

The Stochastic Air Traffic Flow Management Rerouting Problem

by

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Submitted to the Department of Electrical Engineering and Computer
Science

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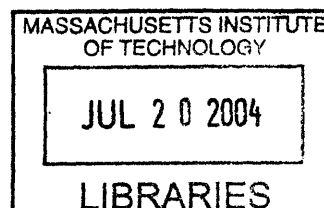
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Abstract

We formulate a model for planning the rerouting of aircraft to alleviate en-route congestion, with system capacity being modeled stochastically. To overcome problems with tractability, we apply a Dantzig-Wolfe decomposition and present an efficient method for solving it. The decomposed formulation is shown to be tractable for real-world problem, and it generates up to a ten percent reduction in cost when compared to an otherwise equivalent deterministic model. We show that even when the decomposed formulation fails to terminate within a reasonable time, a near-optimal solution can still be generated.

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Chapter 1

Introduction

Every year delays cost the airline industry billions of dollars [3]. These delays are, for the most part, caused by severe weather conditions, which reduce the capacity of both airports and en-route airspace. Of these two types of capacity reduction, the reduction in airport capacity has received far more attention.

At present the Federal Aviation Administration (FAA) implements a policy of ground-holding aircraft that would not have an immediately available landing slot upon arrival. This technique of converting airborne delays into less costly ground delays has generated tremendous savings [7]. There is however, at present, no similar method for reducing costs in the case of congestion en-route.

This thesis examines the problem of optimally adjusting flight plans to meet reduced en-route capacities imposed by convective weather. Due to the uncertainty involved in plans based upon weather predictions, it is useful to model the problem stochastically, so that a plan will be robust against several likely developments. Using a model that allows the rerouting of aircraft to be done as needed, makes it possible to use available capacity more efficiently than using a model with fixed flight routes. This is because aircraft can avoid chokepoints, and demand can be more effectively adjusted to meet restrictions when rerouting is an alternative. The model and corresponding solution technique, presented in this thesis are designed to incorporate both of these characteristics to generate plans with full-recourse and dynamically selected routes.

Chapter 2 motivates the problem by first describing how air traffic control works and how the system is impacted in the presence of convective weather. In the end of the chapter, previous work on the problem is presented. Chapter 3 presents the formulation for stochastic planning with rerouting, after first giving the formulation that it was based upon, and also presents a method for generating solutions to the problem is given. Chapter 4 presents some variations on the basic model, which allow the system and airline operations to be modeled with greater accuracy. Chapter 5 provides the results of several computational experiments devised to determine both the effectiveness and the tractability of the model and formulation. Finally, Chapter 6 draws some conclusions about the usefulness of the model and presents directions for future work.

Chapter 2

Problem Description

The objective of this chapter is to explain the operational problem in the airline industry that the model in this thesis addresses. Section 2.1 explains interactions of the major decision makers that affect flight planning and execution. Next, section 2.2 presents a closely related problem. Section 2.3 explains the causes of congestion. Section 2.4 presents the costs that need to be considered when evaluating schedule adjustments. Section 2.5 explains what needs to be considered in constructing an appropriate model. The final section presents previous work that has been done in this area.

2.1 National Airspace System

Due to the competitive nature of the commercial airlines, the primary users of the National Airspace System (NAS), scheduling is typically done in an aggressive fashion with little consideration of system capacity. The main concern of the Federal Aviation Administration (FAA) is the safety of air transportation, and it otherwise tries to interfere with airline operations as little as possible.

Ensuring safety is primarily done through the appropriate direction of aircraft in the air, and rarely involves having the airlines modify their schedules. By enforcing their safety requirements in this manner, rather than with schedule adjustments, the FAA allows for the existence of unnecessary airborne delays, which can be excessively

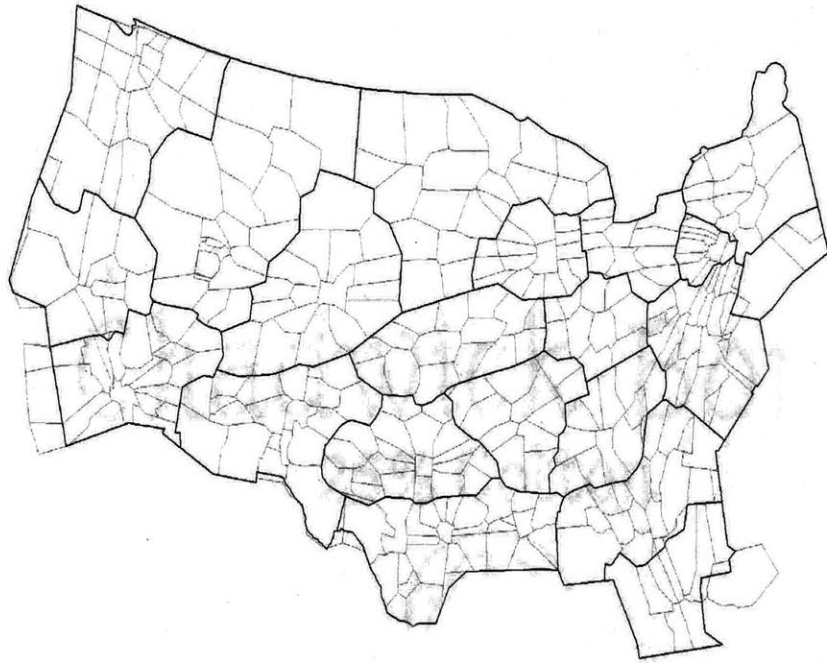


Figure 2-1: The division of the National Airspace System. The darker lines show the boundaries of ARTCC's while the lighter lines show the sectors.

costly.

The NAS is divided up into twenty-two regions, each of which is controlled by an Air Route Traffic Control Center (ARTCC). Each of these regions is in turn divided into about 40 to 60 sectors and each sector has a single controller in charge of directing the aircraft within it.

If the demand in a sector s exceeds its capacity, a controller will place restrictions on aircraft entering sector s . These restrictions can either be a minimum distance or minimum time period between successive aircraft entering the sector. These restrictions result in limited outflows in neighboring sectors, whose controllers must in turn limit their own inflows so that capacity is not exceeded. This ripple effect can result in major airborne delays. Due to the highly distributed nature of air traffic control, in which communication between sector controllers is rather limited, it is difficult to reroute aircraft around congested regions in real time.

The FAA tends to mandate rerouting around congestion only in severe cases when

it is clear that safety is a concern. When doing so there is a standard set of alternative routes that are selected from a playbook, which has limited flexibility.

2.2 Collaborative Decision Making

In the mid-1990s, the FAA and the airline industry started a joint program called Collaborative Decision Making (CDM). This initiative was started to develop effective ways to deal with the inefficient way in which arrival capacity was being used due to the competitive nature of the airlines. The idea behind CDM is that by sharing information about operations, unavoidable system delays can be redistributed in an equitable manner and improve efficiency, and that this is achieved by creating incentives for the airlines to provide information about their operations.

This program has brought about an improvement in the performance of ground delay programs (GDP) that have generated tremendous savings annually compared to GDPs prior to CDM. GDPs are a technique for assigning landing times at congested airports, translating what would normally be an airborne delay into a less costly ground hold. Due to the great success that has been achieved reducing the cost from airport congestion, the CDM program has recently been expanded, with the formation of the Long-Term Collaborative Routing Group, to investigate similar methods, involving natural extensions to the ground delay programs that already exist, for alleviating congestion delays that occurs en-route [5].

2.3 Causes of Congestion

Within the NAS the vast majority of flights are planned well in advance because the heaviest users are the major airlines, which must schedule in advance to accommodate passenger plans. It is not always possible for these schedules to be met, due primarily to unforeseen system congestion. Congestion can occur at and near airports and en-route. The focus of this thesis is developing a methodology for addressing problems that result from en-route congestion.

En-route congestion is a result of the need to impose spacing restrictions on how close aircraft may be to each other, as well as how closely they may approach a dangerous weather formation such as a thunderstorm. These restrictions are imposed in real-time by air traffic controllers that communicate with and appropriately direct pilots.

This spacing restriction on a tactical level results in capacity reductions on the strategic level. The ability to effectively direct traffic within a sector depends on how many aircraft are in it and how close they must get to each other. As the number of aircraft increases the controller has less time to focus on each aircraft, and must pay more attention to maintaining separation minima due to the decreased distances between aircraft. When there is bad weather in a sector, the usable area is reduced. This means that bad weather increases the density for a fixed number of aircraft.

The end result of this line of reasoning is that constraints on the capacity of en-route sectors, whether they result from increased traffic or bad weather, must be considered during the planning of flight trajectories to ensure that controllers will be able to maintain the desired separation among aircraft. It is estimated that approximately 70 to 75 percent of airline delays are caused by weather [5]. For this reason, this thesis focuses on addressing the problem of weather related congestion as opposed to the general congestion problem as a whole. The aspects of the model that make it more weather specific are the incorporation of stochasticity as well as being focused on a small region within the NAS rather than modeling it in its entirety.

2.4 Costs

The cost to an airline of a certain flight plan is determined by several factors. The most tangible of these factors is fuel usage. A slightly more indirect cost is the safety cost, and an even more indirect cost comes from customer dissatisfaction which ultimately can lead to a loss of business.

Fuel and safety costs are primarily accumulated while an aircraft is in the air. This means that if an aircraft cannot land without delay at the arrival airport, given

that it departs on time, then it is better to have the airplane wait on the ground before departure rather than be delayed in the air.

The cost from customer dissatisfaction is present both in delays that take place on the ground and delays in the air. Combining these different types of costs gives a positive cost for each time period that a flight is forced to wait on the ground, and an even greater cost for each time period that a flight must wait in the air.

2.5 Model Characteristics

There are two important characteristics of the model used in this thesis to address en-route congestion problems. These two characteristics are the modeling of stochasticity and the ability for aircraft to be redirected along a route that is different from its nominal route. Section 2.5.1 presents the case for needing stochasticity, while section 2.5.2 explains why rerouting capabilities are necessary.

2.5.1 Dealing with Stochasticity

Due to the highly stochastic nature of weather, it is difficult to generate an accurate high fidelity forecast of the state of the weather within the NAS at a time that is more than two hours away [6], and consequently, an accurate capacity forecast is also unavailable. Planning two hours in advance is not a reasonable option because flights of more than two hours duration could incur significant delays in the air due to plans developed on the basis of incorrect capacity forecasts.

A stochastic model can address problems caused due to severe weather. Although it would be possible to solve a deterministic planning problem and generate flight plans that are feasible under any possible weather development by assuming a worst case capacity for each sector, there is potential for underutilized system capacity when the weather is better than the worst case. A more flexible and less costly approach would be to generate contingency plans for each possible weather scenario, which could better utilize the capacity available in good weather scenarios while still ensuring that the bad weather scenarios do not result in disaster.

2.5.2 The Need for Modeling Rerouting

With the exception of one model presented by Patterson and Bertsimas [10], methods for dealing with enroute congestion do not typically provide for the capability of rerouting aircraft. Instead the costs of delay are mitigated only through the use of ground holding and airborne holding. That approach can neglect a major source for effectively offloading excess demand.

When there is convective weather within the NAS, it is rather unlikely that sector capacities will be reduced uniformly. It is instead likely that some sectors will have their capacities impacted more than others. Effective rerouting of aircraft can utilize available sector capacity and reduce system delay.

2.6 Literature Review

A stochastic model for the dynamic rerouting of aircraft has not been found anywhere in the literature. Patterson and Bertsimas present a model for solving the *Air Traffic Flow Management Rerouting Problem (TFMRP)* [10], but it does not address the issue of stochasticity. Additionally, to deal with dimensionality problems, flights from the same airport were aggregated into single commodities in a multi-commodity flow. This results in a loss of distinction among some flights, as well as fixed travel times between locations that are independent of equipment type. This loss of distinction prevents the use of flight specific delay costs and also limits the ability to correctly model a sequence of flights flown by a single aircraft. The fixed travel times result in model inaccuracies that should be avoided because they either assume aircraft speeds that are not feasible for slower aircraft or neglect the use of higher speeds for faster aircraft.

The Stochastic Air Traffic Flow Management Rerouting Problem (STFMRP) and formulation presented in this paper are based mainly upon the *Air Traffic Flow Management Problem (TFMP)* formulation of Patterson and Bertsimas [3]. This problem, and a corresponding formulation, was introduced by Lindsay, Boyd, and Burlingame. Another formulation was later presented by Helme. The reason that the Patterson-

Bertsimas formulation was used as a basis for the STFMRP formulation is because it has proven to have strong LP relaxations, resulting in tractability for large problems. Although these models do not address rerouting or stochasticity, Patterson and Bertsimas suggest an untested modification to incorporate rerouting into the model. Alonso, Escudero, and Ortuño [1] examined a stochastic variant of the Patterson TFMP formulation, but the need for rerouting was still not addressed.

Chapter 3

Problem Formulation and Solution Technique

This chapter presents the Stochastic Air Traffic Flow Management Rerouting Problem (STFMRP) and presents a technique for solving it. Section 3.1 presents a model for solving the Air Traffic Flow Management Problem that was formulated by Patterson and Bertsimas and used as a basis for the STFMRP. Section 3.2 introduces the STFMRP and describes the model formulation that was generated for solving it. This model, however, proved to be intractable in its initial formulation when experimenting with an integer programming solver. Therefore it was necessary to develop an alternate solution technique. This was accomplished by applying a Dantzig-Wolfe decomposition, the details of which are given in section 3.3.

3.1 Patterson-Bertsimas TFMP Model

The TFMP attempts to solve the operational problem of minimizing air traffic delay costs, but it does so by using deterministic sector capacities and fixed flight routes. Under this restricted set of assumptions, Patterson and Bertsimas were able to develop a model that could solve a problem formulation for the entire NAS modelled over a several hour period, with solution time short enough to be feasible for airline planning. An important key to the success of their model was that the 0-1 formulation had

a strong LP relaxation that required little or no application of branch and bound techniques.

Problem Definition

For a set of flights \mathcal{F} and a set of sectors \mathcal{J} , modeled over time periods \mathcal{T} , the problem is defined with the following data.

N_f	=	number of sectors in the path of flight f
$P(f, i)$	=	the i th sector in the path of flight f
$P(f, 1)$	=	the departure airport of flight f
$P(f, N_f)$	=	the arrival airport of flight f
P_f	=	the set $\{P(f, i) : 1 \leq i \leq N\}$
l_{fj}	=	minimum travel time in sector j for flight f
$C_j(t)$	=	capacity of sector j at time t
d_f	=	scheduled departure time of flight f
r_f	=	scheduled arrival time of flight f
c_f^a	=	cost of holding flight f in the air for one period
c_f^g	=	cost of holding flight f on the ground for one period
T_f^j	=	set of feasible times for flight f to arrive at sector j
\underline{T}_f^j	=	first time period in the set T_f^j
\overline{T}_f^j	=	last time period in the set T_f^j

The two different costs c_f^a and c_f^g are used to represent the greater cost of having an airplane circle in the air as opposed to sitting on the ground. The objective of the model is to minimize the total cost for all flights by deciding how long to hold each plane on the ground and in the air.

3.1.1 Model Formulation

All of the decision variables are of the form w_{ft}^j where

$$w_{ft}^j = \begin{cases} 1 & \text{if flight } f \text{ arrives at sector } j \text{ by time } t \\ 0 & \text{otherwise} \end{cases}$$

Each triple (f, j, t) such that $t \in T_f^j$ contributes one decision variable to the formulation. Note, by variable definition, for each flight f , the expression $w_{ft}^{P(f,1)} - w_{f,t-1}^{P(f,1)}$ can only be equal to one for a single value of t . Specifically that value is the time at which flight f will depart. Thus we can compute the departure time as

$$\sum_{t \in T_f^{P(f,1)}} t(w_{ft}^{P(f,1)} - w_{f,t-1}^{P(f,1)})$$

The arrival time can be computed in a similar manner. Both the departure and arrival are not allowed to happen before the scheduled time. Therefore, the ground delay for a flight can be found by subtracting the scheduled departure time from the modeled departure time. The arrival delay can be computed in the same way, and then the airborne delay is found by subtracting the ground delay from the arrival delay. Multiplying these delays by their respective costs gives the objective function for the model below.

$$\begin{aligned} \text{Min} \sum_{f \in \mathcal{F}} & \left[c_f^g \left(\sum_{t \in T_f^{P(f,1)}} t(w_{ft}^{P(f,1)} - w_{f,t-1}^{P(f,1)}) - d_f \right) \right. \\ & + c_f^a \left(\sum_{t \in T_f^{P(f,N_f)}} t(w_{ft}^{P(f,N_f)} - w_{f,t-1}^{P(f,N_f)}) - r_f \right. \\ & \left. \left. - \sum_{t \in T_f^{P(f,1)}} t(w_{ft}^k - w_{f,t-1}^k) - d_f \right) \right] \end{aligned} \quad (3.1)$$

$$\sum_{f: P(f,i)=j, P(f,i+1)=j', i < N_f} (w_{ft}^j - w_{ft}^{j'}) \leq C_j(t) \quad \forall j \in \mathcal{J}, t \in \mathcal{T} \quad (3.2)$$

$$w_{f,t+l_{fj}}^{j'} - w_{ft}^j \leq 0 \quad \forall f \in \mathcal{F}, j = P(f, i), j' = P(f, i + 1), t \in \bar{T}_f^j, i < N_f \quad (3.3)$$

$$w_{ft}^j - w_{f,t-1}^j \geq 0 \quad \forall f \in \mathcal{F}, j \in P_f, t \in T_f^j \quad (3.4)$$

$$w_{f,\bar{T}_f^j}^j = 1 \quad \forall f \in \mathcal{F}, j \in P_f \quad (3.5)$$

$$w_{ft}^j \in \{0, 1\} \quad \forall f \in \mathcal{F}, j \in P_f, t \in T_f^j \quad (3.6)$$

Constraints 3.2 model the limited sector capacities. If an aircraft has entered the i th sector in its path at time t , but has not yet entered the $i + 1$ st sector, then the difference within the sum will equal one, and it will otherwise be zero. The sum of all of these differences gives the number of aircraft in a given sector at time t , which must be less than the given capacity.

Constraints 3.3 enforce the minimum travel times through sectors in the path, preventing flight f from spending less time than l_{fj} in sector j

Constraints 3.4 represent the connectivity in time between the variables. Thus if a flight has arrived at a sector by time t , then for each $t' > t$, the flight has also arrived by time t' .

Constraints 3.5 ensure that flights arrive at each sector by the last possible time.

Constraints 3.6 force all of the variables to be binary.

3.2 Stochastic Air Traffic Flow Management

Rerouting Problem

The Stochastic Air Traffic Flow Management Rerouting Problem (STFMRP) assumes that a scenario-based stochastic forecast of sector capacities is available. This is the

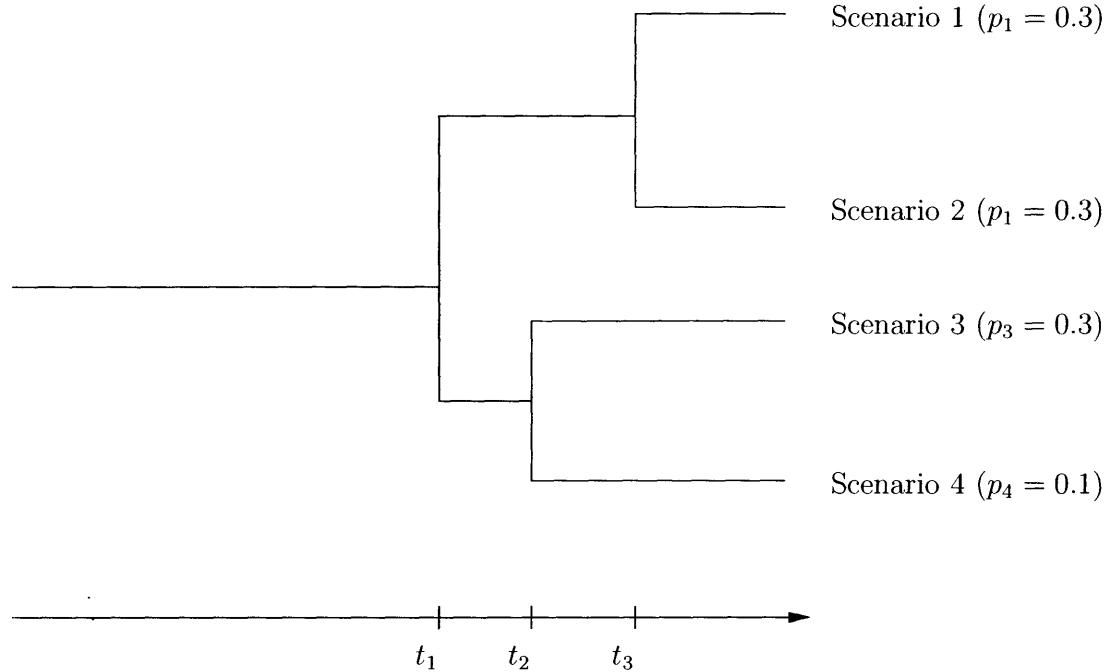


Figure 3-1: A scenario tree example.

only source of stochasticity in the model, as travel times are deterministic.

3.2.1 Definition

When rerouting and stochasticity are added to the problem, the definition requires a few minor changes. Now letting \mathcal{S} be the set of scenarios in the forecast, the changes in the problem data are given below.

$l_{fj j'}$ = minimum travel time from sector j to sector j' for flight f

$N(j)$ = the set of sectors neighboring sector j

$C_{js}(t)$ = capacity of sector j at time t in scenario s

$T_{\Delta}(s, s')$ = earliest time by which the scenarios
 s and s' can be distinguished

p_s = probability that scenario s is the true scenario

An illustration of a scenario forecast tree is shown in figure 3-1. For example in this scenario, we have $T_{\Delta}(1, 2) = t_3$ and $T_{\Delta}(1, 3) = t_1$.

3.2.2 Formulation

As stated previously, the Patterson-Bertsimas model was quickly solvable even when modeling large scenarios. The expectation in using this model as a basis for a STFMRP formulation, was that increasing the complexity by adding in rerouting and stochasticity would be offset by the reduced size of the region being modeled, resulting in a tractable formulation. Although their model does not support the rerouting of aircraft, Patterson and Bertsimas suggest that this can be done by extending the variables to be of the following form:

$$w_{jt}^{jj'} = \begin{cases} 1 & \text{if flight } f \text{ arrives at sector } j' \text{ from sector } j \text{ by time } t \\ 0 & \text{otherwise} \end{cases}$$

By extending the variable definition even further with an additional subscript such that

$$w_{fts}^{jj'} = \begin{cases} 1 & \text{if flight } f \text{ arrives at sector } j' \text{ from sector } j \text{ by time } t \text{ in scenario } s \\ 0 & \text{otherwise} \end{cases}$$

the Patterson-Bertsimas model can be extended to provide the capability for modeling a stochastic scenario in which rerouting is allowed.

The two costs c_f^a and c_f^g are used again as in the original model, except that this time the cost being minimized is an expected cost. Ground delays and air delays are again the basis of this expected cost, and rerouting is now available as well.

Sectors labeled δ_f and ρ_f respectively correspond to the departure and arrival airports as $P(f, 1)$ and $P(f, N_f)$ did in the Patterson-Bertsimas model. For example if $w_{fts}^{j\rho_f} = 0$ then flight f has not arrived at its destination by time t , from sector j , in scenario s . For these airports we define $N(\delta_f)$ and $N(\rho_f)$ to be the sets of sectors that are feasible entrances into the modeled region from the departure and arrival airports respectively. Similarly an airport is included in $N(j)$ if sector j is reachable from the airport without passing through other modeled sectors.

In addition to including variables for each pair of neighboring sectors, the variable $w_{fts}^{\delta_f \delta_f}$ will be used to represent departure by time t as $w_{ft}^{P(f,1)}$ was used earlier. The airborne delays and ground delays are computed in the same as they were before, but now the cost being minimized is a weighted average of the costs for each scenario, with the weights being given by the scenario probabilities p .

$$\text{Min } \sum_{s \in \mathcal{S}} p_s \sum_{f \in \mathcal{F}} \left[c_f^g \sum_{t \in T_f^{\delta_f}} (t - d_{ff}) (w_{fts}^{\delta_f \delta_f} - w_{f,t-1,s}^{\delta_f \delta_f}) \right. \\ \left. + c_f^a \left(\sum_{t \in \bar{T}_f^{\rho_f}, j \in N(\rho_f)} (t - r_f) (w_{fts}^{j \rho_f} - w_{f,t-1,s}^{j \rho_f}) \right. \right. \\ \left. \left. - \sum_{t \in T_f^{\delta_f}} (t - d_f) (w_{fts}^{\delta_f \delta_f} - w_{f,t-1,s}^{\delta_f \delta_f}) \right) \right] \quad (3.7)$$

$$\sum_{f \in \mathcal{F}} \sum_{j' \in N(j)} w_{fts}^{j'j} - \sum_{f \in \mathcal{F}} \sum_{j'' \in N(j)} w_{fts}^{jj''} \leq C_{js}(t) \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \quad (3.8)$$

$$\sum_{j' \in N(j)} w_{f,t+l_{fj'},s}^{jj'} - \sum_{j'' \in N(j)} w_{fts}^{j''j} \leq 0 \quad \forall j \in \mathcal{J}, f \in \mathcal{F}, t \in T_f^j, s \in \mathcal{S} \quad (3.9)$$

$$w_{fts}^{jj'} - w_{f,t-1,s}^{jj'} \geq 0 \quad \forall j \in \mathcal{J}, j' \in N(j), f \in \mathcal{F}, t \in T_f^j, s \in \mathcal{S} \quad (3.10)$$

$$w_{fts}^{\delta_f \delta_f} = 1 \quad \forall f \in F, s \in \mathcal{S}, t = \bar{T}_f^{\delta_f} \quad (3.11)$$

$$\sum_{j \in N(\rho_f)} w_{fts}^{j \rho_f} = 1 \quad \forall f \in F, s \in \mathcal{S}, t = \bar{T}_f^{\rho_f} \quad (3.12)$$

$$w_{fts}^{jj'} - w_{fts'}^{jj'} = 0 \quad \left\{ \begin{array}{l} \forall j \in \mathcal{J}, j' \in N(j), f \in F, \\ s, s' \in \mathcal{S}, t < T_{\Delta}(s, s') + l_{fjj'} \end{array} \right. \quad (3.13)$$

$$w_{fts}^{jj'} \in \{0, 1\} \left\{ \begin{array}{l} \forall j \in J, j' \in N(j), f \in F, \\ s \in S, t \in T \end{array} \right. \quad (3.14)$$

Constraints 3.8, like constraints 3.2, prevent the number of aircraft in a sector from exceeding the forecasted capacity during that time period. The only difference is that to determine arrival by time t , it is necessary to sum the variables corresponding to arrival for all possible entry sectors j' . Similarly, to determine departure from the sector, we sum over all possible exiting sectors j'' .

Constraints 3.9 are similar to constraints 3.3 in that they enforce the minimum travel time requirement for flights traveling between two sectors. They, with 3.14, also prevent a flight from exiting a sector into multiple neighbors. Again it is necessary to sum over all of the entrances and exits. A possibly useful interpretation for these constraints would be to view them each as half of a flow balance constraint that says that the flow out is less than or equal to the flow in.

Constraints 3.10 provide connectivity between consecutive time periods as constraints 3.4 did.

Constraints 3.13 represent the inability to take actions based upon information that is not yet available, which is more formally known as the non-anticipativity principle. Specifically the constraints state that at time t it is not possible to choose to go towards sector j' under scenario s and not do so in scenario s' if the two scenarios are not distinguishable at that time.

Constraints 3.11 and 3.12 specify that exactly one aircraft representing flight f must take off and exactly one must land.

Constraints 3.14 force all variables to be binary.

3.2.3 Problem size

This formulation generates an LP that proved to be intractable in initial testing. The number of variables generated is roughly $|\mathcal{F}||\mathcal{S}||\mathcal{J}|DK$, where D is the maximum delay (this is the same as the number of possible time periods during which a flight may enter a sector), and K is the average number of neighboring sectors. Even for

a relatively small problem with $|\mathcal{F}| = 60$, $|\mathcal{S}| = 3$, $|\mathcal{J}| = 30$, $D = 10$, $K = 3$, the number of variables is nearly 200,000. The number of constraints generated is approximately the same, mainly due to constraints 3.10. This necessitates the need for an alternate solution method that does not simply feed this formulation directly to an MIP solver.

3.3 Decomposition

To deal with the unwieldy size of the formulation, a Dantzig-Wolfe decomposition [4] was used. The master problem is given by the constraints 3.8, the only constraints that bundle the flights together. The remaining constraints define $|\mathcal{F}|$ subproblems, each of which yields a feasible flight plan as a solution.

3.3.1 Master Problem

Let λ_i^f be a 0-1 variable that represents the selection of flight plan i for flight f as generated by a sub-problem for flight f . The following parameters are defined for the master problem.

p_i^f = The expected delay cost of flight plan i for flight f

$\gamma_i^f(j, t, s)$ = Indicator function that is 1 if flight plan i
has the aircraft in sector j at time t in scenario s

I_f = The set of all possible flight plans for flight f

The master problem is defined as

$$\text{Min } \sum_{f \in F} \sum_{i \in I_f} p_i^f \lambda_i^f$$

$$\sum_{i \in I_f} \lambda_i^f = 1 \quad \forall f \in F \tag{3.15}$$

$$\sum_{f \in F} \sum_{i \in I_f} \gamma_i^f(j, t, s) \lambda_i^f \leq C_{js}(t) \quad \forall j \in \mathcal{J}, t \in \mathcal{T}, s \in \mathcal{S} \tag{3.16}$$

$$\lambda_i^f \in \{0, 1\} \quad \forall f \in \mathcal{F}, i \in I_f \quad (3.17)$$

Constraints 3.15 and 3.17 ensure that one flight plan is chosen for each flight while constraints 3.16 are the sector capacity constraints 3.8.

Since there are a huge number of possible flight plans for each flight, column generation must be used to keep the problem size reasonable. Each time the master problem is solved on a restricted subset of flightplans, up to $|\mathcal{F}|$ new flight plans are generated using the dual costs of the current solution to the restricted master problem. If none of the newly generated λ_i^f 's has a negative reduced cost, the algorithm terminates.

3.3.2 Solving the Sub-problems

The master problem generates two different types of dual costs. Cost μ_i corresponds to the cost of the i th flight constraint of type 3.15, and cost ν_{jts} is associated with the sector capacity constraint for sector j at time t in scenario s . Given these dual costs, the goal of the sub-problems is to identify flight plans that will lead to a solution with lower cost.

Instead of using an LP solver to solve the subproblems, it is possible to exploit their time dependent structures to generate optimal solutions with a Dynamic Programming (DP) algorithm. To put this in the form of a DP problem, let z correspond to a segment of the scenario tree. Each segment of the tree represents a possible information state about the true weather scenario. In each information state the set of candidates for the true scenario is a subset of \mathcal{S} . The DP algorithm can iteratively compute the optimal expected cost-to-go for a flight that reaches sector j at time t in segment z , by starting with terminal costs of the various landing times and working backwards in time. This cost-to-go is a minimized expectation of the costs that will be accrued over the remainder of a flight.

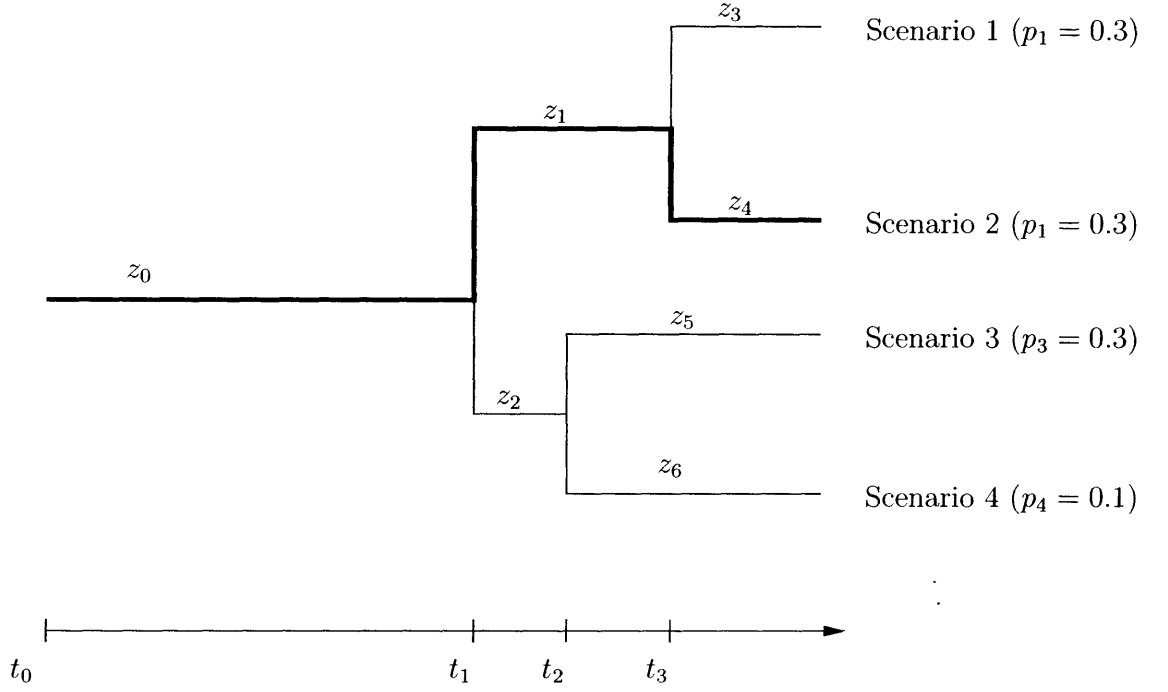


Figure 3-2: A scenario tree with labeled information states.

- $Z(t)$ = the set of segments for time t
- $V_f(t, j, z)$ = the optimal cost-to-go for flight f while in
= time t , sector j , and segment z .
- $S(z)$ = the set of scenarios included in segment z
- \hat{p}_z = the probability that the true scenario is included
= in segment z ($\sum_{s \in S(z)} p_s$)
- $Z(z, t)$ = the subset of $Z(t)$ that is reachable from segment z

As an illustration of this notation, we have the same scenario tree as before, but it now has its segments labeled. Letting z_0 correspond to the information state during planning gives $Z(t_0) = z_0$, $S(z_0) = \mathcal{S}$, $\hat{p}_z = 1$, and $Z(z_0, t) = Z(t)$. In the scenario tree z_0 shows up as the segment coming from the root of the tree at time t_0 .

For the realization of scenario 2, which is depicted with the thick lines in the tree, we start of in information state z_0 , where the true scenario could be any of the four possibilities. We then proceed to information state z_1 , in which we know that the true scenario is either scenario 1 or scenario 2, and finally we end up in state

z_4 , in which we are certain that the true scenario is scenario 2. For segments that start farther along in time, $Z(z, t)$, becomes a reduced subset of $Z(t)$. For example $Z(z_1, t_3) = \{z_3, z_4\}$, while $Z(t_3) = \{z_3, z_4, z_5, z_6\}$ because once it is known that the true scenario is either scenario 1 or scenario 2, it is not possible to know later on that the true scenario is scenario 3 or scenario 4. To begin the computation of the DP, first the terminal costs are set depending on how late the flight reaches its destination, as

$$V_f(t, \rho_f, z) = c_f^a(t - a_f) \text{ for each } t \in T_f^j, z \in Z(t).$$

Using these terminal values, the sector capacity dual costs of ν_{tjs} , it is now possible to compute the cost-to-go functions for previous time periods and other sectors. This cost is computed as

$$V_f(t, j, z) = \min \left[\begin{array}{l} \sum_{z' \in Z(z, t+1)} \frac{\hat{p}_{z'}}{\hat{p}_z} V_f(t+1, j, z') + \sum_{s \in S(z)} \frac{\nu_{tjs}}{\hat{p}_z}, \\ \min_{j' \in N(j)} \left(\sum_{z' \in Z(z, t+l_{fjj'})} \frac{\hat{p}_{z'}}{\hat{p}_z} V_f(t+l_{fjj'}, j', z') \right. \\ \left. + \sum_{t'=t}^{t+l_{fjj'}-1} \sum_{z' \in Z(z, t')} \frac{\hat{p}_{z'}}{\hat{p}_z} \sum_{s \in S(z')} \frac{\nu_{t'js}}{\hat{p}_z} \right) \end{array} \right] \quad (3.18)$$

The first term in the minimization corresponds to the cost-to-go if the aircraft is delayed in the air for one period. This is computed as the expected cost-to-go from being in the sector at the next time period, which is the expectation of $V_f(t+1, j, z')$, plus the cost of occupying the sector for the current time period, which is $\frac{\nu_{tjs}}{\hat{p}_z}$. To compute the expectation of $V_f(t+1, j, z')$, we add the costs for each possible z' scaled by the probability $\frac{\hat{p}_{z'}}{\hat{p}_z}$ of ending up in state z' at time $t+1$ given that at time t , the information state is z . The reason that the sector occupancy costs are scaled by $\frac{1}{\hat{p}_z}$ is because ν_{tjs} is an expected cost of planning to occupy sector j at time t in scenario s given that at time t_0 the information state is z_0 . The cost incurred by this choice is zero for other information states at time t , therefore the cost in state z must be

scaled in this manner.

The second term corresponds to the minimum cost of directing the aircraft towards a neighboring sector. This cost is computed as the expected cost-to-go from the neighboring sector after the specified minimum travel time plus the expected cost of occupying the current sector until that travel time has elapsed.

For the departure airport we compute the cost the same except that the first term, which corresponded to in-place delay is decreased by $c_f^a - c_f^g$ to represent the value gained by substituting ground delay for airborne delay. The value obtained for $V_f(d_f, \delta_f, z_0)$ plus μ_f gives the reduced cost of the flight plan, which is only kept if this cost is less than zero.

3.3.3 Branch and Bound Strategy

The LP relaxation of the formulation does not always yield an integral solution, so it is necessary to formulate a branch and bound methodology to get the optimal integral solution. The typical branch and bound strategy of choosing a single fractional variable to set to 0 or 1 in separate subproblems does not work well in conjunction with column generation, and it also changes the structure of the subproblem, making it impossible to use the DP. To branch effectively while still maintaining a subproblem structure that is solvable by the DP, the following method was employed [2].

For a problem with a fractional solution, the problem is split into two subproblems. Let (t, j, s) be a time-sector-scenario triplet for which some fractional part of flight f is designated to be in sector j at time t in scenario s . One of the subproblems requires that the flight passes through (t, j, s) , while the other forbids it. This is easily implemented in the subproblems by adjusting the prices ν_{tjs} to be either $-\infty$ or ∞ .

As is typically the case, the choice of what to branch upon is an important factor in performance. The most effective strategy found was to branch on a triple (t, j, s) with a maximum dual cost among those that experienced fractional occupation. Because the LP relaxation of the formulation is fairly strong to begin with, we found that this strategy terminates with an optimal solution in a reasonable amount of time.

Chapter 4

Modeling Variations

This chapter introduces several variations to the STFMRP formulation that can be used to model the problem in greater detail. Section 4.1 presents some constraints that are part of the Patterson-Bertsimas model that were not included in the STFMRP. Section 4.2 shows how non-linear cost functions can be used for the delay cost rather than the linear cost functions that are used. Lastly section 4.3 shows how the level of passenger disruption caused by a potential solution can be incorporated into the objective function.

4.1 Patterson-Bertsimas variations

There are three sets of constraints that were part of the Patterson-Bertsimas formulation [10] of the TFMP that were left out of the STFRMP formulation of this thesis. These constraints on legitimate flight plans were not removed because they made the model too complicated. They were simply left out because they were not seen as being as relevant as sector capacities when rerouting aircraft to accommodate convective weather. Two of these types of constraints allow limits to be imposed on departure and arrival capacities at airports. Adding these constraints will be discussed in subsection 4.1.1. The third type of constraint gives the ability to model continued flights, which means that if a single aircraft is used for two consecutive flights, then the departure of the second flight cannot be before a minimum turnaround time has elapsed

after the first flight lands. The implementation of these constraints is discussed in subsection 4.1.2.

4.1.1 Arrival and Departure Capacities

Because the model can only be used for modeling regions of limited size, the flights included will often represent only a fraction of the flights scheduled to use the arrival and destination airports. The exceptions to this would be airports that are actually within the region being modeled. Since this region is most likely experiencing severe convective weather, it would be prudent to model the reduced runway capacity at these airports by limiting their arrival and departure capacities.

For a flight f , the two expressions

$$w_{fts}^{\delta_f \delta_f} - w_{f,t-1,s}^{\delta_f \delta_f}, \quad (4.1)$$

and

$$\sum_{j \in N(\rho_f)} (w_{fts}^{j \rho_f} - w_{f,t-1,s}^{j \rho_f}) \quad (4.2)$$

each only evaluate to 1 if the flight departed or arrived respectively at time t in scenario s and otherwise evaluates to 0. Thus summing each expression over all flights using the same airport gives the numbers of flights departing and arriving at a particular time.

The runway restrictions for airports in the set \mathcal{K} are defined with the following data

$D_{ks}(t)$ = the departure capacity of airport k at time t in scenario s

$A_{ks}(t)$ = the arrival capacity of airport k at time t in scenario s

δ_f = the departure airport of flight f

ρ_f = the arrival airport of flight f

Adding the constraints

$$\sum_{f: \delta_f = k} (w_{fts}^{\delta_f \delta_f} - w_{f,t-1,s}^{\delta_f \delta_f}) \leq D_{ks}(t) \quad \forall k \in \mathcal{K}, s \in \mathcal{S}, t \in \mathcal{T} \quad (4.3)$$

$$\sum_{f:\rho_f=k} (w_{fts}^{j\rho_f} - w_{f,t-1,s}^{j\rho_f}) \leq A_{ks}(t) \quad \forall k \in \mathcal{K}, s \in \mathcal{S}, t \in \mathcal{T} \quad (4.4)$$

generate the desired restrictions on the departure and arrival capacities.

These constraints remain as part of the master problem in the decomposition. The added complexity from each constrained airport should be no greater than the complexity that would come with adding an additional constrained sector. The effect on the subproblems is that there is an added cost for certain departure and arrival times. This can be dealt with appropriately in the DP algorithm by modifying the terminal costs and adjusting how the costs are computed at the departure airport.

It is even possible to model capacities that are not independent of each other, such that more arrivals can be allowed at the cost of reducing the number of departures, or vice versa. For more details on dependent capacities see [10].

4.1.2 Continued Flights

For airports within the modeled region, it could be useful to model the use of a single aircraft for successive arriving and departing flights to ensure greater model accuracy. Let \mathcal{C} be a set of ordered pairs of flights representing all connections, such that for each $(f, f') \in \mathcal{C}$ flight f is continued by flight f' . Letting θ_f be the minimum turnaround time needed for flight f , the needed constraints are as follows.

$$\sum_{j \in N(\rho_f)} w_{fts}^{j\rho_f} - w_{f',t-\theta_f,s}^{\delta_{f'}\delta_{f'}} \geq 0 \quad \forall (f, f') \in \mathcal{C}, t \in T_f^{\delta_{f'}}, s \in \mathcal{S} \quad (4.5)$$

Constraints 4.5 work in a manner similar to constraints 3.9, by preventing a departure of the continuing flight before a minimum amount of time has elapsed after arrival of the continued flight.

As for the airport capacity constraints 4.3 and 4.4, these constraints would remain in the master problem when the problem is decomposed. The changes to the subproblem are again only changes to costs of certain departure and arrival times, which are easily captured in the DP algorithm.

4.2 Non-linear Costs

Although linear cost functions are often used to determine the values of solutions to problems in air traffic flow management, it is not necessary when using the 0-1 formulation of Patterson and Bertsimas. Many costs, such as fuel and safety costs, are in fact accumulated in a linear manner, with an equal value being consumed for each time period spent aloft. There are however some costs that do not accumulate in a linear manner. One example is the cost of missed passenger connections. A flight can most likely arrive several minutes late without having any passengers miss their connections to other flights, but a longer delay indeed results in missed connections. The cost accumulated during the first few minutes of delay is zero, whereas the cost of further delay is positive. Clearly a non-linear cost function can be of use in making the model more accurate.

It is not possible to model a flight cost that is an arbitrary function of the ground delay and the airborne delay, but it is possible to model a cost that can be computed as an arbitrary function of ground delay plus an arbitrary function of total delay. As was noted in section 4.1.1, for each scenario s and flight f , the two expressions 4.1 and 4.2 each only evaluate to one for a single time, and that these times are the departure and arrival times respectively. This means that the objective can be changed to

$$\text{Min } \sum_{s \in \mathcal{S}} p_s \sum_{f \in \mathcal{F}} \left[\sum_{t \in T_f^{\delta f}} c_f^g(t) \left(w_{fts}^{\delta f \delta f} - w_{f,t-1,s}^{\delta f \delta f} \right) + \sum_{t \in T_f^{\rho f}, j \in N(\rho f)} c_f^r(t) \left(w_{fts}^{j \rho f} - w_{f,t-1,s}^{j \rho f} \right) \right] \quad (4.6)$$

where $c_f^g(t_1)$ and $c_f^r(t_2)$ are the costs associated with flight f departing and arriving at times t_1 and t_2 respectively.

This change is all that is needed to use non-linear costs. These costs do not affect the ability to apply the decomposition and use the DP algorithm to generate flight plans. The algorithm only requires a modification to how the terminal costs are

initialized and how the costs are computed at the departure airport.

4.3 Passenger Disruption

The level of passenger disruption that is caused by a schedule change cannot simply be computed from the arrival times of flights. This disruption also depends on when connecting flights leave. If a flight arrives late, passengers might still make a connection if their next flight is also delayed. Therefore, the actual cost of a solution can be computed more accurately if the cost function is not limited to being a sum of costs for individual flights.

To represent the disruption, the model can include variables of the following form:

$$x_{ff's} = \begin{cases} 1 & \text{if passengers from flight } f \text{ do not connect with flight } f' \text{ in scenario } s \\ 0 & \text{otherwise} \end{cases}$$

The constraints used to represent disruptions are similar to the constraints used for continued flights. The difference is that the passenger disruption constraints make the minimum delay between departure and arrival optional, with a cost for not meeting the minimum, while the continuation constraints make the delay mandatory. Let $\kappa_{ff'}$ be the minimum time between arrival and departure that allows passengers from flight f connect to flight f' , and let \mathcal{Q} be the set of possible passenger connections. The constraints to represent making connections are

$$\sum_{j \in N(\rho_f)} w_{fts}^{j\rho_f} - w_{f',t+\kappa_{f,s}}^{\delta_{f'}\delta_{f'}} + x_{ff's} \geq 0 \quad \forall (f, f') \in \mathcal{Q}, t \in T^{\delta_{f'}}$$

The addition of the $x_{ff's}$ term to the minimum delay type constraint allows the original constraint to be violated, but only at the cost of missed connections.

These constraints will be treated the same as the flight continuation constraints in the decomposition. Because they stay in the master problem, it now has $x_{ff's}$ variables in addition to λ_i^f variables.

Letting $\xi_{ff'}$ be the cost incurred when passengers on flight f cannot make their connection to flight f' , the terms $p_s \xi_{ff'} x_{ff's}$ are added to the objective function to represent the expected cost of missed connections. By including these disruption costs in the objective function, it is possible to more accurately evaluate the real cost of a solution and therefore generated solutions perform better.

Chapter 5

Computational Results

This chapter gives the results of several computational experiments performed using the decomposed formulation of section 3.3. There are three goals for these experiments. The first goal is to determine whether problems of a realistic size can be solved with the model in a reasonable amount of time. The second goal is to compare the costs that can be achieved using the model to costs that result from using a deterministic model. The last goal is to examine the quality of plans that are generated if the decomposition algorithm is terminated prior to reaching optimality.

All tests were run on a 2.1 GHz Pentium 4 with 1 GB of RAM. For the decomposition, the Master problem was solved using the XPRESS-MP solver, while the DP subproblems to generate flight plans were implemented in Java.

The scenarios all modeled a region of 40 sectors in and near the Cleveland ARTCC, with model time periods corresponding to 5 minutes. This region is depicted in figure 5. The sectors in the middle of this region are designated as having conditions that are likely to lead to storm activity. The number of flights included in the model was 148. Costs are set such that $c_f^a = 2$ and $c_f^g = 1$ for all flights.

5.1 Running Time

Test 1 models 3 possible scenarios, corresponding to good weather, moderate storms, and heavy storms. The times at which these three scenarios become distinguishable

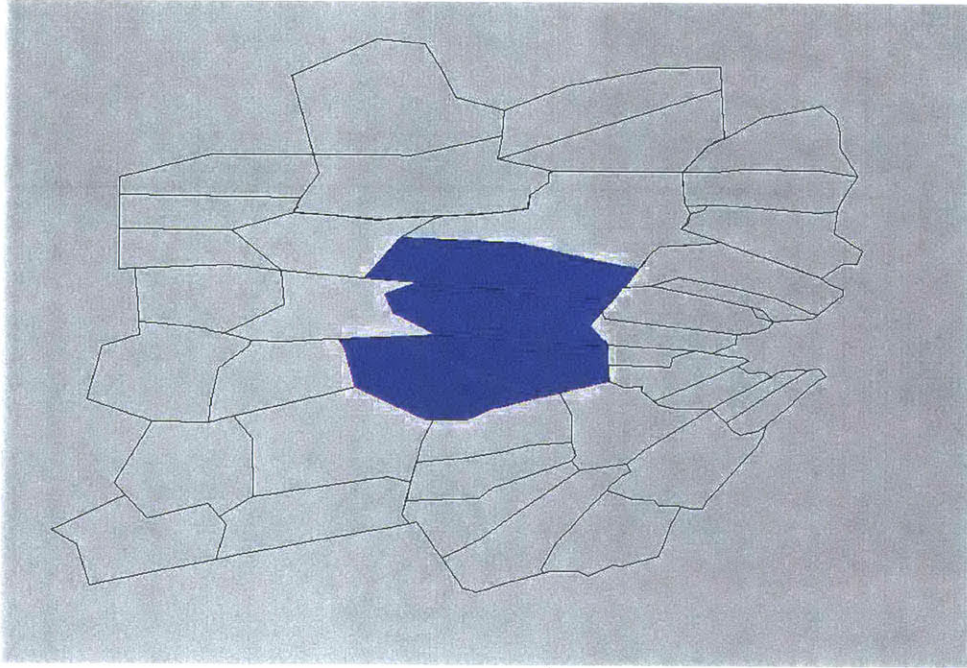


Figure 5-1: Sectors modeled in computational exercises, with the storm impacted sectors shown in blue.

from each other is the same for each pair. The nominal capacity for each sector is set to 10 until the scenario divergence time, at which point the capacities of the central sectors are reduced to 4 and 2 respectively in the moderate storms scenario and the heavy storm scenario, while the capacities remain at 10 in the good weather scenario. The good weather scenario is assigned a probability of .4, and the other two scenarios each occur with probability .3.

The problem is solved after 72 iterations of column generation requiring a total of 12 minutes. Table 5.1 shows how much time is needed to solve the DP subproblems and the LP for the restricted master problem in each of several iterations. The time needed for the master problem increases at first as many columns are generated in the first few iterations, but levels off at around 1 second. The DP time stays relatively constant as one would expect since the algorithm computes the same formula in each iteration with different inputs.

The total time needed to obtain a solution is dominated by the column generation process. This can be alleviated in two ways. One way is by using a more performance

Iteration	DP time (secs)	LP time (secs)
1	9.982	0.168
2	9.652	0.251
3	9.646	0.485
4	9.583	0.725
5	9.555	0.739
6	9.861	0.864
7	9.871	0.804
8	9.65	0.917
9	9.754	0.942
10	9.763	1.005
⋮	⋮	⋮
72	9.496	1.011

Table 5.1: Computation times in seconds for DP subproblems and LP master problem.

oriented language rather than Java. Another way is to parallelize the process. The DP time is actually the time to solve 1 subproblem for each of the 148 flights, rather than the time to solve a single problem. Since the problem solutions are not dependent on each other, these many subproblems can be easily solved in parallel on separate computers. In addition to these two methods for reducing the solution time, the LP time for the master problem can also be reduced by reusing the bases from previous solutions as a starting point in each iteration.

5.2 Stochastic vs. Deterministic

In the second test, the results for the stochastic model are compared with results obtained using deterministic capacities. The scenario used is the same as for the first test. To evaluate the use of deterministic capacities, flights are planned using “fixed” capacities until the divergence time, and replanned to deal with the actual capacities at that time. Table 5.2 summarizes the results from the stochastic plan, and from a deterministic plan using the mean capacities as the fixed capacities.

There is more than a ten percent reduction in cost between the stochastic model and the deterministic model with mean capacities. This is a tremendous cost im-

Model	Cost achieved
Stochastic	127.3
Deterministic with mean capacities	142.0

Table 5.2: Plan costs for stochastic and deterministic planning methods.

provement when considering the amount of money that is annually lost by the airline industry due to delays. The deterministic model is also somewhat optimistic in the sense that it assumes that a global replan is possible while planes are enroute. As was stated in Chapter 2, the highly distributed nature of air traffic control makes the changing of flight plans very difficult. This necessitates proper planning with full recourse prior to takeoff.

5.3 Convergence rates

As was mentioned in section 3.3, the column generation process can be halted prior to satisfaction of the optimality condition. The master problem can then be solved with only a subset of the feasible flight plans for each flight. This premature algorithm termination might be necessary when an answer is needed soon, but the algorithm has not yet terminated. If this is to be used as a fallback plan, then it is useful to know how far from optimal the generated plan actually is.

As the system becomes more constrained, the algorithm's time to termination tends to degrade. For this reason, the capacity of the impacted sectors in the heavy storm scenario is decreased to 1 for this test. The algorithm was terminated prior to completion after running for 800 iterations. The entries in table 5.3 show the optimal values for the restricted master problem obtained after several iterations. The iterations listed are those at which the objective value actually improved, starting with iteration 7 which had the first feasible solution.

The objective value did not improve at all during the last 700 iterations. The bound given by the LP relaxation of the master problem is 173.87 which is within 1 percent of the best solution obtained. When observing the results of the test for

Iteration	Objective Value
7	198.8
8	192.9
9	187.1
10	183.2
11	181.3
12	180.1
13	177.4
14	177.1
15	177
16	176.5
19	176.2
36	175.9
42	175.6
94	175.3

Table 5.3: Objective function for restricted master problem

section 5.1, it is noted that the optimal solution was found long before the branch and bound process was able to prove that the solution was indeed optimal so it is quite possible that 175.3 is that actual objective value even the algorithm did not manage to prove this within a reasonable amount of time.

Chapter 6

Conclusion

6.1 Summary

In this thesis, it is shown that a model that handles both stochasticity and rerouting can deal effectively with en-route capacity reductions due to convective weather. A model that does this, the STFMRP, based upon the TFMP formulation of Patterson and Bertsimas is described along with a method for generating solutions to the STFMRP that deals with the intractability of the large 0-1 MIP that results from the problem formulation. This method was implemented and evaluated in several contexts.

The solution technique is shown to work on a realistically-sized problem, and provide results that are a significant improvement over the results from using a deterministic model for planning. In the case of slow termination, which can happen for problems that are heavily constrained, it is shown that optimal or near-optimal solutions can be obtained without running the algorithm to completion.

6.2 Future Work

A major problem with a global optimization approach determine optimal allocation of en-route capacity is that airlines are reluctant to share information if they do not perceive any benefit. This can result in allocating resources to flights that have been

cancelled, which is of course suboptimal.

Before the formation of the CDM initiative, there was a different methodology used to implement ground delays. Using CDM has proven to be more effective, due to the incentive for airlines to share information. In designing methods to allocate congested resources, it is necessary to consider the willingness of the airlines to participate.

The price based decomposition used in this paper is suggestive of an auction, in that prices are adjusted so that resources go to buyers that value them the most. With very minor changes, the master problem can be recast as a combinatorial auction problem, where each flight or airline bids for the right to occupy certain sectors during certain time periods.

Recently, a large amount of research has been done for effective ways to solve combinatorial auction problems, and just as importantly, the truth revelation that is embedded in these methods [9, 8]. As we see from the computational experiments, the LP relaxation of the STFMRP formulation typically does not yield integral solutions. This indicates that placing prices on individual resources, as was done in the decomposition, fails to capture some interdependence among them. By considering the value of a resource bundle as a whole rather than as the sum of the values of individual resources, it is possible that more insight could be gained into the structure of the problem, and a better solution technique could be generated.

By using an auction approach to slot allocation, it could be shown to airlines that it is in their best interest to show how much they truly value the resources that they are allocated. Airlines would not need to bid with real money. They could instead each be allocated a number of credits based upon their nominal airspace usage. This method of allocation would match well with the CDM ideology, which maintains that negotiation between the FAA and the airlines is a necessary part of efficient planning.

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