ESCHER'S WORLD:

LEARNING MATHEMATICS AND DESIGN IN A DIGITAL STUDIO

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Abstract

In recent years, mathematics educators have begun to call for reforms in mathematics learning that emphasize open environments rather than the traditional pedagogy of exposition and drill. This thesis explores one example of an open learning environment created by combining mathematics and design activities in a "mathematics studio." This thesis looks at: (a) whether students can learn specific mathematics topics in a studio environment, (b) whether learning in such an environment will change the way students solve mathematical problems, and (c) whether learning in such an environment will change students' attitude towards mathematics. Two iterations of the mathematics studio experiment in a project at the MIT Media Laboratory known as Escher's World suggest that: (a) students can learn about the mathematical concept of symmetry in a studio learning environment, (b) students learn to use visual thinking to solve mathematical problems in a studio learning environment, and (c) students develop a more positive attitude towards mathematics as a result of working in a studio learning environment. The thesis uses a qualitative research model, so in addition to reporting the outcomes of the mathematics studio experiment, the thesis also describes the specific characteristics of the mathematics studio that were influential in creating a successful learning environment. In particular, the thesis describes how expression, expressive activities, and expressive computational media give students a sense of control over their learning.

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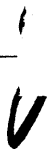
◪ ESCHER'S WORLD

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1 INTRODUCTION

The publication of the National Council of Teachers of Mathematics Standards in 1989 marked the emergence of a coherent reform movement in curriculum (NCTM 1989). Instead of a pedagogy based on exposition by the teacher, reformers propose moving from drill and practice to more open learning environments. They call for the introduction of extended projects, group work, and discussions among students. In many ways, the learning environment these reformers describe seems more similar to a studio course in design or architecture, where students work on extended projects exploring creative solutions to general problems, than to a traditional mathematics class. This thesis describes one such mathematics studio, and evaluates the success of this approach in learning some mathematics concepts.

Specifically this thesis addresses three aspects of mathematics learning:

- Content knowledge: Can students learn to understand a specific mathematics topic in a design studio?

- Skill acquisition: Will learning in a design studio affect the way students solve mathematics problems?

- Change in attitude: Will students feel differently about mathematics after learning mathematics in a digital studio?

In addition, this thesis presents qualitative data to explore the process by which the mathematics studio achieved its results. The focus of this section of the thesis is on the following key questions:

- What specific aspects of a "mathematics studio" make it a valuable environment for learning mathematics?

- What role does the computer play in the success of a "mathematics studio"?

- What implications does the idea of a "mathematics studio" have for mathematics pedagogy or for our understanding of the process of learning more generally?

The answers to these questions suggest that indeed a studio learning environment can be used effectively and profitably in mathematics education.

2 BACKGROUND AND SETTING

Since the time of John Dewey and Francis W. Parker at the turn of the century, educators have understood that the arts can play a fundamental role in education (Sidelnick 1995). Arguments for art in formal education have historically (and logically) have called for two distinct, though not necessarily exclusive, places for art in the curriculum. One role for art in education since the nineteenth century is as a distinct discipline. Justified at different times and in different places by its beneficial effect on students' study skills, as a set of useful job skills, or as a means to self-awareness and the understanding of others, educators have argued that students should take art as a formal course of study in addition to the "academic" subjects of mathematics, history, English, science, and foreign languages (Brown and Korzenik 1993). The other position, which goes back at least as far as the turn of the century, is that art as a mode of expression is a key component in the process of all learning. Following the lead of Colonel Parker's Cook County Normal School, proponents of this view argue that arts learning should be integrated into all of the traditional disciplines (Sidelnick 1995).

This second position--that arts learning should be integrated across the curriculum--presents a tantalizing vision for mathematics educators. The idea of "learning math by making art" appeals to a broad spectrum of students, parents, and others who feel disenfranchised by the "traditional" mathematics curriculum and pedagogy. People get excited by the idea of learning mathematics with the freedom and joy associated with art-making. Since the publication of the National Council of Teachers of Mathematics "Curriculum and Evaluation Standards for School Mathematics" in 1989 (the NCTM Standards), followed by the "Professional Standards for Teaching Mathematics" in 1991, key components of this vision have also been sanctioned by a section of the professional community of mathematics educators. The Standards call for the introduction of

extended projects, group work, and discussions among students--elements of a learning environment that overall seems more similar to a studio course in design or architecture than to a traditional mathematics class (NCTM 1989, NCTM 1991).

In recent years, there have been several examples of interventions that demonstrate the power of this vision of a "mathematics studio." In a qualitative study, Leslie Willett demonstrated that mathematics learning is more effective in the context of arts-based lessons than with standard mathematics pedagogy at the elementary school level (Willett 1992). Arthur Loeb's visual mathematics curriculum (Loeb 1993) has not been studied formally, but substantial anecdotal evidence supports his approach to the study of the formal mathematics of symmetry through a design studio as an effective learning environment for undergraduate students.

This thesis examines another successful attempt to create a "mathematics studio" where students learn about mathematics and art simultaneously in an art studio-like environment. The Escher's World research project at the Massachusetts Institute of Technology Media Laboratory brought twelve high-school students from public schools in Boston, Massachusetts to the Media Laboratory for brief but intensive workshops during the spring and summer of 1995. In these workshops, students created posters and worked on other design projects using mathematical ideas of mirror and rotational symmetry.

The theoretical basis of the Escher's World project is in the constructionist learning paradigm (Papert 1991a, Papert 1993), which suggests that building things is a particularly rich context for building understanding. The theory of constructionism has been supported by investigations into the way students learn through the design and construction of real objects and virtual microworlds (Kafai and Harel 1991, Resnick 1991, Resnick and Martin 1991, Resnick and Ocko 1991). Using a qualitative research methodology based on observations of student behavior and structured interviews, this thesis documents the process by which mathematics learning took place in the Escher's

World design studio: the anatomy, as it were, of a learning environment. The central questions are:

- Was the Escher's World mathematics studio a successful learning environment?

- What aspects of this design studio made it a good venue for learning mathematics?

This thesis presents a detailed account of one specific instance of the "mathematics studio" idea. However, the Escher's World project was not designed only to understand how students learn mathematics through art in one particular studio environment. The goal was also to use this particular learning environment to elaborate a framework for thinking more generally about how students can use design activities as a context for learning. In particular, this thesis demonstrates the role of expressive activities and expressive technology in design activities and in learning more generally.

3 METHODS

This section of the thesis is divided into five sections: Participants, Workshop Activities, Workshop Facilities, Data Collection, and Data Analysis. The purpose of this section is to provide an overview of the Escher's World workshops (see Figure 1). In a qualitative study such as this, a rich sense of the particular participants and activities is essential. This section provides a foundation for thinking about the Escher's World workshops which will be developed in more detail in the Results sections (chapters 4 and 5 below). Similarly, the Data Collection and Data Analysis portions of the Methods section explain the structure of the student interviews and the general process of processing and coding the data. The theoretical frameworks within which the data was interpreted are presented in the Results sections (chapters 4 and 5 below).

Activity	Steps
Pre Interview (thesis section 3.4)	Image Descriptions Word Problems Affect Questions Affect Survey
Workshop (thesis section 3.2)	Mirror Symmetry Investigations Mirror Symmetry Explorations Rotational Symmetry Investigations Rotational Symmetry Explorations
Post Interview (thesis section 3.4)	Image Descriptions Word Problems Affect Questions Affect Survey
Follow-up Interview (thesis section 3.4)	Affect Questions
Data Analysis (thesis section 3.5)	Coding Statistical Analysis

Figure 1: Overview of Escher's World project activities

3.1 Participants

The Escher's World project conducted two workshops during the spring and summer of 1995. At each workshop, six students from Boston public high schools came to the Media Lab for 12 hours on two or three days.[1] The students were all volunteers, and were told that they would be attending a workshop about mathematics and art. Students in the first workshop heard about the workshop from their mathematics teachers, while those in the second workshop were contacted through a local after-school enrichment program for high school students. The students came from a range of backgrounds and ability levels, with five of the students attending "test schools" in the Boston School system.[2] Five of the twelve students in the workshops were foreign-born. Eleven of the twelve students were persons of color, including two Latino students. One student was white, and there were no Asian or Asian-American students. Five of the students were male and seven were female, with all of these categories more or less evenly distributed between the two workshops except that all of the students from test schools attended the second workshop and all of the foreign-born students attended the first workshop.

3.2 Workshop Activities

In both workshops, students spent 12 hours at the Media Lab. In the first workshop, students came for 4 hours a day for three days. In the second workshop, students came for 6 hours a day for two days. Both of the workshops were divided into two sections, each lasting approximately 6 hours. The first section was organized around the concept of mirror symmetry; the second was organized around the concept of

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[1] A seventh student was also present during the second workshop, but she was not able to complete the interview protocol and was dropped from the data analysis.

[2] Boston Latin and Latin Academy in Boston are public high schools that require an admissions test of academic achievement for entry.

rotational symmetry. While the 12 hours of the first workshop were spread over three days rather than two, the sections were still roughly equal in duration, with the break coming in the middle of the second day.

Each section of the workshop began with a "warm-up" activity, which was then followed by "investigations," "explorations," and a final "check-in" with the students. These sections are each described below, but what none of the descriptions of workshop activities can completely capture is the emphasis throughout the workshop on creating an open, studio-like atmosphere for learning. Students were encouraged, particularly during the "exploration" portion of the workshops (which comprised the bulk of time during the day) to sit and work where they liked, to use media of their own choosing, to collaborate or work alone as they wished, to eat, take breaks, go to the bathroom, and change projects at their own discretion.

3.2.1 Warm-Up

Warm-up lasted approximately 1/2 hour and presented students with a short mathematical game or puzzle which was described as an opportunity to "stretch their minds." The games were also chosen to help create a relaxed atmosphere and help students meet each other and become more comfortable working together. One activity, for example, asked students to solve a topological problem by disentangling themselves after being tied together by their wrists. Part of the warm-up time was also spent on administrative and logistical issues related to the workshop.

3.2.2 Investigations

The investigation portion of the workshop lasted approximately one hour. During investigations, students worked on a series of short problems relating to symmetry on their own or in small groups. Students wrote entries in their workshop journals, and discussed their observations as a group with a workshop leader as facilitator. In the first

day of the workshop, for example, students began their investigation of mirror symmetry by making name-tags that read normally when viewed in a mirror. This was followed by a search for words that look the same when viewed in a mirror, and from there to the classification of the letters of the alphabet by their mirror lines. Students worked on each of these problems individually or in pairs or groups at their own discretion, with the group discussing the "results" of each problem. Students conducted a similar sequence of investigations involving rotational symmetry using a telidescope in the second section of the workshop the next day.

3.2.3 Explorations

Based on their investigations, students spent two to three hours working on extended projects in design on their own or with a partner. Students worked on one shorter project (approximately one hour), and then presented their work to the group for discussion, questions, and comments. Following this "peer review," students began a more ambitious project (approximately two hours), integrating ideas about symmetry, principles of design, and feedback from their presentation. In the first day of the workshop, for example, students followed their classification of the alphabet by creating a design of their own choosing that had mirror symmetry. After discussing their designs, students worked for the
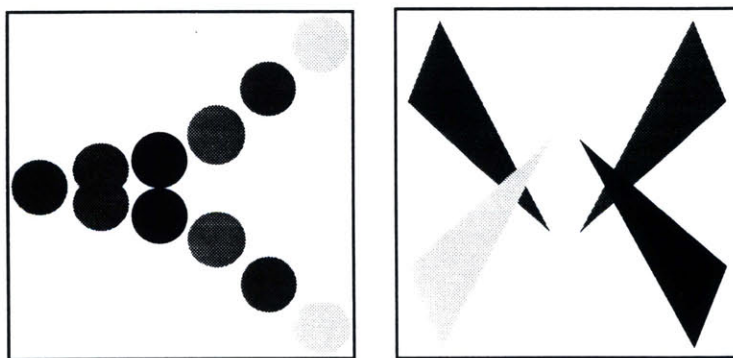

Figure 2: Student work from Escher's World: explorations of composition and symmetry

remainder of the day creating designs that had mirror symmetry but did not place the focus of the composition in the middle of the design. In the second section of the

workshop on rotational symmetry students made designs using rotational symmetry, and then, following the presentation of their work, tried to make designs that used rotational symmetry but presented a lopsided or unbalanced composition.

3.2.4 Check-in

At the end of each day, there was time for reflection on the day's activities lasting approximately 1/2 hour. Students wrote in their workshop journals in response to specific questions about the content and structure of the workshop. There was also time to discuss as a group any problems or concerns that came up during the day, and to make any schedule or logistical adjustments required for the coming day.

3.3 Workshop Facilities

The workshops took place in a conference room at the Media Lab that was modified to resemble an art studio. A variety of works of art by students and professional artists were placed on the walls, and a variety of artistic media were available for students' use. In addition to the author, who acted as workshop leader for both workshops, there were one or two other adults in the studio during the workshops as a resource for students.

Students were provided with Macintosh computers throughout the workshops, with one computer available for two or three students. The computers were connected to flatbed scanners, color printers, and a large format color plotter. Computers were equipped with Aldus Superpaint and Adobe Photoshop (commercially-available drawing and image-manipulation programs; Aldus Corp. 1993, Knoll et al. 1993) and with the Geometer's Sketchpad (commercially available educational software for mathematics; Jackiw 1995). During the investigation portion of the workshops, students were introduced to some of the basic functionality of these programs (particularly the

Geometer's Sketchpad). Students were able to work on the computers or with traditional materials during their explorations; all of the students chose to use a computer for some portion of their work.

3.4 Data Collection

3.4.1 Kinds of Data Collected

Escher's World uses a qualitative model of research (Glesne and Peshkin 1992, Maxwell 1992, Weiss 1994). Qualitative research attempts to understand phenomena by gathering a rich set of data for a limited number of instances. The goal is to create, in Clifford Geertz's phrase, a "thick description" of events, such that the researcher can interpret events from the subjects' perspective (Geertz 1973).

The main source of data for the Escher's World workshops was full, structured pre- and post-interviews conducted with each of the workshop participants immediately before and after the workshop, as well as an additional "affect interview" with each student from two to five months after the completion of the workshop. The format of these interviews and their subsequent analysis was guided by 14 preliminary interviews conducted with students, mathematics and art teachers, and experts in the field of symmetry.

The interviews were supported by videotapes of the workshops and field notes from workshop leaders and other facilitators. All student sketches and designs from the workshops and all student journals were preserved for review and analysis. Students in the second workshop were also given a brief survey about their feelings towards mathematics, art, and computers immediately before and after the workshop.

3.4.2 Structure of Interviews

The structured pre- and post-interviews were divided into three components. The first component was a series of affect questions about mathematics and art, focusing particularly on attitudes towards these disciplines. The second section of each interview was a detailed discussion of four works of art from a set of 7 images (see appendix for images). The works of art were reproduced in standard size and format, and students were given three prompts: (1) How would you describe this to someone who had not seen it? (2) Would it be difficult to make something like this? (3) Do you like this piece? The final section of the interviews consisted of two to four mathematics problems from a set of 16 problems (see appendix for problems). Students were asked to solve the problems, and to describe their thought process as they worked.

Affect interviews (conducted two to five months after the workshops) were similar in structure to the first section of pre- and post-workshop interviews. In affect interviews, students described their attitudes towards mathematics and art.

3.4.3 Structure of Survey

The surveys given to students before and after the second workshop asked students to rate how strongly they agreed or disagreed with a series of statements about mathematics, art, and computers (see appendix for survey questions). Ratings ranged from 5 (agree strongly) to 1 (disagree strongly).

3.5 Data Analysis

Interviews from the workshop were transcribed and broken into excerpts, where each excerpt represented one complete answer to a question, including any follow-up questions or clarification by the student. This was done to preserve the coherence of students' thinking as reflected in their responses, and resulted in extended excerpts that

often were coded in multiple categories. This helped identify ways in which different themes in the students' experience of the workshop were related. Answers to questions about images and solutions to each of the mathematics problems were similarly broken into excerpts, again making every effort to preserve the integrity of student answers to individual questions or problems.

3.5.1 Coding of Interview Data

Each section of the interviews (general questions, image descriptions, and word problems) was coded separately. Excerpts were tagged with information about each student which was hidden during coding. In order to provide consistency across the interviews, excerpts were mixed randomly before coding, and coding within each section of the interview questions (general questions, image descriptions, and word problems) was done by the same person and checked for accuracy.

3.5.1.1 Codes for General Questions

General questions were first coded with key concepts used by students. Borrowing terms from linguistics and phonic analysis, the literature on qualitative research often refers to these concepts used explicitly by interview subjects as "emic" concepts (after the term "phonemic," which refers to meaningful units of sound for the users of a particular language). Following a procedure typical in qualitative students, every effort was made to develop higher-order, analytic categories based on these fundamental emic concepts. Emic concepts were aggregated into larger analytic categories based on fundamental theoretical concepts in the literature of learning theory. A more detailed description of the analytic categories used in the analysis of the Escher's World data is given in the Results section below (see section 5.1, page 30). The main analytic categories used were: Control, Expression, Interaction, Authenticity, Novelty, and Computer. Excerpts were also coded for expressions of affect towards mathematics, art, and the workshop, including students'

like or dislike of these subjects before and after the workshop, and any expression of changes in feelings towards the disciplines as a result of the workshop.

3.5.1.2 Codes for Image Descriptions

Design texts divide the basics of design education into "elements and principles" (Johnson 1995), where elements generally refer to particular physical portions of a design image and principles refer to formal or systematic relations between elements. Excerpts about images were assigned codes for elements of "form" and "color," and principles of "symmetry" and "composition." These categories were based on preliminary interviews with students, symmetry experts, and mathematics and art teachers, where novices used elements of form and color in their descriptions of images and experts used principles of symmetry and composition.

People tend to go through stages in their development of visual and aesthetic understanding across a range of topics (Parsons 1987). That is, there are stages in their understanding of color, or in their interpretation of forms or composition. Codes for the topics identified in preliminary interviews were subdivided into two stages for the purposes of this analysis: "general" comments and "analytical" comments.

For elements (form and color), general comments referred to excerpts that contained catalogs of shapes or colors ("a lot of circles and lines"). Analytical comments about elements were those that referred to specific relationships between elements of the picture ("the green is too dark for the bright blues and purples"), that distinguished between similar elements based on a specific criteria ("the sun is blue, and then [there is] another sun, smaller--it's red"), or that combined elements into larger descriptive units ("diamond shapes all together combined into like a star").

Similarly, general comments about principles referred to excerpts that contained informal descriptions of formal concepts such as symmetry and composition. For

instance, one student's comment that in an Escher print "the heads are all together everywhere you look... so it's like they're standing right beside each other in different [places]" was coded as "symmetry, general." Analytical comments about principles were those that used formal or mathematical descriptions of symmetry ("it's four time radial symmetric") or composition ("it doesn't really have a focus—or it has multiple focuses").

Thus, the coding matrix for images had 8 cells:

	Elements		Principals	
	Form	Color	Symmetry	Composition
General				
Analytical				

Many excerpts were coded in more than one category. The following description, for example, was coded for "form, analytical," "color, general," and "symmetry, analytical":

> I noticed these little blue lines coming out of these little red designs, and I realized that it was angular symmetry. It looks like... whoever made this could have just started out with a block that had on two sides the red and blue base, and then on the rest of it just made the yellow, and these blue dots and red dots and the little line right there, and then made part of the circle, or one fourth of those circles--and then... made four versions of it with different angles and... moved them together.

3.5.1.3 Codes for Word Problems

Word problems were coded for students' use of a visual representation during some portion of the problem-solving process (usually some form of sketch of the problem). Following Rieber, the term visual representation was used broadly to refer to "representations of information consisting of spatial, non-arbitrary (i.e., 'picture-like' qualities resembling actual objects or events), and continuous... characteristics," including both internal and external representations (Rieber 1995). Problems were also coded for correct or incorrect answers to the problem, where "correct" answers included answers

that fit the stated conditions of the problem even if a student's solution was not the "expected" answer.

3.5.2 Statistical Analysis

When coding was complete, frequencies were tallied for each code. Pre- and post-interview totals were compared overall, between workshops, and for individual students. Student responses for affect interviews were included in the analysis of students' attitudes. The statistical significance of observed changes was computed using a t-test with n=12. Results for the survey from the second workshop (with n=6) were similarly tabulated and analyzed.

The analysis of students' experiences during the workshop relied more heavily on the qualitative nature of the data collected. Frequencies were tallied for the codes related to students' experiences, and these were compared between categories using t-tests and correlations to look for relationships in the data. These results were used to inform more detailed qualitative comparisons based on excerpts that were coded in multiple categories.

4 QUANTITATIVE RESULTS: EVALUATION OF STUDENT LEARNING IN THE MATHEMATICS STUDIO

The results section that follows presents a summary of the "effects" of the workshop, showing that students developed their understanding of the mathematical concept of symmetry, and also that students' approach to mathematics changed as a result of the workshop.

4.1 Criteria for Understanding

Mathematics education reformers argue that change in mathematics education needs to affect not only the setting in which learning takes places, but also the goals toward which learning is directed. Traditional approaches to education do not place first priority on students understanding mathematical ideas (Brandt 1994, Perkins & Blythe 1994). Tests stress the coverage of the "content" of the curriculum, or limited skills acquisition. In contrast, theorists emphasize that "understanding" requires that students develop the ability to use ideas in appropriate contexts, to apply ideas to new situations, to explain ideas, and to extend ideas by finding new examples (Gardner 1991, Gardner 1993, Sierpinska 1994). The remainder of this chapter of the thesis describe how results from the Escher's World mathematics studio workshops suggest that students do develop this kind of understanding of mathematics through studio activities.

4.2 Students Learn about Symmetry

4.2.1 Use of Symmetry in Designs

Students were able to use the concept of symmetry to create original designs. During the workshops all of the students (12/12) were able to make designs using mirror

symmetry, and 83% of the students (10/12) were able to make designs using rotational symmetry.

4.2.2 Application of Symmetry to Analysis of Images

Students developed their ability to apply the concept of symmetry to the analysis of images. Before the workshop, students made analytical references to symmetry an average of 0.5 times while looking at 4 images in structured interviews. After the workshop, mean analytical references to symmetry rose to 4.3 references over 4 images (see Figure 3; mean change +3.8, $p<0.01$).
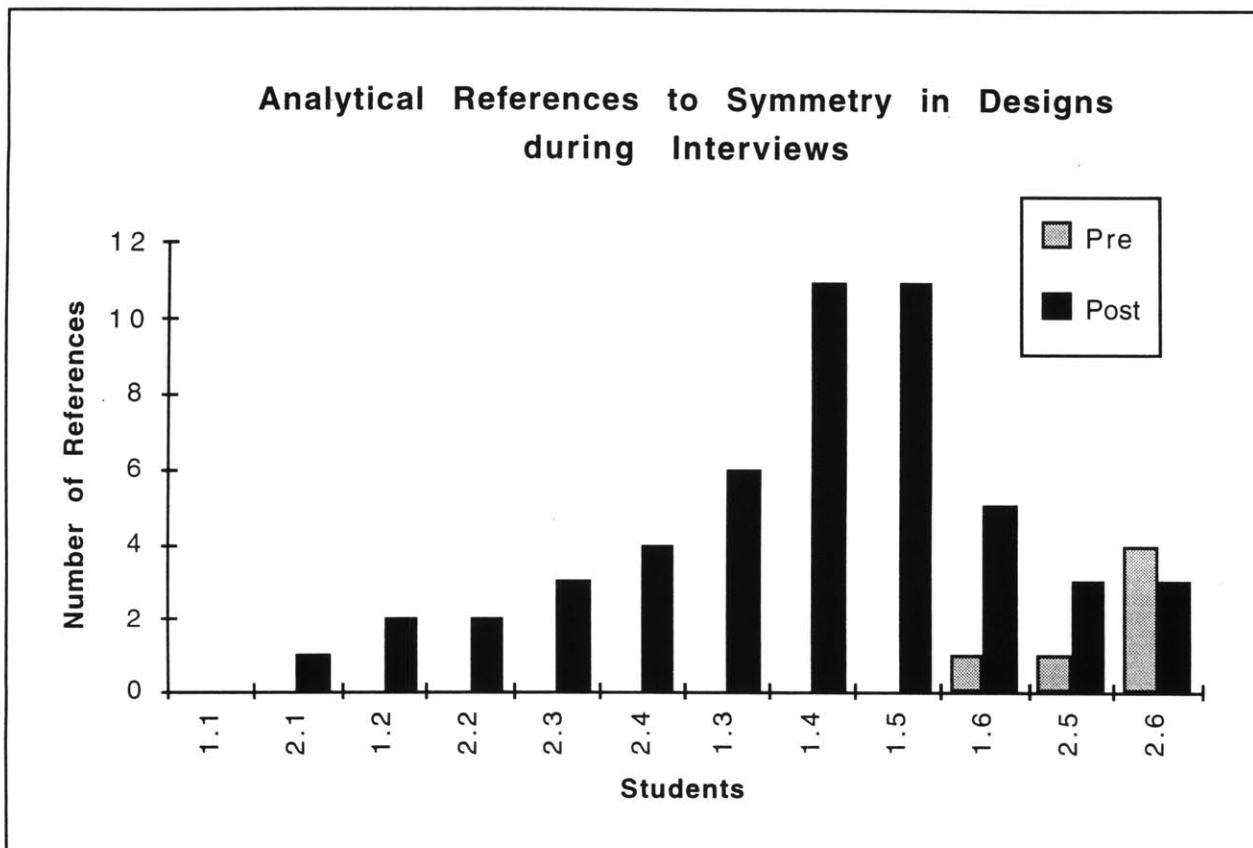

Figure 3: Students learned to use symmetry to analyze images. Students 1.1–1.6 attended the first workshop; students 2.1–2.6 attended the second workshop. Students have been ordered for clarity of presentation.

The change in the total number of analytical comments between pre- and post-interviews was not significant (from 11.1 comments per student before the workshop to 12.6 comments per student after the workshop, mean change +1.4 references; p=0.36),

Analytical References in PRE interview (12 students)

Percent of references in category

- 100%
- 80%
- 60%
- 40%
- 20%
- 0%

4% Symmetry
75% Forms
19% Color
1% Composition

Category

Analytical References in POST interview (12 students)

Percent of references in category

- 100%
- 80%
- 60%
- 40%
- 20%
- 0%

34% Symmetry
46% Forms
10% Color
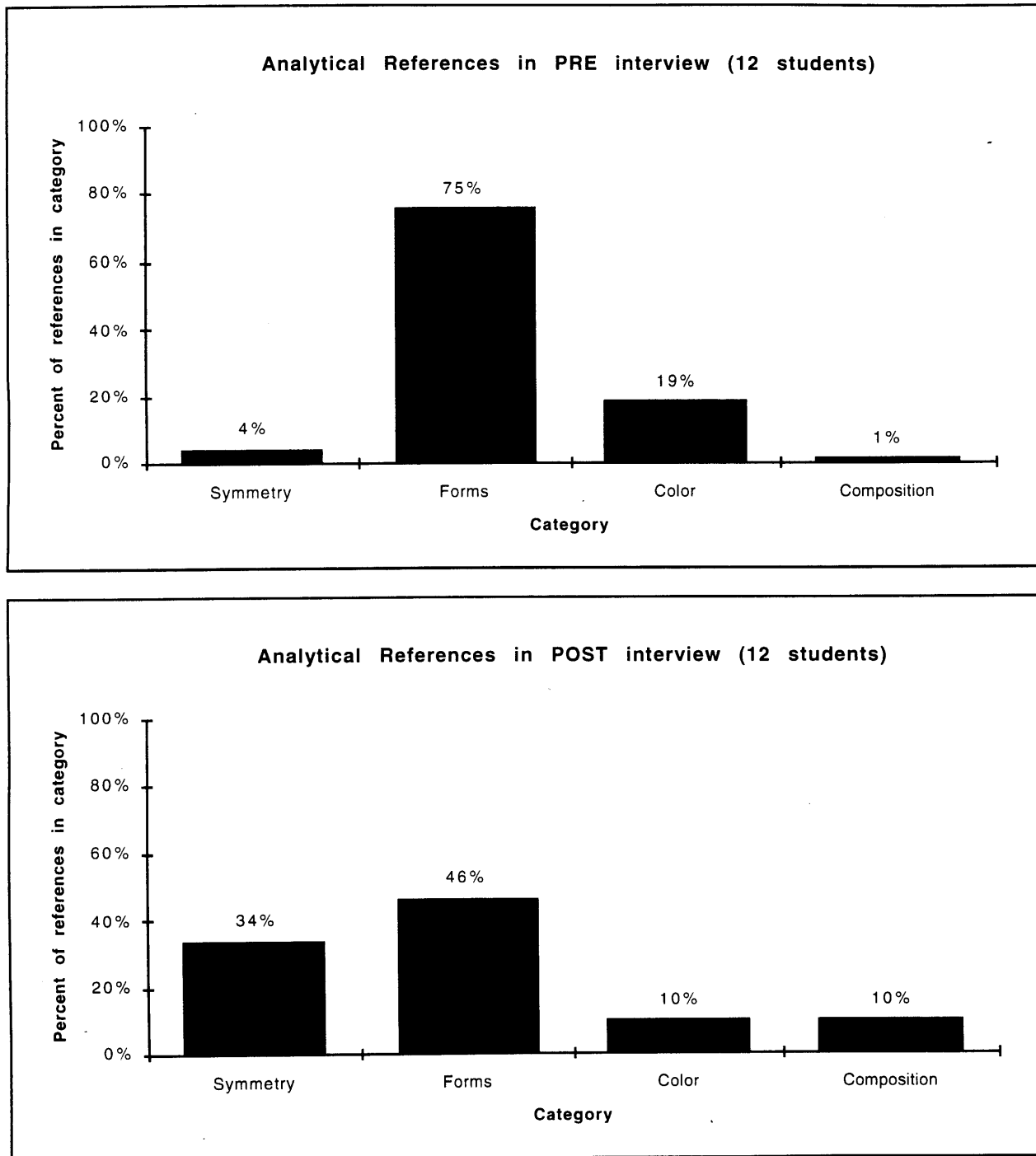10% Composition

Category

Figure 4: Rise in analytical references to symmetry was related to a drop in analytical references to forms. Graph shows aggregate data for 12 students in 2 workshops. Change in symmetry references is +30%; change in references to forms is –29%.

which suggests that students were not becoming more analytical overall; rather, as they began to use the concept of symmetry as a tool for analysis, students replaced some other form of analysis. Change in analytical references to color (see Figure 4, page 23; mean change -0.8 references; $p=0.16$) were not statistically significant. Analytical references to composition were too small in pre-interviews (1% overall) to account for the rise in analytical references to symmetry. This suggests that students replaced an analysis in terms of the elements of form with a more mathematical analysis in terms of the principal of symmetry. This hypothesis is supported by the change in percentage of analytical comments made about forms and change in percentage of analytical comments made about symmetry, which have a coefficient of correlation of 0.76 across all students.

It should be noted here that students also showed an increase in analytical references to composition, which will be discussed in a later paper about arts learning in Escher's World.

4.2.3 Use and Explanation of the Concept of Symmetry

The number of students who could use and explain formal concepts of symmetry rose dramatically over the course of the workshop (see Figure 5, page 25). Some student explanations of symmetry were fairly general descriptions even after completing the workshop, but others are quite specific. For example, one student said: "If you drew a line down the middle—the line of symmetry—the two halves would be identical. They would be exactly the same: mirror images of each other."
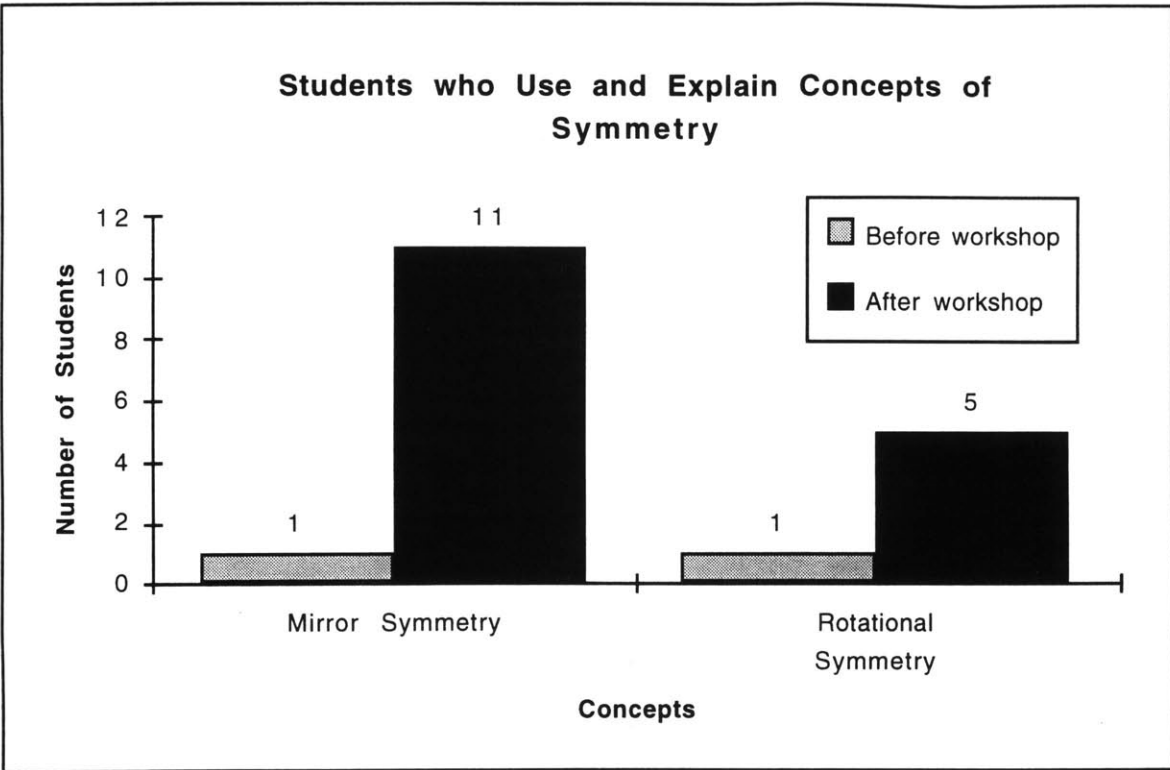
Students who Use and Explain Concepts of Symmetry

Figure 5: Students learned to use and explain symmetry through design. Graph shows data for 12 students in 2 workshops

4.2.4 Finding New Examples of Symmetry

After the workshop, students started to see symmetry in the world around them: 75% of the students (9/12) reported thinking about symmetry beyond the context of the workshop in post interviews or follow-up interviews. Students reported seeing symmetry in drawings, chairs, wallpaper, rugs, video games, flowers, and clothing.

4.3 Students Learn to Solve Mathematics Problems Visually

4.3.1 Use of Visual Representations Shows Mathematical Understanding

The workshop did not deal with mathematics word problems, or explicitly with the use of visual representation as a tool for solving traditional mathematics problems. After the workshop, however, students used visual representations as a successful

Use of visual representation during problem solving before workshop

Percent of Problems where sketches were used

100% ⊤

50% ⊣

0% ⊥

Students: 1.2, 1.4, 1.6, 2.5, 1.3, 1.1, 2.1, 1.5, 2.3, 2.4, 2.2, 2.6

Use of visual representation during problem solving after workshop

Percent of Problems where sketches were used

100% ⊤

50% ⊣

0% ⊥

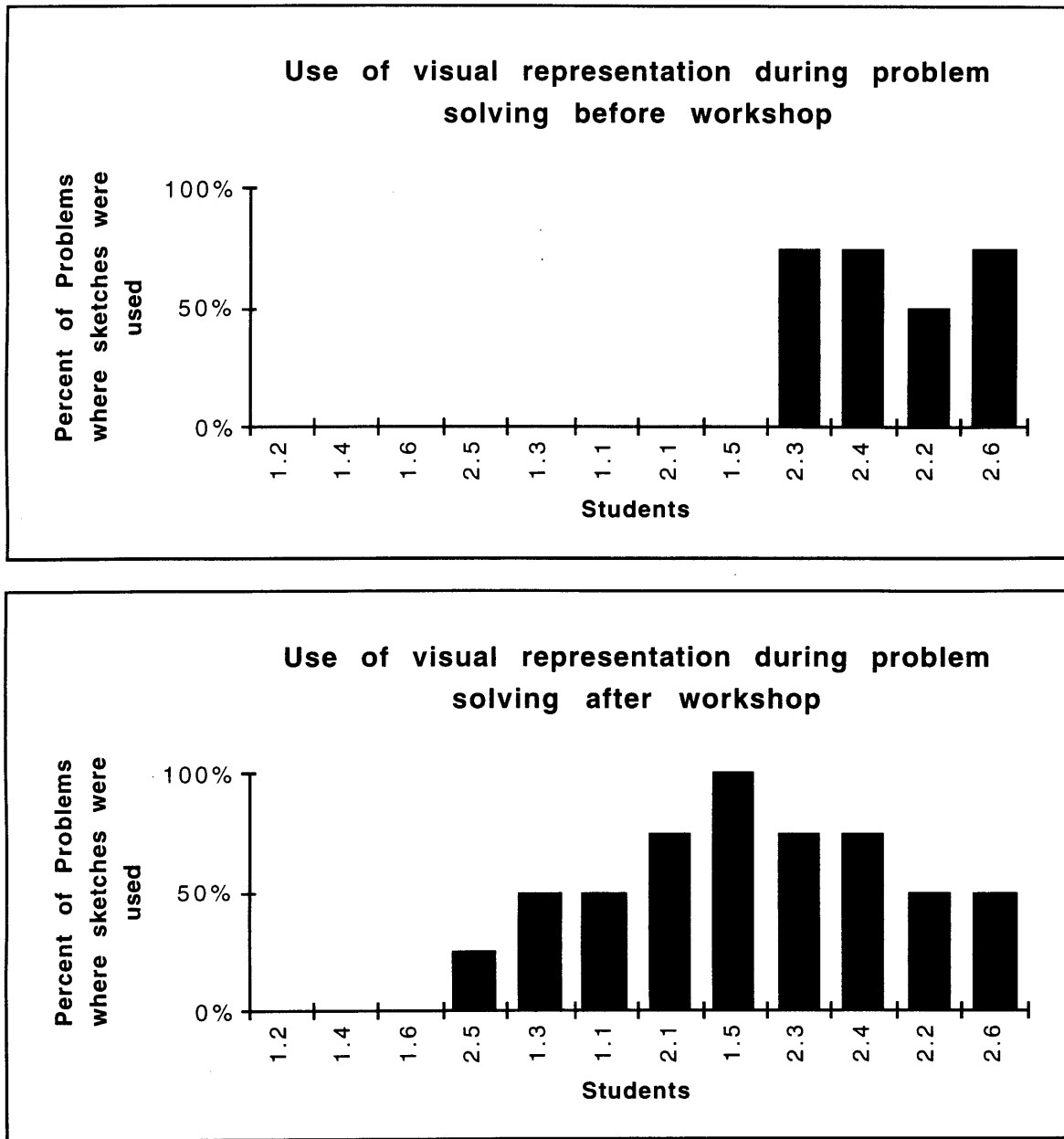Students: 1.2, 1.4, 1.6, 2.5, 1.3, 1.1, 2.1, 1.5, 2.3, 2.4, 2.2, 2.6

Figure 6: Students learned to use visual representations during problem solving. Students 1.1–1.6 attended the first workshop; students 2.1–2.6 attended the second workshop. Students have been ordered for clarity of presentation.

problem solving strategy. Only 33% of the students (8/12) used visual representations to solve word problems before the workshop, while 75% of students did after the workshop (see Figure 6, page 26; $p < 0.06$).

For example, in Figure 7 below, before the workshop the student did not use a visual representation to solve the problem: "One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 3 miles due east. After Julie turns right again and walks for another 2 miles, she decides to go home. How far does she have to go to get back to her home?" After the workshop, the same student working on a similar problem used a visual representation of the problem situation.
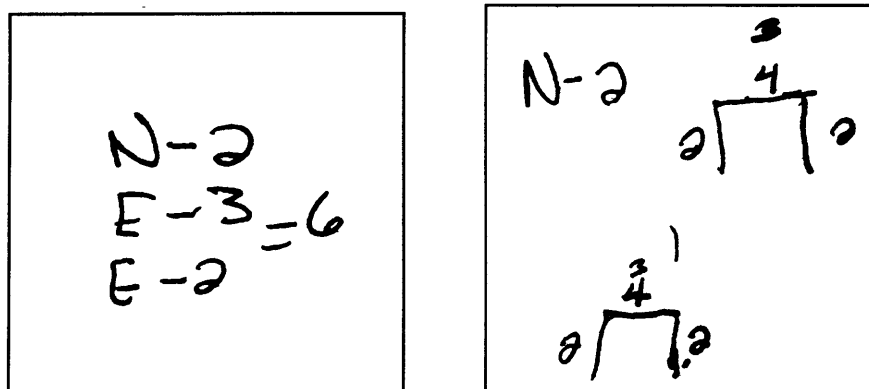

Figure 7: One student's notes while solving a problem during interviews. In the pre-interview (left image), the student did not use a visual representation. While solving a similar problem during her post-interview (right image), the student represented the problem visually and produced a correct solution

4.3.2 Visual Representations as a Successful Problem Solving Strategy

Use of visual representations for word problems after the workshop was correlated with success in problem solving during post interview problems ($r=0.83$). Some students solved problems without using visual representations, and some students used representations but failed to solve problems; however, no student solved more problems overall than the total number of problems they attempted using visual representations.

This is reflected in the absence of data points in the upper portion of the scatter plot

(above the diagonal line) in Figure 8 below.

Visual representations and success in solving word problems
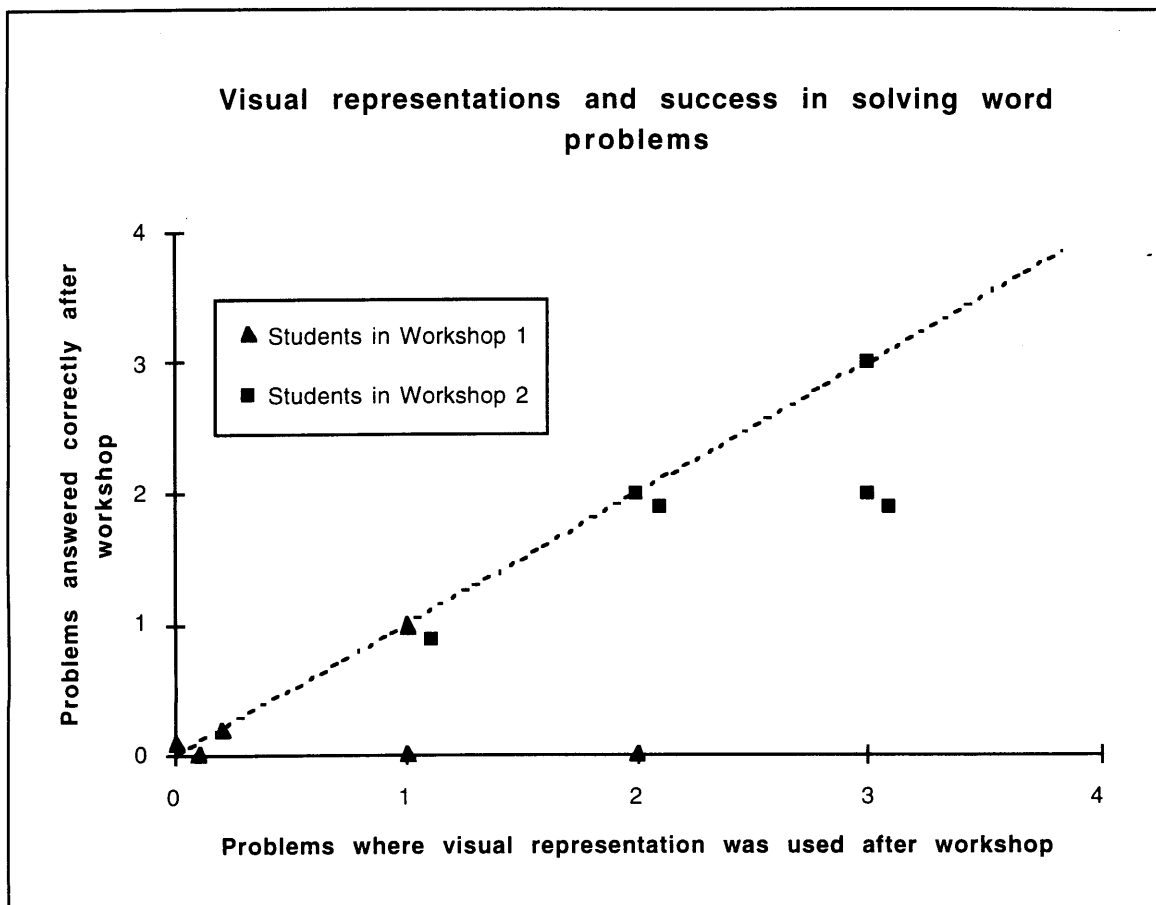
Figure 8: Visual representations helped students solve problems (r=0.83)

4.4 Students Like Mathematics More

In post-interviews and follow-up interviews, 67% of students overall (8/12)

reported feeling more positive about mathematics as a result of the workshop. This

reported change was supported by survey data (surveys were collected for only 6 of the

12 participants, whereas all of the participants were interviewed). In the survey, students

responded to 4 prompts about mathematics:

"I like math class/I don't like math class."
"I like doing math problems/I don't like doing math problems."
"I like thinking about math/I don't like thinking about math."
"I understand math/I don't understand math."

Students marked a scale from 5 (most positive) to 1 (least positive). As shown in

Figure 9, the total rating for the 4 mathematics questions went up from before the

workshop for 67% of students surveyed (4/6). No student's total went down. Change for

"I like math class/I don't like math class" (mean +0.67; $p<0.01$) was particularly striking.
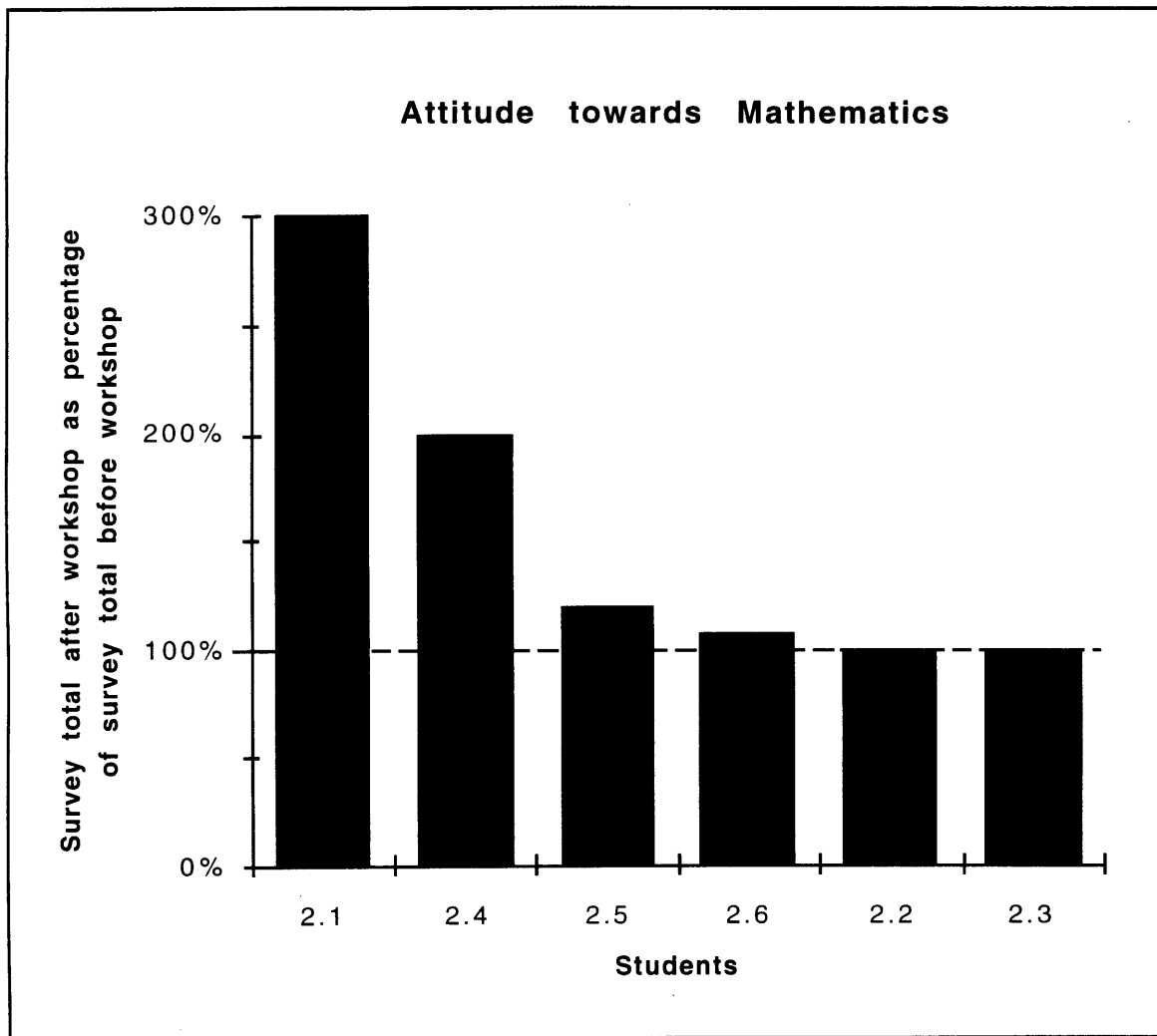
Attitude towards Mathematics

Figure 9: Students felt more positive about mathematics after the workshop. Graph shows data from survey conducted for the second workshop only. Students have been ordered for clarity of presentation.

5 QUALITATIVE RESULTS: HOW LEARNING HAPPENS IN THE MATHEMATICS STUDIO

Students learned about the mathematical idea of symmetry in the Escher's World workshops, and learned to apply visual thinking skills to mathematical problem solving. At the same time they discovered they liked mathematics and liked this new kind of learning environment. One student said simply: "If school was like this, attendance would be perfect!" Certainly something good must have been going on if students were willing to give up time on their spring break or summer vacation to work for four to six hours at a time learning math with a kind of focus rarely seen in school class rooms. The question is: What made this learning environment "work" for students?

In attempting to answer this question, the remainder of the results section describes in some detail students' experiences during the workshop based on their comments in interviews. In particular, the analysis that follows focuses on students' experience of "control" over their learning and learning environment, and on the relationship between control and the expressive nature of the activities in Escher's World. In order to understand the role that Control and Expression play in students' experience of the workshop, however, we must also look at four separate but related concepts: Interaction, Authenticity, Novelty, and role of the Computer in the Escher's World workshops.

This section of the results begins with a brief overview of the theoretical ideas that frame the analysis. This is followed by a description of data from the project showing the way these theories interact in the mathematics studio environment.

5.1 Terms and Conditions

As described above in Codes for General Questions (section 3.5.1.1, page 17), students' comments about the Escher's World workshops were coded in six major analytic

categories: Control, Expression, Interaction, Authenticity, Novelty, and Computer. Of these six categories, Novelty and Computer need little explanation. Both are more or less direct applications of the students' own (emic) concepts. So comments about computers (hardware, peripherals, or software) were coded accordingly.[3] Students did not use the word "novelty" in any of the interviews, but they often talked about things being or seeming "new" or "different," and these comments were coded as comments about Novelty.

The other four categories (Control, Expression, Interaction, and Authenticity) all have a history in the literature of education theory. There is not space here to describe their etymologies in full, but the sections below outline the context in which these ideas apply to the Escher's World mathematics studio.

5.1.1 Control

The educational importance of learners having control over their own learning is certainly not new. Dewey wrote in great detail about the role of freedom and social control in students' development. In particular, Dewey suggested that "freedom" is a necessary (though not sufficient) condition for the development of self-control. By "freedom" Dewey meant not only the physical freedom to move in space, but also the more important freedom to make decisions, to "frame purposes" and to exercise judgment (Dewey 1938). Many theorists similarly emphasize the extent to which learners must control their learning experiences (Sizer 1984, Papert 1991b, Gardner 1993, Prawat 1995).

For purposes of this analysis, excerpts were coded for Control when students referred to freedom of physical as well as intellectual movement, when they talked about making their own choices, judgments, or decisions--in short, when they described in a

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[3] Students used tools other than computers during the workshop, but student did not particularly talk about other kids of tools during interviews.

positive or negative way the effects of their own control (or lack of it) in their learning experience.

5.1.2 Expression

In his "Talks on Pedagogics," Francis Parker first articulated the idea that "expression" is a process fundamental across many disciplines of thought (Parker 1894). Rather than emphasizing individual modes of expression such as singing, writing, dance, or painting, Parker suggested that all of the means of expression were essential vehicles through which ideas in any domain are learned. In Parker's vision, the arts--as means of expression--were integrated throughout the curriculum, much as writing is used in many different disciplines today. In the years since Parker's work, art--and with it expression--has been moved from the center of American education (except in the early primary grades) despite arguments by theorists such as Dewey, Rudolph Arnheim, and Elliot Eisner that art-making is an essential learning experience (Dewey 1915, Arnheim 1969, Eisner 1987). Where art survives, it is as a separate course of study, and the notion that expression is an essential component of disciplines such as mathematics, history, or the sciences has been all but lost.

The key component of Parker's idea of expression is that it represents "the manifestation of thought and emotion" (Sidelnick 1995). That is, expression is the process of taking some part of ones internal being (an idea, a feeling, an impression) and representing it externally. In the analysis of the Escher's World project data, excerpts were coded for Expression when students referred to ways in which they were able to make something that reflected their own ideas or preferences--or to times and places when they are prevented from doing so.

5.1.3 Interaction

As with control and expression, the importance of interpersonal interactions has a prominent place in learning theory. One of the major themes of Lev Vygotsky's work is the idea that social interaction is a critical component in cognitive development. In particular, Vygotsky argued that the immediate potential for cognitive development (what he called the zone of proximal development) could only be fully realized with adequate adult guidance or peer collaboration (Vygotsky 1978). Other theorists similarly argue that an essential part of learning to think is learning to think with others (Bruner 1986, Bruner 1990, Pea 1993).

For purposes of this analysis, excerpts were coded for Interaction when students referred to ways in which their learning experience was affected by the active participation of others (or lack thereof). This includes descriptions of help given to or received from adults or peers, collaborative work, public presentations of ideas or work, conversations or other "purely social" interactions--in short, Interaction refers to the range of students' relations to others people as it connects to their learning experiences.

5.1.4 Authenticity

Like the other concepts described above, the idea of authenticity in learning experiences is not new. Dewey wrote of the need to act "in behalf of purposes that are intrinsically worth while," and as early as 1866, William Ware of the Massachusetts Institute of Technology was writing about the value of having students work on practical projects drawn from genuine problems in the field of study (Ware 1866). The idea of authentic assessment has received particular attention in the past decade or so, forming the core of proposals for reform such as the Coalition of Essential Schools (Sizer 1992), the Teaching for Understanding Project (Perkins and Blythe 1994), and the Connected Mathematics Framework (Wilensky 1995).

Authenticity is used in two very different ways in the literature of education reform. On one hand, it is used to refer to the alignment of curriculum and assessment-- that is, "authenticity" means that students are given a meaningful and "valid" assessment of the extent to which they have developed the skills and understanding required from a particular course of study. The more compelling use of the term--and the one that is used in the context of this analysis--refers to the extent to which goals and expectations for learning are aligned with conditions in the world beyond the immediate learning environment. In this sense of the term, a learning activity is "authentic" if it reflects in some deep way an important part of the wider world in which students operate or will operate in the future. Authentic activities, for example, could ask students to develop their mathematical skills by doing the same things (and using mathematics in the same way) that professional scientists, mathematicians, artists, policy-makers, reporters, or others do.

For purposes of this analysis, excerpts were coded for Authenticity when students referred to ways in which their learning experience reflected things that they understood to be a legitimate part of the world outside of school and the Escher's World workshop. This included, for example, descriptions of the way that their designs were like the work of "real" artists, judgments (especially public judgments) about the quality of work by themselves and others, and discussions of the way outside interests were reflected in their work and their thinking.

5.2 A Framework for Thinking about the Mathematics Studio

All of which brings us to the heart of the matter: namely, trying to understand the particular combination of control, expression, interaction, authenticity, novelty, and use of computers that made the Escher's World a powerful learning environment for the twelve students who participated in the workshops. In a sense, these six ideas outline the

space of students' experiences in the workshop, and the question is how to begin to understand this space (see Figure 10).
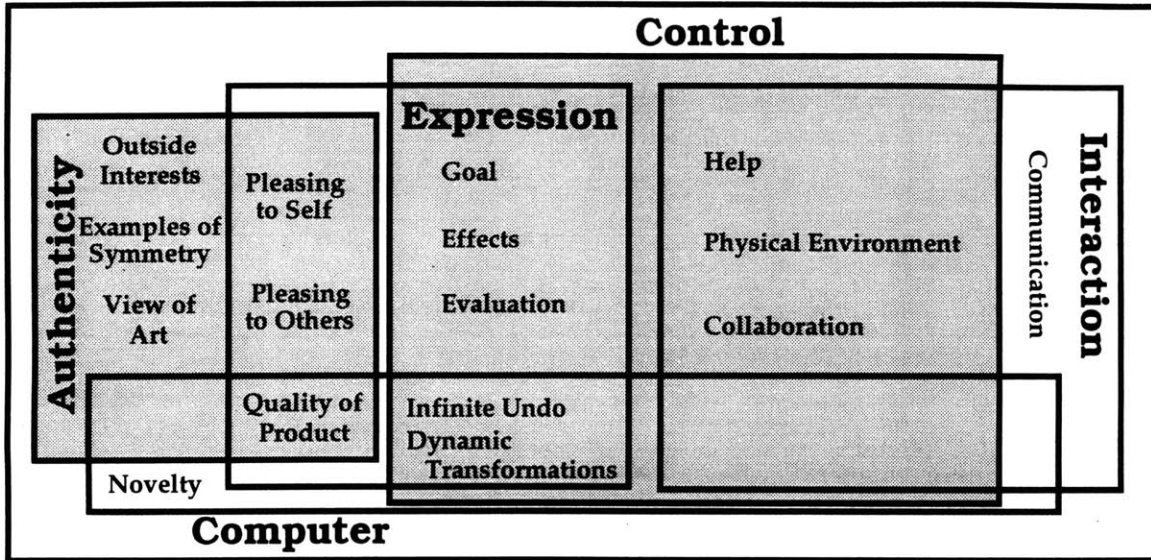

Figure 10: A schematic diagram of the space of student experiences in Escher's World

5.2.1 Novelty

From a practical point of view, the "novelty" of the Escher's World workshop and its effect on students is perhaps the most problematic. Even without looking at the data, one would assume that some portion of students' positive response to Escher's World can be attributed to the fact that the workshop was a very different experience from learning in school--if only because the workshop took place in a very different space, for only a few days, using computers, scanners, and large-format plotters, with new adults, and with video cameras, interviews, and all of the other trappings of research. Escher's World would surely be at least somewhat less exciting if it were a regular part of every students' education.

Looking at students' comments, one important part of the novelty of Escher's World was clearly the technology, for which students mostly use the generic term

"computer." There is a clear relationship between students' use of the computers and the prevalence of their comments about the novelty of the workshop. Students who used computers extensively during the workshop made an average of 0.8 comments about novelty, compared to an average of 0.4 comments for students who did not use computers extensively ($p < 0.01$), and the number of students' comments about computers overall is correlated with the number of comments about novelty with $r = 0.55$. Students made comments like: "I knew you could see pictures in the computer but I never seen it before." And: "I really was interested in how the computer can transfer information [from a scanner]--which I'd never seen before--and get your picture how big you wanted it, what size you wanted, everything."

While there was clearly a significant "novelty" effect in students' reactions to the workshop--especially related to computers--it is also clear that there were more "deep structure" effects of the workshop environment. For example, only a third of the comments about computers (35% or 6/17) referred to their novelty, and only three comments about computers referred to their novelty alone.

5.2.2 Control and Interaction

A look at the relative frequency of student comments in the various categories makes it clear that Control, and with it Interaction, were the most important aspects of the Escher's World mathematics studio for these students (see Figure 11, page 37). Students' comments in these areas show significant overlaps, with 36 joint references to control and interaction--over half of the total comments about control and almost 75% of the comments about interaction. Student comments about interaction are correlated with comments about control with $r = 0.79$.

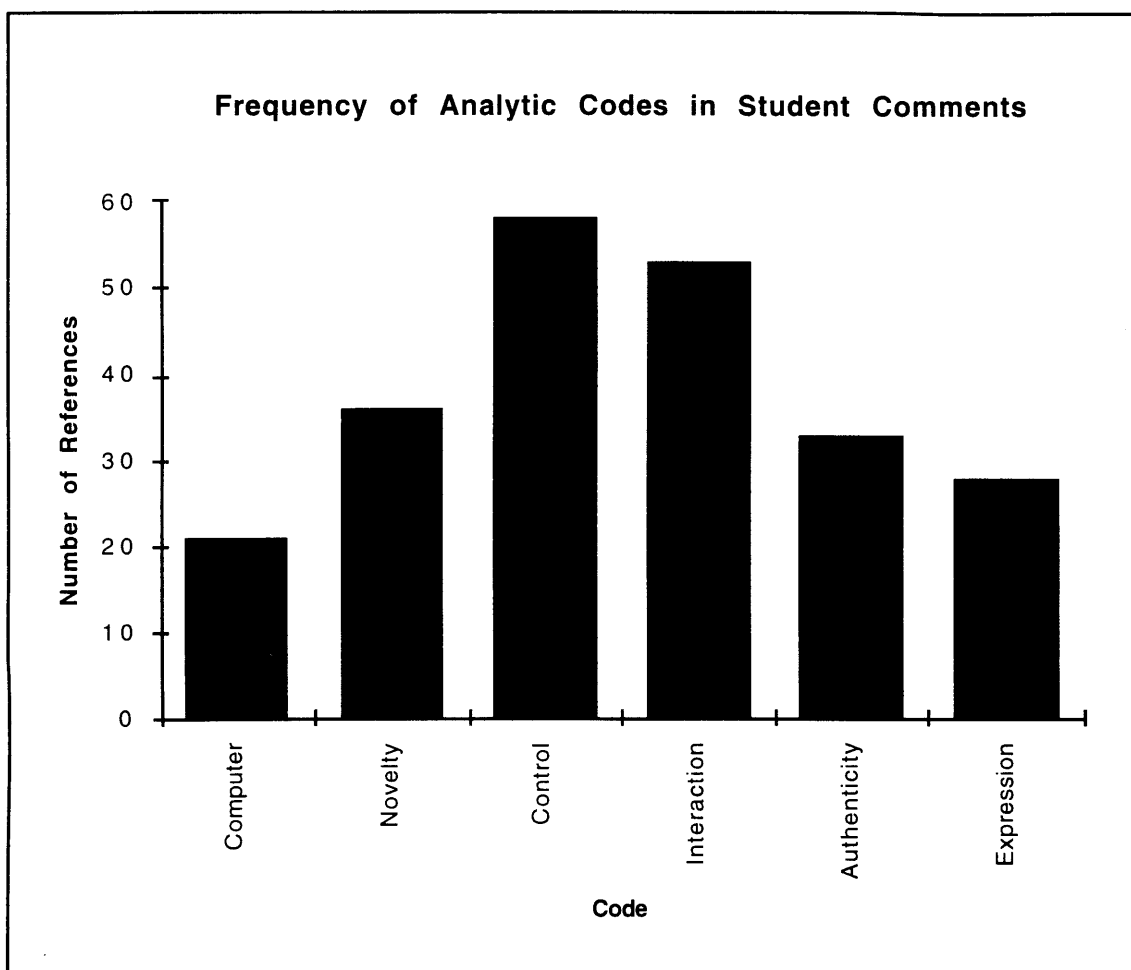**Frequency of Analytic Codes in Student Comments**

Figure 11: Frequencies of student comments, by analytic category.

There were several ways in which students talked about feeling as if they were in control of their interactions during the workshops. Students talked about their ability to decide for themselves when to work alone, when to work with a peer, and when to consult with an adult. One student said simply: "[In the workshop] if I don't know something, I just ask you or other friends to sit by me. In class [at school] you can't talk." Similar sentiments were echoed in two-thirds of the comments where students talked about both control and interaction. In almost all of the comments about working with peers (16/19) and about getting help from adults (16/18) students talked about the fact that in the workshop they were in control of how and when these interactions took place.

The other major way in which students thought about control and interaction with others was in the quality of the physical environment. Several students were impressed by the extent to which adults in the workshop responded to their requests about type and quantity of food. Others commented on the fact that they could take breaks and go to the bathroom *without* having to interact with an adult first.

Of course, students also talked about control and interaction as independent categories. Most of the comments about interaction that did not have to do with control focused on the idea of communication--often referring to public discussions or conversations about students work or about ideas in the workshop. Interestingly, the comments about communication overall were evenly split in terms of control: 11 of 22 comments about communication did not refer to control while 12 of 22 did. This is not surprising, as part of the workshop asked students to present and explain their work on projects to the group, which was a process that not all of them looked forward to.

The comments about control that did not have to do with interaction were of two kinds. A small but meaningful selection of comments were about how students felt that they were more in control of their own learning or looked at mathematics, art, or computers differently as a result of the workshop. But the majority of comments about control that did not deal with interaction were about expression.

5.2.3 Expression

Overall, student comments about expression were clearly connected to the issue of control. The frequency with which students refer to expression shows a clear correlation to their comments about control ($r=0.94$)! And a full two thirds (67% or 19/28) of the comments about expression reflect the fact that the expressive arts-based activities of the workshop put students in control of their own learning.

This control-through-expression was manifested in three different ways: control over goal, control over effects, and control over evaluation. Students were able to adjust, for example, the level of difficulty of the projects they took on (control over goal). "You gave us an idea," one student said, "but we basically made up [the specific problem]. So it could be as hard as you want it to be or as simple as you want it to be." They were able to, as one student put it, "decide what we wanted [and] make it any which way we wanted."

Students also had a sense of control over the emerging products of their work (control over effects). As one student said: "I can change where people's eyes go, and where the focus is, and even change how they look at it.... Before I would draw something, and if I didn't like it, I would just forget it and try something totally new. Whereas now, I know if I change something minor and change it drastically then it can become a whole new picture, but still very similar."

And students felt that the decisions about when to stop working and about what constituted a finished product were in their hands (control over evaluation): "I got to decide how I wanted it to go," said one student. "If I didn't like the way I was going I could stop and change it around so then I'll start over.... You said, 'I want something with symmetry with angular symmetry.' And that still leaves a lot of different ways you can go." Another student explained: "When we had finished putting it altogether we wanted to do it differently. We weren't sure how [to do it differently, but], we could do it differently if we wanted to."

The cumulative effects of this sense of control were obvious when students tried to make images that had rotational symmetry but were not balanced in their overall composition. Students were able to produce designs that fit the constraints of the problem after a short time, but they continued to work despite growing frustration with the difficulty of the problem. They were looking for a solution that *they* liked. Student's sense

of control over the structure of the activities of the workshop was clearly a key element of the workshop. As one student put it: "When you do it for yourself, you can take any time, or you could do it any way you want to, because it's yourself, your own stuff." It may also be worth noting that the frequency of student comments about expression shows some correlation to students performance on word problems ($r=0.56$ for the correlation of frequency of comments about expression and percentage of correct answers on post-interview word problems).

Of course not all of the comments about expression related to students sense of control over their activities. Of the eleven comments about expression that did not reflect how expressive activities gave students control over their learning, all but two were about how expressive activities provided authenticity in the learning environment.

5.2.4 Authenticity

If, as described above, expression, control, and interaction are all closely linked in students' experience of the workshop, then authenticity clearly stands somewhat apart from that nexus. Only 40% of student comments about authenticity (13/23) also refer to some combination of expression, control, or interaction. Furthermore, most of these comments are about authenticity in the somewhat limited sense that a student's work is regarded as pleasing to himself or herself or (in 4 cases) pleasing to someone else. The majority of the comments where authenticity overlaps with expression, control, or interaction are essentially about students validating their own work as interesting beyond the workshop setting, as in: "My mother, she liked it a lot." In some ways, students' feelings of the authenticity of their expressive work is more than anything else a manifestation of their control over the activity, although, of course, the idea of self-validation is clearly an important concept in its own right.

The remainder of the comments about authenticity fall into three fairly distinct and evenly distributed categories. In their comments, students related the workshop to their own outside interests, such as video games, summer jobs, television, and so on. Students described finding symmetry in world around them, both in art and in everyday objects. And students described changed views of art in general, as in this student's explanation of how she thinks about art in relation to the workshop:

> If I draw pictures I'll think about M.C. Escher [and] how his pictures sort of play with your mind.... You thought before, but you didn't think in depth.... You just thought: 'Well, it's art. Look at it, and see.' But I guess [the workshop] made you think about why the artist put that there--because you were able to like put [things] wherever you wanted to. You understood why you put there, and you want other people to understand why you put it there. And I guess that's what you learned about the artist. Like, they put that there, and they want you to understand why. Or they might not want you to understand why.... They put it there to confuse you, to make you think.

The fact that the activities of the workshop reflected parts of the larger world for students--things they or others like and see as valid and important--played a role in the workshop. But authenticity was also to some extent separate from the close connections students made between control, expression and their interpersonal interactions in their experience of the workshop.

5.3 The Role of Computers in Escher's World

Computer is the final analytic category that remains to be discussed. I have left it until last because in many ways, the role of computers in students' experiences of the workshop reflects and reinforces the patterns described in the other analytic categories. Computers certainly played a part in creating an atmosphere that seemed very different from traditional school classes for these students. But it is also clear from interviews that computers contributed to students' sense of control and to their feelings of authenticity about the workshop.

Not surprisingly, students' experiences of computers in the Escher's World workshops reflected to some extent the particular software they were using. As described above in the Methods section, most students who used the computers to any great extent during the workshop used the Geometer's Sketchpad program. The Geometer's Sketchpad allows students to create basic geometric figures such as circles, lines, and arbitrary polygons, and change their size, orientation, and color. More important, students can define mathematical relationships between theses objects: ratios, angles, and geometric transformations. So, for example, a student could create a line and a polygon, and then create the reflection of the polygon in the line.

When objects are moved on the screen in the Geometer's Sketchpad, mathematical relationships are preserved. The display is updated in real time as students "drag" points, lines, and figures on the screen. In this way, students can explore the effects of various mathematical constraints and relations quickly and easily, looking for solutions to mathematical problems that have aesthetic appeal. Sketchpad also preserves a record of all of a students actions during a given session with the program.[4] This provides students with the ability to "undo" their actions, letting them step back to and through previous states in their exploration rapidly.

Several of these features of the program helped students in thinking about and learning about symmetry during the workshop. Students commented explicitly about the ease with which they could play with designs on the computer. Students also described the program's ability to hold an image constant, to let them make very precise changes, and to let them explore the consequences of those changes. In other words, the computer helped by letting students control dynamic and static explorations:

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[4] For technical reasons, the program preserves this history only from the last time a particular working sketch was last saved.

The computer just made everything easy. You didn't have to hold everything right--[the computer] just it did it for you, so... you could concentrate on actually what you were seeing instead of just [thinking:] 'Well, I think I saw that, let me try that again and see if I see the same thing.'

You drew that dog, and then when you got the mirror on the screen you [moved] it around so that you could get a duplicate of it.... When we did it on the computer... I could actually move the mirror around the screen, move it in closer, and make like one picture out of the two, and move further apart.

The infinite undo feature of the Geometer's Sketchpad also gave students a sense of control over their exploration. As one student said: "The computers helped because it was like easier [than working] on paper [if] you'd have to erase it, or start again. You could just undo it, and then try something differently. That was easier because it was much quicker." Overall six of the seven students who worked extensively with computers commented about one or more of these ways in which the computer increased their ability to control their explorations of the mathematical problems of design.

There were also a small number of comments (three overall) about the effects of the computer on the overall look of students' images. One important aspect of the workshop for students was that they thought their designs looked good, and students clearly felt that the computers helped make their work seem more like "real art" in that respect. One student said that doing the workshop without the computers "probably wouldn't have been that much fun because... the outcome wouldn't have been too nice. You wouldn't have liked it too much."

5.4 Differences among students in Escher's World

One of the advantages of collecting qualitative data is that from interviews and close observations of students at work is that one develops a picture of the students as individual learners, rather than as faceless "subjects" or identical elementary particles in a high-energy physics experiment. So for the Escher's World researchers, "student 1.3" is a

sophomore in high school in a low mathematics track. But she says that she loves mathematics because she is good at it and because she knows it will help her become a doctor some day. For student 1.3, the workshop was eye-opening--something she and her friends from school who were also in the workshop think about often. They joke about symmetry in the T-shirts they wear to school. Student 1.3 was one of the students whose descriptions of images changed the most as a result of the workshop. Before the workshop she thought the pictures in the interview were "weird," and after the pre-interview it wasn't clear that she wanted to come to a workshop that was about strange things like that! After the workshop, she laughed at the same images, happily talking about symmetry, delighted to have a way to "understand" what had seemed so strange only days before. Student 1.3 was also one of the students who began to use visual representations in her problem solving as a result of the workshop, finding the interview problems less frustrating and more fun as a result.

"Student 2.6," on the other hand, is a junior in high school who likes mathematics and has always been confident in his mathematical abilities. For student 2.6, the workshop felt like it was more about art than about mathematics. He used concepts of symmetry to describe interview images and used visual representations in his problem solving both before and after the workshop, and felt like the workshop was interesting, but mostly review. "I do think it's a good workshop and it could help," he said at one point, "just not me." At the same time, student 2.6 did get some things out of the workshop. He talked in post-interviews about seeing symmetry in his work at the Computer Museum, and about how thinking visually helps him in his advanced physics course. He was also clearly affected by his explorations of the compositional concept of balance, using the idea to talk about all kinds of images and situations in the world around him, including the solution to mathematics problems.

Still, for all this sense of workshop participants as distinct individuals, 12 hours of workshop and three interviews are not enough to determine with certainty why some students responded with such enthusiasm while others were less forthcoming in their praise of the workshop. There were, however, two patterns among the students worth noting as directions for further research. One is that overall female students seemed to respond more enthusiastically to the workshop than males. Females used symmetry to analyze interview images substantially more in post interviews than males (females added an average of 5.4 references to symmetry between pre and post interviews, males added an average of 1.4 references, $p < 0.01$). In post interviews females used visual representations more often than males (females used visual references in 57% of problems, males used visual representations in 30% of problems, $p < 0.01$). Female students also commented more about the novelty of the workshop, and more about authenticity in the workshop.

The other interesting grouping within the Escher's World students were the students who were foreign born. These students were more likely than other students to say that they liked mathematics, even though they did not do as well on traditional word problems as other students. On the other hand, these students used symmetry to analyze images more than other students in post-interviews. Interestingly, although all of the foreign born students were in the first workshop, these same statistical differences are not significant between workshops overall. Previous research by McKnight and others (McKnight et al. 1987) found that foreign students did better than students in the United States on traditional tests of mathematics achievement, but that students in the United States had more positive attitudes toward mathematics. The experiences of the foreign born students in the Escher's World project suggests an additional level of complexity in applying the results of prior research work to foreign born students living in the United States.

Unfortunately (though not surprisingly), the reasons for the differences among students described above are not clear from the data collected thus far in the project. But these differences (particularly the gender differences) suggest areas for further research.

6 DISCUSSION

This section of the thesis is divided into two parts. The first part is a discussion of the qualitative outcomes of the Escher's World project; this part looks at student learning in Escher's World in the context of other experiments in studio mathematics. The second part is a discussion of the qualitative data from the project; this part describes some of the implications for research in learning suggested by the experience of these students in the Escher's World mathematics studio environment.

6.1 Discussion of Quantitative Outcomes

Escher's World shows that the mathematical concept of symmetry can be explored and learned in an art studio environment. This basic result supports the ideas of numerous theorists who have suggested that learning in traditional academic disciplines can be enhanced or even transformed by the arts (Read 1943, McFee 1961, Arnheim 1969, Field 1970, Silver 1978). In a recent study Willett showed that mathematics learning was more effective in the context of arts-based lessons than with standard mathematics pedagogy at the elementary school level (Willett 1992). Arthur Loeb's visual mathematics curriculum (Loeb 1993) has not been studied formally, but substantial anecdotal evidence supports his approach to the study of symmetry through a design studio as an effective learning environment for undergraduate students.

Willet's research established that elementary students can learn mathematics content effectively in an art studio setting. The results of Escher's World support this same conclusion for high-school students, but also show that after exploring mathematical ideas in an art studio setting, students gain access to an additional mode for thinking about mathematics problems. In the Escher's World workshops, learning mathematics in

the context of visual arts helped students learn to use visual thinking as part of their mathematics problem solving.

Although a number of researchers argue that visual thinking is related to successful mathematical thinking (Piemonte 1982, Hershkowitz and Markovits 1992), recent work by Campbell et al. suggests that students' visualization ability is not necessarily a factor in their success at solving problems (Campbell et al, 1995). Data from the Escher's World study shows that students' use of visual thinking during problem solving was correlated with their success in solving word problems after the workshop. This suggests that the workshop activities helped students make a more effective connection between visual thinking skills and mathematical problem solving.

Data from Escher's World also suggests that students' learning of content and skills in an art studio environment is connected to a positive change in attitude toward mathematics. Willett's study of elementary students did not address students' attitude toward mathematics. In another study of the effect of arts activities and mathematics with fourth grade students, Forseth found that students' attitude towards mathematics improved, but that there was no significant improvement in students test scores compared to a control group (Forseth 1976). Results from Escher's World suggest that under the proper circumstances, positive change in students' attitude towards mathematics can be achieved in combination with meaningful changes in the way students approach mathematics problems.

While the students in the two workshops discussed in this thesis showed significant development in their mathematics knowledge, skills, and attitude, it is important to remember that Escher's World represents a very brief intervention. It was necessarily limited in the number of mathematical topics that students could investigate, and in the depth to which students could explore any one topic. It is not clear that the large positive changes seen in this brief but intensive intervention would continue at the

same rate over a longer intervention; nor it is clear how long the effects of one brief intervention will last for students. These facts, combined with the small sample size of the experiment, suggests that some caution should be used in making sweeping claims based on these data.

6.2 Discussion of Qualitative Data

The qualitative data described above suggest several areas worth hilighting: (1) the importance of control in learning and the role that expression plays in empowering students; (2) the relative importance of control and authenticity in learning; (3) the affective and expressive role of computers in learning; and (4) the implications of the Escher's World research for the idea of expressive mathematics. Each of these ideas is discussed in more detail below.

6.2.1 Empowerment and Expression

The idea of "empowering" students is the topic of much discussion in educational circles. The NCTM Standards explicitly call for the development of students' "mathematical power" (NCTM 1989) and numerous recent books and articles talk about the idea of empowerment (see, for example, Driskill and Polansky 1994, Fibkins 1995, Byron 1995). But there is clearly not a consensus as to exactly what student empowerment means and how to achieve it.

The results from Escher's World suggest a framework for thinking about empowerment in terms of students' sense of control over their learning and learning environment. In the context of Escher's World, students felt that they had control over their own learning in several important ways. Students were able to get help from adults and peers in the amount, manner, and time of their own choosing. Students were able to work collaboratively with peers and adults if and when they chose to. And students had

the freedom to organize their activities in space and time--"freedom of movement" as Dewey described, but also the freedom to regulate the pace and timing of their work.

Escher's World also gave students control over the learning activities they were engaged in. The expressive nature of the activities meant that students were able to decide for themselves how to address the mathematical and artistic problems presented in the workshop. They were able to direct their problem solving process, and to decide what constituted a desirable solution. But this control did not mean that students were simply "doing whatever they wanted to do." Adults and the community of their peers still played an important role in student's learning. Adults and peers--as well as students' sense of the authentic nature of the problems they were working on--formed the context in which students evaluated their efforts. And, of course, the other people in the workshop environment also provided a network of support for students as they worked.

Other authors since Dewey have described a similar relationship between freedom or control on the part of learners and the power of a learning environment. Indeed, this idea is one of the central principles of the constructionist learning paradigm (Harel and Papert 1991, Kafai and Harel 1991). What the Escher's World research emphasizes is the role that personal expression and expressive activities play in creating a sense of control for learners.

One concrete way to see how expressive activities help students feel empowered is to look at the role of mistakes in Escher's World. Despite reassurances from teachers, students know that in a traditional class room, mistakes are bad. As one student said: "If you make a mistake in school they made it seem like you didn't know what you were doing, and everybody else did.... If you made a mistake you were singled out, and the class had to slow down so you could catch up." In contrast, student's in Escher's World felt as if they could learn from their mistakes:

"I made this picture and I thought it would be sidey [mirror symmetric]. I found that it was a 'fake out,' and it really looked like it was a sidey but it was not. So then when I looked at it and I put the mirror in the middle I thought that you can learn from your wrongs . So I made another picture. I made a heart. And I knew that it would be sidey [because I] checked it. I folded the paper. First I drew like half of the heart. And then I folded the paper and I traced it again to get the other side right. And I was always folding the paper all the time [to keep checking my work]."

The interactions of the studio environment of Escher's World helped students feel that mistakes were an opportunity rather than a liability. "It was okay to make mistakes," said one student, "because you could always get help." Similarly, the computers--and particularly some of the functionality described above--helped create a more constructive role for mistakes: "The computers helped because it was [easier to deal with mistakes]. On paper you'd have to erase it, or start again. [On the computer] you could just undo it, and then try something different. " Finally, the expressive nature of the activities of Escher's World gave students the freedom to explore. Mistakes were not "wrong," they were simply steps on the road to finding a better answer: "We could learn from our mistakes--do it totally different if we wanted to.... There wasn't any one way we could do it. We could do it differently. We could make a whole bunch of the same thing, or different things.... We could change [a design], and it didn't have to look exactly like a house, or exactly like a dog, or something like that."

The results from Escher's World make it clear that expressive activities are a powerful context for learning, and provide a particularly powerful vehicle for giving students a sense of control over their own learning. In this sense, the ideas of "expression" and "expressive activities" help elaborate a framework for understanding the success of design activities as a context for learning.

6.2.2 Authenticity

The data from Escher's World clearly shows that students' experience of control plays a powerful role in creating an effective learning environment. The data also suggests

that students' sense of the authenticity of their work helps in this process. However, even on the somewhat simplistic metric of quantity of comments, control had a more meaningful place in students' experiences of the workshop. When the rich relationships between control, expression, and interactions in the workshop are added to the mix, it seems fair to say that for these students, authenticity played something of a supporting role in making Escher's World a successful learning experience.

If students' sense of control is indeed a more significant factor in their learning than authenticity, recent emphasis in education on "authentic assessment" may need to be reevaluated. Authenticity is clearly an important part of students' learning experiences. However, data from Escher's World suggests that "authentic" experiences that do not also provide students with a sense of control may not be as effective for students as they could be. Some reformers, such as the Coalition of Essential Schools, explicitly include the idea of student control in their discussion of authenticity in learning (Sizer 1984). Other projects, such as the Teaching for Understanding project, emphasize authenticity but say little about the role of students control over their own learning (Perkins and Blythe 1994). Without in any way implying that authenticity is unimportant, it seems clear that reform proposals can not avoid the parallel issue of student empowerment.

6.2.3 Computers

A number of authors have written about specific aspects of computational media that make them effective in the process of learning (see, for example, Perkins and Unger 1994, Jackson et al. in press). One trait commonly described is the dynamic quality of computer representations--that is, their ability to reflect change in a continuous and immediate fashion. As Albert Michotte argued in his monograph "The Perception of Causality" (Michotte 1963), when objects respond directly to each other's motions in time and space we attribute causality as a primary percept. That is, a sufficiently powerful

dynamic representation can create direct intuitions about underlying causal relationships. It is not surprising, then that dynamic representations help students understand complex ideas.

What the Escher's World data shows, however, is that dynamic representations also have an affective component for learners. For students in the workshops, the ability to change a design quickly, easily, and in a continuous fashion--to make small and large scale adjustments with equal ease--contributed not only to the development of intuition, but also to a sense of control over their work. They interpreted the representations not only as being dynamic, but also flexible. The process of continuous and immediate change was important in part because it gave students the ability to manage their own explorations of mathematical and artistic relationships.

This sense of control was reinforced by another aspect of the particular software students used in Escher's World. The "infinite undo" feature of the Geometer's Sketchpad meant that students could change their minds--and change mistakes--quickly and easily. The computer provided a forgiving environment in which students could explore freely with little risk of making a catastrophic mistake that would cost them large amounts of time and effort.

Finally, the data from Escher's World suggests that computers played an important role in students' experience of the workshop by raising the aesthetic quality of the work produced. Scanned and enlarged, or printed with pure colors and clean lines, students felt their work looked better when done on the computer, and this, in turn, validated their efforts.

This combination of affective and expressive functions for the computer clearly goes beyond the mere "novelty" of the computer equipment used, and also suggests a richer view of the potential role of computers in education than more cognitively-oriented theories suggest. At the same time, data from Escher's World makes it clear that

computers were neither necessary nor sufficient for the success of this learning environment. Some students were able to learn without using the computers extensively-- just as some students who used computers extensively showed less dramatic changes in their approach to mathematics after the workshop.

A more realistic view of the role of computers in an environment such as Escher's World might be as a "stimulating" (rather than necessary or sufficient) condition. Computers can play a powerful role in empowering students, especially in the service of expressive activities. But computers alone are not enough to empower students, and they certainly can be used in ways that do not help students exercise effective control in their own learning process.

6.2.4 Expressive Mathematics

All of the above suggests that expressive activities are a powerful context for learning, and that one of the benefits of combining art and mathematics education in an environment such as Escher's World is that students are able to think about mathematical ideas in an expressive way. The larger question, however, is how to extend this idea beyond the arts realm. Part of the point of the Escher's World project is to show that mathematics and arts learning can be combined in a way that is productive for both domains. But ultimately the lessons of that "collaboration" should transfer back into each individual domain. A subsequent paper will explore the nature of arts learning in Escher's world, and within that context the role of analytical thinking in the arts. Here the logical question to ask is: What does it mean to have an "expressive" mathematics problem independent of the arts?

Perhaps an example from arts education will be suggestive. When a student learns to draw a human face in a life-drawing studio class, it is quite likely that before starting he or she will look at the work of masters of the craft: a face drawn by Leonardo, one by

Van Gogh, by Picasso, by Matisse. A student may look at ten, twenty, or even a hundred such master works. When he or she sits down to draw a face, the problem is richer, but in every respect just as challenging and as genuine as it was before the student looked at the examples. In fact, even if the student were a perfect mimic and could recreate the masters' drawings perfectly, the drawing of a new face would still be a creative process.

This example from the arts suggests a working definition for an "expressive problem" in any domain. An expressive problem is one that: (1) has multiple valid solutions, where (2) the solutions are more or less interesting, pleasing, beautiful, socially, economically, or politically desirable, or otherwise preferable according to some set of personal criteria. In other words, an expressive problem is a problem where one can say: "Both of these answers are correct, but I prefer the first one because...." The results from Escher's World described above show that students saw the problems and activities of the Escher's World workshops as expressive by this definition--and also that these aspects of the problems and activities contributed directly to students' experience of the workshop as an environment where they were able to control their own learning.

This, in turn, suggests two questions for further study. The first question is whether expressive problems from the arts and from other domains could be the focus of a compelling mathematics education. The second, and perhaps more fundamental question, is about the nature of expressive activities in general--and about the relationship of expression to mathematical thinking in particular.

The data collected thus far from Escher's World shows that expressive activities help students control their own learning. But are there other connections between expression and mathematical thinking? Why, for example, do students begin to use visual thinking in their problem solving as a result of working in Escher's World? Is it just increased familiarity with visual representations as a result of art-making? Or are there

some deeper structural connections between expressive activity and problem solving in mathematics?

More in-depth analysis of this last question will be the subject of a future paper about the Escher's World project. However, the correlation reported above between expression and students' success in problem solving suggests that the idea of expressive mathematics may have implications beyond the integration of mathematics and art education and beyond the importance of empowering students to control their own learning.

7 CONCLUSION

The data from Escher's World reported in this thesis suggest that a studio setting is productive context for learning mathematics. This result supports other research findings regarding connections between mathematics and the visual arts. However, data from Escher's World goes beyond previous work by suggesting that while learning mathematics in visual arts environment, students not only learn specific mathematical concepts, they also develop the ability to use visual thinking as an effective tool for problem solving.

There are many more possible ways to imagine mathematics education than theorists and researchers have yet explored. The results of the Escher's World project suggest that expression can play an important role in mathematics learning. The limited size and scope of workshop results reported in this thesis, combined with the apparent success of the "mathematics studio" as a learning environment suggest that further work should be done to develop the concept of studio mathematics as an example of an expressive learning environment.

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9 APPENDIX

9.1 Interview Guidelines

Pre-interview Guidelines

INTRODUCTION

Description of Research Project

Explanation of Subjects Rights and Protections

Description of Interview Procedure

GENERAL BACKGROUND INFORMATION

Name, age, etc.

What courses has subject taken in math, art?

GENERAL QUESTIONS ABOUT MATH AND ART.

How does subject feel about math and art classes. Does s/he like/dislike them? Can subject give examples or reasons?

How does subject feel about math and art outside of courses? Is there a difference? If so, why?

QUESTIONS ON WORK OF ART

Description

How does subject describe work of art: general description.

Can subject provide a more detailed description?

What does subject see as "key" or most important features in the description?

Technical issues

What does the subject think would be hard about making the piece? What are the key technical issues? What would someone need to understand to create it?

What might students learn in the process of making such a piece?

General Impressions

Determine subject's general response: Does s/he like/dislike it in general?

What in particular does s/he like/dislike about it?

Repeat Questions on Work of Art for each piece

ADDITIONAL COMPARISON QUESTIONS

Are the descriptions of pieces similar or different? Explore similarities or differences with subject.

MATH PROBLEMS

Show subject problem. Ask subject to describe reactions to the problem. Does s/he like/dislike it? Can subject give explanations or reasons?

Ask subject to try to solve problem. Ask him/er to describe his/er thinking while working on the problem. Tell him/er that solution is not as important as understanding how s/he approaches such a problem.

Repeat for additional problems.

QUESTIONNAIRE

Explain instructions

CONCLUSION

How did subject feel about this interview? Were there any surprises?

Description of workshop, and explanation of post-interview process.

Thanks to the subject

Post-interview Guidelines

INTRODUCTION

Explanation of Subjects Rights and Protections

Description of Interview Procedure

GENERAL QUESTIONS ABOUT WORKSHOP

Did subject like the workshop?

Was the workshop what you expected? Were there any surprises?

How would subject describe workshop to parent/teacher/peer?

What did subject think s/he learned?

What in the workshop helped this learning happen? (Be as specific as possible.)

Was workshop like or unlike classes at school?

GENERAL QUESTIONS ABOUT MATH AND ART.

Did workshop change subject's feelings or thoughts about math?

Did workshop change subject's feelings or thoughts about art?

What about workshop helped subject change views? (Be as specific as possible.)

QUESTIONS ON WORKS OF ART

General Impressions

Determine subject's general response: Does s/he like/dislike it in general?

What in particular does s/he like/dislike about it?

Description

How does subject describe work of art: general description.

Can subject provide a more detailed description?

What does subject see as "key" or most important features in the description?

Technical issues

What does the subject think would be hard about making the piece? What are the key technical issues? What would someone need to understand to create it?

What might students learn in the process of making such a piece?

Repeat Questions on Work of Art for each piece

If view changed, why? What happened in workshop to change view? (Be as specific as possible.)

MATH PROBLEMS

Show subject problem. Ask subject to describe reactions to the problem. Does s/he like/dislike it? Can subject give explanations or reasons?

Ask subject to try to solve problem. Ask him/er to describe his/er thinking while working on the problem. Tell him/er that solution is not as important as understanding how s/he approaches such a problem.

Repeat for additional problems.

If view changed, why? What happened in workshop to change view? (Be as specific as possible.)

QUESTIONS ABOUT SYMMETRY AND COMPOSITION

What is symmetry?

What did you know about symmetry before workshop?

Are their different kinds of symmetry?

What in the workshop changed you view? (Be as specific as possible.)

Repeat questions for Composition.

## QUESTIONNAIRE

Explain instructions

CONCLUSION

How did subject feel about this interview? Were there any surprises?

Thanks to the subject

9.2 List of images used in interviews

Students were shown four images in both pre and post interviews. One image was chosen from each of the sets 1-4 below. Note that set 3 contains only one image; all students were shown the same image in both pre and post interviews for this set. All images were re-sized to 5 inch width and reproduced in color.

1a

1b

2a

2b

3a
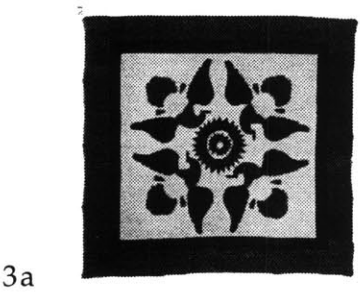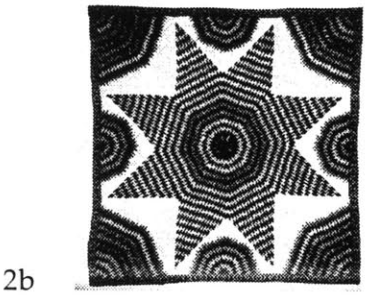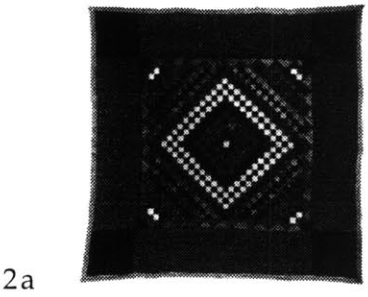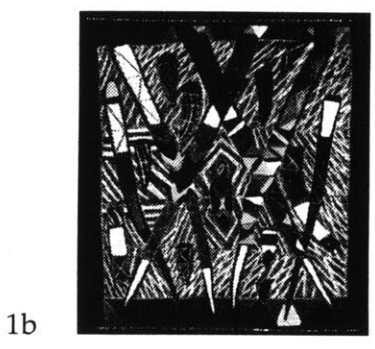
4a
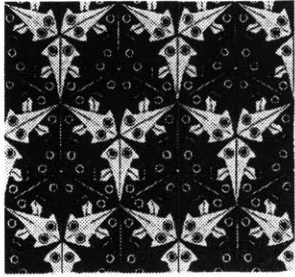
4b

9.3 List of problems used in interviews

During pre and post interviews, students from the first workshop were given two out of four problems 1.1-1.4 below. Students from the second workshop were given 4 problems, one each from sets 2.1–2.4 below. Students were given the problems one at a time, each typed on a separate sheet of paper. Students had unlimited time to work on each problem and were provided with a pad of lined paper and a pen or pencil.

Problem 1.1
Ms. Jones has to 25 pencils and 10 pens to give to her students. She gives each student the same number of pencils. She gives each student the same number of pens. At most how many students does Ms. Jones have?

Problem 1.2
Bob and Tanecka each have a 12-inch pizza pie. Bob cuts his pizza into 8 pieces. Tanecka cuts her pizza into 6 pieces. If you put the pizza's one on top of the other, at most how many cuts in Bob's pizza would be in the same place a cut in Tanecka's pizza?

Problem 1.3
Sally and Juan each have a 14-foot ladder. Sally's ladder has 21 rungs on it. Juan's ladder has 15 rungs on it. If you put the two ladders side by side, how many rungs would be in the same place?

Problem 1.4
A group of students has 12 apples and 15 oranges. They share the apples and oranges so that each students has the same number of whole apples and whole oranges as every other student has. At most how many students could there be in the group?

Problem 2.1a
One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 3 miles due east. After Julie turns right again and walks for another 2 miles, she decides to go home. How far does she have to go to get back to her home?

Problem 2.1b
One day, Julie decides to go for a walk. She leaves her home and walks for 2 miles due north. Then she turns right and walks for 4 miles due east. Julie then realizes that she dropped her watch 1 mile back. She turns around and walks until she reaches her watch. After she picks up her watch, Julie turns right again and walks for another 2 miles due south. Now Julie wants to go home. How far does she have to go to get back to her home?

Problem 2.1c
A leaf falls and lands 5 yards east of the tree it was on. A boy picks up the leaf, and walks 10 yards north. The boy sees a swing set on his left, 15 yards to the west. He runs to the swings, but drops the leaf 10 yards before he reaches the swings. How far is the leaf from its tree after the boy drops it?

Problem 2.2a
One border of Theo's backyard is 20 yards long. Theo wants to put up a fence along this border. If he puts up one fence post every 5 yards, how many posts does Theo need?

Problem 2.2b
One border of Theo's backyard is 20 yards long. Theo wants to put up a fence along this border. If he puts up one fence post every 5 yards, how many posts does Theo need? Problem Number 2 Theo wants to put up fences along two borders of his backyard. One border is 15 yards long, the other is 20 yards. If he puts up one fence post every 5 yards, how many posts does Theo need?

Problem 2.2b
Yolanda has a pipe that is 8 feet long. She needs to cut the whole pipe into pieces that are 2 feet long. How many cuts does she need to make?

Problem 2.3a
A snail is stuck on the inside wall of a well, 5 feet down from the top of the well. It moves 3 feet up the wall every day. But every night, the snail slips 1 foot down the wall. After how many days will the snail reach the top of the well?

Problem 2.3b
A snail is stuck on the inside wall of a well, 5 feet down from the top of the well. It moves 2 feet up the wall every day. But every night, the snail slips 1 foot down the wall. After how many days will the snail reach the top of the well?

Problem 2.3c
Yanni goes to his favorite restaurant and orders an 12 ounce soda. Every time he finishes drinking 4 ounces of his soda, a waiter pours 1 more ounce of soda into his cup. How many times will the waiter pour soda into Yanni's cup before Yanni completely empties his cup?

Problem 2.4a
Leo asks Luanda if she will lend him money for a hamburger. Luanda says, "But you borrowed money for a hamburger from me yesterday! And you still owe me 1 dollar for the soda you bought last week!" So Leo says, "Well, okay -- lend me money again today, and I'll owe you 4 dollars all together." How much does a hamburger cost?

Problem 2.4b
Leo asks Luanda if she will lend him money for a hamburger. Luanda says, "But you borrowed money for a hamburger from me yesterday! And you still owe me 80 cents for the soda you bought last week!" So Leo says, "Well, okay -- lend me money again today, and I'll owe you 4 dollars and 30 cents all together." How much does a hamburger cost?

Problem 2.4c

Two years ago, Janelle was four times as old as Sangita. If Janelle is twenty years old now, how old is Sangita now?

9.4 List of survey questions used in interviews

Students were given the following "sample question":

For each set of statements, circle the number that represents how you feel.
<u>Example:</u>
I like talking on the phone.   5   4   3   2   1   I don't like talking on the phone.
If you really like talking on the phone, circle 5. If you like it most of the time, circle
4. If you like it half of the time and don't like it the other half, circle 3. If you
don't like it most of the time, circle 2. If you really don't like talking on the phone,
circle 1.

The survey consisted of 15 questions, all in the same format as the example:

1. I like school./I don't like school.
2. I like math class./I don't like math class.
3. I like doing math problems./I don't like doing math problems.
4. I like thinking about math./I don't like thinking about math.
5. I understand math./I don't understand math.
6. I like to make art./I don't like to make art.
7. I like looking at art./I don't like looking at art.
8. I like thinking about art./I don't like thinking about art.
9. I understand art./I don't understand art.
10. I like computers./I don't like computers.
11. I understand computers./I don't understand computers.
12. I learn a lot from listening to the teacher./I don't learn a lot from listening to the teacher.
13. I learn a lot from working on a computer./I don't learn a lot from working on a computer.
14. I learn a lot from working with other students./I don't learn a lot from working with other students.
15. I learn a lot from working by myself./I don't learn a lot from working by myself.