Finding a Cycle with Maximum Profit-to-Time Ratio — An Application to Optimum Deployment of Containerships

by

Ronald W.Y. Chu

Submitted to the Department of Ocean Engineering
in partial fulfillment of the requirements for the degree of

Master of Science in Operations Research

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 1994

Author ....
Department of Ocean Engineering
May 6, 1994

Certified by ....
Ernst G. Frankel
Professor of Marine Systems
Thesis Supervisor

Accepted by ....
A. Douglas Carmichael
Chairman, Departmental Graduate Committee

Finding a Cycle with Maximum Profit-to-Time Ratio — An Application to Optimum Deployment of Containerships

by

Ronald W.Y. Chu

Submitted to the Department of Ocean Engineering
on May 6, 1994, in partial fulfillment of the
requirements for the degree of
Master of Science in Operations Research

Abstract

The problem of finding a cycle with maximum Profit-to-Time (PTT) ratio where the total profit is the sum of the arc profits of the cycle is a well known problem first formulated by Dantzig et al. in the 60's. They showed that the problem can be interpreted as finding the optimum tour of a bulk ship.

This thesis addresses the problem of finding the maximum PTT cycle for a containership. The less-than-vessel-load and many-origins-to-many-destinations characteristics of containership make the problem distinct from bulk ship in one very important aspect – the total profit is the sum of the arc *and* chordal profits of the cycle. We prove that the problem is NP-Hard by showing that the Traveling Salesman Problem is a special case of it. However, when the visit sequence of ports is fixed a priori, we find special cases under which the problem admits polynomial algorithm. These uncapacitated cases correspond to ports arranged along a convex shoreline, a river or a canal, or that port time dominates sailing time. For these special cases, we give exact and compact linear program descriptions of the problem which can be used to design optimum vessel capacity. Other than the special cases, we have evidence that a compact LP description is unlikely. We derive new mixed integer program formulations for two capacitated cases: one on ports along a shoreline, the other on ports without the shoreline restriction but separated by a deep-sea. Our formulations are based on the observation that the deployment can be modeled as a simple tour in an extended graph. They do not require the vessel to be empty at any port. For the real-world deployment of a fleet of containerships, a greedy heuristic is developed for finding a pattern with maximum PTT ratio. The algorithm assigns ship to tour sequentially in decreasing value of PTT ratio. It finds the optimum solution for a homogeneous fleet. By investigating the ships and operating characteristics of container trades, we argue that the problem instances encountered often lead to our greedy algorithm finding the optimal solution even when the fleet is non-homogeneous.

Thesis Supervisor: Ernst G. Frankel
Title: Professor of Marine Systems

Acknowledgments

I show my sincerest thanks to Professor Frankel without whose help and understanding this work could not have been completed.

The continual financial support and generosity of Neptune Orient Lines, Inc. to this project is fully appreciated.

I dedicate this thesis to my family in Hong Kong, especially to my Mother. I always remind myself that the work and effort spent here is in no way and at no moment comparable to hers on her children.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

This research deals with finding the optimum way to deploy a fleet of ocean going containerships. Given a group of deep-sea ports, and the estimated weekly amount of containers flows among them, how should the liner operator design periodic regular services to the ports so as to maximize his profit? We call this the Liner Deployment Problem (LDP). The methodology we develop is suitable for medium- to long-term strategic route planning for the liner operator. We focus on routing of deep-sea trades which generate the majority of revenues for the liner operator in an international market. Routing of deep-sea containerships is at the top of a series of hierarchy decisions such as designing feeder services to serve the deep-sea routes, selective cargo booking to maximize the revenue of each voyage leg, and back-hauling of empty containers to balance the inventory at every port.

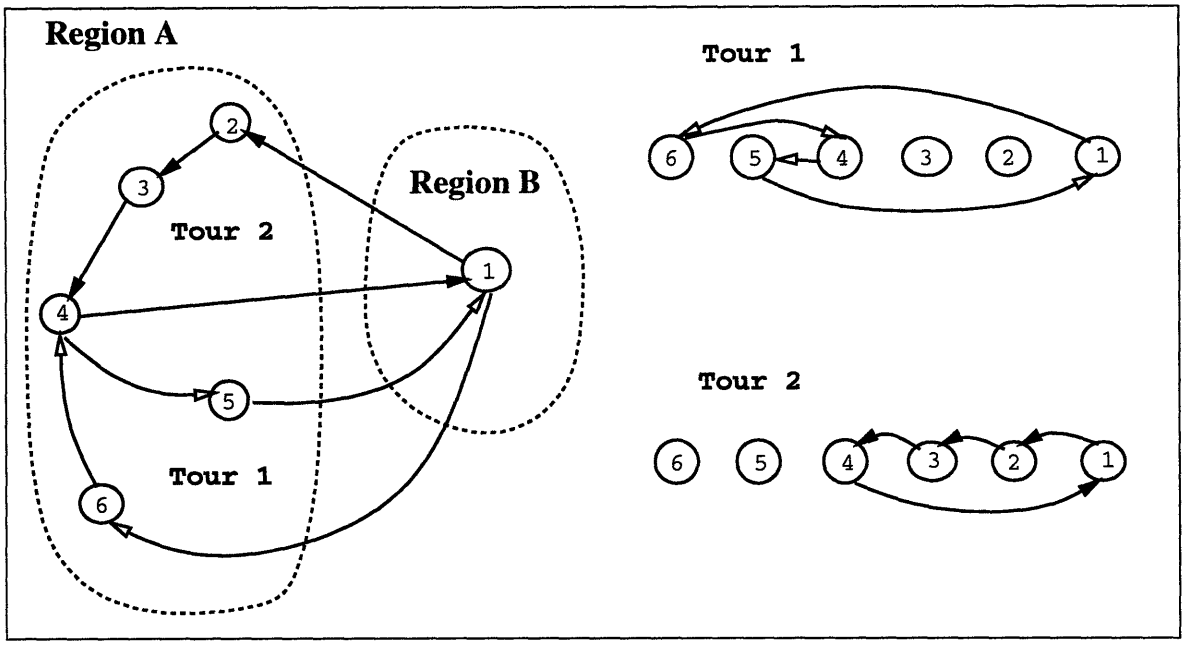
Dantzig et al. [4] give a very elegant formulation to find the optimum deployment of a bulk ship that maximizes the profit made per unit time. Using Dantzig-Wolfe Decomposition, they show how the method can be extended for the optimum deployment of a fleet of bulkers. Bulk ships operate very differently from containerships, and we will see that the decision nature of the latter makes it unlikely to be solved by the same neat approach. Olson et al. [12] delineate various factors that affect containership operations, and suggest a profit maximizing method to design schedules. However, they avoid to discuss the more difficult routing issue. Almogy and Levin [1], and Datz [3] suggest ways to maximize the profit of the liner operator by selective cargo booking, assuming a given route. Boffey et al. [2] provide a methodology that determines both the routes and schedules for a fleet of containerships. The routing part is selected by the planner using an interactive approach, and the profit maximization of each route is based on a heuristic steepest ascent method. Later, Lane et al. [10] extend their search effort for feasible routes by a route generating subroutine. The planner analyzes as many routes as he likes by a profit maximizing forward-looking heuristics subject to all shipping requirements are satisfied. Finally, a set partition technique is used to make sure that no cargo shipment is performed more than once. Perakis and Jaramillo ([14], [9]) formulate a given set of routes explicitly into a linear program which they solve and use rounding to obtain a minimum cost deployment pattern for a fleet of containerships.

Rana and Vickson [17] are probably the first to address the liner routing problem rigorously. They define the problem on a set of ports the visit sequence of which is allowed to switch from one direction (outbound) to another (inbound) exactly once. Exploiting this very special network structure, they present an algorithm based on decomposition to maximize the profits of a fleet of containerships over a fixed period of time. This mixed integer formulation involves a non-linear objective function and constraints. The non-linearity is due to quadratic terms coming from the product of an integer variable with a binary variable and a continuous variable respectively. They resolve the non-linearity by systematically fixing the integer variable at different values within its expected range, and then solving all these as subproblems. Moreover, embedded in their route generating subroutine is a direct enumerative procedure. Therefore the total effort spent is data dependent. The only coupling constraints among ships are through the available cargo quantities in the planning period. Hence they are able to decompose the fleet optimization problem into individual ship optimization problem by the Dantzig-Wolfe Decomposition technique. Each subproblem corresponds to finding the optimum tour for one single ship, and determining what amount of cargo to carry in each voyage. They solve this subproblem by Bender's Decomposition. Solution of all subproblems are returned to the reduced master problem of the Dantzig-Wolfe Decomposition as new proposals. The reduced master problem is an integer problem, since it is in general not possible to combine two different routes to obtain a single feasible route. This reduced master problem is solved by Lagrangean Relaxation on the coupling cargo constraints using subgradient optimization. A special implicit enumeration procedure is used to resolve the duality gap. The final optimality gap of their method is within 3%, and they give examples of a 20 ports 3 ship scenario.

One critical condition under which the above algorithm works is that ports are sequenced a priori. Two important assumptions are made as a result of this condition. First, the ship changes direction exactly twice. Second, the ship is empty at the end-ports where she changes direction. However, Figure 1-1 shows that in deep-sea deployment, this critical condition may not hold. In other considerations of the Liner Deployment Problem, the issue of the level-of-service, namely the frequency of call and the transportation time, is not addressed. From the shippers' perspectives, a carrier who calls a port weekly is more attractive than another carrier who calls biweekly, even though the latter can provide double carrying capacity per call. The drawback of modeling the problem over a long period of time is that the available cargo amount is treated as a lump-sum in the whole period, indifferent to when it is picked up. Another unexplored area of the Liner Deployment Problem is trans-shipment. A trans-shipment takes place if a container travels on more than one ship from the origin to the destination. This is a common phenomenon in deep-sea container transportation when the cargo is originated from a small sea-port which is not visited by any main-haul deep-sea containerships.

This thesis first delineates the characteristics of deep-sea liner trades from which some useful guidelines for formulating our mathematical model are obtained. Despite the efforts of various talented researchers, the LDP is yet far from being fully studied. Chapter 3 explores the fundamental reason underlying the difficulty of the problem:

even with grossly simplified assumptions, the LDP is computationally equivalent to the Traveling Salesman Problem. But we are still able to identify a special case that the problem can be solved efficiently. It is the case in which distances are measured along a rigid shoreline, and that neither the transportation time nor the ship capacity are binding constraints. Exploiting the deep-sea trades characteristics, Chapter 4 describes a heuristic algorithm for the general capacitated case. Our algorithm is a greedy assignment of ships to tours in non-decreasing order of Profit-to-Time (PTT) ratio. We prove that this method is optimum if the fleet is homogeneous. Furthermore, even for non-homogeneous fleet, we argue that under most operating conditions one likely to encounter in deep-sea trades, the method can still give a solution with maximum overall PTT most of the time. Chapter 5 is devoted to explaining the mixed integer linear mathematical formulation we use to find the maximum PTT ratio of each ship when the port visit sequence is fixed a priori, a situation which we called the Shoreline LDP. Instead of modeling the cargo as a lump-sum for the whole period, we propose to specify a certain amount of cargo over a shorter period, say, one week. If the ship calls within that period, she is eligible to collect the cargo, else that fraction of cargo is lost forever to other competitors. Our model does not require the vessel to be empty at any port. In Chapter 6, we extend the mixed integer capacitated model to capture more realistic operating conditions. In particular, we give a model without restriction to a shoreline geometry. Chapter 7 presents some computational results of the formulation, which is solved by the Branch and Bound method. Lastly, we discuss some exciting further research based on ideas developed here that may lead to a more promising approach to handle a real-world containership routing decision.



In deep-sea trades, if we assume an a priori visit sequence and empty vessel at end-ports, we may not be able to find the real optimum. The figure shows the ports numbered in an a priori sequence and a hypothetical optimum tours had there been no restriction on the sequence of ports. In Tour 1, the ship changes direction from outbound to inbound twice, once at 6, the other at 5. This is not allowed under the assumption. In Tour 2, if we impose the condition that the ship must be empty at the end port 4, cargo can not be transported from 3 to 1.

Figure 1-1: Drawbacks of Common Assumptions in Liner Deployment: (1) Fixed A Priori Port Visit Sequence (2) Empty Vessel at End-Ports

Chapter 2

Characteristics of Deep-Sea Liner Shipping

This Chapter serves two purposes. First, through describing the characteristics of deep-sea liner trades, we simultaneously illustrate the nature of the optimization problem we are addressing. As we will show in Chapter 3, this problem belongs to the NP-hard class, which is notoriously difficult to solve. The second purpose for understanding these characteristics is that we may make use of them to narrow down the problem instances, concentrating on cases one likely to encounter in real-world, and hopefully reduce the number of possible feasible solutions one needs to consider. We then exploit these observations in formulating the Liner Deployment Problem, and in designing an algorithm for solving it in Chapter 4.

Deep-sea container trades refer to shipment of containers across the Pacific and Atlantic Oceans. Examples of nowadays deep-sea services are the trans-Pacific trades between Asia and North America West Coast, the trans-Atlantic trades between North America East Coast and Europe, and the Round-the-World trades which start from Asia, cross Pacific Ocean to North America West Coast, sail through Panama Canal to North America East Coast, and eventually return back to Asia via Suez Canal, proceeding either east or west bound.

Sea-borne container trade is a many-to-many-pick-up-and-delivery type of transportation. This means that one port may have cargoes for several other ports, while itself also being the destination from several other origin ports. Unlike bulk ship, the cargoes loaded on board or discharged from a containership in a port usually constitute only a fraction of the ship's capacity. In other words, the cargoes available between a single pair of ports is seldom sufficient enough to fill the ship[1]. Therefore, to gain more revenues instead of running near-empty ship is one reason why a containership usually calls more than two ports in the tour she serves. However, the more ports the ship calls in a tour, the longer is the time she needs to complete the tour, which means that the on-board transportation time for some cargoes becomes unacceptably long. This reduces the competitiveness of the carrier and the amount of cargoes he can collect. Also, another factor that affects availability of cargoes is the frequency of call. The operator who serves a tour once every month is less attractive to a customer than another who serves it every week. Hence to ensure an acceptable visit frequency, the longer the time for the tour, the more ships the operator needs to deploy in the tour. The exact relationship between the level-of-service and cargo availability is not very well known, and is not the objective of this research. But the norm in the industry has been to provide weekly service. In order to achieve this, liner operators frequently pool their ships together to form consortia. This phenomenon of rationalization becomes more and more common nowadays and sees no trend of reversal [18]. Throughout this thesis, we will use the term liner operator, containership operator, carrier, and consortium interchangeably.

Liner operators who involve in deep-sea trades usually commit to a fixed regular service for a long period of time. For example, the deployment patterns of the trans-Pacific service of Neptune Orient Line, Inc., and its consortia shown in Table 2.1 have been maintained for 3 years. Since we are dealing with deep-sea, long-haul containers transportation, the primary interest of the operator is to obtain revenues

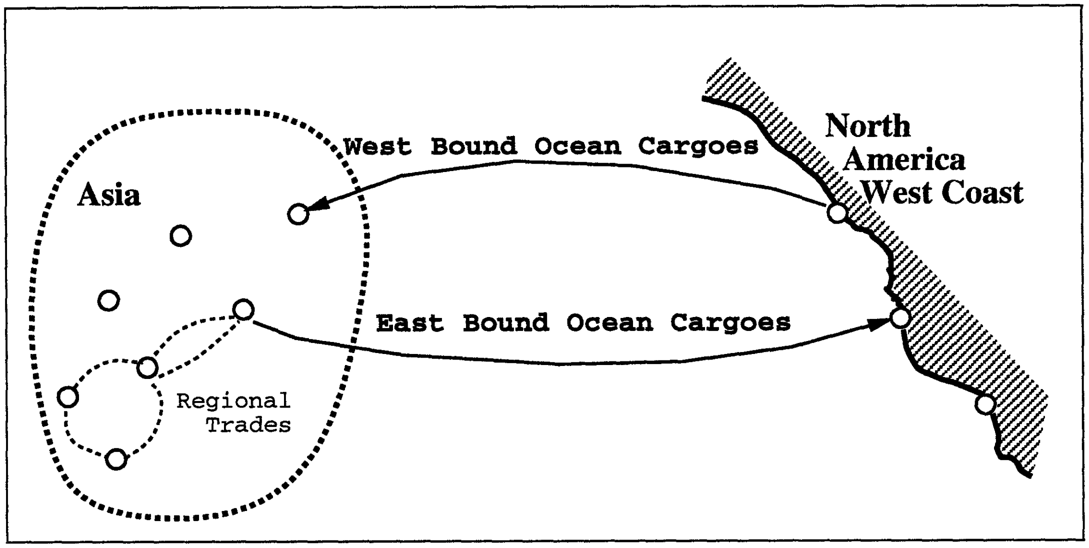
[1] Had this been the case, the Liner Deployment Problem becomes exactly the same as the tramper deployment problem which can be easily formulated and solved (see formulation (BSP) in beginning of Chapter 5).

Table 2.1: Neptune Orient Line, Inc. and Consortia Deployment Patterns for Trans-Pacific and East Bound Round-the-World Trades

Deployment	Route	Number of Ships
Loop 1	LGB-OAK-HKG-KAO-LGB	5
Loop 2	LGB-OAK-YOK-NGO-KOB-HAK -BSN-KOB-SHI-YOK-LGB	5
Loop 3	SEA-VAN-YOK-KOB-HKG-KAO-KOB -NGO-YOK-SEA	5
EBRW	SUZ-SGP-HKG-KAO-BSN-OSA-TKY -LGB-PAN-CRS-NFK-NYK-HFX-SUZ	9

by transporting containers across the ocean. Thus it is natural to group ports into Regions separated by deep-sea. Container trades among ports within the same Region are either (1) cabotage, meaning they are usually restricted to local flag ships, or (2) are of secondary importance, meaning that the revenues they generate can be regarded as additional bonus should there be spare capacity to carry some cargoes belonging to the Regional trade during voyages within the Region. Hence the profit gaining ability of the deployment tour of a ship mainly depends on how fast and frequent she can traverse the ocean separating the Regions, while loading herself with as many containers on board as possible (Figure 2-1). Each time the ship calls a port, she definitely wastes some time in harbor-in, harbor-out, berth or pilotage waiting, etc. The poorer the facilities of the port, the more will such time be wasted, and the less likely the port will be called by a deep-sea liner. Hence within each Region, only the well equipped container ports needed be considered as candidates for visiting. Note also that because of time spent in ports, the total travel time from port to port satisfies the strict triangle inequality even if the ports are all arranged on an Euclidean straight line.

In the case of the trans-Pacific trade between Asia and North America West Coast, the deep-sea trade is between two sets of ports, one set in Asia, and the other along North America West Coast. Here are the Asian ports that are likely to be called: Hakata, Tokyo, Yokohama, Nagoya, Kobe, Osaka, Shimonoseki, Moji, Shimizu, Busan, Inchon, Hong Kong, Kaohsiung, Keelung, and Singapore. North

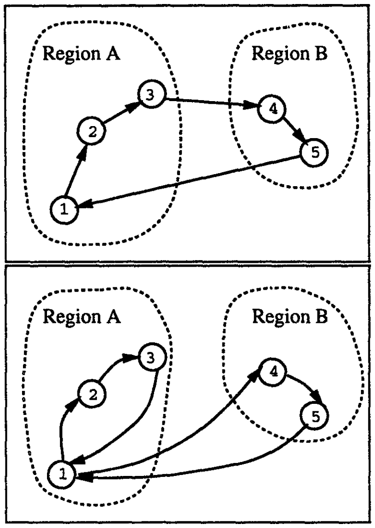


The primary interest of the deep-sea liner operator is to transport as much ocean going cargoes as possible.

Figure 2-1: Cargo Transportation in Deep-Sea Container Trades

America West Coast has the following candidate ports: Vancouver, Seattle, Portland, Oakland, Los Angeles, and Long Beach.

From the above discussion, and emphasizing the fact that additional revenues from carrying cargoes within the same Region are less important than among different Regions, the only reason to justify a ship calling a port more than once per tour is to shorten the on-board time for the ocean cargoes from that port to another Region, or to avoid over capacity in some intra-regional voyage legs (Figure 2-2). Extending this argument, one can see that it is rare that a deep-sea containership will call the same port more than two times per tour. Thus we have a very important observation on the tour pattern of deep-sea liner trades — that if each port is modeled as two nodes, all deep-sea deployment patterns are simple tours in this extended graph; further, if transportation time and ship capacity are not binding constraints, the deployment will be a simple tour in the original graph where each port is modeled as only one node.

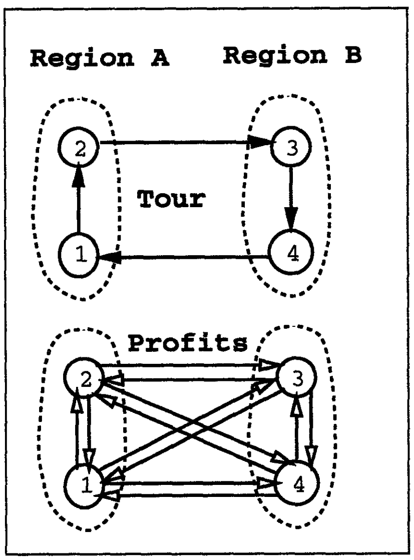


(a) Simple Tour
The tour of the ship is 1-2-3-4-5-1. If she picks up cargo at 1, discharges cargo at 4, the sea-borne transportation time of this ocean cargo is lengthened by stops at 2 and 3.

(b) Non-Simple Tour
The tour is 1-2-3-1-4-5-1: port 1 is called two times. The cargo A_{14} can be loaded on board at the second call, its transportation time to Region $\mathcal{B}$ is shorter. Another reason to call port 1 a second time may be that at the first call, there is not enough capacity on board for cargo A_{14} because the ship is still carrying cargo from Region $\mathcal{B}$ to ports 2 and 3.

Figure 2-2: Possible Reasons for a Non-Simple Tour

The settings here are similar to that of Dynamic Network Flow problems studied by operations researchers. Each arc (sea-link) on the network of nodes (sea ports) has a profit (cargo revenue) as well as time (sea- plus port-time) associated with it. But unlike the Dynamic Network Flow problem which can be efficiently solved ([5] and [13]), the many-origins-to-many-destinations and the less-than-full-ship nature of container trades complicate the way to calculate arc profits in a simple tour. In the Liner Deployment Problem, the cargo revenues obtainable per tour can be imagined as the sum of the arc profits of those arcs *enclosed* by the simple tour that spans the ports visited (Figure 2-3). The time to complete the tour is the sum of the time coefficients associated with only those arcs defining the tour. Thus the efficiency of the ship in making profits can be calculated as the ratio of the sum of the enclosed arc profits to the sum of the arc time of the enclosing tour. This idea of Profit-to-Time ratio will be formally discussed in the next Chapter.



Let r_{ij} be the profits the ship can gain for carrying cargo from i to j. Suppose the ship travels on a simple tour 1-2-3-4-1. She is eligible for gaining the profits of the arcs enclosed by the tour:

$$r_{12}+r_{13}+r_{14}+r_{21}+r_{23}+r_{24}+r_{31}+r_{32}+r_{34}+r_{41}+r_{42}+r_{43}$$

$$\begin{aligned}\text{PTT ratio} &= \frac{\text{sum of enclosed arc profits}}{\text{length of enclosing tour}} \\ &= \frac{\sum r_{ij}}{t_{12}+t_{23}+t_{34}+t_{41}}\end{aligned}$$

Note that if there is cabotage, or there is intra-region trade restrictions, $r_{12}=r_{21}=r_{34}=r_{43}=0$

Figure 2-3: Using Enclosed Arcs to Determine the Profit-to-Time Ratio

Chapter 3

Some Theoretical Insights of Liner Deployment Problem

The objective of the design of a containership deployment pattern is to maximize the profits of the liner operator over a period of time. Since the route of a containership is maintained for a relative long planning horizon, the long-term average of profit over time, namely the Profit-to-Time (PTT) ratio, is a convenient performance measure for the objective function. The PTT ratio for a ship is the total profit per tour she makes divided by the time that is required to complete the tour assigned to her. The sum of PTT ratios of all ships in the fleet is the overall PTT ratio of the operator, which is the ratio of the total profit to the time needed to gain this profit.

Finding the maximum PTT ratio for each containership turns out to be an important step in solving the problem. To have a better understanding on the Liner Deployment Problem (LDP), we will first study the complexity of finding the PTT ratio in the context explained in Figure 2-3.

Let us consider an extremely simplified version of the LDP:

(1) There is only one ship to deploy;

(2) The vessel has capacity big enough for all cargoes in the system;

(3) The on-board transportation time does not affect the amount of cargo the operator can get. This means that shippers are indifferent to the level-of-service with respect to transport time;

(4) Cargo quantity between any port pair is expressed as TEU per week, which is the maximum amount of cargo the operator can get whenever he serves that port pair. Such quantity of cargo is not affected by his frequency of service or transport time;

(5) No trans-shipment is allowed.

(1)-(5) together with the fact that the travel times satisfy straight triangle inequality necessarily imply that the deployment must be a simple tour. We call this the Uncapacitated Linear Deployment Problem (ULDP).

3.1 Constant Travel Time Uncapacitated LDP

In the constant time uncapacitated LDP, in addition to the assumptions stated above, the travel time for any arc (i, j) is a positive constant, which can be rescaled to unity. Finding a tour with maximum PTT is equivalent to finding a subset of nodes $\mathcal{I}$ from the given set $\mathcal{N}$ such that the total sum of arc profits in the set $\mathcal{I}$ divided by the cardinality of $\mathcal{I}$ is maximized:

$$\max \text{PTT} \iff \max_{\mathcal{I} \subseteq \mathcal{N}} \frac{\sum_{i,j \in \mathcal{I}} p_{ij}}{|\mathcal{I}|}$$

where p_{ij} is the arc profit of (i, j).

Proposition 3.1 *When profits p_{ij} can be any real numbers, any algorithm that solves* *the constant time ULDP also solves the Maximum Clique Problem.*

Proof

Given an instance of the Maximum Clique Problem, transform it into the constant time ULDP by assigning a profit of $-(N^2 - N)$ to arc (i, j) if it does not exist in the original graph, and a profit of unity if the arc exists. □

Proposition 3.1 motivates us to consider only non-negative profits. In the context of LDP, a non-negative arc profit corresponds to the case that the expenditure of traveling from port i to j is at least compensated by the revenues of cargo that one picks up at i and delivers at j. This assumption is not unrealistic.

3.2 Constant Time Non-Negative Profits ULDP

Proposition 3.2 *When profits p_{ij} are non-negative and travel time along any arc is a constant, the ULDP is solvable in polynomial time.*

Proof

The proof is by constructing a compact linear program formulation for this problem:

$$z^* = \max \sum_{i,j \in \mathcal{N}} p_{ij} x_{ij} \tag{P1}$$

Subject to:

$$\sum_i x_i = 1 \tag{3.1}$$

$$x_{ij} \leq x_i \quad \forall (i, j) \tag{3.2}$$

$$x_{ij} \leq x_j \quad \forall (i, j) \tag{3.3}$$

$$x_i, x_{ij} \geq 0 \tag{3.4}$$

As will be seen later, the variable x_i has the interpretation of the reciprocal of the cardinality of the selected set $\mathcal{I}$ if i is an element of $\mathcal{I}$, and zero otherwise. Its dual is:

$$\lambda^* = \min \quad \lambda \tag{D1}$$

Subject to:

$$\lambda - \sum_j \alpha_{ij} - \sum_j \beta_{ji} \geq 0 \tag{3.5}$$

$$\alpha_{ij} + \beta_{ij} \geq p_{ij} \quad \forall (i,j) \tag{3.6}$$

$$\alpha_{ij}, \beta_{ij} \geq 0 \tag{3.7}$$

where λ is the dual variable for (3.1), α_{ij} is the dual variable for (3.2), β_{ij} is the dual variable for (3.3), x_i is the dual variable for (3.5), and x_{ij} is the dual variable for (3.6) respectively.

Let (x_i^*, x_{ij}^*) and $(\lambda^*, \alpha^*, \beta^*)$ be the primal and dual optimum respectively. Constraint (3.1) and non-negativity of x_i guarantee that at least one x_i^* must be strictly positive. Let us arrange the strictly positive values of x_i^* in descending order. Without loss of generality, let:

$$\underbrace{x_1^* = x_2^* = \ldots = x_{|\mathcal{A}_1|}^*}_{\mathcal{A}_1} > \underbrace{x_{|\mathcal{A}_1|+1}^* = x_{|\mathcal{A}_1|+2}^* = \ldots = x_{|\mathcal{A}_1|+|\mathcal{A}_2|}^*}_{\mathcal{A}_2} > x_{|\mathcal{A}_1|+|\mathcal{A}_2|+1}^* = \ldots \geq 0$$

By complementary slackness condition, we have:

$$x_i^* > x_j^* \Rightarrow \alpha_{ij}^* = 0, \;\; \beta_{ij}^* \geq p_{ij}, \;\; \alpha_{ji}^* \geq p_{ji}, \;\; \beta_{ji}^* = 0 \tag{3.8}$$

By the same reasoning, and by making use of the fact that p_{ij} is non-negative:

$$x_i^* > x_j^* > 0 \Rightarrow \alpha_{ij}^* = 0, \;\; \beta_{ij}^* = p_{ij}, \;\; \alpha_{ji}^* = p_{ji}, \;\; \beta_{ji}^* = 0 \tag{3.9}$$

Next we will argue that the set $\mathcal{A}_1$ is actually an alternate optimal solution. For each x_i^* in $\mathcal{A}_1$, using relations (3.8) and (3.9), the dual constraint (3.5) must be satisfied at equality:

$$\lambda^* - \sum_{x_j^* \in \mathcal{A}_1, j \neq i} \alpha_{ij}^* - \sum_{x_j^* \in \mathcal{A}_1, j \neq i} \beta_{ji}^* = 0 \quad \forall x_i^* \in \mathcal{A}_1 \tag{3.10}$$

Since x_i^* is strictly greater than zero for all x_i^* in $\mathcal{A}_1$, and we are maximizing the objective function which coefficients are non-negative, we must have x_{ij}^* strictly greater than zero whenever x_i^* and x_j^* are both in $\mathcal{A}_1$. By applying the complementary slackness condition again on (3.6), we have:

$$(\alpha_{ij}^* + \beta_{ij}^* - p_{ij}) x_{ij}^* = 0$$

for all x_i^* and x_j^* both in $\mathcal{A}_1$. This implies $(\alpha_{ij}^* + \beta_{ij}^*)$ equals p_{ij}. Adding the $\mathcal{A}_1$ equations (3.10), we have:

$$|\mathcal{A}_1|\lambda^* = \sum_{x_i^*, x_j^* \in \mathcal{A}_1} p_{ij} \tag{3.11}$$

(3.11) is equivalent to saying that the set $\mathcal{A}_1$ has a PTT ratio same as the primal optimum. Hence it must be an alternate optimum. In other words, we have shown that by solving the compact primal program (P1), a feasible optimum solution to the constant time, non-negative profits ULDP can be found easily by inspection. □

3.3 Node-Oriented Time Non-Negative Profits ULDP

Proposition 3.3 *When profits p_{ij} are non-negative and travel time to node i along any arc is a positive constant t_i which may be different among different i, the ULDP is also solvable in polynomial time.*

Proof

The proof of Proposition 3.2 can be repeated with constraint (3.1) replaced by:

$$\sum_i t_i x_i = 1 \tag{3.12}$$

where t_i is the strictly positive node-oriented travel time. □

When the travel time from port i to port j is negligible compared to the time spent in i or j for all arc (i, j), we have a situation similar to a node-oriented travel time. At this point, it is interesting to notice that the constant time, non-negative profits ULDP has been known in the operations literature as the Maximum Density Subgraph Problem. Picard et al. [15] are the first to study it. Goldberg [8] gives a polynomial algorithm. However, his algorithm does not generalize to our node-oriented travel time problem. Gallo et al. [6] interpret the problem from a boarder perspective of parametric problems. They do not propose a compact formulation, but their algorithm is able to solve the node-oriented non-negative profits ULDP.

3.4 Arc-Oriented Time Non-Negative Profits ULDP

Proposition 3.4 *When profits p_{ij} are non-negative and travel time from node i to j along arc (i, j) is a positive number t_{ij}, which may be different among different arcs, any polynomial time algorithm that finds a tour with a maximum PTT ratio also solves the optimization version of the $\{1, 2\}$-Traveling Salesman Problem.*

Proof

Given any instance of the $\{1,2\}$-TSP optimization problem with distances c_{ij} equal either 1 or 2, we construct an instance of our arc-oriented, non-negative profits ULDP as follow:

$$t_{ij} = c_{ij} \qquad \forall (i,j)$$

$$p_{ij} = i \qquad \forall (i,j)$$

Then, the solution to our problem must be a cycle that touches all nodes exactly once, with total profit $\frac{N(N^2-1)}{2}$. Since the PTT ratio is maximized, the travel time around this cycle must be minimized, that is, the cycle is the $\{1,2\}$-TSP optimal solution.

□

The recognition version of $\{1,2\}$-TSP problem is NP-complete [7], therefore Proposition 3.4 implies that it is quite unlikely to find an efficient polynomial time algorithm that solves this simplified Liner Deployment Problem.

3.5 Rigid Shoreline Non-Negative Profits ULDP

Proposition 3.4 suggests that the Liner Deployment Problem is difficult in an Euclidean graph, and explains why researchers have considered graphs in which the nodes are presequenced: such case corresponds to the ports being arranged along the same shoreline ([16], [17], and [19]). Perhaps the most exciting discovery we obtain from the above studies is that when distances are measured along a *rigid* shoreline, the non-negative profits, uncapacitated version of LDP is solvable in polynomial time.

Definition 3.1 *Ports i, j, and k are said to be on a shoreline if:*

$$t_{ik} \leq t_{ij} + t_{jk} \tag{3.13}$$

whenever k is situated in between i and j on the shoreline, where t_{ij} is the non-negative travel time from i to j.

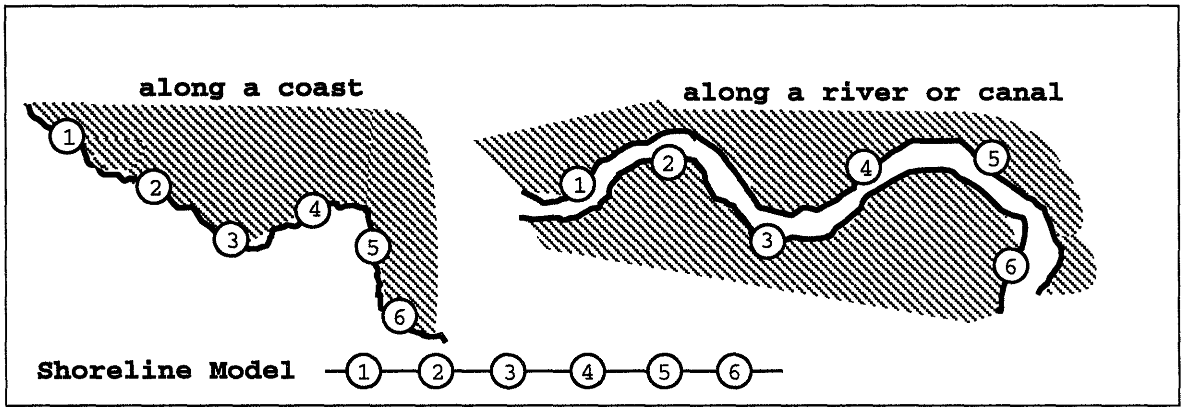


Figure 3-1: Real-World Examples of Shoreline Problem Instance

Definition 3.2 *When the shoreline relationship (3.13) is satisfied at equality for all i, j, k, we say that the nodes are arranged along a rigid shoreline.*

A prerequisite for either the shoreline or the rigid shoreline case is that the nodes are arranged and numbered in a sequence. An analogy to the rigid shoreline case are ports along a river or canal (Figure 3-1).

Proposition 3.5 *When profits p_{ij} are non-negative and travel time t_{ij} is measured along a rigid shoreline, the non-negative profits ULDP is solvable in polynomial time.*

Proof

Notice that because of the assumption of the ULDP stated in the beginning of this Chapter, in the optimum tour, the vessel must change direction on the shoreline exactly twice: once at each end-port of the tour. For each port pair (i, j) along the shoreline, find the tour with the maximum PTT ratio such that the two extreme ports of the tour are respectively i and j. This subproblem is easily solvable because given fixed end-ports i and j, the total sailing time of the tour equals twice t_{ij}. Whether the ship should stop at an intermediate port k between i and j or not depends on the additional non-negative port time t_k at port k. Thus each subproblem reduces to the node-oriented time non-negative profits ULDP which is solvable in polynomial time by Proposition 3.3. All we need to do is to solve this $O(N^2)$ subproblems and find the port pair with the maximum ratio. $\square$

3.6 Shoreline Non-Negative Profits ULDP: an Open Problem

Proposition 3.5 motivates us to push the limit further and explore the complexity of the non-negative profits ULDP when relation (3.13) is not necessarily satisfied at equality. Such case is one step nearer to the real-world deep-sea scenario when the geographical locations of ports cause operators to prefer a fixed sequence of visits. Unfortunately, no fruitful results is obtained along this direction.

Conjecture 3.6 *When profits p_{ij} are non-negative, the visiting sequence of ports is fixed a priori, and non-negative travel time t_{ij} is measured along a shoreline given by inequality (3.13), the non-negative profits ULDP is NP-Hard.*

Proposition 3.4 and Conjecture 3.6 justify us to adopt some heuristic technique to handle the real-world Liner Deployment Problem. To this goal we will proceed in the next Chapter of the thesis.

Chapter 4

Algorithm for Containership Fleet Deployment

We will present our algorithm, which iteratively assigns ships in decreasing order of PTT ratio to tours, updates the remaining cargo matrix, and proceeds until no ship remains. The algorithm calls a subroutine which evaluates the maximum PTT ratio. Depending on which case we consider, the complexity of the subroutine is given by the appropriate Propositions in Chapter 3. The detail solution methodology of the subroutine is left to Chapter 5. We show that this greedy algorithm is optimum if the fleet is homogeneous, and that it gives close to optimal deployment pattern most of the time even when the fleet is non-homogeneous.

4.1 Input data

The algorithm requires the following input data:

(1) The trade regions the liner operator or the consortia is interested in, and the ports in each trade region;

(2) Estimated weekly available containers in TEU between all port pairs that are in different trade regions, and the average revenue per TEU for each of this port pair;

(3) For each ship in the fleet, its capacity in TEU, the sailing time in days between every port pair, and the average port time in days at each port;

(4) For each ship in the fleet, the average daily operating costs at sea and in port, and the cargo handling cost in $ per TEU at each port.

4.2 The Greedy Algorithm

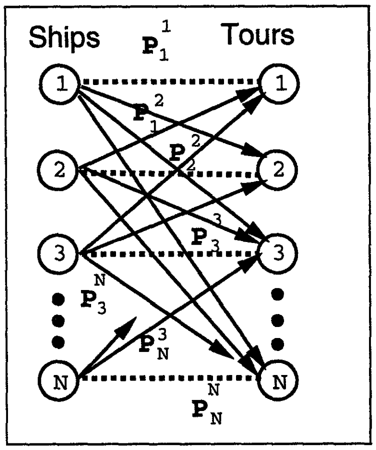
The algorithm runs as follows:

Step 1: While there are unassigned ships, find for each ship, a tour which has the maximum Profit-to-Time ratio;

Step 2: Find the ship/tour assignment in Step 1 that has the biggest ratio and assign the ship to that tour;

Step 3: Check if there are enough ships of the same type to provide weekly service to the tour found in Step 2. If necessary, readjust service speeds of other ships so that enough ships can be assigned to this tour to ensure a weekly call. Compare the overall Profit-to-Time ratio of such adjustment with the second best or third best assignment found in Step 1 to determine which assignment to select eventually;

Step 4: Update the weekly cargo matrix as a result of the assignment decided in Step 3. If there remain ships in the fleet, go to Step 1, else quit.



P_i^j = PTT ratio of assigning ship i to tour j.
The dotted arc from each ship represents her final optimal assignment. The number of tours are much more than those listed. We only show those tours that are eventually chosen in the optimum solution. Tours and arc profits shown are highly symbolic. For example, the arc coefficients are not constant but depend on all previously made assignment. It may happen that two tours with the same ports in the same sequence are listed twice with different numbers. This condition corresponds to we trying to assign ship to it at different moment, therefore its incoming arc coefficients may be different depending on which previous assignments have been made already.

Figure 4-1: Liner Deployment Problem Interpreted as Maximum Assignment Problem

4.3 Explanation of the Algorithm

We can view the Liner Deployment Problem as a very special type of Assignment Problem — the problem of assigning ships to tours. This is a Maximum Assignment Problem in which the profit of arc (i, j) corresponds to the PTT ratio P_i^j of assigning ship i to tour j (Figure 4-1). We assume that there is no restriction on ships to tours, i.e., every tour is accessible to every ship. This may not be true if some ports have draft restriction to some ships, or if the ship is of post-Panamax design such that she can not go through Panama Canal. As shown in Table 4.1, the fleets of the consortia under our study do not show any draft or breadth restriction. The main difference between our assignment problem and the classical assignment problem is that the arc profits are neither constants nor known a priori, but are functions of other assignments. Moreover, the total number of nodes is expoentially large, so that one may not even write down all of them. These two facts make it impossible to solve our problem by any of the efficient Assignment Problem algorithm. Nevertheless, it does not prevent us from studying the problem from this perspective.

Figure 4-1 shows the solution of the Liner Deployment Problem solved by the approach of our algorithm — assigning ships to tours in decreasing order of PTT

Table 4.1: Neptune Orient Line, Inc. and its Consortia Vessel Characteristics for Trans-Pacific and East Bound Round-the-World Deployment in 1991

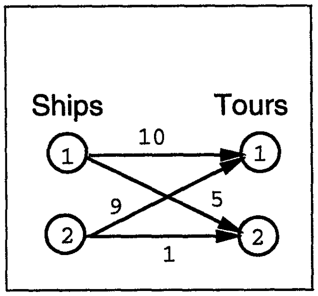
Vessel	Lgth (m)	Brth (m)	Dght (m)	TEU (C)	Speed ($\frac{1}{T}$)	T	$\frac{C}{T}$	$\frac{C}{T^2}$	Tour
OOCL Freedom	241.0	32.2	12.5	3161	21.5	0.04651	67962	1461172	EBRW
OOCL Fortune	241.0	32.2	12.5	3161	21.5	0.04651	67962	1461172	EBRW
OOCL Faith	241.0	32.2	12.5	3161	21.5	0.04651	67962	1461172	EBRW
OOCL Fair	241.0	32.2	12.5	3161	21.5	0.04651	67962	1461172	EBRW
Rainbow Bridge	241	32.2	12.5	2901	22.4	0.04464	64982	1455606	EBRW
Ambassador Bridge	241	32.2	12.5	2901	22.4	0.04464	64982	1455606	EBRW
Neptune Jade	244.0	32.3	12.5	2966	21.5	0.04651	63769	1371034	EBRW
Neptune Garnet	244.0	32.3	12.5	2966	21.5	0.04651	63769	1371034	EBRW
OOCL Friendship	241	32.2	12.5	2706	22.5	0.04444	60885	1369913	EBRW
Yamaaki Maru	230	32.2	10.5	2832	21.0	0.04762	59472	1248912	L1
China Container	250.5	32.2	11.5	2430	23.0	0.04348	55890	1285470	L1
OOCL Exporter	270.8	30.6	11.5	2466	22.5	0.04444	55484	1248413	L1
Neptune Amber	231.0	32.2	12.5	2216	23.0	0.04348	50968	1172264	L1
Neptune Diamond	233.5	32.3	12.5	2158	23.0	0.04348	49634	1141582	L3
Neptune Crystal	231.0	32.3	12.5	2084	23.0	0.04348	47932	1102436	L3
Oriental Explorer	252.2	30.5	10.9	2394	19.5	0.05128	46683	910319	L3
Oriental Executive	252.2	30.5	10.9	2394	19.5	0.05128	46683	910319	L3
OOCL Educator	252.2	30.5	10.9	2394	19.5	0.05128	46683	910319	L1
Japan Apollo	227	31.2	10.9	1919	22.5	0.04444	43178	971494	L3
Neptune Coral	222.4	32.3	11.5	1863	23.0	0.04348	42849	985527	L2
Shin Kashu Maru	221.5	31.2	10.9	1834	22.8	0.04386	41815	953387	L2
Neptune Pearl	222.4	32.3	11.5	1757	23.0	0.04348	40411	929453	L2
Shin Beishu Maru	204.4	32.2	11.5	1928	20.3	0.04926	39138	794510	L2
Japan Alliance	220	32.2	11.5	1692	22.3	0.04494	37647	837646	L2

Sources: [18], [11], and courtesy of Neptune Orient Line, Inc.

Table 4.2: Neptune Orient Line, Inc. Controlled Containership Fleet in 1989

Vessel	Lgth (m)	Brth (m)	Dght (m)	TEU (C)	Speed ($\frac{1}{T}$)	T	$\frac{C}{T}$	$\frac{C}{T^2}$
Neptune Zicron	275.1	32.3	12.5	3327	24.0	0.04167	79848	1916352
Neptune Topaz	275.8	32.3	12.5	3327	24.0	0.04167	79848	1916352
Neptune Ruby	275.8	32.2	12.5	3300	23.0	0.04348	75900	1745700
Neptune Jade	244.0	32.3	12.5	2966	21.5	0.04651	63769	1371034
Neptune Garnet	244.0	32.3	12.5	2966	21.5	0.04651	63769	1371034
Neptune Amber	231.0	32.2	12.5	2216	23.0	0.04348	50968	1172264
Neptune Diamond	233.5	32.3	12.5	2158	23.0	0.04348	50968	1172264
Neptune Crystal	231.0	32.3	12.5	2084	23.0	0.04348	47932	1102436
Neptune Coral	222.4	32.3	11.5	1863	23.0	0.04348	42894	985527
Ace Concord	207	32.2	11.5	1948	21.0	0.04762	40908	859068
Neptune Pearl	222.4	32.3	11.5	1757	23.0	0.04348	40411	929453
Omex Pioneer				1408	21.0	0.04762	29568	620928
Neptune Emerald	225	27.2		1543	18.0	0.05556	27774	499932
Neptune Ivory	225	27.2		1543	18.0	0.05556	27774	499932
Neptune Beryl	161	25.0	9.7	859	17.6	0.05682	15118	266084
Neptune Jasper	161	25.0	9.7	859	17.6	0.05682	15118	266084
Supanya	152	20.1	9.5	610	17.5	0.05714	10675	186813
Paithoon	152	20.1	9.5	608	15.0	0.06667	9120	136800
Perkasa	126.5	20.1	8.3	369	14.0	0.07143	5166	72324

Sources: [18], [11]



P_1^1 is the maximum. After assigning ship 1 to tour 1, P_2^2 is the maximum.

$$\begin{array}{lrcl} \text{PTT by greedy:} & P_1^1 + P_2^2 & = & 10 + 1 = 11 \\ \text{PTT by other:} & P_1^2 + P_2^1 & = & 5 + 9 = 14 \end{array}$$

The greedy algorithm fails. Notice that criteria in Assumption 4.1 is not satisfied because:

$$\frac{1}{5} = \frac{P_2^2}{P_1^2} \not\geq \frac{P_1^1}{P_2^1} = \frac{9}{10}$$

Figure 4-2: Example of an Assignment Problem in which Greedy Algorithm Fails

ratio. Without loss of generality, we assume the assignment is ship 1 to tour 1, ship 2 to tour 2,..., and so on in that order. That is, we assume that P_1^1 is the biggest arc profit, and after tour 1 is occupied by ship 1, P_2^2 is the biggest arc profit for the remaining assignment graph with the two node 1's and all their incident arcs removed, and so on. Also, without loss of generality, we can assume that there are only N tours available for us, and all these N tours are already listed in the Figure (one can imagine that the unlisted tours are all less profitable and therefore need not be considered). To reiterate the condition described above in mathematical language, we assume:

$$\begin{aligned} P_1^1 &= \max\{P_i^j : 1 \leq i,j \leq N\} \\ P_2^2 &= \max\{P_i^j : 2 \leq i,j \leq N\} \\ &\vdots \qquad\qquad \vdots \\ P_{N-1}^{N-1} &= \max\{P_i^j : N-1 \leq i,j \leq N\} \end{aligned}$$

Our question is: under what condition will the algorithm obtain the optimal solution, given it uses the above greedy approach to solve the Assignment Problem, i.e., it assigns ships to tour as per the maximum P_i^j? For the generic Assignment Problem in which the arc profits are arbitrary, the greedy approach easily fails to find the optimum. The reason is that it is too short-sighted so that toward the end of the assignment, it is forced to assign people/ships to very bad jobs/tours so that the overall gain is much degraded. Figure 4-2 shows an example.

However, the problem instances of our Liner Deployment Problem is not that arbitrarily to discourage the use of a greedy approach. For example, notice that if the fleet is homogeneous, i.e., the performance of the ship is same for every tour: P_i^j equals P^j for all ship i, then, the greedy approach guarantees optimum. In this special case, the challenge is not solving the Assignment Problem, but to find the best tour for the remaining unassigned ships among the many possible tours. But as seen in Table 4.1 and 4.2, the fleet of a liner operator is seldom homogeneous. We next explore under what circumstances this greedy approach still gives an optimum solution.

4.4 Performance of Greedy for Non-Homogeneous Fleet

For a non-homogeneous fleet, let us first understand what are the main factors that cause the PTT ratio to differ from tour to tour. To make our argument more tractable, we will concentrate on the revenue per unit time of the tour, and assume that the cost per unit time is constant. From the liner operator's perspective, this assumption is justified because the operating costs per day of a ship is roughly fixed as long as the operator is committed to keeping his ship running at her design speed. Let us consider the case of two ships, ship 1 and ship 2, and two possible tours, tour 1 and tour 2. Let V_i be the capacity of ship i, s_i be the speed of ship i, T_i^j be the time for ship i to complete tour j, u_i^j be the utilization[1] of ship i in tour j, and r_j be the average revenue per container in tour j. Suppose the PTT ratio of ship 1 in tour 1 is bigger than in tour 2, i.e., $P_1^1 > P_1^2$. We an express the PTT ratios as:

$$P_1^1 = \frac{r_1 u_1^1 V_1}{T_1^1}$$

[1] Utilization is the ratio of the amount of loaded containers on-board to the vessel capacity.

$$P_2^1 = \frac{r_1 u_2^1 V_2}{T_2^1}$$

By assumption, P_1^1 is bigger than P_1^2. The possible reasons for this to occur are either one or a combination of the followings:

(1) in tour 2, ship 1 has to travel a longer distance to gain the same utilization whereas the average revenue per container in both tours are the same;

(2) utilization and average revenue are the same, but the ports visited in tour 2 has an inferior service, causing ship 1 to waste more time in the ports;

(3) less cargo is available in tour 2 than in tour 1, but travel time and average revenue per container of both tours are the same;

(4) the average revenue per container in tour 2 is smaller, but both tours have the same tour travel time and same utilization.

Non-Homogeneity Assumption

We assume that our non-homogeneous fleet satisfies the following condition: If P_1^1 is the maximum ratio, then under each of the case (1) through (4), provided that the speed/capacity ratio of the container ships satisfied certain relationship explained below, we have:

$$\frac{P_2^2}{P_1^2} \geq \frac{P_2^1}{P_1^1} \tag{4.1}$$

The physical interpretation of the above relationship is: when ship 2 performs worse than ship 1 in a tour 1 (i.e., $P_2^1 < P_1^1$), then in tour 2 in which ship 1 has a poorer performance than the previously mentioned tour (i.e., $P_1^2 < P_1^1$), the relative performance of ship 2 to the ship 1 will not deteriorate (i.e., inequality (4.1)). Furthermore, we will show that even when (4.1) is not satisfied, the greedy approach to assign ship 1 to tour 1, ship 2 to the remaining tour 2 results in the highest cumulative PTT ratio for most of the time compared with the assignment otherwise (ship 1 to tour 2, ship 2 to tour 1).

To justify this assumption this, let us consider separately case (1) to (4).

4.5 Justification of Non-Homogeneity Assumption

Case (1)

Let the travel time of ship 1 in tour 2 be increased by δ from that of tour 1. Under constant utilization and average revenue assumption, we have :

$$\begin{aligned}
\frac{P_2^2}{P_1^2} &= \frac{r_2 u_2^2 V_2}{(T_1^1+\delta)\frac{s_1}{s_2}} \frac{(T_1^1+\delta)}{r_1 u_1^1 V_1} \\
&= \frac{V_2 s_2}{V_1 s_1} \\
&= \frac{P_2^1}{P_1^1}
\end{aligned}$$

Case (2)

Let the total increase in port time in tour 2 be δ. Then, under constant utilization and average revenue assumption:

$$\begin{aligned}
\frac{P_2^2}{P_1^2} &= \frac{V_2}{(T_2^1+\delta)} \frac{(T_1^1+\delta)}{V_1} \\
\frac{P_2^2}{P_1^2} - \frac{P_2^1}{P_1^1} &= \frac{V_2(T_1^1+\delta)}{V_1(T_2^1+\delta)} - \frac{V_2 T_1^1}{V_1 T_2^1} \\
&= \frac{V_2(T_2^1-T_1^1)\delta}{V_1 T_2^1 (T_2^1+\delta)}
\end{aligned}$$

The above difference will be negative if T_2^1 is less than T_1^1, i.e., ship 2 is faster than ship 1. Since we have assumed P_1^1 is bigger than P_2^1, the only possibility for this to happen is V_2 is less than V_1, i.e., ship 2 is smaller than ship 1. Although it is possible to have a faster and smaller ship, contemporary design characteristics for container vessels is bigger ships usually travel faster (Table 4.1 and 4.2). Nevertheless, let us also consider the case of a smaller but faster ship. The difference between the cumulative PTT ratio of the greedy assignment versus the other assignment is:

$$\begin{aligned}
\text{greedy:} \quad P_1^1 + P_2^2 &= \frac{V_1}{T_1^1} + \frac{V_2}{(T_2^1 + \delta)} \\
\text{other:} \quad P_2^1 + P_1^2 &= \frac{V_2}{T_2^1} + \frac{V_1}{(T_1^1 + \delta)} \\
\text{greedy - other:} \quad z(\delta) &= \frac{(P_1^1 - P_2^2)\delta}{(T_1^1 + \delta)(T_2^1 + \delta)} + \frac{(V_1(T_2^1)^2 - V_2(T_1^1)^2)\delta}{(T_1^1 + \delta)(T_2^1 + \delta)T_1^1 T_2^1}
\end{aligned}$$

By assumption, the first term is always positive. The second term is negative if $V_1(T_2^1)^2$ is less than $V_2(T_1^1)^2$. To see how likely the second term will be negative, we look into the fleet composition of the deep-sea trades of the NOL consortia and other independent operators (Table 4.1 and 4.2). By arranging the ships in decreasing order of $V_i s_i$ (which is a rough estimate of the PTT ratio), we plot the quantity $V_i s_i^2$ versus $V_i s_i$ in Figures 4-3, 4-4, and 4-5. As seen from the figures, only a few ships — those that correspond to the small valleys in the graphs — can possibly have a negative second term. Thus we conclude that under this condition, the cumulative PTT ratio of the greedy assignment still gives better overall profits for most of the real-world cases.

Case (3)

Assume that in tour 2, the weekly cargo available is ρ less than in tour 1, and the total tour time is the same. Therefore:

$$\begin{aligned}
P_1^2 &= \frac{V_1 - \rho}{T_1^1} \\
P_2^2 &= \frac{\min\,(V_1 - \rho, V_2)}{T_2^1}
\end{aligned}$$

Subcase (a): when $V_1 - \rho > V_2$, then:

$$
\begin{aligned}
\frac{P_2^2}{P_1^2} - \frac{P_2^1}{P_1^1} &= \frac{V_2}{T_2^1}\frac{T_1^1}{V_1-\rho} - \frac{V_2}{T_2^1}\frac{T_1^1}{V_1} \\
&= \frac{V_2}{T_2^1}\left(\frac{T_1^1}{V_1-\rho} - \frac{T_1^1}{V_1}\right) > 0
\end{aligned}
$$

Subcase (b): when $V_1 - \rho < V_2$, then:

$$
\begin{aligned}
\frac{P_2^2}{P_1^2} - \frac{P_2^1}{P_1^1} &= \frac{V_1-\rho}{T_2^1}\frac{T_1^1}{V_1-\rho} - \frac{V_2}{T_2^1}\frac{T_1^1}{V_1} \\
&= \frac{T_1^1}{T_2^1}\left(1 - \frac{V_2}{V_1}\right) \text{ which is } \begin{cases} > 0 & \text{if } V_2 < V_1 \\ < 0 & \text{if } V_2 > V_1 \end{cases}
\end{aligned}
$$

Hence for subcase (3b), the condition (4.1) is not satisfied if the ship that performs worse in the best tour is a bigger ship. Unfortunately, we can not rule out the possibility of this occurring, and *this is a case in which the greedy approach may lead to a suboptimal.*

Case (4)

If the average revenue per container in tour 2 is smaller than that of tour 1 by a factor of f, and assuming both tours have the same utilization and travel time, it is straightforward to see that:

$$
\begin{aligned}
\frac{P_2^2}{P_1^2} &= \frac{fV_2}{T_2^1} / \frac{fV_1}{T_1^1} \\
&= \frac{P_2^1}{P_1^1}
\end{aligned}
$$

Hence to conclude, we can say that relationship (4.1) holds most of the time given the current trend in the design of ocean going containerships and if the assumption that ships have similar utilization in all tours is valid. By exploiting this special problem instance commonly encountered in deep-sea container trades, our next Proposition guarantees that the greedy approach to assign ships to tours gives an optimum solution.

Proposition 4.1 *Let P_a^b be the arc profit of assigning node a to b in an $n \times n$ Maximum Assignment Problem. Suppose the arc profits satisfy these criteria:*

(a) Let P_i^j be the maximum of all feasible arc profits, i.e.,
$P_i^j = \max \{P_s^t : \text{ for all feasible assignment } s \text{ to } t \}$;

(b) For all $s \neq i, t \neq j$, we have:

$$\frac{P_s^t}{P_i^t} \geq \frac{P_s^j}{P_i^j};$$

(c) Both conditions (a) and (b) apply to all square subsets of the Assignment graph.

Then, the greedy algorithm which assigns sequentially by using the remaining feasible arc with maximum arc profit correctly solves the Maximum Assignment Problem.

Proof

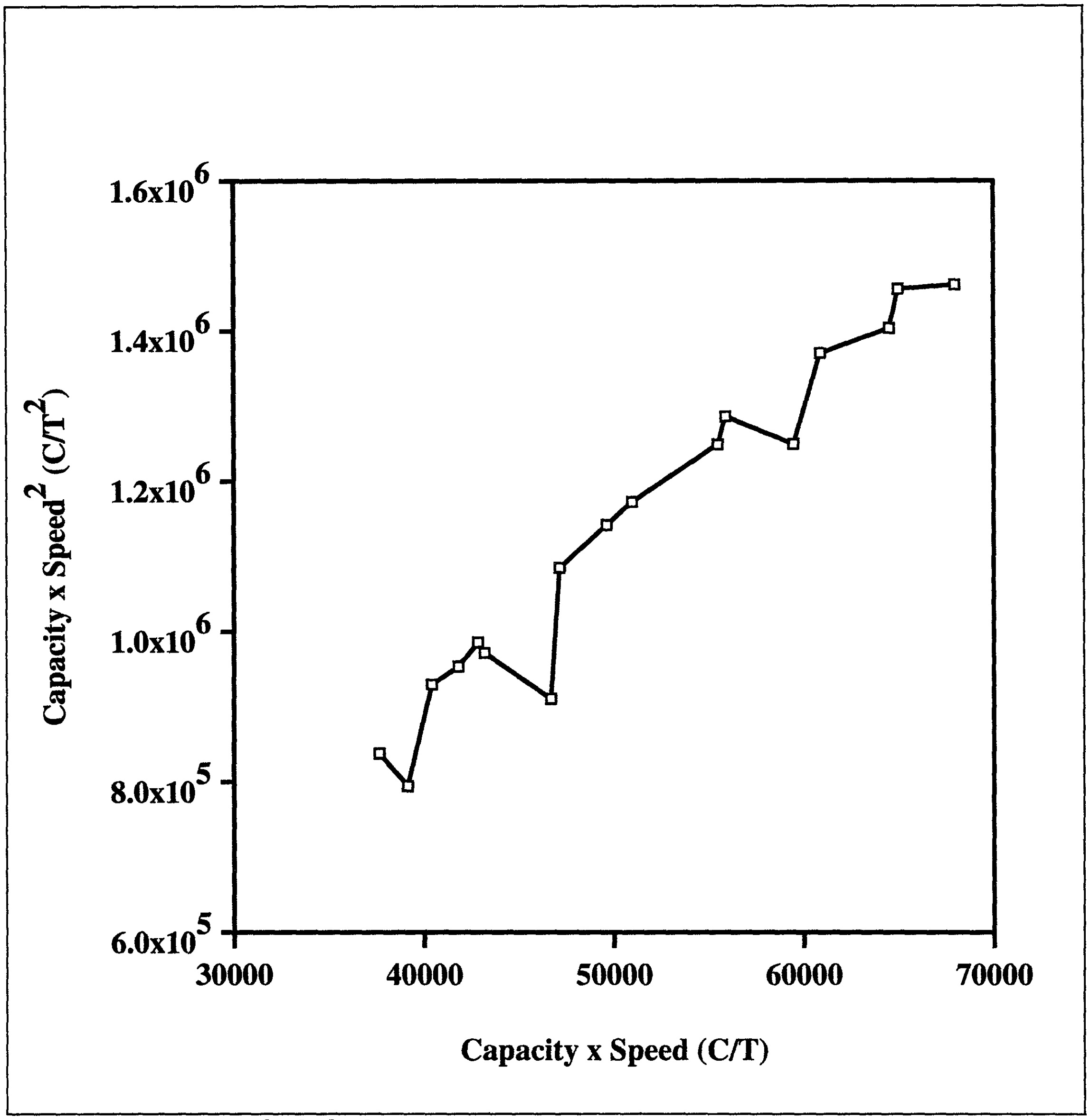
The proof by mathematical induction is given in Appendix A. □

Let us see how Proposition 4.1 can be used in our Liner Deployment Problem. First, as we have argued above, the Liner Deployment Problem can be viewed as an Assignment Problem, although the arc profits are unknown a priori. Second, we can apply Assumption 4.1 to establish criteria (a) and (b). Third, if we assign the ship to the tour that has the maximum PTT ratio, the remaining fleet forms another deployment problem, which by Assumption 4.1, also satisfies criteria (a) and (b) most of the time. Thus this ship by ship approach solves the Assignment Problem optimally, provided that:

(1) the problem instances of the deployment problem always obey Assumption (4.1), and

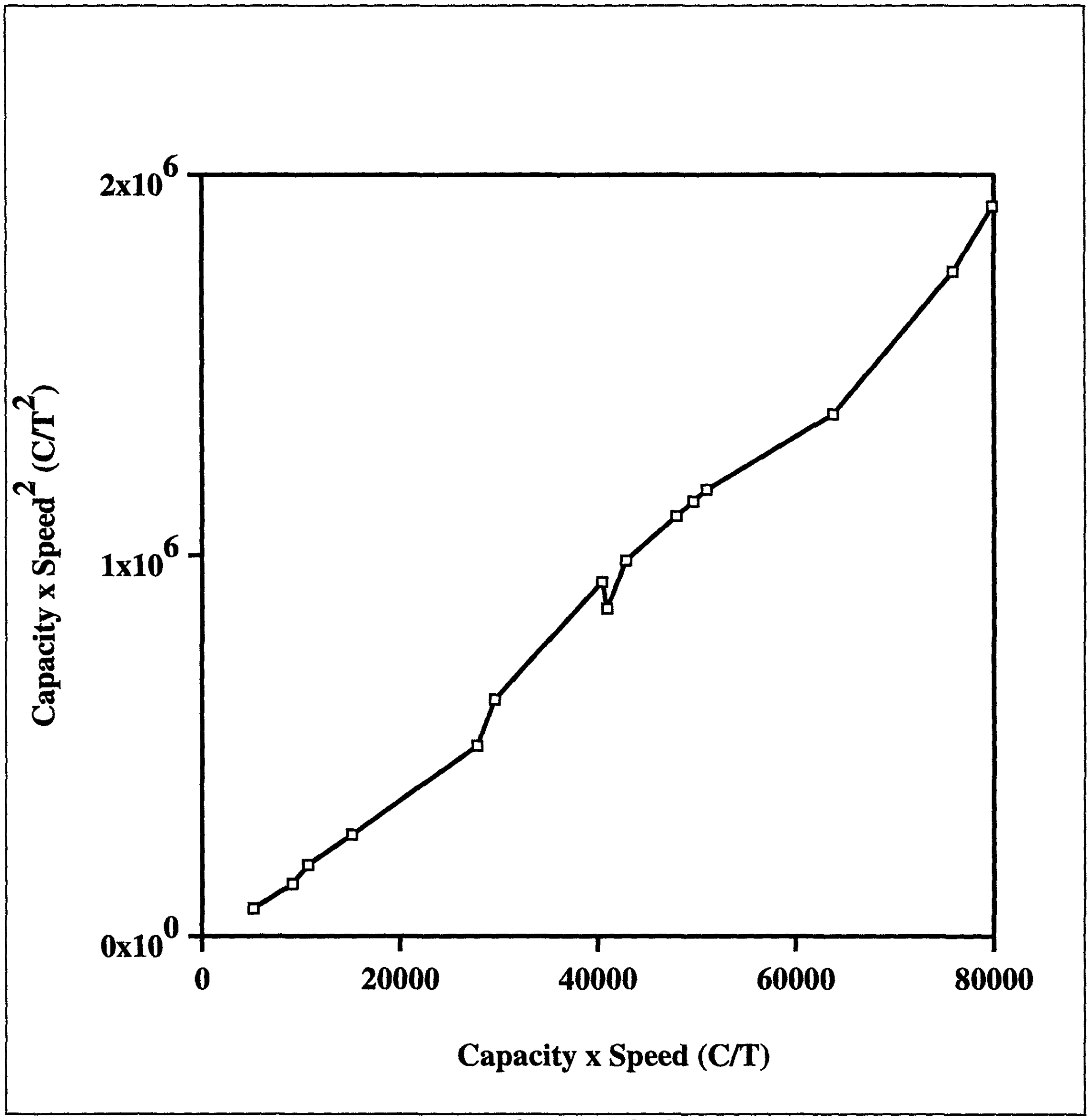
(2) we can calculate the best assignment for each ship at every iteration.

From now on, we assume condition (1) is satisfied, so that our greedy algorithm still gives an optimum fleet deployment so long as each subproblem is solved to optimum. We probably need some post-optimization heuristics to improve our solution if we find a major violation of relation (4.1) in our data. In the next Chapter, we concentrate on presenting our formulation for solving the best assignment for each ship. i.e., for finding a tour with maximum PTT ratio.



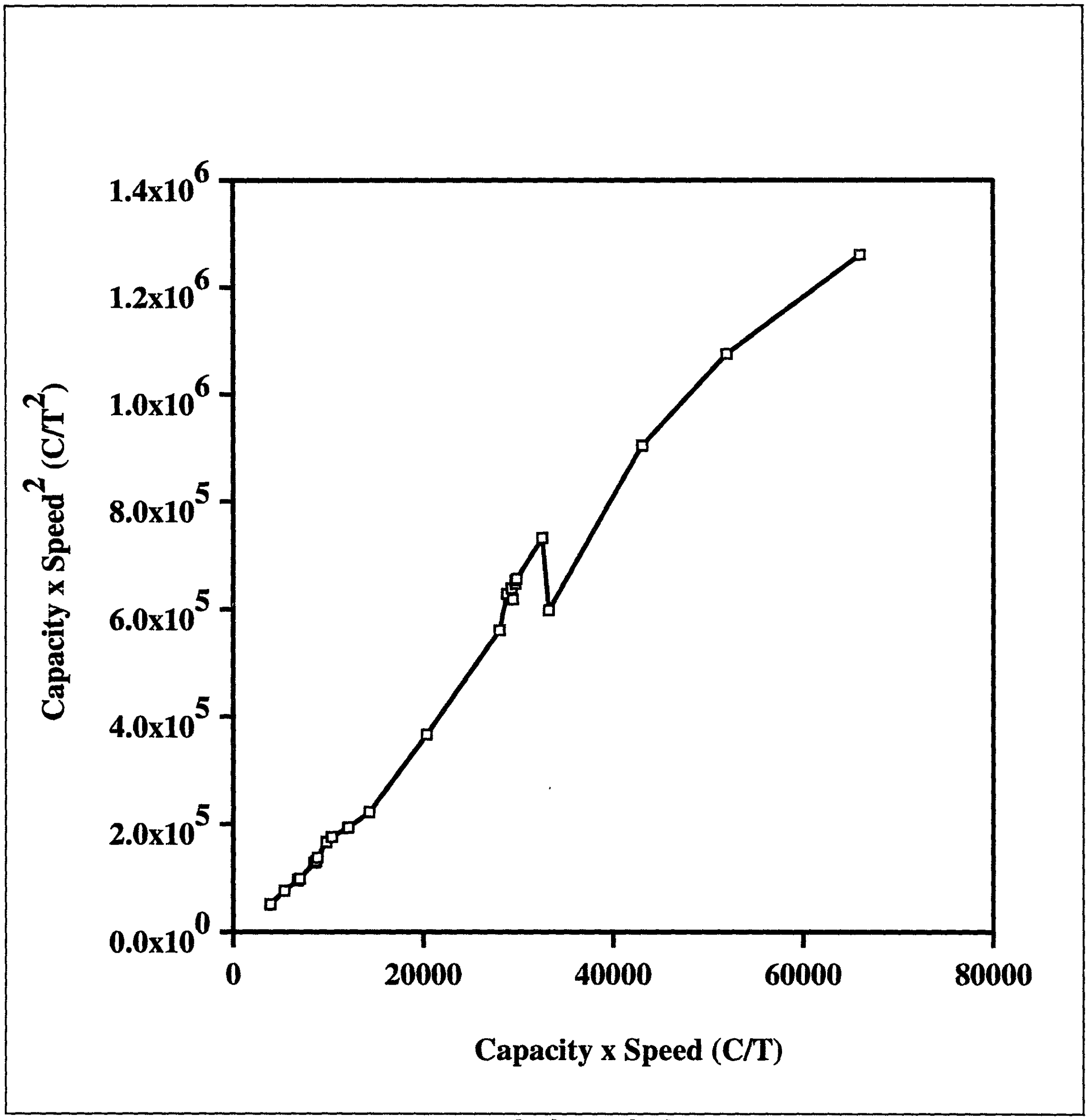
Sources: [18], [11], and courtesy of Neptune Orient Line, Inc.

Figure 4-3: Fleet Characteristics of Trans-Pacific and Round-the-World Trades of Neptune Orient Line, Inc. and Consortia in 1991



Sources: [18] and [11].

Figure 4-4: Characterisitcs of Neptune Orient Line, Inc. Controlled Containerships in 1991



Sources: [18] and [11].

Figure 4-5: Characteristics of Sea-Land Service Inc. Controlled Containerships in 1989

Chapter 5

Finding Maximum PTT Ratio for Shoreline Capacitated LDP

Proposition 3.4 highlights the intrinsic difficulty of the LDP and explains why researchers have developed more efforts to study the case in which ports are arranged in a sequence. The rigorous mathematical programming approach in [17] is based on the assumption that the ports are arranged on a shoreline such that in each tour, the ship changes direction exactly twice at the end-ports. We call this the Shoreline Capacitated LDP. In this Chapter, we provide a new mathematical formulation for finding a tour with maximum PTT ratio. Our formulation does not require the vessel to be empty at her end-ports. This overcomes the drawback shown in Figure 1-1.

Dantzig et al. [4] gave a very elegant method for finding the maximum PTT ratio for a tramper, or bulk ship operator:

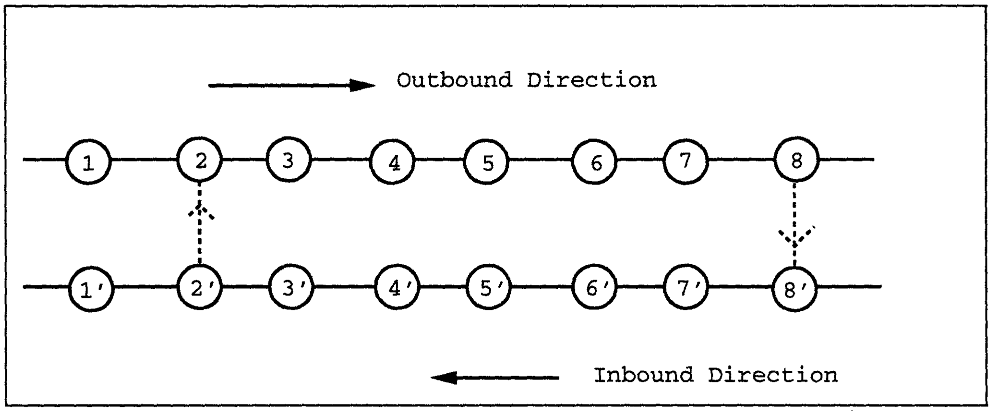
$$\max \quad \sum_i \sum_j p_{ij} x_{ij} \qquad \text{(BSP)}$$

Subject to:

$$\sum_i \sum_j t_{ij} x_{ij} = 1 \tag{5.1}$$

$$\sum_j x_{ij} - \sum_j x_{ji} = 0 \qquad \forall \text{ port } i \tag{5.2}$$

$$x_{ij} \geq 0 \quad \forall \text{ port pairs } (i, j) \tag{5.3}$$



Whenever the ship changes direction from outbound to inbound, an (i, i') arc is used.

Figure 5-1: Each Port Modeled as Two Nodes

where p_{ij} = profit if tramper travels from port i to j

t_{ij} = time for tramper to travel from port i to j

This problem can be solved in polynomial time, and the solution is guaranteed to be a simple tour which has the maximum PTT ratio. In the solution, x_{ij} equals the reciprocal of the tour time if the arc from i to j is included in the solution, and zero otherwise. Unfortunately, it can not be applied directly to the LDP because the revenue of a containership is generated differently from that of a tramp ship (see Figure 2-3). This slight variation of the problem is enough to make it extremely difficult to solve as implied by Proposition 3.4: pure linear programming formulation probably does not work, we need to use zero/one integer variables.

An assumption that frequently accompanies the Shoreline geometry is that the ports are numbered consecutively along the shoreline in an outbound direction from port 1, and that each port is visited at most twice per tour, once in the outbound voyage, once in the inbound. A natural approach to model this situation is to create two nodes for each port. Thus i and i' represent the same port, one for the outbound visit, the other for the inbound visit respectively (Figure 5-1).

5.1 Nomenclature

Parameters

A_{ij} : estimated weekly available cargo from port i to port j assuming there is a weekly service from i to j (TEU per week);

c_1^s : daily operating costs at sea for ship s ($ per day);

c_{2i}^s : daily operating costs at port i for ship s ($ per day);

c_{3i}^s : port due at port i for ship s ($);

c_{4i}^s : cargo handling cost at port i for ship s ($ per TEU);

r_{ij} : average revenue of cargo from port i to port j ($ per TEU);

t_i^s : average port time for ship s at port i (days);

t_{ij}^s : sailing time for ship s from port i to port j (days);

V_s : capacity of ship s (TEU);

$\mathcal{N}$: the given set of ports;

$\mathcal{N}'$: the duplicate set of ports of $\mathcal{N}$ to model a second visit;

N : number of ports along the shoreline, i.e., the cardinality of $\mathcal{N}$ or $\mathcal{N}'$.

For sake of simplicity, the superscript s which stands for ship s is dropped hereafter.

Continuous Variables

$$x_{ij} : \begin{cases} 0 & \text{if ship } s \text{ does not sail } \textit{directly} \text{ from port } i \text{ to port } j \\ y & \text{if ship } s \text{ sails } \textit{directly} \text{ from port } i \text{ to port } j; \end{cases}$$

y : is the reciprocal of the time needed by ship s to complete her tour (days^{-1});

$\bar{x}^{ij}$: this quantity divided by y is the fraction of weekly cargo from port i to port j that is carried by ship s;

Binary Variables

$$z_{ij} : \begin{cases} 0 & \text{if ship } s \text{ does not sail directly from } i \text{ to } j \\ 1 & \text{if ship } s \text{ sails directly from } i \text{ to } j \end{cases}$$

$$z_i : \begin{cases} 0 & \text{if ship } s \text{ does not visit port } i \\ 1 & \text{if ship } s \text{ visits port } i \end{cases}$$

5.2 Mixed Integer Formulation

Objective Function – maximization (SLDP)

$$\sum_{i,j\in\mathcal{N}} \left\{(r_{ij} - (c^s_{4i} + c^s_{4j}))A_{ij}\right\} \left[\bar{x}^{ij} + \bar{x}^{ij'} + \bar{x}^{i'j} + \bar{x}^{i'j'}\right] \tag{5.4}$$
$$- \sum_{i\in\mathcal{N}} \{(c^s_{2i} - 1)t^s_i + c^s_{3i}\} \sum_{j\in\mathcal{N}} x_{ij} - \sum_{i'\in\mathcal{N}'} \{(c^s_{2i} - 1)t^s_i + c^s_{3i}\} \sum_{j'\in\mathcal{N}} x_{i'j'} - c^s_1 \sum_{i\in\mathcal{N}} z^s_{ii'}$$

Network Constraints

$$z_{ii'} + \sum_{j\in\mathcal{N}}^{j>i} z_{ij} = z_i \quad \forall i \in \mathcal{N} \tag{5.5}$$

$$z_{i'i} + \sum_{j'\in\mathcal{N}}^{j'<i'} z_{i'j'} = z_{i'} \quad \forall i' \in \mathcal{N}' \tag{5.6}$$

$$z_{i'i} + \sum_{j\in\mathcal{N}}^{j<i} z_{ji} = z_i \quad \forall i \in \mathcal{N} \tag{5.7}$$

$$z_{ii'} + \sum_{j'\in\mathcal{N}}^{j'>i'} z_{j'i'} = z_{i'} \quad \forall i' \in \mathcal{N}' \tag{5.8}$$

Subtour Breaking Constraint

$$\sum_{i\in\mathcal{N}} z_{ii'} \leq 1 \tag{5.9}$$

Integer Constraints

$$z_i, z_{i'} \in \{0,1\} \quad \forall i \in \mathcal{N} \tag{5.10}$$

Imposing Integer Constraints on Variables x

$$x_{ij} \leq z_{ij} \quad \forall i,j <\in \mathcal{N}, i < j \tag{5.11}$$

$$x_{i'j'} \leq z_{i'j'} \quad \forall i',j' \in \mathcal{N}', i' > j' \tag{5.12}$$

$$x_{ii'} \leq z_{ii'} \qquad \forall i \in \mathcal{N} \tag{5.13}$$

$$x_{i'i} \leq z_{i'i} \qquad \forall i' \in \mathcal{N}' \tag{5.14}$$

Network Constraints on Variables x

$$x_{ii'} + \sum_{j \in \mathcal{N}}^{j>i} x_{ij} = x_{i'i} + \sum_{j \in \mathcal{N}}^{j<i} x_{ji} \quad \forall i \in \mathcal{N} \tag{5.15}$$

$$x_{i'i} + \sum_{j' \in \mathcal{N}'}^{j'<i'} x_{i'j'} = x_{ii'} + \sum_{j' \in \mathcal{N}'}^{j'>i'} x_{j'i'} \quad \forall i' \in \mathcal{N}' \tag{5.16}$$

$$\sum_{i \in \mathcal{N}} x_{ii'} = y \tag{5.17}$$

$$\sum_{i,j \in \mathcal{N}}^{j>i} t_{ij} x_{ij} + \sum_{i',j' \in \mathcal{N}'}^{j'>i'} t_{ji} x_{j'i'} + \sum_{i \in \mathcal{N}} t_i \sum_{j \in \mathcal{N}}^{j>i} x_{ij} + \sum_{i' \in \mathcal{N}'} t_i \sum_{j' \in \mathcal{N}'}^{j'<i'} x_{i'j'} = 1 \tag{5.18}$$

Direct Shipment Constraints

$$\bar{x}^{st} + \bar{x}^{st'} \leq \sum_{j \in \mathcal{N}}^{j>s} x_{sj} \quad \forall s, t \in \mathcal{N} \tag{5.19}$$

$$\bar{x}^{s't} + \bar{x}^{st} \leq \sum_{i \in \mathcal{N}}^{i<t} x_{it} \quad \forall s, t \in \mathcal{N} \tag{5.20}$$

$$\bar{x}^{s't} + \bar{x}^{s't'} \leq \sum_{j' \in \mathcal{N}'}^{j'<s'} x_{s'j'} \quad \forall s, t \in \mathcal{N} \tag{5.21}$$

$$\bar{x}^{st'} + \bar{x}^{s't'} \leq \sum_{i' \in \mathcal{N}'}^{i'>t'} x_{i't'} \quad \forall s, t \in \mathcal{N} \tag{5.22}$$

Capacity Constraints

$$\sum_{j,k\in\mathcal{N}}^{j<i<k} A_{jk}\bar{x}^{jk} + \sum_{j\in\mathcal{N}k'\in\mathcal{N}'}^{j<i} A_{jk}\bar{x}^{jk'} + \sum_{j\in\mathcal{N},j'\in\mathcal{N}'}^{j\neq i} A_{ij}\left[\bar{x}^{ij} + \bar{x}^{ij'}\right]$$

$$+ \sum_{j',k'\in\mathcal{N}'}^{j'<k'} A_{jk}\bar{x}^{j'k'} + \sum_{j'\in\mathcal{N}',k\in\mathcal{N}}^{k>i,k\neq j} A_{jk}\bar{x}^{j'k} \leq Vy \quad \forall i \in \mathcal{N} \tag{5.23}$$

$$\sum_{j',k'\in\mathcal{N}'}^{k'>i'>j'} A_{jk}\bar{x}^{j'k'} + \sum_{j'\in\mathcal{N}',k\in\mathcal{N}}^{j'>i'} A_{jk}\bar{x}^{j'k} + \sum_{j\in\mathcal{N},j'\in\mathcal{N}'}^{j'\neq i'} A_{ij}\left[\bar{x}^{i'j'} + \bar{x}^{i'j}\right]$$

$$+ \sum_{j,k\in\mathcal{N}}^{j>k} A_{jk}\bar{x}^{jk} + \sum_{j\in\mathcal{N},k'\in\mathcal{N}'}^{k'<i',k'\neq j'} A_{jk}\bar{x}^{jk'} \leq Vy \quad \forall i' \in \mathcal{N}' \tag{5.24}$$

Non-Negativity Constraints

$$x_i, x_{ij} \geq 0 \quad \forall i, (i,j) \tag{5.25}$$

$$\bar{x}^{ij} \geq 0 \quad \forall (i,j) \tag{5.26}$$

5.3 Explanation of Model

Integer Solution is a Simple Tour in Extended Graph

The network constraints (5.5)-(5.8) set up the shoreline network structure. The shoreline property implies that whenever the ship changes direction from outbound to inbound, she must use one of the artificial arcs (i, i'). Hence (5.9) and (5.10) are enough to ensure that the integer solution represents a simple tour in the extended graph $\mathcal{N} \cup \mathcal{N}'$, and that there is no subtour. Next we use the idea of the bulker model (BSP) at the beginning of the Chapter to construct the PTT requirement. Constraints (5.15),(5.16), and (5.17) resemble the constraints (5.1) and (5.2). Constraints (5.11)-(5.14) guarantee that when all z's attain integer values, a simple tour solution on the x_{ij} variables is imposed. Hence those values of x's that are strictly positive must be identical to the reciprocal of the tour time, which, according to (5.18) is the sum of sailing time and port time. Since the tour uses exactly one of the arcs (i, i') to change sailing direction from outbound to inbound, (5.17) implies that y

must be the reciprocal of the tour time.

Cargoes Pick-up and Delivery of without Trans-shipment

Constraints (5.19)-(5.22) mean that cargoes can not be shipped from port s to t unless both are visited by the same ship. Hence this model does not allow trans-shipment. The available cargo between port s and t is expressed in terms of a weekly estimated quantity A_{st}. Here, we try to model the situation that if there is a weekly service by a fleet of ships in a tour containing both ports s and t, then, the maximum amount of the weekly cargo from s to t that can be gained by each ship in the fleet is A_{st}. Even if the containership operator succeeds to arrange more than one visit per week to s and t in this particular tour, the maximum combined amount of weekly cargo gained is still A_{st}. Thus A_{st} can be interpreted as the amount of weekly cargo that can be reserved for the liner operator by his local agents given that he can provide weekly regular service. Of course, A_{st} varies with the level-of-service and market demand. We will neither explore nor model this relationship in the thesis. We just assume that in order to maintain this maximum available amount of cargo, the service must be maintained at a weekly frequency. That is why in Step 3 of our greedy algorithm, we request a post-optimization adjustment. This procedure intends to model the common practice among containership operators to maintain weekly calls for staying competitive.

Amount of Cargo On-Board Can Not Exceed Vessel Capacity

Constraints (5.23) and (5.24) ensure that the amount of cargo on-board does not exceed the ship capacity. We first argue that each term in the LHS represents a cargo on its way to the final destination that is on-board the ship when the latter leaves port i. To see this, if the ship calls port i, the shoreline network structure (5.15)-(5.17) imply that the sum of the x_{ij} terms over j plus the term $x_{ii'}$ equals y. By constraints (5.19)-(5.22), none of the $\bar{x}$'s values can be bigger than y. Thus $\bar{x}$ divided by y represents the fraction of weekly cargo gained by the ship. Dividing both sides of (5.23) by y, one can see that the LHS is exactly the total amount of cargo on-board the ship when she leaves i. This quantity must be less than or equal to the vessel capacity which is the RHS bound. When the ship does not call port i, (5.23) becomes redundant. Thus applying this inequality for each node in the graph guarantees that the vessel capacity constraint is not violated.

Non-Empty Vessel Allowed at End-Ports

Notice that we do not require that the ship be empty at the end-ports. For example, $\bar{x}^{st'}$ can be strictly positive with s in $\mathcal{N}$ (an outbound port), and t' in $\mathcal{N}'$ (an inbound port).

Objective Function

The objective is to maximize the Profit-to-Time ratio of the ship. Since the ship is committed to serve the fixed tour for a relatively long time, the PTT ratio is essentially the profit gained per tour divided by the time to complete the tour. As explained above, the tour time is y^{-1}. Recall that the weekly cargo is A_{ij}, and that if there is a service in a tour, we will arrange ships to serve the tour weekly. Also, recall that $\bar{x}^{ij}$ divided by y is the fraction of cargo captured per tour by the ship. For clarity, we will omit the superscripted ports which are located on the inbound direction of the model. Then the revenue of the tour is:

$$\sum_{(i,j)} r_{ij} A_{ij} \frac{\bar{x}^{ij}}{y}$$

Revenue per unit time is the above expression divided by y^{-1} and equals:

$$\sum_{(i,j)} r_{ij} A_{ij} \bar{x}_{ij}$$

The costs of the tour consist of these terms:

$$
\begin{aligned}
\text{ship operating cost at sea} &= \frac{\text{sea-cost}}{\text{day}} \text{ x sea-time} &&= c_1^s\left(\frac{1}{y} - \sum_i t_i z_i\right) \\
\text{ship operating cost at ports} &= \sum \frac{\text{port-cost}}{\text{day}} \text{ x port-time} &&= \sum_i c_{2i}^s t_i z_i \\
\text{port dues} &= \sum \text{ port dues} &&= \sum_i c_{3i}^s z_i \\
\text{cargo handling costs} &= \frac{\text{handling costs}}{\text{TEU}} \text{ x cargo} &&= \sum_{(i,j)} \left(c_{4i}^s + c_{4j}^s\right) A_{ij} \frac{\bar{x}_{ij}}{y}
\end{aligned}
$$

The total operating costs per unit time is the sum of the above costs divided by y^{-1}. By the shoreline network structure again, when z_i are integers, the product of z_i and y is simply the sum of $x_{ii'}$ and $\sum_j x_{ij}$. Thus the operating cost per unit time can be simplified to:

$$
c_1^s - \sum_i t_i \left[x_{ii'} + \sum_j x_{ij}\right] + \sum_i c_{2i}^s t_i \left[x_{ii'} + \sum_j x_{ij}\right] + \sum_i c_{3i}^s \left[x_{ii'} + \sum_j x_{ij}\right]
$$

$$
+ \sum_{(i,j)} \left(c_{4i} + c_{4j}^s\right) A_{ij} \bar{x}^{ij} - (c_{2i}^s + c_{3i}^s) t_i z_{ii'} y
$$

The last term in the above expression corrects the double counting of daily port charge c_{2i}^s and port due c_{3i}^s for the end-ports. The PTT ratio is the difference of these two terms and thus equals our objective function:

$$
\sum_{(i,j)} \left\{(r_{ij} - (c_{4i}^s + c_{4j}^s))A_{ij}\right\} \bar{x}^{ij} - \sum_{i \in \mathcal{N}} \{(c_{2i}^s - 1)t_i + c_{3i}^s\} \sum_{j \in \mathcal{N}} x_{ij} - c_1^s
$$

Update of Weekly Cargoes

Whenever we assign a ship to a tour, part or all of the weekly available cargoes A_{ij} of the ports that the ship visits is taken. Since we will make sure that enough ships are assigned to this tour in order to maintain weekly service, the remaining available weekly cargo is the same in every week, i.e., it equals either zero in case all the weekly cargo is taken by the ship, or it equals the fraction of A_{ij} that is not loaded on-board. However, it may happen that the operator runs out of the same type of vessel to assign to the tour, and the remaining ships are of so different in design characteristics that it is impossible to adjust speed to fit the service requirement. In this case, the operator may only be able to provide less than weekly service, say, biweekly service to the ports in the tour. This may cause a difficulty to define the new weekly cargo quantities A_{ij} for the next iteration. Because if we are considering a week in which no ship in the fleet provides service to port i in a tour that goes through i and j, we have, according to our assumption, the full availability of the old weekly cargo A_{ij}. But if we are considering a week in which a ship in the fleet is visiting port i, we are only left with a fraction or none of A_{ij}.

Strictly speaking, since the level-of-service in this case is degraded (from expected weekly to actual biweekly), some customers may turn to other carriers and thus the available cargo is less than A_{ij}, which is estimated on a weekly service base. We will resort to this simplified method to model this real world phenomenon: simply averaging the fraction of cargo left. Thus for the particular example in which the service is degraded from weekly to biweekly, we assume that the new weekly cargo equals half of the remaining fraction of cargo left by our previous assignment. This implies that if we are able to deploy additional ships to this port such that weekly service is provided, then we are able to capture all the left-over cargoes. Else part of the cargoes is lost to other competitors for ever.

5.4 Valid Inequalities to Strengthen Formulation

The following valid inequalities are found to be useful in reducing the integrality gap of the formulation (SLDP).

$$\bar{x}^{st} + \bar{x}^{st'} + \bar{x}^{s't} + \bar{x}^{s't'} \leq y \qquad \forall s, t \in \mathcal{N} \tag{5.27}$$

$$x_{ii'} \leq \sum_{j \in \mathcal{N}}^{j<i} x_{ji} \qquad \forall i \in \mathcal{N}, i \neq 1 \tag{5.28}$$

$$x_{i'i} \leq \sum_{j' \in \mathcal{N}'}^{j'>i'} x_{j'i'} \quad \forall i' \in \mathcal{N}', i' \neq N' \tag{5.29}$$

Constraints (5.27) help to prevent the same (s, t) cargo pair be counted several times in the objective function when both ports s and s' or t and t' are visited. They are useful to generate tighter gap when the weekly available cargo amount A_{st} is very small compared with the vessel capacity V. In this case, (5.27) dominates the capacity constraints (5.23) and (5.24). Constraints (5.28) and (5.29) help prevent both $x_{ii'}$ and $x_{i'i}$ are positive.

Chapter 6

Extensions of Shoreline Capacitated LDP Model

The goal of the Chapter is to develop a model closer to the real-world operating scenario. Such model can then be used in the PTT ratio optimization subroutine of our greedy algorithm in Chapter 4 to handle a fleet deployment problem. We will consider two extensions of our shoreline capacitated LDP model of Chapter 5: (1) imposing travel time constraint between some specified port pairs; and (2) routing without the shoreline restriction.

6.1 Imposing Transportation Time Constraint

Between big container ports competition among operators is high because shippers have more choices among carriers. Our thesis does not address the issue of competition, which is measured by the level-of-service of the operator. One important element of the level-of-service is the transportation time. When the operator designs his route, he may want to introduce a minimum bench mark transport time between certain port pairs so that his level-of-service is not too small compared with his competitors. For each port pair (s, t) which the operator wants to impose a travel time upperbound T^{st}, add the following constraints:

$$\sum_{(i,j)} t_{ij}\tilde{z}_{ij}^{st} \leq T^{st} \tag{6.1}$$

$$\tilde{z}_{ij}^{st} \leq z_{ij} \quad \forall (i,j) \neq (s,s'),(s',s),(t,t'),(t',t) \tag{6.2}$$

$$\sum_{j} \tilde{z}_{ij}^{st} - \sum_{j} \tilde{z}_{ji}^{st} = \begin{cases} -1 & \text{if } i = s \\ 0 & \text{otherwise} \\ 1 & \text{if } i = t \end{cases} \tag{6.3}$$

$$\tilde{z}_{ij}^{st} \in \{0,1\} \qquad \forall (i,j) \tag{6.4}$$

By constraints (6.2)-(6.4), the integer variables $\tilde{z}$ which equal unity represent a path from s to t. Constraints (6.2) only allow those voyage legs served by the ship to define the path. Finally (6.1) guarantees that the transport time along the (s,t) path is less than or equal to the bench mark value T^{st}. Preventing constraints (6.2) to act on the artificial links $(s,s'),(s',s),(t,t')$, and (t',t), the shortest path time between the nodes created for s and t respectively will be used to satisfy the travel time bench mark constraint (6.1).

6.2 Relaxing Shoreline Network Restriction

We face two difficulties for a routing formulation without the shoreline geometry. The first difficulty is how to eliminate subtours. The second difficulty is how to guarantee the vessel capacity constraint is obeyed at every voyage leg. Central to the new formulation are two key observations of deep-sea containership deployment: (1) a port is not visited more than twice in a tour; (2) the ship crosses the deep-sea from one trade Region to the other once in each direction per tour (see Chapter 2) (see Chapter 2. From the first observation, we can keep on using two nodes to model every given port, and we call the graph with these extra nodes an extended graph. In the Shoreline model (SLDP), the assumption derived from this observation is that the ship changes direction at most twice. In the model below in which we relax the Shoreline geometry, the weaker assumption adopted is that each arc in the extended graph is not used more than once. This w eaker assumption is sufficient to allow our formulation give a simple tour solution without the shoreline network. The second observation enables us to derive a mixed integer description of the problem. We overcome the first difficulty by using a modified Traveling Salesman subtour breaking constraint, at the expense of increasing the size of the formulation expoentially. We overcome the second difficulty by introducing a dummy cargo that fills the ship at her deep-sea and consequent voyage legs.

6.2.1 Subtour Breaking Constraints

We use the same nomenclature as in Chapter 5.1. The following model is constructed for a containership which trades between Region $\mathcal{A}$ and Region $\mathcal{B}$ separated by deep-sea. The objective function is similar to (5.4).

Network Constraints

$$z_{ii'} + \sum_{j \in \mathcal{N}} z_{ij} = z_i \quad \forall i \in \mathcal{N} \tag{6.5}$$

$$z_{i'i} + \sum_{j' \in \mathcal{N}} z_{i'j'} = z_{i'} \quad \forall i' \in \mathcal{N}' \tag{6.6}$$

$$z_{i'i} + \sum_{j \in \mathcal{N}} z_{ji} = z_i \quad \forall i \in \mathcal{N} \tag{6.7}$$

$$z_{ii'} + \sum_{j' \in \mathcal{N}} z_{j'i'} = z_{i'} \quad \forall i' \in \mathcal{N}' \tag{6.8}$$

Subtour Breaking Constraint

$$\sum_{i \in \mathcal{A} \cup \mathcal{A}'} \sum_{j \in \mathcal{B} \cup \mathcal{B}'} z_{ij} \leq 1 \tag{6.9}$$

$$\sum_{(i,j) \in \mathcal{S}} z_{ij} \leq \sum_{i \in \mathcal{S}} z_i - z_k \quad \forall \mathcal{S} \subseteq \mathcal{A} \cup \mathcal{A}', \text{ any } k \in \mathcal{A} \cup \mathcal{A}'$$
$$\forall \mathcal{S} \subseteq \mathcal{B} \cup \mathcal{B}', \text{ any } k \in \mathcal{B} \cup \mathcal{B}' \tag{6.10}$$

Integer Constraints

$$z_{ij}, z_{i'i} \in \{0, 1\} \quad \forall (i,j), \forall i \tag{6.11}$$

The integer constraints imposed on x variables, the network constraints on x variables, the direct shipment constraints, and the non-negativity constraints are similar to (5.11)-(5.22), and (5.25)-(5.26) respectively.

Constraint (6.9) prevents more than one tour to span Region $\mathcal{A}$ and $\mathcal{B}$. It is built on the observation that each tour crosses the deep-sea from $\mathcal{A}$ to $\mathcal{B}$ exactly once. Constraints (6.10) prevents subtours to form within each Region. The difference between our constraint and the classical TSP subtour breaking constraint is that for each subset $\mathcal{S}$ under consideration, we do not know a priori how many nodes in $\mathcal{S}$ are visited. Therefore, the RHS of (6.10) is a variable rather than the cardinality of $\mathcal{S}$ minus one. If all the nodes in $\mathcal{S}$ are visited, (6.10) correctly prevents those nodes to form a subtour. If some nodes in $\mathcal{S}$ are not visited, (6.10) for $\mathcal{S}$ is redundant.

6.2.2 Vessel Capacity Constraints

To make sure that every voyage leg satisfies the capacity constraint, we introduce the following new variables:

$\bar{B}_{ij}$: a dummy cargo with no revenue and with abundant supply between any port pair (i, j);

w_i : $\begin{cases} 1 & \text{if } i \text{ is the last port in Region } \mathcal{B} \text{ that the ship visits} \\ 0 & \text{otherwise} \end{cases}$

These constraints make sure that the vessel can not be overloaded on any voyage leg:

<u>Vessel Capacity Constraints</u>

$$w_i = \sum_{j \in \mathcal{A}} z_{ij} \qquad \forall i \in \mathcal{B} \tag{6.12}$$

$$\bar{B}^{ij} \leq V x_{ij} \quad \forall i \in \mathcal{B}, \forall j \in \mathcal{A} \tag{6.13}$$

$$\sum_{s \in \mathcal{B}} \sum_{t \in \mathcal{A}} A_{st} \bar{x}^{st} + (w_i - 1)V + \sum_{s \in \mathcal{B}} \sum_{t \in \mathcal{A}} \bar{B}^{st} \leq V \sum_{s \in \mathcal{B}} \sum_{t \in \mathcal{A}} x_{st} \qquad \forall i \in \mathcal{B} \tag{6.14}$$

$$V \sum_{s \in \mathcal{B}} \sum_{t \in \mathcal{A}} x_{st} - \sum_{\forall s} A_{sj} \bar{x}^{sj} - \bar{B}^{ij}$$
$$+ \sum_{\forall t} A_{jt} \bar{x}^{jt} + \sum_{\forall t} \bar{B}^{jt} + (z_{ij} - 1)V \leq V \sum_{s \in \mathcal{B}} \sum_{t \in \mathcal{A}} x_{st} \qquad \forall (i,j) \qquad (6.15)$$

Imagine inequality (6.15) is divided throughout by y. As discussed in Chapter 5.3 when we explain the objective function, each term can now be interpreted as a cargo quantity. When voyage leg (i, j) is used, z_{ij} equals one and constraint (6.15) is activated. Let us track each voyage leg of the vessel starting from that deep-sea leg (i, j) which crosses from Region $\mathcal{B}$ to $\mathcal{A}$. The first term in the LHS of (6.15) represents the total amount of cargo onboard the vessel when she crosses the deep-sea from Region $\mathcal{B}$ to $\mathcal{A}$. The second term represents the amount of cargo unloaded at port j, the first port of Region $\mathcal{A}$. The third term is the amount of dummy cargo carried on the deep-sea leg (i, j). Hence the sum of the second and third term represents the new space available onboard the vessel after she unloads at j. The forth and fifth term in the LHS represent the amount of new

cargoes loaded on-board at j. Hence sum of all terms in the LHS should not exceed the vessel capacity. However, the RHS will be less than the vessel capacity unless she is fully loaded when she crosses the deep-sea. This can always be achieved because she can be loaded with dummy cargoes $\bar{B}_{ij}$ if necessary. In fact, since i is the last port in Region $\mathcal{B}$, (6.14) guarantees that the ship can not be overloaded by dummy cargoes. The above argument leads to the fact that the vessel capacity is obeyed at the deep-sea leg (i, j) between Region $\mathcal{B}$ and $\mathcal{A}$, and that we know exactly how much empty space is available onboard after the vessel unloads at j, the first port of call in Region $\mathcal{A}$. Using these two information, one can repeat similar reasoning and conclusion for the next and subsequent voyage legs after (i, j). Thus this recurring recurring nature of constraints (6.14) and (6.15) eventually guarantee that the ca pacity constraint is always satisfied. Finally, notice that the first term in the LHS of (6.15) is same as the RHS term. They cancel out each other in the final formulation, but are shown here to help the explanation.

Chapter 7

Computation Results

The purpose of test computations is to see how our formulation behaves. We test the extended version of the capacitated LDP with two Regions. The first Region consists of 11 ports, the second consists of 3. We did not include all the subtour breaking constraints. But we have all those constraints that prevent a subtour between two nodes within each region. The rest of the constraints are added whenever we encounter subtours. Once added, a subtour breaking constraint is kept in the formulation. Any CPU time reported below refers to the CPU time for a solution where subtours have already been eliminated, using the number of subtour breaking constraints up to that moment.

We want to choose bad input instances to test our formulation. Since the optimum solution is a tour with maximum Profit-to-Time ratio, we adjust the denominator – the distance input – in order to get some difficult instances. The distance matrix for this test computation given in Table 7.1 is generated as follow. We first generate a 14×14 asymmetric distance matrix at random from the interval [0,10]. For a distance pair between the two Regions, we increase its value by 10. We solve the LP relaxation of the classical TSP formulation using this distance matrix. Then we deliberately adjust the distance so that this LP relaxation solution has fewer integer variables. We iterate the process till we got a highly non-integral solution of the relaxed problem. At the end of this process, we have a distance matrix that is a very bad problem instance for the 14 nodes TSP problem. It obeys neither the triangular inequality nor symmetry. Being a denominator of a ratio, its role is to imitate the randomness of the cost and revenue input data.

The cargo matrix, and port dues are selected at random as shown in Table 7.2 and 7.3. Given that we deliberately use the distance matrix to create a bad input instance, we do not spend further effort in generating a random cargo revenue matrix. All cargo revenues r_{ij}'s are set to unity, all other costs are set to zero except a port due for each port. This is equivalent to imposing a fixed charge whenever a port is called. The vessel capacity for the base case is 3000 TEU.

The mixed integer program shown in Chapter 6 is solved using Version 1.2 (1991) of the mixed-integer optimization package CPLEX installed in a SUN 3-280 workstation. The optimum tour of the base case is shown in Figure 7-1, which is of length 34.9 days. For this particular instance, it happens that the ship visits quite a number of ports in her tour, which is a simple tour in the original graph.

To speed up the Branch-and-Bound process in finding integer solution, we make use of the user interface ability of CPLEX to input valid lower cut-off bound generated heuristically. At the end of the LP optimization process, we check which solutions $\bar{x}^{ij}$'s are strictly positive. For those that are positive, the ship must visit both ports i and j. Then we set the z_i's of those ports to one. This process fixes most of the influential undecided 0/1 integer variables except the w's, i.e., from which ports the ship should exit Region $\mathcal{B}$. We determine this using the CPLEX Branch-and-Bound process again. This is relatively easy to solve and we got a valid lower cutoff fairly quickly. Without this accelerated way to generate the first lower cutoff, as shown in Table 7.4, the search process is much longer. The reason this heuristics is very helpful in this case is explained as follows.

From the computation experience we gained, the Branch-and-Bound process usually sets most of the z_i's to zero initially in a depth first search, thus leading to a tour with a very small number of ports to begin with. As shown in Figure 7-1, if the optimum tour happens to have quite a number of ports, the poor lower cutoff found at the beginning of the Branch-and-Bound process creates a lot of unfathomed nodes to be explored. In the exploration of each of these nodes, the search again starts with a very small tour first, thus lengthening the optimum search.

As seen in Table 7.4, the valid inequalities (5.27)-(5.29) reduce dramatically the gap between the LP and IP from 18% to 4%, and speed up the calculation process.

Table 7.5 and 7.6 show the performance of the model with respect to various vessel capacity and cargo availability A_{ij}. The solution time seems to increase with decreasing vessel capacity. When the capacity is decreased to 2300, a more complicate tour pattern emerges: port 3 is visited two times to compromise some violated capacity constraint within Region $\mathcal{A}$. As seen from Table 7.6, solution time also increases with cargo quantity, but the tour gets simpler. The reason of a longer solution time is that when the A_{ij}'s are bigger, the valid inequalities (5.27)-(5.29) become less and less binding, hence their presence does not help reducing the integrality gap. In fact, when A_{ij}'s tend to infinity, we no longer have the less-than-vessel-load condition and the problem reduces to the bulk ship problem (5.1)-(5.3). In that case, our complicate fo rmulation is highly redundant and hence a longer time is needed for finding the optimum.

If we can estimate the real-world cargo quantity and revenue between all port pairs, it is useful to repeat similar calculations to see the change in deployment pattern vis-à-vis the change in operating conditions. Testing with real-world data is the only way to judge how our model behaves in practice.

Table 7.1: Distance Matrix for a Fictitious 2 Regions Problem Without Shoreline Network Restriction

t_{ij} (in days)

D O	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	-	5.8	0.8	1.2	5.9	6.0	2.9	0.8	4.6	1.5	5. 7	12.1	14.1	14.4
2	1.2	-	2.7	4.1	4.5	4.9	5.4	2.0	1.2	1.0	5.9	14.9	14.6	17.5
3	6.0	4.6	-	0.5	2.4	0.6	2.4	0.8	0.9	0.8	5.3	12.6	15.2	13.9
4	4.1	3.3	3.8	-	1.9	1.9	1.8	3.8	2.8	2.4	4.6	13.8	14.9	15.1
5	0.5	0.8	0.8	4.0	-	0.9	1.2	2.1	2.5	1.6	3.5	14.9	15.2	17.7
6	3.4	2.0	3.5	5.3	5.1	-	1.8	2.0	3.4	1.7	1.2	16.5	15.0	17.8
7	1.9	3.7	2.1	3.7	4.2	3.2	-	1.7	3.1	4.4	3.6	12.6	12.7	11.4
8	0.5	2.0	5.0	1.2	5.6	4.3	2.0	-	3.7	1.9	3.1	14.9	15.6	15.1
9	1.5	4.2	5.0	4.3	4.2	3.1	3.8	4.1	-	1.9	2.9	13.5	12.6	13.4
10	3.7	3.2	1.2	2.5	1.8	1.5	1.3	2.7	2.6	-	1.2	11.6	16.7	14.6
11	4.8	2.5	2.8	4.5	2.2	2.7	2.4	3.3	1.9	3.1	-	16.3	18.1	14.8
12	12.3	16.7	15.9	13.7	17.7	19.5	13.9	17.5	12.9	15.2	13.6	-	5.2	4.8
13	15.3	15.9	16.1	15.1	17.9	14.7	16.7	15.2	14.2	14.2	17.1	2.8	-	2.6
14	12.6	13.5	12.4	14.2	14.6	14.9	13.3	14.1	15.1	13.9	14.2	4.0	3.9	-

Table 7.2: Cargo Matrix for a Fictitious 2 Regions Problem

A_{ij} (TEU per week)

D O	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0	0	0	0	0	0	0	0	0	0	0	50	100	150
2	0	0	0	0	0	0	0	0	0	0	0	100	200	450
3	0	0	0	0	0	0	0	0	0	0	0	100	100	100
4	0	0	0	0	0	0	0	0	0	0	0	80	200	100
5	0	0	0	0	0	0	0	0	0	0	0	50	50	100
6	0	0	0	0	0	0	0	0	0	0	0	0	0	100
7	0	0	0	0	0	0	0	0	0	0	0	20	70	50
8	0	0	0	0	0	0	0	0	0	0	0	130	100	80
9	0	0	0	0	0	0	0	0	0	0	0	150	200	400
10	0	0	0	0	0	0	0	0	0	0	0	300	200	400
11	0	0	0	0	0	0	0	0	0	0	0	50	300	330
12	30	40	20	100	150	10	30	50	100	200	190	0	0	0
13	10	50	100	180	200	50	100	100	200	190	200	0	0	0
14	50	100	300	310	200	70	50	10	400	350	210	0	0	0

Table 7.3: Cost Matrix for a Fictitious 2 Regions Problem

Only port dues are non-zero. All other costs are assumed to be zero.
The port dues c_{3i} are:

1	2	3	4	5	6	7	8	9	10	11	12	13	14
50	200	150	180	180	130	110	80	70	100	110	100	80	80

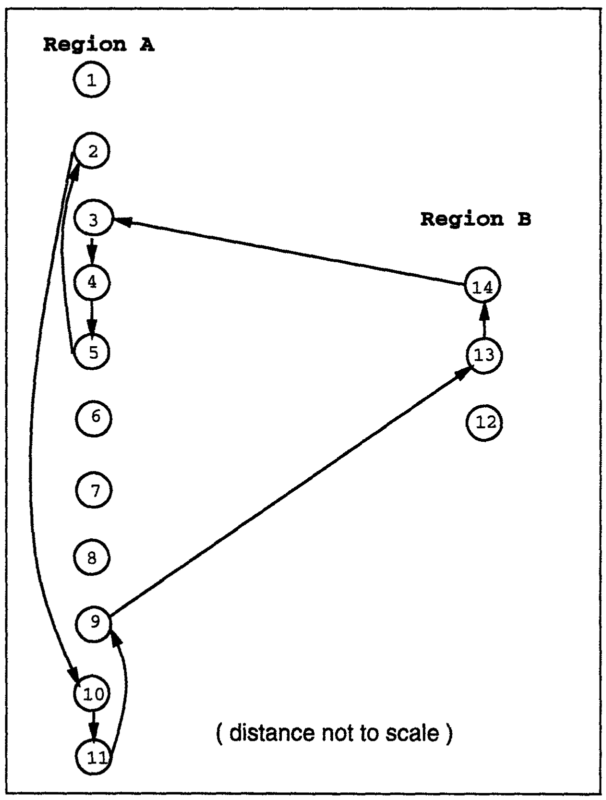


Figure 7-1: Optimum Tour of a Two Regions Capacitated LDP without Shoreline Network Restriction

Table 7.4: Test Results: Base Case

Base Case, Vessel Capacity = 3000

Scenario	Optimum Tour	Tour Length	LP Value	IP Value	CPU Time
no lc, no (5.27)-(5.29)	-	-	1625.7	1246*	> 2 hrs
with lc, no (5.27)-(5.29)	3-4-5-2-10-11-9-13-14-3	34.9	1625.7	1378.2	32.0 min
both lc and (5.27)-(5.29)	3-4-5-2-10-11-9-13-14-3	34.9	1427.0	1378.2	2.6 min

lc = heuristic lower cutoff implemented;
* = this is the best IP solution after 2 hours of computation

Table 7.5: Variation of Maximum PTT Ratio with Vessel Capacity

Scenario	Optimum Tour	Tour Length	LP Value	IP Value	CPU Time
$V = 1000$	1-3-9-13-14-1	31.3	636.0	530.4	8.9 min
$V = 1900$	3-10-11-9-13-14-3	31.5	1077.6	1009.5	2.6 min
$V = 2300$	3-8-1-10-3-13-14-3	37.9	1264.3	1170.2	5.7 min
$V = 3000$	3-4-5-2-10-11-9-13-14-3	34.9	1427.0	1378.2	2.6 min
$V = 3400$	1-3-10-11-5-2-9-13-14-12-1	38.5	1529.9	1466.8	2.6 min
$V = 4000$	1-3-4-5-2-10-11-9-13-14-12-1	39.6	1616.1	1616.1	1.0 min

Computations are done with the heuristics for finding a lower cutoff earlier, and the valid inequalities (5.27)-(5.29).

Table 7.6: Variation of Maximum PTT Ratio with Cargo Quantity

Scenario	Optimum Tour	Tour Length	LP Value	IP Value	CPU Time
Base Case	3-4-5-2-10-11-9-13-14-3	34.9	1427.0	1378.2	2.6 min
$\uparrow$ by 1000	1-10-14-1	28.7	145.5	113.6	3.5 min
$\uparrow$ by 2000	1-10-14-1	28.7	72.3	34.3	4.1 min

Computations are done with the heuristics for finding a lower cutoff earlier, and the valid inequalities (5.27)-(5.29).

We make an across the board increase in the cargo matrix. The solution value appears smaller than the basecase because the cargo revenues r_{ij} are not rescaled.

Chapter 8

Conclusions and Further Research

8.1 Summary of Findings

Despite containership routing is on the top hierarchy of a series of operational decisions, the efforts done on it are far from satisfactory. This thesis views the problem from the perspective of finding a deployment pattern with maximum Profit-to-Time (PTT) ratio. We identify the fundamental difficulty of the problem from which we obtain useful insights. There are two special cases under which the problem can be efficiently solved. Without surprise, both correspond to the uncapacitated version. Our vision of the problem to model the cargo on a weekly available basis leads to a useful application of the uncapacitated version – in designing the optimum vessel capacity. The first special case corresponds to the situation when port time dominates sailing time. Mathematically, it is also known as the Weighted Maximum Density Subgraph Problem among operational researchers. Our studies lead to a compact linear program description of this problem. The second special case can be interpreted as the ports being located along a rigid shoreline, such as the convex coast of a continent, or along a river or canal. We propose an $O(N^2)$ algorithm to solve it, N being the number of ports.

For the remaining cases, we propose new mathematical programming models to find the maximum PTT ratios. Following the common assumption that the visit sequence of ports is determined a priori, the problem becomes which port to select for visit along the sequence. Apparently, this version is non-trivial. Our model does not require the containership to be completely empty at any port, capturing a more realistic scenario than current models. In deep-sea trades, ports can be grouped in trade Regions, using this fact, we extend the capacitated model to one without restrictions on visit sequence.

We examine the fleet deployment problem from an engineering perspective by relating the maximum PTT ratio of a tour with the vessel's principal characteristics, namely her speed and capacity. We deduce a relationship which if satisfied by the ships and the system, implies that the problem can be solved by a greedy assignment procedure. This relation is always satisfied for a homogeneous fleet. The non-homogeneous fleet characteristics of the operators we studied suggest that such relationship is still fulfilled most of the time, provided that the utilization of the vessel is the same for all tours.

8.2 Further Research

This work opens our view to the Liner Deployment Problem in several interesting areas.

From the theoretical perspective, Conjecture 3.6 is still unsettled. The Shoreline Non-Negative Profits Uncapacitated LDP is challenging because of its intrinsic simplicity yet the decision version of it appears to be in the border of the class P and NP. Resolving it may lead to new insight to some other problems, and perhaps a better understanding of the boundary between the two classes.

From the practical perspective, there are other important issues of the Liner Deployment Problem that this research does not address. Trans-shipment is an important practice in liner shipping, yet as far as we know, no routing model ever takes into account of it. We have attempted the issue without much success. The other issue is the quality of service of the deployment pattern.

The theme of our research has been using mathematical programming approach to help containership operator design a better deployment pattern. Throughout the thesis, we recognize the importance of the level-of-service of the operator in his deployment pattern, but our emphasis have been placed on the routing issue per se, isolating the effect of routing on the level-of-service provided to customers. The two strategies we have adopted to capture the interaction between routing and level-of-service are (a) the amount of cargo available is modeled as a weekly available quantity, which will be foregone to other competitors if the operator is not able to provide a weekly service; and (b) a total transport time upperbound between some selected port pairs to ensure the service quality between these port pairs is not inferior to that provided by other competitors.

However, we strongly feel the need of a more comprehensive treatment of level-of-service in a strategic routing decision such as the LDP. For example, the level-of-service consists not only of the transport time, but also the total transit time, frequency of service, and reliability of service, etc. It affects the amount of cargo available to the operator, and the amount of available cargo determines where to deploy the ships. Unfortunately, pure mathematical programming approach similar to ours has treated the problem in the reverse direction: assume a fixed cargo available pattern, then find the optimum deployment. This seems not an adequate method.

Thus our next effort is to develop an integrated framework such that the level-of-service of each route is included as a feed back to the selection of the route itself. Such treatment of the routing decision necessarily expands the problem to a larger domain, for example, inclusion of other competitors in the model. Though it requires a new methodology, our current studies provide useful building blocks.

Appendix A

One Necessary Condition that the Greedy Approach Solves the Assignment Problem

We want to prove Proposition 4.1 in this Appendix.

Proposition 4.1

Let P_a^b be the arc profit of assigning node a to b in an $n \times n$ Maximum Assignment Problem. Suppose the arc profits satisfy these criteria:

(a) Let P_i^j be the maximum of all feasible arc profits, i.e.,
$P_i^j = max\ \{P_s^t :$ *for all feasible assignment s to t* $\}$;

(b) For all $s \neq i, t \neq j$, we have:

$$\frac{P_s^t}{P_i^t} \geq \frac{P_s^j}{P_i^j};$$

(c) Both conditions (a) and (b) apply to all square subsets of the Assignment graph.

Then, the greedy algorithm which assigns sequentially by using the remaining feasible arc with the maximum arc profit correctly solves the Maximum Assignment Problem.

Proof

We will establish the proof by induction on n. First consider n equals 2. We have a

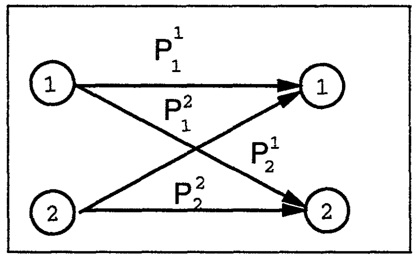


Figure A-1: A Two by Two Assignment

2×2 Assignment Problem as shown in Figure A-1.

$$\text{By (a), we can assume: } \begin{cases} P_1^1 &> P_1^2 \\ P_1^1 &> P_2^1 \\ P_1^1 &> P_2^2 \end{cases}$$

$$\text{By (b), we have: } \quad \frac{P_2^2}{P_1^2} \geq \frac{P_2^1}{P_1^1}$$

If P_2^1 is less than P_2^2, then, since P_1^1 is the largest, we have:

$$\text{total PTT by greedy } = P_1^1 + P_2^2 > P_1^2 + P_2^1$$

and the case is proved. So let us assume that P_2^1 is bigger. In this case, we can rewrite the above relationships as:

$$\left.\begin{array}{rcl} P_1^1 &=& aP_1^2 \\ P_1^1 &=& bP_2^1 \\ P_1^1 &=& cP_2^2 \end{array}\right\} \quad \text{where } a, b, c > 1$$

$$\text{from (b), } a = kc \quad \text{where } k > 1$$

The total PTT ratio of the two assignments can be calculated as follows:

$$\begin{array}{rrcl} \text{greedy assignment:} & g & = & P_1^1 + P_2^2 \\ \text{other assignment:} & \bar{g} & = & P_2^1 + P_1^2 \\ \text{The ratio of total PTTs, i.e., } \dfrac{g}{\bar{g}}: & z & = & \left(\dfrac{1+bc}{a+b}\right)\dfrac{a}{c} \\ & & = & \left(\dfrac{1+bc}{b+kc}\right)k \\ & \dfrac{\partial z}{\partial b} & = & \dfrac{k}{(b+kc)^2}\left(kc^2-1\right) > 0 \\ & \dfrac{\partial z}{\partial c} & = & \dfrac{k}{(b+kc)^2}\left(b^2-k\right) \qquad \begin{cases} < 0 & \text{if } b^2 < k \\ > 0 & \text{if } b^2 > k \end{cases} \end{array}$$

Thus to minimize z, b is set at a minimally constant, and c is increased to maximum, i.e., b tends to 1, c tends to infinity imply that z is minimized. Plugging these values into the above expression for z, we find that z is greater than or equal to 1, proving that the greedy assignment is optimal for n equals 2.

Next, let us use a similar technique to prove the case for n equals 3. We refer to the 3×3 Assignment Problem shown in Figure A-2.

$$\text{By (a), we can assume:} \quad \left\{ \begin{array}{lllllll} P_1^1 & > & P_2^1, & \text{or} & P_1^1 & = & aP_2^1 \\ P_1^1 & > & P_2^2, & \text{or} & P_1^1 & = & bP_2^2 \\ P_1^1 & > & P_2^3, & \text{or} & P_1^1 & = & cP_2^3 \\ P_1^1 & > & P_3^1, & \text{or} & P_1^1 & = & dP_3^1 \\ P_1^1 & > & P_3^2, & \text{or} & P_1^1 & = & eP_3^2 \\ P_1^1 & > & P_3^3, & \text{or} & P_1^1 & = & fP_3^3 \\ P_1^1 & > & P_1^2, & \text{or} & P_1^1 & = & gP_1^2 \\ P_1^1 & > & P_1^3, & \text{or} & P_1^1 & = & hP_1^3 \end{array} \right.$$

$$\text{By (b), we have:} \quad \left\{ \begin{array}{lllllll} \frac{P_2^2}{P_1^2} & \geq & \frac{P_2^1}{P_1^1}, & \text{or} & \frac{g}{b} & \geq & \frac{1}{a} \\ \frac{P_2^3}{P_1^3} & \geq & \frac{P_2^1}{P_1^1}, & \text{or} & \frac{h}{c} & \geq & \frac{1}{a} \\ \frac{P_3^2}{P_1^2} & \geq & \frac{P_3^1}{P_1^1}, & \text{or} & \frac{g}{e} & \geq & \frac{1}{d} \\ \frac{P_3^3}{P_1^3} & \geq & \frac{P_3^1}{P_1^1}, & \text{or} & \frac{h}{f} & \geq & \frac{1}{d} \end{array} \right.$$

$$\text{By (c), and WLOG, we have:} \quad \left\{ \begin{array}{lllllll} P_2^2 & \geq & P_2^3, & \text{or} & \frac{1}{b} & \geq & \frac{1}{c} \\ P_2^2 & \geq & P_3^2, & \text{or} & \frac{1}{b} & \geq & \frac{1}{e} \\ P_2^2 & \geq & P_3^3, & \text{or} & \frac{1}{b} & \geq & \frac{1}{f} \\ \frac{P_3^3}{P_2^3} & \geq & \frac{P_3^2}{P_2^2}, & \text{or} & \frac{c}{f} & \geq & \frac{b}{e} \end{array} \right.$$

where $a, b, c, d, e, f, g, h \geq 1$.

Let g_i be the total PTT ratio by the greedy assignment:

$$g_1 \quad = \quad P_1^1 + P_2^2 + P_3^3 \quad = \quad P_1^1 \left(1 + \frac{1}{b} + \frac{1}{f}\right)$$

Using the result for n equals 2, the total PTT ratio by other potential better assignments are:

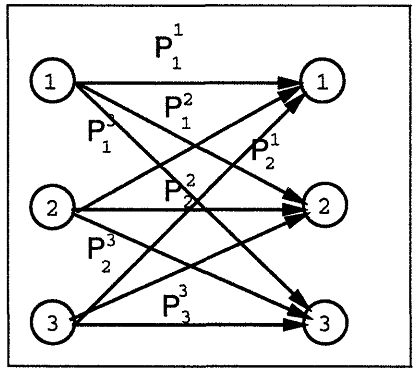


Figure A-2: A Three by Three Assignment

$$
\begin{aligned}
g_2 &= P_1^2 + P_2^1 + P_3^3 &&= P_1^1\left(\frac{1}{g} + \frac{1}{a} + \frac{1}{f}\right) \\
g_3 &= P_1^2 + P_2^3 + P_3^1 &&= P_1^1\left(\frac{1}{g} + \frac{1}{c} + \frac{1}{d}\right) \\
g_4 &= P_1^3 + P_2^2 + P_3^1 &&= P_1^1\left(\frac{1}{h} + \frac{1}{b} + \frac{1}{d}\right) \\
g_5 &= P_1^3 + P_2^1 + P_3^2 &&= P_1^1\left(\frac{1}{h} + \frac{1}{a} + \frac{1}{e}\right)
\end{aligned}
$$

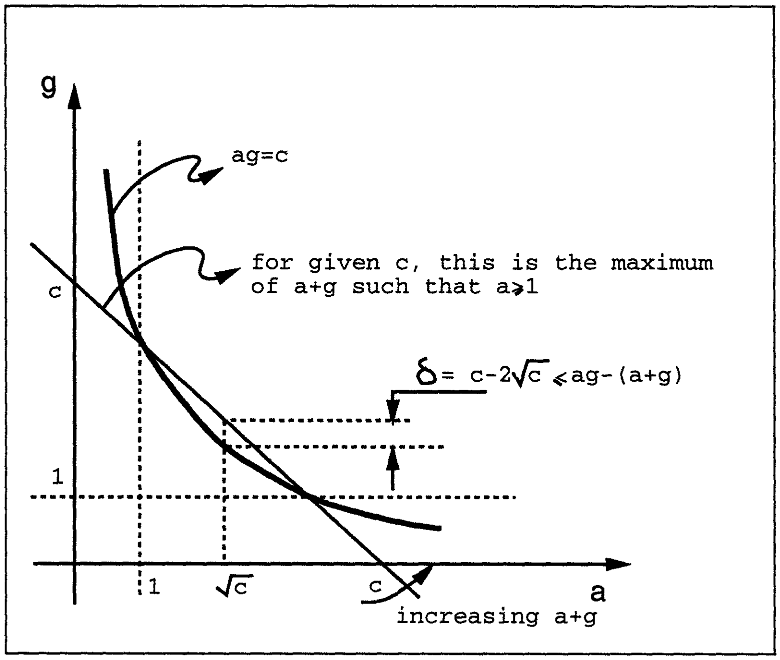
Now we argue that the greedy approach gives at least as great a PTT ratio as any of the above assignments.

<u>Proof of $g_1 \geq g_2$</u>

$$
\begin{aligned}
g_1 \geq g_2 \iff 1 + \frac{1}{b} + \frac{1}{f} &\geq \frac{1}{g} + \frac{1}{a} + \frac{1}{f} \\
\iff \frac{b+1}{b} &\geq \frac{a+g}{ag} \\
\text{since } ag &\geq b, \text{ let } b = \frac{ag}{k}, \text{ where } k \geq 1 \\
\iff ag + k &\geq a + g
\end{aligned}
$$

Hence the proof reduces to showing that the optimum value of the following mathematical programming is non-negative:

$$
\begin{aligned}
\min \quad & ag + k - (a+g) \\
\text{subject to} \quad & a \geq 1 \\
& g \geq 1 \\
& k \geq 1
\end{aligned}
$$



From the figure, given the constraint set,

$$\begin{aligned} ag-(a+g) &\geq \delta \\ &= c-2\sqrt{c} \\ ag-(a+g)+k &\geq c-2\sqrt{c}+k \\ &\geq c-2\sqrt{c}+1 \\ &= (\sqrt{c}-1)^2 \\ &\geq 0 \end{aligned}$$

Figure A-3: Graphical Proof of $g_1 \geq g_2$

The result follows by inspection from Figure A-3.

<u>Proof of $g_1 \geq g_4$</u>

$$\begin{aligned} g_1 \geq g_4 \iff 1+\frac{1}{b}+\frac{1}{f} &\geq \frac{1}{h}+\frac{1}{b}+\frac{1}{d} \\ \iff \frac{f+1}{f} &\geq \frac{d+h}{dh} \\ \text{since } hd &\geq f, \text{ let } f=\frac{hd}{k}, \text{ where } k \geq 1 \\ \iff hd+k &\geq h+d \end{aligned}$$

Hence the proof can be established using exactly the same argument as above.

<u>Proof of $g_1 \geq g_3$</u>

$$g_1 \geq g_3 \iff 1+\frac{1}{b}+\frac{1}{f} \geq \frac{1}{g}+\frac{1}{c}+\frac{1}{d}$$

Here the proof reduces to showing that the optimum value of the following mathematical programming is non-negative:

$$\min \underbrace{\left(1+\frac{1}{b}+\frac{1}{f}\right)}_{g_1}-\underbrace{\left(\frac{1}{g}+\frac{1}{c}+\frac{1}{d}\right)}_{g_3}$$

Subject to:

$$\begin{aligned}
ag &\ge b && \text{(A.1)}\\
ah &\ge c && \text{(A.2)}\\
dg &\ge e && \text{(A.3)}\\
dh &\ge f && \text{(A.4)}\\
c &\ge b && \text{(A.5)}\\
e &\ge b && \text{(A.6)}\\
f &\ge b && \text{(A.7)}\\
ce &\ge bf && \text{(A.8)}\\
a,b,c,d,e,f,g,h &\ge 1 && \text{(A.9)}
\end{aligned}$$

The above problem can be solved by a non-linear programming software package. But we proceed to solve it by inspection as follows.

We want to make g_1 as small as possible,while maintaining g_3 as big as possible within the constraint set. Suppose we let $f \to \infty$. (A.4) implies either d or h should go to infinity. To maximize g_3, we choose to let $h \to \infty$ instead. Moreover, by (A.8), either c or e should go to infinity. Again, since g_3 contains the term $\frac{1}{c}$, we choose to set e to infinity. But in this case, (A.3) forces us to set d or g to infinity. Selecting either one of this will make one term in g_3 vanishes. Suppose we have selected g to be at infinity. Then, we have:

$$g_1 \to 1+\frac{1}{b}$$

$$g_3 \to \frac{1}{c}+\frac{1}{d}$$

But (A.5) implies

$$\frac{1}{c} \le \frac{1}{b}$$

and since d is greater than or equal to one, it follows that g_3 is less than or equal to g_1. Similar conclusion can be obtained when we try to set different variables to infinity.

Proof of $g_1 \geq g_5$

The proof of this case again involves solving another non-linear optimization problem and can be done by the inspection method shown above, or by a non-linear software package.

At this point, we have proved the Proposition for the case of $n = 2, 3$. Now, we proceed to complete the main induction proof. Assume the greedy approach works for $n = 1, 2, \ldots, N$. For $n = N + 1$, without loss of generality, let us assume :

$$P_1^1 = \max \{P_i^j : 1 \leq i, j \leq N + 1\}$$

meaning that the greedy will assign ship 1 to tour 1.

Case 1: None of ships 2, 3, ..., $N, N + 1$ has tour 1 as her best tour

Then obviously, it is correct to assign ship 1 to tour 1. After this assignment, we are left with an $N \times N$ assignment subgraph which the greedy solves optimally by induction.

Case 2: Some of the ships 2, 3, ..., $N, N + 1$ has tour 1 as her best tour

Without loss of generality, assume that the greedy approach assigns ship 1 to tour 1, ship 2 to tour 2,..., and so on. Now consider any other feasible solution. If there exist assignments in this alternate solution that are the same as some greedy assignments, we can remove them from the assignment graph (Figure A-4). The remaining square subgraph, by criteria (c) and our induction assumption, is solvable to optimum by the greedy approach. Hence the feasible assignment can not outperform our greedy solution.

On the otherhand, if none of the assignment in the feasible solution is the same as our greedy assignment, we have a situation as shown in Figure A-5. In that case, the $N + 1 \times N + 1$ Assignment Problem can be considered as the union of two smaller assignment problems, each of which the greedy is able to solve to optimum by induction assumption and by criteria (c). Hence, in this case, the feasible solution still can not outperform the greedy solution.

This concludes the Proposition by mathematical induction for all values of n. $\square$

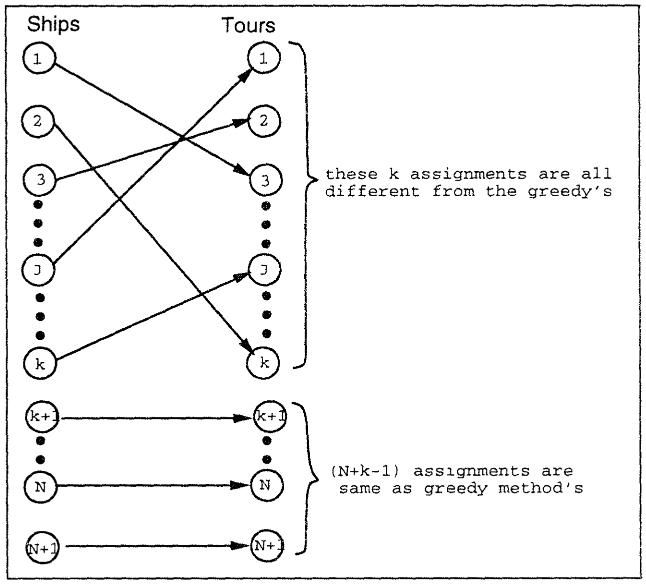


Figure A-4: Some Assignments Same as Greedy's

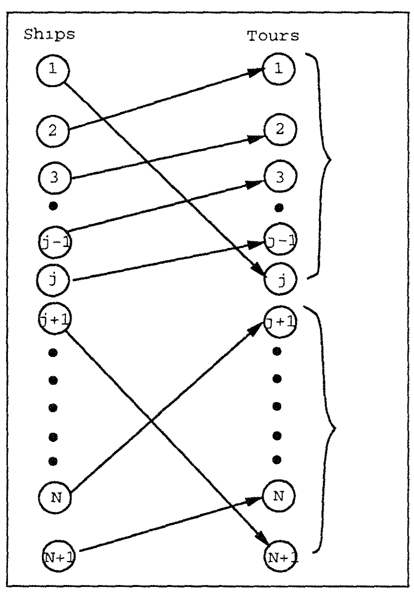


Figure A-5: No Assignment Same as Greedy's

References

[1] Almogy, Y., Levin, O., 1970,
Parametric Analysis of a Multi-Stage Stochastic Shipping Problem, *IFOFS, OR 69: Proceedings of Fifth International Conference on OR*, Tavistock, London, p 359-370.

[2] Boffey, T.B., Edmond, E.D., Hinxman, A.I., Pursglove C.J. 1979
Two Approaches to Scheduling Container Ships with an Application in the North Atlantic Route, *Journal of the Operational Research Society*, 30, p 413-425.

[3] Datz, I.M., 1971
Planning Tools for Ocean Transportation, Cornell Maritime Press, Cambridge, Maryland.

[4] Dantzig, G., Blattner, W.O., and Rao, M.R., 1966
Finding a Cycle in a Graph with Minimum Cost to Time Ratio with an Application to a Ship Routing Problem, *Theory of Graphs: International Symposium*, Rosemtiehl P ed., Rome, Italy.

[5] Ford, L.R. & Fulkerson, D.R., 1962
Flows in Networks, Princeton University Press, NJ, U.S.A.

[6] Gallo, G., Grigoriadis, M.D., and Tarjan, R.E., 1989
A Fast Parametric Maximum Flow Algorithm and Applications, *SIAM Journal for Computation*, Vol 18, No.1, p 30-55.

[7] Garey, M.R. & Johnson D.S., 1979
Computers and Intractability: a Guide to the Theory of NP-Completeness, Freeman, San Francisco, CA, U.S.A.

[8] Goldberg, A.V., 1984
Finding a Maximum Density Subgraph, Technical Report No. UCB CSD 84/171, Computer Science Division (EECS), University of California, Berkerly, CA.

[9] Jaramillo, D.I., and Perakis, A.N., 1991
Fleet Deployment Optimization for Liner Shipping Part 2: Implementationand Results, *Maritime Policy and Management*, Vol 18, p 235-262.

[10] Lane, D.E., Heaver, T.D., and Uyeno, D., 1987
Planning and Scheduling for Efficiency in Liner Shipping, *Maritime Policy and Management*, Vol 14, No. 2, p 109-125.

[11] Lloyd's Register of Shipping, 1992
Register of Ships 1992-93, Lloyds Register of Shipping, London, U.K.

[12] Olson, C.A., Sorenson, E.E., Sullivan, W.J., 1969
Medium Range Scheduling for Freighter Fleet, *Operations Research*, 17, p 565-582.

[13] Orlin, J.B., 1984
Minimum Convex Cost Dynamic Network Flows, in *Mathematics of Operations Research*, Vol 9 ,1984, p 190-207.

[14] Perakis, A.N. and Jaramillo, D.I., 1991
Fleet Deployment Optimization for Liner Shipping Part 1: Background, Problem Formulation, and Solution Approaches, *Maritime Policy and Management*, Vol 18, p 183-200.

[15] Picard, J.C. and Queyranne, M., 1982
A Network Flow Solution to Some Nonlinear 0-1 Programming Problems, with Applications to Graph Theory, *Networks*, Vol 12, p 141-159.

[16] Psaraftis, H.N., Solomon, M.M., Magnanti, T.L., and Kim, T.U., 1990
Routing and Scheduling on a Shoreline with Release Time, *Management Science* Vol 36 No.2, p 212-223.

[17] Rana, K., Vickson, R.G., 1991
Routing Container Ships Using Lagrangean Relaxation and Decomposition, *Transportation Science*, Vol 25 No.3, p 201-214.

[18] The National Magazine Co. Ltd., 1989
Containerisational Yearbook International 1989, the National Magazine Co. Ltd., London, U.K.

[19] Tsitsiklis, J.N., 1992
Special Cases of Traveling Salesman and Repairman Problems with Time Windows, *Networks*, Vol 22, p 263-282.