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THE PHYSICAL PROCESSES ACCOMPANYING DETONATION IN THE INTERNAL COMBUSTION ENGINE

by

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FOREWORD

The study of detonation described in the present report is a continuation of work started in 1927 as the result of a discussion of the factors determining the sound of detonation. In the course of this discussion Professor E. S. Taylor suggested that an investigation of the sound from various engines might lead to useful results and proposed the start of such a project. The work has been carried out in the Internal Combustion Engine Laboratory at the Institute with the encouragement and general supervision of Professors C. F. Taylor and E. S. Taylor. In the course of the work up until 1933 several instruments were developed and a number of articles were published in addition to the writer's thesis for the degree of Master of Science. The present subject was chosen by the writer for his Doctor's Thesis with the advice of Professor P. M. Morse. Throughout the course of a long and often discouraging investigation Professor Morse has supplied inspiration and help especially in connection with theoretical aspects of the problem.

Mr. J. H. Lancor and Mr. Russell Fanning have given assistance and valuable suggestions on the design and construction of the amplifier-oscillograph system. Mr. Edward Gugger has been particularly helpful throughout the

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BIOGRAPHY

Charles Stark Draper was born October 2, 1901 at Windsor, Missouri. He attended the Windsor Grade School and the Windsor High School, graduating in 1917. After two years study at the University of Missouri he transferred to Stanford University and graduated with an A.B. in Psychology in June, 1922. Draper entered M.I.T. in the fall of 1922 and received his S.B. in electrochemical engineering in June, 1926. After working for one year in a commercial laboratory, he returned to M.I.T. as Sloan Automotive Fellow and received a master's degree without specification in 1928. Draper held the Crane Automotive Fellowship for the 1928-29 school year, acted as research assistant in aeronautical engineering from 1929 until 1931, and was research associate in the Internal Combustion Engine Laboratory from 1932 until 1935. In 1935 he was appointed assistant professor of aeronautical engineering to teach subjects related to aeronautical instruments and served in this position until his appointment as associate professor of aeronautical engineering starting with the fall term of 1938.

ABSTRACT

Detonation or "knocking" in the internal combustion engine is considered as a problem in engineering and as a problem in physics. Information collected from a general review of the pertinent literature is correlated to show that the general nature of detonation is known but the details of the chemical and physical processes involved are still obscure. It is further shown that in the present state of the art, detonation is an important factor in limiting the output and efficiency of internal combustion engines and for this reason forms a major problem in engine design and fuel production.

The present report describes the development of apparatus and an analytical method for studying the physical aspects of detonation by means of pressure variations within the combustion chamber. Sample records from two engines are included and a quantitative analysis is carried out for one case as an illustration of the general method.

The extensive literature of gaseous combustion is reviewed with special reference to the mode of flame propagation known as the "detonation wave." Two general regimes of flame travel in a combustible gas are distinguished: (1) <u>Normal Combustion</u>, in which the reaction proceeds by means of heat transfer and mechanical trans-

port of burning particles into the active medium ahead of the flame front, resulting in flame speeds of a few meters per second. (2) Detonation, in which combustion proceeds so rapidly that the reaction is completed before any substantial amount of energy can leave the reaction zone either by mechanical expansion or by heat transfer processes so that high local pressures are produced and flame travels through the medium with a velocity which may be several times greater than the velocity of sound in the unburned gases.

Internal combustion engines ordinarily operate with the normal type of combustion in the cylinder charge. However, with commercial fuels, as the engine output is increased either by raising the compression ratio or by supercharging, the "detonating" type of combustion appears. When this occurs, some part of the charge burns so rapidly that a localized region of high pressure is produced. The mechanical energy associated with this region immediately spreads over the combustion chamber volume in the form of pressure waves. The frequency of these waves is determined by the size and shape of the combustion chamber and by the velocity of sound in the enclosed medium. Vibrations are set up in the combustion chamber walls both by the initial excitation and by the pressure waves which follow. These vibrations of the combustion chamber walls produce sound waves in the free air near the engine which are heard as the most prominent immediate evidence that an engine is detonating.

Detonation has the effect of producing an abnormally high temperature in some local region of the charge. This high temperature is associated with relatively high density and particle velocity and often results in overheating near the region involved in the initial pressure rise. In addition to this effect, the gas motion due to intense pressure waves tends to scrub away the layer of stagnant gas which normally protects the walls from excessive heat transfer and may cause pre-ignition due to high temperature of poorly cooled parts such as spark plug points. Engines with aluminum alloy surfaces exposed inside the combustion chamber may be mechanically damaged after a certain period by the combination of high temperatures and high pressures which accompany detonation.

Methods of measuring detonation as described in the literature are usually based on an arbitrary procedure carried out with a specified instrument and a particular engine. The results are usually reported in terms of the composition of the mixture of standard fuels required to produce a detonation intensity identical with that observed in a given case. The standard mixtures are composed of octane and heptane, the results being reported in terms of the "octane number" which is identical with the percentage of octane in the standard mixture. The "Midgley Bouncing Pin" which is used for establishing detonation intensities is satisfactory only for laboratory work so various other methods, usually based on temperature measurements or estimates of sound intensity, are employed in work on full scale engines. No attempt is made in any case to systematically relate the results of such detonation studies to events inside the combustion chamber or to determine the physical magnitudes involved.

In the present report an analytical method is developed for reconstructing the general pressure changes due to detonation from records made at two or more points on the combustion chamber walls. This method consists in finding a mathematical expression for the possible pressure vibrations by solving the wave equation for sound with proper boundary conditions to fit the given combustion chamber, and then determining the proper coefficients in this equation to account for the observed pressure variations. By this process the size, shape and location of the exciting pressure rise can be found by placing time equal to zero in the equation. Results from this procedure will not be exact due to the failure of large pressure variations to fulfill the assumptions of the theory of sound and the difficulty of exactly accounting for all the modes of vibration present in practice. However, it is certain that the method as described supplies a more powerful tool for studying detonation than any of the schemes now in common use.

Apparatus for measuring rapid pressure changes is discussed in some detail and an electromagnetic rate of change of pressure indicator suitable for detonation studies is described. The results of calibrations carried out under actual operating conditions are given and a number of records made in detonating engines are shown.

As an example of the suggested procedure for studying detonation, the case of an engine with a flat topped piston and flat cylinder head is considered. The various modes of free vibration in the gas enclosed by such a chamber are found by solving the wave equation in cylindrical coordinates. The corresponding wavelength data are expressed in terms of plots suitable for engineering use and certain factors useful in calculating energy densities associated with the wave motion are tabulated.

Experimental records made with the recording apparatus on a CFR engine are discussed from the standpoint of the theory. The results are in general agreement with the analytical predictions although the records are too complicated for exact analysis. The energy associated with the pressure waves set up by detonation is calculated for a typical case. It is shown that this energy could be liberated by approximately one per cent of the total charge. Particle velocities and displacements are calculated with reasonable results.

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SECTION I

DETONATION AS AN ENGINE AND FUEL PROBLEM

WHAT IS DETONATION?

Origin of the term "Detonation."

Detonation in the internal combustion engine is a phenomenon which occurs in the cylinder charge under certain operating conditions. The most prominent immediate evidence of detonation is the presence of a ringing sound. This characteristic sound has lead to the names of "knocking" or "knock," "pinging," and "pinking."(1,2,3,4) These terms occur in the literature without definite distinction although recently there has been an attempt to classify detonation into types on the basis of the externally audible noise.(4,5) The application of the term "detonation" to a cylinder process of the internal combustion engine is based on results from flame studies first carried out by Bertholet and Vieille in 1881.(6,7,8) The results of these investigators and subsequent researches along the same lines by Mallard and Le Chatelier,(9) and H. B. Dixon(10,11) showed that flame is propagated through an imflammable mixture by two distinctly different processes: 1) ordinary combustion with relatively low velocity and

2) "l'Onde Explosive" or "detonation" with a constant velocity often considerably in excess of sound velocity in the unburned mixture. The possible occurrence of "explosive waves" inside the cylinder of an internal combustion engine was discussed by the physical chemist, W. Nernst, in a lecture before the forty-sixth general meeting of the Vereines deutscher Ingenieure in 1905.(12) Nernst recognized that the general results obtainable by applications of the first and second laws of thermodynamics would have to be supplemented by detailed studies of flame propagation process and said in part: "--- 4) Die Fortpflanzung der Entzündung in einem explosiven Gasgemisch erfolgt teils durch Wärmeleitung auf dem Wege der langsamen Verbrennung, teils rein hydrodynamisch durch Selbstzündung infolge der Fortpflanzung der Druckes sehr kräftiger Kompressionswellen (Berthelots Explosionswelle). Den Mechanismus der Fortpflanzung beider Arten von Entzündung kann man wohl als im wesentlichen klargestellt ansehen."

"5) In hinreichend rascher Verbrennung fähigen Gemischen geht die langsame Verbrennung von selbst nach Durchlaufen eines mehr oder minder langen Weges in die Explosionswelle über; die Ausbildung der Explosionswelle kann durch Reflexion von Kompressionswellen, Auftreffen der langsamen Verbrennung auf Hindernisse and dergl. beschleunigt werden; die weitere Klarstellung dieser Frage und auch der sweiten nach der Selbstenzündlichkeit von Gasgemischen infolge der Kompression, welche eng mit der ersten zusammenhängt, bedarf aber jedenfalls dringend noch weiterer experimenteller Untersuchungen."

Nernst also recognized that interaction effects between pressure wave reflections and combustion would exist inside a closed chamber such as an engine cylinder and outlined the essentials of the process by which standing waves would be set up. Lacking definite experimental data, he cautioned designers against destructive mechanical effects due to the pressure waves, emphasized the necessity for further research.

"Theories" of Detonation

Approximately fifteen years elapsed after Nernst's speech in Germany before the problem of "detonation" became definite enough for engine designers to realize that in "spark knock," as the phenomenon was generally called, they were dealing with one of the most important factors limiting the efficiency and power of internal combustion engines. Increasing the compression ratio enabled an engine to derive more useful work from a particular fuel but in the case of commercial fuels, detonation always intervened as some high limit was reached and forced a compromise between reliable operation and economy.(1) Many hypotheses were advanced to explain the observed facts of detonation. The writer, in work for his Master's Thesis in 1928, found eighteen more or less distinct "theories" on the subject.(13) Many investigators with different backgrounds of knowledge had a hand in the matter and produced ideas varying from a simple statement that actual impact between metal parts was responsible for the observed noise(14) to complicated explanations for the chemical processes involved.(15,16,17,18) Many of the "theories" dealing with chemical aspects of the problem had been advanced before 1925 and are briefly summarized by Clark and Thee in a short article.(19) In most cases the statements were vague and there was considerable confusion between the physical and the chemical aspects of the problem. Much discussion and the accumulated evidence of many investigations gradually clarified the situation until the quotation given below from D. R. Pye's "The Internal Combustion Engine"(1) is a fair summary of the opinions existing at the present time:

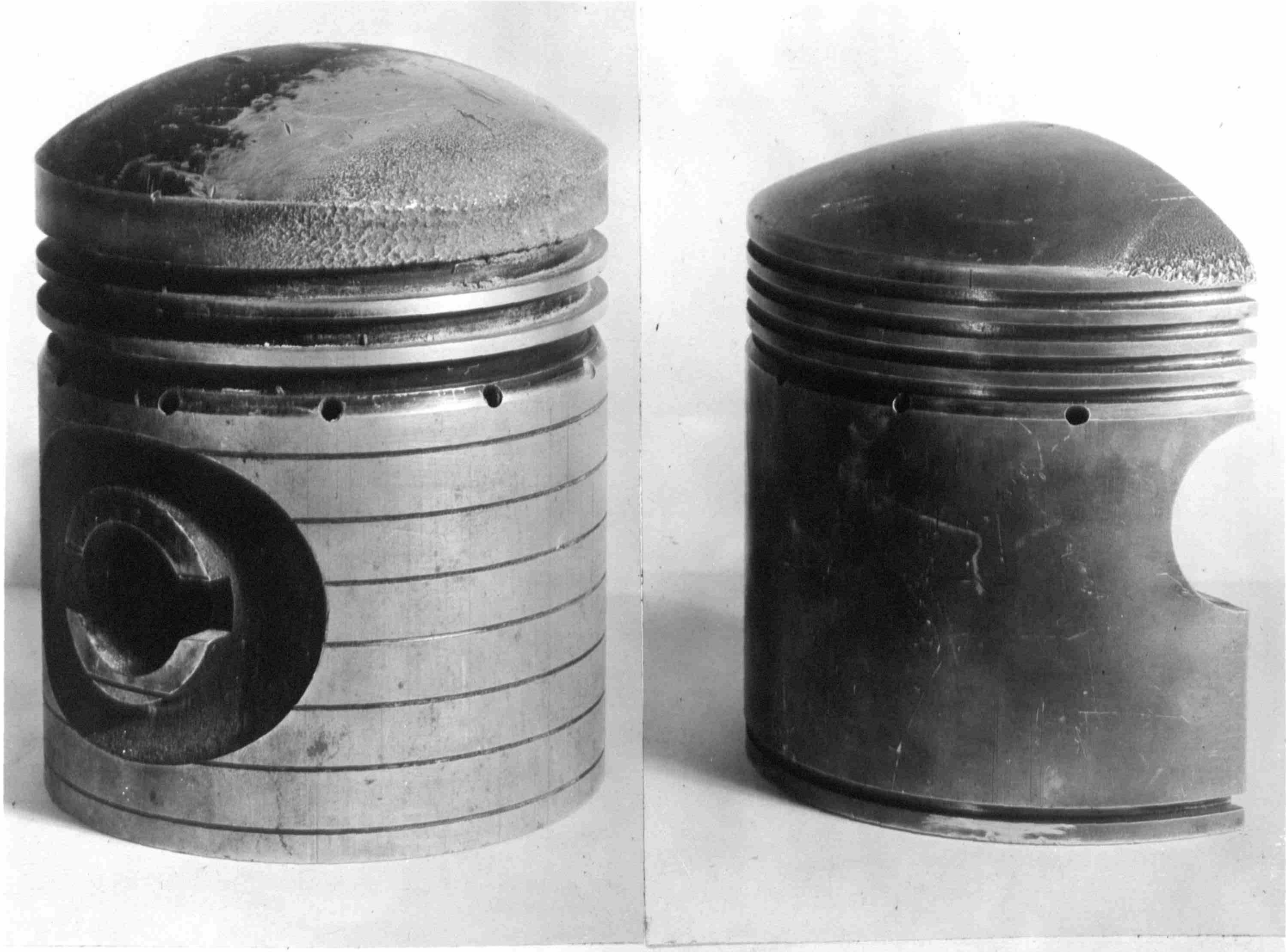
"The noise which gives the name of 'knocking' to detonation is like that produced by a sharp ringing blow upon the metal of the engine cylinder. Formerly

It was supposed to be actually due to some such mechanical cause, which might be derived, it was supposed from a looseness between moving parts, Experiment soon proved this supposition to be untenable, and in place of a mechanical blow between two solid parts of the engine, we now imagine the noise to be caused by a blow delivered against the cylinder wall by a wave of high pressure travelling at great speed through the gas which forms the working substance. If it is difficult to imagine a noise being set up in this way, it should be remembered that the noise of a hammer blow is due to the vibration set up by the sudden and <u>local</u> high pressure produced under the hammer; and the fact that sharp <u>local</u> high pressures are in fact produced in the gaseous contents of a cylinder when detonation occurs has often been demonstrated.

"In an engine cylinder, combustion is initiated during normal working by an electric spark timed to occur at a definite instant toward the end of the compression stroke. The correct timing varies according to engine speed and other factors, but ignition should never start before the spark has passed. If it does do so, on account of overheated sparking plug points, or scraps of stray incandescent carbon, then pre-ignition is occurring: a state of affairs which never persists for long without violent thumping, and excessive pressures being developed before the end of the compression stroke which very soon bring the engine to a standstill. Detonation on the other hand, is something which follows after ignition has been started by the spark in the normal manner, and if it is not violent it may persist for long periods without adversely affecting the running of the engine. If the detonation becomes violent, however, it is very apt to lead to conditions in which the sparking plug points become overheated, and so to prepare the way for pre-ignition to occur. The two phenomena, however, are totally distinct from one another. Detonation follows, while pre-ignition, as its name implies, precedes the spark."

"So long as no detonation takes place, the flame started by the spark spreads steadily throughout the combustion space at a speed which depends mainly upon gas turbulence, as well as upon the pressure and temperature before ignition. Under certain conditions, which depend upon the chemical nature of the fuel, the rate of spread of combustion becomes accelerated and a 'detonation wave' is set up, in the front of which the gas pressure may rise locally to a figure far in excess of the average pressure in the cylinder. Pistons have actually been broken from time to time by the hammer blow which this detonation wave is capable of delivering, and from its less violent manifestations is derived the characteristic 'knock' when the wave front reaches the cylinder wall."

Pye's summary of the detonation process is clear that a <u>local</u> pressure rise always exists although there is still some vagueness both on the physical and chemical sides of the problem. No attempt is made to explain the details of the chemical reaction, while the idea of a noise due to an "impact wave" alone(without mention of the standing waves produced when the initially localized energy spreads over the cylinder volume) overlooks a point already mentioned in the analysis of Nernst. One aspect of the problem is rightly given prominence by Pye, i.e. that normal operation will continue indefinitely if the detonation is sufficiently mild while violent detonation will often induce pre-ignition due to the overheating of some part inside the combustion chamber. This pre-ignition will either result in loss of power or actual stoppage of the engine due to the release of combustion energy too early in the cycle. Damage to engine parts due to purely mechanical causes is given a very minor part



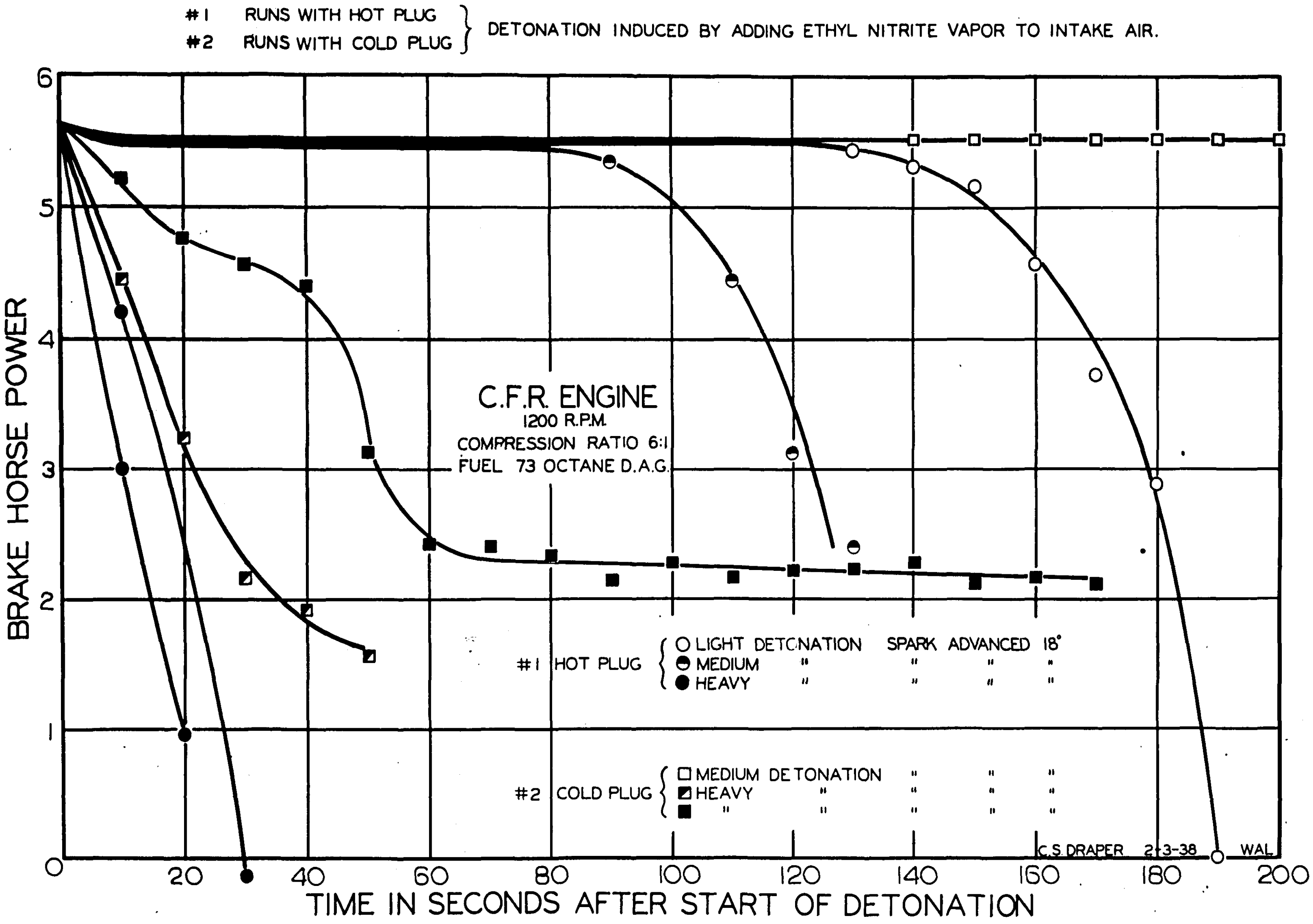
EFFECT OF DETONATION ON ALUMINUM ALLOY PISTONS

PLATE I

in the discussion.

Physical Damage due to Detonation

On the basis of a personal experience with engines extending over the past ten years and many discussions with engine manufacturers, the writer believes that failures due directly to abnormal pressures accompanying detonation seldom occur in practice. For example, a C. F. R. Engine in the internal combustion engine laboratory at M. I. T. has been run intermittently with extremely violent detonation over a period of years without any sign of damage due to the abnormal conditions. This is common experience with well designed engines using cast iron pistons. On the other hand, aluminum alloy pistons taken from engines which have been subjected to detonation often show eroded areas at the probable locations of the last parts of the charge to burn. Plate 1 shows this effect on two pistons taken from aircraft engines. It has been found possible to duplicate the observed erosion by application of intense local heating from a welding torch flame to the piston material, so it is not unreasonable to suppose that the high local temperature produced by the detonating part of the charge is mainly responsible for the mechanical damage due to detonation in a properly designed engine.

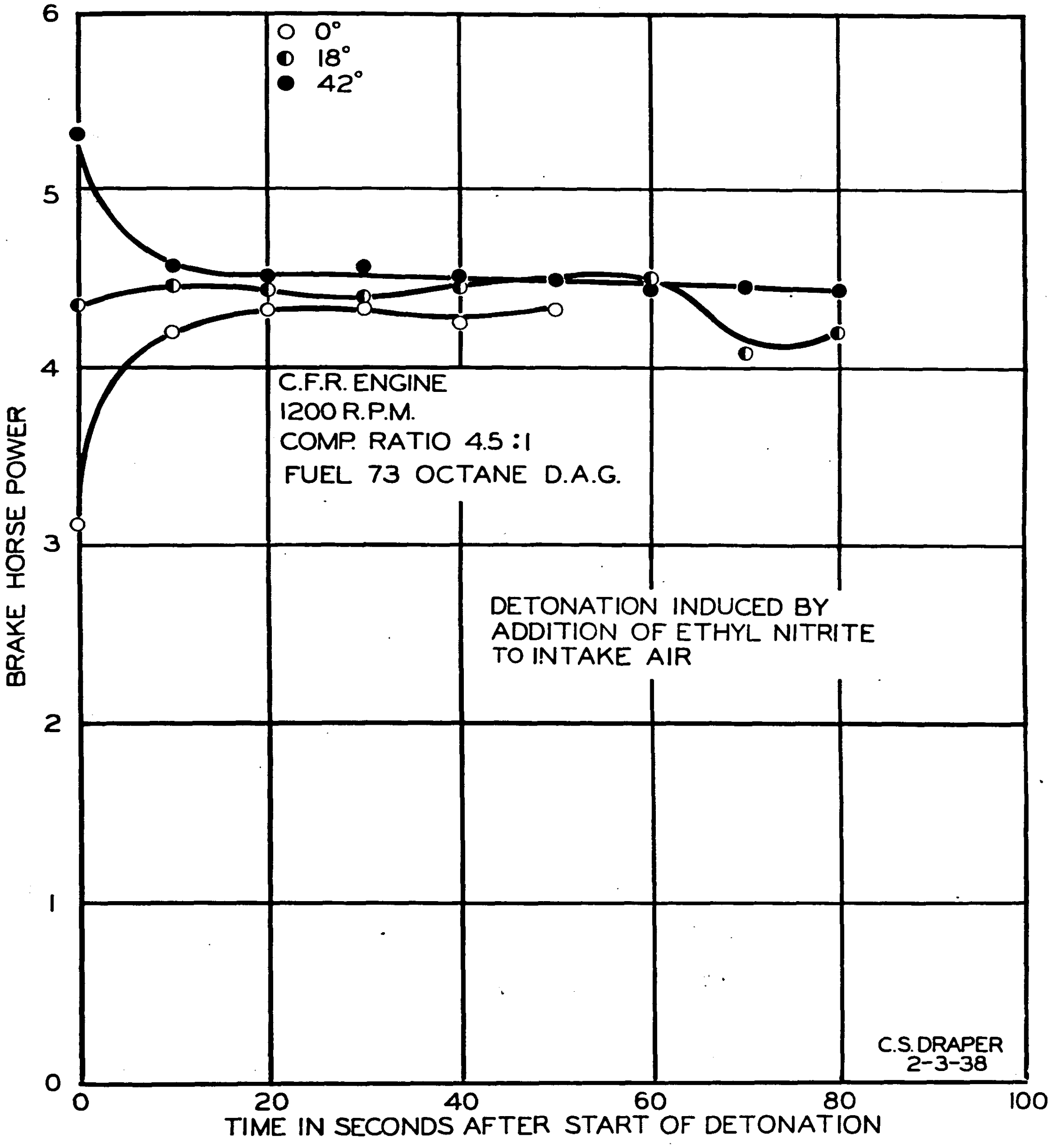


EFFECT OF DETONATION ON ENGINE OPERATION

FIG. 1

constant as judged by ear and the power output of the engine was measured at ten second intervals until the engine either stopped or a stable condition was reached. Light, medium and heavy detonation intensities were used.

The curves of figure 1 summarize the results of the experiments described above. With the automobile spark plug, the start of detonation in all cases was followed in a short time by complete failure in output of the engine. Under heavy detonation the engine stopped within thirty seconds while with mild detonation engine operation continued for over two minutes before a serious power decrease appeared. When an aircraft engine spark plug was used, mild detonation produced no measurable decrease in output while heavy detonation soon reduced the power to less than half the original value. Frequent tests for pre-ignition were made by opening the ignition switch. If the engine continued to run without the spark, it was certain that pre-ignition was present. It was found in every case that pre-ignition appeared whenever a power decrease was observed. These results conform to the statements made by Pye that an engine can operate normally with detonation present and that pre-ignition is a prominent factor in the



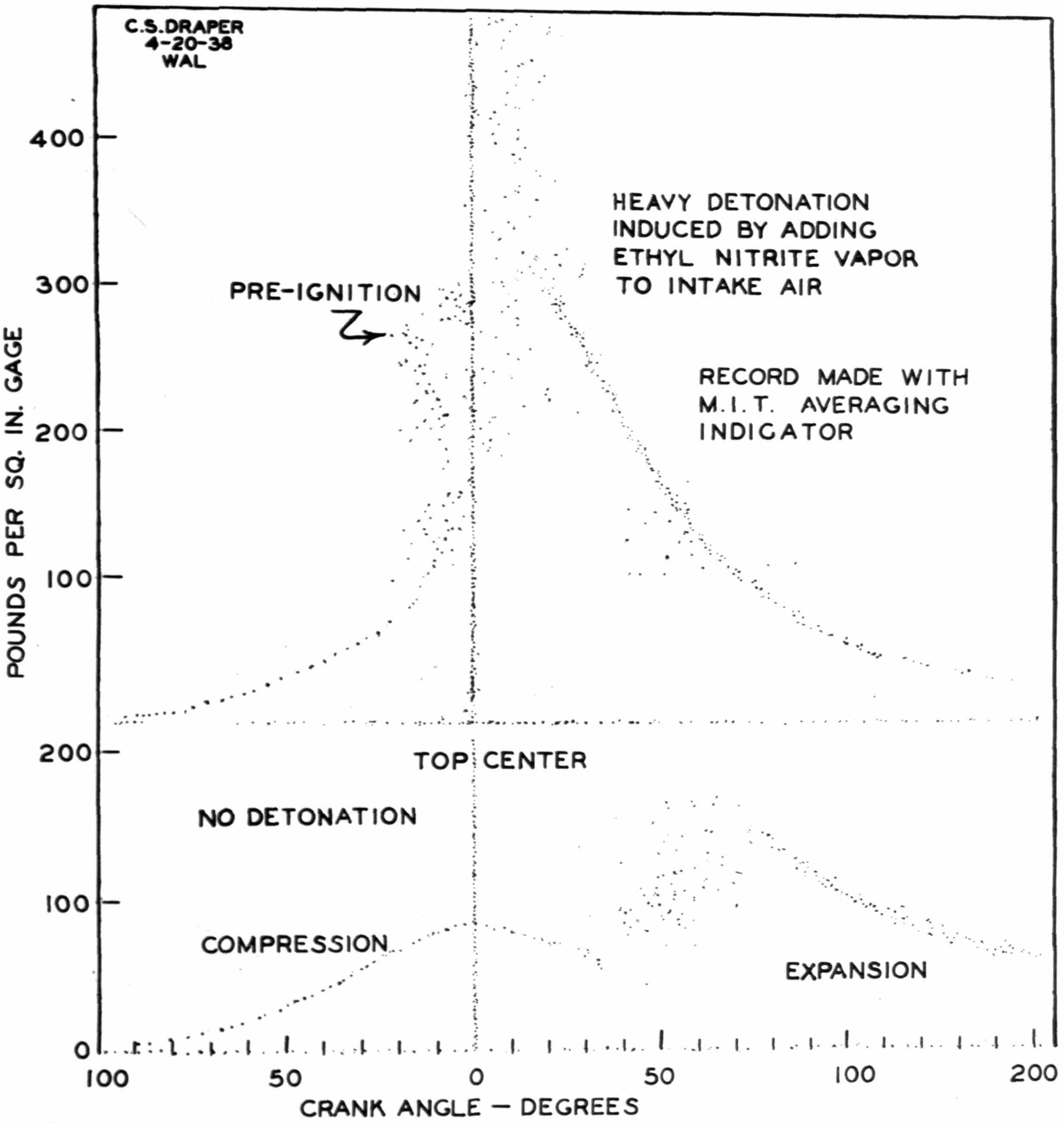
#5. RUNS WITH VERY COLD PLUG FOR SPARK ADVANCE OF 0°, 18°, AND 42°.

EFFECT OF DETONATION ON ENGINE OPERATION

FIG. 2

disturbances of engine operation which accompany detonation. The fact that changing only the spark plug produced a considerable difference in the ability of the engine to resist detonation is very strong evidence that thermal rather than mechanical effects establish the practical limits of engine operation.

Pre-ignition is not the only change in engine operation which may accompany detonation, in fact it is necessary to have a rather complete picture of the attendant circumstances to predict the effect of detonation in any given case. As an extreme but typical example of detonating operation, the results from a second series of tests on the C. F. R. engine are summarized in figure 2. For these runs a very well cooled spark plug was used and the compression ratio was reduced to a point where violent detonation could continue indefinitely without serious pre-ignition. Three spark advances were used, 42 degrees, which was slightly greater than that for best operation, 18 degrees, which was somewhat less than the best operation point and 0 degrees with which the engine output fell off to about two-thirds of the best power. The curves show that a <u>decrease</u> in power occurred with greatest spark advance while an <u>increase</u> in power accompanied detonation with the spark set at top center. For 18



C.F.R. ENGINE SPEED 1200 R.P.M. FUEL 73 OCTANE D.A.G.
COMPRESSION RATIO 4.5:1 SPARK ADVANCE 0 DEGREES

EFFECT OF DETONATION ON CYLINDER PRESSURE

FIG. 3

degrees advance no substantial change in power took place.

The apparently contradictory results of figure 2 can be explained with the aid of the pressure-time curves of figure 3 made with the M. I. T. High Speed Engine Indicator.(20) The retarded spark run was selected as the effects in question are most clearly demonstrated in this case. Without addition of the knock inducer to the intake air, maximum pressure was not developed until some 60 degrees after top center when the piston was already well down on the expansion stroke, so that the resulting thermal efficiency was lower than normal. Ethyl nitrite added to the intake air not only produced violent detonation as judged by ear, but also speeded up the general process of combustion until maximum pressure occurred near top center. The longer effective expansion stroke derived more useful work from the thermal energy and the power output of the engine increased. A few pressure points on the compression side of top center are out of line with the general trend of the pressure curve and were probably due to erratic pre-ignition. With the hint that the essential action of the knock inducer was to cause a more rapid burning of the fuel, the other two cases of figure 2 can be explained. With 42 degrees spark advance, the maximum pressure was occurring so near top center that any increase in speed of combustion could only produce higher pressures to work against the piston coming up on the compression stroke and thus reduce the net power output of the engine. For the intermediate case of 18 degrees spark advance, the maximum pressure was occurring slightly later than the crank angle for best power. The effect of a shorter combustion interval was to move the maximum pressure to a point slightly too early for best power with the net result that the engine output remained substantially unchanged. In the cases just considered it is fairly certain that the characteristic sound waves of detonation were merely a by-product and played no essential part in changing the power output of the engine.

DETONATION DEFINED

The preceding section has shown that certain physical effects are associated with the phenomenon known as detonation in the internal combustion engine.

1) A characteristic ringing sound audible outside the engine.
2) An alteration in power output of the engine.
3) A tendency toward overheating the walls of the combustion chamber and pre-ignition.
4) Mechanical damage to inside surfaces of the combustion chamber, especially in the case of aluminum alloy parts.

The external audibility of detonation is always dependent upon the general noise level near the engine. Thus the sound of detonation can be more readily detected in a quiet engine with muffled exhaust than in a noisy engine with open exhaust. However, it is certain that the characteristic noise appears in any case if detonation is sufficiently violent.

The change in power output due directly to detonation is generally small in practice but can often be detected under test stand conditions. On the other hand, the disturbance in operation which accompanies detonation so severe that pre-ignition occurs, will effectively stop the engine in a short time. This development of pre-ignition is the most serious immediate consequence of detonation. It has already been noted that pre-ignition can be controlled to some extent by the use of well cooled spark plugs.

Moderate detonation intensities do not ordinarily produce mechanical failure of an engine until after a considerable period of operation. In conventional engine designs the characteristic pitting of aluminum surfaces will usually be apparent only when the engine is dismantled for overhaul. However, the writer has seen pistons in experimental engines so severely damaged by detonation that operation was impossible after four or five hours of running. These failures all seemed to be due to high temperatures rather than excessive pressures. The dangerous weakening of aluminum alloy cylinder heads from increased temperature due to detonation has already been noted.

All the effects described above fall into a consistent picture if detonation is defined as a process in which there is such a rapid combustion of some part of the cylinder charge that a localized region of high pressure is produced.

The above definition of detonation agrees in general with accepted ideas on the subject as summarized by Pye.(1) No attempt is made to explain the chemical processes involved. The question of whether the rapid combustion is in the form of a traveling wave or a reaction occurring simultaneously over the region concerned is regarded as immaterial. Much additional evidence tending to confirm the general picture of detonation as outlined here will be discussed later in a review of past investigations.

Using the concept stated in the definition, the four physical manifestations of detonation can be explained as noted below

1) The externally audible noise is due to cylinder wall vibrations produced by a combination of the initial shock and the standing wave system

within the cylinder enclosure which follows the local pressure rise.

2) The alteration in power output directly due to detonation is produced by a shift in the timing of cylinder pressures with respect to piston position when part of the charge burns at an increased rate. This effect is essentially a decrease in the time required for combustion and will be equivalent to a change in spark advance for a given case.

3) Pre-ignition is induced by overheating of spark plug points or other poorly cooled internal surfaces not necessarily located in contact with the region of abnormally rapid combustion. Surfaces in contact with the initial disturbance will receive heat for a short time at an extremely high rate due to high temperature, high density and violent gas motion as the localized energy of the high pressure gas spreads into the cylinder enclosure. Surfaces out of contact with the initially rapid combustion will be heated by a reduced resistance of the gas-solid surface film to heat flow rather than an increased average temperature of the cylinder gases. This change in

the thermal conductivity of the surface film will be a natural consequence of the scrubbing action produced by violent pressure waves in the gas and will be responsible for the general increase in cylinder head temperature. Surfaces near the outer end of a cavity in the combustion chamber wall, such as the space between the central electrode and the outer shell of a spark plug, will be especially affected since the oscillatory gas motion past such parts will be intensified. This idea of increased heat transfer due to pressure waves has been discussed by Withrow and Rassweiler(21) who showed that such an action is required to explain observed results.

4) Mechanical damage to aluminum alloy surfaces results from the high temperatures and steep temperature gradients produced near the region of rapid combustion. These thermal effects in a metal consisting of crystals separated by a matrix of different constitution and known to lose strength rapidly with increasing temperature(22) will certainly account for the damage produced by an extended period of operation with severe detonation.

DETONATION AS AN ENGINEERING PROBLEM

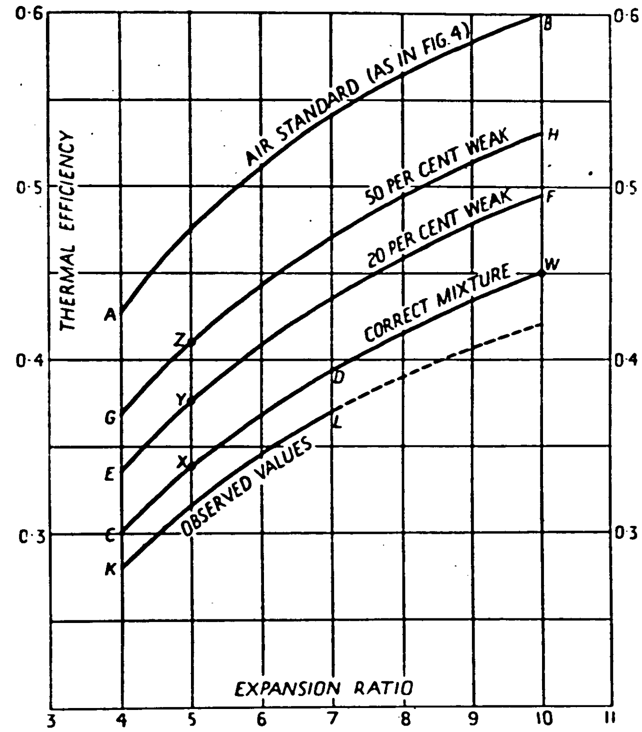
Factors controlling Engine Output

Power output from an internal combustion engine is a function of the mass of air flowing through the engine in unit time, the chemical energy content of a unit mass of the fuel-air mixture, the efficiency with which heat is converted into mechanical energy, and the mechanical losses in the engine. In practice the chemical energy content of the mixture varies somewhat with the fuel-air ratio but is substantially the same for all commercial fuels in ~~side~~ use at the present time. The mechanical efficiency is about the same for well designed engines, so that fuel energy content and mechanical losses can be disregarded in a generalized discussion of the factors affecting the output from modern engines. Air consumption in an engine of given size depends upon the number of cycles per unit time and the charge density when the intake valve closes. Thus the power can be increased either by increasing engine speed or by supercharging to supply the mixture at increased pressure. A more or less definite upper limit is placed on speed by mechanical strength considerations and the restriction to air flow in rapidly operating valves. The remaining expedients of supercharging and increasing thermal efficiency are the most practical methods for obtaining more power from an engine of fixed size.

Detonation as a Limiting Factor on Engine Performance

The performance of modern transportation equipment is limited by the power output available from engines of practical size and weight. This statement is particularly true of aircraft, for which it is also essential that the power plant should have the highest possible thermal efficiency in order to reduce the fuel load required for long range flights. It follows that a factor which limits the efficiency and intake pressure of internal combustion engines will play an extremely important part in modern power plant design. The discussion which is to follow will outline the way in which the tendency toward detonation establishes both the maximum practical fuel economy and output of modern high performance engines.

Thermal efficiency is dependent upon many design details but the most important factor is the ratio of charge volume at the end of expansion to charge volume at the start of expansion. The effect



Variation of thermal efficiencies, theoretical and observed, with expansion ratio. **REF. 1**

FIG. 4

of this "expansion ratio" or "compression ratio" on thermal efficiency is discussed in detail by Pye(1) who gives calculated values with air as the working medium and with actual fuel mixtures of three fuel-air ratios. Pye's results are summarized in the curves of figure 4. The lower curve, KL, based on the mean of many observed values under optimum operating conditions, shows that it is possible to realize a high percentage of the theoretical efficiency in an actual engine. Gains obtained by increasing the compression ratio are especially desirable since fuel economy and engine output are both improved while the amount of waste heat to be dissipated by the cooling system is reduced.

Supercharging does not greatly affect thermal efficiency if the compression ratio is held constant but the general cycle pressures of the working medium are increased as the intake pressure becomes higher.

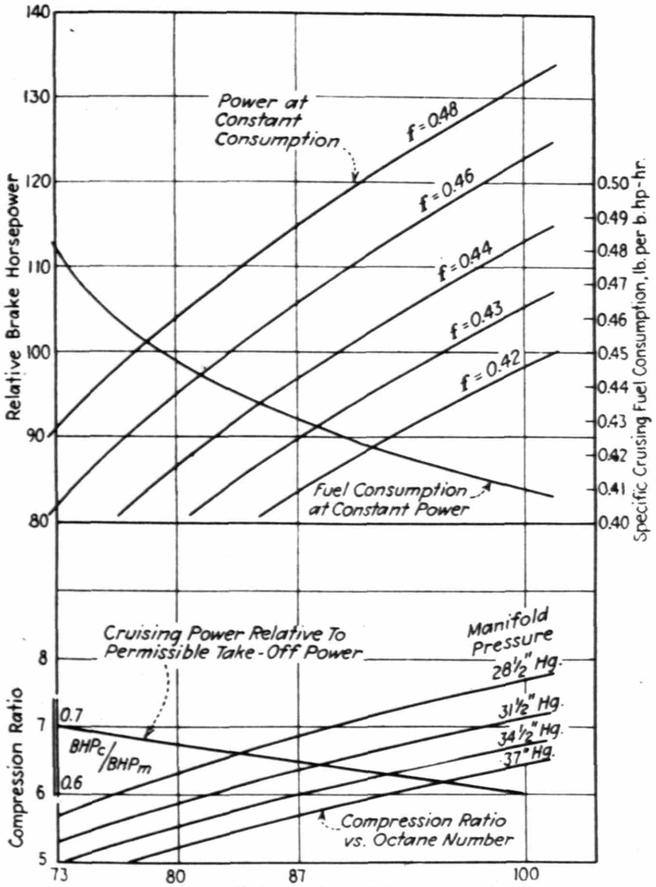
One necessary result of increasing either the compression ratio or the supercharger pressure is that the last part of combustion occurs at a higher temperature. The work of many investigators has shown that there is a close relationship between the tendency toward detonation and the temperature of the last part of the charge to burn. In a review of recent detonation studies, C. F. Taylor and E. S. Taylor(23) summarize the situation in the statement: "--- the significant temperature in determining whether or not a given mixture detonates, is the highest temperature reached by the last part of the charge before its combustion. If this is above its minimum 'self-ignition' temperature, and if the normal flame does not pass through it too soon, it will detonate." It follows that both the most effective methods for increasing the output from an engine of given size <u>inevitably produce detonation</u> which in turn will lead to overheating of the combustion chamber walls and to pre-ignition. In an engine with an aluminum alloy cylinder head the overheating will cause complete failure of the engine if it is allowed to continue.

The practical limit on fuel economy due to detonation in a given engine is set by mixture ratio requirements; that is, a rich mixture tends to suppress detonation, and as the mixture is leaned out, a point may be reached where detonation occurs and causes overheating of the cylinder head. This effect is so serious that the cylinder head temperature rise which occurs as the mixture is leaned out is often taken as indicating the presence of detonation.(62)

The chain of cause and effect outlined above has resulted in a universal opinion that detonation is a very serious limitation in modern engine design. In regard to this situation Domonoski and Finch(2) say: "--- Detonation at present has to be accepted as a limit to the efficiency of the internal combustion engine. If it did not exist, there would be no reason why brake mean effective pressures far in excess of those employed at present could not be attained by means of increased compression ratio augmented by supercharging, with resultant increase in power output per cubic inch of displacement and in fuel economy." Dr. H. C. Dickinson(24) of the National Bureau of Standards nicely sums up the effort of designers to improve engine performance in the statement, "--- A given engine will deliver more power and, incidently, use less fuel if the compression ratio is raised. The designer, therefore, raises the compression ratio until the engine begins to knock whenever the temperature or the carbon deposits increase. The user then demands an antiknock fuel. Having stopped the knock in this way, the designer may again raise the compression and repeat the process so long as suitable fuels can be found. The problem of antiknock fuels, therefore, is likely to increase in importance."

Detonation began to assume the aspects of a major engineering problem in the early years of the decade following 1920 after Midgly and Boyd(25,26) demonstrated that the evils of detonation could be controlled to a considerable extent by the addition of certain compounds to commercial fuels. This pioneer work started a series of elaborate investigations directed toward the development of fuels with less tendency toward detonation in high performance engines. It is certain that the engine designer must give a large share of the credit for the high performance of modern power plants to the chemical engineer and his laboratory collaborators.

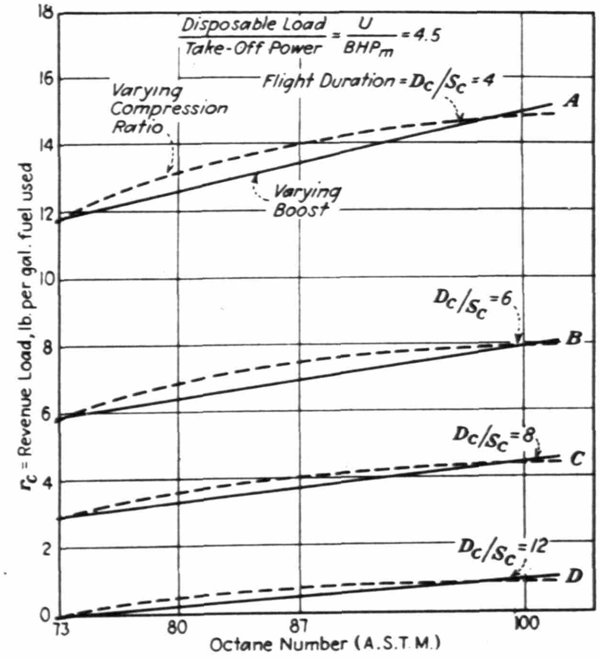
The vital importance of fuels to suppress detonation in modern aviation equipment has been discussed by D. P. Barnard(27) who uses octane number, the accepted quantitative measure of detonation tendency, as the independent variable with power output, fuel consumption, revenue load per gallon of fuel and other quantities of engineering interest as dependent variables. Typical results from Barnard's analysis are given in figures 5 and 6. Keeping in mind that octane number increases with increasing resistance to detonation, the lower curves of figure 5 show that the practical limit to manifold pressure (supercharging)



Relative Output and Economy as Determined by Octane Number

REF. 27

FIG. 5



Revenue Load Vs. Octane Number

REF. 27

FIG. 6

increases for a given compression ratio if fuels with higher octane numbers are used. The possibilities of increased power and fuel economy are indicated in the top curves of figure 5. Figure 6 shows that considerable increases in revenue load per gallon of fuel burned are possible if the detonation resistance of the fuel is improved and the power plant is modified to take full advantage of the possible gain. The heavy lines show the results if compression ratio is held constant and boost (supercharging) is increased to the limit in each case. Somewhat better results can be obtained by using the highest possible compression ratio rather than raising the manifold pressure except for octane numbers near 100.

Summary

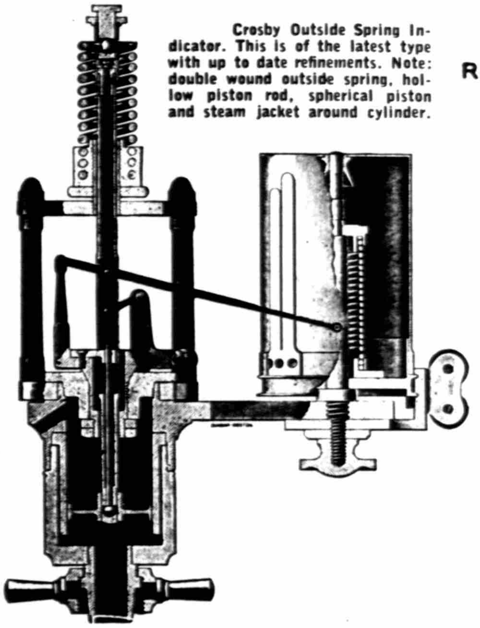
The evidence cited above shows that detonation is the factor which limits both economy and output of modern internal combustion engines using spark ignition. This fact is universally recognized and many agencies are at present studying detonation as a major engineering problem.

ENGINE INDICATORS AND DETONATION

Mechanical Indicators

Detonation produces marked changes in engine operation so that its presence can be detected more or less surely by most of the devices required in a routine engine test. However, detonation especially affects combustion chamber pressures and temperatures so it is logical that detonation measuring instruments should develop as specialized types of pressure indicators and thermometers. DeJuhaz(28) in his book, "The Engine Indicator," gives a general discussion of the many devices developed for measuring pressures in engine cylinders since Watt constructed his first indicator about 1790.

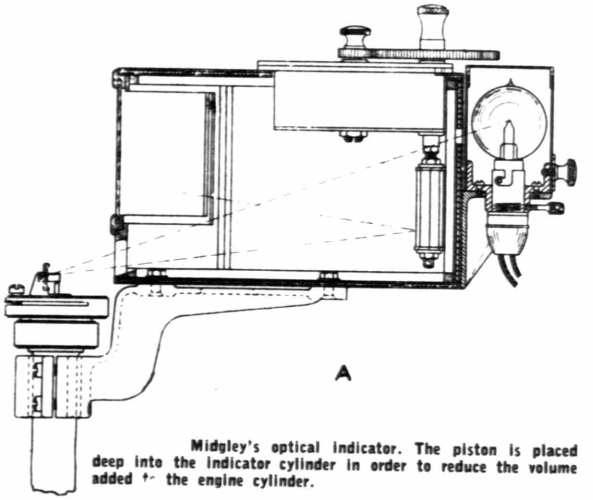
Figure 7 shows the essential parts of a present day mechanical indicator suitable for use on low speed engines. The instrument is essentially a spring restrained piston operating in a small cylinder connected to the combustion chamber of the engine. A pointer connected to the piston by means of a linkage, gives a pencil record on paper wrapped arounda drum



Crosby Outside Spring Indicator. This is of the latest type with up to date refinements. Note: double wound outside spring, hollow piston rod, spherical piston and steam jacket around cylinder.

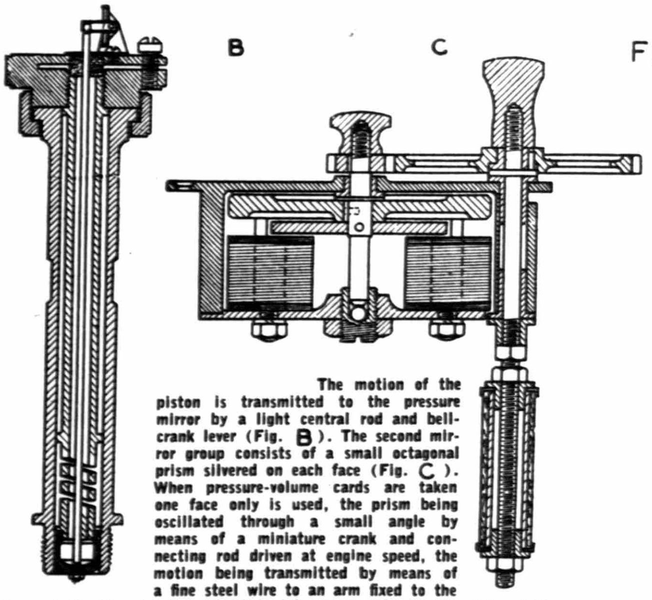
REF. 28

FIG. 7



Midgley's optical indicator. The piston is placed deep into the indicator cylinder in order to reduce the volume added to the engine cylinder.

FIG. 8



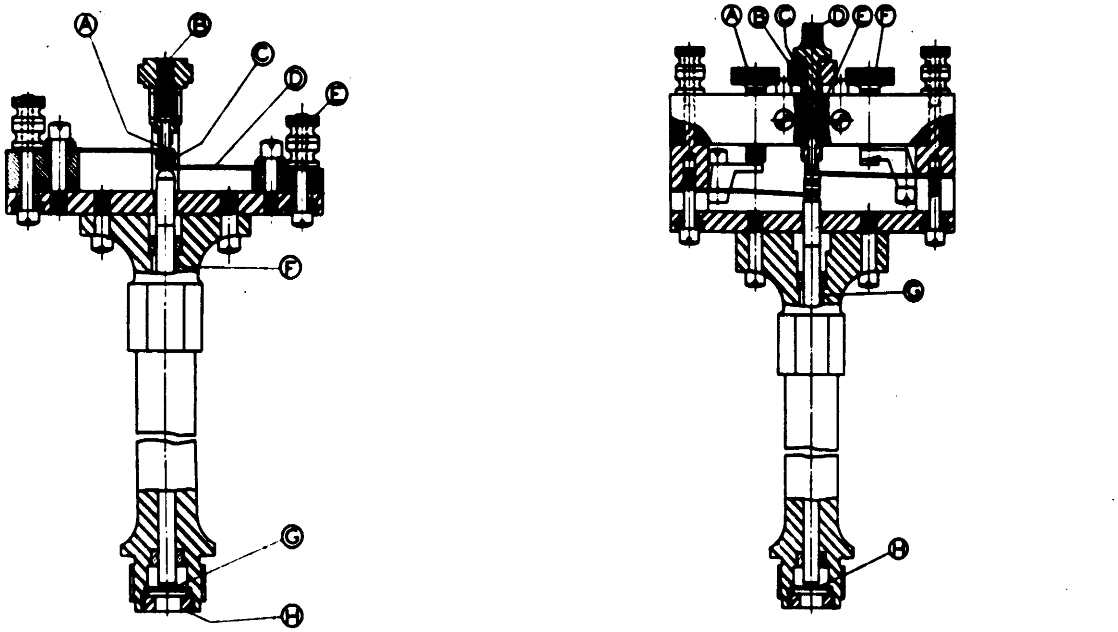
The motion of the piston is transmitted to the pressure mirror by a light central rod and bell-crank lever (Fig. B). The second mirror group consists of a small octagonal prism silvered on each face (Fig. C). When pressure-volume cards are taken one face only is used, the prism being oscillated through a small angle by means of a miniature crank and connecting rod driven at engine speed, the motion being transmitted by means of a fine steel wire to an arm fixed to the mirror shaft, the wire being kept taut by means of a spring. When taking pressure-time cards the octagonal mirror is given a rotary motion, being driven by a small electric motor (Fig. C) synchronised with the engine by means of a distributo driven at engine speed.

REF. 28

beams as in the class of instruments known as <u>Optical Indicators</u>.(31,32,33) The Midgley Indicator (33) shown and described in figure 8 is especially interesting since this device is a direct ancestor of the <u>Midgley Bouncing Pin</u> which is today widely used in detonation measurements.

<u>Bouncing Pin Detonation Indicator</u>

With normal engine conditions and for speeds lower than some upper limit introduced by the inertia of solid elements, the mechanical indicators give satisfactory records. However, detonation is always accompanied by pronounced vibrations in the expansion line of the engine power stroke. Many experiments have shown that the recorded frequencies are usually characteristic of the indicator rather than the cylinder process itself. Midgley(34) took advantage of the fact that the initial shock is the most important effect of detonation on mechanical indicators. He placed a loose rod or "pin" on a diaphragm freely exposed to the cylinder gases. For normal operation the rate of change of pressure is so slow that with proper adjustments the pin does not leave the diaphragm but the shock accompanying detonation causes the pin to "bounce" away from the diaphragm and close a pair of electrical contacts. The time integral of the current



Sectional View of Bouncing Pin

A—Spring plunger in upper stop screw. B—Upper stop adjusting screw. C—Contact points normally open by approximately .005". D—Lower contact spring. E—Lower contact terminal. F—Bouncing pin; steel rod with bakelite insulating button in upper end. G—Diaphragm which responds to detonation, and imparts bouncing action to pin which rests upon it. H—Diaphragm retaining nut.

Sectional View of Adjustable Bouncing Pin

A—Lower leaf adjusting screw. B—Center adjusting screw. C—Locating spring. D—Bumper spring adjusting screw. E—Bumper spring. F—Upper leaf adjusting screw. G—Bouncing pin. H—Diaphragm.

REF. 35

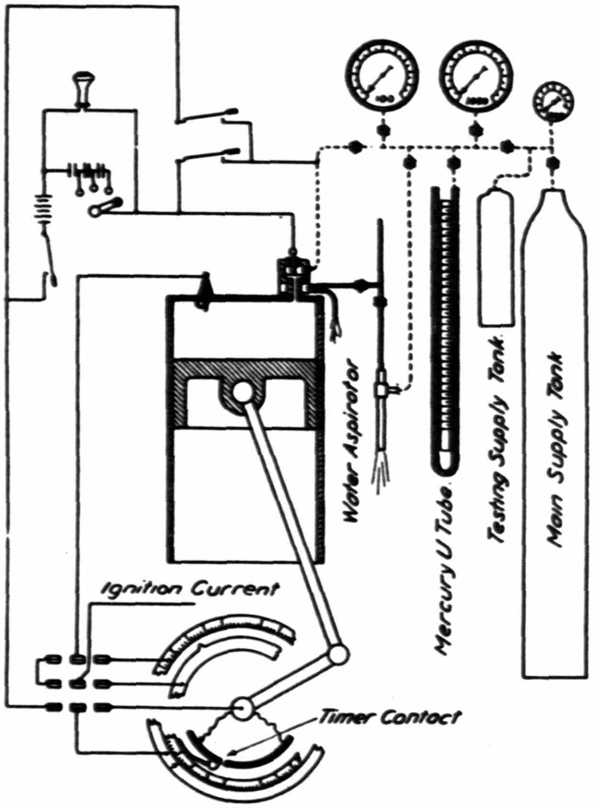
FIG. 9

flowing across the contacts from a constant potential source is taken as a measure of detonation intensity. In Midgley's original arrangement, the current was averaged by means of an electrolytic cell and pipette for measuring the gas evolved from a sulphuric acid solution in a given time. Later improvements have replaced the gas coulombmeter by a heating coil and thermocouple system with which the average current can be read directly from a heavily damped meter. No essential changes have been made in the Bouncing Pin since it was first revealed by Midgley in 1922 so that figure 9, taken from a recent instruction book on detonation measurements,(35) is a representative description of the instrument at all stages of its development. The procedure for measuring detonation with the Bouncing Pin will be considered in the next section of the present report.

Averaging Indicators

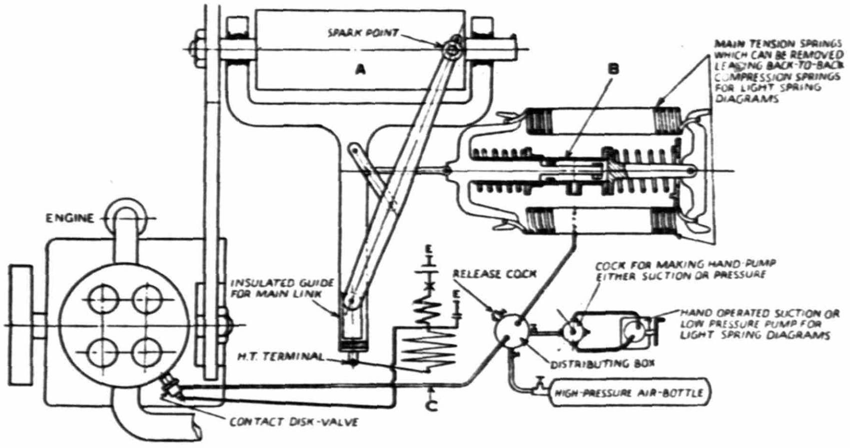
Successive cycles of an internal combustion engine show such great variations that an estimate of power developed based on a single card from a mechanical or optical indicator would be liable to serious error. This engine characteristic coupled with the indicator dificulties already mentioned has led to the development of instruments using balanced pressure systems. In such indicators a thin diaphragm or a small disc valve is the only moving part and the required displacement is limited to a few thousandths of an inch. With such an arrangement inertia troubles are considerably reduced and the recording system gives a result based on a number of successive engine cycles. Two recording schemes are used: 1) a known pressure is established outside the balancing element and the crank angles at which this pressure exists are determined, and 2) a known crank angle is established and the pressure occurring at this crank angle is determined.

The essential parts of the Balanced Pressure Diaphragm Indicator as developed at the Bureau of Standards(36) are shown in figure 10. The sensitive unit is connected to the right-hand side of the cylinder head. Pressure from the engine cylinder is applied to the lower side of the diaphragm while the upper side is exposed to a controllable pressure above or below atmospheric depending upon whether one of the compressed air tanks or the aspirator is connected to the indicator. A water jacket is used to prevent damage from the high engine temperatures. An insulated contact is mounted inside the unit just above the center of the diaphragm. If the cylinder pressure is higher than the external



Balanced Diaphragm Indicator of the Bureau of Standards (American Instrument Co.). **REF. 28**

FIG. 10



The 'Farnboro' Indicator. **REF. 1**

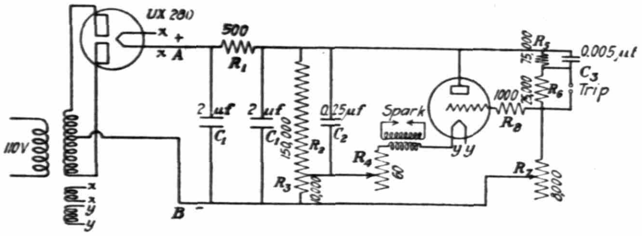
FIG. 11

pressure an electrical circuit is completed, while this circuit is open if the cylinder pressure is lower than the outside pressure. The indicator contact is in series with a battery, a telephone receiver and an adjustable timing contact mounted on the engine crankshaft. In operation, the timing contact is set to a selected crank angle so a click will be heard in the receiver only if both contacts are closed at the same time. The outside pressure is then reduced until a series of clicks in the telephone receiver shows that the cylinder pressure is higher than the applied pressure over part of the cycle. Equality between the internal and external pressures is determined as the point at which the telephone receiver clicks become irregular and finally disappear entirely. The Bureau of Standards Balanced Diaphragm Indicator provides one of the most accurate methods for measuring cylinder pressures in high speed engines. However, the necessity of exploring a considerable number of individual points in the cycle makes the process of taking an indicator card laborious and slow in practice.

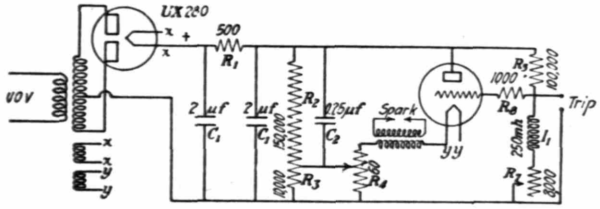
In the Farnborough Indicator(37,38) a free disc valve is used instead of a diaphragm to detect the instant of equality between cylinder pressure and a controlled external pressure. This valve floats between two seats and acts as a switch in the primary circuit of an induction coil in such a manner that a spark jumps between a pointer and the metal of a rotating drum whenever the valve is forced away from either seat. The essential parts of the Farnborough Indicator are shown in figure 11. The pointer is connected to a spring controlled piston sliding inside a cylinder and so designed that the displacement of the pointer is proportional to the external pressure applied to the indicator disc valve. The drum is connected to the engine by a rigid drive so that each line parallel to the axis of rotation corresponds to a definite crankshaft angle. If the external pressure is held fixed, a piece of paper wrapped around the drum will be perforated by the spark at the crank angles for which the cylinder pressure is equal to the external pressure. By slowly varying the external pressure a complete <u>pressure-crank angle</u> curve will be formed by the successive spark holes in the paper. The process of making an indicator card can be completed in less than one minute so the Farnborough Indicator is very convenient in actual use.

Extended tests of the Farnborough Indicator by various agencies brought out certain defects in the instrument:(39,40) 1) for points near the peak pressure, the time of contact with the upper seat is too short for the primary current to reach the value necessary for a satisfactory spark, 2) pitting of the valve seats occurred after a short period of operation due to arcing in the primary coil circuit, 3) the areas of the disc exposed to pressure on the two sides are not equal due to the requirement of a finite seat width, and 4) errors were introduced due to inertia effects on the disc. A systematic attack on these difficulties at the National Advisory Committee for Aeronautics(39) resulted in an improved instrument similar to the Farnborough but with the errors reduced by refinements in design.

Professor E. S. Taylor(20) and the writer working in the Internal Combustion Engine Laboratory at M. I. T. developed an instrument which combined the convenient recording system of the Farnborough Indicator with a small balanced pressure diaphragm cylinder similar to that of the Bureau of Standards. Electrical pitting was eliminated by using the diaphragm contact to control only the grid current of a General Electric Thyratron in a special relay circuit. The circuits and cylinder unit are shown in figure 12. The diaphragm thickness ranges from 0.001 to 0.005 in. and the diameter is 0.500 in. A total motion of the

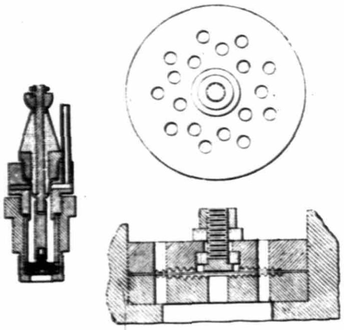


(*a*) Make



(*b*) Break

Circuits of the Massachusetts Institute of Technology Balanced Diaphragm Indicator. Two circuits are used, with one the spark occurring with the "make" (ascending branch) with the other the spark occurring with the "break" of the diaphragm contact (descending branch). By means of a double-throw switch it is possible to change from one circuit to the other.



REF. 28

Balanced - Pressure Diaphragm Unit for the M.I.T. Indicator showing enlarged detail of diaphragm and seats at right.

FIG. 12

center of the diaphragm of about 0.001 inch is allowed by the perforated and grooved backing plates which support the diaphragm against the high pressures involved. One circuit is arranged to produce sparks only when the diaphragm leaves the contact and the other circuit gives sparks only when the diaphragm first touches the contact. With this scheme, ample time is always available to charge the condenser supplying power for the spark so that indicator cards with no "gaps" near the peak pressure are obtained.

All balanced pressure indicators show the effect of detonation as a series of points near the peak which correspond to pressures much higher than the start of the expansion line. These high points are due either to the initial detonation shock or to the first pressure disturbance as it spreads over the cylinder. The indicator card of figure 3 taken under detonating conditions shows very plainly the extraneous points due to detonation.

DeJuhasz and others(28) have developed a type of high speed indicator distinct from any of the instruments described above. This so-called <u>Sampling Valve Indicator</u> uses a conventional low speed cylinder and piston indicator to record the cycle pressures. This low speed indicator is connected to the engine cylinder through a small valve which stays open for a very short time and which is timed with respect to the engine crank position by means of a controllable phasing system. This phasing gear also determines the position of the indicator drum. The phase is changed so slowly that a pressure equal to the cylinder pressure is built up inside the indicator for each crank angle and recorded at the proper position on the indicator drum.

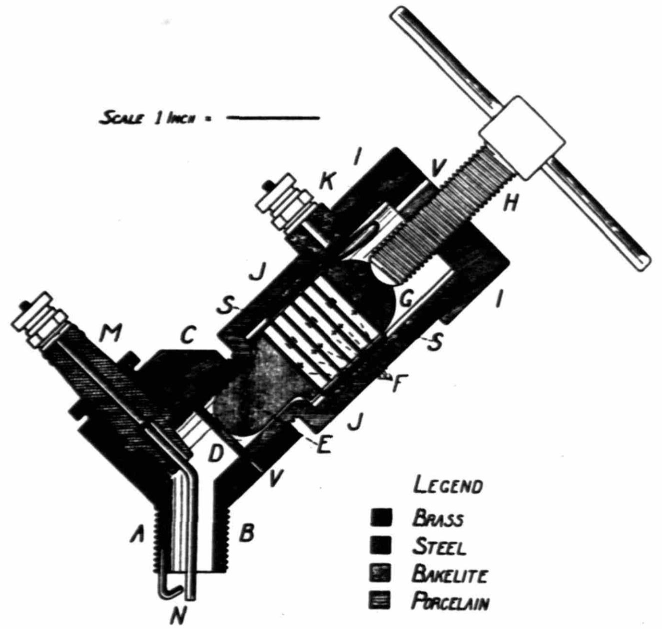
Detonation produces erratic deviations from a smooth curve near the maximum pressure point of records from a sampling valve indicator. This effect is less marked than in the case of balanced pressure indicators since the slow action of the recording system will necessarily average out high pressures existing for a very short time.

Instantaneous Pressure Indicators

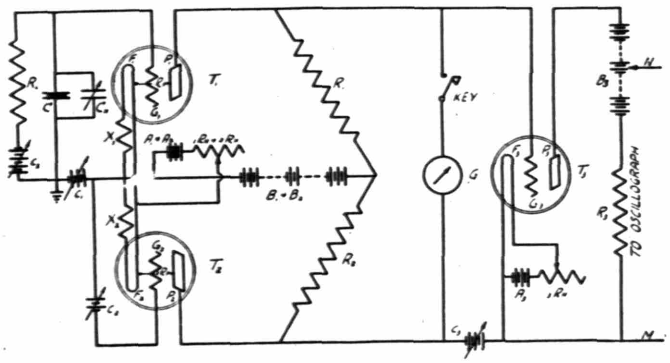
Pressure changes occurring in the cylinders of high speed engines are so rapid that accurate continuous records are very difficult to obtain with a purely mechanical system. Many devices for making continuous records of rapidly varying pressures have been developed and are known as Instantaneous Pressure Indicators. Such instruments are combinations of electrical and mechanical elements which include; 1) a pickup element to transform pressure variations into electrical changes, 2) a coupling system which receives electrical input from the pickup unit and produces an output suitable for recording or indicating purposes and 3) an indicating system or a recording system which changes the output of the coupling system into a form suitable for examination by an observer.

Detonation produces such fast pressure changes that the only accurate indicating or recording element is a cathode ray oscillograph.(41,42) However, for normal engine operation and qualitative studies of detonation, conventional moving wire oscillographs or discharge tube instruments are satisfactory.(43,44,45) The various types of indicators incorporate standard units for the recording or indicating system so that no special attention will be given to this phase of the problem in the discussion which follows.

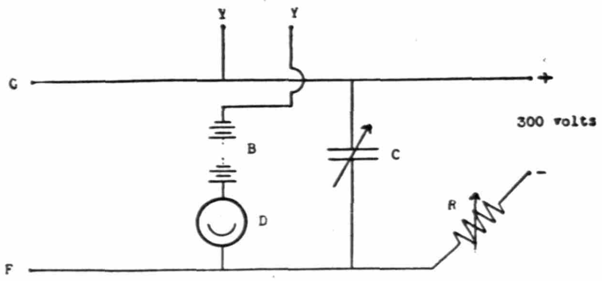
One problem common to instantaneous indicators is that of changing pressures into mechanical displacements to act on some type of electrically sensitive element. At the same time it is necessary to protect the electrical parts from high cylinder temperatures. These conditions are almost universally met by the use of a diaphragm exposed to the cylinder pressures. The essential requirement beside the ruggedness to withstand engine operation is that the complete system must have



Piezo-electric indicator of Watson and Keys. M spark plug; E and G end blocks; F quartz disks; K terminal; D gasket plate; S Bakelite lining.



The amplifying unit of the Watson and Keys indicator.



Time scale circuit of the Watson and Keys indicator. **REF. 28**

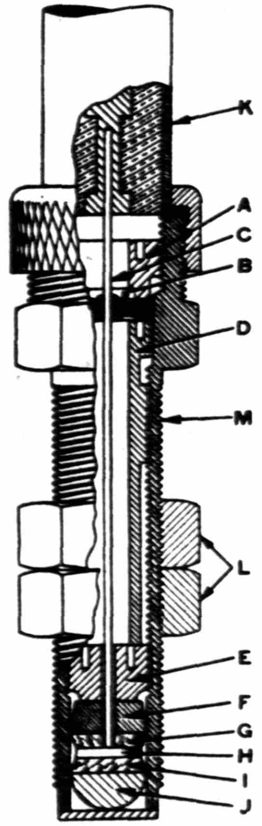
FIG. 13

a natural frequency much higher than the component frequencies of the processes to be recorded. There is a definite lack of quantitative information on dynamic errors in the literature on indicators so that the material which follows will be descriptive rather than analytical.

Piezo-Electric Indicators

Many designers have used the piezo-electric property of tourmaline or quartz as a means for transforming mechanical displacements into voltage changes.(46,47, 48,49) Figure 13, showing the instrument of Watson and Keyes (48) is typical of the arrangements used. A number of quartz discs are electrically connected in parallel and placed under an initial tension in mechanical contact with an exposed diaphragm. It is necessary to make sure that force is applied uniformly over the crystal faces to prevent breakage. This is accomplished by using spherical elements at some point in the force transmitting system. An amplifier is required between the piezo-electric element and the cathode ray tube indicator. A sweep circuit is used to spread out the deflections due to pressure into a pressure-time card for visual observation. The Radio Corporation of America(49) has recently announced a commercial piezo-electric

THE R.C.A. PIEZO-ELECTRIC INDICATOR.



A—Top Nut
B—Insulator Disc
C—Center Electrode Lead
D—Stop Nut
E—Bottom Nut
F—Top Electrode
G—Top Crystal
H—Center Electrode
I—Bottom Crystal
J—Bottom Electrode
K—Cable Contact Assembly
L—Lock Nut
M—Shell

REF. 49

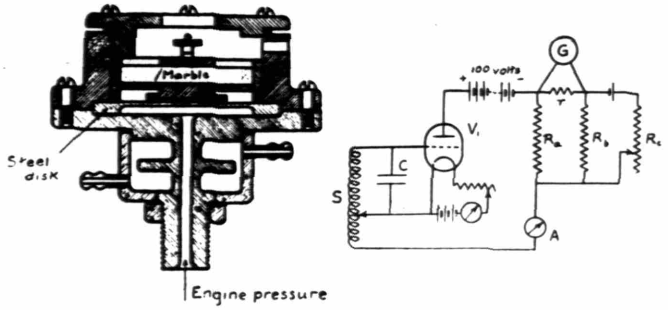
FIG. 14

indicator to be used in connection with a standard cathode ray oscillograph. Figure 14 shows the essential parts of the R. C. A. Indicator.

Electro-Static Capacity Indicators

Mechanical displacements of a diaphragm can be made to affect an electrical circuit by changing the electro-static capacity between the plates of a condenser. A number of engine indicators have been designed to use this principle.(50,51) Figure 15 shows the arrangement of Obata and Josida. A relatively large diaphragm forms one plate of the condenser and is exposed to the cylinder gases through such a small connecting tube that the unit can be screwed into a spark plug hole. This condenser forms part of an oscillating circuit and modulates the input to a vacuum tube which acts as amplifier and detector. The changes in plate current of the vacuum tube are recorded by an electro-magnetic oscillograph. The same principles are used in the D. V. L. Indicator(51) as shown in figure 16. In the D. V. L. instrument a sealing diaphragm is used to protect the deflecting condenser plate from hot gases in the cylinder.

Both piezo-electric and variable capacity indicators require that the coupling amplifier have a high input impedance and low capacity in the connecting leads.



The pressure unit and the electrical circuit used in Juichi Obata's electrical indicator. The steel disc is 2 mm. (3/32 in.) thick and of 2 in. dia. The indicator body is water cooled. An Einthoven string galvanometer G is used to record the current variations, which are proportional to the pressure. REF. 28

FIG. 15

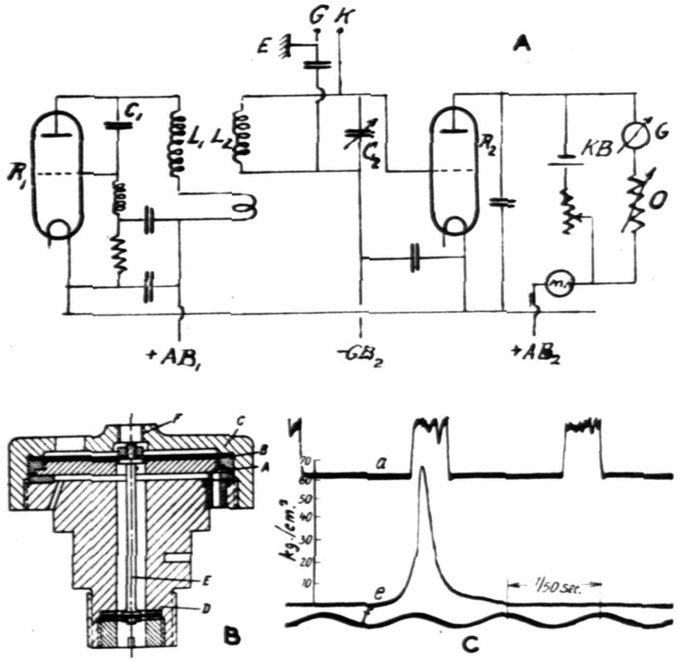


Fig. B Indicating arrangement of the Deutsche Versuchsanstalt fur Luftfahrt. D is a weak diaphragm connected by means of rod E with the pressure measuring stiff diaphragm B. This latter forms a condenser together with the stationary and insulated plate A. The electrical circuit is shown on Fig. A incorporating vacuum tubes, balance coils and an oscillograph for recording the current variations.

Fig. C is a specimen card showing also the time scale and the position of the dead centers.

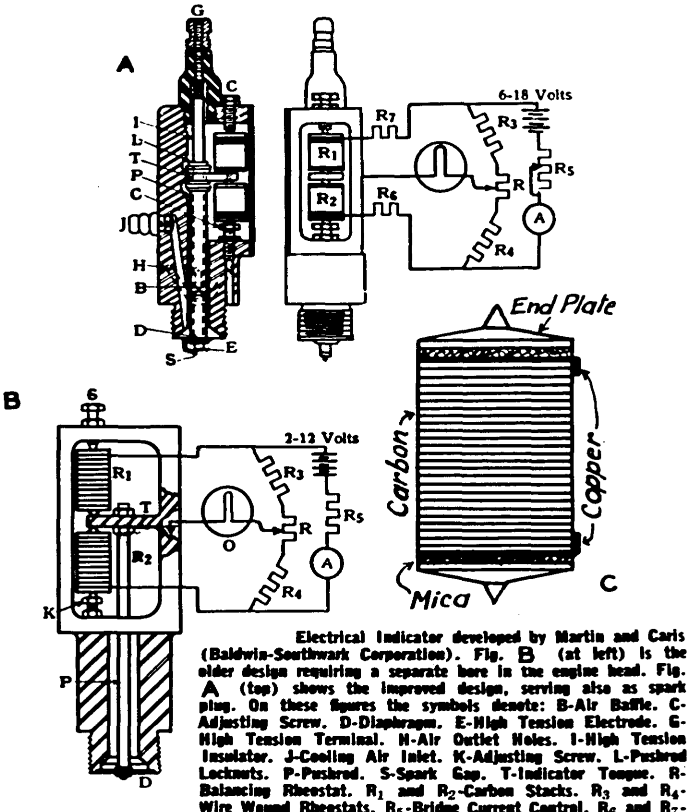
REF. 28

FIG. 16

For this reason the first tube of the amplifier must be as close to the pickup unit as possible with connections carefully shielded against extraneous electrical or magnetic fields. Satisfactory results can be obtained at the expense of careful design and maintenance under laboratory conditions, but trouble would be expected under severe test house or field conditions with large engines.

Carbon Resistance Indicators

Units formed of a stack of carbon discs have the property of changing electrical resistance with applied force. Such elements have been studied by many investigators and in particular have been developed to give quantitative results by McCollum and Peters(52) who were particularly interested in distant reading strain gages. The carbon disc stacks of McCollum and Peters were adapted by Martin and Caris(53) to the engine indicator problem in an instrument offered commercially by the Baldwin-Southwark Corporation of Philadelphia. Figure 17 shows the essential features of this indicator. Two differentially operated carbon stacks are used as arms of a Wheatstone Bridge which is supplied from a battery. The effects obtained are so large that a conventional electromagnetic oscillograph can be connected across the mid-points of the bridge and



Electrical Indicator developed by Martin and Caris (Baldwin-Southwark Corporation). Fig. B (at left) is the older design requiring a separate bore in the engine head. Fig. A (top) shows the improved design, serving also as spark plug. On these figures the symbols denote: B-Air Baffle. C-Adjusting Screw. D-Diaphragm. E-High Tension Electrode. G-High Tension Terminal. H-Air Outlet Holes. I-High Tension Insulator. J-Cooling Air Inlet. K-Adjusting Screw. L-Pushrod Locknuts. P-Pushrod. S-Spark Gap. T-Indicator Tongue. R-Balancing Rheostat. R_1 and R_2-Carbon Stacks. R_3 and R_4-Wire Wound Rheostats. R_5-Bridge Current Control. R_6 and R_7-Temperature Compensators.

C shows the construction of the carbon pile according to McCollum and Peters.

REF. 28

FIG. 17

used to indicate or record pressures. Variations in resistance of the carbon due to temperature changes are compensated by metallic resistors mounted adjacent to the stacks inside the indicator body. A sealing diaphragm at the lower end of the threaded body protects the internal parts of the indicator against cylinder gases while the necessary elasticity is supplied by a cantilever spring between the carbon stacks.

Martin and Caris, working in the General Motors Research Laboratory at Detroit have continued to improve the carbon stack indicator and this instrument has been used for many investigations to be discussed later in the present report. The writer examined several of the later models during a visit to the General Motors laboratory and observed that the double carbon stack had been replaced by a single stack working with smaller displacements than were used in the original indicator. It is interesting to note that other investigators(41,54) who carried out independent carbon stack indicator developments also arrived at designs using single resistance elements.

In practice the carbon stack indicator must be calibrated frequently and is subject to indefinite thermal effects. The response to a given pressure change depends upon the rate at which the change is made so calibrations should be made with conditions approximating those of actual operation. Detonation produces a vibration on the expansion line of the record from a carbon stack indicator. In the ordinary case, the frequency of pressure waves accompanying detonation lies in the range between 3000 cycles per second and 15000 cycles per second(56) so that a carbon stack indicator with natural frequency between 3500 and 11000 cycles per second(53) will respond to the detonation frequencies but will not be uniformly sensitive over the required range. The most serious limitation of carbon stack indicators in studying detonation is that the severe shock accompanying the initial disturbance will mechanically damage the carbon discs and produce a considerable erratic change in sensitivity. A sample record showing this effect has been given by the writer in reference 56.

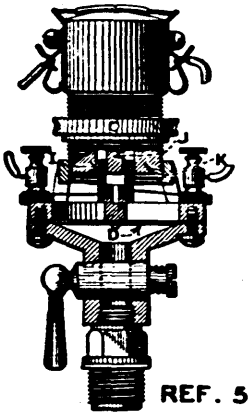
Electro-magnetic Indicators

Augustus Trowbridge(43,55) in 1921 first called attention to the advantages of recording rate of change of pressure rather than pressure and constructed an electromagnetic generator type of pickup unit for this purpose. Figure 18 shows the essential parts of the Trowbridge Indicator. With regard to the general problem Trowbridge said:

"There is no simple means of electrically transmitting a record of the actual position of a moving body, but it is an extremely simple matter to transmit a record of its velocity at every instant of its motion. From a record of its position at any known instant and its varying velocity at each and every succeeding instant, it is a simple geometrical matter to find its position at each and every instant. Electromagnetic induction furnishes a simple means of recording the velocity at which a body is moving the current induced in a small coil attached to the moving diaphragm, and so mounted that any one of the circular turns of the coil lies in a magnetic field radiating from the center of the circle, is proportional to the velocity with which the turn (or turns) of wire move in the direction at right angles to the plane of the magnetic field."

"-----Fig. 2* shows an elevation (partly in section) of the indicator; the coil referred to is shown at I and the diaphragm to which the coil is rigidly attached at D. The internal pole of the magnet providing the field which runs radially through the coil is shown at J and the other (ring-shaped) at K. As will be seen from the figure, an up and down motion of the diaphragm D causes the coil I to move across the radial magnetic

*Figure 18 of the present report)



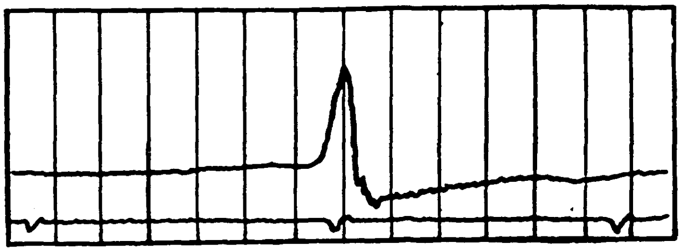
REF. 55

FIG. 2. THE
INDICATOR

FIG. 18

(FIG. 12)

REF. 55



(FIG. 13)

FIG. 19

field and by this motion to induce currents in the circuit of the coil which are in one direction as the diaphragm rises, in the opposite direction when the diaphragm falls, and zero when the diaphragm is at rest in any position."

Trowbridge was particularly interested in his indicator as a means for studies of engine operating conditions and showed that the rate of change of pressure diagrams could be interpreted without integration. The following passage is taken from one of his articles:(55)

"-----The experience of the writer has been, however, that this geometrical transformation is by no means necessary, as one quickly acquires a facility in interpreting the original record so that quite as much, if not more, information may be obtained from it as from a record of the pressure with the time. For quantitative, as distinguished from comparative work, however, such as measurement of indicated horsepower, it would be necessary to convert by a simple geometric construction the record directly obtained into the standard form of indicator diagram. It if is not desired to obtain a quantitative value of the indicated horsepower but merely to compare the explosions in a cylinder under different working conditions or to compare the explosions in the several cylinders of an engine, or to determine accurately the effect of slight changes in the timing, lift, etc., of the valves, or in the timing of the ignition, it is not only unnecessary to convert the original experimental records into the standard indicator-diagram form -- it is actually easier and more accurate to make measurements on the original records."

At the time of Trowbridge's experiments, detonation had not yet emerged as a major factor in limiting engine performance so that "knocking" records appear only incidentally in his discussion. Figure 19 shows two engine records (Trowbridge's figures 12 and 13) cited as typical for an engine in operation. With regard to these records Trowbridge says:

"Fig. 12 shows a record of pressure variations in a cylinder at 540 r.p.m., with retarded spark. The indications on the extreme lower edge of the record show the 'in center' and 'out center' positions of the piston. The sudden rapid rate of rise of the pressure shown on the main trace of the record comes slightly after the explosion 'in center' position. The attainment of maximum pressure occurs when this trace crosses the horizontal line drawn through the undisturbed portions of the trace.-----"

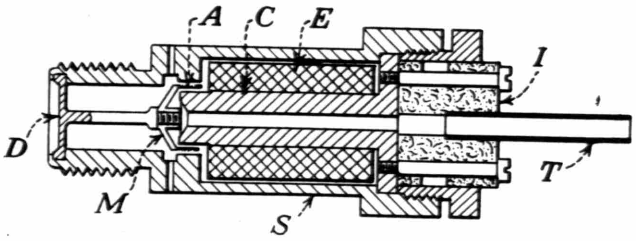
"Fig. 13 shows the same engine running at approximately the same speed of revolution but with the spark more advanced. The rate of rise of the pressure is seen to be more abrupt. Maximum pressure occurs 14.5 deg. of crankshaft after explosion 'in center.' Violent and rapid pressure variations occur during the portion of the cycle just after 'in center,' as shown by the broken character of the trace. This is the record of the 'knock' due to an abnormally advanced spark.-----"

There is not doubt that Trowbridge saw the possible application of an electromagnetic indicator to the study of detonation but his ideas of the detonation process were vague and his indicator certainly did not respond properly to "knock." Many later investigations(5,42, 56,59) have shown that detonation is accompanied by oscillatory variations in the rate of change of pressure with amplitudes of the same order of magnitude as the general cycle changes. The records taken by Trowbridge show oscillations during the expansion stroke both with and without "knock." There is no marked difference in either amplitude or frequency between the two cases. <u>It is thus obvious that the Trowbridge Indicator did not give a true record of the pressure variations accompanying detonation.</u>

The exact conditions of Trowbridge's experiments are unknown but it is certain that the use of a large diaphragm mounted on the end of a relatively long and narrow connecting passage would have distorted the pressure changes in the cylinder before they could reach the diaphragm. Another possibility is that the electromagnetic oscillograph used for recording had such a natural frequency that it was incapable of responding to the high frequency vibrations due to detonation. It is interesting to note that a test made in 1938 by the writer with one of Trowbridge's original indicators showed that detonation strongly excited a natural frequency of about 900 cycles per second. The actual detonation frequency of 6500 cycles per second for the engine used could not be found in the record. From figure 19, the recorded vibration frequency is about 700 cycles per second (the vertical lines are 0.01 second apart). As it is probable that the instruments used in the two cases were almost identical, the oscillations on Trowbridge's sample records could have been caused by the continual excitation of natural frequency vibrations in his indicator.

After the pioneer work of Trowbridge, no further development of the electromagnetic indicator was reported in the literature for some ten years. This lack of interest in a potentially valuable instrument was probably due to an aversion on the part of engineers to the use of a derived quantity instead of direct pressure values and to the lack of a marked difference between detonating and non-detonating records. The writer started the study of detonation as a physical process in his Master's Thesis submitted in June 1928. After several years work with resistance type indicators and preliminary tests on various other instruments he decided that electromagnetic pickup units possess definite advnatages in simplicity, sensitivity, reliability, and ruggedness. In addition, the electrical circuit impedance is so low that extraneous interference can easily be eliminated. In addition, detonation was found to have a much greater effect on the rate of change of pressure record than on the direct pressure card.(56) For these reasons a project for development of electromagnetic indicators was started in 1931 with the collaboration of the staff of the Internal Combustion Engine Laboratory at M. I. T. under the supervision of Professor C. F. Taylor and Professor E. S. Taylor.

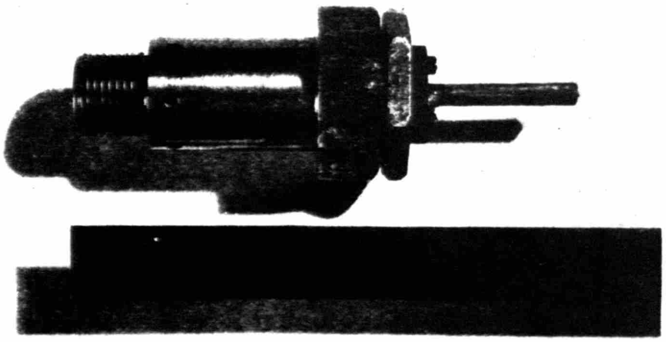
The first electromagnetic indicators constructed at M. I. T. were of the moving coil type as reported in two publications.(56,57) Figure 20 is a diagram of one of the early indicators taken from reference 57. A coil of fine wire A, was wound on a magnesium spider M, which was supported in a magnetic field by the steel diaphragm D.



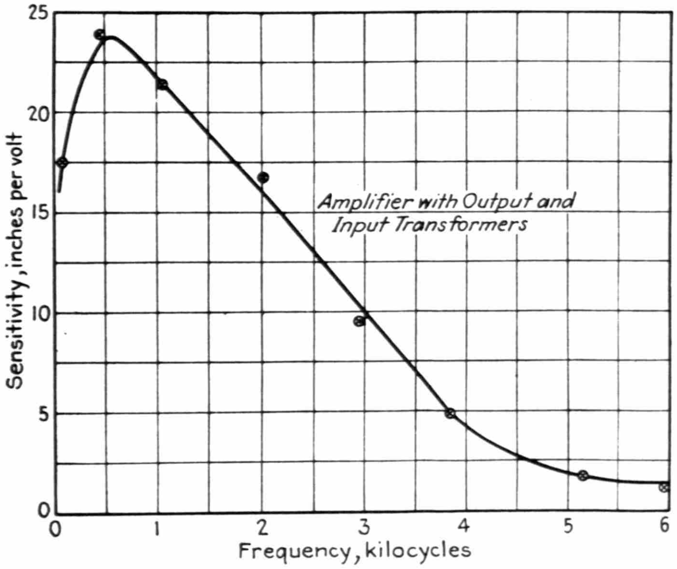
1—Sectional View of Diaphragm Element of Rate of Change of Pressure Indicator

REF. 57

FIG. 20



2—External View of Diaphragm Element



Frequency Response Curve of Amplifier and Oscillograph Combination

REF. 57

FIG. 21

A magnetic field was maintained across the radial air gap which contains the spider M. This field was supplied by the coil E, wound on the soft iron core C. The magnetic path was completed through the outer shell of the instrument, S. The leads to the coils were brought in through the Bakelite block I, and cooling air was introduced through the tube T.

Errors due to connecting tubes and chambers were avoided by directly exposing the diaphragm to cylinder pressures. With the coil structure connected to the center of the diaphragm, the natural frequency was necessarily low, varying between 10,000 and 20,000 cycles per second for different models of the instrument. With the small motions used, mechanical losses due to vibration were so low that the diaphragm was substantially undamped. The detonation frequencies as measured were between 3500 and 7000 cycles per second so that the instrument sensitivity obviously was not independent of frequency. However, the response characteristic of the amplifier-oscillograph combination used for recording also varied greatly with frequency as shown in figure 21. The non-linearity of the complete system was so bad that only frequency data were used in drawing conclusions from the results. The general development of electromagnetic indicators has been continued by the writer as part of his thesis for the doctorate in physics.

A revival of interest in the possibilities of electromagnetic indicators for engine tests has occurred within the last year. In particular, Beale and Stansfield(58) have recognized the usefulness of the rate of change of pressure record both for general performance work and for investigations of detonation. These authors describe (60) the application of an electromagnetic generator to an instrument for measuring detonation called the "Sunbury Knock Indicator" which includes a vacuum tube amplifier and a meter to read knock intensity.

Summary

The records from all types of engine indicators are affected in some way by detonation.

Piston and cylinder instruments with either mechanical or optical indicating systems will show any gross changes in the pressure card in addition to vibrations in the expansion line. In general, these vibrations have appeared to be due to shock excitations of natural frequencies in the indicator mechanism rather than directly to the pressure waves accompanying detonation.

Averaging Indicators show the presence of detonation by erratic disturbances of the combustion line and the expansion line near the peak pressure. With many cycles required to produce a single indicator card it will be impossible to obtain true records from an averaging indicator of the instantaneous pressure disturbances produced by detonation.

Instantaneous Indicators using electromechanical pickup units and oscillographic recording are capable of producing true records of detonation pressure effects. In any case reliable results can only be expected if the instrument is properly designed for the high frequencies encountered in detonation work.

Electromagnetic Indicators which produce direct records of the rate of change of pressure are particularly suited for detonation research work. The impedance of the electrical circuit is so low that extraneous pickup effects are minimumized while the units are simple and rugged with good sensitivity.

The only instrument designed specifically for measuring detonation and in general use at the present time is the Midgley Bouncing Pin. In this device a metal rod rests freely on a metal diaphragm exposed to the cylinder gases. When detonation occurs the high velocity imparted to the diaphragm by the rapid pressure rise causes the pin to jump off the diaphragm and close a pair of spring controlled electrical contacts. The average current flowing in a circuit connected across the contacts is taken as a measure of detonation over small ranges of intensity.

With the exception of two articles by the writer (42,56) the literature does not record any attempts to establish quantitative relationships between experimental data from any indicator and the instantaneous pressure variations accompanying detonation. The observed results have been accepted as pertinent information and interpreted on the basis of experience rather than by the use of analytical methods.

METHODS FOR MEASURING DETONATION

General Problem

Early investigators found detonation measurements a difficult and complex task matter and to make matters worse the problem became more complicated as the general fund of knowledge increased. The tendency of a given fuel to detonate depends upon the particular engine used, the compression ratio, number of spark plugs, location of spark plugs, engine speed, cylinder temperature, spark advance, intake pressure, intake temperature, fuel-air ratio, homogeneity of the mixture, humidity and probably several other factors. Ricardo(61) was among the first to recognize that reproducible measurements of detonation requires the use of a particular engine in connection with a very carefully specified test procedure. However, data determined in this way for a particular fuel was of no practical value to a person who required information with regard to another engine using a different fuel or even the same fuel. Efforts to overcome this difficulty led to the development of the procedure known as knock rating in which the detonation tendency of a fuel is measured in terms of an arbitrary scale by means of a standard procedure on a specified engine. With such a scale established, the tendency of any fuel to detonate in a given engine under any conditions can be predicted more or less exactly if the behaviour of some reference fuel in the given engine is known. This rather indirect method has proved to be the only workable system for engineering measurements of detonation.

In order to rate fuels in an engine some means of comparing detonation intensities must be used. Four methods for estimating particular detonation intensities were found to be practical.(1,34,63)

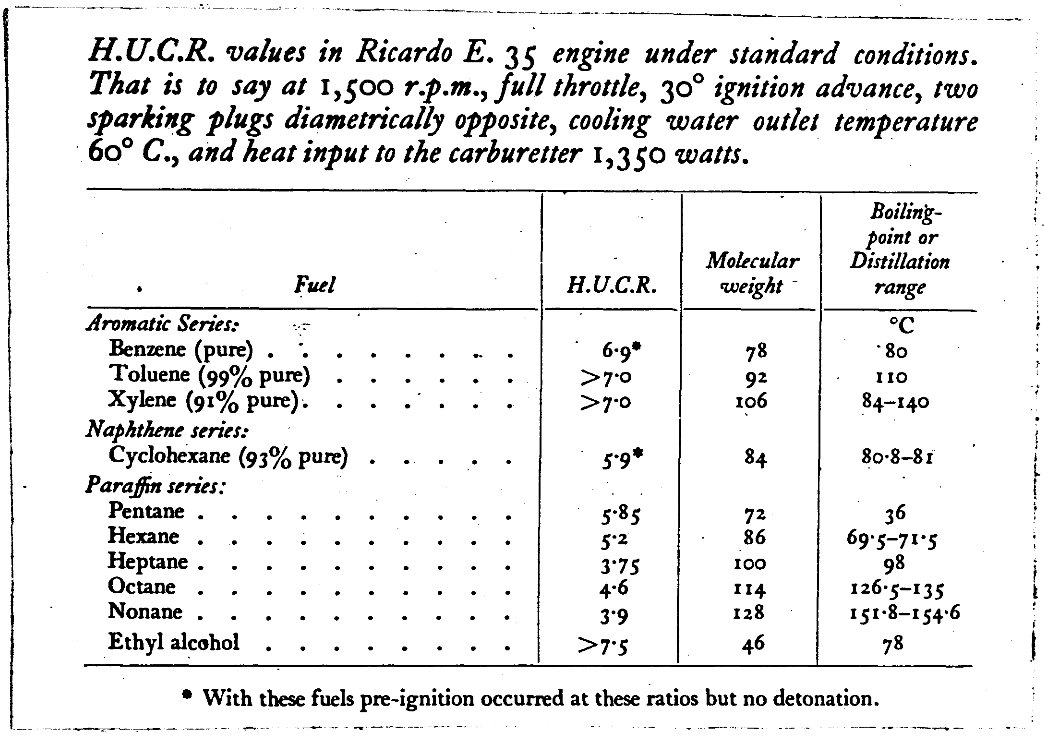
1) Aural detection of the first sound of detonation to establish the condition called incipient detonation, 2) Appearance of the first few abnormally high pressure points on the record of an averaging indicator such as the Farnborough or M.I.T. instruments, 3) Equal readings on the indicator of a Bouncing Pin, and 4) A sudden increase in cylinder head temperature as measured by a suitably mounted thermo-couple. This condition is known as the temperature runaway point (62).

Ricardo 's H.U.C.R.

Ricardo(61) established the first definite system of knock rating fuels and used as the measure of detonation tendency 'the Highest Useful Compression Ratio, or H.U.C.R. The H.U.C.R. was defined as "--the compression at which detonation first became audible with certain definite temperature conditions and with both the ignition and mixture strength adjusted to give the highest efficiency. It is not the highest compression which can be employed, but the highest it is worth while to employ." This figure of merit still leaves a considerable latitude to the individual operators in judging the point when the 'highest compression ratio it is worth while to employ" has been reached. In this connection Pye (1) says "--With the Ricardo E. 35 engine which has been largely used for comparing H.U.C.R. values through its having been the first in the field, the bouncing pin cannot be used for reasons of design; and although the maximum pressure indicator could be used, it is scarcely necessary to do so because auditory methods are quite satisfactory. A skillful observer can repeat observations of H.U.C.R. with fair certainty by auditory methods to 0.05 of a ratio.---"

A list of H.U.C.R. values determined by Ricardo for various fuels are shown in Table I.

TABLE 1



H.U.C.R. values in Ricardo E. 35 engine under standard conditions. That is to say at 1,500 r.p.m., full throttle, 30° ignition advance, two sparking plugs diametrically opposite, cooling water outlet temperature 60° C., and heat input to the carburetter 1,350 watts.

Fuel	*H.U.C.R.*	*Molecular weight*	*Boiling-point or Distillation range*
Aromatic Series:			°C
Benzene (pure)	6·9*	78	80
Toluene (99% pure)	>7·0	92	110
Xylene (91% pure)	>7·0	106	84–140
Naphthene series:			
Cyclohexane (93% pure)	5·9*	84	80·8–81
Paraffin series:			
Pentane	5·85	72	36
Hexane	5·2	86	69·5–71·5
Heptane	3·75	100	98
Octane	4·6	114	126·5–135
Nonane	3·9	128	151·8–154·6
Ethyl alcohol	>7·5	46	78

* With these fuels pre-ignition occurred at these ratios but no detonation.

The table not only shows the range of values found in using Ricardo's method but also demonstrates that the paraffin series compounds which form the largest part of commercial gasoline are the worst offenders from the standpoint of detonation.

Although of great value for identifying desirable and undesirable chemical constituents of internal combustion engine fuels, the H.U.C.R. never came into wide use for commercial purposes. With good detonation resisting properties worth considerable premiums in money it became necessary to find a method which would give consistent results in the hands of any trained laboratory staff. A system of expressing detonation tendencies in terms of a fuel mixture has proved to be better suited for commercial purposes than the engine adjustment scheme of Ricardo.

The Reference Fuel Method

Midgley and Boyd (25) in 1922 found that in certain cases the detonation tendency of a mixture of two fuels varies in an approximately linear manner from one pure fuel to the other pure fuel. By 1927 the technique of knock rating had developed to such a point that fuel mixtures could be used quantitatively as references in detonation measurements (64,65,66). Edgar(64,65) proposed a universal knock rating scale in terms of iso-octane and normal heptane, two compounds of like volatility in the proper range and both of which could be obtained in a state of high purity. Heptane showed a very pronounced tendency to detonate while iso-octane was practically a non-detonating fuel.

The CFR Committee

The matter of accurate knock rating of fuels had become so important that the problem was attacked systematically by a group of interested organizations in the United States. A brief history of the so-called CFR committee is given in a bulletin of the manufacturer (67) who supplies knock rating equipment:

"Recognition of the interdependence of engine performance upon compression ratio and fuel anti-knock property, as taught by Ricardo, prepared the way for a co-operative development by the fuel and engine producers. The advent of modern cracking processes disclosed the wide range of anti-knock quality existing in fuels and emphasized the necessity for a standard test method. In 1928, a far-sighted group of fuel producers and engine manufacturers who had formed the Co-operative Fuel Research Committee, assigned the problem of developing a test method and equipment for knock rating to a group designated as the Detonation Sub-Committee.

"The first meeting of this latter group disclosed three items essential to the pursuit of a comprehensive motor fuel research: (1) a standardized engine and accessories; (2) a common reference fuel and rating scale; (3) a uniform testing procedure. By 1931, all three had become realities. Waukesha Motor Company had developed a fuel testing engine meeting the approval of the Co-operative Fuel Research Committee. Dr. Graham Edgar of the Ethyl Gasoline Corporation had suggested Iso-Octane and Normal Heptane, both stable hydrocarbons as reference fuels. These were made commercially obtainable and a rating scale based upon them was adopted and a uniform test procedure was tentatively approved.

"In 1932, a correlation of laboratory knock ratings with the behavior of motor fuels in service was undertaken. Road tests at Uniontown, Pennsylvania, and subsequent correlation at the Waukesha Motor Laboratories brought about a modification and improvement of the standard testing unit and its operating technique. This improved test method was named the "CFR Motor Method"; but the old method was not discarded, since it had proved itself indispensable from the experimental standpoint as an aid to the refiner in developing suitable gasolines and to the automotive engineer in improving the combustion characteristics of his engine. It was retained under the name "CFR Research Method."

"Since the problem of correlating road knock ratings with those found in the laboratory was of primary importance, and since improvement in commercial engine designs and in fuels was constant, additional road checks were made in 1934. These later tests, also held at Uniontown, with nearly twice as many motor car builders participating as in the first program, confirmed the findings of the 1932 tests.

"To maintain the greatest possible degree of accuracy, a group of the Co-operative Fuel Research member laboratories now conducts co-operative tests each month. Results are tabulated and circulated so that each of the co-operating laboratories may compare its results with the entire group's average. The CFR Detonation Sub-Committee with the co-operation of the U.S. Bureau of Standards is working continually on research projects to improve knock rating technique. This continuous development program insures the maintenance of accuracy in the CFR unit as well as in the testing technique, and the improvements are made available to CFR engine owners. The proved reproducibility of results by the CFR method has compelled its acceptance and recognition as the industry's yardstick for gasoline motor fuel rating. Approval by the American Society for Testing Materials resulted in its receiving ASTM designation D-357-36T.

"At the present time, the following organizations have formally endorsed the ASTM CFR unit and technique, and given their official acceptance: The U. S. Bureau of Standards, American Society for Testing Materials, Society of Automotive Engineers, American Petroleum Institute, and the Automobile Manufacturer's Association (formerly National Automobile Chamber of Commerce).

"In October, 1936, the Secretary of the Chemical Standardization Committee of the Institution of Petroleum Technologists of London reported that the CFR Motor Method was officially adopted by the Standardization Committee and published in the 1935 edition of the Institution's 'Standard Methods for Testing Petroleum and Its Products' under the I. P.T. Serial Designation G.39(T). Slight modifications of the ASTM testing technique were incorporated, but no amendments were made with regard to the engine itself. Thus, the ASTM CFR Octane Rating Unit is the international yardstick for gasoline fuel rating. The ASTM-CFR Knock Rating Method".

The historical outline above has been included in full because it gives a concise picture of the reasons for knock rating, the general methods employed and a hint of some of the troubles encountered. Before further discussion of the general problem it is desirable to review certain parts of the detailed technique for knock rating as described in the ASTM publication D 357-36T (68). Certain pertinent sections will be quoted below.

> "1. This method is intended for determining the knock characteristics, in terms of an arbitrary scale of octane numbers, of gasolines and equivalent fuels for use in spark-ignition engines,

other than engines for aircraft."

ASTM Octane Number

"2. The ASTM octane number of a motor fuel is the whole number nearest to the octane number (note 1) of that mixture of iso-octane and normal heptane which the motor fuel matches in knock characteristics when compared by the procedure specified herein.

"Note 1. Octane number is defined by and is numerically equal to the percentage by volume of iso-octane (2,2,4-trimethylpentane) in a mixture of iso-octane and normal heptane, used as a primary standard for measurement of knock characteristics. Thus, by definition, normal heptane has an octane number of zero and iso-octane of 100."

Apparatus

"3. The knock-testing unit described in this section shall be used without modification. The engine shall be known as the CFR Engine and shall be marked by plate or other approved means------"

"The apparatus shall consist of a continuously-variable-compression motor together with suitable loading and accessories as follows --------"

"Knock intensity is measured by a bouncing pin, in conjunction with a knockmeter. (The knockmeter is a damped hot-wire ammeter which indicates the effective current in the circuit, thus permitting instantaneous readings). Current is supplied from a small direct-current generator, belt driven from the power absorbing unit. ------------"

Reference Fuels

"5. Primary Reference Fuels.-- The primary reference fuels shall be iso-octane (2,2,4-trimethylpentane), and normal heptane (note 1). Both shall be certified for suitability as primary reference fuels by the National Bureau of Standards ."

"Note 1. Secondary Reference Fuels.--Mixtures of normal heptane and iso-octane required for referee testing are expensive. For this reason secondary reference fuels may be used for routine determinations. Such secondary reference fuels may be straight-run or other stable gasolines suitable for the purpose. One of the reference fuels should be of low knock rating and the other of high knock rating, or if a sufficiently high knock-rating fuel is not available, a mixture of the higher-knock-rating fuel plus a knock suppressor may be used. These secondary reference fuels shall be calibrated on the octane-number scale against normal heptane and iso-octane sufficiently often to insure accuracy of calibration and for every case, whether a fuel is rated by secondary reference fuels or by means of normal heptane and iso-octane, the result shall be recorded as an octane number. --------- "

Bouncing Pin Assembly.--"The gap setting shall be 0.003-in. to 0.005-in.* ----------"

Preliminary Adjustment of Compression Ratio

"8. Using a mixture of 65 parts of iso-octane and 35 parts of normal heptane, the compression ratio for first audible knock shall be obtained by increasing the compression ratio, by increments of two turns of the crank, from a point where there is no knock to the compression ratio at which audible knock is first detected. The proper knock intensity for use in making knock ratings shall be the knock intensity obtained with the mixture of 65 parts of iso-octane and 35 parts of normal heptane when the compression ratio is in-

*At this point detailed instructions for the mechanical procedure in adjusting the Bouncing Pin.

creased one unit over that compression ratio giving first audible knock. Then the numerical indication of knock intensity obtained from the knockmeter shall be recorded. This procedure is necessary for the first adjustment only.

Note. This knock intensity should be equivalent to the intensity obtained with a mixture of 65 per cent of iso-octane and 35 per cent of normal heptane at a compression ratio of 5.3±0.05 to 1 when testing at a barometric pressure of 760 mm.

For subsequent test on fuel samples the compression ratio shall be set to duplicate the knock intensity as recorded above, provided no change has been made in the bouncing pin adjustment in the meantime. In no case shall the knock intensity be such that the engine does not cease firing when ignition is interrupted."

Outline of Procedure.

"9. The octane number of a fuel shall be ascertained by comparing the knock intensity for the fuel with those for various blends of the reference fuels until two blends differing in knock rating by not more than two octane numbers are found, one of which gives a higher knock intensity shall be measured by a bouncing pin indicator in conjunction with a knockmeter."

"Before the test sample and the blends of the reference fuels can be compared, the compression ratio must be set to give the proper knock intensity and the carburetor adjusted to give the maximum knock for each fuel. ---------"

"11. Finally, adjust the compression ratio to give the same reading on the knockmeter when using the fuel under test as was obtained in the first adjustment under section 8. --------- "

Octane Number Determination

"13. With the carburetor set for the air-fuel ratio of maximum knock, alternate series of readings of knock intensity shall be taken on the fuel under test and a reference fuel blend. The knockmeter needle shall be allowed to reach equilibrium before the final reading is recorded.

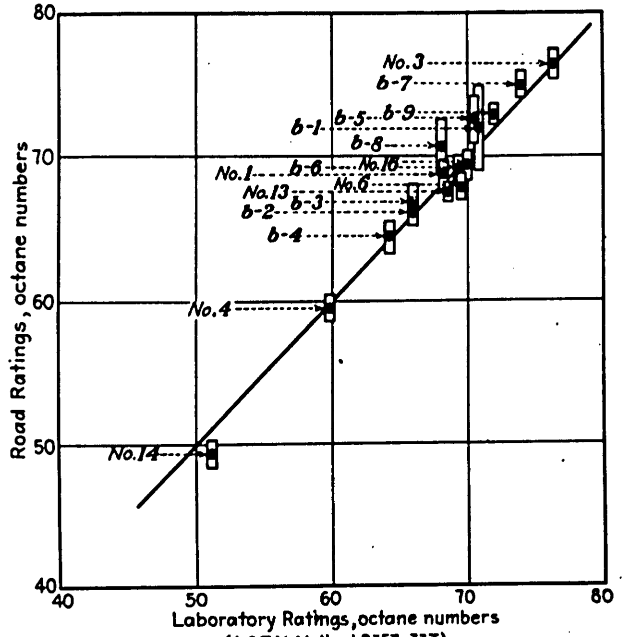
"At least 3 alternate series of readings shall be taken on each fuel. After changing from one fuel to another, at least one minute shall be allowed for the engine to reach equilibrium. With some fuels, an appreciably longer time interval may be required.------

> "The test shall be continued in this manner until the knock intensity for the fuel sample is definitely higher than one blend and lower than another blend of the reference fuels. The difference between these two final reference fuel blends shall not be more than two octane numbers."
>
> "14. The knock rating of the fuel sample shall be obtained by interpolation from the figures so recorded and the nearest whole number reported as ASTM octane number."

A long series of tests using the ASTM-CFR technique in many laboratories has shown that the gneral method will produce sufficiently consistent results for ordinary commercial purposes. However, Veal(69) in reporting on the 1934 Road Tests notes that "----- The Coöperative Fuel Research at present embraces a number of projects directed toward the goal of securing more accurate laboratory knock ratings,----- (1) Preparation of a more precise definition of the procedure for setting test conditions, particularly in respect to the knock intensity employed. (2) Determination of the influence of altitude upon knock ratings. (3) Determination of the influence of exhaust-system characteristics upon knock ratings. (4) Determination of the effect upon ratings of Uniontown fuels of reducing mixture temperatures by stages from 300 to 200° Fahr. (5) Improvement of the bouncing-pin indicator.---" This list of research projects nicely illustrates the apparently trivial things which are important in the art of knock rating.

Correlation of Laboratory and Road Test Octane Ratings

In spite of recognized possibilities for improvement, the laboratory rating of fuels seems to be in a satisfactory state as an empirical process. However the correlation of laboratory tests with the behaviour actually observed in engines under service conditions has always given trouble.(69) That is, a fuel rated inferior to another in the laboratory may prove to give better performance in a given engine under certain conditions, while these positions may be reversed in the same engine under other conditions. This matter was thoroughly investigated with automobiles in two series of road tests as mentioned above. Figure 22 from reference 69 shows the correlation between laboratory octane numbers and octane numbers determined by a specified procedure in the road tests. The letter and number designations refer to particular fuels and the rectangles represent the spread of values found in various automobiles. In these tests detonation



Correlation between Road and Laboratory Ratings

REF. 69

FIG. 22

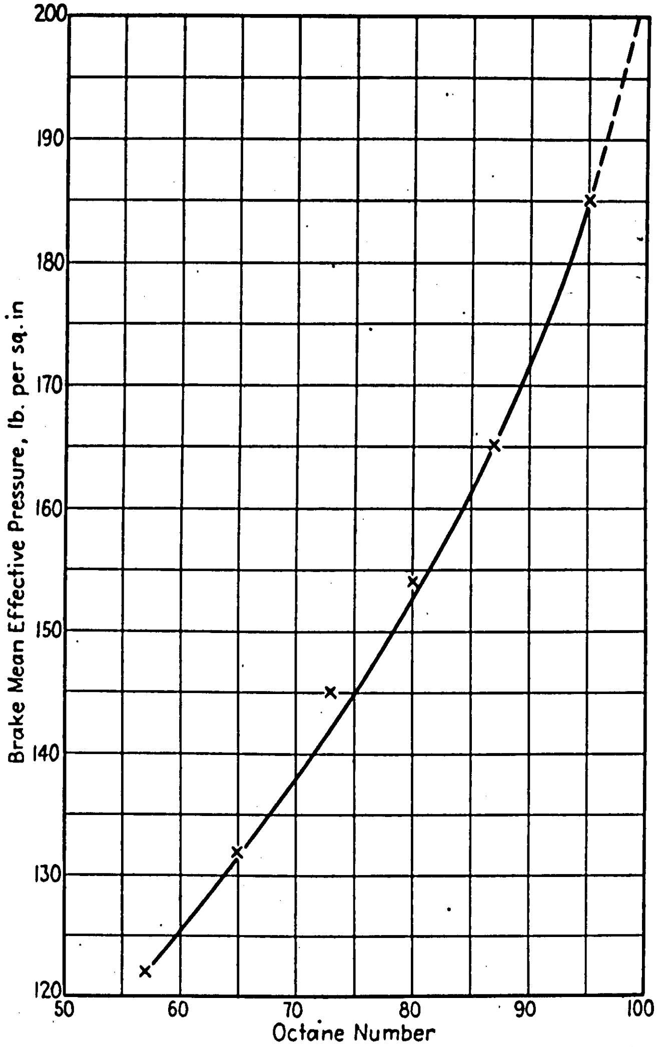


FIG. 23

Limitation Imposed on B.M.E.P. by Octane Number of Fuel with Full-Scale Air-Cooled Engine Normal Operation at Sea Level REF. 62

intensities were determined with an auditory method by a number of observers and the recorded result taken as an average.

Correlation of Laboratory and Full-Scale Octane Ratings for Aviation Engines

With the correlation of laboratory and road test octane numbers fairly well settled, the CFR Committee has turned to the more serious problem of detonation in aviation engines. In the first report of the CFR Committee on the rating of fuels in full-scale aircraft engines Veal(62) says: "-----The penalty of an invalid knock rating for automobile fuel is small. It may be expected that the automobile engine will knock and, under conditions of continued use of such fuel, damage to the engine will undoubtedly result. With the aviation engine the problem is not so simple. If we have an aviation engine designed to give its maximum at take-off on fuel of 87 octane number and the fuel used rates 87 octane number by the CFR Motor Method, actually rates but 83 octane number on the full-scale engine, almost certain disaster is invited. In the relatively short period of high-output operation at take-off, the detonation induced may be of sufficient intensity to cause engine failure and a crash result."

To stress the importance of accurate fuel knock rating, Veal(62) gives a plot (figure 23) showing the relation between the possible brake mean effective pressure for continued operation with a certain engine and octane number of the fuel used. He says: "----With a slope of this character is it not to be expected that the engine user will willingly sacrifice one octane number nor will he be satisfied with a test method which may require a tolerance of three or four octane numbers for lack of correlation with the full-scale engine in addition to the normal tolerance of two octane numbers allowed for test errors.

It should be borne in mind that:

(1) Engine performance is given and guaranteed contingent upon the use of a fuel having a specified minimum knock rating.

(2) Requirements of modern aviation practice do not permit large sacrifices in critical take-off power to compensate for uncertain knock values. Neither do they permit the purchase at increased cost, of fuels of higher knock value offering a substantial factor of safety for unreliable knock values."

Air Corps Method

Before the problem of fuel performance in aviation engines was considered by the CFR Committee, the U. S. Army Corps had found that laboratory octane ratings did not give reliable information on the order of knocking tendencies for fuels in actual service. To overcome this difficulty, the Air Corps developed a technique of knock rating on full-scale engines and issued specifications based on this method.(70) With the severe vibration and the high noise level always accompanying the operation of aviation engines, it was impossible to use either the bouncing pin or auditory methods for determination of knock intensities. In addition, the detonation noise considered objectionable in automobile engines, was of no interest in aircraft engines, the stress being entirely upon power output and fuel economy. These new requirements produced a method of knock rating entirely different in its mechanical details from the CFR scheme but retaining the reference fuel scale of octane numbers. The essential features of the aircraft engine knock rating procedure and evaluation of results are outlined below in quotations from Veal's(62) article:

CFR Method for Full-Scale Knock Rating in Aviation Engines

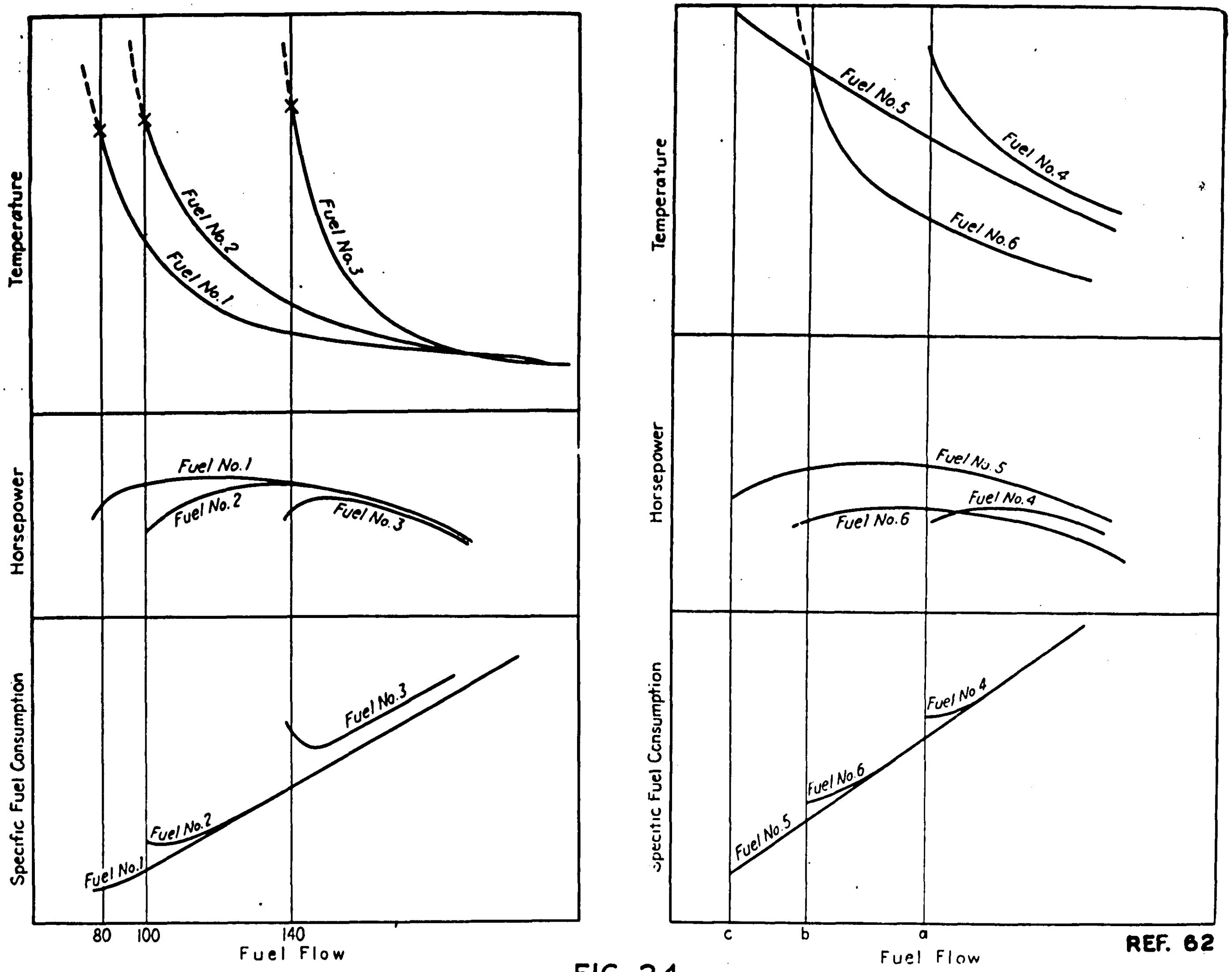
"The original test procedure directed the operator to adjust the engine by throttling, if necessary, until it just gave satisfactory performance on one of the reference fuels, and with the throttle locked in this position, to make mixture control runs on the reference fuel and on a group of test fuels until the fuel having performance characteristics most like those of the reference fuel was determined. After the first tests were completed, it was found desirable to modify this procedure to the extent of recommending that each reference fuel be bracketed between two test fuels -- one higher and one lower than the reference fuel. The method of comparison was to vary the mixture control with each fuel from full-rich to minimum-allowable total fuel flow and to record cylinder temperatures, fuel flow, and power output at each mixture-control position.-------"

"The general principle followed in the appraisal of the test data has consisted fundamentally in determining the point (as indicated by cylinder temperatures) beyond which difficulties due to fuel limitations would definitely be encountered. This point is usually well defined for any single run on a given fuel. The events immediately preceding the attaining of this point in operation varied considerably from run to run, particularly with different engines. It, therefore, becomes impossible to establish a simple rule for locating the "knocking" point on the various data sheets. Rather the method of actually locating the "knocking" point must be determined from a thorough appreciation of the factors affecting operation of aircraft engines under the test conditions employed.------"

"In a few instances it has been possible to 'bracket' one or more test fuels between two reference fuels; while in most other instances the converse has been followed-in which a reference fuel has been bracketed between two test fuels. It would have been desirable, of course, to bracket all test fuels between suitably-spaced reference fuels. Operation under critical conditions, with fuels of widely varying knock rating at any one manifold pressure, was obviously impractical. The fuel of low octane number in such a group suffers such an abrupt transition to severe knocking that actual measurement of any considerable temperature rise is impractical."

"In a sense this limitation simplified the problem of evaluating differences, as the greatest range to be so covered was rarely more than three or four octane numbers. The effects of small variations in output and fuel consumption rarely amounted to more than one octane number.----"

"Within the foregoing general principles at least two different appraisal procedures may be used: (1) simple inspection of individual 'case records.'


Temperature
Horsepower
Specific Fuel Consumption
Fuel No.1
Fuel No.2
Fuel No.3
80
100
140
Fuel Flow
Fuel No.4
Fuel No.5
Fuel No.6
c
b
a
Fuel Flow

REF. 62

FIG 24

(2) Use of an indexing 'formula' in which the various pertinent factors are assigned definite relative weights in advance, and all results evaluated in accordance with the rule so established.----"

"The plots of figure 24 are samples of the data taken during knock rating runs on full-scale engines. It will be noted that as the mixture is leaned out, producing a reduced fuel flow to the engine, the cylinder head temperature starts to rise, at first slowly and then very rapidly until the 'runaway point" is reached and the cylinder head will fail from thermal weakening of the material if detonation is allowed to continue.

"Match Number" Method of Correlation

"One method of analysing the data involved the use of a "match number". In regard to this quantity, Veal(62) says;

"If two fuels (which have been run at a single throttle opening without interruption of the run and on a mixture-control curve from full-rich mixture to the leanest mixture possible without encountering excessive head temperatures) are to be compared at either a single fuel-flow rate or at a particular head temperature, it is obviously necessary that such a basic datum point for the comparison must be determined by the limitations of the poorer fuel. As the match-number method is based on performance differences at a fixed fuel flow, the leanest mixture attainable with the poorer fuel is employed. The fuel first reaching a critical head temperature is the poorer fuel, as it is not possible further to lean out the mixture with it, and the other fuel or fuels are compared with it at its leanest point of useful performance. In rare cases neither fuel reaches a critical head temperature, but the poorer fuel will reach its maximum head temperature at a higher fuel flow than the other, and the comparison is made at this fuel flow."

"With the datum point thus chosen, an empirical equation is used to correlate the data.

Equation for Match Number:

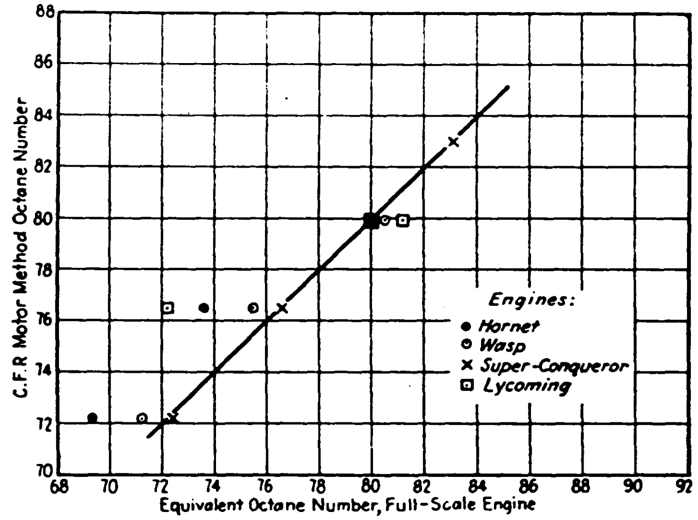
$$100/\text{specific horsepower output} + \text{maximum head temperature} + \text{s.f.c.}(1000) = \text{match number} \qquad (1)$$

where

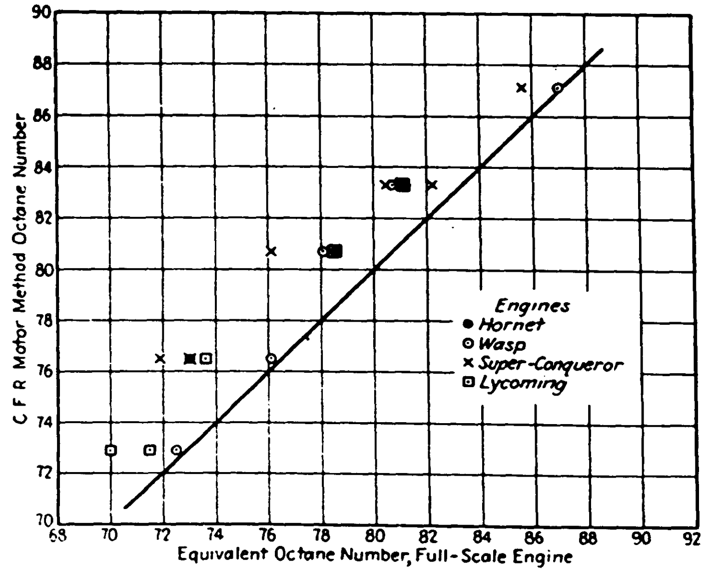
specific horsepower output = Horsepower/displacement in cu. in.

s.f.c. = specific fuel consumption in lb. per horsepower-hour.

"The lower the match number, the better the fuel. Experience with stable fuels and use of the match number for a large number of data from these as well as other tests indicates that a difference in fuels of one octane



Correlation Chart for Test Fuels (b) and (b) Plus Lead—Full-Scale Engine Ratings by Inspection



Correlation Chart for Test Fuels (c) Plus Lead —Full-Scale Engine Ratings by Inspection REF. 62

FIG. 25

number is substantially equivalent to 15 points difference in the match numbers. If this ratio is accepted for the range of fuels covered in these tests, it is possible to make quantitative comparisons between reference and test fuels for each run."

Figure 25 shows the correlation between octane numbers determined from the CFR Motor Method and full-scale engine tests interpreted by the procedure outlined above. The correlations indicate that the full-scale octane numbers are usually lower than the CFR Motor Method octane numbers with a maximum difference of about five octane numbers. Such a condition is recognized not only as undesirable but dangerous and the CFR Committee is now carrying out a program of intensive research on the knock rating of aviation fuels.

Summary

In reviewing the methods which have been successful for detonation measurements, one striking fact is that no use is made of physical or chemical analysis in any part of the process. The investigators have used the tools which worked in practice and obtained consistent results by detailed specification of the equipment and procedure to be used in making measurements on an arbitrary scale.

Despite the lack of perfect correlation between octane numbers determined in the laboratory and octane numbers from full-scale engine tests, the whole matter of knock rating seems to be on a solid foundation both as regards consistency and test procedure. The whole system is so firmly intrenched in the petroleum and automotive industries that any suggested change in apparatus or method would have to be a great improvement over the CFR Method before it could expect to be adopted for commercial use.

SECTION II

DETONATION AS A PHASE OF COMBUSTION

DETONATION AS A PHASE OF COMBUSTION

Historical Background

Combustion has played a long and honorable role in the history of science. The story of this role is told in an interesting book by Bone and Townend (71) called "Flame and Combustion in Gases." This book will often be used as a reference in the following pages. A discussion of combustion researches, including an extended bibliography of pertinent literature, is given by Rosecrans and his collaborators (72, 73) in Bulletins of the Engineering Experiment Station of The University of Illinois. Other brief reviews of combustion work have been presented by Clark and Thee (74) and Duchêne (75). A general technical discussion of gaseous combustion is given by Haslam and Russell in a book entitled "Fuels and Their Combustion" (76). It is impossible to cover completely such an extensive subject as combustion in the present report so the treatment will be limited to a discussion of those aspects which are related to internal combustion engine problems.

Combustion experiments started as a phase of chemical research in the years 1660-1670 when Robert Boyle, Robert Hooke and John Mayow worked toward a clarification of certain ideas handed down from the Alchemists. Although stated in somewhat different terms, the concept of combustion expounded by these early workers was substantially in accord with the results of modern research. In the years 1680 to 1800 the 'phlogiston' theory of combustion came into general acceptance even in the face of definite contradictory evidence. All combustible bodies were held to contain at least two principles, one of 'combustibility' called 'phlogiston', which escapes during combustion and the other of 'incombustibility', which remains behind. A metal was supposed to be composed of 'phlogiston' and an earthy residue which remained behind upon heating the metal. Substances, such as coal, charcoal, sulphur or phosphorus were supposed to consist so largely of 'phlogiston' that after its escape by combustion little or no residue remained.

With regard to the overthrow of the phlogiston theory, Bone and Townend(71) say: "------It was Lavoisier, however, who first realised that the discoveries of Black, Priestley, Scheele and Cavendish were incompatible with the old theory of phlogiston, and using their discoveries supplemented by his own experiments, he finally overthrew it and established his oxygen theory of combustion. -------In 1772 he commenced to experiment for himself on combustion, and found that when sulphur and phosphorus are burnt, so far from there being any loss, a great increase in weight occurs. ------Lavoisier therefore concluded that a large quantity of air becomes fixed in combustion and in order to confirm this view, he reduced litharge with charcoal, finding that a large amount of air was liberated. ------Also he found that only a portion of the enclosed air was taken up by the molten metal, the residual air being inert".

The technique of measuring gas volume changes at constant pressure which had been developed to decide between fundamentally different theories of matter was used in 1804 by John Dalton for studying combustion of simple hydrocarbons. The results of these experiments enabled Dalton to illustrate the law of multiple proportions as required by his new atomic theory.

Sir Humphrey Davy was first to study combustion from an engineering standpoint when he investigated the inflammability of coal mine gases between the years 1815 and 1819. He applied the already available methods of gas analysis and extended the previous work by a systematic study of combustion in gaseous mixtures. He established two important facts: 1) each inflammable gas has a definite limited range of mixture strength with air or oxygen within which explosions are possible, and 2) some inflammable mixtures are more easily ignited than others. In addition to his general contributions, Davy invented the Miner's Safety Lamp and discovered catalytic combustion.

Bunsen(82), working in the years 1836 to 1880, brought the art of gas analysis by means of combustion and selective absorption almost to its present-day state. In

1857 he made the first recorded measurements of flame speeds in explosive mixtures. In his experimental method he lighted the explosive mixture as it issued at a known velocity from an orifice. The velocity of flow was then gradually reduced until the flame just struck back through the orifice and ignited the mixture in the tube. Bunsen also receives credit for making the first observations on the pressure developed by a gaseous explosion in a chamber at constant volume. His vessel was a strong glass tube closed at one end by a valve held in place by a weight. The mixture was ignited by an electric spark and the resulting pressure estimated by finding the limiting value of the weight which would just allow the valve to open. This crude experiment was the prototype of many elaborate investigations which were later conducted on combustion at constant volume. By the end of the 'Bunsen Era', general methods were well established for accurately determining the overall effects of explosions in changing the chemical composition of gaseous mixtures and the stage was ready for the opening of the 'Modern Period' which started in 1880 with the work of Mallard and Le Chatelier(9).

With regard to the Modern Period of combustion research Bone and Townend(71) say: "The first decade of the period was ushered in by the brilliant work of four French investigators. This was marked by the application of photographic methods by Mallard and Le Chatelier to the investigation of the development and propagation of flame in explosive mixtures, and by the discovery of the phenomenon of detonation by them and by Berthelot and Vielle simultaneously, revealing flame speeds far in excess of any previously contemplated, and comparable with the velocity of sound through gaseous media.

"Mallard and Le Chatelier chiefly concerned themselves with the systematic investigation of phenomena associated with the initial stages of gaseous explosions up to the point when detonation is set up; they discovered and investigated (1) what has been termed the initial period of 'uniform slow velocity' in which the flame is propagated from layer to layer of the explosive mixture in an open tube 'mainly by conduction' uninfluenced by compression waves or other accelerating causes, and (2) the 'vibratory period' which in certain definable circumstances intervenes between the said initial phase and that of detonation. The beautiful photographic method which they introduced and developed for this purpose constituted an entirely new and fruitful line of experimental attack, and is still being successfully applied to-day in research laboratories the world over.

"Berthelot and Vieille not only discovered the development of 'detonation' in explosions, but were the first to devise and successfully apply a method for measuring accurately the high uniform flame speeds involved. Not only so, but Berthelot essayed the first theoretical exposition of the nature of what he termed 'l'onde explosive' a problem of great intricacy which has since occupied the attention of several emminent mathematical-physicists, although it is not yet entirely solved. Berthelot likened the velocity of 'l'onde explosive' to that of a sound wave passing through the gaseous mixture, with, however, the important difference that, whereas a sound wave is propagated from layer to layer with a small compression and velocity determined solely by the physical condition of the vibrating medium, it is an abrupt change in chemical condition which is propagated in the explosion wave, and which generates an enormous force as it passes through each successive layer of the medium.

"All the four French investigators successfully applied themselves to the determination of the pressures developed during gaseous explosions in closed vessels, a problem which had remained untouched since Bunsen's experiments in 1867. They confirmed his observations that the pressures realized fall far short of those calculated on the assumptions of adiabatic combustion conditions and constancy of the specific heats of the products; but they rejected his interpretation of the discrepancy, and ascribed it mainly to an increase in specific heats of the products with temperature, which is now recognized as being a potent contributory cause.

"The problem was also taken up in this country independently by Dugald Clerk, who, applying thermodynamical methods to the pressure-time curves, greatly advanced our knowledge of pressure development in such explosions; and, while questioning the conclusion of the French investigators concerning increase in the specific heats of products with temperature, he suggested 'delayed combustion' as a probable contributory factor to the so-called pressure suppression' in gaseous explosions.------

"It was in 1880 that H. B. Dixon, in repeating Bunsen's experiments on the division of oxygen between carbonic oxide and hydrogen, when both are present in excess, discovered that a mixture of carbonic oxide and oxygen, dried by long contact with phosphoric anhydride, would not explode when sparked in the usual way in an eudiometer over mercury, although the presence of even a trace of moisture at once rendered the mixture explosive. These experiments, proving as they did the complexity of what at first sight would appear to be one of the simplest cases of combustion, opened up an entirely new field of scientific investigation, and propounded questions of the highest theoretical importance which are still occupying the attention of chemists.------

"H. B. Dixon's other principal investigations, which have extended uninterruptedly over a period of nearly fifty years have comprised (i) the systematic determination of 'rates of explosion' in which he greatly extended the work of Berthelot and Vieille, (ii) the photographic analysis of explosion flames, in which he followed up and extended the earlier work of Mallard and Le Chatelier, and (iii) the determination of ignition temperatures of gaseous explosive mixtures.

"The second decade of the period under review was marked by new discoveries and controversies concerning the structure and luminosity of flames and the mechanism of hydrocarbon combustion. The rediscovery (1892), in H. B. Dixon's laboratory in Manchester, of what originally Dalton had found concerning the explosion of ethylene with its own volume of oxygen, together with the simultaneous observations of Smithells and Ingle (1892) upon the interconal gas of hydrocarbon flames, and the work of Lean and Bone (1892) and of Bone and Cain (1895) upon the explosion of ethylene or acetylene, respectively, with less than its own volume of oxygen, gave the time-honoured doctrine of the ' preferential combustion of hydrogen' in hydrocarbon flames its death blow. The mechanism of the combustion of hydrocarbons was afterwards systematically studied by W. A. Bone and his various collaborators (1902-1912), whose results proved it to involve essentially the 'hydroxylation' of the hydrocarbon molecule followed by secondary decompositions, according to temperature conditions, of the primary hydroxylated products.------

"During the past fifteen years R. V. Wheeler and his collaborators have paid great attention to the measurement of the speeds of the initial uniform flame propagation through gaseous explosive mixtures. Applying with certain refinements, the photographic method originally devised by Mallard and Le Chatelier, as well as a 'screen-wire' electrical method on similar lines to those of previous workers, they have corrected some errors in the earlier French results, and published new data and curves for a large number of combustible gas and air mixtures.------

"During the same period much attention has been directed to the electrical ignition of gaseous explosive mixtures, chiefly as the outcome of W. M. Thornton's work, in which it was proved that for given sparking conditions (e.g. electrodes, type of discharge, voltage, etc.,) a certain minimum spark energy is required to ignite a given explosive mixture and the question has naturally arisen as to whether (or how far) ignition and flame propagation are conditioned by ionization.------

"Since 1921, W. A. Bone, in conjunction first of all with the late W. A. Haward, and afterwards with D. M. Newitt and D. T. A. Townend, has published a series of memoirs upon "Gaseous Combustion at High Pressures" in which the behaviour of hydrogen-air and carbonic oxide-air mixtures, on explosion in specially designed bombs at initial pressures up to 175 atmospheres, has been investigated and new factors brought to light.------"

The historical outline of Bone and Townend serves to suggest the problems which have appeared in connection with combustion in gases but does not mention any of the extensive researches which have been recently carried out on combustion in the internal combustion engine. Since 1920 so many investigators have simultaneously worked on various phases of engine combustion that a satisfactory discussion based on historical precedence is difficult for this period. For this reason the treatment of combustion that follows will be divided according to subjects rather than chronological order.

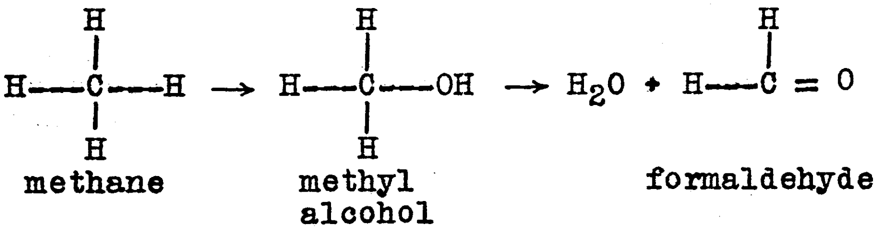
Chemical Processes in the Combustion of Hydrocarbons

Most of the precise work on detonation has been carried out with definite chemical compounds since this procedure was necessary in order to obtain reproducible results. However, commercial internal combustion engine fuels are almost all hydrocarbons of indefinite and variable composition. For this reason, the combustion process involved in the explosion of typical hydrocarbons is of great interest. The work of Bone and his collaborators in this connection has already been mentioned. These investigators(71) observed the chemical changes occuring at temperatures between 300 and 500 degrees C in mixtures of oxygen and various pure hydrocarbons. Two experimental methods were used; (1) the gaseous reagents were sealed in carefully cleaned glass bulbs and placed in a constant temperature oven and (2) the reagents were continuously circulated in a closed system including a tube heated to the desired temperature. With these procedures, the combination of oxygen with the fuel was so slow that the process could be followed by taking samples at intervals and using the ordinary methods of gas analysis. Similar controlled experiments for explosive reactions were carried out by the same workers using; (1) closed vessels of constant volume, (2) 'detonation' in a long tube under conditions where the combustion would be caused by adiabatic compression in the explosion wave and (3) steel bombs in which high ? initial pressures were possible. From the evidence gathered in these experiments the 'hydroxylation' theory of hydrocarbon combustion was evolved.

Bone and Townend(71) explain the 'hydroxylation' theory as follows:

"The idea underlying this theory is that, when a hydrocarbon is oxidised, there is a natural tendency for its hydrogen atoms to be successively converted into OH groups, thus producing 'hydroxylated' molecules with consequent heat evolution. The stability of such molecules would vary very greatly with temperature and other circumstances; some of them, more particularly those containing two such OH groups attached to one and the same carbon atom, would be very unstable in any circumstances, and would quickly decompose into stabler products. Thus, for example, the almost instantaneous decomposition of dihydroxy-methane into formaldehyde and steam is readily understood. The theory, therefore, supposes that the course of the slow oxidation of such hydrocarbons as methane, ethane and ethylene is essentially one of successive 'hydroxylation' stages, with evolution of heat, accompanied by (according to circumstances) the thermal decomposition of unstable 'hydroxylated' molecules into simpler products, which latter may undergo further oxidation in like manner. And, according to such a view, neither steam nor oxides of carbon are immediate oxidation products, but arise as the result of thermal decomposition of unstable hydroxylated molecules.---------The theory will, perhaps, be best understood by considering the following scheme showing how it would explain the slow combustion of methane-------at temperatures between 300 and 400 degrees C:------"

"1. At temperatures below the ignition point, the hydroxylation of methane is as follows:



$$CH_4 \rightarrow CH_3OH \rightarrow H_2O + H{-}CH{=}O \qquad (9)$$

Depending on the amount of oxygen present, the formaldehyde thus formed may either:

(a) Break down thermally to CO and H_2:

$$CH_2O \rightarrow CO + H_2. \qquad (10)$$

(b) React with oxygen to form CO and H_2O:

$$CH_2O + \frac{1}{2} O_2 = CO + H_2O. \qquad (11)$$

(c) React with sufficient oxygen for complete combustion to CO_2 and H_2O:

$$CH_2O + O_2 = CO_2 + H_2O. \qquad (12)$$

"2. The combustion of a chain hydrocarbon is similar, i.e., one of the CH_3 groups is oxidized in the same manner. The following diagram shows the mechanism for the slow combustion of ethane at temperatures below its ignition point;

$$\underset{\text{ethane}}{H_3C.CH_3} \longrightarrow \underset{\text{ethyl alcohol}}{CH_3.CH_2OH} \longrightarrow [CH_3.CH\ (OH_2)] \longrightarrow \underset{\text{acetaldehyde}}{CH_3.CHO} +$$

$$H_2O \longrightarrow CO + H_2 + \underset{\text{formaldehyde}}{H.CHO} \longrightarrow \underset{\text{formic acid}}{HCOOH} \longrightarrow \underset{\text{carbonic acid}}{CO(OH)_2} \longrightarrow$$

$$CO_2 + H_2O \qquad (13)"$$

The results of the explosion experiments did not lead to such definite conclusions as the work at lower temperatures. However, Bone and Townend feel that the process of combustion is essentially the same in both cases for they say: "------It (the 'hydroxylation theory') certainly affords what we believe to be a true explanation of slow combustion; ------ the immediate result of the initial encounter between hydrocarbon and oxygen is probably much the same in the two cases, namely, the formation of a 'hydroxylated' or 'oxygenated' molecule. At the higher temperature of flames, secondary thermal decompositions and interactions undoubtedly come into operation at an earlier stage, and play a more important rôle, than in slow combustion; they do not, however, precede the onslaught of the oxygen upon the hydrocarbon but arise in consequence of it."

The 'hydroxylation' theory is accepted by Haslam and Russell(76) who discuss the extension to heavier hydrocarbons:

"The heavier hydrocarbons usually undergo reaction other than simple hydroxylation, and the same is true of the lighter hydrocarbons if conditions are unfavorable for the formation of hydroxylated compounds. The heavy hydrocarbons may be 'cracked' to give saturated and unsaturated lighter hydrocarbons, or they may be decomposed completely into carbon and hydrogen.-----------"

"Thus in the ordinary combustion of hydrocarbons, there is a race between thermal decomposition and hydroxylation. If the conditions favor hydroxylation (such as preheating the hydrocarbons and air, and allowing time for the entrance of oxygen into the hydrocarbon molecule), there will be no soot. If, however, conditions favor cracking, as, for example, if the hydrocarbons and oxygen from the air are not thoroughly mixed together, the heat from the combustion of part of the hydrocarbon decomposes or cracks the remainder."

The "hydroxylation" theory is consistent with the facts of combustion as they are known today and is commonly accepted by investigators in the field. Stevens(77,78,79,80) of the United States Bureau of Standards in the years 1922 to 1930 carried out an extended series of experiments using soap bubbles for constant pressure bombs and recorded the progress of combustion photographically. Stevens was interested in the chemical side of the general problem and particularly in the kinetics of combustion. With regard to the actual processes in the burning of complex fuels Stevens(80) says:

> "But in the case of nominally higher reaction orders, where large numbers of molecules may be involved in the transformation, the actual processes taking place are rarely found to follow a mechanism indicated by the reaction order of the stoichiometric equation, although the equilibrium condition agrees with it and the reaction constant is expressed in its terms. The processes taking place in these high-order reactions are found to follow simpler orders. It is found that even trimolecular reactions are rare and that the processes taking place in polymolecular transformations proceed by simultaneous uni, bi, rarely, trimolecular orders."

* "On the assumption that the resulting reaction is the summation of simultaneous reactions each following its own order and mechanism independently of other reactions occurring at the same time within the reaction zone, a stoichiometric equation for complete combustion of a composite fuel of known components may be written as a simple reaction:

$$n_1 F + n_2O_2 = n_3CO_2 + n_4H_2O$$

(F represents the fuel molecule)
(n_1, n_2, n_3, n_4 are equal to the number of molecules involved)

Stevens(80) gives the following conclusions as part of the last report of his series;

"1. In the case of the gaseous explosive reaction at constant pressure, the data given in this report show that the statistical expression, $\Gamma = [F]^{n_1} [O_2]^{n_2}$, derived from the order of the stoichiometric equation written for complete combustion of a fuel, is proportional to the spatial rate at which an equilibrium is established in the gaseous explosive system, and that this relation is found to hold for high reaction orders where very complex hydrocarbon fuels are involved in the transformation."

"2. The above relation, since it is based solely upon the initial and final condition of the transformation, is independent of the microprocesses, whatever these may be, resulting in the final union of the initial active components in the proportions required by the reaction constant K for the temperature and pressure at which the reaction takes place."

"3. The data also provide interesting confirmation of the assumption that high-order reaction processes consist of many simultaneous simpler ones each running its course within the reaction zone according to its own order

---

*(Note con'd.) The equivalent order of this reaction, $n_1 + n_2$, is determined from the relative importance of the order of the separate components of the fuel mixture. The statistical expression derived from this order is

$$\Gamma = [F]^{n_1} \quad [O_2]^{n_2} ."$$

and mechanism independently of any other reactions occurring at the same time. The probability of the correctness of this assumption is chiefly shown by the fact that the equivalent reaction order of a composite fuel may be determined from the reaction orders of its components, and further, that the velocity constant, k_F, may also be determined from the velocity constants of those components."

It is plain from the foregoing discussion that combustion of hydrocarbon fuels in gaseous form is a very complex process in which the final products of water and oxides of carbon are formed as the result of somewhat indefinite chain reactions involving various degrees of oxidation. However, the overall effect of combustion follows ordinary chemical principles and can be analysed with satisfactory precision.

Thermodynamic Aspects of Combustion

Thermodynamically the effect of combustion is to change chemical energy into the form of heat and mechanical energy. The most interesting case is that in which the released chemical energy is converted into potential energy of compressed gas which in turn can be partly converted into mechanical work. The simplest procedure both analytically and experimentally is to burn the gaseous mixture at constant volume in a strong chamber. Theoretically, if the heat of combustion is known for any conditions and the specific heats of both the initial gases and the reaction products are available, the pressure developed by any given reaction at constant volume can be calculated.(82) This general problem has interested investigators since the days of Bunsen.

Bunsen found that the pressure developed in a gaseous explosion is less than that to be expected if there were no heat losses, the reaction went to completion, and specific heats were independent of temperature. This work was taken up by Mallard and Le Chatelier(83) and Berthelot and Vieille(84) who estimated maximum temperatures and mean values of specific heats from pressure-time records. The general procedure was to use a mechanical or optical pressure indicator connected to bombs of various sizes and shapes. It was found that the size and configuration of the bomb influenced heat losses and the time interval required for the pressure to reach its maximum after ignition.

Of particular interest for detonation work is the recorded fact that Mallard and Le Chatelier found that one of their indicators responded to an instantaneous local pressure much higher than that possible from the energy content of the explosive mixture as a whole. They identified this effect with "l'onde explosive" in the burning gases. This observation shows that a phenomenon, which was later to develop into a major obstacle for engine designers, was first discovered before 1883 by investigators who had a very definite idea as to the reasons for its occurrence.

Clerk(85,86), working about 1885, was particularly interested in the thermodynamics of internal combustion engines. Using a mechanical indicator connected to a cylindrical vessel, he made pressure-time records of hydrogen-air and coal gas-air mixtures. He studied particularly the effect of mixture ratio on the maximum pressure and the time required for burning. Clerk found that the highest pressure was developed by a mixture with an excess of combustible gas present, a result that is still important in the operation of internal combustion engines.

Bone and Townend(71) summarize the results of the early experiments on the thermodynamics of gaseous combustion at constant volume:

"-----in the early work on gaseous explosions in closed vessels, and in particular in that carried out during the years 1880-1885, it was definitely established that the pressures so developed were always considerably below those calculated on then known considerations. The great disparities between the found and 'calculated' maximum pressures have been attributed by various investigators to one or the other of the following causes:"

"1. To the marked increases in the specific heats of steam and carbon dioxide with temperature. This is now generally admitted to be one of the most important factors in such connection,-------"

"2. To loss of energy by direct radiation. Thus in the explosion of a mixture of hydrogen and oxygen it seems probable that the inital action results in the formation of an intensely vibrating molecular complex from which steam issues as the first recognizable product.------"

"3. To dissociation of products (steam and carbon dioxide). In the case of two combining gases producing a dissociable product, it is clear that if the average temperature of the system exceeds that at which dissociation begins, a percentage of the heat of combustion will not be available at the moment of maximum pressure."

"4. To heat loss by conduction. This is not nearly so great as was supposed by Hirn, and is probably quite small in very fast burning mixtures."

"5. To the fact that combustion may not be completed within the period required for attainment of maximum pressure, with the consequent occurrence of 'after-burning'. In certain cases doubtless this factor is operative, but whether it is generally so, or counts much in the burning of turbulent mixtures, may be questioned."

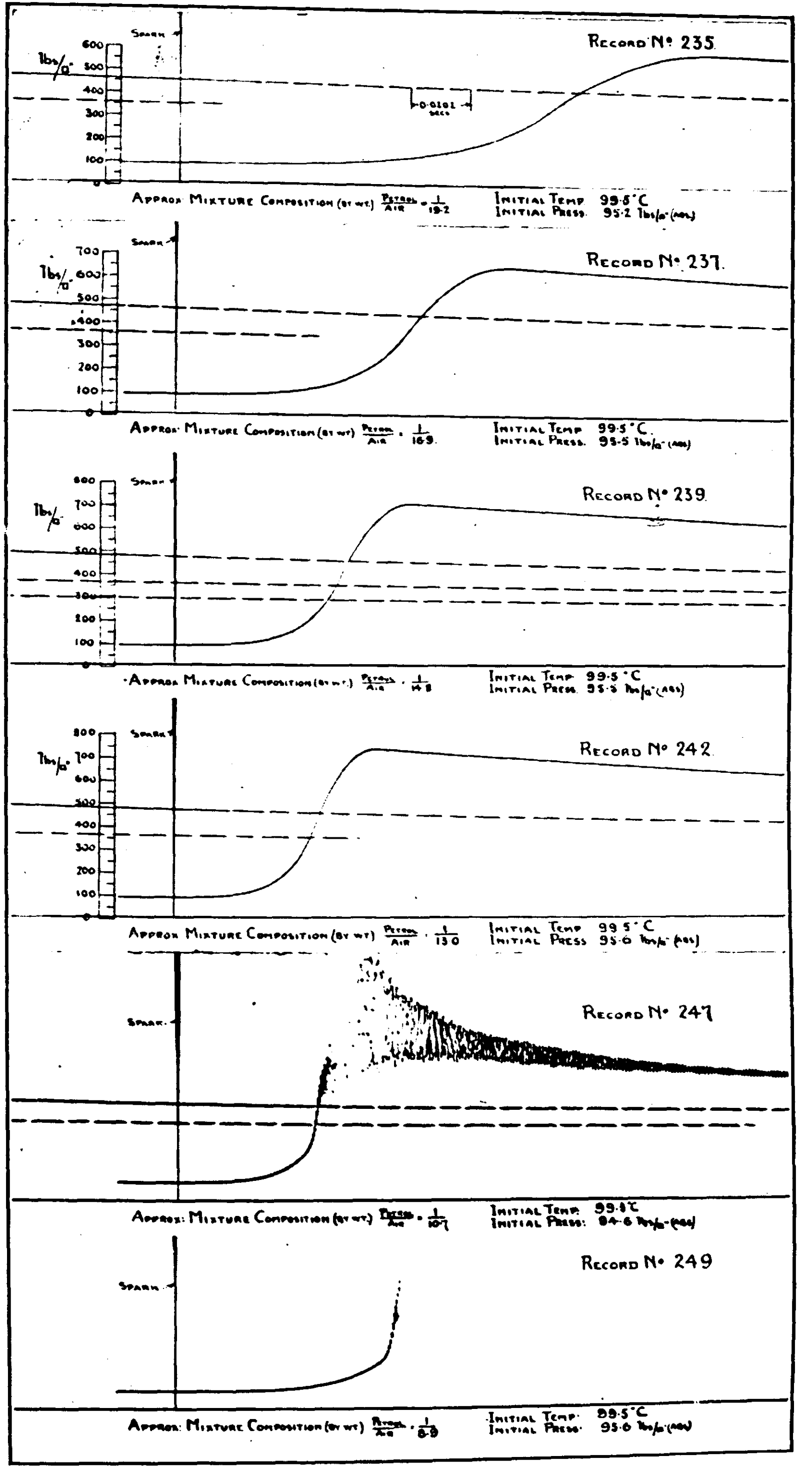
"No one cause by itself is capable of accounting for the observed facts, and all may play more or less important parts according to circumstances.------"

This list of factors influencing pressure development in constant volume explosions was quite definite by 1885 and has been shown by later research to be substantially complete. The method of constant volume combustion for studying the thermodynamic aspects of gaseous reactions has been improved by the application of modern instrument technique and is still one of the most powerful tools in the field.

The value of the constant volume explosion method was recognized particularly in Germany where Langen(87) used ignition at the center of a spherical bomb to reduce heat losses and simplify the flame path. Nerst(88) and his collaborators Pier(89), Bjerrum(90) and Siegel(91) were very active in eliminating errors from the experimental procedure and in interpretation of the results from the standpoint of chemical equilibrium and thermodynamics. Hopkinson(92) added temperature measurements to his observations by placing electrical resistance thermometers at various points in the explosion vessel.

David(93) carried on the study of heat losses with resistance grids placed inside the bomb and a bolometer exposed through a fluorite window to radiation from the burning gases. The methods introduced by David have been generally adopted in later work on radiation from the charge inside engine cylinders.

By 1920 the amount of information collected on gaseous combustion in closed chambers had reached such proportions that the fundamental properties of simple gaseous mixtures were known. Later experiments have either increased the precision of measurements already carried out or have been used to determine the behaviour of fuels such as benzol, gasoline and special hydrocarbon mixtures when combined with air in various ratios under simulated engine conditions. Fenning(94) refined the constant volume method to give results consistent to about one part in five hundred and extended his experiments to include commercial fuels. Plate II shows a series of pressure records made by Fenning showing the effect of mixture ration on pressure development at constant volume. The upper record for a "lean" mixture shows a comparatively slow pressure rise, while the lower two records for richer mixtures indicate high rates of pressure increase followed by the strong oscillations characteristic of detonation.



PRESSURE-TIME RECORDS OF PETROL-AIR EXPLOSIONS. (Fenning.)

REF. 71

PLATE II

Tizard and Pye(95) investigated the effects of dissociation on pressure development in a long series of experiments directed toward the special problems of the internal combustion engine.

Soon after 1925, the properties of fuel-air mixtures were well enough known for use by designers as engineering data. Ricardo(3) outlined a method for estimating cycle temperatures, pressures and thermal efficiencies from thermodynamic information but did not give a discussion of the effects of varying mixture ratio. Goodenough and Baker(96) calculated the history of the working fluid in internal combustion engines for various mixture ratios but did not express their results in a form suitable for general use.

In 1936, Hershey, Eberhardt and Hottel(97) presented a series of charts which combined in a form convenient for rapid use, the information necessary to estimate temperatures, pressures, and efficiencies under any engine operating conditions. These writers include a detailed list of references used as sources of fundamental data for their charts. With regard to these diagrams they say:

"The working fluid in an internal-combustion engine, from the time of completion of intake to the time of firing, has a chemical composition fixed by the fixed operating conditions of the engine. Consequently, its thermodynamic properties are susceptible to presentation in diagrammatic form in a manner analogous to that used in showing the properties of steam. However, from the time of ignition to the time of expulsion of the working fluid from the engine, chemical change is going on within the mixture. In the ideal engine this change is assumed to proceed so rapidly that the mixture is at all times in both physical and chemical equilibrium. The properties of a mixture of fixed atomic composition that is always in complete equilibrium also may be represented diagrammatically although, of course, the additional requirement of chemical equilibrium introduces complications in the computation. Since the compostion of the mixtures both before and after combustion is dependent on the fuel-air ratio, one diagram of each type is necessary for each air-fuel ratio that it is desired to study. Diagrams of these two types, constructed for various air-fuel ratios, are presented and discussed in some detail------."

With the publication of the Hottel Charts, the long story of gaseous reactions in at least one type of explosive mixture (the charts are based on a hydrocarbon fuel with the same carbon-hydrogen ratio as octane) is complete from a thermodynamic standpoint. In practice, the data from such charts will always be approximations to experimental results since no account is taken of heat losses, lack of homogeneity in the mixture, improper ignition timing and other uncertain factors. It is improbable that future research will bring any new variables into the problem but the precision of present knowledge may be increased by future experiments.

Ignition of Explosive Mixtures

With regard to the general problem of starting a self-supporting reaction, Nernst(88) says:

"Let us first consider the case where the reaction progresses in the sense which is associated with the development of heat. The progress of the reaction causes an elevation of temperature, which accelerates the velocity. But this accelerated velocity means a quicker decomposition, and therefore in turn causes an increased development of heat, which again reacts to hasten the decomposition. Thus it is evident how a very extraordinary acceleration of the reaction velocity may take place under favourable circumstances. In this way we can explain the 'stormy reactions'. It will be found that these are invariably associated with a development of heat."

"The reaction velocity in many systems at the ordinary temperature is very slight, and perhaps may have no appreciable value. In such cases the mutual acceleration of the reaction velocity and the development of heat does not come into play, because the slight amount of heat developed is conducted away to the environment before any perceptible rise in temperature occurs."

"Now it is by no means necessary to bring the whole system to a temperature at which the reaction velocity is sufficiently great; to ignite the gases it is only necessary to heat them locally to a certain extent, as can be done by means of the electric spark."

"Let us consider again for the sake of simplicity a homogenous system, such as an electrolytic gas mixture; then in every case, at that point where the temperature reaches a sufficient limit, the reaction between two gases will progress more quickly, and therefore the temperature of the point will rise. One of two events will then happen: either the heat developed will be taken away from the environment of the point by radiation and conduction more quickly than it can be generated anew, and therefore after a short time the temperature will sink, and the reaction velocity will again return to a minimal value; or, the heat developed at the point considered will be sufficiently great to heat the surroundings to a temperature of lively activity; in this case the high temperature causes the rapid reaction between the gases to spread over the whole system, and a combustion takes place, resulting in the almost complete union of all gases in the system which are capable of reaction."

"That temperature, to which a point of the system must be heated in order to cause combustion, is called the "ignition temperature." It is obvious from the preceding considerations that its value depends on a large number of factors, such as the heat of the reaction, the thermal conductivity, the capacity for diffusion possessed by the gas,, and the dependence of the reaction velocity upon the temperature; it will also vary with the temperature of the surroundings, and with the pressure of the system."

"Thus, the ignition temperature has quite a secondary nature; it is clear that it cannot be described as the point where the mutual action of the gas begins. This would be as absurd as to say that the boiling-point of a liquid is the point where vaporization begins."

Nernst's exposition of the process by which combustion is initiated in a combustible mixture shows the complexity of the general problem. Four methods of gaseous ignition are discussed by Bone and Townend(71); 1) contact with flame or a sufficiently heated surface, 2) passage into a heated enclosure, 3) heating by adiabatic compression and 4) exposure to a sufficiently powerful electric spark. Of these means, the electric spark is not of particular interest in detonation research but one or all of the purely thermal processes may be active in a detonating engine. Pre-ignition in an engine is directly due to combustion started by contact with hot surfaces or passage of gas into the heated space between the central electrode and the shell of a spark plug, while ignition by adiabatic compression is important in detonation itself.

Bone and Townend(71) outline the work of several investigators on spark ignition who studied among other things, the amount of current required for ignition as dependent upon mixture ratio. The general theory of electrical ignition was considered by Finch and Cowen(98) who say:

"When an electric discharge, powerful enough to bring about ignition, is passed through such a gaseous medium, chemical combination occurs in both the cathode and inter-electrode zones. The rate of such combustion is directly proportional to the current; that in the cathode zone being proportional to the total number of ions arriving in unit time at the cathode, and that occurring in the inter-electrode zone (positive column) being probably proportional to the number of suitable ions formed by the passage of the current. The heat liberated by the gases on combination contributes to the number of ions formed by the passage of the current. With an igniting current this heat source of ionisation produces ions more rapidly than they recombine to electrically neutral atoms or molecules, or are removed from the zone of the discharge by convection currents in the gas. Thus the source of ionization is cumulative and leads to ignition and explosion when a sufficient concentration of suitable ions has been attained. Furthermore, ignition does not occur at the actual moment of initial passage of the igniting current, but only after the lapse of a definite time interval, or lag, during which the cumulation of suitable ions which is necessary to bring about ignition is gradually attained by the cumulative addition of these ions which are formed by the heat of combination of the combustible gases to the ions due to the passage of the current."

The first controlled experiments on ignition temperatures were carried out by Mallard and Le Chatelier(9) who allowed combustible mixtures to flow into evacuated porcelain vessels previously heated to a known temperature. In this work the existence of a time delay up to ten seconds was observed in certain cases. This is the first record of "lag" in the process of ignition which will appear later as a factor in determining whether or not a mixture will detonate under given engine conditions.

Dixon and Coward(99) improved on the earlier method by heating the components of the combustible mixture separately and then bringing them into contact in concentric streams. This method gave much more consistent results than the old procedure. Dixon and Coward found the time lag in ignition and observed that lower temperatures were accompanied by longer lags. Bone and Townend(71) summarize the lag problem:

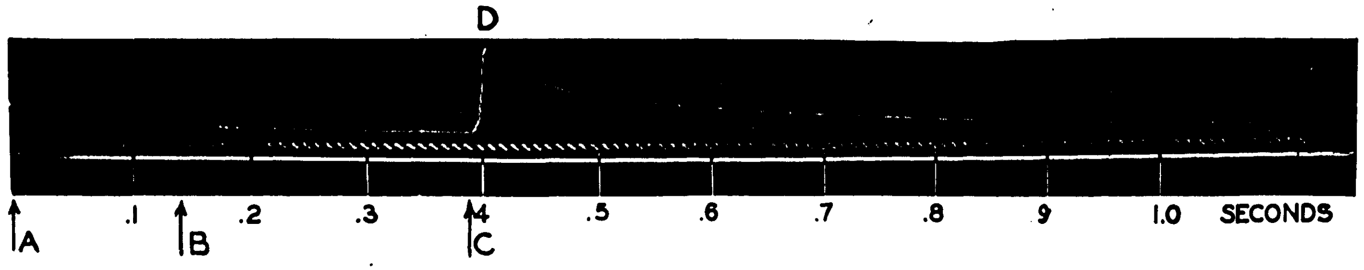
> "It seems probable that such effects of 'lag' (the variation of 'lag' with temperature) are primarily due to 'pre-flame' combustion producing heat more rapidly than it is dissipated, with the result that, if sufficient time be allowed, the temperature of the system is raised to that at which flame appears. In other words, the longer the time that can be allowed for the inflammation to occur, the lower is the temperature to which the gaseous medium need be heated in the first instance. Hence, for many practical purposes, the <u>minimum temperature</u> to which a particular mixture need be heated initially, in order to ensure eventual inflammation after an unlimited time interval, should be regarded as the effective ignition temperature rather than that to which it must be heated initially in order to ensure instantaneous inflammation. -------"

Ignition temperatures of fuel-air mixtures found by the conventional methods were unsatisfactory when they were applied to ignition by adiabatic compression. At the suggestion of Nernst, Falk(100) attacked the problem of ignition by adiabatic compression in a steel cylinder fitted with a piston. This piston was driven in by a weight released from a variable height. From the energy required to produce ignition, the corresponding pressure and temperature were estimated. Dixon and his co-workers continued the experiments of Falk with improved apparatus and obtained consistent results for various simple gaseous mixtures.

Tizard and Pye(101,102) made an extended study of compression ignition in hydrocarbon-air mixtures. Conditions similar to those existing in engine cylinders were obtained by using a special compression machine suggested by Ricardo. Figure 26 shows the essential parts of this machine. The mixture was compressed in the cylinder A by the piston B which was then held stationary at the top of compression. The initial temperature before compression could be varied by the cylinder jacket temperature, and the compression ratios could be controlled by screwing the whole cylinder and head up or down with respect to the crankcase. The piston was driven through an internal expanding friction clutch fitted to a flywheel rotating at constant speed. When the clutch was engaged the piston was driven upward by means of the toggle system shown, and when it reached the top of its stroke it was automatically locked into position. At the same time, the outside sheath of the telescopic connecting rod disengaged from the central rod on which it moved freely so long as the clutch was acting.

Plate III shows a pressure-time record from the compression machine. With regard to records of this type Pye(1) says:

"When the initial temperature and ratio of compression were such that the temperature reached by the adiabatic compression was a little above the self-ignition temperature of the particular fuel used, it was found that a record such as that illustrated was obtained. The surprising nature of this diagram will be clear when it is understood that the portion AB represents the pressure rise due to the motion of the piston, and that this is followed by a period of constant or slightly <u>falling</u> pressure BC, which in some experiments lasted no less than 3/4 second. In the record illustrated it was 1/4 second, the wavy line, which gives the time scale, being the trace of a tuning fork making 100 vibrations per second. At the point C, which is the end of the delay, explosion takes place without any further outside influence whatever. The rapid rise of pressure shown at CD is followed by subsequent cooling, everything from the point B onwards having happened at constant volume, and with no appreciable gas leakage meanwhile."



Typical Pressure-time Records shewing Ignition of Gaseous Explosive Mixtures on Adiabatic Compression. (Tizard and Pye.)

REF. 71

PLATE III

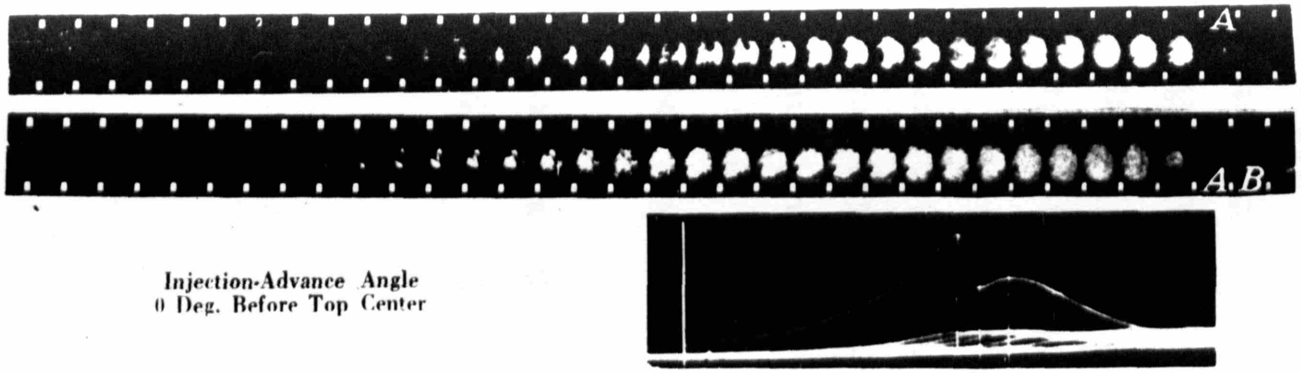
"During the delay period BC, cooling to the cylinder walls must have been going on, and one can argue therefore that ignition in some form started at B, for after that the average temperature must at once have started to fall. During all the delay period minute nuclei of combustion must have been smouldering, and then, after an appropiate period of incubation, burst out into a full-fledged explosion."

"It must be understood that this period of incubation was not a matter of chance. With all liquid hydrocarbon fuels it was found to be very consistent and to depend upon the amount by which the compression temperature exceeded a certain minimum below which ignition would not occur at all. As the compression temperature was raised above this minimum, so the delay period was shortened."

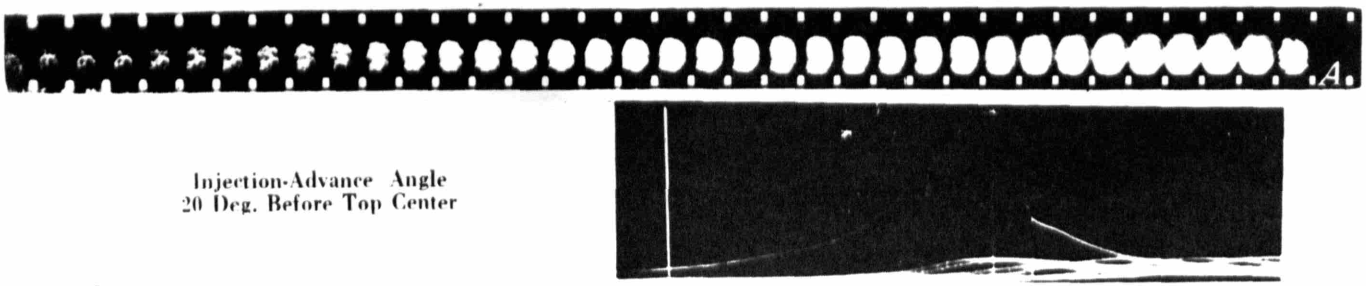
"This phenomenon of delayed combustion which was demonstrated so clearly in the compression machine is evidently the explanation of the period of 15 to 20 degrees crank angle after the spark has passed in a petrol engine during which combustion has no effect upon cylinder pressure. In an engine, of course, the mixture during this period is violently turbulent, whereas in the compression machine it must have been practically stagnant. But turbulence, although it has a profound effect as soon as combustion is fairly started, seems to have little or none during the delay period. We must imagine a string of nuclei carried in a stream of turbulent air from the sparking plug points, each nucleus hatching independently until the general conflagration sets in."

Rothrock(103) of the National Advisory Committee for Aeronautics studied the ignition of hydrocarbon sprays in a compression machine fitted with glass sides in the combustion chamber. Motion pictures at rates up to 2000 frames per second were taken of the injection process and the following combustion. During injection, illumination was furnished by a series of powerful electric sparks while light from the flame itself exposed the film during combustion. A special optical indicator was used to make a continuous record of pressures inside the cylinder. The machine itself was a single cylinder engine fitted with a release valve maintaining atmospheric pressure inside the chamber until the compression stroke before the injection of fuel.

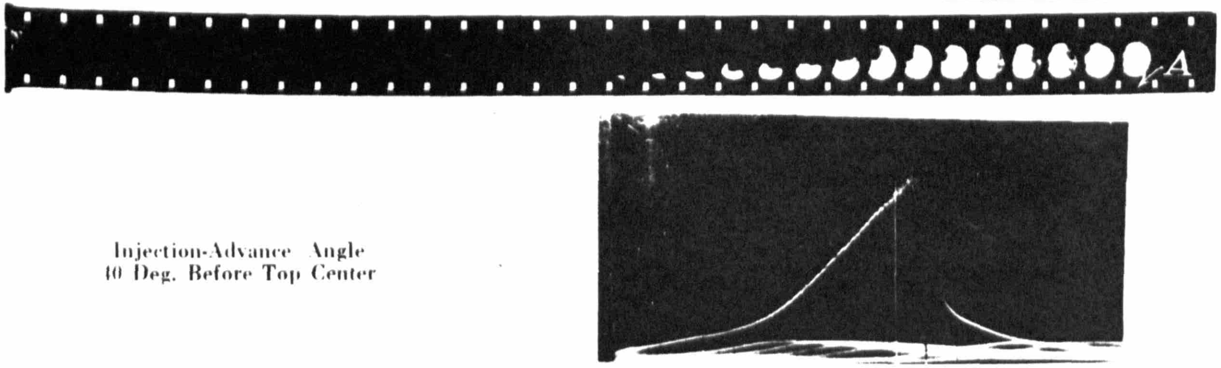
Plate IV shows a series of pictures taken at 1300 frames per second showing the course of combustion and pressure variation for a series of injection-advance angles. Enlargements of selected frames showing injection,



Injection-Advance Angle
0 Deg. Before Top Center



Injection-Advance Angle
20 Deg. Before Top Center



Injection-Advance Angle
40 Deg. Before Top Center

Effect of Injection-Advance Angle on Flame Propagation and Pressure Rise in a Compression-Ignition Engine

The letter *A* indicates the start of the flame; the letter *B*, the fuel spray. The engine-coolant temperature was 150 deg. fahr. and the engine speed, 570 r.p.m.

REF. 103

PLATE IV

and the course of combustion are given in Plate V. The start and spreading of the flame evidently took place in about the manner predicted by Pye for the case of compression ignition in a homogeneous mixture. Detonation is indicated in the lower picture of the series by vibrations in the expansion line of the pressure record. Rothrock draws the following conclusions from his experiments;

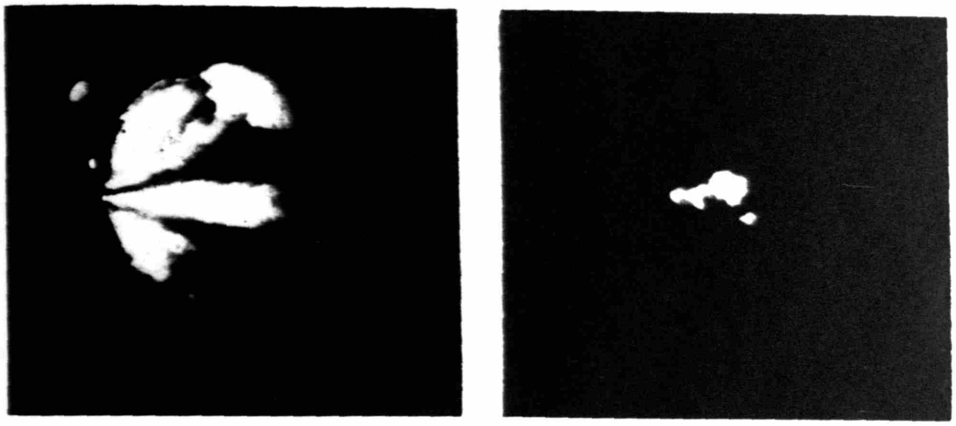
"1) With a short ignition-lag in a quiescent combustion chamber the burning starts from around the spray envelope and from there spreads throughout the combustion chamber. With a long ignition-lag the burning may start at any point in the chamber. In either case, the burning may start at one, or more than one, point."

"2) The course of combustion, aside from the original chemical properties of the fuel, is controlled by the

(a) Time interval between injection and the start of combustion

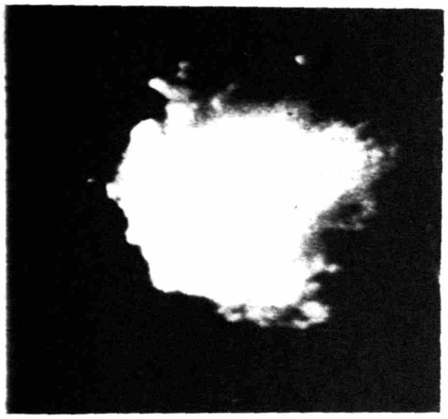
(b) Temperatures and pressures existing in the combustion chamber during this time interval

(c) Distribution of the fuel at the start of combustion."

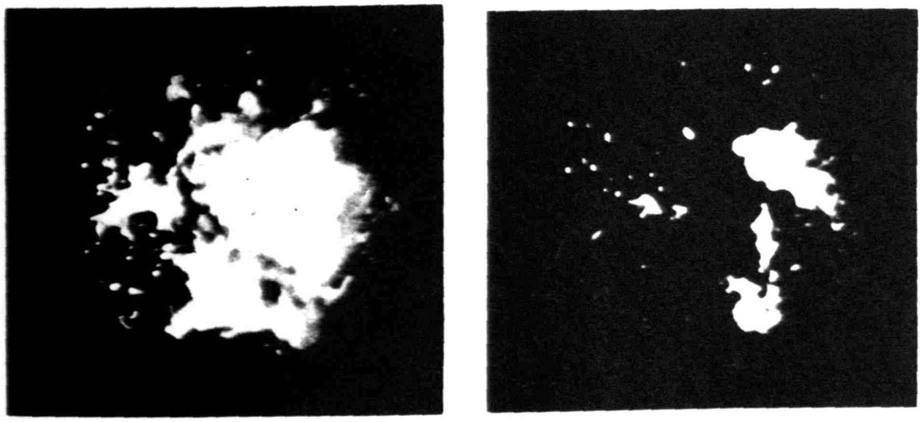


Second Frame
Silhouette of the spray

Third Frame
Start of the flame



Fourth Frame
Note the two main sprays illuminated by combustion



Twentieth Frame
"After-burning"

Twenty-Fifth Frame
Continuation of "after-burning"

Enlargements of Frames from the Second Series of Photographs Shown in PLATE IV.

REF. 103

PLATE V

Summarizing results from the various studies of ignition, it is apparent that:

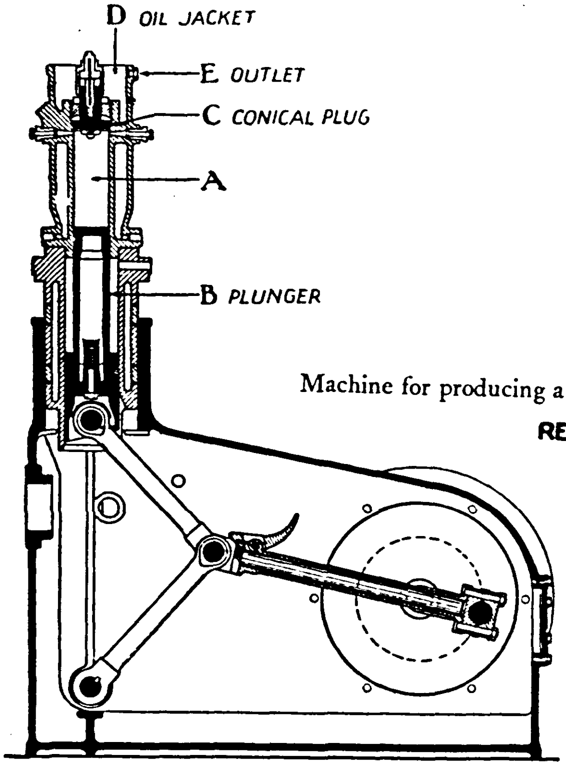
1) Combustible mixtures can be ignited thermally either by conduction or by compression.

2) A time delay occurs after ignition and before general inflammation takes place.

3) The time delay depends upon temperature of the mixture and the nature of the ignition process but is generally reduced by an increase in temperature at the time of ignition.

Flame Movements at Constant Pressure

Flame in a gaseous mixture indicates a sharply localized moving region of reaction which divides the system into three zones: the region of unburned gases, the zone of explosive reaction and the region of reaction products behind the flame. A divergence of opinion exists as to the completeness of the reactions in the flame front but there is no doubt that a major part of the total change occurs in this region. In many cases the flame front emits such intense radiation that the progress of combustion can be recorded photographically. This otherwise excellent method has the fundamental difficulty that flame photographs show movements of the reaction zone with respect to space rather than with respect to the gases



Machine for producing a single rapid compression.
REF. 1

FIG. 26

Apparatus for Measuring Flame Speeds by Photographic Method. (Bone, Fraser and Winter.)
REF. 71

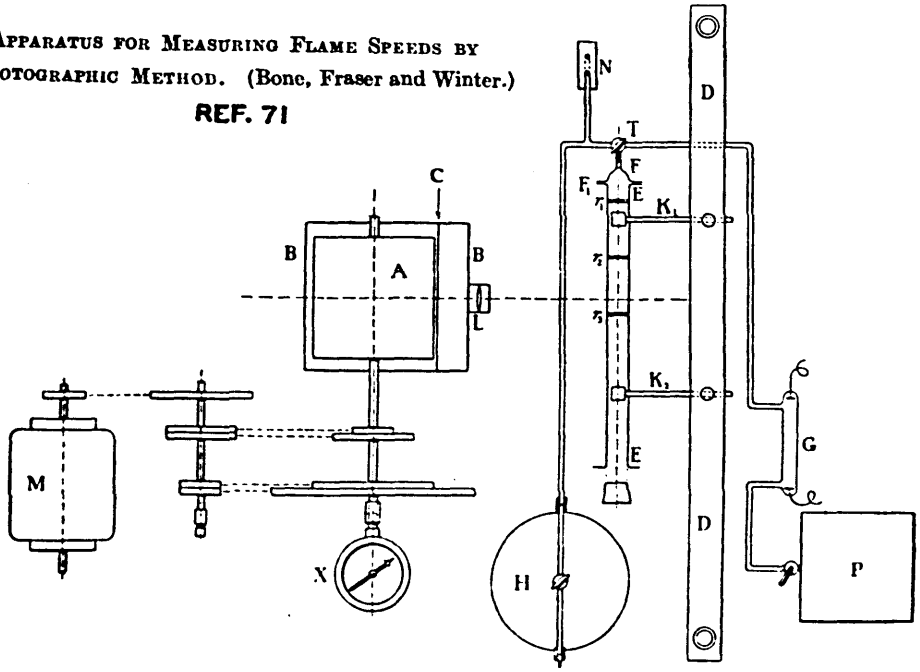
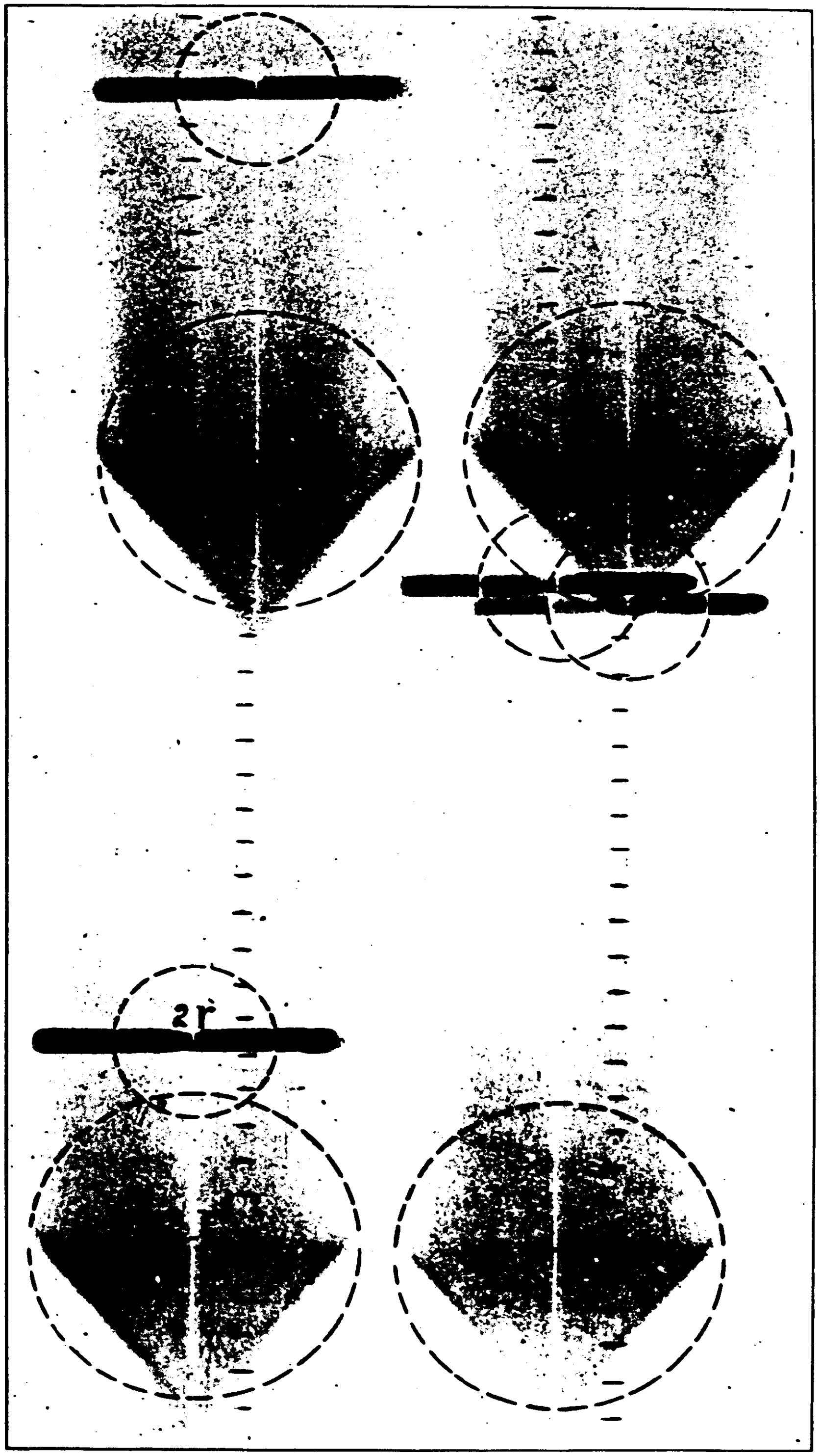


FIG. 27

involved. Since the rapid temperature changes caused by combustion are accompanied by violent motions which mechanically displace the flame front, the true progress of combustion in the gas can be determined only under special experimental conditions.

Stevens(77,78,79,80) using ignition at the center of his "constant pressure bomb" realized an almost ideal case. By application of the principles of hydrodynamics he was able to show that the change in pressure ahead of the flame front could be neglected for reaction velocities small compared to the velocity of sound. He also found an expression showing that the pressure inside the flame surface was smaller than the pressure outside in the unburned mixture. For reactions falling within the limits of his assumptions, Stevens deduced a relationship between the velocity of flame with respect to the mixture and the observed velocity in space. Plate VI shows typical flame photographs from the constant pressure combustion process. It was found that all the records for velocities not too near the velocity of sound showed a flame velocity constant with respect to space.

Stevens did not study detonation but rather avoided this type of reaction by using only low flame velocities. With regard to higher velocity reactions he



Shows a photographic time-volume record of four gaseous explosive reactions at constant pressure. The figures are ½ actual size. 2r is the sphere of initial components considered. 2r′ the sphere of its reaction products at the instant the reaction is complete

REF. 78

PLATE VI

says:

> "For reactions of high velocity, there are indications that the soap film reflects the impulse wave (or at least a part of it) starting with the ignition. Where this reflected wave meets with the outgoing flame surface, an almost instantaneous increase in velocity takes place following the corresponding increase at this point in the concentration of the gases the flame is entering."

With regard to flame propagation, the results of Stevens show that a constant velocity in space is to be expected for symmetrical propagation in a mixture at constant pressure for sufficiently slow velocities. Definite data are lacking but Stevens hints very strongly that marked disturbances in flame propagation are to be expected from the interaction between pressure waves and the reaction zone at high flame velocities.

Slow Flame Movements in Tubes

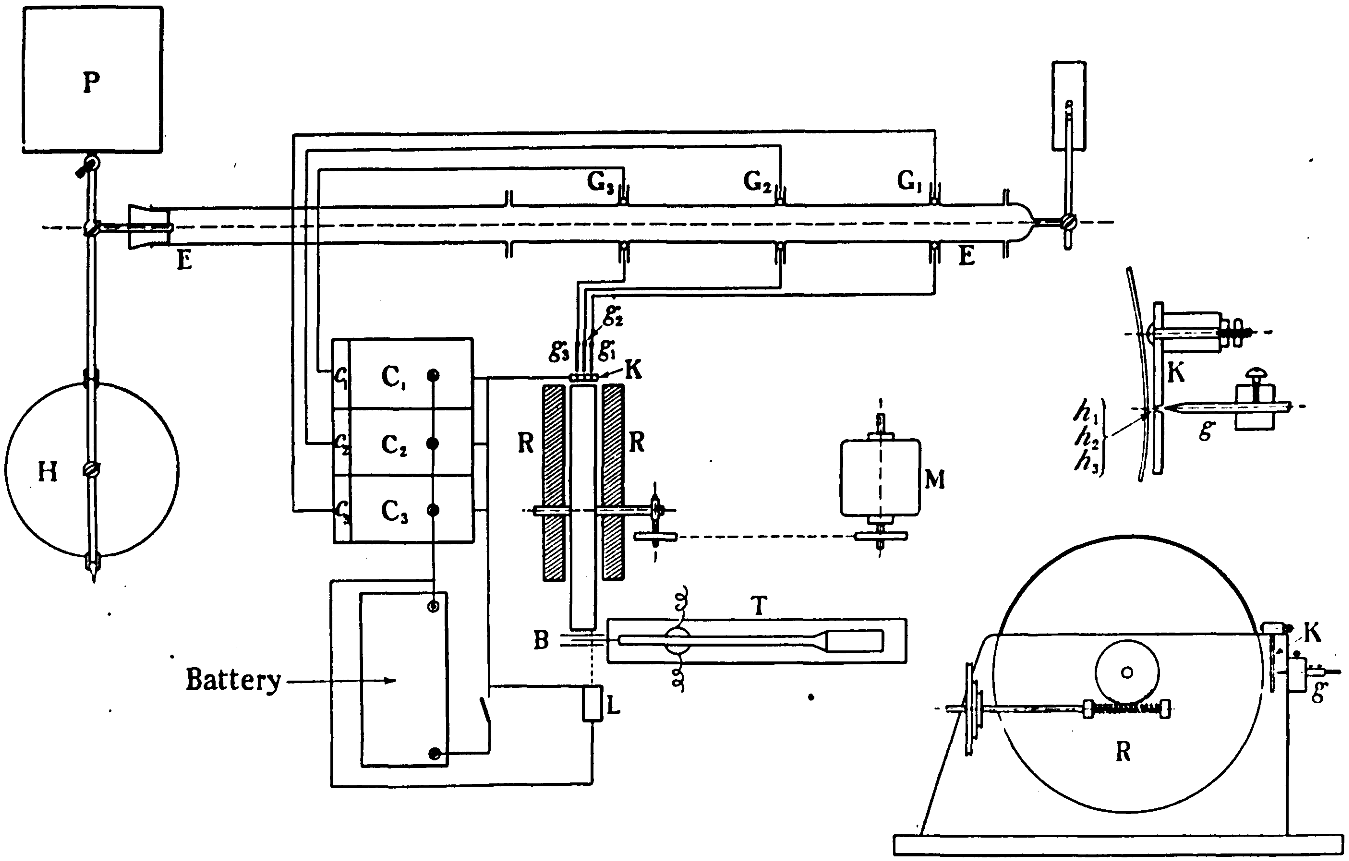
At first thought the simplest case of flame propagation seems to be that in which the problem is reduced to one dimension by confining the burning gases in a tube. This idea was the guiding principle in many of the early investigations of combustion and is used for certain purposes at the present time. However, the unpredictable effects of heat conduction and viscous action near tube walls have always introduced uncertainties. In addition to wall effects, observers have consistently found strong disturbances from pressure wave reflections and general flow in the mass of gas. Only in the case of the very rapid flame propagation called 'l'onde explosive' or 'detonation' has there been close agreement in data from various sources. The succeeding discussion will outline the history of tube experiments and the generally accepted results.

Mallard and Le Chatelier(9) originated the methods still in general use for studying flame motions. Where the flame is sufficiently bright the best method is to record photographically on moving film, while electrical methods are available in cases of low flame luminosity. In the electrical scheme terminals are placed at known intervals in the flame path and a) connected with a fine wire to be fused by the flame,

b) connected with a light piece of conducting material to be thrown off by the mechanical disturbance accompanying the flame, or c) simply immersed in the gas and used as ionization gaps. In each case some type of electrical chronograph is used to record the time intervals required for the flame to move between stations in the tube.

Figure 27 shows a typical arrangement for use of the photographic method as described by Bone and Townend(71). The drum A, covered by photographic film is driven at constant speed by the motor M. The length of the transparent combustion tube EE is imaged on the moving film by means of the lens L. Opaque bands r_1, r_2, r_3--- wrapped around the tube identify positions along the tube to be used in interpreting the records. The other parts shown connected to the tube are used for support and for supplying controlled combustible mixtures in the experiments.

Bone and Townend also show the essential parts of an electrical system for measuring flame speeds. In figure 28 the combustion tube is EE with ionization gaps at G_1, G_2 and G_3. These internal gaps are subjected to a high voltage from the induction coils C_1, C_2, C_3 and connected in series with external gaps g_1, g_2, g_3. A high frequency interruption is used on the induction



Apparatus for Measuring Flame Speeds by the Electrical Method. (Bone, Fraser and Winter.)

REF. 71

FIG. 28

coils and the condensers c_1, c_2, c_3 across the coil secondary serve to keep a substantially constant voltage on the gaps. The system is so adjusted that the voltage across the gaps is insufficient to cause breakdown with uninflamed gas in the gaps but high enough to produce a spark across the external gaps successively as ionization in the flame front reduces the total circuit resistance. When a spark jumps across one of the external gaps a photographic spot is produced on a film attached to the outside of a rotating wheel in the recorder R. The tuning fork T and the lamp L are arranged to impress a timing record on the film.

Comparable tests carried out by the two methods showed that flame velocities measured by the two methods agreed within an experimental error of 2.5 per cent.

The classical experiments of Mallard and Le Chatelier produced records like those shown in Plate VII. Discussing the important features of the flame records Bone and Townend say:

"The behavior of these mixtures was found to differ according as they were ignited at or near (a) the open, or (b) the closed end of a tube. In the case of (a) it was always observed that the flame proceeded for a certain distance along the tube at a practically uniform slow velocity, which Mallard and Le Chatelier regarded as the true rate of propagation 'by conduction'. With CS_2 + 6NO mixtures this uniform movement was succeeded by an 'oscillatory period', the flame swinging backwards and forwards with increasing amplitudes, and finally either dying out altogether, or giving rise to 'detonation', according to circumstances. With the 'oxygen' mixtures the initial period of uniform velocity was shorter, and appeared to be succeeded abruptly by 'detonation', without passing through any intermediate 'oscillatory period'. When, however, the mixtures were ignited near the closed end of the tube, the forward movement of the flame was continuously accelerated until finally 'detonation' was set up."

"These important features of their experiments are well shown in Figs. 15 to 17 respectively (Plate VII), which need no further explanation. Fig. 15 is the graph from the explosion of a mixture CS_2 + 6NO in a tube 3 cms. in diameter and 3 meters long, composed of three sections, each 1 metre long, connected in series by means of caoutchouc rings which in the graph serve as reference marks, ignition having been effected near the open end. Here the portion ab of the graph shows the 'uniform movement' which lasted until the flame had traversed nearly all the first metre section of the tube,

Note:
Figure numbers refer to discussion of Bone and Townend.

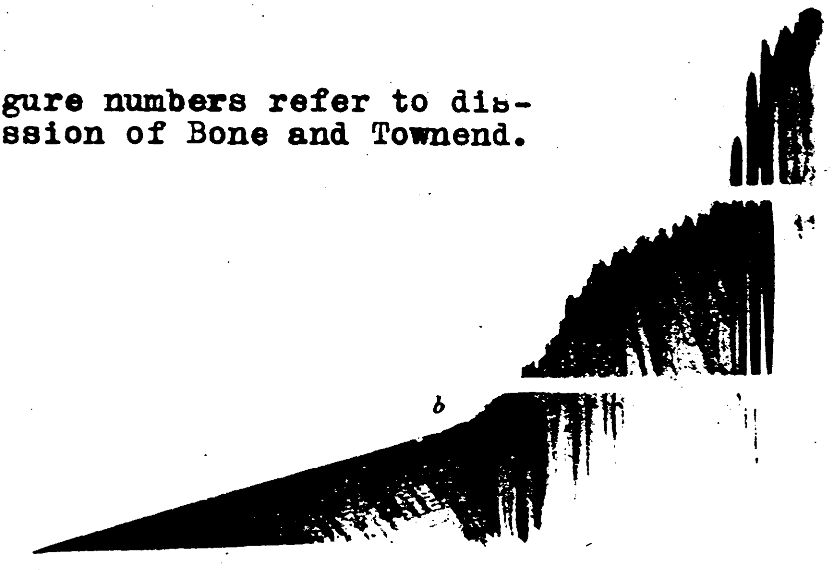


FIG. 15.

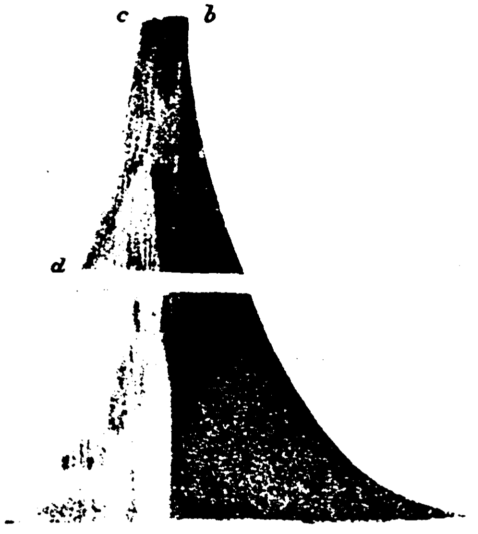


FIG. 17.

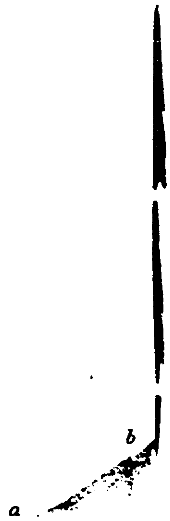


FIG. 16.

MALLARD AND LE CHATELIER'S PHOTOGRAPHIC RECORDS OF FLAME MOVEMENTS THROUGH GASEOUS EXPLOSIVE MEDIA IGNITED IN GLASS TUBES.

REF. 71

PLATE VII

whilst the remainder shows the 'oscillatory period' which extended over the next 1 3/4 meters or so, when the flame was extinguished. Fig. 16 shows the graph from a $CS_2 + 3\ O_2$ explosion in a tube, 1 cm in diameter and 3 metres long, ignition having been from the open end, in which the initial 'uniform movement' indicated by the portion ab was abruptly succeeded by 'detonation'. Fig. 17 shows the graph from a $CS_2 + 6NO$ explosion originated by spark ignition near the <u>closed</u> end of a tube 3 cm. in diameter and 3 metres long. It was noticed that whereas the initial 'slow uniform movement' exhibited no signs of violence, the final 'detonation' was characterised by an exceedingly high uniform velocity, great brilliance of flame, as well as by the shattering effects usually associated with the popular idea of explosions. The contrasts between these initial and final stages are well brought out in Fig. 16."

"Although Mallard and Le Chatelier found the velocity of the initial 'slow uniform movement' to be independent of the material composing the walls of the tube, they shewed that it may be retarded by their cooling influence unless a certain limiting diameter is exceeded. This 'limiting diameter' seemed not to be fixed but to vary with the composition of the explosive mixture and with the velocity of flame propagation through it. They found that, in general, the limiting diameter necessary to any retardation of the flame is greater the slower the flame-velocity; and, conversely, that the constriction of tube requisite to extinguish the flame is greater the greater the flame-velocity."

"Mallard and Le Chatelier regarded the initial uniform flame-movement through a stagnant explosive mixture as being governed by the transference of the heat of combustion from one layer to the next 'by conduction' in the sense that hot products streaming out from the burning layer mix with the cold unburnt gases in the next layer, which is thereby raised to its ignition temperature. From such point of view, provided that the tube diameter exceeds the aforesaid limit, and that ignition is at the <u>open</u> end of the tube by a source of heat not greatly exceeding in temperature the ignition temperature of the mixture, and does not appreciably disturb it, the speed of the uniform movement would depend only on the composition of the mixture, its temperature and pressure."

"Mallard and Le Chatelier recognised that the moderate flame speeds (less than 30 meters per second) which characterise the initial uniform movement could be accelerated by such influences as 'turbulence', which would assist in the transmission of heat; and they attributed the 'vibratory movement', which in many of their 'open tube' experiments immediately succeeded the initial 'uniform movement', to the explosive mixture being thrown into a state of rapid vibration as the result of hot burnt gases being ejected by expansion from the open end---."

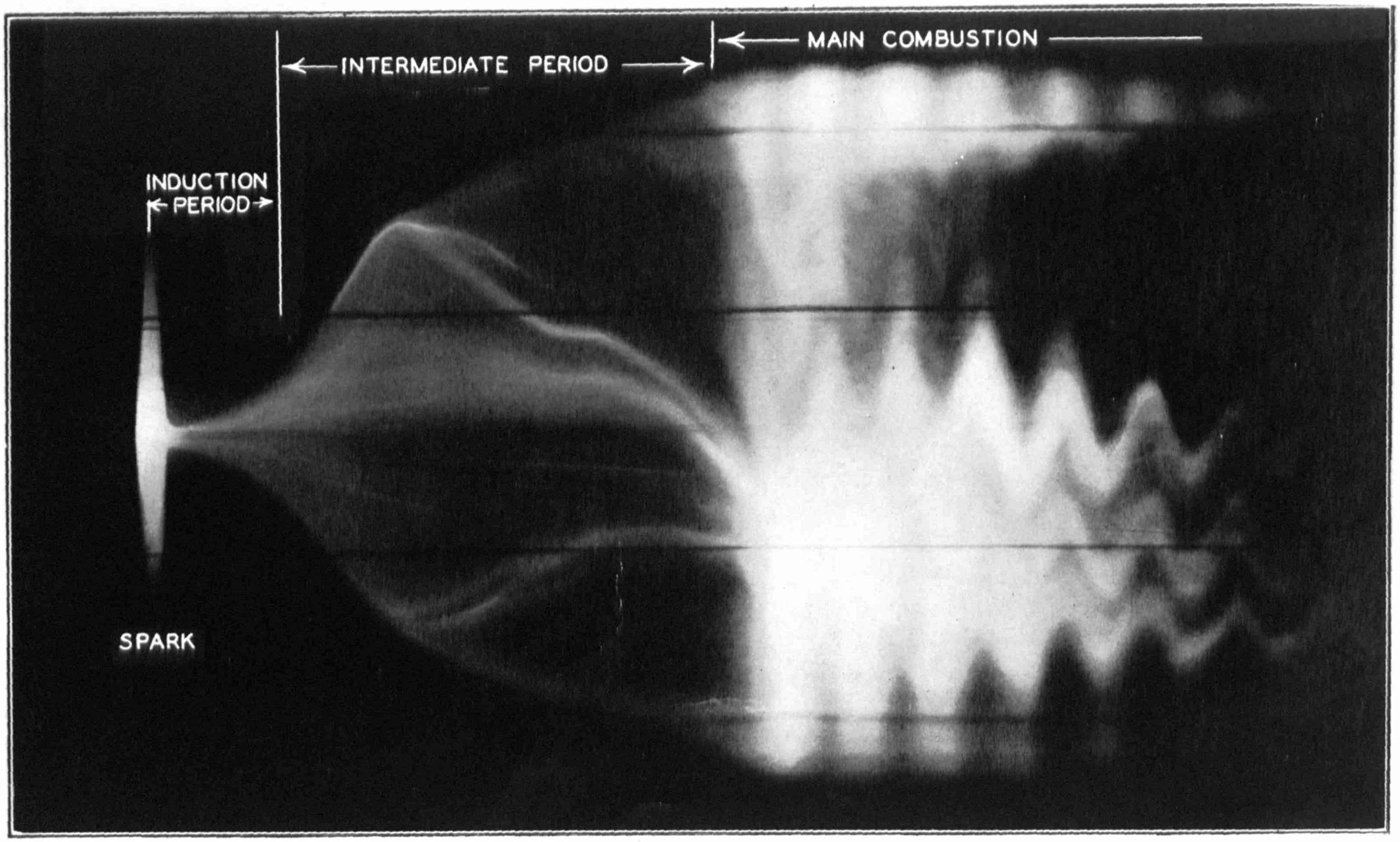
"During the 'vibratory period' the flame may either be extinguished, if during a backward swing it is (so to speak) asphyxiated, or give rise to 'detonation', if sufficiently accelerated during a forward swing. It was shewn that when extinction occurred it was invariably associated with a backward swing during a vibration of great amplitude; in such circumstances the extinction might not be final, a new inflammation of reduced intensity supervening. Indeed, the photographic records showed the combustion intensity to be very variable during different phases of the same vibration."

Following the early work described above many extensive investigations of the initial stages of flame propagation were carried out. Dixon(11), Wheeler and his collaborators(105,106,107), Morgan(108,109), Georgeson and Hartwell(110), Coward and Jones(111) all made contributions to the knowledge of flames. Bone and his co-workers(71,112,113) were especially active both in making careful experiments and in critical examination of existing information. Plate VIII is a typical photograph discussed by Bone and Townend for the case of spark ignition of an explosive mixture at the mid point of a transparent tube closed at both ends. Discussing the general features of the record they say:

"The evidence of the experiments lies so much in the photographs themselves, that little need be said about them beyond indicating the precise conditions under which they were obtained. It is left to each reader to study them for himself, because, while their main features will be obvious to all, their interpretation leaves room for discussion, which it is hoped their publication will provoke.

"To us they suggest such possibilities as (a) the occurrence under ordinary sparking conditions, of what seems to be much like a definite 'induction period' as a preliminary to the actual combustion; (b) an initial propagation through the medium of a 'ghost-like flame' conditions involving only a very partial combination of the gases; and (c) the main combustion following later as the result of superposing of a compression wave, or the like, upon a system which during the phase (b) has already become sensitive to chemical changes."

The presence of standing pressure waves is very evident in the record of the final combustion stages.



Enlarged Record of Flame Movements in a $CH_4 + O_2$ Mixture. REF. 71

PLATE VIII

Such waves are typical of 'detonation' in closed vessels.

In spite of the vast amount of time and effort expended on the subject of slow flame movements, the situation has grown more indefinite rather than becoming clarified as additional evidence has accumulated. After specializing in this field for many years, Bone and Townend summarize their opinions in the paragraphs below:

"It may be said that, whereas it seems probable that 'conduction' plays an important, and possibly in some cases predominant, part in the propagation of flame from layer to layer during the initial period of uniform flame movement, other factors, such as convection current and turbulence, which by producing movements _en masse_ of the combining gases quicken the combustion, must also come into operation, more or less according to circumstances. Also, the 'intensity' of the source of ignition may play a considerable part. Indeed the propagation of the flame during the initial phase of uniform slow movement may not be entirely a thermal phenomenon, as many have hitherto supposed, but thermionic influences may also come in. From this point of view it may perhaps even be doubted whether, strictly speaking, the expression 'flame-speed' ought to be used in connection with the initial uniform slow movement in the sense of being a 'natural constant' for each particular gas-air mixture, so much does it vary with circumstances other than pressure and temperature."

"Such evidence as the foregoing (records from careful experiments) has led us to doubt whether it is possible any longer rigidly to maintain either (i) that all quiescent explosive mixtures necessarily develop an initial uniform flame movement on ignition by means of a flame at the open end of a horizontal tube, or (ii) that even when a 'uniform movement' is initially set up in such circumstances its velocity is necessarily quite the same for the same tube diameter. In the latter case, doubtless, it most frequently happens that the observed initial uniform velocity for a given explosive mixture, and with one and the same tube diameter, will not differ very much from a certain mean value, which therefore may be regarded as having some significance relative both to the properties of the gaseous medium and to its environment. But it seems impossible to regard such mean uniform speed as a physical constant of the mixture in the same sense as we regard its 'rate of detonation'."

" We think that a careful study of the photographic evidence included in this chapter will convince readers that, notwithstanding all the scientific investigation of the matter since Mallard and Le Chatelier first took it up in the year 1880, we have still much to learn about the initial stages of gaseous explosions. Indeed a systematic reinvestigation of the whole subject now seems called for,------."

In the final analysis it appears that the initial stages of flame propagation in tubes is in a very unsatisfactory state because the reduction of the problem to one dimension has been accompanied by many difficulties due among other things to the large tube wall surface in contact with the confined gases. Thus the indeterminate effects of heat conduction, viscous friction, turbulence, mass motions in the gas, convection, pressure waves, etc., more than overbalance the advantage gained from geometrical simplifications.

It is interesting to note that the uncertainties inherent in tube experiments were eliminated at one stroke by Stevens when he replaced rigid containing walls by soap bubble films in his constant pressure bomb. As a reward for this improvement Stevens found his "initial flame velocities" to be constant and reproducible within a small experimental error.

Detonation in Tubes

Berthelot and Vieille(6,7,8) and Mallard and Le

Chatelier(9) discovered the very high velocity regime of flame propagation called "l'onde explosive" about 1881. In later investigations, Berthelot and Vieille specialized in "l'onde explosive" just as Mallard and Le Chatelier were particularly interested in the initial stages of flame propagation. The relatively high velocity found in the "explosion wave" or "detonation" as the process was soon named, required such long distances of travel that the photographic measuring technique had to be replaced by an electrical method. The mixture was fired in a closed tube of lead, so arranged that after a run long enough to set up the explosion wave, the flame broke a bridge of tinfoil and started an electrical chronograph. This chronograph was stopped by the rupture of a second tinfoil bridge placed in the tube at a known distance from the first.

From the very first experiments, velocity measurements on the explosive wave gave consistent results in sharp contrast to the initial stages of combustion.

Bone and Townend(71) summarize the results of Berthelot and Vieille:

"In their experimental work Berthelot and Vieille proved that the velocity of the 'explosion wave' is quite independent of the length of the column of gas traversed, and of the material and diameter of the tube employed, provided a certain small limiting diameter is exceeded;

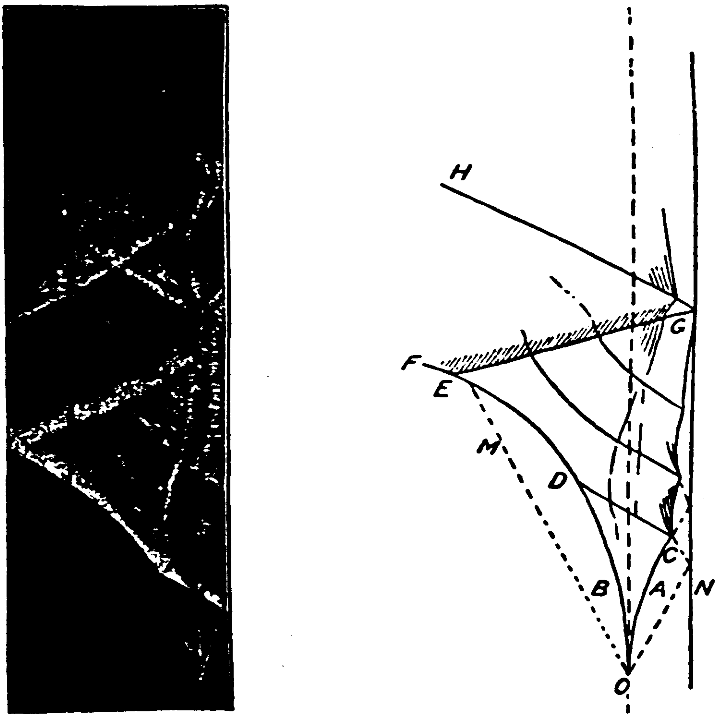
also that it is immaterial whether the tube is laid out straight, coiled round a drum, or even zigzaged. They also concluded that the velocity is independent of the pressure, but this is not strictly correct for, as H. B. Dixon subsequently shewed, the rate increases slightly with pressure up to about two atmospheres, when it becomes nearly constant. They term it 'une propriété fondamentale; car elle établit que la vitesse de propagation de l'onde explosive est régie par les mêmes lois générales que la vitesse du son.'

"Berthelot and Vielle distinguished between two limiting conditions of gaseous combustion, namely; (1) ordinary flame propagation, in which the heat developed by the chemical change is mainly lost by radiation, conduction, and by contact with inert gas, except the small part required to raise the next layer to its ignition temperature, and (2) detonation in which the heat of chemical change is transmitted adiabatically to the next layer. Between such limits a whole series of intermediate states may conceivably intervene, (mais elles ne constituent acun régime regulier... En effect, le passage d'un régime a l'autre est accompagnie, comme il arrive en général dans les transitions de cette espèce, par de mouvements violents des déplacements de matière ètendus et irreguliers, pendant les quels la propagation de la combustion s'opere avec une vitesse de plus en plus considerable.'

"The principal conclusions reached by Berthelot and Vieille may be summarized in two sentences, namely: (i) that the velocity of the explosion wave is a true physical constant for each particular inflammable mixture, and (ii) that the wave is propagated by the impact of the products of combustion of one layer upon the unburnt gases in the next at the mean velocity of translation of the burnt gaseous molecules, retaining the whole heat developed in the reaction, which, however, is to be regarded as a <u>limit</u> representing the <u>maximum</u> possible rate of propagation, and therefore subject to diminution in particular cases."

Dixon and his collaborators (10,11,114,115) began work on detonation soon after the work of Berthelot and Vieille and continued their efforts for many years. They used not only the electrical method for measuring flame velocities but adapted the photographic arrangement of Mallard and Le Chatelier by speeding up the film motion to a possible maximum of 100 meters per second. Figure 29 is a record taken by Dixon. Bone and Townend (71) describe the analysis of this record as given in the following quotation:

"----The igniting spark in starting the explosion at O sends out invisible compression waves in both directions



Development of an Explosion in a CS_2+5O_2 Mixture.

Analysis of Same.
REF. 71

FIG. 29

along the tube; these traverse the unburnt gases in advance of the flame with the velocity of sound, as indicated by the dotted lines OM, ON in the diagram. The flame itself, travelling at first more slowly than the compression waves, traces the curves OA and OB. The compression wave ON, on reaching the closed end of the tube, reflected back again as NC, and, on meeting the flame (which is still travelling in the direction OA), retards it and passes thence through the hot and still burning gases as the visible wave CD. An instant later it overtakes, at D, the front of the flame travelling in the direction OB, thereby accelerating it, and increasing its luminosity in consequence of the quickened combustion. The flame then continues to move forward from D with rapidly accelerated velocity until 'detonation' is finally set up at the point E, near where the flame catches up with the compression wave OM. Indeed, it is the forward 'kick' received by the already accelerated flame front on its overtaking the compression wave OM which finally determines 'detonation.' At the instant when 'detonation' is set up the flame attains its final constant velocity, and the combustion and luminosity their maximum intensities; simultaneously, a strongly luminous wave of compression EG (called the 'retonation wave') is sent backwards through the still burning gases, which on reaching the near end of the tube is reflected back as HG. The 'detonation' wave EF passes onwards through the mixture with its characteristic high velocity and intense luminosity."

Bone and Townend note that Le Chatelier independently discovered that a 'retonation' wave is always set up when a 'detonation' wave starts. They say further:

"Thus in studying gaseous explosions, it is necessary to distinguish between (i) the 'detonation wave' -----, (ii) the 'retonation wave'---, (iii) the 'reflection wave'--- and (iv) the 'collision wave'---, whose velocities through the medium are in the order given, that of 'detonation' being highest, as the following determinations by Le Chatelier for an equimolecular mixture of acetylene and oxygen (C_2H_2 + O_2) indicate:

	Metres per second
Detonation-wave	2990
Retonation-wave	2300
Reflection-wave	2250
Collision-wave	2050

Except in special circumstances (e.g., when it is reinforced by another reflected wave) the velocity of the 'retonation wave' is always inferior to that of the 'detonation wave'. When, however, the 'retonation wave' is developed just at the closed end of a tube it may be reinforced by a 'reflection wave' in which case its velocity is indistinguishable from that of true detonation."

"Taking the evidence as a whole, therefore, there can be but little doubt that during the inital stages of an explosion the combustion <u>in the flame front</u> is comparatively slow, and much combination goes on behind it. In detonation on the other hand, the chemical changes <u>concerned in the propagation of the wave fronts</u> are practically instantaneous; albeit, when a gas burns in <u>stages</u>, only the first of them may actually be concerned in the wave front, as in the combustion of cyanogen which definitely proceeds in two stages. There can also be no doubt as to the important rôle played by compression waves in determining 'detonation' in gaseous explosions, and as to 'collision waves', etc., being largely responsible for their violent shattering effects."

Nernst(81), who devoted much attention to the detonation problem, substantially agrees with the summary of Bone and Townend for, with respect to flame propagation in gases, he says:

"<u>Slow</u> combustion consists in the layer of gas first ignited passing on its heat by conduction to the next layer, and thus bringing the latter to the point of ignition; the rate of propagation depends, therefore, firstly, on the amount of heat conducted, and secondly, on the velocity with which a moderated heated layer begins to react chemically, and so to bring itself to a high temperature, i.e. the rate depends in general on the change of reaction velocity with temperature."

"Combustion may also be propagated in a second, entirely different way, depending on the phenomenon just discussed, that an explosive mixture of gas can be ignited by strong compression, or more correctly----by the resulting rise in temperature. The increase in pressure causes an increase in concentration, and therefore, according to the law of mass action, also an increase in reaction velocity. Hence it is extraordinarily favourable to the rate at which the heat of combustion is developed. We see, therefore, that a very powerful wave of compression produced in a gas can start as well as propagate the combustion, and, moreover, with extraordinarily great velocity."

"A compression wave of this kind is produced in a gaseous mixture brought to a very high temperature by combustion; it must travel considerably faster than the ordinary compression wave, because in the compressed (still unburnt) layer ignition causes a very strong development of pressure, which, according to the theory of waves, must increase the rate of propagation."

"On the basis of these considerations can be calculated the absolute velocity of the explosion wave, --------. It is clear, however, that it must be considerably greater than the velocity of sound in the mass of gas (heated to a high temperature by the explosion). The measurements given below confirm this; they show that the velocity of the explosion wave is one and a half to two times the velocity of sound at the temperature of combustion. The processes taking place after ignition in a combustible gas contained in a long tube, can now be presented as follows: the first condition is that of slow combustion; the heat is conducted to the next layer of gas, and thus combustion is propagated at the rate of but a few metres a second. As, however, a strong increase of pressure is produced by the combustion, so at the same time the neighboring, still unburnt, layers are compressed; this causes an increase in the reaction velocity, as has been already shown, and ignition takes place more rapidly. But the result of this is to cause the next layers to be still more strongly compressed, and in this way we can see that, provided we have a mixture which burns sufficiently fast, the rate of combustion must constantly be increased. As soon as the compression in the unburnt layers becomes so great that self-ignition follows, the resulting extraordinarily powerful compression wave is propagated with very great velocity and with simultaneous ignition, i.e. we have the spontaneous development of the 'explosion wave'.

Campbell and Woodhead(116) studied the ignition of an explosive mixture by a detonation wave in a different mixture. When mixtures having different detonation velocities were used, the velocity in the ignited mixture was found to change rapidly and reach its characteristic value within a short distance of the ignition point. It was also found that the presence of inert gas between the igniting and ignited mixture did not affect the result, thus proving that the observed effect could be produced by the pressure wave alone without the aid of heat conduction.

The experiments described so far were for the most part carried out with inorganic gases. Since engine fuels are ordinarily hydrocarbons, it is of interest to consider certain results of Dixon's on detonation in a hydrocarbon mixture as reported by Bone and Townend:

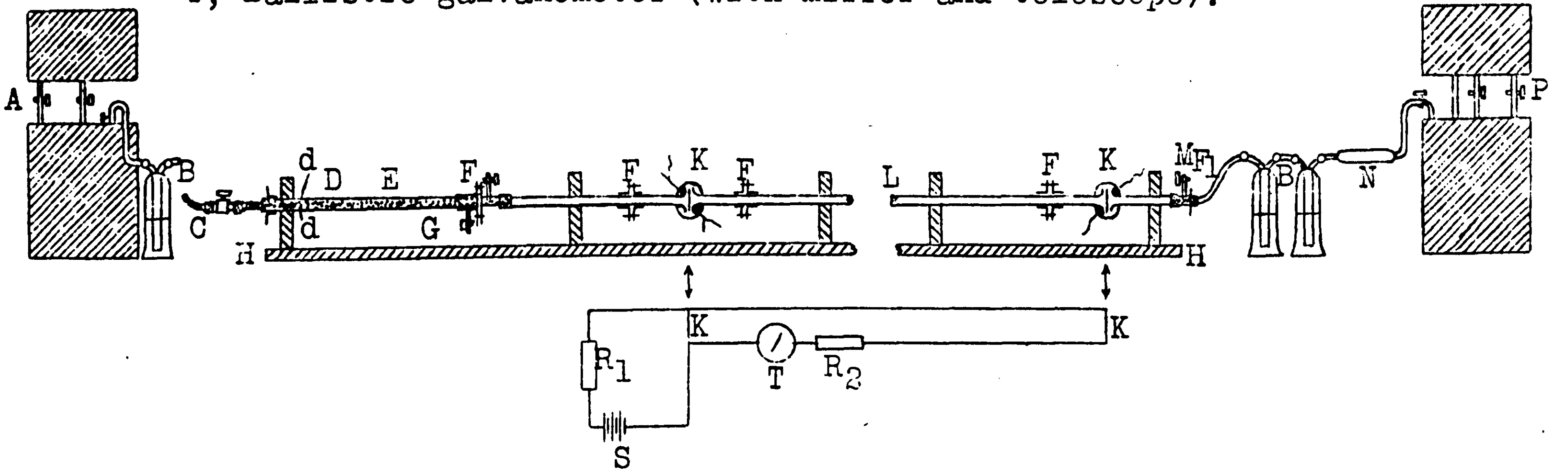
"-----------They shewed conclusively (i) that there are distinct stages in the combustion of hydrocarbons in explosions, the first alone being concerned in the propagation of the explosion wave, and (ii) <u>that in the wave</u> the carbon burns primarily to carbonic oxide, which in turn burns to carbonic anhydride in the rear of the wave. The rate of explosion for a mixture containing sufficient oxygen for <u>complete</u> combustion is always much less than the rate for one containing only sufficient oxygen to burn the carbon to carbonic oxide."

Egerton and Gates(117,118,119), Morgan(120) and Maxwell and Wheeler(121) studied detonation in tubes with particular reference to knocking in internal combustion engines. The results are somewhat confusing since Egerton and Gates concluded that engine fuel mixtures tested under conditions similar to working conditions did not develop detonation waves, while Maxwell and Wheeler held that knocking in engines is produced by detonation waves rather than auto-ignition ahead of the flame front. Morgan showed that it was possible to obtain strong pressure vibrations inside a short tube accompanied by a noise similar to knocking in engines outside the tube. These effects occurred only if the rate of burning became fast enough to set up natural frequency vibrations inside the tube.

Wendlandt(122,123), a student of Nernst, reported the results of a quantitative study of the effects of fuel-air ratios on detonation velocities in long tubes. The essential features of his apparatus are shown in figure 30. The reaction took place in the long glass tube L fitted with aluminum foil bridges K. When the left hand bridge was broken by the wave, a constant current started to flow into a fixed condenser. This current was stopped by the rupture of the second bridge after the wave had moved over a known length of the tube. The time interval involved was estimated from the accumulated charge on the condenser as measured by a calibrated ballistic galvanometer. Ignition of the gases under study was accomplished by a detonation wave in "Knallgas" ($2H_2 + O_2$). This igniting wave was generated in a steel tube extension connected to the reaction tube in which combustion was started by an electric spark. Sufficient distance was allowed between the Knallgas-combustion mixture boundary to allow transient effects to subside. In order to determine the effect of distance traveled on detonation velocity in the mixture under observation, a second glass tube with foil bridges was attached to the end of a long tube placed in series with the first measuring tube.

Tests on the ignition system showed that the impulse supplied at the start of the reaction tube was constant within experimental errors. This was determined by substituting air for combustible gas in the measuring system and measuring the resulting pressure wave velocity.

A, Copper gasometer containing Knallgas $(2H_2+O_2)$. B, Washbottles, H_2SO_4 conc. C, Pressure tubing to connect with oil air pump, or with the lead tubing, or with the Gasometer A. D, Spark gap, 3 mm dd. Connections to secondary coil of inductor. E, Steel tube (130 cm long, 21 mm diameter). F, Flanges 5 cm diameter; F_1 to attach tube extension. G, Stopcock at side of tube. H, Wood support, 5 m long. K, Glass extensions 15 mm long, 11 mm inside diameter to hold the aluminum foil. L, Jene hard glass tube, 21 mm diameter, about 3 mm wall thickness. M, Stopcock to manometer. N, Calcium chloride tube. P, Gasometer with gas mixture to be studied. R, Resistance ($R_1=R_2$ about 10^4 ohms). S, Accumulator (2 - 8 volts). T, Ballistic galvanometer (with mirror and telescope).



REF. 122

Sketch of the short length of the explosion tube showing connections.

FIG. 30

Results from several trials are given below:

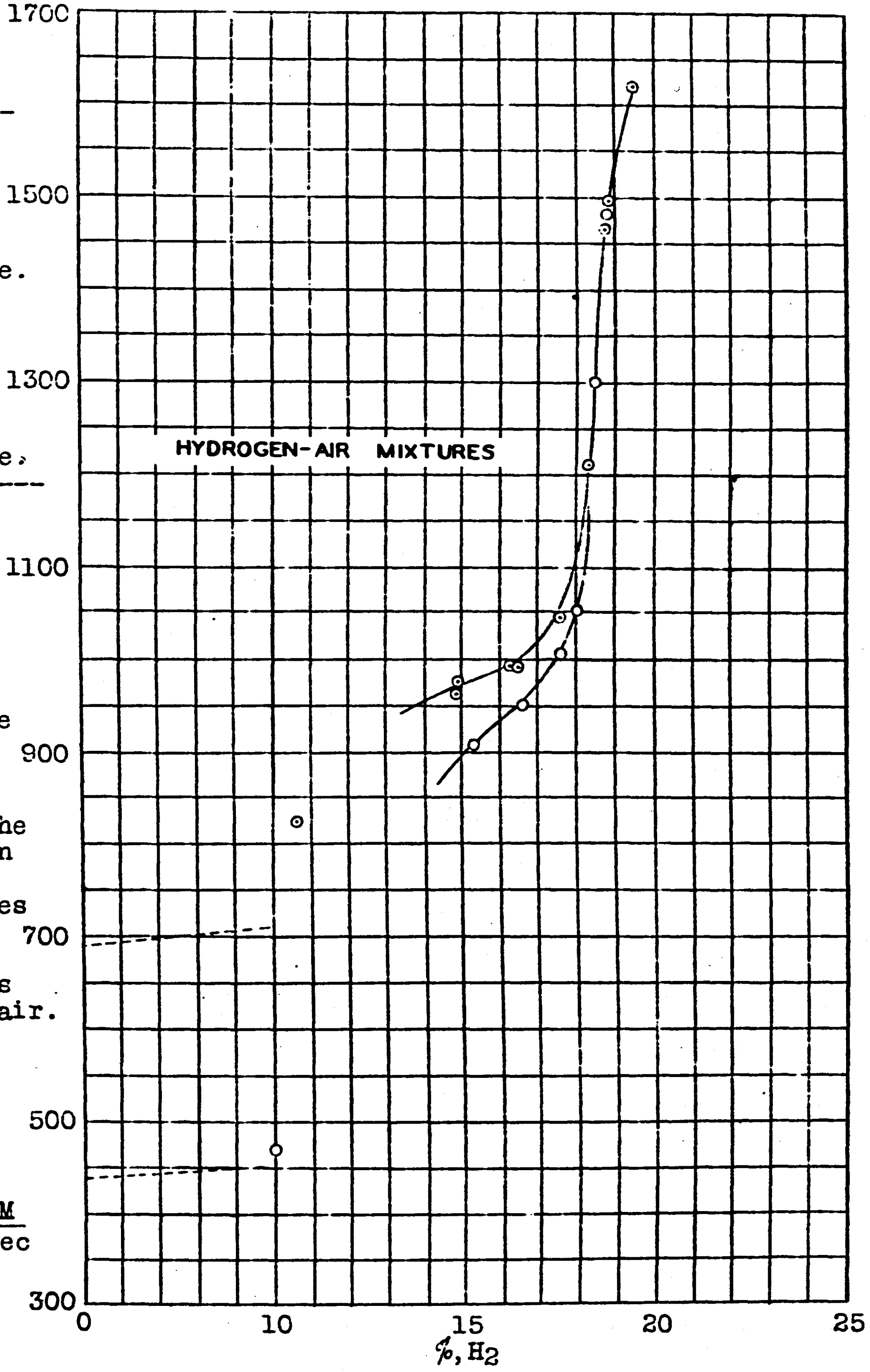
Velocity in air of the impact wave from the detonation of

$2H_2 + O_2$, 1 atmosphere, 18 deg. C.

Length measured	80 to 320 cm	652 to 893 cm
Velocity, meters/sec.	675	423
	680	432
	656	427

This observed decrease in wave velocity with distance is in good agreement with the results of other observers and in accord with the theory of waves of finite amplitude. The velocity of the initial wave was well above the characteristic detonation wave velocity for any of the mixtures used in the experiments.

Figure 31 gives the results from hydrogen-air mixtures. The wave velocity varied greatly with mixture strength, especially in the region between 15 and 20 percent hydrogen. Near the 20 percent mixtures, the wave velocity was the same in both measuring sections, while in weaker mixtures, the velocity was less at the far end than at the near end of the combustion tube. This constancy of velocity over long lengths of the path is taken by Wendlandt as characteristic of Normal Detonation: "By Normal Detonation is to be understood every stable quasi-stationary detonation process with plane wave front; its criterion is



The mark ⊙ indicates observed results over the first measuring distance of the tube. The mark o shows results over the second measuring distance of the tube. The mark ---- indicates the velocity the initial (ignition) impact wave would have in the gaseous mixture if not influenced by the reaction in the gases. These values are calculated from the results found for air.

REF. 122

FIG. 31

the constant rate of propagation over extended distances." The data of figure 31 indicate that normal detonation occurred not at the begining of the sharp rise in velocity from normal burning but further on in the course of this rapid velocity increase.

Discussing his experimental results, Wendlandt says:

"Detonation processes are characterized by intense compression behind the flame (high pressure) and a high rate of propagation (km/s). In distinction from the thermal reaction process (normal burning) there is in detonation the formation of a quasi-stationary wave characteristic of detonation. In long tubes of not too small diameter, the detonation wave is independent of the length of the tube, of its material and of its diameter. Its velocity is a characteristic constant of the gaseous mixture (a mixture is defined by its chemical composition, pressure and temperature)."

"From the proceeding experimental investigation---, it may be seen that by gradually diluting a detonating mixture, the velocity of the detonating wave will gradually decrease until a mixture ratio is reached below which a very rapid fall in velocity is observed for only a slight decrease in the value of the mixture ratio F/O_2. The region where this sharp change takes place corresponds to a definite mixture ratio. -----------."

"Below a certain velocity of propagation, that is, below a certain mixture ratio value, the propagation of the explosive wave no longer shows the characteristic of a quasi-stationary wave. Instead, the wave shows a steady decrease in velocity with increase of distance traversed. At a definite mixture ratio the flame goes out while the initial ignition wave, unaccompanied by chemical change, goes on at decreasing velocity. In the case of hydrogen-air mixtures, this limiting condition lies between an 18 and 19 volume per cent mixture. The corresponding limiting velocity is 1250 m/s. Above this limiting ratio and its corresponding velocity of propagation, the velocity curve in the coordinate figure represents mixture ratios resulting in normal detonation."

"2. Interpretation of Results.-At very high velocity a volume element contained within the zone of a detonation wave has an exceedingly short period in which transformation may take place. This period is at most less than 10^{-5} second. Compared with this the period of reaction in a process at constant pressure and ignition temperature is great. Cassel------ gives for this period in the case of $H_2 + O_2$ mixtures, 10^{-2} second. For $CO + O_2$ mixtures the period may be much longer. Transformation within the explosive wave must take place at temperatures and pressures that decrease with greater and greater dilution of the active components, till finally the period required for their transformation within the wave is not completed and the flame goes out."

"From this viewpoint it could well follow that the rate of propagation of the detonation wave should be constant for each detonating mixture; the reaction period for every such mixture in a quasi-stationary wave is definite, and would be expected to fall off rapidly when by progressive dilution the detonation limit is approached; that is, when the reaction period of the mixture ratios become large."

"In passing from a 19 percent to an 18 percent hydrogen-air mixture, the temperature attained within the wave would, according to calculations to follow sink by about 80 degrees. If we assume that in both these cases the reaction period is small, the transformation in the wave will be complete. But suppose that already in the 19 percent mixture the reaction is only 4/5 what it should be for completion, then for the 18 percent mixture it must be still slower and less complete because the temperature for this mixture lies lower. At still lower temperatures the reaction period would be still greater. With decreasing mixture ratios, decreasing amounts are transformed and decreasing temperatures and longer reaction periods result, until a quasi-stationary wave can no longer be supported."

"Obviously the rate of increase in the velocity of the detonation wave, above this limit, with increase of the fuel will be the greater the more rapidly the reaction period decreases with rise of temperature. This agrees with experimental results. In the carbon-monoxide-oxygen mixtures this increase is steeper than in the hydrogen-air mixtures. Likewise it may be seen why the limit of a normal detonation wave does not lie exactly at the end of this short range of mixture ratios but lies rather in the course of this steep change in the velocity of the impact wave."

The studies outlined above have shown that detonation in gaseous mixtures has given consistent experimental results which can be logically explained by analysis. The essential feature of detonation is that the processes involved are adiabatic. This is because combustion in the wave front takes place in such a short time that substantially no energy can be removed from the seat of the reaction either by mechanical expansion or by heat transfer. With this in mind it is not surprising that consistent results are obtained with any reasonable experimental apparatus since the environment can have no effect on the course of such a reaction. The high propagation velocity of detonation is due to the high temperature and pressure produced when all the chemical energy associated with a given element of the mixture is released in this element without loss to the environment.

The similarity between the detonation wave and an ordinary sound wave is obvious since a detonation wave with a negligible amount of chemical energy released in the wave front would be identical with the sound wave. Interactions between sound waves and detonation waves have been illustrated in a number of the records displayed above. It is apparent that standing pressure waves must result from the initial distrubance set up by a detonation wave in a closed vessel. The high velocity and pressure of the true detonation wave will gradually be lost by dissipative processes and the waves will degenerate into sound waves.

Using the ideas developed from tube experiments, a fundamental definition of detonation can be established, i.e. Detonation is a process within the body of a combustible gas which occurs so rapidly that no mechanical or thermal energy is lost from the reaction zone during the course of the reaction. This definition is compatible with the ideas of detonation in the internal combustion engine already advanced in Section I, includes the results of tube experiments and distinguishes detonation from simple pressure waves in a gaseous medium.

Flame Movement and Pressure Development in Closed Vessels

Experiments under laboratory conditions had fairly well completed the general picture of gaseous combustion by 1920. However, it was certain that the laboratory technique of tube experiments was powerless to answer the question of whether or not a particular fuel would knock under given engine conditions. In answer to this situation several investigators started work with closed vessels in an effort to gain engineering information by approximating the conditions within engine combustion chambers. The object was not to measure thermodynamic quantities but rather to trace out the course of flame travel and pressure development as affected by various operating variables.

Gaseous combustion inside a closed vessel is complicated by the changing conditions under which successive increments of the mixture burn. Thus the first small nucleus of flame will develop under substantially constant pressure at the initial temperature while the last part of the mixture may react almost at constant volume under the high temperature and pressure produced by compression from the already burned gas. These changing conditions are accompanied by mass motions of the mixture and possibly by intense pressure waves. With such an indefinite environment for individual elements it is not surprising that many important problems have not been solved even after intensive effort during the last two decades.

Kratz and Rosecranz(124) using a technique essentially similar to that of Fenning studied pressure development in closed vessels of various shapes. Illuminating gas-air mixtures were used for combustion at various mixture strengths. A number of their conclusions are listed below:

"(2) In general, the effect of turbulence during explosion is to cause an increase of maximum pressure and a decrease in the time of explosion. This effect is greater for lean mixtures than for the richer ones. The maximum pressure is also produced with a slightly greater air-gas ratio than for the case where no turbulence exists."

"(3) The effect of turbulence seems to be due to the more intimate mixing of the gas and air before inflammation, thus bringing more molecules into contact, rather than to the projection of the flame into unburned parts of the mixture."

"(4) The position of the source of ignition has a considerable influence on the rate of inflammation and on the maximum pressure. In vessels patterned after the L-head type of combustion space used in internal combustion engines, ignition in the valve chamber results in a maximum pressure of about 10 per cent less than that obtained at the center of the head."

"(5) Both turbulence and variations in the position of ignition seem to have more influence upon the rate of inflammation than upon the maximum pressure.----"

"(6) In certain cases there is some evidence of the formation of pressure waves which travel smoothly through the mixture and produce a higher maximum pressure than if the inflammation had proceeded in the usual way. These pressure waves differ in character from true explosion waves."

"(7) The maximum pressure and time of explosion are materially affected by the shape of the explosion vessel. This seems to be caused by variations in the ratio of surface to volume for the different vessels. From the standpoint of maximum pressure produced, the spherical explosion vessel is best. The cylindrical and conical vessels give about the same results, but the maximum pressure is about 8 per cent lower than that for the spherical vessel. The L-head type gives results about 16.5 per cent lower than the spherical."

"(8) The combustion of the gases in any pocket in the vessel is more or less incomplete, due to the cooling effect of the walls. The incomplete combustion results in a reduction of the maximum pressure."

"(9) The cooling of the gases for a given mixture during any time after the attainment of maximum pressure varies directly with the ratio of surface to volume of the explosion vessel, for the particular vessels used."

"(10) Radiation from the gaseous mass is an important factor in the cooling of the mixture. Variations in the character of the inner surface of the walls of the vessel cause variations in the cooling curves."

The results cited above are necessarily qualitative and are chiefly of interest in that they show the many variables which must be controlled in order to obtain consistent results from combustion studies.

Woodbury, Lewis and Canby(125,126) combined pressure measurements with flame photographs taken through a transparent window in the side of a cylindrical bomb. Although some measurements were made with detonation mixtures, the chief result of the experiments was to show the way in which flame spreads in a closed chamber. In particular, it was found that the flame proceeds at an approximately constant velocity over part of the path and then proceeds more slowly after being almost stationary for a short time. The pressure-time curve showed a break similar to that of the velocity curve at the time of the "flame arrest". Experiments with artificial turbulence inside the chamber showed that the "flame arrest" could be reduced by this method. The work of Woodbury, Lewis and Canby served as an immediate stimulus to further experimental and analytical studies of flame propagation in closed vessels.

Midgley(127,128) applied the principles of thermodynamics and hydrodynamics to data from the closed vessel experiments. He noted that the motion of each gas particle was reversed in direction by passage of the reaction zone so that the motion was always away from the flame front with a rapid change in velocity at the front. This change in velocity requires a pressure difference across the front. Midgley calculated the pressure difference ~~different~~ from the experimental data and showed that it was small except just before the "flame arrest".

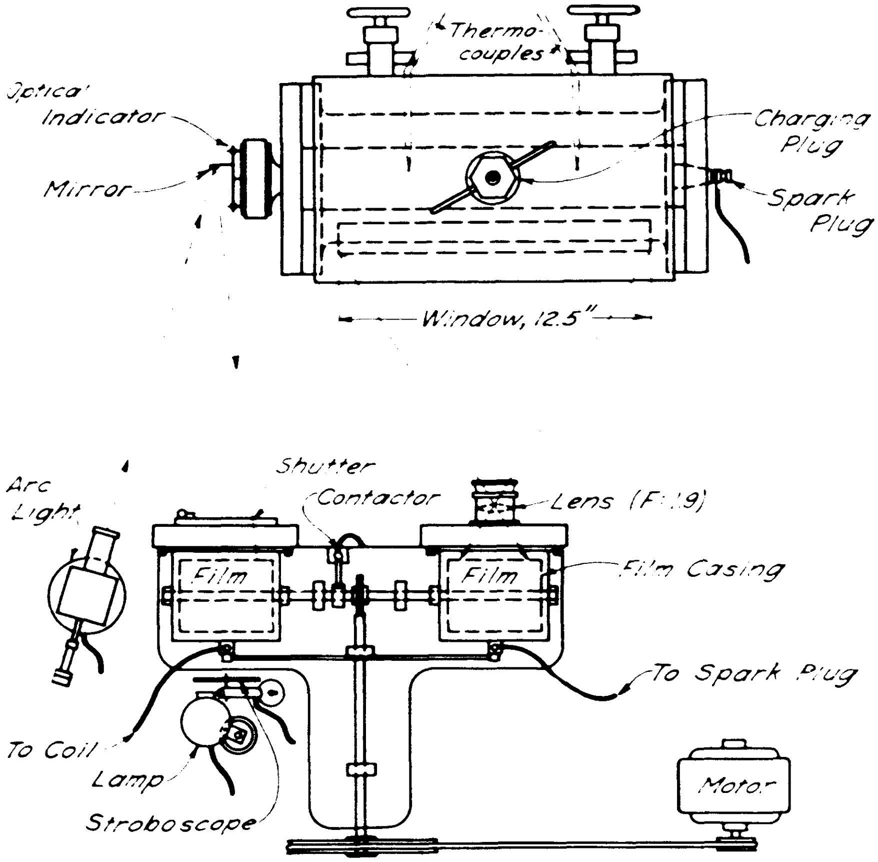


DIAGRAM OF RECORDING APPARATUS

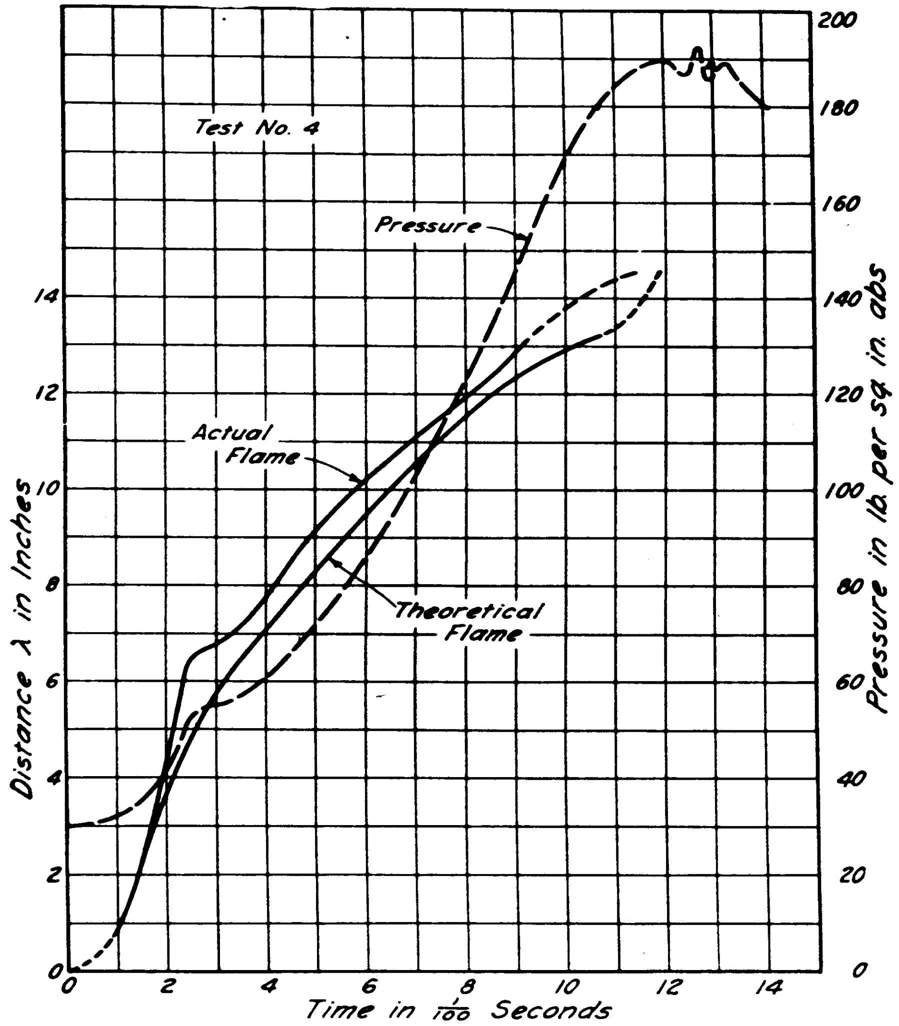
REF. 73

FIG. 32

Midgley and Janeway(129) extended the analysis already started by Midgley by including an expression involving the chemical reaction velocity. The resulting equation for the pressure difference across the flame front indicated the existence of a critical pressure. At this critical pressure, a sudden very great increase in reaction velocity was predicted which would correspond to the start of a detonation wave. This detonation wave was considered as probably identical with the knocking process in internal combustion engines.

Rosecrans(130) investigated pressure development and flame travel in a closed vessel by a method similar to that of Woodbury, Lewis and Canby. In all of his work, ether-air mixtures were used as the combustible gas. Figure 32 shows the arrangement of apparatus with the various parts identified. Figure 33 is a graphical representation of typical records showing flame front position and pressure as a function of time after ignition. The "flame arrest" is plainly shown at about 0.025 seconds.

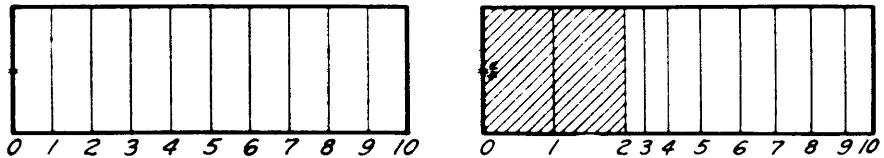
Using a method developed by Nagel(131), Rosecrans analysed the physical processes accompanying combustion inside the closed vessel. Figure 34 shows graphically the changes in volume experienced by various parts of a stagnant mixture during combustion. If uniform ignition over a cross section is assumed to occur at the left hand side



PRESSURE AND FLAME CURVES FOR TEST NO. 4

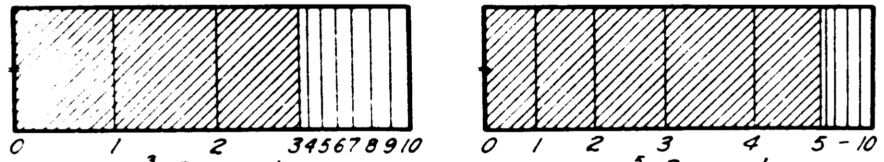
REF. 73

FIG. 33



Before Ignition

2/10 Burned



3/10 Burned

5/10 Burned

DIAGRAM OF EXPLOSION PROCESS

REF. 73

FIG. 34

of a bomb, the layer of gas nearest the end burns and expands. This action compresses all the unburned layers if ordinary combustion is assumed. The second layer is ignited by contact and mechanical spread of the flame, and burns, further compressing the unburned mixture. This action is repeated as shown in the figure until combustion has been completed throughout the enclosed gas. The increased pressure and density of the last portions to burn is apparent.

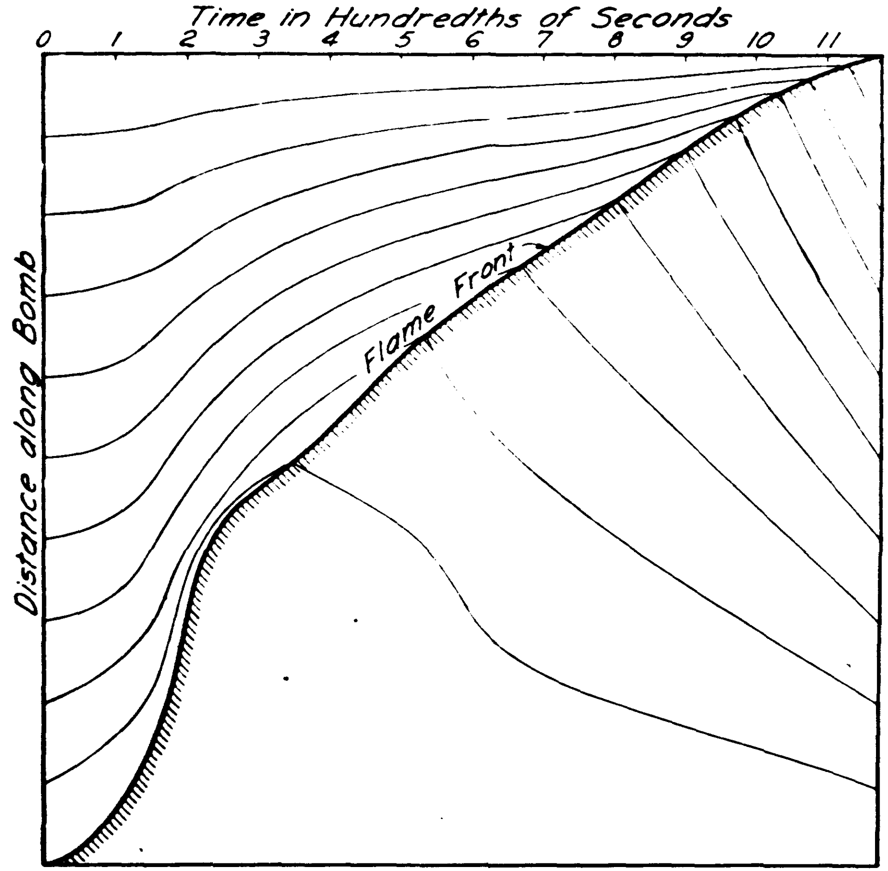
Rosecrans uses the case described above to specify the process he considers in his analysis;

"A combustion such as described is what might be termed a 'normal explosion', since it is not of the 'slow combustion' type which presumably occurs immediately after ignition, nor is it a detonation which might be called 'abnormal explosion'."

Using the method of Midgely, and experimental data from his own work, Rosecrans made the diagram of figure 35, showing the positions of various planes in the mixture during a typical explosion. For example, a plane of gas initially located half way down the bomb at first moves slowly forward ahead of the flame front, then increases in velocity, and finally burns at 0.09 second after ignition. It is then thrown to the rear with a relatively high velocty and finally at the end of the combustion process returns to its original position. The diagram shows how the density behind the flame front is decreased particularly during the intial stages of burning.

After considering several hypdtheses which he rejects, Rosecrans suggests an explanation for the "flame arrest";

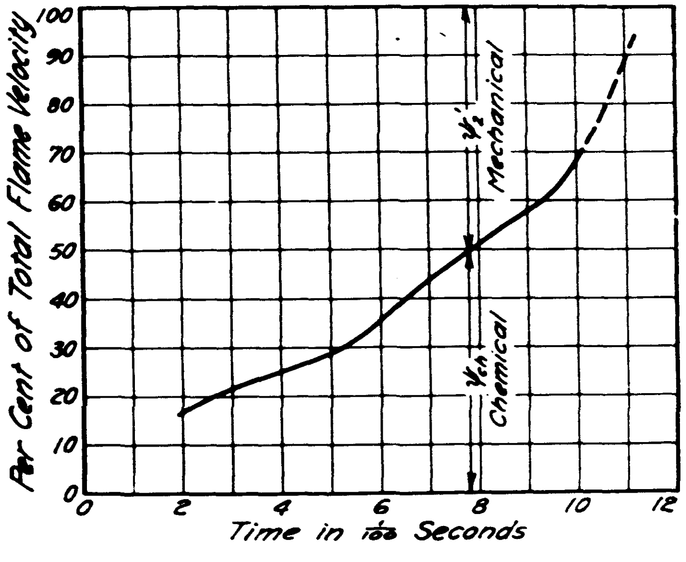
"The results of Woodbury, Lewis and Canby show that when turbulence is produced by a rapidly rotating fan, the flame arrest still exists in practially the same position. However, as the turbulence is increased, by increasing the speed of the fan, the flame arrest becomes less pronounced. These facts suggest the hypothesis that a possible cause of the flame arrest is the partial extinction of the flame on account of low density of the gas. ----- a diagram of motion of various planes of gas during the explosion, shows clearly the great decrease in density of the gas behind the flame front. It is entirely possible that this region of low density extends into the combustion zone itself, and that when the density reaches a certain minimum, partial extinction of the flame occurs. The momentary cessation of the motion of the flame affords an opportunity for pressure equalization, as shown by the



Gas Movement Diagram for Test No. 4

FIG. 35

REF. 73



Curve Showing the Division of the Total Flame Velocity for Test No. 4

FIG. 36

REF. 73

pressure diagram at the instant of flame arrest, and the consequent increase in density in the combustion zone allows the flame to proceed again."

The hypothesis of Rosecrans as to the cause of "flame arrest" is essentially the same as that advanced many years before to explain the extinction of flame in tubes during the "vibratory motion" period.

Rosecrans analysed the spatial motion of flame down the length of his bomb into one component due to mass motion of the gas and a second component due to spreading of the flame within the gas. Figure 36 is a plot showing the division of velocity into these two components during an explosion.

"Figure--- shows, for Test No. 4, the division of the total velocity into forward velocity of the molecules ψ_z' (mechanical velocity) and reaction velocity ψ_{ch} (chemical). The chemical reaction velocity at 0.02 seconds after ignition (the earliest point of the explosion at which computations could be made) was 16 per cent of the total flame velocity. The reaction velocity then increased until at 0.10 second it was 68 per cent of the total flame velocity. The curve naturally trends upwards from the last calculated point at 0.10 seconds, and can be extended to 100 per cent at the time the flame fills the bomb. That is, at the end of the explosion process, when the end wall of the bomb is reached, the total flame velocity is entirely due to chemical reaction velocity."

Rosecrans summarizes his results in the following conclusions:

> "(a) The theoretical analysis, as proposed by Nägel and as adapted to the particular cylindrical bomb explosion apparatus used, gives results which conform approximately to the experimental results as determined by photographing the flame propagation."
>
> "(b) The theoretical flame propagation curve in most cases differs from the experimentally determined curve by amounts which can be accounted for by the heat loss which occurs in the experimental determinations."
>
> "(c) The experimental results on the whole confirm the results of Woodbury, Lewis and Canby, that the flame fills the bomb at the time that maximum pressure is attained. However, this does not always appear to be the case."
>
> "(d) The experimental results do not furnish any evidence of the dependence of the velocity of flame propagation on the initial temperature."

"(e) The flame arrest phenomenon appears in all the flame photographs, and occurs quite uniformly at about one half the length of the bomb from the ignition point. At the time of the flame arrest the pressure had in general risen to about 25 per cent of the maximum."

"(f) The hypothesis that the flame arrest was caused by spherical pressure waves, or such waves reflected from the side walls of the bomb, has been disproved."

"(g) The hypothesis that the flame arrest was caused by a sound wave reflected from the end of the bomb opposite the ignition point has been disproved."

"(h) A suggested hypothesis for the cause of the flame arrest is that the great decrease in density of the gases within and behind the flame front causes a momentary extinction of the flame. This explanation at least does not conflict with any known facts regarding the flame arrest."

"(i) From the characteristics of the expansion and compression of the burned and unburned gases, respectively, during an explosion, the existence of afterburning and adjustment of chemical equilibrium behind the flame front has been demonstrated."

"(j) The relative proportion of the total flame velocity which is due to chemical reaction velocity, for the mixtures investigated, varies from 15 to 85 per cent during the progress of the explosion."

"(k) The results indicate that the reaction velocity is a function of the pressure. An equation expressing this relation has been suggested."

The work of Rosecrans does not deal directly with detonation but it does illustrate the general process of flame propagation in closed vessels and proves that the conditions of high temperature and density which favor the high reaction rates required for detonation tend to be built up in the last part of the charge to burn.

Wheeler and his co-workers(121,132,133) investigated combustion in a closed vessel using simultaneous flame pictures and pressure records. In general they checked the results already reported by Rosecrans with regard to "normal explosions" and extended the work by observing knocking explosions of pentane in a 15 inch steel cylinder. The usual phenomenon of flame arrest after an initial acceleration was found. An increase in the initial pressure caused the characteristic "pinking sound" accompanied by vibrations in the luminous gas and in the pressure record. In these cases there was a sudden acceleration in the flame motion near the end of the combustion chamber. The "afterglow" was of shorter duration and started only after flame had reached the end of the cylinder. A reduction in length of the cylinder caused a detonating mixture to burn without detonation. Tetraethyl lead vapor produced no effect on the explosion unless it was first decomposed.

Brown and his collaborators(134 to 142 inclusive) used the closed vessel technique for a series of investigations starting in 1925. These workers were interested in chemical phases of the detonation problem and in particular studied the correlation between the tendency of a fuel to detonate and its behaviour in their combustion chamber. The "autoignition temperature" and the maximum rate of rise of pressure during combustion in a closed vessel were found to be important factors in determining the tendency of a fuel to detonate. They found that for certain mixtures there is a critical initial temperature above which an increased initial temperature causes a reduction rather than an increase in the rate of rise of pressure. It was shown that flame velocity and the rate of pressure rise varied together with changes in conditions inside the bomb.

By exploding fuel-oxygen mixtures with added nitrogen and judging the tendency to detonate by the amount of nitrogen required to reduce detonation to an arbitrary level, it was concluded that the rate of rise of pressure was an important factor in measuring the detonation tendency in particular cases. Aromatic and paraffin hydrocarbons with similar rates of pressure rise and therefore supposedly with the same tendency to set up the detonation wave, were shown to have different tendencies to knock in engines. For this reason it was suggested that engine knocking was not due to detonation waves.

Summarizing their efforts to analyse the causes of engine knock, Brown and Watkins(138) said;

"The facts summarized in this paper indicate that both the rate of rise of pressure and the auto-ignition must be considered in estimating the tendency of a fuel to knock in an internal combustion engine, and suggest auto-ignition of the unburned part of the charge caused by adiabatic compression against heated surfaces as the mechanism of combustion causing engine knock. The rate of flame travel and the rate of rise of pressure in explosions may approach the rate accompanying the detonation wave, but the mechanism of auto-ignition is so radically different from the mechanism initiating the detonation wave that a precise distinction should be made."

"------ It was found that if the maximum rate of rise of pressure as determined in a progressive homogenous reaction under constant initial conditions be divided by the auto-ignition temperature on the absolute scale, a number is obtained which varies directly as the knocking tendency of that particular fuel in an engine, or inversely as Ricardo's 'highest useful compression ratio' for that fuel."

Carr and Brown(140) and Souders and Brown(141) studied the effect of tetraethyl lead (the most widely used anti-knock agent) on flame propagation and pressure rise in their bomb apparatus. The lead tetraethyl was added both as a vapor and after decomposition. When added as a vapor the material was effective only when combustion was slow enough for decomposition to take place ahead of the flame. These effects were a decrease both in flame velocity and the rate of pressure rise. If auto-ignition was induced ahead of the flame by means of a hot surface, the lead tetraethyl decreased the velocity of the auto-ignition flame and also increased the temperature necessary for auto-ignition to take place.

Duchêne(143) designed an apparatus for flame study to give a closer approximation to engine conditions than a chamber with rigid walls. He used hydrocarbon vapor-air mixtures in a tube closed by a movable piston. By a sudden motion of this piston the combustible mixture was compressed to a point below its auto-ignition temperature and ignited by a spark while in this condition. A number of hydrocarbons similar to commercial engine fuels were investigated under varying conditions. The results were in general similar to those already described by Rosecrans for ether-air mixtures. Duchêne applied the analytical method proposed by Mach(144) and was able to explain the essential features of his experimental results.

The above account of closed vessel experiments with simultaneous records of flame position and pressure shows that the general features of combustion within rigid boundaries are known. Within such an enclosure, burning in a combustible gas starts at practically constant pressure and the initial temperature, while the last part of the reaction takes place under high density and temperature conditions which are particularly favorable to rapid reactions. Since rapid combustion is a pre-requisite for the appearance of detonation, it is logical to expect detonation in a closed vessel only after a large portion of the charge has burned. That detonation usually starts in the latter stages of combustion in closed vessels, has been a common observation of all investigators.

Although a qualitative relationship between auto-ignition temperature, rate of pressure rise in a closed vessel and the tendency of a given fuel to detonate in engines has been established, no precise method for predicting the performance of fuels under operating conditions was developed. It was this failure of laboratory studies of combustion to produce a reliable method for knock rating fuels that resulted in the use of engine test procedures as already described.

SECTION III

DETONATION AS A COMBUSTION CHAMBER PROCESS

DETONATION AS A COMBUSTION CHAMBER PROCESS

Introduction

Combustion in an engine takes place under conditions which differ from those of bomb experiments in several ways:

(1) The fresh charge is mixed with hot combustion products from the preceding cycle.

(2) The gases are ignited immediately after rapid compression in a hot chamber which follows high velocity flow through a restricted valve port.

(3) Burning proceeds in an enclosure of irregular shape with walls of uneven temperatures and changing volume due to piston motion.

Because of these differences it was found impossible to apply directly either the analytical or laboratory results of bomb experiments to engine problems. By 1925 the limitations of classical methods were so apparent that a new technique appeared which uses engines in actual operation. Improvements both in instrumentation and analysis rapidly appeared and are still in progress.

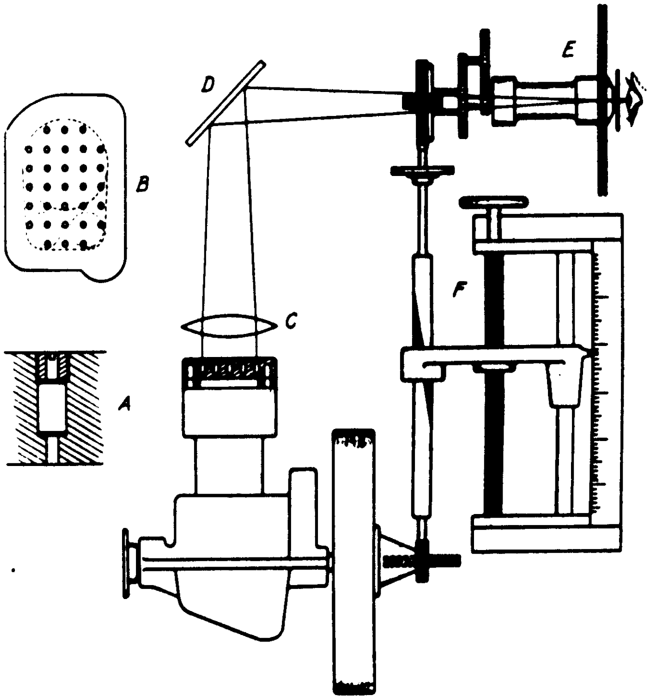
Flame movement and pressure development in engines can be investigated by means of instruments similar to those used in work with bombs and tubes. Pressure indicators, flame cameras, radiometers and ionisation gaps have all been adapted to engine conditions. In one respect the availability of a large number of successive cycles for study has simplified the technique of engine experiments. This repetitive nature of the process makes it possible to use the stroboscopic method of measurement. An instrument using this method is designed to indicate the value of some physical quantity at a single point of the cycle. By making this point definite and controllable the complete history of the quantity under study can be traced. With the rapid succession of cycles in an engine direct visual observations with a flashing light can be used in addition to recording processes.

Investigators of engine combustion have all found erratic variations of considerable magnitude between cycles. For this reason, data taken by stroboscopic instruments do not give precise lines but rather dispersed points through which average curves must be drawn. This unavoidable averaging effect of stroboscopic devices makes this type of instrument particularly suitable for studies of quantities associated with general engine performance. For example, it is better to judge mean cylinder pressure from an averaged record than from the instantaneous pressures of a single cycle. On the other hand, a composite record made up of points from many cycles is worthless for following the rapid changes of high frequency pressure waves inside the combustion chamber. The possibilities and limitations of particular instruments will appear in the following discussion.

Detonation will play a large part in many of the investigations to be reported. The desire to understand and then to control this troublesome phenomenon has always been in the background even though a particular research was directed toward another specific object. It will be found that the results of engine experiments confirm the notions of detonation already advanced without giving material assistance toward a quantitative analysis of the processes involved.

Flame Movement in the Combustion Chamber

Ricardo(145) and Glyde(146) placed a single row of windows across the cylinder head of an engine and used a stroboscopic shutter to determine the crank angle at which the flame appeared in each. Simultaneous records made with an indicator showed the relationship between



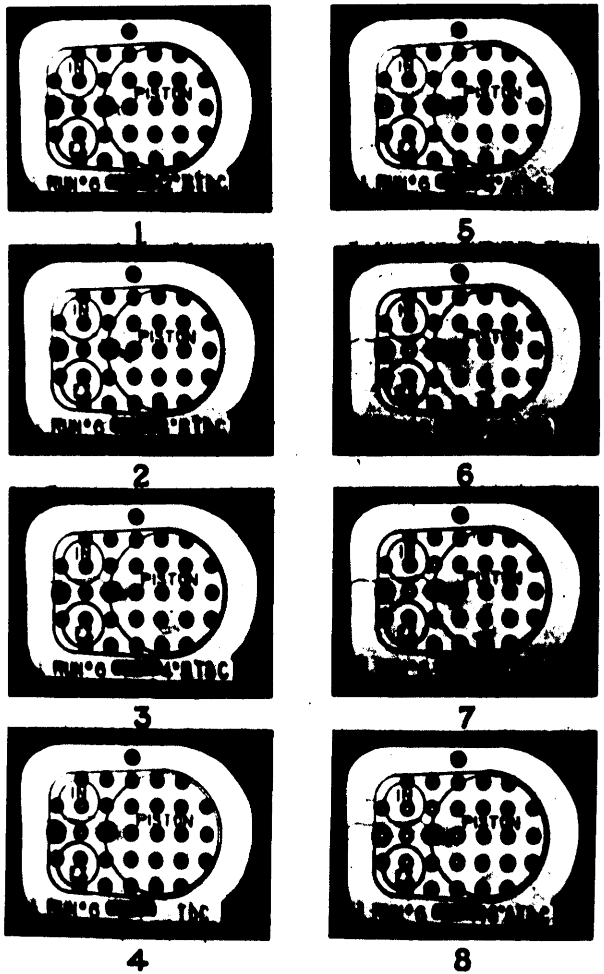
Apparatus Used in Studying Flame Movement

Special engine heads with numerous windows are observed through a stroboscope, which permits a momentary view at the same point in successive cycles. By changing the timing of the view, the progress of the flame can be followed.

A=Detail of Window
B=Plan View of Head
C=Lens
D=Mirror
E=Stroboscope
F=Phase-Changing Device

REF. 148

FIG. 37



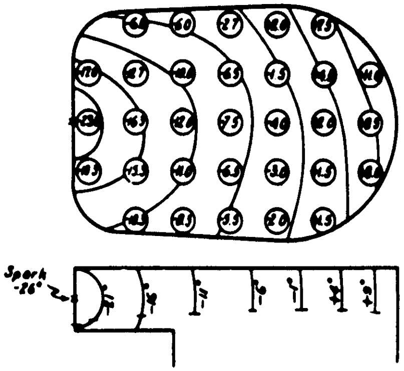
Typical Photographs Showing Progress of Inflammation REF. 148

The spark plug—with wire attached—is just over the left-hand edge of the piston. The spark advance was 20 deg., and picture 1 shows the first trace of flame in the window to the left of the plug. Succeeding pictures show the spread of flame to all parts of the combustion chamber.

FIG. 38

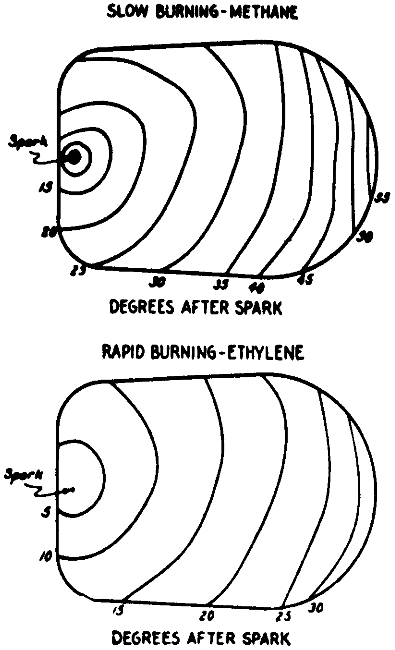
flame front position and cylinder pressure. Their results showed that the flame moved across the combustion chamber in the same general manner as in a closed bomb except for a pronounced effect of turbulence.

Marvin and his collaborators(147,148,149) have carried out an extensive investigation of the effects of engine design and operating conditions on flame movements in the combustion chamber. A number of small windows were distributed over the top of the combustion chamber and observations were made with a stroboscopic shutter. Figure 37 shows the experimental arrangement for visual observations. Two discs with properly shaped holes were used in the shutter so the period of observation at each crank angle was very short. Permanent records were made by placing a camera at the eye position. The extent of the burned gases at a given instant could be approximately determined by observing the illuminated windows in the photograph. Figure 38 is a typical series of photographs showing the progress of inflammation in a particular case. From the photographic record, flame diagrams like those of figure 39 were plotted and were used in studying the effect of engine variables. Pressure records corresponding to each flame diagram were taken with the Bureau of Standards Balanced Diaphragm Indicator.

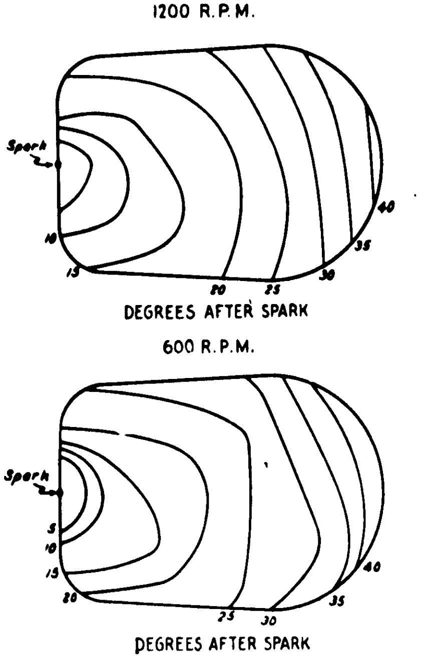


Typical Flame Diagram. The Position of the Engine Crank When Flame Arrives Under Each of the Windows, as Determined from Photographs or Visual Observations, Is Used in Estimating the Position of the Flame Front at Successive 5-Deg. Intervals after Occurrence of the Spark

REF. 149



Effect of a Difference in "Reaction Velocity", That Is, Linear Rate of Advance of the Flame Front into the Unburned Charge

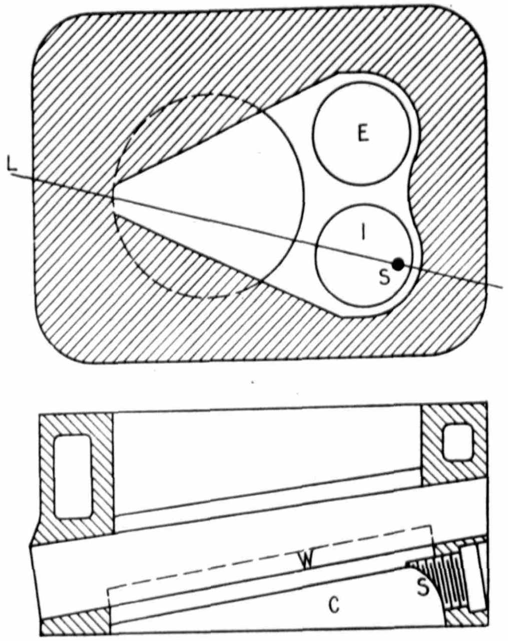


Flame Velocities Are Nearly Proportional to Engine Speed; So, the Time Required for Complete Inflammation of the Charge, Expressed in Degrees of Engine-Crank Rotation, Remains Nearly Constant Over a Wide Range of Speeds

FIG. 39

Marvin, Wharton and Roeder(149) summarize the results of their flame studies in the following conclusions:

> "(1) Under all conditions covered, the flame spreads in a roughly concentric pattern about the point or points of ignition."
>
> "(2) It follows that with single ignition, the shortest combustion time will be obtained by placing the spark plug near the center of the combustion chamber."
>
> "(3) Still shorter combustion times can be secured by using two or more plugs."
>
> "(4) Flame velocities are dependent upon the character of the fuel and are reduced by addition of residual gases to the charge or by departures from the mixture ratio giving maximum power."
>
> "(5) For normal explosions in engine, flame speeds appear to be independent of pressure and, while they probably increase with temperature, evidence of pronounced effects is lacking."
>
> "(6) Flame speeds and the rate of pressure development increase nearly as fast as engine speed, which explains why engines can be operated at very high speeds with only a moderate

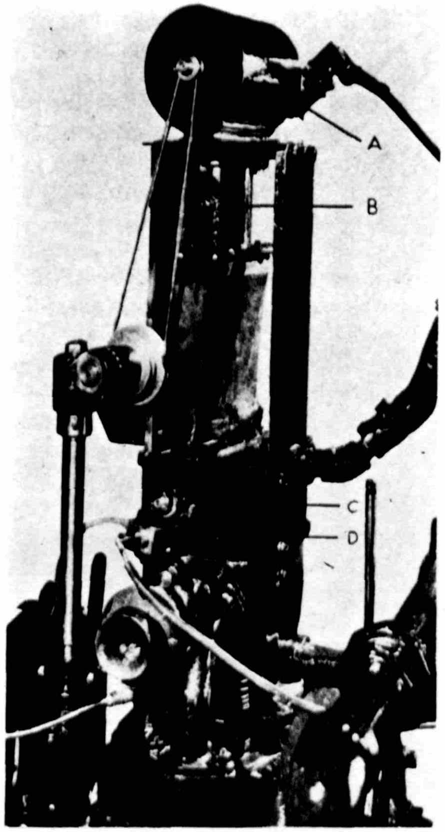


Schematic diagram of combustion chamber B.

S, spark plug location. L, centreline of window. E, exhaust valve. W, window. I, intake valve. C, combustion chamber.

REF. 151

FIG. 40



Combustion Camera Mounted upon Engine

A—Film drum
B—Cylindrical tube which supported lens carrier
C—Nut and washer on end of window retainer
D—Spark plug indicator

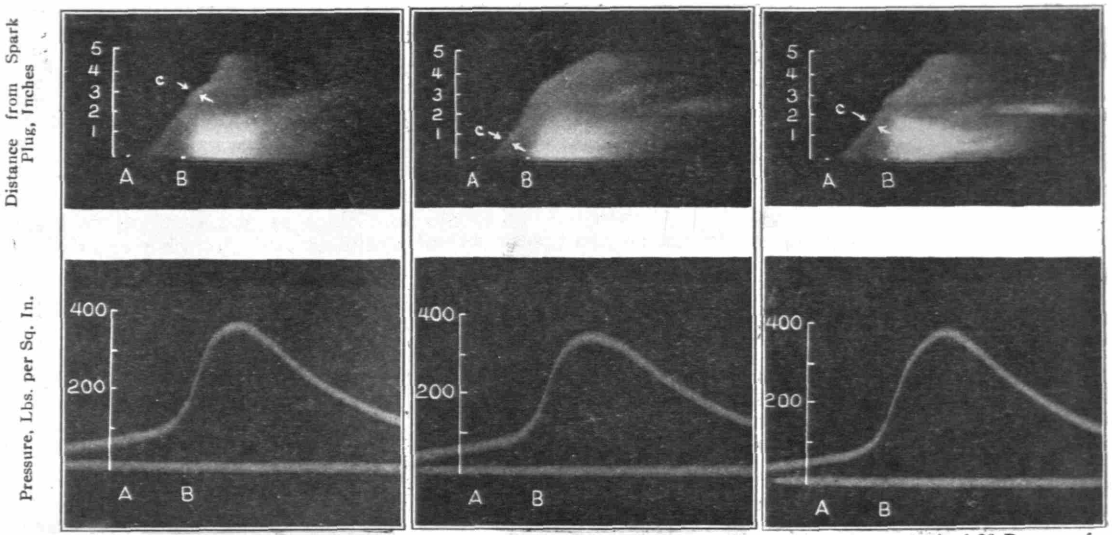
REF. 150

FIG. 41

> composition and pressure and temperature in the low temperature range."
>
> "(12) In the engine, preflame reactions alter the composition and temperature of the unburned charge to an unknown extent, which probably varies for different fuels. Since these reactions are associated with relatively high temperatures and extremely short heating periods, it would appear that they can be produced and their effects studied with certainty only in an engine or a high-speed compression machine."

Withrow and Boyd(150) and Withrow and Rassweiler(151,152) substituted a narrow continuous window across the combustion chamber for the discrete windows of the stroboscopic method and used a rotating drum camera to study flame propagation. Figure 40 shows an elevation and plan view of their combustion chamber and window. Figure 41 is a photograph of the experimental engine with the film drum in place. A General Motors Indicator(53) was used to record instantaneous pressures.

Sample flame and pressure records for normal combustion are shown in Plate IX. The narrow region identified as the combustion zone is marked with arrows in each case. The reillumination of the burnt gases after



Charge Ignited 20 Degrees of Revolution before Top Dead Center

Charge Ignited 25 Degrees of Revolution before Top Dead Center

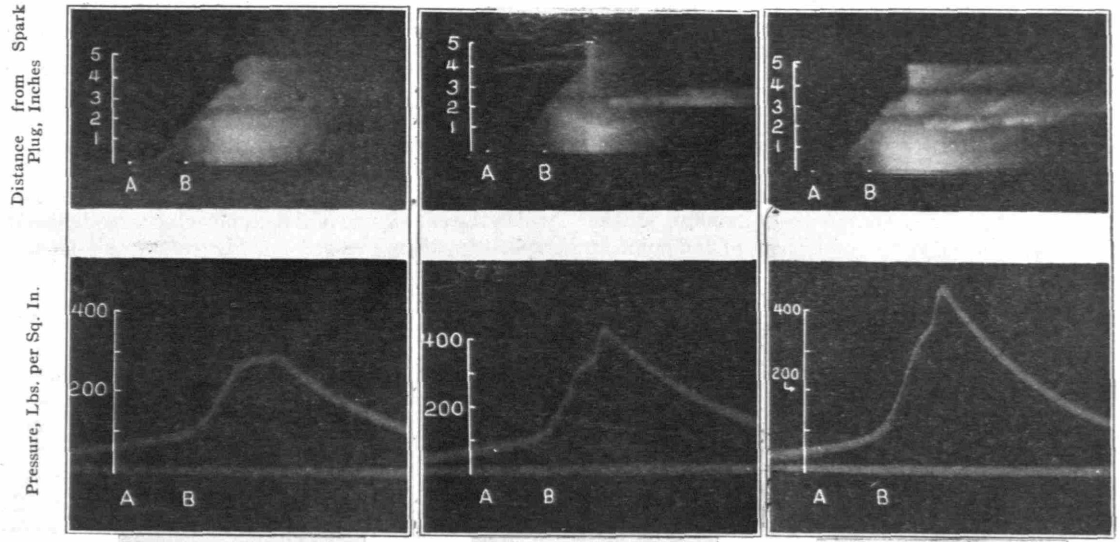
Charge Ignited 30 Degrees of Revolution before Top Dead Center

Flame Movement in a Mixture of Benzene and Air

REF. 150

NON KNOCKING EXPLOSIONS

PLATE IX



Spark Advance of 20 Degrees

Spark Advance of 25 Degrees

Spark Advance of 30 Degrees

Flame Movement in Engine When Operated with a Fuel Composed of 25 Per Cent Benzene and 75 Per Cent Gasoline

KNOCKING EXPLOSIONS

REF. 150

PLATE X

the flame has completed its path is apparent in all three pictures. This phenomenon is called the <u>afterglow</u>. Plate X shows three records with increasing intensities of detonation from left to right. These flame pictures are characterized by a very rapid inflammation of the last part of the charge to burn while the pressure records show a sudden increase in pressure at the instant the flame acceleration occurs.

Withrow and Boyd were especially interested in flame propagation during detonation and discuss the experimental results in detail:

"One of the most significant facts that is brought out by these flame and pressure records is that the phenomenon of knock which heretofore has been recognized in the gasoline engine by its sound and the characteristic shapes of the pressure records is due to a many-fold increase in the rate of inflammation within the latter portion of the charge. In other words, just at the time that knock begins there is a many-fold increase in the amount of charge beginning to burn, with the emission of light, per unit time. During the course of the present work this characteristic of the knock has been observed in hundreds of knocking explosions in the engine both photographically and visually. The visual observations were made by replacing the film drum with a rotating mirror. Further, the absence of this characteristic increase in the rate of inflammation has been observed in hundreds of non-knocking explosions in the engine."

"In regard to the way in which inflammation takes place at the time of knock there seem to be three possibilities. First, at the beginning of the knock spontaneous ignition may occur at a point or a number of points in advance of the normal combustion zone. This flame then spreads rapidly through the remaining unburned portions of the charge. Second, all of the unburned portions of the charge visible through the window may burst into flame simultaneously at the beginning of the knock, instead of at one or several points in advance of the normal combustion zone. Third, the velocity of propagation of the normal combustion zone may increase sharply or to substantial infinity as it passes through the last part of the charge to burn. The records in this paper offer evidence that all three of these types of inflammation occur in various knocking explosions in the gasoline engine."

Several other aspects of the combustion problem were considered by Withrow and Boyd in their conclusions;

"(1) Starting at the spark plug, a flame, or narrow combustion zone, moves progressively through the charge. Oxidation of the fuel is apparently complete within this narrow zone of combustion, but the products of combustion to the rear of the flame front continue to emit light for some time."

"(2) During normal or non-knocking combustion, the time required for the average flame to travel across the combustion space (under the conditions of these tests) was about 40 degrees of crankshaft revolution."

"(3) A knocking explosion differs from a non-knocking one only in the way the last portion of the charge burns. The difference is this: Whereas in a non-knocking explosion the flame continues to move at a comparatively constant velocity clear to the end of the combustion space, in a knocking explosion the latter portion of the charge inflames at a much higher rate than normal. This rate is often so high that at the instant of knock the flame appears simultaneously throughout the whole of the portion of the charge still remaining to be burned."

"(4) The extremely high rate of inflammation in that portion of the charge which burns at the instant of knock is apparently due to auto-ignition occuring within it. This may be caused by temperature induced within that part of the charge by adiabatic compression."

"(5) The violence of the knock is determined by how large a portion of the total charge is involved in the spontaneous inflammation, or the amount of it still remaining to be burned at the instant knock occurs."

"(6) The very rapid, and often substantially instantaneous, inflammation that occurs within the portion of the charge which burns at the instant of knock is accompanied simultaneously by a very rapid rise in cylinder pressure. The magnitude of this pressure rise increases along with the intensity of the knock."

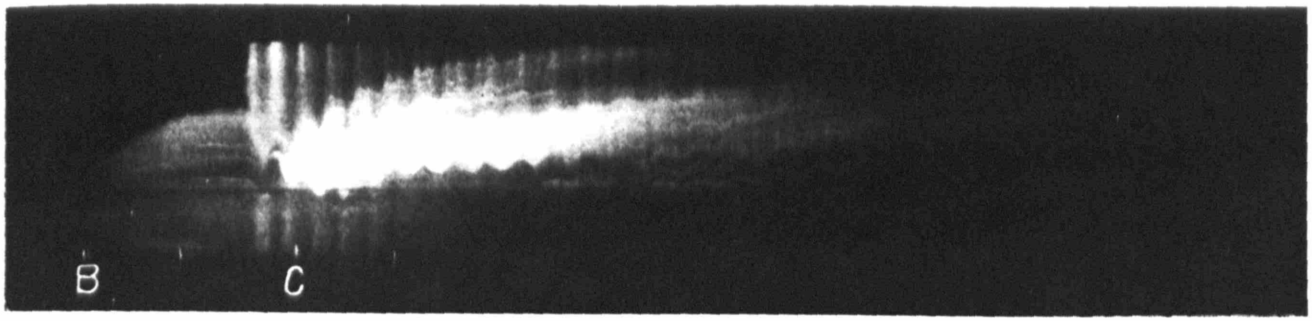
"(7) The one effect upon combustion of the presence of lead tetraethyl in the gasoline is to prevent the extremely rapid inflammation of the latter portion of the charge, and the accompanying pressure rise which is the knock. Lead tetraethyl has no effect upon the velocity

> or the character of the flame prior to the time at which knock would have occured in its absence."

The flame pictures so far displayed have shown evidence of a sudden pressure rise but no sign of pressure waves within the cylinder charge. However, Rassweiler and Withrow(152) note that cyclic motions of glowing particles within the burned gases appear after the initial disturbance due to a severe knock. Plate XI showing wave motion in the burned gases is one of the first published records of this phenomenon in a flame picture. With regard to this picture the investigators say:

"When these flame records were taken, the knock was so violent that sound or shock waves were set up in the freshly burned gases. These shock waves reveal themselves both in the detonating zone and in the vicinity of the spark plug because of the fact that the intensity of the luminosity fluctuated as the peak of the compression wave moved back and forth through the combustion space."

"Several of the shock-wave pictures have been obtained, and they are very interesting for a number of reasons, chief among which is the possibility of estimating engine temperatures by making the assumption that these



Flame Record of Knocking Explosion, Showing Shock Waves

B. 20 degrees after ignition *C.* 40 degrees after ignition

REF. 152

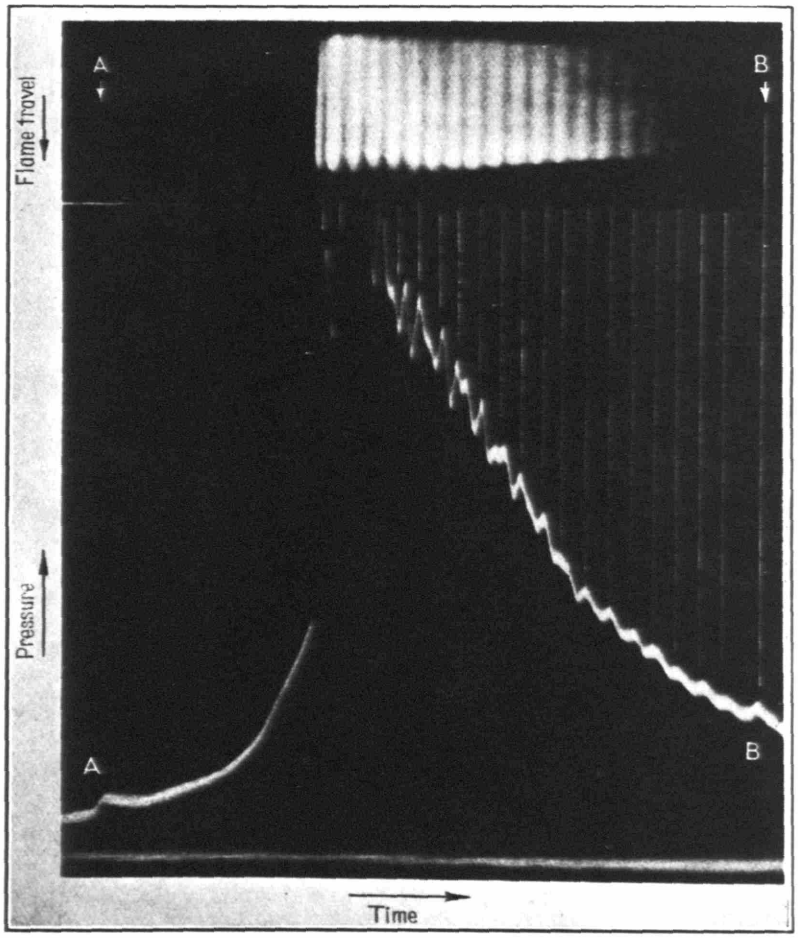
PLATE XI

waves behave as sound waves in a perfect gas. The average velocity of the waves in Fig.---, for example, is 1040 meters per second during a period of time equivalent to 15 degrees of crank-shaft revolution just before the knock occurred. On the basis of the foregoing assumption, this corresponds to a temperature of about 2250 degrees C."

It is interesting to note that flame pictures similar to those of Rassweiler and Withrow were being made at M.I.T. at about the same time (1932-33) by J. P. Elting. One of Elting's pictures was included in N.A.C.A. Report No. 493 by the writer(56) and used for the purpose of estimating pressures in combustion chamber waves.

To clinch the argument that pressure waves were responsible for the cyclic gas disturbances, Withrow and Rassweiler(151) made simultaneous records of gas motion and cylinder pressures. One of these combined records is given in Plate XII and shows the correspondence between oscillations in the gases and fluctuations in the pressure. Using evidence of this type Withrow and Rassweiler concluded:

"The fact that the frequency of the vibrations changes with time and is nearly independent of engine



The relationship between the frequencies of the shock waves on the flame photographs and the vibrations recorded on the pressure cards at an engine speed of 1,500 r.p.m. Knock induced with isopropyl nitrite.

Fuel, petrol. Full throttle. Spark advance, 30 deg.
A ignition. B, 91 deg. after ignition.

REF. 151

PLATE XII

speed, indicates that the disturbances on the pressure cards and the flame pictures are not induced by movement of parts of the engine. As further evidence of this fact the non-knocking flame picture of figure--- is presented. This record was photographed under the same engine conditions as the flame picture in figure---- except that no isopropyl nitrite was added. The picture is typical for non-knocking combustion and no shock waves can be discerned."

"After seeing the correlation between flame and pressure records the possibility seems rather remote that the vibrations on the pressure cards arise from some natural frequency in the indicator or the recording oscillograph. However, there is other evidence on this matter; namely, the frequency characteristics of the indicator and the oscillograph. The natural frequency of the tongue of the indicator as determined by Martin and Caris is between 5 and 10 kilocycles per sec., a value well above that shown on the pressure records. The natural frequency of the oscillograph element is around 1 kilocycle per sec."

"It should be emphasised that the frequency of the pressure fluctuations is correctly registered even though the natural frequency of the oscillograph element is only around 1 kilocycle per second. However, the wave-form, phase and magnitude of the pressure fluctuations are not correctly registered."

"On the basis of this evidence, the conclusions are: (1) that the dominant pressure fluctuations observed on the indicator cards taken with the carbon-stack indicator result from real pressure fluctuations in the engine, and (2) that the inference previously drawn from the flame pictures alone that pressure waves are sometimes set up in an engine running under knocking conditions, is correct."

The work of Withrow and Rassweiler clarified current ideas about detonation by replacing more or less reasonable speculations with experimental facts. Their conclusion that burning is completed in the flame front while the "afterglow" is not due to further rapid chemical reaction but to thermal effects in the products of combustion, does not agree with the idea of Bone and Townend which was cited earlier. Withrow and Rassweiler regard the "main combustion" of Bone and Townend as "afterglow" and the "ghostlike flame" of the earlier investigators as the main combustion. With regard to detonation, Withrow and Rassweiler definitely proved the existence of pressure waves in the cylinder gases and showed the nature of the excitation responsible for these pressure waves.

Bouchard, C. F. Taylor and E. S. Taylor(153) used the method of flame photography through a narrow window for quantitative determination of the effects of engine operating conditions. These investigators were especially interested in changes due to altitude and supercharging. A summary of their conclusions is given below:

> "(1) Under conditions of normal combustion without detonation, the general nature of the movement of the flame front remains the same, that is, a period of slow burning exists after the initiation of combustion during which the flame speed is gradually increasing until it reaches a maximum at a point approximately 10 per cent of the distance across the combustion-chamber. The flame front then continues to travel on at a nearly uniform maximum speed to a point approximately 90 per cent of the distance across the chamber. From this point, as the flame approaches the opposite side of the chamber, its speed decreases rapidly."

"(2) The speed of the flame front increases, and the time for 10 and 95 per cent flame travel are reduced by the following:

Increase in the pressure level at which combustion occurs.

Decrease in initial temperature.

Decrease in proportion of residual gas.

Increase in small-scale turbulence.

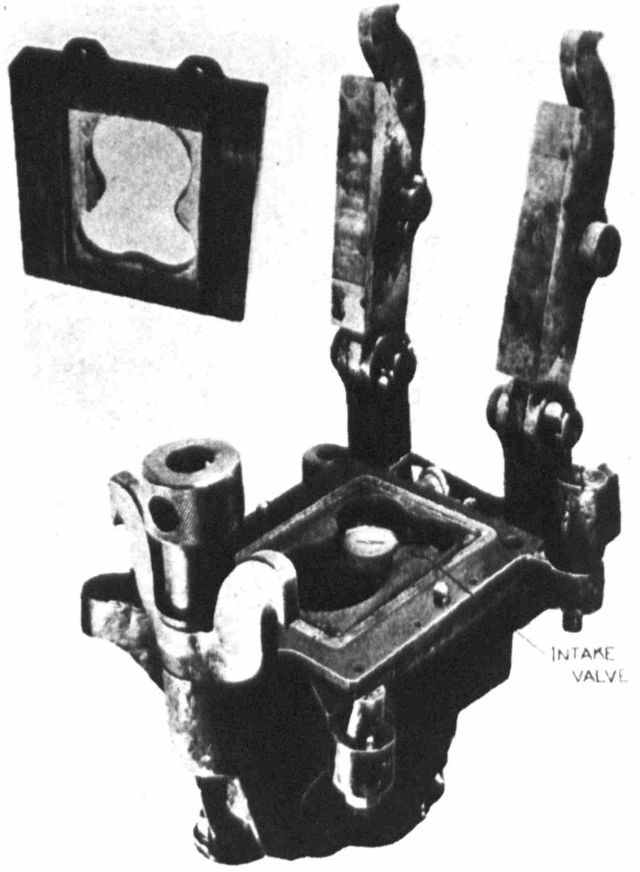
"(3) The average flame speeds during the initial period, the period of rapid flame travel, and the period of low flame speed near the end of the process, tend to vary in the same direction over a wide range of engine variables."

"(4) Under most operating conditions, the initial period of slow flame travel occupies from 25 to 30 per cent of the total time required for combustion."

These conclusions are in general similar to those of Marvin and his co-workers except for the effect of initial temperature for which Marvin found no consistent effect. It will be noticed that the "flame arrest" which attracted so much attention in bomb experiments does not appear strongly enough in engine tests to be a subject for discussion. This disappearance of the "flame arrest" is probably due to strong turbulence in the charge set up by passage of the charge through the intake valve.

Even with the excellent results obtained from photography through narrow windows in the combustion chamber, certain points were still matters for conjecture since a complete description of a three dimensional combustion process could not be obtained from records with a single space coordinate. This difficulty was attacked by Rassweiler and Withrow(154,155,156) as part of their extended research program at the General Motors Research Laboratory in Detroit. These investigators obtained two space coordinates on photographic flame records by constructing a special high speed motion picture camera to view all parts of the combustion chamber and to operate in synchronism with their experimental engine. Figure 42 shows the cylinder head and window mounting arrangement for a direct view of every part of the combustion chamber. The window was a fused quartz plate cemented in a heavy invar frame designed to be easily removed for cleaning.

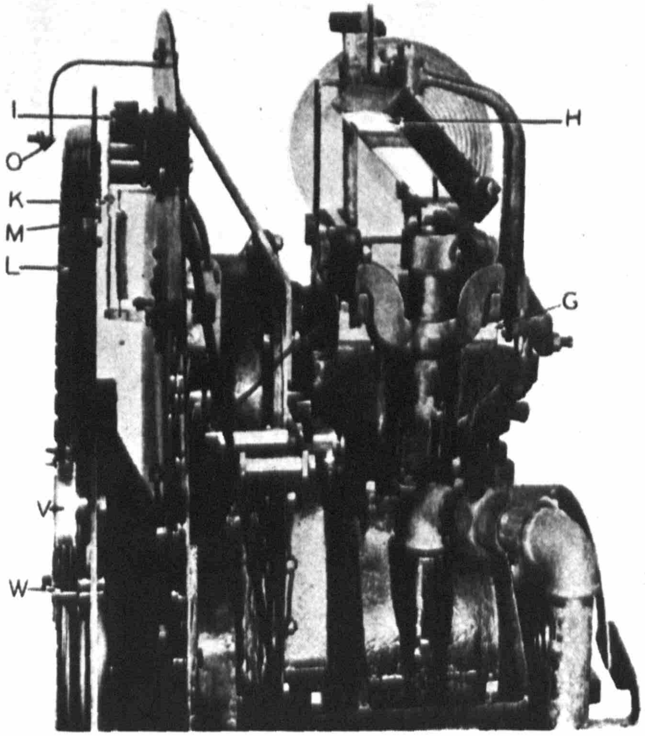
Figure 43 shows the engine and flame photography apparatus assembled for use. The engine is shown



Cylinder-Block and Window Used for Obtaining an Unobstructed View of the Combustion in a Gasoline Engine

REF. 154

FIG. 42



High-Speed Motion-Picture Camera Mounted on Engine

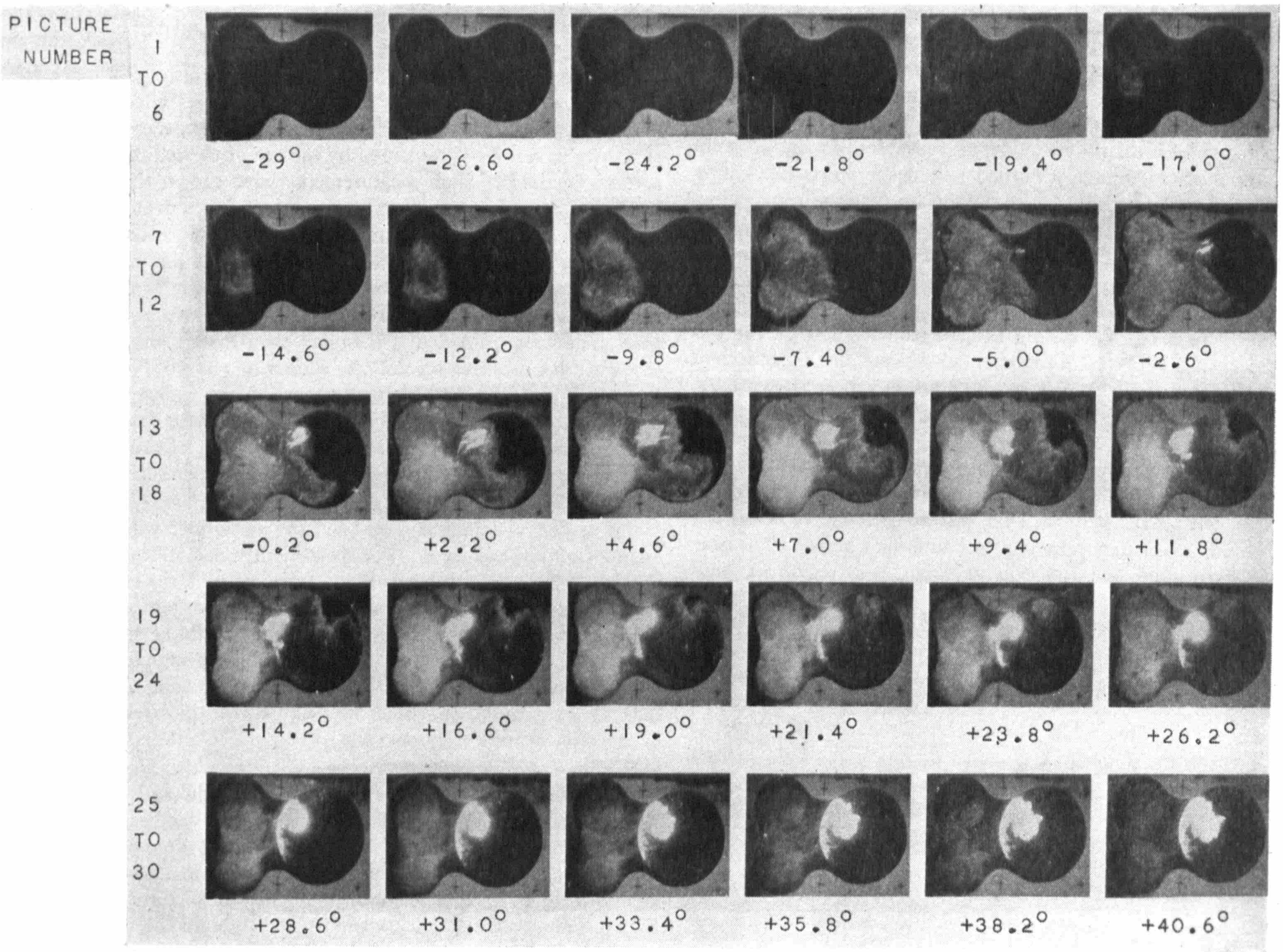
G – window retainer
H – mirror
I – field lens
O – focal-plane shutter
K – shutter
L – rotating disc
V – metering-sprocket
W – take-up spool

REF. 154

FIG. 43

at the right with the window mounted in place. Light from the engine flame comes up through the window, is reflected by the mirror H into the stationary lens I, passes through the moving lenses M, and is again reflected up to the film which is carried inside the overhanging rim of the disc L. The disc is rotated at engine speed by the crankshaft. There are 30 small lenses M. As each small lens passes the stationary lens, a separate picture of the engine flame is recorded. With this arrangement of the optical parts, the image formed on the film by the lenses moves with the film during the exposure of each picture. K is a shutter which opens for one engine explosion only. The "focal-plane" shutter O controls the exposure time of each picture. Extra film is stored on the spool W and, by means of the metering device V, a fresh piece of film can be pulled into place after each set of pictures is photographed. When in operation, the camera is covered with a light tight housing.

Plate XIII is a complete series of flame pictures for one cycle as given by Rassweiler and Withrow. As this display illustrates all the important features of combustion in a spark ignition engine, the author's description and discussion of the series will be given in full.



A Non-Knocking Explosion in a Gasoline Engine Running at 2000 R.P.M.

REF. 154

PLATE XIII

"Below each flame photograph is the crankshaft angle at which the focal-plane shutter stopped the exposure, the minus (-) signs denoting angles before top dead-center and the plus (+) signs denoting angles after top dead-center. The duration of each exposure (not to be confused with the time interval between exposures) was 2.2 crankshaft deg.; for example, picture No. 1 was exposed from 31.2 deg. before, until 29 deg. before top dead-center. The time interval between each pair of exposures was 2.4 deg. of crankshaft revolution which, at 2000 r.p.m. amounted to 1/5000 sec.; thus, the entire set of 30 pictures was exposed in 0.006 sec."

"The ignition spark occurred in the second picture, which was exposed from 28.8 deg. before until 26.6 deg. before top-dead-center. The spark continued to oscillate during the exposures of pictures Nos. 2 to 5, inclusive. In the fifth picture the spark was surrounded by a small globule of flame about 1/2 in. in diameter. During the exposure of pictures Nos. 4 to 9, inclusive (from 24 deg. before until 9.8 deg. before top dead-center), this globule of flame increased in size in a fairly regular manner showing a slight tendency to travel faster toward the exhaust valve than toward the intake valve."

"In the tenth picture the flame front began to spread through the throat of the combustion space and, at this time, the flame propagated faster through the portion of the combustion-chamber located near the bottom of the picture than through that portion near the top of the picture. The increased flame velocity along this one side of the combustion-chamber continued during the rest of the explosion with the result that the last part of the charge to burn was located in the portion of the combustion-chamber near the upper side of the picture. It is believed that this unsymmetrical flame propagation was caused by mass movements of the charge induced during the intake stroke."

"Just after the flame front reached the edge of the piston, in picture No. 11, a bright spot made its appearance close to the edge of the piston and thereafter increased in size until it was the most prominent feature shown in pictures Nos. 25 to 30. This brilliant luminosity was produced by incandescent carbon left by partially burned and decomposed lubricating oil. Apparently, this oil was thrown up into the combustion-chamber by the upward motion of the piston because the spot first appeared before top dead-center. In pictures photographed after top dead-center there appeared, close to the cylinder walls, additional luminosity which was probably due to incandescent carbon formed from oil left in the cylinder walls as the piston moved downward. The incandescent carbon is of particular interest because, as will be shown later, it allows the gas movements behind the flame front to be followed."

"At the forward edges of the flames in each of pictures Nos. 13 to 22 inclusive, well-defined regions appear. Such a region is shown particularly well on picture No. 17, for example. Behind this flame front is a less luminous region and, near the spark-plug and the exhaust valve, is a region of high luminosity called the afterglow."

"There is already considerable evidence to indicate that the gasoline is burned in the flame front in so far as permitted by the available oxygen and by chemical equilibrium under the prevailing conditions. This evidence consists of experiments with the sampling valve, which showed that the free oxygen at a point in the combustion-chamber disappeared almost as soon as the flame arrived; and also of experiments with the spectrograph, which showed that the spectrum of the flame fronts was characteristic of burning hydrocarbons while the spectrum of the afterglow was characteristic of carbon dioxide. If the gasoline is burning in the flame front, then the flame-front portion of the charge should expand, thereby compressing the gases both ahead of, and behind, the flames."

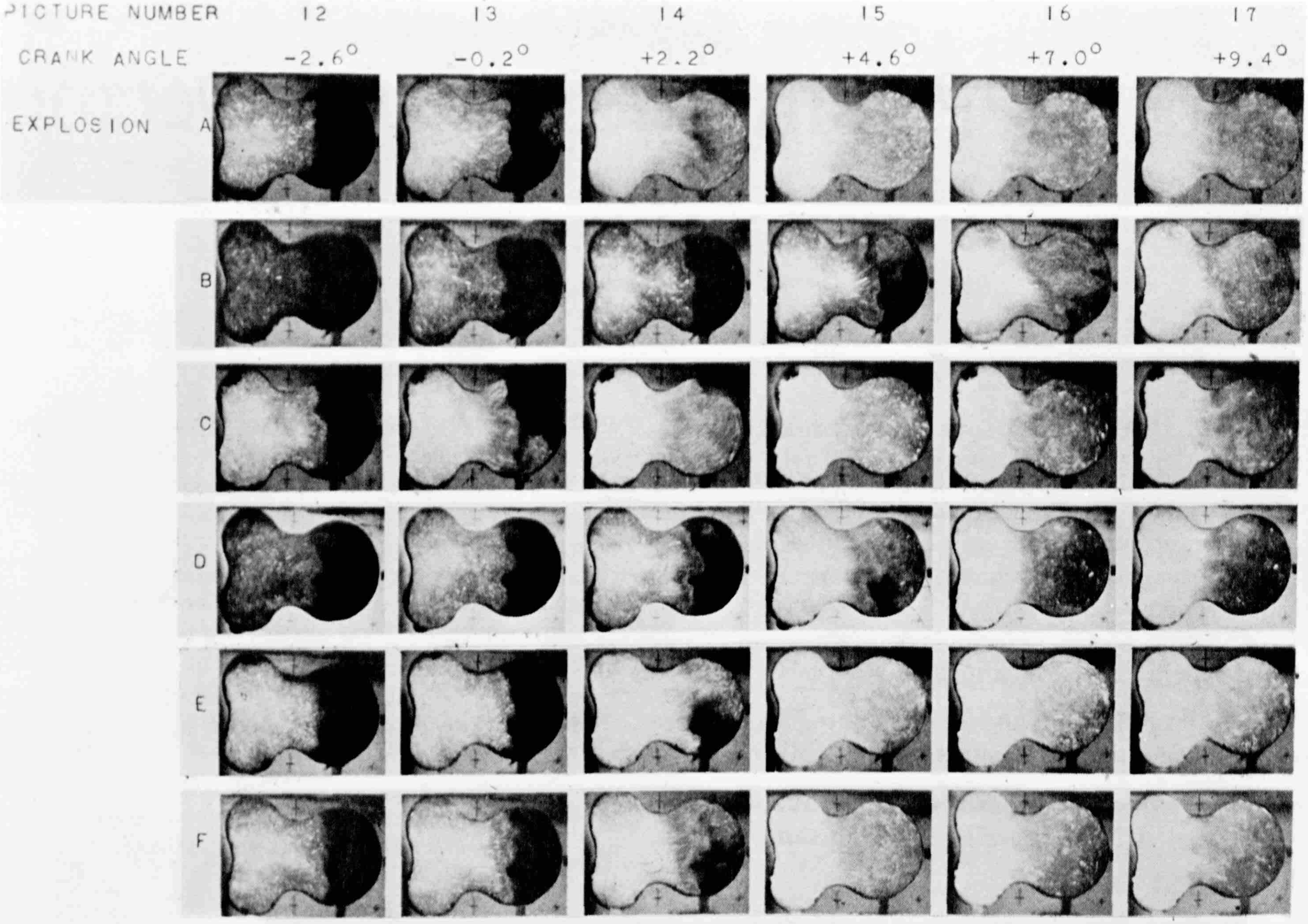
"Referring again to Fig.--- and examining the luminous cloud of incandescent carbon, it will be noted that in pictures Nos. 13 to 18 the luminous area was pushed backward toward the spark-plug by the expansion of the burning gases in the flame front. In pictures Nos. 19 to 30, the motion of the cloud of incandescent carbon was reversed, the gas being pulled into the cylinder by the descent of the piston. The point of reversal of this motion in picture No. 19 occurred at the time of peak pressure in this explosion. At peak pressure a considerable portion of the charge was still unburned. In fact, combustion was not completed until the twenty-fifth or twenty-sixth picture, which was approximately 15 deg. of rotation past the time of peak pressure."

"The afterglow in Fig.--- is noteworthy. In pictures Nos. 13 to 18 the afterglow increased rapidly in brilliance in those portions of the charge that burned first and reached its maximum intensity at, or slightly before, peak pressure. The greater intensity of the afterglow over the exhaust valve as compared with its intensity over the intake valve in these pictures is probably connected with the mass movements of the charge already mentioned in connection with the unsymmetrical flame propagation. As the pressure decreased during the exposure of pictures Nos. 20 to 30, the afterglow decreased rapidly in intensity."

"Now in connection with the behaviour of the afterglow, it is interesting to consider the temperature changes which occur in those portions of the charge that burn first, these temperatures having been measured by the use of the sodium-line reversal method. Such measurements have shown that the temperature in the gases that burn first continues to increase until slightly before maximum pressure. The continued temperature increase results from compression of the initially burned gas during the combustion of the remainder of the charge. At the end of combustion, the temperature in the gases that were burned first is higher than that in the gases that burn last. Simultaneously, as shown by pictures Nos. 27 to 30 in Fig.---, the gases that were burned first radiate light of greater intensity than those gases that burned last. It therefore appears that the changes in the intensity of the afterglow and in the magnitudes of the gas temperatures are closely related."

One of the major objectives in the work of Withrow and Rassweiler was to determine precisely the processes accompanying knock in an engine. For this purpose they ran an extended series of tests under knocking conditions. Plate XIV is made up of pictures 12 to 17 inclusive, from six different engine cycles. With regard to the top explosion sequence they say:

"--------In the twelfth picture the flame front was over the inside edge of the piston, and the combustion process appeared to be normal in every respect. It will be noted that the flame front was at approximately the same position as in the corresponding picture of non-knocking combustion in Fig.---. But in the thirteenth picture of Fig.--- (the sequence being described), auto-ignition occurred near the end of the combustion space over the piston at a point well separated from the normal flame front. This auto-ignition is the first evidence of knock in this explosion. In picture No. 14, exposed less than 1/2000 sec. later, the spontaneous inflammation has spread all through the remaining charge except for a small dark area, which apparently is not quite inflammed and which lies just ahead of the original flame front. Most



Occurrence of Knock in Six Different Explosions Photographed under Similar Engine Conditions

REF. 154

PLATE XIV

striking is the fact that the form and position of this original flame front are not greatly changed from those in the non-knocking explosion."

"Thus, in this case and perhaps in all knocking explosions, the knock is definitely not a result of a sudden increase in velocity of the advancing flame. While this point has been made before in connection with time-displacement records of knocking explosions, it has never been so convincingly demonstrated as in Fig.---."

"On comparing the sets of pictures in Fig.---, it is at once apparent that knock did not occur at the same crankshaft angle of revolution in all six explosions. In each case picture No. 12 was exposed between 4.8 and 2.6 deg. before top dead-center. At this time explosion C showed some evidence of the inception of auto-ignition in the lower right hand corner of picture C-12, but none of the other explosions gave any indications of the beginning of auto-ignition. During the next 2.4 crankshaft deg. auto-ignition appeared distinctly in explosions A and C as is evidenced by pictures Nos. 13 of these explosions. When the exposure of pictures Nos. 14 was completed at 2.2 deg. after top-dead-center, there were evidences of auto-ignition in all six explosions. In pictures Nos. 15 for which the exposures ended at 4.6 deg. after top dead-center, inflammation appears to be complete in explosions A and C, is almost at an end in explosions E and F, and is well under way in explosions B and D, requiring about 5 more crankshaft deg. for completion in explosion B (pictures Nos. 16 and 17) and about 2 more crankshaft deg. completion in explosion D."

"It will be noted also that auto-ignition did not begin at the same position in the combustion space in every explosion, the variation being particularly striking in explosions A, B, and C. Furthermore, in some of the pictures, flame appeared at several points in the charge even after knock was well under way. For example, in picture No. 14 of explosion F, the flame has started to spread from two points distinctly separated from both the original flame front and the area of initial spontaneous ignition. These observations are significant because, if knock were produced by a hot-spot or hot-spots on the combustion-chamber walls, auto-ignitions should always occur at the same position or positions in consecutive explosions."

"The possibility of following the movements of the gases behind the flame front by observing the motion of a luminous cloud of incandescent carbon particles suspended in the inflamed gases was mentioned in the description of Fig.---. Similarly in Fig.--- (showing the knocking explosions), the gas movements behind the flame front may be followed by observing the many luminous spots which appear on the pictures of the inflamed gases. When at rest, these spots appear to be approximately spherical but, when their motion is appreciable during the 1/2500 sec. in which the picture is being exposed, the spots lengthen out in streaks, the lengths of which are determined by the distances moved during the exposure."

"But the important feature in Fig.--- (showing knocking explosions) is the tremendous increase in velocity of these luminous spots at the time of knock. This phenomenon, while evident in every explosion, is particularly striking in pictures Nos. A-14, B-15 and E-14. The gas motion is of course, induced by the sudden burning and expansion of the portion of the charge that burns last."

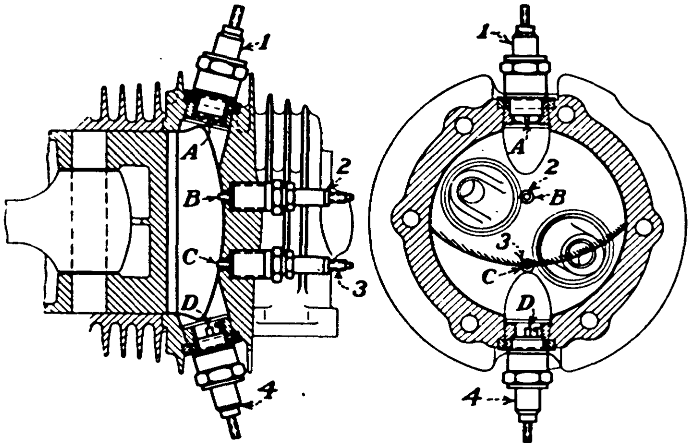
The direction of the gas movements caused by knock varies with the position in the combustion space where auto-ignition occurs. This effect is illustrated particularly well by pictures Nos. 14 exposed from top dead-center until 2.2 deg. past top dead-center in explosions A, C, E, and F. For example, in explosion A the gases move directly toward the spark-plug and valves, while in explosion C the gases are reflected by the wall of the combustion-chamber shown in the lower side of the picture. Similar directional gas movements can be observed in other knocking explosions."

"The violent gas movements evident in Figs.--- just after knock occurs appear to be closely related to the three undesirable consequences of knock: the sound; the increased jacket temperature, particularly in air-cooled engines; and the loss of power. The relationship between the gas movements and the sound of knock has already been discussed in some detail. The increased jacket temperature and loss of power both undoubtedly result from increased heat transfer from the gases to the walls. It is apparent that this heat transfer would be greatly augmented by the 'scrubbing action' of the high gas velocities induced by knock."

The results described above leave no doubt as to the general nature of the chemical and physical processes accompanying detonation in the internal combustion engine. The initial distrubance is due to a rapid combustion of the last part of the charge to burn which sets up a high local pressure and starts pressure waves within the combustion space. This region of abnormally rapid combustion is not associated with a definite position in the combustion chamber nor does it occur at the same crank angle in successive cycles. With constant engine conditions there is a considerable variation in the intensity of the initial disturbance as judged from flame photographs. These results must be given primary consideration in any study of detonation as a physical process and they will play an important part in later discussions.

Ionization gaps have been applied to the study of combustion chamber processes. In this scheme two insulated electrodes are exposed inside the combustion space and connected to a circuit for detecting the change in resistance accompanying strong ionization in the gas near the points.

MacKenzie and Honaman(157) in 1920 described a method for measuring flame velocities in engine cylinders by the use of ionization gaps. The ionization was detected by external series spark gaps with an applied voltage just below that necessary for breakdown. In 1926, Charch, Mack and Boord(158) studied the effect of knock suppressors on ionization in burning gases. They

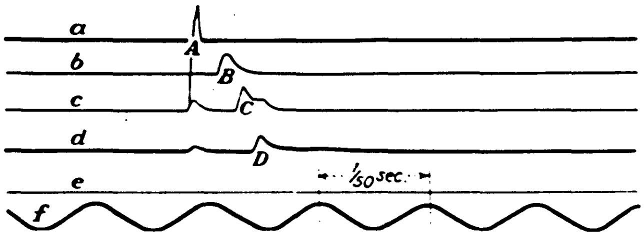


Flame-Spread Indication by Means of Ionization Currents

The combustion chamber is provided with spark plug 1, and measuring electrodes or ionization gaps 2, 3 and 4. The ignition current at *A* and the ionization currents at *B*, *C* and *D*, set up by the arrival of the flame at these points, are recorded by an oscillograph

REF. 161

FIG. 44



Oscillograph Record of Flame Spread

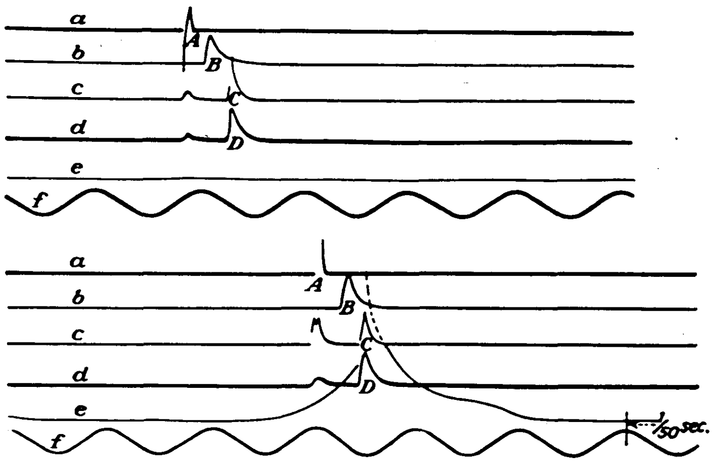
The deflection at point *A* marks the moment of ignition: those at points *B*, *C* and *D*, the arrival of the flame at the respectively lettered ionization gaps of Fig. 44. The bottom line is the time record of a calibrated tuning fork.

REF. 161

FIG. 45

used two well separated insulated electrodes in the combustion chamber and measured the current flowing in the circuit under a constant applied potential by means of a galvanometer. It was found that the average current flowing in the circuit was increased by the presence of detonation.

Schnauffer(159,160,161) refined the ionization gap method into a practical tool for approximate measurements of flame velocities. Figure 44 shows the application of Schnauffer's scheme to the head of an air-cooled engine. The gaps were connected to vacuum tube amplifiers designed to operate individual electromagnetic oscillograph elements. Figure 45 shows a typical record of normal flame travel in the combustion chamber of figure 44. The arrival of the flame at each gap was recorded as a sudden break in the trace from the corresponding oscillograph element. The presence of detonation was accompanied by a reduction in the interval between inflammation at the last two gaps as shown in figure 46. As an improvement on the oscillographic method of recording a later procedure connected a small neon tube to the output from each amplifier and modified the circuits so that the initial appearance of flame at any gap would be marked by a flash in the tube. The neon tubes were arranged on a large scale model at positions similar to the positions of the gaps inside the combustion chamber. With this scheme it was possible to

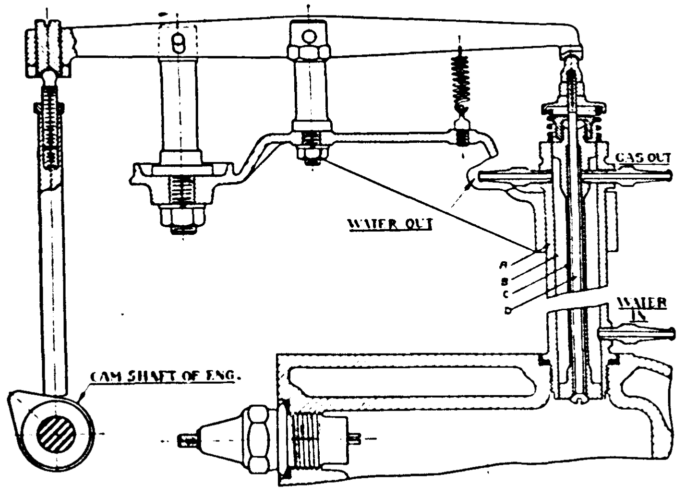


Oscillograms Taken During Detonating Combustion

The lower sections of these oscillograms show that in detonation a certain part of the fuel commences to burn simultaneously, since evidently the flame arrives at points C and D at the same time. In may also be seen the simultaneous record of cylinder pressure

REF. 161

FIG. 46



Sampling Valve

REF. 163

FIG. 47

make records of flame motions by photographing the neon lights on a moving photo-sensitive surface. Schnauffer investigated the general field of engine variables and obtained results similar to those of Marvin and his co-workers.

Chemical Changes During Combustion

Chemical reactions in combustion chambers have not been considered in the preceding discussion. Actually the methods of gas analysis had been applied to engine processes before the technique of flame photography was developed. Of necessity, the first analytical work was done on combustion products issuing from the engine exhaust. The extensive literature on this subject will not be considered here beyond quoting a summary given by Lovell, Coleman and Boyd(163):

"------- For all conditions, with the possible exception of those prevailing during extremely high speeds, the fuel is completely burned, so far as the amount of air available will permit. In the case of a fuel composed of carbon and hydrogen, the exhausted products consist of carbon dioxide, and water. When insufficient air for complete combustion is present, as is nearly always true in practice, the exhaust contains, in addition to these, carbon monoxide and hydrogen. But the oxygen is always completely used up---except a trace which persists possibly as a result of a chemical equilibrium condition of secondary importance--unless there is a lean mixture and the air is in excess."

"From the composition of the exhaust it is possible to calculate the fuel-air ratio and to obtain thereby a value that agrees closely with direct measurements upon the fuel and air fed to the engine. Furthermore, if the ratio of fuel to air and the additional fact that there is an apparent equilibrium for the water-gas reaction

$$H_2 + CO_2 = H_2O + CO$$

corresponding to some temperature are known, the composition of the exhaust may be calculated from the fuel-air ratio to check direct analysis. Hence, the major factor in determining the exhaust composition is the ratio of air to fuel."

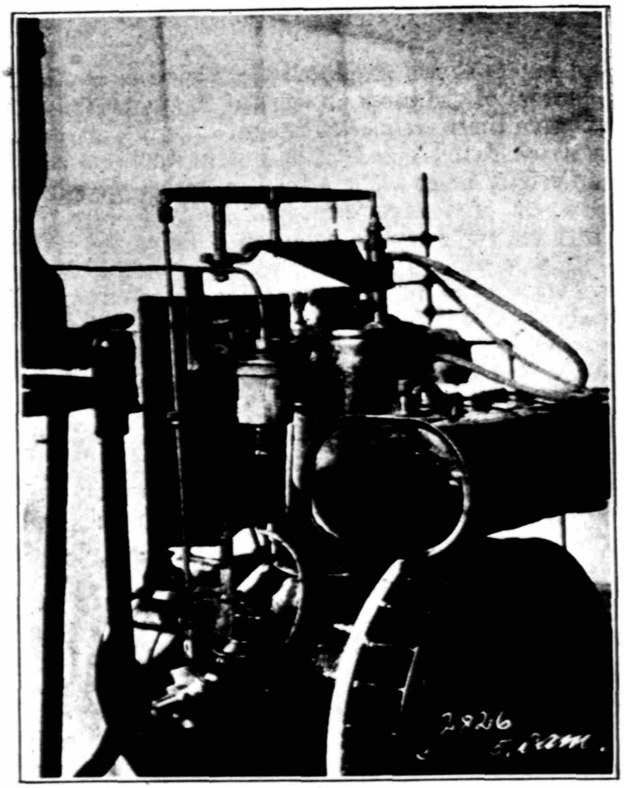
"It appears, therefore, that the 'over-all' reaction of combustion is quite a definite thing, and that it is conditioned by the fuels used, and by the presence or absence of knock. Under normal conditions, the burning doubtless takes place in such a small fraction of the explosion and scavenging strokes that such minor differences and disturbances as occur during combustion are lost sight of when the exhaust is the basis of consideration. At any rate, by the time the gases have reached the exhaust ports of the engine, they have had time to come to what is an apparent condition of equilibrium. Consequently, the composition of the exhaust relates to the burning as a whole. For investigating the combustion itself it is necessary, therefore, to examine the gases in the cylinder during the actual combustion period, in order to find out, first, how fast they are being consumed, and, second, by what chemical reactions they are burning."

The desire to trace out the instantaneous progress of combustion reactions was the underlying motive of several research projects. Quick acting valves designed to remove samples with reactions arrested at various stages of combustion, spectrographic analysis and radiometric methods have all ~~be~~ been applied to the problem. Particular attention has been directed toward the study of possible preflame reactions in an effort to interpret engine processes in terms of the hydroxylation theory of combustion. As in other phases of engine research, a solution to the problem of detonation has been the major goal before most of the investigators.

Boyd and his collaborators(163,164,165) working in the General Motors Research Laboratory described their first experiments with a sampling valve in 1927. Figure 47 is a diagram of their valve. An outer shell, A, was threaded to fit into a spark plug hole so that the valve could be used in various locations on the cylinder head. Inside the shell and surrounding the gas space C, was a water jacket, B, through which a stream of cold water was passed. The poppet valve, D, was held against its seat by means of a strong spring. The valve stem extended through the gas space and was sealed with packing at its upper end. The valve was operated by a rocker arm, which was actuated through a push rod by a cam attached to the camshaft of the engine in such a way that its angular position could be varied at will. The cam was quick-acting, opening and closing the valve in about two degrees of revolution. The valve allowed about 1 c.c. of gas, as measured at atmospheric pressure, to pass during each opening period. Figure 48 shows the sampling valve installed on the experimental engine.

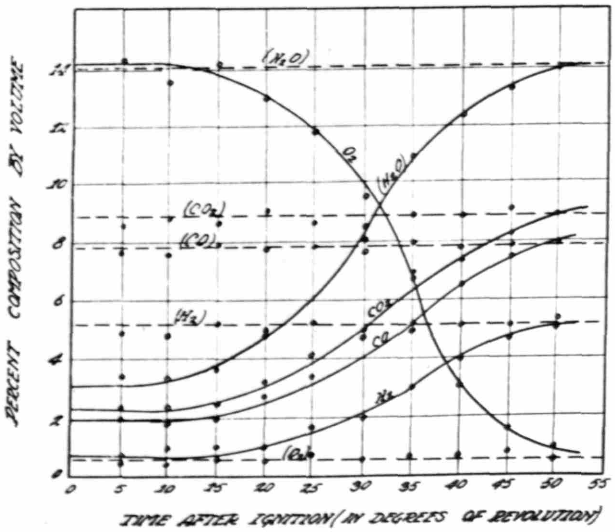
Gas samples were collected for various stages of the engine cycle and analysed in a Burrell precision gas apparatus for CO_2, unsaturation, O_2, H_2, CO, and paraffin hydrocarbons. Water was estimated from the gas composition from the oxygen and nitrogen balances.

Figure 49 shows the results from an engine test. With regard to these graphs the authors say:



REF. 163

FIG. 48



Composition of Gases from Engine Cylinder at Various Times after Ignition

Full lines, cylinder gases. Dotted lines, exhaust gases, sampled simultaneously. Delco-light engine—1200 r. p. m. 5.0: 1 compression ratio. Gasoline fuel—75 per cent of theoretical air

REF. 163

FIG. 49

"The solid line curves in Figure--- show how the gas taken from the burning mixture at various points in the cycle gradually changes its composition from that present before iginiton to that corresponding to the regular exhaust, or to complete combustion so far as the oxygen present permits."

"In interpreting these data, it is recognized, first of all, that they may not represent exactly the composition of the gases in the combustion chamber at the instant of sampling; for it is probably not possible to stop the combustion reaction by the method used, so far as any individual molecule is concerned, except at certain stages or equilibrium conditions. But from the results presented in figure---, it is apparent that it is possible to obtain a gas consisting of partially burned fuel in various stages of combustion, as is shown by the presence of oxygen, together with some of the products of combustion--or, in other words, a sample which represents the charge when only partially burned. It is apparent, also that the sampling valve operates rapidly enough to yield a sample which, although taken from a burning mixture, is cooled quickly enough to arrest, in part at least, the combustion in progress at the instant. The reaction is stopped, or greatly retarded, in different ways. The gases are cooled by contact with the cold walls of the sampling valve chamber, and this cooling may be increased to some extent by expansion through the valve. The reaction is also interrupted by the considerable decrease in the density of the gas as its pressure is reduced from that of the combustion chamber to that of the outside atmosphere."

"It may be seen from the figure that the points representing zero time after ignition show some products of combustion. This is due to the fact that since scavenging of the burned gases is never complete, a part of the new charge consists of exhaust gas from the previous explosion. It is possible to estimate the efficiency of the scavenging of the previous charge from these data."

"The form of the curves in the figure indicates that the mixture in the combustion chamber is probably almost homogeneous. There is no abrupt break in the curve, such as might be expected to be present if there were a narrow zone of flame moving progressively across the cylinder, within which the combustion reaction completed itself, so far as the air permitted. This does not mean at all that a 'flame' did not proceed through the chamber, as there must have been a spread of something through the charge that marked the beginning of combustion.

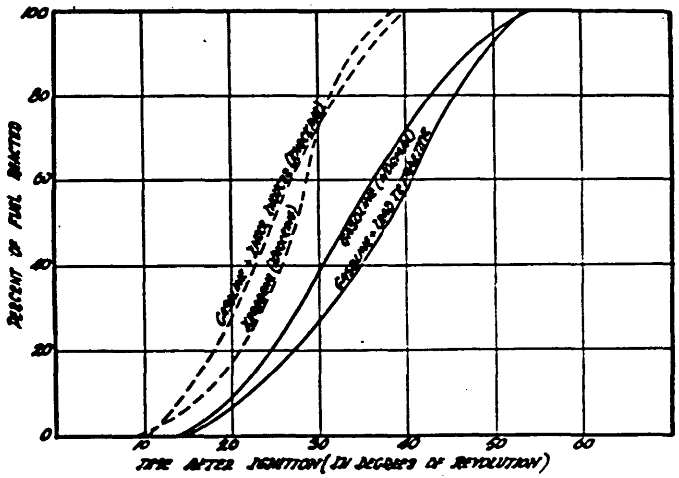
But the data do seem to show that the combustion reaction was of considerable duration, and that during this period it was apparently going on substantially throughout the combustion chamber."

"Another important observation is that from the curves of figure--- it is possible to estimate the rate at which the burning is progressing. This measurement of rate may be made directly; for from the curves information may be obtained, (1) as to what percentage of the oxygen has been consumed at any given time, and (2) as to how much of the products of combustion have been formed at the same time. It is possible, therefore to plot a curve showing the portion of the fuel which has been burned at various times during the combustion stroke. ?

------"

The curves of figure 50 were given as examples showing the way combustion proceeds to completion. It was noted that such curves could be used for the study of detonation:

"The relative rates under these various conditions are apparent from the chart. It is seen that when the engine knocks, whether the detonation is caused by the presence of kerosene in the gasoline or by a chemical knock inducer, the gasoline burns at a rate more rapid

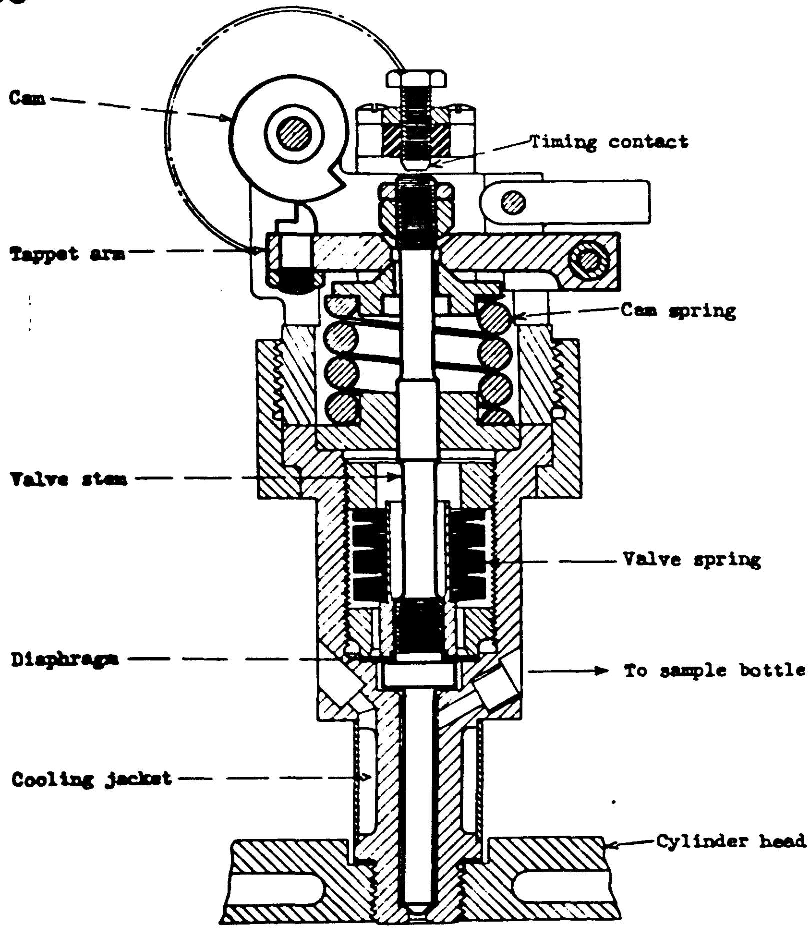


Amount of Fuel Burned at Various Times after Ignition

For normal combustion and for knocking combustion caused by different means. Delco-light engine—1200 r. p. m. 75 per cent of theoretical air

REF. 163

FIG. 50



Schematic diagram of N.A.C.A. gas-sampling valve.

REF. 166

FIG. 51

than normal. It may be seen, also, that the presence of tetraethyl lead in a knocking combustion brings the rate of burning back to normal, within the probable limit of error."

The discussion of sampling valve results shows that in general the method gave a general picture of the increased combustion speed accompanying detonation. However, the uncertainty of the point at which reaction was halted in the sampled gas and the lack of flexibility in the method were such serious handicaps that the sampling valve was soon replaced by other methods for detonation studies. It is interesting to note that the sampling valve data indicated the conclusion that the chemical reactions of combustion were not completed in a narrow zone, a result which is not in accord with later investigations.

Spanogle and Buckley(166) of the National Advisory Committee for Aeronautics in 1933 described an improved form of sampling valve operated by the inertia of a spring loaded tappet arm. The essential parts of this valve are shown diagrammatically in figure 51. The cam is driven by a flexible shaft which gradually compresses a spring through a tappet arm. When the cam reaches the release point, the cam spring accelerates the rocker arm to such a velocity that the valve is lifted off its seat for a short time when the rocker strikes the valve stem. The valve is returned to its seat by a strong spring acting directly on the stem. A sealing diaphragm is used to prevent escape or contamination of the gas sample. Under test, the valve was found to have an opening period of 0.0004 second at all operating speeds and had a variation in the time of opening of 0.00005 second at 750 cycles per second. The movement of the top of the valve stem was 0.004 inch.

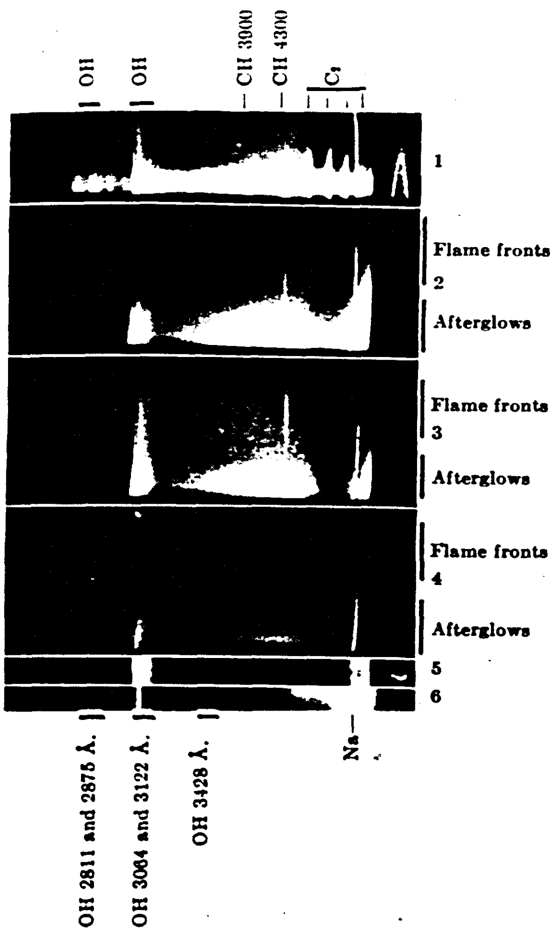
The N.A.C.A. sampling valve was used to investigate the combustion of gases in the combustion-chamber of a high speed compression ignition engine with two different types of injection nozzles. The usefulness of this valve for general engine work was noted but no additional results were given.

Egerton, Smith and Ubbelohde(167) in 1935 described some experiments with a sampling valve located at various points of the cylinder head. Some evidence was found of reactions before passage of the flame. In particular, nitrogen peroxide and formaldehyde were detected. It was suggested that in detonation, these preflame reactions sensitize the gas so that a condition favorable to rapid combustion is produced.

This detection of oxidation products before the main combustion reaction is consistent with the hydroxylation theory of burning and agrees with spectrographic results to be discussed later.

Spectrographic methods for investigation of combustion in engines were first suggested by Midgley and Gilkey(168) in 1922. These writers outlined experiments in which pressures would be measured by means of wavelength changes in the sodium D lines, temperatures determined with the aid of Wien's law and chemical processes traced by studying characteristic emission spectra of the compounds involved. No actual experiments were described.

In 1926, Clark and his co-workers(169,170,171) at the Massachusetts Institute of Technology started a series of investigations based on the use of a spectrograph to record the ultra-violet radiation passing through a small quartz window mounted in the combustion chamber wall of an engine. In the engine used, the combustion chamber had the form of a circular cylinder and the window was mounted on the diameter at right angles to the spark plug position. No attempt was made to vary the portion of the charge observed but a shutter rotating in synchronism with the crankshaft served to divide the combustion stroke into four quarters. With this procedure only general conclusions were possible: (1) Without knocking on ordinary straight run gasoline, the various quarters of the explosion have substantially the same short wave limit at about 3500 Å. The only distinguishable lines were from the spark plug electrodes; (2) Under knocking conditions the quarters of the combustion process were different. For the first quarter the limit was 2360 Å, for the second 3064 Å and the last two quarters 3500 Å. Band limits were found at 4314 Å, 3064 Å and 2811 Å in the first quarter, and 3064 Å alone in the second quarter. No bands were observed in the last two quarters. Absorption lines were nearly invisible in the first quarter, noticable in the second and clearly evident in the last two quarters; (3) The addition of an antiknock material such as lead tetraethyl equalized the width of the spectrum in the four quarters and in general returned conditions characteristic of non-knocking explosions. Although the characteristic OH band at 3064 Å due to water vapor, was identified, the experimental records were too obscured by the general background radiation to permit a thorough analysis of the combustion process. The general principles involved and the remaining problems were discussed



A.—Comparison of Spectrum of Gas-Air Bunsen Burner Flame with Spectra of Several Fuels Burning in Engine under Nonknocking Conditions

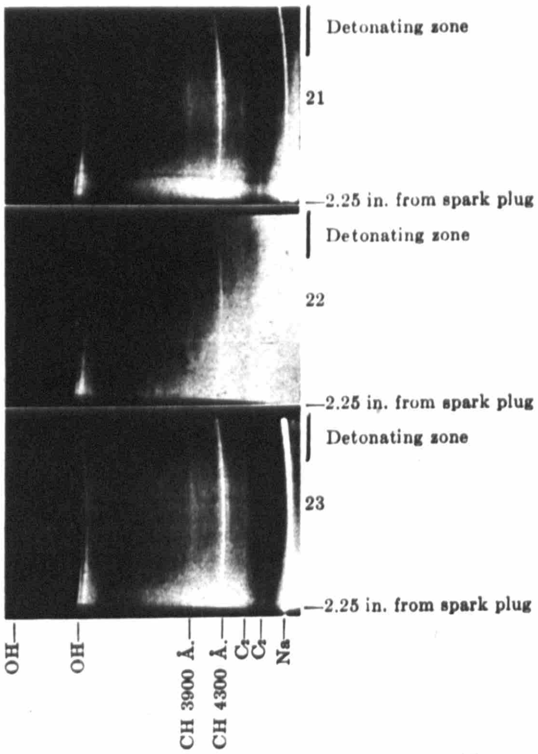
1. Bunsen burner
2. Benzene in engine
3. Isoöctane in engine
4. Gasoline in engine
5. Hydrogen afterglow in engine
6. Hydrogen in engine; no disk

REF. 152

PLATE XV

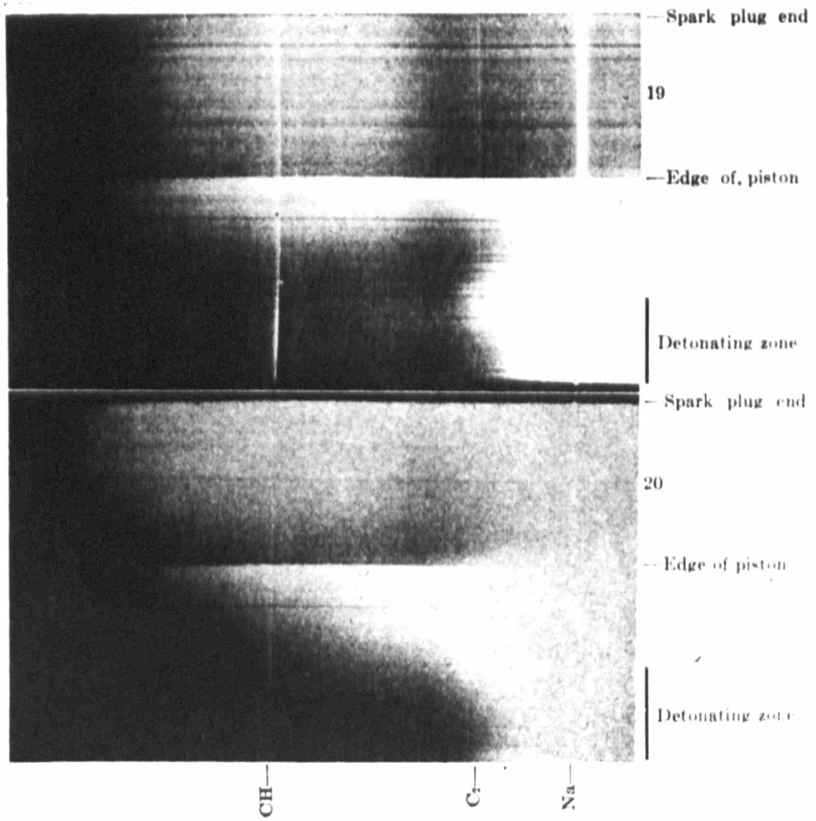
by Thee(172) who cooperated in much of the laboratory work.

Withrow and Rassweiler(152,173) improved the spectrographic method by using a window extending completely across the combustion chamber and a rotating shutter to separate flame front radiation from radiation due to afterglow. Their apparatus included the engine already used for making the narrow window flame photographs described above. The optical system was so arranged that the entire combustion chamber window was imaged through the slit onto the spectrograph plate. This procedure made it possible to observe simultaneous differences between combustion reactions in different parts of the chamber. In order to interpret the records, comparison spectra from suitable flames were exposed on the plates taken from the engine. The spectra of part A, Plate XV show characteristic lines in ultra-violet from non-knocking engine explosions and comparison flames as noted. The upper section of each spectrum was exposed during the flame passage while the lower section was exposed during the afterglow period. The wavelengths and corresponding chemical compounds are noted on the plate. Part B of Plate XVIshows a comparison of knocking and non-knocking explosions in the engine with the essential data noted.



B.—Comparison of Ultra-Violet Flame Spectra of Nonknocking and Knocking Combustion in Engine

21. Gasoline not knocking; spark advance 10°
22. Gasoline knocking; spark advance 25°
23. Isoöctane not knocking; spark advance 25°



C.—Comparison of Visible Flame Spectra of Nonknocking and Knocking Combustion in Engine

19. Gasoline not knocking 20. Gasoline knocking

REF. 152

PLATE XVI

Radiation in the visible for knocking and non-knocking explosions is recorded in the spectra of part C, Plate XVI. The absence of the CH and C_2 lines in the knocking explosion is to be especially noted.

Discussing the results of their spectrographic studies, Rassweiler and Withrow say in part:

"One very important fact that has been established by these spectroscopic studies of engine combustion is that, as the flame fronts travel through the detonating zone, the intensities of the CH and C_2 bands decrease if the engine is knocking, and, conversely, if the engine is not knocking the densities of these bands increase. These changes in spectral intensities are brought about either by a change in the number of potential emitters formed per unit volume of the detonating zone during the passage of the flame fronts or by a change in the average number of transitions which these molecules experience during the course of their existence.---------Thus there appear to be four chief factors to be considered: first, the densities of the gases through which the flame fronts pass; second, the temperatures of the gases in the flame fronts; third, the reactions that form the CH and C molecules; and fourth, the reactions that consume these molecules.

At present the relative importance of the various factors is not at all clear, but some consideration should be given to each of them."

After a discussion of the factors mentioned, Rassweiler and Withrow conclude:

"It seems likely then that the decreased intensities of the CH and C_2 bands in knocking explosions are produced largely by a change in the chemical reactions that either form or consume the CH and C_2 radicals. A change in the rate of formation of these radicals may result if, prior to the occurrence of knock, a considerable amount of oxidation and thermal decomposition takes place in the unburned gases ahead of the flame fronts. However, if this is the cause of the decreased intensities of the CH and C_2 bands in knocking explosions, it is probable that the reaction products are not hydrocarbons, because hydrocarbons generally emit the CH and C_2 bands during their combustion."

Withrow and Rassweiler(174,175,176,177) followed up the problem of preflame reactions in knocking combustion by a modification of the engine used for flame photography. Their general method was to study absorption spectra of the mixture just before explosion in different parts of the combustion chamber. Figure 52

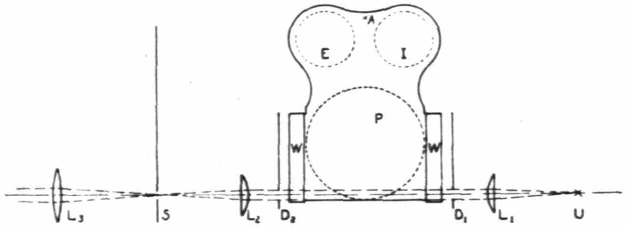


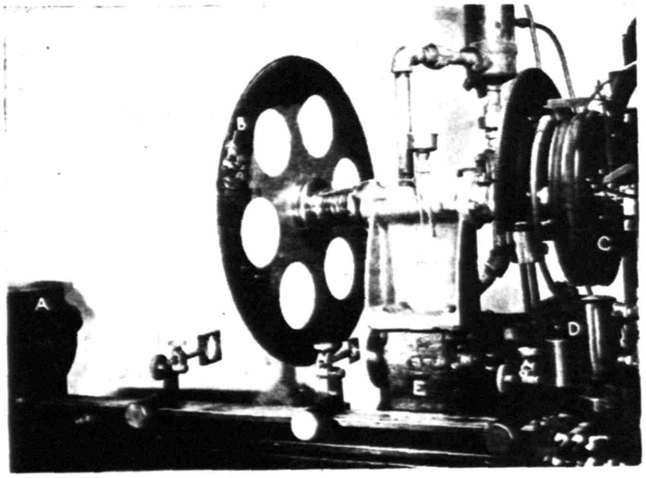
FIG. 52

COMBUSTION CHAMBER AND OPTICAL SYSTEM

A. Spark plug
*D*₁, *D*₂. Diaphragms
E. Exhaust valve
I. Intake valve
*L*₁, *L*₂, *L*₃. Lenses
P. Piston
S. Stroboscopic disk
U. Underwater spark
W. Window

REF. 174

FIG. 53



ENGINE AND OPTICAL SYSTEM FOR PHOTOGRAPHING ABSORPTION SPECTRA

A. Spectrograph
B. Stroboscopic-disk slot
C. Synchronous switch
D. Underwater spark
E. Cylinder head

REF. 174

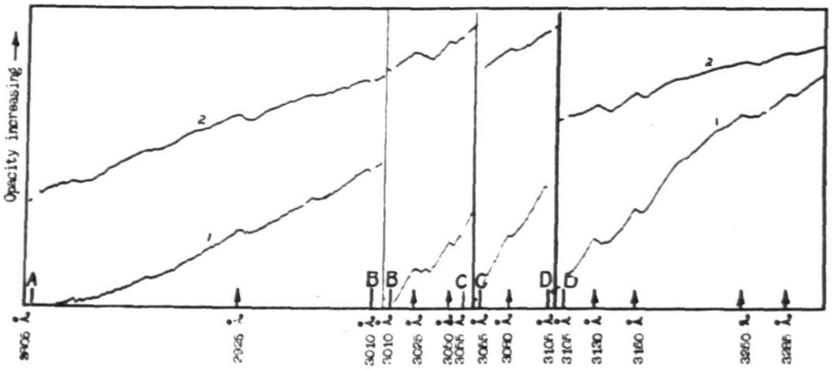


FIG. 54

PHOTOMETRIC RECORDS IDENTIFYING FORMALDEHYDE VAPOR IN THE NONINFLAMED GASES IN THE ENGINE WHEN OPERATING ON AN ISO-OCTANE–DIETHYL ETHER BLEND

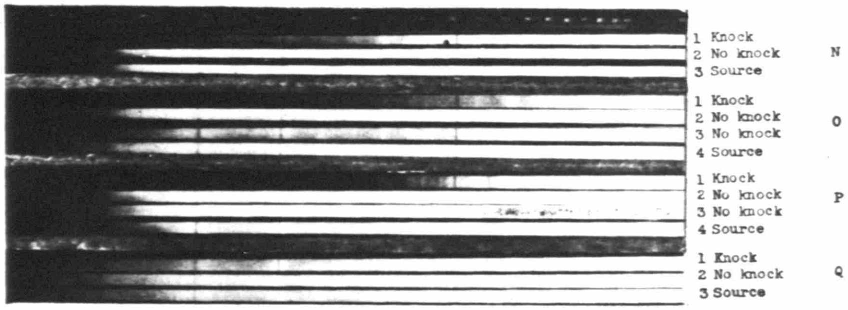
1. Gases in engine
2. Formaldehyde

REF. 175

shows the arrangement used for taking absorption records with the various parts labelled.

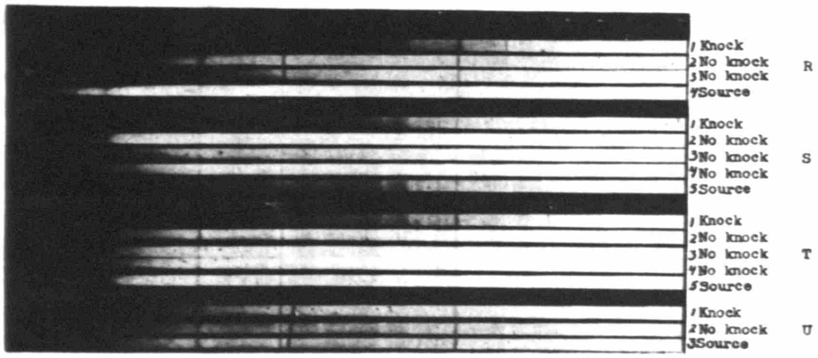
The optical system consisted of three lenses which were mounted with the light source and the spectrograph on an optical bench so the whole system could be shifted along the combustion chamber windows. The source, U, was placed at the principal focus of the lens L_1, and the light passed through the engine in a parallel beam. The lens L_2 formed an image of the source in the plane of the stroboscopic disk, S, and the lens L_3 formed a second image on the slit of the spectrograph. The diaphragms, D_1 and D_2 limited the width of the light beam. The rectangular apertures in these diaphragms were 0.62 cm. wide and 1.27 cm. high, and were used in either of two positions. The first position was at the ends of the windows farth~~er~~est from the spark plug as shown in figure 52. Here the beam was bounded by the end wall of the combustion chamber on the one side, by the ceiling of the combustion chamber above, and by the top of the piston, when at top center. The region viewed by the optical system in this position was called the "knocking zone". The second position of the optical bench placed the apertures at the opposite end of the windows where the light passed through the central region of the combustion space and therefore through portions of the gaseous charge outside of the knocking zone. The source of light was an underwater spark between beryllium electrodes immersed in circulating distilled water. The timing was controlled by a synchronous switch driven at camshaft speed by the same shaft which carried the stroboscopic disk. This switch could be adjusted to close the spark circuit at any desired angle of crankshaft revolution. The duration of the spark was very short compared to the time allowed for exposure by the disk so the only functions of the disk were to cut out light from the flames and to assist in timing the exposures with respect to flame front positions. Figure 53 is a photograph showing the apparatus set up for a test.

Rassweiler and Withrow expended much time and effort in eliminating spurious effects due to absorption in the fuel vapor and from other sources. Great care was taken to accurately reproduce engine conditions of pressure, density and temperature when making exposures for direct comparison. Plate XVIIshows absorption spectra taken with various fuels for knocking and non-knocking explosions. With the knocking conditions used, strong absorption was shown by the gases in the knocking zone just before inflammation. Under comparable conditions



Comparison of Absorption by Charges Just Prior to Inflammation under Knocking and Nonknocking Conditions When No Heat Is Added to Fuel-Air Mixture before Entering Engine

N_1. Gasoline A
N_2. 75 per cent gasoline C + 25 per cent isoöctane
O_1. 80 per cent heptane + 20 per cent isoöctane
O_2, O_3. Isoöctane
P_1. 60 per cent isoöctane + 40 per cent ether
P_2, P_3. Isoöctane
Q_1. 60 per cent isoöctane + 40 per cent ether
Q_2. Isoöctane
(*N*, *O*, *P* taken through knocking zone; *Q*, through center of combustion chamber)



Comparison of Absorption by Charges Just Prior to Inflammation under Knocking and Nonknocking Conditions with Fuel-Air Mixture Heated before Entering Engine

R_1. Gasoline A
R_2, R_3. Gasoline C
S_1. 80 per cent heptane + 20 per cent isoöctane
S_2, S_3, S_4. Isoöctane
T_1. 70 per cent isoöctane + 30 per cent ether
T_2, T_3, T_4. Isoöctane
U_1. 70 per cent isoöctane + 30 per cent ether
U_2. Isoöctane
(*R*, *S*, *T* taken at knocking end; *U*, at center of combustion chamber)

REF. 174

PLATE XVII

with different fuels and in the absence of knock, no absorption is apparent at the same positions ahead of the flame front.

In discussing the results of their experiments, Withrow and Rassweiler say:

"--------The data indicate, therefore, that the chemical change in the knocking zone ahead of the flame fronts is greater in degree or different in kind in knocking than in nonknocking combustion."

"In explaining the nature of knock, the absorption spectra supplement the previously reported flame photographs and emission spectro. To the writers' knowledge it has not been previously demonstrated experimentally in an engine that chemical change takes place ahead of the flame fronts, although such a hypothesis seemed to explain best the low intensity of emission of the CH and C_2 bands when the last part of the charge is inflamed in knocking combustion. These bands are characteristic of the flames of hydrocarbon fuels such as gasoline. The additional weight of evidence from the absorption spectra adds considerable strength to the conclusion that the observed weakening of CH and C_2 bands in the knocking zone when an engine knocks is due to the disappearance of a large

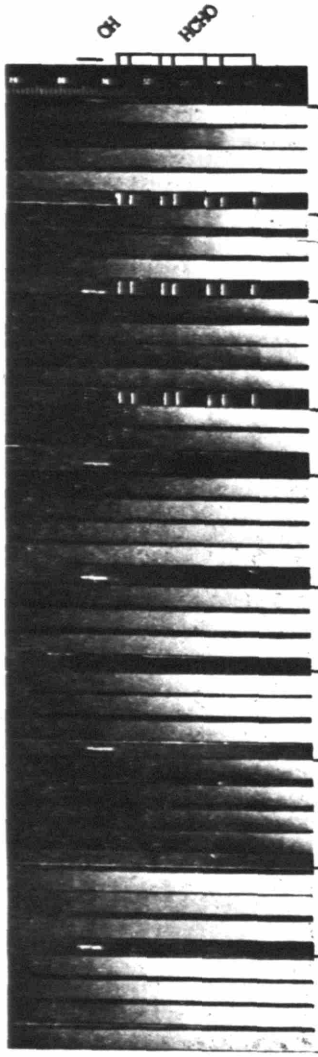


PLATE	SPECTRUM NUMBER	PER CENT OF FLAMES VISIBLE THROUGH DISK	FUEL	DEGREE OF KNOCK
F	1	100	GASOLINE E	MODERATE
	2	50 to 75		
	3	0		
	4	0		
G	1	75 to 90	GASOLINE D	MODERATE
	2	0 to 10		
	3	0		
H	1	100	UNREFINED GASOLINE C	LIGHT
	2	100	GASOLINE C	
	3	75	UNREFINED GASOLINE C	
	4	75	GASOLINE C	
I	1	0 to 10	GASOLINE A	SEVERE
	2	50	UNREFINED GASOLINE A	
J	1	50 to 100	ETHANOL	NONE
	2	75		
	3	10		
	4	0		
K	1	75	METHANOL	NONE
	2	0		
	3	0		
L	1	0	BENZENE	NONE
	2	25		
	3	100		
M	1	0	ISOÖCTANE (2,2,4-TRIMETHYL-PENTANE)	NONE
	2	50		
	3	90		
	4	100		
N	1	0	A COMMERCIAL GASOLINE	NONE
	2	25 to 50		
	3	0 to 10		
O	1	0	GASOLINE C	NONE
	2	25		
	3	90		
	4	TUNGSTEN LAMP		

Spectra Showing that Absorption by Formaldehyde in the Engine Is Characteristic of Knocking Fuels Only

Engine speed, 600 r. p. m.

REF. 175

PLATE XVIII

portion of the original hydrocarbons before the arrival of the flames with the formation of compounds which do not form CH and C_2 radicals when they inflame."

"In conclusion, the data presented here support the theories of knock which include the idea of spontaneous ingition ahead of the flame front, preceded by relatively slow reactions in the noninflamed gases."

With preflame reactions definitely proved from absorption experiments, it remained for Rassweiler and Withrow(175) to identify the reaction products appearing as a result of these processes. Following a hint from the hydroxylation theory of hydrocarbon combustion, absorption spectra from the engine were taken on the same plate with comparable spectra from formaldehyde vapor. Photometric curves from such a plate are shown in figure 54. A comparison of the positions and general shapes of the absorption bands, as shown by the curves, definitely indicates that formaldehyde is responsible for these bands in the engine spectra. Plate XVIII is a series of spectrograms showing the absorption prior to burning for a number of different fuels. The increased absorption in knocking combustion of gasoline as compared with non-knocking combustion is very apparent.

In summarizing their work on absorption, Rassweiler and Withrow say:

"The previous work gave little or no information regarding the nature of the reactions that take place in the noninflamed charge prior to knock; but now the identification of formaldehyde as one of the reaction products shows that at least a part of the hydrocarbons undergoes oxidation just prior to knock. In all knocking combustion that has been examined in the manner described in this paper, such evidence of oxidation has been found. Conversely, under engine conditions well outside the range of knock, no evidence of the presence of formaldehyde has been discovered. In those exceptional cases where formaldehyde was detected during non-knocking combustion, the engine conditions were near the threshold of knock and the intensities of the formaldehyde bands were low."

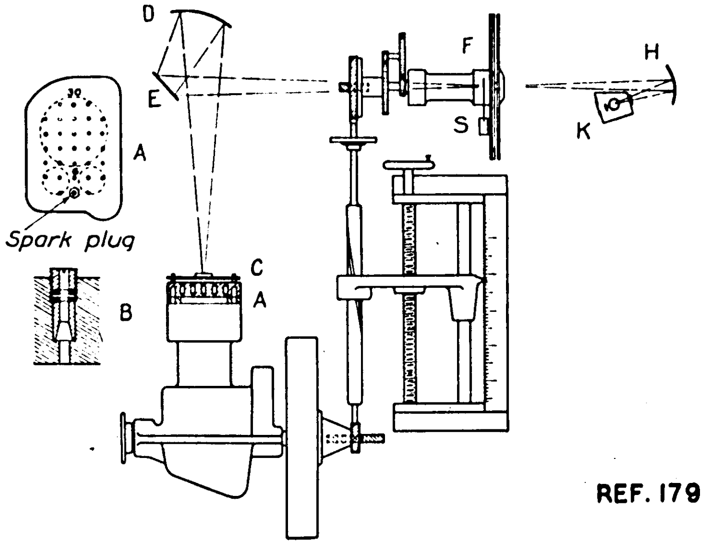
Although formaldehyde is formed ahead of the flame fronts when the engine is knocking on a wide variety of fuels, this compound does not produce knock when it is introduced into the engine with the intake air. (It has been demonstrated by means of absorption spectra that at least a part of the formaldehyde introduced in this manner remains intact until the time of knock.) On this account it appears that formaldehyde alone is not responsible for knock. Furthermore, there is ample evidence on the absorption spectra that compounds other than formaldehyde are present in the fuel-air mixture just prior to the occurrence of knock. It is not yet determined whether these other compounds are precursors of the formaldehyde, whether they are formed from formaldehyde, or whether they are connected with an entirely different reaction sequence."

"The results outlined in this paper are not in disagreement with certain theories of knock based upon studies of the slow oxidation of hydrocarbons outside the engine. However there is considerable uncertainty in applying the results of such studies to the engine. One reason for this difficulty is that all the conditions existing in the fuel-air mixture prior to knock are not known. Even such an essential factor as the maximum temperature attained by the noninflamed gas has not yet been measured. Accordingly, speculations on the mechanism of the reactions preceding knock will be postponed until further experiments are completed."

After the investigations described above had been completed, Rassweiler and Withrow carried out a number of other studies with their spectrographic equipment but most of the work was directed toward the specific effects of lead tetraethyl and did not add any fundamental knowledge to the general subject of detonation.

In 1922 Midgley and McCarthy(178) used a thermopile to measure the total radiation passing through a window placed in the combustion chamber wall of an engine. They observed that the total radiation rose to a higher peak and decreased more rapidly in the presence of detonation than with normal operation. No attempt was made to analyse the radiation.

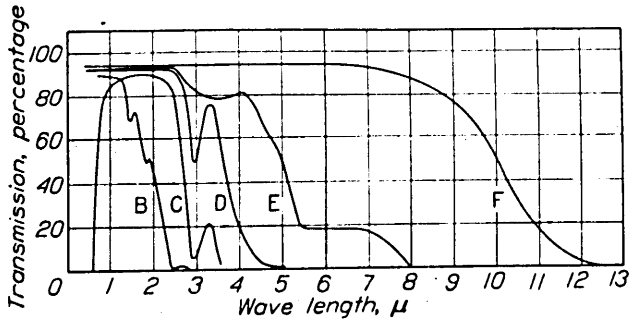
Marvin, Caldwell and Steele(179) reported an investigation during 1934 in which they placed a fluorite window in one of the cylinder head openings of the engine already described in connection with Marvin's stroboscopic studies of flame travel. The radiant energy passing through this window was analysed by means of a thermocouple and filter system with the object of following the course of the reaction taking place in the burning gases. Figure 55 shows the experimental arrangement and figure 56 gives the transmission curves of the five filters used to analyse the radiation.



REF. 179

Diagram of apparatus. A, Engine head showing location of windows; B, Detail of fluorite window mounting; C, Radiation shield and filter; D, E, H, Surface silvered mirrors; F Stroboscope; S, Electromagnetically operated shutter; K, Thermocouple.

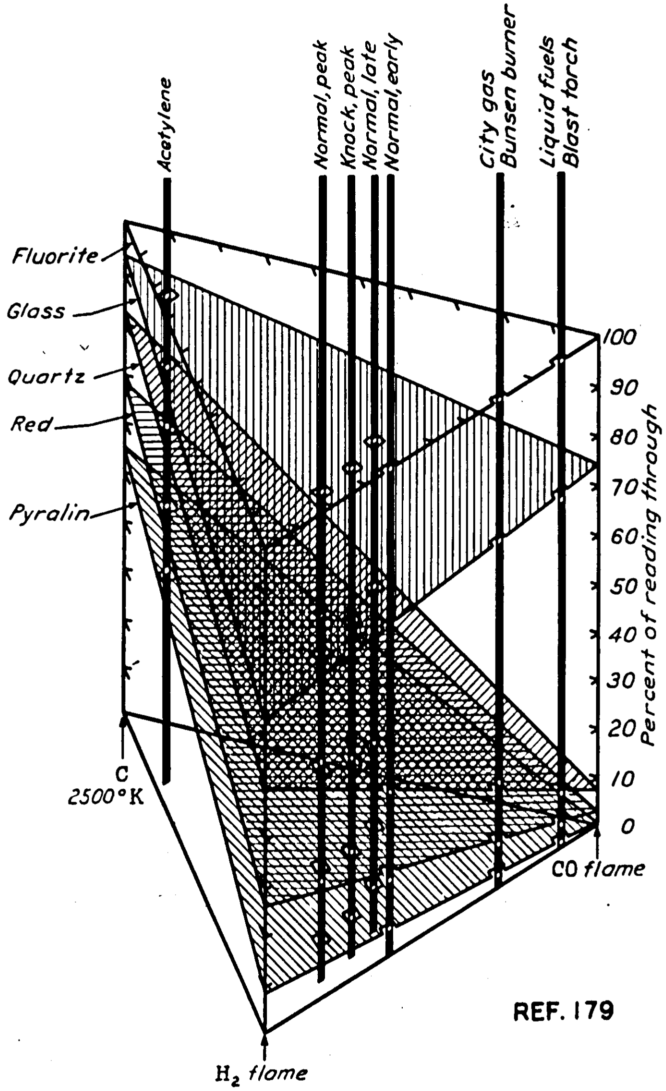
FIG. 55



Filter characteristics. B, Pyralin (thickness=2.13 mm); C, Schott's monochromatic red (t=3.18 mm); D, Quartz (t=8.94 mm); E, Microscope cover glass (t=0.15 mm); F, Fluorite (t=3.00 mm).

REF. 179

FIG. 56



REF. 179

Diagram for analyzing spectral distributions.

FIG. 57

"It was found that:

"Nearly all the energy radiated by the flame in an engine is in the infrared portion of the spectrum and is apparently emmitted almost exclusively by water vapor and carbon dioxide formed in the combustion process, radiation from incandescent carbon being relatively very weak.

"In treating the data:

"Observations through filters were analysed to show the relative intensity of the radiation from these 3 substances by 2 independent methods that produced satisfactory agreement. In one method, a conventionalized spectrum was made up of geometrical figures having locations and shapes simulating the prominent emissions of the three principal radiators, and the relative amounts of energy required in these regions to account for the readings obtained through the filters were calculated. In the other method, the characteristic spectral distributions for the three principal radiators were determined directly in terms of filter readings by observations through the filters of flames burning separately hydrogen and carbon monoxide, thus producing separately only water vapor and carbon dioxide as radiators; and by similar observations of a black-body furnace simulating radiation from heated carbon particles. The relative extent to which the characteristics of the three principal radiators are exhibited in the observations on composite flames in burners and in the engine are then determined graphically."

Figure 57 shows the diagram used for graphical analysis. It was only necessary to find a location on the simulated three dimensional diagram corresponding to the relative amounts of radiation transmitted by the various filters and then to read off the flame composition from the triangular scale.

In addition to the results already cited, the radiometric method led to the following conclusions:

"In a normal explosion, radiation from a given element of charge begins to increase upon arrival of the visible flame and continues to rise for about 20 degrees of crank travel thereafter, indicating that reactions producing water vapor and carbon dioxide persist for at least this period and probably longer after inflammation."

"When fuel-knock occurs, flame appears earlier in the region remote from the spark plug and radiation reaches a maximum much sooner after the appearance of flame. This phenomenon indicates not only earlier ignition of the last portion of the charge to burn but more rapid reaction following inflammation than in the normal type of burning."

"Measurements of total radiation thus provide a convenient means of determining the effect of engine operating conditions on the depth of the reaction zone behind the flame front and the duration of combustion in a given element of charge."

"Although total radiation varies greatly during the engine cycle and considerably for different engine operating conditions, spectral distribution shows only small changes over a wide range of operating conditions. The significance of these small changes is obscured by lack of fundamental data regarding the effects of flame depth, density, temperature, and pressure on the spectral distributions of H_2O and CO_2 and upon the relative potency of these substances as radiations. It is believed that observations through filters of burner and engine flames using as fuels H_2 and CO separately and in known mixtures would provide basic data for a more adequate interpretation."

The above conclusions are in general similar to those of other investigators except with regard to the thickness of the reaction zone accompanying the flame front. Conclusions from flame photographs and pressure records led Rassweiler and Withrow to believe that combustion is complete in a thin region identical with the flame, while the radiometric method suggested that burning persisted for a much longer period. However, the reactions occurring after passage of the flame-front are of little importance for detonation studies so no further attention will be given to this problem in the present report.

Flame Temperature Measurements

Temperature of the working fluid is one of the most important thermodynamic properties in studies of engine combustion. However, it is also one of the most difficult quantities to measure directly. This difficulty, rather than a lack of interest, has been responsible for the relatively few investigations of temperature in the combustion chamber. The conditions are so severe, both as to maximum temperature and rapid variations that ordinary methods are powerless and some type of radiation analysis must be used. This procedure is not particularly desirable since the seat of radiation effects will always be a distributed mass of gas rather than a point. In addition to these practical aspects of the problem, there is some uncertainty as to what the concept of temperature is to mean when it is applied to nonuniform conditions in a rapidly burning mass of gas.

Hershey(180) recognized the difficulties to be expected when he undertook the temperature measurements which he reported in 1922. In his discussion of the general problem he said:

"Any investigation or discussion of flame radiation and temperature is beset with many difficulties. The source of excitation causing radiation has not been definitely determined, nor is the mechanism of emission clearly understood. As to the meaning of flame temperature, an exact definition is difficult to formulate because of the absence of thermal equilibrium in regions of intense chemical activity. These considerations apply to flame as it occurs in continuous combustion in a stationary flame and in the cyclic combustion in an internal-combustion engine. In the case of the latter, however, the situation is further complicated by the flame movement, the lack of homogeneity in the combustible, and the short duration of the cycle."

"One method of dealing with the difficulty of defining flame temperature is to employ an operational concept of temperature and define it by describing the operation of temperature measurement. Since the operation of measuring flame temperature may be performed in several different ways, all of which may be regarded as defining the temperature, all should be applied and those selected as being satisfactory which give equivalent results."

"Three distinct methods of temperature measurement may be considered. These are:

(1) Thermometric measurement with a solid in thermal equilibrium with the flame, corrections being made for the effect of the solid on the flame.

(2) Radiometeric measurement from the flame radiation, corrections being made for the imperfections of the radiator.

(3) Thermal and chemical measurement (calculated temperatures) from the heat liberated by the combustion reaction and the specific heat of the products of combustion, correction being made for dissociation and losses.

All of these methods of measurement have been used in determining the temperature of continuous flame. It is here proposed to compare the last two methods as applied to a determination of flame temperature in an internal-combustion engine. The radiometric temperatures are the results of the present investigation and were obtained from measurements of the intensity of radiation from the flame during combustion in an engine. The calculated temperatures used for comparison are those found by Goodenough and Baker."

Hershey gives a discussion of the theory used to correct for the imperfection of burning gas as a radiator

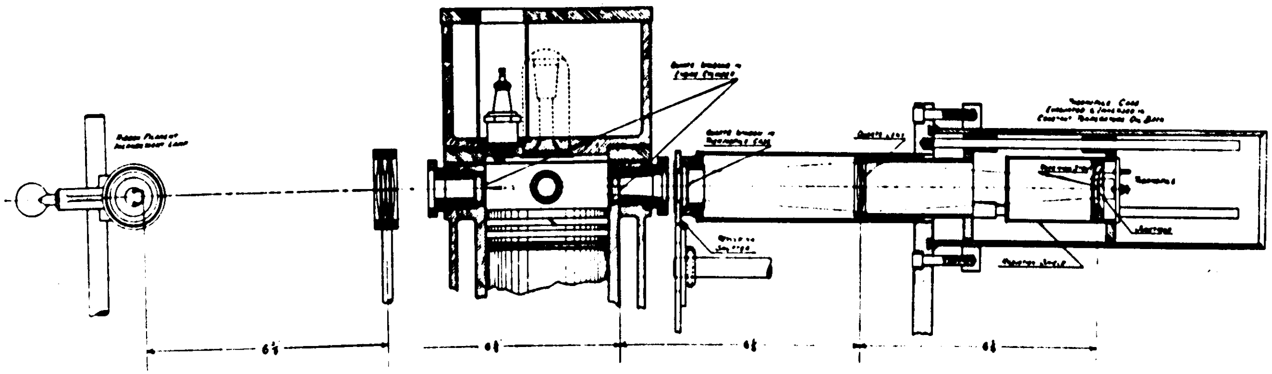


DIAGRAM OF ENGINE, THERMOPILE, AND OPTICAL SYSTEM

REF. 180

FIG. 58

when compared with a black body. As actually applied, his method was based upon determinations of radiant absorption in the flame by measuring radiation from the flame alone, radiation from a continuous source and radiation from the flame superimposed on the source. Figure 58 is a diagram of the apparatus used by Hershey and shows means for projecting radiation from an incandescent filament through quartz windows on either side of the combustion chamber on to the sensitive element of a thermopile. A stroboscopic shutter was used to isolate various ~~variations~~ portions of the cycle.

From his results Hershey concluded:

" The agreement between the calculated temperatures of Goodenough and Baker and the observed temperatures of the present investigation is reasonably close for a narrow range of air-fuel ratios in the region of 90 per cent of the theoretical ratio. For rich or lean air-fuel ratios, the observed temperatures are considerably lower."

" The two principal sources of error in radiometric temperature measurements are the inaccuracy of the flame absorption measurements, due to their small absolute value, and the assumption of pure thermal excitation of the radiation from the flame. Both would lead to final temperatures which are too high. The error due to incorrect absorption should not exceed five per cent. At present there is no means of determining the error due to not having pure thermal radiation; but, since the observed temperatures are, for the most part, lower than the calculated temperatures, this error should not be excessive."

The results of temperature measurements by the method of direct radiometry were so uncertain that this scheme was replaced by the spectral line reversal method. Flame measurements in engines with this method were pioneered by Hershey and Paton(181) and Watts and Lloyd-Evans (182,183). Both groups of investigators introduced a small amount of some sodium compound into the engine with the intake air and isolated the D lines for study by means of a spectroscope. The general discussion of this method given by Hershey and Paton is reproduced below:

"To obtain the instantaneous values of the effective temperature, the thermometric substance must be distributed throughout the entire region where the temperature is desired, and must have a heat capacity negligible as compared with that of the working substance. These conditions can be satisfied by introducing in very small quantities, compared with the total amount of gas present, a thermometric substance which is easily vaporized. The temperature of this substance can then be found by comparing its brightness at any given wavelength with that of a continuous radiator whose brightness temperature at that wave-length is known. The amount of vapor present must be sufficient to radiate and absorb light according to Kirchhoff's Law, and this condition can be attained with extremely small quantities of vapor if comparison is made at a wave-length corresponding to some resonance radiation of the vapor. Under these circumstances, investigation has shown* that the thermometric substance has a negligible effect on flame temperature. So also in the engine no effect on combustion was observed and the temperature thus determined may be interpreted as the effective temperature at the instant of comparison."

"When such effective temperatures are known, corresponding concentrations can be computed, assuming chemical equilibrium. These concentrations will likewise be effective values and, while their uncertainty will be at least as great as that of the temperature measurements, they should indicate in a general way the progress of the chemical reactions."

"--------The method of temperature measurement just outlined is known as the line-reversal method, and has been used extensively in measuring the temperatures

* See reference 184.

of a wide variety of flames. The principles involved can be readily understood by considering the application of the method in determination of stationary flame temperatures. In figure*---F represents a bunsen flame into which a metallic vapor such as sodium is introduced. The vapor must be that of an element whose resonance radiation is in a wave-length region which can be easily observed. Radiation from the tungsten ribbon filament lamp at L passes through the flame to the slit of the spectroscope at S. The flame is practically transparent to radiation in the visible region except at λ, the wave-length of the resonance radiation of the metallic vapor in the flame. At this wave-length, due to the presence of the vapor, strong absorption as well as emission occur. Let E_λ and E'_λ be the monochromatic emissive power at the wave-length λ for the lamp filament and the flame respectively, and let a'_λ be the absorptivity of the flame at the same wave-length. The energy which reaches the slit of the spectroscope at this wave-length will be $E'_\lambda + (1 - a'_\lambda)E_\lambda$, assuming that reflection at the flame surface is negligible. If

$$E'_\lambda > a'_\lambda E_\lambda$$

a bright line will be observed, crossing the continuous

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* Figure 59 of present report.

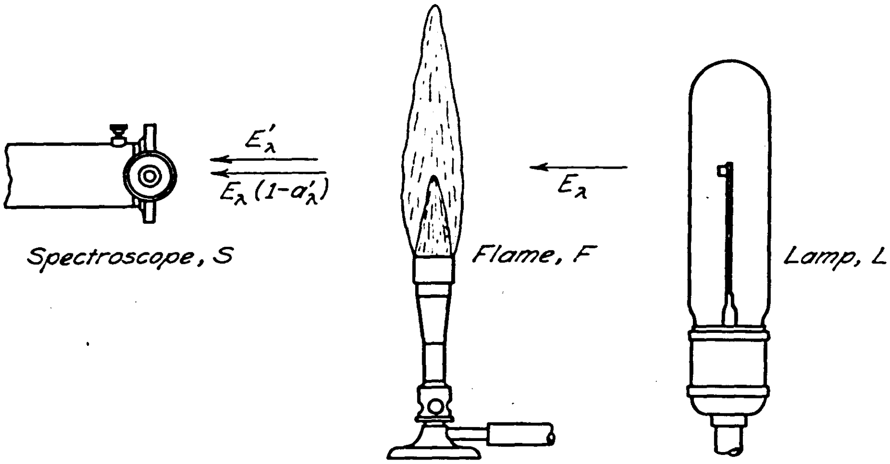


DIAGRAM OF LINE REVERSAL APPARATUS

REF. 181

FIG. 59

spectrum of the filament radiation at λ. If

$$E_\lambda' < a_\lambda' E_\lambda$$

a dark or reversed line will appear at this wave-length. For the particular case when

$$E_\lambda' = a_\lambda' E_\lambda \tag{1}$$

neither the bright nor the reversed line will be seen, and the flame will be emitting as much energy of wave-length λ as it is absorbing."

"If the flame satisfies Kirchhoff's Law for radiation of wave-length λ,* then

$$E_\lambda' / a_\lambda' = \bar{E}_\lambda \tag{2}$$

where $\bar{E}_\lambda$ is the monochromatic emissive power of a perfect radiator at the wave-length λ, the true temperature of this radiator being the same as that of the vapor in the flame. Therefore,

$$E_\lambda = \bar{E}_\lambda \tag{3}$$

From Wien's Equation

$$E_\lambda = \frac{C_1}{\lambda^5} \text{ exp. } (- C_2/\lambda S_\lambda) \tag{4}$$

where S_λ is the brightness temperature of the lamp filament at the wave-length λ. Also

$$\bar{E}_\lambda = \frac{C_1}{\lambda^5} \text{ exp. } (-C_2/\lambda T') \tag{5}$$

* This is proved to be true for stationary flames colored by alkali-metal vapors in reference 185.

where T' is the true temperature of the vapor in the flame. Hence

$$T' = S_\lambda \qquad (6)$$

If it is assumed that the vapor is in thermal equilibrium with the gases in the flame, and that its radiation is the result of purely thermal excitation, then the brightness temperature S_λ of the lamp filament must be the same as the true flame temperature. The agreement between the values found for the temperature of stationary flames by the line-reversal method and by other independent methods is satisfactory proof of the validity of these assumptions."

"The brightness temperature of the lamp filament can be measured with an optical pyrometer, so that the temperature of the flame can be readily determined. However, when the effective wave-length of the pyrometer screen is not the same as that of the resonance radiation of the vapor in the flame, it is necessary to make a slight correction for the variation in emissivity of the lamp filament between these two wave-lengths. If ε_λ is the monochromatic emissivity of the filament at wave-length λ then

$$E_\lambda = \varepsilon_\lambda \cdot \frac{C_1}{\lambda^5} \cdot \exp.(-C_2/\lambda T) \qquad (7)$$

when T is the true filament temperature. Combining this equation with (4) gives

$$1 / T - 1 / S_\lambda = (\lambda / C_2) \ln \varepsilon_\lambda \qquad (8)$$

Evaluating this equation for the two wave-lengths, and eliminating T, gives, finally,

$$1 / S_{\lambda_2} = (\lambda_1 / C_2) \ln \varepsilon_{\lambda_1} - (\lambda_2/C_2) \ln \varepsilon_{\lambda_2} + 1/S_{\lambda_1} \qquad (9)$$

where λ_1 is the effective wave-length of the pyrometer screen and λ_2 is the wave-length of the radiation from the vapor. The pyrometer measurement give the value of S_{λ_1}, the constant C_2 is the same for all temperatures, the emissivity for a tungsten filament is known over a wide range of wave-lengths and temperatures, and hence the value of S_{λ_2}, which is equal to the true flame temperature, can be calculated from equation (9)."

"In measuring the flame temperature in an internal combustion engine by the line reversal method it is only necessary to pass the lamp radiation through the flame in the engine by means of suitable windows on opposite sides of the cylinder, and to introduce sufficient sodium into the combustible to make it possible to observe both the bright and reversed line. A stroboscopic shutter, driven from the engine crankshaft, limits the observation to a short interval in successive cycles and

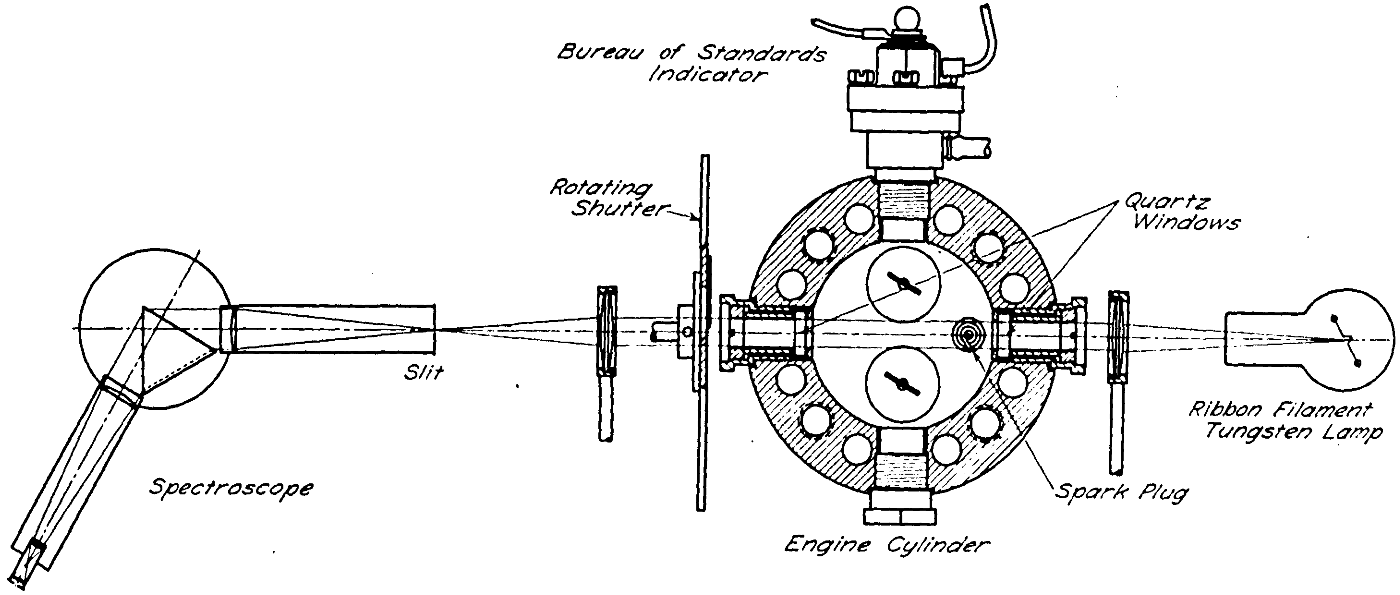


Diagram of Optical System

REF. 181

FIG. 60

thus gives an approximation to the instantaneous values of the effective flame temperatures."

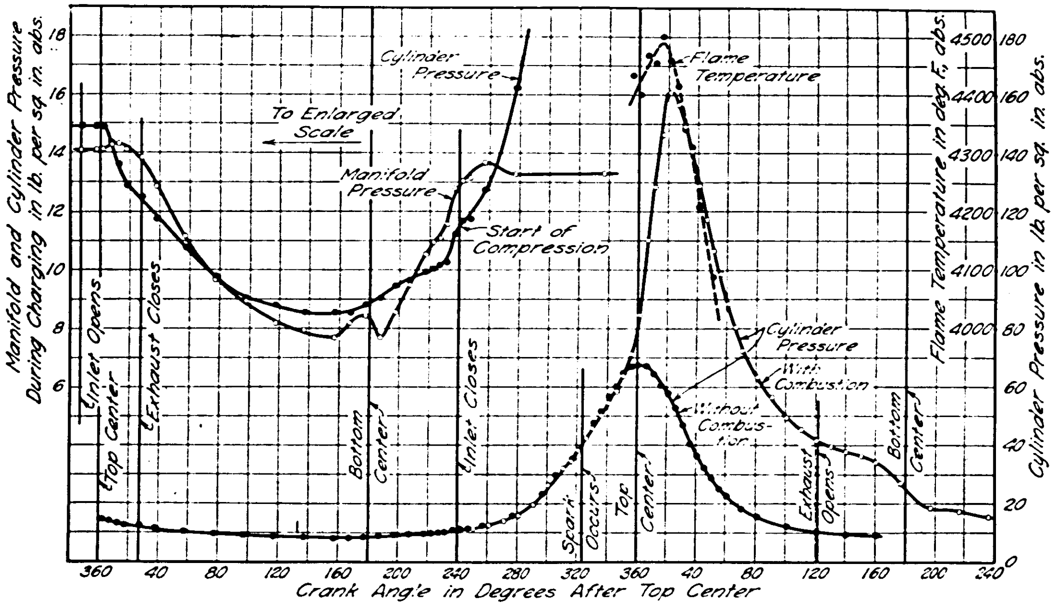
Figure 60 shows the essential parts of the apparatus used by Hershey and Paton. The stroboscopic shutter was used with an opening of about 18 degrees of crankshaft revolution. Sodium was introduced in the form of NaOH solution. Many tests were made to check the effects of various adjustments and conditions on the observed results.

The curves of figure 61 show the data taken for each complete run in studying the general problem.

Hershey and Paton give the following conclusions from their investigation:

"(1) The flame temperature found from a line-reversal measurement is characteristic of thermal equilibrium, established throughout the gases in the engine cylinder early in the process of burning and maintained during the subsequent expansion."

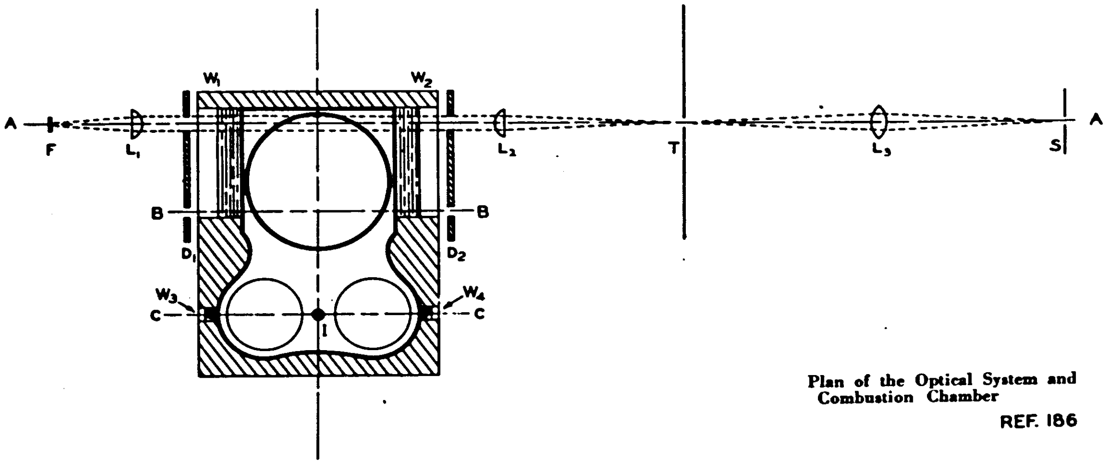
"(2) The maximum flame temperatures for a compression ratio 3.86 to 1 have been found to vary from 3750 deg. F. abs., with the richest and leanest air-fuel mixtures, to 4450 deg. F. abs. with an air-fuel ratio of 13.9 to 1."



Cylinder and Manifold Pressures
Air Fuel Ratio 13.7:1

REF. 181

FIG. 61



Plan of the Optical System and Combustion Chamber

REF. 186

FIG. 62

"(3) The maximum observed flame temperatures and the corresponding calculated temperatures were found to be in closest agreement for the normal operating range of air-fuel ratios between 12 to 1 and 14 to 1. Throughout this range the calculated values are approximately 600 degrees higher than the observed values. With either richer or leaner mixtures the difference increases and reaches a maximum of 1000 degrees at the lean combustion limit."

"(4) Since some of the discrepancy between the calculated and the observed maximum temperatures is undoubtedly due to experimental errors, the most likely of these have been considered and their upper limits estimated. If the observed values were increased by these amounts they would still be from 200 to 600 deg. F. below the calculated temperatures. It would, therefore, seem probable that this remaining difference is due to an inadequate analysis of the actual combustion process."

"(5) The concentrations of the gases in an engine cylinder may be calculated when the temperature and pressure are known, hence the measurement of temperature, independent of the other thermodynamic variables, makes it possible to study the progress of the chemical reactions and equilibrium at high temperatures."

Hershey and Paton proved the usefulness of the line reversal procedure as a general method but made no attempt to study temperature differences in the cylinder or to discover the effects of detonation. These phases of the combustion problem were attacked by Rassweiler and Withrow (186) using a modification of their absorption apparatus. In particular they measured temperatures in three zones and examined the changes characteristic of detonation. Figure 62 shows the essential parts of their apparatus in a plan view of the combustion chamber.

Two windows, W_1 and W_2 exposed the "knocking end" and "center" of the combustion chamber while two circular windows 1/4 inch in diameter permitted observations of the gas below the spark plug I. The lens L_1 was so placed that light from a point on the incandescent filament F passed through the engine as a pencil of parallel rays. An image of the filament was focussed on the stroboscopic shutter T by the lens L_2

and lens L_3 refocussed the image on the spectrograph slit S. The diaphragms D_1 and D_2 limited the extent of the beam passing through the combustion chamber. The stroboscopic shutter was a disc with a narrow adjustable slot near its outer edge. Uncertainties in phasing were eliminated by driving the shutter directly from the crankshaft. During the intake stroke, light from the source was cut off by a second slotted disc which was rotated at camshaft speed in a plane parallel to the first disc.

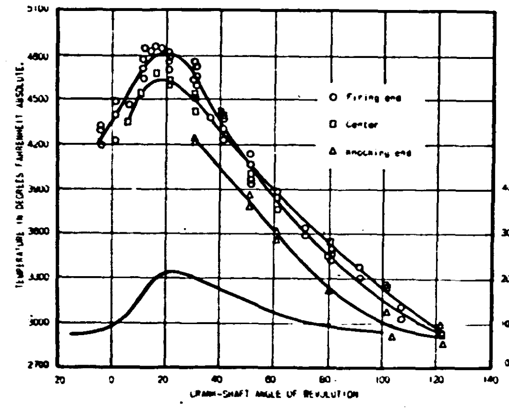
With regard to the procedure in making a temperature reading the writers say:

"The visual determination of the reversal point in the engine is complicated by the flicker from the stroboscopic disc and by the variation in temperature from explosion to explosion. Near the reversal point the observer sees the line changing back and forth from emission to absorption because of the fact that some flames are hotter and some are cooler than the source. The observed variation of the temperature in successive explosions changes with the crankshaft angle and with the engine conditions. This fluctuation may amount to 200 deg. fahr. In the experiments to be described, the limits for this range were determined and the mean value taken. To decrease errors in judgement, independent determinations at each point were made by two observers."

"Measurements have been made at three different positions in the combustion space with the optical paths normal to the general direction of flame travel as indicated in the figure. With the optical path in the position CC, temperature measurements could be made as soon as the flames had spread about an inch away from the spark plug or about 10 deg. of crankshaft revolution after ignition occurred. Subsequently, measurements could be made along line CC at any desired angle during the expansion stroke. Under the conditions of these experiments the flames reached position BB about 20 deg. of crankshaft revolution after ignition. With the optical path at position AA, measurements could not be made until the flames neared the end of their travel or, in terms of crankshaft revolution, not until about 45 degrees after ignition. It is important to note that at positions BB and CC, temperatures could be measured before the end of flame travel and therefore before maximum pressure was reached."

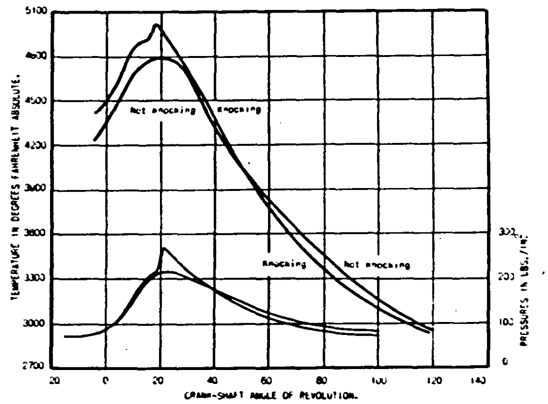
Figure 63 shows the temperature variation with crank angle for each of the three positions under non-knocking conditions. Among other things it is particularly interesting to note that the gases near the region of initial burning around the spark plug are hotter than the last part of the charge to burn except during the last part of the expansion stroke. Rassweiler and Withrow explain this result with the aid of a theory advanced by Hopkinson:

"The explanation of the temperature gradient in a bomb, as given by Hopkinson, can be applied to the engine and is therefor included at this point. As the gas near the center of the bomb--near the point of ignition--burns, it expands against a low pressure doing work and losing energy. Subsequently, when the rest of the charge burns, the gases at the center are recompressed to approximately their original volume. Since the recompression takes place at a higher pressure than the expansion, the work done on the inflamed gas near the spark plug is greater than the work done by the gas and it gains energy adiabatically. But now consider a small mass of gas which burns last. After being compressed at a relatively low pressure, it burns and expands to approximately its original volume against a high pressure. Inasmuch as the amount of work done by the gas is greater than the work done on the gas, it loses energy. Thus the adiabatic expansion and compression of the burning gases in closed vessels results



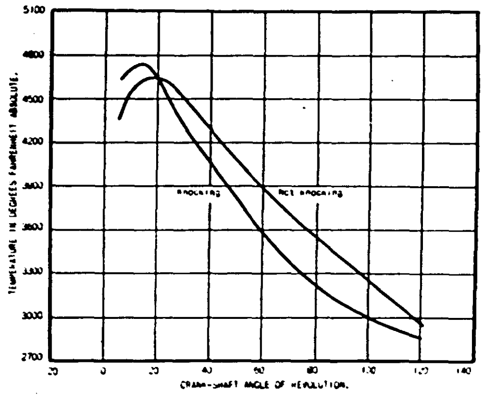
Temperatures Measured at Three Different Positions in the Combustion Chamber under Non-Knocking Conditions REF. 186

FIG. 63



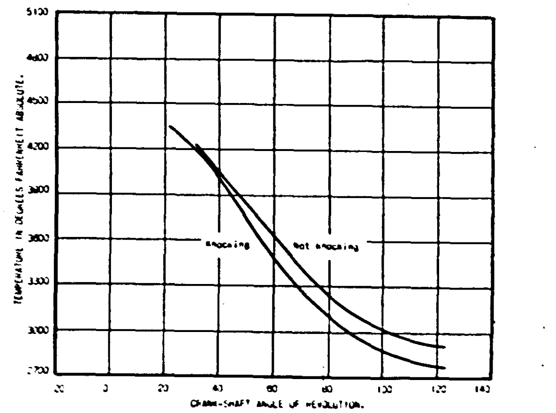
Comparison of Temperatures at the Firing End under Knocking and Non-Knocking Conditions REF. 186

FIG. 64



Comparison of Temperatures in the Center of the Combustion Chamber under Knocking and Non-Knocking Conditions REF. 186

FIG. 65



Comparison of Temperatures in the Knocking Zone under Knocking and Non-Knocking Conditions REF. 186

FIG. 66

in an unequal distribution throughout the combustion space of the thermal energy released by combustion; and when pressure equilibrium is established, the gases which burned first are hotter than the gases which burned last."

Figures 64, 65 and 66 show comparisons of temperature as a function of crank angle for knocking and non-knocking combustion at each of the three charge regions. Discussing these curves Rassweiler and Withrow say:

"(1) Under knocking conditions, the temperatures reach a maximum several degrees earlier than under non-knocking conditions. This difference is produced by rapid inflammation of the portion of charge which burns when the engine knocks."

"(2) At all three positions the maximum temperatures measured under knocking conditions are higher than under non-knocking conditions. The difference in maximum temperature may be due, in part at least, to the fact that, in the knocking case, inflammation is complete at an earlier angle, at which time less energy has been absorbed by the piston and less energy lost to the walls."

"(3) During most of the expansion stroke, the rate of cooling is greater under knocking than under non-knocking conditions. This observation is most significant.

Consider, for example the two curves in figure[*]-- which cross at 50 deg. past top dead-center. Obviously, the different rates of cooling at this angle cannot be attributed to different rates of expansion under the two conditions. Therefore there must have been greater heat loss to the walls in the knocking case. But at the point of intersection at 50 deg. after top dead-center, the greater heat loss to the walls in knocking combustion must be accounted for by differences between knocking and non-knocking explosions other than a temperature difference of the gases. In the course of the general investigation of the combustion phenomenon at this laboratory there have been observed two differences between knocking and non-knocking combustion which may partially account for the greater heat loss to the walls in knocking explosions. First, emission spectra have shown that the continuous radiation at the red end of the spectrum is stronger under knocking than under non-knocking conditions; that is there appears to be more black-body radiation in the former case. However, on the basis of quantitative measurements, Marvin, Caldwell and Steele have recently concluded that there is very little black-body radiation from either knocking

[*] Fig. 64 of the present report.

or non-knocking combustion. Second, flame pictures show appreciable mass movements of the gases as the pressure waves set up by knock pass back and forth through the combustion space. The increased degree of turbulence resulting from this cause undoubtedly augments the rate of heat transfer to the walls. The effect on the engine of the higher rate of heat transfer to the jacket walls is a higher jacket temperature and a greater rate of pressure decrease which results in a loss of power as compared with non-knocking combustion. The fact that the two temperature curves cross at a somewhat later angle in the figure than do the two pressure curves is not significant because of the limited portion of the combustion space covered by the temperature measurements shown in this one chart."

"(4) The temperatures of the exhaust gases are lower when the engine is knocking than when it is not knocking. The greater heat loss through the cylinder walls during knocking combustion appears to be responsible for this difference in exhaust-gas temperatures."

Rassweiler and Withrow consider a number of other points in their treatment of experimental results but for the most part these other problems are still subjects for controversy and do not change the fundamental picture as regards detonation.

Summary

Normal combustion and detonation as they occur inside the engine cylinder have been investigated by several methods. The results obtained are in agreement as to the general nature of the phenomena with a few points still under discussion. Details of the chemical and physical processes involved are at present subjects for speculation, although there is a definite trend toward closer experimental examinations of all phases of the problem. The literature does not reveal any serious attempts toward a quantitative analysis of the photographically recorded particle motions which accompany detonation.

Under normal conditions in an engine combustion chamber, flame fronts start slowly from each point of ignition and accelerate to an approximately constant velocity which is maintained until the fronts are slowed up near the chamber walls. This combustion process is accompanied by an increase in general pressure which reaches its maximum in about the time interval required for the flame fronts to pass through all parts of the mixture. Both the pressure and flame front position vary with time without apparent vibrations.

Under detonating conditions the combustion chamber processes are identical with those of normal burning until a critical point is reached after the flame fronts have passed through a substantial portion of the unburned mixture. At this instant, very rapid combustion occurs in the unburned gas which produces violent motion within the charge. This initial disturbance is followed by vibratory particle displacements due to standing waves of pressure within the combustion chamber. The rapid local combustion which is characteristic of detonation apparently follows a period of partial oxidation in the unburned gases and is erratic both as to location and timing. Several centers of disturbance may appear almost simultaneously ahead of the progressive flame fronts or there may be a sudden continuous increase in speed of the reaction zones already in existence.

All the investigations reported in the literature were carried out with the aid of cumbersome apparatus applied to special engines. This situation leaves a definite need for equipment suitable for use on any engine under any operating conditions. With direct radiation from the burning charge not available for examination, instantaneous pressure recording seems to be the only feasible means for studying detonation under routine conditions. Such pressure records offer at best an indirect method for determining the position and intensity of disturbances within the combustion chamber. It is therefore essential to develop a quantitative theory and a technique for interpreting pressure variations taken at possible locations in terms of the complete pattern associated with these variations. This aspect of the general problem will be especially considered in the present report.

SECTION IV

PRESSURE DISTURBANCES ACCOMPANYING DETONATION

PRESSURE DISTURBANCES ACCOMPANYING DETONATION

Introduction

Detonation has been considered from several viewpoints in the previous sections. Pressure disturbances accompanying detonation have often been in the background but have never formed the main theme of any investigations. This was partly due to difficulties in obtaining quantitative pressure records and partly to a reasonable opinion among investigators that combustion research offered the shortest road to tangible results. However, even with a dearth of quantitative information on the disturbances, it is certain that detonation is characterized on the physical side by:

(1) A sudden local pressure rise in some part of the burning charge.

(2) Intense pressure waves within the charge which follow the initial excitation.

It has already been noted in Section I that engine indicators are affected by the pressure disturbances accompanying detonation, but in general do not produce records suitable for quantitative work. Since pressure measurements can be carried out when other methods are not available, the development of a reliable indicator for rapid pressure fluctuations has long been a goal before investigators of detonation. This problem of an indicator to record the pressure phenomena of detonation was attacked by the writer with the results to be described later in the present report. In the usual case indicators can be located only at a few positions in the combustion chamber wall. This limitation makes it necessary to develop a theory of the physical processes involved before any available information can be interpreted in terms of the general pattern of events inside the combustion chamber. In the succeeding pages, the writer will present a systematic analysis of the pressure variations accompanying detonation based on the theory of sound.

General Pressure Disturbances and Sound Waves

Pressure disturbances are possible in any compressible medium. That is, if a local compression in some region of the medium is suddenly released, the energy due to the displacement of particles from their equilibrium positions will spread away from the region of excitation in the form of a pressure wave. The nature of this wave depends upon the interaction of force gradients and inertia reactions in the body of the medium. A theoretical analysis of the problem can be based on three equations:

(1) The equation of motion of an element of the medium within the body of the medium;

(2) The equation of continuity for an element of volume within the body of the fluid;

(3) The ratio of pressure changes to density changes within the body of the medium.

For gases such as the working fluid in an engine cylinder, the ability to support shearing forces is so low that tangential forces can be neglected in setting up the equation of motion. For the gaseous medium also, the perfect gas law can be used to express the relationship between pressure and density. Even with the gas law available, it is still necessary to specify the type of process involved. In pressure wave phenomena, the changes occur so quickly that the effect of heat transfer is negligible and the processes can be treated as adiabatic. As derived in Appendix A, the three fundamental equations of hydrodynamics for rapid processes are:

$$\partial\bar{v}/\partial t + (\bar{v} \cdot \nabla)\bar{v} + \nabla P/\rho - \bar{F} = 0 \qquad (2)$$

$$\rho\nabla\cdot\bar{v} + \bar{v}.\nabla\rho + \partial\rho/\partial t = 0 \qquad (3)$$

$$dP/d\rho = \gamma P/\rho = c^2 \qquad (4)$$

Where $\bar{v}$ = vectorial particle velocity

P = pressure

ρ = density

$\bar{F}$ = vectorial body force

$\gamma = \dfrac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$

No general solution of equations (2), (3) and (4) is possible but the very useful wave equation for sound is obtained by limiting consideration to sufficiently small disturbances. To be "sufficiently small" a disturbance must produce a fractional change from the equilibrium density which is small compared to unity. Such disturbances are said to be "infinitesimal" and any disturbances which do not meet this condition are said to be "finite".

The initial pressure disturbance of detonation probably has very often the same order of magnitude as the instantaneous equilibrium pressure in the combustion chamber. It follows that a satisfactory analysis should extend to disturbances of finite size. However, such a general treatment is impossible with the present limitations of mathematics but certain useful facts can be illustrated by considering the special case of a one dimensional disturbance of finite size.

Lord Rayleigh (187) and Lamb(188) devote some attention to pressure waves of finite amplitude and in particular review the work of Earnshaw and Riemann (references to the original articles are given by Rayleigh and Lamb). The essential features of Riemann's treatment of the subject are outlined below in the notation of the present report.

If body forces are neglected, the equation of motion for a disturbance which varies only with distance along the x axis becomes

$$\partial v_x/\partial t + v_x \cdot \partial v_x/\partial x = - 1/\rho \; \partial P/\partial x \qquad (5)$$

while the equation of continuity is

$$\partial \rho/\partial t + v_x \cdot \partial \rho/\partial x = - \rho \; \partial v_x/\partial x \qquad (6)$$

Now define an integral associated with pressure and density changes as

$$\bar{\omega} = \int_{\rho_0}^{\rho} dP/\rho \qquad (7)$$

Then

$$\partial \bar{\omega}/\partial x = 1/\rho \cdot \partial P/\partial x = 1/\rho \cdot dP/d\rho \cdot \partial \rho/\partial x \qquad (8)$$

and

$$\partial \bar{\omega}/\partial t = 1/\rho \cdot \partial P/\partial t = 1/\rho \cdot dP/d\rho \cdot \partial \rho/\partial t \qquad (9)$$

Substituting in the value of c as defined in equation (4), equations (8) and (9) become

$$\partial\rho/\partial x = \rho/c^2 \;.\; \partial\bar{\omega}/\partial x \qquad (10)$$

$$\partial\rho/\partial t = \rho/c^2 \;.\; \partial\bar{\omega}/\partial t \qquad (11)$$

Using these relations in (5) and (6) gives

$$\partial v_x/\partial t + v_x \;.\; \partial v_x/\partial x = -\,\partial\bar{\omega}/\partial x \qquad (12)$$

$$\partial\bar{\omega}/\partial t + v_x \;.\; \partial\bar{\omega}/\partial x = -\,c^2 \;.\; \partial v_x/\partial x \qquad (13)$$

Now introduce a new variable, ω, defined by the relation

$$d\omega = d\bar{\omega}/c \qquad (13)$$

so that equations (12) and (13) become

$$\partial v_x/\partial t + v_x \;.\; \partial v_x/\partial x = -\,c \;.\; \partial\omega/\partial x \qquad (14)$$

$$\partial\omega/\partial t + v_x \;.\; \partial\omega/\partial x = -\,c \;.\; \partial v_x/\partial x \qquad (15)$$

Adding and subtracting equations (14) and (15)

$$\left\{ \partial\,/\partial t + (v_x + c)\,\partial\,/\partial x \right\} (\omega + v_x) = 0 \qquad (16)$$

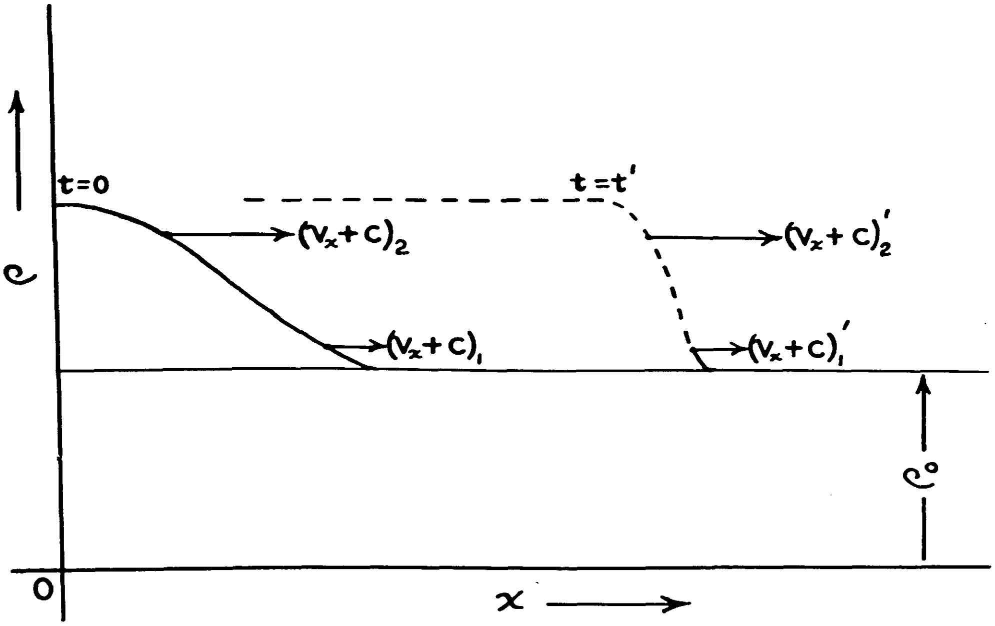
$$\left\{ \partial\,/\partial t + (v_x - c)\,\partial/\partial x \right\} (\omega - v_x) = 0 \qquad (17)$$

It follows that the rate of change of $(\omega + v_x)$ with time is equal in magnitude to the rate of change of $(\omega + v_x)$ with distance multiplied by $(v_x + c)$. This is equivalent to the statement that any given value of $(\omega + v_x)$moves in the positive x direction with the velocity $(v_x + c)$. Similarly any given value of $(\omega - v_x)$ moves along the x direction with the velocity $(v_x - c)$.

Now consider, that at a given instant, a disturbance can be represented on a plot with density as ordinates and x as abscissae. The diagram for t = 0 of figure 67 indicates such a case. Under the adiabatic law, the ratio will increase with increasing density so that for like values of v_x, the velocity of propagation of a disturbance will increase with density. It follows that regions in which the disturbance is stong will overtake regions of lesser intensity until eventually the disturbance front becomes perpendicular to x. When this occurs the derivatives of equations (16) and (17) become infinite and the mathematical treatment is no longer valid. Two stages of this process are indicated in figure 67.

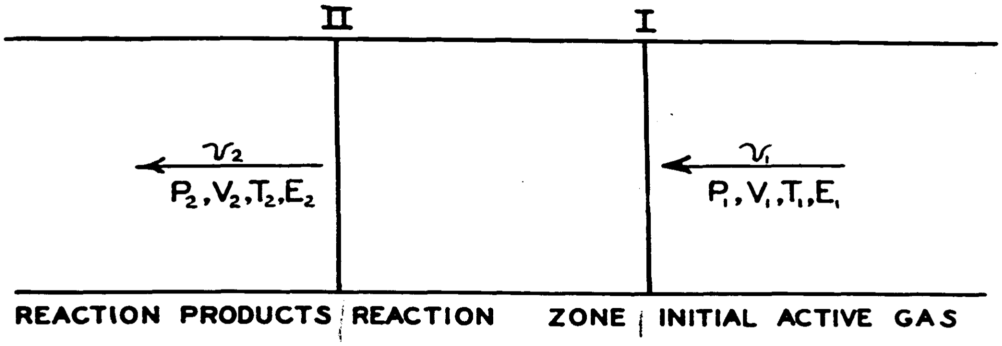
Lord Rayleigh sums up the process described above from a simple physical viewpoint:

"But this law of derivation cannot hold good indefinitely. The crests of the velocity curve gain continually on the troughs and must at last overtake them. After this the curve would indicate two values of u (velocity) for one value of x, ceasing to represent anything that could actually take place. In fact we are not at liberty to push the application of the integral beyond the point at which the velocity curve becomes discontinuous, or the velocity curve has a vertical tangent.------"



PROPAGATION OF A WAVE OF FINITE AMPLITUDE.

FIG. 67



$W = v_1 - v_2$ = FLOW VELOCITY OF REACTION PRODUCTS AWAY FROM REACTION ZONE.

$\bar{c}_{v_2}$ = MEAN SPECIFIC HEAT OF REACTION PRODUCTS BETWEEN T_1 AND T_2

QUASI-STATIONARY DETONATION WAVE.

FIG. 68

"When discontinuity sets in, a state of things exists to which the usual differential equations are inapplicable; and the subsequent progress of the motion has not been determined. It is probable as suggested by Stokes, that some sort of reflection would ensue. In regard to this matter we must be careful to keep purely mathematical questions distinct from physical ones. In practice we have to do with spherical waves, whose divergence may of itself be sufficient to hold in check the tendency to discontinuity. In actual gases too it is certain that before discontinuity could enter, the law of pressure would begin to change its form, and the influence of viscosity could no longer be neglected. But these considerations have nothing to do with the mathematical problem of determining what would happen to waves of finite amplitude free from viscosity, whose pressure is under all circumstances exactly proportional to its density; and this problem has not been solved."

"It is worthy of remark that, although we may of course conceive a wave of finite disturbance to exist at any moment, there is a limit to the duration of its previous independent existence. By drawing lines in the negative instead of in the positive direction we may trace the history of the velocity curve; and we see that as we push our inquiry further and further into past time the forward slopes become easier and the backward slopes steeper. At a time, equal to the greatest positive value of dx/du, antecedent to that at which the curve is first contemplated, the velocity would be discontinuous."

Both Riemann and Earnshaw found formal integrals for a one dimensional pressure disturbance of finite amplitude. However, the results could be applied only to special cases. The simplest example considered was that of a surface of discontinuity propagated in one dimension with constant velocity. This situation forms the basis for analysis of the detonation wave to be considered later so the present discussion will be limited to a remark of Lord Rayleigh:

"It would lead us too far to follow out the analytical development of Riemann's method, for which the reader must be referred to the original memoir; but it would be improper to pass over in silence an error on the subject of discontinuous motion into which Riemann and other writers have fallen. It has been held that a state of motion is possible in which the fluid is divided into two parts by a surface of discontinuity propagating itself with constant velocity, all the fluid on one side of the surface of discontinuity being in one uniform condition as to density and velocity, and on the other side in a second uniform condition in the same respects. Now, if this motion were possible, a motion of the same kind in which the surface of discontinuity is at rest would also be possible, as we may see by supposing a velocity equal and opposite to that with which the surface of discontinuity at first moves, to be impressed upon the whole mass of fluid. In order to find the relations what must subsist between the velocity and density on the one side (u_1, ρ_1) and the velocity and density on the other side (u_2, ρ_2), we notice in the first place that by the principle of conservation of matter $\rho_2 u_2 = \rho_1 u_1$. Again, if we consider the momentum of a slice bounded by parallel planes and including the surface of discontinuity, we see that the momentum leaving the slice in the unit of time is for each unit of area $(\rho_2 u_2 = \rho_1 u_1) u_2$, while the momentum entering is $\rho_1 u_1^2$. The difference of momentum must be balanced by the pressures acting at the boundaries of the slice, so that

$$\rho_1 u_1 (u_2 - u_1) = p_1 - p_2 = a^2(\rho_1 - \rho_2)$$

whence

$$u_1 = a\sqrt{\rho_2/\rho_1} \quad , \quad u_2 = a\sqrt{\rho_1/\rho_2}$$

The motion thus determined is, however, not possible; it satisfies indeed the conditions of mass and momentum, but it violates the condition of energy*------------"

It would be of little use here to describe the work of Hugoniot who replaced the simple law of adiabatic

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*The equation of energy referred to is Bernouilli's Equation for a compressible medium.

compression by a general equation including the intrinsic energy of the fluid. This treatment is discussed in some detail by Lamb(188) in his chapter on Waves of Expansion and will appear as an essential part of the analysis of detonation waves given below. However, certain remarks of Becker(189) with regard to waves of finite amplitude are of interest:

"-----------When the wave front has reached a certain steepness, the counter forces of friction and heat conduction oppose the tendency to further compression. A condition will be reached where these two tendencies compensate each other and from this point on a quasi-stationary wave form will be propagated along the tube."

"----------with the change in temperature difference (in the wave front)* there has followed a change in pressure and density difference which are in themselves a source of wave formation thrown back from the original wave front------. In this way the actual procésses in the formation of compression impulses are seen to be so complicated that at present a complete theoretical treatment of their formation seems out of the question. Only after the impulse wave has become quasi-stationary do we again find conditions more satisfactory for theoretical

---

*(Writer's Note) The only ideas of practical importance to be gained from the theory of waves of finite amplitude are;

analysis."

Mathematically, the important case for pressure wave analysis is that in which the fractional changes in density and pressure are so small that $dP/d\rho = c^2$ can be treated as a constant and the maximum value of v_x is small in comparison with c. Under these conditions equations (16) and (17) can be written

$$\left\{ \partial/\partial t + c \cdot \partial/\partial x \right\} (\delta P/_{\rho_o} + v_x) = 0 \qquad (18)$$

$$\left\{ \partial/\partial t - c \cdot \partial/\partial x \right\} (\delta P/_{\rho_o} - v_x) = 0 \qquad (19)$$

Addition and subtraction of equations (18) and (19) show that given values of v_x and δP are propagated along the x direction at constant velocity and without change in form. This result is a special case of the wave equation for sound which is derived in Appendix A. In its general form for pressure disturbances, the equation of sound is

$$\partial^2 p/\partial t^2 = c^2 \nabla^2 p \qquad (20)$$

Where p = excess pressure = actual pressure - equilibrium pressure

$c = \gamma P_o/\rho_o$

---

* (Footnote con'd. from previous page) (1) that such waves change their form as they are propagated and (2) the mathematics involved is not suitable for application to complex cases.

$\gamma = \frac{\text{Specific heat at constant pressure}}{\text{Specific heat at constant volume}}$

P_o= Total pressure with no disturbance present

ρ_o= Density with no disturbance present

From the argument outlined above, it appears that c is the constant velocity of a sound wave.

Solutions to fit many cases of practical importance can be found for equation (20). Even with some loss in accuracy it is better to use the mathematically exact wave equation solution for intense disturbances and apply reasonable corrections based on physical principles, than to attempt a solution for the complete hydrodynamical equations. The essentials of the wave equation are; (1) the density changes involved must be so small that the coefficient $dP/d\rho$ is substantially constant and (2) the maximum particle velocity must be small compared to the velocity of sound in the medium. The validity of these assumptions must necessarily be tested by experiment in any particular case.

The Detonation Wave in One Dimension

Detonation waves in one dimension have been studied theoretically by several investigators since Riemann established a method of attack and Hugoniot(189) corrected Riemann's assumption with regard to the relationship between pressure and density. Chapman(190) was first to use the method of Riemann for a derivation of the rate of propagation of the detonation wave. Chapman considered that all parts of the wave front traveled with the same velocity at the minimum velocity consistent with this assumption and that at the point of maximum pressure, the chemical change concerned in the propagation of the wave is complete with the unburned gases immediately in front of the wave fired by compression. Calculated detonation wave velocities were in good agreement with the measured values of Dixon but his formula depended upon details of the chemical reaction which were later found to be unnecessary. Jouguet(191) and Crussard(192) working independently of Chapman presented an extended analyses and discussion of the detonation problem which resulted in satisfactory predictions of detonation wave velocities.

Becker(193,194) carried through an analysis of one dimensional wave fronts taking into account the effects of heat conduction which proved that the fronts involved in an intense disturbance are so thin that the assumption that a gas is a continuous medium is no longer valid. The theory of detonation waves as outlined below is based on the discussion given by Becker.

In general, the processes occurring in a thin slice of the active medium must fulfill the equation of continuity, the conservation of energy and the second law of motion. For the case considered, heat conduction and viscous action are unimportant so these effects will be neglected in the following analysis. If λ is the thickness of an element of unit area, v is the velocity along the x direction, E is the specific energy, ρ is the density and P is the pressure, the mass, the momentum and the energy associated with the element are:

$$\text{Mass} = \rho\lambda \tag{21}$$

$$\text{Momentum} = \rho\lambda v \tag{22}$$

$$\text{Energy} = \rho\lambda\ (E + v^2/2) \tag{23}$$

Now the time rate of change of mass must be zero. The time rate of change of momentum must be equal to the difference in pressure between the faces of the element and the time rate of change of energy (if no chemical reaction is present) is equal to the difference in power flow across the two faces. Expressing these conditions mathematically gives

$$d(\rho\lambda)/dt = 0 \tag{24}$$

$$d(\rho\lambda v)/dt = -\lambda\partial P/\partial x\ . \tag{25}$$

$$d\left[\rho\lambda\ (E + v^2/2)\right]/dt = -\lambda\partial(Pv)/\partial x \tag{26}$$

These equations of a traveling wave can not be integrated in general, but the case where the disturbance has assumed the form of a quasi-stationary wave can be analysed. In such a wave the form of the disturbance is constant and moves with constant velocity along the x direction. It is convenient to use a coordinate system moving with the wave front so the system can be treated as if it were stationary. In this case the time derivatives of (24), (25) and (26) vanish and the equations can be integrated by writing $v\partial/\partial x$ for d/dt. By excluding the wave front itself and considering conditions as uniform before the front and immediately behind the front as indicated in figure 68, the equations become

$$v_1/V_1 = v_2/V_2 \tag{27}$$

$$v_1{}^2/V_1 + P_1 == v_2^2/V_2 + P_2 \tag{28}$$

$$E_1 + v_1^2/2 + P_1V_1 = E_2 + v_2^2/2 + P_2V_2 \tag{29}$$

Where $V = 1/\rho$ is the specific volume of the medium. Solving these equations for the velocities and the energy difference gives:

$$v_1^2 = V_1^2 \cdot (P_2 - P_1)/(V_1 - V_2) \tag{30}$$

$$v_2^2 = v_2^2 \cdot (P_2 - P_1)/(V_1 - V_2) \tag{31}$$

$$E_2 - E_1 = 1/2 \cdot (P_1 + P_2)\ (V_1 - V_2) \tag{32}$$

The last equation of this set is the Hugoniot equation which takes the place of the adiabatic relationship between pressure and density as used in sound theory.

In order to apply equations (30), (31) and (32) to the case of a detonation wave in a combustible gas, the energy difference must be equated to the difference between the heat of reaction, Q, which is liberated in the reaction zone, and the change in intrinsic energy of the gaseous medium in passing through the reaction zone, i.e.

$$E_2 - E_1 = \bar{C}_V(T_2 - T_1) - Q \qquad (33)$$

Where $\bar{C}_V$ is the average specific heat of the reaction products between the temperatures T_1 and T_2.

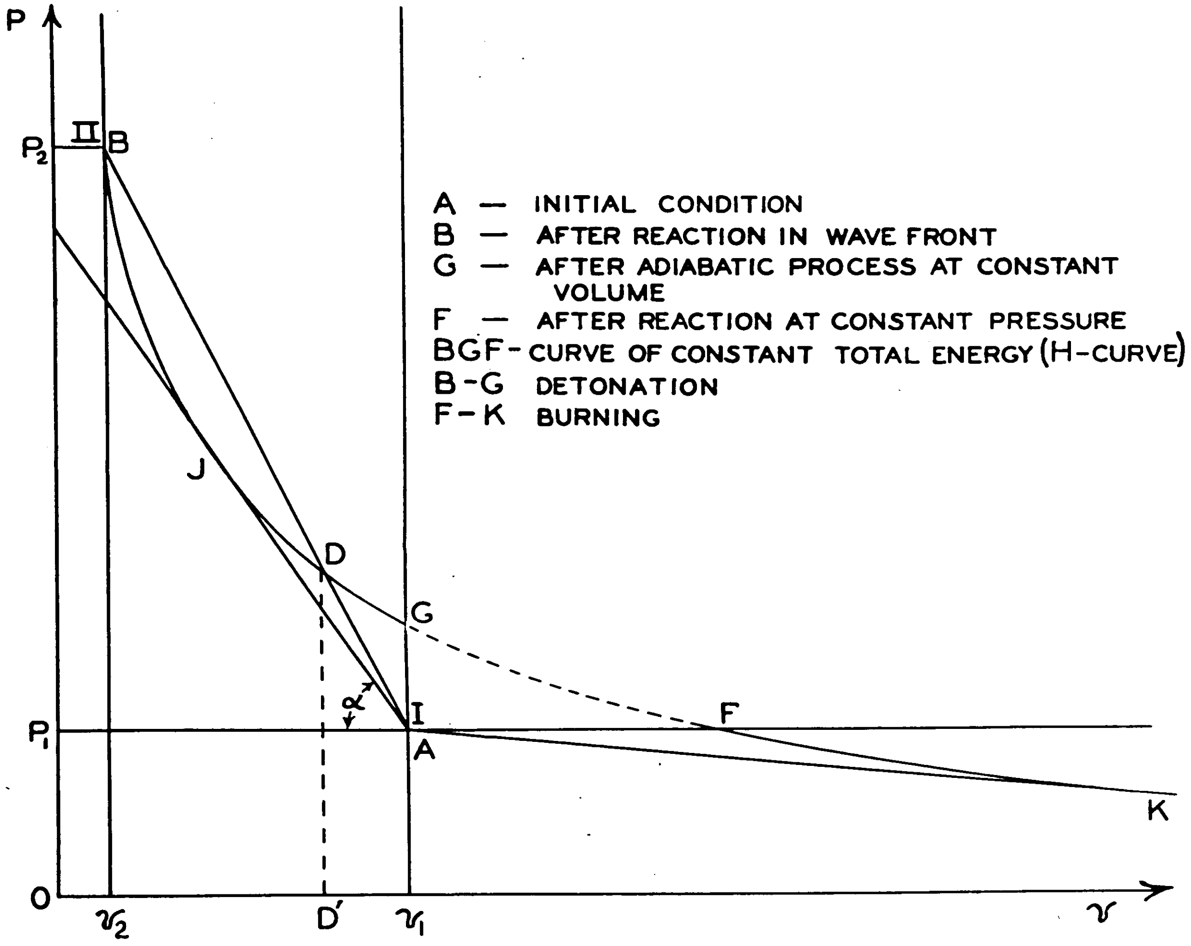
It is convenient to study the detonation wave in terms of $D = v_1$, the velocity with which the front moves into the unburned gases and $W = v_1 - v_2$, the flow velocity of the reaction products away from the reaction zone. In terms of the other variables, D and W are

$$D = V_1 \sqrt{\frac{P_2 - P_1}{V_1 - V_2}} \qquad (34)$$

$$W = (V_1 - V_2) \sqrt{\frac{P_2 - P_1}{V_1 - V_2}} \qquad (35)$$

Becker discusses the detonation wave with the aid of a P-V diagram like that of figure 69. The point A corresponds to the initial conditions P_1V_1 of the unburned gases. A curve of constant total energy, BGF, is then drawn in accord

FOR CASE OF ONE DIMENSIONAL DISTURBANCE



PRESSURE-VOLUME DIAGRAM FOR PROCESSES IN REACTION ZONE OF DETONATION WAVE

FIG. 69

with equations (32) and (33). The point G is a point on this ''H-curve'' corresponding to adiabatic reaction at constant volume while the point F is that for combustion at constant pressure. The point B is a general point corresponding to conditions in the reaction products.

If the location of the point B on the H-curve can be determined, both the velocity of propagation D, and the flow velocity W, can be found since the slope of a line from A to the H-curve is just $(P_2 - P_1)/(V_1 - V_2)$. If α is the angle between the V axis and the line AB, then

$$D = V_1 \sqrt{\tan \alpha} \qquad (36)$$

$$W = (V_1 - V_2) \sqrt{\tan \alpha} \qquad (37)$$

Becker's discussion of the P-V diagram is outlined below:

But $\sqrt{\tan \alpha}$ for that portion of the H-curve between G and F is imaginary and corresponds to no actual natural process. The H-curve, therefore, consists of two entirely different modes of a chemical transformation which will be distinguished by the names detonation and normal burning.

From the course taken by the H-curve we can make the following qualitative statements concerning these modes of transformation:

Detonation.-(The portion of the H-curve, BDG) In the reaction zone, intense increase of pressure and concentration. Direction of flow of combustion products, positive. A high propagation velocity, D.

Burning.-(The portion of the H-curve, FK). In the reaction zone, decrease of pressure and dilatation. Direction of flow of combustion products W, negative. Relatively slow rate of propagation D."

"By the above consideration there has been gained a qualitative idea of the processes under consideration. The figure might convey the impression, however, that along the part of the curve BG there could be varying velocities of detonation D. But experience has shown that under careful adjustment and measurement, the detonation velocity of a definite substance has a constant value characteristic of that substance. This value remains constant over the entire length of the tube, whether the tube be only a few centimeters in length or a hundred meters. In what follows we shall show that the theory here set forth also predicts this fact and that the detonation becomes stable only for a definite point of the H-curve.

To substantiate this we shall show the following statement to be true: *The entropy is a minimum for the point J on the curve and a maximum for the point K. These are the two points of contact between the H-curve and the tangents drawn to the curve from the point A.*

At this point Becker gives a discussion in which he compares the slope, $\varphi = -(dP_2/dV_2)_{ad.}$, of an adiabatic on the P-V plane with the slopes of the H-curve and of a straight line from the point A to a point on the H-curve. With regard to this result he says:

"-------- Our result may therefore be expressed as

Above J(onJ,B), $\varphi > \tan \alpha$

Below J(on J,B), $\varphi < \tan \alpha$

" With the help of this relationship, we obtain at once a valuable criterion for the *stability of the detonation wave*. The density of the reaction products $1/V_2$, is greater at the wave front than the density $1/V_1$ of the initial explosive components at this point. A little later, i.e. behind the wave front, the combustion products will expand since the pressure, due to heat losses will sink below P_2. There is thus formed immediately behind the wave front a rarefied region whose tendency must be opposed to the progress of the compression wave. Whenever the wave front is affected by the conditions just stated it will not maintain its pressure P_2 and as a consequence, its rate of propagation will be reduced. The detonation wave will only maintain its velocity characteristic of the explosive substance as long as the velocity of this rarefied region in the reaction products and the normal velocity of the detonation wave are the same.

The velocity of the detonation wave is given by the expression $D = V_1 \sqrt{\tan \alpha}$. The velocity of the rarefied area behind the wave front may be conceived of as an additive quantity made up of the velocity of sound in reactions products, $V_2 \sqrt{-(dP_2/dV_2)_{ad}}$ and the mass velocity of the reaction products $W = (V_1 - V_2) \sqrt{\tan \alpha}$. The detonation wave is therefore instable as long as

$$V_2 \sqrt{\varphi} + (V_1 - V_2) \sqrt{\tan \alpha} - \sqrt{V_1 \tan \alpha} > 0$$

that is, as long as

$$\varphi > \tan \alpha \text{"}$$

"We saw from equation --- that for those points B of the H-curve above J, φ is greater than tan α; and concluded therefrom that no detonation represented by such points could be stable. They will be continually weakened by the effect of the expansion of the reaction products (from cooling) behind the wave front. Any such designated point on the H-curve will slip back until it reaches J, and at this point the detonation wave will become stable; for at J, $\varphi = \sqrt{\tan \alpha}$. At this point, too, the mass velocity of the reaction products have the same velocity as the detonation wave. The detonation wave will from this point on propagate itself within the gases at a constant velocity corresponding to the point J. What are the conditions represented by the lower part of the H-curve? It seems that here for any given detonation velocity V_1 $\sqrt{\tan \alpha}$, the velocity of movements of the reaction products will adjust itself to the condition at D or B. A reference to equations ------ shows that D and B stand in the same relation to each other as the gas conditions in front of and behind the impulse wave, viz., that the entropy is always greater for B than it is for D. When it is recalled that the reaction products at the moment of their formation represent the condition of maximum probability in the sense of statistical mechanics for the given order of reaction, it would be reasonable to conclude that they would favor the condition indicated by B, and that for this reason the lower part of the detonation region of the H-curve represents no real condition. By this reasoning we come to the following conclusions."

"On the basis of thermodynamic probability, only detonation waves of the B-type above J in the coordinate figure, are possible. Mechanically, these waves are instable and pass over at once to the normal detonation condition indicated at J."

"The point J according to the above considerations is determined by the relation

$$\frac{P_2 - P_1}{V_1 - V_2} = -(dP_2/dV_2)_{ad} \text{"}$$

"According to the above consideration it would indeed be possible (as by intense initial ignition) to induce detonation waves of greater velocity than normal. They would, however, soon sink to the normal rate and there remain constant. --------------"

"In case no chemical transformation accompanies the impact wave, the wave degenerates to an ordinary sound wave, in which case the points J,G,K,V_2,V_2'* fall together with V_1"

Becker notes that the expression for the detonation wave velocity was given first by Chapman and later by Jouguet. He gives a table comparing the calculated detonation wave velocities with measured velocities for a number of gaseous mixtures. The average difference between the

*Becker uses V_2' as the specific volume at the tangent to the lower part of the H-curve.

calculated and measured values is about 1 per cent with a maximum disagreement of 2 per cent.

Detonation wave theory as applied to one dimension is of little direct use in connection with the problem of detonation in an internal combustion engine. However, it is reasonable to expect that the processes involved are essentially similar in the two cases. In the one dimensional wave, the reaction is so rapid that the chemical energy is released as heat in the reaction zone before any energy is lost from the reaction zone either as heat or as mechanical work. In the engine combustion chamber, three space coordinates are involved and the distances are too short to be certain whether or not a progressive reaction of constant velocity occurs. These circumstances would make it very difficult to decide with certainty whether or not the engine process is identical with normal detonation in the sense that all the chemical energy of the fuel is released in a wave front to produce a condition of stability. However, it seems to the writer that this question is largely of academic interest since the difference between stable and unstable detonation waves would lie only in velocity variations of a process already so rapid as to have undesirable effects on the engine. *Fundamentally, the detonation process in the engine is identical with the process* occurring in the reaction zone of a detonation wave in the sense that both are characterized by such a high rate of reaction that an intense local pressure disturbance is produced.

Do The Pressure Waves Accompanying Detonation in Engines Obey the Laws of Sound?

Pressure disturbances which meet the requirements of sound theory can be analysed with much greater facility than disturbances which can not be handled by this method. In fact, any complicated problem involving changes too large to be treated as sound waves is almost certainly impossible of solution with the mathematics available at the present time. This state of affairs makes it important to determine whether or not the theory of sound can be applied to the pressure variations produced by detonation inside the combustion chamber. Very probably the initial local disturbance of detonation will often be beyond the limitations of sound theory. However, this local high pressure is soon replaced by a system of standing waves completely filling the chamber. This spreading will be accompanied by a substantial reduction in the maximum excess pressure present in the chamber. In addition to the spreading effect, viscous action and heat transfer processes will tend to rapidly reduce the amplitude of pressure waves to a level where sound theory can be applied. With no theoretical method available for determining whether or not the pressure amplitude reaches sound wave proportions soon enough for the theory to be useful, any conclusion with regard to the matter must be based upon experimental data. The results of investigators who have considered sound waves in connection with detonation will be reviewed below.

Nernst(12) in 1905 discussed the effect of detonation in the internal combustion engine and showed very definitely that standing waves of pressure were to be expected in the combustion chamber under certain conditions. Many investigators found evidence of pressure waves both in closed vessels and in engines but no definite discussion of this phase of detonation appeared until the work of Maxwell and Wheeler(121,195) was reported in 1929. These authors considered the striated appearance of flame photographs from closed vessels which were accompanied by vibrations in the pressure record and studied the effect of different cylinder lengths. Their views on the subject of pressure waves are summarized on the quotation below:

"The effect of change of length of cylinder on the period of the vibrations, the appearance of the records and the effect of variations in initial pressure to density ratios support the view that the striations are due to the establishment of a stationary longitudinal wave in the column of gases. The initiation of such a stationary wave may possibly be due to the compounding of reflected progressive waves, of the same period and amplitude such as those recorded by Payman----as occurring in the hot products behind a flame in a closed vessel."

Maxwell and Wheeler carried out a series of experiments to determine the effect of different variables on the formation of stationary waves in a combustion chamber in an effort to find the controlling factors. They state the following conclusions:

"The most important factor appears to be the character of the reactions proceeding in the wake of the flame.--------"

"The acoustic properties of the charge of gaseous mixture, as determined by the size and shape of the combustion space, play a part in affecting the resonance.---"

"It would seem that, for resonance to be induced, a certain minimum temperature must be exceeded at some point or points in the burnt and burning gases, the stationary wave being formed by the compounding, with their reflections from the ends of the cylinder, such progressive waves (originating behind the flame-front) as have been noted by Payman. These progressive waves appear to be initiated by a sudden disturbance at a point, or at several points, in the wake of the flame."

Although Maxwell and Wheeler studied the problem of standing waves in a combustion chamber experimentally and unquestionably used the methods of acoustics for a qualitative discussion, they did not report any quantitative analysis of the wave systems they found nor any data to prove that sound theory was valid for their working conditions.

In 1932 Serruys(196,197,198,199,200) reported the first of a series of investigations particularly devoted to the pressure aspects of detonation. His initial step was to study the duration of the initial high pressure accompanying detonation in an engine. For this purpose he measured the length of time required for the loose valve of a Farnboro Indicator to move from the lower to the upper seats under the peak pressure and calculated the desired result from known properties of the valve. The results of this procedure showed that the detonation pressure existed for less than 1/10,000 second. No data on the indicator location or the size and shape of the combustion chamber were given.

Serruys(197) continued his work with the Farnboro indicator and estimated the maximum pressures from his experimental results. His conclusions were:

"Nous sommes donc conduits à admettre que le cognement correspond à la combustion très rapid et à <u>volume sensiblement constant</u> de la portion des gaz que brule la dernière, portion dont las pression avant la combustion est déjà tres élevée en raison de la dilatation des gaz déjà brules (30 kg/cm^2 et plus). Le calcul précédent montre en effect que si l'équilibre des pressions était maintenu, même approximativement dans la chambre de combustion, les pressions mesurées n'auraient pu être atteintes."

"Nous pensons donc que le cognement est un phenomene <u>local</u> d'une durée comprise entre 1/10,000 and 1/20,000 de seconde, se produisant à la <u>fin</u> de la combustion, c'est-à-dire lorsque la très grande majorité des calories que celle-ci peut fournir a déjà été mise en jeu et dont l'intensité est telle qu'elle peut aisément doubler et même tripler la pression, le maximum absolu pouvant dépasser 100 kg/cm^2."

With the idea of detonation as a sharp local pressure rise firmly established, Serruys(198) further investigated the matter by taking simultaneous pressure records at two points within the combustion chamber. He was particularly interested in estimating the time interval required for the disturbance to travel a known distance within the chamber. No details of the indicators or the oscillograph system are described but the records indicate that some sort of a rotating drum was used to establish the time coordinate. It was noted that one indicator had a natural frequency of 19,600 cycles per second while the other had a natural frequency of 26,400 cycles per second. The indicators were located 5 centimeters apart in the chamber.

Records taken without detonation showed no difference in the two pressure records, but with detonation, the initial pressure disturbance had unlike magnitudes at the two indicators and arrived at measurably different times. Several diagrams taken under mild detonation conditions showed that for the points studied there was:

(1) an excess pressure of about 5 kg/cm^2 lasting from 1/10,000 to 1/20,000 second.
(2) the pressure rise reached one indicator about 1/10,000 second before the other, corresponding to a propagation velocity of about 500 meters per second.
(3) the initial disturbance was followed by a series of pressure waves of decreasing

intensity and frequency varying from 3,300 cycles per second at the start of expansion to 2,000 cycles per second at the end of expansion.

The decrease in frequency observed during the expansion stroke was cited as evidence to show that the observed results were not due to natural frequency effects in the indicators. The decrease in frequency was explained as the effect of reduced sound velocity in the charge as the gases cooled on the expansion stroke. Serruys summarized his results from the instantaneous pressure records in the following conclusions:

(1) Detonation occurs near the end of combustion;

(2) A small part of the total charge is involved in the detonating combustion;

(3) The duration of the final burning is in general less than 1/10,000 second which is so short that a very high pressure is built up in a localized region;

(4) The gas which has detonated and exists momentarily at high pressure expands immediately into other portions of the combustion with the formation of pressure waves;

(5) The intensity of these pressure waves diminishes rapidly with distance from the initial disturbance and also with the number of successive reflections;

(6) The velocity of propagation of these waves in the burnt gases appeared to be about 500 m/sec. with a normal compression ratio of 5.5 to 1.

The results of Serruys confirm the information on detonation from other sources and are based on data from pressure measurements alone. His general picture of the pressure disturbances involved is reasonable and he interprets his data in terms of sound velocity. His measured velocity of 500 meters per second for sound in the burnt charge of an engine with a compression ratio of 5.5 to 1 is about half of that found by the writer for similar conditions. Without more complete information on the experimental arrangement used by Serruys no explanation of this apparent discrepancy can be given.

Sound waves in the free air outside the engine have often been investigated in connection with detonation and in some cases correlated with the pressure waves inside the combustion chamber. Huf, Sabina and Hill(201,202) studied the amplified output from a microphone placed near the engine as a means of knock rating. They found the sound energy distributed over a wide range of frequencies and used a filter system to eliminate the band below 2000 cycles per second. The indicator was an ordinary output meter suited to the amplifier which was operated by various types of microphones. It was found that this arrangement was satisfactory for knock rating under laboratory conditions. Paul and Albert(203) using an arrangement similar to that of Huf, Sabina and Hill investigated a frequency range from 50 to 10,000 cycles per second and found that the detonation noise was not confined to narrow frequency limits nor peaked at a single frequency but rather existed over a wide band with varying intensity. Carpenter and Stansfield(204) recognized that mechanical noises from engine parts contributed a relatively high intensity noise background to the microphone output and reduced this difficulty by designing the "Strobophonometer". This instrument consisted of a conventional microphone, amplifier and meter system with the addition of a stroboscopic contact operated by the engine which reduced the sensitivity of the apparatus to zero except during a short interval of each cycle which could be selected at will. With regard to the microphone used, the writers say:

"For the majority of applications a simple telephone receiver earpiece is highly satisfactory as the microphone. When this is held in a sponge rubber mounting against the source of sound the diaphragm is damped by the air pocket formed between the machine, the rubber and the diaphragm housing. This damping is sufficient to give the necessary frequency range, and the range may easily be changed when required. As has been explained above, the selection of high frequencies does not appear to be of value in most cases, and may definitely be harmful. If necessary, however, diaphragms of various thicknesses held near to the machine, but not enclosed to give air damping, may be substituted, the thickness being such as to respond to a desired frequency.---------"

Using the "Strobophonometer" Stansfield and Carpenter found that the sound due to detonation was concentrated during the first part of the expansion stroke while the general noise level was relatively constant throughout the stroke. No attempt was made to make accurate frequency determinations or to correlate the observed results with internal processes of the engine.

MacCoull and Stanton(205) continued the development of the microphone as a device for studying detonation. They connected a sound-level meter and a frequency analyzer to their electro-static microphone and analysed the frequency spectrum from several engines in the laboratory and on the road. With regard to the CFR engine they concluded:

(1) Knocking is accompanied by a sound energy increase at frequencies up to 8000 cycles per second.

(2) For the particular engine used, the increased sound energy due to knock was especially noticeable in the frequency band between 3000 and 4000 cycles per second and 6000 and 7000 cycles per second.

(3) Under similar conditions, cylinder head material had only a minor effect of the noise spectrum.

MacCoull and Stanton constructed a special electro-static indicator unit to be exposed to cylinder gases through a suitable opening in the combustion chamber wall. With this indicator they applied the method of frequency analysis and also took oscillograms with a cathode ray tube. It was found that the frequency spectrum given by the internal indicator was similar to that found by means of an external microphone. No attempt to treat detonation as an acoustic problem inside the combustion chamber was discussed.

Wawrziniok(206) developed a piezo-electric indicator for the purpose of measuring the pressure variations accompanying detonation in engines. In order to test the reliability of his instrument, tests were first carried out in a closed bomb fitted with electrodes for locating the flame front by ionization effects. A microphone was placed in the free air outside the bomb. It was found that the external microphone records contained the same frequencies as simultaneous records from the indicator. To check the source of the frequencies, experiments with bombs of different size were carried out. Sound velocity within the exploded gases during the period of the vibrations was calculated by the usual equation. Wawrziniok's results led to the following conclusions:

The combustion noise began even before the flame reached the bottom. The time between the visible beginning of the noise in the oscillogram and the maximum pressure was 20 to 30 per cent of the total combustion time.

It was recognized that the noise was a secondary phenomenon of the gas vibrations in the bomb. The maximum amplitude, as experimentally verified, was 4 atmospheres (As to how far, due to the high frequency of the vibrations, the inertia forces of the indicator affected the records, no conclusion can yet be made.) The noises consisted throughout of nonsinusoidal tones.

In rich mixtures the noise developed exponentially with close approximation. Since, even in the engine, a similar beginning of the nonsinusoidal detonation noises was found, there seemed to be some relationship between the two phenomena.

By tests with three bombs of different combustion chamber lengths, a direct proportion was established between the length of the combustion chamber and the reciprocal of the Hertz number of the gas vibrations, from which it was concluded that the vibrations of the combustion gases in the bomb were longitudinal.-------"

For a sound velocity of 900 m/s at $T_{abs} = 2000$ deg. C., the fundamental wave length was twice the length of the combustion chamber.

Applying his indicator to an engine in operation, Wawrziniok obtained data which he discussed as below:

"------The noise begins with the frequency of 3,200 Hertz, i.e., with the frequency of the detonation as determined for the engine. The noise then dies away through 2,500 Hertz to 2,100 Hertz. This agreement indicates that the maximum length of the combustion chamber seems to be the determining factor for the development of the noise. In the present case this was 13.3 cm. as measured perpendicular to the cylinder axis which almost exactly agrees therefore with the bomb length of 13.8 cm. This seems to justify the assumption that the results obtained in the bomb tests correspond in some degree to those obtained in the engine tests and that the detonation noise of the engines is identical with the combustion noises of bombs. From this it follows that the detonation noises are likewise attributable to longitudinal gas vibrations, which are produced by pressure variations and become audible externally."

The work of Wawrżiniok showed that the theory of sound could be applied to both bomb experiments and engine tests. Calculated sound velocities used in connection with chamber dimensions gave computed frequencies in satisfactory agreement with observed results. The investigation described did not include a wide enough range to permit a general conclusion but the evidence presented indicates that the theory of sound can be applied to at least some cases of detonation in engines.

Boerlage and his collaborators(5) applied a single piezo-electric indicator to the investigation of the pressure waves accompanying detonation in an engine. A number of openings for the indicator were provided in various parts of the combustion chamber and it was also possible to shift the spark plug position. By this means both the detonation zone and the point of measurement could be changed. The chamber was of an irregular shape and no analytical method of predicting frequencies was discussed. Data from a number of experiments led to the conclusion that the pressure waves due to detonation were made up of the normal modes for the particular chamber. Shifting the position of the detonation zone had the effect of changing the relative magnitudes of the various modes and the maximum amplitude depended upon the severity of the detonation. Fluctuations as high as ten atmospheres were reported but no sudden disturbances were detected in the detonation zone.

The results discussed in the last paragraph are in general agreement with those of other investigators except for the observed absence of a local pressure rise in the detonation zone and the apparently excessive pressure increase of ten atmospheres. Both of these effects could easily be due to defects in the measuring equipment. Since a description of the apparatus was not given, no estimate of the possible causes of error can be discussed here.

In a preceding section on combustion experiments, it was noted in several places that the vibratory displacements of luminous particles after detonation could be ascribed to sound waves. In particular, Withrow and Rassweiler(151) showed that velocity estimates made from their flame photographs gave results in agreement with the velocity of sound as computed from pressure and density data. A similar result was found in the case of the flame photograph discussed by Elting(56). As a matter of common experience, flame photographs do not reveal particle motion until relatively severe detonation is present, so information from this source can be applied to a wider range of conditions than that from pressure indicators. On the basis of evidence from particle motions it appears that reasonably accurate results can be expected from the use of sound theory for analysing the effects of detonation in engines.

In 1931 the writer(56) undertook an investigation of the pressure waves accompanying detonation in engines with the definite object of determing whether or not quantitative results could be obtained from the theory of sound. Two engines with circular combustion chambers, flat cylinder heads and flat pistons were chosen for the work since the resonant wave systems in such chambers could be conveniently analysed by mathematics. Conventional methods were applied to solve the wave equation in cylindrical coordinates and fit the result to sound waves in a right circular cylinder with flat ends. This analysis was used to predict the frequencies for the various types of resonant wave systems in terms of cylinder dimensions and sound velocity in the reaction products of the fuel-air mixture. The experimental problem was to determine equilibrium pressure, equilibrium density and pressure wave frequency as functions of crank angle. The pressure and density measurements were made by the usual methods. Pressure wave frequencies were found with the arrangement shown in figure 70. A CFR engine and an N.A.C.A. Universal Test Engine with a special flat cylinder head were used in the experiments.

Sound velocity was computed by the formula

$$c = \sqrt{\gamma P_o/\rho_o} \tag{38}$$

In addition to the measured values of pressure and density it was necessary to estimate the ratio of specific heats for various conditions during the expansion stroke. This ratio was calculated from the best specific heat data available at the time. The results are given in the plots of figure 71.

The M.I.T. rate of change of pressure indicator, as described in Section I, was used with a direct current amplifier and an electro-magnetic oscillograph to record the pressure wave frequencies. The sensitivity of the indicating system varied so badly with frequency that no estimates of pressure amplitudes were attempted. However, frequencies were recorded accurately and could be used for

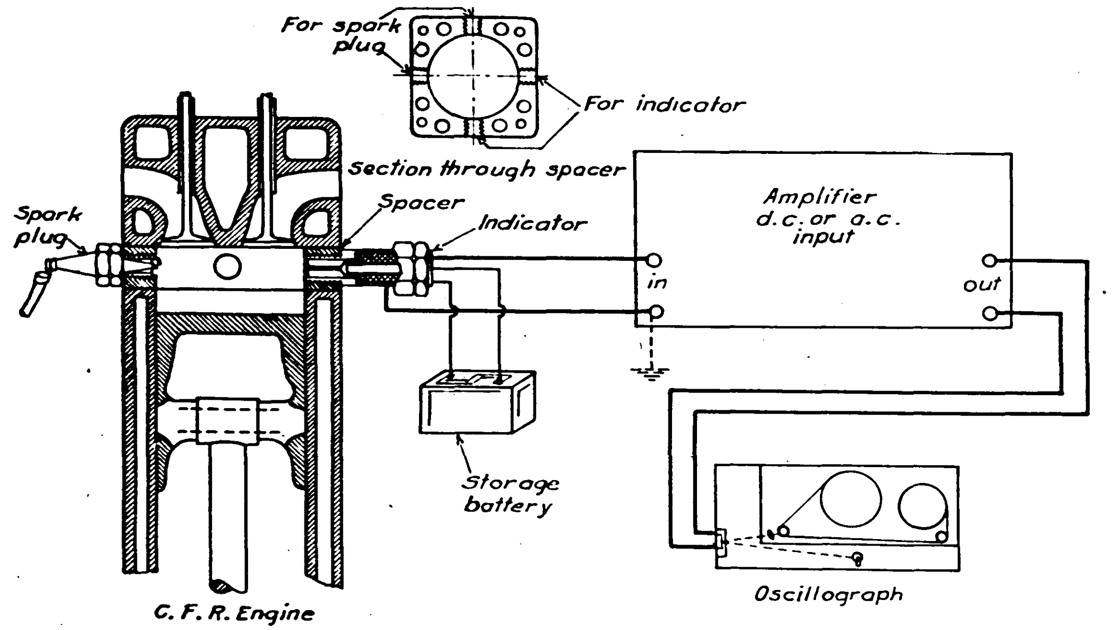
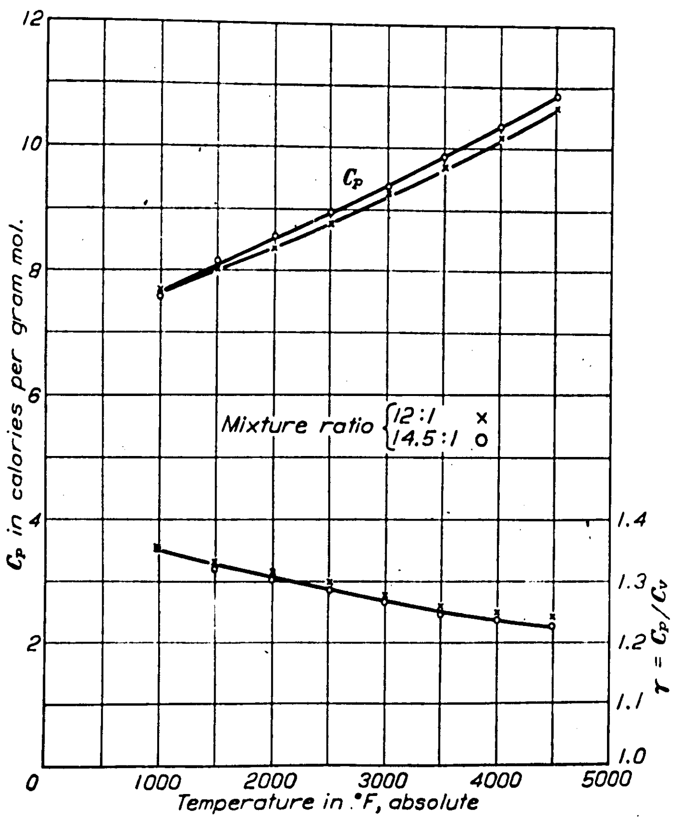


Diagram of connections for M.I.T. rate of change pressure of indicator.

REF. 56

FIG. 70

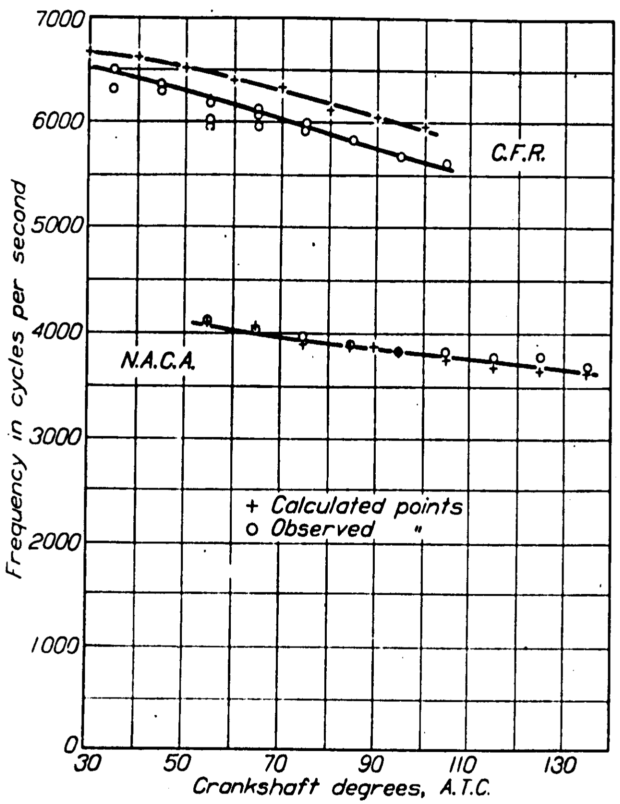


Variation of C_p and γ with change of temperature and mixture ratio. Data taken from reference 15. Gas composition:

Mixture	12:1	14.5:1
CO_2	8.2	11.7
O_2	.5	1.0
CO	6.1	1.0
CH_4	.6	.1
H_2	2.8	.2
N_2	68.8	73.4
H_2O	13.0	12.6

FIG. 71

REF. 56



Comparison of calculated and observed detonation sound frequencies. Waukesha C.F.R. engine: Bore=3.25 in.; stroke=4.5 in. N.A.C.A. engine: $\frac{\text{con. rod length}}{\text{crank throw}}$=3.32 in.; compression ratio=5.5; bore=5 in.; stroke=7 in.; speed=1,540 r.p.m. Air consumption=4.89 lb./min. Fuel consumption=0.313 lb./min. Assumed molecular weight of exhaust gas=28; temperature of exhaust gas=900° C.

REF. 56

FIG. 72

comparisons with calculated values. Figure 72 is a plot showing the variation of measured and computed frequencies with crank angle in two engines. The close agreement between the predictions of theory and the results of experiment certainly justify further applications of the theory of sound to pressure disturbances accompanying detonation in engines.

A Method for Studying the Pressure Disturbances Accompanying Detonation

Detonation studies would be greatly assisted by a method for determining the location, size, shape and magnitude of the initial pressure disturbance. To be of wide use, such a method must be based on measurements which can be conveniently carried out on a conventional engine. This last requirement excludes the use of apparatus depending upon radiant energy from the burning mixture and favors instruments actuated by pressure. Pressure measuring devices also have the possibility of indicating instantaneous values which represents an advantage over instruments for measuring temperatures. In particular, if a rate of change of pressure indicator is used, high frequency pressure waves can be clearly differentiated from the relatively slow pressure changes of normal operation. From a practical standpoint, pressure actuated instruments can easily be made simple and rugged enough to withstand the severe temperature and pressure conditions of engine operation. All of these considerations lead to the conclusion that the most convenient instruments for studying detonation will probably supply direct data on instantaneous pressures at one or more locations in the combustion chamber. The analytical problem is to secure the maximum amount of knowledge with regard to general combustion chamber events from a given amount of information on instantaneous pressures.

Assuming the availability of pressure data, a satisfactory analysis of detonation in a particular case must start from a knowledge of instantaneous variations at a few indicators arbitrarily located and give a complete picture with respect to time and space of the pressure wave system responsible for the observed variations. The possibility of realizing such a scheme in practice was first suggested to the writer by Professor Morse of the Physics Department at M.I.T. By a combination of analytical and experimental procedures, Professor Morse proposed the determination of a mathematical expression describing the complete wave system within the combustion chamber for a short time after the initial excitation by detonation. This equation was to be based on the theory of sound and the assumption that damping effects could be neglected over a few cycles. The arbitrary coefficients of the general equation were to be adjusted to fit the actual pressures recorded by instantaneous indicators at possible locations in the combustion chamber walls. By placing time equal to zero in the resulting particular expression for pressure waves the extent, location, size and magnitude of the initial disturbance would be given directly.

Several difficulties, both practical and theoretical appear to hamper execution of the pressure wave analysis outlinedabove. The theory of sound would be applied to the initial stages of a disturbance which involves relatively large local pressure changes occurring at a finite rate and which is subject to damping after the initial excitation. The waves within the chamber travel in a medium of non-uniform temperature at any given instant which varies rapidly in pressure and density as time goes on. The chamber enclosing the wave system continuously changes in size and shape during the disturbance.

The finite size of the pressure changes involved and the uncertain rate at which the initial excitation develops can not be accurately taken into account in the theory of sound. It follows that any results obtained by the wave analysis method as outlined will necessarily be somewhat approximate. Corrections could probably be made for damping although the advantage gained might not be worth the additional trouble when the other inherent inaccuracies are considered.

Temperature variations in the medium are important only in so far as they affect the velocity of sound, and this means that the percentage effect of such variations are reduced to half by the square root sign on temperature in the velocity equation. Remembering that the measured temperature differences across the combustion chamber have been of the order of ten per cent, it seems probable that the net effect on sound velocity will be less than five per cent at most.

With the frequencies found to accompany detonation in engines, the piston motion will have a relatively small effect on the volume of the combustion chamber over a period of several cycles. This means that conditions in the system are substantially constant at any instant so far as pressure calculations are concerned. For this reason, the disturbance accompanying detonation can be considered as <u>quasi-stationary</u> at any instant.

From a practical viewpoint, the wave analysis equation will involve so many coefficients to be determined that it will probably be very difficult to carry out direct computations. In the absence of experience to the contrary, the best method of attack may be to find the theoretical wave systems produced by a series of assumed initial pressure distributions and find by comparison a distribution giving variations at the indicators similar to those found by experiment. Such a process will necessarily be based on trial and error tempered with judgment rather than a routine computation procedure.

All the factors noted above combine to reduce the accuracy of results from wave system analysis. However, the available information indicates that considerable differences will exist between succeeding engine cycles so even results of high precision would have to be interpreted in terms of averages. It is probable that the resulting general information would not be much more definite than the picture obtainable from the actual application of the wave system analysis.

The writer accepted the practical realization of Professor Morse' wave system analysis scheme as his object in the work described by the present report. The investigation naturally divides into two major parts:

A. The design and construction of a system to accurately record instantaneous pressure variations at two or more points in the combustion chamber.

B. The development of analytical methods for interpreting experimental pressure records in terms of a general wave system in the combustion chamber.

The experimental problem required an instrument to produce accurate data on pressure variations from the low frequencies of normal cycle changes to some indefinite upper limit which appeared from the writer's previous experience to be higher than 20,000 cycles per second. Three component parts combine to make up such an instrument:

(1) A pickup unit to transform pressure fluctuations into electrical output which not only respond accurately over the required frequency range but must also be rugged enough to withstand engine operation for long periods.

(2) A recording oscillograph able to produce satisfactory photographic traces at film speed high enough to permit reasonably accurate wave form analysis.

(3) An amplifier with sufficient gain and a sensitivity substantially independent of frequency over the required range.

The design of these units will be considered in a separate section.

The principal steps used in analysing the pressure wave system for a given detonating cycle are listed below:

(1) Solve the wave equation for sound in terms of coordinates suitable for the particular combustion chamber considered.

(2) Determine the normal modes of [illegible] vibration by requiring that particle velocity remain single valued and finite at all points of the chamber and vanish in a direction perpendicular to the walls at the boundaries.

(3) Substitute in particular coordinates for the indicator locations and obtain an expression in terms of arbitrary coefficients, sound velocity and time for the pressure variations at each indicator.

(4) Compute the velocity of sound as a function of crank angle from measurements of equilibrium pressure and density.

(5) Calculate frequencies for the normal modes of vibration from sound velocity and the chamber dimensions.

(6) Substitute these data into the pressure equation and find particular values of the arbitrary coefficients by adjusting the computed resultant pressure variations at the indicators to fit the experimental records.

(7) Determine the size, location, shape and magnitude of the initial pressure disturbance by placing time equal to zero in the pressure wave equation.

It was found in practice that the procedure outlined above could not be rigidly followed on account of the inherent complexity of the problem but it served as a useful guide in following out the analysis of any particular case.

Detonation measurements have always been carried out in terms of arbitrary readings of a particular instrument when used in a particular manner on a particular engine. Such a procedure has greatly hampered investigations and discussions of the subject. It would be more reasonable to base a system of measurement on the average energy density in the combustion chamber associated with the pressure waves produced by detonation. Such a scheme would have the advantage of dealing with a definite quantity independent of the method of measurement. Whether or not, the energy associated with detonation would be a satisfactory measure of the harmful effects of the phenomenon can only be determined by experience.

The remainder of the present section will be devoted to finding analytical expressions for pressure variations and energy densities in an engine with a flat topped, cylindrical combustion chamber. Several other useful relations between quantities associated with wave motion in such a chamber will be discussed as incidental to the general problem.

Pressure Waves in a Circular Cylinder With Flat Ends

Appendix A outlines the development of the wave equation from sound from fundamental principles. This result has been given in its general form as equation (20). With the wave equation available it is necessary to find a solution and then fit this solution to particular boundaries before numerical results can be acheived. In the present case, it was elected to solve the wave equation for the case of a circular cylinder with flat ends since such a geometrical configuration could be realized in combustion chambers of practical engines. To carry out this solution, cylindrical coordinates r, θ and z were chosen as most convenient since the cylinder boundaries could be expressed as surfaces of constant r and constant z.

Details of the procedure for solving the wave equation in cylindrical coordinates and introducing the boundary conditions are given in Appendix B. The resulting expression for pressure waves in the cylinder is given in equation (39)

$$p = \sum p_{n_\theta n_r n_2} J_{n_\theta} \left[\alpha_{n_\theta n_r}(r/a)\right] \cos(n_\theta \theta - \varphi_{n_\theta n_r}) \;\mathrm{X}$$

$n_\theta = 0,1,2 \ldots$

$n_r = 0,1,2 \ldots$

$n_z = 0,1,2 \ldots$

$$\cos(n_z \pi z/h)\cos(2\pi\nu_{n_\theta n_r n_z} t - \varepsilon_{n_\theta n_r n_z}) \quad (39)$$

Where

n_θ = number of azimuthal nodes (nodal planes containing the cylinder axis and a diameter).

n_r = number of radial nodes (nodal cylinders).
(this disregards nodal line coinciding with axis for $n_\theta > 0$)

n_z = number of axial nodes (nodal planes perpendicular to the cylinder axis).

$p_{n_\theta n_r n_2}$ = absolute amplitude of pressure associated with the mode $n_\theta n_r n_2$ (mode with nodal surfaces $n_\theta n_r n_2$).

$\alpha_{n_\theta n_r}$ = number associated with the nodal surfaces $n_\theta n_r$ (the method for determining these numbers is outlined in App. B).

r = radial distance from cylinder axis.

a = radius of cylinder.

θ = azimuth angle.

$\varphi_{n_\theta n_r}$ = phase angle associated with azimuth.

z = distance from bottom of cylinder measured parallel to the cylinder axis.

h = height of cylinder.

$\nu_{n_\theta n_r n_2}$ = frequency of the mode $n_\theta n_r n_2$.

t = time.

$\varepsilon_{n_\theta n_r n_2}$ = phase angle associated with time.

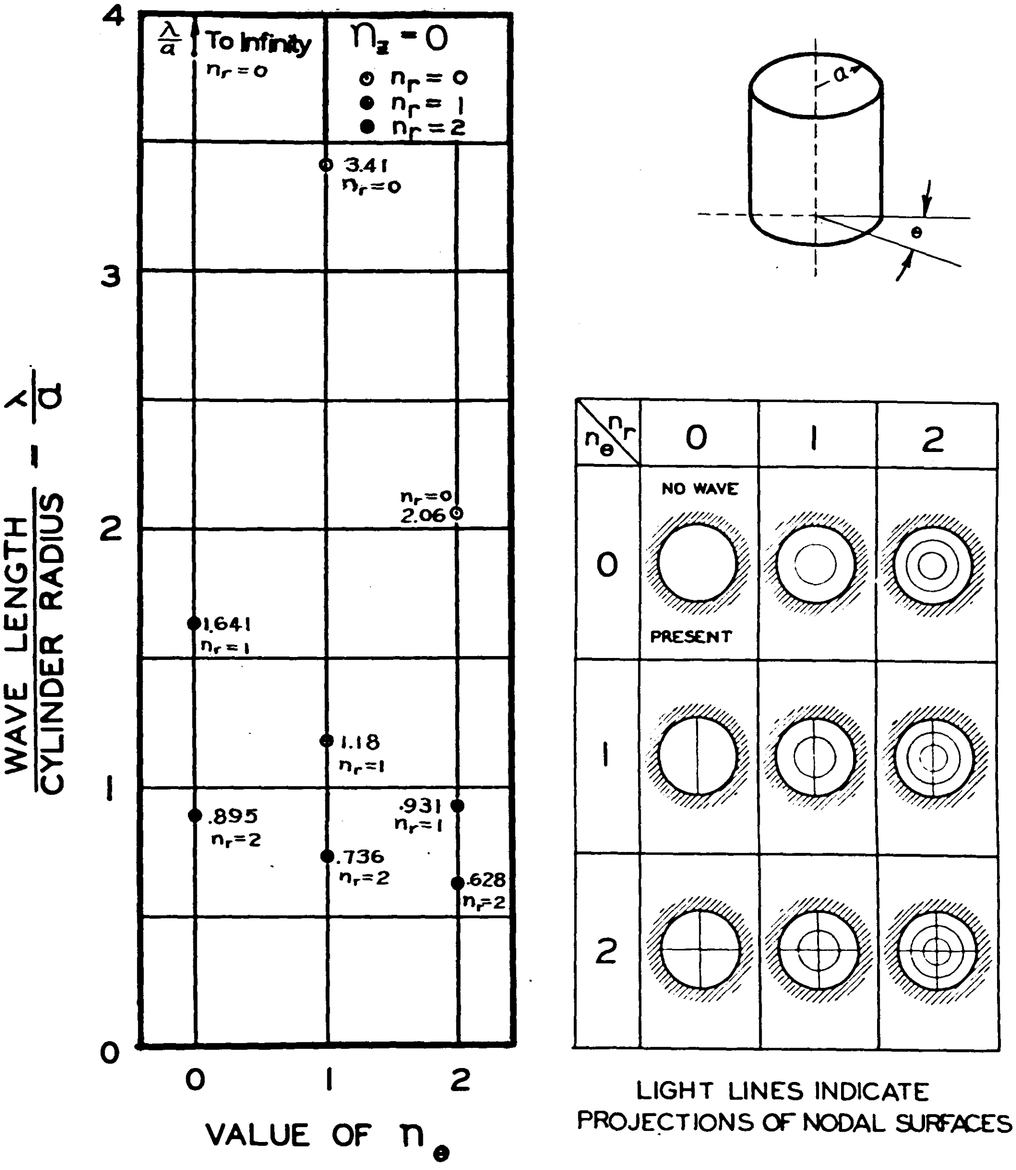
The frequency $\nu_{n_\theta n_r n_z}$ is related to the velocity of sound c, and a distance $\lambda_{n_\theta n_r n_z}$, which will be called the wavelength, by the equation

$$\nu_{n_\theta n_r n_2} = c/\lambda_{n_\theta n_r n_2} \qquad (40)$$

As the term will be used in the present report, a nodal surface is a region of zero pressure variation for the particular mode considered. This definition differs from the conventional usage in which a node is defined as a region of zero particle velocity.

The scheme of identifying modes of vibration by means of the number of nodal surfaces in θ, r and z has proved to be very convenient in practice. In the succeeding pages of this report the identifying subscripts will be ommitted whenever there is no chance of ambiguity.

Figure 73 shows the ratio of wavelength to cylinder radius for several simple modes of vibration with no axial nodes. These modes correspond to cases in which the wave problem has been reduced to two dimensions. Figures 74 and 75 show the wavelength expressed in terms of cylinder height



LIGHT LINES INDICATE PROJECTIONS OF NODAL SURFACES

FIG. 73 SOUND-WAVE-LENGTH OF WAVES TRANSVERSE TO THE AXIS IN A CYLINDRICAL CHAMBER

W.A.S. AUG. 1937

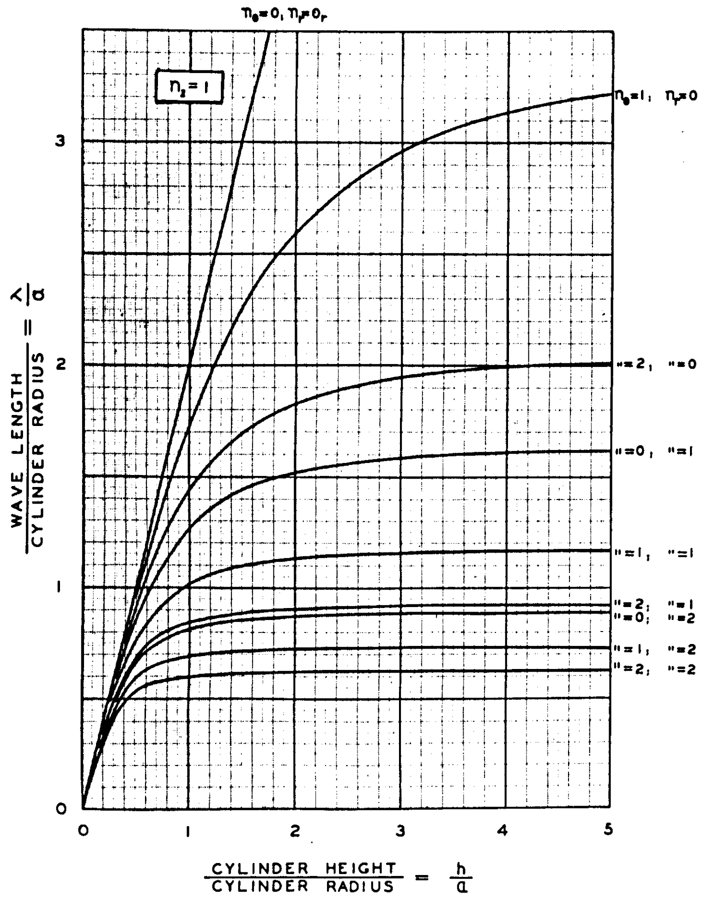


FIG. 74

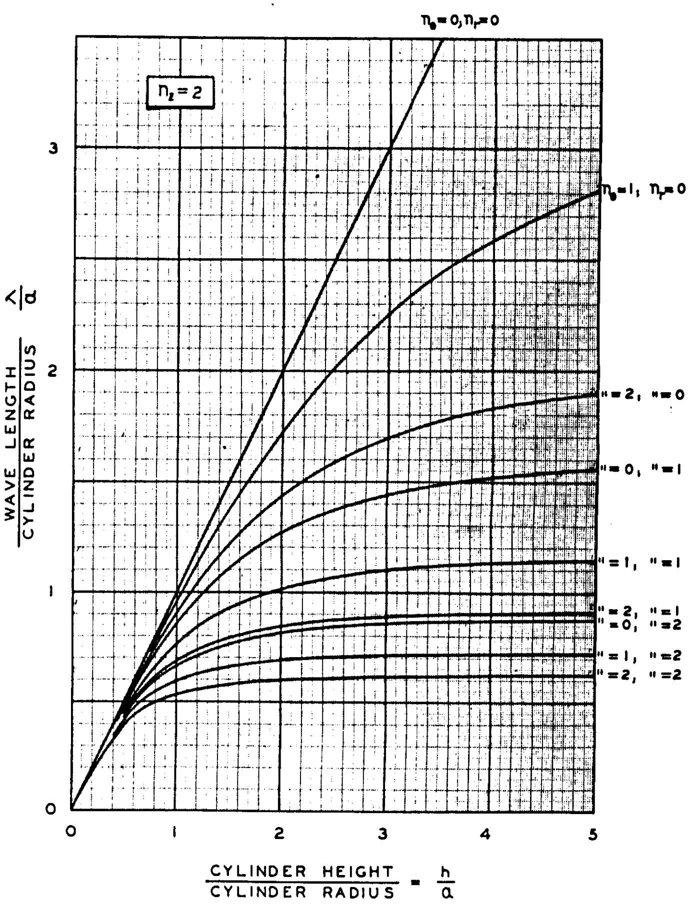


FIG. 75

SOUND-WAVE-LENGTH OF AXIAL WAVES IN A CYLINDRICAL CHAMBER and cylinder radius for modes with one and two axial nodes respectively. With these figures available and the velocity of sound known, it is a simple matter to calculate the frequency corresponding to any given mode of vibration by means of equation (40).

Figures 73, 74 and 75 give some hint of the difficulties encountered in attempting to identify the particular modes of vibration excited in a given case. Frequencies for the simplest modes are well separated but rapidly become closer together as the nodal systems grow more complicated. This means that vibrations in two dimensions with one or two azimuthal nodes can be easily picked out, but when one or two axial nodes are combined with one or two azimuthal nodes, the problem of identification is extremely troublesome. The fifth column of Table II lists frequencies for various modes of vibration in the CFR engine with a sound velocity of 3000 feet per second. The small differences between frequencies for the more complicated modes is very apparent.

Energy Associated with Pressure Waves in a Cylindrical Chamber

Equation (50-a) of Appendix A is a general expression for the total energy associated with pressure waves in an enclosed volume. In Appendix C this energy equation is specialized to fit the case of each particular mode of vibration of a circular cylinder with flat ends. The result of this analysis in terms of average energy density is given in equation (41)

$$M_{n_\theta n_r n_2} = E_{n_\theta n_r n_z} / (p_{n_\theta n_r n_z} / P_0) \qquad (41)$$

where $E_{n_\theta n_r n_z}$ is the average energy density associated with the mode $n_\theta n_r n_z$.

Values of the ratio $M_{n_\theta n_r n_z}$ for the simplest modes of vibrations are given in Table II. In practical use, if the mode excited can be identified by the frequency, and the pressures measured with proper indicators, the energy density associated with the particular mode can be calculated by means of equation (41). It is to be noted that the ratio M tends to decrease as the mode of vibration becomes more complicated.

Pressure Waves Produced by an Arbitrary Initial Disturbance

The modes of vibration excited by a given knock will depend upon the shape, location and magnitude of the initial pressure disturbance. Actually this initial rise in pressure will take place at a finite rate which can be determined only by experiment. Even if the rate of rise of pressure were known, it would be difficult to account for this effect in the analysis. For these reasons,

TABLE 2

BASED ON SOUND VELOCITY OF 3000 FT./SEC. IN C.F.R. ENGINE

AZIMUTHAL NODES n_θ	RADIAL NODES n_r	AXIAL NODES n_z	$M_{n_\theta n_r n_z}$	CYCLES SEC $n_{n_\theta n_r n_z}$
0	0	0	1.0000	0
0	0	1	.5000	15430
0	0	2	.5000	30900
0	1	0	.1622	13500
0	1	1	.0811	20570
0	1	2	.0811	33700
0	2	0	.0900	24800
0	2	1	.0450	29210
0	2	2	.0450	39700
1	0	0	.1220	6500
1	0	1	.0610	16780
1	0	2	.0610	31500
1	1	0	.0583	18800
1	1	1	.0292	24200
1	1	2	.0292	36200
1	2	0	.0368	30200
1	2	1	.0184	34000
1	2	2	.0184	46200
2	0	0	.0666	10800
2	0	1	.0333	18800
2	0	2	.0333	32700
2	1	0	.0433	23900
2	1	1	.0217	28300
2	1	2	.0217	39000
2	2	0	.0315	35400
2	2	1	.0158	38600
2	2	2	.0158	47000
3	0	0	.0457	14830
3	1	0	.0360	28300
4	0	0	.0353	18950
4	1	0	.0307	32900
5	0	0	.0272	22600
5	1	0	.0267	37100
6	0	0	.0230	26600
6	1	0	.0232	41550
7	0	0	.0186	30200
7	1	0	.0205	45800
8	0	0	.0161	34000
8	1	0	.0185	50000
9	0	0	.0142	38000
9	1	0	.0169	53500

a simplified form was chosen for the initial excitation in the hope that the analytical results would agree closely with experimental observations to justify the general method. For computation, it was assumed that the initial disturbance could be approximated by assigning a constant pressure p_0, to a region between r_1 and a, 0 and z_1, $-\theta_1$ and θ_1. Appendix D outlines details of the process in which both sides of equation (39) are multiplied by the trancendental functions on the right hand side and integration over the entire cylinder volume is carried out for t = 0. This procedure gives an expression for the pressure amplitude of each mode of vibration in terms of the shape of the initial disturbance and the magnitude of the local pressure.

$$p_{n_\theta n_r n_z} = \frac{p_0}{\cos \epsilon_{n_\theta n_r n_z} M_{n_\theta n_r n_z} V} \int_{r_1/a}^{1} \int_{-\theta_1}^{\theta_1} \int_{0}^{z_1/h} J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)] \cos(n_\theta \theta - \varphi_{n_\theta n_r}) \cos(n_z \pi z/h) r dr d\theta dz \qquad (42)$$

Where V = total volume of the charge.

In finding amplitudes of particular modes, the time phase angle can be taken as zero so that $\cos \epsilon$ becomes unity. The ratios M are known from the energy calculations so it is only necessary to evaluate the triple integral for various shapes, sizes and locations of the initial disturbance in order to study the effect of these variables on the relative magnitudes of the pressure amplitude factors. Where magnitudes alone are considered the phase angles in azimuth are immaterial and may be taken as zero in each case. The general integral of equation (42) as found in Appendix D is given below

$$p_{n_\theta n_r n_z}/p_0 = 2/\ M_{n_\theta n_r n_z}(A \cdot R \cdot Z) \qquad (43)$$

Where

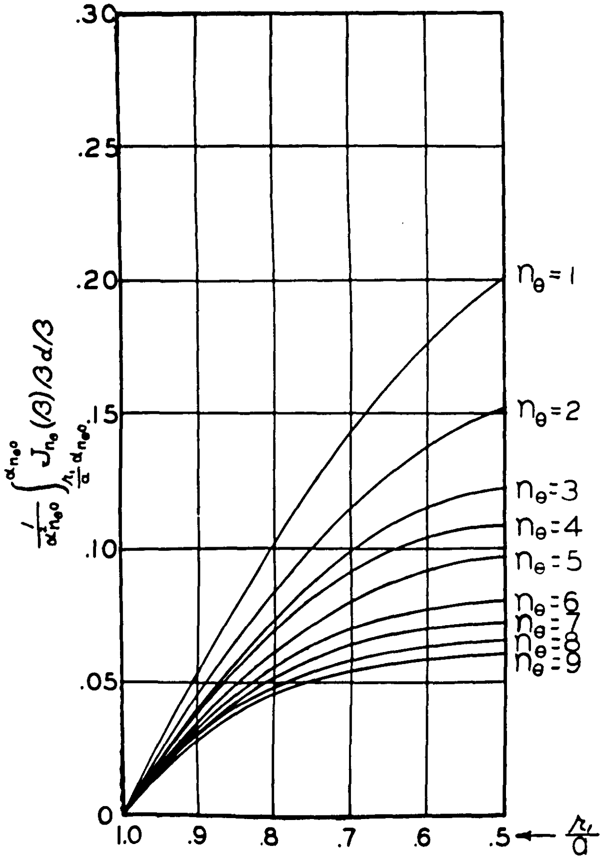
$$A = \frac{\sin n_\theta \theta}{n_\theta} \qquad (44)$$

$$R = 1/\alpha^2_{n_\theta n_r} \int_{(r_1/a)\alpha_{n_\theta n_r}}^{\alpha_{n_\theta n_r}} J_{n_\theta}(\beta)\,\beta\, d\beta \qquad (45)$$

$$Z = \frac{\sin(n_z \quad {}_1/h}{n_z} \qquad (46)$$

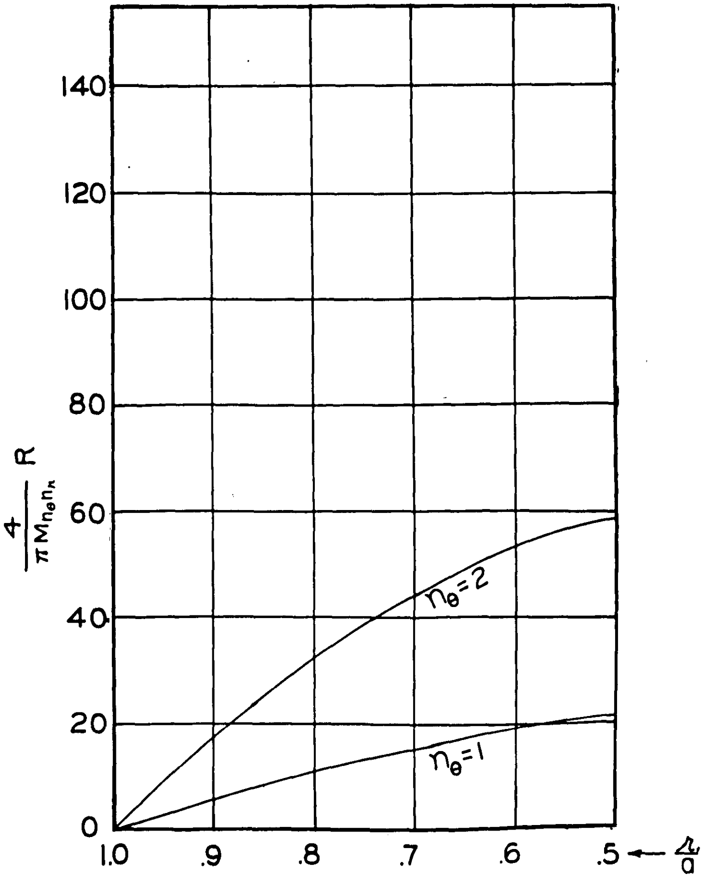
Appendix D also gives the special forms of equation (43) which arise when nodes are absent from one or more of the coordinates. The curves of figure 76 show the variation of R with the radial extent r_1/a of the exciting pressure rise if no radial nodes exist. This case has considerable practical importance since the detonation zone is usually near one side of the cylinder and this location will be favorable toward unsymmetrical wave forms with one or more azimuthal nodes.

As an example of some practical importance, wave systems with no axial or radial nodes and one or two



VARIATION OF RADIAL INTEGRAL WITH RADIAL EXTENT

FIG. 76

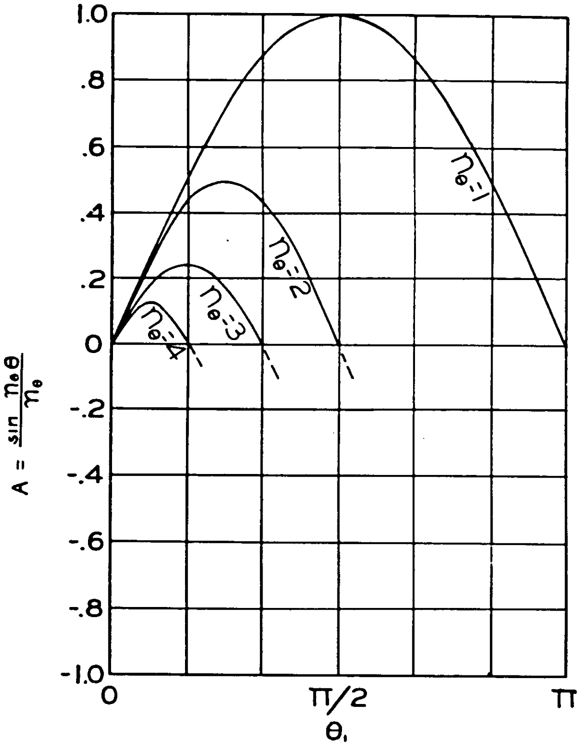


EFFECT OF RADIAL FACTOR ON PRESSURE AMPLITUDE RATIO

FIG. 77

azimuthal nodes will be considered. In addition it will be assumed that the axial extent of the excitation includes the whole cylinder length. Figure 77 shows the factor R multiplied by the coefficient depending upon M plotted as a function of the radial extent of the initial disturbance. Both terms increase almost linearly with the radial extent, the higher values being associated with two azimuthal nodes. From these curves, it appears that increasing thickness of the disturbance would produce higher pressure amplitudes in both cases considered.

Figure 78 is a plot of the azimuthal term , A, in equation (43). It is apparent that increasing azimuthal extent at first brings greater amplitude in all unsymmetrical modes of vibration. This increase continues for any particular system of azimuthal nodes until a maximum is reached at $\theta_1 = \pi 2 n_\theta$. Past this maximum, the amplitude ratio in each case decreases and becomes zero when $\theta_1 = \pi n_\theta$. Beyond this point the pressure amplitude ratio becomes negative. This corresponds to a difference in algebraic sign between the pressure wave and the initial excitation. This discussion leads to the general conclusion that a pressure rise of small radial extent will tend to excite complicated modes of vibration. As the azimuthal extent of the excitation is increased, simpler modes will



Effect of Azimuthal Extent on Pressure Amplitude Ratio

FIG. 78

become more prominent than the complicated modes. To illustrate this point, if θ_1 is about $\pi/12$, the modes with two or more azimuthal nodes have about the same factor A, as the mode with one azimuthal node. Referring to figure 77 for the radial factor shows that the two noded vibration will be present more strongly than the vibration with a single node. On the other hand, if θ_1 is near $\pi/2$ (corresponding to a total angular extent of about 180 degrees) the mode of vibration with one azimuthal node will appear very strongly while the mode with two azimuthal nodes will be small or absent entirely. It will be seen later that the conditions suggested above have actually been found by experiment.

Recalling that the frequencies of various modes of vibration are and can be identified with the aid of wave length data and the velocity of sound, it is apparent that the number of azimuthal nodes can be determined from experimental data in a given case. If the number of azimuthal nodes is known, a rough estimate of the angular extent of the initial disturbance can be made by the method outlined above. For more complicated wave systems this procedure must be modified to fit each particular case.

Summary

The preceding discussion has shown that the theory of sound can be applied to an approximate analysis of the pressure disturbances which accompany detonation in the internal combustion engine. These disturbances consist of a rapid pressure rise in some localized part of the burning gases which is followed by resonant frequency pressure waves within the combustion chamber. The method of attack depends upon a theoretical analysis of the wave system which comes into existence after the initial pressure rise has subsided. By comparing experimental pressure records with the pressure wave forms predicted by theory for a series of arbitrarily assumed excitations, an approximate description of the events accompanying detonation in a given case can be developed.

High pressures in the initial disturbance and the resonant wave system, damping effects, and the finite rate of rise of pressure in the localized excitation region, all tend to make the results uncertain even if a complete analysis is possible. In spite of these difficulties, it is reasonable to expect that the methods developed here will be useful in increasing the general knowledge of detonation as an engine phenomenon.

SECTION V

APPARATUS FOR RECORDING HIGH FREQUENCY PRESSURE DISTURBANCES IN ENGINE CYLINDERS

APPARATUS FOR RECORDING HIGH FREQUENCY PRESSURE DISTURBANCES IN ENGINE CYLINDERS

General Requirements

Equipment for quantitative studies of the pressure disturbances accompanying detonation must fulfill two major requirements. It must give an accurate representation of the pressure changes over a frequency range extending from about five cycles per second to a high limit above 20,000 cycles per second, and it must be able to withstand considerable periods of continuous operation under severe engine conditions. For convenience in interpreting complex records, the overall sensitivity of the apparatus should be substantially constant over the range of operation. To facilitate installation, the unit actually exposed to the cylinder pressures should be small and adaptable to openings of spark plug size in the cylinder head.

Mechanical reasons definitely eliminate conventional engine indicators using mechanical or optical recording systems. Such instruments are too large to install in the restricted space available on modern engines and in addition are seriously affected by engine vibration. Averaging indicators overcome these objections but can not supply any data on high frequency pressure waves in the cylinder. These considerations lead to the choice of an electrical indicator with oscillographic recording. On the basis of an extended experience with various types of electrical indicators, the writer decided that the electro-magnetic generator offered the best means for converting mechanical motion of a diaphragm into electrical changes. The writer realizes that many investigators would differ with this opinion. However, further space devoted to discussion of a point which has been in controversy for many years will not settle the matter. An instrument must be judged on the basis of results, and a device which gives satisfactory performance without excessive care will be as good as any other means for the purpose at hand. Electro-magnetic generators are simple, rugged and produce a relatively high output with low internal impedance. Such generators give an output voltage which is proportional to the rate of change of pressure and this is a definite advantage for studies of high frequency pressure waves of relatively low amplitude. Another important factor is that thermal deflection effects in the diaphragm are minimized since a rate of change instrument does not require an accurate adjustment of the zero position. Other considerations enter into the general problem but the writer is content to have his wisdom in the matter judged on the basis of final results.

Cathode ray oscillographs have all the characteristics required for recording the pressure variations accompanying detonation. The electron beam does not introduce any difficulties at either high or low frequencies for detonation work. However, a high frequency limit for recording is established by the combination of spot intensity, photographic emulsion speed and lens aperture. With modern motion picture film and a standard cathode ray tube the recording problem can be satisfactorily solved by adjustment of a node potential in the tube. The oscillograph system used in the present investigation was essentially a conventional installation.

Amplifiers are required to raise the pickup unit output voltage to the level necessary for operation of the oscillograph. In practice, the performance of this coupling system establishes both the high and low frequency limits for satisfactory operation of the complete indicator. The criterion to be applied is that the record must be a faithful representation of the rate of change of pressure in the cylinder. It has been found from experience that after the very brief excitation period the amplitude changes between successive cycles of the pressure waves are so small that the problem can be treated by steady state methods. The amplifier system actually used will be described later.

Selection of Generator Type for Rate of Change of Pressure Indicator

With the decision to use a rate of change of pressure (dP/dt) indicator definitely made, it was necessary to choose between several possible schemes in designing the electro-magnetic generator system. Under the restrictions of size and form imposed by the engine, three generator types appeared to be feasible:

(1) A cylindrical coil moving perpendicular to a radial magnetic field. Such an instrument has already been described in Section I.

(2) A cylindrical shell of magnetic material in a radial magnetic field and arranged to shift flux from one side to the other of a stationary coil when moved in the axial direction. Generators with this construction will be referred to as of the <u>shell inductor type</u>.

(3) A flat diaphragm of magnetic material so arranged that its motion will vary the reluctance of a magnetic circuit in which the flux links a stationary coil. Generators of this type are essentially similar in construction to the conventional telephone receiver and are designated as of the flat diaphragm type.

Diagrams showing the working parts of each generator type are given in figures A-3, A-4 and A-5 of Appendix E.

Each of the generator types mentioned can be reduced to practice, so it is necessary to make the best choice for the problem at hand. Such a selection can reasonably be based on three criteria:

(1) Simplicity and ruggedness of construction.

(2) Voltage output for a given diaphragm velocity.

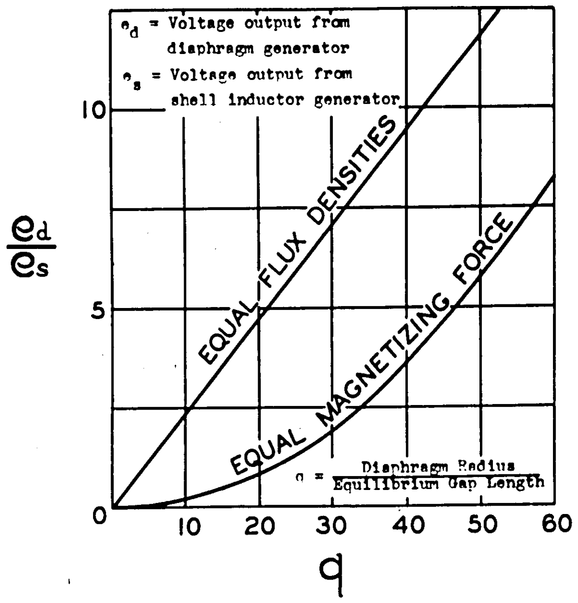
(3) Susceptibility to mechanical vibration.

Flat diaphragm generators do not require mechanical attachments to the diaphragm and for this reason are definitely superior to the other two types from the standpoint of simplicity.

Appendix E gives a detailed examination of the voltage output available with reasonable designs of the three types. On the basis of equal magnetizing forces and equal diaphragm velocities, the shell inductor generator was found to produce an output about 35 times as great as that from a moving coil generator. On the basis of equal flux densities at the sruface of the working gap, the voltage output from the shell generator will be about twice the voltage output from the moving coil generator.

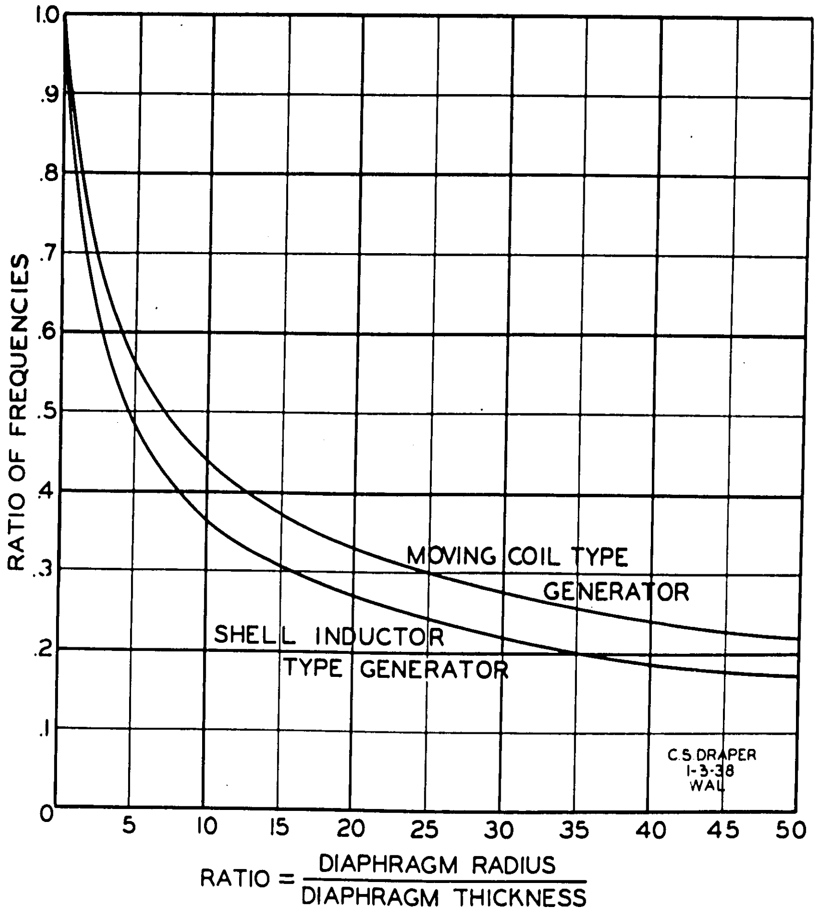
Figure 79 gives curve which compare the output voltages from the shell inductor generator and the flat diaphragm generator. In this case, the voltage ratio depends upon the gap length used for the flat diaphragm generator. The ratio of this gap length to diaphragm radius is taken as the independent variable in figure 79, with the ratio of flat diaphragm generator voltage to shell inductor generator voltage as ordinates. For all gap lengths less than about 1/20 of the diaphragm radius, the flat diaphragm unit is seen to be definitely superior. In practice the radius/gap ratio can easily be made near 100 so the flat diaphragm generator is better than the other types by a wide margin.

Indicator discussions usually note that mechanical vibration introduces extraneous effects on pressure records, but to the writer's knowledge the literature does not contain a systematic treatment of this subject.



Ratio between Generated Voltage from Diaphragm and Shell Inductor Generators for Equal Velocities

FIG.79



Effect of Attached Parts on Natural Frequency of Generating System - Comparison with Flat Diaphragm

FIG.80

There has been only a vague recognition of the fact that any system which is sensitive to pressure by virtue of the elastic deflection of some mechanical part will necessarily be sensitive to vibration. Another general fact is that a mechanical system of complicated shape is liable to excitation at various natural frequencies by motion in any direction. This action is due to coupling effects within the system. From this it follows as a logical consequence, that a simple mechanical shape in which a single natural frequency can be controlled by design is more desirable than any complicated structure. This again is a very strong argument for the flat diaphragm generator as compared with the other two types.

Many laboratory experiments by the writer and his collaborators have shown that moving parts of high speed indicators are substantially undamped in operation. Several attempts were made to reduce natural frequency effects by the introduction of damping forces. These attempts failed almost completely because it was not feasible to obtain high damping forces with the very small displacements involved. The remaining possibility is to use an indicator natural frequency very much higher than the forcing frequencies found in operation. The conventional theory of vibration(209) shows that an undamped system must have a natural frequency about four times the highest steady state exciting frequency to follow a forcing motion without appreciable errors. In the case of an indicator for studying detonation, this condition requires a natural frequency for the indicator of about 100,000 cycles per second. With this high frequency the effects of mechanically excited transients were never detected in practice. This was probably due to a greater decrement per cycle and the completion of a large number of cycles in a short time combined with reduced amplifier sensitivity in this range.

The last paragraph has demonstrated the necessity of a high natural frequency for the elastic element of an indicator for recording effects. Sensitivity of the indicator will increase as the elastic coefficient of the diaphragm decreases. It is therefore desirable to use the smallest elastic coefficient possible in a diaphragm strong enough to withstand the pressure of engine operation. Combining the natural frequency and strength requirements, it is obvious that the best arrangement will be that giving the highest possible natural frequency for a given elastic coefficient. In general, the natural frequency of any simple system will be proportional to the square root of the ratio of the elastic coefficient to the effective mass. For a diaphragm, the elastic coefficient will depend upon linear dimensions while the effective mass will be a function of the size and mass attached rigidly to the center of the diaphragm. Obviously, the natural frequency for a plain diaphragm will be higher than that for the same diaphragm with a coil or inductor attached. A quantitative study of this phase of the indicator problem is given in Appendix F and the results are summarized in the curves of figure 80. In any practical unit, the diaphragm radius will be at least seven times the thickness, so a reduction of natural frequency to approximately one half that for a flat diaphragm can be expected with either the shell inductor or the moving coil generator.

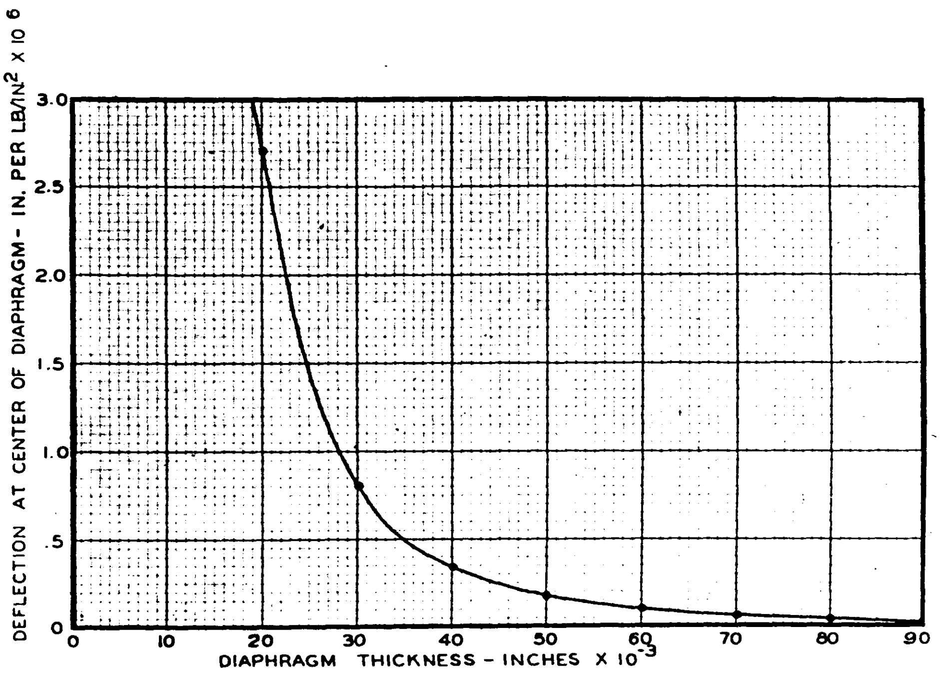
For the reasons detailed above, there is no doubt that a generator of the flat diaphragm type is very much superior to either of the other types considered. In accord with this conclusion, the indicator development was concentrated on the flat diaphragm type of unit.

Diaphragm Theory

Appendix G contains a detailed calculation of the elastic coefficient for a flat circular diaphragm of uniform thickness. Figure 81 is a plot of the elastic coefficient for steel diaphragms of one half inch diameter expressed as a function of thickness. The half inch diameter is chosen since this is the largest practical size for indicators to fit into holes for conventional 18 millimeter spark plugs.

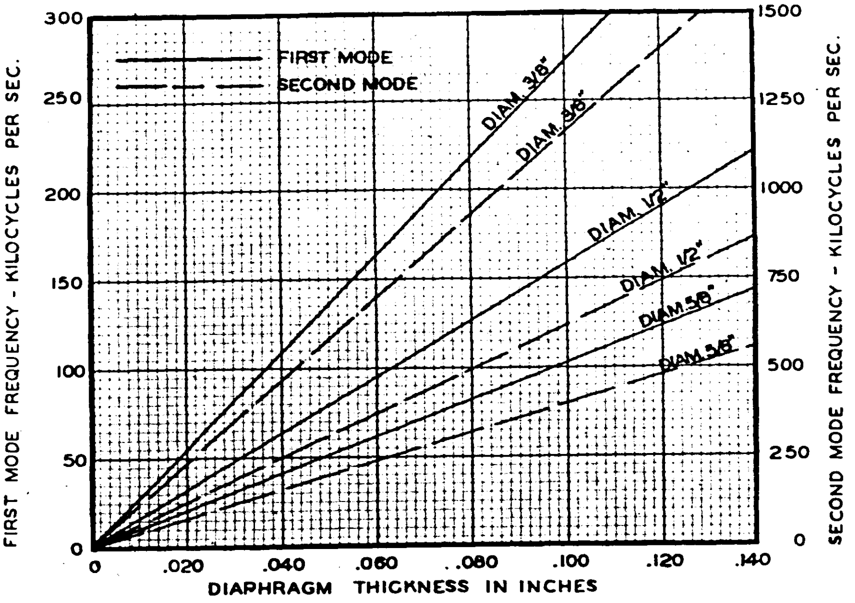
Natural frequencies for a circular diaphragm clamped at the edges are calculated in Appendix H. Figure 82 shows plots of natural frequency as a function of thickness for steel diaphragms of different diameters with either the first or second modes excited. A diaphragm one half inch in diameter and 0.060 inch in thickness will have a natural frequency of vibration about 95,000 cycles per second.

Appendix I considers the relative motion between the center and edge of a circular diaphragm with clamped edges and the boundaries subjected to a simple harmonic vibration perpendicular to the diaphragm surface. One derivation was based on the complete diaphragm theory, while another treated the diaphragm as an equivalent system of a mass connected to the vibrating edges by means of a massless spring. Figure 83 shows the results of these derivations plotted with the ratio of relative motion amplitude to forcing motion amplitude as ordinates



ELASTIC CONSTANT
OF
CIRCULAR STEEL DIAPHRAGMS

FIG. 81



NATURAL FREQUENCY vs. THICKNESS
FOR CIRCULAR STEEL DIAPHRAGMS

FIG. 82

A.B.S.-OCT.'37

and the ratio of forcing frequency to natural frequency as abscissae. It will be noted that each point on the exact theory curve is higher than the corresponding point of the equivalent system curve. The ratio of these ordinates is constant at 1.63 which is just equal to a factor $64/(g_n a)^4$ which appears from elastic theory considerations. The writer is uncertain as to whether or not the factor mentioned is present because of an error, but did not review the derivations for the necessary check.

Motion of a diaphragm subjected to a pressure which varies sinusoidally with time, is discussed in Appendix J. The results are summarized in figure 84 with the ratio of relative motion between the center and edge of the diaphragm to the relative motion at zero frequency plotted as ordinates and forcing frequency ratios as abscissae. In this case, there is very good agreement between results from the exact theory and the equivalent system theory. The plotted curve verifies the statement previously made that pressure frequencies will be satisfactorily followed by the deflections.

Diaphragms are sensitive to both pressure and vibration. It follows that there is a ratio between the pressure amplitude to produce the same deflection amplitude as a given forcing vibration, all the quantities being sinusoidal at a given frequency. This problem is treated in Appendix K with the result that for a diaphragm of 0.5 in. diameter and 0.060 in. thickness

$$P_{vm}/\zeta_{am} = 1.64 \times 10^7 \cdot \beta^2 \cdot \varepsilon^{-i\delta} \qquad (47)$$

Where

ζ_{am} = amplitude of forcing vibration applied to edge of diaphragm in inches.

P_{vm} = pressure amplitude producing the same effect as the vibration amplitude in pounds per square inch.

δ = phase angle between the pressure and the vibration.

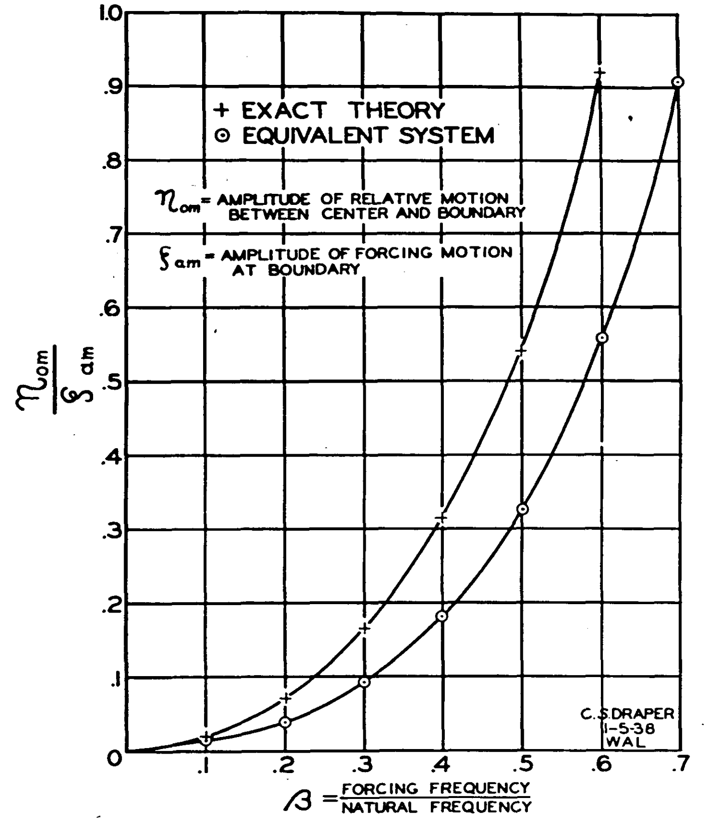
$$\beta = \frac{\text{Forcing frequency}}{\text{Natural frequency of diaphragm}}$$

This equation contains implicitly the ratio between exact theory and equivalent theory results as noted in connection with figure 83. If this former result is found to be in error the proper correction should be applied to equation (47).

The Effect of Temperature on a Flat Diaphragm Generator

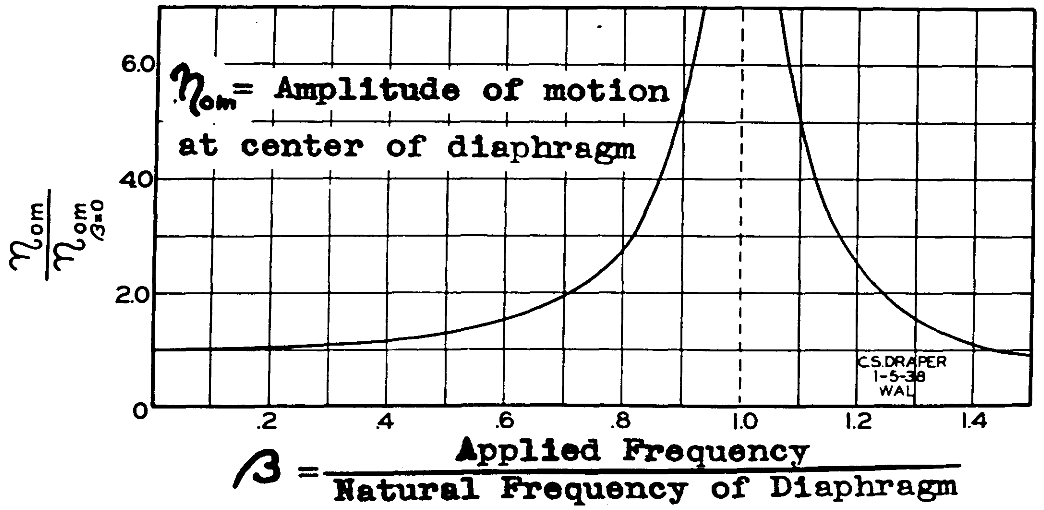
Mechanical aspects of the flat diaphragm generator problem have been considered without regard to the effects of temperature. These effects may take several forms:

(1) actual damage to the insulation may occur through deterioration.



Motion of Clamped Diaphragm Due to Simple Harmonic Motion of Boundary.

FIG.83



Effect of Frequency on Diaphragm Deflection under Uniform Pressure Varying Sinusoidally

FIG.84

(2) resistance of the generating winding will increase.

(3) the elastic coefficient of the diaphragm will change due to the effect of temperature on Young's Modulus.

(4) the magnetic permeability of the diaphragm will change.

(5) diaphragm deflection will occur due to the uneven expansion between inner and outer surfaces under themal gradients.

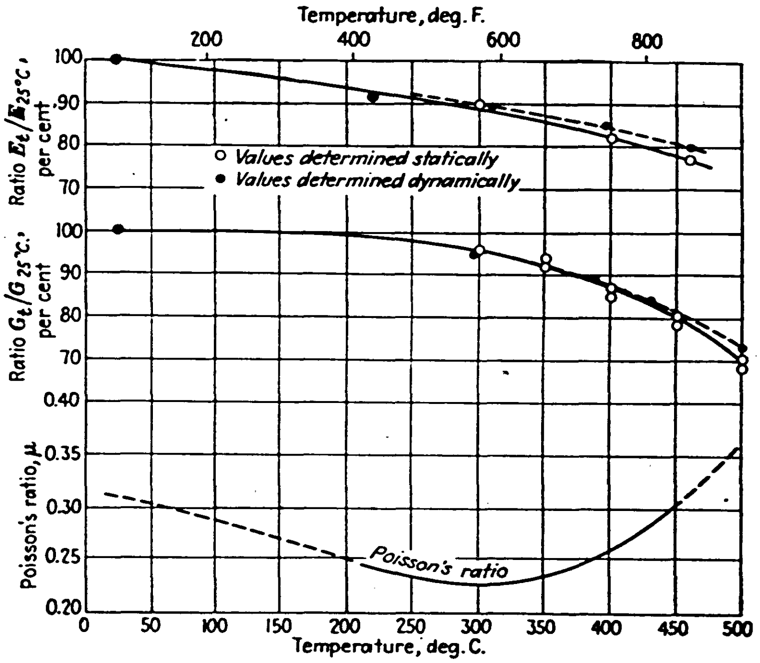
Experience over a period of years has shown that indicator operating temperatures are near the practical upper limit for enamel insulation. Failures due directly to thermal damage of insulation have occurred in a few cases only.

As a matter of necessity in obtaining proper overall indicator characteristics, pickup units are always connected in series with a resistance many times greater than the winding impedance. In such a circuit, any resistance changes due to thermal effects in the winding will have a negligible effect on operation of the system.

The elastic properties of a diaphragm depend upon Poisson's Ratio, σ, and Young's Modulus, E, by a relationship derived in Appendix G. Both of these fundamental properties depend upon temperature. Figure 85 (from reference 210) shows the approximate magnitudes of these variations for a steel similar to that used in constructing the indicators.

Many examinations of the indicators after extended periods of operation in an engine showed that a ''temper color'' usually appeared on the inside diaphragm surface. This ''temper color'' was a dark blue under the most severe conditions. This color corresponds to about 600 degrees Farenheit as noted in the ''Machinist's Handbook.'' Reference to figure 85 shows that the variation in E between room temperature and 600 degrees Farenheit is about 10 percent. This would correspond to 10 percent variation in elastic coefficient and 5 percent variation in natural frequency. The variation in Poisson's Ratio is considerably larger but has a somewhat smaller effect on the elastic behaviour due to the form of the equation involved.

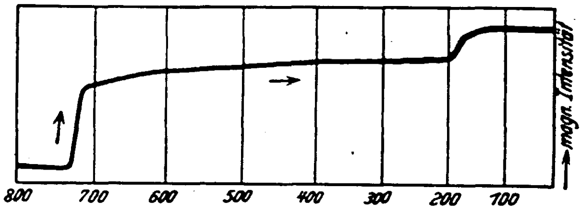
The diaphragm necessarily carries the working flux in a flat diaphragm generator so that any change in diaphragm permeability will be reflected as a change in reluctance of the magnetic circuit. The curve of figure 86 (taken from reference 211) shows that over the temperature range involved, the permeability of a steel diaphragm will decrease but not by a large percentage. In a practical design, the diaphragm reluctance will be a small



Variation of modulus of elasticity E, modulus of rigidity G, and Poisson's ratio μ with temperature, for a medium-carbon steel. (*Versé.*[319])

REF. 207

FIG. 85



Magnetisierungs-Temperaturkurve für Stahl mit 1,2% C (Esser).

REF. 208

FIG. 86

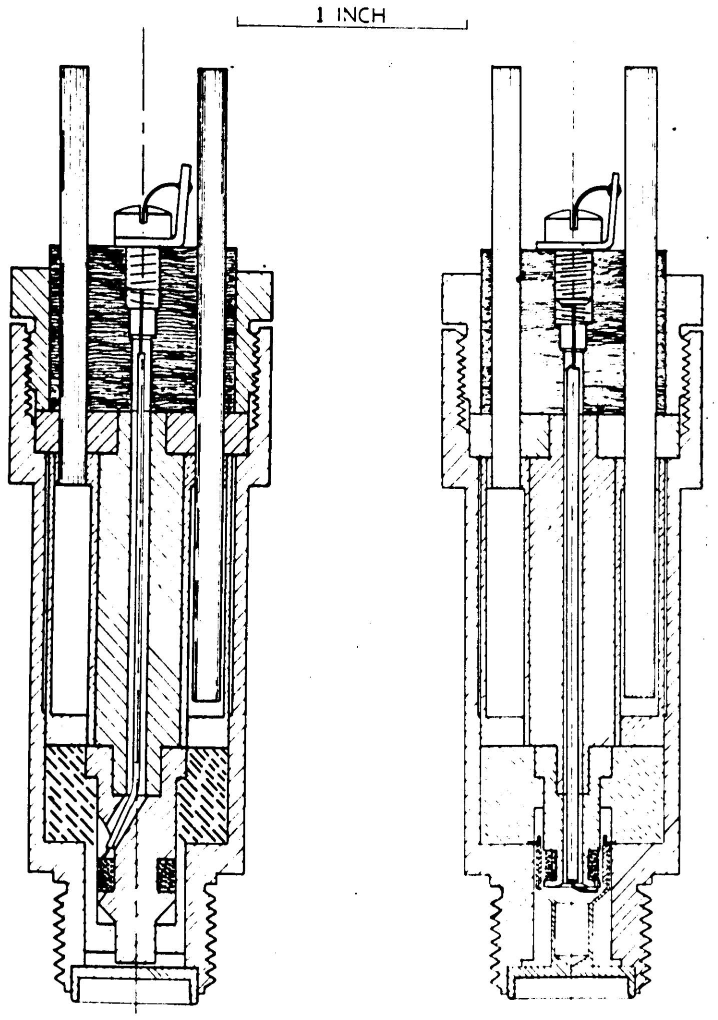
percentage of air gap reluctance, so that the net effect of temperature on total flux in the circuit can probably be neglected.

Temperature gradients in the diaphragm will necessarily exist under operating conditions. These gradients produce a measurable but somewhat indefinite effect on the air gap length. With the overall sensitivity dependent upon the equilibrium gap length, it follows that thermal effects which distort the diaphragm will affect the pickup unit performance. This effect can be reduced by using a longer equilibrium gap length but this is undesirable since the pickup sensitivity is also reduced. No quantitative estimate of the effect of thermal deflections on pickup operation can be made from the data now available.

The discussion above has shown that thermal effects on indicator sensitivity are too large to be neglected and difficult to estimate from theoretical considerations. This leaves calibration under actual operating conditions in the engine as the only reliable method for accurately determining the overall sensitivity of an electro-magnetic indicator.

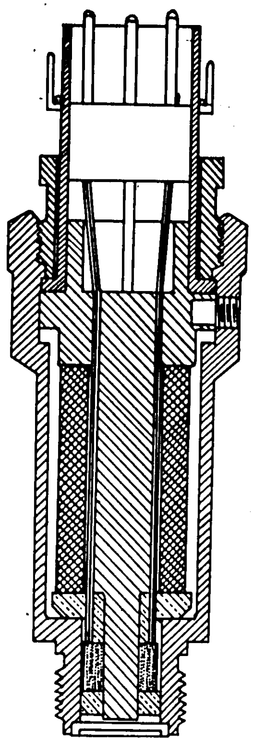
Complete Pickup Unit

Figure 87 shows a cross-sectional diagram of a



Rate of Change of Pressure Indicators
with Permanent Magnet Excitation

FIG.87



Rate of Change of Pressure Indicator
with Electro-Magnetic Excitation

FIG.88

moving shell inductor pickup unit on the right. The shell inductor is machined as an integral part of the diaphragm and overlaps the stationary coil a small distance on either side. One side of the coil is grounded and the other connection runs through a hole to the external terminal. A central core of magnetized cobalt steel supplies the necessary magnetic excitation. This core is compressed against the lower shoulder of the indicator shell by a threaded bushing at the top. A water jacket is placed around the permanent magnet to cool the unit. It was found by experiment that this cooling jacket was unnecessary. The magnet retained its strength satisfactorily after an initial "shake-down period" and the insulated coil was not damaged by the operating temperatures without the water jacket.

The left hand diagram of figure 87 shows the essential parts of a pickup unit with a flat diaphragm generator. The simplicity of a flat diaphragm and straight cylindrical pole piece is evident.

Figure 88 is a diagram of the flat diaphragm type unit showing the proportions adopted after tests on a number of experimental designs. This particular unit was excited by a coil carrying direct current for the purpose of studying the effect of variations in magnetizing force on sensitivity. The final design was similar to figure 88 with a cobalt steel permanent magnet substituted for the electro-magnet.

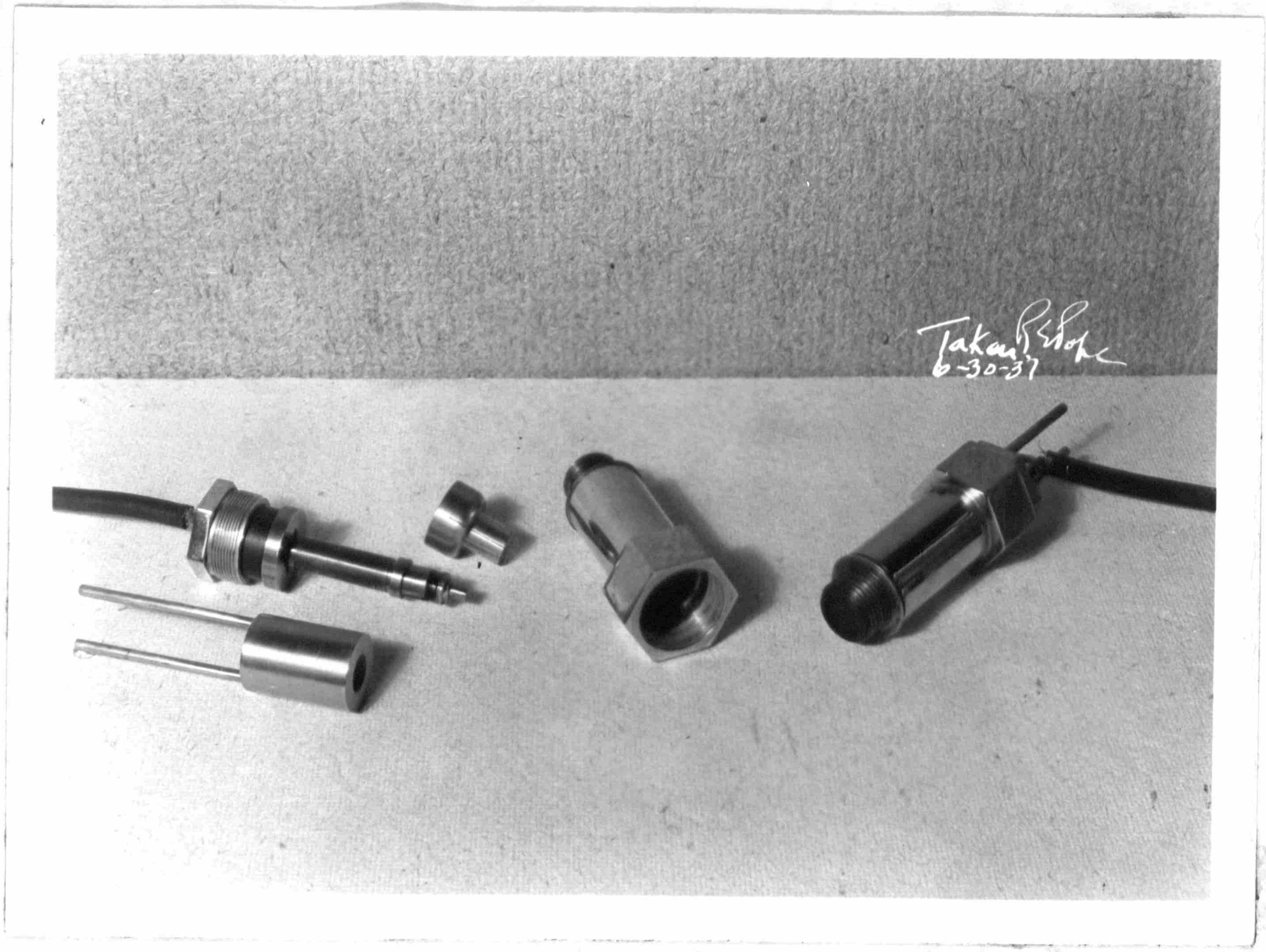
Plate XIX is a photograph showing the actual appearance of the left hand unit of figure 87.

The records which will be discussed later were made with units with construction details similar to those of the diagram of figure 88. These units produced an output voltage of about 10^{-7} volts per pound per square inch per second. The maximum rate of change of pressure encountered in engine tests under average conditions is about 10^{5} pounds per square inch per second. It follows that the output voltage of the unit available for driving an amplifier is about 0.01 volts to produce full deflection of the cathode ray oscillograph spot.

CathodeRay Oscillograph and Recording System

Three component units are required for a complete cathode ray recording system: a cathode ray tube, a power supply and sweep circuit and a camera designed for continuous film motion. The first of these units was avilable as a commercial article while the other two were specially adapted to the work at hand.

Cathode ray tubes specifically designed for photographic recording with high spot speeds are commer-



Photograph of dP/dt Indicator.
Plate XIX.

cially available. After an extended trial of tubes with three inch screens, it was decided to use the RCA type 907 tube with a five inch screen. This decision was based on the smaller ratio of spot diameter to spot deflection and the greater spot intensity available with the larger tube. It was found that the spot intensity was too low for the highest spot speed encountered if the recommended anode potential of 2000 volts was used. In practice this anode potential was increased to 3000 volts with no apparent ill effects on the tube. With this accelerating potential, a voltage swing of 660 volts on the deflecting plates was necessary to produce the full spot motion of four inches.

Power for the cathode ray tube anodes was furnished by a unit similar to that recommended by the tube manufacturers(210). A special feature of the power supply was the use of a General Radio Variac on the primary of the high voltage transformer to control the anode potential. A meter to read this voltage was incorporated as a permanent part of the unit. The total range available was 0 to 6500 volts at the anodes. Transformers with special high voltage insulation and low inter-winding capacity were obtained from the Raytheon Manufacturing Company for the power supply.

The sweep circuit used for examination of the cathode ray trace before making photographic records, was identical with that given by the tube manufacturer(210).

Photographic records were taken on 35 millimeter motion picture film by means of a camera already available in the Internal Combustion Engine Laboratory at M.I.T. This camera had been used for making the flame photographs described by Bouchard and his collaborators(153). Details of this camera are given in the reference cited. A film speed of 400 inches per second was used throughout the experimental work. This speed permitted accurate measurements on frequencies below about 15,000 cycles per second and the identification of frequencies up to 50,000 cycles per second.

The two cathode ray tubes were mounted side by side with their axes vertical. The fluorescent screens were placed with the rounded edges of the glass in contact. The distance between the camera and the tube faces was adjusted so that a deflection of four inches of the spot produced a motion of the image equal to one half inch.

Satisfactory photographic density for observations from the negatives with about three quarters of the maximum swing was obtained with a lens aperture of

1.9 and 3000 volts anode potential. Agfa Ansco "Ultra Speed" motion picture film was found to be superior to any other emulsion tested for the photographic records.

Amplifiers

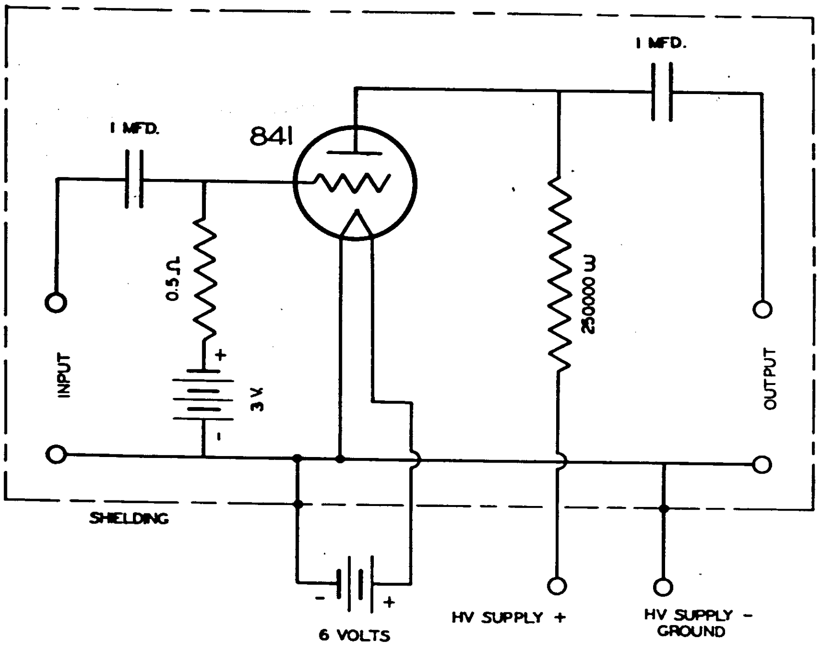
Based on the pickup unit output for average conditions, it was assumed that a maximum instantaneous potential of 0.01 volts would be available as input to the amplifying system. The cathode ray tubes required a swing of 660 volts for full spot deflections. From these data it was estimated that the amplifiers should produce an overall voltage gain of about 10^5. A safety factor of ten was introduced and the goal in the amplifier design was placed at a gain of 10^6.

To meet the requirement of accurate records over the entire range of frequency components encountered in engine work, the amplifiers were designed for a sensitivity variation of less than five percent from five cycles per second to 40,000 cycles per second. The allowable time delay for the complete system was placed at less than 10^{-5} seconds in any part of the frequency range.

Vacuum tubes capable of producing a high amplification are not well suited to a large swing in the output voltage. For this reason it was found necessary to divide the amplifiers into two sections, one to give a high voltage amplification with a normal output voltage swing and the other to produce the output voltage range required to cause full scale deflection of the cathode ray spots. Figure 89 shows the single triode stage used as the high voltage amplifier, the output terminals were connected to the cathode ray tube plates and the input terminals to the high gain amplifier. A special power supply was used for the high voltage section as shown in the diagram of figure 90. The direct current output potential was about 1100 volts.

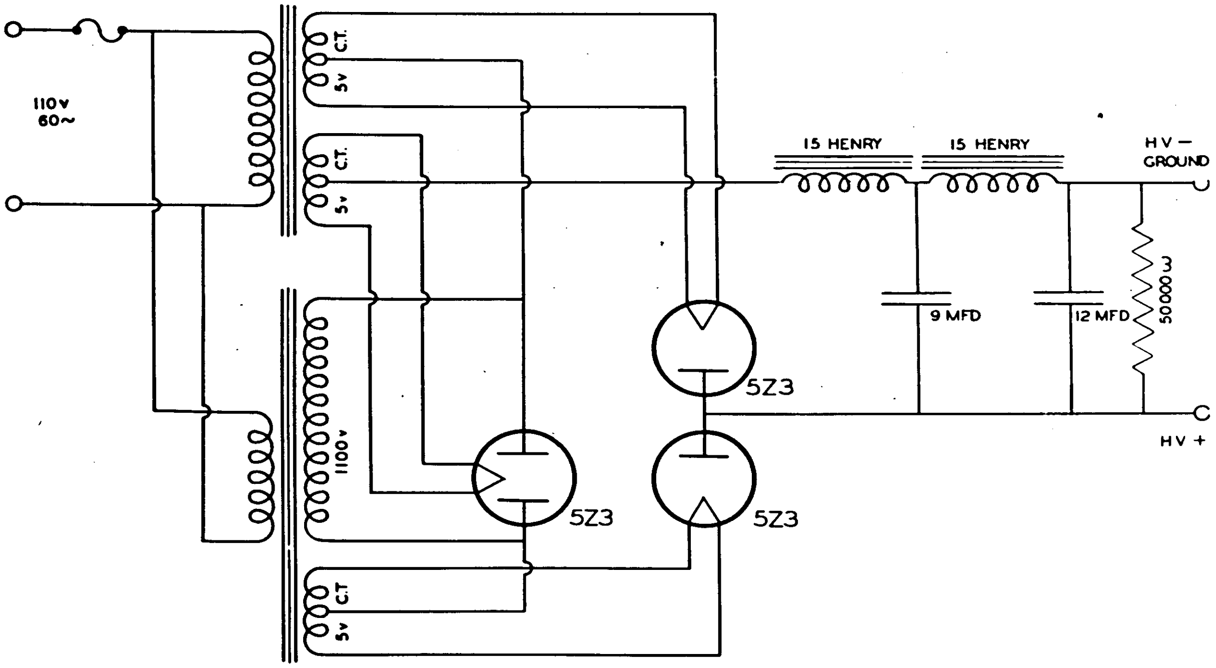
Plates XX and XXI are photographs of the high voltage amplifier and the high voltage power supply.

Figure 91 is a circuit diagram of the high gain amplifier. This unit was designed with resistance-capacity interstage coupling in accord with the conventional principles of amplifier design(211). Pentode connections were used for the first two stages because the characteristic low inter-electrode capacity was favorable to a wide frequency response. The third tube was connected as a triode since it was impossible to obtain a sufficiently great undistorted voltage swing from the pentode connection. "Decoupling" circuits between stages were required to permit operation of the complete amplifier from a



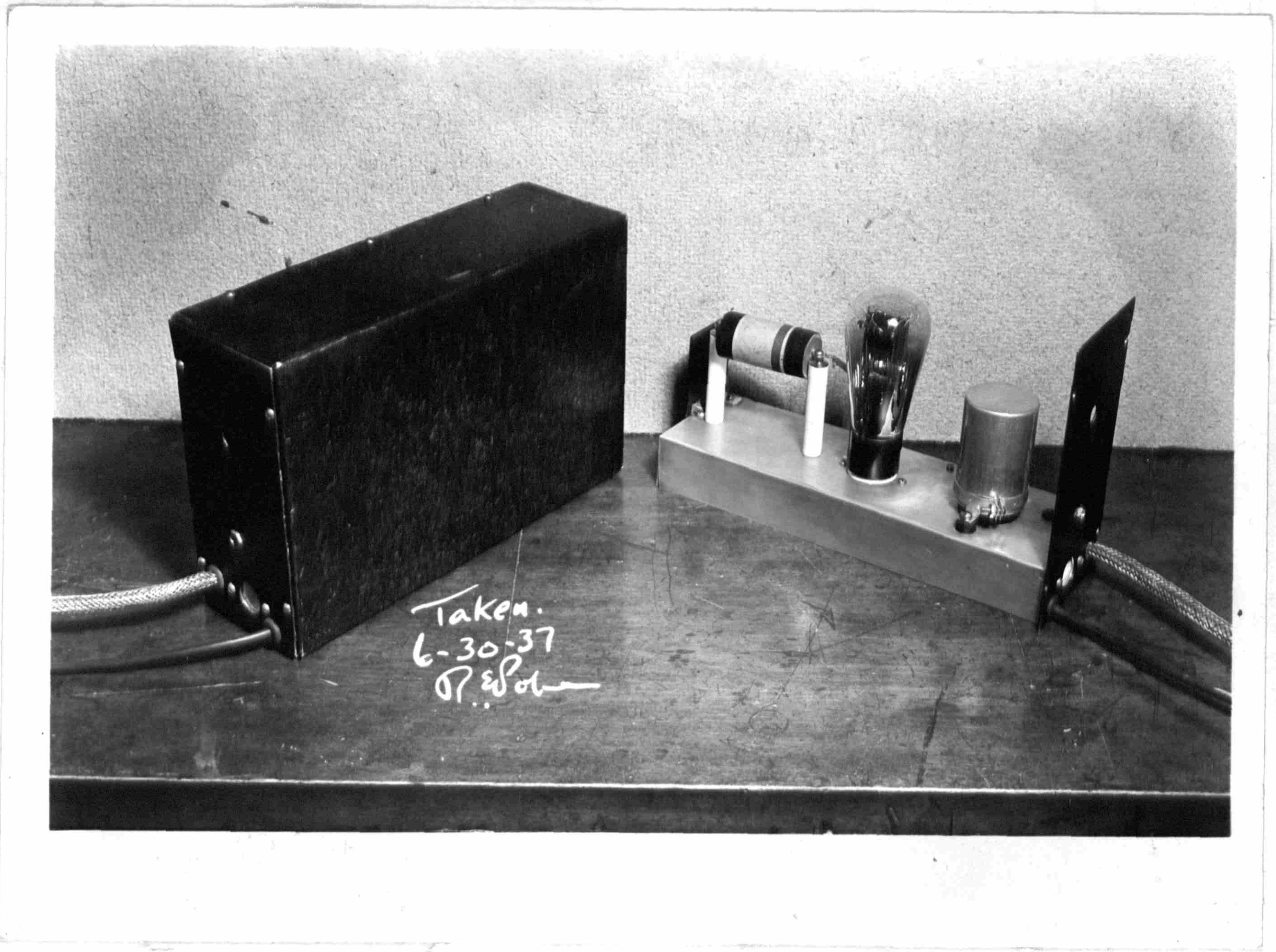
CIRCUIT DIAGRAM OF HIGH VOLTAGE AMPLIFIER

FIG.89



CIRCUIT DIAGRAM OF HIGH VOLTAGE POWER SUPPLY

FIG.90



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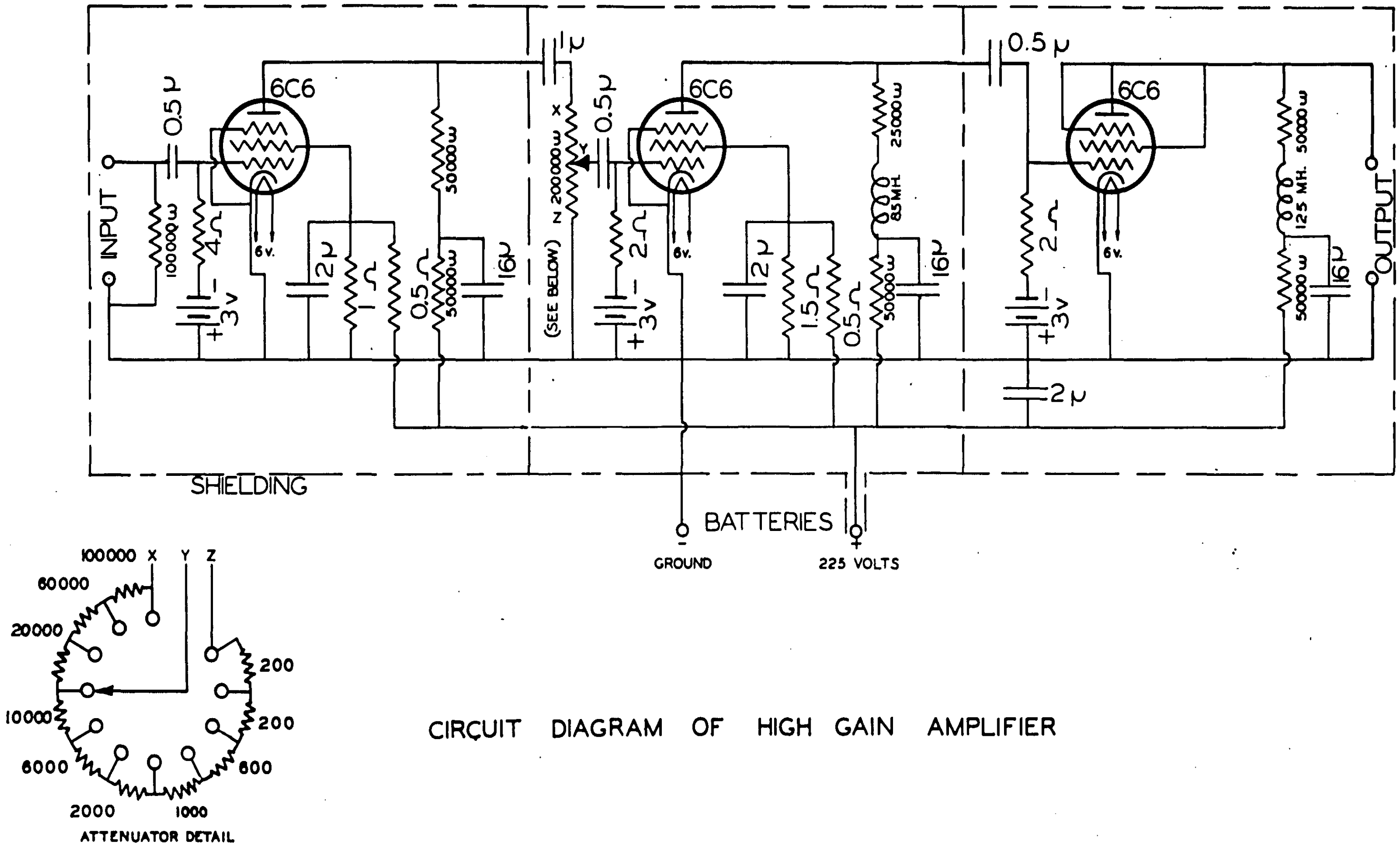
Photograph of High Voltage Amplifier.
Plate XX.



CIRCUIT DIAGRAM OF HIGH GAIN AMPLIFIER

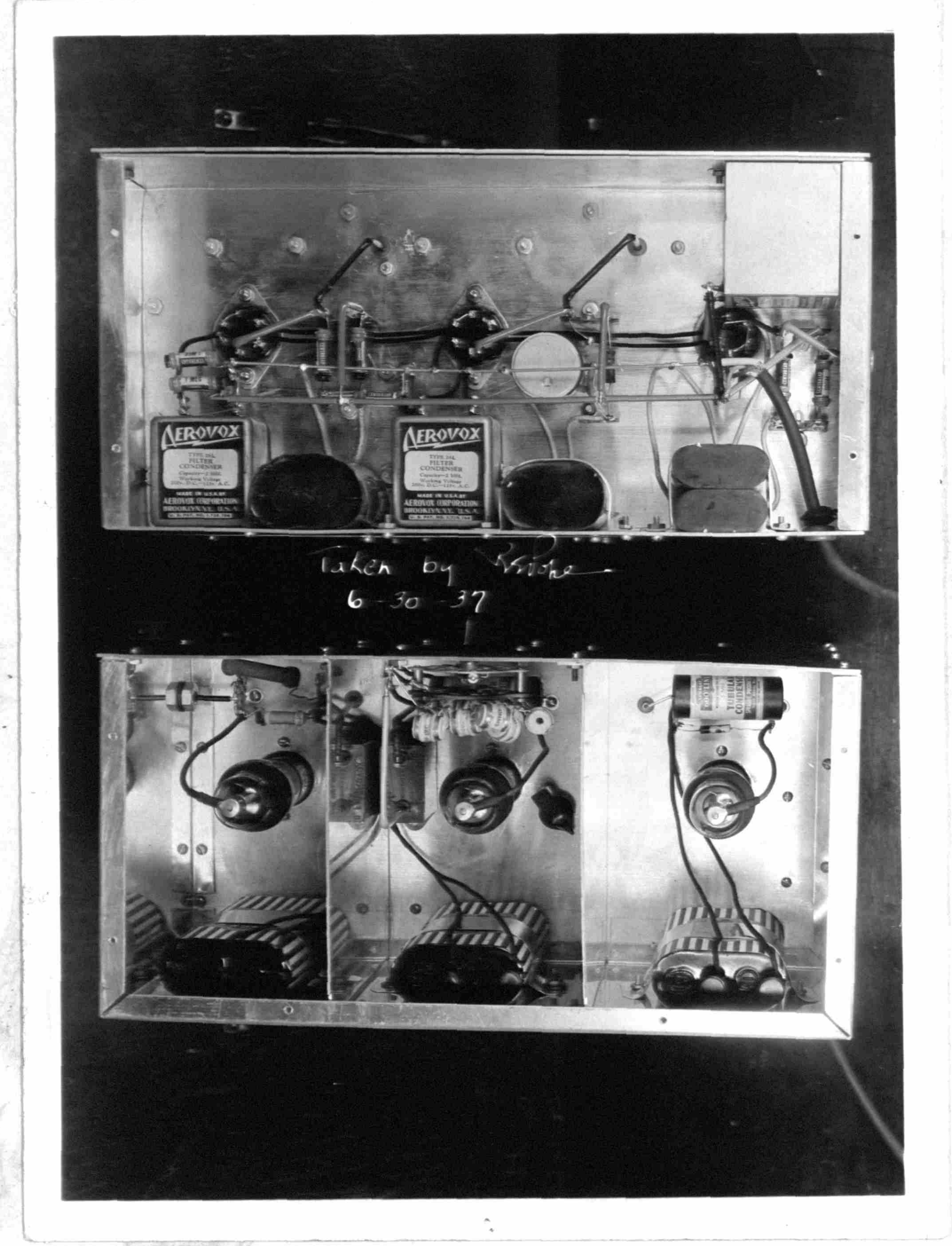
FIG. 91

W. A. S. JUNE 1937

common plate supply battery. The condensers and resistors used in the decoupling circuits were adjusted to compensate for the normal decrease in sensitivity and the phase shift with low frequencies. Inductances were placed in the plate loads of the last two stages to compensate for the effects of shunting capacity at high frequencies. A precision wire wound attenuator was placed between the first and second stages. Plate XXII is a photograph showing details of the two high gain amplifiers.

Plate XXIII is a photograph of the complete amplifier-oscillograph system.

Overall electrical calibrations were carried out by means of a General Radio Beat Frequency Oscillator with the pickup unit connected in the input circuit. The standard signal was applied to the input by means of a series resistance. This resistance was always adjusted to a small percentage of the total input circuit resistance. Figure 92 shows a sensitivity curve for each system. It is apparent the requirements established for the response characteristic are fulfilled by the actual apparatus.



Taken by R. Roche
6-30-37

Photograph of High Gain Amplifiers.
Plate XXII.

Taken
6-30-37
R Pope

Complete Amplifier-Oscillograph System.
Plate XXIII.

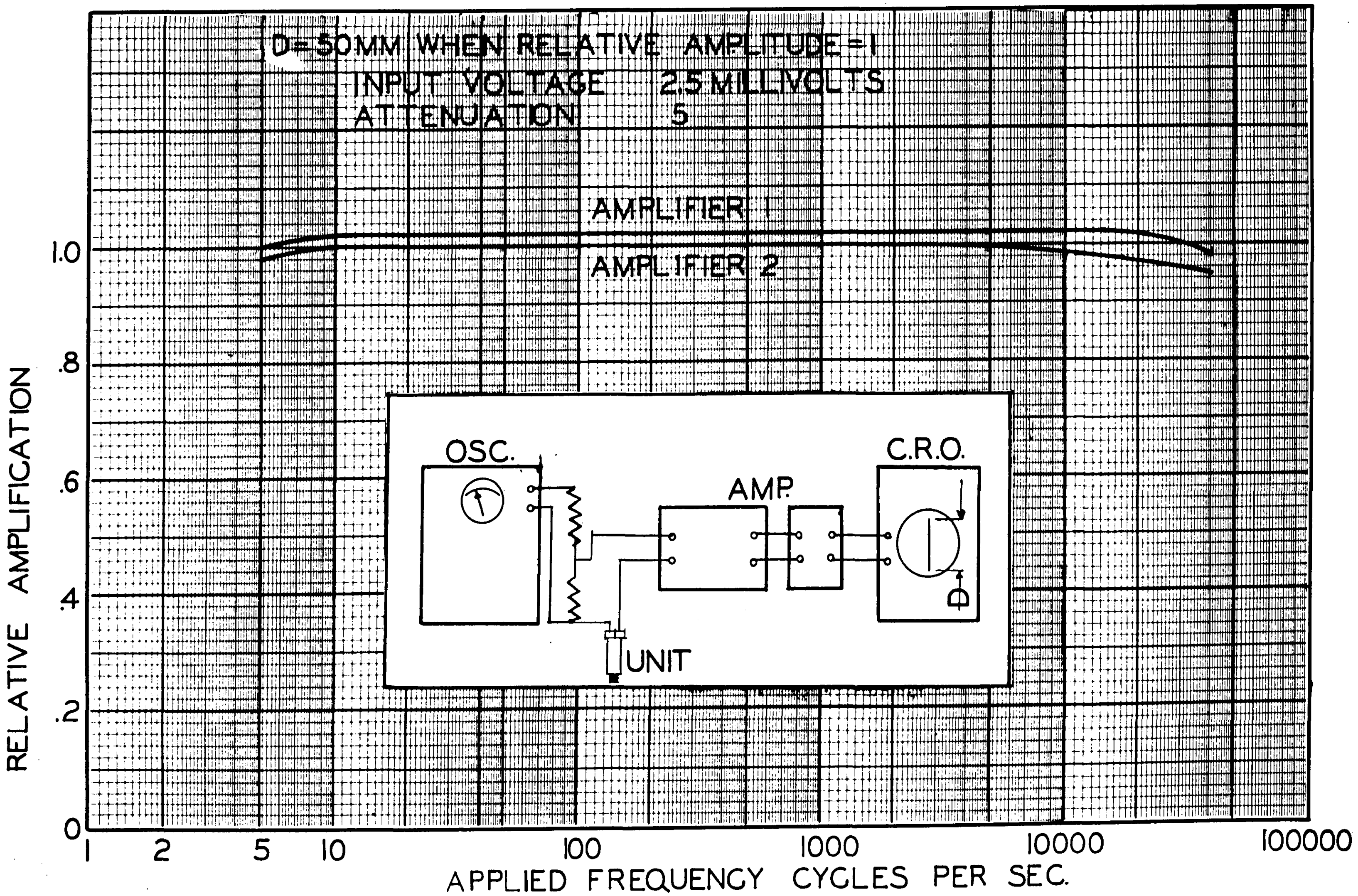

D=50MM WHEN RELATIVE AMPLITUDE=1
INPUT VOLTAGE 2.5 MILLIVOLTS
ATTENUATION 5
AMPLIFIER 1
AMPLIFIER 2
OSC.
AMP.
C.R.O.
UNIT
RELATIVE AMPLIFICATION
1.0
.8
.6
.4
.2
0
1
2
5
10
100
1000
10000
100000
APPLIED FREQUENCY CYCLES PER SEC.

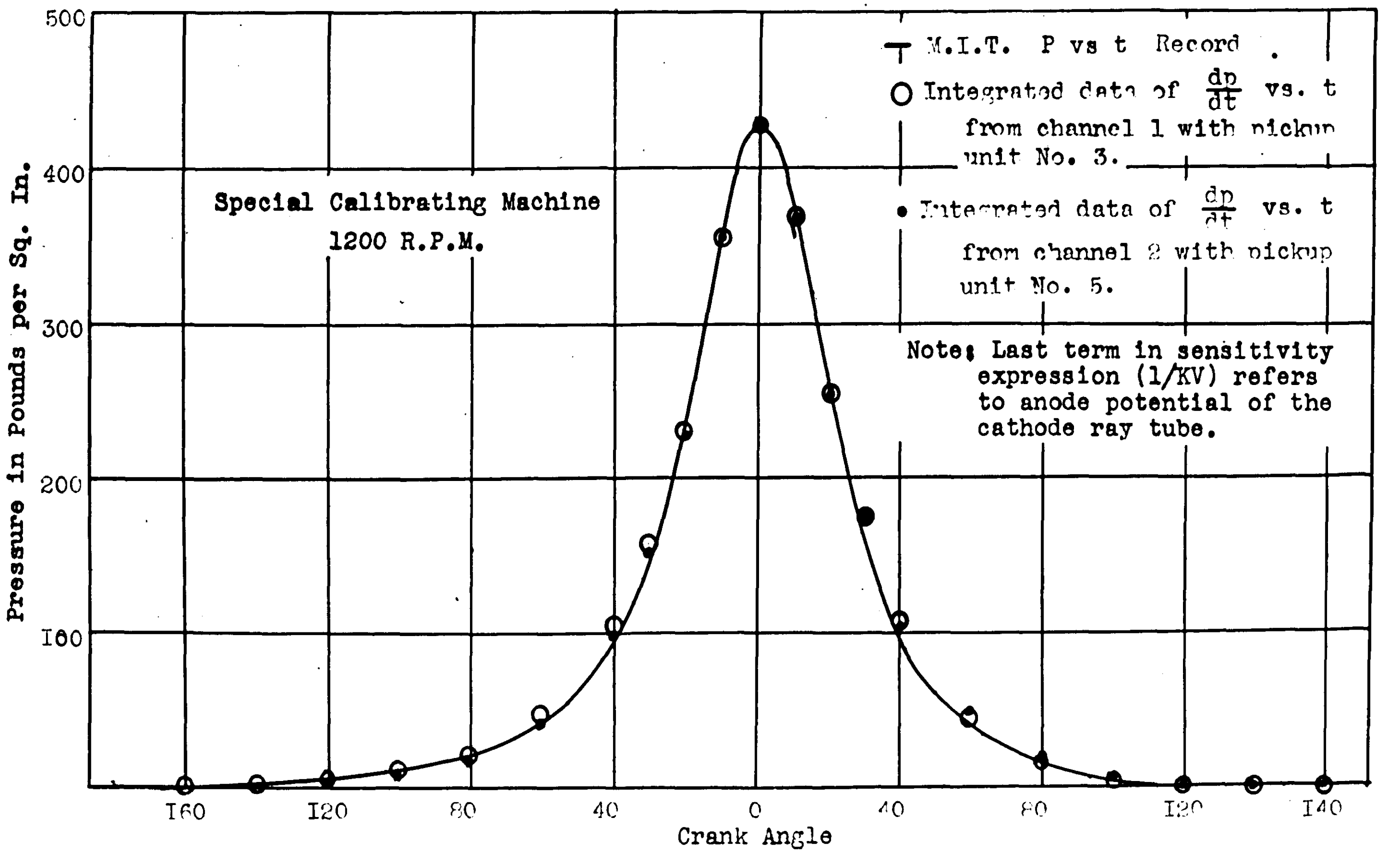
FIG. 92

SECTION VI

CALIBRATION AND ENGINE TESTS

Sensitivity of pickup unit No. 3 with channel No. 1 = 3.25×10^6 lbs./sq.in./sec./in. per $\frac{1}{kv}$

Sensitivity of pickup unit No. 5 with channel No. 2 = 2.53×10^6 lbs./sq. in./sec./in. per $\frac{1}{kv}$

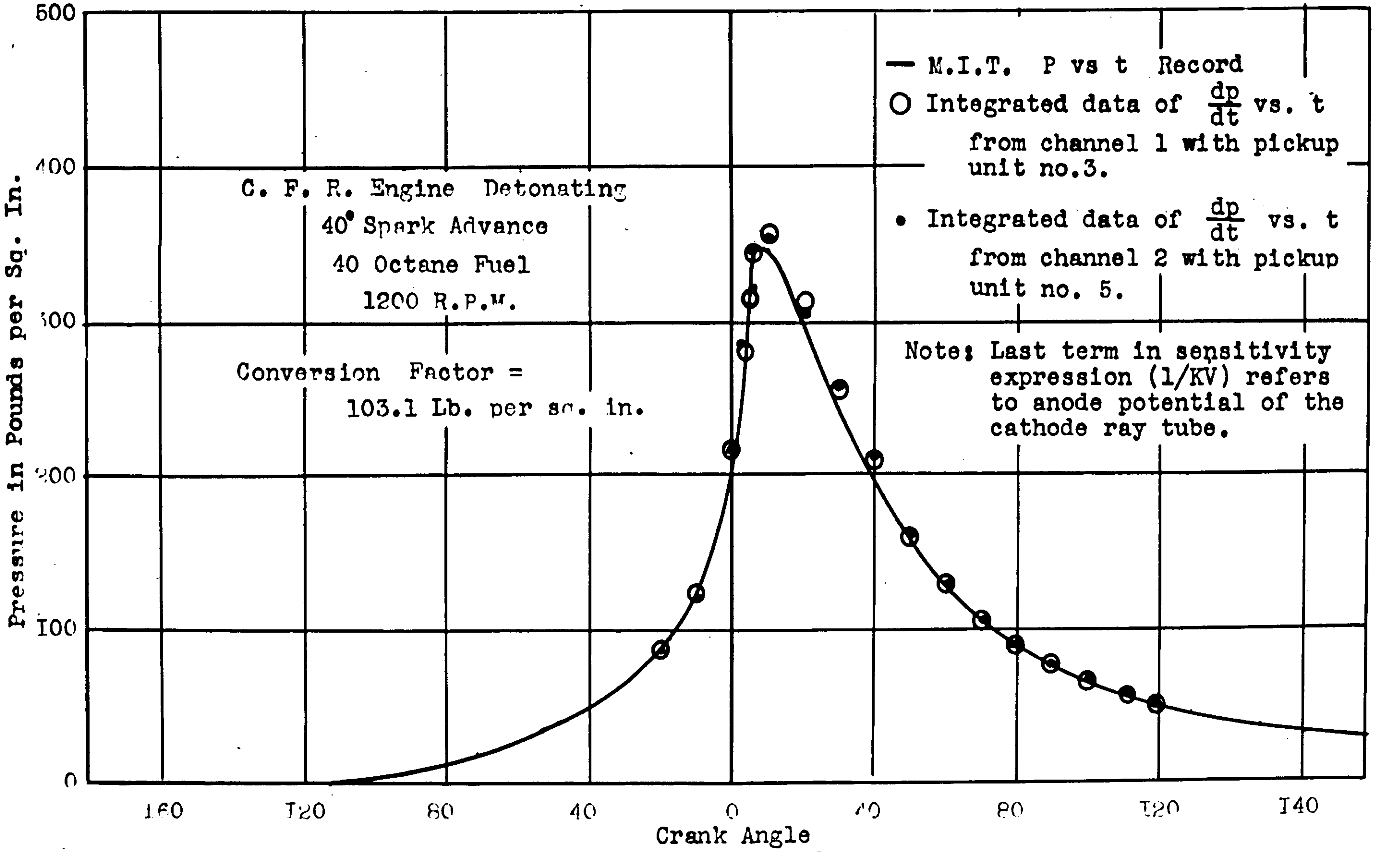


OVERALL CALIBRATION OF $\frac{dp}{dt}$ INDICATOR.

Fig. 93

Sensitivity of pickup unit No. 3 with channel No. 1 = 3.25 X 10^6 lbs./sq.in./sec./ in. per $\frac{1}{kv}$

Sensitivity of pickup unit No. 5 with channel No. 2 = 2.53 X 10^6 lbs./sq.in./sec./in. per $\frac{1}{kv}$

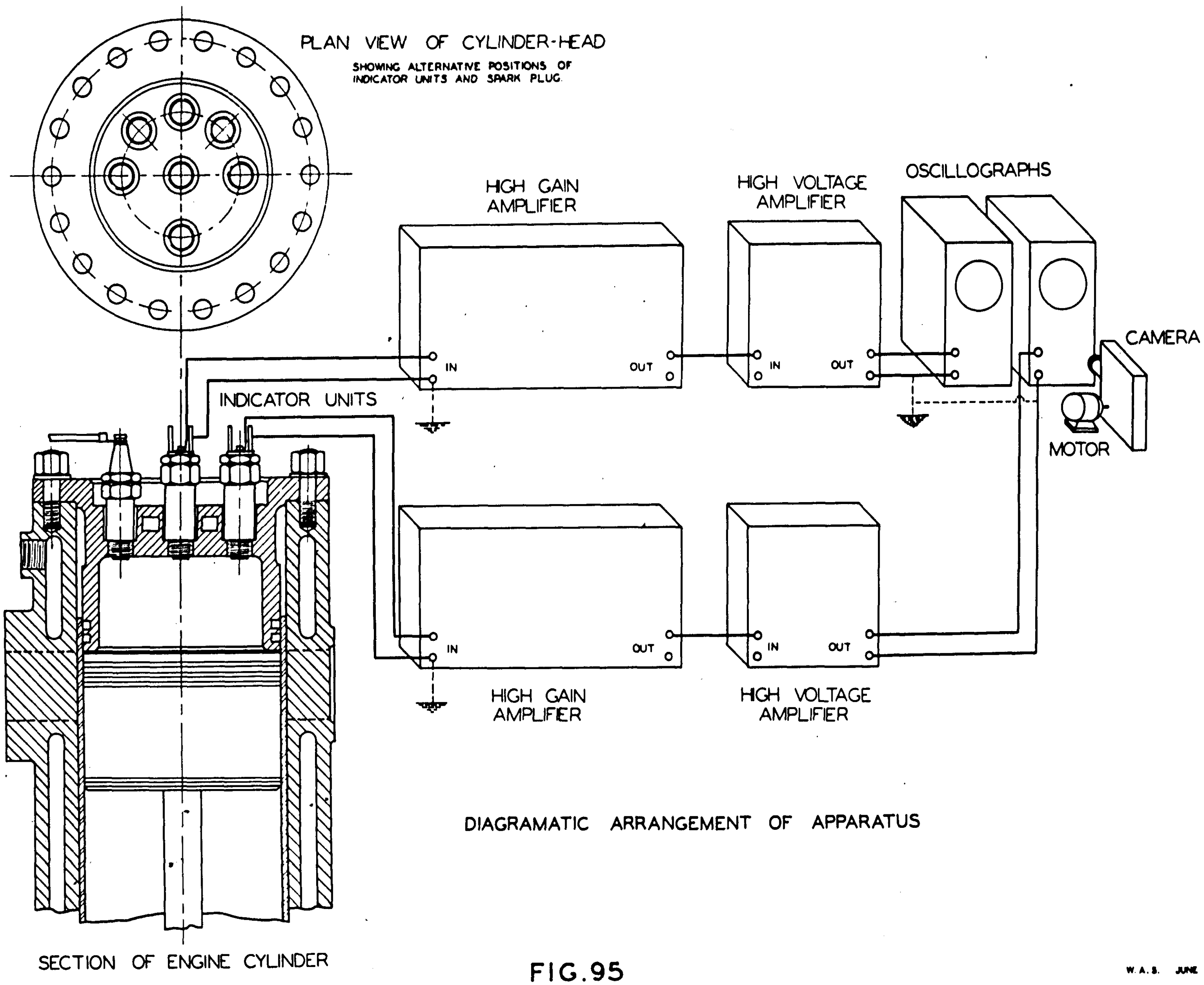


OVERALL CALIBRATION OF $\frac{dp}{dt}$ INDICATOR.

Fig. 94

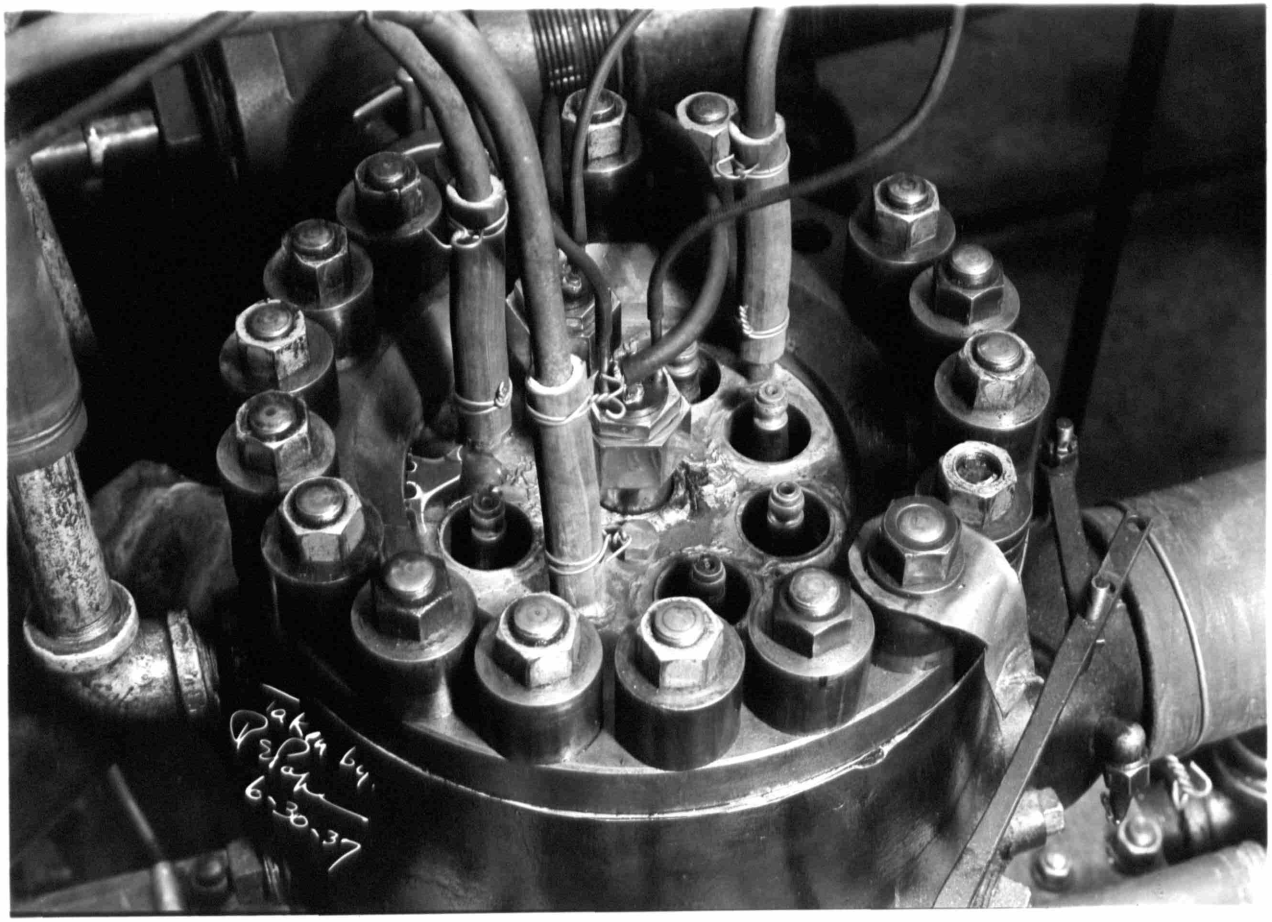
the CFR engine, the writer decided to make use of a special sleeve valve engine which was available in the Internal Combustion Engine Laboratory. This engine had a bore of 5 inches as compared to the 3.25 inch bore of the CFR engine. The sleeve valve construction was particularly desirable since by the design of a special head, indicators could be located in any one of seven positions, as desired. Figure 95 is a diagram of the installation for recording pressure variations in the sleeve valve engine. Plate XXIV is a view of the head showing two indicators in place.

Experiments showed that the pressure wave records from the Sleeve Valve Engine were not only complicated but were composed largely of frequencies higher than those found in the CFR engine. This was evidently due to the excitation of higher harmonics in a chamber with height and radius approximately equal. Another difficulty entered because of severe vibrations in the flat cylinder head which caused strong disturbances of the pressure records. For these reasons the work was carried back to the CFR engine fitted with a special spacing block below the cylinder head in which indicators could be placed simultaneously on a diameter opposite the spark plug and a diameter at right angles to the spark plug. Figure 96 is a diagram showing the application of apparatus to the CFR engine. Plate XXV is a photograph showing the installation on the CFR engine.



DIAGRAMATIC ARRANGEMENT OF APPARATUS

FIG.95



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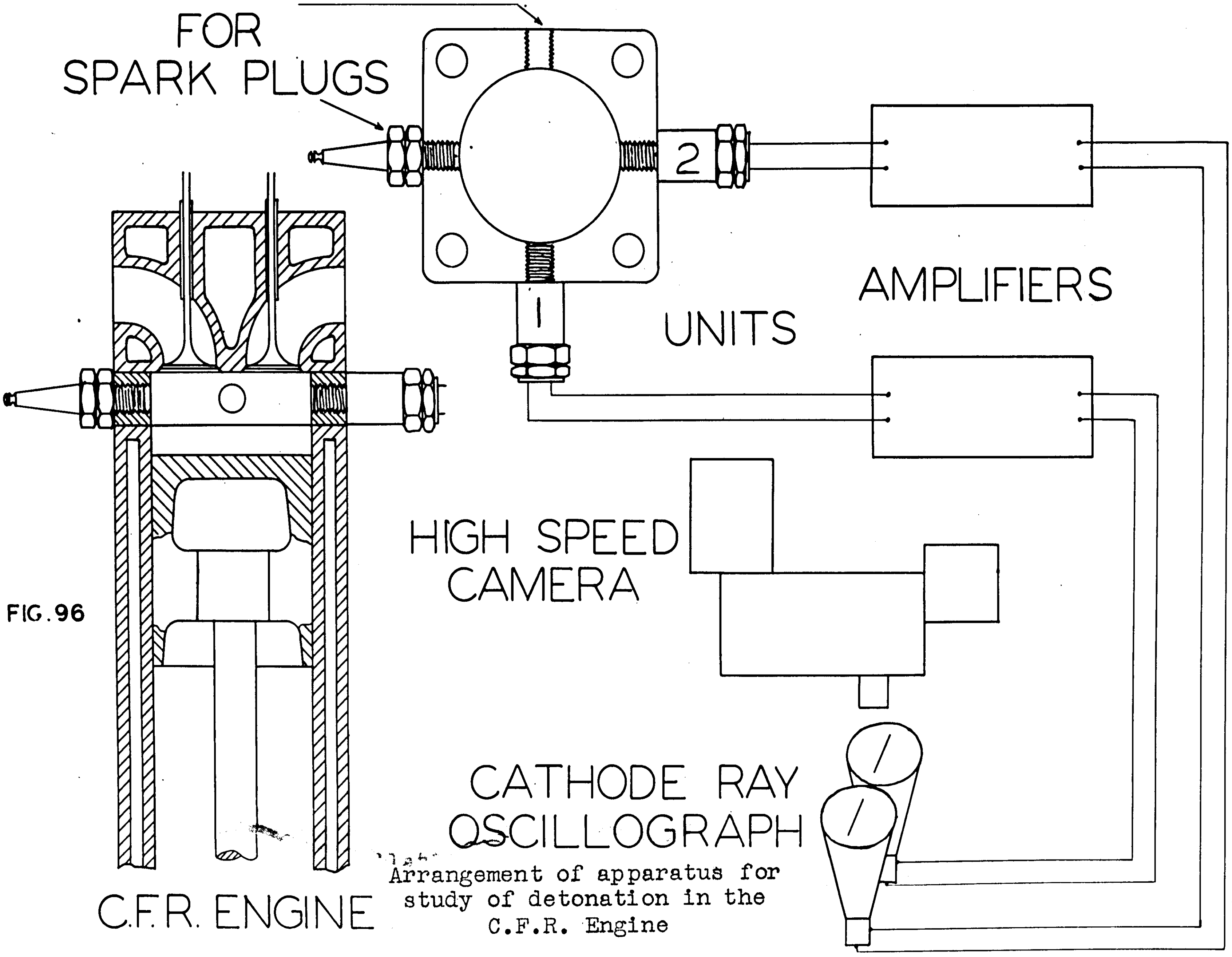
Head of Sleeve Valve Engine Showing Indicators in Center and One Side Position with Alternate Spark Plug Positions

Plate XXIV

FIG. 96



Arrangement of apparatus for study of detonation in the C.F.R. Engine

SECTION VII

DISCUSSION OF RESULTS AND CONCLUSIONS

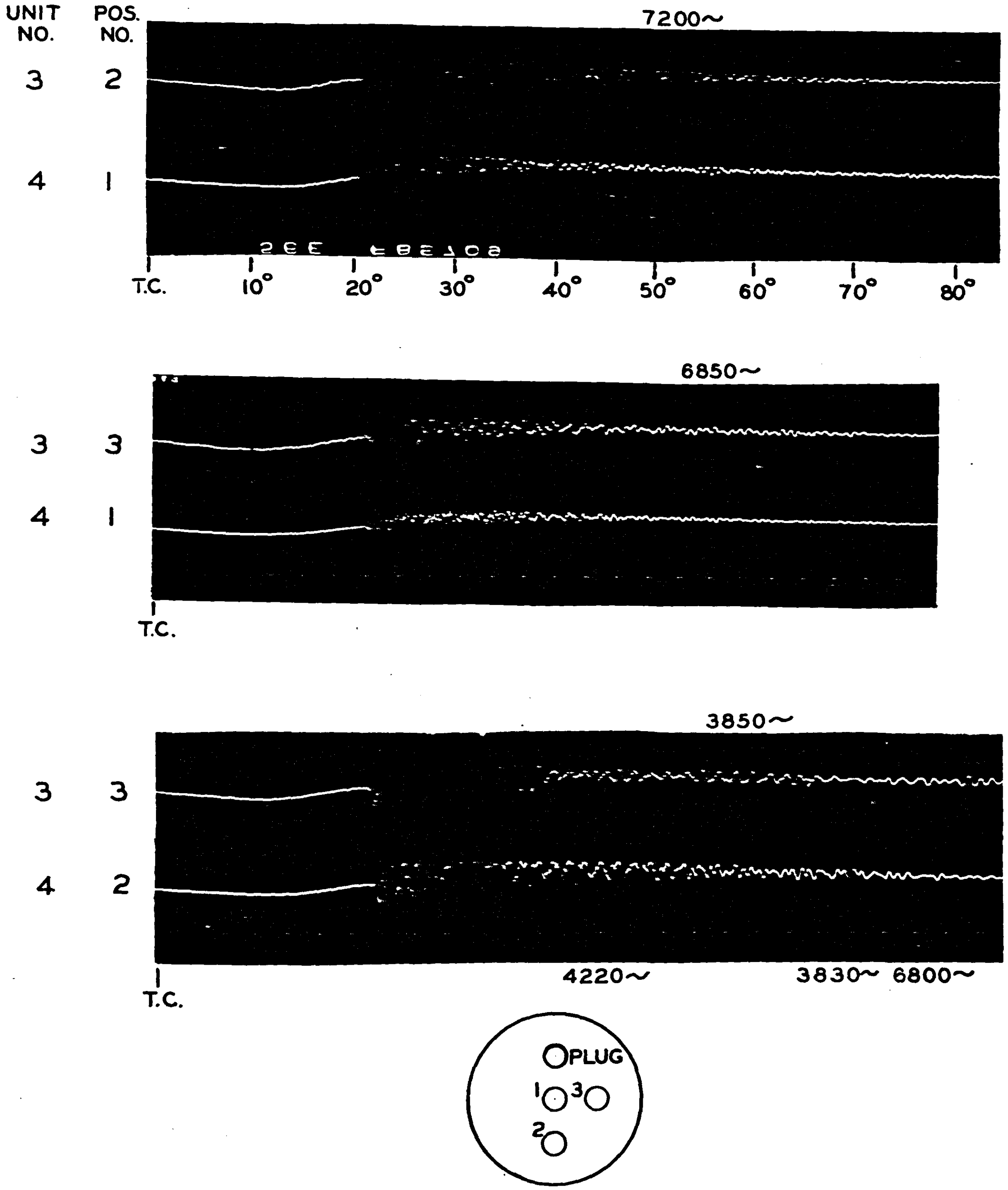
SECTION VII

DISCUSSION OF RESULTS AND CONCLUSIONS

Sleeve Valve Engine Results

Plate XXVI shows three dP/dt records taken with moderate detonation in the Sleeve Valve Engine. Indicators were located in the central position and two side positions opposite and at right angles to the spark plug as indicated. These records are typical of many hundreds of cycles taken with this engine. The complicated nature of the curves for a given record is apparent and the erratic changes from cycle to cycle are very pronounced. From such curves it was possible to tentatively identify certain modes of vibration, usually with one axial node and one or more azimuthal nodes; however any definite comparison between cycles was found to be very difficult.

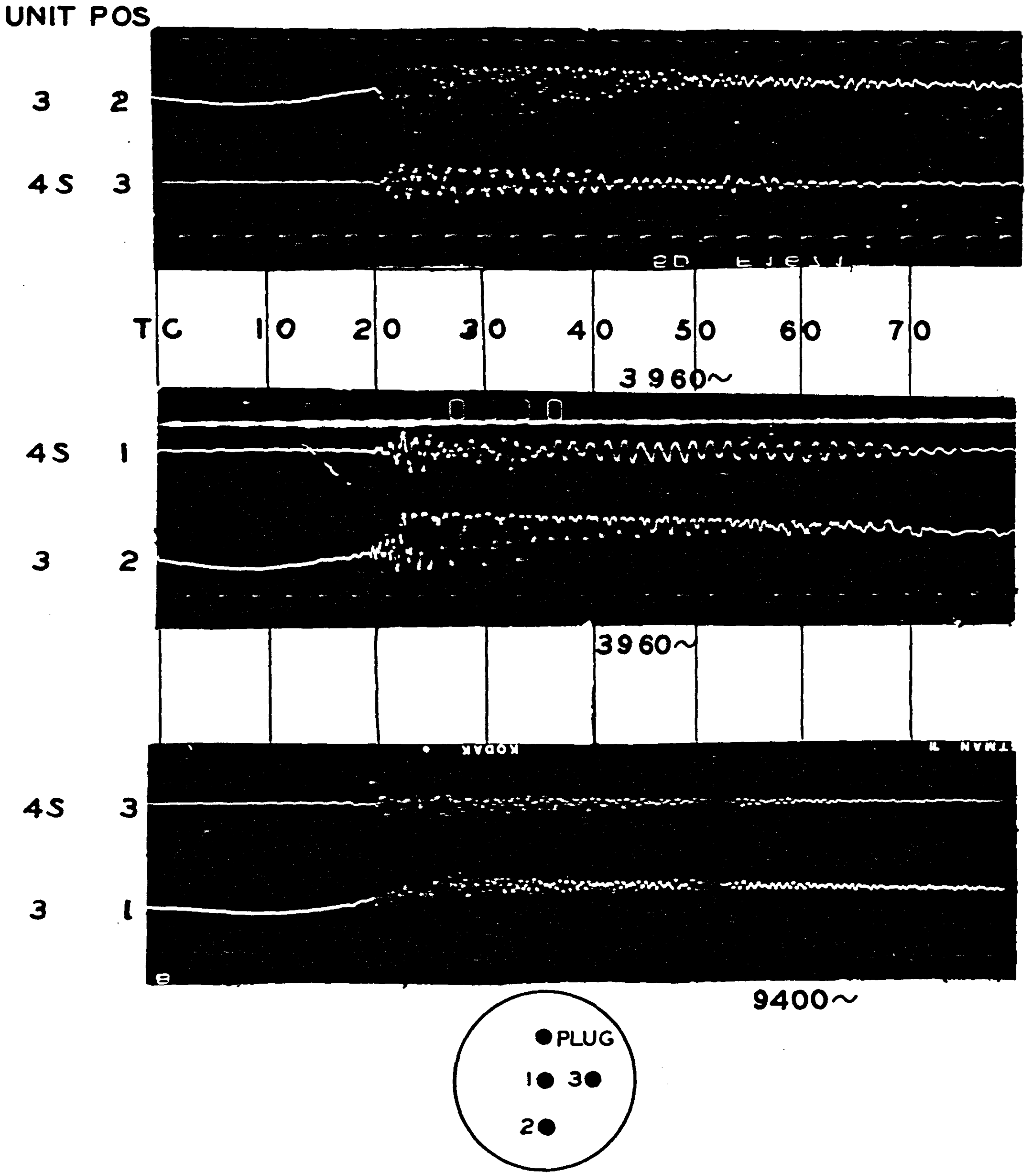
Plate XXVII is a series of three records taken to test the effect of mechanical vibration on the oscillograph records. In each case, one indicator of the pair was rigidly attached to the cylinder head in the designated position but shielded from gas pressures by means of a special fitting. This condition is shown by an S beside the unit number. The records show that the displacements due to mechanical vibration are of the same



MODERATE DETONATION RECORDS

Sleeve Valve Engine; 1050 R.P.M.; Spark Advance 30°; 40 Octane Fuel; Max. Power Mixture; Film Speed 400 inches per second.

Plate XXVI



MODERATE DETONATION RECORDS

Sleeve Valve Engine; 1050 R.P.M.; Spark Advance 30°; 40 Octane Fuel; Maximum Power Mixture; Film Speed 400 inches per second. Units marked (S) are shielded [illegible].

plate XXVII

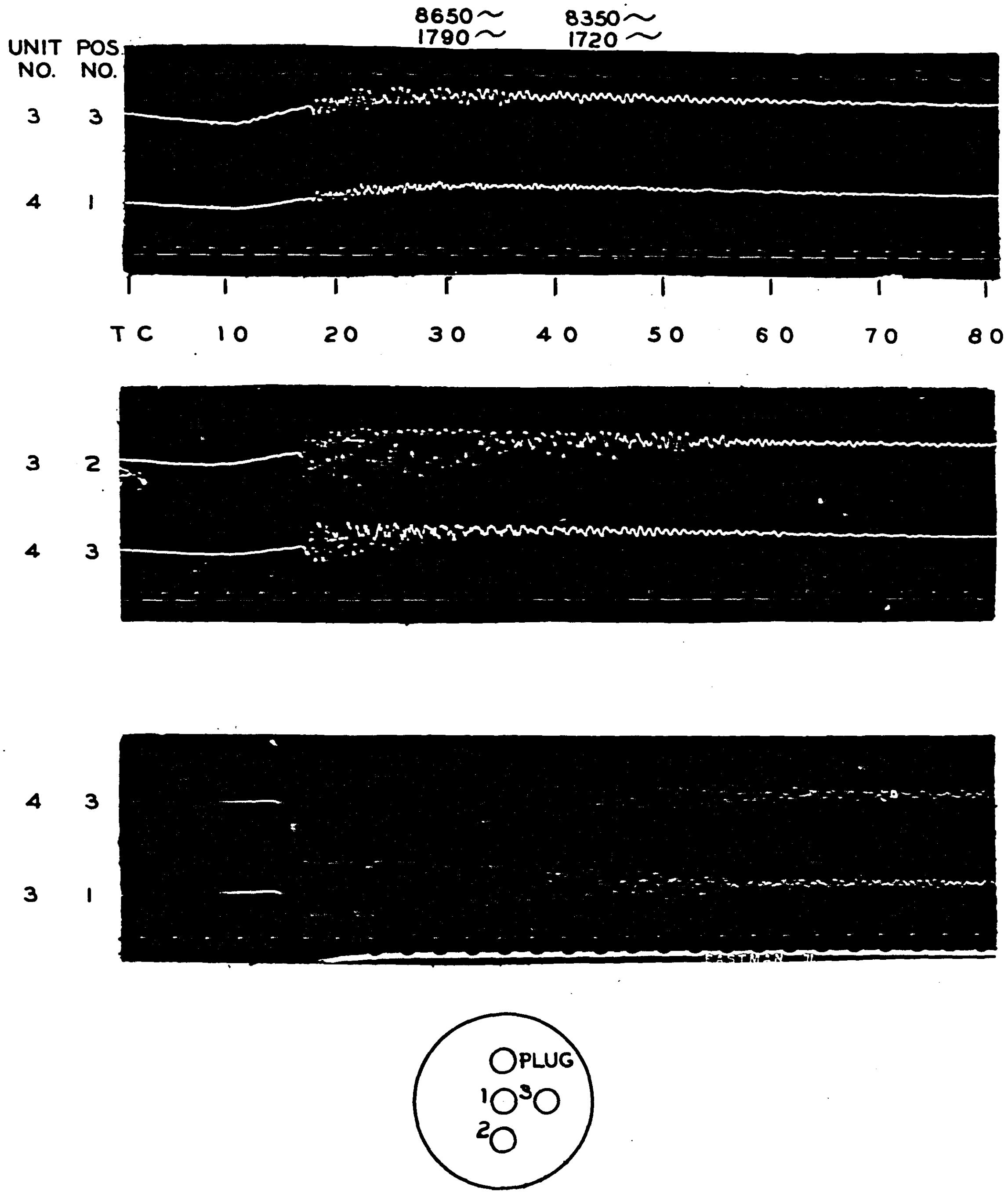
order of magnitude as the combined pressure and vibration effects and have complicated wave forms. Considerable effort was expended in an attempt to reduce these vibration effects without success.

The records of Plate XXVIII show a comparison between records taken with "light", "moderate" and "severe" detonation. One indicator was always located in the position at right angles to the spark plug for comparison purposes, while the other indicator was shifted between the center and edge position. The severe detonation record was taken with indicator sensitivity reduced to 1/10 that used for the other two records. Increasing record amplitude with increasing detonation intensity is apparent but the wave form is too complicated for reasonable analysis. One difficulty was that the engine produced records of widely varying amplitude and wave form for a constant setting of the engine controls.

The records displayed in Plates XXVI, XXVII and XXVIII indicate the troubles encountered in work on the Sleeve Valve Engine. These difficulties were so great that the attempt to obtain quantitative results from the Sleeve Valve Engine were abandoned.

CFR Engine Results

Experiments in the CFR Engine with indicators shielded from gas pressure showed that by using soft



LIGHT, MODERATE, AND SEVERE DETONATION RECORD

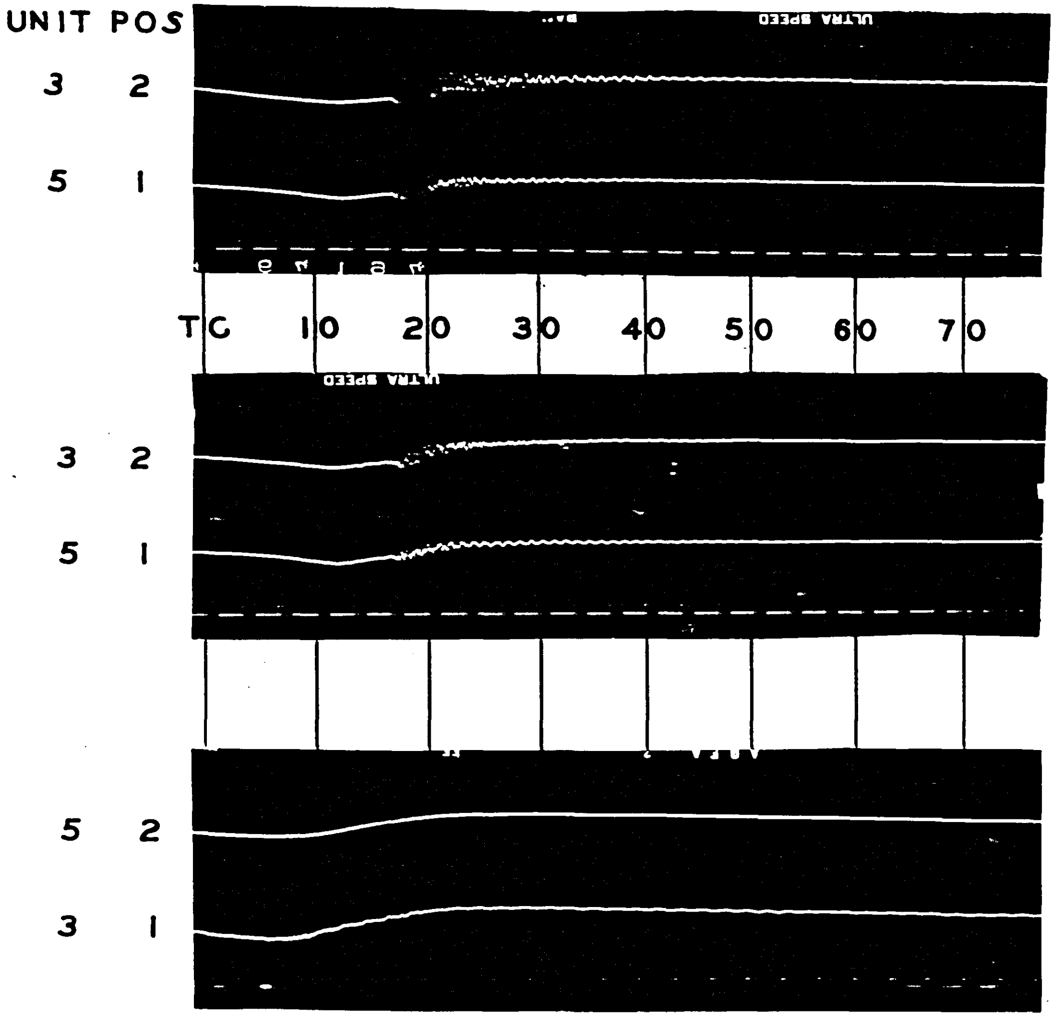
Sleeve Valve Engine; 1050 R.P.M.; 40 Octane Fuel; Film Speed 400 inches per second.

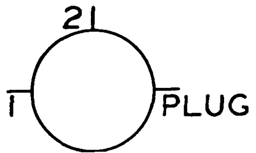
Plate XXVIII

copper-asbestos gaskets and screwing the units loosely into the threaded holes, mechanical vibration effects on the records could be completely eliminated. This was probably due to the rigidity of the heavy walled cylinder used for the spacer and the fact that large mechanical forces could not be transmitted through the aluminum threads. By elimination of vibration effects, the way was opened to quantitative measurements of pressure variations in the gases.

A systematic study of the effect of indicator position with respect to spark plug position showed that the records were completely erratic from cycle to cycle as regards the wave form and the amplitude varied within certain limits depending upon the engine conditions in use at the time. It was possible to establish a regime of "light" detonation, "moderate" detonation or "severe" detonation but the records of succeeding cycles were far from identical with a given condition.

Plate XXIX shows three cycles taken with "light" detonation as the engine condition. Differences between successive cycles and between the two indicator positions of the same cycle are apparent. Comparison of these records with records taken with greater detonation intensities shows that the frequencies





Sensitivity of unit 3 = 9.75 x 10^6 lbs./sq. in./sec./in.
Sensitivity of unit 5 = 7.60 x 10^6 lbs./sq. in./sec./in.

SUCCESSIVE CYCLES WITH LIGHT DETONATION AND A CYCLE WITH NO DETONATION.

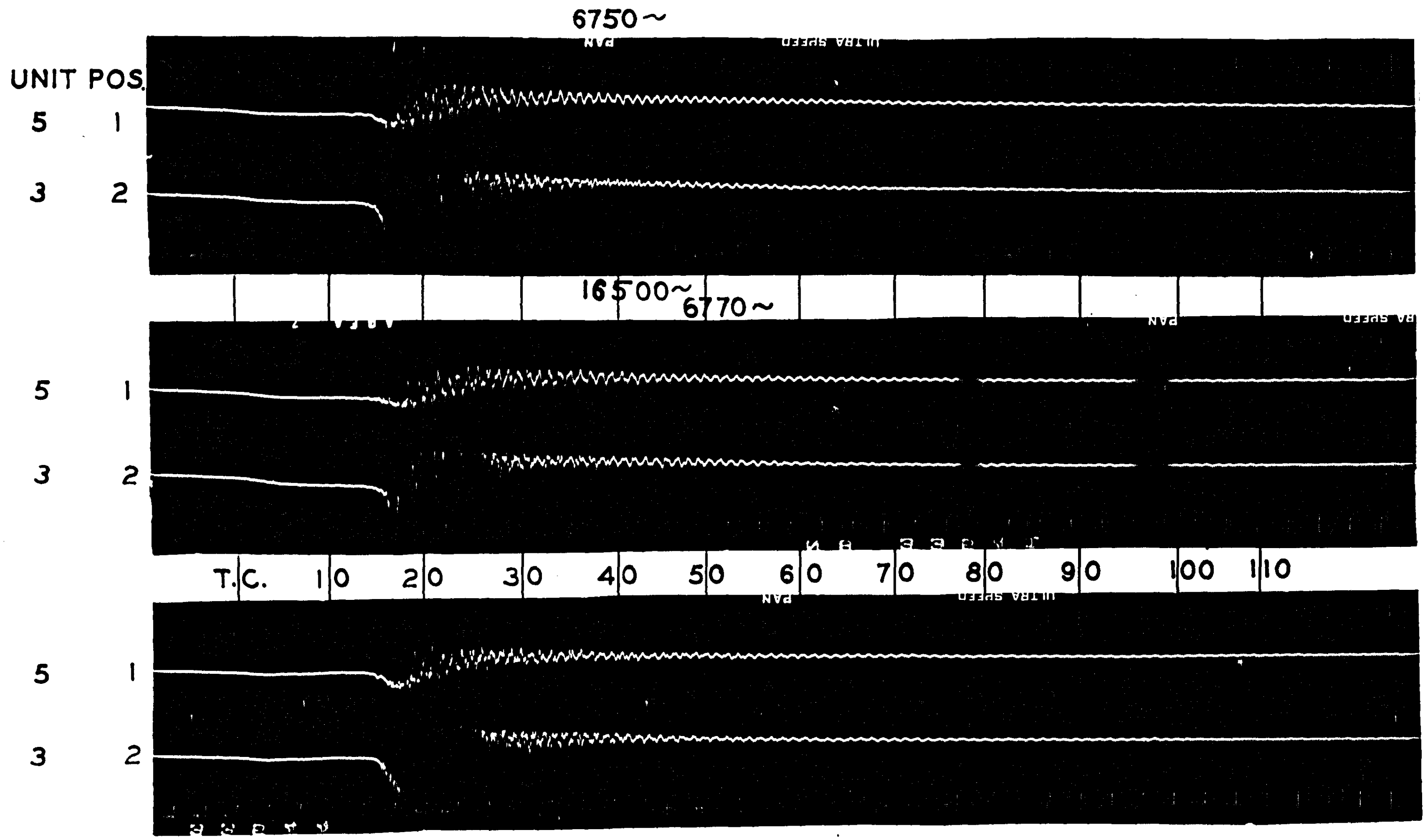
C.F.R. Engine; 1200 R.P.M.; Spark Advance 40°; 73 Octane Fuel, Compression Ratio 4.35 :1; Maximum Power Mixture; Film Speed 400 inches per second.

Plate XXIX

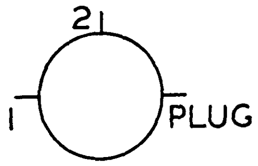
are higher and the wave form more complicated than for more severe conditions. The bottom record of the group shows that both pressure wave and vibration effects were negligible in the absence of detonation.

Plate XXX shows three successive cycles with "heavy" detonation induced by the use of a low octane fuel. In each case the records are essentially similar with the most pronounced frequency component being lower than that found with 'light' detonation. The waves for the position at right angles to the spark plug seem to start with a higher frequency and amplitude than the waves for the position opposite the plug. The wave form is too complicated for a precise analysis but does give the general information that the location, size and shape of the initial pressure rise was similar in all three cycles.

Plate XXXI shows two cycles with moderate detonation induced by use of a fuel with a low octane rating. The pressure wave system builds up gradually into a complicated wave form which resolves almost completely into a simple wave form of lower frequency than that found for "light" detonation. In the upper record, the waves at the position opposite the spark plug shows very complicated vibrations of high frequency throughout



SUCCESSIVE CYCLES WITH HEAVY DETONATION.

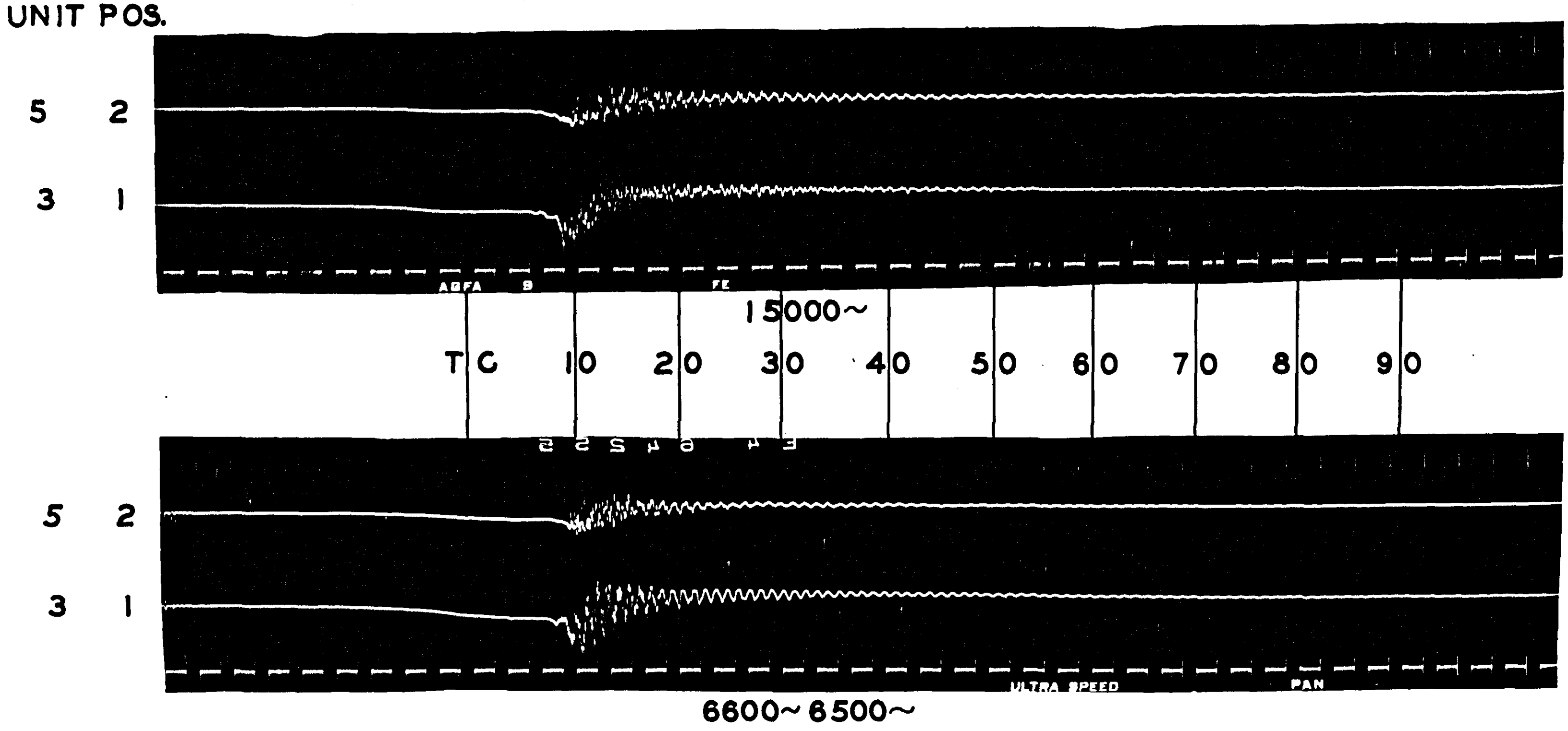


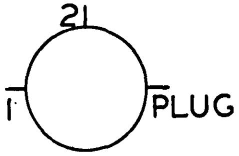
C.F.R. Engine, 1200 R.P.M.; Spark Advance 40°; 40 Octane Fuel; Compression Ratio 4.85 : 1; Maximum Power Mixture; Detonation induced by ethyl nitrite; Film Speed 400 inches per second.

Sensitivity of unit 3 = 9.75 x 10^6 lbs./sq. in./sec./in.
Sensitivity of unit 5 = 7.60 x 10^6 lbs./sq. in./sec./in.

Plate XXX

Sensitivity of unit 3 = 9.75 x 10^6 lbs./sq. in./sec./in.
Sensitivity of unit 5 = 7.60 x 10^6 lbs./sq. in./sec./in.





SUCCESSIVE CYCLES WITH MODERATE DETONATION.

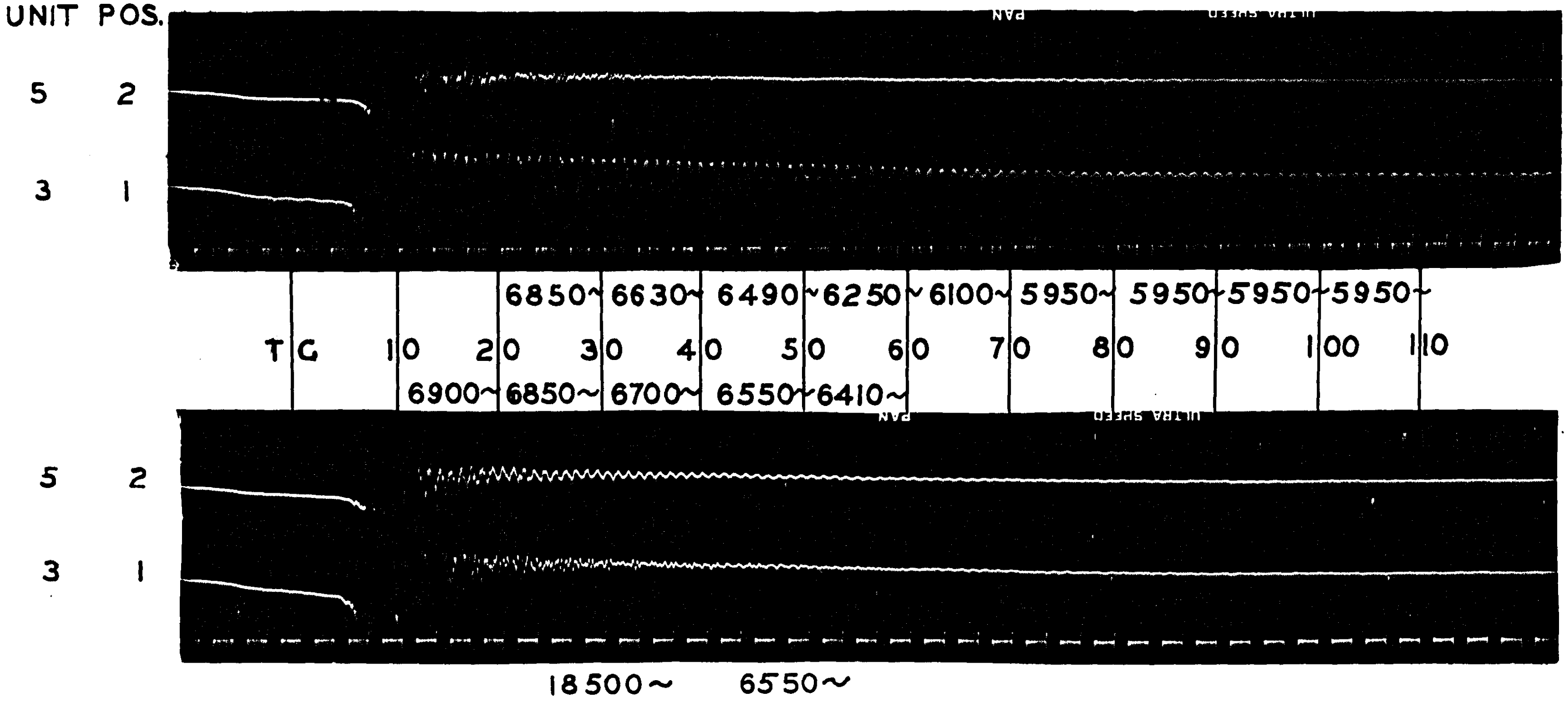
C.F.R. Engine; 1200; Spark Advance 40°; 40 Octane Fuel; Compression ratio 4.85:1; Maximum Power Mixture; Film Speed 400 inches per second.

Plate XXXI

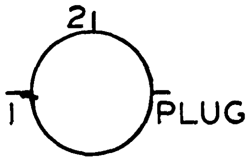
the entire range while the waves for the right angle position are complicated at the start and become simpler as time goes on. In the lower record this behaviour is just reversed with the position at opposite the spark plug showing a relatively simple form after a short period at the start and the right angle position showing high frequencies at the start and changing into a low amplitude simple wave form. It will be shown later that the observed lower frequencies correspond to the lowest frequency mode of vibration with one azimuthal node. With this in mind, the observed results can be explained as due to a shift in position of the detonating region between successive cycles.

Plate XXXII shows two records taken with 'severe' detonation induced by the combination of a low octane rating fuel and a knock inducer added to the intake air. The upper record is very striking in that the position at right angles to the spark plug shows a relatively low amplitude, high frequency vibration of complicated wave form while the record taken opposite the spark plug shows a strong vibration of simple wave form and decreasing frequency as the piston descends. In the lower record, the indicator located opposite the plug shows a complicated wave form of high frequency with almost no

Sensitivity of unit 3 = 9.75 x 10^6 lbs./sq. in./sec./in.
Sensitivity of unit 5 = 7.60 x 10^6 lbs./sq. in./sec./in.



SUCCESSIVE CYCLES WITH SEVERE DETONATION.



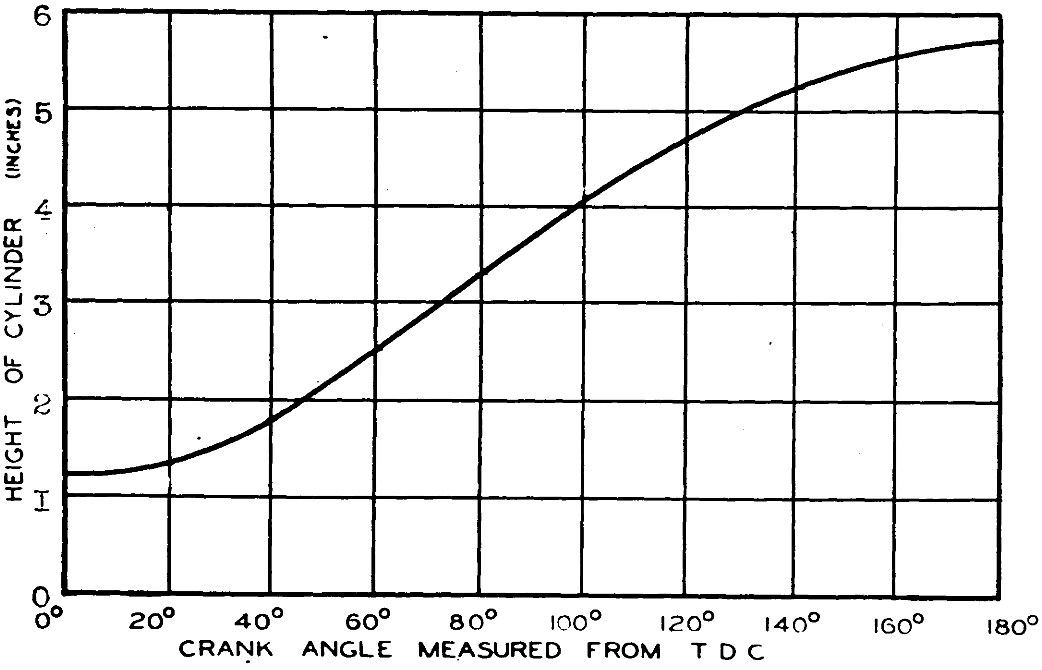
C.F.R. Engine; 1200 R.P.M.; Spark Advance 40°; 40 Octane Fuel; Compression Ratio 4.85 : 1 ; Maximum Power Mixture; Detonation induced by ethyl nitrite; Film Speed 400 inches per second.

Plate XXXII

trace of the simple low frequency found at right angles to the spark plug. These records will be used for examples in the discussion which is to follow.

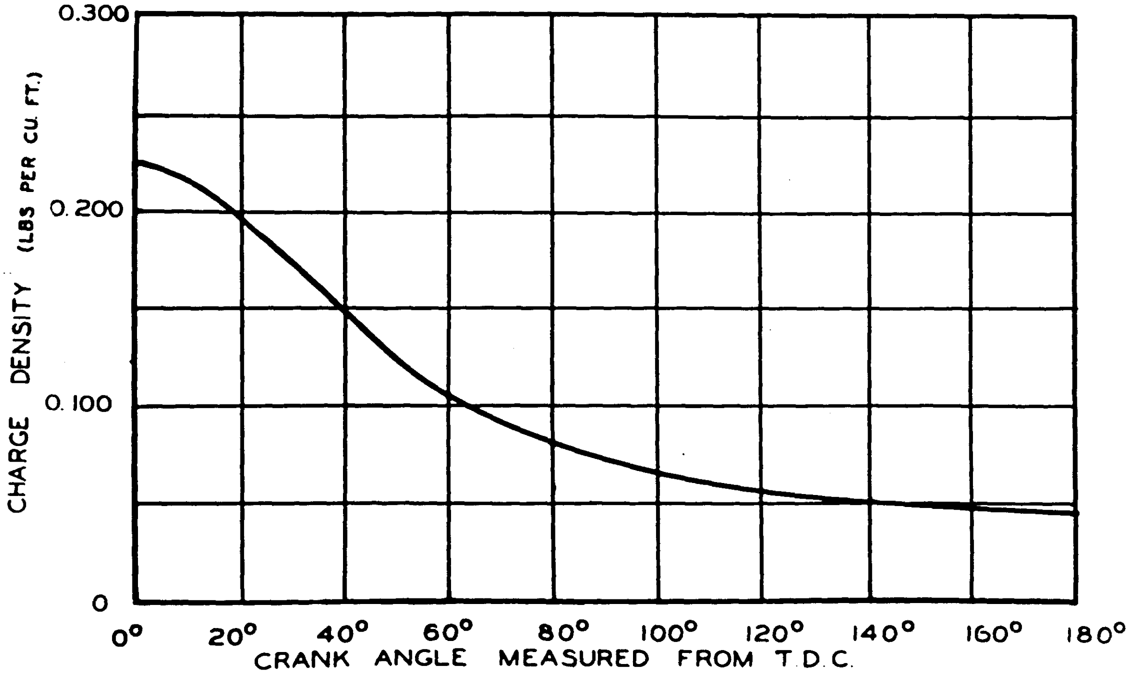
Figure 97 is a curve of cylinder height as a function of crank angle and figure 98 is a plot of charge density against crank angle. Figure 99 is a plot showing the variation of charge temperature with crank angle. The information for this plot was taken from the Hottel Chart using crude data from the indicator card previously given and air consumption measurements made during the engine test. Figure 100 gives curves of sound velocity and wave frequency from Plate XXXII as functions of crank angle. These figures will be used in the correlation of experimental results with the predictions of theory.

Figure 101 is a plot taken from Lamb's "Hydrodynamics" of the equal pressure amplitude contours for vibrations with one azimuthal node and no axial nodes in a circular cylinder. For such a system it is apparent that an indicator located as at B_1 will show a maximum pressure amplitude while an indicator located at B_2 will show no vibrations of the frequency corresponding to the pressure amplitude system shown. Indicators located as at A_1 and A_2 will show approximately equal amplitudes but the records will be of opposite phase since the locations



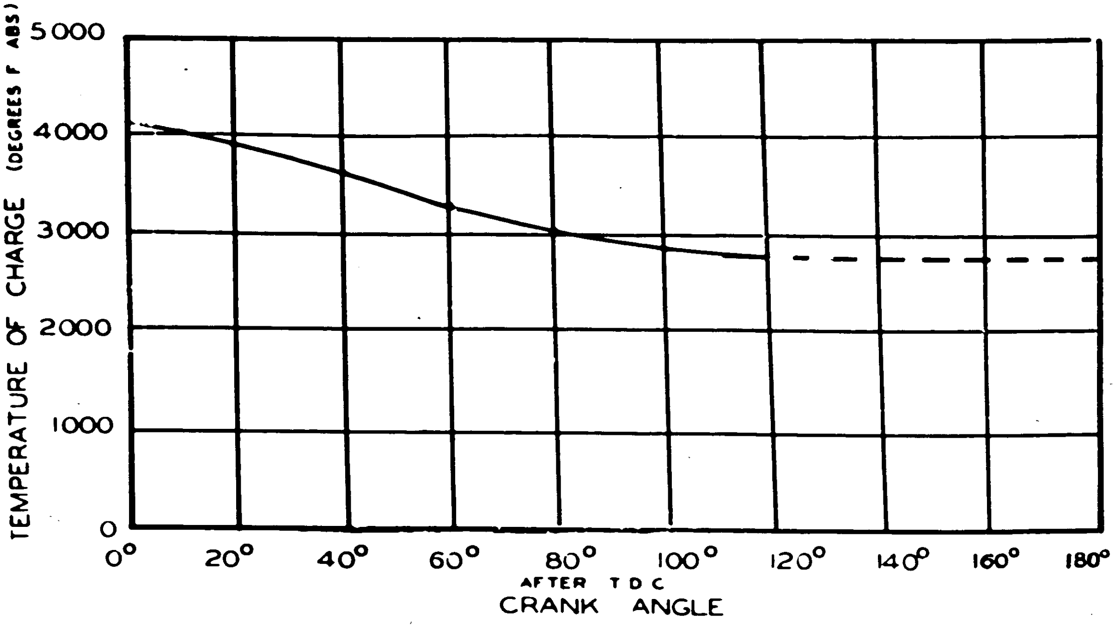
CYLINDER HEIGHT vs. CRANK ANGLE

Fig. 97



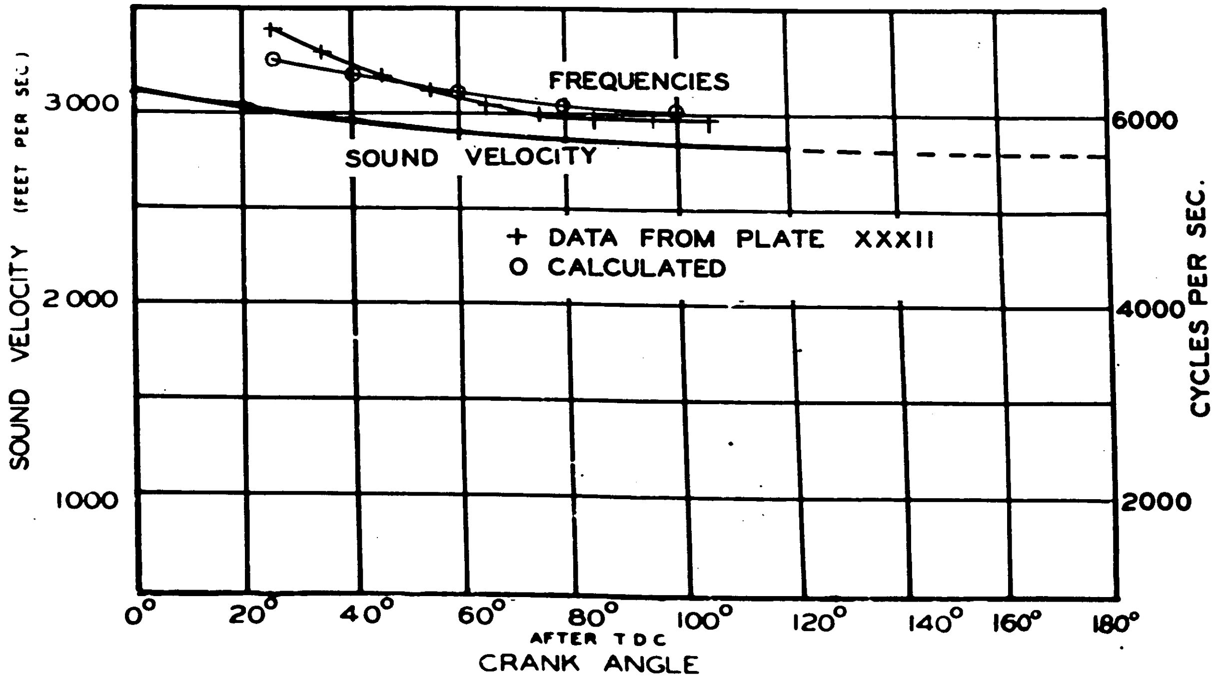
CHARGE DENSITY vs. CRANK ANGLE

Fig. 98



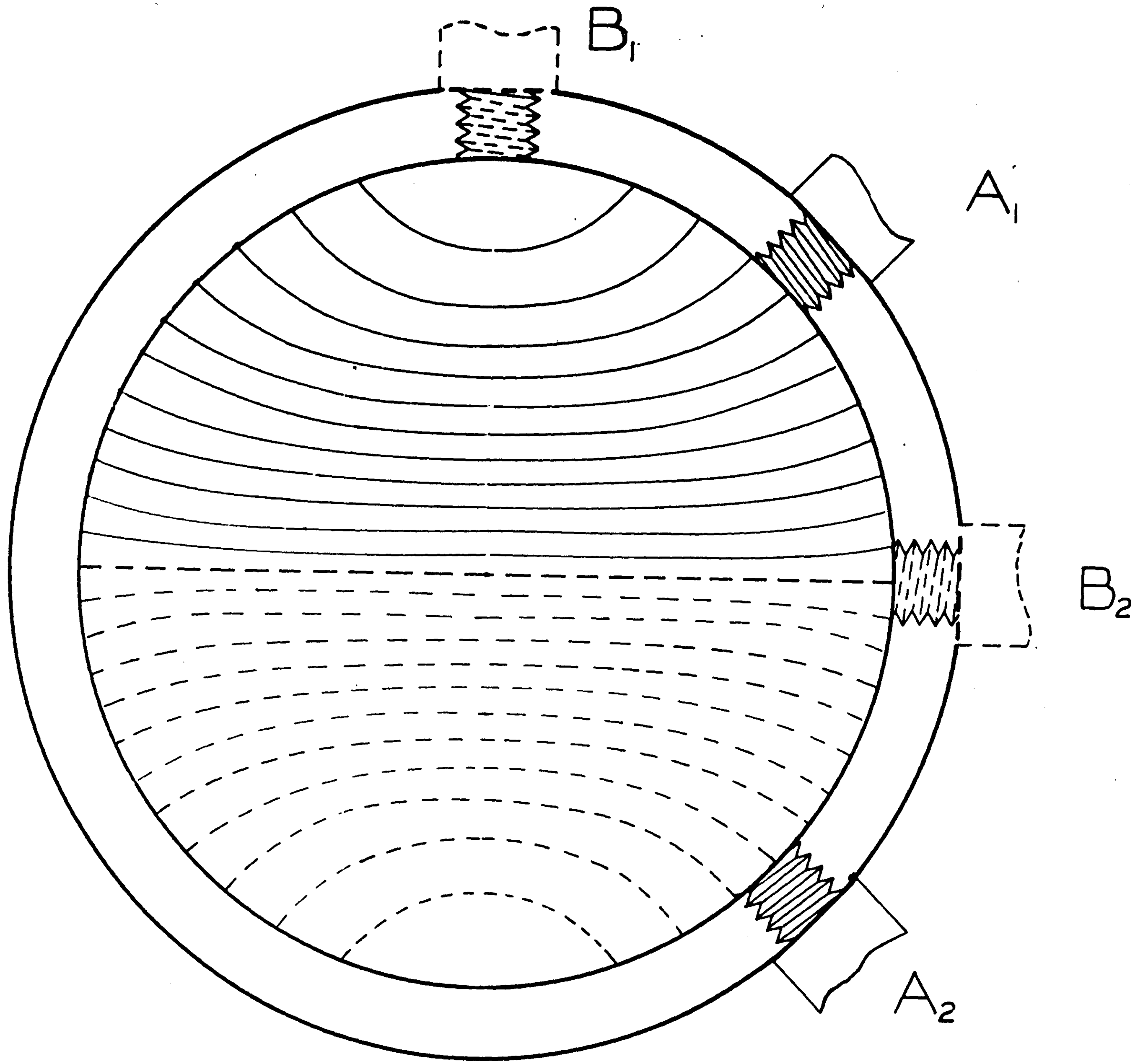
TEMPERATURE vs CRANK ANGLE

FIG. 99



FREQUENCY AND SOUND VELOCITY vs. CRANK ANGLE

FIG. 100



Lines of equal pressure amplitude for the lowest frequency mode of vibration in a circular cylinder. Fig. 101.

CONCLUSIONS

Qualitative Discussion of Results

Detonation of increasing severity from "light" to "severe" is illustrated in Plates XXIX to XXXII. In each case, two or more successive cycles are given. For each engine condition, the records have the same general features but are different in details. According to the analysis of Section IV such variations in the pressure wave systems must be due to differences in the initial disturbances. Thus the wave system from a pressure rise almost 180°in azimuthal extent will contain a strong component of the lowest frequency possible in the cylinder. On the other hand, pressure suddenly developed in a region of small azimuthal extent will produce modes of vibration with more than one azimuthal node and with frequencies higher than the lower limit. This trend is clearly shown in the oscillograph records. For "light" detonation, only high frequencies appear and the lowest frequency mode becomes stronger as detonation becomes more severe.

The records of Plate XXX, XXXI and XXXII which were taken with "moderate," "heavy" and "severe" detonation show relatively strong components of the one azimuthal node type of vibration. For the same engine conditions, erratic variations usually appear in the relation between the amplitudes found at the positions opposite and at right angles to the plug. Reference to the discussion of figure 101 shows that this observed result corresponds to changes in position of the initial disturbance. This observed erratic behaviour of the excitation region is in agreement with the reported results of the investigators whose work has been reviewed in Section IV.

A summary of information from a qualitative examination of the oscillograph records, shows that the results of the present experiments are in general agreement with the predictions of sound theory and with the observations of previous investigators.

Comparison of Calculated and Observed Frequencies

Conclusions with regard to the usefulness of the analytical method described above must be based on quantitative comparisons of theoretical and experimental results. The work described in the present report was concerned with the development of procedure and equipment rather than an extended investigation of detonation. For this reason the following discussion will be limited to a single example as an illustration of the method and a demonstration of the approximate magnitudes involved. On account of the simple wave form and because a relatively high pressure amplitude offers the most severe test of the theory, the lower trace of the upper record in Plate XXXII will be analyzed.

As a general check on the theory, calculated frequencies are compared with measured frequencies in figure 100. The calculated points are based on the λ/a ratio for the mode of vibration with one azimuthal node and no radial or axial nodes. This ratio is 3.41 as taken from figure 73. Equation 40 and sound velocities from figure 100 were used to calculate frequencies for various crank angles for the CFR Engine (bore = 3.25 inches). The greatest discrepancy between measured and calculated frequencies is 5% and occurs at the first measurable point after the initial disturbance. From 40° crank angle until the last point given at 100° after top center, the calculated and experimental curves agree so closely that the results constitute an almost perfect check. The difference between the theoretical and computed frequencies at the first point can be due either to the effect of finite wave amplitude or to an error in estimating the frequency from the somewhat distorted wave form in the region concerned.

Considering the relatively high pressure wave amplitude in the case selected for analysis, it is certainly to be expected that the theory of sound will give quantitative results when applied to the pressure waves which usually accompany detonation.

Calculation of the Energy Associated with the Pressure Waves

On the basis of the proof given above that sound theory can be applied to detonation disturbances, the theory can reasonably be used to calculate the energy density, the particle velocity and the particle displacements associated with a given system of pressure waves.

In the case previously considered as an example, several frequencies are present but by far the largest amplitude occurs for the simplest mode. Remembering that the record amplitude for a given mode must be divided by the corresponding frequency in calculating the actual pressure, it is apparent that a substantially correct value for the energy density can be found by considering only the contribution of the component with one azimuthal node. In addition to this, Table 2 shows that the energy ratio, $M_{100} = 0.119$ is over twice as large as this ratio for two azimuthal nodes and is greater than for any of the other more complicated modes.

From Section IV, the relationship between energy density, excess pressure, equilibrium pressure and the ratio of specific heats is

$$E_{n_\theta n_r n_z} = M_{n_\theta n_r n_z} (p^2_{n_\theta n_r n_z} / \gamma P_o) \qquad (41)$$

As a typical example, conditions at 20 degrees after top center will be considered. For a single frequency, the excess pressure can be written

$$p = p_a \sin(2\pi\nu t) \qquad (48)$$

and

$$dp/dt = p_a\, 2\pi\nu \cos(2\pi\nu t) \qquad (49)$$

The rate of change of pressure amplitude can be found from the record and the known indicator sensitivity. In the present case, the record amplitude is 0.135 inch, the sensitivity is 9.75×10^6 lb/in^2/sec/in. and the frequency is 6900 cycles per second. Using these data

$$p_a = (0.135)(9.75 \times 10^6)/(2\pi 6900) = 30.4 \text{ lb/in}^2 \qquad (50)$$

It is interesting to note that this amplitude is about 1/10 of the equilibrium pressure. This fraction is "small" compared to unity as required by the theory of sound.

From figure 99, the charge temperature is 3900 degrees F. From figure 71 the ratio of specific heats is 1.24. From figure 91 the equilibrium charge pressure is 315 lb/in^2 absolute. Substituting these data in equation (41) gives

$$E_{100} = (0.119)(30.4)^2(144)^2/(1.24)(315)(144) = 40.6 \frac{\text{ft.}}{\text{lb./ft}^3} \qquad (51)$$

From figure 97 the height of the chamber at 20 degrees after top center is 1.3 inches. The bore of the CFR Engine is

3.25 inches so that the total energy associated with the wave motion is

$$\text{Total Energy} = (40.6)\pi(3.25)^2(1.3)/(4)(1728) = 0.254 \text{ ft. lb.} \quad (52)$$

At 20 degrees after top center some damping of the wave amplitude has occurred so that the value of 0.254 ft. lb. must be somewhat lower than the energy associated with the initial disturbance.

A comparison of the pressure wave energy with the chemical energy of the charge leads to a rough estimate of the fraction of the charge which was involved in the initial disturbance. By assuming that the detonating part of the charge burns instantaneously at constant volume, Professor E. S. Taylor has estimated that the highest possible efficiency for the production of pressure waves is about 13% for a fuel-air ratio of 0.078 and a compression ratio of 6. This wave production efficiency probably has about this same value for the present case so the amount of chemical energy released by the detonating part of the charge will be approximately ten times the energy associated with the pressure wave system.

An estimate of the total chemical energy released per cycle can be made from a knowledge of the available energy per unit mass of air in the cylinder charge and the mass of the charge. Hershey and his collaborators(97) give the chemical energy per pound of air (if sufficient fuel to burn the air is present) as

$$\text{B.T.U./lb. of air} = 1507\ (1 - f) + 300\ f \qquad (53)$$

where

f = weight fraction of combustion products in the unburned mixture.

As a reasonable assumption f can be taken as 0.05. On this basis the chemical energy associated with one pound of air is 1445 B.T.U. From figure 98 the charge density at top center is 0.225 lb/ft^3 and the chamber height is 1.17 inches from figure 97. Taking into account the area of the cylinder cross-section the volume can be computed and the weight of charge per cycle is found to be 1.26×10^{-3} pounds. The mixture had 0.085 pounds of fuel per pound of air so that the actual weight of air per cycle was 1.15×10^{-3} pounds.

Using the above data shows that the energy per cycle was 1.66 B.T.U. or $1.66 \times 778 = 1290$ ft. lb. Comparing this result with the computed pressure wave energy of 0.25 ft. lb. shows that

$$(0.25)(10)(100)/1290 \cong 0.2 \text{ per cent} \qquad (54)$$

of the total charge was involved in the initial disturbance. This estimate is unquestionably low due to the disregard of all components except that of lowest frequency and the use of an amplitude reduced by damping from the maximum. The writer feels that the result of equation (54) is somewhat better than an order of magnitude. It is reasonable to conclude that about one per cent of the total acted as the detonating part of the charge in the case considered.

Calculation of Particle Velocity

Particle velocities due to pressure waves are interesting in connection with the magnitude of the "scrubbing action" at the cylinder wall which has been advanced as an explanation of the increased heat transfer accompanying detonation. Such velocities can be calculated by means of equation (65-a) as given in Appendix B for a simple harmonic wave of frequency ν.

$$\bar{v}_a = - \frac{\Delta p_a(r,\theta,z)}{\rho_0 \, 2\pi\nu} \tag{65-a}$$

where the subscript a refers to amplitudes only.

From equation (39) the expression for excess pressure amplitude in the 100 mode is

$$p_a = p_{100} \, J_1 \left[\alpha_{10}(r/a)\right] \left[\cos\,\theta\right] \tag{55}$$

the azimuthal phase angle being reduced to zero by a proper location of the reference axis. At the cylinder boundary

$$p_a = p_{100} \, J_1 \, (\alpha_{10}) \cos \theta \tag{56}$$

From Table Ia of Appendix B $\alpha_{10} = 1.841$.

Now in cylindrical coordinates

$$p = i_1 \frac{\partial p}{\partial r} + i_2 \frac{1}{r} \frac{\partial p}{\partial \theta} + i_3 \frac{\partial p}{\partial z} \tag{57}$$

where i_1, i_2, i_3 are unit vectors along r, along the tangent at r and along z respectively.

At the circular boundary in the present case

$$\frac{\partial p}{\partial z} = 0 \text{ since } n_z = 0 \tag{58}$$

$$\frac{\partial p}{\partial r} = 0 \text{ since radial motion is zero at the cylinder wall,} \tag{59}$$

so that the tangential velocity is

$$v_{ta} = - \frac{p_{100}}{a} \; \frac{J_1(1.841)}{\rho_o \, 2\pi\nu} \; \partial \, (\cos \theta)/\partial\theta \tag{60}$$

In the present case

$$\rho_o = \frac{0.225}{32} = 0.703 \times 10^{-2} \text{ slugs/ft}^3$$

$$\nu = 6900 \text{ cycles/second}$$

$$a = \frac{3.25}{(2)(12)} = 0.135 \text{ ft.}$$

$$p_{100} = (30.4)(144) = 4{,}370 \text{ lb/ft}^2$$

The tangential velocity becomes

$$v_{ta} = \frac{(4,370)(0.5819)\ \sin\theta}{(0.135)(0.703)(10^{-2})(2\pi)(6900)} = 61 \text{ ft/second} \quad (61)$$

Taking the case of $\theta = 0$ and integrating to find the particle displacement δ_{ta} gives

$$\delta_{ta} = \frac{12 \times 61}{2\pi\ 6900} = 168 \times 10^{-4} = 0.017 \text{ inch.} \quad (62)$$

Both the velocity and amplitude and the displacement amplitude appear to have reasonable values. So far as the writer knows there is no quantitative information available on the effect of high frequency sinusoidal particle motion on heat transfer from hot gases to a solid surface. However, it is reasonable to expect that a violent disturbance would tend to reduce the thickness of the surface film and favor an increased heat flow.

General Conclusions

The illustrative example discussed above shows that the methods of analysis developed in Section IV can be conveniently applied to practical cases.

The indicating equipment will withstand actual engine conditions and produce satisfactory records of the pressure disturbances accompanying detonation.

Comparison of experimental frequencies with the predictions of sound theory shows that the theory will give substantially correct results when applied to the pressure waves accompanying detonation.

Calculated values of pressure amplitude, energy density, particle velocity amplitude and particle displacements all appear to be reasonable in the case considered.

The results obtained certainly justify further quantitative studies of detonation by means of the equipment and methods described in the present report.

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APPENDIX A

EQUATION OF MOTION OF A FLUID PARTICLE WITHIN THE BODY OF A FLUID

(References - P. M. Morse - "Vibration and Sound", McGraw-Hill, 1936

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Pressure wave phenomena are governed by the interplay between inertial reactions of particles and pressure gradients within the body of a continuous medium. Mathematical treatments of wave motion are based on a combination of three simple relationships:

1). The equation of motion of a particle within the medium.

2). The equation of continuity.

3). The equation of density as a function of pressure.

Fluid media are characterized by their inability to support shear. All the important cases to be considered in the present study deal with waves in a gas so it will be sufficient to study wave theory as applied to fluids. In the usual derivations a rectilinear coordinate system is used since this gives the least complicated physical picture. It is a simple matter to step from the rectilinear to any other type of coordinates by a mathematical transformation.

EQUATION OF MOTION

Consider an elementary cube of dimensions dx, dy, dz with its center at (x,y,z) located within the body of a fluid medium. The pressure P within the fluid will be considered as a function of position. If tangential forces are neglected in accordance with the assumption of a fluid, the force on the left-hand face of the cube is (in figure A-1)

$$\left(P - \partial P/\partial y \cdot dy/2 \right) dx\, dz \qquad (1\text{-a})$$

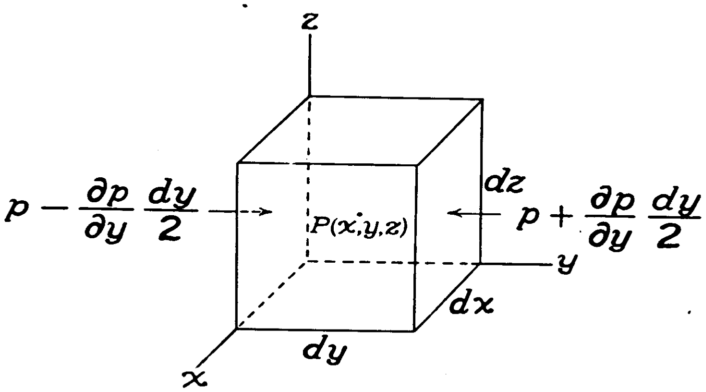
Similarly the force on the opposite face of the cube is

$$-\left(P + \partial P/\partial y \cdot dy/2\right) dx\, dz \qquad (2\text{-a})$$

Taking the algebraic sum of these forces gives the net pressure force acting in the direction of the positive y axis as

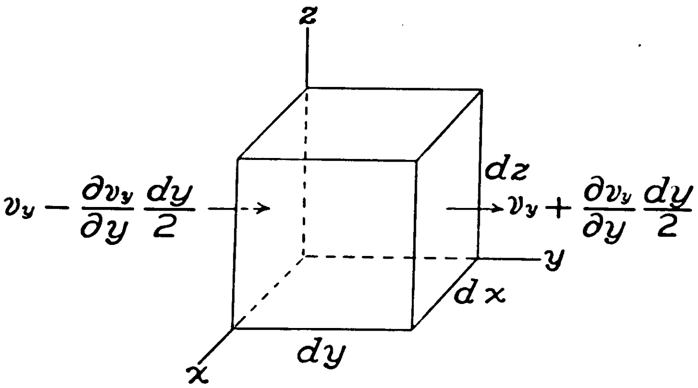
$$- \partial P/\partial y \cdot dx\, dy\, dz \qquad (3\text{-a})$$

In general the fluid particle will also be acted upon by body forces such as gravity which are proportional to the



PRESSURE ON FACES OF VOLUME ELEMENT

FIG. A-1



VELOCITY INTO VOLUME ELEMENT

FIG. A-2

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mass of the cube. If the component of body force acting along the positive y axis is taken as F_y, the positive net force in this direction is

$$F_y \rho \, dx \, dy \, dz - \partial P/\partial y \, dx \, dy \, dz \qquad (4\text{-a})$$

Where ρ is the density.

This force acts to accelerate the elementary cube in the positive y direction. This acceleration must be found by considering the velocity a function both of time and of position in the fluid. If the y component of velocity is called v_y, then

$$v_y = f(x,y,z,t) \qquad (5\text{-a})$$

so that, a_y, the corresponding component of acceleration is

$$a_y = dv_y/dt = \partial v_y/dx. \, dx/dt + \partial v_y/\partial y.dy/dt + \partial v_y/\partial z.dz/dt + \partial v_y/\partial t \qquad (6\text{-a})$$

But the velocity components v_x, v_y, v_z

$$v_x = dx/dt \;,\; v_y = dy/dt \;,\; v_z = dz/dt \qquad (7\text{-a})$$

and the y component of acceleration can be written

$$a_y = v_x \, \partial v_y/\partial x + v_y \, \partial v_y/\partial y + v_z \, \partial v_y/\partial z + \partial v_y/\partial t \qquad (8\text{-a})$$

Multiplying the y-component of acceleration by the mass within the cube gives the inertia reaction in this direction. Combining this expression with equation (4) gives the equation for motion along the y axis.

$$(v_x\partial v_y/\partial x + v_y\partial v_y/\partial y + v_z\partial v_y/\partial z + \partial v_y/\partial t)\ \rho\ dx\ dy\ dz = -\partial P/\partial y\ .\ dx\ dy\ dz + F_y\ \rho\ dx\ dy\ dz \qquad (9\text{-a})$$

The two expressions for the x and z axes are similar to equation (9-a). All three of these equations can be combined into a single vector equation.

$$\partial\bar{v}/\partial t + (\bar{v}.\nabla)\ \bar{v} + \nabla P/\rho - \bar{F} = 0 \qquad (10\text{-a})$$

In equation (10-a) a bar above a quantity indicates a vector. In equation (9-a) the first three terms in the parenthesis are present because of acceleration effects due to motion of a fluid element between regions of varying velocity. For the case of small displacements from equilibrium in a medium without strong velocity gradients, the space derivative terms can be neglected in comparison with the time derivative term. Also over short distances in a medium of low density the effects of body forces are negligible under ordinary circumstances. Making these approximations, the equation of motion becomes

$$\partial\bar{v}/\partial t + 1/\rho\ .\nabla P = 0 \qquad (11\text{-a})$$

This equation has the form used in the ordinary theory of sound. It states that the quantities which determine motion in sound phenomena are the pressure gradient and direct accelerations due to small particle displacements.

EQUATION OF CONTINUITY

The equation of continuity is a statement of the balance between the rate of change of mass within an elementary volume of a fluid and the rate at which mass flows across the boundaries of this volume.

Referring to figure A-2 the amount of mass flowing in across the left-hand face in a time dt is

$$\left[\rho\, v_y - \partial(\rho\, v_y)/\partial y \,.\, dy/2\right] dx\, dz\, dt \qquad (12\text{-a})$$

Similarly the mass flowing in across the right-hand face during the same time interval is

$$-\left[\rho\, v_y + \partial(\rho\, v_y)/\partial y \,.\, dy/2\right] dx\, dz\, dt \qquad (13\text{-a})$$

The net inflow across faces perpendicular to the y axis is thus

$$-\,\partial(\rho\, v_y)/\partial y \,.\, dx\, dy\, dz\, dt \qquad (14\text{-a})$$

Including similar equations for the flow in along the other two axes and equating this to the increase in mass within the boundaries for the time dt gives

$$-\left[\partial(\rho\, v_x)/\partial x + \partial(\rho\, v_y)/\partial y + \partial(\rho\, v_z)/\partial z\right] dx\, dy\, dz\, dt = \partial\rho/\partial t \,.\, dx\, dy\, dz\, dt \qquad (15\text{-a})$$

In vector form this becomes

$$\nabla(\rho\, \bar{v}) + \partial\rho/\partial t = 0 \qquad (16\text{-a})$$

or

$$\rho\nabla\bar{v} + \bar{v}\nabla\rho + \partial\rho/\partial t = 0 \qquad (17\text{-a})$$

For the case of ordinary sound waves the velocities are so small and the density gradients are so slight that $\bar{v}\nabla\rho$ can be neglected in comparison with the other two terms. Under this assumption the equation of continuity becomes

$$\rho\nabla\bar{v} + \partial\rho/\partial t = 0 \qquad (18\text{-a})$$

PRESSURE AS A FUNCTION OF DENSITY

Changes occurring in a sound wave are so rapid that the effect of heat conduction between regions of different temperature does not play an essential part in the process. The truth of this condition is checked by experimental results. It is thus satisfactory in ordinary cases to assume that the relation between pressure and density follows the adiabatic law for which the derivation is outlined below.

For a given system of unit mass the law of energy conservation is

$$\delta Q = C_V \, dT + A \, P \, dV \qquad (19\text{-a})$$

where

Q = heat energy added to the system.

T = Absolute Temperature.

V = Volume.

C_V = Specific heat at constant volume.

A = Reciprocal of the mechanical equivalent of heat.

If the medium is a perfect gas

$$P \, V = R \, T/M \qquad (20\text{-a})$$

where

R = Universal gas constant for one mole.

M = Molecular weight of the gas.

Differentiating (20-a) gives

$$P \, dV + V \, dP = R/M . \, dT \qquad (21\text{-a})$$

By substitution in equation (19-a)

$$\delta Q = (C_v + A \, R/M) \, dT - A \, V \, dP \qquad (22\text{-a})$$

In this equation the coefficient of the temperature term is C_P, the specific heat of the gas at constant pressure.

For an adiabatic process (no heat interchange with the surroundings) $\delta Q = 0$

and after substitution for T in terms of P and V

$$C_P/C_V \cdot dV/V = dP/P \tag{23-a}$$

If $\gamma = C_P/C_V$, the adiabatic equation becomes

$$dP/d\rho = \gamma \, P/\rho = c^2 \tag{24-a}$$

where c will later prove to be the velocity of sound.

EQUATION OF SOUND

Equations (18-a) and (24-a) can be obtained to give

$$\nabla \bar{v} = - 1/\rho \cdot d\rho/dP \cdot \partial P/\partial t = - 1/(\rho \, c^2) \cdot \partial P/\partial t \tag{25-a}$$

With time and displacement as independent variables, equation (25-a) can be differentiated and written

$$\nabla(\partial \bar{v}/\partial t) = - \partial\left(1/(\rho \, c^2) \cdot \partial P/\partial t\right)/\partial t \tag{26-a}$$

Applying the operator ∇ to equation (11-a) gives

$$\nabla(\partial \bar{v}/\partial t) = - \nabla(1/\rho \cdot \nabla P) \tag{27-a}$$

Equating the right-hand sides of these expressions gives

$$\partial\left(1/(\rho \, c^2) \cdot \partial P/\partial t\right)/\partial t = \nabla (1/\rho \cdot \nabla P) \tag{28-a}$$

No generally useful solution of this equation is possible. However, if consideration is limited to changes which are so small that the rate of change of pressure with density can be treated as constant (c^2 = constant) and variations in density are so small that

$$(\rho - \rho_o)/\rho_o << 1, \qquad (29\text{-a})$$

where ρ_o is the density with no disturbance present, then equation (28-a) becomes

$$\partial^2 P/\partial t^2 = c^2 \nabla P \qquad (30'\text{-a})$$

This is the Wave Equation which forms the basis for the theory of sound.

It will be noted that if $p = P - P_o$, where P_o is the pressure with no disturbance present (equilibrium pressure), $\partial P/\partial t = \partial p/\partial t$ and $\nabla P = \nabla p$. Thus equation (30'-a) can be written

$$\partial^2 p/\partial t^2 = c^2 \nabla p \qquad (30\text{-a})$$

This pressure p will be called the excess pressure.

For one dimensional disturbance such as plane waves moving along an x axis, the general solution can be written as

$$p = F(ct - x) + f(ct + x) \qquad (31\text{-a})$$

Thus a pressure disturbance whose shape is once established will maintain its form but will move along the x direction with the velocity c required to keep the arguments of F and f constant.

EFFECTS NEGLECTED IN THE EQUATION OF INFINITESIMAL WAVES

Dissipative influences such as viscosity, heat conduction and radiation have been neglected in the derivation of equation (30-a). In general the effect of dissipation will be to cause attenuation of a pressure wave as it moves through an elastic medium. However, the greater mathematical difficulties introduced in a treatment which already contains a number of approximations can be justified only in special cases. In practice attenuation effects can be neglected over short time intervals and small distances without serious errors.

The distinction between infinitesimal waves as studied above and waves of finite amplitude lies in the departure of an actual medium from the ideal assumptions of linearity. That is, the coefficient $dP/d\rho$ which determines the velocity of a disturbance varies from its equilibrium value if the changes are too great. Various investigators have studied waves of finite amplitude but the results are too clumsy for application to complex physical problems.

In the present study of pressure waves accompanying detonation, the various complicating effects will be kept in mind for interpretation of results but as a matter of practical necessity will be disregarded in the mathematical treatment.

SIMPLE HARMONIC VIBRATIONS

Actual pressure waves can often be analysed into simple harmonic components of various frequencies. For the case of a single frequency this corresponds to the assumption

$$p = \varphi(x,y,z,)\ \varepsilon^{-i\omega t}$$

where (32-a)

$\omega = 2\pi\nu = 2\pi c/\lambda = 2\pi/T$

ν = frequency

T = period

λ = wavelength in case of plane waves and is defined as above in terms of c and ν for other cases.

Substituting the expression p into equation (30-a).

$$(\nabla^2 + \omega^2/c^2)\varphi = 0 \qquad (33\text{-a})$$

The various useful results found below will depend upon finding solutions of equation (33-a) to fit specific physical cases.

ENERGY IN PRESSURE WAVES

In any elementary volume like that of figure I the energy associated with the wave motion can be considered as associated with the particles within the volume. The equation for particle velocity can be found by differentiating equation (11-a) with respect to time and applying the operator to equation (25-a). Combining the resulting expressions gives

$$\partial^2\bar{v}/\partial t^2 = c^2 \nabla^2\bar{v} \qquad (34\text{-a})$$

It follows that the particle velocity also satisfies the wave equation.

Equation (11-a) states that

$$\partial\bar{v}/\partial t = - 1/\rho_o \cdot \nabla p \qquad (11\text{-a})$$

so that if pressure is a simple harmonic function of time, particle velocity must also be a simple harmonic function of time having a like frequency but with a phase difference of $\pi/2$. Thus if at a point x_o,y_o,z_o

$$p = p_a(x_o,y_o,z_o) \cos \omega t \qquad (35\text{-a})$$

then

$$\partial\bar{v}/\partial t = - 1/\rho_o \cdot \nabla p_a(x_o,y_o,z_o) \cos \omega t \qquad (36\text{-a})$$

When time is reckoned as zero from a maximum of the pressure cycle, the particle velocity at the point in question will be

$$\bar{v} = -1/\rho_o\omega \cdot \nabla p_a(x_o,y_o,z_o) \int \cos \omega t \, d(\omega t) \qquad (37\text{-a})$$

or

$$\bar{v} = - 1/\rho_o\omega \cdot \nabla p_a(x_o,y_o,z_o) \sin \omega t \qquad (38\text{-a})$$

where the constant of integration is of no importance since only disturbances from equilibrium are being considered.

Another integration with respect to time gives the particle displacement δ as

$$\delta = 1/\rho_o\omega^2 \cdot \nabla p_a(x_o,y_o,z_o) \cos \omega t \qquad (39\text{-a})$$

As a check on these results it is noted that equations (38-a) and (39-a) have the proper dimensions.

Equations (35-a) and (38-a) show that the maximum particle velocity occurs when the excess pressure is zero while the particle velocity is zero when the excess pressure is at its maximum. From an energy standpoint all the energy associated with the wave motion is kinetic when maximum particle velocity exists and all the energy is potential when maximum excess pressure exists. For this reason an integration extending from an instant of maximum excess pressure to an instant of zero excess pressure will give all the energy associated with the wave motion in a small volume containing the point x_o,y_o,z_o. This corresponds to an integration extending over one quarter of a wave cycle.

The rate at which work is done <u>by</u> the expansion is given by

$$dW/dt = + p\ (dV/dt) \qquad (40\text{-a})$$

where W is the energy involved.

Consider the rate of change of the elementary volume in figure A-2.

The rate of decrease in volume due to motion of the left-hand face of the figure is

$$(v_y - \partial v_y/\partial y \;.\; dy/2)\; dxdz \qquad (41\text{-}a)$$

The *increase* due to motion of the right-hand face is

$$(v_y + \partial v_y/\partial y \;.\; dy/2)\; dxdz \qquad (42\text{-}a)$$

The *net* increase in volume is thus

$$\partial v_y/\partial y \;.\; dxdydz \qquad (43\text{-}a)$$

Applying similar reasoning to the other faces of the elementary volume and combining the results in a single vector equation gives

$$\text{Rate of Increase of Volume} = dV/dt = \nabla \bar{v}\; dxdydz \qquad (44\text{-}a)$$

The rate at which work is done *on* material within the elementary volume becomes

$$dW/dt = -\, p\; \nabla\; \bar{v}\; dxdydz \qquad (45\text{-}a)$$

Substituting the value of $\nabla\; \bar{v}$ from equation (25-a) gives

$$dW/dt = 1/(\rho\; c^2) \;.\; p \;.\; \partial p/\partial t \;.\; dxdydz \qquad (46\text{-}a)$$

The total energy associated with the motion in the elementary volume becomes

$$W = 1/(\rho_o c^2) \;.\; dxdydz \int_0^{T/4} p \;.(\partial p/\partial t)\; dt \qquad (47\text{-}a)$$

Using the expression of equation (35-a) for p as a function of space coordinates

$$W = 1/(\rho_o c^2) \cdot dxdydz \cdot p_a^2(x_o,y_o,z_o) \int_0^{\frac{\omega T}{4}} \cos \omega t \sin \omega t \, d(\omega t)$$

or

$$W = p_a^2(x_o,y_o,z_o)/2\rho_o c^2 \cdot dxdydz \qquad (48\text{-a})$$

To get the total energy associated with pressure waves in an enclosure with fixed boundaries it is only necessary to carry out a triple integration with the proper limits.

$$W_V = 1/2\rho_o c^2 \cdot \iiint_V p_a^2(x,y,z) \, dxdydz \qquad (49\text{-a})$$

or

$$W_V = 1/2\gamma P_o \cdot \iiint_V p_a^2(x,y,z) \, dxdydz \qquad (50\text{-a})$$

Note End of App. I.

APPENDIX B

SOLUTION OF THE WAVE EQUATION FOR A CIRCULAR CYLINDER WITH FLAT ENDS

(References - P. M. Morse - "Vibration and Sound", McGraw-Hill, 1936

Lord Rayleigh - "Theory of Sound", MacMillan and Co., 1926

Horace Lamb - "Hydrodynamics", Cambridge University Press, 1932

Jahnke-Emde - "Tables of Functions", B. G. Teubner, 1933

G. N. Watson - "Theory of Bessel Functions", Cambridge, 1922)

Pressure waves within a closed container must fulfill the condition of zero particle velocity along the normal to any rigid boundary and in addition must satisfy the wave equation (30-a).

Assuming that the expression for pressure is a simple harmonic function of time

$$p = p_a(x,y,z)\ \varepsilon^{-i\omega t} \qquad (32\text{-a})$$

the function of position $p_a(x,y,z)$ must satisfy

$$(\nabla^2 + \omega^2/c^2)p_a = 0 \qquad (51\text{-a})$$

In order to obtain solutions of this equation to fit the case of a circular cylinder with flat ends it is necessary to express the operator ∇^2 in terms of the cylindrical coordinates r, z and θ. When this is carried out, equation (33-a) becomes

$$(\partial^2/\partial r^2 + 1/r \cdot \partial/\partial r + 1/r^2 \cdot \partial^2/\partial\theta^2 + \partial^2/\partial z^2 + \omega^2/c^2)p_a = 0 \qquad (52\text{-a})$$

Now assume that the p_a can be expressed as a product of three functions which each depend upon a single variable, i. e., that

$$p_a = R(r)Z(z)\Theta(\theta) \qquad (53\text{-a})$$

Substitution in equation (34-a) gives

$$\partial^2R/\partial r^2 . Z\Theta + Z\Theta/r \cdot \partial R/\partial r + RZ/r^2 \cdot \partial^2\Theta/\partial\theta^2 + R\Theta . \partial^2Z/\partial z^2 + 4\pi^2/\lambda^2 \cdot RZ = 0 \qquad (54\text{-a})$$

Dividing by $RZ\Theta$ gives

$$1/R \cdot \partial^2R/\partial r^2 + 1/R\,r \cdot \partial R/\partial r + \overset{1}{\cancel{\Theta}}/r^2\Theta . \partial^2\Theta/\partial\theta^2 = - 1/Z \cdot \partial^2Z/\partial z^2 - 4\pi^2/\lambda^2 = 4\pi^2/\lambda_z^2 - 4\pi^2/\lambda^2 \qquad (55\text{-a})$$

The left-hand member of this equation depends upon r and θ only while the middle member depends only upon z. Since r, z and θ are all independent variables, it follows that the right-hand member must be equal to a constant.

The constant terms combined with the terms depending only upon z give the expression

$$\partial^2 Z/\partial z^2 + (4\pi^2/\lambda_z^2)\, Z = 0 \qquad (56\text{-a})$$

The solution of this equation is

$$Z = (\mathrm{Const})\ \varepsilon^{-i2\pi z/\lambda_z} \qquad (57\text{-a})$$

Separating the terms depending upon θ from the terms depending upon r in equation (55-a) gives

$$r^2/R \,.\, \partial^2 R/\partial r^2 + r/R \,.\, \partial R/\partial r + (4\pi^2/\lambda^2 - 4\pi^2/\lambda_z^2) r^2 =$$

$$- 1/\Theta \,.\, \partial^2 \Theta/\partial \theta^2 = n_\theta^2 \qquad (58\text{-a})$$

An argument similar to that given above shows that n_θ must be constant.

The equation in θ alone is

$$\partial^2 \Theta/\partial\theta^2 + n_\theta^2 \Theta = 0 \qquad (59\text{-a})$$

of which the solution is

$$\Theta = (\mathrm{Const})\ \varepsilon^{-in_\theta \theta} \qquad (60\text{-a})$$

Now define λ_{n_θ, n_r} by the equation

$$1/\lambda^2_{n_\theta n_r} = 1/\lambda^2_{n_\theta n_r n_z} - 1/\lambda_z^2 \qquad (61\text{-a})$$

where $\lambda_{n_\theta n_r n_z}$ is written for the symbol previously called λ. The left-hand side of equation (58-a) can be written as

$$\partial^2R/\partial r^2 + 1/r \cdot \partial R/\partial r + (4\pi^2/\lambda^2_{n_\theta n_r} - n^2_\theta/r^2)R = 0 \quad (62\text{-a})$$

This is a form of Bessel's Equation which is solved by Bessel's and Neuman functions, i.e.,

$$R = (\text{Const})\, J_{n_\theta}(2\pi r/\lambda_{n_\theta n_r}) + (\text{Const})\, N_{n_\theta}(2\pi r/\lambda_{n_\theta n_r}) \quad (63\text{-a})$$

In this solution, n_θ is restricted to integral values.

Combining equations (57-a), (60-a) and (63-a) with trignometric function substituted for the exponentials gives

$$p_a(r,\theta,z) = (\text{Const}) \begin{bmatrix} J_{n_\theta}(2\pi r/\lambda_{n_\theta n_r}) \\ N_{n_\theta}(2\pi r/\lambda_{n_\theta n_r}) \end{bmatrix} \begin{bmatrix} \sin n_\theta\theta \\ \cos n_\theta\theta \end{bmatrix} \begin{bmatrix} \sin 2\pi z/\lambda_z \\ \cos 2\pi z/\lambda_z \end{bmatrix} \quad (64\text{-a})$$

Where either or both terms in the individual brackets can be used. It remains to fit this equation to the particular conditions of a circular cylinder with flat ends normal to the cylinder axis. This is done by considering the requirements on the pressure equation at the cylinder boundaries.

From equation (38-a) the amplitude $\bar{v}_a$ of the particle velocity is

$$\bar{v}_a = - \nabla p_a/\rho_o\omega \qquad (65\text{-a})$$

It follows that the velocity amplitude in a certain direction is proportional to the gradient of the excess pressure amplitude along the same direction. Thus at a fixed point, the radial velocity will be proportional to the gradient of the excess pressure amplitude along the radius. This relation will be used in fitting the expression for excess pressure to the boundary conditions.

Geometrically the cylinder radius identified with any value of θ is the same as that corresponding to θ with the addition of an integral number of complete revolutions. For this reason, n_θ must be restricted to integral values if the physical requirement that p_a be a single valued function of position is fulfilled.

When n_θ is taken as zero there will be variation of p_a with angle which corresponds to a symmetrical pressure distribution. Within $n_\theta = 1$ there will be a plane through diameter over which no pressure variation exists. Such a plane is called _an azimuthal node_. In general there will be a number of azimuthal nodes equal to the value of n_θ.

At the bottom ($z = 0$) and top ($z = h$) of the cylinder the pressure gradient in the z direction must be zero, i.e.,

$$\partial\rho_a/\partial z = \left\{----\right\}\begin{bmatrix} 2\pi/\lambda_z \cos 2\pi z/\lambda_z \\ 2\pi/_{\lambda_z} \sin 2\pi z/\lambda_z \end{bmatrix} \begin{matrix} = 0\ ;\ z = 0 \\ = 0\ ;\ z = h \end{matrix} \qquad (66\text{-a})$$

The first requirement is fulfilled if the sine term is chosen. The second condition is satisfied if

$$2\pi h/\lambda_z = n_z\pi \qquad (67\text{-a})$$

where n_z is restricted to integral values. The wavelength λ_z associated with axial waves is thus

$$\lambda_z = 2h/n_z \qquad (68\text{-a})$$

Substituting this result in the part of equation (64-a) which is dependent upon z gives

$$p_a = \left\{-------\right\}\cos \pi n_z z/h \qquad (69\text{-a})$$

Thus if n_z is zero there will be no variation in pressure in an axial direction and the wave problem reduces to one of two dimensions. With $n_z = 1$ there will be a single nodal plane normal to the cylinder axis at h/2. The number of axial nodes will be equal to the value of n_z. At the circular walls of the cylinder the radial pressure gradient must be zero. Differentiating equation (64-a) with respect to r gives

$$\partial p_a/\partial r = \left\{-----\right\}\begin{bmatrix} \partial J_{n_\theta}(2\pi r/\lambda_{n_\theta n_r})\ /\partial r \\ \partial N_{n_\theta}(2\pi r/\lambda_{n_\theta n_r})\ /\partial r \end{bmatrix} \qquad (70\text{-a})$$

The slope of the Neuman function becomes infinite as the argument approaches zero. Since an infinite pressure gradient is impossible, the Neuman function is excluded and the radial boundary condition becomes at a, the radius of the cylinder wall

$$\partial p_a/\partial r = \left\{ \text{-----} \right\} \partial J_{n_\theta}(2\pi a/\lambda_{n_\theta n_r}) = 0 \qquad (71\text{-a})$$

In this equation n_r is used to designate the ordinal number of the root considered. When $n_r = 0$, no radial node exists. For $n_r = 1$ there is one nodal cylinder and in general the value of n_r is equal to the number of radial nodes (this disregards nodal line coinciding with axis for $n_\theta > 0$)

A table of values of the argument which solves equation (71-a) is given in Table 1-a for different numbers of radial and diametrical nodes.

		NUMBER OF RADIAL NODES		
		0	1	2
Number of Azimuthal Nodes	0	$\alpha_{00} = 0$	$\alpha_{01} = 3.832$	$\alpha_{02} = 7.016$
	1	$\alpha_{10} = 1.841$	$\alpha_{11} = 5.332$	$\alpha_{12} = 8.536$
	2	$\alpha_{20} = 3.05$	$\alpha_{21} = 6.75$	$\alpha_{22} = 10.00$

Where $\alpha_{ij} = 2\pi a/\lambda_{ij}$

For the calculation of frequencies it is convenient to tabulate $\beta_{n_\theta n_r}$ defined as the ratio of $\lambda_{n_\theta n_r}$ to the cylinder radius, a, i.e.

NUMBER OF RADIAL NODES

Number of Azimuthal Nodes		0	1	2	
	0	$\beta_{00} = 00$	$\beta_{01} = 1.641$	$\beta_{02} = 0.895$	
	1	$\beta_{10} = 3.41$	$\beta_{11} = 1.18$	$\beta_{12} = 0.736$	
	2	$\beta_{20} = 2.06$	$\beta_{21} = 0.931$	$\beta_{22} = 0.628$	(72-a)

It remains to develop the relationship between frequency, sound velocity and cylinder dimensions. From equation (61-a)

$$1/\lambda^2_{n_\theta n_r n_z} = 1/\lambda^2_{n_\theta n_r} + 1/\lambda^2_z \qquad (73\text{-a})$$

Substituting from equations (68-a) and (72-a) gives

$$1/\lambda^2_{n_\theta n_r n_z} = 1/\beta^2_{n_\theta n_r} a^2 + n_z^2/4h^2 \qquad (74\text{-a})$$

Placing equation (74-a) in non-dimensional form gives

$$\lambda_{n_\theta n_r n_z} /a = 1/ \sqrt{1/\beta^2_{n_\theta n_r} + (n_z^2/4)(a/h)^2} \qquad (75\text{-a})$$

The frequency corresponding to a certain nodal configuration in a cylinder with a certain ratio of radius to height can be calculated from the equation

$$\nu_{n_\theta n_r n_z}(a/h) = (c/a) \frac{a}{\lambda_{n_\theta n_r n_z}(a/h)} \qquad (76\text{-a})$$

For convenience the subscripts will be dropped from the symbols ν and λ when they are used in the various formulae. It will always be understood that the values of ν and λ depend upon conditions in a manner given by equation (76-a). Values of $\frac{\lambda_{n_\theta n_r n_z}(a/h)}{a}$ can be taken from the curves and tables specially prepared for this purpose and given in the body of this report.

Summarizing the various points discussed above leads to an expression for the instantaneous excess pressure in a circular cylinder with flat ends.

$$p = \sum_{n_\theta n_r n_z} p_{n_\theta n_r n_z} J_{n_\theta}(2\pi r/\lambda_{n_\theta n_r}).\cos(n_z \pi z/h).\cos(n_\theta \theta + \varphi).\sin(2\pi\nu t + \varepsilon) \qquad (77\text{-a})$$

Where

φ = phase angles associated with azimuth

ε = phase angles associated with time.

This expression takes into account all the allowed frequencies and modes of vibration possible in an elastic medium enclosed by a circular cylinder with flat ends normal to the cylinder axis.

APPENDIX C

RELATION BETWEEN PRESSURE WAVE AMPLITUDE AND ENERGY DENSITY

In Appendix A it is shown that the energy associated with a pressure wave system of a single frequency is

$$W = 1/\alpha P_0 \iiint \rho^2 (x,y,z)\, dx\, dy\, dz \qquad (78\text{-a})$$

In Appendix B the expression for excess pressure in a circular cylinder with flat ends is given as equation (77-a). For calculation of energies only the space dependent part of each term is required. Thus for each component

$$W_{n_\theta n_r n_z} = \frac{\rho^2_{n_\theta n_r n_z}}{\gamma P_0} \int_0^{r/a=1} \int_0^{\theta = 2\pi} \int_0^{z/h=1} J^2_{n_\theta} \left[\alpha_{n_\theta n_r} (r/a) \right] \cos^2(n_\theta \theta) \cos^2(n_z \pi z/h)\, r\, dr\, d\theta\, dz \qquad (79\text{-a})$$

Integration of the term dependent upon the radius r gives

$$\frac{a^2}{\alpha^2_{n_\theta n_r}} \int_0^{\alpha_{n_\theta n_r}} J^2_{n_\theta}(x)dx = \frac{a^2}{\alpha^2_{n_\theta n_r}} \left\{ x^2/2 \left[J^2_{n_\theta}(x) - J_{n_\theta - 1}(x) J_{n_\theta + 1}(x) \right] \right\} \Bigg|_0^{\alpha_{n_\theta n_r}} \qquad (80\text{-a})$$

Substituting the limits

$$\int_0^{r/a=1} J^2_{n_\theta}(\alpha_{n_\theta n_r}\ r/a)r\,dr = a^2/2\left[J^2_{n_\theta}(\alpha_{n_\theta n_r}) - J_{n_\theta - 1}(\alpha_{n_\theta n_r})\ J_{n_\theta + 1}(\alpha_{n_\theta n_r})\right] \tag{81-a}$$

The term depending upon azimuth is

$$\int_0^{2\pi} \cos^2(n_\theta\theta - \varphi_{n_\theta n_r})d\theta \begin{aligned} &= \pi \text{ if } n_\theta \neq 0 \\ &= 2\pi \text{ if } n_\theta = 0 \end{aligned} \tag{82-a}$$

Similarly

$$\int_0^{z/h=1} \cos^2(n_z\pi\ z/h)dz \begin{aligned} &= h/2 \text{ if } n_z \neq 0 \\ &= h \text{ if } n_z = 0 \end{aligned}$$

Thus

$$W_{n_\theta n_r n_z} = (1/4)(\rho^2_{n_\theta n_r n_z}/\gamma P_0)(\pi a^2 h) \cdot \left\{J^2_{n_\theta}(\alpha_{n_\theta n_r}) - J_{n_\theta - 1}(\alpha_{n_\theta n_r})\ J_{n_\theta + 1}(\alpha_{n_\theta n_r})\right\} \tag{83-a}$$

if $n_\theta \neq 0$ and $n_z \neq 0$.

If either n_θ or n_z is zero the energy will be twice that given by equation (83-a). If both n_θ and n_z

are zero the energy will be four times that given by equation (83-a).

Taking into account that $\pi a^2 h$ is just the total volume of the cylinder equation (83-a) can be written

$$M_{n_\theta n_r n_z} = \frac{E_{n_\theta n_r n_z}}{\frac{\overline{\rho^2}_{n_\theta n_r n_z}}{\gamma P_0}} = K \left[J^2_{n_\theta} (\alpha_{n_\theta n_r}) - J_{n_\theta-1} (\alpha_{n_\theta n_r}) \, J_{n_\theta+1} (\alpha_{n_\theta n_r}) \right] \qquad (84\text{-a})$$

where $E_{n_\theta n_r n_z}$ is the average energy density associated with the particular frequency considered.

Table II gives values of the ratio $M_{n_\theta n_r n_z}$ for a number of the lowest frequency terms.

APPENDIX D

PRESSURE WAVES DUE TO AN ARBITRARY INITIAL DISTURBANCE

The problem of determining details of an initial disturbance from pressure records has been discussed in the text. Experience has shown that it is more practical to work from assumed excitations and calculate the resulting wave system than to attempt analysis in the other direction. Thus the waves at a given location due to a series of different excitations can be calculated and the proper type of excitation to fit a given case selected by comparison with the actual pressure records.

As a reasonable approximation, it is assumed that the initial disturbance can be represented as a region of constant excess pressure, p_o, with the boundaries r_1 and a, $-\theta_1$ and θ_1, 0 and z_1. Starting with these data, the analytical procedure is intended to determine magnitudes of the pressure amplitude coefficients, $p_{n_\theta n_r n_z}$, in equation (77-a) for the given case.

Both sides of equation (77-a) are multiplied by the trancendental functions of the right hand side and integration is carried out over the entire cylinder volume for t = 0. At t = 0 the excess pressure is zero everywhere except in the region of the initial disturbance, so the limits of the left hand integral are just the disturbance boundaries. The resulting equation is

$$p_{n_\theta n_r n_z} = \frac{\int_{r_1/a}^{1}\int_{-\theta_1}^{\theta_1}\int_{0}^{z_1/h} p_0 J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)]\cos(n_\theta\theta - \varphi_{n_\theta n_r})\cos(n_z\pi z/h)\,r\,dr\,d\theta\,dz}{\cos\epsilon_{n_\theta n_r n_z}\int_{0}^{1}\int_{0}^{2\pi}\int_{0}^{1} J^2_{n_\theta}[\alpha_{n_\theta n_r}(r/a)]\cos^2(n_\theta\theta - \varphi_{n_\theta n_r})\cos^2(n_z\pi z/h)\,r\,dr\,d\theta\,dz} \qquad (85\text{-a})$$

The integral in the denominator has already been considered in the discussion of energy density in Appendix C and expressed as the ratio $M_{n_\theta n_r n_z}$. If this ratio is used, equation (85-a) becomes

$$p_{n_\theta n_r n_z} = \frac{p_0}{\cos\epsilon_{n_\theta n_r n_z} M_{n_\theta n_r n_z} V}\int_{r_1/a}^{1}\int_{-\theta_1}^{\theta_1}\int_{0}^{z_1/h} J_{n_\theta}[\alpha_{n_\theta n_r}(r/a)]\cos(n_\theta\theta - \varphi_{n_\theta n_r})\cos(n_z\pi z/h)\,r\,dr\,d\theta\,dz \qquad (86\text{-a})$$

Where V is the total cylinder volume.

The integrals involving z and θ can be written down immediately, but the Bessel's Function integral is more difficult to evaluate. Now introduce a new variable, β, as defined below

$$\beta = a_{n_\theta n_r n_z}(r/a) \qquad (87\text{-a})$$

so that

$$r\,dr = a^2/\alpha^2_{n_\theta n_r} \cdot \beta\,d\beta \qquad (88\text{-a})$$

Making this substitution in the integral on r

$$\int_{r_1/a}^{1} J_{n_\theta}\, \alpha_{n_\theta n_r} (r/a) r \, dr = a^2/\alpha^2_{n_\theta n_r} \int_{(r_1/a)\alpha_{n_\theta n_r}}^{\alpha_{n_\theta n_r}} J_{n_\theta}(\beta)\, \beta \, d\beta \qquad (89\text{-a})$$

The phase angle of the azimuthal integral depends upon an arbitrary selection of the reference axis and is unimportant for the integration, so that the azimuth integral becomes

$$\int_{-\theta_1}^{\theta_1} \cos (n_\theta \theta) \; d\theta = 2/n_\theta \; . \sin \; n_\theta \theta_1 \qquad (90\text{-a})$$

Similarly the integral in z is

$$\int_{0}^{z_1/h} \cos (n_z \pi z/h) \; dz = (h/n_z \pi) \sin (n_z \pi \; z_1/h) \qquad (91\text{-a})$$

Now define A, R and Z by equations (92-a), (93-a) and (94-a), where

$$A = \frac{\sin (n_\theta \theta)}{n_\theta} \qquad (92\text{-a})$$

$$R = 1/\alpha^2_{n_\theta n_r} \int_{(r_1/a)\alpha_{h_\theta n_r}}^{\alpha_{n_\theta n_r}} J_{n_\theta}(\beta)\ \beta\ d\beta \qquad (93\text{-a})$$

$$Z = \frac{\sin(n_z \pi z_1/h}{n_z} \qquad (94\text{-a})$$

Remembering that $V = \pi a^2 h$, substituting A, R and Z, and taking the time phase angle as zero since only magnitudes are being considered, equation (86-a) can be written

$$p_{n_\theta n_r n_z}/\ p_o = 2/\pi^2 M_{n_\theta n_r n_z}\ (A\ .\ R\ .\ Z) \qquad (95\text{-a})$$

Special cases arise when no nodes exist in one or more of the coordinates. These are listed below.

n_θ	n_r	n_z	$p_{n_\theta n_r n_z}/\ p_o$				
0	n_r	n_z	$2/\pi M$	$(\theta/\pi$	R	$Z)$	(96-a)
n_θ	n_r	0	$4/\pi M$	$(A$	R	$z_1/h)$	(97-a)
n_θ	0	0					
0	0	n_z	$2/\pi M$	$(\theta/\pi$	$(1 - r_1^2/a^2)$	$Z)$	(98-a)
0	n_r	0	$2/M$	$(\theta/\pi$	R	$z_1/h)$	(99-a)

In practice the initial disturbance of detonation takes place adjacent to one wall of the cylinder and the wave pattern therefore tends to be unsymmetrical. This means that radial nodes will not usually occur and the case represented by equation (97-a) is the most generally useful. Figure ~~77~~ 76 of the text shows the variation of the integral, R, as the radial extent of the initial disturbance varies. The effect of changing the azimuthal spread is evidently sinusoidal. The amplitude ratio increases directly with the axial extent of the excitation. The simple cases of one and two azimuthal nodes are illustrated in figures ~~78~~ 77 and ~~79~~ 78 of the text.

APPENDIX E

DESIGN OF ELECTROMAGNETIC SYSTEMS FOR RATE OF CHANGE OF PRESSURE INDICATORS

Diaphragm motions can be converted into electrical voltages by means of an electromagnetic generator. Three general types of generators can be adapted to meet the rigid requirements of size and ruggedness necessary for an instrument to operate in the internal combustion engine.

Each type must include a source of magnetomotive force which establishes flux across an air gap. A coil of wire is so positioned that a change in air gap causes a change in the flux linkages so that a voltage proportional to the rate of linkages is generated. This coil may be rigidly attached to the diaphragm as in the type described below as the moving coil or the coil may be stationary as in the generators designated below as inductor types. Three possible generator types are considered below with respect to their suitability for engine indicators. The basis of comparison will be the voltage output produced by a given diaphragm velocity.

A. MOVING COIL GENERATOR

1) General

The essentials of this generator type are shown in figure A-3.

Let

r_d = radius of the diaphragm taken as the radius of the outer air gap surface.

r_c = radius of inner core of the air gap.

L = effective width of the air gap (somewhat greater than the physical gap length).

φ = total effective flux in air gap.

B = flux density at inner surface of the air gap.

N = number of turns in the generating coil.

Δx = motion of diaphragm in a time interval Δt.

F = magneto-motive force applied across the air gap.

p = r_d/r_c

The number of flux lines per unit length of the air gap is φ/L.

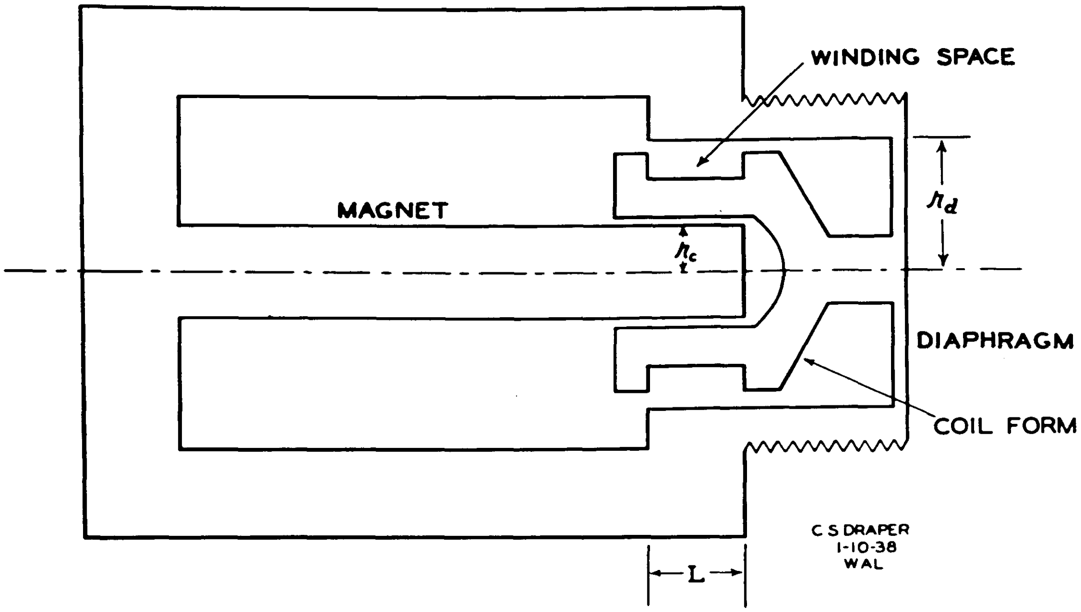
In a motion Δx the N turns of wire will cut

$$N\varphi\Delta x/L \qquad (100\text{-a})$$

lines of flux. If this motion takes place in time Δt the generated voltage will be

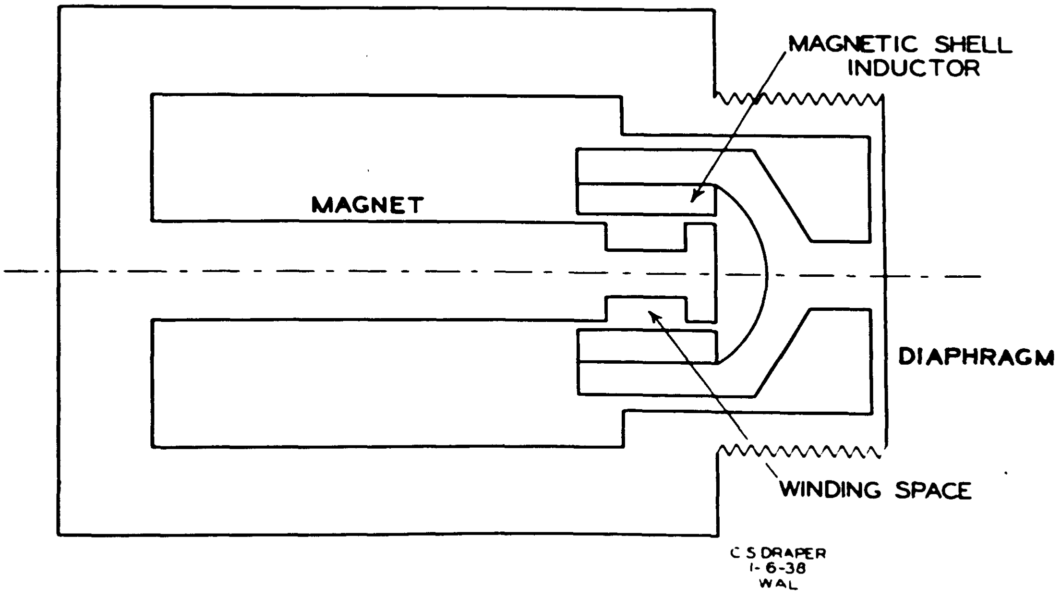
$$e = -(N\varphi/L)(dx/dt) \times 10^{-8} \qquad (101\text{-a})$$

for the limit of small motions.



ELEMENTS OF INDICATOR WITH MOVING COIL GENERATOR

FIG. A-3



ELEMENTS OF INDICATOR WITH SHELL INDUCTOR GENERATOR

FIG. A-4

In terms of flux density equation (101-a) becomes

$$e = -2\pi NBr_c\ (dx/dt \times 10^{-8} = -\left(2\pi\right)\left(r_d/p\right) NB\ (dx/dt) \times 10^{-8} \quad (102\text{-a})$$

2) Calculation of Flux Density

The air gap reluctance is

$$R = \left(1/2\pi L\right) \ln\ (r_d/r_c) \qquad (103\text{-a})$$

In the case of values of (r_d/r_c) which are near unity the linear term of the logarithmic expansion will give sufficient accuracy so that equation (102-a) becomes

$$R = 1/2\pi L\ (r_d/r_c - 1) = 1/2\pi L\ (p - 1) \qquad (104\text{-a})$$

It is reasonable to adjust the effective gap width to such a value that the flux density at the gap surface is equal to the flux density in the core, i.e.

$$r_c^2 = 2r_cL \qquad (105\text{-a})$$

$$L = r_c/2 = r_d/2p \qquad (106\text{-a})$$

The gap reluctance becomes

$$R = 1\ p\ (p - 1)/\pi r_d \qquad (107\text{-a})$$

and the flux density is

$$B = F\ p/(p - 1)r_d \qquad (108\text{-a})$$

3) Calculation of the Number of Generating Turns

In practice about 75 per cent of the gap volume can be filled with generating turns. The winding cross section for a band of length L is

$$A_w = .75\ r_d\ (p - 1)\ L/p \tag{109-a}$$

If the external diameter of the wire is d, the number of turns is

$$N = \frac{(.75) r_d\ (p - 1)}{d^2 p}\ L \tag{110-a}$$

or using the assumption of equation (105-a)

$$N = \frac{3(p - 1)\ r_d^2}{4\ \ 2p^2\ \ d^2} \tag{111-a}$$

4) Generated Voltage

Using the various relations derived above the expression for generated voltage becomes

$$e = - \frac{2.36\ F}{p^2}\ (r_d/d)^2\ \ dx/dt \times 10^{-8} \tag{112-a}$$

$$e = - \frac{2.36\ (p - 1)}{p^3}\ r_d^3/d^2\ \ B\ dx/dt \times 10^{-8} \tag{113-a}$$

B. CYLINDRICAL SHELL INDUCTOR TYPE GENERATOR

1). General

A second type of generator uses a cylindrical shell of magnetic material to shift flux from one side to the other of a stationary coil. The magnetic elements of such a generator are shown in figure A-4.

Let

λ = width of overlap of inductor shell on either side of winding.

The flux per unit width of air gap is

$$\varphi/2\lambda \tag{114-a}$$

A motion Δx will cause an amount of flux $\varphi\ \Delta x/2\lambda$ to be moved from one side of the winding to the other so that the generated voltage corresponding to a velocity $\Delta x/\Delta t$ becomes in the limit

$$e = - N(\varphi/2)\ (1/\lambda)\ (dx/dt) \times 10^{-8} \tag{115-a}$$

In terms of flux density this voltage is

$$e = - N\ B\ (2\pi r_d/p)(dx/dt) \times 10^{-8} \tag{116-a}$$

2) Calculation of Flux Density

As a reasonable assumption the air gaps inside and outside the shell inductor can be taken as

$$(r_d - r_c)/25 = (r_d/25)(p - 1)/p \tag{117-a}$$

The reluctance of the inner gap is

$$R_i = (p - 1)/25\ (2\pi)(2\lambda) \tag{118-a}$$

The corresponding reluctance for the outer gap is

$$R_0 = (p - 1)/p(\pi\lambda)(200) \tag{119-a}$$

where the assumption has been made that the width of the winding is equal to the sum of the inductor overlaps on both ends. The total gap reluctance is

$$R = \frac{(p - 1)}{100\,\pi\,\lambda} \left[\frac{2p + 1}{2p}\right] \qquad (120\text{-a})$$

Using this value of reluctance the flux density on the inductor overlap area is

$$B = 50\ F\ p^2/(2p + 1)(p - 1)\ r_d \qquad (121\text{-a})$$

3) Calculation of the Number of Generating Turns

In order to keep the flux density at the gap surface equal to that in the core

$$\lambda = r_c/4 \qquad (122\text{-a})$$

If the flux is to be equally divided between the two ends of the coil the core area inside the winding for constant flux density will be

$$r_c' = 0.707\ r_c \qquad (123\text{-a})$$

The winding depth will thus be

$$(1 - .707)\ r_c = 0.293\ r_c \qquad (124\text{-a})$$

Taking the winding width as $r_c/2$ in accord with the assumption already made, the number of turns becomes

$$N = (.586/4p^2)(r_d^2/d^2) \qquad (125\text{-a})$$

4) Generated Voltage

Substituting the values of N and φ gives

$$e = 46\ (r_d/d)^2 \left[F/p(p - 1)(2p + 1)\right] (dx/dt) \times 10^{-8} \qquad (126\text{-a})$$

In terms of flux density the generated voltage is

$$e = .92\ r_d^3/d^2\ (1/p^3)\ B\ dx/dt \times 10^{-8} \qquad (127\text{-a})$$

C. FLAT DIAPHRAGM INDUCTOR TYPE GENERATOR

1) An inductor type generator can be made to use the variations in reluctance between a pole piece and a flat diaphragm for generating voltage in a stationary coil. This type of instrument is similar to the ordinary diaphragm type telephone receiver. The essential elements of the telephone type generator are shown in Fig. A-5.

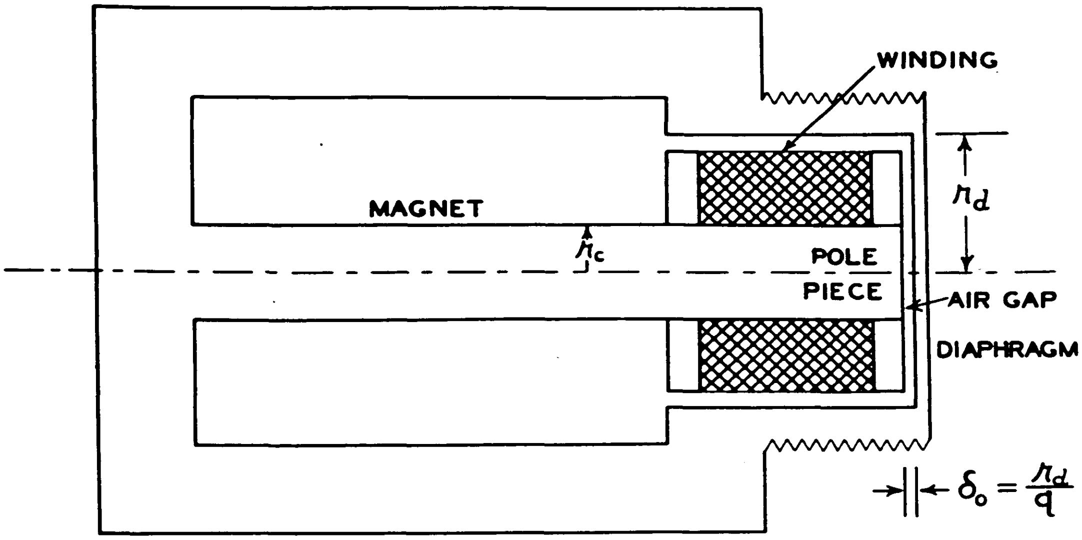
The reluctance of the air gap will be inversely proportional to the length δ, i.e., $\varphi \alpha \theta\ 1/\delta$. If the actual length δ differs from the equilibrium value by an effective distance Δx_e, the flux is

$$\varphi_{\Delta x} = \varphi\ \delta_0/\delta_0 - \Delta x_e = \varphi_0/(1 - \Delta x_e/\delta_0) \qquad (128\text{-a})$$

where φ_0 is the flux for $\Delta x_e = 0$.

If the deflection Δx_e is always a small fraction of δ_0 the flux can be written as

$$\varphi_{\Delta x} = \varphi_0 + \varphi_0\ \Delta x_e/\delta_0 \qquad (129\text{-a})$$



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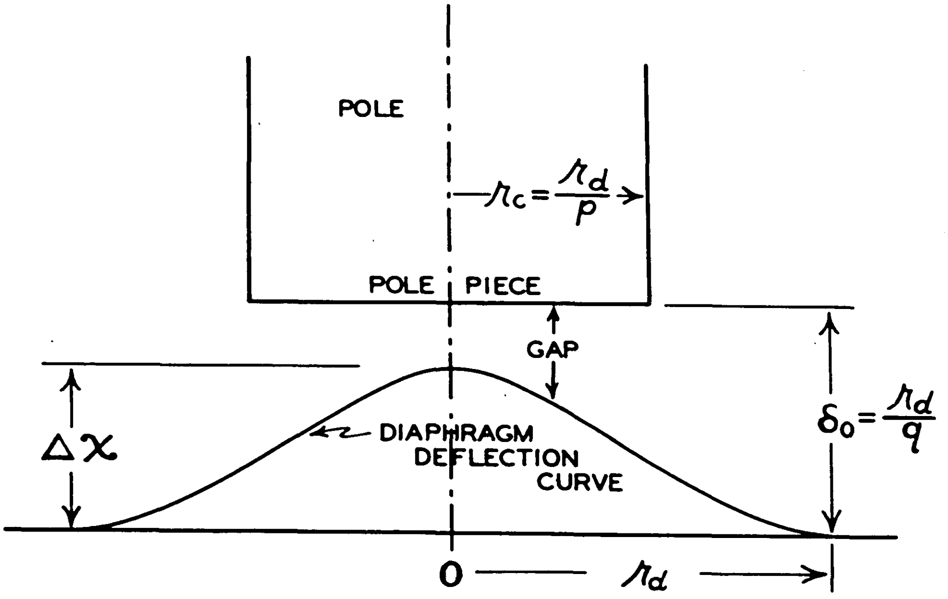
ELEMENTS OF INDICATOR WITH FLAT DIAPHRAGM GENERATOR

FIG. A-5



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WAL

AIR GAP BETWEEN DIAPHRAGM AND FLAT POLE PIECE

FIG. A-6

so that $\Delta\varphi$, the change in flux corresponding to Δx_e is

$$\Delta \varphi = \varphi_0 \, \Delta x_e/\delta_0 \tag{130-a}$$

The generated voltage if Δx_e occurs in a time Δt becomes in the limit

$$e = - N \, \varphi_0/\delta_0 \;\; dx_e/dt \;\; x \; 10^{-8} \tag{131-a}$$

In terms of flux density equation (131-a) is

$$e = - \frac{N \, B_0 \, \pi \, r_{de}^2}{\delta_0 p^2} \, (dx_e/dt) \; x \; 10^{-8} \tag{132-a}$$

where r_d is taken as the effective radius of the core which will be greater than the actual core radius by about the gap length δ_0.

2) Calculation of Flux Density

The gap reluctance for $\Delta x = 0$ is

$$R_0 = \delta_0/\pi \, (r_c + \delta_0)^2 \tag{133-a}$$

Or in terms of r_d

$$R_0 = 1/q\pi r_d(1/p + 1/q)^2 \tag{134-a}$$

where $q = r_d/\delta_0$.

The flux density is

$$B_0 = Fq/r_d \tag{135-a}$$

The effective change in gap length differs from the deflection at the diaphragm center because of the curvature of the diaphragm. This deflection curve for the case of symmetrical deflections without radial nodes is shown in Fig. A-6.

The method of attack will be to calculate the change in reluctance for each annular ring and find the integrated effect by considering all the reluctances of all the rings in parallel. Consider the diaphragm radius r_d to be unity for purposes of the derivation.

Using the plot of Fig. A-7, the reluctance in terms of the central diaphragm deflection is

$$R = R_0 - \Delta R$$

$$= R_0 (1 - K\Delta x/\delta_0) \qquad (136\text{-a})$$

where $K\Delta x$ corresponds to Δx_e of equation (130-a). The flux will be

$$\varphi = F/R_0(1 - K\Delta x/\delta_0)$$

$$= F/R_0 + (F/R_0)(K\Delta x/\delta_0) \qquad (137\text{-a})$$

and

$$\Delta\varphi = F/R_0 K\Delta x/\delta_0 \qquad (138\text{-a})$$

so that equation (131-a) becomes

$$e = - N\ (F\pi r_d^2/\delta_0^2 p^2)\ \ (K)\ (dx/dt)\ x\ 10^{-8} \qquad (139\text{-a})$$

or

$$e = - N\ (B\pi r_d^2/\delta_0 p^2)\ K\ (dx/dt)\ x\ 10^{-8} \qquad (140\text{-a})$$

3) Calculation of the Number of Generating Turns

The generating coil should be wound on the pole piece as near the air gap as possible in order that the flux shifted from the gap by reluctance changes can cut the maximum number of turns. Taking the length of the winding as equal to its depth, the number of turns will be

$$N = (.9)\, r_d^2\, (p - 1)^2/d^2p^2 \qquad (141\text{-a})$$

The factor of 0.9 is introduced to take care of clearances and insulation.

4) Generated Voltage

Introducing the expression for the number of turns, the generated voltage becomes

$$e = -\, 2.83\, (r_d^2/d^2)\, \left[(p - 1)^2q^2/p^4\right] F\, K\, (dx/dt) \times 10^{-8} \qquad (142\text{-a})$$

$$e = -\, 2.83\, (Br_d^3/d^2) \left[(p-1)^2q/p^4\right] K\, dx/dt \times 10^{-8} \qquad (143\text{-a})$$

C. COMPARISON OF THREE GENERATOR TYPES

1) Comparison of Moving Coil Generator with Moving Shell Generator

In practice, a lower limit to the ratio between outer and inner radii of the air gap is fixed by the mechanical requirements of a substantial thickness for the shell or coil form. In a reasonable design for engine work, the radius r_d will be about 7/16 inch while the space between the core and the outer shell can be about .035 inches. This gives 1.2 as the value of the ratio p.

On the basis of equal magnetizing forces the ratio between the output voltage from the shell inductor type and the moving coil type for equal velocities is

$$\frac{e_{shell}}{e_{m.c.}} = \left(46/2.36\right)(p/(p - 1)(2p + 1) = 34 \qquad (144\text{-a})$$

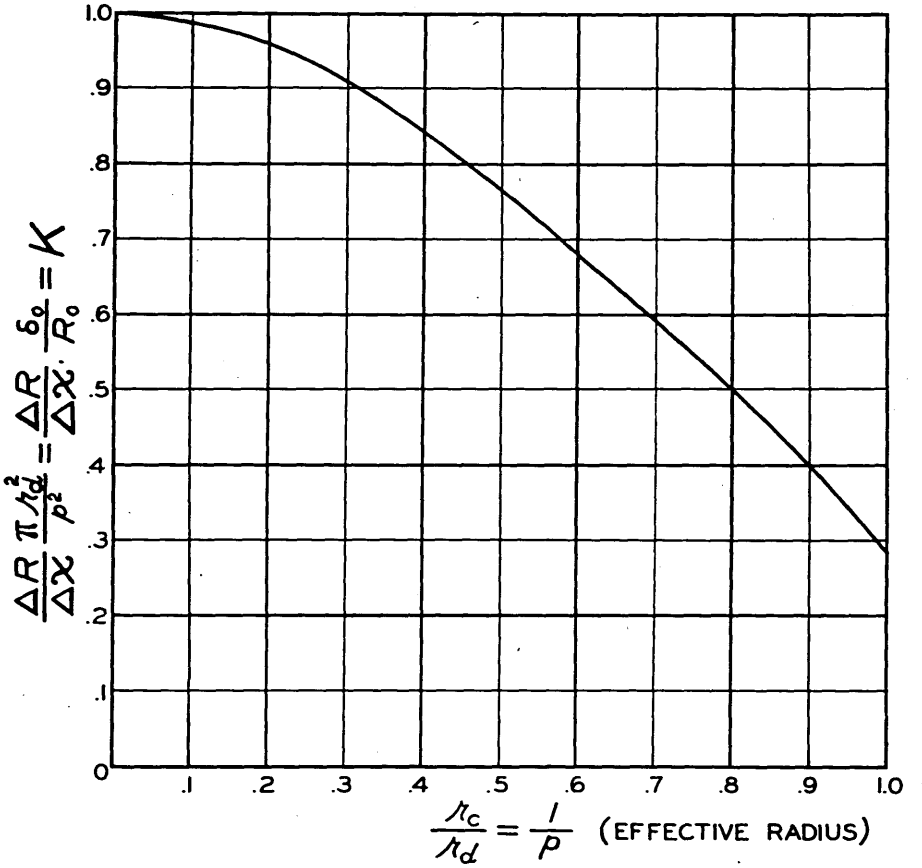
On the basis of equal flux densities the corresponding ratio is

$$\frac{e_{shell}}{e_{m.c.}} = .92/2.36\ (1/(p - 1) \cong 2 \qquad (145\text{-a})$$

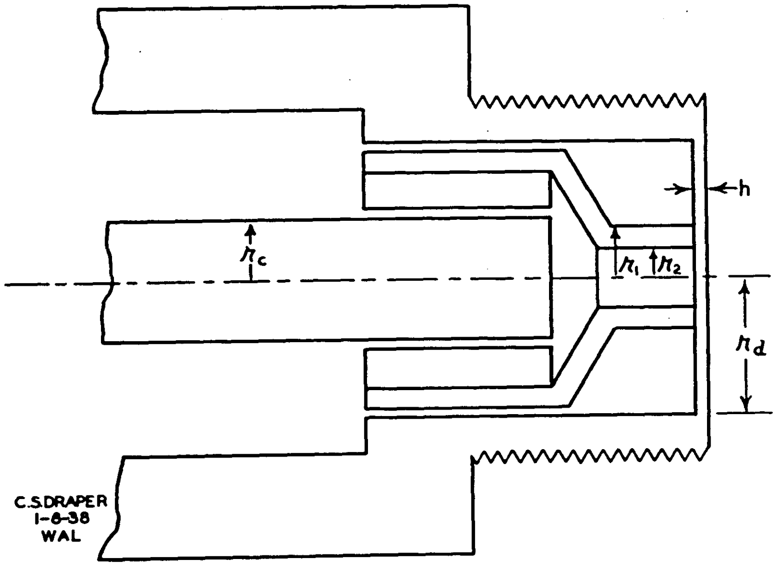
2) Comparison of Diaphragm Inductor Type with Shell Inductor Type

Reference to Fig. A-7 shows that the effective reluctance change decreases rapidly as the core radius becomes larger with respect to the diaphragm radius. With the core radius 40 per cent of the diaphragm radius the effective reluctance change is 80 per cent of the change for the same motion of a flat piston. Taking this as a reasonable working value the ratio p_d becomes 2.5. The ratio q, of equilibrium gap length to diaphragm radius will be left as a parameter.

The ratio of generated voltage from the diaphragm inductor type to the generated voltage from the



Effect of Pole Piece Diameter on Reluctance Changes
Fig. A-7



Essential Mechanical Parts of Moving Coil
and Shell Inductor Generators
Fig. A-8

shell type for equal magnetizing forces and velocities.

$$\frac{e_d}{e_s} = \frac{2.83 \dfrac{\{p_d-1\}^2}{p_d^4} q^2 K}{46 \dfrac{1}{p_s(p_s-1)(2p_s+1)}} \tag{146-a}$$

$$= 2.31 \times 10^{-3} \quad q^2$$

where subscripts d and s refer to diaphragm and shell inductor units, respectively.

On the basis of equal flux densities the corresponding ratio is

$$\frac{e_d}{e_s} = \frac{2.83 \left[(p_d-1)^2/p_d^4\right] q K}{.92 \, (1/p_s^3)} \tag{147-a}$$

$$\frac{e_d}{e_s} = .24 \, q \tag{148-a}$$

APPENDIX F

COMPARISON OF INDICATOR TYPES ON BASIS OF NATURAL FREQUENCY OF MOVING SYSTEM

Moving coil and shell inductor type generators require the attachment of parts to the diaphragm whose combined mass is of the same order of magnitude as the effective mass of the diaphragm itself. This added mass will have a considerable effect on the natural frequency of the moving system. The discussion below outlines a quantitative study of the reduction in natural frequency due to generator parts.

Fig. A-8 shows the essential features of the physical problem.

Let

$$p = r_d/r_c \qquad (149\text{-a})$$

$$s = r_d/r_1 \qquad (150\text{-a})$$

$$u = r_d/r_2 \qquad (151\text{-a})$$

$$v = r_d/h \qquad (152\text{-a})$$

1) Volume of Shell Within the Air Gap

The actual clearances are small compared to the radial distance between the inner and outer gap surfaces so the shell cross section can be written as

$$A_s = \pi (r_d^2 - r_c^2) \qquad (153\text{-a})$$

$$= \pi r_d^2 (p^2 - 1)/p^2$$

Under the assumption of a coil width equal to $r_c/2$ and a flux density at the gap surface equal to the flux density in the core, the total axial length of the shell will be r_c, and the shell volume will be

$$V_x = \pi r_d^3/p^3 (p^2 - 1) \qquad (154\text{-a})$$

For the moving coil generator, the axial length of the coil form will be about 30% over the coil length $r_c/2$. The corresponding volume is

$$V_{smc} = 0.65 \pi r_d^3 (p^2 - 1)/p^3 \qquad (155\text{-a})$$

2) Volume of Conical Section

Assumption of a cone angle of 45° gives an axial cone length of

$$r_d - r_1 = r_d (s - 1)/s \qquad (156\text{-a})$$

The cone surface area will be

$$A_c = \sqrt{2} \pi r_d^2 (s^2 - 1)/s^2 \qquad (157\text{-a})$$

If the conical section has the same thickness as the straight cylinder attachment to the diaphragm the cone volume is approximately

$$V_c = \sqrt{2} \pi r_d^3 \left[(s^2 - 1)/s^3 u\right] (u - s) \qquad (158\text{-a})$$

3) Volume of Cylindrical Section

Taking the distance from the diaphragm to the shell as r_d, the length of the cylindrical section is

$$r_d \ (s-1)/s \qquad (159\text{-a})$$

The cross section of this section will be

$$A_{cl} = \pi r_d^2 \ (u^2 - s^2/s^2u^2) \qquad (160\text{-a})$$

The corresponding volume becomes

$$V_{cl} = \left[\pi r_d^3/s^3u^2\right] (u^2-s^2) \ (s-1) \qquad (161\text{-a})$$

4) Mass of Attached Parts

The mass of each component part can be found by use of the proper density. In the shell inductor type generator the sheel itself will be of magnetic material having a density equal to that of the diaphragm material. In the moving coil type generator, the shell will probably be of aluminum with a copper winding. It follows that the shell mass must be computed separately for the two cases, i.e.

$$m_s = \rho_s \left[\pi r_d^3 \ \ (p^2 - 1)/p^3\right] \qquad (162\text{-a})$$

$$m_{smc} = \rho_{mc} \left[0.65\pi r_d^3 \ \ (p^2 - 1)/p^3\right] \qquad (163\text{-a})$$

The conical and cylindrical sections will have the same density as the shell in the inductor type so the corresponding mass becomes

$$m_{cc} = \rho\,(\pi r_d^3/us^3)\left[\sqrt{2}\;(s^2-1)\,(u-s) + \frac{(u^2-s^2)\,(s-1)}{u}\right] \qquad (164\text{-a})$$

5) Effect of Attached Parts on Natural Frequency

The undamped natural frequency of a flat diaphragm is

$$n_m = 1/2\pi\ \sqrt{k/m_e}$$

where k and m_e are the effective elastic constant and mass respectively. The mass is increased by the addition of m_a due to attached parts but the elastic constant remains unchanged, the ratio of the flat diaphragm frequency to the new frequency is

$$n_n/n_a = \sqrt{(m_e + m_a)/m_e} \qquad (165\text{-a})$$

Using the expression of Appendix H for the equivalent mass

$$m_e = 0.614\ \rho\pi\ r_d^3/v \qquad (166\text{-a})$$

For the shell inductor type of generator, the density of the attached parts will be approximately equal to the diaphragm density so the frequency ratio becomes

$$n_n/n_a = \sqrt{1+v/.614\left\{(p^2-1)/p^3+1/s^3u^2\left[\sqrt{2}(s^2-1)(u-s)u+(u^2-s^2)(s-1)\right]\right\}} \qquad (167\text{-a})$$

$$\frac{n_n}{n_a} = \sqrt{1.1 + v/.614 \left\{ \rho_{mc}/\rho \left[.65(p^2-1)/p^3 + \sqrt{2}(s^2-1)(u-s)/s^3u \right] + (u^2-s^2)(s-1)/s^3u^2 \right\}} \qquad (168\text{-a})$$

where a mass equal to 10% of diaphragm mass is allowed for the screw fastening the coil shell to the steel post.

For a practical engine indicator, the following values are reasonable for the various ratios

$$p = 1.27; \; s = 2.30; \; u = 2.60$$

In a moving coil indicator the coil form will usually be of aluminum wound with copper. Allowing 25% of the total volume for copper, the effective density will be about 4 gms/cm^2. With a steel diaphragm ρ_{mc}/ρ is about 0.5.

Substitution of the ratios gives for equation (167-a)

$$(n_a/n_n)_s = 1/\sqrt{1 + 0.612v} \qquad (169\text{-a})$$

The corresponding result from equation (168-a) is

$$(n_a/n_n)_{mc} = 1/\sqrt{1.1 + 0.380v} \qquad (170\text{-a})$$

These two ratios are plotted as functions of v in figure 80 of the text.

APPENDIX G

DEFLECTION OF CIRCULAR DIAPHRAGM UNDER UNIFORM PRESSURE

(References: "The Mathematical Theory of Elasticity - Love; "Partial Differential Equations of Mathematical Physics"- Webster)

In the theory of elasticity it is shown that pressure P and normal displacement ζ for a thin diaphragm must fulfill the equation

$$\nabla^4 \zeta \quad \cancel{\nabla^4}\zeta = \frac{P}{N} \tag{171-a}$$

where

$$N = \frac{E h^3}{12 (1 - \sigma^2)} \tag{172-a}$$

h = diaphragm thickness

E = Young's modulus

σ = Poisson's ratio

For the case of a thin diaphragm clamped at the edge, the deflection and slope must vanish at the boundary. For a circular diaphragm it is necessary to use cylindrical coordinates in which the operator ∇^2 becomes

$$\nabla^2 = 1/r \left\{ \partial/\partial r \, (r \, \partial/\partial r) + \partial/\partial\theta \, (1/r \, \partial/\partial\theta) + \partial/\partial \overset{\zeta}{\cancel{z}} \, (r \, \partial/\partial \overset{\zeta}{\cancel{z}}) \right\} \tag{173-a}$$

From symmetry there is no variation in z with θ.

In addition the variation of displacement with z is negligible and equation (171-a) becomes

$$(1/r)\,\partial/\partial r\left\{ r\left(\partial/\partial r\right)\left[1/r\ (\partial/\partial r)\, r\left(\partial/\partial r\right)\right]\right\} = P/N \qquad 174\text{-a}$$

It is found that the equation

$$\overset{\zeta}{\cancel{z}} = 3/16\ \frac{(1 - \sigma^2)\,(a^2 - r^2)^2}{E\,h^3}\ P \qquad 175\text{-a}$$

is a solution of (174-a) and fulfills the boundary conditions. The deflection at the center of the disk becomes

$$\overset{\zeta}{\cancel{z}} = 3/16\ \frac{(1 - \sigma^2)\,a^4}{E\,h^3}\ P \qquad 176\text{-a}$$

For a given diaphragm this expression has the form

$$\overset{\zeta}{\cancel{z}} = K\,P \qquad 177\text{-a}$$

In the case of a steel diaphragm

$$3/16\ \frac{(1 - \sigma^2)}{E} = 8 \times 10^{-14}\ \text{c.g.s. units} \qquad 178\text{-a}$$

With a diaphragm 0.5 inch (1.27 cm.) in diameter the constant K varies with thickness as shown in figure 81 of the text.

APPENDIX H

NATURAL FREQUENCY OF CIRCULAR DIAPHRAGMS

(References: "Electrical Vibration Instruments" - A. E. Kennelly. "Vibration and Sound" - Morse)

Free vibrations of a diaphragm are studied by equating the elastic forces to the inertia reaction of the diaphragm per unit area. Replacing the pressure term in equation (169-a) by an inertia reaction term gives

$$N\nabla^4\zeta = -h\rho\ \partial^2\zeta/\partial t^2 \qquad 179\text{-a}$$

or

$$\nabla^4\zeta + \frac{12\ (1 - \sigma^2)\rho}{E\ h^2}\ \partial^2\zeta/\partial t^2 = 0 \qquad 180\text{-a}$$

Let

$$\zeta = Z\ (r,\theta)\ e^{-i2\pi\nu_n t} \qquad 181\text{-a}$$

Substitution in equation (180-a) gives

$$\left(\nabla^4 - \frac{12(1 - \sigma^2)\ \rho 4\pi^2\nu_n^2}{E\ h^2}\right)\ Z = 0 \qquad 182\text{-a}$$

Placing

$$g_n^4 = \frac{12(1 - \sigma^2)\rho 4\pi^2\nu_n^2}{E\ h^2} \qquad 183\text{-a}$$

equation (182-a) can be written

$$(\nabla^2 - g_n^2)\,(\nabla^2 + g_n^2)\,Z = 0 \tag{184-a}$$

Thus Z can be a solution of the expression inside either of the brackets. The general solution will be a linear combination of both expressions. Using polar coordinates the second parenthesis becomes

$$1/r \,.\, \partial(r\,\partial Z/\partial r)/\partial r + 1/r \,.\, \partial\left[1/r\,\partial Z/\partial\theta\right]/\partial\theta + g_n^2\,Z = 0 \tag{185-a}$$

where the variation of deflection with Z is neglected.

Placing

$$Z = R\,(r)\,\Theta\,(\theta) \tag{186-a}$$

equation (185-a) becomes

$$r^2/R\;\partial^2 R/\partial r^2 + r/R\;\partial R/\partial r + g_n^2 r^2 = -(1/\Theta)\;\partial^2\Theta/\partial\theta^2 = n_\theta^2 \tag{187-a}$$

Equation (187-a) separates into

$$\partial^2\Theta/\partial\theta^2 + n_\theta^2\,\Theta = 0 \tag{188-a}$$

and

$$\partial^2 R/\partial r^2 + 1/r \,.\, \partial R/\partial r + (g_n^2 - n_\theta^2/r^2)\,R = 0 \tag{189-a}$$

Equation (188-a) has solutions of the form

$$\Theta = (\text{const}) \cos (n_\theta \theta + \alpha) \qquad (190\text{-a})$$

This result requires that n_θ have only integral values in order for the function to be single valued.

The solution of equation (188-a) has the form

$$R = (\text{const})\, J_{n_\theta} (g_n r) + (\text{const})\, N_{n_\theta} (g_n r) \qquad 191\text{-a}$$

Since the solution must remain finite at the center of the diaphragm the Neuman function must be eliminated.

An argument similar to that outlined above applies to the first parenthesis of equation (184-a) if g_n is replaced by ig_n. The required solutions are known as <u>hyperbolic Bessel Functions</u> defined as

$$I_{n_\theta} (g_n r) = i^{-n_\theta} J_{n_\theta} (ig_n r) \qquad 192\text{-a}$$

The complete solution of equation (184-a) becomes

$$\zeta = (\text{const}) \left[J_{n_\theta} (g_n r) + \lambda I_{n_\theta} (g_n r) \right] \cos(n_\theta \theta + \alpha)$$

$$\cos (2\pi\nu t + \varepsilon) \qquad 193\text{-a}$$

For a diaphragm clamped at the edge, $\zeta = 0$ and $\partial\zeta/\partial r = 0$ at $r = a$. The first condition requires that

$$\lambda = \frac{-J_{n_\theta}(g_n a)}{I_{n_\theta}(g_n a)} = \frac{-J_{n_\theta}(g_n a)}{i^{-n_\theta} J_{n_\theta}(i g_n a)} \qquad \text{194-a}$$

Differentiating equation (193-a) gives

$$\partial\zeta/\partial r = (\text{const})\left\{\ \right\}\left\{ d\left[J_{n_\theta}(g_n a)\right]/dr + \lambda \cdot d\left[I_{n_\theta}(g_n a)\right]/dr \right\} \qquad \text{195-a}$$

so that

$$\lambda = -\frac{d\left[J_{n_\theta}(g_n a)\right]/dr}{d\left[J_{n_\theta}(i g_n a)\right]/dr} \qquad \text{196-a}$$

Combining the expressions for λ

$$\frac{J_{n_\theta}(g_n a)}{\dfrac{d\left[J_{n_\theta}(g_n a)\right]}{dr}} = \frac{I_{n_\theta}(g_n a)}{\dfrac{d\left[I_{n_\theta}(g_n a)\right]}{dr}} \qquad \text{197-a}$$

In practice the most important case is that in which the diaphragm vibrates without radial or diametrical nodes. Mathematically this corresponds to making $n_\theta = 0$. Equation (197-a) becomes

$$\frac{J_o(g_n a)}{J_1(g_n a)} = \frac{J_o(ig_n a)}{iJ_1(ig_n a)} \qquad 198\text{-a}$$

This equation has a root

$$g_n a = 3.196 \qquad 199\text{-a}$$

and

$$\lambda = -\frac{J_o\ (3.196)}{J_o(i\ 3.196)} = +.05571 \qquad 200\text{-a}$$

The natural frequency corresponding to this mode of vibration can be found by solving equation (183-a) for n_n

$$\overset{\nu}{\cancel{n}}_n = \frac{(ga)^2\ \sqrt{E}}{4\pi\ \sqrt{3\rho(1-\sigma^2)}} \cdot h/a^2 \qquad 201\text{-a}$$

The table below shows a number of additional values of $g_n a$ which satisfy equation (197-a)

<u>NUMBER OF RADIAL NODES</u>

Number of Diametrical Nodes	0	1	2
0	$g_n a = 3.196$	$g_n a = 6.306$	$g_n a = 9.425$
1	$g_n a = 4.60$	$g_n a = 7.80$	$g_n a = 10.96$
2	$g_n a = 5.90$	$g_n a = 9.40$	$g_n a = 12.60$

The next frequencies higher than that for $g_n a = 3.196$ are

$$\frac{\nu_{n_{10}}}{\nu_{n_{00}}} = \frac{(4.60)^2}{(3.196)^2} = 2.1 \qquad 202\text{-a}$$

and

$$\frac{\nu_{n_{01}}}{\nu_{n_{00}}} = \frac{(6.306)^2}{(3.196)^2} = 3.91 \qquad 203\text{-a}$$

Thus the higher modes of vibration for a diaphragm have frequencies well separated from the lowest mode.

For a steel diaphragm vibrating in the first symmetrical mode, equation (201-a) becomes

$$\nu_n = 2.52 \times 10^5 \, h/a^2 \qquad 204\text{-a}$$

where h and a are in cm.

Or

$$\nu_n = 99.3 \times 10^3 \, h/a^2 \qquad 205\text{-a}$$

if h and a are in inches.

The relationship between natural frequency and thickness for practical indicator diaphragms is given in figure 82 of the text.

APPENDIX I

MOTION OF A CIRCULAR DIAPHRAGM CLAMPED AT THE BOUNDARY DUE TO SIMPLE HARMONIC MOTION APPLIED TO THE BOUNDARY

A. DIAPHRAGM THEORY

For the case of a diaphragm in an electrical pickup unit sensitive to relative motion between the edges and center of the diaphragm, interest is primarily in

$$\eta = \zeta_0 - \zeta_a \qquad 206\text{-a}$$

where

ζ_a = forced motion applied at the edge of the diaphragm.

ζ_0 = motion at the center of the diaphragm.

The differential equation of the diaphragm motion is

$$\nabla^4 \zeta + \frac{12\,(1 - \sigma^2)\,\rho}{E\,h^2}\,\partial^2\zeta/\partial t^2 = 0 \qquad 207\text{-a}$$

For the case of symmetrical deflections with no radial nodes, the solution for ζ will have the form

$$\zeta = \zeta_m(r)\,e^{i\omega_f t} \qquad 208\text{-a}$$

where

$\zeta_m(r)$ = amplitude of vibration which depends only upon r

ω_f = $2\pi\overset{\nu}{\cancel{n}}_f$

$\overset{\nu}{\cancel{n}}_f$ = frequency of the forcing motion.

Substitution in equation (207-a) gives

$$\left(\nabla^4 - \frac{12\,(1 - \sigma^2)\rho}{E\,h^2}\,\omega_f^2\right)\zeta_m = 0 \qquad 209\text{-a}$$

For convenience in studying the effect of frequency on the motion let

$$\beta = \omega_f/\omega_n = \overset{\nu}{\cancel{n}}_f/\overset{\nu}{\cancel{n}}_n \qquad 210\text{-a}$$

and

$$g_f^4 = \frac{12\,(1 - \sigma^2)\,\rho\,\omega_n^2}{E\,h^2}\,\beta^2 \qquad 211\text{-a}$$

By comparison with equation (183-a)

$$g_f = \sqrt{\beta}\;g_n \qquad 212\text{-a}$$

Equation (209-a) becomes

$$(\nabla^2 - \beta\overset{q_f^2}{g_f^2})(\nabla^2 + \beta g_f^2)\,\zeta_m = 0 \qquad 213\text{-a}$$

This equation has the same form of solution as equation (184-a) for symmetrical diaphragm displacements, i.e.,

$$\zeta_m = C \left[J_0 \left(\sqrt{\beta} g_n r \right) + \lambda I_0 \left(\sqrt{\beta}\ g_n r \right) \right] \qquad 214\text{-a}$$

Treatment of the problem is sinplfied by using the ratio r/a in the argument and placing $g_n a = 3.196 = \alpha$, which will be true for frequencies well below ν_n. Including the term depending upon time

$$\zeta = C \left[J_0 \left(\alpha \sqrt{\beta} . r/a \right) + \lambda I_0 \left(\alpha \sqrt{\beta} . r/a \right) \right] e^{i\omega_f t} \qquad 215\text{-a}$$

Assuming simple harmonic motion of frequency $\overset{\nu}{\cancel{\nu}}_f$ for the edges of the diaphragm

$$\zeta_a = \zeta_{am}\, e^{i\omega_f t} \qquad 216\text{-a}$$

The general expression for relative motion between the edge and points on the diaphragm

$$\eta = - \left\{ \zeta_{am} - C \left[J_0 (\alpha \sqrt{\beta} . r/a + \lambda I_0 \left(\alpha \sqrt{\beta} . r/a \right) \right] \right\} e^{i\omega_f t} \qquad 217\text{-a}$$

At the boundary of the diaphragm

$$d\eta/\partial r = 0 \text{ at } r = a \qquad 218\text{-a}$$

$$\eta = 0 \text{ at } r = a \qquad 219\text{-a}$$

Applying condition (218-a)

$$\partial\eta/\partial r = -C \left[-\frac{\alpha \sqrt{\beta}}{a} J_1 \left(\alpha \sqrt{\beta} . r/a \right) + \frac{\lambda \alpha \sqrt{\beta}}{a} I_1 \left(\alpha\ \sqrt{\beta}\ . r/a \right) \right] \qquad 220\text{-a}$$

gives

$$\lambda = J_1 (\alpha \sqrt{\beta})/I_1 (\alpha \sqrt{\beta}) \qquad 221\text{-a}$$

Using condition (219-a)

$$C = \frac{\zeta_{am} I_1 (\alpha \sqrt{\beta})}{J_0 (\alpha \sqrt{\beta}) I_1(\alpha \sqrt{\beta}) + J_1(\alpha \sqrt{\beta}) I_0(\alpha \sqrt{\beta})} \qquad 222\text{-a}$$

So that finally the expression for relative motion becomes

$$\eta/\zeta_{am} = - \left[1 - \frac{J_1(\alpha \sqrt{\beta}) J_0(\alpha \sqrt{\beta} . r/a) + J_1(\alpha \sqrt{\beta}) I_0(\alpha \sqrt{\beta} . r/a)}{J_0(\alpha \sqrt{\beta}) I_0(\alpha \sqrt{\beta}) + J_1(\alpha \sqrt{\beta}) I_0(\alpha \sqrt{\beta})} \right] e^{i\omega_f t}$$

223-a

For the case of $r = 0$.

$$\eta_a/\zeta_{am} = - \left[1 - \frac{I_1(\alpha \sqrt{\beta}) + J_1 (\alpha \sqrt{\beta})}{J_0(\alpha \sqrt{\beta}) I_1(\alpha \sqrt{\beta}) + J_1(\alpha \sqrt{\beta}) I_0(\alpha \sqrt{\beta})} \right] e^{i\omega_f t}$$

224-a

A plot of η_{am}/ζ_{am} vs. β from equation (221-a) is given in figure 83 of the text and identified as the "exact theory" curve.

B. EQUIVALENT SYSTEM THEORY

An actual diaphragm can be replaced by an equivalent system consisting of a concentrated mass, m_e, attached to the boundary by a spring of equivalent constant k.

The equation of motion is

$$m_e \, d^2\zeta_0/dt^2 + k\,(\zeta_0 - \zeta_a) = 0 \qquad 225\text{-a}$$

To correspond with the previous work change the variable to relative motion

$$\eta_0 = \zeta_0 - \zeta_a \qquad 226\text{-a}$$

Then the absolute motion of the equivalent mass becomes

$$\zeta_0 = \eta_0 + \zeta_a = \eta_0 + \zeta_{am}\, e^{i\omega_f t} \qquad 227\text{-a}$$

if the boundary is subjected to a simple harmonic motion of frequency $\omega_f = 2\pi\nu_f$.

Substitution in the equation of motion gives

$$d^2\eta_0/dt^2 + \omega_n^2\eta_0 = \zeta_{am}\,\omega_f^2\, e^{i\omega_f t} \qquad 228\text{-a}$$

The solution of this equation is

$$\eta_0 = \zeta_{am}\,\frac{\beta^2}{1 - \beta^2}\, e^{i\omega_f t} \qquad 229\text{-a}$$

so that the relation between the amplitude of relative motion and the amplitude of forcing motion is

$$\eta_{0m}/\zeta_{am} = \beta^2/(1 - \beta^2) \qquad 230\text{-a}$$

This function is plotted as the "equivalent system" curve of figure 83.

APPENDIX J

MOTION OF A CIRCULAR DIAPHRAGM CLAMPED AT THE BOUNDARY UNDER A SIMPLE HARMONIC FORCING PRESSURE

A. EXACT DIAPHRAGM THEORY

For the case in which a diaphragm with fixed and clamped edges is subjected to uniformly distributed pressure varying so rapidly that inertia effects must be considered, the equation of motion becomes

$$\nabla^4\eta + h\rho/N \, . \, \partial^2\eta/\partial t^2 = P/N \tag{231-a}$$

η = relative motion between a point on the diaphragm ~~center~~ and the boundary.

If the pressure is varying sinusoidally with time

$$P = P_m e^{i\omega_f t} \tag{232-a}$$

Assuming a sinusoidal solution for diaphragm deflection $\eta = \eta\ (r)\ e^{i\omega_f t}$, and substituting in the equation of motion gives

$$M \left[\nabla^4\eta(r) - (h\rho\omega_f^2/N).\eta(P)\right] = P_m/N \tag{233-a}$$

where M is to be determined.

Using the relation of equation (209-a)

$$\nabla^4 \eta(r) - \beta^2 g_n^4 \, \eta(r) = P_m/MN \qquad 234\text{-a}$$

The solution of equation (231-a) must satisfy the conditions for a clamped circular boundary in addition to the differential relation. By reference to Appendix J it is reasonable to assume a solution of the form for the dase of symmetrical deflections.

$$\eta = M\Big[I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}.r/a) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta}.r/a) - I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}) - J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})\Big] \qquad 235\text{-a}$$

The bracket obviously becomes zero at r = a while the reversal in sign produced by differentiation of one term but not of the other fulfills the slope condition.

Applying ∇^4 to $I_0(\alpha\sqrt{\beta}.r/a)$ and $J_0(\alpha\sqrt{\beta}.r/a)$ has the effect of multiplying each of these functions by $\beta^2 g_n^4$ so that substitution in equation (231-a) gives

$$M\beta^2 g_n^4\Big[I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})\Big] = P_m/N \qquad 236\text{-a}$$

so that

$$M = (P_m/N\beta^2 g_n^4)\,\frac{1}{\Big[I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta})+J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})\Big]} \qquad 237\text{-a}$$

The solution for diaphragm deflection is

$$\eta = \left(P_m/N\beta^2 g_n^4\right) \frac{\left[I_1(a\sqrt{\beta})J_0(\alpha\sqrt{\beta}r/a) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta}r/a) - I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}) - J_1(\alpha\sqrt{\beta})\, I_0(\alpha\sqrt{\beta})\right]}{\left[I_1(\alpha\sqrt{\beta})\, J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})\, I_0(\alpha\sqrt{\beta})\right]}\, e^{i\omega_f t} \qquad 238\text{-a}$$

Placing r = 0 to determine the deflection at the center of the diaphragm

$$\eta_0 = \left(P_m/N\beta^2 g_n^4\right) \frac{\left[I_1(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta}) - I_1(\alpha\sqrt{\beta})\, J_0(\alpha\sqrt{\beta}) - J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})\right]}{\left[I_1(\alpha\sqrt{\beta})\, J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})\right]}\, e^{i\omega_f t} \qquad 239\text{-a}$$

At very low frequencies the deflection will be equal to that for a constant pressure as given by equation (174-a)

$$\eta_{\substack{am\\ \beta=0}} = P_m a^4/64N \qquad 240\text{-a}$$

Using this relationship in equation (236-a) gives

$$\eta_0 = (P_m a^4/64N)\;\; 64/(g_n a)^4\; \beta^2 \frac{[\;\;]}{[\;\;]} \qquad 241\text{-a}$$

$$\eta_0 = \eta_{\substack{am\\ \beta=0}} \frac{64}{(g_n\alpha)^4\beta^2} \left[\frac{I_1(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})}{I_1(\alpha\sqrt{\beta})\, J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})} - 1\right] e^{i\omega_f t} \qquad 242\text{-a}$$

Using the value of ~~α~~ $qn\alpha$ = 3.196 for symmetrical deflections gives

$$\eta_0 = \underset{\beta=0}{\eta_{0m}} \frac{0.614}{\beta^2}\left[\frac{I_1(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})}{I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})} - 1\right] e^{i\omega_f t} \tag{243-a}$$

The effect of varying frequency on the diaphragm can be expressed as

$$\eta_{0m}/\underset{\beta=0}{\eta_{0m}} = \frac{0.614}{\beta^2}\left[\frac{I_1(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})}{I_1(\alpha\sqrt{\beta})J_0(\alpha\sqrt{\beta}) + J_1(\alpha\sqrt{\beta})I_0(\alpha\sqrt{\beta})} - 1\right] \tag{245-a}$$

A series of values for the bracket has already been computed and plotted in figure 83.

A. EQUIVALENT SYSTEM THEORY

Considering the diaphragm as an equivalent system having one degree of freedom with equivalent mass m_e and equivalent elastic constant k_e the equation of motion for the center of the diaphragm is

$$m_e\, d^2\eta_0/dt^2 + k_e\eta_0 = \pi a^2 P_m\, e^{i\omega_f t} \tag{246-a}$$

Dividing through by m_e gives

$$d^2\eta_0/dt^2 + k_e/m_e\ \eta_0 = \pi a^2/m_e\ P_m\, e^{i\omega_f t} \tag{247-a}$$

Substitution of

$$\eta_0 = \eta_{0m}\, e^{i\omega_f t} \tag{248-a}$$

reduces the equation to

$$(-\omega_f^2 + \omega_n^2)\ \eta_{Om} = (\pi a^2/m_e)\ P_m \tag{248-a}$$

Dividing by $\omega_n^2 = \sqrt{k_e/m_e}$ makes the frequency variable dimensionless, i.e.,

$$\eta_{Om} = \pi a^2 P_m/k_e(1 - \beta^2) \tag{249-a}$$

For very low frequencies $(\beta \rightarrow 0)$ the deflection is equal to that for static conditions

$$\underset{\beta=0}{\eta_{Om}} = (\pi a^2/k_e)\ P_m \tag{250-a}$$

Comparison of equation 247-a with equation 174-a gives the value of k_e in terms of diaphragm constants

$$k_e = 64\pi N/a^2 = 16\pi Eh^3/3(1 - \sigma^2)a^2 \tag{251-a}$$

Similarly the equivalent mass can be found in terms of the actual diaphragm mass by use of equation 181-a as

$$m_e/m = 64/(g_n a)^4 = 0.614 \tag{252-a}$$

From equations (246-a) and (247-a) the effect of frequency on diaphragm deflection can be expressed by

$$\eta_{Om}/\underset{\beta=0}{\eta_{Om}} = 1/(1 - \beta^2) \tag{253-a}$$

Figure 84 of the text is a plot showing the variation of diaphragm motion amplitude with frequency. In this case the ~~exact~~ equivalent theory gives results identical with the exact theory.

A P P E N D I X K

DIAPHRAGM SUBJECTED TO BOTH VIBRATION AND PRESSURE

The results of Appendix I and Appendix J can be combined directly since any sum of solutions of the equation of motion will be a solution of the equation. In practice the problem will usually be that of separating the effects of vibration from the effects of pressure on a record produced by an instrument sensitive to both vibration and pressure. Since the pressure will probably also force the vibration it will be useful to assume that the pressure and vibration have equal frequencies. The previous discussion has shown that for frequencies well below the natural frequency of the first symmetrical mode, the equivalent theory will produce satisfactory results.

Combining equations (229-a) and (248-a) gives

$$\eta = \left[\frac{\zeta_{am} \, \beta^2 \, (1.63)}{(1 - \beta^2)} \, e^{-i\delta} + \frac{\pi a^2 \, P_m}{k_e \, (1 - \beta^2)} \right] e^{i\omega_f t} \qquad 254\text{-a}$$

This equation can be written as

$$\eta = \frac{\pi a^2}{k_e (1-\beta^2)} \left[\frac{\beta^2 (1.63) k_e}{\pi a^2} \, \zeta_{am} e^{-i\delta} + P_m \right] e^{i\omega_f t} \qquad 255\text{-a}$$

The effect of vibration can thus be treated as if a pressure, P_v, were acting where

$$P_{vm} = \frac{\beta^2 (1.63) \; ke}{\pi a^2} \zeta_{am} \; e^{-i\delta} \qquad 256\text{-a}$$

For a steel diaphragm 0.5 inch in diameter and 0.060 inch thick equation (253-a) is

$$P_{vm} = \beta^2 \zeta_{am} \; 4.44 \times 10^{11} \; e^{-i\delta} \qquad 257\text{-a}$$

where pressure is in dynes per square centimeter and deflection is in centimeters. If pressure is in pounds per square inch and deflection is in inches, equation (254-a) becomes

$$P_{vm} = \beta^2 \; 1.64 \times 10^7 \; \zeta_{am} \; e^{-i\delta} \qquad 258\text{-a}$$