

Distance Information Transmission Using First Order Reflections

by

Douglas S. Brungart

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science

at the

Massachusetts Institute of Technology

September, 1994

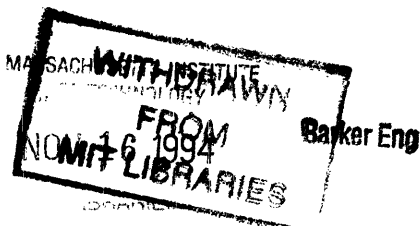
©1994 Douglas S. Brungart
All Rights Reserved

The author hereby grants to MIT permission to reproduce and to distribute
publically paper and electronic copies of this thesis in whole or in part.

Signature of Author:.....
Department of Electrical Engineering and Computer Science
August 26, 1994

Certified By:.....
Nathaniel I. Durlach, Thesis Supervisor
Senior Research Scientist, Research Laboratory of Electronics

Approved By:.....
Frederic R. Morgenthaler
Chairman, Committee on Graduate Students



DISTANCE INFORMATION TRANSMISSION
USING FIRST ORDER REFLECTIONS

by

DOUGLAS S. BRUNGART

Submitted to the Department of
Electrical Engineering and Computer Science
on August 26, 1994 in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

ABSTRACT

Although audio virtual reality systems have improved substantially in recent years, they still do not adequately address the problem of simulating distance for virtual sound sources. Systematic variations in intensity provide a powerful cue for simulating changes in the relative distance of a sound source, but they fail to give the listener any information about the absolute distance to the source unless there is a priori information about its intensity. First order reflections provide one possible way to code absolute distance information in a virtual audio display without any prior knowledge about the intensity of the source. Two parameters of these reflections, the delay τ between the primary and reflected signals and the ratio m of the intensity of the reflected signal to the intensity of the primary signal, can be manipulated to encode the absolute distance information. Five experiments were performed to evaluate the upper limit on the amount of information that variations in the parameters τ and m of a first order reflection can provide to a listener. The first two experiments examined the information transmission in each parameter when the other parameter was fixed. The second two experiments measured the information transmission in each parameter when the other parameter was randomly varied, and the last experiment measured the information transferred by both parameters simultaneously. The results show a maximum average information transfer of approximately 1.74 bits for both parameters, which would allow a listener to reliably place a sound in one of three distance categories. The data also show large variations in the performance of the different subjects which seem to be related to musical experience. Although the information transfer measured for the reflection filters used was not as high as expected, there is some indication that the results could be improved with modifications to the values of m used for the stimuli. Further research is needed to explore this possibility.

Thesis Supervisor: Nathaniel I. Durlach
Title: Senior Research Scientist

Acknowledgments

Although theses cannot list more than one author on the title page, this document and the research that went into it would not have been possible without the support of many friends and colleagues, new and old. Material support was provided in the form of funding by the Air Force Office of Scientific Research and the Office of Naval Research, in the form of equipment by AL/CFBA, and in the form of salary and tuition by the Air Force's Palace Knight program. Most of the credit for intellectual support goes to my advisor, Nat Durlach, who somehow managed to steer me clear of all major obstacles without ever taking control of the direction of the research. It has been a privilege and an honor to work with someone so widely respected as a scientist and as a person. Valuable insights were also provided by Lou Braida, Bill Rabinowitz, and Barbara Shinn-Cunningham. Ted Knox performed admirably in his dual roles of Palace Knight mentor and supervisor, and his support and understanding have been a great comfort to me. I cannot overemphasize the importance of Rich McKinley, Mark Ericson, Ron Dallman, Dave Ovenshire, and others too numerous to mention at Wright-Patterson who prepared me so well for my adventures in Boston and continued to assist me throughout my thesis. Finally, I must thank Gilkey, who first brought me to MIT and convinced me that I could succeed here.

Moral support plays such an important role in every endeavor that I cannot end these acknowledgments without giving some credit to all those people who made the past year at MIT fun as well as educational. Paul and Jay (the Sandal Boys) were always more than willing to provide a distraction from my work. This contrasts with Lou, who was able to make me feel guilty for not working. The Friday Afternoon Club always provided succor in my weakest moments, while the Head Graduate Student and his seventh floor subjects always made life in the office interesting. Jeanie and Aaron have been invaluable friends and they have made the transition to MIT infinitely easier. They also very generously donated their time to be subjects in my experiment. My parents deserve some credit for dutifully keeping me posted on all the developments back home in Beavercreek. And, last but not least, I must thank Lisa for all her long distance love and support, and for giving me a reason to finish up and go home.

Table of Contents

<i>1. Introduction</i>	6
<i>2. Background</i>	10
2.1 Audio Distance Cues	10
2.2 Rippled Noise	14
2.3 Information Transmission	18
2.4 The Decision Model	22
<i>3. Theoretical Development</i>	24
<i>4. Experimental Setup</i>	27
<i>5. Experiment Design</i>	30
5.1 Preliminary Experiments	30
5.1.1 Experiment 1: Identify τ with m fixed	31
5.1.2 Experiment 2: Identify m with τ fixed	32
5.2 Single Parameter Experiments	34
5.2.1 Experiment 3: Identify τ with m roved	37
5.2.2 Experiment 4: Identify m with τ roved	37
5.3 Supplemental Experiments 1 and 2	38
5.3.1 Identify τ , m fixed	38
5.3.2 Identify m , τ fixed	39
5.4 Two Parameter Identification Experiment	39
5.4.1 Stimulus	39
5.4.2 Training	40
5.4.3 Experiment	40
<i>7. Discussion</i>	48
<i>8. Concluding Remarks</i>	56
<i>9. Appendix A: Bias in Information Transfer Estimates</i>	58

10. *Appendix B: Data Processing in the Two Dimensional Experiment* _____61

11. *Appendix C: Comparison of Trial Selection With and Without Replacement* ____69

12. *Appendix D: Information Transfer and Correctness Correlation Within Trials*__70

13. *Appendix E: Response Biases*_____74

14. *Appendix F: Confusion Matrices*_____77

15. *References* _____114

1. Introduction

In recent years a great deal of work has been done to create realistic virtual audio displays. These virtual audio displays focus on adding directionality to single-channel sound sources by electronically processing the sound into two output channels (left and right ear) which can be listened to with stereo headphones. The signal is processed with head related transfer functions (HRTFs) that recreate the characteristics of the sounds that reach the left and right eardrums when listening to a sound source in an anechoic environment. Many of these virtual audio displays are also connected to some device that measures head position and allows the listener to interact with the synthesized sound source (i.e., the sound source seems to stay in the same position in the room as the listener moves his head). A full description of these virtual audio displays can be found in Wenzel (1991).

In general, these audio displays can manipulate only two parameters of the sound source, azimuth and elevation. Peculiarly, these audio displays are often called "three-dimensional", despite the fact that they are clearly only two-dimensional. Very little work has been done to make these devices truly three-dimensional by adding realistic distance cues to the directional cues.

There are many obvious uses for audio distance cues in a virtual environment. Distance coding could be used to provide more complete spatial information in navigational or weapon displays. It could also provide a method for systematically prioritizing information in a multiple channel communications system or warning system. Perhaps most importantly, it could greatly enhance the situational awareness and sense of immersion associated with virtual environment systems. There is no question that an effective technique for adding distance information to a virtual sound source would benefit the ongoing effort to create better and more realistic virtual environments.

An overview of the audio cues believed to be relevant to distance perception in the real world is located in the background section below. Unfortunately, these cues either do not provide very accurate distance information or they rely heavily on a priori

information about the source. All of the data available indicate that humans are simply not very good at determining the absolute distance of an unfamiliar sound source.

In a case such as this one where human performance in the real world is not particularly good, it may be possible to replace the real world information with a modified version that provides considerably better performance in the virtual world. Shinn-Cunningham (Shinn-Cunningham et al., 1994) has done considerable work in the area of super-localization, which is an attempt to improve localization performance by modifying the head related transfer functions which provide directional information in the real world. In her experiment, the localization filters were remapped to provide enhanced resolution directly in front of the listener at the cost of somewhat worse resolution to the sides of the listener. The results show that subjects were able to adapt to these modified cues after some exposure to the virtual environment as long as visual information consistent with the location of the sound sources was provided.

The Shinn-Cunningham study demonstrates that subjects are able to effectively use modified auditory spatial information after a period of adaptation, at least when the information is based on the actual cues present in the real world. An important aspect of the adaptation seems to be the correlation between the changes in the auditory characteristics of a sound source and the visual position of the source. It is likely that listeners in a virtual environment will learn to use modified auditory information to perceive the location of an object as long as it systematically varies with the visual location of the object during a period of adaptation, and appropriate proprioceptive/kinesthetic information relating to head movements is provided. It is also probable that the adaptation will progress more quickly if the auditory information is based on some cue that is correlated with spatial location in real world environments. There is some indication, for instance, that human adults are more likely to associate higher sound intensity with a closer sound source than infants are (Litovsky and Clifton, 1992), implying that this association may be learned rather than inborn. A study by Gardner (1968) shows that listeners tend to perceive a whispered voice as being closer than its actual position, and a shouted voice as being farther than its actual position. This also seems to be a learned behavior involving distance perception. Thus there is no reason to believe that distance perception in a virtual audio environment cannot be

improved by choosing an audio cue, varying it systematically with distance, and allowing the subject to adapt to the cue by interacting with sound sources in a virtual world.

Of the possible distance cues to use for distance coding, one logical choice is reflections. Reflections and reverberation have been shown to be an important element of distance perception, and they can be implemented without drastically changing the character of a sound. Furthermore, reflections in the real world provide absolute distance information without a priori information about the loudness of the source (although some familiarity with the room is required). And previous work examining the discriminability of white noise with a reflection shows a reasonable ability to discriminate changes in both the delay of the reflection and the strength of the reflection (Yost and Hill, 1978). Thus reflections seem to be a reasonable choice for providing systematic audio distance information.

The purpose of this thesis is the evaluation of first order reflections to determine their suitability for distance coding in a virtual audio system. A distance coding scheme using first order reflections might be an effective way for providing information about the absolute distance of a sound source without any a priori information about the intensity of that sound source. Furthermore, because reflections are present in almost every real-world listening environment and humans are accustomed to listening to sounds with reflections present, there is reason to believe such coding can be achieved without making familiar types of sounds unrecognizable. A distance coding scheme using reflections requires listeners to perform an identification task, where they must correctly choose the distance associated with a particular reflection from a number of possible distances. Thus, this research will focus on identification experiments involving first order reflections. This differs significantly from previous work involving reflection and broadband noise, often referred to as rippled noise (see background section), because the listener must be able to remember the reflection characteristics at each distance over time and not just compare two temporally proximate signals. In particular, the principles of information theory are used to quantitatively measure the maximum amount of information provided by changes in the parameters of a first order reflection. This maximum information transfer represents a channel capacity for reflection information and should establish an upper bound on the effectiveness of reflection-based distance

coding in virtual audio displays. No attempt is made to define an optimal coding scheme or deal with the problems of adaptation related to the implementation of distance coding.

2. Background

This thesis builds on prior work in three four areas: audio distance cues, rippled noise, information transmission in audio displays, and the decision model for psychoacoustics. A brief overview of the relevant literature in each of these fields is provided in this section.

2.1 *Audio Distance Cues*

The best overall summary of various audio distance cues is the review by Coleman (1963). He lists a number of possible sources of distance cues, including the well known correlation between the distance of a sound source and its apparent intensity. Coleman covers only distance cues relevant to an anechoic listening environment. In reverberant environments, another cue may be in effect: the ratio of the intensities of the primary and reflected sounds (Mershon & King, 1975).

These distance cues can be separated into exocentric and egocentric categories. Exocentric cues provide information only about the relative distances of two sounds. Egocentric cues provide information about the absolute distance from the sound source to the listener. Many of the most important distance cues, including intensity cues, are of the exocentric variety, and they provide no absolute distance information unless the listener is very familiar with the sound source a priori.

The intensity cue is a powerful exocentric cue, and it has a tendency to dominate distance perception. This cue is based on the inverse first power law, which states that the amplitude of a sound is inversely proportional to the distance from the source. This law can be expressed as (Coleman, 1963):

$$"(1/R) \text{ loss}" \text{ in dB} = 20 \log_{10}(R/R_0) \quad (2.1.1)$$

where R is the distance from the source to the listener and R_0 is the distance from a reference point to the listener. If this cue is truly dominant, than the just noticeable change in distance should be related to the minimum audible change in intensity, which is about 0.4 dB for broadband noise. This would correspond to approximately a 5% change in distance, according to the inverse first power law. This hypothesis was

examined by Strybel and Perrot (1984). Their findings were consistent with this hypothesis for distances greater than 3m, but they found that at 3m or less a change in distance much larger than 5% was necessary to provide accurate discrimination by the subjects. It is not clear what combination of distance cues the subjects were using to evaluate these near sound sources, but it is obvious that they are much less accurate than judgments based on intensity alone.

The dominance of intensity cues in adults was shown by Litovsky and Clifton. They compared the abilities of adults and six month old infants to determine whether a sound stimulus was located 15cm or 1m away (Litovsky & Clifton, 1992). They found that adults were far more likely to base their distance judgments on intensity than the infants. This result indicates that the use of the intensity cue is based at least in part on listening experience, and implies that it may be possible to learn to use an artificially created distance cue as naturally as the well-known intensity cue after a period of training.

Gardner performed a number of experiments involving the egocentric distance perception of sources directly in front of the listener. He found that distance judgments of human speech amplified through loudspeakers were based primarily on the amplitude of the speech presentations and not on the distance to the speaker (Gardner, 1968). In contrast, he found that the absolute distance judgments to actual human speakers were far more accurate and tended to be based on the type of speech used. When the live talker whispered, the subject tended to underestimate the distance. When the talker shouted, the distance was overestimated. This phenomenon is most likely a result of the expectations of the subjects that a talker would whisper only when close to the listener and would shout only when far away. Low level and conversational level speech generated relatively accurate distance judgments. These results indicate that his subjects used a priori information about the intensity of human speech to estimate the distance of the sound source based on the perceived attenuation of the speech. When the speech was presented electronically at an abnormally loud or soft level, the distance judgments were incorrect. When the speech originated from a human speaker, these judgments were far more accurate. Clearly intensity provides a dominant cue for the determination of relative distances, but it provides no absolute distance information unless the intensity of

the source is known beforehand. Some other cue must be used to allow egocentric distance perception.

Reflections, which occur in almost all realistic listening environments, offer one possible egocentric cue. If the only reflecting surface involved is a floor there will be a direct mapping between the distance of the source and the parameters of the reflection, including the delay of the reflection, the relative intensity of the reflected signal, and the angle of incidence of the reflection. In more complicated reverberant environments, the characteristics of the reflections should still vary systematically with distance, but there will be a large number of variables involved and they will vary in a very complex manner with the location of the source and the listener. Still, there will be a mapping of reflection characteristics to source distance that does not rely on source characteristics and should provide a means of evaluating absolute distance if the listener is familiar with the environment.

The effects of reflections on distance perception were studied by Mershon and King (1975). They placed subjects in an anechoic chamber and in a reverberant tunnel and asked them to listen to various sound sources. The distance estimates of the subjects who listened to the sounds in a reverberant environment were much larger than those of the subjects who listened in the anechoic chamber. These data are reinforced by later experiments by Mershon and Bowers (1979) and Butler, Levy and Neff (1980). The Mershon and Bowers study found a correlation between the actual and perceived distances of a sound source when the listeners were both blindfolded and unfamiliar with the reverberant environment. This implies that reflections provide some absolute distance information even when there is little or no a priori information about the detailed listening environment. The 1980 study showed that binaural recordings made in a reverberant environment appeared to be up to three times as far from the listener as those made in an anechoic environment. The importance of reflections in distance perception was further verified in a study by McMurtry and Mershon (1985). This study examined the effects of noise and of hearing protection on distance judgments. The distance judgments made when the reflection components were masked out by noise or hearing protection were considerably closer than those made with unmasked reflection

components. It is clear from these results that the reflection cue is a very important component of distance perception.

At least two studies have used virtual audio displays to examine the effects of reflections on distance perception. D'Angelo and Ericson (1993) used several 3-D Audio Display generators to compare distance %JND with no reflections, a single floor reflection, left and right wall reflections, and floor and wall reflections. In each case, the intensity of the signal was also adjusted for distance. They found %JND with no reflections was 7%, and %JND with reflections was 6%. Thus, reflections provided a very modest improvement in performance.

Brungart (1993) also performed a study examining the effects of reflections on distance perception. He had untrained subjects identify the absolute distance of sound sources (distance to the source in feet) with intensity distance cues and with and without a floor reflection under three conditions- listening directly to loudspeakers, listening to binaural recordings of loudspeakers, and listening to sound synthesized by a 3-D Audio Display generator. In the direct loudspeaker presentation, he found a modest increase in the perceived distance of sources when the floor reflection was added. In the other two conditions, the addition of the reflection produced very minimal changes in perceived distance.

Each of these studies combined overall intensity cues with reflection cues, and clearly in such cases the overall intensity cues dominate. None of these studies, however, have attempted to determine the amount of information provided by reflections when no a priori information about the intensity of the transmitted signal is available. This situation is frequently encountered in real word situations, and merits further investigation. This thesis examines the amount of information transmitted by reflections in order to determine their viability as an absolute distance cue in virtual audio systems.

2.2 Rippled Noise

When a sinusoid is delayed by one half period and added back to itself, the delayed signal will be exactly out of phase with the original signal and the sum of the two signals will be zero. Similarly, if the sinusoid is delayed by a full period and added to itself, the two signals will be exactly in phase and the resulting signal will be a sinusoid with the same frequency and twice the amplitude of the original signal. These are the two extreme frequency responses of a delay and add filter: if the delayed time is anything between zero and one half or between one half and one full period, the amplitude of the resulting sinusoid will lie somewhere between zero and twice the amplitude of the original signal.

If the delay time is greater than the period of the sinusoid then we find that a delay of any number of full periods result in a doubling of amplitude, and a delay of any number of full periods plus one half period results in zero amplitude. Thus a delay of 1ms would double sinusoids of 1000Hz, 2000Hz, 3000Hz, 4000Hz, etc., and would zero sinusoids of 500Hz, 1500Hz, 2500Hz, 3500Hz, etc.

A broad band noise signal passed through such a filter with delay τ will have alternating, linearly spaced peaks and notches in its power spectrum starting with a peak at 0Hz, followed by a notch at $1/2\tau$, followed by a peak at $1/\tau$, and extending infinitely with peaks at n/τ for all n and notches at $2n+1/\tau$ for all n . When there is no attenuation in the delayed signal, the power spectrum can be described as

$$|Y(\omega)|^2 = 2 + 2 \cos(\omega\tau) \quad (2.2.1)$$

where $Y(\omega)$ is the frequency spectrum of the filtered noise, ω is the radian frequency, and τ is the delay time of the filter. If the delayed signal is also attenuated, then a more complex equation will describe the power spectrum, but the alternating peaks and notches will still occur in the same places.

Broadband noise that has been processed by a delay-and-add filter is often referred to as ripple noise because of the ripples of the peaks and notches in the frequency spectrum. Human listeners tend to associate rippled noise stimuli with pitches. A number of experimenters (Bilsen, 1966; Yost, Hill, and Perez-Falcon, 1978) have

shown that subjects asked to adjust the frequency of a periodic signal (square wave or pulse train) until the pitch matches the pitch of the rippled noise will match to a frequency of $1/\tau$ Hz. These pitches produced by the rippled noise are frequently called repetition pitches.

Several studies by Yost and Hill have explored the ability of listeners to discriminate between two bursts of rippled noise with slightly different characteristics. In one experiment (Yost, Hill, and Perez-Falcon, 1978), they asked observers to listen to two 500 millisecond bursts of rippled noise created by passing white noise and random-interval pulse trains through simple delay and add filters. One of the filters had delay τ ms, and the other had a slightly greater delay $\tau+\Delta\tau$ ms. Neither filter attenuated the delayed signal. They used a same-different forced-choice discrimination procedure to determine the change in the delay of the filter $\Delta\tau$ necessary to distinguish the test filter from the original filter 75% of the time. They determined the Weber fraction for pitch discrimination, defined as

$$\frac{\Delta(1/\tau)}{1/\tau} = \frac{\Delta\tau}{\tau+\Delta\tau} \quad (2.2.2)$$

which is the ratio of the just noticeable change in repetition pitch to the repetition pitch, to be 3% for values of τ between 1.5ms and 5ms, and 5% for a τ of 1ms. This is about ten times as great as the Weber ratio for pitch discrimination in square waves, which is approximately 0.3% for frequencies above 400Hz. In the same paper, Yost predicted that repetition pitch is determined by a dominant frequency region located approximately at $4/\tau$ Hz.

A later study by Yost and Hill (1978) tested discriminability of two other variations in rippled noise. The first was the change in the attenuation of the delayed signal necessary to correctly discriminate between two rippled noise stimuli. In this experiment, the subject listened to two signals, one with the delayed signal attenuated by A dB and one with the delayed signal attenuated by slightly greater ($A+\Delta A$ dB) attenuation. The discrimination threshold was defined as the amount of additional attenuation ΔA required for the subject to correctly discriminate 70% of the trials under the same-different forced-choice paradigm. This was found as a function of the

attenuation A and the results are shown in Figure 1. The threshold values increase significantly as the baseline attenuation A is increased, and the thresholds are much higher for the smallest value of τ , 0.66 ms, than for the other two delay values tested.

Figure 1: Discrimination of Attenuation in Delayed Signal

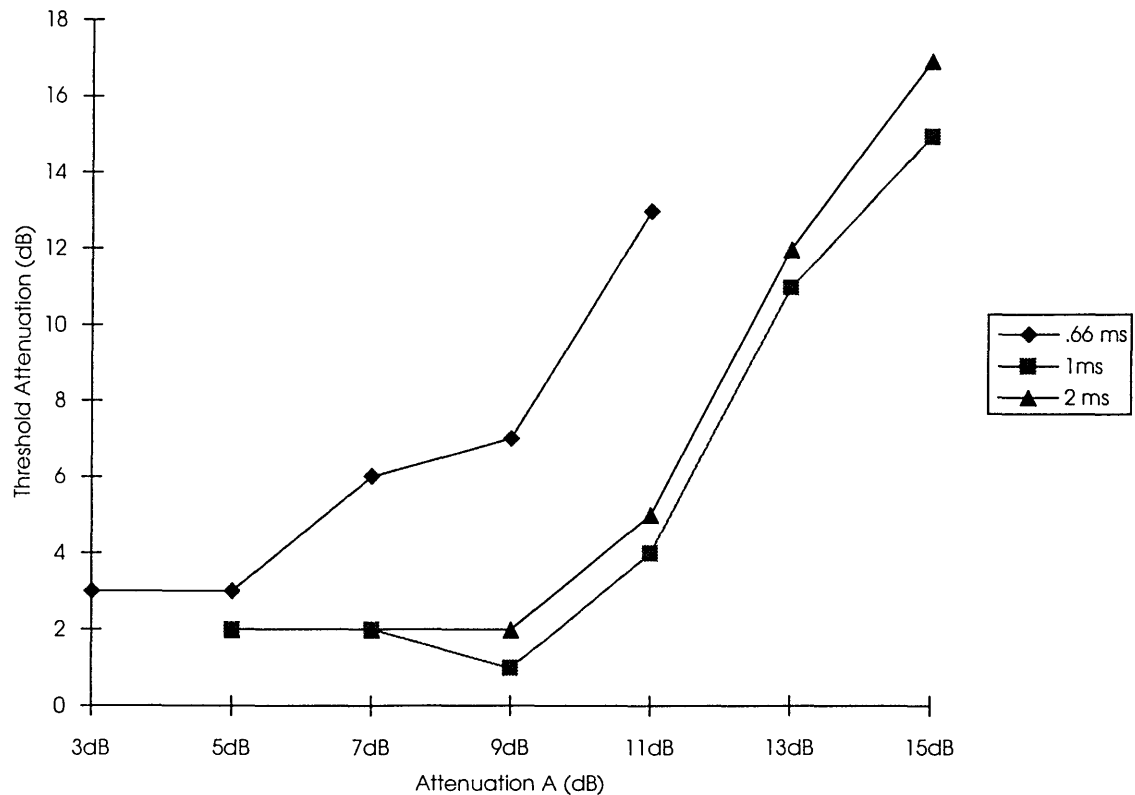


Figure 1: This chart (from Yost & Hill, 1977) plots the amount of additional attenuation ΔA (vertical axis) in the reflected signal required to discriminate a stimulus with 70% accuracy from a signal with attenuation A (horizontal axis) in the reflected signal. The results are shown for the three values of τ shown in the legend. A signal with τ of 1 ms and 7 dB of attenuation in the delayed signal, for instance, has a threshold attenuation of approximately 2 dB. This means that it can be discriminated with 70% accuracy from a signal with 9 dB or more of attenuation in the delayed signal, but not one with 8 dB of attenuation in the delayed signal.

Yost and Hill also measured the pitch strength of rippled noise under various conditions. The pitch strength of a rippled noise sample is the maximum amount of attenuation A in the delayed signal that still allows a listener to discriminate (70% correct) between a signal with delay τ and signal with delay 1.1τ . His results show that pitch is strongest (A 10% change in τ could still be discriminated with 23dB of attenuation in the delayed signal) around $\tau = 2$ ms and that it monotonically decreases in strength as τ increases above or decreases below 2ms. The results also show that pitch

strength approaches zero (discrimination of a 10% change in τ was not possible even with no attenuation in the delayed signal) for $\tau < 5\text{ms}$ and $\tau > 20\text{ms}$. Another interesting feature of Yost and Hill's data is the wide variability in performance among the eight subjects used in the study who were not systematically trained. The thresholds for those subjects ranged over approximately 10dB. Interestingly, the best performers in the study were two subjects with extensive musical training.

2.3 Information Transmission

The measurement of information transmission is a convenient way to quantitatively evaluate the amount of information provided by a message, signal, or other communication that is based on the principles of Information Theory. Information Theory was first developed by Shannon in 1949, and has since been expanded into an important branch of communication theory.

The quantitative theory of information is based on an assumed probabilistic distribution of possible outcomes. Of these outcomes, the amount of information provided by each is determined by the unexpectedness of the outcome. If we know for certain that the sun will rise every day, and someone tells us that the sun will rise tomorrow, that does not provide any information at all- there was no uncertainty of the outcome before the communication. If someone tells us there will be a total eclipse tomorrow, that message will provide much more information, since we do not in general expect an eclipse to occur.

Information theory places a quantitative value on information, defined as the negative of the log of the probability of a given outcome. In general, the logarithm is base 2 and the resulting value is measured in bits of information. Another important measure is the average information of a distribution of outcomes. The average information of a distribution, or entropy, is defined as:

$$H = -\sum_x p(x) \log p(x) \quad (2.3.1)$$

where each value of x is a possible outcome and $p(x)$ is the probability of outcome x . A fair coin, for instance, has two possible outcomes, each with probability 0.5, so the entropy of this distribution is $-.5\log_2 .5 + -.5\log_2 .5 = 1$ bit of entropy. It turns out that the average information is greatest for uniform distributions. If the coin were weighted, and landed tails up with probability .6, the entropy would be $-.6\log_2 .6 + -.4\log_2 .4$, or .97 bits.

One useful property of entropy is that the number of bits of entropy is equivalent to the average number of yes and no questions (or binary digits) necessary to determine

the outcome. It may be necessary to pool a number of outcomes together to approach this limit in practice, but it is interesting that such a simple calculation can quickly determine a limit on the most efficient possible coding system for a distribution of outcomes.

One interesting use for information theory is measuring the amount of information transferred by a signal or communication. Essentially, the information transfer is the difference between the uncertainty of the outcome before the signal is received and the uncertainty of the outcome after the signal is received. If the outcome is known for certain when the signal is received, the a posteriori uncertainty is zero, so the information transfer is equal to the entropy of the input. For instance, if you look at a coin after you flip it, you are sure of the outcome, so the entire entropy of the trial (1 bit) is transferred as information. In general, complete transfer does not occur, and it is necessary to find the entropy of the outcome X given the communication received Y , and subtract that from the entropy of the input. Therefore information transfer T is :

$$T(X;Y) = -\sum_x p(x)\log p(x) - \sum_x p(x|y)\log p(x|y) \quad (2.3.2)$$

Where X is the input distribution and $p(x|y)$ is the probability that input x occurred when output y is known.

The experiments for this thesis were designed to measure information transfer in an identification experiment. This is done by setting up a confusion matrix, with the N actual stimuli presented along the i axis and the N possible responses along the j axis. In this case the information transfer T can be measured directly:

$$T = \sum_{i,j} p_{ij} \log \frac{p_{ij}}{p_i p_j} \quad i=1\dots N; j=1\dots N; \quad (2.3.3)$$

Where p_i is the marginal probability of input i , p_j is the marginal probability of output j , and p_{ij} is the probability of the joint event ij . A comprehensive analysis of the information transfer in identification experiments can be found in Garner and Hake

(1952). Appendix A provides an analysis of the bias of the maximum likelihood estimator of information transmission.

A number of experiments have been performed to measure the amount of information transmitted in stimuli of various types. Pollack (1952), for instance, measured the amount of information transferred when a listener was asked to identify the frequency of a tone with a randomized amplitude. The frequencies of the tones were equally spaced on a logarithmic scale from 100 Hz to 8000 Hz. Pollack found that the information transfer increased rapidly as the number of tones increased from two to four. The information transfer for more than four tones, however, leveled off at approximately 2.3 bits. This implies that a listener cannot reliably identify more than approximately five different tones, and that the use of more than five tones as stimuli does not significantly increase the amount of information provided. Another Pollack study (1953) found that extending the range of frequencies did not add much information, but that the presentation of a reference tone before each trial could moderately increase information transfer.

The apparent limit on the number of different stimuli with variations in a single parameter that can be reliably identified is not limited to the frequency of tones. Miller (1956) lists a large number of different types of stimuli that exhibit the same property. Independent of the range of stimuli or number of stimuli used, the maximum information transfer was found to be 2.3 bits for the loudness of a tone, 1.9 bits for the saltiness of a solution, 3.25 for the position of a pointer in a linear interval, and 2.2 bits for the size of a square. A number of other types of unidimensional identification experiments are listed, but all have a maximum information transfer between 1.6 bits and 3.9 bits. Miller equates this maximum information transfer with the channel capacity for a human observing unidimensional changes in a stimulus. He found the mean channel capacity for a one dimensional identification experiment to be 2.6 bits, with a standard deviation of 0.6 bits. This is equivalent to reliable identification of approximately 6.5 different stimuli. Miller refers to the tendency for a wide variety of stimuli to have a maximum information transfer of about 2.6 bits as the “seven plus or minus two” effect.

The 1953 study by Pollack also showed that information transfer could be substantially increased by adding another dimension to the identification experiment. In

this case he asked the listeners to identify the sound level and the frequency of the tone, and he was able to increase the information transfer to 2.9 bits from 1.8 bits for frequency alone and 1.7 bits for sound level alone. Pollack and Ficks (1953) found that the median performers in their subject pool increased from 2.1-2.3 bits of information transfer in unidimensional experiments to 5.3-7.2 bits for six or eight dimensional experiments. His findings suggest that additional information is transferred when dimensions are added to the stimulus, but that the total information transfer is less than the sum of the unidimensional information transfers.

2.4 The Decision Model

The preliminary theory of intensity perception developed by Durlach and Braida (1969) uses a model based on internal noise that can be adapted to many different types of identification experiments. For this model, N different stimuli are used, each with the parameter under investigation varied so the value of that parameter in S_1 is less than that of S_2 and the stimuli are ranked in this order up to the stimulus with the largest value of the parameter, labeled S_N . The subject is required to identify each stimulus with one of N different numerical responses, labeled R_1 through R_N . The model assumes that there is a unidimensional continuum X (representing the decision axis), and that each stimulus presentation generates a particular value of X . Furthermore, it is assumed that the subject uses $N+1$ "criteria" (labeled C_i where $-\infty = C_0 < C_1 < \dots < C_{N-1} < C_N = \infty$) to identify each stimulus, so that he gives response R_m if and only if $C_{m-1} < X \leq C_m$. The conditional probability distribution of X given stimulus S_i ($p(X|S_i)$) is assumed to be gaussian with mean $\mu(S_i)$ and a standard deviation σ that is independent of S .

Thus each different stimulus will generate a value of X with a normal probability distribution, and the expected value of X is determined by the stimulus but the variance of the distribution is independent of the stimulus. The values of the criteria may be independent of the location of these expected values, but the minimum error probability is achieved if each criterion is placed halfway between the expected values of X associated with two adjacent stimuli (i.e. $C_i = (\mu(S_i) + \mu(S_{i+1}))/2$). The spacing between the expected values of X generated by two stimuli is normalized by dividing by σ to allow its interpretation using the unit normal gaussian distribution. The resulting value, d' , is called the sensitivity index for the two stimuli, and is defined for stimuli S_i and S_j as

$$d'(S_i, S_j) = (\mu(S_i) - \mu(S_j))/\sigma. \quad (2.4.1)$$

The sensitivity determines how well the subjects are able to distinguish between the two stimuli. The sensitivities are additive ($d'(S_i, S_k) = d'(S_i, S_j) + d'(S_j, S_k)$) and are independent of the criteria.

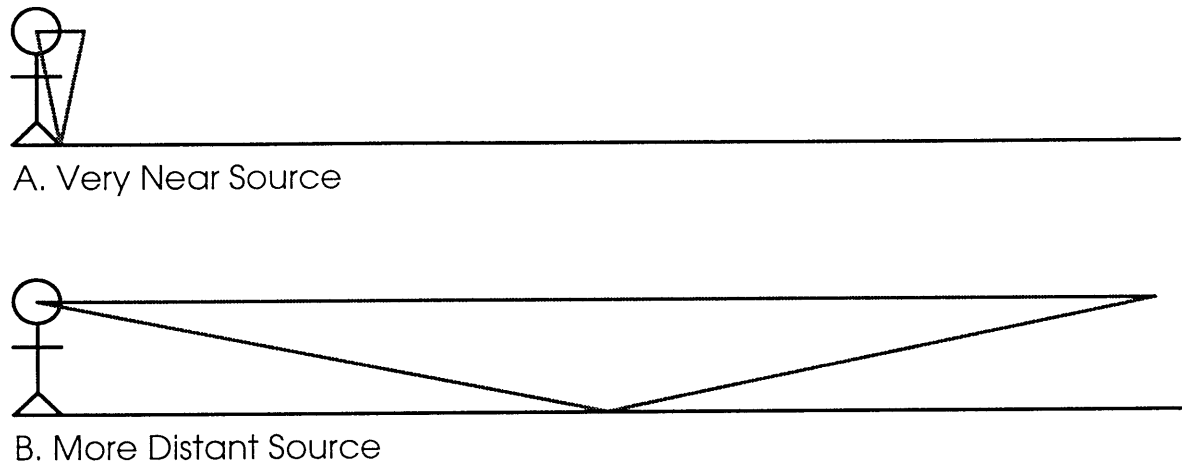
Another interesting property of d' is the sensitivity edge effect (Braidá and Durlach, 1972). This is the tendency for resolution to increase (d' is larger) at the edges of the range of stimuli used in the experiment. This is believed to be a result of the use of the extremes in the stimulus range as "perceptual anchors".

The decision model gives us another way to look at the data in the confusion matrices of an identification experiment. It has the advantage of examining the performance of the subjects for each stimulus presented. In contrast, the information transfer measure gives us only a single quantitative measure of performance for the entire matrix. One drawback is that the model was designed for intensity experiments and may not be completely applicable to experiments involving reflection delay and reflection strength (although it has been successfully applied to a number of dimensions other than intensity, including sound source azimuth). It may also be difficult to generate an accurate estimate of the parameters of the models with as few trials as are necessary for estimating information transfer. Nevertheless, these models can give some insight into the perceptual resolution of differences in first order reflections.

3. Theoretical Development

This research addresses the feasibility of using reflection based algorithms to provide distance coding in a virtual audio display. The study focuses on the simplest possible reverberant environment, a single floor reflection. Figure one shows typical sound paths in two such single-reflection environments. Two things are apparent in this illustration that are generally true for floor reflections on a flat surface. First, the ratio of distance traveled by the primary signal to distance traveled by the reflected signal approaches one as the distance goes to infinity. Since sound intensity is inversely proportional to distance traveled, this implies that the ratio of intensities of the primary and reflected signals approaches unity as the distance approaches infinity. Second, the intensity of the reflected signal is always less than that of the primary signal, and the ratio of the intensity of the reflection to the intensity of the primary signal increases with source distance.

Figure 1: Single floor reflections, near and far sources



All systems of this nature can be characterized by a single echo or comb filter equation:

$$S_{A,m,\tau} = A(1 + me^{-j\omega\tau}). \quad (3.1)$$

In this equation, A is the overall amplitude of the signal, m is the ratio of the amplitudes of the reflected and primary signals, τ is the time delay between the primary and reflected signals, and ω is the angular frequency in radians per second. If the sound source and the listener are assumed to be the same height h off the ground, and the distance D between them is much larger than h , then it is possible to approximate τ as a function of D and h , i.e.

$$\tau = \frac{2}{V} \sqrt{h^2 + \frac{D^2}{4}} - \frac{D}{V}, \quad (3.2)$$

where V is the velocity of sound. When $D \gg h$, as is usually the case, this can be reduced further to

$$\tau = \frac{2h^2}{VD}. \quad (3.3)$$

In general, the amplitude ratio m will tend to increase with increased distance, but if we fix m at one for simplicity, we get a filter with the following frequency response:

$$|S_{A,1,D}(\omega)|^2 = 4A^2 \cos^2\left(\frac{\omega h^2}{VD}\right) \quad (3.4)$$

$$\text{phase}[S_{A,1,D}(\omega)] = \frac{\omega h^2}{VD}. \quad (3.5)$$

It is likely that echo coding could provide an easily implemented way to generate distance information in a virtual audio display. Furthermore, reflections are so common in real-world listening environments that such cues might be quite natural sounding and

probably would not interfere too greatly with the ability to recognize familiar sounds. The question that has not been addressed in previous studies is whether or not such coding will provide a genuine benefit to the listener. How well can humans identify different reflection filters? This thesis will attempt to answer that question.

4. Experimental Setup

This research is based on a set of psychoacoustic experiments to determine the amount of information provided by changes in the different parameters of the simple comb filter described in the previous section. The experiments involve the manipulation of three parameters: the overall intensity A , the relative intensity of the reflection m , and the time delay of the filter τ . The overall intensity A was randomized in order to prevent the listener from determining the strength of the reflection from the overall amplitude of the signal.

A total of five experiments were performed: two preliminary experiments to identify τ with m fixed and m with τ fixed; two experiments to identify τ with m roved (i.e. randomly varied) and m with τ roved; and one experiment to identify τ and m simultaneously. The preliminary experiments were used to help train the subjects and give some insight into the general range of expected results. The other experiments provided more useful data about the amount of information transmitted in first order reflections. The exact particulars of each of the experiments and the results of those experiments are described in the sections that follow. This section is devoted to describing the hardware and software setup used to generate the stimuli used in the experiment and collect the responses from the subject.

A Gateway 2000 4SX-33V computer controlled all of the experiments. This PC was equipped with a Digital Audio Laboratories CardD DA/AD board, which was used to generate the sound stimuli. The CardD has two 16bit digital to analog converters that operate at sampling rates from 32KHz to 48KHz. All of these experiments used the 48KHz sample rate.

From the D/A board, the signal was sent to an Auditory Localization Cue Synthesizer (ALCS) for reflection processing. The ALCS is a special purpose digital signal processor that was originally designed to generate virtual audio environments by processing sound with head related transfer functions that were updated by the subjects head motions (McKinley & Ericson, 1988). The synthesizer has two audio input channels and a stereo headphone output channel. The input signals are sampled at a

40KHz rate and passed to the digital signal processing (DSP) board, which consists of 4 TMS-320C25 DSP microprocessors. Two of the processors are dedicated to the left ear output, and two are dedicated to the right ear. For each ear, the input signals are processed in two stages. The first stage adds directional information by convolving the signal with a finite impulse response filter representing the head related transfer function of a particular direction relative to the listener. The second stage adds reflection information. The signal is then converted back to analog at a 40KHz sample rate, amplified, and sent to a standard 0.25 inch stereo headphone jack.

The ALCS communicates with the PC through a standard RS-232 communications port. This port is monitored by a fifth TMS-320C25 that controls all I/O functions and synchronizes the operations of the four signal processing microprocessors.

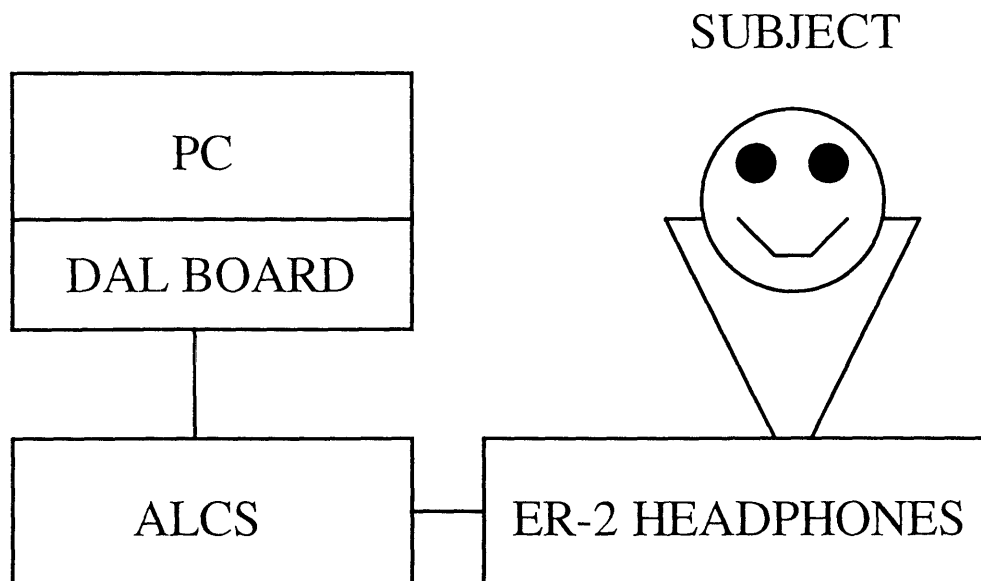
For these experiments, the first stage was disabled by replacing the normal head related transfer function FIR with an impulse, causing a passthrough in that stage. The second stage was used to generate reflections of varying delays and amplitudes and to control the overall attenuation of the signal. For each trial, three parameters were sent to the ALCS by the PC. The first was an overall amplitude scaling factor from 0-31. The input signal was multiplied by this factor and then divided by 32 to provide a range of overall amplitude scaling factor from 0 to 0.96875 in increments of 0.03125. A similar scaling factor from 0-31 was used to adjust the amplitude of the reflection, which was extracted from a delay line in the processor with a delay in samples (25 μ s resolution) requested by the PC.

The subjects were trained initially with AKG K240DF headphones. The final training and data collection were done with Etymotic Research ER-2 headphones. These headphones provide a nearly flat frequency response at the eardrum of the subject from 100Hz to 10KHz (as measured by the manufacturer with a Zwislocki coupler). They use foam eartips that are inserted into the ear canal, so they also provide between 32dB and 42dB of attenuation of environmental noise.

A program written in Borland C++ was used to run the experiment. It allows easy manipulation of parameters through a window interface and allows a well-trained

subject to perform the experiment with minimal supervision. When running the experiment, the subject is presented with a stimulus and asked to identify that stimulus through a numerical response on the keyboard. A training option is also provided that allows the subject to choose which stimulus he wants to hear. During the experiment, a log is kept of the important parameters and subject response for each trial presented, and a separate log is kept of the confusion matrices produced in each run. The control program has an option that allows the examination of the overall performance and information transfer of a given subject. Figure 2 shows the configuration of the experimental setup.

Figure 2: Experimental Setup



A total of four subjects participated in the study. Three were graduate students and one was an undergraduate student. One of four subjects was female. All four reported normal hearing. Three of them had recent pure tone audiograms demonstrating normal hearing, and the fourth was quickly screened for threshold problems at 100Hz, 250Hz, 1KHz, 2KHz, and 4KHz, and was normal in each frequency range. Only one of the four subjects had any prior experience in localization experiments.

5. Experiment Design

Data were collected from a total of five regular experiments, plus two supplementary experiments. The design of each experiment is described in this section. The actual results of each experiment are shown and discussed in the Results chapter. The confusion matrices for each experiment are shown in Appendix F.

5.1 Preliminary Experiments

The first two experiments were designed to measure the information present in either the delay τ with the reflection strength m fixed or in the reflection strength m with the delay τ fixed.

Stimulus

The stimulus used in these experiments was a 521 millisecond burst of white gaussian noise. The waveform was created using ISPUD, a signal processing package developed at MIT, and stored on the hard drive of the control PC. The effective bandwidth of the noise was limited by the low-pass antialiasing filters of the ALCS system, which have a cutoff frequency of 10KHz. The exact same waveform was used in every trial, so for these experiments the stimulus was effectively frozen.

Overall Amplitude

The overall amplitude of the stimulus was controlled by the ALCS. The control computer generated a random number from 8 to 31 and this number was divided by 32 to determine the overall attenuation of the signal before it was passed through the delay-and-add filter. Thus the voltage level of the output was effectively multiplied by a scaling factor ranging from 0.25 (12 dB of attenuation) to 0.96875 (0.28 dB of attenuation). Under this linearly roving paradigm, the decibel attenuation of the stimulus tends to be smaller on average than it would be if the attenuation were roved on a decibel scale. Note that the first two steps on the scale are separated by only 0.28 dB, but the last two steps are separated by 1 dB. The baseline amplitude was adjusted with the volume control on the ALCS to place the loudest stimuli at a loud but comfortable listening level.

5.1.1 Experiment 1: Identify τ with m fixed

Identified Parameter

In the first experiment the subjects listened to trials with the reflection strength m fixed and they were asked to identify the associated delay τ . Ten different values of τ were used, ranging from 0 ms to 9 ms in 1 ms intervals. The delay values associated with each stimulus are shown below in Table 1, along with the repetition pitch ($1/\tau$), associated with rippled noise with that delay value (See background section).

Table 1: Experiment 1 Stimuli

Stimulus Number	Delay τ	Repetition Pitch
0	0 ms	-
1	1 ms	1000 Hz
2	2 ms	500 Hz
3	3 ms	333 Hz
4	4 ms	250 Hz
5	5 ms	200 Hz
6	6 ms	167 Hz
7	7 ms	143 Hz
8	8 ms	125 Hz
9	9 ms	111 Hz

Training

The subjects were initially trained with a special option in the software that allowed the subjects to choose one of the ten response numbers in Table 1. The signal was then presented with the selected delay and the fixed value of m , but with the amplitude varied according to the linear randomization described above.

When the subjects had trained until they felt they could comfortably identify the ten delay filters, they were asked to perform a number of short training blocks, each containing 200 trials (20 for each possible delay value). The information transfer of each

of these blocks was computed to give a rough estimate of the subject's proficiency, and when the subject performance stopped improving and stabilized within 0.2 bits for several blocks in a row, the data collection began.

Experiment

The actual experiment consisted of 5 blocks of trials per subject. Each block consisted of 200 trials, with 20 for each of the 10 possible delay values. The values of τ for the trials in each block were chosen randomly without replacement. Thus there were a total of 1000 trials in the experiment for each subject, with 100 for each of the ten values of τ , or a total of 10 for each box in the 10 by 10 confusion matrix.

The experiment was performed for two fixed values of m . In the first condition, reflection strength m was fixed at 0.97 (0.28 dB attenuation from primary signal). In the second condition, the reflection strength m was fixed at 0.50 (6.0 dB attenuation). In both cases the subject was given the correct stimulus number after each trial, and after each block of trials they were shown their confusion matrix and the associated information transfer.

5.1.2 Experiment 2: Identify m with τ fixed

Identified Parameter

In the second experiment the subjects listened to trials with the reflection delay τ fixed and were asked to identify the reflection strength m . Eight different values of m were used, ranging from 0% reflection strength to 87.5% reflection strength (ratio of reflection amplitude to primary amplitude). The reflection strength m relative to the primary signal is shown as an amplitude ratio in percent and as a power ratio in decibels for each of the eight stimuli in Table 2.

Table 2: Experiment 2 Stimuli

Stimulus Number	Amplitude Ratio of m	Power Ratio of m
0	0.0%	-
1	12.5%	-18.1 dB
2	25.0%	-12.0 dB
3	37.5%	-8.5 dB
4	50.0%	-6.0 dB
5	62.5%	-4.1 dB
6	75.0%	-2.5 dB
7	87.5%	-1.2 dB

Training

The training for this experiment was essentially the same as that for the first experiment. The subjects were first allowed to choose the reflection strength of the filter and then were played a stimulus with that reflection strength, and with the fixed value of τ and the overall amplitude randomized. They were then asked to perform 160 trial blocks until their performance stabilized (information transfer stopped increasing and two trials were within a 0.2 bit range), before the formal experiment was performed.

Experiment

The actual experiment consisted of 5 blocks of trials per subject. Each block consisted of 160 trials, with 20 for each of the 8 possible values of m . As in the first experiment, the values of m for the trials in each block were chosen randomly without replacement. Thus there were a total of 800 trials in the experiment for each subject, with 100 for each of the eight values of m , or a total of 12.5 for each box in the 8 by 8 confusion matrix.

Two values of τ were used for this experiment. In the first condition, the reflection delay τ was fixed at 5 ms (repetition pitch $1/\tau = 200$ Hz). In the second condition, the reflection delay τ was fixed at 9 ms (repetition pitch $1/\tau = 111$ Hz). As in

Experiment 1, the subjects were given feedback about the correct stimulus for each trial and the confusion matrix and information transfer for each block of trials.

5.2 Single Parameter Experiments

The third and fourth experiments measured the information transfer in either the delay τ or the reflection strength m when the other parameter was varied.

Stimulus

The stimuli used in the second experiment were not frozen. Instead, the white gaussian noise sample was randomly selected from ten different noise waveforms created from the same statistical distribution with the ISPUD program. The waveforms were all exactly one second in length. As in the first experiment, the bandwidth was limited to 10KHz by the anti-aliasing filters in the ALCS.

Overall Amplitude

As in the first two experiments, the overall amplitude was varied. In the third and fourth experiments, however, the amplitude was varied in only ten steps, rather than 23, and the steps were spaced logarithmically at approximately 2 dB intervals, rather than the linear spacing used in the first experiment. The actual attenuation voltage multipliers and decibel attenuation levels of the ten steps are shown in Table 3. The base overall level was the same as the first and second experiments. It was selected with the volume control of the ALCS to place the loudest stimuli at a level that was somewhat loud yet still comfortable for the subjects.

Table 3: Randomized Amplitude Steps

Amplitude Step	Voltage Multiplier	Decibel Attenuation
1	0.13	18.0 dB
2	0.16	16.1 dB
3	0.19	14.5 dB
4	0.25	12.0 dB
5	0.31	10.1 dB
6	0.41	7.8 dB
7	0.50	6.0 dB
8	0.63	4.1 dB
9	0.78	2.1 dB
10	0.97	0.3 dB

Identified Parameters

In the third and fourth experiments, the reflection delay τ and the voltage ratio of the reflection signal to the primary signal m were placed on logarithmic scales, in contrast to the linear scales used in the first experiment. This was done in an attempt to make the steps between the different stimulus types perceptually equal, since the Yost work on rippled noise indicated that JND's for reflection strength and for delay tended to follow Weber's law. Furthermore, the maximum information transfer of 0.90 bits measured in Experiment 2 indicated that a reduction in the number of stimuli from 8 (3 bits of input information) to 6 (2.6 bits of input information) would not limit the information transmission in the experiment, so the number of steps used for m was reduced to six.

The ten values of τ used for the stimuli were spaced equally logarithmically from 0.5 ms to 15 ms. The ten steps, along with the stimulus number used to identify that step when the subject was identifying τ (Experiment 3) and the repetition pitch associated with the delay ($1/\tau$), are shown in Table 4.

Table 4: Experiment 3-5 Stimulus Delay Values

Stimulus Number	Delay τ	Repetition Pitch
0	0.50 ms	2000 Hz
1	0.73 ms	1370 Hz
2	1.07 ms	939 Hz
3	1.55 ms	644 Hz
4	2.27 ms	441 Hz
5	3.31 ms	302 Hz
6	4.82 ms	207 Hz
7	7.04 ms	142 Hz
8	10.28 ms	97 Hz
9	15.00 ms	67 Hz

Similarly, the six values of m used for the stimuli were chosen to be approximately equally spaced on a logarithmic scale at intervals of approximately 3 dB. The actual voltage ratios and decibel attenuations for the six values of m are shown in Table 5.

Table 5: Experiments 3-5 Stimulus Reflection Depth

Stimulus Number	Amplitude Ratio of m	Decibel Attenuation of m
0	18.8%	14.5 dB
1	25.0%	12.0 dB
2	34.4%	9.3 dB
3	50.0%	6.0 dB
4	78.1%	2.1 dB
5	96.9%	0.3 dB

For each trial in each experiment the software randomly selected 1 of the 10 samples of noise, 1 of the 10 amplitude steps, 1 of the 10 delay values, and 1 of the 6 reflection strength values, with all 4 parameters independent. In Experiment 3, the subject was asked to choose one of the delay steps from 0 to 9, and in Experiment 4 the subject was asked to choose one of the reflection strength steps from 0 to 5.

5.2.1 Experiment 3: Identify τ with m roved

Training

The training for Experiment 3 was similar to that for the first two experiments. The subjects first used a training mode where they could select the delay stimulus value (0-9, see Table 4) and the amplitude, noise sample, and reflection strength were determined randomly. When they were comfortable with the stimuli, they performed a number of 200 trial blocks until their information transfer stabilized before beginning the experiment.

Experiment

As in the first experiment, five blocks of two hundred trials were collected for each of the four subjects. Each block had twenty trials for each of the ten values of τ listed in Table 4, and the trials in each block were chosen randomly without replacement. In each trial the noise sample was randomly determined, the overall amplitude level was randomly chosen from the steps shown in Table 3, and the reflection strength m was randomly chosen from the values shown in Table 5. After each trial the subjects were shown the correct stimulus number, and after every block they were shown their confusion matrix and information transfer.

5.2.2 Experiment 4: Identify m with τ roved

Training

The training for Experiment 4 was nearly identical to the training for Experiment 3. The training mode used by the subjects allowed them to choose the reflection strength m (0-5, see Table 5), while the delay τ , the amplitude, and the noise sample were randomly determined. This training continued until the subjects felt they were familiar

with the six stimuli. Then they performed a number of 120 trial blocks. When the information transfer in the blocks stabilized, the experiment was started.

Experiment

Five blocks of trials, each 120 trials in length, were collected for each of the four subjects. Each block had 20 trials for each of the 6 values of reflection strength listed in Table 5, and the trials in each block were chosen randomly without replacement. In each trial the noise sample was randomly determined, the overall amplitude level was randomly chosen from the steps shown in Table 3, and the reflection delay τ was randomly chosen from the values shown in Table 4. The subjects were given the correct stimulus number after each trial, and were shown their confusion matrix and information transfer after each 120 trial block.

5.3 Supplemental Experiments 1 and 2

There are a number of significant differences between the stimuli used in Experiments 1 and 2 and the stimuli used in Experiments 3-5. In the first two experiments, the amplitude, the delay τ , and the reflection strength m were all varied on a linear scale, while in the last three experiments, these parameters were varied on a logarithmic scale. In addition, the actual noise waveform used in the first two experiments was frozen, rather than randomly picked from ten samples, and the length of the noise waveform was approximately half as long as the noise samples used in the later experiments (521 ms vs. 1000 ms). The effects of these changes on identification performance are not easily predictable, so two short studies were designed to directly compare the results from the first two experiments to those of the last three experiments.

5.3.1 Identify τ , m fixed

The first supplementary study repeated the first condition of the first experiment, where the subject was asked to identify the delay τ with the reflection strength m fixed at 0.9675. This experiment differed from the first experiment by using the longer, randomized stimuli from experiments 3-5 (See section 5.2), and by using the logarithmically varied overall amplitude level shown in Table 3. The ten values of delay

τ that the subject was required to identify are the logarithmically spaced values in Table 4. In all other ways (feedback, trial block size, number of trials, training, etc.) the experiment was identical to the first condition of Experiment 1.

5.3.2 Identify m , τ fixed

The second supplementary study repeated the first condition of the second experiment, where the subject was asked to identify the reflection strength m with the delay τ fixed at 5 ms. As in the first supplementary experiment, the longer, randomized stimuli of experiments 3-5 were used, and their overall amplitude was varied logarithmically by randomly selecting one of the ten steps shown in Table 3. The eight linearly spaced values of m used in the first experiment (see Table 2) were replaced with six logarithmically spaced values of m used in experiments four and five (shown in Table 5). The training, block size, number of trials, and feedback were all the same as in Experiment 2.

5.4 Two Parameter Identification Experiment

The fifth experiment essentially combined the two identification tasks involved in experiments three and four into a single, two parameter identification experiment. For each trial, the subjects were required to identify both the reflection strength m and the delay τ of the stimulus. The much larger number of possible responses (60 versus 10 or 6 for experiments three and four) required a much larger number of trials and a different system for calculating information transfer.

5.4.1 Stimulus

The stimuli used for the experiment were the same as those used in the third and fourth experiments. The noise sample was chosen randomly from the ten 1000 ms files used for Experiments 3 and 4. The overall amplitude was varied randomly among the ten steps listed in Table 3. Every trial was presented with one of the six logarithmically spaced values of reflection strength m in Table 5 and one of the ten logarithmically spaced values of delay τ listed in Table 4.

5.4.2 Training

The training for Experiment 5 was somewhat different from the training for the other experiments because of the very large number of possible stimuli. The training was simplified by the use of the same stimulus values as in Experiments 3 and 4, so the subjects only needed to learn to identify both of the parameters simultaneously and they did not have to learn any new stimuli. A training mode allowed the subjects to select both reflection strength and delay from the six and ten possible values, so only the amplitude was randomized in the resulting presentation. When the subjects were comfortable with their ability to identify both parameters simultaneously, several training runs of one hundred trials each were run in order to allow the subjects to become accustomed to the two parameter identification. Unfortunately, there was no way to simply evaluate performance for such a small number of trials in the 60 by 60 confusion matrix so there was no way to tell if subject performance had stabilized before beginning the experiment. The inability to verify that sufficient training had occurred may have caused the information transfers measured in the experiment to be underestimates.

5.4.3 Experiment

In this experiment it was impossible to present enough trials to use the maximum likelihood estimate of information transfer, which requires five trials per box of the confusion matrix or 18,000 trials in this experiment. Due to time constraints, each subject participated in only 2,400 trials, or 0.667 per box of the confusion matrix. Although this was not enough trials for the direct maximum likelihood estimator, it was enough to make a rough estimate of the information transfer using a method developed by Houtsma (1983). This method of data processing is described in Appendix B. The trials were presented in 24 blocks of 100 trials each. In the earlier experiments, the stimuli in each block were chosen without replacement in order to control the number of presentations of each stimulus in the experiment. The large number of possible stimulus combinations in this experiment would have required at least 3600 trials in a block to have an equal number of presentations for each stimulus using this no-replacement strategy. Therefore, in every trial of Experiment 5 both the reflection strength m (chosen

from the six possibilities in Table 5) and the delay τ (chosen from the ten possibilities in Table 4) were picked randomly with replacement. The impact of this change in trial selection strategy was believed to be small, and is discussed in detail in Appendix C. After each trial the subjects were shown the correct stimulus numbers of the delay and the reflection strength, and after each 100 trial block they were shown two confusion matrices representing reflection strength and delay.

6. Results

This chapter summarizes the results of all the experiments. All of the one dimensional data was analyzed to measure information transfer and interstimulus sensitivity. The values of information transfer were calculated from the confusion matrices in each experiment using the maximum likelihood estimator, and were adjusted for the bias of that estimator as discussed in Appendix A. The second analysis was based on the Decision Model developed by Durlach and Braida (1969). Each of the confusion matrices was processed to determine the maximum likelihood estimates of the interstimulus sensitivity values ($d'(S_i, S_{i+1})$) and of the criterion values ($C_0 \dots C_M$) with the assumption that σ was constant. The total sensitivity Δ' ($d'(S_{\max}, S_{\min})$) and response bias β for each subject in each experiment were derived from these estimates. The response biases are of only marginal interest. They are shown in Appendix D. The total sensitivities are more interesting. They were used in conjunction with Braida and Durlach's model (1972) to predict the information transfer for each subject in each experiment based on the Decision Model. These predicted information transfers, which were compared to the empirically measured information transfers, assume equal interstimulus sensitivities and no response bias.

Table 1 summarizes the conditions in each of the 9 unidimensional analyses. This includes the two supplementary experiments and the m and τ projections derived from the data in Experiment 5. The projections are the confusion matrices obtained by summing all of the trials in the two dimensional experiment with the same stimulus and response values for one parameter and ignoring the other parameter. Table 2 gives the empirically determined information transfer, the empirically determined total sensitivity, and the predicted information transfer based on the total sensitivity in each of these 9 one dimensional cases. The next two pages show the interstimulus sensitivity data for each subject in each experiment. The results of the two dimensional experiment, along with some of the results already shown in Table 2 (included for comparison purposes), are shown in Table 4.

Table 1: Summary of Experiments

Experiment	1 Cond. 1	1 Cond. 2	Supp. 1	2 Cond. 1	2 Cond. 2	Supp. 2	3	4	5 τ projection	5 m projection	5 m, τ
Identification Parameter	τ	τ	τ	m	m	m	τ	m	τ	m	m, τ
Scale	linear	linear	log	linear	linear	log	log	log	log	log	log
Background Parameter	m	m	m	τ	τ	τ	m	τ	m	τ	-
Status of Background	fixed=0.97	fixed=0.5	fixed=0.97	fixed=5ms	fixed=9ms	fixed=5ms	roved	roved	identified	identified	-
Amplitude Roving Scale	linear	linear	log	linear	linear	log	log	log	log	log	log
Stimulus Waveform	frozen	frozen	random	frozen	frozen	random	random	random	random	random	random
Stimulus Duration	521ms	521ms	1000ms	521ms	521ms	1000ms	1000ms	1000ms	1000ms	1000ms	1000ms
Number of Stimuli	10	10	10	8	8	6	10	6	10	6	60
Number of Trials	1000	1000	1000	800	800	600	1000	600	2400	2400	2400
Trials/Box in Matrix	10	10	10	12.5	12.5	16.7	10	16.7	24	66.7	0.7

Table 1 summarizes the setup of each of the experiments. The identification parameter is the variable the subjects were asked to identify, and the spacing of the identification parameter in the stimuli is shown as either linear or logarithmic (log). The Other Parameter is the background parameter in the experiment. The Other Condition category tells whether this parameter was fixed at a particular value, logarithmically roved, or also identified. The Amplitude Roving category tells whether the overall amplitude was linearly roved or logarithmically roved.

Table 2: Values of IT and Δ'

Experiment	Subject	IT	Δ'	$IT\Delta'$
Experiment 1 Condition 1	DB	1.35	7.4	1.3
	AS	2.61	13.9	2.0
	JK	1.84	9.0	1.6
	CG	0.98	9.1	1.6
Experiment 1 Condition 2	DB	1.31	5.7	0.7
	AS	2.77	13.8	2.0
	JK	1.81	11.0	1.8
	CG	1.02	6.8	1.3
Experiment 2 Condition 1	DB	0.75	3.8	0.7
	AS	0.90	4.7	0.8
	JK	0.75	3.8	0.7
	CG	0.44	2.5	0.3
Experiment 2 Condition 2	DB	0.77	2.7	0.4
	AS	0.88	4.7	0.8
	JK	0.48	3.1	0.5
	CG	0.43	3.2	0.5
Experiment 3	DB	0.94	4.9	0.9
	AS	1.76	10.4	1.6
	JK	0.98	4.9	0.9
	CG	0.58	3.3	0.6
Experiment 4	DB	0.22	2.1	0.3
	AS	0.55	3.4	0.6
	JK	0.40	2.9	0.4
	CG	0.27	2.5	0.3
Experiment 5, τ Projection	DB	0.80	4.3	0.8
	AS	2.09	12.2	1.8
	JK	1.25	6.1	0.7
	CG	0.43	2.6	0.3
Experiment 5, m Projection	DB	0.15	2.2	0.3
	AS	0.55	3.6	0.6
	JK	0.31	1.9	0.2
	CG	0.08	1.2	0.1
Supplemental Experiment 1	DB	1.47	8.7	1.6
	AS	2.60	25.3	2.6
Supplemental Experiment 2	DB	0.54	3.2	0.5
	AS	0.84	4.5	0.8

Table 2 shows a comparison between the information transfer (IT), the total sensitivity (Δ'), and the information transfer calculated from the total sensitivity ($IT\Delta'$) using the data from Braida and Durlach (1972) for each subject in each experiment.

Figure 1: Sensitivity in Experiment 1, Condition 1

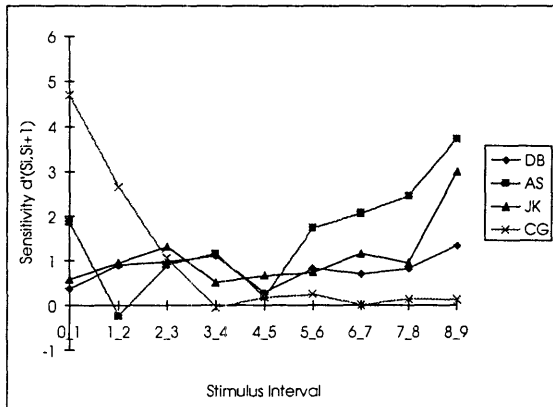


Figure 2: Sensitivity in Experiment 1, Condition 2

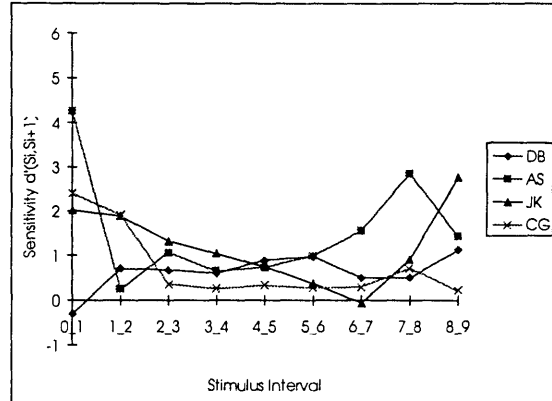


Figure 4: Sensitivities in Experiment 1 and Supplementary Experiment 1

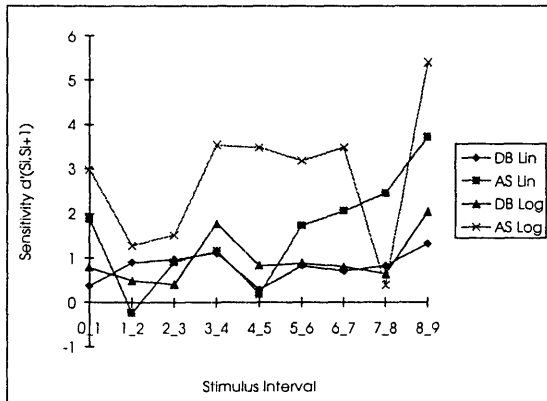


Figure 3: Sensitivity in Experiment 3

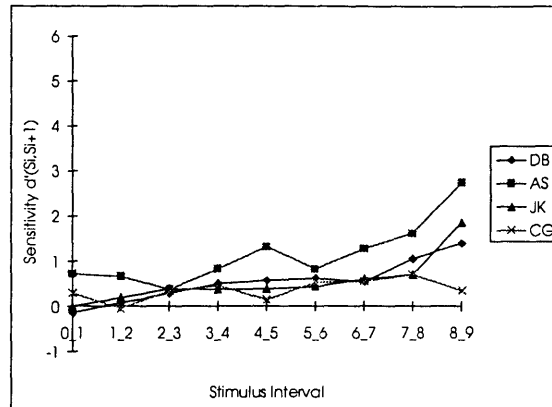
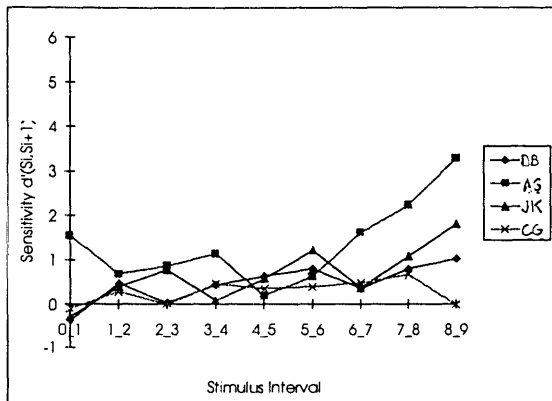


Figure 5: Sensitivity for τ in Experiment 5



These figures show the sensitivity values for the τ identification experiments. The data from Supplemental Experiment 1 are superimposed on the data from Experiment 1, Condition 1 to allow an easy comparison of the sensitivity data on the linear and logarithmic τ scales for subjects DB and AS.

Figure 6: Sensitivity in Experiment 2, Condition 1

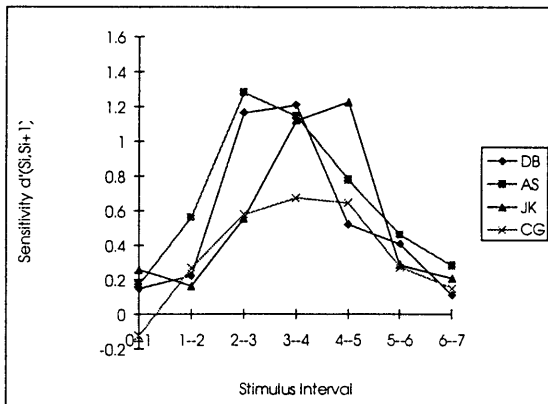


Figure 7: Sensitivity in Experiment 2, Condition 2

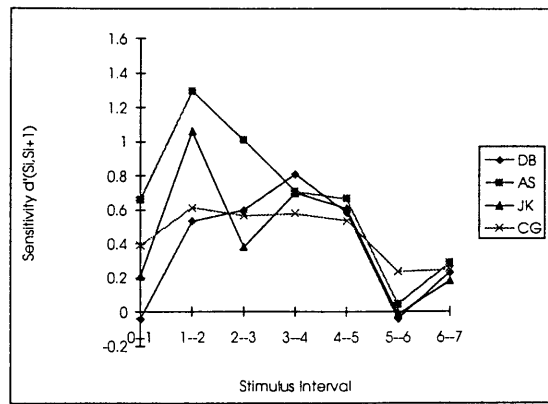


Figure 8: Sensitivities in Experiment 2 and Supplementary Experiment 2

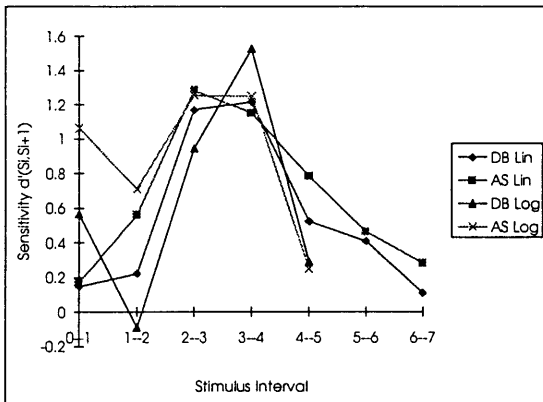


Figure 9: Sensitivity in Experiment 4

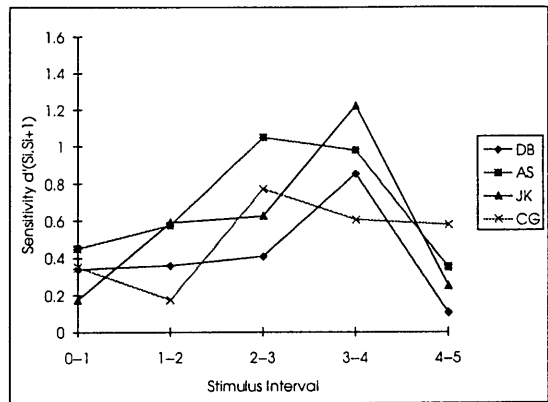
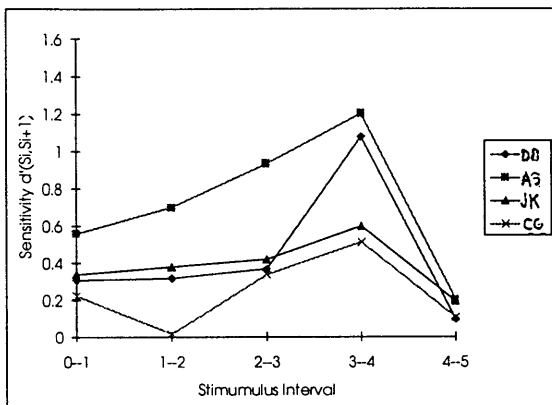


Figure 10: Sensitivity for m in Experiment 5



These figures show the sensitivity data for the m identification experiments. The data from Supplementary Experiment 2 are provided, and they are superimposed on the data from Experiment 2, Condition 1 to allow an easy comparison of the sensitivity data on the linear and logarithmic scales for subjects DB and AS. Note that the ordinate scale in these graphs differs from the scale used in the graphs of τ identification results.

Table 4: Comparison of Information Transfer

Subject	Experiment 3	Experiment 4	Sum	τ -project	m-project	Sum	Experiment 5
DB	0.94	0.22	1.16	0.80	0.15	0.95	1.28
AS	1.76	0.55	2.31	2.09	0.55	2.64	3.09
JK	0.98	0.40	1.38	1.25	0.31	1.56	2.07
CG	0.58	0.27	0.85	0.43	0.08	0.51	0.52
Average	1.07	0.36	1.43	1.14	0.27	1.42	1.74

The predicted reliability of the data from these experiments should be discussed. Overall, the information transmissions in the τ identification experiments are believed to be quite reliable. This is evidenced by the relative stability in these measurements when small changes in the stimuli occurred (Conditions 1 and 2 of Experiment 1, Supplemental Experiments 1 and 2, and the brief experiment described in Appendix C). The information transmissions in the m identification experiments are probably not as reliable. These values are stable for the two conditions of Experiment 2 (Except for JK), but in the other experiments the information transfer was relatively poor and the information transfers do not seem to be very stable. The m projection confusion matrix in Experiment 5 has an exceptionally large number of trials per box in the matrix (approximately 75), so the m-projection information transfers from Experiment 5 are probably relatively accurate. Time constraints did not allow a thorough examination of the statistical significance of these results, but statistical analyses should be a priority in any future extension of this work.

7. Discussion

A. Wide variation of performance among subjects:

One of the most striking features of the data is the wide variation in the performance of the different subjects. In the experiments where the delay τ was identified the proficiency of the four subjects was clearly differentiated. Subject AS was always best, both in information transfer and in total sensitivity. The overall performance of JK and DB was comparable in both areas, and subject CG was consistently poor in information transfer. The sensitivities for subject CG, however, are not as bad in comparison to the other subjects as his information transfer, and in the first experiment, where the delay was varied on a linear scale, he showed extremely high sensitivity for the lowest stimulus values. Yost and Hill (1978) also found wide variations in subject performance, but they were not quite as dramatic as these. Musical experience seems to be the dominant factor in determining the performance of the four subjects. Subject AS is an avid musician who plays both the piano and the electric bass and, most importantly, has perfect pitch. He is able to instantly identify musical notes, and he was able to associate the repetition pitch of the different delay values with standard musical pitches. When asked to describe the stimuli in the experiment, he was able to tell the experimenter the musical notes associated with each of delay values. It was clear that perfect pitch gave subject AS a considerable advantage over the other subjects in identifying the delay.

Subject JK, who was second best in the τ -identification experiments, was a piano player who had received some formal training in identifying musical intervals. She said she memorized the sound of one of the delay values and made her judgments based on the interval between the sound and this perceptual anchor. Subject DB played the clarinet in the past but had never received any formal training in pitch identification. His identification ability for τ was somewhat worse than subject JK's. Subject CG's performance was consistently lower than that of the other subjects. He was not a musician but claimed to be an avid listener of music and something of an audiophile.

Subject performance in the m-identification tasks seemed to be much less hierarchical than in the τ identification experiments. Although AS was always the best performer, his information transfer was in some cases only 0.11 bits higher than the second best subject. The other subjects were somewhat mixed. Subject DB was the worst performer in Experiment 4, and subject JK was very poor in the second condition of Experiment 2. The changes in m-identification ability for the different experiments exhibited by subjects DB and JK are somewhat puzzling. Subject DB performed much worse with the logarithmically spaced reflection strengths, while JK was inexplicably poor when τ was fixed at 9 ms. In general, however, it is clear that the ability to identify parameter changes in reflections varies greatly across the population of listeners with normal hearing, and that it improves with musical training.

B: Predictions of Information Transfer from Total Sensitivity

Table 2 shows the total sensitivities measured for each subject in each experiment, as well as the information transfers measured in each case and the information transfers predicted from the total sensitivity from the graph in Braida and Durlach (1972). This graph plots the information transfer as a function of total sensitivity based on the decision model with ideally located criteria. The predicted and actual information transfers are quite similar in nearly every case. There are some notable exceptions, however. In some cases the prediction disagrees with the measured information transfer by a wide margin. This is the case for AS and CG in Experiment 1, Condition 1, for AS and DB in Experiment 1, Condition 2, for DB in Experiment 2, Condition 2, and for JK in the τ projection of Experiment 5. These discrepancies do not seem to be correlated with response bias, and their cause is not clear from these data.

C: Comparison of Results to Existing Literature:

When compared to the information transfers measured in previous experiments involving unidimensional audio displays, the information transfers obtained for m and τ are quite disappointing. Miller (1956) cites data by Pollack that indicates that the maximum information transfer achieved in the identification of tone frequency is

approximately 2.5 bits independent of the range of stimuli and number of stimuli used. He also cites data by Garner showing the maximum information transfer for variations in amplitude is approximately 2.3 bits. The average of 1.72 bits of information transfer measured for τ with m fixed is on the low end of the range of values found in most unidimensional identification experiments. The even lower values measured for m identification (0.73 bits average for τ fixed at 5 ms was the highest in any m identification experiment) are very low in relation to other unidimensional experiments. The m data indicate that channel capacity is either much lower for reflection strength than for other audio parameters or that capacity was not reached in this experiment due to the selection of stimuli used. The issue is discussed further below.

D: Difference in performance in Experiments 1 and 2 and 3-5:

There are a number of differences between the stimuli used in Experiments 1 and 2 and those used in Experiments 3-5. The noise waveforms were frozen in the first two experiments, and randomly selected in the last three. The stimulus durations were different. The scale used to rove the overall amplitude was linear in Experiments 1-2 and logarithmic in Experiments 3-5. And, perhaps most importantly, values of the identified parameter used in the stimuli were linearly spaced in the first two experiments and logarithmically spaced in the last three experiments. The data from Yost and Hill (1978) show that the discrimination of repetition pitch seems to obey Weber's law. Thus perceptually equally spaced values of τ in the stimuli (which should generate a relatively constant value of d' in an identification experiment) would be spaced logarithmically. The Yost and Hill data also imply that reflection strength discrimination is logarithmic, at least for lower values of m with τ of 1 ms or 2 ms. In the linear scale used in Experiments 1 and 2, the logarithmic spacing of the lower-numbered stimuli (the logarithmic spacing can be viewed as the ratio of the values of the parameter in adjacent stimuli) is greater than the logarithmic spacing of the higher-numbered stimuli. This should cause the interstimulus sensitivity d' to be greater for the lower-numbered stimuli and smaller for the higher numbered stimuli in Experiments 1 and 2 relative to the sensitivities for those stimuli in Experiments 3-5. The sensitivity data for the

supplemental experiments, which were designed to explore the differences in performance caused by the changes in the stimulus spacing, do not show much difference in the interstimulus sensitivity for the two scales.

In the first supplementary experiment, the information transmission performance of the subjects improved slightly over their performance in the first experiment. The sensitivity was nearly identical for subject DB, but subject AS showed a very large improvement in resolution with the logarithmic scale. It is unfortunate that CG could not be used in this experiment. The data from Experiments 1 and 3 indicate that he has a much greater resolution for small values of τ with the linear scale, a phenomenon not seen in the data of any other subject. The reasons for this anomaly are not known.

In the second supplementary experiment, the information transfer of each subject degraded, although the reduction was much greater for subject DB. The sensitivity data show that the resolution for subject AS was increased, but his total sensitivity was not quite as good in the six step log scale as in the eight step linear scale. Subject DB showed no improvement in resolution with the logarithmic scale. The sensitivity data do not show why DB's information transfer was so low for the logarithmic scale.

Overall, it appears that the modified stimuli and responses used in Experiments 3-5 slightly increased information transfer in the delay identification tasks, but decreased it in the reflection strength identification task. The reduction in reflection strength identification seems to vary in magnitude by subject, and it appears that subject DB was most adversely affected.

E: Comparison of Results for τ and m :

The information transfer and total sensitivity for τ identification were considerably higher than those for m identification. It is obvious that it was much easier to identify τ than it was to identify m in these experiments. This is not an unreasonable result, however, considering the discrimination data for rippled noise by Yost and Hill (1978). These data (which are shown in Figure 1 in the background section) indicate that the total range of m used would span no more than 4 or 5 JNDs (just noticeable differences) when τ was fixed at 0.66 ms and no more than 6 or 7 JNDs when τ was

fixed at 1ms or 2ms. In contrast, the Weber ratio of about 5% for τ would indicate that the range of delays used in the logarithmic scale spans more than 50 JNDs with m fixed at 1. This would explain the overall poor performance in reflection strength identification relative to delay identification.

The shapes of the sensitivity curves for m and τ are also radically different. Braida and Durlach (1972) found that interstimulus sensitivity tends to increase in the vicinity the extreme values of the stimulus range, a phenomenon referred to as the “resolution edge effect”. The effect is believed to be a result of the subject using the extreme values of the stimuli as “perceptual anchors”. (The details of a model based on perceptual anchors are found in Braida and Durlach (1972)). This effect is found in the data for τ identification in almost every experiment. The values of d' for the subjects tend to increase around Stimulus 0 in the first two experiments (especially for CG and AS). Although the results of Supplementary Experiment 1 did not indicate such a trend, it definitely seems that the linear delay scale used in the first two experiments provides greater sensitivity around Stimulus 0 than the logarithmic delay scale used in later experiments. The values of d' tend to increase around Stimulus 9 in all of the experiments. These effects could be caused by differences in the underlying sensitivity for changes in τ , but the data by Yost and Hill (1978) indicate that pitch strength, which is closely related to delay sensitivity, is greatest for $\tau=2$ ms, which is not on the edge of either stimulus scale. Therefore this increase in sensitivity appears to be consistent with the resolution edge effect.

A more unexpected result is the very strong decrease in interstimulus sensitivity at the extreme values of m in the stimulus range. The Yost and Hill data indicate that sensitivity for reflection strength should decrease for the smallest values of m . Although the Yost and Hill data do not show the discriminability of m at values of m higher than 0.7, the general trend in the data indicates that discriminability for higher values of m (and therefore the sensitivity) should be relatively good. Yet every experiment shows a general downward concavity in the d' data for m . This is an extremely strange phenomenon that is not documented in the literature, and none of the data in this experiment indicate a cause for this behavior. A thorough study examining the

underlying JNDs for m at various values of τ , particularly in the vicinity of $m=1$, may help to explain this behavior in the sensitivities for m .

F: Interaction between m and τ :

Each subject's information transfer was much lower in Experiments 3-4, where the background parameter was roved, than in Experiments 1-2, where the background parameter was fixed. The two supplemental experiments show that some of the decrease in m identification ability may have been caused by the changes in stimuli between those experiments, but the degradation in performance is much higher than the results of the supplementary experiments would predict. It is very likely that this decrease in performance is caused by interaction between m and τ . This interaction between m and τ indicates that they are integral parameters, rather than separable parameters (Durlach, Tan, et al., 1989).

G: Projections from Experiment 5 versus Experiments 3-4:

Table 3 summarizes the results pertinent to the information transfer in the two dimensional experiment. The m and τ projections were generated by summing together all the trials in Experiment 5 with the same stimulus and response value in one parameter while ignoring the other parameter. These data can be used to compare subject performance in identifying m when τ was also identified to the performance in identifying m when τ was randomly varied but not identified. In these data, none of the subjects improved at identifying reflection strength m when they also had to identify the delay τ . The information transfers in both cases appear quite low the estimates may be noisy, but to the extent that this trend is significant, it shows that the process of identifying τ interferes slightly with and degrades m identification performance. The data for identifying τ with m roved or identified are mixed among the subjects. Subjects AS and JK improved when both parameters were identified, and subjects CG and DB degraded when both parameters were identified. The average of the sum of the information transfers in Experiments 3 and 4 is very close to the average of the sum of the information transfers in the m and τ projections of Experiment 5. Overall, it appears

that the information transfers obtained in the m and τ projections of Experiment 5 are roughly comparable to those obtained in Experiments 3 and 4. In other words, the as long as the secondary parameter is roved, the additional burden of having to identify the second parameter has little effect on performance.

H: Two Dimensional Information Transfer:

In the first four experiments, the calculation of information transfer was relatively straightforward. It could be calculated directly from the confusion matrices with the maximum likelihood estimator and then adjusted for bias using a simple correction equation, as described in Appendix A. Unfortunately the bias correction equation does not work if there are fewer than five trials per box in the confusion matrix, a condition that would have required 18,000 trials in Experiment 5. Therefore an alternative method of estimating the information transfer in the two dimensional experiment was necessary. One method of estimating information transfer for a small number of trials is based on the matching of the subject data with information transmission curves generated by a computer simulation of the experiment. The method, which was developed by Houtsma (1983), is described fully in Appendix B. This method estimates information transfer indirectly, and is certainly not as accurate as a direct measure based on a very large number of trials would be, but it does measure the information transfer without making any assumptions about the interaction of the two stimulus parameters. The two dimensional information transfer estimates in Experiment 5, calculated with Houtsma's method, are shown in Table 4 along with the data from Experiments 3 and 4 and the information transfers from the m and τ projections in Experiment 5.

The information transfers found in Experiment 5 using Houstma's method are believed to have some discrepancies that are discussed in the Appendix B describing the data processing for that experiment. In summary, it is believed that DB's information is accurate, JK and AS's are overestimates, and CG's is an underestimate. This would indicate that the information transfer in the two dimensional experiment was slightly higher than the sum of the information transfers of the two projection matrices associated with the two dimensions. This can be accounted for in part by a tendency for the

subjects' m and τ identification performance to be positively correlated within the individual trials. The Yost and Hill (1978) data indicate that both reflection strength discrimination and delay discrimination are maximized when τ is approximately 2 ms. This would cause relatively good performance for both parameters in trials with certain delay values. An analysis of the effects of such a correlation on information transfer is provided in Appendix D.

8. Concluding Remarks

The goal of the thesis was to evaluate the possibility of using delay-and-add filters with two adjustable parameters, the length of the delay τ and the ratio of the amplitude of the delayed signal to the primary signal m , to provide absolute audio distance cues in a virtual environment. The results show that such filters can transmit, on average, a maximum of approximately 1.7 bits of information. This is sufficient to allow a well-trained listener to reliably place a broadband sound into one of three distance categories (close, medium, and far, perhaps). Also, this information can be obtained without destroying the character of the signal and without a priori information about the intensity of the sound source. Although the information transfer that is achievable with reflection based distance coding seems to be limited, and the usefulness of such coding is restricted to broadband stimuli, it still provides an improvement over the nonexistent absolute distance cues in current virtual audio displays.

The strange downward concavity in the interstimulus sensitivities for m may provide a clue for significantly increasing information transfer in the two dimensional case. If, by changing the interstimulus spacing of the m values, the sensitivity at the edges of the stimulus range could be increased to make it equal to the sensitivity in the center of the stimulus range, or even to increase it above the sensitivity in the center of the stimulus range as would be expected from the resolution edge effect, a considerable increase in information transfer could be realized. The question is whether or not this downward concavity can be reduced by increasing the stimulus intervals at the edges of the range without substantially decreasing sensitivity in the center of the stimulus range. The downward concavity in the d' data for m is puzzling, and a priority of any further extension of this work should be a closer examination of this behavior.

It should be noted that the average information transfer with two dimensional changes in the reflection characteristics was no higher than the changes in delay only with a fixed value of m . If two parameter stimuli are used for distance coding, the range of reflection strengths should be carefully chosen to ensure that reflection strength values that substantially degrade discrimination in τ are not used. If the distance coding

is attempted with variations in one dimension only (τ), it may be possible to increase the information transfer by carefully choosing the value of m associated with each stimulus. It is probably possible to obtain more information transfer with two dimensional variations in the stimulus than with one dimensional variations in the stimulus, but the range of values used for m and τ must be carefully chosen.

A number of suggestions for further research on this subject can be made based on these results. The first issue that needs further exploration is the underlying discriminability of the reflection strength, particularly for values of m close to 1. The JNDs for m should be measured for a wide range of values of m and τ . The results of such a study might explain the strange downward concavity of the sensitivity data for m . Once these JNDs are determined, it should be possible to choose an appropriate stimulus spacing for m and measure the resulting information transfers for the m variable with τ fixed or roved. If this information transfer for m increases substantially, then the two dimensional information transfer with the new reflection strength stimulus spacing can be measured and compared to the results achieved for identifying τ with m fixed. Another possible topic for further research would be the careful selection of the m values associated with each stimulus in the one dimensional τ identification experiment in order to maximize the information transfer. A number of questions still remain unanswered, and more work is required in order to obtain a thorough understanding of the issues involved in the identification of first order reflections.

Although the results of these experiments are not exceptionally encouraging, they do show that reflection based coding can provide some egocentric audio distance information. It is also likely that different stimulus values for reflection strength and delay values could be found that would provide better sensitivity and information transfer than were achieved in this experiment. However, further research is required to determine how much improvement can be obtained in this manner.

9. Appendix A: Bias in Information Transfer Estimates

In Experiments 1-4, the information transfer was calculated from the confusion matrix using the maximum likelihood estimator of information transfer, which is directly based on the maximum likelihood estimator of entropy. This maximum likelihood estimator was also used for determining the information transfer of each simulation curve after 32,400 trials in Experiment 5. These maximum likelihood estimators are biased, however, so the data were adjusted using the estimates of the bias developed by Miller (1954). A brief discussion of the equations used to calculate the adjusted estimates of information transfer in the experiments follows. Recall that the entropy H of a distribution is:

$$H = -\sum_x p(x) \log p(x) \quad (\text{A.1})$$

where each value of x is a possible outcome and $p(x)$ is the probability of outcome x . Thus the maximum likelihood estimator of the entropy for a distribution where each value of i represents one of k possible outcomes is:

$$\hat{H} = -\sum_{i=1}^k \frac{n_i}{n} \log_2 \frac{n_i}{n} \quad (\text{A.2})$$

where each n_i is the number of observations of outcome i and n is the total number of trials. Miller (1954) shows that this is a biased estimator of the entropy. Thus the expected values of H and \hat{H} are not equal. In fact:

$$H - E[\hat{H}] = E \left[\sum_{i=1}^k \frac{n_i}{n} \log_2 \frac{n_i}{np(i)} \right]. \quad (\text{A.3})$$

The term on the right has a chi-square distribution, and some mathematical manipulations, combined with the elimination of second order terms, yield an easily evaluated estimate of the bias of the maximum likelihood estimator of entropy:

$$H - E[\hat{H}] = \frac{\log_2 e}{2n} (k-1). \quad (\text{A.4})$$

Thus the maximum likelihood estimator \hat{H} underestimates the entropy H , and a correction term based on the number of possible outcomes and the number of trials must be added to \hat{H} to get an unbiased estimate of the entropy. The information transfer T also has a biased maximum likelihood estimator. Recall that it is defined by the following equation:

$$T = \sum_{i,j} p_{ij} \log \frac{p_{ij}}{p_i p_j} \quad i=1\dots r; \quad j=1\dots c; \quad (\text{A.5})$$

Where p_i is the marginal probability of input i , p_j is the marginal probability of output j , and p_{ij} is the probability of the joint event ij . The values r and c are related to the number of rows and columns in the resulting confusion matrix. The maximum likelihood estimator of information transfer is:

$$\hat{T} = \sum_{i,j} \frac{n_{ij}}{n} \log_2 \frac{n \cdot n_{ij}}{n_i n_j} \quad (\text{A.6})$$

where n is the total number of trials, n_i is the number of trials observed with outcome i , n_j is the number of trials observed with outcome j , and n_{ij} is the number of trials observed with the joint outcome ij .

The bias in the maximum likelihood estimator of information transfer can be derived from the bias of the maximum likelihood estimator of the entropy. It can be shown that:

$$\hat{T} = \hat{H}(x) + \hat{H}(y) - \hat{H}(x, y). \quad (\text{A.7})$$

consequently, the bias of the information transfer estimate is:

$$T - E[\hat{T}] = -\frac{\log_2 e}{2n}(r-1)(c-1). \quad (\text{A.8})$$

Therefore the maximum likelihood estimate of information transfer is an overestimate and a correction factor based on the number of possible stimuli r , the number of possible responses c , and the number of trials n must be subtracted from the maximum likelihood estimate to generate an unbiased estimate of information transfer.

This correction factor was used for the information transfer data shown for each of the experiments. Table 10 shows the values of r , c and n in each of these experiments, and the correction factor that was subtracted from the maximum likelihood estimate of information transfer in each case.

Table 5: Bias Correction Factors

Experiment Number	r	c	n	Correction Factor
1	10	10	1000	0.0584
2	8	8	800	0.0442
3	10	10	1000	0.0584
4	6	6	600	0.0300
5	60	60	32400	0.0775

10. Appendix B: Data Processing in the Two Dimensional Experiment

Because of the large number of boxes in the confusion matrix of the two dimensional experiment, it is not possible to accurately measure the information transfer in this experiment with the maximum likelihood estimator used in the first four experiments (see Appendix A). That estimator cannot be accurately corrected for bias with fewer than five trials per box in the confusion matrix. In this experiment, that would have required 18,000 trials, or 18 times the data of any of the other conditions. There was not enough time to collect that quantity of data, so an alternative method for finding an unbiased estimate of information transfer, originally developed by Houtsma (1983), was used.

This method uses a computer simulation of an identification experiment to approximate the behavior of information transmission for small numbers of actual trials compared to the size of the confusion matrix. The simulation parameters can be modified until the simulated experiment closely matches the actual data collected for small numbers of trials. The information transfer can then be determined by continuing the simulated experiment for a large number of trials and calculating the information transfer of the simulation with the maximum likelihood estimate.

The simulated experiment generated a confusion matrix where the stimulus in each trial was a randomly chosen integer N from 0 to 59, and the response was equal to N plus another random integer from $-S$ to S that represented the error in identification. The parameter S could be varied between 0 and 59, with 0 representing perfect performance and 59 representing completely random performance. If the sum of the stimulus N and the randomly generated error was greater than 59 or less than 0, another random error between $-S$ and S was chosen until a valid response between 0 and 59 was generated.

The simulation was run for each value of S from 0 to 59. In each case, the information transfer in the simulation was calculated with the maximum likelihood

estimator after every 100 trials for the first 2400 trials, and after every 1000 trials for the next 30,000 trials. Thus a total of 32,400 trials were simulated for each value of S .

Figure 13 shows the results from the simulated experiment for selected values of the error parameter S . Note that the curve for $S=0$ increases as trials are added and the diagonal of the matrix fills, but that in all other cases the curve decreases rapidly for small number of trials and then asymptotically approaches a horizontal line as the number of trials increases. Also note that information transfer in the simulation approaches its limiting value more rapidly for lower values of S . This occurs because, for small S values, only a small number of the boxes in the confusion matrices close to the diagonal will have non-zero values, so the effective size of the confusion matrix is smaller and it fills more rapidly. Also note that the information transfers of adjacent values of S are more closely spaced as S increases.

Figure 13: Simulated Information Transfer Curves

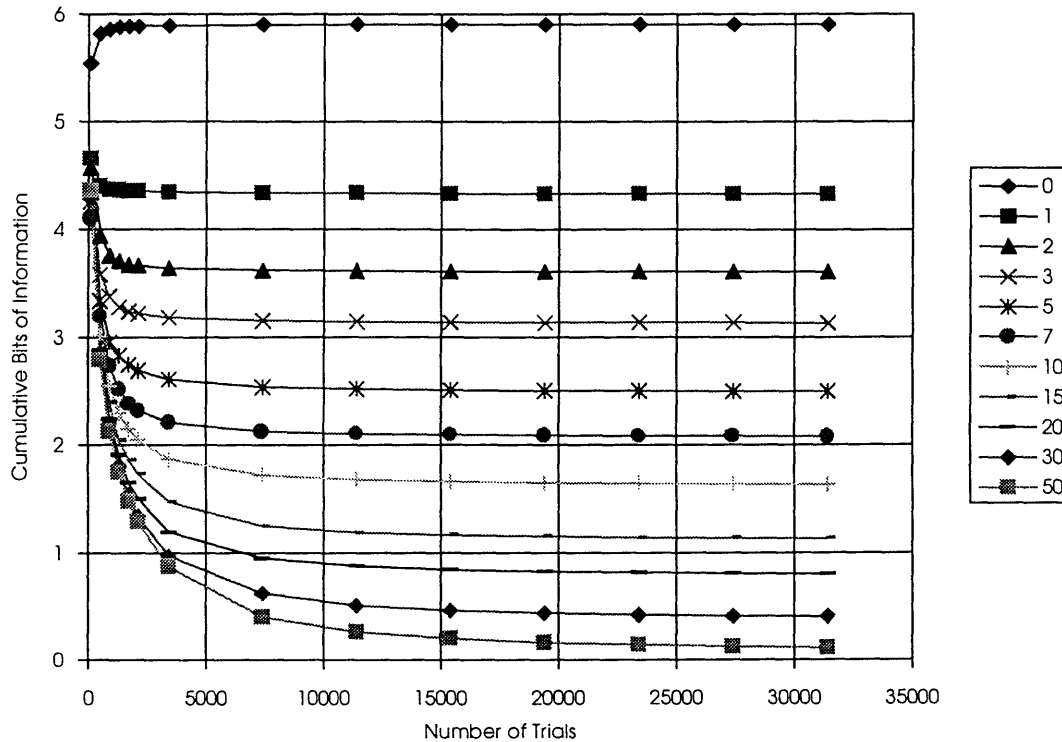


Figure 13: This graph shows the information transfer at various points in the simulated experiment. The horizontal axis shows the number of trials completed, and the vertical axis shows the information transfer in bits of information. The different curves represent different values of the error parameter S , from 0 to 50, as shown in the legend.

In order to determine the information transfer for each subject, the data was used to calculate the cumulative information transfer after each 100 trial block, and the resulting data points were compared to each of the curves generated by the simulation. Because of the instability of the information transfer in the early trials, only the last 12 one hundred trial increments were used for the curve fitting. The simulation and the actual data were compared for twelve points representing 1300 total trials, 1400 total trials, etc., up to 2400 total trials.

The simulation curve with the smallest root mean square error when compared to the subject data at these 12 points was determined to be the closest match to the subject data, and the error parameter S of that curve was used to decide which simulation curves would be used to interpolate the information transfer of the subject. The mean error and the mean magnitude of error for the three curves with error parameters $S-1$, S , and $S+1$ were used to interpolate the information transfer of the subject. If the mean value of the curve fell above the curve with error parameter S , the mean error of curves S and $S-1$ were used to linearly interpolate the information transfer of the subject data. Otherwise the curves S and $S+1$ were used for the interpolation. In each case, the information transfers for the simulation curves with error parameter S and $S+1$ or $S-1$ with 32,400 trials (9 per box in the confusion matrix) were used for the interpolation. The information transfers of the two closest curves were interpolated by multiplying the difference between the 32,400 point information of both curves by the ratio of the mean distance between the curve for error parameter S and the subject data divided by the mean distance between the curves with error parameters S $S+1$ (or $S-1$), and then adding this value to the information transfer for the curve for parameter S . For subject AS, for example, the data curve was above the simulation curve of $S=3$ (the closest RMS value), so the information transfer was interpolated by finding the difference in information transfer for $S=2$ and $S=3$ (.47 bits), multiplying it by the ratio of the mean difference between the data and $S=3$ (.031576) and the mean difference between $S=3$ and $S=2$ (0.43512), yielding $0.47 \times .031576 / 0.43512 = .0341$ bits, which is added to the information for the $S=3$ curve (3.132), giving an interpolated information transfer of 3.17 bits. This was then corrected for bias in the 32,400 simulated trial experiment (See

Appendix A), so .078 bits were subtracted, giving an adjusted information transfer value of 3.09 bits.

The following graphs show the curves of the actual subject data, the simulation curve with the smallest RMS error for each subject, and the two simulation curves immediately above and below the minimum RMS error curve. Each graph is accompanied by a table that shows the mean error, mean magnitude of error, root mean square error, and information transfer for 32,400 trials for each of the three closest simulation curves, as well as the interpolated information transfer of the subject after correction for bias. In the first three graphs, the position of the data curve relative to the simulation curves is very obvious and the information transfer estimates are probably relatively accurate. Subject DB's data seems to almost perfectly fit the model, but the data curves for subjects AS and JK seem to have a slightly more negative slope than closest simulation curve, so it is likely that the information transfer values determined for those subjects are overestimates. The simulation curves surrounding subject CG's curves are quite noisy after 2400 trials, so more trials are probably necessary to get a really good estimate of his information transfer. Also, his data curve has a more positive slope than the surrounding simulation curves, so his information transfer of 0.52 bits is almost certainly an underestimate. Although these estimates are clearly not perfect, they do provide a way of estimating the information transfer of the 60 by 60 matrix without ignoring any possible interactions between the two stimulus dimensions.

Figure 14: Subject DB; Experiment 5 Information Transmission

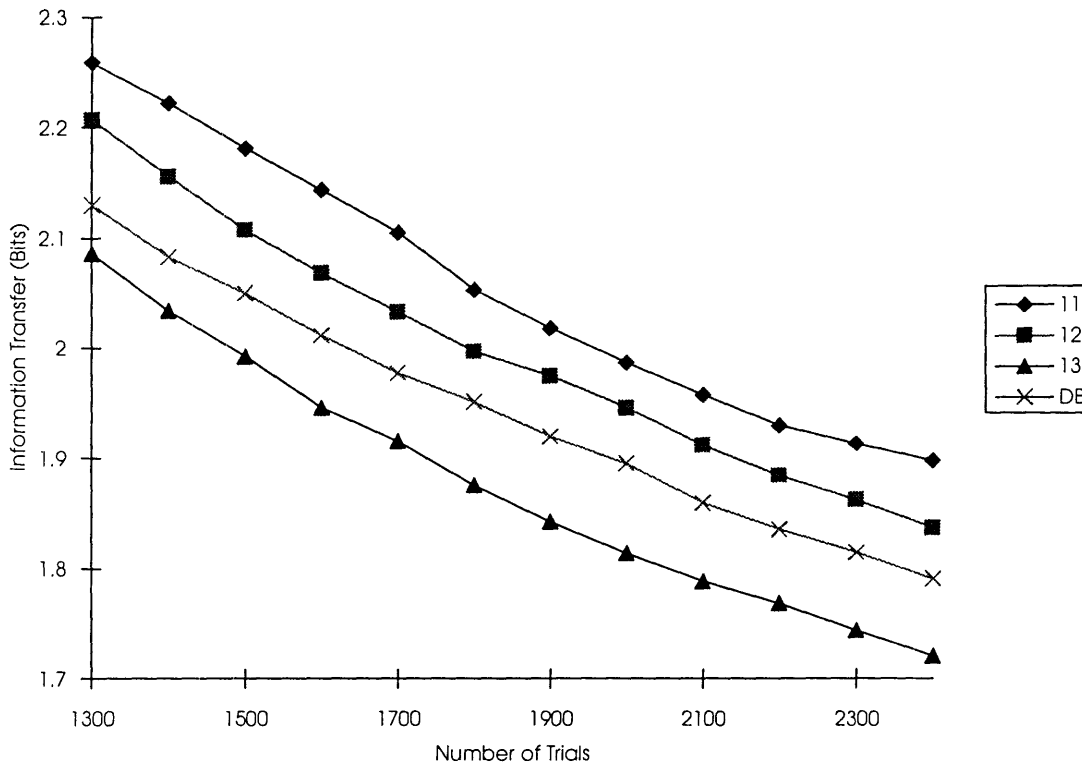


Figure 14: This graph shows the cumulative information transmission from 1300 to 2400 trials for subject DB and for the three simulation curves with error parameter $S=12$ (Min RMS error), $S=11$, and $S=13$. The horizontal axis shows the number of trials, and the vertical axis shows information transmission in bits.

Table 10: Interpolation Statistics- Subject DB

Error Parameter	Mean Error	Mean Magnitude Error	Root Mean Square Error	Information Transfer (32,400 trials)
12	0.055676	0.055676	0.056477	1.403745
11	0.112606	0.112606	0.113887	1.513605
13	-0.065967	0.065967	0.066848	1.301950

Table 10: This table shows the statistics used to interpolate information transmission for Subject DB. The error parameter is the value of S used for the simulation; The mean error is the average difference between the simulation curve and the actual data for the 12 points from 1300 trials to 2400 trials; the mean magnitude error is the average magnitude of error for the 12 points; and the RMS error is the root mean square error for the 12 points. The information transfer is the value for the simulated experiment with 32,400 trials (not corrected for bias).

Information Transmission: 1.28 Bits

Figure 15: Subject AS; Experiment 5 Information Transmission

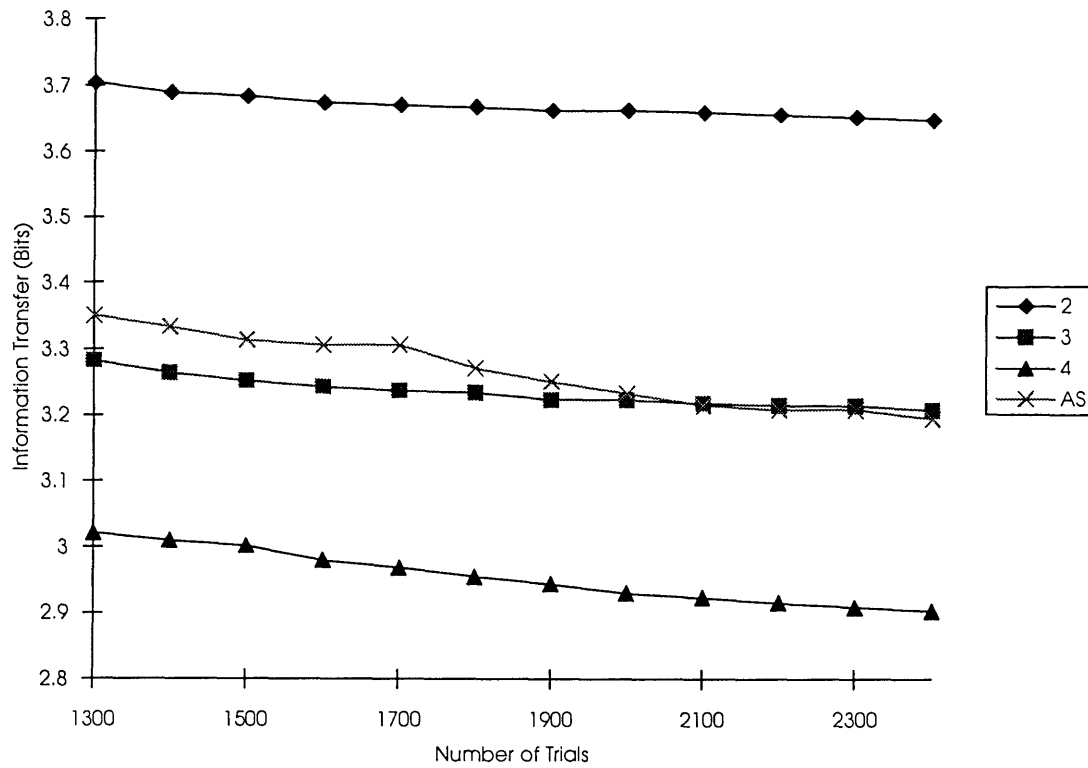


Figure 15: This graph shows the cumulative information transmission from 1300 to 2400 trials for subject AS and for the three simulation curves with error parameter $S=3$ (Min RMS error), $S=2$, and $S=4$. The horizontal axis shows the number of trials, and the vertical axis shows information transmission in bits.

Table 11: Interpolation Statistics- Subject AS

Error Parameter	Mean Error	Mean Magnitude Error	Root Mean Square Error	Information Transfer (32,400 trials)
3	-0.031576	0.036294	0.045284	3.131790
2	0.403566	0.403566	0.405351	3.606067
4	-0.311272	0.311272	0.311630	2.778457

Table 11: This table shows the statistics used to interpolate information transmission for Subject AS. The error parameter is the value of S used for the simulation; The mean error is the average difference between the simulation curve and the actual data for the 12 points from 1300 trials to 2400 trials; the mean magnitude error is the average magnitude of error for the 12 points; and the RMS error is the root mean square error for the 12 points. The information transfer is the value for the simulated experiment with 32,400 trials (not corrected for bias).

Information Transmission: 3.09 Bits

Figure 16: Subject JK; Experiment 5 Information Transmission

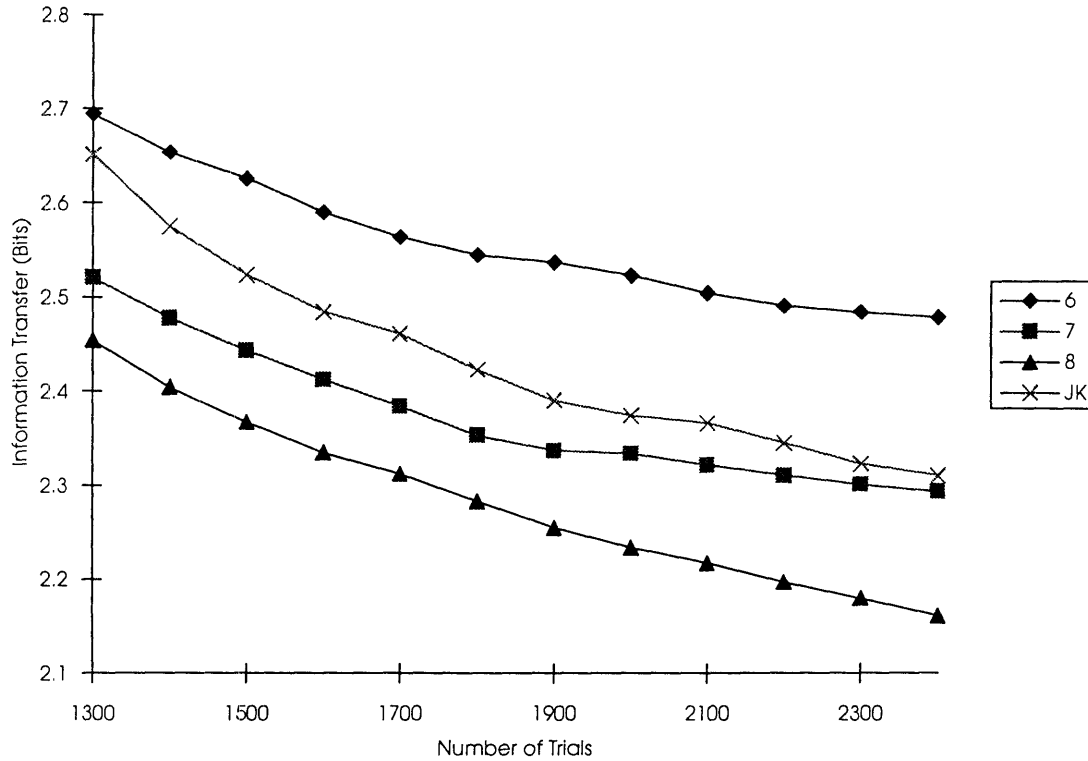


Figure 16: This graph shows the cumulative information transmission from 1300 to 2400 trials for subject JK and for the three simulation curves with error parameter $S=7$ (Min RMS error), $S=6$, and $S=8$. The horizontal axis shows the number of trials, and the vertical axis shows information transmission in bits.

Table 12: Interpolation Statistics- Subject JK

Error Parameter	Mean Error	Mean Magnitude Error	Root Mean Square Error	Information Transfer (32,400 trials)
7	-0.061611	0.061611	0.069098	2.080919
6	0.121733	0.121733	0.126727	2.272476
8	-0.152558	0.152558	0.153361	1.905569

Table 12: This table shows the statistics used to interpolate information transmission for Subject JK. The error parameter is the value of S used for the simulation; The mean error is the average difference between the simulation curve and the actual data for the 12 points from 1300 trials to 2400 trials; the mean magnitude error is the average magnitude of error for the 12 points; and the RMS error is the root mean square error for the 12 points. The information transfer is the value for the simulated experiment with 32,400 trials (not corrected for bias).

Information Transmission: 2.07 Bits

Figure 17: Subject CG; Experiment 5 Information Transmission

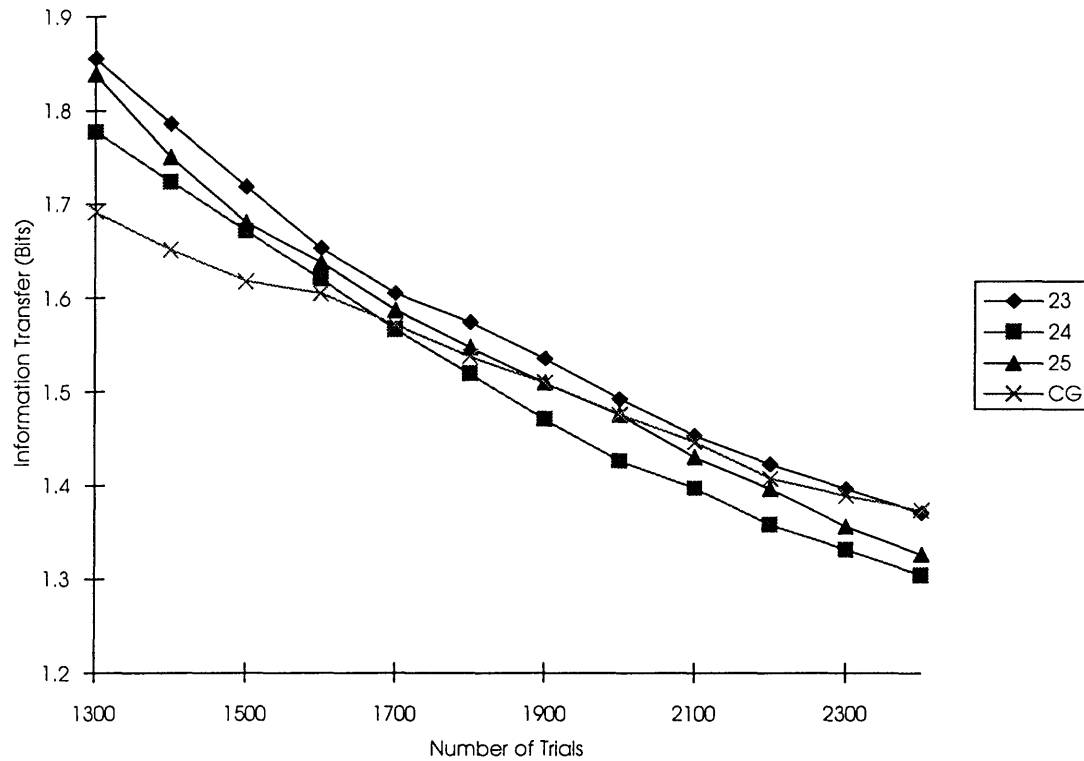


Figure 17: This graph shows the cumulative information transmission from 1300 to 2400 trials for subject CG and for the three simulation curves with error parameter $S=23$ (Min RMS error), $S=24$, and $S=25$. The horizontal axis shows the number of trials, and the vertical axis shows information transmission in bits.

Table 13: Interpolation Statistics- Subject CG

Error Parameter	Mean Error	Mean Magnitude Error	Root Mean Square Error	Information Transfer (32,400 trials)
24	-0.009319	0.047056	0.052428	0.607550
23	0.048948	0.049311	0.071534	0.652733
25	0.021590	0.039493	0.058034	0.567736

Table 13: This table shows the statistics used to interpolate information transmission for Subject CG. The error parameter is the value of S used for the simulation; The mean error is the average difference between the simulation curve and the actual data for the 12 points from 1300 trials to 2400 trials; the mean magnitude error is the average magnitude of error for the 12 points; and the RMS error is the root mean square error for the 12 points. The information transfer is the value for the simulated experiment with 32,400 trials (not corrected for bias).

Information Transmission: 0.52 Bits

11. Appendix C: Comparison of Trial Selection With and Without Replacement

In the first four experiments all of the data was collected in five blocks, and the identified parameter in each of these blocks each value of the identified parameter was randomly selected without replacement so that each stimulus occurred twenty times in each block. In the fifth experiment, the large number of blocks prevented a no replacement strategy for trial selection, so the data were collected in 100 trial blocks and the stimuli were randomly selected with replacement. In theory, it should be possible for the subjects to do slightly better in the no replacement experiments if they are able to keep track of the frequency of appearance of each stimulus. It was believed that this effect would be small for the number of occurrences for each stimulus (twenty) and the number of stimuli (six to ten) involved.

In order to verify that this effect was in fact small, subject DB repeated experiment three with a different trial selection strategy. The data were still collected in five 200 trial blocks, but in each block the stimuli were randomly selected with replacement. The resulting information transfer was 0.96 bits (adjusted for bias), which was nearly identical to the original information transfer from experiment four of 0.94 bits. Although this is only a single data sample, it does indicate that there is not a large difference in performance for subjects in the two trial selection strategies.

12. Appendix D: Information Transfer and Correctness Correlation Within Trials

The information transfer results in Experiment 5 which were determined using Houtsma's method are somewhat higher than those from the other two methods. If the information transfers for AS and JK are assumed to be slight overestimates, and the information transfer for CG is assumed to be an underestimate (this is indicated by the slopes of their data relative to the simulation curves), the information transfer in the two dimensional experiment (calculated using Houtsma's method) appears to be slightly higher than the sum of the two projection matrices from Experiment 5 for all of the subjects.

It was hypothesized that such an increase in the two dimensional information transfer could result if the subject performance in identifying m and τ tended to be good (a small difference between the response number and the stimulus number) during the same trials. Such a correlation between correctness in the two dimensions could be a result of inattention, if the subjects became fatigued during some of the trials and they performed poorly in both dimensions during those trials. It could also be caused by the actual stimulus values used in certain trials. In this experiment, the data by Houtsma and Yost (1978) indicate that discrimination of delay with attenuation in the reflection (referred to as pitch strength) and discrimination between different attenuations in the reflection at a given value of τ (shown in Figure 1 of the background section) both tend to be best at values of τ around 2 ms, and to degrade as τ decreases in both cases and as τ increases in the τ discrimination case. Furthermore, a large attenuation in the reflected signal tends to make both discrimination between attenuations in the reflected signal and the discrimination of delay more difficult. Thus it is likely that identification was easiest for both dimensions in trials when certain combinations of delay and reflection strength occurred in the stimulus.

The correlation between correctness (the magnitude of the difference between the stimulus number and response number) in m and τ for individual trials in Experiment 5 was verified by examining all of the trials by the 4 subjects. The average error

(magnitude difference between stimulus and response) in m was plotted for each of the 10 possible errors in τ . In other words, all of the trials in which the subject gave an exact response for τ were examined, and the average magnitude of error in m for those trials was computed. This was repeated for all of the trials where the subject response for τ was one different from the actual stimulus, and for all trials where the subject response was two different, and so on. Figure 18 shows the resulting graph. Magnitudes of error in τ greater than 5 are not plotted because only a very small number of trials had responses in that range. The graph clearly shows a correlation between the magnitude of error in the response for m and the magnitude of error in the response for τ . The same procedure was performed to plot the average magnitude of error in the response for τ versus the magnitude error in the response for m . These results are shown in Figure 19, and they show a reduction in the average error of the response for τ at the error for the response for m increases from 0 to 1, but a clear increase in the error in the delay response as the error in the reflection strength response increases beyond 1. This correlation between τ identification performance and m identification performance occurred in the data for all four subjects.

Figure 18: Average Magnitude Error in m vs. Error in τ

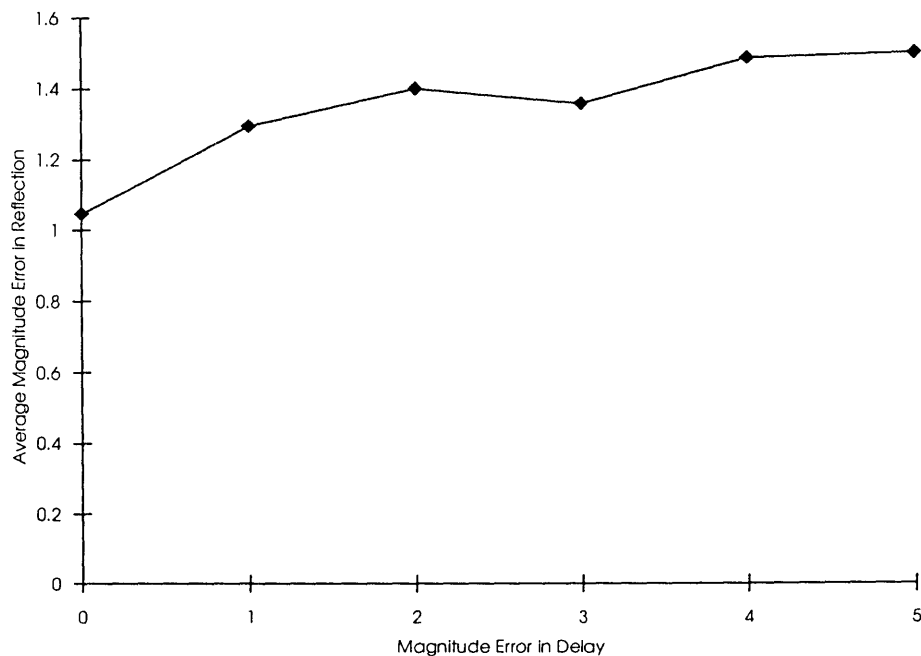
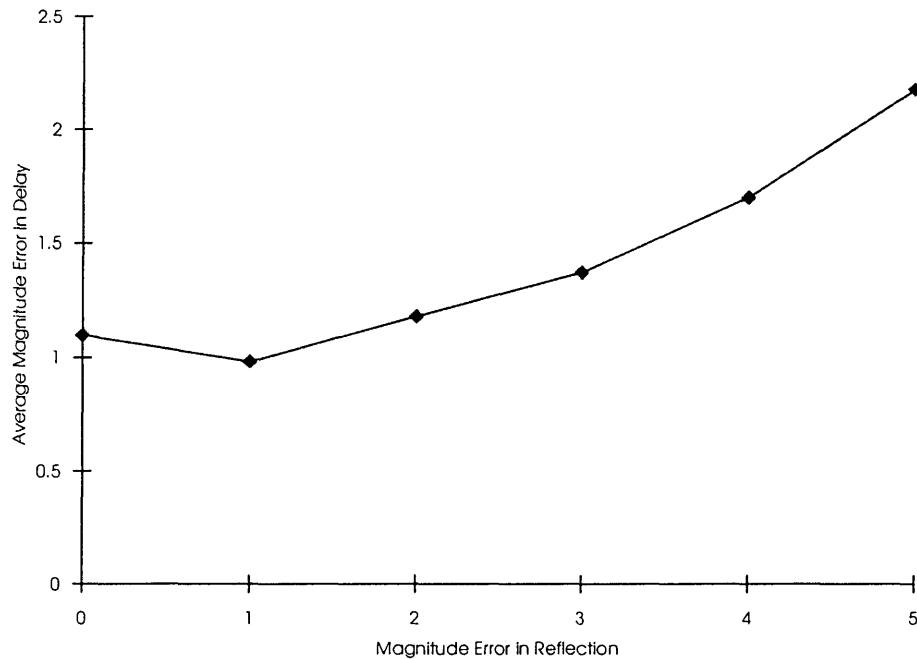


Figure 19: Average Magnitude Error in τ vs. Error in m



Once the correlation between the errors in the two dimensions within individual trials was established, it was necessary to determine what the impact of such a correlation would be on the information transmission of a two dimensional experiment. A number of simulated experiments were executed to determine how correlated errors interfere with information transmission. In each experiment, 32,400 trials were executed in a two dimensional experiment with 6 response categories in one dimension and 10 response categories in the other dimension. This parallels the conditions in Experiment 5. Also, in each trial the subjects were equally likely to perform poorly or well in the identification task for each parameter. One experiment was run where the performance in each dimension was independent (two random variables were determined in for each trial, one to choose whether there would be good or bad performance in each dimension), one experiment was run where good performance was positively correlated (one random variable established whether performance in both dimensions would be good or bad), and one experiment was run where good performance was negatively correlated (one random variable determined whether performance would be good in the 6 category dimension and bad in the 10 category dimension, or bad in the 6 category dimension and good in the

10 category dimension). These three experiments were run with two different definitions of good and bad performance. The first time good performance was defined as no error in the responses for a particular dimension and bad performance was defined as a totally random response for a particular dimension. The second time good performance was an error randomly chosen from +1, 0, and -1 (errors were always chosen repeatedly until a valid response number was achieved), and bad performance was an error equally distributed between -5 and 5 for the 10 category dimension and -3 and 3 for the 6 category dimension. The resulting information transfers (uncorrected for bias) are shown in Table 14. These information transfers are the average of two runs for each experiment. In all experimental runs the results fell within 0.02 bits of the average value shown in the table, so the results are fairly repeatable. The results clearly show that a positive or negative correlation between identification performance for the two dimensions within the individual trials results in a larger information transfer than when the correctness of the two parameters is independent.

Table 14: Information Transfer and Correctness Correlation

Good Performance Error Range	± 0	± 1
Bad Performance Error Range (10/6)	$\pm 9/\pm 5$	$\pm 5/\pm 3$
Uncorrelated	1.64	1.05
Positive Correlation	2.08	1.17
Negative Correlation	2.09	1.19

At first it seems odd that the information transfer increases when the correctness in the two dimensions is negatively correlated. The reason for this increase is the additional knowledge about τ which is gained by knowing that the response for m is incorrect. For instance, in the first experiment where good performance was perfect and poor performance was random, any value of m other than the actual value was always paired with a perfect response for τ , and any incorrect response for τ was always paired with an exactly correct response for m . This will increase the information transfer significantly.

13. Appendix E: Response Biases

This appendix shows the response biases of the subjects in each of the experiments. The biases are calculated from the maximum likelihood estimates of the criteria $C_0 \dots C_m$. These maximum likelihood estimates are made by the same program that finds the values of the sensitivity d' . The biases are the difference between the actual criterion separating two responses and the minimum error location of that criterion, which would be halfway between the adjacent means $\mu(S_i)$ and $\mu(S_{i+1})$. In the 0-1 interval, a negative value indicates a bias in favor of response 1, and a positive value indicates a bias in favor of response 0. A strong tendency to choose (or not choose) a particular response is indicated by the slope of the line across a stimulus. If the line from the 1-2 interval to the 2-3 interval goes from a positive to a negative value (as it does in this case for subject CG in Experiment 1), this indicates that the subject is biased in favor of Stimulus 1 over Stimulus 2 more than he is biased in favor of Stimulus 2 over Stimulus 3. In other words, the lower cutoff criterion for Response 2 (C_2) is pushed to the right on the X axis (assuming that the X axis increases from left to right), and the upper cutoff criterion for Response 2 (C_3) is pushed to the left on the X axis. This decreases the range of values of the internal variable X that result in Response 2, thereby causing a general bias against response 2. Similarly, a strong positive slope indicates a bias in favor of a particular response.

Figure 20: Criterion Biases in Experiment 1, Condition 1

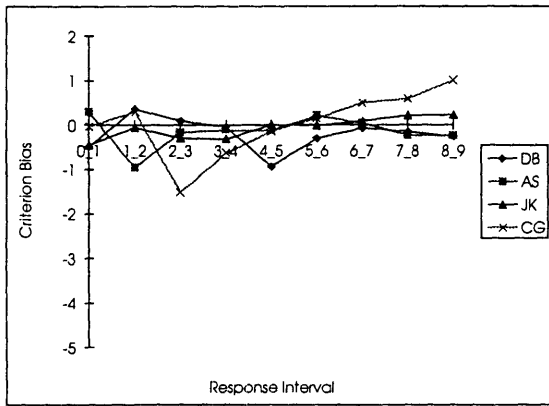


Figure 21: Criterion Biases in Experiment 1, Condition 2

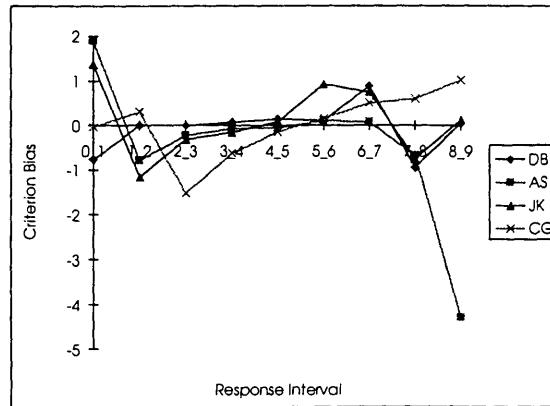


Figure 22: Criterion Biases in Supplementary Experiment 1

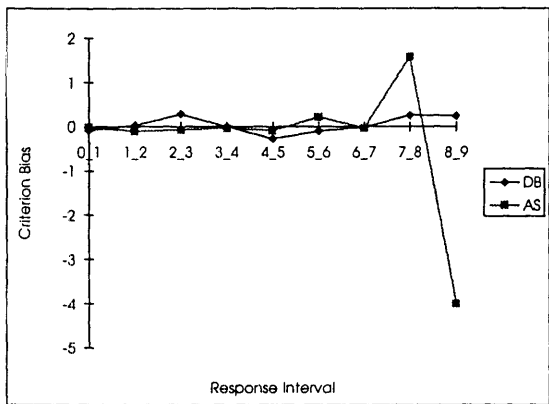


Figure 23: Criterion Biases in Experiment 3

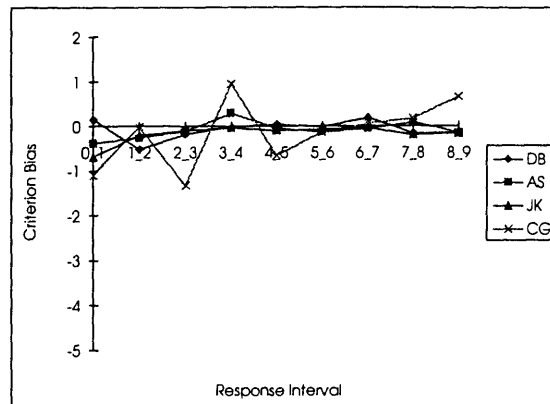
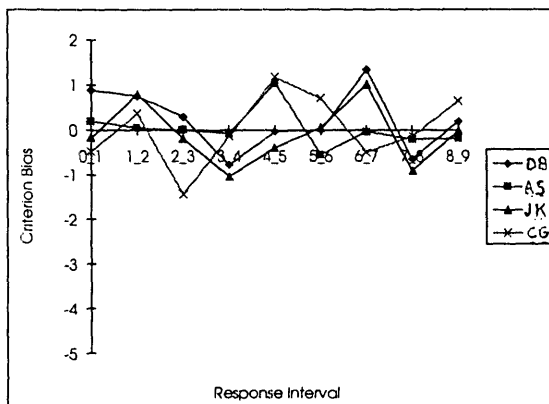


Figure 24: Criterion Biases for τ in Experiment 5



These figures show the response biases for each subject in each of the 5 τ -identification experiments. A positive bias favors the response on the left of the interval (the lower numbered response), and a negative bias favors the response on the right of the interval (the higher numbered response).

Figure 25: Criterion Biases in Experiment 2, Condition 1

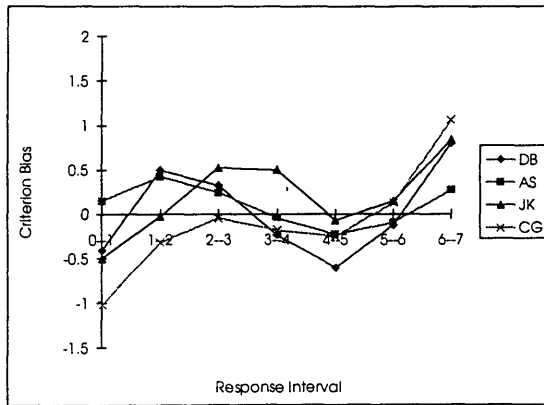


Figure 26: Criterion Biases in Experiment 2, Condition 2

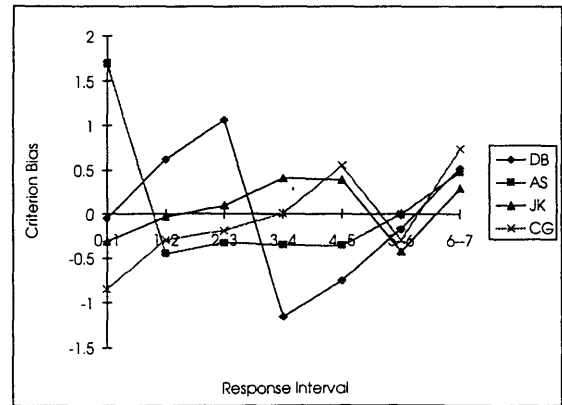


Figure 29: Criterion Biases in Supplemental Experiment 2

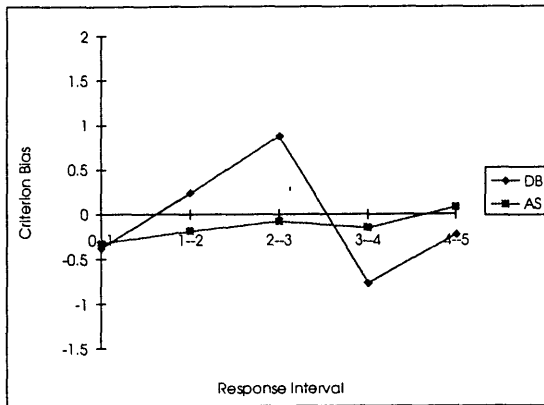


Figure 27: Criterion Biases in Experiment 4

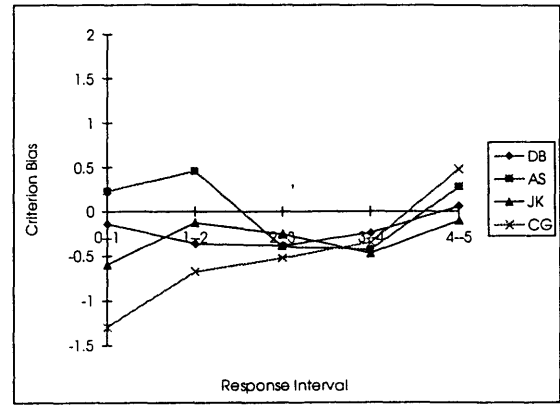
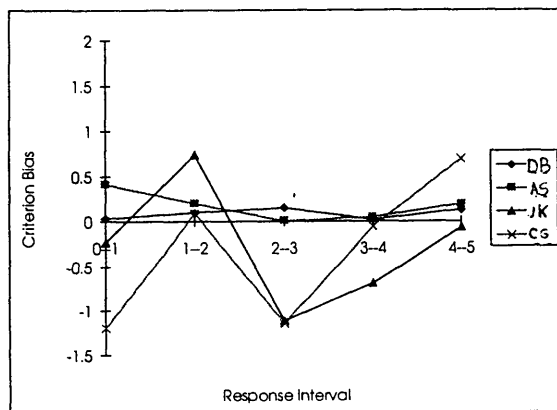


Figure 28: Criterion Biases for m in Experiment 5



These figures show the response biases for each subject in each of the 5 m-identification experiments. A positive bias favors the response on the left of the interval (the lower numbered response), and a negative bias favors the response on the right of the interval (the higher numbered response). Note that the ordinate scale is different than the ordinate scale used in the τ identification experiments.

14. Appendix F: Confusion Matrices

This appendix contains the confusion matrices for Experiments 1-5 and for Supplemental Experiments 1 and 2. Note that the matrices shown for Experiment 5 are actually projections of the two-dimensional confusion matrix, created by summing together all trials with the same stimulus and response values in one parameter and ignoring the other parameter. Each confusion matrix is accompanied by a graph showing a three dimensional representation of that confusion matrix, and by the information transfer value associated with that confusion matrix. The information transfers were calculated using the maximum likelihood estimator and are adjusted for bias, as described in Appendix A.

Table 15: Subject DB; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	44	19	4	1	-	-	-	-	-	1
	1	40	66	31	11	2	3	-	-	-	-
	2	7	10	42	6	3	1	1	-	-	-
	3	2	5	12	48	6	2	2	-	-	-
	4	1	-	7	21	47	11	6	1	3	-
	5	5	-	3	6	26	49	11	6	3	1
	6	-	-	1	5	10	23	56	17	4	2
	7	-	-	-	-	4	6	12	45	12	2
	8	-	-	-	2	1	3	7	24	52	10
	9	1	-	-	-	1	2	5	7	26	84

Table 15: This table shows the confusion matrix for subject DB in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 30: Subject DB; Identify τ , $m=.97$

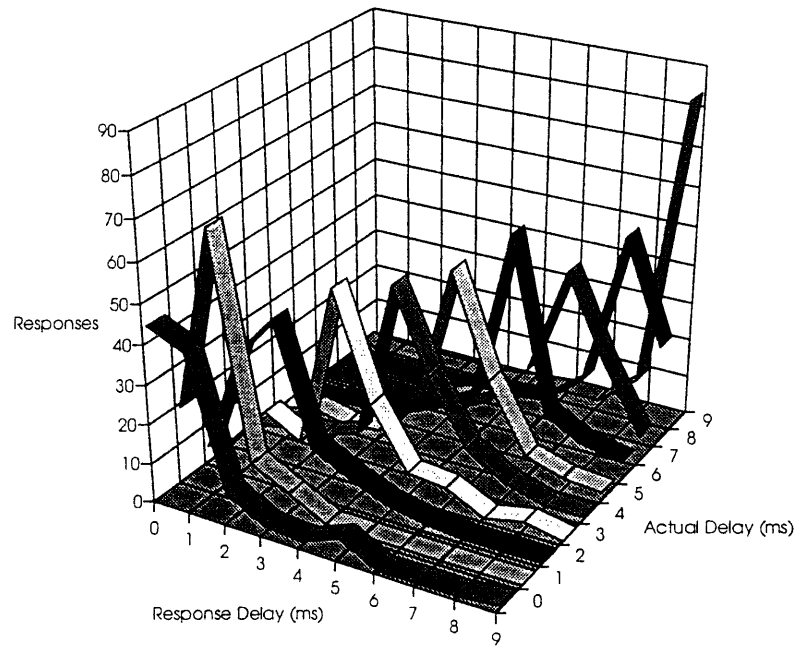


Figure 30: This shows a three dimensional representation of the confusion matrix for subject DB in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.35 bits

Table 16: Subject AS; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	94	-	-	-	2	6	-	-	-	-
	1	-	72	21	-	-	-	-	-	-	-
	2	1	19	65	3	1	3	-	-	-	-
	3	1	5	10	92	-	2	5	-	-	-
	4	-	1	2	1	67	8	-	-	-	-
	5	4	2	2	3	25	81	-	-	-	-
	6	-	-	-	1	-	-	95	1	-	-
	7	-	-	-	-	-	-	-	98	-	1
	8	-	-	-	-	5	-	-	-	100	-
	9	-	1	-	-	-	-	-	1	-	99

Table 16: This table shows the confusion matrix for subject AS in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 31: Subject AS; Identify τ , $m=.97$

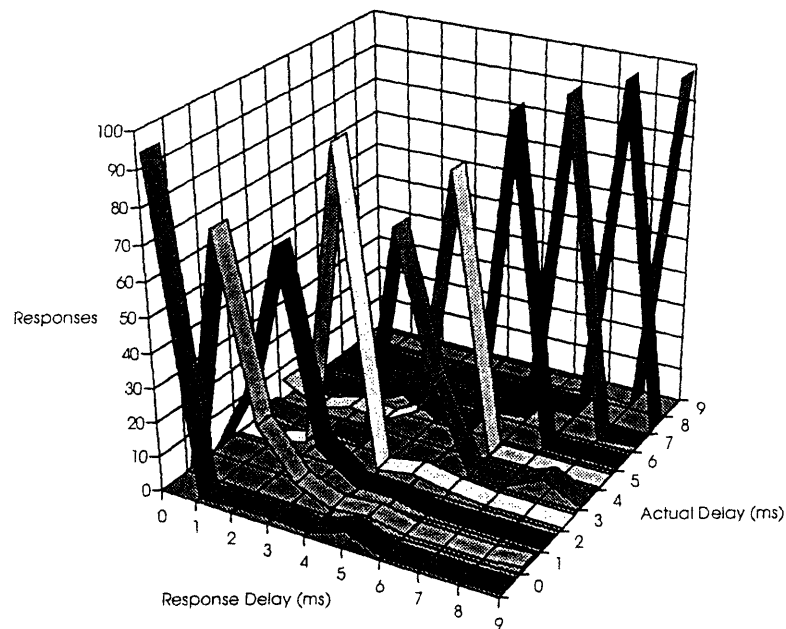


Figure 31: This shows a three dimensional representation of the confusion matrix for subject AS in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 2.61 bits

Table 17: Subject JK; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	63	2	4	1	-	-	-	-	-	-
	1	4	83	23	1	-	-	-	-	-	-
	2	19	13	47	5	3	-	-	-	-	-
	3	1	1	12	62	14	3	3	-	1	-
	4	6	1	9	8	67	15	5	5	3	1
	5	1	-	-	10	5	71	5	5	-	-
	6	1	-	1	11	6	8	81	6	4	1
	7	1	-	2	2	3	2	6	67	21	1
	8	1	-	2	-	2	-	-	17	71	6
	9	3	-	-	-	-	1	-	-	-	91

Table 17: This table shows the confusion matrix for subject JK in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 32: Subject JK; Identify τ , $m=.97$

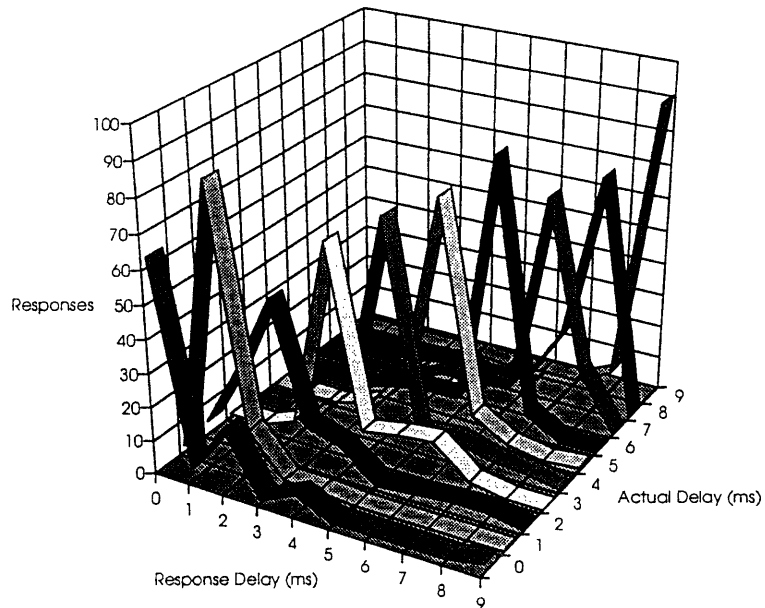


Figure 32: This shows a three dimensional representation of the confusion matrix for subject JK in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.84 bits

Table 18: Subject CG; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	95	1	-	1	-	-	1	-	-	2
	1	-	84	3	-	-	-	-	-	-	-
	2	1	11	35	4	3	1	2	-	1	-
	3	1	2	28	26	5	6	4	3	4	4
	4	1	1	17	16	23	9	14	10	13	8
	5	-	1	11	21	27	46	21	19	20	22
	6	-	-	6	23	26	24	41	28	34	32
	7	-	-	-	7	15	13	16	36	15	20
	8	-	-	-	2	-	1	1	4	12	7
	9	2	-	-	-	1	-	-	-	1	5

Table 18: This table shows the confusion matrix for subject CG in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 33: Subject CG; Identify τ , $m=.97$

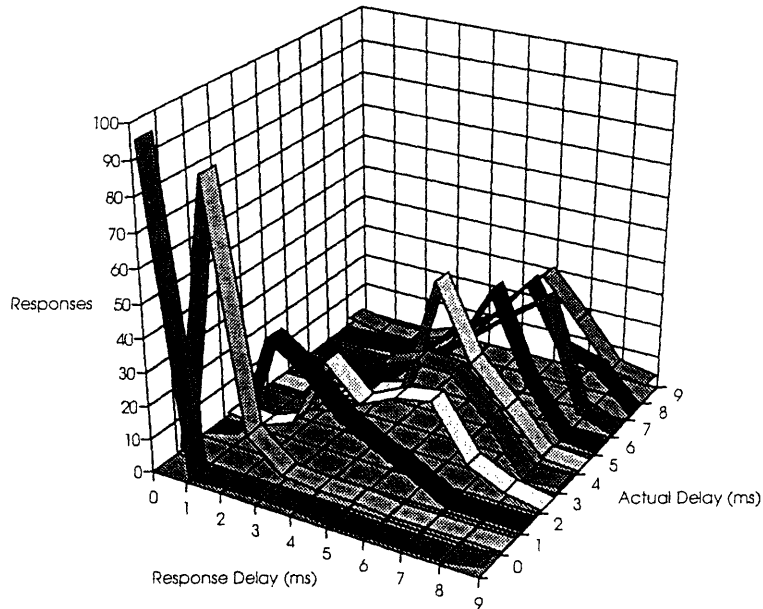


Figure 33: This shows a three dimensional representation of the confusion matrix for subject CG in the first condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.98 bits

Table 19: Subject DB; Identify τ , $m=.5$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	36	18	4	1	-	-	-	1	-	1
	1	19	55	25	10	4	1	-	1	-	-
	2	16	13	47	19	7	1	-	-	-	-
	3	12	9	14	42	18	7	2	1	-	1
	4	5	4	6	18	54	17	8	2	3	-
	5	9	-	3	8	12	56	9	1	1	1
	6	3	-	-	-	2	13	68	13	10	2
	7	-	1	-	1	2	2	7	49	14	2
	8	-	-	1	1	1	3	6	29	51	23
	9	-	-	-	-	-	-	-	3	21	70

Table 19: This table shows the confusion matrix for subject DB in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 34: Subject DB; Identify τ , $m=.5$

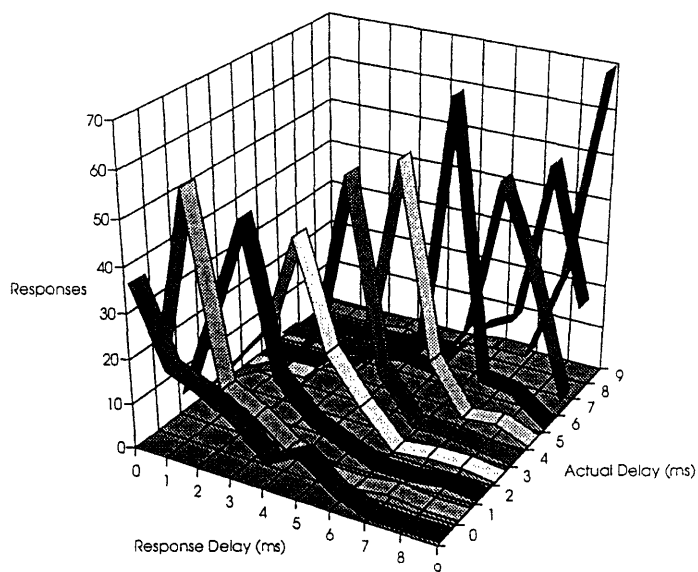


Figure 34: This shows a three dimensional representation of the confusion matrix for subject DB in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.31 bits

Table 20: Subject AS; Identify τ , $m=.5$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	99	1	1	1	1	2	-	-	-	-
	1	-	84	18	-	-	-	-	-	-	-
	2	-	14	69	1	3	-	-	-	-	-
	3	-	1	5	88	-	-	8	-	-	-
	4	-	-	1	1	81	3	-	-	3	-
	5	1	-	6	-	14	95	-	1	-	-
	6	-	-	-	8	-	-	91	-	-	-
	7	-	-	-	-	-	-	-	99	-	-
	8	-	-	-	-	1	-	1	-	97	-
	9	-	-	-	1	-	-	-	-	-	100

Table 20: This table shows the confusion matrix for subject AS in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject

Figure 35: Subject AS; Identify τ , $m=.5$

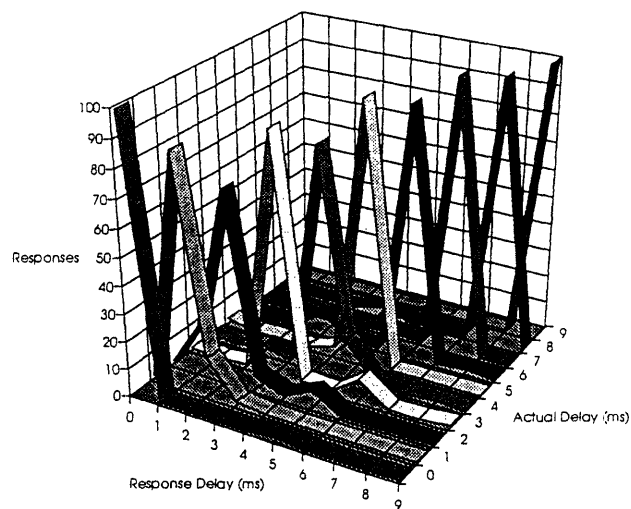


Figure 35: This shows a three dimensional representation of the confusion matrix for subject AS in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 2.77 bits

Table 21: Subject JK; Identify τ , $m=.5$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	92	16	-	-	-	-	-	-	-	-
	1	2	75	9	-	1	-	-	-	-	1
	2	1	5	65	6	2	2	-	2	-	-
	3	-	1	17	70	2	4	3	1	-	-
	4	-	1	4	8	66	10	3	4	5	-
	5	-	-	4	10	10	54	2	4	2	-
	6	-	-	-	5	10	13	74	27	7	2
	7	1	2	1	-	6	14	8	52	18	1
	8	2	-	-	1	3	3	8	10	65	10
	9	2	-	-	-	-	-	2	-	3	86

Table 21: This table shows the confusion matrix for subject JK in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject

Figure 36: Subject JK; Identify τ , $m=.5$

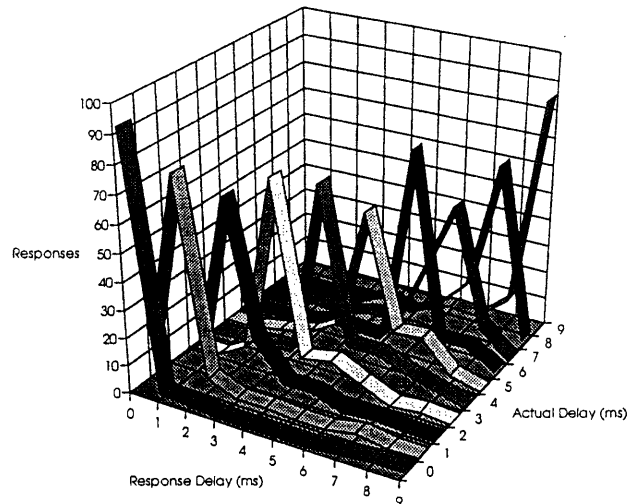


Figure 36: This shows a three dimensional representation of the confusion matrix for subject JK in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.81 bits

Table 22: Subject CG; Identify τ , $m=.5$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	92	2	-	-	-	-	-	-	-	2
	1	3	82	7	3	1	-	1	-	-	-
	2	-	14	38	10	3	4	6	5	6	2
	3	1	1	21	30	15	9	10	6	2	3
	4	3	1	18	17	39	20	15	13	9	5
	5	1	-	10	20	20	35	18	17	5	7
	6	-	-	5	17	12	21	29	22	13	11
	7	-	-	1	3	7	8	12	23	19	21
	8	-	-	-	-	3	3	7	10	37	26
	9	-	-	-	-	-	-	2	4	9	23

Table 22: This table shows the confusion matrix for subject CG in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject

Figure 37: Subject CG; Identify τ , $m=.5$

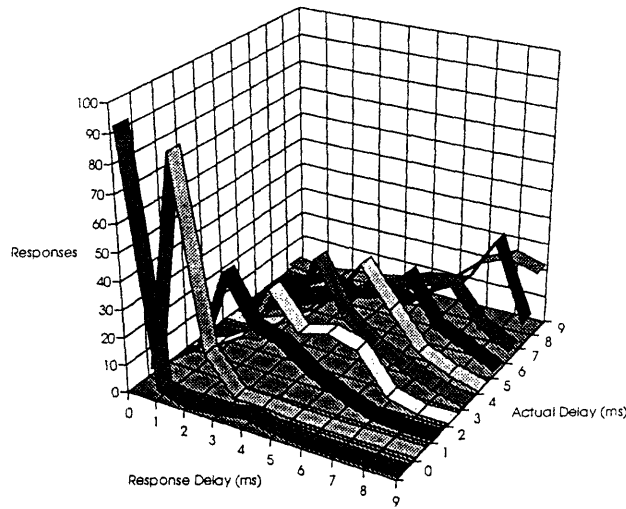


Figure 37: This shows a three dimensional representation of the confusion matrix for subject JK in the second condition of Experiment 1, where the subject is asked to identify the delay τ with m fixed at 0.5. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.02 bits

Table 23: Subject DB; Identify m , $\tau=5$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	34	31	23	4	2	-	-	-
	1	47	41	38	22	4	1	-	-
	2	12	15	25	13	4	2	2	1
	3	6	10	11	25	10	6	1	1
	4	-	3	1	15	14	13	6	7
	5	-	-	2	15	37	32	30	27
	6	-	-	-	6	23	39	45	41
	7	1	-	-	-	6	7	16	23

Table 23: This table shows the confusion matrix for subject DB in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 38: Subject DB; Identify m , $\tau=5$ ms

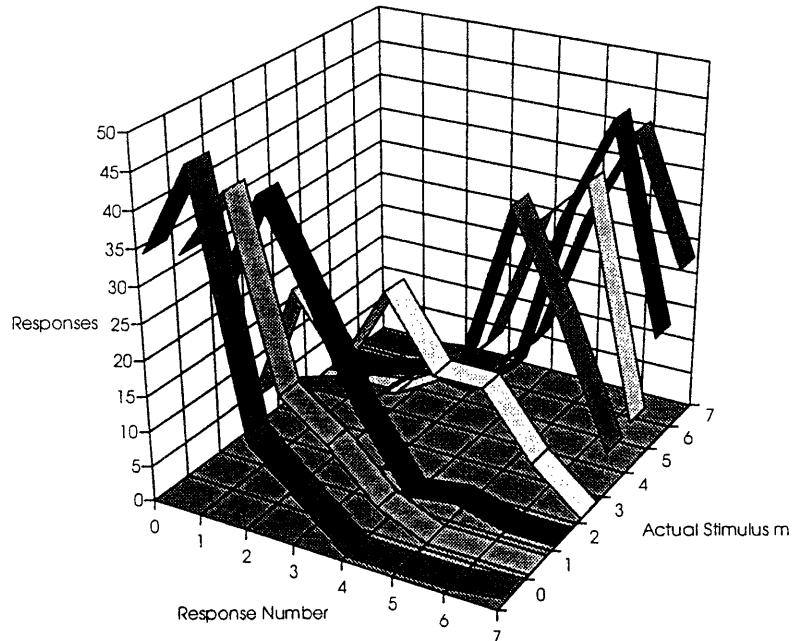


Figure 38: This shows a three dimensional representation of the confusion matrix for subject DB in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.75 bits

Table 24: Subject AS; Identify m , $\tau=5$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	55	49	35	7	1	-	-	-
	1	25	28	21	9	1	-	-	-
	2	19	19	22	19	2	1	-	-
	3	-	4	17	33	27	6	1	1
	4	-	-	4	20	27	22	10	7
	5	-	-	1	9	21	25	29	21
	6	-	-	-	2	14	30	28	26
	7	1	-	-	1	7	16	32	45

Table 24: This table shows the confusion matrix for subject AS in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 39: Subject AS; Identify m , $\tau=5$ ms

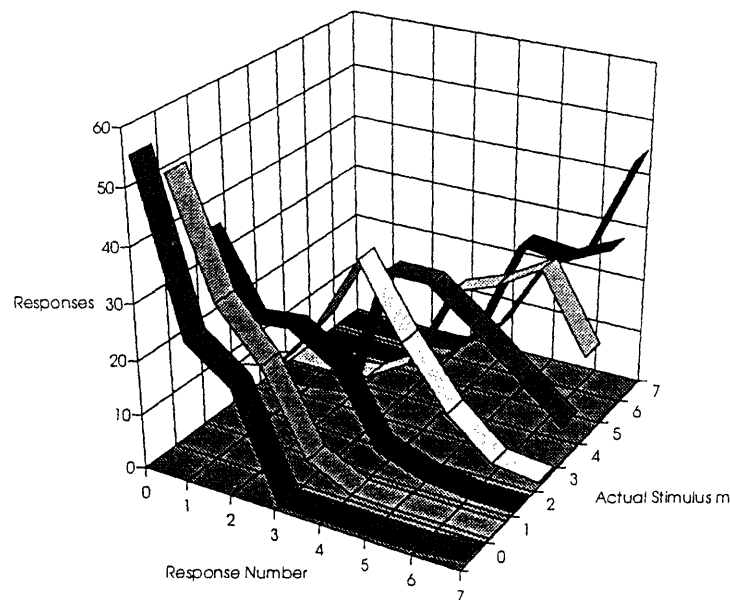


Figure 39: This shows a three dimensional representation of the confusion matrix for subject AS in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.90 bits

Table 25: Subject JK; Identify m , $\tau=5$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	30	25	15	16	5	-	-	-
	1	32	25	31	10	6	-	-	-
	2	32	34	37	32	9	4	1	-
	3	6	15	14	23	28	5	3	5
	4	-	1	3	14	16	17	11	9
	5	-	-	-	3	23	44	37	28
	6	-	-	-	2	13	19	34	33
	7	-	-	-	-	-	11	14	25

Table 25: This table shows the confusion matrix for subject JK in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 40: Subject JK; Identify m , $\tau=5$ ms

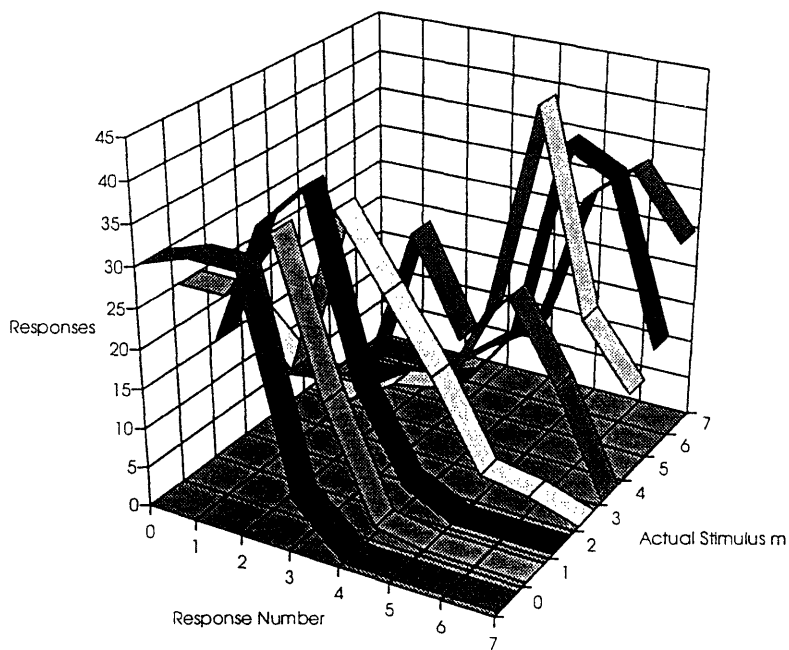


Figure 40: This shows a three dimensional representation of the confusion matrix for subject JK in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.75 bits

Table 26: Subject CG; Identify m , $\tau=5$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	11	18	16	3	-	-	-	-
	1	32	22	19	12	4	1	-	1
	2	29	29	25	22	12	4	3	-
	3	8	17	13	21	14	9	7	4
	4	9	8	13	22	27	17	11	12
	5	9	6	11	14	25	34	29	28
	6	1	-	3	4	14	30	41	40
	7	1	-	-	2	4	5	9	15

Table 26: This table shows the confusion matrix for subject CG in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 41: Subject CG; Identify m , $\tau=5$ ms

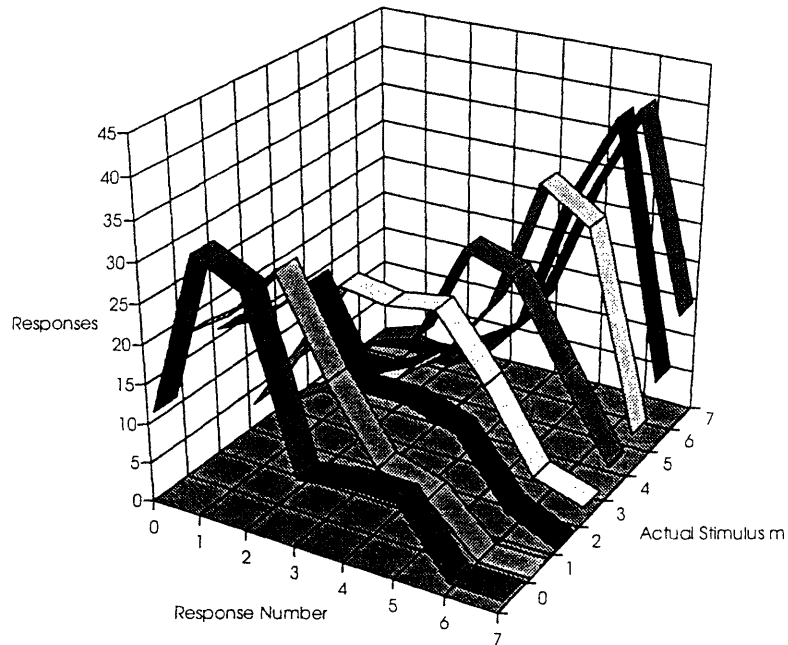


Figure 41: This shows a three dimensional representation of the confusion matrix for subject CG in the first condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.44 bits

Table 27: Subject DB; Identify m , $\tau=9$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	41	43	26	13	2	-	-	-
	1	42	36	24	12	-	-	-	-
	2	14	16	27	17	2	-	1	-
	3	2	5	10	17	10	3	2	1
	4	-	-	9	15	26	14	19	9
	5	1	-	1	16	32	29	23	28
	6	-	-	2	9	18	28	29	29
	7	-	-	1	1	10	26	26	33

Table 27: This table shows the confusion matrix for subject DB in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 9 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 42: Subject DB; Identify m , $\tau=9$ ms

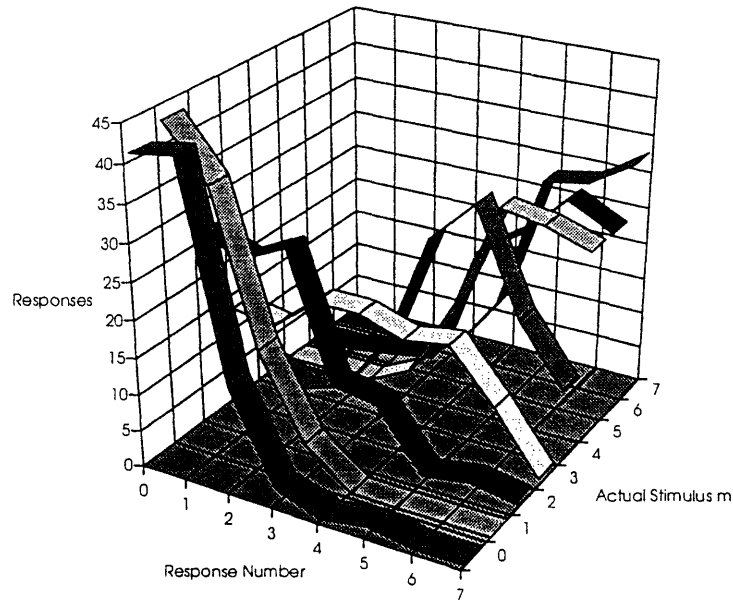


Figure 42: This shows a three dimensional representation of the confusion matrix for subject DB in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.77 bits

Table 28: Subject AS; Identify m , $\tau=9$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	92	57	18	3	-	1	-	1
	1	6	31	24	9	1	-	-	-
	2	-	11	28	22	8	2	2	-
	3	2	1	23	22	13	7	8	3
	4	-	-	4	19	26	15	13	17
	5	-	-	2	12	29	26	30	22
	6	-	-	1	11	15	29	21	16
	7	-	-	-	2	8	20	26	41

Table 28: This table shows the confusion matrix for subject AS in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 9 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 43: Subject AS; Identify m , $\tau=9$ ms

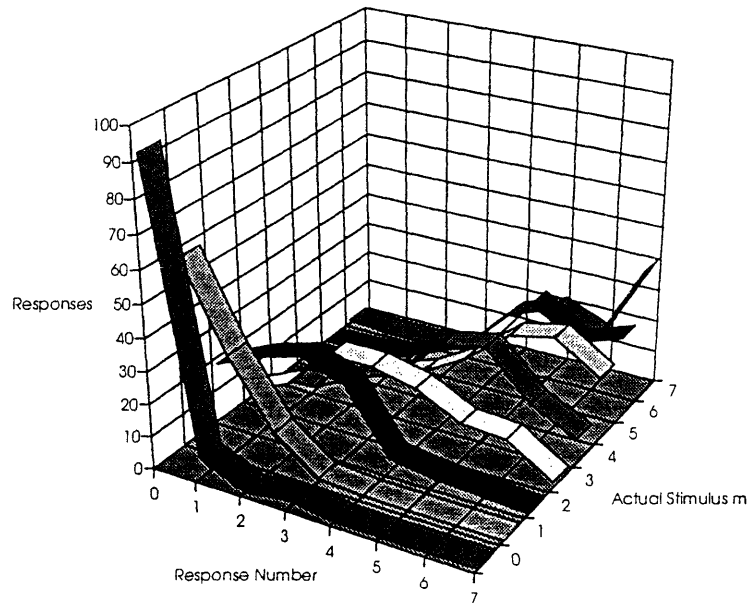


Figure 43: This shows a three dimensional representation of the confusion matrix for subject AS in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.88 bits

Table 29: Subject JK; Identify m, $\tau=9$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	42	30	8	6	-	-	-	-
	1	35	41	17	12	3	2	1	1
	2	16	22	33	24	15	10	7	9
	3	3	4	17	20	26	14	21	14
	4	1	2	7	14	18	15	14	18
	5	1	1	10	12	11	19	16	14
	6	2	-	6	6	16	24	20	17
	7	-	-	2	6	11	16	21	27

Table 29: This table shows the confusion matrix for subject JK in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 9 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 44: Subject JK; Identify m, $\tau=9$ ms

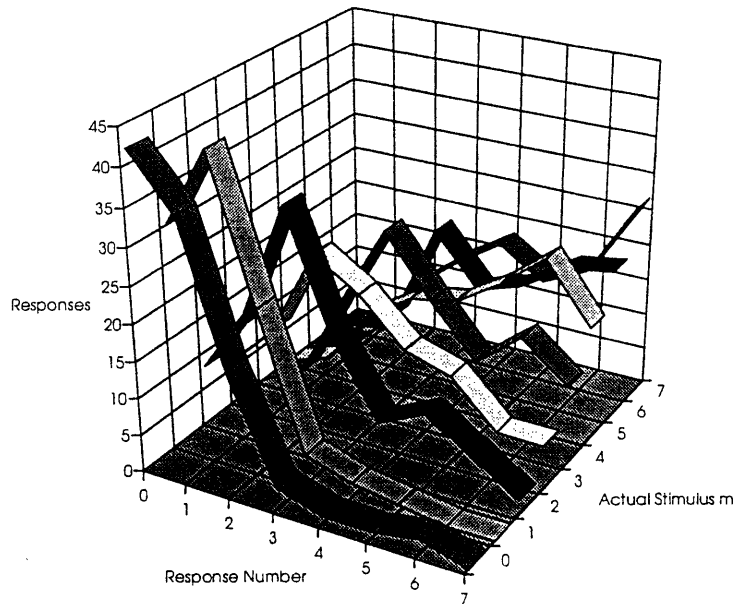


Figure 44: This shows a three dimensional representation of the confusion matrix for subject JK in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.48 bits

Table 30: Subject CG; Identify m , $\tau=9$ ms

		S t i m u l u s							
		0	1	2	3	4	5	6	7
R e s p o n s e	0	24	16	6	-	-	-	-	-
	1	42	32	19	8	3	3	-	6
	2	15	25	28	18	7	9	3	3
	3	9	11	19	26	22	11	6	7
	4	5	8	19	23	24	20	26	17
	5	5	6	6	17	21	32	25	24
	6	-	2	3	8	20	19	30	24
	7	-	-	-	-	3	6	10	19

Table 30: This table shows the confusion matrix for subject CG in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 9 ms. The vertical columns show the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 45: Subject CG; Identify m , $\tau=9$ ms

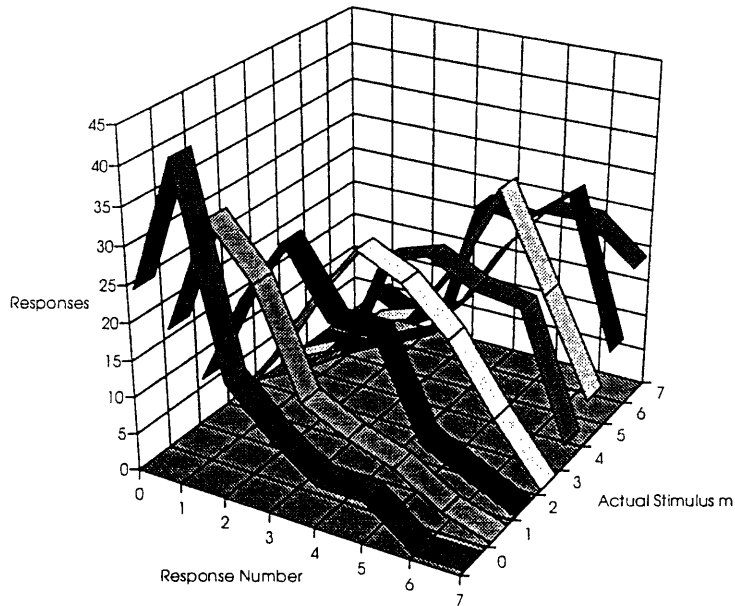


Figure 45: This shows a three dimensional representation of the confusion matrix for subject CG in the second condition of Experiment 2, where the subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The bottom right axis shows the stimulus number presented (see Table 2 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.43 bits

Table 31: Subject DB; Identify τ , m roved

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	37	27	27	16	10	3	1	2	1	1
	1	32	40	29	25	13	12	5	7	1	-
	2	8	10	16	11	9	4	4	1	-	-
	3	5	11	13	28	11	2	3	2	2	-
	4	4	6	5	9	37	15	9	2	2	2
	5	3	2	7	5	8	42	10	6	1	-
	6	1	1	1	5	6	17	52	25	4	1
	7	1	-	2	-	2	1	7	38	4	3
	8	6	2	-	1	3	4	9	13	63	9
	9	3	1	-	-	1	-	-	4	22	84

Table 31: This table shows the confusion matrix for subject DB in the third experiment, where the subject is asked to identify the delay τ with m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 46: Subject DB; Identify τ , m roved

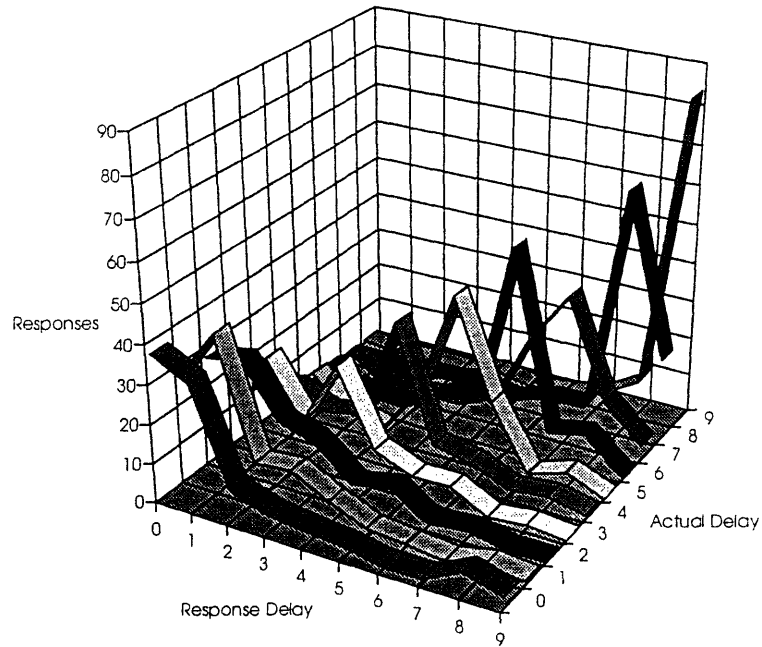


Figure 46: This shows a three dimensional representation of the confusion matrix for subject DB in the third experiment, where the subject is asked to identify the delay τ with m roved. The bottom right axis shows the stimulus number of τ (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.94 bits

Table 32: Subject AS; Identify τ , m roved

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	62	13	4	3	3	-	1	1	2	1
	1	3	49	15	15	-	-	-	1	-	1
	2	7	9	50	18	5	1	-	2	-	-
	3	16	17	15	48	25	6	2	2	6	1
	4	4	3	10	8	48	-	1	1	-	-
	5	1	-	-	1	5	77	3	4	1	-
	6	1	1	-	2	5	5	92	-	3	-
	7	-	-	2	3	2	8	-	85	3	1
	8	1	3	1	1	-	1	1	3	85	-
	9	5	5	3	1	7	2	-	1	-	96

Table 32: This table shows the confusion matrix for subject AS in the third experiment, where the subject is asked to identify the delay τ with m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 47: Subject AS; Identify τ , m roved

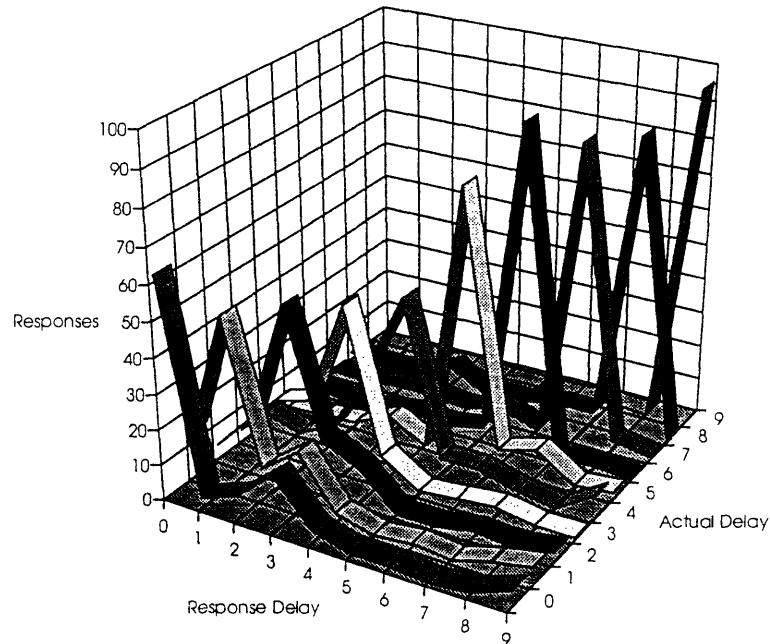


Figure 47: This shows a three dimensional representation of the confusion matrix for subject AS in the third experiment, where the subject is asked to identify the delay τ with m roved. The bottom right axis shows the stimulus number of τ (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.76 bits

Table 33: Subject JK; Identify τ , m roved

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	30	26	15	12	6	2	2	0	0	1
	1	22	28	22	11	11	1	1	3	2	0
	2	14	14	26	11	10	3	0	2	0	0
	3	8	14	14	30	15	9	2	1	0	0
	4	6	1	7	14	23	8	5	2	1	0
	5	1	1	1	5	13	51	15	5	0	0
	6	2	3	1	7	9	15	60	13	6	1
	7	3	4	8	4	2	5	7	50	4	4
	8	9	7	3	4	5	3	7	20	84	6
	9	5	2	3	2	6	3	1	4	3	88

Table 33: This table shows the confusion matrix for subject JK in the third experiment, where the subject is asked to identify the delay τ with m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 48: Subject JK; Identify τ , m roved

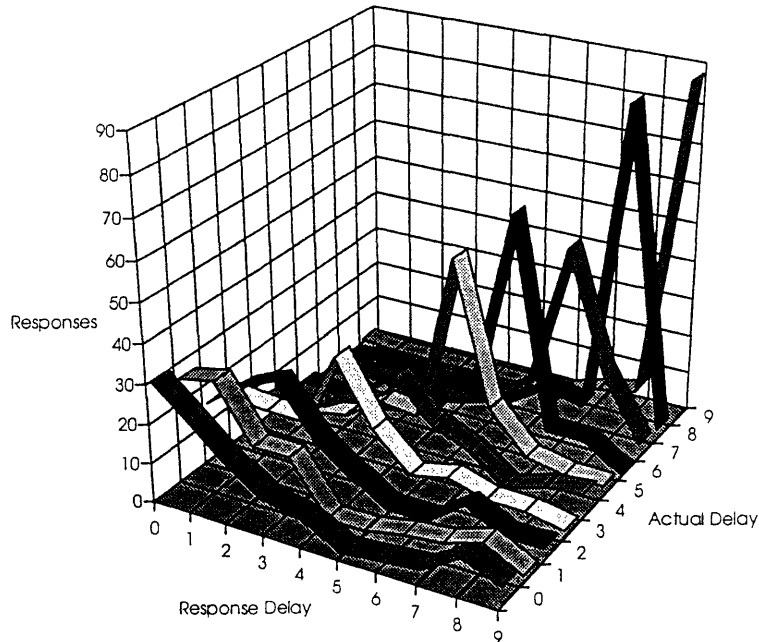


Figure 48: This shows a three dimensional representation of the confusion matrix for subject JK in the third experiment, where the subject is asked to identify the delay τ with m roved. The bottom right axis shows the stimulus number of τ (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.98 bits

Table 34: Subject CG; Identify τ , m roved

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	16	5	3	1	-	-	-	-	-	-
	1	22	26	15	9	4	5	2	2	1	3
	2	19	24	40	20	19	10	12	3	1	2
	3	14	16	14	37	14	7	7	8	1	4
	4	10	10	15	13	36	24	13	2	7	2
	5	8	8	9	13	15	30	18	10	6	1
	6	4	7	3	5	7	17	27	20	8	12
	7	6	4	1	2	4	5	15	31	16	15
	8	-	-	-	-	1	2	5	19	48	24
	9	1	-	-	-	-	-	1	5	12	37

Table 34: This table shows the confusion matrix for subject CG in the third experiment, where the subject is asked to identify the delay τ with m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 49: Subject CG; Identify τ , m roved

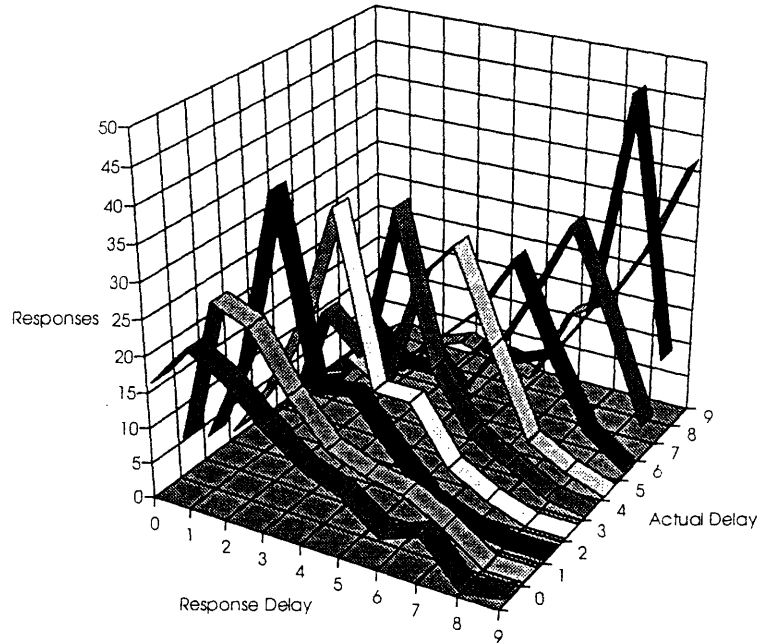


Figure 49: This shows a three dimensional representation of the confusion matrix for subject CG in the third experiment, where the subject is asked to identify the delay τ with m roved. The bottom right axis shows the stimulus number of τ (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.58 bits

Table 35: Subject DB; Identify m , τ roved

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	51	42	27	15	8	3
	1	9	3	3	3	2	-
	2	12	17	13	13	4	5
	3	17	17	28	27	26	16
	4	8	13	20	22	23	37
	5	3	8	9	20	37	39

Table 35: This table shows the confusion matrix for subject DB in the fourth experiment, where the subject is asked to identify the reflection strength m with delay τ roved. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 50: Subject DB; Identify m , τ roved

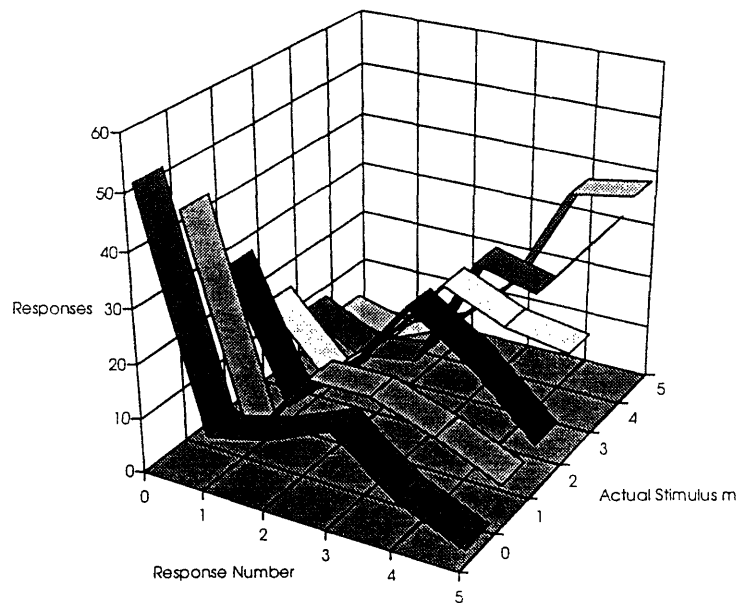


Figure 50: This shows a three dimensional representation of the confusion matrix for subject DB in Experiment 4, where the subject is asked to identify the reflection strength m with delay τ roved. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.22 bits

Table 36: Subject AS; Identify m , τ roved

		S t i m u l u s					
R		0	1	2	3	4	5
e	0	56	40	24	4	-	-
s	1	20	23	18	10	1	1
p	2	16	20	24	18	6	6
o	3	6	12	21	26	21	12
n	4	2	5	13	39	40	37
s	5	-	-	-	3	32	44
e							

Table 36: This table shows the confusion matrix for subject AS in the fourth experiment, where the subject is asked to identify the reflection strength m with delay τ roved. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 51: Subject AS; Identify m , τ roved

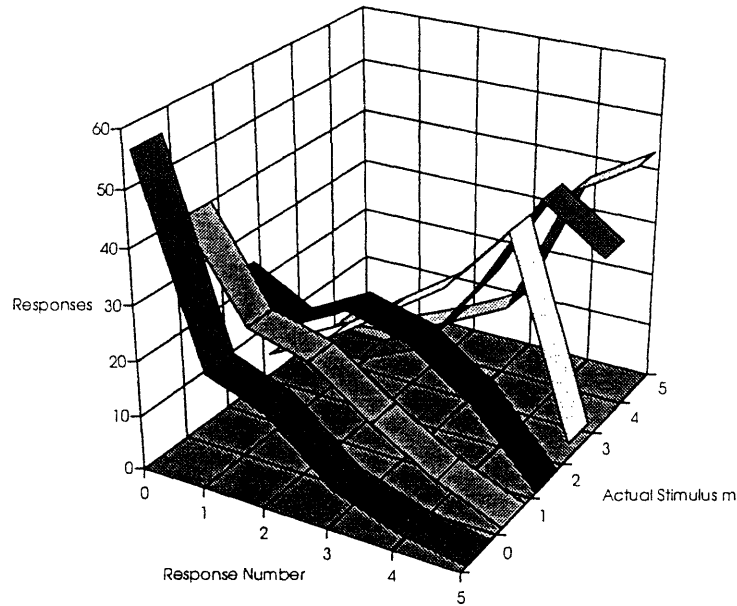


Figure 51: This shows a three dimensional representation of the confusion matrix for subject AS in Experiment 4, where the subject is asked to identify the reflection strength m with delay τ roved. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.55 bits

Table 37: Subject JK; Identify m , τ roved

		S t i m u l u s					
R		0	1	2	3	4	5
e	0	25	25	13	6	3	-
s	1	42	32	22	8	2	2
p	2	15	18	19	15	9	7
o	3	14	14	22	25	10	10
n	4	4	11	18	34	37	27
s	5	-	-	6	12	39	54
e							

Table 37: This table shows the confusion matrix for subject JK in the fourth experiment, where the subject is asked to identify the reflection strength m with delay τ roved. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 52: Subject JK; Identify m , τ roved

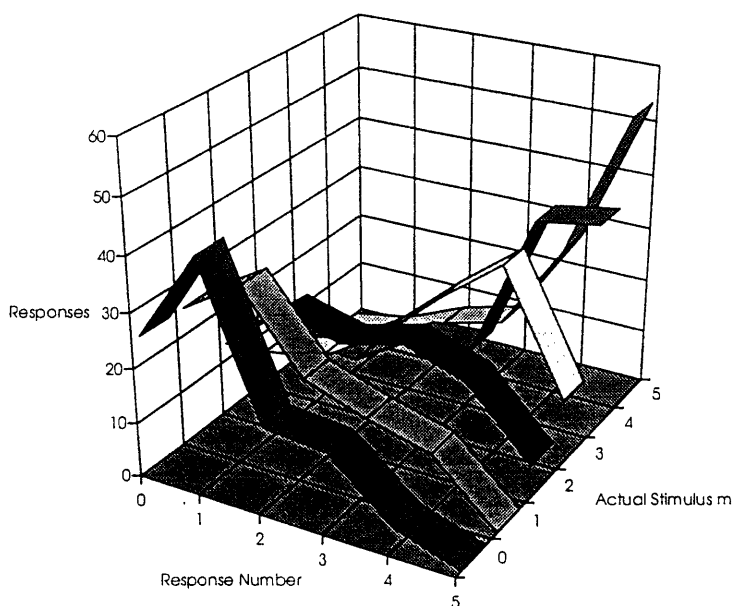


Figure 52: This shows a three dimensional representation of the confusion matrix for subject JK in Experiment 4, where the subject is asked to identify the reflection strength m with delay τ roved. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.40 bits

Table 38: Subject CG; Identify m , τ roved

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	13	9	3	2	-	1
	1	30	22	18	4	3	3
	2	25	20	28	11	8	6
	3	17	28	29	33	21	12
	4	15	19	19	40	54	39
	5	-	2	3	10	14	39

Table 38: This table shows the confusion matrix for subject CG in the fourth experiment, where the subject is asked to identify the reflection strength m with delay τ roved. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 53: Subject CG; Identify m , τ roved

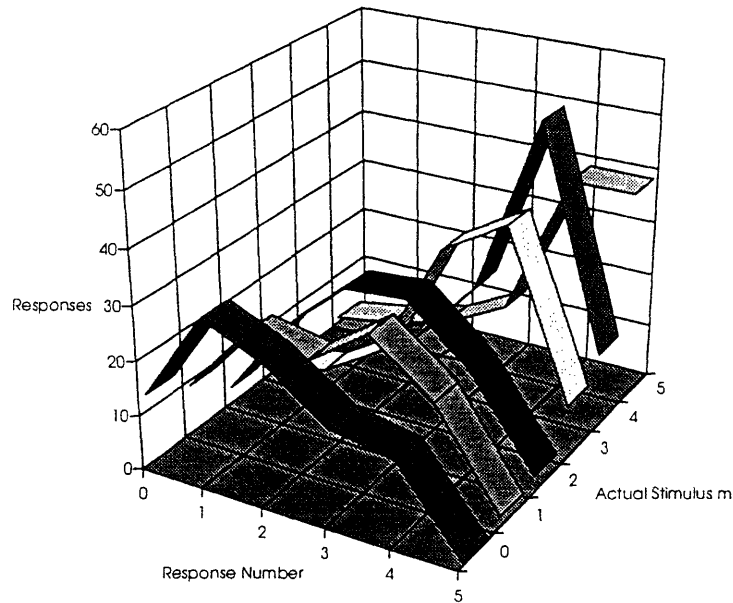


Figure 53: This shows a three dimensional representation of the confusion matrix for subject CG in Experiment 4, where the subject is asked to identify the reflection strength m with delay τ roved. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.27 bits

Table 39: Subject DB; Results for τ , Identify m and τ

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	125	111	97	80	49	22	21	15	11	7
	1	14	26	18	11	11	4	2	0	4	2
	2	29	39	37	26	14	8	8	3	5	-
	3	18	26	30	76	23	21	9	2	1	-
	4	14	10	34	34	97	49	27	9	4	4
	5	9	9	12	14	27	86	25	19	8	-
	6	11	3	7	11	28	56	110	59	8	4
	7	3	2	3	2	3	13	21	99	22	4
	8	6	1	6	-	1	4	15	36	143	55
	9	2	-	2	-	1	1	2	6	37	117

Table 39: This table shows the confusion matrix of subject DB for τ in the fifth experiment, where the subject is asked to identify the delay τ and the reflection strength m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 54: Subject DB; Results for τ , Identify m and τ

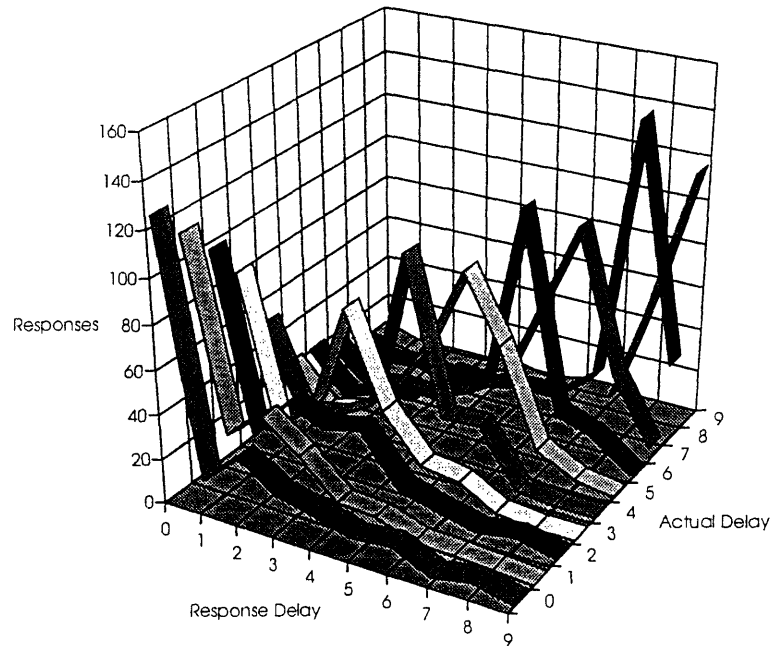


Figure 54: This shows a three dimensional representation of the confusion matrix of subject DB for τ in the fifth experiment, where the subject is asked to identify the delay τ and the reflection strength m . The bottom right axis shows the stimulus number of τ (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.80 bits

Table 40: Subject DB; Results for m , Identify m and τ

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	270	189	130	92	68	60
	1	56	76	65	41	39	21
	2	36	59	82	64	43	27
	3	37	41	51	72	75	72
	4	22	28	46	77	91	74
	5	20	23	29	46	81	97

Table 40: This table shows the confusion matrix of subject DB for parameter m in the fifth experiment, where the subject is asked to identify the reflection strength m and the delay τ . The vertical columns show the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 55: Subject DB; Results for m , Identify m and τ

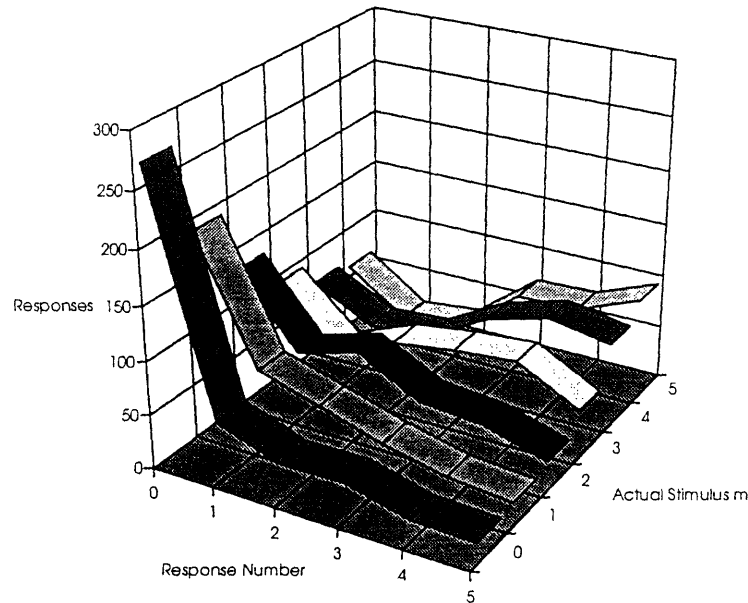


Figure 55: This shows a three dimensional representation of the confusion matrix of subject DB for parameter m in Experiment 5, where the subject is asked to identify the reflection strength m and the delay τ . The bottom right axis shows the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.15 bits

Table 41: Subject AS; Results for τ , Identify m and τ

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	205	42	15	17	3	-	-	-	-	1
	1	17	127	52	23	1	-	1	-	-	3
	2	7	26	104	40	8	2	-	1	3	1
	3	7	24	42	110	18	-	1	1	1	1
	4	1	2	11	45	174	5	2	-	1	1
	5	-	3	5	9	6	207	2	5	-	-
	6	1	1	3	7	6	3	231	2	4	1
	7	-	1	-	4	4	4	-	227	1	-
	8	-	5	2	8	1	-	4	-	218	4
	9	9	12	10	7	5	3	-	1	9	219

Table 41: This table shows the confusion matrix of subject AS for τ in the fifth experiment, where the subject is asked to identify the delay τ and the reflection strength m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 56: Subject AS; Results for τ , Identify m and τ

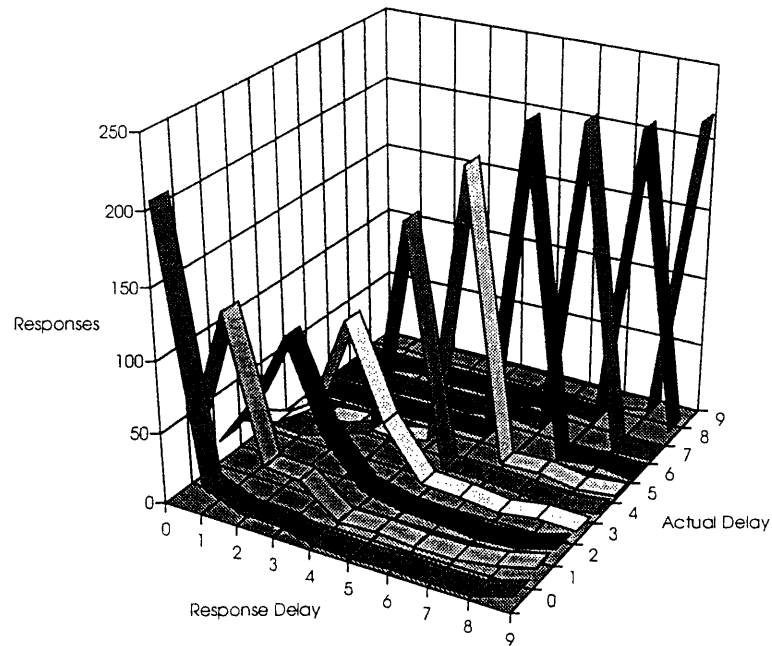


Figure 56: This shows a three dimensional representation of the confusion matrix of subject AS for t in the fifth experiment, where the subject is asked to identify the delay t and the reflection strength m . The bottom right axis shows the stimulus number of t (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 2.09 bits

Table 42: Subject AS; Results for m, Identify m and τ

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	279	228	105	38	10	7
	1	47	64	58	29	4	4
	2	27	70	102	83	44	24
	3	13	35	83	156	114	89
	4	2	11	17	90	131	102
	5	1	0	7	35	131	160

Table 42: This table shows the confusion matrix of subject AS for parameter m in the fifth experiment, where the subject is asked to identify the reflection strength m and the delay τ . The vertical columns show the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 57: Subject AS; Results for m, Identify m and τ

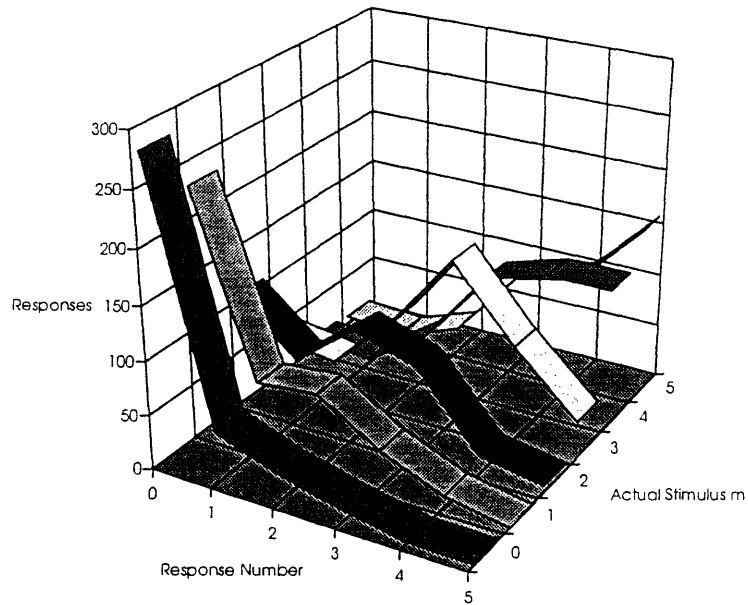


Figure 57: This shows a three dimensional representation of the confusion matrix of subject AS for parameter m in Experiment 5, where the subject is asked to identify the reflection strength m and the delay τ . The bottom right axis shows the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.55 bits

Table 43: Subject JK; Results for τ , Identify m and τ

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	62	61	32	11	7	2	2	3	5	1
	1	52	61	47	19	10	1	2	1	-	1
	2	77	64	95	27	16	4	4	10	3	4
	3	15	26	49	90	30	8	3	2	1	-
	4	7	10	15	33	85	28	9	3	3	-
	5	8	1	10	36	70	191	32	5	2	-
	6	7	4	5	7	16	27	165	26	2	2
	7	4	3	7	3	3	1	18	148	25	7
	8	7	1	1	4	2	1	10	31	172	31
	9	3	1	6	1	-	1	2	-	33	157

Table 43: This table shows the confusion matrix of subject JK for τ in the fifth experiment, where the subject is asked to identify the delay τ and the reflection strength m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 58: Subject JK; Results for τ , Identify m and τ

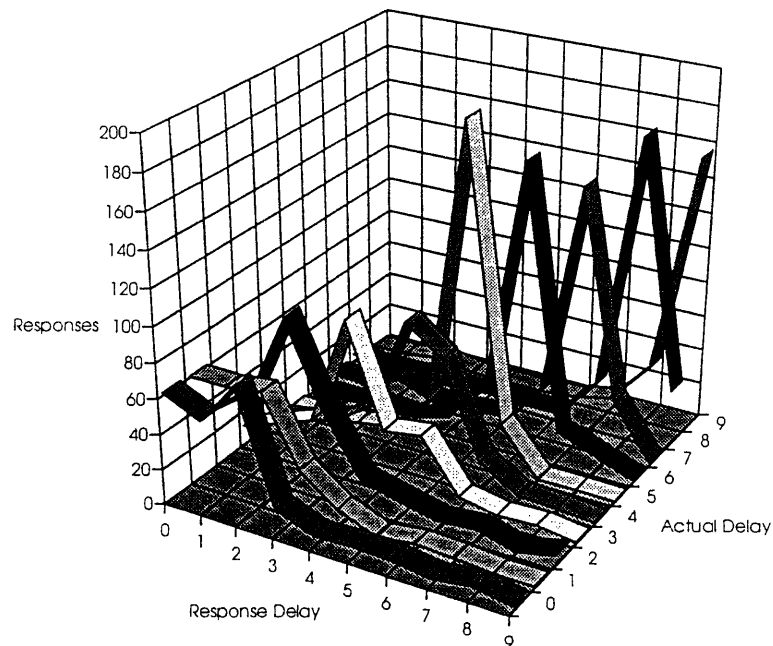


Figure 58: This shows a three dimensional representation of the confusion matrix of subject JK for t in the fifth experiment, where the subject is asked to identify the delay t and the reflection strength m . The bottom right axis shows the stimulus number of t (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.25 bits

Table 44: Subject JK; Results for m, Identify m and τ .

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	138	82	48	18	8	3
	1	122	129	106	42	30	11
	2	80	74	83	61	34	23
	3	36	40	86	67	54	41
	4	22	38	83	112	122	119
	5	8	15	44	73	154	194

Table 44: This table shows the confusion matrix of subject JK for parameter m in the fifth experiment, where the subject is asked to identify the reflection strength m and the delay τ . The vertical columns show the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 59: Subject JK; Results for m, Identify m and τ

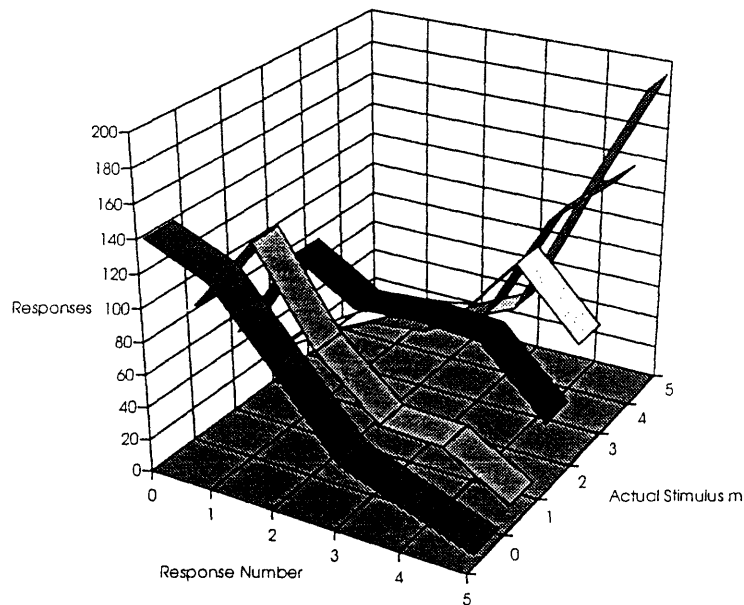


Figure 59: This shows a three dimensional representation of the confusion matrix of subject JK for parameter m in Experiment 5, where the subject is asked to identify the reflection strength m and the delay τ . The bottom right axis shows the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.31 bits

Table 45: Subject CG; Results for τ , Identify m and τ

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	38	33	19	10	6	1	2	2	1	2
	1	38	39	31	19	6	11	3	2	3	1
	2	42	34	36	27	22	15	4	8	10	4
	3	26	27	24	58	33	14	15	10	7	10
	4	38	34	41	47	76	52	36	28	11	12
	5	40	36	41	43	61	88	43	37	19	14
	6	14	21	16	19	33	46	74	41	29	13
	7	9	4	7	4	10	27	40	64	40	33
	8	3	1	2	4	1	5	15	47	103	66
	9	2	1	-	-	2	1	3	12	47	51

Table 45: This table shows the confusion matrix of subject CG for τ in the fifth experiment, where the subject is asked to identify the delay τ and the reflection strength m roved. The vertical columns show the stimulus number of τ (see Table 4) presented, and the horizontal rows show the response given by the subject.

Figure 60: Subject CG; Results for τ , Identify m and τ

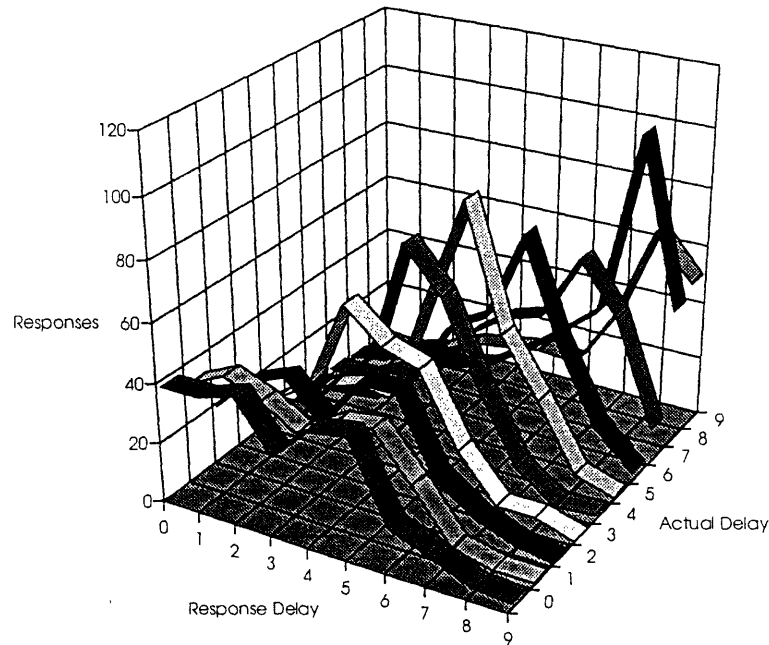


Figure 60: This shows a three dimensional representation of the confusion matrix of subject CG for t in the fifth experiment, where the subject is asked to identify the delay t and the reflection strength m . The bottom right axis shows the stimulus number of t (see Table 4) presented, and the bottom left axis shows the response given by the subject.

Information Transfer: 0.43 bits

Table 46: Subject CG; Results for m , Identify m and τ

		S	t	i	m	u	l	u	s
R		0	1	2	3	4	5		
e	0	26	16	14	5	4	3		
s	1	78	79	69	48	19	25		
p	2	145	132	141	129	98	78		
o	3	97	108	125	123	96	103		
n	4	34	55	53	88	112	109		
s	5	11	16	16	27	47	71		
e									

Table 46: This table shows the confusion matrix of subject CG for parameter m in the fifth experiment, where the subject is asked to identify the reflection strength m and the delay τ . The vertical columns show the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 61: Subject CG; Results for m , Identify m and τ

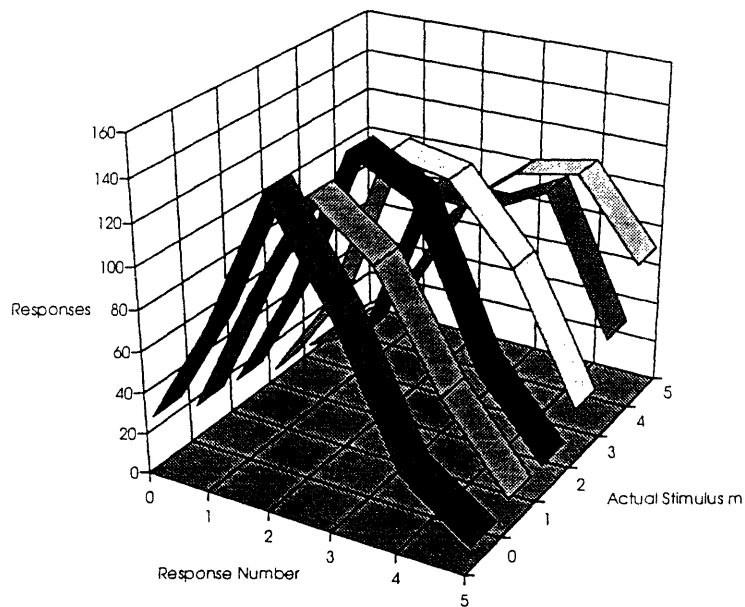


Figure 61: This shows a three dimensional representation of the confusion matrix of subject CG for parameter m in Experiment 5, where the subject is asked to identify the reflection strength m and the delay τ . The bottom right axis shows the stimulus number presented (see Table 5) for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.08 bits

Table 47: Subject DB; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	32	7	6	2	1	-	-	-	-	-
	1	36	57	19	13	7	1	-	-	-	-
	2	14	23	48	18	3	3	-	-	-	-
	3	18	12	18	53	16	2	-	-	-	-
	4	-	1	6	8	68	19	10	4	2	1
	5	-	-	3	5	5	54	15	6	3	1
	6	-	-	-	1	-	11	55	17	2	-
	7	-	-	-	-	-	3	6	38	5	1
	8	-	-	-	-	-	7	14	33	66	11
	9	-	-	-	-	-	-	-	2	22	86

Table 47: This table shows the confusion matrix for subject DB in Supplemental Experiment 1, which compares Experiment 1 and Experiment 3. The subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 62: Subject DB; Identify τ , $m=.97$

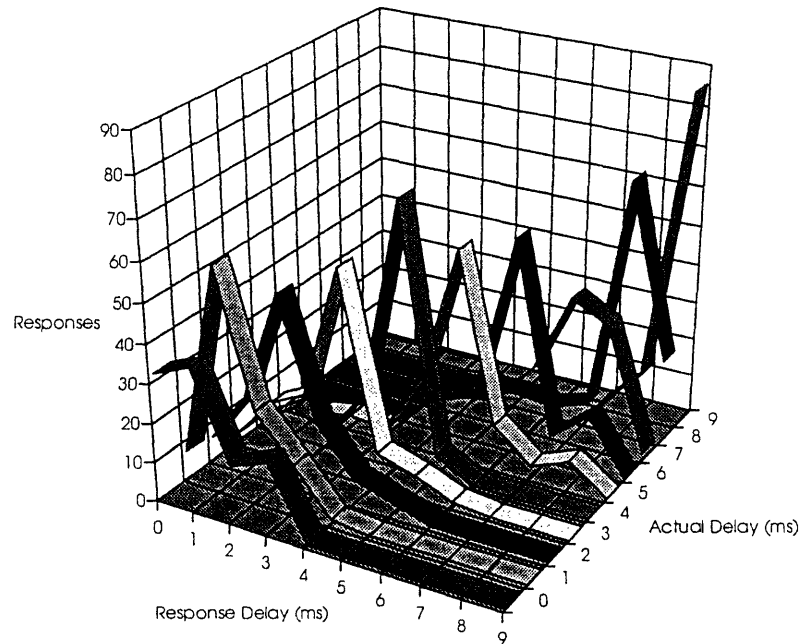


Figure 62: This shows a three dimensional representation of the confusion matrix for subject DB in Supplemental Experiment 1, which compares Experiment 1 with Experiment 3. The subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 1.47 bits

Table 48: Subject AS; Identify τ , $m=.97$

		S t i m u l u s									
		0	1	2	3	4	5	6	7	8	9
R e s p o n s e	0	95	7	-	-	-	-	-	-	-	-
	1	3	81	17	7	-	-	-	-	-	-
	2	-	8	76	10	-	-	-	-	-	-
	3	2	3	7	82	2	-	-	-	-	-
	4	-	1	-	1	98	2	-	-	-	-
	5	-	-	-	-	-	93	-	-	-	-
	6	-	-	-	-	-	5	100	1	-	-
	7	-	-	-	-	-	-	-	97	-	-
	8	-	-	-	-	-	-	-	2	100	1
	9	-	-	-	-	-	-	-	-	-	99

Table 48: This table shows the confusion matrix for subject AS in Supplemental Experiment 3, which compares Experiment 1 and Experiment 3. The subject is asked to identify the delay τ with m fixed at 0.97. The vertical columns show the actual value of τ presented in ms, and the horizontal rows show the response given by the subject.

Figure 63: Subject AS; Identify τ , $m=.97$

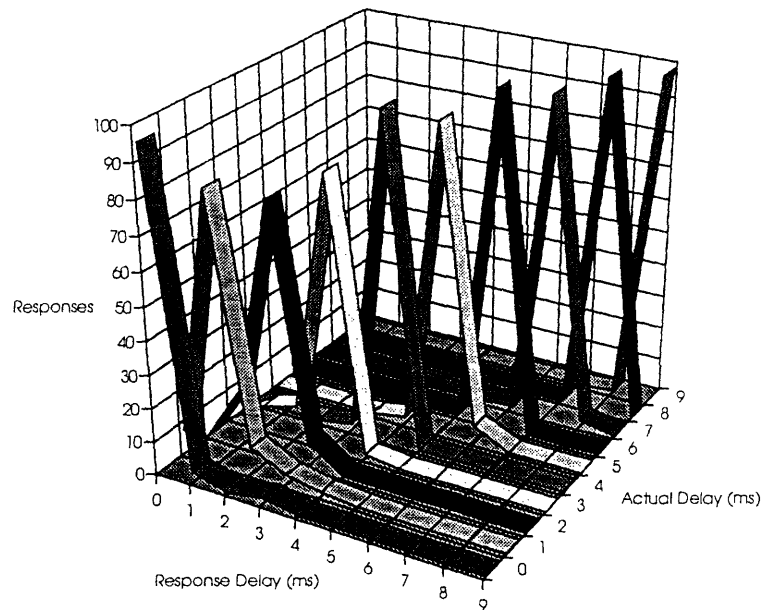


Figure 63: This shows a three dimensional representation of the confusion matrix for subject AS in Supplemental Experiment 1, which compares Experiment 1 with Experiment 3. The subject is asked to identify the delay τ with m fixed at 0.97. The bottom right axis shows the actual value of τ presented in ms, and the bottom left axis shows the response given by the subject.

Information Transfer: 2.85 bits

Table 49: Subject DB; Identify m , $\tau = 5$ ms

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	47	42	23	8	-	-
	1	22	37	28	8	5	4
	2	24	12	22	12	6	2
	3	5	6	13	22	10	6
	4	1	2	14	36	56	51
	5	1	1	-	14	23	37

Table 49: This table shows the confusion matrix for subject DB in Supplemental Experiment 2, which was designed to compare Experiment 2 with Experiment 4. The subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 64: Subject DB; Identify m , $\tau = 5$ ms

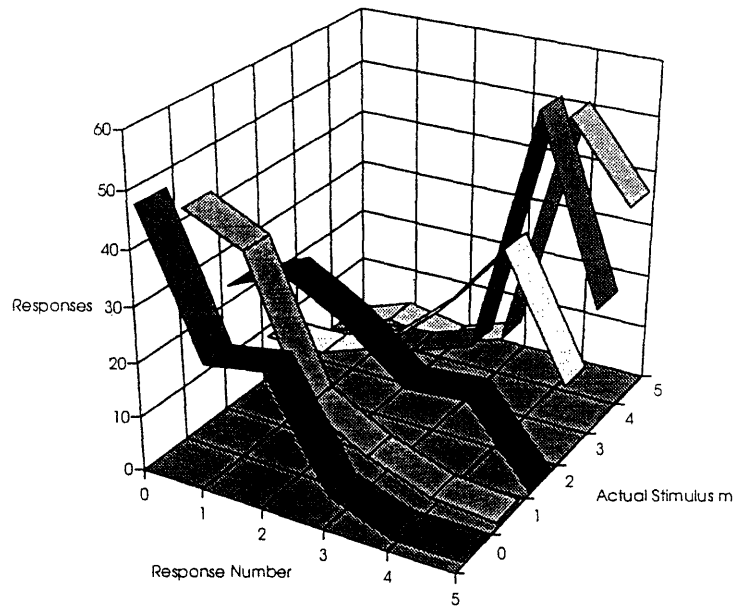


Figure 64: This shows a three dimensional representation of the confusion matrix for subject DB in Supplemental Experiment 2, which was designed to compare Experiment 2 with Experiment 4. The subject is asked to identify the reflection strength m with delay τ fixed at 5ms. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.54 bits

Table 50: Subject AS; Identify m , $\tau = 5$ ms

		S t i m u l u s					
		0	1	2	3	4	5
R e s p o n s e	0	73	48	11	-	-	-
	1	22	30	39	6	2	-
	2	4	16	25	16	2	1
	3	1	5	19	44	17	13
	4	-	1	6	22	46	42
	5	-	-	-	12	33	44

Table 50: This table shows the confusion matrix for subject AS in Supplemental Experiment 2, which was designed to compare Experiment 2 with Experiment 4. The subject is asked to identify the reflection strength m with delay τ fixed at 5 ms. The vertical columns show the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the horizontal rows show the response given by the subject.

Figure 65: Subject AS; Identify m , $\tau = 5$ ms

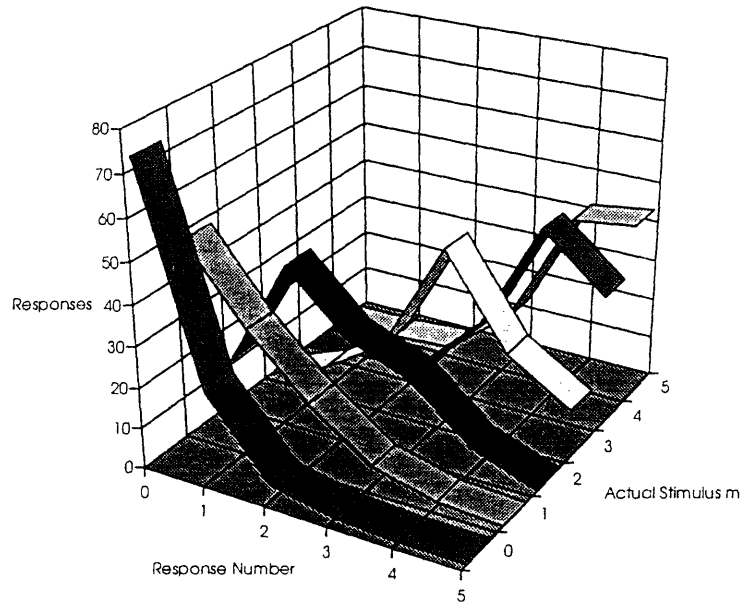


Figure 65: This shows a three dimensional representation of the confusion matrix for subject AS in Supplemental Experiment 2, which was designed to compare Experiment 2 with Experiment 4. The subject is asked to identify the reflection strength m with delay τ fixed at 5ms. The bottom right axis shows the stimulus number presented (see Table 5 for m value associated with each stimulus number), and the bottom left axis shows the response given by the subject.

Information Transfer: 0.84 bits

15. References

- Berliner, J.E., Durlach, N.I. 1973. Intensity Perception. IV. Resolution in roving-level discrimination. *J. Acoust. Soc. Am.* 53:1270-1287
- Braida, L.D., Durlach, N.I. 1972. Intensity Perception. II. Resolution in one-interval paradigms. *J. Acoust. Soc. Am.* 51:483-502
- Bilsen, F.A. 1966. Repetition pitch: Monaural interaction of a sound with the same, but phase shifted sound. *Acustica* 17: 295-300
- Butler, R.A., Levy, E.T., Neff, W.D. 1980. Apparent distance of sounds recorded in echoic and anechoic chambers. *J. Exp. Psycho., Hum. Percept. Perform.* 6:745-50
- Braida, L.D., Durlach, N.I. 1970. Intensity Perception II. Resolution in One-Interval Paradigms. *J. Acoust. Soc. Am.* 51:483-502
- Brungart, D.S. 1993. Distance Simulation in Virtual Audio Displays. NAECON conference proceedings, May 1993.
- Coleman, P.D. 1963. An analysis of cues to auditory depth perception in free space. *Psychol. Bull.* 60:302-15
- D'Angelo, W.R., Ericson, M.A., Scarborough, E.L. 1993. Measurement of distance perception using virtual audio. NAECON conference proceedings, May 1993
- Durlach, N.I., Braida, L.D. 1969. Intensity Perception. I. Preliminary Theory of Intensity Resolution. *J. Acoust. Soc. Am.* 46:372-383
- Durlach, N.I., Held, R.M. 1994. Further Research on Super Auditory Localization for Improved Human-Machine Interfaces. Proposal to the Air Force Office of Scientific Research
- Durlach, N.I., Tan, H.Z., MacMillan, N.A., Rabinowitz, W.M., Braida, L.D. 1989. Resolution in one dimension with random variations in background dimensions. *Perception & Psychophysics.* 46:293-296
- Gardner, M.B. 1969. Distance estimation at 0 degree or apparent 0 degree oriented speech signals in anechoic space. *J. Acoust. Soc. Am.* 54:1489-95
- Garner, W.R., Hake, H.W. 1952. The amount of information in absolute judgements. *Psychol. Rev.* 58:446-459
- Houtsma, A.J. 1983. Estimation of mutual information from limited experimental data. *J. Acoust. Soc. Am.* 74:1626-1629

- Litovsky, R.Y., Clifton, R.K. 1992. Use of sound pressure level in auditory distance discrimination by 6-month old infants and adults. *J. Acoust. Soc. Am.* 92: 794-802
- Malme, C.I. 1959. Detectability of small irregularities in a broadband spectrum. Q. Prog. Rep., MIT Research Laboratory of Electronics, Cambridge, MA, 139-142
- McKinley, R.L., Ericson, M.A. 1988. Digital synthesis of binaural auditory localization cues using headphones. *J. Acoust. Soc. Am.* 83:S18
- McMurtry, P.L., Mershon, D.H. 1985. Auditory distance judgement in noise, with and without hearing protection. *Proceedings of Human Factors Society-29th Annual Meeting* 811-813
- Mershon, D.H., Bowers, J.N. 1979. Absolute and relative cues for the auditory perception of egocentric distance. *Perception* 8:311-22
- Mershon, D.H. King, L.E. 1975. Intensity and reverberation as factors in the auditory perception of egocentric distance. *Percept Psychophys.* 18:409-415
- Middlebrooks, J.C., Green, D.M. 1991. Sound localization by human listeners. *Annu. Rev. Psychol.* 42:135-159 ,
- Miller, G.A. 1956. The magical number seven, plus or minus two: some limits on our capacity for processing information. *Psychol. Review* 63:81-97
- Miller, G.A., Madrow, W.G. 1954. On the maximum likelihood estimate of the Shannon-Wiener measure of information. AFCRC-TR-54-75, 1954
- Moore, C.J., Oldfield, S.R., Dooley, G.J. 1989. Detection of spectral peaks and notches at 1 and 8 kHz. *J. Acoust. Soc. Am.* 85:820-836
- Pollack, I. 1952. The Information of Elementary Audio Displays. *J. Acoust. Soc. Am.* 24:745-749
- Pollack, I. 1953. The Information of Elementary Audio Displays II. *J. Acoust. Soc. Am.* 25:765-769
- Pollack, I. Ficks, L, 1955. The Information of Elementary Multidimensional Audio Displays. *J. Acoust. Soc. Am.* 26:155-158
- Shinn-Cunningham, B.G., Lehnert, H., Kramer, G., Wenzel, E.M., Durlach, N.I. 1994. Auditory Displays. In R. Gilke & T. Anderson (Eds.), *Spatial and Binaural Hearing*. New York: Erlbaum
- Wenzel, E.M. 1991. Localization in Virtual Acoustic Displays. *Presence* 1:80-107

- Yost, W.A. 1982. The dominance region and ripple noise pitch: A test of the peripheral weighting model. *J. Acoust. Soc. Am.* 72:416-425
- Yost, W.A., Hill, R. 1978. Strength of pitches associated with ripple noise. *J. Acoust. Soc. Am.* 64:485-492
- Yost, W.A., Hill, R. 1979. Models of the pitch and pitch strength of ripple noise. *J. Acoust. Soc. Am.* 66:400-410
- Yost, W.A., Hill, R., Perez-Falcon, T. 1978. Pitch and pitch discrimination of broadband signals with rippled power spectra. *J. Acoust. Soc. Am.* 63:1166-1173