

# Generation of dielectrophoretic force under uniform electric field

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**Abstract** — Effective dipole moment method has been widely accepted as the de facto technique in predicting the dielectrophoretic force due to the non-uniform electric field. In this method, a finite-particle is modeled as an equivalent point-dipole that would induce a same electric field under the external electric field. This approach is only valid when the particle size is significantly smaller than the characteristic length of interest. This assumption is often violated in a microfluidic device, where the thickness or width of the microchannel can be as small as the particle. It is shown in this numerical study that when the dimensions of the particle were in the same order of magnitude as the characteristic length of the device, dielectrophoretic force can be induced even in a uniform electric field. This force arose due to the disturbance of the particle and the bounding wall.

**Index Terms** — electrostatic force, dipole moment, dielectrophoresis

## I. INTRODUCTION

**D**IELECTROPHORESIS has been demonstrated to be able to manipulate bio-particles having characteristic length in micrometers. For instance, the separation of viable and non-viable yeast cells [1], the separation of mixtures of *Bacillus subtilis*, *Escherichia coli* and *Micrococcus luteus* bacteria [2], the separation of human

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leukemia cells [3] and the separation of human breast cancer cells from blood [4].

Effective dipole moment method [5] has been commonly employed to predict the magnitude and direction of the dielectrophoretic forces in such devices. According to the effective dipole method, the dielectrophoretic force,  $\mathbf{F}$ , on a particle can be approximated as the dot product of the equivalent dipole induced on the particle,  $\mathbf{p}$ , with the gradient of the applied electric field,  $\mathbf{E}$ , [5]

$$\mathbf{F} = (\mathbf{p} \cdot \nabla) \mathbf{E} \quad (1)$$

where  $\nabla$  is a del operator. For a more rigorous approach, multipolar moment method [6] can be used to find the dielectrophoretic force induced by the higher order electric field terms [7].

There are two major assumptions in using the effective moment method. Firstly, it is assumed that the characteristic length of interest is significantly larger than the particle size. Secondly, it is assumed that the equivalent dipole or multipoles induced on the particle under non-uniform electric field is the same as the dipole or multipoles induced under a uniform electric field. In microfluidic devices where the microchannel thickness or width can be as small as 100  $\mu\text{m}$ , and the nominal cell diameter in the range of 1~20  $\mu\text{m}$ , these assumptions are often violated. Some attempts [8] have been made in proposing Maxwell Stress Tensor analytical approach to replace the dipole moment method, which was later found to be the same exact solution as the multipolar moment method [9]. In addition, there were also numerical approaches using the Maxwell Stress Tensor [10,11]. This investigation extends the numerical approach to include the case of uniform electric field.

## II. BOUNDARY CONDITIONS

The problem space consists of a dielectric rod with radius  $r$ , position at a distance  $a$  in a vacuum space, as shown in Figure 1. The length of the problem space is arbitrarily defined as twice the height. A potential with a normalized value 1 unit is applied to the left edge, and a zero potential is maintained at the right edge. The dielectric

rod has a permittivity of  $\varepsilon_2 = 7\varepsilon_1$ , where  $\varepsilon_1$  is chosen as the permittivity of the free space, for simplicity. The normal components of the potential are zero at the top and bottom surfaces. The applied potential generated an electric field that induces a polarization charge on the dielectric rod surface. This surface charge interacts with the electric field to give rise to electrical forces acting on the dielectric rod. The problem is formulated and solved using a partial differential equation solver (FlexPDE).

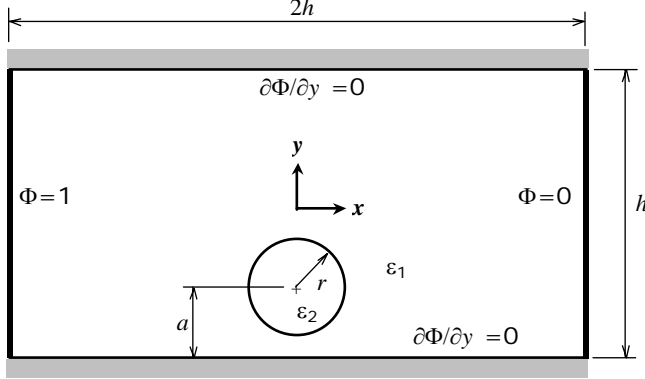


Fig. 1. Two-dimensional model simulating the electric field and dielectrophoretic force. The potential is applied on the left and right edges to generate an uniform electric field.

### III. ELECTROSTATIC FORCE

#### A. Electric field

The calculation of the electric field often starts with the Gauss' law, which relates the electric field intensity to its source. In the differential form, the Gauss' law is [12]

$$\nabla \cdot \varepsilon_0 \mathbf{E} = \rho \quad (2)$$

where  $\varepsilon_0$  is the absolute permittivity of free space, and  $\rho$  is the total free charge in space. Equation (2) is readily valid for material having relative permittivity,  $\kappa$ , by taking the multiplication of  $\kappa$  on  $\varepsilon_0$ . For convenient, the product is usually denoted as  $\varepsilon$ . In micrometer length scale, the total free charge can be assumed to be zero. In addition, the Faraday's law relates the circulation of  $\mathbf{E}$  to the time rate of change of the magnetic flux,  $\mathbf{H}$ , which in differential form is [12]

$$\nabla \times \mathbf{E} = \frac{\partial}{\partial t} \mu_0 \mathbf{H} \quad (3)$$

where  $\mu_0$  is the permeability of free space. In a microfluidic device, the characteristic length of the particle is much smaller than the electric field length, such that the right-hand side term is approximately zero. Thus, the vector electric field can be redefined as the gradient of a scalar  $\Phi$ ,

$$\mathbf{E} = -\nabla \Phi \quad (4)$$

The governing equation is obtained by combining equation (4) into equation (2), taking into account the

simplification for micrometer length scale,

$$\nabla \cdot \varepsilon (-\nabla \Phi) = 0 \quad (5)$$

which is readily reduced to the Laplace equation for a homogenous material. In this study, the electric field on the problem space is obtained by solving the Laplace equation.

#### B. Dielectrophoretic force

The electric field exerted a force on the surface charge. Employing the Maxwell Stress Tensor method, the total force acting on the rod surface,  $\mathbf{F}_{\text{mst}}$ , is found by integrating the tangential and normal component of the electric field,  $\mathbf{E}_{r2}$  and  $\mathbf{E}_{n2}$ , respectively, over the dielectric rod surface,  $s$ , [10,13]

$$\mathbf{F}_{\text{mst}} = \frac{1}{2} (\varepsilon_2 - \varepsilon_1) \oint_s \left( \mathbf{E}_{r2}^2 + \frac{\varepsilon_2}{\varepsilon_1} \mathbf{E}_{n2}^2 \right) \mathbf{n} ds \quad (6)$$

where index 1 refers to the surrounding material, and index 2 refers to the object.

### IV. NUMERICAL RESULT

The electric field and electrostatic force acting on the dielectric rod are obtained by solving Equation (5) and (6) subjected to the boundary conditions in Figure 1. For the purpose of comparison, a simulation without the dielectric rod is also performed. The result in Figure 2 shows the electric field distribution when the dielectric rod is omitted. This condition corresponds to the dipole moment approach, where the dielectrophoretic force is obtained by taking the product of the equivalent dipole with the gradient of the electric field as depicted in equation (1). Using this approach, the dielectrophoretic force is predicted to be zero throughout the problem space, since the gradient of a uniform electric field is zero.

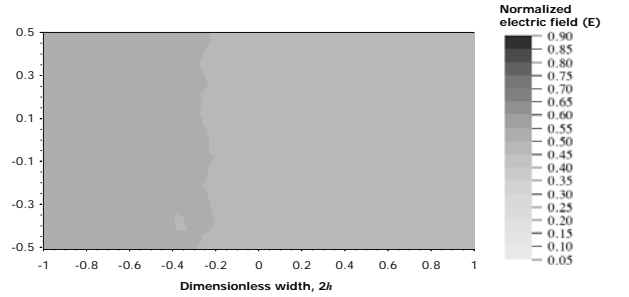


Fig. 2. Electric field distribution without considering the dielectric rod.

The electric field distribution when the dielectric rod is included in the calculation of the electric field is shown in Figure 3. Due to the existence of the dielectric rod, a non-uniform electric field is generated. The non-uniform electric field give rise to the dielectrophoretic force acting on the dielectric rod surface.

The normalized dielectrophoretic force for different value of dielectric rod radius and position is shown in

Figure 4. The result predicts that the force on the dielectric rod increases significantly when the rod is near to the wall. The force is small when the particle size is an order of magnitude smaller than the channel thickness, and zero when the dielectric rod is at the center of the channel.

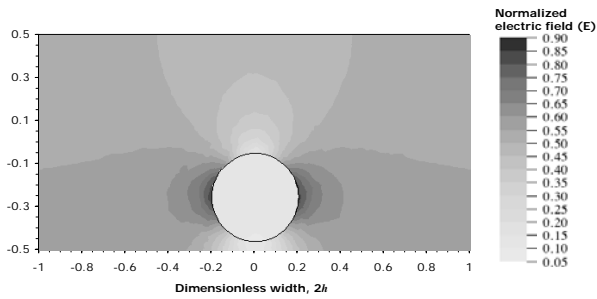


Fig. 3. Electric field distribution taking into account of the dielectric rod. All variables have been normalized.

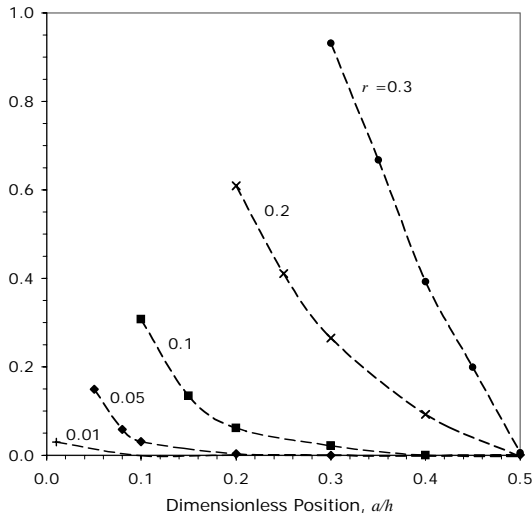


Fig. 4. The dependence of electrostatic force on the particle diameter to microchannel height,  $a/h$ , and the particle radius,  $r$ . All variables have been normalized.

## V. DISCUSSIONS AND CONCLUSION

This simulation shows that the particle has a significant effect on the electric field distribution when the particle diameter is in the same order of magnitude as the microchannel thickness. More specifically, the result highlighted the shortfall of the current effective dipole method in determining the dielectrophoretic force when the size of the particle cannot be ignored. The dielectrophoretic force can be generated even under an initially uniform electric field.

Hitherto, the biggest hindrance in using the numerical approach is that it is not able to account for the case of a lossy material. The simulation is only readily available for a perfect conductor or perfect insulator. Without this capability, the dependence of dielectrophoretic force on the applied voltage frequency cannot be included in the simulation.

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