

Analysis and Comparison of Connections in Steel Structures

by

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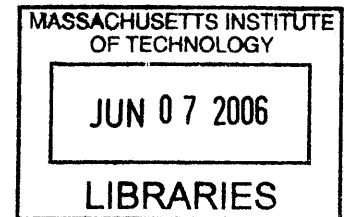
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ABSTRACT

The topic of connections is often given only limited attention in structural analysis and design of buildings, despite the fact that they can play a critical role in the structure. It is customary practice in the U.S. for the structural engineer to design the structural members, but leave the connection details to the steel fabricator. While this practice is more efficient and pragmatic in some instances, it is also necessary for structural engineers to have a good knowledge of connection behavior, especially when dealing with newer or atypical connections. Both the theoretical modeling of connections and the physical geometry and components of connections are considered in this thesis.

For the theoretical modeling of connections, the concept of semi-rigid connections as an alternative to the conventional idealizations of perfectly pinned or rigid connections will be addressed. Included will be methods to model and design semi-rigid connection behavior within frames. The effects of connection behavior on frames will also be considered.

With regard to the physical components of connections, an overview of types of fasteners and joints will be presented, including types and methods of installing bolts and welds, shop-welded, field-bolted column trees, and comparisons between these options. A final section presents options for connections between hollow structural sections of both circular and rectangular cross sections.

Finally, the topics covered for connections will be applied to the design project for the MIT class 1.562 as part of the MEng program in high performance structures.

Thesis Supervisor: Jerome J. Connor

Title: Professor of Civil and Environmental Engineering

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1. Introduction to Connections in Steel Buildings

Most design tasks in structural engineering are based both on theoretical models and empirical evidence. The theory is developed into a design procedure, and this procedure is carried out and tested. The results of these tests are then used to formulate new and more accurate analysis procedures. So, to some extent, this is an iterative process. Centuries of experience with building structures as well as new developments in analytical procedures have led to a greater understanding of how structures work. This is especially true in the area of connections in structures. There is a fairly high degree of uncertainty in the behavior of connections, which makes their analysis and design difficult, and makes experimental verification so important. Many of the elements of the connections have variability in their properties, which, when analyzed together, further increase the uncertainties in the connection. The consequence of this is that careful and accurate design of connections can be crucial to the design of the structure. This thesis attempts to understand and compare various types of connections in structural steel buildings, including both the modeling theory of connections as well as the physical components of connections.

1.1 Connection Design Methods

In the United States, a typical design engineer specifies the loads to be transferred through the connection, but does not design the connection completely. The steel fabricator is responsible for designing the exact location, number, and type of bolts and welds, producing shop drawings with connection details. The design engineer then checks these shop drawings. In many ways this is the most practical and efficient method of design, but in some ways it might be beneficial for structural engineers to have a greater understanding of connections. The general design procedure which structural engineers use can be summarized in four essential steps:

1. Calculate the distribution of forces within the connection.

This calculation starts with a determination of the distribution of forces within the elements to be connected. For beams carrying shear and moment, it is almost always assumed that the beam web carries the shear force while the beam flanges carry the bending moment. Once these are determined, the

flexibility of the parts of the connection are assessed. The flexibility is important for determining the flow of forces within the connection.

2. Design the components of the connection such that the each force path has the required strength to transmit the forces.
3. Ensure adequate ductility in the connection. This amounts to taking compatibility into account, because the above steps account only for force equilibrium.
4. Check that serviceability requirements are met. Also check other factors such as fatigue resistance, fracture for welds, and corrosion protection.

According to one evaluation, “This is self-evident, and yet it is surprising how frequently designers leave a weak link somewhere in a connection” [4]. There are a number of possible reasons for this omission. One possible source is that designers sometimes use more than one analysis to determine the distribution of forces. For example, when both welds and bolts are present in a beam, many sources recommend against looking at these connections separately. A more preferable method is to determine the most critical part of the connection, and then use this to determine the forces in other elements of the connection. Another possible source of error in connection design comes from lack of experience. Because connection design does depend so heavily on learning from past experiences and test results, inexperienced engineers can sometimes naively follow code guidelines without regard to common practices and history. And finally, another major source of error is that codes often focus on the actual connector strength, such as the bolt or weld strength, without focusing on other factors, such as the plates or angles which connect the members, and construction procedures. [4]

One of the most interesting facts about the design of structural connections is that, in practice, many checks are not actually carried out. “Most connection design is very straightforward and satisfies the preceding criteria by implication rather than by specific calculation.” [14]. The elements of the connection, such as geometry, bolt spacing, types of bolts, and weld geometries are often determined such that these checks are not required. The assumption, then, is that these requirements have already been met. By designing the geometry of the connection to meet, for example, code requirements on minimum edge distance between the bolt and the plate, or by using symmetrical

connections, an exact structural analysis is not actually carried out. Furthermore, often heuristic methods are used, without a true understanding of why these guidelines work. For example, plate thickness might be designed to be a certain percentage of the bolt diameter. While in most cases these simplifications and guidelines are quite acceptable, it does not help engineers gain experience and develop a full understanding of connections, which could be detrimental when it comes to more irregular structures, such as hollow sections, eccentric loadings, or fatigue-critical zones. For this reason a greater focus on connections in academia and in practice may be very beneficial to structural engineers.

1.2 Characterization of Connections

Connections are classified by their strength as well as their ductility, where ductility is a description of the rotation capacity. The strength classification of connections is based on the relative moment resistance of the connection compared to the moment resistance of the beam. A full strength connection has a strength that is greater than the plastic moment capacity of the beam. Both strength and rotation capacity are essential for the connections. To illustrate the importance of both strength and rotation capacity in connections, full-strength connections designed by plasticity theory can be considered. In this case, one possible complication may occur when the yield stress of the beam is actually higher than the specified value, which is often the case. The structural system is designed so that a plastic hinge theoretically forms in the beam before it forms in the connection for full-strength connections. However, when the yield strength of the beam is significantly higher than the specified value, the beam will not reach the fully plastic moment until it reaches a stress higher than predicted in design. In this case, sufficient rotation capacity at the connection is required in order to ensure that the beam develops its full plastic strength. [10]

1.2.1 Idealized Pinned and Moment Connections

Structural connections are typically categorized into one of two idealized types: pin connections or fixed connections. A pin connection is one which fully transmits

forces but not moments between the connected elements. Therefore, there may be a difference in the rotations of the connected elements at the point of the connection. Although in practice pin connections usually provide some rigidity, this is ignored in design. A moment, or rigid, connection is one which fully transmits both moments and forces between the connected elements. In this case there is no difference in rotation between the connected elements at the point of the connection; the original angle between the connected elements remained unchanged and the joint is said to be “rotationally stiff”. However, both of these connections are idealizations which are used in design but sometimes do not accurately describe the real behavior of the connection. [6]

1.2.2 Semi-Rigid Connections

In U.S. steel design codes, there is some mention of types of connections which fall between these two idealized types. In the ASD Specifications, the connection labeled as “Type 3” is considered semi-rigid, meaning that it can transfer vertical shear and also some moment. In the LRFD Specifications, the two types of connections are fully restrained (FR) and partially restrained (PR). The PR type can include both ideally pinned and also semi-rigid connections. The differences between the two obviously change the moment distribution along the connected members, but also affect other concerns such as frame drift and P-delta effects. The LRFD code provides fairly detailed design guidelines for the FR connections, but, “in contrast to this, the design specification provides for Type PR construction only broad principles for analysis and design. It is left to the engineer to implement these basic principles in a qualitative manner.” [6] Because this design process is not widely used and highly developed, some design principles and examples will be presented and analyzed in this thesis.

Semi-rigid connections are both ductile and partial-strength. Ductility, in this sense, is not the same concept as material ductility, and refers instead to a relatively high degree of rotation capacity. Connections are generally classified as ductile if the maximum rotation under the maximum moment is greater than .003 radians. For special moment frames in seismic zones, this minimum is increased to .004 radians because greater ductility is desired. [5] Partial strength implies that the connection strength is less than the plastic moment of the beam. The connection strength can be chosen by the

designer, and in practice this strength is typically between 30% and 50% of the plastic moment resistance of the beam. [10]

1.3 Connections in a Frame

The type of connections used in the structure relates to the type of frame used. Moment frames refer to frames which have rigid or moment connections between structural elements. These types of frames are often used when lateral bracing is unfeasible or architecturally undesirable. Moment frames are also generally more flexible under lateral loads. For global elastic analysis of a moment frame, rigidity of the connections is assumed so that elastic beam theory can apply. On the other hand, for plastic global analysis of a moment frame, full strength of the connections is assumed, so that the plastic hinge occurs in the beam before it occurs in the connection. In contrast to moment frames, braced frames are frames which have some type of bracing between structural elements. These types of frames only require pinned connections because the bracing elements provide the necessary lateral stability. In general, moment connections cost more to fabricate and erect. In the design process, braced frames, which usually have pinned connections, are designed with regard to strength requirements, and unbraced frames, which rely on moment connections, are designed with regard to stability and deformation limits. Semi-rigid frames have some continuity of rotation in the connections, but do not achieve the full bending rigidity of the members. Semi-rigid frames tend to be used for shorter buildings when lateral forces are low, or in beams where some amount of rigidity at the ends helps to reduce deflections. [11]

In addition to these distinctions, frames are also characterized as “sway” or “non-sway.” While moment frames are often referred to as sway frames and braced frames are referred to as non-sway frames, this does not define the boundary between sway and non-sway frames, which actually depends on the loading. A frame is called non-sway for a

given load case if $\frac{V_{sd}}{V_{cr}} \leq 0.1$, where V_{sd} is the design value for the total vertical load and

V_{cr} is the elastic critical load for failure of the frame in sway mode. However, for typical frames, the non-sway classification can be simplified. In this case, a frame is characterized as non-sway if

$\frac{\delta}{H} * \frac{V}{H} \leq 0.1$ where δ is the interstory drift, h is the story height, V is the vertical reaction at the bottom of a story, and H is the horizontal reaction at the bottom of a story. [17]

2. Semi-Rigid Connections and Frames

Semi-rigid frames can be, in some instances, a more economical choice of framing system than the other alternatives. In some sense they offer a solution to the problem of choosing between a moment frame and a braced frame. With braced frames, the connections are cheap, but the structural members need to be larger than they would be in a moment or semi-rigid frame. The reason is that, for typical loadings of a distributed load along the beam, fixity of the connections shifts the moment diagram upward from the simply supported case. This means that the total moment is shared between the positive moment at midspan and the negative moment at the supports, thereby decreasing the design moment. For example, the moment diagram for a simply supported beam with a factored distributed load of 1 k/ft and W18x35 section is shown in Figure 1. For this simply supported beam, the design moment which the member must be able to resist is 74.5 k-ft., and the midspan deflection is .04329 ft.

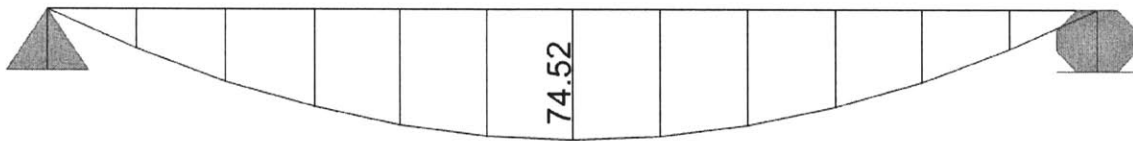


Figure 1: Moment Diagram of Simply Supported Beam

In comparison, the moment diagram for a fixed-fixed beam with rigid connections is shown in Figure 2. The fixity at the supports shifts the moment diagram upward, and now the design moment for the beam is only 49.7 k-ft., as compared to 74.52 k-ft. for the simply supported case. The midspan deflection is .008465 ft., as opposed to .04329 ft. for the simply supported case.

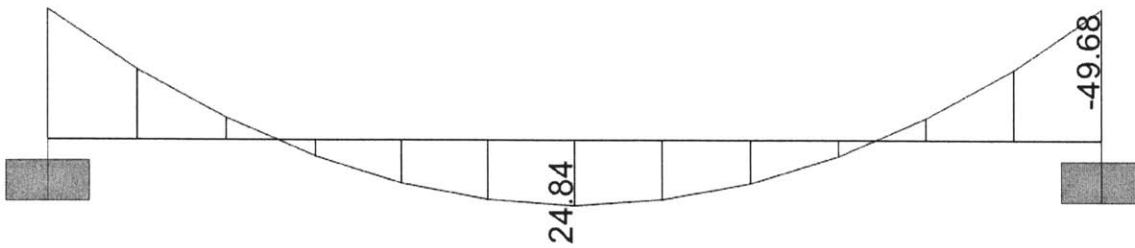


Figure 2: Moment Diagram of Fixed-Fixed Beam

By comparing these diagrams, the fixed-fixed beam has a smaller maximum moment, which is approximately 50 k-ft., compared to the maximum moment of the simply supported span. Therefore, the fixed-fixed beam has a smaller requirement for the flexural strength of the section. However, this design is not as economical as it could be, because the difference between the negative moment and the positive moment is not minimized. To reach the optimal design moment condition, the moments at the support and at midspan should be equal or close to equal. A semi-rigid beam with a partial fixity factor, which is the relationship between moment and rotation, of 20,000 k-ft./radian gives favorable results, as shown in the moment diagram in Figure 3. In this design, the positive moment at the support and the negative moment at midspan are very similar. Therefore, the semi-rigid beam need only be designed for a flexural strength of 40 k-ft. as opposed to 75 k-ft. for the pinned-pinned case and 50 k-ft. for the fixed-fixed case.

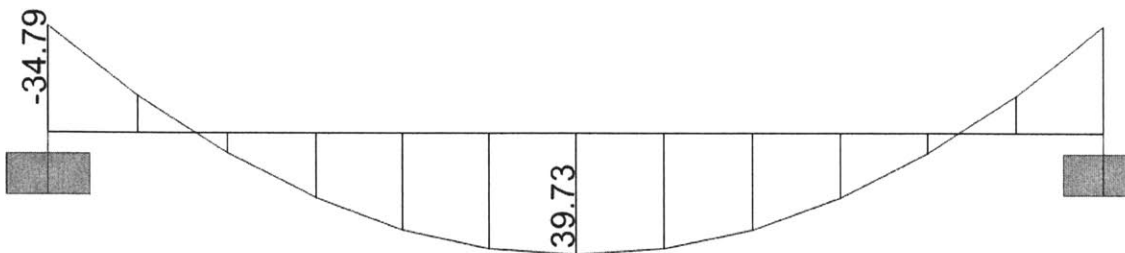


Figure 3: Moment Diagram for Semi-Rigid Beam

However, this may not always be the most desirable case. For example, if deflections are the critical parameter and govern the design, a fixed-fixed beam may be preferable because it will yield the smallest midspan deflections. In particular, semi-rigid connections give rise to high lateral deflections, which may be undesirable. Depending

on the scenario, the designer may perform an economic analysis and discover that the potential savings in beam size for a semi-rigid frame do not offset the extra cost of designing semi-rigid connections, and therefore decide to use a braced frame with simpler, cheaper connections. If stiffeners are required in the connection or if the beam must be haunched, then it is unlikely that the savings in beam size will be greater than the cost of these additional connection details. It is usually the case that increasing the depth of a column or beam is cheaper than welding stiffeners to the connection. Nevertheless, the semi-rigid frame is beneficial in some cases in that it presents another option for the design which may be optimal.

2.1 The Moment-Rotation Relationship

The concept of semi-rigidity has been around for a long time, but one of the reasons why this concept is not more often used in practical design of frames is that it is difficult to model. One of the key elements in semi-rigid connections is the moment-rotation curve. This curve is not as important for other types of connections. For

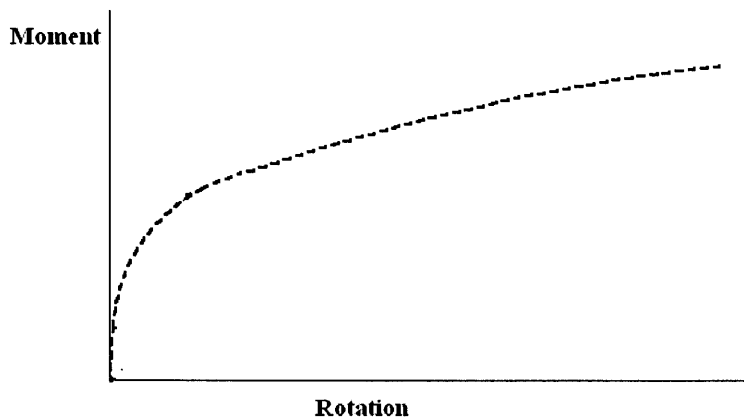


Figure 4: Typical Moment-Rotation Curve
Courtesy of ESPED [10]

example, in a beam column connection, moment connections are modeled with the assumption that no relative rotation between the beam and column occurs, whereas for pinned connections, it is assumed that the beam is free to rotate from the

column. For semi-rigid frames, the response is somewhere between these two cases, and this is why the moment-rotation curve is so crucial.

Moment-rotation curves are not linear. For smaller values of moments the rotation increases more quickly than it does for higher values. This nonlinear behavior is due mostly to local yielding of parts of the connection, and also to material discontinuity,

stress concentrations, local buckling in the vicinity of the connection, and changes in the geometry under the loading. The nonlinear behavior can be exacerbated in certain types of connections, especially bolted connections, where slip can cause a higher rotation than that caused by the moment. [7]

2.1.1 Modeling the Moment-Rotation Curve

The important parameters of a moment-rotation curve are the initial stiffness, R_{ki} , which is the initial slope of the curve, the secant stiffness, R_{ks} , which is the effective stiffness of the connection, the tangent stiffness, R_{kt} , which is the instantaneous stiffness, which decreases with increasing moment, and the unloading stiffness, R_{ku} , which is the result of load removals or reversals. Figure 5 shows these stiffness parameters.

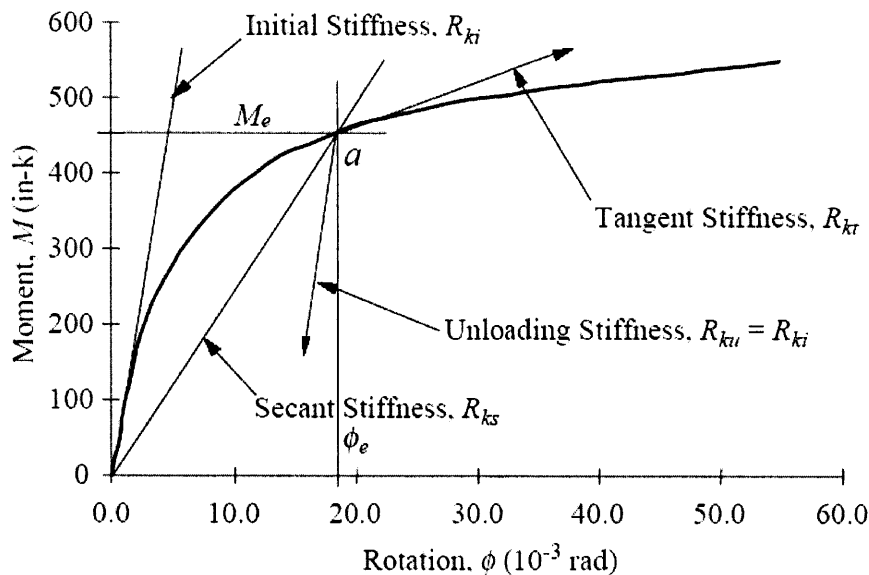


Figure 5: Moment-Rotation Curve
 Courtesy of Bjorhovde and Christopher [18]

This curve can be obtained by a number of different methods. One method is by experiment. In this case, care must be taken to ensure that the test conditions are an accurate representation of the design problem. The moment-rotation curve of the actual connection being designed is not known exactly, and the amount of variability in connection behavior makes this approach uncertain. In order to convert the moment-rotation empirical data into a format that is useful for design, curve-fitting techniques are necessary which approximate the actual curve are necessary. One such example is the

three-parameter power model, which models the moment-rotation curve for beam-column connections using double angles to make the connection. The curve is approximated as:

$$M = \frac{R_{ki} * \theta}{\left[1 + \left(\frac{\theta}{\theta_0} \right)^n \right]^{\frac{1}{n}}}$$

where R_{ki} is the initial stiffness of the connection, n is a shape factor,

and θ_0 is the reference rotation of M_u/R_{ki} . [5]

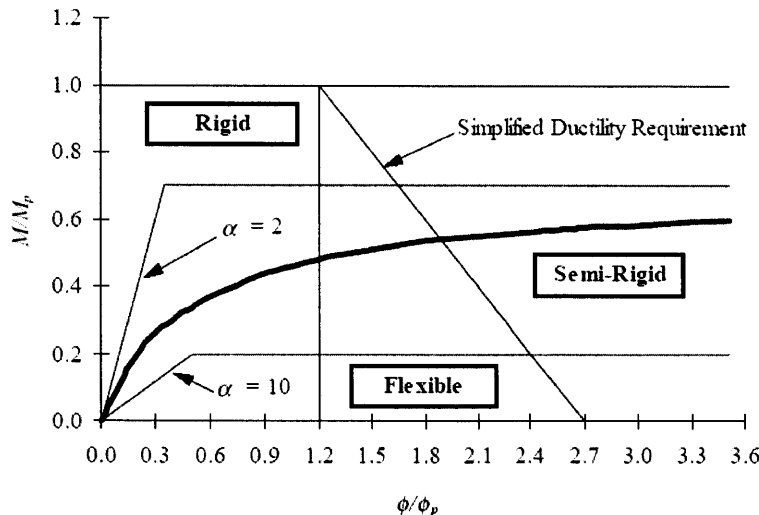
2.1.2 Classification of Connections by Rigidity

With the concept of semi-rigidity, it is more important to be able to classify connections as “rigid”, “semi-rigid”, or “flexible”. One classification developed by Bjorhovde compares the stiffness of the connection to the stiffness of the connected members. The non-dimensional factor for the connection stiffness is

$$\alpha = \frac{EI}{R_{ki}d}$$

where d is the depth of the beam. [2] Using this parameter can help classify

connections based on their stiffness. The chart which classifies connections based on this



factor is shown in Figure 6.

Figure 6: Bjorhovde Classification of Connections
Courtesy of Bjorhovde and Christopher [8]

Eurocode 3 gives guidelines to approximate the moment-rotation curve, also taking into account that semi-rigid connections exhibit different behavior in braced frames than they do in unbraced frames. The parameters

$\bar{m} = \frac{M}{M_p}$ and $\bar{\theta} = \frac{\theta}{\theta_p}$ where M_p is the full plastic moment capacity of the beam

and $\theta_p = \frac{M_p}{\frac{EI_b}{L_b}}$ are used to classify connections.

These parameters define the Moment-rotation curve, which helps engineers to classify different types of connections, as described in Eurocode 3. [10]

For unbraced frames, where sidesway is permitted [10],

$$\bar{m} = 25 * \bar{\theta} \quad \text{for } \bar{m} \leq 2/3$$

$$\bar{m} = \frac{25\bar{\theta} + 4}{7} \quad \text{for } 2/3 \leq \bar{m} \leq 1.0$$

For braced frames, where sidesway is prohibited [10],

$$\bar{m} = 8 * \bar{\theta} \quad \text{for } \bar{m} \leq 2/3$$

$$\bar{m} = \frac{20\bar{\theta} + 3}{7} \quad \text{for } 2/3 \leq \bar{m} \leq 1.0$$

The plots for these equations are shown in Figures 7 and 8.

The curve for unbraced frames is:

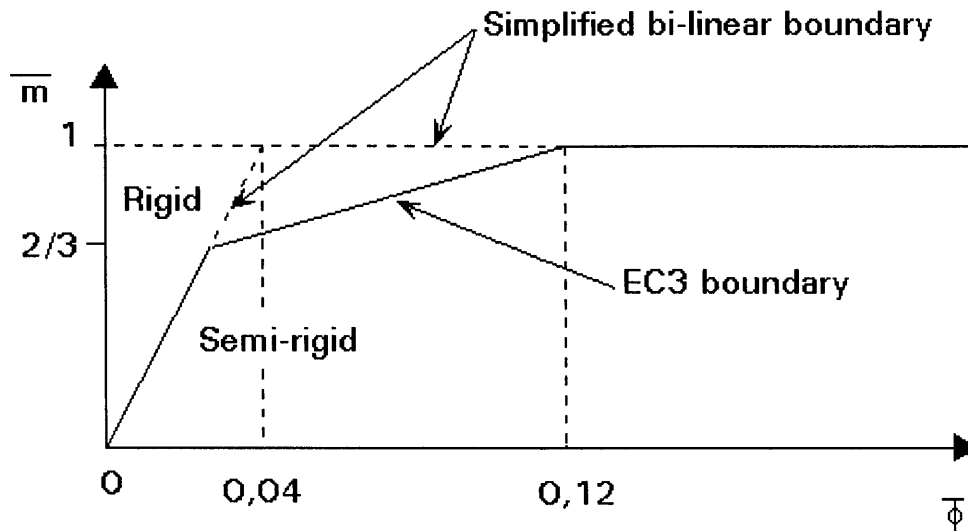


Figure 7: Boundary Between Rigid and Semi-Rigid Connections for Unbraced Frames
Courtesy of ESDEP [10]

The curve for braced frames is:

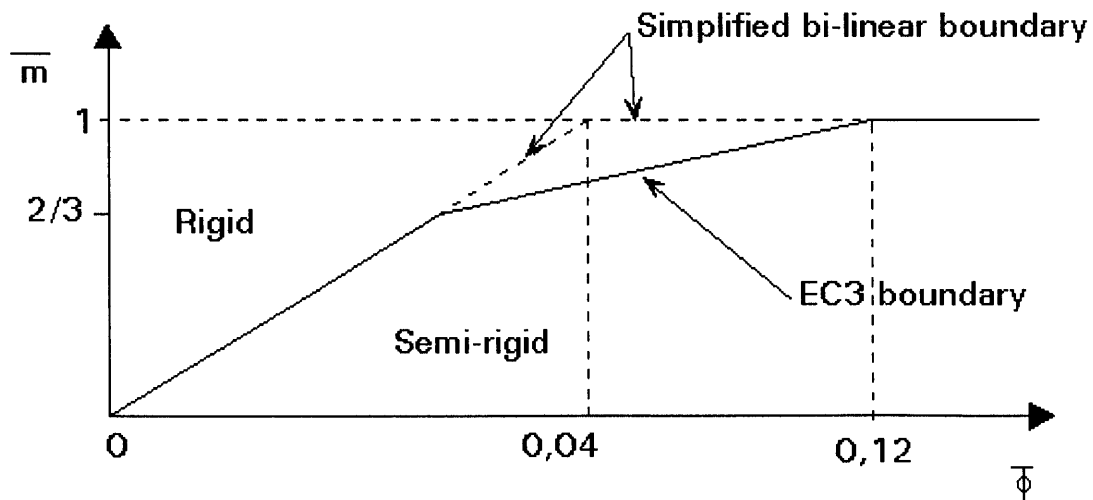


Figure 8: Boundary Between Rigid and Semi-Rigid Connections for a Braced Frame
Courtesy of ESDEP [10]

An alternative to the empirically obtained $M-\phi$ curve is to use finite element analyses to model the connection and determine the relationship. The benefit of this

technique is that costly materials and laboratory time do not have to be used. Another benefit is that parameters in the model can be changed more easily, and the exact connection details can be modeled. The downsides to this approach are that it is time-consuming and requires a high level of understanding of finite element modeling. This method is suitable for welded connections but is especially difficult for bolts, as bolts have a higher amount of variability and complexity. For instance, with a geometric lack-of-fit, the bolt hole in the plate may be larger than the bolt diameter to an extent that allows more rotation than expected. [17]

2.2 Design of Semi-Rigid Structures

One of the conventional methods of analyzing a beam whose end restraints fall in between the idealized pin and the idealized fixed end is the concept of the “end-fixity factor”. [6] This end-fixity factor, r , is defined as:

$$r = \frac{\alpha}{\phi} = \frac{1}{1 + \frac{3EI}{RL}}$$

This factor, r , defines the stiffness of the end-connection relative to the attached element. It is the rotation of the end of the beam divided by the combined rotation of the beam and the connection due to a unit end-moment, which is expressed as $\frac{\alpha}{\phi}$. Therefore, for pinned connections, the end-fixity factor is 0 whereas for fixed connections, the factor is 1. For a semi-rigid connection, then, the end-fixity factor lies somewhere between 0 and 1. This end-fixity factor, r , can be a more useful parameter for characterizing beams than the rotational stiffness, R , or the connection stiffness ratio, RL/EI , because it has an approximately linear relationship with the moments and deflections.

2.2.1 Moment Distribution Method for Semi-Rigid Design

The end-fixity factor is also useful when applying the moment distribution method to semi-rigid frames. The moment distribution method is a way to solve for the moments in an indeterminate structure by fairly simple hand calculations, using successive approximations that converge to the actual moment distribution. Chen

outlined a method for applying the moment distribution method to semi-rigid frames. [5]

This approach is summarized here, with an example to follow:

Expression for the member stiffness factor K_{ij} [5]:

$$K_{ij} = s_{ij} \frac{4EI}{L} \text{ and } K_{ji} = s_{ji} \frac{4EI}{L}$$

where the end connection flexibility factors s_{ij} are defined as [5]:

$$s_{ij} = \frac{3r_i}{4 - r_i r_j} \text{ and } s_{ji} = \frac{3r_j}{4 - r_i r_j}$$

The distribution factor, DF, which is defined as the stiffness factor of the member divided by the total stiffness factor for the joint [5]:

$$DF_{ij} = \frac{K_{ij}}{\sum K_{ij}}$$

The carry-over factor determines the moment to be carried to the far end from the near end, and is defined as the ratio between the far-end moment to the near-end moment. The carry-over factor, C, for semi-rigid frames is defined as [5]:

$$C_{ij} = \frac{r_j}{2} \text{ and } C_{ji} = \frac{r_i}{2}$$

To illustrate the use of this method, it is applied to a simple one-story, two-bay frame, where all connections are defined to be partially fixed, with equivalent spring constants of 20,000 k-ft./radian, as shown in Figure 9, with the joints labeled as shown. The bay widths are 24 ft. and the column heights are 12 ft. A 1 k/ft gravity load was applied to the beams, and the members are all W18x35.

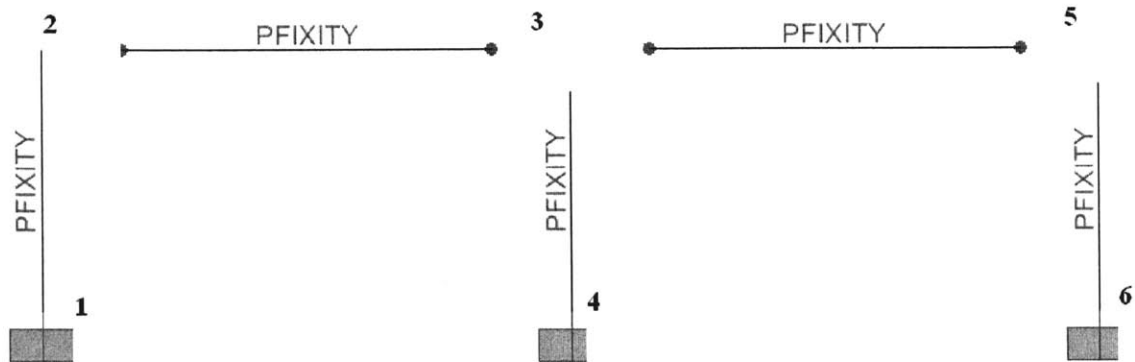


Figure 9: SAP2000 Model of Semi-rigid Frame

Using the parameters of the moment distribution method for semi-rigid frames as outlined above, the method was applied to this frame.

For the beams,

$$r = \frac{1}{1 + \frac{3 * 102,730}{20,000 * 24}} = .609$$

For the columns, $r = 1$

$$K_{23} = K_{32} = K_{35} = K_{53} = \frac{3r_{beam}}{4 - r_{beam}^2} * \frac{4EI_{beam}}{L_{beam}} = 8620$$

$$K_{21} = K_{34} = K_{65} = \frac{3r_{column}}{4 - r_{beam}r_{column}} * \frac{4EI_{column}}{L_{column}} = 18450$$

$$\text{For joint 2: } DF_{21} = \frac{K_{21}}{K_{21} + K_{23}} = .682$$

$$DF_{23} = 1 - DF_{21} = .318$$

$$\text{For joint 3: } DF_{32} = \frac{K_{32}}{K_{32} + K_{34} + K_{35}} = .242$$

$$DF_{35} = .242 \text{ by symmetry}$$

$$DF_{34} = 1 - 2(.242) = .516$$

$$\text{For joint 5: } DF_{56} = \frac{K_{56}}{K_{56} + K_{53}} = .682$$

$$DF_{53} = 1 - DF_{56} = .318$$

For the beams, the fixed end moments (FEM) which serve as the starting point in the moment distribution method were calculated to be

$$M_1 = -M_2 = \frac{wL^2}{12} \left[\frac{3r(2-r)}{4-r^2} \right] = 33.6 \text{ k-ft.}$$

The moment distribution method is shown in Figure 10.

Joint	1	2		3			4	5		6
Member	12	21	23	32	34	35	43	53	56	65
DF	0	0.682	0.318	0.242	0.516	0.242	0	0.318	0.682	0
FEM (k-ft)			33.6	-33.6		33.6		-33.6		
		-22.92	-10.68					+10.69	+22.92	
	-11.46			-3.25		3.25				11.46
Total Moment (k-ft)	-11.46	-22.92	22.92	-36.85	0	36.85	0	-22.92	22.92	11.46

Figure 10: Moment Distribution Method Example for Semi-rigid Frames

These values are similar to the results from the SAP2000 model of the same frame in Figure 11.

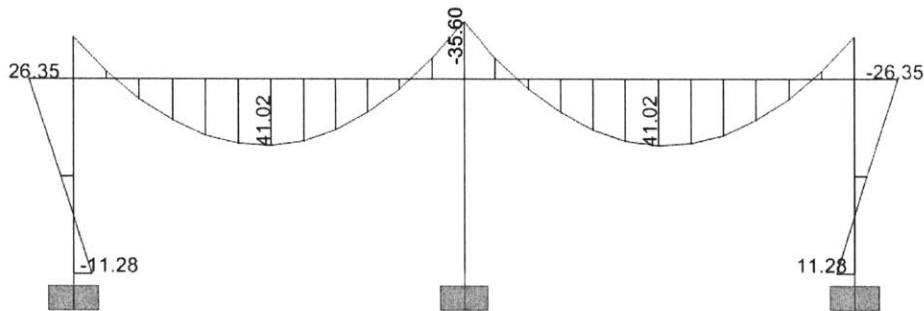


Figure 11: SAP2000 Moment Diagram for Semi-Rigid Frame

The accuracy of the moment distribution method to the moment diagram of the SAP2000 model shows that the moment distribution method can be used for semi-rigid frames as well as for rigid frames. This is a powerful tool that can be used for simple hand calculations and verification of results.

2.2.2 Frame Design

The overall frame and connections can be designed either from an elastic or a plastic perspective. In general, the calculations for frames and connections need not be analyzed by the same method. However, a connection which remains in the elastic range until failure should be used in frames that are also designed elastically, unless the connections are moment connections, and have a higher moment resistance than the connected beams. When elastic theory is used to determine the force distribution in a

connection, it will result in a stiffer connection than the value given by plastic theory. On the other hand, when the connections are designed plastically but the frame is designed elastically, second-order effects should be considered if the ratio between the Euler buckling load and the design load is small. For a global elastic analysis, the spring behavior is considered to be linear, and the connections are modeled as rotational springs. For purposes of simplicity, the elastic design moment is considered to be 2/3 of the ultimate moment, because experimental results show that most beam-column connections behave elastically through 2/3 of their full moment strength.

2.2.3 Stability Design of Semi-Rigid Frames

The stability design of semi-rigid frame is similar to that of rigid frames, where the traditional use of alignment charts for braced and unbraced frames can be modified to account for the semi-rigid nature of the joints, as proposed by Chen and Lui [6]. For columns connected to beams at both ends with semi-rigid connections with rotational stiffness R , the modified stiffness distribution factor, G , is given as

$$G = \frac{\sum \frac{I_c}{L_c}}{\sum \frac{\alpha_b I_b}{L_b}}$$

Where properties with subscript “c” are for the column and properties with subscript “b” are for the beam. The parameter α is a modification factor for the beams which effectively reduces the moment of inertia, I_b , of the beam.

For braced frames, $\alpha_b = \frac{1}{1 + \frac{2EI_b}{RL_b}}$

For unbraced frames, $\alpha_b = \frac{1}{1 + \frac{6EI_b}{RL_b}}$ where R is the rotational stiffness at the ends of the

beam. [5]

2.2.4 Structural Computer Modeling of Semi-Rigid Buildings

An analysis program, SAP2000, can model structures with semi-rigid connections. When choosing joint releases, the user can either choose to completely release the moment transfer between connected elements or to specify a partial fixity factor. This is a feature of the newer version of SAP2000 (version 9), which perhaps reflects the trend toward using semi-rigid connections in practical design scenarios. The partial fixity factor is the relationship between the moment and the rotation at the connection, or the equivalent rotational spring constant. For example, the moment diagram shown above in Figure 3 shows the moment diagram for a beam with partial fixities at both ends of the beam of 20,000 k-ft/radian. In order to check that the results are accurate, it is possible to determine the equation for the moment diagram, integrate it to find the rotation equation, and check that the moment and the rotation at the ends of the beam are related by the specified partial fixity factor. For the beam as shown above in Figure 3, this check was performed. With the loading of 1.0 k/ft plus the dead load of the W18x35 section, the shear and moment can be expressed as:

$$V = 1.035x - 12.42$$

$$M = -\int V(x)dx = -\frac{1.035}{2}x^2 + 12.42x + C_1$$

Using the relation that $M(x) = EI * \frac{d^2v}{dx^2}$ and $M(x) = EI * \frac{d\beta}{dx}$,

$$\beta(x) = \frac{1}{EI} \int M(x)dx = \frac{1}{EI} \left[\frac{-1.035}{6}x^3 + \frac{12.42}{2}x^2 + C_1x + C_2 \right]$$

$$v(x) = \int \beta(x)dx = \frac{1}{EI} \left[\frac{-1.035}{24}x^4 + \frac{12.42}{6}x^3 + \frac{C_1}{2}x^2 + C_2x + C_3 \right]$$

For the W18x35 section used, $I_{xx} = .0246 \text{ ft}^4$ and $E_{\text{steel}} = 4,176,000 \text{ ksf}$.

Using the boundary conditions that

$$\beta(0) = \frac{M(0)}{20,000}$$

$$v(0) = v(L) = 0$$

the three constants C_1 , C_2 , and C_3 can be solved, yielding the following equations:

$$M = -\frac{1.035}{2}x^2 + 12.42x - 34.79$$

$$\beta(x) = \frac{1}{EI} \int M(x)dx = \frac{-1.035}{6}x^3 + \frac{12.42}{2}x^2 - 34.79x - 178.74$$

$$\beta_0 = \frac{M(0)}{EI * \text{partialfixityfactor}} = \frac{34.79}{.0246 * 29,000 * 12^2 * 20,000} = .00174 \text{ radians}$$

This gives $\beta(0) = .00174$ radians, and $M(0) = 34.79$ k-ft., which agrees with the SAP2000 results.

In addition to studying the properties of the connections, it is important to determine what effect the connections have on the overall behavior of the structure. In order to compare frames with different types of fixity, three SAP2000 models were created. All three models had the same geometry, but the joint fixities as well as bracing members were altered between the models. The geometry of these buildings was a 2-story, 3-bay frame, with a story height of 12 ft. and a bay width of 24 ft. A distributed gravity load of 1.0 kips/ft was

applied to all of the beams, and a

0.25 k/ft distributed wind load

was applied laterally to the

beams in the y-direction, which

is the critical direction because

the frame is only two bays in the

y-direction as opposed to three

bays in the x-direction. The

beams and column were designed

to be W21x83 sections. Three

models were created and analyzed: a rigid-framed model, a semi-rigid model, and a

braced frame. The basic geometry of the building is shown in Figure 12.

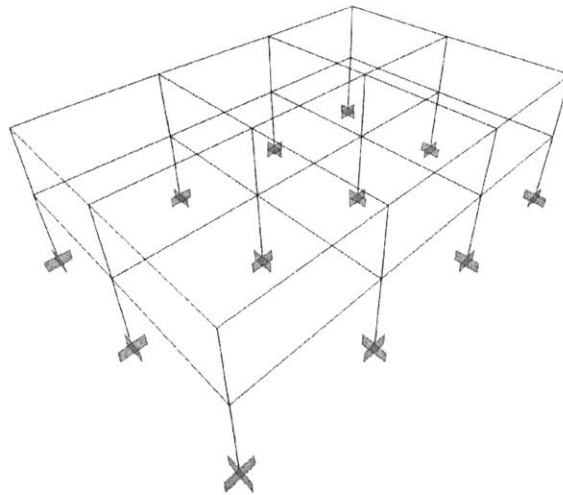


Figure 12: Geometry of SAP Model

The braced frame model is braced in the direction of wind loading, as shown in Figure 13. The braces are defined to be the same sections as the other members for simplicity. All of the connections, including beam-beam, beam-column, and beam-brace, are idealized pinned connection, meaning that there is no moment transfer. The model for the semi-rigid frame is the same as that of the rigid frame, except that all connections were assigned a partial fixity factor, which is a new feature of the newer versions of SAP 2000.

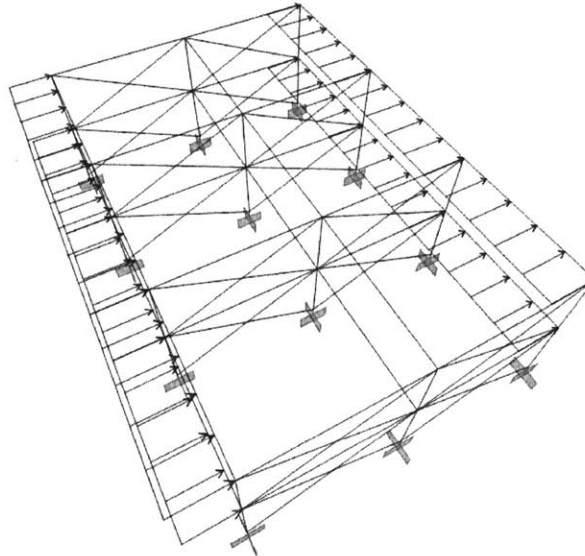


Figure 13: SAP Model of Braced Frame

The partial fixity factor for this frame was chosen to be 200,000 k-ft/radian, which yields a moment of approximately 83 k-ft. It is a reasonable middle point between a rigid connection and a pinned connection, and produces moments intermediate between the two.

With both gravity loads and lateral loads, a static analysis of the three frames was performed. The maximum values for moment, vertical deflection, and interstory drift were calculated and compared for the three frames.

	GRAVITY LOADS ONLY		GRAVITY AND LATERAL LOADS		
	Maximum Moment (k-ft)	Maximum Deflection (ft)	Maximum Moment (k-ft)	Maximum Deflection (ft)	Maximum Interstory Drift (ft)
Rigid Frame	70.5	0.00484	103.1	0.00756	0.0642
Semi-Rigid Frame	57.3	0.005963	89.86	0.008596	0.06783
Braced Frame	77.9	0.012621	77.9	0.012621	0.0006633

Table 1: SAP Results for Three Framing Types

These results show the effect that lateral loads have on the system. As predicted by a traditional understanding of braced frames, bracing elements substantially limit the maximum interstory drift. The braced frame exhibited an interstory drift approximately 100 times less than the interstory drift for the other frames. Braced frames are generally thought to be the most efficient structural type for simple frames, but architecturally they are not always feasible. This analysis also illustrates the difference between rigid and semi-rigid frames. With lateral loads, the semi-rigid frame showed a 13% decrease in moment and an approximately 13% increase in the beam deflections. The decision to use one frame over another is based on which aspect of design governs in each particular case. If the flexural strength of the beams governs the design, it might be beneficial to use the semi-rigid frame. The semi-rigid frame has lower design moments, and the savings in beam size for this smaller moment might be significant. On the other hand, if serviceability limit states govern the design, especially interstory drift, it might be more efficient to use the rigid frame, which has lower deflections.

For the purposes of comparison, the same analysis was run for the same building geometry, but with a total of 12 stories instead of 2 stories. The members, geometries, and loads were kept the same except for the additional stories added, as shown in Figure 14. Once again, the three frames were a braced frame, a rigid frame, and a semi-rigid frame with partial fixity factors of 200,000 k-ft/radian.

The results from this model were somewhat different from the results for the 2-story model. The result for the rigid frame and the semi-rigid frame became more similar at this higher building height. The moments for the rigid frame and the semi-rigid frame were three times greater than the values for the two-story building, whereas the moments for the braced frame were, as expected, essentially

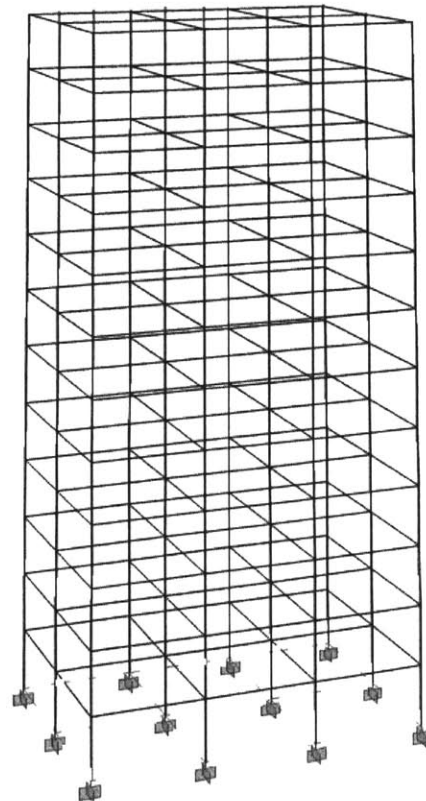


Figure 14: 12-story SAP model

the same as for the two-story building. Typically, semi-rigid frames are not used for buildings over a few stories unless some other type of lateral system is added, such as shear walls. In this scenario, it is possible that the combined system of shear wall and semi-rigid, rather than fully rigid, connections would result in savings for the structure.

3. Structural Connections

The previous section describes the modeling of connections and their role in frame design, but it is important to see also how these theoretical models, such as pinned connections and fixed connections, actually exist in the physical structure.

3.1 Pinned and Fixed Connections

Generally, connections between beams or from beams to columns which only connect to the webs of the beams are considered pinned connection. That is, since the shear force is carried in the beam web, only shear is assumed to be transmitted to the connected element if only the web is connected. One example of this is Figure 15. Using a single angle, as shown above, or a double angle, where the channel section is on both sides of the web, are the most typical

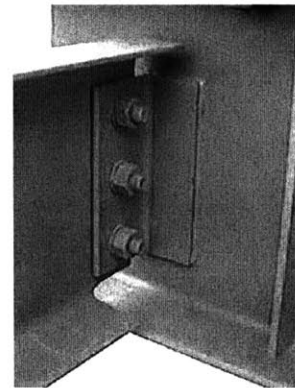


Figure 15: Example of Pinned Connection
Courtesy of ESDEP [10]



Figure 16: Example of Moment Connection
Courtesy of University of Wyoming [18]

methods of making a pinned connection. On the other hand, if the flanges of the beam are connected to the adjacent column or beam, then a moment connection is assumed. One example, where the beam web is bolted to the column and the beam flanges are welded to the column, can be seen in Figure 16. These idealizations are usually accurate, although there is usually some amount of moment transfer with pinned

connections which is neglected in design.

In order to achieve the idealized moment connection, the connection should be both rigid and full-strength, relative to the connected members. Attaining the full-strength requirement is fairly routine and easily met, but attaining the rigidity requirement can be more difficult. Rigidity implies that there is minimal flexibility. In order to achieve this, direct load paths through the connection are preferred; it is better to carry loads in axial tension or compression than in bending. [16]

Some of the ways in which moment connections are achieved are as follows:

End plate connections:

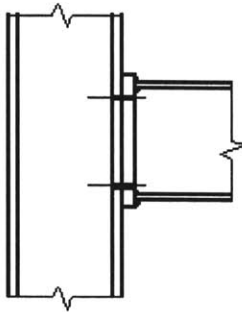


Figure 19: End Plate Moment Connection

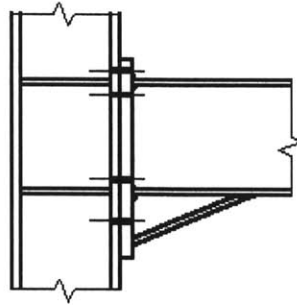


Figure 18: Haunched Moment Connection
Courtesy of ESDEP [10]

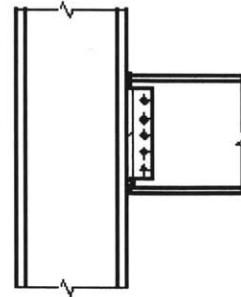


Figure 17: Hybrid Moment Connection
Courtesy of ESDEP [10]

The hybrid connection, where the beam web is bolted and the beam flanges are welded to the column is one of the most common in North America. [10]. Although moment

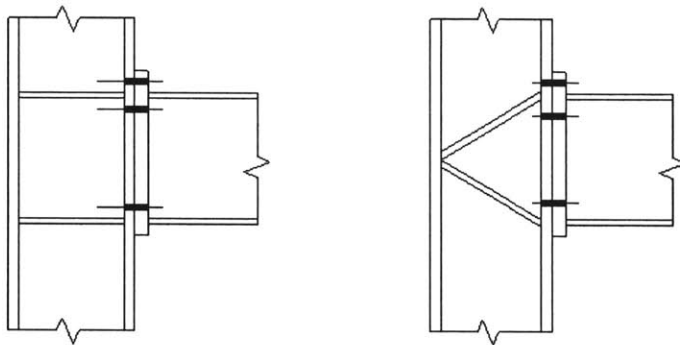


Figure 20: Examples of Stiffened Moment Connections
Courtesy of ESDEP [10]

connections are required to resist shear and sometimes axial forces, the moment is usually dominant for strength. Typically, moment is transferred by axial compression at the bottom flange and axial tension at the top flange. When the

strength in some region of the connection is insufficient, stiffeners can be added to strengthen the connection. Some examples of stiffened connections can be seen in Figure 20.

3.2 Semi-Rigid Connections

The physical joints for semi-rigid connections are more complicated to design than simple pinned or moment supports. If pinned joints are those which provide connection to the web of the beam only, and moment connections are those which connect to both the web and flanges of the beam, then semi-rigid connection fall

somewhere between the two. True semi-rigid connections provide a small amount of fixity between the beam flange and the column. However, it is often the case that idealized pinned connections actually have more rotational rigidity than expected; in this case the frame should perhaps have been modeled as a semi-rigid one. For example, joints which involve clip angles either welded or bolted from the column flange to the beam web are usually classified by AISC-ASD as pinned connections, but actually can develop up to 20% of the full fixed-end moment of the beam. Therefore, they provide more rigidity than the structural analysis assumed. Another type of connection, which provides more moment restraint than predicted by AISC-ASD, are the top and seat-angle connections, where an angle is connected from the column flange to the beam flange. According to AISC specifications, “The seat-angle transfers only vertical reaction and should not give significantly restraining moments on the end of the beam.” [7]. However, experiments show that this is not the case; the seat-angle does aid the connection in transferring moment between the beam and the column. The stiffest of semi-rigid connections is considered to be T-stub connections, where T-stubs are bolted to the beam and column at the top and bottom flanges of the beam. [7]

3.3 Welded Joints

There are two main types of welds: groove welds, which are often called butt welds, and fillet welds. Other types of welds including plug, slot, and spot welds are also used. For building structures in general, approximately 80% of the welds are fillet welds, 15% are butt welds, and an additional 5% are some other type of weld. [3] The assumptions underlying the structural design of welds are that welds are homogeneous and isotropic, residual stresses and stress concentrations in the welds are negligible, and the connected parts are held rigid with negligible deformations. This implies that there is a uniform stress distribution in the weld, which is not actually the case. Although this simplifying assumption is not completely accurate, the material ductility leads to a redistribution of stresses from residual stresses and stress concentrations, resulting in an overall stress reduction. Fillet welds are often preferred to butt welds because they require less equipment, fewer skills of the welder, and less preparation of the elements to be joined. Butt welds, in contrast, require more labor and preparation. There are also

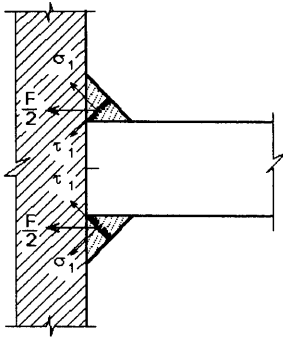


Figure 22: End Fillet Weld
Courtesy of ESDEP [10]

many types of welding processes, many of which are used in the shop. However, field welding usually employs metal arc welding, because it can be performed in a number of different positions and orientations.

The orientation of the welds relative to the connected elements can be important in determining the weld strength. Side fillet welds carry an axial force parallel to the weld length, as seen in Figure 21. In

contrast, end fillet welds transfer axial loads perpendicular to the weld length, as in Figure 22. An end fillet weld is generally stronger than a side fillet weld. The main reason for this is that the failure plane for an end fillet weld is larger, due to the geometry of the connection.

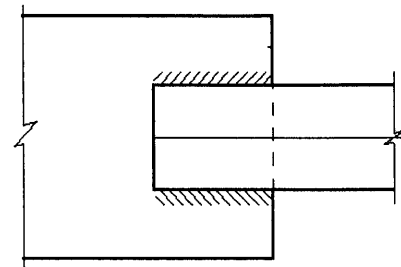


Figure 21: Side Fillet Weld
Courtesy of ESDEP [10]

3.4 Bolted Joints

The strength of a bolted connection depends on the resistance of the individual components of the connection. There are two main types of bolts: “ordinary” bolts, which are also called bearing bolts, and pre-tensioned or high-strength bolts, which are used in slip critical conditions. Ordinary bolts are cheaper and preferred when higher strength bolts are not needed, although in many design scenarios, it is common for engineers to specify high strength bolts.

There are a number of bolt sizes and strengths available. For a bearing bolt that is not pre-tensioned, the bearing strength depends on the shear resistance of the bolt and the bearing resistance of the plates. If the bearing resistance of the plates is less than the shear resistance of the bolts, then the connection has a larger joint rotation capacity. This is more likely to occur when the bolt hole is larger than necessary and when the joint is longer; to protect against this, the bolt hole diameter in the plate is usually limited to the bolt diameter plus one eighth of an inch. [16] In addition to non-pre-tensioned bolts, high strength friction grip (HSFG) bolts are also used. These are used when the stiffness and strength requirements are higher, and especially when the connection conditions are

slip-critical and subject to fatigue. The shear strength of the HSFG bolts depends on the coefficient of friction of the materials and the amount of pretension.

The primary function of bolts is to transfer forces from one plate to another. Bolts usually carry both axial tension or compression and transverse shear. Generally, axial forces through the bolt are preferred, because experimental results show that the shear strength is usually only about 60% of the tensile strength for ordinary bolts. The reason that the shear strength is reduced is due to secondary bending actions caused by lack of symmetry of loading, and by additional rotation of the bolt due to the larger hole diameter than the bolt diameter. Increasing the number of bolts in a joint, which lengthens the joint, can mitigate the reduced shear resistance. However, long joints have undesirable effects on shear resistance too. With long joints, the bolts at the ends of the joint fail first, before the other bolts have developed their full strength. For cases where the joint length is long, the shear strength of the fasteners is reduced by a certain factor to account for this. Especially at low moments, the bolt flexibility causes higher rotations at the connection than the moment would induce. In order to minimize the slippage of the bolt, it is preferable to subject the bolts to axial loads instead of shear whenever possible. It is also beneficial to place the bolts as closely as possible to the beam, provide additional stiffeners close to the bolts as needed, and maximize the moment arm of the connection, using a haunch if necessary.

In many cases, ordinary or bearing bolts are not permissible, and HSFG bolts are used. The pre-tensioning of the bolts, which must have an initial pre-tension of at least 70% of their ultimate tensile strength, makes the bolt exert a compressive force on the connected plates. This clamping pressure facilitates load transfer through friction forces. A connection which uses HSFG bolts is much stiffer than an ordinary bolt, although this is true only until slip occurs. Slip occurs when the applied loads are higher than what the plates and bolt interfaces can carry in friction, and the plates begin to move relative to each other. After this occurs, the HSFG bolt behaves as a bearing bolt. Until this point, though, HSFG bolts have very good properties under alternating forces, fatigue loads, and good stiffness properties. They are higher in cost than ordinary bearing bolts.

For pre-tensioned bolts, it can be difficult to determine and control the amount of pre-tension. This is an issue of installation. One method, called “turn-of-nut”

installation, involves prescribing an amount of torque past the point where the bolt is snug-tight. Typically, bolts are turned one half turn past the snug tight condition, but this may be hard to measure. One of the drawbacks of this method is that it is difficult to measure exactly how much pretension is in the bolt and it is difficult to inspect, although overall it is a reliable method. Another installation method, “calibrated wrench” installation, is based on the assumption that there is a relationship between torque and pre-tension. Although this is theoretically true, there is a very complex relation between the two due to the complex geometries, including thread pitch and angle, and variance in friction conditions. This complex relationship is accounted for by testing the connections first to determine how much tension is in the bolts. Then the wrench is calibrated to produce this amount of tension in the actual connection. Another solution to control the amount of pre-tension is to use tension control (TC) bolts. In this type of bolt, the end of the threaded part of the bolt has a torque control groove which can determine the amount of torque in the bolt, as seen in Figure 23. Once a pre-determined value of torque is reached, the end of the screw breaks off and the bolt is not tightened beyond this point. This greatly simplifies construction and inspection of bolts, and allows for a greater degree of certainty in the analysis of bolt behavior.

[15]

Other practices and types of bolts can be used in non-typical circumstances. One such example, when deformations and rotations are to be minimized, is the use of injection bolts. In this type of bolt, the space between the bolt and the walls of the bolt hole in the plate are filled with resin. A small hole in the head of the bolt is drilled so that the resin can be injected after the bolt is put in place. This procedure is used to repair riveted joints in

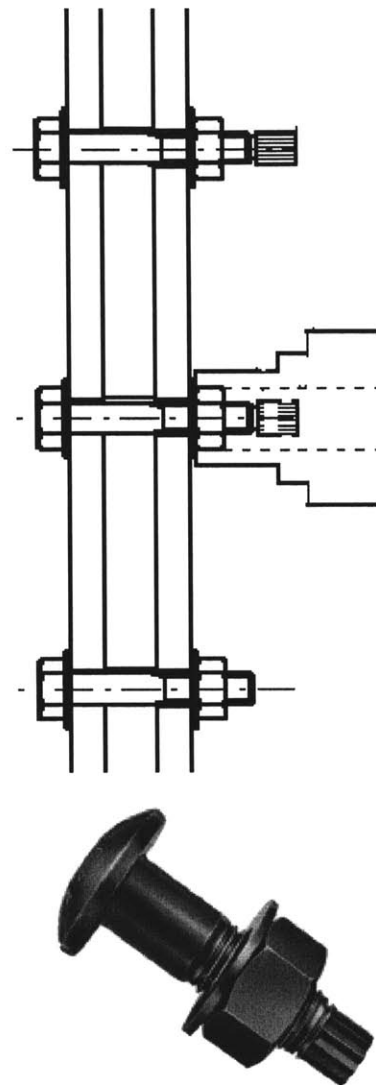


Figure 23: Tension Control Bolt
Courtesy of ESDEP [10]

maintenance of historic structures, as well as for situations with high loads, such as railroad bridges.

In addition to strength and deformation concerns, bolts should also be designed for proper corrosion protection. To protect the plates, zinc-coated, or galvanized, bolts may be used. The zinc coating adds friction to the bolt threads and provides protection to weathering corrosion. However, it can increase the possibility of stress corrosion and hydrogen embrittlement. This is a special concern with high strength bolts, which use a higher strength steel. Even a small amount of hydrogen, which is introduced and absorbed during the galvanizing process or during corrosion reactions, can cause stress-corrosion cracking, a type of failure which occurs under corrosion and high stress.

3.5 Failure of Connections

Some parts of a connection are often designed to yield first, to ensure that if yielding does occur, it happens in a controlled way. This is particularly important in semi-rigid connections, where the connections are more likely to be stressed beyond the yield point and undergo plastic deformations. Sudden yielding, as well as sudden failure, should be avoided in the design. The fasteners, which include welds and bolts in tension, are brittle, and their failure could be catastrophic. Therefore failure of the welds or bolts in tension should be avoided. In semi-rigid connections, all-welded connections, unless they are very standard and certain, are generally avoided. [14]

If the connection is subjected to fluctuating stresses, fatigue may be an issue for the connection. Fatigue is the mechanism of crack growth and propagation under these fluctuating stresses. Local stress concentrations, due to structural loads and geometric discontinuities, are the main cause of fatigue cracking; however most structural design considers only average stresses at the macroscopic level, ignoring the possible stress concentrations that may form at connections. In particular, welds, which have very small discontinuities in the weld metal, are susceptible to crack initiation and growth. To minimize the chance of fatigue cracking, the joint should be designed to have good fit-up, minimal discontinuities, and a smooth profile. In special circumstances where fatigue stresses are critical, this may involve grinding of the weld surface to achieve a smoother profile. This practice helps to remove the small cracks on the weld surface that facilitate

crack growth. Another practice to reduce the possibility of fatigue cracks is the use of peening, or repeating hammering. This causes local yielding of the steel, and is usually applied at the weld toe where cracks are likely to initiate. [4]

Furthermore, to reduce the risk of sudden failure, if necessary, one or more components of the connection are purposely designed as a “weak link”. This weak link is usually the end plate, which is the plate between the column and the beam as shown in Figure 24. One way this is achieved is by limiting the maximum plate thickness to 60% of the bolt diameter. [10] Although a thicker end plate could transmit more moment, this would increase the possibility that the bolts might fail before the plate, which would be a brittle and abrupt failure. This is a different approach from strength-based design, in which the end plate thickness is likely to be equal to the bolt diameter. In order to keep the end plate strong, it is preferable to use compact,

closely spaced bolts which are as close to the structural elements as possible. It is usually preferred to have ductile failure. The failure modes which are considered ductile are slippage of bolts in the flanges, yielding of gross area of plates, and yielding of the gross area of the girder. Failure modes which have some ductility are local buckling of the flange plates, local buckling of girder flanges. The brittle failure modes, which are undesirable, are block shear of flange splice plates, fracture of flange plate welds, shear fracture of web bolts, and fracture of the net area of the web splice plate or the girder.

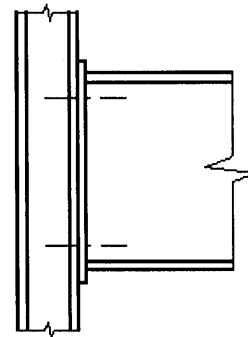


Figure 24: Typical Connection with End Plate
Courtesy of ESDEP [10]

3.6 Comparison and Choice of Joints

It is informative to study the different types of connections, but ultimately a choice must be made as to which type of connection to use in a structure. This decision is based on many factors, including the effectiveness of the connection, the ease of assembly, the availability of resources, and the economic factors. In general, under controlled conditions, it is easier and cheaper to make a welded connection than a bolted one. This means that for joints which are prefabricated in a shop, welded connections are usually preferred for typical and regular connections. However, on site this is not necessarily

true, because welding requires better access to the structure, which might not be easily obtained. Welding also requires more skill and licensing of the workers than bolting, as well as temporary bolts and cleats to hold members together. Also, in the field bolts may be preferred because there is less variability than welding, and because bolts are more cheaply and easily inspected. However, both bolts and welding are often used in many different types of structures, depending on the goals and constraints of the connection. [4]

3.7 Column Trees

One common type of framing often used in seismic zones is the special moment resisting frame. The difference between this type of frame and the ordinary moment resisting frame is based on the amount of ductility in the system. Special moment-resisting frames are designed to have a greater amount of ductility, which acts as a damping mechanism during earthquakes. Because of this, some codes allow for special moment-resisting frames to be designed for smaller seismic loads than ordinary moment frames. [1]

The Northridge earthquake in 1994 in particular, as well as the 1995 Kobe earthquake and others have resulted in a greater concern with field-welded moment frames. Moment frames are often preferred in seismic regions because of their ductility, but the many failures that occurred in these types of structures during the earthquakes led to a re-evaluation of field-welded moment connections. After these

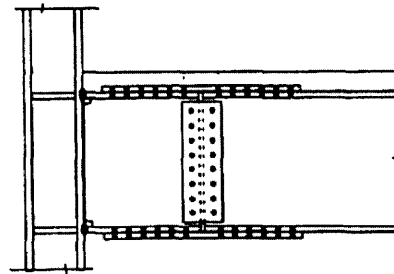


Figure 25: Girder Splice for Moment-Resisting Column Tree

earthquakes, emphasis was placed on designing ductile connections for seismic zones, and the minimum rotation to meet this ductility requirement was chosen to be at least .004 radians. [1] One of the methods of fabrication that structural engineers have used more often since the investigations of these earthquakes is connections that are shop-welded and field bolted. This allows for greater control of the welds in the shop, and greater certainty in the field, since bolts, especially

those with snap-off ends, are more easily inspected than welds in the field. One option for shop-welded, field-bolted connections is the column tree, where short sections of the girders are welded to the columns in the shop. The column trees are transported to the site, where they are erected. The middle spans of the girders are then field-bolted to the girder stubs which have been pre-welded to the columns in the shop. These connections are efficient because most of the complicated connection design is done in the shop, so field work is reduced and simplified. A typical field-bolted girder splice is shown in Figure 25.

The first column-tree systems were designed to be rigid frames, where the connections from the girders to the column trees were stronger than the design strength of the beams. However, it was proposed that making these connections semi-rigid would increase the seismic performance of the structure by increasing

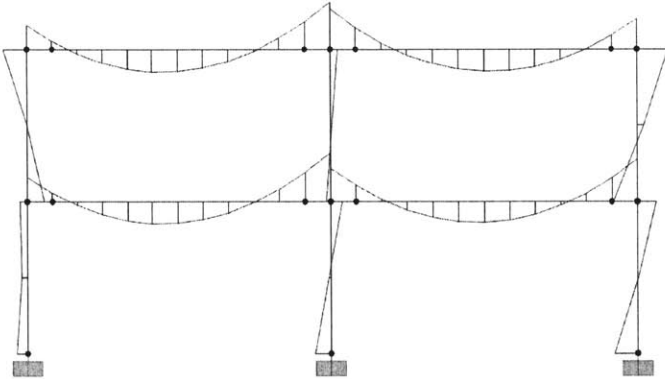


Figure 26: Moment Diagram for Rigid Frame with Column Trees

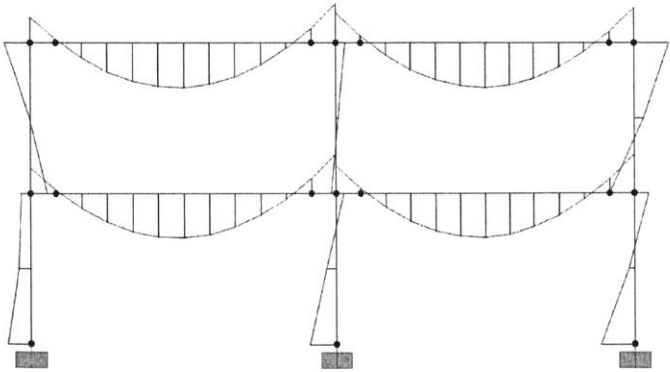


Figure 27: Moment Diagram for Semi-Rigid Frame with Column Trees

reduce the moments that are transmitted to the column tree, as in Figure 27. The girder splice where the main span of the girder meets the column tree limits the moments that is

damping, increasing the period of vibration, and reducing the stiffness in a way that would reduce the effect of seismic loads. [1] Also, the semi-rigid connection from the girder to the column tree can be used to

transferred to the column tree, thereby ensuring that failure does not occur at the beam-column connection. By designing the girder splices as bolted connections, the shop-welded, field-bolted benefit can be maintained. Additionally, for seismic design, the bolted connection allows for slip, which can be an important energy-dissipating mechanism during earthquakes. Slip should not be allowed to occur under ordinary service loads, but during extreme loadings such as earthquakes, it can be an important component of the structure's response.

4. Connections for Hollow Structural Sections

Hollow structural sections are in common use in structural engineering. They provide many advantages over traditional rolled sections such as W-shapes, including greater lateral-torsional stability buckling resistance for a given volume of material, and greater aesthetic appeal. The radius of gyration which is defined as $r = \sqrt{\frac{I}{A}}$ is usually higher for a hollow section than for an open section, such as a W-section about its weak axis. Therefore, hollow sections are useful in instances where buckling about an arbitrary axis is important to control. Both circular hollow sections (CHS) and rectangular hollow sections (RHS) are often used. For members subject primarily to uni-axial bending, rectangular sections with their long side in the plane of bending are most efficient. For bending about two planes, square sections are usually used, and for bending about many planes, circular hollow sections are chosen. Hollow sections also provide better corrosion protection because there are fewer corners and complex geometric configurations where corrosion is likely to occur.

One of the issues with this type of element is the difficulty in connecting members. With more conventional members, the surfaces are mostly planar and so bolted and welded joints are easier to accomplish. Furthermore, joints connecting hollow members often have several elements framing into one joint location, sometimes at irregular angles, as in a space truss. In contrast, conventional sections generally are more likely to form right angles. These irregular angles, which may not be constant through the structure, further complicate the issue of connections between hollow elements.

An instance in which hollow circular sections (CHS) were used is in the design proposal for the Boston Maritime Museum, which was part of the 1.562 class at MIT for the 2005-2006 year. In the Fort Point Channel area of Boston, this Maritime Museum was proposed as a floating structure with a diagrid exterior. In Section 5.2, the design methods presented are applied to the design of this proposed structure.

One association, the International Committee for the Development and Study of Tubular Construction (CIDECT), seeks to promote knowledge of steel hollow sections.

This association is led by the leading manufacturers of hollow sections and pipes, so it focuses more heavily on practical design. The perspective of the manufacturer, which CIDECT promotes, is important because of the rising concern for design for assembly. If design took manufacturing and assembly into account more often, structures would be more efficient. One of the publications which CIDECT has made available to design engineers is a design and analysis guide entitled “Hollow Sections in Structural Applications”, authored by Prof. J. Wardenier, which gives much attention to connections. [20]

4.1 Types of Connections

For hollow sections, there is a choice of several types of connections to use. These include connections with special prefabricated connectors, end plates to be bolted together, welds to plates, and welds directly to an adjacent structural element. The first three are considered indirect connections, meaning that the structural elements are connected by means of an additional member, whereas the last type is considered a direct connection. The last type is the simplest solution, especially for RHS, where the sides of the element into which the adjacent element frames are planar. For CHS, this type of connection is more problematic because the perimeter of the connected area has a highly nonlinear stiffness distribution, which complicates the design. The first type of connection, using prefabricated joints, is often preferred for space trusses where the elements frame into the joint at irregular angles, and is used more often for CHS. [10]

To begin with, the overall joint behavior is analyzed. Joints can generally be classified into two types: indirect connections, where the structural elements are joined together through plates, angles, or some other member, and direct connections, where structural elements are directly joined together. In general, direct connections are favored because there are fewer locations at which failure can occur. This also makes the design simpler.

It is also generally preferred to have no eccentricity in the joints. This means that the centerlines of the connected joints should form a point, as in Figure 28. However, for

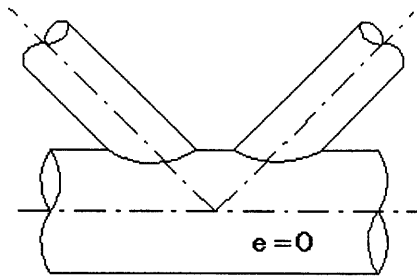


Figure 28: Non-Eccentric CHS Connection
Courtesy of ESDEP [10]

ease of fabrication or erection, it may be necessary to have some amount of eccentricity, as in Figure 29. If the eccentricity, e , is kept below one half of the outer diameter of a CHS or one half of the width of a RHS, the moments that result from the eccentric connection can be neglected.

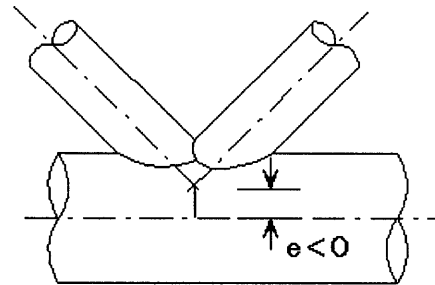


Figure 29: Eccentric CHS Connection
Courtesy of ESDEP [10]

4.2 Strength of Connection

The strength of the connection is taken to be the lowest strength for each of the connected parts, including the plate, the weld, and the different parts of the hollow section which are stressed. In general, yielding of the plate is unlikely because it is usually designed to have a higher stiffness than the section. However, if the hollow section is small with a large thickness, then yielding of the plate could occur. Another potential failure is the weld, but for almost all structures, the welds are designed to be of higher strength than the elements, because failure at the connections would generally be more catastrophic than failure in the middle of the elements. A third potential failure mode is lamellar tearing of the hollow element. This is a type of cracking which forms beneath the weld due to a lack of ductility in the thickness direction of the base metal, which in this case is the hollow section. The cause of lamellar tearing is high shrinkage stresses induced by the weld. In order to avoid for this, the hollow section should have adequate ductility, minimal internal defects, and a low sulfur content.

The strength of CHS sections is usually described in terms of possible failure modes. The failure modes which determine the limit states of circular hollow sections, there are six generally recognized types of failure. These are as follows:

- 1) Plastic failure of the side wall of the member, as shown in Figure 31.
- 2) Local buckling or instability due to bearing of

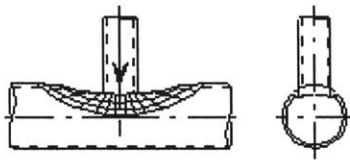


Figure 31: Plastic Failure of CHS Wall
Courtesy of ESDEP [10]

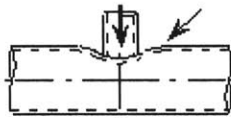


Figure 32: Local Buckling of CHS Wall
Courtesy of ESDEP [10]

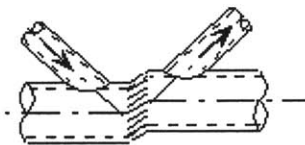


Figure 33: Shear Failure of CHS
Courtesy of ESDEP [10]

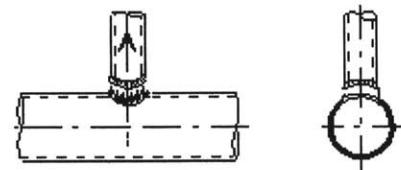


Figure 34: Punching Shear
Courtesy of ESDEP [10]

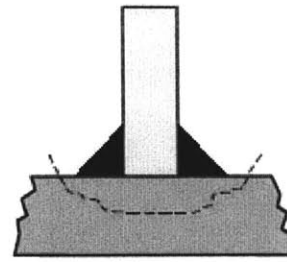


Figure 30: Lamellar Tearing
Courtesy of CIDECT [20]

the side wall of the member, as in Figure 32.

- 3) Shear failure of the member, as in Figure 33.
- 4) Punching shear, shown in Figure 34.

This type of failure can occur when the plate width, b_1 in Figure 35, is less than the width of the hollow section, b_0 . In this case, punching shear can be the governing failure mode.

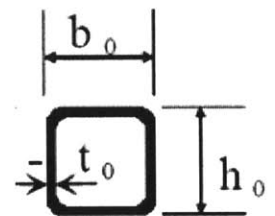


Figure 35: RHS Cross Section
Courtesy of CIDECT [20]

Two of the primary models used to determine the strength of CHS joints are the ring model and the punching shear model. The punching shear model is applied in Section 5.2 to a practical design problem.

4.2.1 Welded Joints

Welding of hollow sections is the most common method of making connections. The welding procedures for hollow sections are generally the same as for open sections. For CHS, fillet welds can be used to directly join two sections, if the ratio of the

diameters of the connected members is less than 1/3. [10]

RHS can also be joined by fillet welds, as shown in Figure 37. If the cross section of one RHS is to be connected to the side wall of another RHS, and there is insufficient width to make a fillet weld, the side walls should be beveled, or slightly cut on a diagonal, to ensure full penetration. This is also the case if the members are connected at angles less than 60° , as shown in Figure 36.

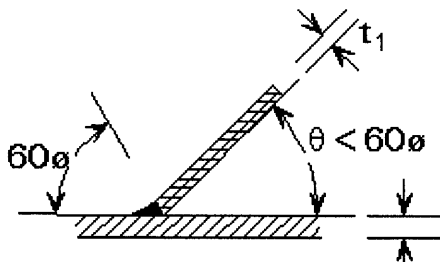


Figure 36: Beveled Edges for RHS Connection
Courtesy of ESDEP [10]

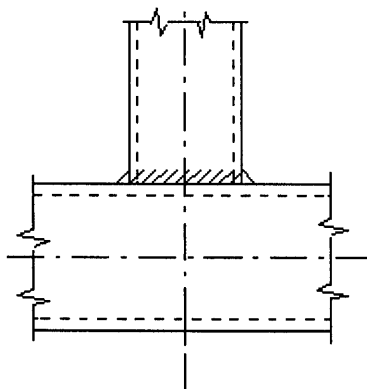


Figure 37: Direct Welded Connection for RHS
Courtesy of ESDEP [10]

4.2.2 Bolted Joints

In general, direct bolting between hollow sections is difficult. Because hollow sections are closed sections, the inside of the section is not accessible, except in cases where hand holes are specified or if the connection is made at the end of the member. However, these cases are rare and in general bolted connections directly from one member to another with the use of plates are not made. The typical use of bolts involves welding plates or angles to the hollow sections, and then bolting these plates together. This type of assembly is sometimes preferred to fully welded connections, because it allows the welding to occur in the shop and the bolting to occur in the field. This complies with the guideline that, whenever possible, it is preferable to shop-weld and field-bolt as much as possible. Some types of bolted connections are shown in Figures 38 and 39.

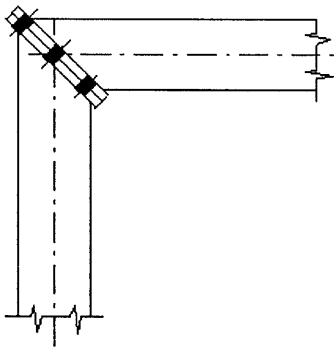


Figure 39: Bolted Knee Joint Connection
Courtesy of ESDEP [10]

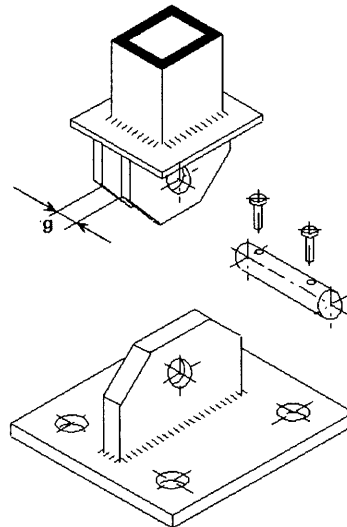


Figure 38: Pinned Connection Using Bolts for RHS
Courtesy of ESDEP [10]

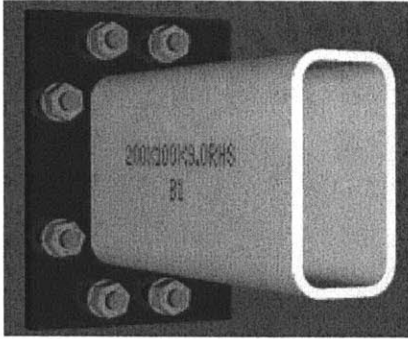


Figure 40: RHS Connection to End Plate
Courtesy of CIDECT [20]

For a typical bolted connection, the cross section of the hollow element is attached to an end plate, which acts as a flange of the section. This plate may be circular or rectangular, and solid or a ring. An example of a RHS welded to an end plate which is bolted to another member is shown in Figure 40. The possible failure

modes for this type of connection are plastification of the hollow section, rupture of the weld between the flange plate and the section, rupture of the bolts, or rupture of the flange plate. The first mode, plastification of the hollow section, can be avoided simply by choosing sufficient member dimensions and material properties. The second mode, rupture of the weld, can be avoided by correct welding design and procedure. The weld should have fillet welds as well as full or partial penetration groove welds. According to one design guide, the thickness of the fillet weld should be at least $.4 \times$ wall thickness of the hollow section for areas in compression, and at least the full wall thickness of the hollow section for areas in tension. [9] To avoid the third and fourth failure modes, the diameter of the plate, placing, number, and arrangement of bolts can be designed in a beneficial way.

Sometimes stiffening ribs are attached perpendicularly to the flange plates, which would allow the thickness of the flange plate to be reduced. The problem with this practice is that it requires extra fabrication, protection, and maintenance, and so it is preferred to use a thicker plate without stiffeners. It is preferable to minimize the distance between the outer edge of the section and the ring of bolts and to minimize the distance between the bolt axis and the edge of the flange plate. This means also that the flange plate should only be as wide and as tall as necessary. This is done in order to prevent the two flange plates which are bolted together from pulling apart in tension.

However, the distance between the bolts and the member must take into account the constructability issue of providing space for the socket wrench to tighten the bolts.

To summarize the findings of a number of tests, the general design issues for bolted connections are:

1. The number of bolts, evenly spaced, should be increased in the tensile zone of the connection.
2. It is preferable to use many small diameter bolts rather than few large diameter bolts.
3. The bolts should be positioned close to the hollow section
4. The thickness of the end flange plate should be at least 1.5 times the bolt diameter.

4.3 Connection Details for RHS

A typical joint for a RHS section is one in which the hollow section is connected to a plate perpendicular to its longitudinal axis. In this case, the plate carries the load through the weld, and to the RHS member. For the rectangular hollow section, the stiffness is higher at the corners

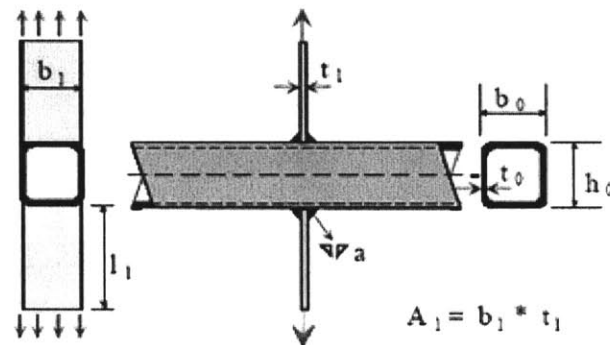


Figure 41: RHS Welded to Plate

of the cross section than in the middle of the sides, because at the corners, the load is transferred axially to the long dimension, h_0 , rather than by bending through the short dimension, t_0 . For this reason, connections between rectangular hollow sections and plates should attempt to apply stresses to the corners of the hollow sections rather than to the centers of the cross section. Because the wall thickness of the section is fairly small, it is not very stiff in bending.

One of the problems of this result is the difference in deformation between the RHS and the plate. The plate will be subjected to loads along its large surface, so it is

very stiff. The difference in deformation between the two could cause problems for compatibility relations, and this must be taken into account in joint design.

Because of the sensitivity of the section to loads not placed at the corners, circular hollow sections may be preferable to rectangular hollow sections in some applications. For space structures, prefabricated connectors, as in Figures 42 and 43, which the structural elements frame into are often preferred. Direct connection between structural members is sometimes used as the method of joining the elements, but since this gives rise to non-linear stiffness, sometimes connections with flange end plates are preferred.

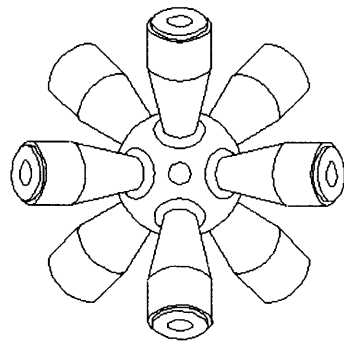


Figure 43: Okta-s Joint for Space Structures

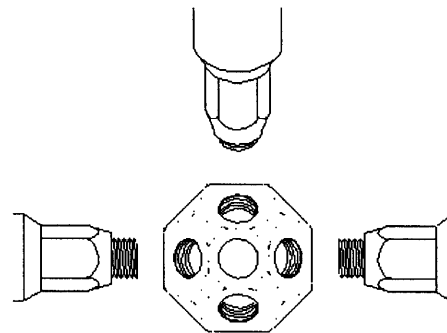


Figure 42: Mero Joint for Space Structures

5. Application of Study of Connections to MEng Course 1 Fort Point Channel Project

To see the connection between theoretical principles and design, some of the topics covered will be applied to actual projects. As part of the design project of the MEng program at MIT, the Old Northern Avenue bridge in the Fort Point Channel of Boston was examined.

5.1 Repair and Rehabilitation of Old Northern Avenue Bridge

This steel bridge was constructed in 1905 and is currently closed to vehicular traffic, as it was deemed unsafe for cars. The bridge now serves as a pedestrian walkway for commuters who park in South Boston and work in the financial district as well as for visitors and tourists to the many attractions in this area. It is an important historic bridge, and it is favored by pedestrians as an alternative to the noisier and less pedestrian-friendly new Northern Avenue bridge close to it.

After assessing locals' opinions about it, it was decided to keep and rehabilitate the bridge instead of tearing it down. This is in definite agreement with

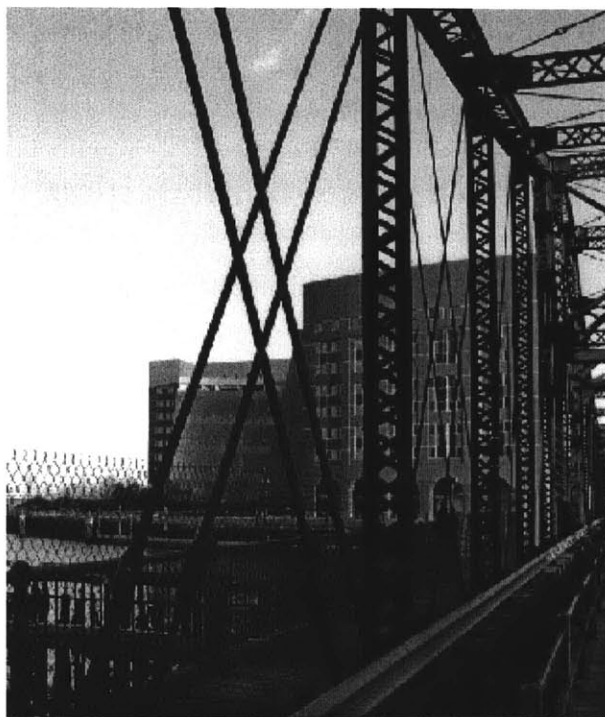


Figure 44: Photograph of Old Northern Avenue Bridge in Boston

Historic Boston Incorporated, which states that the bridge is a “rare surviving example of a steel-framed operable swing bridge”, “an important connector between downtown Boston and South Boston Seaport”, and the bridge has won “popular support for preservation”.

It is important to note that the design processes described here are only rough guidelines. A thorough analysis of the bridge has not been undertaken, so some

assumptions have to be made which may be inaccurate. However, this analysis could potentially be useful to other rehabilitation projects involving historic steel structures.

Among the many potential concerns with regard to the Old Northern Avenue Bridge, one might be its connections. The bridge's connections, as seen in Figure 45, are made through the use of rivets, which are not often used today. In renovating the bridge, it might be necessary to make welded connections to the existing material. In order to determine the risks involved with this process, it is important to take into account the effects of welding to old steels. Two of the materials contained in steel which are important to consider in its weldability are sulfur and phosphorus. Low percentages of phosphorus improve the machinability of steel. In structural steels manufactured today, the amount of phosphorus is limited to .05% of the volume of steel. Sulfur can also improve machinability, but the level is generally kept at .06% or below. A high sulfur content can cause weld porosity and brittleness, which is sometimes referred to as a "hot short" because it occurs more at high temperatures. For good weldability, the range of phosphorus and sulfur content which are acceptable are .03% and .035%, respectively. [12]



Figure 45: Connection on Old Northern Avenue Bridge

The first ASTM Specifications for structural steel were adopted in the U.S. in 1901. The two types were A9-Structural Steel for Buildings and A7-Structural Steel for Bridges. A7 was required to be made by open hearth process. For these two types of steel, the composition was defined as shown in Figure 46.

	Phosphorous	Sulphur
A9 Structural Steel		
Open-Hearth	0.06%	
Bessemer	0.10%	
A7 Structural Steel		
Acid Process	0.06%	0.05%
Basic Process	0.04%	0.05%

Figure 46: Steel Content per ASTM 1901 Specifications

From this figure it is clear that the contents are above the levels considered acceptable for weldability according to today's standards. This problem is compounded by the fact that an increase of 25% of the stated limits was permitted in the finished steel product. This was not a problem at the time, because welding was rarely used. [12]

In the Old Northern Avenue Bridge, it is possible that A7 steel was used, but engineers often do not have full knowledge of the composition of old structures. In order to determine the content and weldability of old steel, a new weld can be made to the old structure with a plate or angle, and then broken off. If the base metal, which is the old structure, pulls out with the new plate, this is an indication that preheat is needed before new welds are made to the old structure. Cracking would indicate that the existing structure is not suitable for welding. Another method to determine the content of the existing steel would be to obtain a small material sample to be tested at a materials testing lab. This would result in the determination of a CE (carbon equivalent), which measures the susceptibility of the weld to cracking. This is a commonly used parameter in welding of steels, and although it is called the carbon equivalent, it also takes into account the effect of other materials, including manganese, nickel, copper, and silicone. [12]

When riveted connections are strengthened, the welding increases the stiffness of the joint. This limits its ability to deform under thermal stresses, thereby creating additional stresses and possible load redistribution. These should be considered in the design. Another concern is the common tendency of both engineer and welder to oversize and over-design the weld, thinking that they are adopting a conservative

approach to design. However, this leads to a larger heat affected zone and an additional intermixing of the base metal and weld filler metal. This can cause deformations, as well as increase the possibility of cracks occurring in the heat affected zone. Furthermore, the geometry and fit-up of the connection are another factor in weldability. Due to rust, roughness, and general deterioration of the existing structure, proper fit-up with new support members may be difficult. The existing structure should have all coating, including rust, removed before welding occurs. The presence of rust in the weld can be dangerous, as it contains moisture which can help introduce hydrogen in the weld. [12]

5.2 Circular Hollow Section Connections for Maritime Museum Proposal

In addition to addressing the issue of the Old Northern Avenue Bridge, the Fort Point Channel Project

also proposed the design for a maritime museum.

This museum consists of three sail-shaped structures. Each has CHS members making up the exterior of the structure.

These sections are

inclined horizontally and vertically, creating a doubly curved surface which resembles a

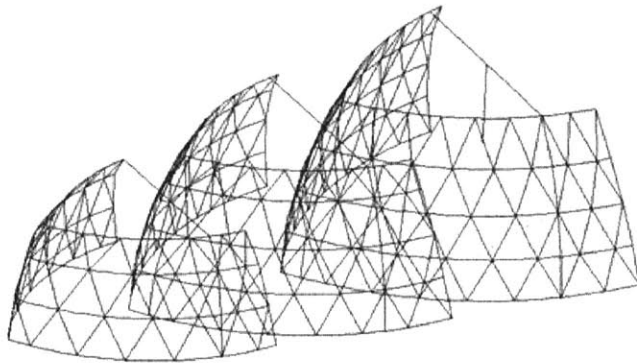


Figure 48: Diagrid Structure for Boston Maritime Museum Proposal

The typical CHS sections used in this design are 10" in outer diameter with a .5" thickness. One option for connecting these CHS section is to weld them together directly. This is a physically simple option, and one that is often preferred if possible. Based on the geometry of the model, two K-type joints would have to be

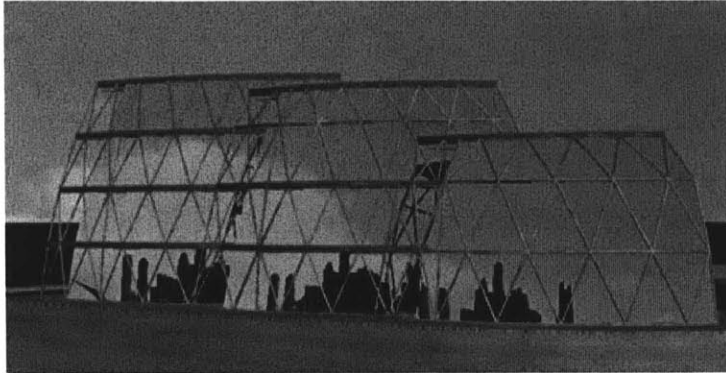


Figure 47: Rendering of Boston Maritime Museum Proposal

sail. The CHS sections were chosen for their ability to create an architecturally aesthetic structure. A rendering of this design is shown in Figure 47. The CHS members are oriented as shown in Figure 48.

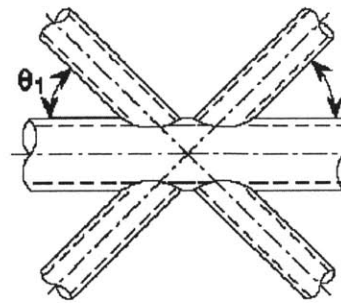


Figure 49: Directly Welded Connection

used. The K-joint is named for its resemblance to the letter “K” and has the geometry as shown in Figure 49. The punching shear model can be used to model the connection of directly welded CHS members. This model for failure criterion assumes that the

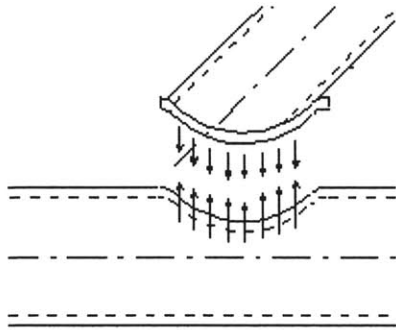


Figure 50: Punching Shear Model for Directly Welded CHS Connection

punching shear stress is uniformly distributed over the area where the members are connected, and the method of failure is shown in Figure 50. The strength of a connection, based on the punching shear model, is

$$N_1 = .58 * f_y * \pi * d_0 * t_0 * \frac{1 + \sin(\theta_1)}{2(\sin(\theta_2))^2}$$

where f_y is the yield strength of the material, d_0 is the diameter of the CHS section, t_0 is the thickness of the CHS

section, and θ_1 is the acute angle between the connected members. [9]

This formula, applied to the member in the proposed design for the maritime museum, results in a design strength for the connection of

$$N_1 = .58 * 50 * \pi * 10 * .5 * \frac{1 + \sin 30^\circ}{2(\sin 30^\circ)^2} = 1366.5 \text{ kips}$$

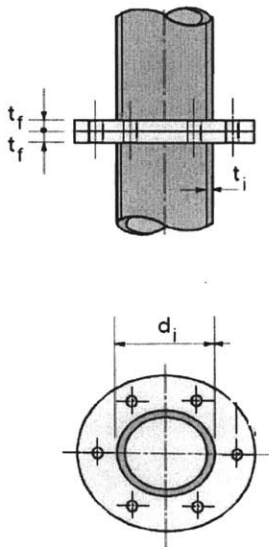


Figure 51: Bolted Flange Plates for CHS Connection

Alternatively, indirectly joined connections could be used, by attaching flange plates to the CHS members, as in the design in Figure 51. It is preferable to minimize the distance between the outer edge of the section and the ring of bolts, d , and also to minimize the distance between the bolt axis and the edge of the flange plate, b . This means also that the flange plate should only be as wide and as tall as necessary. However, the distance a must take into account the constructability issue of providing space for the socket wrench to tighten the bolts. This minimum distance, d , is suggested to be

$$a = \sqrt{2} * a + \frac{D}{2} - t_u \quad [9]$$

Where a is the weld thickness, D is the outer diameter of the socket wrench, and t_u is the thickness of the plate between the bolt head and the flange plate.

The required minimum flange plate thickness, is

$$t_f = \sqrt{\frac{k * T}{f_{y,p}}} \quad [9]$$

Where $f_{y,p}$ is the yield stress of the plate material, k is a coefficient to be read from a table presented in the design guide, and T is the axial tensile force. To determine the coefficient, k , the ratio of the diameter of the ring of bolts to the diameter of the CHS member must be decided. It is preferable to minimize this ratio, so the value of 1.2 is chosen. The k value in this equation for a diameter ratio of 1.2 is approximately .18. [9] So for the proposed maritime museum typical section, with an applied factored axial load of 1000 kips based on the SAP2000 model results,

$$t_f = \sqrt{\frac{k * T}{f_{y,p}}} = \sqrt{\frac{.18 * 1000}{50}} = 1.8 \text{ inches}$$

So the flange plates should be chosen to be thicker than 1.8 inches.

The number of bolts n to be used in the connection is given by

$$n = \frac{T}{T_u} * \left[1 - \frac{1}{f_3} + \frac{1}{f_3 * \ln(r_1 / r_2)} \right] * PSF \quad [9]$$

Where T is the axial tensile force, T_u is the ultimate design tensile strength of a bolt, r_1 is the distance from the center of the section to the edge of the plate, r_2 is the distance from the center of the section to the center of the bolt, PSF is a partial safety factor, and f_3 is an empirical factor to be read from a design table 9]. A typical value for f_3 is 5, and a typical value for r_1/r_2 is 1.5. The partial safety factor typical value is taken to be 1.25. The strength of a typical A325 bolt, from LRFD design, with a clear distance between bolts of L_c of 2 inches, is

$$R_n = \phi * 1.2 * L_c * t * F_u = .75 * 1.2 * 2 * 2 * 65 = 234 \text{ kips} \quad [16]$$

For the proposed design of the maritime museum, the number of bolts should be

$$n = \frac{1000}{234} * \left[1 - \frac{1}{5} + \frac{1}{5 * \ln(1.5)} \right] * 1.25 = 6.9$$

Therefore a minimum of 7 bolts are need in a ring for the flange plate connection of the CHS members.

6. References

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