MATERIAL EVALUATION AND SELECTION PROCESSES TO ENABLE DESIGN FOR MANUFACTURE

By

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Abstract

In order to optimize product designs it is necessary to quickly evaluate many candidate materials in terms of performance and processing costs. Evaluation using physical prototypes yields concrete results but is time intensive and costly when dealing with multiple optimization objectives. As an alternative, computer aided simulation is a reliable means of material evaluation and selection, is increasingly available to smaller companies due to the shrinking cost of computation, and is essential for handling the dual optimization objectives of manufacturability and performance in a timely and cost effective manner. To support this thesis, the author first examines iRobot Corporation's current process of experimental trial and error for evaluating and selecting a polymer material for use in the wheels of its robotic military vehicles. The author then demonstrates that the experimental derived performance results can be reasonably predicted using the viscoelastic properties of polymers, as captured in such models as the standard linear solid model, and that this predictability can be used to quickly simulate wheel performance with computer aided engineering (CAE) tools. Finally, the author performs a cost analysis of the current material evaluation/selection process versus the CAE approach to show the best path forward for incorporating CAE tools into the design process of smaller corporations like iRobot.

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Chapter 1: Introduction and Overview

Companies that design product must develop a design process. Inherent in this development is a decision on where to place the prototyping versus modeling boundary. This decision is one of how to best utilize both experimentation through physical prototyping and that of analytical modeling [1,2]; it has significant ramifications on a company's ability to rapidly and cost effectively optimize designs and therefore impacts competitiveness in terms of price, quality, and time to market. The degree of impact varies as a company progresses through different stages of growth and maturation, so the boundary location itself must also be dynamic and the decision on where to place it must be reassessed continually.

On one end of the spectrum is design through prototyping where all determination of performance is made through physical mock-ups of the product. This approach is advantageous in that it provides concrete performance results for a specified design concept and given set of operating conditions, and can require low resource investment if the complexity of the design optimization problem is low. For example, if designers are only optimizing for one objective, it can be less costly in terms of engineering time and materials to optimize through iterative prototyping. However, resource investment can grow rapidly as additional objectives are added to the optimization problem, making it difficult to rigorously determine if a design is truly optimal.

On the other end of the spectrum is design through modeling and simulation[i] where determination of performance is predicted through computer aided engineering (CAE) tools. Using CAE tools, engineering time remains low with increasing optimization complexity. This allows for rapid optimization given multiple objectives and for a more rigorous determination of global design optima. The disadvantages of design through modeling include the relatively large capital investment required to purchase and operate the necessary software tools and the fact that modeling can never fully eliminate the need to prototype. At a minimum, confirmation prototyping is still required to establish concrete results for the final design.

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[i] The literature sometimes refers to analytical models as "analytical prototypes" but the author will not do so in this text. Rather, the author will reserve the term prototype to refer only to physical mock-ups of a product.

Small companies in the start-up phase often wind up on the prototyping end of the spectrum. Such companies are often concerned about survival and as such are predominantly worried about making a product that works in the face of an unstable customer base. In this environment prototyping is viable as the optimization space contains only one objective (performance) and modeling tools are an investment that can only be justified based on payoffs from future products, designs, or orders that may never come.

As a company succeeds and grows out of the start-up phase, the prototyping vs. modeling boundary must be pushed out to include more modeling if the company is to remain competitive. This is because success brings with it a more stable customer base demanding more product with increasing quality and lower costs. This increases the complexity of the optimization space. Products must not only work but manufacturability becomes a larger concern. Modeling allows for cost effective optimization given these multiple demands. Additionally, the stability of the customer base ensures that investments in modeling tools can be recouped in future products and designs.

iRobot, specifically the Small Unmanned Ground Vehicle (SUGV) group of the Government and Industrial Robots (G&IR) division, is at such a crossroads where the boundary between prototyping and modeling must be reevaluated.

1.1 iRobot Government and Industrial Robots Division Overview and Background

The flagship product of the G&IR division is the PackBot, shown in Figure 1. The PackBot traces its roots back to 1997 when the then seven year old iRobot bid on the DARPA "Point Man" Broad Agency Announcement (BAA). This BAA was looking for a robotic scout vehicle for use primarily in an urban combat environment. iRobot won the contract and developed a tracked vehicle platform. The original contract went through several iterations, resulting in numerous versions of the vehicle ranging from what was called the Urbie to the PackBot Scout.

Shortly after the issuing of the "Point Man" BAA, the British government issued a call for proposals under a project named ICECAP. The purpose of this project was to develop a next generation explosive ordinance disposal (EOD) robot for use in the British military. iRobot responded, leveraging the work done for the "Point Man" BAA. The idea was to simply develop

EOD payloads for the tracked vehicle platform already in existence. iRobot won the contract but was ultimately unsuccessful in satisfying the ICECAP requirements. However, the work done resulted in what was to become the PackBot EOD variant.

Figure 1 - PackBot Explorer

Following the lead of the British, the U.S. government issued a call for proposals for a next generation EOD robot of their own under the MTRS contract. iRobot responded to this call, leveraging the PackBot EOD development. Near simultaneously, the U.S. government issued an emergency needs contract to immediately supply EOD robots to the troops fighting the war on terror in Afghanistan and Iraq. Again, iRobot responded by leveraging the PackBot EOD development. iRobot won a substantial piece of both contracts and deployed several PackBot EODs to combat theaters and is currently producing more.

Finally, feedback on the performance of the PackBot Scout, originally developed back as part of the "Point Man" BAA, led to the design of the PackBot Explorer variant, which is not yet in production.

Throughout the PackBot development the design process relied heavily on iterative prototyping. The project was living contract to contract and lacked the stable customer base and product demand that would require reliably repeatable production at low cost. Performance was the dominant metric. The product did perform, rather successfully, and this success would necessitate a change in the design process boundary between prototyping and modeling.

1.2 Project Motivation

The success of the PackBot was largely responsible for iRobot securing, in 2004, an estimated $32 million contract to deliver a SUGV to the Army as part of its Future Combat Systems (FCS) initiative [3]. The importance of this contract to iRobot cannot be understated. It allowed iRobot to grow its G&IR engineering department by over 15% in 2004 alone; propelled iRobot to a tier one defensive supplier with the likes of Honeywell, Lockheed Martin, General Dynamics, and Raytheon; and will help fund the growth of infrastructure necessary to bid and win large defense contracts in the future.

Success on the SUGV contract is critical to iRobot's future growth, and depends, in part, on the ability to meet aggressive cost targets. SUGV, as what one could consider a next generation PackBot, needs to be about half the price of the most comparable PackBot variant and requires a paradigm shift away from a design process emphasizing functionality alone, to one that balances functionality and manufacturability.

In general, the problem becomes one of pushing the design process boundary between prototyping and modeling out to include more complex modeling and simulation so as to allow designers to quickly and reliably optimize designs given the multiple optimization objectives of performance and manufacturability. A subset of this problem is analyzing the applicability of modeling and simulation processes for evaluating and selecting materials for use in various parts. Computer aided simulation is a reliable means of material evaluation and selection, is increasingly available to smaller companies transitioning out of the start-up stage of maturation due to the shrinking cost of computation, and is essential for optimizing across multiple objectives in a timely and cost effective manner

To support this thesis, the author begins in Chapter 2 by examining the current process of iterative prototyping for evaluating and selecting a polymer material for use in the wheels of the SUGV. The author then demonstrates that the experimental derived performance results can be reasonably predicted using the viscoelastic properties of polymers, as captured in such models as the standard linear solid model, and that this predictability can be used to quickly simulate wheel performance with CAE tools. In Chapter 3, the author performs a cost analysis of the current material evaluation/selection process versus the CAE approach to show the best path forward for incorporating CAE tools into the design process of younger corporations, like iRobot. Finally, the author concludes in Chapter 4 with potential areas of further research.

Chapter 2: Material Evaluation and Selection for Wheel Design – A Case Study

2.1 Wheel Design Overview

The SUGV is a track driven vehicle with two drive wheels and two idler wheels. These wheels support and propel the tracks, but also act as principal shock absorbers in the event of shock loads. Shock absorption is important because of the sensitive electronic and optical payloads and the rugged military operating environment. Installation of a dedicated shock absorbing system is impractical because of weight and space constraints coupled with operating conditions that could impose shock loads from any arbitrary direction. As a result, the bulk of the shock absorption task is left to the wheels. This places the wheels in an interesting design space. Ideally they should be stiff so as to support the driving load, efficiently transmit torque from the axles to the tracks, and absorb large amounts of energy for a given deflection. However, in order to reduce deceleration forces on the robot under impact, the wheels should be ductile so as to allow relatively large deflections without fracture of permanent deformation.

These design conflicts place the wheel in a difficult middle ground of impact problems. On one end there is the sacrificial cage problem. In this case the cage must absorb large amounts of energy over a relatively large deflection distance in order to preserve the contents. If a driver drives his car into a wall at 60 miles per hour and walks away, he accepts that he can't reuse his car. On the other end is the problem of the indestructible water bottle. If someone drops their water bottle, they want it to survive and be reusable but care not about "preserving" the water inside. In the middle, the SUGV wheels must absorb significant amounts of energy over large deflections in order to protect the SUGV payloads, but they must also survive without fracture or significant permanent deformation so that the robot can drive away after impact.

This middle ground, combined with the stringent weight requirements of the SUGV, restricts the wheels to the polymer class of materials, as shown in Figure 2. In this figure, Young's modulus is plotted versus elongation (both normalized by density). Good materials, i.e. those that are both stiff (high Young's modulus) and ductile (high elongation) for their weight,

are found in the upper right quadrant. Figure 2 reveals that it is the polymer class of materials that resides in the upper right quadrant.

Figure 2 - Specific Stiffness vs. Specific Elongation

Part performance (P) in all applications, including impact, is a function of functional requirements (F), geometry (G) and material (M) [4]:

$$P = f(F,G,M) \, .$$

If material is restricted to one class of materials, then only a certain subset of geometries is available and that subset of geometries is typically available to all materials in the class. In this case, the factors of the part performance equation become independent. An optimal geometry for a particular application tends to be optimal regardless of the material used within the class. (i.e. an I-beam has a larger bending stiffness to weight ratio than a comparable rectangular beam regardless of whether the beams are made of nylon, polyethylene, etc.). Similarly, properties of

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the materials within the class tend to affect performance independent of geometry. For example, nylon has a larger specific stiffness than polyethylene regardless of its shape. This independence that exists when the material class is restricted makes the complex middle ground of impact problems in which the SUGV wheel lies a bit less inhospitable. Since the material class is restricted to polymers, designers can optimize geometry and material selection separately. Thus, if g represents the number of possible geometries and m represents the number of possible materials within the polymer class, the number of combinations to search in seeking an optimal design grows with $g + m$ rather than $g * m$.

2.2 Legacy Design Approach – Experimental Trial and Error

The PackBot wheel served as a baseline design for the SUGV. Wheel design for the PackBot relied heavily on iterative prototyping, or what the author terms experimental trial and error. Wheels of varying geometries and materials were prototyped and underwent a drop test to determine their performance. While the independence property of part performance greatly reduces the number of combinations to be tested ($g + m << g * m$ if g and m are large), practically speaking, $g + m$ is prohibitively too large in its own right to systematically search for an optimum through trial and error. As a result, designers were time constrained to simply tweak whatever functional design combination they stumbled upon, potentially resulting in a local optima. After months of effort, the resulting design is one that functions well but relies on the machining of cast polymers, bringing the cost per wheel up to the order of $100.

Moving forward, SUGV designers believed $100 per wheel was likely to be a local optima and therefore an opportunity to significantly reduce costs through redesign. Both geometry and material were to be reinvestigated; however, due to the independent nature of these two factors, as discussed in section 2.1, the author will focus solely on materials for now. In terms of material selection, the design methodology remained unchanged from the PackBot. The experimental hunt and peck search was simply reopened, focusing on previously untried polymer materials with less expensive processing techniques (i.e. injection moldable). Ironically, this methodology arose, in part, from a lack of time necessary to truly understand the physics behind the dynamics of wheel impact. In the long run, though, the experimental trial and error approach is so time consuming that the resources invested in an alternative material search could very

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easily outweigh the return through production cost savings. An alternative process for material evaluation and selection is necessary.

2.3 Desired Future Design Approach – Prediction Modeling

A better design approach involves performance prediction. It would be far more efficient if given a set of easily obtainable inputs, designers could accurately predict wheel performance without any prototyping. Intuition leads us to believe that this is possible as there ought to be a constitutive relationship between some set of physical properties and performance. As a check of this assertion, an easy first step is to attempt to experimentally determine a prediction model.

2.3.1 Building a Prediction Model through Experiments

The goal is to build a prediction model that uses a set of material properties to explain impact performance. The best data collection strategy to do so would use designed experiments to systematically vary the material property inputs and observe the resulting performance [5]. Such a strategy is not practical in this case as it assumes that the material property inputs to the prediction model can be independently controlled. Rather, the only independent choice available is material, which dictates a certain combination of strongly correlated properties. Therefore, performance data is unplanned relative to material property input factors and any resulting prediction model is thus subject to many potential pitfalls in terms of the usefulness of the results [6,7]. Despite its shortcomings, a prediction model using material properties still provides some understanding of which properties contribute significantly to impact performance and the manner in which they do so. The first step in building such a prediction model is defining the material data set, the response variable, and the input factors.

2.3.1.1 Defining Material Data Set, Response Variables, and Input Factors

Eight different materials were tested, as outlined in Table 1.

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Material	Type	Manufacturing Process
A	Type 6 Nylon	Casting
B	Type 6/12 Nylon copolymer	Casting
C	Thermoplastic polyester elastomer	Injection Molding
D	Polycarbonate	Injection Molding
E	Polyetherimide	Injection Molding
F	TPU - Polyether (Urethane)	Injection Molding
G	Toughened Type 6/6 Nylon	Injection Molding
H	Thermoplastic polyester elastomer	Injection Molding

Table 1 - Material Data Set

Materials C through H were selected because they are representative of the full range of impact performance seen in the new polymers tested. Additionally, materials A and B were included as representative of a class of polymers that performed well in PackBot designs in order to provide a performance baseline against which to compare new materials under consideration.

The response variable is taken as the failure height (in inches) of the prototype wheels in a specified drop test. For this test the prototype wheels were affixed to a 14 lbs. slide hammer which is mounted to a six foot rail. This setup allows the wheels to drop onto an anvil from heights varying from zero to 70 inches. The test apparatus is show in Figure 3.

Figure 3 - Drop Test Fixture with Prototype Wheel

Two wheels of each material were dropped every two inches from six inches to 70 inches (or until catastrophic failure) and the failure height was recorded for each sample. The wheels were not rotated between drops, but rather were allowed to impact on the same spot each drop. Failure is defined as either 1) at least one crack or tear all the way through the rim or any spoke, 2) permanent deformation of 5/32" or more as measured along the radius, or 3) bottoming out on the urethane stop collar. These failure criteria are motivated by three failure modes for wheels on an actual robot:

1. Material failure, either through fracture or plastic deformation.
2. Failing to absorb/dissipate enough energy prior to the robot chassis bottoming out.
3. Decelerating too quickly (i.e. material is too stiff), thereby transmitting too large a shock load to the robot's payloads.

In practice, the last failure mode is not a concern because the material class is limited to polymers. Extremely stiff polymers tend to shatter before exceeding a tolerable deceleration level. Therefore, failure criteria one and two are designed to capture the first failure mode. Failure criteria three partially addresses the second failure mode and eliminates artificially low permanent deformation numbers at higher drop heights. It is important to note that at this stage in the discussion the author is not attempting to correlate these failure criteria with any sort of absolute measure of adequate performance on an actual robot. Rather, the criteria were chosen to get an adequate measure of relative performance among the materials tested. Using the specified testing procedure and failure criteria, the response variable values for each material tested is given in Table 2.

Material	Failure Height (in)
A	58
A	60
B	58
B	66
C	42
C	48
D	24
D	26
E	10
E	12
F	52
F	54
G	30
G	34
H	40
H	44

Table 2 - Response Variable Values

It would be ideal if the inputs factors were derived from published material data sheets. However, manufactures vary greatly in the data they publish. A quick scan of material data sheets for any property that one could reasonably believe to be linked to impact performance reveals only two properties published across all eight materials tested; flexural modulus and tensile elongation to failure. So, the input factors are taken to be flexural modulus and tensile elongation to failure. This is perhaps not unreasonable since, as per the wheel design overview discussion in section 2.1, one would expect stiffness (read flexural modulus) and ductility (read tensile elongation to failure) to be critical to performance. The values for the input factors are listed in Table 3.

Material	Flexural Modulus (MPa)	Elongation (%)
A	2930	50
B	2310	52.5
C	1116	400
D	2230	98
E	3510	60
F	483	350
G	1793	55
H	550	450

Table 3 - Input Factors

2.3.1.2 Model Fit Results

The prediction equation for a model investigating only the main effects of the input factors takes the form below:

$$failureheight = b_0 + b_1 FM + b_2 Elong \ ,$$

where *FM* and *Elong* are the flexural modulus and tensile elongation to failure input factors, respectively, and *failureheight* is the failure height response variable. Using least squares regression to determine the coefficients b_0, b_1, and b_2 yields the results in Table 4.

	Coeff	Std Error	t-stat	P-value
Intercept	70.49325	24.15349	2.918553	1.20%
FM	-0.011472	0.008099	-1.416422	18.02%
Elong	-0.041995	0.050069	-0.838738	41.68%
R2	0.165358			

Table 4 - Model Fit Results: Material Data Sheet Input Factors

To understand these results in fuller detail, the reader is encouraged to refer to Appendix A for a brief least squares regression primer. Suffice to say, however, these results are not good. First and foremost, they violate physical intuition. Negative coefficients for both flexural modulus and elongation to failure indicate that drop performance should increase as the material becomes less stiff and less ductile. This is exactly opposite to what was argued in section 2.1. Of course this may not be such a problem seeing as neither the *FM* nor the *Elong* coefficient is particularly significant. In fact, the high P-values for *FM* and *Elong* , coupled with the low R^2 indicate that a prediction equation based upon the input factors chosen is not significantly better at predicting failure height than is the average failure height alone.

2.3.1.3 Adding Strain Rate Dependencies to the Prediction Model

The input factors used in the previous model are typically derived from tests utilizing low strain rates. As such, they do not capture the strain rate dependencies on material properties exhibited by polymers. An impact is a high strain rate event so it follows that strain rate

dependencies would play a significant factor in drop test performance. Therefore, additional input factors predicated on these dependencies are required to increase the accuracy of the prediction model.

The strain rate dependencies of polymers are often modeled using viscoelastic elements. One such simple model is the standard linear solid (SLS) model [8], illustrated in Figure 4.

E_1 and E_2 = Elastic coefficients
η = Viscous coefficient
σ = Stress
ε = Strain

Figure 4 - Standard Linear Solid Model

The SLS has a constitutive stress/strain relationship shown below:

$$\frac{d\varepsilon}{dt} + \frac{E_1}{\eta}\varepsilon = \frac{E_1 + E_2}{\eta E_2}\sigma + \frac{1}{E_2}\frac{d\sigma}{dt}.$$

To check if such a strain rate dependent stress/strain relationship is substantially better for modeling an impact event, the author approximated a wheel impact in two ways. First, using a simple elastic relationship (linear, but no strain rate dependencies – representative of a stress/strain relationship captured by the input factors used in section 2.3.1.1) as shown in Figure 5.

Figure 5 - Elastic Approximation of Wheel Impact

Here, x represents displacement of the hub towards the rim. In this model the governing equation is

$$m\frac{d^2x}{dt^2} + kx = 0.$$

The second approximation uses a viscoelastic relationship (linear, with strain rate dependencies – representative of the SLS model), depicted in Figure 6.

Figure 6 - Viscoelastic Approximation of Wheel Impact

Here x again represents the displacement of the hub towards the rim. In this case, the governing equation is

$$cm\frac{d^3x}{dt^3} + m(k_1 + k_2)\frac{d^2x}{dt^2} + ck_2\frac{dx}{dt} + k_1k_2x = 0.$$

These two approximations were compared against actual acceleration data. In this test, a prototype wheel of each material was dropped from six inches and the resulting acceleration values were captured with a Silicon Designs 200G accelerometer (model 2210-200) mounted to the slide hammer of the drop test fixture. The test results are shown for Material A in Figure 7. Similar results were achieved for the remaining seven materials. As defined in the two governing equations, the acceleration acting upon the mass upon impact is negative. So, for ease of viewing, Figure 7 plots deceleration. The elastic and viscoelastic curves in Figure 7 represent the best fit of the appropriate governing equation to the actual data. Appendix B covers in greater detail the derivation of these best fit curves.

Figure 7 – Wheel Drop Test Deceleration Data

It is fairly apparent that the viscoelastic approximation is better at matching actual data. This approximation deals with forces and displacements as opposed to stresses and strains. Therefore, it incorporates the SLS parameters and the geometry of the prototype wheels. Since

the prototype wheel geometry is held constant, it reasonably follows that the SLS parameters alone should accurately reflect the stain rate dependencies of the polymer materials tested. Therefore, a material's SLS parameters might be valuable input factors to a drop test failure height prediction equation. Unfortunately, SLS parameters are not commonly published so they must by extracted through material tests.

2.3.1.4 Deriving Standard Linear Solid Parameters through Acceleration Measurements

One possible method of deriving SLS parameters starts with the parameters of the viscoelastic impact approximation. As stated earlier, the governing equation for this approximation is in terms of force and displacement, while the constitutive equation for the SLS model describes stresses and strains. Thus, to get from the viscoelastic impact approximation parameters to the SLS values, one must convert force and acceleration data to stress and strain data. Doing so on the prototype wheels is impractical for two reasons. First, the contact area and axis of displacement during the course of impact is not straightforward to calculate. Second, and more importantly, performing such a conversion on a prototype wheel eliminates the advantage of developing a prediction equation in the first place. If, in order to determine the input factors for a drop test failure height prediction equation, one must prototype a wheel and drop it at least once in order to gather acceleration data, there is no point in having a prediction equation as it would only save the 15 minutes or so it takes to drop the prototype wheel to failure. Fortunately a conversion of force / acceleration data to stress / strain data is practical if manufacturer test plaques are used. Test plaques come in simple rectangular solid geometries, allowing for straightforward calculations of contact areas and original lengths along displacement axes. Additionally, they are available as free samples, obviating the need for time consuming prototyping.

The procedure for using test plaques to determine the SLS parameters begins with a simple modification to the drop test figure as pictured in Figure 8.

Figure 8 - Drop Test Fixture with "Nose"

The "nose" affixed to the slide hammer is used to impact a material test plaque placed upon the anvil. Acceleration measurements from the impact can then be used to determine the k_1, k_2, and c of the viscoelastic impact approximation. The appropriate conversion factor is L/A, where L is the original length along the axis of displacement, and A is the force contact area. Therefore,

$$E_1 = \frac{k_1 L}{A},$$

$$E_2 = \frac{k_2 L}{A},$$

and

$$\eta = \frac{cL}{A}.$$

The origin of this conversion factor is outlined in Appendix C.

In practice, this approach for determining SLS parameters is not viable because deceleration data from different materials is virtually indistinguishable. Figure 9 reveals this fact for materials A and H. These two materials differ significantly in terms of published material

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properties but, as the figure shows, the deceleration data measured from their test plaques is practically identical.

Figure 9 - Test Plaque Drop Test Deceleration Data - 1/8" Drop

The exact reason for these indistinguishable results is not fully understood, nor was it fully explored. The author's theory, however, is that in rectangular solid form the materials tested are all sufficiently stiff so that any differences between them are washed out by the compliance of the test apparatus itself. Evidence to support this theory is shown in Figure 10, where one can see that the response from dropping the slide hammer directly on the anvil, with no material present, is not radically different from the response with material present.

Figure 10 - 1/8" Drop Test Deceleration Data - No Material vs. Test Plaque

2.3.1.5 Deriving Standard Linear Solid Parameters through Time-Temperature Superposition Measurements

A second method of extracting the SLS parameters can be seen by recognizing that the SLS model acts as a mechanical filter. In response to a sinusoidal stress, there is a phase lag in the resulting strain. The tangent of this phase lag is often referred to as the loss tangent, and is given by

$$\tan(delta) = \frac{\eta E_2 \omega}{E_1(E_1 + E_2) + (\eta\omega)^2},$$

where ω is frequency measured in rad/s. Refer to Appendix D for a derivation of this result. Plotted against frequency, the loss tangent has a resonant peak that falls off on either side, as depicted in Figure 11.

Figure 11 - Ideal Loss Tangent Function vs. Frequency [rad/s]

Therefore, one could derive SLS parameters by measuring the actual loss tangent function of a material and fitting the best idealized curve. Taking such measurements falls under a class of testing referred to as Dynamic Mechanical Analysis (DMA), and many techniques and test equipment exist for this purpose. The author used TA Instrument's Q800 in single-point, cantilever mode. Refer to Appendix E for test setup and data.

Even with access to appropriate test equipment and techniques, measuring loss tangent functions versus frequency is problematic. This is because while the idealized loss tangent function shown in Figure 11 is normalized to have a peak equal to one that occurs at one rad/s, actual loss tangent function peaks vary greatly in amplitude and frequency depending on the material. However, most measurement devices, including the Q800, have a relatively small frequency range (on the order of 1 - 1000 rad/s), making it very probable that the loss tangent peak falls outside the dynamic range of the test instrument. Instead, one must rely on the time-temperature superposition (TTS) principle [8]. Because of the relationship between temperature and molecular mobility, the TTS principle allows one to trade temperature for time (1/frequency)

in measuring dynamic mechanical behavior. In other words, mechanical effects present at high temperatures and high frequencies (small time intervals) can also be observed at low temperatures and low frequencies (large time intervals). Measurements made at the low temperatures and low frequencies can be translated into their high temperature/high frequency equivalents with a multiplicative shift factor.

Specifically, for a given loss tangent value, we can define a shift factor a_T as

$$a_T = \frac{f(T_{ref})}{f(T)}.$$

Therefore, if one makes a loss tangent measurement at a certain temperature T and finds a specified loss tangent value to occur at frequency $f(T)$, then the frequency at which this same loss tangent value will occur at a reference temperature T_{ref} is given by

$$f(T_{ref}) = a_T f(T).$$

In general, one can suppose the characteristic frequency of any time and temperature dependent process to be described with an Arrhenius relationship:

$$f(T) = C e^{-A/RT},$$

where A is the activation energy of the process [J/mol], R is the gas constant (8.3 J/K mol), and T is the absolute temperature [K]. The shift factor then becomes

$$a_T = \exp\left[\frac{A}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right].$$

In log space, the shift factor becomes additive:

$$\ln(f(T_{ref})) = \ln(f(T)) + \ln(a_T),$$

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where

$$\ln(a_T) = \left[\frac{A}{R} \left(\frac{1}{T} - \frac{1}{T_{ref}} \right) \right].$$

One can therefore take low frequency sweep measurements (from ~1 – 100 rad/s) at several discrete temperature steps and shift them left or right in log space until they line up in a master curve. The extent to which the individual frequency sweep curves "line up" can be qualitatively judged by how well the resulting shift factors follow an Arrhenius curve. Performing the TTS measurements for the eight materials in the data set yields the loss tangent functions pictured in Figure 12.[ii] Appendix E discusses the quality of the master curves and the reason behind the choice of 100°C as a reference temperature.

The curves in Figure 12, while they have the characteristic loss tangent shape, are not ideal. The ideal loss tangent function, in log space, is symmetric about the peak and has ascending and descending slopes equal to one. This is not the case for the experimentally derived curves, making any attempt to fit an ideal loss tangent function to these curves a relatively futile exercise. However, these curves certainly contain information about the individual materials. The author has chosen to quantify this information through the peak loss tangent value and the frequency at which this peak occurs. In defense of this choice, the reader should note that these values are easy to obtain and relate to the original SLS parameters in the following way:

$$\omega_{peak} = \sqrt{\frac{E_1(E_1 + E_2)}{\eta^2}},$$

$$\tan(delta)_{\omega_{peak}} = \frac{E_2}{2\eta\omega_{peak}}.$$

[ii] The data in Figure 12 is given in Log_{10}. This is an inconvenient and potentially confusing switch from the natural logarithms used to derive the shift factor, a_T, but it remains mathematically valid and is consistent with the way in which data is presented in TTS analysis software and in the literature.

Figure 12 - Loss Tangent Functions vs. Frequency [rad/s] (100°C Reference Temp)

2.3.1.6 Redefined Input Factors and Resulting Model Fit

Adding ω_{peak} and $\tan(delta)_{\omega_{peak}}$ to the flexural modulus and elongation input factors defined in section 2.3.1.1, the author obtained the revised input factors shown in Table 5.

| Material | Flexural Modulus [Mpa] | Elongation [%] | 100C Ref Temp | |
			log(w [rad/s])	log(tan delta)
A	2930	50	9.771625843	-0.990191419
B	2310	52.5	14.28828845	-0.968476695
C	1116	400	9.293901913	-0.855560129
D	2230	98	-10.38832401	0.260576689
E	3510	60	-23.14789589	0.364807365
F	483	350	6.210411091	-0.456009206
G	1793	55	5.254963231	-0.851217185
H	550	450	15.91689276	-0.942419026

Table 5 - Input Factors: Revision 1

Using a prediction equation of the form

$$failureheight = b_1 FM + b_2 Elong + b_3 \log(\omega) - b_4 \log^2(\tan delta)$$

results in a much better model fit. Table 6 reveals incredibly significant results that explain over 99% of drop failure height variability, as opposed to just over 16% before.

	Coeff	Std Error	t-Stat	P-value
FM	0.030879	0.002358	13.09518	0.00%
Elong	0.068393	0.00632	10.8222	0.00%
log(w)	4.001448	0.344369	11.61967	0.00%
log^2(tan delta)	-74.12129	9.967347	-7.436412	0.00%
R2	0.992248			

Table 6 - Model Fit Results: Input Factors Revision 1

Table 7 shows that this prediction equation also predicts failure height fairly well (within 20% worst case).

While this model allows failure height prediction without the need to prototype wheels, it still suffers from a couple drawbacks. To begin with, manufacturers don't speak in terms of ω_{peak} and $\tan(delta)_{\omega_{peak}}$, so the data must be obtained independently. Also, obtaining ω_{peak} and $\tan(delta)_{\omega_{peak}}$ is cumbersome. Taking the numerous frequency sweep measurements at discrete temperature steps necessary to perform TTS shifting requires six hours on average per material sample tested. In order to address these drawbacks, it is best to revise the input factors one more time.

Material	Average Failure Height [in]	Predicted Failure Height [in]	% Error
A	59	60.3211027	2.24%
B	62	62.572884	0.92%
C	45	44.7517025	0.55%
D	25	28.9612911	15.85%
E	11	9.99902964	9.10%
F	53	48.289698	8.89%
G	32	26.4488343	17.35%
H	42	45.6199089	8.62%

Table 7 - Predicted Failure Heights: Input Factors Revision 1

Due to the time and temperature trade off exploited for the TTS measurements, one might expect loss tangent data versus frequency (fixed temperature) to have a similar behavior to loss tangent data versus temperature (fixed frequency). This is exactly what is observed. Figure 13 shows that virtually the same information is present in loss tangent data versus temperature as was present in the loss tangent versus frequency curves of Figure 12. In a similar fashion to that used before, the author has quantified this data in terms of T_{peak}, the temperature at which the peak loss tangent occurs, and peak loss tangent, $\tan(delta)_{T_{peak}}$.

Replacing ω_{peak} and $\tan(delta)_{\omega_{peak}}$ with T_{peak} and $\tan(delta)_{T_{peak}}$ addresses the disadvantages of the former input factors. Firstly, T_{peak} corresponds to the glass transition temperature, T_g. An alternative expression for the loss tangent is the ratio of the loss modulus to the storage modulus: G''/G' [9]. At the glass transition temperature, the storage modulus drops precipitously relative to the loss modulus, leading to a peaking of the loss tangent [10]. Glass transition temperatures of polymers are well understood and often measured by manufacturers. Therefore, flexural modulus, elongation at failure, glass transition temperature, and loss tangent (or loss coefficient as it sometimes named) at the glass transition comprise a set of input factors based upon material properties commonly understood by manufacturers.

Figure 13 - Loss Tangent Functions vs. Temperature [C] (10 Hz Reference Frequency)

Secondly, directly as a result of the first point, the inputs factors are all easily obtainable. Even if T_g and $\tan(delta)_{T_g}$ are not available from the manufacturer, they are easier to obtain through DMA measurements than ω_{peak} and $\tan(delta)_{\omega_{peak}}$. Collecting loss tangent data versus temperature requires only a single temperature sweep at one frequency, which can reduce testing time from the six hours previously quoted to less than one hour.

Revision two of the input factors leads to the data in Table 8.

Material	Flexural Modulus [Mpa]	Elongation [%]	10 Hz Reference Frequency	
			Tg [C]	log(tan delta)
A	2930	50	41.3	-0.990191419
B	2310	52.5	18.6	-0.968476695
C	1116	400	52.4	-0.855560129
D	2230	98	163.5	0.260576689
E	3510	60	235.6	0.364807365
F	483	350	60	-0.456009206
G	1793	55	76	-0.851217185
H	550	450	24.3	-0.942419026

Table 8 - Input Factors: Revision 2

The resulting model fit, shown in Table 9, is based upon a prediction equation of the form:

$$ failureheight = b_0 + b_1 FM + b_2 Elong + b_3 T_g - b_4 \log^2(\tan delta). $$

	Coeff	Std Error	t-Stat	P-value
Intercept	69.28176	4.793976	14.45184	0.00%
FM	0.019813	0.002421	8.183769	0.00%
Elong	0.042497	0.01022	4.158029	0.16%
Tg	-0.531116	0.035824	-14.82588	0.00%
log^2(tan delta)	-48.91906	5.62998	-8.689028	0.00%
R2	0.978007			

Table 9 – Model Fit Results: Input Factors Revision 2

Material	Average Failure Height [in]	Predicted Failure Height [in]	% Error
A	59	59.558731	0.95%
B	62	61.518022	0.78%
C	45	44.753013	0.55%
D	25	27.469797	9.88%
E	11	9.7330428	11.52%
F	53	51.68572	2.48%
G	32	31.333205	2.08%
H	42	42.948469	2.26%

Table 10 - Predicted Failure Heights: Input Factors Revision 2

This fit is also very good. In fact it is arguably better than the fit of revision 1 in terms of prediction performance, shown in Table 10.

The prediction model looks good but in order to more firmly establish confidence in it one must ask two pertinent questions. How well does it predict failure heights for materials outside of the data set with which it was created and does it make good physical sense? To address the first question, the author capitalized on the fact that the model uses a data set of eight materials to fit five coefficient estimates. This results in more degrees of freedom than are needed, so we can drop materials out of the data set, re-fit the model, and then see how well the prediction equation predicts the failure heights of the excluded materials. The author chose to exclude materials D and E, the materials with the poorest predicted failure heights in terms of percent error. Table 11 and Table 12 tabulate the results. The fit remains significant and the prediction performance is promising. Materials D and E, although not used to fit the model, have predicted percent errors of less than 20% in the worst case.

	Coeff	Std Error	t-Stat	P-value
Intercept	72.6254	8.520789	8.523318	0.01%
FM	0.019599	0.003166	6.190218	0.04%
Elong	0.039367	0.013758	2.861479	2.43%
Tg	-0.533358	0.061577	-8.661633	0.01%
log^2(tan delta)	-51.61031	6.345302	-8.133626	0.01%
R2	0.947115			

Table 11 - Model Fit Results: Materials D & E Excluded

Material	Average Failure Height [in]	Predicted Failure Height [in]	% Error
A	59	59.386973	0.66%
B	62	61.636611	0.59%
C	45	44.518399	1.07%
D	25	29.479709	17.92%
E	11	11.250587	2.28%
F	53	53.136437	0.26%
G	32	32.000239	0.00%
H	42	42.321341	0.77%

Table 12 - Predicted Failure Heights: Materials D & E Excluded

To address the second question, consider a normalized prediction model. To normalize the model, the author divided each input factor by the largest magnitude within the range. For example, from Table 8, the largest of the $\tan(delta)_{T_R}$ input factors by magnitude is that of material A at 0.99. Therefore, each $\tan(delta)_{T_R}$ input factor was divided by 0.99. This method led to the normalized input factors depicted in Table 13.

| Material | Flexural Modulus (MPa) | Elongation | 10Hz Ref Freq | |
			Tg	log(tan delta)
A	0.834758	0.11111111	0.175297	-1
B	0.65812	0.11666667	0.078947	-0.978070175
C	0.317949	0.88888889	0.222411	-0.864035088
D	0.635328	0.21777778	0.693973	0.263157895
E	1	0.13333333	1	0.368421053
F	0.137607	0.77777778	0.254669	-0.460526316
G	0.510826	0.12222222	0.322581	-0.859649123
H	0.156695	1	0.103141	-0.951754386

Table 13 - Normalized Input Factors

By normalizing the input factors, the magnitude of the coefficient estimates then provides a measure of their importance in impacting the response value. Figure 14 shows the coefficient magnitudes.

Figure 14 – Magnitude of Coefficient Estimates for Normalized Model

The results show T_g and *FM* to be the most important effects, which makes good physical sense. The glass transition temperature is the temperature at which a polymer transitions from the glassy plateau to the rubbery plateau. As such, below this temperature a polymer is more glassy and brittle and above this temperature a polymer is more rubbery and ductile. Therefore, the glass transition temperature is inversely proportional to ductility. So T_g speaks to ductility and *FM* speaks to stiffness. Since the sign of the T_g estimate is negative and the sign of the *FM* estimate is positive, the model indicates that high ductility (low T_g) and high stiffness (high *FM*) are the most important for determining failure height performance. This jibes well with the argument made in section 2.1 that wheels must be ductile enough to undergo large deformation without fracture or permanent deformation, yet stiff enough to absorb large amounts of energy for a given deformation. The only difference now as compared to this earlier argument is that T_g is apparently a much better indication of ductility than is tensile elongation to failure.

2.3.1.7 Experimental Prediction Model Conclusions and Observations

There are a few conclusions to draw from the experimentally derived prediction model:

- It provides good insight into what governs wheel impact performance. Specifically, stiff, ductile materials perform best. This is not an exceptionally revealing statement, however capturing both of these qualities in a single material appears to be something unique to materials A and B, explaining their superior performance.

- Prediction is possible.

- If prediction is possible, then computer aided engineering (CAE) tools can be brought to bear in order to model and simulate complex design problems such as wheel impacts far more quickly than even experimental model fitting.

It is this last point that will be explored further in the following sections.

2.3.2 Prediction Modeling using Computer Aided Engineering Tools

The quest for a prediction model started with some assumptions on the constitutive relationship between stress and strain and experimentally derived some material properties that play a significant role in governing drop failure height performance. If instead one can determine the appropriate constitutive stress/strain relationship or the appropriate series of stress/strain relationships that govern deformation through an entire wheel impact event, one can treat the complex wheel geometry as a series of infinitesimally small, but finite in number, elements, each governed by the aforementioned stress/strain relationship(s). Solving for stresses and strains at any given point in space or time then becomes a case of solving a system of thousands, simultaneous stress/strain equations, something at which computers excel.

The analysis technique described above is a numerical technique known as the Finite Element Method (FEM) and is the foundation of Finite Element Analysis (FEA). FEA was first developed in the nineteen forties to analyze relatively simple structural problems that defied closed form solution [11]. Developments in FEM knowledge and computing power over the years have led to countless FEA software packages today that are capable of accurately modeling incredibly complex problems. Included in these developments is the derivation of numerous models to capture the stress/strain behavior of materials under a wide range of applications. Previously in this document the author has explored simple linear elastic and linear viscoelastic models but there are several stress/strain models for use in FEA software such as non-linear viscoelastic, elastoplastic, and even models relying on equations of state.

To test the suitability of FEA, the iRobot contracted with an outside company to perform analysis on the SUGV wheel design. This outside design firm was given performance specifications, to include:

- Peak impact energy.
- Maximum allowable deceleration.
- Target wheel weight.

In addition, volumetric constraints were specified (i.e. the wheel must be round and fit with the specified dimensions). The design firm was free to vary material used and geometry particulars within the volumetric constraints. Working under these conditions, the design firm was able to iterate through multiple design combinations until the performance specifications were met. The

iterative process relied solely on FEA to model and simulate performance of each of the designs and took no longer than 2 man-weeks to execute. This is significantly less time than the many man-months of effort to develop performance prediction through the regression model fitting discussed in section 2.3.1. Additionally, the FEA model was able to predict the effects of both material and geometry on performance. These time savings and the increased capabilities of the resulting model are arguments enough for the benefits of FEA, but to strengthen the argument the author will examine the business case for FEA in greater detail in the following chapter.

Chapter 3: Building a Business Case for CAE

3.1 Estimated Cost of Experimental Trial and Error Methodology

Previous sections have shown CAE tools such as FEA to have large potential time savings in performance prediction. To better understand how these time savings translate into costs, the author wishes to directly compare the cost of design using the legacy experimental trial and error approach with the desired future design approach of performance prediction using FEA. For sake of comparison, the author will attempt to capture all costs from the specification of performance requirements to the point in the design phase were enough confidence exists to prototype a full scale working part for confirmation tests.

This time window is hard to define for the experimental trial and error approach due, in part, to the evolving nature of the performance requirements themselves. However, as an estimate, the author has chosen to include the design time of the PackBot wheel, which led to the geometry and material baseline for the SUGV wheel, and the design time spent to optimize this baseline in order to find a material that can be manufactured more cost effectively. The PackBot design took approximately four man-months of engineering time and cost \$5,000 in materials. SUGV optimization led to enough confidence in material F to move forward with full sized prototype production. This effort required another five and a half man-months of time and cost \$12,000 in materials.[iii] Engineering time for both the PackBot and SUGV design phases was estimated to be divided among senior and junior engineers in a one-third to two-thirds ratio. iRobot's burdened hourly rate for those two groups is \$174.18 and \$109.47 respectively. So, for engineering time, the resulting cost is

$$\frac{1}{3}(9.5man-months)\left(173.33\frac{man-hours}{man-month}\right)\left(\$174.18\frac{}{man-hour}\right)+$$

$$\frac{2}{3}(9.5man-months)\left(173.33\frac{man-hours}{man-month}\right)\left(\$109.47\frac{}{man-hour}\right)=\$215,775.05$$

[iii] This time estimate does not include the time spent by the author in developing the performance prediction equations of section 2.3.1. The author's time was omitted because confidence in material F was developed by parallel work of other engineers.

Here, $173.33 \, ^{man - hours}\!/\!_{man - month}$ was derived by taking

$$\left(\left(52 \, man - weeks/man - year\right)\left(40 \, man - hours/man - week\right)\right)/\left(12 \, man - months/man - year\right).$$

Adding material expenditures to the engineering time costs leads to a total design cost of

$$\$215,775.05 + \$5,000.00 + \$12,000.00 = \$232,775.05 \, .$$

Therefore, the total design effort, leading to enough confidence in material F to prototype a full size part, required 9.5 man-months of engineering time at a cost of ~$230k.

3.2 Estimated Cost of FEA Approach

In contrast, the author spent an estimated half of a man-month coordinating the efforts of the design firm, efforts that resulted in a geometry and material selection ready for prototyping. The design firm charged $7,083.00 for their design study and took 2 man-weeks to complete it. A conservative estimate bills the author's time at a senior engineer's burdened hourly rate. Therefore, the cost of engineering time to iRobot is

$$\left(0.5 \, man - months\right)\left(173.33 \, ^{man - hours}\!/\!_{man - month}\right)\$174.18\!/\!_{man - hour} = \$15,095.31$$

Adding the $7,083.00 charge from the design firm and the total cost to iRobot is $22,178.31. Thus, the FEA design effort required one man-month of engineering time (one half from iRobot and one half from the design firm) at a cost of ~$22k.

These numbers are about an order of magnitude lower in terms of both time and money, making a very convincing case for the use of FEA in the design process.

3.3 Applicability of FEA to Other Aspects of the SUGV Design

Similar savings to those discussed above could be realized in other areas of the SUGV design. One potential use for FEA that immediately stands out involves thermal analysis. A problem faced by the SUGV designers is heat dissipation within the main electronics housing. As no dedicated cooling system exists, heat dissipation occurs solely as the result of conduction through the chassis and convection to the environment. In the PackBot design, an all aluminum chassis provides sufficient thermal conductive capacity and surface area to cool the electronics during normal operation. In the SUGV, as weight is a critical design parameter, it would be nice to replace some chassis elements with lighter weight materials such as plastics. However, such a design decision raises the question of impact on thermal dissipation. Will the electronics get too hot if plastic materials of lower thermal conductivity are used?

To answer this question, designers relied on the tried and true approach of experimental trial and error. A chassis mock-up was made using part aluminum and part plastic and a sample power load was placed in the electronics housing. The temperature within the housing was measured to see if it exceeded acceptable levels. Determining performance using a different material or different chassis configuration would require a new chassis mock up and an additional experimental run.

Heat transfer is generally well understood and thermal properties of materials are well known, so this is a problem that lends itself even more readily to FEA than the wheel design problem. In fact, the simple FEA software owned by iRobot as an add-on module to its computer aided design (CAD) package is very capable of handling this thermal problem. The original experimentation, including time to machine the chassis mock-up and collect temperature data, required two man-weeks to complete. The results from this experiment were replicated using FEA to within 8% (32°C final air temperature within the electronics box for experimental results versus 32.5°C in FEA) in three man-days time [12].

3.4 FEA – Required Investment

Sections 3.1 through 3.3 show multiple aspects of the SUGV design where using CAE tools such as FEA could have potentially large returns in terms of time and money. How best to proceed in implementing FEA more widely throughout the design process therefore depends on

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the level of investment required. A first level of investment is in the software packages needed to perform FEA.

3.4.1 Cost of FEA Software

Many FEA software packages exist on the market today ranging from basic tools capable of handling a small subset of problems to very robust tools that can solve a wide range of problems. Part of what separates FEA tools on this spectrum is their ability to solve linear versus non-linear problems and their inclusion of implicit versus explicit solvers.

Linear tools can only incorporate linear constitutive relationships. This precludes their ability to solve problems requiring complex, non-linear models in order to accurately capture real world behavior. Implicit versus explicit solvers differ in their ability to solve for steady-state versus transient behavior. To illustrate this difference let's assume a quantity Q that we want to evaluate at time $t = (n+1)dt$ in terms of its value at time $t = ndt$, i.e.

$$Q_{n+1} = Q_n + Sdt,$$

where S is the rate of change in Q. An implicit solver would evaluate S in terms of some quantities known only at the new time step $t = (n+1)dt$, while an explicit solver evaluates S solely in terms of known quantities at the old time step $t = ndt$ [13]. Implicit solutions are unconditionally stable, meaning they are stable for any size dt. This makes them attractive for steady-state problems as large time steps can be used in order to reach steady-state quickly. However, implicit methods require iteration to advance the solution for each time step towards a final, converged state. This becomes computationally intensive for small time steps, making implicit solvers impractical for capturing transient or dynamic behavior. Explicit solutions, on the contrary, are conditionally stable, meaning there is a limit to the size of dt before the solution explodes. This makes explicit solvers a poor choice for solving steady-state problems of a large time scale, as a prohibitively large number of time steps would be required to reach a solution. However, explicit solutions do not require iteration to converge to a solution for each time step, making an explicit method ideal for solving for transient behavior.

FEA packages vary greatly in price depending of where they are on the scale of robustness. Pro/ENGINEER Structural and Thermal, the FEA package that iRobot has as an add-on to its Pro/ENGINEER CAD package, is a linear FEA package with only an implicit solver, placing it in the basic tool category. As such, it is inexpensive (a license costs $5000 plus $840 annually for maintenance, where maintenance includes access to technical support). However, it is not well suited to solve more complex design problems like the SUGV wheel.

On the more robust end of the FEA software spectrum are packages like ANSYS and ABAQUS. These packages can solve both linear and non-linear problems and have implicit and explicit solvers. They are well suited to solve design problems like the SUGV wheel. In fact, the design firm utilized by iRobot to perform an analysis of the SUGV wheel design used ABAQUS. As shown in Table 14, the robustness of these packages comes at a cost. This table summarizes the cost for all three FEA packages.

| FEA Package | Type of Solver | Capabilites | Cost | | Lease |
| | | | Purchase | | |
			Price	Annual Maintenance	Annual Fee
Pro/ENGINEER Structural and Thermal	Implicit	Linear Mechanical and Thermal	5,000.00	850.00	N/A
ANSYS Mechanical	Implicit	Linear and Non-Linear Mechanical and Thermal	30,390.00	6,080.00	17,470.00
ANSYS LS-DYNA	Explicit	Transient	12,600.00	2,520.00	7,250.00
Total			42,990.00	8,600.00	24,720.00
ABAQUS Standard	Implicit	Linear and Non-Linear Mechanical and Thermal			
ABAQUS Explicit	Explicit	Transient			
Total			54,312.00	13,035.00	19,750.00

Table 14 - FEA Software Costs

3.4.2 Cost of FEA Expertise

Of course FEA is only as good as the engineer using it, so a substantial piece of the cost in investing in FEA capabilities is the cost of the people to use them. As Pro/ENGINEER Structural and Thermal is a basic tool, it can be fairly well understood and utilized by engineers working with it on a part time basis (the three man-days of effort to conduct the thermal analysis on the chassis discussed in section 3.3 included time taken by the engineer to figure out how to

use the software). Therefore, iRobot would incur no additional labor cost in utilizing Pro/ENGINEER Structural and Thermal.

ANSYS and ABAQUS, by contrast, are far more complex, as are the problems they are capable of solving. Thus, getting good utility out of these packages would require at least one full time engineer with FEA experience. Such an engineer, due to the experience required, would bill at iRobot's senior engineer burdened rate. This leads to an annual labor expense of

$$(\$174.18/man - hour)(2080man - hours/man - year) = \$362,294.40/man - year.$$

3.4.3 Recouping FEA Investment

To solve a complete range of FEA problems, the Pro/ENGINEER Structural and Thermal package iRobot already has in house is inadequate by itself. iRobot must utilize a robust package like ANSYS or ABAQUS. Figure 15 estimates the net present costs of operating either ANSYS or ABAQUS through both the lease or purchase options. This figure shows that leasing ABAQUS is the least expensive option up until the five year mark. As this is on the time scale of software obsolescence, leasing ABAQUS is arguably the best option for acquiring a robust FEA software package. Therefore, total annual operating costs include software lease fees of $\$19,750/year$ and labor expenses of $\$362,294.40/year$. Sections 3.1 and 3.2 suggest a potential savings of approximately $200,000 in using FEA for the SUGV wheel design problem. Thus, a full time FEA engineer would have to work $(\$362,294.40 + \$19,750.00)/\$200,000 = 1.91$ wheel design sized jobs per year in order to recoup the investment in software and labor.

Figure 15 – FEA Software Net Present Operating Costs

Another option is to use the Pro/ENGINEER Structural and Thermal package iRobot already owns for simple problems and use FEA consulting firms, like the design firm used to analyze the SUGV wheel design problem, to solve problems requiring a robust tool. This would obviate the need to acquire robust software like ANSYS or ABAQUS, as well as the talent to use it. Pricing various FEA consulting firms yields a typical hourly rate around $175.00/$man - hour$. This rate results in an annual labor cost of

$$(\$175.00/man - hour)(2080man - hours/man - year) = \$364,000/man - year .$$

Here the only additional costs are the administrative costs of coordinating the contracted work. These costs include the time spent generating statements of work and purchase orders and paying invoices. Let's assume this administrative overhead adds a 9% time increase for every man-hour of FEA consulting work used. This translates to an extra two man-days per man-month of FEA work of coordination over what would be required if the work was done in house. We know from sections 3.4.1 and 3.4.2 that the cost of doing any amount of ABAQUS level FEA work (from one to 2080 man-hours) in house would cost $382,044.40 annually ($19,750 for software lease and $362,294.40 for labor). The question then becomes how many hours of contract FEA work must be done before it is more cost effective to bring this work in house. This question is answered by solving the following equation:

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$$(\$175.00)x + 0.09x(\$109.47) \geq \$382{,}044.40.$$

In the above equation it is assumed that the required coordination effort could be accomplished at a lower burdened rate, equivalent to a junior engineer. Solving for x yields 2067 man-hours, or 0.99 man-years.

These results show that utilizing FEA to optimize designs through performance prediction offers a potentially large return on investment over the experimental trial and error approach used in the past, on the order of 1000% using the SUGV wheel design problem as a guide (~$230,000 saved for ~$22,000 invested). The best way to incorporate FEA tools is to utilize the existing software for basic problems and contract out more complex problems to FEA consulting firms up until the point of one man-year of consulting work. After this threshold, it is more cost effective to invest in the robust tools and expertise and bring them in house.

Such conclusions confirm the belief held by some of the SUGV design engineers that the experimental trial and error design approach had grown too costly. However, skepticism over the usefulness of CAE tools like FEA as an alternative still exists. Specifically there are concerns over whether FEA will actually reduce the number of prototypes required or will require more prototypes to verify that simulation was accurate. This skepticism can only be overcome by actually trying to use tools such as advanced FEA to incorporate more modeling and simulation into the design process. The potential cost savings calculated and the recommendations made in this chapter provide an impetus and direction for doing just that.

Chapter 4: Conclusion

For historical and cultural reasons, iRobot's SUGV group has come to rely on extensive prototyping to optimize and prove out designs. Aggressive cost targets for the SUGV require a design optimized not only for performance but also for manufacturability. Achieving this balance necessitates a paradigm shift away from the process of experimental trial and error to one involving more use of performance prediction. To demonstrate that performance prediction is possible and reliable, the author used regression fitting to develop a prediction model for determining material performance in a wheel drop test. The result of this effort was a prediction model that explains over 97% of the variability in failure drop height using readily obtainable material properties and provides some good intuition for the physics governing wheel impact performance.

These results provide an insight into the possibilities of performance prediction that opened the door for exploration of computer aided engineering tools like finite element analysis software packages that rely on the finite element method. While performance prediction through regression fitting provides some relief from the resource intensity required by the experimental trial and error approach, the use of FEA provides a real competitive advantage in design optimization by providing significant savings in time and money required. Again, using the wheel design problem as a test case, the author was able to show that FEA tools can offer savings up to an order of magnitude in the time and money required to optimize a design. Finally, the author was able to concretely show that the best path forward for iRobot in terms of implementing the use of FEA more widely in its design process is to make better use of the basic tools it already has for simple problems and contract out more complicated problems to FEA consulting firms.

Future research in this area could occur in three major areas:

Combining high stiffness with a low glass transition temperature in the same material: Section 2.3.1.7 concludes that materials with a low T_g and high stiffness provide the best performance in wheel drop tests. However materials A and B appear to be unique in achieving both of these properties in the same material. Determining how these two materials do so could provide the

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knowledge necessary to replicate a low T_g and high stiffness in an injection moldable material. This would allow iRobot to manufacture wheels of comparable performance to those of the current PackBot design at a fraction of the cost. This could be beneficial because while there is confidence that material F, the lead contender of the injection moldable materials, performs adequately, the material used in the current PackBot design still significantly outperforms it. Further work along these lines is of little strategic value, however, as it only addresses the particular problem of the wheel design, a problem that is arguably already solved to a level that does not warrant such further investigation.

Implementing lessons learned: Of more strategic importance is the implementation of the lessons learned regarding the power of utilizing FEA in the design process. Work has just begun to expand the knowledge of the capabilities of the Pro/ENGINEER Structural and Thermal tool already in house, to understand the boundary between problems this tool can solve and problems more robust tools like ANSYS and ABAQUS can solve, and to develop a relationship with a FEA consulting firm capable of utilizing robust tools to solve complex problems. As a first step in this process, the author has worked with several of the design engineers to develop three structural test cases that were presented to both a firm specializing in Pro/ENGINEERING Structural and Thermal training and a firm specialized in the use of ABAQUS. The idea behind this process is to use the test cases as a way to understand the capabilities and limitations of each FEA tool, while at the same time to get useful information needed for design that would otherwise be obtained through prototyping. Results from this process will have to be incorporated into a new design methodology that can intelligently determine what work is best suited for Pro/ENGINEERING Structural and Thermal versus more robust tools versus work that still must be performed through prototyping. Additionally, this modified design methodology needs to be incorporated into the design processes used on other product lines across both divisions of iRobot to solidify the corporation's competitive advantage in product design moving forward into the future.

Incorporating intangible costs into the CAE cost analysis: Key to implementing the lessons learned in a manner discussed above is expanding the CAE cost analysis to include intangible costs. Chapter 3 analyzed costs from a purely accounting perspective. The conclusion stating

that it is best to outsource FEA work up until the threshold of one man-year of work could change drastically if costs on intangible items like intellectual property are factored in. To use the wheel design as an example, one could argue that the wheels and the impact survivability they enable are at the core of iRobot's competitive advantage in military robots. The intellectual property of their design should therefore be guarded and not shared. If there are enough similar design components that could benefit from FEA but do not amount to one man-year of effort, it still might make sense to invest in the resources to perform the complex analysis in house.

Appendix A: Least Squares Regression Primer

Response variables can be expressed as a combination of input factors plus some random error:

$$y_i = \beta_0 + \beta_1 x1_i + \beta_2 x2_i + \varepsilon_i,$$

where y_i is the ith response value; β_0, β_1, and β_2 represent the coefficients; $x1_i$ and $x2_i$ represent the ith input factors; and ε_i is a random error [5]. This example illustrates a linear combination of two input factors, though this doesn't have to be the case. Non-linear terms such as interaction terms ($x1_i x2_i$) or polynomial terms ($x1_i^2$) can also be explored. In addition, while there are only two input factors in the above example, in general, the number of input factors is only limited by the number of response value data points in the data set.

Let b_0, b_1, and b_2 be the estimates of the coefficients β_0, β_1, and β_2. Then $\hat{y}_i$, the predicted value of the ith response value, is given by the prediction equation below:

$$\hat{y}_i = b_0 + b_1 x1_i + b_2 x2_i.$$

The ith residual, e_i, then becomes

$$e_i = y_i - \hat{y}_i.$$

Least squares regression generates coefficient estimates such that the sum of the squares of the residuals is minimized:

$$\min(SS_{res}) = \min\left(\sum_{i=1}^{n} e_i^2\right).$$

As b_0, b_1, and b_2 are estimates of the true coefficients, they are random variables with a mean and variance. It can be shown that the mean of an estimate is the true coefficient it is estimating. Therefore, one quality measure of a prediction equation involves determining each input factor's significance by utilizing the variance of the estimate of its coefficient to develop a

level of confidence that the estimate's mean, or the true coefficient, is non-zero. Evaluating an estimate's variance requires at least one degree of freedom. For example, if one wishes to fit a straight line prediction equation, consisting of an intercept term and slope term, to a response value data set consisting of only two data points, there are no remaining degrees of freedom with which to evaluate variance of the coefficient estimates. In this simple case, the lack of any degrees of freedom is readily apparent because two points define a line. There are no residuals with which to evaluate variance. The number of degrees of freedom for variance analysis is equal to the number of response value data points minus the number of coefficients estimated. Since, as stated before, at least one degree of freedom is required for variance analysis, the number of coefficients that can be estimated is therefore equal to the number of response value data points minus one. This imposes a limit, as discussed earlier in this appendix, on the number of input factors one can include in a prediction equation.

Another quality metric is the coefficient of determination, R^2, that provides a measure of the overall adequacy of a prediction equation. The coefficient of determination is given by:

$$R^2 = 1 - \frac{SS_{res}}{SS_{total}},$$

where

$$SS_{res} = \sum_{i=1}^{n} e_i^2,$$

$$SS_{total} = \sum_{i=1}^{n} (y_i - \bar{y})^2,$$

and

$$\bar{y} = \frac{1}{n} \sum_{i=1}^{n} y_i.$$

In this way, R^2 varies from zero to one. An R^2 of zero indicates that the prediction equation explains none of the variability in the response. An R^2 of one indicates that the prediction equation explains all of the variability in the response. Generally speaking, the higher the R^2, the greater the overall adequacy of the prediction equation.

The regression results from Table 4 (reprinted here for convenience) offer an example with which to gain a more concrete understanding of regression and regression metrics.

	Coeff	Std Error	t-stat	P-value
Intercept	70.49325	24.15349	2.918553	1.20%
FM	-0.011472	0.008099	-1.416422	18.02%
Elong	-0.041995	0.050069	-0.838738	41.68%
R2	0.165358			

Table 4 - Model Fit Results: Material Data Sheet Input Factors

In this table, the Coeff column gives the coefficient estimates for the prediction equation. Thus, the prediction equation for failure height becomes:

$$failureheight = 70.49325 - 0.011472 * FM - 0.041995 * Elong .$$

The standard error of the coefficient estimate is the square root of the variance. Dividing the coefficient estimate by the standard error yields a test statistic (t-stat) for the significance of the coefficient estimate. This t-stat is a measure of how many standard errors the coefficient estimate is away from zero. Armed with this statistic, and knowledge that coefficient estimates follow a t-distribution, it is possible to determine the probability (P-value) of observing a given coefficient estimate if the true coefficient is actually zero. As a rule of thumb, 5% is usually taken as a significance level cutoff, i.e. a P-value less than 5% indicates that there is less than a 5% chance that the true coefficient is zero given the observed coefficient estimate.

One can immediately see the poor quality of these regression results. The two input factors used, *FM* and *Elong* , both have a P-value much larger than 5%. Additionally, the R^2 indicates that the prediction equation less than 17% of the failure height variability.

Appendix B: Derivation of Best Fit Viscoelastic and Elastic Wheel Impact Approximation Curves

From section 2.3.1.3, the governing equation for the viscoelastic approximation of wheel impacts is

$$cm\frac{d^3x}{dt^3} + m(k_1 + k_2)\frac{d^2x}{dt^2} + ck_2\frac{dx}{dt} + k_1k_2x = 0,$$

where, as depicted in Figure 6, x is the displacement of the wheels hub towards its rim and m represents the mass of the axle load acting on the wheel hub. This equation can be rearranged in the following form:

$$x''' = a_1x'' + a_2x' + a_3x,$$

where

$$a_1 = -\frac{(k_1 + k_2)}{c},$$

$$a_2 = -\frac{k_2}{m},$$

and

$$a_3 = -\frac{k_1k_2}{cm}.$$

Acceleration, x'', during impact can be measured directly through the accelerometer mounted to the slide hammer of the drop test fixture. Taking the derivative of acceleration yields jerk, x'''. Integrating acceleration once gives velocity, x', and integrating acceleration twice gives displacement, x. Therefore, armed with jerk, acceleration, velocity, and displacement data versus time, one can estimate a_1, a_2, and a_3 using least squares regression where x''' is the response and x'', x', and x are the inputs. From a_1, a_2, and a_3 it is possible to calculate k_1, k_2, and c:

$$k_2 = -ma_2,$$

$$c = \frac{-k_2}{a_3 + \dfrac{a_1}{a_2}},$$

and

$$k_1 = \frac{ca_1}{a_2}.$$

The calculated k_1, k_2, and c can then be plugged back into the governing equation to yield a viscoelastic approximation to the measured acceleration data.

In practice, this approach does not work well. The governing equation for the viscoelastic approximation is only valid for the first half cycle of the impact event (i.e. only up until the point where the wheel rebounds off of the impacting surface). The equation assumes the model is anchored to the impacting surface and would therefore provide invalid results past this point. Running a regression on the first half cycle of the jerk, acceleration, velocity, and displacement vectors can sometimes leads to negative values for k_1, k_2, or c. This is mathematically possible because while negative values provide an unstable system, the exploding errors are not captured within the first half cycle and therefore do not impact the regression fit. Figure 16 illustrates this phenomenon. Here the reader can see the unstable nature of the viscoelastic approximation, due in this case to a negative k_1. However, the fit for the first half cycle is pretty good, revealing how negative values for k_1, k_2, or c could still produce a good fit when performing a least squares regression on only the first half cycle.

To constrain the values of k_1, k_2, or c to be positive, the author took a different approach beginning with breaking up the governing differential equation into a series of differences equations:

$$x[n+1] = x[n] + x'[n](t[n+1] - t[n]),$$

$$x'[n+1] = x'[n] + x''[n](t[n+1] - t[n]),$$

$$x''[n+1] = x''[n] + x'''[n](t[n+1] - t[n]),$$

and

$$x'''[n] = -\frac{(k_1 + k_2)}{c} x''[n] - \frac{k_2}{m} x'[n] - \frac{k_1 k_2}{cm} x[n];$$

The initial conditions for these differences equations are

$$x[1] = 0,$$

$$x'[1] = \sqrt{2gh},$$

and

$$x''[1] = 0,$$

where g is the acceleration due to gravity measured in $m/_{s^2}$, and h is the drop height measured

in m.

Figure 16 - Unstable Deceleration Data Approximation

These difference equations were programmed in a Microsoft Excel spreadsheet, depicted in Figure 17, which calculates the acceleration value for a given k_1, k_2, and c at each time step, compares it the to the value from the accelerometer data at that same time step, and calculates the square error. The square errors from each time step in the first half cycle of the impact event are then summed and Microsoft Excel's Solver add-in is used to solve for the optimal k_1, k_2, and c that minimizes the summed square errors subject to the constraint that k_1, k_2, and c must be non-negative. Assuming the time steps ($t[n+1]-t[n]$) are sufficiently small, this approach yields good results.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	Experimental Data								Model					k1	k2	c
2	Time(s)	Accel(volts)	Time(adj)	Accel(G)	Stotal		x	x'	x"	x"'	Accel(G)	Sres		235309.8	727708.4	527.8663
3	-0.000544	0.016	0	0.8	258.690905		0	1.728305529	0	-197920	0	0.64				
4	-0.00054	0	4E-06	0	285.065099		6.91E-06	1.728305529	-0.79168	-196829	0.0807837	0.006526		mass =	6.3546	kg
5	-0.000536	0.016	8E-06	0.8	258.690905		1.38E-05	1.728302362	-1.57899	-195745	0.1611219	0.4081653		drop hgt =	6	in
6	-0.000532	0.036	0.000012	1.8	227.523163		2.07E-05	1.728296046	-2.36197	-194669	0.2410177	2.4304258		initvel =	1.728306	m/s
7	-0.000528	0.024	0.000016	1.2	245.983809		2.77E-05	1.728286598	-3.14065	-193600	0.3204743	0.7735655				
8	-0.000524	0.02	2E-05	1	252.297357		3.46E-05	1.728274035	-3.91505	-192538	0.3994946	0.3606067		Data pnts	4216	
9	-0.00052	0.036	0.000024	1.8	227.523163		4.15E-05	1.728258375	-4.6852	-191485	0.4780817	1.7474679		SSres =	9490.929	
10	-0.000516	0.052	0.000028	2.6	204.02897		4.84E-05	1.728239634	-5.45114	-190438	0.5562387	4.1769602		ybar =	16.88387	
11	-0.000512	0.028	0.000032	1.4	239.75026		5.53E-05	1.72821783	-6.21289	-189399	0.6339685	0.5868043		SStotal =	262897.9	
12	-0.000508	0.036	0.000036	1.8	227.523163		6.22E-05	1.728192978	-6.97049	-188367	0.7112741	1.1853242				
13	-0.000504	0.056	0.00004	2.8	198.355421		6.91E-05	1.728165096	-7.72395	-187342	0.7881584	4.0475067		R^2 =	0.963899	
14	-0.0005	0								-186324	0.8646244	3.0115286				
15	-0.000496	0								-185313	0.940675	0.7384395				
16	-0.000492	0								-184310	1.016313	2.5080645				
17	-0.000488	0								-183313	1.0915414	3.6422143				
18	-0.000484	0								-182323	1.166363	0.4014959				
19	-0.00048	0								-181340	1.2407806	0.0579753				
20	-0.000476	0								-180364	1.314797	0.8368535				
21	-0.000472									-179395	1.388415	1.9276962				
22	-0.000468	-0								-178432	1.4616374	7.0843138				
23	-0.000464	-0								-177477	1.534467	3.7421624				
24	-0.00046	0								-176527	1.6069063	1.4566229				
25	-0.000456	-0								-175585	1.6789583	5.1936509				
26	-0.000452									-174649	1.7506255	3.0646896				
27	-0.000448	0								-173719	1.8219106	0.1429516				
28	-0.000444	0.052	0.0001	2.6	204.02897		0.000173	1.727594182	-18.5496	-172796	1.8928163	0.5001087				
29	-0.00044	0.032	0.000104	1.6	233.596712		0.00018	1.727319984	-19.2408	-171879	1.9633452	0.1320198				

Solver Parameters

Set Cell: $O$9
Equal To: Max, Min, Value of:
By Changing Variable Cells: $N$2:$P$2
Subject to the Constraints: $N$2:$P$2 >= 1
Standard GRG Nonlinear

Buttons: Solve, Close, Guess, Options, Add, Change, Delete, Standard, Reset All, Help

Figure 17 - Spreadsheet for Finding Optimal Viscoelastic Approximation Parameters

In essence, the author used the spreadsheet in Figure 17 to run a manual least squares optimization in order to constrain the decision variables to be positive. Mathematically this optimization problem is represented as follows:

$$\min\left[\sum_{n=1}^{N}\left(x''_{actual}[n]-x''_{approximation}[n]\right)^2\right],$$

by varying k_1, k_2, and c; subject to the constraint

$$k_1 > 0,$$

$$k_2 > 0,$$

and

$$c > 0.$$

Here, $x''_{actual}[n]$ are the acceleration values from the accelerometer measurements and $x''_{approximation}[n]$ are the acceleration values calculated from the difference equations outlined above.

A similar approach was used to derive the elastic approximations. For these, the governing equation, given in section 2.3.1.3 and is reprinted here:

$$m\frac{d^2x}{dt^2} + kx = 0,$$

can be reduced to the following difference equations:

$$x[n+1] = x[n] + x'[n](t[n+1] - t[n]),$$

$$x'[n+1] = x'[n] + x''[n](t[n+1] - t[n]),$$

and

$$x''[n] = -\frac{k}{m}x[n].$$

These difference equations were programmed into a Microsoft Excel spreadsheet identical in nature to the one described above, thereby solving the optimization problem

$$\min\left[\sum_{n=1}^{N}(x''_{actual}[n] - x''_{approximation}[n])^2\right],$$

by varying k; subject to the constraint $k > 0$.

Appendix C: Derivation of Geometric Conversion Factor

To convert from k_1, k_2, and c to E_1, E_2, and η, let's begin by defining some terms:

F = force (N),

σ = stress (N/m^2),

x = displacement (m),

ε = strain (%),

k = elastic coefficient (N/m),

E = elastic coefficient (N/m^2),

c = viscous coefficient ((N)(s)/m),

η = viscous coefficient ((N)(s)/m^2),

A = contact area (m^2),

L = original length along axis of displacement (m).

The constitutive relationship of the elastic elements in the viscoelastic approximation is

$$F = kx.$$

Recognizing that

$$F = \sigma A$$

and

$$x = \varepsilon L,$$

we can substitute as follows

$$\sigma A = k\varepsilon L.$$

Gathering terms leads to

$$\sigma = \frac{kL}{A}\varepsilon.$$

The constitutive relationship for the elastic elements in the SLS model is

$$\sigma = E\varepsilon,$$

so

$$E = \frac{kL}{A},$$

as stated in section 2.3.1.4.

The constitutive relationship for the viscous elements in the viscoelastic approximation is

$$F = c\frac{dx}{dt}.$$

Applying the same substitution as before yields

$$\sigma A = cL\frac{d\varepsilon}{dt}.$$

Gathering terms results in

$$\sigma = \frac{cL}{A}\frac{d\varepsilon}{dt}.$$

The constitutive relationship for the viscous elements in the SLS model is

$$\sigma = \eta\frac{d\varepsilon}{dt},$$

so

$$\eta = \frac{cL}{A},$$

as stated in section 2.3.1.4.

Appendix D: Derivation of SLS Constitutive Stress/Strain Relationship and Loss Tangent Function

The SLS model from Figure 4 is represented again in Figure 18, which shows the stress/strain relationships for each of the individual elements.

Figure 18 - SLS Model with Stress/Strain Relationships for Each Individual Element

Equilibrium and compatibility give the following relationships:

$$\sigma = \sigma_2 = \sigma_1 + \sigma_3,$$

$$\varepsilon = \varepsilon_1 + \varepsilon_2,$$

and

$$\varepsilon_1 = \varepsilon_3.$$

The subscripted terms can be written in terms of σ and ε:

$$\varepsilon_2 = \frac{\sigma}{E_2},$$

$$\varepsilon_1 = \varepsilon - \frac{\sigma}{E_2},$$

$$\sigma_1 = E_1\varepsilon_1 = E_1\varepsilon - \frac{E_1}{E_2}\sigma,$$

$$\sigma_3 = \sigma - \sigma_1 = \left(1 + \frac{E_1}{E_2}\right)\sigma - E_1\varepsilon,$$

$$\frac{d\varepsilon_3}{dt} = \frac{\sigma_3}{\eta} = \frac{1}{\eta}\left[\left(1 + \frac{E_1}{E_2}\right)\sigma - E_1\varepsilon\right],$$

and

$$\frac{d\varepsilon_3}{dt} = \frac{d\varepsilon}{dt} - \frac{1}{E_2}\frac{d\sigma}{dt}.$$

Equating the last two equations yields

$$\frac{1}{\eta}\left[\left(1 + \frac{E_1}{E_2}\right)\sigma - E_1\varepsilon\right] = \frac{d\varepsilon}{dt} - \frac{1}{E_2}\frac{d\sigma}{dt},$$

which is equivalent to the form of the constitutive equation in Figure 4:[iv]

$$\frac{d\varepsilon}{dt} + \frac{E_1}{\eta}\varepsilon = \left(\frac{E_1 + E_2}{\eta E_2}\right)\sigma + \frac{1}{E_2}\frac{d\sigma}{dt}.$$

The loss tangent function is in response to a sinusoidal input in stress. Therefore, $\sigma(t)$ takes the form $\sigma(t) = \sigma_0\sin(\omega t)$. The resulting strain will also be sinusoidal but will lag the stress in phase: $\varepsilon(t) = \varepsilon_0\sin(\omega t - \phi)$. The constitutive equation then becomes

$$\varepsilon_0\omega\cos(\omega t - \phi) + \frac{E_1}{\eta}\varepsilon_0\sin(\omega t - \phi) = \left(\frac{E_1 + E_2}{\eta E_2}\right)\sigma_0\sin(\omega t) + \frac{1}{E_2}\sigma_0\omega\cos(\omega t).$$

Using the following trigonometric identities:

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

[iv] This form includes an σ term omitted, erroneously in the author's opinion, from Powell's [8] derivation.

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b),$$

and collecting terms leads to

$$\sin(\omega t)\left(\varepsilon_0 \omega \sin(\phi) + \frac{E_1}{\eta}\varepsilon_0 \cos(\phi)\right) + \cos(\omega t)\left(\varepsilon_0 \omega \cos(\phi) - \frac{E_1}{\eta}\varepsilon_0 \sin(\phi)\right)$$
$$= \left(\frac{E_1 + E_2}{\eta E_2}\right)\sigma_0 \sin(\omega t) + \frac{1}{E_2}\sigma_0 \omega \cos(\omega t)$$

Equating $\sin(\omega t)$ and $\cos(\omega t)$ terms from the left and right side of the equation results in the following two equations:

$$\varepsilon_0 \omega \sin(\phi) + \frac{E_1}{\eta}\varepsilon_0 \cos(\phi) = \left(\frac{E_1 + E_2}{\eta E_2}\right)\sigma_0,$$

and

$$-\frac{E_1}{\eta}\varepsilon_0 \sin(\phi) + \varepsilon_0 \omega \cos(\phi) = \frac{1}{E_2}\sigma_0 \omega.$$

Solving this system of equations for $\sin(\phi)$ and $\cos(\phi)$ gives

$$\sin(\phi) = \frac{\sigma_0}{\varepsilon_0}\left[\frac{\dfrac{E_1\omega}{E_2\eta} - \dfrac{\omega(E_1 + E_2)}{E_2\eta}}{\left(\dfrac{E_1}{\eta}\right)^2 + \omega^2}\right],$$

and

$$\cos(\phi) = \frac{\sigma_0}{\varepsilon_0}\left[\frac{\dfrac{\omega^2}{E_2} + \dfrac{E_1(E_1 + E_2)}{E_2\eta^2}}{\left(\dfrac{E_1}{\eta}\right)^2 + \omega^2}\right].$$

Finally, solving for $\tan(\phi)$:

$$\tan(\phi) = \frac{\sin(\phi)}{\cos(\phi)} = \frac{\eta E_2 \omega}{E_1(E_1 + E_2) + (\eta\omega)^2} = \tan(delta),$$

as given in section 2.3.1.5.

Appendix E: DMA Test Setup and Data Analysis

The DMA used to derive loss tangent data was TA Instrument's Q800, pictured in Figure 19.

Figure 19 - Q800 DMA

This machine is capable of measuring loss tangent in one of several modes, to include tension, compression, and bending. The expression of the loss tangent function derived in Appendix D remains unchanged for any of these modes. Deciding which mode is most suitable therefore relies mostly on sample size and geometry.

For the eight materials tested, the author started with manufacturer test plaques. These test plaques typically come in solid rectangular form in something like 3" x 5" x 1/8" dimensions. Test plaques were chosen as they can be obtained at no cost and relatively quickly, typically seven to ten days. These plaques lend themselves well to testing in bending mode because they can be readily cut with unsophisticated tooling into rectangular beams small enough to fit in the Q800 (on the order of 1" x 1/8" x 1/8").

Testing in bending mode requires the Q800's cantilever clamp, shown in Figure 20. This clamp has two outer anchor points that are fixed and a movable anchor point in the center. Longer material samples are anchored at both ends and deflected in the center. This is the dual point cantilever method. Shorter material samples pass through only the center, movable anchor point and one of the outer, fixed anchor points. In this way they are anchored at one end and deflected at the opposite end. This is the single point cantilever method and is the method used

by the author for his testing. In either case the loss tangent is calculated by measuring the phase lag between the applied force on the movable anchor point and the resulting deflection.

Figure 20 - Q800 Cantilever Clamp

TTS shifting requires frequency sweep measurements made at several discrete temperatures; the larger the frequency range in the sweep measurements and the more temperatures used, the more accurate the generated master curve. The frequency range is limited by inertial effects. As test frequency increases, the inertia of the movable anchor begins to dominate the measured phase lag between applied force and displacement. The frequency at which the inertial effects begin to dominate depends on the material being tested (the stiffer the material, the higher the frequency) and can be easily observed by a rapid increase in loss tangent. The frequency range must therefore be chosen appropriately for the material under test to avoid inertial effects. For the eight materials tested by the author, a frequency range in the neighborhood of one to 20 Hertz sufficed.

Practically speaking, the number of discrete temperatures tested is only limited by the allowable time to run a test. Stepping in temperature requires significant time to equilibrate the test chamber and material sample. Thus, one wants to use the minimum number of temperatures that will generate a master curve which accurately captures the peaking behavior of the loss tangent function. As discussed in section 2.3.1.6, the peak in the loss tangents of polymers occurs as a result of the glass transition. So to capture the rising and falling edges of the peak, one would want to test temperatures around the glass transition temperature. The author tested every 5°C from approximately $T_g \pm 50°C$.

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Material	Tested Frequency Range (Hz)	Tested Temperature Range (5C steps)
A	1 thru 20	20 thru 120
B	1 thru 20	0 thru 100
C	1 thru 20	-10 thru 90
D	1 thru 20	100 thru 200
E	1 thru 20	165 thru 265
F	1 thru 15	0 thru 100
G	1 thru 20	0 thru 100
H	1 thru 20	0 thru 100

Table 15 - Test Conditions for TTS Data

Table 15 summarized the frequency and temperature ranges used for collected the TTS data. This data was imported into TA Instrument's Rheology Advantage Data Analysis software to perform the TTS shifting and master curve generation. In frequency space, the frequency at which the peak loss tangent occurs depends greatly on the reference temperature chosen. Thus, a common reference temperature must be chosen for all eight material master curves in order to get comparable data. The author chose 100C somewhat arbitrarily, as it was a good middle ground temperature to which all eight master curves would converge in the software.

As mentioned in section 2.3.1.5, the quality of a master curve can be measured by how well the shift factors for each frequency sweep curve follow an Arrhenius curve given by

$$a_T = \exp\left[\frac{A}{R}\left(\frac{1}{T} - \frac{1}{T_{ref}}\right)\right].$$

Figure 21 plots this comparison for material A. The other seven materials exhibit similarly good fits, as summarized in Table 16, which shows the activation energy, A, and the R^2 of the Arrhenius fit for each of the eight materials tested.

Figure 21 - Material A Shift Factors versus Arrhenius Curve

Material	A (kJ/mol)	R^2
A	308.02	0.9921
B	317.52	0.9937
C	363.19	0.9962
D	598.97	0.9851
E	668.91	0.9875
F	268.93	0.9512
G	361.56	0.9944
H	391.84	0.9980

Table 16 - Activation Energy and R^2 of Arrhenius Fit to TTS Shift Factors

Measuring the loss tangent versus temperature is a far easier measurement to make on the Q800. One simply needs to fix frequency and sweep across the desired temperatures. As the dynamic temperature range of the test chamber is very large, any temperature one could practically want to measure is directly obtainable. Additionally, the time required to run this test is far less than that required to collect the TTS data.

Since the objective in this case is still to capture the loss tangent peak, $T_g \pm 50°C$ remains a good temperature range across which to test. The reference frequency is again restricted by inertial effects. One needs to choose a reference frequency that is below the critical inertial effects frequency for all materials tested. The author chose a reference frequency of 10 Hz for this purpose.

As an alternative to retesting all samples for loss tangent versus temperature, it is important to recognize that the TTS data already captures the loss tangent value at 10 Hz across the desired temperature range. The Rheology Advantage Data Analysis software can tease out these values and piece together master curves versus temperature for the desired reference frequency. The author used this approach to generate the master curves shown in Figure 13.

Appendix F: Useful Acronym Definitions

CAE – Computer Aided Engineering

SUGV – Small Unmanned Ground Vehicle

G&IR – Government and Industrial Robots

BAA – Broad Agency Announcement

EOD – Explosive Ordinance Disposal

FCS – Future Combat Systems

SLS – Standard Linear Solid

DMA – Dynamic Mechanical Analysis

TTS – Time-Temperature Superposition

FEM – Finite Element Method

FEA – Finite Element Analysis

CAD – Computer Aided Design

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