Benefits of Postponement for Fashion Products with Forecast Updates

by

Huiling Gong

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Author ....
Engineering Systems Division
May 6, 2005

Certified by ....
Yossi Sheffi
Professor of Civil and Environmental Engineering
Professor of Engineering Systems
Thesis Supervisor

Accepted by ....
Yossi Sheffi
Professor of Civil and Environmental Engineering
Professor of Engineering Systems
Director, MIT Center for Transportation and Logistics

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Abstract

This thesis examines the benefit of postponement of fashion products by considering the overage cost of the intermediate product and the correlation between the demand for each end products produced from it. The benefit of postponement is measured by the percentage increase in maximum expected profit after demand is realized. The production process is modelled as a two stage newsvendor problem and the forecast update path follows an additive martingale. An optimal solution and a myopic solution are proposed to solve this problem. Numerical results indicate that the benefit of postponement decreases with the overage cost of the intermediate product and the correlation between demand for each end products. It becomes less sensitive to the overage cost of the intermediate product when end products are more negatively correlated. It is also less sensitive to the demand correlation between end products when the overage cost of the intermediate product is low. In addition, the benefit of postponement is sensitive to the additional unit costs introduced by postponement. A case study to NFL Jerseys purchase planning indicates that an increase of unit cost by 10% can reduce the benefit of postponement by over 50%.

Thesis Supervisor: Yossi Sheffi
Title: Professor of Civil and Environmental Engineering
Professor of Engineering Systems

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Chapter 1

Introduction

In today's competitive environment, customers expect broader product variety and novelty and easy availability of the product with the configuration they desire. This customer expectation has caused higher market dynamics and inventory risk. *Postponement* is a powerful strategy that can help manufactures to alleviate inventory risk without sacrificing customer expectations. Fashion products, that is, products with a short life cycle, generally have higher inventory risk than products with a long life cycle; therefore, postponement strategies tend to be more valuable. This thesis develops a quantitative model to analyze the benefit of postponement for fashion products.

1.1 Background

Higher customer expectations have led to a general trend of product proliferation, short product life cycle, and shorter required delivery time. "'Consumer electronics have become almost like produce,' says Michael E. Fawkes, senior vice-president of HP's Imaging Products Div. 'They always have to be fresh.'"(Engardio and Einhorn, 2005) Product proliferation increases demand uncertainty of each end product, resulting in higher forecast error for each end product. Shorter product life cycle also contributes to higher forecast error and increases inventory obsolescence risk. Lee and Billington (1994) notes that it is common for high technology products to have a forecast error of 400%. Shorter required delivery time excludes the option of make-to-order if the time to "make" exceeds the time a customer is willing to wait.

Postponement is a strategy that delays product differentiation decisions until a later point in the supply chain. By doing so, it allows the decision maker to gather more information about customer demand and therefore make better decisions. Postponement, a term first coined by Alderson (1950), is also called delayed differentiation, end of line configuration, and late point differentiation (Lee and Carter, 1993) in some literature. Postponement strategies can take different forms. Zinn and Bowersox (1988) describes four different ways of implementing postponement in a supply chain: labelling, packaging, assembly, and manufacturing postponement. Swaminathan and Lee (2003) describes three enablers of postponement in product and process design: component standardization, process standardization and process re-sequencing. The numerous successful implementations of postponement includes the oft-cited examples such as the localization postponement in HP's DeskJet printers (Lee and Billington, 1994), the vanilla box approach used in IBM's final assembly stage of RS6000 server machines (Swaminathan and Tayur, 1998), the front-end process standardization in the production of Xilinx's integrated circuits (Brown et al., 2000), the process standardization and vanilla box approach in Zara's product design (Harle et al., 2001), Beneton's resequencing of dying and knitting process (Dapiran, 1992), and Xilinx's adoption of field-programming integrated circuits to postpone the differentiation to the customer site (Brown et al., 2000). More examples can be found in Venkatesh and Swaminathan (2001).

Rockhold and Hall (1998) classify costs affected by postponement in the the following categories: asset-driven costs, logistics costs, material costs, and location-specific costs. Asset-driven costs include inventory costs, depreciation of equipments and other fixed assets. Postponement may lead to change in inventory holding locations or the form of the inventory. Semi-finished products are often in a more compact form, resulting in lower transportation costs. On the other hand, postponement can lead to loss of economies of scale due to decentralized production of the finished products. Postponement affects material cost due to a change in the location where the material is used, and the location affects material procurement. Location-specific costs include labor costs, duties and taxes, which can be influenced by tax-haven status, currency exchange rates, or local content rules.

In general, postponement strategies can help to reduce inventory cost, but often incur higher total costs in manufacturing and assembly. Additional costs incurred by postponement include loss of economies of scale, additional variable costs associated with standardizing a process(Lee and Tang, 1997) or re-sequencing a process(Dapiran, 1992), and initial investment and training costs.

From the certainty of information's point of view, postponement related costs can be deterministic and stochastic. The deterministic costs are those not directly affected by the uncertainty of demand, such as fixed asset costs, training costs, transportation costs, labor costs, duties and taxes, etc. The stochastic costs are inventory costs which is directly affected by the uncertainty of demand. Postponement reduced stochastic costs by using better information, therefore stochastic costs reflect the value of information. Since the evaluation of deterministic costs are straight-forward, the focus of this thesis is to develop a quantitative model to analyze the stochastic costs related to postponement.

1.2 Existing Models on the Benefit of Postponement

Among the existing general analytical models of postponement benefits, the following three models are most relevant to my work: the basic model in Lee (1996) which captures the risk pooling effect of postponement, the extended model in Lee and Whang (1998) which considers forecast improvement over time, and Aviv and Federgruen (2001)'s model which analyzes the learning effect on forecast improvement.

Lee (1996) models the production process with postponement into two-stages: a standard production process and a differentiated production process. This model assumes negligible inventory of intermediate product, periodic review policy, identical and independently distributed (iid) demands over time, and equal-fractal allocation rule, meaning all end products share a common safety stock factor. The mean and variance of steady state inventory level are derived as a function of service level, from which inventory holding costs and fill rates can be calculated. The major influence of Lee's model to this thesis is that the model captures the risk pooling effect, that is, the variance of aggregated demand is smaller than the sum of variance of the individual demands, but the model does not consider forecast improvement over time.

Lee and Whang (1998) extends the above model by adding demand correlation across different time periods. In particular, the future demand is modelled as a random walk from current demand. This model captures the value of forecast improvement over time.

Aviv and Federgruen (2001) studied the value of postponement when the demand distribution evolves in a Bayesian framework, that is, the parameters of the demand distributions are random variables whose distribution is characterized via prior distributions. The model considers the ordering, inventory holding, and backorder costs, and concludes that the learning effect from the Bayesian framework increases the benefits of postponement.

All of the above models are based on safety stock analysis and inventory levels are derived from a common safety factor, rather than to optimize the total profit. In addition, they do not consider the inventory obsolescence cost at the end of a product life cycle, therefore, are not suitable for products with short life cycles, particularly fashion goods, where inventory obsolescence cost is a major concern.

Parsons (2004) proposed a news-vendor model with risk pooling to plan inventory for NFL Replica Jerseys. The news-vendor model with risk pooling is demonstrated to offer higher expected profit than a traditional news-vendor model at comparable service level, but this model does not provide a general approach to quantify how forecast improves over time, and the second stage is make-to-order which is a special case of forecast improvement.

This thesis will develop a newsvendor model with forecast updates to analyze the benefit of postponement for fashion products.

Chapter 2

Basic Model

In this thesis, the benefit of postponement is analyzed by comparing the maximum profit of M end products under two different production settings: producing the end products with and without postponement. The model consists of three parts: demand realization, forecast evolution and production planning.

2.1 Demand Realization Model

The model assumes *terminal demands*, meaning all demands are realized instantly at the beginning of the selling season. This is equivalent to assuming no replenishment opportunities once the selling season starts. In reality, this assumption often holds when the selling season is short relative to the delivery and production lead time, or when replenishment fixed cost is too high.

Table 4.1 summarizes the notations used in this thesis.

Category	Notation	Description
Products	M	Number of end products
Lead Time	L_0	Lead time to produce the common intermediate product (stage one)
	L_1	Lead time to convert the intermediate product into end products (stage two)
	L	Lead time of total production, $L = L_0 + L_1$
Time	T	Point in time of demand realization, which is also the end of planning horizon
	t_0	Point in time of initial production of the intermediate product $t_1 = T - L_1 - L_0$
	t_1	Point in time of initial product differentiation, $t_1 = T - L_1$
Price	P_i	Selling price of end product i (Subscript i is omitted when P_i is the same across different end products.)
	C_i	Unit cost of converting the intermediate product to end product i (Subscript i is omitted when C_i is the same across different end products.)
	V_i	Salvage value of end product i (Subscript i is omitted when V_i is the same across different end products.)
	O_i	Overage cost of end product i, $O_i = C_0 + C_i - V_i$ (Subscript i is omitted when O_i is the same across different end products.)
	U_i	Underage cost of end product i, $U_i = P_i - C_i - C_0$ (Subscript i is omitted when U_i is the same across different end products.)
	O_0	Overage cost of the intermediate product, $O_0 = C_0 - V_0$
Inventory	$u_{t,i}$	Production quantity of product i starting at time t, $i \in \{0, 1, 2, ..., M\}$, where $i = 0$ represents the intermediate product
	$x_{t,i}$	Inventory on hand of product i at time t, $i \in \{0, 1, 2, ..., M\}$, where $i = 0$ represents the intermediate product
Forecast	$w_{t,i}$	Forecast at time t for product i's terminal demand
	ε_t	Multi-variant Random variable indicating the difference between forecast at time $t + 1$ and that of time t for the end products
	Σ_t	Covariance matrix of ε_t
	Φ	Cumulative Distribution Function (CDF) for the standard normal distribution with mean 0 and standard deviation 1
	Φ^{-1}	The inverse of Φ

Table 2.1: Notations

2.2 Forecast Evolution Model

The forecast evolution model follows the Martingale Model of Forecast Evolution (MMFE) developed independently by Graves and Qui (1986) and Heath and Jackson (1994). It is a generic probabilistic model, which can accommodate both judgemental forecasts and time series analysis. In particular, the forecast evolution is modelled as an additive model, that is,

$$w_{t+1} = w_t + \varepsilon_t \tag{2.1}$$

$$\text{where} \quad \varepsilon_t \sim N(0, \Sigma_t) \tag{2.2}$$

where w_t and w_{t+1} are demand forecasts of time t and $t+1$ respectively, and ε_t's are independent, identically distributed multivariate normal random vectors with mean 0 and a covariance matrix Σ_t. If the total number end products is M, then w_t, w_{t+1} and ε_t are vectors of dimension M. This model suggests that the next demand forecast *evolves* from the current forecast by some random amount. Note that the time intervals between adjacent forecast updates do not need to have equal length.

In reality, the covariance matrix Σ_t of random vector ε_t can be obtained by analyzing historical forecast evolution patterns or according to expert knowledge. We use two examples from apparel industry to illustrated how to derive the covariance matrix of the random vector.

2.2.1 Example 1: Derive Covariance Matrix from Historical Forecasts

In the apparel industry, manufacturers often have very little idea about the final demand when they start production, but as time progresses, the forecast becomes increasingly accurate due to information gathered from trade shows, street trends, or early customer orders. Assume that a manufacturer has the forecast data for similar products of the past ten years, as summarized in Table 2.2:

Year	Initial Forecast at time t_0	Updated Forecast at time t_1	Difference
1	(1000 2000)	(500 2300)	(-500 300)
2	(1500 1500)	(1400 1600)	(-100 100)
3	(800 2000)	(1500 1500)	(700 -500)
4	(1200 1000)	(1000 1400)	(-200 400)
5	(2000 2000)	(1600 2400)	(-400 400)
6	(1500 2500)	(1200 2800)	(-300 300)
7	(2000 1000)	(2000 1000)	(0 0)
8	(1500 1000)	(1100 1500)	(-400 500)
9	(2500 1000)	(2800 200)	(1000 -800)
10	(2000 2500)	(2200 1800)	(200 -700)

Table 2.2: Historical Forecast Data

The covariance between the change in forecast of the two end products is

$$cov(\varepsilon_1, \varepsilon_2) = \sum_{i=1}^{10} (\varepsilon_1(i) - \bar{\varepsilon}_1)(\varepsilon_2(i) - \bar{\varepsilon}_2)/10 = -198000 \tag{2.3}$$

The sample variances of the change in forecast is (248888 237777). Hence, the covariance matrix is

$$\Sigma = \begin{pmatrix} 248888 & -198000 \\ -198000 & 237777 \end{pmatrix} = \begin{pmatrix} 499 \\ 488 \end{pmatrix} (499\ 488) \begin{pmatrix} 1 & -0.8 \\ -0.8 & 1 \end{pmatrix}$$

2.2.2 Example 2: Derive Covariance Matrix according to Expert Knowledge

Sometimes historical forecast data are not available, due to a variety of reasons, such as introduction of new products, or simply bad record keeping. In this case, Delphi method can be used by asking industry experts to generate an initial forecast and how much the updated forecast would deviated from the initial forecast after each major events, such as trade shows or reception of early customer orders. For example, assume there are two end products. If an industry expert provides the following guess

1. Initial forecast is $(1000, 2000)$;

2. According to his/her experience, he/she is 95% confident that after the trade show 6 months later the forecast would be within range $(1000 \pm 500, 2000 \mp 500)$;

3. The total demand forecast is unlikely to change.

Then we can formulate this forecast update after the trade show as

$$
\begin{aligned}
w_{t_1} &= (1000, 2000) + \varepsilon \\
\text{where } \varepsilon &\sim N(0, \begin{pmatrix} 500 \\ 500 \end{pmatrix} (500\ 500) \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix})
\end{aligned}
$$

The model assumes discrete time, with a total planning horizon of T time periods. At the beginning of each time period, the forecast is revised with the latest information and decisions on the production quantity of each product is made based on the latest forecast. The objective is to maximize the expected profit at time T, which is the end of the planning horizon, or the start of the selling season.

2.3 Production Planning Model

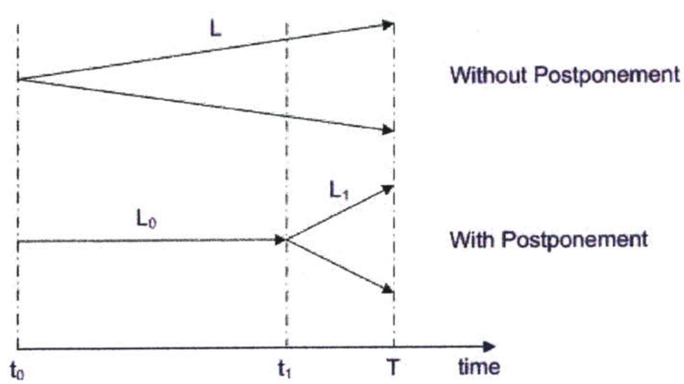


Figure 2-1: Overview of production process: postponement v.s. non-postponement

As illustrated in Fig 2-1, without postponement, the products are differentiated at the beginning of the entire production process. With postponement, the production process is separated into two stages: during the first stage, only those operations that are common to all the end products are carried out, and product differentiation decisions are made at the beginning of the second stage. It is assumed that the total production lead time with and without postponement is the same, the second stage production lead time (in the postponed production setting) for different end products are the same, and there is no capacity constraint for both the intermediate product and the end products with and without postponement. Let L be the total production lead time, L_0 and L_1 be the stage one and stage two lead time with postponement ($L = L_1 + L_2$). Let t_0 represent the point in time L periods before the demand realization time, and t_1 be the point in time L_1 periods before the demand realization time, then under the above assumptions, it is optimal to make production decisions at time t_0 without postponement and at time t_0 and t_1 when postponement is implemented.

Chapter 3

Problem Formulation and Analysis

Using the models described in chapter 2, we formulate the production planning problem with and without postponement into a single-stage and two-stage stochastic programming problem respectively. This chapter analyzes these problems and proposes optimal and suboptimal solutions to them.

3.1 Without Postponement

Without postponement, the production planning problem can be formulated as a simple single-stage stochastic programming problem. At time $t_0 = T - L$, the production quantity of each end product is decided.

$$\max_{u_{t_0}} J_{u_{t_0}}(u_{t_0}) \tag{3.1}$$

where

$$
\begin{aligned}
J_{u_{t_0},w_{t_0}}(u_{t_0}, w_{t_0}) &= \sum_{i=1}^{M} J(u_{t_0,i}) \\
&= \sum_{i=1}^{M} \mathrm{E}_{w_{T,i}}\left[-C_i u_{t_0,i} + P_i \min\{x_i, w_{T,i}\} + V_i \max\{x_i - w_{T,i}, 0\} | w_{t_0}\right] \\
&= \sum_{i=1}^{M} \left\{(P_i - C_i)u_{t_0,i} - (P_i - V_i)\int_{-\infty}^{u_{t_0,i}} (u_{t_0,i} - w_{T,i}) f_{w_{T,i}}(w_{T,i}) dw_{T,i}\right\} \\
&= \sum_{i=1}^{M} \left\{(U_i)u_{t_0,i} - (U_i + O_i)\int_{-\infty}^{u_{t_0,i}} (u_{t_0,i} - w_{T,i}) f_{w_{T,i}}(w_{T,i}) dw_{T,i}\right\} \\
w_T &\sim N(w_{t_0}, \sum_{k=1}^{L} \Sigma_{T-k})
\end{aligned}
$$

The optimal production quantity for each end product can be solved independently as a standard news-vendor problem.

$$
\begin{aligned}
u^*_{t_0,i} &= w_{t_0,i} + z^*_i \sigma_i \\
\text{where } z^*_i &= \Phi^{-1}\left(\frac{U_i}{U_i + O_i}\right) \\
\sigma_i &= \sqrt{\sum_{k=1}^{L} \Sigma_{T-k}(i,i)}
\end{aligned}
$$

where function Φ is the Cumulative Distribution Function (CDF) for the standard normal distribution with mean 0 and standard deviation 1, and Φ^{-1} is its inverse.

The corresponding maximum profit is

$$
J^* = J(u^*_{t_0}) = \sum_{i=1}^{M}(U_i)u^*_{t_0,i} - (U_i + O_i)\sigma_i(z^*_i\Phi(z^*_i) + \phi(z^*_i))
$$

3.2 With Postponement

With postponement, the production planning problem can be formulated as a two-stage stochastic programming problem. Depending on whether postponement introduces additional variable cost, the problem can be formulated differently. In this chapter, we analyze the basic case, that is, the total production cost is the same with and without postponement. In chapter 5, the more general case is formulated and the effect of additional variable cost introduced by postponement is discussed.

When postponement does not introduce additional costs, it is optimal to produce only the intermediate product at the first stage. Let $t_0 = T - L_1 - L_0$ and $t_1 = T - L_1$. At time t_0, the production quantity of the common intermediate product $u_{t_0,0}$ is decided. At time t_1, the production quantity of each end product $u_{t_1,i}$ is decided given the production decision at time t_0, where $i = 1, 2, \ldots, M$. The total stochastic programming problem is formulated as

$$\max_{u_{t_0,0}} \mathrm{E}_{w_{t_1}} \left[\max_{u_{t_1,i}} J_{t_1}(u_{t_0,0}, u_{t_1,i}, w_{t_1}) \right]$$

such that

$$x_{t_1,0} - \sum_{i=1}^{M} u_{t_1,i} \geq 0 \tag{3.2}$$

where

$$\begin{aligned} x_{t_1,0} &= u_{t_0,0} && (3.3) \\ x_{T,0} &= x_{t_1,0} - \sum_{i=1}^{M} u_{t_1,i} && (3.4) \\ w_T &\sim N(w_{t_1}, \sum_{k=1}^{L_1} \Sigma_{T-k}) && (3.5) \\ w_{t_1} &\sim N(w_{t_0}, \sum_{k=1}^{L_0} \Sigma_{t_1-k}) && (3.6) \end{aligned}$$

$$
\begin{aligned}
J(u_{t_0,0}, u_{t_1,i}, w_{t_1}) &= \mathrm{E}_{w_T}\left[\sum_{i=1}^{M}(-C_i u_{t_1,i} + P_i \min\{u_{t_1,i}, w_{T,i}\} + V_i \max\{u_{t_1,i} - w_{T,i}, 0\})\,|w_{t_1}\right] \\
&\quad -C_0 u_{t_0,0} + V_0 x_{T,0} \\
&= \sum_{i=1}^{M}\left\{\begin{array}{l} (P_i - C_i)u_{t_1,i} \\ -(P_i - V_i)\int_{-\infty}^{u_{t_1,i}}(u_{t_1,i} - w_{T,i})f_{w_{T,i}}(w_{T,i})dw_{T,i} \end{array}\right\} \\
&\quad -C_0 u_{t_0,0} + V_0(u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i}) \\
&= \sum_{i=1}^{M}\left\{\begin{array}{l} (P_i - C_i - V_0)u_{t_1,i} \\ -(P_i - V_i)\int_{-\infty}^{u_{t_1,i}}(u_{t_1,i} - w_T)f_{w_T}(w_T)dw_T \end{array}\right\} \\
&\quad -(C_0 - V_0)u_{t_0,0} \\
&= \sum_{i=1}^{M}\left\{\begin{array}{l} (U_i + O_0)u_{t_1,i} \\ -(U_i + O_i)\int_{-\infty}^{u_{t_1,i}}(u_{t_1,i} - w_T)f_{w_T}(w_T)dw_T \end{array}\right\} \\
&\quad -(O_0)u_{t_0,0}
\end{aligned} \tag{3.7}
$$

3.2.1 Optimal Solutions by Considering Forecast Updates at Stage One (Optimal Approach)

Before we derive the optimal solution, we first point out a simple fact that the overage cost of the intermediate product is always less than or equal to that of any end products.

Proposition 1

$$O_0 \leq \min\{O_i\} \tag{3.8}$$

The reason is that we always have the option to salvage the extra intermediate products by converting them to the end product with the minimum overage cost, in which case, the overage cost of the intermediate product is equal to the overage cost of that end product. When the costs of the end products are the same, we have $O_0 \leq O$.

To solve this two-stage stochastic programming problem, we start with stage two and solve backwards.

Decision at Stage Two

At stage two (time t_1), the problem is a multi-variable constrained optimization problem described as follows:

$$\max_{u_{t_1,i}} J_{t_1}(u_{t_0,0}, u_{t_1,i}, w_{t_1}) \tag{3.9}$$

such that

$$u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i} \geq 0 \tag{3.10}$$

where $J_{t_1}(u_{t_0,0}, u_{t_1,i}, w_{t_1})$ is defined in equation (3.7).

We first propose the solution to the degenerate case, that is, when perfect demand information is available at stage two, or

$$w_T = w_{t_1},$$

which corresponds to the "make-to-order" case. And then we derive the solution to a general case, that is, when the demand information is uncertain at stage two.

To facilitate discussion, we assume the overage and underage costs across different products are the same, which is referred to as *"balanced cost"* for the rest of the thesis. In reality, this assumption often holds when the end products belong to the same product family.

When Perfect Demand Information Is Available at Stage Two

Having perfect demand information when the decision at stage two has to be made corresponds to the so called 'make-to-order' or 'customize-to-order', which has broad applications.

In this case, the optimal solution of stage two is to meet as much demand as possible, and salvage extra intermediate products. The maximum expected profit for this degenerate case is stated in the following proposition.

Proposition 2 *If perfect demand information is available at the second stage, then the* *maximum expected profit is*

$$J^*_{t_1}(u_{t_0,0}, w_{t_1}) = \begin{cases} Uu_{t_0,0} - (U+O_0)(u_{t_0,0} - \sum_{i=1}^{M} w_{t_1,i}), & \text{if } u_{t_0,0} > \sum_{i=1}^{M} w_{t_1,i}; \\ Uu_{t_0,0}, & \text{otherwise.} \end{cases} \quad (3.11)$$

The proof is straightforward, therefore is omitted.

When Demand Information Is Uncertain at Stage Two

When demand information is uncertain at stage two, the problem can be solved using lagrange multiplier method as described below.

$$L(u_{t_0,0}, u_{t_1,i}, w_T, \lambda) \quad (3.12)$$

$$= \sum_{i=1}^{M} \left\{ \begin{array}{l} (U_i + O_0)u_{t_1,i} \\ -(U_i + O_i) \int_{-\infty}^{u_{t_1,i}} (u_{t_1,i} - w_T) f_{w_T}(w_T) dw_T \end{array} \right\}$$

$$-(O_0)u_{t_0,0} \quad (3.13)$$

$$-\lambda(u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i})$$

$$\frac{\partial L}{\partial u_{t_1,i}} = -(U_i + O_i)F_{w_{i,T}}(u_{t_1,i}) + U_i + O_0 - \lambda = 0 \quad (3.14)$$

$$\lambda \geq 0 \quad (3.15)$$

$$u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i} \geq 0 \quad (3.16)$$

$$\lambda(u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i}) = 0 \quad (3.17)$$

Solving equation (3.14), we get

$$u^*_{t_1,i} = F^{-1}_{w_{i,T}}\left(\frac{U_i + O_0 - \lambda}{U_i + O_i}\right) = w_{t_1,i} + z^*_i \Phi^{-1}\left(\frac{U_i + O_0 - \lambda}{U_i + O_i}\right) \quad (3.18)$$

In general, the optimal policy $u^*_{t_1,i}$ can only be solved numerically by searching for λ such that conditions (3.16) and (3.17) are satisfied. However, as demonstrated below, closed formula for $u^*_{t_1,i}$ can be derived when the end products have "*balanced costs*". Let $O = O_i$ and $U = U_i$ for any i. Then equation 3.18 becomes

$$u^*_{t_1,i} = F^{-1}_{w_{T,i}}\left(\frac{U+O_0-\lambda}{U+O}\right) = w_{t_1,i} + z^*_i\Phi^{-1}\left(\frac{U+O_0-\lambda}{U+O}\right) \tag{3.19}$$

The following proposition describes the optimal solution at stage two when demand information is uncertain.

Proposition 3 *When the demand information at stage two is imperfect, the optimal solution for the constraint optimization problem (3.9) is*

$$u^*_{t_1,i}(u_{t_0,0}, w_{t_1}) = w_{t_1,i} + \sigma_i z^*(u_{t_0,0}, w_{t_1}) \text{ where} \tag{3.20}$$

$$\sigma_i = \sqrt{\sum_{k=1}^{L_1} \Sigma_{T-k}(i,i)} \text{ and} \tag{3.21}$$

$$z^*(u_{t_0,0}, w_{t_1}) = \begin{cases} \Phi^{-1}\left(\frac{U+O_0}{U+O}\right), & \text{when } O_0 < O \text{ and} \\ & u_{t_0,0} \geq \sum_{i=1}^M w_{t_1,i} + \Phi^{-1}\left(\frac{U+O_0}{U+O}\right)\sum_{i=1}^M \sigma_i; \\ \frac{1}{\sum_{j=1}^M \sigma_j}(u_{t_0,0} - \sum_{j=1}^M w_{t_1,j}), & \text{otherwise.} \end{cases} \tag{3.22}$$

And the maximum profit is

$$\begin{aligned} J^*_{t_1}(u_{t_0,0}, w_{t_1}) = {} & (U+O_0)\sum_{i=1}^M w_{t_1,i} + [(U+O_0)z^* - (U+O)(z^*\Phi(z^*) + \phi(z^*))]\sum_{i=1}^M \sigma_i \\ & -(O_0)u_{t_0,0} \end{aligned} \tag{3.23}$$

Proof For a constrained optimization problem with inequality constraints, the constraints can be active or inactive with respect to the optimal solution.

When $\lambda = 0$

In this case, constraint (3.10) is inactive, which corresponds to the case when some of the intermediate product is not converted to any end product. This happens when $u_{t_0,0}$ is high and $\frac{U+O_0}{U+O} < 1$ $(O_0 < O)$, that is, the inventory on hand of of intermediate product is high (because too many intermediate products were produced in stage one) and the overage cost of the intermediate product is smaller than those of the end products, which happens when the intermediate can be reused for other products or in the next product life-cycle. Then the optimal policy is

$$u^*_{t_1,i} = F^{-1}_{w_{T,i}}\left(\frac{U+O_0}{U+O}\right) \ \forall i \in \{1, 2, ..., M\}$$

and $u^*_{t_1,i}$ satisfies the following inequality.

$$u_{t_0,0} \geq \sum_{i=1}^{M} u^*_{t_1,i}$$

Let

$$z_i = \frac{u_{t_1,i} - w_{t_1,i}}{\sigma_i},$$

Since $w_T \sim N(w_{t_1}, \sum_{k=1}^{L_1} \Sigma_{T-k})$, we have $z \sim N(0,\ 1)$. Hence, the optimal solution is

$$\begin{aligned} z^*(u_{t_0,0}, w_{t_1}) &= \Phi^{-1}\left(\frac{U+O_0}{U+O}\right) \\ u^*_{t_1,i}(u_{t_0,0}, w_{t_1}) &= w_{t_1,i} + \sigma_i z^*(u_{t_0,0}, w_{t_1}) \\ \text{when } O_0 < O \text{ and } u_{t_0,0} &\geq \sum_{i=1}^{M} w_{t_1,i} + \Phi^{-1}\left(\frac{U+O_0}{U+O}\right)\sum_{i=1}^{M}\sigma_i \end{aligned}$$

When $\lambda > 0$

In this case, constraint (3.10) is active, that is

$$u_{t_0,0} - \sum_{i=1}^{M} u_{t_1,i} = 0. \tag{3.24}$$

This corresponds to the case when all the intermediate products are converted to end products. This happens when $u_{t_0,0}$ is low, or $\frac{U+O_0}{U+O} = 1$ $(O_0 = O)$. Low $u_{t_0,0}$ means that the inventory on hand of the intermediate product is low because too little intermediate product was produced in stage one. $O_0 = O$ means that the salvage value of the intermediate product is the same as those of the end products. For example, this can happen when the intermediate product is unstable or cannot be reused in the next product life cycle. In this case, we have

$$z_i^* = \Phi^{-1}(\frac{U + O_0 - \lambda}{U + O}) \;\forall\, i$$

Notice that z^* is independent of i. Let $z^* = z_i^*$. We have,

$$u_{t_1,i}^* = w_{t_1,i} + \sigma_i z^*$$

Substituting $u_{t_1,i}$ in 3.24 by 3.21, we get

$$\begin{aligned} \sum_{i=1}^{M} (w_{t_1,i} + \sigma_i z^*) &= u_{t_0,0} \\ \Rightarrow \quad z^*(u_{t_0,0}, w_{t_1}) &= \frac{u_{t_0,0} - \sum_{j=1}^{M} w_{t_1,i}}{\sum_{j=1}^{M} \sigma_j} \end{aligned}$$

From z^*, we get,

$$\begin{aligned} \lambda^* &= U + O_0 - (U + O)\Phi(z^*) \\ u_{t_1,i}^*(u_{t_0,0}, w_{t_1}) &= w_{t_1,i} + \sigma_i z^* \\ &= w_{t_1,i} + \sigma_i \frac{u_{t_0,0} - \sum_{j=1}^{M} w_{t_1,j}}{\sum_{j=1}^{M} \sigma_i} \end{aligned}$$

And the maximum profit is

$$
\begin{aligned}
J_{t_1}^*(u_{t_0,0}, w_{t_1}) &= \sum_{i=1}^{M} \left[(U+O_0)u_{t_1,i}^* - (U+O)(z^*\Phi(z^*)+\phi(z^*))\sigma_i\right] - (O_0)u_{t_0,0} \\
&= (U+O_0)\sum_{i=1}^{M} u_{t_1,i}^* - (U+O)(z^*\Phi(z^*)+\phi(z^*))\sum_{i=1}^{M}\sigma_i - (O_0)u_{t_0,0} \\
&= (U+O_0)\sum_{i=1}^{M}(w_{t_1,i}+\sigma_i z^*) - (U+O)(z^*\Phi(z^*)+\phi(z^*))\sum_{i=1}^{M}\sigma_i - (O_0)u_{t_0,0} \\
&= (U+O_0)\sum_{i=1}^{M} w_{t_1,i} + \left[(U+O_0)z^* - (U+O)(z^*\Phi(z^*)+\phi(z^*))\right]\sum_{i=1}^{M}\sigma_i - (O_0)u_{t_0,0}
\end{aligned}
$$

Decision at Stage One

With the formula for $J_{t_1}^*(u_{t_0,0}, w_{t_1})$ from proposition 2 and 3, the optimal solution of stage one (time t_0) can be derived. The decision problem of stage one is a single variable optimization problem stated as follows:

$$\max_{u_{t_0,0}} J_{t_0}(u_{t_0,0})$$

where

$$
\begin{aligned}
J_{t_0}(u_{t_0,0}) &= \mathrm{E}_{w_{t_1}}[J_{t_1}^*(u_{t_0,0}, w_{t_1})|w_{t_0}] \\
&= \int_{-\infty}^{\infty} J_{t_1}^*(u_{t_0,0}, w_{t_1}) f_{w_{t_1}|w_{t_0}}(w_{t_1}) dw_{t_1}
\end{aligned}
$$

Again the solution to the stage one problem depends on whether or not perfect demand information is available at stage two.

Proposition 4 *$J_{t_0}(u_{t_0,0}, w_{t_0})$ is concave with respect to $u_{t_0,0}$.*

Proof

1. When Perfect Demand Information Is Available at Stage Two

Let

$$y = \sum_{i=1}^{M} w_{t_1,i} \tag{3.25}$$

Since w_{t_1} is a multivariate normal, and y is a summation of random variables with a joint-normal distribution, therefore, y must be normal. From the joint distribution of $w_{t_1,i}$, we can derive the distribution of y as follows

$$\begin{aligned} y &\sim N(\mu, \sigma^2), \\ \text{where } \mu &= \sum_{i=1}^{M} w_{t_0,i}, \\ \sigma &= \sqrt{\mathbf{1}' \sum_{k=1}^{L_0} \Sigma_{t_1-k} \mathbf{1}}. \end{aligned}$$

We can rewrite the profit function $J_{t_0}(u_{t_0,0})$ as follows.

$$\begin{aligned} J_{t_0}(u_{t_0,0}) &= \int_{-\infty}^{\infty} J^*_{t_1}(u_{t_0,0}, w_{t_1}) f_{w_{t_1}|w_{t_0}}(w_{t_1}) dw_{t_1} \\ &= U u_{t_0,0} - (U + O_0) \int_{-\infty}^{u_{t_0,0}} (u_{t_0,0} - y) f_y(y) dy \\ &= \begin{cases} U u_{t_0,0} - (U + O_0) \max\{u_{t_0,0} - \mu, 0\}, & \text{if } \sigma = 0; \\ U u_{t_0,0} - (U + O_0)\sigma(z\Phi(z) + \phi(z)), & \text{otherwise.} \end{cases} \quad (3.26) \\ \text{where } z &= \frac{u_{t_0,0} - \mu}{\sigma}. \quad (3.27) \end{aligned}$$

(a) If $\sigma > 0$,

$$\begin{aligned} \frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}) &= U - (U + O_0)\sigma(z\phi(z)/\sigma + \Phi(z)/\sigma + \phi'(z)/\sigma) \\ &= U - (U + O_0)\Phi(z) \quad (3.28) \\ \frac{\partial J^2_{t_0}}{\partial^2 u_{t_0,0}}(u_{t_0,0}) &= -(U + O_0)\phi(z)/\sigma \quad (3.29) \end{aligned}$$

Since $\frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) \leq 0 \; \forall \; u_{t_0,0}$, $J_{t_0}(u_{t_0,0})$ is concave with respect to $u_{t_0,0}$.

(b) If $\sigma = 0$,

$$\frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}) = \begin{cases} U, & \text{if } u_{t_0,0} < \sum_{i=1}^{M} w_{t_0,i}; \\ -O_0, & u_{t_0,0} > \sum_{i=1}^{M} w_{t_0,i}. \end{cases} \tag{3.30}$$

Thus $J_{t_0}(u_{t_0,0})$ is monotonically increasing when $u_{t_0,0} < \mu$, and monotonically decreasing when $u_{t_0,0} > \mu$. And it is straightforward to prove that $J_{t_0}(u_{t_0,0})$ is continuous at $u_{t_0,0} = \mu$. Hence, function $J_{t_0}(u_{t_0,0})$ is concave with respect to $u_{t_0,0}$ when $\sigma = 0$.

2. When Demand Information Is Uncertain at Stage Two

Notice that z^* in equation 3.22 only depends on $\sum_{i=1}^{M} w_{t_1,i}$ instead of each $w_{t_1,i}$, that is, z^* is a function of y defined in 3.25. To simply notation, let

$$\begin{aligned} \hat{\sigma} &= \sum_{i=1}^{M} \sigma_i \\ \hat{z} &= \Phi^{-1}\left(\frac{U + O_0}{U + O}\right) \\ G(u_{t_0,0}) &= u_{t_0,0} - \hat{z}\hat{\sigma} \\ H(u_{t_0,0}, y) &= \frac{1}{\hat{\sigma}}(u_{t_0,0} - y) \end{aligned}$$

Notice that $\hat{z}$ is a constant. Now we can rewrite equation 3.22 as

$$z^*(u_{t_0,0}, y) = \begin{cases} \hat{z}, & \text{when } O_0 < O \text{ and } y \leq G(u_{t_0,0}); \\ H(u_{t_0,0}, y), & \text{otherwise.} \end{cases}$$

Next, we analyze the profit function in two cases: $O_0 < O$ and $O_0 = O$.

(a) When $O_0 < O$

In this case, we have

$$
\begin{aligned}
J_{t_0}(u_{t_0,0}) &= \int_{-\infty}^{\infty} J_{t_1}^*(u_{t_0,0}, w_{t_1}) f_{w_{t_1}}(w_{t_1}) dw_{t_1} \\
&= \int_{-\infty}^{\infty} \left\{ \begin{array}{c} [(U+O_0)z^* - (U+O)(z^*\Phi(z^*) + \phi(z^*))] \sum_{i=1}^{M} \sigma_i \\ +(U+O_0)y - (O_0)u_{t_0,0} \end{array} \right\} f_y(y)dy \\
&= \int_{-\infty}^{G(u_{t_0,0})} [(U+O_0)\hat{z} - (U+O)(\hat{z}\Phi(\hat{z}) + \phi(\hat{z}))] \hat{\sigma} f_y(y)dy \\
&\quad + \int_{G(u_{t_0,0})}^{\infty} [(U+O_0)H - (U+O)(H\Phi(H) + \phi(H))] \hat{\sigma} f_y(y)dy \\
&\quad +(U+O_0)\mu - O_0 u_{t_0,0} \\
&= [-(U+O)\phi(\hat{z})\hat{\sigma}] F_y(G(u_{t_0,0})) \\
&\quad + \int_{G(u_{t_0,0})}^{\infty} [(U+O_0)H - (U+O)(H\Phi(H) + \phi(H))] \hat{\sigma} f_y(y)dy \\
&\quad +(U+O_0)\mu - (O_0)u_{t_0,0}
\end{aligned} \tag{3.31}
$$

When $\sigma = 0$,

$$
J_{t_0}(u_{t_0,0}) = \begin{cases} U u_{t_0,0} - (U+O)(u_{t_0,0} - \mu)\Phi((u_{t_0,0} - \mu)/\hat{\sigma}) & \\ \quad -(U+O)\phi((u_{t_0,0} - \mu)/\hat{\sigma})\hat{\sigma}, & \text{when } u_{t_0,0} - \hat{z}\hat{\sigma} \leq \mu; \\ -(U+O)\phi(\hat{z})\hat{\sigma} + (U+O_0)\mu - O_0 u_{t_0,0}, & \text{otherwise.} \end{cases}
$$

H is a function of $u_{t_0,0}$ and y in the above equations. So the first derivative of J_{t_0} over $u_{t_0,0}$ is

$$
\begin{aligned}
& \frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}) \\
= & -(U+O)\phi(\hat{z})\hat{\sigma} f_y(G(u_{t_0,0}) \\
& \int_{G(u_{t_0,0})}^{\infty} [(U+O_0) - (U+O)\Phi(H)] f_y(y)dy \\
& +(U+O)\phi(\hat{z})\hat{\sigma} f_y(G(u_{t_0,0}) - O_0 \\
= & (U+O_0)(1 - F_y(G(u_{t_0,0}))) - (U+O) \int_{G(u_{t_0,0})}^{\infty} \Phi(H) f_y(y)dy - O_0
\end{aligned} \tag{3.32}
$$

When $\sigma = 0$,

$$\frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}) = \begin{cases} U - (U+O)\Phi((u_{t_0,0}-\mu)/\sigma), & \text{when } u_{t_0,0} - \hat{z}\hat{\sigma} \leq \mu; \\ -O_0, & \text{otherwise.} \end{cases}$$

The second derivative is

$$\begin{aligned} & \frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) \\ = \; & -(U+O_0)f_y(G(u_{t_0,0})) - (U+O)\int_{G(u_{t_0,0})}^{\infty} \phi(H)\frac{1}{\hat{\sigma}}f_y(y)dy \\ & +(U+O)\Phi(\hat{z})f_y(G(u_{t_0,0})) \\ = \; & -(U+O)\left(\int_{G(u_{t_0,0})}^{\infty} \phi\left(\frac{u_{t_0,0}-y}{\hat{\sigma}}\right)\phi\left(\frac{y-\mu}{\sigma}\right)\frac{1}{\hat{\sigma}}dy\right) \leq 0 \end{aligned} \tag{3.33}$$

When $\sigma = 0$,

$$\frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) = \begin{cases} -(U+O)\phi((u_{t_0,0}-\mu)/\hat{\sigma})/\hat{\sigma}, & \text{when } u_{t_0,0} - \hat{z}\hat{\sigma} \leq \mu; \\ 0, & \text{otherwise.} \end{cases}$$

Since $\frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) \leq 0 \; \forall \; u_{t_0,0}$, function $J_{t_0}(u_{t_0,0})$ must be concave with respect to $u_{t_0,0}$.

(b) When $O_0 = O$

In this case, we have

$$\begin{aligned} J_{t_0}(u_{t_0,0}) \; = \; & \int_{-\infty}^{\infty} \left[(U+O_0)H - (U+O)(H\Phi(H) + \phi(H))\right]\hat{\sigma} f_y(y)dy \\ & +(U+O_0)\mu - (O_0)u_{t_0,0} \end{aligned} \tag{3.34}$$

So the first derivative of J_{t_0} over $u_{t_0,0}$ is

$$\frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}) = U - (U+O)\int_{-\infty}^{\infty} \Phi(H) f_y(y) dy \tag{3.35}$$

The second derivative is

$$\begin{aligned} & \frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) \\ = & -(U+O)\left(\int_{-\infty}^{\infty} \phi\left(\frac{u_{t_0,0}-y}{\hat{\sigma}}\right)\phi\left(\frac{y-\mu}{\sigma}\right)\frac{1}{\hat{\sigma}}dy\right) \leq 0 \end{aligned} \tag{3.36}$$

Since $\frac{\partial J_{t_0}^2}{\partial^2 u_{t_0,0}}(u_{t_0,0}) \leq 0 \ \forall \ u_{t_0,0}$, function $J_{t_0}(u_{t_0,0})$ must be concave with respect to $u_{t_0,0}$.

Q.E.D.

When perfect demand information is available at stage two, the optimal solution can be computed analytically, as shown in the following proposition.

Proposition 5 *If perfect demand information is available at stage two, the optimal initial production quantity of the intermediate product is*

$$\begin{aligned} u_{t_0,0}^* &= \mu + z^*\sigma \qquad (3.37) \\ \text{where } \mu &= \sum_{i=1}^{M} w_{t_0,i} \\ \sigma &= \sqrt{\mathbf{1}'\sum_{k=1}^{L_0} \Sigma_{t_1-k}\mathbf{1}} \\ z^* &= \Phi^{-1}\left(\frac{U}{U+O_0}\right) \end{aligned}$$

And the maximum expected profit is

$$J_{t_0}^* = U\mu - (U + O_0)\sigma\phi(z^*) \tag{3.38}$$

$$\tag{3.39}$$

Proof

1. when $\sigma > 0$

   Since, $J_{t_0}(u_{t_0,0})$ is concave with respect to $u_{t_0,0}$ (Proposition 4), the optimal production quantity of the intermediate product $u_{t_0,0}^*$ can be derived by setting the first derivative to 0. That is,

$$\frac{\partial J_{t_0}}{\partial u_{t_0,0}}(u_{t_0,0}^*) = U - (U + O_0)\Phi(z^*) = 0. \tag{3.40}$$

   Hence,

$$\begin{aligned} z^* &= \Phi^{-1}(\frac{U}{U + O_0}) \\ u_{t_0,0}^* &= \mu + \sigma z^* \qquad (3.41) \\ J_{t_0}^* &= U u_{t_0,0}^* - (U + O_0)\sigma(z^*\Phi(z^*) + \phi(z^*)) \\ &= U(\mu + \sigma z^*) - (U + O_0)\sigma(z^*\frac{U}{U + O_0} + \phi(z^*)) \\ &= U\mu - (U + O_0)\sigma\phi(z^*) \qquad (3.42) \end{aligned}$$

2. when $\sigma = 0$

   $J_{t_0}(u_{t_0,0})$ is not differentiable at $u_{t_0,0} = \mu$, but $J_{t_0}(u_{t_0,0})$ is monotonically increasing when $u_{t_0,0} < \mu$, and monotonically decreasing when $u_{t_0,0} > \mu$. And $J_{t_0}(u_{t_0,0})$ is continuous at $u_{t_0,0} = \mu$. Therefore, the optimal solution is

$$u_{t_0,0}^* = \mu, \tag{3.43}$$

$$J_{t_0}^* = U u_{t_0,0}^* = U\mu \tag{3.44}$$

Notice that the above case (when $\sigma = 0$) is a limiting case for when aggregated demand information is uncertain at stage one. In other words, as $\sigma \to 0$ in stage one, we get this case.

Q.E.D.

When perfect demand information is available at stage two, the optimal solution can be computed analytically, as shown in the following proposition.

Proposition 6 *If the demands for end products are perfectly negatively correlated, the optimal initial production quantity of the intermediate product at stage one is*

$$\begin{aligned} u^*_{t_0,0} &= \mu + z^*\hat{\sigma} &(3.45)\\ \text{where } \mu &= \sum_{i=1}^{M} w_{t_0,i} \\ \hat{\sigma} &= \sum_{i=1}^{M}\sigma_i = \sum_{i=1}^{M}\sqrt{\sum_{k=1}^{L_1}\Sigma_{T-k}(i,i)} \\ z^* &= \Phi^{-1}(\frac{U}{U+O}) &(3.46)\end{aligned}$$

And the maximum expected profit is

$$\begin{aligned} J^*_{t_0} &= U\mu - (U+O_0)\sigma\phi(z^*) &(3.47)\\ & &(3.48)\end{aligned}$$

When demand information is uncertain at stage two and the demands are not perfectly negatively correlated, the closed-form formula for the optimal solution at stage one ($u^*_{t_0,0}$) does not seem to exist. The optimal solution can be computed numerically using standard nonlinear optimization methods(Bertsekas, 1999). In the numerical examples in the next chapter, we used the 'fminbnd' function in matlab to find the optimal solution.

3.2.2 Suboptimal Solution by Ignoring Forecast Updates at Stage One (Myopic Approach)

In this section, we propose a heuristic approach, which is referred to as the Myopic Approach in the rest of the thesis. The myopic policy chooses the optimal production quantity of the intermediate product at time t_0 by ignoring future forecast updates. In this case, the stage two decision is still a constrained optimization problem with the same formulation as before, but the decision at stage one becomes a standard single variable news-vendor problem, regardless of the decision to be made at stage two. Hence, at stage one, the optimal production quantity of the intermediate product is

$$\begin{aligned} u^*_{t_0,0,w_{t_0}} &= \mu + \sigma\Phi^{-1}\left(\frac{U}{U+O}\right), \\ \text{where } \mu &= \sum_{i=1}^{M} w_{t_0,i}, \\ \sigma &= \sqrt{\mathbf{1}'\sum_{k=1}^{L}\Sigma_{T-k}\mathbf{1}}. \end{aligned}$$

The expected profit can be calculated numerically using equation 3.31.

Chapter 4

Numerical Results and Discussion

This chapter demonstrates applications of the model to a series hypothetical settings. The optimal solutions are computed and the benefit of postponement is discussed and compared.

This thesis restricts to the cases when the end products have *balanced costs* (see Chapter 3), which is a reasonable assumption for products within the same product family. In reality, products within the same product family often have similar costs and price. For example, different colors of iPod are priced the same in almost all stores. Different sizes and colors of the blouses of the same style usually have similar costs and the same initial price. Here we consider only the regular price. Promotion prices are out of the scope of this thesis.

4.1 Description

In the numerical experiments carried out in this chapter, it is assumed that the total cost for producing an end product, including the cost of producing the intermediate product and the cost of converting it to an end product, be unit 1, regardless of the postponement point. The following table summarized the parameters used in the numerical experiment.

In this example, the number of end products is 2. The total production lead time is 10 periods. The cost for producing the intermediate product is 50% of the total cost. The price

Category	Parameter	Value
Products	M	2
Costs	C_0	0.5
	C	0.5
	$C_{total} = C_0 + C$	1
	V_0	0.30, 0.40, 0.45, 0.49
	V	0.8
	P	1.3
	$O_0 = C_0 - V_0$	0.20, 0.10, 0.05, 0.01
	$O = C_0 + C - V$	0.2
	$U = P - C_0 - C$	0.3
Forecasts	μ	[10 10]
	σ	[1 1]
	ρ	-1, -0.5, 0, 0.5, 1 (correlation between demands for end products)
Lead Times	L	10
	L_1	0, 2, 4, 6, 8, 10
	$\frac{L_1}{L}$	0, 0.2, 0.4, 0.6, 0.8, 1 (percentage of the first stage lead time)

Table 4.1: Input Parameters (See table 4.1 in chapter 2 for parameter descriptions.)

of each end product is 1.3 times the total cost. The salvage value of each end product is 80% of the total cost. From the price and cost information, we can derive the underage cost U and overage cost O of the end products.

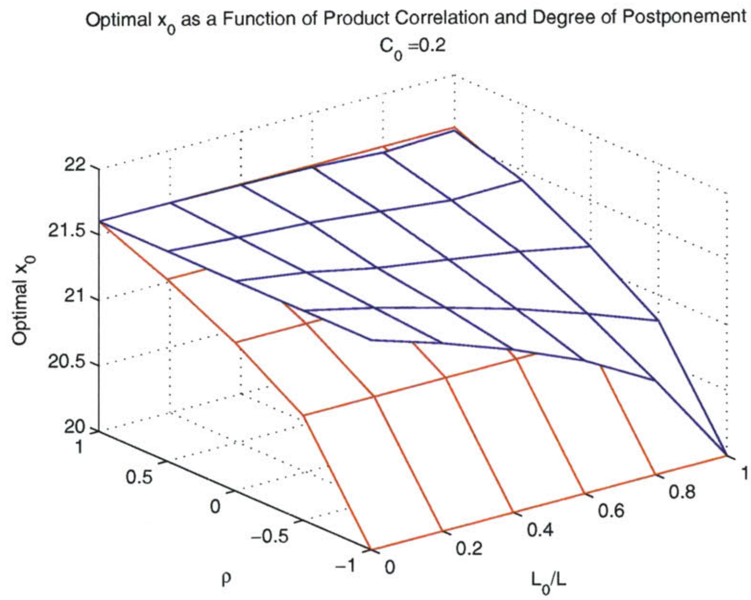
$$U = P - C = 1.3 - 1.0 = 0.3 = 30\%C$$

$$O = C - V = 1.0 - 0.8 = 0.2 = 20\%C$$

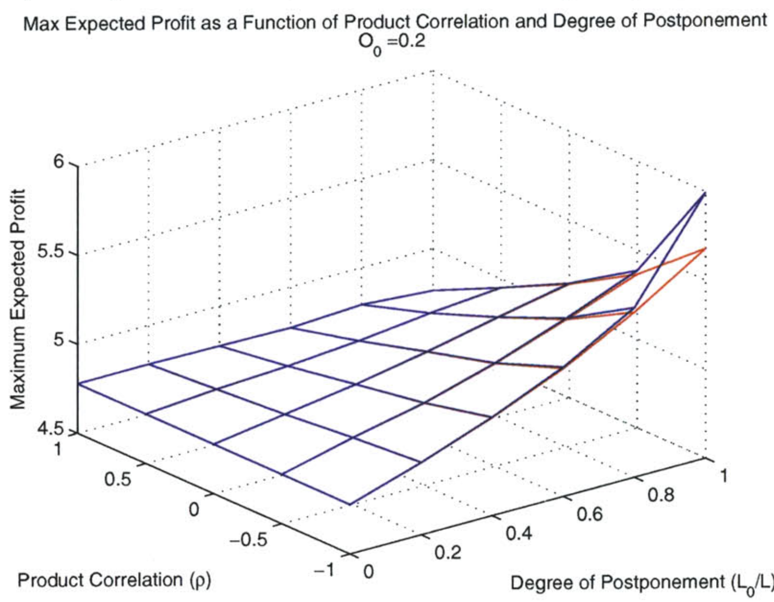
The initial forecast for both end products is 10. The standard deviation introduced in each additional forecast period is 1. $\frac{L_1}{L}$, the percentage of the first stage lead time is an indicator of the degree of postponement. $\frac{L_1}{L} = 0$ means that products are differentiated at the very beginning, that is, no postponement is implemented. When $\frac{L_1}{L} = 1$, maximum postponement is achieved, which is equivalent to the case when the decision of stage two is based on perfect demand information. In other words, when $\frac{L_1}{L} = 1$, stage two is make-to-order. $0 < \frac{L_1}{L} < 1$ corresponds to the case when postponement is implemented, but stage two is still make-to-forecast. As $\frac{L_1}{L}$ increases, that is, the degree of postponement increases, stage two is making to increasingly better forecast.

4.2 Results

Both the optimal and myopic solutions are computed for different degrees of postponement, product correlation, and the overage cost of the intermediate product. The results are shown in the following figures.

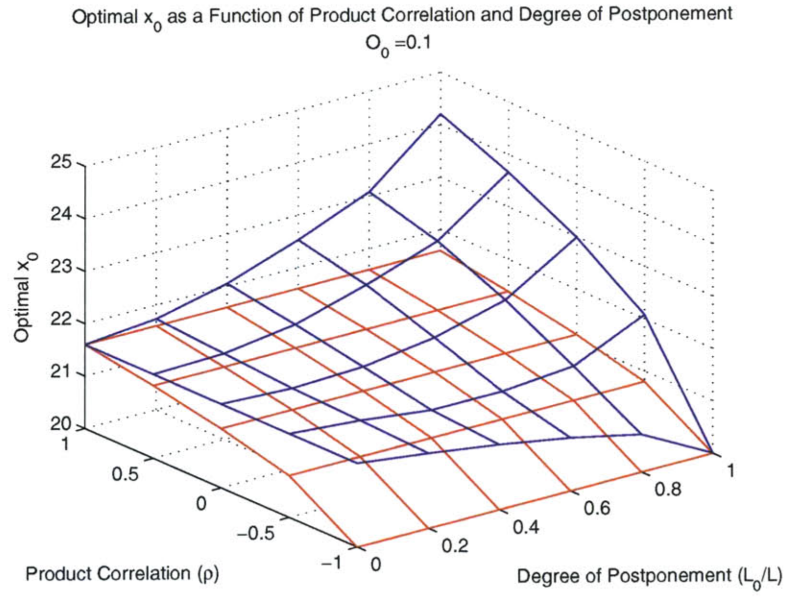


(a) optimal x_0 as a function of product correlation and degree of postponement

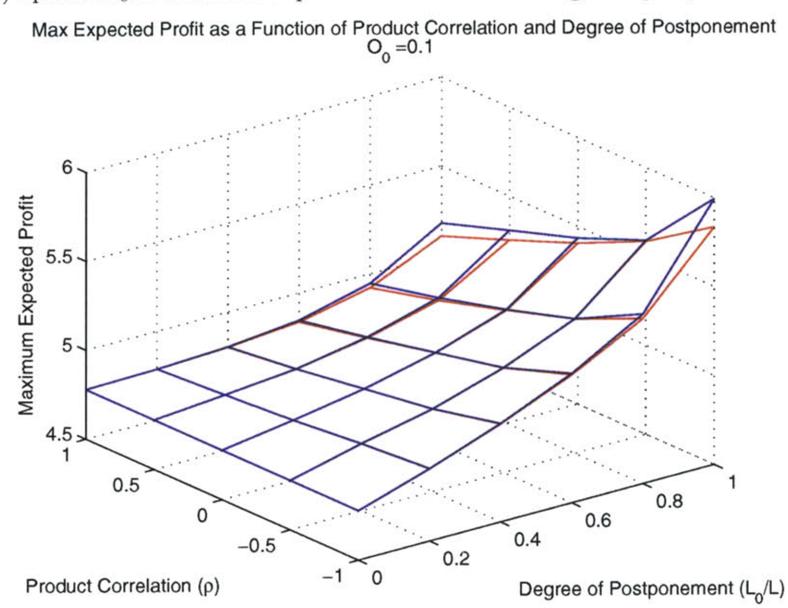


(b) maximum profit as a function of product correlation and degree of postponement

Figure 4-1: Optimal v.s. Myopic Solutions when $O_0 = O = 0.2$

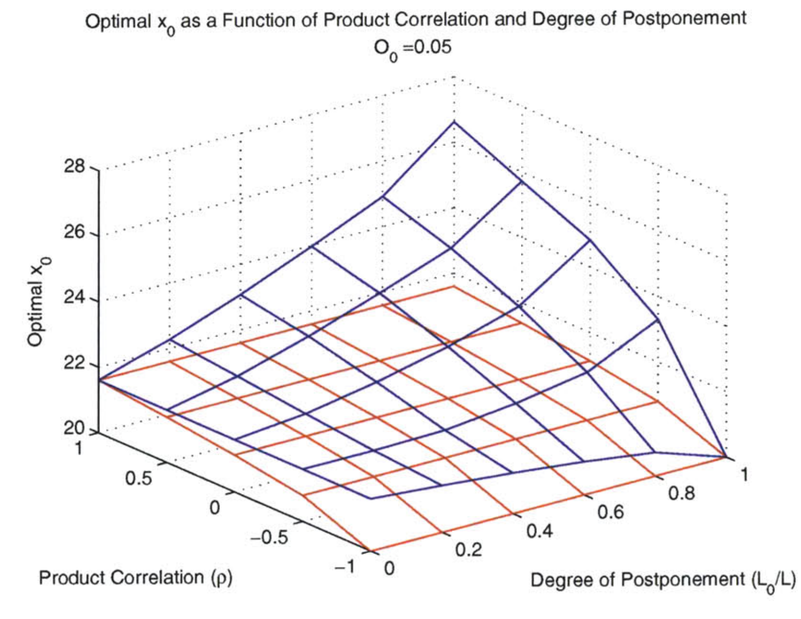


(a) optimal x_0 as a function of product correlation and degree of postponement

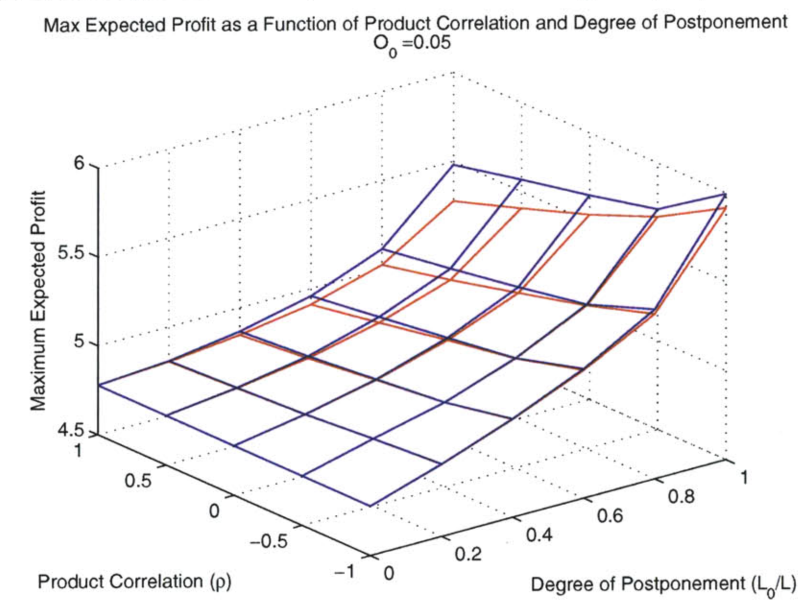


(b) maximum profit as a function of product correlation and degree of postponement

Figure 4-2: Optimal v.s. Myopic Solutions when $O_0 = 0.1$ and $O = 0.2$

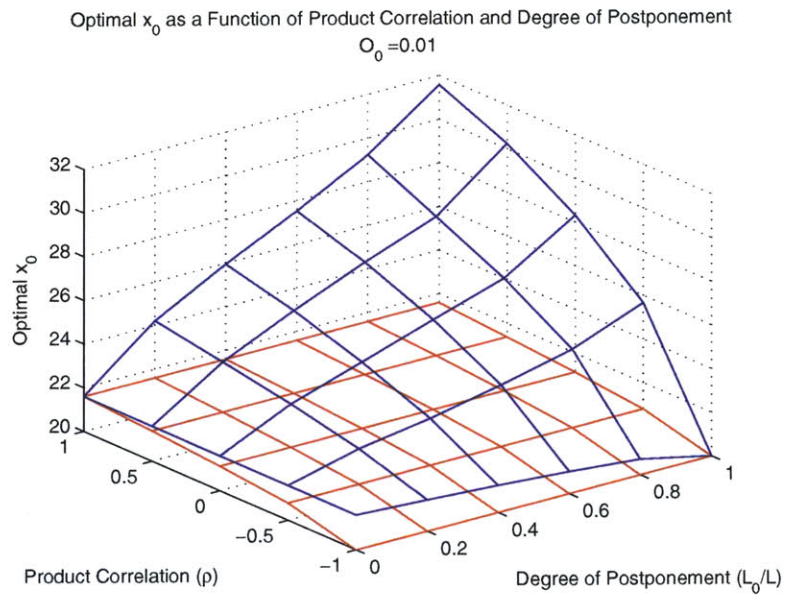


(a) optimal x_0 as a function of product correlation and degree of postponement

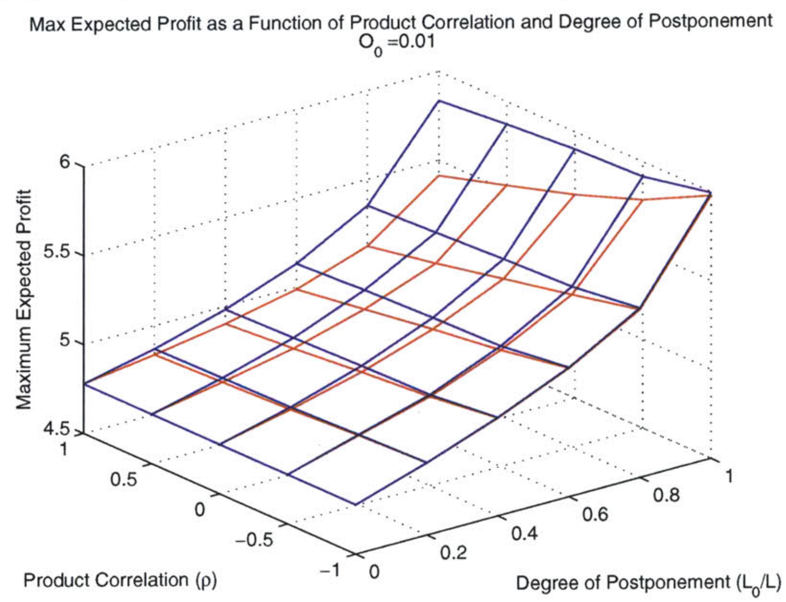


(b) maximum profit as a function of product correlation and degree of postponement

Figure 4-3: Optimal v.s. Myopic Solutions when $O_0 = 0.05$ and $O = 0.2$

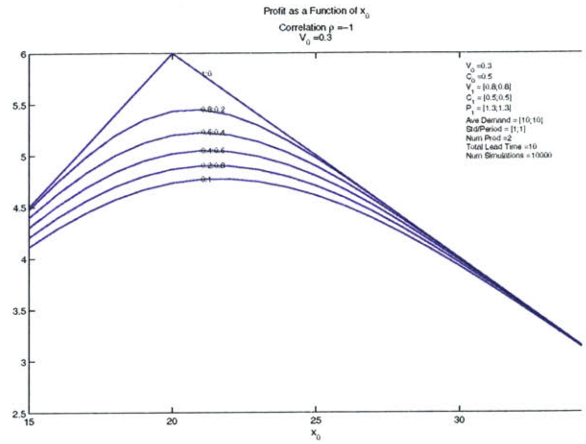


(a) optimal x_0 as a function of product correlation and degree of postponement

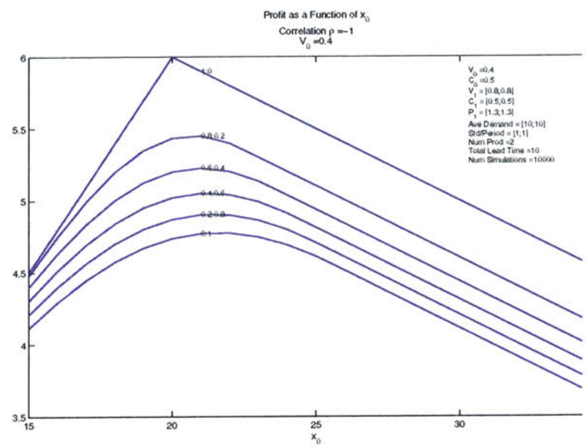


(b) maximum profit as a function of product correlation and degree of postponement

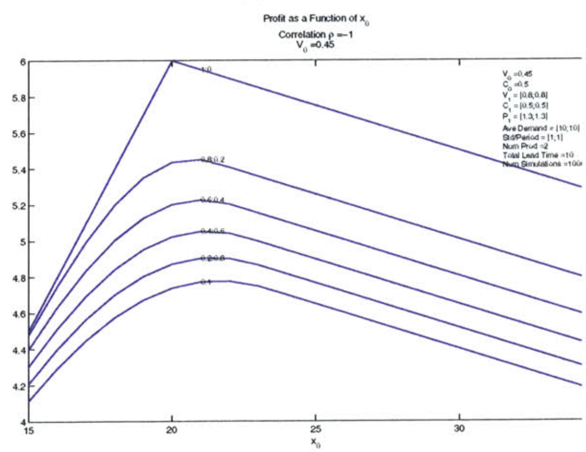
Figure 4-4: Optimal v.s. Myopic Solutions when $O_0 = 0.01$ and $O = 0.2$



(a) $O_0 = 0.20$

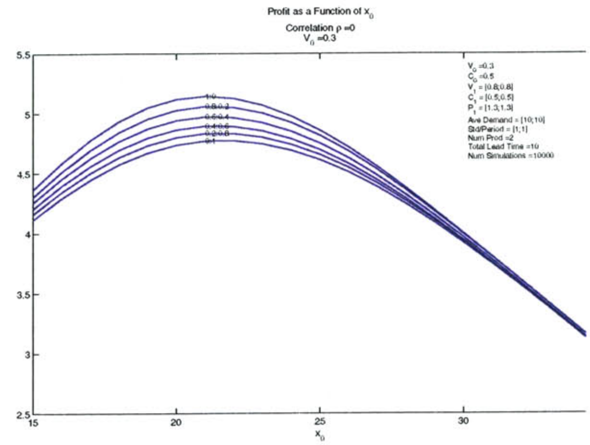


(b) $O_0 = 0.1$

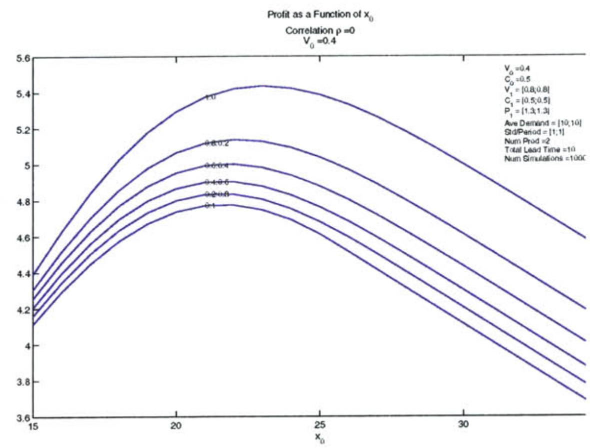


(c) $O_0 = 0.05$

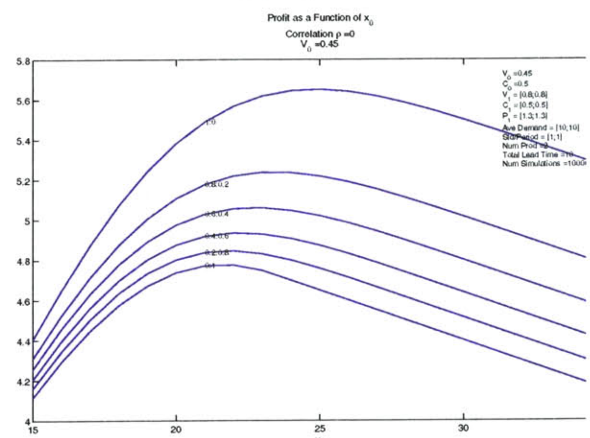
Figure 4-5: Profit as a Function of x_0, when $\rho = -1$



(a) $O_0 = 0.20$

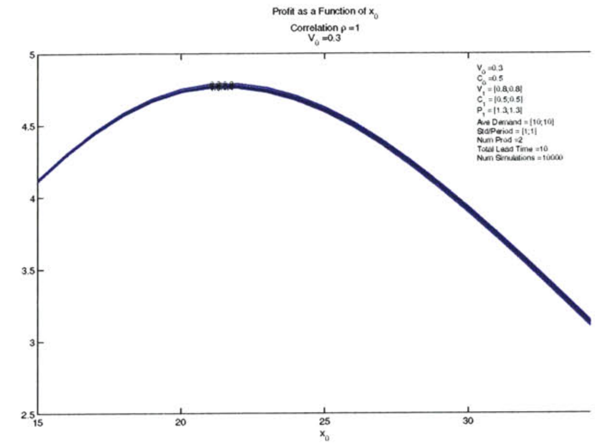


(b) $O_0 = 0.1$

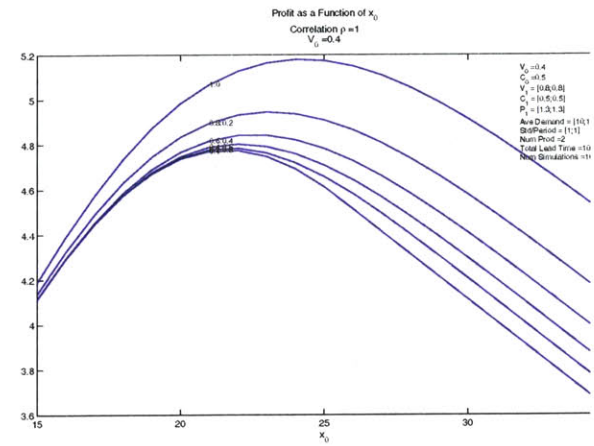


(c) $O_0 = 0.05$

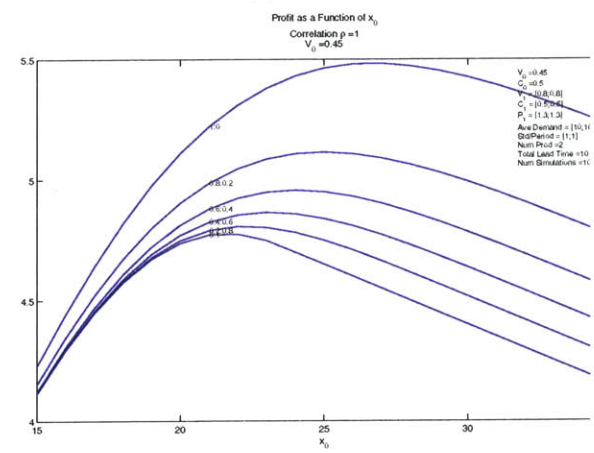
Figure 4-6: Profit as a Function of x_0, when $\rho = 0$



(a) $O_0 = 0.2$



(b) $O_0 = 0.1$



(c) $O_0 = 0.05$

Figure 4-7: Profit as a Function of x_0, when $\rho = 1$

4.3 Discussion

Computational results give us the following observations:

1. Negative correlation between the end products increases the benefit of postponement. This finding is consistent with the conclusion from Lee (1996).

   Negative correlation between the products means that when the demand for one product increases, the demand for the other products tend to decrease, resulting in a smaller variance of the the total demand. The fact that the aggregated demand has a smaller variance than individual demand is called *risk-pooling.* The more negatively correlated the end products, the more risk-pooling we get. When the end products are perfectly positively correlated, we get no risk-pooling.

   In the above numerical experiments, when the overage cost of the intermediate product is the same as that of the end products 4-1, postponement leads to a profit increase of 25% when products are negatively correlated, compared with no profit change when products are positively correlated. In reality, examples of risk-pooling are often observed and utilized. For example, in the Reebok NFL case (Parsons, 2004), the demand for each NFL Jerseys is poor at the start of a football season, but the aggregated demand is far more accurate, therefore, it is beneficial for Reebok to produce blank jerseys at early stages of the season and convert the blank jerseys to dressed jerseys after the demand information is more accurate as the season progresses.

2. Under the costs considered in this model, as the degree of postponement increases, the maximum profit never decreases.

   (a) When the overage cost of the intermediate product (O_0) is equal to that of the end products (O), postponement is not beneficial if the end products are perfectly correlated.

   (b) When the overage cost of the intermediate product (O_0) is less than that of the end products (O), the maximum profit strictly increases with the degree of postponement, even when the end products are perfectly correlated. In addition, other conditions equal, the maximum profit decreases with O_0.

Intuitively, when the intermediate product has a higher salvage value, or lower overage cost than the end products, then postponement offers additional benefits due to reduced obsolescence costs. In reality, the life-cycle of an intermediate product is often longer than that of end products, which means that the intermediate product can be reused in the next product life cycle of the end products, resulting in higher salvage value of the intermediate products. For example, in the Reebok NFL case, the blank jerseys can be reused for the next season(Parsons, 2004). In pharmaceutical industry, a commonly used FDA approved frozen chemical sometimes can last ten years resulting in an overage cost close to zero, while a pill made of this chemical usually lasts only a few months to a couple of years. A packaged medicine usually has an even earlier expiration date, and must be discarded or repackaged to sell in other regions with different regulations when the expiration date comes. Using the two-stage news-vendor model, the reduction in obsolescence cost can be quantified.

(c) When O_0 is close to zero, the maximum profit becomes insensitive to the demand correlation between the end products.

This observation suggests that if the overage cost of the intermediate product is really low, the manufacture does not need to worry about the demand correlation much. Even if the the demands are perfectly correlated, postponement will achieve similar savings as when the demands are negatively correlated. The rationale is that there is an upper bound in the amount of savings postponement can achieve, and when the overage cost of the intermediate product is close to zero, savings due to reduced obsolescence cost itself is getting close to the total savings. In other words, obsolescence savings become dominant in the total savings, therefore, the savings from risk pooling have little effect on the total benefit of postponement.

Note that the maximum savings is achieved when the products are perfectly negatively correlated and the second stage production is make-to-order. In this case, decisions at both stage one and stage two are made based on perfect demand information - at stage one, perfect demand information for the total demand, that is, the demand for the intermediate product is deterministic, although demand information for each end product is uncertain; at stage two, perfect demand information for each end products is available. Therefore, postponement generates maximum savings.

3. The maximum profit becomes less sensitive to O_0 as the negative correlation between end products increases. When the end products are perfectly negatively correlated, the optimal solution does not change with O_0.

   This observation suggests that if the manufacturer has good knowledge of the aggregated demand at the initial production stage (even if individual demand is highly uncertain), then the manufacturer does not need to worry much about the salvage value of the intermediate product. The rationale is similar as the previous observation 2c, but in this case, the cost savings is dominated by the savings due to risk-pooling. When the production quantity of the intermediate product can be determined with high accuracy, there would be very little overage cost introduced by the intermediate product, therefore the overage cost of the intermediate does not play an important role in the total savings.

4. Myopic solution generates lower maximum expected profit, but the difference from the optimal solution is small.

   In the above numerical experiments, the largest difference between the maximum expected profit obtained from the myopic solution and the optimal solution is about 5%. This observation can be explained by the shallow curvature of the curve of $J(u_{t_0,0})$ at the neighborhood of the optimal $u^*_{t_0,0}$ which can be observed from figure .

   This finding tells us that the myopic solution does not sacrifice too much of the profit. Since the myopic solution is much more computationally efficient - the computation complexity of the myopic solution is linear to the number of simulations - therefore, the myopic approach can be used to solve a much more complex problem, when computing the optimal solution is too time-consuming.

Chapter 5

Limitations and Extensions of the Model

The previous analysis gives us insight in the joint effect of overage cost of the intermediate product and the negative correlation on the benefit of postponement. However, the model ignores the additional costs introduced by postponement and must be extended in order to adapt to real cases. This chapter demonstrates the effect of postponement costs on the choice of optimal solutions.

5.1 Effect of Postponement Cost on the Optimal Solution

In reality, the total cost is often higher when postponement is implemented. For example, in the Reebok NFL case, the total cost of producing a dressed Jersey is \$11.9 if a blank Jersey is produced first, as compared with \$10.9 if the dressed Jerseys are produced directly. When the cost with postponement is higher, the pure postponement strategy is not necessarily optimal.

5.1.1 Problem Formulation

We first introduce some additional notations to represent the costs without postponement.

Notation	Description
$\hat{C}$	total cost without postponement for any end product
$\hat{O}$	overage cost of end product without postponement for any end product
$\hat{U}$	underage cost of end product without postponement for any end product

Let $u_{t_0,i}$ be the planned direct production quantity for end product i (without producing the common intermediate product first) at time t_0, where $i = 1, 2, ..., M$. Same as before, let $u_{t_0,0}$ be the planned production quantity of the intermediate product at time t_0.

Decision at Stage Two

Proposition 7 *If perfect demand information is available at the second stage, then the maximum expected profit is*

$$J^*_{t_1}(u_{t_0,0}, u_{t_0,i}, w_{t_1}) = \begin{cases} P\sum_{i=1}^M w_{t_1,i} - C_0 u_{t_0,0} - \hat{C}\sum_{i=1}^M u_{t_0,i} - C\sum_{i=1}^M \max\{w_{t_1,i} \\ -u_{t_0,i}, 0\} + \hat{V}\sum_{i=1}^M \max\{u_{t_0,i} - w_{t_1,i}, 0\} \\ +V_0 \max\{u_{t_0,0} - \sum_{i=1}^M \max\{w_{t_1,i} - u_{t_0,i}, 0\}, 0\}, \\ \qquad \textit{if } u_{t_0,0} + \sum_{i=1}^M \min\{u_{t_0,i}, w_{t_1,i}\} > \sum_{i=1}^M w_{t_1,i}; \\ U u_{t_0,0} + \hat{U}\sum_{i=1}^M u_{t_0,i} - (\hat{U}+\hat{O})\sum_{i=1}^M \max\{u_{t_0,i} - w_{t_1,i}, 0\}, \\ \qquad \textit{otherwise.} \end{cases} \tag{5.1}$$

The proof is straightforward, therefore is omitted.

Decision at Stage One

Similar to the case when postponement does not introduce additional cost, the decision problem at stage one is

$$\max_{u_{t_0,0},u_{t_0,i}} J_{t_0}(u_{t_0,0}, u_{t_0,i}) \tag{5.2}$$

where

$$\begin{aligned} J_{t_0}(u_{t_0,0}, u_{t_0,i}) &= \mathrm{E}_{w_{t_1}} J^*_{t_1}(u_{t_0,0}, u_{t_0,i}, w_{t_1}) &(5.3)\\ &= \int_{-\infty}^{\infty} J^*_{t_1}(u_{t_0,0}, u_{t_0,i}, w_{t_1}) f_{w_{t_1}}(w_{t_1}) dw_{t_1} &(5.4)\end{aligned}$$

This problem is much more difficult than 3.37. First, the policy space at stage one is much larger. Not only the production quantity for the intermediate product ($u_{t_0,0}$) needs to be decided, but also that of each end products ($u_{t_0,i}$). In addition, the profit function $J^*_{t_1}(u_{t_0,0}, u_{t_0,i}, w_{t_1})$ is in a much more complex form (5.1), making it difficult to find the optimal solution efficiently. In general, some search algorithm can be used to find the best solution within the search space, but it is time-consuming, and might not guarantee an optimal solution. Parsons (2004) proposed an effective heuristic algorithm to find a suboptimal solution. Next, we provide a brief review on Parsons's risk-pooling algorithm, and compare the maximum expected profit under three different policies - full postponement strategy with no end products produced at the first stage, the hybrid policy proposed in Parsons (2004) and the policy without postponement.

The data used in this chapter are taken from Parsons (2004).

5.1.2 NFL Jerseys Case

In this case, Reebok has two ways of ordering NFL Jerseys shirts from its supplier located overseas - to order directly the dressed Jerseys, or to order blank Jerseys, which can be converted to dressed Jerseys in North America. The lead time between the order receival to

Parameter	Value
C_0: cost of blank	9.5
V_0: salvage value for blank	8.46
C: cost of decorate in North America	2.40
P: wholesale sales price	24
V: salvage value for dressed	7
$\hat{C}$: cost of dressed	10.9
$\hat{O} = \hat{C} - V$: overage cost of dressed if produced directly	3.9
$\hat{U} = P - \hat{C}$: underage cost of dressed if produced directly	13.1
$O = C_0 + C - V$: overage cost of dressed if produced from blank	4.9
$U = P - C_0 - C$: underage cost of dressed if produced from blank	12.1
$O_0 = C_0 - V_0$: overage cost of blank	1.04

Table 5.1: NFL Jerseys: Production Costs (Parsons, 2004)

Notation	Desc	Mean (μ_i)	Standard Deviation (σ_i)
$w_{t_0}(total)$	NEW ENG PATRIOT Total	87679.5	19211.26701
$w_{t_0}(0)$	Other Players	23274.9	10473.705
$w_{t_0}(1)$	BRADY, TOM #12	30763.2	13843.44
$w_{t_0}(2)$	LAW, TY #24	10569	4756.05
$w_{t_0}(3)$	BROWN, TROY #80	8158.8	3671.46
$w_{t_0}(4)$	VINATIERI, ADAM #04	7269.6	4361.76
$w_{t_0}(5)$	BRUSCHI, TEDY #54	5526.3	3315.78
$w_{t_0}(6)$	SMITH, ANTOWAIN #32	2117.7	1270.62

Table 5.2: NFL Jerseys: Forecast as of March 1st, 2003 (Parsons, 2004)

order shipment is about 30 days, while the time for converting the blank Jerseys to dressed Jerseys is very short. For detailed description of the case, see Parsons (2004). The following table summarizes the pricing and cost information.

The following table provides the forecast information as of March 1st, 2003 (stage one):

The first stage decision is make-to-stock, while the second stage decision is customize-to-order, which means that the blank Jerseys are converted to dressed Jerseys based on perfect demand information.

Three different policies are compared.

With the full postponement policy, only blank Jerseys are produced at the first production stage, and they are converted to dressed Jerseys when perfect demand information is available. The maximum expected profit can be obtained using equation 3.47.

Without postponement, at the first stage, the production quantity for each type of dressed Jerseys are decided separately using standard newsvendor model; at the second stage the blank Jerseys (produced to satisfy the demand for other players) can also be used to fulfill the unmet demand for named players. The maximum expected profit is computed by simulation.

The hybrid policy first computes the critical ratio (the probability the terminal demand can be fulfilled by blank Jerseys) of the blank Jerseys, r. And this critical ratio is used to compute the estimated underage cost of dressed Jerseys, $\bar{U}$. From $\bar{U}$ the optimal production quantity for the dressed Jerseys ($u_{t_0,i}$) are derived. In the end, $u_{t_0,i}$ are used to adjust the demand for the blank Jerseys (μ_0, σ_0) by considering the possibility of fulfilling the unmet demand of the named Jerseys using the bland Jerseys. In summary,

$$
\begin{aligned}
r_1 &= U/(U+O_0) \\
\bar{U} &= r_1(C+C_0-\hat{C}) + (1-r_1)(P-\hat{C}) \\
r_2 &= \frac{\bar{U}}{\bar{U}+\hat{O}} \\
z &= \Phi^{-1}(r_2) \\
u_{t_0,i} &= \mu_i + \sigma_i z \\
E[LostSales](i) &= \mu_i - u_{t_0,i} + \sigma_i(z\Phi(z) + \phi(z)) \\
\mu_0 &= \mu_0(old) + \sum_{i=1}^{6} E[LostSales](i) \\
\sigma_0^2 &= \sum_{i=0}^{6} \sigma_i^2 \\
u_{t_0,0} &= \mu_0 + \sigma_0\Phi^{-1}(r_1)
\end{aligned}
$$

The maximum expected profit is computed by simulation, and equation 5.1 is used.

To observe the effect of the additional costs introduced by postponement, we also include the optimal solution for a hypothetical case, that is, the cost of converting a blank Jersey is reduced by \$1, making the total cost with and without postponement the same. The computation results are summarized in the following table:

The difference between the maximum profits in the above table and the corresponding

Method	Same Cost With and Without Postponement (C_0 = \$8.5)	Same Cost With and Without Postponement ($C = \$1.4$)	Full Postponement	No Postponement	Hybrid (Parsons(2004))
$u_{t_0,0}$: Optimal production quantity of blank Jerseys	$140,376$	$115,532$	$114,783$	$38,051$	59809
$u_{t_0,i}$: Optimal production quantity of dressed Jerseys	0	0	0	$\begin{pmatrix} 41,018 \\ 14,092 \\ 10,879 \\ 10,501 \\ 7,983 \\ 3,059 \end{pmatrix}$	$\begin{pmatrix} 24852 \\ 8538 \\ 6591 \\ 5407 \\ 4110 \\ 1575 \end{pmatrix}$
$u_{t_0,0} + \sum_{i=1}^{7} u_{t_0,i}$: Total initial production quantity	$140,376$	$115,532$	$114,783$	$118,565$	$110,883$
J^*: Maximum Profit (\$)	$1,146,261$	$1,110,717$	$1,023,696$	$987,000$	$1,043,000$
J^*: Relative Maximum Profit (\$)	1.16	1.13	1.04	1	1.06

Table 5.3: NFL Jerseys: Policy Comparison

ones in Parsons (2004) are caused by different ways of evaluating the profit. We computed the maximum profit using simulation, while Parsons (2004) estimated the maximum profit by assuming all the unmet demand of the dressed Jerseys are fulfilled by the blank Jerseys, which is an optimistic estimation. The above results suggest that a hybrid solution is better than the full postponement solution by about 2%, which is quite significant, since the profit increase by the fully postponed solution is only 4%. In addition, this example demonstrates that the variable cost of postponement has significant impact on the total benefit of postponement. By increasing production cost by about 10%, the profit increase achieved by postponement is reduced by about 50% (6% v.s. 13%) if the additional cost is incurred on the product differentiation phase, or about 62% (6% v.s. 16%) if the additional cost is imposed in the common production phase.

Chapter 6

Conclusion

This thesis analyzes the benefit of postponement for fashion products using a two stage newsvendor model with forecast evolution. The benefit of postponement is measured by the percentage increase in maximum expected profit. Analytical and numerical results indicate that

1. Other costs equal, the benefit of postponement decreases as the salvage value of the intermediate product increases, or the correlation between the demand for end products increases. However, for a set of perfectly negatively correlated products, benefit of postponement becomes insensitive to the salvage value of the intermediate product. And if the salvage value of the intermediate product is close to its cost, then the benefit of postponement is insensitive to the correlation of the end products.

2. If postponement introduces additional costs, then the hybrid policy proposed by Parsons (2004) performs much better than a pure postponement policy. In addition, reducing costs in the common production phase leads to a higher profit than reducing the same amount of cost in the differentiated production phase.

The model and findings in this thesis can be used to analyze the benefit of postponement for fashion products related to uncertainty reduction, and can be integrated into more complex models that includes other values and costs described in chapter 1.

Chapter 7

Future Work

To simplify notation and gain analytically insights, the analysis in this thesis assumes that the demand evolution follows a multi-variant normal distribution (additive martingale (Heath and Jackson, 1994)). However, multi-variate lognormal distribution (multiplicative martingale) might be a more realistic approximation (Heath and Jackson, 1994), because it captures the effect of demand size on its variance. The demand evolution model can be easily adapted to any other distributions, but analytical solutions as proposed in chapter 3 might not exist.

Capacity constraints are ignored in the model proposed in this thesis. Adding capacity constraint will push production decisions further back from demand realization time, forcing productions to be made at higher uncertainty, hence, reduces optimal expected profit. It would be interesting to analyze the effect of capacity on the benefit of postponement.

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