Response of Venice Storm Gates to Incident Waves

by

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Abstract

For the prevention of flooding of Venice a system of gates to close the inlets of the lagoon has been designed. Each system is composed of a series of 20 hollow gates hinged at the bottom. In the present work a linear theory is developed first to study the motions of the gates forced by a monochromatic incident wave. The gates are assumed to be vertical and the fluid domain is approximated to a channel of infinite length on the Adriatic side and to a semi infinite space on the lagoon side. Several theorems based on Green's formula are developed to get a deeper understanding of the physics of the problem and to check both the theory and the numerical computations performed. In particular, the law of energy conservation is derived. The amplitude of gates motion, added mass and radiation damping are reported for a large interval of periods. Synchronous resonce of the gates is found and the occurence of negative added masses is reported and discussed.

Thesis Supervisor: Chiang C. Mei
Title: Donald and Martha Harleman professor

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Contents

Chapter 1

Introduction

For the prevention of flooding of Venice a system of gates to close the inlets of the lagoon has been designed. Each system is composed of a series of 20 hollow gates. Each gate has a shape of a box and is hinged at the bottom along a common axis across the inlet. The gates rest on the sea bottom during normal time and are raised by buoyancy when a storm is forecasted. When in operation, the gates will be inclined at about 45° from the horizontal.

Each set of gates is a dynamical system subject to the forcing of incident waves. Its response to monochromatic waves and narrow banded spectra has been studied in the past [3],[7],[6],[5] by assuming the gates to span across a channel. Thus the lagoon and the sea sides (Adriatic sea) of the gates are replaced by two very long channels. Under these conditions the natural modes of the gates ([3],[2],[1]) can be excited only nonlinearly by subharmonic resonance. This happens because the natural mode is perfectly trapped; there is no energy radiation. Conversely, no energy can be fed by a linear mechanism to the systems by incident waves. However, if energy leakage is allowed then energy can also be fed to the system by a linear mechanism and synchronous resonance can be expected.

In this study we investigate the response of the gates to monochromatic incident waves when the geometry on the Lagoon side is replaced by a semi infinite space. This change allows radiation of energy and synchronous resonance can occur. After developing a linear theory in chapter 2, some mathematical identities are discussed in chapter 3. These identities provide not only a deeper understanding of the physics, but also a way to check both the theory and the numerical computations. Results are discussed in chapter 4 and conclusions are drawn in chapter 5.

Chapter 2

Problem formulation

A typical inlet of Venice lagoon with the proposed gates is here approximated by an infinite channel on the Adriatic side and by a semi-infinite space on the Lagoon side (Figure: 2-1). For computational simplicity we assume the gates to be vertical when in static equilibrium (Figure: 2-2). They are set into motion by the incident waves from the Adriatic. As a result waves are radiated both in the Adriatic and in the lagoon from the gates.

2.1 Gate dynamics

2.1.1 Equation of motion for a set of gates

Assuming the fluid to be inviscid, incompressible, and in irrotational motion, its velocity field can be described by a potential $\Phi(x, y, z, t)$ that satisfies Laplace Equation

$$\nabla^2 \Phi = 0$$

and the boundary conditions. In the following we will use a complex potential ϕ such as $\Phi = \mathrm{Re}\left(\phi e^{-i\omega t}\right)$. For brevity we will omit both the Re operator and the time factor exponential.

A gate can be regarded as a rigid body with one degree of freedom (rotation about the hinge), while a set of n gates located in one of the inlets of the lagoon can be regarded as a body with n degrees of freedom. The motion of the gates must obbey Newton's law:

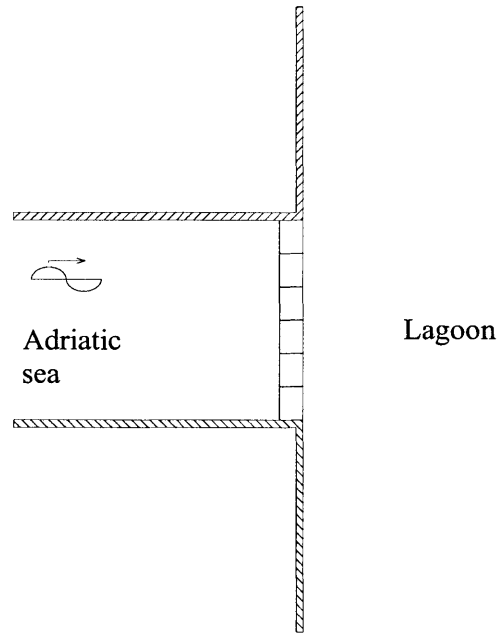

Figure 2-1: Plan view of the simplified geometry assumed.

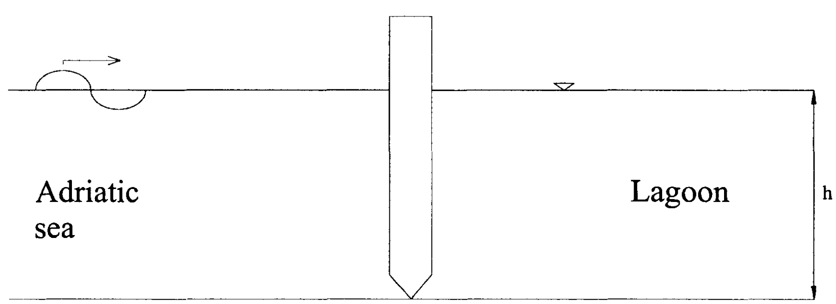

Figure 2-2: Cross section of the simplified geometry assumed.

$$I\left\{\ddot{\Theta}\right\}+C\left\{\Theta\right\}=-\rho\iint\limits_{S_\alpha}dS\ \Phi_t^{(1)}\left\{n\right\} \tag{2.1}$$

where: I is the inertia of each gate $\iiint\limits_{V_s}\left(x^2+(z+h)^2\right)\rho_s dV$; h the water depth at the gate's site (the subscript s denotes the solid part of the gates). $\{\Theta\}=\{\theta_1,\theta_2,...,\theta_\alpha,...,\theta_n\}e^{-i\omega t}$ is the vector representing the rotation of each gate . C is the buoyancy restoring torque on each gate,

$$C=\left[\rho g\left(I_{xx}^A+I_z^V\right)-Mg\left(z^c+h\right)\right] \tag{2.2}$$

with

$$I_{xx}^A=\iint\limits_{S^A}x^2dx \qquad I_z^V=\iiint\limits_{V}\left(z+h\right)dV \tag{2.3}$$

S^A denotes the cross sectional area of the gate at the water line (at rest), V the displaced volume the gate, and z^c is the depth of the center of mass. $-\rho\iint\limits_{S_\alpha}\Phi_t\left(h+z\right)dS$ is the moment due to the dynamic water pressure acting on the gate α. This integral has to be carried out on both the lagoon and the sea side of the gate. The gates are ssumed to be all identical.

2.1.2 Floating body dynamics: scattering and radiation

When a wave train strikes a gate, two physical phenomena occur: scattering and radiation. "Scattering" means that the wave changes direction because of the collision ; "radiation" means that the body emits waves because it oscillates under the action of the incident wave. An approach to study the behavior of the floating body (Haskind, 1944 as in [4]) is to decompose the wave potential into two parts, one that takes into account the incident and the scattered wave, and the other that takes into account the radiated waves. The sum of this two parts is the total velocity potential.

For the special geometry of the problem, the fluid domain can be split into two parts to be treated separately. On the Adriatic side all the phenomena described above occur: we have radiation and scattering because of the incident waves coming from the Adriatic. In the lagoon side only radiation takes place (the gates emit waves into the lagoon). For this reason we distinguish the potential ϕ by:

$$\phi = \begin{cases} \phi^{+} & \text{in the Lagoon} \\ \phi^{-} & \text{in the Adriatic} \end{cases}$$

We introduce a complex amplitude of motion so that $\dot{\Theta}_\alpha = \mathrm{Re}\left(V_\alpha e^{-i\omega t}\right)$, ($V_\alpha = -i\omega\theta_\alpha$ denote the the angular velocity of the α^{th} gate) where α represents a generic degree of freedom of the body ($\alpha = 1, ..., number\ of\ d.o.f$), we can write

$$\phi = \phi^D + \sum_\alpha V_\alpha \phi_\alpha \tag{2.4}$$

ϕ^D represents the sum of the diffracted (or scattered) potential and the incident potential ($\phi^D \equiv \phi^{D-}$), it has to satisfy the following conditions:

$$\begin{array}{ll} \nabla^2\phi^D = 0 & \textit{in the fluid} \\ \frac{\partial\phi^D}{\partial n} = 0 & \textit{on all solid boundaries} \\ & \textit{including the moving body} \\ \frac{\partial\phi^D}{\partial z} - \frac{\omega^2}{g}\phi^D = 0 & \textit{on the free surface} \\ \phi^D - \phi^I & \textit{outgoing at infinity} \end{array} \tag{2.5}$$

where ϕ^I represents the wave potential of the incident wave. ($\phi^S = \phi^D - \phi^I$ represents the potential of the scattered wave).

The radiation potential ϕ_α represents the effect due to the rotation of gate α while the all the others are at rest, it has to satisfy:

$$\begin{array}{ll} \nabla^2\phi_\alpha = 0 & \textit{in the fluid} \\ \frac{\partial\phi_\alpha}{\partial n} = \begin{cases} 0 & \textit{on the sea bottom and} \\ & \textit{on the stationary gate} \\ z + h & \textit{on the moving body} \end{cases} & \\ \frac{\partial\phi_\alpha}{\partial z} - \frac{\omega^2}{g}\phi_\alpha = 0 & \textit{on the free surface} \\ \phi_\alpha & \textit{outgoing at infinity} \end{array} \tag{2.6}$$

From the above it follows that the total potential $\phi = \phi^D + \sum_\alpha V_\alpha \phi_\alpha$ satisfies the boundary condition on the floating body:

$$\frac{\partial \phi}{\partial n} = V_\alpha (h + z), \quad \textit{on the moving body} \tag{2.7}$$

is the kinematic condition on the body surface.

The advantage of this decomposition is that the velocity $\frac{\partial \phi}{\partial n}$ is known and so both the radiation and diffraction problem for the normalized potential ϕ_α can be solved. Once the two solutions are found, the unknown velocities can be computed by using the equation of the body motion (eq. (2.1)). It is important to note that this is possible because of the linearity and the fact that the set of differential equations of the body motion are reduced to a set of linear equation $\left(\frac{d}{dt}(\cdot) = -i\omega(\cdot)\right)$.

In order to solve the problem we have to find first the potentials ϕ_α and ϕ^D. So that we can calculate the hydrodynamic force and finally solve the equation of motion of the gates.

2.1.3 Added mass and radiation damping

The hydrodynamic force on the β^{th} gate due to the radiated potential by gate the α^{th} gate can be written as :

$$
\begin{aligned}
-\rho \iint_{S_\beta} p\,(h+z)\,dS &= \mathrm{Re}\left[\left(i\rho\omega V_\alpha \iint_{S_\beta} \phi_\alpha\,(h+z)\,dS\right) e^{-i\omega t}\right] \\
&= \mathrm{Re}\left[\left[\mathrm{Re}\left[i\rho\omega \iint_{S_\beta} \phi_\alpha\,(h+z)\,dS\right] + i\,\mathrm{Im}\left[i\rho\omega \iint_{S_\beta} \phi_\alpha\,(h+z)\,dS\right]\right] V_\alpha e^{-i\omega t}\right] \\
&= \mathrm{Re}\left[\left[-\rho\omega \iint_{S_\beta} \mathrm{Im}\,\phi_\alpha\,(h+z)\,dS + i\rho\omega \iint_{S_\beta} \mathrm{Re}\,\phi_\alpha\,(h+z)\,dS\right] V_\alpha e^{-i\omega t}\right] \\
&= \left[-\rho\omega \iint_{S_\beta} \mathrm{Im}\,\phi_\alpha\,(h+z)\,dS\right] \mathrm{Re}\left(V_\alpha e^{-i\omega t}\right) \\
&+ \left[-\rho \iint_{S_\beta} \mathrm{Re}\,\phi_\alpha\,(h+z)\,dS\right] \mathrm{Re}\left[\frac{d}{dt}\left(V_\alpha e^{-i\omega t}\right)\right] \\
&= \left[-\rho\omega \iint_{S_\beta} \mathrm{Im}\,\phi_\alpha\,(h+z)\,dS\right] \dot{X}_\alpha + \left[-\rho \iint_{S_\beta} \mathrm{Re}\,\phi_\alpha\,(h+z)\,dS\right] \ddot{X}_\alpha
\end{aligned}
\tag{2.8}
$$

The integral has to be carried out on both sides of gate β, with the potential α. Let us define:

$$
\begin{aligned}
\mu_{\beta\alpha} &= \rho \iint_{S_\beta} \mathrm{Re}\,\phi_\alpha\,(h+z)\,dS = \tfrac{1}{\omega}\,\mathrm{Im}\left(F^A_{\beta\alpha} - F^L_{\beta\alpha}\right) \\
\lambda_{\beta\alpha} &= \rho\omega \iint_{S_\beta} \mathrm{Im}\,\phi_\alpha\,(h+z)\,dS = -\,\mathrm{Re}\left(F^A_{\beta\alpha} - F^L_{\beta\alpha}\right)
\end{aligned}
\tag{2.9}
$$

and

$$
F^A_{\beta,\alpha} = i\rho\omega \iint_{S_\beta} \phi^-_\alpha\,(h+z)\,dS \tag{2.10}
$$

$$F^L_{\beta,\alpha} = i\rho\omega \iint\limits_{S_\beta} \phi^+_\alpha \left(h+z\right) dS \tag{2.11}$$

$F^A_{\beta,\alpha}$ and $F^L_{\beta,\alpha}$ are respectively the force on the adriatic side and on the lagoon side of gate β, due to the oscillation of gate α. The superscript A denotes the Adriatic side and the superscript L the lagoon side.

The total hydrodynamic force on gate α due to the motion of all gates is then

$$F_\alpha = -\sum_\beta \mu_{\beta\alpha}\ddot{\Theta}_\beta - \sum_\beta \lambda_{\beta\alpha}\dot{\Theta}_\beta \tag{2.12}$$

$\mu_{\beta\alpha}$ is called "added mass" because it appears as a coefficient of the body acceleration and has the dimensions of mass. $\lambda_{\beta\alpha}$ is called "radiation damping" because is in front of a velocity and has dimension of mass/time. $\mu_{\beta\alpha}$ is the added mass of gate β due to the motion of gate α. Radiation damping represents the energy lost in the form of radiated waves by the oscillating body. This can be seen by observing the work done over a period, to the fluid by all gates is:

$$\overline{\dot{E}} = \sum_\alpha \overline{F^R_\alpha \dot{\Theta}_\alpha} = \sum_\alpha\sum_\beta \mu_{\beta\alpha}\overline{\ddot{\Theta}_\beta\dot{\Theta}_\alpha} + \sum_\alpha\sum_\beta \lambda_{\beta\alpha}\overline{\dot{\Theta}_\beta\dot{\Theta}_\alpha} \tag{2.13}$$

Since $\mu_{\beta\alpha} = \mu_{\alpha\beta}$ (to be proven later) the first term can be written as:

$$\sum_\alpha \frac{1}{2}\sum_\beta \mu_{\beta\alpha}\overline{\ddot{\Theta}_\beta\dot{\Theta}_\alpha + \dot{\Theta}_\alpha\ddot{\Theta}_\beta} = \sum_\alpha \frac{1}{2}\sum_\beta \mu_{\beta\alpha}\overline{\frac{d}{dt}\dot{\Theta}_\beta\dot{\Theta}_\alpha} = 0 \tag{2.14}$$

and it vanishes because of periodicity. Thus we have:

$$\overline{\dot{E}} = \sum_\alpha\sum_\beta \lambda_{\beta\alpha}\overline{\dot{\Theta}_\beta\dot{\Theta}_\alpha} \tag{2.15}$$

and the matrix $[\lambda]$ has to be positive semidefinite.

In general added mass and radiation damping are matrices and they correspond to the imaginary and the real parts of the complex potential (see eq.(2.9)).

2.2 The gate response

In order to solve the equation of motion of the gates we need to find first the radiation potentials ϕ_α so that we can compute the hydrodynamic forces.

Recall that ϕ_α^- and ϕ_α^+ are the potentials generated by the unit angular velocity oscillation of the α^{th} gate (i.e. $V_\alpha = 1$) in the Adriatic and lagoon side respectively, the total potential on each side is given by:

$$\phi^- = \phi^{-D} + \sum_\alpha V_\alpha \phi_\alpha^- \qquad \phi^+ = \sum_\alpha V_\alpha \phi_\alpha^+ \tag{2.16}$$

The hydrodynamic force on a typical gate (say the $\beta^{th} gate$) can be written as:

$$-\rho \iint_{S_\beta} \phi_t \,(h+z)\, dS \tag{2.17}$$

$$\begin{aligned} &= \left[-\rho \iint_{S_\beta} -i\omega\phi^D \,(h+z)\, dS \right] + \sum_\alpha \left[-\rho V_\alpha \iint_{S_\beta} -i\omega\phi_\alpha^- \,(h+z)\, dS \right] \\ &- \sum_\alpha \left[-\rho V_\alpha \iint_{S_\beta} -i\omega\phi_\alpha^+ \,(h+z)\, dS \right] = \\ &= F_\alpha^D + \sum_\alpha F_{\beta,\alpha}^A V_\alpha - \sum_\alpha F_{\beta,\alpha}^L V_\alpha = \\ &= F_\alpha^D + (-i\omega) \sum_\alpha F_{\beta,\alpha}^A \theta_\alpha - (-i\omega) \sum_\alpha F_{\beta,\alpha}^L \theta_a \end{aligned} \tag{2.18}$$

where $\theta_\beta = \frac{V_\beta}{-i\omega}$ denotes the amplitude of rotation of the β^{th} gate. We have a minus sign in

front of $F_{\beta,\alpha}^L$ because the action of a positive pressure on the gate in the lagoon side determines a negative moment on the gate.

Using the fact that the time dependence is given by the exponential $e^{-i\omega t}$, we have from eq.(2.1):

$$\left(-\omega^2 I + C\right)\{\theta\} = \left\{F^D\right\} + (-i\omega)\left[F^A\right]\{\theta\} - (-i\omega)\left[F^L\right]\{\theta\}$$

or, rearranging:

$$\left[\left(-\omega^2 I + C\right)[D] + i\omega\left[F^A\right] - i\omega\left[F^L\right]\right]\{\theta\} = \left\{F^D\right\} \tag{2.19}$$

In the above $\{\theta\} = \{\theta_1, \theta_2, ..., \theta_n\}$ is a vector containing the angular displacement of each of the n gates. $\left[F^A\right]$ and $\left[F^L\right]$ are matrices with generic terms $F^A_{\beta,\alpha}$ and $F^L_{\beta,\alpha}$,or in other words, the moments acting on the β^{th}gate due to the wave potential generated by the α^{th} gate. $\left[F^A\right]$ is for the Adriatic side and $\left[F^L\right]$ is for the Lagoon side.$[D]$ is the identity matrix of size n.

2.2.1 Radiation potential in the Adriatic side

The wave potential ϕ^-_α due to the oscillating gate α with unit angular displacement in the Adriatic has to satisfy

$$\nabla^2 \phi^-_\alpha = 0$$

and the boundary condition cf.(2.6):

$$\begin{array}{ll}
\frac{\partial \phi^-_\alpha}{\partial z} = 0 & on\ z = -h \\
\frac{\partial \phi^-_\alpha}{\partial y} = 0 & on\ sidewalls\ of\ the\ channel \\
\frac{\partial \phi^-_\alpha}{\partial n} = 0 & on\ all\ the\ still\ gates \\
\frac{\partial \phi^-_\alpha}{\partial x} = (h+z) & on\ the\ moving\ gate\ \alpha \\
\frac{\partial \phi^-_\alpha}{\partial z} - \frac{\omega^2}{g}\phi^-_\alpha = 0 & on\ z = 0 \\
\phi^-_\alpha\ outgoing\ at\ -\infty &
\end{array} \tag{2.20}$$

The radiation condition represents the requirement that the moving gate generates only outgoing waves (waves travelling from the gates to infinity). ϕ^-_α is computed by expansion of the potential in a channel of infinite lenght and a Fourier series rappresentantion of the boundary condition on the gate.

Eigenfunction expansion on the Adriatic side

Because of the presence of lateral breakwaters the Adriatic side of Venice Storm gates can be approximated as a channel with impermeable vertical walls on the sides. By using separation of variables, the wave potential for a channel of infinite length can be written with the following expansion (as in [3]):

$$\begin{aligned}\phi_\alpha = {} & A_{00}^{(\alpha)} e^{\pm i\alpha_{00}x} \cosh[k_0(z+h)] + \sum_{n=1}^{\infty} A_{0n}^{(\alpha)} e^{\mp \alpha_{0n}x} \cos[k_n(z+h)] \\ & + \sum_{m=1}^{M} A_{m0}^{(\alpha)} e^{\pm i\alpha_{m0}x} \cos\frac{m\pi y}{a} \cosh[k_0(z+h)] \\ & + \sum_{m=M+1}^{\infty} A_{m0}^{(\alpha)} e^{\mp \alpha_{m0}x} \cos\frac{m\pi y}{a} \cosh[k_0(z+h)] \\ & + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(\alpha)} e^{\mp \alpha_{mn}x} \cos\frac{m\pi y}{a} \cos[k_n(z+h)]\end{aligned} \tag{2.21}$$

where k_0 is the positive real root and ik_n are the positive imaginary roots of dispersion relationship,

$$\omega^2 = gk \tanh[kh] \tag{2.22}$$

and α_{mn} is given by

$$\begin{aligned} \alpha_{0,0} &= k_0 && for\ n = 0,\ m = 0 \\ \alpha_{m0} &= +\sqrt{(k_0)^2 - \left(\tfrac{m\pi}{a}\right)^2} && for\ n = 0,\ m \leq M \\ \alpha_{m0} &= +\sqrt{(k_0)^2 + \left(\tfrac{m\pi}{a}\right)^2} && for\ n = 0,\ m > M \\ \alpha_{mn} &= +\sqrt{(k_n)^2 + \left(\tfrac{m\pi}{a}\right)^2} && for\ n > 0,\ \forall m \end{aligned} \tag{2.23}$$

for $m, n \in \mathbb{N}$. Here h is the water depth and a the width of the channel. M represents the highest value of m that gives $\alpha_{m0} = \sqrt{(k_0)^2 - \left(\frac{m\pi}{a}\right)^2} \in \mathbb{R}$. The reference frame is assumed as in Figure (2-3) with the zero of the x-axis on the Adriatic face of the gate.

The first two terms of eq.(2.21) represent respectively long crested propagating and evanes-

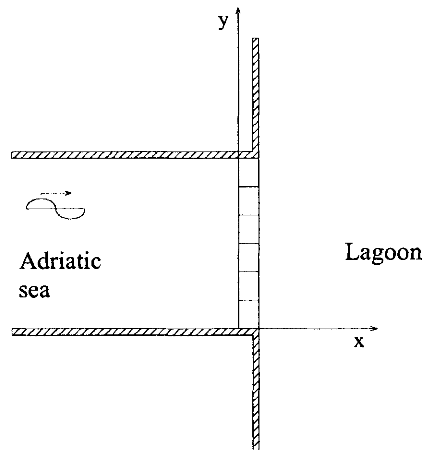

Figure 2-3: Reference frame on the Adriatic side. The x-axis has its zero on the Adriatic face of the gates.

cent modes, the third represents short crested propagating modes and the last two short crested evanescent modes.

We shall introduce normalizing factors

$$c_0 = \frac{\sqrt{2}}{(h + (g/\omega^2)\sinh^2 k_0 h)^{1/2}}; \qquad c_n = \frac{\sqrt{2}\cos k_n(z+h)}{(h - (g/\omega^2)\sin^2 k_n h)^{1/2}} \tag{2.24}$$

and the normalized eigenfunctions in the $z - direction$ are,

$$f_0 = \frac{\sqrt{2}\cosh k_0(z+h)}{(h + (g/\omega^2)\sinh^2 k_0 h)^{1/2}}; \qquad f_n = \frac{\sqrt{2}\cos k_n(z+h)}{(h - (g/\omega^2)\sin^2 k_n h)^{1/2}} \tag{2.25}$$

so that:

$$\int_{-h}^{0} f_p^2(z)dz = 1$$

$$\int_{-h}^{0} f_p(z) f_q(z)dz = 0, \quad p \neq q$$

Making use of the radiation condition, the expansion in eq.(2.21) becomes:

$$\phi_\alpha^- = A_{00}^{(\alpha)} e^{-i\alpha_{00}x} f_0 + \sum_{n=1}^{\infty} A_{0n}^{(\alpha)} e^{+\alpha_{0n}x} f_n + \sum_{m=1}^{M} A_{m0}^{(\alpha)} e^{-i\alpha_{m0}x} \cos\frac{m\pi y}{a} f_0 \tag{2.26}$$
$$+ \sum_{m=M+1}^{\infty} A_{m0}^{(\alpha)} e^{+\alpha_{m0}x} \cos\frac{m\pi y}{a} f_0 + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(\alpha)} e^{+\alpha_{mn}x} \cos\frac{m\pi y}{a} f_n$$

Observe that expansion (2.26) can be written in a more compact form. If we let κ_0 be the positive real root and κ_n the positive immaginary root of the disperdion relationship (eq. 2.22) we have

$$\phi_\alpha^- = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}^{(\alpha)} e^{-i\gamma_{mn}x} \cos\frac{m\pi y}{a} \cosh[\kappa_n(z+h)] \tag{2.27}$$

with

$$\gamma_{mn} = +\sqrt{(\kappa_n)^2 - \left(\frac{m\pi}{a}\right)^2}$$

that can be eitherpositive real or positive immaginary. Eq.(2.27) is identical to eq.(2.26). In fact for an evanescent mode γ_{mn} is a positive imaginary and so the exponent $-i\gamma_{mn}$ is positive real (evanescent mode, $x < 0$ in the Adriatic side); for a propagating mode γ_{mn} is a positive real and so the exponent $-i\gamma_{mn}$ is negative imaginary (left going wave). When κ_n is immaginary the hyberbolic cosine reduces to cosine.

Eq.(2.27) is used for the numerical computations.

The boundary condition on the gate is:

$$\frac{\partial \phi_\alpha^-}{\partial x}\Big|_{x=0} = g(y,z) = \begin{cases} (h+z) & for\ y \in [y_{\alpha-1}; y_\alpha] \\ 0 & for\ y \notin [y_{\alpha-1}; y_\alpha] \end{cases} \tag{2.28}$$

where $y_{\alpha-1}; y_\alpha$ are the coordinates of the extremities of the α^{th} moving gate.

In order to find the coefficients $A_{mn}^{(\alpha)}$ of eq.(2.26) we consider a series expansion of the boundary condition (2.28):

$$g(y,z)=\sum_{m=0}^{\infty}C_{m0}^{(\alpha)}\cos\frac{m\pi y}{a}f_0(z)+\sum_{m=0}^{\infty}\sum_{n=1}^{\infty}C_{mn}^{(\alpha)}\cos\frac{m\pi y}{a}f_n(z) \tag{2.29}$$

Since

$$\begin{aligned}\frac{\partial\phi_\alpha}{\partial x}|_{x=0}=f(y,z)&=A_{00}^{(\alpha)}(-i\alpha_{00})f_0(z)+\sum_{n=1}^{\infty}A_{0n}^{(\alpha)}(\alpha_{0n})f_n(z)\\&+\sum_{m=1}^{M}A_{m0}^{(\alpha)}(-i\alpha_{mn})\cos\frac{m\pi y}{a}f_0(z)+\sum_{m=M+1}^{\infty}A_{m0}^{(\alpha)}(\alpha_{m0})\cos\frac{m\pi y}{a}f_0(z)\\&+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}A_{mn}^{(\alpha)}(\alpha_{mn})\cos\frac{m\pi y}{a}f_n(z)\end{aligned}$$

once $C_{mn}^{(\alpha)}$ is known, $A_{mn}^{(\alpha)}$is found from:

$$\begin{aligned}A_{m0}^{(\alpha)}&=\tfrac{C_{m0}^{(\alpha)}}{-i\alpha_{m0}}; && for\ n=0\ and\ m\le M\\A_{m0}^{(\alpha)}&=\tfrac{C_{m0}^{(\alpha)}}{\alpha_{m0}}; && for\ n=0\ and\ m>M\\A_{mn}^{(\alpha)}&=\tfrac{C_{mn}^{(\alpha)}}{\alpha_{mn}}; && for\ n>0\ and\ \forall m\end{aligned}$$

Calculation of the coefficients $C_{mn}^{(\alpha)}$

We can compute $C_{mn}^{(\alpha)}$ by exploiting the orthogonality of the eigenfunctions:

$$\int_0^a\cos\frac{m\pi y}{a}\cos\frac{p\pi y}{a}dy=\begin{cases}\frac{a}{\varepsilon_m} & m=p\\0 & m\neq p\end{cases}$$

with ε_m equal to 1 for $m=0$, and equal to 2 for $m\neq 0$. Multiplying both sides of (2.29) by $\cos\frac{p\pi y}{a}$and $f_q(z)\,,(q=0,1,2...)$ and integrating we get:

$$\int_0^a \int_{-h}^0 g(y,z) \cos\frac{p\pi y}{a} f_q(z)\, dydz =$$

$$= \sum_{m=0}^{\infty} C_{m0}^{(\alpha)} \int_0^a \cos\frac{m\pi y}{a} \cos\frac{p\pi y}{a} dy \int_{-h}^0 f_0(z) f_q(z)\, dz +$$

$$+ \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} C_{mn}^{(\alpha)} \int_0^a \cos\frac{m\pi y}{a} \cos\frac{p\pi y}{a} dy \int_{-h}^0 f_n(z) f_q(z)\, dz$$

While the right hand side of the above expression (RHS) is simply equal to

$$RHS = C_{mn}^{(\alpha)} \frac{a}{\varepsilon_m}$$

the left hand-side (LHS) is

$$LHS = \int_0^a \int_{-h}^0 g(y,z) \cos\frac{m\pi y}{a} \begin{pmatrix} f_0(z) & ;(n=0) \\ f_n(z) & ;(n>0) \end{pmatrix} dydz$$

$$= \int_{y_{\alpha-1}}^{y_\alpha} \cos\frac{m\pi y}{a} dy \int_{-h}^0 (z+h) \begin{pmatrix} \frac{\sqrt{2}\cosh k_0(z+h)}{(h+(g/\omega^2)\sinh^2 k_0 h)^{1/2}} & ;(n=0) \\ \frac{\sqrt{2}\cos k_n(z+h)}{(h-(g/\omega^2)\sin^2 k_n h)^{1/2}} & ;(n>0) \end{pmatrix} dz$$

$$= \begin{cases} c_0 \left[\frac{h\sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2}\right] L & ;\ n=0, m=0 \\ c_0 \left[\frac{h\sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2}\right] \frac{2a}{m\pi} \cos\frac{m\pi\overline{y_\alpha}}{a} \sin\frac{m\pi L}{2} & ;\ n=0, m>0 \\ c_n \left[\frac{h\sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2}\right] L & ;\ n>0, m=0 \\ c_n \left[\frac{h\sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2}\right] \frac{2a}{m\pi} \cos\frac{m\pi\overline{y_\alpha}}{a} \sin\frac{m\pi L}{2} & ;\ n>0, m>0 \end{cases}$$

where $\overline{y_\alpha}$ and L represent respectively the coordinate of the middle point and the width of the moving gate (the gates have all the same width). See Figure(2-4). In the last integral we made use of the subtraction formulas for circular functions.

Finally we get for $A_{mn}^{(\alpha)}$:

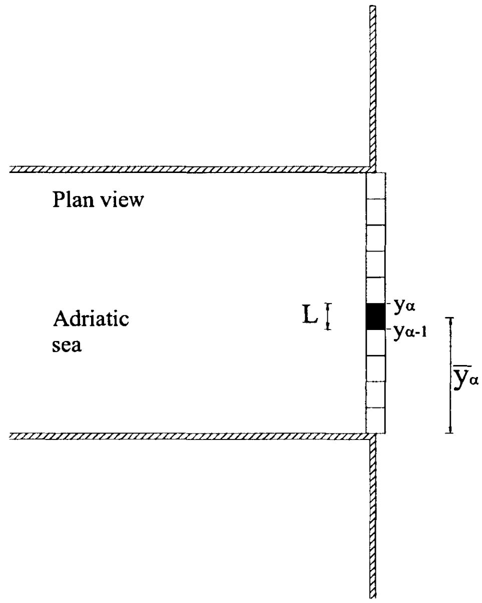

Figure 2-4: Schetch of the symbols used to compute the coefficients of the expansion of the radiation potential on the Adriatic side

$$A_{mn}^{(\alpha)} = \begin{cases} \frac{1}{-i\alpha_{00}} \frac{\varepsilon_0}{a} \left[c_0 \left[\frac{h \sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2} \right] L \right]; & n = 0, m = 0 \\ \frac{1}{-i\alpha_{m0}} \frac{\varepsilon_m}{a} \left[c_0 \left[\frac{h \sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2} \right] \frac{2a}{m\pi} \cos \frac{m\pi \bar{y}_\alpha}{a} \sin \frac{m\pi L}{2} \right]; & n = 0, 1 < m \leq M \\ \frac{1}{\alpha_{mn}} \frac{\varepsilon_m}{a} \left[c_0 \left[\frac{h \sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2} \right] \frac{2a}{m\pi} \cos \frac{m\pi \bar{y}_\alpha}{a} \sin \frac{m\pi L}{2} \right]; & n = 0, m > M \\ \frac{1}{\alpha_{m0}} \frac{\varepsilon_0}{a} \left[c_n \left[\frac{h \sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2} \right] L \right]; & n > 0, m = 0 \\ \frac{1}{\alpha_{mn}} \frac{\varepsilon_m}{a} \left[c_n \left[\frac{h \sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2} \right] \frac{2a}{m\pi} \cos \frac{m\pi \bar{y}_\alpha}{a} \sin \frac{m\pi L}{2} \right]; & n > 0, m > 0 \end{cases} \tag{2.30}$$

Observe that it blows up for $\alpha_{mn} \to 0^+$ (i.e. when $k_n \to (\frac{m\pi}{a})$),this situation is associated with the decrease of the number of propagating modes (see further).

2.2.2 Radiation potential on the lagoon side

The lagoon side is modelled as a semi-infinite space (Figure: 2-1). Let us look first for the velocity field due to a single moving gate with unit angular velocity. The boundary value problem is similar to the Adriatic side, now the radiation condition can be expressed with the more general Sommerfel form:

$$
\begin{array}{ll}
\nabla^2 \phi_\alpha^+ = 0 & \\
\frac{\partial \phi_\alpha}{\partial z} = 0 & on\ z = -h \\
\frac{\partial \phi_\alpha}{\partial x} = 0 & on\ sidewalls\ of\ the\ semi-space \\
\frac{\partial \phi_\alpha}{\partial x} = 0 & on\ all\ the\ still\ gates \\
\frac{\partial \phi_\alpha^+}{\partial x} = (h+z) & on\ the\ moving\ gate\ \alpha \\
\frac{\partial \phi_\alpha^+}{\partial z} - \frac{\omega^2}{g}\phi_\alpha^+ = 0 & on\ z = 0 \\
\left(\frac{\partial \phi_\alpha^+}{\partial r} - ik\phi_\alpha^+\right)\sqrt{kr} \to 0 & kr \to 0
\end{array}
$$

The potential ϕ_α^+ can be found by using an adequate Green's function as follows.

Green's function

Define the function $G(x, y-y', z)$ by

$$G_{xx} + G_{yy} + G_{zz} = 0, \quad x > 0. \tag{2.31}$$

$$G_z - \sigma G = 0, \quad z = 0, \quad \sigma = \omega^2/g$$

$$G_z = 0, \quad z = -h$$

and

$$G_x = \delta(y-y')(z+h), \quad x = 0.$$

and G is outgoing at infinity. The reference frame is here assumed as in fig.(2-5)[1]. The x-axis has its axis on the lagoon face of the gate.

[1]Note that with different reference frames on the Adriatic and Lagoon side practically we are not accounting fot the gates thickness. The two fluid domains are physically separated, they are coupled only by the kinematic boundary condition that is independent from thickness (in the linear approximation). Ultimately, we are interested in the forces on the gates that are integral qualtities. This procedure leads to the same result we would have gotten if we had considered only one reference frame and we had assumked zero thickness of the gates.

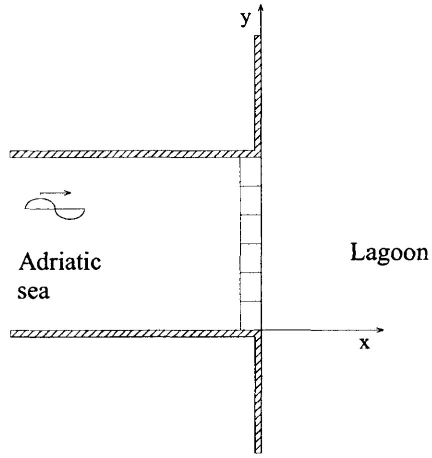

Figure 2-5: Reference frame on the Lagoon side. The x-axis has its zero on the lagoon face of the gates.

If the α^{th} gate $y \in Y_\alpha$ is moving at unit angular velocity, then

$$\phi_\alpha(x,y,z) = \int_{Y_\alpha} G(x, y-y', z)dy'$$

which satisfies

$$\frac{\partial \phi_\alpha}{\partial x} = \begin{cases} (z+h)\,, & y \in Y_\alpha \\ 0, & y \notin Y_\alpha. \end{cases} \tag{2.32}$$

By radial symmetry we can replace the half space by the whole space $0 \leq \theta < 2\pi$, and (2.32) by

$$\lim_{r \to 0} \pi r G_r = (z+h)\,. \tag{2.33}$$

where

$$r = (x^2 + (y-y')^2)^{1/2}$$

Making use of the normalized eigenfunctions defined in eq.(2.25) we expand

$$(z+h) = B_0 f_0 + \sum_{n=1}^{\infty} B_n f_n$$

and so:

$$B_0 = \int_{-h}^{0} (z+h) f_0 dz = c_0 \left(\frac{h \sinh(k_n h)}{k_n} + \frac{1 - \cosh(k_n h)}{k_n^2} \right) \tag{2.34}$$

$$B_n = \int_{-h}^{0} (z+h) f_n dz = c_n \left(\frac{h \sin(k_n h)}{k_n} + \frac{1 + \cos(k_n h)}{k_n^2} \right) \tag{2.35}$$

where c_0 and c_n are defined by (2.24).

Assume

$$G(r,z) = \sum_{n=0}^{\infty} F_n(r,z) f_n(z). \tag{2.36}$$

Then from (2.31),

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial F_0}{\partial r} \right) + k^2 F_0 = 0; \quad \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial F_n}{\partial r} \right) - k_n^2 F_n = 0;$$

with the BC:

$$\lim_{r \to 0} \pi r \frac{\partial F_0}{\partial r} = B_0.$$

$$\lim_{r \to 0} \pi r \frac{\partial F_n}{\partial r} = B_n$$

The solutions are:

$$F_0 = A_0 H_0^{(1)}(kr), \quad F_n = A_n K_0(k_n r)$$

For small r

$$F_0 \approx A_0 \left[1 + \frac{2i}{\pi} \ln(kr) \right],$$

$$\lim_{r \to 0} \pi r \frac{\partial F_0}{\partial r} = 2iA_0 = B_0.$$

Hence

$$A_0 = \frac{B_0}{2i}$$

Also, for small r,

$$F_n \approx A_n(-\ln(k_n r))$$

Hence

$$\lim_{r \to 0} \pi r \frac{\partial F_n}{\partial r} = -\pi A_n = B_n$$

and

$$A_n = -\frac{B_n}{\pi}$$

This completes the Green Function, so the potential generated by the α^{th}moving gate is

$$\phi_\alpha(x,y,z) = \int_{Y_\alpha} G(x, y-y', z)dy' = \int_{Y_\alpha} \sum_{n=0}^{\infty} F_n(r) f_n(z) dy' = \tag{2.37}$$

$$= \frac{B_0}{2i} \int_{Y_\alpha} H_0^{(1)}(k_0 r) f_0(z) dy' + \sum_{n=1}^{\infty} -\frac{B_n}{\pi} \int_{Y_\alpha} K_0(k_n r) f_n(z) dy'$$

2.2.3 The amplitude of motion of the gates

Once $\left[F^A\right]$ and $\left[F^L\right]$ are known from the solution of the radiation problem, they can be used (along with the forcing term $\left\{F^D\right\}$) to calculate the amplitude of motion $\{\theta\}$ of the n gates by solving eq.(2.19):

$$\left[\left(-\omega^2 I + C\right)[D] + i\omega\left[F^A\right] - i\omega\left[F^L\right]\right]\{\theta\} = \left\{F^D\right\}$$

We describe next the calculation of the matrices $\left[F^A\right]$, $\left[F^L\right]$ and $\left\{F^D\right\}$.

Calculation of $\left[F^A\right]$

The elements of $\left[F^A\right]$ are calculated with eq.(2.10) :

$$F^A_{\beta,\alpha} = i\omega\rho \iint_{S_\beta} \phi^-_\alpha \,(h+z)\, dS$$

where ϕ^-_α is evaluated at $x = 0$ and represents the wave potential due to the unit velocity oscillation of the α^{th} gate. Recall that ϕ^-_α is given by eq. (2.26)(ϕ^-_α is evaluated at $x = 0$), so the above integral is :

$$F_{\beta,\alpha}^{A} = \iint_{S_\beta} \phi_\alpha^- (h+z)\, dS = \sum_{m=0}^{\infty} A_{m0,\alpha}^{(\alpha)} \int_{y_{\beta-1}}^{y_\beta} dy \cos\frac{m\pi y}{a} \int_{-h}^{0} (h+z)\, f_0(z)\, dz +$$

$$+ \sum_{m=0}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(\alpha)} \int_{y_{\beta-1}}^{y_\beta} dy \cos\frac{m\pi y}{a} \int_{-h}^{0} (h+z)\, f_n(z)\, dz =$$

$$= A_{00}^{(\alpha)} \left[\frac{h\sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2}\right] L + \sum_{n=1}^{\infty} A_{0n}^{(\alpha)} \left[\frac{h\sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2}\right] L +$$

$$+ \sum_{m=1}^{\infty} A_{m0}^{(\alpha)} \left[\frac{h\sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2}\right] \frac{a}{m\pi} 2\cos\frac{m\pi\bar{y}_\beta}{a} \sin\frac{m\pi L}{2a} +$$

$$+ \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}^{(\alpha)} \left[\frac{h\sin(hk_n)}{k_n} + \frac{\cos(hk_n)}{k_n^2} - \frac{1}{k_n^2}\right] \frac{a}{m\pi} 2\cos\frac{m\pi\bar{y}_\beta}{a} \sin\frac{m\pi L}{2a}$$

whit $y_{\beta-1}, y_\beta$ coordinates of the edges of the β^{th} gate on whose surface the integral is calculated, $\bar{y}_\beta$ coordinate of the middle point of the β^{th} gate, L width of the β^{th} gate (the gates are assumed to have all the same width).

Recalling the expression of $A_{mn}^{(\alpha)}$ given by eq.(2.30), it is readily verified that the value of the above integral doesn't change by switching $\bar{y}_\alpha$ with $\bar{y}_\beta$, so the matrix $\left[F^A\right]$ is symmetric.

Calculation of $\left[F^L\right]$

The elements of $\left[F^L\right]$ are calculated with eq.(2.11):

$$F_{\beta,\alpha}^{A} = i\omega\rho \iint_{S\beta} \phi_\alpha^+ (h+z)\, dS$$

where ϕ_α^+ is evaluated at $x=0$ and is given by eq.(2.37), that is:

$$\phi_\alpha(x,y,z) = \int_{Y_\alpha} G(x, y-y', z)dy' = \int_{Y_\alpha} dy' \sum_{n=0}^{\infty} F_n(r) f_n(z)$$

Thus we have:

$$F^L_{\beta,\alpha} = -\rho \iint\limits_{S_\beta} - i\omega\phi_\alpha \,(h+z)\, dS = i\rho\omega \iint\limits_{S_\beta} dS\,(h+z) \int_{Y_\alpha} G(x, y-y', z) dy' =$$

$$= i\rho\omega \iint\limits_{S_\beta} dS\,(h+z) \left[\frac{B_0}{2i} \int_{Y_\alpha} H_0^{(1)}(k_0 r) f_0(z) dy' + \sum_{n=1}^{\infty} -\frac{B_n}{\pi} \int_{Y_\alpha} K_0(k_n r) f_n(z) dy' \right]$$

While the z integral can be evaluated analitically,the y integral must be evaluated numerically as follows.

Referring to fig.(2-6), let the α^{th} gate be the the only moving gate. For $\beta \neq \alpha$ the radiated potential ϕ^+_α is divided into several small pieces (say P) of width δp. The β^{th} gate upon whitch the integral is computed, is divided into many small pieces (say P') of width $\delta p'$. On the α^{th} gate each piece represents an elementary source. The idea is to compute $F^L_{\beta,\alpha}$ as a sum of the effects of each small part δp (seen as a source) on all the other parts $\delta p'$ of β^{th} gate. (See Figure(2-6)). The integral becomes:

$$F^L_{\beta,\alpha;(\alpha\neq\beta)} = i\rho\omega \int_{Y_\alpha} dy' \sum_n \int_{y_{\beta-1}}^{y_\beta} dr F_n(r) \int\limits_{-h}^{0} f_n(z)\,(h+z)\, dz \qquad (2.38)$$

$$\simeq i\rho\omega \left[\sum_{p'=1}^{P} \delta p' \sum_{n=1}^{N} \sum_{p=1}^{P} \delta p F_n(p,p') B_n \right]$$

in which use is made of (2.34) and (2.35). $F_n(p,p')$ represents the value of the $F_n(k_n r)$ when r is equal to the distance between the mid point of element p and the mid point of element p', N represents the number of terms taken into account in the truncated expansion of the Green Function (eq.(2.36)).

For $\beta = \alpha$ the Green function is singular and special care is needed. For the part near the singular point ($p = p'$) the integral is evaluated analytically by making use of the approximation of Hankel and Kelvin functions.

In particular we have for $\beta = \alpha$

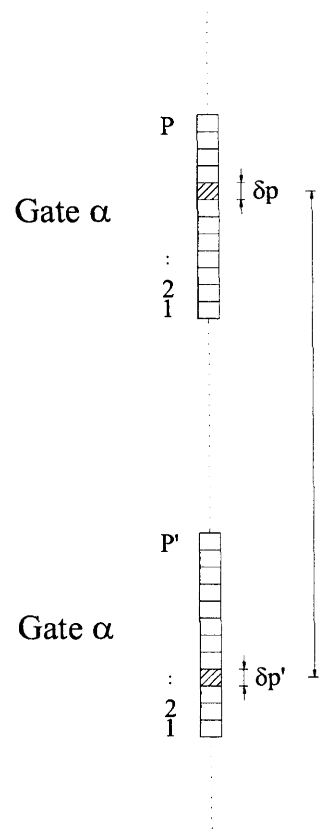

Figure 2-6: Calculation of the Hydrodinamic force on the lagoon side.

$$F^L_{\alpha\alpha} = i\rho\omega \int_{Y_\alpha} dy' \sum_n \int_{y_{\alpha-1}}^{y_\alpha} dr F_n(r) \int_{-h}^{0} f_n(z)\,(h+z)\,dz \tag{2.39}$$

$$\simeq i\rho\omega \left[I_{sp} + \sum_{\substack{p'=1 \\ p'\neq p}}^{P} \delta p' \sum_{n=1}^{N} \sum_{p=1}^{P} \delta p F_n(p,p') B_n \right]$$

I_{sp} represents the contribution to the integral around the singular point evaluated as follows. For $k_n r \ll 1$, we can use the approximation of Hankel and Kelvin function for small values of the argument:

$$H_0^{(1)}(k_0 r) \simeq 1 + \frac{2i}{\pi}\ln(kr); \qquad K_0(k_n r) \simeq -\ln(kr)$$

Let $s = \delta p/2$, then:

$$2\int_0^s dy' 2 \int_0^s H_0^{(1)}(k_0 r) dr = 4 \int_0^s dy' \int_0^s dy H_0^{(1)}(k_0 \left|y-y'\right|) =$$

$$\simeq 4 \int_0^s dy' \int_0^s dy \left[1 + \frac{2i}{\pi}\ln(k_0\left|y-y'\right|)\right] =$$

$$= 4 \int_0^s dy' \left\{ y + \frac{2i}{\pi}\left[(s-y')\ln(k_0\left|s-y'\right|) - s^2 + y'\ln\left(k_0 y'\right) \right] \right\} =$$

$$= 4 \left\{ s + \frac{2i}{\pi} \left[\int_0^s dy' \left(s-y'\right)\ln(k_0\left|s-y'\right|) - s^2 + \int_0^s dy' y' \ln\left(k y'\right) \right] \right\} =$$

$$= 4 \left\{ s + \frac{2i}{\pi} \left[\frac{s^2}{2}\ln(k_0 s) - \frac{s^2}{4} - s^2 + \frac{s^2}{2}\ln(k_0 s) - \frac{s^2}{4} \right] \right\} =$$

$$= 4 \left\{ s + \frac{2i}{\pi} \left[s^2 \ln(k_0 s) - \frac{3}{2} s^2 \right] \right\}$$

In reaching the 3^{rd} equality we have made use of the following result:

$$\int_0^s dy \ln(k\,|y-y'|) = \int_0^{y'} dy \ln(k\,(y'-y)) + \int_{y'}^s dy \ln(k\,(y-y')) =$$

$$= (let:\ k\,(y'-y) = p;\ k\,(y-y') = q) =$$

$$= \int_{ky'}^0 -\frac{dp}{k}\ln p + \int_0^{k(s-y')} \frac{dq}{k}\ln q =$$

$$= \frac{1}{k}\,|p\ln p - p|_0^{ky'} + \frac{1}{k}\,|q\ln q - q|_0^{k(s-y')} =$$

$$= y'\ln ky' + (s-y')\ln\left[k\,(a-y')\right] - s$$

In the same manner we can calculate the integral of $K_0(kr)$ as:

$$2\int_0^s dy' 2\int_0^s K_0(k_n r)dr = 4\int_0^s dy' \int_0^s dy K_0(k_n\,|y-y'|) =$$

$$\simeq 4\int_0^s dy' \int_0^s dy\left[\ln(k_n\,|y-y'|)\right] =$$

$$= 4\left\{\left[s^2\ln(k_n s) - \frac{3}{2}s^2\right]\right\}$$

Thus we have:

$$I_{sp} \simeq \frac{B_0}{2i}4\left\{s + \frac{2i}{\pi}\left[s^2\ln(ks) - \frac{3}{2}s^2\right]\right\} + \sum_{n=1}^{N}\frac{-B_n}{\pi}4\left\{\left[s^2\ln(k_n s) - \frac{3}{2}s^2\right]\right\} =$$

$$= \frac{B_0}{2i}4\left\{\frac{\delta p}{2} + \frac{2i}{\pi}\left[\frac{\delta p^2}{4}\ln(k\frac{\delta p}{2}) - \frac{3}{2}\frac{\delta p^2}{4}\right]\right\} + \sum_{n=1}^{N}\frac{-B_n}{\pi}4\left\{\left[\frac{\delta p^2}{4}\ln(k_n\frac{\delta p}{2}) - \frac{3}{2}\frac{\delta p^2}{4}\right]\right\} =$$

$$= \frac{B_0}{2i}\left\{2\delta p + \frac{2i\delta p^2}{\pi}\left[\ln(k\frac{\delta p}{2}) - \frac{3}{2}\right]\right\} + \delta p^2\sum_{n=1}^{N}\frac{-B_n}{\pi}\left[\ln(k_n\frac{\delta p}{2}) - \frac{3}{2}\right]$$

Forcing term $\{F^D\}$

We choose as incident wave a plane wave with amplitude A, wave number k (given by the dispersion relationship and equal to k_0) and frequency ω. We assume this incoming wave is fully reflected by the gates, so:

$$\phi^D = \phi^I + \phi^R = \frac{-igA}{\omega} \frac{\cosh[k_0(z+h)]}{\cosh(k_0 h)} \left(e^{+ik_0 x} + e^{-ik_0 x} \right)$$

On the α^{th} gate the hydrodynamic action is then (ϕ^D is evaluated at $x = 0$):

$$F_\alpha^D = i\rho\omega \int_{y_{\alpha-1}}^{y_\alpha} dy \int_{-h}^{0} dz \frac{-2igA}{\omega} \frac{\cosh[k_0(z+h)]}{\cosh(k_0 h)} (h+z) = \frac{2\rho g A L}{\cosh(k_0 h)} \frac{k_0 h \sinh[k_0 h] - \cosh(k_0 h) + 1}{k_0^2} \tag{2.40}$$

being L the length of gate α.

Chapter 3

General identities

It is possible to derive identities that involve some of the quantities discussed so far. They have several advantages. First, they provide a deeper understanding of the physics of the problem and a deeper meaning of the involved quantities. Second, they can be used to check the numerical results. Here we shall derive some identities. In the next chapter we describe how they are used to check the numerical results. The identities discussed in this section are obtained from applications of Green's theorem.

Green's theorem states that given two functions f and g ,both $f, g \in C^2$ in a closed 3D domain the following identity holds:

$$\int_V f\nabla^2 g - g\nabla^2 f = \int\limits_S \left[f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n} \right] dS$$

where V is a closed region bounded by the closed surface S , and n the outer normal. If f and g satisfy the no flux condition thorough the rigid walls, the free surface boundary condition and the field equation $\nabla^2 f = \nabla^2 g = 0$, then the integral reduces to:

$$\int\limits_{S_g+S_\infty} \left[f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n} \right] dS = 0 \tag{3.1}$$

where S_g is the surface of all gates; S_∞ is the distant surface that bounds the volume V at infinity.

Moreover, if f and g both further satisfy the radiation condition the integral on S_∞ vanishes too and therefore we have:

$$\int_{S_g} \left[f\frac{\partial g}{\partial n} - g\frac{\partial f}{\partial n} \right] dS = 0$$

On the Adriatic side we shall derive the so called Haskind theoreme (as in [4]), then we shall verify the symmetry of the added mass and radiation damping matrices. Finally we shall derive the law of energy conservation in the entire domain (lagoon plus Adriatic).

3.1 Haskind's theorem for the Adriatic side

Haskind theorem provides a relationship between the hydrodynamic force due to the incident and diffracted wave, the radiation potential ϕ_α^- in the far field, and the potential of the incident wave ϕ^I.

Let's consider a volume in the channel in the Adriatic side bounded by a surface S_∞ at $x = -\infty$, the moving gate S_α, all other still gates S_{sg}, the free surface S_f, the sides of the channel S_s and the sea bottom S_b. Applying Green's theorem (eq.(3.1)) to this volume using the incident potential $\phi^I$and the radiated potential ϕ_α, and reminding that this potentials satisfy the field equation $\nabla^2\phi^I = \nabla^2\phi_\alpha = 0$ and the boundary conditions on the free surface, on the bottom and on the solid lateral walls, we have:

$$\oint \left[\phi^I\frac{\partial \phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial \phi^I}{\partial n} \right] dS = \iint_{S_\infty + S_\alpha + S_{sg}} \left[\phi^I\frac{\partial \phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial \phi^I}{\partial n} \right] dS = 0 \tag{3.2}$$

By definition the hydrodynamic force on gate α is given by:

$$F_\alpha^D = \iint_{S_\alpha} p\,(h+z)\,dS = i\rho\omega \iint_{S_\alpha} (\phi^I + \phi^S)\,(h+z)\,dS = i\rho\omega \iint_{S_\alpha + S_{sg}} (\phi^I + \phi^S)\frac{\partial \phi_\alpha}{\partial n} dS$$

Note that $\iint_{S_{sg}} (\phi^I + \phi^S)\frac{\partial\phi_\alpha}{\partial n} dS = 0$ because $\left.\frac{\partial\phi_\alpha}{\partial n}\right|_{S_{sg}} = 0$. Since ϕ^S (potential of the diffracted wave) and ϕ_α are outgoing at infinity, Green's theorem applied to this two potentials gives:

$$\iint\limits_{S_\infty+S_\alpha+S_{sg}} \left[\phi^S\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^S}{\partial n}\right] dS = \iint\limits_{S_\alpha+S_{sg}} \left[\phi^S\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^S}{\partial n}\right] dS = 0$$

implying

$$\iint\limits_{S_\alpha+S_{sg}} \phi^S\frac{\partial\phi_\alpha}{\partial n}dS = \iint\limits_{S_\alpha+S_{sg}} \phi_\alpha\frac{\partial\phi^S}{\partial n}\, dS \tag{3.3}$$

F_α^D can be written as:

$$F_\alpha^D = i\rho\omega \iint\limits_{S_\alpha+S_{sg}} (\phi^I\frac{\partial\phi_\alpha}{\partial n} + \phi_\alpha\frac{\partial\phi^S}{\partial n})dS = i\rho\omega \iint\limits_{S_\alpha+S_{sg}} (\phi^I\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^I}{\partial n})dS \tag{3.4}$$

where the last equality follows from $\frac{\partial\phi^I}{\partial n} = -\frac{\partial\phi^S}{\partial n}$. From eq.(3.2) and eq.(3.4) we have:

$$F_\alpha^D = i\rho\omega \iint\limits_{S_\alpha+S_{sg}} (\phi^I\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^I}{\partial n})dS = -i\rho\omega \iint\limits_{S_\infty} (\phi^I\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^I}{\partial n})dS \tag{3.5}$$

This is Haskind's theorem. It relates the diffraction force on gate α to far field radiation potential of that gate and the potential of the incident wave. The advantage is that it is possible to compute the diffraction force without knowing the diffraction potential.

Let's carry out the integral on S_∞ of eq.(3.5) by using the expressions found for the radiated and the incident potential, and recalling that only propagating modes have to be taken in the far field (evanescent modes do not contribute any more)

$$\begin{aligned} I = &\iint\limits_{S_\infty} (\phi^I\frac{\partial\phi_\alpha}{\partial n} - \phi_\alpha\frac{\partial\phi^I}{\partial n})dS = \\ = &\int_{-h}^0 dz \int_0^a dy \frac{igA}{\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)} e^{+ikx} \sum_{m=0}^{M} A_{m0}^{(\alpha)} \left(-i\alpha_{mn}\right) e^{-i\alpha_{mn}x} \cos\frac{m\pi y}{a} f_0(z) + \\ + &\int_{-h}^0 dz \int_0^a dy \frac{-igA}{\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)} ike^{+ikx} \sum_{m=0}^{M} A_{m0}^{(\alpha)} e^{-i\alpha_{m0}x} \cos\frac{m\pi y}{a} f_0(z) \end{aligned}$$

Employing orthogonality of eigen functions $f_0(z)$ and $f_n(z)$ the hyperbolic cosine and making use of the normalization factor c_0 given in eq.(2.24) we have:

$$I = \frac{igA}{\omega} e^{+ikx} c_0 \int_{-h}^{0} dz \frac{\cosh^2[k_0(z+h)]}{\cosh(k_0 h)} \sum_{m=0}^{M} A_{m0}^{(\alpha)} \left((-i\alpha_{m0})\, e^{a_{m0}\alpha_{m0}x} - ik_0 e^{-i\alpha_{m0}x} \right) \int_0^a dy \cos\frac{m\pi y}{a}$$

since

$$\int_0^a dy \cos\frac{m\pi y}{a} = \begin{cases} a, m = 0 \\ 0, m \neq 0 \end{cases}$$

and $\alpha_{00} = k_0$; the above expression reduces to:

$$\begin{aligned} I &= \frac{igA}{\omega} a e^{+ik_0 x} A_{00}^{(\alpha)} e^{-ik_0 x} \left(-ik_0 - ik_0\right) c_0 \int_{-h}^{0} dz \frac{\cosh^2[k_0(z+h)]}{\cosh(kh)} = \\ &= \frac{-igA}{\omega} a A_{00}^{(\alpha)} 2ik_0 c_0 \frac{\sinh(2k_0 h) + 2k_0 h}{4k_0 \cosh(kh)} \end{aligned}$$

Using eq.(3.5) and the expression of $A_{00}^{(\alpha)}$:

$$\begin{aligned} F_\alpha^D &= -i\rho\omega I = -i\rho\omega \iint_{S_\infty} (\phi^I \frac{\partial \phi_\alpha}{\partial n} - \phi_\alpha \frac{\partial \phi^I}{\partial n}) dS = \qquad (3.6) \\ &= -i\rho\omega \frac{-igA}{\omega} a 2ik_0 \underbrace{\frac{1}{-ik_0 a} Lc_0 \left[\frac{h \sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2} \right] p}_{A_{00}^{(\alpha)}} \frac{\sinh(2k_0 h) + 2k_0 h}{4k_0 \cosh(kh)} \\ &= \frac{2\rho g A L}{\cosh(kh)} \frac{k_0 h \sinh[k_0 h] - \cosh(k_0 h) + 1}{k_0^2} \end{aligned}$$

This is the explicit expression of Haskind's theorem eq.(3.5) which is exactely what we obtained in the direct calculation of F_α^D in eq.(2.40) . The fact that the two expressions are the same provides a check for the theory elaborated so far.

3.2 Symmetry of added mass and radiation damping matrices

With the choice $f = \phi_\alpha$ and $g = \phi_\beta$ and using the boundary conditions we have:

$$\iint_{S_\alpha} \phi_\beta \, (h+z) \, dS = \iint_{S_\beta} \phi_\alpha \, (h+z) \, dS$$

In view of the definitions of the added mass and radiation damping (eq.(2.8) and eq.(2.9)) we have:

$$\operatorname{Im}\left[\iint_{S_\alpha} \phi_\beta \, (h+z) \, dS\right] = \operatorname{Im}\left[\iint_{S_\beta} \phi_\alpha \, (h+z) \, dS\right] \Longrightarrow \mu_{\alpha\beta} = \mu_{\beta\alpha} \tag{3.7}$$

$$\operatorname{Re}\left[\iint_{S_\alpha} \phi_\beta \, (h+z) \, dS\right] = \operatorname{Re}\left[\iint_{S_\beta} \phi_\alpha \, (h+z) \, dS\right] \Longrightarrow \lambda_{\alpha\beta} = \lambda_{\beta\alpha} \tag{3.8}$$

Thus the matrices $[\mu]$ and $[\lambda]$ are both symmetric.

3.3 Energy conservation

Applying Green's theorem to the Adriatic side first, and then to the Lagoon side (Figure(3-1)), and making use of the dynamic boundary condition on the gates, we shall find an identity for the entire problem that represents energy conservation. On both sides we use as f and g the total potential ϕ and its complex conjugate ϕ^*. In this way the derived expression has also a physical meaning because the quantity $\phi\frac{\partial \phi^*}{\partial n}$ integrated over a surface is proportional to time averaged power.

3.3.1 Adriatic side

For the Adriatic side we get:

$$\int_{S_\infty^-} \left[\phi \frac{\partial \phi^*}{\partial n} - \phi^* \frac{\partial \phi}{\partial n}\right] dS + \int_{S_g} \left[\phi \frac{\partial \phi^*}{\partial n} - \phi^* \frac{\partial \phi}{\partial n}\right] dS = 0 \tag{3.9}$$

where S_g is represents all the gates and

$$\phi = \phi^I + \phi^S + \sum_\alpha V_\alpha \phi_\alpha$$

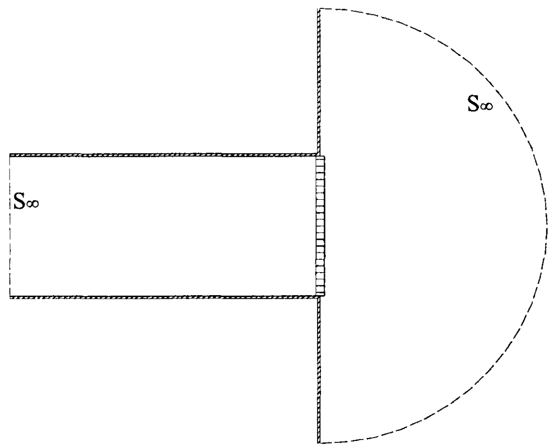

Figure 3-1: Schetch of the two domains (Adriatic-Lagoon) used for energy conservation identy.

It is understood that in this section $\phi \equiv \phi^-$, we omit the minus sign for brevity. The evaluation of the integrals is as follows on S_∞^-

$$\int_{S_\infty^-} \left[\phi\frac{\partial\phi^*}{\partial n} - \phi^*\frac{\partial\phi}{\partial n}\right] dS = -\int_{S_\infty^-} 2\,\mathrm{Im}\left[\phi\frac{\partial\phi^*}{\partial x}\right] dS$$

and:

$$\phi\frac{\partial\phi^*}{\partial n} = \left(\phi^I + \phi^S + \sum_\alpha V_\alpha\phi_\alpha\right)\frac{\partial}{\partial n}\left(\phi^{I*} + \phi^{S*} + \sum_\alpha V_\alpha^*\phi_\alpha^*\right) =$$
$$= \phi^I\frac{\partial\phi^{I*}}{\partial n} + \phi^I\frac{\partial\phi^{S*}}{\partial n} + \phi^I\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} + \phi^S\frac{\partial\phi^{I*}}{\partial n} + \phi^S\frac{\partial\phi^{S*}}{\partial n}$$
$$+ \phi^S\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} + \frac{\partial\phi^{I*}}{\partial n}\sum_\alpha V_\alpha\phi_\alpha + \frac{\partial\phi^{S*}}{\partial n}\sum_\alpha V_\alpha\phi_\alpha + \sum_\alpha V_\alpha\phi_\alpha\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n}$$

Let

$$\not{\phi} = \frac{\cosh[k(z+h)]}{\cosh(kh)}$$

and recall that:

$$\phi^I = \frac{-igA}{\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)}e^{+ikx} = \frac{-igA}{\omega}\not{\phi}e^{+ikx}$$

$$\phi^S = \frac{-igA}{\omega}\frac{\cosh[k(z+h)]}{\cosh(kh)}e^{-ikx} = \frac{-igA}{\omega}\phi\!\!/ e^{-ikx}$$

and that in ϕ_α only propagating modes are found on S_∞ so that

$$\phi_\alpha = \sum_{m=0}^{M} A_{m0}^{(\alpha)} e^{-i\alpha_{m0}x}\cos\frac{m\pi y}{a} f_0$$

(M is the greatest integer for which α_{m0} is real), thus we have:

$$\begin{aligned}
\phi\frac{\partial\phi^*}{\partial n} = {} & \frac{-igA}{\omega}\phi\!\!/ e^{+ikx}(+k)\frac{gA}{\omega}\phi\!\!/ e^{-ikx} + \frac{-igA}{\omega}\phi\!\!/ e^{+ikx}(-k)\frac{gA}{\omega}\phi\!\!/ e^{+ikx} + \frac{-igA}{\omega}\phi\!\!/ e^{+ikx}\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} + \\
& \frac{-igA}{\omega}\phi\!\!/ e^{-ikx}(+k)\frac{gA}{\omega}\phi\!\!/ e^{-ikx} + \frac{-igA}{\omega}\phi\!\!/ e^{-ikx}(-k)\frac{gA}{\omega}\phi\!\!/ e^{+ikx} + \frac{-igA}{\omega}\phi\!\!/ e^{-ikx}\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} + \\
& + k\frac{gA}{\omega}\phi\!\!/ e^{-ikx}\sum_\alpha V_\alpha\phi_\alpha - k\frac{gA}{\omega}\phi\!\!/ e^{+ikx}\sum_\alpha V_\alpha\phi_\alpha + \sum_\alpha V_\alpha\phi_\alpha\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} \\
& = -ik\left(\frac{gA}{\omega}\phi\!\!/\right)^2 + ik\left(\frac{gA}{\omega}\phi\!\!/\right)^2 e^{+2ikx} + \frac{-igA}{\omega}\phi\!\!/ e^{+ikx}\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} \\
& - ik\left(\frac{gA}{\omega}\phi\!\!/\right)^2 e^{-2ikx} + ik\left(\frac{gA}{\omega}\phi\!\!/\right)^2 + \frac{-igA}{\omega}\phi\!\!/ e^{-ikx}\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n} + \\
& + k\frac{gA}{\omega}\phi\!\!/ e^{-ikx}\sum_\alpha V_\alpha\phi_\alpha - k\frac{gA}{\omega}\phi\!\!/ e^{+ikx}\sum_\alpha V_\alpha\phi_\alpha + \sum_\alpha V_\alpha\phi_\alpha\sum_\beta V_\beta^*\frac{\partial\phi_\beta^*}{\partial n}
\end{aligned}$$

The last term can be written as:

$$\begin{aligned}
\sum_\alpha V_\alpha\phi_\alpha\sum_\beta V_\beta^*\frac{\partial\phi_\beta^*}{\partial n} = \sum_\alpha\sum_\beta V_\alpha V_\beta^*\phi_\alpha\frac{\partial\phi_\beta^*}{\partial n} = \\
= f_n^2(z)\sum_\beta[V_\alpha V_\beta^*\left(\sum_{m=0}^{M} A_{m0}^{(\alpha)} e^{-i\alpha_{m0}x}\cos\frac{m\pi y}{a}\right) \\
\left(\sum_{m=0}^{M} A_{m0}^{(\beta)*}(+i\alpha_{m0})e^{+i\alpha_{m0}x}\cos\frac{m\pi y}{a}\right)]
\end{aligned}$$

Upon integration along y and z, exploiting the orthogonality of the cosine function it reduces to:

$$\int_0^a dy \sum_\alpha V_\alpha \phi_\alpha \sum_\beta V_\beta^* \frac{\partial \phi_\beta^*}{\partial n} = f_n^2(z) \sum_\alpha \sum_\beta V_\alpha V_\beta^* \left(\sum_{m=0}^{M} A_{m0}^{(\alpha)} A_{m0}^{(\beta)*} (i\alpha_{m0}) \frac{a}{\epsilon_m} \right)$$

Denoting

$$q = \left(\frac{gA}{\omega} \frac{1}{\cosh(kh)} \right) \sqrt{\frac{2}{h + (g/\omega^2)\sinh^2 k_0 h}}$$

and

$$w = \int_{-h}^{0} dz \cosh^2[k(z+h)] = \left[\frac{\sinh[2kh] + 2kh}{4k} \right]$$

we have:

$$\int_{S_\infty^-} \left[\phi \frac{\partial \phi^*}{\partial n} \right] dS = \int_{-h}^{0} dz \int_0^a dy \left(\phi \frac{\partial \phi^*}{\partial n} \right) = \tag{3.10}$$

$$\begin{aligned}
=& w[ika \left(\frac{gA}{\omega} \frac{1}{\cosh(kh)} \right)^2 e^{+2ikx} - ie^{+ikx} q \sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} (+i\alpha_{00}) e^{+i\alpha_{00}x} a \\
& - ika \left(\frac{gA}{\omega} \frac{1}{\cosh(kh)} \right)^2 e^{-2ikx} - ie^{-ikx} q \sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} (+i\alpha_{00}) e^{+i\alpha_{00}x} a \\
& + ke^{-ikx} q \sum_\alpha V_\alpha A_{00,\alpha}^{(\alpha)} e^{-i\alpha_{00}x} a - ke^{+ikx} q \sum_\alpha V_\alpha A_{00}^{(\alpha)} e^{-i\alpha_{00}x} a] + \\
& + \sum_\alpha \sum_\beta V_\alpha V_\beta^* \left(\sum_{m=0}^{M} A_{m0,\alpha}^{(\alpha)} A_{m0}^{(\beta)*} (i\alpha_{m0}) \frac{a}{\epsilon_m} \right) \\
=& w[+ika \left(\frac{gA}{\omega} \frac{1}{\cosh(kh)} \right)^2 e^{+2ikx} + ake^{+2ikx} q \sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} \\
& - ika \left(\frac{gA}{\omega} \frac{1}{\cosh(kh)} \right)^2 e^{-2ikx} + kaq \sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} \\
& + kae^{-2ikx} q \sum_\alpha V_\alpha A_{00}^{(\alpha)} - kaq \sum_\alpha V_\alpha A_{00}^{(\alpha)}] + \\
& + \sum_\alpha \sum_\beta V_\alpha V_\beta^* \left(\sum_{m=0}^{M} A_{m0}^{(\alpha)} A_{m0}^{(\beta)*} (i\alpha_{m0}) \frac{a}{\epsilon_m} \right)
\end{aligned}$$

Therefore

$$\mathrm{Im}\int_{S_\infty^-}\left[\phi\frac{\partial\phi^*}{\partial n}\right]dS =$$

$$= \mathrm{Im}\{w\left[ake^{+2ikx}q\sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} + akq\sum_\alpha V_\alpha^* A_{00}^{(\alpha)*} + kae^{-2ikx}q\sum_\alpha V_\alpha A_{00}^{(\alpha)} - kaq\sum_\alpha V_\alpha A_{00}^{(\alpha)}\right] +$$

$$+\sum_\alpha\sum_\beta V_\alpha V_\beta^*\left(\sum_{m=0}^{M} A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\left(i\alpha_{m0}\right)\frac{a}{\epsilon_m}\right)\} =$$

$$= \mathrm{Im}\left\{w\left[+akq\sum_\alpha V_\alpha^* A_{00}^{(\beta)*} - kaq\sum_\alpha V_\alpha A_{00}^{(\alpha)}\right] + \sum_\alpha\sum_\beta V_\alpha V_\beta^*\left(\sum_{m=0}^{M} A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\left(i\alpha_{m0}\right)\frac{a}{\epsilon_m}\right)\right\}$$

We have used the fact that $\alpha_{00} = k_0$ and the fact that the sum of conjugate terms has no imaginary part.

In terms of θ_α the above integral can be written as:

$$\mathrm{Im}\int_{S_\infty^-}\left[\phi\frac{\partial\phi^*}{\partial n}\right]dS = \mathrm{Im}\{w\left[akq\sum_\alpha i\omega\theta_\alpha^* A_{00}^{(\alpha)*} + kaq\sum_\alpha i\omega\theta_\alpha A_{00}^{(\alpha)}\right] + \tag{3.11}$$

$$+\sum_\alpha\sum_\beta \omega^2\theta_\alpha\theta_\beta^*\left(\sum_{m=0}^{M} A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\left(i\alpha_{m0}\right)\frac{a}{\epsilon_m}\right)\}$$

which is the first part of eq.(3.9)

Now we turn to the integral on all the gates (S_g) which is the second part of eq.(3.9)

$$\int_{S_g}\left[\phi\frac{\partial\phi^*}{\partial n} - \phi^*\frac{\partial\phi}{\partial n}\right]dS = \int_{S_g} 2\,\mathrm{Im}\left[\phi\frac{\partial\phi^*}{\partial n}\right]dS$$

now,

$$\int_{S_g} dS\phi\frac{\partial\phi^*}{\partial n} = \int_{S_g} dS\left(\phi^I+\phi^S+\sum_\alpha V_\alpha\phi_\alpha\right)\left[\frac{\partial}{\partial n}\left(\phi^{I*}+\phi^{S*}\right)+\frac{\partial}{\partial n}\sum_\alpha V_\alpha^*\phi_\alpha^*\right] =$$

$$= \int_{S_g} dS\left(\phi^I+\phi^S+\sum_\alpha V_\alpha\phi_\alpha\right)\frac{\partial}{\partial n}\sum_\alpha V_\alpha^*\phi_\alpha^*$$

because

$$\frac{\partial}{\partial n}\left(\phi^{I*}+\phi^{S*}\right)=0$$

everywhere on the gates. Next,

$$\int_{S_g} dS\left(\phi^I+\phi^S+\sum_\alpha V_\alpha\phi_\alpha\right)\frac{\partial}{\partial n}\sum_\alpha V_\alpha^*\phi_\alpha^* = \sum_\beta\int_{S_\beta}\left(\phi^I+\phi^S+\sum_\alpha V_\alpha\phi_\alpha\right)\sum_\alpha V_\alpha^*\frac{\partial\phi_\alpha^*}{\partial n}dS =$$

$$= \sum_\beta\int_{S_\beta}\left(\phi^I+\phi^S+\sum_\alpha V_\alpha\phi_\alpha\right)V_\beta^*\frac{\partial\phi_\beta^*}{\partial n}dS$$

because

$$\frac{\partial\phi_\beta^*}{\partial n}=\begin{cases}(h+z) & on\ S_\beta\\ 0 & else\end{cases}$$

follows that:

$$\int_{S_g} dS\phi\frac{\partial\phi^*}{\partial n} = \sum_\beta\int_{S_\beta}\left[i\omega(h+z)V_\beta^*\left(\phi^I+\phi^S\right)+i\omega(h+z)V_\beta^*\sum_\alpha V_\alpha\phi_\alpha\right]dS = \tag{3.12}$$

$$= \sum_\beta\left[V_\beta^*\int_{S_\beta} i\omega(h+z)\left(\phi^I+\phi^S\right)dS+V_\beta^*\sum_\alpha V_\alpha\int_{S_\beta} i\omega(h+z)\phi_\alpha dS\right] =$$

$$= \sum_\beta\left[V_\beta^*\frac{F_\beta^D}{i\rho\omega}+V_\beta^*\sum_\alpha V_\alpha\frac{F_{\alpha,\beta}^a}{i\rho\omega}dS\right] = i\omega\left\{\theta\right\}^{T*}\frac{\left\{F^D\right\}}{i\rho\omega}+(-i\omega)(i\omega)\left\{\theta\right\}^{T*}\frac{\left[F^a\right]}{i\rho\omega}\left\{\theta\right\} =$$

$$= \left\{\theta\right\}^{T*}\frac{\left\{F^D\right\}}{\rho}-i\omega\left\{\theta\right\}^{T*}\frac{\left[F^A\right]}{\rho}\left\{\theta\right\}$$

Substituting into eq.(3.9) eq.(3.11) and eq.(3.12) we have:

$$\mathrm{Im}\left\{ w\left[akq\sum_{\alpha} i\omega\theta_{\alpha}^{*}A_{00}^{(\alpha)*} + kaq\sum_{\alpha} i\omega\theta_{\alpha}A_{00}^{(\alpha)}\right] + \sum_{\alpha}\sum_{\beta}\omega^{2}\theta_{\alpha}\theta_{\beta}^{*}\left(\sum_{m=0}^{M} A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\left(i\alpha_{m0}\right)\frac{a}{\epsilon_{m}}\right)\right\} =$$

$$= \mathrm{Im}\left\{ \{\theta\}^{T*}\frac{\{F^{D}\}}{\rho} - i\omega\{\theta\}^{T*}\frac{[F^{A}]}{\rho}\{\theta\}\right\} \quad (3.13)$$

3.3.2 Lagoon side

On the Lagoon side:

$$\int_{S_{\infty}^{+}}\left[\phi\frac{\partial\phi^{*}}{\partial n} - \phi^{*}\frac{\partial\phi}{\partial n}\right]dS + \int_{S_{g}}\left[\phi\frac{\partial\phi^{*}}{\partial n} - \phi^{*}\frac{\partial\phi}{\partial n}\right]dS = 0 \quad (3.14)$$

The second integral can be written as

$$\int_{S_{g}}\left[\phi\frac{\partial\phi^{*}}{\partial n} - \phi^{*}\frac{\partial\phi}{\partial n}\right]dS = \int_{S_{g}} 2\,\mathrm{Im}\left[\phi\frac{\partial\phi^{*}}{\partial n}\right]dS \quad (3.15)$$

Here

$$\phi = \sum_{\alpha} V_{\alpha}\phi_{\alpha} \quad \text{with} \quad \phi_{\alpha} = \int_{Y_{\alpha}} G(x, y-y', z)dy'$$

It is understood that in this section $\phi = \phi^{+}$.Also on this side we have:

$$\frac{\partial\phi_{\beta}^{*}}{\partial n} = \begin{cases} (h+z) & on\ S_{\beta} \\ 0 & else \end{cases}$$

Thus:

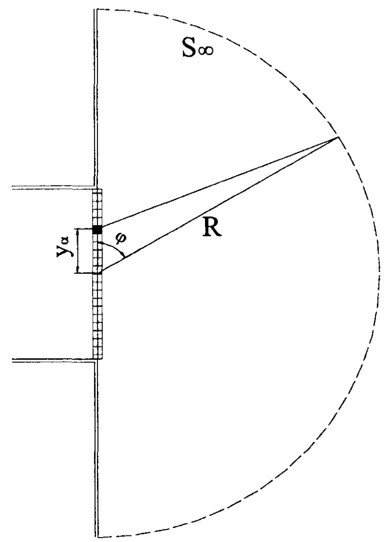

Figure 3-2: Schetch of the lagoon side

$$\int\limits_{S_g} dS\phi\frac{\partial\phi^*}{\partial n} = \int\limits_{S_g} dS\sum_\alpha V_\alpha\phi_\alpha\frac{\partial}{\partial n}\left(\sum_\alpha V_\alpha\phi_\alpha\right)^* = \int\limits_{S_g} dS\sum_\alpha\sum_\beta V_\alpha V_\beta^*\phi_\alpha\frac{\partial\phi_\beta^*}{\partial n} = \tag{3.16}$$

$$= \sum_\beta\int\limits_{S_\beta} dSV_\beta^*\sum_\alpha V_\alpha\phi_\alpha(h+z) = \sum_\beta V_\beta^*\sum_\alpha V_\alpha\int\limits_{S_\beta}(h+z)\phi_\alpha dS =$$

$$= \sum_\beta V_\beta^*\sum_\alpha V_\alpha\frac{F_{\alpha,\beta}^L}{i\rho\omega} = -i\omega\left\{\theta\right\}^{T*}\frac{\left[F^L\right]}{\rho}\left\{\theta\right\}$$

For the first integral on the distant surface S_∞^+ in eq.(3.14) we have:

$$\int\limits_{S_\infty^+}\left[\phi\frac{\partial\phi^*}{\partial n} - \phi^*\frac{\partial\phi}{\partial n}\right]dS = \int\limits_{S_\infty^+} 2\,\mathrm{Im}\left[\phi\frac{\partial\phi^*}{\partial n}\right]dS$$

Now on S_∞ $r = R$ is very large,

$$\int_{S_\infty^+} \phi \frac{\partial \phi^*}{\partial n} dS = \int_{S_\infty^+} \left(\sum_\alpha \theta_\alpha \phi_\alpha \right) \left(\sum_\beta \theta_\beta^* \frac{\partial \phi_\beta^*}{\partial n} \right) dS =$$

$$= \sum_{\alpha,\beta} \int_{-h}^{0} dz \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} R d\varphi \; \theta_\alpha \theta_\beta^* \int_{Y_\alpha} G(x, y-y', z) dy' \int_{Y_\beta} \frac{\partial G^*(x, y-y', z)}{\partial n} dy'$$

For small y_α / R, the integral can be approximated as follows. With $\epsilon_\alpha = \frac{y_\alpha}{R} \ll 1$ and referring to Figure (3-2)

$$r_\alpha = \sqrt{R^2 + y_\alpha^2 - 2Ry_\alpha \cos(\varphi)} = R\sqrt{1 + \epsilon_\alpha^2 - 2\frac{y_\alpha}{R}\cos(\varphi)} = R\,(1 - \epsilon_\alpha \cos(\varphi)) \; + O(\epsilon_\alpha^2)$$

We can write the Hankel function for large values of the argument:

$$H_0^{(1)}(z) = \sqrt{\frac{2}{\pi z}} e^{i\left[z - \frac{\pi}{4}\right]} (1 + O(\frac{1}{z}))$$

so that the integral of the Green function on the α^{th}−gate can be approximated as follows.

$$H_0^{(1)}(kr_\alpha) = H_0^{(1)} \left[kR \left(1 - \frac{y'}{R} \cos(\varphi) \right) \right]$$

and the radiated potential is:

$$\phi_\alpha = \int_{Y_\alpha} G(x, y-y', z) dy' = A_0 f(z) \int_{\bar{y}_\alpha - \frac{L}{2}}^{\bar{y}_\alpha + \frac{L}{2}} \sqrt{\frac{2}{\pi kr}} e^{i\left[kr - \frac{\pi}{4}\right]} (1 + O(\frac{1}{kr})) dy' = \quad (3.17)$$

$$= A_0 f(z) \int_{\bar{y}_\alpha - \frac{L}{2}}^{\bar{y}_\alpha + \frac{L}{2}} \sqrt{\frac{2}{\pi kR \left(1 - \frac{y'}{R} \cos(\varphi)\right)}} e^{i\left[kR\left(1 - \frac{y'}{R} \cos(\varphi)\right) - \frac{\pi}{4}\right]} (1 + O(\frac{1}{kr})) dy'$$

In the above $\frac{y'}{R} = O(\epsilon)$, so we can use Taylor expansion:

$$\sqrt{\frac{1}{\left(1 - \frac{y'}{R} \cos(\varphi)\right)}} = 1 - \frac{y'}{R} \cos(\varphi) + O(\epsilon^2)$$

$$\frac{1}{\left(1 - \frac{y'}{R} \cos(\varphi)\right)} = 1 + \frac{y'}{R} \cos(\varphi) + O(\epsilon^2)$$

Substitute the above expansion into eq.(3.17) and retaining terms up to and including $O(\frac{y_\alpha}{R})$:

$$\phi_\alpha = A_0 f(z)\sqrt{\frac{2}{\pi kR}}\int_{\bar{y}_\alpha-\frac{L}{2}}^{\bar{y}_\alpha+\frac{L}{2}}\left(1-\frac{y'}{R}\cos(\varphi)\right)e^{i\left[kR\left(1-\frac{y'}{R}\cos(\varphi)\right)-\frac{\pi}{4}\right]}\left(1+O(\frac{1}{kr})\right)dy' =$$
$$= A_0 f(z)\sqrt{\frac{2}{\pi kR}}e^{i\left[kR-\frac{\pi}{4}\right]}\int_{\bar{y}_\alpha-\frac{L}{2}}^{\bar{y}_\alpha+\frac{L}{2}} e^{-iky'\cos(\varphi)}\left[1+O(\frac{1}{kr})\right]dy'$$

Evaluating the integral:

$$\int_{\bar{y}_\alpha-\frac{L}{2}}^{\bar{y}_\alpha+\frac{L}{2}} e^{-iky'\cos(\varphi)}dy' = \frac{1}{-ik\cos(\varphi)}\left|e^{-iky'\cos(\varphi)}\right|_{\bar{y}_\alpha-\frac{L}{2}}^{\bar{y}_\alpha+\frac{L}{2}} = -2i\frac{e^{-ik\bar{y}_\alpha\cos(\varphi)}}{-ik\cos(\varphi)}\sin\left(\frac{L}{2}k\cos(\varphi)\right) =$$
$$= 2\frac{e^{-ik\bar{y}_\alpha\cos(\varphi)}}{k\cos(\varphi)}\sin\left(\frac{L}{2}k\cos(\varphi)\right)$$

it follows that the radiated potential is:

$$\phi_\alpha = A_0 f(z)\sqrt{\frac{2}{\pi kR}}e^{i\left[kR(1-\frac{\bar{y}_\alpha}{R}\cos(\varphi))-\frac{\pi}{4}\right]}\left[\frac{2\sin\left(\frac{L}{2}k\cos(\varphi)\right)}{k\cos(\varphi)}+O\left(\frac{\bar{y}_\alpha}{R},\frac{1}{kR}\right)\right]$$

and:

$$\frac{\partial\phi^*}{\partial n} = \left(\frac{\partial}{\partial R}\phi_\alpha\right)^* = \left[\frac{\partial}{\partial R}\left(A_0 f(z)\sqrt{\frac{2}{\pi kR}}e^{i\left[kR(1-\frac{\bar{y}_\alpha}{R}\cos(\varphi))-\frac{\pi}{4}\right]}\frac{2\sin\left(\frac{L}{2}k\cos(\varphi)\right)}{k\cos(\varphi)}\right)\right]^* =$$
$$= A_0 f(z)e^{-i\left[kR(1-\frac{\bar{y}_\alpha}{R}\cos(\varphi))-\frac{\pi}{4}\right]}\sqrt{\frac{2}{\pi kR}}(-i)\frac{2\sin\left(\frac{L}{2}k\cos(\varphi)\right)}{\cos(\varphi)}+\sqrt{\frac{1}{kR}}O\left(\frac{\bar{y}_\alpha}{R},\frac{1}{kR}\right)$$

Thus we have:

$$\int\limits_{S_\infty^+} \phi_\alpha \frac{\partial \phi_\alpha^*}{\partial n} dS = \int_{-h}^{0} dz \int_0^\pi \phi_\alpha \frac{\partial \phi_\alpha^*}{\partial R} R d\varphi = |A_0|^2 \int_{-h}^{0} f^2(z)\, dz =$$

$$= |A_0|^2 \int_{-h}^{0} \left(\frac{\sqrt{2}\cosh k(z+h)}{(h + (1/\sigma)\sinh^2 kh)^{1/2}} \right)^2 dz \int_0^\pi \frac{-8i}{k^2\pi} \frac{\sin^2\left(\frac{L}{2} k \cos(\varphi)\right)}{\cos^2(\varphi)} d\varphi =$$

$$= |A_0|^2 \frac{-8i}{k^2\pi} \int_0^\pi \frac{\sin^2\left(\frac{L}{2} k \cos(\varphi)\right)}{\cos^2(\varphi)} d\varphi$$

In general (for $\alpha \neq \beta$)we have:

$$\int\limits_{S_\infty^+} \phi_\alpha \frac{\partial \phi_\beta^*}{\partial n} dS = \int\limits_{S_\infty^+} \left\{ A_0 f(z) \sqrt{\frac{2}{\pi k R}} e^{i\left[kR(1 - \frac{y_\alpha}{R}\cos(\varphi)) - \frac{\pi}{4}\right]} \frac{2\sin\left(\frac{L}{2} k\cos(\varphi)\right)}{k\cos(\varphi)} \right\}$$

$$\left\{ A_0 f(z) e^{-i\left[kR(1 - \frac{y_\beta}{R}\cos(\varphi)) - \frac{\pi}{4}\right]} \sqrt{\frac{2}{\pi k R}} (-i) \frac{2\sin\left(\frac{L}{2} k\cos(\varphi)\right)}{k\cos(\varphi)} \right\} dS =$$

$$= |A_0|^2 \frac{-8i}{k^2\pi} \int_0^\pi e^{ik(y_\beta - y_\alpha)\cos(\varphi)} \frac{\sin^2\left(\frac{L}{2} k \cos(\varphi)\right)}{\cos^2(\varphi)} d\varphi$$

In the above $r = \left[x^2 + (y - y')^2\right]^{0.5} = O(R)$. The integral on the far surface in the Lagoon is:

$$\int\limits_{S_\infty^+} \phi \frac{\partial \phi^*}{\partial n} dS = 2\,\mathrm{Im}\left[\sum_{\alpha,\beta} \omega^2 \theta_\alpha \theta_\beta^* |A_0|^2 \frac{-8i}{k^2\pi} \int_0^\pi e^{ik(y_\beta - y_\alpha)\cos(\varphi)} \frac{\sin^2\left(\frac{L}{2} k \cos(\varphi)\right)}{\cos^2(\varphi)} d\varphi \right] \tag{3.18}$$

Substituting into eq.(3.14) eq.(3.16) and eq.(3.18) we have:

$$-i\omega \{\theta\}^{T*} \frac{\left[F^L\right]}{\rho} \{\theta\} = \mathrm{Im}\left[\sum_{\alpha,\beta} \omega^2 \theta_\alpha \theta_\beta^* |A_0|^2 \frac{-8i}{k^2\pi} \int_0^\pi e^{ik(y_\beta - y_\alpha)\cos(\varphi)} \frac{\sin^2\left(\frac{L}{2} k \cos(\varphi)\right)}{\cos^2(\varphi)} d\varphi \right] \tag{3.19}$$

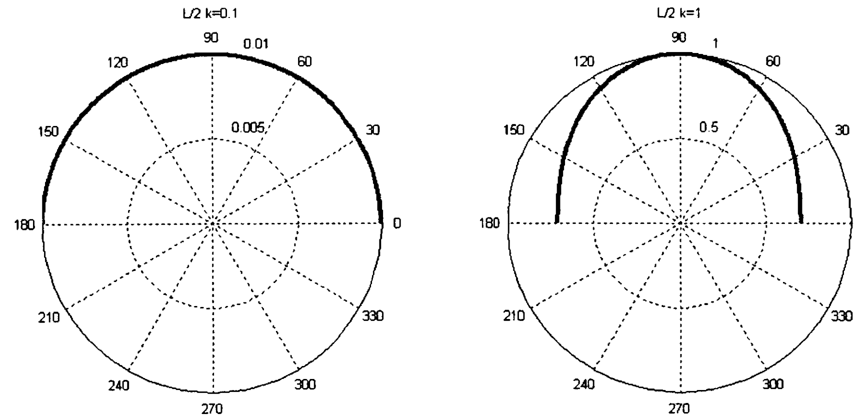

Figure 3-3: Polar plot of $\frac{\sin^2\left(\frac{L}{2}k\cos(\varphi)\right)}{\cos^2(\varphi)}$ for $\frac{L}{2}K = 0.1$; $\frac{L}{2}K = 1$ vs. φ

It is interesting to observe the polar plot (Figure 3-3) of the integrand on the last expression, because it shows how energy is spread along the angle. In the case $y_{\beta -} y_\alpha = 0$, for Venice gates $k \sim 0.1 - .01$,so $\frac{L}{2}k \sim 1 - 0.1$.

3.3.3 Use of the Dynamic condition

After analyzing Green's formula in this two domains, we can use the dynamic boundary condition on the gates to couple the two domains, that is

$$\left(-\omega^2 I + C\right)\{\theta\} = \left\{F^D\right\} - i\omega\left[F^A\right]\{\theta\} + i\omega\left[F^L\right]\{\theta\}$$

Multiplying both sides on the left by $\{\theta\}^{T*}$, rearranging, and taking the imaginary parts of both sides we get:

$$\underbrace{\operatorname{Im}\left[\left(-\omega^2 I + C\right)\{\theta\}^{T*}\{\theta\}\right]}_{\text{Power stored in the gates}} = \underbrace{\operatorname{Im}\left[\{\theta\}^{T*}\left\{F^D\right\} - i\omega\{\theta\}^{T*}\left[F^A\right]\{\theta\}\right]}_{\text{Power flux -Adr side}} + \underbrace{\operatorname{Im}\left[-i\omega\{\theta\}^{T*}\left[F^L\right]\{\theta\}\right]}_{\text{Power radiated-Lagoon side}} \tag{3.20}$$

Recall from the application of Green's theorem to the two sides we have found from eq.(3.13):

$$\int_{S_\infty^-} 2\,\mathrm{Im}\left[\phi^- \frac{\partial \phi^{-*}}{\partial x}\right] dS = 2\,\mathrm{Im}\left[\{\theta\}^{T*}\frac{\{F^D\}}{\rho} - i\omega\{\theta\}^{T*}\frac{[F^a]}{\rho}\{\theta\}\right]$$

and eq.(3.19)

$$\int_{S_\infty^+} 2\,\mathrm{Im}\left[\phi^+ \frac{\partial \phi{+}^*}{\partial r}\right] dS = 2\,\mathrm{Im}\left[-i\omega\{\theta\}^{T*}\frac{[F^l]}{\rho}\{\theta\}\right]$$

Substituting these into eq.(3.20) we get:

$$\mathrm{Im}\left[\left(-\omega^2 I + C\right)\{\theta\}^{T*}\{\theta\}\right]\frac{1}{\rho} = \mathrm{Im}\int_{S_\infty^-} \phi^- \frac{\partial \phi^{-*}}{\partial x} dS + \mathrm{Im}\int_{S_\infty^+} \phi^+ \frac{\partial \phi{+}^*}{\partial r} dS \tag{3.21}$$

where ϕ^-, ϕ^+represent respectively the wave potential in the adriatic and the lagoon side.

Since $\left(-\omega^2 I + C\right)$ is real:

$$\left(-\omega^2 I + C\right)\{\theta\}^{T*}\{\theta\} = \left(-\omega^2 I + C\right)\sum_{i=1}^{n}|\theta_i|^2$$
$$\Longrightarrow \mathrm{Im}\left[\left(-\omega^2 I + C\right)\{\theta\}^{T*}\{\theta\}\right] = 0$$

where n is the length of the vector $\{\theta\}$ .Thus eq.(3.21) becomes:

$$\mathrm{Im}\int_{S_\infty^-} \phi^- \frac{\partial \phi^{-*}}{\partial x} dS + \mathrm{Im}\int_{S_\infty^+} \phi^+ \frac{\partial \phi^{+*}}{\partial r} dS = 0$$

or

$$\iint_{S_\infty^-}\left[\phi^- \frac{\partial \phi^{-*}}{\partial n} - \phi^- \frac{\partial \phi^-}{\partial n}\right] dS = \iint_{S_\infty^+}\left[\phi^+ \frac{\partial \phi^{+*}}{\partial n} - \phi^{+*} \frac{\partial \phi^+}{\partial n}\right] dS \tag{3.22}$$

This expression represents a balance of power flux, in the far fields.

Substituting into eq.(3.22) eq(3.11) and eq.(3.18) we have the explicit energy theorem:

$$\mathrm{Im}\{\left[\frac{\sinh[2kh]+2kh}{4k}\right]\left(\frac{gA}{\omega}\frac{1}{\cosh(kh)}\right)\sqrt{\frac{2}{h+(g/\omega^2)\sinh^2 k_0 h}}\left[ak\sum_{\alpha} i\omega\theta_{\alpha}^{*}A_{00}^{(\alpha)*}+ka\sum_{\alpha} i\omega\theta_{\alpha}A_{00}^{(\alpha)}\right]+$$
$$+\sum_{\alpha}\sum_{\beta}\omega^2\theta_{\alpha}\theta_{\beta}^{*}\left(\sum_{m=0}^{M}A_{m0}^{(\alpha)}A_{m0}^{(\beta)*}\left(i\alpha_{m0}\right)\frac{a}{\epsilon_m}\right)\}=$$
$$=\mathrm{Im}\left[\sum_{\alpha,\beta}\omega^2\theta_{\alpha}\theta_{\beta}^{*}\left|A_0\right|^2\frac{-8i}{k^2\pi}\int_0^{\pi}e^{ik(y_{\beta}-y_{\alpha})\cos(\varphi)}\frac{\sin^2\left(\frac{L}{2}k\cos(\varphi)\right)}{\cos^2(\varphi)}d\varphi\right] \quad (3.23)$$

For given geometry and frequency, the above identity depends only on the amplitudes θ_α of motion of the gates. If θ_α from numerical computations are correct they must satisfy this identity.

Chapter 4

Numerical results and discussion

The theory described is implemented in a computer program written in the Matlab environment (see Appendix). Input data are the geometrical parameters of the domain and of the gates , the amplitude and period of the incident wave (table 4.1), the number of terms taken into account in the wave expansions and the number of subdivisions used for some integrations (table 4.2). Of physical interest are, the added mass and radiation damping matrices and the complex amplitude of motion of each gate, for a wide range of frequencies. The code takes advantage of the built-in functions of the Matlab environment.

The correctness of the code is first checked by making use of the identities and properties discussed.

Symbol	**Desciption**	**Value**
L	Gate's width (m)	20
h	Water depth at the gates (m)	14
a	Channel width (Adriatic) (m)	20
M	Gate's mass (Kg)	2.8*10^5
ρ	Water density (Kg/m^3)	1000
g	Gravity acceleration (m/s^2)	9.8
I	Gate's inertia $(Kg * m^2)$	33.337*10^6
C	Gate's bouyancy restoring torque (m^2/s^2)	60.246*10^6
A	Amplitude incident wave (m)	1

Table 4.1: Geometrical parameters assumed

Description	Value
N of terms in the wave expansion - Adriatic side: n=	20
N of terms in the wave expansion - Adriatic side: m=	300
N of terms in the wave expansion - Lagoon side: n=	35
N of subdivision for the integral in eq.(2.38)	300
N of subdivision for the integral in eq.(2.39)	300

Table 4.2: Values assumed for the numerical computations

4.1 Checking the correctness of the results

4.1.1 Adriatic side

Haskind's theorem provides a check for the analytical expressionof the external forcing term.

4.1.2 Added Mass and Radiation Damping

The symmetry of the Added Mass and the Radiation Damping matrices has been verified and then this property is used to speed up the computation time.

The positive semidefiniteness of the Radiation Damping matrix is always verified by checking that the eigenvalues are always non-negative.

Figure (4-1) shows the minimum of the eigenvalues of the radiation damping matrix agains period.

4.1.3 Energy check

The energy law eq.(3.23) provides an excellent way to check the correctness of the numerical computations.The two sides of the eq (3.23) depend on the motion of the gates that is influenced by the local evanescent modes. So the energy identity can be satisfied only by correct results even though we compare only far field quantities.

The difference of the two sides of the eq.(3.23) is calculated for periods from 4 to 30 seconds (every 0.1s). Figure (3.23) show that the identity is always satisfied (the difference is in the order of 1%). Green's theorem applied to the two domains (lagoon and adriatic) separately is satisfied by the numerical results too. The integral with respect to the angle φ that appears

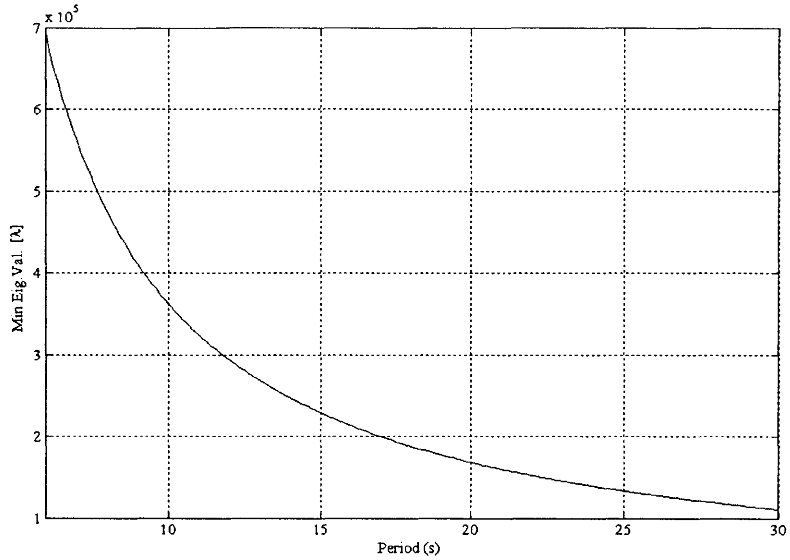

Figure 4-1: Minimun eigenvalue of the added mass matrix [λ] against period (s)

in the expression of the lagoon far field potential is calculated by quadrature exploiting an available Matlab function ("quadl").

4.2 Prediction of gates motion

Extensive computations were carried out to explore the behavior of the gates for different values of period of incident wave. The intervall of periods from 4 to 30 seconds has been choosen because it is of engineering interest for the gates design. Table (4.1) summarizes the data assumed for the gate.

The overall result can be summarized by Figure(4-4) showing how the average amplitude computed as

$$\Theta = \frac{\sum_{\alpha=1}^{N} |\theta_\alpha|}{N}$$

changes with period. We can observe there are some values for which the amplitude becomes very large and the gates oscillate out of phase Figure(4-5). This can be reduced by taking into

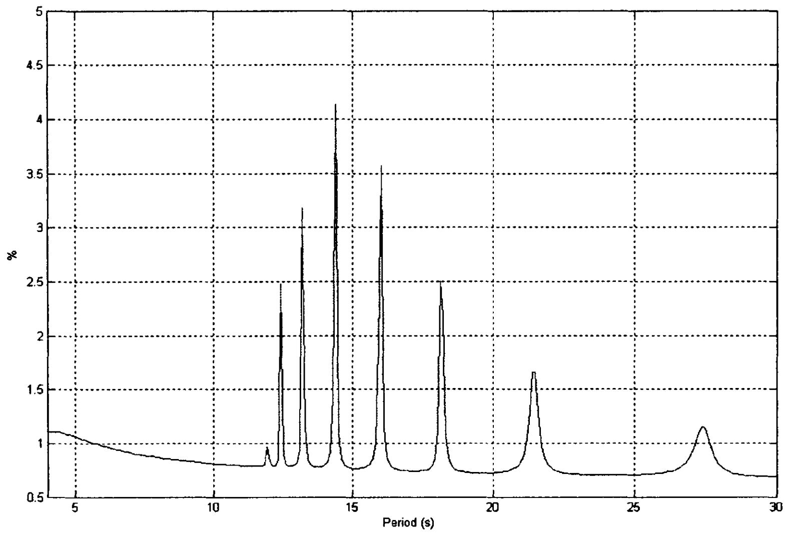

Figure 4-2: The top figure shows the values of the lefth hand side and the right hand side of the global identity eq.(3.22). The bottom figure shows the difference in percentage of the 2 sides of the identity.

account broad band of the icident sea, vortex shedding and nonlinearity.

From Li &Mai [1] the shapes of the natural modes for 20 gates in a channel of infinite length have been found. Upon comparison an excellent agrement is found in the gate displacement between the periods of resonance. Li and Mei report both even and odd modes, but here only the even ones are excited because the forcing is a plane wave with wave front parellel to the gates (i.e. symmetric forcing). Note that the computed amplitudes cannot fulfill the condition

$$\int_0^a \theta(y)\, dy = 0$$

which is the condition required for the existance of the natural modes [3]. This happens because in this theory, radiation to both Adriatic and Lagoon side are different (unlike their theory where all gates are in the middle of a very long channel).

A closer look at Figure (4-4) shows that besides the resonant periods, the gate amplitude increases with increasing period (i.e. longer waves are more resonated).

The most interesting results are close to resonance. The recorded values of resonant periods

	Resonant period (s)													
	11.9		12.4		13.2		14.4		16		18.2		21.5	
	Ampl	Phase/)	Ampl	Phase/)	Ampl	Phase/)	Ampl	Phase/)	Ampl	Phase/)	Ampl	Phase/)	Ampl	Phase/)
Gate 1	0.242	0.477	0.550	0.456	0.852	0.437	1.045	0.074	1.253	-0.015	1.301	0.055	1.503	0.342
Gate2	0.181	0.419	0.587	-0.531	0.978	-0.553	1.434	0.973	1.157	2.773	0.384	2.331	0.569	0.571
Gate 3	0.185	0.314	1.114	0.430	0.978	0.414	0.193	0.382	1.183	2.782	1.384	2.844	0.747	3.101
Gate 4	0.274	0.531	0.477	-0.565	0.511	0.436	1.663	0.061	1.297	0.017	0.470	2.426	1.197	-3.087
Gate 5	0.169	0.153	0.435	0.404	0.873	-0.555	0.867	0.924	1.332	0.040	1.312	0.109	0.497	2.895
Gate 6	0.325	0.553	0.527	0.445	1.244	0.427	0.899	0.939	1.173	2.726	1.352	0.111	0.913	0.508
Gate 7	0.169	0.125	0.555	-0.546	0.085	0.429	1.630	0.060	1.196	2.748	0.440	2.310	1.696	0.388
Gate 8	0.299	0.530	1.106	0.433	0.652	-0.552	0.253	0.388	1.290	0.029	1.406	2.832	1.305	0.435
Gate 9	0.162	0.269	0.518	-0.553	1.388	0.428	1.457	0.969	1.299	0.027	0.461	2.394	0.253	1.760
Gate 10	0.218	0.453	0.484	0.423	0.314	-0.542	1.063	0.081	1.187	2.758	1.318	0.102	1.101	-3.132
Gate 11	0.218	0.453	0.484	0.423	0.314	-0.542	1.063	0.081	1.187	2.758	1.318	0.102	1.101	-3.132
Gate 12	0.162	0.269	0.518	-0.553	1.388	0.428	1.457	0.969	1.299	0.027	0.461	2.394	0.253	1.760
Gate 13	0.299	0.530	1.106	0.433	0.652	-0.552	0.253	0.388	1.290	0.029	1.406	2.832	1.305	0.435
Gate 14	0.169	0.125	0.555	-0.546	0.085	0.429	1.630	0.060	1.196	2.748	0.440	2.310	1.696	0.388
Gate 15	0.325	0.553	0.527	0.445	1.244	0.427	0.899	0.939	1.173	2.726	1.352	0.111	0.913	0.508
Gate 16	0.169	0.153	0.435	0.404	0.873	-0.555	0.867	0.924	1.332	0.040	1.312	0.109	0.497	2.895
Gate 17	0.274	0.531	0.477	-0.565	0.511	0.436	1.663	0.061	1.297	0.017	0.470	2.426	1.197	-3.087
Gate 18	0.185	0.314	1.114	0.430	0.978	0.414	0.193	0.382	1.183	2.782	1.384	2.844	0.747	3.101
Gate 19	0.181	0.419	0.587	-0.531	0.978	-0.553	1.434	0.973	1.157	2.773	0.384	2.331	0.569	0.571
Gate 20	0.242	0.477	0.550	0.456	0.852	0.437	1.045	0.074	1.253	-0.015	1.301	0.055	1.503	0.342
RHS*10^4	6.682		6.950		7.403		7.955		8.672		9.531		10.010	
LHS*10^4	6.627		6.777		7.167		7.626		8.362		9.323		9.843	
diff(%)	0.008		0.025		0.032		0.041		0.036		0.022		0.017	

Figure 4-3: Results at resonance.Values of amplitude and phase/π of motion are given for each one of the 20 gates. The table reports also the value of the LHS and RHS of eq.(3.23) and their difference in percentage.

and respective amplitudes are reported in figure (4-3). Figures (4-5,4-6) show the modal shapes at resonance.

4.2.1 Resonance of the forced oscillations in the adriatic side

We can observe that the power radiated in the adriatic by the gates moving with forced oscillation and unit angular velocity (namely when we are studying the potential ϕ_{α}^{-}) becomes infinite for certain values of period. In other words, the amplitude of the waves generated by a gate's motion with unit angular velocity becomes infinite. If we consider how the number of propagating

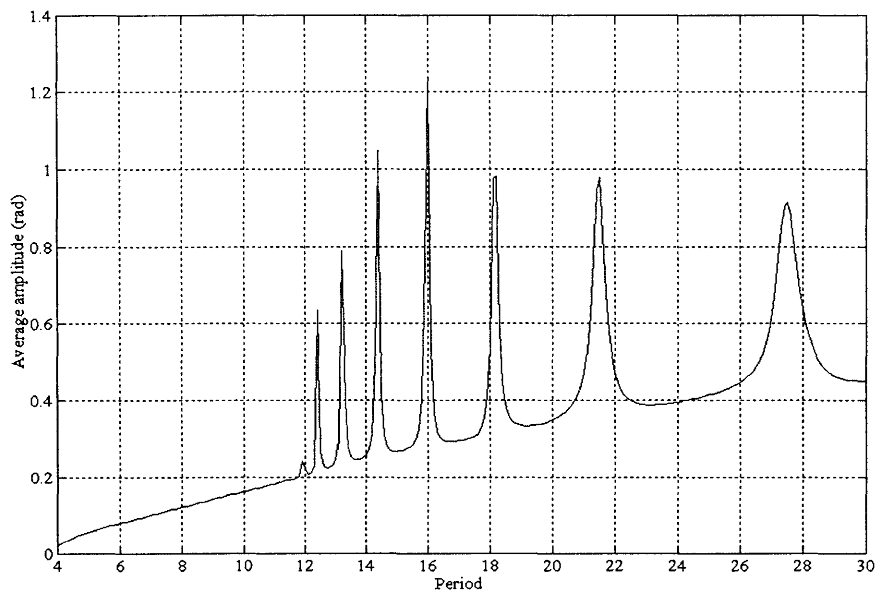

Figure 4-4: Average amplitude of motion of the gates against period of incident wave.

modes changes with the period (see Figure(4-7)-bottom graph). When this number decreases the radiated power jumps from an infinite value to a finite one. These values of periods correspond to cut-off periods. The condition under which one of the harmonics of the radiated waves has amplitude going to infinity (and so does the power) is given by $\alpha_{mn} = \sqrt{K_n^2 - \left(\frac{m\pi}{a}\right)^2} \rightarrow 0^+$ (it is shown afterwards). After that, when Kn slightly decreases, $\alpha_{mn} = \sqrt{k_n^2 - \left(\frac{m\pi}{a}\right)^2}$ would be complex and so the correct α_{mn} is given by $\alpha_{mn} = \sqrt{K_n^2 + \left(\frac{m\pi}{a}\right)^2}$ and that mode becomes an evanescent one.

Here we want to show that when $\alpha_{mn} \rightarrow 0^+$ the radiated power diverges. In order to do that let's calculate first the radiated power through a far surface:

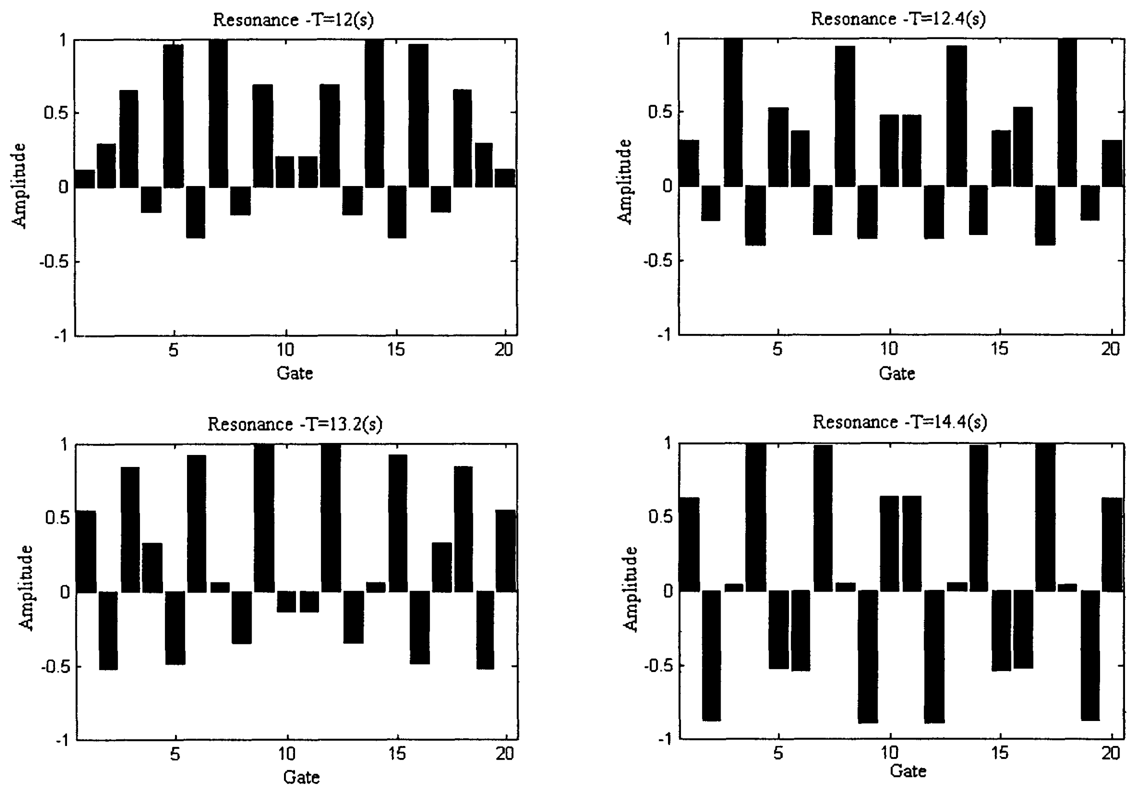

Figure 4-5: Normalized position of the gates at $t = 0$ when resonance occurs. (Periods between 11-15 s)

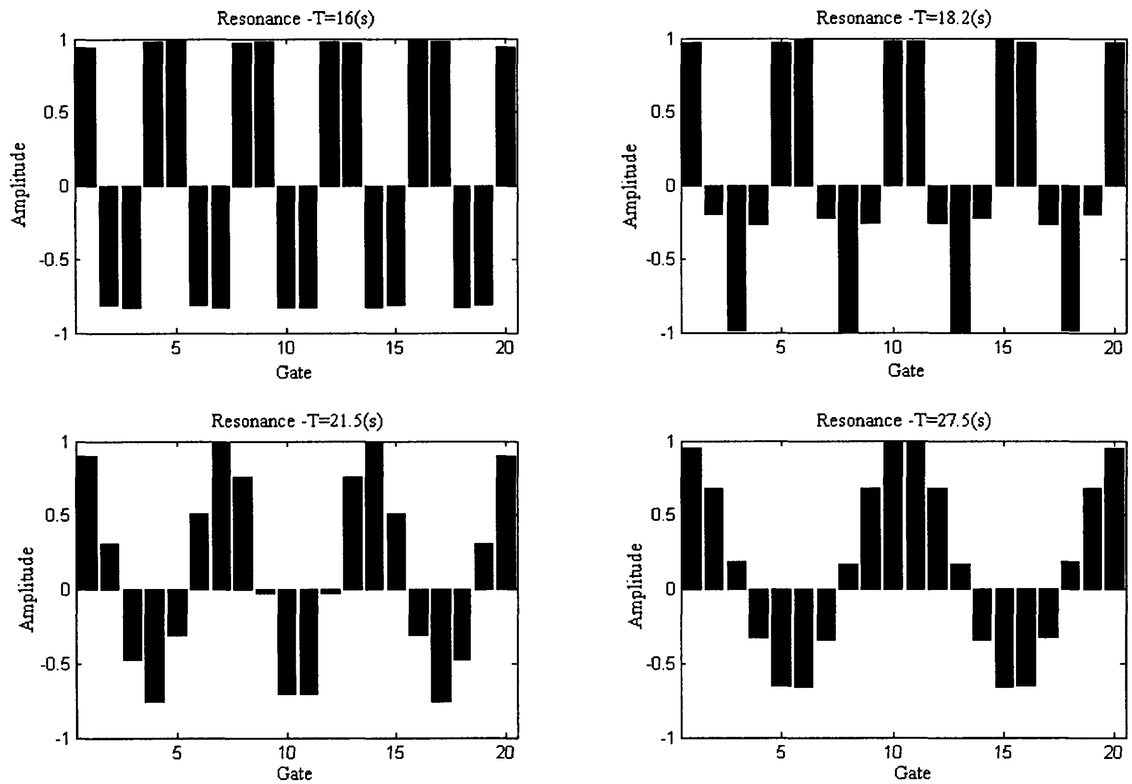

Figure 4-6: Normalized position of the gates at $t = 0$ when resonance occurs. (Periods between 15-30 s)

$$P = \mathrm{Im} \int\limits_{S_\infty} \left(\phi_\alpha^- \frac{\partial \phi_\alpha^{-*}}{\partial n} \right) dS =$$

$$= \mathrm{Im} \int\limits_{S_\infty} dS \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}^{(\alpha)} e^{-i\alpha_{mn}x} \cos \frac{m\pi y}{a} \cosh[k_n(z+h)]$$

$$\underbrace{\left[- \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}^{(\alpha)} \left(-i\alpha_{mn}\right) e^{-i\alpha_{mn}x} \cos \frac{m\pi y}{a} \cosh[k_n(z+h)] \right]^*}_{\frac{\partial \phi_\alpha}{\partial n} = -\frac{\partial \phi_\alpha}{\partial x}}$$

taking advantage of the orthogonality of cosh and cos and recalling that evanescent modes are vanished (so $n = 0$ and $m = 0, 1, ..., M$):

$$P = \mathrm{Im} \int\limits_{S_\infty} - dS \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn}^{(\alpha)} e^{-i\alpha_{mn}x} \cos \frac{m\pi y}{a} \cosh[k_n(z+h)]$$

$$+ \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left(A_{mn}^{(\alpha)} \right)^* \left(-i\alpha_{mn}\right)^* \left(e^{-i\alpha_{mn}x} \right)^* \cos \frac{m\pi y}{a} \cosh[k_n(z+h)] =$$

$$= \mathrm{Im} \sum_{m=0}^{M} - \left| A_{m0}^{(\alpha)} \right|^2 \left(-i\alpha_{m0}\right)^* e^{-i\alpha_{m0}x} e^{i\alpha_{m0}x} \int_0^a dy \cos^2 \frac{m\pi y}{a} \int_{-h}^0 dz \cosh^2[k_0(z+h)]$$

When $\alpha_{m0} \longrightarrow 0^+$,$\left| A_{m0}^{(\alpha)} \right|$ goes to ∞. In order to understand the behavior of the radiated power we can focus only on the term of the summation for which $\alpha_{m0} \longrightarrow 0^+$. Plugging into the above expression eq.(2.30) for $A_{m0}^{(\alpha)}$ we get:

$$P = \lim_{\alpha_{m0} \longrightarrow 0^+} \mathrm{Im} \left[- \left| A_{m0}^{(\alpha)} \right|^2 \left(-i\alpha_{m0}\right)^* \int_0^a dy \cos^2 \frac{m\pi y}{a} \int_{-h}^0 dz \cosh^2[k_0(z+h)] \right] =$$

$$= \lim_{\alpha_{m0} \longrightarrow 0^+} \mathrm{Im} \left\{ - \left| \frac{1}{-i\alpha_{m0}} c_0 \frac{2}{m\pi} \cos \frac{m\pi \overline{y_\alpha}}{a} \sin \frac{m\pi L}{2} \left[\frac{h \sinh\left(hk_0\right)}{k_0} - \frac{\cosh\left(hk_0\right)}{k_0^2} + \frac{1}{k_0^2} \right] \right|^2 \right.$$

$$\left. \left(-i\alpha_{m0}\right) \int_0^a dy \cos^2 \frac{m\pi y}{a} \int_{-h}^0 dz \cosh^2[k_0(z+h)] \right\} =$$

$$= \lim_{\alpha_{m0} \longrightarrow 0^+} \mathrm{Im} \left[- \left(\frac{1}{\alpha_{m0}} \right)^2 \left(-i\alpha_{m0}\right) \right] = +\infty$$

So when the frequency of the oscillation is such as $k_0 \to \left(\frac{m\pi}{a}\right)^+$ and $\alpha_{m0} \longrightarrow 0^+$,the radiated

power goes to ∞. In the last passage we dropped the terms that are uninfluenced by $\alpha_{m0} \longrightarrow 0^+$ because they all have finite real values.

4.2.2 Added mass and radiation damping

We showed in the last paragraph that the amplitude of the radiated waves in the Adriatic can become unbounded and so are the added mass and radiation damping (defined by eq.(2.9)). In fact when the radiated power diverges, i.e. $\alpha_{m0} \longrightarrow 0^+$ and the imaginary part of the complex amplitude $A^{(\alpha)}_{m0}$ of the m^{th} mode tends to become infinity, the integral of the potential $\phi_{\alpha,m}$ (the m indicates that we are considering only the m^{th} mode) on the generic β^{th} gate

$$i\rho\omega \iint_{S_{B,\beta}} \phi^-_{\alpha,m}(h+z)dS$$

has its imaginary part going to infinity and null real part (on that mode). This means that when approaching the resonance condition for that mode there is an infinite radiation damping and no added mass, so the energy of the gate is entirely given away to generate waves in the canal (on that mode). On the other hand, just after the cut off period, we have one propagating mode less but now,even if α_{m0} is still very small, A_{mn} is has real value (eq.2.30):

$$A^{(\alpha)}_{m0} = \frac{1}{\alpha_{m0}} \frac{\varepsilon_m}{a} \left[c_0 \left[\frac{h \sinh(hk_0)}{k_0} - \frac{\cosh(hk_0)}{k_0^2} + \frac{1}{k_0^2} \right] \frac{2a}{m\pi} \cos \frac{m\pi \bar{y}_\alpha}{a} \sin \frac{m\pi L}{2} \right]; \qquad n = 0, m > M$$

So now that mode has real part going to infinity and null imaginary part. This means that there is no radiation on that mode (it is now an evanescent one) and the added mass tends to infinity.

The described behaviour influences the values of added mass and radiation damping for the overall system. Figure(4-7) shows how this two quantities vary over wave period (we plotted the maximum of the elements of matrices against period). The graph includes the plot of the number of propagating modes against period. It is possible to recognize that the added mass

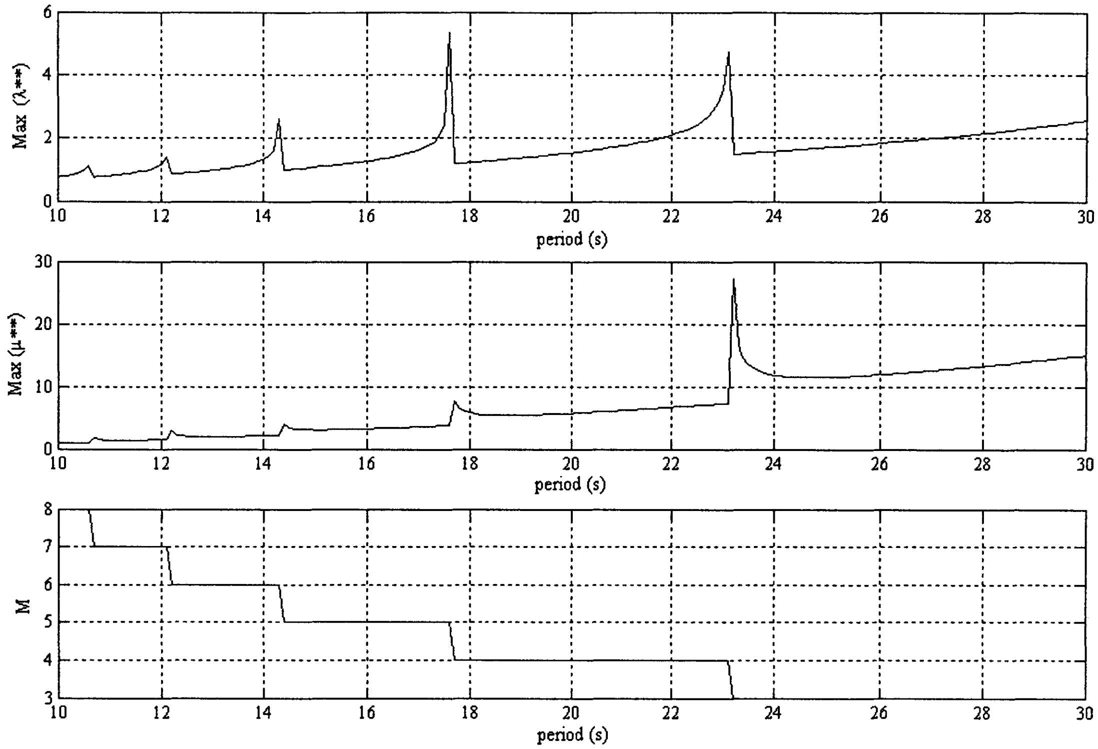

Figure 4-7: Top graph: normalized max value of the radiation damping matrix (λ^{**})vs. period of incident wave. Middle graph: normalized max value of the added max matrix (μ^{**})vs. period of incident wave. The normalization factor is $\frac{1}{\rho\omega^2 L h^4}$. Bottom graph: number of propagating modes in the adriatic vs. period of incident wave. (interval 10-30 s)

increases just before the decrease of the number of propagating modes and the added mass increases just after.

A plot of $\mu_{\beta\alpha}$ and $\lambda_{\beta\alpha}$ for $\alpha = 1, 2, .., 20$; $\beta = 1, 2, .., 20$ against period of the incident wave is given in Figure(4-9;4-25).

On the sign of $[\mu]$ and $[\lambda]$

Observing the added mass matrix $[\mu]$ it is possible to notice that some terms are negative. This can be explained by noting that

$$\mu_{\beta\alpha} = \rho \operatorname{Re} \iint\limits_{S_{\beta th_{gate}}} \phi_\alpha^- (h+z)\, dS = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn,\alpha} \int_{y_{\beta-1}}^{y\beta} dy \cos \frac{m\pi y}{a} \int_{-h}^{0} (h+z)\, f_n(z)\, dz$$

two different gates have different added mass because of the different value of the integral along the $y - direction$. This means that there is a strong connection between the value of the added mass (but the same applies to the radiation damping) and $\phi_\alpha^- (y)$. The plot of the real part of $\phi_\alpha^- (x, y)|_{\substack{x=0 \\ z=0}}$ behaves like a modulated sinusoid, and its $y-$integral is clearly negative on some gates.Figure (4-8). This corresponds to the occurrence of negative signs in the added mass matrix.

This occurence of negative coefficients in observed in the radiation matrix $[\lambda]$ too. Since the radiation matrix is proportional to radiated energy it has to be positive semidefinite. But this doesn't imply that some terms can't be negative. The positive semidefiniteness of $[\lambda]$ is always verified by checking that the eigenvalues are always positive.

Figure (4-8) shows what described above. The minus signs points out the gates on whitch the added mass and the radiation damping are found to be negative.

The occurrance of negative added masses and radiation damping appears then related to the peculiar geometry we have been studying.

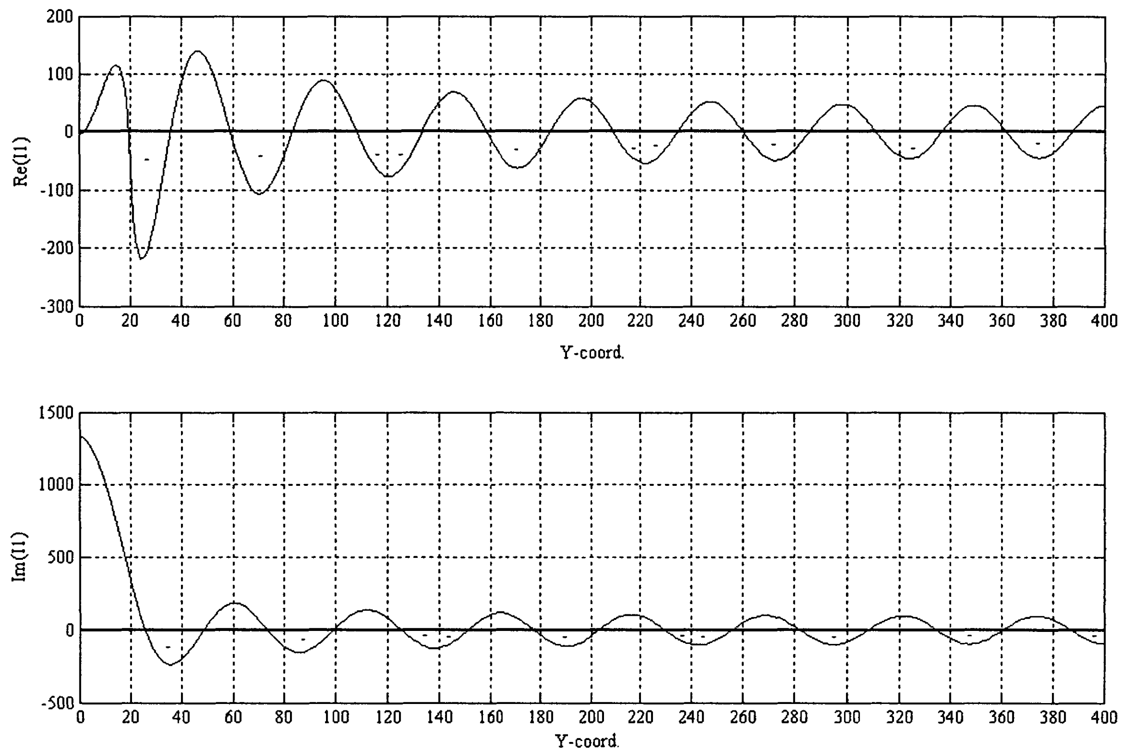

Figure 4-8: Plot of the real and immaginary part of $I1 = \int_{-h}^{0} dz \; \phi_1^- \big|_{x=0}$ against $y-coordinate$. T=6s. Each gate has a width of 20m. The minus sign shows where the integral is negative, this corresponds to what is found in the matrices.

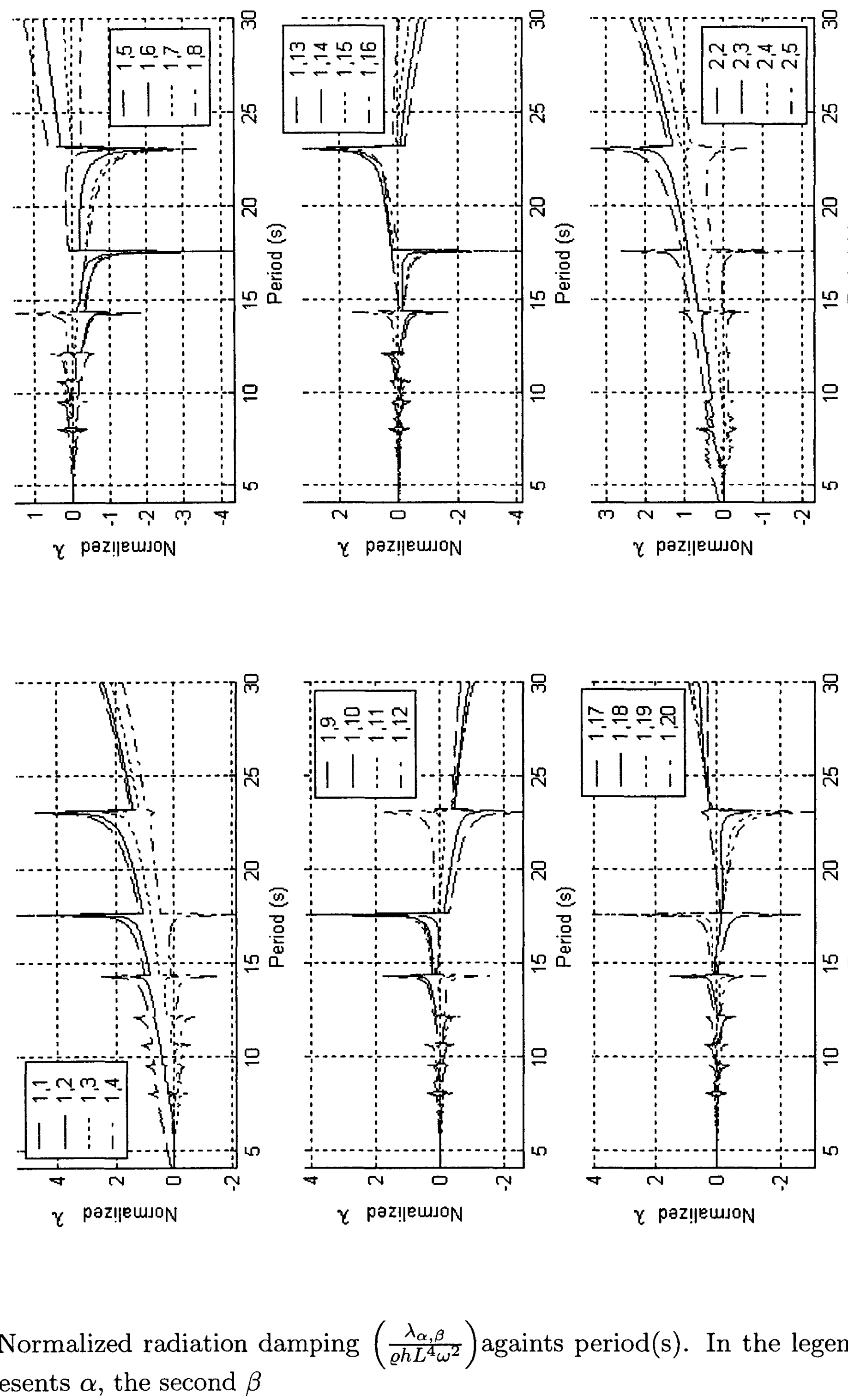

Figure 4-9: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

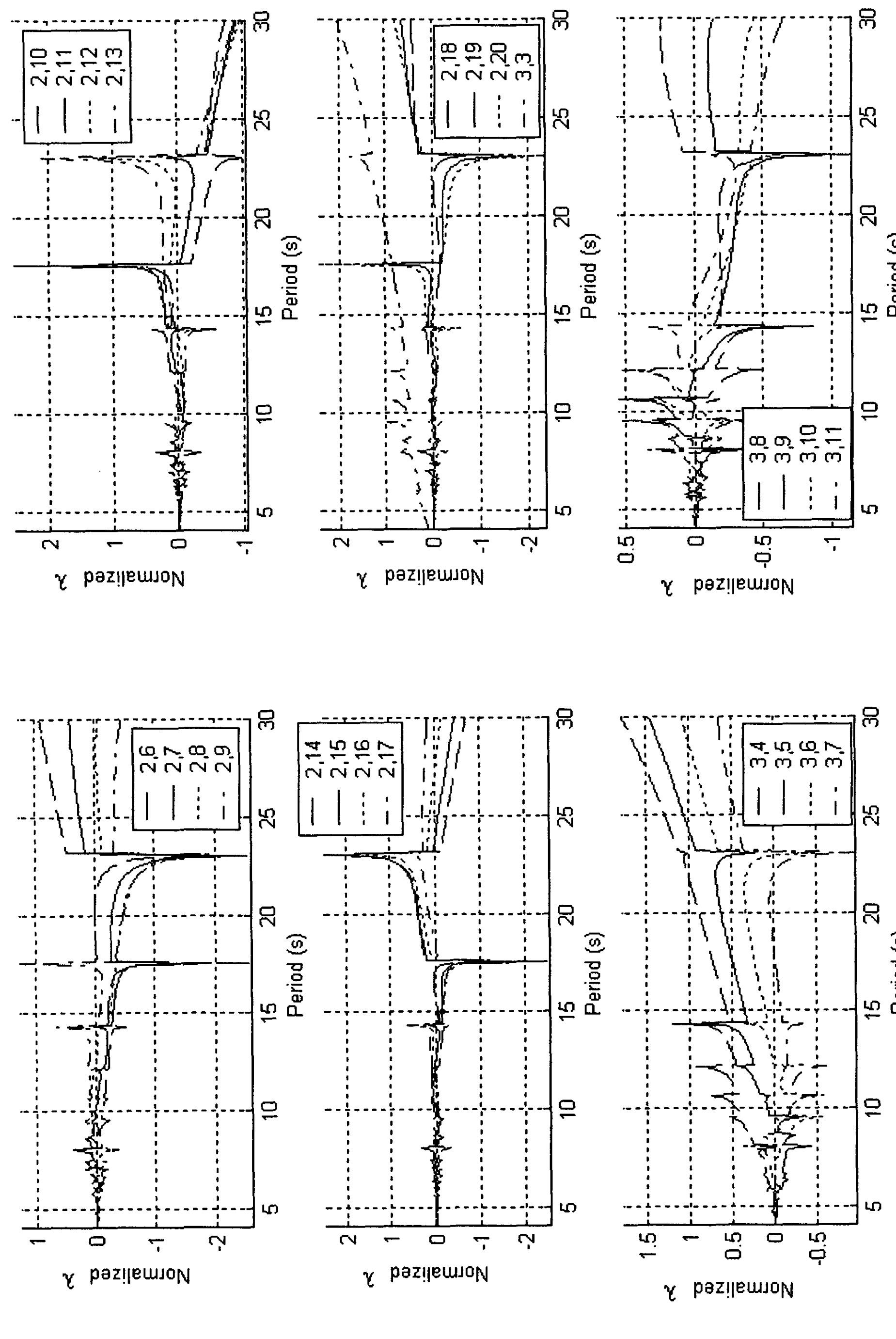

Figure 4-10: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

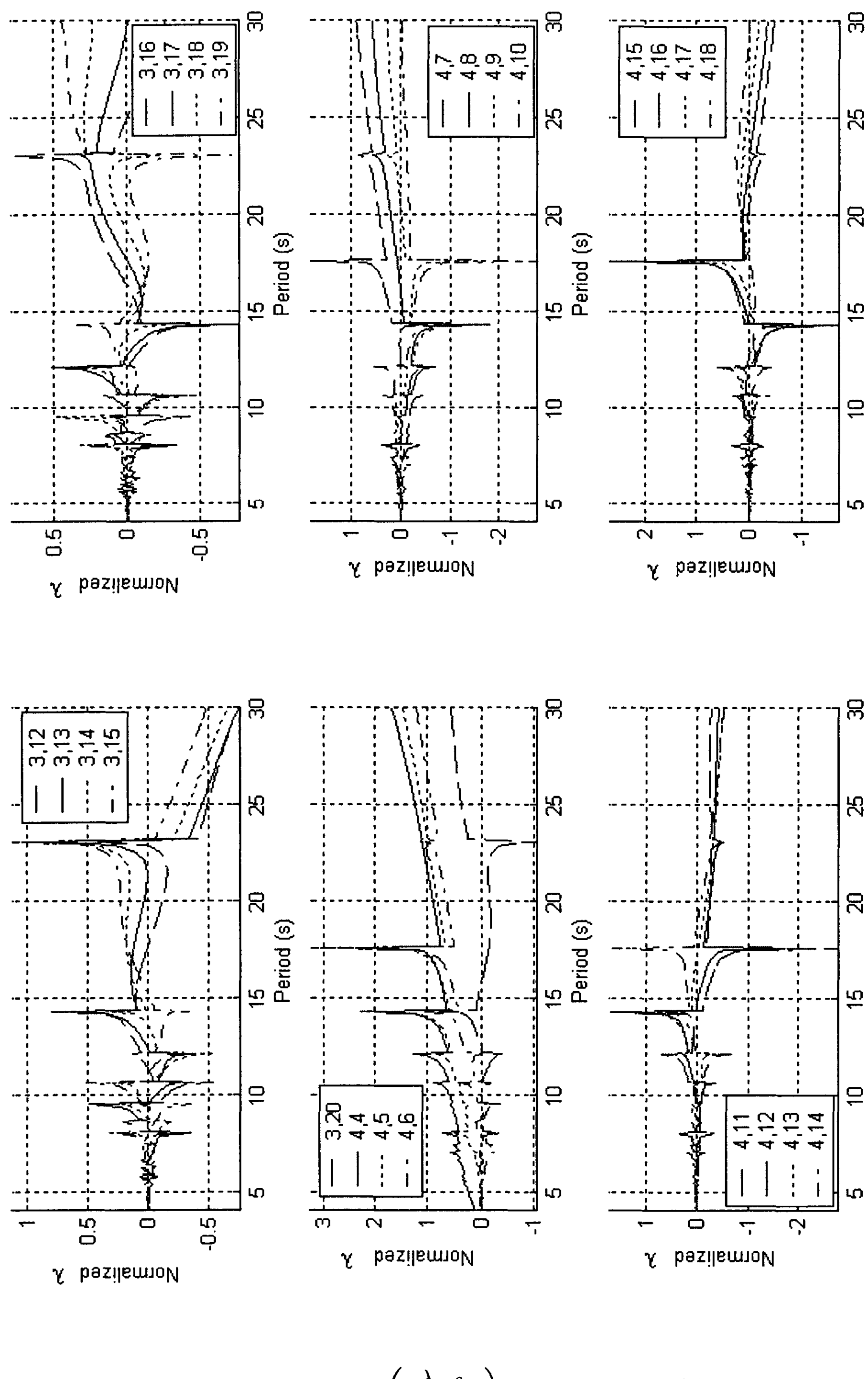

Figure 4-11: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

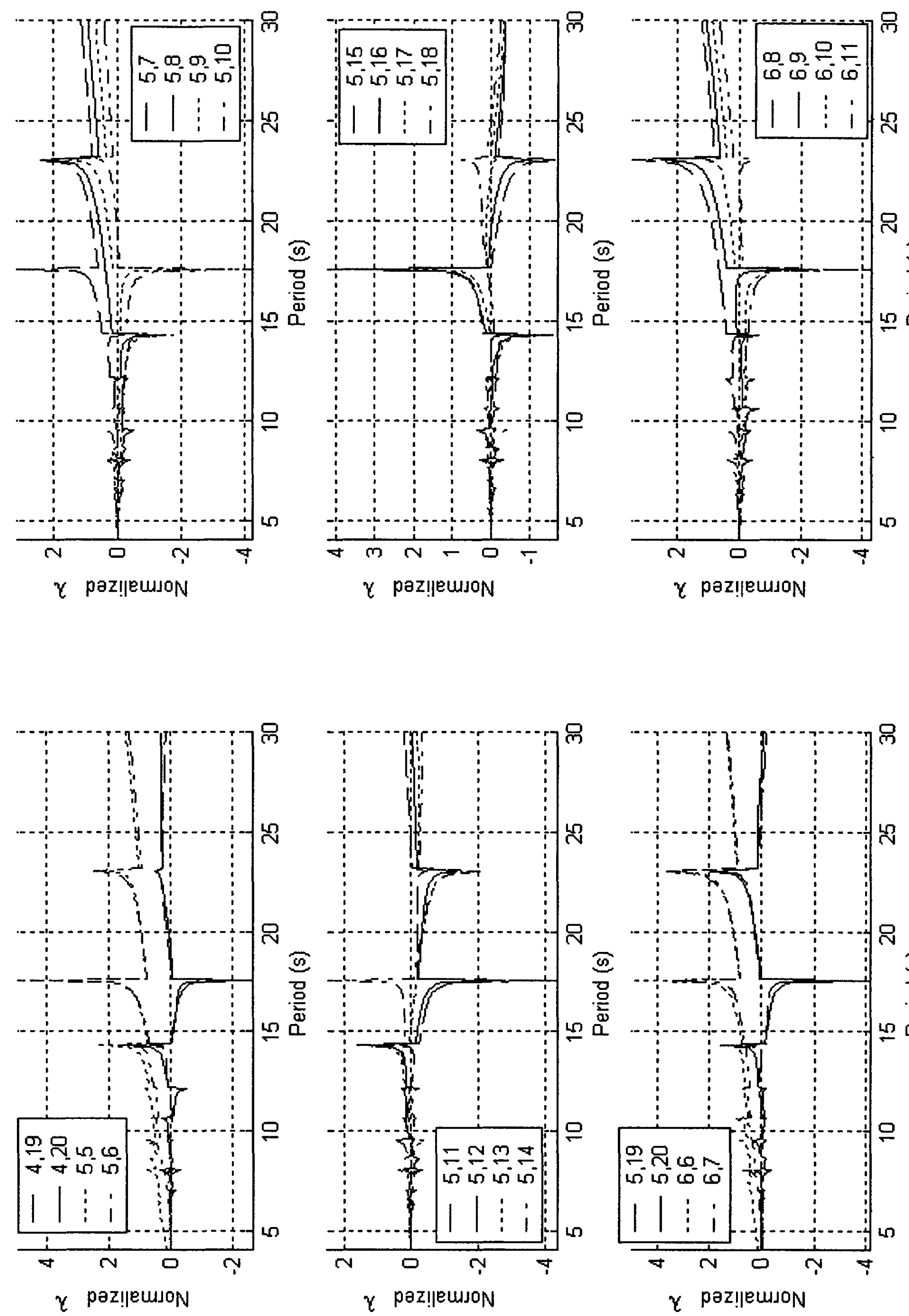

Figure 4-12: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

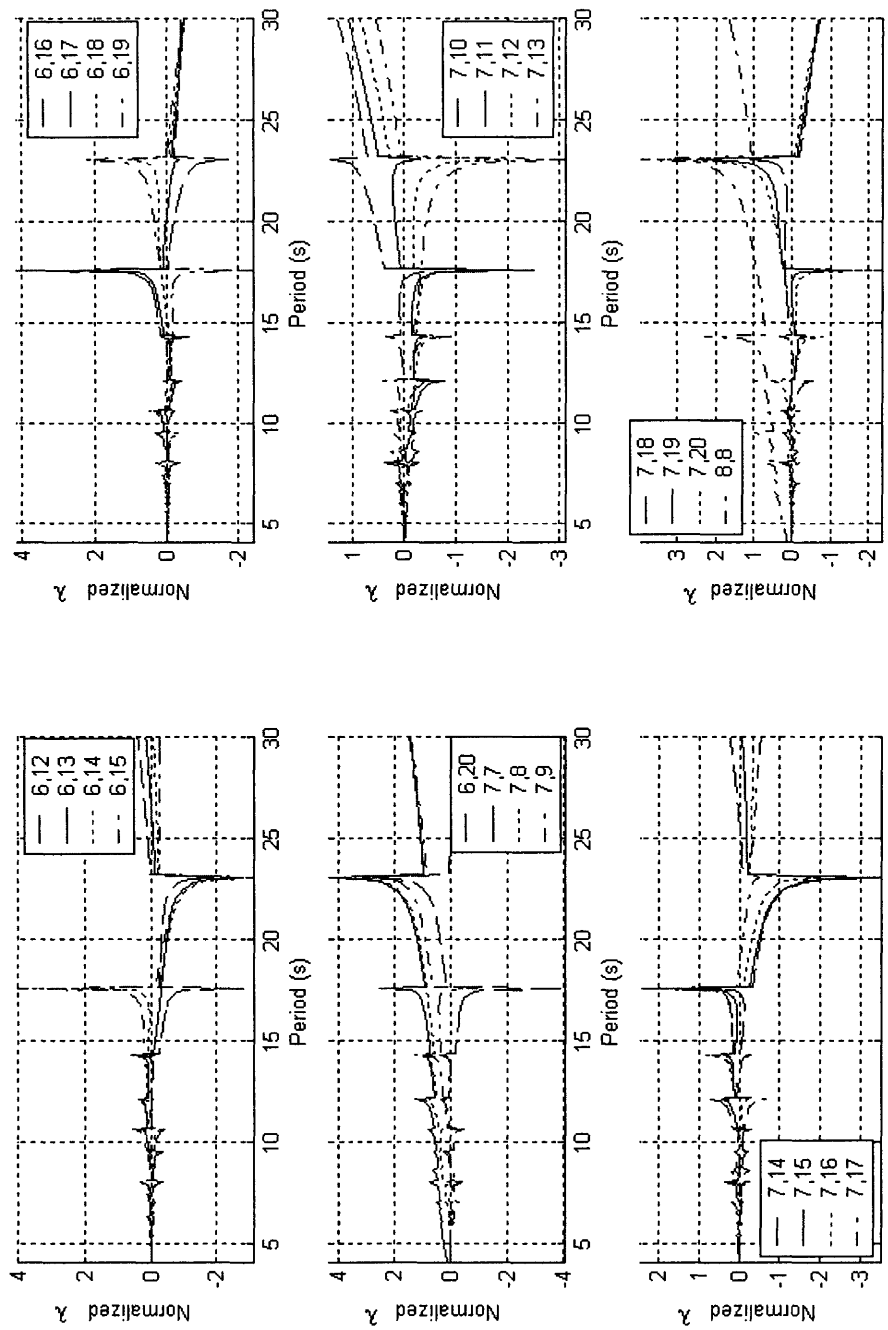

Figure 4-13: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

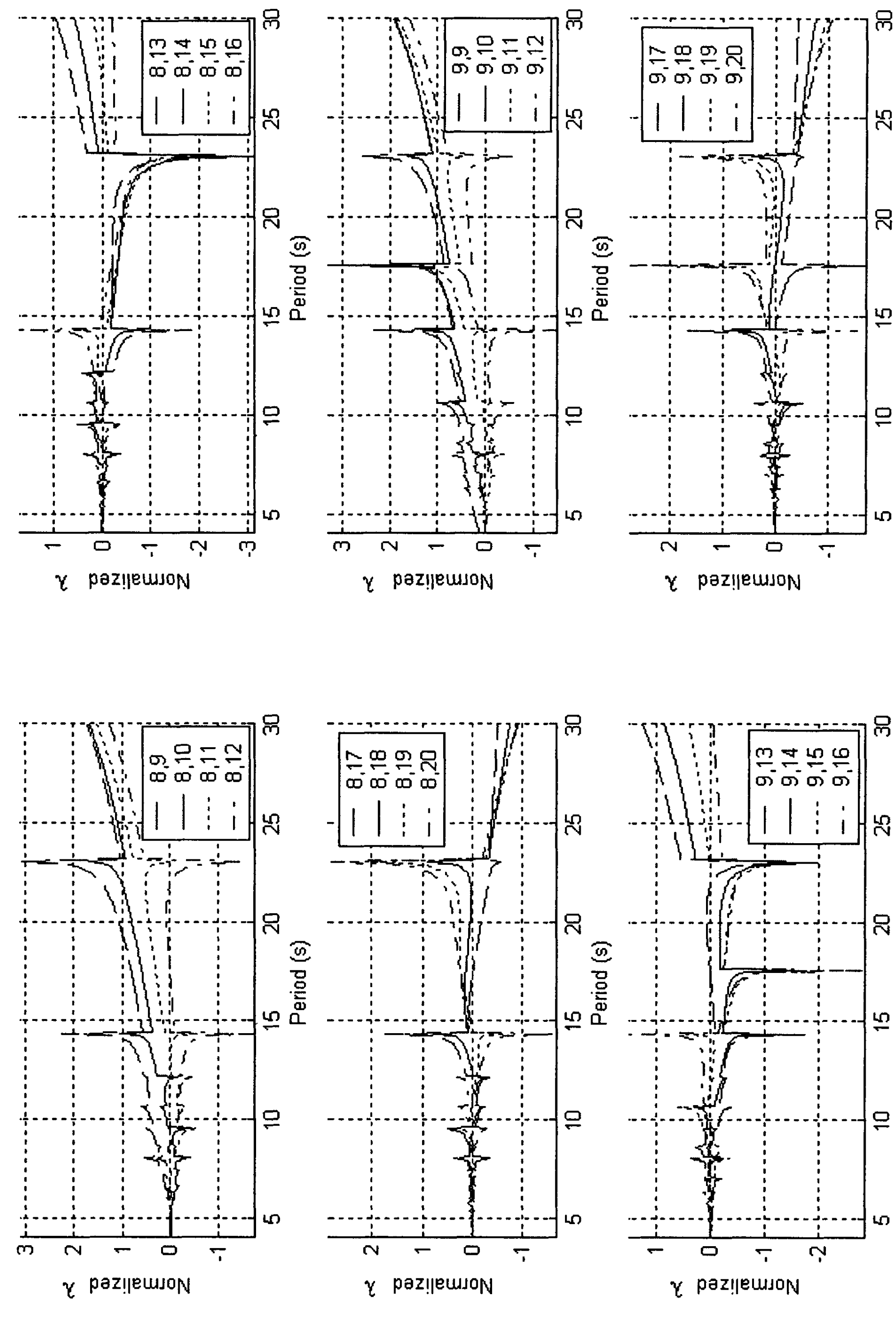

Figure 4-14: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

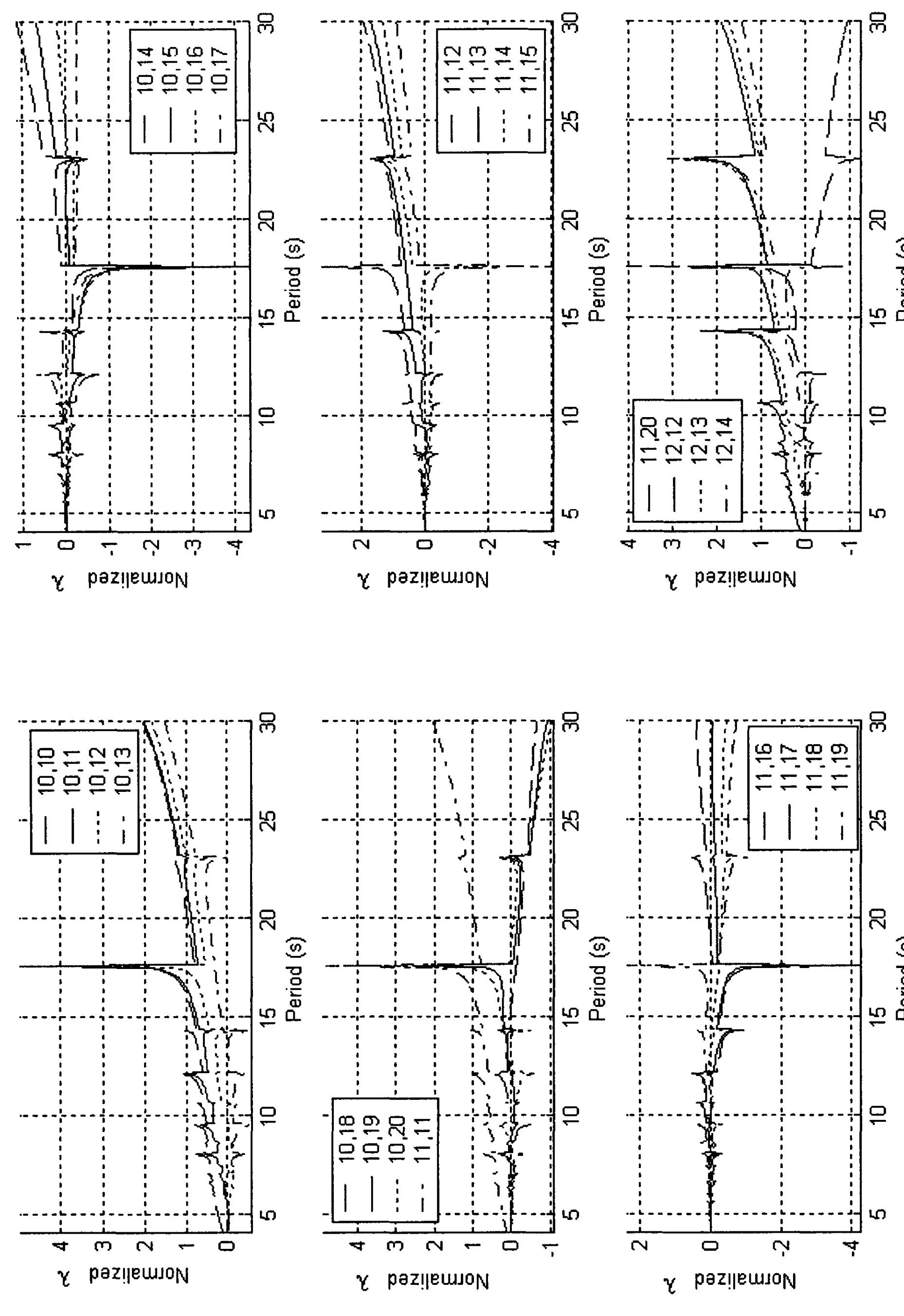

Figure 4-15: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

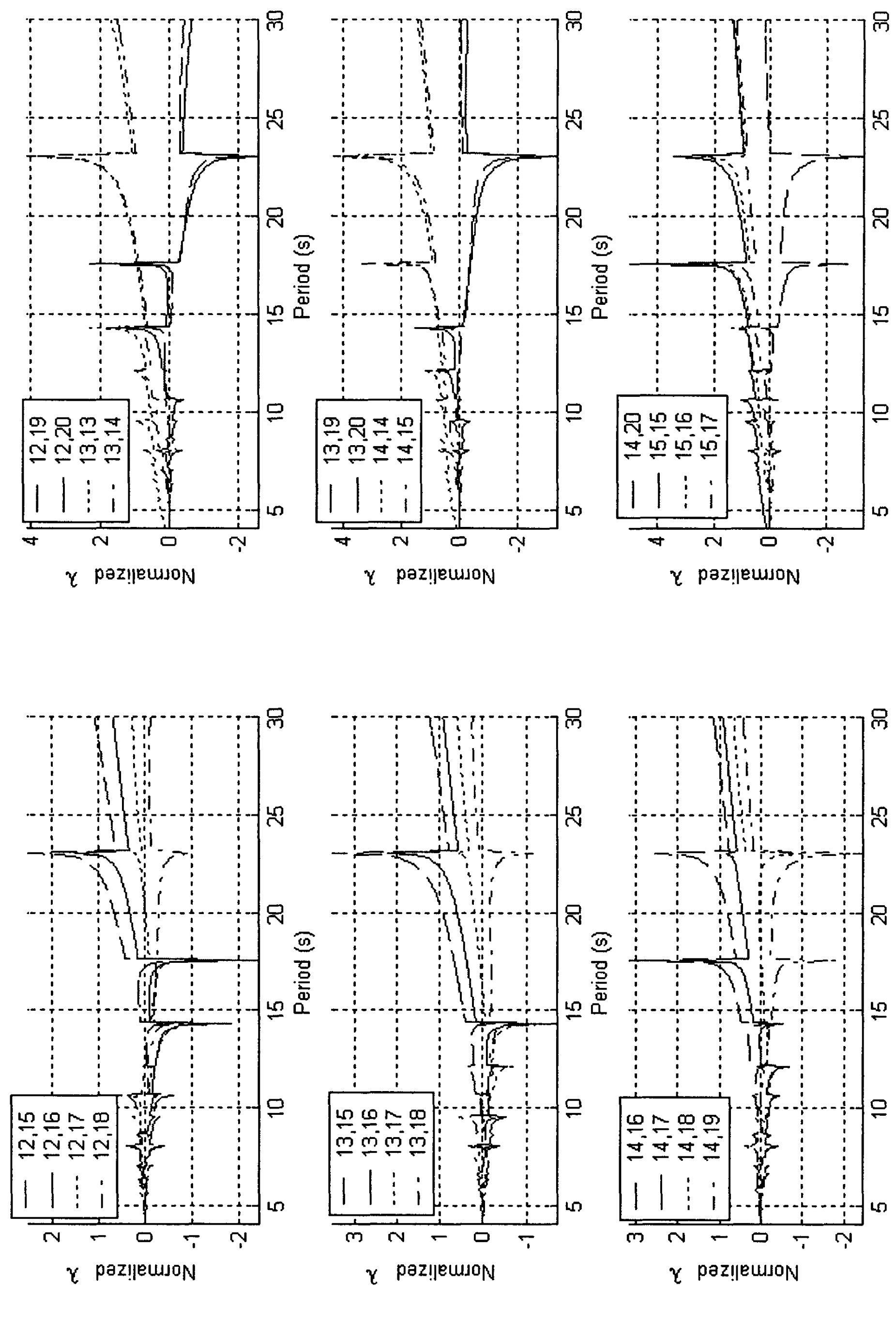

Figure 4-16: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

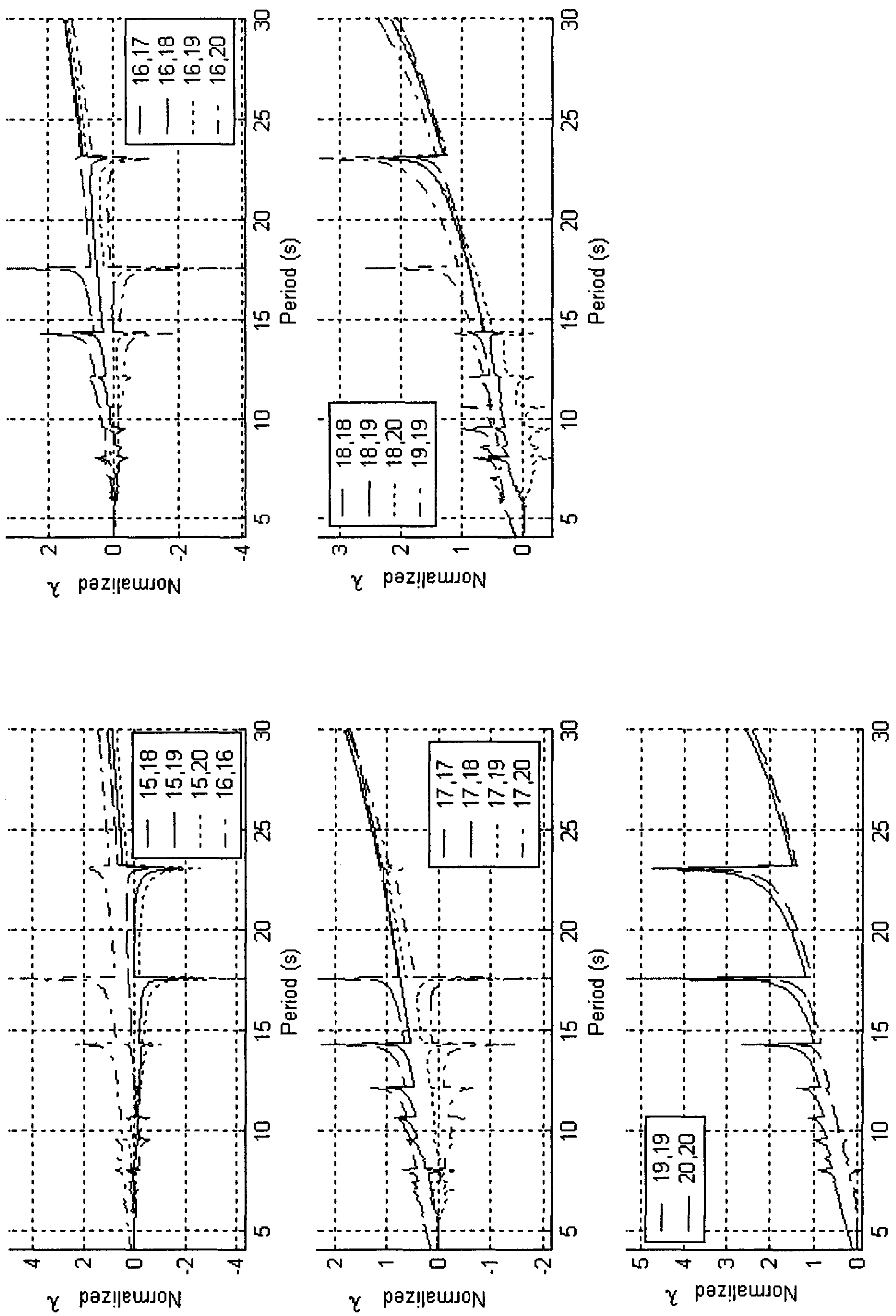

Figure 4-17: Normalized radiation damping $\left(\frac{\lambda_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

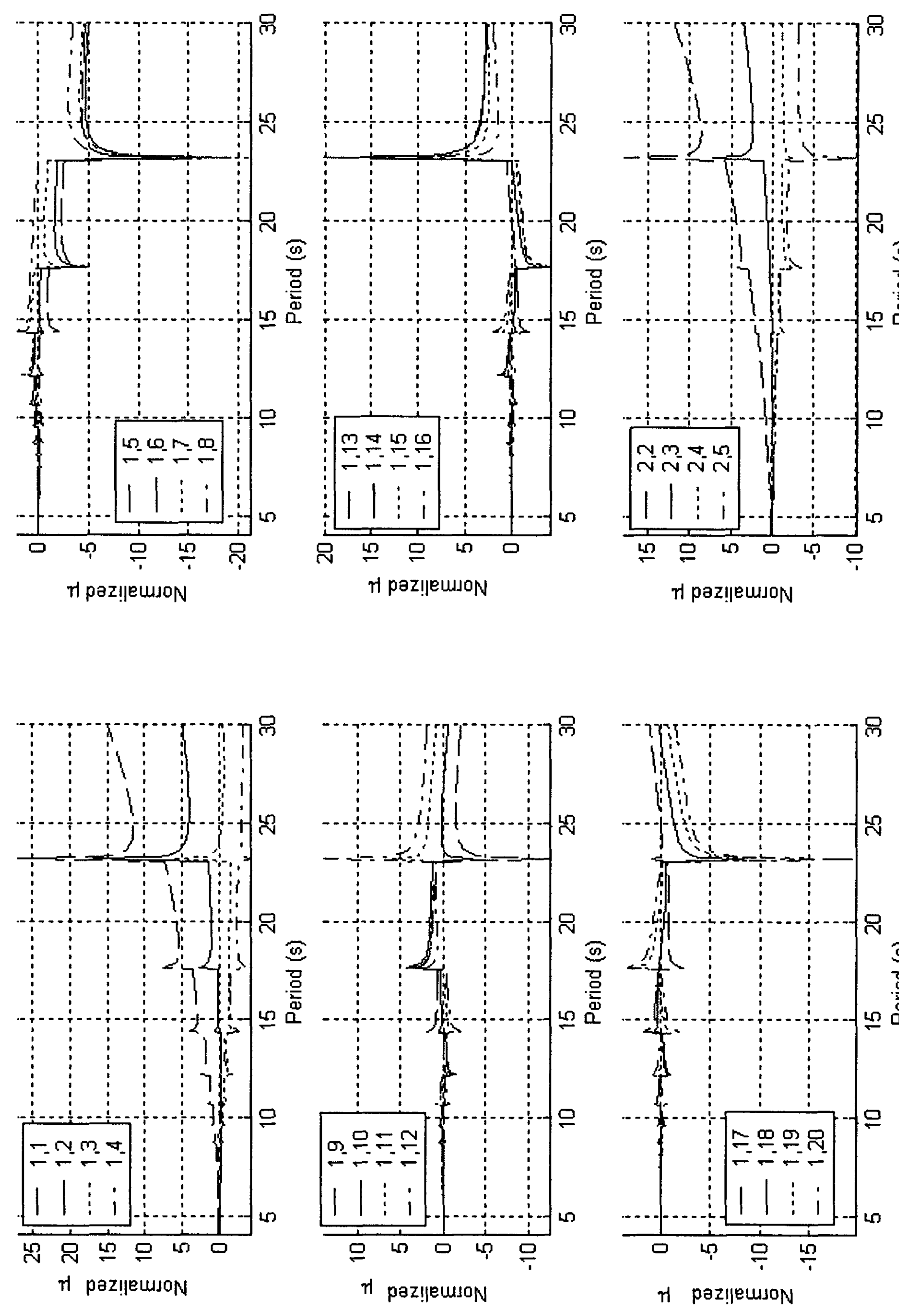

Figure 4-18: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

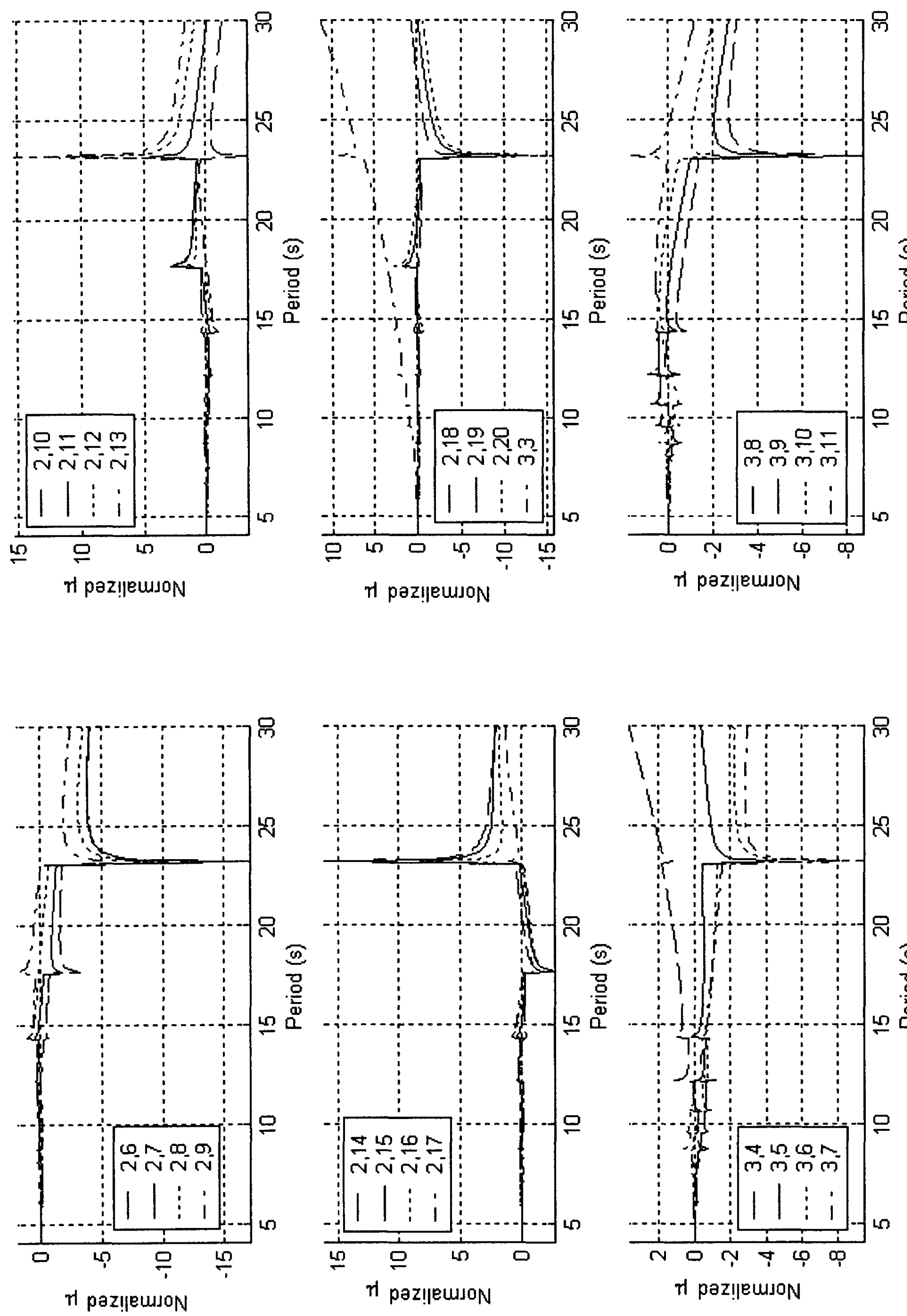

Figure 4-19: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

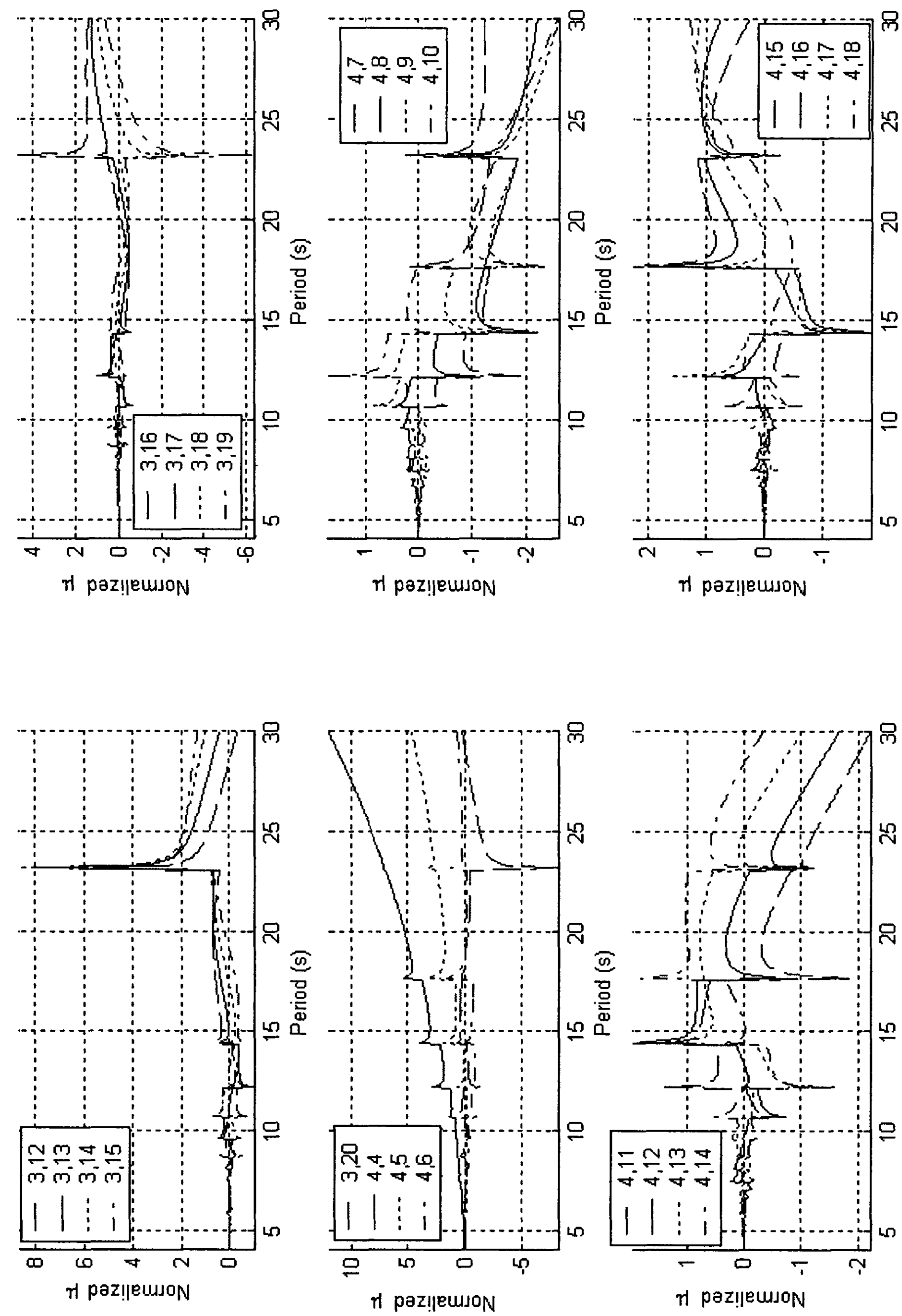

Figure 4-20: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

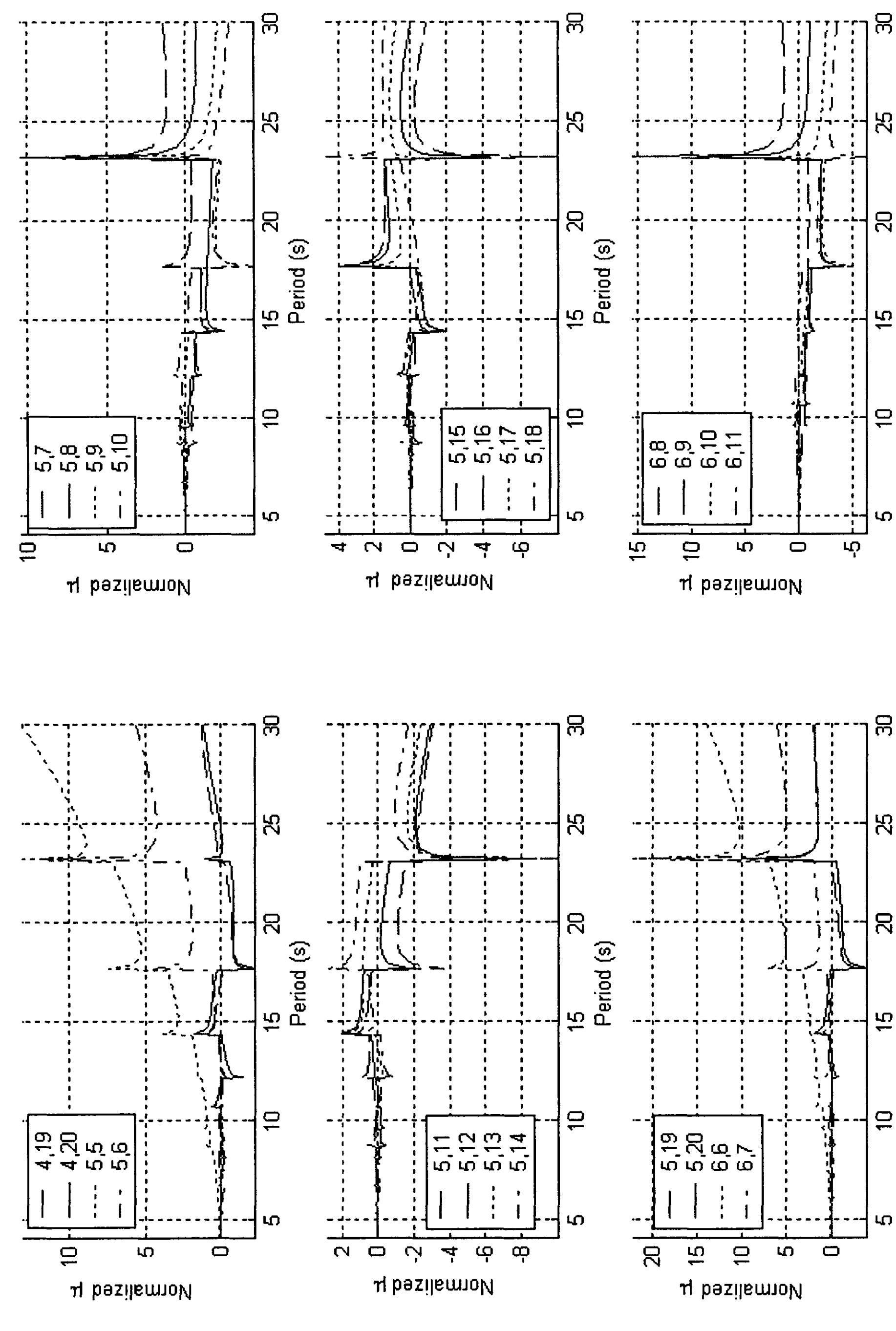

Figure 4-21: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

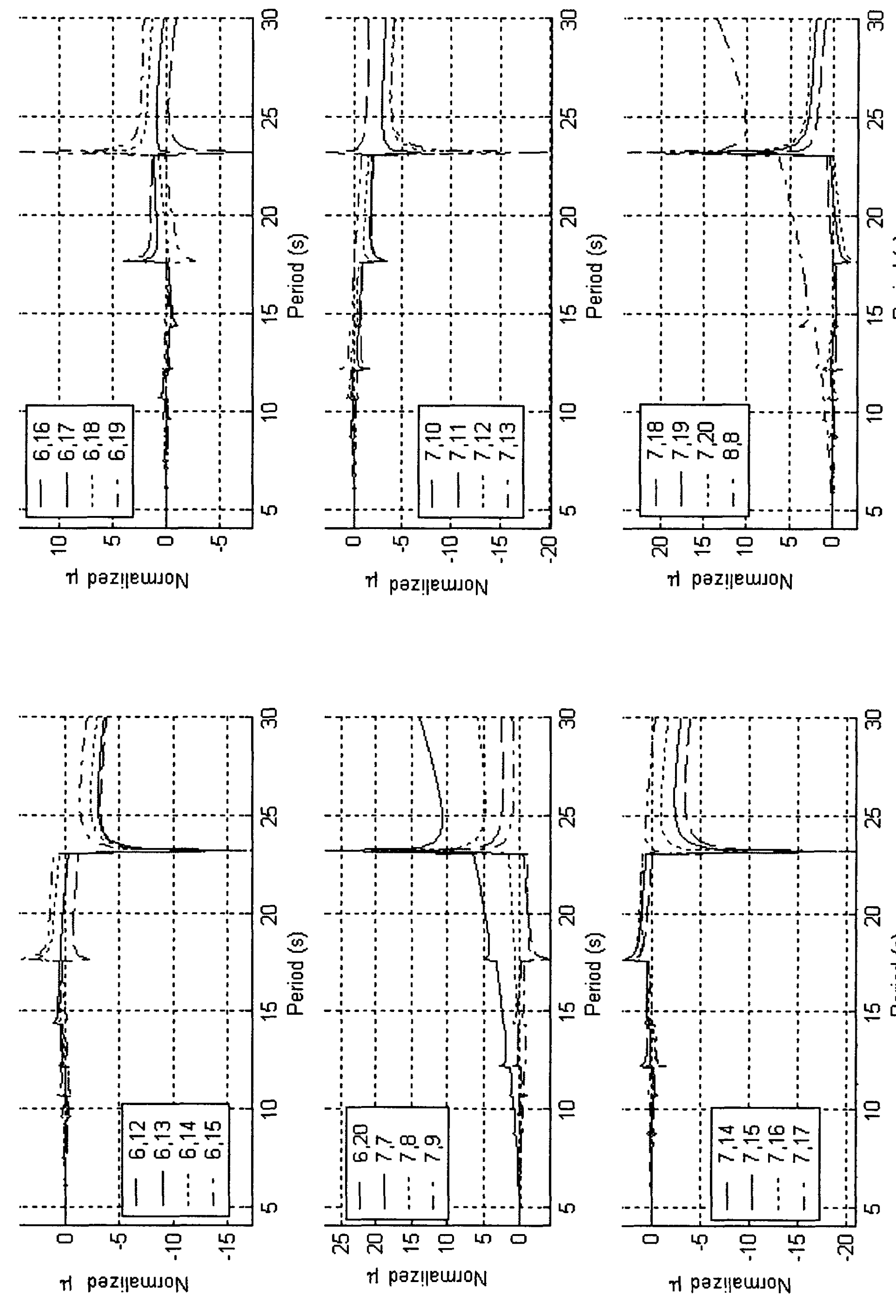

Figure 4-22: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

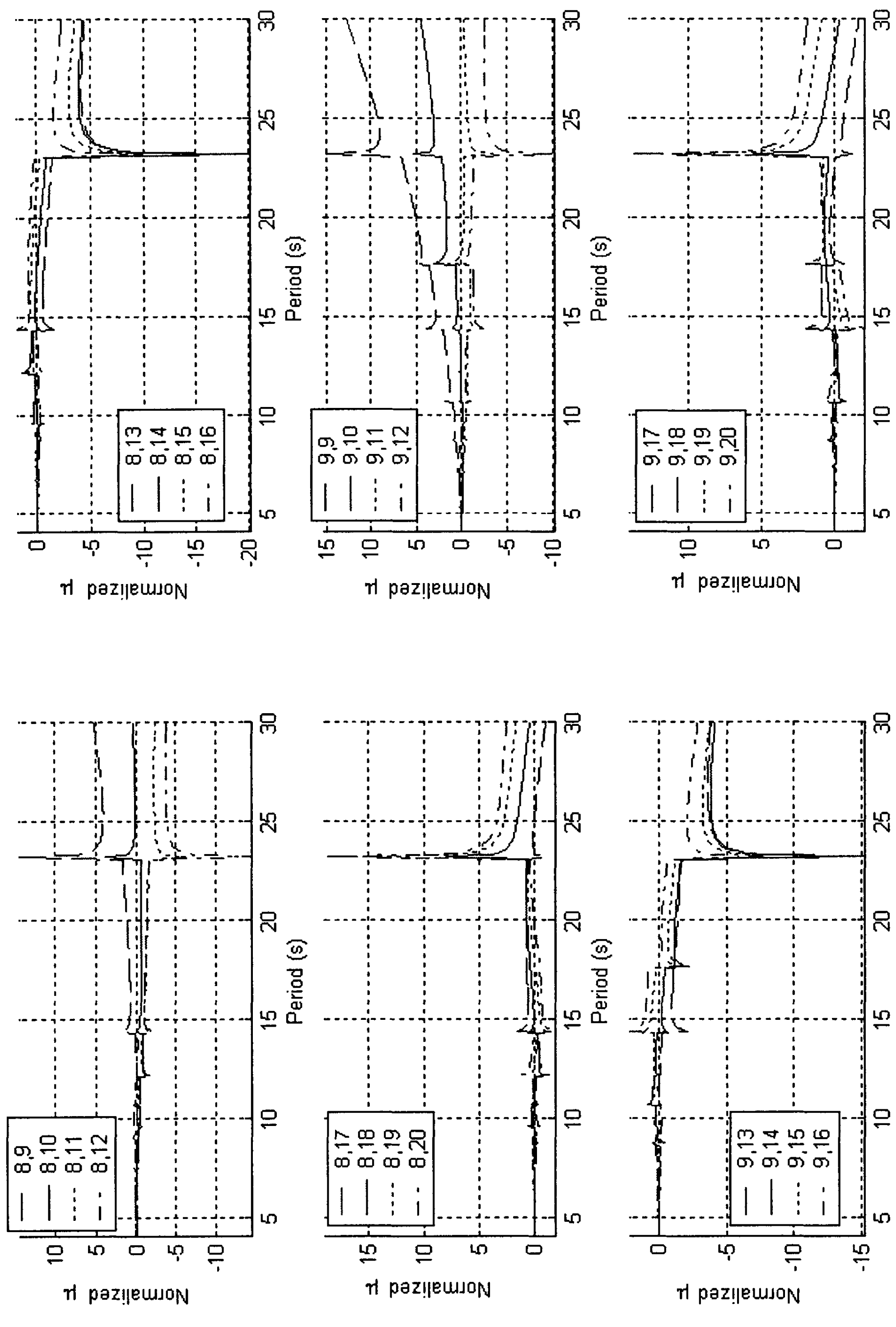

Figure 4-23: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$ againts period(s). In the legend the first number represents α, the second β

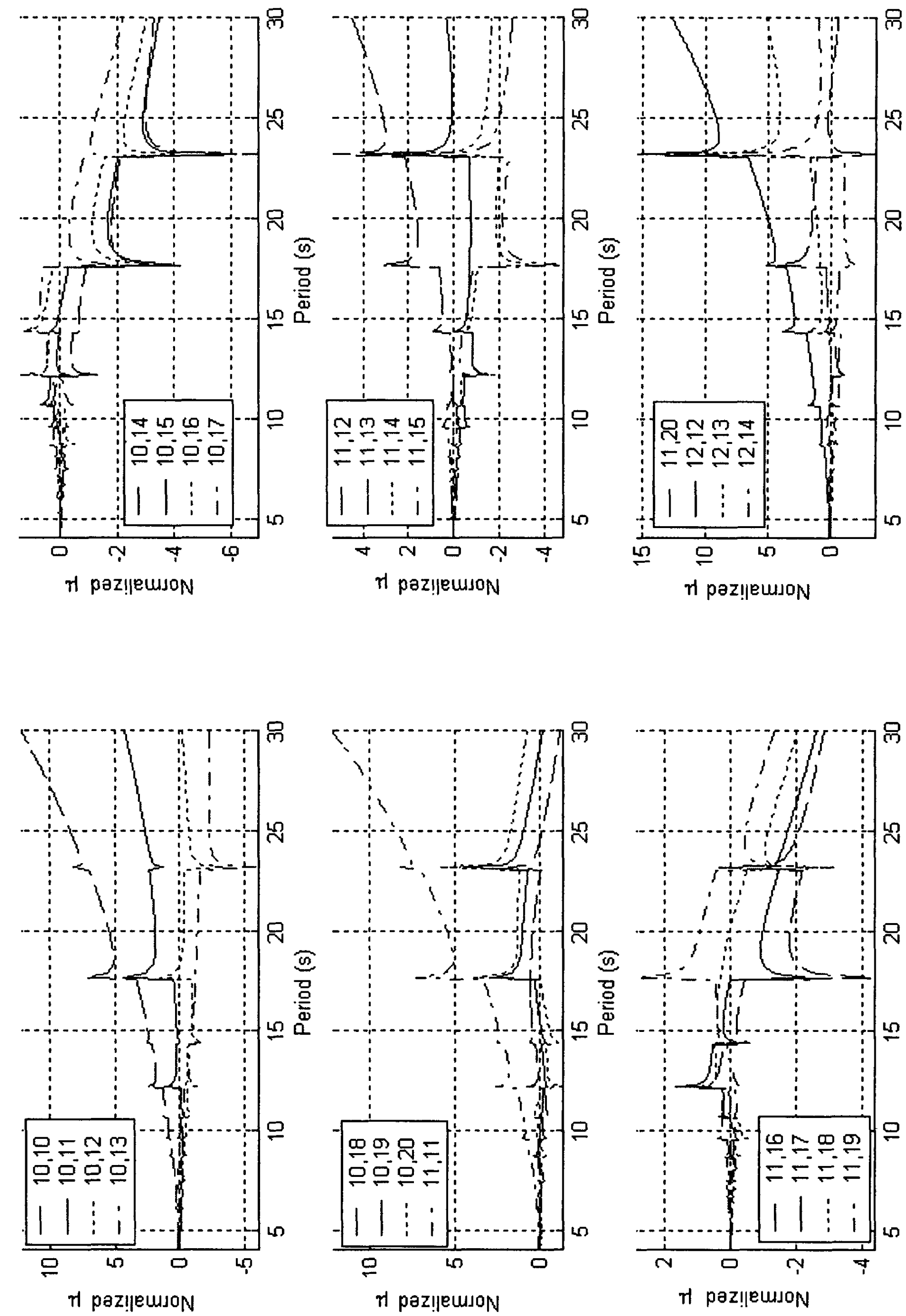

Figure 4-24: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

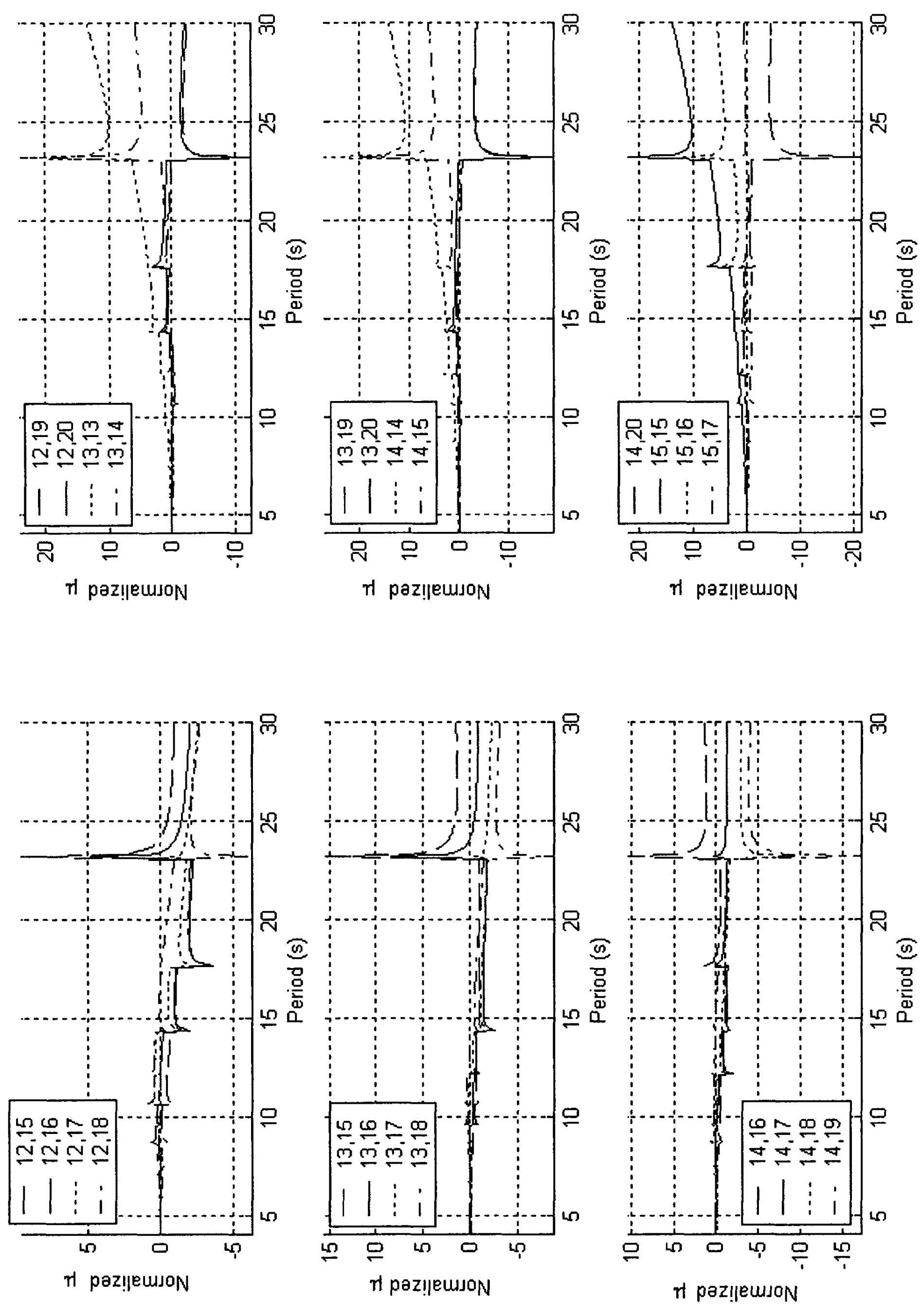

Figure 4-25: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

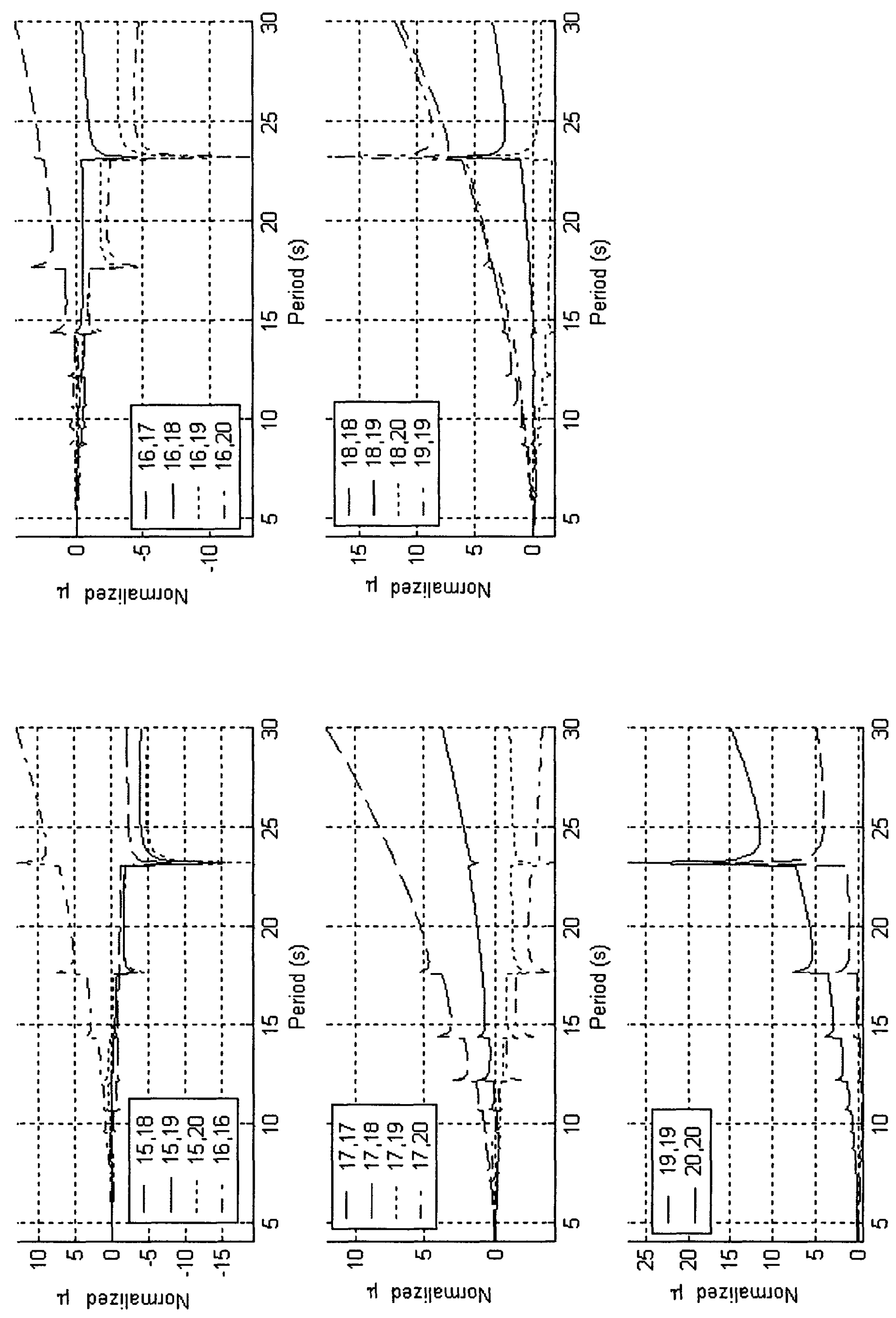

Figure 4-26: Normalized Added mass $\left(\frac{\mu_{\alpha,\beta}}{\varrho h L^4 \omega^2}\right)$againts period(s). In the legend the first number represents α, the second β

Chapter 5

Conclusions

In the present work we have developed a linear theory to study the motion of Venice storm gates forced by a monochromatic incident wave. The gates are assumed to be vertical and the fluid domain is approximated by a channel of infinite length on the Adriatic side and a semi infinite space on the lagoon side. Then, we have discussed some mathematical identities based on Green's theoreme. These are usefull to provide a deeper understanding of the physics of the problem and to provide a way to check both the theory and the numerical computations performed. In particular, the law of energy conservation is derived. After checking the reliability of the numerical computations we finally discussed the amplitude of motion of the gates, added mass and radition damping for an intervall of periods of engineering interest.

The numerical results validated with the derived identities show that synchronous resonance of the gates occurs. The modal shapes and resonant periods have an excellent agreement with those reported by Li & Mei [1]. The occurence of negative added masses is reported and discussed.

Further study that takes into account non linearity, vortex shedding and broad forcing band can give a more realistic prediction of the gates response.

Appendix A

Appendix: computer program

We give here the listing of the various subroutins written in the Matlab enviroment. The code is composed by:

Name of the routine	Short description
many_frequencies.m	main loop on the incoming wave period
amplitudes.m	call for the routine tha assemple $[F^A]$ and $[F^L]$. Computes θ_α
one_frequency_adriatic.m	computes $[F^A]$ (and calles Calculation_of_Amn)
one_frequency_lagoon_green.m	computes $[F^L]$
Calculation_of_Amn.m	Computes the coefficients of the wave expansion in the adriatic side
global_energy_check	Computes the RHS and LHS of the energy conservation, eq(3.23)
f1.m	used to solve dispersion relationship
f2.m	used to solve dispersion relationship
myfunff.m	used to calculate the energy equation terms

Listing of the code follows.

many_frequencies.m

```
%this code computes the amplitude of motion of Venice Gates under plane
%monochromatic incident wave attach
%written by -Andrea Adamo- 2003
clear
```

```
global om h a g ro Ygate Ngates Nn Nm;
%==========DATA============
%computational quantities
%computions have to be done in the intervall of periods [tmin,tmax] with time stepping tstep
tmin=6;
tmax=6;
tstep=.1;
%=========Adriatic parameters
Nmax_a=2; %max value of index n in the Adriatic expansion
Mmax=3; %max value of index m in the Adriatic expansion
%========Lagoon parameters
Nmax_l=3; %number of n coefficents(lagoon expansion)
P=3%number of subdivisions of the fixed gate
Pprime=5%number of number of subdivisions of the moving gate
%geometrical quantities
%==========GLOABAL PARAM
%coord of gates(the number indicates where the gate ends)
Ygate=[20,40,60,80,100,120,140,160,180,200,220,240,260,280,300,320,340,360,380,400];
Ngates=20; % variable with the total number of gates
L=20; %width of a gate(m)
h=14; %water depth (m)
a=400; %with of the channel in front of the gates (m)
g=9.8; %acceleratio of gravity (m/s^2)
ro=1000; %water density (Kg/m^3)
A=1; %amplitude of the incident wave (m);
M=33.337*10^6; %generic term of the inertia matrix
C=60246000;% generic term of the ouyancy restoring matrix
global om
tic
```

```
as=1;
for bb1=tmin:tstep:tmax
TT=0+bb1;
om=(2*pi)/TT;
amplitudes
Period(as)=TT;
%storage of variables for plotting purposes
Power(as,1:2)=Ptot;
Faa(1:20,1:20,as)=Fa;
Fll(1:20,1:20,as)=Fl;
Amplitude(1:20,as)=Ampl;
%---generation of the number of prop modes for plotting purposes---
xo=[0,1];
Kn=fzero('f1',xo);
p=1;
m=p-1;
cc=sqrt(Kn^2-(m*pi/a)^2);
while angle(cc)==0
alfa(p)=cc;
p=p+1;
m=p-1;
cc=sqrt(Kn^2-(m*pi/a)^2);
end
Npropmodes(as)=max(size(alfa));
alfa=0;
%--------------------------
as=as+1;
end
save dati4_30
disp('-----------------------------------------------')
```

disp('total computational time is: ')

toc

disp('———————————— ')

amplitudes.m

%this part calls for the other parts and then assembles the matrix equation for the solution of

%the body motion amplitudes and solves it.

%global om h a g ro Ygate Ngates Nn Nm R Theta Z;

%==========GLOABAL PARAM

%Ygate=[20,40,60,80,100,120,140,160,180,200,220,240,260,280,300,320,340,360,380,400];%coord of gates

%Ngates=20; % variable with the total number of gates

%L=20; %width of a gate(m), this value is used only in this routine to calculate the forcing term

%h=14; %water depth (m)

%a=400; %with of the channel in front of the gates (m)

%g=9.8; %acceleratio of gravity (m/s^2)

%ro=1000; %water density (Kg/m^3)

%A=1; %amplitude of the incident wave (m);

%=========ADRIATIC PARAM

%Nmax=20; %number of n coefficents

%Mmax=300; %number of m coefficents

%=======GATES PARAM

%M=280*10^3 ; %gate's mass(Kg)

%hh=20 ;%gate's height(m)

%”””””””””””””””””””’

%————CALCULATIONS————

%calculation of Kn

```
for p=1:Nmax_a
n=p-1;
if n==0
xo=[0,1];
k=fzero('f1',xo);
Kn(p)=k;
else
%I'm using the non dimensional expression for the dispersion relationship to exploit the interval 0.5pi/1.5pi
xo=[(n-.5+30*eps)*pi,(n+.5-30*eps)*pi];
KAPPA=fzero('f2',xo);
k=KAPPA/h;
Kn(p)=k*i;
end
end
%—-
disp('computing the ADRIATIC Side');
%call for the calculation of the matrice Fa on the adriatic side
[Fa]=one_frequency_adriatic(Nmax_a,Mmax,Kn);
%————
disp('computing the LAGOON Side');
%call for the calculation in the lagoon side with Green's functions
[Fl]=one_frequency_lagoon_green(Nmax_l,P,Pprime); % k is the first (real) value on the roots of disperdion relationship, used for the forcing term
%————
disp('computing amplitudes');
%mass matrix
%M=33.337*10^6;
MassMat=M*eye(Ngates,Ngates);
%Bouyancy restoring matrix
```

```
%C=60246000;
CMat=C*eye(Ngates,Ngates);
%construction of the vector Fd (forcing term vector)
k=Kn(1);
Fd=(2*ro*g*A)*(L/cosh(k*h))*((k*h*sinh(k*h)-cosh(k*h)+1)/(k*k))*ones(Ngates,1); %new
%Assemblage of the system to solve
KK=(-(om^2)*MassMat)+CMat+sqrt(-1)*om*Fa-sqrt(-1)*om*Fl;
%Solution.
Ampl=KK\Fd;
%call for the global energy check
Ptot=global_energy_check(Ngates,Ampl,L,ro,om,h,g,A,a,Ygate,MassMat,CMat,Fa,Fl,Fd);
```

one_frequency_adriatic.m

```
function [Fa]=one_frequency_adriatic(Nmax,Mmax,Kn)
%in this routine we want to calculate, for a given frequency, Fa
%INPUTs are a the coordinates of the edges of the gates, and the number of gates.
global om h a g ro Ygate Ngates
disp('begin one_frequency routine');
Pot(Ngates,Ngates)=0;
for cc1=1:Ngates %loop on the gates that generate waves
if cc1-1==0 Yg=0;
else Yg=Ygate(cc1-1); end
L=Ygate(cc1)-Yg; %with of a gate (m)
Y=(Ygate(cc1)+Yg)*.5; %y coordinate of the j-th gate's middle point
calculation_of_amn;
for cc2=cc1:Ngates %loop to calculate the integral of the potential on the other gate (I exploit the fact that the matrix is gonna be symmetic)
%various constant needed to run "calculation_of_amn"
if cc2-1==0 Yg=0;
else Yg=Ygate(cc2-1); end
L=Ygate(cc2)-Yg; %with of a gate (m)
```

```
Y=(Ygate(cc2)+Yg)*.5; %y coordinate of the j-th gate's middle point
for p=1:Mmax
m=p-1;
for q=1:Nmax
n=q-1;
if m==0
Pot(cc1,cc2)=Pot(cc1,cc2)+Amn(p,q)*L*((Kn(q)*h*sinh(Kn(q)*h)-cosh(Kn(q)*h)+1)/(Kn(q)^2));
else
Pot(cc1,cc2)=Pot(cc1,cc2)+Amn(p,q)*(a/(m*pi))*2*cos(m*pi*Y/a)
    *sin(m*pi*L/(2*a))*((Kn(q)*h*sinh(Kn(q)*h)-cosh(Kn(q)*h)+1)/(Kn(q)^2));
end
end
end
end
end
%at this point Pot has to be a matrix filled only in the upper triangular part.
%now we fill it completely exploiting the fact that this matrix gas to be symmetric.
Pot=Pot+(Pot.')-diag(diag(Pot));
Fa=sqrt(-1)*ro*om*Pot;
disp('end of one_frequency routine');
```

one_frequency_lagoon_green.m

```
function [Fl]=one_frequency_lagoon_green(Nmax_l,P,Pprime);
global h om g ro Ngates Ygate
%this function calculates Fl on the lagoon side using Green function.
%——————————
%constants and variables
%==Ndisp=35; %number of roots of the dispersion relationship taken into account
%==NN=Ndisp; %number of terms taken into account in the series expansion of (z+h)
and G
```

```
L=20; %with of a gate (m)
%Kn vector of length Ndisp containing the roots of the dispersion relationship
%Bn vector with the coefficients of the series expansion (z+h),see prof Mei's notes
%An vector with the coefficients of G
%fz vector with the value of the function f used for the expansions
%vk counter
%--------------------------------------------------
%calculate roots of dispersion relationship: kn
for p=1:Nmax_1
n=p-1;
if n==0
xo=[0,1];
k=fzero('f1',xo);
Kn(p)=k;
else
%I'm using the non dimensional expression for the
%dispersion relationship to exploit the interval 0.5pi/1.5pi
xo=[(n-.5+30*eps)*pi,(n+.5-30*eps)*pi];
KAPPA=fzero('f2',xo);
k=KAPPA/h;
Kn(p)=k;%*i;
end
end
%--------------------------
%Computation of Bn ( I add the correction for unit amplitude rotation)
sig=(om^2)/g;
Bn=(sqrt(2)./sqrt((h-((sin(Kn.*h)).^2)./sig))).*(((h*sin(Kn.*h))./Kn)+((1+cos(Kn.*h))./(Kn.^2)));
Bn(1)=(sqrt(2)./sqrt((h+((sinh(Kn(1).*h)).^2)./sig))).
        *(((h*sinh(Kn(1).*h))./Kn(1))+((1-cosh(Kn(1).*h))./(Kn(1).^2)));
%Computation of An
```

```
An=-Bn./pi;
An(1)=Bn(1)/(2*sqrt(-1)); %this because An(1) differs from the others in its definition
%-----------------------
%------for the moving gate ------
%-----------------------------------
NNstep=30; %number of subdivisions of the moving gate
IIss=L*.5;
IIii=-L*.5;
ddss=(IIss-IIii)/NNstep;
Y=[IIii:ddss:IIss]+(ddss*.5);
Nstep=NNstep;
Is=L*.5;
Ii=-L*.5;
ds=(Is-Ii)/Nstep;
yy=[Ii:ds:Is]+(ds*.5);
int_fi=0;
for ff=1:length(Y)
y=Y(ff);
int_G=0;
for fd=1:length(yy)
rr=abs(y-yy(fd));
if rr>0
KK=besselk(0,(Kn*rr));
KK(1)=besselh(0,1,(Kn(1)*rr)); %because the first term is with an Hankel function
int_G=int_G+ds*sum(An.*Bn.*KK);
else
a=ds*.5; %in this part KK is contains already the integral evaluated in an analitycal way
KK=-4*(a*a*log((Kn(1)*a))-1.5*a*a);
KK(1)=4*(a+2*sqrt(-1)*(a*a*log((Kn(1)*a))-1.5*a*a)/pi);
int_G=int_G+sum(KK.*An.*Bn);
```

```
end
end
int_fi=int_fi+int_G*ddss;
end
recordpot(1)=int_fi;
%------------------------------------
%------------------------------------
%------------------------------------
%computation of the integral on the hankel and kelvin function on a gate
%---======+++++ LOOP
for vk=2:Ngates
%---======+++++
%P=300; %number of subdivisions of the fixed gate
IIss=Ygate(vk)-10;
IIii=Ygate(vk-1)-10;
ddss=(IIss-IIii)/P;
Y=[IIii:ddss:IIss]+(ddss*.5);
%Pprime=350; %number of number of subdivisions of the moving gate
if P==Pprime; Pprime=Pprime+10; end
Is=L*.5;
Ii=-L*.5;
ds=(Is-Ii)/Pprime;
yy=[Ii:ds:Is]+(ds*.5);
int_fi=0;
for ff=1:length(Y)
y=Y(ff);
int_G=0;
for fd=1:length(yy)
rr=y-yy(fd);
KK=besselk(0,(Kn*rr));
```

```
KK(1)=besselh(0,1,(Kn(1)*rr)); %because the first term is with an Hankel function
int_G=int_G+ds*sum(An.*Bn.*KK);
end
int_fi=int_fi+int_G*ddss;
end
%—————————————
%—=====+++++
recordpot(vk)=int_fi;
end
%—=====+++++
%assemblage of the matrix Fl
Pot(Ngates,Ngates)=0;
Pot(1,:)=recordpot;
%at this point Pot has to be a matrix filled only in the first row.
%now we fill it completely exploiting the fact that this matrix gas to be symmetric
for cc1=2:Ngates
for cc2=cc1:Ngates
Pot(cc1,cc2)=Pot(cc1-1,cc2-1);
end
end
Pot=Pot+(Pot.')-diag(diag(Pot));
Fl=sqrt(-1)*om*ro*Pot;
%as an output Kn(1) is requested for the prosecution of the code
%k=Kn(1);
```

Calculation_of_amn.m

```
%this calculates Kn, alfa_nm, Amn, Cmn given omega, and geometry
%———other variables————
%Kn: vector with the kn values
%Amn: matrix with the coefficients of the series expansion
```

```
%alfa: matrix with the alfa_mn coefficients of the serie expansion
%x,y: vectors with the coordinates for plotting
%n,m,p,q,r,s:Counters
%k,KAPPA,l1,l2,l3,l4,eta,bc:working variables
%eps_m: is 1 if m=0;it is 2 otherwise
%BC: the value of the boundary condition (for a control)
%++++++++++++++++++++++++++
%Calculation of alfa_mn (p is the counter for m; q is the counter for n)
for p=1:Mmax
m=p-1;
for q=1:Nmax
n=q-1;
alfa(p,q)=sqrt(Kn(q)^2-(m*pi/a)^2);
end
end
%++++++++++++++++++++++++++
%Calculation of Amn and Cmn
%a_n represents the coefficient in front of the exponent in the x term, that keeps a sign or
an i
%eps_m represents a constant that depends on m
for p=1:Mmax
m=p-1;
for q=1:Nmax
n=q-1;
a_n=-sqrt(-1);
if m==0
eps_m=1;
        l4=L;
else
eps_m=2;
```

```
l4=((2*a)/(m*pi))*cos(m*pi*Y/a)*sin(m*pi*L/(2*a));
end
l1=eps_m/(alfa(p,q)*a_n*a);
l2=(4*Kn(q))/(sinh(2*Kn(q)*h)+(2*Kn(q)*h));
l3=(h*sinh(h*Kn(q))/Kn(q))-(cosh(h*Kn(q))/(Kn(q)^2))+(Kn(q)^-2);

Cmn(p,q)=(eps_m/(a))*l2*l3*l4;
Amn(p,q)=l1*l2*l3*l4;
end
end
```

global_energy_check.m

```
function [Ptot]=global_energy_check(Ngates,Ampl,L,ro,om,h,g,A,a,Ygate,MassMat,CMat,Fa,Fl,Fd
global om h g
global Kn ybya L
%———-Lagoon side part: variables
%Yy=[190:-20:-200]; %array with the coordinate of the middle point of each gate for hte calculation in the lagoon side;
asi=(a*.5-L*.5);
ssi=-L
ass=-a*.5;
Yy=[asi:ssi:ass];%array with the coordinate of the middle point of each gate for hte calculation in the lagoon side;
%^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
% check on the LAGOON side
%%^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
%==============================
%Compute power flux throu the all gates
%==============================
Plag_gates=imag(-sqrt(-1)*om*conj(Ampl.')*(Fl/ro)*Ampl);
```

```
%==============================
%Compute power flux (throu a far surface) due to all the gates moving with the computed amplitude.
%==============================
xo=[0,1];
Kn=fzero('f1',xo);
%Computation of Ao (for the unit amplitude case)
sig=(om^2)/g;
Bo=(sqrt(2)/sqrt((h+((sinh(Kn*h))^2)/sig)))*(((h*sinh(Kn*h))/Kn)+((1-cosh(Kn*h))/(Kn^2)));
Ao=Bo/(2*sqrt(-1));
%=====================++++++++
%Calculation of a part of the power
P1=0;
for ca=1:Ngates
for cb=ca:Ngates
ybya=Yy(cb)-Yy(ca);
I1=0;
I1=quadl(@myfunff,0,pi,10^-10);
P1(ca,cb)=(om^2)*Ampl(ca)*conj(Ampl(cb))*I1;
end
end
P1=P1+(P1.')-diag(diag(P1));
I2=sum(sum(P1))*((abs(Ao))^2)*(-8*sqrt(-1)/(pi*Kn^2));
Plag_far=imag(I2);
%============================
%============================
%^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
% check on the ADRIATIC side
%^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
%============================
```

```
%Compute power flux throu the gates
%================================
Padr_gates=imag(conj(Ampl.')*(Fd/ro)-sqrt(-1)*om*(conj(Ampl.'))*(Fa/ro)*Ampl);
%================================
%Compute power flux throu a far surface (due to all the gates moving with the computed amplitude)
%================================
%calculation of alfa (doens't depend on the gate- I calculate only those related to prop modes)
p=1;
m=p-1;
cc=sqrt(Kn^2-(m*pi/a)^2);
while angle(cc)==0
alfa(p)=cc;
p=p+1;
m=p-1;
cc=sqrt(Kn^2-(m*pi/a)^2);
end
Npropmodes=max(size(alfa));
%calculation of A_mn (in the rows values for different m,in the colums values for different gate)
for q=1:Ngates %counter on the gates
if q-1==0 Yg=0;
else Yg=Ygate(q-1); end
%L=Ygate(cc1)-Yg; %with of a gate(m)-fixed in this routine
Y=(Ygate(q)+Yg)*.5; %y coordinate of the j-th gate's middle point
for p=1:Npropmodes %counter on the value of m
m=p-1;
a_n=-sqrt(-1);
if m==0
```

```
eps_m=1;
l4=L;
else
eps_m=2;
l4=((2*a)/(m*pi))*cos(m*pi*Y/a)*sin(m*pi*L/(2*a));
end
l1=eps_m/(alfa(p)*a_n*a);
l2=(4*Kn)/(sinh(2*Kn*h)+(2*Kn*h));
l3=(h*sinh(h*Kn)/Kn)-(cosh(h*Kn)/(Kn^2))+(Kn^-2);
A_mn(p,q)=l1*l2*l3*l4;
end
end
%calculation of II6
II6=0;
for ra=1:Ngates
II6=II6+sqrt(-1)*om*conj(Ampl(ra))*conj(A_mn(1,ra));%*exp(alfa(1)*sqrt(-1)*X);
end
II6=(Kn*g*A/om)*(1/cosh(Kn*h))*II6;%*exp(-sqrt(-1)*Kn*X)
%calculation of II8
II8=0;
for ra=1:Ngates
II8=II8-sqrt(-1)*om*Ampl(ra)*A_mn(1,ra);%*exp(-alfa(1)*sqrt(-1)*X);
end
II8=(-Kn*g*A/om)*(1/cosh(Kn*h))*II8;%*exp(sqrt(-1)*Kn*X)
%calculation of II9
II9=0;
for ra=1:Ngates
for rb=1:Ngates
Ip=0;
for p=1:Npropmodes
```

```
eps_m=2; if p==1 eps_m=1;end
Ip=Ip+(A_mn(p,ra)*conj(A_mn(p,rb))*sqrt(-1)*alfa(p)/eps_m);
end
II9=II9+(om^2)*Ampl(ra)*conj(Ampl(rb))*Ip;
end
end
%the net radiated power is:
Padr_far=((sinh(2*Kn*h)+2*Kn*h)/(4*Kn))*imag(II6+II8+II9)*a;
%==============================
%==============================
Ptot=[Padr_far,Plag_far]
```

f1.m

```
function y =f1 (k)
global h om g
a=-om^2;
y=g*k*tanh(k*h)+a;
```

f2.m

```
function y =f2 (KAPPA)
global h om g
OM=(h*om^2)/g;
y=-tan(KAPPA)-OM/KAPPA;
```

myfunff.m

```
function y=myfunff(x);
global Kn ybya L
y=exp(sqrt(-1)*Kn*ybya*cos(x)).*(sin(L*.5*Kn*cos(x)).^2)./(cos(x).^2);
```

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