

Essays in Auction and Market Design

by

Andrei Bremzen

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Economics

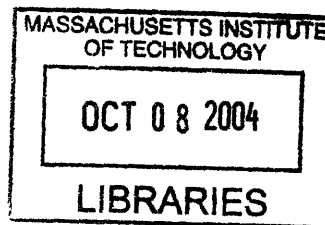
at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

September 2004

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Abstract

This thesis consists of three essays in auction and market design.

Chapter 1 studies sequential auctions with potential entry between rounds. In a simple model with two rounds, two initial bidders and one potential entrant, it is shown that every symmetric equilibrium first round bidding function must feature some degree of pooling. In one such equilibrium, the symmetric bidding function is a step function, reflecting the desire of present bidders to hide information from the potential entrant in order to deter entry. Extensions of the simple model to multiple incumbents and uncertain presence of the entrant are discussed.

Chapter 2 studies the choice between two modes of trade: selling at a posted price or bargaining. It is shown that the choice of one of the regimes may serve as a signal of quality of the good, otherwise unobservable to buyers. The main result of this chapter is that both modes can coexist on the same market. This result holds both when sellers can choose the quality is given exogenously and when they can not.

Chapter 3 examines origins of rules restricting the set of auction formats available to the seller in an auction. While wider set of possible auction formats available to the seller may increase his expected revenue, choice of one of the formats discloses seller's private information; the seller may want to commit to an auction format *ex ante* to avoid this disclosure. The value of commitment is analyzed in the context of announced versus hidden reservation value choice. A policy of conditional disclosure is introduced, which generates revenue higher than that generated by either of the unconditional policies. In the context of public procurement auctions, implications of expected and unexpected favoritism on the part of the auctioneer are analyzed.

Thesis Supervisor: Bengt Holmström
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Acknowledgments

When I started with my PhD program at MIT, I did not believe that I would ever graduate. Completing the program and especially writing a thesis sounded nearly unrealistic. I would never have accomplished this goal without unconditional and continuous support from people around me.

I am infinitely indebted to my advisors, Bengt Holmström and Sergei Izmalkov.

Despite his heavy workload as the Department Head, Bengt has always been ready to find time to meet with me. I benefited immensely from talking to him, both in immediate connection with my work and on broader economic issues. Bengt's ongoing encouragement was indispensable for successful completion of this thesis.

Sergei has been generous with his time and particularly careful with details when reading my drafts; his continuous editing significantly refined my writing. Besides providing expert professional advice, Sergei has also been very friendly and helpful on the personal level.

I have greatly benefited from discussions with MIT faculty. Early in the process of writing the thesis Paul Milgrom provided valuable guidance. Comments and suggestions by Glenn Ellison, Haluk Ergin, Jean Tirole, Birger Wernerfelt and theory lunch participants substantially improved thesis chapters.

It is unlikely that I would have been admitted to MIT without support of the New Economic School in Moscow, where I received my first degree in economics. Faculty and staff of NES were not only highly helpful throughout the application process, but also very encouraging throughout my years at MIT. It is their confidence in me that helped me maintain focus on my work throughout years and ultimately complete it.

My experience at MIT would be significantly less pleasant if not for my fellow graduate students. I greatly enjoyed the social life at MIT and particularly at MIT Russian House. I am thankful to Department of Economics staff in general and Gary King in particular for always extending a helpful hand to assist me with large and small tasks.

My parents have shown extraordinary patience awaiting me home for years.

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Chapter 1

Sequential auctions with entry deterrence

1.1 Introduction

This chapter studies sequential auctions with potential entry between rounds. Although sequential auctions have been thoroughly studied, the common assumption in the existing literature is that the set of bidders does not change from one round to the next. It is natural to extend the analysis to the case where new bidders may enter the auction in future rounds, if they find it attractive, possibly at positive entry costs. The purpose of this chapter is to show how potential entry threat changes standard equilibrium predictions.

I consider a repeated English auction with two rounds and three bidders with independent private valuations. Two identical items are for sale, one in each round. Each bidder wants at most one item, so the winner of the first round gets the item and leaves the auction. There are two bidders in the first round. The third bidder can not participate in the first round but can decide to enter in the second round, at a positive entry cost. The third bidder's entry decision is based not only on his own valuation, but also on the observed outcome of the first round (the price and the identity of the winner). This opens the door for strategic behavior in the first round: the decision of a bidder to drop out in the first round not only means that she is not getting the item today but also sends a signal about her valuation to the potential entrant, which may affect his entry decision. Since the entrant's decision whether to enter is relevant for the payoff to the first-round bidder, the first round

bidder will take the entrant's reaction into consideration when deciding at what price to drop out. In particular, an incumbent has an incentive to pretend to have higher valuation than she actually does, in order to deter entry; the costs of staying too long are, however, that an incumbent may end up winning the first round, at a price higher than she would be willing to pay. The main focus of the analysis is on how these strategic considerations affect bidders' behavior.

This chapter is, to the best of my knowledge, the first paper to study endogenous entry decisions in the context of sequential auctions. Sequential auctions with a constant number of bidders were first studied by Ortega-Reichert ([11]). He developed a two-person two-stage model of competitive bidding and recognized that the strategies of the bidders may respond not only to the fact that there will be more rounds in the future, but also to the information about their rivals' valuations that has been revealed in previous rounds (such as their bids). He considered a common value auction, where in stage two each bidder updates her estimate of her own valuation of the good based on the first-period bid of her rival (this setup is further analyzed in [8]). Milgrom and Weber ([10]) develop a general model of multistage auctions and compare different procedures for sequential auctions, as well as various information structures, i.e., what information about the bids in past auctions is revealed to the remaining bidders; in their setup, too, all the bidders participate from the first round.

One other area of research of relevance to the present study explores strategic motives of bidding; the idea is that not only does a bid represent a claim to win the object, but it also conveys some information that the bidder possesses, which may be relevant to the other bidders. This idea builds on an auction design that allows for sequential moves: traditional one-stage sealed bid auctions provide no opportunity for signaling. Avery ([1]) studies strategic bidding in English auctions when bidders' valuations are correlated (and thus bids reveal payoff relevant information to competitors). Daniel and Hirshleifer ([5]) build a two-bidder model of an ascending price auction, in which the bidders incur positive costs every time they submit a bid; jump bidding serves to signal a high valuation to the opponent in order to force him to quit early.¹ In none of these papers strategic motives for bidding include entry deterrence, since all of them assume an exogenous set of bidders.²

¹See also Fishman ([6]).

²There also are a few studies of bidding environments in which submitting or revising a bid is costly and entry decisions are endogenous; this is not an unreasonable assumption, for instance in procurement

In this chapter I find that in the presence (even uncertain) of potential entrant induces the first round bidders to bid strategically, and their equilibrium bidding functions are radically different from bidding functions in auction without strategic considerations. I show first that, as long as entry costs are strictly positive, in no equilibrium are first round bidding functions monotone in valuation; this implies that the two-stage auction is generally inefficient. Next, I show that for any level of entry costs there exists a subgame perfect equilibrium which involves step functions as first round bidding functions, with the number of steps decreasing in entry costs. Therefore, strategic concealing the information about their valuation is an important issue for first round bidders.

The rest of the chapter is organized as follows. Section 1.2 describes the model and shows that every symmetric first-round equilibrium bidding function must feature some pooling. Sections 1.3 and 1.4 provide full characterization of a symmetric equilibrium in which the (first-round) bidding functions are step functions. Section 1.5 looks at an extension with exogenous entry barriers, and Section 1.6 discusses multiple first round bidders. Section 1.7 concludes.

1.2 The Model

Two identical units of a good are offered for sale by means of an ascending price auction; the two rounds of auction are conducted sequentially and the outcome of the first round (the final price and the identity of the winner) is publicly observable before the second round starts. In each round, in case of a draw (last active bidders dropping out at the same price), the good is assigned to each of them with equal probability.

There are three risk-neutral players (or bidders), each demanding at most one unit of the good. In the first round only players 1 and 2 participate; the winner obtains the good and quits, the loser passes on to the second round. Upon observing the outcome of the first round, player 3 decides whether to enter in the second round. If he enters, he incurs entry cost $c > 0$ and then bids against the loser of the first round. If he does not enter, the loser of the first round obtains the item for free³. The bidders' valuations of the good are

or construction auctions. These works too are relevant to the current study setup; however, in all of them only single round auctions are considered. See, for example, Gal, Landsberger and Nemirovski ([7]) and Landsberger and Tsirelson ([9]).

³According to this formulation, players are inherently asymmetric in that the player 3 faces entry costs while those for his rivals are assumed to be sunk. An alternative formulation in which all three bidders have to incur costs to entry results in a similar solution.

independently drawn from uniform distribution on $[0, 1]$ and are private information.

I am looking for a subgame perfect Bayesian symmetric equilibrium in weakly undominated strategies of this game. Such an equilibrium consists of

- A drop out strategy $b(v)$ that players 1 and 2 follow in the first round;
- Belief of player 3 about the type of his rival given the outcome of period 1;
- An entry decision of player 3 as a function of observed result of period 1;
- Symmetric bidding strategies of player 3 and his rival in the second round.⁴

As usual, belief of player 3 must be consistent with the prior distribution and strategies played by players 1 and 2 in the first round, and his entry decision must be optimal given his belief.

It is straightforward to conclude that if player 3 decides to enter, the only pair of weakly undominated strategies for the bidders in the second round is to drop out at prices that are equal to their true valuations. Therefore the nontrivial part of the analysis is to characterize first round bidding and the third player's entry decision.

1.2.1 Costless entry

Assume first that the entry is costless for the third player, i.e., $c = 0$. I want to calculate $b(v)$ – the equilibrium strategy of each of the bidders in the first round as a function of her valuation.

In this case the only weakly undominated strategy for player 3 is to enter and then stay in until the price reaches his valuation; therefore, his strategy is trivial and the game reduces to a two player game, which apparently has a solution in weakly dominant strategies.

A bidder with valuation v will choose to drop out in the first round at price $b(v)$, at which she is indifferent between winning now and receiving $v - b(v)$ and dropping out, with expected gain from winning the item in the next round $\int_0^v [v - x] dx = \frac{v^2}{2}$. Therefore, each bidder's dominant strategy in the first round is to drop out at $b(v) = v - \frac{v^2}{2}$.

This function is strictly monotone in the valuation, implying that the entrant will be able to deduce the valuation of his opponent. If that valuation is higher than his own, he

⁴Since the game is symmetric with respect to bidders 1 and 2, I focus on symmetric equilibria.

will be indifferent between entering and not, since he has no chance of winning anyway; to stay out is a weakly dominated strategy and hence is excluded. However, facing any positive entry cost he will strictly prefer not to enter if he knows that he will lose, so the analysis above does not extend to the case of $c > 0$.

1.2.2 Costly entry: an impossibility result

Lemma 1 *For any positive level of entry costs there exists no subgame perfect symmetric equilibrium bidding function $b(v)$ which is strictly monotone in valuation.*

Proof: Assume the converse, and let $b(v)$ be strictly monotone strategy of players 1 and 2 in the first round. Then the potential entrant will be able to correctly deduce valuation v of his opponent from the price that he observes; he will, therefore, enter if and only if his own valuation is at least $v + c$.

Consider the optimization decision of player 1 in the first round; suppose that she has high valuation $v > 1 - c$. One particular deviation from $b(v)$ that player 1 may consider is playing $b(\tilde{v})$ for some \tilde{v} , i.e., to pretend that she is of type \tilde{v} ; consider small deviations, such that $\tilde{v} > 1 - c$. Such a deviation will give her the payoff of

$$W(v, \tilde{v}) = \int_0^{\tilde{v}} (v - b(x)) dx + v(1 - \tilde{v}).$$

The first term is player 1's expected payoff in the first round, given that player 2 adheres to $b(v)$. Player 1 loses in the first round if player 2 has valuation higher than \tilde{v} , i.e., with probability $1 - \tilde{v}$; in this case player 3 believes that player 1 has valuation \tilde{v} and does not enter, since his own valuation is less than $\tilde{v} + c > 1$; player 2 wins the item for free. Note that, since $b(v)$ is strictly monotone, $W(v, \tilde{v})$ is absolutely continuous in \tilde{v} and has left and right derivatives in \tilde{v} everywhere.

If $b(v)$ is an equilibrium bidding function, then for any v function $W(v, \tilde{v})$ must attain its maximum in \tilde{v} at $\tilde{v} = v$. The first order condition for that its left derivative is nonnegative at $\tilde{v} = v$. We have

$$0 = \frac{d}{d\tilde{v}} W(v, \tilde{v}) \Big|_{\tilde{v}=v-0} = v - b(v) - v = -b(v). \quad (1.1)$$

Clearly this can not be the case, since $b(v)$ is assumed monotone and hence can not be

identically zero. QED.

Remark 1. Lemma 1 holds for any continuous distribution with positive density, not necessarily uniform. For a distribution with continuous density $f(\cdot)$ first order condition (1.1) will read $-b(v) \cdot f(v) = 0$ and, given, $f(v) > 0$, the same argument applies.

Remark 2. An analogue of Lemma 1 holds even if there is more than two incumbents. There exists no separating equilibrium, that is, no equilibrium drop out schedule such that the entrant (in case he observes the complete outcome of the first round, i.e., who dropped out at what price) can deduce valuations of all the bidders from the outcome of the first round. Indeed, if such an equilibrium existed, then when all but two first-round bidders have zero valuations, the other two would be in a situation similar to that of Lemma 1 and the same logic would rule out the possibility of a separating equilibrium.

1.3 Step function as an equilibrium bidding function

Now I turn to constructing a symmetric equilibrium strategies $b(v)$ for bidders in the first round.

Consider first the case of very high entry cost, $c \geq \frac{1}{2}$. Suppose both player 1 and player 2 drop out immediately at price 0, irrespective of their valuations: $b^{(0)}(v) \equiv 0$. Player 3 always observes price zero but can make no inference about his opponent's valuation; he still thinks it is distributed uniformly on $[0, 1]$. Even if he himself has the highest possible valuation of 1, his average payoff if he enters will on average be only $\frac{1}{2}$, not enough to cover entry costs, so he will not enter. Therefore, for $c \geq \frac{1}{2}$ both incumbents bidding $b^{(0)}(v)$ and player 3 never entering is an equilibrium.

If entry costs are not so high ($c < \frac{1}{2}$), then playing $b^{(0)}(v) \equiv 0$ is no longer an equilibrium: each of the first two players expects player 3 to enter with positive probability, so both of them find winning the item in the first period more attractive and will prefer to outbid each other in the first round.

Instead of $b^{(0)}(v)$, consider the following $b^{(1)}(v)$ for some v^* :

$$b^{(1)}(v) = \begin{cases} 0, & 0 \leq v \leq v^*, \\ b^*, & v^* < v \leq 1. \end{cases}$$

To complete the description of the suggested equilibrium strategies, I must specify entry

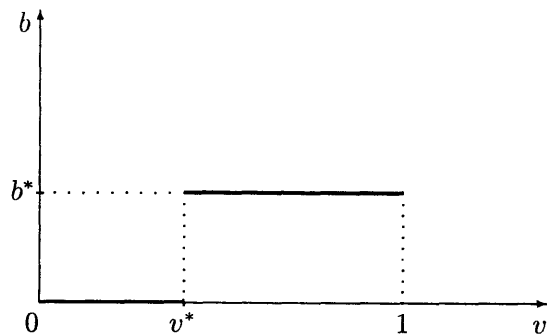


Figure 1-1: Equilibrium step bidding function $b_1(v)$.

decision of player 3 based on the price p he observes. The entry strategy for player 3 is:

- if $p = b^*$, enter if $v > \bar{v} = v^* + \sqrt{2c(1 - v^*)}$ (or never if $\bar{v} > 1$);
- if $p = 0$, enter if $v > \frac{v^*}{2} + c$;
- otherwise, enter if $v > c$.

The latter condition specifies beliefs of player 3 off the equilibrium path: he believes that the deviator is of the lowest possible type, zero⁵.

Proposition 1 *The suggested entry strategy is a best reply of the entrant to strategy $b_1(v)$, as long as $v^* \leq 2c$.*

Proof: If the entrant believes that both first-round bidders adhere to $b^{(1)}(v)$, he will conclude, upon observing $p = 0$, that the loser of the first period has valuation less than or equal to v^* . This gives conditional density $f(x) = \frac{1}{v^*}$ on $[0, v^*]$ and zero elsewhere. Suppose now that player 3 enters and his valuation is $v \geq v^*$. His expected payoff is then

$$\int_0^{v^*} \frac{1}{v^*} [v - x] dx = v - \frac{v^*}{2}.$$

He will choose to enter only if this is at least c , which is the case when $v \geq \frac{v^*}{2} + c$. If $v^* \leq 2c$ then indeed $\frac{v^*}{2} + c \geq v^*$.

Assume now that the entrant observes $p = b^*$. He concludes that his rival is of type higher than v^* , with conditional density $f(x) = \frac{1}{1 - v^*}$ on $[v^*, 1]$ and zero elsewhere. Let his

⁵This specification of beliefs is consistent with Cho-Kreps intuitive criterion as in [3].

own type be $v > v^*$ (obviously he does not enter if $v \leq v^*$); his expected payoff is then

$$\int_{v^*}^v \frac{1}{1-v^*} [v-x] dx = \frac{(v-v^*)^2}{2(1-v^*)}.$$

He will enter if that payoff is at least c , that is, if $v > \bar{v}$, QED.

Now pick b^* to make a player with valuation v^* indifferent between bidding zero and b^* . Assume that player 1 adheres to $b^{(1)}(v)$ and consider player 2 who values the object at some $v \leq \frac{v^*}{2} + c$. When player 2 bids zero, with probability v^* player 1 will bid zero, in which case with probability $\frac{1}{2}$ player 2 gets the object for free and with probability $\frac{1}{2}$ (or for sure if player 1 has valuation $v > v^*$) player 2 passes on to the second round in which the entrant of type $\frac{v^*}{2} + c$ or below does not enter and player 2 gets the object for free, while against the other types of entrant she only wins if her type is higher than the entrant's. Player 2's total expected payoff from bidding zero is therefore equal to

$$\pi_0(v) = \begin{cases} \frac{v^*}{2}v + (1 - \frac{v^*}{2}) (\frac{v^*}{2} + c) v, & v \leq \frac{v^*}{2} + c, \\ \frac{v^*}{2}v + (1 - \frac{v^*}{2}) \left(\frac{(\frac{v^*}{2} + c)^2 + c^2}{2} \right), & v \geq \frac{v^*}{2} + c. \end{cases}$$

If instead she bids b^* , her payoff is

$$\pi_{b^*}(v) = \begin{cases} v^*v + \frac{1-v^*}{2}(v-b^*) + \frac{1-v^*}{2}v\bar{v}, & v \leq \bar{v} \\ v^*v + \frac{1-v^*}{2}(v-b^*) + \frac{1-v^*}{2}\frac{v^2+\bar{v}^2}{2}, & v \geq \bar{v}. \end{cases}$$

Equating $\pi_{b^*}(v^*)$ to $\pi_0(v^*)$ gives

$$\frac{v^*}{2}v^* + \left(1 - \frac{v^*}{2}\right) \left(\frac{v^*}{2} + c\right) v^* = v^* - \frac{1-v^*}{2}b^*, \text{ or}$$

$$b^* = \frac{2v^*}{1-v^*} \left[1 - \frac{v^*}{2} - \left(1 - \frac{v^*}{2}\right) \left(\frac{v^*}{2} + c\right)\right] = \frac{v^*(2-v^*)(1-c-\frac{v^*}{2})}{1-v^*}.$$

To complete the description of $b(v)$ I now have to specify the value of v^* . I do it by considering a particular deviation from $b(v)$ (namely, bidding above zero but below b^*) which must not be profitable for any value.

If player 2 (of a particular type v) bids above zero but below b^* , then with probability v^* player 1 will bid zero and player 2 will win the object for free; however with probability $1 - v^*$ player 1 will bid b^* in which case not only will player 2 lose in the first round, but

also player 3 will believe that player 2 is of type zero and will enter whenever player 3's own valuation is above c . Hence if the valuation v of player 2 is below c , she will on average get vc in the second round, while if her valuation is above c , she will get on average

$$vc + \int_c^v [v - x]dx = \frac{v^2 + c^2}{2}.$$

Her total payoff is therefore equal to

$$\pi_{<b^*}(v) = \begin{cases} v^*v + (1 - v^*)vc, & v \leq c \\ v^*v + (1 - v^*)\frac{v^2+c^2}{2}, & v \geq c. \end{cases}$$

Note that the actual bid does not matter, provided that it is above zero but below b^* .

Proposition 2 *The slope of $\pi_{<b^*}(v)$ at $v = 0$ is below that of $\pi_0(v)$.*

Proof: By inspection. Follows from $v^* < 2c$.

Proposition 2 implies that bidding above zero is not a profitable deviation for small (below c) values of v . However, at larger v bidding positive delivers superior payoff to bidding zero; choose breakpoint v^* to be the valuation such that a bidder with valuation v^* is indifferent between bidding zero and slight positive amount: $\pi_0(v^*) = \pi_{<b^*}(v^*)$.

Proposition 3 *The threshold value v^* chosen in the previous paragraph is below $2c$.*

Proof: It suffices to show that $\pi_{<b^*}(2c) > \pi_0(2c)$ for $v^* = 2c$. This is done by inspection.

Now by construction we have $\pi_0(v^*) = \pi_{<b^*}(v^*) = \pi_{b^*}(v^*)$. Note also that $\pi_0(0) = \pi_{<b^*}(0) (= 0)$, and that $\pi_0(v)$ is linear in the range $[0, v^*]$ while $\pi_{<b^*}(v)$ is convex in that range. This implies that bidding above zero is not profitable for any $v \leq v^*$. We also have to check that bidding anything other than b^* is not a profitable deviation for $v > v^*$.

Proposition 4 *The slope of $\pi_{<b^*}(v)$ is less than that of $\pi_{b^*}(v)$.*

Proof: By inspection.

Corollary. For all $v \geq v^*$ bidding below b^* is inferior to bidding b^* .

Finally, I have to consider bidding above b^* . If $v^* \geq 1 - 2c$, then player 3 never enters upon observation of price b^* in the first round (i.e., $\bar{v} \geq 1$) and hence $\pi_{b^*}(v) = v - \frac{1-v^*}{2}b^* > v - (1 - v^*)b^* = \pi_{<b^*}(v)$, so bidding above b^* is always inferior to bidding b^* (because in

each case the bidder gets the item for sure, but bidding above b^* increases chances of paying b^* rather than zero for the item). On the other hand, if $c \rightarrow 0$, then $\pi_{>b^*}(1) \rightarrow 1$ while $\pi_{b^*}(1) \rightarrow \frac{3}{4}$, so there exists minimum value c_{\min} for which there exists v^* such that $b^{(1)}(v)$ is an equilibrium pair of strategies. Numerical simulations show that $c_{\min} \approx 0.1467146$.

Remark 1. Note that $\pi_{b^{(1)}(v)}(v) - \pi_{<b^*}(v)$ reaches its minimum at $v = 0$, so a bidder of type zero loses the least by deviating from $b^{(1)}(v)$. It is, therefore, not unreasonable for the entrant to presume the deviator to have valuation zero.

Remark 2. For c strictly above c_{\min} (but below $\frac{1}{2}$) there exists a degree of freedom in the choice of the equilibrium $b^{(1)}(v)$, namely, the specification of the breakdown point v^* – it can be moved to the left from the solution of $\pi_{<b^*}(v) = \pi_0(v)$, provided that $\pi_{>b^*}(v)$ is still not greater than $\pi_{>b^*}(v)$.

Example. Let $c = \frac{1}{3}$ and pick $v^* = c = \frac{1}{3}$. Then $\pi_{b^*}(v) = v - \frac{1}{3}b^*$ and $\pi_0(v) = \frac{1}{6}v + (1 - \frac{1}{6})\frac{1}{2}v$, which gives $b^* = \frac{5}{12}$. Payoff from deviation is

$$\pi_{<b^*}(v) = \begin{cases} \frac{5}{9}v, & v \leq \frac{1}{3} \\ \frac{1}{27} + \frac{1}{3}v + \frac{1}{3}v^2, & v \geq \frac{1}{3}. \end{cases}$$

It is readily checked that $\pi_{<b^*}(v) < \pi_{b^*}(v)$ both at $v = 1$ and $v = \frac{1}{3}$, and, consequently,

$$b(v) = \begin{cases} 0, & 0 \leq v \leq \frac{1}{3}, \\ \frac{5}{12}, & \frac{1}{3} < v \leq 1, \end{cases}$$

is a symmetric equilibrium for $c = \frac{1}{3}$.

1.4 Small entry costs equilibrium

In this section I finish up construction of equilibrium by showing that for any entry costs $c > 0$ there exists a step function which is an equilibrium first round bidding function. In particular, I will show how to construct points $0 = v_0^* < v_1^* < \dots < v_{n+1}^* = 1$ and values $0 = b_0^* < b_1^* < \dots < b_n^*$ in such a way that

$$b^{(n)}(v) = \begin{cases} b_k^*, & v_k^* \leq v < v_{k+1}^*, k = 0, \dots, n \end{cases}$$

is an equilibrium first round bidding function.

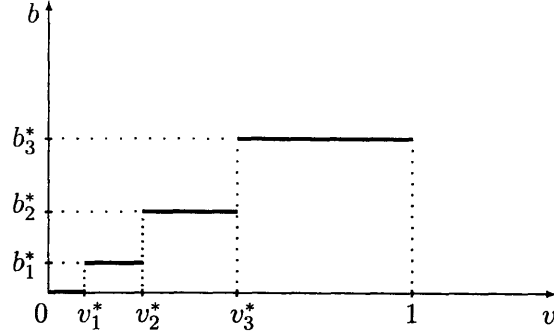


Figure 1-2: Equilibrium step bidding function $b_3(v)$.

Indeed, suppose that player 1 has valuation v and player 2 adheres to the above strategy. Player 1 may choose any of the following strategies: bid b_k^* , or bid above b_k^* but below b_{k+1}^* (I use subscript $< b_{k+1}^*$ to denote this latter strategy). Her expected payoffs from following these strategies are:

$$\pi_{b_k^*}(v) = \begin{cases} \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - b_i^*) + \frac{v_{k+1}^* - v_k^*}{2}(v - b_k^*) + \left(1 - \frac{v_{k+1}^* + v_k^*}{2}\right) v \bar{v}_k, & v \leq \bar{v}_k, \\ \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - b_i^*) + \frac{v_{k+1}^* - v_k^*}{2}(v - b_k^*) + \left(1 - \frac{v_{k+1}^* + v_k^*}{2}\right) \frac{v^2 + \bar{v}_k^2}{2} & \text{otherwise} \end{cases}$$

and

$$\pi_{< b_{k+1}^*}(v) = \begin{cases} \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - b_i^*) + (v_{k+1}^* - v_k^*)(v - b_k^*) + (1 - v_{k+1}^*)vc, & v \leq c, \\ \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - b_i^*) + (v_{k+1}^* - v_k^*)(v - b_k^*) + (1 - v_{k+1}^*) \frac{v^2 + c^2}{2} & \text{otherwise,} \end{cases}$$

where $\bar{v}_k = \min\left\{\frac{v_{k+1}^* + v_k^*}{2} + c, v_k^* + \sqrt{2c(v_{k+1}^* - v_k^*)}\right\}$ is the minimum valuation of player 3 which induces him to enter, conditional on him observing the price of b_k^* in the first round. We assume that if the entrant observes any deviation from $b^{(n)}(v)$, he believes the deviant to be of type 0 and enters whenever his valuation is above c .

To make $b^{(n)}(v)$ symmetric equilibrium, I must ensure that it is optimal for a bidder to play $b^{(n)}(v)$ if she expects the other first round bidder to do so. I do it by choosing v_k^* and b_k^* appropriately. In particular, choose these values in such a way that the following series of equations is satisfied:

$$\pi_{b_k^*}(v_{k+1}^*) = \pi_{< b_{k+1}^*}(v_{k+1}^*) = \pi_{b_{k+1}^*}(v_{k+1}^*), \quad k = 0, 1, \dots, n. \quad (1.2)$$

That is, if a player has valuation v_{k+1}^* , she is just indifferent between bidding b_k^* , bidding $< b_{k+1}^*$, and bidding b_{k+1}^* . The number of steps n is chosen in such a way that one more equality of this kind results in the value of v_{n+1}^* above 1 (it will follow that there will be finitely many steps).

Note that these equations have sequential nature. That is, value v_1^* can be determined from one single equation $\pi_0(v_1^*) = \pi_{<b_1^*}(v_1^*)$. Then values v_2^* and b_1^* can be simultaneously determined from equations $\pi_{b_1^*}(v_1^*) = \pi_0(v_1^*)$ and $\pi_{b_1^*}(v_2^*) = \pi_{<b_2^*}(v_2^*)$ and so on.

Proposition 5 $v_1^* > c$.

Proof: Assume the converse. Then $\pi_0(v_1^*) = \pi_{<v_1^*}(v_1^*)$ becomes

$$\frac{v_1^{*2}}{2} + (1 - \frac{v_1^*}{2})(\frac{v_1^*}{2} + c)v_1^* = v_1^{*2} + (1 - v_1^*)v_1^*c,$$

which has the only nonzero solution $v_1^* = 2c$, contradicting the assumption, QED.

Proposition 6 For $k > 1$, $v_k^* \geq v_{k-1}^* + 2c$ and hence $\bar{v}_{k-1} = v_{k-1}^* + \sqrt{2c(v_k^* - v_{k-1}^*)}$.

Proof: Assume the converse, i.e., that $v_k^* < \bar{v}_{k-1}$. Equation $\pi_{<b_k^*}(v_k^*) = \pi_{b_k^*}(v_k^*)$, after cancelling common terms, becomes

$$(1 - v_k^*)\frac{v_k^{*2} + c^2}{2} = -\frac{v_k^* - v_{k-1}^*}{2}(v_k^* - b_{k-1}^*) + \left(1 - \frac{v_k^* + v_{k-1}^*}{2}\right)\bar{v}_{k-1}v_k^*,$$

and equation $\pi_{b_{k-1}^*}(v_{k-1}^*) = \pi_{<b_{k-1}^*}(v_{k-1}^*)$ becomes

$$(1 - v_{k-1}^*)\frac{v_{k-1}^{*2} + c^2}{2} = \frac{v_k^* - v_{k-1}^*}{2}(v_{k-1}^* - b_{k-1}^*) + \left(1 - \frac{v_k^* + v_{k-1}^*}{2}\right)\bar{v}_{k-1}v_{k-1}^*.$$

Adding the last two equations gives, after some rearrangement,

$$v_k^{*2} + v_{k-1}^{*2} - v_k^*v_{k-1}^* - \frac{v_{k-1}^{*3} + v_k^{*3}}{2} = \left(1 - \frac{v_k^* + v_{k-1}^*}{2}\right)[\bar{v}_{k-1}(v_k^* + v_{k-1}^*) - c^2],$$

and, after cancelling $\left(1 - \frac{v_k^* + v_{k-1}^*}{2}\right)$,

$$v_k^{*2} + v_{k-1}^{*2} - v_k^*v_{k-1}^* = \bar{v}_{k-1}(v_k^* + v_{k-1}^*) - c^2.$$

Now by assumption $\bar{v}_{k-1} < v_k^*$ and hence

$$v_k^{*2} + v_{k-1}^{*2} - v_k^* v_{k-1}^* > v_k^* (v_k^* + v_{k-1}^*) - c^2,$$

$$v_{k-1}^{*2} - v_k^* v_{k-1}^* > v_k^* v_{k-1}^* - c^2,$$

which is clearly a contradiction since the value on the left is negative and the value on the right is positive (by Proposition 5). This completes the proof.

Corollary. There are finitely many steps in $b(v)$.

Proposition 7 *Solving (1.2) results in $b_k^* < b_{k+1}^*$.*

Proof: In view of Proposition 6, equation (1.2), after cancelling common terms, takes the form

$$\begin{aligned} \frac{v_{k+1}^* - v_k^*}{2} (v_k^* - b_k^*) + \left(1 - \frac{v_k^* + v_{k+1}^*}{2}\right) v_k^* \bar{v}_k &= (1 - v_k^*) \frac{v_k^{*2} + c^2}{2} = \\ -\frac{v_k^* - v_{k-1}^*}{2} (v_k^* - b_{k-1}^*) + \left(1 - \frac{v_k^* + v_{k-1}^*}{2}\right) \frac{\bar{v}_{k-1}^2 + v_k^{*2}}{2}. \end{aligned} \quad (1.3)$$

Dividing through by $(1 - v_k^*)$ gives

$$\alpha_k A_k + (1 - \alpha_k) B_k = M_k = \beta_k C_k + (1 - \beta_k) D_k$$

for some $\alpha_k \in [0, 1]$ and $\beta_k < 0$, where $A_k = v_k^* - b_k^*$, $B_k = v_k^* \bar{v}_k$, $C_k = v_k^* - b_{k-1}^*$, $D_k = \frac{\bar{v}_{k-1}^2 + v_k^{*2}}{2}$ and $M_k = \frac{v_k^{*2} + c^2}{2}$. Note that $D_k > M_k$ which, together with $\beta_k < 0$ implies $C_k > D_k > M_k$. On the other hand, $B_k > v_k^{*2} > M_k$, which, together with $\alpha_k \in [0, 1]$ implies $A_k < M_k$. Hence $A_k < C_k$ and $b_k^* > b_{k-1}^*$, QED.

Note that inequalities $A_k < B_k$ and $C_k > D_k$ also prove the following

Proposition 8 *Following $b(v)$, a player with valuations close to the left end of a segment $[v_k^*, v_{k+1}^*]$ (for $k \geq 1$) strictly prefers to lose in the first round and then to pass on the the second round. On the contrary, a player with valuation close to the right end of a segment, prefers to win right away rather than to pass on to the second round.*

Finally, I have to prove that no deviation from $b^{(n)}(v)$ makes any player better off. This is done in the following simple steps.

Step 1. Note that $\pi'_{b_k^*}(v) < \pi'_{b_{k+1}^*}(v)$ for any v . This implies that since, by construction, $\pi_{b_k^*}(v_{k+1}^*) = \pi_{b_{k+1}^*}(v_{k+1}^*)$, bidding any b_i^* other than b_k^* does not give higher payoff than bidding b_k^* if $v \in [v_k^*, v_{k+1}^*]$.

Step 2. Note that $\pi_{<b_{k+1}^*}(0) < \pi_{b_k^*}(0)$ for any v . This, together with $\pi_{<b_{k+1}^*}(v_{k+1}^*) = \pi_{b_k^*}(v_{k+1}^*)$, implies that bidding more than b_k^* is no better than bidding $b^{(n)}(v)$ for any $v < v_{k+1}^*$.

Step 3. Note that $\pi'_{<b_k^*}(v) < \pi'_{b_k^*}(v)$ for any $v > v_k^*$. This, together with $\pi_{<b_{k+1}^*}(v_k^*) = \pi_{b_k^*}(v_k^*)$ implies that bidding less than b_k^* is no better than bidding $b^{(n)}(v)$ for any $v > v_k^*$.

Proposition 9 *The three steps above prove that $b_n(v)$ is best response for $b_n(v)$.*

Proof is omitted.

1.5 Uncertain entry

In this section I extend the analysis to the case of uncertain entry. I now introduce a probability q that player 3 is present and able to compete (if he chooses to). With the complementary probability $1 - q$ entry is impossible for exogenous reasons, regardless of the first round bidders' strategies.

The following lemma characterizes equilibria in this game.

Lemma 2 *Consider a step function*

$$b_n(v) = \begin{cases} b_k^*, & v_k^* \leq v < v_{k+1}^*, k = 0, \dots, n \end{cases}$$

that is an equilibrium bidding function for $p = 1$ (i.e., with no exogenous barriers to entry).

Then function

$$b_n^p(v) = \begin{cases} p \cdot b_k^*, & v_k^* \leq v < v_{k+1}^*, k = 0, \dots, n \end{cases}$$

is an equilibrium bidding function for any $p \in [0, 1]$.

Proof: It is straightforward to compute the profit functions for both equilibrium bidding $b^{(n)}(q, v)$ and any deviation from it. The only term that changes in the expressions, compared to the case of $q = 1$ studied above, is the expected payoff in round two, which is now a weighted average of what it was before and the valuation of the incumbent (corresponding to the case when the entry does not happen for exogenous reasons). We have

$$\pi_{q \cdot b_k^*}(v) = \begin{cases} \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - qb_i^*) + \frac{v_{k+1}^* - v_k^*}{2} (v - qb_k^*) \\ \quad + \left(1 - \frac{v_{k+1}^* + v_k^*}{2}\right) ((1-q)v + qv\bar{v}_k), & v \leq \bar{v}_k, \\ \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - qb_i^*) + \frac{v_{k+1}^* - v_k^*}{2} (v - qb_k^*) \\ \quad + \left(1 - \frac{v_{k+1}^* + v_k^*}{2}\right) \left((1-q)v + q\frac{v^2 + \bar{v}_k^2}{2}\right) & \text{otherwise,} \end{cases}$$

and

$$\pi_{<q \cdot b_{k+1}^*}(v) = \begin{cases} \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - qb_i^*) + (v_{k+1}^* - v_k^*)(v - qb_k^*) \\ \quad + (1 - v_{k+1}^*) ((1-q) + qvc), & v \leq c, \\ \sum_{i=0}^{k-1} (v_{i+1}^* - v_i^*)(v - qb_i^*) + (v_{k+1}^* - v_k^*)(v - qb_k^*) \\ \quad + (1 - v_{k+1}^*) \left((1-q) + q\frac{v^2 + c^2}{2}\right) & \text{otherwise.} \end{cases}$$

By observation, all inequalities that support $b^{(n)}(q, v)$ as an equilibrium follow from respective inequalities for $b^{(n)}(v)$, QED.

The above lemma shows that even if the entry is uncertain, the equilibrium as a step function still exists. Moreover, there is a simple characterization of its parameters: the break points v_i^* are the same, while the equilibrium bids are proportional to the exogenous probability of entry. The intuition is that if the entry is less probable, the incumbents will be reluctant to take risk of winning the item in the first round by bidding high, since their hope is now stronger that the entrant will not be around and they will get the item for free in the second round. That is why the equilibrium bids are decreasing when the probability of no entry (equal to $(1-q)$) increases. In particular, if $q = 0$, the equilibrium bid is identically equal to zero – if there is no entry threat, the first-round bidders can get one unit each at zero price.

1.6 Multiple incumbents

In this section I extend the analysis to the case of more than two incumbents (and no exogenous barriers to entry). Assume that there are two rounds of ascending price auction with identical items for sale one in each round and $n + 1$ players, first n of which are present for both rounds and the $(n + 1)$ th one can only enter to the second round, if he chooses to do so, at costs c . Each player wants at most one item and the winner of the first round quits after she gets it. All players' valuations are independently drawn from the uniform distribution on $[0, 1]$. I am looking for an equilibrium in this new game.

For this scenario it is important to know what exactly the entrant observes. Note that in the case of two first-round bidders, all the available information relevant for the entry decision is contained in the final price of the first round. It coincides with the strategy of the loser, which is as much as the entrant can know at best. Moreover, the ascending price auction is isomorphic to the second price sealed bid auction. Here I assume that player $n + 1$ observes the entire outcome of the first round, i.e., who dropped out at what price.

In case of multiple incumbents, a bidding strategy in the first round not only specifies the price at which to drop out if nobody else dropped out before, but also how to react to the dropping out of other incumbents (and that is where the ascending price auction is different from the second price auction). The final price (at which the last incumbent drops out) does not convey all available information about the strategy of the first-round bidders, because it does not show who dropped out at what price before. Therefore, I assume that the entrant not only observes the outcome (the final price and the identity of the winner) of the first round of the auction, but also the bidding process (who dropped out at what price).

A subgame perfect symmetric equilibrium in weakly undominated strategies that I am looking for will therefore consist of:

- A 'first drop out' function $d_1(v)$ specifying the price at which an incumbent of type v drops out, provided that nobody dropped out before;
- A 'second drop out' function $d_2(v, d_1)$ that specifies the price at which an incumbent drops out if the only other incumbent who dropped out before did it at price d_1 ; and so on, to

- A ‘last drop out’ function $d_{n-1}(v, d_1, d_2, \dots, d_{n-2})$ that specifies the price at which an incumbent drops out if there is only one other incumbent left:
- Beliefs of player $n+1$ about valuations of the incumbents as a function of v, d_1, \dots, d_{n-1} ;
- An entry decision of player $n + 1$ as a function of his beliefs.

As before, once the entry decision is made and the game proceeds to the second round, the only weakly undominated strategies that remain to the players are to drop out at their valuations.

Lemma 3 *A subgame perfect symmetric equilibrium in weakly undominated strategies exists, in which*

- $d_1(v) = v$,
- $d_2(v, d_1) = v$ and so on to $d_{n-2}(v, d_1, \dots, d_{n-3}) = v$,
- $d_{n-1}(v, d_1, d_2, \dots, d_{n-2})$ is a step function with its breakpoints and step bids being functions of d_{n-2} .

Lemma 3 asserts that all incumbents except for two last ones drop out at their true valuations and the two who remain play strategies similar to those of the model with two incumbents.

Proof: Clearly none of the first $n - 2$ players can gain by deviating from the suggested strategies – none of them can win neither of the two items except at price that exceeds their valuations. Suppose now that the price reached d_{n-2} in the first round and there are two players left. Each of them believes that the other is following the strategies suggested above, so she updates the distribution of his rival to uniform with support $[d_{n-2}, 1]$. With probability d_{n-2} the valuation of player $n + 1$ will not exceed d_{n-2} and he will not enter. With the complementary probability $1 - d_{n-2}$ his valuation will be at least d_{n-2} , in which case the game will be similar to the one with two incumbents, for which we know that there exists a step function equilibrium. Therefore, the whole reduced form game is equivalent to the game with two incumbents and exogenous entry barriers discussed in previous section, so Lemma 2 applies. This completes the proof.

1.7 Conclusion

The chapter has studied how the desire to deter future entrants affects present bidding behavior in a sequential ascending price auction. The equilibrium that I identified - with bidding strategies being simple step-functions - suggests that bidders will be very conscious about the potential for future entry and that they will want to conceal information about their private valuations in an effort to deter entry. As a result, the outcome of the auction is not efficient: the two objects do not necessarily go to the bidders who value the objects the most. There are two sources of inefficiency, both caused by the coarseness of the information communicated through the step-function strategies. One source is that the potential entrant may not enter even though he has a higher valuation than the bidder that dropped out in the first round. The other source of inefficiency is that both first-round bidders may drop out at the same time in which case the wrong first-round bidder may get the object.

These conclusions resemble in some respects those in Bhattacharyya [2]. Bhattacharyya studies a two-person, two-stage auction in which entry is endogenous in the first stage: first one player bids and then the other player decides whether to enter. If the other player enters an ascending auction begins, else the first bidder gets the item at the price he bid. Assuming that the second player doesn't enter if he is indifferent (a crucial assumption), there exists a non-trivial first-round bid (equal to half the true value when the distribution is uniform). This result is similar to mine in that the first bidder behaves strategically to deter entry. However, the setting is strategically very different from mine, since in an ascending auction the winning bidder only pays the second highest bid. This opens the door for bluffing, which indeed occurs in my equilibrium, but not in Bhattacharyya's.

The resulting family of step function equilibria also resemble partition equilibria in Crawford and Sobel [4]. In their paper they consider a wide class of games where an informed party (sender) sends a signal to an uninformed party (receiver) about sender's type which is of relevance to the payoffs of both. They find that under some assumptions on the structure of the payoffs the optimal signal for the sender consists of a subset of the range of types to which his type belongs. However, they assume away any uncertainty about the receiver's own type, which precludes their result from being at least directly applicable to the context of private value auctions. In their model equilibria are also multiple; they can, however, rank them in terms of coarseness of the signal. In my model equilibria typically

can not be ranked in a similar way, since not only there typically exist equilibria with different number of steps in the bidding function, but also there is some leeway in the way that break points are chosen.

Many questions remain to be studied in sequential auctions with potential entry in later rounds. First, I do not know whether the step function equilibrium I have identified is the only equilibrium. Second, it would be of interest to analyze the optimal entry cost that an auctioneer should charge (and whether later entry should be cheaper than early entry). More generally, the question of an optimal auction design, including optimal information transmission (what should potential entrants be allowed to see), remains open.

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Chapter 2

Bargain or Post the Price?

2.1 Introduction

This chapter studies pricing behavior of sellers on markets where not only price but the trading rule is a choice variable. The question that is addressed here is why some retailers choose to post the prices of their merchandize and others choose not to post prices, but instead bargain with potential buyers.

The phenomenon that originally motivated this study is pottery selling in the Old City of Jerusalem (also observed on many other similar markets). There are many shops that sell similar pottery; sometimes these shops are located next door to each other, but still follow different pricing policies. Some sellers have prices of their goods posted, while others do not have posted prices and expect a potential buyer to inquire about price and then bargain on it. The question of particular interest in this setup is how the two modes of trade can coexist within the same market, in shops operating in the same location.¹

There are two empirical observations about pottery shops in Jerusalem, registered by anyone who ever tried to buy merchandize in both. First, shops that post prices are usually bigger than those that do not. Second, although the goods look very similar to each other, in fact those sold at fixed price turn out to be of higher quality. The theory that I build in this chapter helps explain both these phenomena.

There has been a literature on the choice of price posting versus bargaining. The emphasis in these studies is put mostly on comparing the two modes of trade under various

¹Guide books on Jerusalem describe haggling as the prevailing mode of trade on a bazaar, but also specifically mention a few shops that commit to posted prices.

specifications of costs and benefits. The present chapter is the first study, to the best of my knowledge, where the coexistence of both modes of trade is addressed.

Another market that exhibits a similar phenomenon is the market for cars, both new and used. In this market there are also retailers who bargain and retailers who sell at fixed prices; however, a number of differences make it difficult to apply the logic usually put forward in the context of the car market to the pottery market.

One obvious difference is that a car is a significantly more important purchase than a piece of pottery; consequently, a buyer is likely to have spent time thinking about what car she wants, what its quality parameters (e.g., features and performance) are, etc.; therefore a buyer has a good idea about her value for a car of a specific model. In contrast, such prior analysis is hardly possible in a pottery market, since the merchandise is to a large extent specific to the place and the buyer typically possesses little information about its quality.

Second, on the pottery market, there are little or no opportunities for the seller to advertise his goods and to build any reputation of selling high quality merchandise – each buyer is likely to be there only once and there is little chance that she will remember what shop she visited once she gets home, so most likely a new buyer will be ignorant about the reputation of each place.² The one-off nature of trade limits the applicability of price-quality or advertising signaling associated with repeated interactions (such as in Nelson [7], Schmalensee [8], Shapiro [9], Kihlstrom and Riordan [5]). On the other hand, some buyers may in fact be experts and be able to assess the quality of goods prior to buying. I show that under certain conditions even if the fraction of informed buyers is very small, sellers of different types will be willing to choose different pricing policies.

Finally, the entire pottery market is located within a very small area, so a typical customer may well be able to inspect many shops prior to making a decision. This is not so much the case in the car market, where convenient dealership location has been mentioned by 49% of the buyers as one of the reasons for first visiting the dealership where they actually purchased the car³.

A number of papers has studied the seller's choice between bargaining and price posting under assumptions that are arguably more relevant for car markets than for pottery markets. Wernerfelt ([12]) focuses his analysis on heterogeneous buyers facing transportation and

²Although tourist guides sometimes recommend specific shops, none of them explicitly claims that merchandise in these shops has superior quality.

³I am grateful to J.D.Powers and the International Motor Vehicle Program at MIT for this data.

inspection costs that a buyer has to incur whether or not a purchase actually happens. This creates opportunities for holdup on the part of sellers. Similarly, Bester ([2]) emphasizes costs that a buyer faces when switching from one shop to another, while buyers are assumed to be homogeneous in their preferences and the quality is assumed to be observable once a buyer enters a shop. Consequently, there is no scope for quality signaling and the resulting predominant trade structure (bargaining or posting the price) is determined by the force of the lock-in effect, i.e., by the relative bargaining power of the parties and buyer's switch costs; no arrangements are considered with both modes of trade simultaneously present in the market. Wang ([11]) compares the two modes of trade based on the costs of each, i.e., displaying costs versus costs of bargaining. However, his study is focused on the case of a single seller and does not aim at explaining the coexistence of the two modes of trade.

Wolinsky ([13]) builds a model in which buyers are different in their willingness to pay for quality and prices serve as signals of quality (which is not directly observable to consumers) at the separating equilibrium; to establish the existence of a separating equilibrium, he assumes that buyers get noisy signals on quality (other than prices). His setup does not allow for bargaining.

There has also been a number of studies that compare and contrast posting a price with running an auction (in connection with e-Bay buyout option). Zeithammer ([14]) argues that posting a price is more appropriate than auctioning when the seller has a large number of sufficiently uniform items of a good, since the costs of posting the price (in particular, of choosing the price) is only incurred once whereas auctioning has per unit costs.⁴

The main purpose of this chapter is to study price posting versus bargaining choice as well as coexistence of the two modes in a setting that models souvenir markets in tourist areas. The key ingredients of the model that I build are the following: buyers do not observe the quality of the good until after they buy it; buyers are heterogeneous in their taste for quality; buyers can costlessly observe pricing policy of all operating shops; finally, sellers are professionals in the sense that they can estimate buyer's valuation of the good upon seeing him and they bargain better than buyers (in the sense of having higher bargaining power). Under these assumptions I show how the choice to post the price may be a signal of high quality and how sellers voluntarily adopt different modes of trade.

The rest of the chapter is organized as follows. The model is presented in Section

⁴See also [10].

2.2. Section 2.3 studies the case of a single seller in full detail. In Section 2.4 the case of two sellers is studied and sufficient conditions are derived for existence of a separating equilibrium. Section 2.5 refers to large markets with endogenous quality choice. Section 2.6 concludes.

2.2 The Model

There is a unit mass of potential buyers and a finite number of sellers in the market for a certain good, differentiated in quality. The good may be either of high quality (or type) q_H or of low quality q_L , such that $q_H > q_L > 0$. Denote $\Delta q = q_H - q_L$.

Sellers are risk neutral and each seller may carry only one type of good. Supplying a unit of good of quality q_H costs $c_H = 1$ to the seller; supplying a good of quality q_L does not cost anything: $c_L = 0$. A seller incurs costs only if trade takes place.

Buyers are risk neutral. Each buyer needs at most one unit of the good. All buyers prefer a high quality good to a low quality good, but they differ in their willingness to pay for quality. In particular, each buyer has a type θ , a taste parameter that I assume is uniformly distributed on $[0, 1]$. The buyer's utility of obtaining a good of quality q (either q_H or q_L) and paying p for it is equal to $\theta q - p$; the buyer's reservation utility is zero. Assume that $\Delta q > 1$, that is, if a buyer is of low type it is efficient to supply her a low quality good, whereas if she is of high type it is efficient to supply her a high quality good.

Informational assumptions are as follows: each buyer knows her type; each seller knows the quality of his good. With the exception of the section on large markets, I assume that a buyer does not know the quality of the good until after she has bought it (when I discuss large markets, I assume that some fraction of buyers are informed and observe the quality of the good). Once a seller sees a buyer, the seller observes her type θ .

The timing is as follows. First, the quality of goods that all sellers have is chosen simultaneously. Under the endogenous quality specification, sellers choose quality of their goods themselves; under the exogenous quality specification, the quality of the good each seller has is picked randomly and independently, with λ being the probability that the quality is high ($q = q_H$). Denote average quality as $\bar{q} = \lambda q_H + (1 - \lambda)q_L$. Next, sellers observe each other's types and choose their pricing policy, that is, whether to post the price and, if so, at what level. Then buyers arrive and costlessly observe all posted prices. Each

buyer chooses whether to buy the good and, if so, at what shop. If the chosen shop has a posted price, she buys at that price (for simplicity I assume that a posted price can not be negotiated). Otherwise, she bargains with the seller.

I consider the following very simple bargaining procedure: either party is randomly selected to make a take it or leave it offer to the other party. With probability α the buyer gets to make the offer; with probability $1 - \alpha$ the seller makes the offer. Under full information and risk neutrality, the outcome of such bargaining is represented by the generalized Nash bargaining solution, in which the buyer receives share α of the gains from trade and the seller receives share $1 - \alpha$ on average. I assume that the seller is more skilled in bargaining and appropriates a larger share of bargaining surplus: $\alpha < \frac{1}{2}$. There are three factors that determine the outcome of the bargaining. First, the final price depends on the buyer's type θ ; other things equal, the higher θ , the higher the expected payoff to both parties. Second, the outcome depends on seller's perceived quality q and costs c ; although these are not directly observable to buyers, in equilibrium a buyer may be able to correctly infer them. Finally, the disagreement point of the buyer of type θ depends on the other sellers' pricing policies: if there is another seller that offers a good that a buyer believes to be of quality q at (posted) price p , that buyer's disagreement point is $\theta q - p$ (or the best of such offers if there is more than one). This is the way in which sellers impose an externality on each other: the more aggressive a price poster is, the lower the expected revenue of a bargainer will be.

If there is more than one seller who does not post the price, the buyer can only bargain with one of them. That is, if the buyer approaches one of the sellers, begins bargaining with him but does not reach an agreement, all other sellers who bargain observe it and do not trust this buyer anymore, so she can not subsequently bargain with anybody else (she can still buy at a posted price though).⁵ I need this assumption to make sure that the buyer does not default by refusing to trade if he is not chosen to make a take the offer (in the hope that he will have luck with another seller).⁶

The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Although I mention pooling equilibria (in which sellers of both types choose the same pricing policy) the main

⁵Guide books specifically suggest that you do not engage in bargaining unless you are prepared to buy the good.

⁶An alternative assumption could be that each buyer can try his luck with each seller who posts no price, resulting in his effective bargaining power being equal to $1 - (1 - \alpha)^k$ where k is the number of sellers who post no price. This specification will complicate algebra without significantly altering the conclusions.

focus is on equilibria in which buyers are able to correctly infer the quality of the good from the seller's pricing policy. To distinguish between two alternative specifications of quality choice, I refer to *separating* equilibria in environments where the quality of the good (or the type of the seller) is exogenous and sellers of different types choose different strategies. I refer to *signaling* equilibria in environments where sellers are ex ante identical, but some of them choose high and others low quality and, importantly, subsequently adopt different pricing policies that signal their choice.

2.3 Single seller and exogenous quality

In this section I study equilibria in a game where there is only one seller who does not strategically choose the quality of his good.

Proposition 10 *There always exists a pooling equilibrium in which the seller of neither type posts the price.*

Proof. Let all buyers believe that any good with a posted price is of low quality and that if the price is not posted, the good is of high quality with probability λ and low quality with probability $1 - \lambda$. Clearly these beliefs are consistent with the seller of neither type posting the price. All there is to show is that the seller of neither type wants to deviate and post a price instead.

Under these beliefs, the buyer of type θ is expecting the surplus of $\theta\bar{q}$ and with probability $1 - \alpha$ the seller will get to make the offer and thus to expropriate this surplus. If the seller is of high type, he will only be trading with buyers of type $\theta = \frac{1}{\bar{q}}$ or above, making his total expected profit equal to

$$(1 - \alpha) \int_{\frac{1}{\bar{q}}}^1 (\theta\bar{q} - 1) d\theta_L = \frac{1 - \alpha}{2} \cdot \frac{(\bar{q} - 1)^2}{\bar{q}}.$$

Alternatively he may post some price p ; by assumption, he will then be believed to have low quality good and only buyers with type $\theta = \frac{p}{q_L}$ or above will buy the good,⁷ making his profit equal to

$$\left(1 - \frac{p}{q_L}\right) (p - 1).$$

⁷If $q_L < 1$, the high type seller will be making no profit if he posts a price, since he can only be making sales at prices below his marginal costs $c = 1$.

The seller's profit is maximized at $p = \frac{q_L+1}{2}$ and equals $\frac{(q_L-1)^2}{4q_L} < \frac{1-\alpha}{2} \cdot \frac{(\bar{q}-1)^2}{\bar{q}}$. The last inequality follows from $\alpha < \frac{1}{2}$ and $q_L < \bar{q}$ (and also from the fact that $g(x) = \frac{(x-1)^2}{x}$ is an increasing function for $x \geq 1$). Therefore, high type seller will not be willing to deviate.

The low type seller's profit from bargaining is equal to

$$(1 - \alpha) \int_0^1 \theta q_L d\theta = \frac{(1 - \alpha)q_L}{2},$$

while his profit if he deviates and posts a price p is

$$\left(1 - \frac{p}{q_L}\right)p,$$

maximized at $p = \frac{q_L}{2}$ at the level $\frac{q_L}{4} < \frac{(1-\alpha)q_L}{2}$, so he will not be willing to deviate either.

This completes the proof.

Proposition 11 *Assume $\alpha < \frac{1}{2}$, i.e., the seller has higher bargaining power than the buyer. Then in any separating equilibrium, i.e., in any equilibrium where pricing behavior of the seller fully reveals his type, a seller of high type posts the price and a seller of low type does not post a price.*

Proof: Assume the converse. That in a separating equilibrium buyers must believe that the good with no posted price is of high quality. Then the low seller posts some price p , and by assumption that price signals low quality. The low type seller will then serve buyers with type θ greater than or equal to $\frac{p}{q_L}$, earning total profit of $(1 - \frac{p}{q_L})p$, which is not greater than $\frac{q_L}{4}$. On the other hand, if he deviates and posts no price, by assumption he will be believed to be of high type and a buyer with type θ will be ready to pay up to θq_H for the good. With probability $(1 - \alpha)$ the seller will be making the offer and will be able to extract the entire surplus θq_H (with the remaining probability α the seller will be earning no profit). The total expected profit of the low type seller if he posts no price is therefore equal to

$$(1 - \alpha) \int_0^1 \theta q_H d\theta = \frac{(1 - \alpha)q_H}{2} > \frac{q_L}{4},$$

so the low type seller will be willing to deviate and post no price, which contradicts the assumption of an equilibrium.

Therefore, if the seller posts no price, in any separating equilibrium he is believed to be of low type. His expected profit from not posting the price is then equal to $\frac{(1-\alpha)q_H}{2}$, which is greater than $\frac{q_L}{4}$. Hence, the low type seller will post no price in any separating equilibrium, QED.

It follows that in any separating equilibrium the high type seller will be posting some price p and the low type seller will be posting no price. Equilibrium conditions require that the seller of neither type wants to imitate the other type. The profit that the low type seller earns if he imitates the high type seller is equal to $(1 - \frac{p}{q_H})p$. If instead the high type seller posts no price his good will be believed to have low quality and be valued at θq_L by a buyer with type θ . In particular, if $q_L < 1$ he will not come to an agreement with any buyer and will be earning zero profit; if $q_L \geq 1$ he will trade with buyers with types $\theta \geq \frac{1}{q_L}$ and only if he will get to make the take it or leave it offer (which happens with probability $1 - \alpha$) and his expected profit will be equal to

$$(1 - \alpha) \int_{\frac{1}{q_L}}^1 (\theta q_L - 1) d\theta = \frac{1 - \alpha}{2} \cdot \frac{(q_L - 1)^2}{q_L}.$$

Therefore, the necessary condition for existence of a separating equilibrium are that there exists p such that both incentive compatibility constraints are satisfied:

$$IC_H : \quad \left(1 - \frac{p}{q_H}\right) (p - 1) \geq \frac{1 - \alpha}{2} \cdot \frac{(\min\{0, q_L - 1\})^2}{q_L}, \quad (2.1)$$

$$IC_L : \quad \frac{(1 - \alpha)q_L}{2} \geq \left(1 - \frac{p}{q_H}\right) p. \quad (2.2)$$

It is easy to see that this condition is also sufficient: take such p and let buyers believe that goods priced at p are of high quality and all other goods are of low quality. Then the high type seller will choose to post price p and the low type seller will choose to post no price; and conditions (2.1) and (2.2) ensure that the seller of neither type is willing to deviate, i.e., this is a separating equilibrium.

The following two propositions help to restate the necessary and sufficient condition derived above in terms of q_H , q_L and α .

Proposition 12 *If $q_L \leq 1$, a separating equilibrium always exists.*

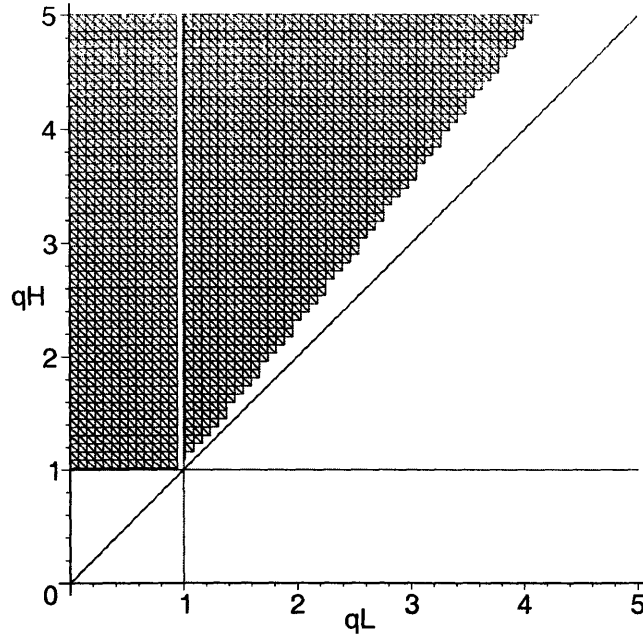


Figure 2-1: $\{q_L, q_H\}$ pairs for which a separating equilibrium exists on the market with a single seller (drawn for $\alpha = \frac{1}{3}$).

Proof: If $q_L \leq 1$, incentive compatibility for high type (2.1) is satisfied for any $p \in [1, q_H]$. The other incentive compatibility (2.2) is satisfied for p close enough to q_H , QED.

Proposition 13 *Suppose $q_H > q_L > 1$. Then a separating equilibrium exists if and only if*

$$\frac{(q_H - 1)^2}{4q_H} \geq \frac{1 - \alpha}{2} \cdot \frac{(q_L - 1)^2}{q_L}. \quad (2.3)$$

Proof: Necessity. At the price posted by the high type seller, condition (2.1) must be satisfied; hence it must at least be satisfied for price $p^M = \frac{1+q_H}{2}$ that maximizes the left hand side of (2.1); this gives (2.3).

The proof of *sufficiency* is more technical and is relegated to the appendix.

Therefore, if the single seller has substantially higher bargaining power than the buyer then in any separating equilibrium the high type seller posts the price and the low type seller bargains. Moreover, if either high quality good is costly to produce compared to the value of the low quality good or the quality of both types of goods are sufficiently valuable compared to the high type production costs, a separating equilibrium exists. High type seller does not want to pretend to be of low type, because he would then get few buyers ready to cover even his costs, and low type seller does not want to pretend to be of high

type, because he does not want to lose the large low type segment of the market.

To address the welfare properties of the equilibria found above, one must first calculate the expected surplus of the buyer with type θ . Consider pooling equilibrium first, with no price posted. If the seller (who observes buyer's type) gets to make the offer, the buyer will receive no surplus. If the buyer gets to make the offer, he faces a choice: either to make offer zero, which only the low type seller will accept, or to make offer one, which the seller of either type will accept. In the first case, his expected payoff is $\alpha(1 - \lambda)\theta q_L$, in the second case it is $\alpha(\theta\bar{q} - 1)$. It is easily verified that a low type buyer will choose the first option and a high type buyer will choose the second option. Buyer's expected payoff in the pooling equilibrium is equal to

$$\Pi_p^*(\theta) = \begin{cases} \alpha(1 - \lambda)\theta q_L, & \text{for } \theta \leq \frac{1}{\lambda q_H}, \\ \alpha[(1 - \lambda)\theta q_L + \lambda\theta q_H - 1], & \text{for } \theta > \frac{1}{\lambda q_H}. \end{cases} \quad (2.4)$$

In contrast, in a separating equilibrium with posted high type seller price p the buyer's expected payoff equals

$$\Pi_s^*(\theta) = \begin{cases} \alpha(1 - \lambda)\theta q_L, & \text{for } \theta \leq \frac{p}{q_H}, \\ \alpha(1 - \lambda)\theta q_L + \lambda(\theta q_H - p), & \text{for } \theta > \frac{p}{q_H}. \end{cases} \quad (2.5)$$

Now I can compare the two types of equilibria in terms of their payoffs to the buyer. If $\lambda p \leq 1$, then all buyers (weakly) prefer separating equilibrium to pooling equilibrium with no posted price. If $\lambda p > 1$, then low type buyers are indifferent, medium type buyers (with $\theta \in [\frac{1}{\lambda q_H}, \frac{p}{q_H}]$) prefer pooling equilibrium and the preferences of high type buyers depend on α . In general, the higher the price in the separating equilibrium and the higher the probability that the seller is of low type, the more likely that buyers will prefer pooling equilibrium.

As for the seller, the low type seller obviously prefers pooling equilibrium. The high type seller gets $(1 - \alpha)\frac{(\bar{q}-1)^2}{2\bar{q}}$ in the pooling equilibrium and $\frac{(q_H-1)^2}{4q_H}$ in the best separating equilibrium. Therefore, the lower α , the higher λ and the further signaling price p away from his optimal price $p^M = \frac{1+q_H}{2}$, the more likely he will prefer the pooling equilibrium to the separating equilibrium with no price.

To complete the description of the equilibria in a single seller case, one must also consider the possibility of a pooling equilibrium in which both types post the same price. This is done in the following proposition.

Proposition 14 *A necessary condition for the existence of a pooling equilibrium with posted price is $\bar{q} \geq 2(1 - \alpha)q_L$. If it is satisfied, then it is necessary and sufficient that at least one of the following two conditions are satisfied:*

$$\frac{\bar{q}2 - 1}{4\bar{q}} \geq \frac{(1 - \alpha)q_L}{2},$$

$$\frac{\bar{q} + \sqrt{\bar{q}^2 - 2(1 - \alpha)q_L\bar{q}}}{2} \geq \frac{q_L^2}{2q_L - 1}.$$

Proof is similar to that of Proposition 13 and is omitted.

Note that in the single seller case the low quality good seller is better off in equilibrium than his high quality counterpart. The intuition for this is that low type seller can always mimic high type seller by posting price and then make more money since his costs are lower. In particular, this observation implies that if the unique seller can choose the quality of the good upfront, he will choose low quality good; even if it is socially efficient to produce high quality good.⁸

In the next section I explore a market with two sellers and show that in a separating equilibrium the low type seller does not necessarily earn higher profits than the high type seller.

2.4 Two sellers: coexistence of the two modes of trade

In this section I show how the two modes of trade – posting the price and bargaining – can coexist on the market. I stick to the assumption of exogenous quality of the good.

Assume that there are two sellers who observe the quality of the each other's good and then choose their pricing policy. I am looking for a separating equilibrium, in which the seller of one type posts the price and the seller of the other type bargains. I will derive sufficient conditions on the parameters for existence of a separating equilibrium in which a seller of low type bargains and a seller of high type posts the price.

⁸This adverse selection phenomenon parallels findings in Bester ([1]).

One important insight that this section conveys is that in principle the range of parameters that support the equilibrium will expand if buyer's beliefs about the seller's type can be conditional on *both* sellers' strategies. To see this, I focus first on the more restrictive (but also more realistic) case when beliefs about seller's type are restricted to be a function of this seller's pricing policy alone; at the end of this chapter I remark on the more general case.

Assumption 1 *Buyers beliefs about the type of a seller depend on this seller's pricing policy alone and do not depend on the other seller's pricing policy.*

First I show that the result parallel to Proposition 11 holds: it can not be the case that in a separating equilibrium a low type seller posts the price and a high type seller bargains.

Proposition 15 *Suppose Assumption 1 holds. Then in any separating equilibrium a seller who does not post the price has the low quality good.*

Proof: Assume the converse. Consider an equilibrium in which the high type seller posts no price and the low type seller posts some price p .

The high type seller will be serving buyers with θ between $\frac{p}{q_L}$ and $\frac{1-p}{q_H-q_L}$. Buyers with θ below $\frac{p}{q_L}$ will not be buying at all and buyers with θ higher than $\frac{1-p}{q_H-q_L}$ will be bargaining, since the joint surplus from trade with the high type seller, $\theta q_H - 1$, will be above that for the low type seller, $\theta q_L - p$.

Therefore, the low type seller will be making profit equal to

$$\pi_L = \left(\frac{1-p}{q_H-q_L} - \frac{p}{q_L} \right) p, \quad (2.6)$$

which is maximized over p at $p = \frac{q_L}{2q_H}$ where the profit is equal to $\frac{q_L}{4q_H(q_H-q_L)}$.

However, the low type seller may consider mimicking the high type seller: post no price and bargain with buyers of type $\frac{1}{q_H}$ or above. By Assumption 1, he will then be believed to be of high type and earn profit equal to

$$\pi_{dev} = \frac{(1-\alpha)}{2} \int_{\frac{1}{q_H}}^1 \theta q_H d\theta = \frac{(1-\alpha)(q_H^2 - 1)}{2q_H} \quad (2.7)$$

The low type seller will be willing to deviate as long as expression $\pi_{dev} > \frac{q_L}{4q_H(q_H - q_L)}$; this inequality easily follows from $\alpha < \frac{1}{2}$ and $q_H - q_L > 1$. Therefore, the low type seller will be willing to pretend to be of high type, which is inconsistent with the definition of a separating equilibrium, QED.

For the rest of this section I will restrict attention to the case $q_L \leq 1$. This assumption implies that the high type seller will never be willing to signal the low type, since that way he will not be making any sales (perceived value of his good will not exceed his costs) and hence the only incentive compatibility constraint that remains to be checked is that the low type seller does not find it profitable to pretend to be of high type.

Consider a pair of strategies and beliefs such that the low type seller does not post a price, the high type seller posts price $p^* \leq \Delta q$ and the buyer believes that a seller has high quality good if and only if he posts price p^* ;⁹ if a buyer is indifferent between buying from two sellers, he buys from either with equal probabilities. For this pair of strategies and beliefs to constitute an equilibrium, it is necessary and sufficient that a low type seller does not want to deviate and post some price p , whether or not the other seller turns out to be posting price p^* or posting no price.

Proposition 16 *If buyers believe that a good is of high type is and only if its price is posted and is equal to p^* , a low type seller is better off posting no price than posting any price other than p^* .*

Proof: If the other seller posts no price, than posting any price other than p^* will result in no sales. Indeed, by assumption about buyers' beliefs, both sellers are then believed to be of low type. A buyer will always choose to bargain, holding the posted price p as his threat point: she will have a chance to win the entire surplus θq_L if she gets to make the offer or $\theta q_L - p$ otherwise, which is on average better than just $\theta q_L - p$ if she buys at posted price. Therefore, posting any price other than p^* is a bad idea for a low price seller if the other seller does not post a price.

If the other seller posts price p^* , thus signaling high type, posting any price other than p^* is not profitable either. The proof parallels that of Proposition 11 and is omitted.

⁹The high type seller will be making no sales if $p^* > \Delta q$ since even with $\theta = 1$ the buyer only gets the increment utility of Δq from buying high quality good rather than low quality good and a separating equilibrium with low type seller posting no price can only be sustainable if the high type makes some sales.

Therefore, the only deviation that may potentially be profitable is for a low type seller to pretend to be of high type and to post price p^* . If both sellers are of low type, than posting no price results in their splitting the market equally and getting $\frac{(1-\alpha)q_L}{4}$ each. If one of them deviated and posts price p^* , he is believed to be of high type and sells to all buyers with $\theta q_H - p^* \geq \theta q_L$ or $\theta \geq \frac{p^*}{\Delta q}$. His profit is then equal to $(1 - \frac{p^*}{\Delta q})p^*$ and a necessary condition for an equilibrium is

$$\frac{(1-\alpha)q_L}{4} \geq \left(1 - \frac{p^*}{\Delta q}\right)p^*. \quad (2.8)$$

If instead one seller is of high type and posts price p^* , then the other seller who is of low type and posts no price serves all buyers with $\theta < \frac{p^*}{\Delta q}$; his profit equals $\frac{(1-\alpha)q_L p^{*2}}{2\Delta q^2}$. If instead he deviates and posts price p^* , buyers will believe that both sellers are of high type and will buy from either with equal probability as long as $\theta q_H \geq p^*$, so that the low type seller will earn $\frac{1}{2}(1 - \frac{p^*}{q_H})p^*$. Hence low type seller's incentive compatibility condition is

$$\frac{(1-\alpha)q_L p^{*2}}{2\Delta q^2} \geq \frac{1}{2} \left(1 - \frac{p^*}{q_H}\right)p^*. \quad (2.9)$$

Therefore, a separating equilibrium exists (for $q_L \leq 1$) if and only if there exists $p^* \leq \Delta q$ such that both (2.8) and (2.9) are satisfied. Note that condition (2.9) is linear in p^* ; so the higher p^* the more is it likely that it is satisfied. In particular, if there exists $p^* \leq \Delta q$ that satisfies both (2.8) and (2.9), then so must $p^* = \Delta q$ (note that $p^* = \Delta q$ trivially satisfies (2.8)). Plugging $p^* = \Delta q$ into (2.9) results in condition $\alpha q_H \leq q_L$.

The following proposition summarizes the above considerations.

Proposition 17 *Suppose that beliefs of the buyer about the quality of the good each seller has can depend on this seller's pricing policy only. Then it a necessary condition for a separating equilibrium on the market with two sellers and exogenous quality to exist is $\alpha q_H \leq q_L$. If further $q_L \leq 1$ this condition is also sufficient.*

Proposition 17 can be easily generalized to the case of n sellers. If $k \geq 1$ sellers turn out to have high quality good and $l \geq 1$ sellers turn out to have low quality good, then it is straightforward to derive the relevant condition, which turns out to be $\Delta q \leq \frac{k+1}{2l}(1-\alpha)q_H$, and it must hold for any k and l such that $k+l = n$. Clearly the most restrictive is the case when $k = 1$ and $l = n - 1$ which, after rearrangement, reads $q_L \geq \frac{n+\alpha-2}{n-1}q_H$. In particular,

for fixed q_L and q_H no separating equilibrium can exist for large n in which buyers form their beliefs about the quality of goods in a particular shop based on pricing policy of this shop only. In the next section I will show that this negative result may no longer hold as long as at least a tiny fraction of buyers can directly observe the quality of goods.

The timing assumption that I make is essential for results of this section: it is important that sellers observe each other type prior to choosing their pricing policies. Note that I check incentive compatibility for a low type seller both if his rival is of high type and if he is of low type; this gives two constraints as opposed to one on his ‘average’ gains from deviation if he did not observe his rival’s type.

This timing assumption also allows for the following construction which concludes this section. I show that if buyers’ beliefs about the quality of seller’s good are not bound to be the function of this seller’s policy only, there always exists a separating equilibrium, at least for $q_L \leq 1$. To see this, assume the following beliefs for some $p_1, p_2 \in [1, \Delta q]$ such that $p_1 \neq p_2$: if one seller posts the price and the other seller does not, the price posting seller is believed to have high quality good if and only if his price equals p_1 (sellers who post no price are assumed to have low quality goods). If both sellers post the price, they both are assumed to have high quality good if and only if they *both* post p_2 ; otherwise they *both* are assumed to have low quality good. Then in equilibrium a high type seller, after learning that his rival is of low type, will post either p_1 ; given this, any price the low type seller post will be signaling low type (since $p_1 \neq p_2$) and hence the best strategy for a low type seller is to post no price. Likewise, if both sellers turn out to be of high type, none of them alone can post any price other than p_2 without being believed to be of low type and hence making no sales, so both will prefer to stick to posting p_2 . The only potentially profitable deviation could be for a low type seller to post p_1 if the other seller is also of low type; this can be precluded by setting p_1 close enough to Δq . Therefore, with such beliefs a separating equilibrium always exists. This construction can easily be expanded onto arbitrarily many sellers.

The assumption that buyer’s beliefs about seller’s type can only be a function of this seller’s pricing policy helps to narrow the range of potential equilibria. A slightly stronger version of it is introduced and studied in the next section in the context of large markets.

2.5 Large markets and endogenous quality

By *large* I mean markets in which there are at least two sellers of high type and at least two sellers of low type. In this section I will study how signaling equilibria can arise in large markets when sellers endogenously choose the quality of their goods.

When all sellers are uninformed and the choice of quality is endogenous, a high type seller can always deviate and produce low quality good, while mimicking the pricing strategy of a seller with high quality good. Since low quality goods are cheaper to produce, it is always a profitable deviation, so in any equilibrium all goods will be of low quality. Therefore, a mechanism is needed to keep high quality sellers from shifting to low quality good while maintaining high quality good price.

As I showed in the previous section, if buyers' beliefs about seller's type may depend on other sellers' strategies, this problem is easily resolved: a low type seller will not choose to mimic a high type seller since this will result in a strategy profile that will activate buyer's 'punishment' mode. Such argument has the following obvious problem: while it is plausible that the sellers engage in complicated strategies in order to maintain market segregation, it is arguably unlikely that buyers's beliefs are so sophisticated. To keep the analysis realistic, I impose the following restriction on buyers' beliefs about seller's types and focus on equilibria that involve beliefs satisfying this restriction.

Definition 1 *Buyer's beliefs are said to be monotonic in price if they have the following form: a seller who does not post any price or posts a price that is below some price $p^* \geq 1$ is believed to have low quality good; otherwise he is believed to have high quality good.*

Assumption 2 *Buyers' beliefs (both on and off equilibrium path) are always monotonic.*

Given monotonicity assumption on buyers' beliefs, the problem of enforcing high quality production of the buyers who signal high quality price needs fixing, and an additional structure must be imposed. I do it by introducing a small fraction of buyers who are informed about the quality of the good and who are particularly valuable customers (i.e., their valuation is high). In this setup a high quality good seller will be willing to keep those customers and hence not willing to switch to low quality product; therefore, informed customers must constitute a significant share of all customers that he has. On the other hand, he must not be willing to undercut another high quality seller. He may want to refrain

from that only in fear of losing uninformed customers because they will no longer believe that his good is of high quality; therefore, uninformed customers must also constitute a significant share of all customers that he has. Proposition 20 establishes conditions under which the two incentive compatibility constraints outlined above can simultaneously be met. Informally, the condition is going to be that producing high quality good should not be too expensive relative to benefits that high quality good brings to the consumers.

Assume that there is a (small) fraction ε of buyers who can correctly identify the quality of the good once they enter the shop, and, moreover, these are exactly the buyers with the highest value of type θ , i.e., those with $\theta \in [1 - \varepsilon, 1]$.¹⁰ Denote by n the total number of sellers, by h the number of sellers who choose to sell high quality good and by l the number of sellers who choose to sell low quality good, so that $h + l = n$.

Proposition 18 *Given $\varepsilon > 0$, a necessary condition for existence of a signaling equilibrium with monotonic buyer's beliefs in large markets is that the threshold value p^* is greater than or equal to $p(\varepsilon) = \frac{\Delta q}{1 + \varepsilon \Delta q}$.*

Proof. Consider a signaling equilibrium with more than two sellers producing high quality good and posting price at or above threshold p^* . First, note that they must post the same price, otherwise one of them will be making no sales. Second, the price they both post must be equal to p^* , otherwise one of them will be willing to undercut the other and have twice as many sales. The only reason why none of the sellers is willing to cut price must be that the uninformed buyers would not buy at a lower price because they would not believe that the good is of high quality.

If a seller offers high type good at price p^* , i.e., adheres to suggested equilibrium strategy, his profits are equal to

$$\Pi(H, p^*) = \frac{1}{h} \left(1 - \frac{p^*}{\Delta q} \right) (p^* - 1). \quad (2.10)$$

If instead he deviates and produces low quality good, while still posting price p^* , he will make more on each sale (p^* instead of $p^* - 1$) but will be making less sales because informed buyers will not be buying at him. His profits will then be

$$\Pi(L, p^*) = \frac{1}{h} \left(1 - \varepsilon - \frac{p^*}{\Delta q} \right) p^*. \quad (2.11)$$

¹⁰Perfect correlation is not necessary for qualitative results, but it simplifies algebra significantly. Any strong enough correlation will work.

Note that for p^* above monopoly price $p^M = \frac{\Delta q + 1}{2}$ profit $\Pi(H, p^*)$ is a decreasing function of p^* , whereas $\Pi(L, p^*)$ is an increasing function of p^* and that they are equal at $p^* = p(\varepsilon)$. Therefore, a necessary condition for existence of a signaling equilibrium is $p^* \geq p(\varepsilon)$, QED.

Note that $p(\varepsilon) \rightarrow \Delta q$ as $\varepsilon \rightarrow 0$, i.e., when the share of informed buyers is small, signaling price p^* is almost as high as the maximum price at which even the buyer with highest possible type θ is willing to buy high quality good; the sales at this price are naturally very low. Any seller would be happy to cut the price since this would increase his sales and ultimately profits; he does not do it only for fear of losing uninformed customers, who would not believe that such price cut is consistent with high quality good.

Proposition 19 *Given $\varepsilon > 0$, a necessary condition for existence of a signaling equilibrium on a large market with h high quality sellers is $p^* \leq \Delta q(1 - h\varepsilon)$.*

Proof is similar to that of Proposition 18: I have to rule out a specific deviation of a seller of high quality good. This time the deviation is to slightly undercut his high-type competitors while maintaining high quality of his good. The deviant will then lose all uninformed buyers, since they will no longer believe that he has high quality good, but will gain the entire market of informed buyers, which will give him profits of approximately

$$\Pi(H, p^* - 0) = \varepsilon(p^* - 1), \quad (2.12)$$

which should not exceed $\Pi(H, p^*)$. This immediately implies the desired inequality, QED.

Now I can formulate the main result of this section.

Proposition 20 *A signaling equilibrium with monotonic buyers' beliefs exists for small enough ε if and only if $\Delta q > 2$.*

Proof. Necessity follows from propositions 3 and 4: in any such equilibrium threshold value p^* must not be lower than $p(\varepsilon) = \frac{\Delta q}{1 + \varepsilon \Delta q}$ and must not be higher than $\Delta q(1 - h\varepsilon)$. Combining the two inequalities gives $\frac{\Delta q}{1 + \varepsilon \Delta q} \leq \Delta q(1 - h\varepsilon)$ and then, after simplification, $h\varepsilon \leq \Delta q - h$. This implies $\Delta q > h \geq 2$.

Sufficiency: assume $\Delta q > 2$, pick h to be any integer lower than r but not lower than 2. Take ε small enough so that $h\varepsilon \leq \Delta q - h$. Pick $p^* = p(\varepsilon)$. Calculate profits of each high

type seller:

$$\Pi(H, p^*) = \frac{1}{h} \left(1 - \frac{p(\varepsilon)}{\Delta q} \right) (p(\varepsilon) - 1) = \frac{\Delta q(\Delta q - 1 - \Delta q \varepsilon)}{h(1 + \Delta q \varepsilon)^2} \varepsilon.$$

In equilibrium all l low quality sellers bargain, which gives each of them the profit of

$$\frac{B}{l} = \frac{(1 - \alpha)q_L p^{*2}}{2lq_H \Delta q}.$$

Note that if ε is small enough, the profit of each of the high quality sellers will be small too (of the order ε), so that the total profit B of all low type sellers will exceed that. Choose $l \geq 2$ in such a way that

$$\frac{B}{l+1} < \frac{\Pi}{h} \leq \frac{B}{l}.$$

Then there is a signaling equilibrium with h sellers choosing high quality good and posting price p^* and l sellers choosing low quality good and bargaining.

To verify that it is indeed an equilibrium, consider possible deviations. A high quality seller is not willing to offer low quality good at high quality price since he would lose informed buyers and that is unprofitable by choice of p^* . He is not willing to undercut other high quality sellers because then he would lose credibility in the eyes of uninformed buyers and that is not profitable since $h < r$ and ε is small enough. He is not willing to switch to producing low quality good and bargaining since $\frac{B}{l+1} < \frac{\Pi}{h}$. Finally, a low quality seller is not willing to switch to producing high quality good since $\frac{\Pi}{h+1} < \frac{\Pi}{h} \leq \frac{B}{l}$. This completes the proof.

Note that, although the maximum total profits made by all price posting shops goes to zero as $\varepsilon \rightarrow 0$, the volume of trade of an individual price posting shop relative to that of a bargaining shop converges to a final limit. Indeed, this ratio is approximately equal to

$$\frac{h}{l} \frac{1}{\Delta q \varepsilon} = \frac{2q_H(\Delta q - p^*)(p^* - 1)}{(1 - \alpha)q_L p^{*2} \Delta q \varepsilon} \longrightarrow \frac{2q_H(\Delta q - 1)}{(1 - \alpha)q_L \Delta q} = \frac{2q_H(\Delta q - 1)}{(1 - \alpha)q_L \Delta q} \geq \frac{q_H}{(1 - \alpha)q_L} > 1.$$

Therefore, in any signaling equilibrium a price posting shop has higher sales compared to a bargaining shop. This is one explanation of the empirical observation that price posting shops tend to be larger than bargaining shops.

2.6 Conclusion

In this chapter I compared two modes of selling: selling at a posted price and bargaining. In particular, I focused on the question how the two modes of trade can coexist on the same market. Besides building a theory behind the phenomenon itself, this chapter also explains two facts associated with it, namely, the difference in size and quality of good between shops operating in the two modes.

Motivated by the market of souvenirs in a touristy area, I abstracted from issues such as choice or transportation costs and uncertainty about the price. However, uncertainty about the quality of the good being sold is the key factor in explaining the existing structure. Reputation concerns of a seller, which are usually viewed as a functional mechanism that induces sellers to supply high quality goods even when the quality is not directly observable by the buyer, do not work here, since I assume that the market under consideration is small and anonymous, and that each buyer makes a purchase at most once.

I show, first, that with only one seller on the market with exogenous quality of the good, there exists a separating equilibrium, in which sellers of different types choose different pricing strategies. Next, I demonstrate how a separating equilibrium may emerge when there are two sellers, one of each type.

When a seller may choose the quality of his good endogenously, a mechanism is needed to prevent him from choosing low quality and mimicking the price strategy of the high quality seller. I do it by assuming that a fraction of buyers who value the good the most also observe the quality of the good. It is shown that, no matter how small this fraction, a signaling equilibrium exists under simple restrictions on parameters.

This last finding may be of particular interest in helping to explain markets for 'exclusive' products. In particular, I show that the price in such equilibrium may be well above the full information monopoly price. This apparent paradox arises because a high quality seller has to credibly signal his type to uninformed buyers and the way to do it is to choose price high enough so that the only way for him to make profit is to have informed buyers buy at him. Realizing this, uninformed buyers will believe the signal and assume him to be of high quality.¹¹

¹¹In a multiperiod setup the price charged by a firm may also be above the monopoly price for a different reason, although related, reason: a seller signals high quality by committing to substantial losses if low quality is discovered by the buyers; see Klein and Leffler [4] and Milgrom and Roberts [6].

One inherent feature of the model is that typically there will exist multiple equilibria, both pooling and separating, for any number of sellers. A natural idea would be to rule out some of them by means of refining the equilibrium concept, for example, to apply the intuitive criterion of Cho and Kreps ([3]). However, the intuitive criterion will not work in this case for the following reason: the high type good is *more* valued by buyers, while the high type seller is *worse off* playing any given strategy than is the low type seller. Therefore, it is always cheaper for a low type seller to send any signal (price in this case) than it is for a high type seller to send the same signal; hence, there is no chance to conclude that some signals can only be meaningfully sent by high type sellers.

There is a number of issues that are left open in this study. First, in each case the number of sellers is assumed exogenous; it is interesting to study whether conclusions of the analysis continue to hold when the number of active sellers is determined endogenously. Second, I assume that a posted price is a commitment not to renegotiate it; no specific mechanism to support this commitment is discussed. Finally, I do not address the question of costs of bargaining per se, both for the seller and for the buyer. While seller's costs are likely to be represented only by time lost, the costs for the buyer are more complex: it is a widely observed phenomenon that some buyers refuse to bargain almost at any costs. These issues remain to be studied.

2.7 Appendix

Proof of Proposition 13: Sufficiency. I have to show that as soon as (2.3) is satisfied, there exists p that satisfies both (2.1) and (2.2). First try $p = p^M = \frac{1+q_H}{2}$. For this value of p condition (2.1) becomes (2.3) and condition (2.2) becomes

$$\frac{(1-\alpha)q_L}{2} \geq \frac{q_H^2 - 1}{4q_H}. \quad (2.13)$$

If (2.13) is satisfied, then the proof is complete. Assume it is not; this can be rewritten as

$$q_H^2 - 2(1-\alpha)q_Hq_L > 1. \quad (2.14)$$

Take p equal to p^* – the higher value at which (2.2) is satisfied with equality, i.e., $p^* = \frac{q_H + \sqrt{q_H^2 - 2(1-\alpha)q_Lq_H}}{2}$ so that

$$\left(1 - \frac{p^*}{q_H}\right)p^* = \frac{(1-\alpha)q_L}{2}.$$

At $p = p^*$ condition (2.1) becomes, after some rearrangement,

$$\frac{q_H + \sqrt{q_H^2 - 2(1-\alpha)q_Lq_H}}{2} \geq \frac{q_L^2}{2q_L - 1} \quad (2.15)$$

Because of (2.14), left hand side of (2.15) is greater than or equal to $\frac{q_H+1}{2}$. It now suffices to show that $\frac{q_H+1}{2} \geq \frac{q_L^2}{2q_L-1}$ or $2q_L^2 \leq 2q_Lq_H - q_H + 2q_L - 1$. The last inequality easily follows from assumption $q_H > q_L > 1$, QED.

2.8 References

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Chapter 3

Auction Rules as a Commitment Device

3.1 Introduction

Auctions that are used in practice, both in private and in public contexts, are almost universally conducted according to standard predetermined rules in one of the established and well known formats. Sellers are typically not free to choose any selling format they want but must follow one of the suggested patterns for soliciting bids and selecting winners.

Ex ante EBay lists only three selling formats for sellers to choose from, of which only one (called ‘on-line auction’) is a genuine auction in the economic sense of the word; the other two (‘Fixed Price’ and ‘eBay Stores Inventory’) amount to simply selling items at predetermined prices.¹ Once a seller selects ‘on-line auction,’ he does not have a freedom to choose between, for example, first price, second price or all pay auctions; neither is he allowed to discriminate between bidders in any way. Other auction sites have similar highly restrictive limitations on selling formats.

Rules governing public acquisition or procurement auctions are also highly restrictive. For example, Federal Acquisition Regulation, ‘established to codify uniform policies for acquisition of supplies and services by executive agencies,’ is about 2000 pages long. Similarly, Contracting Policy of the Treasury Board of Canada is over 1500 pages long and EU Procurement Legislation, combines 15 different acts over 1000 pages long in total. Al-

¹There is one other available format but it only applies to real estate sales.

though most of provisions of above mentioned legislature do not specifically concern auction formats, explicit limitations on auction practices are present.

In view of observed restrictions on auction formats for both public and private sellers a natural question arises: why do such restrictions exist? What makes it desirable to restrict auction formats a priori? Would not it be natural to leave the choice of the format and rules of an auction to the seller?²

The objective of this chapter is to show that it may be optimal for the seller to *not* have freedom of auction format choice. The basic intuition is that the choice of the format itself may serve as a signal to potential bidders of important characteristics of the seller that he may want to conceal. One way for the seller to avoid sending such a signal is to commit ex ante to a specified auction format, provided such commitment is common knowledge.

To make this argument, I present a model in which the choice of format is limited to one decision: to disclose the true reservation value or not. Without any claim to generality itself, this single dimension of discretion is sufficient to illustrate a much more general logic.

In my model, there is one seller and two bidders who compete for a single item via sealed bid first price auction. The seller has a reservation value for the item; buyers have valuations that are independent from each other and from the seller's reservation value. The seller may ex ante (before he learns his reservation value) choose to commit to a disclosure policy that specifies whether the seller must disclose the reservation value; one way to do it is to hire an agent (an auctioneer or an auction house) to conduct the auction on the seller's behalf, according to explicit disclosure rules. I show that it may turn out to be optimal for the seller to commit to such a policy. This argument could justify existence of predetermined auction rules, such as eBay selling procedures or public procurement regulations: these rules, when they are common knowledge, may help to enhance seller's ex ante revenue.

Since I work in the context of independent private values, I have to make an extra assumption in order to avoid the famous result of Myerson ([8]) and Riley and Samuelson ([9]), who show that the optimal (in terms of revenue) auction is an auction (for example, a first price auction) with reservation price set at the optimal level (and disclosed). The assumption that I make is that the seller can not commit to any reservation price above his true reservation value. This assumption is natural at least in the public procurement

²Throughout the chapter I am focusing on auctions to sell, not auctions to buy. All the arguments can be easily translated for auctions to buy.

context: a public agency (be it a school board, county officials or a federal office) acting on behalf of the public, will have a hard time explaining to the public why they have rejected a bid that was above their reservation value.³ Since the agency is aware of such a possibility, it has limited power in departing from the true reservation value in announcing minimum bids; in my model I assume the extreme situation where the only announced reservation price that the agency may commit to is the true reservation value. Therefore, the only discretion that the agency may potentially have is whether to disclose its reservation value or not, and I study whether it is optimal to leave this discretion to the agency in the public auction context.

Although the relevance of the above assumption is more doubtful in a private auction context, the assumption that the only credible reservation price for the seller is the true reservation value can still be justified. One further argument in support of it is that the resulting auction procedure is renegotiation proof: if all submitted bids are below seller's reservation value, he has no incentive to further negotiate the sale. On the contrary, if the seller posts a reservation price above the true reservation value and no bid meets it, he may be tempted to arrange a side deal with one of the bidders. If such renegotiation is expected by bidders, they will take the possibility of it into account when choosing their bids, so the optimality of the initial auction will depend on the feasibility of side trading.⁴

In public procurement context seller's commitment takes the form of ex ante restrictions on the auction format that a procurement agency must adhere to. In the existing literature, these regulations are analyzed within a principal-agent framework, where the 'public' is viewed as the principal who hires an agent to procure on its behalf. The focus in these studies is inevitably made on the divergent interests between the seller (or the buyer in the procurement context) and the auctioneer. For example, Laffont and Tirole ([4]) start by assuming that the government and the agency have conflicting objectives in that the agency favors one of the bidders, but unlike the government the agency possesses information about non-price dimensions of each bid (referred to as quality of the good provided), so agency services are indispensable, and derive optimal restrictions to be imposed by the government

³For example, in case of a uniform $[0, 1]$ distribution of private values (to which I limit my attention) the optimal (in the sense of maximizing the expected revenue) reservation price v^* is equal to $\frac{1+r}{2}$, where r is the true reservation value. In particular, the optimal reservation price is always above $\frac{1}{2}$. A bidder may submit a bid that is above r but below $\frac{1+r}{2}$ and when it is rejected, initiate a media campaign and accuse the agency for incorrect allocating of taxpayer's money.

⁴For optimal auctions when the seller can not commit see McAfee and Vincent ([5]) and most recent Skreta ([10]).

on the auction design that the agency may choose. Vagstad ([11] and [12]) further develops their analysis and discusses the choice between centralized (with the government directly carrying out the auction) and decentralized (with the government creating a special agency to carry out the auction) environment. The tradeoff is that the agency has better information about the quality of the product that each firm offers but also may favor local firm over foreign one, which causes inefficiencies.

The message that I want to convey in this study is that the delegation of the auction format choice to the agency in charge of conducting the auction may not be optimal even if the interests of the government and the agency are perfectly aligned. The reason for that is that the government may want to commit to a specific procedure *ex ante*, so that the agency can not signal to the bidders project-specific information that it has before the bidding starts. Published instructions for carrying out procurement auctions serve as a commitment device for the auctioneer and this commitment has value.⁵

The rest of the chapter is organized as follows. Section 3.2 describes the model and compares the revenue for the seller in two cases: under announced and hidden reservation value regimes. Section 3.3 discusses the value of commitment (predetermined auction format). Section 3.4 introduced conditional disclosure as an improvement over both announced value and hidden value regimes. Sections 3.5 and 3.6 focus on public procurement: in Section 3.5, I drop the assumption of the benevolent auctioneer and study what happens when one bidder is favored by the auctioneer; strategic response to favoritism on the part of discriminated bidder is discussed in Section 3.6. Section 3.7 concludes.

3.2 The Model

There is a risk neutral seller who owns one indivisible good. He hires a risk neutral auctioneer to sell it for him, by means of a first price sealed bid auction. The good has reservation value r for the seller, but the seller does not know this reservation value until after he issues the instruction to the auctioneer. The seller's reservation value for the good is uniformly distributed on $[0, c]$, where $0 < c < 1$.

The seller may issue instructions to the auctioneer whether or not to disclose r . If the auctioneer discloses r , it is verifiable.

⁵Ferschtman et al ([2]) emphasize the importance of the provision that delegation contracts are publicly observed. albeit in a different context.

For now I assume that the interests of the seller and the auctioneer are perfectly aligned: they both maximize the revenue from the auction (later I will relax this assumption).

There are two risk neutral bidders, their valuations are independent from each other and from seller's reservation value r and uniformly distributed on $[0, 1]$. They simultaneously submit sealed bids and the bidder who submits a high bid wins the good and pays his bid (it does not matter how ties are broken). Each buyer maximizes her profit, $\pi = p \cdot v - b$, where v is her valuation, b is her bid and $p(b)$ is her probability of winning the good with bid b .

Here is the time line of the game:

1. The seller issues instructions to the auctioneer, prescribing him to disclose r or not, possibly contingent on r itself; the seller may choose to leave discretion to the auctioneer. These instructions become common knowledge.
2. The auctioneer learns r ; he (credibly) discloses r if so instructed, keeps it hidden if so instructed and chooses whether to disclose it if the instruction leaves it to his discretion. If disclosed, r becomes common knowledge.
3. Bidders submit sealed bids.
4. The bids are opened and the high bidder gets the good and pays her bid, provided her bid is greater than or equal to r .

I solve first for the equilibrium bidding functions and expected payoffs (to the seller and to each bidder) in two benchmark cases, when the auctioneer is instructed never to disclose r or always to disclose r . Superscript a stands for announced price, and superscript h - for hidden. Subscript s refers to the seller, subscript b - to the buyer.

Proposition 21 *The symmetric equilibrium bidding function for announced reserve value r is*

$$b^a(v, r) = \begin{cases} 0, & v \leq r \\ \frac{v}{2} + \frac{r^2}{2v}, & v > r. \end{cases} \quad (3.1)$$

Proof. It is straightforward to show that the equilibrium symmetric bidding function $b(v)$ is monotonic in v for $v \geq r$.⁶ Now, if bidder 1 believes that bidder 2 is bidding according

⁶Bidding strategy for $v < r$ is irrelevant since there is no chance to earn positive payoff anyway. Without affect to anyone's payoff I assume that $b(v, r) = 0$ for $r < v$.

to strategy $b(\cdot)$ and the valuation of bidder 1 herself is v , she chooses her bid b to maximize $b^{-1}(b) \cdot (v - b)$, so at the optimum $b^{-1}(b) = (v - b)(b^{-1}(b))'$. Since I am looking for symmetric equilibrium bidding function, at the equilibrium it must be the case that $b = b(v)$ and then $b^{-1}(b) = v$. By the inverse function theorem $(b^{-1}(b))' = \frac{1}{b'(v)}$, so that the differential equation for $b(v)$ is

$$b'(v) = \frac{v - b}{v}.$$

General solution to this differential equation is $b(v) = \frac{v}{2} + \frac{k}{v}$. It is readily shown that at the optimum $b(r) = r$, so that $k = \frac{r^2}{2}$ and the expression for $b(v)$ follows.

Given equilibrium bidding strategies $b^a(v, r)$ it is straightforward to calculate expected revenue of the seller as a function of his reservation value r :

$$\pi_s^a(r) = \frac{1}{3} + r^2 - \frac{r^3}{3}. \quad (3.2)$$

Its average with respect to r is

$$\pi_s^a = \frac{1}{3} + \frac{c^2}{3} - \frac{c^3}{12}.$$

Ex ante expected payoff to either bidder is equal to

$$\pi_b^a = \frac{1}{6} - \frac{c^2}{6} + \frac{c^3}{12}.$$

Next consider the alternative regime, in which the seller does not announce his reservation value.

Proposition 22 *The symmetric equilibrium bidding function, for hidden reserve value, is*

$$b^h(v) = \begin{cases} \frac{2v}{3}, & v \leq \frac{3c}{2} \\ \frac{v}{2} + \frac{3c^2}{8v}, & v > \frac{3c}{2}. \end{cases} \quad (3.3)$$

Proof is similar to that of Proposition 21 and is omitted.

The revenue of the seller, as a function of his reservation value r , is equal to

$$\pi_s^h(r) = \frac{1}{3} + \frac{3c^2}{4} + \frac{3r^3}{4} - \frac{3c^3}{4}.$$

Its average with respect to r is

$$\pi_s^h = \frac{1}{3} + \frac{3c^2}{4} - \frac{9c^3}{16}. \quad (3.4)$$

The expected ex ante payoff to each bidder is

$$\pi_b^h = \frac{1}{6} - \frac{3c^2}{8} + \frac{9c^3}{32}.$$

Comparing (3.2) to (3.4) it is easy to verify that the seller prefers to disclose the reservation value if $c > \frac{20}{23}$ and to keep it hidden if $c < \frac{20}{23}$, and also that ex ante buyers always prefer announced price. The outcome of the auction is efficient if the price is announced but not necessarily so if it is not. In either case, the bidder with the lower valuation never gets the good.

3.3 Value of Commitment

In view of the Propositions 21 and 22 I can now address the value of committing to a disclosure policy. The question that I address in this section is what the seller would prefer: to leave the choice of the auction format (announced vs. hidden reservation value) to the auctioneer's discretion, or rather to ex ante issue specific instructions to the auctioneer. Typically, ex ante restrictions on auctioneer's choice are justified by introducing moral hazard, that is assuming that the auctioneer's objective are different from those of the seller, i.e. the auctioneer may not in fact be maximizing seller's payoff. In this section I show that in fact ex ante restrictions on the information disclosure may be optimal even if the interests of the seller and the auctioneer are perfectly aligned. Ex ante chosen disclosure policy serves as a commitment device to prevent the auctioneer from disclosing the secret reservation value when it is high, in which case the bidders will assume that if the auctioneer does not disclose the reservation value then it must be low, and bid conservatively. Commitment not to disclose the reservation value even when it is high results in bidders bidding more aggressively which ultimately increases revenue on average. The following proposition illustrates the point.⁷

⁷This result reflects insights of Grossman ([3]) and Milgrom ([6]).

Proposition 23 *When the decision whether to disclose the reservation value is left to the discretion of the auctioneer, the only subgame perfect symmetric Bayesian equilibrium involves disclosing reservation value r for any level of r , except possibly one.*

Proof. Assume the converse and consider an equilibrium in which the reservation value is not disclosed for more than one value of r . Denote by r_M the maximal of such values. I want to show that when the actual realization of the reservation value is r^M , the seller is strictly better off disclosing it.

If the seller discloses r_M , his revenue in equilibrium will be $\frac{1}{3} + r_M^2 - \frac{r_M^3}{3}$ by Proposition 21. If he does not, there are two possibilities: either in any symmetric equilibrium bidders will always be bidding below r_M and the object will not be sold (in which case the assertion is trivial, as disclosing r_M will definitely improve expected payoff to the seller) or bidders will be bidding r_M at some valuation $\alpha r_M < 1$. Note that α can not be lower than 1 since then a bidder with valuation αr_M will be making negative profit on average. Neither can α be equal to 1, since then a bidder with valuation r_M will be making zero profit and bidding $r_M - \varepsilon$ for small enough $\varepsilon > 0$ will yield him positive profit on average: he will be winning at least ε if both seller's reservation value and his rival's valuation are below $r_M - \varepsilon$, which has positive probability for small enough ε . Therefore, the only case to be considered is $\alpha > 1$.

The proof of Proposition 21 can be used to establish that the symmetric equilibrium bidding function for $v \geq \alpha r_M$ is $b(v) = \frac{v}{2} + \frac{\alpha(2-\alpha)}{2v} r_M^2$. The seller's expected revenue in this case equals

$$(1 - \alpha r_M) \alpha (2 - \alpha) r_M^2 + \frac{1}{3} - \frac{(\alpha r_M)^3}{3} + \alpha^2 r_M^3. \quad (3.5)$$

On the other hand, if the seller announces r_M , his expected revenue equals, by Proposition 21

$$\frac{1}{3} + r_M^2 - \frac{r_M^3}{3} \quad (3.6)$$

It is easily verified that for $\alpha > 1$ value (3.6) exceeds value (3.5). Therefore, it is not an equilibrium strategy for the seller not to disclose r_M , which contradicts the assumption that r_M is not disclosed. This contradiction completes the proof.

Therefore, the auctioneer, if he shares seller's optimality goals, will ex post choose to disclose the reservation value in any subgame perfect Bayesian equilibrium, unless ex ante committed not to do so. However, as I showed in the previous section, always disclosing the

reservation value is not optimal ex ante, for small enough spread of reservation values c ; hence, the seller will find it optimal to ex ante limit auctioneer's discretion. This may be a reason why the restrictions on the choice of auction format exist in the first place.

3.4 Policy of Conditional Disclosure

In this section I suggest a disclosure policy that is superior to both keeping the reservation value secret and always announcing it. This suggested policy is to announce reservation value r when it is high (higher than some specified threshold y) and to keep it hidden when it is low. I calculate the seller's profit if he adopts this rule.

For announced reservation values $r > y$, Proposition 21 applies. The symmetric equilibrium bidding function for announced reserve value $r \geq y$ is

$$b^{>y}(v, r) = \begin{cases} 0 & v \leq r \\ \frac{v}{2} + \frac{r^2}{2v} & v > r. \end{cases} \quad (3.7)$$

The revenue of the seller, as a function of his reservation value r , is equal to

$$\pi_s^{>y}(r) = \frac{1}{3} + r^2 - \frac{r^3}{3}.$$

Its average with respect to $r \in [y, c]$ is

$$\pi_s^{>y} = \frac{1}{c-y} \left[\frac{c^3}{3} - \frac{y^3}{3} - \frac{c^4}{12} + \frac{y^4}{12} \right].$$

If r is not disclosed prior to the bidding, the bidders realize that it is below y and hence update their prior beliefs $r \sim U[0, c]$ to posterior beliefs $r \sim U[0, y]$. Therefore, Proposition 22 applies and the formulas below follow.

The symmetric equilibrium bidding functions are

$$b^{<y}(v) = \begin{cases} \frac{2v}{3} & v \leq \frac{3y}{2} \\ \frac{v}{2} + \frac{3y^2}{8v} & v > \frac{3y}{2}. \end{cases} \quad (3.8)$$

The revenue of the seller, as a function of his reservation value $r < y$, is equal to

$$\pi_s^{<y}(r) = \frac{1}{3} + \frac{3y^2}{4} + \frac{3r^3}{4} - \frac{3y^3}{4}.$$

Its average with respect to $r \in [0, y]$ is equal to

$$\pi_s^{<y} = \frac{1}{3} + \frac{1}{3} + \frac{3y^2}{4} - \frac{9y^3}{16}.$$

Expected revenue of the seller is therefore equal to

$$\pi_s^y = \frac{y}{c} \pi_s^{<y} + \frac{c-y}{c} \pi_s^{>y} = \frac{c^2}{3} - \frac{c^3}{12} + \frac{y^3}{c} \left(\frac{5}{12} - \frac{23y}{48} \right).$$

With $y = 0$ and $y = c$ the last expression coincides with those for public and secret reservation values, respectively. It is maximized over y at $y = \frac{15}{23}$. Consequently, for $c \leq \frac{15}{23}$ it is optimal for the seller to always keep the reservation value secret; for $c > \frac{15}{23}$ the optimal strategy is to keep the reservation value secret if it is below $\frac{15}{23}$ and to reveal it otherwise.

The intuition behind the result of this section is that when the actual reservation value is high, an uninformed bidder is likely to bid below it while her valuation is actually above it (because she averages her bid over the entire range of possible values of the reservation value) and revealing the reservation value can help sell the good which otherwise may be unsold. On the other hand, when the reservation value is low, uninformed bidders are likely to bid above it anyway, and keeping it secret results in more aggressive bidding.

Conditional disclosure policy is superior to no disclosure only for high enough uncertainty about the seller's reservation value (for $c > \frac{15}{23}$). Besides even in the extreme case of $c = 1$ it only increases seller's ex ante revenue by about 0.2 percent. Nevertheless it is important to introduce this option in order to illustrate the point that optimal auction format need not take the simple form of either always or never disclosing the reservation value.

In the next two sections I depart from the assumption that the auctioneer has the same objective as the seller. I introduce favoritism on the part of the auctioneer and study how predictions of the above analysis change.

3.5 Corrupt Auctioneer

In previous sections I have argued that it is optimal for the seller to ex ante commit not to disclose his reservation value; in the private context the mechanism for such a commitment is provided by preestablished restrictions on auction format of a particular auction house. In a public (for instance, procurement) auction context this commitment is achieved by hiring an auctioneer who conducts the auction on seller's behalf.

Commitment benefits of having the auctioneer rather than the seller conduct the auction are established above; however, there can naturally be agency costs associated with hiring an agent. In the context of my setup I model these costs by assuming that the auctioneer favors one of the bidders by disclosing seller's secret reservation value to her.

Note that even if the auctioneer secretly discloses the seller's reservation value to one of the bidders, the argument of Proposition 23 still applies: there is no perfect Bayesian equilibrium in which the seller does not ex post want to disclose the reservation value to the uninformed bidder when this value is high. Therefore, as long as keeping the reservation value hidden (at least from one of the bidders) is superior to publicly disclosing it, the auctioneer's service is still of value. On the other hand, now there are also costs of these services: the bigger the range of reservation values which are supposed to be kept hidden, the higher the advantage of the favored bidder. Below I derive the optimal partial disclosure threshold.

In this section I assume that the uninformed bidder is not aware of the fact that the other bidder knows the secret reservation value and sticks to her strategy $b^h(v)$ (I consider strategic response of the uninformed bidder to favoritism in the next section): I call this situation 'unexpected favoritism' and use superscript uf to refer to it. I now calculate the best response to $b^h(v)$ on the part of the informed bidder as a function of her own valuation v and reservation value r that she learns, as well as the expected profit of the seller and each bidder. Throughout this section I assume that $c \leq \frac{1}{\sqrt{3}}$.

If informed bidder's own valuation v is below seller's reservation value r , the informed bidder can not win the auction without making negative profits. Her exact bid is not important, provided it is below r ; as above, I specify it at zero.

Consider the case when informed bidder's valuation v is above r . She can choose either $b \leq c$ or $b > c$. If she bids $b \leq c$ she will be winning the auction whenever her bid is above

that of the uninformed bidder, equal to two thirds of the uninformed bidder's valuation (according to $b^h(v)$). Hence the probability that the informed bidder will win the auction if she bids $b \in [r, c]$ is equal to $\frac{3b}{2}$ and she chooses $b \in [r, c]$ to maximize her expected payoff $\frac{3b}{2}(v - b)$. If she decides to bid below c , she will choose $b(v) = r$ for $v \in [r, 2r]$ and $b(v) = \frac{v}{2}$ for $v > 2r$. Her expected payoff will be $\frac{3r(v-r)}{2}$ and $\frac{3v^2}{8}$.

If the informed bidder decides to bid above c , then her being informed about the reservation value is irrelevant and she does not have any advantage over the uninformed bidder. Hence, by virtue of $b^h(v)$ being the symmetric equilibrium in the case of no favoritism, informed bidder's optimal bidding strategy against $b^h(v)$ is $b^h(v)$ itself. She will be winning with probability v and receiving payoff of $v - b^h(v) = \frac{v}{2} - \frac{3c^2}{8v}$, so that her total expected payoff is $\frac{v^2}{2} - \frac{3c^2}{8}$.

To complete the description of the optimal strategy of the informed seller, I must specify the cutoff point below which she bids $b \leq c$ and above which she bids $b > c$. This is done by comparing the expected payoffs derived above. It is easily verified that for small r (such that at $v = 2r$ bidding above c is not profitable) this cutoff equals $c\sqrt{3}$, while for large r it is $\frac{3r + \sqrt{3c^2 - 3r^2}}{2}$. Therefore, best response bidding function of the informed bidder takes the following form:

$$b_{fb}^{uf}(v, r) = \begin{cases} 0, & v \leq r, \\ r, & r \leq v < 2r, \\ \frac{v}{2}, & 2r \leq v \leq c\sqrt{3}, \\ \frac{v}{2} + \frac{3c^2}{8v}, & v > c\sqrt{3}. \end{cases} \quad (3.9)$$

for $r \leq \frac{c\sqrt{3}}{2}$, and

$$b_{fb}^{uf}(v, r) = \begin{cases} 0, & v \leq r, \\ r, & r < v \leq \frac{3r + \sqrt{3c^2 - 3r^2}}{2}, \\ \frac{v}{2} + \frac{3c^2}{8v}, & v > \frac{3r + \sqrt{3c^2 - 3r^2}}{2}. \end{cases} \quad (3.10)$$

for $r > \frac{c\sqrt{3}}{2}$.

Now it is straightforward to compute expected profits of the seller and both bidders. The seller gets on average

$$\pi_s^{uf} = \frac{1}{3} + \frac{3c^2}{4} - .7394592495c^3.$$

The informed (favored) bidder gets on average

$$\pi_{fb}^{uf} = \frac{1}{6} - \frac{3c^2}{8} + .3706359226c^3,$$

the uninformed bidder gets on average

$$\pi_{db}^{uf} = \frac{1}{6} - \frac{3c^2}{8} + .3176340828c^3.$$

Profits of the seller and the uninformed bidder are lower compared to hidden reservation value no favoritism case, while the profit of the informed bidder is higher.

It is easy to compute the optimal threshold value for conditional disclosure policy, $y^{uf} = 0.47$. Note that it is well below the threshold level for no favoritism case, which is equal to $\frac{15}{23}$. The intuition behind this finding is clear: if the seller suspects that the auctioneer is going to favor one of the bidders, he worries that this favored bidder, upon learning r , will not compete aggressively when r is high but rather will just bid r leaving the seller with no profit. Publicly announcing r restores competition and ultimately improves seller's expected profit.

Note that the efficiency of this auction (defined as the sum of the expected payoffs to the seller and both bidders) is lower than that for hidden reservation value. There are two kinds of inefficiency associated with keeping the reservation value hidden. The first kind of inefficiency is that it may not be the bidder with the higher valuation who gets the item. The second kind of inefficiency is that the item may remain in the seller's hands even though one or both of the bidders have valuations above seller's reservation value but fail to bid above it. Without favoritism on the auctioneer's part, bidder's strategies are symmetric and monotone in valuations, so the item, if sold, always goes to the more efficient bidder; on the other hand, the inefficiency of the second kind is high. If the auctioneer favors one of the bidders, the inefficiency of the second kind is partly remedied, but the inefficiency of the first kind is introduced. The analysis above shows that on the balance the efficiency declines, so favoritism is not justified from the efficiency standpoint.⁸

⁸If efficiency, rather than optimality, is the seller's priority, his optimal policy is to always disclose the reservation value, as this leads to fully efficient outcome. However, as I showed earlier, optimality

In the context of favoritism, one may also consider the possibility that the interests of the seller are associated with those of the favored bidder. If the seller is the government, one of the bidders is a domestic firm and the other bidder is a foreign firm, than the government may be more interested in having a domestic rather than a foreign firm to win the object. The government may actually prefer to forego some of its own revenue in favor of that of the domestic firm, and hence tacitly sponsor favoritism on the part of the auctioneer. Whether or not it will actually want to do so depends on the weight with which the government values profit of the domestic firm versus its own profit. If this weight is low, the government will not want the auctioneer to engage in favoritism, since government's revenue is lower under favoritism. On the other hand, it is easy to see that the sum of the expected profits of the seller and the favored bidder is higher with favoritism than without it. Therefore, if the government's concern about the profit of the domestic firm is high enough, it will prefer hidden reservation value with favoritism regime over both announced and hidden reservation value regimes.

The analysis in this section was based on the assumption that the uninformed bidder is unaware of the fact that the other bidder is favored by the auctioneer and is secretly informed about hidden reservation value. In the next section I show how the analysis is modified if the uninformed bidder is aware of favoritism.

3.6 Strategic response to favoritism

In this section I analyze how the strategy of the uninformed bidder changes if she takes into account that the other bidder is being favored, i.e., that the other bidder knows the secret reservation value r ; I refer to this situation as 'expected favoritism' and use superscript *ef*. The question I want to answer is whether this strategic response calls for more aggressive or more conservative bidding on the part of the uninformed bidder; a priori it is even conceivable that the seller (especially when the seller is the government concerned not only with its own profits but also with that of the favored bidder) could find it in its interest to sponsor favoritism and to keep it common knowledge that one bidder is being favored. The

motivation calls to keep the reservation value secret. If it so happened that efficiency gains from favoritism exceeded efficiency costs, then it would be conceivable that the seller, driven by some mixed optimality and efficiency concerns, would be *interested* in the auctioneer secretly favoring one of the bidders. However, the above efficiency result precludes such possibility: unexpected favoritism is always detrimental for the seller, whatever his objectives.

latter, it turns out, is never the case: the seller does not benefit from favoritism and when favoritism takes place, it is not in the seller's interest to let the discriminated bidder know that she is being discriminated.

I now solve for the equilibrium pair of strategies; throughout this section I assume $c \leq \frac{5}{8}$. I start with the uninformed bidder. In view of the results of the previous sections I conjecture that she follows a linear strategy for low valuations: $b_{db}^{ef}(v) = \lambda v$ for $v \leq v^*$ (the value of v^* to be determined). If that is her strategy, then, for small enough r , the best response of the informed bidder is to bid r for $v \in [r, 2r]$ and $\frac{v}{2}$ for $v \in [2r, c]$. Uninformed bidder's optimization problem is then easily solved: her optimal bidding strategy is $b(v) = \frac{2v}{3}$ for $v \leq v^*$, confirming the linearity conjecture.

Also, when v is large (higher than some v^{**} to be determined), both bidders will bid above c , in which case they are symmetric as the information that the favored bidder possesses has no value. In this case, as was shown above, the equilibrium bidding functions are $b(v) = \frac{v}{2} + \frac{2cv^{**} - (v^{**})^2}{2} \cdot \frac{1}{v}$.

I drop technical details that help to derive optimal bidding strategies in the medium range of v and also to solve for v^* and v^{**} . In equilibrium the bidding strategy of the uninformed bidder must be a solution to the differential equation shown below. I used numerical methods to solve for values v^* and v^{**} . It turns out that $v^* = \frac{6c}{5}$ and $v^{**} = \frac{8c}{5}$. Below are the equilibrium strategies for both the informed and the uninformed bidder.

The equilibrium strategy of the uninformed bidder is

$$b_{db}^{ef}(v) = \begin{cases} \frac{2v}{3} & v \leq \frac{6c}{5}, \\ \tilde{b}(v) & \frac{6c}{5} < v \leq \frac{8c}{5}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \frac{8c}{5}, \end{cases} \quad (3.11)$$

where $\tilde{b}(v)$ is a monotonic function, such that, after change of variables $x = \frac{5b}{4c}$ and $t = \frac{5v}{4c}$, function $x(t)$ is the solution to differential equation

$$x'(t) = \frac{(2t - x)\sqrt{1 - 2tx + t^2} + 2t^2 - 4tx + x^2 + 1}{2t\sqrt{1 - 2tx + t^2} + 3t^2 + 2 - 5tx}$$

with initial condition $x(\frac{3}{2}) = 1$. Function $x(t)$ is very well approximated by linear $\tilde{x}(t) = \frac{t}{2} + \frac{1}{4}$.

The equilibrium strategy of the informed (favored) bidder is

$$b_{fb}^{ef}(v, r | r \leq \frac{4c}{5}) = \begin{cases} 0 & v \leq r, \\ r & r \leq v < 2r, \\ \frac{v}{2} & 2r \leq v \leq \frac{8c}{5}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \frac{8c}{5}, \end{cases} \quad (3.12)$$

for $r \leq \frac{4c}{5}$ and

$$b_{fb}^{ef}(v, r | r > \frac{4c}{5}) = \begin{cases} 0 & v \leq r, \\ r & r < v \leq \tilde{b}^{-1}(r) + \sqrt{\frac{16}{25}c^2 - 2r\tilde{b}^{-1}(r) + [\tilde{b}^{-1}(r)]^2}, \\ \frac{v}{2} + \frac{8c^2}{25v} & v > \tilde{b}^{-1}(r) + \sqrt{\frac{16}{25}c^2 - 2r\tilde{b}^{-1}(r) + [\tilde{b}^{-1}(r)]^2} \end{cases} \quad (3.13)$$

for $r > \frac{4c}{5}$.

Bid of the uninformed bidder is the same as in the case of no strategic response for $v \leq \frac{6c}{5}$ and is more *conservative* for $v > \frac{6c}{5}$. This result answers the concern of this section: since the uninformed bidder bids more conservatively when she knows that the other bidder is informed, the revenue of the seller is unambiguously lower. Therefore, it is in the interest of the seller to keep the uninformed bidder unaware of favoritism.

The expected profit of the seller is equal to

$$\pi_s^{ef} = \frac{1}{3} + \frac{16c^2}{25} - 0.541867c^3.$$

The expected profit of the informed bidder is equal to

$$\pi_{fb}^{ef} = \frac{1}{6} - \frac{8}{25}c^2 + 0.281467c^3,$$

and the expected payoff of the discriminated bidder is

$$\pi_{fb}^{ef} = \frac{1}{6} - \frac{8}{25}c^2 + 0.280548c^3.$$

It is easy to compute the optimal threshold value for conditional disclosure policy, $y^{uf} = 0.50$. This optimal disclosure threshold value is close to that for unexpected favoritism case

but below that for no favoritism case.

Note that the efficiency of the auction is higher for expected favoritism than it is for unexpected favoritism. This is the case because uninformed bidder now bids more conservatively and therefore it is less often the case that the uninformed bidder wins whereas the informed bidder has in fact higher valuation. Note also that the sum of profit of the seller and the informed bidder is not only lower than that for the case of unexpected favoritism, but also than the profit for hidden reservation value, although still above that for announced reservation value. Two conclusions follow from this last observation. First, if the seller favors one of the bidders but can not privately disclose secret information to her without the other bidder being aware of such information leak, the seller should not disclose the information (and keep the two bidders in symmetric positions). Second, even in the worse possible scenario for the seller, i.e., if the auctioneer is corrupt and that he is corrupt is publicly known, still it is in the seller's interest to engage in the relationships with the auctioneer (rather than to unconditionally mandate disclosure of his reservation value or, equivalently, sell the object on his own). Indeed, it is easy to verify that Proposition 23 still applies, which means that without the commitment (or auctioneer's services) the seller will in any subgame perfect equilibrium be disclosing the reservation value. This last observation implies that benefits from commitment outweigh costs of agency: even when the auctioneer is known for malpractice, it is worth for the seller to draw on his services.

3.7 Conclusion

When I started this study, I was convinced that it is always optimal for the seller to disclose his reservation value in an auction, as well as all other information relevant to the bidding that he possesses. It is commonly argued that the main reason why the auctioneer may not want to do so is the fear of collusion among bidders. In particular, when all the bidders collude, the seller will be making no profit if he discloses his reservation value, whereas if he keeps it secret he gets positive profit with positive probability. On the other hand, if the auctioneer is not supposed to reveal the information that he possesses, this opens the door for favoritism as long as one departs from the assumption of benevolent auctioneer, i.e., the auctioneer may still privately reveal the secret reservation price to one of the bidders, thus undermining the idea to keep it secret and, worse, creating asymmetry between bidders.

potentially deteriorating seller's profit.

However, as the results of this study suggest, in the independent private value setting,⁹ it is not necessarily true that the profit-maximizing seller is always better off revealing the secret price rather than concealing it, under the important assumption that the seller can not commit to a reservation price above his true reservation value (this assumption invalidates optimality results of Myerson ([8]) and Riley and Samuelson ([9])). In fact it turns out that in many cases the reverse is true: concealing the true reservation value may, while compromising efficiency of the allocation, actually improve the seller's revenue. Therefore, it is unnecessary to appeal to the threat of collusion between bidders to justify keeping the reservation price hidden.

Neither is it necessary, it turns out, to introduce moral hazard in seller-auctioneer relationships in order to justify instructions limiting auctioneer's discretion on whether to disclose the reservation price or not. In fact, the auctioneer and the seller, both maximizing seller's expected ex ante payoff, may find it in their interest to commit to specific rules regarding information disclosure (it turns out that conditional disclosure of a particular form may improve their payoff compared to both full disclosure and no disclosure). This commitment (observed by bidders) keeps bidders from making adverse inferences from the fact that the auctioneer keeps his reservation value hidden and induce them to bid aggressively enough that the seller's profit becomes on average higher than in the case of full auctioneer's discretion.

While the basic model built in this chapter suffices to illustrate the insights specified above, there are issues that are left behind. Effects of possible collusion, originally a motivating factor, are not studied, and neither is the mechanism of corruption modelled explicitly. Finally, an open question is how the findings of this analysis generalize to multiple bidders or multiple dimensions of the seller's objectives (e.g., quality). These issues deserve further scrutiny.

3.8 References

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⁹If bidders' valuations are affiliated, that makes the case stronger for revealing the information rather than concealing it, as pointed out by Milgrom and Weber ([7]).

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