# INSIGHT IN PROBLEM SOLVING: DEVELOPING A NEURAL NETWORK THEORETICAL ACCOUNT OF THE PROCESSES INVOLVED IN ATTAINING INSIGHT

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# ABSTRACT

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Insight has enjoyed the reputation of an elusive phenomenon in psychology and insight problems are very difficult to solve. Only very specific hints concerning their solution have been found to significantly increase the number of problem solvers who are able to solve insight problems. The result of this has been to suggest that insight does not exist, that it is a mysterious phenomenon, or that it is an aspect of problem solving which we have so far failed to understand. Insight in problem solving is investigated from the perspective that the phenomenon needs explanation and it is argued that, while insight has been operationally defined and a clear set of key empirical findings have been established, the conceptual explanation of insight has been largely ignored. It is suggested that a conceptual account of insight is needed so that this aspect of cognitive processing can be incorporated into the main body of cognitive research on problem solving. The current tension in cognitive science and cognitive psychology is examined and it is argued that writing a conceptual account of insight in neural network theoretical terms will not only advance our understanding of insight, but will also reflect on the debate in cognitive theory. This is a result of its status as an aspect of problem solving and as a phenomenon which symbolic theory has so far failed to offer a clear explanation for. A conceptual account of insight in neural network terms is advanced which offers a comprehensive account of the key empirical findings on insight. It is suggested that insight can be understood as the recognition of a pattern to insight problems. Predictions derived from the theory suggest that overcoming the effects of past learning, employing conceptual transfer, and fostering expertise at insight problem solving will significantly facilitate insightful problem solution. These predictions are submitted to experimental testing with 152 participants who are required to solve the nine-dot problem. Facilitation was measured in terms of time to solution and the number of participants who correctly solved insight problems. $\chi^2$ analyses revealed no significant differences in the number of participants who successfully solved the nine-dot problem following interventions to overcome the effects of past learning ($\chi_{(1)}^2 = 2{,}087$, $p>0{,}05$), or to facilitate conceptual transfer ($\chi_{(3)}^2 = 3{,}542$, $p>0{,}05$). Pearson's correlation coefficient revealed no significant correlation in ability to solve insight problems, offering no support for the prediction that it is possible to display expertise at insight problem solving. The findings are not interpreted as a rejection of the viability of the neural network account of insight in problem solving due to the limited number of participants who were able to solve the nine-dot problem in the facilitated and unfacilitated conditions. It is suggested that the neural network account of insight needs further investigation and that the conceptual account offers the most comprehensive account of insight to date.

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# CHAPTER 1

# A JOYFUL FASCINATION WITH COGNITIVE PROCESSING: THESIS OVERVIEW

## 1.1. A joyful fascination with cognitive processing

This thesis emerges out of a joyful fascination with the fact that, not only do people breathe, eat, walk and talk, they actually think! Is it not amazing that there is something going on inside your head right now as you read these words, perhaps hear them in your head, reflect on them, compare them to your own experience and decide that the writer is in the throes of some bizarre, thesis induced mania! You could communicate all of these thoughts and attempt to express the subjective experience of thinking without having the slightest idea of where these thoughts are located, how you have them, or how you express them. You can use them to hold a conversation with your mother, write a shopping list, grade a student's essay, or solve complex and difficult problems.

There are many, many people who can accomplish these feats, these truly amazing feats. These accomplishments have been accorded a mystical status, particularly when people display skills which they cannot explain, such as occurs in the solution of insight problems. Such accomplishments **are** amazing, but they are only mystical because we cannot yet explain comprehensively the process by means of which they are accomplished. There cannot be something mystical about a process which everyone can follow. This thesis hopes to take one tiny, tottering step toward demystifying one aspect of cognitive processing - insight in human problem solving - and reducing it to the sphere of the truly amazing.

## 1.2. A tiny, tottering step

The aim of this chapter is to briefly detail why and how this tiny, tottering step will be taken. This will be achieved by:

1. Briefly detailing the long-standing tradition of interest in cognition and cognitive processing, and thus demonstrating the relevance tradition accords this thesis topic;
2. Framing this thesis against the backdrop of the disciplines of cognitive science and cognitive psychology, the two research disciplines primarily interested in cognitive processing;
3. Highlighting the current theoretical upheaval within these disciplines and the resultant need for work which positions itself within the divide between symbolic theory and neural network theory[i], the two major theoretical frameworks used to explain cognitive processing in cognitive science and cognitive psychology;
4. Drawing attention to the central place of research on problem solving in this upheaval, and the paucity of our knowledge concerning the processes involved in one fascinating aspect of problem solving - insight;
5. Posing the writer's contention that re-writing processes such as insight in terms of neural network theory is not only necessary with respect to the theoretical debate in cognitive science and cognitive psychology, but is also potentially highly beneficial for advancing our understanding of the processes involved in insight;
6. Finally, outlining the chapter structure of this thesis, by means of which a neural network theory of insight will be advanced, and by means of which its central tenets will be tested.

### 1.2.1. Interest in cognitive processing

It is difficult to define exactly what we mean by the terms cognition and cognitive processing, without framing them in terms of a particular theoretical approach to the study of mind. Thus, for the purposes of this overview, Bechtel and Abrahamsen's (1991) broad definition of cognition as a range of mental processing, which includes reasoning, memory, language, perception, and motor control, will be accepted. To define the mental or cognitive processing involved in these cognitive activities, would once more require a descent to the level of a particular theoretical approach to the study of these activities. As one of the primary aims of this thesis is to question the sweeping terms in which traditional approaches to cognition have written such mental activities, by highlighting the potential of re-writing cognitive processing in alternate terms, such theoretically biased definitions must be avoided at this stage.

Cognition and cognitive processing have fascinated philosophers for centuries. As Leiber (1991) points out, it was Aristotle and Plato who offered the fundamental questions and debates with which researchers studying cognition concern themselves today. Researchers in psychology have been conducting the scientific study of human behaviour, and of mind, for approximately the last hundred years.

More recently, this interest in cognition and cognitive processing has become common to philosophers, psychologists, linguists, computer scientists, neuroscientists, biologists, mathematicians, and statisticians, amongst others. This broad interest has led to the development of an umbrella discipline or, in terms used by Bechtel and Abrahamsen (1991), an active cross-disciplinary research cluster, which embraces this pursuit. This umbrella discipline is known as Cognitive Science, and it is comprised of the sub-disciplines of psychology, computer science (or, more specifically, artificial intelligence), linguistics, philosophy, and neuroscience. It therefore seems necessary that a thesis which concerns itself with the study of some aspect of cognitive processing, should reflect on its position relative to the endeavours of cognitive science.

Definitions of cognitive science appear to be fairly uniform. Stillings, Weisler, Chase, Feinstein, Garfield and Rissland (1995) define cognitive science as the science of the mind, and assert that cognitive scientists seek to understand such phenomena as perception, thought, memory, language comprehension, learning, and other mental phenomena. The nature of the mind is accepted as being computational or information processing. This important feature of the orientation of cognitive science, its conception of the mind as an information processing system, is echoed in Kaplan and Simon's (1989) definition of cognitive science as the study of intelligent behaviour or intelligent systems, with particular reference to intelligent behaviour as computation. It would appear that the development of the digital computer not only resulted in what has become perhaps **the** dominant conception of the nature of mind, but also led to the founding of an entire research discipline.

Before briefly considering the impact of the digital computer on theories of mind, it seems important to highlight the fact that researchers in psychology concerned themselves with the nature of cognition and cognitive processing long before the advent of the computer age. Barsalou (1992) delineates concisely the course which this study has followed. It began in the late 19th century with the introspectionists, who attempted to describe the content and composition of conscious experience systematically, as well as the psychophysicists, who were concerned with describing the systematic relations between conscious experience and physical information in the environment. Thus, interest in human thought has an established tradition within **psychology**, which pre-dates its subsumption under the rubric of cognitive science and which operated outside of the realms of information processing.

This tradition continued with the Gestalt psychologists in the early 20th century, though a concurrent theoretical development - Behaviourism - was to dominate psychology from approximately 1910 to 1950, and interest in human thought ceased during this time. It re-emerged with the advent of Cognitivism in psychology during the late 1950's, and Chomsky's critique of the behaviourist position on learning, advances in information theory, and the computer metaphor, all helped lead to the cognitive revolution and the "overthrow" of behaviourism. Indeed, the advent of cognitivism could be seen as a Kuhnian shift in paradigm, since which interest in cognition has been here to stay.

> *Led by a new paradigm, scientists adopt new instruments and look in new places. Even more important, during revolutions scientists see new and different things when looking with familiar instruments in places where they have looked before. It is rather as if the professional community has been suddenly transported to another planet where familiar objects are seen in a different light and joined by unfamiliar ones as well.* [1]
>
> (Kuhn, 1962, p.111).

It could certainly be argued that the shift from behaviourism to cognitivism constitutes such a revolution.

### 1.2.2. Cognitive psychology or cognitive science?

Now that the scope of cognitive science, and the lengthier tradition of cognitive study within psychology has been outlined, it seems necessary to position this thesis in terms of these research pursuits.

[1] It is interesting that this shift in paradigm sounds very similar to the operation of insight as the recognition of a new pattern of relationships. The reader is referred to chapters two and four for an explication of insight, and insight as pattern recognition.

This is a cognitive psychology thesis, and not a cognitive science thesis. This distinction is very important. First of all, cognitive psychology forms part of cognitive science and, arguably, no one discipline can accomplish the goals of cognitive science. Second, the goals of cognitive science and cognitive psychology overlap, but are distinct. Cognitive science seeks to develop machine intelligence, and is thus interested in developing cognitive architectures (which are computer based) that simulate human intelligence. Within this global framework, cognitive psychology seeks to determine what theoretical understandings of the mind apply to human behaviour. As Lycan (1990) points out, the question for cognitive psychology is: given that a computer can do X, Y, and Z, does it do so in the same manner as human beings? It would be impossible for one thesis to address this entire question, and therefore this thesis will tackle one aspect of human behaviour: insight in problem solving. Once we have considered the two major, competing conceptions of computation, and have outlined the choice of insight in problem solving, it will be evident that this is a cognitive psychology thesis, which assumes its place under the general banner of research in cognitive science.

### 1.2.3. Symbolic theory versus neural network theory

Let us now return to a consideration of the impact of the development of the digital computer on theories of mind, and the clear position this provides for this thesis. The development of the digital computer clearly led to the dominant conception of cognition in terms of the operation of computer hardware and computer software - symbolic theory. [Bechtel and Abrahamsen (1991) and Lycan (1990), offer convincing evidence for the domination of symbolic theory. Also, Posner's (1989) tome on cognitive science includes only one chapter which does not couch cognition in terms of symbolic theory.] In these terms, the brain is likened to the hardware, and the mind is framed in terms of the software, which operates by means of the rule-based manipulation of symbols which have their own semantics and syntactics, as does a computer programme.

Recently, cognitive science (and in particular its sub-branches of philosophy and cognitive psychology) has had to contend with a theoretical framework which threatens to challenge symbolic theory for its supremacy in explaining cognitive processing. Indeed, this challenge has been issued in such strong terms that appeals to a further Kuhnian paradigmatic shift are being issued. (See for instance Bechtel & Abrahamsen, 1991). Although the concept of Neural Network processing (the notion that cognition operates on massively connected networks of elementary units that represent objects of thought in some way, by means of their differential relationships and primacies in awareness) has a well-established pedigree[2], it is only recently that neural network theory has enjoyed renewed research interest. This interest is of such magnitude that important features on the face of cognition are rapidly being re-written in neural network terms. It seems far too early, however, to accept the proposition that this upheaval represents a Kuhnian shift in paradigm, and this is certainly not the position adopted within this thesis. Instead, the proposition endorsed by this thesis, is that it is only by re-writing cognitive phenomena in terms of neural network theory, that we can begin to assess the soundness of this theoretical framework. After all, it is not feasible to assert that neural network theory should replace symbolic theory as a conception of cognitive phenomena until its account of these phenomena has been written and evaluated.

### 1.2.4. The place of problem solving and insight

One area of cognition that is at the forefront of the sparring match between symbolic theory and neural network theory, is human problem solving. As Newell

[2]The origins of neural network theory can arguably be seen in associationism as a component of behaviourism, and can clearly be traced as far back as McCulloch and Pitts (1943) paper on cognitive architectures modelled on the network of neurons in the brain, as Bechtel and Abrahamsen (1991) illustrate.

and Simon (1972) point out, for any theory of cognition to be taken seriously, it must be able to account for the empirically well-established features of human problem solving. To further echo this sentiment, Clark (1989) points out that problem solving and scientific creativity (both of which are closely linked to the main topic of study for this thesis - insight) are areas which symbolic theory has claimed as its own. Of substantial significance in this theoretical debate is the clear indication that neural network theory is enjoying great success in offering a theoretical explanation of problem solving, debateably, greater success than symbolic theory (see for instance Peng & Reggia, 1989; Lesser, 1991; Ye & Salvendy, 1991). This, surely, is crucial to the continued development of a rival theory - why should researchers expend valuable research time and money on a theory which cannot offer a more comprehensive account of consistently observed empirical findings than the currently spoken theoretical framework?

One aspect of problem solving which symbolic theory has so far not been able to offer an adequate explanation of, is insight. There is a large group of problems which, it is widely accepted, can only be solved by the application of a process dubbed insight. However, a consideration of the literature clearly reveals that there is no clear consensus concerning what insight is, how it happens, what circumstances precipitate or inhibit insightful problem solution, why there are individual differences in the application and experience of insight, what processes are utilised in manifesting insight or, indeed, whether insight actually exists as a distinct cognitive process.

### 1.2.5. The need to re-write insight in terms of neural network theory

For work that wishes to position itself within the divide between symbolic theory and neural network theory, what better area of investigation could there be than one which falls within the rubric of a traditionally highly important aspect of cognitive theorising, and which has been so poorly accounted for by established theory?

What is most intriguing, is the fact that Simon and Kaplan (1989) identify the origin of network models with Hebb's proposal of a conceptual nervous system - the same work within which he offers a tentative position on insight (see Hebb, 1949). A further intriguing link between neural network theory and insight, lies in the fact that insight originates in Gestalt psychology, and a useful synthesis has been proposed between Gestalt psychology and neural network theory. What also makes the re-writing of insight within neural network terms such an exciting prospect, is the potential which this framework has to offer a process level explanation of insight, and it is precisely the cognitive processing involved in insight which is so poorly understood.

Neural network theory has been enthusiastically embraced for two main reasons: its biological plausibility and its capability of using a parallel hardware environment to conceptualise cognitive processing on a scale remotely similar to human cognition (Bechtel & Abrahamsen, 1991). It embodies the potential for closer approximation of the human mind than we have, until now, enjoyed. Another exciting feature of neural network theory, particularly in the area of insight where the explanation of the phenomenon is so unclear, is the possibility of a process explanation at the level of the microstructure of cognition. In other words, the theory embodies the promise of detailing exactly the cognitive processes that could be involved in attaining insight.

This is especially promising when one considers that people who display insight cannot express **verbally** how they achieved insight. The process seems to be beyond words; in fact, verbalising during insight problem solving actually impedes insight (Stanley, Mathews, Buss & Kotler-Cope, 1989; Schooler, Ohlsson & Brooks, 1993). When one considers that symbols are essentially linguistic structures, the re-writing of insight in neural network terms appears to be a particularly profitable pursuit, not only for our understanding of insight (and this alone would be a worthwhile venture), but also in terms of illuminating the current theoretical sparring match in cognition. As Clark (1989) states, the symbolic model of the mind can be characterised as just the mind's own talk, its explanation of what it is doing. This talk explains much, but not how the mind accomplishes what it does. Neural network theory, as a process level explanation of the mind's work, has the potential to offer an explanation of the manner in which cognitive processes such as insight are achieved. We can only **begin** to assess the importance of this explanation once prominent features on the face of cognition have been re-written in terms of neural network theory.

In terms of the statement that this is a cognitive psychology thesis, this dissertation must address the issue of whether a neural network theoretical account (an alternative to the traditional symbolic form of computation) of the processing involved in attaining insight, offers a viable account of the manner in which a human problem solver displays reaches a problem solution by insight. In this way, the question for cognitive psychology posed in section 1.2.2. (whether the manner in which a computer processes information is the same as the manner in which a human processes information), can, on one small count, be addressed.

The aim of this thesis, therefore, is to attempt a re-writing of insight in problem solving in terms of neural network theory. To clearly illuminate this model of insight, attention will be focused on one particular insight problem which has clearly been more extensively researched and written about than any other - the so-called nine dot problem[3]. To offer support for this conception of insight in problem solving, some of the central tenets of this neural network postulation of insight will

[3]In the nine-dot problem, the problem solver is presented with three straight rows of three dots, and is instructed to connect all nine dots with only four straight lines, without lifting their pen from the paper. Most problem solvers attempt to connect all of the dots by staying within the boundaries suggested by the square shape of the problem presentation, not realising that the rest of the space on the page is also available for drawing lines. It is only by extending the length of the lines beyond this boundary that the problem can be solved, and the problem solver generally realises that this boundary is self-imposed before the problem is solved. This realisation is thought to occur by insight. Please refer to appendix 1 for a copy of the nine-dot problem and its solution.

be subjected to an empirical test with human problem solvers. The viability of a neural network theoretical account of insight can then be examined.

### 1.2.6. Chapter structure

In developing this model, Chapter 2: *Staking Out Insight*, will review literature on insight in problem solving, and will demonstrate the surprisingly clear and consistent nature of the key empirical findings, despite the poor definitions and serious gaps in our current theoretical understanding of insight in problem solving. It will be argued, however, that the literature clearly suggests that insight is the process (or processes) by means of which a problem whose formulation is ambiguous, and for which only the non-obvious conceptualisation is appropriate, is solved. In support of this, it will also be demonstrated that the literature on insightful problem solving clearly suggests that recognition of this non-obvious conceptualisation is paramount, and that the application of the important concept of expertise in problem solving, has not been clearly applied to insight and should thus be explored.

Chapter 3: *The Divide Between Symbolic Theory and Neural Network Theory*, will explore the tensions between the two major theoretical explanations of cognitive processing, will explore the tension between the symbolic and neural network theories of cognition, and will compare and contrast the two theoretical frameworks. This will serve to position this enterprise within the field of debate between these two theories, as well as within the re-writing of cognitive phenomena in neural network terms. The viability of a neural network theoretical account of insight will also be considered.

Chapter 4: *A Neural Network Theoretical Model of Insight in Problem Solving*, represents the development of a neural network explanation of insight in problem solving, with particular reference to the well-utilised nine dot problem. It will be posited that a neural network model of insight in problem solving is particularly worthy of investigation. An attempt will be made to demonstrate that this theoretical model can account for the key empirical findings in insightful problem solving, and it will be suggested that this theoretical model is perhaps more comprehensive than any conception of insight in problem solving posed to date. Some predictions concerning the performance of human problem solvers made by the central tenets of this model will also be outlined, and a means of testing them will be suggested.

Chapter 5: *Method*, will outline the experiment which tested the predictions generated by the theoretical explanation of insight in neural network terms. The experiment is an independent groups design, with six levels of the independent variable. This empirical test involves exposing 152 participants to various problem solving tasks which the theoretical model predicts will facilitate solution of a target insight problem. This target insight problem is the nine-dot problem.

Chapter 6: *Results*, details the findings concerning the human test of predictions made by the neural network model of insight. These results demonstrate the surprising finding that only 14,02% of participants were able to solve the target nine-dot problem following facilitation, and that none of the facilitation conditions led to a significant increase in correct solutions over the unfacilitated problem solving conditions.

Chapter 7: Discussion, explores the implications posed by these findings and will suggest that, although the findings do not support the neural network theoretical model of insight, they cannot be used to refute it. This argument will be based on a theoretical and methodological evaluation of the human test. Conclusions regarding the viability of the neural network account of insight will be drawn, and suggestions will be made for further research.

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[i]There is some debate concerning the appropriate terms to describe these theoretical approaches to characterising cognition. In using symbolic theory, the writer refers to the traditional, information processing, computer metaphor of cognition. By so doing, the writer concurs with Bechtel and Abrahamsen's (1991) conclusion that this theory is epitomised by its approach to cognition as symbol manipulation. The choice of the term neural network theory is less clear cut, though it is selected on the basis that it is more appropriate than the alternatives. The term connectionism is too broad, covering associationism as it is understood in behaviourist terms. Parallel distributed processing refers too specifically to a particular architecture for modelling cognition, in which representations are not only processed in parallel, but are also distributed. Finally, the term neural networks can be understood at a purely biological level. Thus, the term neural network theory is chosen, to suggest a structure of the mind that has its origins in the physical structure of the brain, but is taken to refer to a theoretical understanding of the operation of mind that extends beyond the purely biological level. The reader is referred to chapter 3 for an explication of the neural network theoretical position. These terms will be used consistently throughout this thesis.

# CHAPTER 2
# STAKING OUT INSIGHT

## 2.1 Introduction

> *The persistent lack of a mechanism for insight, linked with the charge that the notion of insight is somehow supernatural, has shackled researchers who would explore this most important of cognitive processes. ... We don't yet understand insight. But to say that we do not yet understand is quite different from saying that the phenomenon is caused by divine intervention or, perhaps worse, that there is no phenomenon.*
>
> Metcalfe (1995, p.x)

As this chapter will illustrate, the above quotation clearly captures the current state of research exploring insight in problem solving: the process (or processes) by means of which insight is achieved have yet to be explicated, and researchers in the field adopt three distinct approaches to the study of insight.

There are those researchers who adopt the premise that no special process which can be characterised as insight exists, and that insight is merely a normal part of some species of problem solving. This premise, however, seems to emerge from a particular theoretical orientation toward cognition and, more specifically, from an emphasis on learning theory. A second approach adopted with respect to the study of insight could be viewed as the mystification of insight. From this perspective, insight is seen as a phenomenon which defies explanation and there is a tendency to hold that insight can only be displayed by a select group of intellectually superior individuals. This point of view emerges from a particular interpretation of the original Gestalt perspective on insight, and seems to be adopted only by some researchers in creativity and scientific discovery in more recent times. A final orientation toward the study of insight can be discerned amongst those researchers who believe that insight does exist, that it is displayed during the solution of a particular class of problems, and that it is a cognitive process which is currently inadequately explained. A critical review of literature on insight in problem solving, which constitutes the work of this chapter, suggests that the first two approaches mentioned above be dismissed, and that the third approach to insight be adopted.

## 2.2 Staking out insight

An integrated review of the literature on insight in problem solving is difficult to conduct, for several reasons. First, although three basic approaches to the study of insight can be identified, they are by no means unitary. Researchers who can be grouped together on the basis of their general approach to the study of insight, still utilise distinctly different formulations of insight. In particular, those researchers who believe that insight exists as a process which we have just not managed to explain as yet (the third approach mentioned above) differ widely in their conception and definition of insight. This leads us to the second reason for the difficulty in conducting an integrated review of the literature on insight: researchers adopt a variety of definitions of insight, most of which are theoretically impoverished. In fact, a case will be made that these definitions should be viewed as purely operational[1]. A further difficulty faced when integrating this literature, is that insight has a very chequered career. Its popularity has waxed and waned with the theoretical upheavals experienced within psychology, and with the various theoretical conceptions within which problem solving has been studied. There has

[1]Although her work is not used directly, the distinction between the operational and theoretical elaboration of insight is similar to Hornstein's (1988) contention that intelligence has been operationally defined within psychology, but that its theoretical explanation has been seriously ignored.

been no consistent perspective from which insight has been viewed, probably because the processes underlying insight are so poorly understood.

These difficulties must be reflected within this review of literature on insight. To accomplish this, the review will be divided into several sections which deal with distinct aspects of the literature on insight, rather than conducting a review in any sort of date or study chronology. Structuring the review in this way will also demonstrate why this thesis works within the approach to insight which characterises it as a part of problem solving which we do not yet understand, as well as demonstrating why it is argued that the definitions of insight in the literature are poor and should only be accepted operationally. It will also make clear the reasons for suggesting that the explanations of insight in the literature are theoretically impoverished, as well as the reasons for proposing that the key empirical findings are sufficiently clear and consistent to support a theoretical explication of insight,

A review of insight in problem solving must begin with a consideration of problem solving in general before distinguishing insight as characteristic of a particular type of problem solving. Thus:

1. a very brief historical overview of problem solving will be conducted, and;
2. definitions and the scope of research on problem solving will be explored.
3. Insight as a special type of problem solving distinct from insight characterised as intuition, creativity, or scientific discovery, will then be considered, and;
4. a brief historical overview of insight will be conducted.
5. Definitions of, and theoretical explanations for insight will then be explored;

6. Following this, the key empirical findings concerning insight in problem solving, with particular reference to empirical findings based on the well-researched nine-dot problem, will be outlined.
7. Methodological considerations raised by these studies will be considered, and;.
8. the perspective on insight which can be drawn from this integrated review, and which will then be utilised as a basis for the development of a theoretical perspective on insight, will be outlined as a conclusion to this chapter.

This review will not be exhaustive - it does not have to be - but it does have to be comprehensive enough to reveal the extent of our knowledge concerning insight in problem solving. The body of research on this topic is not vast, and this is perhaps a further indication of the limited extent of our knowledge concerning the processes involved in insight.

### 2.2.1. A brief historical overview of problem solving

Historical overviews of research on a topic tend to lay bare the influence of theoretical assumptions, and thus allow us to choose with care our perspective on that topic. This overview of research on problem solving will be no different.

The history of research on problem solving can be understood in terms of a pendulum swing between the two conceptions of problem solving Anderson (1993) identifies within the literature. The first conception is that the cognitive activities involved in problem solving can be understood in terms of principles of cognition which are far more general than problem solving; in terms of learning theory, perhaps, where obstacles are overcome by trial and error. The second is that problem solving is fundamental to all higher level cognition.

Learning experiments involving cats escaping from puzzle boxes, were conducted by Thorndike (1898, in Anderson, 1993) at about the same time as the birth of modern psychology. Thorndike concluded that the cats did not display any behaviour which could be characterised as problem solving, and that there was merely a learned strengthening of correct responses. Thus, early psychology viewed problem solving and learning as one and the same.

The pendulum carried out its reverse swing with Gestalt psychologists such as Kohler (1927), Ellis (1938), Maier (1940) and Duncker (1945), who considered problem solving to be one of the most prominent features of consciousness and its explanation to be one of the most important tasks facing psychology.

Behaviourism saw problem solving consumed once more by learning theory, and it was only with the rise of cognitivism that problem solving celebrated its existence once more. This resulted in the authoritative work on problem solving: Newell and Simon's 1972 tome. Since then, problem solving has enjoyed a central place in cognitive science with the assertion that offering a comprehensive account of its features should be the goal of any credible theory of the mind's activity (Newell & Simon, 1972).

### 2.2.2. Defining problem solving and outlining its scope

Many researchers in problem solving seem to believe that individuals reading their research papers understand exactly what problem solving is, as they do not bother to define the central concept with which they are working. It could be suggested that they must, therefore, be utilising a layperson's definition of problem solving, one which any reader would understand. It cannot be because they assume a type of unified field theory toward the study of problem solving, if we accept van Lehn's (1989) proposal that there is no coherent theory of problem solving. While it is true that there appears to be no coherent theory of problem solving, viable theoretical explications have been developed, and there is substantial agreement concerning operational definitions and the scope of problem solving.

Frequent reference will be made in this chapter to the distinction between theoretical explanations and operational definitions, a distinction illustrated by Rosenthal and Rosnow (1991). They define a theoretical explanation, as the definition of a term at an abstract or conceptual level. An operational definition, on the other hand, is the meaning assigned to a term at the level of the empirical conditions or operations needed to measure it. The operational definition, therefore, renders a concept measurable.

#### 2.2.2.1. Toward a unitary definition of problem solving

A lay view of problem solving could be that it is the resolution of something that is difficult to deal with, or that constitutes some kind of impediment to progress. This suggests that a wide range of activity could be characterised as problem solving, and that people could be solving problems in much of their everyday tasks. This view is echoed by van Lehn (1989) who endorses what has become the traditional cognitive science perspective, in the statement that virtually any human activity could be viewed as problem solving.[2] Van Lehn, however, distinguishes this activity from the tasks studied in problem solving research. These tasks take minutes or hours to perform and are made up of externally observable actions, as well as a verbal protocol. The verbal protocol represents the problem solver's talk as they work, and is seen to represent an internal sequence of actions. Thus, the definition of problem solving is constrained by time and by its multi-step nature.

This verbalisable, serial processing falls within the rubric of symbolic theory, and represents part of the traditional position on problem solving in cognitive science.

[2]This could be seen as support for the notion that traditional cognitive science and symbolic theory constitute folk psychology. This notion is explored in chapter 3.

It characterises the approach adopted by much research on problem solving. This can be seen in Kotovsky and Simon (1990), Carlson, Khoo and Yaure (1990), Carlson and Yaure (1990), Elio and Scharf (1990) and Priest and Lindsay (1992), a few examples of researchers whose work has been published in a variety of prestigious cognitive psychology journals. This demonstrates the widespread acceptance of this approach.

Of course, one cannot hope to reflect the symbolic perspective on problem solving, nor indeed define problem solving regardless of theoretical framework, without considering Newell and Simon's (1972) conception of problem solving. In their view, problem solving is the activity of intelligent adults who are confronted by a desired object and do not know immediately what series of actions to perform to acquire it.

What is most interesting, is that the conception of problem solving formulated within neural network, connectionist, and parallel distributed processing research concurs with Newell and Simon's definition (see for instance Peng & Reggia, 1989; Lesser, 1991; Zualkernan & Johnson, 1992). The difference between these two perspectives lies in the theoretical conception of the cognitive processing involved in reaching problem solution and, to a lesser degree, in what is considered to be the appropriate province of research in problem solving.

There is thus agreement in the literature concerning an operational definition of problem solving, as well as many of the features of problem solving and why they occur (as we will see more clearly in the following section). The concept has been explicated beyond an operational definition in terms of problem solution, but there are still differences in the theoretical definitions of problem solving. However, both the symbolic and neural network theoretical perspectives on problem solving have been fairly well developed, as we will consider in more detail in chapter three. What is important to note here, is that research in problem solving is unitary at the operational level and has also seen the development of viable theoretical explanations to account for its phenomena. As we will see in sections 2.2.5 and 2.2.8, this is not the case with insight.

2.2.2.2. Some comments on the scope of problem solving

It now seems appropriate to consider what is regarded as the province of research in problem solving - by sketching an outline of the scope of problem solving - before moving on to distinguish insight as a special type of problem solving.

The most fitting place to begin an outline of the scope of problem solving is probably with Newell and Simon's (1972) work. They suggest that problem solving research should focus on short tasks, that it should be concerned mainly with performance, only a little with learning, and not at all with development, and that it should focus on the integrated activities that lead to problem solution. This brief seems to have been followed for more than 20 years in the research literature.

Within this framework for problem solving, Newell and Simon also suggest that there are three main tasks covered by problem solving: chess, symbolic logic, and cryptarithmetic. This conception of the scope of problem solving has been expanded to cover such tasks as puzzle problems (see for e.g., Kotovsky & Simon, 1990; Falk, 1992; Anderson, 1993), geometrical problems (see Metcalfe & Wiebe, 1987), incremental problems (in Metcalfe & Wiebe, 1987), logico-deductive problems (Best, 1990; Billman & Shaman, 1990; Johnson-Laird & Byrne, 1990; Rips, 1990), physics problems (Hardiman, Dufresse & Mestre, 1989; Elio & Scharf, 1990; Robertson, 1990), and insight problems (Kaplan & Simon, 1990). Some indication of the expansion of scope is given by this list, though it is probably far from comprehensive.

Newell and Simon (1972) also suggest that problems of moderate difficulty are most appropriate for study. This has certainly not remained the case and the consideration of how people solve difficult problems has led to the study of differences in expert and novice problem solving, as well as a renewal of interest in insight problems. As van Lehn (1989) points out, the study of expert - novice differences dominated work on problem solving in the 1980's. (for some indication of the progress achieved here see, Elio & Scharf, 1990; Priest & Lindsay, 1992).

A list of other features of problem solving which have been the subject of intense research since 1972, will serve to complete this very brief outline of the scope of this research are. Apart from a consideration of expertise, modern research on problem solving has concerned itself with questions concerning the use of domain specific knowledge (e.g., Ross, Ryan & Tenpenny, 1989; Bassok, 1990; Carlson & Yaure, 1990), transfer of knowledge from one problem or context to another (Ross & Kennedy, 1990; Reed & Bolstad, 1991; Gorrel, 1993,), the use of analogies (see, Novick & Holyoak, 1991), the use of heuristics (Durnin, 1991), incubation (Smith & Blankenship, 1991), and the difference between problem solving in knowledge rich and knowledge lean domains (e.g., Siegler, 1989). It is apparent from this list that the scope of problem solving has expanded greatly since the early 1990's and that it has seen the application of diverse concepts from cognitive theory.

### 2.2.3 Distinguishing insight as a special type of problem solving

Now that problem solving has been defined and its scope has been outlined, it becomes necessary to distinguish what we mean by insight in problem solving as opposed to insight as it is used in other contexts. This will involve delimiting the concept of insight as it is used in this thesis.

The most common use of the term insight in psychology, is to refer to insight achieved within the therapeutic context. This is not the conception of insight referred to here, and the discussion of insight in this thesis cannot be applied to the therapeutic context.

Insight within psychology can also be taken to imply intuition, creativity, and scientific discovery, as well as insight in problem solving. The final implication is the one investigated by this thesis. Intuition is not usually regarded to be within the realm of mainstream psychology. Creativity implies something novel (Martindale, 1981), whereas insight is unusual, but is not necessarily original. Scientific discovery is the concept closest to the formulation of insight endorsed by this thesis. It incorporates the shifts of paradigm referred to in chapter one and involves the reconception of known information (Jabri, 1988, 1991; Lamb, 1991). However, insight in problem solving is preferred as a topic of investigation for several reasons.

First of all, it could be argued that much of scientific discovery depends on creativity and, as has already been highlighted, creativity and insight are different. Secondly, there is an established research tradition which investigates insight in problem solving and, although there has been some systematic research conducted on scientific discovery (e.g., Lamb, 1991), no such claim can be made for insight in the context of discovery. The proposal of a viable theoretical model for a phenomenon must be based on a reasonably clear set of key empirical findings. This is possible for insight in problem solving, but not for scientific discovery. Finally, there are also severe methodological difficulties in studying scientific discovery. Almost anyone can potentially display insight in problem solving for a researcher to study, but moments of breakthrough in scientific thinking are reserved for individuals regarded as gifted scientists, whose sudden conceptual leaps are often not obligingly displayed for laboratory replication. This also suggests that insight in problem solving and insight in scientific discovery are different, and thus that the application of findings obtained on scientific discovery, for instance, cannot be extended to insight.

For these reasons, this thesis concerns itself with insight as a special type of problem solving. Van Lehn (1989) provides a neat conception of this. He suggests that, if problems are multi-step tasks in which no one step is critical, insight problems are multi-step tasks in which only some steps are crucial and difficult.

The formulation of insight developed in this thesis does not agree with this position entirely, as it does not endorse the connotation that problem solving involves serial processing. However, the distinction does convey the manner in which insight is delimited for this undertaking. To more clearly distinguish insight as a special type of problem solving, a very brief historical overview of this topic is required as well as a more detailed consideration of definitions.

### 2.2.4. A brief historical overview of insight in problem solving

It is to be expected that the popularity of research in insight would wax and wane with interest in problem solving, particularly as we have confined ourselves here to a consideration of insight in problem solving. There are, however, certain interesting differences in the historical fortunes of these two research areas. These historical differences can, once more, be seen in the light of shifts in theoretical perspectives within psychology.

Where research in problem solving can be traced back to the origins of modern psychology, this is not the case with the concept of insight. As Mayer (1995) points out, the theory of associationism (very briefly that the mind comprises pre-established associations between ideas and that thinking one idea causes an individual to think another idea) dominated the conception of problem solving around the turn of the century. He therefore suggests that researchers such as

Thorndike would not have allowed any space for concepts such as insight within their research[3].

Interest in insight in problem solving can be traced back to Gestalt psychologists such as Kohler (1927), Petermann (1932), Maier and Schneirla (1935), Ellis (1938), Duncker (1945), and Kohler (1947). Insight, as defined by Gestalt psychology, was taken to be one of the central features of the Gestalt perception of human and animal mental functioning. Indeed, Ellis (1938) considered the mapping of the processes involved in insight to be one of the most important research tasks in psychology.

The dominance of behaviourism squashed the central place insight had enjoyed within psychology and even Kohler (1947), though he still insisted that there was more to human psychology and thus more to insight than stimulus response learning, conceded that questions concerning the nature of insight no longer seemed to have a place within psychology.

It seems inevitable, though, that with the substantial interest in problem solving provoked by cognitivism, the unresolved debate concerning the existence of insight should surface once more. The most vociferous voices in this era are those of Robert Weisberg, Joseph Alba, and Roger Dominowski (e.g., Weisberg & Alba, 1981a, 1981b; Dominowski, 1981). From approximately 1980 onwards, more work on insight seems to have been conducted than in the previous 65 years. Most of the work conducted in the last fifteen years seems to accept the existence of insight as a process that we still need to explain. This will be more clearly illustrated in the following section, where definitions of insight used in the literature will be examined and where theoretical explanations of insight in the literature will be reviewed.

[3]While it is true that the theory of associationism cannot account for insight - because presentation of a problem will always cause the same response and, therefore, novelty is not really possible - the tendency to learn associations between particular concepts can account for some of the features of insight (such as fixation), as we will see in chapter 4.

### 2.2.5. Definitions of insight, theoretical explanations, and the case of operationalism

The task of this section of the review of literature on insight in problem solving is to consider definitions and theoretical explanations of insight offered by researchers working within the three approaches to the study of insight, identified in section 2.1. It may seem strange to consider definitions and theoretical explanations of a concept before reviewing the empirical findings of studies on insight. There are, however, two reasons for doing so.

First of all, though researchers can be divided into different groups on the basis of the assumptions they endorse in their approach to insight, assumptions which are clearly tied to the definitions of and theoretical explanations for insight which they offer, they are united by the key empirical findings they present. As will be demonstrated, the empirical findings are consistent regardless of the underlying approach to insight. Where differences are offered within the literature they can be accounted for by interpretation or methodological considerations. To illuminate this for the reader, it is necessary to consider the definitions and theoretical explanations within each approach before proceeding to a unified summary of the key empirical findings on insight in problem solving.

The second reason for structuring the review of literature on insight in this way, is because there are very few differences between the definitions and the theoretical explanations offered for insight. What is meant by this is that there is no clear distinction between what has been offered as a definition of insight (which one would expect to be operational in nature) and the theoretical or conceptual explication of insight. The argument which will be presented here is that explanations of insight are theoretically impoverished, that the exploration of insight at the conceptual level is not sound, and that only an operational definition of insight can be accepted from the literature.

Now that the reasons for structuring this section in the shape that the reader will now find it have been made clear, a review of the definitions and theoretical explanations of insight within the three approaches can be conducted. A case will also be made for dismissing the first two approaches as unfeasible and for positioning the theoretical and empirical work of this thesis clearly within the third approach to insight: namely, that insight does exist as a process which as yet remains unexplained.

#### 2.2.5.1. Insight? What insight?

Researchers whose work has been grouped together here, share the perspective that insight as a special process does not exist and that the term itself is useless. This is most clearly demonstrated by the work of Weisberg and Suls (1973), who avoid using the term insight altogether, even though they present a series of experiments investigating what they refer to as Duncker's candle problem. This problem is later referred to by Weisberg and Alba (1981a) as one of the typical so-called insight problems. The remaining work positioned here explicitly states that insight does not exist and that further use of the term is pointless (Weisberg & Alba, 1981a, 1981b, 1982; Lamb, 1991; Weisberg, 1992, 1995).

This assertion concerning the futility of pursuing insight is supported in two ways [and this is most clearly evident in Weisberg and Alba (1981a) where they present their findings from a series of ten experiments]. The first of these is by defining an insight problem instead of defining insight, and the second is by adopting a particular theoretical approach to the study of these problems based on learning theory.

Insight problems can be defined as: difficult; problems which seem to require the application of a particular type of past experience, but in fact cannot be solved by the application of this experience; and as problems which are only solved when appropriate past experience is brought to bear on them (Weisberg & Alba, 1981a). This definition is used within the literature to argue that the so-called insight problems are merely slightly different to the norm, but are solved by the same processes as those involved in any form of problem solving. These are taken to be the application of **past experience** (my emphasis) in a step by step fashion, within the boundaries of particular problem conceptions. In other words, when presented with a problem, the problem solver draws on past experience to categorise the problem and then works systematically through his/her knowledge to reach a solution. If a solution does not occur, past experience is consulted to see whether the original categorisation is in fact correct and, if there is a suspicion that it is not, an alternative categorisation is applied and explored on the basis of past experience, in a systematic manner.

What we see here, in this explanation of the solution of insight and other problems, is a borrowing of concepts from Newell and Simon's (1972) concept of problem solving (the notion of problem domains and the serial search through that problem space, which we will explore in section 2.2.5.3), within a formulation of problem solving that clearly rests on learning theory. Thus, problems are solved only by the application of past experience.

As we saw in the historical overview of problem solving, this approach belongs to the conception of problem solving as explained by far more general principles of cognition, and which has its origins in behaviourism. If this conception has no place for insight it also has no real place for the study of problem solving at all. The importance which we have seen so many researchers since 1972 accord to problem solving as a fundamental aspect of human cognition suggests that we cannot take this approach seriously. Added to this, is the fact that no explanation is given of the manner in which past experience is selected, applied, and analysed, in which alternative past experience is selected, applied, and analysed, or in which conclusions concerning the initial problem are drawn. Also, Metcalfe (1986b) has provided strong evidence which suggests that insightful problem solving does not occur by means of the step by step application of past experience. This explanation of insight based on past experience is clearly theoretically impoverished, and does no more than support the definition of an insight problem.

In conclusion, the approach to insight which suggests the term be abandoned, rests on a theoretical approach which can never account comprehensively for the features of problem solving and which can therefore not be endorsed for the purposes of this thesis. What it does provide, however, is an operational definition of insight. Insight problems are clearly defined, and we can therefore identify problems that can be placed in this class. The solution of these problems must be achieved by whatever processes are required to display insight, as we have clearly rejected the notion that insight does not exist. The solution of one of these problems therefore provides us with an operational definition of insight.

#### 2.2.5.2. A mystical phenomenon

It seems rather strange to accord space within a psychology thesis to any conception of cognitive processing phrased in mystical terms. The inclination is to dismiss such a claim out of hand. However, this approach to the study of insight must be accorded some space for three reasons. First of all, this approach is what characterises the Gestalt perspective on insight and it was, after all, the Gestalt psychologists who pioneered research in problem solving. Secondly, Kohler (1947) has asserted that the claim to a mystical or supernatural phenomenon was not what he intended. Finally, this conception of insight continues to be given voice amongst researchers in more recent times. Seifert, Meyer, Davidson, Patalano and Yaniv (1995) define insight as seeing and understanding the inner nature of things (like problems) clearly, especially by intuition. Intuition is taken to be the immediate knowing of something without the conscious use of reasoning. The mystical conception of insight has also recently been endorsed by researchers interested in creativity and scientific discovery, albeit only in their claims that insight is the province of great scientific and creative minds. This renders insight a seemingly mystical and supernatural feat (e.g., Csikszentmihalyi & Sawyer, 1995). Therefore, let us briefly consider Gestalt definitions and theoretical explanations for insight.

Dominowski (1981) highlights the fact that Gestalt psychologists have referred to insight as a mystical process which somehow precedes solution to a difficult problem and, indeed, this is one of the main criticisms Weisberg and Alba (1981a) level at the use of the term insight. While there does not appear to be a clear statement in the Gestalt literature that insight is mystical, it is true that even when refuting the claim that Gestaltists had rendered insight mystical, Kohler (1947) makes his re-definition of insight mysterious. Here, he defines insight as the direct awareness of determination, where determination refers to the state of one part of a whole being caused by its position within and relation to that whole. This is not dissimilar to his 1927 definition of insight as the appearance of a complete solution with reference to the whole lay-out of the field, a definition endorsed by Tsai (1981a, 1981b), where insight is defined as the perception of relationships between parts of a complex field leading to a sudden solution to a problem. These definitions are more clearly framed in problem solving terms by Scheerer (1963) who defines insight as the proper perception of the requirements of a problem leading to a subjective aha! experience and problem solution, and Gardner (1978) who defines insight as an aha! experience, or a sudden hunch leading to an elegant solution to a problem.

The theoretical explanation offered by Gestalt psychologists for insight is poor. The manner in which determination is recognised, in which perception of the whole field is achieved, in which proper perception of the requirements of the problem are reached, and indeed the means by which a solution is reached, are all left unexplained. This conception of insight may rest on the vital recognition that the whole operates at a level beyond the functioning of its parts, but this entire process remains mysterious. To suggest that this process is considered mystical or supernatural must, however, represent a subjective interpretation, as this is not clearly suggested by the Gestalt literature.

What we can draw from this literature, is further support for our operational definition of insight in terms of problem solution. If proper perception of the problem must occur before insight can be displayed, it means that there must be a tendency to first adopt an improper perception. Thus, these problems lead the problem solver to first view them in one way, a manner in which solution is not possible, before they are viewed in a manner from which solution is possible. This is very similar to the operational definition derived in section 2.2.5.1.

#### 2.2.5.3. Insight ... to be explained

The work which is grouped here may represent different conceptions of insight, but it does share the fundamental assumption that insight as a cognitive process does exist and that it still needs to be properly explained. There is also a general feeling that an account of the processes involved in insight should be sought amongst theories of cognitive processing embraced by cognitive psychology and cognitive science, in particular from aspects of symbolic theory. This is probably a result of the dominant position symbolic theory has enjoyed in cognitive science. It will be demonstrated, by considering definitions and theoretical explanations of insight provided by the work reviewed here, that the explanations offered from within this perspective have so far proved to be inadequate. This is not to suggest that a symbolic theoretical account of insight has been properly written - it has not. However, it will be strongly suggested that this theoretical conception of cognition is not the best place to begin an explanation of insight, given the nature of the key empirical findings on insight and given that the strengths and weaknesses of symbolic theory do not render it particularly amenable to a conception of insight (as we will see in chapter three).

Three definitions of insight are evident in the work grouped here. All three utilise the same theoretical conception of insight, one which does little to advance the formulation of insight beyond the operational level. The first of these definitions is most clearly operational: insight is the process by means of which insight problems are solved (Metcalfe, 1986a, 1986b, Metcalfe & Wiebe, 1987; Schooler, Ohlsson & Brooks, 1993). Insight has also been defined as the cognitive processes involved in the subjective aha! experience during problem solving (Kaplan & Simon, 1990; Davidson, 1995). This definition is also clearly measurable and the processes involved in insight are left unexplained. The third definition of insight rests on James's (1890, in Lockhart, Lamon & Gick, 1988) notion of sagacity.

Sagacity is interpreted as the skill of constructing or selecting appropriate representations (of problem formulations in this case), and thus insight is defined as the achievement of a problem solution following an initial failure of sagacity (Lockhart, Lamon & Gick, 1988; Gick & Lockhart, 1995). This definition of insight is also clearly measurable. It also alludes to the conceptual definition of insight utilised by researchers placed in this group. As well as proposing a definition of insight in terms of problem solution, this definition also refers to the conceptual definition of insight - that insight can be understood in terms of problem representations. Let us now consider the extent of this theoretical conception of insight.

It has been repeatedly stated in the literature that we know very little concerning the processes involved in achieving insight (e.g., Kaplan & Simon, 1990; Metcalfe, 1995). Even amongst work which assumes that insight can be explained, little real theoretical advance has occurred.

Traditional theories of cognitive processing have been drawn on to suggest that, when confronted with an insight problem, problem solvers select a problem domain or representation based on the formulation of the problem. This problem space is then systematically worked through in a serial fashion, in an attempt to solve the problem. When the solution is not found within this representation (remember that this must happen by definition) the problem solver casts around for an alternative representation and begins the solution attempt once more. A period of incubation may precede selection of the correct problem space. Following the selection of the problem representation which contains the solution, the obviousness of this solution strikes the problem solver with such force that it is experienced as an aha! experience. This conception of insight is most clearly stated by Kaplan and Simon (1990).

If this conception is examined more closely, however, it does little to further the explanation of insight beyond the operational level. The processes involved in selecting, searching, and changing representation, as well as assessing possible problem solutions, remain unexplained. Suggestions such as the switch of representation depending on dissatisfaction with the current representation and appropriate cues to guide the search for a new representation (Kaplan & Simon, 1990) do not really advance our understanding of the manner in which insight is achieved. What we see here is an elaboration of the definition of insight - it is selecting a correct problem representation after first selecting an inappropriate one. It is accepted that this has occurred when the problem is solved.

It is possible to offer a counter argument here. This argument would suggest that the detailed explanation of cognitive processes is not the goal of symbolic theory (see Bechtel and Abrahamsen, 1991), and that it is perhaps only necessary to describe the manner in which semantic rules govern cognition. Surely what we have seen as an explanation of insight from within symbolic theory does not even accomplish this. Stating that insight involves the selection of an appropriate problem representation by means of a search constrained by perceptual cues of the problem, hints provided by the experimenter, prior knowledge belonging to the problem solvers, and heuristics hardly lays bare the rule-based fashion in which cognition operates. Also, if it is possible to achieve an explanation of the processes involved in cognitive processing, why should we not attempt to do so? This explanation seems particularly worthy of pursuit, as it could provide us with the potential to teach more people to think insightfully.

In conclusion, we have seen that three approaches have been adopted to the study of insight, two of which seem unacceptable (the notion that insight does not exist and the notion that insight is a mystical process). The approach which suggests that insight is a process which is potentially explainable is the only one which seems compatible with an attempt to understand insight. All three approaches have provided us with an operational definition of insight, one which is stated at the level of the insight problem. In these terms insight can be operationally defined as the process (or processes) by means of which problems whose formulation is ambiguous, and for which only the non-obvious conceptualisation is appropriate, are solved. It will be the work of chapter four to propose a theoretical explanation of the process of insight. Before doing so, it is necessary to outline the key empirical findings concerning insight. It is on the basis of these findings that researchers have proposed the existence of insight as a cognitive process, and it is for these findings which a theoretical explanation of insight will have to account.

### 2.2.6. Empirical findings

It has already been suggested that the key empirical findings on insight in problem solving are consistent across the three approaches to defining and explaining insight theoretically, and that the findings will therefore be integrated in this review. Most of the empirical findings reported here are based on experiments conducted with the nine-dot problem. This is because it has been the most well-researched of all of the insight problems, as well as forming the focus of the debate that saw the renewal of interest in insight in the 1980's (see Weisberg & Alba, 1981a, 1981b; Dominowski, 1981; Ellen, 1982). Where findings are not based on the nine-dot problem, or where this problem produces different data to other insight problems, this will be indicated.

2.2.6.1. Problem difficulty

Problem difficulty is one of the defining features of insight problems. In fact, they are so difficult that the majority of problem solvers do not reach problem solution even when given an extended period of time in which to work on them. For example, Weisberg and Alba (1981a) report that none of their subjects correctly solved the nine-dot problem within 20 attempts, while Lung and Dominowski (1985) report that 9,38% of their subjects correctly solved the problem within the same number of attempts. Kaplan and Simon (1990) present similar findings concerning the difficulty of the mutilated checkerboard problem[4]. In particular, they report on a graduate student in chemical engineering who spent 18 hours working on the problem and still did not manage to solve it.

[4]The mutilated checkerboard problem involves presenting the problem solver with a checkerboard from which two opposing corners have been removed. The problem solver is informed that there are 62 squares remaining on the board, and that they must attempt to cover these squares with 31 dominoes, where a domino can cover two adjacent squares, but cannot cover squares which are on diagonals. The problem solver is required to cover all of the remaining squares on the board with the dominoes, or to prove that it is impossible to do so. The solution to the problem is that it is impossible to cover all of the squares with the 31 dominoes and this rests on the recognition that a domino must cover one black and one white square and as the two opposite corners are the same colour, there are an uneven number of black and white squares left to cover. The reader is referred to appendix 1 for a copy of the problem and its solution.

It has been suggested (Weisberg & Alba, 1981a) that the nine-dot problem is distinct from other insight problems in that, even after the appropriate problem representation is obtained, the problem is still difficult. Indeed, they suggest that the problem is so difficult that even when problem solvers are familiar with the problem and its solution, they struggle to solve the problem. While it is true that there is still some work to do to solve the nine-dot problem (please refer to appendix 1 for a copy of the problem and its solution) once the problem solver realises that solution depends on extending the lines outside the square connoted by the dots, it has not been demonstrated that the processes involved in solving this problem are any different to other insight problems. It is particularly unlikely that this will be demonstrated when one considers that the ambiguous formulation and the need to realise a non-obvious solution, the defining features of an insight problem, are present in the nine-dot problem.

#### 2.2.6.2. Inappropriate problem conception

Linked to the difficulty of insight problems, is the finding which most clearly distinguishes insight problems from non-insight problems - the consistency with which problem solvers adopt an initial conception of the problem which makes it difficult to solve. Indeed, Dominowski and Dallob (1995) stress the invariant nature of this feature of the nine-dot and other insight problems, in defining insight. Also, no studies could be found in the literature which contradicted this aspect of the empirical findings.

#### 2.2.6.3. Persistence

Persistence refers to the tenacity with which problem solvers pursue a problem solution within their initial problem conception. The term persistence is preferred to the term fixation, as fixation implies the Gestalt notion that the problem solver is perceptually stuck on the shape of the square connoted by the nine-dot problem, for. There is evidence that suggests it is not necessarily the shape of the square which impedes problem solution (see Weisberg & Alba, 1981a, 1981b). There is, however, ample evidence to support the notion of persistence, a substantial volume of which is reviewed by Seifert et al (1995). As an example of persistence, Kaplan and Simon (1990) report that in their experiments with the mutilated checkerboard problem, subjects would persist with their initial problem conception until they were told that it was impossible to solve the problem in this way.

#### 2.2.6.4. Appropriate problem conception

In reaching problem solution, persistence is followed by the selection of an appropriate problem conception. It is consistently reported that problem solvers who solve insight problems do so by changing the manner in which they understand the problem requirements. This is seen in the nine-dot problem with the realisation that the space outside of the dots is also available for so-called line-extensions (Weisberg & Alba, 1981a; Lung & Dominowski, 1985). It is also evident with the mutilated checkerboard problem, where Kaplan and Simon (1990) manipulate the salience of the colour of the squares, encouraging subjects to shift their understanding of the problem from the number of squares in total to the number of squares of different colours.

#### 2.2.6.5. Incubation

Problem solution, or appropriate problem conception, is often preceded by a period of incubation. Incubation has been defined as a period of inactivity between active work on the problem and achievement of resolution, during which progress is somehow made nonetheless (Perkins, 1995). The least consistent of the findings reported here are linked to incubation and insight. Dominowski and Jenrick (1972) and Olton and Johnson (1976) failed to find laboratory evidence for an incubation effect, while Dreistadt (1969) and Smith and Blankenship (1991) did, a finding which supports anecdotal evidence (Smith, 1995). The contradictions in findings could be due to differences in method and Smith (1995) provides a strong case for this, while suggesting that incubation effects are very real in producing insight.

2.2.6.6. Facilitation

Research attention has been given to the means by which solution on insight problems can be facilitated. This has involved studying the effects of fairly specific hints for particular insight problems. For example, Weisberg and Alba (1981a) and Lung and Dominowski (1985) consider the effect of dot problems which encourage line extensions, on solution of the nine-dot problem. This involves presenting problem solvers with a problem that requires them to connect dots, but makes the need to draw lines beyond the dots on the page far more obvious than for the nine-dot problem. The four dot problem is an example which places four dots in the shape of an incomplete triangle. Problem solvers are asked to connect the dots with four straight lines, and the triangular shape of the problem facilitates the realisation that the problem can only be solved by drawing lines beyond the dots. It was found that specific hints did facilitate solution of the nine-dot problem. Indeed, the percentage of problem solvers who correctly solved the nine-dot problem rose from 0-9 %, to between 43% and 100%. Kaplan and Simon (1990) also found that drawing attention to the parity of the squares in the mutilated checkerboard problem significantly facilitated solution on that problem.

While the effect of problem specific hints on facilitation of insight problem solutions has been studied, the effect of more general facilitation has not. No consideration has, for instance, been accorded the notion that making explicit the manner in which insight problems prompt an inappropriate problem conceptualisation could facilitate solution. In other words, would facilitation on insight problems occur by telling problem solvers that insight problems are ambiguous and that they require a non-obvious solution? Also, no consideration has been given to the effect of presenting problems whose characteristics require a problem solution that might overcome the assumptions usually brought to bear on a particular insight problem, other than by providing an alternative form of a given problem. This would involve presenting problem solvers with a problem which demonstrates the possibility of using blank spaces on the page, but is not itself a dot problem.

#### 2.2.6.7. Transfer

Linked to the concept of facilitation is the notion of transfer of information one problem to another solution attempt. Lockhart, Lamon and Gick (1988) state that most studies have found no effect of transfer for insight problems. Based on their experimental evidence, they conclude that transfer from one problem to a target problem can only be facilitated when puzzlement has been induced prior to presentation of solution for the transferable problem. They use this finding to suggest that the mere presentation of information content is not significantly transferred, but that the conceptual processing involved in insight problem solving is. Again, there has been no clear study of the notion that the processing of one insight problem can be transferred to another insight problem.

#### 2.2.6.8. Expertise

Linked to this are the findings concerning expertise in insight problem solving. Data is available on the effect of being an expert in a knowledge domain that might be activated by an insight problem. For instance, Kaplan and Simon (1990) found that individuals who were highly proficient in mathematics tended to take longer to solve the mutilated checkerboard problem. This is because they spend more time considering the number of squares in total prior to recognising the implications of the colour of the squares. It is as if this expertise reinforces the initial, incorrect problem representation. What has not been considered, is the effect of expertise in the cognitive processing involved in solving insight problems. Perhaps it is possible to be an expert at solving insight problems, particularly if you are immediately looking for the non-obvious problem conception.

#### 2.2.6.9. Inability to predict success or failure

In a convincing series of studies Janet Metcalfe has found that while problem solvers can predict impending success or failure at solving non-insight problems, they cannot do so for insight problems (Metcalfe, 1986a, 1986b; Metcalfe & Weibe, 1987). What has been found, is that predictions of imminent success usually predict failure. This has been used to provide clear support for the existence of insight as a process which is distinct from ordinary problem solving.

#### 2.2.6.10. Suppression of insight by verbalisation

Two recent studies have provided substantial evidence that verbalising while attempting to solve an insight problem significantly inhibits the ability to achieve insight (Mathews, Buss & Kotler-Cope, 1989; Schooler, Ohlsson & Brooks, 1993). This complements the evidence reported by Kaplan and Simon (1990) which suggests that the crucial cognitive steps taken to achieve insight are not present in the verbal protocol of subjects who solve the mutilated checkerboard problem. When verbal protocols produced by individuals who have solved non-insight problems are given to new problem solvers they can follow the protocol in a step by step fashion and solve the problem themselves. This cannot be accomplished with insight problems. The processes involved in attaining insight are somehow not verbalisable.

### 2.2.7. Some methodological considerations

Now that the key empirical findings on insight have been outlined, it seems necessary to raise some methodological considerations with respect to the studies on which these findings are based.

First of all, the empirical data comes from experimental studies. However, the data are often not interpreted strictly on the basis of the experimental findings. Protocol analysis is used to provide an over-riding interpretation of the data. Verbal protocols are taken to be a report of the subject's own mental states and mental processing. Their analysis is taken as a statement of the cognitive processing involved in tasks such as problem solving (Ericsson & Simon, 1993). Given that the processes involved in insight are not verbalisable, these interpretations must be suspect. It is perhaps necessary to explore the phenomenon of insight without the use of protocol analysis.

Secondly, findings are often based on a limited number of subjects. For example, Kaplan and Simon's (1990) conclusions are based on only 23 subjects and Weisberg and Alba's (1981a) strong contention concerning the lack of evidence for the existence of insight is partly based on the data from only 15 subjects who were required to solve the nine-dot problem in its original format. It seems to be necessary to increase the subject numbers used in studies on insight in problem solving. This will allow us not only to describe the features of insight in more depth.

The limited subject numbers in research on insight seem to be due to the perceived necessity of testing subjects individually. This is most time consuming and costly, and it is therefore understandable that subject numbers for these studies are limited. This should not, however, allow us to ignore the need to base our conclusions on adequate subject numbers and perhaps it is viable to consider testing subjects in groups, particularly if we consider the collection of verbal protocols to be unnecessary in furthering our understanding of insight.

A final striking methodological feature of the empirical studies on insight is the focus on solution attempts to determine success at problem solving. How can one possibly define what constitutes one solution attempt? Surely the time taken to solve a problem, or the failure to solve a problem within an allotted time period, is a far more objective and useful measure to use in the study of insight problem solving. Indeed, many studies take this time in to consideration, but their data are still reported in terms of number of solution attempts. My solution attempt might constitute far more or far less processing than your solution attempt.

Despite these methodological considerations, it must be acknowledged that the empirical findings on insight in problem solving are surprisingly clear and strikingly consistent, particularly when one considers the disparate nature of the conceptions of insight utilised by various researchers. These findings certainly beg the formulation of a theoretical account which hopes to explain the processing involved in insightful problem solution.

### 2.2.8. Chapter conclusions

This review of literature on insight in problem solving has demonstrated the disparate nature of the formulations of insight, as well as the poor theoretical explanations of insight, despite the clear and consistent nature of the empirical findings. This provides support for the notion that we have no adequate understanding of the processes involved in attaining insight. A tenable theory of insight would surely see researchers reaching some degree of agreement concerning a perspective on insight, such as we see in the literature on problem solving. This should particularly be the case as it is not really possible to disagree widely on the empirical findings produced by research on insight in problem solving.

It is therefore postulated that the important work still to be conducted on insight is to propose a theory of insight in problem solving which can account for the empirical findings detailed in this chapter. This theory should also incorporate important aspects of the general literature on problem solving which have not been clearly applied to insight in the past, such as the concept of expertise.

Although no comprehensive theory of any aspect of cognition can be established in one piece of work[5] and theories should always be open to critique and re-writing, this is the work which the rest of this thesis endeavours to tackle. This will by no means be put forward as **the** formulation of insight in problem solving, but it will be tentatively suggested that with this work we take one tiny step closer to the mark.

It now becomes necessary to decide on a theory of cognition, within which to write a conceptual account of insight. After all, any aspect of cognition must fit within a general framework of cognitive processing. The selection of this general framework constitutes the agenda for chapter three.

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[5]It could also be suggested that no adequate theory of any aspect of cognition can be offered until we have a comprehensive theory of the mind or a true artificial intelligence which we can explore piecemeal.

# CHAPTER 3

# THE DIVIDE BETWEEN SYMBOLIC THEORY AND NEURAL NETWORK THEORY

## 3.1 Introduction

> *Almost everyone who is discontent with contemporary cognitive psychology and current 'information processing' models of the mind has rushed to embrace 'the Connectionist alternative'. When taken as a way of modeling* ***cognitive architecture,*** *Connectionism really does represent an approach that is quite different from that of the Classical cognitive science that it seeks to replace. Classical models of the mind were derived from the structure of Turing and Von Neumann machines. ... In contrast, Connectionists propose to design systems that can exhibit intelligent behaviour without storing, retrieving, or otherwise operating on structured symbolic expressions. The style of processing carried out in such models is thus strikingly unlike what goes on when conventional machines are computing some function.*
>
> Fodor and Pylyshyn (1988, pp.4-5)

The review of literature on insight in problem solving which constituted chapter two, illustrated that the theoretical explication of insight is very poor despite both the long history which the study of insight has enjoyed and the clear and consistent nature of the empirical findings on insight. It therefore seems evident that the work still to be conducted in the study of insight in problem solving, is the writing of a viable theoretical account of the processes that lead to insight. As this thesis endeavours to take a tiny step toward this goal by developing a possible theoretical explanation for the processes involved in insight, it becomes necessary to consider how the cognitive processing involved in insight might be explained. Thus, the purpose of this chapter is to outline a general theoretical framework of cognitive processing within which to write a conceptual account of the processes involved in attaining insight in problem solving. At the same time, by considering the current theoretical upheaval in cognitive theory, this work will be positioned within the endeavour of re-writing cognitive processes in neural network theoretical terms.

The quotation with which this section opened clearly demonstrates that there are only two real contenders at present for the title of Most Viable Theory of Cognition. These are: the current title holder, "Classical cognitive science", which we have seen (in chapter one) constitutes Symbolic theory; and the number one contender, "Connectionism" or, in the terminology endorsed by this thesis, Neural Network theory. As this quotation also suggests, though rather less directly, there is a distinct degree of tension between adherents of these two approaches. This tension has led to a theoretical sparring match in cognitive science and cognitive psychology, between the dominant conception of cognition and the would-be usurper. This tension must be examined before a theoretical account of insight can be written.

It is proposed that a consideration of the literature which reflects the current sparring match in cognitive theorising reveals that the dominance of symbolic theory as a theory of cognition is the result of historical processes. Thus, the dominance of symbolic theory as an explanation of cognition can be seen as historically coincidental. This dominance has many theoretical assumptions associated with it which are carried over to the study of cognitive phenomena, and these assumptions have not been seriously questioned. It will also be suggested that the renewed interest which neural network theory is enjoying is not only a result of more recent historical factors, but also reflects a recognition of the need to question the assumptions symbolic theory makes concerning cognition. This is over and above a recognition of the special characteristics of neural network theory. It is from a combination of these factors that the current tension between these two theories seems to emerge.

Although neural network theory brings its own theoretical assumptions to bear on the study of cognitive phenomena, no resolution of this theoretical tension can be contemplated before the potential of neural network theory has been explored. It is therefore proposed that the re-writing of cognitive phenomena in neural network terms is necessary in the light of this theoretical tension, as well as offering the potential of advancing our understanding of phenomena which have been inadequately explained by symbolic theory. One of these phenomena is insight.

## 3.2. The sparring match in cognitive science and cognitive psychology

In order to propose that the re-writing of insight in neural network theoretical terms is necessary and that it will provide a viable account of insight, it is important to understand the source of the tension between symbolic theory and neural network theory. Insight has only really been explored in symbolic theoretical terms within cognitive theorising and an exploration of the tension between symbolic theory and neural network theory will provide some justification for the suggestion that insight can be profitably conceptualised in neural network terms. Examining the tension between these two theories will involve:

1. a consideration of the histories of the two contenders in the cognitive science arena.
2. An elaboration of the symbolic theoretical perspective on cognition and;
3. an elaboration of the neural network theoretical perspective on cognition.
4. A consideration of the differences between these two perspectives and;

5. motivation for the suggestion that neural network theory is an appropriate theoretical framework within which to write a viable account of insight.

Although it is not the purpose of this thesis to suggest that one of these theories should be accepted as **the** account of cognition, it is impossible to remain neutral in the light of this tension. After all, an explanation of a cognitive phenomenon cannot be atheoretical, particularly if we accept Danziger's (1986) contention that we cast our theoretical net over everything we observe. This review of cognitive theorising will reflect this lack of theoretical neutrality, in such literal features as the space given to a consideration of each of the cognitive theories, as well as in the suggestion that neural network theory is an appropriate theoretical framework to conceptualise cognition.

### 3.2.1. A brief historical account of the sparring match in cognitive science

Examining the historical processes by means of which symbolic theory has become enthroned as the dominant conception of cognitive processing will render the theoretical assumptions which this dominance carries with it available for scrutiny. It will be suggested that the dominance of symbolic theory is a result of historical factors associated with the development of the digital computer, the paradigmatic shift from behaviourism to cognitivism and the development of symbolic theory at this time, and the rise and fall of neural network theory. The coincidental nature of these historical factors has gone largely unquestioned and it is, therefore, necessary to explicitly state these historical processes and the related theoretical assumptions symbolic theory inherited. It is only with the return of interest in neural network theory that these assumptions have been questioned, and this has contributed to the current tension in cognitive theory.

#### 3.2.1.1. The dominance of symbolic theory

A clear perspective on the recent historical developments in the study of cognition which led to cognitive science becoming synonymous with symbolic theory, is offered by Johnson-Laird (1988). It begins with the concurrent existence of Behaviourism and Gestalt psychology, with a clear strand being drawn from Gestalt psychology to symbolic theory. Gestalt psychology accorded prime importance to the structural relations in perception and sought laws to explain this structure, rather than conducting an exploration of mental processes.

At the same time in linguistics, however, de Saussure developed the notion of the signifier and the signified. A sign or symbol, usually a word, signified an object in the real world. These two theoretical perspectives came together in the framework of Structuralism and within this framework, Piaget proposed that thought developed from internalising one's own actions. Structuralism, in these terms, was not formal enough to study mental processing. However, this was changed with the development of the digital computer and the rise of information processing theory which it was to herald.

By the mid 1950's, researchers Herbert Simon and Allen Newell had developed programmes to run on digital computers which could conduct logical proofs, something that had been the province of the human mind until then. With the death knell being rung by Chomsky's attack on the behaviourist conception of learning in language acquisition and use, psychology returned once more to the study of the mind. The computer, with its newly developed programmes which could perform some of the functions of human cognition, provided an exciting new metaphor which tied mind and body together in computer hardware and software and which allowed the formal study of mental processes which structuralism had, until then, been unable to provide. The computer operated as a symbol processor, running programmes which provided instructions for locating symbol tokens in various physical locations and processing them in a rule-based fashion. This metaphor was transferred directly to the operation of the brain and the mind, with the brain providing the physical locations upon which the mind could operate as a symbol processor. Therefore, symbolic theory and cognitive theory became one and the same with the development of the digital computer.

This is only one historical perspective on the rise of cognitivism, one offered by a proponent of symbolic theory. Let us go beyond this account of the co-occurence of the invention of the digital computer and the rise of cognitive psychology and consider how these developments led to the theoretical assumptions of this conception of cognition.

The fact that symbolic, computer based architectures could display some of the features of human cognition was taken to mean that this must provide an accurate account of the manner in which human cognitive processing occurs. It is entirely conceivable that, although computer and mind achieve similar end results, they do so in a very different fashion. This is a general criticism of the methods used in cognitive science and we will see that this criticism can also be levelled at neural network theory. However, add to this the fact that the structure of computer hardware is very unlike the structure of the brain and the assumption that cognitive processing must occur in the same fashion as the computer is rendered more open to question. Also, consider what would have happened within cognitive theorising if the computer had been developed in some fashion other than the von Neumann machine, or had not been developed at all. Cognitive theory could possibly have taken an entirely different shape and the supremacy of symbolic theory might never have occurred.

### 3.2.1.2. The rise, fall, and re-birth of neural network theory

Given that the dominance of symbolic theory as **the** conception of cognition seems to be, to a large degree, due to historical factors which saw a re-birth of interest in cognition coincide with the development of the von Neumann computer, it is not surprising that there is currently considerable attention being offered to an alternative conception of cognition. There has also been widespread criticism of the biological and, more specifically, the neural implausibility of the symbolic conception of the mind (see for e.g. McClelland, Rumelhart & Hinton, 1986; Smolensky, 1988). This criticism has led to considerable interest in neural network theory.

The intensive research which this interest sparked off in the second half of the 1980's has led to the current theoretical debate in cognitive science, and proponents of symbolic theory have lined up against what Fodor and Pylyshyn (1988) refer to as the Connectionist alternative. This recent challenge to the cognitive science establishment demands that the history of neural network theory be charted so that its theoretical assumptions are also laid bare for examination. This is necessary to illuminate the tension in cognitive theorising, and so that one theory is not blindly accepted in the place of another.

A consideration of the historical background of neural network theory is offered by Bechtel and Abrahamsen (1991). They suggest that, although the neural network theoretical perspective is not associationist, its roots can be traced back to associationism and thus, in some measure, to behaviourism. The perspective of associationism is based on contiguity: the notion that, because two ideas occur in close proximity, some sort of connection is forged between them and one idea will, in the future, bring to mind the other. Early neural networks based on associationism, were developed by researchers such as McCulloch and Pitts (1947, in Bechtel & Abrahamsen, 1991) during the early years of the cognitive revolution to model pattern recognition (in particular recognition of patterns based on partial information) and memory . These networks were based on statistical rather than logical principles, employed representations, but not symbols, and were self-training. These cognitive models were eschewed, not because their basis in associationism undermined the ground gained by cognitivism as Fodor and Pylyshyn (1988) suggest, but rather because of the absence of powerful mathematical and processing tools upon which to implement them. The combined force of logic, linguistics, and the symbolic, serial processing platform afforded by the digital computer led a focus on symbolic theory at the expense of development in neural network theory.

It seems rather interesting then that interest in neural networks should re-surface with such vigour during the 1980's and Bechtel and Abrahamsen (1991) offer several reasons for this. First of all, they suggest that powerful new approaches to network modelling were developed including new architectures, new training techniques and advances in mathematical descriptions. Secondly, researchers attracted to neural network theory were highly credible and amongst them was a distinguished physicist, John Hopfield. Third, neural network theory offered the opportunity to bring cognitive science closer to neuroscience with a model of cognition which was inspired by the neural structure of the brain. Fourth, this interest in neuroscience was related to a more general concern with parsimony - symbol systems were becoming more diverse, more complex, and more *ad hoc*. Finally, a number of cognitive science researchers had become concerned with the limitations of symbolic theory in particular their limitations in accounting for human behaviour. Rule-based systems were seen to be hampered by their brittleness, inflexibility, difficulty, learning from experience, inadequate generalisation, domain specificity, and inefficiency due to serial searches through large systems.

This criticism of symbolic systems heralds the contrasts between symbolic theory and neural network theory which we will explore in sections 3.2.4 and 3.2.5., but will suffice for now to illustrate some of the reasons for the sudden rise in interest in neural network models. This renewed interest, then, has led to the current sparring match within cognitive science.

Before moving on to a further consideration of the sparring match in cognitive science and thus to the proposal that neural network theory is an appropriate theoretical model of cognitive processing within which to write a viable account of insight, let us examine the theoretical assumptions which this brief historical account of the development of neural network theory has revealed.

The most obvious theoretical assumption is the one accorded by the interest in bringing cognitive science and neuroscience closer together. This suggests that cognitive processing operates in a fashion which is similar to the operation of the brain at the neural level and this is not necessarily the case. It does, however, seem easier to envision cognition following the principles of brain processing rather than following the operation of computer processing.

Another theoretical assumption lies in the notion that neural network theory is an elaboration of associationism. This suggests that the proximity of objects, events, and ideas is enough to establish a learnt relationship between them, a perspective which has clear roots in behaviourism and the concept of essences. Whether we do indeed learn in this manner is open to question and this assumption must therefore merely be acknowledged. To place this in perspective, consider that symbolic theory seems not to have offered an adequate explanation of the manner in which humans learn and that an extension of the principles of associationism seems to have the potential to do so.

A final theoretical assumption can be seen in the use of mathematical and statistical theory to explain behaviour. This is common throughout most of the history of psychology and has, in particular, influenced the nature of traditional research methods used to study human behaviour[1]. It must, therefore, merely be acknowledged as a possible source of criticism of any theoretical conception of psychological phenomena advanced within a neural network theoretical explanation.

Now that a brief historical account of the tension between symbolic theory and neural network theory has been offered, it is necessary to continue our examination of the current sparring match in cognitive science and cognitive psychology by outlining the theoretical conceptions provided by these two approaches to the study of cognition. We begin with a consideration of symbolic theory.

### 3.2.2. An outline of symbolic theory

In this attempt at defining the symbolic theoretical conception of cognition and cognitive processing, the lack of theoretical neutrality in our presentation of cognitive theory will be evident. Less space will be accorded to the outline of symbolic theory, than to the outline of neural network theory, or to the examination of the differences between the two.

There are several reasons for this. First, symbolic theory is traditionally established within cognitive science and cognitive psychology and thus its tenets are better known and more clearly established. This extensive research tradition and theoretical dominance within cognitive theorising (as has already been established in section 3.2.1.) means that it is not necessary to give an elaborate outline of symbolic theory in order to argue the merits of its case as a viable cognitive theory.

[1]See Kurt Danziger (1986) for a convincing argument concerning the manner in which psychological theories have been shaped by a reliance on statistical methodologies.

Second, as the title of this chapter suggests, the focus here is on the differences between symbolic and neural network theory as a means of establishing that neural network theory could provide a more viable framework for conceptualising insight, as well as highlighting the current theoretical debate in cognitive theorising.

Finally, neural network theory will be utilised as the theoretical framework within which to write a conceptual account of insight, a decision which will be clearly motivated in sections 3.2.4 and 3.2.5 and which falls within the enterprise of re-writing cognitive phenomena in neural network terms. It is therefore necessary to provide a far fuller exposition of the neural network theoretical position than the symbolic position. Now that the bias in the structure of this theoretical review has been made evident, let us move on to outlining the symbolic account of cognition.

A broad definition of symbolic theory is offered by Hatfield (1990). He suggests that symbolic theory views representations as symbols in an internal representational system or language of thought. Psychological processes are thus computations defined over these representations. From Bechtel (1988) we can add to this that the traditional computer is used as a model for this conception of the mind and its cognitive functioning. In other words, the operation of the traditional computer is the same as the operation of the mind, both of which use symbols as representations of thoughts, ideas, objects, events and behaviours (to name a few) which can be manipulated by means of computations in an internal language of thought. Both mind and computer employ rules to direct the manipulation of these representations, and both mind and computer are physical devices which store knowledge at physical locations (Bechtel, 1988).

Bechtel and Abrahamsen (1991) note that there are some, slight differences in conceptions of symbolic theory due to the existence of two strands leading from two of the disciplines contributing toward cognitive science. One strand leads directly from the philosophy of logic to cognitive science and the other strand leads from linguistics, through cognitive psychology to cognitive science. The strand originating in logic views computers as symbol manipulation devices, while the strand from linguistics and cognitive psychology sees human cognition as consisting in symbol manipulation. These two strands are most often brought together to conceptualise human cognition in terms of the operation of the traditional computer, and as our purpose is to reflect on human cognitive processing, this unified position is the one which will be presented here.

According to Newell (1990), symbolic theory assumes that the mind is a universal computational system and, as symbol systems are universal computational systems, humans are assumed to be symbol systems. The human symbol system is made up of various components which constitute its defining features. It has memory, which is made up of a structure which contains symbol tokens and which is independently modifiable at some grain size, or some level of the cognitive processing system. The system is made up of symbols which are patterns that provide access to distal structures, or structures elsewhere in memory. Symbol tokens are the occurrence of a pattern in such a structure. Newell, Rosenbloom and Laird (1989) explain the need for symbols. This need arises because it is not possible for all of the structure involved in computation to be assembled ahead of time at the physical site of computation. (To understand this one needs to remember that cognition is symbol manipulation in a physical computational device and that knowledge is therefore stored at physical locations.) Thus, it is necessary to travel out to other (distal) parts of the memory to obtain the additional memory / knowledge structure.

Further components of the symbol system are its operations, interpretations, and capacities. Operations are processes that take symbol structures as input and produce symbol structures as output. These operations need interpretations, which are processes that take symbol structures as input and execute the relevant operations. The symbol system cannot function without certain capacities. These include sufficient memory and sufficient symbols, complete composability (so that operators can construct any symbol structure), and complete interpretability (so that interpretable symbol structures are available for any arrangement of operations). These symbol systems are intelligent and are built up of multiple levels which are hierarchical and interactive. Posner's (1986) much critiqued modularity theory is a more extreme example of this.

Now that the symbol system has been described and the centrality of symbols has been highlighted, let us consider in a little more detail how the rule-based manipulation of symbols functions as cognition. Basically, cognition is seen to operate as a language of thought in which mental representations have a combinatorial syntax and semantics (Fodor & Pylyshyn, 1988). Thus, the semantic content of a representation is a function of the semantic contents of its syntactic parts, together with its constituent structure. The manipulation of these symbols is rule based and, as Bechtel and Abrahamsen (1991) point out, a representational role is assigned to a particular symbol by virtue of the manner in which it is treated by the rules of the system. Rules are usually applied in a serial fashion. Therefore, to have a conscious thought involves the rule based retrieval of the symbol tokens, from various physical locations, that will make up the thought and combining them in a manner which is sensitive to their semantics and syntax. The representational function of these symbols is granted by the fact that they are selected and combined in this way.

This is a brief outline of symbolic theory and critique of this conception of cognition and cognitive processing will be reserved for section 3.2.4. It is necessary, however, to add that this theory has been posed as the only viable framework from which to understand human cognition (Fodor & Pylyshyn, 1988) and that it has seen great success in accounting for such cognitive processes as language acquisition and use (Hatfield, 1990), reasoning (Bechtel & Abrahamsen, 1991; Frensch, 1991; Nakamura, Kleiber and Kim, 1992) and problem solving

(Newell, Rosenbloom & Laird, 1989; Newell, 1990 for a discussion of the SOAR architecture; Quinn, 1991).

### 3.2.3. An outline of neural network theory

Neural network theory presents a radically different conception of cognition and cognitive processing to the one offered by symbolic theory. Extensive use will be made of the texts by Rumelhart, McClelland and the PDP Research Group (1986), McClelland, Rumelhart and the PDP Research Group (1986), and Bechtel and Abrahamsen (1991), as well as personal experience with neural network simulations in order to present the neural network theoretical position outlined here. Where other sources are included, this will be indicated.

Neural network theory is based on a model of the mind which is brain inspired. At the neural level, the brain is made up of massively interconnected neurons which form neuronal nets that are believed to fire in parallel (Bechtel, 1988). Thus, this conception of the mind suggests that cognitive processing can be approximated by a type of neural network structure that is made up of massively connected, simple processing units ("neurons") that operate in parallel. Cognition can be understood in terms of the operation of many neural networks, each made up of individual units, which operate together.

Two features which neural network theory shares with symbolic theory, are the notion that cognition can be modelled in a computer and the idea that cognitive models should be representational models. However, the perspective offered by neural network theory on both of these counts is quite different to the symbolic account. Although neural network theoretical simulations can be run on a traditional, serial processor, this is not considered to be ideal. It is the realisation of parallel processing in a serial processing environment and until parallel hardware on the scale of the human brain is developed, neural network cognitive simulations can never be fully realised. On the issue of representation, neural network theory does not make use of symbols as a representational device in cognition.

Within neural network theory representation can be either local or distributed. In local representation individual units are assigned a representational function within the interconnected array. Their representational function must usually be supplied by an interpretation on the part of the network designer (Diederich, 1992) and is not a feature of the structure of the cognitive architecture - the same unit can be supplied with more than one representational meaning. To make this more concrete, let us consider an example. Within a network architecture, one unit can be supplied with the representational function CUP. Whenever that unit displays activity above its threshold (this will be explained in due course) the concept CUP is signified, and is only signified by interpretation on the part of the network designer who decided that this particular unit stood for CUP. The very same unit can signify the concept BIRD in another simulation (although the manner in which this unit now responds to input will probably be quite different), if that is the representational function which the network designer has assigned to activity of this unit at that time. Thus, representation is local when an individual unit is equated with a single concept. Distributed representation is very different.

Representations are distributed across a network architecture when more than one unit is used to signify a concept. Let us use the CUP example once more. To convey the concept of a cup we may use one unit to represent HANDLE, one unit to represent PORCELAIN, one unit to represent ROUND, and another unit to represent CONTAINER. Thus, there is no one unit which can signify the concept of a cup. All, or some, of these units are used to represent the concept. Distributed representation in the absence of symbols is one of the features which makes neural network theory so different from symbolic theory. This allows a network architecture to display some of the features which are characteristic of human cognition and which a symbolic system cannot convey (see for e.g., Stone &

van Orden, 1989). This will be explored when we contrast the two theories in section 3.2.4.

The terms activity and threshold have already been mentioned in relation to these units and it therefore seems fitting that these features of the theory now be elaborated on. Cognitive processes are approximated by the flow of activation across the connections between units in a neural network simulation. Thus, an initial activation is applied to the units, which represents the stimulation of thought, perception, or some other sensory input, for example, and this activation is then spread to other units by means of connections. The manner in which the activation is spread, as well as the resultant pattern of activity across all of the units, is taken to represent the result of cognitive processing (O'Brien, 1991).

The connections between units are either excitatory or inhibitory - initial activation is either spread over units to increase their resting activation or it actually leads to a decrease in activation below resting point. Activation is spread by means of various learning and updating procedures of a mathematical nature, which we will consider a little later on. These interconnected units can also have certain thresholds. This means that units will only contribute to the representational function of the network when their current activation exceeds a certain minimum level.

Not all units that make up the network architecture are the same, and not all connections within the architecture are the same. There are input units, which are specialised to receive input from an external source (either the external world, sensory organs, or other networks). There are also output units which represent the external output, or result of cognitive processing within the network structure. Hidden units occur between input and output units, and fulfil the major representational function of the network.

An example of the representational function of a simple network will make this description of neural network architectures a little easier to understand.

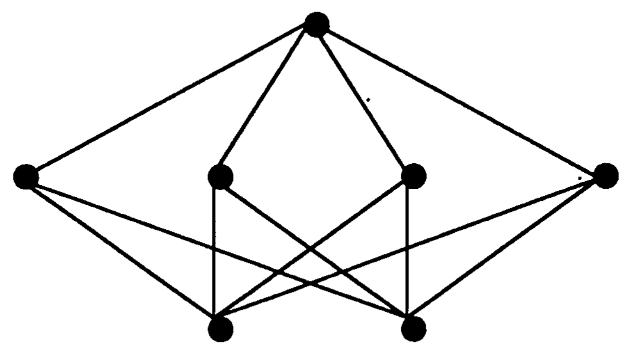



**Figure 3.1. Diagrammatic representation of a simple neural network architecture**

The above network has a layer of two input units, four hidden units, and one output unit. The hidden units could be assigned the representational function of the components of the CUP concept referred to earlier. The first unit in the hidden layer could represent HANDLE, the second unit could represent PORCELAIN, the third ROUND, and the fourth CONTAINER. When the input units receive stimulation from the sensory system, when the individual has viewed a cup for instance, activation will be spread from these units to the hidden units in the next layer. The increase in activation which these units experience corresponds to the recognition of the features of the concept CUP. This activation is fed to the output unit and if this unit climbs in activation to the level required for recognition, CUP will be represented and the individual who has viewed the cup will recognise it as such.

With respect to the connections between units, these can either be feedforward or continuous. Feedforward connections can only spread activation in one direction through the network, from the input units, through the hidden units, to the output units, whereas continuous connections can feed activation both forwards and backwards through the network. Our example above could make use of either type of connection. The units in the hidden layer could also be connected to each other so that spreading activation between the different features of the CUP concept could occur.

Connections also have different strengths which can either be set at the outset, or which can be allowed to modify themselves during processing. The activation that will therefore be passed along to any unit depends on the type of connection leading to that unit, the strength of the connection, and the threshold activation level of that unit. Different network designs can be achieved by means of various combinations of these features of network architecture (see for e.g. Lippmann, 1987).

Mathematical and statistical principles enter into the network architecture by means of different equations for propagating activation and different equations for determining learning. Various equations which specify the manner in which activation is spread between units in the network are available. They specify different contributions of activation depending on the net input to a unit, the weight of the connection, the position of the unit in the network and thus what other units it receives activation from, and a decay rate which specifies the decrease in activation which can be expected across time. Activation equations can either be applied asynchronously, in which each unit decides randomly when to update its activation, or synchronously, where activations are updated during each processing sweep. A stochastic activation function can also be applied in updating activation levels and this is usually used in conjunction with simulated annealing.

The pattern of activation that the network settles into is taken as supplying a representation at the cognitive level. Sometimes, networks settle into activation patterns for purely statistical reasons which do not best represent the cognitive state of affairs the network is being used to portray. Simulated annealing involves overcoming these local minima by adding more variability to the activation levels in terms of a temperature function. Thus, activation equations can be very different across network architectures.

One of the most exciting features of neural network theory is that its cognitive architectures are self-teaching and that learning is viewed as a central part of the cognitive system (Estes, 1991). It has also been suggested that neural network theory is particularly important because it allows the systematic examination of the interaction between learning and representation (Hanson & Burr, 1990). Cognitive learning can be simulated at the network level without the intervention of the network designer and under entirely local control at the level of information available to individual units (Roitblat & von Fersen, 1992).

Learning occurs by adjusting the weighted connections between units and thus changing the activation patterns across the network, which performs the representational function of the network. Thus, the network can demonstrate the learning that occurs when an individual learns that a cup is made up of a round container with a handle, by modifying the weights between these signifying units so that their activation levels will be higher when the concept CUP is presented to the network. This usually happens by means of association, so that a cup is more readily recognised in the future when any one, or combination of, the cup signifiers are present. Learning equations can operate on the basis of strengthening connections between units that represent associated objects or concepts, by comparing the current activation of the network with the goal activations and feeding the difference back through the network, or by modifying those weights which are contributing most significantly to the failure in representational function (Shoemaker, Carlin & Shimabukuro, 1991).

What has also been alluded to here, is a difference in control of the representational function of the network. Networks can be allowed to settle into a pattern of activation which conveys a cognitive representation, or they can have a desired activation output imposed upon them. In either case, neural network theory has proved to offer a powerful account of learning (see for e.g., Doyle, 1990; Parks et al, 1991; Choi, McDaniel & Busemeyer, 1993; Elman, 1993; Quartz, 1993), as well as behaviour which forms an exception, two areas where symbolic theory has been particularly weak.

Neural network theory has also proved quite powerful in other areas. Network models have offered an account of pattern recognition generally, which is accepted as being more accurate than symbolic theory (see for e.g., Navon, 1990; Treisman, 1990; Greenwood, 1991; Metcalfe, 1991; Shanks, 1991). They have also offered convincing accounts of category learning (see for e.g., Gluck & Bower, 1988,1990; Pazzani, 1991; Kruschke, 1992), word recognition (see for e.g., Seidenberg & McClelland, 1989; Allen & Madden, 1990; Besner, Twilley, McCann & Seergobin, 1990; van Orden, Pennington & Stone, 1990; Fera & Besner, 1992) and face recognition (Burton, Bruce & Johnston, 1990).

Neural network models have been used to successfully simulate case-based reasoning (see for e.g., Barnden & Srinivas, 1992), automaticity (Cohen, Servan-Schreiber & McClelland, 1992) and the cognitive functioning of an idiot savant (Norris, 1990). Neural network architectures have also been developed to offer a competitive account of language acquisition and use (see for e.g., Rumelhart & McClelland, 1986; Plunkett & Marchman, 1991, 1993), as well as semantic memory (Kroll & Klimesch, 1992).

Perhaps most important for the purposes of this thesis, neural network models have been used to offer an account of the processes involved in problem solving (see for e.g., Ye & Salvendy, 1991; Zualkernan & Johnson, 1992), an account which seems to be superior to that proposed by symbolic theory. This is particularly the case with optimisation problems (see for e.g., Abe, Kawakami & Hirasawa, 1992), such as the travelling salesman problem (see for e.g., Xu & Tsai, 1991) and diagnostic problem solving (see for e.g., Peng & Reggia, 1989; Lesser, 1991).

This consideration of the successful application of neural network models to various aspects of cognition, heralds a consideration of the contrasts between symbolic theory and neural network theory, the purpose of the following section.

### 3.2.4. Contrasting symbolic theory and neural network theory

The discussion of some of the criticisms and special areas of application of symbolic theory and neural network theory which we saw in the previous section, precedes a review of the contrasts between these two theoretical approaches, as well as a consideration of their advantages and disadvantages for advancing our understanding of human cognition. It is necessary to explore these contrasts so that the theoretical sparring match in cognitive theory can be more fully understood.

As has already been suggested, the dominance of symbolic theory can be seen as an emergent feature of certain historical processes, and a recognition of the theoretical assumptions which this dominance has bequeathed to the study of cognition begs the consideration of an alternative conception of human cognitive processing. This, together with the special characteristics which neural network theory provides, suggests that this framework for explaining human cognition must be evaluated. Hence, part of the work of this thesis is to re-write insight in neural network terms to see if this theory can account for the well known empirical features of insight in problem solving. Before moving on to chapter four and the work of writing a conceptual account of insight, it is necessary to contrast symbolic theory and neural network theory as explanations of cognitive processing and to consider how the special characteristics of neural network theory will make for a viable account of the processing involved in attaining insight in problem solving.

One of the most widely discussed differences between symbolic theory and neural network theory can be found in their biological plausibility. Symbolic theory has been criticised for its biological implausibility (Bechtel, 1988; Rumelhart, McClelland & the PDP Research Group, 1986; Wolters & Phaf, 1990), while neural network theory is seen as possessing the distinct advantage of neural plausibility (Bechtel & Abrahamsen, 1991). Symbolic theory has also been criticised for its brittleness and inflexibility (Bechtel & Abrahamsen, 1991). This theory cannot account for graceful degradation[2]; if a rule-based system is damaged it tends to cease functioning completely. This is not what happens with human cognition, and the gradual decline in functioning is something which neural network theory explains particularly well (Bechtel & Abrahamsen, 1991). Part of the inflexibility of symbolic theory is that it cannot account for the manner in which humans deal with exceptions. Symbolic theory explains behaviour which follows the rule, but cannot account particularly well for behaviour which is unusual. Neural network theory, on the other hand, offers a highly plausible account of this satisfaction of soft constraints (Bechtel & Abrahamsen, 1991).

Neural network theory also offers an account of content addressable memory, a feature of human cognition which symbolic theory struggles to explain (Wolters & Phaf, 1990). Spreading activation along learnt connections between characteristics accounts well for the sudden surfacing into memory of an item, on the basis of presentation of some (or even one) of its features[3]. This also accounts for part of the resilience to damage which neural network theory can account for. Neural network theory can also offer an explanation for the distinctly human capacity to

[2]Graceful degradation is a feature of the human cognitive system. When this system is damaged it does not cease functioning completely, but instead displays a gradual decline in the efficiency of its processing.

[3]To understand this it is necessary to use a brief illustration of the neural network position on learning and memory. A cup might be defined by learnt associations between such features as handle, porcelain, round, and container. When a network is presented with the feature handle, for instance, the activity of this unit will spread along the connections to the other cup features learnt by association. The unit representing the concept cup is then likely to receive sufficient activation from these units to surface into awareness, without having been directly activated itself.

learn from experience (Bechtel & Abrahamsen, 1991). Symbolic theory has traditionally struggled to account for learning (Hanson & Burr, 1990) and, although it can offer some explanation for learning on the basis of experience, the establishment of different weighted connections between representations seems to offer a more plausible account of human cognitive learning. A cognitive property which links well to this is generalisation.

These special characteristics have led Bechtel (1988) to suggest that this theory offers a more realistic account of human cognitive performance than symbolic theory, and that it has the potential to explain all cognitive phenomena. There are, however, theorists who would dispute this. Hatfield (1990) suggests that symbolic theory and neural network theory each have their own special sphere of application and that they should stick to offering an account of the cognitive processing associated with that sphere. He suggests that the special sphere of application for symbolic theory is linguistics, while for neural network theory it is pattern recognition and neural implementation of symbolic processing. Based on this he raises doubt concerning the ability of neural network theory to guide research in to higher cognition. Let us pursue the source of this doubt.

What Hatfield (1990) is alluding to is one of three perspectives on the place of neural network theory in cognitive theorising and the doubt he expresses is common to two of these perspectives. The first of these is that symbolic theory is the only viable conception of human cognition and that the neural network enterprise is completely misguided (Pinker & Prince, 1988). The second of these perspectives, and the one which Hatfield (1990) alludes to, is that the place of neural network theory is as an implementation of a rule-based, higher cognitive system (Bechtel, 1988; Fodor & Pylyshyn, 1988; Anderson, 1990; Chater & Oaksford, 1990; Dawson & Schopflocher, 1992; Goodman, Higgins, Miller & Smyth, 1992). This perspective has received a lot of support in the literature, but has not been explored at a conceptual level or at the level of a simulation. The third perspective on the place of neural network theory, is that it is a viable replacement for symbolic theory as a complete account of human cognition (Rumelhart, McClelland & the PDP Research Group, 1986; Smolensky, 1988).

The source of these differences in opinion concerning the place of neural network theory in an account of cognition and cognitive processing lies in the debate concerning the character of thought. For proponents of the symbolic approach and the neural network implementation approach, the character of thought is linguistically based; in other words, cognition is fundamentally based on a language of thought. For proponents of the neural network approach cognition rests fundamentally on pattern mapping and, therefore, on pattern recognition. It is impossible, at present, to resolve this debate within cognitive science, never mind within the scope of one cognitive psychology thesis. It is necessary, though, to examine the foundations of this debate as they are presented in the literature, and it will be suggested that the nature of the symbolic criticism of neural network theory certainly does not render the neural network account of cognition unlikely.

Bechtel (1988) provides a good overview of the nature of the defence utilised by the language of thought proponents. They argue that our cultural products, such as our linguistic expression of information, utilises a serial, rule-based structure of reasoning and that we acquire much of the information which we use in subsequent thought from these products. This supports the apparent seriality and logical character of conscious thought, in which we sometimes employ rule-based operations and sometimes teach people explicit rules for performing tasks.

None of these points of argument can be taken to suggest that the cognitive activity performed by the mind must be in the form of a language of thought. The first point to consider is the apparent serial character of thought. The brain clearly does not process in serial, so why should we assume that the mind does? It seems entirely possible that we impose a serial character upon our cognitive activity in relation to other people; that perhaps this character of thought is a social creation. Serial processing certainly cannot account for the processing which occurs during insight. The second point to consider is that what is being discussed here is conscious thought. If Clark's (1989) rather telling suggestion that the language of thought is merely the mind's commentary on the processing it has performed, a suggestion which likens symbolic theory to folk psychology, is to be taken seriously, it is entirely possible that this conscious mind talk does not employ the same character as the unconscious cognitive processing which supports thought. A third point to consider is the rule-based nature of thought. Rumelhart (1989) suggests that neural network systems can display the regularities captured by rule processing systems as an emergent feature of the processing within this system. Therefore, just because cognition looks as if it is governed by rules, does not mean that the underlying features of cognition are those of a rule processor.

It is, therefore, clear that none of the features of cognition which are used to support the language of thought position convincingly suggest that cognitive processing must occur in this fashion. Once again, it can be suggested that it is only the dominance of symbolic theory which has prevented an alternative conception of cognition from gaining ground. The debate, however, does not stop there and symbolic theorists have suggested that there are aspects of cognition which neural network theory will never be able to offer an adequate account of. One of these is language processing.

Rumelhart and McClelland (1986) provide an account of a neural network simulation which models the developmental phases children go through in learning the past tense of English verbs. This simulation was taken to offer a neural network theoretical account of language acquisition. This account was heavily criticised (see for e.g., Lachter & Bever, 1988; Pinker & Prince, 1988), but Miikkulainen and Dyer (1991) have once again produced data which suggests that neural network systems can model language processing. Thus, there has been no conclusion concerning the ability of neural network theory to account for language acquisition and use, and it is entirely possible that the development of powerful parallel hardware and improved learning algorithms will see neural network theory offering a highly competitive account of this aspect of cognitive processing. Also, this debate has rested on computer simulations, a method which has already been criticised (see section 3.2.1.1.), and it seems necessary that predictions made by neural network theory be tested with human participants before any conclusions are drawn.

The challenge of an incomplete account of cognitive functioning does not begin and end with language processing. There are other aspects of human cognition concerning which symbolic theorists have asserted neural network theory is inadequate. These are the features of recursion, systematicity, inferential coherence, and productivity. Some researchers (see for e.g., Fodor & Pylyshyn, 1988; Fodor & McLaughlin, 1990) have asserted that these features of cognition are only displayed by symbolic systems and that architectures which do not utilise the rule-based manipulation of symbols cannot account for these aspects of cognitive processing. This has been countered by Smolensky (1988) and by Macdonald (1995), who state that the debate focuses around whether neural network architectures can display compositionally structured representations which are genuinely non-classical (i.e. not symbolic) and that the likelihood is that they can.

Although no resolution can be reached concerning the debate between these two theoretical approaches to cognition, what is clear is that symbolic theory has not succeeded in squashing the challenge offered by neural network theory in accounting for cognitive processing. Given the special characteristics which neural network theory lends to the account of cognition, it seems necessary that this account be advanced and that important features on the face of cognition be re-written in neural network terms. The viability of this theory can then be tested. It also seems very important that predictions made by neural network conceptions of cognitive phenomena be closely linked to human data, particularly as most of the debate centres around data produced by computer simulations of cognitive functioning.

Before moving on to chapter four and the task of re-writing insight in neural network theoretical terms, and chapters five, six and seven, the report of a human test of some of the predictions made by this conceptual account, it seems important to address one further question. If the debate concerning which is the most suitable theoretical account of cognition is unresolved, why write and test a neural network account of insight and not a symbolic account of insight? The answer to this is that neural network theory seems to offer a more viable account of insight than symbolic theory does. Let us briefly consider why this answer is made.

### 3.2.5. Neural network theory and a viable account of insight

The most obvious reason for considering a neural network theoretical account of insight is only indirectly related to the viability of this account, and that is that symbolic theory has so far been unable to offer a conceptual account of the empirical findings associated with attaining insight in problem solving. Symbolic theory has so far proved incapable of offering a processing explanation of insight. Neural network theory has the potential to offer precisely such a process level account. It could be suggested that a process level explanation of insight is unnecessary, that we should be focusing on cognition more globally. How can we reflect on insight at all, demonstrate insight, or teach people to think insightfully without understanding the processes involved in attaining insight? It does, therefore, seem necessary to understand such things as why insight is so difficult, how it happens so suddenly, and what people do to attain insight.

Neural network theory has had particular success in conceptualising problem solving and, as insight is a particular feature of some problem solving, it seems conceivable that this theory may be able to successfully account for insight as well. This approach has also provided a good account of the manner in which new problems are learned or negotiated (Hanson and Burr, 1990) and insight problems are certainly not familiar problems. It is also striking that people who display insight cannot verbalise the processes involved in solving insight problems. This suggests that insight may not employ the same processing that language use does and neural network theory certainly offers the potential to account for processing which is not linguistically based. Neural network theory is also fully equipped to explain some of the puzzling features of insight, such as its difficulty, the propensity to select the wrong approach to the problem, and the incubation which precedes correct problem solution. Finally, van Leeuwen (1989) has suggested that neural network theory and Gestalt psychology are highly compatible and, as Gestalt psychology originated interest in insight, pursuing a neural network theoretical account of insight seems promising. It is therefore high time that we consider what a conceptual account of the processes involved in insight in problem solving would look like.

# CHAPTER 4

# A NEURAL NETWORK THEORETICAL MODEL OF INSIGHT IN PROBLEM SOLVING

## 4.1 Introduction

From the review of literature on insight in problem solving (which constituted chapter two), it was suggested that a conceptual account of the processing necessary to attain insight still needs to be formulated. This is because insight has only really been defined in operational terms despite the relatively clear and consistent nature of the empirical findings that have been reported. In chapter three (with its consideration of the sparring match in cognitive science and cognitive psychology) we saw that there are really only two conceptual frameworks for cognitive processing within which a conceptual account of insight can be written. The choice of neural network theory not only seems particularly beneficial for advancing our understanding of insight, but will also reflect on the theoretical debate between symbolic theory and neural network theory.

It is, therefore, now the work of this chapter to write a conceptual account of insight in neural network terms. Central features of this model, which can tested in order to provide some illumination on the viability of this account of the processing involved in insight, will be highlighted. It will be suggested that the operational definition of, and empirical data on insight, are consistent with this conceptual account of insight written in neural network terms.

## 4.2 A conceptual account of insight

This model of insight will suggest that insightful problem solution consists of a recognition of the pattern which characterises insight problems. Insight problems present the problem solver with an ambiguous task, one in which the type of problem suggested by the manner in which the problem presented is unclear. Past learning on the part of the problem solver elicits the recognition that the current problem is characteristic of a particular problem type or task. This recognition also elicits the information associated with that problem type.

To illustrate this, consider an example. If you are presented with a task in which you are required to connect dots with lines this is likely to invoke the familiar task of drawing lines from one dot to another and not the task of drawing lines which start or end at blank spaces. The information that lines can begin and end at blank spaces is not associated with the problem type which has been recognised. The recognition of this inappropriate problem type and the subsequent work on this problem within the structure which this type suggests, will not lead to solution. The problem solver needs to recognise that this problem belongs to a non-obvious problem type. This is the recognition that there is a pattern to insight problems, that they are a type of problem which require a non-obvious solution.

Insight problems are a problem type or task which can be recognised and which, following appropriate recognition, elicit the information that the problem solver must overcome past learning and look for a non-obvious solution to the problem. Insight problems are an exception and this is why it is difficult to correctly recognise them, to recognise the pattern to insight problems and thus to display insight.

It will not only be demonstrated that this conception of insight in problem solving as a recognition of the pattern which characterises insightful problem solution is consistent with the literature, but that it also allows the well-researched notion of expertise in problem solving to be applied to insight. This will be used to suggest that insight can be facilitated by teaching people to become experts at recognising the pattern to insight problems and thus at insight processing. This is characterised by the recognition of the ambiguous nature of the presented problem and thus applying the non-obvious solution. It will also be suggested that, if this conceptual account of insight in problem solving proves to be viable, important implications can be drawn to the theoretical debate in cognitive science and cognitive psychology.

This conceptual account of insight will be advanced by:

1. Utilising the operational definition of insight derived from the literature to frame insight in problem solving in neural network terms as a recognition of the pattern to insight problems.
2. Explaining the empirical findings on insight in terms of neural network theory, and;
3. Applying the notion of expertise to insight processing.
4. Drawing implications from this to the debate in cognitive theory.
5. Outlining some of the central tenets of this conceptual model of insight which can be subjected to an empirical test with human problem solvers to provide an initial evaluation of the viability of this model.
6. Formulating the empirical testing which will address these research questions.

Before moving on to an outline of this model it is important to address one issue. This neural network theoretical model will not be based on a computer simulation. This is particularly important to discuss as all of the successful applications of symbolic theory and neural network theory reported in the previous chapter were based on computer simulations of human cognitive functioning. The absence of a computer simulation may seem a little strange given the tradition in cognitive science of explaining cognitive phenomena by means of the results of a computer model. Instead, the principles of neural network theory will be used to inform an explanation of the cognitive processing which might be involved in attaining insight and thus to write a conceptual account of insight.

There are several reasons for eschewing a computer simulation and instead applying the principles of cognitive functioning which seem to lie behind the processing displayed by the computer programme to explain insight. First of all, as has been stressed before, this is a cognitive psychology thesis and not a cognitive science thesis. Thus, the aim is to determine whether a computer based account of cognition can explain insight, not to develop a computer model which displays insight. Much of the research in cognitive psychology does not rely on computer simulations. Instead, its aims are to apply the postulates of a theory to construct a conceptual account of some aspect of psychological functioning and then to subject this account to empirical testing. Given the rather arbitrary nature of simulation data this seems to be an approach worth considering and applying to neural network theory.

In respect of the rather arbitrary nature of simulation data, it has already been suggested (in chapter three) that, just because a computer and a human produce the same results, does not mean that they do so by employing the same processes. Also, a computer simulation can be biased to produce whatever results the researcher expects in the name of discovering what constraints may apply to human cognition. Where does one draw the line between biasing the simulation to produce the expected results and imposing real constraints? Surely this is not particularly good methodology.

Linked to this is the rather arbitrary decision concerning which architecture to use in the simulation. This applies particularly to neural network theory, where there are many different architectures to choose from based on the number of units, the number of layers, the type of connection, the type of learning algorithm, the type of activation function, *etc*. Which of these architectures represents the manner in which human cognition functions? Do they all, and if so, which architecture best represents the manner in which humans process insight problems? Add to this the fact that most simulations are the instantiation of parallel processing on serial hardware, hardly what is meant by brain-style processing. It thus seems that the degree to which a simulation can provide information on human cognitive processing is somewhat limited, as it is quite unlikely that our artificial cognitive architectures resemble mind functioning.

Many neural network researchers conduct simulations of cognitive phenomena despite these problems. Simulations remain an effective tool for explaining cognitive processing and they provide a tangible platform for the development of theoretical accounts and for theoretical debate. It is, therefore, clear that the claim concerning the benefits of eschewing a computer simulation is a contentious one.

Despite the contentious nature of this claim, a simulation will not be conducted for the reasons outlined above, and the general principles of neural network theory will be used as a framework from which to understand cognitive processing and within which to construct a theoretical account of insight. This account will be subjected to empirical testing to attempt an initial evaluation of the viability of this account. And now, at last, we get to the real business of this chapter - formulating a conceptual account of insight.

### 4.2.1. Insight as recognition of the pattern to insight problems

To write a conceptual account of insight it is necessary to begin with the operational definition of insight derived from the literature. That was, that insight is the process by means of which a problem whose formulation is ambiguous, and for which only the non-obvious conceptualisation is appropriate, is solved. What is necessary, then, is to determine what this process is. The conceptual understanding of this process which will be proposed here is that it involves the recognition of a pattern.

This is not the same thing as pattern recognition. Pattern recognition is usually where an image, such as the number 3, is presented to an individual who then recognises the symbol as representing the concept three. However, it could be suggested that pattern recognition shares features with the processing that occurs when a problem is presented to an individual which they must recognise. This is recognition of a pattern at a different level of the cognitive system. Hence, insight will be described as the recognition of the pattern which characterises insight problems.

Categorisation could be understood to characterise the first steps in insight problem solving. The problem is presented to an individual who must categorise the problem within a general class of problems so that a solution strategy can be selected and applied. The term categorisation was also considered as a description of the process of insight, but it was deemed to apply too strictly to concepts, such as bird, which have defining features, such as wings, flies, and has feathers (see for e.g., Shanks, 1991). A problem could be seen as a category with defining features, but this understanding cannot really be extended to a problem solution. Classification was also considered, but this seems to apply to the assignment of an object / problem to a particular class, with no further activity. Classification can be seen as a part of the recognition of a pattern (see for e.g., Lippmann, 1987).

Within a neural network theoretical conception, however, there is no need to offer support for cognitive processing such as insight to rest on something akin to pattern recognition. All thought is believed to operate according to the recognition of patterns, just as symbolic theory suggests that all thought operates on the basis of language processing. This, however, seems like a rather blind acceptance, particularly given the current theoretical debate in cognitive theorising. Therefore, the proposal that insight can be understood as the recognition of a pattern, will be explored in terms of the operational definition of insight above, before moving on to consider whether an explanation of the empirical findings are consistent with this account. To make this clearer, the nine-dot problem will be used as an example. Remember that the nine-dot problem involves presenting the problem solver with three rows of three dots and the instruction that all of the dots must be connected with four straight lines, without lifting pen or pencil from the paper. Most problem solvers fail to realise that the space on the page beyond the dots must be utilised to reach problem solution and that lines cannot begin and end on dots only. (Please refer to appendix 1 for a visual presentation of the nine-dot problem and its solution.)

When a problem solver is presented with the nine-dot problem, a problem which is known to be solvable by the application of insight, he or she will generally attempt to solve the problem by drawing lines within the boundary suggested by the square shape of the problem presentation. As we saw in chapter two, the Gestalt psychologists understood this to mean that the problem solver was stuck on the properties of the visual field - the square shape of the problem. This makes little sense conceptually and information-processing theorists suggested that this tendency to stay within the boundaries of the square was due to the choice of a particular problem space, a suggestion which also proved to be conceptually weak. It was, however, suggested that the choice of problem space is based on past experience although the manner in which this experience is applied, was not explained. What seems highly appealing conceptually and what is informed by the principles of neural network theory, is that the problem is approached on the basis of **past learning**.

What could this past learning be in the case of the nine-dot problem? Well, one source of past learning which would be immediately available when an individual is presented with dots to connect by drawing lines on paper, are the connect-the-dot drawings most children (and some adults!) are exposed to. This problem could, therefore, be recognised as a member of this class of tasks and hence the tendency to connect dots within the boundary of the square. After all, if you are connecting dots you have to connect **dots**, not blank spaces on the paper. In this case, the square shape of the dots is irrelevant; the dots could be in a triangular shape or in a circular shape.

Another source of past learning which springs quickly to mind, is the group of so-called optimisation problems. We are all exposed, whether it be in mathematics or some other discipline, to the concept of taking the shortest possible number of steps to reach a specified goal. Indeed, there are a whole class of problems which require optimisation and the travelling salesman problem is an example of this. In this problem, a salesman has, for example, forty cities on his sales route and the problem solver must suggest how the salesman could visit all of these cities while covering the shortest possible distance. The specification in the nine-dot problem that **four** lines be used and the problem solver's first experience with the problem suggesting that it would take at least five lines to connect all of the dots, could lead the problem solver to recognise this problem as a member of the optimisation class. This would suggest that as many dots as possible must be covered by each line. Including blank spaces on the page does not count as covering a dot. Add to this the fact that by extending lines there are occasions when only two dots are covered by one line and utilising the additional space on the page seems even more unlikely.

What we have seen here is that past learning can be applied inappropriately to the nine-dot problem and the problem is incorrectly recognised as belonging to a particular class of problem tasks. This is facilitated by the ambiguous nature of the problem presentation. It is possible to go one step further and suggest that the problem is only ambiguous because these sources of past learning are available. This sounds very much like the recognition of a pattern. Patterns which link tasks are established on the basis of past learning and new information is recognised as being a member of a particular pattern class on the basis of its similarity to a familiar member of that class. The response which this class of patterns usually evokes is then applied to the new information. In other words, a new problem is classified on the basis of the recognition of the pattern which links this problem to connect-the-dot tasks, for instance. Once recognition of this pattern has occurred the information which has been associated with this task in the past is activated in memory. Information which has been associated with other tasks, is suppressed. Thus, the recognition of a problem type leads to the availability of information which has been associated with that problem type in the past. For example, that lines can be drawn between dots. The information that lines can begin and end in blank space, is suppressed.

So, what happens when a problem is solved by insight? Well, remember that what is crucial, according to the operational definition of insight, is that only the non-obvious conceptualisation of the problem is appropriate. Thus, the problem solver needs to recognise that the problem is not a member of the class of problems suggested by past learning, but that it is a member of a class of problems that require a non-obvious solution.

It is then necessary to recognise that the problem is a member of tasks in which lines can be drawn on blank paper spaces. It is, of course, quite conceivable that the problem can be solved by the appropriate recognition of membership to some other class of problems, such as problems in which lines have to be extended beyond dots. This is not what is important. What is crucial to attaining insight, is that the problem is recognised as belonging to a class of insight problems, which display the pattern of requiring a non-obvious problem solution. Once problem solvers have recognised the pattern to insight problems, any new insight problems can be recognised as members of this group and the information that they require a non-obvious solution will be available.

Now that insight has been conceptualised according to the principles of neural network theory, as the recognition of a pattern which characterises insight problems, the empirical findings derived from the literature can be examined. It can be determined whether these findings are consistent with this account and are explainable in neural network terms. This will also allow for a description of the manner in which this recognition is achieved. If the empirical findings are consistent with this account, the explanation of these findings will advance our conceptual account of insight and will suggest that insight can be understood in neural network terms.

### 4.2.2. Explaining the empirical findings

To explain the empirical findings on insight in terms of the principles of neural network theory and to explore whether this data is consistent with a conceptual account of insight as the recognition of the pattern to insight problems, the features of insight outlined in chapter two will be explored.

#### 4.2.2.1. Problem difficulty

It was determined that one of the defining features of insight problems is their difficulty. How can the fact that insight problems are so difficult that very few people manage to solve them, be accounted for conceptually? If we apply the concept of insight as a form of recognition this can be explained quite easily. Recall that insight consists of recognising the non-obvious nature of the required problem solution and that past learning leads to the problem being approached in an inappropriate manner. In neural network terms, learning consists of establishing heavily weighted connections between items that have occurred together in the past and by establishing inhibitory connections to other items. The more often these items occur together, the more heavily weighted the connections between these items and the more inhibitory the nature of the connections to other items. Thus, the presentation of one of these items leads almost immediately to a response in terms of the item which has occurred with it often in the past and it is extremely difficult to bring to awareness a third item because of the suppression this third item experiences.

Consider a simple example to make this clearer. If you look out of your bedroom window every morning for a year and on 364 mornings you see birds which are sparrows, and on only 1 morning do you see pigeons, it is extremely likely that you will think of sparrows when you hear the word bird and extremely unlikely that you will think of a pigeon.

The same principles apply in making insight problems difficult. Because their formulation is ambiguous, they allow the application of a well-learnt response and it is extremely difficult to even consider an alternative response. If, as a child, you learnt repeatedly that being presented with dots and being asked to draw lines meant that you were dealing with a connect-the-dots problem, this will be the heavily biased response and all other responses will be inhibited. As the insight problem is attempted, the heavily biased responses surface more clearly into awareness (receive more positive activations) and the other responses are suppressed (receive more negative activations). Repeated attempts to solve the problem lead to more inhibitory activation being fed to other problem types. It will be difficult to even call to mind one of those other responses, because they are prevented from emerging into awareness based on the effects of past learning. As the solution to an insight problem requires just this, we have just accounted very nicely for the fact that very few people are able to solve insight problems and thus for the difficulty in attaining insight.

#### 4.2.2.2. Inappropriate problem conception

The adoption of an initial approach to the problem from which it is impossible to solve the problem, is exactly what we have been referring to when explaining the difficulty of insight problems. It is precisely the inappropriate recognition, the well-learnt one, which is heavily biased to emerge into awareness and stay there.

#### 4.2.2.3. Persistence

Persistence refers to the extended period of time over which problem solvers attempt to solve the presented problem, as if it is a member of the class of problems which they have inappropriately recognised. Again, this is explainable by the mechanisms presented above. The connections between the presented problem and the chosen problem task are so strong, and the connections to non-obvious problem types are so heavily inhibited, that it is extremely difficult to change the problem classification. Activation is continually fed from the presented problem along the connections to familiar problems. The inappropriate problem types and their associated elements keep receiving the highest activation and other problem types are inhibited more and more. The problem solver is caught in a loop where their solution attempts are not going to be successful, but they cannot select an alternative approach.

#### 4.2.2.4. Appropriate problem conception

This is only likely to have occurred if the problem has been solved; it is quite unlikely that someone would recognise the type of problem with which they are presented and not be able to solve the problem. Usually the only impediment to problem solution in insight problems, is the tendency to misrecognise the problem. It has been suggested in the literature (Weisberg & Alba, 1981a) that the nine-dot problem is different and that it is difficult even after the appropriate problem conception is recognised. This does not seem particularly tenable, especially if it is remembered that there are two inappropriate problem types which can be applied to the nine-dot problem (connect-the-dot tasks and optimisation problems) and that moving from recognising the problem as an example of one of these, to the other, does not constitute appropriate recognition. It might be necessary to realise that one can utilise the space on the page outside of the dots, as well as to recognise that one can cover fewer than the maximum possible number of dots with a drawn line before the problem can be solved. Perhaps this does make the nine-dot problem slightly different from the other insight problems, but only in as far as there are two inappropriate problem types which are suggested by past learning instead of only one.

What is of interest here, though, is what happens when the presented problem is recognised as being a member of the class of problems which require a non-obvious solution. This is usually accompanied by the realisation that the initial problem types applied to the insight problem were inappropriate. These two factors, and the fact that the recognition of the problem solution is almost immediately accompanied by a realisation of the problem solution, lead to the subjective a-ha! experience which is often thought to be the central feature of insight.

It must be assumed that connections to problem types other than the ones biased by past learning do exist. After all, everybody knows that you can draw lines on blank pieces of paper and that problems do not always require optimisation. The difficulty is that the nature of these connections is highly inhibitory. This is what makes insight in problem solving so rare. So how do we explain the fact that these inhibitory connections are sometimes changed?

Well, we know from our definition of insight that you have to recognise the need for a non-obvious solution. However, this surely only happens when the strongly inhibitory nature of the weights has already changed. More activation can then be sent along these connections to make them less negative, until they eventually become positive. This is likely to be a very long and difficult process, though the recognition of the need to cast around for a non-obvious solution would certainly speed up the process of modifying these inhibitory weights. What could also happen, is something that will make these connections less inhibitory - the positive connections to the problem types biased by past experience can gradually become less positive.

It was illustrated in chapter three that some learning algorithms, back propagation for example, operate by comparing the obtained output from a processing cycle to the desired output and then feed the difference back through the network to adjust the weights on the connections. What also happens, is that the weights which are contributing most to the error of the network are the ones which are adjusted most.

Now it is conceivable that the inhibitory connections to other problem types could be recognised as contributing most to the error in problem solution and that they could therefore receive the greatest adjustment, leading them to eventually become positive. What is far more likely, is that the positive connections to the inappropriate problem types are recognised as contributing most to the error and therefore receive the greatest adjustment, leading them to become less and less positive. As these weights become less positive, their inhibitory force on other connections becomes weaker. These weights could eventually become positive and activation can then be sent along them to other problem types. The newly available patterns of information could then climb to activation levels which lead them to enter into awareness.

This will take a long time and it is quite possible that the problem solver will lose interest in the problem before this happens. However, this is consistent with the fact that insight problems are difficult, that very few people manage to solve them, and that if they do, this solution is slow. If solution were to occur, it would entail appropriate recognition, in which the presented problem is appropriately recognised as being a member of the class of insight problems. This, then, is the essential processing involved in insight and it can be described as the recognition of the pattern to insight problems.

4.2.2.5. Incubation

Incubation seems to precede insight, if indeed insight does occur. Incubation is understood to be a period of inactivity, between active work on the problem and achievement of resolution. During this time progress is somehow made nonetheless. Incubation can be understood, in neural network theoretical terms, as below threshold processing. This is precisely what was discussed in the previous section, when activation levels are adjusted before the results of this processing are brought to awareness. The processing required to render the weights to the inappropriate problem types less positive and the processing which results in the connections to the appropriate problem types changing from inhibitory to positive, all occurs outside of awareness. It is dubbed below threshold processing, because units are provided with a threshold level of activation below which they will not respond.

This applies particularly to binary units, which are either on or off, and only activation above the threshold value can change their status. The principle does carry over to continuous units, where the threshold can be understood as an activation level which will bring the representation of the unit to awareness. This is a significantly positive state of the unit. Thus, the processing which is necessary to change the negative activation level of the appropriate problem type, to a positive activation level, occurs during the incubation process. The principles of neural network theory account rather nicely for this feature of insight in problem solving.

#### 4.2.2.6. Facilitation

The literature on insight has suggested that fairly specific hints can be used to encourage problem solution. As an example, providing practice on extending lines beyond dots, on to blank spaces of the page that contains the problem, increased the likelihood of correctly solving the nine-dot problem (Weisberg & Alba, 1981a). It is possible, within this neural network theoretical conception of insight, to understand why this facilitation would increase the likelihood of correct solution. It is past learning which has led the problem solver to incorrectly classify the presented problem and the effect of providing training on tasks which are similar to the required problem solution provides a competing source of learning. Information from the training tasks is especially available for application as these training sessions are usually presented in close temporal proximity to the target problem. Thus, when the target problem is presented to the problem solver the weights to the appropriate problem type may already have been adjusted to a positive level. Also, the activation of the associated units may still be at a higher level than resting activation as a result of the processing which has just occurred while solving these facilitation problems. Therefore, these specific hints could lead to a facilitation of insight.

It has been suggested in the literature that less specific hints do not lead to a facilitation effect. This is consistent with our account of insight. The problem solver has to recognise the pattern which links the facilitation and target problems. If the hints are vague, they will not be sufficient to overcome the biasing effect of past learning and recognition will fail.

What has not been considered in the literature, is something which this conceptual account of insight suggests might lead to insight - the explicit demonstration that there is a class of problems which are ambiguous and which prompt an inappropriate problem conception. In other words, there has been no investigation of the effect of making explicit the nature of insight and the nature of the realisation which is essential in insight processing. The application of neural network theory to insight suggests that this could lead to a significant facilitation effect, if the target problem is recognised as being a member of this class. Appropriate recognition is still necessary. However, by explicitly stating the ambiguous nature of insight problems and highlighting the biasing effects of past learning on insight problem solving, the connection weights to problem types suggested by past learning can be weakened. The effect of this, as has already been suggested, is to decrease the inhibition associated with less obvious problem types, and casting around for a non-obvious solution will add to the likelihood that activation will be sent along these connections. This notion of making explicit the nature of insight processing will be explored further when the concept of expert insight processing is discussed (section 4.2.3.).

#### 4.2.2.7. Transfer

This perspective on facilitation is supported by the findings on transfer in insight. Transfer is the application of information from one problem to another. The literature suggests that there is no transfer of content for insight problems, whereas there is for other problems. This is understandable as it is not the content of the insight problem which is vital, but the necessity of recognising that the presented problem requires a non-obvious solution. This finding in the literature has been used to suggest that it is the conceptual processing involved in insight problem solving which is transferred to produce significant facilitation. This is precisely what was suggested in section 4.2.2.6. Over and above this, we saw how the application of knowledge concerning insight processing might lead to a facilitation of problem solution.

#### 4.2.2.8. Expertise

The expertise reported in the literature on problem solving is quite different to the expertise at insight processing alluded to above. What has been considered in the literature is the effect of being an expert in a knowledge domain that might be activated by an insight problem. The effect of this expertise is to retard insight. Once again, this is completely consistent with a neural network theoretical account of insight. The inhibiting effect of past learning on insight has already been explored. Imagine the far greater inhibiting effect of being an expert in a domain that is linked to an inappropriate problem type. The weights on the connections to that problem type are likely to be even more positively set, and the weights to the appropriate problem type will thus be highly inhibitory. Also, it is likely that the threshold activation associated with the problem types elicited by expertise will be quite low.

Let us use the mutilated checkerboard problem as an example. Remember that in this problem the problem solver is presented with a checkerboard whose opposite corners have been removed, and the requirement is to cover the all of the remaining 62 squares with 31 dominoes, or to prove that this is impossible. The most obvious problem type suggested by the problem presentation is a purely mathematical one, in which 31 multiplied by two, is 62. The problem solver is amazed when their attempts at covering the squares fail and they seek a mathematical proof for this failure based on the total number of squares. What is crucial, however, is the recognition that this problem requires a non-obvious solution and the subsequent realisation that the opposite corners of the checkerboard are the same colour. As any domino covers a black and a white square the covering is impossible, because there are an unequal number of squares of each colour remaining.

Past experience heavily biases the mathematical problem type and insight is therefore unlikely. If the problem solver also happens to be a mathematician, the biasing effects of this approach will be so great that no other problem type is likely to be considered. The persistence with which the problem solver is likely to pursue the mathematical conception will probably be daunting[1]. The positive weights between this problem and the mathematical problem type will be extremely high and the recognition of the colour of the squares will thus be heavily inhibited. It would appear that a conceptual account of insight in neural network terms can account for yet another feature of the empirical data on insight.

#### 4.2.2.9. Inability to predict success or failure

This feature of the insight literature can be understood in very much the same terms as incubation. The processing which occurs below threshold, or before a problem type is brought to awareness, is not conscious. Before an item receives positive, or above threshold activation, the individual is not aware that any processing is occurring. The recognition of the appropriate problem type, and thus the realisation of the problem solution, seems to be sudden. Because the processing involved in this is below awareness, the emergence of the correct solution cannot be predicted.

The failure to solve an insight problem which usually follows the prediction of imminent success can also be accounted for by this conceptual account. When a problem solver is working on an inappropriately recognised problem type, one biased by past experience, the consistently positive nature of the connection weights means that the processing occurs consciously or in awareness. The individual concerned also believes that the problem they are working on is a member of this likely problem type. Thus, if a problem solver states that they are about to successfully solve the problem they are working on they could be applying an inappropriate problem type. As the insight problem cannot be solved by the

[1]Recall, for instance, the engineering student who spent 18 hours attempting to provide mathematical proof for the impossibility of the problem, based on the total number of squares (see section 2.2.6.1.).

application of this problem type, failure will follow the prediction of impending success.

4.2.2.10. <u>Suppression of insight by verbalisation</u>

It is quite easy to understand, in the terms of this conceptual account, why the crucial steps taken to achieve insight are not present in the verbal protocols of problem solvers. The cognitive processing involved in attaining insight occurs outside of awareness as we have seen. We can talk about the cognitive processing that we are aware of, but not the processing that we are unaware of. It is also entirely possible that the processing involved in insight occurs in a form which is very different from verbal expression. Perhaps we can talk about our thinking, but not how we think. This is Clark's (1989) understanding of the language of thought. It is also possible that, due to the suddenness of insight, problem solvers just do not have time between the recognition of the appropriate problem type and the problem solution to reflect on the manner in which they reached that point. This is the least likely explanation, however, because there would certainly be time for some discussion of the processes by means of which the appropriate problem type is selected.

What is more difficult to explain, is the finding that verbalisation during problem solving suppresses insight. The most likely explanation for this is that verbalising while working on a problem focuses attention. If a problem solver is discussing the solution attempt which they are working on, their attention is probably more fully focused on the problem type they are employing. This could heighten awareness of the problem type which has been recognised and thus suppress awareness of an alternative type. Information which is not associated with the well-learnt pattern will be suppressed.

### 4.2.3. Expert insight processing

We now move on to a consideration of the manner in which being an expert at insight processing could facilitate insight problem solving. It is important to recall that what is important in the attainment of insight during problem solving, is the processing that is necessary to produce insight. Therefore, if features of the literature on problem solving in general are to be applied to insight problem solving, they should be applied to the processing involved. It has been noted (in chapter two) that the concept of expertise has been inadequately applied to insight. We have seen that some consideration has been given to the effect of expertise in a knowledge domain related to the insight problem, but no application has been made to insight itself. The suggestion which this conceptual account has already made regarding the application of expertise to insight is that individuals can become experts at insight processing. It is necessary to examine what this means.

It is quite conceivable that people can become experts at displaying insight in problem solving, at recognising the ambiguous nature of the problem presentation, at recognising the inappropriate problem types which are suggested by past learning, and at recognising the pattern which suggests a non-obvious solution. This would involve an explicit awareness of the manner in which insight problems function and practice at the types of solutions which are required. Thus, expertise can be developed at recognising insight problems.

This would mean that, either the effects of past learning must be circumvented in the existing weighted connections, or that recognition of the insight problem leads to an immediate change in the weights to problem types that are potentially associated with the current problem. It is perhaps also necessary that information which is not usually associated with the recognised pattern is not heavily inhibited. The immediate activation of other information would involve the evaluation that there is a far greater difference between the current problem solution attempt and the desired solution than most people make. This would lead to a more substantial change in the existing weights. In either case, the notion of expert insight processing is entirely consistent with a neural network theoretical conception of insight and is a fascinating result of the application of general problem solving data to insight problem solving.

### 4.2.4. Reflecting on the sparring match in cognitive theory

It is now necessary to consider what this neural network theoretical account of insight implies for the debate in cognitive science and cognitive psychology. If this account proves to be a viable conception of the processes involved in attaining insight during problem solving, and insight is taken to be a higher cognitive process, there are grounds for suggesting that the neural network account may have application as a general theory of cognition. Insight is only one small feature of cognitive processing and strong conclusions cannot be drawn on the basis of one account. As has been stressed before, however, it is only by re-writing cognitive phenomena in neural network terms that we can begin to assess the strength of this theory. If this re-writing is successful for insight, there is little reason to believe that it will not prove to be successful for other features of cognition. We can then consider testing the supposition that some form of pattern recognition forms the basis of thought.

Should a neural network theoretical account not prove to be viable (and this cannot be decided on the basis of one set of empirical tests, but only by open critique and substantial testing), the notion that the basic character of thought can be understood by the recognition of patterns is not supported. This does not reflect in any way, however, on the proposal that neural network theory is applicable at the implementation level. It would then be necessary to consider an account of insight which uses a neural network implementation of a rule-based, symbolic processor. However, as symbolic theory has not proved itself to be successful in explaining insight to date, the potential of this approach would still be open to question. By positioning this work on insight within the divide between symbolic theory and neural network theory, no matter what the verdict concerning the viability of the account proves to be, some reflection on the debate in cognitive theory can be cast.

It could be argued from the perspective of symbolic theory that this entire enterprise is not worth pursuing, that this explication of the processing involved in attaining insight is not necessary, and that we should only be interested in outlining the general principles of cognition. The reply to this could run as follows. It is impossible to teach people to think insightfully if we do not know how insight processing occurs. Insightful thinking has great application in creativity, in research, and in advancing knowledge generally. We should surely want to advance our knowledge by any means possible. Also, we have seen the result of a lack of explanation for phenomena such as insight - they are reduced to the sphere of the mystical. Finally, why should we not want to explain the processing involved in insight if the explanation is available? The explanation must, however, be viable, and it is the task of the rest of this chapter to outline some of the central tenets of this theoretical model of insight which can be subjected to empirical testing. The formulation of this empirical test will also be outlined. This will constitute the first step toward establishing the potential of this conceptual account of insight. These central tenets will be posed as research questions, to render them amenable to testing.

### 4.2.5. Some research questions

The main contention which this account of insight has made, is obviously that insight is a form of recognition. This is not particularly amenable to direct testing. How do you decide experimentally, whether someone is recognising a the pattern to insight problems or not? However, it is indirectly testable on the basis of the predictions which this account of insight has generated. If these predictions hold, some credence is lent to this conceptual explanation of insight.

The first of these predictions concerns past learning. It was suggested (in section 4.2.1.) that insight only occurs when the biasing effects of past learning are overcome. Until this happens, the problem is recognised as an example of an inappropriate type and information which is associated with an insightful solution will not be applied to the problem. The prediction concerning past learning thus generates three related questions. Is information associated with the insightful solution applied to an insight problem without facilitation? Is insight facilitated by conditions which overcome the effects of past learning? Does the attempt to overcome past learning lead to the application of information related to the insightful solution?

A second prediction of this neural network theoretical account of insight is the notion of conceptual transfer. It was suggested (in section 4.2.2.4.) that making explicit the nature of insight, as well as making explicit the need to recognise the non-obvious problem type, will lead to significant facilitation of insight. This generates a clear question. Will making explicit knowledge concerning insight problems and the realisation which is necessary to solve them lead to a significant facilitation of insight?

A third and final prediction generated by the conceptual account of insight which is amenable to empirical testing, concerns expertise at insight processing. The suggestion was made (in section 4.2.3.) that if insight consists of a necessary recognition, it should be possible for people to be experts at recognising the pattern to insight problems. This produces the question: Is it possible to display expertise at insight processing?

Now that we have posed the research questions which can be addressed in order to begin an initial evaluation of the conceptual account of insight outlined in this chapter, it is necessary to move on to formulating the empirical testing of these questions.

### 4.2.6. Formulating an empirical test

The task of this final section of chapter four is to formulate the empirical method by means of which the research questions derived from the conceptual account of insight will be tested. The results of this study will provide an initial evaluation of the viability of this account of insight.

Data from the study will be produced in terms of number of problem solutions and, where problems are solved, time taken to solution. Number of solution attempts will not be used, because of the difficulty in defining what constitutes one solution attempt. This was discussed in chapter two (section 2.2.7.). The target problem will be the nine-dot problem. The results generated by problem solving with the nine-dot problem will be used to address the research questions posed in section 4.2.5. This problem is selected for several reasons which have already been established (in chapters two and four). These reasons will be summarised here. There is a well-established tradition of using the nine-dot problem in research on insight. As a result of this, there is little question that it can be regarded as a classic insight problem. It is therefore safe to suggest that the processing which problem solvers display in solving this problem, constitutes insight. Most of the key empirical findings which were highlighted in chapter two, were based on the nine-dot problem. Finally, this problem was also used as a basis on which to construct the conceptual account of insight.

However, as we saw in chapter two, there has been a suggestion that the nine-dot problem is atypical among insight problems. In particular, it has been suggested that the nine-dot problem is more difficult than other insight problems (see section 2.2.6.1.). Add to this the potential accusation that the use of one problem as the target for insight problem solving represents mono-operation bias and it is clearly necessary to address the issue of whether the nine-dot problem is similar to other insight problems.

To consider whether the nine-dot problem is atypical it is necessary to ask two questions. Is the nine-dot problem more difficult than other problems? In other words, are people more likely to solve other insight problems than they are to solve the nine-dot problem? And, is the nine-dot problem still difficult to solve even after problem solvers realise that they can utilise blank spaces on the page? Recall that the failure to recognise this possibility was one of the two impediments that could prevent insightful solution and that most insight problems possess only one impediment to solution. Once these questions have been addressed, the research questions derived from predictions made by the neural network theoretical account of insight in problem solving can be tested. Before outlining the method by means of which these questions are operationalised it is necessary to address the issue of sampling in relation to a methodological consideration raised in chapter two.

In chapter two (section 2.2.7.) the need to use a larger sample size and the feasibility of testing subjects in groups, were highlighted. When the opportunity arose to test an entire third year psychology class during the course of one cycle in their tutorial programme, this was considered an ideal opportunity to tackle the problem of sample size. This is due to the fact that approximately 180 students were registered for the course at that time. It was hoped that the use of this larger sample size would mean that sufficient numbers of participants would display insight, particularly following facilitation, so that the conditions which led to insightful problem solution could be more fully investigated than they have been in past studies.

As facilitation has increased the percentage of problem solvers who successfully solve the nine-dot problem from between zero and nine percent, to anywhere between forty three and one hundred percent, even if the sample is divided into different conditions there should still be sufficient correct solutions for meaningful comparisons. This would, in particular, allow the comparison of solution times. Given the detail concerning the processing necessary to reach insight which the neural network theoretical account provides, such comparisons could be especially meaningful. Quicker solution times could, for instance, suggest conditions which facilitate significant negation of past learning and the immediate availability of information associated with the insightful solution. It would be most interesting to compare conditions which produce slower solution times, to conditions which produce quicker solution times. The use of this larger sample size should make these comparisons possible. It does, however, present several other problems.

Participants would have to be assigned to conditions at the level of a tutorial group (approximately 12 people). It would be impossible to run different conditions within one group, particularly given the differences in procedure which would be necessary for various conditions. The methodological problem posed by this is that students had elected to be members of particular tutorial groups and that this would render these groups potentially non-equivalent. This can be partially overcome by randomly assigning entire tutorial groups to conditions, reducing the potential methodological problem and thus allowing the larger sample size to be utilised.

Another problem posed by the use of this class is the time limit imposed on tutorial sessions and thus on the length of time for an experimental condition. This places limits on the length of time which can be given to work on a problem. This is particularly important with the nine-dot problem which can take some time to solve. The length of problem solving time usually utilised by studies on insight, varies between ten and twenty minutes. Fifteen minutes would seem to be long enough to provide participants with the opportunity to solve the problem without prompting people to give up. It is, however, possible that this could reduce the number of participants who reach a solution to this problem. More time cannot be provided as this would not allow enough time to introduce other manipulations within the course of a session. Allowing fifteen minutes to solve the nine-dot problem means that the time which can be allotted to solve other problems is severely limited. For some problems, this time must be reduced to five minutes. However, as the nine-dot problem is the target problem, a minimum of fifteen minutes must be given to the solution of this problem. The limits which this places on solutions of other problems must be borne in mind. A second session at a later time cannot be considered, because it is far too likely that participants would discover the solution to the nine-dot problem before then, or that they would become familiar with other insight problems during this time.

The questions concerning the nine-dot problem can, therefore, be addressed by means of an experiment conducted with this larger sample size. The question of whether the nine-dot problem is more difficult than other problems can be answered by requiring participants to attempt to solve the nine-dot and other insight problems and then comparing the number of correct solutions across these problems. It is not expected that the nine-dot problem will prove to be more difficult than other insight problems. We have seen from the theoretical conception of insight that it is difficult in any form, because of the heavily biasing effects of past learning. Also, the model predicts that an individual who can solve one insight problem is likely to be able to solve others. This is due to the similarity of processing required to display insight across different types of problems. The content of the problem is not relevant, but the conceptual processing is.

The other insight problems selected for comparison to the nine-dot problem are the mutilated checkerboard problem, the horse and rider problem, the card problem, and the tower problem. These have all been clearly identified in the literature as classic insight problems. Kaplan and Simon (1990) make extensive use of the mutilated checkerboard problem, Scheerer (1963) uses the horse and rider problem as typical of insight problems, and Metcalfe and Weibe (1987) include the card problem and the tower problem as clear examples of insight problems in their studies. Also, these problems meet the operational definition of insight endorsed by this thesis. In particular, they all require a non-obvious solution. Please refer to appendix 1 for an example of these problems and their solutions.

The second question relating to the possible atypical nature of the nine-dot problem, asks whether this problem is still difficult after problem solvers have recognised that they can utilise blank spaces on the page. This question will be addressed by providing problem solvers with problems which overcome the effects of this source of past learning and which then lead to the application of information not associated with connect-the-dot problems. These problems will also make available information which would not usually be brought to bear on the nine-dot problem, because of the biasing effects of past experience. Recall that a possible source of the particular difficulty of the nine-dot problem might lie in the need to negate two sources of past learning. This will be addressed by considering whether people who extend the lines they draw as a result of facilitation are more likely to solve the nine-dot problem than people who have not displayed line extensions. No prediction is made concerning this finding. The problems selected for this facilitation are an adaptation of Weisberg and Alba's (1981a) problem facilitating line extensions, and a problem based on the Necker cube.

The facilitation problem used by Weisberg and Alba (1981a) consists of four dots positioned to suggest a triangle. Connecting the dots requires problem solvers to extend lines they draw so that a line will begin and end on a blank space of the page. The triangular shape of the problem makes it quite likely that they will produce line extensions. This effect is likely to be transferred to the nine-dot problem and forty three percent of the problem solvers in Weisberg and Alba's (1981a) study who solved the facilitation problem, solved the nine-dot problem.

This was in comparison to the zero percent who solved the nine-dot problem without facilitation. This facilitation problem does not require the problem solver to extend lines back through the problem in a fashion similar to the nine-dot problem. To accomplish this an adapted problem is designed which involves turning the problem on its side and adding an additional line of dots which will require the extension of lines back through the dot presentation. This should also help to reinforce the beginning and ending of lines at points beyond dots. Therefore, the problem is adapted for this study to provide a clearer facilitation of the nine-dot problem. Please refer to appendix 3 for a copy of the problem.

The second problem chosen to address the difficulty of the nine-dot problem is a version of the Necker cube. It presents problem solvers with a two dimensional square and asks them to make the square three dimensional (a copy of the problem is provided in appendix 3). This problem should encourage problem solvers to specifically draw lines beyond the square shape suggested by the dots of the nine-dot problem. It not only suggests that lines can begin and end in blank spaces, but also demonstrates transcending the shape of the initial problem presentation.

Now that we have outlined the manner in which the typicality of the target problem as an insight problem can be addressed, it is necessary to set out the manner in which the research questions can be tested. Let us first consider the research questions relating to the effects of past learning. The two questions dealing with the application of information associated with the insightful solution can easily be addressed by considering the features of the problem attempts which participants demonstrate. In particular, the number of problem solvers who attempt line extensions can be considered. It is expected that problem solvers will not extend lines into blank spaces of the page during without facilitation, because of the biasing effects of past learning, but that the removal of this bias will lead to a significant increase in the number of line extensions displayed.

The question relating to past learning which asks whether insight is facilitated by conditions which overcome the effects of past learning, is essentially the same as the question which asked whether solution of the nine-dot problem is significantly facilitated when problem solvers realise that they can use blank spaces on the page. A response to this question can also take the form of a consideration of the solution times following the presentation of facilitation problems designed to overcome the effects of past learning. These problems are the adapted line extension problem and the adapted Necker cube problem. Participants will be required to solve both of these problems to ensure that the effects of this source of past learning have been overcome. As it is possible that the effect these problems have in overcoming past learning will be different based on their order of presentation, this must be counterbalanced. In other words, some participants must receive the line extension problem first and some participants must receive the adapted Necker cube problem first. If there is no difference between these two groups, the data can be combined. It is predicted that solution times which follow either combination will be far quicker than solution times produced by subjects who receive no cues for negating past learning.

The mutilated checkerboard, horse and rider, tower, and card problems will be used to test the question concerning conceptual transfer. This question asked whether making explicit, knowledge concerning insight problems and the realisations which are necessary to solve them, will facilitate insight. The theoretical account of insight which generates this question predicts that merely providing a solution to an insight problem will not promote transfer, but that knowledge concerning the processing necessary to attain insight will be transferred and will facilitate problem solution. Problem solvers thus need to be provided with either an explanation of insight solutions to various insight problems before they are asked to solve the target problem, or they need to be given an explicit statement concerning the recognition which is necessary to achieve insight, together with the problem solution, before attempting the target problem. Problem solvers will be required to work on the example insight problems, before the solutions are presented in order to produce puzzlement. It was noted in chapter two that significant facilitation of insight only occurs after an initial attempt at problem solution. This will demonstrate to the problem solver that they are misrecognising-recognising the problem and will make the enhance the effect of the conceptual facilitation. It can also be predicted that the solution times for subjects who receive an explicit statement concerning the nature of insight problems will be far quicker than the solution times for subjects who are provided with only the problem solutions.

To provide a clearer picture of the facilitation effect lent by making explicit the nature of insight processing, solution times for the nine-dot problem following all facilitation problems can be compared.

The final research question generated by this conceptual account of insight which will be tested empirically, asks whether it is possible to display expertise at insight processing. If this is possible, the explicit statement of the manner in which insight problems operate should facilitate solution of the nine-dot problem. Also, those people who are able to solve one insight problem, should be able to solve others. In this case, participants should tend to solve none of the insight problems or all of them.

Now that the empirical testing of the research questions generated by the neural network conceptual account of insight has been formulated, it is necessary to outline this empirical test. We begin with an explanation of the method of the study.

# CHAPTER FIVE

# METHOD

## 5.1 Sample

The participants for this study were 152 students at the University of Cape Town. These students were all registered for the third year course in psychology at that university. Both biological sexes were included in the sample, though the sample is predominantly female. This reflects the fact that more female students choose to major in psychology than do male students. It was not deemed necessary to ascertain how many individuals belonged to each sex as this was not expected to influence the results in any way. Participants also belonged to different so-called race groups. Race was also not expected to influence the results in any way and therefore no information was collected on the racial composition of the sample. The participants also ranged quite considerably in age, but as they were all students majoring in psychology this was not expected to have any significant effect on the results.

Participants were required to take part in the study during the course of one of their regular tutorial sessions. Of the 180 students who were registered for the course at the time of the study, 28 did not attend their tutorial meeting in the week during which the study was conducted and therefore did not form part of the sample. Individual participants were not assigned to different conditions. Assignment took place at the level of the tutorial groups and was random. Each tutorial group consists of approximately 12 people, though some tutorial groups were slightly smaller due to student withdrawals from the course and absence from sessions. The number of students who would be attending each tutorial group could not be predicted as absence from groups is never uniform. As tutorial groups were tested as a whole, the number of participants tested together ranged from 6 to 12. This also meant that the number of participants in different conditions was not equal.

The study was divided into two parts and 4 tutorial groups were assigned to part one of the study. Due to absence from tutorial sessions, 34 participants in total provided data for this part of the study. Part two of the study collected data from 12 tutorial groups and this produced data for 118 participants. Part one of the study was divided into two conditions, with two tutorial groups assigned to each condition. Data was produced for 20 participants in condition one and 14 participants in condition two. Part two of the study was divided into four conditions, with 3 tutorial groups assigned to each condition. Data was contributed by 31 participants in condition one, 29 participants in condition two, 31 participants in condition three, and 27 participants in condition four.

## 5.2 Experimental design

The study was a post-test only control group design, to use Campbell's (1957) classification. There was one independent variable with 6 levels. The need to address different research questions led to the inclusion of independent groups design features, as well as the generation of frequency and categorical data. The design meets Kies and Bloomquist's (1985) criteria for a true experiment, having an active independent variable and equivalent groups created by randomisation.

Part one of the study followed an independent groups design, with position of the nine-dot problem as the independent variable. Both groups were required to solve the nine-dot, mutilated checkerboard, horse and rider, card, and tower problems. However, one group received the target problem (the nine-dot problem) first, and one group received the target problem last. The dependent variable for this design was time taken to solve the nine-dot problem. Data was also gathered on a second dependent variable, namely number of insightful solutions. Descriptive data on the features of the nine-dot solution attempts was also generated by both groups. These included the number of individuals who incorrectly claimed to have reached a solution, number of individuals who attempted to extend lines beyond dots, number of individuals who were near a solution, number of individuals who had seen the problem before, and number of individuals who had remembered the solution to the problem.

Part two of the study followed a post-test only design with one independent variable made up of four levels. The independent variable was type of facilitation provided. In the first facilitation condition, problem solvers were provided with the four insight problems (the mutilated checkerboard, horse and rider, card, and tower problems) and their solutions. In the second facilitation condition, problem solvers were provided with the same four insight problems and their solutions, but were also provided with an explicit statement of the nature of the processing required to solve each of these problems. The third facilitation condition involved presentation of the nine-dot problem, followed by the line extension problem, and then the adapted Necker cube problem. The fourth facilitation condition involved presentation of the nine-dot problem, followed by the adapted Necker cube problem, and then the line extension problem. Both groups from part one of the study were added to these four groups to provide two further levels of the independent variable. The group of participants who received the nine-dot problem first acted as a control group, while those who attempted solution of the mutilated checkerboard, horse and rider, tower, and card problems first served as a further type of facilitation. The logic behind including this as an additional facilitation condition rests on the possibility that participants who solve one of these problems may recognise the process which underlies their insightful solution, and apply this information to solution of the target problem.

Although conditions three and four of part two of the study look as though they should constitute a dependent groups design, this is not the case. Although participants received the nine-dot problem in the position of a pre-test, it is obvious that comparison of solution times between this first testing and the post-test would be meaningless. If participants solve the problem on the initial presentation, their second solution "attempt" is of no interest. The initial presentation of the nine-dot problem constitutes part of the facilitation condition, as well as an opportunity to gather more data concerning the nine-dot problem. The use of pre-testing, in a study on problem solving, is often rather pointless. Instead, solution times on the nine-dot problem, the dependent variable for this design, must be compared between participants.

Data was also gathered on a second dependent variable, namely number of insightful solutions of the nine-dot problem. This frequency data was gathered on the basis of a classification variable, namely, type of facilitation provided for solution of the nine-dot problem. Descriptive data was also generated from solution attempts made on the nine-dot problem. These included the number of individuals who incorrectly claimed to have reached a solution, number of individuals who attempted to extend lines beyond dots, number of individuals who were near a solution, number of individuals who had seen the problem before, and number of individuals who had remembered the solution to the problem.

## 5.3 Materials

Part one of the study used the nine-dot, mutilated checkerboard , horse and rider, card, and tower problems. These are all considered to be classic insight problems. (The problems only were presented to the participants in part one of the study. The nature of the insight required to solve the problem is provided for the reader's benefit and to demonstrate why these problems are all considered to be insight problems. Copies of the problems and their solutions can be found in appendix 1, while a complete collection of the materials for the study can be found in appendix 3.)

The nine-dot problem consists of three rows of three dots, which must be connected by drawing four straight lines without lifting the pen or pencil from the page. The insight required to reach problem solution is that the lines must be extended beyond the dots forming the square shape of the problem presentation, so that lines begin and end on blank pieces of the page

The mutilated checkerboard problem consists of presenting the problem solver with a checkerboard that has two of its opposite corners removed. The problem instructions state that there are 62 squares remaining on the board and that the problem solver must cover these squares with 31 dominoes, or prove that a complete covering is impossible. Each domino must cover two squares which are horizontally or vertically, but not diagonally adjacent. The insight required to reach problem solution is that the two opposite corners are the same colour. Therefore, there are an unequal number of squares of each colour left. As each domino must cover a black and a white square, a complete covering is impossible.

The horse and rider problem consists of two pictures. One picture shows two horses, one upside down on top of the other. The second picture shows two riders in an orientation opposite to the picture with the horses. The problem instruction requires the problem solver to place the riders on the horses so that two complete horses and riders are formed. The insight necessary to reach problem solution is that each of the new horses must be made up of parts from both of the original horses. The original horses cannot be used in their initial form.

The card problem consists only of an instruction. The problem solver is required to describe how to cut a hole in an 8X13 centimetre card that is big enough to put his or her head through. Obviously this card is not big enough to fit over the problem solver's head and the insight necessary to reach problem solution is that the word hole must not be understood in its usual sense. The problem solver needs to realise that a spiral can be cut out of the card which, when unfolded, will provide a "hole" big enough to fit his or her head through

The tower problem also consists only of a problem instruction. The problem solver is told that a prisoner is attempting to escape from a tower. He finds a rope in his cell which is half long enough to permit him to reach the ground. He divides the rope in half, ties the two halves together, and escapes. The problem solver is asked to suggest how he could have done this. The insight required to reach problem solution is that the rope can be cut in half vertically, not horizontally, and that two halves will be produced which are each half long enough to reach the ground. They can be tied together to permit escape.

Part two of the study also used these problems. Several other materials were used in addition to these problems. The solutions to these problems were used, as well as these solutions combined with a statement which made explicit the nature of the realisation necessary for insightful problem solution. Use was also made of two facilitation problems and their solutions. These were a line extension problem adapted for the study, and a version of the Necker cube problem adapted for the study.

The line extension problem presented the problem solver with three rows of three dots positioned in a zigzag. The instruction required the problem solver to connect these nine dots by means of three straight lines, without lifting pen or pencil from the paper. The solution requires that the lines be extended beyond the dots so that lines begin and end on blank spaces of the page

The adapted version of the Necker cube problem presented the problem solver with a two dimensional square which they were required to turn into a three dimensional square. The solution is to draw a version of the Necker cube

The materials used in the study were presented to participants on photocopied sheets of paper, either as individual sheets or as a stapled collection of sheets. Features such as the size of the problems or the typeface used were not considered to be particularly important for studying insightful problem solution and it was decided that they should just be reasonable. The mutilated checkerboard problem, for instance, had to be of sufficient size for participants to count the number of squares and distinguish the colour of the squares with ease. There is no reason to suspect that detailed features of the problem layout would have any effect on the attainment of insight.

## 5.4 Procedure

The description of the conduct of the study will be fairly detailed, given the rather unusual procedure of testing participants in groups on insight problem tasks.

Sixteen tutorial groups of approximately twelve students each were available during one week of the third year psychology tutorial programme, for inclusion in the study. The tutorial groups were randomly assigned to the different conditions and the tutors for the groups served as experimenters. The writer was one of the tutors.

Experimenters were provided with a sheet of instructions for each of the conditions, a copy of the information sheet that was designed for the participants, a scoring sheet that was marked for each tutorial session (all of which can be found in appendix 2), and the prepared package of problems ready for distribution to the participants for each tutorial session. Each experimenter would note down time at solution of a problem, for participants who solved problems, in hours, minutes and seconds.

As students arrived for their tutorial session they were assigned a participant number. Once all participants were seated, experimenters began the experimental session in accordance with the appropriate procedural instructions. For each condition participants received the first batch of problems prefaced by the participant instructions. Participants were asked to read the instructions, but not to turn to the next page until they were told to do so.

These instructions informed participants that they would be taking part in a study on problem solving and that they would be contributing data toward a research project. They were told that the time taken for them to solve the problems would be recorded, but that this was not a test and would not reflect on their abilities. Participants were asked not to look at what other members of the group were doing and not talk to other psychology students concerning the tasks they completed during the session. They were also asked to write on the problem sheets as much as they wanted to and, in particular, to write down the solution to the problem if they could. Participants were also instructed to clearly indicate to the experimenter that they had completed a problem so that the time at their solution could be noted.

Experimenters emphasised the need not to talk to other students about the study, as well as the importance of indicating to them when they had solved a problem. Participants turned to the next sheet in front of them and the start time for the experimental session was noted.

For part one of the study, the next sheet contained an insight problem. In condition one participants had only the nine-dot problem in front of them and were given 15 minutes to work on the problem. If a participant indicated that they had solved the problem within this time, the time at their solution was noted. Any participant who solved the problem within 15 minutes waited until the entire time period for work on this problem had elapsed. All participants were stopped after 15 minutes and the nine-dot problem was removed.

Participants were given four additional insight problems to solve. Participants indicated when they had solved a problem so that their solution time could be recorded, and they were free to move on to the next problem as soon as they had solved the problem they were working on. Participants were instructed to move on to the next problem every 5 minutes, regardless of whether they had solved the problem they were busy with, to ensure that each participant attempted all of the problems within the time available for the experimental session. Therefore, the second part of the experiment involved 20 minutes problem solving time in total.

The problems participants were given to solve during this 20 minute session, were the mutilated checkerboard, horse and rider, card, and tower problems. The order of these problems was random as it was felt that an order effect might otherwise occur. This could particularly be the case if a participant solved the first or second problem they received. It is conceivable that the ability to solve this problem would lead the participant to recognise the non-obvious nature of the required solution and apply this recognition to subsequent problems. As there were not enough participants available to counter-balance the order of all of these problems, it was decided to randomise the order to control for the order effect across participants. The consideration of order effects led to the addition of a second condition for part one of the study.

Although the possible order effects for all of the problems used for part one of the study could not be counterbalanced, there were enough participants available to counterbalance the order of the target problem, the nine-dot problem, if one considered the tasks for the experiment to be divided at the level of the nine-dot problem and other insight problems. It was particularly important to consider the order of the nine-dot problem as, to assess the difficulty and typicality of the nine-dot problem, it was necessary to have a group of participants working on this problem without having attempted another insight problem prior to this in the experimental session. However, it was also necessary, in order to address one of the research questions, to assess solution of the nine-dot problem after attempts with the other insight problems. This could best be addressed by positioning the nine-dot problem last in the sequence. In this way, the order effect for the nine-dot problem could effectively be counter-balanced. The positioning of the nine-dot problem after the other problems constituted condition two for part one of the study.

The procedure for condition two was exactly the same as the procedure for condition one, except that the order of problems presented to participants was different. Participants were given the randomly ordered collection of four insight problems, followed by the nine-dot problem.

Following completion of both of the conditions for part one of the study, participants were asked whether they had seen the nine-dot problem before, and whether they had remembered the solution to the problem. Participants were also asked whether they had seen any of the other problems before, if so which one(s), and once again whether they had remembered the solution(s). This completed the experimental session.

The procedure for conditions one and two of part two of the study was somewhat similar to the procedure for part one. Following the initial instructions from the experimenter, participants were presented with the mutilated checkerboard problem. They were given 5 minutes to work on the problem and were told that they could move on to the next problem as soon as they had solved the problem and had indicated that they had done so. The experimenter noted down the time at which any participant indicated they had solved a problem and, after 5 minutes, instructed participants who had not solved the mutilated checkerboard problem to turn the sheet and read the solution to the problem.

For condition one, this contained a statement of the problem solution only. For condition two, this contained the same solution statement as well as a statement which detailed the nature of the recognition which was necessary to solve the problem. Participants were given one minute to view this solution and were then asked to turn to the next problem. This was the card problem, followed by a statement of the solution (condition one) or a statement of the solution as well as an explicit statement of the recognition necessary to solve the problem (condition two). The same procedure applied for this problem, as well as for the two problems which followed it, namely the horse and rider problem and the tower problem.

The order of these problems was the same for all participants, though that order had been selected randomly. It was felt that the order of presentation of the problems and their solutions could effect transfer of the conceptual processing to the target problem (the nine-dot problem). As there was no interest in the ability of the participants to solve the four problems for which they would be given solutions, and participants were only required to work on the problems to induce the puzzlement which would facilitate conceptual transfer, it was felt that the order of these problems should be the same for all participants. In this way, the cumulative effect of conceptual transfer which would then be applied to the nine-dot problem would not be different as a result of the methodology of the study.

After 24 minutes of work on these problems and their solutions participants were instructed to move on to the final sheet in the batch of problems they had been provided with, if they had not already done so. This sheet contained the nine-dot problem. The procedure with respect to this problem was the same as that employed in part one of the study. Participants were given 15 minutes to work on the problem, were requested to indicate when they had solved the problem so that their time at solution could be noted, and were asked whether they had seen the problems before and remembered the solutions.

The procedure for conditions three and four of part two of the study were somewhat different. The participants received the same experimental instructions and began their problem solving work on the nine-dot problem. They were given 15 minutes to work on the problem and were allowed to move on to the next sheet in front of them as soon as they had completed the problem they were busy with and had indicated their solution of the problem.

The 15 minutes of problem solving time on the nine-dot problem was used to rule out participants who could solve the problem without facilitation and to induce puzzlement for those participants who would require facilitation. The use of 15 minutes might seem a little lengthy for this, but it was felt that these participants could also be used to provide information on the characteristics of problem solving with the nine-dot problem, without any intervention. These participants could be added to the participants in condition one, part one of the study, which would significantly increase the number of participants from whom this information could be derived. For this reason, the length of time given for problem solving on the nine-dot problem had to be the same for these participants.

At the end of the 15 minutes work on the nine-dot problem, participants who had not indicated that they had solved the problem were asked to move on to the next problem in the batch in front of them. This contained either the adapted line extension problem (condition three) or the adapted Necker cube problem (condition four). Participants were given 5 minutes to work on this problem. This was merely to induce puzzlement and, although participants were asked to indicate when they had solved a problem in order to maintain procedural consistency, there was no real interest in this time. After 5 minutes, those participants who had not indicated that they had solved the problem were instructed to move on to the next sheet. This contained either the solution to the adapted line extension problem (condition three) or the solution to the adapted Necker cube problem (condition four). After 1 minute of viewing time for this solution, participants were instructed to move on to the next sheet.

This contained the adapted Necker cube problem (condition three) or the adapted line extension problem (condition four). The same procedure applied and the corresponding solutions were viewed. The order of these problems was counter-balanced even though there was no interest in solution of these problem *per se*. These problems were included in an attempt to overcome a feature of the past learning associated with the nine-dot problem and it was felt that the cumulative effect of these cues might differ based on order of presentation

Once the procedure with both the adapted line extension and the adapted Necker cube problems was complete, participants who had not moved on to the final problem were instructed to do so. This was the nine-dot problem once more and 15 minutes problem solving time was allowed to enable an assessment concerning the effects of these facilitation problems. This would also make the final problem solving work on the nine-dot problem comparable across all four conditions of part two of the study. Participants were once more asked to indicate to the experimenter that they had solved the problem. The experimental session was completed by questioning participants concerning prior knowledge with any of the problems they had just seen.

This concludes the description of the experimental procedure for the study and it just remains to outline the scoring procedure for the main aspects of the data which was collected. As experimenters had as many as 12 participants to run in any one experimental session, they merely noted down the time at which the experimental session started and the time at which the participant solved a problem. The time to solution could then be calculated by working back to the start time, on the basis of how much time was allowed per problem and whether the participant had solved the problem prior to the one for which a solution time was noted. Solutions were judged to be correct against the standard insightful solutions used for these problems. These solutions are the ones which are included in appendix 1, and which participants received in the respective facilitation conditions. Solution times were not given to participants who indicated that they had solved a problem when in fact they had not.

Features of the problem solving attempts were judged according to the problem attempts participants had drawn on the problem sheets. Where participants had not written or drawn anything on the problem sheet, the problem was scored as unattempted. This happened very infrequently. Features of the participants problem solving attempts on all problems were scored according to whether they violated the problem instructions. In addition to this, features of the problem solving attempts on the target problem (the nine-dot problem) were scored on the following criteria: whether the problem solver had retraced a line, whether they had attempted line extensions beyond dots, whether their attempt was near a solution, and whether they had missed a dot.

For part one of the study, participants also received a score for the total number of correct problem solutions they produced. Following an interesting finding concerning the tower problem, all participants were re-scored on the tower problem for the numerical position in which they had received this problem within the batch of insight problems. We now move on to a consideration of this interesting finding, as well as the other findings of the study, in chapter six.

# CHAPTER 6
# RESULTS

## 6.1. Introduction

This chapter presents the findings of the empirical study described in chapter five. The study was designed to test predictions derived from the central tenets of the neural network theoretical account of insight in problem solving, developed in chapter four. This chapter will begin with a brief summary of the findings of the study, before moving on to a more detailed presentation of the results.

Of the 152 participants in total who produced data for the study, 14 participants were familiar with the nine-dot problem and were thus excluded from the main analyses. Of the 92 participants who produced problem solving attempts with no facilitation on the nine-dot problem, 83 were unfamiliar with the nine-dot problem and 8,43% of them were able to solve the nine-dot problem in the time allotted. Of the 118 participants who produced problem solving attempts on the nine-dot problem following facilitation, 107 were unfamiliar with the nine-dot problem and 14,02% of them were able to solve the nine-dot problem within the time provided. (Recall that the total number of problem solving attempts with the nine-dot problem does not equal the number of participants, as 58 participants - those in conditions three and four of part two of the study - produced problem solving attempts on the nine-dot problem prior to **and** following facilitation, where the initial attempt served as an unfacilitated attempt as well as forming part of the facilitation intervention.) The unexpectedly small number of participants who managed to solve the nine-dot problem meant that the research questions could not be addressed in terms of differences in solution times. Instead, differences in number of solutions had to be considered.

In terms of the typicality of the nine-dot problem, familiarity with the problem did decrease solution time and findings for the nine-dot problem were no different to findings for the mutilated checkerboard, horse and rider, and card problems. The tower problem, however, produced significantly more correct solutions.

The facilitation provided to overcome past learning did not produce a significantly greater number of solutions on the nine-dot problem. There was also no effect of facilitation on the nine-dot problem on the basis of conceptual transfer. There was no evidence to suggest that it is possible to display expertise at insight processing and only solution of the horse and rider problem was significantly correlated to solution of the nine-dot problem. The presentation of the results for the study is concluded with a description of two of the main features of attempts to solve the nine-dot problem. Alpha was set at $\alpha = 0{,}05$ for all statistical tests of significance.

## 6.2. Typicality of the nine-dot problem

In chapter four, the contention that the nine-dot problem is atypical was explored and it was suggested that the typicality of the nine-dot problem as an insight problem would have to be explored. This section reports the findings concerning the issue of typicality and the questions which this generated.

We will begin the presentation of the results of the study by considering the findings based on the 14 participants who were familiar with the nine-dot problem. They will then be excluded from the rest of the analyses. Of the 14 participants who were familiar with the nine-dot problem, 9 correctly solved the problem within the time available for problem solution. It was decided to conduct a comparison of the solution times for participants who were familiar with the solution to the nine-dot problem and participants who solved the nine-dot problem without facilitation. If solution times for participants familiar with the problem are no different to solution times for participants who were unfamiliar with the problem, and solved it without facilitation, the problem is so difficult that prior familiarity with it does not decrease solution time. This would reflect on the typicality of the nine-dot problem, as insight problems are generally easier to solve following prior exposure to the solution.

However, of the 9 participants who correctly solved the nine-dot problem, 3 belonged to groups who had received the nine-dot problem after facilitation while 6 had received the nine-dot problem without, or prior to, facilitation. It is important to address any possible difference between these two groups in terms of solution time, reported in seconds.

**Table 6.1: Mean time to solution for problem solvers familiar with the nine-dot problem, with and without facilitation**

| | 9-dot After Facilitation | 9-dot No Facilitation |
|---|---|---|
| Mean Time | 527 | 168,556 |
| Standard Deviation | 505,64 | 146,057 |

A two-tailed independent t-test revealed no significant difference in solution times between these two groups: $t_{(7)} = 2{,}038$, $p > 0{,}05$. However, given that the power of the test is unacceptably low ($\delta = 0{,}896$, with a medium effect size selected on the basis that the effect of insight facilitation on recall was unknown), the highly unequal variances, and the proximity of the t-value to the critical t-value for this test ($t_{crit} = 2{,}365$), it does not seem safe to combine these groups. Although it cannot be concluded that prior facilitation has any effect on recall of solution for the nine-dot problem, it will not be stated that groups which have had facilitation on insight are the same as groups which have had no facilitation on insight problem solving. Participants who recalled the nine-dot solution after facilitation at insight problem solving will therefore not be used in further analysis.

For the comparison of solution times for the 6 problem solvers who were familiar with the nine-dot problem, with those for the 7 problem solvers who were unfamiliar with the nine-dot problem, the following descriptive statistics were obtained.

**Table 6.2: Mean times to solution for problem solvers familiar and unfamiliar with the nine-dot problem**

| | Familiar 9-Dot | Unfamiliar 9-Dot |
|---|---|---|
| Mean Time | 168,556 | 568,572 |
| Standard Deviation | 146,057 | 189,678 |

A two-tailed independent t-test revealed a significant difference in solution times between these two groups: $t_{(11)} = -4{,}797$, $p<0{,}001$. As the sample size was very small, based on the fact that very few people were able to solve the nine-dot problem, consideration was given to the power of the test. Power was 0,36 ($\delta = 1{,}587$, $\alpha = 0{,}05$). A large effect size was selected for the determination of power, as it was expected that familiarity with the nine-dot problem would have a significant impact on solution time. Following this decision, the calculated power of the test was considered to be acceptably high. It can therefore be concluded that familiarity with the nine-dot problem does significantly decrease time needed for solution. This would suggest that the nine-dot problem is not atypical as an insight problem.

However, this comparison of solution times would be deceiving if a significant number of participants who were familiar with the nine-dot problem failed to recall the solution. Thus, a further reflection on the typicality of the nine-dot problem, which familiarity with this problem could provide, is based on a comparison of the number of individuals who are familiar with the problem and manage to recall the solution, with the number of individuals who are unfamiliar with the problem and manage to reach a solution. The 14 participants who were familiar with the nine-dot problem will therefore be compared to the 78 participants who, of the 92 participants who produced unfacilitated problem solving attempts on the nine-dot problem, were unfamiliar with the problem.

**Table 6.3: Number of solutions for problem solvers familiar and unfamiliar with the nine-dot problem**

| | 9-Dot Solved | 9-dot Not Solved |
|---|---|---|
| Familiar | 9 | 5 |
| Unfamiliar | 7 | 71 |

Analysis of the above Table revealed $\chi_{(1)}^2 = 21{,}573,\ p<0{,}005$. Therefore, a significantly greater number of problem solvers who are unfamiliar with the nine-dot problem, fail to solve the problem than do problem solvers who are familiar with the problem. This suggests that the nine-dot problem is not atypical and is significantly easier following familiarity with the problem.

Now that those individuals familiar with the nine-dot problem have been excluded from the analysis, we can continue. We begin with a consideration of the typicality of the nine-dot problem.

Before addressing the issue of typicality directly, it is necessary to answer an initial question which concerns the position of presentation of the nine-dot problem in the unfacilitated conditions. Recall that the problem was either the first problem participants saw, or the last in a sequence of insight problems. Although solution attempts on other insight problems cannot be considered a form of facilitation, it is conceivable that, where problem solvers reach an insightful solution, this recognition could carry over to subsequent problem solving attempts and act as facilitation. Although very few people managed to solve any of the insight problems, as can be seen from Table 6.4, it still seems prudent to assess possible differences in frequencies of solutions for position of the nine-dot problem. Of the 78 participants who atttempted to solve the nine-dot problem prior to any other problem solving attempts ( participants in condition one of part one of the study, and conditions three and four of part two of the study), 71 were unfamiliar with the nine-dot problem. Of the 14 participants in condition two of part one of the study who attempted to solve the nine-dot problem following problem solving attempts with the other insight problems, 11 of them were unfamiliar with the nine-dot problem.

**Table 6.4: Number of solutions for the nine-dot problem, based on position**

| | 9-Dot Solved | 9-dot Not Solved |
|---|---|---|
| 9-Dot First | 6 | 65 |
| 9-Dot Last | 1 | 10 |

Analysis of table 6.4 revealed $\chi_{(1)}^2$ = 0,2592, p>0,05. Therefore, no significant difference exists for number of solutions on the nine-dot problem in terms of position of the problem. These two groups will, therefore, be combined. This means that consideration of the nine-dot problem will be made in terms of those participants who attempted the problem without facilitation, and this includes those problem solvers who saw the nine-dot problem after working on other insight problems, and those participants who attempted the problem after facilitation. The problem solvers who received different types of facilitation can be divided according to the three types of facilitation.

*Question 1: Is the nine-dot problem more difficult than other insight problems?*

This question was addressed by a consideration of the number of correct solutions across the different insight problems to ascertain whether there were more correct solutions for other insight problems than there were for the nine-dot problem.

**Table 6.5: Number of problem solutions across insight problems in study part one**

| | 9-Dot Problem | Mutilated Checker-board Problem | Horse and Rider Problem | Card Problem | Tower Problem |
|---|---|---|---|---|---|
| Solved | 1 | 1 | 2 | 3 | 21 |
| Not Solved | 27 | 27 | 26 | 25 | 7 |

The 28 participants in part one of the study who were unfamiliar with the nine-dot problem as well as the other insight problems (6 of the 34 participants in part one of the study were familiar with at least one of the problems), have been included in table 6.5. Given the fact that so few people were able to solve the nine-dot, mutilated checkerboard, horse and rider, and card problems, a consideration of solution times across problems is impossible. We must, therefore, restrict our consideration of the difficulty of the nine-dot problem relative to other insight problems, to frequency and categorical data in terms of number of solutions and whether a problem is solved or not.

It is obvious from Table 6.5 that the mutilated checkerboard, horse and rider, and card problems are as difficult as the nine-dot problem and that there are not significantly more solutions on one of these problems than on the nine-dot problem. There is, therefore, no point in conducting a statistical test of significance of this question.

The tests which are conducted must be chosen with care to avoid conducting multiple $\chi^2$ comparisons on the same data. The only result of conducting statistical tests on data such as that in Table 6.5, would be to increase the Type I error rate. The only insight problem which shows a substantial number of solutions is the tower problem. This raises the question of whether the nine-dot problem is substantially more difficult than the tower problem. The performance on the nine-dot problem for the 11 participants in condition two of part one of the study who were unfamiliar with the insight problems, will therefore be compared to the performance on the tower problem for the 17 participants in condition one of part one of the study who were unfamiliar with the insight problems.

**Table 6.6: Number of solutions for the nine-dot problem presented last, and the tower problem when the nine-dot problem was presented first**

| | 9-Dot Problem | Tower Problem |
|---|---|---|
| Solved | 1 | 12 |
| Not solved | 10 | 5 |

Analysis of Table 6.6 revealed $\chi_{(1)}^2 = 8{,}516$, $p < 0{,}005$. Therefore, there is a significant difference in the number of problems solvers who were able to solve the nine-dot and tower problems. More participants were able to solve the tower problem than the nine-dot problem and it would appear that the nine-dot problem is significantly more difficult than the tower problem. The data for this analysis came from the two different conditions in part one of the study to avoid the problem of comparing frequency data from the same participants. We saw in the analysis of Table 6.4 that there was no difference in the ability to solve the nine-dot problem based on the position of the problem, and the tower problem occurred in random order across both conditions. The two conditions can therefore be considered to be equivalent for this comparison.

*Question 2: Is the nine-dot problem still difficult to solve even after problem solvers realise that they can use blank spaces on the page?*

This question essentially asks whether the nine-dot problem is still difficult even after one source of past learning has been removed as an impediment to solution. This will be addressed by considering the number of people who extended lines and then solved the nine-dot problem following facilitation in the form of the adapted line and Necker cube problems. Of the 58 participants in conditions three and four of part two of the study, 26 extended lines in attempting to solve the nine-dot problem. However, 2 of the participants extended lines prior to facilitation, and will therefore be excluded from the consideration of this question.

**Table 6.7: Number of solutions for the nine-dot problem following facilitated extension of a line**

| | 9-Dot Solved | 9-Dot Not Solved |
|---|---|---|
| Extended Line | 14 | 10 |

Analysis of Table 6.7 revealed $\chi_{(1)}^2 = 0{,}375$, $p>0{,}05$. Therefore, there was no significant difference in the number of solutions on the nine-dot problem for participants who extended a line. It would seem that the nine-dot problem is still difficult even after participants realise that they can use the blank spaces on the page. This is supported by the fact that only 2 people who extended lines during the first presentation of the nine-dot problem went on to solve the problem following facilitation with the adapted line and Necker cube problems.

## 6.3. Effects of past learning

The question of whether overcoming the effects of past learning will facilitate insight, can be considered by addressing three subsidiary questions.

*Question 1: Is information associated with the insightful solution applied to an insight problem without facilitation?*

**Table 6.8: Number of problem solvers who extended a line for the nine-dot problem without facilitation**

| | Line Extensions | No Line Extension |
|---|---|---|
| No Facilitation | 4 | 79 |

Only 4 of the 83 people who attempted to solve the nine-dot problem without faciliation, and who were unfamiliar with the problem, extended lines in their problem solving attempts with the nine-dot problem without any facilitation for line extensions. It is therefore obvious that the information that it is possible to extend lines into blank space, which is associated with the insightful solution, is not applied to the nine-dot problem without any facilitation. Again, a test for significant differences is not really necessary here to address the question posed, and would only serve to increase the chances of making a type I error, because of multiple chi-square tests on the same data.

*Question 2: Does the attempt to overcome past learning lead to the application of information which is associated with the insightful solution?*

This question can be addressed by comparing the number of people who extend a line in their problem solving attempt with the nine-dot problem following presentation of the adapted line and Necker cube problems, with those who extend a line without facilitation. The facilitation condition is made up of the 58 participants in conditions three and four of part two of the study, excluding the two participants who extended lines prior to faciliation and the 6 participants who were familiar with the nine-dot problem. The condition without facilitation consists of the 34 participants in part one of the study, excluding the four who were familiar with the nine-dot problem.

**Table 6.9: Number of problem solvers extending lines for the nine-dot problem following facilitation to overcome past learning, and no facilitation**

| | Facilitation | No Facilitation |
|---|---|---|
| Line Extensions | 16 | 4 |
| No Line Extension | 34 | 26 |

Analysis of Table 6.9 revealed $\chi_{(1)}^2 = 2{,}56$, $p > 0{,}05$. Therefore, there is no significant increase in the number of participants extending a line in their problem solving attempts with the nine-dot problem following presentation of the adapted line and Necker cube problems. This would suggest that these problems do not overcome the effects of past learning which inhibits the use of blank spaces on the page, and which is brought to bear on the nine-dot problem.

The power of this test is obviously quite low. The test of interest is based on only 20 people who extended lines in their problem solving attempts, and it is highly unlikely that significant differences would be detected on the basis of such limited participant numbers. As Blalock (1960) argues, the power of the chi-square test to detect significant differences doubles with the number of participants in the test. Although the measure of power is based on observed frequencies in all cells, it is apparent that the limited subject numbers for subjects who extended lines would be a problem here. The low power, combined with an apparent trend in the data toward extending lines following facilitation, suggests that the no difference finding could be a result of the sample size for this test. Strong conclusions to the effect that attempts to overcome past learning do not lead to the application of information associated with the insightful solution can thus not be made.

*Question 3: Is insight facilitated by conditions which overcome past learning?*

This question can be addressed by comparing the number of participants who solve the nine-dot problem following facilitation designed to overcome past learning, with the number who do not solve the problem following this facilitation. Before conducting this comparison, it is important to consider whether there is any order effect for the adapted line and Necker cube problems. Recall that the adapted line problem appeared first in condition three and that the adapted Necker cube problem appeared first in condition four of part two of the study. The 47 participants who were unfamiliar with the nine-dot problem (11 participants were familiar with the problem) will be used to address this question.

**Table 6.10: Number of solutions for nine-dot problem in condition 3 and condition 4**

| | Condition 3 | Condition 4 |
|---|---|---|
| 9-Dot Solved | 4 | 5 |
| 9-Dot Not Solved | 22 | 16 |

Analysis of Table 6.10 revealed $\chi_{(1)}^2$ = 2,1696, p>0,05. Therefore, there was no significant difference in the number of solutions based on order of the facilitation problems. These two groups can therefore be combined, and these participants can be compared to those participants in part one of the study.

**Table 6.11: Number of solutions for the nine-dot problem following facilitation designed to overcome past learning, and no facilitation**

| | Facilitation | No Facilitation |
|---|---|---|
| 9-Dot Solved | 9 | 1 |
| 9-Dot Not Solved | 38 | 27 |

Analysis of Table 6.11 revealed $\chi_{(1)}^2$ = 2,087, p>0,05. Therefore, there was no significant increase in the number of participants who solved the nine-dot problem following presentation of the adapted line and Necker cube problems. This would suggest that there is no significant increase in the number of participants displaying insightful problem solving following conditions which attempt to overcome past learning.

However, a trend can once again be detected in the data, with more people tending to solve the nine-dot problem following facilitation designed to overcome past learning. Once more, the power of the test is expected to be low, based as it is on only 10 participants who were able to solve the problem. This suggests that strong statistical conclusions concerning no effect of facilitation cannot be drawn. If the facilitation designed to overcome past learning does indeed have a significant effect, there is very little chance that this test would have detected the effect.

## 6.4. Conceptual transfer

One of the predictions generated by the theoretical model, was that making explicit the nature of the processing necessary to reach insight would significantly facilitate insight.

*Question 1: Is there a significant facilitation of insight by conceptual transfer?*

This was answered by comparing the number of participants who solved the nine-dot problem following conditions which facilitated conceptual transfer, with the number of participants who solved the nine-dot problem in the other facilitation conditions, and the no facilitation conditions. Of particular interest was the comparison between the number of participants who solved the problem following conceptual transfer and the number of participants who solved the problem following presentation of insight problems and their solutions. The 26 participants in condition two of part two of the study who were unfamiliar with the nine-dot problem, will be compared to the 29 participants in condition one of part two of the study who were unfamiliar with the nine-dot problem, to address this particular question.

**Table 6.12: Number of participants solving the nine-dot problem across all facilitation conditions**

| | No Facilitation | Insight Solution | Insight Solution and Statement | Line and Necker cube facilitation |
|---|---|---|---|---|
| Solved | 1 | 3 | 4 | 9 |
| Not Solved | 27 | 26 | 22 | 38 |

Analysis of Table 6.12 revealed $\chi_{(3)}^2 = 3{,}542$; $p > 0{,}05$. Therefore, there is no significant difference in the number of participants who solved the nine-dot problem in any of the conditions. Facilitation had no effect on number of participants solving the problem and, in particular, there was no effect for conceptual transfer over presentation of insight problems and their solutions. Once again, this question could have been more clearly addressed by a comparison of solution times, but the very limited number of participants who were able to solve the nine-dot problem in any condition, renders this comparison pointless. This contributes to the apparent lack of power of the test once more. It is not possible to determine whether an apparent trend revealed by the data is significant or not.

## 6.5 Expertise at insight processing

The prediction generated by the theoretical account of insight in terms of expertise, was that it should be possible to exhibit expertise at insight processing, as displayed in insight problem solving.

*Question 1: Is it possible to display expertise at insight problem solving?*

This question could be addressed by considering the percentage of participants who solved the nine-dot problem following the condition for conceptual transfer (based on the four participants who solved the nine-dot problem in condition two of part two of the study), with the percentage of participants who solved the nine-dot problem following presentation of insight problems and their solutions (based on the 3 participants who solved the nine-dot problem in condition one of th part two of the study). The conceptual transfer condition should make explicit the nature of the processing necessary to solve an insight problem.

**Table 6.13: Percentage of participants solving the nine-dot problem following conceptual transfer, and presentation of insight solutions**

| | Conceptual transfer | Insight Solutions |
|---|---|---|
| % Solved 9-Dot | 18,18 | 11,54 |

As we saw from the analysis of Table 6.12, there was no significant difference in the number of people who solved the nine-dot problem across the different conditions ($\chi^2_{(3)}$ = 3,542, p>0,05), and therefore no significant difference between the conditions presented in Table 6.13. There is thus no evidence to suggest that it is possible to display expertise at insight processing following conceptual transfer.

*Question 2: Is there a relationship between ability to solve one insight problem and ability to solve another?*

If participants display a consistent ability to solve or not solve insight problems, this suggests that people are able to develop expertise at insight processing. This question was addressed by correlating solutions across all of the problems in part one of the study, namely the nine-dot, mutilated checkerboard, horse and rider, card, and tower problems, where solution on each problem was coded dichotomously.

**Table 6.14: Correlation matrix for ability to solve an insight problem, across insight problems**

| | 9-Dot Problem | Mutilated Checker-board Problem | Horse and Rider Problem | Card Problem |
|---|---|---|---|---|
| Mutilated Checker-board Problem | $\phi$ = -0,036<br>$\chi^2$ = 0,036 | | | |
| Horse and Rider Problem | $\phi$ = 0,694<br>$\chi^2$ = 13,486 | $\phi$ = -0,051<br>$\chi^2$ = 0,073 | | |
| Card Problem | $\phi$ = -0,064<br>$\chi^2$ = 0,115 | $\phi$ = -0,064<br>$\chi^2$ = 0,115 | $\phi$ = -0,092<br>$\chi^2$ = 0,237 | |
| Tower Problem | $\phi$ = 0,117<br>$\chi^2$ = 0,383 | $\phi$ = 0,117<br>$\chi^2$ = 0,383 | $\phi$ = -0,136<br>$\chi^2$ = 0,518 | $\phi$ = 0,210<br>$\chi^2$ = 1,235 |

Phi coefficients are reported in each case in table 6.14, and are then converted to chi-square values for the test of significance[1]. A consideration of the correlation matrix reveals that only the nine-dot and horse and rider problems are significantly correlated ($\chi_{(1)}^2$ = 13,486, $p<0{,}005$). This would suggest that those people who are able to solve the nine-dot problem, are also able to solve the horse and rider problem, and vice versa. However, this is not enough to suggest that there is a relationship in ability to solve insight problems. Though the correlation was based on 28 people, only 1 person had solved the nine-dot problem, and 2 people had solved the horse and rider problem. This finding can therefore not be used to support the statistical claim that there is a

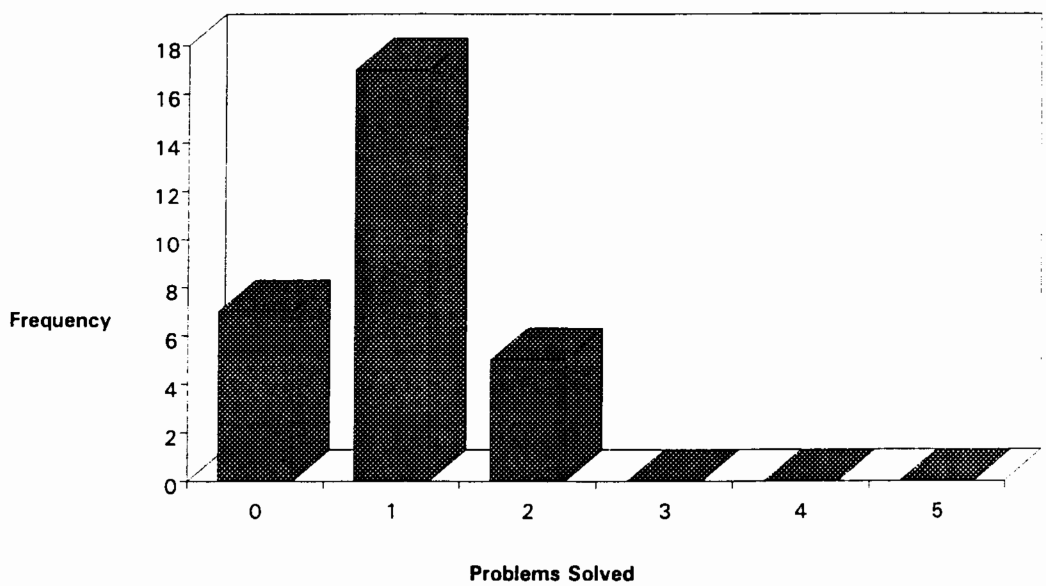
[1] This was considered more expedient than drawing up 10 two-by-two chi-square tables to adress this question. The use of chi-square to test the phi coefficients was deemed to be equivalent to the use of two-by-two tables with chi-square.

relationship in ability to **solve** these two problems. Rather, it demonstrates a correlation in **not** solving the nine-dot and horse and rider problems. This just confirms that these problems are both very difficult to solve.

*Question 3: Is a person who solves one insight problem more likely to solve another?*

The notion of expertise at insight processing suggests that an individual who is able to solve one insight problem should be able to solve another insight problem as well. This can be addressed by considering how many insight problems participants are able to solve across the problems in part one of the study. If participants display expertise at insight processing they should be clustered, according to the number of insight problems they solve, at either 0 or close to 5.

**Figure 6.1.: Participants grouped by number of problems solved**




As can be seen from the figure above, it is clear that there is an overall low ability to solve the insight problems in the study. Number of problems solved tends to cluster close to one, because a substantial number of participants were able to solve the tower problem, but they tended not to solve any of the other problems.

## 6.6 Descriptive data

This presentation of the findings for the study will close with a presentation of some descriptive data for solution attempts on the nine-dot problem.

**Table 6.15: Percentage of participants incorrectly claiming solution on an insight problem**

| 9-Dot Problem | Mutilated Checker-board Problem | Horse and Rider Problem | Card Problem | Tower Problem |
|---|---|---|---|---|
| 8,57 | 29,41 | 61,18 | 67,65 | 14,71 |

It was therefore quite likely, particularly on the horse and rider and card problems, for participants to claim that they had solved a problem when in fact they had not.

**Table 6.16: Number of participants near solution on the nine-dot problem, relative to the number who solved the problem**

| | 9-Dot Unfacilitated | 9-Dot Facilitated |
|---|---|---|
| Near Solution | 4 | 8 |
| Solved | 1 | 8 |

It would appear that few of the participants approached a solution to the nine-dot problem, without managing to solve the problem. Now that the findings for the study have been presented, we move on to the discussion and an interpretation of these findings in terms of the theoretical account of insight generated in chapter four.

# CHAPTER 7
# DISCUSSION

## 7.1. Introduction

In the final chapter of this thesis, an attempt will be made to understand the findings reported in chapter six and to explore the implications they pose for the neural network theoretical account of insight proposed in chapter four. This perspective on insight suggested that the processing involved in reaching insight during problem solving can be understood as a recognition of the pattern to insight problems. This proposal generated several predictions which were empirically testable. The results of this test (the findings reported in chapter six) then reflect on the viability of this account of insight. Although they cannot offer a definitive conclusion concerning the viability of this neural network account of the processing involved in insight, the conclusions which can be drawn from the study will be explored. This will entail an attempt to understand the findings in terms of the theoretical predictions which were made by this account of insight, as well as in terms of a methodological evaluation of the empirical test, and a statement of the conclusions which can be drawn concerning the viability of the neural network theoretical account of insight. As it is difficult to separate the exploration of the findings in theoretical and methodological terms, this will be conducted simultaneously.

## 7.2. Understanding the findings

Before considering what implications the findings reported in chapter six might hold for the theoretical predictions concerning insight, it is important to consider that the findings do perhaps not represent a precise test of these predictions. This is because the effect of any of the conditions which the theoretical model suggested should significantly facilitate insight, could not be judged in terms of a comparison of solution times. This was the result of a very limited number of participants who were able to solve the nine-dot problem. It seems important, then, to begin with a consideration of this most surprising finding.

The 8,43% of the sample who were able to solve the nine-dot problem unaided is comparable with previous studies. The 14,02% who were able to solve the problem following facilitation is extremely surprising, being far lower than the smallest percentage of correct solutions (43%) reported in the literature. The absence of a significant facilitation effect held across all facilitation conditions. As the findings concerning the unfacilitated condition are not dissimilar to results in the literature, this would suggest that the surprising finding in respect of facilitation is not a consequence of sample characteristics, particularly as participants were randomly assigned to experimental conditions at the level of a tutorial group (approximately 12 people). Although it is still possible that there are some minimal effects of pre-selection bias to these tutorial groups, this should be small and it is extremely unlikely that it could account for the substantial difference to past findings in terms of facilitation which this study reports.

It also suggests that differences in experimenters and in experimenter conduct cannot be used to explain the insignificant effect of facilitation. Experimenters were assigned along with participants at the level of the tutorial group. Although this could introduce some effects on the basis of pre-selection, once again, as tutorial groups were randomly assigned to experimental conditions it is highly unlikely that these effects could have been systematic enough to account for the consistent lack of effect of facilitation. It can therefore be suggested that the small percentage of participants who solved the nine-dot problem following facilitation in this study, was a consequence of the facilitation conditions although there is no direct evidence for this . Whether the absence of a facilitation effect suggests that the predictions derived from the theoretical account of insight are inaccurate, and thus that the theoretical account of insight is not viable, or whether this finding is the consequence of methodological factors will be explored next.

There were essentially three main predictions generated by the neural network theoretical account of insight and they all suggested conditions which would facilitate insight in problem solving. It is therefore necessary to consider each of these predictions and their associated findings in turn, to establish whether the lack of significant facilitation has major theoretical or methodological implications. We begin by considering the prediction that overcoming the effect of past learning will significantly facilitate insight.

The absence of a significant facilitation effect was the result of testing the prediction concerning the effects of past learning. This was based on the presentation of two problems designed to overcome a source of past learning thought to impede insight on the nine-dot problem. This was the familiar group of connect-the-dot tasks which were thought to teach problem solvers that lines drawn for the nine-dot problem should begin and end on dots. It was therefore suggested that problem solvers do not realise that they can use blank spaces on the page and that providing them with problems which would require them to do so, would overcome this source of past learning. The adapted line and Necker cube problems were chosen for this, but showed no significant effect. This could be due to the fact that the nine-dot problem is thought to elicit a second source of past learning which would serve as an impediment to solution - the group of optimisation problems. This would be consistent with the theoretical account of insight presented in this thesis. As there was no intervention to overcome the impediment posed by optimisation, this question cannot be addressed and no clear conclusions can be drawn concerning the viability of the account on the basis of the predictions made concerning past learning.

It is also conceivable that the adapted line and Necker cube problems used in this study were not successful in demonstrating to participants that they could extend lines beyond dots. This is supported by the finding that there was no significant increase in the number of line extensions produced during problem solving attempts with the nine-dot problem following presentation of these problems. However, as the line problem is merely a slightly adapted version, one which more closely resembles the nine-dot problem, of the four dot problem which was used successfully by Weisberg and Alba (1981a) to facilitate line extensions, it seems unlikely that this finding is a consequence of the adapted line problem.

It is possible that the Necker cube problem suppressed line extensions. This seems unlikely, however, especially as one would expect that the effect of suppression might be greater when this problem is presented last, a finding which did not hold as there was no effect of order of the two facilitation problems. What is possible, methodologically speaking, is that the limited number of individuals who extended lines when attempting to solve the nine-dot problem, bequeaths a lack of power to the statistical test of the effects of overcoming past learning. Therefore, if the attempt to overcome the learning derived from tasks such as connect-the-dot drawings does have a significant facilitation effect, this could not be detected by the statistical testing conducted by this study

The findings concerning the effects of past learning neither support nor refute the theoretical model. From a methodological point of view, it is possible that participants were frustrated by being presented with the nine-dot problem a second time after they had failed to solve it initially, and therefore did not concentrate on the attempt to solve the problem. The risk of producing frustration was justified by using the initial presentation of the nine-dot problem to induce puzzlement and to provide a larger number of subjects who attempted the nine-dot problem without facilitation.

From a theoretical point of view, although the absence of a facilitation effect suggests that the contention that insight is pattern recognition does not hold, there are two interpretations of the findings which are consistent with the model. These are that solution on the nine-dot problem is impeded by a second source of past learning, and that overcoming the effects of past learning and thus fostering the recognition of the pattern which links two problems is extremely hard. It must be, because even inducing puzzlement on top of the presentation of facilitation problems had no effect on ability to solve the problem. This is despite the fact that the research literature reports that inducing puzzlement prior to facilitation leads to significant facilitation of insight problem solving. The findings concerning the lack of a significant effect of facilitation designed to overcome past learning could therefore be interpreted as consistent with the theoretical account of insight in neural network terms. The statement that displaying the recognition necessary for insight is difficult, can certainly be applied to the understanding of the findings concerning conceptual transfer.

These findings demonstrated no significant facilitation as a result of making explicit the nature of the recognition necessary to reach insight. Again, this could be understood methodologically or theoretically. The methodological question here, however, seems to be only one of operationalisation. Does the statement concerning the solution to the problems, the impediments based on past learning which people usually bring to bear on the problem, and the fact that insight problems cannot be solved on the basis of the most obvious solution, make explicit the recognition necessary to reach insight? Possibly not, especially as there is no clear evidence to suggest exactly what insight processing is.

The theoretical perspective on the findings concerning conceptual transfer has a lot more to offer. Even if the nature of the intervention does make explicit the recognition necessary to reach insight the problem solver still has to make that recognition. This will be difficult to do especially as their past experience will be inhibitory and they will be recognising a new pattern. Insight problems are, after all, exceptional. This generates a more fundamental theoretical question, one which questions the assumption that the pattern requiring recognition is the one which links all insight problems. In other words, is it necessary to recognise that insight patterns are linked by the fact that they require a non-obvious solution, or is there a pattern to each individual insight problem? If so, this could account for the difficulty of insight problems. It would render them all unique. Linked to this is the possibility that recognition of the pattern that links insight problems cannot be learnt on the basis of an instruction, but only on the basis of experience with the pattern. Thus, an individual would have to solve an insight problem to be able to recognise the pattern to insight problems. This is essentially the prediction concerning expertise.

Although the findings did not support the prediction that it is possible to display expert insight processing, the data available did not constitute a fair test of this prediction. This was a result of the very limited number of participants who were able to solve the nine-dot problem without facilitation. For this reason it was not possible to draw any real conclusions concerning the significant correlation between ability to solve the nine-dot problem and ability to solve the horse and rider problem. It was also not possible to accept at face value the finding that there was no significant correlation between solutions of the other insight problems, a finding which would otherwise suggest that there was no demonstration of expertise at insight processing.

This was probably the strongest prediction of the neural network account of insight, and it is unfortunate that it did not receive a fair test. The ceiling effect which seems to occur in the number of people who are able to solve the nine-dot, and most of the other insight problems used in this study, must be addressed by increasing the initial sample size. Although the sample size employed by this study was substantially higher than is usually used to study insight in problem solving, it was obviously not large enough. It would seem fair to estimate, given the fact that this study has reasonable external validity to insight in problem solving in terms of its sample size, that approximately 8-9% of any sample will solve the nine-dot or similar insight problems. The number of participants needed for a sufficiently powerful test of hypotheses concerning insight can be calculated and the sample size necessary to provide this number of correct solutions can, therefore, be established. The characteristics which enable these individuals to achieve insight in problem solving can then be explored.

Let us attempt the calculations to establish the required sample size for a valid test of hypotheses concerning insight in problem solving, making a few decisions regarding the required power and the effect size which may vary between researchers. Imagine that we wish to conduct a t-test on solution times for participants who solve the nine-dot problem unaided and participants who solve the nine-dot problem following an intervention designed to overcome the effects of past learning. If we set our required level of power at a moderate 50% and use $\alpha$ = 0,05, we will need 31 participants per sample who solve the nine-dot problem if we assume that the effect size for past learning is medium (0,5 - we are being moderate throughout). Based on the prediction that 9% of all participants will solve the nine-dot problem without facilitation, we need at least 344 participants who attempt to solve the nine-dot problem without facilitation.

Should we decide to use a more stringent, 80% level of power and on top of this decide that the effect of facilitation will be large ($\gamma$ = 0,8), we will need a minimum of 25 participants who solve the nine-dot problem unfacilitated to address our question concerning past learning. This represents an initial sample size of 278 participants. Assuming that we will not want to run the study to address this question on the basis of one type of intervention and that we will therefore require additional participants who can be assigned to various facilitation conditions, the size of the required pool of participants becomes extremely large for an experimental design.

It could be argued that the external validity of this study is not sufficient to draw conclusions concerning the number of people in any sample who will solve the nine-dot problem, given the 0-9% range suggested by the literature and the fact that this study used students. The use of students was motivated by the notion that students are not substantially different to other possible participants in respect of problem solving, and therefore the external validity of the conclusion concerning the number of participants who will solve the nine-dot problem unfacilitated is not damaged. Students were also selected as the participants for the test of theoretical predictions in this thesis, because they represent a fairly stringent test of a theory by virtue of their homogeneity as a sample (Rosenthal & Rosnow, 1991). A feature on which they are particularly homogenous is education. This would certainly imply a fairly uniform source of past learning, something which was particularly vital to this study.

To round off our consideration of the findings relating to expertise at insight problem solving it is necessary to consider the result that problem solvers who solved one insight problem did not go on to solve many more. From the prediction that it was possible to be an expert insight problem solver it was expected that people would show a uniform ability to solve insight problems. The finding that most people tended to cluster around one insight problem solved did not support the notion of expertise. However, if we look at the results more closely it is possible to argue that participants did demonstrate a uniform ability at insight problem solving.

Most of the participants solved one insight problem and the majority of participants solved the tower problem. This suggests that the tower problem is atypical among the insight problems included in the study. If we were to remove the tower problem from the data we would see that most people cluster around 0 problems solved, a uniform ability at insight processing. This is consistent with the theoretical model. It is still noticeable, however, that people who did manage to solve one insight problem did not solve more. In fact, if we were to remove the tower problem we would be left with no participants who solved more than one problem. There is, however, a possible methodological explanation for this. Due to the time limitations imposed by using experimental sessions with a fixed, maximum length, the solution times for problems other than the nine-dot problem had to be reduced to 5 minutes. It is possible that the limited number of solutions is due to this time restriction. A fair test of the prediction concerning expertise would have required more time.

We have seen that the findings cannot be used to either support or refute the neural network theoretical account of insight. Consideration was given to the typicality of the nine-dot problem so that the external validity of any findings concerning the model based on this problem, could be assessed relative to insight in problem solving more generally. Although the finding that the nine-dot problem was not different to most of the insight problems included in the study is not useful in terms of extending findings, the consideration of typicality did reveal the atypicality of the tower problem. Significantly more people were able to solve this problem than any of the other problems. This was not a result of the position of the tower problem in the sequence of problems, even though the random ordering happened to place most of the tower problems in position two or four in the sequence. The atypicality of the tower problem must, therefore, be due to the nature of the problem itself.

The tower problem does meet the operational definition of insight - its solution is non-obvious. However, there are degrees of “non-obviousness”. Most people have access to the information that it is possible to cut things vertically. We do so often, think of fruit and vegetables for instance. These are objects that, even if we do not cut them ourselves, we encounter sliced in this fashion on a regular basis. It is also fairly easy to think of situations in which we unravel objects such as string.

Therefore, the past learning which would facilitate solution of the tower problem is readily available. The significant number of participants who solved this problem is therefore consistent with the theoretical account of insight offered by this thesis. However, as the empirical findings produced to test the theoretical model of insight did not support the predictions generated by this model, this finding alone cannot be used to support the model. It is also conceivable that this finding could be explained in alternate theoretical terms.

## 7.3. Viability of the neural network account of insight

It has been established during the course of this discussion that the empirical findings reported in this thesis neither support nor refute the theoretical account of insight in neural network terms. This was established on a methodological and a theoretical basis. Although it was suggested (in chapter four) that one set of empirical tests were certainly not enough to test the viability of a theory and that the empirical testing in this thesis was merely an initial commentary on a newly written conceptual account of insight, it is now even more difficult to pronounce on the viability of the neural network account of insight.

None of the empirical tests provided support for the theoretical account of insight in neural network terms. However, it has been argued that this lack of support can be explained on methodological and theoretical grounds. There are also many other questions which the empirical work of this thesis has raised, and which can be tested in order to provide further illumination on the viability of this account.

These include questions such as whether the removal of optimisation as a source of past learning would facilitate solution on the nine-dot problem, whether it is possible to overcome the effects of past learning with more intensive interventions or longer time periods, and whether expertise at insight problem solving could be demonstrated if a sufficient number of correct solutions were available. This is, of course, apart from an attempt to address the questions which the empirical work of this thesis did tackle, but which it could not pronounce on due to the very limited number of participants who were able to solve the nine-dot problem. Providing answers to these questions would require an initial sample size at least three times the size of the sample utilised by the empirical work of this thesis. Furnishing definitive answers was, however, never thought to be part of this work.

What is still of prime importance, however, is the question of whether insight is best understood in neural network terms. It is clearly evident that the neural network account of the key empirical findings from the literature on insight in problem solving is superior to any of the explanations offered so far from a competing theoretical standpoint. Phenomena such as incubation and facilitation which have not been accounted for by previous theoretical explanations, are clearly explainable by the neural network perspective. This perspective has also allowed the notion of expertise to be transferred from the general literature on problem solving and incorporated into the realm of insight in a most intriguing fashion. Arguably superior accounts are offered for the phenomena of insight which symbolic theory has attempted to explain in the past, such as problem difficulty, initial problem conception, persistence, and selection of the appropriate conception. The neural network account has certainly offered a richer and more consistent description of these key findings.

The only point on which the neural network account of insight written in this thesis can be seriously challenged, is on the interpretation of insight as the recognition of the pattern to insight problems. This is a particular understanding of the processing which might be involved in insight, which was informed by the neural network account of insight. It is not necessary to postulate that this recognition is what characterises insight for the neural network account to be viable. Indeed, if we were to adopt the stance of a simulation researcher, an over-arching conceptual account of the processing involved in displaying findings which characterise insight would not be necessary. This account is provided by the operation of the neural network, and hence by neural network theory itself.

It would seem that the only means of determining whether the neural network account is the most viable option currently available for conceptualising insight, would be to write an account of insight in symbolic terms, and in neural network implementation terms, which are of similar scope to this neural network account. All three accounts would have to be empirically tested with adequate sample sizes to render statistical tests of hypotheses internally valid. Despite this contention, however, the consistency with which the neural network account explains insight in problem solving cannot be denied. A small set of empirical tests which were rendered inconclusive by virtue of the limited number of people who were able to solve the target insight problem, cannot be used to refute this account which it was the major work of this thesis to provide.

## 7.4. Conclusion

This thesis reviewed literature on insight in problem solving and argued that the conceptual explanation of insight in the literature is poor. This was despite the existence of a clear operational definition of insight in problem solving and a body of consistent key empirical findings amongst the research on insight. The conceptual explanation of insight has clearly been hampered by claims that it is a mystical phenomenon, that it does not exist, as well as by its primary association with Gestalt psychology and the historical fortunes which this theoretical orientation experienced. As a result, the study of insight in problem solving has only resurfaced on the fringes of the cognitivist revolution. The dominance of symbolic theory in cognitive science and cognitive psychology, a theoretical orientation which seems limited in its ability to provide a conceptual explanation of the processing which leads to insight in problem solving, only contributed to the marginal interest which insight received.

As a result, it was decided to undertake the work of writing a conceptual account of insight in neural network terms. This would not only provide scope for advancing our understanding of insight, but would also provide commentary on the current debate in cognitive theory which sees symbolic theory lined up against the would-be usurper in the form of neural network theory. The dominance of symbolic theory and the current tension in cognitive science and cognitive psychology was also traced to historical factors. This was used to suggest that there is a need to re-write cognitive phenomena in neural network terms, to assess whether the assumptions which symbolic theory applies to cognitive functioning are indeed necessary to explain cognition. Insight seemed to be a prime candidate for this re-writing, given its association with problem solving and the poor conceptual explication which it has suffered in the past.

The key empirical findings which were identified in the research literature on insight in problem solving proved to be easily, clearly, and consistently explained by neural network theory. In fact, it was clear that an argument could be made for suggesting that neural network theory offered a superior account of these key findings and therefore of insight. The neural network theoretical position was used to develop a conceptual understanding of insight which suggested that insight in problem solving requires a recognition of the pattern which links insight problems. The conceptual explanation of insight in neural network terms was used to derive predictions which would reflect on the central tenets of the neural network account of insight and which could be subjected to empirical testing. These predictions suggested experimental manipulations which could be used to facilitate insightful problem solution.

The experimental testing of these predictions revealed the surprising finding that there was no effect for any of the facilitations expected to produce a significant increase in the number of correct solutions produced on the target nine-dot problem. The consideration of solution times had to be abandoned due to the limited number of participants who were able to solve the nine-dot problem. A consideration of the number of participants who solved the nine-dot problem revealed no significant effect for attempts to overcome past learning, conceptual transfer, or expertise at insight processing.

These results were understood to be the result of a lack of power for the statistical tests conducted, bequeathed by the low sample size for correct solutions. They could therefore not be used to support or refute the viability of the neural network account of insight. It was also possible to interpret these findings in a manner which was conceptually consistent with the neural network account of insight. A surprising finding concerning the atypicality of the tower problem among the other insight problems, offered some support for the conceptual understanding of insight in neural network terms. The only means by which the neural network account of insight which it was the task of this thesis to write can be assessed, is by extensive additional empirical testing with considerably larger sample sizes. Some commentary can then be offered on the viability of the neural network account of cognition.

As the end of this thesis is reached, the viability of a neural network account of insight remains an open question. It is necessary to submit the account to further testing and possibly to develop and test symbolic and neural network implementation theories of insight before any clear conclusions can be drawn. What is evident from the work of this thesis, however, is that it is possible to account for the key empirical findings concerning insight in conceptual terms within a comprehensive theoretical account. This is something which has not been accomplished to date. The fact that a neural network theoretical framework was used to write an account of a phenomenon which must be seen as a higher cognitive process, has offered some reflection on the sparring match in cognitive science and cognitive psychology.

This thesis can be concluded by offering some commentary on the future of research on insight. The neural network account of insight written in this thesis has the conceptual tools to account convincingly for the fact that insight is difficult to achieve, even though the facilitations for insight which were based on this account did not prove to be effective. However, we still do not know why some people are better at displaying insight than others. Once more, the neural network account of insight has the potential to offer an explanation for this, but this needs to be more fully investigated. When this investigation has taken place, perhaps we can isolate the factors which lead some people to think insightfully and which prevent other people from doing so. This seems to be the future of research on insight. When we know what underlies insightful thinking, we can begin to teach people to think insightfully. Insightful thinking defies the norm. It does not rely on what has already been learnt, but makes use of a non-obvious perspective to reach unusual and insightful conclusions. It is possible that if we can teach people the techniques which will allow them to think insightfully, the Kuhnian shift in paradigm will become a more frequent occurrence and the elusive phenomenon of insight will be rendered commonplace.

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# APPENDIX 1

## THE INSIGHT PROBLEMS AND THEIR SOLUTIONS

The Nine-dot Problem

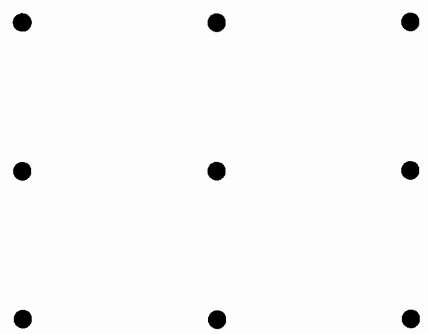
The Mutilated Checkerboard Problem

The Horse and Rider Problem

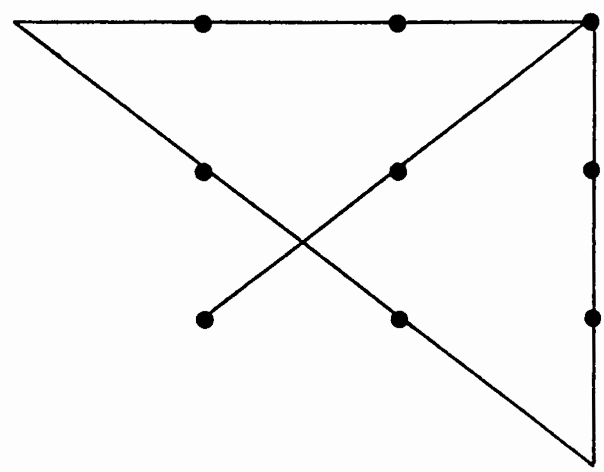
The Card Problem

The Tower Problem

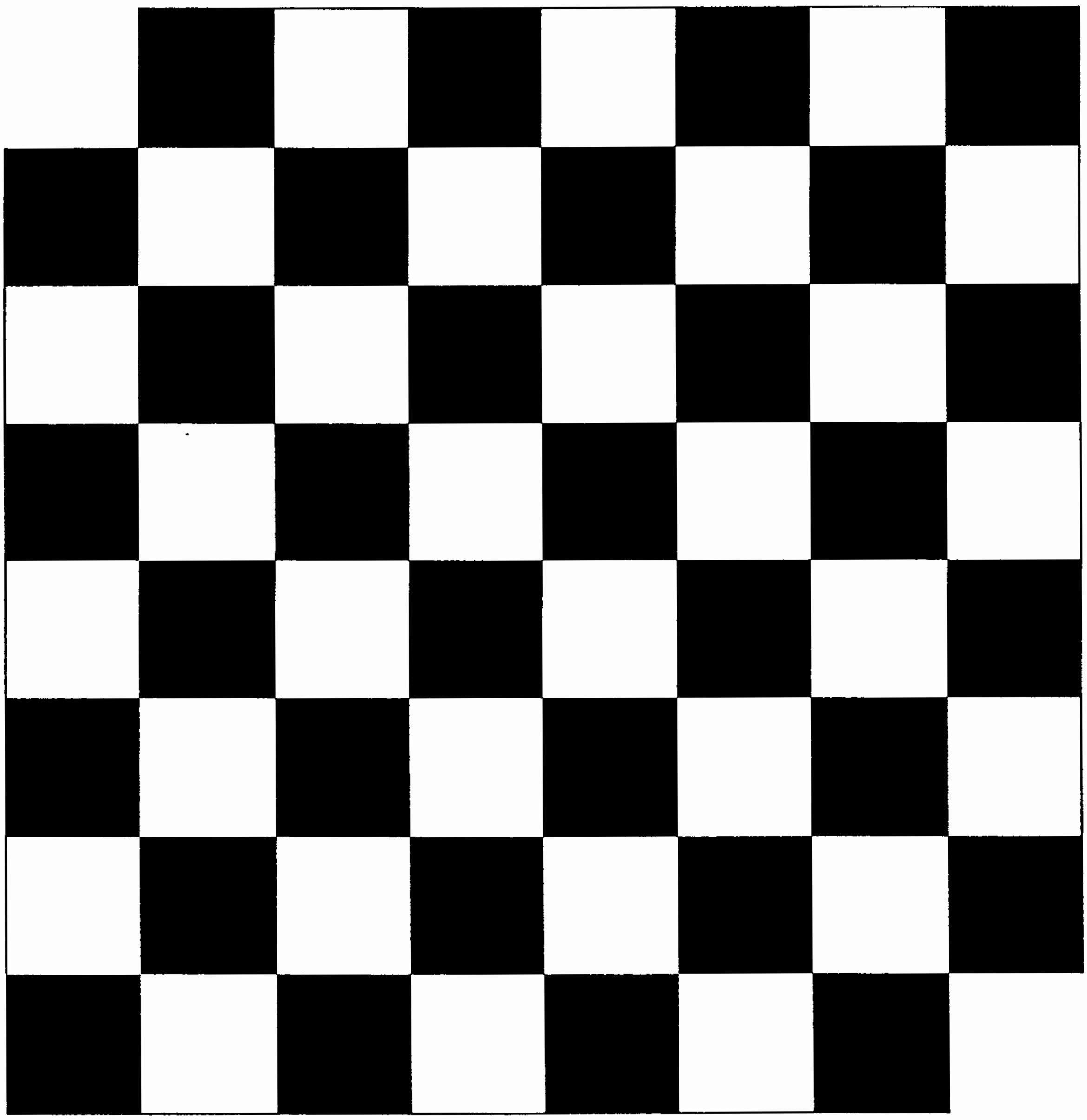
**Below you will see three rows of three dots. Connect the nine dots with four straight lines without lifting your pen or pencil from the paper.**



**Below you will see three rows of three dots. Connect the nine dots with four straight lines without lifting your pen or pencil from the paper.**



**Below you will see a checkerboard whose diagonally opposite corners have been removed. Imagine placing dominos on the board so that one domino covers two horizontally or vertically (but not diagonally) adjacent squares. Show how 31 dominos would cover the 62 squares remaining on the board, or prove logically that a complete covering is impossible.**

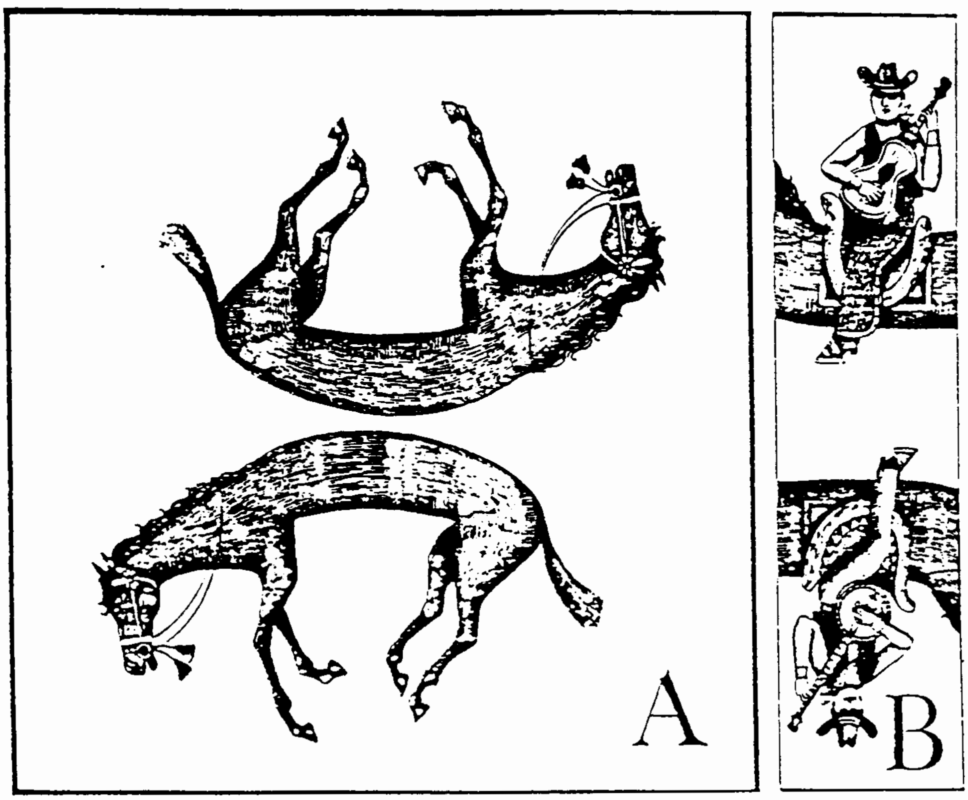


**Solution to the Mutilated Checkerboard problem.**

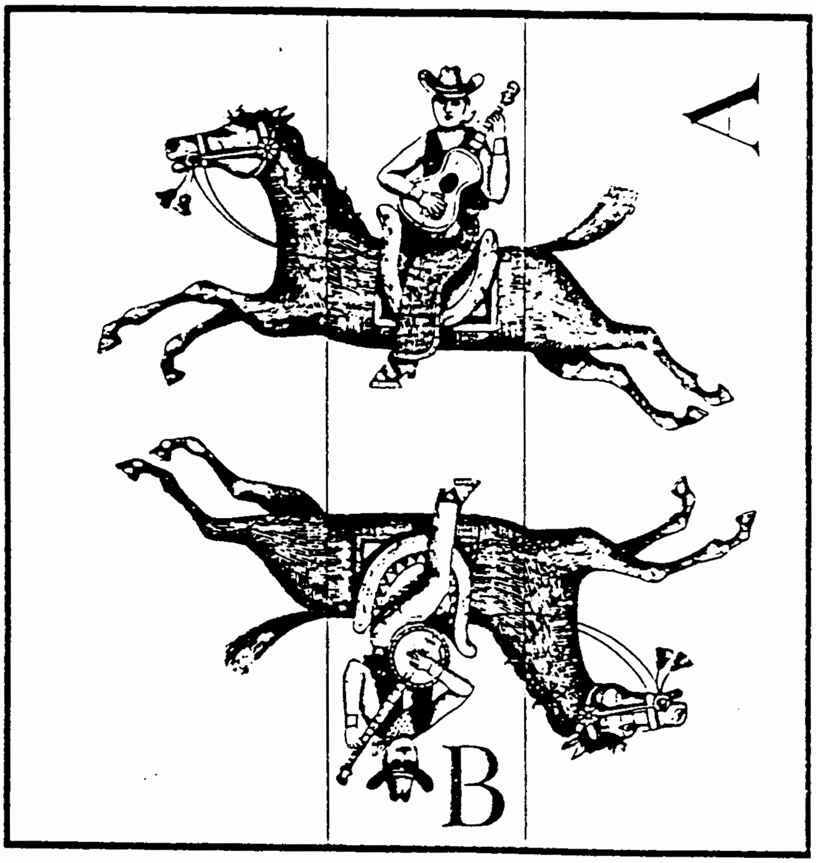
You would have noticed that the problem instructions demanded that you cover two adjacent squares with each domino. Therefore, each domino covers a black and a white square. Diagonally opposite corners on a checkerboard are the same colour. Thus it is not possible to cover the 62 remaining squares with 31 dominos, given that a domino must cover a black and a white square, and there are not 31 white and 31 black squares left.

Below you will see two drawings, one labelled A and one labelled B. Mentally place the drawing labelled B over the drawing labelled A, so as to create two complete horses and riders.

A



**B**



**Describe how to cut a hole in an 8 X 13 cm. card that is big enough for you to put your head through.**

**Solution to the Card Cutting problem**

There are several possible solutions to the card cutting problem. The most common one, however, is the notion of cutting a circle out of the card in a spiral fashion, thus creating a "hole" more than large enough to put your head through.

**A prisoner was attempting to escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How could he have done this?**

**Solution to the Tower problem.**

You would have noticed that the problem instructions stated that the prisoner cut the rope in half. If he cuts the rope down the middle (in half down its length), the prisoner will have two pieces of rope each half of the length to the ground. By tieing these together, he will be able to escape.

# APPENDIX 2

## INSTRUCTIONS TO PARTICIPANTS AND EXPERIMENTERS

Participant Instructions

Experimenter Instructions

Experimenter Scoring Sheet 1 (Part 1, Part 2 Conditions 1 & 2)

Experimenter Scoring Sheet 2 (Part 2 Conditions 3 & 4)

# STUDY ON PROBLEM SOLVING

During today's tutorial you will be taking part in a study concerned with problem solving, and you will be asked to solve a number of problems.

The aim of this exercise is two-fold. Firstly, some of the data which you contribute today will be used to illustrate some of the statistical and methodological principles you are currently learning. Perhaps by applying these principles to a study which you have taken part in, they will become somewhat clearer than they are in the abstract. To this end, you will be provided with data and worked examples from this study in a couple of weeks time.

Secondly, the data which is gathered today will be used as part of a research project which hopes to contribute to our understanding of the mechanisms of problem solving. The findings of this research project will be made availble to anyone who wishes to see them.

Your tutors will be presenting problems to you and timing how long you take to solve these problems. This is not a test and the length of time it takes you to solve these problems does not reflect on your abilities. We would, however, ask you to take this exercise seriously and not to look at what other people are doing - it is quite likely that they have different problems to solve than the ones you are working on. **We would also ask that you not tell any other psychology student what you did during the course of this tutorial.** If your classmates know what to expect before they have their tutorial, it will invalidate the research findings.

You will now be given problem sheets to work on. Write on these sheets as much as you want to and, in particular, write down the problem solution where you can. Indicate decisively to your tutor each time you solve a problem, so that your solution time can be noted.

Thank you in advance for your co-operation, and happy problem solving.

STUDY 1 CONDITION 1

- Hand out instructions and 9-dot problem.
- Tell students to read the instructions but not to turn to the next page until instructed.
- Emphasise the feedback they will be receiving, the need not to tell other students about the study, and the importance of indicating when they have solved a problem.
- Tell them to turn to the next sheet and then you can note down the start time on the subjects' sheet.
- Allow 15 minutes of problem solving on the 9 dot problem.
- Students will be letting you know when they have solved the problem (decide whether you would like them to raise a hand or whatever). Note down the time on the sheet under problem 1. Any student who has reached a solution will have to wait for the end of the 15 minutes.
- After 15 minutes, stop everyone and take in the sheets.
- Hand out the next batch of problems.
- Tell students that they now have four problems to solve - they can move on to the next problem as soon as they have completed a problem (indicating their completion to you), but after 5 minutes you will be instructing them to move on to the next problem even if they have not completed the one they are busy with.
- After each 5 minute block (there will be 20 minutes total problem solving) tell students to move on to the next problem if they have not reached a solution on the current problem. i.e. After 5 minutes "if you have not solved your first problem, move on to the second one", after 10 minutes "if you have not solved your second problem, move on to the third one now", etc.
- After 20 minutes, stop everyone and take in the sheets.
- Ask whether anyone had seen the 9 dot problem and its solution before, and note down those subjects who had on your subject sheet.
- Ask whether anyone had seen any of the other problems and their solution before, and note down which ones they had seen on the subject sheet.

P.S. There is no problem with you showing people how the 9 dot problem is solved at the end of the session.

STUDY 1 CONDITION 2

Hand out instructions and the batch of problems.

- Tell students to read the instructions but not to turn to the next page until instructed.
- Emphasise the feedback they will be receiving, the need not to tell other students about the study, and the importance of indicating when they have solved a problem.
- Tell them to turn to the next sheet and then you can note down the start time on the subjects' sheet.
- Tell students that they now have four problems to solve - they can move on to the next problem as soon as they have completed a problem (indicating their completion to you), but after 5 minutes you will be instructing them to move on to the next problem even if they have not completed the one they are busy with.
- Students will be letting you know when they have solved the problem (decide whether you would like them to raise a hand or whatever). Note down the time on the sheet under problem 1.
- After each 5 minute block (there will be 20 minutes total problem solving) tell students to move on to the next problem if they have not reached a solution on the current problem. i.e. After 5 minutes "if you have not solved your first problem, move on to the second one", after 10 minutes "if you have not solved your second problem, move on to the third one now", etc.
- After 20 minutes, stop everyone and take in the sheets.
- Hand out the 9 dot problem.
- Allow 15 minutes of problem solving on the 9 dot problem.
- Students will be letting you know when they have solved the problem. Note down the time on the sheet under problem 5. Any student who has reached a solution will have to wait for the end of the 15 minutes.
- After 15 minutes, stop everyone and take in the sheets.
- Ask whether anyone had seen the 9 dot problem and its solution before, and note down those subjects who had on your subject sheet.
- Ask whether anyone had seen any of the other problems and their solution before, and note down which ones they had seen on the subject sheet.

P.S. There is no problem with you showing people how the 9 dot problem is solved at the end of the session.

STUDY 2 CONDITION 1a and 1b

- Hand out instructions and the batch of problems.
- Tell students to read the instructions but not to turn to the next page until instructed.
- Emphasise the feedback they will be receiving, the need not to tell other students about the study, and the importance of indicating when they have solved a problem.
- Tell them that they have four problems in front of them to solve, and that they must not turn to the following sheet until they have solved the problem on the present sheet, or until they are instructed to turn the sheet.
- Tell them to turn to the next sheet and then you can note down the start time on the subjects' sheet.
- Allow 5 minutes of problem solving.
- Instruct anyone who has not solved the first problem to stop solving that problem and to turn the sheet.
- Allow 1 minute viewing time and then instruct them to turn the sheet to the next problem.
- Repeat this until they have been through all four problems.
- After 24 minutes stop all problem solving and take in the sheets.
- Hand out the 9 dot problem and allow 15 minutes of problem solving, asking students to indicate to you when they have solved the problem.
- After 15 minutes, stop problem solving and take in the sheets.
- Ask whether anyone had seen the 9 dot problem and its solution before, and note down those subjects who had on your subject sheet.
- Ask whether anyone had seen any of the other problems and their solution before, and note down which ones they had seen on the subject sheet.

P.S. There is no problem with you showing people how the 9 dot problem is solved at the end of the session.

STUDY 2 CONDITION 2a and 2b

- Hand out instructions and the batch of problems.
- Tell students to read the instructions but not to turn to the next page until instructed.
- Emphasise the feedback they will be receiving, the need not to tell other students about the study, and the importance of indicating when they have solved a problem.
- Tell them that they have several problems in front of them, but that they are not to turn to the following sheet until they have completed the problem on the current sheet, or until they are instructed to turn to the next sheet.
- Allow 15 minutes problem solving on the 9 dot problem, timing when a student completes the problem.
- After 15 minutes, tell anyone who has not completed the problem to turn to the next sheet.
- Allow 5 minutes problem solving, then instruct anyone who has not solved the problem to turn to the next sheet.
- Allow 1 minute viewing and then instruct anyone currently viewing the solution to turn to the next sheet.
- Allow 5 minutes problem solving and then instruct anyone who has not solved the problem to turn to the next sheet.
- Allow 1 minute to view the solution and then instruct people to turn to the next sheet.
- Allow 10 minutes problem solving on the final presentation of the 9 dot problem and then take in the sheets.
- Ask whether anyone had seen the 9 dot problem and its solution before, and note down those subjects who had on your subject sheet.
- Ask whether anyone had seen any of the other problems and their solution before, and note down which ones they had seen on the subject sheet.

P.S. There is no problem with you showing people how the 9 dot problem is solved at the end of the session.

Start time:

| | | |
|---|---|---|
| Subject 1: | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 2 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 3 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 4 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 5 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |

| | |
|---|---|
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 7 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 8 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 9 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 10 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot......................... |
| | Familiar other......................... |
| Subject 11 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |

| | |
|---|---|
| | Problem 5.............................. |
| | Familiar 9 dot.......................... |
| | Familiar other.......................... |
| Subject 12 | Problem 1.............................. |
| | Problem 2.............................. |
| | Problem 3.............................. |
| | Problem 4.............................. |
| | Problem 5.............................. |
| | Familiar 9 dot.......................... |
| | Familiar other.......................... |

| | | |
|---|---|---|
| Start Time...................................... | | |
| Subject 1 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 2 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 3 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 4 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 5 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 6 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 7 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |
| Subject 8 | problem 1...................... | |
| | problem 2...................... | |
| | problem 3...................... | |
| | problem 4...................... | |

| | |
|---|---|
| Subject 9 | problem 1...................... |
| | problem 2...................... |
| | problem 3...................... |
| | problem 4...................... |
| Subject 10 | problem 1...................... |
| | problem 2...................... |
| | problem 3...................... |
| | problem 4...................... |
| Subject 11 | problem 1...................... |
| | problem 2...................... |
| | problem 3...................... |
| | problem 4...................... |
| Subject 12 | problem 1...................... |
| | problem 2...................... |
| | problem 3...................... |
| | problem 4...................... |

# APPENDIX 3

## ALL MATERIALS FOR THE STUDY

The Insight Problems

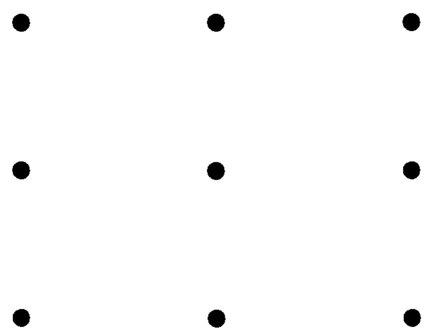
The Standard Solutions to the Insight Problems

The Solutions to the Insight Problems for Conceptual Transfer

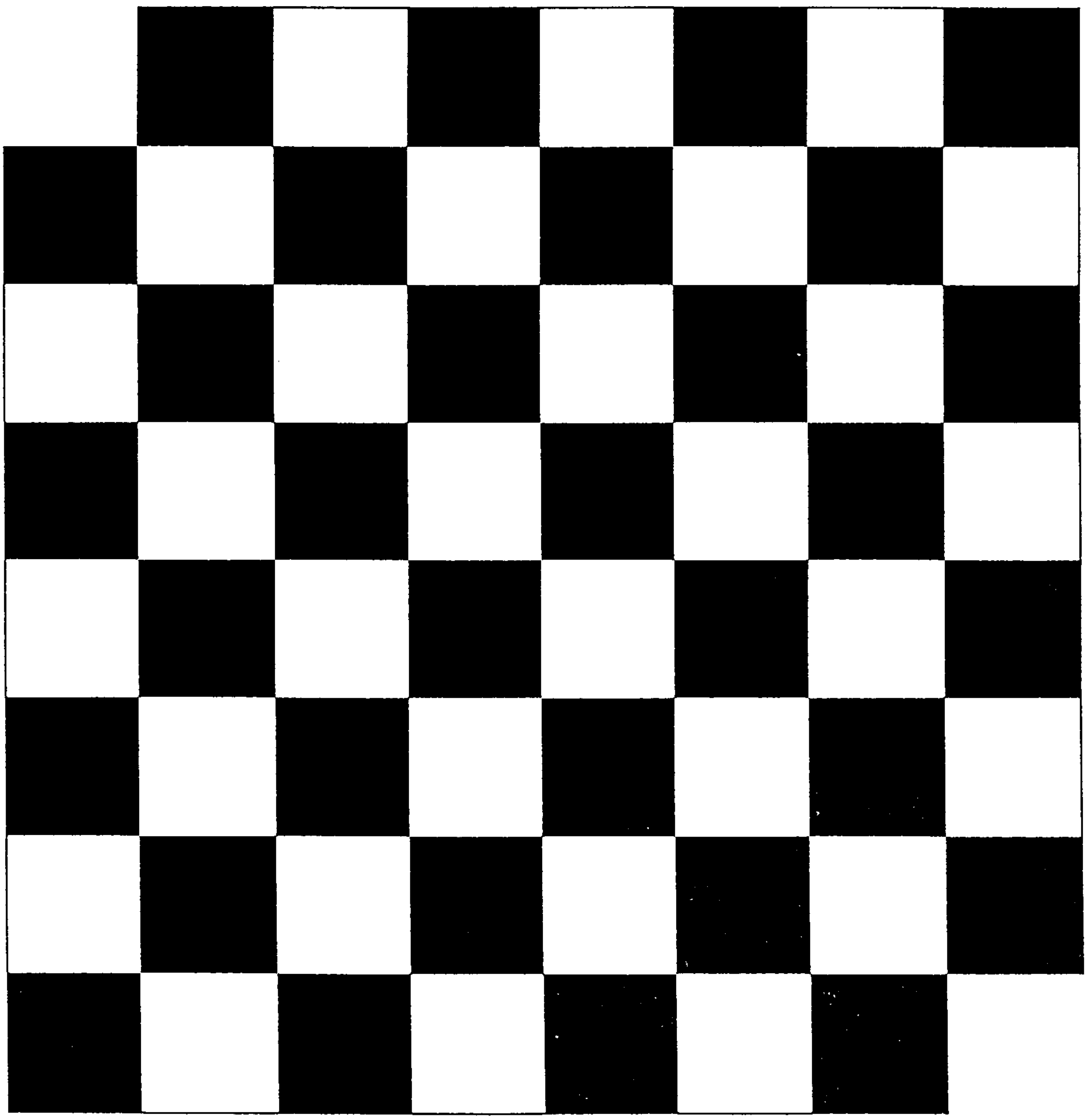
The Adapted Line Problem and its Solution

The Adapted Necker Cube Problem and its Solution

Below you will see three rows of three dots. Connect the nine dots with four straight lines without lifting your pen or pencil from the paper.

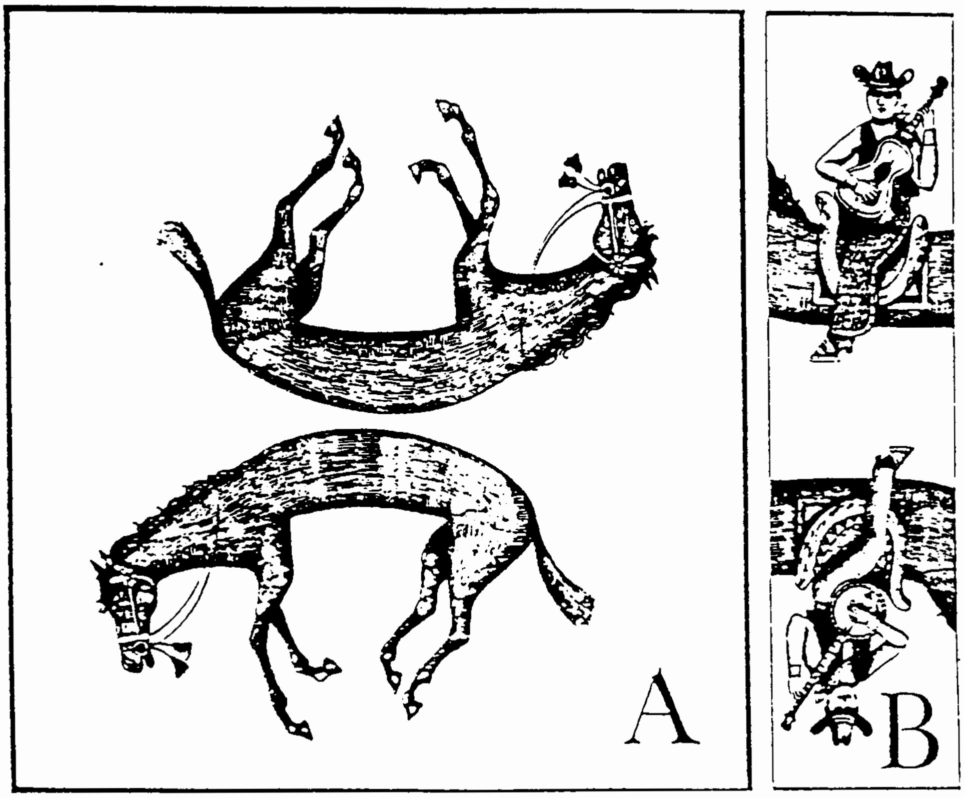


**Below you will see a checkerboard whose diagonally opposite corners have been removed. Imagine placing dominos on the board so that one domino covers two horizontally or vertically (but not diagonally) adjacent squares. Show how 31 dominos would cover the 62 squares remaining on the board, or prove logically that a complete covering is impossible.**



Below you will see two drawings, one labelled A and one labelled B. Mentally place the drawing labelled B over the drawing labelled A, so as to create two complete horses and riders.

**A**



**Describe how to cut a hole in an 8 X 13 cm. card that is big enough for you to put your head through.**

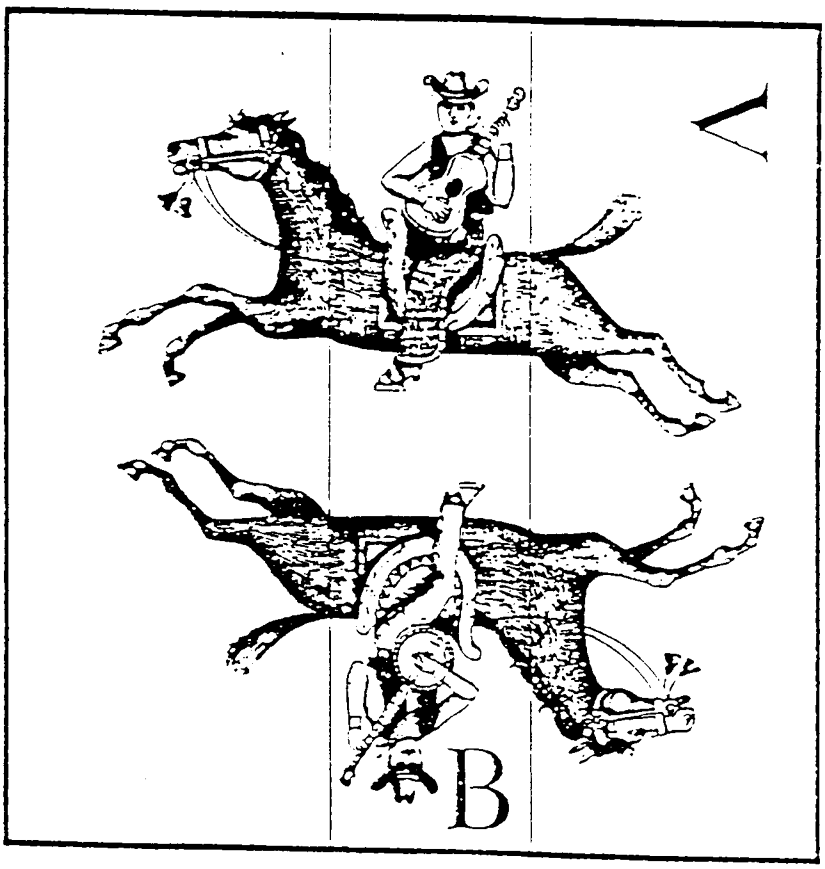
**A prisoner was attempting to escape from a tower. He found in his cell a rope which was half long enough to permit him to reach the ground safely. He divided the rope in half and tied the two parts together and escaped. How could he have done this?**

**Solution to the Mutilated Checkerboard problem.**

You would have noticed that the problem instructions demanded that you cover two adjacent squares with each domino. Therefore, each domino covers a black and a white square. Diagonally opposite corners on a checkerboard are the same colour. Thus it is not possible to cover the 62 remaining squares with 31 dominos, given that a domino must cover a black and a white square, and there are not 31 white and 31 black squares left.

Solution to the Horse and Rider problem.

B



**Solution to the Card Cutting problem**

There are several possible solutions to the card cutting problem. The most common one, however, is the notion of cutting a circle out of the card in a spiral fashion, thus creating a "hole" more than large enough to put your head through.

**Solution to the Tower problem.**

You would have noticed that the problem instructions stated that the prisoner cut the rope in half. If he cuts the rope down the middle (in half down its length), the prisoner will have two pieces of rope each half of the length to the ground. By tieing these together, he will be able to escape.

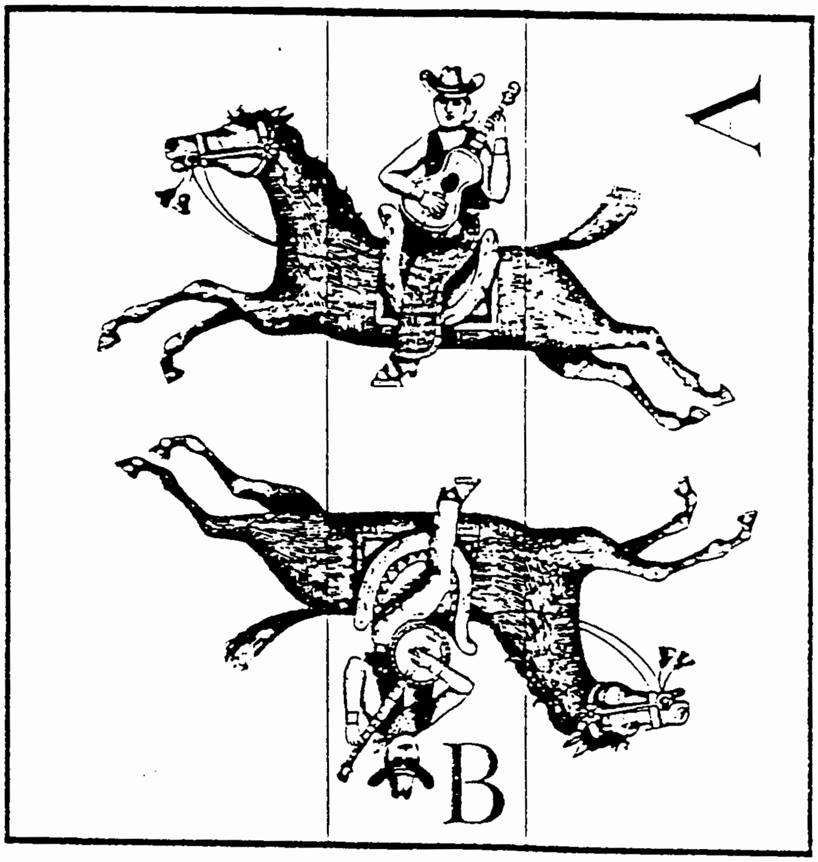
**Solution to the Mutilated Checkerboard problem.**

You would have noticed that the problem instructions demanded that you cover two adjacent squares with each domino. Therefore, each domino covers a black and a white square. Diagonally opposite corners on a checkerboard are the same colour. Thus it is not possible to cover the 62 remaining squares with 31 dominos, given that a domino must cover a black and a white square, and there are not 31 white and 31 black squares left.

Most people do not pay attention to the colour of the squares on the checkerboard. Instead, they attempt to work out mathematically whether it is possible to cover 62 squares with 31 dominos. It is only when people notice the colour of the squares on the checkerboard and realise that the opposite corners are the same colour, that they are able to solve the problem. There are a group of problems, known as insight problems, for which the most obvious solutions do not work. The mutilated checkerboard is one of these problems.

**Solution to the Horse and Rider problem.**

B




Most people attempt to solve this problem by placing part B over the existing horses in part A. This solution will not work, and it is only after rotating part A and using the existing horses as pieces of the horse and rider problem, that the problem can be solved. This problem is another insight problem, for which an obvious solution will not work.

**Solution to the Card Cutting problem**

There are several possible solutions to the card cutting problem. The most common one, however, is the notion of cutting a circle out of the card in a spiral fashion, thus creating a "hole" more than large enough to put your head through.

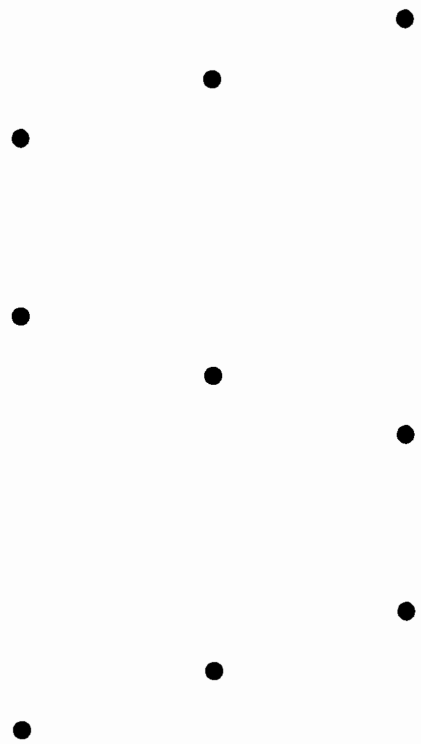
Most people approach this problem with the obvious definition of a hole, and it is not possible to cut one hole out of an 8 X 13 cm card which is large enough to put your head through. It is only when people take a less obvious definition of a "hole" that they are able to solve this problem. The card cutting problem is also an insight problem for which the most obvious solution will not work.

**Solution to the Tower problem.**

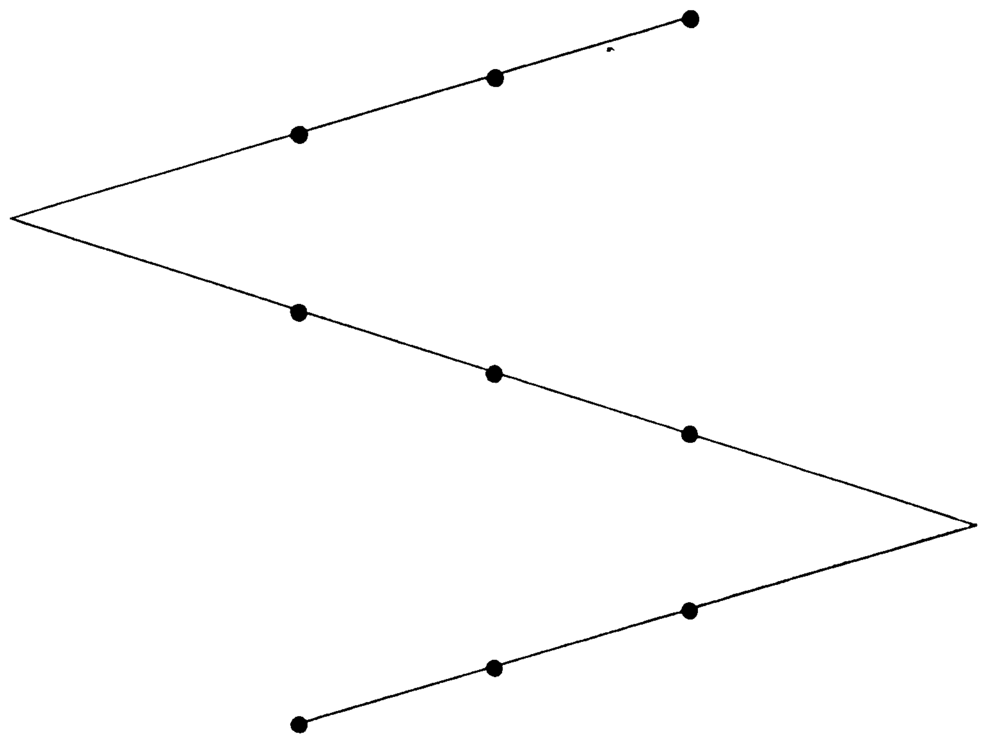
You would have noticed that the problem instructions stated that the prisoner cut the rope in half. If he cuts the rope down the middle (in half down its length), the prisoner will have two pieces of rope each half of the length to the ground. By tieing these together, he will be able to escape.

The obvious approach to this problem is to take the rope and cut it in half across its length, thus providing two pieces of rope one quarter of the necessary length. This approach will not work, and it is only by taking a less obvious definition of cutting a rope in half, that the problem can be solved. The tower problem is also an insight problem for which the most obvious approach will not work.

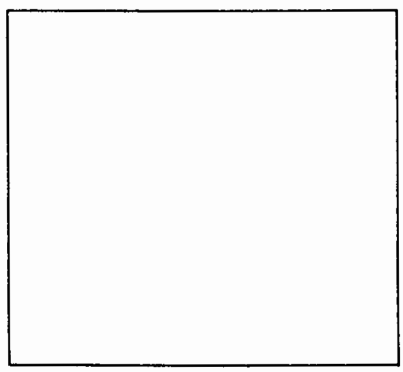
Below you will see three rows of three dots. Connect the nine dots with three straight lines without lifting your pen or pencil from the paper.



**Below you will see the solution to the three line dot problem.**

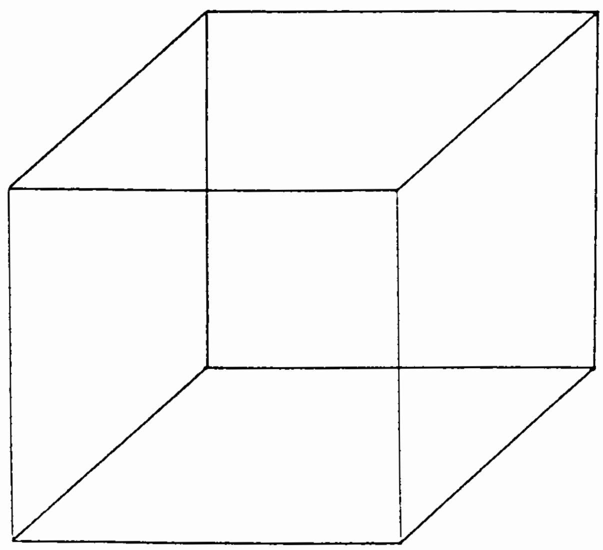


**Below you will see a two dimensional square. Change this square in to a three dimensional figure, using any space available on the page.**



Below you will see one possible solution to the three dimensional square problem.

There are many variations on this problem.



# APPENDIX 4

A COPY OF THE RAW DATA IS AVAILABLE FROM THE AUTHOR ON REQUEST