

A Method For Analyzing The Delivery Frequency From A Distribution Center To A Retail Grocery Store

by

Christopher Wayne Kerslake

Bachelor of Science, Mechanical Engineering
University of Minnesota

Submitted to the Engineering Systems Division in Partial Fulfillment of the
Requirements for the Degree of

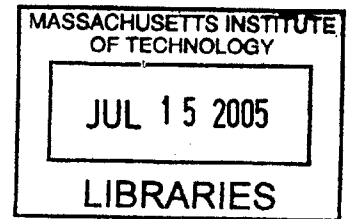
Master of Engineering in Logistics

at the

Massachusetts Institute of Technology

June 2005

© 2005 Christopher Kerslake
All rights reserved



The author hereby grants to MIT permission to reproduce and to distribute publicly paper and electronic copies of this thesis document in whole or in part.

Signature of Author

[Handwritten Signature]
Engineering Systems Division
6-May, 2005

Certified by

[Handwritten Signature]
Chris Caplice
Executive Director, Master of Engineering in Logistics Program
Thesis Supervisor

Accepted by

[Handwritten Signature]
Yossi Sheffi
Professor of Civil and Environmental Engineering
Professor of Engineering Systems
Director, MIT Center for Transportation and Logistics

BARKER

A Method For Analyzing The Delivery Frequency From A Distribution Center To A Retail Grocery Store

by

Christopher Wayne Kerslake

Submitted to the Engineering Systems Division

On 6-May, 2005 in Partial Fulfillment of the

Requirements for the Degree of

Master of Engineering in Logistics

Abstract

Currently, no adequate method exists for determining how frequently a retail store in a supermarket chain should receive deliveries from its distribution center. Existing methods neglect many crucial constraints, such as the necessity for deliveries to fall on fixed days of the week, severely limited shelf space, and the inability for many stores to hold additional overstock product in a backroom. This paper addresses the problem by outlining a new method for determining the delivery frequency by developing a simulation model for the replenishment process of a supermarket chain. The model can also be used to provide insight into other aspects of the replenishment process, such as shelf space allocation, and reorder rules. Using this model, we were able to show that significant cost savings were available to the supermarket chain we worked with on the project by changing the delivery schedules for their stores.

Acknowledgements

First, I would like to thank my thesis advisor, Chris Caplice, for his guidance, insight, and motivation throughout the entirety of this project. I have learned a great deal from you during the past year, thanks.

I would also like to thank Ed Rodricks and his team at the supermarket chain for their undying support of this project. This project would not have been possible without your cooperation.

Table of Contents

| | |
|--|-----------|
| Abstract | i |
| Acknowledgements | ii |
| List of Tables | v |
| List of Figures | v |
| | |
| Chapter 1: Overview | 1 |
| 1.1 - Introduction | 1 |
| 1.2 - Literature Review | 3 |
| 1.3 - Roadmap | 4 |
| Chapter 2: Current Operations | 6 |
| 2.1 - Grocery Chain X Overview | 6 |
| 2.2 - Replenishment Process | 7 |
| 2.3 - Store Operations | 8 |
| 2.4 - Delivery Schedules | 10 |
| 2.5 - Distribution Center Operations..... | 12 |
| Chapter 3: Sample Data for 3 Retail Stores | 14 |
| 3.1 - The High-Volume Store | 14 |
| 3.2 - The Low-Volume Store | 21 |
| 3.3 - The Medium-Volume Store..... | 23 |
| Chapter 4: Methodology | 26 |
| 4.1 - General Assumptions..... | 26 |
| 4.2 - Simulation..... | 30 |
| 4.3 - Model Validation | 41 |
| Chapter 5: Results and Analysis | 45 |
| 5.1 - The High-Volume Store | 45 |
| 5.2 - The Low-Volume and Medium-Volume Stores..... | 50 |
| 5.3 - General Procedure for Choosing Days Without Deliveries..... | 52 |
| 5.4 - Effects on System of Changing Delivery Schedule for One Store..... | 55 |
| 5.5 - The Effects of Changing Re-Order Points and Shelf-Allocation | 57 |
| Chapter 6: Conclusions | 64 |
| Chapter 7: Areas for Future Research | 66 |
| Bibliography | 67 |

List of Tables

| | |
|---|----|
| Table 1: Percentage of SKU's Accounting for Sales – The High-Volume Store | 15 |
| Table 2: Sales Statistics for Each Day of Week | 20 |
| Table 3: Comparison of Three Stores | 25 |
| Table 4: Actual vs. Simulated Sales and Deliveries | 43 |
| Table 5: Store-Specific Inputs | 46 |
| Table 6: Simulation Results – The High-Volume Store..... | 46 |
| Table 7: Simulation Results - The High-Volume Store (continued)..... | 49 |
| Table 8: Simulation Results – The Low-Volume Store | 51 |
| Table 9: Simulation Results - The Medium-Volume Store | 52 |
| Table 10: Simulation Results for Adjusted ROP's..... | 58 |
| Table 11: Simulation Results for Adjusted ROP's and Shelf Capacity | 61 |

List of Figures

| | |
|--|----|
| Figure 1: Number of Deliveries per Week | 11 |
| Figure 2: Number of Deliveries Each Day | 12 |
| Figure 3: Percentage of SKU's Accounting for Sales – The High-Volume Store..... | 15 |
| Figure 4: Number of Products with a Given Days-worth of Inventory–The High-Volume Store.. | 16 |
| Figure 5: Monthly Demand for Hershey's Cocoa – The High-Volume Store | 17 |
| Figure 6: Average Total Daily Sales by Month – The High-Volume Store..... | 18 |
| Figure 7: Average Sales on Each Day of the Week – The High-Volume Store..... | 19 |
| Figure 8: Average Delivery Size – The High-Volume Store | 21 |
| Figure 9: Average Sales on Each Day of the Week – The Low-Volume Store | 22 |
| Figure 10: Average Delivery Size – The Low-Volume Store..... | 23 |
| Figure 11: Average Sales on Each Day of the Week – The Medium-Volume Store | 24 |
| Figure 12: Average Delivery Size – The Medium-Volume Store..... | 24 |
| Figure 13: Effect of Shelf Space Adjustment Factor on Percentage of SKU's in Backroom..... | 29 |
| Figure 14: Simulation Diagram..... | 38 |
| Figure 15: Actual Deliveries Compared with Simulated Deliveries | 42 |
| Figure 16: Actual Sales Compared with Simulated Sales..... | 43 |
| Figure 17: Size of Daily Deliveries Resulting from Skipping 2 Days | 50 |
| Figure 18: Combining Deliveries | 54 |
| Figure 19: Ratio of Actual-to-Expected Miles Removed from Route..... | 56 |
| Figure 20: Distribution of How ROP's Changed | 59 |
| Figure 21: Distribution of How Shelf Space Allocation Changed | 62 |

1 Overview

Recently, many companies have been trying to reduce inventory carrying costs at each stage in the supply chain while at the same time trying to increase customer service. At retail stores, this generally includes reducing the amount of merchandise on the shelves and in the back room, an area which is not on the sales floor and is used for storing excess product. Companies that have successfully implemented programs reducing store inventory have realized large savings in carrying costs. In doing so, however, stores have often had to increase the frequency of deliveries from their distribution centers in order to keep product on the shelves, resulting in increased transportation costs. Particularly with the uncertainty in fuel prices, there is rising interest in taking a closer look at the trade-off between carrying and product handling costs and transportation costs, and in trying to determine the delivery frequency that will result in the lowest overall costs. Constraints such as limited physical space in the store and the necessity of a fixed delivery schedule further complicate the issue. This paper addresses this problem by providing a method for analyzing the delivery frequency from a distribution center to a retail store.

Section 1.1: Introduction

The rising cost of transportation is of particular concern to the retail grocery industry, which moves large volumes of low-margin goods. Most grocery stores are severely constrained

in the number of deliveries they require each week because of factors such as limited shelf space, little or no back-room storage, large demand uncertainty, and an increasing number of Stock Keeping Units (SKU's) being sold at each store. Some stores have such large SKU count to shelf space ratios that they are unable to stock even an entire case of some products on their shelves. These tight constraints make reducing the delivery frequency to these stores quite difficult. Some research has been done in this area, but existing models do not incorporate the constraints of both limited physical space at the store and the necessity for deliveries to fall on fixed days of the week.

For this thesis, we worked with a retail grocery chain (from this point forward referred to as Grocery Chain X) to develop a method for determining the delivery frequency for each individual store based on a set of characteristics including shelf-space, transportation costs, inventory costs, and product handling costs. Since retail grocery stores generally receive deliveries from multiple warehouses, both their own and those of their vendors, the project scope was narrowed to deliveries arriving at the stores from one specific location. The chosen location was a distribution center (DC) owned and operated by Grocery Chain X, which warehoused both Dry Grocery and Frozen products. Because the two types of products did not share common transportation, the scope of this project was narrowed further to include only Dry Grocery items. Dry Grocery products were the simpler choice because shelf-life considerations did not need to be included in the model.

Section 1.2: Literature Review

A significant amount of research has been done in finding the Economic Order Quantity (EOQ) for products. The EOQ takes into account all costs which are impacted by the order size, namely inventory holding costs, ordering costs, purchase costs (including volume discounts), and stock-out costs. Transportation costs are generally included in the ordering costs if there is a fixed charge per delivery. If all or part of the transportation cost is based on the number of items ordered, the variable portion of the cost is generally added to the purchase price. Even though the EOQ is appropriate in many applications for finding the optimal order size, and therefore the order frequency, it does have its limitations (Silver, Pyke, and Peterson, 1998). The limitation of the EOQ model that becomes particularly apparent when trying to apply it to a retail grocery store is that does not take into account non-financial considerations, such as delivery time windows and labor availability. The EOQ model also becomes difficult to use when looking at the several thousand SKU's that are shipped on a single truck, each with a different demand pattern.

Balintfy (1964) and others have done work in determining the replenishment schedule by looking at it in terms of a Joint Replenishment Problem (JRP). The JRP refers to a situation where several different products can be ordered together for one fixed cost for the entire order (usually referred to as a major setup cost) and an additional charge per product (minor setup cost). In the case of transporting inventory from a warehouse to a retail store, the transportation cost would be the major setup cost, and there would be little or no minor setup cost. This is case because the delivery cost remains the same (to the point until the truck is filled) regardless of the size of the delivery, with the possible exception of small incremental order-picking costs, depending on how the orders are picked. Under Balintfy's method, each product is assigned a

can-order, and a *must-order* level. When one product drops below its must-order level, all products below their can-order level are ordered. Enough of each product is ordered to raise its level to an order-up-to level. For grocery retailers, this is not a logical replenishment method because with several thousands of SKU's and in many cases very limited shelf-space, the can-order and must-order numbers will be very close to the same. This method also does not lend itself to a fixed delivery schedule, nor does it incorporate truck capacity constraints.

Cachon (2001) considers a method for determining delivery frequencies which dispatches a truck once the total order size reaches a given threshold. For this method, continuous review of shelf inventory is needed, and Cachon is able to show that this method performs better than comparable methods which use periodic review. He assigns a dollar value to shelf space but assumes that shelf space is unlimited and determines the optimal allocation for each product. Again, grocery retailers are often severely constrained by shelf space limitations and generally do not have total freedom to reallocate shelf space. With a product mix that is continually changing, reallocating shelf space based on optimal numbers for thousands of SKU's is not practical. Also, dispatching a truck after it reaches a given threshold means that the delivery schedule will not be fixed, which makes it very difficult for grocery stores to schedule their stocking labor.

Section 1.3: Roadmap

Three stores with varying physical sizes and daily demand volumes were chosen as test locations along with a single distribution center which supplied these stores with Dry Grocery products. We first became familiar with how the stores were currently managing their

inventories and how orders were generated at the distribution center and eventually delivered to the store. We also noted the constraints that were going to place limits on the number of deliveries the stores would need to receive each week.

Once we understood the basics of the current store replenishment process, we were able to build a generic model in MS Excel that could simulate the existing process for a given store. The construction of the model is described in Chapter 4. In Chapter 5 we look at how changing different parameters, such as the delivery schedule, re-order points of products, and shelf space allocation, affect the overall costs associated with a particular store. In addition to looking at stores individually, we also looked at the effects on the entire system of changing the delivery frequency for one store.

2 Existing Operations

This section will not cover all operations of Grocery Chain X's supply chain but will instead focus only on how the stores place orders with the distribution center (DC), how the orders are received at the store, and how the orders are picked at the DC and delivered to the store. Since no two stores operate in exactly the same manner, this section will focus on general operations which pertain to the majority of the stores.

Section 2.1: Grocery Chain X Overview

Grocery Chain X is a supermarket chain with approximately 200 retail stores throughout New England. The stores are located in metropolitan areas, such as Boston, MA and Providence, RI, as well as many rural areas. The stores vary greatly in physical size and sales volume, and therefore have a wide range of delivery schedules. The 200 stores are supplied with product from two distribution centers (DC's) and one cross-dock facility, all of which are owned and operated by Grocery Chain X. The stores also receive product directly from four DC's owned and operated by a grocery wholesaler, as well as numerous vendors who provide direct delivery to each individual store.

The product moving from the Grocery Chain X-owned warehousing facilities to the stores are transported primarily by Grocery Chain X's private fleet. The fleet is comprised of

approximately 150 tractors, 200 trailers, and 300 refrigerated trailers. Product delivered to the stores from non-Grocery Chain X facilities is transported by the suppliers of the product.

Section 2.2: Replenishment Process

The replenishment process is handled by a forecasting software package in conjunction with a Supervised Reorder (SRO) system, which calculates how much of each product is actually needed at the store and places the order with the DC. There are still a few stores, generally smaller ones, that have not yet migrated to an automated replenishment system and still perform this function manually. This section will focus on the operations of the stores which have automated this process, but the basic ideas apply to the other stores as well.

The forecasting software gathers Point-of Sale (POS) data and uses three years' worth of these data to forecast the sales for each stock keeping unit (SKU) individually on a daily basis. The forecasting package makes adjustments to the forecast based on day-of-the-week and seasonality factors, price reductions, and whether or not a particular SKU is placed in the weekly advertisement.

Once the forecasting software has generated the forecast, the SRO system takes this forecast and calculates how much of each product should be delivered to the store. To make these calculations, the SRO system must also know the current amount of product on the shelf (this information is also supplied by forecasting software), the shelf space allotment, the reorder point, and case size for each SKU. This information is entered into the system once and remains there until a manual change is made. Based on the above information, the SRO system

calculates the amount of product expected to be left on the shelf at the time the next delivery arrives. If the amount projected to be left on the shelf is less than the reorder point, enough product is ordered in increments of one case to restock the shelf. (Store management does have the ability to manually change the amount ordered as they deem necessary.) Any product that will not fit on the shelf must be placed in the backroom. Because reducing the number of deliveries per week means that the size of each delivery is increased, fewer deliveries means that there is a greater likelihood that additional product will need to be stored in the backroom. This becomes the major tradeoff when determining the optimal delivery schedule.

In addition to the regular orders generated by the automated reorder process, each store also receives an extra order (which is generated separately) each week for promotional items. This order of promotional items is combined with the store's regularly scheduled delivery on either Wednesday or Thursday so that the promotional product will be on hand early enough to display it on the sales floor by Friday, the first day of the promotional week.

Section 2.3: Store Operations

Each store carries about 20,000 SKU's, (this number can vary somewhat depending on the size of the store,) which are broken down into three main categories, Dry Grocery, Frozen, and Perishable (which includes meat, dairy, and produce.) Each category of products is delivered separately because each has different transportation requirements and comes from a different location. Because this project focused on Dry Grocery, the process described below and in subsequent sections will pertain to Dry Grocery only. The other categories are handled in a similar manner, but there are slight differences due to the perishable nature of the products.

When a load arrives at the store dock, it may contain product for only one store, or it may contain product for up to four stores. Each situation is handled differently. If the truck contains product for only a single store, the trailer is left at the store and the tractor picks up the other trailer that was left during the last delivery for dunnage and returns it to the DC. Dunnage is the term used for waste that needs to be returned to the DC for recycling and includes things such as corrugated packaging material, plastic tubs, pallets, etc. When the trailer can be left at the store, it will generally sit full until the stocking personnel arrive. However, if the load contains product for several stores, it must be live-unloaded. During a live unload, the driver has to wait at the store while the product is moved from the trailer to a staging area where it remains until the night stock, which results in double-handling of the product. When a truck is live-unloaded, the trailer cannot be switched with the dunnage trailer, so the dunnage trailer remains at the store until either the next delivery, in which that store will ideally be the last stop, or until a separate trip can be made to switch the dunnage trailer for an empty one. Because of the added complications that arise due to live unloads, the stores prefer to have the trailer left at the store.

Once the delivery has arrived at the store, it generally sits either on the truck or in the staging area until the night stockers arrive (the start time varies by store.) The night stockers then load the product onto “U-boats” (carts shaped like the letter “U” which are used for taking the product onto the sales floor.) The stockers fill the shelves to capacity (or until they run out of product) and put any remaining product in the backroom. Product stored in this area is moved to the sales floor throughout the following day. Any product which has to be moved to the backroom has to be handled an additional time, so for that reason, stores try to minimize the amount of product that must be stored there.

Section 2.4: Delivery Schedules

Currently, Grocery Chain X determines the number of deliveries per week from the DC to a store based on its average weekly sales volume. There is, however, some room for the stores to negotiate on the number of deliveries per week and on which days of the week the deliveries will be made. Stores generally prefer to receive deliveries as frequently as possible (up to daily) for several reasons. First, assuming that the forecast error of demand is normally distributed, when the time period between deliveries is decreased, the variability of demand over that time period is decreased by a factor of the square root of the proportional decrease in the time period. For example, if the time period between deliveries is decreased from four days to two days, the standard deviation of the demand is reduced by square root four, or two. When the variability of demand is reduced, the chance of stocking out of any given product with a fixed amount of safety stock is also reduced. The amount of required safety stock is of particular concern for stores with limited shelf space because a greater amount of safety stock means that more product will need to be stored in the backroom, which, as discussed earlier, leads to additional handling of the product.

Second, when stores do stock out of a product for any reason, more frequent deliveries mean that the stores have to wait for a shorter time period until the next delivery arrives, which decreases the amount of time the store is without a particular product.

The third reason that stores prefer more frequent deliveries, and in some cases the determining factor, is that some stores do not have the physical capacity to store enough product, on the shelves or in the backroom, to allow them to skip an additional delivery day. When this is the case, delivery frequencies of these stores cannot be altered. Also, when the dunnage trailer is

full, the dunnage has to be stored in the backroom, which further limits the amount of backroom space available.

Currently, there are 21 different delivery schedules for the 122 stores which receive deliveries from the Dry Grocery DC. Just under a third of the stores receive deliveries everyday, and the number of deliveries per week for the other stores can be seen in Figure 1.

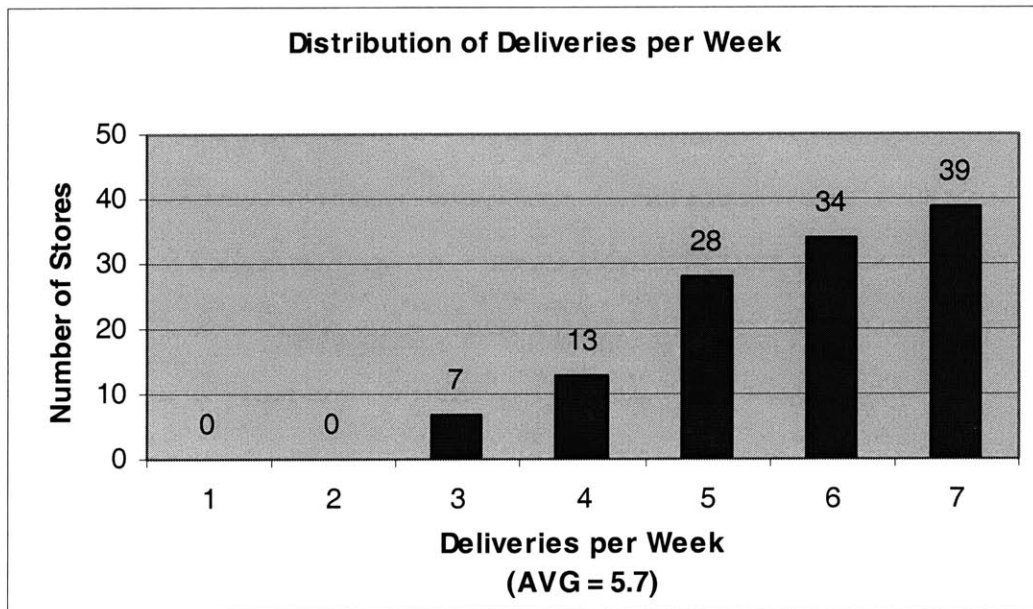


Figure 1: Number of Deliveries per Week

The deliveries are distributed relatively evenly throughout the week, with Wednesdays seeing the fewest scheduled deliveries and Saturday seeing the greatest number. See Figure 2. Grocery Chain X tries to keep deliveries evenly spaced throughout the week in order to keep driver and equipment utilization as high as possible.

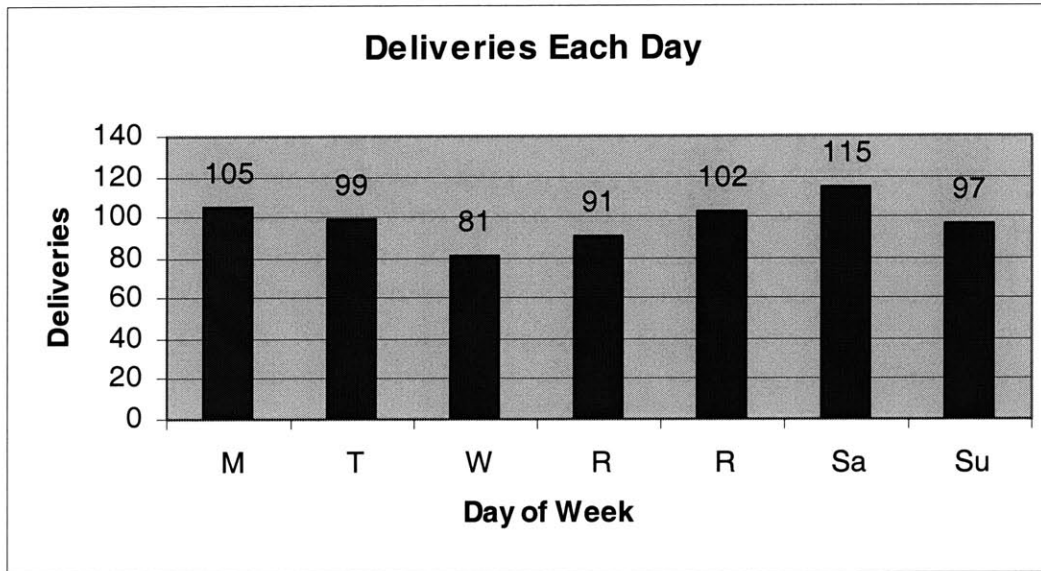


Figure 2: Number of Deliveries Each Day

Section 2.5: Distribution Center Operations

The Dry Grocery distribution center is somewhat centrally located and averages about 110 miles from each store. The DC warehouses both Dry Grocery and Frozen products and serves as a cross-dock for slow-moving grocery items, which are not stored at that location and are instead delivered from a wholesaler's warehouse. Because this paper focuses on Dry Grocery only, the operations described in the section will relate specifically to this product category.

Inbound shipments of product are received and put away throughout the day. However, the DC operations that are particularly relevant to the store deliveries are those that are associated with picking an order (combining individual cases of different products onto a pallet

which will be delivered to an individual store) and loading it onto a truck. Orders are received by 4:00pm the day before they need to be delivered and are picked on the day of delivery in an order which depends on the scheduled delivery time for each store.

The order picking is directed by a pick-to-voice system, which receives the order information from the Supervised Re-Order (SRO) system and reads it off to the picker on an item-by-item basis. The system tells the picker which product needs to be picked and in which bay he can find the product. The picker then drives the forklift (with a pallet in place) to the specified bay and recites to the system a random number which is placed on the bay to ensure that he is at the correct location. (This random number is manually changed every few days to prevent the pickers from memorizing the number and, therefore, defeating its purpose.) The system then tells the picker the quantity of that particular product to load onto the pallet. The picker loads the product and tells the system that he has finished, and the system then tells him the information for the next product. This process is repeated until the pallet is full. The full pallet is then loaded onto the empty trailer and the picker gets an empty one and repeats this process until the entire order for the store is picked and put on pallets. Deliveries are picked and dispatched throughout the day from approximately 6am to 6pm, although the first orders to be delivered each day are generally picked the afternoon before.

3 **Sample Data for 3 Retail Stores**

Grocery Chain X provided us with a year's worth of sales information, data on shelf space allocation for each product, and reordering data for three stores, which from this point forward will be referred to as the High-Volume Store, Low-Volume Store, and Medium-Volume Store. The stores were chosen to have a variety of delivery schedules and a wide variety of average weekly sales volumes. This section presents general patterns seen in the data.

Section 3.1: Large-Volume Store

For the model, we looked at one year's worth of daily sales data for each SKU sold in the Large-Volume Store from 7-Dec03 to 4-Dec04. The data in Table 1 and Figure 3 show that, as is the case for most retail stores, a small percentage of SKU's accounts for a disproportionately large amount of sales, which means that certain SKU's move through the store much more quickly than others. The fast moving SKU's are more likely to be ordered and delivered much more frequently than others, so there is a greater number of opportunities for them to need to be stored in the backroom.

| % SKU's | % of Total Sales (Units) | % of Total Sales (Dollars) |
|----------------|---------------------------------|-----------------------------------|
| 1% | 12% | 13% |
| 2% | 18% | 20% |
| 3% | 23% | 25% |
| 5% | 31% | 33% |
| 10% | 45% | 47% |
| 20% | 63% | 64% |
| 30% | 74% | 75% |
| 50% | 88% | 89% |

Table 1: Percentage of SKU's Accounting for Sales – The High-Volume Store

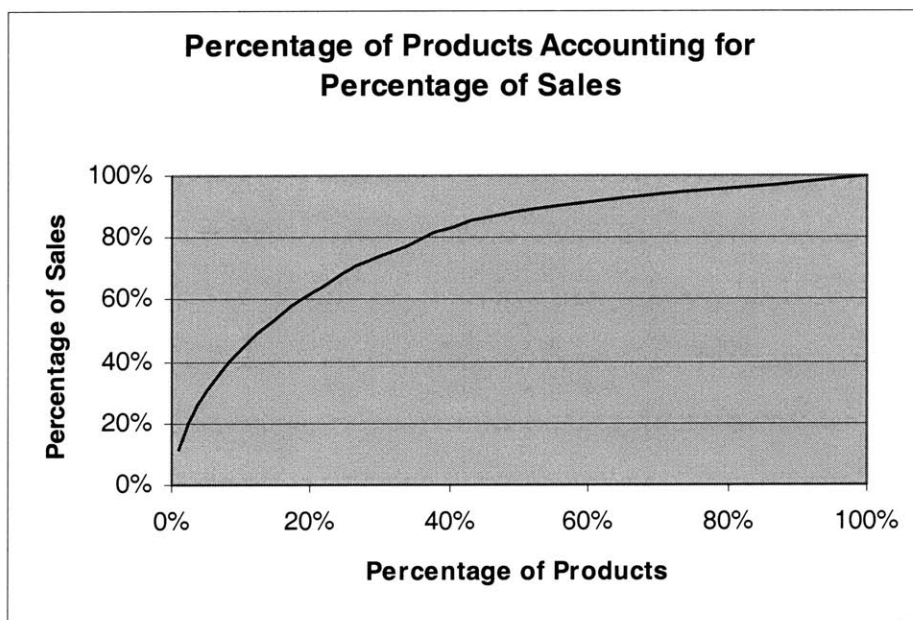


Figure 3: Percentage of SKU's Accounting for Sales – The High-Volume Store

Of the 9456 total Dry Grocery SKU's, 8069 had at least seven days' worth of inventory on the shelves, based on average sales. 92 SKU's were not even capable of holding one full day's worth of inventory on the shelves, meaning these products will have to be stored in the backroom

and restocked during the day. The High-Volume Store receives deliveries everyday, and because of this, the products with one or two days' worth of inventory on the shelves are going to be the products most likely affected by skipping delivery days. The products with space allocations which cannot even fit one days' worth of product will be placed in the backroom regardless of delivery schedule changes (although more cases may need to be stored there for some products), and products with more than two days' worth of inventory will most likely be able to absorb the daily variations in sales even if deliveries become less frequent. The distribution of shelf space allocations can be seen in Figure 4. One possible conclusion that could be drawn from this data is since there are relatively few products with inadequate shelf space and 85% of products with more than enough, re-allocating shelf space may greatly reduce the need for storing product in the backroom. With less product in the backroom initially, there is greater opportunity for skipping delivery days without adversely affecting store operations or overall costs. This will be discussed further in Section 5.4

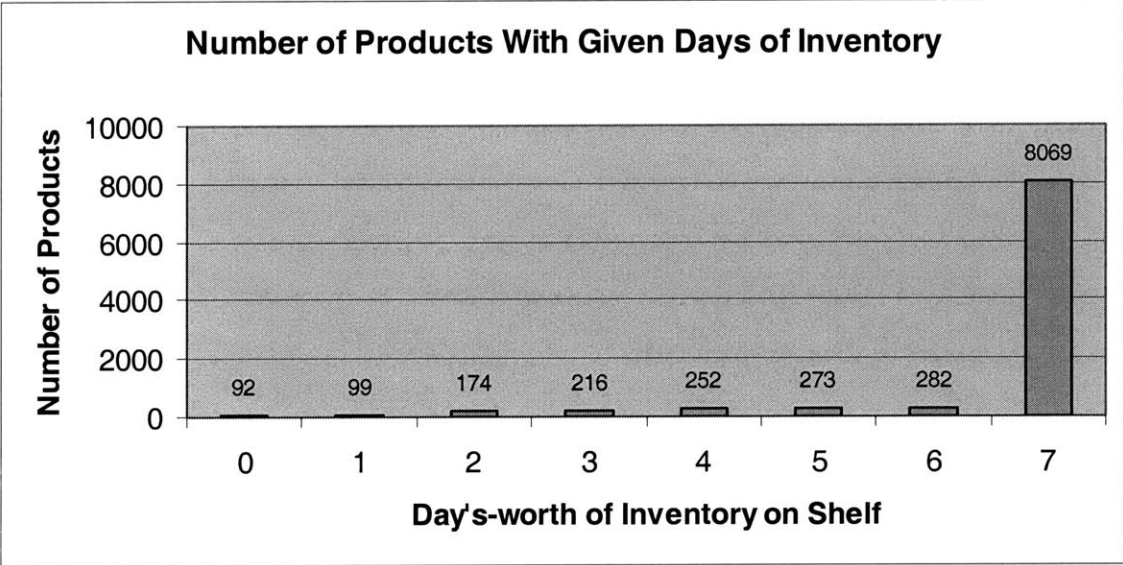


Figure 4: Number of Products with a Given Days-worth of Inventory – The High-Volume Store

In a typical supermarket, there are many products which have demands that vary considerably with the time of year, for instance, the demand for Hershey's Cocoa, shown in Figure 5. However, while the demand for individual products experience significant seasonality, the overall sales for The High-Volume Store remain relatively consistent throughout the year, although there is usually a slight dip during the summer months (See Figure 6.) This fact becomes important when determining the number of deliveries per week a given store will require. If the overall sales for a store experience significant seasonality, the possibility arises that a store may require more deliveries during certain times of the year than it does during others. The other possibility would be that the number of deliveries per week would need to be able to accommodate the times of year that experience the highest demand, which will not necessarily be the best case for the times of year with lower demands.

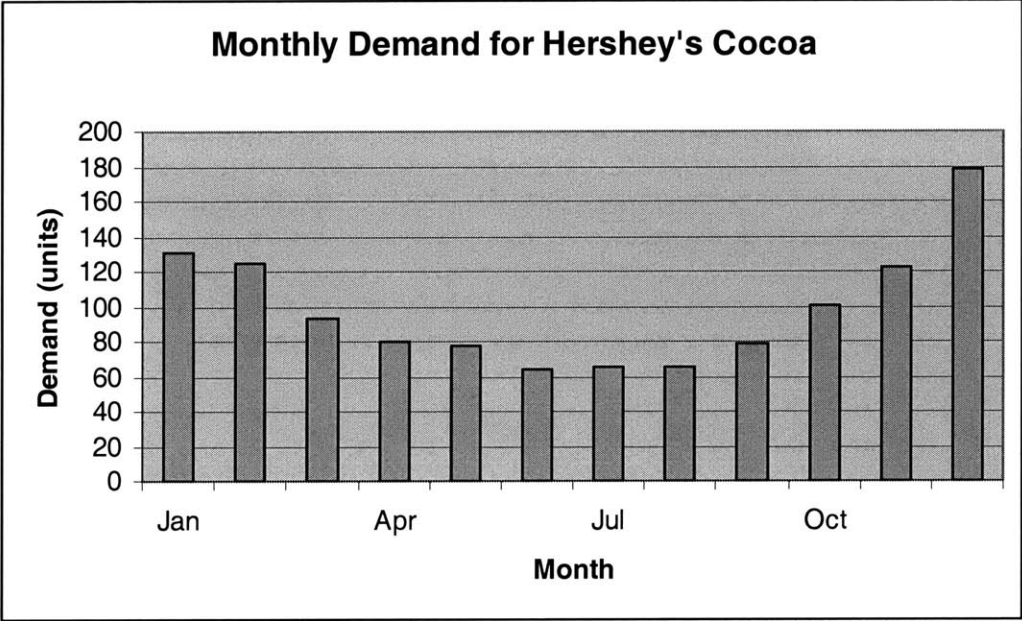


Figure 5: Monthly Demand for Hershey's Cocoa – The High-Volume Store

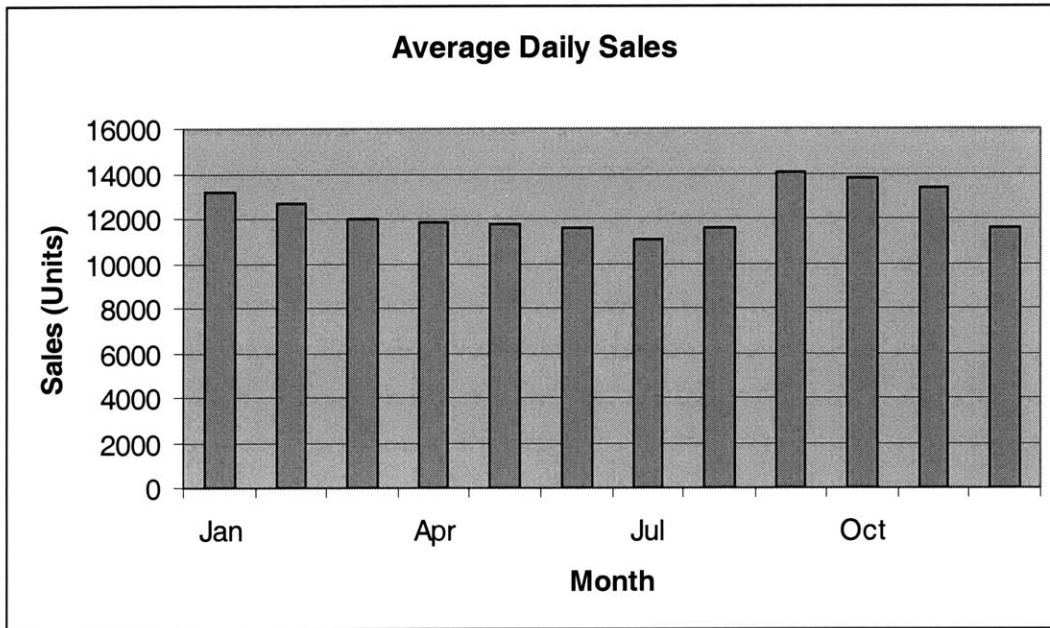


Figure 6: Average Total Daily Sales by Month – The High-Volume Store

The average daily sales (units) for The High-Volume Store vary with the day of the week as shown in Figure 7. These variations need to be taken into account when building the model of the replenishment system because forgoing a delivery on a particular day of the week now becomes much easier than on others. If a delivery is going to be skipped, enough product has to be delivered during the previous delivery to cover the demand until the next delivery. Therefore, skipping a day with lower sales reduces the risk of needing to deliver more product than will fit on one truck in the previous day's delivery.

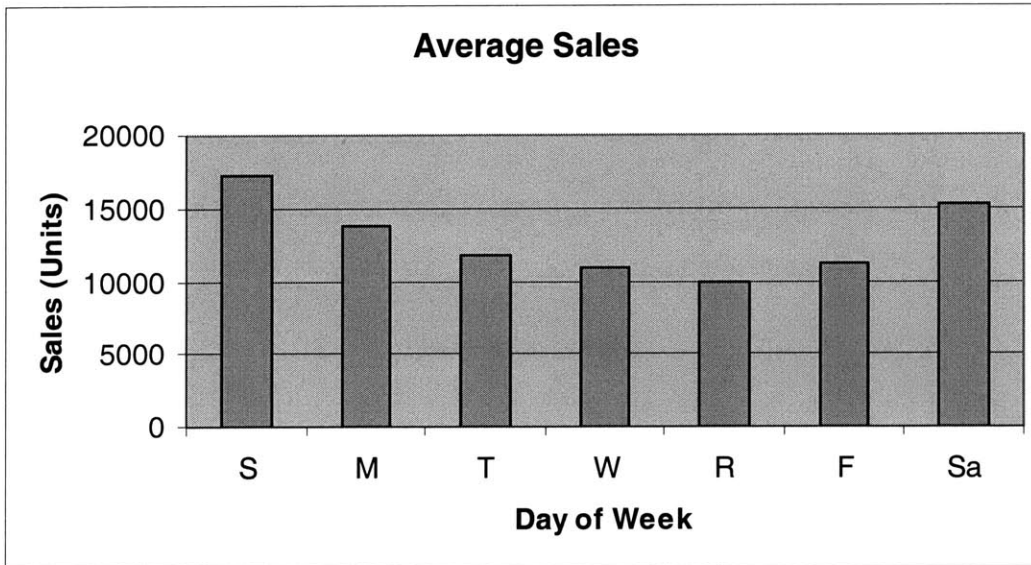


Figure 7: Average Sales on Each Day of the Week – The High-Volume Store

It is also important to note that although average sales vary throughout the week, the sales on any given day of the week remain relatively stable, which means that there is not a significant shift of sales from one day of the week to another during certain times of the year. The day-of-week statistics are shown in Table 2. This table omits the two weeks in the past year when the store was closed for one day each week.

| Day of Week | Mean Total Sales | Standard Deviation | Coefficient of Variation |
|--------------------|-------------------------|---------------------------|---------------------------------|
| Sunday | 17262 | 2638 | 0.15 |
| Monday | 13746 | 1700 | 0.12 |
| Tuesday | 11735 | 1317 | 0.11 |
| Wednesday | 10716 | 1208 | 0.11 |
| Thursday | 10359 | 1098 | 0.11 |
| Friday | 11325 | 1090 | 0.10 |
| Saturday | 15480 | 2080 | 0.13 |

Table 2: Sales Statistics for Each Day of Week

With the exception of Wednesdays' deliveries, which have the promotional products added to them (in the case of The High-Volume Store, about 1300 ft³), the pattern for the amount of product delivered to The High-Volume Store closely corresponds to the amount of sales seen on the same day, with lower sales and deliveries seen in the middle of the week. See Figure 8.

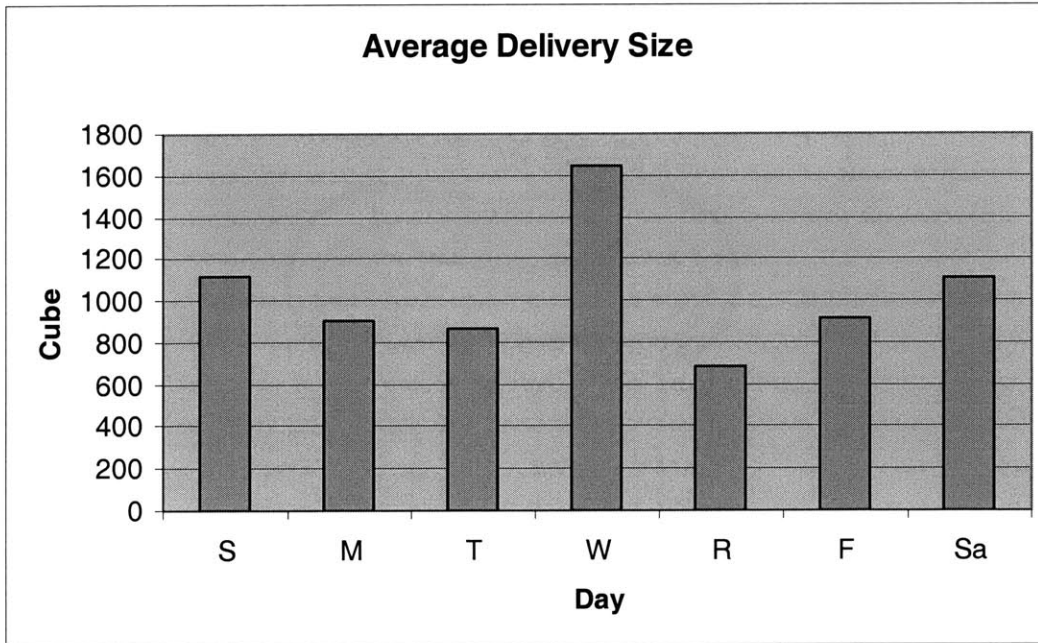


Figure 8: Average Delivery Size – The High-Volume Store

Section 3.2: The Low-Volume Store

The Low-Volume Store saw very similar patterns in sales data to those seen in The High-Volume Store, but the total average sales volumes were about 20% of those seen in The High-Volume Store. The average daily sales for The Low-Volume Store are shown in Figure 9.

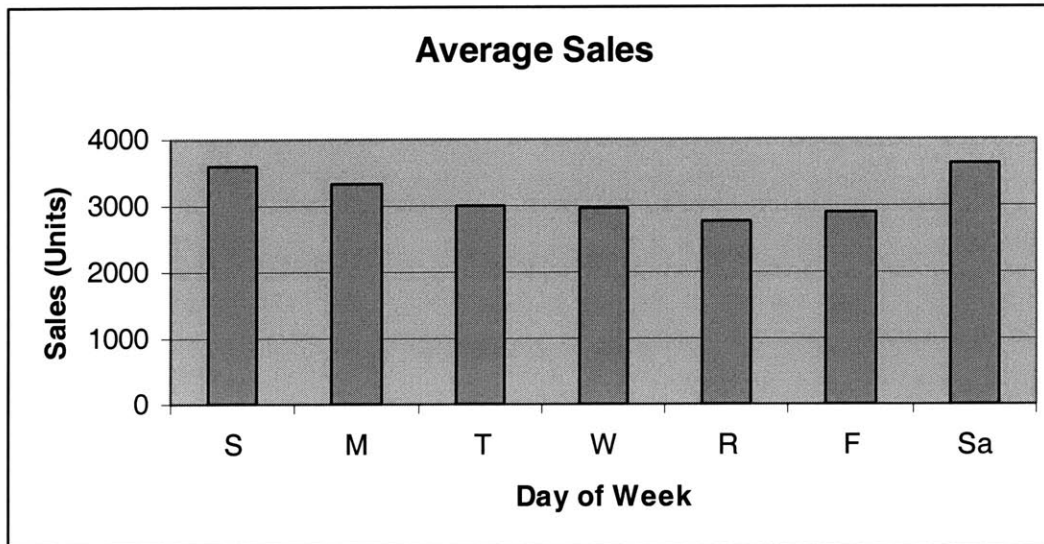


Figure 9: Average Sales on Each Day of the Week – The Low-Volume Store

The Low-Volume Store currently receives only three deliveries per week, and the average sizes of these deliveries can be seen in Figure 10. Each delivery is approximately one-third of a full truckload, which becomes important when trying to combine loads to reduce the number of deliveries per week.

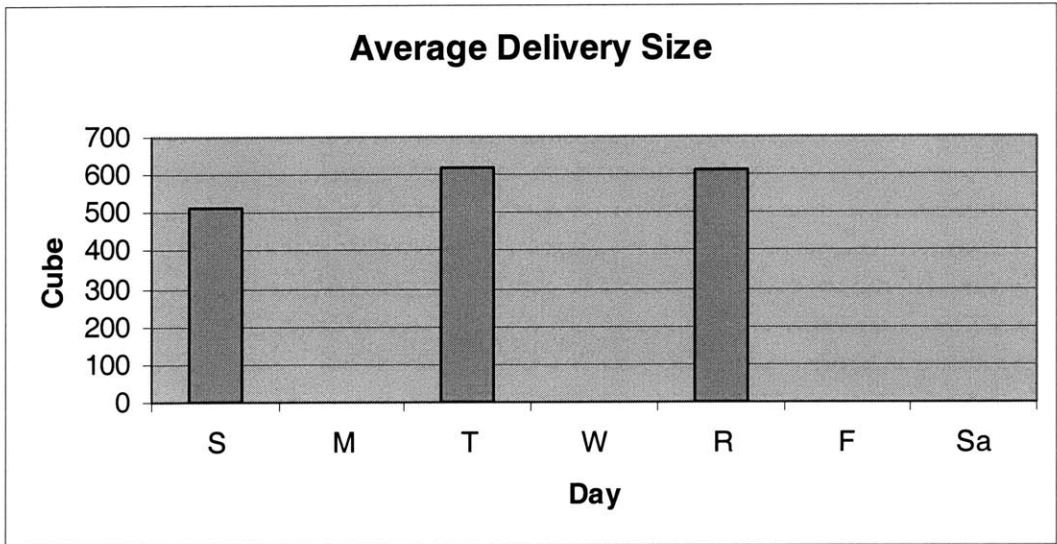


Figure 10: Average Delivery Size – The Low-Volume Store

Section 3.3: The Medium-Volume Store

The sales, and therefore the deliveries for The Medium-Volume Store are in between the sizes of those of the High and Low Volume Stores, and can be seen in Figures 11 and 12, respectively.

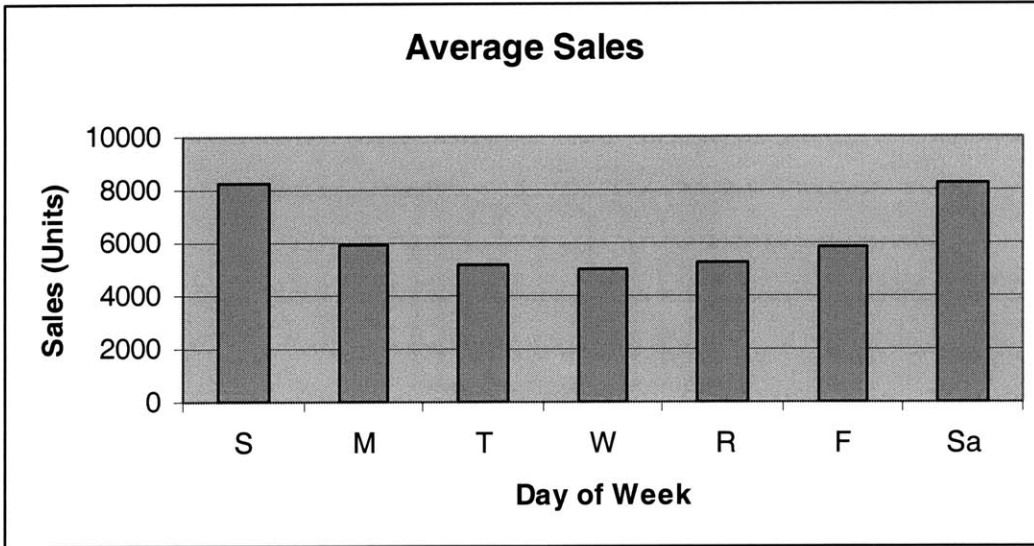


Figure 11: Average Sales on Each Day of the Week – The Medium-Volume Store

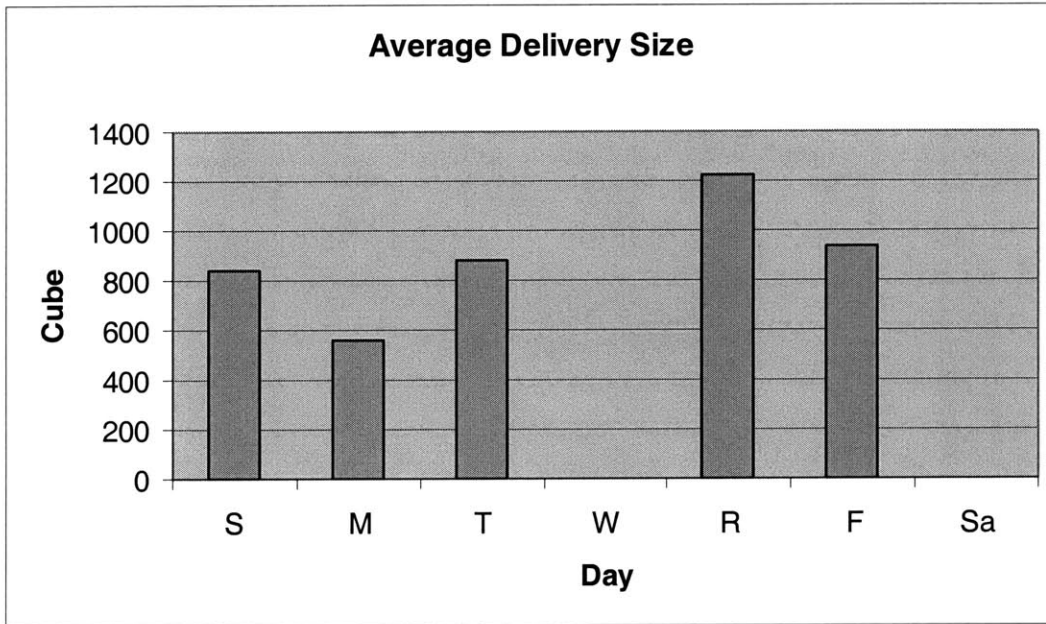


Figure 12: Average Delivery Size – The Medium-Volume Store

A comparison of some key statistics for the three stores can be seen in Table 3. From these statistics it can be seen that the number of SKU's carried by each store is not necessarily correlated to the sales volume of the store, but is instead based on factors such as, physical store size, store location (rural vs. urban, for example) and demographics of the local clientele. For example, The High-Volume Store is located in an urban area, and has limited sales floor space compared to its sales volume. The Medium-Volume Store, on the other hand, has much more sales floor space, and can therefore carry a much greater number of SKU's without having to store extra product in the backroom.

| Store | Size of Sales Floor (ft²) | Number of Dry Grocery SKU's | Average Weekly Dry Grocery Sales (Units) | Number of Deliveries / Week | Average Delivery Size (cubic feet) |
|----------------------|---|------------------------------------|---|------------------------------------|---|
| High-Volume | 30,920 | 9450 | 87,000 | 7 | 1040 |
| Medium-Volume | 56,882 | 11,200 | 43,800 | 5 | 890 |
| Low-Volume | 18,708 | 8020 | 22,000 | 3 | 580 |

Table 3: Comparison of Three Stores

4 Methodology

To determine the optimal delivery frequency for an individual store, we built a model that emulated Grocery Chain X's replenishment process. We then validated the model by comparing the model's behavior to actual data we had for the system. Once we had a model that we believed realistically simulated what was happening in the system, we tried different delivery schedules until we found the schedule that resulted in the lowest overall costs for that store. To make the model manageable, we had to make certain assumptions.

Section 4.1: General Assumptions

In order to model the replenishment system of Grocery Chain X, we made the following assumptions:

1. Every time a store orders a certain product, that product will arrive on the next delivery. The DC always has every product in stock, and all orders are picked without errors. This is not totally unrealistic because the products which are most often out of stock at the DC are the slow moving products, which have a small effect on the overall replenishment system.
2. By skipping a delivery to a store and holding the extra product in the backroom, the DC would not be forced to order product from its suppliers any earlier than it

normally would have. We took this to be the case because stores would not all skip deliveries on the same day of the week so the effects would be minimal. This assumption could be incorrect if the overall number of deliveries per day for all stores is changed so that deliveries are no longer evenly distributed throughout the week.

3. The delivery schedule of one store has no effect on the transportation routing of the system as a whole, and removing a delivery from one store results in the savings of the transportation costs for one truck traveling from the DC to the store and back. This is not actually the case because if one delivery was removed from the system, the transportation routing system would reroute the entire system and the resulting savings could range from zero up to the costs we assumed for the model. This will be discussed further in Section 5.3
4. Promotions are forecasted separately from the regular sales, but all promotional items are added to the delivery on the same day each week. The way promotions are actually handled is close to our assumption, however only about 80% of promotional items are delivered on the scheduled promotional delivery day. The remaining 20% is added as needed throughout the following week.
5. A stock-out occurs only when the actual demand of a given product is greater than the amount of that product in the store (both on the shelves and in the backroom.) This assumption does not necessarily perfectly model what happens in the store because if there is product in the store, it does not necessarily mean that it is on the shelf and available to customers. However, by skipping delivery days, we are delivering the product earlier and placing it in the backroom rather than leaving it at the DC. The

chance of stock-outs occurring while there is product in the store generally will not be affected by changing the delivery schedule. The only possible change in the number of stock-outs that could result from skipping delivery days would arise under the following scenario:

- a. A delivery is made and no delivery is scheduled for the following day.
 - b. During the day of the delivery, there is unusually high demand for a given SKU.
 - c. If there had been a delivery scheduled for the following day, more product could be ordered for the next delivery to cover for the unusually high demand for that day, but this would not now be possible.
6. All stores restock overnight. Most stores, including the stores for which we ran the simulation do stock overnight; however, this is not the case for all stores. The forecasting and SRO systems currently used by Grocery Chain X do not account for different stocking times among stores either.
7. Shelves are physically capable of holding a greater number of units of each SKU than the shelf space allocation states. It is against store policy to put a greater number of units of a certain product on the shelf than the stated capacity. However, if a stocker is left with only an item or two in the case, many SKU's have the needed flexibility to accommodate the extra product without having to place it in the backroom. For the simulation, we assumed that an additional 25% of each product could be stored on the shelves. Therefore, we used a shelf space adjustment factor of 1.25. Grocery Chain

X verified that this was a reasonable assumption, but the number is changeable in the model. We also performed a sensitivity analysis on this variable. The affect that the shelf space adjustment factor had on the percentage of SKU's stored in the backroom is shown in Figure 13. This adjustment factor had no significant impact on the number of stockouts.

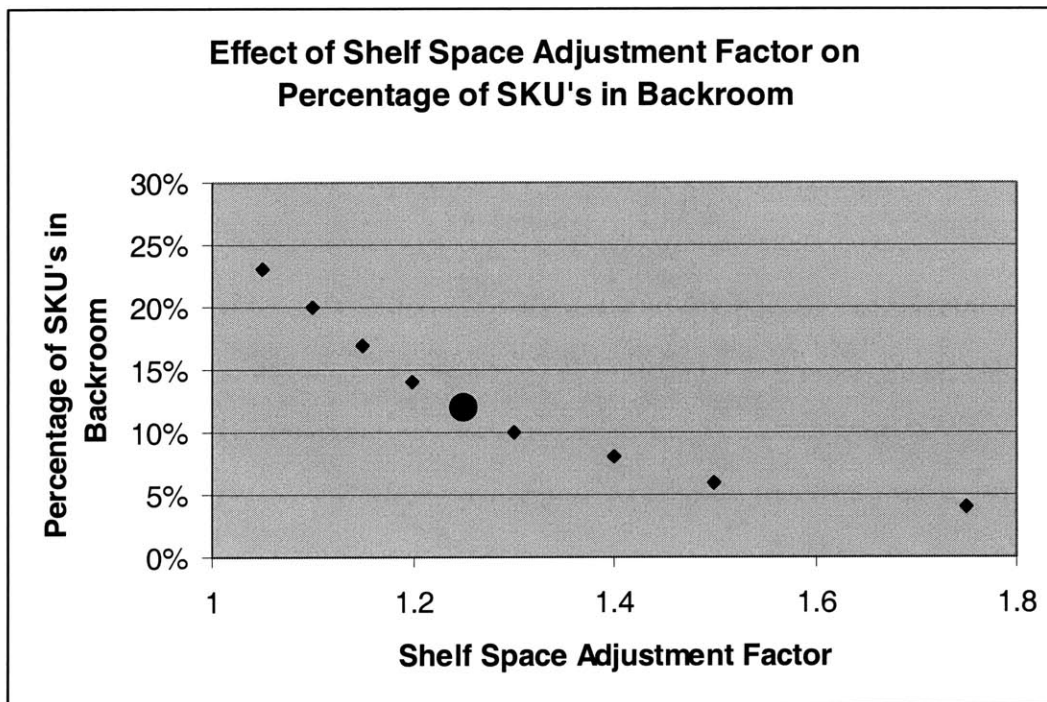


Figure 13: Effect of Shelf Space Adjustment Factor on Percentage of SKU's in Backroom

8. There are no additional inventory holding costs associated with storing product in the backroom. This assumption was made because if the product was not stored in the backroom, it would be stored in the DC; it would just be moved to the store a day earlier than it otherwise would have. Holding costs for the two locations are

identical. The model does, however, have the ability to account for any differences in holding costs if it was determined that there actually was a difference in the costs of holding product in the DC instead of the stores.

Section 4.2: Simulation

To create a simulation that could accurately model the reorder and replenishment systems of Grocery Chain X, we first had to fully understand what decisions were being made by the automated systems and how and when these decisions were being made. The two systems we would have to emulate in the model were the forecasting system and the Supervised Reorder System (SRO). The Transportation Management System (TMS) actually routed the trucks, but vehicle routing was outside the scope of this project.

The forecasting system collects point-of-sale data for each SKU at each store and stores this information on a daily basis for a rolling three-year period. With three years' worth of data, the forecasting system is able to account for factors that affect sales, such as the day of the week and the time of the year, and can forecast the sales for the following days until the next delivery is scheduled to arrive. At 4pm each day before a scheduled delivery, the SRO systems looks at the amount of each product in the store (both on the shelves and in the backroom) and subtracts the amount of that product forecasted to be sold before the delivery following the one that is currently being sized. For example, if a store is scheduled to receive a delivery on both Tuesday and Thursday, at 4pm on Monday the SRO will determine how much of each product to delivery on Tuesday by looking at the total amount of each product currently in the store and subtracting from it the amount of product forecasted to be sold between 4pm on Monday and Thursday

evening (the time when the next delivery will be moved to the sales floor.) The time of delivery for each store is different, but the shelves are generally restocked at night, so the forecast has to cover demand through the evening of the following delivery day. The system does not currently have the ability to account for the different stocking times of each store. If the final projected amount is less than the Reorder Point (ROP) for that product, the SRO system orders enough cases to fill the shelf to capacity, generally a single case.

The amount of product stored in the backroom is determined by comparing the amount of each SKU in the store with the physical amount of shelf-space allocated to that SKU. If the total amount of a given SKU at the store is greater than the allotted shelf-space (adjusted according to the shelf space adjustment factor discussed in Section 4.1) the remainder is stored in the backroom. The situation where there are too many units of a certain SKU in the store to fit on the shelf arises due to one of two reasons, either the forecasting system predicts a greater number of sales than actually seen (so too much is ordered,) or there is insufficient shelf-space allocated to that product, and the shelf is incapable of holding enough units to cover sales until the next delivery is scheduled to be stocked.

Each time product needs to be taken to the backroom, there is a cost associated with that case. The cost for each case is calculated based on the following assumptions:

1. There is a cost associated with having to physically take the product that will not fit on the shelf to the backroom (cost of stocking labor.)
2. If a portion of a case will not fit on the shelf, the partial case is counted as an entire case, because for the purpose of this model, whether or not the case is full does not impact how it is handled when taken to the backroom.

3. The product that will not fit on the shelf is stored in the staging area until the end of the stocking cycle. Then, all left-over product is taken to the backroom in as many trips as are necessary.
4. There is an additional cost of having to bring all product from the backroom to the staging area before the next stocking cycle (cost of stocking labor.)

The cost will vary from store to store based primarily on two factors: the cost of store labor and time required to move one load of extra product to the backroom. Factors that impact the time required to move product to the backroom include the distance from the sales floor to the backroom, whether or not the backroom is on the same level as the sales floor, and how obstructed the route to the backroom is. (Some routes require the stocker to travel through areas congested with product, equipment, etc.) The cost we used for The High-Volume Store was \$.27/case, and was calculated based on store time studies as follows:

| | |
|--|------------------------|
| Time required to handle backstock case: | .636 min / case |
| Travel time onto sales floor and back: | .067 min / case |
| Transport of backstock pallet: | .057 min / case |
| <u>Elevator time to get to and from back room</u> | <u>.048 min / case</u> |
| Total time required to handle 1 backstock case: | .808 min / case |

Store labor: \$20.30 / hr (\$14.50 + 40% for benefits)

Total Cost / Case = (.808min/case / 60 min/hr) * \$20.30/hr → **\$.27 / case**

Using sales data from 7-Dec03 to 4-Dec04, we built a simple model using MS Excel to emulate the forecasting and reorder rules used by Grocery Chain X's systems. Because of the large number of SKU's for which we had to provide forecast information, we used a simplified method of calculating the forecast. We accounted for the day-to-day variations in sales, but neglected the seasonality seen in some products. We assumed that simplifying the forecasting calculations would not greatly affect the accuracy of the model for two reasons:

1. When we generated an actual demand number for the product (this is explained further on the following page) we also did not take seasonality into account. In actuality, the store will experience seasonality in sales for some products, but the forecasting system will be able to account for this and adjust its forecast accordingly, hopefully negating the seasonality affects on the forecast accuracy. Also, inaccuracies in the forecast increase the amount of stockouts and backstock, so this represents a worst case scenario.
2. The variation in overall store sales is much more closely tied to the day of the week, than to the week of the year. Different products experience seasonally higher and lower demands at different times, canceling out most of the overall effects on store sales. This was discussed previously in Section 3.1.

Once we had the reorder rules built into the model, we had to build in the capability to “randomly” generate demand which would closely match the demand seen for each product over the past year. We first tried using the following function in Excel, because of its simplicity: **NORMINV(Rand(), mean, standard_dev)**, which generates a pseudo-random number based on a Normal distribution using the mean and standard deviation for each product, based on the

sales data we had. This function posed two problems for our application, however. First, the Normal function will potentially produce negative demands for some products. Second, the Normal distribution is a continuous distribution, which means that it will produce demands of partial products. To try get around these shortfalls, we set any negative demand equal to zero and rounded all demands up to the nearest integer. This function generated demands similar to the actual demands seen in the sales data for products with higher sales per day (generally over 15 units) because the chance that the Normal distribution would produce a negative demand for these products was relatively low. However, since the percentage of products with high demands was so low, the overall demand for the store which was generated by the model did not closely match the actual data. Therefore, the normal distribution was abandoned for this application.

We found that a Poisson distribution more closely represented what an individual product was actually experiencing in the store. A Poisson distribution describes the number of times an event occurs during a given time interval, in this case, the number of times a product is sold during one day. Because a Poisson distribution describes the number of times an event occurs, it can be neither a negative nor a non-integer number, which solves the two main problems we experienced with the Normal distribution. A POISINV function is available in a statistical add-in for Excel called SIMTOOLS. With this function (using the mean sales values for each day of the week), we were able to generate demands which much more closely matched the actual demands seen in the sales data. For those SKU's that experienced very limited demand, particularly the products that sold either zero or one product each day, the Poisson distribution could produce individual demands higher than demands actually seen in the past. This is due to the fact that there is theoretically no limit to the tail on a Poisson distribution, which means that there is no limit to the sales that the simulation could theoretically produce. However, the mean

would still be accurate, and the infrequent higher demand would produce a worse-than-actual case because higher demands increase the chance of a stockout. Also, the fact that these are slow-moving products means that their overall affect on the system will be minimal. For these reasons, we felt that the Poisson distribution would be appropriate for the very slow-moving products as well.

Based on the simulated demand, the model also needed the capability to handle a stock-out event. A stock-out occurs when the demand for a given product exceeds the amount of that product in the store. (A stock-out can also occur if there is product in the backroom, but it is not moved to the shelf when the shelf is empty. The model did not account for this because these stock-outs can be avoided with proper stocking procedures, which are not necessarily affected by the delivery frequency.) A cost was assigned to a stock-out, but an accurate cost was extremely difficult to determine because what actually happens during a stock-out is influenced greatly by consumer behavior. For example, if a store is out of a specific product, one of several things can happen. The consumer could wait and buy the product from that store the next time it is in stock, the customer could buy it at another store, the customer could buy a similar product instead, or the customer could not buy that product or a similar one. There is also the possibility that if the customer experiences a significant number of stock-outs at a certain store, he will no longer shop at that store, resulting in a loss of future sales. We used a stock-out cost of \$.25 for all products, certainly not an accurate number for every product, but stock-outs were not greatly affected by the delivery frequency because reorder points for Grocery Chain X are set high enough to account for the extra variations seen when the forecast time is extended by a day or two (the case when delivery days are skipped.) If changing the delivery schedule had greatly affected the

number of stockouts, the model could have easily been modified to set a stockout price for each product individually based on its sales price.

The total delivery costs are comprised of transportation costs (\$1.72 per mile, provided by Grocery Chain X) and the cost of picking and loading an order at the DC (\$40.80 per delivery, based on labor costs at the DC and time studies performed by Grocery Chain X.) The cost per delivery is calculated by adding the transportation cost to the delivery cost where the costs are defined as follows:

| | |
|----------------------|---|
| Transportation cost: | $2 * (\text{distance from DC to Store}) * \$1.72 / \text{mile}$ |
| Order-picking cost: | \$40.80 / delivery |

Because each delivery truck has a fixed capacity, in our case 1750 cubic feet, the model also needed to be able to determine how many cubic feet of product would need to be delivered each day. We knew the physical size of a case of each product, so we just needed to add the total cubic feet for all products for each delivery. If more than 1750 cubic feet of product was required on a given day, an extra delivery would be needed, and therefore, an additional transportation cost would be incurred. Since we modeled only one store at a time, we assumed that if a day's delivery exceeded 1750 cubic feet, two deliveries would be needed and the transportation cost would double; however, the entire order would still be picked as 1 order, so the order-picking cost would remain unchanged.

Once the general rules for the simulation were defined, we also had to assign values to the variables in the model. The following variables were held constant during the simulation for each store:

- Distance from the DC to the store
- Capacity of the delivery trucks
- Physical size of the backroom
- Product data (number of units per case, physical size of the case)
- Re-order rules

The following were assumed to be unchangeable during the first run of the simulation, but we also looked at the affects of changing these variables in later runs.

- Re-order point for each SKU
- Shelf space allocation for each SKU

Different delivery schedules were tried holding the above fixed to try to find the schedule that would minimize the overall costs. A pictorial representation of the simulation model can be seen in Figure 14.

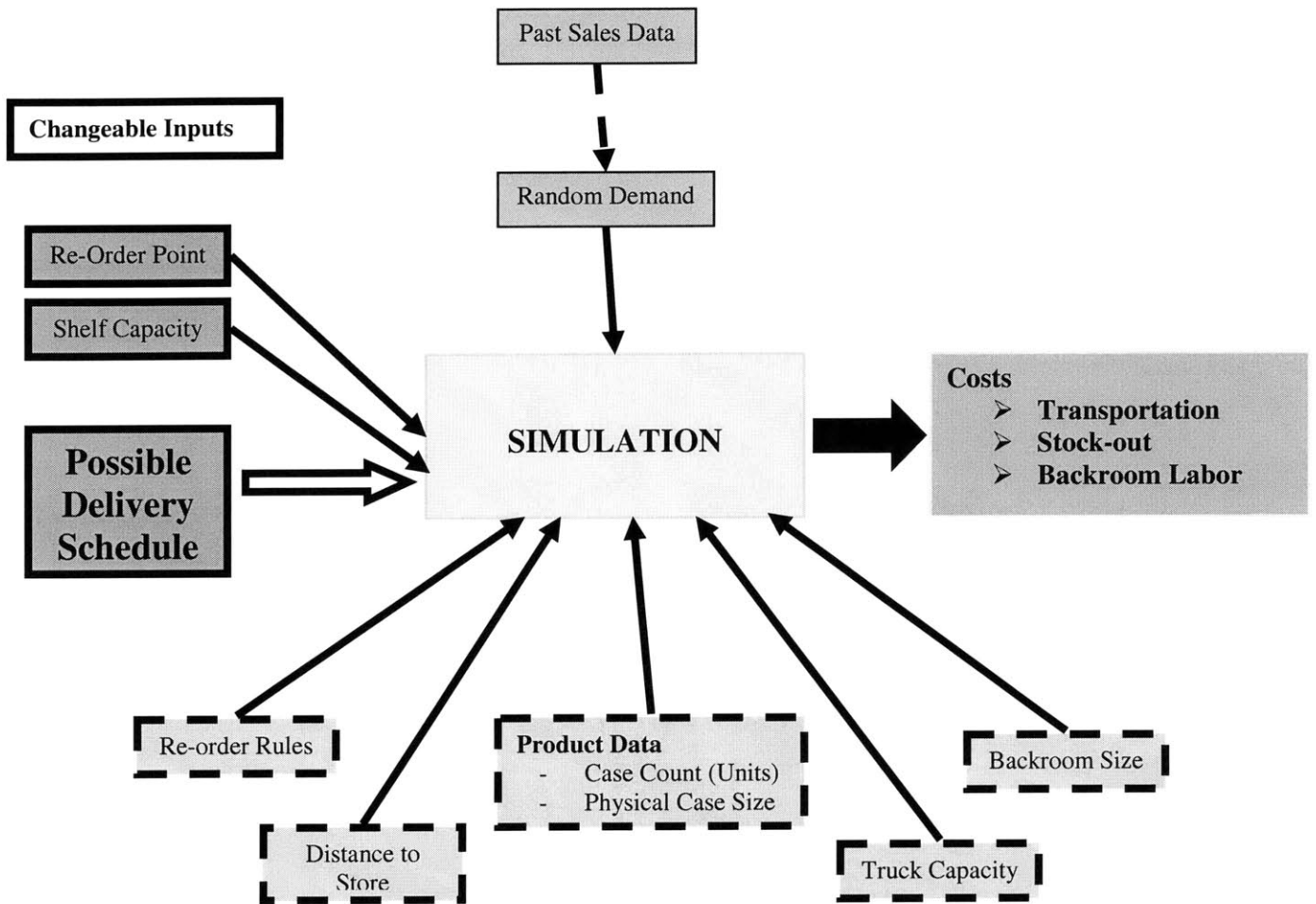


Figure 14: Simulation Diagram

The following describes the operations of the simulation:

1. Initial values were entered for all variables held constant throughout the simulation (boxes shown with a dashed border in Figure 14.) The reorder rules are incorporated into the model, and the reorder point and shelf capacity are imported from existing store data.
2. The desired delivery schedule was entered. We started with the schedule currently used, in the case of The High-Volume Store, delivery everyday.
3. An initial value for the total amount of product in the store was entered for each SKU. For simplicity, we assumed a full store and started with every shelf filled to capacity, but no product in the backroom. Doing this causes the model to show a lower than realistic cost for the first run. After initializing the model, however, the model stores the ending values from the previous simulation and uses those for the next run.
4. Once initialized, the model calculates the values for the first day, in our case Monday, based on the rules outlined earlier in this section. Specifically:
 - a. The model determines whether or not each product needs to be ordered. For the products that are ordered, the total number of cubic feet for that day's delivery is computed.
 - b. The model determines how much of each product will be at the store after the delivery arrives by adding the amount delivered (if any) to the amount in the store at the end of the previous day.

- c. The model generates a demand for each product, which simulates the actual demand seen at the store.
 - d. The model determines if any products see an actual demand greater than the amount of product in the store. If so, a stock-out is assumed to have occurred, and a stock-out cost is charged to the store.
 - e. Based on the sales, the model determines how much shelf space is currently available for each product and how much needs to be stored in the backroom.
5. The total number of stock-outs and cases moved to the backroom are computed along with the total costs.
 6. Delivery costs are calculated.
 7. Steps 4, 5 and 6 are repeated until the end of the week is reached (Sunday.)
 8. At the end of the week, the total costs are computed along with the total number of stock-outs, total cases sent to the backroom, and the total number of deliveries (all scheduled plus any extra trucks required if one delivery exceeds 1750 cubic feet.) These totals are then captured and copied into another worksheet.
 9. After 1 week has been simulated, the next week begins using the ending values from the previous week. Steps 4 – 9 are repeated 52 times, simulating a one-year run time.

Section 4.3: Model Validation

Once the model was built and tested, we needed to make sure that the results closely resembled what was truly happening in the real world. To accomplish the validation we ran the simulation using The High-Volume Store's current delivery schedule and compared the results to the data we had for the last year. Figure 15 shows the average daily deliveries predicted by the model next to the actual deliveries from the past year. Wednesday had the greatest variance, but we felt this was due to the fact that the promotional items were added to this delivery separately from the regular replenishment system, and we didn't have good data showing how much of Wednesdays' actual deliveries were part of the regular replenishment and how much was promotional product. In the model, we used 1300 ft³ of promotional product, this may have been higher than the amount actually seen at the store, but we preferred to err on the high side to provide the worst-case scenario. By using a higher amount of promotional product, the truck would fill up faster, and there would be a greater likelihood of needing a second truck, which would raise the transportation costs.

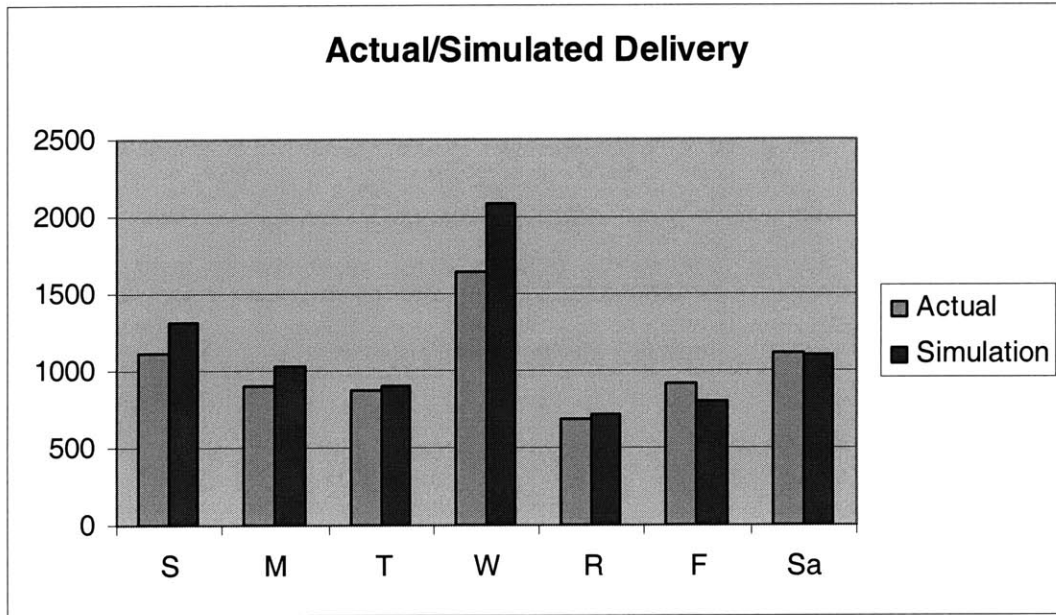


Figure 15: Actual Deliveries Compared with Simulated Deliveries

Figure 16 shows the simulated sales from one run compared to the actual sales seen for the past year. The difference between the actual and simulated results was 5% or less for each day of the week. Table 4 shows the difference between the simulated and the actual sales and delivery data.

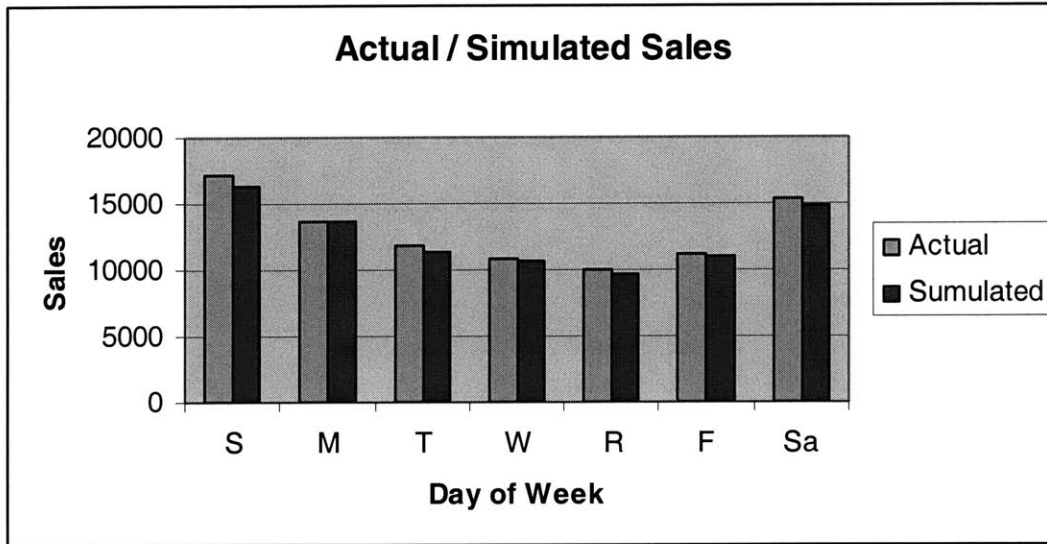


Figure 16: Actual Sales Compared with Simulated Sales

| Day | Simulated Delivery (cubic feet) | Actual Delivery (cubic feet) | Percent Difference | Simulated Sales (units) | Actual Sales (units) | Percent Difference |
|--------------|---------------------------------|------------------------------|--------------------|-------------------------|----------------------|--------------------|
| Sunday | 1311 | 1117 | 17% | 16,400 | 17,245 | -5% |
| Monday | 1034 | 905 | 14% | 13,667 | 13,747 | -1% |
| Tuesday | 890 | 870 | 2% | 11,415 | 11,825 | -3% |
| Wednesday | 2080 | 1650 | 26% | 10,600 | 10,883 | -3% |
| Thursday | 720 | 685 | 5% | 9600 | 9961 | -4% |
| Friday | 806 | 917 | -12% | 11,020 | 11,216 | -2% |
| Saturday | 1100 | 1108 | -1% | 14,820 | 15,292 | -3% |
| Total | 7940 | 7252 | 9% | 90,167 | 87,522 | -3% |

Table 4: Actual vs. Simulated Sales and Deliveries

Once we determined that the model was creating results comparable to the data we had for last year, we showed the model to the several members of Grocery Chain X's transportation and forecasting staff to get an idea of whether or not the numbers intuitively made sense to them. After confirming that the model was producing reasonable results, we ran different scenarios for each store by changing their delivery schedules.

5 Results and Analysis

Once we were convinced that the model was a reasonable representation of the actual replenishment process, we simulated this process for three different stores in the grocery chain. The chosen stores were all roughly 100 miles from the Distribution Center (DC) and had different delivery schedules and weekly sales volumes. First, we looked at each store individually, and then we looked at how changing the delivery schedule of one store would impact the delivery system as a whole. The results are displayed and discussed in this section.

Section 5.1: The High-Volume Store Results

Originally, The High-Volume Store received a delivery every day, and since Wednesday's delivery also included the promotional products, it was often large enough to require two trucks. First, we ran the model with the original delivery schedule so that we would have a baseline to which we could compare the results of other possible delivery schedules. We then ran the simulation removing one delivery per week, varying which delivery day we skipped. The inputs we used for The High-Volume Store, which were defined in Section 4.2, are shown in Table 5.

| ----- FIXED INPUTS ----- | | |
|----------------------------------|------|-------------------|
| Max cube/del | 1750 | ft^3 |
| Cost of Stockout | 0.25 | per day |
| Backroom labor cost | 0.27 | per case |
| Transportation Cost | 1.72 | per mile |
| Order Pick Cost | 40.8 | per pick |
| Number of Miles from DC to Store | 82 | per one-way del. |
| Avg Size of Promotional Del. | 1300 | Cube |
| Shelf Space Adjustment | 1.25 | correction factor |
| Inventory Carrying Cost Dif. | 0 | Yearly Rate |

Table 5: Store-Specific Inputs

We found that the day of the week we skipped had a significant impact on the overall weekly costs. The results are shown in Table 6.

| Days Without Delivery (promotional order = Wed) | Average Total Weekly Cost (dollars) | Average Amount of Product in Backroom (partial cases) | Maximum Amount of Product in Backroom (partial cases) | Average Number of Trucks per Week | Minimum Delivery Volume (cubic ft) | Maximum Delivery Volume (cubic ft) | Average Weekly Stockouts (occurrences) |
|---|---|--|--|--|---|---|---|
| None (As-is) | 4660 (+0%) | 1100 | 1150 | 8.0 | 790 | 2000 | 45 |
| Sunday | 4850 (+4%) | 1200 | 1730 | 7.9 | 800 | 2377 | 55 |
| Monday | 4820 (+3%) | 1180 | 1540 | 8.0 | 770 | 2150 | 35 |
| Tuesday | 4730 (+2%) | 1160 | 1465 | 7.8 | 750 | 2100 | 55 |
| Wednesday (promo- Th) | 4490 (-4%) | 1160 | 1450 | 7.1 | 830 | 2140 | 50 |
| Thursday | 4460 (-4%) | 1145 | 1380 | 7.1 | 920 | 2800 | 50 |
| Friday | 4430 (-5%) | 1145 | 1475 | 7.0 | 800 | 2300 | 50 |
| Saturday | 4840 (+4%) | 1190 | 1625 | 8.0 | 800 | 2050 | 50 |

Table 6: Simulation Results – The High-Volume Store

For instance, by removing Sunday's delivery, the total average weekly cost increases, while removing Friday's delivery reduces the cost. Recall that the major tradeoff in this model is increasing backroom inventory to eliminate the costs associated with a delivery. (This was the case for Grocery Chain X, but is not necessarily the case for all stores.) By not scheduling a delivery on Sunday, Saturday's delivery becomes large enough that it will generally not fit on one truck, so two trucks are needed. Hence, the average weekly number of trucks arriving at the store only decreases from 8.0 to 7.9, rather than to 7.0. The average number of cases or partial cases in the backroom also increases by 100, which cannot be offset by reducing the average number of deliveries by 0.1. By removing Friday's delivery, however, the average number of deliveries per week drops to 7.0 and the average number of cases needing to be stored in the backroom only increases by 45. In this case, the extra cost of having to place more products in the backroom is more than offset by removing a delivery to the store, so the average total weekly cost decreases by over 200 dollars. It should be noted, however, that although the average weekly cost is calculated using the average amount of product going to the backroom, generally the delivery day before a day without a scheduled delivery sees a much larger amount going to the backroom. This number needs to be taken into account when making sure the backroom is large enough to handle the extra product, and when scheduling stocking labor for that day. By skipping a delivery on Friday, for example, although the average amount of product in the backroom only increases to about 1145 cases, this amount increases to about 1400 cases on Thursday. (When Friday's delivery is not skipped, the maximum amount stored in the backroom is only about 1150 cases.)

The number of stockouts does not seem to be significantly impacted by the delivery schedule. This can be explained by the fact that there are two competing factors that are

affecting the number of stockouts. (Recall that we defined a stockout as a situation where the demanded number of a specific product is greater than the amount of that product in the store, both on the shelf and in the backroom.) The first factor is, that by removing a delivery from the schedule, the product that would normally be delivered on the day which is now skipped, is now delivered with the previous delivery. This means that now the product is actually in the store before it normally would have been there. The competing factor which balances the effects of delivering the product earlier is by skipping a delivery day, the forecasting system is forced to forecast one day further into the future, which decreases the accuracy of the forecast. If the delivery day was not skipped, the order for that day would be placed one day later, which means that an extra day's sales will be known, rather than having to be forecasted.

We also ran the simulation defining a stockout as the situation where the demand for a product is greater than the amount of product at the store, less the amount that arrived on that day's delivery. We did this because different stores receive their deliveries and stock their shelves at different times of the day, and it is therefore possible that the product is at the store, but still on the truck, and not readily available to the customers. The model showed that although the number of stockouts increases, it increases for all of the different delivery schedules relatively uniformly. Therefore, the definition of a stockout has little effect on which delivery schedule should be used. The definition of a stockout only affects the overall number of stockouts reported by the model.

After running the simulation for the seven different scenarios for removing one day from the delivery schedule, we looked at removing two days from the schedule. In most cases, removing an extra day's delivery simply resulted in the delivery for the day before becoming too large to fit on a truck, therefore eliminating any potential benefits. However, there was one

scenario in which two days' worth of deliveries could be added to other deliveries during the week without making those deliveries so large as to require an extra truck. By eliminating deliveries on Wednesday and Friday (and moving the promotional delivery to Thursday), neither Tuesday's delivery nor Thursday's delivery would need to add an additional truck. Tuesday's and Wednesday's product, when combined, was still usually less than the capacity of one truck. Since Thursday's delivery would include the promotional product, it would already require part of a second truck, and Friday's delivery could be added to it without creating the need for a third truck. The results for this scenario are shown in Table 7.

| Days Without Delivery (promotional order = Wed) | Average Total Weekly Cost (dollars) | Average Amount of Product in Backroom (partial cases) | Maximum Amount of Product in Backroom (partial cases) | Average Number of Trucks per Week | Minimum Delivery Volume (cubic ft) | Maximum Delivery Volume (cubic ft) | Average Weekly Stockouts (occurrences) |
|---|---|--|--|--|---|---|---|
| None (As-is) | 4660 (+0%) | 1100 | 1150 | 8.0 | 790 | 2000 | 45 |
| Wed, Fri (promo – Th) | 4200 (-10%) | 1190 | 1450 | 6.0 | 1090 | 2850 | 55 |

Table 7: Simulation Results - The High-Volume Store (continued)

The maximum delivery, now on Thursday, increases to 2850 ft³, but the capacity of two trucks is 3500 ft³. The two delivery schedules (current schedule and two skip days) and the amounts of each delivery are shown side-by-side in Figure 17.

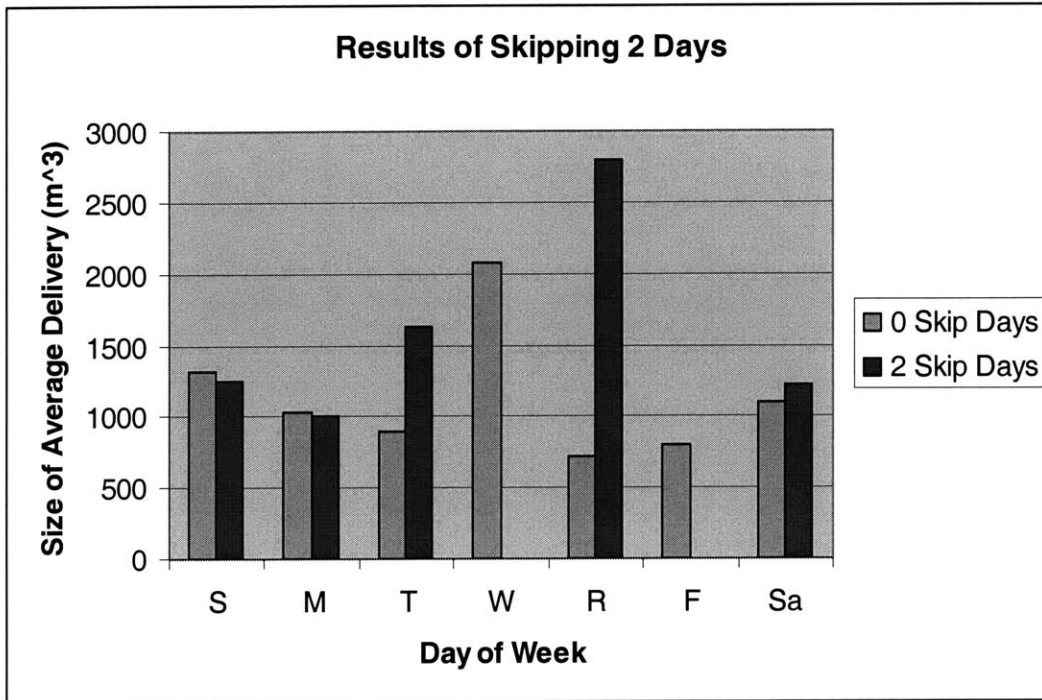


Figure 17: Size of Daily Deliveries Resulting from Skipping 2 Days

By decreasing the delivery frequency to five days per week, the number of cases stored in the backroom spikes up by about 300 cases on days before the days with no delivery, but the average number of cases for the week in the backroom only increases by about 90. The increase in backroom storage costs are more than offset by the decrease in transportation costs.

Section 5.2: The Low-Volume and Medium-Volume Stores

Results similar to those for The High-Volume Store were attained for the other two stores by running the model with the parameters specific those stores. The model showed that the lowest cost option for The Low-Volume Store would be to deliver to the store two times per week instead of three. The Medium-Volume Store currently receives five deliveries per week,

but the model showed that the lowest cost option would be to remove two of the deliveries. For all three stores, the delivery schedule could be found where additional backroom costs were outweighed by the savings in transportation costs. The results for The Low and Medium-Volume Stores are shown in Tables 8 and 9, respectively.

| Days Without Delivery (promotional order = Thur) | Average Total Weekly Cost (dollars) | Average Amount of Product in Backroom (partial cases) | Maximum Amount of Product in Backroom (partial cases) | Average Number of Trucks per Week | Minimum Delivery Volume (cubic ft) | Maximum Delivery Volume (cubic ft) | Average Weekly Stockouts (occurrences) |
|---|--|--|--|--|---|---|---|
| M, W, F, Sa (As-Is) | 2200 (+0%) | 670 | 710 | 3.0 | 4700 | 900 | 25 |
| M, Tu, W, F, Sa | 1970 (-10%) | 700 | 830 | 2.0 | 910 | 1000 | 40 |
| M, Tu, Th, F, Sa | 1975 (-10%) | 713 | 800 | 2.0 | 820 | 1140 | 36 |
| Su, Tu, W, F, Sa | 1980 (-10%) | 715 | 820 | 2.0 | 800 | 1130 | 40 |
| Su, M, Tu, Th, F | 1960 (-11%) | 700 | 780 | 2.0 | 970 | 980 | 36 |

Table 8: Simulation Results – The Low-Volume Store

| Days Without Delivery (promotional order = Thur) | Average Total Weekly Cost (dollars) | Average Amount of Product in Backroom (partial cases) | Maximum Amount of Product in Backroom (partial cases) | Average Number of Trucks per Week | Minimum Delivery Volume (cubic ft) | Maximum Delivery Volume (cubic ft) | Average Weekly Stockouts (occurrences) |
|--|---|--|--|--|---|---|---|
| W, Sa (As-Is) | 4140 (+0%) | 1180 | 1260 | 5.0 | 550 | 1100 | 10 |
| M, W, Sa | 3820 (-8%) | 1210 | 1310 | 4.0 | 850 | 1220 | 10 |
| Su, W, F | 3790 (-8%) | 1200 | 1330 | 4.0 | 760 | 1460 | 10 |
| T, R, Sa (promo- Wed) | 3780 (-9%) | 1190 | 1290 | 4.0 | 830 | 1250 | 15 |
| Su, T, W, F | 3420 (-17%) | 1200 | 1360 | 3.0 | 930 | 1420 | 20 |
| Su, M, W, F | 3830 (-7%) | 1240 | 1420 | 3.7 | 900 | 1850 | 15 |
| M, T, R, Sa (promo- Wed) | 3450 (-17%) | 1202 | 1390 | 3.1 | 790 | 1600 | 20 |
| Su, T, R, Sa (promo- Wed) | 3840 (-7%) | 1260 | 1470 | 3.8 | 790 | 1880 | 20 |
| M, T, W, F, Sa | 4000 (-4%) | 1310 | 1510 | 4.0 | 1970 | 2070 | 30 |

Table 9: Simulation Results - The Medium-Volume Store

Notice that skipping Su, T, W and F needs an average of 3.0 trucks per week and has a weekly cost of \$3420. Skipping M, T, R and Sa requires an average of 3.1 trucks per week and the cost is slightly higher. This means that every one in ten weeks, an extra truck will be required because one of the deliveries will be large enough to require an extra truck.

Section 5.3: General Procedure for Choosing Days Without Deliveries

One method for finding the best delivery schedule for a store is by mass enumeration, which means running every possible delivery schedule through the model. The total number of possible delivery schedules for a store (not including the promotional delivery) is two to the seventh power, or 128 possible schedules. Since running this model 128 times would be quite

time-consuming, we were able to establish a general procedure for eliminating certain groups of delivery schedules which did not make sense. This procedure is specific to stores that follow the replenishment process described in Section 2.1, and is outlined below.

1. Run the simulation with deliveries on every day of the week, without including the promotional delivery. This will provide the approximate amount of product required each day of the week.
2. Determine the total amount of product required each week by adding the delivery sizes for each day found in Step 1.
3. Determine the minimum number of truckloads of product required each week. This is accomplished by taking the total amount of product required each week (found in Step 2) plus the promotional product, and dividing this sum by the capacity of 1 truck. In the case of Grocery Chain X, this number is 1750 ft³. The number of deliveries must be rounded up to the nearest integer. This will provide the lower limit to the number of truckloads required each week. All possible delivery schedules which provide fewer deliveries than this lower limit can be eliminated. This minimum number of truckloads may or may not actually be possible, depending on how the product requirements are distributed throughout the week.
4. Look at the results of the simulation from Step 1 and try to find daily deliveries that can be combined, onto one truck. Remember that the promotional product must be combined onto a truck on either Wednesday or Thursday. Figure 18 shows the delivery sizes from The Medium-Volume Store, and one possible way of combining them into three deliveries, the minimum number required found in Step 3. There may

be several ways of doing this. It is possible that there will be no way to combine the deliveries into only three. If this is the case, combine deliveries into the minimum number possible.

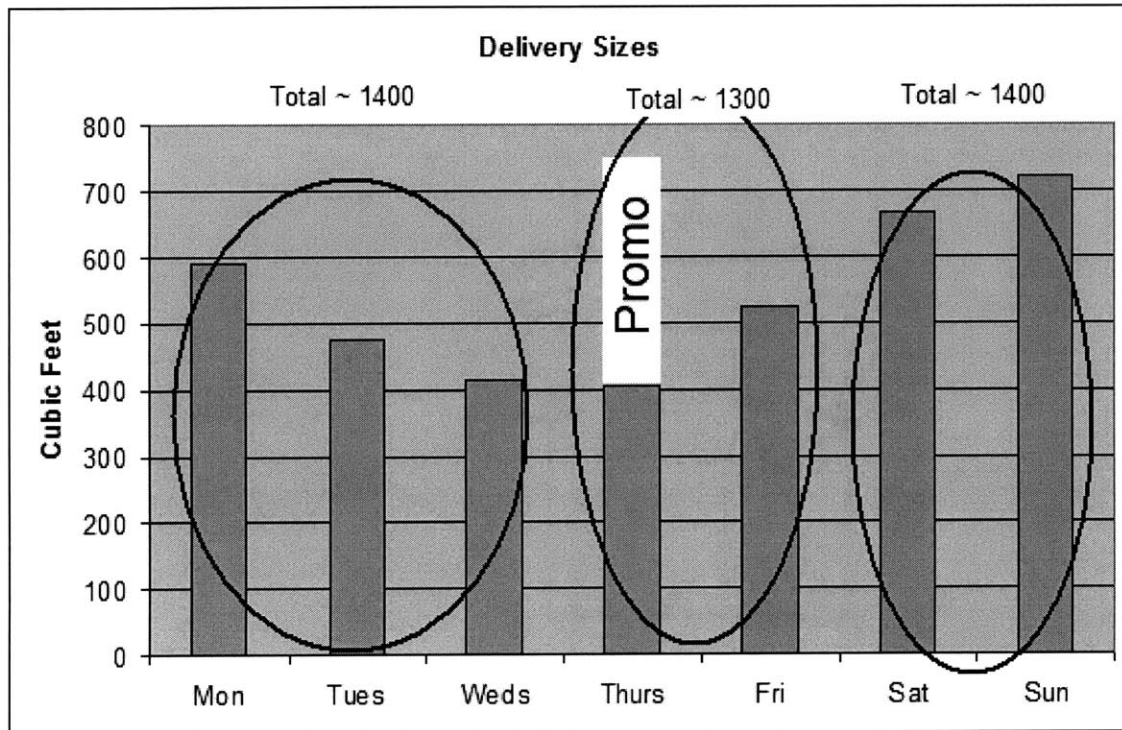


Figure 18: Combining Deliveries

5. If either Wednesday's or Thursday's delivery plus the promotional product, or one day's delivery alone is greater than the capacity of one truck, try to combine another day's delivery with this one if it can be done without requiring a third truck.
Combine the remaining deliveries for the week in the same manner as in Step 4.

6. Run the simulation for all of the delivery schedules found in Steps 4 and 5 to determine the lowest cost option.
7. It is possible that making the minimum number of deliveries per week (from Steps 4 and 5) is not the lowest cost option, depending on how much product will need to be stored in the backroom. For this reason, repeat Step 4, but this time, add one more delivery per week.
8. Run the simulation for all delivery schedules found in Step 7 and compare the results to those from Step 6. Find the lowest cost option.

Section 5.4: Effects on System of Changing The Delivery Schedule for One Store

After looking at each store individually, we needed to determine how changing the delivery schedule of one store would impact the transportation system as a whole. Specifically, because a delivery to a given store is often combined with the deliveries to one or more other stores, we wanted to know whether or not removing a delivery to one store would actually result in any savings in transportation, and if so, the amount of the savings. The transportation costs calculated in the model assume that loads are not combined with the deliveries from any other stores and that the total distance from the DC to the store and back will be saved by removing a delivery.

To investigate the impacts on the system as a whole, we ran different scenarios on Grocery Chain X's Transportation Routing System (TRS), which actually designs and schedules the delivery routes for each truck. First, we ran the TRS with the current delivery schedule to get

a baseline. Then we re-ran the simulation removing one, two, six, and ten stores. The results are shown in Figure 19.

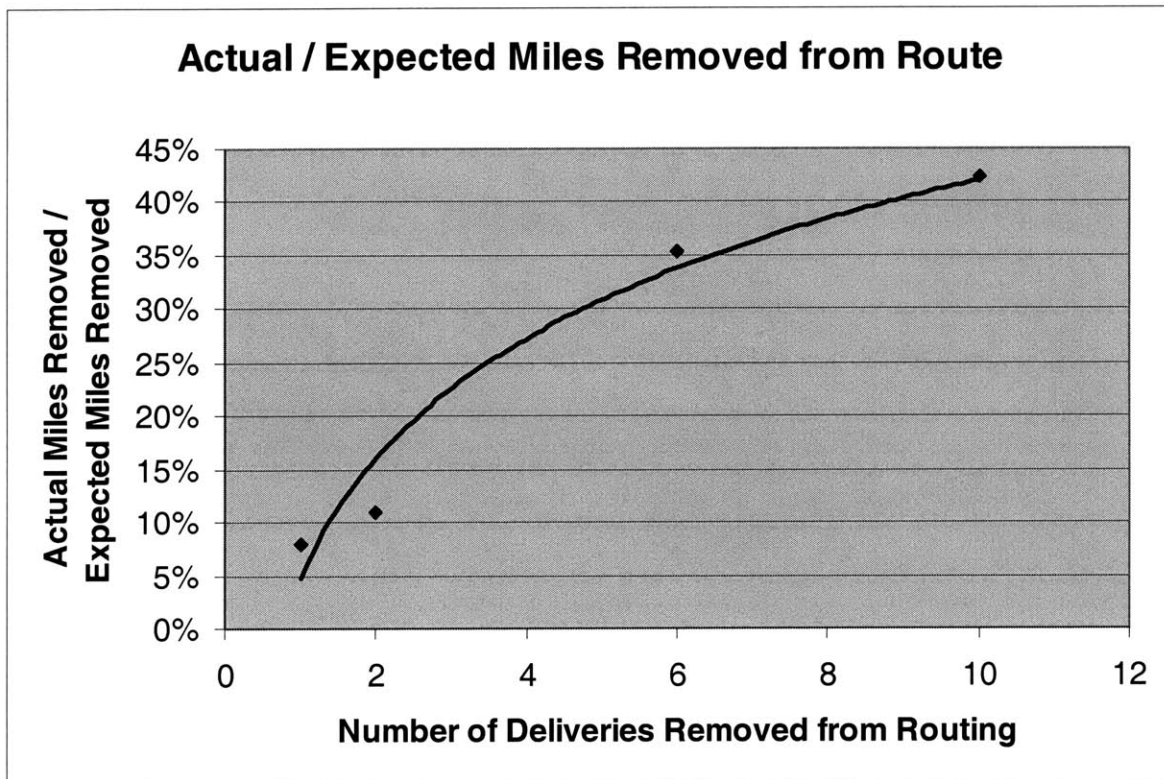


Figure 19: Ratio of Actual-to-Expected Miles Removed from Route

By removing only one store delivery from the system, the mileage reduction is only about ten percent of the expected reduction (distance from the DC to the store.) However, as the number of stores removed from the delivery schedule on a single day approaches ten stores, this number increases to over 40%. Because the TRS routes the deliveries based on which stores need to receive deliveries and the size of the deliveries, the routing will be different each day. This is just one sample routing and actual daily results may differ substantially, depending on

how the removed deliveries were combined with other loads. If a scheduled delivery was a dedicated truck going from the DC to the store, and that load was removed, the entire mileage would be dropped from the routing. Generally, this is not the case, however, so removing one delivery results in the TRS system rerouting several of the deliveries. Overall, from the TRS test runs we performed, we found that there is a better chance of seeing greater savings in total miles traveled when a greater number of deliveries can be removed on the same day, compared to the case where the removed deliveries are scattered throughout the week. Depending on the number of stores and deliveries in the system, this fact could have an impact on driver utilization. However, in Grocery Chain X's system, there are over 200 stores, so distributing the days which are removed from the delivery schedule throughout the week should not be difficult.

Section 5.5: The Effects of Changing Re-Order Points and Shelf-Allocation

After running the simulation for all three stores using actual Re-Order Points (ROP) and Shelf-Allocation numbers used currently by the stores, we looked at how changing these parameters would change the overall results. First, we changed the ROP's using several different methods, all of which were simple formulas applied to all products, and all of which produced similar results. For example, when we set the ROP for each product in The High-Volume Store according to Formula 1, the overall costs were reduced and are shown in Table 10.

Formula 1: $ROP_i = \text{Max}[2, \text{Max}(\sigma_{D-i})]$, where:

ROP_i = the re-order point for Product i

σ_{D-i} = Daily standard deviation of demand for Product i

| SKIP DAYS (Promotional order = Thursday) | Original ROP's | | ROP's Adjusted According to Formula 1 | | Percentage Change | |
|--|----------------------------------|---|---------------------------------------|---|----------------------------------|---|
| | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) |
| None | 4660 | 1100 | 3700 | 600 | -21% | -45% |
| Sunday | 4850 | 1200 | 3840 | 650 | -21% | -46% |
| Monday | 4820 | 1180 | 3800 | 640 | -21% | -46% |
| Tuesday | 4730 | 1160 | 3780 | 620 | -20% | -47% |
| Wednesday (promo - Th) | 4490 | 1160 | 3480 | 620 | -22% | -47% |
| Thursday | 4460 | 1145 | 3440 | 620 | -23% | -46% |
| Friday | 4430 | 1145 | 3460 | 630 | -22% | -45% |
| Saturday | 4840 | 1190 | 3800 | 640 | -21% | -46% |
| Wed, Fri (promo - Th) | 4200 | 1190 | 3170 | 640 | -25% | -46% |

Table 10: Simulation Results for Adjusted ROP's

Table 10 shows that the best solution remains to skip deliveries on Wednesday and Friday, and that the relative order of the best days to skip remains essentially the same. The number of products stored in the backroom, however, is reduced by over 45% for each delivery schedule, which leads to overall cost reductions of over 20%.

By adjusting the Re-Order Points in this manner, 11% of the SKU's saw increases in their Re-Order Points, but the majority (80%) of the SKU's saw decreases in their Re-Order Points, for many of the slower moving products, significant decreases. 9% of the Re-Order Points remained unchanged. The distribution of the ratios of New Re-Order Points to Current Re-Order Points is shown in Figure 20. The SKU's which saw significant increases in their Re-Order points were the very fast-moving products which currently have ROP's which are set lower than they otherwise would be because of limited shelf space.

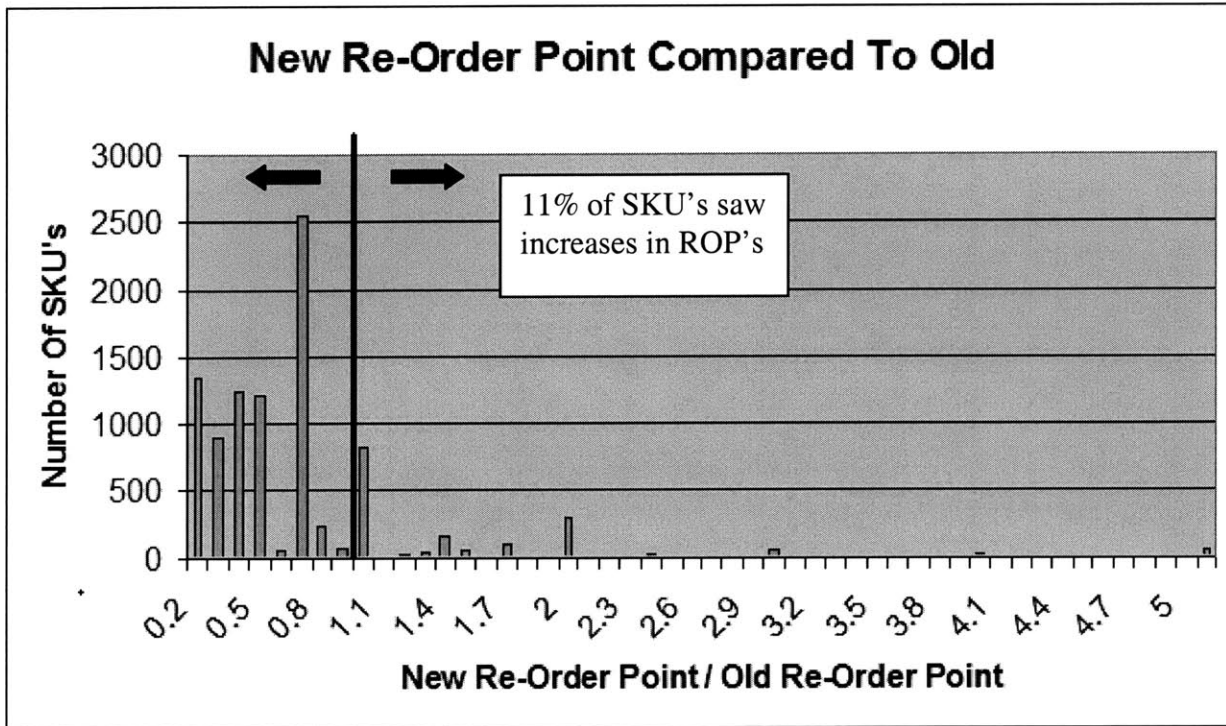


Figure 20: Distribution of How ROP's Changed

The method we used for determining ROP's is not the best way to accomplish this and was just an attempt to find the magnitude of the effect of changing this parameter. A more sensible approach to setting the ROP's may be to group products by factors such as daily sales, standard deviation, and/or shelf-space allocation, and then to set the ROP's for each group of products using a separate formula for each group. How the ROP's should optimally be set remains a possibility for future research.

After adjusting the Re-Order Points to try to lower the number of cases stored in the backroom, we looked at how shelf-space was allocated to each SKU. Currently, shelf-space allocation is influenced by sales and marketing factors much more so than by operational or supply chain factors. This is typical of the way most retail stores allocate their shelf-space

because it has been shown in numerous studies, Cox (1970), Curhan (1972), Dubelaar et al. (2001), and countless others, that where and how much of each product is displayed on the shelves can have a significant impact on sales. In our model, we took a cursory look at how the backroom stock would be affected if shelf-space was allocated based on trying to reduce the number of cases stored in the backroom, rather than by the factors currently being used.

To try to determine the affect of shelf-space allocation on the number of cases stored in the backroom, we used The High-Volume Store as our case study. We first needed to get a rough idea of how much usable shelf-space was actually available in the store. We assumed that shelf height was adjustable and therefore chose to measure the amount of shelf-space in cubic feet rather than in square feet since product height differs greatly among SKU's. This may or may not be an appropriate way to look at this problem, depending on how flexible shelf height truly is.

To find the total amount of shelf-space available in the store, we took the current shelf space allocation in number of units for each product and converted this number into number of cases. We then took the number of cases times the physical size of a case. After doing this for each product, we took the sum of the amounts of shelf-space allocated to each product to arrive a total amount of shelf-space available in the store. After we knew the total amount of shelf-space in the store, we allocated to each product an amount of shelf-space according to Formula 2.

Formula 2: $SS_i = ROP_i + PS_i$, where:

SS_i = Shelf Space Allocated to Product i in number of units

ROP_i = Re-Order Point of Product i in number of units

PS_i = Number of units contained in 1 case of Product i

The idea behind this formula was that when the Balance On Hand (BOH) of a product dropped below its Re-Order Point, a case would be ordered, and theoretically the entire case should fit on the shelf, thereby basically eliminating product in the backroom. This process worked for the vast majority of the products and nearly eliminated the necessity of storing product in the backroom. However, there were a few fast-moving products which had daily sales greater than one case plus the Re-Order Point, and these products would need to be restocked throughout the day. The results of this simulation are shown in Table 11.

| SKIP DAYS (Promotional order = Thursday) | Original ROP's and Shelf Space Allocation | | ROP's and Shelf Space Allocation Adjusted | | Percentage Change | |
|--|---|---|---|---|----------------------------------|---|
| | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) | Avg. Total Weekly Cost (dollars) | Avg. Amount in Backroom (partial cases) |
| None | 4660 | 1100 | 2610 | 10 | -44% | -99% |
| Sunday | 4850 | 1200 | 2660 | 40 | -45% | -97% |
| Monday | 4820 | 1180 | 2670 | 30 | -45% | -97% |
| Tuesday | 4730 | 1160 | 2650 | 30 | -44% | -97% |
| Wednesday (promo - Th) | 4490 | 1160 | 2350 | 25 | -48% | -98% |
| Thursday | 4460 | 1145 | 2300 | 20 | -48% | -98% |
| Friday | 4430 | 1145 | 2300 | 20 | -48% | -98% |
| Saturday | 4840 | 1190 | 2640 | 30 | -45% | -97% |
| Wed, Fri (promo - Th) | 4200 | 1190 | 2010 | 30 | -52% | -97% |

Table 11: Simulation Results for Adjusted ROP's and Shelf Capacity

When both the Re-Order Points and the Shelf-Space Allocations were adjusted, the overall costs we cut by about 50% when compared to the results using the current values because the amount of product needing to be stored in the backroom was reduced to almost zero. However, this

model assumes that sales will not be altered by the changes in shelf-space allocation, which has been shown in numerous studies not to be the case. Also, using this model allocated very large amounts of shelf-space to certain large, fast-moving products. For example, one-gallon bottled water received almost 15 times the amount of shelf-space currently allotted to that product, probably not a practical solution for a store. There were a few outliers like the water, that saw increases to their shelf space allocation of greater than 100%, but these products comprised less than three percent of the total number of SKU's. 66% of SKU's saw no change or a decrease in their allocations, while only 34% saw increases. Figure 21 shows the distribution of the ratios of new allocations to current allocations for all products.

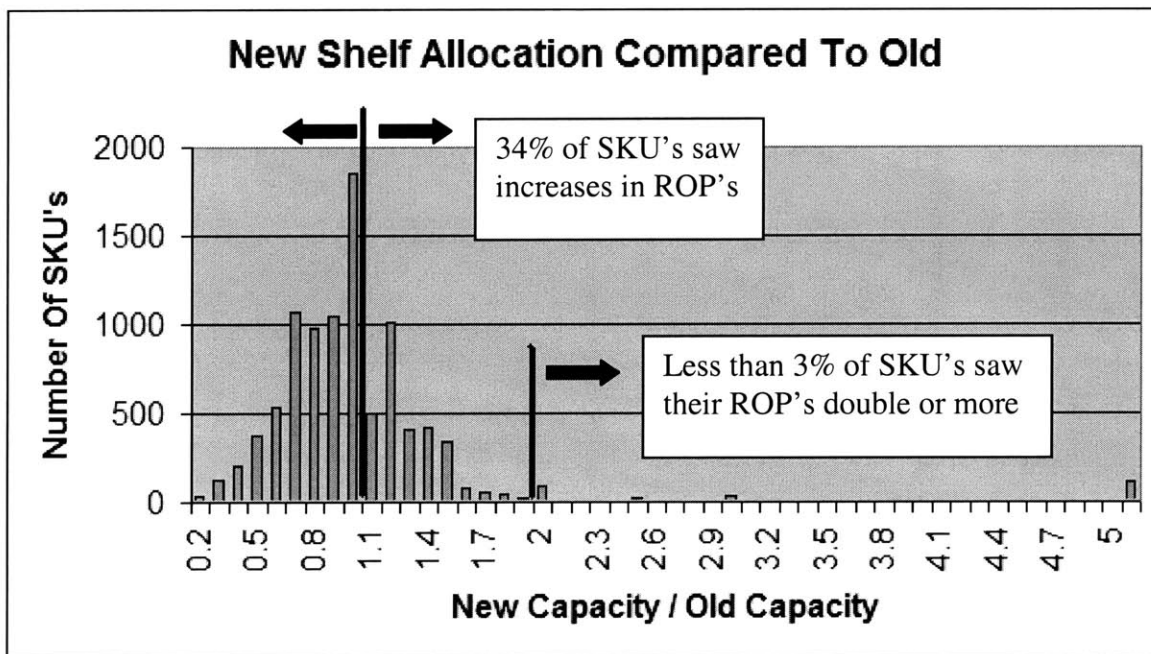


Figure 21: Distribution of How Shelf Space Allocation Changed

Using this model, the overall shelf-space requirement for The High-Volume Store decreased from 9200 ft² to 8700 ft², so the remaining 500 ft² could be re-allocated by the store on a discretionary basis.

This thesis does not advocate re-allocating shelf space based solely on the method we used, which does not take into account the effect this re-allocation would have on sales. It only shows that shelf space allocation can have a significant impact on store operations and, therefore, overall costs. Incorporating the findings of this thesis into existing methods for allocating shelf space remains a possibility for future research.

6 Conclusions

We were able to draw several conclusions from the simulation, some specific to Grocery Chain X's operations, some more widely applicable. We were able to show that the primary trade-off for determining the delivery schedule for a store in Grocery Chain X, is the cost of transportation versus the labor cost associated with storing product in the backroom. For Grocery Chain X, we found that the method they are currently using to determine the number of deliveries per week results in more deliveries than are necessary and that transportation savings (and overall cost savings) are available by reducing this number. Even The High-Volume Store, which has one of the highest ratios of sales volumes to available shelf space of any of the stores, can remove two deliveries per week. We also discovered that the day or days of the week that do not receive deliveries can have a significant impact on overall costs. Therefore, determining only the number of deliveries per week each store should receive is not enough, but the days of delivery must also be specified to realize the lowest overall cost. The day or days of the week that are the best choice for one store to skip, are not necessarily the best for another store, so the simulation must be run for each store individually.

Other than the delivery schedule, we found that the Re-Order Points and Shelf Space Allocations for each product can also have a significant impact on how much product needs to be stored in the backroom, and therefore, the costs as well. We showed that by using a very simple method to reset the Re-Order Points and Shelf Space Allocation, we could reduce the amount of product which needs to be stored in the backroom to nearly zero. The method we used to reset

these parameters is by no means the best way to do this, and better results are, no doubt, possible. Also, any savings resulting from adjusting the delivery schedule are independent of the Re-Order Point and Shelf Space Allocation for each product, and the lowest cost delivery schedule does not change when the Re-Order Points and/or Shelf Space Allocations are changed; costs for each scenario are just adjusted up and or down accordingly.

We also found that the way in which the deliveries are routed with the other deliveries in the system can greatly affect the transportation savings, compared to the savings predicted by the model when looking at a store individually. The savings predicted by the model will match the actual savings when the deliveries removed from the schedule were to be delivered directly from the DC to the store, with no other stops scheduled on the route. If this is not the case, and deliveries for multiple stores are combined, actual savings will vary depending on how the Transportation Management System combines and routes the deliveries.

7 Areas for Future Research

In analyzing the delivery frequencies for three retail grocery stores, this thesis has uncovered two primary areas for future research, both of which can have significant impacts on the operations and costs for a retail grocery chain independent of the delivery schedules of its stores. These two areas are: 1) determining how to set the re-order point for each product, and 2) determining how to allocate shelf space within the store.

We have shown that re-order points can have a sizable impact on how much product needs to be stored in the backroom. We have also shown that reducing the amount of product in the backroom lowers the costs associated with handling that product. This thesis does not, however, determine the optimal way to set these re-order points, and this remains an area for future research.

This thesis also explored the impact that shelf space allocation has on the amount of product stored in the backroom. However, it does not incorporate into the model the effect that shelf space allocation has on sales. Many studies have been done which show the impact that shelf space allocation has on sales, but they fail to include the impact on store operations, particularly the labor associated with double-handling the product that cannot fit on the shelves. Research that looks at both factors simultaneously could yield a better method for allocating shelf space in retail grocery stores.

Bibliography

- Azuma, N. and Fernie, J. (2001). Retail marketing strategies and logistical operations at a Japanese grocery supermarket chain - case study of Summit Inc. International Journal of Retail & Distribution Management, 19, 282-297.
- Baker, R. C. and Urban, T. L. (1998). A Deterministic Inventory System With An Inventory-Level-Dependent Demand Rate. Journal of the Operational Research Society, 39, 9, 823-31.
- Balintfy, J. (1964). On a basic class of multi-item inventory problems. Management Science, 10, 287-297.
- Bultez, A. and Naert, P. (1988). SHARP: Shelf Allocation For Retailers' Profit. Marketing Science, 7, 3, 211-231.
- Cachon, G. (2001). Managing a retailer's shelf space, inventory, and transportation. Manufacturing & Service Operations Management, 3, 211-229.
- Chaouch, B. (2001) Stock levels and delivery rates in vendor-managed inventory programs. Production and Operations Management, 10, 31-44.
- Corstjens, M. and Doyle, P. (1981). A Model For Optimizing Retail Space Allocations. Management Science, 27, 7, 822-33.
- Cox, K.K. (1970). The Effect Of Shelf Space Upon Sales Of Branded Products. Journal of Marketing Research, 7, 55-58.
- Curhan, R. (1972). The Relationship Between Shelf Space And Unit Sales. Journal of Marketing Research, 9, 406-412.
- Çetinkaya, S. and Lee, C. (2000). Stock replenishment and shipment scheduling for vendor-managed inventory systems. Management Science, 46, 217-232.
- Dreze, X. and Hoch, S. J. and Purk, M. E. (1994). Shelf Management And Space Elasticity. Graduate School of Business, University of Chicago.
- Dubelaar, C. and Chow, G. and Larson, P. D. (2001). Relationship Between Inventory, Sales And Service In A Retail Chain Store Operation. International Journal of Physical Distribution and Logistics Management, 31, 2, 96-108.
- Graves, S. (1996). A multiechelon inventory model with fixed replenishment intervals. Management Science, 42, 1-18.
- Hall, R. W. and Racer, M. (1995). Transportation With Common Carrier And Private Fleets: System Assignment And Shipment Frequency Optimization. IIE Transactions, 27, 217-225.

Gallego, G. and Ozer, O. (2003). Optimal Replenishment Policies For Multiechelon Inventory Problems Under Advanced Demand Information. Manufacturing & Service Operations Management, 5, 157-175.

Silver, E. A. and Pyke, D. F. & Peterson, R. (1998). Inventory management and production planning and scheduling. New York, NY: John Wiley & Sons, Inc.

Wang, Y. and Gerchak, Y. (2001). Supply Chain Coordination When Demand Is Shelf-Space Dependent. Manufacturing and Service Operations Management, 3, 1, 82-87.