AN OLIGOPOLISTIC PRICING MODEL OF THE U. S. COPPER INDUSTRY: A PROBABILITY MODEL APPROACH

by

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The analysis of competitive industries and/or competitive sectors of industries is theoretically and econometrically straightforward. Determinant solutions occur at the intersection of supply and demand curves, where supply curves represent the horizontal summation of the marginal cost curves of the members of the industry (or sector thereof). Economic rents may accrue in light of differential cost conditions across members. However, no member of a competitive sector can affect price; they are all price-takers. Short-run production decisions on the part of a given firm are made by comparing the market price with production costs.

In oligopolistic markets, the micro theory no longer supports the use of a supply curve. Deterministic market solutions, based upon costs (i.e., the supply curve) and demand alone, are no longer possible. The reason is, of course, that the members of an oligopolistic industry are no longer price-takers; they are price-setters. An individual member or group of individuals in an oligopolistic industry have pricing discretion. Costs, particularly short-run variable costs, provide a reasonably solid lower bound to pricing behavior;[1] however, many factors can contribute to pricing behavior and pricing strategies well above marginal cost.[2]

The pricing behavior and pricing strategies that are utilized by an oligopoly reflect the pecuniary and nonpecuniary objectives of the members of the oligopoly and their ability to achieve those objectives, given the conditions of the markets(s) in which they operate. In an unconstrained world, it is probable that the members of an oligopoly would impose the collusive monopolistic pricing solution with explicit collusive profits/sales sharing agreements if necessary. In the constrained environment that faces real world oligopolies, such short-run collusive profit-maximizing behavior is constrained by the conditions of that market. For example, for an

[1] In a dynamic model, even this lower bound could be broken.

[2] Witness the details of R. F. Lanzillotti, "Pricing Objectives in Large Companies," American Economic Review, December, 1958, pp. 921-940.

oligopoly characterized by mature price leadership, it will be easier for the group to maintain pricing/production decisions near the collusive monopolistic level in the face of tight demand and high capacity utilization than under market conditions of slack demand and low operating rates, ceteris paribus. This is merely George Stigler's "urgency of purchase" effect upon the seller's ability to price discriminate.[1] Likewise, the presence of government intervention (e.g., stockpiles of primary commodities, vigorous antitrust action, environmental and health regulations) and dynamic considerations (limit-pricing barriers to entry of new participants and/or new products) will constrain the pricing/production decisions of the oligopoly. For example, an oligopoly will have difficulty maintaining a collusive monopolistic price in the face of significant government sales from strategic stockpiles and substantial entry (if the collusive price is above the limit price). Finally, the number and size of the sellers, the number and size of the buyers and their ability to collusively avoid price-shaving will affect the pricing/production decisions of the oligopoly in ways explored thoroughly by Stigler.

This paper discusses a model for analyzing the oligopolistic pricing behavior of the U.S. primary producers of copper. While it can potentially be extended to other oligopolies, I do not discuss its generalization here. The model identifies a number of alternative pricing strategies available to the U.S. copper industry, as a whole, in the short run. In the face of considerable market share stability over the historical period (1950-1974), long-term purchasing contracts, stable price leadership patterns and little evidence of price-shaving (at least through 1974), the inter-oligopoly pricing issues pursued by Stigler[2] have little relevance for the U.S. copper producers over 1950-1974. As a result, the model developed here does not examine the causal factors of price-shaving. On the contrary, the model attempts to analyze the pricing behavior of the "price-led" U.S. primary producers of

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[1] See G. Stigler, "A Theory of Oligopoly," Journal of Political Economy, February, 1964, Vol. LXXII, #1, pp. 44-61. These examples are elaborated more thoroughly in Section B when the forces that constrain oligopoly pricing are formalized into a probability model.

[2] As Stigler states, "Fixing market shares is probably the most efficient of all methods of combatting secret price reductions." See ibid, p. 46.

copper, as a whole, and the market forces that help determine that group pricing behavior. The model differs considerably from a competitive pricing model in ways discussed in Section A.

The paper posits alternative pricing strategies. Given these pricing strategies, the paper discusses how the constraining market forces operating upon the U.S. primary producers of copper affect its actual pricing behavior. The technique identifying the alternative pricing strategies has been developed elsewhere[1] and has been labelled a "parametric analysis." These "parametric" pricing strategies are *ex ante*; actual *ex post* pricing behavior will depend upon the *ex ante* pricing strategies considered, the desires of the oligopolists, and the market forces constraining the ability of the oligopolists to implement their desires. Utilizing the parametric analysis of alternative pricing options, this paper endogenizes the oligopolistic pricing decisions by utilizing a probability model to analyze the effects of market conditions upon the oligopolistic pricing decision.

The parametric analysis of oligopolistic pricing in the U.S. copper industry is discussed in Section A below and compared with the competitive paradigm. Some empirical results are presented supporting the oligopolistic pricing model. The probability model formulation of oligopolistic pricing in the face of market constraints is discussed in Section B. Section C presents some further empirical results from the probability modelling. Some concluding remarks are offered in Section D.

A. A PARAMETRIC ANALYSIS OF OLIGOPOLY PRICING IN THE U.S. COPPER INDUSTRY

For means of comparison, let us briefly examine the pricing behavior implied by the competitive model under conditions of perfect certainty. In this case, the supply curve summarizes the horizontal summation of the marginal cost curves of the participants in the industry and the intersection of that supply curve with market demand yields instantaneous market equilibrium price and quantity. If demand should shift, in this example, along the summation of the marginal cost curves, the new equilibrium is instantaneously achieved by

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[1] See Arthur D. Little, Inc. (ADL), *Economic Impact of Environmental Regulations on the U.S. Copper Industry* (August, 1976); and R. Hartman, *An Oligopolistic Pricing Model of the U.S. Copper Industry* (M.I.T., Ph.D. Dissertation) February, 1977.

by the addition of new producers when demand shifts out or the elimination of the marginal producers when demand shifts in. Demand is known with perfect certainty and the market clearing price and quantity are known with certainty.

Needless to say, such a competitive, perfect certainty model is a textbook case. The world is characterized by incomplete information and non-competitive market structures. There has been a long tradition of models aimed at analyzing price-setting behavior in non-competitive situations (many, unfortunately, deal purely in the restrictive world of duopoly). Most of the theoretical literature deals with profit maximizers acting under different (relatively rarefied) behavioral assumptions. The mere assumption of profit maximizing behavior *per se* flies in the face of much of the literature.[1] However, the theoretical models cling tenaciously to such assumed behavior for a given producer.

The models combine it with behavioral assumptions directed at fellow producers already in an industry (Bertrand, Cournot, Von Stackelberg) or directed at outsiders who are potential entrants (Sylos-Labini, Bain, Modigliani and Bhagwhati).[2] The use of limit pricing to prevent entry has been developed into dynamic programming models by Gaskins[3] and others.

In view of this brazenly short discussion of a very broad literature, it should be clear that the number of possible short-run and long-run oligopolistic pricing strategies is quite large. Given a wide range of assumed behavior and corporate goals, possibilities for alternative pricing behavior abound. The

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[1] Witness Lanzillotti, *op. cit.*; Herman Simon, "Theories of Decision-Making in Economics and Behavioral Science," *American Economic Review*, June, 1959, pp. 253-283; W.J. Baumol, *Business Behavior, Value and Growth*, 1959; Oliver Williamson, *The Economics of Discretionary Behavior*, 1964, *Corporate Control and Business Behavior*, 1970, and "Managerial Discretion and Business Behavior," *AER*, December, 1963, pp. 1032-1057; Robin Morris, "A Model of the 'Managerial' Enterprise," *QJE*, May, 1963; Cyert and March, *A Behavioral Theory of the Firm*, Prentice-Hall, 1963; and so on.

[2] For an excellent review of this literature plus references, see Paul Joskow, "Firm Decision-Making Processes and Oligopoly Theory," *American Economic Review, Papers and Proceedings*, May, 1975, pp. 270-279.

[3] Darius Gaskins has also applied his thinking to the aluminum industry. See "Dynamic Limit Pricing: Optimal Pricing Under Threat of Entry," *Journal of Economic Theory*, March, 1971, pp. 306-322, and "Alcoa Revisited: The Welfare Implications of a Secondhand Market," *Journal of Economic Theory*, July, 1974, pp. 254-271.

discussion here attempts to utilize more simplified short-run pricing strategies and more simplified assumptions concerning the interactive behavior of the members of the oligopoly. The interactive behavioral assumptions implicit here are that a well-defined sub-group of the oligopolists dominate price-setting activities (i.e., the oligopoly is characterized by mature price leadership) and that relatively stable market-share patterns characterize all members of the oligopoly over time.[1]

The pricing strategies or modes of pricing behavior utilized here are chosen with two purposes: firstly, to identify a "most likely" or "normal" pricing strategy of the U.S. primary copper producers in a given year, and secondly, to identify reasonably solid bounds around that "most likely" or "normal" pricing behavior. Three modes of ex ante pricing behavior have been identified to accomplish these purposes.[2] They include:

1. Collusive monopolistic pricing (MR = MC);
2. Full-cost pricing (P = ATC); and
3. Average variable cost pricing (P = AVC).

"Full-cost pricing" can be assumed to characterize the "normal" behavior of some oligopolies in a normal year.[3] By this formulation, price is set equal to average total cost (ATC), where average total cost includes average operating costs (i.e., average variable costs) plus average fixed costs (which include a target or desired rate of return on investment). The desired

[1] These assumptions will not be valid for all oligopolies. They are for the primary producers of copper in the United States. See ADL, op. cit.; Hartman, op. cit., for a full treatment of these issues and their effect on the analysis.

[2] Certainly, many other ex ante solutions could be identified. For example, alternative target rates of return could be built into the full-cost pricing solution or P = MC could be identified. The identification of the full-cost pricing solution as most likely or "normal" reflects the realities of the U.S. copper industry rather than all oligopolies. The discussion is easily generalizable by assuming all pricing strategies have unknown probabilities and letting the probability model developed in Section B indicate which solution is "most likely."

[3] This seems to be true for the primary copper producers. For a discussion and bibliography on "full-cost pricing," see Scherer, op. cit., Chapters 6 and 7; and Edwin Mansfield, Microeconomic Theory and Applications (Norton, New York, 1971).

or target rate of return on investment can be thought of as reflecting competitive rental rates.[1] The exact meaning of full-cost pricing shall be clarified below.

The purpose of this paper is not to examine the validity of the full-cost pricing hypothesis itself. The literature contains much support for a number of variations of it. Hall and Hitch[2] found that 30 of 38 firms used some variant of it; Kaplan, Dirlam and Lanzillotti[3] found 10 of their 20 firms used some variant of it. Cyert, March and Moore[4] were able to predict, to the penny, the price of 188 of 197 randomly chosen items in a department store using a simple wholesale cost mark-up rule. Scherer[5] examines the "full-cost pricing" tools of General Motors including "standard volume" and "standard price." The use of "full-cost pricing" and the "representative firm" as a coordinating device for loose-knit cartels and oligopolies is examined in varying contexts by Machlup, Scherer, Hall and Hitch, Cyert and March, Kaplan, Dirlam, and Lanzillotti and Fog.[6] Furthermore, Fog and Hall and Hitch[7] have examined how their full-cost pricing firms have broken away from the full-cost price in the face of recessionary pressures.

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[1] For greater elucidation of the use of a target or desired rate of return, see Lanzilotti, op. cit.; Kaplan, Dirlam, and Lanzilotti, Pricing in Big Business; Scherer, op. cit., Chapters 6-9.

[2] R. Hall and C. Hitch, "Price Theory and Business Behavior," Oxford Economic Papers, May, 1939, pp. 12-45.

[3] A. Kaplan, J. Dirlam and R. Lanzilotti, Pricing in Big Business (Washington: Brookings, 1958), p. 130; and R. Lanzilotti, "Pricing Objectives in Large Companies", American Economic Review (December, 1958), pp. 923 and 929.

[4] See R. Cyert and J. March, A Behavioral Theory of the Firm (Englewood Cliffs: Prentice-Hall, 1963), pp. 146-147.

[5] F. Scherer, Industrial Market Structure and Economic Performance (Chicago: Rand McNally, 1970), p. 174.

[6] See F. Machlup, The Economics of Sellers' Competition (Baltimore: Johns Hopkins Press, 1952); F.M. Scherer, op. cit., pp. 173-179, 223-224, 290 and 305-306; Hall and Hitch, op. cit., pp. 27-28; Cyert and March, op. cit., p. 120; Kaplan, Dirlam and Lanzilotti, op. cit., p. 16; and B. Fog, "How are Cartel Prices Determined," Journal of Industrial Economics (November, 1956), pp. 16-23 and Industrial Pricing Policies: An Analysis of Pricing Policies of Danish Manufacturers (Amsterdam: North Holland, 1960).

[7] Hall and Hitch, op. cit.; Fog, op. cit.

As a result, it can be stated that the "full-cost pricing" behavior appears to characterize the pricing strategy of some oligopolistic firms in "normal" years,[1] with "normal" demand/supply conditions. Furthermore, such pricing behavior is the competitive solution for a market in long-run equilibrium. However, administered pricing behavior will arise if an oligopoly can maintain price above the full-cost pricing level or at the full-cost pricing level in periods of slack demand.

There are, however, going to be short-run conditions that could deviate the pricing strategy of an oligopoly from its full-cost pricing strategy (if that is its target). For example, for the U.S. primary producers of copper, unforeseen factors, including some strikes, collapse of world demand, nationalization of ore deposits, and/or overheating of world demand during the Vietnamese War years, can impinge themselves[2] upon pricing and production decisions and prevent primary producers from realizing the "full-cost pricing" strategy. These market and nonmarket conditions could make it feasible for the oligopoly to raise prices to a collusive monopolistic pricing strategy; they could also make it impossible for the oligopoly to avoid lowering prices below the "full-cost pricing" target. To take account of the effects of these phenomena in the short run, the "full-cost pricing" solution can be bounded by the "average variable cost" pricing solution or strategy and the "collusive monopolistic" pricing solution or strategy.

The lower bound on price in a given year has been identified as the equality of price and average variable costs (AVC). At this point, the unit price covers average operating costs alone without any fixed cost coverage. While this lower bound reflects real short-run pricing options, a firm or group of firms cannot price at this lower bound for long.

The short-run upper bound upon price identified for the analysis is the "collusive monopolistic" point determined by the intersection of oligopoly's

[1] As is the case with the primary producers of copper. For reasons, see ADL, op. cit.; Hartman, op. cit.

[2] As most certainly they will impinge upon the competitive sub-sectors of the copper industry. However, since these sectors are competitive, discretionary pricing decisions are not open to them in the first place.

marginal cost[1] and marginal revenue curves. In an unconstrained world of robber barons, this might be the "normal, most likely" solution. In reality, this is an upper bound only in the sense that perfect collusion exists, that profit-sharing agreements are operative and work perfectly and that the marginal cost curve fully articulates this collusive behavior.[2]

These three pricing options for an oligopoly are schematicized in Figure 1A for the traditional U-shaped cost curves and in Figure 1B[3] for the horizontal marginal cost (MC) curve case. Let us identify the three pricing options as parametric solutions 1, 2 and 3. Clearly the parametric solutions 1 and 3 bound the "full-cost pricing" parametric solution 2. As seen in Figures 1A and 1B, bounds exist for both price ($P_1 - P_3$) and quantity ($Q_3 - Q_1$) solutions. The width of the bounds for the primary producers of copper in a given year is determined by many things: the elasticity of demand, the elasticity of supply in the competitive fringe sectors, the level of total copper demand, the assumptions regarding the oligopolistic structure of the primary producers as reflected in the shape and level of the cost curves for that group.

Using the facts of the U.S. copper industry and the cost curves of Figure 1B, Figure 1C indicates the change in the bounds for parametric solution 2 in a year characterized by extreme demand pressures ("demand crunch"). Figure 1D examines the comparative statics in a year characterized by collapsed demand ("demand slack"). As would be expected in a demand crunch case, the primary producers would be pushing against capacity along the steeply rising segments of the relevant cost curves (MC, ATC, AVC). In this case, all three

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[1] Such marginal cost curves are explained more fully in Figure 1 and Hartman, op. cit., and ADL, op. cit.

[2] While the prices of the U.S. primary producers of copper move together, it is not felt that such price leadership activity indicates "collusive monopolistic" pricing. As a matter of fact, historical model results indicate that the collusive monopolistic price solutions are considerably above the "full-cost pricing" solution and the actual solution in most years. The exceptions are a few years when upward shifts in demand appear to have bestowed considerable market power (in a scarcity sense) upon the primary producers. Because of the real limit-pricing concern about long-run substitution in demand to aluminum, it was not expected that the primary producers would gravitate consistently to a short-run collusive monopolistic price solution.

[3] A familiarity with the representative of cost curves (ATC, AVC, MC) and demand (DD) and marginal revenue curves (MR-MR) is assumed. The bemused reader should consult Henderson and Quandt, op. cit., Chapter 3; or Edwin Mansfield, op. cit.

price solutions rise while the bounds around parametric solution 2 narrow. See Figure 1C. In slack demand situations, the primary producers will not be pushed to capacity.[1] Price solutions for parametric solutions 2 and 3 will be lower, while the bounds around parametric solution 2 will widen considerably. See Figure 1D. Figures 1C and 1D demonstrate principally the effect of alternative demand levels rather than demand elasticity on the parametric solutions.[2] However, such "demand slack" and "demand crunch" alternatives do characterize the cyclical copper industry at different times during the historical periods.[3]

An analysis of an oligopoly, in general, and the U.S. primary producers of copper, in particular, must take its price-setting behavior explicitly into account. The three pricing options introduced above are developed to analyze explicitly that potential price-setting behavior.[4]

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[1] Nor will the competitive fringe, secondary copper refiners and copper scrap producers/sellers be pushed to capacity.

[2] As drawn, the demand curve in Figure 1C is more elastic in the relevant range, which also contributes to narrowing the parametric solution bounds.

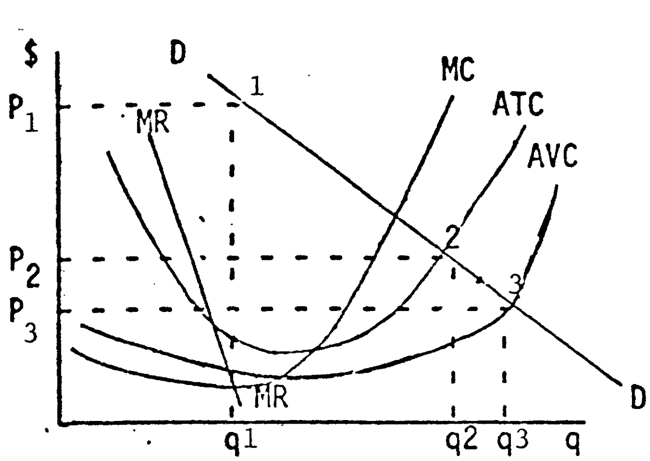
[3] The comparative statics of the model are examined more fully in Hartman, op. cit., Chapter 4.

[4] The econometric simulation model for the U.S. copper industry discussed in Hartman, op. cit., and ADL, op. cit., embeds these three parametric solutions into a simulation of the domestic copper industry on a year-to-year basis. The model simulates the entire copper industry for each mode of price-setting behavior on the part of the oligopolistic primary producers. The "normal" mode of pricing behavior on the part of the primary producers will generate model solutions for all endogenous variables under "normal" conditions.

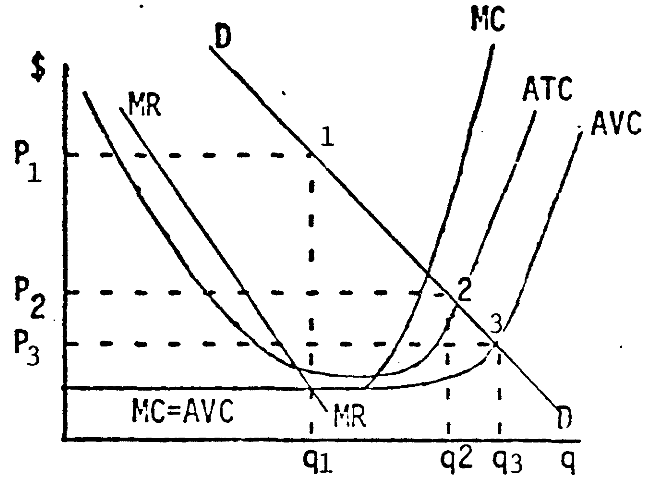
However, severe demand crunch (late 1960's, 1974) and demand slack (1975-1976) can drive the primary sector and the industry toward non-"normal" strategies or solution bounds. The simulation model mentioned above will estimate those solution bounds for the entire industry. For example, under severe demand crunch, parametric solution 1 (collusive monopolistic solution) could become relevant; the primary producers are assumed to act in unison restricting output to that level dictated by the intersection of their joint MR and MC curves (and raising price accordingly). The model simulates how such pricing activity raises copper prices generally, hence generating greater levels of production in the two competitive sectors. On the other hand, in severe demand slack situations (1975), parametric solution 3 may become relevant. For parametric solution 3 (price set at average variable cost) in a given year, the model will indicate the production and pricing behavior of the primary producers and how these pricing and production actions will affect copper prices and production in the competitive sectors.

The detailed discussion of the model indicates these facets more clearly (see sources above). Suffice it to say that the pricing behavior of the oligopolistic primary refiners has important impacts upon the rest of the domestic copper industry. As a result, the oligopolistic pricing model of the copper industry simulates in a given year the entire industry for each mode of pricing behavior on the part of the primary producers.

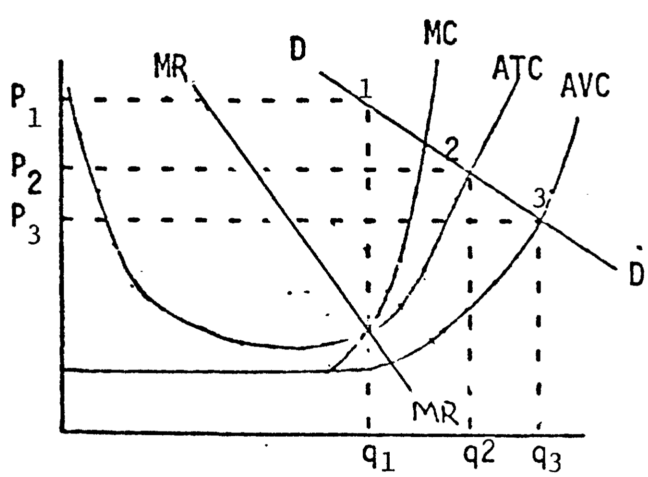
FIGURE 1: MODES OF PRICING BEHAVIOR -- PARAMETRIC SOLUTIONS



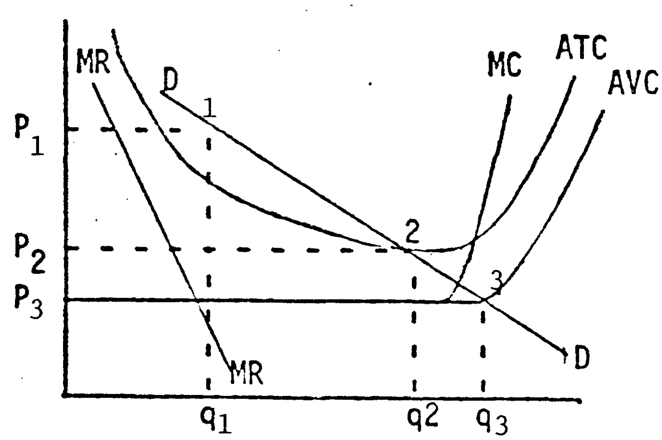
A) Traditional U-Shaped Costs



B) Horizontal MC

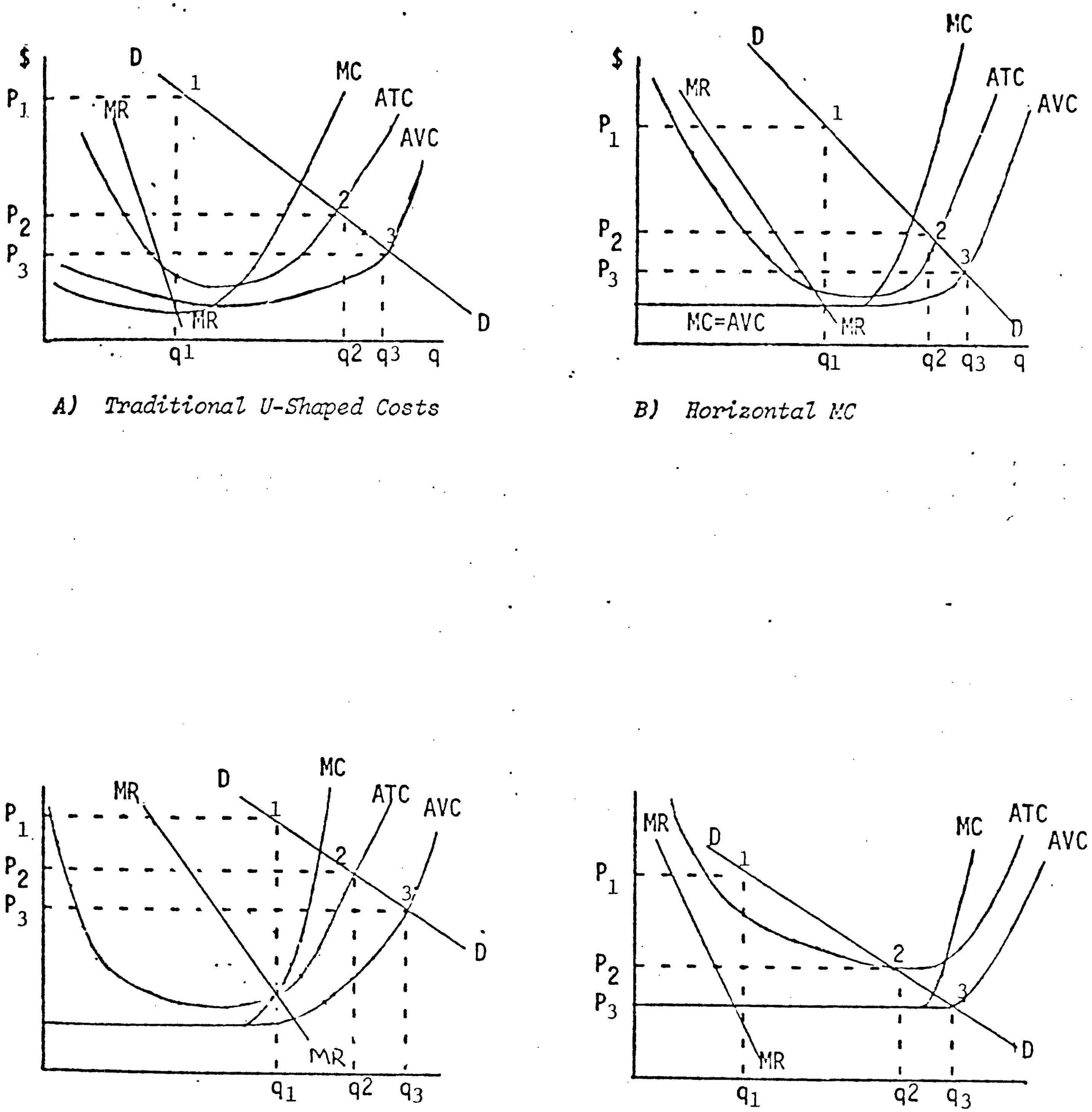


C) Demand Crunch



D) Demand Slack

FIGURE 1: MODES OF PRICING BEHAVIOR -- PARAMETRIC SOLUTIONS



A) Traditional U-Shaped Costs

B) Horizontal MC

C) Demand Crunch

D) Demand Slack

However, the three parametric solutions are clearly ex ante. To make an actual price/production forecast, one can identify the "most probable" parametric solution. As discussed above, the "full-cost pricing" mode of pricing behavior characterizes some oligopolies in "normal" years. This is certainly true for the primary copper producers. However, in years characterized by extremely slack demand, it is likely the oligopoly will not be able to support a price that includes a normal rate of return to investment.[1] In these cases, the pricing/production decision will fall below the full-cost price level and parametric solution 3 will represent the most probable pricing/production behavior. Likewise, as discussed above, in years of extreme demand crunch, the parametric bounds narrow and parametric solution 1 will be the "most probable" industry simulation. In cases of extreme "demand crunch" and slack demand, parametric solutions 1 and 3 may become the most probable solutions.

Just when parametric solutions 1, 2 or 3 become adequate approximations of oligopoly pricing behavior is determined by market conditions. In an unconstrained world, parametric solution 1 would always be chosen. In an uncertain world, parametric solution 2 may represent the long-run profit maximizing solution.[2] In the real world the pricing strategy utilized by an oligopoly is determined by goals of the oligopoly and the market forces constraining the oligopoly's ability to affectuate those goals.

I shall turn, in Section B, to the formulation of a probability model which will combine the three pricing strategies identified above with the market conditions in which the U.S. primary copper producer oligopoly exists in order to quantify how market conditions will, in actuality, affect oligopolistic pricing decisions. Before doing that, however, let me emphasize the differences between this pricing model and the competitive model.

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[1] 1975 and 1976 being prime examples for the copper industry. Historically, the dispersion of actual rates of return to gross book value of assets has been quite large. Of course, the extent to which the oligopoly can maintain full-cost pricing in slack years indicates the "strength" of the oligopoly.

[2] General Motors use of full-cost pricing is to obtain "over a protracted period of time a margin of profit which represents the highest attainable return commensurate with capital turnover and the enjoyment of wholesale expansion with adequate regard to the economic consequences of fluctuating volume." See Donaldson Brown, "Pricing Policy in Relation to Financial Control," Management and Administration (1924), p. 197.

In the beginning of this section the discussion of the competitive, perfect certainty model indicated price/production solutions occurred along the marginal cost curves of the competitive producers in response to instantaneously perceived shifts in demand. The three pricing strategies introduced above for the U.S. primary producers of copper and the bounding of the oligopoly's pricing/production decision within parametric solutions 1 and 3 imply that the pricing/production solutions occur along the demand curve; just where they occur depends upon the desires of the oligopoly and the ability of the oligopoly to effectuate those desires in the face of constraining market forces. In a world of uncertainty, the econometrically estimated demand curve facing the oligopoly reflects the rational expectations demand and the knowledge of oligopoly costs generates the three parametric pricing solutions. However, it should be noticed that in Figures 1A through 1C that the full-cost pricing solution and average variable cost pricing solution fall below the marginal cost solution (the competitive case). If the oligopoly is covering its ATC[1] and it desires to do so, then the fact that the full-cost pricing solution falls below the marginal cost pricing solution is acceptable behaviorally. The assumption that parametric solution 3 is realistic implies that in the face of collapsed demand (which is the only time parametric solution 3 is relevant) the oligopoly does not restrict output and try to price at marginal cost. On the contrary, the oligopoly is assumed to attempt to maximize revenues and cover at least some of its fixed costs (until P = AVC along D).

Table 1 presents some limited results which suggest that the actual pricing/production decisions of the U.S. primary producers of copper do lie along the demand curve bounded by parametric solutions 1 and 3. These results come from simulations utilizing the model discussed in footnote 4, page 8A. The actual solution is also given. These simulation results are derived from relatively crude cost estimates for the primary producers over 1964-1973.[2]

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[1] A full understanding of the composition of the oligopoly cost curves would be helpful in understanding what this P = ATC solution implies for the members of the oligopoly. See ADL, op. cit., and Hartman, op. cit., Chapter 4.

[2] See ADL, op. cit., and Hartman, op. cit.

In spite of the crude cost estimates, the results support the oligopolistic pricing schema outlined here. In 1964, 1965, 1970 and 1972 the actual price/production decision was between parametric solutions 2 and 3--below the full-cost pricing levels along the demand curve. These years were characterized by comparatively slack demand. In 1967 and 1968, the actual solution is approximately at full-cost pricing levels. In 1973, the actual solution is above parametric solution 2 between 1 and 2 along the demand curve. In the three remaining years, the actual solution is ambiguous, however, none of the ambiguous years suggest that marginal cost pricing was pursued.

B. A PROBABILITY MODEL FORMULATION OF THE PARAMETRIC ANALYSIS: MARKET POWER AND ITS EFFECTS ON PRICING

Maximum Likelihood Formulation

The discussion above identified three *ex ante* pricing options, parametric solutions 1, 2 and 3. Table 1 indicated how the actual pricing solution lay between parametric solutions 1 and 3 over the time. Ignoring the actual solution, consider the selection of the actual pricing option in a given year to be a single drawing from a trinomial distribution $F_x(X) = F_x(X; P_1, P_2)$, where

$$F_x(X) = \begin{cases} P_1 \text{ for } X = X_1 \\ P_2 \text{ for } X = X_2 \\ P_3 \text{ for } X = X_3. \\ 0 \text{ for otherwise} \end{cases} \quad (1)$$

where X_1, X_2, and X_3 are column vectors of the (price, quantity) solutions in parametric solutions 1, 2 and 3, respectively. We have, therefore,

$$\overline{X} = P_1X_1 + P_2X_2 + P_3X_3 = E(X) \quad (1a)$$

$$VAR_X = P_1(X_1-\overline{X})(X_1-\overline{X})' + P_2(X_2-\overline{X})(X_2-\overline{X})' + P_3(X_3-\overline{X})(X_3-\overline{X})'$$

TABLE 1: U.S. PRIMARY COPPER PRODUCERS PRICING AND PRODUCTION DECISIONS, ACTUAL (A) AND PARAMETRIC SOLUTIONS 1, 2 AND 3 (1964-1973)

	1964				1965				1966				1967			
	1	2	3	A	1	2	3	A	1	2	3	A	1	2	3	A
Q	1,329	1,528	1,943	1,741	1,434	1,583	1,958	1,864	862	1,346	1,858	1,896	927	1,265	2,603	1,251
P	68	62	48	50	74	69	59	54	69	54	38	55	66	56	44	57

	1968				1969				1970				1971			
	1	2	3	A	1	2	3	A	1	2	3	A	1	2	3	A
Q	1,338	1,891	2,117	1,547	1,633	2,174	2,281	1,937	1,422	1,519	2,204	1,975	873	1,109	1,745	1,733
P	76	60	53	60	86	70	67	64	83	80	59	77	69	62	43	65

	1972				1973			
	1	2	3	A	1	2	3	A
Q	1,208	1,542	2,379	2,011	1,613	2,281	2,259	2,027
P	78	67	43	62	92	70	70	72

NOTES: Q, in 1,000 short tons; production for U.S. primary producers.

P, the Engineering and Mining Journal Price of Copper (1974 $).

Nos. 1, 2 and 3 are parametric solutions 1, 2 and 3. A gives the actual values.

The likelihood function L_t for a given drawing in year t is given by

$$L_t = P_{1t}^{n_{1t}} P_{2t}^{n_{2t}} P_{3t}^{n_{3t}}$$

where $P_{1t} + P_{2t} + P_{3t} = 1$, $n_{1t} + n_{2t} + n_{3t} = 1$, and n_{1t}, n_{2t}, n_{3t} can only be 0 or 1.

To maximize L_t in a given year with respect to X we choose $X = X_i$ such that $P_i = \underset{j}{\text{Max}}(P_j)$.[1]

If we assume that the drawings each year are time-wise[2] independent and that (P_{1t}, P_{2t}, P_{3t}) can vary each year, the likelihood function of observing over t = 1 . . . . τ, a particular pattern of random drawings (i.e., pattern of pricing regimes) is

$$L = \prod_{t=1}^{\tau} L_t = \prod_{t=1}^{\tau} P_{1t}^{n_{1t}} P_{2t}^{n_{2t}} P_{3t}^{n_{3t}} \qquad (2)$$

$$= \prod_{t\epsilon\tau_1} P_{1t} \prod_{t\epsilon\tau_2} P_{2t} \prod_{t\epsilon\tau_3} P_{3t}$$

where $P_{1t} + P_{2t} + P_{3t} = 1$, for all t; $n_{1t} + n_{2t} + n_{3t} = 1$, for all t; $n_{it} = 0$ or 1 for all i and all t; τ_i is the group of years t for which $n_{it} = 1$ (i.e., the years for which parametric solution i was chosen); and $\tau_1 U \tau_2 U \tau_3 = \tau$.

The P_{it} in equation 2 can be expressed in terms of variables expressing market conditions so that $P_{it} = F_i(B_i Z_t)$, where Z_t is a vector of variables in year t and B_i is assumed constant over time. By specifying $P_{it} = F_i(B_i Z_t)$,

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[1] This is essentially how the model of choice operates in Hartman, op. cit., and ADL, op. cit.

[2] This is not a bad assumption when the ability to exert market power is usually a short-run phenomenon (as is the case for the copper industry) reflecting short-run shifts in demand and sudden scarcities; witness, for example, the copper industry in 1974 and 1975. The probability that solution 1 was chosen in 1974 is high; the probability that solution 3 was imposed in 1975 is high; and finally, the imposition of solution 3 in 1975 depended entirely upon events in 1975, not upon the choice in 1974.

and utilizing equation 2, maximum likelihood estimates (MLE) of B_i can be obtained. Utilizing these estimates of B_i and F_i will yield MLE $\hat{P}_{it}$ for all i and t. As a result, the effects of Z upon the P_i can be quantified. Furthermore, $\hat{P}_{it}$ can be utilized in equations 1a), yielding a single endogenous solution $\overline{X}_t$ with VAR_{X_t}. The functional forms F_i and Z_t are developed below.

The Functional Determination of the Probabilities (P_{it}): Definition of Market Power

It has been indicated in Section A that for some oligopolies, parametric solution 2 can be the normal, "most probable" long-run competitive equilibrium solution. Parametric solutions 1 and 3 bound that solution. However, which solution best approximates ex post the pricing behavior of an oligopoly in a given year depends upon market conditions and constraints upon the ability of the oligopoly to exert market power.

The following discussion develops for the U.S. primary producers of copper, a continuous variable expressing the market power for oligopolists. This market power variable will be utilized for determining the probabilities (P_{it}) introduced above. However, before plunging into that specific analysis, some general comments are in order. The discussion in Section A indicated that in "demand crunch" periods, the collusive monopolistic solution (MC=MR) becomes a real possibility. Likewise, in "demand slack" periods, parametric solution 3 (P=AVC) becomes probable. Such qualitative statements merely imply that as demand shifts suddenly and robustly upwards in the short run against fixed capacity, the sudden excess demand yields high capacity utilization rates and greater market power to raise prices to desired levels. As mentioned in the introduction, this is merely Stigler's "urgency of purchase"[1] phenomenon. Likewise, sudden and significant decreases in demand, given fixed capacity, generate low capacity utilization, stockpiling of inventories, and decreased market power (i.e., ability to maintain price at desired full-cost pricing levels).

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[1] G. Stigler, op. cit.

While such statements hold generally for an oligopoly, the extent to which each factor is important in determining the market power of an oligopoly differs from oligopoly to oligopoly and may differ for a given oligopoly over time. The ability of a given oligopoly to take advantage of market power and impose pricing regime 1 depends upon such factors[1] as: the number of participants in the oligopoly, the stability and "maturity" of the oligopoly with respect to price leadership and following roles, the relationship of the collusive monopolistic price to perceived limit prices, the discount rate of the oligopolists, the elasticity of supply in competitive fringe sectors, the expected impact of federal regulations upon the current and future state of the industry,[2] the presence of anti-trust measures, etc.

These insights can be made more specific for the copper industry. Prolonged periods of demand crunch in the 1960's were characterized by prices above full cost pricing levels; however, they were still well below collusive monopolistic price solutions.[3] The constraints upon market power (i.e., pricing) appear to have included the real concern about long run substitution from aluminum, (a limit pricing argument) the fact that the vertically-integrated oligopolistic primary producers were capable of obtaining a good portion of the short-run monopoly profits through market power at the semi-fabricating and fabricating stages of production,[4] and the use of the copper stockpiles by the federal government. By 1974, however, some concerns about long run substitution in demand from aluminum were diminished because aluminum had indeed captured some of the markets where substitution had been feared. As a result, in the face of a sudden and great increase in demand, the primary producers did appear to price at para-

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[1] The relevance of these considerations is made clearer in Hartman, op. cit., Chapter 4, Section Aix.

[2] For example, if Environmental Protection Agency legislation is expected to limit domestic copper capacity, domestic producers may raise prices until foreign supply expands enough to compete through sizable levels of imports.

[3] These insights are based upon historical simulation of the copper industry utilizing the model discussed in Hartman, op. cit. See Appendix B of that discussion for a presentation of historical simulation results.

[4] This is David McNicols' insight. See D. McNicol, "The Two-Price System in the Copper Industry." MIT PhD dissertation, (February, 1973).

metric solution 1. Furthermore, the government stockpiles of copper, utilized to exert downward pressure on copper prices in the 1960's, were effectively exhausted by 1972; hence, they did not provide a constraining force in 1974.[1]

To summarize such insights for the copper industry, we may say that although significant shifts against fixed capacity may be necessary to generate market power, constraining influences may limit its use. However, high and suddenly increasing demand in the face of low government stockpiles has seemed to generate the unconstrained market power necessary for the primary producers to impose discretionary pricing option 1.

In a more general equational form we have:

$$MP_t = G(CPU_t, DEMAND_t, INVENTORIES_t, LIMIT\ PRICE_t, INSTITUTIONAL\ FACTORS_t) \quad (3)$$

where MP_t is the level of market power experienced in year t by the oligopoly. It is hypothesized in (3) to be a function of five factors. CPU_t is the capacity utilization rate for the oligopolists; it summarizes the interaction of demand, capacity expansion and the cost curves of the oligopolists in year t; it is assumed $G_1 > 0$. $DEMAND_t$ is the measure of the robustness of demand increases in a given year t; it can be proxied for copper by the level or the changes in world copper prices (ΔRPLME), in U.S. industrial production (ΔYUD), or in primary producers production/sales (ΔQPR). It is expected that $G_2>0$. The presence of inventories will constrain the ability of the oligopoly to effectuate demand crunch conditions into market power and price increases. While the determination of inventory behavior is complicated in itself, involving a combination of buffer stock and speculative motives[2] and, while its effect requires some formulation of the costs of holding inventories, it can be safely assumed that $G_3<0$. Clearly, the ability to translate market conditions into market power and higher prices depends upon the height of the limit price; hence $G_4>0$. Finally, a range of institutional issues will determine the extent that current market conditions can be translated into market power. For example, imposition of import quotas will

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[1] See Hartman, op. cit.; ADL, op. cit.; Charles River Associates (CRA), Economic Analysis of the U.S. Copper Industry (March. 1970).

[2] For a discussion of inventory modeling for the primary producers of copper, see Hartman, op. cit.

increase market power, ceteris paribus. The elimination of import quotas will lower market power ceteris paribus. The formation of a cabinet committee such as the Houthakker Committee for the analysis of copper will diminish market power. Government stockpiling policies will affect market power. Clearly, the effect will depend upon the particular institutional arrangement; if we are to believe the Stigler "capture" theory of regulation and government intervention, then $G_5>0$. If on the other hand, the populist, antagonistic relationship between government institutions and a given oligopoly holds, we expect $G_5<0$.

The initial specifications for the copper industry can take the following forms:

$$MP_t = G_1(CPU_t, \Delta RPLME_t, INV_t, GOV_t) \quad (4a)$$
$$MP_t = G_2(CPU_t, \Delta YUD_t, INV_t, GOV_t) \quad (4b)$$
$$MP_t = G_3(CPU_t, \Delta QPR_t, INV_t, GOV_t) \quad (4c)$$

where MP_t is the level of market power in year t

CPU_t is the capacity utilization rate at the full-cost pricing solution for year t

$\Delta RPLME_t$, ΔYUD_t, are changes in world copper prices, and the index of U.S. industrial production over t-1 to t (levels of YUD are also used).

ΔQPR_t is the proportional change in physical primary producers' production for parametric solution 2 from year t-1 to t.

INV_t is the level of final product inventories of the primary producers at the beginning of year t.

GOV_t is the level of government stockpiles of copper at the beginning of year t.

Equation 4b) will be used in Section C.

Final Statement of the Likelihood Function

Before turning to estimation, it is necessary to link the market power variables to the probabilities of selecting parametric solutions 1, 2 or 3. It is clear from the discussion above that $\partial P_{1t}/\partial MP_t>0$ while $\partial P_{3t}/\partial MP_t<0$. Therefore, we have

$$P_{1t} = F_1(MP_t), \; \partial F_1/\partial MP_t>0$$
$$P_{3t} = F_3(MP_t), \; \partial F_3/\partial MP_t<0 \quad (5)$$
$$\text{and} \quad P_{2t}=1-F_1(MP_t) - F_3(MP_t) = F_2(MP_t)$$

Let us furthermore define the F_i to be the logit specification. As seen above, we assume that X_t is a multinomial random variable described by 3 probabilities $Pr(X_t = X_{it}) = P_{it}$

where $\sum_{i=1}^{3} P_{it} = 1$ and $0 < P_{it} < 1$, for all i and t.

Using the logit specification, we obtain:

$$Pr(X_t = X_{it}) = P_{it} = \frac{e^{\phi i(MP_t)}}{\sum_{j=1}^{3} e^{\phi j(MP_t)}} \tag{6}$$

Using linear versions of the equational specifications of MP_t in (4b), we may specify equation 6 for i = 1, 2 and 3 by determining the $\phi_i(MP_t)$ as follows:

$$\phi_1(MP_t) = \alpha Y_t = \alpha_1 CPU_t + \alpha_2\ \Delta YUD_t + \alpha_3 INV_t + \alpha_4 GOV_t \tag{7a}$$

$$\phi_3(MP_t) = \beta Y_t = \beta_1 CPU_t + \beta_2\ \Delta YUD_t + \beta_3 INV_t + \beta_4 GOV_t \tag{7b}$$

$$\phi_2(MP_t) = \gamma Z_t = \gamma_1 (CPU_t - \overline{CPU})^2 + \gamma_2 (\Delta YUD_t - \overline{\Delta YUD})^2 + \gamma_3 (INV_t - \overline{INV})^2 + \gamma_4 (GOV_t - \overline{GOV})^2 \tag{7c}$$

As a result 6 becomes for i = 1, 2 and 3:

$$P_{1t} = \frac{e^{\alpha Y_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Z_t}} \tag{8a}$$

$$P_{2t} = \frac{e^{\gamma Z_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Z_t}} \tag{8b}$$

$$P_{3t} = \frac{e^{\beta Y_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Zt}} \tag{8c}$$

where $Y_t = (CPU_t, \Delta YUD_t, INV_t, GOV_t)'$, $Z_t = ((CPU_t - \overline{CPU})^2, (\Delta YUD_t - \overline{\Delta YUD})^2, (INV_t - \overline{INV})^2, (GOV_t - \overline{GOV})^2)'$

and where the elements of Y_t are defined above with equation (4). Clearly, Z_t is a vector of the squares of the difference of the variables in Y from their means.

The forms of equation 8 deserve some comment. Clearly, it is expected that $\partial P_{1t}/\partial MP_t > 0$ and $\partial P_{3t}/\partial MP_t < 0$. As a result and in light of the discussion preceding equation 4, my priors in 8a) are: $\alpha_1 > 0$ (increased capacity utilization rates increase market power and the probability of effecting collusive monopolistic pricing); $\alpha_2 > 0$ (the greater the short run increase in demand(ΔYUD) the greater the market power, etc.); $\alpha_3 < 0$ (the greater the "overhang" of inventories, the less the market power and the probability of effecting collusive monopolistic prices); $\alpha_4 < 0$ (the greater the overhang of government stockpiles, the less the market power etc.). In equation 8c) the priors are reversed; increased market power decreases the probability that the oligopoly is forced to average variable cost pricing. Hence, I believe that $\beta_1 < 0$, $\beta_2 < 0$, $\beta_3 > 0$, $\beta_4 > 0$.

Equation 8b) expresses the nonlinear relationship of P_{2t} to market power (MP_t). As market power increases above "normal" levels, or decreases below "normal" levels, P_{2t} declines. Only when market power is near "normal" levels, will the normal full cost pricing solution occur. I have specified this divergence from "normality" as the squared differences from the mean (over the sample period) of the constitutent variables in Y_t. The priors for 8b) are γ_1, γ_2, γ_3, $\gamma_4 < 0$; as market power diverges from normal levels, P_{2t} declines.

The form of equations 8a-8c is identical to that of

$$P_{1t} = \frac{e^{\alpha Y_t + \alpha' Z_t}}{\Sigma e^{\phi j(Y_t,\ Z_t)}} \tag{9a}$$

$$P_{2t} = \frac{e^{\gamma' Y_t + \gamma Z_t}}{\Sigma e^{\phi j(Y_t,\ Z_t)}} \tag{9b}$$

$$P_{3t} = \frac{e^{\beta Y_t + \beta' Z_t}}{\Sigma e^{\phi j(Y_t,\ Z_t)}} \tag{9c}$$

where α', γ', β' are constrained to be zero. Equations 9a-9c are the general multinomial logit formulation.

Likelihood techniques are used here and utilizing equations 8a-8c the likelihood function 2 becomes:

$$L = \prod_{t=1}^{\tau} L_t = \prod_{t\epsilon\tau_1} P_{1t} \prod_{t\epsilon\tau_2} P_{2t} \prod_{t\epsilon\tau_3} P_{3t} = \prod_{t\epsilon\tau_1} \frac{e^{\alpha Y_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Z_t}} \quad (10a)$$

$$\prod_{t\epsilon\tau_2} \frac{e^{\gamma Z_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Z_t}} \qquad \prod_{t\epsilon\tau_3} \frac{e^{\beta Y_t}}{e^{\alpha Y_t} + e^{\beta Y_t} + e^{\gamma Z_t}}$$

subject to the same constraints.

The more generalized formulation resulting from not imposing the constraints on α', β' and γ' (i.e., using equations 9a-9c) is

$$L = \prod_{t\epsilon\tau_1} P_{1t} \prod_{t\epsilon\tau_2} P_{2t} \prod_{t\epsilon\tau_3} P_{3t} = \prod_{t\epsilon\tau_1} \frac{e^{\alpha Y_t + \alpha' Z_t}}{e^{\alpha Y_t + \alpha' Z_t} + e^{\beta Y_t + \beta' Z_t} + e^{\gamma' Y_t + \gamma Z_t}}$$

$$\prod_{t\epsilon\tau_2} \frac{e^{\gamma' Y_t + \gamma Z_t}}{e^{\alpha Y_t + \alpha' Z_t} + e^{\beta Y_t + \beta' Z_t} + e^{\gamma' Y_t + \gamma Z_t}} \quad 10b)$$

$$\prod_{t\epsilon\tau_3} \frac{e^{\beta Y_t + \beta' Z_t}}{e^{\alpha Y_t + \alpha' Z_t} + e^{\beta Y_t + \beta' Z_t} + e^{\gamma' Y_t + \gamma Z_t}}$$

This section has formally equated market power with the probability that an oligopoly (the U.S. primary producers of copper) can impose desired pricing regimes upon the domestic market in the face of constraining market conditions and government response (through strategic stockpiles of copper).

The estimation of the parameters of 10a or 10b will indicate how market conditions, inventory positions, and governmental action affect the market power of an oligopoly by affecting its ability to impose those alternative pricing regimes (1, 2 and 3).

C. SOME EMPIRICAL RESULTS

The technique utilized to estimate the model parameters translates the generalized multinomial logit formulation of Section B into the usual choice problem.[1] The reader interested in the details of the maximum likelihood technique should consult the relevant documentation.[2] The technique utilizes the convexity of the conditional logit function to converge quickly through a Newton-Raphson algorithm if a maximum exists.[3] The existence of the maximum is virtually certain in samples of more than 10-20 observations.[4] However, as discussed below, the need for 10-20 observations generated convergence difficulties given the limited size of my sample.

Equation 10b is the likelihood function initially utilized. The purpose of estimating the general likelihood function is to test the hypothesized constraints: $\alpha' = \beta' = \gamma' = 0$.

The parameters of the system 9a-9c are identified to a normalization. The program utilized for the estimation normalizes one set of coefficients--(α, α'), (β, β') or (γ', γ)--to zero. I have normalized (α, α') to zero.[5]

The data is detailed in Table 2. The variables have been defined in Section B; they are redefined in the notes to Table 2. Since the variables in the Table were fundamental to a more detailed analysis of the U.S. copper industry, they are discussed in greater length in the sources mentioned above.[6] Y_{2t}, the level of U.S. industrial production in year t, and Y_{4t} the level of government stockpiles of copper at the beginning of year t are treated as exogenous in that analysis. The level of final product inventories at the beginning of year t, Y_{3t}, is a predetermined variable for year t. The capacity

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[1] For an analysis of the choice problem see T. Domencich and D. McFadden, Urban Travel Demand, A Behavioral Analysis (North Holland/American Elsevier, 1976).

[2] The computer program utilized has been developed by C. F. Manski. See "The Conditional/Polytomous Logit Program: Instructions for Use," an unpublished mimeo (Carnegie-Mellon University, 1974). For a greater discussion of the choice problem, see T. Domencich and D. McFadden, op. cit.

[3] Manski, op. cit.

[4] Domencich and McFadden, op. cit., p. 111.

[5] See Manski, op. cit., Domencich and McFadden, op. cit., and Nerlove and Press, op. cit. The normalization does not matter; I have chosen (α, α') because $P_1 = 1$ only once. See Table 1.

[6] To wit, ADL, op. cit., and Hartman, op. cit.

utilization rate for the full-cost pricing solution for year t, Y_{1t}, is an endogenous variable determined by demand, production capacity and costs of the price setting oligopoly (the primary copper producers) in year t. $Z_1 - Z_4$ are defined in terms of $Y_1 - Y_4$ and their means.

The discussion of this paper has referred specifically to analyses of the U.S. copper industry. The P_{it} are determined by the historical simulation results of those analyses.[1] $P_{it} = 1$ or 0 is determined by observing the squared prediction error for parametric solutions 1, 2 and 3 from the actual historical values. $P_{it} = 1$ when the sum of the squared prediction errors for all endogenous variables in parametric solution i were smallest for year t. That is, $P_{it} = 1$, $P_{jt} = 0$, $j \neq i$ when $\sum_{k=1}^{K} (\hat{Y}_{jkt} - Y^A_{kt})^2$ is minimized for $j = i$, where $\hat{Y}_{jkt}$ is the predicted value of endogenous variable k[2] in year t under parametric solution j. Y^A_{kt} is the actual value of endogenous variable k in year t. As seen in Table 2, parametric solution 1 provided the best estimate of the actual variables in 1974; the full-cost pricing parametric solution 2 provided the best estimate in 1967, 1968, 1969, 1972 and 1973; and parametric solution 3 (P = AVC) provided the best estimate in 1964-1966, 1970, 1971 and 1975.

Clearly, the assignment of probabilities in this fashion is arbitrary. It implies that in the year t when $P_{it} = 1$, parametric solution i provides the best central tendency for prediction. It is equally true the actual solution could be in the tail of parametric solutions j, $j \neq i$. However, the purpose of this effort is to indicate how market forces constrain and affect oligopoly pricing. The arbitrary assignment of probabilities in this fashion merely positions the actual solution along the continuum of solutions between parametric solutions 1 and 3. The functional relationship of that position to constraining market conditions will indicate how those constraints affect market power, i.e., the probability of imposing desired pricing/production strategies.

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[1] Hartman, op. cit., and ADL, op. cit.

[2] In the full model of the U.S. copper industry the endogenous variables include five demands for refined copper, three sources and four price variables. See Hartman, op. cit., and ADL, op. cit.

The data in Table 2, particularly the P_{it} resulted from detailed historical simulation of the U.S. copper industry that drew heavily from engineering cost estimates for the primary producers across four stages of production over the 12 years, 1964-1975. As a result, it would require much effort to expand the sample in Table 2 if accurate estimates of the P_{it} and Y_{1t} were sought for earlier years. This is mentioned because the 12 observations for 1964-1975 restricted the empirical analysis as is discussed below.

After normalization, initial attempts to estimate the 16 parameters β, β', γ' and γ were nonconvergent. The sample is just not large enough to support estimation.[1] Restricting $\gamma' = \beta' = 0$ and estimating β and γ led to convergent estimates which are given in Table 3. However, given the small sample size for estimating eight parameters, the variances of the estimates are extremely large in equation A1. None of the parameters are significantly different from zero; however, one can reject H_0: all parameters = 0 (see below). The signs of $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$ and $\hat{\beta}_4$ are all as hypothesized. As market power increases with higher capacity utilization rates (Y_1) and higher levels of demand (Y_2), the probability declines that parametric solution 3 (P = AVC) will be imposed (i.e., $\hat{\beta}_1 < 0$, $\hat{\beta}_2 < 0$). Likewise, the greater are the overhangs of private sector copper stockpiles (Y_3) and government stockpiles (Y_4), the smaller will be the market power of the primary producers and they will be less capable of imposing parametric solutions 1 or 2 ($\hat{\beta}_3 > 0$, $\hat{\beta}_4 > 0$).

It was hypothesized in Section B that γ_1, γ_2, γ_3 and γ_4 are all less than zero. In Table 2 only $\hat{\gamma}_4 < 0$; however, $\hat{\gamma}_1$, $\hat{\gamma}_2$ and $\hat{\gamma}_3$ are not significantly different from zero. Such evidence may suggest that it is either inappropriate to relate the probability of imposing the full-cost pricing regime to squared deviations of $Y_1 - Y_4$ from sample means or that the sample size is too small. The inappropriateness of the Z_i seems to be particularly true for Y_2 which increases relatively monotonically.

Table 3A presents several other sets of results with combinations of the β_i and $\gamma_1 - \gamma_3$ constrained to be zero. Also presented in Columns B are results when absolute changes (ΔYUD) in the index of industrial production are utilized for Y_2. As can be seen, as the number of estimated parameters is reduced,

[1] The problem is numerical. With such a small sample, the Hessian becomes singular preventing the use of the Newton-Raphson algorithm.

the significance of estimates rise. Furthermore, in the logit formulation, the Newton-Raphson estimation process is sensitive to multicollinearity; this sensitivity becomes severe in small samples. In the sample Y_1 and Y_2 are collinear given the fact that they both attempt to represent market conditions in each year. Equations A3-A5 document parameter estimates suppressing one market condition variable (Y_1 or Y_2) and one inventory overhang variable (Y_4 is used). Furthermore, elements of $\gamma_1 - \gamma_3$ are suppressed. As a result, the variances of the parameter estimates in equations A3-A5 are lowered considerably. Again, the estimates for β_1 and β_2 in A3-A5 indicate that conditions of excess demand (Stigler's "urgency of purchase") decrease the probability that the oligopoly will price at AVC. The presence of government stockpiles decrease the market power of the oligopoly and increase the probability that the AVC solution obtains. Likewise, as government stockpiles diverge from "normal levels" the probability parametric solution 2 will obtain are decreased. In words, as government stockpiles are above (below) normal, the probability that average variable cost pricing (collusive monopolistic pricing) will obtain increases.

In Table 3B, where changes in demand (ΔYUD) are used to proxy market conditions rather than levels of demand (i.e., YUD), the significance of β_2 and β_4 are near the 90% level. Again when equations B are estimated with both β_1 and β_2, the data is too collinear to obtain convergence.

For all equations in Table 3 a likelihood ratio test for H_0: all parameters = 0 can be rejected consistently at levels around 90% and sometimes above 95%.

It should be noticed in Table 2 that historical simulation suggests parametric solution 1 was obtained only once, in 1974. Of course, we could treat the actual solution for 1974 as being in the tail of parametric solution 2. In that case, all solution values can be conceived of as being either full-cost pricing (parametric solution 2) or average variable cost pricing (parametric solution 3). As a result, we have a binary logit model where the probability that parametric solution 2 is imposed is a positive function of market power (i.e., $\partial P_2/\partial Y_1 > 0$; $\partial P_2/\partial Y_2 > 0$; $\partial P_2/\partial Y_3 < 0$; $\partial P_2/\partial Y_4 < 0$) and the probability that parametric solution 3 is imposed is a negative function of market power (i.e., $\partial P_3/\partial Y_1 < 0$; $\partial P_3/\partial Y_2 < 0$; $\partial P_3/\partial Y_3 > 0$; $\partial P_3/\partial Y_4 > 0$).

This reinterpretation implies that as the market power of the oligopoly increases, it can obtain its full targeted rate of return P = ATC and sometimes more (in 1974). Likewise, as market power declines, the oligopoly has less power to maintain its targeted rate of return and P = AVC becomes the pricing regime forced on it by market conditions and inventory positions (private sector stocks and particularly strategic government stockpiles). This reinterpretation also reduces the number of parameters to be estimated.

The results of the binary logit treatment in Table 4 agree with those in Table 3; however, the standard errors and t statistics for (H_0: parameter = 0) are more respectable. Again in this case, Y_1 and Y_2, where Y_2 is changes in industrial production, are too collinear to permit convergent estimates for $\hat{\beta}_1$ and $\hat{\beta}_2$ together. However, since β_1 and β_2 are both proxying the same phenomenon, estimates of a differentiated effect is unnecessary.[1]

In equations 1 and 2 we see that $\hat{\beta}_1 = \partial P_3/\partial Y_1 < 0$ and $\hat{\beta}_3 = \partial P_3/\partial Y_3$ and $\hat{\beta}_4 = \partial P_3/\partial Y_4$ are greater than zero. Hence, for the binary case, $\partial P_2/\partial Y_1 > 0$ and $\partial P_2/\partial Y_3$ and $\partial P_2/\partial Y_4$ are less than zero. In equations 3 and 4 we see that $\partial P_3/\partial Y_2 < 0$ and $\partial P_3/\partial Y_4 > 0$ while $\partial P_2/\partial Y_2 > 0$ and $\partial P_2/\partial Y_4 < 0$. H_0: all parameters = 0 can be rejected at acceptable levels; in equations 3 and 4 it can be rejected above 95%.

D. CONCLUSION AND ECONOMIC INTERPRETATIONS

The discussion in Section A indicated that while movements along a supply curve (the horizontal summation of marginal cost) provide a useful summary of pricing and production decisions/behavior in a workably competitive sector or industry, they are not useful for an oligopoly. The reason is that not only costs but also the discretionary pricing decisions of the oligopolists must be analyzed. An alternative model based upon common oligopoly costs and movements along the expected demand curve was posited. To that end, Section A introduced three potential pricing/production strategies or regimes along the demand curve for an oligopoly: MR=MC, P=ATC; and P=AVC.[2]

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[1] See Domencich and McFadden, op. cit., Chapter 5.

[2] Of course, other parametric solutions can be specified such as P=MC. They can be estimated and included in a probability model formulation.

The ability for a group of oligopolists to impose a pricing/production strategy will be restricted by its market power. While an oligopoly would like to impose MR=MC and may be satisfied with P=ATC, there will be times when market conditions may force that oligopoly to P=AVC. The market power (ability to impose desired prices) is described in Section B to be determined by market conditions (excess demand, high capacity utilization vs. excess supply, low capacity utilization), inventory "overhangs," government actions (anti-trust, stockpiling activities) and limit pricing considerations, to name a few. To ease empirical work, three factors were identified: market conditions as proxied by levels and changes in demand (index of industrial production) and by capacity utilization rates, stockpiles of inventories held by producers and users of copper, and strategic government stockpiles of copper.

In Section C it was found that the probability of the occurrence of a parametric solution, or the market power of the oligopolists to impose a particular pricing/production strategy, was indeed related to market conditions. The results indicate particularly how market conditions and government stockpiling affect the copper oligopolies' pricing/production policies. For example, as market power increased due to excess demand and high capacity utilization, the probability decreased that the oligopoly would be forced to P=AVC, while the probability increased that P=ATC (Table 4) or that MR=MC or P=ATC (Table 3). Likewise, when strategic government stockpiles were low, market power increased with the same probabilistic effects: the oligopoly could impose P=ATC or MR=MC (Table 3) or P=ATC (Table 4). On the other hand, when market power declined due to market conditions and government stockpiling activities, the probability that the oligopoly would be forced to price at average variable cost increased.

These results differ crucially from a competitive model because they suggest that the oligopoly's pricing/production decisions are not summarized by movements along the supply (marginal cost) curve. On the contrary, the oligopoly's pricing/production decisions lie along the oligopoly demand curve (Table 1). Where along that demand curve they occur depends upon the costs of the oligopoly (hence, parametric solutions 1, 2 and 3), the pricing/production targets of the oligopoly, and crucially upon the ability of the oligopoly to effectuate those goals (as parametrized by the estimates in Tables 3 and 4).

Given the fact (in Table 2) that parametric solution 1 seldom provides a reasonable approximation of the primary copper producers' pricing/production decisions, let us utilize the binary strategy case in Table 4, equation 1, to give some economic interpretation to the discussion. Recall that in the binary strategy case, the full-cost pricing and average variable cost pricing strategy are utilized to bound the pricing/production decisions of the oligopoly. In Table 5 the binary pricing/production strategies are full-cost pricing (P_2) and average variable cost pricing (P_3). Clearly, the primary producers' desire to impose the full-cost pricing strategy in all years, in the binary case. However, their ability to effectuate those desires is constrained by market conditions. The market power (i.e., the probability of imposing the desired pricing/production strategy) increases from P_2 = 49% when market variables (capacity utilization, primary producer stocks and government stockpiles) are at their mean levels. If we were to take an expectation of the pricing/production levels utilizing P_2 and P_3 at the market variable means, we would find that in average years the primary producers, as a group, are not attaining their desired rate of return on investment incorporated into P=ATC. As capacity utilization increases 5% above the mean and copper stockpiles are lowered 5% below the mean, market power increases in that P_2 rises from .49 to .57. When these market conditions move 10%, the market power (i.e., P_2) increases from .49 to .64. In 1973, capacity utilization at the full-cost pricing strategy was 94.3%, and primary producers' stocks and government stockpiles of refined copper were at relatively low historical levels (49,000 and 217,800 short tons, respectively--see Table 2). In light of the discussion in the paper, market power should have been quite high in this year. Using the values, P_2 = .89 for 1973, P_2 was even higher in 1974.

On the other hand, when slack capacity exists and government stockpiles are high, P_3 rises from .51 at the means to .58 at 5% deviation from the means and to .65 at 10% deviation from the means of the market variables.

TABLE 2: DATA FOR INITIAL ESTIMATES

Year	Y_1	Y_2	Y_3	Y_4	Z_1	Z_2	Z_3	Z_4	P_1	P_2	P_3
1975	.717	78.0	97.45	0	.000144	46.10	41.09	120,721.5			1.0
1974	.792	100.0	194.9	35.3	.003969	231.34	10,786.9	97,437.62	1.0		
1973	.943	101.08	49.1	217.8	.045796	265.36	1,758.96	16,809.12		1.0	
1972	.620	89.81	157.4	251.6	.011881	25.20	4,403.65	9,187.22		1.0	
1971	.468	82.35	103.0	251.6	.068121	5.95	143.04	9,187.22			1.0
1970	.640	84.09	160.6	253.4	.007921	.49	4,838.59	8,845.4			1.0
1969	.929	91.14	45.9	253.4	.04	40.32	2,037.62	8,845.4		1.0	
1968	.856	87.41	56.6	261.4	.016129	6.86	1,186.11	7,404.60		1.0	
1967	.599	82.85	55.4	275.2	.0169	3.76	1,270.21	5,220.06		1.0	
1966	.640	82.02	65.7	452.4	.007921	7.67	642.12	11,014.50			1.0
1965	.785	73.32	60.8	897.4	.003136	131.56	914.46	302,445.			1.0
1964	.758	65.45	45.6	1,019.9	.000841	374.04	2,064.79	452,189.			1.0
$\overline{Y}_i$	.729	84.79	91.04	347.45							

Y_1 is the estimated capacity utilization for the full-cost pricing solution for year t.

Y_2 is the level of U.S. industrial production in year t.

Y_3 is the level of final product inventories of the oligopolistic primary producers at the beginning of year t (in 1,000 short tons).

Y_4 is the level of government stockpiles of copper at the beginning of year t (in 1,000 short tons).

$Z_{1t} = (Y_{1t} - \overline{Y}_1)^2$

$Z_{2t} = (Y_{2t} - \overline{Y}_2)^2$

$Z_{3t} = (Y_{3t} - \overline{Y}_3)^2$

$Z_{4t} = (Y_{4t} - \overline{Y}_4)^2$

P_{it} = 1 if parametric solution i is assumed to hold in year t.

Scaling of variables as follows: $Y_1 = Y_1*10$; $Y_2 = Y_2/10$; $Y_3 = Y_3/10$; $Y_4 = Y_4/100$; $Z_1 = Z_1*100$; $Z_2 = Z_2/100$; $Z_3 = Z_3/100$; $Z_4 = Z_4/10000$.

TABLE 3: PARAMETER ESTIMATES

	A) Table 1 Data					B) Table 1 Data and Alternative Y_2	
	A1	A2	A3	A4	A5	B1	B2
β_1	-.45816 (1.166) (.39)	-.16924 (.72) (.24)		-.20024 (.18) (1.11)			
β_2	-.024176 (1.309) (.018)	-.1388 (.77) (.18)	-.10248 (.14) (.73)		-.13543 (.14) (.97)	-.32051 (.197) (1.63)	-.27969 (.197) (1.42)
β_3	.21867 (.298) (.73)	.1089 (.19) (.57)					
β_4	.91293 (.96) (.95)	.58975 (.49) (1.204)	.45504 (.41) (1.11)	.60539 (.45) (1.35)	.53306 (.45) (1.18)	.60578 (.35) (1.73)	.54232 (.328) (1.65)
γ_1	.66426 (.647) (1.03)						
γ_2	12.282 (14.4) (.853)	5.0884 (7.2) (.706)	6.0329 (6.97) (.87)			.010202 (.011) (.93)	
γ_3	.11158 (.079) (1.41)						
γ_4	-4.4086 (4.01) (.25)	-1.3134 (1.7) (.77)	-1.5507 (1.67) (.93)	-.1602 (.195) (.82)	-.15329 (.195) (.786)	-.10656 (.28) (.38)	-.04318 (.17) (.254)
$-2 \text{ Log } \lambda$	14.812	9.7808	9.14	6.114	5.59	9.93	9.18
Reject: H_0: All Coefficients = 0	∿93%	∿90%	∿95%	∿90%	∿90%	Above 95%	Above 95%

NOTES: Standard errors in first parentheses; t statistics for H_0: parameter = 0 in second parentheses; $\lambda = \frac{Lo}{L}$ where Lo is the value of the likelihood function for H_0: All parameters = 0.

TABLE 4: BINARY LOGIT TREATMENT

	1	2	3	4
β_1	-.41425 (.268) (-1.54)	-.2555 (.17) (-1.48)		
β_2			-.39118 (.252) (-1.55)	-.2975 (.199) (-1.49)
β_3	.10749 (.125) (.86)		-.19565 (.189) (-1.03)	
β_4	.59664 (.377) (1.58)	.55204 (.395) (1.39)	.88626 (.654) (1.35)	.43553 (.289) (1.50)
$-2 \text{ Log } \lambda$	4.9652	4.1652	8.3834	5.8312
Reject H_0: All Coefficients = 0	∿83%	∿90%	∿96%	∿95%

NOTES: Standard errors in first parentheses; t statistics for (H_0: parameter = 0) in second parentheses;

$\lambda = \frac{Lo}{L}$ where Lo is the value of the likelihood function for (H_0: All parameters = 0).

TABLE 5: THE EFFECTS OF MARKET CONDITIONS AND GOVERNMENT STOCKPILES UPON THE MARKET POWER OF THE U.S. PRIMARY PRODUCERS

Equations defining market power (i.e., probability of being able to impose desired pricing/production strategy given market conditions): Table 4

$$P_3 = \frac{e^{\beta Y}}{e^{\beta Y} + e^{\alpha Y}} = \frac{1}{1 + e^{(\alpha-\beta)Y}} = \frac{1}{1 + e^{.41425Y_1 - .10749Y_3 - .59664Y_4}}$$

$$P_2 = 1 - P_3$$

Market Variable at Means		Increased Market Power		Decreased Market Power	
		Y_1 5% above mean Y_3 5% below mean Y_4 5% below mean	Y_1 10% above mean Y_3 10% below mean Y_4 10% below mean	Y_1 5% below mean Y_3 5% above mean Y_4 5% above mean	Y_1 10% below mean Y_3 10% above mean Y_4 10% above mean
P_3	.51	.43	.36	.58	.65
P_2	.49	.57	.64	.42	.35

where: Y_1 is the capacity utilization rate for the full-cost pricing solution;
Y_2 is the level of copper inventories of the oligopolistic primary producers;
Y_3 is the level of government stockpiles of copper.