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The Diffusion of Photovoltaics: Background, Modeling and Initial Reaction of the Agricultural-Irrigation Sector

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Abstract

This paper deals with the background, development and calibration of a model of innovation-diffusion, designed to help allocate government field test and demonstration resources in support of a photovoltaic technology across sectors, regions and over time. The paper reviews current work in the area of diffusion and substitution models, and gives a brief review of current theory in the buyer behavior area.

A model is developed, drawing upon concepts in these areas, and its computer implementation is reviewed. The measures needed to calibrate the model are performed in the agricultural-irrigation sector in conjunction with a field installation in Mead, Nebraska. The analysis of those results indicated that

- only three to four demonstration projects are needed to eliminate new product risk-perception among farmers;
- exposure to a working PV site makes farmers more aware of potential energy savings than does a description of the system;
- key factors associated with PV are
    - newness/expense
    - complexity of the system and use of untried concepts
    - independence from traditional fuel sources.
- exposure to the site has little effect on preference;
- PV is acceptable to a wide range of farmers;
- a premium would be paid for the product.

Additional model developments and the potential of a model-use to support decision-making for government programs are reviewed.

1. PV Diffusion and the Development of Government Programs

The objectives of this paper are to

- motivate and describe a new approach toward modeling the diffusion of new technologies;
- calibrate that model in one market sector;
- show how that model can be used to help the government in allocating field test and demonstration resources across sectors, across regions and over time.

The results are aimed at developing support for Photovoltaic (PV) development programs, but the methodology developed is general.

Our approach has a "marketing" orientation -- we develop a diffusion model based on concepts from the literature on buying behavior. We rely on field measurements (if attitudinal), to calibrate the model.

Our approach marries the economic modeling experience present in the technological forecasting literature with the theory of buyer behavior found in the marketing literature and is structured to develop information to support government (and private sector) decision-making (normative modeling).

There is strong motivation for asking people what they think or seeing how people react to a new product.

There is a growing body of evidence (see Rothwell [22], Von Hippel [27], Utterback [26], for example) that this type of user-feedback early in the R&D process both accelerates product development and improves product success rates. If we incorporate this concept into the structure of a diffusion model (see Bass [5], Mansfield [18]), we can incorporate market controls into established models of new technology growth.

We view the government as a potential diffusion accelerator in this process through its actions of

- lowering costs (through incentive programs and by acting as a guaranteed buyer, encouraging learning curve development) and by

- reducing uncertainty (through providing visible evidence of effectiveness -- demonstration programs -- and by assuring satisfaction by providing government warranties).

By the nature of the material, considerable background material is developed and reviewed here to set the stage for the modeling task. The next section briefly reviews current work in the area of diffusion and substitution models. Several are developed in more detail, and some simple policy implications are derived from one, in an associated appendix. Our approach is positioned relative to this mode of model development.

Section 3 offers a brief excursion into the arcane literature of buyer behavior and the adoption of new products. The model developed in that section -- static, and calibrated in a single sector at a time -- is the basis of the PV diffusion model developed in the next section. The diffusion model is time-dependent and explicitly incorporates interactions between developments in different sectors. A brief outline of how the model can be used to support government decision-making problems along with a description of the computer implementation of the model is included.

Sections 5 to 9 describe the application and results of the field measurements associated with model calibrations in the agricultural-irrigation sector. The results of that analysis tell under what conditions PV could successfully enter the market for irrigation equipment. Specifically we find:

- only 3-4 demonstration projects are needed to eliminate new product risk-perception among farmers;

- exposure to a working PV site makes farmers more aware of potential energy savings than does a description of the system;

- key factors associated with PV adoption are
    - newness/expense
    - complexity of system and use of untried concepts
    - independence from traditional fuel sources.
- exposure to a PV demonstration site affects the way people think about irrigation but does not affect people's preference for the system. (This means that a carefully designed advertising program can have the same effect on system preference as a demonstration program would).

Finally, a few caveats. First, the work presented here is preliminary in nature. The model will be developed further, calibrated on additional sectors (residential is currently underway) and the computer program will be completed. The incentive for this work is to help develop government program guidance; that use is still to come. And, finally, as the paper treats a diverse set of material, some sections may be hard going for some readers. Bear with us; it's worth the effort.

2. Diffusion and Substitution Models

There is currently much interest in the development of models to forecast the diffusion of new technologies, particularly in the alternative energy area. Economic and technological forecasters have done considerable work in the area of trying to describe the time path of adoption. As we will see, these models, while useful descriptive tools, are not designed to help develop program guidance (what can be done to accelerate adoption of the innovation?) nor can they be calibrated except after the fact, when government or private sector controls are no longer important.

A frequently referred-to diffusion model is the Fisher-Pry [13] model. The model hypothesizes that the adoption of a new product is proportional to the fraction of the old one (the one being substituted) that is still in use. The mathematical form of this model is as follows:

$$(1) \qquad \frac{df}{dt} = bf(1-f)$$

where

f = fraction of the market captured by the innovation, and

b = initial rate of adoption of the product.

Integration of (1) yields

$$(2) \qquad f = \frac{1}{1 + \exp b(t-to)}$$

where

to = time at which f = 1/2.

This model - a logistic curve - has been shown to describe the adoption process quite well in 17 product areas investigated by Fisher and Pry [13], confirming much of Mansfield's earlier, similar work.

Blackman [6] develops a model which does not assume 100% penetration will be reached. The form of that model is

(3) $$\frac{df}{dt} = bf(F-f)$$

where F = fraction of the market penetrated.

Floyd [14] adds a linear "patch" to the Blackman and Fisher-Pry models which tend to over-estimate near the end of the forecast period. His model under-estimates. Sharif and Kabir [24] modify the patch.

Stapleton [25] suggests the use of a cumulative normal curve (the so-called Pearl [20] curve) to model the s-shaped phenomena. He argues that most models of technological changes assume:

(a) an s-shaped pattern over time, with early growth being exponential in character,

(b) an upper limit on growth, and

(c) a symetric curve with a transition point half way between the extremes.

Thus, as the cumulative normal has these properties, and is, indeed as flexible as the logistic, he argues for its case.

Nelson, Peck and Calacheck [19] postulate that the s-shape follows from a gradual movement from one form of equilibrium to another when a new product enters the market. A simple mathematical statement of this phenomenon assumes that the percent adjustment in any one period is proportional to the percent difference between the actual level of adoption of the innovation and the level corresponding to the new equilibrium. Thus, we obtain:

(4) $$\log Y_t - \log Y_{t-1} = b(\log k - \log Y_t)$$

where Y is the level of adoption and k, the new equilibrium. If we replace $\log Y_t - \log Y_{t-1}$ by $\frac{d\log Y}{at}$ and $b = -\log\beta$, (4) becomes

(5) $$\frac{d\log Y}{at} = -\log\beta(\log k - \log Y)$$

which is the differential equation associated with the Gompertz function.

Mansfield [18] implies that it is the lagged response to constant economic stimulus which leads to s-shaped response, developing the logistic formulation.

These are several behavioral explanations for the s-shaped phenomena; Babcock [3], Hägestrad [15], and Stapleton [25] all suggest that it is the distribution of resistance to innovation that causes the s-shaped pattern.

Sahal [23] reviews the qualitative aspects of the diffusion process conforming to s-shaped trends, reviewing economic, learning process and selection process explanations. He notes that it is easy to show, after the fact, that one or another form of s-shaped curve will describe the phenomena well. He concludes that "... the value of such a model is limited because it sheds little light on the nature of the underlying mechanism. More important, such a model is likely to be of little help in prediction because of the difficulty in choosing, (especially at an early stage in the process of diffusion) a specific form from a variety of s-shaped curves that would be appropriate." ([23], page 230)

We agree. Our problem, indeed, focuses on

(a) early stages of innovation diffusion and

(b) controls of the process.

Thus, we investigate the diffusion of innovations, not so much to predict as to control. We wish to determine how the government or a private sector decision-maker can affect the rate and ultimate penetration of an innovation diffusion. As Sahal points out, existing technological forecasting methodology is unsuitable for this purpose, both because of the lack of controllable variables imbedded in the models and because no measureable, casual phenomena are included in the model which can be empirically verified.

We thus look to incorporate normative, new product acceptance models from the marketing literature, with their associated measurement procedures, into new product diffusion models. We will base our projections of demand on data (even if attitudinal) from potential adopters as opposed to on assumptions from economic model builders. We feel it is better to have empirical foundations, even if those foundations are subject to bias and change, than to base model calibrations on conjecture.

(Appendix 3 contains a detailed, technical development of several of the models that are important to us here.)

3. The Adoption of New Products

This section presents an overview of a model of organizational adoption of new capital equipment. While the methodology is developed for organizations, it is appropriate for consumers and other, single decision-makers as well. In that case "organization" and "decision participant", the terms used below, will be synomynous with buyer or consumer. The material presented here is developed in more detail in Choffray and Lilien [11] and in Lilien et al [16].

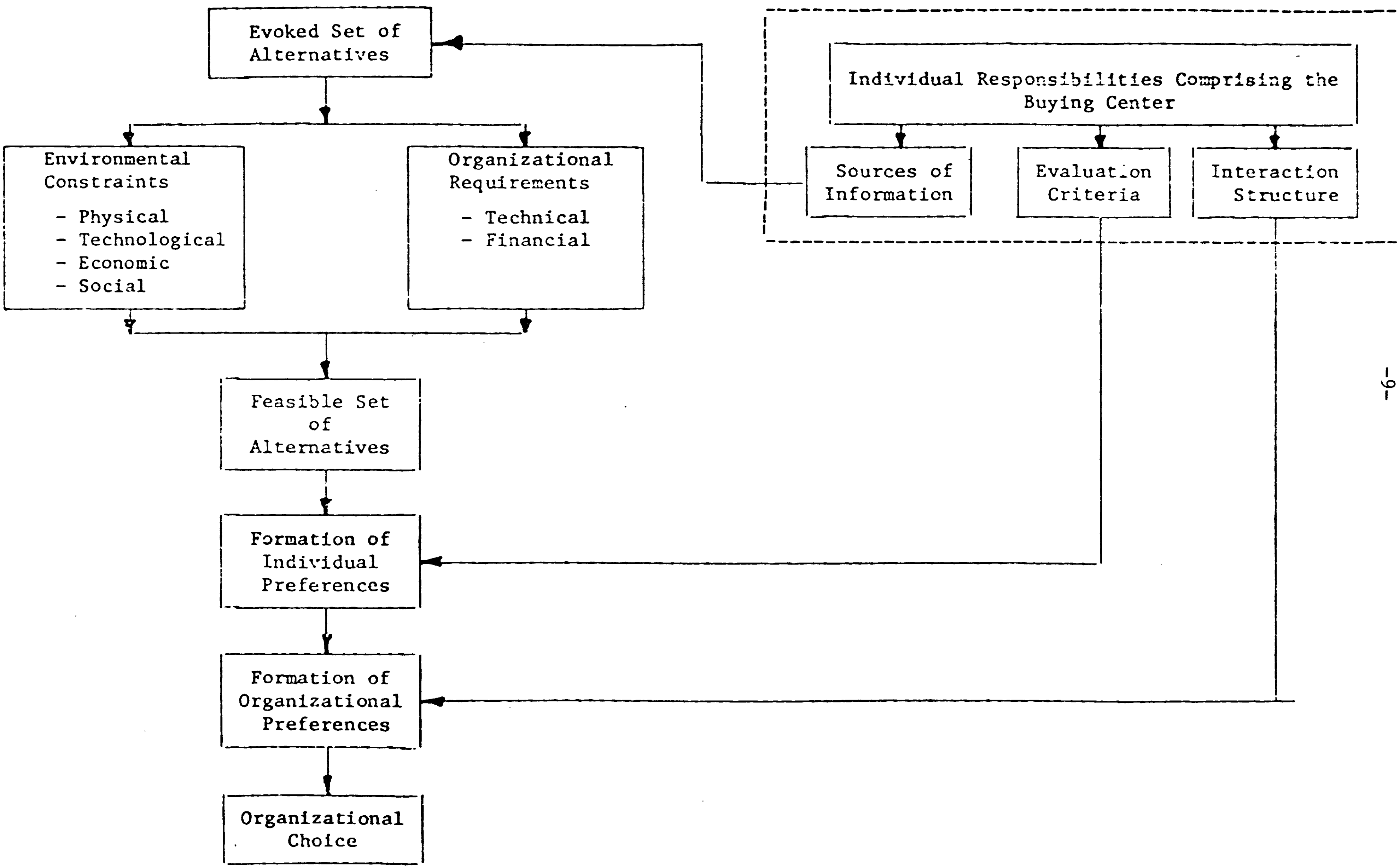
3.1 Industrial Adoption of Capital Equipment

For industrial adoption of capital equipment - such as photovoltaic electric generation - a multiperson decision process is the normal mode of behavior. This decision process is characterized by the involvement of

- several individuals, with different organizational responsibilities, who
- interact with one another in a decision-making structure specific to each organization, and
- whose choice-alternatives are limited by environmental constraints and organizational requirements.

Figure 1 illustrates a framework developed to describe the organizational adoption process. It focuses on the links between the characteristics of an organization's buying center (those individuals involved in the purchase decision) and the three major stages in the industrial purchasing decision process: (a) the elimination of alternatives which do not meet organizational requirements, (b) the formation of decision of decision participants' preferences, and (c) the formation of organizational preferences.

Figure 1: Major Elements of Organizational Buying Behavior



Although simple, this conceptualization of the industrial purchasing decision process is consistent with current state of knowledge in the field. It operationalizes the concept of the "buying center" and explicitly deals with the issues of product feasibility, individual preferences, and organizational choice.

3.2 The operational Model

A complete, operational model of industrial response requires that organizational differences be explicitly handled. This model addresses the following issues:

1. Potential customer organizations differ in their "need specification dimensions", that is, in the dimensions they use to define their requirements. They also differ in their specific requirements along these dimensions.
2. Potential customer organizations differ in the composition of their buying centers: in the number of individuals involved, in their specific responsibilities, and in the way they interact.
3. Decision participants, or individual members of the buying center, differ in their sources of information as well as in the number and nature of the evaluation criteria they use to assess product alternatives.

The consideration of these sources of organizational heterogeneity in an aggregate model of industrial response requires that members of the buying center be grouped into meaningful "populations". Here we use "decision participant category" to refer to a group of individuals whose responsibilities in their respective organizations are essentially similar. Examples of such participant categories are "production and maintenance engineers", "purchasing officers", "plant managers", etc.

Our objective with this analysis is to gain leverage by analyzing similar situations together -- hence, we focus on areas where individual or organizational homogeneity allows meaningful aggregation. To this end, we assume:

A1. Within potential customer organizations, the composition of the buying center can be characterized by the categories of participants involved in the purchasing process.

A2. Decision participants who belong to the same category hare the same set of product evaluation criteria as well as information sources.

Both of these assumptions have been empirically verified in the solar air conditioning study (Lilien et al [16]).

Figure 2 presents the general structure of the industrial market response model. Four submodels comprise this structure, each of whose purpose, structure and method of calibration is briefly described below.

3.2.1 The Awareness Model

3.2.1a Purpose

The awareness model links the level of marketing support for the industrial product investigated -- measured in terms of spending rates for such activities as Personal Selling (PS), Technical Service (TS), and ADvertising (AD) -- to the probability that a decision participant belonging to category i, (engineer, say) will evoke it as a potential solution to the organizational purchasing problem. Let

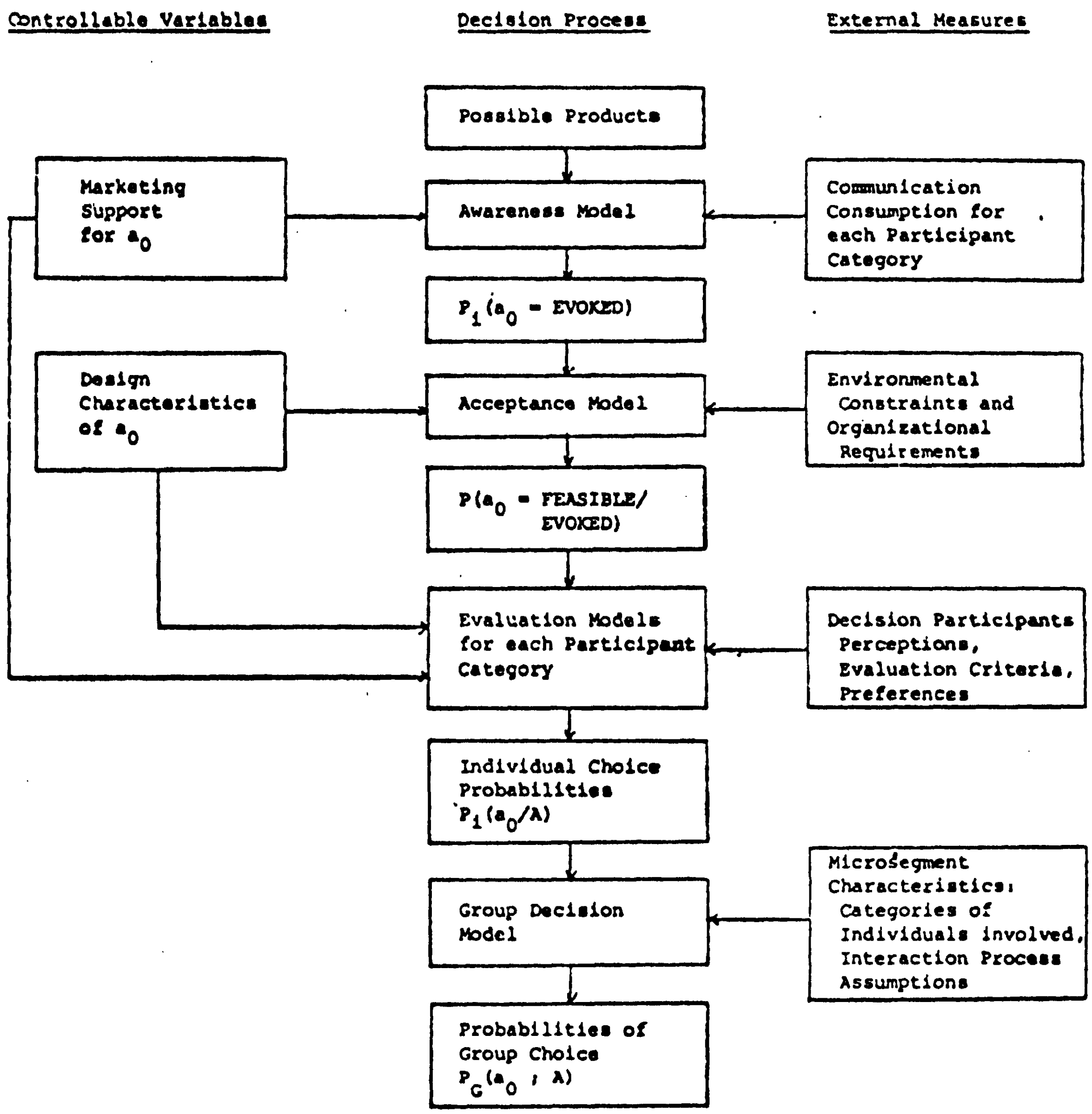
$$P_i\ (a_0 = \text{EVOKED})$$

denote this probability. Hence we assume that

$$P_i\ (a_0 = \text{EVOKED}) = f_i\ (PS, TS, AD). \qquad (6)$$

Implicit in this formulation is assumption A2 above which states that individuals who belong to the same participant category share essentially the

Figure 2: GENERAL STRUCTURE OF AN INDUSTRIAL MARKET RESPONSE MODEL



same sources of information.

When several decision participant categories are involved in the purchasing process for each of a group of customer organizaitons, the probability that the product will be evoked as an alternative is the probability that at least one memeber of the buying center will evoke it.

In the analysis which follows, we will assume awareness at different levels to demonstrate the effect on market penetration. Again, the analysis simplifies when each organization has only a single decision-participant.

3.2.2 The Acceptance Model

3.2.2a Purpose

The acceptance model relates the design characteristics of the product to the probability that it will be acceptable to a potential customer. This submodel accounts for the process by which organizations in the potential market screen out "impossiblies" by setting product selection requirements (e.g., limits on price, reliability, payback period, number of successful prior installations, etc.).

Although organizations in the potential market may differ in their need specification dimensions, as well as in their requirements along these dimensions, the acceptance model assumes that the process by which organizations eliminate infeasible alternatives is essentially similar across potential customer organizations.

3.3.2b Analytical Structure

Several models can be used to approximate the process of organizational elimination of infeasible alternatives. Choffray and Lilien [11] develop two convergent approaches to specify the form of the acceptance model. Both approaches require information about the maximum (or minimum) requirement along each relevant need specification dimension from a sample of organizations in the potential market. The first approach is probabilistic and derives the multivariate distribution of organizational requirements from the values observed in the sample. The second approach uses simulation and logit regressions to relate the fraction of organizations for which an alternative is feasible to its design characteristics.

Independent of the approach followed, the elimination function, once specified, can be input to a simulation which (1) provides insight into product design trade-offs, and (2) allows accurate prediction of the rate of market acceptance.

3.2.3 The Individual Evaluation Models

3.2.3a Purpose

Individual evaluation models relate evaluation of product characteristics to preferences for each category of decision participants. The models permit the analysis of industrial market response to changes in product positioning. They therefore feed back important information for the development of industrial communication programs that address the issues most relevant to each category of participants.

3.2.3b Analytical Structure

The development and calibration of individual preference models assume an n-dimensional "evaluation space" common to each category of decision participants. The axes in this space are independent and express how individuals in that group structure product attributes into fewer, higher-order evaluation criteria. An individual's evaluation of a product can then be represented as a vector of coordinates in that space.

Choffray and Lilien [10] develop new methods to analyze the evaluation space of categories of decision participants. That analysis indicates that participant categories differ substantially in the number and composition of their evaluation criteria. Moreover, they show that preference regressions estimated for each participant category provide substantially different results than would have been obtained from a more aggregate analysis.

Once calibrated, models of individual preference formation are used to predict preference for product alternatives. These preferences, in turn, are transformed into individual probabilities of choice.

3.2.4 The Group Decision Model

3.2.4a Purpose

The last element of the industrial market response model is the group decision model which maps individual choice probabilities into an estimate of the group probability of choice.

3.2.4b Analytical Structure

Choffray and Lilien [8] develop four classes of descriptive probabilistic models of group decision-making. They include a Weighted Probability Model, a Proportionality Model, a Unanimity Model, and an Acceptability Model. These models encompass a wide range of possible patterns of interaction between decision participation categories and offer representation of this process for most industrial buying decisions. Depending on the manager's understanding of the interaction process within a particular microsegment, any of these models, or a combination of them can be used to assess group choice.

3.2.5 Linking the Submodels

Combining the four submodels just presented, we get a general expression for the unconditional probability of organizational choice of product a_0:

$$\text{(7)} \quad \text{PR }[a_0 = \text{ORGANIZATIONAL CHOICE}] = \text{PR }[a_0 = \text{GROUP CHOICE} | \text{INTERACTION, FEASIBLE, EVOKED}] \times \text{PR }[a_0 = \text{FEASIBLE} | \text{EVOKED}] \times \text{PR }[a_0 = \text{EVOKED}]$$

3.3 Implementation of the Industrial Market Response Model

Implementation of the structure described above requires a measurement methodology which provides input to the various models. This section reviews the measurement steps involved in a typical implementation, many of which have been performed in the study. These measurements are summarized in Figure 3.

3.3.1 Measurement at the Market Level

The first measurement step, called macrosegmentation, specifies the target market for the product. The purpose of macrosegmentation is to narrow the scope of the analysis to those organizations most likely to purchase the product. Bases for macrosegmentation might be as general as S.I.C. code classification, geographic location, etc. The output of this measurement step is an estimate of the maximum potential market for the product. Let Q denote that maximum potential.

Figure 3: MAJOR MEASUREMENTS NEEDED FOR CALIBRATING THE INDUSTRIAL MARKET RESPONSE MODEL

DATA TYPE	SOURCE	TARGET
1. Hard	1.1. Market	1.1.1 Macrosegmentation: Target Market Definition
	1.2. Organizations	1.2.1 Identification of Need Specification Dimensions; Measurement of Organizational Requirements. (Focus Groups)
		1.2.2 Microsegmentation: Grouping of Organizations on the Basis of Buying Center Composition (Decision Matrix)
	1.3. Decision Participants	1.3.1 Product Awareness and Communication Consumption Patterns (Questionnaire)
		1.3.2 Product Evaluations and Preferences (Questionnaire)
2. Soft	2.1. Industrial Marketing Manager	2.1.1 Judgmental Estimates of Interaction Process

3.3.2 Measurements at the Customer-Organization Level

Two major types of measurements have to be obtained at this level. If, as in our case, the potential market for the product contains a large number of customers, a representative sample can be drawn.

Organizations' need specification dimensions have to be identified first, and then the requirements of each firm in the sample along these dimensions must be assessed. Identification of these dimensions follows discussions with potential decision participants. Group interview methods are particularly suitable for this purpose. We found that such interviews with members of the buying center of a few (3-5) potential customers are generally sufficient to identify the set of relevant specification dimensions.

Survey questions are developed next. We developed questions requesting the maximum (or minimum) value along each specification dimension beyond which the organization would reject a product out of hand. These answers are the main input to the acceptance model.

Next, information is collected on the composition of the buying center and the respective organizational responsibilities of its members. This information allows the development of a decision matrix (see Choffray and Lilien [10]) which requests the percentage of the task responsibilities for each stage in the purchasing process associated with each category of decision participant.

Choffray [8] develops methodology based on cluster analytic procedures to use this information to identify microsegments of potential customers which are relatively homogeneous in the composition of their buying centers. Call the microsegments identified at this stage $S_1 \ldots S_n$ and the percentage of companies in the potential market that fall in each $V_1 \ldots V_n$.

Within each microsegment, the general structure of the buying center's composition is assessed. Let microsegment S_q be characterized by the set of participant categories, $DEC_q = \{D_i, i = 1...r_q\}$, that are usually involved in the purchasing process. For instance, in segment S_1, corporate managers along with design engineers might be the major categories of participants involved. In S_2, production engineers are involved too, etc.

3.3.3 Measurement at the Decision Participant Level

For each category of decision participant, product awareness, perceptions and preferences are measured at the individual level.

Product awareness can be obtained through survey questions asking each potential decision participant what product(s) or brand(s) of product they think of in a specific product class. In our case, we can use awareness variation for sensitivity analysis in our penetration estimates.

The measurement of individual perceptions, evaluations, and preferences for product alternatives makes use of concept statements, accurately describing each product. The use of product concepts is most desireable when a physical sample product is unavailable. When it is, individuals, if possible, should be exposed to that physical product. Individual product perceptions or reactions to these concepts are then recorded along each of a set of perceptual scales which include the relevant attributes used by individuals to assess products in this class.

3.3.4 Measurement at the Managerial Level

The measurements described above are used to the three frist components of the industrial market response model. Development of group choice models, however, requires assumptions about the type of interaction which takes place between decision participant categories.

As suggested earlier, the measurement methodology relies on experience with the product class. The final input to the industrial response model consists of the decision-maker's specification of those models if interaction which best reproduce his understanding of the purchasing decision process for the companies which fall in each microsegment.

3.4 Assessing Market Response: Integrating Measurements and Models

The information provided by the measurement methodology and fed into the components of the various models leads to an estimate of market response. Let $M_q(a_0)$ denote the estimated share of microsegment S_q that finally purchase product a_0, photovoltaics in our case. Hence

$$(8) \qquad M_q(a_0) = \sum_e \alpha_{eq} \; P_r[a_0; A|MOD_e, DEC_q]$$

where $P_r[a_0; A|MOD_e, DEC_q]$ denotes the probability that a_0 is the organizational choice, given the involvement of decision categories DEC_q and an interaction model MOD_e.

Given a maximum potential unit sales of Q (generally obtained from product-class consumption figures) for product a_0, we can estimate expected sales of a_0 by computing

$$(9) \qquad \text{Sales}(a_0) = Q[\sum_{q=1}^{S} V_a M_q(a_0)]$$

The projections developed here use this model as a base; the next section demonstrates how this model can be made dynamic.

4. The PV Diffusion Model

The model described in the last section is static: it predicts the sales of a new product at a given point in time under a given set of conditions. Conceptually, we lay out a model such as that in Figure 2 for each of a number of years into the future; the dynamics of the market place change perceptions of the products and the feasibility requirements of potential customers vary. Cost variability in the marketplace is modeled to change with cumulative production. The number of prior successful installations changes receptivity of potential customers.

Thus, the PV diffusion model considers the following controlling influences:

1. Cost per unit of energy produced
2. Relative cost of other energy sources
3. The perception of risk in adopting the innovation
4. External factors, such as government policy
5. Non-economic factors such as aesthetics and "energy-saving"

We are interested in predicting total cumulative demand as a function of controllable parameters. For example, What is the effect of a marketing policy that reduces perceptions of risk? How can the government best allocate resources to stimulate demand? There are two related assumptions underlying this, first - order PV model:

1. Cost (price) declines by an experience curve as production increases, increasing demand;
2. The likelihood of adoption in a sector is increased as the number of successful installations increases. In this model, demonstration

projects are assumed to affect potential adoptors in the same environment or 'sector,' suggesting that we split the market into different sectors, each with separate response.

Note that the learning curve, (Andress [2]) as a model for predicting cost declines with production, uses certain restrictive assumptions. Clearly, better cost/supply models are needed (see Abernathy and Wayne [1]); the development of such models is beyond the scope of this work. Our developments are general, however, and could incorporate a superior cost/supply model easily.

Since the cost of production is not highly dependent on application, the cost decline function is assumed to be common to sectors, i.e., market-wide. Influences such as competing technologies are not considered in this model; rather, a few key variables are included to give a first approximation of likely market response.

Early in the life of a product, the only dynamic input to the decision process is government investment. By acting as a large, guaranteed consumer through pilot projects in various sectors it increases installations, thereby increasing likelihood of adoption in those sectors, and causes a cost decline through increased production. Figure 4 outlines the decision process in a model with two sectors. That figure incorporates a simplification of Figure 2 in each of two sectors, with time-dependent feedback.

As discussed in the previous sections, to obtain meaningful results from this model, it must be calibrated accurately. We need to know not only the form of the adoption likelihood function, but also how it changes with customer attitudes. This will allow us to run model-based sensitivity analyses on the effect of market-place changes.

The PVO model (so-called to distinguish it from later more sophisticated models) is discrete time and deterministic. PVO models the increase in

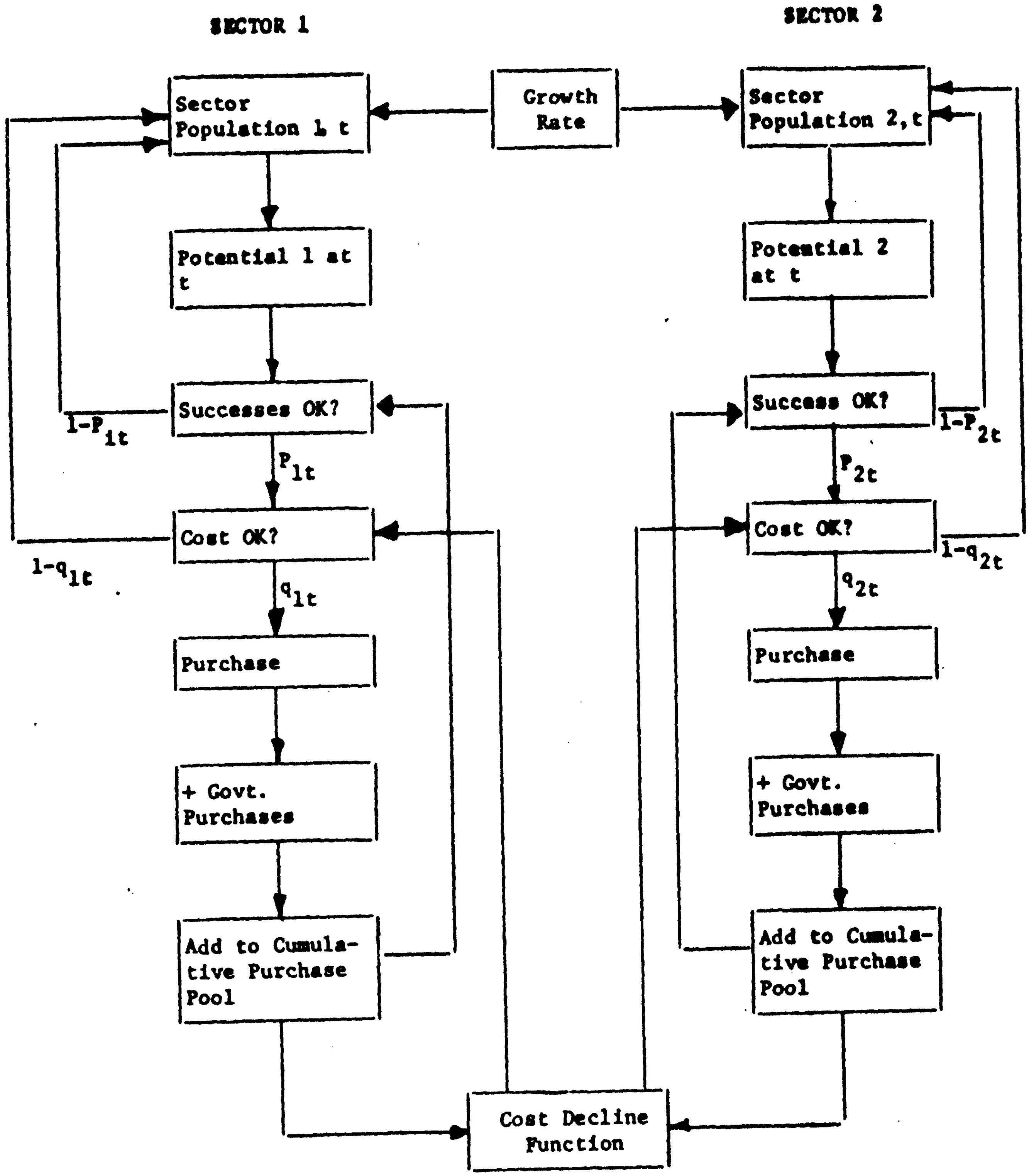


Figure 4: PV_0 Model Outline

installations, and unit cost decreases, over time. All variables related to number of installations are split into private and government parts. The variables that we want to follow are:

X_{it} = number of square feet of government installations in sector i at time t

Y_{it} = number of square feet of private installations in sector i at time t

C_t = cost per square foot of PV at time t

PVO models cost as an exponentially decreasing function of the cumulative number of square feet in all installations.

Let $N_{it} = \sum_{\tau=0}^{t-1} (X_{i\tau} + Y_{i\tau})$ be cumulative square feet installed in section i before t, and

$N_t = \sum_i N_{it}$ be cumulative square feet across sectors.

Let Z_i be the average installation size in sector i, so

N_{it/Z_i} = number of installations in sector i.

A standard form for a cost decline is "constant doubling," where cost is discounted by a fraction λ when production doubles, or (where C_0) is initial cost):

(10) $$C_t = C_0 \, (N_t/N_0)^{\log_2 \lambda}$$

The question then is, at the next time step how many additional square feet will be bought as a function of this cost? Our assumptions imply that the fraction of consumers who will buy are those who find the cost low enough and the number of prior successful installations high enough (subject to their perception of PV). Data from the solar air-conditioning study (see Lilien et al. [16]) suggests that the cost and number of successful installation components of the decision are approximately independent, and so can be described as independent probability distributions, say f_c for cost and f_{si} for successes in sector i. Note that the model we develop below in no way requires independence; however, it does make our notation a bit clearer to assume independence at this point. Thus,

(11) $$\int_{C_t}^{\infty} f_C(p)dp = 1 - F_C(C_t) = \text{probability that } C_t \text{ is acceptable, and}$$

(12) $$\int_{0}^{N_{it}/Z_i} f_{S_i}(p)dp = F_{S_i}(N_{it}/Z_i) = \text{probability that } N_{it}/Z_i$$

successes are acceptable in sector i. Also define P_{i0} = total potential installations in sector i at time 0 and let g_i be the growth rate of number of installations, so at time t there are $p_{it} = (1+g_i)^t P_{i0}$ potential installations in sector i. Before time t, S_{it}/Z_i installations have already been made, and X_{it}/Z_i new installations are made at time t in sector i by the government. The remaining private purchase potential is decreased by three factors. The two acceptability criteria of cost and successes have already been discussed. Given acceptance on that basis, there is then a product-perception/probability of choice: $h(\vec{A})$, where $\vec{A}$ is a vector of product/market characteristics. Referring to the models in the previous section, (equation (7)) $h(\vec{A})$ = Prob(organizational choice/awareness and acceptability).

Thus, the number of private square feet of PV purchases at time t in sector i is expected to be:

$$(13) \quad Y_{it} = (P_{it} - N_{it}/Z_i - X_{it}/Z_i)\ (1-F_C(C_t)) \cdot F_{Si}(N_{it}/Z_i) \cdot h(\vec{A}) \cdot Z_i \ .$$

Now that we have Y_{it} we can find N_{it} and N_t, setting the stage for advancing to t+1.

We can now use this model to formulate a simple decision-problem for the government. The government's problem is to decide how much to spend and how to allocate demonstration project resources. This becomes

find $\{X_{it}\}$

to maximize $$\sum_i \sum_t Y_i (1/1+\lambda)^t,$$

where λ is a discount factor,

subject to $$\sum_i X_{it} C_t \leq Z_t, \quad \text{for all } t,$$

where Z_t = annual government budget constraint.

Later versions of the model will incorporate the more complex problems arising when governmental action can include subsidies, tax credits, etc.

But what happened to the S-curve? If the F_c and F_{Si} distributions are reasonably symmetric, and no erratic behavior occurs in the marketplace, an S-shaped response will, indeed, be generated by equation (13). Note, however, that this is an output of our model, completely dependent on input assumptions. Erratic market activities or bi-modal F_C or F_{Si} distributions (all perfectly reasonable possibilities) will lead to double S-curve (an S curve with three inflection points) or more complex behavior. We thus let the theory of buyer behavior and our market measurements, filtered through the model, generate the response curve. This is a very different view of the process from those reviewed in section 2; we feel it is a more reasonable one, however.

PVO has been implemented as a system of computer programs written in PL/I written on the Multics System at MIT. Within the framework of a SUPERVISOR program there are commands to:

1. CREATE a block of sector-specific or model-wide parameters,
2. MODIFY such a block,
3. COPY such a block under a new name,
4. DELETE such a block, or the output of a run of a model,
5. DISPLAY such a block or the output of a run,
6. LIST the names of the existing sectors, models or runs, and
7. RUN a model.

These commands are user-oriented in that the programs will prompt the user for missing or erroneaous parameters. In addition, typing "help" in response to any system prompt will produce an explanation of appropriate actions, and "return" will end processing.

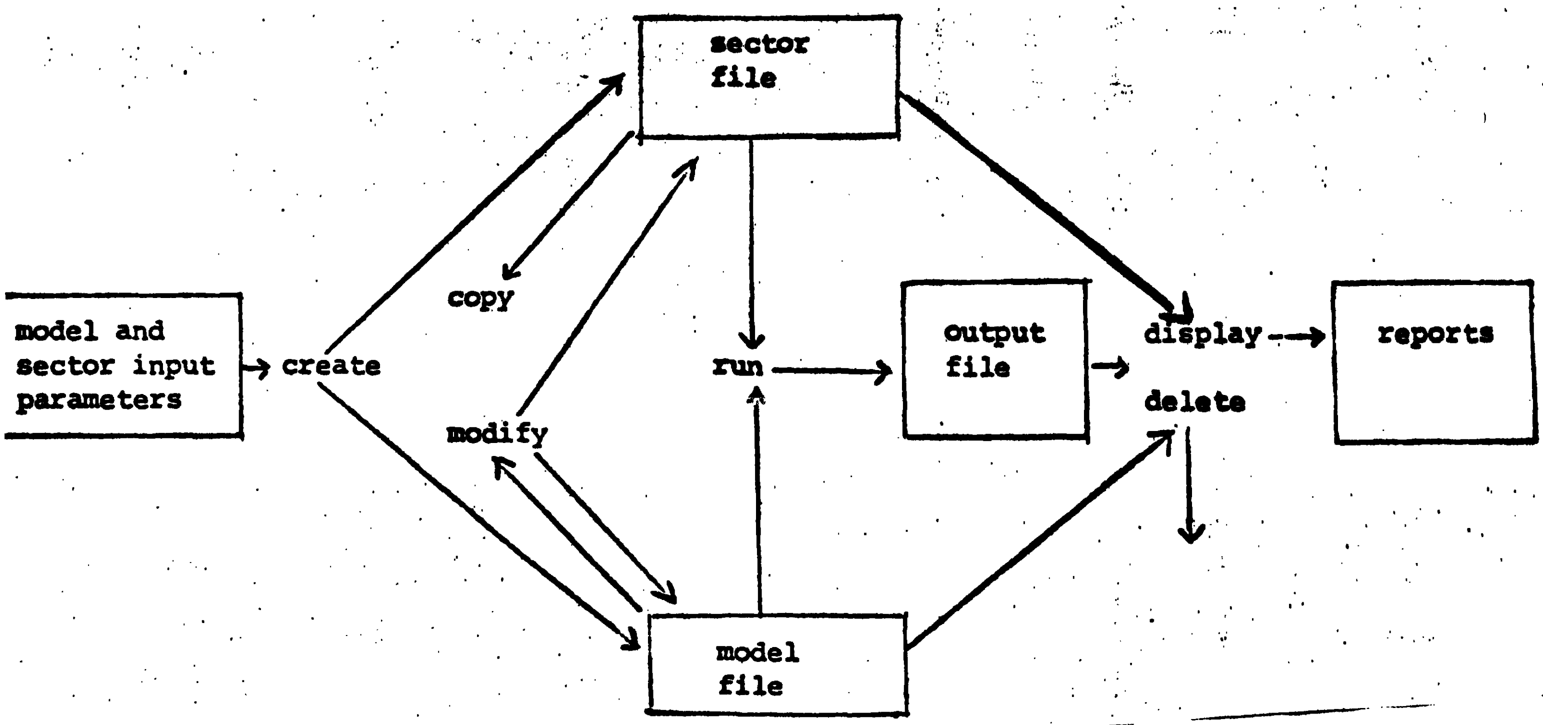


Figure 5

Figure 5 outlines the general protocol used to interact with the system and to run models. More complete system documentation will be included in a later document.

5. Data Acquisition: Agricultural Sector

The agricultural sector -- especially for remote irrigation pumping applications -- is considered to be a likely early adopter of PV because

(a) energy is needed in locations remote from central power supplies;

(b) pricing behavior by utilities (a leveling or annualizing of rates) effectively burdens the farmer for any seasonal energy usage.

(c) pumping for irrigation is needed most when solar energy is most abundant;

(d) farmers find long pay back periods more acceptable than many other sectors (see Figure 19).

Thus, we reacted quickly to collect data in conjunction with a PV experimental installation in Mead, Nebraska on July 21, 1977.

To test the reaction of farmers to the demonstration installations, relative to their reaction to a description, we collected data from two groups of respondents. These respondents were randomly selected from farmers who were attending Farm Tractor and Safety Day at the University of Nebraska's experimental farm at Mead. It was here that the PV irrigation demonstration was to take place and be viewed for the first time.

The two groups of respondents were:

1. Farmers who had not been exposed to the PV demonstration;
2. Farmers who had just been through the demonstration.

A third set of measurements was taken from those who had been interviewed prior to viewing the demonstration and who returned to be reinterviewed after they had viewed the demonstration.

The actual sample design is summarized in Table 1.

TABLE 1: Sample Design

	Measurement	Demonstration	Measurement		
Group 1	0	x	0	87	(exposed and re-exposed)
Group 2	0			104	(measured only)
Group 3		x	0	105	(exposed first)
				296	

5.1 Questionnaire Design and Issue Development

An unstructured questionnaire was developed, including the measurements we needed on sector demographics, price-acceptance distribution, number of successes prior to adoption, cost decline factors and average sizes of installations.

In addition, we used this opportunity to probe other areas and issues that would assist in future demonstration designs and marketing research in other sectors.

As part of this design, two project members went to Lincoln, Nebraska to interview a cross-section of individuals who were involved and knowledgeable about farm management and irrigation.

The people interviewed were:

1. A farm business writer, who also owned a small farm;
2. A large farm owner-operator;
3. A farm-extension county agent for the state;
4. A farm machinery dealer;
5. A bank farm loan officer;
6. An official of the Farm Bureau;

7. The Department Head of Agricultural Engineering at the University of Nebraska;
8. University of Nebraska Professor of Agriculture and Water Resources;
9. University of Nebraska Public Relations and Communications editor in charge of the PV demonstration project;
10. Radio and T.V. station farm editors in Lincoln.

These interviews provided the basis to develop a set of questions addressing important irrigation issues.

A questionnaire draft was completed after these interviews. This draft was tested on farms in rural Massachusetts and New Hampshire. Modifications to the questionnaire and instructions to interviewers were made and the final questionnaire/interviewer instructions were finalized and printed.

5.2 Logistics

Field interviewers were contracted through a market research interviewing firm. This provided the project with 21 professional interviewers, from Lincoln or Omaha, Nebraska. Three members of the project staff acted as interviewer trainers and supervisors.

During the initial trip to Nebraska, arrangements for site location and facilities were made in cooperation with the Lincoln Labs of MIT and University of Nebraska. The best location would be one close to the demonstration site but on a route that would enable prospective respondents to the inercepted before arriving and after leaving the PV demonstration. The traffic pattern for the PV demonstration enabled this selection without problem (see Figure 6).

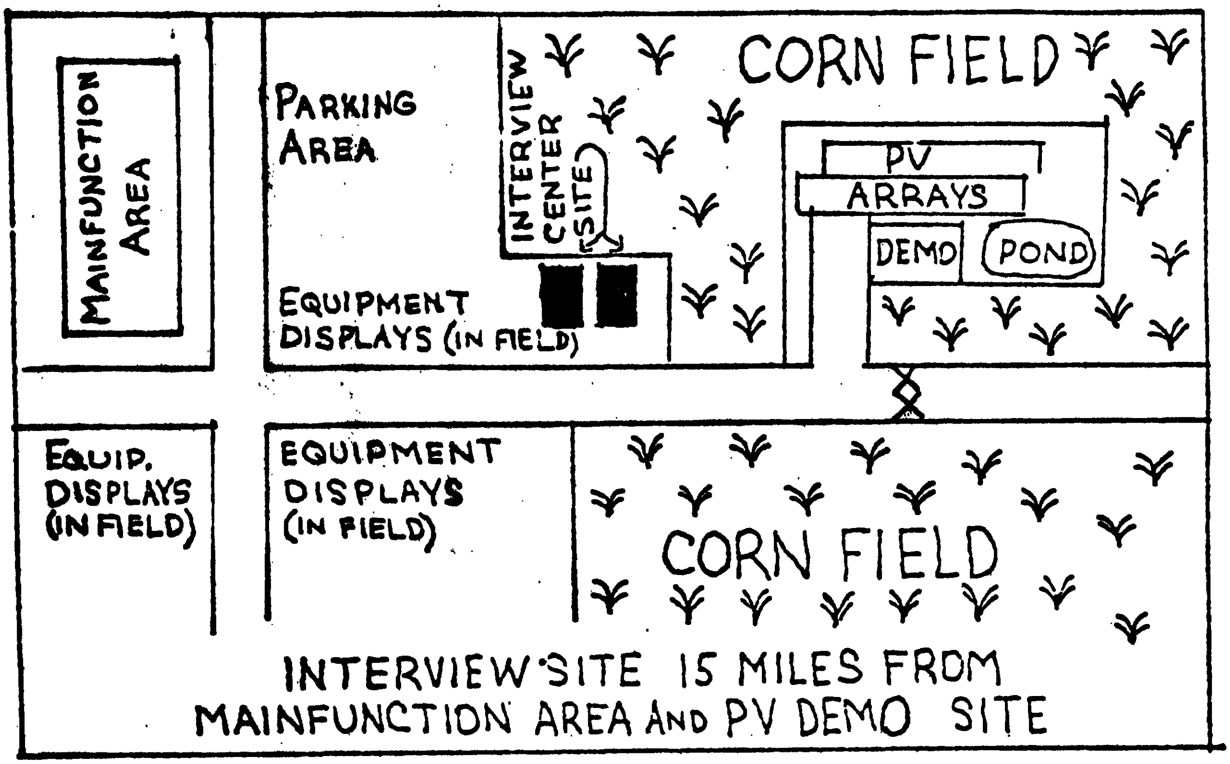


Figure 6: PV, Mead, Nebraska Demonstration Site

The temperature was in the high 90's at the time and our air conditioned trailers aided in obtaining cooperation.

5.3 Execution and Supervision

The trailers were clearly identified with signs indicating their association with the PV project:

"M.I.T. - U. of N.
INTERVIEW CENTER"

The interview required 20-30 minutes to complete depending upon the group being interviewed.

In order to insure understanding and proper execution of the interviewing, three project members circulated among the interviewers as they worked with respondents both inside and outside of the trailers.

Completed interviews were spot checked for completion. A running tab of interviews, by group, was maintained. Interviewers were directed from time to time to focus on specific groups in order to provide proper balance.

During the morning, announcements were made at the Main Function area promoting cooperation of the farmers with the interview center. Interviewers were identified with a tag and red "Interviewer" ribbons indicating their MIT-University of Nebraska PV association, their name and interviewer number.

A total of 296 interviews were completed between 8:30 a.m. and 4:00 p.m., as indicated in Table 1.

6. Data Analysis

As indicated earlier, this document gives a quick read of the results of the Mead experience. This section reviews the data collected. The next two shows how the data can be used to calibrate the model for the agriculture sector.

The farmers in the field study were a diverse group. Figure 7 shows a wide distribution of ages, with an average of 42 years old. Figure 8 indicates that nearly half have graduated only from high school, though 38.5% have some college experience. From Figure 9 we see that most of this education is farm related as 30.4% of the total, or 79% of those with college courses, have taken an agriculture course. These numbers are 25.5% and 66% respectively for business courses. For 76% of the farmers, according to Figure 10, farming is their only occupation, and from Figure 11, 72% have spent more than 10 years on their farm, and average of 24 years on the farm. There is a wide range of sizes of farms according to Figure 12 with 52.2% of the farms 400 acres or less, but enough very large farms to pull the average up to 60%. Farming income is rather low, with 45% of the farms grossing under $40,000, according to Figure 13. Figure 14 shows 46.3% of the farms in this sample are irrigated, by a variety of methods, (Figure 15), using a variety of fuels (Figure 16), with more than half of those who irrigate using diesel fuel at least part of the time. Because the distribution of acreage is skewed, the distribution of irrigation fuel costs is also. Though 65.0% of the irrigators spend $3000 or less on fuel, the average expenditure is $4920, according to Figure 14.

Figure 7. AGE IN YEARS

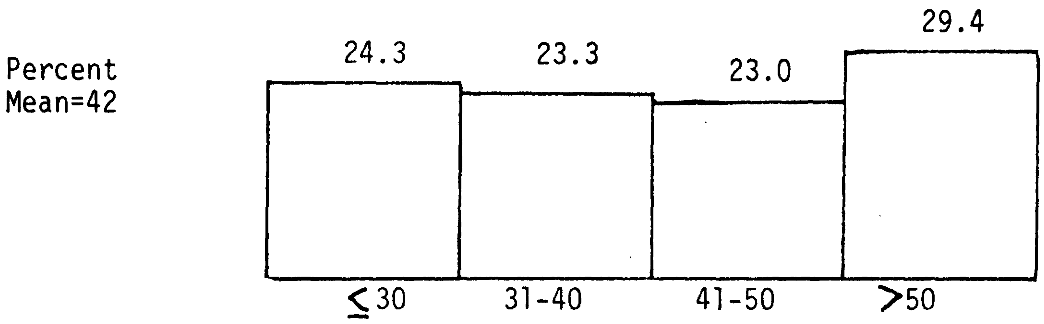


Figure 8. EDUCATION

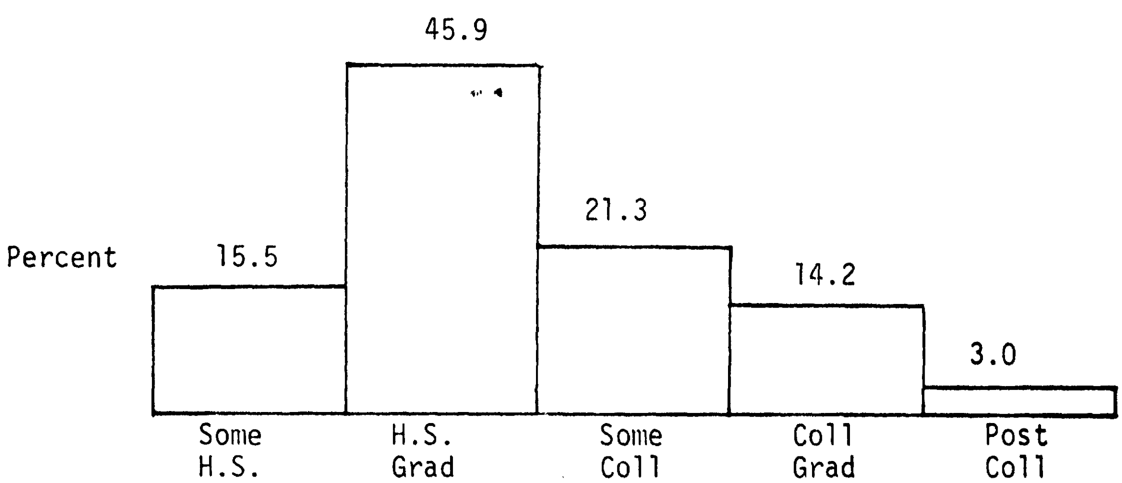
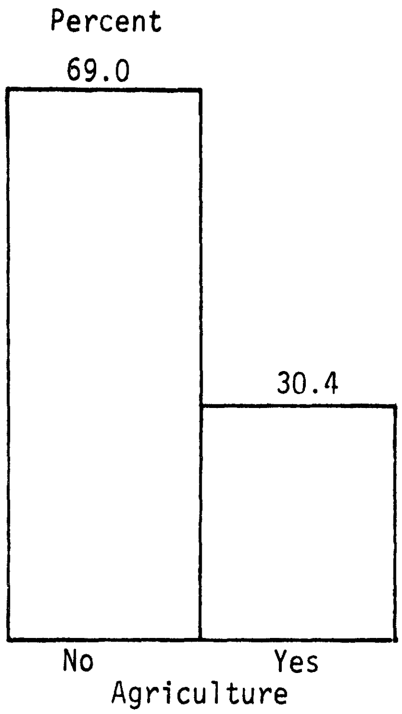


Figure 9. COLLEGE COURSE



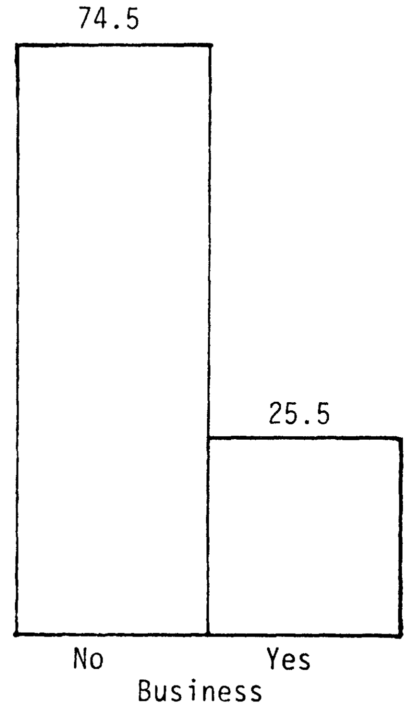


Figure 10. FARMING AS SECOND OCCUPATION?

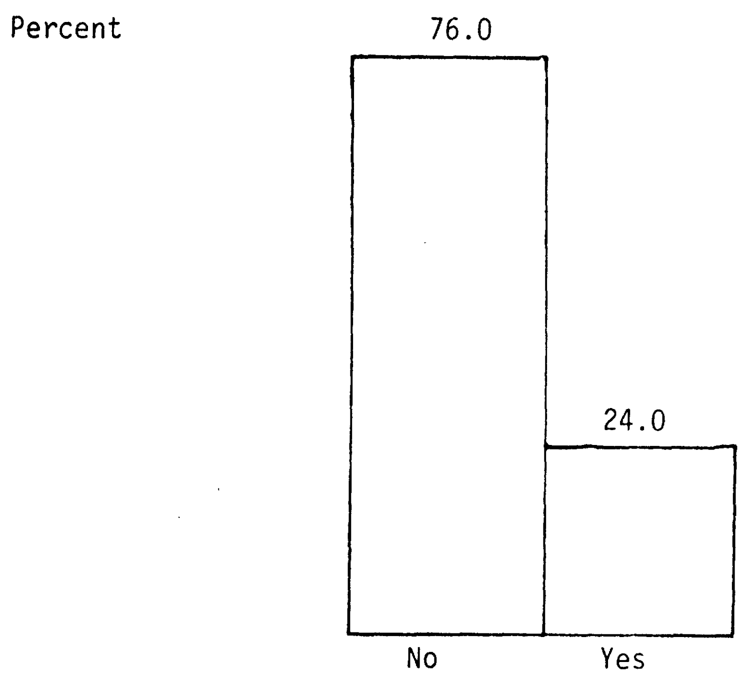


Figure 11. YEARS ON FARM

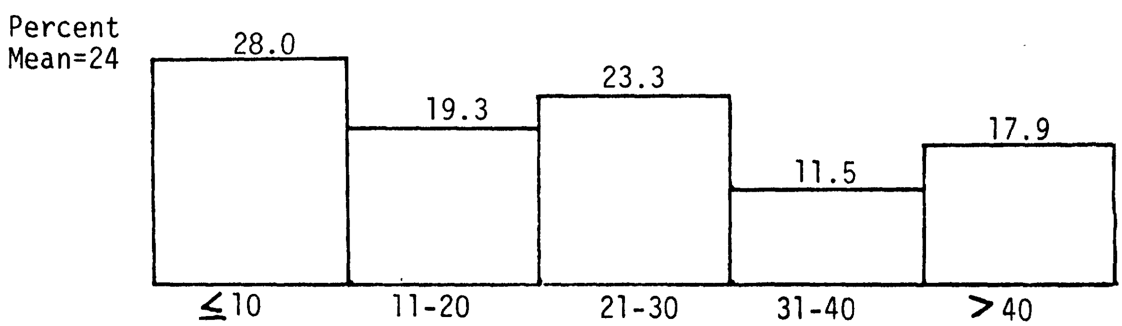


Figure 12. ACRES ON FARM

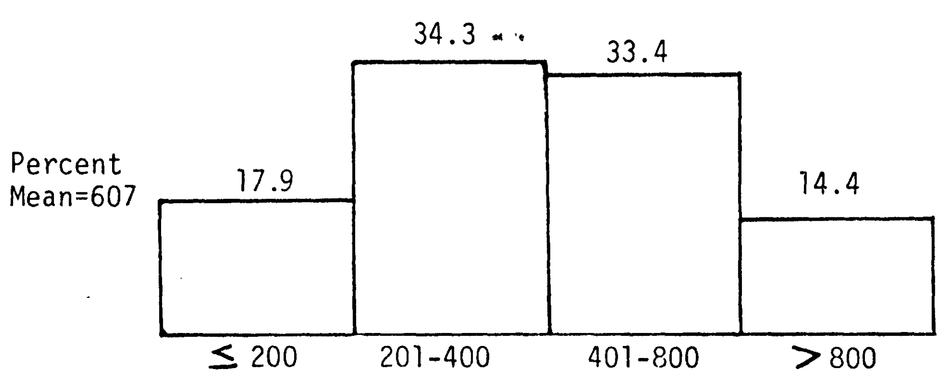


Figure 13. GROSS FARM INCOME (K$)

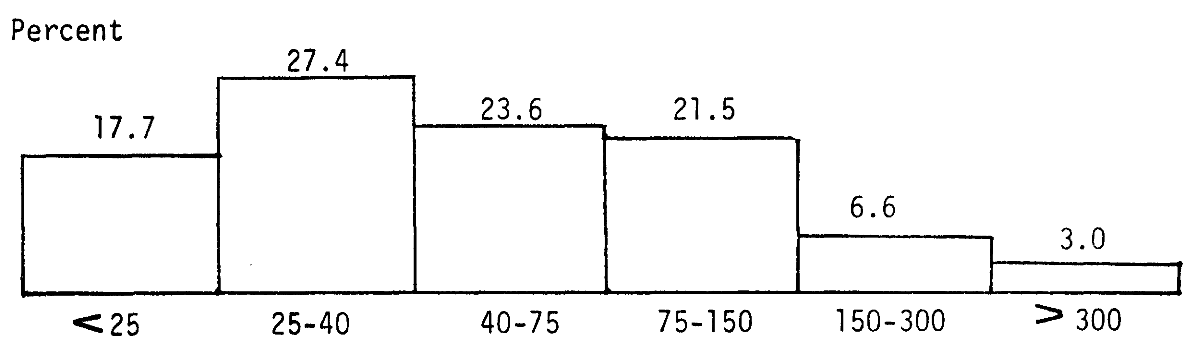


Figure 14. Irrigate?

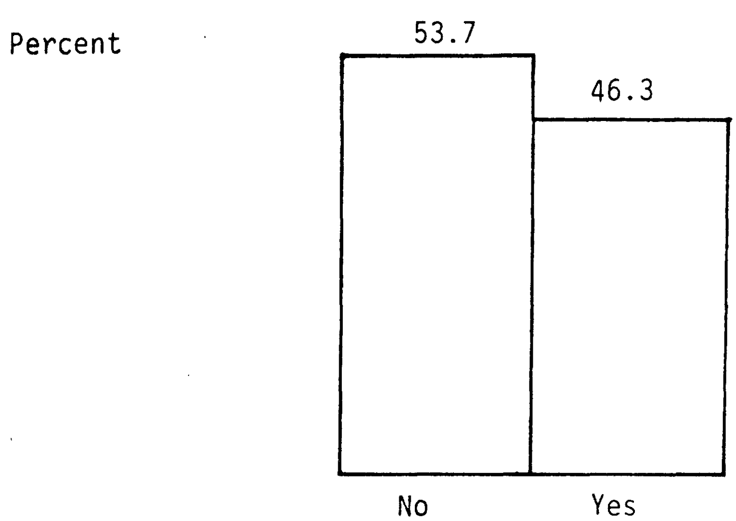


Figure 15. IF IRRIGATE, METHOD USED
(percent adds up to more than 100 due to multiple responses)

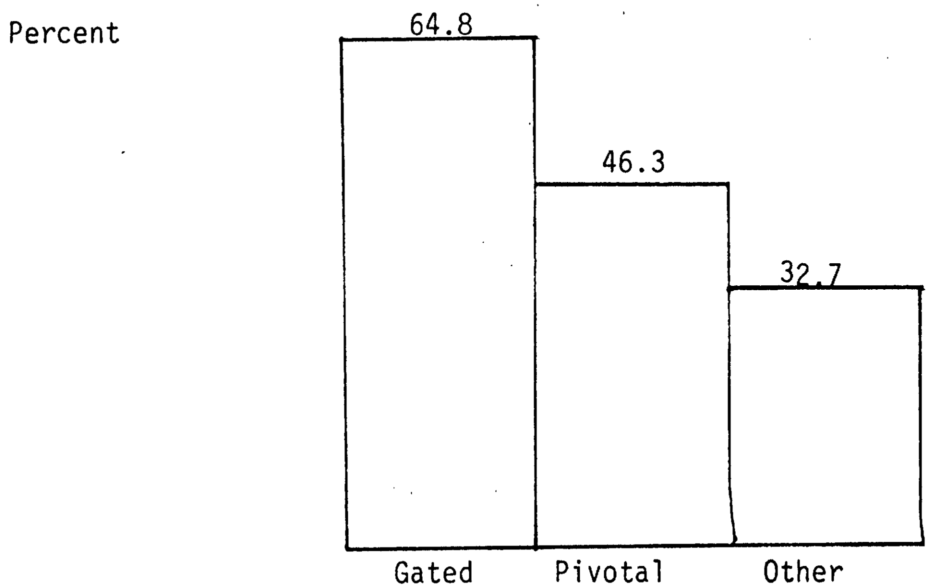


Figure 16. IF IRRIGATE, FUEL USED
(percent adds up to more than 100 due to multiple responses)

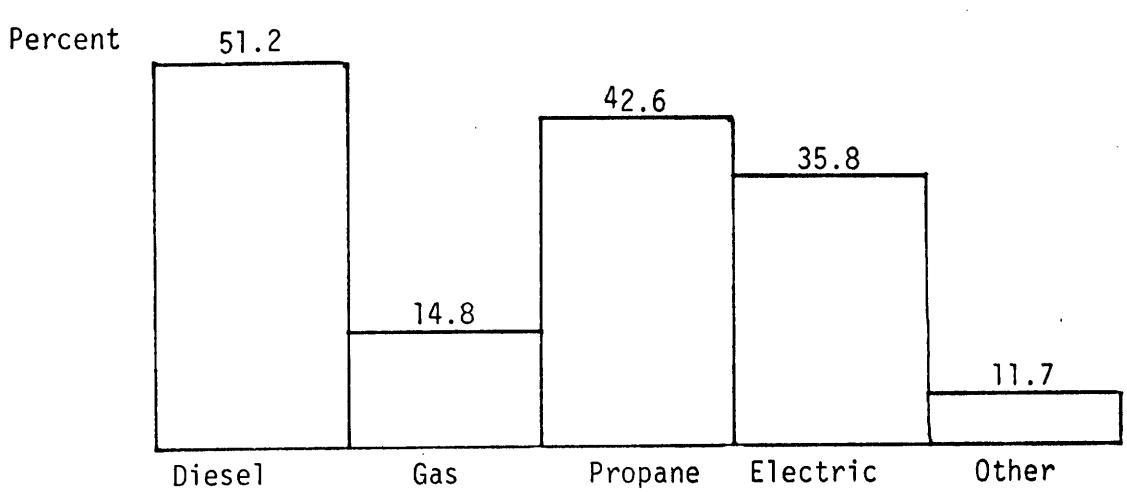


Figure 17. FUEL COST ($) IF FARM IS IRRAGATED

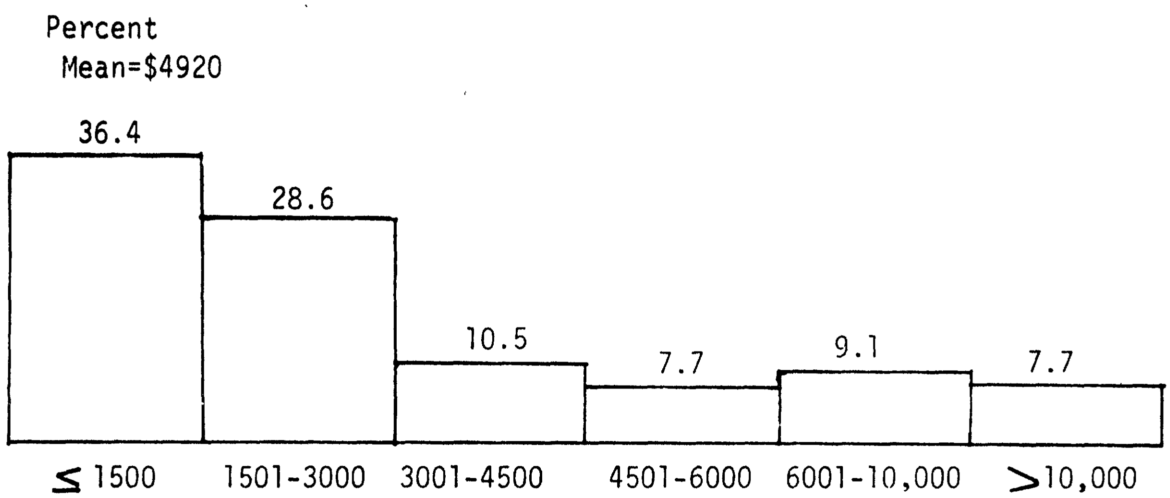


Figure 18. MINIMUM SYSTEM DURABILITY IN YEARS NECESSARY FOR ADOPTION

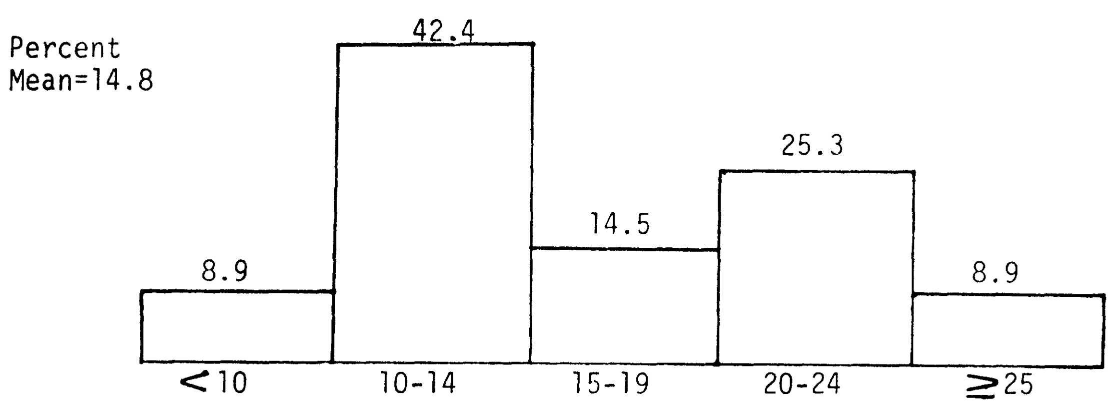


Figure 19. MAXIMUM PAYBACK PERIOD IN YEARS NECESSARY FOR ADOPTION

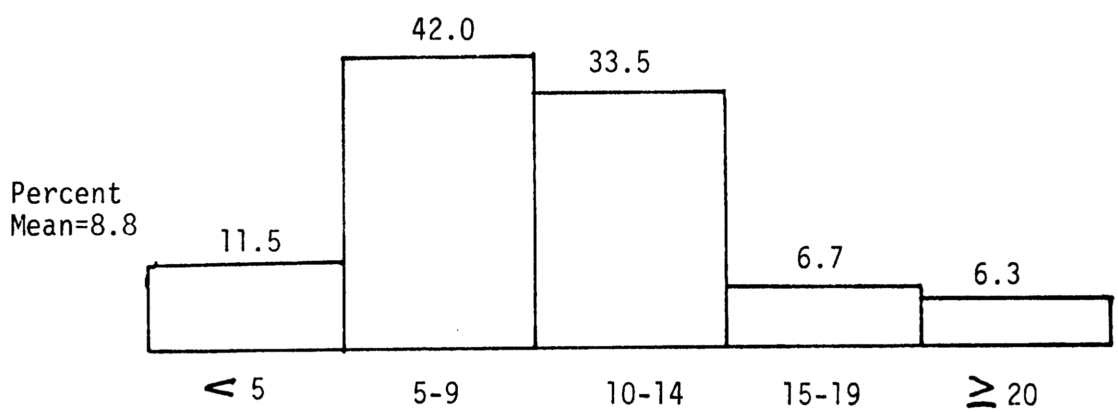
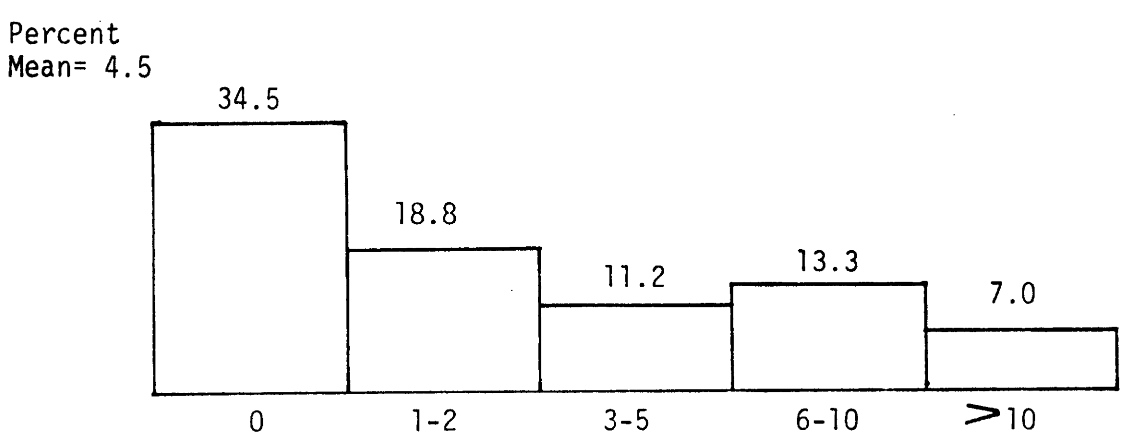


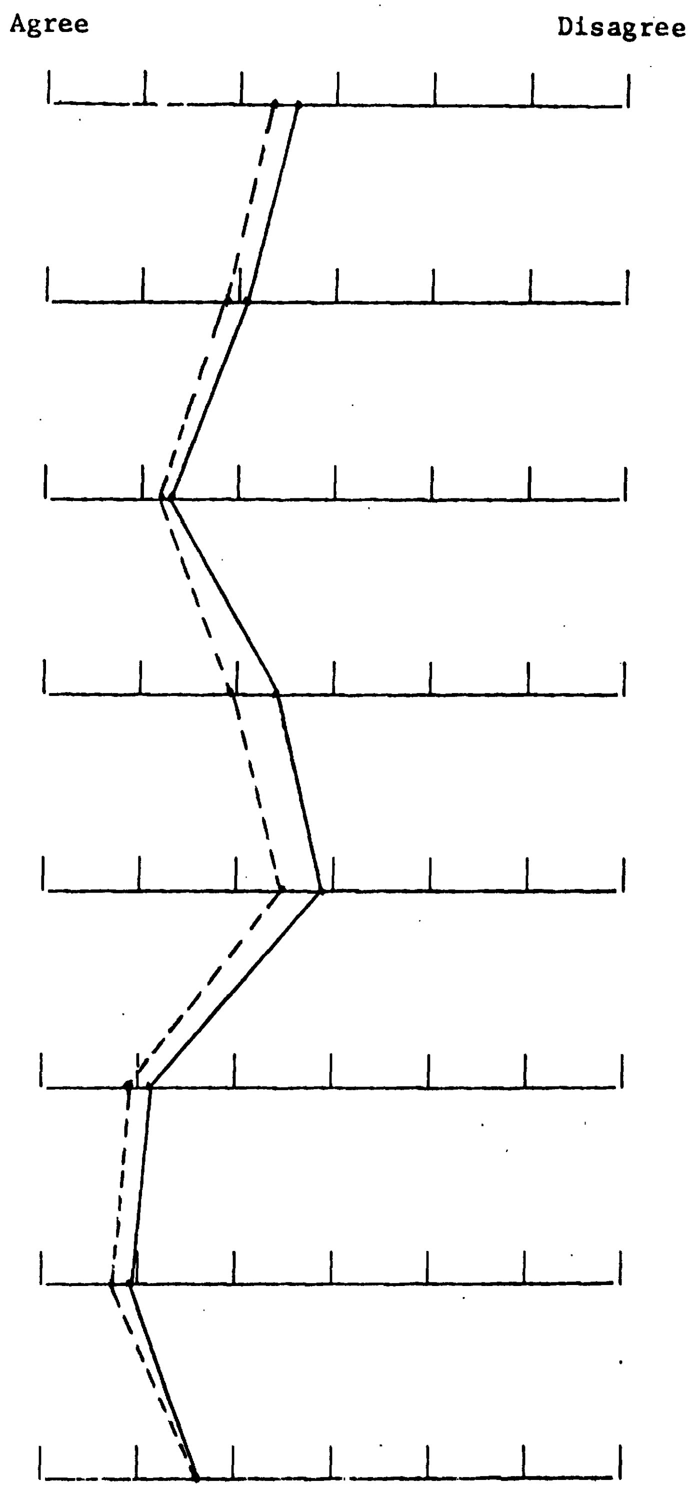
Figure 20. MINIMUM NUMBER OF PRIOR SUCCESSFUL INSTALLATIONS NECESSARY FOR ADOPTION



What do the farmers as a group expect from an irrigation system? These are key questions needed to calibrate equations such as (13) in our acceptability model. 56.9% of the farmers expect it to last at least 10-19 years under normal use (Figure 18), with an average of 14.8 years. More than half of them, 53.5%, expect a system to pay itself back in less than 10 years, 8.8 years on average (Figure 19). Most of the farmers need to see only a few successful installations before they would be willing to buy a system themselves (Figure 17). 34.5% are willing to be first in their area to buy a system, and 53.3% would be convinced by 2 or fewer successful installations. However, there are enough skeptics to push the average up to 4.5 installations.

Perceptions of irrigation systems (displayed in Figure 21-23) offer few surprises. Photovoltaic systems score very well in reducing pollution, saving resources and protection against fuel rationing, and very poorly on cost, weather sensitivity and technical maturity. By contrast, the conventional systems score well on being simple, mature technologies. The perceptual differences between farmers exposed to the demonstration project and those who were not are not great. Though feelings that photovoltaics saves resources, reduces pollution, etc., were enhanced, there was no change in the degree to which the farmers were willing to consider such a system. Similarly, though perceptions of the conventional systems' societal value fell after seeing the demonstration, the degree of consideration didn't significantly change. It is interesting to note that the responses to questions 6, 7, and 14, all about energy savings, pollution and rationing had much less variation for PV than for combustion or electric, or indeed, other PV questions.

Figure 21a. Perceptions of Photovoltaics System



1. The system provides reliable power for irrigation.

2. Adoption of the system protects against power failures.

3. Performance of the system is sensitive to weather conditions.

4. The system is flexible enough for multiple uses on the farm.

5. The system is more expensive to maintain than other systems.

6. The system protects against farm fuel rationing.

7. The system allows us to do our part in reducing pollution.

8. The system uses too many concepts that have not been fully tested.

———— = Pre Exposure   - - - - - - = Post Exposure

Figure 21b. Perceptions of Photovoltaics System
(Continued)

9. Installation of the system is too costly.

10. The system is too complex.

11. The system is visually unattractive.

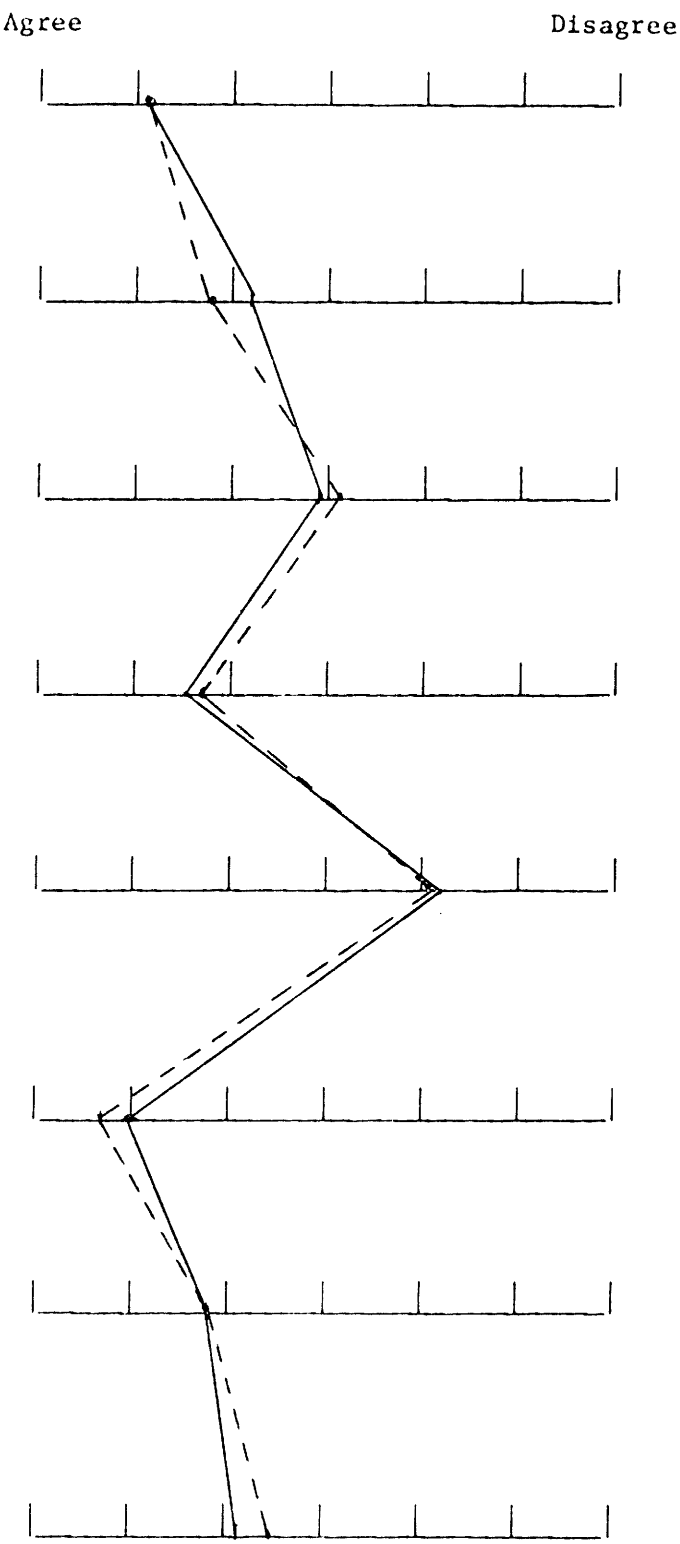
12. The system is subject to weather damage.

13. Other forms of energy will be available in the future which make the system unnecessary.

14. The system leads to considerable savings of energy resources.

15. If the experts approved and recommended a system, I would seriously consider it.

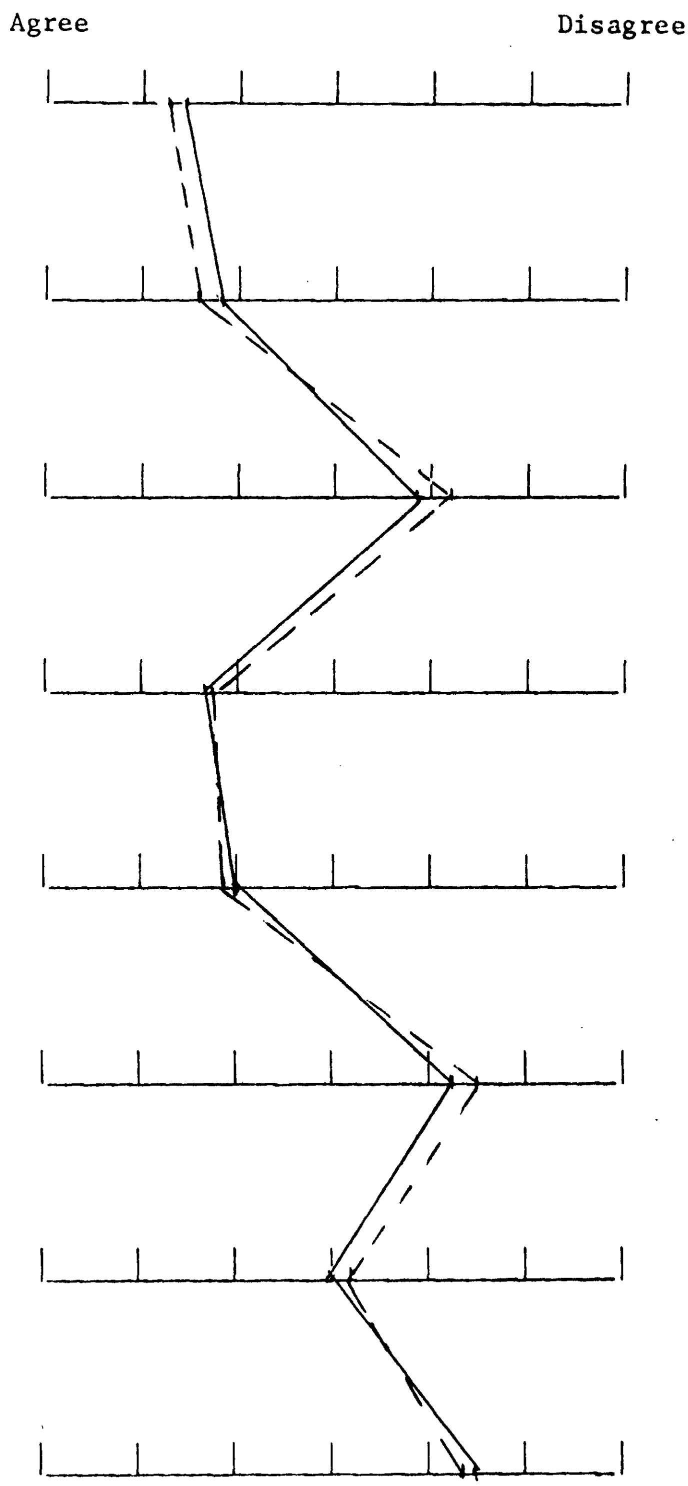
16. The system requires changes in conventional farm practices.



———————— = Pre Exposure

------------ = Post Exposure

Figure 22a. Perceptions of Combustion System



1. The system provides reliable power for irrigation.
2. Adoption of the system protects against power failures.
3. Performance of the system is sensitive to weather conditions.
4. The system is flexible enough for multiple uses on the farm.
5. The system is more expensive to maintain than other systems.
6. The system protects against farm fuel rationing.
7. The system allows us to do our part in reducing pollution.
8. The system uses too many concepts that have not been fully tested.

———————— = Pre Exposure   ------------ = Post Exposure

Figure 22b. Perceptions of Combustion System (Continued)

9. Installation of the system is too costly.

10. The system is too complex.

11. The system is visually unattractive.

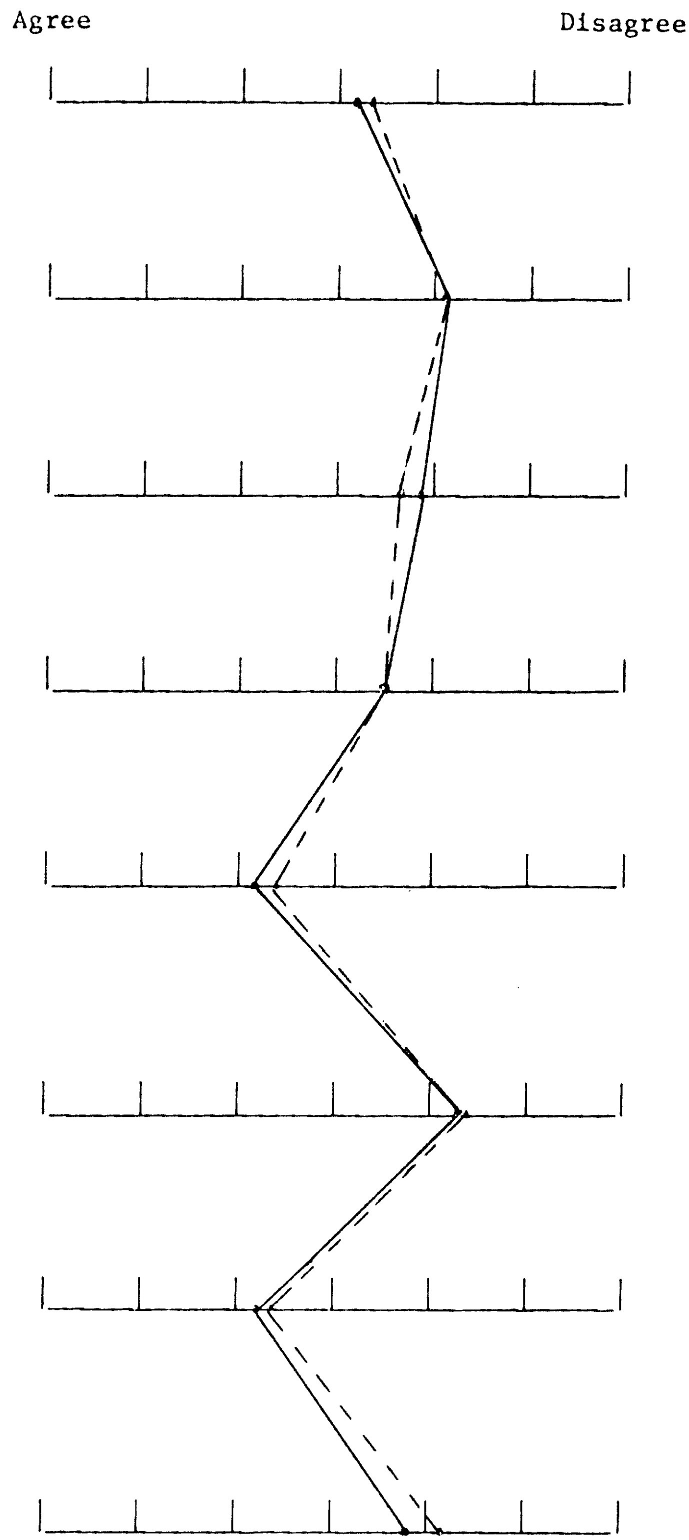
12. The system is subject to weather damage.

13. Other forms of energy will be available in the future which make the system unnecessary.

14. The system leads to considerable savings of energy resources.

15. If the experts approved and recommended a system, I would seriously consider it.

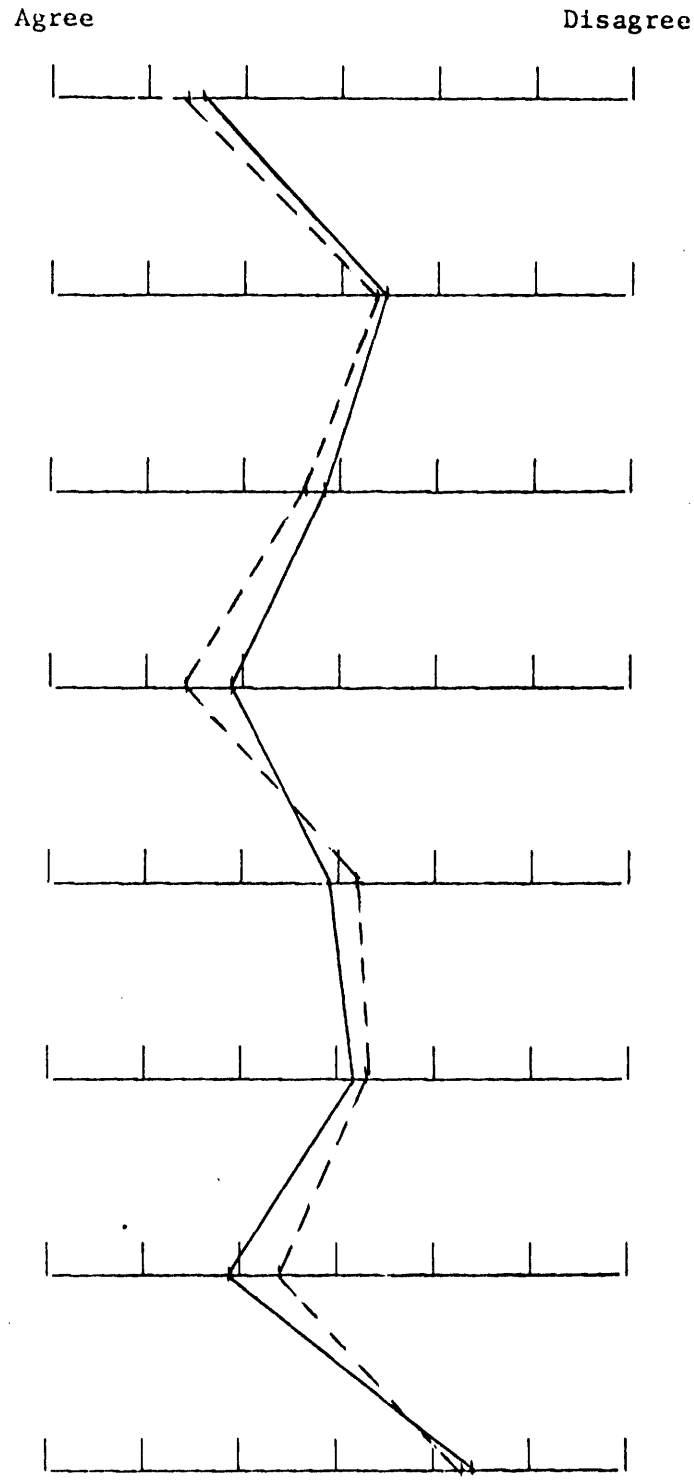
16. The system requires changes in conventional farm practices.



——————— = Pre Exposure ------------ = Post Exposure

Figure 23a. Perceptions of Electric System

1. The system provides reliable power for irrigation.
2. Adoption of the system protects against power failures.
3. Performance of the system is sensitive to weather conditions.
4. The system is flexible enough for multiple uses on the farm.
5. The system is more expensive to maintain than other systems.
6. The system protects against farm fuel rationing.
7. The system allows us to do our part in reducing pollution.
8. The system uses too many concepts that have not been fully tested.

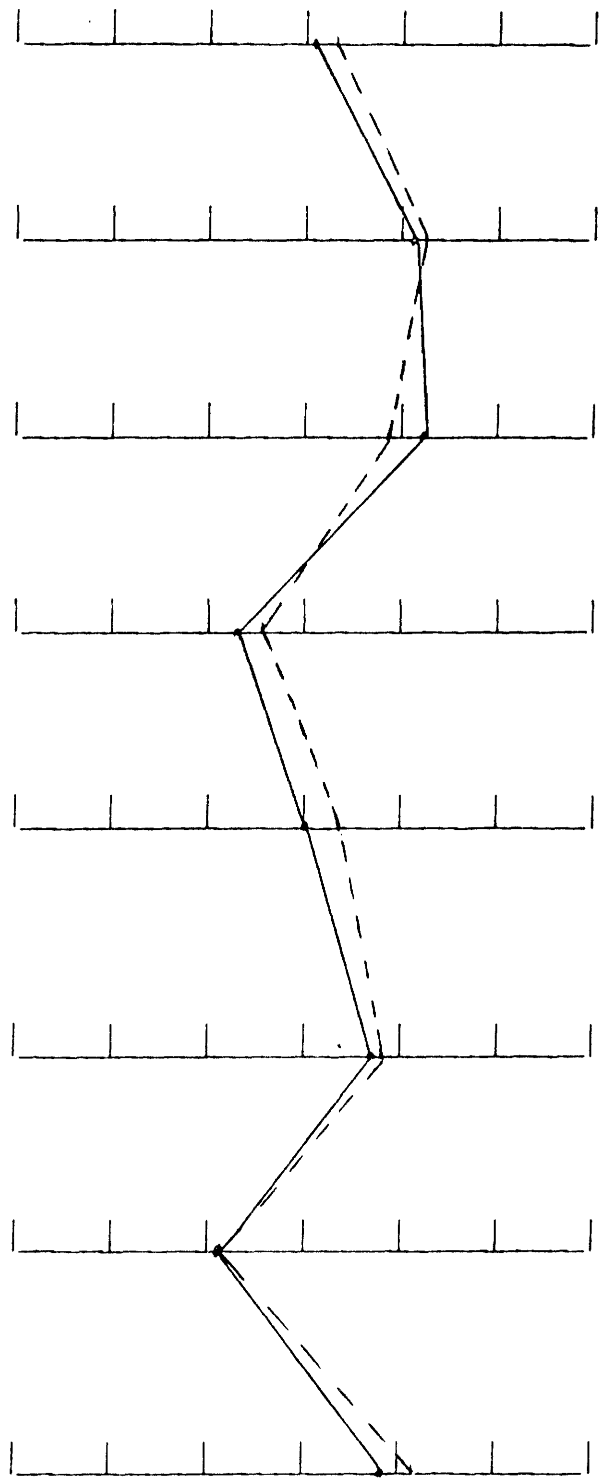


———————— = Pre Exposure - - - - - - - - - - - = Post Exposure

Figure 23b. Perceptions of Electric System
(Continued)

Agree Disagree

9. Installation of the system is too costly.
10. The system is too complex.
11. The system is visually unattractive.
12. The system is subject to weather damage.
13. Other forms of energy will be available in the future which make the system unnecessary.
14. The system leads to considerable savings of energy resources.
15. If the experts approved and recommended a system, I would seriously consider it.
16. The system requires changes in conventional farm practices.



———————— = Pre Exposure ------------ = Post Exposure

Figure 24. PV CHOICE

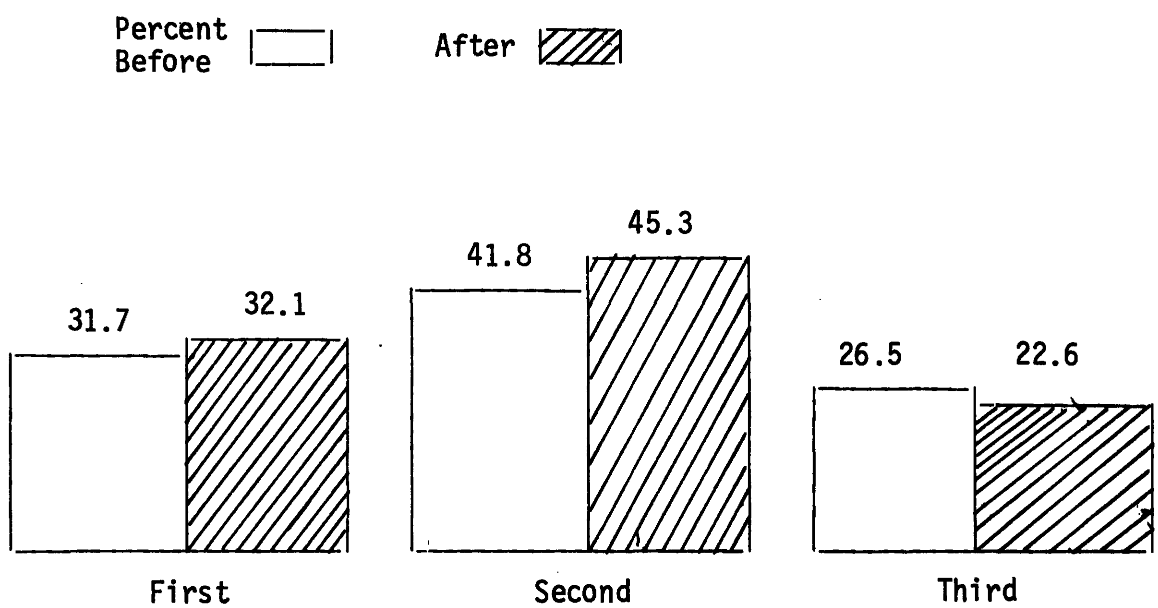


Figure 25. PERCENT INCREASE WILLING TO PAY FOR PV

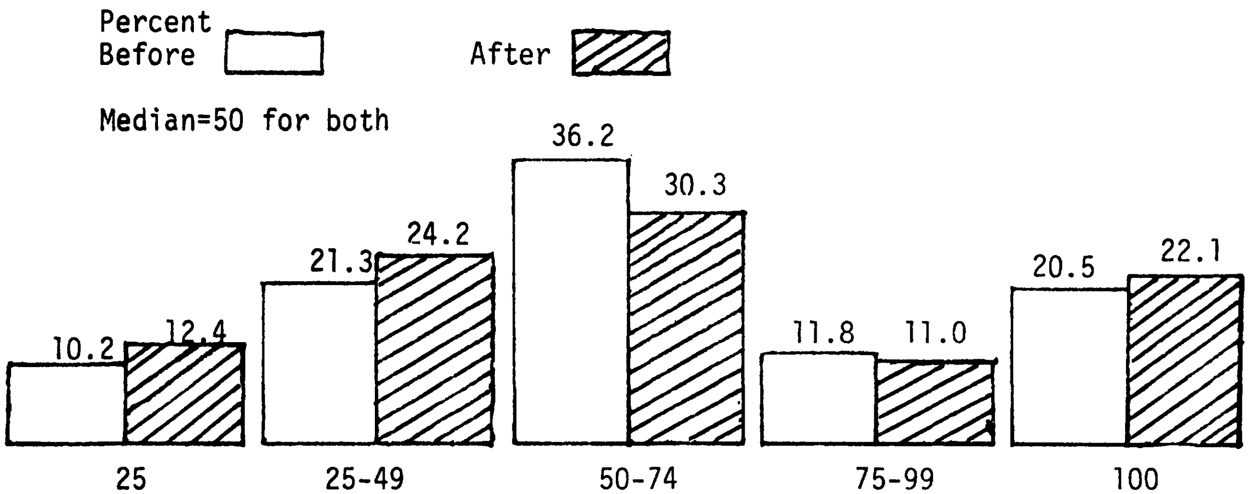
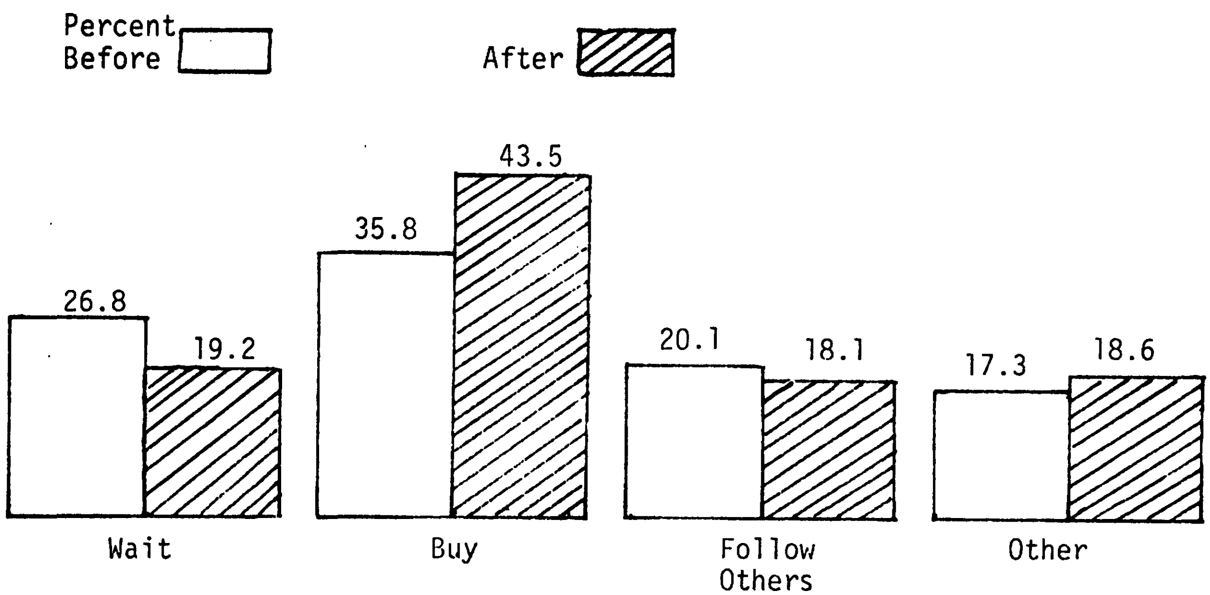


Figure 26. ACTION IN ENERGY SHORTAGE IF PV AVAILABLE



When it came to choosing among the three systems, exposure to the demonstration seems to have made little difference. The farmers generally chose photovoltaics about the same after exposure, (Figure 24). The percent increase in initial cost the farmers are willing to pay also seems unaffected by exposure (Figure 25). For both pre- and post-exposure the median increase is 50 %.

Finally, Figure 26 shows that, following exposure, nearly half the sample would buy photovoltaics in an energy shortage.

These data show that:

- PV is understandable;
- PV is acceptable to a wide variety of farmers;
- A premium would be paid for the product;
- Field exhibits have little influence on preference.

7. PV Preference and Perceptual Analysis

Factor analysis is used to determine the key perceptual dimensions farmers use to assess irrigation systems. Consumers may be able to respond to an unlimited number of questions about a topic; yet, in their minds, they may structure information into only a few, key underlying dimensions. We refer to these dimensions as evaluation criteria. We must test to see if these evaluation criteria were equal for the three groups that were sampled: those responding only to the post-test survey, those responding only to the pre-test survey, and the group responding to the pre-test and post-test surveys.

The first step is to factor analyze the results for all respondents within the respective subgroups. From the results of the factor analysis, we can determine whether the dimensionalities of the evaluation spaces are the same for each of the subgroups. If the dimensionalities are indeed equal, we can determine whether the evaluation criteria are similar across subgroups by employing the test discussed in Choffray and Lilien [10]. The test measures whether the factor score coefficients used to measure the scores for each individual on a given factor are the same for the two groups.

Item responses for each of the three subgroups were factor analyzed. Factors were extracted using the criterion that eigenvalues must be greater than or equal to 1.0. In addition, certain combinations of these subgroups were factor analyzed to be consistent with the requirements for the test. For the following discussion we denote:

pre-test and post-test survey	= Group A
pre-test survey only	= Group B
post-test survey only	= Group C

Our analysis suggested that all groups seemed to have perceptual spaces of dimensionality 3, the number of factors extracted for each group.

First we test Group A vs Group C. This comparison reveals if there is a significant effect from exposure to the pre-test questionnaires. The results of this comparison (summarized in Appendix 1) indicate that the evaluation criteria exhibited by Groups A and C are essentially similar.

Analysis of the similarity between Groups B and C was undertaken to assess the effect of respondents inspecting a PV site on their evaluation criteria. The results of comparing factors that load on similar variables between B and C (summarized in Appendix 1) reveal significant dissimilarities. Therefore, we can state that the evaluation criteria of the group that was only exposed to a concept statement are different from the evaluation criteria of the group that was exposed to the PV site.

For simplicity we can roughly name the factors for all these groups as:

1. Newness/Expense
2. Complexity/Untried Concepts
3. Independence from Traditional Fuel Sources

The preference regressions, discussed below, suggest that factor 2 is the factor of key importance in explaining preference.

We are now concerned with how perceptions are related to preferences; thus, we use preference regression analysis to determine how these evaluation criteria are related to system preferences.

Individuals were asked to rank their system preferences assuming that each of the alternatives satisfied the respondent's minimum requirements for system payback period, system life and the number of prior locations. These rank preferences were regressed against the corresponding factor scores using ordinary least squares, for the sample groups B and (A+C). The regression equations for both groups are shown in Appendix 2. The preference paramters for both equations are significantly different from zero at the 90% confidence level and all appear to have the correct sign except for the preference parameter for factor 1 for group (A+C). Analysis of the relative importance of the questionnaire items for group (A+C) revealed that the negative sign of the preference parameter for factor 1 is explained by the fact that the importance of protection against fuel rationing outweighs the negative characteristics of high initial cost, system complexity and system newness which are also found in factor 1.

Using the regression equation we predicted the system preferences for each of the respondents (predictions were only made for individuals who had ranked all three systems). The percentage of correct predictions for first preference recovery are shown in Figure 27. The results indicate that the model is useful in predicting individual preferences.

To determine the relative importance of each of the items to the respondents in groups B and (A+C), the preference parameters were multiplied by the corresponding factor score and summed up across each item. This tells what the effect of a unit change in one of the original items is predicted to do to preference. (If an item were only to load on one factor, at a unit level, this would be equivalent to examining the size of the regression coefficient.) The results of these calculations are shown in Figures 28 and 29.

The results indicate that by exposing potential users to an operational PV powered irrigation system we are able to lower their concerns that a PV system contains "too many untried concepts" and is "visually unattractive." Also, after exposure to an operating PV system, protection against fuel rationing and system reliability become the two most important criteria. Since a PV system provides greater protection against fuel rationing than either of the other two systems under consideration, one would assume that a higher percentage of individuals in group (A+C) would prefer the PV system than the individuals in group B. This hypothesis, however, is not supported by the data from the questionnaire. 32.1% of the respondents in group (A+C) ranked the PV system first in comparison to 31.7% for group B. The percentage of respondents ranking PV second was 45.3% for group (A+C) and 41.8% for group B.

From the above data we can conclude that exposure to an irrigation system powered by a PV energy source will change a potential user's evaluation criteria. However, it will not have an impact on their system preferences even when it is assumed that the PV system satisfied their minimum system requirements.

Additional preference analyses were performed including age, farm size and education of the farmer. These variables did <u>not</u> add to the predictive or explanatory power of the model, suggesting that the perceptual (factor analysis) data does a good job in explaining individual preferences.

Figure 27

Model Predicted Preference Recovery

GROUP B (Not Exposed to Site)

1st Preference Recovery	Full Preference Recovery
.597 (expected = 0.337)	.388 (expected = 0.167)

GROUP (A + C) (Exposed to Site)

1st Preference Recovery	Full Preference Recovery
.528 (expected = 0.334)	.352 (expected = 0.167)

Figure 28

Rank of Variables in Order of Importance to Respondents

GROUP B

(Not Exposed to Site)

RANK	DESCRIPTION
1	leads to considerable savings of energy
2	system is too complex
3	system contains too many untested concepts
4	system is visually unattractive
5	system protects against power failures
6	expert approval & recommendations prior to consid.
7	system reduces pollution
8	provides reliable power for irrigation
9	protects against fuel rationning
10	system installation too costly
11	system flexible for multiple use
12	sys. requires change in conventional farm practices
13	future forms of energy will make system unnecessary
14	system more expensive than others to maintain
15	system is subject to weather damage
16	system sensitive to weather conditions

Figure 29

Rank of Variables in Order of Importance to Respondents

GROUP (A + C)

(Exposed to Site)

RANK	DESCRIPTION
1	protects against fuel rationing
2	provides reliable power for irrigation
3	system is too complex
4	expert approval & recommendations prior to consideration
5	system reduces pollution
6	leads to considerable savings of energy
7	system is visually unattractive
8	system requires change in conventional farm practices
9	system flexible for multiple uses
10	future forms of energy will make this system unnecessary
11	system installation too costly
12	system more expensive to maintain than others
13	system protects against power failures
14	system contains too many untested concepts
15	system is subject to weather damage
16	system sensitive to weather conditions

8. Feasibility and Design Analysis

We are now concerned with the requirements potential adopting farmers have prior to adopting PV for irrigation. This will give us design clues as well as calculations for the rest of equation (13). There are several dimensions to this problem:

1. By examining the minimum standards for system life, payback period, and prior number of successes specified by farmers, we can determine the proportion of users for whom various systems will be acceptable.
2. Using preference data, we can estimate market share. By examining the total market, we can project total dollar sales.
3. Using the results of (1), we can reach some conclusions regarding minimum standards and optimum mix of system design characteristics.

8.1 Calibration of Feasibility Function

The questionnaire asked farmers to specify the minimum values for system life, payback period, and number of prior installations they would demand before considering a photovoltaic-powered irrigation system. We can thus calculate for any given value of a parameter, the proportion of farmers who found any level of a variable acceptable. For a set of values for the three dimensions, the proportions finding each dimension acceptable were multiplied to calculate the acceptability of the system. (This assumes independence, which was empirically verified.) This is the probability that a farmer will find the system acceptable. Mathematically:

$$Pr_{ac} = Pr_{sl} \times Pr_{pp} \times Pr_{pi}$$

where Pr_{ac} = the probability of acceptance of the system

Pr_{sl} = the proportion that find system life acceptable

Pr_{pp} = the proportion that find payback acceptable

Pr_{pi} = the proportion satisfied with the number of prior installations.

A sample of design trade-off output is tabulated in Figure 30.

8.2 Calcuation of Market Potential, Near Term

In a section of the questionnaire, farmers ranked photovoltaics and two other power systems by preference, assuming they all met the minimum requirements. Photovoltaics was the first choice of 32%. By multiplying the probability of acceptance by the probability of preference given acceptance, we can predict the market share of various photovoltaic systems (assuming that everyone is made aware of the product.)

Sales of self-propelling irrigation systems were listed as $37.4 million for SIC 35238 93 (center pivot), and $6.9 million for SIC 34238 95 (others), in 1972 (Census of Manufacturers [7]). Given some inflation and an increase in price due to photovoltaics, we will estimate the current market to be $50 million. From this figure and the marekt share, sales for photovoltaic-powered systems with various characteristics can be calculated. A sample is given in Figure 31.

8.3 Design Targets

These projections are interesting, but they are not the most important use of the feasibility data. For the designer of a system the crucial question

Figure 30

Probability of Acceptance of PV
for Various System Characteristics

Payback Period	Necessary Life	Number of Prior Installations	Probability of Acceptance
5	18	3	.68
5	18	8	.81
5	8	3	.36
5	8	8	.42
13	18	3	.11
13	18	8	.13
13	24	3	.12
13	24	8	.15

Figure 31

Estimated Near Term PV Market Potential - Irrigation

Payback Period	Expected Life	Probability of Acceptance	Preference	Market Share	Potential Annual Estimated Sales
5	8	.44	.32	.15	$ 7.5 M
5	18	.84	.32	.28	$14.0 M
5	24	.93	.32	.32	$16.0 M
8	8	.33	.32	.11	$ 5.5 M
8	18	.52	.32	.18	$ 9.0 M
8	24	.60	.32	.20	$10.0 M
13	18	.14	.32	.05	$ 2.5 M
13	24	.15	.32	.05	$ 2.5 M

is: how much will acceptance increase with an incremental change in payback period or system life? When this information is compared with incremental cost, then a rational decision on system design can be made. For the government, the question is: how many pilot programs must be launched before farmers will accept photovoltaics as a proven technology? As above, the action is on the margin.

To answer these questions, iso-acceptance curves were drawn for each pair of system characteristics (Figures 32,33,34). Each chart assumes that the third characteristic is acceptable. The probability of acceptance of each pair of data points is plotted, then curves are sketched through all points with the same probability. These curves represent the trade-off between characteristics; in economic terms, these are indifference curves. A farmer is equally satisfied with each pair of values along the line. Inflection points represent a key trade-off for that level of acceptance. Target values can then be determined for a given level of acceptance (probably driven by break-even market share or sales). The level of acceptance can be determined for any given set of parameters.

Let's examine Figure 32. Two points, A and B are marked. A represents a 9 year payback period with 4 prior, successful installations. B represents a 3 year payback with 1 prior, successful installation. Were either of these conditions to occur, 50% of the farmers would find PV acceptable on these two dimensions. Thus, farmers, on average, are willing to pay a 9-3 or 6 year payback "premium" to obtain the risk reduction associated with seeing 4-1 or 3 additional, successful installations. Figures 32 and 33 demonstrate that the government need only sponsor 3 or 4 pilot projects. Beyond that, acceptability is dominated by other factors.

Figure 34 would be important to a marketer or a design engineer. Payback period is a simple function of the price and the cost of fuel. Expected life is determined by design and materials. This chart indicates that although low values of payback and high values of life are needed to get high acceptance (5 and 17 years respectively for 80%), less stringent values will still capture some market (e.g. 11 and 11 is acceptable for 25%).

Figure 32: Prior Locations vs. Payback Acceptability Curves

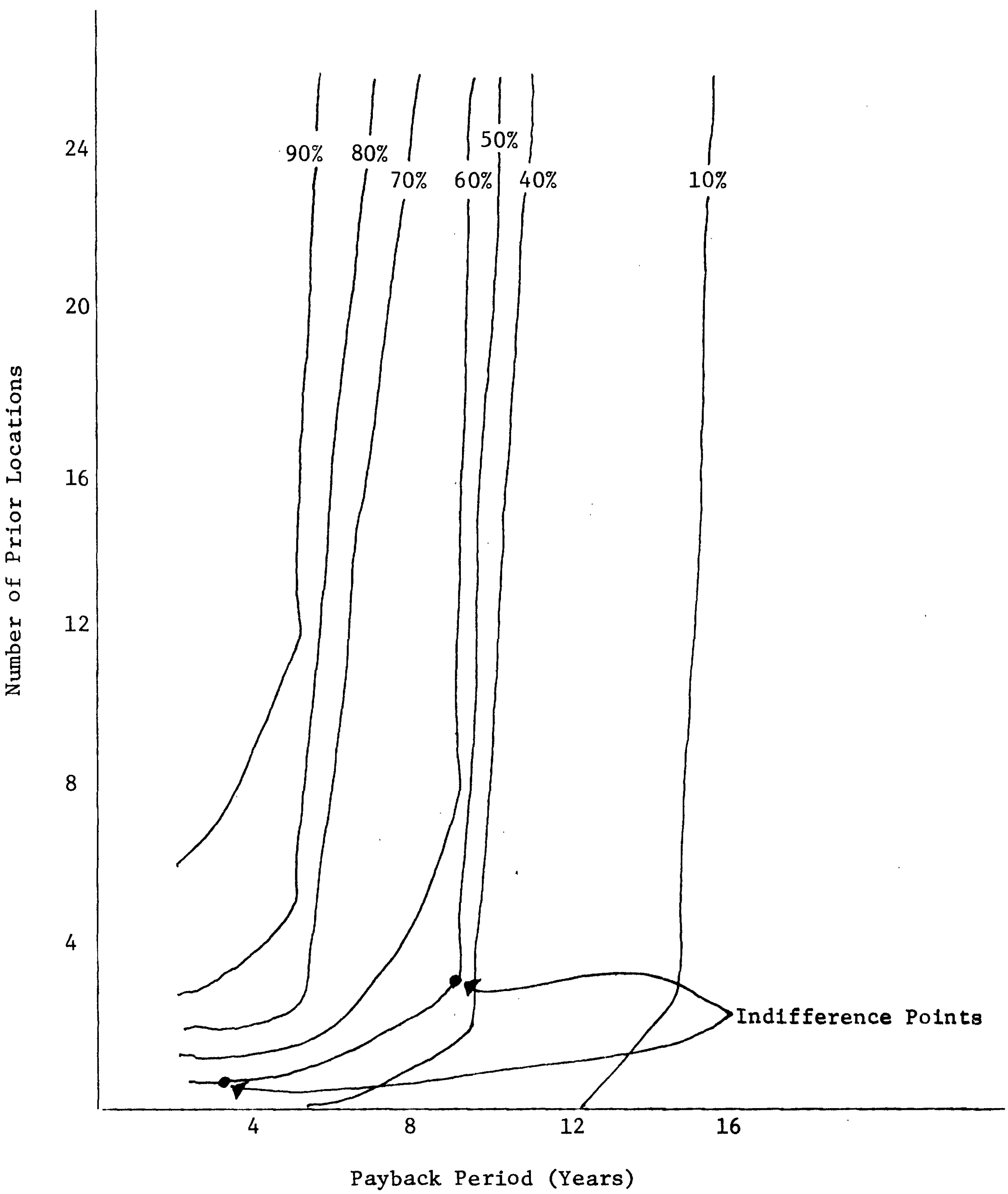


Figure 33: Prior Locations vs. Necessary Life Acceptability Curves

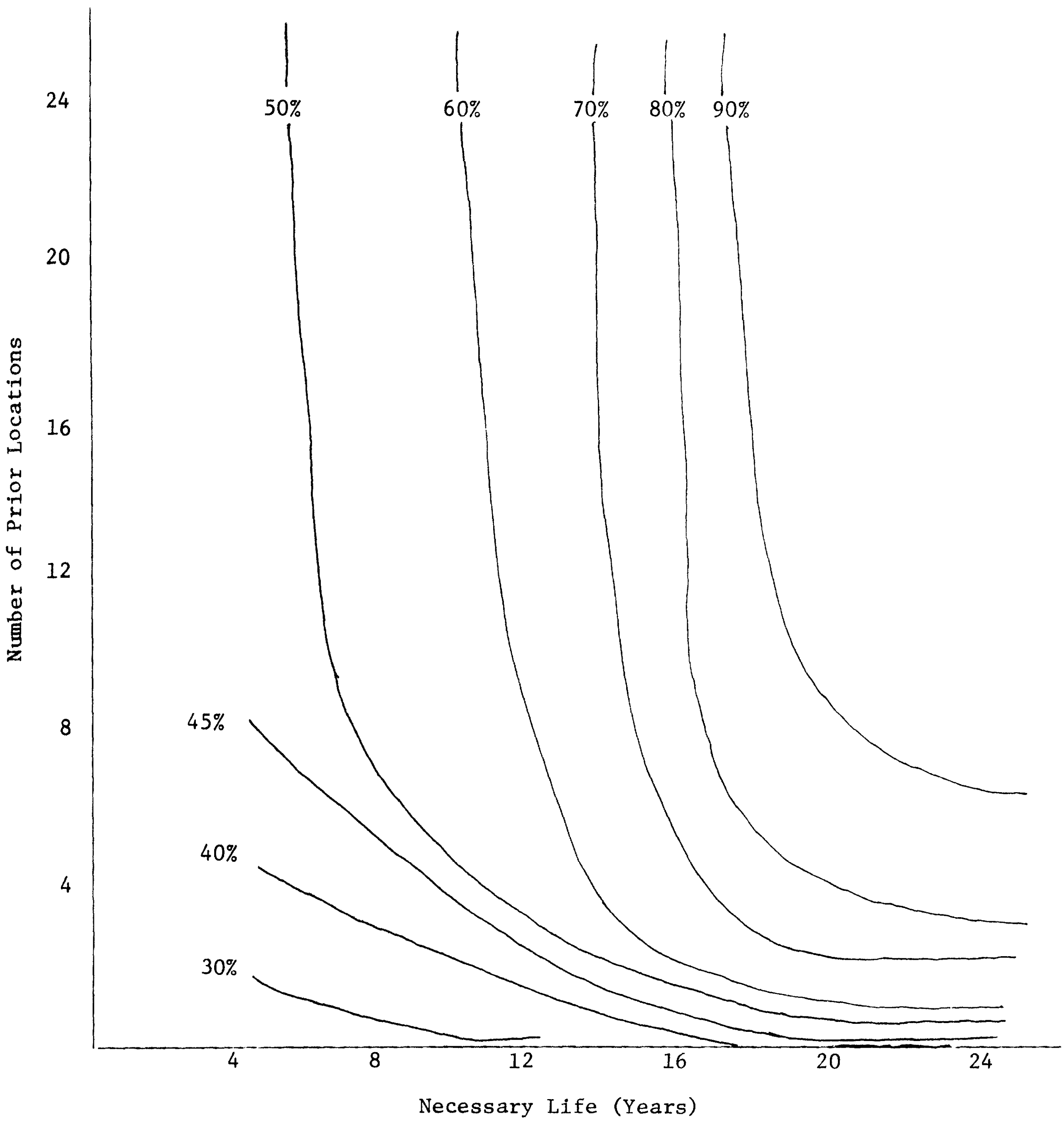
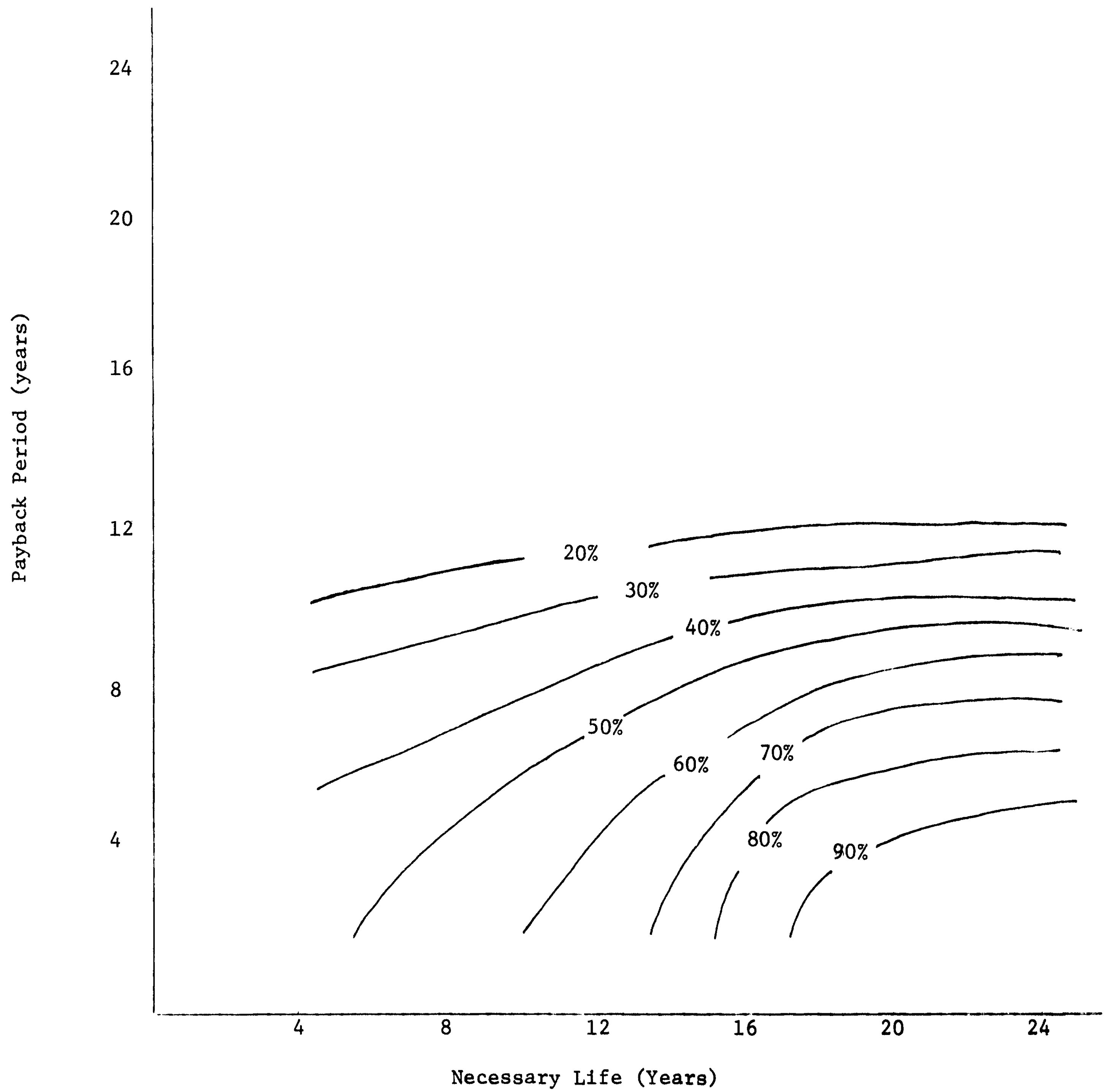


Figure 34: <u>Payback Period vs. Necessary Life Acceptability Curves</u>



9. Summary of Irrigation - Sector Results

The analysis in the last 2 sections permits calibration of equation 13. The use of that model to evaluate the impact of government programs is the subject of later publications.

However, the agriculture sector results stand on their own in importance. We can summarize the key results as follows:

- Exposure to an operational PV site (as opposed to simply reading a concept statement) had a significant impact on:
  - the way people think about irrigation systems,
- but not on:
  - conditional preference ranking of combustion, electric or PV powered systems.
- Key factors were identified to be:
  - newness/expense
  - complexity/untried concepts
  - independence from traditional fuel sources
- Rank order of evaluation items was more readily explainable when the respondents had been exposed to the PV site
- Exposure to the working PV site made respondents more aware of the energy savings potentials of the PV system
- The expected payback period for PV systems is the most significant factor limiting its marketability at this time
- Only 3-4 demonstrations are needed effectively to eliminate new product risk perception among farmers.

10. Status and Future Work

As noted earlier, the material reported on here is in an early, developmental state. More analysis is needed to integrate fully the results of our Mead experience into the computer model, and to evaluate the effect of various government policies.

Our next steps after that are to:

- calibrate the model on a second sector (residential);
- complete the programming and documentation of a user-oriented computer program to integrate these calibrations into an operational model;
- develop that model further.

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APPENDIX 1

With Choffray and Lilien's [9] modification of the Chow test, when we compare the evaluation criteria of A against C we must also look at the evaluation criteria of the combined group, (A + C). Actually, we are comparing A and C with (A + C) to determine equality; our null hypothesis is that the groups are, in fact, equal. Defining,

$$C_p = \left\{ \frac{N(1-R_p^2)}{n_1(1-R_{p1}^2) + n_2(1-R_{p2}^2)} - 1 \right\} \frac{N-2q}{q}$$

where,

R_{p1}^2, R_{p2}^2 = squared multiple correlations associated with the estimation of factor p in sample 1 and 2 respectively

R_p^2 = squared multiple correlation associated with factor p in pooled sample

n_1, n_2 = number of responses in sample 1 and 2 respectively

N = $n_1 + n_2$

q = number of items

When the value of C_p exceeds the corresponding F statistic at the appropriate level of significance, the null hypothesis of equality is rejected.

APPENDIX 1 (cont'd.)

For comparison of A and C:

Factor	C_p	$F_{.90}$	$F_{.95}$	$F_{.99}$
1	0.41	1.72	2.01	2.75
2	0.42	1.72	2.01	2.75
3	1.79	1.72	2.01	2.75

null hypothesis is accepted at .95 level.

For comparison of B and C:

Factor	C_p	$F_{.90}$	$F_{.95}$	$F_{.99}$
1	2.62	1.72	2.01	2.75
2	2.53	1.72	2.01	2.75
3	0.22	1.72	2.01	2.75

null hypothesis is rejected at the .95 level.

APPENDIX 2

Preference Regression Equations

Group B

$$\text{Pref} = 1.949 + .236F_{1i} - .188F_{2i} - .130F_{3i} + \varepsilon_i$$

$$(38.4) \quad (3.88) \quad (-3.14^{2i}) \quad (-1.78)^{3i}$$

$$n = 230$$

Group (A + C)

$$\text{Pref} = 1.99 - .086F_{1i} + .318F_{2i} - .069F_{3i} + \varepsilon_i$$

$$(56.9) \quad (-2.32) \quad (7.58) \quad (-1.35)$$

$$n = 484$$

numbers in parenthesis are t statistics

Appendix 3

A.1 Introduction

This appendix gives a detailed, technical analysis of two models of innovation diffusion that are particularly relevent for our discussion. These two models examine the pace at which a new technique or a new product is accepted and used by the market once it has been introduced. The theoretical framework of these models stems mathematically from contagion models which are used in applications of epidemiology. The basic deterministic contagion models are discussed in section A2. Section A3 describes Mansfield's adoption model which is expressed in terms of the number of firms that have introduced the innovation. The Bass model which is expressed in terms of cumulative sales of the innovation is covered in section 4.

A modification of the Bass model is introduced which allows us to explore the implication for government demonstration programs associated with a new innovation. A theorem is proven concerning optional government policies and other conjectures are stated which will be proven more generally in a later report. The appendix then discusses the limitation of these models, and the extensions needed for our purposes.

A.2 Contagion Models

Contagion models (Bailey [4]) are used to study the spread of epidemics. They are the basis of the two adoption models that follow; thus we examine them first. One should note the close similarity between the basic equations of the contagion model and those of the Mansfield and Bass models (eqn. (2.1) with eqn. (3.3) and eqn. (4.1); eqn. (2.3) with eqn. (3.5) and eqn. (4.8); eqn. (2.4) with eqn. (4.7); figure 3 with figure 4).

Assumptions

It is assumed that the population under study is homogeneous and that all possible contacts between individuals are made. It is also assumed that the population is of constant size N+1 throughout the time of the study. In general, the population is divided into four groups:

(a) susceptible	-	those individuals that are uninfected and susceptible to infection.
(b) exposed	-	those that are infected but are not yet contagious (in an incubation period).
(c) infectious	-	those individuals that are infected and are contagious.
(d) removed	-	those that are not susceptible to further infection; those in this group remain in this group for all future times.

For the study of the adoption models, it is only necessary to look at the simplest of contagion models where the population is divided into two groups rather than four -- susceptible and infectious. It is assumed that initially there is only one infectious individual and N susceptible people. Another assumption is that there is no transition from infectious to susceptible. The rate of growth of the infectious population is assumed to be proportional to the number of contacts between those that are susceptible and those that are infectious.

We introduce the following notation. Let

$i(t)$ = number of infectious at time t.

$s(t)$ = number of susceptible at time t.

N = initial number of susceptibles.

k = probability that contact between <u>one</u> infectious and <u>one</u> susceptible will cause the susceptible individual to become infectious.

(Note: although k is assumed to be constant, the model can be extended to allow k to be time dependent.)

Mathematically the rate of growth of the epidemic is $-(ds(t)/dt)$ where:

$$ds(t)/dt = -ks(t)i(t) \tag{2.1}$$

Using the assumption that the population size is fixed at N+1, equation (2.1) becomes:

$$ds(t)/dt = -ks(t)\ (N+1 - s(t))$$

Solving for k:

$$-k = \frac{ds(t)/dt}{s(t)(N+1-s(t))} = \frac{ds(t)/dt}{(N+1)s(t)} + \frac{ds(t)/dt}{(N+1)(N+1-s(t))}$$

Integrating both sides:

$$-kt + c_o = \frac{1}{N+1}\,[\ln(s(t)) - \ln(N+1 - s(t))]$$

or equivalently:

$$\frac{s(t)}{N+1-s(t)} = c_1 e^{-k(N+1)t}$$

solving for s(t):

$$s(t) = \frac{c_1(N+1)}{c_1 + e^{k(N+1)t}}$$

Using the initial conditions ($s(0) = N$, $i(0) = 1$) to evaluate the constant c, it is found that $c_1 = N$. Thus

$$s(t) = \frac{N(N+1)}{N + e^{k(N+1)t}} \tag{2.2}$$

Now i(t) can be obtained by using the assumption that $i(t) = N+1-s(t)$:

$$i(t) = \frac{N+1}{1 + Ne^{-k(N+1)t}} \tag{2.3}$$

Thus equations (2.2) and (2.3) fully characterize this simple model. The graphs of s(t) and i(t) as a function of time are shown in figures 1 and 2 respectively.

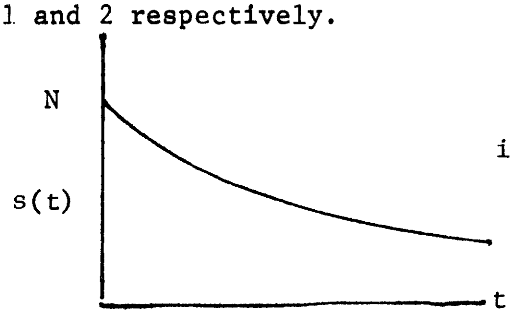

figure 1

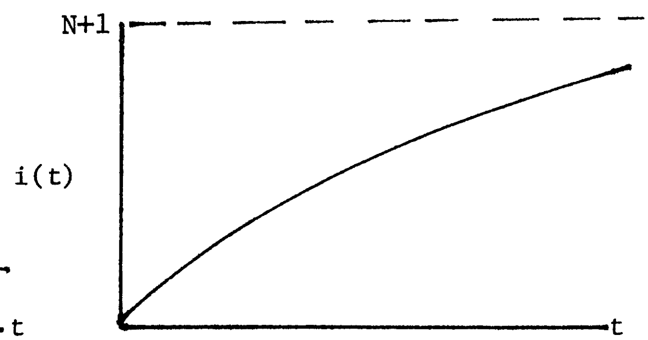

figure 2

Also of interest is the graph of the rate of growth of the epidemic as a function of time. First, a mathematical expression for $-(ds(t)/dt)$ is derived by inserting equations (2.2) and (2.3) into equation (2.1):

$$-\frac{ds(t)}{dt} = \frac{kN(N+1)^2 \, e^{k(N+1)t}}{[N + e^{k(N+1)t}]^2} \tag{2.4}$$

Thus the graph is as shown in figure 3.

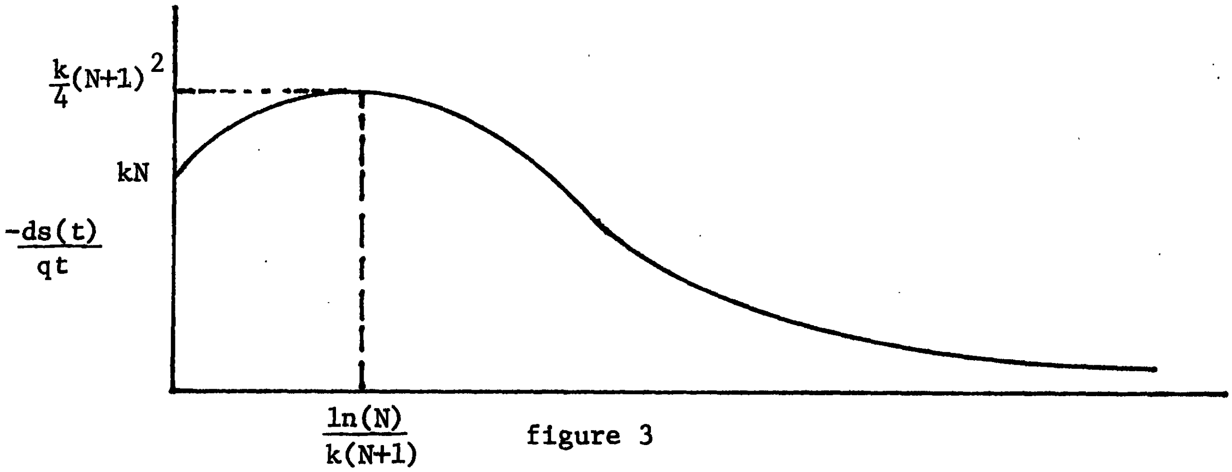

figure 3

In applying the contagion model to the innovation process think of s(t) as those individuals that have not yet adopted the innovation and i(t) as those who have adopted the innovation and have an effect on adoption behavior. A contagion model can be developed to include the case where there is a third group -- those who have adopted, but too recently to have any effect on the behavior of others. In such a case the "exposed" group is included in the contagion model.

A.3 Mansfield Model

The basis of the Mansfield [18] model is the assumption that the probability that a firm will introduce a new technique is (1) an increasing function of the proportion of firms already using it, (2) an increasing function of the profitability of doing so, and (3) a decreasing function of the size of the investment required. It is assumed that all firms in the market will eventually adopt the innovation. It is also assumed that the profitability of installing the innovation relative to that of alternative investments is appreciably greater than unity.

Notation

$m(t)$ = number of firms having introduced the innovation at time t.

$h(t)$ = proportion of firms not using the innovation at time t but that introduce the innovation by time t+1; (proportion of "hold outs").

N = total number of firms considered eligible to adopt the innovation.

P = profitability of installing the innovation relative to that of alternative investments.

C = investment required to install the innovation as a percent of average total assets.

ε = error term for Taylor expansion of h(t).

r = rate of imitation; sum of the coefficients of all terms in the Taylor expansion of h(t) which contain m(t)/N.

s = rate of "innovation"; sum of all the terms in the Taylor expansion of h(t) which do not contain m(t)/N.

k = integration constant.

Mansfield defines h(t) as

$$h(t) = \frac{m(t+1) - m(t)}{N-m(t)} \tag{3.1}$$

But by assumption,

$$h(t) = f\left[\frac{m(t)}{N}, P, C\right]$$

If it is assumed that h(t) is approximately continuous then it can be approximated by its Taylor expansion up to and including the second order terms. Assuming that the coefficient of $(\frac{m(t)}{N})^2$ term is zero (this assumption was found to be true for the twelve innovations studied) then:

$$h(t) = \alpha_1 + \alpha_2(\frac{m(t)}{N}) + \alpha_3 P + \alpha_4 C + \alpha_5 P(\frac{m(t)}{N}) + \alpha_6 C(\frac{m(t)}{N}) + \alpha_7 PC + \alpha_8 P^2 + \alpha_9 C^2 + \varepsilon \tag{3.2}$$

Combining equations (3.1) and (3.2):

$$m(t+1) - m(t) = [N-m(t)]\ [\alpha_1 + \alpha_2(\frac{m(t)}{N}) + \ldots + \alpha_9 C^2 + \varepsilon]$$

Next assume that time is measured in small increments. Then the preceding equation is equivalent to:

$$dm(t)/dt = [N-m(t)]\ [s + r(\frac{m(t)}{N})] \quad \text{, where} \tag{3.3}$$

$$r = \alpha_2 + \alpha_5 P + \alpha_6 C$$

$$s = \alpha_1 + \alpha_3 P + \alpha_4 C + \alpha_7 PC + \alpha_8 P^2 + \alpha_9 C^2 + \varepsilon$$

Solving the differential equation for m(t):

$$m(t) = \frac{N[\exp(k+(r+s)t - \frac{s}{r})]}{1 + \exp(k+(r+s)t)} \tag{3.4}$$

Now assume that as one goes backward in time, the number of firms having the innovation tends to zero. That is $\lim_{t \to -\infty} m(t) = 0$. Then equation (3.4) becomes:

$$m(t) = N[1 + e^{-(k + rt)}]^{-1} \tag{3.5}$$

(Equation 3.5 is the form of a logistic function.)

Note that an immediate consequence of the above assumption is that s equals zero.

The value of k and r can be estimated using least squares, since from equation (3.5) it follows that:

$$\ln\left[\frac{m(t)}{N-m(t)}\right] = \ln[h(t)] = k + rt$$

which can be treated as a regression equation.

Had Mansfield assumed that time could only have non-negative values then the earlier assumption becomes $\lim_{t \to 0} m(t) = 0$. From this assumption it follows that $k = \ln(\frac{s}{r})$ and

$$m(t) = \frac{N(\frac{s}{r})\,[e^{(r+s)t} - 1]}{1 + (\frac{s}{r})\, e^{(r+s)t}} = \frac{N[e^{(r+s)t} - 1]}{\frac{r}{s} + e^{(r+s)t}}$$

Note that now s does not necessarily equal zero. Unfortunately, obtaining values for k, r and s is not as straightforward as before since the corresponding transformation is:

$$\ln\left[\frac{m(t) + N(\frac{s}{r})}{N - m(t)}\right] = k + (r+s)t$$

In conclusion, it can be shown that the number of firms having introduced the innovation, plotted against time can be approximated by an S-shaped curve such as the logistic function graph. It should also be noted that the rate of imitation, r, is a linear function of the relative profitability P and the initial investment C.

A.4 Bass Model

A.4.1 Assumptions

The basic premise of Bass' (5) model is that the timing of the initial purchase is related to the number of previous buyers. Thus the probability that an initial purchase occurs at time t given that no such purchase has yet been made is assumed to be a linear function of the number of previous buyers. Potential adopters are divided into two groups: (1) the innovators -- those individuals who decide to adopt a new product independently of the decisions of others, and (2) imitators -- those who are influenced in their timing of adoption by the decisions of other individuals. Since the products that are applicable for the model are infrequently purchased and/or of high cost, replacement sales are excluded from the total sales of the product. Thus, the model only deals with initial purchases of the product.

A.4.2 Notation

- $P(t)$ = probability that an initial purchase occurs at time t given that no such purchase has yet been made.
- $f(t)$ = probability of a purchase at time t.
- $F(t)$ = probability of a purchase by time t; cumulative of $f(t)$.
- $s(t)$ = sales at time t.
- $Y(t)$ = cumulative sales by time t.
- p = coefficient of innovation; probability of a purchase at $t = 0$.
- q = coefficient of imitation; the effect of previous buyers on the rate of adoption.

  Note: Although both p and q are assumed to be constant, the model can be extended to allow them to be time dependent. This time dependency could reflect other factors in the adoption process such as changes in profitability or changes in the initial investment required.

N = number of ultimate purchases; potential market size.

The mathematical formulation of the basic assumption is:

$$P(t) = f(t)/(1 - F(t)) = p + (\frac{q}{N})\ Y(t) \tag{4.1}$$

Sales at time t is equal to number of possible purchases multiplied by probability of a purchase at time t:

$$s(t) = Nf(t) \tag{4.2}$$

Thus cumulative sales at time t becomes

$$Y(t) = \int_0^t s(t)dt = \int_0^t Nf(t)dt = N\int_0^t f(t)dt = NF(t) \tag{4.3}$$

However sales at time t is also equal to the probability that a purchase occurs at time t given that no purchase has yet been made multiplied by the number of purchases yet to be made.

$$s(t) = p(t)[N - Y(t)]$$

Substituting equation (4.1) into the above equation:

$$s(t) = [p + (q/N)\ Y(t)]\ [N - Y(t)] = pN + (q-p)\ Y(t) + (q/N)\ (Y(t))^2 \tag{4.4}$$

Using equations (4.3) and (4.4) it is now possible to form the differential equation:

$$Nf(t) = pN + (q-p)(N)F(t) - (q/N)\ (NF(t))^2$$

or equivalently:

$$f(t) = \frac{d}{dt}F(t) = p + (q-p)F(t) - q(F(t))^2$$

The solution of the differential equation is:

$$F(t) = \frac{q - p\ \exp(-(t+c)(p+q))}{q[1 + \exp(-(t+c)(p+q))]}$$

Using the fact that $F(0) = 0$, the value of the constant c can be obtained: $c = -[\ln\ (q/p)]/(p+q)$.

Thus the probability of purchase by time t becomes:

$$F(t) = \frac{1 - e^{-t(p+q)}}{1 + (q/p)e^{-t(p+q)}} = \frac{e^{t(p+q)} - 1}{e^{t(p+q)} + (q/p)} = \frac{p(e^{t(p+q)} - 1)}{pe^{t(p+q)} + q} \tag{4.5}$$

Differentiating equation (4.5), f(t) becomes:

$$f(t) = [\frac{(p+q)^2}{p}]\ [\frac{e^{-t(p+q)}}{(1 + (q/p)\ e^{-t(p+q)})^2}] = \frac{p(p+q)^2\ e^{t(p+q)}}{(pe^{t(p+q)} + q)^2} \tag{4.6}$$

Using equations (4.2) and (4.6) we find s(t) to be:

$$s(t) = [\frac{N(p+q)^2}{N}]\ [\frac{e^{-t(p+q)}}{(1 + (q/p)\ e^{-t(p+q)})^2}]$$

$$= \frac{Np(p+q)^2\ e^{t(p+q)}}{(pe^{t(p+q)} + q)^2} \tag{4.7}$$

Now, using equations (4.3) and (4.5), we find Y(t) to be:

$$Y(t) = (N)\ (\frac{1 - e^{-t(p+q)}}{1 + (q/p)\ e^{-t(p+q)}}) = (N)\ (\frac{e^{t(p+q)} - 1}{e^{t(p+q)} + (q/p)}) \tag{4.8}$$

Thus equations (4.5) through (4.8) determine the values of the variables of the Bass Model.

The graph of sales at time t plotted against time t is shown in figure 4.

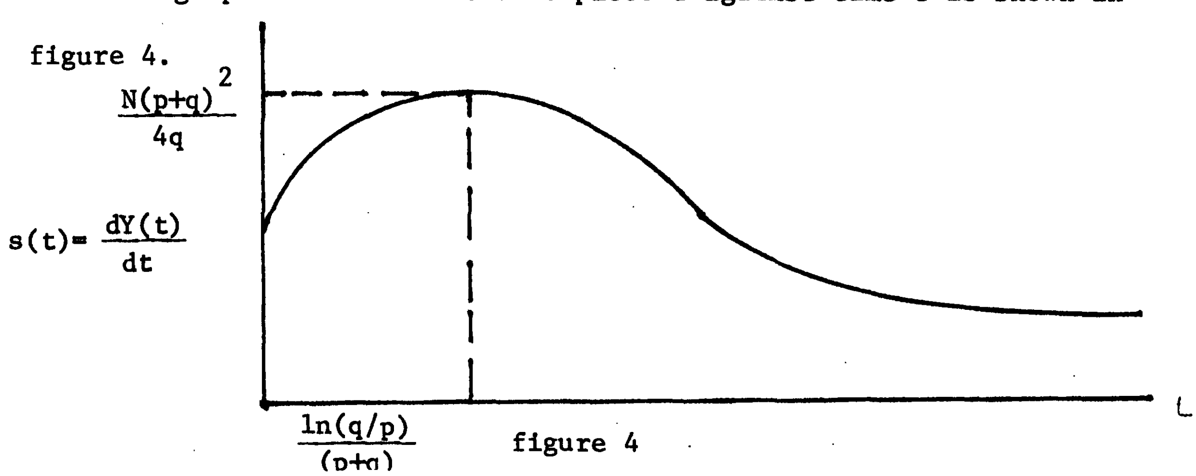

figure 4

To determine the value of the parameters we use discrete time series analysis. Let equation (4.4) be the basic equation of the model. Since this model is in terms of discrete time, let $Y(t) = \sum_{t=0}^{t-1} s(t)$. Notice that equation (4.4) is then of the form $s(t) = a + b\ Y(t) + c(Y(t))^2$ where $a = pN$, $b = q-p$, and $c = q/N$. Having determined the values of a,b, and c through regression in the time sense of sales, it is possible to solve for p and q.

$$q = -cN \qquad \text{and} \qquad p = a/N$$

Then $$b = q-p = -cN - a/N \qquad \text{or} \qquad cN^2 + bN + a = 0$$

(In practice, strong multicollinearity between $Y(t)$ and $Y(t)^2$ makes estimates of b and c quite difficult.)

Thus solving for N:

$$N = (-b + \sqrt{b^2 - 4ac})/2c$$

Assume that the effect of a government demonstration program is the same as that of an "innovative" purchase. How can this enlarge the market? In this case we must assume that the potential market size no longer will be considered as constant. Rather, assume it can be increased by externally increasing sales at one particular time. This external increase in sales will also accelerate the rate at which the innovation is adopted by the market. Solving equation (4.7) for N, the effect of a unit increase in sales at time t on the potential market size can be calculated:

$$N = \frac{(1 + (q/p)\ e^{-t(p+q)})^2}{((p+q)^2/p)\ e^{-t(p+q)}}\ s(t) = \frac{(pe^{t(p+q)} + q)^2}{p(p+q)^2\ e^{t(p+q)}}\ s(t)$$

$$= \frac{1}{f(t)}\ s(t)$$

From the above equation we see that the effect of a unit increase in sales at time t is the coefficient of the s(t) term.

Theorem: In general, it is best to have the external increase in sales as early as possible.

Let d be the time at which the increase in sales occurs.

Theorem(restated): The effect on potential market size of an increase in sales at time d will be greater than the effect of an increase in sales at time t for all t greater than d.

Proof: We want to find t such that (1) t > d and (2) the effect of increase in s(t) > the effect of increase in s(d).

Thus as stated earlier:

$$\frac{(pe^{t(p+q)} + q)^2}{p(p+q)^2\ e^{t(p+q)}} > \frac{(pe^{d(p+q)} + q)^2}{p(p+q)^2\ e^{d(p+q)}}$$

or:

$$e^{d(p+q)}\ (pe^{t(p+q)} + q)^2 > e^{t(p+q)}(pe^{d(p+q)} + q)^2$$

Expanding the squared term and simplifying:

$$p^2e^{(2t+d)(p+q)} + q^2e^{d(p+q)} > p^2e^{(2d+t)(p+q)} + q^2e^{t(p+q)}$$

Then:

$$p^2e^{(d+t)(p+q)}(e^{t(p+q)} - e^{d(p+q)}) > q^2(e^{t(p+q)} - e^{d(p+q)})$$

Since t >d, the above equation can be simplified without any change of the inequality.

$$e^{(d+t)(p+q)} > (q/p)^2$$

Taking logarithms of both sides and solving for t:

$$t > 2\ \frac{\ln\ (q/p)}{p+q} - d \qquad \text{(a)}$$

Now we want to find a value of d that maximizes the effect of the increase. By calculus the value that minimizes the effect can be determined ($d_{min} = \frac{\ln(q/p)}{p+q}$; d_{min} is the time of peak sales). Thus it follows from equation (a) that it is best to increase sales either as early as possible or after sales has peaked and has declined (a late life-cycle boost, irrelevent for us here).

Given that sales was increased initially (i.e. d=0), the marginal increase in the potential market size is 1/p. In this case, it will not be worthwhile to externally increase sales until the probability of purchase by this time is $1 - \frac{p}{q}$. Since p is generally much smaller than q, this other time will occur at the end of the adoption process. Therefore we conclude it is best to increase sales as early as possible.

An alternate proof is given below for the case when the external increase is assumed to increase cumulative sales rather than increase sales for that time. The effect of increasing cumulative sales at a particular time can be determined in the same manner as before using equation (4.8). The effect of a marginal increase in cumulative sales at time d is $[pe^{d(p+q)} + q]/[p(e^{d(p+q)} - 1)]$. Thus we want to find t such that (1) t > d and (2) the effect of increase in Y(t) > the effect of increase in Y(d). Stated mathematically:

$$\frac{pe^{t(p+q)} + q}{p(e^{t(p+q)} - 1)} > \frac{pe^{d(p+q)} + q}{p(e^{d(p+q)} - 1)}$$

or equivalently:

$$(pe^{t(p+q)} + q)(e^{d(p+q)} - 1) > (pe^{d(p+q)} + q)(e^{t(p+q)} - 1)$$

Expanding terms and simplifying:

$$qe^{d(p+q)} - pe^{t(p+q)} > qe^{t(p+q)} - pe^{d(p+q)} \quad \text{, or}$$

$$(q+p)e^{d(p+q)} > (q+p)e^{t(p+q)}$$

Since both p and q are positive the above equation can be simplified without any change in the inequality:

$$e^{d(p+q)} > e^{t(p+q)}$$

Taking logarithms of both sides:

$$d(p+q) > t(p+q)$$

Again simplifying:

$$d > t$$

This leads to a contradiction with the assumption that t > d for all t. Therefore, we conclude it is best to increase sales as early as possible.

A.5 Discussion

Bass and Mansfield have developed models to trace the rate at which an innovation is adopted. Bass uses as his measure of adoption cumulative sales while Mansfield uses the number of firms having introduced the innovation. Both models use a rate of imitation and a rate of innovation to describe the behavior of the market. Mansfield assumes that these coefficients are functions of relative profitability and required initial investment. Bass, however, makes no explicit assumptions of the underlying factors of the coefficients.

The models share many of the same limitations. Both assume that the potential market size is predetermined and that the entire market will eventually adopt the innovation. Neither Bass nor Mansfield considers changes in the requirements for adoption. This can be remedied, as pointed out earlier, by making the appropriate parameters time dependent rather than constant. The Bass model assumes that there is no competition to draw off buyers. While Mansfield does take into account competition, he assumes that the profitability of adopting the innovation is greater than alternative investments. This seems unlikely for many new technologies where the costs have not had the time to decrease. Mansfield points out the fact that his model "would not be applicable to some innovations, like an entirely new product, where, as more firms produce it, it becomes less profitable for others to do so." There may not be a sufficient amount of data for calibration of the parameters since the models deal with innovation which, in general, generate only sparse data.

It seems clear that these models, despite their shortcomings, are good for tracing the rate at which an innovation is adopted, retrospectively, but are not as useful to project the growth rate of an innovation, or to analyze controls on that growth rate, before introduction.

A significant modification of Bass' model is provided by Robinson and Lakhani [21] who modify Bass' coefficient of imitation (q) to be dependent on price and assume price declines with a learning or experience curve. They then show that a dynamic pricing policy can produce significantly greater manufacturers' profits than a policy which equates marginal revenues with marginal costs each year. That dynamic policy is also shown to accelerate the innovation.

A.6 Conjectures

Two conjectures which are currently being explored are:

Conjecture 1: A policy that allocates demonstration projects evenly across geographic areas can always be improved if the potentials, imitation rates and innovation rates of those areas differ. An optimal policy will be a policy of geographic concentration, then expansion.

Conjecture 2: A policy that allocates demonstration projects evenly across economic sectors (residential, agricultural, etc.) can be improved as in conjecture 1 by concentration in a single sector. An optimal policy will concentrate in a sector, and then expand.

By optimal in the above conjectures we mean that given demonstration resources do the most to accelerate the diffusion of the innovation.

Determination of the conditions under which these conjectures hold is the subject of current work.