientational variations. The
resulting explicit (and potentially irregular) three dimensional
construction may then be "embedded" in an implicit regular three
dimensional array of atomic interaction points of unlimited size.
The minimum set of energetic interations may then be determined and
computed and summed for use by a standard non-derivative minimiza-
tion routine. The code as written here is specific for the simula-
tion of isomers of the family of ring-substituted poly- (p-phenylene
terephthalamide) as isolated chains or as identical chains packed in
a regular lattice. Program options include scanning of intramolec-
ular (conformational) and/or intermolecular (crystal packing)
variables and optimization of same. A fixed strain may be imposed
on the unitcell by means of the inut file us.dt, which assumes one
of twelve predefined unitcell representations for the general lattice.
COMMON blocks and PARAMETER statements are set up to enable easy
variation of the scale of simulation. Important dimensions of the
simulation are shown below; change of the appropriate dimension in
all PARAMETER statements will introduce the change throughout the
simulation:
Description of Program Dimension Parameters
NKIND $=$ Numger of atom types of interest
NIBB $=$ Number of internal building blocks per repeat unit
MNRU $=$ Maximum number of repeat units per chain
MNCH = Maximum number of chains (a square number)
MNAT $=$ Maximum number of atoms per chain +1

```
    MNIC = Maximum number of independently orientable chains
    MCOP = MNIC*5
    MOPT = MCOP+6
Some parameters are specific to the PPTA polymer family
    NRNG = Number of rotatable rings per repeat unit = 2
    NSUB = Number of substitutions possible per repeat unit = 2
    MNSB = NSUB*MNRU
    NDOF = NIBB + NRNG
For the Head-to-Head, Tail-to-Tail isomer, it is possible to
reconfigure the code to consider a 56 atom repeat unit:
    NRNG --> 4
    NIBB --> 16
    NDOF --> 20
    Arrays G,H,W dimension --> 5995
    LMV (2,9) --> LMV (2,18)
    VARS(97) --> VARS(109)
    OMVARS (97) --> OMVARS (109)
    GRAD(97) --> GRAD(109)
    disable "NUNIQ = 2" option in DATARD subroutine
```


The simulation control proceeds as follows:
Input Variables, Ranges,
and Control Intructions
Inititialize Parameters
Loop Over All Lattices:
Loop Over All Conformations:
MAIN
Input Variables, Ranges,
Inititialize Parameters
$1<--$ DATARD <-- RDSTRAIN
1
|
Execute Model: or VELGEN
Load Current Parameters
| <-- LDVAR
Generate Parent Chain $\quad \mid<--$ COORD <--PHENYL
Generate Parent Chain $\quad \mid<--$ COORD <--PHENYL
Determine Helix $\quad<--$ HELIX (GYRA)
Determine Helix | <-- HELIX (GYRA)
Generate Lattice $\quad \mid<--$ UNITCEL <--STRAIN
Initialize Charge and
$\mid<--$ NONBOND
Connectivity
(Perform Ring Substitution] | <-- SUBSET <-- RANDS1
Load Lattice Multiplicity
$\mid<-=$ SUBSET <-- RANDSI $\mid$
Calculate Model Energy
| <-- ENERGY <-- ELJ,
Output Results
Execute Model: or VA10
EES !
| <-- RESWR
End
1

CHARACTER*10 FNAME
PARAMETER ( $\mathrm{NKIND}=9, \mathrm{NIBB}=8, \mathrm{MNRO}=6, \mathrm{MNCH}=9, \mathrm{MNAT}=179$, $\mathrm{NRNG}=2$, $\mathrm{NSUB}=4$,
$+\quad \mathrm{MNSB}=24, \mathrm{NDOF}=10, \mathrm{MNIC}=9, \mathrm{MCOP}=45, \mathrm{MOPT}=51$ )
COMMON /CHAINS/ PHI (NIBB),THETA (NIBB), BL (NIBB), PHIR (NRNG),
$+\quad \operatorname{KSB}(4), \mathrm{PCH}(4, \mathrm{MNAT}), \mathrm{NARU}$, NRUS, NCH,
$+$
+
+
+
+
$+$
COMMON /SCAN/ NSTOT, NSANG,NSCAN (NDOF), SCANO(NDOF), SCAND(NDOF),
NCTOT, NCMUT, NMUT (MOPT), CMUTO (MOPT), CMUTD (MOPT)
COMMON /CELL/ $\operatorname{CELD}(3), \operatorname{CELA}(3), \operatorname{CHSPEC}(5, \operatorname{MNCH}), \mathrm{CHLOC}(3, \mathrm{MNCH})$,
COMMON /CELL/ CELD (3), CELA (3), CHSPEC (5,
NUNIV, KONDENS, TEMP, DENSC
COMMON /CNTR/ IENTRY1,IENTRY2

```
        COMMON G(97),H(4753),W(291)
        COMMON /MINIM/ LMV (2,9),LCV (5,11),OMVARS (97)
        COMMON /OPRS/ NVAR,VARS(97),GRAD(97),GRADNM
        COMMON /STRN/ KUC,KASE,ST(3,3),SUCV (3,3),DINIT,DRRIME,DCON
        DIMENSION PHISTR(NDOF),INDSC(NDOF),SPECSTR(MOPT),INDMU (MOPT),
    +
    DIMENSION PHISTRTP(NDOF),INSTR(5)
    EXTERNAL CELGEN
    DEGRAD = 3.1415926535/180.
    IENTRY1 = 0
    IENTRY2 = 0
Read variable/instruction input file and open output file
    MODE = 1
    CALL DATARD( NVAR,INSTR,MODE )
    WRITE(*,*) ' Input PCSP Results Filename (up to 10 characters)',
    +
    READ(*,2) FNAME
    FNAME = 'U.res'
    OREN (UNIT=11, FILE=FNAME )
    WRITE(11,3) FNAME
C
C Initial values of variables which may be scanned must first be
C stored for future use. These variables are the torsion angles
C (PHI(8) and PHIR(2)) of the repeat unit, loaded into PHISTR and
C PHISTRTP, and the crystal specifications (CHSPEC(3,J) for the first
C nine chains,CELD(3),CELA(3)), loaded 1nto SPECSTR.
    DO }100\mathrm{ I=1,NDOF
        IF (I.LE.NIBB) THEN
            PHISTR(I) = PHI(I)
            PHISTRTP(I) = PHI(I)
            THESTR(I) = THETA(I)
        ELSE
            PHISTR(I) = PHIR(I-NIBB)
            PHISTRTP(I) = PHIR(I-NIBB)
        END IF
        INDSC(I) = 1
    CONTINUE
C
    DO 200 J=1,MNIC
        DO 200 I=1,5
                KK = I+(J-1)*5
                SPECSTR(KK) = CHSPEC(I,J)
                INDMU (KK) = 1
            CONTINUE
    noconcur
    DO 250 I=1,3
        KK = MCOP+I
        SPECSTR(KK) = CELD(I)
        INDMU(KK) = 1
    CONTINUE
    noconcur
    DO 260 I=1,3
        KK = MCOP+3+I
        SPECSTR(KK) = CELA(I)
        INDMU (KK) = 1
    CONTINUE
C
C If optimization is desired, set parameters for VAlOA
    IF (NVAR.EQ.O) GO TO }30
    TENRGY = 0.
    MODE = 1
    MAXCL = 10000
    IPRINT = 1
    STEPL =0.001
    EPSVA =0.001
300 CONTINUE
```

```
C
****************************************************************************
Here begin the nested loops for scanning of variables. The complete
scan space is divided into two nested loops: LOOP 1000 scans through
all conformational variables; LOOP 1100 scans through all cell
packing variables.
C
C
C
Test for inadmissable angle combinations in CELA(3):
        CELANEW = CELA (3)
        TLIM = SIN(CELA (1)*DEGRAD)*SIN(CELA (2)*DEGRAD) +
                        COS (CELA (1) *DEGRAD)*COS (CELA (2)*DEGRAD)
            BLIM = -SIN(CELA (1)*DEGRAD)*SIN (CELA (2)*DEGRAD) +
                COS (CELA (1)*DEGRAD)*COS (CELA (2)*DEGRAD)
                IF( COS((CELA (3)-1.)*DEGRAD).LT.TLIM ) THEN
                    IF( COS((CELA(3)+1.)*DEGRAD).GT.BLIM ) THEN
                    GO TO 480
                    ELSE
                    CELANEW = ACOS( BLIM )/DEGRAD - 5.
                    END IF
                ELSE
                    CELANEW = ACOS (TIIM )/DEGRAD + 5.
                END IF
                IF( ABS (CELANEW-CELA (3)).LE.15.) GO TO 470
                    WRITE(11,*) 'Exit due to Inadmissable Cell Angles'
                            GO TO 600
470 CELA (3) = CELANEW
        WRITE(11,*)'Inftial Gamma Angle Correction:Gamma= ',CELA(3)
    480 CONTINUE
C
Call Case Simulation using either optimization or no-optimization
mode:
        IF (NVAR.EQ.0) THEN
    No optimization desired:
            CALL CELGEN(NVAR,VARS,TENRGY)
        ELSE
    Optimization desired:
    Load cell parameters into VARS vector and call CELGEN once
    to get initial values for DEE, RHO, THT before starting VA1OA
            CALL LDVAR( NVAR,VARS, +1,KMOL )
            CALL CELGEN (NVAR, VARS,TENRGY)
            CALL VAIOA(CELGEN,NVAR, VARS,TENRGY,G,H,W,
        + O.,OMVARS,STEPL,EPSVA,MODE,
        + MXCALL,IPRINT,IEXIT)
            SQGRAD = 0.
            DO 650 I=1,NVAR
                GRAD(I) = G(I)
                SQGRAD = SQGRAD + G(I)**2
            GRADNM = SQRT (SQGRAD)
        END IF
C
C End Case Simulation
    CALL LDVAR( NVAR,VARS, -1, KMOL )
        IF (IEXIT.NE.1) WRITE(6,4) IEXIT
        IF(TENRGY.LE.40.) CALL RESWR( INSTR )
        CALL RESWR( INSTR )
```



```
Set specifications for next cell structure
```

```
C
    600 CONTINUE
        DO 400 NC = 1,MOPT
    N = MOPT+1-NC
    IF(NMUT (N).EQ.0) GO TO 400
    INDMU(N) = INDMU(N) + 1
    IF(INDMU (N).LE.NMUT(N)) GO TO 410
    INDMU (N) = 1
    CONTINUE
    400
    DO 450 K = 1,MOPT
    IF(K.GT.MCOP) GO TO 420
    J = (INT((K-.01)/5)) + 1
    I=K - (J-1)*5
    CHSPEC(I,J) = SPECSTR(K)
    IF(NMUT (K).EQ.0) GO TO 450
    CHSPEC(I,J) = CMUTO(K) + (INDMU(K)-1)*CMUTD(K)
    GO TO 450
    4 2 0
    IF(N.GF.(M
    I = K-MCOP
    CELD(I) = SPECSTR(K)
    IF(NMUT(K).EQ.O) GO TO 450
    CELD (I) = CMUTO (K) + (INDMU (K) -1)*CMUTD (K)
    GO TO 450
    I = K-MCOP-3
    CELA(I) = SPECSTR(K)
    IF (NMUT (K).EQ.O) GO TO 450
    CELA(I)' = CMUTO(K) + (INDMU (K) - I)*CMUTD (K)
    CONTINUE
c
C Reinitialize conformation parameters:
    DO }700I=1,NDO
    IF(I.LE.NIBB) THEN
                PHI(I) = PHISTRTP(I)
                THETA(I) = THESTR(I)
                    ELSE
                                    PHIR(I-NIBB) = PHISTRTP(I)
                            END IF
        CONTINUE
        CONTINUE
1100
c
C *************** End of Cell Packing Scan LOOP 1100 ****************
C}\mathrm{ C Set angles for next conformation:
C
    DO 500 ND=1,NDOF
                            N = NDOF+1-ND
                        IF(NSCAN (N).EQ.O) GO TO 500
                        INDSC (N) = INDSC(N) + 1
                        IF(INDSC(N).LE.NSCAN(N)) GO TO 510
            INDSC (N) = 1
    CONTINUE
    DO 550 I=1,NDOF
            IF(I.LE.NIBB) THEN
                PHI(I) = PHISTR(I)
                    THETA(I) = THESTR(I)
                    ELSE
                    PHIR(I-NIBB) = PHISTR(I)
            END IF
            PHISTRTP(I) = PHISTR(I)
            IF(NSCAN(I).EQ.O) GO TO 550
            IF(I.LE.NIBB) THEN
                    PHI(I) = SCANO(I) + (INDSC(I) - 1)*SCAND (I)
                    PHISTRTP(I) = PHI(I)
                    ELSE
                    PHIR(I-NIBB) = SCANO(I) + (INDSC(I) -1)*SCAND (I)
                    PHISTRTP(I) = PHIR(I-NIBB)
                END IF
                CONTINUE
1000 CONTINUE
C
```

```
* E************* End of Conformation Scan LOOP 1000 ***************
C
        CLOSE (UNIT=11)
    2 FORMAT( A10 )
    3 FORMAT(IX,' PCSP Simulation Results in File: ',A10)
    4 FORMAT (1X,'IEXIT = ',I3)
        END
        SUBROUTINE CELGEN( NVAR,VARS,TENRGY )
C
C Function routine for Polymer Crystal Simulation. Calls routines
C required to build a parent chain, determine its primary axis and
C align the chain along this axis, perform chemical substitutions
C on the chain, replicate the parent chain in a regular lattice, and
C calculate a total potential energy for the construction.
C
    PARAMETER( NKIND=9,NIBE=8,MNRU=6,MNCH=9,MNAT=179,NR
    COMMON /CHAINS/ PHI (NIBB),THETA (NIBB),BL (NIBB),PHIR(NRNG),
                KSB (4), PCH (4, MNAT), NARU, NROS,NCH,
                A (4, 4, MNAT) , XYZ (4, MNAT, MNCH) ,KIND (MNAT, MNCH),
                CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT) , DEE, THT, RHO,
                NUNIQ.
    COMMON /SCAN/ NSTOT,NSANG,NSCAN (NDOF),SCANO (NDOF),SCAND (NDOF),
                NCTOT, NCMUT, NMUT (MOPT) , CMUTO (MOPT) , CMUTD (MOPT)
    COMMON /CELL/ CELD (3),CELA (3),CHSPEC (5,MNCH),CHLOC (3,MNCH),
                NUNIV, KONDENS, TEMP, DENSC
    COMMON /ENRG/ EDEF,EDEL,ENOB1,ENOB2,ECOU1,ECOU2,EULJ,EUES,
                            EOLJP, EUESP, ECFF, ETOT
COMMON /CNTR/ IENTRY1,IENTRY2
DIMENSION VARS (NVAR),KCMV (56)
C
C For the helix calculation, NHLXI identifies the first atom of the
C repeat unit for purposes of helix aligmnent, and the KCMV vector
C contains a list of atom identities for use in centering the helix
C In the case of THT = 0 (either by HELIX or GYRA):
C
    NHLXI = 13
    KCM = 0
cvd$ noconcur
    DO 200 I=1,NARU/28
        DO 200 J=1,28
        IX = (I-1)*28
        IF( (J.EQ.1).OR.(J.EQ.2).OR.(J.EQ.11).OR.(J.EQ.13).OR.
                (J.EQ.15).OR.(J.EQ.16).OR.(J.EQ.25).OR.(J.EQ.27) ) THEN
            KCM = KCM + 1
            KCMV (KCM) = IX + J
            ELSE
            END IF
    200 CONTINUE
C
    CALL LDVAR (NVAR,VARS,-1,KMOL)
    IF ({NSTOT.EQ.1).AND.(NVAR.EQ.O)) KMOL = 0
    IF ((KMOL.EQ.O).AND.(TENRGY.NE.O.)) GO TO 100
    NFLAG = 0
    CALL COORDN
    NHLXF = NHLXI + NARU
    CALL HELIX( A(1,1,NHLXI),A(1,1,NHLXF),KCM,KCMV,NFLAG )
    IF (NFLAG.EQ.O) CALL GYRA( KCM,KCMV )
    IF (KMOL.EQ.O) CALL LDCHN ( 1 )
    100 IF (KMOL.EQ.O) CALL LDCHN( -1 )
    CALL UNITCEL
    CALL NONBOND( IENTRY2 )
    CALL SUBSET( IENTRY1 )
    CALL ENERGY
    TENRGY = ETOT
    RETURN
    END
```

SUBROUTINE LDVAR( NVAR, VARS, LDIR, KMOL )

```
C
C If LDIR > 1 : Loads PHI'S, THETA'S, CELD'S, CELA'S, and CHSPEC'S
    into the variable vector VARS according to the
    information in LMV and LCV and sets their proper
    order of magnitude
If LDIR < 1 : Loads vARS into the above mentioned arrays according
                        to the information in LMV and LCV
Normalizes VARS to the range -180. to +180, where appropriate:
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
                MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
    COMMON /CHAINS/ PHI(NIBB),THETA(NIBB),BL (NIBB),PHIR(NRNG),
                        KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
                        A (4, 4,MNAT), XYZ (4,MNAT,MNCH),KIND (MNAT,MNCH),
                        CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT), DEE, THT, RHO,
                    NUNIQ
        COMMON /CELL/ CELD(3),CELA(3), CHSPEC (5,MNCH), CHLOC (3,MNCH),
                        NUNIV, KONDENS,TEMP, DENSC
            COMMON /MINIM/ LMV (2,9),\operatorname{LCV}(5,11),OMVARS (97)
            COMMON /SV2/ TVARL
            DIMENSION VARS(NVAR),TROT(9),KCORR(9)
            DO 50 J=1,MNIC
    50 KCORR(J) = 0
C NORMALIZE, WHERE APPROPRIATE:
            IF (LDIR.GT.O) GO TO 300
            DO 400 I=1,NVAR
            IF (OMVARS(I).NE.100.) GO TO 400
            VARS(I) = MOD(VARS(I), 360.)
            IF (VARS (I).LT. -180.) VARS (I) = VARS (I) + 360.
            IF (VARS(I).GT.+180.) VARS(I) = VARS(I) - 360.
            CONTINUE
C LOAD VECTOR/ARRAYS:
    NV = 1
    KMOL = 1
    NIBBL = NIBB + NRNG/2
    DO 100 J=1,NIBBL
        DO 100 I=1,2
            IF (LMV(I,J).EQ.O) GO TO 100
            IF (LDIR.LT.0) GO TO }12
                    OMVARS (NV) = 100.
                    IF (J.GT.NIBB) VARS (NV) = PHIR( (J-NIBB-1)*2 + I )
                    IF (J.GT.NIBB) GO TO 130
                    IF (I.EQ.1) VARS(NV) = PHI(J)
                    IF (I.EQ.2) VARS(NV) = THETA(J)
            GO TO 130
                    IF (J.GT.NIBB) PHIR( (J-NIBB-1)*2 + I ) = VARS(NV)
                    IF (J.GT.NIBB) GO TO 130
                    IF (I.EQ.I) PHI(J) = VARS (NV)
                    IF (I.EQ.2) THETA(J) = VARS (NV)
            NV = NV+1
            IF (NV.GT.NVAR) GO TO 500
    IF (NV.EQ.1) KMOL = 0
    DO 200 J=1,11
            DO 200 I=1,5
            IF (LCV(I,J).EQ.0) GO TO 200
            IF (IDIR.LT.0) GO TO 220
                IF (J.GT.MNIC) GO TO 260
                    VARS (NV) = CHSPEC(I,J)
                    IF (I.EQ.1) OMVARS (NV) = 1.
                IF (I.EQ.2) OMVARS(NV) = 0.I
                IF (I.EQ.2) TVARL = CHSPEC(I,J)
                IF (I.EQ.3) OMVARS (NV) = 100.
                IF (I.EQ.4) OMVARS(NV) = 100.
                IF (I.EQ.5) OMVARS(NV) = 100.
                GO TO 230
                IF (J.GT.(MNIC+1)) GO TO 270
                VARS (NV) = CELD (I)
                OMVARS (NV) = 10.
                GO TO 230
                VARS (NV) = CELA(I)
```

```
                    OMVARS (NV) = 100.
                    GO TO 230
    220
                    IF (J.GT.MNIC) GO TO 620
            IF (I.NE.2) GO TO 450
    Potential energy minimization appears to be very sensitive to the chain
    translation variable; VAIOA tends to make large steps in CHSPEC(2,J) if
    the initial guess creates atomic overlap. For this reason, steps in
    this variable are limited in magnitude. The value returned to VAlOA is
    altered; only when VA1OA begins to take steps less than STPMX will the
    corresponding Gradient and Hessian be correct.
    Even then, there may accumulate a significant deviation between the
    chain translation variable and the correponding VARS variable. Thus,
    where the initial condition may be poor and large steps are likely,
    SFRAC should be restricted, perhaps to 5% (SFRAC = 0.05); where small
    steps are expected and the corresponding minimization results are
    Intended for reuse, this restriction should be lifted (SFRAC = 1.0)
        SFRAC = 1.0
        IF(TVARL.GE.999.) THEN
            CHSPEC(I,J) = VARS (NV)
            TVARL = VARS (NV)
        ELSE
            STPMX = SFRAC*REAL (NUNIQ)
            VDIF = VARS (NV) - TVARL
            STP = SIGN( MIN(STPMX,ABS (VDIF)),VDIF )
            IF( (SFRAC.GT.O.9).AND.(ABS(STP-VDIF).GT.1E-5) )
                    WRITE(11,*) 'WARNING: translation has been restricted'
            CHSPEC(I,J) = CHSPEC(I,J) + STP
            END IF
C
Chain translation along the chain axis must be restricted for
model chains of finite length; here, helix symmetry may be used
to limit translation without imposing constraints on optimization.
The appropriate range should center about zero and equal the number
of repeat units required to define all unique segments (ie: range
= -0.5 to 0.5 for symmetric repeat units or isomers of head-to-tail
alignment, = -1.0 to 1.0 for isomers of head-to-head, tail-to-tall
alignment.
Correct translation to the range -0.1 to 0.1:
    Translation correction may be disabled by setting TURNS = 0.
    TURNS = AINT( CHSPEC(I,J) )
    TURNS = 0.
    CDIF = CHSPEC (I,J)-TURNS
    IF (NUNIQ.NE.1) GO TO 280
C For range -0.5 to 0.5, enable the following line:
    IF (ABS(CDIF).GT.0.5) TURNS = TURNS+SIGN(1.,CDIF)
    CONTINUE
    CHSPEC(I,J) = CHSPEC(I,J) - NUNIQ*TURNS
    TROT(J) = NUNIQ*TURNS*THT
    KCORR(J) = 1
    VARS (NV) = CHSPEC(I,J)
    TVARL = VARS (NV)
    GO TO 230
620
6 3 0
450
230
200
500
5 1 0
    IF (J.GT.(MNIC+1)) GO TO 630
    CELD(I) = VARS (NV)
    GO TO 230
    IF (J.GT.(MNIC+2)) GO TO 230
    CELA(I) = VARS (NV)
    IF (CELA(I).LT.O.) CELA(I) = CELA(I)+180.
    GO TO 230
        CHSPEC(I,J) = VARS(NV)
        NV = NV+1
        IF (NV.GT.NVAR) GO TO 500
    DO 510 J=1,MNIC
    IF (KCORR(J).EQ.1) CHSSPEC(5,J)= CHSPEC(5,J)+TROT(J)
    RETURN
    END
```

```
    SUBROUTINE LDCHN( LDIR )
C
C If KMOL = 0, it is not necessary to regenerate the parent chain
C for each iteration; LDCHN is a computational expedient which stores
C the coordinates of the parent chain for use by later iterations.
C If LDIR > 0 : loads parent into PCH
C If LDIR > 0 : loads PCH back into XYZ
C
PARAMETER ( \(\mathrm{NKIND}=9\), \(\mathrm{NIBB}=8, \mathrm{MNRU}=6, \mathrm{MNCH}=9, \mathrm{MNAT}=179\), NRNG \(=2\), \(\mathrm{NSUB}=4\), MNSB \(=24\), NDOF \(=10, \mathrm{MNIC}=9, \mathrm{MCOP}=45, \mathrm{MOPT}=51\) )
COMMON /CHAINS/ PHI (NIBB), THETA(NIBB), BL(NIBB),PHIR(NRNG), KSB (4) , PCH (4, MNAT) , NARU, NROS, NCH,
A ( 4,4, MNAT) , XYZ (4, MNAT, MNCH) , KIND (MNAT, MNCH), CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT) , DEE , THT, RHO, NUNIQ
DO 100 I=1,NRUS*NARU+1
DO \(100 \mathrm{~J}=1,4\)
IF (LDIR.GT.0) \(P C H(J, I)=X Y Z(J, I, 1)\)
\(\operatorname{IF}(L D I R . L T .0) X Y Z(J, I, I)=P C H(J, I)\)
RETURN
END
REAL FUNCTION RANDSI (SEED)
This is a special function for Random number generation on \(32-b 1 t\) machines that do not support long integer multiplication and truncation. The technique used is to do the multiplication and addition in parts, by splitting all integers in a 'high' and a 'low' part. The algorithm is exact, and should give machine-independent results.
INTEGER*4 SEED,IA,IB,I1,I2,I3
The algorithm implemented is (following D.E. Knuth): seed \(=\) seed*1592653589 + 453816691 if (seed.1t.0) seed \(=\) seed \(+1+2147483647\) Note that \(1592653589=48603 * 2 * * 15+30485\)
\(I A=\operatorname{SEED} / 32768\)
\(I B=\operatorname{MOD}(S E E D, 32768)\)
I1 \(=\) IA*30485
\(I 2=I B * 30485\)
I3 = IB*48603
\(I 1=M O D(I 1,65536)\)
\(I 3=\operatorname{MOD}(I 3,65536)\)
\(I 1=I 1+I 3+13849+I 2 / 32768+\operatorname{MOD}(I A, 2) * 32763\)
\(I 2=\operatorname{MOD}(I 2,32768)+12659\)
\(I 1=I 1+I 2 / 32768\)
\(I 2=\operatorname{MOD}(I 2,32768)\)
\(I 1=\operatorname{MOD}(I 1,65536)\)
SEED = I1*32768 + I2
RANDS \(1=\) SEED*4.65661287308D-10 RETURN
END
```

SUBROUTINE COORDN

```
C
Builds a Poly(p-phenylene terephthalamide) chain consisting of
alternating diacid and diamine units. The phenyl rings are assumed
to be hexagonal planar, and the amide carbons and nitrogens are
assumed to form sp2 planar bonds but angles are not required to be
120 degrees.
*****************************************************************************
Numbering of internal angles
used to place atoms:
PHIR(1)
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
    +
    COMMON /CHAINS/ PHI (NIBB),THETA(NIBB),BL(NIBB),PHIR(NRNG),
                    KSB (4), PCH (4, MNAT),NARO,NRUS,NCH,
                        A (4, 4,MNAT), XYZ (4,MNAT, MNCH), KIND (MNAT, MNCH),
                    CHRG (MNAT, MNCH) , KONEC (MNAT , MNAT) , DEE, THT, RHO,
                    NUNIQ
        DIMENSION AL (4, 4), KATOM(11), XYZL (4,11)
        DATA DCN,DNH,DCO/1.39,1.00,1.24/
        DO 50 I=1,NIBB/8
            KRD = (I - 1)*8
            BL (1+KRU) = DCO
            BL}(2+KRU) = DCN
            BL}(3+KRO) = DN
            BL}(5+KRU) = DN
            BL}(6+KRU)= DC
            BL}(7+KRU)=DC
        CONTINUE
        NST = 3
        NTYP = 1
C
C Start building:
C First phenyl ring lies along the X axis; ring rotation angle must be
C corrected for PHI(2) = 0 :
    PHI2 = PHIR(NTYP)-PHI (2)
    CALL PHENYL( NTYP,PHI2,KATOM,XYZL )
    NTYP = 2
    DO 100 I=1,11
            KIND(I, 1) = KATOM(I)
            DO 100 J=1,4
            XYZ (J,I,I) = XYZL(J,I)
100 IF(J.EQ.1 ) XYZ (J,I,I) = XYZ(J,I,1) - XYZL(J,1)
C
First amide bond has PHI = 0, first carbonyl has PHI - 180, by
default; since no previous A-matrix exits, load both directly:
    CALL ATRNS( 180.,THETA(1),XYZ(1,11,1),AL )
    DO 120 I=1,4
            DO 120 J=1,4
120 A(I,J,12)=AL(I,J)
    XYZ(1,12,1) = BL(1)
    XYZ (2,12,1) = 0.
```

```
        XYZ (3,12,1) = 0.
        XYZ (4,12,1) = 1.
        KIND (12,1)=6
        CALL ATRNS( 0.,THETA(2),XYZ(1,11,1),AL )
        DO 140 I=1,4
        DO 140 J=1,4
        A(I,J,13) = AL(I,J)
        XYZ(1,13,1)=BL(2)
        XYZ (2,13,1) = 0.
        XYZ (3,13,1)=0.
        XYZ (4, 13,1) = 1.
        KIND (13,1) = 5
        NUMA = 14
        DO 160 I=12,13
            CALI MV (A (1, 1, I) , 4, 3, XYZ (1,I, 1), 4,XYZ (1,I,1) )
160
C Loop over all repeat units:
c
    DO 500 NR=1,NRUS
C
C Loop over all points in repeat unit:
            DO 400 NF=NST,NIBB-1,2
            PHI(NF) = PHI (NF+1) + 180.
            NUMAM = NUMA-1
            DO 400 NA=1,2
                NMKR = NF+NA-1
            IF( .NOT.(MOD(NMKR,4).EQ.0) ) GO TO 200
            CALL PHENYL( NTYP,PHIR(NTYP),KATOM,XYZL )
            NTYP = 1 + MOD (NTYP,NRNG)
            BL(NMKR) = XYZL (1,11)
            DO 220 I=1,11
                    KIND (NUMA, 1) = KATOM(I)
                    DO 240 J=1,4
                    XYZ (J,NUMA,I) = XYZL (J,I)
                    CALL ATRNS( PHI (NMKR),THETA (NMKR),BL (NF-1),AL )
                    CALL MM( A(1,1,NUMAM),AL,A(1,1,NUMA),4,4,4,4,4,4 )
                    CALL MV ( A (1,1,NUMA) , 4,3,XYZ (1,NUMA,1),4,XYZ (1,NUMA,1) )
    220
            NUMA = NUMA +1
            GO TO 400
            200 XYZ (1,NUMA,1) = BL (NMKR)
            XYZ (2,NUMA,1) = 0.
            XYZ (3, NUMA, 1) = 0.
            XYZ(4,NUMA,1) = 1.
            KIND (NOMA,1) = 4
            IF ( MOD (NMKR+2,8).EQ.O ) KIND(NUMA,1) = 3
            IF ( MOD (NMKR+6,8).EQ.0 ) KIND(NUMA,1) = 5
            IF ( (MOD (NMKR+7, 8).EQ.0).OR. (MOD (NMKR+1, 8).EQ.0) )
                    KIND (NUMA, 1) = 6
C
C For first element of second and successive repeat units, draw upon
C A-matrix of last atom positioned in previous repeat unit:
C
            IF (NF.EQ.1) THEN
                    CALL ATRNS( PHI (NMKR), THETA (NMKR),BL (NIBB),AL )
                    CALL MM( A (1, 1,NUMAM),AL,A (1, 1,NUMA),4,4,4,4,4,4 )
            ELSE
                    CALL ATRNS( PHI (NMKR),THETA (NMKR),BL (NF-1),AL )
                    CALL MM( A (1, 1, NUMAM), AL, A (1, 1, NUMA), 4,4,4,4,4,4)
            END IF
C
C Transform to Frame 0:
            CALL MV ( A (1, 1, NOMA) , 4, 3, XYZ (1, NOMA, 1), 4,XYZ (1,NUMA, 1) )
            NUMA = NUMA+1
        CONTINUE
        NST = 1
        CONTINUE
        RETURN
    END
```

SUBROUTINE PHENYL( NTYP,PHI,KATM,XLOC)

```
Builds a phenyl ring within the frame of reference of the first
atom preceeding the phenyl ring and returns atom types and local
```

coordinates


DIMENSION KATM(12), RGBL (12), RGPH (12), RGTH(12), XLOC (4, 12)
DIMENSION ALOC $(4,4)$
DATA DPH, DCPH, DNEH, DHPH/1.40,1.50,1.41,1.10/
IF ( MOD (NTYP, 2).EQ.1 ) THEN
RGBL (1) $=\mathrm{DCPH}$
ELSE
RGBL (1) $=\mathrm{DNPH}$
END IF
RGC $=$ RGBL (1) + DPH
$\operatorname{RGBL}(2)=\operatorname{RGC}+\mathrm{DPH}$
RGBL (11) $=$ RGBL (2) + RGBL (1)
DO $100 \mathrm{I}=1,2$
$\operatorname{KATM}(I)=1$
$\operatorname{KATM}(11)=3$
IF ( MOD (NTYP, 2).EQ.O ) KATM(11) $=5$
DO $200 \mathrm{I}=3,9,2$
$\operatorname{KATM}(I)=1$
RGBL (I) $=\mathrm{DPH}$
RGPH (I) $=$ PHI
IF ( (I.EQ.5).OR.(I.EQ.9) ) RGPH(I) $=$ RGPH(I) +180.
RGTH(I) $=120$.
$\operatorname{IF}($ (I.EQ.7).OR.(I.EQ.9) ) RGTH (I) $=60$.
DO $300 \mathrm{I}=4,10,2$
$\operatorname{KATM}(I)=2$
RGBL (I) $=$ DPH + DHPH
RGPH (I) $=$ PHI
$\operatorname{IF}($ (I.EQ. 6).OR. (I.EQ.10) ) RGPH(I) $=\operatorname{RGPH}(I)+180$.
RGTH $(I)=120$.
IF( (I.EQ.8).OR.(I.EQ.10) ) RGTH(I) $=60$.
DO $500 \mathrm{I}=1,11$
$\operatorname{XLOC}(1, I)=\operatorname{RGBL}(I)$
$X \operatorname{XIOC}(2, I)=0$.
$\operatorname{XIOC}(3, I)=0$.
$\operatorname{XIOC}(4, I)=1$.
DO $600 \mathrm{I}=3,10$
CALL ATRNS ( RGPH (I) , RGTH (I) , RGC, ALOC )
CALL MV ( ALOC, 4, 3, XLOC (1, I) , 4, XLOC (1, I) )
RETURN
END

SUBROUTINE HELIX ( AO, AX, KCM, KCMV, NFLAG )
C
C Calculates Helix Parameters (Pitch, Twist, and Radius) and converts
$C$ XYZ coordinates into a frame of reference wherein the helix axis
$C$ corresponds to the $Z$ axis. For this purpose, $A O$ and $A X$ are sectors
$C$ of the A-matrix locating corresponding atoms in successive molecular
$c$ repeat units, retrieved for the purpose of calculating the net
$C$ transformation (AH) per repeat unit and the translation vector (BH)
$C$ per repeat unit; these are equivalent to the $A$ matrix and $B$ vector
C of Sugeta My yazawa (BIOPOLYMERS,5,673,1967)

```
c >>>>>> MODIFIED TO RETURN COORDINATES UNTRANSFORMED <<<<<<<<
C >>>>>>> FOR VALUES OF THT BELOW THTCRIT <<<<<<<
C
    PARAMETER (EPSILON = 1.E-6)
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
    + MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
    COMMON /CHAINS/ PHI(NIBB),THETA(NIBB),BL (NIBB),PHIR(NRNG),
                    KSB (4) , PCH (4, MNAT) , NARU, NRUS, NCH,
                        A (4, 4, MNAT) , XYZ (4,MNAT, MNCH), KIND (MNAT, MNCH),
                        CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT) , DEE, THT, RHO,
                    NUNIS
        DIMENSION AO (4,4), AX (4,4), AH (4,4), BH (4), AP (4, 4), AZ (4, 4)
        DIMENSION EMAT (4,4),AT (4,4),CA(4),CB(4),BN(3,2)
        DIMENSION TX(3),TY(3),TZ(3),T(4,4)
        DIMENSION KCMV(KCM)
        THTCRIT = 30.*(3.1415926535/180.)
        KK = 0
c
C Determine AH and BH:
C Expeditious choice of AO such that AO = Inverse of AO enables the
C following shortcut:
    100 BH(I) = AX(I,4) - AO(I,4)
C
C Determine helix:
C
    200
        DO 200 I=1,4
            DO 200 J=1,4
                EMAT (I,J) = 0.
            IF (I.EQ.J) EMAT(I,J) = 1.
        CALL MT( AH,AT, 3,4 )
        CALL MA( AT, EMAT, AT, 3, 4, -1. )
        CALL MA( EMAT, AH,AP, 3,4, -1. )
        CALL MV( AT,4,3,BH,3,CA )
        CALL MV( AP, 4, 3, BH, 3,CB )
        CASQ = CA(1)**2 + CA(2)**2 + CA(3)**2
        CBSQ = CB(1)**2 + CB(2)**2 + CB(3)**2
        CSQ = MAX(CASQ,CBSQ)
        BSQ = BH(1)**2 + BH(2)**2 + BH(3)**2
        CDC = CA (1)*CB(1) +CA (2)*CB(2)+CA(3)*CB (3)
        IF ( ((1.-AH (1,1)*AH(2,2)*AH(3,3)).GE. (100.*EPSILON)).OR.
                ((AH(1,1)+AH(2,2)+AH(3,3)).LT.O.)) GO TO 300
        +
    Special Case: THT = 0; straight line helix
        THT = 0.
        DEE = SQRT( BSQ )
        RHO = 0.
        GO TO 360
C
C Special Case: BH vector corresponds to helix axis but THT is nonzero
C ("twisted rod"); for this case, determine THT by applying AH
C rotation to an arbitrary unit vector normal to axis (valid for BH(1)
c not equal to zero)
C
300 CONTINUE
    IF (CSQ.GT.EPSILON) GO TO 320
    IF( BH(1).LT.EPSILON ) WRITE(*,*) 'Invalid Assumption: HELIX'
    BRTO = BH(3)/BH(1)
    BN(1,1) = SQRT( BRTO**2/(1.+BRTO**2) )
    BN (2,1) = 0.
    BN(3,1) = - BN(1,1)/BRTO
    CALL MV( AH,4,3,BN(1,1),3,BN(1,2) )
    BN1SQ = BN (1,1)**2 + BN (2,1)**2 + BN (3,1)**2
    BN2SQ = BN (1,2)**2 + BN (2,2)**2 + BN (3,2)**2
    THT = ACOS( (BN (1, 1)*BN (1,2) + BN (2,1)* BN (2,2) + BN (3,1)*BN (3,2))
    + /SQRT(BN1SQ*BN2SQ) )
    RHO =0.
    DEE = SQRT( BSQ )
```

```
C
    360 CONTINUE
        DO 350 I=1,3
        TZ(I) = (XYZ (I,NARU+1,1)-XYZ(I, 1,1))/DEE
        TX(I) = TZ(I)
        IF (ABS(TZ(3)).LT.EPSILON) THEN
        TX(1) = 0.
        TX(2) = 0.
        TX(3)=-1.
    ELSE IF (ABS(TZ(3)-1.).LT.EPSILON) THEN
        TX(1) = 1.
        TX(2) = 0.
        TX(3) = 0.
        ELSE
        ALPHA = ACOS (TZ (3))
        TX(3) = -1.*SQRT (TX (1)**2+TX (2)**2)*TAN(ALPHA)
        END IF
        TXL = SQRT(TX(1)**2 + TX(2)**2 + TX(3)**2 )
        DO 330 I=1,3
        TX(I) = TX(I)/TXL
        GO TO 500
        CONTINUE
        IF (ABS (CDC/CSQ-1.).IT.EPSILON) THEN
        CDC = CSQ
        ELSE IF (ABS(CDC/CSQ+1.).LT.EPSILON) THEN
        CDC = -CSQ
    ELSE
    END IF
    THT = ACOS( CDC/CSQ )
    RHO = SQRT (CSO)/( 2*(1.-COS (THT)) )
C
C Special Case: THT=180; Planar zigzag helix
C
    IF (ABS(THT-3.1415926535).GT.EPSILON) GO TO 400
    CALL MA( AH, EMAT,AZ, 3,4,+1. )
    CALL MV( AZ,4,3,BH,3,TZ )
    TZM2 = 0.
    DO 365 I=1,3
        TZM2 = TZM2 + TZ (I)**2
        TZM = SQRT(TZM2)
    DEE = TZM/2.
    DO 370 I=1,3
        TX(I) = CA(I)/SQRT( CSQ )
        TZ(I) = TZ(I)/TZM
    GO TO 500
    CALL VV( CA,CB,TZ,3)
    BCC = 0.
    DO 310 I=1,3
        BCC = BCC + BH(I)*TZ(I)
    DEE = BCC/(CSQ*SIN(THT))
    TZL = SQRT(TZ(1)**2 + TZ (2)**2 + TZ (3)**2 )
    DO 450 I=1,3
        TZ(I) = TZ(I)/TZL
        TX(I) = CA(I)/SQRT( CSQ )
    CALL VV( TZ,TX,TY,3 )
    DO 550 I=1,3
        T(1,I) = TX(I)
        T(2,I) = TY(I)
        T(3,I) = TZ(I)
            T(4,I) = 0.
        T(1,4) = RHO
        T(2,4)=0.
        T(3,4)=0.
        T(4,4)=1.
C
C >>>>>>> MODIFIED RETURN FOR THT < THTCRIT <<<<<<<
        IF( THT.LT.TGTCRIT ) RETURN
\>>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
Convert from initial coordinates to helix coordinates:
```

```
    DO }600\mathrm{ J=1,NARU*NRUS+1
        CALL MV( T,4,3,XYZ (1,J,1),4,XYZ (1,J,1) )
        IF (ABS (THT).GT.EPSILON) GO TO 800
C
For THT = O helix radius is arbitrary; for this case, center
molecule about the average projection of the KCM atoms previously
specified in the KCMV vector:
    IF (KK.EQ.1) GO TO 800
    NN =0
    XI =0.
    YI=0.
    DO }700\textrm{I}=1,NAR
            DO }740\textrm{J}=1,\textrm{KCM
                    IF( I.EQ.KCMV(J) ) THEN
                    XI = XI + XYZ(1,I,1)
                YI = YI + XYZ (2,I,1)
                    NN = NN + 1
                    ELSE
                END IF
            CONTINUE
        CONTINUE
        XI = XI/NN
        YI = YI/NN
        RHO = SQRT(XI**2 + YI**2 )
        TX(1) = -XI/RHO
        TX(2) = -YI/RHO
        TX(3) = 0.
        TY(1) = YI/RHO
        TY(2) = -XI/RHO
        TY(3)=0
        TZ(1) = 0.
        TZ(2) = 0.
        TZ(3) = 1.
        KK = 1
        GO TO 750
    800 THT = THT*180./3.1415926535
        NFLAG = 1
        RETURN
        END
            SUBROUTINE GYRA( KCM, KCMV )
C
C Calculates the Radius of Gyration matris for a polymer chain and
C then transforms coordinates to a frame aligning the primary axis
C of the point distribution along the Z axis
C
PARAMETER (EPSILON=1.E-6)
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
        MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
    +
            COMMON /CHAINS/ PHI(NIBB),THETA(NIBB),BL(NIBB),PHIR(NRNG),
                        KSB (4), PCH (4, MNAT), NARU, NRUS,NCH,
                A (4, 4, MNAT), XYZ (4,MNAT,MNCH) ,KIND (MNAT,MNCH),
                CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT), DEE, THT, RHO,
                NUNIQ
            DIMENSION RG(4,4),RA(4),EVC (4,4),EVT (4,4), EVL (3), BVI (3),
            BV2 (3),2V(3),PV1 (3) , PV2 (3)
            DIMENSION KCMV (KCM)
            PI = 3.1415926535
            DO 100 I=1,4
            RA(I) = 0.
            DO 100 J=1,4
            EVC(I,J) = 0.
            EVT(I,J) = 0.
            RG(I,J) = 0.
        EVT(4,4)=1.
C
C Determine RG of backbone atomic coordinates.
C
    M = 0
```

```
        RGSCALE = 0.
        NBBA = NIBB*NRUS
        DO 200 I=1,NRUS
        DO 200 J=1,NARU
        M=M+1
        DO 200 JCMP=1,KCM
            IF( J.EQ.KCMV(JCMP) ) THEN
            DO 220 K=1,3
                RA(K) = RA (K) + XYZ (K,M,1)/NBBA
                    DO 220 L=K,3
                    RG(K,L) = RG(K,L)+XYZ (K,M, I)*XYZ (L,M, 1)/NBBA
            CONTINUE
            ELSE
            END IF
    200 CONTINUE
    RG(3,1)= RG(1,3)
    RG(2,1)=RG(1,2)
    RG(3,2) = RG(2,3)
    DO 250 K=1,3
    DO 250 L=1,3
    RG(K,L)=RG(K,L)-RA(K)*RA(L)
C
C Determine eigenvalues and eigenvectors by method of Jacobi
C Transformations (Press et.al., NUMERICAL RECIPES, Cambridge
C University Press, 1986, P342.
    CALL JACOBI( RG, 3,4,EVL,EVC,NROT )
C Sort eigenvalues in order EVL3 > EVL1 > EVL2 and rearrange eigen-
vectors accordingly:
    DO 300 I=1,2
        K= I
        T=EVL(I)
        DO 320 J=I+1,3
            IF(EVL(J).LE.T) THEN
                K=J
                T = EVL (J)
            END IF
        CONTINUE
        IF (K.NE.I) THEN
            EVL(K) = EVL(I)
            EVL(I) = T
            DO 340 J=1,3
                    T=EVC(J,I)
                    EVC(J,I) = EVC(J,K)
                    EVC (J,K) = T
        END IF
    CONTINUE
    T = EVL(1)
    EVL(1) = EVL(2)
    EVL(2) = T
    DO 360 J=1,3
            T=EVC (J,1)
            EVC(J,1) = EVC(J,2)
            EVC(J,2) = T
360
C
C Ensure right-handedness of rotation matrix:
    DET = EVC (1,1)*EVC (2,2)*EVC (3,3)+\operatorname{EVC}(2,1)*EVC (3,2)*\operatorname{EVC}(1,3)+
    + EVC(3,1)*EVC (1,2)*EVC (2,3) - EVC (1,1)*EVC (3,2)*EVC (2,3)-
    + }\operatorname{EVC}(2,1)*\operatorname{EVC}(1,2)*\operatorname{EVC}(3,3)-\operatorname{EVC}(3,1)*\operatorname{EVC}(2,2)*\operatorname{EVC}(1,3
        IF((ABS (DET)-1.).GT.EPSILON) WRITE(*,*)
        + 'ERROR: Eigenvector Matrix'
            IF(ABS(DET-1.).LE.EPSILON) GO TO 410
            DO 400 I=1,3
            EVC}(I,2)=-\operatorname{EVC}(I,2
    410 CONTINUE
C
C Transform coordinates to Primary Axis frame of reference
```

```
        DO 520 I=1,4
    DO 520 J=1,4
        EVT (J,I) = EVC(I,J)
        DO 500 I=1,NARU*NRUS+1
        DO 540 J=1,3
            XYZ(J,I,I)= XYZ (J,I,1) - RA(J)
        CALL MV( EVT,4,3,XYZ(1,I,1),4,XYZ(1,I,1) )
        CONTINUE
500
C
C Determine comparable "helix" parameters:
    (NOTE: backbone atoms only, code is model specific)
        DEE = XYZ (3,NARU+1,1) - XYZ (3,1,1)
        GRHO = 0.
        DO 600 I=1,3
        IF (ABS(EVL(3)-EVL(I)).GT.EPSILON) GRHO = GRHO+EVL(I)
        CONTINUE
        RHO = SQRT (GRHO)
C
C For GTHT determination, the relative rotation of the vertical plane
C intersecting an arbitrary bond vector (carbonyl C=O) is chosen
    N1 =27
    N2 = 28
    N3 = N1+NARU
    N4 = N2+NARU
    DO 650 I=1,3
        BV1(I) = XYZ(I,N2,1) - XYZ(I,N1,1)
        BV2(I) = XYZ(I,N4,1) - XYZ(I,N3,1)
        ZV}(I)=0
    zV(3) = 1.
    CALL VV( BV1,ZV,PV1,3 )
    CALL VV( BV2,2V,PV2,3 )
    DOT = 0.
    PV1M = 0.
    PV2M = 0.
    DO }660\textrm{I}=1,
        DOT = DOT + PV1(I)*PV2(I)
            PV1M = PV1M + PV1(I)**2
            PV2M = PV2M + PV2(I)**2
    DOT = DOT/SQRT(PV1M*PV2M)
    IF ( ABS (ABS (DOT)-1.).LE.EPSILON ) DOT = SIGN(1.,DOT)
    GTHT = ACOS( DOT )
    THT = GTHT*180./PI
    RETURN
    END
    SUBROUTINE UNITCEL
Creates a unit cell consisting of a regular array of identical chains
based on an initial parent chain. The configuration of the array of
chain centers is determined by the dimensions (CELD(1),CELD(2))
between the parent (P) and two neighboring chains (N1,N2) and the
angle (CELA(3)) between these neighbors (N1-P-N2). The array of
centers is oriented relative to the parent axis (Z-axis) by the
angles CELA(1) (Z-P-N1) and CELA(2) (Z-P-N2). Finally, each chain
may be rotated about its center and/or translated along its local
chain axis independently according to the Eulerian angles and
fractional translation values in CHSPEC.
    (Z axis extends out of paper):
        M,
        Angles: 214 = ALPHA [ CELA(1) ]
        Z12 = BETA [ CELA (2) ]
        Dimensions: 1-2 = A [ CEID(1)]
        1-4 = B [CELD(2) ]
    1-Z =C [CELD(3)]
```



```
>>>>>>>>>>>> Modified to impose the following constraints: <<<<<<<<<<
    1. All odd-number designated chains translated identically
        according to CHSPEC(2,1); all even-number designated chains
        translated according to CHSPEC (2,2)
    2. All odd-number designated chains rotated identically
        according to CHSPEC(3-5,1); all even-number designated chains
        rotated according to CHSPEC(3-5,2)
>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>>><<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
    PARAMETER( EPSILON=1.E-6 )
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
    +
    COMMON /CHAINS/ PHI(NIBB),THETA(NIBB),BL(NIBB),PHIR(NRNG),
                    KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
                    A (4, 4, MNAT), XYZ (4, MNAT, MNCH), KIND (MNAT, MNCH),
                    CHRG (MNAT, MNCH), KONEC (MNAT, MNAT), DEE, THT, RHO,
                    NUNIQ
        COMMON /CELL/ CELD(3),CELA(3),CHSPEC(5,MNCH),CHLOC(3,MNCH),
                    NUNIV, KONDENS,TEMP, DENSC
        COMMON /STRN/ KUC,KASE,ST (3, 3),SOCV (3, 3),DINIT,DPRIME,DCON
        DIMENSION CROT (4,4),UCV (3,3)
        DEGRAD = 3.1415926535/180.
        CELD(3) = DEE
C
C
C
>>>>>>>>>> Modified to allow only two chain types <<<<<<<<<<<<<<<<<<<<
    DO 50 I=3,NCH,2
        DO 50 J=1,5
                CHSPEC(J,I) = CHSPEC}(J,1
                CHSPEC(J,I-1) = CHSPEC(J,2)
            IF (MOD (NCH, 2).NE.O) GO TO 55
            CHSPEC(1,NCH) = CHSPEC}(1,2
            CHSPEC (2,NCH) = CHSPEC (2,2)
            CHSPEC (3,NCH) = CHSPEC (3,2)
            CHSPEC(4,NCH) = CHSPEC(4,2)
            CHSPEC (5,NCH)=CHSPEC(5,2)
            CONTINUE
c
C >>>>>>>>>> End Modification <<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<<
C Convert unit cell parameters CELD and CELA into placement vectors
used to locate chains on a regular periodic lattice: these are the
C vectors 1-2 and 1-4 (see dlagram above) stored in CHLOC (1,2) and
C CHLOC (1,4), of which all other chain locations are linear
C combinations
C
    DO 100 I=1,3
        DO }100\textrm{J}=1,\textrm{NCH
            CHLOC (I,J) = 0.
            CALL VECTORIZ( CELD, CELA,UCV, KERR )
            IF(KERR.EQ.1) WRITE(11,*) 'VECTORIZ ERROR: Invalid Angle Set'
            DO 200 I=1,3
            CHLOC(I,2) = UCV (I, 1)
            CHLOC(I, 4) = UCV (I, 2)
C
C If specified, fix lattice density:
            IF (NCH.GT.1)
        + CALL DENS( CELD,CHLOC (1,2),CHLOC (1,4),TEMP,KONDENS,DENSC )
C
C Determine location vector CHLOC (I,J) of each chain J up to J = NCH,
the total number of chains, in a spiral counterclockwise protocol:
Here the cell strain is imposed. CELD and CELA should not be
optimized. This is triggered by KUC not equal to 0.
    IF (KUC.NE.0) CALL STRAIN(KUC,KASE,DINIT,ST,SUCV,CHLOC,DPRIME)
```

c

```
    DO 300 I=1,3
        CHLOC(I,3)=CHLOC(I,2) + CHLOC(I,4)
        IF(NCH.GT.4) THEN
        ITRAN = IFIX( (SQRT(FLOAT (NCH))+0.1)/2 )
        JTRAN = 1
        JTRK = -1
        KTRK = 1
        DO 320 NTRK=5,NCH
            DO 340 I=1,3
                CHLOC(I,NTRK) = JTRK*CHLOC (I, 2) + KTRK*CHLOC (I, 4)
                IF(NTRK.GE. ((2*JTRAN+1)**2)) JTRAN = JTRAN+1
            IF( ( JTRAN.EQ.JTRK).AND.( JTRAN.NE.KTRK)) THEN
                KTRK = KTRK+1
            ELSE IF((-JTRAN.NE.JTRK).AND.( JTRAN.EQ.KTRK)) THEN
                JTRK = JTRK-1
            ELSE IF((-JTRAN.EQ.JTRK).AND.(-JTRAN.NE.KTRK)) THEN
                KTRK = KTRK-1
            ELSE IF(( JTRAN.NE.JTRK).AND.(-JTRAN.EQ.KTRK)) THEN
                JTRK = JTRK+1
            ELSE
                JTRK = JTRK+1
            END IF
            CONTINUE
        ELSE
        END IF
For each chain desired, duplicate parent chain and apply
specified rotation/translation:
-- For Location I: CHSPEC(1,I)= 0. Position not occupied
                    = 1. Position occupied
                CHSPEC(2,I)= Chain Translation (given as a fraction
                    of the C dimension)
                CHSPEC(3,I)= the first Eulerian angle (LAMBDA)
                CHSPEC(4,I)= the second Eulerian angle (XI)
                CHSPEC(5,I)= the third Eulerian angle (OMEGA)
The order of operations to place each chain is:
            duplicate chain
            center about origin
            rigid body rotation about Eulerian angles
            translation along crystallographic (CELD) dimensions
            translation (CHSPEC(2,I)) along chain axis
    ZSHFT = (XYZ (3,NARU*NRUS +1,1) +XYZ (3,1,1) )/2.
    DO 450 I= NCH,1,-1
        ROM = CHSPEC (3,I)*DEGRAD
        RPS = CHSPEC(4,I)*DEGRAD
        RXI = CHSPEC (5,I)*DEGRAD
        CROT (1,1) = COS (RXI)*COS (RPS)*COS(ROM) -SIN (RXI)*SIN (ROM)
        CROT (1,2) = - COS (RXI)*COS(RPS)*SIN (ROM) -SIN(RXI)*COS (ROM)
        CROT (1,3)=-\operatorname{COS (RXI)*SIN (RPS)}
        CROT(1,4) = CHLOC(1,I)
        CROT (2,1) = SIN (RXI)*COS (RPS)*COS (ROM) +COS (RXI)*SIN (ROM)
        CROT (2,2) = -SIN(RXI)*COS (RPS)*SIN (ROM) +COS (RXI)*COS (ROM)
        CROT (2,3) = -SIN(RXI)*SIN(RPS)
        CROT (2,4)= CHLOC (2,I)
        CROT (3,1) = SIN(RPS)*COS (ROM)
        CROT (3,2)=-SIN(RPS)*SIN(ROM)
        CROT (3,3) = COS(RPS)
        CROT (3,4) = CHLOC (3,I)
        CROT (4,1) = 0.
        CROT (4,2) = 0.
        CROT (4,3)=0.
        CROT (4,4) = 1.
        DO 400 J=1,NARU*NRUS
            KIND(J,I) = KIND(J,1)
            XYZ(1,J,I) = XYZ (1,J,1)
            XYZ(2,J,I) = XYZ (2,J,1)
            XYZ(3,J,I) = XYZ(3,J,1) - ZSHFT
            XYZ(4,J,I)=1.
```

```
                        CALL MV( CROT, 4, 3,XYZ(1,J,I), 4,XYZ(1,J,I) )
            XYZ(1,J,I) = XYZ(1,J,I)+CHSPEC(2,I)*DEE*COS (RXI)*SIN(RPS)
            XYZ (2,J,I) = XYZ (2,J,I) +CHSPEC (2,I)*DEE*SIN (RXI)*SIN (RPS)
            XYZ(3,J,I) = XYZ (3,J,I) +CHSPEC (2,I)*DEE*COS (RPS)
            XYZ(4,J,I) = 1.
        CONTINUE
    CONTINUE
        REIURN
        END
    SUBROUTINE DENS( D,V1,V2,TEMP,KD,DENSC )
c
C If KD = 1 : llalculates polymer density based on group additivities 
If KD=1 : 年 Calculates polymer density based on group additivities 
If KD=1 : 年 Calculates polymer density based on group additivities 
If KD=1 : 年 Calculates polymer density based on group additivities 
If KD = 1 : 年 Calculates polymer density based on group additivities 
PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
                    MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
    COMMON /CHAINS/ PHI(NIBB),THETA (NIBB),BL (NIBB),PHIR (NRNG),
                        KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
                        A (4, 4,MNAT), XYZ (4,MNAT,MNCH),KIND (MNAT,MNCH),
                        CHRG (MNAT, MNCH), KONEC (MNAT, MNAT), DEE, THT, RHO,
                        NUNIQ
        DIMENSION V(2),D(3),W(2),V1(3),V2(3),V3(3),V4(3),WX(9),VX(9)
        DATA WPH,WAM /74.,43./
        DATA WX /0.,1.,0.,0.,0.,0.,19.,35.5,79.9 /
        DATA VPH,VAM / 56.3,24.9/
        DATA VX /0.,3.3,0.,0.,0.,0.,10.9,19.9,33.6 /
            DATA GLCR,AA,BB/ 1.07,0.914,0.00029 /
        DEGRAD = 3.1415926535/180.
C
Calculate molecular weight and molar volume of glassy repeat unit:
    WGO = 2*(WPH + WAM + 2*WX(2) ) + KSB(4)*WX(KSB(1)) -
            KSB (4) *WX (2)
        VGO = 2*( VPH + VAM + 2*VX(2) ) + KSB(4)*VX(KSB(1)) -
        + KSB(4)*VX(2)
            WGO = REAL (NARU/28)*WGO
            VGO = REAL (NARU/28)*VGO
            VG1 = GLCR/(VGO*(AA +BB*TEMP))
C
Compute cell volume:
C
    400
    450
    400
    +
    +
    +
    DO 200 I=1,3
        V1(I) = V1(I)/D(1)
        V2(I) = V2(I)/D(2)
        V3(I) = 0.
    V3(3)=1.
    CALL VV( V1,V2,V4,3 )
    VVOL = 0.
    DO 300 I=1,3
        VVOL = VVOL + V3(I)*V4(I)
    VVOL = ABS (VVOL)
    IF( KD.NE.1 ) GO TO 500
C
C Two Cases for fixed density calculations:
1) If D(1).eq.D(2) : fix density such that this equality is
                maintained (eg. hexagonal packing calculations
2) If D(1).ne.D(2) : (usual case) fix D(2) and density while varying
                        D(I)
    AB}=1./(D(3)*VVOL*.602205*VG1
    IF(ABS(D(1)-D(2)).LT.1.E-6) GO TO 400
    D(1) = AB/D(2)
    GO TO 500
400 D(1) = SQRT (AB)
    D(2) = D(1)
5 0 0 ~ C O N T I N U E
```

```
        DENSC = VG1*WGO
        DO 600 I=1,3
            V1(I) = V1(I)*D(1)
            V2(I) = V2(I)*D(2)
    CONTINUE
c
Variable density case:
    IF(KD.NE.1) DENSC = WGO/(VVOL*D (1)*D (2)*D(3)*.602205)
    RETURN
    END
    SUBROUTINE NONBOND( IENTRY )
C
C Creates Table of Connectivity and assigns partial atomic charges to
atoms of the parent polymer form (PPTA)
C Need be executed only once for a given cell composition, but must
be called after all chains have been created
    MRMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
                MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
            COMMON /CHAINS/ PHI (NIBB),THETA (NIBB),BL (NIBB),PHIR(NRNG),
                KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
                A (4, 4,MNAT), XYZ (4,MNAT,MNCH),KIND (MNAT, MNCH),
                CHRG (MNAT, MNCH), KONEC (MNAT,MNAT), DEE,THT, RHO,
                NUNIQ
            DATA CHARGC, CHARGO, CHARGN, CHARGH/. 38,-.38,-.28,.28/
            DATA CHARGC,CHARGO,CHARGN, CHARGH/.0,.0,.0,.0/
            DATA CHARGPPC,CHARGNPC,CHARGPH/ -0.06,-0.12,0.15 /
c
C
            DO }100\mathrm{ I=1,NARU*NRUS
                DO 120 J=1,NCH
            CHRG(I,J) = 0.
            DO 130 J=1,NARO*NRUS
            KONEC(I,J) = 0.
    CONTINUE
C
C All atoms within a given phenyl ring or amide bond plane are fixed
C relative to one another; atoms directly bonded to each ring/plane
C are fixed relative to that ring/plane
C
C Amide atom charges are assigned:
C
            NMER = NARU/28
            DO 500 LL = 1,NCH
            KTYP = 10
            MARK = 0
            DO }200\mathrm{ I=1,NMER*NRUS
                DO 200 II=1;4
                    DO 300 J=MARK+1,MARK+KTYP
                    IF (KTYP.EQ.10) GO TO }31
                    IF (KIND (J,LL).EQ.3) THEN
                                    CHRG(J,LL) = CHARGC
                    ELSE IF(KIND(J,LL).EQ.4) THEN
                                    CHRG(J,LL) = CHARGH
                                    ELSE IF(KIND(J,LL).EQ.5) THEN
                                    CHRG(J,LL) = CHARGN
                                    ELSE IF(KIND(J,LL).EQ.6) THEN
                                    CHRG(J,LL) = CHARGO
                    ELSE
                    END IF
    310 CONTINUE
c
C Three Independent charges allowed: carbons 1&4, carbons 2,3,5&6,
C and all hydrogens
C
```

```
        IF(KTYP.EQ.4) GO TO 315
        IF(J.LE.(MARK+2)) THEN
        CHRG(J,LL) = CHARGPPC
        ELSE
        IF(KIND(J,LL).EQ.1) CHRG(J,LL) = CHARGNPC
        IF(KIND (J,LL).EQ.2) CHRG(J,LL) = CHARGPH
        END IF
        CONTINUE
c
                                    IF(LL.NE.1) GO TO 300
            DO }320\textrm{K}=\textrm{J}+1,MARK+KTYP+
                KONEC(J,K) = 1
        CONTINUE
        IF(LL.EQ.1) THEN
            MARK = MARK+KTYP
            IF(KTYP.NE.10) GO TO 400
            KTYP = 4
            KONEC (MARK-9,MARK+2) = 1
            KONEC (MARK-9,MARK+3) = 1
            KONEC (MARK-9,MARK+4) = 1
            KONEC (MARK-8,MARK+2) = 1
            KONEC (MARK-8,MARK+3) = 1
            KONEC (MARK-8,MARK+4) = 1
            IF( MARK.GE. (NRUS*NARU-4)) GO TO 317
            KONEC (MARK-9,MARK+5) = 1
            KONEC (MARK-9,MARK+6) = 1
            KONEC (MARK-8,MARK+5) = 1
            KONEC (MARK-8,MARK+6) = 1
            IF( MARK.GE.(NRUS*NARU-18)) GO TO 317
            KONEC (MARK-9,MARK+15) = 1
            KONEC (MARK-8,MARK+15) = 1
            CONTINUE
            IF( MARK.LE.10 ) GO TO 200
            DO }370 L=1,1
                KONEC (MARK-11, MARK-10+L) = 1
            KONEC (MARK-10,MARK-9) = 1
            KONEC (MARK-10, MARK-8) = 1
            KONEC (MARK-10,MARK+1) = 1
            IF(MARK.GE. (NARU*NRUS-4)) GO TO 318
            KONEC (MARK-11,MARK+5) = 1
            KONEC (MARK-11,MARK+6) = 1
            IF (MARK.GE. (NARU*NRUS-18)) GO TO 318
            KONEC (MARK-11,MARK+15) = 1
            CONTINUE
            GO TO 200
            KTYP = 10
            IF( MARK.EQ.(NARU*NRUS) ) GO TO 200
            KONEC (MARK-2,MARK+2) = I
            KONEC (MARK-2,MARK+11) = 1
            KONEC (MARK-3,MARK+1) = 1
            KONEC (MARK-3,MARK+2) = 1
            KONEC (MARK-3,MARK+11)=1
    ELSE
            MARK = MARK+KTYP
            IF(KTYP.EQ.10) THEN
                KTYP = 4
            ELSE
                KTYP = 10
            END IF
        END IF
CONTINUE
CONTINUE
    IENTRY = 1
    RETURN
    END
```

```
    SUBROUTINE SUBSET( IENTRY )
C
Generates a key (KSUB) for phenyl ring substitution based on the
info in KSB, and then recalculates substituent positions and
assigns partial atomic charges.
>>>> NOTE: IT IS CURRENTLY ASSUMED THAT THE QHAL DATA <<<<<<
>>>> STATEMENT CONTAINS THE CORRECT MONO-OR DI-SUB VVALUES <<<<<<
All substitution occurs on one ring per repeat unit.
        KEY: KSB(1): Substituent atom/group ID;
                    2 = H (1e. no substitution)
                    7=F,8=Cl, 9 = Br
                    KSB(2): Substition location (1 = acid, 2 = amide)
                    KSB(3): Substitution order ( }0=\mathrm{ random, 1 = head-to-head,
                        tall-to-tail, 2 = head-to-tail)
            KSB(4): Substitution type ( }0=\mathrm{ none, 1 = mono-sub at the
                        2 position, 2 = di-sub at the 2,6 positions)
        INTEGER*4 SEED
        PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
    MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
        COMMON /CHAINS/ PHI(NIBB),THETA(NIBB),BL(NIBB),PHIR(NRNG),
                    KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
                    A(4, 4,MNAT), XYZ (4,MNAT, MNCH), KIND (MNAT, MNCH),
                    CHRG (MNAT, MNCH), KONEC (MNAT, MNAT), DEE, THT, RHO,
                    NUNIQ
        COMMON /SUBS/ WTED(4)
        DIMENSION KSUB (MNSB,MNCH),DX (9),PV (3), QHAL (10,3)
        SAVE KSUB
        DATA QHAL/ -0.22,0.22,0.22,0.,0.,0.,0.,0.,0.,0.,
    + -.02,-.06,-.12,-.12,-.03,-.03,+.17,+.17,+.02,+.02,
    + +.02,-.06,-.09,-.09,-.14,-.14,+.17,+.17,+.08,+.08/
    DATA DX/1.,1.,1.,1.,1.,1.,1.42,1.70,1.87/
If no substitution required, normalize phenyl ring EDEL weightings
and RETURN immediately:
    IF( (KSB (1).NE.2).AND.(KSB (4).NE.0) ) GO TO 30
    DO 40 I=1,4
        WTED(I) = 1.
    RETURN
    30 CONTINUE
C
C
    DO 50 I=1,4
            WTED(I) = 0.
    SEED = i
    DO 100 J=1,NCH
        DO 100 I=4,2*NRNG*NRUS,4
            KM = 3
            IF( KSB(2).EQ.2 ) KM = 1
            IF( KSB(3).EQ.O) THEN
                    KS = I - KM + NINT( RANDS1 (SEED) )
            ELSE IF( KSB(3).EQ.1 ) THEN
                KS = I - KM + MOD (KS,2)
                IF( I.EQ.4 ) KS = I - KM
            ELSE IF( KSB(3).EQ.2) THEN
                    KS = I - KM
            ELSE
                    WRITE(*,*) 'ERROR: Invalid KSB value in SUBSET'
            END IF
            DO 200 K=I-3,I
                KSUB(K,J) = 0
                IF( K.EQ.KS ) KSUB(K,J)=KSB(1)
                    IF( K.NE.KS ) WTED(K-I+4) = WTED (K-I+4) +
                            2./REAL (NRNG*NRUS*NCH)
                CONTINUE
    CONTINUE
```

```
        IENTRY = 1
C
    400 CONTINUE
        DO 500 J=1,NCH
        DO 500 I=1,2*NRNG*NRUS
            KSTEP = MOD (I,2)
            IF( KSUB(I,J).EQ.O ) GO TO 500
C
C Perform substitution:
C 1) Recalculate group coordinates:
C
    KNRU = (I-1)/4
    KLOC = KNRU*28 + (I-4*KNRU) /3*14 + 3 + MOD (I, 2)*4
    DO 300 L=1, KSB (4)
            LLOC = KLOC+(L-1)*2
            PVM2 = 0.
            DO 350 K=1,3
                PV(K) = XYZ(K,LLOC+1,J) - XYZ(K,LLOC,J)
                    PVM2 = PVM2 + PV (K)**2
                    PVM = SORT( PVM2 )
                    DO 360 K=1,3
                    XYZ(K,LLOC+1,J) = XYZ (K,LLOC,J) +
                            PV(K)*DX(KSUB (I,J))/PVM
            KIND(LLOC+1,J) = KSUB(I,J)
            CONTINUE
C
C 2) Reassign ring charges according to substitution:
C Ten independent charges allowed, in the following vector order:
    C1 C4 C3 C5 C2 C6 H3 H5 X2 X(Or H)6
            IF(KSTEP.EQ.1) THEN
                    CHRG (KLOC-6,J) = QHAL (1,KSUB (I,J)-6)
                    CHRG (KLOC-5,J) = QHAL (2,KSUB (I,J) -6)
                    CHRG (KLOC-4,J) = QHAL (3, KSUB (I,J) -6)
                    CHRG (KLOC-3,J) = QHAL (7, KSUB (I,J)-6)
                    CHRG (KLOC-2,J) = QHAL (4,KSUB (I,J) -6)
                    CHRG (KLOC-1,J) = QHAL (8,KSUB (I,J) -6)
                    CHRG (KLOC ,J) = QHAL (5,KSUB (I,J) -6)
                    CHRG (KLOC+1,J) = OHAL (9,KSUB (I,J) -6)
                    CHRG (KLOC+2,J) = QHAL (6,KSUB (I,J) - 6)
                    CHRG (KLOC+3,J) = QHAL (10,KSUB (I,J) -6) .
            ELSE
                    CHRG (KLOC-1,J) = QHAL (1,KSUB (I,J) -6)
                    CHRG (KLOC-2,J) = QHAL (2,KSUB (I,J) -6)
                        CHRG (KLOC +4,J) = QHAL (3,KSUB (I,J) -6)
                    CHRG (KLOC+5,J) = QHAL (7, KSUB (I, J) -6)
                    CHRG (KLOC +6,J) = QHAL (4,KSUB (I,J) - 6)
                    CHRG (KLOC + 7,J) = QHAL (8, KSUB (I,J) -6)
                    CHRG(KLOC ,J) = QHAL (5,KSUB (I,J) -6)
                    CHRG (KLOC+1,J) = QHAL (9,KSUB (I,J) -6)
                    CHRG(KLOC+2,J) = QHAL ( }6,\operatorname{KSUB}(I,J)-6
                CHRG (KLOC + 3,J) = QHAL (10,KSUB (I,J) -6)
            END IF
    5 0 0 ~ C O N T I N U E ~
        RETURN
        END
        SUBROUTINE ENERGY
C
Calculates the total potential energy per residue of the simulation
stucture, based on a polymer of the PPTA family
Contributions: EDEF (Bond Angle Deformation, Valence FF)
    EDEL (Conjugated Bond Interaction Energy)
    ENOB1 (Intramolecular LJ Interaction)
    ECOU1 (Intramolecular Coulombic Interactions)
    ENOB2 (Intermolecular LJ INTERACTIONS)
    ECOU2 (Intermolecular Coulombic Interactions)
    EULJP (Intramolec. LJ Lattice Correction)
    EUESP (Intramolec. ES lattice Correction)
```

```
C
C
The reference unit for calculating the potential energy depends on
the choice of RBASE and the size of the Helical Repeat Unit (HRU):
Case 1: Single chain simulation: disregard "helical basis region";
    calculate interaction of first RBASE residues with rest of
    chain.
Case 2: THT = 0: HRU is identical to chemical repeat unit, thus
    similar to Case l
Case 3: HRU consists of NRUS-2 or fewer structural repeat units;
    region may be treated exactly
Case 4: HRU exceeds NRUS-2 or is incommensurate: Region is
    arbitrarily set equal to the chemical repeat unit
ANGERR is the angular discrepancy allowed in determining the number
of turns of the helix required to obtain an integral number of
complete 360 degree rotations
This version of ENERGY provides a compiled toggle option between the
two lattice sum methods: 3D Periodicity (LSTOGG=0) or Helical Period-
Idity (LSTOGG=1, enable MV call in lattice sum section). On a
representative minimization (eg. 9 PPTA chains w/22 degrees of freedom) C the following
comparison times were obtained
C
    LOGICAL FIRST,REDUND
    PARAMETER( EPSILON=1E-6 )
    PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
    +
    COMMON /CHAINS/ PHI (NIBB),THETA (NIBB),BL (NIBB),PHIR(NRNG),
                    KSB (4),PCH (4,MNAT),NARU,NRUS,NCH,
                        A (4, 4, MNAT) , XYZ (4, MNAT, MNCH) , KIND (MNAT, MNCH),
                    CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT) , DEE , THT, RHO,
                    NUNIQ
    COMMON /CELL/ CELD (3), CELA (3),CHSPEC (5,MNCH), CHLOC (3,MNCH),
                        NUNIV, KONDENS,TEMP,DENSC
    COMMON /ENRG/ EDEF,EDEL,ENOB1, ENOB2,ECOU1,ECOU2,EULJ,EUES,
                        EULJP,EUESP,ECFF,ETOT
    COMMON /SUBS/ WTED(4)
    COMMON /STRN/ KOC,KASE,ST (3, 3),SUCV (3,3),DINIT,DPRIME,DCON
    DIMENSION ANGZ (12), ANGO(12),DCL (3,3)
    DIMENSION FMULT (MNCH,MNCH)
    DIMENSION VDWR(NKIND),ALPHA (NKIND), EFFNE (NKIND)
    DIMENSION UX(3),VY(3),UZ (3),DVECT(4),R(4,4,21)
    DIMENSION SUM1 (3136),SUM6(3136),SUM12(3136),SUM1P(3136),
                        SUM6P (3136), SUM12P(3136), TMP1 (3136),TMP6 (3136),
                    TMP12 (3136), TMP1P(3136), TMP6P(3136),TMP12P (3136)
    DATA VDWR/ 1.96,1.37,2.03,1.37,1.96,1.60,1.35,1.80,1.95/
    DATA ALPHA/0.768,0.330,2.16,0., 1.95,1.02,0.38,1.41,3.34/
    DATA EFFNE/5.2, 0.9, 5.2, 0.9.6.1, 7., 8., 12., 21./
    DATA ANGZ/ .088,.088,.050,.088,.050,.088,.088,.088,
    + .088,.050,.050,.088 /
    DATA ANGO/ 120.,120.,120.,120.,120.,120.,120.,120.,
    + 120.,120.,120.,120./
    DATA BO,B00,CO,C00,B1,B2/9.0,13.0,.2,.4,2.8,21.2/
    DATA BXO,BXOO, CXO, CXOO /0.0,32.0,.0,.8/
    DATA DE/ 3.5 /
    DATA ANGERR/ 5./
    DATA BWDIST/ 3.3 /
C
    FIRST = .TRUE
    DEGRAD = 3.1415926535/180.
    LSTOGG = 0
Initialize VDWR, ALPHA, and EFFNE arrays:
    CALL ELJ( NKIND,1,1,VDWR,ALPHA,EFFNE,3.,3.,FIRST,POTINIT )
C
    RBASE = REAL( NUNIQ )
    ECFF=0.
```

```
        EDEF = 0.
        EDEL = 0.
        ENOB1 = 0.
        ECOO1 = 0.
        ENOB2 = 0.
        ECOU2 = 0.
        EULJ = 0.
        EUES =0.
        EUIJP = 0.
        EUESP = 0.
        NATOMS = NARU*NRUS
        NM1 = NATOMS-1
C
C Initialize intermolecular interaction multiplier matrix:
c
c
    50 CONTINUE
C WRITE (11,1)
c 1 FORMAT(1X,'helical repeat length exceeds chain length')
            NER = NRUS-2
            NER = 1
            GO TO 80
            NER = M
            GO TO 80
    100 NER = 1
    80 CONTINUE
            LATM = INT (RBASE*NER*NARU+1.01)
C
C Bond angle deformation energy:
    L = 1
    IF(ABS (CHSPEC(1,L)).NE.1.) GO TO 900
    DO 300 K=1,NARO/28
            KADD = (K-1)*NARU/2
            KADE = (K-1)*NIBB/2
        DO 300 J=1,12
            IF( J.EQ.9 ) THEN
            KA = 12 + KADD
            KB = 13 + KADD
            KC = 11 + KADD
            ELSE IF( J.EQ.10) THEN
            KA = 14 + KADD
            KB = 15 + KADD
            KC = 13 + KADD
            ELSE IF(J.EQ.11) THEN
            KA = 26 + KADD
            KB = 27 + KADD
            KC = 25 + KADD
        ElSE IF( J.EQ.12 ) THEN
            KA = 28 + KADD
            KB = 29 + KADD
            KC = 27 + KADD
            ELSE
            END IF
            IF( J.LE.8 ) THEN
                EDEF = EDEF+0.5*ANGZ (J)*(THETA (J+KADE) -ANGO (J))**2
            ElSE
                CSQ = (XYZ (1, KA, L) -XYZ (1,KB, L))**2 +
                                    (XYZ (2,KA, L) -XYZ (2,KB,工))**2 +
                                    (XYZ (3, KA, L) -XYZ (3,KB,L))**2
            ASQ = (XYZ (1,KB,L) -XYZ (1,KC,L))**2 +
                        (XYZ (2,KB, L) -XYZ (2,KC, L))**2 +
                        (XYZ (3, KB,L) -XYZ (3,KC,L))**2
```

```
            BSQ = (XYZ (1,KC,L)-XYZ (1,KA,L))**2 +
    + (XYZ(2,KC,L) -XYZ(2,KA,L))**2 +
    +
                        XYZ (3,KC,L) -XYZ (3,KA,L)) **2
            ANGT= ACOS( (ASQ+BSQ-CSQ)/(2*SQRT (ASQ*BSQ)) )
            ANGT= ANGT/DEGRAD
            EDEF = EDEF+0.5*ANGZ (J)*(ANGT - ANGO(J))**2
        END IF
    300 CONTINUE
C
C Conjugated bond interaction energy:
    Note: with the introduction of substitution on the ring (potentially
    in a random sequence and without equal occurance at the two 'ends'
    of the ring) with different delocalization potentials, it becomes
    necessary to compute EDEL welghted by the appropriate loci of
    substitutions:
        NRT = NARU/28
        DO 400 K=1,NRT
            NRR = (K-1)*NRNG/NRT
            NRP = (K-1)*NIBB/NRT
            EDEL = EDEL + WTED (2)*CURTINV ( (PHIR (1+NRR) -PHI (2+NRP)),
                                    BO,C0 )
            EDEL = EDEL + WTED (3)*CURTATE ( PHIR (2+NRR),B00,COO )
            EDEL = EDEL + WTED (4)*CURTATE ( (PHIR (2+NRR)-PHI (6+NRP)),
                    B00,COO
            EDEL = EDEL + WTED(1)*CURTINV ( PHIR(1+NRR),BO,CO )
            EDEL = EDEL + (1. - WTED(2))*CURTATE{ (PHIR(1+NRR) -
                    PHI (2+NRP)), BXO,CXO )
            EDEL = EDEL + (1.- WTED(3))*CURTATE( PHIR(2+NRR),BXOO,CXOO )
            EDEL = EDEL + (1. - WTED (4))*CURTATE ( (PHIR(2+NRR) -
                PHI(6+NRP)),BXOO,CXOO )
            EDEL = EDEL + (1. - WTED(1))*CURTATE( PHIR(1+NRR), BXO,CXO )
            EDEL = EDEL + B1/2*(1.-COS(PHI(4+NRP) *DEGRAD))
                    + B2/2*(1.-COS(2*PHI(4+NRP) *DEGRAD))
            EDEL = EDEL + B1/2* (1.-COS (PHI (8+NRP) *DEGRAD))
            + B2/2*(1.-COS(2*PHI(8+NRP) *DEGRAD))
    4 0 0 ~ C O N T I N U E ~
        DO 500 L1=1,L1MAX
            IF(L1.NE.1) GO TO 530
C
C Intramolecular nonbonded interactions:
        DO 520 I=1,NM1
            IP1 = I+1
            DO 510 J=IP1,NATOMS
                    IF( KONEC(I,J).EQ.1 ) GO TO }51
                    IF( I.GE.LATM ) GO TO 510
            D2 = (XYZ (1,I,L1)-XYZ (1,J,L1))**2 +
    + (XYZ(2,I,I1)-XYZ (2,J,L1))**2 +
    + (XYZ(3,I,I1)-XYZ (3,J,L1))**2
            DIS = SQRT (D2)
            CALL ELJ( NKIND,KIND (I,L1),KIND (J,L1),VDWR,ALPHA,EFFNE,
                        D2,D2,FIRST,POTLJ )
            ENOB1 = ENOB1 + POTLJ
            CALL EES( BWDIST,DIS,DE,CHRG(I,L1),CHRG(J,L1),POTES )
            ECOU1 = ECOU1 + POTES
            CONTINUE
520 CONTINUE
530 CONTINUE
        IF (NCH.EQ.1) GO TO 500
        LP1 = L1+1
Intermolecular interactions:
DO 550 L2=LP1, NCH
    IF( INT(FMULT(L1,L2)*100.).EQ.0 ) GO TO 550
    IF( INT(ABS(CHSPEC(1,L2))+.01).NE.1 ) GO TO 550
    ENOBT = 0.
    ECOUT = 0.
    DO 570 I=1,NATOMS
            DO 560 J=1,NATOMS
```

```
            IF (I.LT.LATM) THEN
            D2 = (XYZ(1,I,L1)-XYZ(1,J,L2))**2 +
    + (XYZ(2,I,L1)-XYZ(2,J,L2))**2 +
    + (XYZ (3,I,L1) -XYZ (3,J,L2))**2
            DIS = SQRT(D2)
            CALL ELJ( NKIND,KIND (I,L1),KIND (J,L2),VDWR,ALPHA, EFFNE,
            D2,D2,FIRST,POTLJ )
            ENOBT = ENOBT + POTLJ
            CALL EES( BKDIST,DIS,DE,CHRG(I,L1),CHRG(J,L2),POTES )
            ECOUT = ECOUT + POTES
            ELSE
            IF (J.LT.LATM) THEN ,
            D2 = (XYZ(1,I,L1)-XYZ(1,J,L2))**2 +
    + (XYZ (2,I,L1)-XYZ (2,J,L2))**2 +
        + (XYZ (3,I,L1)-XYZ (3,J,L2).)**2
            DIS = SORT(D2)
            CALL ELJ( NKIND,KIND(I,L1),KIND(J,L2),VDWR,ALPHA, EFFNE,
                                    D2,D2,FIRST,POTLJ )
            ENOBT = ENOBT + POTLJ
            CALL EES( BNDIST,DIS,DE,CHRG(I,L1),CHRG(J,L2),POTES )
            ECOUT = ECOUT + POTES
            ELSE
            END IF
            END IF
            CONTINUE
            continue
            ENOB2 = ENOB2 + 0.5*ENOBT
            ECOU2 = ECOU2 + 0.5*ECOUT
        CONTINOE
        CONTINUE
c
C Infinite Lattice Correction:
        (specific for two-chain-type lattices, with interchain interactions
        1-1, 1-2, 2-1, and 2-2 only; of these, only half of the 1-1 and 2-2
        interactions are unique, and the 1-2 interactions are identical to
        the 2-1 interactions. NUNIV represents "Infinite" lattice size)
            IF(NUNIV.EQ.O ) GO TO 630
            DO 610 K=1,3
                UX(K) = CHLOC (K,2) - CHLOC (K,4)
                UY(K) = CHLOC (K,2) + CHLOC (K,4)
                    UZ (K) = 0.
                    DVECT(K) = 0.
            UZ(3) = RBASE*REAL (NER) *DEE
            DVECT(4) = 1.
            NCM2 = NINT(SORT (REAL (NCH))-2)
            NNCM = -NCM2-1
            ESPWR = BWDIST*LOG(DE)
C
C Set up helix periodicity transforms beforehand:
        IF(LSTOGG.EQ.0) GO TO 625
        DO 620 LZN = 1,2*NUNIV+1
            LZ = LZN-NUNIV-1
            RTN = RBASE*REAL (LZ*NER)
            RTH = RTN*THT
            RDE = RTN*DEE
            CALL RTRANS( RTH,RDE,R(1,1,L2N) )
    620 CONTINUE
    625 CONTINUE
C
        DO 600 L1=1,2
        DO 600 L2=L1,2
            KFIJ = L1+L2-1
            LXS = -NUNIV
            IF( KFIJ.NE.2 ) LXS = 0
            NNLX = NUNIV-LXS+1
            NNLY = 2*NUNIV+1
            NNLZ = NNLY
C
C zero all atom-atom summations:
```

C

```
        DO 645 I=1,LATM-1
        DO 645 J=1, LATM-1
    NTMP = (I-1)*(LATM-1) + J
    SUM1P (NTMP) = 0.
    SUM6P (NTMP) = 0.
    SUM12P(NTMP) = 0.
    SUM1 (NTMP) = 0.
    SUM6 (NTMP) = 0.
    SUM12 (NTMP) = 0.
    CONTINUE
c
C Loop over all lattice points, checking for redundancy with explicit
C calculation scheme for I1-L2:
C
cvd$ noconcur
    DO }680\mathrm{ LX=LXS,NUNIV
cvd$ noconcur
    DO 680 LY=-NUNIV,NUNIV
cvd$ noconcur
    DO }680\textrm{LZ=-NUNIV,NUNIV
        NRED = (LX-LXS)*NNLY*NNLZ + (LY+NUNIV)*NNLZ +
                        (LZ+NUNIV) + 1
            IF( (KFIJ.NE.2).AND.(LX.EQ.O).AND.(LY.LE.0) ) THEN
                    REDUND = .TRUE.
                    GO TO 680
                ELSE
                    REDUND = .FALSE.
                END IF
                IF( ABS(LZ).LE.(NRUS-1) ) THEN
                    IF (KFIJ.NE.2) THEN
                    REDUND = ( (ABS (LX) +ABS (LY)).LE.NCM2 )
                    ELSE
                        IF( (LX+LY).NE.NCM2 ) THEN
                            REDUND = ( ((ABS (LX) +ABS (LY)).LE.NCM2).OR.
                            ((LX.LT.O).AND. (LY.LT.O).AND.((LX+LY).EQ.NNCM)) )
                END IF
                END IF
                END IF
                IF ( REDUND ) GO TO 680
C
C Loop over all atoms of the units of L1 and L2 at this lattice point
C to get inverse distances:
C
        DO 650 I=1, LATM-1
        DO 650 J=1, LATM-1
        NTMP = (I-1)*(LATM-1) +J
C ESPWR = (VDWR(KIND (I,L1))+VDWR(KIND(J,L2)) )*LOG(DE)
        IF( LSTOGG.EQ.O ) THEN
                        DVECT (1) = XYZ(1,J,L2) - XYZ (1,I,L1)
                        DVECT(2) = XYZ (2,J,L2) - XYZ(2,I,L1)
            DVECT(3) = XYZ (3,J,L2) - XYZ (3,I,L1)
            DVV = (DVECT(1)+LX*UX(1)+IY*UY(1)+LZ*UZ(1))**2
        + +(DVECT (2)+LX*UX(2)+LY*UY(2)+LZ*UZ (2))**2
        ELSE
            LZN = LZ + NUNIV + 1
                DVECT(1) = XYZ(1,J,L2) - CHLOC (1,L2)
                    DVECT (2) = XYZ (2,J,L2) - CHLOC (2,L2)
                            DVECT (3) = XYZ(3,J,L2) - CHLOC (3,L2)
C Next line disabled only to allow for better concurrency optimization; C must be enabled
for use of helical periodicity calculations
C CALL MV ( R (1,1,LZN),4,3,DVECT,4,DVECT )
    DVECT(1) = DVECT(1) + CHLOC(1,L2) - XY2 (1,I,L1)
    DVECT (2) = DVECT (2) + CHLOC (2,L2) - XYZ (2,I,L1)
    DVECT (3) = DVECT (3) + CHLOC (3,L2) - XYZ (3,I,L1)
    DVV = (DVECT(1) +LX*UX(1)+IY*UY(1))**2
    + (DVECT (2)+LX*UX(2) +IY*UY (2))**2
    +(DVECT (3)+LX*UX (3)+LY*UY (3))**2
            END IF
            DVRCOT = SQRT (DVV)
```

```
C Note: discontinuity in S1 is possible, may have to spline (5/16/88)
            S1 = EXP(ESPWR/DVROOT)/DVROOT
            IF( DVROOT.LE.BWDIST ) S1 = DE/DVROOT
            S6 = 1./DVV**3
            S12 = S6**2
            IF( (KFIJ.NE.2).AND.(LX.EQ.0).AND.(LY.EQ.O) ) THEN
                TMP1P(NTMP) = S1
                TMP6P(NTMP) = S6
                TMP12P(NTMP) = S12
                    ELSE
                TMP1(NTMP) = S1
                TMP6(NTMP) = S6
                TMP12(NTMP) = S12
                    END IF
    CONTINUE
c
C Add this element to summation
            DO 655 I=1, LATM-1
            DO }655\textrm{J}=1\mathrm{ , LATM-1
                NTMP = (I-1)* (LATM-1) +J
                SUM1P(NTMP) = SUM1P(NTMP) + TMP1P(NTMP)
                SUM6P(NTMP) = SUM6P(NTMP) + TMP6P(NTMP)
                    SUM12P(NTMP) = SUM12P(NTMP) + TMP12P(NTMP)
                        SUM1 (NTMP) = SUM1 (NTMP) + TMP1 (NTMP)
                        SUM6(NTMP) = SUM6(NTMP) + TMP6(NTMP)
                        SUM12(NTMP) = SUM12(NTMP) + TMP12(NTMP)
        CONTINUE
        CONTINUE
680
c
C Finally, calculate the energies for each atom-atcm pair:
C
        DO 665 I=1, IATM-1
        DO 665 J=1,LATM-1
            NTMP = (I-1)* (LATM-1) +J
            CALL ELJ( NKIND,KIND (I,L1),KIND (J,L2),VDWR,ALPHA,EFFNE,
                                    SUM6 (NTMP), SUM12 (NTMP), FIRST,POTLJ )
        +
                            SUM6 (NTMP),SUM
                EULJ = EULJ + 0.5*POTLJ
                EUES = EUES + 166.*SUM1 (NTMP)* CHRG(I,L1)* CHRG (J,L2)/DE
                IF( (SUM6P (NTMP).EQ.O.).AND.(SUM12P(NTMP).EQ.O.) ) GO TO 665
                CALL ELJ( NKIND,KIND (I,L1),KIND (J,L2),VDWR,ALPHA, EFFNE,
                                    SUM6P(NTMP),SUM12P (NTMP),FIRST, POTLJP)
            +
                                    EULJP = EULJJP + 0.5*POTLJP
                                    EUESP = EUESP + 166.*SUM1P(NTMP)*CHRG (I,L1)*CHRG(J,L2)/DE
        CONTINUE
    CONTINUE
    CONTINUE
C Here is calculated the penalty function that fixes chain pitch DEE
C
    IF( KUC.NE.0 ) ECFF = DCON*( DEE - DPRIME )**2
C
C Total energy per residue:
C
    ENOB1 = ENOB1/NER/RBASE
    ENOB2 = ENOB2/NER/RBASE
    ECOU1 = ECOU1/NER/RBASE
    ECOU2 = ECOU2/NER/RBASE
    EULJ = EULJ/NER/RBASE
    EUES = EUES/NER/RBASE
    EULJP = EULJP/NER/RBASE
    EUESP = EUESP/NER/RBASE
    ETOT = EDEF + EDEL + ENOB1 + ENOB2 + ECOU1 + ECOU2 + ECFF +
    +
        RETURN
900 CONTINUE
    WRITE(11,*) 'ERROR: No parent chain available'
    RETURN
    END
```

SUBROUTINE RTRANS (TH,D,R )
$C$ Invokes a fixed strain on the chain locations. KUC and KASE specify
$C$ the unitcell convention and relative orthogonal axes, for frame of
reference purposes. ST is the orthogonal strain tensor (elements of
which are fractional change; eg. 0.01 ). CHIOC(1,2) and CHLOC(1, 4) are
used to relate unitcell and are returned in strained form. Changes
along the chain axis dimension must be handled using a penalty
function, such that the undeformed length CEE becomes the deformed
length CEEP
DIMENSION $\operatorname{ST}(3,3), \operatorname{STP}(3,3), \operatorname{CHLOC}(3,9), \operatorname{CRT}(3,3), \operatorname{CV}(2), \operatorname{DV}(2)$,
$+$
$\operatorname{SFR}(3,3), \operatorname{SFRI}(3,3), \operatorname{UCV}(3,3), \operatorname{SUCV}(3,3)$
C
C First, define placement of unit cell in lattice (Type 1-12: see key)
C and establish coordinate rotation tensor
KONV $=1$
$\operatorname{UCV}(1,3)=0$.
$\operatorname{UCV}(2,3)=0$.
$\operatorname{UCV}(3,3)=\operatorname{CEE}$
500 CONTINUE
IF ( KUC.EQ. 1 ) THEN
C Type 1: $a=1-9, b=1-3$
IF ( KONV.EQ. 1 ) THEN
DO $110 \mathrm{I}=1,3$
$\operatorname{UCV}(I, 1)=\operatorname{CHLOC}(I, 2)-\operatorname{CHLOC}(I, 4)$
$\operatorname{OCV}(I, 2)=\operatorname{CHLOC}(I, 2)+\operatorname{CHLOC}(I, 4)$
ELSE
DO $115 \mathrm{I}=1,3$
$\operatorname{CHLOC}(I, 2)=0.5 *(\operatorname{UCV}(I, 2)+\operatorname{UCV}(I, 1))$
$\operatorname{CHLOC}(I, 4)=0.5 *(\operatorname{UCV}(I, 2)-\operatorname{UCV}(I, 1))$
END IF
ELSE IF (KUC.EQ. 2 ) THEN
C Type 2: $a=1-9, b=1-51$
IF ( KONV.EQ.1 ) THEN
DO $120 \mathrm{I}=1,3$
$\operatorname{OCV}(I, 1)=\operatorname{CHLOC}(I, 2)-\operatorname{CHLOC}(I, 4)$
$\operatorname{UCV}(I, 2)=3 * \operatorname{CHLOC}(I, 2)-\operatorname{CHLOC}(I, 4)$
ELSE
DO $125 \mathrm{I}=1,3$
$\operatorname{CHLOC}(I, 2)=0.5 *(\operatorname{UCV}(I, 2)-\operatorname{UCV}(I, 1))$

```
125 CHLOC(I,4) = 0.5*(UCV(I,2) - 3.*UCV(I,1))
    END IF
        ELSE IF (KUC.EQ.3) THEN
C Type 3: a=1-9, b=1-35
            IF( KONV.EQ.1 ) THEN
            DO 130 I=1,3
                UCV(I,1) = CHLOC(I,2) - CHLOC(I,4)
                            UCV}(I,2)=3*CHLOC(I,4) - CHLOC(I,2
            ELSE
            DO 135 I=1,3
                        CHLOC(I,2)=0.5*(UCV (I,2) + 3.*UCV(I,1))
                        CHLOC}(I,4)=0.5*(UCV(I,2)+\operatorname{UCV}(I,1)
            END IF
        ELSE IF (KUC.EQ.4) THEN
C Type 4: a=1-9, b=1-11
            IF( KONV.EQ.1 ) THEN
                DO 140 I=1,3
                            UCV(I,1) = CHLOC(I,2) - CHLOC(I,4)
140 UCV (I,2) = 2*CHLOC (I,2)
            ELSE
                DO 145 I=1,3
                        CHLOC(I,2) = 0.5*UCV(I,2)
                        CHLOC (I,4) = 0.5*UCV (I,2) - UCV (I,I)
            END IF
            ELSE IF (KUC.EQ.5) THEN
C Type 5: a=1-7, b=1-11
            IF( KONV.EQ.1 ) THEN
                DO 150 I=1,3
                                    UCV}(I,1)=-\operatorname{CHLOC}(I,2) - CHLOC(I,4
                                    UCV}(I,2)=2*\operatorname{CHLOC}(I,2
            ELSE
                DO 155 I=1,3
                                    CHLOC (I,2) = 0.5*UCV(I,2)
1 5 5
                                    CHLOC (I,4) = -UCV (I,1) - 0.5*UCV(I,2)
            END IF
            ELSE IF (KUC.EQ.6 ) THEN
C Type 6: a=1-9, b=1-15
            IF( KONV.EQ.1 ) THEN
                DO 160 I=1,3
                                    UCV}(I,1)=\operatorname{CHLOC}(I,2)-\operatorname{CHLOC}(I,4
                                    UCV}(I,2)=2*CHLOC(I,4
            ELSE
                    DO 165 I=1,3
                        CHLOC (I, 2) = UCV (I,1) + 0.5*UCV (I, 2)
                        CHLOC(I,4) = 0.5*UCV(I,2)
            END IF
        ELSE IF( KUC.EQ.7 ) THEN
C Type 7: a=1-3, b=1-15
            IF( KONV.EQ.1 ) THEN
                DO }170\textrm{I}=1,
                    UCV}(I,1)=\operatorname{CHLOC}(I,2)+\operatorname{CHLOC}(I,4
                    UCV}(I,2)=2*\operatorname{CHLOC}(I,4
                        ELSE
                DO 175 I=1,3
                        CHLOC(I,2) = UCV(I,1) - 0.5*UCV (I,2)
                        CHLOC (I,4) = 0.5*UCV (I,2)
            END IF
        ELSE IF (KUC.EQ.8) THEN
C Type 8: a=1-9, b=1-2
    IF( KONV.EQ.1 ) THEN
                DO 180 I=1,3
                    UCV(I,1) = CHLOC(I,2) - CHLOC(I,4)
                        UCV}(I,2)=\operatorname{CHLOC}(I,2
            ELSE
                DO }185\textrm{I}=1,
                        CHLOC}(I,2)=\operatorname{UCV}(I,2
1 8 5
                                    CHLOC(I,4) = UCV(I,2) - UCV(I,1)
            END IF
            ELSE IF (KUC.EQ.9) THEN
C Type 9: a=1-7, b=1-2
            IF( KONV.EQ.1 ) THEN
```

```
            DO 190 I=1,3
                UCV(I,1) = -CHLOC(I,2) - CHLOC(I,4)
                        UCV(I,2)=CHLOC(I,2)
        ELSE
            DO }195\textrm{I}=1,
                        CHLOC(I,2) = UCV(I,2)
                        CHLOC(I,4) = -UCV(I,2) - UCV(I,1)
            END IF
        ELSE If (KUC.EQ.10) THEN
C Type 10: a=1-9, b=1-4
            IF( KONV.EQ.1 ) THEN
            DO 200 I=1,3
                    UCV(I,1) = CHLOC(I,2) - CHLOC(I,4)
                    UCV(I,2) = CHLOC(I,4)
            ElSE
                DO 205 I=1,3
                    CHLOC(I,2) = UCV(I,2) + UCV(I,1)
                    CHLOC(I,4) = UCV(I,2)
            END IF
            elSe IF (kUC.EQ.11) then
C Type 11: a=1-3, b=1-4
            IF( KONV.EQ.1 ) THEN
                DO 210 I=1,3
                    UCV(I,1) = CHLOC(I,2) + CHLOC(I,4)
                    UCV(I,2) = CHLOC(I,4)
            ElSE
                    DO 215 I=1,3
                    CHLOC(I,2) = UCV(I,1) - UCV(I,2)
                    CHLOC(I,4) = UCV(I,2)
            END IF
        ELSE IF (KUC.EQ.12 ) THEN
C. Type 12: a=1-2, b=1-4
            IF( KONV.EQ.1 ) THEN
                    DO 220 I=1,3
                    UCV(I,1) = CHLOC(I,2)
                    UCV(I,2)=CHLOC(I,4)
                        ELSE
                        DO 225 I=1,3
                    CHLOC(I,2)=UCV(I,1)
                    CHLOC(I,4) = UCV(I,2)
            END IF
        ELSE
            WRITE(*,*) 'ERROR: invalid cell definition in STRAIN'
        END IF
        IF( KONV.EQ.2 ) GO TO 600
c
By convention, strain tensor is given in a Unit Cell Frame of
Reference (UCV):
2 possibilites: UCV(1,1) lies in ST xz plane (KASE 1)
                            or:= UCV(1,2) lies in ST yz plane (KASE 2)
Rotate UCV to correspond to strain tensor F.O.R., deform UCV,
rotate back into original F.O.R., then decompose back to CHLOC
vectors
C
    DO 300 I=1,3
    DO 300 J=1,3
    SFR(I,J)=0.
    SFR(3,3)=1.
    CV(1) = UCV(1,KASE)
    CV(2)}=\operatorname{UCV}(2,KASE
    CVM = SQRT(CV(1)**2 + CV (2)**2)
    KOTHER = MOD (KASE, 2) +1
    SFR(1,1) = CV(KASE)/CVM
    SFR(1,2) = REAL (KOTHER-KASE)*CV (KOTHER)/CVM
    SFR(2,1) = -SFR(1,2)
    SFR(2,2)=SFR(1,1)
C
    DO 400 I=1,3
            DO 450 J=1,3
450 STP(I,J) = ST(J,I)
400 STP(I,I) = STP (I,I) + 1.
```

C
c

Read output format instruction codes, mode for Hessian in minimization, simulation dimensions, and ring substitution
$\operatorname{READ}(10,2) \quad(\operatorname{INSTR}(I), I=1,5)$
$\operatorname{READ}(10,2)$ MODE
READ $(10,1)$ NARU, NRUS, NCH, NUNIV, KONDENS, TEMP
IF (NRUS.GT.MNRU) NRUS $=$ MNRU
$\operatorname{READ}(10,2) \quad(\operatorname{KSB}(I), I=1,4)$
The polymer generation routines create a single polymer type (unsubstituted poly (p-phenylene terephthalamide). Modifications of the chemistry are intructed by $K S B(4)$, which is translated into a ring substitution sequence by the subroutine SUBSET.
Substitution on the ring also affects the number of monomer units required to describe a unique structural unit (ie. one repeat unit if unsubstituted or head-to-tail, two if head-to-head, tall-to-tail); this information is used by LDVAR to limit chain translation during optimization and is passed by NUNIQ. Here, determine NUNIQ:

```
If the 56 atom HHTT isomer representation is used, disable NUNIQ = 2:
```

NUNIQ $=1$
IF ( KSB (3).EQ.1 ) NUNIQ = 2
Read conformation paramters THETA, PHI, optimization flags for these two variables, and scan instructions for PHI. Same for PHIR.

DO $100 \mathrm{I}=1$, NIBB
$\operatorname{NSCAN}(I)=0$
NVAR $=0$
NSTOT $=1$
NSANG $=0$

```
    DO 200 I=1,NIBB
        READ (10,5) THETA(I),PHI (I), (LMV (J,I),J=1, 2),
                        ISCAN,SCO,SCX,SCD
        IF (LMV (1,I).GT.O) NVAR=NVAR+1
        IF (LMV (2,I).GT.0) NVAR=NVAR+1
        IF (ISCAN.LT.1) GO TO 200
        PHI(I) = SCO
        SCANO(I) = SCO
        SCAND (I) = SIGN (SCD,SCX-SCO)
        NSCAN(I) = INT(1.01+(SCX-SCO)/SCAND (I))
        NSANG = NSANG + 1
        NSTOT = NSTOT*NSCAN(I)
    CONTINUE
        DO 210 I=1,NRNG
            KTAG = MOD( (I+1),2 ) + 1
            ITAG = NIBB + I/3 + 1.
            JTAG = NIBB + I
            READ (10,9) PHIR(I), LMV (KTAG,ITAG), ISCAN, SCO, SCX, SCD
            IF (LMV (KTAG,ITAG).GT.0) NVAR=NVAR+1
            IF (ISCAN.IT.1) GO TO 210
            PHIR(I) = SCO
            SCANO(JTAG) = SCO
            SCAND (JTAG) = SIGN(SCD,SCX-SCO)
            NSCAN(JTAG) = INT(1.01+(SCX-SCO)/SCAND (JTAG))
            NSANG = NSANG + 1
            NSTOT = NSTOT*NSCAN(JTAG)
        CONTINUE
210
C
C Read cell packing parameters CELD (with optimization flag and scan
intructions), CELA (with flag and scan), and CHSPEC (with flag and
scan, up to a maximum of first nine chains)
C
    DO 400 I=1,MOPT
    NMOT(I) = 0
    NCTOT = 1
    NCMUT = 0
    DO 420 I=1,3
        READ (10,6) CELD (I), LCV (I,MNIC+1),IDVAR, DVO, DVX,DVD
        IF(IDVAR.LT.1) GO TO 420
        KK = MCOP+I
        CELD(I) = DVO
        CMUTO(KK) = DVO
        CMUTD (KK) = SIGN (DVD,DVX-DVO)
        NMUT (KK) = INT(1.01+(DVX-DVO)/CMUTD (KK))
        NCMUT = NCMUT+1
        NCTOT = NCTOT*NMUT (KK)
    CONTINUE
    DO 440 I=1,3
        READ (10,6) CELA (I), LCV (I,MNIC+2),IDVAR,DVO,DVX,DVD
        IF(IDVAR.LT.1) GO TO 440
        KK = MCOP+3+I
        CELA(I) = DVO
        CMUTO(KK) = DVO
        CMUTD (KK) = SIGN(DVD,DVX-DVO)
        NMUT (KK) = INT(1.01+(DVX-DVO)/CMUTD (KK))
            NCMUT = NCMUT+1
            NCTOT = NCTOT*NMUT (KK)
        CONTINUE
        DO 460 I=1,3
            IF(CELD(I).LT.1.) CELD(I) = 1000.
    DO 300 J=1,MNIC
            DO 300 I=1,5
                READ (10,7) CHSPEC(I,J), LCV (I,J),ICVAR,CVO, CVX, CVD
                IF (ICVAR.LT.1) GO TO 300
                KK = I +(J-1)*5
                CHSPEC(I,J) = CVO
                CMUTO(KK) = CVO
                CMOTD (KK) = SIGN(CVD,CVX-CVO)
                NMUT (KK) = INT(1.01+(CVX-CVO)/CMUTD (KK))
                NCMUT = NCMUT+1
                NCTOT = NCTOT*NMUT (KK)
```

```
    300 CONTINUE
        DO 500 J=1,MNIC+2
            DO }500I=1,
    500 IF (LCV (I,J).GT.O) NVAR=NVAR+1
    CLOSE (UNIT=10)
C
Call strain Input file
    CALL RDSTRAIN( MODE )
C
    1 FORMAT( 5(I4),F8.3)
    2 FORMAT( 5(I4))
    5 FORMAT( 2(F8.3),3(I4, 4X),3(F8.3))
    6 FORMAT(F8.3,2(I4,4X),3(F8.3))
    7 FORMAT( F8.3,2(I4,4X),3(F8.3) )
    9 FORMAT(F8.3,2(I4,4X),3(F8.3))
    10 FORMAT( 4(I4))
    END
    SUBROUTINE RESWR( INSTR )
C
C Output simulation results to a file opened in the main program,
C according to instructions contained in INSTR
C
        COMMON /CELL/ CELD(3),CELA(3),CHSPEC(5,MNCH),CHLOC (3,MNCH)
            NUNIV,KONDENS,TEMP,DENSC
        COMMON /ENRG/ EDEF,EDEL,ENOB1,ENOB2,ECOU1,ECOU2,EULJ,EUES,
            EULJP,EUESP,ECFF,ETOT
        COMMON /OPRS/ NVAR,VARS (97),GRAD (97),GRADNM
        COMMON G(97),H(4753),W(291)
        COMMON /STRN/ KUC,KASE,ST (3,3),SUCV (3,3),DINIT,DPRIME,DCON
        DIMENSION INSTR(5)
        CHARACTER*2 KAR(NKIND)
    DATA KAR/'C','H','C','H','N','O','F','CL','BR'/
C
    NHELEM = NVAR* (NVAR +1)/2
    IF(INSTR(1).NE.1) GO TO 210
    WRITE(11,1)
    WRITE (11, 2)
    NIBB2 = INT(NIBB/2)
    DO 110 J=1,NIBB2
        WRITE(11,20) THETA(J),PHI(J),THETA(J+NIBB2), PHI(J+NIBB2)
    WRITE(11,21) (PHIR(I),I=1,NRNG)
    WRITE (11, 23) (KSB(I),I=1,4)
    IF(INSTR(2).NE.1) GO TO 220
    WRITE (11,4) DEE, RHO,THT
220 IF(INSTR(3).NE.1) GO TO 230
    WRITE (11,5)
    WRITE(11,24) TEMP,DENSC,NUNIV
    WRITE (11, 6) (CELD (I), I=1,3)
    WRITE(11,7) (CELA(I), I=1,3)
    WRITE (11,8)
    DO 200 J=1,2
        IF(ABS(CHSPEC(1,J)).EQ.O.) GO TO 200
        WRITE (11,9) J,(CHSPEC(I,J),I=2,5)
    CONTINUE
    IF(KUC.EQ.O) GO TO 235
    WRITE(11,31)
    WRITE(11,32) KOC,KASE
    WRITE (11, 33) (ST(I,I), I=1,3)
    WRITE(11,33) (ST(2,I),I=1,3)
    WRITE(11,33) (ST(3,I),I=1,3)
```

WRITE $(11,34)$
$\operatorname{WRITE}(11,33) \quad(\operatorname{SUCV}(1, I), I=1,3)$
WRITE $(11,33) \quad(\operatorname{SUCV}(2, I), I=1,3)$
$\operatorname{WRITE}(11,33) \quad(\operatorname{SUCV}(3, I), I=1,3)$
WRITE (11,*)
IF(INSTR(4).NE.1) GO TO 240
EINTRA $=\mathrm{EDEF}+E D E L+E C F F+E N O B 1+E C O U 1+E U L J P+E U E S P$
EINTER = ENOB2+ECOU2+EULJ+EUES

```
Keep
```

EDEF $=$ MAX ( $-999 ., \operatorname{MIN}(999 ., E D E F)$ )
EDEL $=\operatorname{MAX}(-999 ., M I N(999 ., E D E L))$
ECFF $=\operatorname{MAX}(-999 . . \operatorname{MIN}(999 ., E C F F))$
ENOB1 $=\operatorname{MAX}(-999 ., \operatorname{MIN}(999 .$, ENOB1 ) )
ECOU1 $=\operatorname{MAX}(-999 ., \operatorname{MIN}(999 ., E C O U 1))$
EOLJP $=$ MAX ( -999., MIN ( 999., EULJP) )
EUESP $=\operatorname{MAX}(-999 ., \operatorname{MIN}(999 ., E U E S P))$
EINTRA $=$ MAX ( -999., MIN( 999., EINTRA) )
ENOB2 $=\operatorname{MAX}(-9999 ., \operatorname{MIN}(9999 .$, ENOB2 ) )
ECOU2 $=$ MAX ( $9999 .$. MIN ( 9999.., ECOU2 ) )
EULJ $=\operatorname{MAX}(-9999 ., \operatorname{MIN}(9999 .$, EULJ $)$ )
EUES $=$ MAX ( -9999., MIN ( 9999., EUES ) )
EINTER $=$ MAX ( $-9999 .$, MIN ( 9999., EINTER ) )
ETOT = MAX ( -9999., MIN ( 9999., ETOT ) )
WRITE $(11,10)$
WRITE $(11,11)$
WRITE $(11,27)$ EDEF, EDEL, ECFF, ENOB1, ECOU1, EULJP, EUESP, EINTRA
$\operatorname{WRITE}(11,12)$
WRITE $(11,28)$ ENOB2,ECOU2,EULJ, EUES,EINTER,ETOT
IF (NVAR.NE.0) WRITE $(11,22)$ NVAR, GRADNM
IF(INSTR(S).NE.1) GO TO 250
WRITE $(11,13)$
LCNT $=0$
DO $300 \mathrm{~L}=1, \operatorname{MIN}(9, \mathrm{NCH})$
$\operatorname{IF}(\operatorname{ABS}(\operatorname{CHSPEC}(1, L))$. NE.1.) GO TO 300
LCNT $=$ LCNT +1
IO $=1+($ LCNT-1) *NARU*NRUS
IX $=$ LCNT*NARU*NRUS
DO $350 \mathrm{I}=\mathrm{IO}, \mathrm{IX}$
$I M=I-I O+1$
IF ((I.LE.9).AND. (KIND (IM, L).LE.7))
WRITE (11,14) $\operatorname{KAR}(\operatorname{KIND}(\operatorname{IM}, L)), I,(\operatorname{XYZ}(J, I M, L), J=1,3)$
IF (( (I.GT.9).AND. (I.LE.99)).AND. (KIND (IM, L).LE.7))
WRITE (11,15) $\operatorname{KAR}(\operatorname{KIND}(I M, L)), I,(X Y Z(J, I M, L), J=1,3)$
IF (( (I.GT.99).AND. (I.LE.999)).AND. (KIND (IM, L).LE.7))
$\operatorname{WRITE}(11,16) \operatorname{KAR}(\operatorname{KIND}(I M, L)), I,(\operatorname{XYZ}(J, I M, L), J=1,3)$
IF ((I.GT.999).AND. (KIND (IM, L).LE.7))
WRITE (11, 25) KAR(KIND (IM, L)), I, (XYZ (J, IM, L) , J=1, 3)
IF ((I.LE.9).AND. (KIND (IM, L).GT.7))
WRITE (11,17) $\operatorname{KAR}(\operatorname{KIND}(I M, L)), I,(X Y Z(J, I M, L), J=1,3)$
IF (( (I.GT.9).AND. (I.LE.99)).AND. (KIND (IM, L).GT.7))
WRITE (11, 18). KAR(KIND (IM, L)), I, (XYZ (J, 工M, L), J=1, 3)
IF (( (I.GT.99).AND. (I.LE.999)).AND. (KIND (IM, L).GT.7))
WRITE (11, 19) $\operatorname{KAR}(\operatorname{KIND}(I M, L)), I,(X Y Z(J, I M, I), J=1,3)$
IF ((I.GT.999).AND. (KIND (IM, L).GT.7))
WRITE $(11,26) \operatorname{KAR}(\operatorname{KIND}(I M, L)), I,(X Y Z(J, I M, L), J=1,3)$
CONTINUE
CONTINUE
CONTINUE
FORMAT ( / 1X, '***** RESIDUE INTERNAL PARAMETERS **************' )
FORMAT ( 6X,'THETA',5X,' PHI ',9X,'THETA', 5X,' PHI ')
FORMAT ( 1X,'***** HELIX PARAMETERS; DEE,RHO,THT:',2X,
3(1X,F9.5))

FORMAT ( 3X,' PRINCIPLE AXES; A,B,C :',10X,3(1X,F9.5) )
FORMAT ( 3X,' PRINCIPLES ANGLES; ALPHA, BETA, GAMMA:', 3(IX,F9.5) )
FORMAT ( $3 X$, ' CHAIN NUMBER TRANSL ORIENT/ROTATION')
FORMAT ( 6X, I3, 9X,F7.5, 3X, F6.1, IX, F6.1, 2X,F8.3)
FORMAT ( 1X,'***** POTENTIAL ENERGIES **********************')
FORMAT ( $1 \mathrm{X},{ }^{\prime} \star$ INTRA: EDEF', 4X,'EDEL', 4X,'ECFF', 4X,
FORMAT( A1, I1,5X, 3F12.4)
FORMAT( Al, I2, 4X, 3F12.4 )
FORMAT( A1,I3, 3X, 3F12.4 )
FORMAT( A2,I1,4X,3F12.4 )
FORMAT( A2,I2,3X,3F12.4 )
FORMAT( A2,I3,2X,3F12.4 )
FORMAT( A1,I4,3X,3F12.4 )
FORMAT( A2,I4,2X,3F12.4 )
FORMAT( 5X, 3(F8.3,2X),5X,3(F8.3,2X) )
FORMAT( 5X,F8.3,2X,F8.3,6X,F8.3,2X,F8.3)
FORMAT( 4X,'RING ROTATION ANGLES: ',F8.3,2X,F8.3,4X,F8.3,2X,
F8.3)
FORMAT( 4X,'RING SUBSTITUTION CODE: ',4I3 )
FORMAT( 3X,' TEMP(K), DENS(G/CC), CELL SUM:',8X,2(F9.5,1X),I4)
FORMAT( /IX,' ** IMPOSED STRAIN: CELL TYPE, CASE, STRAIN ',
'TENSOR **' )
FORMAT( 10X,I4,4X,I4 )
FORMAT ( 10X, 3(F9.4,2X) )
FORMAT( /1X,' ** STRAINED UNIT CELL VECTORS ** ')
FORMAT( 5E15.7 )
FORMAT( 2I4 )
RETURN
END
SUBROUTINE RDSTRAIN( MODE )
C
C Reads special file for unitcell definition and strain tensor
PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
MNSB=24, NDOF=10,MNIC=9,MCOP=45,MOPT=51 )
COMMON /CHAINS/ PHI (NIBB),THETA (NIBB),BL(NIBB),PHIR(NRNG),
KSB (4) , PCH (4,MNAT) ,NARU,NRUS,NCH,
A (4, 4, MNAT), XYZ (4, MNAT, MNCH) , KIND (MNAT, MNCH),
CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT) , DEE, THT, RHO,
NUNIQ
COMMON /CELL/ CELD (3),CELA(3),CHSPEC (5,MNCH),CHLOC (3,MNCH),
NUNIV, KONDENS,TEMP,DENSC
COMMON /STRN/ KUC,KASE,ST (3, 3),SUCV (3, 3),DINIT,DPRIME,DCON
COMMON /MINIM/ LMV (2,9),LCV(5,11),OMVARS(97)
COMMON /OPRS/ NVAR,VARS(97),GRAD(97),GRADNM
COMMON G(97),G(4753),W(291)
COMMON /SV2/ TVARL
NHELEM = NVAR*(NVAR+1)/2
OPEN (UNIT=10,FILE='US.dt',STATUS='OLD')
READ (10,*) KUC, KASE
READ (10,*) DINIT,DCON
READ (10,*) (ST (1,I), I=1, 3)
READ (10,*) (ST (2,I),I=1,3)
READ (10,*) (ST (3,I),I=1,3)
CLOSE (UNIT=10)
C
C Read Existing Hessian, if it exists
C
IF( MODE.EQ.1 ) GO TO }10
OPEN (UNIT=10, FILE=' HESS', STATUS='OLD')
READ (10,1002) ( H(J),J=1,NHELEM )
CLOSE( UNIT=10 )
c
C If Hessian exists, read in corresponding geometry

```

IF ( MODE.EQ. 1 ) GO TO 300
OPEN (UNIT=10, FILE=' GEOM' ,STATUS \(={ }^{\prime}\) OLD')
```

        READ (10,1002) (VARS (J),J=1,NVAR )
        CLOSE( UNIT=10 )
        TVARL = 1000.
        CALL LDVAR( NVAR,VARS,-1,KMOL)
        FORMAT( 5E15.7)
        FORMAT( 2I4 )
        CONTINUE
        RETURN
        END
    SUBROUTINE ATRNS( PHI,THETA,BL,A )
    C
C "A" Matrix Generation
"T"-Matrix as per Flory, STAT. MECH. OF CHAIN MOLECULES, p. 21

```

```

    DIMENSION A(4,4)
    COMPL (X) = (180.-X)*3.1415926535/180.
    TH = COMPL (THETA)
    PH = PHI*3.1415926535/180.
    CT}=\operatorname{COS(TH)
    ST = SIN(TH)
    CP = COS (PH)
    SP}=SIN(PH
    A(1,1) = CT
    A(1,2)=ST
    A(1,3) = 0.
    A(1,4) = BL
    A(2,1) = ST*CP
    A(2,2) = -CT*CP
    A(2,3) = SP
    A(2,4) = 0.
    A(3,1) = ST*SP
    A(3,2) = -CT*SP
    A(3,3) = -CP
    A(3,4)=0.
    A(4,1) = 0.
    A(4,2)=0.
    A(4,3) = 0.
    A(4,4)=1.
    RETURN
    END
    SUBROUTINE MV( A,NFORMA,NA,V,N,R )
    C
C - Multiplies matrix with vector: R = A * V
C
C
C
C
- Input : A NA x N Matrix
NFORMA First dimension of A in calling program
NA Number of rows in A
V Vector of length N
N Length of vector v
- Output: R Resultant vector of length NA, can be v
DIMENSION A(NFORMA,NFORMA), V(N), R(NA), RR(20)
IF (NA.GT.NFORMA) RETURN
DO 100 I=1,NA
x=0.
DO 200 K=1,N
X=X + A(I,K)*V(K)
200
100 RR(I) = X
DO 300 I=1,NA
R(I) = RR(I)
RETURN
END

```
```

SUBROUTINE VV( $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{N})$
C
Calculates the vector cross product C = A X B, where A,B, and C
have length N
DIMENSION A(N),B(N),C(N),K(2)
DO }100 I=1,
DO 200 J=1,2
K(J) = MOD( (I+J-1),N ) +1
C(I) = A(K(1))*B(K(2))-A(K(2))*B(K(1))
CONTINUE
RETURN
END
SUBROUTINE MM( A,B,C,NFORMA,NFORMB,NFORMC,NA,MA,MB )
C
- Multiplies matrix with matrix: C = A X B
(C can be A or B)
DIMENSION CC (20,20),A (NFORMA,NFORMA),B (NFORMB,NFORMB)
DIMENSION C(NFORMC, NFORMC)
DO 100 I=1,NA
DO }100\textrm{J}=1,\textrm{MB
X = 0.
DO 200 K=1,MA
X=X+A(I,K)*B(K,J)
CC(I,J) = X
DO }300\textrm{I}=1,N
DO 300 J=1,MB
C(I,J)=CC(I,J)
RETURN
END
SUBROUTINE MT ( A, B,N,NP )
C
C Calculates the transpose of the N x N matrix A contained within
C
the physical array NP x NP, and returns as the matrix B
DIMENSION A(NP,NP),B(NP,NP)
DO 100 I=1,NP
DO 100 J=1,NP
B(I,J) = A(I,J)
DO 200 I=1,N
DO 200 J=1,I
B(J,I) = A(I,J)
B(I,J)=A(J,I)
RETURN
END
SUBROUTINE MA( A,B,C,N,NP,S )
C
C If S = 1 : Adds matrices A and B to form C
C If S =-1 : Subtracts matrix B from A to form C
C. All matrices are N x N, contained within arrays having physical
C dimensions NP x NP
c
DIMENSION A(NP,NP),B(NP,NP),C(NP,NP)
DO 100 I=1,N
DO 100 J=1,N
C(I,J) = A(I,J) + S*B(I,J)
RETURN
END

```
```

C
C Converts scalar data (lengths, angles) to three placement vectors
C Convention: SLEN(3) lies along z axis
SLEN(1) lies within X-Z plane
SLEN(2) placement determined by above
SANG(i) is the angle between SLEN(1) and SLEN(k), etc.
KERR normally equals 0; KERR = 1 if the set of input angles do not
make a viable set
DIMENSION SLEN(3),SANG (3),VECT (3,3), ARA (3,3), ARB (3,3)
COMPL (X) = (90.-X)*3.1415926535/180.
EPSILON = 1E-6
DEGRAD = 3.1415926535/180.
KERR = 0
DO 100 I=1,3
DO 100 J=1,3
ARA (I,J) = 0.
ARB (I,J) = 0.
100 VECT(I,J)=0.
VECT (1,1) = SLEN (1)
VECT (1, 2) = SLEN (2)
VECT (3,3) = SLEN(3)
COSGP = ( COS (SANG (3)*DEGRAD) - COS (SANG (1)*DEGRAD)*COS (SANG (2)
+
*DEGRAD) )/( SIN(SANG (1)*DEGRAD)*SIN (SANG (2)*DEGRAD) )
IF((ABS (COSGP)-1.).GT.EPSILON) GO TO 900
IF(ABS (ABS(COSGP)-1.).LE.EPSILON) COSGP = SIGN(1.,COSGP)
GP = ACOS (COSGP)
CA = COMPL(SANG (1))
CB = COMPL(SANG(2))
ARA (1,1) = COS (CB)
ARA (1,3) = -SIN (CB)
ARA (2,2) = 1.
ARA(3,1) = SIN(CB)
ARA (3,3) = COS(CB)
ARB (1,1) = cosGP*COS (CA)
ARB (1,2) = -SIN(GP)
ARB(1,3)=-COSGP*SIN(CA)
ARB (2,1) = SIN(GP)*COS (CA)
ARB (2,2) = CosGP
ARB (2,3) = -SIN(GP)*SIN(CA)
ARB (3,1) = SIN (CA)
ARB (3,3)=\operatorname{cos}(CA)
CALL MV( ARA, 3, 3,VECT (1, 1),3,VECT (1,1) )
CALL MV(ARB, 3,3, VECT (1,2),3,VECT (1,2))
RETURN
900 CONTINUE
KERR = 1
RETURN
END
SUBROUTINE ELJ(NKIND,NUC1,NUC2,VDWR,ALPHA,EFFNE,R2,S2,FIRST,
+
Calculates the 6-12 attraction/repulsion potential between two nuclei
separated by the distance R (R2 = R**2)
A shifted 3-segment-potential is used:

- For R .LE. RCRIT-H/2 : 6-12 potential with R, shifted
- For R.GE. RCRIT+H/2 : 0.
- For RCRIT-H/2 < R < RCRII+H/2 : A spline-like cubic polynomial
between these two potentials
(NKIND must not exceed MAXA)
LOGICAL FIRST
PARAMETER( MAXA=9,EPSILON=1E-6 )
DIMENSION VDWR(NKIND),ALPHA (NKIND), EFFNE (NKIND)
COMMON /SV1/ A (MAXA,MAXA), C (MAXA, MAXA), EMLH (MAXA,MAXA),
+
YLH (MAXA,MAXA), YRH (MAXA,MAXA), FSTRL (MAXA, MAXA),
RR,RL,RR2,RL2
c

```
```

Note: spline portion not in use...
Modified to pass two distances, R2 and S2. If R2=S2=distance**2, the
routine operates normally, and splining is possible; otherwise, R2
represents the inverse 6th order distance sum and S2 represents the
inverse 12th order distance sum, as supplied by infinite lattice sum
calculations.
DATA RCRIT,H/ .1,1./
IF (FIRST) GO TO 500
100 POTENT = 0.
IF (ABS(R2/S2-1.).GE.EPSILON) GO TO 300
IF (R2.GE.RR2) RETURN
IF (R2.LE.RL2) GO TO 200
R = SQRT( R2 )
D1 = RR - R
D2 = R - RL
POTENT = EMLH(NUC1,NOC2)*D2*(D1**2)
+ + YLH(NUC1,NOC2)*(D1**2)*(2.*D2+H)
+ RETURN
200 D1 = R2**3
D2 = D1**2
POTENT = A(NUC1,NUC2)/D2 - C(NUC1,NUC2)/D1 - FSTRL(NUC1,NUC2)
RETURN
300 POTENT = A(NUC1,NUC2)*S2 - C(NUC1,NUC2)*R2
RETURN
C
C Initialize potential parameters on first pass
500 CONTINUE
IF (RCRIT.LT.I.) RCRIT = 500.
IF (H.LT.0.1) H=0.1
C
C Calculate A(I,J) and C(I,J):
C
DO }600I=1,NKIN
DO 600 J=1,NKIND
IF( ((ALPHA(I)-0.).LT.EPSILON).AND.
((ALPHA(J)-0.).LT.EPSILON) ) THEN
C(I,J) = 0.
ELSE
C(I,J) = 365.*ALPHA(I)*ALPHA(J)/
(SQRT (ALPHA (I)/EFFNE (I)) +SQRT (ALPHA (J)/EFFNE (J)))
END IF
A(I,J) = 0.5*C(I,J)*(VDWR(I)+VDWR(J))**6
600 CONTINUE
C
C Calculate polynomial parameters:
RL = RCRIT - H/2.
RR=RCRIT + H/2.
RL2 = RL**2
RR2 = RR**2
RC2 = RCRIT**2
R6 = RL**6
R12 = R6**2
R7 = R6*RL
R13 = R12*RL
RC6 = RC2**3
RC12 = RC6**2
DO }700\textrm{I}=1,NKIN
DO }700\textrm{J}=1\mathrm{ ,NKIND
FSTRL(I,J) = A(I,J)/RC12 - C(I,J)/RC6
EMLH(I,J) = (-12.*A(I,J)/R13 + 6.*C(I,J)/R7)/(H**2)
YLH(I,J) = (A(I,J)/R12 - C(I,J)/R6)/(H**3)
YRH(I,J) = FSTRL(I,J)/(H**3)
700 CONTINUE
FIRST = .FALSE.
GO TO 100
END

```
```

    SUBROUTINE EES( BWD,DIST,EBULK,AC1,AC2,EEL )
    Calculates Coulombic interaction between two charge centers using
a distance-dependent dielectric constant:
DIST < Critical Dist (BWD) : EPS = 1
SVDW < DIST < (1+FACT)*BWD : EPS = Quintic spline
DIST > (1+FACT)*BWD. : EPS = Block-Walker Approx.
DATA FACT/ 0.1 /
P12 = BWD*LOG (EBULK)
RC = BWD + (FACT * BWD)
IF (DIST.LE.BWD) THEN
EPS=1.
ELSEIF ((DIST.GT.BWD) .AND. (DIST.LT.RC)) THEN
DELTA = BWD*FACT
S = EBULK*EXP(-P12/RC)
SP = (EBULK*P12/RC**2)*EXP(-P12/RC)
SDP = ((P12/RC)-2.)* (EBULK*P12/(RC**3))*EXP (-P12/RC)
D=(10.*(S-1.)/(DELTA**3))-(4*SP/(DELTA**2))+(0.5*SDP/DELTA)
E=(-15.* (S-1.)/(DELTA**4))+(7.*SP/(DELTA**3))-(SDP/DELTA**2)
F=(6.*(S-1.)/(DELTA**5))-(3.*SP/(DELTA**4))+(.5*SDP/DELTA**3)
ETA = DIST - BWD
EPS = 1.+D*ETA**3+E*ETA**4+F*ETA**5
ELSE
EPS = EBULK*EXP(-P12/DIST)
ENDIF
EEL = 332.*AC1*AC2/EPS/DIST
RETURN
END
SUBROUTINE JACOBI (A,N,NR,D,V,NROT)
C
C Jacobi Transformation of a symmetric matrix A, having dimensions
C N X N, stored in an array NP x NP. On output, upper elements of
C A are destroyed. D contains the eigenvalues and V contains the
C normalized eigenvectors
From: Press et.al., NUMERICAL RECIPES, p346.
PARAMETER (NMAX=100,EPSILON=2E-7)
DIMENSION A(NP,NP),D(NP),V(NP,NP),B(NMAX),Z (NMAX)
DO 12 IP=1,N
DO 11 IQ=1,N
V(IP,IQ)=0.
CONTINUE
V (IP,IP)=1.
CONTINUE
DO 13 IP=1,N
B(IP)=A(IP,IP)
D(IP)=B(IP)
Z(IP)=0.
CONTINUE
NROT=0
DO 24 I=1,50
SM=0.
DO 15 IP=1,N-1
DO 14 IQ =IP+1,N
SM=SM+ABS (A (IP,IQ))
14 CONTINUE
14 CONTINU
C *** Modified for PC where underflow is not set to zero : GCR 10/87 ***
IF(SM.EQ.O.)RETURN
IF(SM.LT.EPSILON) RETURN
C ***********************************************************************
IF(I.LT.4)THEN

```
```

        TRESH=0.2*SM/N**2
    ELSE
        TRESH=0.
    ENDIF
    DO 22 IP=1,N-1
        DO 21 IQ =IP+1,N
            G=100.*ABS (A (IP,IQ))
            IF((I.GT.4).AND.(ABS (D(IP))+G.EQ.ABS (D(IP)))
                    .AND.(ABS (D(IQ))+G.EQ.ABS (D (IQ)))) THEN
                A(IP,IQ)=0.
            ELSE IF(ABS(A(IP,IQ)).GT.TRESH)THEN
                H=D(IQ) -D (IP)
                    IF (ABS (H)+G.EQ.ABS (H)) THEN
                    T=A(IP,IQ)/H
                    ELSE
                    THETA=0.5*H/A (IP,IQ)
                    T=1./(ABS (THETA) +SQRT (1.+THETA**2))
                    IF(THETA.LT.O.)T=-T
                    ENDIF
                C=1./SQRT (1+T**2)
                S=T*C
                TAU=S/(1.+C)
                H=T*A(IP,IQ)
                Z(IP)=Z(IP)-H
                Z(IQ)=Z(IQ) +H
                D (IP) =D (IP) -H
                D(IQ) =D (IQ) +H
                A(IP,IQ)=0.
                DO 16 J=1,IP-1
                    G=A(J,IP)
                    H=A (J,IQ)
                    A(J,IP)=G-S* (H+G*TAU)
                    A(J,IQ)=H+S* (G-H*TAU)
            CONTINUE
            DO 17. J=IP+1,IQ-1
                    G=A(IP,J)
                    H=A(J,IQ)
                    A(IP,J)=G-S* (H+G*TAU)
                    A (J,IQ) =H+S* (G-H*TAU)
                CONTINUE
                DO 18 J=IQ+1,N
                    G=A (IP,J)
                    H=A (IQ,J)
                    A(IP,J)=G-S* (H+G*TAU)
                    A (IQ,J) =H+S* (G-H*TAU)
                CONTINUE
                DO 19 J=1,N
                    G=V (J,IP)
                    H=V(J,IQ)
                    V(J,IP)=G-S* (H+G*TAU)
                    V(J,IQ)=H+S* (G-H*TAU)
                CONTINUE
                NROT=NROT+1
            ENDIF
        CONTINUE
    CONTINUE
    DO 23 IP=1,N
        B(IP)=B(IP)+Z(IP)
        D(IP)=B(IP)
        Z(IP)=0.
    CONTINUE
    CONTINUE
PAUSE '50 iterations should never happen'
RETURN
END

```

REAL FUNCTION CURTATE ( X,AP,CP )
C
```

C Returns the Y value of X of a curtate function described by

```
AP (magnitude) and CP (curvature), \(X\) in degrees
\(E P S=1.0 E-6\)
TMAX \(=5000\)
DEGRAD \(=3.1415926535 / 180\).
\(C\) Find \(T\) by iteration:
XRAD \(=X * D E G R A D\)
\(T=X R A D\)
DO \(100 \mathrm{I}=1\), IMAX
TINC \(=2 . *\) XRAD \(-T+C P * S I N(T)\)
IF ( ABS (TINC).LE.ERS ) GO TO 200
\(100 \mathrm{~T}=\mathrm{T}+\) TINC
200 CONTINUE
IF ( I.GE.IMAX ) WRITE(*,*) 'WARNING: Iteration Iimit in Curtate:
+ TINC \(=\), ,TINC
C Calculate curtate function value:
\(\operatorname{CURTATE}=(A P / 2) *.(1 .-\operatorname{COS}(T))\)
RETURN
END

REAL FUNCTION CURTINV ( X,AP,CP )
C
\(C\) Returns the \(Y\) value of \(X\) of an inverted curtate function described \(C\) by \(A P\) (magnitude) and \(C P\) (curvature), \(X\) in degrees.
\(E P S=1 . E-6\)
IMAX \(=5000\)
\(P I=3.1415926535\)
DEGRAD \(=\) PI/180.
\(C\) Find \(T\) by iteration:
\(\mathrm{XRAD}=\mathrm{X} * D E G R A D\)
\(T=X R A D-P I\)
DO \(100 \mathrm{I}=1\), IMAX
TINC \(=2 . *\) XRAD \(-T+C P * S I N(T)+P I\)
IF (ABS (TINC). LE.EPS) GO TO 200
\(100 \mathrm{~T}=\mathrm{T}+\mathrm{TINC}\)
200 CONTINUE
IF ( I.GE.IMAX ) WRITE(*,*) 'WARNING: Iteration Limit in InvCurt:
+ TINC \(=1\), TINC
C Calculate inverted curtate function value:
CURTINV \(=(\mathrm{AP} / 2.) \star(1 .+\operatorname{COS}(\mathrm{T}))\)
RETURN
END

SUBROUTINE MULMAX (MNCH,NCH, NCH2, L1MAX, FMULT )
C
C For a regular lattice packing geometry having chains of identical
\(C\) placement parameters, intermolecular potential calculations may be C expedited by recognition of multiplicative interactions. For a
\(C\) lattice consisting of only two chain types (designated by odd or even
C numbering) in an counterclockwise alternating protocol, this routine generates the multiplicity matrix FMULT by which unique energetic interactions must be multiplied to determine relative probabilities of occurance.

The use of the intermoiecular multiplier matrix assumes periodicity within the lattice of the form:

Two types of chains: all even numbered chains alike all odd numbered chains alike
Chain 1 forms lattice center; chains 2-9 form first shell about the center, proceeding counterclockwise from a position "east" of chain 1; chains 10-25 form second shell about center, proceeding ccw form a position "east" of chain 9, etc. For a given lattice size (eg. \(3 \times 3,4 \times 4,5 \times 5\) ) all edge-to-edge intermolecular interactions are ignored (ie. in the \(3 x 3\)
```

lattice, interactions between chains separated by a third chain
are ignored, in the 4x4 lattice, those separated by two chains
are ignored, etc.)
NOTE: The Lattice Sum Method does not require an interaction
multiplicity correction; however, it does recognize repetitive
interactions within the "explicit cell" in order to minimize
computation time. Statements which only serve to correct the
(non-zero) MAGNITUDE of the multiplicity correction have been
disabled and marked by an asterix
(NCH must not exceed MAXC, MAXC2 = MAXC**2)
PARAMETER( MAXC=25,MAXC2=625 )
DIMENSION FMULT (MNCH,MNCH), LOC (2,MAXC),IA (5,MAXC2),FA(MAXC2)
DO 100 I=1,NCH
DO 100 J=1,NCH
FMULT (I,J) = 0
DO 120 I=1,NCH**2
FA(I) = 0.
DO 120 J=1,5
120 IA(J,I)=0
NCPS = INT(SQRT (REAL (NCH))+.1)
RUPC = REAL ((NCPS-1)**2)
Build a test array to locate chains:
DO 150 I=1,4
DO 150 J=1,2
LOC (I,J) = 0
LOC (1,2)=1
LOC (1,3) = 1
LOC (2,3) = 1
LOC (2,4) = 1
IF(NCH.GT.4) THEN
ITRAN = INT((REAL (NCPS)+0.1)/2)
JTRAN = 1
JTRK = -1
KTRK = 1
DO 200 NTRK=5,NCH
DO 250 I=1,3
LOC (I,NTRK) = JTRK*LOC(I, 2) + KTRK*LOC(I, 4)
IF(NTRK.GE. ((2*JTRAN+1)**2)) JTRAN = JTRAN+1
IF( ( JTRAN.EQ.JTRK).AND.( JTRAN.NE.KTRK)) THEN
KTRK = KTRK+1
ELSE IF((-JTRAN.NE.JTRK).AND. ( JTRAN.EQ.KTRK)) THEN
JTRK = JTRK-1
ELSE IF ((-JTRAN.EQ.JTRK).AND. (-JTRAN.NE.KTRK)) THEN
KTRK = KTRK-1
ELSE IF(( JTRAN.NE.JTRK).AND.(-JTRAN.EQ.KTRK)) THEN
JTRK = JTRK+1
ELSE
JTRK = JTRK+1
END IF
200 CONTINUE
ELSE
END IF
C
c. Loop through all interactions to determine multiplicities of all
C unique interactions:
NINT = 1
IA(1,1)=1
IA (2,1) = 0
IA}(3,1)=
IA (4,1) = 1
IA(5,1)=2
FA(1) = 1.
IF( NCH.EQ.4 ) FA(1)=.5
DO 300 I=1,NCH-1
IP1 = I+1

```
```

        DO 300 J=IP1,NCH
            IF((I.EQ.1).AND.(J.EQ.2)) GO TO 450
    C
c Detemmine orientation of present interaction:
NXS = LOC (I,J) - LOC (I,I)
NYS = LOC (2,J) - LOC (2,I)
IF ( ABS (NXS).LT.ABS (NYS) ) THEN
NDS = NXS
NYS = SIGN( (ABS (NYS)-ABS (NXS)),NYS )
NXS = 0
ELSE IF( ABS(NXS).GT.ABS(NYS) ) THEN
NDS = NYS
NXS = SIGN( (ABS (NXS) -ABS (NYS)),NXS )
NYS =0
ELSE
NDS = SIGN( NXS,(NXS*NYS) )
NXS =0
NYS =0
END IF
C
C Discard if too distant (ie. edge-to-edge)
IF( NCH.EQ.4 ) GO TO 470
ND = ABS (NXS) + ABS (NYS) + ABS (NDS)
IF( ND.GT.(NCPS-2) ) GO TO 450
CONTINUE
C****
C**Determine scale factor (1 if interior, 1/2 if along border of finite
C**cube) :
C**
C** IF( (I.GT.((NCPS-2)**2)).AND.((J-I).IT.NCPS).AND.
C** + (NDS.EQ.O) ) THEN
SCALE = . 5
C** ELSE
C** SCALE = 1.
C** END IF
C** IF( (J.EQ.NCH).AND.(NDS.EQ.O) ) SCALE = . 5
C****
C Compare to previous interactions for multiplicity:
C
DO 400 K=1,NINT
IF(MOD ((I+IA(4,K))/2.,I.).EQ.O.) THEN
IF ((NXS.EQ.IA(1,K)).AND. (NYS.EQ.IA (2,K)).AND.
(NDS.EQ.IA(3,K))) THEN
FA(K) = FA (K) +SCALE
GO TO 450
ELSE
END IF
ELSE
END IF
IF(MOD((I+IA(5,K))/2., 1.).EQ.O.) THEN
IF((-NXS.EQ.IA(1,K)).AND.(-NYS.EQ.IA (2,K)).AND.
(-NDS.EQ.IA(3,K)).AND.((NXS.NE.O).OR.(NYS.NE.O))) THEN
FA(K) = FA (K) +SCALE
GO TO 450
ELSE
END IF
ELSE
END IF
CONTINUE
NINT = NINT+1
IA(1,NINT) = NXS
IA (2,NINT) = NYS
IA (3,NINT) = NDS
IA(4,NINT) = I
IA (5,NINT) = J
FA(NINT) = SCALE
CONTINUE
450 CONTIN
C

```
```

C
Load Multiplier matrix:
C
LIMAX = 1
DO 500 I=1,NINT
L1 = IA (4,I)
L2 = IA (5,I)
L1MAX = MAX (LIMAX,I1)
500 FMOLT(L1,L2) = FA(I)
C**500 FMULT(L1,L2) = FA(I)/RUPC
RETURN
END
SUBROUTINE VAIOA (FUNCT,N,X,F,G,H,W,DFN, XM, HH, EPS,MODE,MAXFN,
1
IPRINT,IEXIT)
C HARWELL - LIBRARY ROUTINE VAIOA: OPTIMIZATION WITHOUT DERIVATIVES
C USING A FLETCHER-POWELL ALGORITHM. ROUTINES NEQUIRED: MC11A.
C ARRAY-DIMENSIONS IN THE CALLING ROUTINE:
C X, XM : LENGTH N
C G, H, W : LENGTH N* (N+1)/2 OR 3*N, WHICHEVER IS LARGER
c
C CALLING PROCEDURES HAVE BEEN CHANGED FROM THE ORIGINAL ICL-FORTRAN
C EXTERNAL DECLARATON ADDED FOR PORTABILILTY GCR 19.4.89
C
REAL X(1),G(1), H(1),W(1),XM(1)
EXTERNAL FUNCT
IF(IPRINT.NE.0)PRINT }100
1000 FORMAT('1ENTRY TO VA1OA'/)
NN=N* (N+1)/2
IG=N
IGG=N+N
IS=IGG
IDIFF=1
IEXIT=0
IR=N
IF (MODE.EQ.3)GOTO15
IF (MODE.EQ. 2)GOTOIO
IJ=NN+1
DO }5\textrm{I}=1,
DO }6\textrm{J}=1,
IJ=IJ-1
6 H(IJ)=0
5 H(IJ)=1.
GOTO15
10 CONTINUE
CALL MC11B (H,N,G, DUMMY,G,IR, IDUMMY, DUMMY)
IF(IR.LT.N) RETURN
15 CONTINUE
Z=F
ITN=0
C
CALL FUNCT (N, X,F)
C
IFN=1
DF=DFN
IF (DFN.EQ.O.)DF=F-Z
IF (DFN.LT.O.)DF=ABS (DF*F)
IF(DF.LE.O.)DF=1.
17 CONTINUE
LINK=1
IF (IDIFF-1) 100,100,110
18 CONTINUE
IF (IFN. GE . MAXFN) GOTO9O
2O CONTINUE
IF (IPRINT .EQ.0) GOTO21
IF (MOD (ITN, IPRINT) .NE.0) GOTO21
PRINT 1001,ITN,IFN
1001 FORMAT(24I5)
PRINT 1002,F
1002 FORMAT((5E15.7))

```
```

        IF (IPRINT.LT.0)GOTO21
        PRINT 1002,(X(I),I=1,N)
        PRINT 1002,(W(IG+I),I=1,N)
    C ******* Statements inserted by GCR to save updates of Hessian *******
NHELM = NN
IF(N.LT.5) NHELM = 3*N
OPEN(UNIT=12,FILE='GEOM')
WRITE (12, 1002) (X (I), I=1,N)
CLOSE (UNIT=12)
OPEN(UNIT=12,FILE='HESS')
WRITE(12,1002).(H(J),J=1,NHELM)
CLOSE (UNIT=12)
C *******************************************************************************
21 CONTINUE
ITN=ITN+1
DO 22 I=1,N
22W(I)=-W(IG+I)
CALL MC1IE(H,N,W,DUMMY,G,IR,IDUMMY,DUMMY)
Z=0.
GSO=0.
DO 29 I=1,N
W(IS+I)=W(I)
IF (Z*XM(I).GE.ABS(W(I)))GOTO29
Z=ABS (W (I))/XM(I)
29 GSO=GSO+W(IG+I)*W(I)
AEPS=EPS/Z
IEXIT=2
IF(GSO.GE.0.)GOTO92
ALPHA=-2.*DF/GSO
IF (ALPHA.GT.1.) ALPHA=1.
FF=F
TOT=0.
INT=0
IEXIT=1
30 CONTINUE
IF (IFN. GE . MAXFN) GOTO90
DO }31\textrm{I}=1,
31 W(I)=X(I) +ALPHA*W(IS+I)

```

```

    CALL FUNCT(N,W,F1)
    C
IFN=IFN+1
IF(F1.GE.F)GOTO4O
F2=F
TOT=TOT+ALPHA
32 CONTINUE
DO }33\textrm{I}=1,
33 X(I)=W(I)
F=F1
IF (INT-1) 35,49,50
35 CONTINUE
IF (IFN.GE .MAXFN) GOTO90
DO }34\textrm{I}=1,
34 W(I)=X(I) +ALPHA*W(IS+I)
C
CALL FUNCT (N,W,F1)
C
IFN=IFN+1
IF(F1.GE.F)GOTO50
IF(F1+F2.GE.F+F.AND . 7.*F1+5.*F2.GT.12.*F)INT=2
TOT=TOT+ALPHA
ALPHA=2.*ALPHA
GOTO32
40 CONTINUE
IF (ALPHA.LT. AEPS)GOTO92
IF (IFN.GE.MAXFN) GOTO90
ALPHA=.5*ALPHA
DO 41 I=1,N
4I W(I)=X(I)+ALPHA*W(IS+I)
c

```
```

C
IFN=IFN+1
IF (F2.GE.F)GOTO45
TOT=TOT+ALPHA
F=F2
DO 42 I=1,N
42 X(I)=W(I)
GOTO49
4 5 CONTINUE
Z=.1
IF (F1+F.GT.F2+F2) 2=1.+.5* (F-F1)/(F+F1-F2-F2)
IF(Z.LT..1) Z=.1
ALPHA=2*ALPHA
INT=1
GOTO3O
49 CONTINUE
IF (TOT . LT . AEPS) GOTO92
5 0 ~ C O N T I N U E ~
ALPHA=TOT
DO 56 I=1,N
56W(I)=W(IG+I)
LINK=2
IF (IDIFF-1) 100,100,110
54 CONTINUE
IF (IFN.GE . MAXFN) GOTO90
GYS=0.
DO }55I=1,
GYS=GYS+W(IG+I)*W(IS+I)
55W(IGG+I)=W(I)
DF=FF-F
DGS=GYS-GSO
IF (DGS.LE.O.) GOTO2O
IF (DGS+ALPHA*GSO.GT . O. ) GOTO70
C COMPLEMENTARY DFP FORMULA
SIG=1./GSO
IR=-IR
CALL MCIIA(H,N,W,SIG,G,IR,1,0.)
DO }60 I=1,
60 G(I)=W(IG+I)-W(IGG+I)
SIG=1./(ALPHA*DGS)
IR=-IR
CALL MCI1A(H,N,G,SIG,W,IR,O,O.)
GOTO2O
70 CONTINUE
DFP FORMULA
ZZ=ALPHA/(DGS-ALPHA*GSO)
SIG=-2Z
CALL MCI1A(H,N,W,SIG,G,IR,1,1E-7)
Z=DGS*ZZ-1.
DO }71\textrm{I}=1,
71 G(I)=W(IG+I)+Z*W(IGG+I)
SIG=1./(22*DGS**2)
CALL MC11A(H,N,G,SIG,W,IR,0,0.)
GOTO2O
90 CONTINUE
IEXIT=3
GOTO94
92 CONTINUE
IF (IDIFF.EQ . 2) GOTO94
IDIFF=2
GOTO17
94 CONTINUE
DO }95\textrm{I}=1,
95G(I)=W(IG+I)
C ******************* ATTENTION, NEXT STATEMENT INSERTED BY UWS **********
MAXFN=IFN
C ****************O PASS NUMBER OF FUNCTION CALLS BACK TO CALLER*******
IF(IPRINT.EQ.O) RETURN
PRINT 1001,ITN,IFN,IEXIT
PRINT 1002,F
PRINT 1002,(X(I),I=1,N)

```

PRINT 1002, (G(I), I=1,N)
```

C ******* Statements inserted by GCR to save updates of Hessian ********
NHELM = NN
IF(N.LT.5) NHELM = 3*N
OPEN(UNIT=12,FILE='GEOM')
WRITE(12,1002) (X(I),I=1,N)
CLOSE (UNIT=12)
OREN (UNIT=12,FILE='HESS')
WRITE (12,1002) (H(J),J=1,NHELM)
CLOSE (UNIT=12)
C ************************************************************************
RETURN
100 CONTINUE
DO }101\textrm{I}=1,
Z=HH*XM(I)
ZZ=X(I)
X(I) = 2Z Z Z
C
CALL FUNCT (N,X,F1)

```

```

    W(IG+I)=(F1-F)/Z
    101 X(I)=2Z
    IFN=IFN+N
    GOTO (18,54),LINK
    110 CONTINUE
        DO }111\mathrm{ I=1,N
        Z=HH*XM(I)
        22=X(I)
        X(I)=2Z+Z
    C
CALL FUNCT (N,X,F1)
C
X(I)=ZZ-Z
C
CALL FUNCT (N,X,F2)
C
W(IG+I)=(F1-F2)/(2,*Z)
111 X(I)=2Z
IFN=IFN+N+N
GOTO (18,54),IINK
RETURN
END

```
            SUBROUTINE MC11A(A,N, Z,SIG,W,IR,MK,EPS)
C ***********ENTRY STATEMENTS CHANGED FROM ORIGINAL ICL FORTRAN**UWS***
    DIMENSION A(1), Z(1),W(1)
C UPDATE FACTORS GIVEN IN A BY SIG*Z*ZTRANSPOSE
    IF (N.GT.1) GOTOI
    \(\mathrm{A}(1)=\mathrm{A}(1)+\) SIG \(\star \mathrm{Z}(1) * * 2\)
    IR=1
    IF (A(1).GT.O.)RETURN
    \(A(1)=0\).
    IR=0
    RETURN
    1 CONTINUE
    \(\mathrm{NP}=\mathrm{N}+1\)
    IF (SIG.GT.0.) GOTO40
    IF (SIG.EQ.O. OR.IR.EQ.O)RETURN
    TI=1./SIG
    \(I J=1\)
    IF (MK.EQ. O) GOTOIO
    DO \(7 \mathrm{I}=1, \mathrm{~N}\)
    \(\operatorname{IF}(\mathrm{A}(\mathrm{IJ}) . \mathrm{NE} .0) \mathrm{TI}=.\mathrm{TI} \mathrm{I}+\mathrm{W}(\mathrm{I}) * * 2 / \mathrm{A}(\mathrm{IJ})\)
    7 IJ=IJ+NP-I
    GOTO2O
    10 CONTINUE
    DO \(11 \mathrm{I}=1, \mathrm{~N}\)
    11 W(I) \(=2(I)\)
    DO \(15 I=1, N\)
    \(I P=I+1\)
```

    V=W(I)
    IF(A(IJ).GT.O.)GOTO12
    W(I)=0.
    IJ=IJ+NP-I
    GOTO15
    1 2 continue
TI=TI+V**2/A(IJ)
IF (I.EQ.N)GOTO14
DO 13 J=IP,N
IJ=IJ+1
13W(J)=W(J)-v*A (IJ)
14 IJ=IJ+1
15 contINUE
20 CONTINUE
IF(IR.LE.O )GOTO2
IF(TI.GT.O.)GOTO22
IF (MK-1) 40, 40,23
21 TI=0.
IR=-IR-1
GOTO23
22 TI=EPS/SIG
IF(EPS.EQ.O.)IR=IR-1
23 contINUE
MM=1
TIM=TI
DO 30 I=1,N
J=NP-I
IJ=IJ-I
IF(A(IJ).NE.O.)TIM=TI-W(J)**2/A(IJ)
W(J)=TI
30 TI=TIM
GOT041
40 CONTINUE
MM=0
TIM=1./SIG
4 1 ~ c o n t i n u e ~
IJ=1
DO 66 I=1,N
IP=I+1
v=2(I)
IF(A(IJ).GT.0.)GOTO53
IF(IR.GT.O .OR.SIG.LT.O..OR.V.EQ.O.)GOTO52
IR=1-IR
A(IJ)=V**2/TIM
IF(I.EQ.N)RETURN
DO 51 J=IP,N
IJ=IJ+1
51 A(IJ)=2(J)/V
RETURN
5 2 ~ C O N T I N U E ~
TI=TIM
IJ=IJ+NP-I
GOT066
53 CONTINUE
AL=V/A(IJ)
IF (MM) 54,54,55
54 TI=TIM+V*AL
GOTO56
55 TI=W(I)
56 CONTINUE
R=TI/TIM
A(IJ)=A(IJ)*R
IF(R.EQ.0.)GOTO7O
IF (I.EQ.N)GOTO70
B=AL/TI
IF(R.GT.4.)GOTO62
DO 61 J=IP,N
IJ=IJ+1
Z(J)=Z (J) -V*A (IJ)
61 A(IJ)=A(IJ)+B*2(J)
GOT064

```
```

    62 GM=TIM/TI
        DO 63 J=IP,N
        IJ=IJ+1
        Y=A(IJ)
        A(IJ)=B*Z(J)+Y*GM
    63 2(J)=Z(J)-V\starY
    6 4 \text { CONTINUE}
        TIM=TI
        IJ=IJ I 1
    6 6 ~ C O N T I N U E ~
    7 0 \text { CONTINUE}
        IF(IR.LT.0)IR=-IR
        RETURN
    C
FACTORIZE A MATRIX GIVEN IN A
ENTRY MC11B(A,N,2,SIG,W,IR,MK,EPS)
IR=N
IF(N.GT.1)GOTO100
IF (A (1).GT.O.) RETURN
A(1)=0.
IR=0
RETURN
100 CONTINUE
NP=N+1
II=1
DO 104 I=2,N
AA=A (II)
NI=II+NP-I
IF(AA.GT.0.)GOTO101
A(II)=0.
IR=IR-1
II=NI+1
GOTO104
101 CONTINUE
IP=II+1
II=NI+1
JK=II
DO }103\mathrm{ IJ=IP,NI
V=A (IJ)/AA
DO }102\mathrm{ IK=IJ,NI
A (JK)=A(JK) -A (IK)*V
102 JK=JK+1
103 A (IJ)=V
104 CONTINUE
IF (A (II).GT.O.) RETURN
A(II)=0.
IR=IR-1
RETURN
C MULTIPLY OUT THE FACTORS GIVEN IN A
ENTRY MCI1C(A,N,Z,SIG,W,IR,MK,EPS)
IF(N.EQ.1)RETURN
NP=N+1
II=N*NP/2
DO 202 NIP=2,N
JK=II
NI=II-1
II=II-NIP
AA=A(II)
IP=II+1
IF(AA.GT.0.)GOTO2O3
DO 204 IJ=IP,NI
204 A(IJ)=0.
GOTO2O2
203 CONTINUE
DO 201 IJ=IP,NI
V=A(IJ) *AA
DO 200 IK=IJ,NI
A(JK)=A(JK) +A(IK)*V
200 JK=JK+1
201 A(IJ)=V
202 CONTINUE
RETURN

```

C MULTIPLY A VECTOR \(Z\) BY THE FACTORS GIVEN IN A
ENTRY MC11D(A,N, Z,SIG,W,IR,MK,EPS)
IF (N.GT.1) GOTO300
\(\mathrm{Z}(1)=\mathrm{Z}(1) * \mathrm{~A}(1)\)
\(W(1)=Z(1)\)
RETURN
300 CONTINUE
\(\mathrm{NP}=\mathrm{N}+1\)
\(I I=1\)
\(\mathrm{N} 1=\mathrm{N}-1\)
DO \(303 \mathrm{I}=1\), N1
\(Y=2\) (I)
IF(A(II).EQ.O.)GOTO3O2
\(I J=I I\)
IP=I+1
DO \(301 \mathrm{~J}=I P, N\)
\(I J=I J+1\)
\(301 \mathrm{Y}=\mathrm{Y}+\mathrm{Z}\) (J) *A(IJ)
\(302 \mathrm{Z}(\mathrm{I})=Y * A(I I)\)
\(W(I)=Z(I)\)
303 II=II+NP-I
\(\mathrm{Z}(\mathrm{N})=\mathrm{Z}(\mathrm{N}) \star \mathrm{A}(\mathrm{II})\)
\(W(N)=Z(N)\)
DO \(311 \mathrm{~K}=1\), N1
\(\mathrm{I}=\mathrm{N}-\mathrm{K}\)
\(I I=I I-N P+I\)
IF (Z (I).EQ.0.) GOTO311
\(I P=I+1\)
\(I J=I I\)
\(\mathrm{Y}=\mathrm{Z}\) (I)
DO \(310 \mathrm{~J}=I P, N\)
\(I J=I J+1\)
\(310 \mathrm{Z}(\mathrm{J})=\mathrm{Z}(\mathrm{J})+\mathrm{A}(\mathrm{IJ}) * Z(I)\)
311 CONTINUE
RETURN
C MULTIPLY A VECTOR \(Z\) BY THE INVERSE OF THE FACTOKS UIVEN IN A
ENTRY MCIIE(A,N, Z, SIG, W, IR, MK, EPS)
IF (IR.LT.N) RETURN
\(\mathrm{W}(1)=2(1)\)
IF (N.GT.1) GOTO400
\(\mathrm{Z}(1)=\mathrm{Z}(1) / \mathrm{A}(1)\)
RETURN
400 CONTINUE
DO \(402 \mathrm{I}=2, \mathrm{~N}\)
\(\mathrm{IJ}=\mathrm{I}\)
II=I-1
\(V=Z(I)\)
DO \(401 \mathrm{~J}=1\), I1
\(V=V-A(I J) * Z(J)\)
401 IJ \(x I J+N-J\)
\(W(I)=V\)
\(402 \mathrm{Z}(\mathrm{I})=\mathrm{V}\)
\(\mathrm{Z}(\mathrm{N})=\mathrm{Z}(\mathrm{N}) / \mathrm{A}(\mathrm{IJ})\)
\(\mathrm{NP}=\mathrm{N}+1\)
DO 411 NIP=2,N
I=NP-NIP
\(I I=I J-N I P\)
\(V=2\) (I) \(/ \mathrm{A}\) (II)
\(I P=I+1\)
IJ=II
DO \(410 \mathrm{~J}=I \mathrm{IR}\), N
\(I I=I I+1\)
\(410 \mathrm{~V}=\mathrm{V}-\mathrm{A}(\mathrm{II}) * \mathrm{Z}\) (J)
\(4112(I)=V\)
RETURN
C COMPUTE THE INVERSE MATRIX FROM.FACTORS GIVEN IN A
ENTRY MC11F(A,N, Z, SIG, W, IR, MK, EPS)
IF (IR.ITT.N) RETURN
\(\mathrm{A}(1)=1 . / \mathrm{A}(1)\)
IF (N.EQ.1) RETURN
\(\mathrm{NP}=\mathrm{N}+1\)
```

        N1=N-1
        II=2
        DO 511 I=2,N
        A(II) =-A(II)
        IJ=II+1
        IF(I.EQ.N)GOTOSO2
        DO 501 J=I,N1
        IK=II
        JK=IJ
        V=A(IJ)
        DO 500 K=I,J
        JK=JK+NP-K
        V=V+A(IK)*A(JK)
    500 IK=IK+1
    A(IJ) =-V
    501 IJ=IJ+1
    502 CONTINUE
        A(IJ)=1./A(IJ)
        II=IJ+1
        AA=A(IJ)
        IJ=I
        IP=I+1
        NI=N-I
        DO 511 J=2,I
        V=A(IJ)*AA
        IK=IJ
        K=IJ-IP+J
        Il=IJ-1
        NIP=NI+IJ
        DO 510 JK=K,I1
        A(JK)=A (JK)+V*A(IK)
    510 IK=IK+NIP-JK
    A(IJ)=V
    511 IJ=IJ+NP-J
RETURN
C---------------------------- END OF OPTIMIZING ROUTINE ---------------------
END

```

```

    COMMON blocks and PARAMETER statements are set up to enable easy
    variation of the scale of simulation. Important dimensions of the
    simulation are shown below; change of the appropriate dimension in
    all PARAMETER statements will introduce the change throughout the
    simulation:
    Description of Program Dimension Parameters
        NKIND = Numger of atom types of interest
            NIBB = Number of internal building blocks per repeat unit
    MNARU = Maximum number of atoms per repeat unit
    MNRO = Maximum number of repeat units per chain
    MNCH = Maximum number of chains (a square number)
    MNAT = Maximum number of atoms per chain + 1
    MNIC = Maximum number of independently orientable chains
    MCOP = MNIC*5
    MOPT = MCOP+6
    Some parameters are specific to the PPTA polymer family
    NRNG = Number of rotatable rings per repeat unft = 2
    NSUB = Number of substitutions possible per repeat unit = 2
    MNSB = NSUB*MNRU
            NDOF = NIBB + NRNG
    MDOF = NNARU = 28
    ```

In the modified chain description, the method of chain description and the numbering of atoms in the chain have been altered. These changes are contained primarily in the routines COORDN, NONBOND, and ENERGY, to reflect the new construction, atom numbering, degrees of freedom, and force field. However, the following routines are also effected through the dimensioning of arrays, the control of DO loop executions, and atomic numbering, but do not differ sufficiently from the previous versions to justify reproduction in their entirety. (*) indicates that only changes to PARAMETER and COMMON statements are required
\begin{tabular}{ll} 
CMODEL & CELGEN \\
LDVAR & LDCHN \(\star\) \\
DATARD & RESWR \\
RDSTRAIN * & HELIX \\
GYRA \(\star\) \\
DENS \(\star\) & UNITCELL * \\
& SUBSET
\end{tabular}

SUBROUTINE COORDN
C
C Generates a chain of poly(p-phenylene terephthalamide) with all
\(C\) bond lengths, valence angles, and torsion angles independently
C specified.
C For use in generating elastic constants: This routine replaces COORDN and PHENYL in previous versions.

```

Numbering of Atoms:
C
PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,

+ MNSB=24,NDOF=10,MNIC=9,MCOP=45,MOPT=51,MNARU=28 )
COMMON /CHAINS/ PHI(MNARU),THETA(MNARU),BL(MNARU),PHIR(NRNG),
KSB (4), PCH (4,MNAT),NARU,NRUS,NCH,
A(4,4,MNAT), XYZ (4,MNAT,MNCH),KIND (MNAT, MNCH),
CHRG (MNAT, MNCH), KONEC (MNAT, MNAT) , DEE, THT, RHO,
NUNIQ
DIMENSION AL (4,4), KATOM(11), XYZL (4,11)
C
Constraints:
a) planar sp2 hydridization at amide nitrogens and carbons
PHI(11) = PHI(12) + 180.
PHI(13) = PHI(14) + 180.
PHI(25) = PHI(26) + 180.
PHI(27) = PHI(28) + 180.
b) 1-parameter (angle) ring deformation
DO 110 I=7,21,14
ALPHA = THETA(I)
BETA = 180. - 0.5*ALPHA
GAMMA = 90. + 0.25*ALPHA
THETA(I-6) = BETA
THETA(I-4) = BETA
THETA(I-2) = BETA
THETA(I-1) = BETA
THETA(I+2) = BETA
THETA(I-5) = GAMMA
THETA(I-3) = GAMMA
THETA(I+1) = GAMMA
THETA(I+3) = GAMMA
110 CONTINUE
C c) Selected bond deformation (ring symmetric)
DO 120 I=1,15,14
BL}(I+5)=BL(I+1
BL}(\textrm{I}+7)=\textrm{BL}(\textrm{I}+1
BL(I+9) = BL(I+3)
120 CONTINUE
C
C Start building...
C All atoms added pair-wise, one "backbone" and one "pendant"
DO 2100 NREP=1,NRUS+1
DO 2100 NODD=1,MNARU-1,2
DO 2100 NPAIR=1,2
C
C Add only first atom of N+1st repeat unit:
IF( (NREP.EQ. (NRUS+1)).AND.((NODD.NE.1).OR.(NPAIR.NE.1)) )
GO TO 2100
NODD1 = NODD - 1
NLOC = NODD1 + NPAIR

```
```

    NLOC1 = NLOC-1
    NCURR = NLOC+NARU* (NREP-1)
    NCURR1 = NCURR-1
    NATD1 = NCURR-NPAIR
    IF(NLOC1.EQ.O) NLOC1 = 28
    C
XYZ (1,NCURR, 1) = BL (NLOC)
XYZ(2,NCURR,1) = 0.
XYZ (3,NCURR,1) = 0.
XYZ(4,NCURR,1) = 1.
KIND (NCURR,1) = 1
C
C First atom fixes Cartesian space to colncide with first bond; no
C further manipulation necessary
IF( NCURR.EQ.1 ) GO TO 2100
C
C Determine atom type:
IF(MOD(NLOC,2).EQ.1) KIND(NCURR,1) = 2
IF((NLOC.EQ. 1).OR. (NLOC.EQ.15)) THEN
KIND (NCURR,1) = 1
ELSE IF((NLOC.EQ. 7).OR.(NLOC.EQ.27)) THEN
KIND (NCURR, 1) = 3
ELSE IF((NLOC.EQ.14).OR.(NLOC.EQ.26)) THEN
KIND (NCURR,1) = 4
ELSE IF((NLOC.EQ.13).OR.(NLOC.EQ.21)) THEN
KIND (NCURR, 1) = 5
ELSE IF((NLOC.EQ.12).OR.(NLOC.EQ.28)) THEN
KIND (NCURR, 1) = 6
END IF
C
C Second atom placed in xy plane: PHI=0. Load A matrix directly.
C Otherwise, obtain A matrix from previous connection
C
IF (NCURR.EQ.2) THEN
CALL ATRNS( 0.,THETA(1),BL(1),AL).
DO 2320 I=1,4
DO 2320 J=1,4
2320 A(I,J,NCURR)=AL(I,J)
ELSE
IF( ((NLOC.GE.12).AND. (NLOC.LE.16)).OR.
+ (NLOC.GE.26).OR.(NLOC.LE.2) ) THEN
IF( (NLOC.EQ.12).OR.(NLOC.EQ.26) ) THEN
CALL ATRNS( PHI (NLOC1),THETA (NLOC1),BL (NLOC-5),AL)
CALL MM( A (1, 1,NCURR-5), AL, A (1, 1, NCURR), 4, 4, 4, 4, 4, 4)
ELSE IF( (NLOC.EQ.13).OR.(NLOC.EQ.27) ) THEN
CALL ATRNS( PHI (NLOC1),THETA(NLOC1),BL(NLOC-6),AL)
CALL MM( A (1,1,NCUKR-6),AL,A(1,1,NCURF),4,4,4,4,4,4)
ELSE
NLOT = NLOC-MOD (NCURR,2)-1
IF(NLOC.EQ.1) NLOT = 27
NATT = NCURR-MOD (NCURR, 2)-1
CALL ATRNS( PHI(NLOC1),THETA(NLOC1),BL(NLOT),AL)
CALL MM( A(1,1,NATT),AL,A(1,1,NCURR), 4, 4,4,4,4,4)
END IF
ELSE
CALL ATRNS( PHI(NLOC1),THETA(NLOC1),BL(NODD1),AL)
CALL MM( A (1,1,NATD1),AL,A(1,1,NCURR), 4,4,4,4,4,4)
END IF
END IF
C TRANSFORM TO FRAME O:
CALL MV( A(1, 1,NCURR),4,3,XYZ(1,NCURR,1),4,XYZ(1,NCURR,1) )
CONTINUE
RETURN
END

```

SUBROUTINE NONBOND ( IENTRY )
```

Creates Table of Connectivity and assigns partial atomic charges to
atoms of the parent polymer form (PPTA)
Need be executed only once for a given cell composition, but must
be called after all chains have been created
Replaces NONBOND in previous versions involving alternate chain
description
PARAMETER( NKIND=9,NIBB=8,MNRU=6,MNCH=9,MNAT=179,NRNG=2,NSUB=4,
+
COMMON /CHAINS/ PHI(MNARU),THETA(MNARU),BL (MNARU),PHIR(NRNG),
KSB (4), PCH (4, MNAT), NARU, NRUS, NCH,
A (4, 4,MNAT) , XYZ (4, MNAT, MNCH), KIND (MNAT, MNCH),
CHRG (MNAT, MNCH) , KONEC (MNAT, MNAT), DEE, THT, RHO,
NUNIQ
DATA CHARGC, CHARGO, CHARGN, CHARGH/. 38,-.38,-.28,.28/
DATA CHARGPPC, CHARGNPC, CHARGPH/ -0.06,-0.12,0.15/
IF( IENTRY.EQ.1 ) RETURN
C
120
100 I=1,NARU*NRUS
DO 120 J=1,NCH
CHRG(I,J) = 0.
DO 130 J=1,NARU*NRUS
KONEC (I,J) = 0.
CONTINUE
100
C
All atoms within a given phenyl ring or amide bond plane are fixed
relative to one another; atoms directly bonded to each ring/plane
are fixed relative to that ring/plane
Amide atom charges are assigned:
NMER = NARU/28
DO 500 LL = 1,NCH
KTYP = 10
MARK = 0
DO 200 I=1,NMER*NRUS
DO 200 II=1,4
DO 300 J=MARK+1,MARK+KTYP
IF(KTYP.EQ.10) GO TO 310
JXX = J
IF(J.EQ.(MARK+1)) JXX = J - 4
IF (KIND(JXX,LL).EQ.3) THEN
CHRG (JXX,LL) = CHARGC
ELSE IF(KIND (JXX,LL).EQ.4) THEN
CHRG (JXX,LL) = CHARGH
ELSE IF (KIND (JXX,LL).EQ.5) THEN
CHRE (JXX,LL) = CHARGN
ELSE IF (KIND (JXX,LL).EQ.6) THEN
CHRG (JXX,LL) = CHARGO
END IF
310 CONTINUE
C
C Three independent charges allowed: carbons 1\&4, carbons 2,3,5\&6,
and all hydrogens
IF(KTYP.EQ.4) GO TO }31
IF((J.EQ.(MARK+1)).OR. (J.EQ.(MARK+6))) THEN
CHRG(J,LL) = CHARGPPC
ELSE IF (J.EQ. (MARK+7)) THEN
CHRG (J+4,LL) = CHARGPH
ELSE IF (KIND(J,LL).EQ.1) THEN
CHRG (J,LL) = CHARGNPC
ELSE
CHRG(J,LL) = CHARGPH
END IF
315
CONTINUE

```
```

        CONTINUE
        MARK = MARK+KTYP
        IF(KTYP.EQ.10) THEN
            KTYP = 4
        ELSE
        KTYP = 10
        END IF
    CONTINUE
    CONTINUE
MARK = 0
KEND = 0
DO 700 I=1,NMER*NRUS
DO }720\textrm{L}=0,14,1
DO }710\textrm{J}=1,
DO }710\textrm{K}=1,
IF( (KEND.EQ.1).AND. ((J+K+L).GT.28)) GO TO 710
KONEC (MARK+J+L,MARK+J+K+L)=1
CONTINUE
KONEC (MARK+4+L, MARK+12+L) = 0
KONEC (MARK+4+L, MARK+13+L) = 0
KONEC (MARK+8+L,MARK+12+L) = 0
KONEC (MARK+8+L, MARK+13+L) = 0
KONEC (MARK}+7+L,MARK+16+L) = 0
KONEC (MARK+6+L, MARK+15+L) = 0
KONEC (MARK+8+L,MARK +17+L) = 0
KONEC (MARK+1+L,MARK +11+L) = 1
KONEC (MARK+1+L, MARK+7+L) = 0
KONEC (MARK+2+L,MARK+9+L) = 0
KONEC (MARK+4+I, MARK +11+L) = 0
IF((KEND.EQ.O).AND. (L.EQ.O)) KONEC (MARK+8+L,MARK+16+L) = 0
KONEC (MARK+7+L,MARK+11+L) = 0
DO }730\textrm{J}=1,
KONEC (MARK+3+L, MARK+6+J+L) = 0
KONEC (MARK + 3+L,MARK+10+J+L) = 0
KONEC (MARK+5+L, MARK+8+J+L) = 0
KONEC (MARK+5+L,MARK+11+J+L) = 0
IF( (KEND.EQ.1).AND. ((J+L).GT.15) ) GO TO 730
KONEC (MARK+8+L, MARK+13+J+L)=0
CONTINUE
KONEC (MARK+9+L,MARK+10+L) = 1
KONEC (MARK+9+L, MARK +1I+L) = I
KONEC (MARK+10+L,MARK+11+L) = 1
DO }740\textrm{J}=1,
IF((KEND.EQ.1).AND. ((J+L).GT.16)) GO TO 732
KONEC (MARK+12+L,MARK+12+J+L) = 1
KONEC (MARK+13+L,MARK+12+J+L) = 1
IF((KEND.EQ.1).AND.((J+L).GT.15)) GO TO 734
KONEC (MARK+14+L,MARK+13+J+L) = 1
IF(KEND.EQ.1) GO TO 740
KONEC (MARK+13+L,MARK+15+J+L) = 1
CONTINUE
KONEC (MARK+14+L,MARK+16+L) = 0
IF((KEND.EQ.1).AND.(L.EQ.14)) GO TO }72
KONEC (MARK+13+L,MARK+22+L) = 1
KONEC (MARK+13+L, MARK+24+L) = 1
KONEC (MARK+13+L,MARK+25+L) = 1
CONTINUE
MARK=MARK+NARU
IF(I.EQ.(NRUS-1)) KEND = 1
CONTINUE
IENTRY = 1
RETURN
END

```

SUBROUTINE ENERGY
two lattice sum methods: 3D periodicity. (LSTOGG=0) or Helical period-
idity (LSTOGG=1, enable MV call in lattice sum section).

LOGICAL FIRST, REDUND
PARAMETER ( EPSILON=1E-6 )
PARAMETER ( \(\operatorname{NKIND=9,~} \mathrm{NIBB}=8, \mathrm{MNRU}=6, \mathrm{MNCH}=9\), \(\mathrm{MNAT}=179\), \(\mathrm{NRNG}=2, \mathrm{NSUB}=4\),
MNSB \(=24\), NDOF \(=10, \mathrm{MNIC}=9, \mathrm{MCOP}=45, \mathrm{MOPT}=51, \mathrm{MNARO}=28\) )
COMMON /CHAINS/ PHI (MNARU),THETA (MNARU), BL (MNARU), PHIR (NRNG), KSB (4) , PCH (4, MNAT), NARU, NRUS, NCH,
A ( \(4,4, \mathrm{MNAT}), \mathrm{XYZ}(4, \mathrm{MNAT}, \mathrm{MNCH}), \mathrm{KIND}\) (MNAT, MNCH) ,
CHRG (MNAT, MNCH), KONEC (MNAT, MNAT), DEE, THT, RHO, NUNIQ
COMMON /CELL/ CELD (3), CELA (3), CHSPEC ( \(5, \mathrm{MNCH}\) ), CHLOC ( \(3, \mathrm{MNCH}\) ), NUNIV, KONDENS, TEMP, DENSC
COMMON /ENRG/ EDEF1,EDEL,ENOB1,ENOB2,ECOU1,ECOU2,EULJ,EUES,
EULJP, EUESP, ECFF, ETOT, EDEF2
COMMON /SUBS/ WTED (4)
COMMON /STRN/ KUC, KASE, ST \((3,3), \operatorname{SUCV}(3,3)\), DINIT, DPRIME,DCON
common /atmp/ BxO, Bx00, \(\mathrm{CxO}, \mathrm{Cx} 00\)
DIMENSION ANGZ (12), ANGO (12), DCL \((3,3)\)
DIMENSION BNDZ (30), BNDO (30), VALZ (48), VALO (48)
DIMENSION FMOLT (MNCH, MNCE)
DIMENSION VDWR (NKIND), ALPHA (NKIND) , EFFNE (NKIND)
DIMENSION UX (3), UY (3) , UZ (3), DVECT (4), R(4, 4, 21)
DIMENSION REDUND (125)
DIMENSION SUM1 (3136), SUM6(3136), SUM12 (3136), SUM1P (3136),
TMP12 (3136), TMP1P (3136), TMP6P (3136), TMP12P (3136)
DATA VDWR/ 1.96,1.37,2.03,1.37,1.96,1.60,1.35,1.80,1.95/
data alpha/o. \(768,0.330,2.16,0 ., 2.95,1.02,0.38,1.41,3.34 /\)
DATA EFFNE/5.2, 0.9, 5.2, 0.9.6.1, 7., 8., 12., 21./
DATA BO, BOO,CO, COO, B1, B2/9.0,13.0, 2, .4, 2.8,21.2 /
DATA BNDZ/ 648.,925.,900.,925.,900.,925.,648.,925.,900.,925.,
900., 1263., 880., 861., 648., 925., 900., 925., 900., 925.,
648.,925.,900.,925., 900., 861., 880., 1263., 925., 925./
```

    DATA BNDO/ 1.48,1.40,1.10,1.40,1.10,1.40,1.48,1.40;1.10,1.40,
        1.10,1.24,1.37,1.00,1.38,1.40,1.10,1.40,1.10,1.40,
        1.38,1.40,1.10,1.40,1.10,1.00,1.37,1.24,1.40,1.40 /
    DATA VALZ/ 0., 0.,.041, 0., 0.,.041,.041,.041,.041, 0.,
        0.,.041, 0., 0.,.041,.088,.088,.088,.050,.050,
        .088,.041,.041,.041, 0., 0.,.041, 0., 0.,.041,
        .041,.041,.041, 0., 0.,.041, 0., 0.,.041,.050,
        .050,.088,.088,.088,.088,.041,.041,.041 /
    DATA VALO/ 120.,120.,119.,120.,120.,119.,119.,119.,122.,120.,
        120.,119.,120.,120.,119.,120.,120.,120., 120.,120:,
        120.,119.,123.,119.,120.,120.,119.,120.,120.,119.,
        119.,119.,123.,120., 120.,119., 120., 120., 119., 120.,
        120.,120.,120.,120.,120.,119.,122.,119./
    DATA PCARBZ,PCARBO/ .0131,180. /
    DATA NHZ,COZ/ .004,.025 /
    DATA BXO,BXOO,CXO,CXOO /0.0,32.0,.0,.8/
    DATA DE/ 3.5 /
    DATA ANGERR/ 5./
    DATA BWDIST/ 3.3/
    FIRST = .TRUE.
    DEGRAD = 3.1415926535/180.
    LSTOGG = 0
    C
C
Initialize intermolecular interaction multiplier matrix:
NCH2 = NCH**2
LIMAX = 1
IF( NCH.NE.1 ) CALL MULMAX( MNCH,NCH,NCH2,L1MAX,FMULT )
IF ( (NCH.EQ.1).OR.(ABS(THT).LT.ANGERR) ) GO TO 100
DO 50 M=1,NRUS-2
PANG = M*THT/360.
IPANG = INT (PANG+0.5)
IF ( ABS (PANG-IPANG) .LE. (M*ANGERR/360.) ) GO TO 70
CONTINUE
NER = 1
GO TO }8
NER = M
GO TO 80
NER = 1
CONTINUE
LATM = INT (RBASE*NER*NARU+1.01)
C
Bond length deformation: (two extra for ring closure)
DO 3100 I=1,28
IF( KIND(I,1).EQ.8 ) THEN
BNDO(I) = 1.70
RCCL = SQRT( (XYZ (1,I,1)-XYZ(1,I-1,1))**2 +
(XYZ(2,I,1)-XYZ(2,I-1,1))**2 +

```
```

    +
            BL(I) = RCCL
            END IF
            EDEF1 = EDEF1 + 0.5*BNDZ(I)*(BL(I) - BNDO(I))**2
    CONTINUE
    DO 3150 I=1,2
        II = (I-1)*14 + 1
        RCB = SQRT( (XYZ (1,II,1)-XYZ (1,II+9,1))**2 +
                            (XYZ(2,II,1)-XYZ(2,II+9,1))**2 +
                            (XYZ(3,II, 1)-XYZ (3,II+9,1))**2 )
            EDEF1 = EDEF1 + 0.5*BNDZ (I+28)* (RCB - BNDO (I+28))**2
        CONTINUE
    c
C Valence angle deformation
C
MKR = 0
DO 3200 I=1,16
IHAF = MOD (I, 8)
IA = (I/9)*14 + 2*IHAF
IB = IA-2
IC = IA+1
ID = IA +2
IF( IHAF.EQ.1 ) THEN
IB = IA-1
ELSE IF( IHAF.EQ.5 ) THEN
ID = IA-9
ELSE IF( IHAF.EQ.6 ) THEN
IA = IA-5
IB = IA-1
IC = IA +5
ID = IA +6
ELSE IF( IHAF.EQ.7) THEN
IA = IA-1
IB = IA-6
IC = IA +1
ID = IA +2
ELSE IF( IHAF.EQ.O ) THEN
IA = (I/9)*14 + 15
IB = IA-2
IC = IA +1
ID = IA+9
ELSE
END IF
MKR = MKR + 1
CALL ANGL( XYZ(1,IB,1),XYZ(1,IC,1),XYZ(1,IA,1),VA )
EDEF2 = EDEF2 + 0.5*VAL2 (MKR)*( VA - VALO(MKR) )**2
MKR = MKR + 1
CALL ANGL( XYZ (1,IC,1),XYZ (1,ID,1),XYZ (1,IA,1),VA )
EDEF2 = EDEF2 + 0.5*VALZ (MKR)*(VA - VALO(MKR) )**2
MKR = MKR + 1
CALL ANGL( XYZ (1,ID, 1),XYZ (1,IB,1),XYZ (1,IA, 1),VA)
EDEF2 = EDEF2 + 0.5*VALZ (MKR)*( VA - VALO(MKR) )**2
CONTINUE
C
C Confugated bond interaction energy:
C Note: with the introduction of substitution on the ring (potentially
C In a random sequence and without equal occurance at the two 'ends'
C of the ring) with different delocalization potentials, it becomes
C necessary to compute EDEL weighted by the appropriate loci of
C substitutions:
NRT = NARU/28
DO 400 K=1,NRT
NRR = (K-1)*NRNG/NRT
NRP = (K-1)*NIBB/NRT
EDEL = EDEL + WTED (2)*CURTINV (-PHI (12+NRP),BO,CO )
EDEL = EDEL + WTED (3)*CURTATE ( (PHI (15+NRP) +180.), B00,C00 )
EDEL = EDEL + WTED (4)*CURTATE (-PHI (26+NRP), B00,C00)
EDEL = EDEL + WTED (1)*CURTINV ( (PHI (1+NRP) +180.),B0,C0 )
EDEL = EDEL + (1. - WTED (2))*CURTATE (-PHI (12+NRP),BXO,CXO )
EDEL = EDEL + (1. - WTED (3))*CURTATE ( (PHI (15+NRP)+180.),

```
```

                                    B00,C00 )
            EDEL = EDEL + (1.- WTED (4))*CURTATE (-PHI (26+NRP),BOO,C00 )
            EDEL = EDEL + (1.- WTED (1))*CURTATE((PHI (1+NRP) +180.),
                    BO,CO )
                            EDEL = EDEL + B1/2* (1.-COS (PHI (14+NRP) *DEGRAD )
                            +B2/2*(1.-COS(2*PHI (14+NRP) *DEGRAD))
                            EDEL = EDEL + B1/2*(1.-COS (PHI (28+NRP) *DEGRAD))
                            + B2/2*(1.-COS(2*PHI (28+NRP) *DEGRAD))
        CONTINUE
    C
Out-of-plane bending for amide N-H and C=O
OPB1 = ABS(PHI(12) - PHI(11) + SIGN(180.,PHI(11)) )
OOPB2 = ABS( PHI(14) - PHI(13) + SIGN(180.,PHI(13)) )
OOPB3 = ABS( PHI(26) - PHI(25) + SIGN(180.,PHI(25)) )
OOPB4 = ABS(PHI(28) - PHI(27) + SIGN(180.,PHI(27)) )
IF (OOPB1.GT.180.) OOPB1 = 360. - OOPB1
IF (OOPB2.GT.180.) OOPB2 = 360. - OOPB2
IF (OOPB3.GT.180.) OOPB3 = 360. - OOPB3
IF (OOPB4.GT.180.) OOPB4 = 360. - OOPB4
EDEF2 = EDEF2 + 0.5*NHZ* (OOPB2**2 + OOPB3**2 )
EDEF2 = EDEF2 + 0.5*COZ*( OOPB1**2 + OOPB4**2 )
C
C Add phenyl-cargon torsions (6 per ring)
DO 3300 I=1,NRNG
IC1 = (I-1)*14
DO 3300 J=1,6
IF(J.EQ.1) THEN
IA = IC1 + 1
IB = IA + 1
IC = IB + 2
ID = IC + 2
ELSE IF(J.EQ.4) THEN
IA = J*2 - 2 + IC1
IB = IA + 2
IC = IB + 2
ID = IC - 9
ELSE IF(J.EQ.5) THEN
IA = J* 2 - 2 + IC1
IB = IA + 2
IC = IB - 9
ID = IC + 1
ELSE IF(J.EQ.6) THEN
IA = J*2 - 2 + IC1
IB = IA - 9
IC = IB + 1
ID = IC + 2
ELSE
IA = J*2 - 2 +.IC1
IB = IA + 2
IC = IB + 2
ID = IC + 2
END IF
CALL TORSION( XYZ(1,IA,1),XYZ(1,IB,1),XYZ(1,IC,1),
XYZ(1,ID,1),BTR )
IF(BTR.LE.-180.) BTR = BTR + 360.
IF(BTR.LE.O.) BTR = BTR + 180.
EDEL = EDEL + 0.5*PCARBZ* (BTR - PCARBO)**2
CONTINUE
C
DO 500 L1=1,L1MAX
IF(L1.NE.1) GO TO 530
C Intramolecular nonbonded interactions:
c
DO 520 I=1,NM1
IP1 = I+1
DO 510 J=IP1,NATOMS
IF( KONEC(I,J).EQ.1 ) GO TO 510
IF( I.GE.LATM ) GO TO }51

```
```

                            D2 = (XYZ(1,I,L1) -XYZ (1,J,L1))**2 +
                        (XYZ (2,I,L1)-XYZ(2,J,L1))**2 +
                            (XYZ(3,I,L1)-XYZ(3,J,L1))**2
            DIS = SQRT(D2)
            CALL ELJ( NKIND,KIND(I,L1),KIND(J,L1),VDWR,ALPHA,EFFNE,
                                    D2,D2,FIRST,POTLJ )
            ENOB1 = ENOB1 + POTLJ
                            CALL EES( BWDIST,DIS,DE,CHRG(I,L1),CHRG(J,LI),POTES )
                            ECOU1 = ECOU1 + POTES
            CONTINUE
        CONTINUE
    CONTINUE
        IF (NCH.EQ.1) GO TO 500
        LP1 = L1+1
    C
Intermolecular interactions:
C
DO 550 L2=LP1,NCH
IF( INT(FMULT(L1,L2)*100.).EQ.0 ) GO TO 550
IF( INT (ABS (CHSPEC(1,L2))+.01).NE.1 ) GO TO 550
ENOBT = 0.
ECOUT = 0.
DO 570 I=1, NATOMS
DO }560\textrm{J}=1,NATOM
IF (I.LT.LATM) THEN
D2 = (XYZ(1,I,L1) -XYZ (1,J,L2))**2 +
+ (XYZ(2,I,L1)-XYZ (2,J,L2))**2 +
+ (XYZ(3,I,L1)-XYZ(3,J,LL2))**2
DIS = SQRT(D2)
CALL ELJ( NKIND,KIND (I,L1),KIND (J,L2) ,VDWR,ALPHA, EFFNE,
D2,D2,FIRST,POTLJ )
ENOBT = ENOBT + POTLJ
CALL EES( BWDIST,DIS,DE,CHRG(I,L1),CHRG(J,L2),POTES )
ECOUT = ECOUT + POTES
ELSE
IF (J.LT.LATM) THEN
D2 = (XYZ (1,I,L1) -XYZ (1,J,I2))**2 +
+
(XYZ(2,I,L1)-XYZ(2,J,L2))**2 +
(XYZ (3,I,L1) -XY2 (3,J,L2))**2
DIS = SQRT(D2)
CALL ELJ( NKIND,KIND(I,L1),KIND (J,L2),VDWR,ALPHA,EFFNE,
D2,D2,FIRST,POTLJ )
ENOBT = ENOBT + POTLJ
CALL EES( BWDIST,DIS,DE,CHRG(I,L1),CHRG(J,L2),POTES )
ECOUT = ECOUT + POTES
ELSE
END IF
END IF
CONTINUE
CONTINUE
ENOB2 = ENOB2 + 0.5*ENOBT
ECOU2 = ECOU2 + 0.5*ECOUT
CONTINUE
CONTINUE
C
Infinite Lattice Correction:
(specific for two-chain-type lattices, with interchain interactions
1-1, 1-2, 2-1, and 2-2 only; of these, only half of the 1-1 and 2-2
interactions are unique, and the 1-2 interactions are identical to
the 2-1 interactions. NUNIV represents "infinite" lattice size)
IF( NUNIV.EQ.O ) GO TO 630
DO 610 K=1,3
UX(K) = CHLOC(K,2) - CHLOC (K,4)
OY(K)=CHLOC(K,2) + CHLOC (K,4)
OZ (K) = 0.
DVECT(K) = 0.
UZ(3) = RBASE*REAL (NER) *DEE
DVECT (4) = 1.
NCM2 = NINT (SQRT (REAL (NCL))}-2
NNCM = -NCM2-1

```
```

    ESPWR = BWDIST*LOG(DE)
    C
C Set up helix periodicity transforms beforehand:
C
IF(LSTOGG.EQ.0) GO TO 625
DO 620 LZN = 1,2*NUNIV+1
LZ = LZN-NUNIV-1
RTN = RBASE*REAL (LZ*NER)
RTH = RTN*THT
RDE = RTN*DEE
CALL RTRANS( RTH,RDE,R(1,1,LZN) )
620 CONTINUE
625 CONTINUE
C
DO 600 L1=1,2
DO }600\mathrm{ L2=L1,2
KFIJ = L1+L2-1
LXS = -NUNIV
IF( KFIJ.NE.2 ) LXS = 0
NNLX = NUNIV-LXS+1
NNLY = 2*NUNIV+1
NNLZ = NNLY
C
C Zero all atom-atom summations:
C
DO }645\mathrm{ I=1, LATM-1
DO }645\textrm{J}=1,LATM-
NTMP = (I-1)* (LATM-1) +J
SUM1P(NTMP) = 0.
SUM6P(NTMP) = 0.
SUM12P (NTMP) = 0.
SUM1 (NTMP) = 0.
SUM6(NTMP) = 0.
SUM12(NTMP) = 0.
645 CONTINUE
C
C Loop over all lattice points, checking for redundancy with explicit
C calculation scheme for L1-L2:
C
cvd\$ noconcur
DO }680\mathrm{ LX=LXS,NUNIV
cvd\$ noconcur
DO 680 LY=-NUNIV,NUNIV
cvd\$ noconcur
DO }680\mathrm{ LZ=-NUNIV,NUNIV
NRED = (LX-LXS)*NNLY*NNLZ + (LY+NUNIV)*NNLZ +
(LZ+NUNIV) + 1
IF( (KFIJ.NE.2).AND.(LX.EQ.0).AND.(LY.LE.O) ) THEN
REDUND (NRED) = .TRUE.
GO TO 680
ELSE
REDUND (NRED) = .FALSE
END IF
IF( ABS(LZ).LE.(NRUS-1) ) THEN
IF (KFIJ.NE.2) THEN
REDUND (NRED) = ( (ABS (LX) +ABS (LY)).LE.NCM2 )
ELSE
IF( (LX+LY).NE.NCM2 ) THEN
REDUND (NRED) = ( ((ABS (LX) +ABS (LY)).LE.NCM2).OR.'
((LX.LT.O).AND.(LY.LT.O).AND.((LX+LY).EQ.NNCM)) )
END IF
END IF
END IF
IF( REDUND (NRED) ) GO TO 680
C
C Loop over all atoms of the units of L1 and L2 at this lattice point
C to get inverse distances:
C
DO 650 I=1, LATM-1
DO 650 J=1,LATM-1
NTMP = (I-1)*(LATM-1) +J

```
```

C ESPWR = (VDWR(KIND(I,I1))+VDWR(KIND(J,L2)) )*LOG(DE)
IF( LSTOGG.EQ.O ) THEN
DVECT (1) = XYZ (1,J,L2) - XYZ (1,I,L1)
DVECT(2) = XYZ(2,J,L2) - XYZ(2,I,L1)
DVECT (3) = XYZ(3,J,L2) - XYZ(3,I,L1)
DVV = (DVECT(1)+LX*UX(1)+LY\starUY(1)+LZ*UZ (1))**2
+ + (DVECT (2)+LX*UX (2) +LY\starUY (2)+LZ*UZ (2))**2
+ + (DVECT (3)+LX*UX(3) +LY*UY(3)+L2*UZ (3))**2
ELSE
LZN = LZ + NUNIV + 1
DVECT(1) = XYZ(1,J,L2) - CHLOC(1,L2)
DVECT (2) = XYZ(2,J,L2) - CHLOC (2,L2)
DVECT(3) = XYZ(3,J,L2) - CHLOC (3,L2)
C CALL MV( R(1,1,L2N),4,3,DVECT,4,DVECT
DVECT(1) = DVECT(1) + CHLOC (1,L2) - XYZ (1,I,L1)
DVECT(2) = DVECT(2) + CHLOC (2,L2) - XYZ (2,I,L1)
DVECT (3) = DVECT (3) + CHLOC (3,L2) - XYZ (3,I,L1)
DVV = (DVECT(1)+LX*UX(1)+LY*UY(1))**2
+ + (DVECT (2)+LX*UX(2)+LY*UY(2))**2
+ + (DVECT(3)+LX*UX(3)+LY*UY(3))**2
END IF
DVROOT = SQRT (DVV)
C Note: discontinuity in Sl is possible, may have to spline (5/16/88)
S1 = EXP(ESPWR/DVROOT)/DVROOT
IF( DVROOT.LE.BWDIST ) S1 = DE/DVROOT
S6 = 1./DVV**3
S12 = S6**2
IF( (KFIJ.NE.2).AND.(LX.EQ.O).AND.(LY.EQ.0) ) THEN
TMP1P(NTMP) = S1
TMP 6P(NTMP) = S6
TMP12P(NTMP) = S12
ELSE
TMP1(NTMP) = S1
TMP6(NTMP) = S6
TMP12(NTMP) = S12
END IF
CONTINUE
C
C Add this element to summation
C
DO 655 I=1, LATM-1
DO 655 J=1,LATM-1
NTMP = (I-1)* (LATM-1) +J
SUM1P(NTMP) = SUM1P(NTMP) + TMP1P(NTMP)
SUM6P(NTMP) = SUM6P(NTMP) + TMP6P (NTMP)
SUM12P(NTMP) = SUM12P(NTMP) + TMP12P(NTMP)
SUM1 (NTMP) = SUM1 (NTMP) + TMP1 (NTMP)
SUM6(NTMP) = SUM6(NTMP) + TMP6(NTMP)
SUM12(NTMP) = SUM12(NTMP) + TMP12(NTMP)
CONTINUE
CONTINUE
c
C Finally, calculate the energies for each atom-atom pair:
C
DO 665 I=1, LATM-1
DO 665 J=1,LATM-1
NTMP = (I-1)* (LATM-1) +J
CALL ELJ( NKIND,KIND(I,L1),KIND (J,L2),VDWR,ALPHA,EFFNE,
SUM6 (NTMP), SUM12 (NTMP),FIRST,POTLJ )
EULJ = EULJ + 0.5*POTLJ
EUES = EUES + 166.*SUM1 (NTMP)*CHRG (I,L1)*CHRG (J,L2)/DE
IF( (SUM6P (NTMP).EQ.O.).AND. (SUM12P (NTMP).EQ.O.) ) GO TO 665
CALL ELJ( NKIND,KIND(I,L1),KIND (J,L2),VDWR,ALPHA,EFFNE,
SUM6P (NTMP),SUM12P (NTMP),FIRST,POTLJP )
EULJP = EULJP + 0.5*POTLJP
EUESP = EUESP + 166.*SUM1P(NTMP)*CHRG(I,L1)*CHRG (J,L2)/DE
665 CONTINUE
600 CONTINUE
630 CONTINUE
C
Here is calculated the penalty function that fixes chain pitch DEE

```
```

C
IF( KUC.NE.O ) ECFF = DCON*( DEE - DPRIME )**2
C
Total energy per residue:
ENOB1 = ENOB1/NER/RBASE
ENOB2 = ENOB2/NER/RBASE
ECOU1 = ECOU1/NER/RBASE
ECOU2 = ECOU2/NER/RBASE
EULJ = EULJ/NER/RBASE
EUES = EUES/NER/RBASE
EULJP = EULJP/NER/RBASE
EUESP = EUESP/NER/RBASE
ETOT = EDEF1 + EDEL + ENOB1 + ENOB2 + ECOU1 + ECOU2 + ECFF +
+
EULJ + EUES + EULJP + EUESP + EDEF2
RETURN
900 CONTINUE
WRITE(11,*) 'ERROR: No parent chain available'
RETURN
END
SUBROUTINE ANGL( AA,BB,CC,ANGLE )
C CALCULATES THE BOND ANGLE AA-CC-BB AND THEN RETURNS THE ENERGY
C ASSOCIATED WITH DEFORMATION OF THIS ANGLE FROM AO:
DIMENSION AA (3), BB (3),CC(3)
ASQ = 0.
BSQ = 0.
CSQ = 0.
DO 100 I=1,3
CSQ = CSQ +(AA (I) -BB(I))**2
ASQ = ASQ +(BB(I) -CC(I))**2
BSQ = BSQ (CC (I) -AA(I))**2
ANGLE = ACOS ( 0.5* (ASQ+BSQ-CSQ)/SQRT (ASQ*BSQ) )
ANGLE = ANGLE*180./3.1415926535
RETURN
END
SUBROUTINE TORSION( AA,BB,CC,DD,TOR )
C CALCULATES THE TORSION ANGLE AA-BB-CC-DD AND THEN RETURNS THE ENERGY
C ASSOCIATED WITH DEFORMATION OF THIS ANGLE FROM TO:
DIMENSION AA (3), BB (3),CC(3),DD(3),AB(3),BC(3),DC(3)
DIMENSION ABC (3),BCD (3)
ABM = 0.
BCM = 0.
DCM = 0.
DOT = 0.
ABCM = 0.
BCDM = 0.
DO }100\textrm{I}=1,
AB(I) = BB(I) - AA(I)
BC(I) = CC(I) - BB(I)
DC(I) = CC(I) - DD(I)
ABM = ABM + AB(I)**2
BCM = BCM + BC(I)**2
DCM = DCM + DC(I)**2
DO 200 I=1,3
AB(I) = AB(I)/SQRT(ABM)
BC(I) = BC (I)/SQRT(BCM)
200 DC(I) = DC(I)/SQRT(DCM)
CALL VV( AB,BC,ABC, 3)
CALL VV( BC,DC,BCD,3 )
DO 300 I=1,3
DOT = DOT + ABC(I)*BCD(I)
ABCM = ABCM + ABC(I)**2
BCDM = BCDM + BCD (I)**2
ARG = DOT/SQRT (ABCM*BCDM)
IF( ABS (ARG-1.).LE.1E-6 ) ARG = 1.
IF( ABS (ARG+1.).LE.1E-6) ARG = -1.
TOR = ACOS( ARG )*180./3.1415926535
C DETERMINE TORSION DIRECTION:

```
```

DTX = 0
DO 400 I=1,3
DTX = DTX + AB(I)*BCD(I)
ARG2 = DTX/SQRT(ABM*BCDM)
IF( ABS(ARG2-1.).LE.1E-6 ) ARG2 = 1.
IF(ABS(ARG2+1.).LE.1E-6) ARG2 = -1.
DRX = ACOS( ARG2 )*180./3.1415926535
TOR = SIGN( TOR, (90.-DRX))
RETURN
END

```

Data Input Filc U.dt:
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline 1 & 11 & 10 & & & & & \\
\hline 1 & & & & & & & \\
\hline 28 & 69 & 20 & 205. & & & & \\
\hline 2 & 22 & 2 & & & & & \\
\hline 120. & 180. & 0 & 0 & 0 & 0. & 180. & 0. \\
\hline 120. & 0. & 0 & 0 & 0 & 0. & 180. & 60. \\
\hline 120. & 180. & 0 & 0 & 0 & 0. & 0. & 0. \\
\hline 122. & 5. & 0 & 0 & 0 & -10. & 10. & 10. \\
\hline 120. & 180. & 0 & 0 & 0 & 0. & 0. & 0. \\
\hline 122. & 0. & 0 & 0 & 0 & 0 . & 180. & 60. \\
\hline 120. & 180. & 0 & 0 & 0 & 0. & 0. & 0. \\
\hline 120. & -5. & 0 & 0 & 0 & -10. & 10. & 10. \\
\hline 150. & 0 & 0 & \(25:\) & 155. & 130. & & \\
\hline 30. & 0 & 0 & 0 . & 180. & 15. & & \\
\hline 4.9 & 0 & 0 & 0. & 0. & 0. & & \\
\hline 4.9 & 0 & 0 & 4.5 & 9. & 1.5 & & \\
\hline 13.1 & 0 & 0 & 0 . & 0. & 0. & & \\
\hline 90. & 0 & 0 & 90. & 60. & -15. & & \\
\hline 90. & 0 & 0 & 90. & 60. & -15. & & \\
\hline 90. & 0 & 0 & 30. & 90. & 15. & & \\
\hline 1. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0.1 & 0.5 & 0.2 & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 120. & 0 & 0 & 0. & 150. & 30. & & \\
\hline 1. & 0 & 0 & -2. & 2. & 1. & & \\
\hline 0.4 & 0 & 0 & -0.4 & 0.5 & 0.1 & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & 0. & 180. & 180. & & \\
\hline -60. & 0 & 0 & -180. & 150. & 30. & & \\
\hline 1. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0 . & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 1. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0 . & 0 & 0 & 0. & 180. & 10. & & \\
\hline 1. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0 . & 0. & & \\
\hline 1. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0. & 0 . & 0. & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0 . & 0 . & & \\
\hline 0. & 0 & 0 & 0. & 0. & 0. & & \\
\hline 0. & 0 & 0 & -1. & 1. & 2. & & \\
\hline 0. & 0 & 0 & 0. & 0 . & 0. & & \\
\hline 0. & 0 & 0 & 0 . & 0 . & 0. & & \\
\hline 0. & 0 & 0 & 0 . & 0 . & 0 . & & \\
\hline 0. & 0 & 0 & 0 . & 0 . & 0. & & \\
\hline
\end{tabular}
\begin{tabular}{cccccc}
0. & 0 & 0 & -1. & 1. & 2. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & -1. & 1. & 2. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0. \\
0. & 0 & 0 & 0. & 0. & 0.
\end{tabular}

Data Input File US.dt:
\begin{tabular}{ccc}
1 & 1 & \\
13.1066 & 0 & \\
0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0 \\
0.0 & 0.0 & 0.0
\end{tabular}

\section*{APPENDIX H: FORTRAN Listing of program to Calculate and Plot X-ray Scattering Diagrams}

The following is a listing of one version of the program used to simulate x -ray scatering behavior. This version is appropriate for use with the output file from the structure simulation program and contains graphics subroutines for use with the IRIS 4D-GT multicolor display. This listing contains the following routines:
\begin{tabular}{lll} 
UCANAL (main) & XRSHADE & MV \\
UCDRAW & RANDS1 & MM \\
FRCOORD & PALETTE & LUBKSB \\
STRUFA & VECTORIZ & LUDCMP \\
PLORG & LATCON & ATSCAT \\
GOURPLOT & VV &
\end{tabular}

PROGRAM UCANAL
```

*****************************************************************************

*     *         POLYMER CRYSTAL X-RAY DIFFRACTION *
                GREGORY C. RUTLEDGE *
        DEPARTMENT OF CHEMICAL ENGINEERING *
        MASSACHUSETTS INSTITUTE OF TECHNOLOGY *
                NOVEMBER 1989
                        *
                        * * *
    (C) Copywrite 1989: Gregory C. Rutledge, Ulrich w. Suter *
*****************************************************************************
This is the driver program to interpret results output by the polymer
Crystal Simulation Program. Reads simulation crystal variables and
Cartesian coordinates of the first four chains, estimates the best
unit cell type (out of twelve possibilities provided), converts
atom coordinates to fractional coordinates, and outputs a plot file
for use by SCHAKAL88 or CHARON (PLUTO derivative) to draw unit cell
schematic (in subroutine UCDRAW).
If specified, the calculation will proceed to call roratines to
generate a list of the x-ray diffraction positions and intensities
for crystal planes hkl up to a pre-determined limit in h, k, and l;
it is assumed that the chain axes lie along the crystallographic c
axis, which corresponds in turn to the fiber axis. A representative
x-ray fiber diagram may be recreated for display.
The input file is PCS.IN; Output files are UNCL.OUT for unit cell
representation, SF.RES for a listing of the x-ray reflection positions
and intensities, and UCDRAW.OUT for the unit cell plot file.
This version contains markers for modification required to simulate
larger repeat units (eg. HHTT isomers) and antiparallel chain types
(eg. HT(A) isomers). These are also marked in the subroutines
FRCOORD, UCDRAW, and STRUFA.
*** marks modifications for HHTT isomers
\&\&\& marks modications for antiparallel chains
CHARACTER*64 TITLE,RARG
CHARACTER^1 KIND (84, 4),GARB1 (72), KGB
C*** CHARACTER*1 KIND(168,4),GARB1 (72),KGB
INTEGER*2 LNBLNK
DIMENSION UCT (3,3),GCT (3, 3, 12),GCL (3,12),GCA(3,12),GHYB (4),
ARA (3,3), ARB (3,3),SS(3),SA (3)
DIMENSION XYZ (3,84,4),KCTST(12)
DIMENSION XYZ (3,168,4),KCTST(12)
COMPL (X) = (90.-X)*3.141526535/180.
DEGRAD = 3.1415926535/180.
NARU = 28
c*** NARU = 56
WRITE(*,*) 'Input Structure Designation (up to 36 characters) >'
READ (*,1) TITLE
FORMAT( A64 )
140 WRITE(*,*) 'Do you want to generate or plot x-ray info: [NO] >'
READ(*,1) RARG
LEN = LNBLNK(RARG)
IF( LEN.GE.1) THEN
IF( (RARG(1:1).EQ.'Y').OR.(RARG(1:1).EQ.'Y') ) THEN
KXRFD = 0
ELSE IF( (RARG(1:1).EQ.'N').OR.(RARG(1:I).EQ.'n') ) THEN
KXRFD = 1
ELSE
GO TO }14
END IF.
ELSE
KXRFD = 1
END IF
OPEN( UNIT=9,FILE='PCS.IN',STATUS='OLD' )

```
```

        OPEN( UNIT=10,FILE='UNCL.OUT')
        DO 100 I=1,25
        IF(I.EQ.14) THEN
            READ (9,10) (GARB1(J),J=1,40),SS(1),SS(2),SS(3)
        ELSE IF (I.EQ.I5) THEN
            READ (9,10) (GARB1 (J),J=1, 40),SA(1),SA(2),SA(3)
        ELSE IF (I.EQ.17) THEN
            READ (9,11) (GARB1 (J), J=1, 16),TNUL, (GARB1 (J), J=1, 18), OM1
            ELSE IF (I.EQ.18) THEN
            READ (9,11) (GARB1 (J),J=1, 16),TX, (GARB1 (J), J=1, 18), OM2
        ELSE
            READ (9, 12) (GARB1 (J),J=1, 72)
            END IF
    CONTINUE
    c
C For HHTT species, use 56 atom NARU with NRUC=3 and larger arrays
C For ant1-parallel HTHT, must read atoms of even-numbered chains
C from the opposite end
C
C\#\#\# NRUC = 3
NRUC = 6
DO 120 I=1,NARU*NRUC*4
II = I/(NARU*NRUC+1) + 1
III = I - (II-1)*NARU*NRUC
IF(III.LE.84) THEN
IF(III.LE.168) THEN
READ (9,14,ERR=130,END=130) KIND(III,II),(XYZ(J,III,II),J=1, 3)
ELSE
READ (9, 14, ERR=130, END=130) KGB,XGB,YGB, ZGB
END IF
C\&\&\& IF(MOD(II, 2).EQ.1) THEN
C\&\&\& IF(III.IE.84) THEN
C\&\&\& READ(9,14,ERR=130,END=130) KIND(III,II),(XYZ (J,III,II),J=1, 3)
C\&\&\& ELSE
C\&\&\& READ(9,14,ERR=130,END=130) KGB,XGB,YGB, ZGB
C\&\&\& END IF
C\&\&\& ELSE
C\&\&\& IF(III.LE.84) THEN
C\&\&\& READ (9,14,ERR=130,END=130) KGB,XGB,YGB,ZGB
C\&\&\& ELSE
C\&\&\& IIX = NARU*NRUC-III +1
C\&\&\& READ(9,14,ERR=130,END=130) KIND(IIX,II),(XYZ (J,IIX,II),J=1,3)
C\&\&\& END IF
C\&\&\& END IF
120 CONTINUE
130 CONTINUE
CLOSE( UNIT=9 )
C
C Convert Simulation variables into corresponding vectors:
C
C***
c*** ss(3) = 2.*ss(3)
CALL VECTORIZ( SS,SA,UCT )
12 General Cell Formats:
1) 2 CHAINS, FC, GENERAL; a=1-9,b=1-3
2) 2 CHAINS, FC, GENERAL; a=1-9,b=1-51
3) 2 CHAINS, FC, GENERAL; a=1-9,b=1-35
4) 2 CHAINS, EC, GENERAL; a=1-9,b=1-11
5) 2 CHAINS, EC, GENERAL; a=1-7,b=1-11
6) 2 CHAINS, EC, GENERAL; a=1-9,b=1-15
7) 2 CHAINS, EC, GENERAL; a=1-3,b=1-15
8) 1 CHAIN, TRANS = 0,1/2; a=1-9,b=1-2
9) 1 CHAIN, TRANS = 0,1/2;a=1-7,b=1-2
10) 1 CHAIN, TRANS = 0,1/2;a=1-9,b=1-4
11) 1 CHAIN, TRANS = 0,1/2; a=1-3,b=1-4
12) 1 CHAIN, TRANS = 0,1/2;a=1-2,b=1-4
Calculate unit cell vectors:
DO 200 I=1,3

```
```

    GCT(I,1,1) = UCT(I,1) - UCT(I, 2)
    GCT(I, 2,1) = UCT(I,1) + UCT(I,2)
    GCT(I, 3,1)= UCT(I, 3)
    GCT(I,2,2) = 3.*UCT(I,1) - UCT(I,2)
    GCT(I,2,3) = 3.*UCT(I,2) - UCT(I,1)
    GCT(I,2,4) = 2.*UCT(I,1)
    GCT(I,2,5) = 2.*UCT(I,1)
    GCT(I,2,6)=2.*UCT(I,2)
    GCT(I,2,7) = 2.*UCT(I,2)
    GCT(I,2,8) = UCT (I,1)
    GCT (I, 2,9)=UCT(I,1)
    GCT(I,2,10)=\operatorname{UCT}(I,2)
    GCT(I,2,11)=UCT(I,2)
    GCT(I,1,12) = UCT(I,1)
    GCT(I,2,12)=\operatorname{UCT}(I,2)
    GCT (I, 3,12)=UCT(I, 3)
    DO 200 J=2,11
            IF( (J.EQ.5).OR.(J.EQ.9) ) THEN
            GCT (I, 1,J) = -UCT(I,1) - UCT(I,2)
            ELSE IF ( (J.EQ.7).OR.(J.EQ.11) ) THEN
                GCT(I,1,J) = UCT(I,1) + UCT(I,2)
            ELSE
                GCT(I,1,J) = GCT(I,1,1)
    END IF
    GCT}(I,3,J)=\operatorname{UCT}(I,3
    200
    C
C Calculate unit cell scalars: sidelengths and angles:
C
DO 300 K=1,12
DOT1 = 0.
DOT2 = 0.
DOT3 = 0.
DO 320 J=1,3
GCL (J,K) = SQRT(GCT (1,J,K)**2+GCT (2,J,K)**2 +GCT (3,J,K)**2 )
DOT1 = DOT1 + GCT (J,2,K)*GCT (J, 3,K)
DOT2 = DOT2 + GCT (J, 3,K)*GCT (J,1,K)
DOT3 = DOT3 + GCT (J,1,K)*GCT (J, 2,K)
GCA (1,K) = ACOS( DOT1/GCL (2,K)/GCL (3,K) )/DEGRAD
GCA (2,K) = ACOS( DOT2/GCL (3,K)/GCL (1,K) )/DEGGRAD
GCA (3,K) = ACOS( DOT3/GCL (1,K)/GCL (2,K) )/DEGRAD
300 CONTINUE
C
Calculate potential hydrogen bonding orientations, distances, and
translational registers:
DOT1 = 0.
DOT2 = 0.
DOT3 = 0.
DOT4 = 0.
UCT1M = SQRT( UCT (1,1)**2 + UCT (2,1)**2 )
UCT2M = SQRT( UCT (1,2)**2 + UCT (2,2)**2 )
GCT1M = SQRT( GCT (1,1,1)**2 + GCT (2,1,1)**2 )
GCT2M = SQRT( GCT (1,2,1)**2 + GCT (2,2,1)**2)
D1M = SQRT((UCT (1,2)-GCT(1,1,1))**2\div(UCT (2,2)-GCT (2,1,1))**2)
DO 400 J=1,2
DOT1 = DOT1 + UCT(J,1)*GCT (J,1,1)
DOT2 = DOT2 + UCT (J,1)*GCT (J, 2,1)
DOT3 = DOT3 + UCT (J,1)* (UCT (J, 2)-GCT (J,1,1))
DOT4 = DOT4 + UCT(J,1)*UCT (J,2)
GHYB(1)=OM1 + ACOS( DOT1/UCT1M/GCT1M )/DEGRAD
GHYB(2) = OM1 - ACOS( DOT2/UCT1M/GCT2M )/DEGRAD
GHYB(3) = OM1 - ACOS( DOT3/UCT1M/D1M )/DEGRAD
GHYB(4) = OM1 - ACOS( DOT4/UCT1M/UCT2M )/DEGRAD
ASHFT = SS(1)/SS(3)*COS(SA (2)*DEGRAD)
BSHFT = SS (2)/SS(3)*COS (SA (1)*DEGRAD)
TXM1 = ASHFT - BSHFT
TXM2 = ASHFT + BSHFT
TXM3 = TX - TXM1 + BSHFT
TXM4 = TX + BSHFT
C
C Determine the 'best' unit cel] representation:

```
```

C
6 3 0
600
KCELL = 1
KCOUNT = 0
DO }650I=1,1
IF(KCTST(I).LT.KCOUNT) GO TO 650
KCELL = I
KCOUNT = KCTST(I)
6 5 0 ~ C O N T I N U E ~
660 WRITE (*,*) 'KCELL = ',KCELL
WRITE(*,*) 'DO you agree? [Yes] > '
READ(*,1) RARG
LEN = LNBLNK(RARG)
IF( LEN.GE.1 ) THEN
IF( (RARG(I:1).EQ.'Y').OR.(RARG(1:1).EQ.'Y') ) THEN
ELSE IF( (RARG(1:1).EQ.'N').OR.(RARG(1:1).EQ.'n') ) THEN
WRITE(*,*) 'Input desired cell type (1 - 12): >'
READ(*,*) KF
IF((KF.LT.1).OR.(KF.GT.12)) THEN
WRITE(*,*) 'Invalid cell ID ... '
GO TO 660
ELSE
KCELL = KF
END IF
ELSE
GO TO 660
END IF
ELSE
END IF
C
For selected cell configuration, compute chain setting angles
relative to bc facet.
COX = XYZ (1,12,1)-XYZ (1,11,1)
COY = XYZ (2,12,1)-XYZ (2,11,1)
SETX = GCT (1,1, KCELL)*COX
SETY = GCT (2,1, KCELL)*COY
COM = COX**2 + COY**2
GCTM = GCT (1,1, KCELL )**2 + GCT (2,1, KCELL) **2
SETA1 = ACOS ( (SETX+SETY)/SQRT (COM*GCTM) )
SETA1 = SETA1/DEGRAD
COX = XYZ (1, 12, 2)-XYZ (1,11,2)
COY = XYZ (2,12,2)-XYZ (2,11,2)
COX = XYZ (1,1,2)-XYZ (1,2,2)
C\&\&\&
C\&\&\&}\quadCOY=XYZ (2,1,2)-XYZ (2,2,2)
SETX = GCT (1,1, KCELL)*COX
SETY = GCT (2,1, KCELL) *COY
COM = COX**2 + COY**2
GCTM = GCT (1,1, KCELL)**2 + GCT (2,1,KCELL) **2
SETA2 = ACOS( (SETX+SETY)/SQRT (COM*GCTM) )
SETA2 = SETA2/DEGRAD
COX = XYZ(1,12,1)-XYZ (1,11,1)
COY = XYZ (2,12,1)-XYZ (2,11,1)
SETX = GCT (1,2, KCELL)*COX
SETY = GCT (2,2, KCELL)*COY
COM = COX**2 + COY**2
GCTM = GCT (1,2,KCELL)**2 + GCT (2, 2, KCELL)**2
SETA3 = ACOS ( (SETX+SETY)/SQRT (COM*GCTM) )
SETA3 = SETA3/DEGRAD
COX = XYZ (1,12,2)-XYZ (1,11,2)
COY = XYZ (2,12,2)-XYZ (2,11,2)
COX = XYZ (1,1,2)-XYZ (1,2,2)
C\&\&\&
C\&\&\&}\quadCOY=XYZ(2,1,2)-XYZ (2,2,2

```
```

        SETX = GCT (1, 2,KCELL) *COX
        SETY = GCT(2,2,KCELL)*COY
        COM = COX**2 + COY**2
        GCTM = GCT(1,2,KCELL)**2 + GCT(2,2,KCELL)**2
        SETA4 = ACOS( (SETX+SETY)/SORT (COM*GCTM) )
        SETA4 = SETA4/DEGRAD
    C
C Output cell conversion results:
C
WRITE (10,1) TITLE
WRITE (10,13) KCELL
DO 900 K=1,12
WRITE(10,9) K
WRITE (10,2) GCL (1, K),GCL (2, K),GCL (3,K)
900
WRITE (10,3) GCA.(1, K), GCA (2,K), GCA (3, K)
WRITE (10,*)
WRITE (10,4) GHYB (1), GHYB (2), GHYB (3), GHYB (4)
WRITE(10,8) GCT1M,GCT2M,D1M,UCT2M
WRITE (10,7) TXM1,TXM2,TXM3,TXM4
WRITE (10,5) (OM1-OM2)
WRITE (10,6) TX
WRITE (10,*) ,
WRITE(10,*) 'SET ANGLES REL TO GCT(1): ',SETA1,SETA2
WRITE(10,*) 'SET ANGLES REL TO GCT(2): ',SETA3,SETA4
CLOSE (UNIT=10)
C
C Generate data file for CHARON plotting and X-Ray fiber diagram:
GCA(1,KCELL),TITLE, KXRFD )
FORMAT(' Sidelengths a,b \& c :',3(F7.2) )
FORMAT(' Cell Angles al,be,ga :',3(F7.2))
FORMAT(' 4 Potential H-Bondings:',4(F7.1) )
FORMAT(' Relative Chain Rotate :',F7.1)
FORMAT(' Relative Chain Iransl.:',F7.3 )
FORMAT(' H-Bond Register :',4(F7.3) )
FORMAT(' H-Bond Chain Distance :',4(F7.2) )
FORMAT(' General Cell Type ',I3 )
FORMAT ('' Best Cell Representation: ',I2)
FORMAT (40A1, 3F10.5)
FORMAT (16A1, F9.5,18A1,F8.3)
FORMAT (72A1)
FORMAT (A1, 6X, 3(2X, F10.4))
END
SUBROUTINE UCDRAW( KCELL,KIND,XYZ,VECT,SIDE,ANG,PTITLE,KXR )
C
C Routine to create SCHAKAL88B files for drawing unit cells of PPTA
C simulations; used in confunction with UCANAL, which provides
C parameters necessary for unit cell definition.
C
C *** marks modifications for HHTT isomers
C
CHARACTER*1 KIND (84,4),FKIND (28,6)
c*** CHARACTER*1 KIND (168,4),FKIND (56,6)
CHARACTER*8 FILENAME
CHARACTER*72 GARB72
CHARACTER*64 PTITLE
INTEGER HPLOT
DIMENSION XYZ (3, 84,4),VECT (3,3),\operatorname{PXYZ}(3,28,6),VTX (3,8)
c*** DIMENSION XYZ (3,168,4),VECT (3,3),\operatorname{VXYZ}(3,56,6),VIX (3,8)
DIMENSION SIDE(3),ANG(3), FXYZ (3,28,6)
c*** DIMENSION SIDE (3), ANG (3), FXYZ (3,56,6)
NARU = 28
c*** NARU = 56
C WRITE(*,*) 'INPUT TITLE >'
READ(*,7) PTITLE

```
```

C Calculate unft cell vertices and fractional coordinates
C of unique atoms (2 chains)
C
220
DO 220 I=1,3
VTX(I,1) = ( XYZ(I,NARU+11,1) + XYZ(I,NARU+13,1) )/2.
D13 = 0
D24 = 0.
DO 200 I=1,3
VTX}(I,2)=\operatorname{VTX}(I,1)+\operatorname{VECT}(I,1
VTX(I,3) = VTX(I,2) + VECT(I,2)
VTX(I,4)=\operatorname{VTX}(I,1)+VECT(I,2)
VTX(I,5) = VTX(I,1)
VTX(I,6) = VTX(I,2)
VTX(I,7) = VTX(I, 3)
VTX(I,8) = VTX(I,4)
D13 = D13 + (VTX(I,3)-VTX (I,1))**2
D24 = D24 + (VTX(I,4)-VTX(I,2))**2
DO 250 I=5,8
VTX(3,I) = VTX(3,I) + SIDE(3)
IF (KCELL.LE.3) THEN
ZC = 0.5*( VTX(3,3) + VTX(3,1) )
ELSE IF (KCELL.LE.7) THEN
ZC = 0.5* ( VTX (3,4) + VTX (3,1) )
ELSE IF (KCELL.LE.11) THEN
ZC=VTX(3,4)
ELSE
ZC = VTX (3,2)
END IF
KSECOND = 2
IF( MOD(INT(KCELLL/2),2).EQ.1 ) KSECOND = 4
IF(KCELL.EQ.2) KSECOND = 2
DO 240 I=1,3*NARU
IF (KCELL.NE.2) THEN
ZRSFT = 0.
ELSE
2RSFT = VECT (3,1)
END IF
IF((XYZ(3,I,KSECOND)+2RSFT).GE.ZC) GO TO 260
NC = I - 1
IF ((MOD (NC,NARU).NE.1).AND. (MOD (NC,NARU).NE.15)) GO TO 280
C*** IF ((MOD (NC,NARU).NE.1).AND. (MOD (NC,NARU).NE.15) .and.
C*** + (MOD (NC,NARU).NE.29).AND.(MOD (NC,NARU).NE.43)) GO TO 280
IF ( (ZC-ZRSFT-XYZ (3,NC,KSECOND))/(XYZ (3,NC+1,KSECOND) -
XYZ(3,NC,KSECOND)).LT.0.5) THEN
NC = NC - 1
ELSE
NC = NC + 9
END IF
CONTINUE
DO 270 J=1,NARU
J1 = NARU + 12 + J
J2 = NC + J
FKIND(J,1) = KIND(J1,1)
FKIND(J,2) = KIND(J2,2)
PXYZ(1,J,1) = XYZ (1, J1,1)
PXYZ (2,J,1)= XYZ (2,J1,1)
PXYZ(3,J,1) = XYZ (3,J1,1)
PXYZ (1,J,2) = XYZ (1,J2,KSECOND)
PXYZ(2,J,2)= XYZ (2,J2,KSECOND)
PXYZ (3,J,2)= XYZ(3,J2,KSECOND)
CALL FRCOORD( PXYZ,VECT,VTX (1, 1), 3, 28,6,3,3,28,2,FXYZ)
CALL FRCOORD( PXYZ,VECT,VTX(1,1),3,56,6,3,3,56,2,FXYZ)
C
Generate symmetry related coordinates appropriate to cell specified
IF (KCELL.LE. 3) THEN
NCH = 5
DO 400 I=1,NARU

```
```

            FKIND(I, 3) = FKIND(I, 1)
            FKIND(I,4) = FKIND (I,1)
            FKIND(I,5) = FKIND(I, 1)
            DO 410 J=1,3
                DO 410 K=3,5
                FXYZ(J,I,K)= FXYZ(J,I,I)
            IF(KCELL.EQ.2) FXYZ(1,I,2)=FXYZ(1,I,2) + 1.
            FXYZ(1,I,3)= FXYZ(1,I,3) + 1.
            FXYZ(1,I,4)= FXYZ(1,I,4) + 1.
            FXYZ (2,I,4)=FXYZ (2,I,4) + 1.
            FXYZ(2,I,5)=FXYZ (2,I,5) + 1.
        ELSE IF (KCELL.LE.7) THEN
            NCH}=
        DO 420 I=1,NARU
            FKIND(I, 3) = FKIND(I, 1)
            FKIND(I,4) = FKIND(I,1)
            FKIND(I,5) = FKIND(I,1)
            FKIND(I, 6) = FKIND(I, 2)
            DO 430 J=1,3
                FXYZ(J,I, 6) = FXYZ(J,I, 2)
                DO 430 K=3,5
                FXYZ(J,I,K)= FXYZ(J,I, 1)
            FXYZ(1,I,3)= FXYZ(1,I,3) + 1.
            FXYZ (1,I,4)=FXYZ (1,I,4) + 1.
            FXYZ(2,I,4)=FXYZ (2,I,4) + 1.
            FXYZ (2,I,5)=\operatorname{FXYZ}(2,I,5) + 1.
            FXYZ(1,I, 6)=FXYZ (1,I,6) + 1.
            ELSE IF (KCELL.LE.11) THEN
                NCH=4
            DO 440 I=1,NARU
            FKIND (I, 3) = FKIND (I, 1)
            FKIND(I,4) = FKIND(I,2)
            DO 450 J=1,3
                FXYZ (J,I, 3) = FXYZ (J,I,1)
                FXYZ(J,I,4)= FXYZ(J,I,2)
            FXYZ (1, I, 3) = FXYZ (1,I, 3) + 1.
            FXYZ(1,I,4)= FXYZ(1,I,4) + 1.
        ELSE
            NCH}=
            DO 460 I=1,NARU
            FKIND(I, 3) = FKIND(I,1)
            FKIND(I,4) = FKIND(I, 2)
            DO 470 J=1,3
                FXYZ(J,I, 3)= FXYZ(J,I,1)
                    FXYZ(J,I,4)=FXYZ(J,I,2)
            FXYZ (1,I,3)= FXYZ(1,I,3) + 1.
            FXYZ (2,I,3)= FXYZ (2,I,3) + 1.
            FXYZ(1,I,4)=FXYZ(1,I,4)-1.
            FXYZ(2,I,4)=FXYZ(2,I,4) + 1.
        END IF
    c
C If specified, initiate generation of X-Ray Fiber Diagram:
IF(KXR,EQ.0) CALL STRUFA(NARU,1,2,FKIND, FXYZ,
+ PIITLE,VECT,SIDE,ANG )
OPEN(UNIT=11,FILE='UCDRAW.OUT')
This code is specific for the generating SCHAKAL88 plot files
WRITE(11,3) PTITLE
WRITE (11,10) (SIDE (I),I=1,3),(ANG (I), I=1,3)
MARK = 0
DO 500 N=1,NCH
N=0
510 N = N+1
DO 500 I=1,NARU
MARK = MARK + 1
IF (MARK.LE.9) THEN
WRITE(11,4) FKIND(I,N),MARK,(FXYZ(J,I,N),J=1, 3)
ELSE IF(MARK.LE.99) THEN

```
```

                WRITE(11,5) FKIND(I,N),MARK,(FXYZ(J,I,N),J=1,3)
            ELSE
                            WRITE (11, 6) FKIND(I,N),MARK, (FXYZ (J,I,N),J=1, 3)
                END IF
    CONTINUE
    IF(N.EQ.1) THEN
            WRITE(11,*) 'SYMM X+1,Y,Z'
            WRITE(11,*) 'SYMM X+1,Y+1, Z'
            WRITE (11,*) 'SYMM X,Y+1,Z'
            IF(NCH.GT.4) GO TO 510
        ELSE IF(NCH.EQ.6) THEN
            WRITE(11,*) 'SYMM X+1,Y,Z'
        ELSE
        END IF
        WRITE (11,9)
        CLOSE( UNIT=11 )
    C
FORMAT(A1, 7X, 3(2X,F10.4))
FORMAT('TITLE ',A64)
FORMAT('ATOM ',A1, I1,5X, 3(F8.4, 2X))
FORMAT ('ATOM ',A1,I2,4X,3(F8,4,2X))
FORMAT('ATOM ',A1, 13, 3X,3(F8.4, 2X))
FORMAT (A64)
FORMAT (A72)
FORMAT('END')
FORMAT('CELL ',6(1X,F6.2) )
END
SUBROUTINE FRCOORD( CXYZ,UCV,ORIG;ND,NA,NM,NU,N1,N2,N3,FXYZ )
C
C Converts coordinates from Cartesian to Fractional:
C ND x NA x NM is the size of the array containing the Cartesian
C coordinates: first dimension is coordinate, second dimension is
C atoms in a molecule, third dimension is number of molecules.
C N1 x N2 x N3 is the in-use portion of the array.
C UCV is the unit cell lattice vector array, having phsyical dimensions
C NU x NU. ORIG(i) is the Cartesian coordinate of the origin
C
C
DIMENSION CXYZ (ND,NA,NM),FXYZ (ND,NA,NM),UCV(NU,NU),ORIG(NU),
DIMENSION CXYZ (3,28,6),FXYZ (3,28,6),UCV(3,3),ORIG(3),
c*** DIMENSION CXYZ (3,56,6),FXYZ (3,56,6), UCV (3,3),ORIG(3),
+

```

```

            DO 100. K=1,N3
            DO 100 J=1,N2
                DO 100 I=1,3
                    FXYZ (I,J,K)=0.
    c
C Compute inverse lattice vector matrix:
DO 200 I=1,3
DO 200 J=1,3
UCVT (J,I) = UCV (J,I)
DO 220 I=1,3
DO 210 J=1,3
UCVTI (I,J)=0.
UCVTI(I,I) = 1.
CALL LUDCMP( UCVT, 3, 3,INDX,D )
DO 230 J=1,3
CALL LUBKSB(UCVT, 3, 3,INDX,OCVTI (1,J) )
230
C
C Convert to fractional coordinates:
DO 300 K = 1,N3
DO }300\textrm{J}=1,\textrm{N}
DO }310\textrm{I}=1,
CSHFT(I) = CXYZ(I,J,K) - ORIG(I)
CALL MV( UCVTI, 3, 3, CSHFT, 3, FXYZ (1,J,K) )
RETURN
END

```
```

    SUBROUTINE STRUFA( NARU,NRUC,NCH,FKIND,FXYZ,
    + TITLE,VECT,SIDE,ANG)
    Program generates theoretical WAXS data for stuctures predicted
by Polymer Crystal Model program. The required input is the
Number of Atoms per Repeat Unit, Number of Repeat Units generated per
Chain, the Number of chains per cell, the x-ray WaveLength, the range
of H,K,L of interest, and the identity (atomic number) and cartesiar.
coordinates of each atom in the unit cell, the "primary axes" and
"primary interaxial angles" as defined within PCM program.
Also used are the "half-beta angle" (Lorentz factor calculation as
per Wolff [Alexander, "X-Ray Diffraction Methods in Polymer Science",
p41]) and the Bragg angle for monochromator (polarization factor
calculation [Alexander, p40]).
The program calculates the conventional crystallographic axes/angles,
determines the type of unit cell, converts to fractional coordinates,
and calculates the d spacing, 2*theta angie, beta angle, polarization
factor, Lorentz factor, structure factor, and reflection intensity for
each (H,K,L) of interest.
If a fiber diagram is requested, the user is prompted for plotting
information required by subroutine STIPLOT (for Laserjet printing)
or GOURPLOT (for display on IRIS 4D-GT) (via PLORG)
INTEGER H
LOGICAL FRST
CHARACTER*64 TITIE,RARG
CHARACTER*1 FKIND (28,6)
CHARACTER*1 FKIND (56,6)
INTEGER*2 LNBLNK
PARAMETER (ERSILON=1E-6,NSPT=1000)
COMMON /VECT/ CRAX(3,3),RLV (3,3),VOL
COMMON /PEAKS/ NS,TWOTH (NSPT), BETA(NSPT),AMP (NSPT)
COMMON /UNICOM/ IOPORT,MODEL,XCORNER,YCORNER,AXL,TTM,ARCRESOL,
+
DIMENSION CXM(3), CAN (3), RHO (3), VECT (3, 3),SIDE (3), ANG (3)
DIMENSION KIND (28,2),FXYZ (3,28,6)
DIMENSION KIND (56,2), FXYZ (3,56,6)
DIMENSION D (-6:6,-6:6,-6:6), BTA (-6:6,-6:6,-6:6)
C*** DIMENSION D(-9:9,-9:9,-9:9), BTA (-9:9,-9:9,-9:9)
DIMENSION TTA (-6:6,-6:6,-6:6),F(-6:6,-6:6,-6:6)
c*** DIMENSION TTA(-9:9,-9:9,-9:9),F(-9:9,-9:9,-9:9)
DIMENSION P(-6:6,-6:6,-6:6), 2(-6:6,-6:6,-6:6),E(-6:6,-6:6,-6:6)
C*** DIMENSION P(-9:9,-9:9,-9:9),Z(-9:9,-9:9,-9:9),E(-9:9,-9:9,-9:9)
DIMENSION MILLER(3,NSPT)
PI = 3.1415926535
BEE = 5.
FRST = .TRUE.
DO 100 I=1,3
CXM(I) = SIDE(I)
CAN(I) = ANG(I)*PI/180.
DO 100 J=1,3
CRAX}(I,J)=\operatorname{VECT}(I,J
DO 120 I=1,2
DO }120\textrm{J}=1\mathrm{ ,NARU
IF(FKIND(J,I).EQ.'C') THEN
KIND(J,I) = 6
ELSE IF(FKIND(J,I).EQ.'H') THEN
KIND (J,I) = 1
ELSE IF(FKIND(J,I).EQ.'N') THEN
KIND (J,I) = 7
ELSE IF(FKIND (J,I).EQ.'O') THEN
KIND(J,I) = 8
ELSE IF(FKIND(J,I).EQ.'L') THEN
KIND (J,I) = 17
ELSE
END IF
CONTINUE
C

```
```

        OPEN (UNIT=11, FILE='SF.RES')
        OPEN (UNIT=12, FILE='SF.PLT')
    200 WRITE(*,*) 'Do you to plot a fiber diagram? [NO] > '
READ(*;7) RARG
LEN = LNBLNK(RARG)
IF( LEN.GE.1 ) THEN
IF( (RARG(1:1).EQ.'Y').OR.(RARG(1:1).EQ.'Y') ) THEN
KPLOT = 1
ELSE IF((RARG(1:1).EQ.'N').OR.(RARG(1:1).EQ.'n') ) THEN
KPLOT = 0
ELSE
GO TO 200
END IF
ELSE
KPLOT = 0
END IF
WRITE(*,*) 'Input Wavelength, MO, and MX > '
READ (*,*) WVL,MO,MX
WRITE(*,*) 'Input Beta orientation and monochromator angles >'
READ (*,*) BETAHAF,CHROM
TEE = 0.815*BETAHAF*PI/180.
CHROM = CHROM*PI/180.
Generate reciprocal lattice vectors:
CALL LATCON
Determine Space Group (1=Cubic, 2=Tetragonal, 3=Orthorhombic,
4=Hexagonal, 5=Monoclinic, 6=Triclinic) symmetry:
KSG = 1
IF( (CXM(1).NE.CXM(2)).OR. (CXM(1).NE.CXM(3)).OR.
(CXM(2).NE.CXM(3)), KSG = 2
IF((CXM(1).NE.CXM(2)).AND. (CXM(1).NE.CXM(3)).AND.
(CXM(2).NE.CXM(3))) KSG = 3
IF( ABS(CAN(3)-PI/2.).GT.EPSILON ) THEN
IF( ABS(CAN(3)-2.*PI/3.).LE.EPSILON ) THEN
KSG = 4
ELSE
KSG = 5
END IF
ELSE
END IF
IF( (ABS (CAN (1)-PI/2.).GT.EPSILON).OR. (ABS (CAN (2)-PI/2.).GT.
+
EHA = 0.
N = 1
WRITE(*,*) 'Calculating Intensities...'
DO 500 H=MO,MX
DO 500 K=MO,MX
DO 500 L=0,MX
IF( (H.EQ.O).AND.(K.EQ.O).AND.(L.EQ.O) ) GO TO 500
RH = REAL (H)
RK = REAL (K)
RL = REAL (L)
A = 0.
B=0.
DET1 = (RH/CXM(1) +COS(CAN (1))*COS(CAN (2))*RK/CXM (2) +
COS(CAN (3))*COS (CAN (1))*RL/CXM (3))-(RH/CXM (1)*
COS (CAN (1))**2+\operatorname{COS}(\operatorname{CAN}(3))*RK/CXM (2) +
COS(CAN (2))*RL/CXM (3))
DET2 = (RK/CXM (2)+\operatorname{COS (CAN (2))* COS (CAN (3))*RL/CXM (3) +}
COS(CAN (1))*COS (CAN (2))*RH/CXM(1))-(RK/CXM (2)*
COS (CAN (2))**2+COS (CAN (1))*RL/CXM (3) +
COS (CAN (3))\starRH/CXM(1))
DET3 = (RL/CXM (3) +COS (CAN (3))*COS (CAN (1))*RH/CXM(1)+
COS (CAN (2))*COS (CAN (3))*RK/CXM (2))-(RL/CXM (3)*
COS (CAN (3))**2+\operatorname{COS}(\operatorname{CAN}(2))*RH/CXM(1)+
COS (CAN (1))*RK/CXM(2))
DET4 = (1.+2.* COS (CAN (1))* COS (CAN (2))*COS (CAN (3)))-

```
```

                    (\operatorname{COS}(\operatorname{CAN}(1))**2+\operatorname{COS}(\operatorname{CAN}(2))**2+\operatorname{COS}(\operatorname{CAN}(3))**2)
            D(H,K,I) = 1./(SQRT( (RH*DET1/CXM(1)+RK*DET2/CXM(2)+RL*DET3/
                    CXM(3))/DET4))
            IF( (REAL (N)*WVL/2).GT.D(H,K,L) ) GO TO 500
            ORD = REAL (N)/2./D (H,K,L)
            TTA (H,K,L) = ASIN( WVL*ORD )
    c
C Sum over all atoms in the unit cell:
C
DO 550 NCN=1,NCH
DO }550\mathrm{ NAT=1,NAFU
FATM = ATSCAT (KIND (NAT,NCN),ORD)
ARG=RH*FXYZ (1,NAT,NCN) +RK*FXYZ (2,NAT,NCN) +
RL*FXYZ (3,NAT,NCN)
A = A + FATM* COS (2.*PI*ARG)
B=B + FATM*SIN(2.*PI*ARG)
CONTINUE
F(H,K,L)=SQRT(A**2 + B**2)
C
Calculate Lorentz Factor (paratropic, general, or diatropic)
and azimuthal angle of reflection beta:
IF (L.EQ.0) THEN
Z(H,K,L) = 1./((SIN(TTA (H,K,L)))**2*COS(TTA (H,K,L)) )
BTA (H,K,L) = 0.
ELSE
RHOM = 0.
PDOT = 0.
DO 700 I=1,3
RHO(I) = RH*RLV (I, 1) +RK*RLV (I, 2) +
RL*RLV(I,3)
PDOT = PDOT + RHO(I)*CRAX (I,3)
7 0 0
RHOM = RHOM + RHO(I)**2
PARG = PDOT/CXM(3)/SQRT (RHOM)
IF (ABS (PARG-1.).LT.EPSILON) PARG = 1.
PHI = ACOS( PARG )
BTA(H,K,L) = 90. - PHI*180./PI
IF( (H.NE.O).OR.(K.NE.O) ) THEN
Z(H,K,L) = 1./((SIN(TTA (H,K,L)))**2* COS (TTA (H,K,L))
*SIN(PHI))
ELSE
Z(H,K,L) = 1./((SIN(TTA (H,K,L)))**2*COS (TTA (H,K,L))*TEE)
END IF
END IF
P(H,K,L) = (1.+(COS (CHROM))**2*(COS (2.*TTA (H,K,L)))**2)/
(1.+(COS (CHROM))**2)
THRM = -2.*BEE*ORD**2
E(H,K,L) = F(H,K,L)**2*P(H,K,L)*Z(H,K,L)*EXP(THRM)
C
C

```
```

C Additional intensity correction (2x) required for equatorial and

```
C Additional intensity correction (2x) required for equatorial and
C azimuthal reflections due to reflection distribution among half as
C azimuthal reflections due to reflection distribution among half as
C many spots in the Fiber Diagram Representation, and again (2x) for
C many spots in the Fiber Diagram Representation, and again (2x) for
all reflections L > 0, since these have been omitted from the loops
all reflections L > 0, since these have been omitted from the loops
C based on symmerty. Net: Azimuthal reflections doubled...
C based on symmerty. Net: Azimuthal reflections doubled...
```

    IF((90.-BTA (H,K,L)).LE.IO*EPSILON) E(H,K,L) = 2.*E(H,K,L)
    ```
    IF((90.-BTA (H,K,L)).LE.IO*EPSILON) E(H,K,L) = 2.*E(H,K,L)
    EHA = MAX( EHA,E(H,K,L) )
    EHA = MAX( EHA,E(H,K,L) )
5 0 0 ~ C O N T I N U E ~
5 0 0 ~ C O N T I N U E ~
    NS = 0
    NS = 0
    DO }650\textrm{L}=\textrm{MX},0,-
    DO }650\textrm{L}=\textrm{MX},0,-
    DO 650 K=MX,MO,-1
    DO 650 K=MX,MO,-1
    DO 650 H=MX,MO,-1
    DO 650 H=MX,MO,-1
        NS = NS + 1
        NS = NS + 1
        MILLER(1,NS) = H
        MILLER(1,NS) = H
        MILLER (2,NS) = K
        MILLER (2,NS) = K
        MILLER(3,NS) = L
        MILLER(3,NS) = L
        TWOTH(NS) = 2.*TTA(H,K,L)*180./PI
        TWOTH(NS) = 2.*TTA(H,K,L)*180./PI
        BETA(NS) = BTA (H,K,L)
        BETA(NS) = BTA (H,K,L)
        AMP(NS) = E(H,K,L)
        AMP(NS) = E(H,K,L)
        IF( (AMP (NS)/EHA).LT.0.00125) THEN
        IF( (AMP (NS)/EHA).LT.0.00125) THEN
            NS = NS - 1
```

            NS = NS - 1
    ```
```

            GO TO 650
        ELSE
        END IF
        NSS = NS - 1
        IF( NS.EQ.1 ) GO TO 650
        DO 660 NCMP = 1,NSS
            IF ( (ABS (TWOTH (NS) -TWOTH (NCMP)).LT.1E-3).AND.
                        (ABS (BETA(NS)-BETA (NCMP)).LT.1E-3) ) THEN
                AMP (NCMP) = AMP (NCMP) +AMP (NS)
                NS = NSS
                GO TO 650
            ELSE
            END IF
        CONTINUE
    CONTINUE
    EHIGH = 0.
    DO 600 N=1,NS
        EHIGH = MAX( EHIGH,AMP (N) )
    WRITE (11, 14)
    WRITE(11,20) KSG
    WRITE (11,15) (CXM(I),I=1,3)
    WRITE(11,16) ((CAN(I)*180./PI), I=1,3)
    WRITE(11,18) EHIGH
    WRITE (12,6) TITLE
    WRITE (11,11)
    DO 610 N=1,NS
        AMP (N) = AMP (N)/EHIGH*100.
        IF( AMP(N).GT.0.25) THEN
        H=MILLER(1,N)
        K=MILLER (2,N)
        L = MILLER (3,N)
    WRITE(12,13) B, K,L,P(H,K,L), Z(H, K,L),F(H,K,L), AMP (N)
        WRITE (11, 17) H, K, L, TWOTH (N) , BETA (N), AMP (N)
        WRITE (12, 17) H, K, L, TWOTH (N) , BETA (N), AMP (N)
        ELSE
        END IF
    CONTINUE
    C
C Plot if desired:
ENDFILE(12)
REWIND (12)
IF( KPLOT.GT.O ) CALL PLORG
C
CLOSE (UNIT=11)
CLOSE (UNIT=12)
FORMAT( 3I4,F8:4,2I4 )
FORMAT( 3F8.4)
FORMAT( I4,3F8.4 )
FORMAT( 2F8.4)
FORMAT( A72 )
FORMAT( 1X,A72 )
FORMAT( A64)
FORMAT( 1X,'Theoretical Structure Factors F(H,K,L)' )
FORMAT( 1X,3('F(',II,',',II,',',II,') ',F9.3,3X) )
FORMAT( 1X,'H,K,L:M,P,Z,F,E ',3I2,I4,4F9.3 )
FORMAT( 1X,'Crystallographic Unit Cell Parameters')
FORMAT( 1X,'Sidelengths: ',3F8.3 )
FORMAT( IX,'Internal Angles: ',3F8.3)
FORMAT( 1X,3I3,3F9.3 )
FORMAT( 1x,'Max Absolute Intensity: ',ElO.3)
FORMAT( 1X,'Space Group Symmetry: ',I2 )
END

```
```

Reformats simulation-generated peaks (discrete intensities at
various 2*theta,beta) to a 2*theta,beta grid of intensity
appropriate for STIPLOT (for Laserjet output) or GOURPLOT (for IRIS
4D-GT display)
Reflections are converted from Dirac spikes of intensity I to
Gaussian distributions through 2 dimensions (beta and two*theta)
having volumes equal to I. The peak shape and resolution to
low intensity are controlled by WB and WT (peak widths at 50%
amplitude) and CO (cut-off intensity fraction, 0 < CO < 1 )
Gaussian: Y = A* exp( - ln2 * ((X - P)/W)**2 )
Peaks are mapped onto a grid having resolution RB degrees in beta
and RT degrees in two*theta

```
    PARAMETER ( NMAX \(=1001\), NSPT=1000)
    COMMON /DPLOT/ NT, NB, TGO, TGX, BGO, BGX, 2 (NMAX, NMAX)
    COMMON /PEAKS/ NS, TWOTH (NSPT), BETA (NSPT), AMP (NSPT)
    WRITE (*,*) ' ,
    WRITE(*,*) 'Input Gaussian Widths (degrees): Two*Theta,Beta >'
    READ (*,*) WT, WB
    WRITE (*,*) 'Input Gaussian cutoff \% (between 0 and 1) >'
    READ(*,*) CO
    WRITE(*,*) 'Input Diagram grid stepsize (2T,B) (degrees) >'
    READ (*, *) RT, RB
    \(\mathrm{PI}=3.1415926535\)
    DELBT \(=\) WB*SQRT( LOG (1./CO)/LOG(2.) )
    DELTT \(=\) WT*SQRT( LOG(1./CO)/LOG(2.) )
    DO \(100 \mathrm{~J}=1\), MMAX
        DO \(100 I=1\), NMAX
            \(2(I, J)=0\).
    \(T G O=10\).
    \(\mathrm{TGX}=0\).
    \(\mathrm{BGO}=0\).
    \(\mathrm{BGX}=0\).
    DO \(200 \mathrm{I}=1\),NS
        IF ( AMP (I).LT. 0.01 ) GO TO 200
        AMPMAX \(=\) MAX ( AMPMAX, AMP (I) )
        \(\mathrm{TGO}=\mathrm{MIN}(\mathrm{TGO}, \mathrm{TWOTH}(\mathrm{I})\) )
        TGX \(=\operatorname{MAX}(T G X, T W O T H(I))\)
        \(\operatorname{BGO}=\operatorname{MIN}(\operatorname{BGO}, \operatorname{BETA}(\mathrm{I}))\)
        \(\operatorname{BGX}=\operatorname{MAX}(\mathrm{BGX}, \operatorname{BETA}(\mathrm{I}))\)
    continue
    SCI \(=\) AMPMAX*4.*LOG(2.)/PI/WB/WT/100.
    TGX \(=\) MIN ( TGX,50. )
    TGO \(=\operatorname{MAX}(T G O, 0\).
    WRITE(*,*) ' WARNING: Two Theta limit in PLORG is 50 degrees'
    \(B G X=90\).
    \(B G O=\operatorname{MAX}(B G O, 0\).
    NT \(=\operatorname{NINT}((\operatorname{TGX}-\mathrm{TGO}) / \mathrm{RT})+1\)
    \(\mathrm{NB}=\operatorname{NINT}((\mathrm{BGX}-\mathrm{BGO}) / \mathrm{RB})+1\)
    NSZB \(=\) NINT ( DELBT/RB )
    NSZT = NINT (DELTT/RT)
    WRITE(*,*) ' Number of Peaks Predicted : ',NS
    WRITE(*,*) ' Calculated Grid Size NTxNB : ',NT,NB
    WRITE(*,*) ' Calculated Peak Grid Size NSZTxNSZB: ',NSZT,NSZB
    DO \(500 \mathrm{I}=1, \mathrm{NS}\)
    AG \(=\) AMP \((I) * 4 . \star\) LOG \((2.) / W B / W T / P I / S C L\)
    KTT \(=\operatorname{NINT}(\operatorname{REAL}(N T) *(\) TWOTH \((I)-T G O) /(T G X-T G O) ;)+1\)
    \(\operatorname{KBT}=\operatorname{NINT}(\operatorname{REAL}(N B) *(B E T A(I)-B G O) /(B G X-B G O))+1\)
    DO 520 JB=1,NSZB
        JBG \(=\mathrm{KBT}-\mathrm{NINT}(\operatorname{REAL}(\mathrm{NS} 2 \mathrm{~B}) / 1.99)+\mathrm{JB}\)
        IF ( JBG.LE.O ) GO TO 520
        IF ( JBG.gT. MMAX ) GO TO 520
        DBSQ \(=(\operatorname{REAL}(J B G-K B T) * R B) * * 2\)
        BFUNC \(=\operatorname{EXP}(-\operatorname{LOG}(2:) * 4 . * D B S Q / W B * * 2)\)
        DO \(540 \mathrm{JT}=1, \mathrm{NSZT}\)
            JTG \(=\) KTT - NINT(REAL(NSZT)/1.99) + JT
```

                    IF( JTG.LE.O ) GO TO 540
                    IF( JTG.GT.NMAX ) GO TO 540
                    DTSQ = ( REAL (JTG-KTT)*RT )**2
                    TFUNC = EXP( -LOG(2.)*4.*DTSQ/WT**2 )
                    Z(JTG, JBG) =Z (JTG,JBG) +BFUNC*TFUNC*AG
                CONTINUE
        CONTINUE
    CONTINUE
    C
XRC.DAT is the calculated intensity grid file for later comparison
to experiment
OREN( UNIT=10,FILE='XRC.DAT')
WRITE(10,101) TGO,TGX,NT
WRITE (10,101) BGO, BGX,NB
DO 400 J=1,NB
DO 400 I=1,NT,10
IP = MIN (NT,I+9)
WRITE (10,102) (Z (K,J),K=I,IP)
CONTINUE
FORMAT (1X, 2F8.2,I4)
FORMAT (10(1X,F7.2))
CLOSE( UNIT=10)
C
C CALL STIPLOT
CALL GOURPLOT
RETURN
END
SUBROUTINE GOURPLOT
C This routine is modified for use with Gouraud shading display on
c the IRIS workstation
C Generates a stipled-pattern fiber diagram from regulary-gridded
C X-ray data in 2*theta vs beta format. The diagram appearance and
C sensitivity may be adjusted by means of ZCH, ZCL, and NREF; ZCH is
C the upper critical percent intensity, above which the diagram is
C saturated, while ZCL is the lower critical percent intensity, below
c which intensity is not recorded. Between these two intensities
C stipling is controlled by a cubic spline function of raw intensity.
C NREF is a reference linear dot density at half the radius of the
plot.
Generates all four quadrants
Data passed in common block DPLOT
CHARACTER*72 TITLE
CHARACTER*1 QTITLE(72)
EQUIVALENCE (OTITLE(1),TITLE)
PARAMETER (NMAX=1001)
COMMON /DPLOT/ NT,NB,TGO,TGX, BGO,BGX, Z (NMAX,NMAX)
DIMENSION X (NMAX, NMAX), Y (NMAX, NMAX)
DIMENSION KCC, ZCLK (8), ZCHK (8), JSTIP (4)
DIMENSION CDEF(3,8),ZCDF(8), ZKEY(8)
DEGRAD = 3.1415926535/180.
c
DEFINE PLOTTING PARAMETERS:

```
```

WRITE(*,*) ' Enter TITLE of Plot > '

```
WRITE(*,*) ' Enter TITLE of Plot > '
READ(*,3) TITLE
READ(*,3) TITLE
WRITE(*,*) ' Enter Origin of Window (X,Y) (pixels) >'
WRITE(*,*) ' Enter Origin of Window (X,Y) (pixels) >'
READ (*,*) XWO, YWO
READ (*,*) XWO, YWO
WRITE(*,*) ' Enter Window Size (pixels, square) > '
WRITE(*,*) ' Enter Window Size (pixels, square) > '
READ (*,*) WSL
READ (*,*) WSL
WRITE(*,*) ' Enter (Fractional) Origin of plot in Window (X,Y) > '
WRITE(*,*) ' Enter (Fractional) Origin of plot in Window (X,Y) > '
READ (*,*) XCO,YCO
READ (*,*) XCO,YCO
    WRITE(*,*) ' Enter Desired Axis Length (Fractional) > '
    WRITE(*,*) ' Enter Desired Axis Length (Fractional) > '
    READ(*,*) AXL2
    READ(*,*) AXL2
    WRITE(*,*) ' Enter Maximum Two*Theta (degrees) > '
```

    WRITE(*,*) ' Enter Maximum Two*Theta (degrees) > '
    ```
```

    READ(*,*) TTM
    WRITE(*,*) ' Enter Palette Color Code, ZCL AND ZCH >.'
    READ(*,*) KCC, 2CL, 2CH
    C
ZMAX = -1.E6
2MIN = 1.E6
DO 100 J=1,NB
DO }100\textrm{I}=1,N
ZMAX = MAX( 2MAX,Z (I,J) )
ZMIN = MIN( ZMIN,Z(I,J) )
100 CONTINUE
IGXA = MIN( TGX,TTM )
TSTP = (TGXA-TGO)/(NT-1)
BSTP = (BGX-BGO)/(NB-1)
WRITE(*,*) ' 2*Theta Range : ',TGO,' to ',TTM
WRITE(*,*) ' Beta Range : ',BGO,' to ',BGX
WRITE(*,*) ' Intensity Range: ', ZMIN,' to ', zMAX
C
DO 200 J=0,NB-1
DO 200 I=0,NT-1
ZLOC = ( Z (I+1,J+1)-2MIN )/(ZMAX-ZMIN)
TLOC = TGO + REAL(I)*TSTP
BLOC = BGO + REAL.(J)*BSTP
XLOC = TLOC*COS( BLOC*DEGRAD )
YLOC = TLOC*SIN( BLOC*DEGRAD )
Z(NT+I,NB+J) = ZLOC
X(NT+I,NB+J) = XLOC
Y(NT+I,NB+J) = YLOC
IF( J.NE.O) THEN
Z(NT+I,NB-J)=ZLOC
X(NT+I,NB-J) = XLOC
Y(NT+I,NB-J) = -YLOC
END IF
200 CONTINUE
NB = 2*NB - 1
DO 250 J=1,NB
DO 250 I=1,NT-1
Z(NT-I,J) = Z(I+NT,J)
X(NT-I,J) = -X(I+NT,J)
Y(NT-I,J) = Y(I+NT,J)
CONTINUE
NT = 2*NT - 1
ZCL = 2CL/100.
2CH = 2CH/100
ZDIF = 2CH - 2CL
ZDF2 = ZDIF**2
ZDF3 = ZDF2*ZDIF
ZCL2 = ZCL**2
AF = -2./2DF3
BF = 6.*2CL/2DF3 + 3./2DF2
CF = -6.*(ZCL2/ZDF3 + ZCL/ZDF2)
DF = 2.*2CL2*ZCL/ZDF3 +3.*ZCL2/2DF2
C
Define specific points on the color scale:
CALL PALETTE( KCC,NCOL,CDEF,2KEY )
DO }610\mathrm{ NC=1,NCOL
ZZ = 2KEY(NC)*100.
ZCDF(NC) = (ZZ-ZCL)/ZDIF
ZCDF (NC) = 2CDF (NC)/100.
6 1 0 CONTINUE
C
C Renormalize grid values with new sensitivity range:
C Loop over grid points:
c
KPRN = 10
DO }700 I=1,N
DO }700\textrm{J}=1,N
IF( Z(J,I).LE.ZCL ) THEN
Z(J,I) = 0.

```
```

            ELSE IF( 2(J,I).GE.2CH ) THEN
                Z(J,I)=1.
            ELSE
                ZZ = Z(J,I)
                    Z(J,I) = (ZZ - ZCL)/ZDIF
            END IF
            CONTINUE
            CALL XRSHADE( XWO, YWO,WSL, XCO, YCO,AXL2, KCC,NT,NB,
                                    X,Y,Z,QTITLE, NCOL, ZKEY, 2CDF, CDEF )
    RETURN

```
SUBROUTINE XRSHADE ( XWO, YWO,WSL, XCO, YCO, CSL, NPL,
+ NPH,NPV,XP,YP, ZP, OTITLE, NCOL, ZKEY, ZCDF, CDEF )
```

Given a rectangular array of values ( $\mathrm{XP}, \mathrm{YP}, \mathrm{ZP}$ ), creates a x-ray fiber diagram using Gouraud shading technique
INTEGER*4 KXO, KYO, KXE, KYE
INTEGER*4 WINOPE
CHARACTER* 72 CHARS
CHARACTER*I QTITLE(72),TLAB1,TLAB2,TLAB3,TLAB4,TLAB5
PARAMETER (NMAX=1001)
REAL BGD (3), CNO (3), $\operatorname{PVTX}(3,4), \operatorname{VTXC}(3,4)$
REAL $\operatorname{CDEF}(3,8), \operatorname{ZCDF}(8), \operatorname{ZKEY}(8)$
REAL XP (NMAX, NMAX) , YP (NMAX, NMAX), ZP (NMAX, NMAX)
REAL $\operatorname{XAX}(2,2), \operatorname{YAX}(2,2), \operatorname{XTIC}(4,15), \operatorname{YTIC}(4,15)$
C
C First, set array to be "colored" and normalize:

```
```

        ZPMAX = -1E6
        2PMIN = 1E6
        XPMAX = -1E6
        XPMIN = 1E6
    ~ YPMAX = -1E6
        YPMIN = 1E6
    C
Renormalize data array (range = 0 - 1):
C
DO 100 J=1,NPV
DO 110 K=1,NPH
ZPMAX = MAX (ZPMAX, 2P (K,J))
ZPMIN = MIN (ZPMIN, ZP (K,J))
XPMAX = MAX(XPMAX,XP (K,J))
XPMIN = MIN (XPMIN,XP(K,J))
YPMAX = MAX(YPMAX,YP(K,J))
YPMIN = MIN(YPMIN,YP(K,J))
CONTINUE
100 CONTINUE
IF(ZPMIN.LT.O.) WRITE(*,*) "ZPMIN less than zero; corrected"
DO 120 J=1,NPV
DO 120 I=1,NPH
IF( ZPMIN.LT.O. ) THEN
ZP(I,J) = (ZP (I,J)-ZPMIN)/(ZPMAX-ZPMIN)
ELSE
ZP(I,J) = ZP(I,J)/ZPMAX
END IF
120 CONTINUE
WRITE(*,*) ' ZPMAX, ZPMIN: ', ZPMAX, ZPMIN
WRITE (*,*) ' XPMAX, XPMIN: ',XPMAX, XPMIN
WRITE (*,*) 'YPMAX, YPMIN: ',YPMAX,YPMIN
C
C Determine axis construction:
C
TICDIST = 10
XCE = XCO + CSL
YCE = YCO + CSL
XAX(1,1) = XCO
XAX(2,1)= YCO + CSL/2.
XAX(1,2) = XCE
XAX (2,2)= XAX (2,1)
YAX(1,1)=XCO + CSL/2.
YAX(2,1) = YCO
YAX(1,2)= YAX(1,1)
YAX (2,2) = YCE
NXTIC = NINT((XPMAX-XPMIN)/TICDIST) + 1
NYTIC = NINT((YPMAX-YPMIN)/TICDIST) + 1
DO 200 I=1,NXTIC
XTIC(1,I) = XCO + REAL(I-1)/REAL (NXTIC-1)*CSL
XIIC (2,I) = XAX(2,1) - CSL/50.
XTIC(3,I)= XTIC(1,I)
200 XTIC (4,I) = XAX (2,1) + CSL/50.
DO 220 I=1,NYTIC
YTIC(1,I) = YAX(1,1) - CSL/50.
YTIC(2,I) = YCO + REAL(I-1)/REAL (NYTIC-1)*CSL
YTIC(3,I) = YAX(1,1) + CSL/50.
YTIC(4,I) = YTIC (2,I)
DO 230 I=1,72
J = 73-I
IF(QTITLE(J).NE.' ') GO TO 240
230 CONTINUE
240 LENT = J
C
C Define background and notation colors:
backgound corresponds to zero intensity;
notation corresponds to complement thereof
DO 250 I=1,3
BGD(I) = CDEF(I,1)
CNO(I) = 0.3*(1. - BGD(I))
250 continue

```
```

C
C Open window and transform coordinates; define picture size and
C configure monitor:
C
WRITE(*,*) 'Ready to set up plot'
KXO = INT (XWO)
KXE = INT (XWO + WSL)
KYO = INT (YWO)
KYE = INT (YWO + WSL)
CALL PREFPO( KXO,KXE,KYO, KYE )
CALL NOBORD
STATUS = WINOPE( "COLARRAY PROGRAM",20)
CALL ORTHO2( 0.,1.,0.,1. )
CALL RGBMOD
CALL GCONFI
CALL C3F (BGD)
CALL CLEAR
C
C Loop over all polygons:
NPHM1 = NPH - 1
NPVM1 = NPV - 1
XSCL = CSL/ (XPMAX - XPMIN)
YSCL = CSL/(YPMAX - YPMIN)
DO }300\textrm{I}=1,\mathrm{ NPHM1
DO 300 J=1,NPVM1
C
C Test for shading required:
IF( (ZP(I,J).LE.ZCDF(1)).AND.(ZP(I+1,J).LE.ZCDF (1)).AND.
+ (ZP(I,J+1).LE.ZCDF(1)).AND.(ZP(I+1,J+1).LE.ZCDF(1)) )
+ GO TO 300
C
C Assign colors to the four vertices:
C
DO 400 NC=1,NCOL-1
IF( ZP(I,J).GE.ZCDF(NC) )
NC1 = NC
IF( 2P(I+1,J).GE.ZCDF(NC) )
NC2 = NC
IF( ZP(I+1,J+1).GE.ZCDF(NC) )
NC3 = NC
IF( 2P(I,J+1).GE.2CDF(NC) )
NC4 = NC
CONTINUE
C
PAR1 = (ZP (I, J) -ZCDF (NC1))/(ZCDF (NC1+1)-ZCDF (NC1))
PAR2 = (ZP (I+1,J)-2CDF (NC2))/(ZCDF (NC2+1)-2CDF (NC2))
PAR3 = (ZP (I+1,J+1)-2CDF (NC3))/(ZCDF (NC3+1)-2CDF (NC3))
PAR4 = (2P(I,J+1)-2CDF (NC4))/(ZCDF (NC4+1)-2CDF (NC4))
DO 450 K=1,3
VTXC (K,1) = CDEF (K,NC1) +PAR1* (CDEF (K,NC1+1)-\operatorname{CDEF}(K,NC1))
VTXC (K,2) = CDEF (K,NC2) +PAR2* (CDEF (K,NC2+1)-CDEF (K,NC2))
VTXC (K,3) = CDEF (K,NC3) +PAR3* (CDEF (K,NC3+1) - CDEF (K,NC3))
VTXC (K,4) = CDEF (K,NC4) +PAR4* (CDEF (K,NC4+1)-CDEF (K,NC4))
CONTINUE
c
Place this polygon:
PVTX(1,1) = (XF (I,J) - XPMIN)*XSCL + XCO
PVTX(2,1) = (YP(I,J) - YPMIN)*YSCL + YCO
PVTX(1,2) = (XP (I+1,J) - XPMIN)*XSCL + XCO
PVTX(2,2)=(YP(I+1,J)-YPMIN)*YSCL + YCO
PVTX(1,3) = (XP(I+1,J+1) - XPMIN)*XSCL + XCO
PVTX (2,3) = (YP (I+1,J+1) - YPMIN)*YSCL + YCO
PVTX(1,4) = (XP (I,J+1) - XPMIN)*XSCL + XCO
PVTX(2,4) = (YP(I,J+1) - YPMIN)*YSCL + YCO
C
C Color Polygon (Gouraud Shading)

```
    CALL BGNPOL()
```

            CALL C3F(VTXC(1,1))
            CALL V2F(PVTX (1,1))
            CALL C3F(VTXC (1,2))
            CALL V2F(PVTX(1,2))
            CALL C3F(VTXC(1,3))
            CALL V2F(PVTX(1,3))
            CALL C3F(VTXC(1,4))
            CALL V2F(PVTX(1,4))
            CALL ENDPOL()
    300 contINUE
    c
c Draw Equatorial Axis
C
CALL C3F(CNO)
CALL BGNLIN
CALL V2F(XAX(1,1))
CALL V2F(XAX (1,2))
CALL ENDLIN
DO 500 I=1,NXTIC
CALL BGNLIN
CALL V2F(XTIC(1,I))
CALL V2F(XTIC(3,I))
CALL ENDLIN
500 CONTINUE
c
C Draw Azimuthal Axis
C
CALL BGNLIN
CALL V2F(YAX(1,1))
CALL V2F(YAX(1,2))
CALL ENDLIN
DO }520 I=1,NYTIC
CALL BGNLIN
CALL V2F(YTIC(I,I))
CALL V2F(YTIC(3,I))
CALL ENDLIN
520 CONTINUE
c
Label Upper Half of Azimuth
c
NLAB = (NYTIC - 1)/2
DO 530 I=1,NLAB
J = I + NYTIC - NLAB
PLAB1 = YTIC (3,J) + CSL/100.
PLAB2 = YTIC (4,J)
TLAB1 = CHAR(I+ICHAR('O'))
TLAB2 = '0'
CALL CMOV2( PLAB1,PLAB2 )
CALL CHARST( TlAB1,1 )
CALL CHARST( TLAB2,1)
530 CONTINUE
C
Draw Title
PLAB1 = XCO + CSL/2. - REAL(LENT/2)*(CSL/WSL)*25.
PLAB2 = YCE + 25./WSL
CALL CMOV2( PLAB1, PLAB2 )
DO 540 I=1,LENT
CALL CHARST( QTITLE(I),1 )
540 continue
c
C Create Key
c
PLAB1 = XCE - 60.*CSL/WSL - 60*CSL/WSL
PLAB2 = YCE - 25.*CSL/WSL
DO 600 I=2,NCOL
CALL C3F(CNO)
CALL CMOV2( PLAB1,PLAB2 )
TKEY = ZKEY(I)*100.
IF( TKEY.GE.100.) THEN
KEY = INT (TKEY)

```
```

        KEY1 = KEY/100
        KEY2 = (KEY-KEY1*100)/10
        KEY3 = KEY-KEY1*100-KEY2*10
        ELSE IF( TKEY.GE.10. ) THEN
            KEY = INT(TKEY)
            KEY1 = ICHAR(' ') - ICHAR('O')
            KEY2 = KEY/10
            KEY3 = KEY-KEY2*10
            ElSE IF( TKEY.GE.1.) THEN
            KEY1 = ICHAR(' ') - ICHAR('0')
            KEY2 = ICHAR(' ,) - ICHAR('O')
            KEY3 = INT(TKEY)
        ElSE
            KEY1 = INT(TKEY)
            KEY2 = ICHAR('.') - ICHAR('0')
            KEY3 = INT(TKEY*10.) - KEY1*10
            END IF
            TLAB1 = CHAR(KEY1 +ICHAR('0'))
            TLAB2 = CHAR(KEY2 +ICHAR('O'))
            TLAB3 = CHAR(KEY3+ICHAR('0'))
            TLAB4 = '%'
            TLAB5 = ',
            CALL CMOV2( PLAB1,PLAB2 )
            CALL CHARST( TLAB1,1 )
            CALL CHARST( TlAB2,1)
            CALL Charst( TlAB3,1)
            CALL CHARST( TlAB4,1)
            CALL ChARST( TlAB5,1)
            PKXLO = PLAB1 + 110.*CSL/WSL
            PKYLO = PLAB2
            PKXHI = PKXLO + 60.*CSL/WSL
            PKYHI = PKYLO + 9.*CSL/WSL
            CALL C3F(CDEF(1,I))
            CALL RECTF( PKXLO,PKYLO,PKXHI,PKYHI )
            PLAB2 = PLAB2 - 40.*CSL/WSL
    6 0 0 ~ C O N T I N U E ~
    C
C Wait 10 seconds (or pause indefinitely)
CALL SLEEP(10)
pause
C and exit
END
SUBROUTINE PALETTE( NPL,N,A,Z )
REAL A(3,8),2(8)
c
DO 75 J=1,4
Z(J) = 0.
DO 75 I=1,3
A(I,J) =0.
75 CONTINUE
GO TO ( 10,20,30,40,50,60,70, 80,90,100,110,120,130,140,
+ 150) NPL
C Black to White:
10 continue
N = 2
Z(2) = 1.
DO }11\textrm{I}=1,
A(I,2) = 1.
11 CONTINUE
GO TO 9999
C Black to Red:
20 CONTINUE
N = 2
z(2) = 1.
A(1,2) = 1.
GO TO 9999
C Black to Green:
30 CONTINUE
N = 2

```
```

        Z(2) = 1.
        A(2,2) = 1.
        GO TO 9999
    C Black to Blue:
    40 CONTINUE
        N = 2
        Z(2) = 1.
        A(3,2)=1.
        GO TO 9999
    C Red to White:
    50 CONTINUE
        N = 2
        Z(2)=1.
        A(1,1) = 1.
        DO 51 I=1,3
            A(I,2) = 1.
        GO TO 9999
    C Green to White:
    60 CONTINUE
        N = 2
        Z(2) = 1.
        A(2,1) = 1.
        DO 61 I=1,3
    61 A(I,2) = 1.
        GO TO 9999
    C Blue to White:
    70 CONTINUE
        N = 2
        Z(2) = 1
        A(3,1)=1.
        DO 71 I=1,3
            A(I,2)=1.
        GO TO 9999
    C Blue to Red:
    80 CONTINUE
        N = 2
        Z(2) = 1.
        A(3,1) = 1.
        A(1,2)=1.
        GO TO 9999
    C Cyan to Magenta:
    90 CONTINUE
        N = 2
        Z(2) = 1.
        A(2,1) = 1.
        A(3,1) = 1.
        A(1,2) = 1.
        A(3,2)=1.
        GO TO 9999
    C Black to Blue to Cyan to Green to Yellow to Red to Magenta to White
        100 CONTINUE
            N = 8
            DO 101 I=2,N
    101 2(I) = REAL(I-1)/REAL (N-1)
        A(3,2) = 1.
        A(3,3) = 1.
        A(2,3) = 1.
        A(2,4) = 1.
        A(2,5) = 1.
        A(1,5) = 1.
        A(I,6)=1.
        A(1,7) = 1.
        A(3,7) = 1.
        A(1,8) = 1.
        A(2,8)=1.
        A(3,8) = 1.
        GO TO 9999
    C Variable distribution of colors: black-grayl-gray2-gray3-white
        110 CONTINUE
            N = 5
            WRITE(*,*) '5 GRAYS: DEFINE INTENSITY LEVELS (0 - 1) >'
    ```
```

        READ(*,*) (Z(I),I=1,5)
        IF(Z(1).GT.0.) 2(1)=0.
        DO 111 I=1,3
        A(I,2) = 0.25
        A(I,3) =0.50
        A(I,4) =0.75
        A(I,5) = 1.
    111 CONTINUE
        GO TO 9999
    C Variable distribution of colors: black-redi-red2-red3-red4
120 CONTINUE
N = 5
WRITE(*,*) '5 REDS: DEFINE INTENSITY LEVELS (0 - 1) >'
READ(*,*) (Z(I), I=1,5)
IF(Z(1).GT.0.) Z(1) = 0.
A(1,2)=0.25
A(.1,3) =0.50
A(1,4) =0.75
A(1,5) = 1.
GO TO 9999
C Variable distribution of colors: black-red-magenta-yellow-white
130 CONTINUE
N = 5
WRITE(*,*) '5 COLORS: DEFINE INTENSITY LEVELS (0 - 1) >'
READ (*,*) (Z(I),I=1,5)
IF(Z(1).GT.0.) Z(1) = 0.
A(1,2) = 1.
A(1,3) = 1.
A(3,3) = 1.
A(1,4) = 1.
A(2,4)=1.
A(1,5) = 1.
A(2,5)=1.
A(3,5) = 1.
GO TO 9999
C Variable distribution of colors: black-blue-cyan-yellow-white
140 CONTINUE
N = 5
WRITE(*,*) '5 COLORS: DEFINE INTENSITY LEVELS (0 - 1) >'
READ (*,*) (Z (I), I=1,5)
IF(Z(1).GT.0.) Z(1)=0.
A(3,2) = 1.
A(2,3)=1.
A(3,3)=1.
A(1,4)=1.
A(2,4)=1.
A(1,5)=1.
A(2,5) = 1.
A(3,5)=1.
GO TO 9999
C
C Variable distribution of colors: black-blue-cyan-yellow-white
150 CONTINUE
N = 5
WRITE(*,*) '5 COLORS: DEFINE INTENSITY LEVELS (0 - 1) >'
READ(*,*) (Z(I), I=1,N)
IF(Z(1).GT.0.) Z(1) = 0.
A(1,1)=.25
A(2,1)=.25
A(3,1) =. .25
A(3,2)=.75
A(1,3)=.8
A(2,3) = . 5
A(3,3)=.85
A(2,4) = 1.
A(3,4)=A(3,3)
A(1,5) = 1.
A(2,5) = A(2,4)
A(3,5) = A(3,4)
GO TO 9999
C

```
```

9999 CONTINUE
RETURN
END
SUBROUTINE VECTORIZ( SLEN,SANG,VECT )
C
C Converts scalar data (lengths, angles) to three placement vectors
C Convention: SLEN(3) lies along Z axis
LEN(1) lies within X-Z plane
SLEN(2) placement determined by above
SANG(1) is the argle between SLEN(j) and SLEN(k), etc.
DIMENSION SLEN (3),SANG (3),VECT (3, 3), ARA (3,3), ARB (3,3)
COMPL(X) = (90.-X)*3.1415926535/180.
DEGRAD = 3.1415926535/180.
DO }100\textrm{I}=1,
DO 100 J=1,3
ARA(I,J) = 0.
ARB (I,J) = 0.
VECT(I,J) = 0.
VECT(1,1) = SLEN(1)
VECT(1,2) = SLEN(2)
VECT (3,3) = SLEN(3)
COSGP = ( COS (SANG (3)*DEGRAD) - COS (SANG (1)*DEGRAD)*COS (SANG (2)
*DEGRAD) )/( SIN(SANG(1)*DEGRAD)*SIN(SANG(2)*DEGRAD) )
GP = ACOS (COSGP)
CA = COMPL(SANG (1))
CB = COMPL(SANG(2))
ARA(1,1) = COS(CB)
ARA(1,3)=-SIN(CB)
ARA (2,2) = 1.
ARA(3,1) = SIN(CB)
ARA (3,3) = COS (CB)
ARB(1,1) = COSGP*COS(CA)
ARB (1,2) = -SIN(GP)
ARB(1,3) = -COSGP*SIN(CA)
ARB (2,1) = SIN (GP)*COS(CA)
ARB (2,2) = COSGP
ARB(2,3) = -SIN(GP)*SIN(CA)
ARB (3,1) = SIN(CA)
ARB (3,3) = COS (CA)
CALL MV ( ARA, 3, 3,VECT (1, 1), 3,VECT (1,1) )
CALL MV( ARB, 3,3,VECT (1, 2), 3,VECT (1,2) )
RETURN
END
SUBROUTINE LATCON
C This subroutine generate the unit cell volume and reciprocal lattice
C vectors from unit cell vectors. Cell vectors should be defined such
C that the c axis coincides with the z axis.
C
COMMON /VECT/ CRAX (3,3), RLV (3,3),VOL
DIMENSION TEMP (3)
CALL VV (CRAX (1, 2), CRAX (1,3),TEMP,3)
VOL = 0.
DO 100 I=1,3
VOL = CRAX(I,I)*TEMP(I) + VOL
DO 300 J=1,3
JP1 = MOD (J,3) +1
JP2 = MOD( (J+1),3 ) +1
CALL VV(CRAX (1,J), CRAX (1,JP1),TEMP, 3)
DO 200 I=1,3
RLV(I,JP2) = TEMP (I)/VOL
CONTINUE
RETURN
END

```
```

        SUBROUTINE VV(A,B,C,N )
    C
C Calculates the vector cross product C =A x B, where A,B, and C
C have length N
DIMENSION A(N),B(N),C(N),K(2)
DO }100\textrm{I}=1,\textrm{N
DO 200 J=1,2
K(J)=MOD((I+J-1),N ) +1
C(I) = A(K(1))*B(K(2))-A(K(2))*B(K(1))
CONTINUE
RETURN
END
SUBROUTINE MV ( A,NFORMA,NA,V,N,R )
C
Multiplies matrix and vector: R = A * V
Input: A NA x N matrix
NFORMA First dimension of A in calling program
NA Number of rows in A
V Vector of length N
N Length of vector }
Output: R Resultant vector of length NA, can be A
DIMENSION A(NFORMA,NFORMA), V(N), R(NA), RR(20)
IF (NA.GT.NFORMA) RETURN
DO }100\textrm{I}=1,N
X=0.
DO 200 K=1,N
X = X + A(I,K)*V(K)
RR(I) = X
DO 300 I=1,NA
300 R(I) = RR(I)
RETURN
END
SUBROUTINE MM( A,B,C,NFORMA, NFORMB,NFORMC,NA,MA,MB )
C
- Multiplies matrix with matrix: C = A X B
(C can be A or B)
DIMENSION CC (20,20),A(NFORMA, NFORMA),B (NFORMB,NFORMB)
DIMENSION C(NFORMC,NFORMC)
DO }100I=1,N
DO 100 J=1,MB
X=0.
DO 200 K=1,MA
X=X + A(I,K)*B(K,J)
200
DO 300 I=1,NA
DO 300 J=1,MB
300 C(I,J)=CC(I,J)
RETURN
END
FUNCTION ATSCAT( KA,X )
C
C Calculates atomic scattering factor, given atom type and ordinate
C function sin(theta)/lambda, where theta is scattering angle and
C lambda is the incident x-ray wavelength. the scattoring factors
C are calculated from a best fit curve of the form:
F(X)=P(1)*EXP (-P(2)*X**2) + P(3)*EXP(-P(4)*X**2) + P(5)
Applied to values provided by L.E. Alexander, 'X-ray Diffraction
Methods in Polymer Science', Wiley-Interscience,1969,p533, taken
In turn from 'International Tables for X-ray Crystallography',
Volume III, p202.

```
```

C
C
C
C
C
C
C
C
C
C
ordinate range = 0.0-1.0
weighting factor = exp( - (x-0.5)**2 )
Data in P array correspond to: P(1,1) = carbon
P(1,2) = hydrogen
P(1,3) = nitrogen
P(1,4) = oxygen
P(1,5) = [empty]
P(1,6) = chlorine
P(1,7) = bromine
DIMENSION P(5,7)
DATA P/1.248,2.517,3.678,23.747,1.044,
+
+
+
+
+
0.3916,7.483,0.5958,30.072,0.0107,
1.778,3.836,3.938,19.226,1.260,
2.836,4.423,3.773,17.229,1.373,
0.,0.,0.,0.,0.,
8.066 ,0.9734,7.983,22.093,0.9458,
19.47 ,2.192,8.390 ,27.804,7.083/
Convert atomic numbers to array identification:
IF( KA.EQ.6 ) THEN
K=1
ELSE IF( KA.EQ.1 ) THEN
K=2
ELSE IF( KA.EQ.7 ) THEN
K=3
ELSE IF( KA.EQ.8 ) THEN
K=4
ELSE IF( KA.EQ.9 ) THEN
K=5
ELSE IF( KA.EQ.17) THEN
K = 6
ELSE IF(KA.EQ.35) THEN
K = 7
ELSE
WRITE(*,*) 'ERROR: Atomic Scattering Curve Not Available'
END IF
ATSCAT=P(1,K)*EXP (-P (2,K)*X**2) +P(3,K)*EXP (-P (4,K)*X** 2) +P (5,K)
RETURN
END

```
    SUBROUTINE LUBKSB ( \(\mathrm{A}, \mathrm{N}, \mathrm{NP}\), INDX, B )
    DIMENSION A(NP,NP), INDX(N), B(N)
    \(I I=0\)
    DO \(12 \mathrm{I}=1, \mathrm{~N}\)
        \(L L=I N D X(I)\)
        SUM=B(LL)
        \(B(L L)=B(I)\)
        IF (II.NE.0)THEN
            DO \(11 \mathrm{~J}=I I, I-1\)
            SUM \(=\) SUM \(-A(I, J)\) *B (J)
            CONTINUE
        ELSE IF (SUM.NE.O.) THEN
            \(I I=I\)
        ENDIF
        \(B(I)=S U M\)
    CONTINUE
    DO \(14 \mathrm{I}=\mathrm{N}, 1,-1\)
        SUM=B (I)
        IF (I.LT.N) THEN
            DO \(13 \mathrm{~J}=\mathrm{I}+1\), N
                \(\operatorname{SUM}=\operatorname{SUM}-A(I, J) * B(J)\)
            CONTINUE
        ENDIF
        \(B(I)=S U M / A(I, I)\)
        CONTINUE
        RETURN
        END
```

    SUBROUTINE LUDCMP (A,N,NF,INDX,D)
    PARAMETER (NMAX=100,TINY=1.OE-20)
    DIMENSION A(NP,NP),INDX(N),VV(NMAX)
    D=1.
    DO 12 I=1,N
    AAMAX=0.
    DO 11 J=1,N
        IF (ABS (A(I,J)).GT.AAMAX) AAMAX=ABS (A (I,J))
    continue
    IF (AAMAX.EQ.O.) PAUSE 'Singular matrix.'
    VV(I)=1./AAMAX
    CONTINUE
DO 19 J=1,N
IF (J.GT.1) THEN
DO 14 I=1,J-1
SUM=A(I,J)
IF (I.GT.1)THEN
DO 13 K=1,I-1
SUM=SUM-A (I,K)*A (K,J)
CONTINUE
A(I,J)=SUM
ENDIF
CONTINUE
ENDIF
AAMAX=0.
DO 16 I=J,N
SUM=A (I,J)
IF (J.GT.1)THEN
DO 15 K=1,J-1
SUM=SUM-A(I,K)*A(K,J)
continue
A(I,J)=SUM
ENDIF
DUM=VV(I)*ABS (SUM)
IF (DUM.GE.AAMAX) THEN
IMAX=I
AAMAX=DUM
ENDIF
CONTINUE
If (J.NE.IMAX)THEN
DO 17 K=1,N
DUM=A (IMAX,K)
A(IMAX,K)=A (J,K)
A(J,K)=DUM
continue
D=-D
VV (IMAX) =VV (J)
ENDIF
INDX(J)=IMAX
IF(J.NE.N)THEN
IF(A(J,J).EQ.O.)A(J,J)=TINY
DUM=1./A(J,J)
DO 18 I=J+1,N
A(I,J)=A(I,J) *DUM
CONTINUE
ENDIF
continue
IF (A (N,N).EQ.O.)A(N,N)=TINY
RETURN
END

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[^0]:    ${ }^{\text {a }}$ values suggested by Hagler, Huler, and Lifson [1974].

