QUANTUM REALITY AND SQUEEZED STATES OF LIGHT

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ABSTRACT

The limitations of the quantum postulates are examined using the self consistent quantum formalism of nonlinear optics. An optics analog of the EPR paradox is presented. This *Gedankenexperiment* uses squeezed vacuum states of light to test Bell's inequality. The generation of squeezed states in the nonlinear Sagnac ring interferometer is reviewed. A new variant of this system, which employs the Sagnac ring as the reflector in a laser resonator, is presented. A unique squeezed state, which we call *modulated vacuum*, results. It is shown that the phase uncertainty of the pump inside the laser cavity does not inhibit the squeezing action. The discussion on squeezed states is applied to Bell's inequality. The conditions for violation of Bell's inequality with squeezed vacuum are derived. It is shown that Bell's inequality is violated in the limit of small photon number. We show how a quantum cryptography system may be set up using pulses of squeezed vacuum, where Bell's inequality is used to test for eavesdropping.

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To My Parents and Sonja

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Chapter 1

Introduction

1.1 Historical Backround

The development of the fundamental quantum postulates did not proceed as smoothly as the current orthodoxy in quantum mechanics would lead one to believe. Although Schroedinger's wave equation proved to be a powerful mathematical tool, capable of explaining Bohr's quantization rules for the hydrogen atom, the scientific community was divided over its physical significance. Schroedinger believed his wave function to be a real physical field, similar to the electromagnetic field [1], while Born thought of the wave function as a "phantom" field whose square magnitude is a probability density [2]. Heisenberg supported Born's interpretation with his uncertainty principle [3]. Einstein, on the other hand, held that description of the wave function is incomplete, and must be generalized by a deterministic theory that brings quantum mechanics in line with classical mechanics. Bohr stressed the wave particle duality as the essence of quantum phenomena. He argued that the wave vs. particle descriptions are complementary in the sense that they allow humans to use classical concepts to explain phenomena outside the scope of human experience [4]. According to Bohr, the wave function is a mathematical structure that embodies this complementarity. It is evident that there was a myriad of varying opinions as to how to interpret the emerging quantum theory.

The foundation of modern quantum theory was formulated by Bohr, Heisenberg and Pauli at a series of meetings in Copenhagen. This so-called Copenhagen interpretation is a synthesis of Bohr's complementarity principle, Born's probabilistic interpretation of the wave function, and Heisenberg's uncertainty principle. Einstein's vigorous opposition to this view was highlited in a famous debate with Bohr at the fifth Solvay Conference on "Electrons and Photons." [5] At this conference, Einstein confronted Bohr with several thought experiments that seemingly "proved" that the uncertainty principle can be violated. But Bohr was able to show in every case that under closer examination there was no violation. It was this victory against Einstein that probably convinced most scientists to embrace the Copenhagen interpretation. Einstein, however, was not deterred by this setback. His most forceful assault on the Copenhagen interpretation came years later with the EPR paradox.

1.2 Conceptual Difficulties

The orthodox quantum theory pictures an elementary particle, like an electron, as a probability distribution without definite position or momentum. Associated with the electron is a wave function $\Psi(x,t)$, which gives the probability $\mid \Psi(x,t) \mid^2$ of finding it at a particular position x. The main conceptual difficulty with this interpretation of the wave function is that the probability density describes a single electron rather then a large ensemble as in classical statistical mechanics. Of course, when a measurement is made, we must find the electron at some point in space. The wave-function is then said to "collapse," so that a subsequent measurement will again find the electron at that point. But we still can say nothing about the position of the electron before the measurement. It could have been anywhere given by the probability $\mid \Psi(x,t) \mid^2$. This would not be strange if we assume that the wave function gives only an incomplete description of nature. In statistical mechanics we have no problems with giving a probabilistic description to systems which cannot possibly be described deterministicaly (a large aggregate of atoms, for example). The strangeness comes about because quantum theory asserts that a probabilistic description is necessary *in principle.* One consequence is Heisenberg's uncertainty principle, which states that it is impossible to know both the position and momentum of a particle at the same time. A measurement of momentum will leave the particle with a completely random position and vice versa. Hence we are led to believe that physical properties come into existence only when they are measured. Many scientists could not accept such a strange conception of reality, a discomfort elegantly expressed by Bernard d'Espagnat [6]:

> The mind demands something more: not necessarily determinism - there is nothing intrinsically irrational about randomness - but at least objective explanations of observed regularities, or in other words causes. Underlying this demand is the intuitive notion that the world outside the self is real and has at least some properties that exist independently of human consciousness.

The group of physicists advocating the position expressed by d'Espagnat became known as realists. The most prominent among them was Albert Einstein.

1.3 EPR and Hidden Variables

In 1935, Einstein, Podolsky and Rosen (EPR) published a classic paper arguing that quantum mechanics fails to provide a complete description of physical reality [7]. EPR described a *Gedankenexperiment* that clearly reveals the paradoxical consequences of quantum mechanics. The thought experiment consisted essentially of two particles that interact such that their positions and momenta become correlated. In quantum language, the two single particle wave functions become "entangled." EPR considered the entangled wave function $\delta(x_1 - x_2 - a)$, which is an eigenfunction of the operator $\hat{x_1} - \hat{x_2}$ with eigenvalue a, and of the operator $\hat{p_1} + \hat{p_2}$ with eigenvalue 0. A measurement of the observable $\hat{x_1}$ will cause the wave function to collapse into an eigenstate of position, thereby determining with certainty the outcome of a position measurement on particle 2. Now the measurements could in principle be done many light years apart. Hence a measurement on one particle should have no influence on the second particle. But this is exactly what happens *unless* we assume that the particles had all along the particular values that were measured. EPR then showed, by analogous arguments on the momentum, that such an assumption leads to a violation of Heisenberg's uncertainty principle, with the conclusion " ... we have thus shown that the wave function does not provide a complete description of the physical reality ... "

Most scientists rejected Einstein's paradox, favoring Bohr's defense of the Copenhagen interpretation [8]. Bohr argued that one must treat both the measuring instrument and the particles as a single system. Accordingly, turning the dial on the measuring instrument (i.e. choosing to measure position versus momentum) alters the state of the whole system. Despite Bohr's arguments, some researchers were influenced by EPR to search for a more "complete" theory. The view that quantum mechanics gives only partial or incomplete statistical information inevitably led to the so called hidden variable theory.

The hidden variable theory is similar to the classical statistical description of atoms in a gas. Since we can never know all the positions and momenta (the "hidden variables" in this case) of all the atoms in a gas, we must accept a partial, but for all practical purposes sufficient, probabilistic description. Hence we are perfectly happy in only knowing the average quantities (pressure, temperature, density, etc.) since that is all we need to design practical devices. Similarly, one may assume the existence of hidden variables in the quantum theory, where the wave function is a result of some average over the hidden variables. In most cases these hidden variables play no part in the physical description of matter. The wave function and the corresponding quantum averages usually give a sufficient description. But as Einstein demonstrated, there are situations where knowledge of these hidden variables would resolve strange paradoxes of the "incomplete" quantum theory. It should be noted that there are also cases in the classical statistical description of matter where knowledge of the average quantities is not a sufficiently accurate description. For example, in high-temperature plasmas the fluid equations sometimes fail to give an accurate description. In this case one must seek a more complete statistical description given by Boltzmann's equation.

Returning to the subject of hidden variables, we note that the lack of knowledge of these variables seems to make the theory superfluous. If it is impossible to make predictions that can be tested with experiments, then the hidden variable vs. the Copenhagen interpretation is just matter of taste. This was the state of affairs until 1965, when J.S. Bell published his seminal paper [9]. In this paper Bell used the hidden variable theory to derive an inequality, now known as Bell's theorem, that allows an experimental comparison between quantum mechanics and hidden variable theory. Furthermore, Bell showed for a simpler version of the EPR experiment that quantum mechanics violates his inequality. This set the stage for theoretical and experimental research that is still very active today. We will not go into an exhaustive review of all the experiments performed to date, except to note the celebrated work of Alain Aspect and co-workers with correlated polarization states of light [10,11]. Excellent reviews of both theory and experiments are given by John F. Clauser and Abner Shimony [12], d'Espagnat [5], and Jim Baggott [13]. Although most of the experiments so far have favored quantum mechanics, many issues have yet to be completely resolved and scientists continue to search for new ways to probe into the quantum world. Bell believed that the search should continue, as he wrote regarding the Aspect experiments [14]:

> ... I certainly hope it is not the end. I think that the probing of what quantum mechanics means must continue, and in fact it will continue, whether we agree or not that it is worth while, because many people are sufficiently fascinated and perturbed by this that it will go on.

1.4 Quantum Optics

In this thesis we will apply the formalism of quantum optics to the weighty issues raised by EPR. The corpuscular or quantum nature of light played a decisive role in the evolution of quantum theory. Indeed, it all began with Planck's description of black body radiation [15] and Einstein's photoelectric effect [16]. The development of quantum electrodynamics greatly enlarged the range of phenomena explained by quantum theory. For example, the mysterious passing of one photon through both slits of Young's interference experiment could now be formally described as single photon interference [17]. The development of laser technology and photon counting techniques led to experiments showing how the interference pattern emerges as a succession of one-photon interference experiments [18].

Quantum optics also predicts two-photon interference. Two-photon interference effects where first demonstrated by Hanbury-Brown and Twiss [19]. They used photon counting techniques to measure the second order correlation or interference between photons of a split beam. Similar methods where later used to demonstrate photon antibunching [20-22], a manifestation of nonlinear two-photon processes. The possibility of applying the quantum properties of light to communication and interferometry led to the investigation of squeezed states [23-25]. Squeezed states of light are also created through multiple photon interactions in nonlinear media. Squeezed light has properties very different from ordinary laser or thermal light. In particular, the photons in squeezed light display large quantum correlations in phase. We will show how such quantum correlations lead to nonlocal interference effects and violation of Bell's inequality.

Much of the material for this thesis comes from the author's work on squeezing [26-28] and its applications to Bell's theorem [29]. The thesis includes a detailed discussion of Bell's theorem and a review of squeezed light and its generation. A new system for generating squeezed light inside a laser resonator will be analyzed. The optics version of the EPR experiment with squeezed light is discussed and the conditions for violation of Bell's inequality will be derived. Bell's theorem will be applied to a quantum cryptography system that uses pulses of squeezed light.

Chapter 2

Bell's Theorem

2.1 EPR Paradox

In this chapter we will discuss in detail a simplified version of the EPR *Gedankenexperiment* due to Bohm [30]. Bohm considered the dissociation of a spin-0 system into two spin-1/2 particles. Conservation of linear momentum requires that the particles fly apart in opposite directions and conservation of a angular momentum is satisfied if the particles are in a singlet state. When the particles are sufficiently separated that no more interactions can take place between them, observers make measurements of their spin components using Stern-Gerlach analyzers. The Stern-Gerlach analyzers are set up independently to measure the spin of particles 1 and 2 in arbitrary directions $\vec{a}$ and $\vec{b}$. A schematic of this system is shown in fig. 2.1.

Let us consider the simple case when both analyzers are aligned in the z-direction. We may expand the wave function in a superposition of eigenstates of

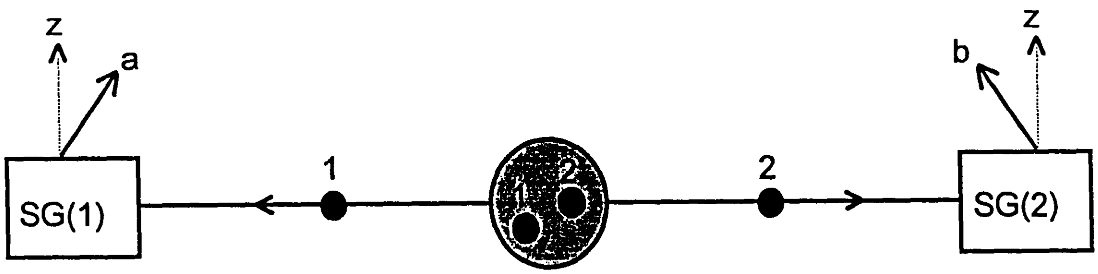

Figure 2.1: Dissociation of a spin-0 system followed by spin measurements.

spin in the z-direction. The singlet state is then given by:

$$| \Psi \rangle = \frac{1}{\sqrt{2}} \{ | +\rangle_1 \, | -\rangle_2 - | -\rangle_1 \, | +\rangle_2 \} \tag{2.1}$$

where + denotes spin up and - spin down. Both particles have an equal probability of being spin up or spin down. A measurement of spin on one of the particles, however, collapses the wave function into a eigenstate corresponding to the particular outcome of the measurement. The result of a spin measurement on the second particle is then determined with certainty. Even when the measurements are made light years apart, if particle 1 measures spin up, we know that particle 2 will measure spin down. It seems that a measurement on particle 1 exerts an influence on the outcome of measurements on particle 2 in violation of the locality principle.

The above analysis is frequently used in physics texts to demonstrate the EPR paradox. Actually, the situation is more subtle. Consider a classical system made up of a gun that shoots red and blue bullets randomly in opposite directions. Further, suppose that the gun is loaded with only two bullets, one red and one blue. Then after it shoots off the two bullets in opposite directions, measurements are made on their colors. Note that here again a measurement on one bullet determines with certainty the outcome of a measurement on the second bullet. Does this classical system then violate the locality principle? There is no paradox here because in classical mechanics particles have definite properties independent of measurements. Hence, even though we don't know the color of the bullet before a measurement is made, we do know that it has a definite color. That is, if we measure blue for bullet 1, then bullet 2 had the color red all along. The measurement on bullet 1 does not really influence the outcome of measurements on bullet 2. We cannot make the same assertion in quantum mechanics. According to the quantum picture, the spin of particle 2 is not *defined* until a measurement is made on the system. Hence the measurement on particle 1 really does exert some kind of nonlocal influence on particle 2.

EPR argued that the only way to avoid violation of the locality principle is to assert that the particles had the particular spin components that were measured all along. But this assertion leads to a contradiction with quantum mechanics. Let us suppose that the particles have definite spin components just like the bullets had definite colors. We can make measurements of either z or x spin components of particle 1, thus determining with certainty the outcome of spin component of particle 2. If we then assume that the particles had those spin components all along, then we must admit that a particle can have well defined spin in both z and x directions at the same time, a violation of Heisenberg's uncertainty principle. EPR thus concluded that quantum mechanics cannot give a complete description of physical reality.

2.2 Hidden Variables

Let us now consider the EPR experiment from the view of hidden variable theory. Since the wave function $\mid \Psi\rangle$ cannot adequately predict the results of measurements, the existence of a more complete specification of the state is postulated. Associated with every possible outcome of a measurement is a hidden variable λ and a normalized probability distribution $\rho(\lambda)$. The average value of an observable $O(\lambda)$ is given by:

$$\langle O(\lambda)\rangle = \int \rho(\lambda) O(\lambda) d\lambda \tag{2.2}$$

This description is very similar to classical statistical mechanics. For example, ρ can be the Maxwellian distribution where the hidden variable corresponds to velocity. Similarly, there may exist hidden variables that describe each outcome of the EPR experiment. The probability distribution ρ is set up by the initial interaction of the particles. Unfortunately, we have no knowledge of the distribution ρ since the hidden variables are truly "hidden," they cannot be directly measured or controlled.

2.3 Bell's Inequality

It turns out that knowledge of ρ is not necessary to derive a result that can be compared with experiment [9]. Consider the correlation between the spins of the two particles in fig. 2.1. According to the quantum rules this correlation is given by:

$$E(\vec{a},\vec{b}) = \langle \Psi \mid \hat{\sigma} \cdot \vec{a}\hat{\sigma} \cdot \vec{b} \mid \Psi \rangle = -\vec{a} \cdot \vec{b} \tag{2.3}$$

where $\hat{\sigma}$ is the Pauli spin operator and we have normalized the units of angular momentum to the quantum of action $\hbar/2$. The "nonlocal" quantum correlations can be perceived from eq. 2.3 when $\vec{a}$ and $\vec{b}$ point in the same direction. In this case the two spin measurements are perfectly anti-correlated:

$$\langle \Psi \mid \hat{\sigma} \cdot \vec{b}\hat{\sigma} \cdot \vec{b} \mid \Psi \rangle = -1 \tag{2.4}$$

Bell used the hidden variable theory together with the locality principle to derive an inequality for the correlation E. Hidden variable theory prescribes that:

$$E(\vec{a},\vec{b}) = \int \rho(\lambda) S1_a \cdot S2_b(\lambda) d\lambda \tag{2.5}$$

where $S1_a \cdot S2_b$ is the observable for the product of spin of the two particles. The locality principle asserts that the measurement of spin of particle 1 should be independent of the measurement of spin of particle 2. This condition is expressed mathematically by:

$$S1_a \cdot S2_b(\lambda) = S1_a(\lambda) \cdot S2_b(\lambda) \tag{2.6}$$

Now let us consider the difference of two correlations involving three independent orientations of the Stern-Gerlach analyzers:

$$E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) = \int \rho(\lambda)\left(S1_a(\lambda) \cdot S2_b(\lambda) - S1_a(\lambda) \cdot S2_c(\lambda)\right) d\lambda \tag{2.7}$$

Manipulating the right side we get:

$$\begin{aligned} E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) = & \int \rho(\lambda) S1_a(\lambda) \cdot S2_b(\lambda)\left(1 + S1_b(\lambda) \cdot S2_c(\lambda)\right) d\lambda - \\ & \int \rho(\lambda) S1_a(\lambda) \cdot S2_c(\lambda)\left(1 + S1_b(\lambda) \cdot S2_b(\lambda)\right) d\lambda \end{aligned} \tag{2.8}$$

Now taking the absolute value and using the triangle inequality yields:

$$\mid E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \mid \leq \ \mid \int \rho(\lambda) S1_a(\lambda) \cdot S2_b(\lambda)\left(1 + S1_b(\lambda) \cdot S2_c(\lambda)\right) d\lambda \mid +$$

$$\mid \int \rho(\lambda) S1_a(\lambda) \cdot S2_c(\lambda) \, (1 + S1_b(\lambda) \cdot S2_b(\lambda)) \, d\lambda \mid \quad (2.9)$$

To proceed further, we use some of the results of quantum mechanics. This is not inconsistent since hidden variable theory is supposedly a generalization of the quantum theory. Hence the spin is still quantized: $S1, S2 = \pm 1$.

The inequality 2.9 is still true when we take the absolute values inside the integrals. Then, noting that $\mid S1 \cdot S2 \mid \leq 1$, we dispose of the absolute values all together:

$$\begin{aligned} \mid E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \mid \leq & \int \rho(\lambda) \, (1 + S1_b(\lambda) \cdot S2_c(\lambda)) \, d\lambda \, + \\ & \int \rho(\lambda) \, (1 + S1_b(\lambda) \cdot S2_b(\lambda)) \, d\lambda \end{aligned} \quad (2.10)$$

If we impose condition 2.4, the second integral vanishes and we are left with Bell's inequality:

$$\mid E(\vec{a},\vec{b}) - E(\vec{a},\vec{c}) \mid \leq 1 + E(\vec{b},\vec{c}) \quad (2.11)$$

We have arrived at this inequality by using the formalism of hidden variables, constrained by the locality principle and some results from quantum mechanics. It is easy to show, however, that there is a range of analyzer orientations where Bell's inequality is violated by quantum mechanics. In particular, for $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = 1/2$ and $\vec{a} \cdot \vec{c} = -1/2$, we have $\mid E(a,b) - E(a,c) \mid = 1$ while $1 + E(b,c) = 1/2$. Thus Bell's inequality provides a concrete test of quantum vs. hidden variable theory.

Although we have used some quantum results in the derivation of Bell's inequality, one might wonder whether it is also possible to prepare a classical system of two particles, similar to the gun shooting red and blue bullets, with statistics that violate Bell's inequality. It can be proved that any system with classical statistics automatically satisfies Bell's inequality. Such a proof has been given by Ekert in the context of quantum cryptography based on Bell's theorem [32]. Ekert proposed using an entangled state of two spin-1/2 particles to send a sequence of random numbers. This sequence can then be used as a "key" in a public-key cryptography system. The security of the key is assured by Bell's inequality. Ekert showed that substitution of a classical pair of particles by an eavesdropper will inevitably give the system "elements of reality" that satisfy Bell's inequality. We will discuss quantum cryptography in more detail in chapter 4.

Chapter 3

Squeezed States of Light

3.1 Quantum Formalism

In the quantum theory of light a formal analogy exists between the oscillation of a harmonic oscillator and a single mode of the electromagnetic field. A particular physical structure (i.e. a cavity) always has a natural set of eigenmodes. An arbitrary electromagnetic disturbance can be described by a sum over these modes or as an ensemble of harmonic oscillators in the quantum picture. Each oscillator is quantized by replacing classical amplitudes with Boson operators [32,33]. Hence the classical plane wave

$$E(x,t) = Ee^{-i\omega t+ikx} + E^{*}e^{i\omega t-ikx} \tag{3.1}$$

becomes

$$\hat{E}(x,t) = \sqrt{\frac{\hbar\omega}{2\epsilon V}}\{\hat{a}e^{-i\omega t+ikx} + \hat{a}^{\dagger}e^{i\omega t-ikx}\} \tag{3.2}$$

where $\hat{a}$ and $\hat{a}^\dagger$ are Boson annihilation and creation operators. They are defined by the commutation relations:

$$\begin{aligned} \left[\hat{a},\hat{a}^\dagger\right] &= 1 \qquad (3.3) \\ \left[\hat{a},\hat{a}\right] &= \left[\hat{a}^\dagger,\hat{a}^\dagger\right] = 0 \end{aligned}$$

The Boson operators are not Hermitian; they cannot be measured experimentally. More relevant for experiments are the in-phase $\hat{q}$ and quadrature-phase $\hat{p}$ components of the electric field. They are physical observables, described by Hermitian operators:

$$\hat{q} = \frac{\hat{a}+\hat{a}^\dagger}{2} \qquad (3.4)$$

$$\hat{p} = \frac{\hat{a}-\hat{a}^\dagger}{2i} \qquad (3.5)$$

$\hat{q}$ and $\hat{p}$ are very similar to position and momentum. Their commutator is given by:

$$[\hat{q},\hat{p}] = \frac{i}{2} \qquad (3.6)$$

The fact that $\hat{q}$ and $\hat{p}$ are physical observables corresponding to the in-phase and quadrature-phase components becomes clear when we write the electric field operator as:

$$\hat{E}(x,t) = \sqrt{\frac{2\hbar\omega}{\epsilon V}}\{\hat{q}Cos(kx-\omega t)+\hat{p}Sin(kx-\omega t)\} \qquad (3.7)$$

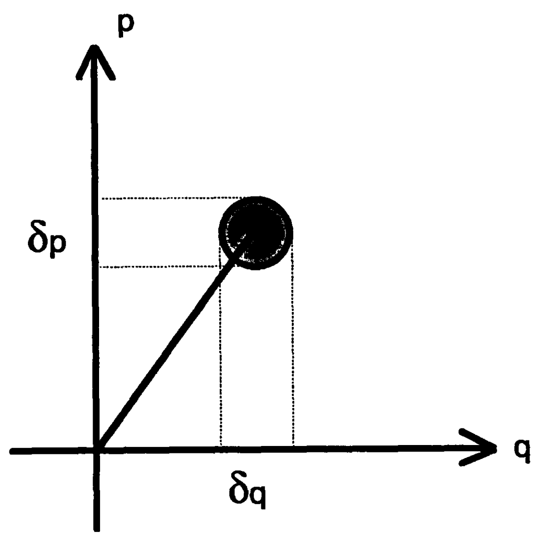

Figure 3.1: Coherent State

3.2 Coherent States

The closest one can come experimentally to a classical monochromatic plane wave is with laser light. Ideal laser light is described quantum mechanically by coherent states [34,35]. Thermal light or black body radiation is also described by statistical mixtures of coherent states. Coherent states can be thought of as classical waves with small quantum fluctuations. The quantum fluctuations in a coherent state are phase independent. They can be visualized by a "noise" circle in phase space (see fig. 3.1). The coherent states are defined mathematically as eigenstates of the annihilation operator [34], i.e.

$$\hat{a} \mid \alpha\rangle = \alpha \mid \alpha\rangle \tag{3.8}$$

The average value of the electric field operator in a coherent state is:

$$\langle\alpha \mid \hat{E}(x,t) \mid \alpha\rangle = \alpha\sqrt{\frac{2\hbar\omega}{\epsilon V}} Cos(kx-\omega t) \tag{3.9}$$

where, without loss of generality, α was taken to be real. Note that this average yields the classical electromagnetic wave.

Using the commutation rules 3.3 it is easy to calculate the fluctuations in the two quadratures of a coherent state:

$$\delta q^2 = \langle\alpha \mid \hat{q}^2 \mid \alpha\rangle - \langle\alpha \mid \hat{q} \mid \alpha\rangle^2 = \frac{1}{4} \tag{3.10}$$

$$\delta p^2 = \langle\alpha \mid \hat{p}^2 \mid \alpha\rangle - \langle\alpha \mid \hat{p} \mid \alpha\rangle^2 = \frac{1}{4} \tag{3.11}$$

The quantum fluctuations in $\hat{q}$ and $\hat{p}$ satisfy the minimum uncertainty product allowed by their commutator. It is also important to notice that δq^2 and δp^2 are independent of the amplitude. The amplitude is related to the average photon number by:

$$\langle\alpha \mid \hat{a}^{\dagger}\hat{a} \mid \alpha\rangle = \mid \alpha \mid^2 \tag{3.12}$$

Hence, for very large average photon number, the quantum fluctuations will be insignificant in comparison. In this limit the coherent state approaches a classical wave. In the other extreme of small average photon number the quantum fluctuations will dominate. This means that even in the absence of an electric field there will be "vacuum" fluctuations. Vacuum fluctuations are also known as virtual photons. They are responsible for such phenomenon as spontaneous emission and the Van der Waals force [32]. We will use photons of squeezed vacuum in the next chapter to test Bell's inequality.

3.3 Squeezed States

Like coherent states, squeezed states are defined as eigenstates of a certain operator [24], i.e.

$$(\mu\hat{a} + \nu\hat{a}^{\dagger}) \mid \alpha, \mu, \nu\rangle = (\mu\alpha + \nu\alpha^{*}) \mid \alpha, \mu, \nu\rangle \tag{3.13}$$

α plays the role of an amplitude, and μ and ν are any two complex numbers satisfying the condition:

$$\mid \mu \mid^2 - \mid \nu \mid^2 = 1 \tag{3.14}$$

The fluctuations in the quadrature components of a squeezed state are calculated by using equations 3.13, 3.14 and the commutation relations 3.3:

$$\begin{aligned} \delta q^2 &= \frac{1}{4} \mid \mu - \nu \mid^2 && (3.15) \\ \delta p^2 &= \frac{1}{4} \mid \mu + \nu \mid^2 && (3.16) \end{aligned}$$

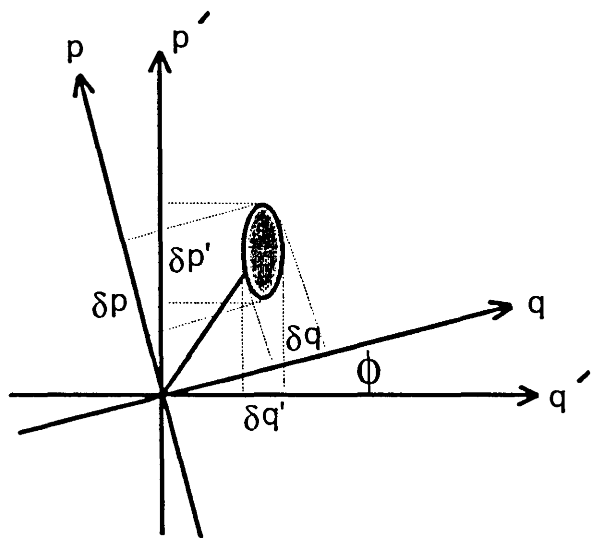

Figure 3.2: Squeezed State

The parameters μ and ν determine the shape and orientation of the squeezed ellipse in figure 3.2. The minimum and maximum axis of the ellipse are along a rotated coordinate system with angle:

$$\phi = Tan^{-1}\left[\frac{i(\mu^*\nu - \mu\nu^*)}{2 \mid \mu \mid\mid \nu \mid + \mu^*\nu + \mu\nu^*)}\right] \tag{3.17}$$

The corresponding minimum and maximum fluctuations are:

$$\delta q'^2 = \frac{1}{4} \mid\mid \mu \mid - \mid \nu \mid\mid^2 \tag{3.18}$$

$$\delta p'^2 = \frac{1}{4} \mid\mid \mu \mid + \mid \nu \mid\mid^2 \tag{3.19}$$

Note that condition 3.14 ensures that the mimimum uncertainty product $\delta q'\delta p' = 1/4$ is satisfied. Hence the area of the noise ellipse of a squeezed state is equal to the area of the noise circle of a coherent state. The average photon number in a squeezed state is given by:

$$\langle \alpha \mid \hat{a}^\dagger \hat{a} \mid \alpha \rangle = \mid \alpha \mid^2 + \mid \nu \mid^2 \tag{3.20}$$

A squeezed vacuum state is obtained when $\alpha = 0$. Unlike the vacuum state, squeezed vacuum has a non zero average photon number.

3.4 Generation of Squeezed States

A squeezed state is generated mathematically by a certain transformation of the wave function in the Schroedinger picture or equivalently by a transformation of the boson operators in the Heisenberg picture [24]. The latter formulation corresponds closely to the transformation of classical amplitudes in optics and often provides the most efficient means of calculation. According to the Heisenberg picture, a squeezed state is obtained when light passes through a system that transforms the Bose operator from $\hat{a}$ to $\mu\hat{a} + \nu\hat{a}^\dagger$ with μ and ν satisfying condition 3.14. Since linear systems preserve coherent states, the squeezing system must be nonlinear. Moreover, the nonlinear interaction must be phase dependent, i.e. it must exert a different force on the two quadrature components. The most succesful experiments have been performend using degenerate parametric amplification [36] and four-wave mixing [37]. Parametric ampification works by amplifying one quadrature while attenuating the conjugate component. Hence the fluctuations in one quadrature are suppressed at the expense of increased fluctuations in the conjugate component. Four-wave mixing can give rise to an intensity dependent index of refraction. The larger amplitude fluctuations experience a greater nonlinear phase shift. Thus the noise circle is deformed or squeezed into an ellipse. In this thesis we will concentrate on degenerate four wave mixing in optical fiber.

The main reason for using degenerate four wave mixing is that the nonlinear interaction is not bandwidth limited by phase matching. Hence ultra-short pulses may be used to enhance the nonlinear effect. The Kerr effect in silica fibers is a convenient means of realizing degenerate four wave mixing experimentally. Furthermore, a fiber ring can be used as a Mach-Zehnder interferometer to separate the pump and signal waves. The pump can then be reused as the local oscillator in homodyne detection. Figure 3.3 shows a schematic of the nonlinear Mach-Zehnder interferometer squeezer

[38]. The two input modes $\hat{a}$ and $\hat{b}$ are the signal and pump, respectively. The input beam splitter mixes these two fields into modes $\hat{c}$ and $\hat{d}$, which then propagate through nonlinear Kerr media. Nonlinear interactions in the Kerr media give rise to squeezing. Interference at the output beam splitter separates the pump $\hat{g}$ from the squeezed signal $\hat{f}$.

All of the quantum statistics of the squeezed light output are contained in its field annihilation operator $\hat{f}$. Thus, we wish to calculate the output operator $\hat{f}$ as a function of the input annihilation operators $\hat{a}$ and $\hat{b}$, by following the evolution of $\hat{a}$ and $\hat{b}$ in the Heisenberg formulation. We use different letters for the annihilation operators so as to indicate their assignments to different reference planes.

Consider first the effect of the input beam splitter. Since a beam splitter is linear and loss-free, the operators obey the same transformation as classical excitation amplitudes. For a 50/50 beam splitter and a particular choice of reference planes, we have the transformation:

$$\begin{aligned} \hat{c} &= \frac{1}{\sqrt{2}}(\hat{a} + i\hat{b}) \\ \hat{d} &= \frac{1}{\sqrt{2}}(i\hat{a} + \hat{b}) \end{aligned} \tag{3.21}$$

The corresponding unitary operator

$$\hat{U} = e^{i\frac{\pi}{4}(\hat{a}^\dagger\hat{b} + \hat{a}\hat{b}^\dagger)} \tag{3.22}$$

describes the evolution of the state function in the Schroedinger formulation. It

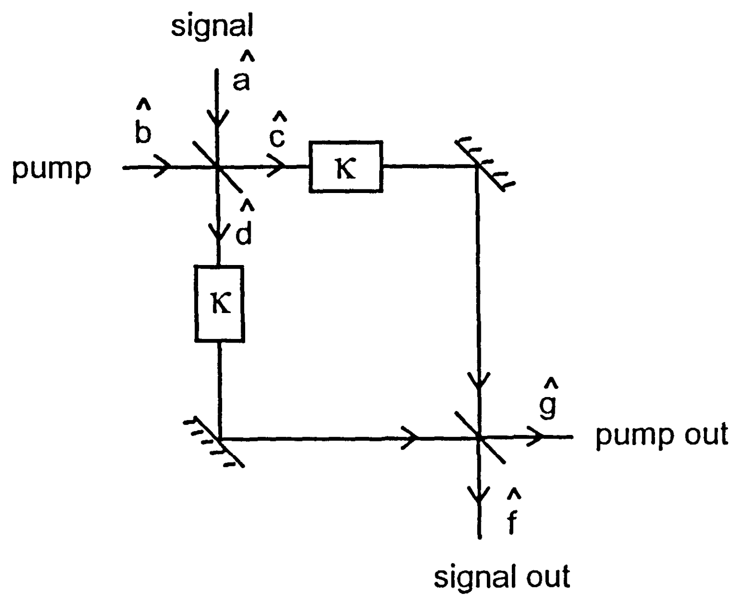

Figure 3.3: Nonlinear Mach-Zehnder Interferometer

follows that a beam splitter maintains coherent states, with the eigenvalues obeying the same transformation relations as the operators.

Now let us see how the operator $\hat{c}$ is changed as it passes through the nonlinear medium. The Hamiltonian of the Kerr media is [39]:

$$\hat{H} = \hbar\omega(\hat{c}^\dagger\hat{c} + \frac{1}{2}) + \frac{\hbar^2\omega^2\chi^{(3)}}{2V^2\epsilon^2}\hat{c}^{\dagger 2}\hat{c}^2 \tag{3.23}$$

where $\hbar$ is Planck's constant, ϵ is the dielectric constant, $\chi^{(3)}$ is the Kerr susceptibility, ω is the carrier frequency and V is the mode volume. The equation of motion in time for $\hat{c}$ is obtained in the usual way, which can be transformed into an equation of motion in space:

$$-i\hbar v_g\frac{d\hat{c}}{dz} = \frac{\hbar^2\omega^2\chi^{(3)}}{V\epsilon^2}\hat{c}^\dagger\hat{c}\hat{c} \tag{3.24}$$

Here v_g is the group velocity. Since $\hat{c}^\dagger\hat{c}$ commutes with the Hamiltonian, it is a constant of motion. Hence we may simply integrate the above equation to get:

$$\hat{c}(L) = e^{i\kappa\hat{c}^\dagger\hat{c}}\hat{c} \tag{3.25}$$

where we took a propagation distance L and $\kappa = \frac{\hbar^2\omega^2\chi^{(3)}L}{V\epsilon^2 v_g}$. A similar analysis applies to operator $\hat{d}$ in the second arm of the interferometer. After performing another (output) beam splitter transformation we obtain the operator $\hat{f}$ of the squeezed light:

$$\hat{f} = \frac{1}{\sqrt{2}}\left\{e^{i\frac{\kappa}{2}(\hat{a}^\dagger - i\hat{b}^\dagger)(\hat{a}+i\hat{b})}\frac{(\hat{a}+i\hat{b})}{\sqrt{2}} + e^{i\frac{\kappa}{2}(-i\hat{a}^\dagger+\hat{b}^\dagger)(i\hat{a}+\hat{b})}\frac{(i\hat{a}+\hat{b})}{\sqrt{2}}\right\} \tag{3.26}$$

To see that $\hat{f}$ does correspond to a squeezing operator we make some approximations. If we assume that the nonlinearity is weak and the pump $\hat{b}$ is much larger than the signal $\hat{a}$, then the exponents can be expanded to first order to yield [26]:

$$\hat{f} = e^{i\frac{\kappa}{2}\hat{b}^\dagger\hat{b}} \left\{ (1 + \frac{i\kappa}{2}\hat{b}^\dagger\hat{b})\hat{a} + \frac{i\kappa\hat{b}^2}{2}\hat{a}^\dagger \right\} \tag{3.27}$$

We have seen that an intense field (i.e. large average photon number) in a coherent state can be well approximated by a classical wave. Thus replacing the operator $\hat{b}$ with a classical c-number β, and ignoring constant phase factors, yields:

$$\hat{f} = \mu\hat{a} + \nu\hat{a}^\dagger \tag{3.28}$$

with $\mu = 1 + \frac{i\kappa}{2} \mid \beta \mid^2$ and $\nu = \frac{i\kappa}{2}\beta^2$. Note that condition 3.14 is satisfied to first order in nonlinear phase shift $\Phi = \kappa\beta^2$. Thus, for small nonlinear phase shift, the output light in mode $\hat{f}$ is squeezed. When the nonlinear phase shift is larger than unity, the squeezed ellipse gets deformed even further into a crescent shape [40]. We will be particularly interested in squeezed vacuum. If the input mode $\hat{a}$ is empty, i.e. if only vacuum fluctuations enter in mode $\hat{a}$, then the output signal will be in a squeezed vacuum state.

3.5 Beating the Shot Noise Limit

One of the most fruitfull applications of squeezed states is the ability to suppress noise in interferometric measurements below the shot noise limit. In this section we will analyze a Mach-Zehnder interferometer to illustrate this property of squeezed states. Fig. 3.4 shows a schematic of a Mach-Zehnder interferometer with homodyne detection. A probe beam $\hat{b}$ passes through both arms of the interferometer. Any disturbance that causes a phase shift between the two paths can be seen in the interference fringes of the output beam $\hat{f}$. Alternatively, the scheme of fig. 5 uses homodyne detection to measure the phase shift.

We will assume that the local oscillator $\hat{g}$ used for homodyne detection can be approximated as a classical signal. Then the operator corresponding to the difference current is given by:

$$\hat{\imath} = 2g\left\{e^{i\psi}\hat{f} + e^{-i\psi}\hat{f}^{\dagger}\right\} \tag{3.29}$$

where ψ is the phase and g is the (classical) amplitude of the local oscillator. Note that, with proper adjustment of the local oscillator phase ψ, $\hat{\imath}$ can measure either the in-phase or quadrature components of $\hat{f}$.

The output operator $\hat{f}$ can be written in terms of the inputs to the interferometer as:

$$\hat{f} = e^{i\frac{\phi}{2}}\left\{\hat{a}Sin(\frac{\phi}{2}) - \hat{b}Cos(\frac{\phi}{2})\right\} \tag{3.30}$$

Mode $\hat{b}$ is the probe beam and mode $\hat{a}$ is usually empty. Since we will normally be looking for small phase changes ϕ, $\hat{f}$ may be expanded as:

$$\hat{f} \;=\; \hat{f}_{\phi=0} + \frac{\partial\hat{f}}{\partial\phi}_{\phi=0}\Delta\phi$$

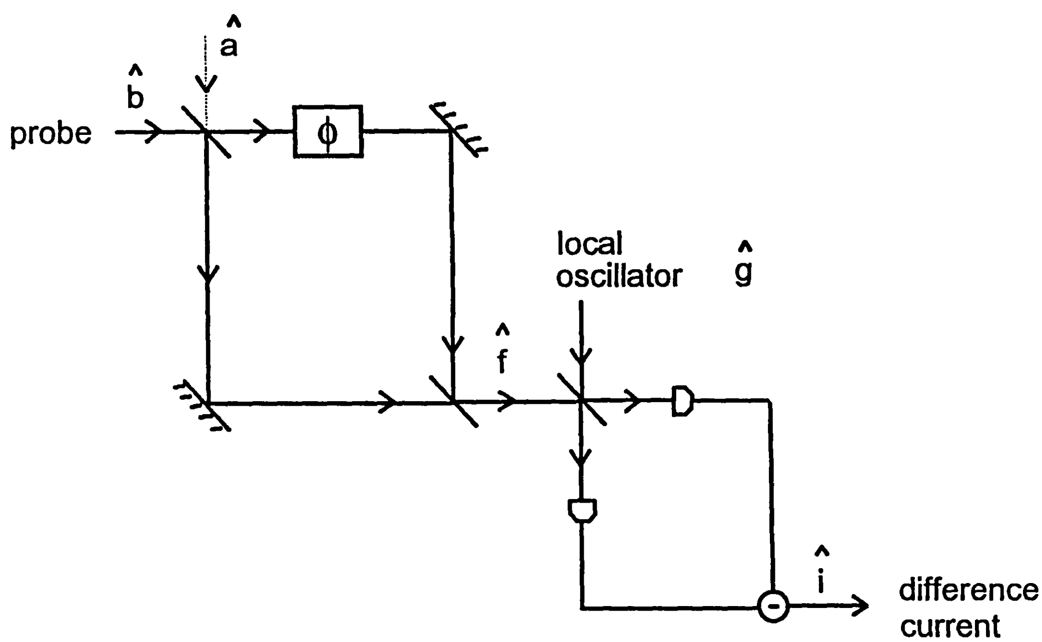

Figure 3.4: Mach-Zehnder interferometer with homodyne detection.

$$\hat{f} \;=\; i\hat{b} + \frac{1}{2}(i\hat{a} - \hat{b})\Delta\phi \tag{3.31}$$

Since the pump may approach a classical signal, all the noise will come from the vacuum fluctuations entering in mode $\hat{a}$. To see this, let us calculate the signal to noise ratio of the difference current $\hat{\imath}$. We fix the phase of the local oscillator so that the in-phase component of mode $\hat{f}$ is measured. The shot noise formula yeilds:

$$\frac{signal}{noise} = \frac{\langle \hat{b}^{\dagger}\hat{b}\rangle}{\delta\hat{q}_a^2} \mid \Delta\phi \mid^2 \tag{3.32}$$

Now, if input port a is empty, phase independent vacuum fluctuations will enter in mode $\hat{a}$ with $\delta\hat{q}_a^2 = 1/4$. In this case we get the usual shot noise formula. This was once thought to be the fundamental quantum limit in photodetection. But, if squeezed vacuum is injected in port a, the fluctuations in the in-phase part of $\hat{a}$ can be made arbitrarily small; the shot noise limit can be overcome. Recent experiments with squeezed states have shown a 5 db noise reduction below the shot noise limit [41,42]. In experiments utilizing the nonlinearities of optical fiber, a major source of classical noise that limits squeezing is Guided Acoustic Wave Brillouin Scattering (GAWBS) [43,44]. One way to limit GAWBS is to use a powerful pump in a short fiber [28]. If the round trip time through the fiber is less than the GAWBS inverse bandwidth, GAWBS noise is suppressed. Another possibility is to use the nonlinear Sangac ring as the reflector in a laser resonator to take advantage of the higher power inside the resonator. We analyze such a scheme in the next section.

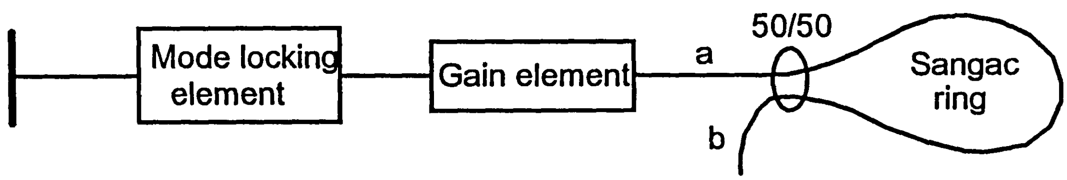

Figure 3.5: Squeezing inside a laser resonator.

3.6 Squeezing with a Number State

It may be advantageous to put the squeezer directly into a modelocked laser resonator, where higher power pulses may be utilized [45,26]. Fig. 3.5 shows a schematic of such a system. The nonlinear Sagnac ring acts as a reflector for the laser. All the power is reflected from the ring back into the laser cavity through port a of the 50/50 coupler. Normally the second port b would just emit vacuum fluctuations. For large enough average power in the laser, squeezing occurs in the fiber ring. In this case squeezed vacuum photons are emited from port b.

The analysis of squeezing for this system does not proceed as for the squeezer of section 3.4. The main difference is that the pump pulse is no longer in a coherent state. Since the pump pulse sees the nonlinearity of the fiber ring on every round trip through the laser cavity, it is transformed into a highly nonclassical state. The noise circle is deformed into a crescent shape with large phase uncertainty [40].

We will take the number state as our model for squeezing with a pump of large phase uncertainty [26]. A number state has total phase uncertainty, and as we shall discover, also presents some interesting quantum features. Fig. 3.6 shows the complete system for squeezing with a number state and homodyne detection. An analysis of the noise characteristics for this system will give us an idea of how squeezing inside a laser cavity will work.

We assume that the pump $\hat{b}$ has a large photon number n, so that the output pump $\hat{g}$ and signal operator $\hat{f}$ have the approximate form:

$$\hat{g} = e^{i\frac{\kappa}{2}\hat{b}^\dagger\hat{b}}\hat{b} \tag{3.33}$$

$$\hat{f} = e^{i\frac{\kappa}{2}\hat{b}^\dagger\hat{b}}\left\{(1+\frac{i\kappa}{2}\hat{b}^\dagger\hat{b})\hat{a}+\frac{i\kappa\hat{b}^2}{2}\hat{a}^\dagger\right\} \tag{3.34}$$

In section 3.4, where the pump was in a coherent state, we where able to linearize the operator $\hat{f}$ by replacing $\hat{b}$ with its classical c-number average. This simplification also showed that $\hat{f}$ was equivalent to a squeezing operator. In the case of a number state pump, we must retain the nonlinear form of the operator $\hat{f}$ shown above. A number state cannot be approximated as a classical wave, even in the limit of large photon number. This also implies that the output of the squeezer is not ordinary squeezed vacuum. We shall call it *modulated vacuum* for reasons that will become clear in what follows.

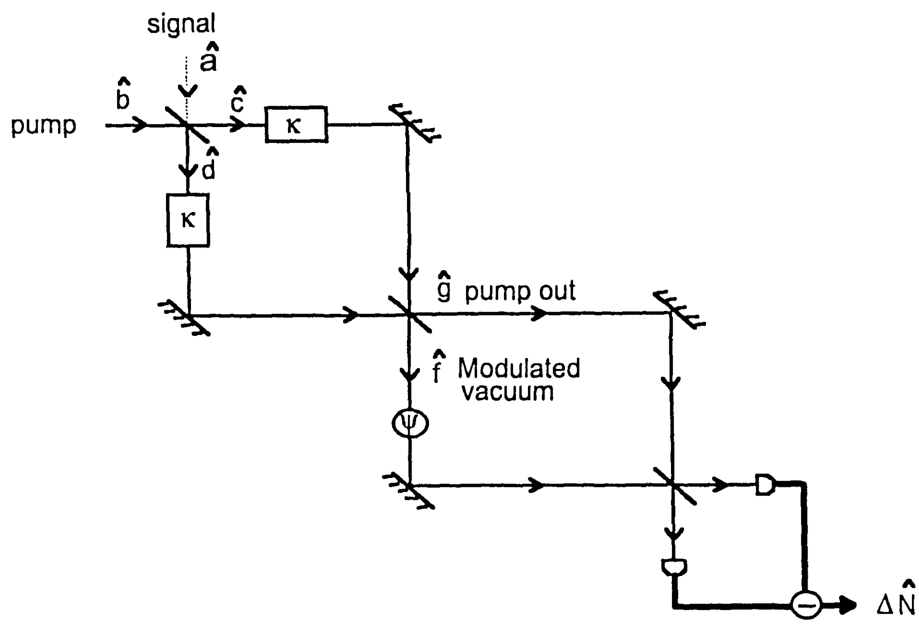

Figure 3.6: Squeezing with a number state.

Noise in the difference current of the homodyne detector is obtained from the matrix elements $\Delta\hat{N}$ and $\Delta\hat{N}^2$, where:

$$\Delta\hat{N} = \hat{g}^\dagger\hat{f} + \hat{g}\hat{f}^\dagger \tag{3.35}$$

$$\begin{aligned}\Delta\hat{N}^2 = \; & \hat{b}^\dagger\left\{(1+\frac{i\kappa}{2}\hat{b}^\dagger\hat{b})\hat{a} + \frac{i\kappa\hat{b}^2}{2}\hat{a}^\dagger\right\} + \\ & \left\{(1-\frac{i\kappa}{2}\hat{b}^\dagger\hat{b})\hat{a}^\dagger - \frac{i\kappa\hat{b}^{\dagger 2}}{2}\hat{a}\right\}\hat{b}\end{aligned} \tag{3.36}$$

Taking averages with input state $\mid\Psi\rangle = \mid 0\rangle_a \mid n\rangle_b$ yeilds:

$$\langle\Psi \mid \Delta\hat{N} \mid \Psi\rangle = 0 \tag{3.37}$$

$$\begin{aligned}\langle\Psi \mid \Delta\hat{N}^2 \mid \Psi\rangle = \; & n + \frac{\kappa^2}{4}(2n^3 - 3n^2) - \kappa(n^2 - n)Sin(2\psi) - \\ & \frac{\kappa^2}{2}(n^3 - 2n^2)Cos(2\psi)\end{aligned} \tag{3.38}$$

The analogous result for a coherent state pump is:

$$\langle\Psi \mid \Delta\hat{N}^2 \mid \Psi\rangle = \bar{n} + \frac{\kappa^2}{4}(2\bar{n}^3 - \bar{n}^2) - \kappa\bar{n}Sin(2\psi) - \frac{\kappa^2\bar{n}^3}{2}Cos(2\psi) \tag{3.39}$$

where $\bar{n}$ is the average photon number. Note that the two results agree to first order in nonlinear phase shift. A more detailed analysis shows that the noise characteristics agree to higher orders [27]. It is interesting that the results for squeezing with a number state are the same as for a coherent state, while the physical mechanism is rather different. A coherent state pump transforms the noise circle in phase space into an ellipse, giving rise to ordinary squeezing. The operation of a number state pump, on the other hand, may be visualized by assigning to each phase component its own probability ellipse. This results in a superposition of many probability ellipses with different orientations of major axes. Hence the amplitude noise of modulated vacuum is actually greater than unsqueezed vacuum fluctuations. But when the pump is reused as the local oscillator in homodyne detection, since it is phase correlated with the modulated vacuum, the minimum noise can be extracted from each ellipse.

Chapter 4

Test of Bell's Inequality With Squeezed Light

4.1 EPR Effect With Photon Interference

In describing quantum interference Feynman wrote that it is "... a phenomenon which is impossible, absolutely impossible, to explain in any classical way ... it contains the only mystery." [46] Although Feynman was refering to one-photon interference in Young's double slit experiment, his remark is even more relevant to nonlocal two-photon interference. Two-particle interferometry employs spatially separated, quantum mechanically entangled states [47]. The interference fringes appear in the coincidence counting rate for detecting both particles. The EPR experiment studied in Ch. 2 is an example of two-particle interferometry. Two-photon interference between parametrically generated photon pairs was experimentally observed by Mandel and co-workers [48,49]. They observed a nonlocal correlation in the simultaneous detection of the signal and idler photons. Subsequently, parametrically generated photon pairs were used in EPR experiments to test Bell's inequality [50-53]. In this chapter we will describe a test of Bell's inequality based on nonlocal interference between photons of squeezed vacuum [29,53]. A schematic of this system is shown in fig. 4.1.

Squeezed vacuum, labeled by mode $\hat{f}$, is split by a 50/50 beam splitter into modes $\hat{h}_1$ and $\hat{h}_2$. These modes propagate in opposite directions to homodyne detectors. Mode $\hat{h}_1$ interferes with coherent local oscillator $\hat{e}_1$ at a 50/50 beam splitter and the two emerging modes $\hat{i}_{1+}$ and $\hat{i}_{1-}$ are detected with photomultipliers. Similarly, mode $\hat{h}_2$ is detected at a second homodyne detector. The difference current of a homodyne detector reflects the phase information of the incident light (see Ch.3). Since photons of squeezed vacuum are quantum correlated in phase, we anticipate an interference effect between the difference currents. This nonlocal interference appears experimentally in the correlation function:

$$E(\phi_1, \phi_2) = \frac{\langle (J_{1+} - J_{1-})(J_{2+} - J_{2-}) \rangle}{\langle (J_{1+} + J_{1-})(J_{2+} + J_{2-}) \rangle} \tag{4.1}$$

The brackets indicate a time average over the experimentally measured detector currents. ϕ_1 and ϕ_2 are the phases of local oscillators $\hat{e}_1$ and $\hat{e}_2$.

The current of a photomultiplier tube is proportional to the electron photoemission rate. The electron emission rate is the same as the photon absorbson rate, which is proportional to the average photon number of the incident light. Hence the quantum mechanical version of 4.1 is given by:

$$E(\phi_1, \phi_2) = \frac{\langle \Psi \mid : (\hat{i}^{\dagger}_{1+}\hat{i}_{1+} - \hat{i}^{\dagger}_{1-}\hat{i}_{1-})(\hat{i}^{\dagger}_{2+}\hat{i}_{2+} - \hat{i}^{\dagger}_{2-}\hat{i}_{2-}) : \mid \Psi \rangle}{\langle \Psi \mid : (\hat{i}^{\dagger}_{1+}\hat{i}_{1+} + \hat{i}^{\dagger}_{1-}\hat{i}_{1-})(\hat{i}^{\dagger}_{2+}\hat{i}_{2+} + \hat{i}^{\dagger}_{2-}\hat{i}_{2-}) : \mid \Psi \rangle} \tag{4.2}$$

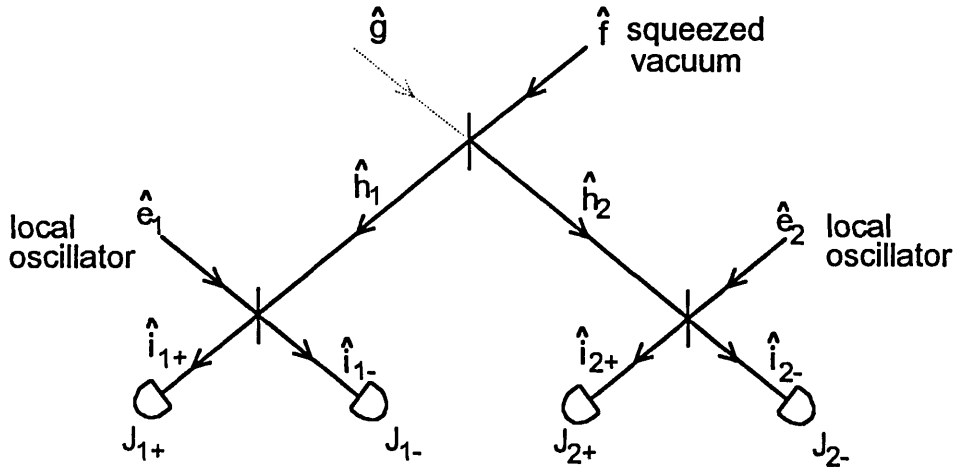

Figure 4.1: Measurement of Bell's inequality.

where :: denotes normal ordering of the operators. Normal ordering follows from the quantum rules for calculating the transition rate for a joint absorption of photons at two different detectors [54].

The experimental scheme described above allows a comparison between the predictions of quantum and hidden variable theory. In particular, nonlocal quantum interference may give rise to a violation of Bell's inequality. A quadrature phase form of Bell's inequality can be derived from a hidden variable theory, as was done in Ch.2 for the system of two spin-1/2 particles [50,55]:

$$-2 \leq E(\phi_1, \phi_2) - E(\phi_1, \phi_{2'}) + E(\phi_{1'}, \phi_{2'}) - E(\phi_{1'}, \phi_2) \leq 2 \tag{4.3}$$

4.2 Conditions for Violation

In this section we will derive the conditions necessary for violation of Bell's inequality with a squeezed vacuum input in mode $\hat{f}$. The calculation is done in Heisenberg's formulation. Averages are taken with respect to the input state:

$$\mid \Psi\rangle = \mid \alpha e^{i\phi_1}\rangle_{b_1} \mid \alpha e^{i\phi_2}\rangle_{b_2} \mid \Phi_s\rangle_f \mid 0\rangle_g \tag{4.4}$$

The local oscillator modes $\hat{b}_1$ and $\hat{b}_2$ are in coherent states. For simplicity, we let them have equal amplitudes α but different phases ϕ_1 are ϕ_2. Since the input beamsplitter has one empty input port, we must take into account the vacuum fluctuations entering this port in mode $\hat{g}$. The input mode $\hat{f}$ is in a squeezed vacuum state, created in the nonlinear Mach-Zehnder squeezer of Ch. 3.

The correlation function E may be simplified by first taking averages with respect to the states of the local oscillators and vacuum mode $\hat{g}$, which yields [29,53]:

$$E(\phi_1, \phi_2) = -\frac{\alpha^2 \left\{ \langle \Phi_s \mid \hat{f}^\dagger \hat{f} \mid \Phi_s \rangle Sin(\phi_1 - \phi_2) + \mid \langle \Phi_s \mid \hat{f}^2 \mid \Phi_s \rangle Sin(\phi_1 + \phi_2 - \epsilon) \right\}}{\alpha^4 + \langle \Phi_s \mid \hat{f}^\dagger \hat{f} \mid \Phi_s \rangle \alpha^2 + \frac{1}{4} \langle \Phi_s \mid \hat{f}^{\dagger 2} \hat{f}^2 \mid \Phi_s \rangle} \tag{4.5}$$

where $\langle \Phi_s \mid \hat{f}^2 \mid \Phi_s \rangle = Re^{i\epsilon}$. It is evident that the correlation function E depends on the statistical properties of the light corresponding to operator $\hat{f}$. We found in Ch. 3 how $\hat{f}$ depends on the signal $\hat{a}$ and pump $\hat{b}$ operators of the nonlinear Mach-Zehnder squeezer:

$$\hat{f} = \frac{1}{\sqrt{2}} \left\{ e^{i\frac{\kappa}{2}(\hat{a}^\dagger - i\hat{b}^\dagger)(\hat{a} + i\hat{b})} \frac{(\hat{a} + i\hat{b})}{\sqrt{2}} + e^{i\frac{\kappa}{2}(-i\hat{a}^\dagger + \hat{b}^\dagger)(i\hat{a} + \hat{b})} \frac{(i\hat{a} + \hat{b})}{\sqrt{2}} \right\} \tag{4.6}$$

Recall that the wave function at the input to the squeezer is $\mid \Phi_s \rangle_f = \mid 0 \rangle_a \mid \beta \rangle_b$. Mode $\hat{a}$ is in a vacuum state and mode $\hat{b}$ is a coherent state pump. The calculations of averages like $\langle \hat{f}^\dagger \hat{f} \rangle$ are considerably simplified by transformation to a Hilbert space spanned by eigenstates of operators $\hat{c}$ and $\hat{d}$, defined by:

$$\begin{aligned} \hat{c} &= \frac{1}{\sqrt{2}}(\hat{a} + i\hat{b}) \\ \hat{d} &= \frac{1}{\sqrt{2}}(i\hat{a} + \hat{b}) \end{aligned} \tag{4.7}$$

Now we have a simple expression for $\hat{f}$:

$$\hat{f} = \frac{1}{2} \left\{ \hat{F}\hat{c} + i\hat{G}\hat{d} \right\} \tag{4.8}$$

where $\hat{F} = e^{i\kappa\hat{c}^\dagger\hat{c}}$ and $\hat{G} = e^{i\kappa\hat{d}^\dagger\hat{d}}$. Averages are now taken with respect to the transformed state function $\mid \Phi_s\rangle = \mid \frac{i\beta}{\sqrt{2}}\rangle_c \mid \frac{\beta}{\sqrt{2}}\rangle_d$. Taking β to be real, we turn next to the calculation of $\langle \Phi_s \mid \hat{f}^\dagger \hat{f} \mid \Phi_s\rangle$. In terms of operators $\hat{c}$ and $\hat{d}$, we have:

$$\hat{f}^\dagger \hat{f} = \frac{1}{2}\left\{\hat{c}^\dagger\hat{c} - i\hat{d}^\dagger\hat{G}^\dagger\hat{F}\hat{c} + \hat{c}^\dagger\hat{F}^\dagger\hat{G}\hat{d} + \hat{d}^\dagger\hat{d}\right\} \tag{4.9}$$

It is clear that

$$\langle\hat{c}^\dagger\hat{c}\rangle = \langle\hat{d}^\dagger\hat{d}\rangle = \frac{\beta^2}{2} \tag{4.10}$$

Using the relations

$$\hat{F}\hat{c} \mid \frac{i\beta}{\sqrt{2}}\rangle_c \mid \frac{\beta}{\sqrt{2}}\rangle_d = \frac{i\beta}{\sqrt{2}} \mid \frac{i\beta}{\sqrt{2}}e^{i\kappa}\rangle_c \mid \frac{\beta}{\sqrt{2}}\rangle_d \tag{4.11}$$

$$\hat{G}\hat{d} \mid \frac{i\beta}{\sqrt{2}}\rangle_c \mid \frac{\beta}{\sqrt{2}}\rangle_d = \frac{\beta}{\sqrt{2}} \mid \frac{i\beta}{\sqrt{2}}\rangle_c \mid \frac{\beta}{\sqrt{2}}e^{i\kappa}\rangle_d \tag{4.12}$$

we obtain

$$\langle\hat{d}^\dagger\hat{G}^\dagger\hat{F}\hat{c}\rangle = -\langle\hat{c}^\dagger\hat{F}^\dagger\hat{G}\hat{d}\rangle = \frac{i\beta^2}{2}\langle\frac{i\beta}{\sqrt{2}}e^{i\kappa} \mid \frac{i\beta}{\sqrt{2}}\rangle\langle\frac{\beta}{\sqrt{2}} \mid \frac{\beta}{\sqrt{2}}e^{i\kappa}\rangle \tag{4.13}$$

Then using a property of coherent states

$$\langle\alpha \mid \gamma\rangle = e^{(-\frac{1}{2}|\alpha|^2 - \frac{1}{2}|\gamma|^2 + \alpha^*\gamma)} \tag{4.14}$$

we finally get

$$\langle\hat{f}^\dagger\hat{f}\rangle = \frac{\beta^2}{2}\left\{1 - e^{\beta^2(cos\kappa - 1)}\right\} \tag{4.15}$$

Similarly, we get

$$\langle \hat{f}^2 \rangle = \frac{\beta^2}{2}\left\{ e^{i\kappa} e^{\frac{\beta^2}{2}(e^{i2\kappa}-1)} - e^{\beta^2(e^{i\kappa}-1)} \right\} \tag{4.16}$$

$$\langle \hat{f}^{\dagger 2} \hat{f}^2 \rangle = \frac{\beta^4}{2}\left\{ \frac{3}{4} - Cos(\kappa) e^{\beta^2(Cos(\kappa)-1)} - \frac{1}{4} e^{\beta^2(Cos(2\kappa)-1)} \right\} \tag{4.17}$$

We may now determine under what conditions Bell's inequality is violated. When the correlation function E is of the form:

$$E(\phi_1, \phi_2) = ASin(\phi_1 - \phi_2) + BSin(\phi_1 + \phi_2 - \epsilon) \tag{4.18}$$

violation occurs if and only if $A^2 + B^2 > 1/2$ [53]. Figure 4.2 shows a plot of $A^2 + B^2$ as a function of local oscillator photon number and nonlinear phase shift $\Phi = \kappa\beta^2$. Violation of Bell's inequality occurs only when the local oscillator photon number and the nonlinear phase shift are less than unity. The latter result also implies small average photon number of the squeezed vacuum since it is roughly equal to $\Phi^2/4$.

4.3 Limit of Small Photon Number

To understand why the limit of small photon number violates Bell's inequality, we expand the squeezed vacuum state in this limit and follow its evolution. The expansion will be truncated to first order in Φ, which corresponds to the limit of small photon number. In this limit, the field operator at the output of the squeezer is given by $\hat{f} = \mu\hat{a} + \nu\hat{a}^\dagger$, where $\mu = 1 + i\Phi/2$ and $\nu = i\Phi/2$. Since $\hat{f}$ annihilates the vacuum

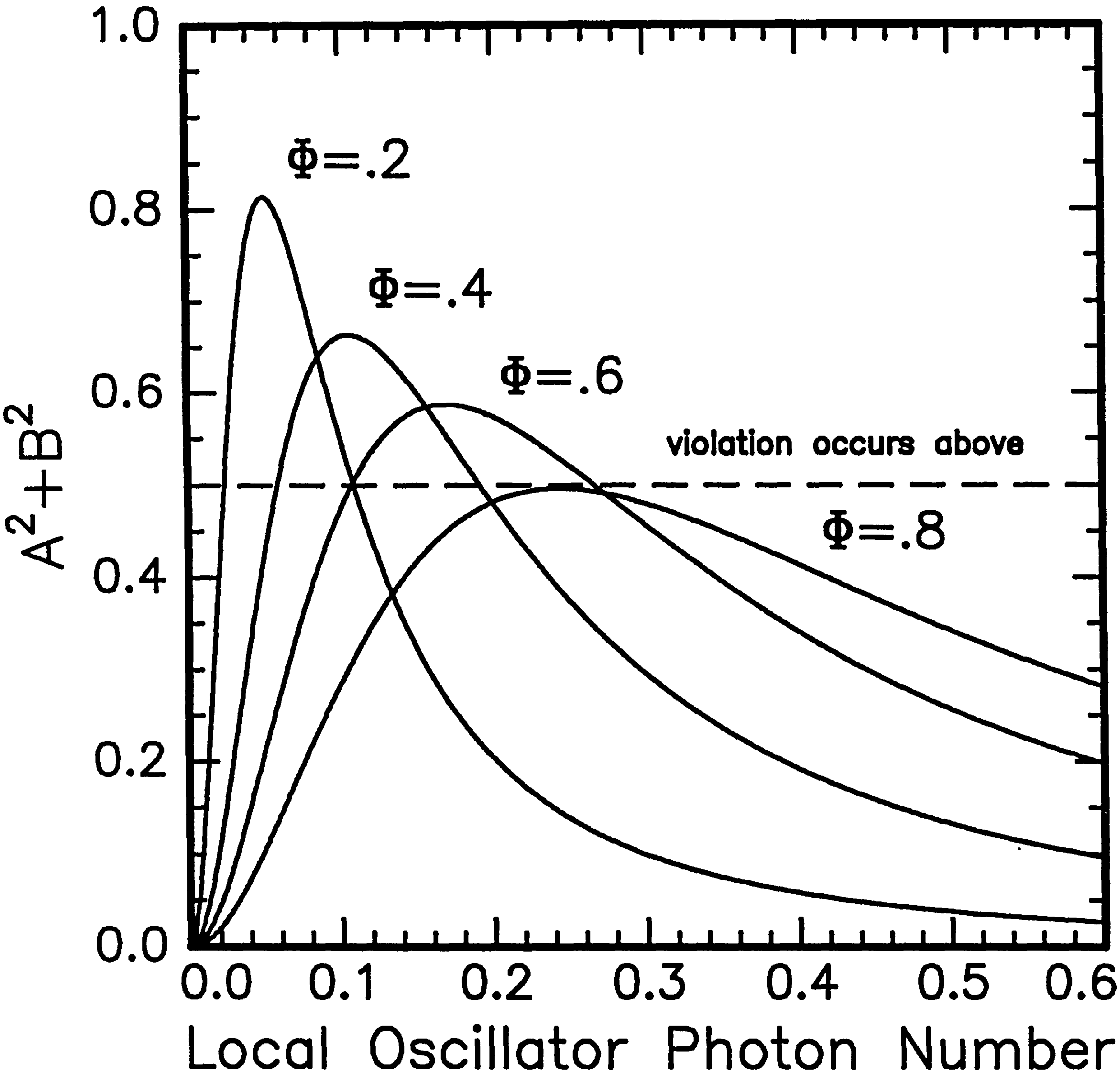

Figure 4.2: Plot of Bell's inequality.

state, we have:

$$(\mu\hat{a} + \nu\hat{a}^{\dagger}) \mid \Phi_s\rangle = 0 \tag{4.19}$$

Expanding $\mid \Phi_s\rangle$ in number states then yields:

$$\begin{aligned} (\mu\hat{a} + \nu\hat{a}^{\dagger})\sum c_n \mid n\rangle &= 0 \\ \sum \left\{\mu c_n\sqrt{n} \mid n-1\rangle + \nu c_n\sqrt{n+1} \mid n+1\rangle\right\} &= 0 \end{aligned} \tag{4.20}$$

This equation provides a recursion relation, which can be used to solve for c_n. For non-zero c_0, only the even terms remain:

$$c_{2n} = (-1)^n \frac{\nu}{\mu} \frac{\sqrt[n]{(2n)!}}{2^n n!} c_0 \tag{4.21}$$

The normalization condition is used to solve for c_0. If we only keep terms up to first order in Φ, then the input state in fig. 4.1 is:

$$\mid \Phi_s\rangle_f \mid 0\rangle_g = \mid 0\rangle_f \mid 0\rangle_g - \frac{i\Phi}{2\sqrt{2}} \mid 2\rangle_f \mid 0\rangle_g \tag{4.22}$$

The state after the input beam splitter is obtained through the operation of the unitary operator $\hat{U}$ (see Ch. 3), which yields:

$$\mid \Phi\rangle_{BS} = \mid 0\rangle_{h_1} \mid 0\rangle_{h_2} - \frac{\Phi}{4} \mid 1\rangle_{h_1} \mid 1\rangle_{h_2} - \frac{i\Phi}{4\sqrt{2}} \mid 2\rangle_{h_1} \mid 0\rangle_{h_2} + \frac{i\Phi}{4\sqrt{2}} \mid 0\rangle_{h_1} \mid 2\rangle_{h_2} \tag{4.23}$$

The first two terms are similar to the state of a twin beam of parametrically generated light in the limit of small parametric gain, which was shown to give maximum violation of Bell's inequality [53]. The off diagonal terms result from the beam splitting process. The beam splitter adds phase independent "noise" that decreases the quantum phase correlations in Bell's inequality. This noise is due to the vacuum fluctuations that enter in the mode $\hat{g}$. It can be suppressed by using two squeezed vacuum inputs. In this case the input state is:

$$\mid \Phi_s\rangle_f \mid \Phi_s\rangle_g = \mid 0\rangle_f \mid 0\rangle_g - \frac{i\Phi}{2\sqrt{2}} \mid 2\rangle_f \mid 0\rangle_g - \frac{i\Phi}{2\sqrt{2}} \mid 0\rangle_f \mid 2\rangle_g \tag{4.24}$$

Operating on this state with the beam splitter transformation $\hat{U}$, we now get:

$$\mid \Phi\rangle_{BS} = \mid 0\rangle_{h_1} \mid 0\rangle_{h_2} - \frac{\Phi}{2} \mid 1\rangle_{h_1} \mid 1\rangle_{h_2} \tag{4.25}$$

The correlation function E is easily calculated for this state to be:

$$E(\phi_1, \phi_2) = \frac{\alpha^2\Phi}{\alpha^4 + \frac{1}{2}\alpha^2\Phi^2 + \frac{1}{4}\Phi^2} Cos(\phi_1 + \phi_2) \tag{4.26}$$

In the limit $\Phi \to 0$ while $\alpha \to \sqrt{\Phi}/2$, E approaches $Cos(\phi_1 + \phi_2)$. This form of E gives maximum violation of Bell's inequality. $A^2 + B^2$ for two squeezed vacuum inputs is plotted in fig. 4.3 for $\Phi = .2$ along with the corresponding result for one squeezed vacuum input (dashed curve). To make a fair comparison, the total average photon number of the squeezed light must be made the same for both cases. This was done by setting $\Phi \to \Phi/\sqrt{2}$ in the results for two squeezed vacuum inputs.

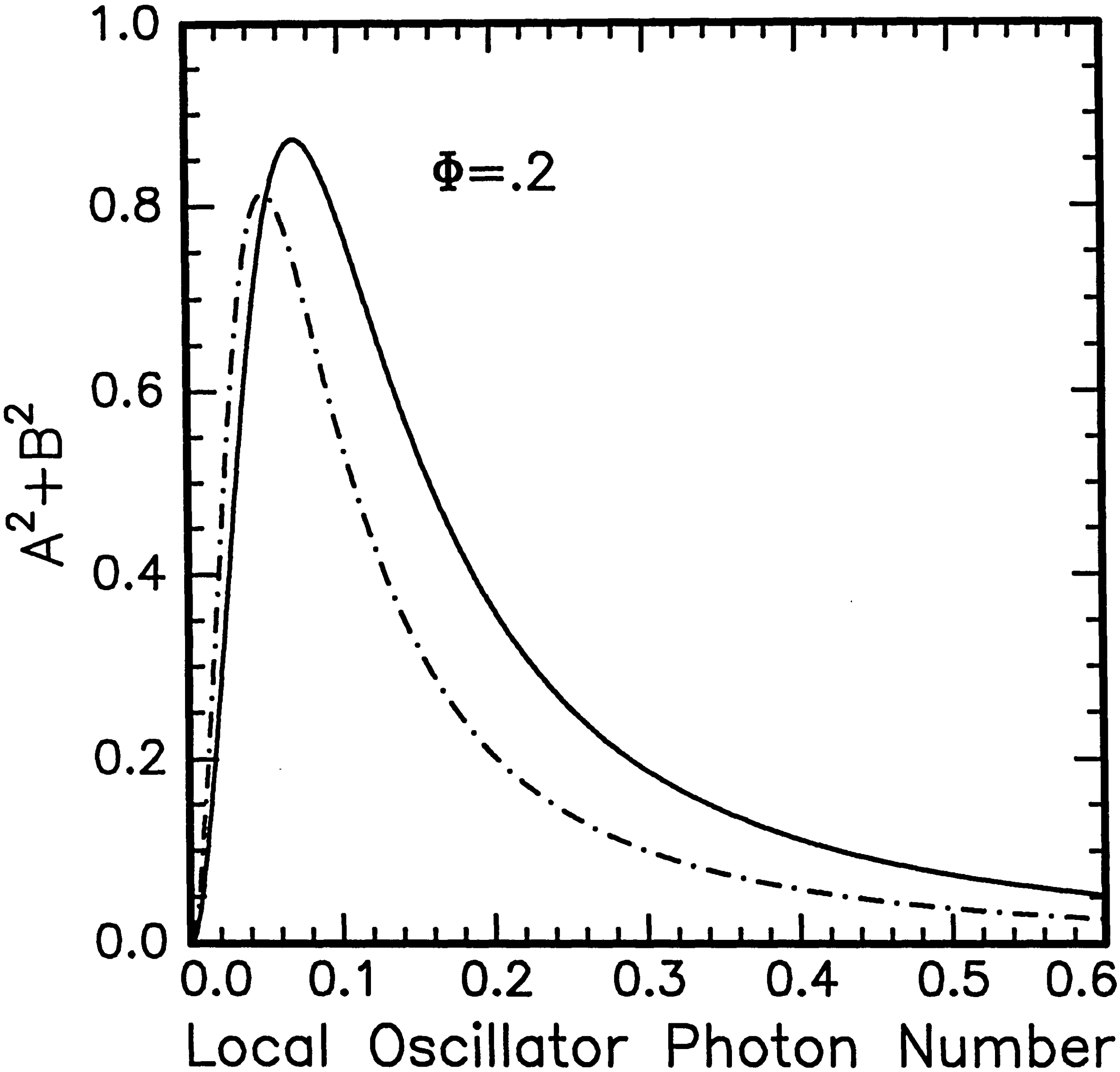

Figure 4.3: Violation with two squeezed inputs.

4.4 Applications to Quantum Cryptography

Quantum physics was first introduced into cryptography by the work of Weisner [56] and Bennett and Brassard [57]. Their idea was based on the fact that quantum systems are inevitably perturbed by even the slightest measurement. The system first proposed by Bennett and Brassard (BB84) encodes a random sequence of 1's and 0's into single quanta in orthogonal polarization states. This random sequence serves as the key in public-key cryptography. The security of the key is ensured by Heisenberg's uncertainty principle. Any measurements on one polarization by an eavesdropper inevitably randomizes the conjugate component, giving him away. A system based on EPR correlated spin-1/2 particles was first proposed by Ekert [32], where Bell's inequality provides the test for eavesdroping. Ekert et. al. [58] also proposed a more practical system using correlated photon pairs. We will consider a similar quantum cryptography system based on non-local phase correlations between squeezed vacuum states of light.

First we review some basic ideas in quantum cryptography. The quantum key distribution system is most simply described in terms of spin-1/2 particles. The objective is to provide two users, say Bob and Alice, with a secret key wich they can use to open cryptograms. The system is very similar to the experiment shown in fig. 2.1. Two spin-1/2 particles are emitted in a singlet state and propagate in opposite directions. Particle 1 is sent to Bob, who measures randomly either the x or y component of spin. Alice makes similar measurements on particle 2. After recording the results of their measurements on a sequence of particles, Alice and Bob communicate publicly to determine when they both measured the same

Bob:	+x	+y	-y	-y	-x
Alice:	-x	+x	+y	+y	+x
	√		√	√	√
Key:	1		1	1	0

Figure 4.4: Quantum key distribution.

component of spin. They then discard all measurements where they did not both measure the same spin component or where either of the detectors failed due to imperfect quantum efficiency. Since the particles are in a singlet state, and assuming the quantum system was not disturbed by eavesdropping, the remaining data will be perfectly anticorrelated. For example, if Bob measures $\hat{S}_x = -1$, then Alice will measure $\hat{S}_x = +1$. Thus a random sequence of 1's and 0's is established (see fig. 4.4).

Any eavesdropping by an adversary will destroy the nonlocal correlations. Bob and Alice can check for this by subjecting a subset of their data to a quantum statistical test. This test involves comparing the correlations between their measurements and the predictions of quantum mechanics. One possibility is to use Bell's inequality.

In the case of spin-1/2 particles it is simpler to just look at the sum of two correlation functions [59]:

$$E(\vec{x},\vec{x}) + E(\vec{y},\vec{y}) = \langle \hat{S}_{x1}\hat{S}_{x2}\rangle + \langle \hat{S}_{y1}\hat{S}_{y2}\rangle \tag{4.27}$$

According to the quantum rules this sum is equal to -2. It was proved by Bennett, Brassard and Mermin [59] that any eavesdropping, no matter how sophisticated, will ruin the quantum correlations.

The EPR optics analog of measuring $\hat{S}_x/\hat{S}_y$ of a spin-1/2 particle is to measure $\hat{q}/\hat{p}$ of a photon. The detection is achieved by replacing Stern-Gerlach analyzers with homodyne detectors as in fig. 4.1. A homodyne detector measures the in-phase component $\hat{q}$, if its local oscillator is in phase with the signal, and the quadrature-phase component $\hat{p}$, if it is $\pi/2$ out of phase. The analogy becomes most transparent when we represent the $\hat{q}/\hat{p}$ operators as matrices in the limit of small photon number. In this limit, we can work in a subspace spanned by photon number states $\mid 0\rangle$ and $\mid 1\rangle$, defined by vectors:

$$\mid 0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; and \; \mid 1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{4.28}$$

The in-phase and quadrature-phase operators are then analogous to the Pauli spin matrices, viz.

$$\hat{q} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{1}{2}\hat{\sigma}_x \tag{4.29}$$

$$\hat{p} = \frac{1}{2}\begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix} = -\frac{1}{2}\hat{\sigma}_y \tag{4.30}$$

We may also define "spin up" and "spin down" eigenstates. For example, the eigenstates of $\hat{q}$ are:

$$| +\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \; and \; | -\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \tag{4.31}$$

The ideal quadrature entangled state, analogous to the spin singlet state, would then be:

$$| \Psi\rangle = \frac{1}{\sqrt{2}}\{| +\rangle_{h_1} \, | -\rangle_{h_2} - | -\rangle_{h_1} \, | +\rangle_{h_2}\} \tag{4.32}$$

where $\hat{h}_1$ and $\hat{h}_2$ refer to the modes of fig. 4.1. In terms of photon number states this wavefunction is given by:

$$| \Psi\rangle = \frac{1}{\sqrt{2}}\{| 0\rangle_{h_1} \, | 1\rangle_{h_2} - | 1\rangle_{h_1} \, | 0\rangle_{h_2}\} \tag{4.33}$$

If it were possible to prepare such entangled states of light, then we could set up our quantum key distribution system just as the one described above, where eq. 4.27 is used to test for eavesdropping. Unfortunately, such states are difficult to produce in the laboratory. The best we can do with squeezed states is the entangled wavefunction:

$$| \Phi\rangle_{BS} = | 0\rangle_{h_1} \, | 0\rangle_{h_2} - \frac{\Phi}{2} \, | 1\rangle_{h_1} \, | 1\rangle_{h_2} \tag{4.34}$$

This wavefunction does not give the perfect correlations required for the quantum statistical test 4.27. But we saw that squeezed vacuum photons do give rise to non-local interference effects that violate Bell's inequality. Hence, by analogy to Ekert's scheme for spin-1/2 particles, we may use Bell's inequality to test for eavesdropping. This scheme may be realized experimentally by a simple modification of the setup in fig. 4.1. The quantum key is encoded into pulses of squeezed vacuum. Bob and Alice measure both the difference and sum currents of their homodyne detectors. The sum current gives the string of 1's and 0's that represent the key. The difference current is used to test for eavesdropping via Bell's inequality. Bob and Alice vary the phases of their local oscillators randomly to prevent the eavesdropper from doing a quantum non-demolition (QND) measurement [60]. A QND measurement can detect an observable without disturbing its free motion. Of course the conjugate observable is still altered. Since the eavesdropper does not know which quadrature component Bob and Alice measure untill after they communicate publicly, he cannot avoid disturbing the quantum statistics of the photons. A general proof that eavesdropping fails is similar to that done by Ekert for spin-1/2 particles.

There are some issues that must be worked out before the ideal scheme described above can be used in practice. The most obvious problem is how to transmit quantum information over long distances. With present technology, losses limit quantum transmissions to a few kilometers. In addition we cannot create perfect squeezed states. One would have to consider the situation where the eavesdropper has better technology. If the eavesdropper can produce 10 db squeezing while we could only get 5 db, then maybe he can think of a measuring scheme that does not effect Bell's inequality enough to be detected. These practical issues, along with an experimental realization of Bell's inequality with squeezed light, are left for future work.

Chapter 5

Conclusions

The hidden variable theory was developed to explain certain "paradoxical" results of quantum mechanics, most notably the EPR paradox. Ever since the famous debates between Einstein and Bohr on the foundations of quantum mechanics, scientists have looked to find new ways to probe into the quantum world. The laser opened up for exploration the world of quantum optics. Extensive experiments were performed on photon interference to understand Feynman's "only mystery." Bohr's complementary principle held up to the test; photons did indeed behave as both waves and particles. This is no longer a mystery but a fact of life.

The development of ultra-short pulse lasers offered even more prospects in quantum optics. Nonlinearities in optical media could now be used to create highly non-classical states of light. This made possible investigations into the nature of quantum measurement. The collapse of the wavefunction has always been a problematic notion in quantum theory. It leads to paradoxical results when applied to two-particle interference experiments. The optics experiment discussed in this thesis presents the same paradox. Photons of squeezed vacuum from a nonlinear Mach-Zehnder interferometer exhibit non-local quantum correlations. The unique properties of squeezed states were reviewed and a new approach for squeezing inside a laser resonator was analyzed. It was shown that nonlocal quantum correlations between photons of squeezed vacuum can be measured and compared with both quantum and hidden variable theory using Bell's inequality. The predictions of quantum mechanics lead to a violation of Bell's inequality in the limit of small quantum numbers.

Squeezed states were originally developed for special applications in interferometry, like gravity wave detection. New ideas for applications are continuing to evolve. We presented a scheme that uses pulses of squeezed vacuum for quantum cryptography. The non-local phase correlations are used to check for eavesdropping. A possibility for future work would be an experimental realization of the quantum cryptography system presented in this thesis. Experiments to improve squeezing performance beyond the 5 db result will also continue at the MIT ultra-fast optics group.

Using the self consistent quantum formalism of nonlinear optics, we were able to do a full analysis of the EPR experiment, including quantization of the measuring operatus. Quantization of the measuring system may be the only way to get around the paradoxes resulting from the "collapse of the wavefuntion" postulate. But we will still have to live with the fact that reality exists only when it is measured.

Bibliography

[1] E. Schroedinger, *Annalen der Physik,* vol. 79, p. 361, 1926.

[2] M. Born, *Zeitschrift fur Physik,* vol. 40, p. 167, 1926.

[3] W. Heisenberg, *The Physical Principles of the Quantum Theory,* University of Chicago Press, 1930.

[4] N. Bohr, *Atomic Theory and the Description of Nature,* Cambridge University Press, 1934

[5] Proceedings of the Fifth Solvay Conference, Brussels, 1927.

[6] B. d'Espagnat, "The quantum theory and reality," Scientific American, p. 158, Nov. 1979.

[7] A. Einstein, B. Podolsky and N. Rosen, "Can quantum-mechanical description of physical reality be considered complete," *Phys. Rev.,* vol. 47, p. 777, 1935.

[8] N. Bohr, "Can quantum-mechanical description of physical reality be considered complete," *Phys. Rev.*, vol. 48, p. 696, 1935.

[9] J. S. Bell, *Physics,* vol. 1, p. 195, 1964.

[10] A. Aspect, P. Grangier and G. Roger, "Experimental realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment:* A new violation of Bell's inequalities," *Phys. Rev. Lett.,* vol. 49, p. 91, 1982.

[11] A. Aspect, J. Dalibard and G. Roger, "Experimental test of Bell's inequalities using time-varying analyzers," *Phys. Rev. Lett.,* vol. 49, p. 1804, 1982.

[12] J. F. Clauser and A. Shimony, "Bell's theorem: experimental tests and implications," *Rept. Prog. Phys.,* vol. 41, p. 1881, 1978.

[13] J. Baggott, *The Meaning of Quantum Theory,* Oxford University Press, 1992.

[14] J. S. Bell in P.W. Davies and J.R. Brown, *The Ghost in the Atom,* Cambridge University Press, 1986.

[15] M. Planck, *Vert. dt. Phys. Ges.,* vol. 2, p. 202, 1900.

[16] A. Einstein, *Phys. Z.,* vol. 17, p. 132, 1905.

[17] D. F. Walls, "A simple field theoretic description of photon interference," *Am. J. Phys.,* vol. 45, p. 952, 1977.

[18] R. L. Pfleegor and L. Mandel, "Interference of independent photon beams," *Phys. Rev.,* vol. 159, p.1084, 1967.

[19] R. Hanbury-Brown and R. Q. Twiss, *Nature,* vol. 177, p. 27, 1956

[20] H. J. Kimble, M. Dagenais and L. Mandel, "Photon antibunching in resonance fluorescence," *Phys. Rev. Lett.,* vol. 39, p. 691, 1977.

[21] H. J. Kimble, M. Dagenais and L. Mandel, "Multiatom and transit-time effects on photon-correlation measurements in resonance fluorescence," *Phys. Rev. A,* vol. 18, p. 201, 1978.

[22] M. Dagenais and L. Mandel, "Investigation of two-time correlations in photon emission from a single atom," *Phys. Rev. A,* vol. 18, p. 2217, 1978.

[23] D. Stoler, "Equivalence classes of minimum uncertainty packets," *Phys. Rev. D,* vol. 1, p. 3214, 1970

[24] H. P. Yuen, "Two-photon coherent states of the radiation field," *Phys. Rev. A,* vol. 13, p. 2226, 1976.

[25] D. F. Walls, "Squeezed states of light," *Nature,* vol. 306, p. 141, 1983.

[26] I. Lyubomirsky, M. Shirasaki and H. A. Haus, "Squeezing with input state of large phase uncertainty," *Technical digest of the OSA topical meating on quantum optoelectronics,* paper QWC2, 1993.

[27] M. Shirasaki, I. Lyubomirsky and H. A. Haus, "Noise analysis of Mach- Zehnder squeezer with nonclassical input," submitted for publication.

[28] C. R. Doerr, I. Lyubomirsky, G. Lenz, J. Paye, H. A. Haus and M. Shirasaki, "Optical squeezing with a short fiber," *Technical digest of QELS conference,* paper QFF3, 1993

[29] I. Lyubomirsky, M. Shirasaki, F. X. Kärtner and H. A. Haus, "Test of Bell's inequality with squeezed light from a Sagnac fibre ring," *Quantum Opt.,* vol. 5, p. 241, 1993.

[30] D. Bohm, *Quantum Theory,* Prentice-Hall, Englewood Cliffs, NJ 1951.

[31] A. K. Ekert, "Quantum cryptography based on Bell's theorem," *Phys. Rev. Lett.,* vol. 67, p. 661, 1991.

[32] R. Loudon, *The Quantum Theory of Light,* Oxford University Press, 1973.

[33] A. Yariv, *Quantum Electronics,* Wiley & sons, NY, 1989.

[34] R. J. Glauber, "Coherent and incoherent states of the radiation field," *Phys. Rev.,* vol. 131, p. 2766, 1963.

[35] E.G. Sudarshan, *Phys. Rev. Lett.,* vol. 10, p. 277, 1963.

[36] L. A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, "Generation of squeezed states by parametric down conversion," *Phys. Rev. Lett.,* vol. 57, p. 2520, 1986.

[37] H. P. Yuen and J. H. Shapiro, "Generation and detection of two-photon coherent states in degenerate four-wave mixing," *Opt. Lett.,* vol. 4, p. 334, 1979.

[38] M. Shirasaki, H. A. Haus and D. Liu Wong, "Quantum theory of the nonlinear interferometer," *Opt. Soc. Am. B,* vol. 6, p. 82, 1989.

[39] N. Imoto, H. A. Haus and Y. Yamamoto, "Quantum nondemolition measurement of the photon number via the optical Kerr effect," *Phys. Rev. A,* vol. 32, p. 2287, 1985.

[40] M. Kitagawa and Y. Yamamoto, "Number-phase minimum-uncertainty state with reduced number uncertainty in a Kerr nonlinear interferometer," *Phys. Rev. A,* vol. 34, p. 3974, 1986.

[41] K. Bergman and H. A. Haus, "Squeezing in fibers with optical pulses," *Opt. Lett.,* vol. 16, p. 663, 1991.

[42] M. Rosenbluth and R. M. Shelby, "Squeezing optical solitons," *Phys. Rev. Lett.,* vol. 66, p. 153, 1991.

[43] R. M. Shelby, M. D. Levenson, S. H. Perlmutter, R. G. DeVoe, and D. F. Walls, "Broad-band parametric deamplification of quantum noise in an optical fiber," *Phys. Rev. Lett.,* vol. 57, p. 691, 1986.

[44] R. M. Shelby, M. D. Levenson, and P. W. Bayer, "Guided acoustic-wave Brillouin scattering," *Phys. Rev. B,* vol. 31, p. 5244, 1985.

[45] K. Watanabe, Y. Ishida, Y. Yamamoto, H. A. Haus, and Y. Lai, "Femtosecond squeezed-vacuum-state generation in mode-locked soliton lasers," *Phys. Rev.,* vol. 42, p. 5667, 1990.

[46] R. P. Feynman, R. B. Leighton and M. Sands, *The Feynman Lectures on Physics III,* Addison-Wesley, 1965.

[47] M. A. Horne, A. Shimony and A. Zeilinger, "Two-particle interferometry," *Phys. Rev. Lett.,* vol. 62, p. 2209, 1989.

[48] R. Ghosh and L. Mandel, "Observation of nonclassical effects in the interference of two photons," *Phys. Rev. Lett.,* vol. 59, p. 1903, 1987.

[49] Z. Y. Ou, X. Y. Zou, L. J. Wang and L. Mandel, "Observation of nonlocal interference in separated photon channels," *Phys. Rev. Lett.,* vol. 65, p. 321, 1990.

[50] P. Grangier, M. J. Potasek and B. Yurke, "Probing the phase coherence of parametrically generated photon pairs: A new test of Bell's inequality," *Phys. Rev. A,* vol. 38, p. 3132, 1988.

[51] M. D. Reid and P. D. Drummond, "Quantum correlations of phase in nondegenerate parametric oscillation," *Phys. Rev. Lett.,* vol. 60, p. 2731, 1988.

[52] S. M. Tan and D. F. Walls, "Violation of Bell's inequality for systems with parametric down-conversion," *Opt. Comm.,* vol. 71, p. 235, 1989.

[53] S. M. Tan, M. J. Holland and D. F. Walls, "Violation of Bell's inequality for systems with quadrature phase coherence," *Opt. Comm.,* vol. 77, p. 285, 1989.

[54] R. Loudon, "Non-classical effects in the statistical properties of light," *Rep. Prog. Phys.,* vol. 43, p. 58, 1980.

[55] M. D. Reid and D. F. Walls, "Violations of classical inequalities in quantum optics," *Phys. Rev. A,* vol. 34, p. 1260, 1986.