

**ANALYSIS OF CRACK PATTERNS IN REINFORCED CONCRETE FRAMES
UNDER SEISMIC EXCITATION**

by

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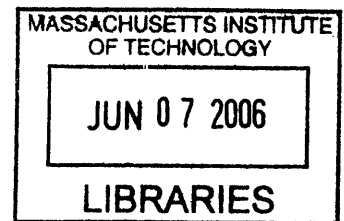
Submitted to the Department of Civil and Environmental Engineering
in Partial Fulfillment of the Requirements for the Degree of

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Abstract

Earthquakes occur as a result of sudden displacements across a fault within the earth. The seismic waves that result from them propagate along the earth's surface and are the cause of multiple damages especially in civil engineering. The main objective of this study is to compare different types of concrete (RC, HSRC and SFRC) in order to determine the one that will contribute the most to increase the steadiness of buildings in seismic regions at the connection between beams and columns.

The first part of this paper consists of introducing these different types of concrete. Then cracks patterns analyses are run at the joint between beam and column. In that part, theoretical approach precedes experimental application. Finally, explanation about benefits and advantages of using SFRC in joints of buildings subjected to seismic excitations will be provided. In fact, concrete reinforced with short steel fibers is associated with its ability to control cracking and enhances properties of concrete.

Thesis Supervisor: Jerome J. Connor

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Introduction

Earthquakes occur as a result of sudden displacements across a fault within the earth. The seismic waves that result from them propagate along the earth's surface and are the cause of multiple damages especially in civil engineering.

Effort in earthquake engineering research became increasingly important in the past years. In fact, studies conducted by the U.S. Geological Survey demonstrate that except for Texas, Florida, the Gulf Coast, and the Midwest, most of the United States is at some risk from earthquakes (USGS, 2002).

To improve constructions quality in terms of seismic resistance, several parameters have to be taken into account. First, soil condition has to be considered in order to minimize ground motion such as landslides, then, risk assessment methods and research need to be more developed and finally, design itself of the various constructions need to be adapted and modified accordingly to the level of seismicity. In that way, many studies have been done in order to develop a better understanding of the behaviour of building systems to ensure that new buildings are designed and old buildings are retrofitted to reduce their vulnerability to excessive damage and large economic losses during earthquakes. Priority issues in building-related earthquake engineering research include prediction of the seismic capacity and performance of existing and new buildings, evaluation of non-structural systems, performance of soil-foundation-structure interaction systems, and determination of the performance of innovative materials and structures. This last aspect will be the centre of attention of the following study comparing different provided solutions.

Innovative materials and structures include clever new uses and configurations of conventional materials and novel developments of smart materials and structures. The challenge to acceptance of innovations is to systematically evaluate the performance of innovative materials and structural systems. Application of innovative materials, including smart materials, to structural systems will provide new cost-effective retrofit, repair, and rehabilitation alternatives.

In the experiments that will be described, earthquakes have been simulated on structures by different types of mechanisms. Figure 1 and Figure 2 illustrate two sorts of system used.

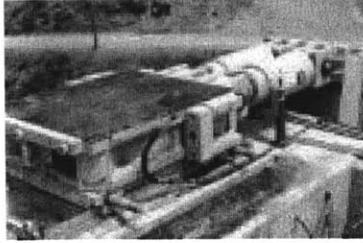


Figure 1: Hydraulic jacks

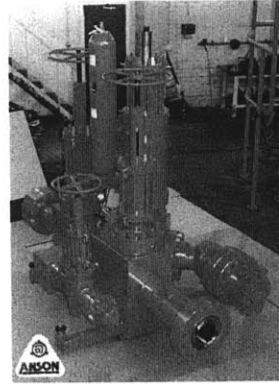


Figure 2: double acting hydraulic actuator

The main goal of this study is to evaluate the application of these innovative concrete materials on beam-column connections subjected to seismic excitation. The comparison will be based on cracks analysis observed on concretes at these specific spots of beams and columns.

In fact, in normal concrete widely utilized as a building structure, deterioration due to continuous load and aging is concentrated on specific parts. It results in large cracks, which accelerate the deterioration of the concrete, although most other regions in the material are preserved well. However, the large crack causes the reinforced concrete to lose its load capacity under seismic deformation. Besides, it reduces long-term durability in a direct or an indirect way.

For example, seismic deformation drives formation of large cracks in concrete in a seismic resistant structure, resulting in the spalling of steel reinforcements inside the concrete. Thus it shortens the life-time of the structure, because it enhances the corrosion of steel reinforcement and failure of concrete. Hence, the crack width is regarded as one of the important factors to govern the durability of structure.

CHAPTER I: Relevant Types of Reinforced Concrete for Seismic Design

The aim of this study is to compare innovative materials in order to analyse how variations in Reinforced Concrete can improve frame joints performance under seismic excitations. The three types of reinforced concrete being analyzed, Normal Strength, High Strength and Steel Fiber Reinforced Concretes, will be described in the following section.

1.1 - Normal Strength Reinforced Concrete (NSRC)

Concrete consists of coarse and fine aggregate, cement, water and, in many cases, different type of admixtures. The materials are mixed together until a cement paste is added, filling most of the void in the aggregates and producing a uniform dense concrete. The plastic concrete is then placed in a mould and left to set, harden and develop adequate strength. NSRC is a plain concrete in which steel bars or webbing has been embedded for strength. Tensile strength of concrete is typically ten percent of its compressive strength. This weakness has always been dealt by using reinforcing bars (rebars) to create reinforced concrete. In the latter, concrete resists compressive stresses and rebars resist tensile and shear stresses.

Generally, the term concrete strength refers to the uniaxial compressive strength as measured by a compression test of a standard test cylinder. This test is used to monitor the concrete strength for quality control or acceptance purposes.

COMPRESSIVE STRENGTH

Since concrete resists compressive stresses and not tensile stresses, compressive strength is the criterion of quality of concrete. Before being tested, the specimens are moist-cured and then tested at 28 days by gradually applying a static load until rupture occurs. The rupture of the concrete specimen may be caused by the applied tensile stress (failure in cohesion), the applied shearing stress (sliding failure), the compressive stress (crushing failure), or combinations of these stresses. Typically, NSRC strength ranges between 3000 and 7000 psi. Many factors can affect the strength and thus the quality of concrete, the following will define the most important parameters that qualify it.

FACTORS AFFECTING CONCRETE STRENGTH

The Water-Cement ratio is one of the most important factors that influence the strength of concrete. To achieve a concrete strength of about 6000 psi, this water-cement ratio has to be about 0.35. This value allows the concrete to be reasonably workable without additives.

Properties and Proportions of concrete constituents also affect significantly the strength of concrete. In fact, an increase in the cement content in the mix and the use of well-graded aggregate will modify this property. Special admixtures are often added to the mix to produce the desired quality of concrete.

Method of mixing and curing can have favourable effects on strength of concrete such as using mechanical concrete mixers and the proper time of mixing. The use of vibrators also decreases the percentage of voids and thus produces a dense concrete. Regarding curing conditions, both moisture and temperature have a direct effect on the hydration of cement.

The age of concrete plays an important role in a way that the strength of concrete increases with age. In practice, the strength of concrete is determined from cylinder or cubes tested at the age of 7 days and 28 days.

Shape and Dimensions of the tested specimen are relevant criteria regarding strength determination. The common sizes of concrete specimens used to predict compressive strength are either 6 by 12 inches cylinders or 6 inches cubes. Sometimes concrete cylinders of non-standard shape are tested. In that case, as the height to diameter ratio increases, the strength indicated by the compression test decreases.

STRESS-STRAIN CURVES OF CONCRETE

The performance of a reinforced concrete member subjected to loads depends on the stress-strain relationship of concrete and steel and on the type of stress applied to this member. Typical stress-strain curves for various concrete strengths are shown in Figure 3. Most structural concretes have f_c values in the 3000 to 5000 psi range. Even though the following figure indicates that the maximum strain that concrete can sustain before it crushes varies inversely with strength, a value of 0.003 is usually taken (as a simplifying measure) for use in the development of design equations.

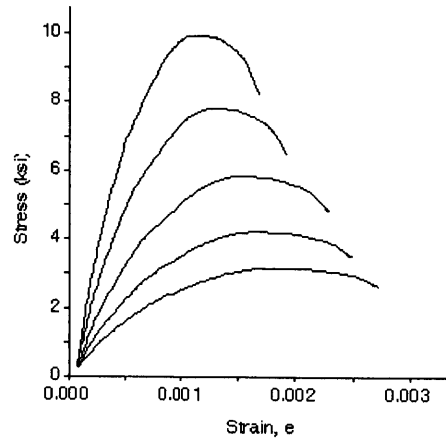


Figure 3: Typical Stress-Strain curves of concrete

TENSILE STRENGTH OF CONCRETE

Concrete is a brittle material, and it cannot resist the high tensile stresses that are important when considering cracking, shear, and torsional problems. Direct tension tests are not reliable for predicting the tensile strength of concrete, due to minor misalignment and stress concentrations in the devices. An indirect tension test called splitting test has been suggested and leads to the two following expressions;

Compressive Stress: $f_c = 2P / (\pi LD) * \{D^2 / (y*(D-y)) - 1\}$

Tensile Stress: $f'_{sp} = 2P / (\pi LD)$

Where P is the compressive load on the cylinder, D and L are the diameter and length of it and y is the distance of the element on the vertical diameter from the top fibers.

FLEXURAL STRENGTH (MODULUS OF RUPTURE) OF CONCRETE

Experiments on concrete beams have shown that ultimate tensile strength in bending is greater than the tensile stress obtained by direct or splitting tests. Flexural strength is expressed in terms of modulus of rupture of concrete (f_r), which is the maximum tensile stress in concrete in bending.

The ACI Code prescribes the value of the modulus of rupture as $f_r = 7.5 \sqrt{f'_c}$ psi.

SHEAR STRENGTH

Shear strength is usually considered as 20 to 30 % greater than the tensile strength of concrete, or about 12% of its compressive strength. However, pure shear is rarely encountered in reinforced concrete since it is typically accompanied by the action of normal forces.

The ACI Code allows an ultimate shear strength of $2 \sqrt{f'_c}$ psi on plain concrete sections.

MODULUS OF ELASTICITY

One of the most important elastic properties of concrete is its modulus of elasticity, which can be obtained from compressive test on cylinders. It can be defined as the change of stress with respect to strain in the elastic range.

$$E_c = \text{unit stress} / \text{unit strain}$$

The ACI Code gives the following formula to calculate the modulus of elasticity of normal concrete considering the secant modulus at a level of stress, f_c , equal to half the ultimate concrete strength, f'_c .

$$E_c = 33 w^{1.5} \sqrt{f'_c} \text{ psi}$$

Where w is the unit weight of concrete. For normal weight concrete, $w = 145$ pcf.

POISSON'S RATIO

It is the ratio of the transverse to the longitudinal strains under axial stresses within the elastic range. This ratio (μ) varies between 0.15 and 0.20 for normal-weight concrete.

SHEAR MODULUS

The modulus of elasticity of concrete in shear ranges from about 0.4 to 0.6 of the corresponding modulus in compression. From the theory of elasticity, the shear modulus is;

$$G_c = E_c / \{2(1+\mu)\}$$

MODULAR RATIO

It is the ratio of the modulus of elasticity of steel to the modulus of elasticity of concrete.

$$n = E_s / E_c$$

Because the modulus of elasticity of steel is considered constant and is equal to 29 000 ksi, for normal-weight concrete, $n = 500 / \sqrt{f'_c}$

1.2 - High Strength Reinforced Concrete (HSRC)

The development of increased concrete compressive strength has been a major focus these past years regarding the improvement of concrete subjected to seismic excitations. In fact, the increase in strengths has played a major role in decreasing member sizes, and hence dead weight, while providing economic advantages for designers, contractors and owners.

High strength concretes can be roughly defined as those with strengths of 8000 psi and above. They tend to exhibit smoother failure surfaces and more brittle behaviour than "normal"

strength concretes. However, as higher strength materials are used in seismic resisting members, more concern will have to be focused on the deformation of the structure and designer's ability to accurately predict performance. In fact, with higher strength concrete, members will be smaller and less stiff leading to increased drift and cracking will become more significant as it affects structural stiffness.

The main concern about the use of HSRC is the reduction in ductility with the increase in compressive strength observed under uniaxial compression. However, it can be shown experimentally and theoretically that high-strength concrete flexural members show a similar or higher ductility when compared to NSRC. Moreover, ductility demands of heavily loaded high-strength concrete compression members can be satisfied by providing extra confining steel.

1.3 - Steel Fiber Reinforced Concrete (SFRC)

The use of reinforced concrete for good composite materials improves its applications. Steel bars, however, reinforce concrete against tension only locally. Cracks in reinforced concrete members extend freely until encountering a rebar. The need for multidirectional and closely spaced reinforcement for concrete arose and that is how the idea of Steel Fiber Reinforced Concrete (SFRC) came up.

Fiber Reinforced concrete is a type of concrete with added fiber materials to reduce shrinkage and increase its toughness. It is the result of a mix of concrete and uniformly distributed fibers, randomly oriented (see Figure 4). Many different kinds of fibers can be used; steel, cellulose, carbon, polypropylene, glass, nylon, polyester... The amount of fibers added to the concrete is defined by the percentage of the total volume of composite that it represents (usually from 0.1 to 3%). The other parameter considered is the Aspect Ratio which is the ratio of fiber length to its diameter.



Figure 4: Steel Fiber Reinforced Concrete

Regarding Steel Fibers, different shapes can be used. As shown on Figure 5 and Figure 6, their length can vary (from ¼ to 3 inches) and their aspect ratio can reach 100.

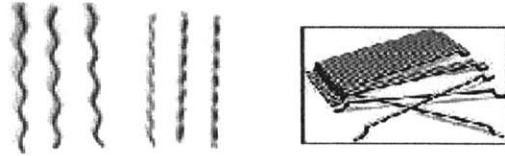


Figure 5: Steel fibers

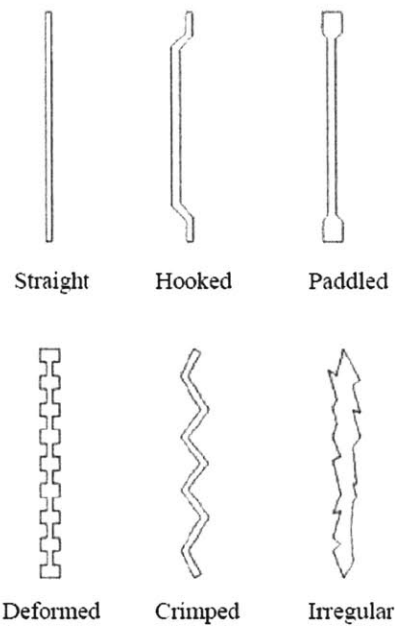


Figure 6: Steel fiber shapes

CHAPTER II: The Beam-Column Connections

Two different categories of beam-column joints exist; type I which refers to ordinary structures subjected to static loading such as gravity and wind loads and type II which is for structures that experience seismic actions, impact or blast loads. In this study, the focus will be on the second category.

In concrete frame structures subjected to earthquake loading, failure often occurs at connections around joints and at base of columns. Numerous examples can be found where beam-column connections with inadequate transverse reinforcement failed; Figure 7 illustrates one of them. Problems have been observed in joints where the members frame eccentric to the joint, and where members having non-coincident longitudinal axes frame into a single joint.

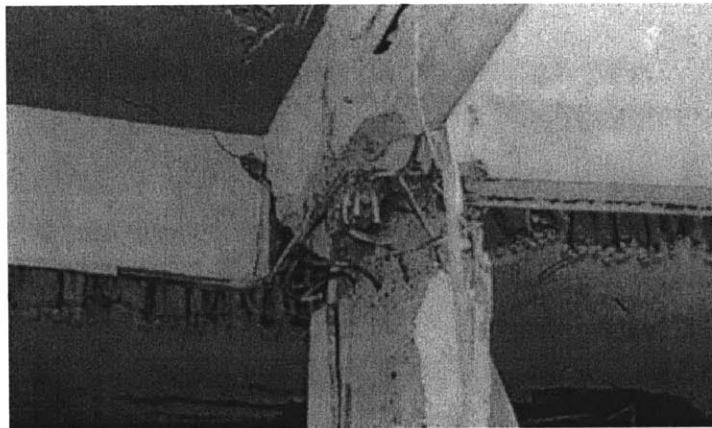


Figure 7: Beam-column joint failure Ansal et Al (1999)

In these types of connections, continuity between members and joints are essential. Following the 1985 Mexico City earthquake, several collapsed buildings were found where columns had not been adequately interconnected through the joints by continuous longitudinal reinforcement. Failures have also been observed where reinforcement splices within members were of insufficient length or were inadequately located.

The seismic load path in a ductile frame flows through the beam-column joint. The traditional design objective for these connections has been to treat them as though they were brittle. Furthermore, the objective of the capacity-based approach is to create a beam-column joint that is stronger than the frame beams that drive it.

Current design recommendations for RC beam-column joints in earthquake-resistant construction given by Joint ACI-ASCE Committee 352 (2002) focus on three main aspects:

- Confinement requirements
- Evaluation of shear strength
- Anchorage of beam-column bars passing through the connection

II.1 - Behaviour mechanisms

Figure 8 illustrates the various connection geometries associated with interior, exterior and corner connections. Connections classified as “interior” have beams framing into all sides of the connection. Exterior connections have at least two beams framing into opposite sides of the column. Beams must be at least three-quarters of the width of the column to meet the requirement for number of beams framing into the column. The shallowest beam also needs to be not less than three-quarter the depth of the deepest beam framing into the connection. All other types of connections are classified as corner connections. “Finally, it is believed that more pairs of horizontal members framing into opposite sides of a joint result in enhanced joint-shear strength” (source [8]).

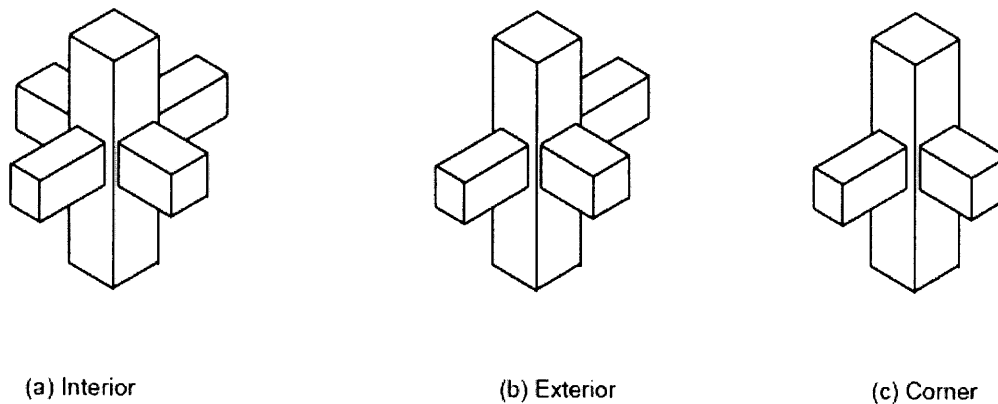


Figure 8: Types of joints in a frame (source [8] fig 1)

The pattern of forces acting on a joint depends on the configuration of the joint and the type of loads acting on it. The following figures show the effect of loads on the three types of joints in terms of stresses and crack patterns.

The forces on an interior connection subjected to gravity loading can be illustrated as on Figure 9. The tension and compression from the ends of the beam can be transmitted directly through the joint such as the axial loading from the column.

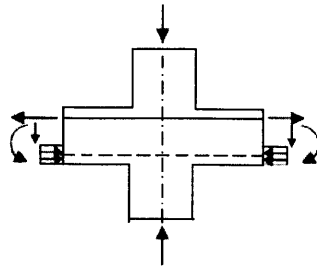


Figure 9: Gravity loading on interior joint (source [8] fig 2a)

The behaviour is different if the connection is subjected to lateral loading such as seismic excitation. In fact Figure 10 describes how the equilibrating forces from the beam and the column develop diagonal tensile and compressive stresses within the joint.

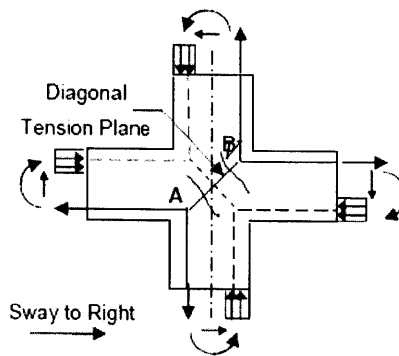


Figure 10: Seismic loading on interior joint (source [8] fig 2b)

Cracks develop perpendicular to the tension diagonal A-B in the joint and at the faces of the joint where the beams frame into it. The dashed lines stand for the compression struts while the solid lines show the tension ties. Transverse reinforcements cross the plane of failure in order to resist the diagonal tensile forces.

Regarding forces acting on an exterior joint, the shear force in the joint creates diagonal cracks (see Figure 11) and thus reinforcement is required.

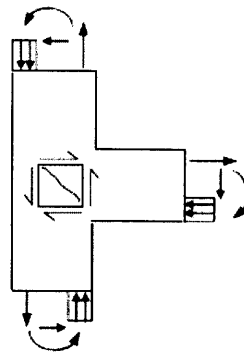


Figure 11: forces on exterior joint (source [8] fig 3a)

Some detailing patterns of longitudinal reinforcements are shown on the following figures. They significantly affect the efficiency of the joints. In fact Figure 12 depicts how the bars bend away from the core of the joint; this configuration provides 25 to 40 % efficiency. On the other hand, when the bars are passing through and are anchored in the joint core, as shown in Figure 13, it can provide up to 100 % efficiency.

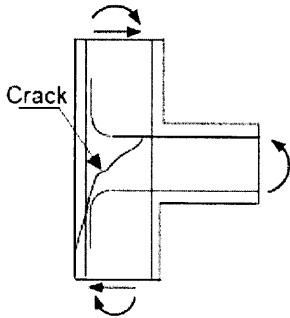


Figure 12: Poor detail in exterior joint (source [8] fig 3b)

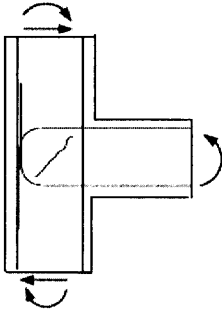


Figure 13: Satisfactory details in exterior joint (source [8] fig 3c)

The forces in a corner joint with a continuous column above work as the ones in an exterior joint with respect to the considered direction of loading. Wall type corners form another sort of joint (also called knee joints or L-joints) wherein the applied moments tend to either open or close the corners. Figure 14, Figure 15, Figure 16 and Figure 17 illustrate the stresses and cracks developed in these kinds of connections.

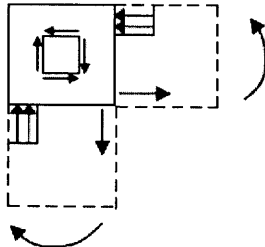


Figure 14: Opening joint (top view) (source [8] fig 4a)

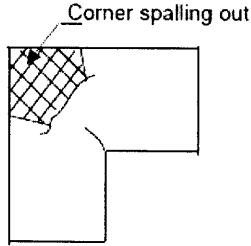


Figure 15: cracks in an opening joint (source [8] fig 4b)

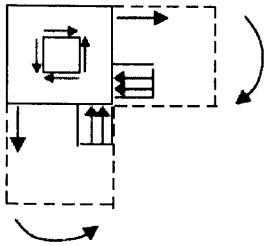


Figure 16: Closing joint (top view) (source [8] fig 4c)

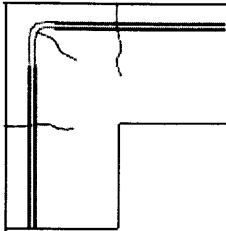


Figure 17: cracks in a closing joint (source [8] fig 4d)

Opening corners tend to develop nascent cracks at the inner corner and failure is marked by the formation of a diagonal tensile crack whereas in a closing joint, the forces developed are exactly opposite; the major crack is oriented along the diagonal of the corner. These joints show better efficiency than the opening joints. During seismic excitations, the reversal of forces is expected and hence the corner joints have to be conservatively designed as opening joints with appropriate detailing.

Failure of opening corner or knee joint is primarily due to the formation of diagonal tension crack across the joint with the outer part of the corner concrete separating from the rest of the specimen. The stress resultants from the framing members are transferred into the joint through bond forces along the longitudinal reinforcement bars passing through the joint and through flexural compression forces acting on the joint faces. The joints should have enough strength to resist the induced stresses and sufficient stiffness to control undue deformations. Large deformations of joints result in significant increase in the storey displacement.

The transverse reinforcement required by the committee 352 provisions for confinement of beam-column joints needs to meet some constraints; single hoops, overlapping hoops or hoops with crossties have to be designed so that the area of steel in each direction is contained between those following values;

$$0.009 \cdot (S_h \cdot h'' \cdot f_c) / f_{yh} < A_{sh} < 0.3 \cdot (S_h \cdot h'' \cdot f_c) / f_{yh} \cdot (A_g / A_c - 1)$$

Where

S_h is the vertical spacing of hoops

h'' is the core dimension measured center-to-center of confining reinforcement

f_{yh} is the yield strength of the hoops

A_g is the cross area of the column section

A_c is the area of the column core measured out-to-out of the transverse reinforcement

II.2 - Application model

In order to get a complete understanding of the behaviour at the joint connection, an analysis on SAP 2000 has been run. The model is a one story building of 5 ft high with one bay of 5 ft width as shown on Figure 18. The joint is a corner joint between a 6*6 in² beam and a 1*1 ft² column (see Figure 19 and Figure 20). The system is subjected to a ground motion in two directions that stands for an earthquake. The following figures illustrate the analysis in order to visualize the deflected shape, shear and moment diagrams and finally to notice the influence of strength variation of concrete on the structure. In fact one first test has been run with a normal strength concrete ($f_c = 4$ Ksi) and the second with high strength concrete ($f_c = 8$ Ksi).

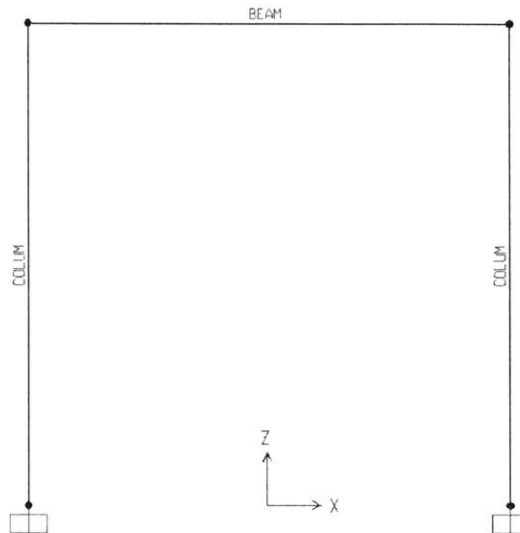


Figure 18: tested structure

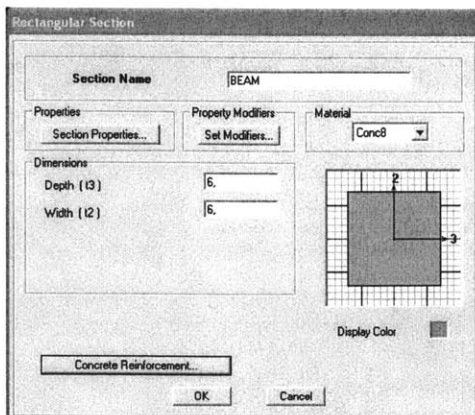


Figure 19: Beam section

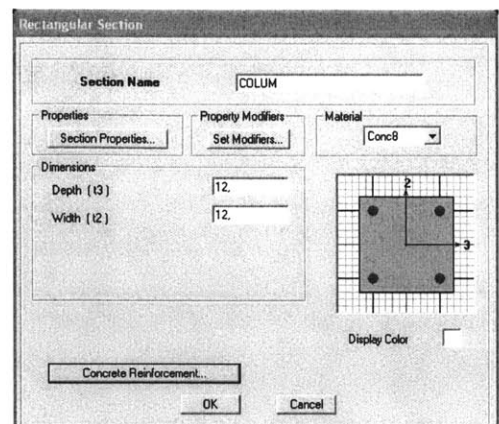


Figure 20: Column Section

For both analyses, one same earthquake has been applied at the bottom of the frame. Its chart is represented on Figure 21. It has a peak ground acceleration equal to 0.2 g and a maximum spectral acceleration equal to 4.459 between 0.2 and 1.0 second. As an example, these specifications are more severe than the accelerations recorded on 17 August 1999 during the Izmit 7.4 Richter scale earthquake.

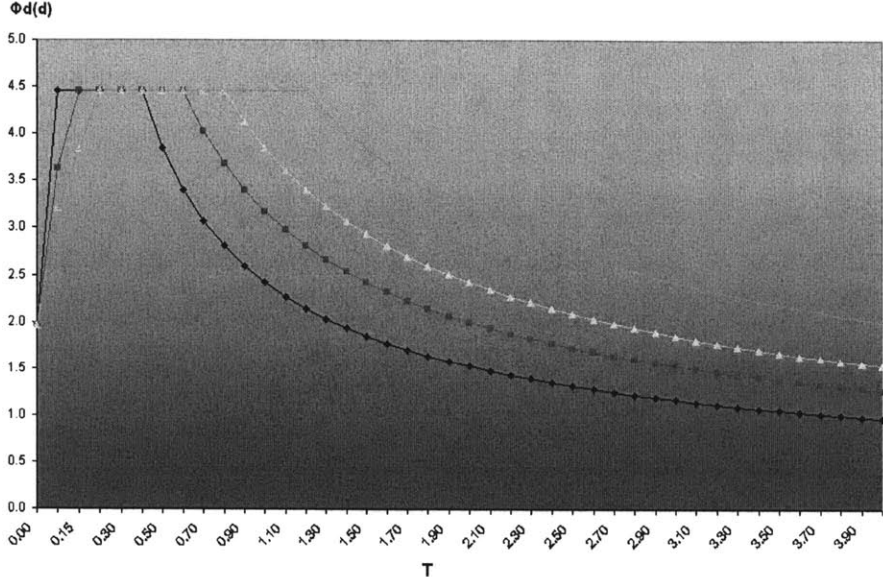


Figure 21: Design Response Spectrum

The first analysis has been run using concrete and steel reinforcement with the properties shown in Figure 22, Figure 23 and Figure 24.

Figure 22: Concrete Property Data (4 Ksi)

Reinforcement Data

Design Type
 Column Beam

Concrete Cover to Rebar Center
 Top: 0,015
 Bottom: 7,000E-03

Reinforcement Overrides for Ductile Beams

	Left	Right
Top	0,	0,
Bottom	0,	0,

OK Cancel

Figure 23: Beam reinforcement

Reinforcement Data

Design Type
 Column Beam

Configuration of Reinforcement
 Rectangular Circular

Lateral Reinforcement
 Ties Spiral

Rectangular Reinforcement
 Cover to Rebar Center: 1,8
 Number of Bars in 3-dir: 2
 Number of Bars in 2-dir: 2
 Bar Size: #6

Check/Design
 Reinforcement to be Checked
 Reinforcement to be Designed

OK Cancel

Figure 24: Column reinforcement

The resulting moment and shear diagrams are shown Figure 25 and Figure 26.

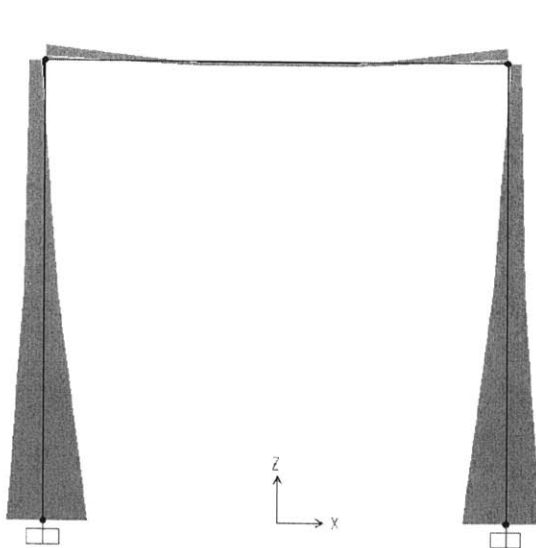


Figure 25: Moment Diagram

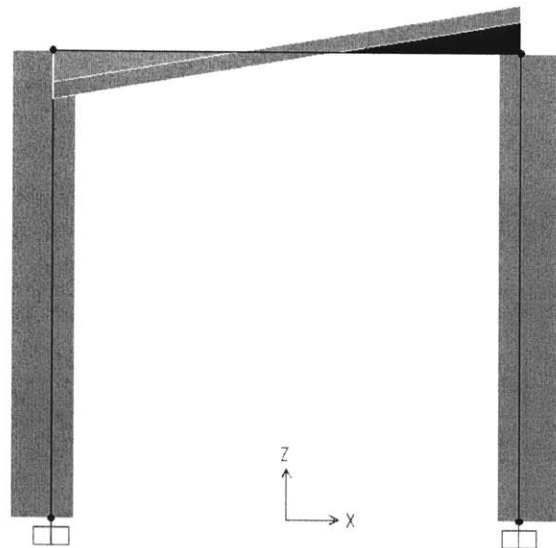


Figure 26: Shear Diagram

The two following figures (Figure 27 and Figure 28) sum up the concrete design data for the different members.

ACI 318-05/IBC 2003 COLUMN SECTION DESIGN Type: Sway Special Units: Kip, ft, F (Summary)

L=5,000
 Element : 1 B=1,000 D=1,000 dc=0,150
 Station Loc : 5,000 E=5472000,0 fc=1152,000 Lt.Wt. Fac.=1,000
 Section ID : COLUM fy=8640,000 fys=5760,000
 Combo ID : EQxCombo RLLF=1,000

Phi(Compression-Spiral): 0,700 Overstrength Factor: 1,25
 Phi(Compression-Tied): 0,650
 Phi(Tension Controlled): 0,900
 Phi(Shear): 0,750
 Phi(Seismic Shear): 0,600
 Phi(Joint Shear): 0,850

AXIAL FORCE & BIAxIAL MOMENT DESIGN FOR PU, M2, M3

Rebar Area	Design Pu	Design M2	Design M3	Minimum M2	Minimum M3
0,010	0,338	0,000	0,569	0,027	0,027

AXIAL FORCE & BIAxIAL MOMENT FACTORS

	Cm Factor	Delta ns Factor	Delta_s Factor	K Factor	L Length
Major Bending(M3)	0,773	1,000	1,000	1,000	5,000
Minor Bending(M2)	1,000	1,000	1,000	1,000	5,000

SHEAR DESIGN FOR V2,V3

	Design Rebar	Shear Vu	Shear phi*Vc	Shear phi*Vs	Shear Vp
Major Shear (V2)	0,002	0,436	0,000	4,927	0,210
Minor Shear (V3)	0,005	18,452	0,000	18,452	18,452

JOINT SHEAR DESIGN

	Joint Shear Ratio	Shear VuTop	Shear VuTot	Shear phi*Vc	Joint Area
Major Shear (V2)	0,015	0,210	1,908	131,373	1,000
Minor Shear (V3)	N/A	N/A	N/A	N/A	N/A

(6/5) BEAM/COLUMN CAPACITY RATIOS

	Major Ratio	Minor Ratio
	0,013	N/A

Notes:

N/A: Not Applicable
 N/C: Not Calculated
 N/N: Not Needed

Figure 27: Column concrete design data

ACI 318-05/IBC 2003 BEAM SECTION DESIGN Type: Sway Special Units: Kip, ft, F (Summary)

L=5,000
 Element : 3 D=0,500 B=0,500 bf=0,500
 Station Loc : 5,000 ds=0,000 dct=0,001 dcb=5,833E-04
 Section ID : BEAM E=5472000,0 fc=1152,000 Lt.Wt. Fac.=1,000
 Combo ID : EQxCombo fy=8640,000 fys=5760,000

Phi(Bending): 0,900
 Phi(Shear): 0,750
 Phi(Seis Shear): 0,600
 Phi(Torsion): 0,750

Design Moments, M3

	Positive Moment	Negative Moment	Special +Moment	Special -Moment
	0,285	-0,569	0,285	-0,569

Flexural Reinforcement for Moment, M3

	Required Rebar	+Moment Rebar	-Moment Rebar	Minimum Rebar
Top (+2 Axis)	1,962E-04	0,000	1,471E-04	1,962E-04
Bottom (-2 Axis)	9,783E-05	7,337E-05	0,000	9,783E-05

Shear Reinforcement for Shear, V2

	Design Rebar	Shear Vu	Shear phi*Vc	Shear phi*Vs	Shear Vp
	8,385E-04	0,745	0,000	1,807	0,316

Reinforcement for Torsion, T

	Rebar At	Rebar A1	Torsion Tu	Critical Phi*Tcr	Area Ao	Perimeter Ph
	0,000	0,000	0,000	0,303	0,037	0,833

Figure 28: Beam concrete design data

The second analysis has been run using concrete with the properties described in Figure 29.

Material Property Data

Material Name:

Display Color:

Type of Material: Isotropic Orthotropic Anisotropic Uniaxial

Type of Design: Design:

Analysis Property Data:

Mass per unit Volume:

Weight per unit Volume:

Modulus of Elasticity:

Poisson's Ratio:

Coeff of Thermal Expansion:

Shear Modulus:

Design Property Data (ACI 318-05/IBC 2003):

Specified Conc Comp Strength, f'c:

Bending Reinf. Yield Stress, fy:

Shear Reinf. Yield Stress, fys:

Lightweight Concrete

Shear Strength Reduc. Factor:

Advanced Material Property Data:

Figure 29: Concrete Property Data (8 Ksi)

By comparing the displacement of the same corner on the deflected shape models it is shown that increasing the strength of concrete indeed improves the resistance of the joints against earthquake (See Figure 30 and Figure 31).

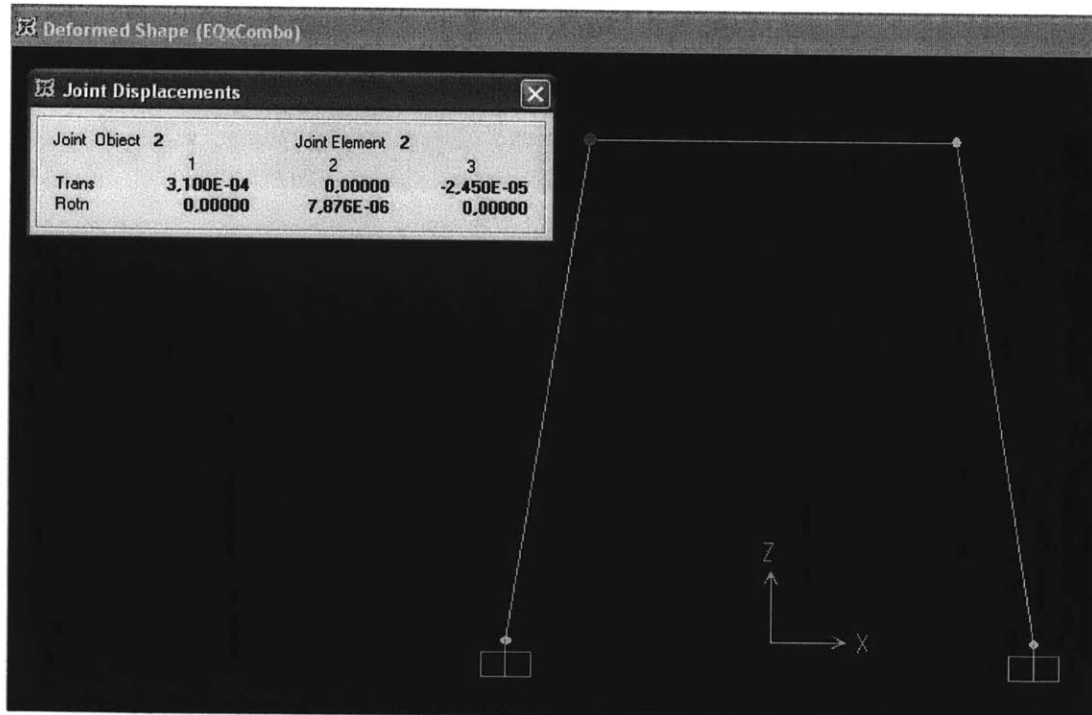


Figure 30: First test Deformed shape

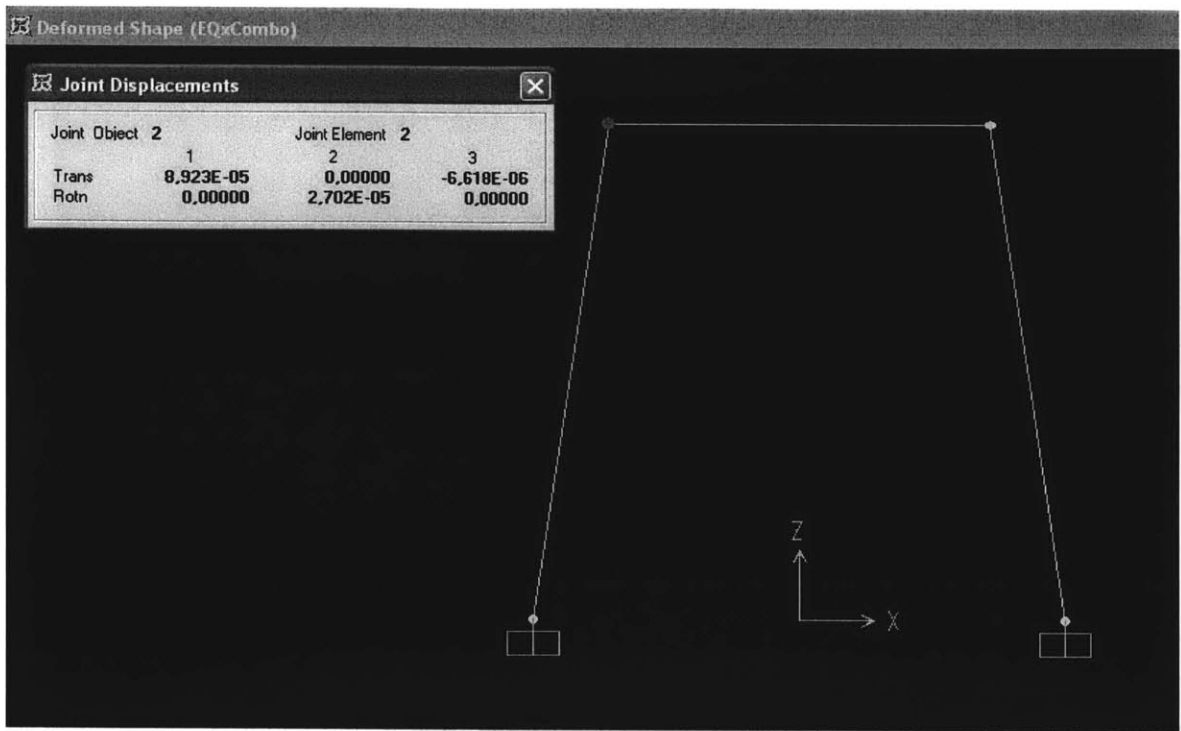


Figure 31: Second test Deformed shape

CHAPTER III: Cracks Analysis on beam-column joints - Experimental application using SFRC

III.1 - Presentation

In 2001, Michael Gebman achieved the following experiment within the framework of his Master Thesis: Application of Steel Fiber Reinforced Concrete in Seismic Beam-Column Joints. The aim of his research was to show that a 2% volume fraction of hooked and steel fibers in a concrete mix could generate an increase of hoop spacing in a seismic beam-column connection.

III.2 - Materials

Plain Concrete: The mix was designed for compressive strengths f'_c and f'_{cr} of 3000 psi and 4060 psi respectively. The unit weight is 142.8 pcf, the slump is 4 in and the water/cement ratio is 0.59.

Steel Fibers: Dramix steel fibers have been used with a 2% volume fraction. They were hooded-end with a length of 1.2 in and a diameter of 0.002 in. The aspect ratio is about 60.

Steel Reinforcing Bars: For both the longitudinal and the lateral reinforcement, Grade 60 ($f_y = 60$ Ksi) deformed steel reinforcing bar has been used.

III.3 - Joint design

The test has been accomplished using three exterior beam-column joints situated in between four strong walls as shown on Figure 32.

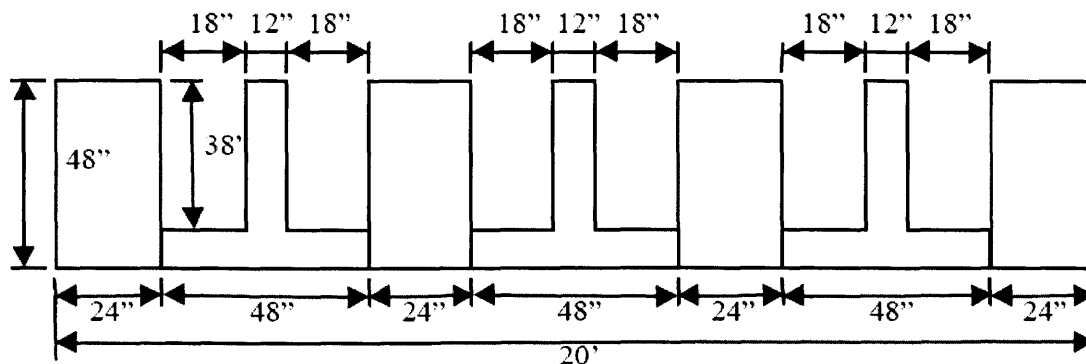


Figure 32: view of specimens (source [11] p23)

The width of the strong walls, beams and columns are 10 in. The other dimensions used are the one shown on the drawing above.

“A quasi-static hysteretic earthquake loading was applied to each beam-column joint test specimen by placing the hydraulic jack in the space between each strong wall and beam (see Figure 33). Loading consisted of six cycles with load point maximum displacements of 1/4 in, 1/2 in, 1 in, 2 in, 4 in and 8 in, corresponding respectively to cycles 1 to 6 as depicted on Figure 34 and Figure 35” (source [11]).

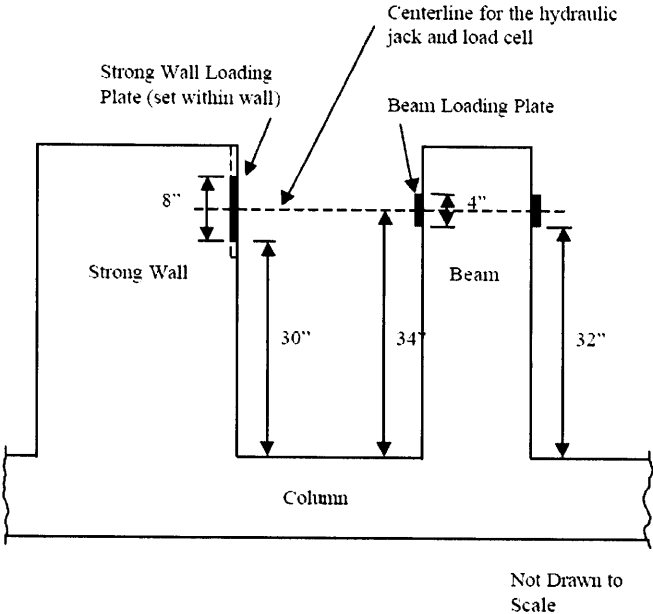


Figure 33: Alignment of loading plates, hydraulic jack and load cell (source [11] p33)

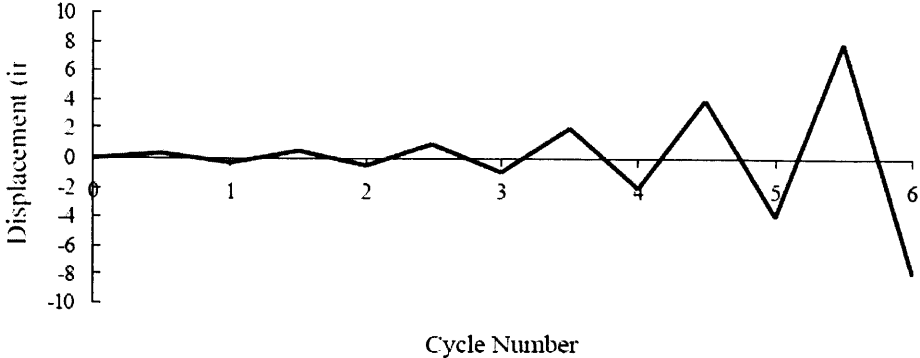


Figure 34: Simulated quasi-static earthquake loading (source [11] p 24)

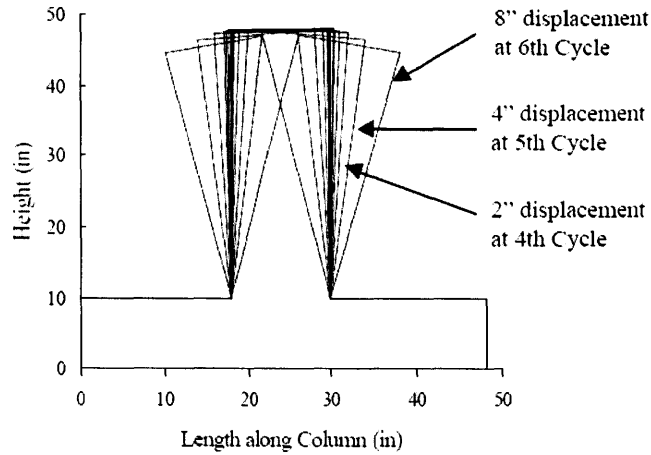


Figure 35: Beam displacement during simulated earthquake (source [11] p25)

“The testing set-up was performed on the specimens described above. Beam top longitudinal reinforcement was 2 #5 and #4 (0.81 in²) and bottom longitudinal reinforcement was 3 #4 (0.60 in²). Column longitudinal reinforcement was 4 #5 (1.23 in²). Ties/hoops were #2 (0.05 in²) at 4 in on the center. Figure 36 depicts this bar arrangement on one specimen” (source [11]).

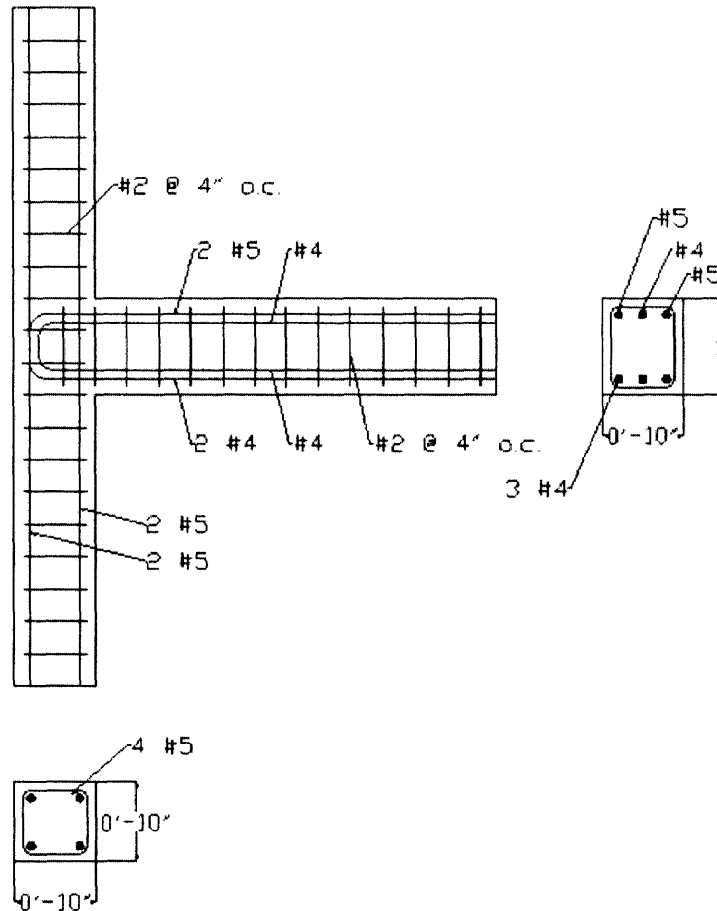


Figure 36: Bar arrangement on specimen (source [11] p28)

To be able to gather the information during the test, 6 strain gages have been placed at different spots of the joint. Then, a load cell was attached in the gap between the jack and the beam. Finally, these devices have been connected to a data acquisition system.

Three different configurations have been analysed in order to conduct the comparison and determine the advantages of variations in the concrete properties.

- RC joint with 4 in spacing
- SFRC joint with 6 in spacing
- SFRC joint with 8 in spacing

III.4 - Results

The data acquisition system provided the pieces of information gathered in the following chart of observed damage patterns. Table 1 is organized in three categories: beam, column and joint, so that the cyclical formation of damage in these three elements can be compared.

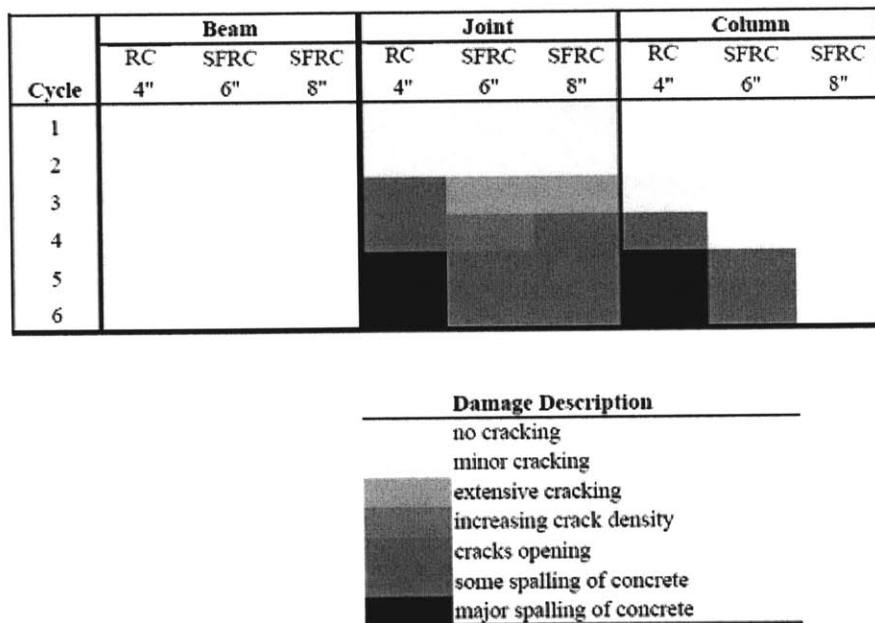


Table 1: Observed damage patterns (source [11] p 71)

DURABILITY

From the chart, it is noticeable that SFRC beam-column joint resists cracking damage better than RC beam-column joint.

Joint cracking was best resisted by the SFRC specimen with 6 in spacing. Minor cracking of the joint began during the first cycle and then joint cracks became more extensive during the

third cycle and increased in intensity during the fourth cycle. The SFRC specimen with 8 in spacing performed almost as well as the one with 6 in spacing. Minor cracking began during the first cycle and became more extensive during the third one. The specimen without steel fibers exhibited inferior performance. Minor cracks began forming during the first cycle. Some of these cracks opened during the third cycle and led to spalling of the joint concrete during the fourth cycle. The spalling became more extensive during the fifth and sixth cycles. Spalling and cracking in the SFRC joints are confined by the steel fibers which allow a better bond between steel and concrete and preserve a good portion of its strength. This increased the effectiveness of joint reinforcement (source [11]).

SEISMIC STRENGTH

By observations made of the testing, the joint with 6 in spacing has a much better seismic resistance than the SFRC joint with 8 in which itself has an improved seismic resistance over the RC joint with 4 in spacing. As a result, these exterior SFRC joints would avoid structural collapse of edifice (source [11]).

SIMPLIFICATION OF CONSTRUCTION

The SFRC joint with 8 in spacing could be advantageous in an area at low risk of very high seismic motion. It would be advantageous because it could reduce difficulty met when placing hoops in a beam-column joint. Hoop spacing can be increased by a factor of 2 thus providing a more simplified beam-column joint construction technique (source [11]).

DESIGN RECOMMENDATIONS

It is recommended that for exterior beam-column joints, in which ease of construction is wished, steel fibers at volume fraction of 2% should be used with code hoop spacing increased by a factor of 2. For exterior joints in high seismic risk areas, an SFRC joint with the same volume fraction and a code hoop spacing increased by a factor of 1.5 should be used. If the hoop spacing increased by a factor less than 1.5, it is likely that an even stronger seismic joint can be produced (source [11]).

CHAPTER IV: Effectiveness of steel fibers in NSC vs HSC - Experimental application

According to the previous experiment, it has been noticed that using steel fiber as shear reinforcement is probably one of the most promising fields for structural applications of SFRC due to the extremely brittle characteristics of the shear failure.

Furthermore, the first analysis run to compare NSRC and HSRC showed the benefits of using concrete with higher strength in a beam-column connection of a frame subjected to seismic excitation.

As a result, the idea would be to compile all the advantages and determine whether using steel fibers as reinforcement of HSRC is much better than using them with NSRC as made in the test above.

IV.1 - Presentation

Barragan, B.E. conducted this study in 2002 as part of his thesis at the Polytechnic University of Catalonia. “The aim of his study was to characterize, at the material level, the failure and toughness of SFRC subjected to direct shear loading. With this aim, the push-off test on a double-notched prism has been used to quantify the shear stress displacement behavior of SFRC” (source [14]). The specimen used is depicted in Figure 37.

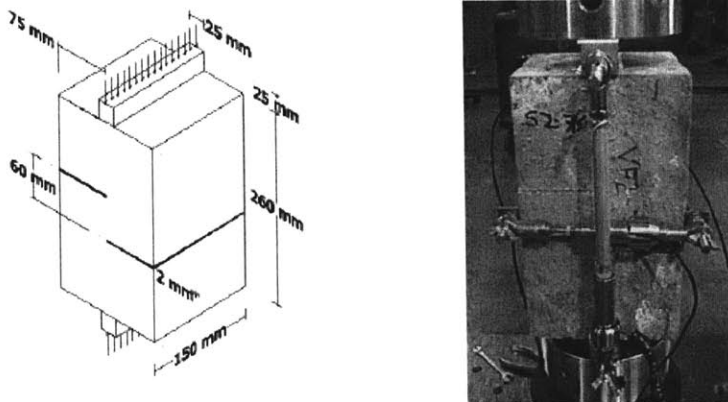


Figure 37: Push-off specimen used (source [14] fig 4.2 & 4.3)

This geometry gives an approximation of the shear forces that can be transferred across a crack and, as a result, indicate the shear toughness.

Two notches of 75 mm length were cut, 60 mm apart, perpendicular to the axis of the specimen. The notch-tips define a vertical plane, along which the load is applied. The shear stresses are larger than the tensile stresses, and consequently, if the tensile cracking can be controlled, shear cracking is expected to dominate the failure.

IV.2 - Materials

Two base concrete mixes have been studied, a normal strength concrete, with a characteristic compressive strength of about 5000 psi and a high strength concrete with a compressive strength of about 10000 psi. In each mix, two dosage of steel fibers have been incorporated, 1.25 pcf and 2.5 pcf. In both cases, the fibers were non-coated, collated and hooked-ended, with circular cross section.

IV.3 - Results

The typical mode of failure for the case of normal strength SFRC and high strength SFRC can be seen in Figure 38 and Figure 39.

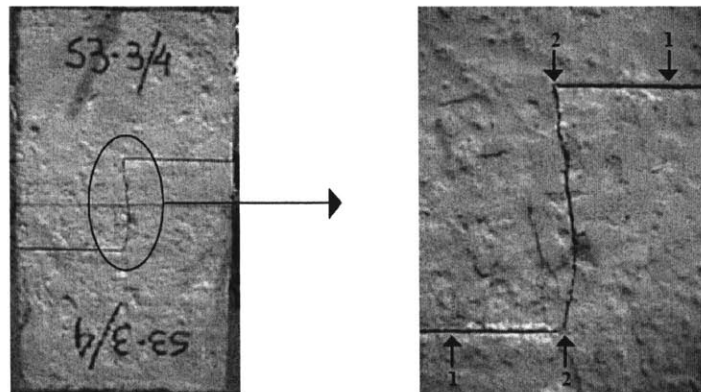


Figure 38: Typical failure mode for NS SFRC (source [14] fig 4.10)

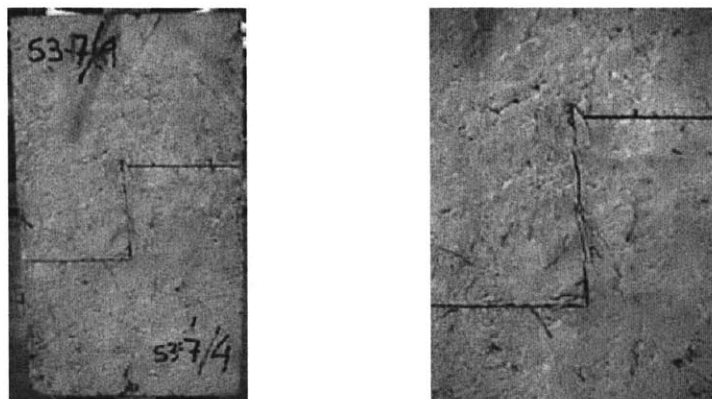


Figure 39: Typical failure mode for HS SFRC (source [14] fig 4.11)

“Failure occurs with the propagation of a single vertical shear crack consisting of a microcracked band that is approximately 0.40 in wide. In the case of the NS SFRC, there is some secondary cracking; a crack starts at point 1 and develops to a length of about 0.4 to 0.6 in. This crack is later arrested and the principal shear crack develops along plane 2-2. The first crack closes once the failure starts to localize on plane 2-2” source [14].

Furthermore, the typical stress versus vertical displacement (or slip) responses for NSC and HSC for plain and fiber concretes are shown in Figure 40 and Figure 41.

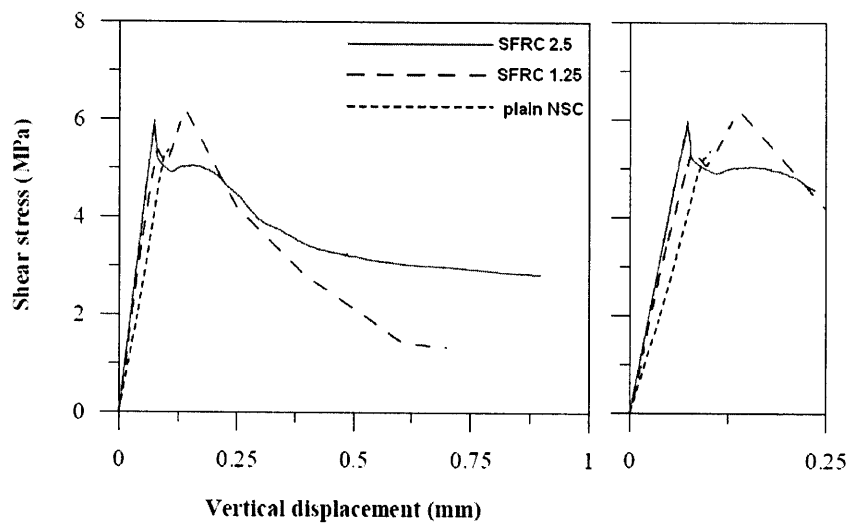


Figure 40: Shear stress vs slip for NSC (source [14] fig 4.12)

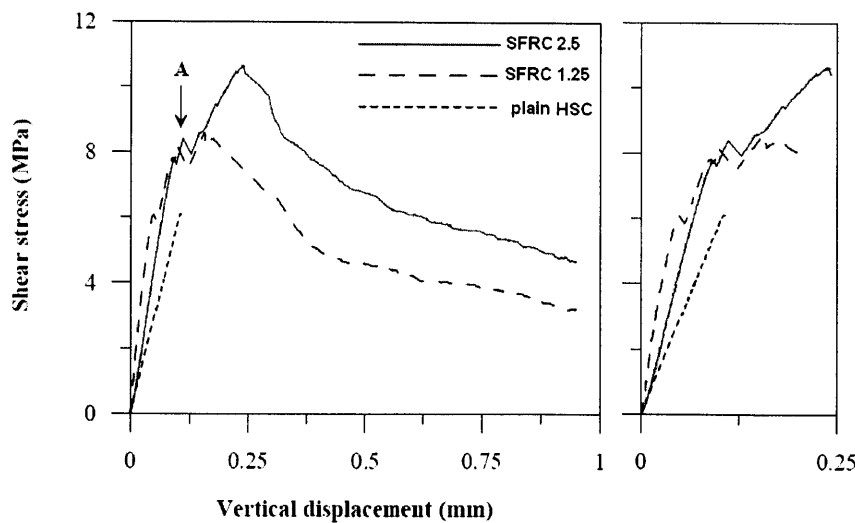


Figure 41: Shear stress vs slip for HSC (figure [14] fig 4.13)

For the plain concretes, a practically linear response up to brittle failure can be observed. The behaviour of the SFRC specimens is notably different; the response is linear up to the first crack (indicated by a peak), followed by a non-linear behaviour. After the first peak, the load in the NSC generally decreases progressively due to the presence of the fibers which harden the structure. In the case of the HSC, there is an increase in the stress after the first peak that is more significant when the volume of fibers is higher.

The improvement of the shear response due to the addition of fibers is influenced by the strength of the concrete and the volume of fibers. In the present work, the higher strength of concrete with the shorter fibers, which gives more fibers per unit area, yields a better performance. As shown by Walraven, 1994, using push-off specimens with traditional reinforcement or external restraint bars, the reinforcement increases the shear friction capacity in HSCs due to better bond, even though there is higher aggregate rupture (source [14]).

From these analyses it can thus be conclude that the most efficient way to strengthen a beam-column joint subjected to a seismic excitation, is to use steel fibers in addition of an high strength reinforced concrete.

CHAPTER V: Numerical Study of strengthening concrete using steel fibers

The previous analysis and experiments confirmed the benefits of using steel fibers to improve the characteristic of concrete in the beam-column joints. The aim of the following study based on numerical experiments using nonlinear finite element analysis is to explain theoretically the results obtained.

According to the graphs on Figure 42, adding steel fibers to concrete increases significantly its tensile toughness and ductility. It is also notable that steel fibers enhance the concrete's ability to resist stresses. In fact, if the modulus of elasticity of the fibers is high with respect to the modulus of elasticity of the concrete, such as steel fibers, the fibers help carry the load, thus increasing the tensile strength of the mix.

As a rule, fibers are generally randomly distributed in the concrete; however, processing the concrete so that the fibers become aligned in the direction of applied stress will result in even greater tensile or flexural strengths.

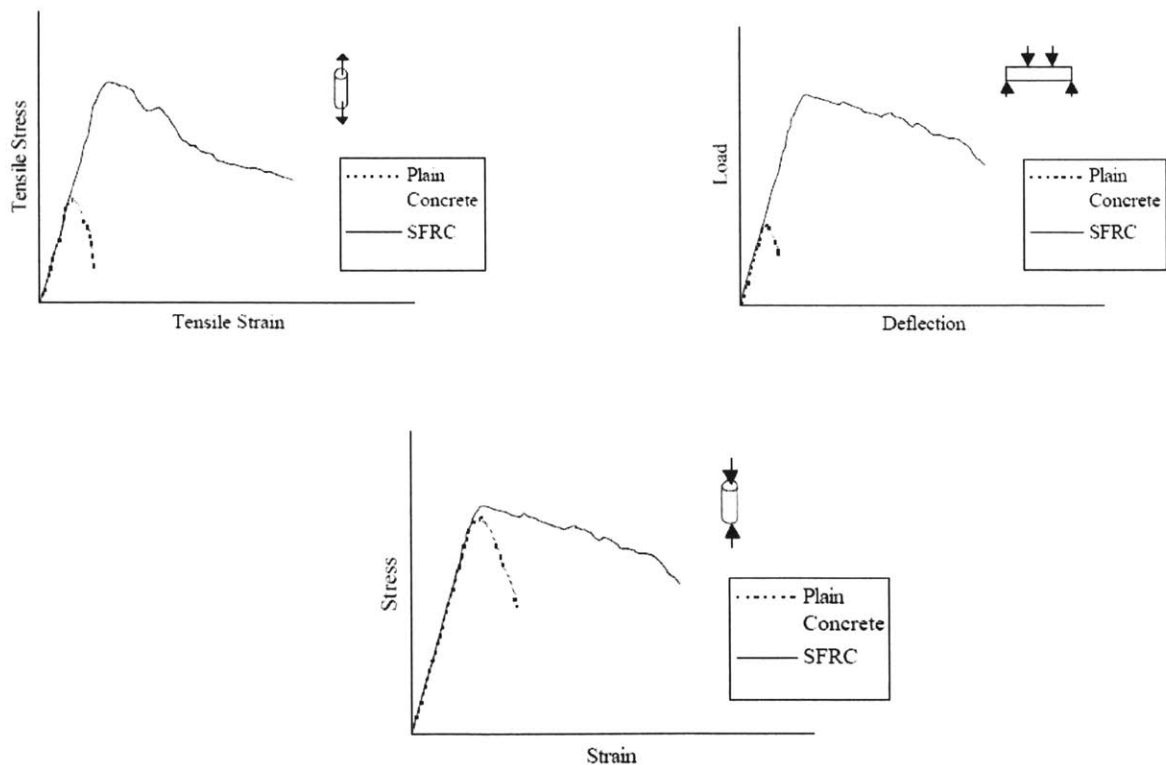


Figure 42: Properties of SFRC (source [11] p4)

A numerical example is solved to show the influence of short steel fibers to improve the strength of concrete in tension. A concrete specimen with dimensions 7.8 * 3.9 * 31.5 in with 4 points loading is analyzed. Steel fibers 1.18 in long and 0.24 in diameter are distributed randomly with several volume contents inside the specimen (source [7]).

V.1 - Effect of volume content

“In this nonlinear analysis, the total load applied is calculated from a series of small displacement increments at the point of loading. At the completion of each incremental solution, the stiffness matrix of the model is adjusted to reflect nonlinear changes in structural stiffness before processing to the next iteration (using Newton-Raphson equilibrium method). The model has no steel bars and rigid bonding between fibers and concrete is considered. The flow chart of this analysis is shown in the appendix” (source [7]).

Several models with different fiber volume contents of 1, 2 and 3 % have been solved to detect the influence of volume content fibers on the tensile behavior of SFRC. The load deflection relation is drawn for each volume content as shown on Figure 43.

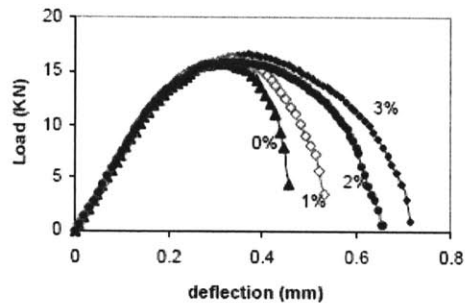


Figure 43: Load-deflection curve with different fiber contents (source [7] fig 9a)

It shows that the fibers do not considerably influence on the flexural strength of concrete. However, the main quality of steel fibers is that it can increase the toughness (energy absorption) and enhance the ductility of concrete. The energy absorption vs the deflection is depicted in Figure 44.

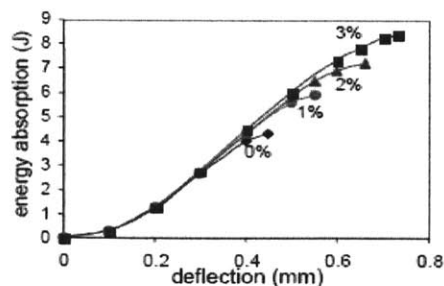


Figure 44: Energy absorption-deflection with different fiber contents (source [7] fig 9b)

Steel fibers increase the energy absorption capacity. It is noticed from the figure that increasing the fiber content keeps the growth rate of the total energy absorption constant. Every one percent of fiber content increases the energy absorption about 50% of the energy absorption capacity of plain concrete and increases the maximum deflection in failure by more than 20%. The enhanced behavior of steel fiber reinforced concrete over its reinforced counter-parts comes from its improved capacity to absorb energy during fracture. "While a plain unreinforced matrix fails in a brittle manner at the occurrence of cracking stresses, the fibers in fiber reinforced concrete continue to carry stress beyond matrix cracking, which helps maintain structural integrity and cohesiveness in the material. If the fibers are aligned parallel to the principal tension stress direction, that will help to improve the tensile strength of the combined material. But it is extremely difficult to align the fibers on this way in the real structures" (source [7]).

V.2 - The growth of cracks

Figure 45 shows an investigation for the percentage of damaged elements in relation with the deflection at the mid span using a concrete with 1% fibers.

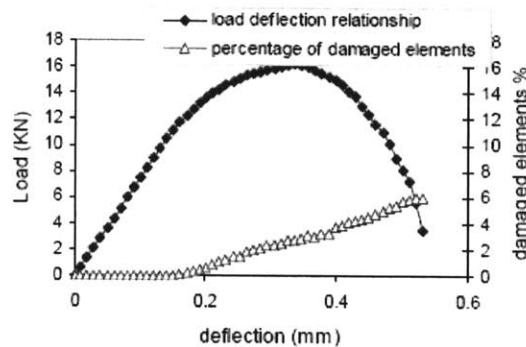


Figure 45: Growth of the damaged elements (source [7])

As soon as any element reaches its maximum elastic tensile strain, cracks start occurring and then the structure enters the plastic phase. The element is damaged when $\omega = 1$. It can be seen from the figure that the damaged elements increased linearly with the deflection at mid span. This means the cracks spread in lines upward while the area around cracks is free of stress and not damaged. The existence of fibers bridges the crack growth making the cracks take a longer way to spread. As a result, cracks take a longer time to reach the failure and thus the ductility of concrete is improved.

Figure 46 shows the crack growth due to different loads. It shows that the propagation of crack grows through lines upward.

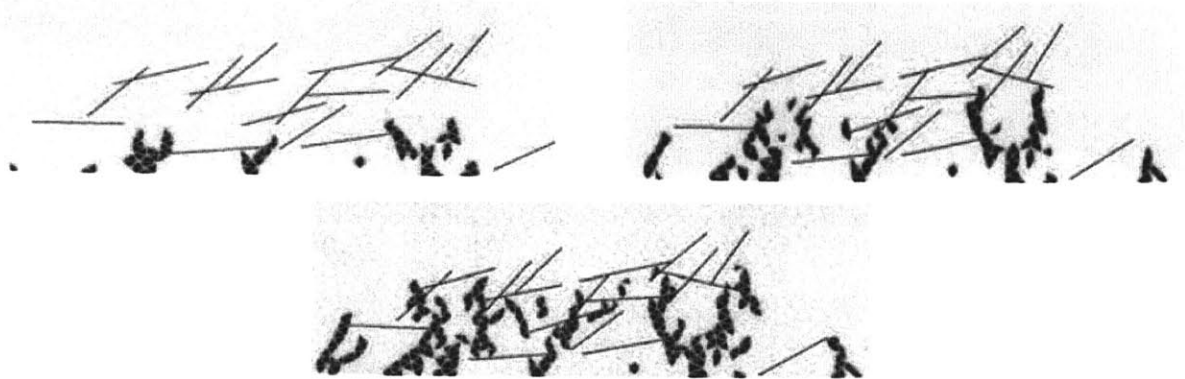


Figure 46: Crack growth (source [7] fig 11a b & c)

“It is visible that the existence of the fibers bridges the extension of cracks. The fiber ability to bridge the crack increase when the fiber is parallel to the tensile stress direction. When the fiber bridges the crack, the crack stops and another crack starts in another weak point so that the fibers help to increase the ductility of the composite material and forbid the sudden failure” (source [7]). Without fibers, the stresses concentrate in the tip of cracks making these location weak points in the structure while the surrounding area of the cracks become free of stress. The fibers help to transfer the stresses along the length of the fiber to the neighbor areas and reduce the stresses at the crack tip. This role of fibers delays the extension of cracks significantly.

V.3 - Suggestion to increase concrete ductility

The ductility of concrete can be improved significantly by increasing the amount of fibers where the tensile cracks are expected. For a 4 point loading beam, the cracks are expected in the middle half of the beam. Figure 47 shows 3 models solved with volume contents 1, 2 and 3%.

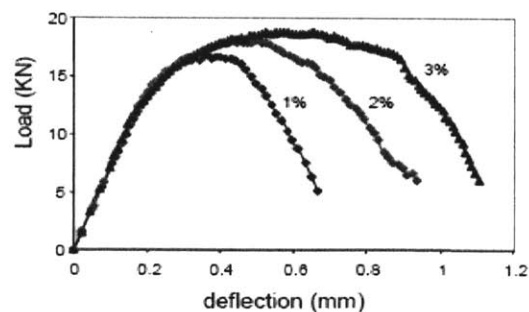


Figure 47: Load-deflection curve with different fiber content distributed in the middle half of the beam (source [7] fig 12a)

All the fibers are distributed in the middle half of the beam in which the cracks are predicted. The ductility of concrete increased significantly by increasing the quantity of fibers in the middle half because their ability to bridge cracks spreading. The energy-absorption curve of Figure 48 shows that strengthening the critical parts of the structure can enhance the energy absorption considerably.

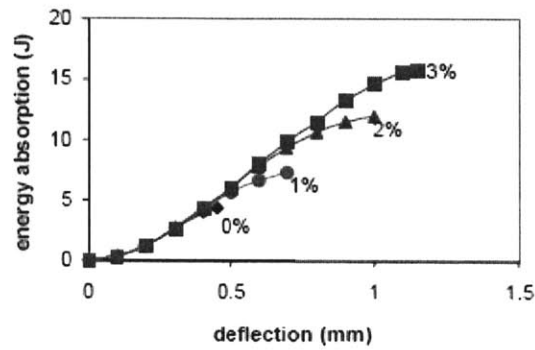


Figure 48: Absorbed energy-deflection curve with different fiber content distributed in the middle half of the beam (source [7])

“Every one percent of fiber content increases the absorption capacity about one time more” (source [7]).

Conclusion

The crack path through composite material such as concrete depends on the mechanical interaction between inclusions (gravel or crushed stone) and the cement-based matrix. Fracture energy depends on the deviations of a real crack from an idealized crack plane. The previous analyses have shown than other factors can influence the growth of cracks in concrete joints under seismic excitation. The comparison between these different concretes in beam-column connections has lead to several conclusions developed below.

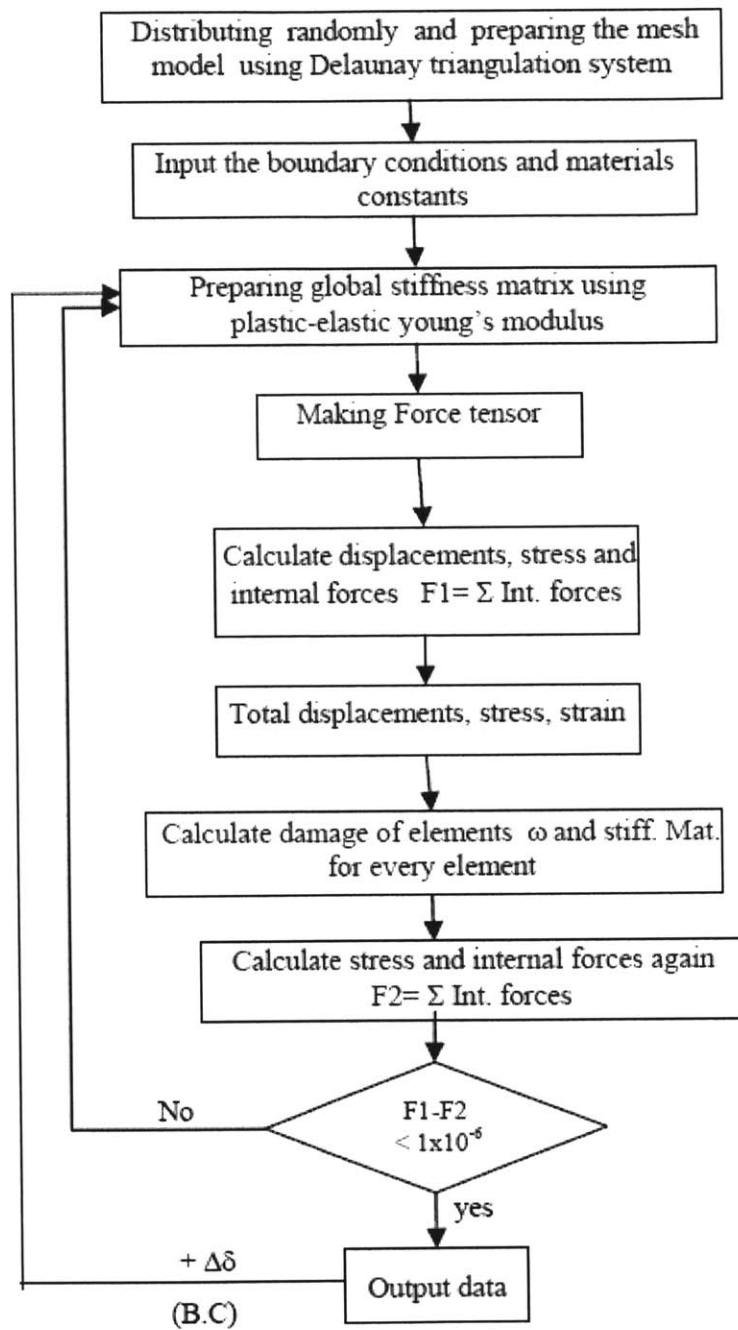
Significant improvements in the ductility of concrete during shear failure and some increase in the shear strength are achieved through incorporation of steel fibers especially using high strength concrete. These fibers provide indeed means of arresting the cracks growth and they are useful if a large amount of energy absorption capacity is required to prevent failure. Moreover, the numerical experiments described previously lead to further inferences; the addition of fibers does not increase the ultimate flexural strength of SFRC beam but it improves the energy absorption capacity of beams. Then, the number of damaged elements was found increased linearly upward while the neighbor area is still not damaged. Finally, the existence of fibers transfers the stress along the fibers to the healthy area helping to delay the extension of cracks. Therefore, it is recommended to strengthen the parts subjected to tensile stress to delay the expected cracks.

References

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APPENDIX



Flow chart of the nonlinear analysis