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THE PRICING OF DURABLE EXHAUSTIBLE RESOURCES\*

by

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October, 1979

MIT - EL 79 - 053WP

\*Research leading to this paper was supported in part by the National Science Foundation under Grant SOC77-24573 to R.S. Pindyck, and that support is gratefully acknowledged. The authors also wish to thank Dennis Epple, Richard Schmalensee, V. Kerry Smith, and Tom Teisberg for helpful comments and suggestions.

## The Pricing of Durable Exhaustible Resources

### 1. Introduction

Hotelling's  $r$ -percent growth rule for the price of an exhaustible resource has had only limited success in explaining the evolution of resource prices. The prices of many exhaustible resources have experienced long secular declines, or more commonly, have fallen over a long period and then later risen, following a U-shaped profile.<sup>1</sup>

The demand characteristics for a large class of exhaustible resources make the simple Hotelling model inadequate as a means of explaining historical price behavior. These resources are durable, so that their demands are functions of the stocks in circulation rather than the flows of production. The best examples of such resources include diamonds, gold, silver, and the other precious metals, but copper and other metals also have durable aspects.<sup>2</sup>

In a recent paper in this Journal, Stewart (1979) used a discrete-time model to argue that competitive producers with constant extraction costs would produce a durable resource at such a rate that Hotelling's  $r$ -percent rule would still apply. For a partially durable resource (i.e. one that depreciates over time) Stewart's result is technically correct but not very meaningful in that it requires an unlimited amount of production in the first period to instantaneously bring the stock of resource in circulation to its maximum value. In this way (if the amount of depreciation each period

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1 - For results of some recent empirical tests of the Hotelling model, see Heal and Barrow (1979), Feige and Geweke (1979), and Smith (1979). Heal and Barrow find U-shaped price profiles to predominate for many resources over the past century. For an excellent source of resource price data, see Manthy (1978).

2 - Salant and Henderson (1978) note that the real price of gold moves with upward surges followed by sharp drops, with price rising much faster than the real rate of interest during the surge. They explain this behavior by introducing anticipations of government gold policies into a standard Hotelling model of production. But while this is an effective way of explaining short-term fluctuations in gold prices, it is not so effective as an explanation of the long-term secular behavior of price.

is larger than the amount of production) the stock in circulation can fall and the price can rise (according to the  $r$ -percent rule) over all succeeding periods. For a perfectly durable resource and constant demand (or a rate of demand growth smaller than  $r$ ), Stewart's result is simply incorrect - clearly the stock in circulation can only increase in such a case, so that (with constant demand) price can only fall.<sup>3</sup>

Here we reformulate the durable resource problem in continuous-time, and show that the problem is in fact ill-defined when extraction cost is constant. We will see that with rising marginal and average extraction cost the competitive price is always falling for a perfectly durable resource and constant demand, and the price profile is U-shaped for a partially durable resource and/or growing demand. Properly accounting for the characteristic of durability may thus help explain the observed behavior of resource prices.<sup>4</sup>

In the next section we present and solve a simple model for the general case of a partially durable resource and autonomous growth in demand. The behavior of the model is discussed in Section 3 for the simplest case of perfect durability and static demand, in Section 4 for the case of partial durability and static demand, and in Section 5 for the general case. Section 6 compares the rate of production in a competitive market with the socially optimal rate.

## 2. The Model

With non-durable resources such as oil or gas, demand is a flow, since once a unit of the resource has been consumed (burned), it no longer provides utility. A unit of a durable resource, on the other hand, continues to provide

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3 - In Stewart's model all production would take place in the first period in this case, so that there would be no relevant inter-temporal price variation. In addition, Stewart has mis-specified the demand relationship by setting price equal to the marginal value of services from a unit of stock in circulation, rather than setting the user cost of holding that unit equal to the marginal value.

4 - There are other explanations for the observed U-shaped profiles of resource prices over the long run. For example, Pindyck (1978) showed that the introduction of exploration and reserve accumulation can result in a U-shaped price profile.

utility as long as it is held, so that demand is a stock relationship. We write the demand relationship as:

$$D(Q) = f(Q)y(t) \tag{1}$$

where  $D(Q)$  is the marginal value of services from a stock of resource in circulation of size  $Q$ ,  $f'(Q) < 0$ , and  $y(t)$  provides for autonomous growth in demand.<sup>5</sup> For simplicity we will assume a constant proportional growth rate, so that  $y(t) = e^{\alpha t}$ .

The user cost of 1 unit of the resource stock is just  $rp - \dot{p} + \delta p$ , where  $r$  is the interest rate and  $\delta$  the rate of depreciation of the stock. Equating this user cost with the marginal value (1) provides the differential equation that the resource price must satisfy at all time:

$$\dot{p} = -f(Q)e^{\alpha t} + (r + \delta)p \tag{2}$$

The remainder of the model looks like a standard Hotelling model, except that we assume that marginal production cost increases with the rate of production  $q$ , i.e.  $C''(q) > 0$ . Letting  $X$  represent cumulative production and  $X_0$  total available reserves, the producer's problem can be written as:

$$\max_q \int_0^{\infty} [pq - C(q)]e^{-rt} dt \tag{3}$$

subject to  $\dot{Q} = q - \delta Q, \quad Q(0) = 0, \tag{4}$

$$\dot{X} = q, \quad X \leq X_0, \quad q \geq 0, \tag{5}$$

together with eqn. (2) for  $\dot{p}$ .

A monopoly producer could control the market price  $p$  as well as the total stock in circulation  $Q$ , so that the monopoly solution is found by explicitly accounting for the three state equations (2), (4) and (5)

5 - It is not clear whether our definition of durability would apply to non-precious metals such as copper or aluminum. In a model of the aluminum market, Gaskins (1974) writes demand as a flow relationship, arguing that aluminum "is durable in the same way that newsprint or steel are durable. It is not a 'normal' durable good like an automobile or a computer...[but] like newsprint must be substantially transformed (fabricated) from its scrap condition to yield useful services." In this paper we are interested in "normal" durable goods for which demand is a stock relationship. If we treat the fabricator as the consumer of copper or aluminum, then these resources might indeed be considered as "normal" durable goods.

when performing the maximization. Competitive producers, on the other hand, have no control over  $p$  or  $Q$ , so that the competitive solution is found by maximizing (3) subject to (5), and then using (2) and (4) to determine the market-clearing price and production levels. Here we present the solution for the competitive market and examine its characteristics.

The solution is obtained by straightforward application of the Maximum Principle. The Hamiltonian is:

$$H = p q e^{-rt} - C(q) e^{-rt} + \lambda q \quad (6)$$

Maximizing (6) with respect to  $q$  and differentiating the resulting equation with respect to time gives:

$$\dot{\lambda} = r[p - C'(q)]e^{-rt} - \dot{p}e^{-rt} + C''(q)\dot{q}e^{-rt} \quad (7)$$

Since  $\dot{\lambda} = -\partial H / \partial X = 0$ , we can rewrite (7) as:

$$\dot{q} = \frac{1}{C''(q)}[\dot{p} - rp + rC'(q)] \quad (8)$$

Finally, substitute eqn. (2) for  $\dot{p}$ :

$$\dot{q} = \frac{1}{C''(q)}[\delta p + rC'(q) - f(Q)e^{\alpha t}] \quad (9)$$

This equation and equations (2) and (4), together with the boundary conditions  $Q(0) = 0$ ,  $q(T) = 0$  (which guarantees that  $H(T) = 0$  as long as  $C(0) = 0$ ), and the condition that  $\int_0^T q(t)dt = X_0$ , determine the equilibrium production and price profiles.

Note that  $\dot{q}$  is bounded only if  $C''(q) \neq 0$ . Suppose  $C''(q) = 0$ . Then production is described by an impulse function at  $t = 0$  that brings  $Q$  to its maximum level. With perfect durability and static demand  $q = 0$  for  $t > 0$ , and price remains constant and equal to  $f(Q)/r$ , i.e. the capitalized marginal value flow from 1 unit of stock. With partial durability price will rise during  $t > 0$  according to the  $r$ -percent rule, as is evident from eqn. (7) (with  $C''(q) = 0$  and  $\dot{\lambda} = 0$ ). But  $\dot{p}$  must also satisfy eqn. (2). To determine the level of output combine eqns. (2) and (7) to eliminate  $\dot{p}$ :

$$(r + \delta)p - f(Q) = r[p - C'(q)] \quad (10)$$

Now differentiate this equation with respect to time, and substitute eqn. (4) for  $\dot{Q}$ :

$$\dot{p} = (q - \delta Q)f'(Q)/\delta \quad (11)$$

Now combine eqn. (7) (the r-percent rule) with eqn. (11) to eliminate  $\dot{p}$ :

$$q(t) = \frac{\delta r[p - C'(q)]}{f'(Q)} + \delta Q \quad (12)$$

Note that  $q < \delta Q$  (which is necessary for  $p$  to rise), and  $q \rightarrow 0$  for all  $t > 0$  as  $\delta \rightarrow 0$ .<sup>6</sup>

It is much more interesting - and much more realistic - to drop the assumption that  $C''(q) = 0$ . In what follows we therefore assume that  $C''(q) > 0$ . As is evident from eqn. (7) the r-percent rule no longer applies, but we will see that the model now provides a better description of historical price behavior.

### 3. Perfect Durability, Static Demand

Here  $\delta = \alpha = 0$ , so that eqn. (9) becomes

$$\dot{q} = -[f(Q) - rC'(q)]/C''(q) \quad (13)$$

The rate of production  $q$  will be non-zero only if  $f(Q)/r > C'(q)$ , i.e. the capitalized marginal value flow from a unit of stock is greater than the marginal cost of extracting the unit (so there is a positive rent), and therefore  $\dot{q} < 0$  up to the terminal time  $T$ . Now consider two cases.

First suppose that the resource constraint is non-binding, i.e.  $X_0$  is very large, so there is a  $Q_0 < X_0$  for which  $f(Q_0) = rp_0 = rC'(0)$ . In this case  $Q \rightarrow Q_0$  asymptotically as  $q \rightarrow 0$  and  $\dot{q} \rightarrow 0$  asymptotically. This is illustrated by the trajectories  $q_A$  and  $Q_A$  in Figure 1.

Now suppose the resource constraint is binding, i.e.  $X_0 < Q_0$ , so that production must cease at time  $T$  before marginal profit falls to zero. Then  $p_T = f(Q_T)/r = f(X_0)/r > C'(0)$ , and  $\dot{q}_T < 0$ . The trajectories for this case are given by  $q_B$  and  $Q_B$  in Figure 1.

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6 - The reader can demonstrate that perfect durability but growth in demand also leads to  $q > 0$  for  $t > 0$  and the r-percent rule for price if  $C''(q) = 0$ .

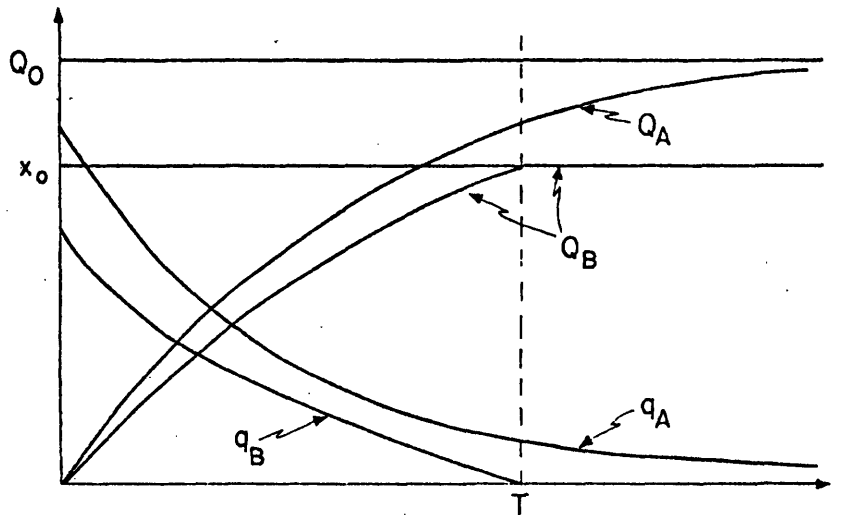


Figure 1 Optimal Trajectories: Perfect Durability, Static Demand

4. Partial Durability, Static Demand

Now  $\alpha = 0$  but  $\delta > 0$  so that eqn. (9) becomes

$$\dot{q} = -[f(Q) - rC'(q) - \delta p]/C''(q) \quad (14)$$

Note from equation (2) (with  $\alpha = 0$ ) that the price profile is U-shaped if the resource constraint is binding, and price is always falling if there is no resource constraint (i.e. if reserves are infinite).

The behavior of production is easiest to see from the phase diagram of Figure 2. Note that initially as  $Q$  rises  $\dot{p}$  grows (i.e. becomes a smaller negative number), and the  $[\dot{q} = 0]$  isocline shifts to the left. Later as  $Q$  falls (if the resource constraint is binding),  $\dot{p}$  begins falling to 0 (as  $Q \rightarrow 0$ ) and the isocline shifts to the right.

When the resource constraint is non-binding,  $\lambda = 0$ ,  $H \rightarrow 0$  asymptotically, and  $q \rightarrow \tilde{q} = \delta \tilde{Q}$  such that  $f(\tilde{Q})\tilde{q}/(r + \delta) = \tilde{p}\tilde{q} = C(\tilde{q})$ . (These relationships determine  $\tilde{Q}$ ,  $\tilde{q}$  and  $\tilde{p}$ .) This case is illustrated by trajectories  $q_A$  and  $Q_A$  in Figure 3, and as can be seen from trajectory A in the phase diagram of Figure 2,  $\dot{q} \leq 0$  always if  $Q = 0$  initially. Note that these trajectories describe a simple stock adjustment model. As the industry "matures" price falls, asymptotically approaching its long-run equilibrium level  $f(\tilde{Q})/(r + \delta)$ .

When the resource constraint is binding  $q$  will typically fall monotonically to 0, with  $Q$  increasing until time  $T_1$  when  $q = \delta Q(T_1)$ , and falling thereafter, as illustrated by trajectory  $B_1$  in the phase diagram and by  $q_B$  and  $Q_B$  in Figure 3. The price profile is therefore U-shaped, i.e. price falls at first as the stock in circulation is built up, and later rises, with  $\dot{p}$  at first rising and then later falling towards zero as  $Q$  depreciates towards zero. The  $r$ -percent rule clearly does not hold, even over the period of rising price. The interval  $(0, T_1)$  over which price is falling is determined largely by  $C''(q)$ . If  $C''(q)$  is small,  $q$  will be high initially (but falling more rapidly), so that  $Q$  approaches its maximum value more rapidly. Finally, note that if  $\delta$  is large,  $q$  can begin falling, then rise, and then fall towards 0 as in trajectory  $B_2$  in the phase diagram.<sup>7</sup>

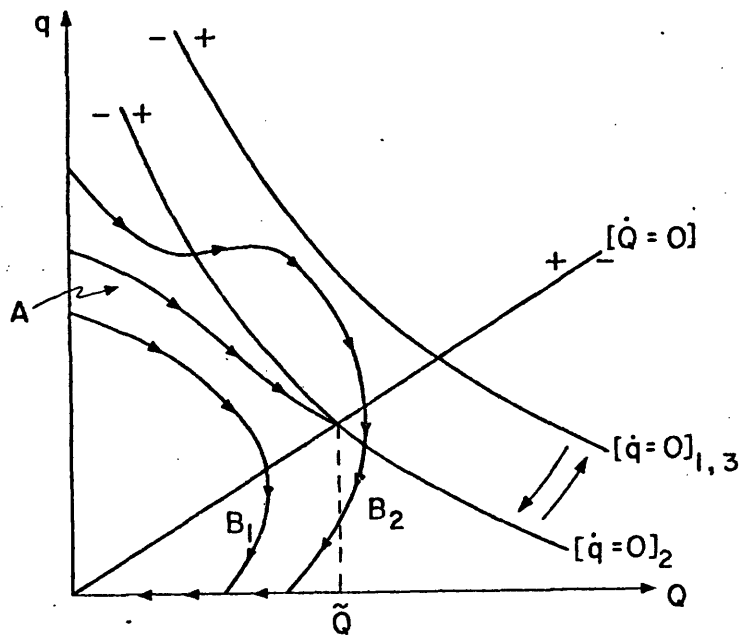


Figure 2 Phase Diagram: Partial Durability, Static Demand

7 -  $q$  begins rising if the  $[\dot{q} = 0]$  isocline shifts leftward enough, and then falls again as the isocline shifts back to the right.



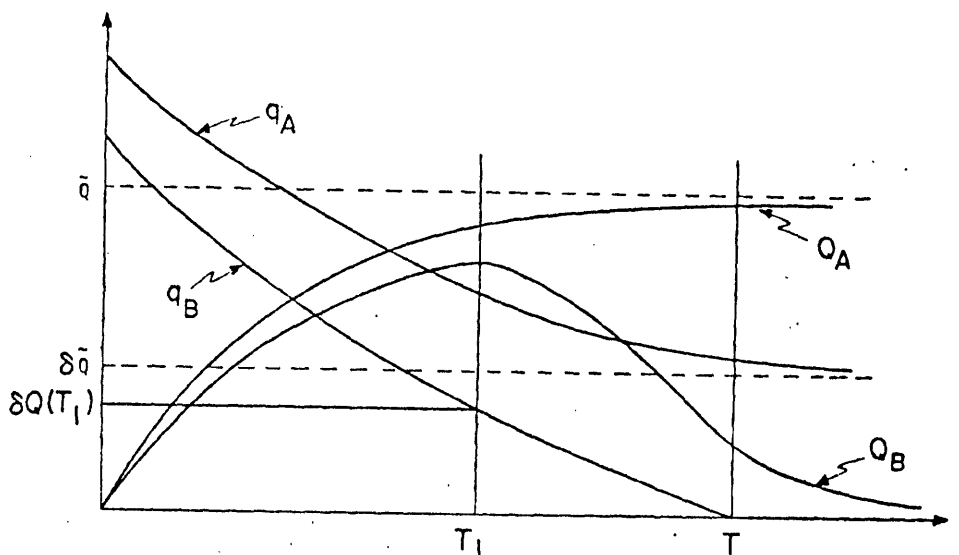


Figure 3 Optimal Trajectories: Partial Durability, Static Demand

5. Partial Durability, Growth in Demand

This is the general case, with the dynamics of production given by eqn. (9). The behavior of price and production will not be very different from the static demand case discussed in the last section. We can again refer to the phase diagram of Figure 2, but note that with  $\alpha > 0$  the  $[\dot{q} = 0]$  isocline will rotate to the right over time around its point of intersection with the  $q$  axis.

When the resource constraint is not binding (reserves are infinite),  $q$  and  $Q$  will approach steady-state equilibrium growth paths in which  $H = 0$ , so that  $p = C(q)/q$ . Thus as  $q$  increases price moves up the average cost curve,  $q > \delta Q$ , and  $Q$  keeps increasing.

When the resource constraint is binding, behavior is again characterized by trajectory  $B_1$  in Figure 2, so that  $q$  falls to zero, and  $Q$  rises and then falls. However, if  $\alpha$  is large enough,  $p$  can always be rising (from a high initial value). Since  $Q \rightarrow 0$  only asymptotically after production has ceased,  $\dot{p}$  need not approach 0 asymptotically as in the case of static demand.

6. Optimality of Competitive Equilibrium

We now show that the rate of production in the competitive market is

socially optimal in that it maximizes the sum of discounted consumer and producer surplus. We demonstrate this for the general case of a partially durable resource with growth in demand.

The total flow of value of services at time  $t$  from a stock in circulation of size  $Q$  is  $e^{\alpha t} \int_0^Q f(S) dS$ . Thus the social maximization problem is:

$$\max_q \int_0^{\infty} \left[ e^{\alpha t} \int_0^Q f(S) dS - C(q) \right] e^{-rt} dt \quad (15)$$

subject to  $\dot{Q} = q - \delta Q, \quad Q(0) = 0 \quad (16)$

and  $\dot{X} = q, \quad X \leq X_0, \quad q \geq 0 \quad (17)$

The solution is again found by straightforward application of the Maximum Principle. The Hamiltonian is

$$H = e^{(\alpha-r)t} \int_0^Q f(S) dS - C(q) e^{-rt} + \lambda q + \mu (q - \delta Q) \quad (18)$$

Maximizing (18) with respect to  $q$  and differentiating the resulting equation with respect to time gives:

$$\dot{\lambda} + \dot{\mu} = C''(q) q e^{-rt} - rC'(q) e^{-rt} \quad (19)$$

Since  $\dot{\lambda} = -\partial H / \partial X = 0$  and

$$\dot{\mu} = -\partial H / \partial Q = -e^{(\alpha-r)t} f(Q) + \delta \mu, \quad (20)$$

we can write (19) as:

$$\dot{q} = \frac{1}{C''(q)} [\delta \mu e^{rt} + rC'(q) - f(Q) e^{\alpha t}]$$

Now note that  $\xi(t) = \mu e^{rt}$  is the (undiscounted) shadow price of a unit of stock in circulation, and can be identified with - and follows the same dynamics as - the competitive market price  $p(t)$ . To see this, differentiate  $\xi(t)$  with respect to time and substitute (20) for  $\dot{\mu}$ :

$$\dot{\xi} = -f(Q) e^{\alpha t} + (r + \delta) \xi$$

This is identical to eqn. (2) if we set  $\xi = p$ , so that the competitive rate of production is indeed socially optimal.

7. Concluding Remarks

We have seen that Hotelling's  $r$ -percent rule will not apply for a resource that is partially or totally durable. Instead, if marginal production cost rises with the rate of production (which it must for the problem to make much sense), the competitive market price will fall initially as the stock in circulation increases, and later rise as the stock decreases and eventually depreciates asymptotically to zero after production has ceased. Even during this later period of rising price, however, the  $r$ -percent growth rule does not hold.

As we mentioned at the outset of this paper, the price profiles for many resources have indeed been U-shaped over the long term (50-100 years). Accounting for durability, as well as the process of reserve discovery and accumulation during the early periods of resource use, may help explain this pattern of price behavior.

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