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A NOTE ON THE USE OF AGGREGATE DATA IN INDIVIDUAL CHOICE MODELS: DISCRETE CONSUMER CHOICE AMONG ALTERNATIVE FUELS FOR RESIDENTIAL APPLIANCES

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INTRODUCTION AND OVERVIEW

The qualitative choice models utilized in many microeconomic analyses incorporate the preferences of a "representative" individual; as a result, these models should be estimated utilizing data on the actual choices of individuals [as in 10, 18, 19, 26, 28, 29]. However, data experiments focusing on individual choices are expensive to gather and not readily available.[1] Furthermore, those experiments that do gather individual data are usually geographically (and sometimes socioeconomically) stratified. As a result, variation in tastes and socioeconomic/demographic characteristics across U.S. regions and groups of individuals may not be fully captured.

On the other hand, compilations of aggregate data are fairly accessible. For example, a body of aggregate data that is readily available and used extensively for analysis of fuel choice [2, 3, 4, 5, 6, 7, 14, 16, 17, 23, 24] consists of pooled time-series of state cross-sections. However, the use of such aggregate data for estimating individual choice models can be defended by its success as an empirical tool rather than its appropriateness for the underlying behavioral model: McFadden and Reid [27] demonstrate that the use of such aggregate data for parameter estimation and prediction with disaggregated choice models leads to serious inconsistencies unless rather restrictive conditions are met.[2]

This paper attempts to bridge the gap between models of individual choice and aggregate data. The reasons for bridging the gap are immediate.

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[1]The Midwest Research Institute [1979] data is one extensively-used example.

[2]Their results are developed for a binary probit model estimated by regression techniques.

The use of choice models has proved very useful in demand analysis, in particular for energy demand. Furthermore, the aggregate pooled time-series/cross-sectional data available for many demand analyses (again particularly energy demand) offer a rich resource to be exploited. The capability to combine such models and such data would be extremely useful.

To that end, Section I below analyzes the effects of using aggregate data to estimate individual choice models. A standard choice model is introduced, the effects of introducing aggregate data are analyzed, and a method suggested for obtaining consistent maximum likelihood estimates. Using the results of Section I, Section II analyzes the choice of fuel for residential appliances using aggregate data. The model of choice is developed fully in [15]. Given a simple version of the model of fuel choice and using the aggregate data, Section III presents estimates of the individual parameters of taste and indicates the extent of asymptotic bias generated when aggregate data are used without correction. Systematic parameter bias toward zero is demonstrated when measurement error is present. Estimates are presented for several appliances where the extent of measurement error is actually estimated.

I) INDIVIDUAL CHOICE MODELS AND AGGREGATE DATA

A. THE STANDARD MODEL WITH INDIVIDUAL DATA

The standard utility formulation for individual choice models is that the utility U_{ij} of alternative j to individual i is

$$U_{ij} = \overline{U}_{ij}(X_j, a_i) + \varepsilon_{ij} \tag{1a}$$

$$= Z_{ij}\overline{\beta} + \varepsilon_{ij}, \text{ where} \tag{1b}$$

X_j is a vector of the observable attributes and levels of consumer services offered by alternative j; a_i is a vector of observable characteristics of the individual i and the environment in which individual i makes his/her choice; $\overline{U}_{ij}$ is the "average" utility of a "representative" individual and ε_{ij} is a random error term induced by purely random behavior, measurement error and/or unobserved characteristics of the individual and/or alternative. Letting Z_{ij} represent combinations of X_j and a_i, then $\overline{U}_{ij}(X_j, a_i) = Z_{ij}\overline{\beta}$ where $\overline{\beta}$ is assumed constant over the entire population (i.e., homogenous tastes).[1]

Given utility function (1), the probability that an individual i chooses alternative k is:

$$\begin{aligned} P_{ik} &= \Pr\ [U_{ik} > U_{ij}, \text{ for all } j \neq k] \\ &= \Pr\ [Z_{ik}\overline{\beta} + \varepsilon_{ik} > Z_{ij}\overline{\beta} + \varepsilon_{ij}, \text{ for all } j \neq k] \\ &= \Pr\ [\varepsilon_{ij} - \varepsilon_{ik} < (Z_{ik} - Z_{ij})\ \overline{\beta}, \text{ for all } j \neq k] \\ &= \Pr\ [\eta_{jk} < (Z_{ik} - Z_{ij})\ \overline{\beta} \text{ for all } j \neq k] \end{aligned} \tag{2}$$

[1] This is the utility formulation utilized by most authors. Hausman and Wise [19] utilize this model for heterogeneous tastes within a probit specification by decomposing the elements of $\overline{\beta}$ into both a constant (mean) and a random taste parameter. McFadden, Tye and Train also develop a heterogenous preference model in [29]. Heterogenous tastes could be introduced here; however for simplicity I focus only on issues of aggregation.

where

$$\eta_{jk} = \varepsilon_{ij} - \varepsilon_{ik}$$

Once the form of the utility function and the distribution $F(\varepsilon)$ of the ε_{ij} are specified, unknown parameters (i.e. $\overline{\beta}$) of $\overline{U}_{ij}$ and the parameters of $F(\varepsilon)$ can be estimated using the likelihood function

$$L(\theta) = \prod_{i}^{m} \prod_{k}^{n} P_{ik}^{\ y_{ik}} \tag{3}$$

where the likelihood function $L(\theta)$ is defined over all individuals $i=1 \ldots m$ and alternatives $k = 1 \ldots n$; $y_{ik} = 1$ if individual i chooses alternative k and =0 otherwise; $\sum_{k=1}^{n} P_{ik} = 1$ for any individual i.

As is well known, the assumptions regarding the distribution $F(\varepsilon)$ determine the functional form of the P_{ik} as follows:

- ε_{ij} distributed as Weibull

This assumption is the basis for logit analyses. In this case $\eta_{jk} = \varepsilon_{ij} - \varepsilon_{ik}$ is distributed logistically [10] and

$$\begin{aligned} P_{ik} &= \Pr\,[\eta_{jk} < \overline{U}_{ik} - \overline{U}_{ij} \text{ for all } j \neq k] \\ &= \frac{e^{\overline{U}_{ik}}}{\sum_{j}^{n} e^{\overline{U}_{ij}}} = \frac{1}{\sum_{j}^{n} e^{(\overline{U}_{ij} - \overline{U}_{ik})}} = \frac{1}{\sum_{j}^{n} e^{(Z_{ij} - Z_{ik})\overline{\beta}}} \end{aligned} \tag{4}$$

- ε_{ij} distributed as normal

In this case, η_{jk} is also distributed normally and equation (2) becomes

$$P_{ik} = \int_{-\infty}^{W^*_{k1}} \cdots \int_{-\infty}^{W^*_{kj}} \cdots \int_{-\infty}^{W^*_{kn}} \phi(r;\ 0;\ \Omega)\,dr_1 \ldots dr_j \ldots dr_n \quad \text{for } j \neq k \tag{5}$$

where $w^*_{kj} = \overline{U}_{ik} - \overline{U}_{ij}$ and $\phi(r; 0, \Omega)$ is the multivariate normal frequency with 0 mean and covariance matrix Ω evaluated at r. If the off-diagonal terms of Ω are zero, "independent" probit results[1]; if Ω is dense, covariance probit results [19].

Substituting (4) or (5) into (3), using data on individuals for their choices and a_i, and maximizing L with respect to θ will yield estimates of the parameters of $\overline{U}_{ij}$ (i.e. $\overline{\beta}$) and $F(\varepsilon)$.

B. THE MODEL USING AGGREGATE DATA

Suppose as before we want to estimate the taste parameters $\overline{\beta}$ in equation (1b) but hope to rely on the pooled time-series/cross-sectional data rather than data on individuals. How will the model and the estimation procedures be altered?

For purposes of discussion let equation 1 become

$$U_{ij} = X_j\overline{\beta}_1 + a_i\overline{\beta}_2 + \varepsilon_{ij} \tag{6}$$

and assume that only three choices are available (j = 1 - 3). Assume further that we hope to use (6), (2) and (3) to estimate $\overline{\beta}_1$ and $\overline{\beta}_2$ with aggregate data consisting of pooled annual time-series of state cross-sections. In that case, we observe collections of individuals i (market shares) who made choices among j = 1, 2 and 3 in a given cross-sectional unit (state) and time period (year) when their utility functions are assumed to be (6). Furthermore, we observe only average state estimates of X_j and a_i

[1]With parameter estimates and predictive performance similar to that of logit when micro data is used.

rather than the actual individual values. For example, rather than observing the "true" price of alternative j (X_j), we observe $\tilde{X}_j = X_j - v_j$ (X_j observed with error v_j) where $\tilde{X}_j$ is the average price of alternative j in a state; and rather than observing the true income of individual i (a_i) we observe $\tilde{a}_i = a_i - v_i$ (a_i observed with error v_i) where $\tilde{a}_i$ is the average income in a state. Using such aggregate data for the characteristics of individual i (a_i) and the characteristics of the choices facing individual i (X_j), let us redefine equation (6) as follows:

$$\begin{aligned} U_{ij} &= X_j\bar{\beta}_1 + a_i\bar{\beta}_2 + \varepsilon_{ij} \\ &= (\tilde{X}_j + v_j)\bar{\beta}_1 + (\tilde{a}_i + v_i)\bar{\beta}_2 + \varepsilon_{ij} \\ &= \tilde{X}_j\bar{\beta}_1 + \tilde{a}_i\bar{\beta}_2 + (\varepsilon_{ij} + v_j\bar{\beta}_1 + v_i\bar{\beta}_2) = \tilde{X}_j\beta_1 + \tilde{a}_i\beta_2 + \tilde{\varepsilon}_{ij} \end{aligned} \tag{6a}$$

where the new error $\tilde{\varepsilon}_{ij}$ is induced by ε_{ij}, in addition to the measurement errors of X_j and a_i ($v_j\bar{\beta}_1 + v_i\bar{\beta}_2$) generated by using the aggregate data rather than the individual data.[1]

Using equation (6a), the probability that an individual i chooses alternative k is given by an altered version of (2) as follows:

$$\begin{aligned} P_{ik} &= \Pr[(\varepsilon_{ij} + v_j\bar{\beta}_1 + v_i\bar{\beta}_2) - (\varepsilon_{ik} + v_k\bar{\beta}_1 + v_i\bar{\beta}_2) < \\ &\qquad (Z_{ik} - Z_{ij})\bar{\beta} \quad \text{for all } j \neq k] \\ &= \Pr[\tilde{\varepsilon}_{ij} - \tilde{\varepsilon}_{ik} < (Z_{ik} - Z_{ij})\bar{\beta} \quad \text{for all } j \neq k] \\ &= \Pr[\tilde{\eta}_{jk} < (Z_{ik} - Z_{ij})\bar{\beta} \quad \text{for all } j \neq k] \end{aligned} \tag{7}$$

[1]This formulation is similar to that of Hausman and Wise [19] except they attribute the additional randomness to tastes rather than measurement error. With individual data, randomness in tastes will probably be most important; when using aggregate data randomness in the measurement error will probably be most important. Of course, both types of randomness could be parameterized and tested if the data existed. Such data would require observations on individuals with repetitions where the measurement errors were capable of characterization.

where $\tilde{\eta}_{jk} = \tilde{\varepsilon}_{ij} - \tilde{\varepsilon}_{ik}$. Once we have characterized $\tilde{\eta}_{jk}$, the P_{ik} will then be substituted into a version of (3) that takes into account the use of the pooled data as follows:

$$L(\Theta) = \prod_{t=t_0}^{t_1} \prod_{s=1}^{50} \prod_{k=1}^{K} \prod_{i=1}^{M_{skt}} P_{ik}^{\,y_{ik}} = \prod_{t=t_0}^{t_1} \prod_{s=1}^{50} \prod_{k=1}^{K} P_k^{\,M_{skt}} \quad (8)$$

where the pooled time-series runs from year t_0 to t_1, covering 50 cross-sectional units (states) for M_{skt}[1] persons in state s in period t that choose alternative k.

As in the case above, the assumptions regarding the distributions of ε_{ij}, v_i, v_k and v_j will determine the functional form of the P_{ik}. However, the possible outcomes are more limited with aggregate data. For example, for $\tilde{\eta}_{jk}$ to be distributed as a logistic curve, the $\tilde{\varepsilon}_{ij}$ must be distributed as Weibull. However, random variables distributed as Weibull are closed under maximization;[2] since the $\tilde{\varepsilon}_{ij}$ consists of the <u>sum</u> of random variables ε_{ij}, v_i and v_j, the distribution of $\tilde{\varepsilon}_{ij}$ is not easily characterized if ε_{ij}, v_i and v_j are Weibull. Likewise, if ε_{ij} is distributed as Weibull and the v_i, v_k and v_j are distributed normally (which seems to be most realistic for v_i, v_k and v_j), $\tilde{\varepsilon}_{ij}$ (hence $\tilde{\eta}_{jk}$) is the sum of both Weibull and normal random variables. The distribution of that sum is extremely complicated. <u>Only</u> when ε_{ij}, v_i, v_k and v_j are distributed normally will $\tilde{\varepsilon}_{ij}$ and $\tilde{\eta}_{jk}$ be distributed normally; in that case the P_{ik} will be analytically tractable.

[1]The use of aggregate data implies that M_{skt} will be determined by shares data; hence, it too is subject to measurement error. I do not examine the effects of that measurement error here.

[2]See Domencich and McFadden [10], pp. 61-62.

We can make this stochastic analysis more precise as follows. Given individual utility in equation (6a) for j=1..3 and the determination of P_{ik} in (7) for k = 1 . . 3, we need to characterize $\tilde{\eta}_{jk}$, where

$$\tilde{\eta}_{jk} = (\varepsilon_{ij} - \varepsilon_{ik}) + (v_j - v_k)\overline{\beta}_1 + (v_i - v_i)\overline{\beta}_2 \tag{9}$$

In (9) it is immediately evident that for personal attributes (a_i) that enter utility additively, <u>measurement errors will have no effect</u>[1] on estimation - $(v_i - v_i) \equiv o$. Thus, if there are no measurement errors in the attributes of the alternatives ($v_j \equiv v_k \equiv o$) and the attributes of the alternatives enter utility additively (i.e. not in interaction with personal attributes) then $\tilde{\eta}_{jk} = \varepsilon_{ij} - \varepsilon_{ik}$ and standard logit or probit analysis is possible given assumptions about ε_{ij} and ε_{ik} [section Ia]. If, however, measurement errors occur in the attributes of the alternatives (X_j) or in interaction terms of the X_j and a_i, then[2]

$$\tilde{\eta}_{jk} = (\varepsilon_{ij} - \varepsilon_{ik}) + (v_j - v_k)\overline{\beta}_1. \tag{9a}$$

Assuming $v_j \sim N(0,\Omega_j)$ and $\varepsilon_{ij} \sim N(0,\sigma_j^2)$ for all j,where Ω_j is a diagonal covariance matrix with the variances of measurement errors of X_j along the diagonal; assuming all covariances among measurement errors (i.e. off-diagonal elements of Ω_j) and between measurement errors and ε_{ij} are zero; and assuming $COV(\varepsilon_{ij}\ \varepsilon_{ik}) = \sigma_{jk}$ for all j and k, we obtain

$$E(\tilde{\eta}_{jk}) = E[(\varepsilon_{ij} - \varepsilon_{ik}) + (v_j - v_k)'\,\overline{\beta}_1] = 0 \tag{9b}$$

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[1]Of course, the individual attributes themselves will have no effect either, in equation (7).

[2]Assuming $\overline{\beta}_1$ includes parameters for attributes of the alternatives and all interaction terms.

and $$V(\tilde{n}_{jk}) = \sigma_j^2 + \sigma_k^2 - 2\sigma_{jk} + \bar{\beta}_1'(\Omega_j + \Omega_k)\bar{\beta}_1 \tag{9c}$$

For further ease of exposition assume $\bar{\beta}_1$ is a scaler; then

$$V(\tilde{n}_{jk}) = \sigma_j^2 + \sigma_k^2 - 2\sigma_{jk} + \bar{\beta}_1^2(\sigma_{v_j}^2 + \sigma_{v_k}^2),$$

and for $\tilde{n}_{21}$ and $\tilde{n}_{31}$ with bivariate normal density function $g_1(\tilde{n}_{21}, \tilde{n}_{31}, \Omega_1)$ and covariance matrix

$$\Omega_1 = \begin{bmatrix} \sigma_1^2+\sigma_2^2-2\sigma_{12}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_2}^2) & \\ \sigma_1^2+\sigma_{23}-\sigma_{12}-\sigma_{13}+\sigma_{v_1}^2\bar{\beta}_1^2 & \sigma_1^2+\sigma_3^2-2\sigma_{13}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_3}^2) \end{bmatrix} \tag{10}$$

we have from equation (5)

$$P_{i1} = \int_{-\infty}^{(\bar{U}_1-\bar{U}_2)/\sqrt{\sigma_1^2+\sigma_2^2-2\sigma_{12}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_2}^2)}} \int_{-\infty}^{(\bar{U}_1-\bar{U}_3)/\sqrt{\sigma_1^2+\sigma_3^2-2\sigma_{13}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_3}^2)}} b_1(\tilde{n}_{21}, \tilde{n}_{31}; \rho_1)\, d\tilde{n}_{21}\, d\tilde{n}_{31} \tag{11}$$

where b_1 is a standard bivariate normal distribution with correlation coefficient ρ_1.[1] The probabilities of P_{i2} and P_{i3} are calculated in the same manner.

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[1] $\rho_1 = (\sigma_1^2+\sigma_{23}-\sigma_{12}-\sigma_{13}+\sigma_{v_1}^2\bar{\beta}_1^2)/\sqrt{(\sigma_1^2+\sigma_2^2-2\sigma_{12}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_2}^2))(\sigma_1^2+\sigma_3^2-2\sigma_{13}+\bar{\beta}_1^2(\sigma_{v_1}^2+\sigma_{v_3}^2))}$

c) MODEL ESTIMATION

For estimation we need the full P_{ij} and Ω_j for j=1..3 and the likelihood function (8). For the three alternatives we have the following covariance matrices for an individual in state s and period t:

$$\Omega_1=\begin{bmatrix} \sigma_1^2+\sigma_2^2-2\sigma_{12}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{1j}}+\sigma^2_{v_{2j}})_{st} & \\ \sigma_1^2+\sigma_{23}-\sigma_{12}-\sigma_{13}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{1j}})_{st} & \sigma_1^2+\sigma_3^2-2\sigma_{13}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{1j}}+\sigma^2_{v_{3j}})_{st}\end{bmatrix}=\begin{bmatrix}\omega_{1,11} & \\ \omega_{1,12} & \omega_{1,22}\end{bmatrix}$$

$$\Omega_2=\begin{bmatrix} \sigma_2^2+\sigma_1^2-2\sigma_{12}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{1j}}+\sigma^2_{v_{2j}})_{st} & \\ \sigma_2^2+\sigma_{13}-\sigma_{21}-\sigma_{23}+\sum_{j=1}^{J}\beta_j^2(\sigma^2_{v_{2j}})_{st} & \sigma_2^2+\sigma_3^2-2\sigma_{23}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{2j}}+\sigma^2_{v_{3j}})_{st}\end{bmatrix}=\begin{bmatrix}\omega_{2,11} & \\ \omega_{2,12} & \omega_{2,22}\end{bmatrix}$$

$$\Omega_3=\begin{bmatrix} \sigma_3^2+\sigma_1^2-2\sigma_{13}+\sum_{j=1}^{J}\bar{\beta}_j(\sigma^2_{v_{1j}}+\sigma^2_{v_{3j}})_{st} & \\ \sigma_3^2+\sigma_{12}-\sigma_{31}-\sigma_{32}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{3j}})_{st} & \sigma_3^2+\sigma_2^2-2\sigma_{32}+\sum_{j=1}^{J}\bar{\beta}_j^2(\sigma^2_{v_{2j}}+\sigma^2_{v_{3j}})_{st}\end{bmatrix}=\begin{bmatrix}\omega_{3,11} & \\ \omega_{3,12} & \omega_{3,22}\end{bmatrix} \quad (12)$$

where $\bar{\beta}$ in (1b) has J elements $\bar{\beta}_j$; $\sigma^2_{v_{1j}}, \sigma^2_{v_{2j}}, \sigma^2_{v_{3j}}$ are the measurement error variances for attributes j of alternative 1, 2 and 3 respectively; $\sigma^2_{v_{1j}}$, $\sigma^2_{v_{2j}}$ and $\sigma^2_{v_{3j}}$ are assumed to be unique for state s and year t--hence

the subscript st for

$(\sigma^2_{v_{1j}}+\sigma^2_{v_{2j}})_{st}$; σ_j^2, σ_{jk} characterize the multivariate residual variance in

$(\varepsilon_{i1}, \varepsilon_{i2}, \varepsilon_{i3})$ which will most likely pick up unobserved factors across states and time.[1]

Generalizing from (11) we have for a given individual with k=1

$$P_1 = \int_{-\infty}^{w_{12}^*} \int_{-\infty}^{w_{13}^*} b(\tilde{\eta}_{21}, \tilde{\eta}_{31}, \rho_1) \, d\tilde{\eta}_{31} \, d\tilde{\eta}_{21} \tag{13}$$

where $w_{12}^* = (Z_1 - Z_2)' \, \bar{\beta}/\sqrt{\omega_{1,11}}$, $w_{13}^* = (Z_1 - Z_3)' \bar{\beta}/\sqrt{\omega_{1,22}}$, b is the standard bivariate normal with $\rho_1 = \omega_{1,12}/\sqrt{\omega_{1,11}\omega_{1,22}}$. Similar derivations hold for P_2 and P_3.

Using the mathematics developed in Appendix A the first order conditions are now apparent. For the log of (8) we have for element Θ_i of Θ

$$\frac{\partial L^*(\Theta)}{\partial \Theta_i} = \sum_{t_0}^{t_1} \sum_{s=1}^{50} \sum_{k=1}^{3} \frac{M_{skt}}{P_k} \frac{\partial P_k}{\partial \Theta_i} \tag{14}$$

where Θ_i can be an element of $\bar{\beta}$, σ_{ij} or σ_{ii} for i and j=1..3.

For any $\bar{\beta}_g$ we have for k=1

$$\begin{aligned} \frac{\partial P_1}{\partial \bar{\beta}_g} &= \phi(w_{12}^*) \, \Phi\left[\frac{w_{13}^* - \rho_1 w_{12}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] \frac{\partial w_{12}^*}{\partial \bar{\beta}_g} \\ &\quad + \phi(w_{13}^*) \, \Phi\left[\frac{w_{12}^* - \rho_1 w_{13}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] \frac{\partial w_{13}^*}{\partial \bar{\beta}_g} \\ &\quad + b(w_{12}^*, w_{13}^*, \rho_1) \frac{\partial \rho_1}{\partial \bar{\beta}_g} \; ; \end{aligned} \tag{15}$$

[1]This randomness could be incorporated through state and time specific variances; this is not done here.

while for any σ_{ij}

$$\frac{\partial P_1}{\partial \sigma_{ij}} = \phi(w_{12}^*)\ \Phi\left[\frac{w_{13}^* - \rho_1 w_{12}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] \frac{\partial w_{12}^*}{\partial \sigma_{ij}}$$

$$+ \phi(w_{13}^*)\ \Phi\left[\frac{w_{12}^* - \rho_1 w_{13}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] \frac{\partial w_{13}^*}{\partial \sigma_{ij}} \tag{16}$$

$$+ b(w_{12}^*, w_{13}^*, \rho_1) \frac{\partial \rho_1}{\partial \sigma_{ij}}$$

Equation (15) indicates how the choice model estimated from aggregate data differs from that estimated from individual data. For example, if individual data is used, $\partial\rho_1/\partial\bar{\beta}_g=0$; and for $\partial w_{12}^*/\partial\bar{\beta}_g$ and $\partial w_{13}^*/\partial\bar{\beta}_g$, $\partial\omega_{1,ij}/\partial\bar{\beta}_g=0$ for i and j=1-2. Hence, the first order condition (15) for $\bar{\beta}_g$ reduces to [see Hausman and Wise (19)],

$$\frac{\partial P_1}{\partial \bar{\beta}_g} = \phi(w_{12}^*)\ \Phi\left[\frac{w_{13}^* - \rho_1 w_{12}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] (Z_1 - Z_2)_g/\sqrt{\omega_{1,11}}$$

$$+ \phi(w_{13}^*)\ \Phi\left[\frac{w_{12}^* - \rho_1 w_{13}^*}{(1-\rho_1^2)^{\frac{1}{2}}}\right] (Z_1 - Z_3)_g/\sqrt{\omega_{1,22}} \tag{17}$$

where $(Z_1 - Z_3)_g = \partial(Z_1 - Z_3)\bar{\beta}/\partial\beta_g$.

Similar derivations are possible for the remaining first order conditions.

The method used to maximize the likelihood function is that developed by Berndt, Hall, Hall and Hausman.[1] The resulting maximum likelihood estimates will be consistent and asymptotically normal. The asymptotic

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[1] Berndt, E.K., B.H. Hall, R.E. Hall, and J.A. Hausman, "Estimation and inference in Nonlinear Structural Models," Annals of Economic and Social Measurement, 3(1974), 653-665. The computer program is adapted from that developed in [19].

covariance matrix of the maximum likelihood estimates is equal to the inverse of the covariance matrix of the gradient of the likelihood function evaluated at the maximum $\Theta^* = (\overline{\beta},\sigma_{ij})^*$.

II CONSUMER CHOICE AMONG ALTERNATIVE FUELS FOR RESIDENTIAL ENERGY-USING APPLIANCES

The analysis of residential appliance and fuel choice offers an example where individual choice models and aggregate data can be usefully combined. The behavior of residential energy demand argues for the application of choice models while the paucity data on individuals necessitates the use of plentiful aggregate data.

As developed elsewhere [12, 14, 15], any analysis of residential energy demand should deal with the fact that fuels and fuel using appliances/equipment are combined in varying ways to produce a particular residential service. This demand behavior can be decomposed into three decisions:

1) The decision to buy an energy-using appliance, capable of providing a particular comfort service(e.g., cooking, heating, lighting, air conditioning, etc.).
2) The decision concerning the technical characteristics of the equipment purchased, the fuel to be used by the equipment and whether the equipment embodies a new technology.
3) Given such equipment, the decision about the frequency and intensity of use.

These decisions span the short run (when the appliance stock is fixed) and the long run (when the size and characteristics of the appliance stock are variable). All three decisions are explicitly modeled in the most current residential energy demand analyses [12].

It is the second decision that is usually analyzed using choice models [3,4, 5, 6, 7, 8, 12, 14, 15, 16, 17, 20, 23, 24]. However, most of these efforts utilize aggregate data without correcting for it. As indicated in Section I, the use of such data in disaggregated behavioral models will generate inconsistent parameter estimates and inconsistent aggregate energy choice estimates [27]. To help overcome this and other difficulties,[1] this Section specifies a choice model for residential appliances which will be estimated using the techniques developed in Section I.

The specification of random utility used here is a version of models found in the literature [see 15, 18]. In that development, the size or capacity of an appliance for household i (X_i) will be determined by a vector of household characteristics a_i where a_i includes weather effects, climate effects and socioeconomic factors such as the size, tastes and age composition of the family, the size of the residence, etc. The capacity of the equipment can then be parameterized $X_i(a_i)$. Conditional on $X_i(a_i)$, the cost of capital and cost of fuels (P_j), rational consumers will identify within each fuel the type of appliance to be purchased (characterized by appliance price C_j, its efficiency EFF_j, and all other characteristics $\tilde{K}_j$) and the desired level of utilization [15]. Finally, consumers will make their fuel/appliance choice based on comparing utility among the possible choices j, j = gas, oil and electricity. Using an indirect utility formulation, we have the utility of alternative

[1] Other deficiencies include the following:

- The analyses have applied the choice models to <u>shares of the stock</u> of home heating equipment. The discrete choice models are really appropriate only for changes (new and replacement purchases) in the stock of capital [See 12, 15].
- The characteristics of the fuels and complementary fuel-burning equipment in the choice set have not been adequately specified. Most choice models in this area have based choice upon only fuel operating costs and have not included capital costs. [see 12, 15].
- Most formulations to date have utilized conditional logit, thereby suffering from the "independence of irrelevant alternatives". [See 12, 13].

All three of these deficiencies are corrected here.

j to household i given as (see [15])

$$\begin{aligned} U_{ij} &= V\left(Y_i,\ C_j,\ P_j/EFF_j,\ \tilde{K}_j;\ X_i(a_i)\right) \\ &= \overline{U}_{ij} + \varepsilon_{ij} = Z_{ij}\overline{\beta} + \varepsilon_{ij} \end{aligned} \tag{18}$$

where Y_i is household income. By specifying the form of V, the exact components of V and $X_i(a_i)$, and the stochastic nature of ε_{ij} and all measurement errors, we can substitute (18) into (7) and (8) to estimate the individual parameters of taste. This is done in Section III.

III EMPIRICAL RESULTS

My purpose here is to explore the estimation results generated by using aggregate data in individual choice models. To that end, I do not specify complicated versions of the indirect utility in equation (18). Rather, a straightforward model of utility is posited that includes operating costs and capital costs for the alternative fuel/appliance options. The end-use equipment for which the choice model is estimated include space heating equipment, clothes dryers, water heaters and kitchen ranges.

Two models are tested for space heating which incorporate different variables for the estimation of operating costs and the cost of capital services. More specifically, the variables[1] include

Space Heating	Capital Costs (α_1)	Operating Costs (α_2)
Model 1	Cost of space heating equipment (C_f)	Fuel price (P_f)
Model 2	Cost of space heating equipment/household income (C_f/y)	Fuel price * heating degree days $(P_f$*HDD$)$

Both models are estimated for choices across natural gas, oil and

[1]Fuel prices are the average price of natural gas per cubic foot (GAS FACTS), marginal price of electricity (developed by Data Resources, Inc. for the Electric Power Research Institute) and #2 fuel oil prices per gallon (AGA househeating survey). Heating degree days are developed by the Natural Oceanic and Atmospheric Administration. Household income is developed from Survey of Current Business data. Finally, detailed equipment costs have been developed for alternative space heating equipment. The data is summarized in M.I.T. Residential Energy Demand Group, "Aggregate Pooled Data Utilized and/or Developed for Residential Energy Demand," M.I.T. Energy Laboratory Working Paper No. MIT-EL 79-047WP, August 1979.

electricity. For clothes dryers, water heaters and kitchen ranges, the variables include[1]

Clothes Drying, Water Heating, and Cooking	Capital Costs (α_1)	Operating Costs (α_2)
	Equipment cost/ household income (C_f/y)	Fuel price/household income (P_f/y)

The models of choice for these three end-uses are estimated for natural gas and electricity.

For all of these end-uses, unit fuel price and equipment costs are appropriate measures of operating costs and capital service costs if weather/climate is fairly constant or irrelevant to use, if equipment lifetimes are similar across fuel-specific equipment and if household discount rates are fairly constant. It is assumed that use of water heaters, kitchen ranges and clothes dryers is invariant to weather; however, it is clear that space heating use and operating costs will be very weather sensitive. Since weather and climate differ across state and furthermore since empirical work suggests that discount rates vary inversely with income [18], space heating model #2 adds weather into operating costs $(P_f{*}HDD)$ and attempts to proxy varying discount rates by dividing by household income (C_f/y). The other end-uses exclude weather but divide both costs by income. As a result, we have from equation (18)

[1] The price and income variables are the same as those used for space heating. The appliance equipment costs are developed by Merchandising Week. Again for greater discussion see M.I.T. Residential Energy Demand Group, *op. cit.*

Space Heating Model 1: $U_{ij} = C_j\alpha_1 + P_j\alpha_2 + \varepsilon_{ij}$

Space Heating Model 2: $U_{ij} = (C_j/y)\ \alpha_1 + (P_j * HDD)\ \alpha_2 + \varepsilon_{ij}$

All Other End-uses: $U_{ij} = (C_j/y)\ \alpha_1 + (P_j/y)\ \alpha_2 + \varepsilon_{ij}$

Pooled annual state averages for P_j, HDD, C_j, and y are utilized.

The results from estimation of space heating models 1 and 2 are given in Tables 1A & 1B. The other end uses are compiled in Table 2. Logit results are given which are equivalent to independent probit results when there are no errors in variables (EIV). For comparison sake in examining the effects of aggregate data, all results utilize independent probit.[1]

For model 1 in Table 1A, measurement error has a substantial effect on the parameter estimates. If measurement error exists at 10% of the average data,[2] consistent parameter estimates (at extremely high levels of significance) are 30% - 70% larger than the inconsistent estimates generated by assuming no errors in variables. If the measurement error is 15% of the variable means, consistent parameter estimates are 7.27 and -11.24 compared with 3.96 and -3.60 generated if no measurement error is assumed. For measurement error above 15%, the likelihood function becomes ill conditioned and estimation becomes impossible. From these results, it is clear that

[1]The full covariance probit could have also been developed.

[2]That is, measurement error variances are 10% of the value of the composite variables appearing in utility.

ignoring measurement error attendent with aggregate data will generate severely inconsistent parameter estimates of household tastes (α_1 and α_2).

The space heating model in Table 1B is better specified and more robust to measurement error. With measurement error at 75% of the variable means,[1] highly significant convergent estimates obtain which differ significantly from those obtained by ignoring such measurement error. However, the inconsistency is only 20% for α_1 and 50% for α_2.

Table 2 presents results for clothes dryers, water heaters and kitchen ranges. These results are much more sensitive to measurement error. For clothes dryers, measurement error at the .10% level would imply parameter inconsistency at the level of 8% for α_1 and 9% for α_2 if uncorrected aggregate data were used. With measurement error variances at .50% of the means, the inconsistencies would be 41% and 44% respectively. For water heaters at .10% EIV, the inconsistency is 8% and 18% while at .50% EIV the likelihood function becomes ill conditioned so that estimation is impossible. Kitchen range choice estimation is somewhat more robust to error. For measurement error at the .10% level, the inconsistency in $\hat{\alpha}_1$ is 1% and in $\hat{\alpha}_2$ it is 4%. At the .50% EIV level, the inconsistency is 7% for $\hat{\alpha}_1$ and $\hat{\alpha}_2$. At levels of measurement error of 1.00% the inconsistency is 14%.

Table 2 also presents estimates of α_1 and α_2 for levels of measurement errors that differ across capital and operating costs--.20% for capital costs and .10% for fuel prices. For clothes dryers the resulting

[1] That is, 75% of C_f/y and P_f*HDD, appropriately scaled. See notes to Table 1.

TABLE 1: LOGIT AND PROBIT RESULTS FOR SPACE HEATING

	$\alpha_1(C_f)$	$\alpha_2(P_f)$	L
A) MODEL 1			
Logit	5.0954 (94.01)	-4.5665 (-57.61)	-46435
Independent Probit No EIV	3.9622 (967)	-3.6048 (-548)	-46629
Independent Probit 10% EIV	5.3741 (590)	-6.4704 (-441)	-46532
Independent Probit 15% EIV	7.2749 (332)	-11.2436 (-261)	-46479
Independent Probit EIV > 15%	Becomes ill-conditioned		

EIV = Errors in variables at x% of means

L = Value of likelihood function

T statistics for Ho: $\alpha_i = 0$ in parentheses

TABLE 1: (cont.)

B) MODEL 2	$\alpha_1(C_f/y)$	$\alpha_2(P_f*HDD)$	L
Logit	.54217 (98.8)	-.55782 (34.5)	-47210
Independent Probit No EIV	.4229 (919.5)	-.4513 (-323.4)	-47370
Independent Probit 10% EIV	.4332 (884)	-.4839 (-329)	-47349
Independent Probit 15% EIV	.4397 (869)	-.5039 (-332)	-47339
Independent Probit 25% EIV	.4528 (836)	-.5436 (-336)	-47322
Independent Probit 50% EIV	.4906 (748)	-.6849 (-342)	-47287
Independent Probit 75% EIV	.5366 (646)	-.9112 (-326)	-47257

EIV = Errors in variables at X% of means

L = Value of likelihood finction

T statistics for Ho: $\alpha_i = 0$ in parentheses

Notes

C_f is system costs in dollars, scaled by 1/100.

y is household income, undeflated

P_f is cents per therm. KWH and gallon

HDD is in degree days, scaled by 1/10000.

See M.I.T. Residential Energy Demand Group, "Aggregate Pooled Data Utilized and/or Developed for Residential Energy Demand," M.I.T. Energy Laboratory Working Paper No. MIT-EL 79-047WP, August, 1979.

TABLE 2: LOGIT AND PROBIT RESULTS FOR WATER HEATERS, KITCHEN RANGES, AND CLOTHES DRYERS

a) Clothes Dryers	$\alpha_1(C_f/y)$	$\alpha_2(P_f/y)$	L
Logit	-5.7468 (-1958.5)	-71.909 (-926.93)	-2131.6×10^4
Independent Probit No EIV	-3.434356 (-43.17)	-42.31929 (-18.38)	-2133.11×10^4
Independent Probit .10% EIV	-3.7089 (-38.6)	-46.6127 (-18.1)	-2130.6×10^4
Independent Probit .50% EIV	-5.768181 (-15.65)	-75.745163 (-11.03)	-2132.9×10^4
Independent Probit .75% EIV	-10.264 (-4.75)	-134.4774 (-4.23)	-2132.41×10^4
Independent Probit .90% EIV	-272.4153 (-.009)	-3526.2268 (-.009)	-2131.9×10^4
Independent Probit .20%/.10%	-3.835122 (-35.99)	-48.291519 (-17.44)	-2130×10^4

b) Water Heaters	$\alpha_1(C_f/y)$	$\alpha_2(P_f/y)$	L
Logit	-4.0667 (-713.46)	-139.05 (-3062.8)	-2784.7×10^4
Independent Probit No EIV	-2.480393 (-21.36)	-80.2258 (-77.20)	-2798.13×10^4
Independent Probit .10% EIV	-2.6932 (-19.75)	-97.9115 (-60.87)	-2778.72×10^4
Independent Probit .30% EIV	-5.1787 (-12.78)	-258.9160 (-12.82)	-2740.35×10^4

TABLE 2: (cont.)

b) (cont.)

Independent Probit .50% EIV	Likelihood function becomes ill-conditioned		
Independent Probit .20%/.10%	-2.7785 (-20.03)	-98.6644 (-61.39)	-2778.1×10^4
c) Kitchen Ranges	$\alpha_1(C_f/y)$	$\alpha_2(P_f/y)$	L
Logit	.26202 (573.93)	.41.678 (-1139.1)	-4111.5×10^4
Independent Probit No EIV	.162983 (16.44)	-25.9708 (-31.77)	-4111.6×10^4
Independent Probit .10% EIV	.165318 (16.33)	-26.3130 (-31.06)	-4111.56×10^4
Independent Probit .50% EIV	.176118 (15.87)	-27.9519 (-38.39)	-4111.3×10^4
Independent Probit 1.00% EIV	.189929 (15.03)	-30.09 (-24.92)	-4111.06×10^4
Independent Probit 2.50% EIV	.283482 (10.91)	-44.5183 (-13.75)	-4110.3×10^4
Independent Probit 5.00% EIV	12.0719 (.007)	-1894.74 (-.007)	-4109.9×10^4
Independent Probit .20%/.10%	.165333 (16.33)	-26.3152 (-31.06)	-4111.6×10^4

Notes

X% EIV is errors in variables at X% of the mean

L is the value of the likelihood function at maximum

t statistics for Ho: parameter = 0 in parentheses

C_f is appliance price for fuel f in dollars

TABLE 2: (cont.)

P_f is fuel price in cents per therm and KWH

Y is real household income

Prices are deflated by the consumer price index regionalized using the state cross-sectional index of Kent Anderson [1].

Number of purchases of each choice (i.e. M_{skt} in equation (8)) scaled by 10^{-4}.

C_f/y and P_f/y scaled by 100.0 to speed convergence

Data fully discussed in M.I.T. Residential Energy Demand Group, "Aggregate Pooled Data Utilized and/or developed for Residential Energy Demand," M.I.T. Energy Laboratory Working Paper No. MIT-EL, 79-047WP, August, 1979.

estimates of α_1 and α_2 are -3.835 and -48.292. For water heaters the extimates are -2.78 and -98.66. For kitchen ranges the estimates are .1653 and -26.32. The extent of inconsistency generated by using uncorrected aggregate data (no EIV) can be easily estimated for the Table.

Tables 1 and 2 indicate the standard result[1] that measurement error systematically biases the parameter estimates toward zero. It is clear from the results that the greater the measurement error, the greater is the bias toward zero.

Finally Table 3 presents actual results for water heater and kitchen range choice. Because Table 2 indicates that consistent estimates can be extremely sensitive to measurement error, it may be impossible to estimate individual taste parameters if the measurement error is too large. This fact should not be surprising. As a result, the contents of Table 3 are interesting. The sources for the estimates of measurement error and the technique for estimating the composite variable variances are indicated in the notes to Table 3. The measurement errors for fuel prices were the smallest, varying from 0.0% of the state mean (for single utility states such as Montana) to 71% for New York. Income measurement variances varied from 150% of the state mean for homogeneous states such as New Hampshire to 863% of the state mean for Alaska. The variances for appliances prices are quite large--a full 8800% of the mean for gas water heaters, for example.

In spite of these large variances, the variances of the composite

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[1] See, for example, E. Malinvaud, Statistical Methods of Econometrics, North Holland, 1970, pp. 144-147.

variables were sufficiently small to permit estimation of individual taste parameters for appliances that revealed the greatest sensitivity to measurement error. The results indicate in Table 3 that severe bias in α_2 for water heaters will occur if uncorrected aggregate data is used. Likewise, for kitchen ranges α_2 is more severely biased than α_1.

In addition to the observation that aggregate data can generate severely inconsistent parameter estimates, a second observation is warranted. Notice in all cases that the operating cost effect is negative. _Ceteris paribus_, we would also expect the capital cost effect to be negative. This is true for water heaters and clothes dryers; however, the capital cost effect for both space heating equipment and kitchen ranges is positive. It would appear that this positive capital cost effect reflects excluded equipment characteristics such as quality, brand effects and optional specialty features. The presence of such specialty features and fuel-specific preferences would exist more convincingly for space heating equipment and kitchen ranges. Clothes dryers and particularly water heaters offer a smaller array of specialty options; hence the choice is truly _ceteris paribus_ and the capital cost effect is negative. Until more extensive aggregate data on the quality and specialty options can be developed, it is difficult to improve upon the positive capital cost effect.

TABLE 3: ACTUAL RESULTS FOR WATER HEATERS AND KITCHEN RANGES

	$\alpha_1(C_f/y)$	$\alpha_2(P_f/y)$	L
Water Heaters	-2.91303 (-23.86)	-374.781 (-28.70)	-2703.09
Kitchen Ranges	0.17343 (15.15)	-28.11302 (-27.51)	-4111.63

NOTES:

1) Using a first-order Taylor approximation and assuming independence of the measurement errors,

$$V(C_f/y) \approx \frac{1}{y^2} V(C_f) + C_f^{\,2}/y^4 \; V(y)$$

and
$$V(P_f/y) \approx \frac{1}{y^2} V(P_f) + P_f^{\,2}/y^4 \; V(y)$$

2) $V(P_f)$ is estimated from state utility data. The estimate for each state was chosen from the year between 1965-1975 for which the greatest number of utilities reported. The appropriate block rate was chosen for an assumed average usage; for example, 200-25 KWH for electricity. The variance was assumed to hold hold for all sample years. For the data source, see Data Resources Inc., The Residential Demand for Energy: Estimates of Residential Stocks of Energy Using Capital, Report to Electric Power Research Institute, EPRI EA-235, January 1977, revised 1979. The raw data files are needed.

3) $V(y)$ is estimated by state for 1975 by the Bureau of the Census, Department of Commerce, Money Income and Poverty Status in 1975 of Families and Persons in the United States and by Region, Divisions and States, Series p. 60, No. 110-113, 1978. It is assumed to be constant across sample years.

4) $V(C_f)$ is estimated from a national sample of appliance distributors and assumed to hold for each state. For example, 125 gas water heaters and 80 electric water heaters were sampled.

APPENDIX A

For the likelihood function the probabilities are in the form

$$F = P_1 = \int_{-\infty}^{a_1(\eta)} \int_{-\infty}^{a_2(\eta)} b\left(X_1, X_2, \rho(\eta)\right) dX_1\, dX_2 \tag{A1}$$

where we arbitrarily work with P_1 and parameterize a_1, a_2 and ρ as functions of η. b is the standard bivariate normal density function. We require $\partial F/\partial\eta$, which by the fundamental theorem of calculus is

$$\frac{\partial F}{\partial \eta} = \left[\frac{\partial F}{\partial a_1}\frac{\partial a_1}{\partial \eta} + \frac{\partial F}{\partial a_2}\frac{\partial a_2}{\partial \eta} + \frac{\partial F}{\partial \rho}\frac{\partial \rho}{\partial \eta}\right]. \tag{A2}$$

We can calculate the components of (A2) as follows. Rewrite F in two equivalent formulations by reversing the order of integration:

$$F = \int_{-\infty}^{a_1(\eta)} \int_{-\infty}^{\frac{a_2(\eta)-\rho(\eta)X_1}{(1-\rho(\eta)^2)^{1/2}}} \phi(X_2|X_1)\,\phi(X_1)\, dX_2\, dX_1 \tag{A3}$$

and $$F = \int_{-\infty}^{a_2(\eta)} \int_{-\infty}^{\frac{a_1(\eta)-\rho(\eta)X_1}{(1-\rho(\eta)^2)^{1/2}}} \phi(X_1|X_2)\,\phi(X_2)\, dX_1\, dX_2 \tag{A4}$$

Then

$$\frac{\partial F}{\partial a_1} = \int_{-\infty}^{\frac{a_2(\eta)-\rho(\eta)\,a_1(\eta)}{(1-\rho(\eta)^2)^{1/2}}} \phi\left(X_2|a_1(\eta)\right)\,\phi\left(a_1(\eta)\right) dX_2$$

$$= \phi(a_1(\eta))\;\Phi\left[\frac{a_2(\eta)-\rho(\eta)a_1(\eta)}{(1-\rho(\eta)^2)^{1/2}}\right] \tag{A5}$$

$$\frac{\partial F}{\partial a_2} = \int_{-\infty}^{\frac{a_1(\eta)-\rho(\eta)a_2(\eta)}{(1-\rho(\eta)^2)^{\frac{1}{2}}}} \phi\left(X_1 \,|\, a_2(\eta)\right) \; \phi\left(a_2(\eta)\right) \; dX_1 \qquad \text{(A6)}$$

$$= \phi\left(a_2(\eta)\right) \; \Phi\left[\frac{a_1(\eta)-\rho(\eta)a_2(\eta)}{(1-\rho(\eta)^2)^{\frac{1}{2}}}\right]$$

Finally using the relation that the derivative of a multivariate normal distribution with respect to an element of the correlation matrix equals the second cross partial derivative with respect to the corresponding ordinates[1] we have

$$\frac{\partial F}{\partial \rho} = \frac{\partial^2 F}{\partial a_1 \partial a_2} = b(a_1,\, a_2, \rho) = b\left(a_1(\eta),\, a_2(\eta),\, \rho(\eta)\right) \qquad \text{(A7)}$$

Combining A5, A6, and A7 into (A2) we obtain

$$\frac{\partial F}{\partial \eta} = \left[\phi\left(a_1(\eta)\right) \; \Phi\left[\frac{a_2(\eta)-\rho(\eta)a_1(\eta)}{(1-\rho(\eta)^2)^{\frac{1}{2}}}\right] \frac{\partial a_1(\eta)}{\partial \eta}\right.$$

$$+ \phi\left(a_2(\eta)\right) \; \Phi\left[\frac{a_1(\eta)-\rho(\eta)a_2(\eta)}{(1-\rho(\eta)^2)^{\frac{1}{2}}}\right] \frac{\partial a_2(\eta)}{\partial \eta}$$

$$\left. + \, b\left(a_1(\eta),\, a_2(\eta),\, \rho(\eta)\right) \frac{\partial \rho(\eta)}{\partial \eta}\right]$$

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[1]See [19].

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