An Agent-based Approach to Modeling Electricity Spot Markets

by

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Abstract

Current approaches used for modeling electricity spot markets are static oligopoly models that provide top-down analyses without considering dynamic interactions among market participants. This thesis presents an alternative model, an agent-based model, and uses it to analyze the markets under various conditions. These markets, in which the participants engage in sealed-bid auctions to sell and/or buy electricity regularly, are viewed as multiagent systems, or as repeated games, played by participants with incomplete information. To represent these market characteristics, the agent-based model is selected, consisting of several power-producing agents with non-uniform portfolios of generating units. These agents employ learning algorithms, including Auer *et al.'s*, softmax action selection, or Visudhiphan and Ilić's model-based algorithms, in determining bid-supply functions from available information.

The simulated outcomes from the agent-based model depend on the choice of non-uniform portfolios and on the learning algorithms that the agents employ. Model verifications against the actual markets are suggested; however, due to a lack of certain confidential information, numerical examples cannot be presented. Nevertheless, the model is used to analyze the effects of market structures and the effect of load-serving entities on the power-producer bidding behavior and market outcomes.

This model could provide one of the main tools for regulators, system planners, and market participants to use scenario simulations to investigate market conditions that could lead to high electricity prices. The model could also be used to analyze market factors (such as new market rules) and their effects on market price dynamics and market participants' behaviors, as well as to identify the "best" response action of one participant against the opponents' actions.

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Introduction

The objective of this thesis is to formulate an electricity market model that closely mimics the dynamics of market prices, as well as the bidding behavior of market participants in the existing electricity spot market over time. State-of-the-art, agent-based modeling is applied to capture the individual behavior of market participants, which contributes to the dynamics of market prices. The application of this model is the analysis of the effects of market structures and the role of active load-serving entities on the market participants' bidding behavior and on price dynamics.

This thesis chooses an agent-based model to formulate electricity spot markets without an competitive-market assumption. Current approaches are static oligopoly models that provide top-down analyses without considering possible dynamic interactions among market participants. Electricity spot markets are dynamic systems with several groups of decision-makers, consisting of power producers, who produce and sell electricity to the market, and in some cases load-serving entities (LSEs), who buy electricity on behalf of their customers. Selling and buying electricity is done through a sealed-bid/double sealed-bid auction.

These auctions occur repeatedly, sometimes as many as twenty-four per day. After each auction, the market participants are not informed of their opponents' quantity dispatched and the prices paid for the dispatched quantity. The repetitive auctions and the information obtained after each auction substantiate the capability of the market participants to learn the other participants' bidding behaviors and adjust their own bids through time. Previous studies have shown patterns of the time-varying bidding behavior of market participants. For example, large bidders tend to submit strategic bids to raise the market prices. Several bidding strategies have been observed, including the capacity withholding and bidding-price raising strategies ([6], [49], and [51]). Figure 0-1 shows the bidding prices for the bidding quantity equal to 2,000 MW of a market participant, denoted by "506459," and the total load in the New England market during January 18-31, 2000, denoted by "Load." Figure 0-2 shows the bidding prices for the bidding quantity equal to 1,000 MW of another market participant, denoted by "218387." These figures suggest that the market participants have adjusted their bids over time, even when demand has been relatively similar. Note that throughout this thesis "load" and "demand," referring to electricity consumptions are used interchangeably.

In addition, an electricity market model designed under the assumption that market conditions allow perfect competition is largely invalid. Several previous studies have confirmed that the competition that exists in the electricity markets is imperfect ([6], [7], [20], [25], and [51]). In cases where there are geographical constraints on the installed capacity and the number of market participants is limited, some power producers will be able to set the market prices. Additionally, hourly market prices frequently exhibit high price-fluctuation. Price spikes occur regularly, especially when demand is large relative to total installed capacity. Prices also vary during different periods, even though demand levels during those periods are similar. Figure 0-3 shows a scatter plot of the sampled hourly market prices and loads from May 1, 1999 to April 30, 2000, and Figure 0-4 shows the histogram of hourly market prices during the same period.

These market characteristics indicate that the electricity market should be viewed as a multiagent system and/or a repeated bidding game by using an agent-based approach. In this game the players or agents represent market participants with different marginal-cost or marginal-utility characteristics, bidding strategies or learning algorithms, and (perhaps) objective functions. This thesis presents methods to formulate a model that closely replicates the market participants' behavior and the resulting price dynamics, to verify the proposed model empirically given the available data, and to extend the model to analyze effects of critical factors on price dynamics and market participants' behavior, such as market structures or demand price-elasticity.

This proposed model consist of agents, power-producing and LSE agents, representing market participants. Each agent submits a bid daily to a system operator who clears the market. A bid is a function of price and quantity, such as a bid-supply function for the power-producing agents and a bid-demand function for the LSE agents. This function indicates the amount of electricity an agent is willing to buy or sell at the specified price. After the bid-submission deadline passes, the system operator clears the market for that hour by matching demand to supply at the least cost and publicly announces market prices and total consumption. This thesis adopts a price merit-order market-clearing mechanism without unit-commitment or network constraints.

After the market clearing price (MCP) is determined, each agent is informed of the total demand, the quantity dispatched, and the price paid for the dispatched quantity or dispatched consumption. In the markets which adopt a uniform-pricing rule, the price paid is equal to the MCP, the maximum bidding price of the supply bids dispatched to meet demand. Conversely, in the markets which adopt a discriminatory-pricing rule, the price paid is the bidding price of the bid that is dispatched. The dispatched quantity of the power-producing agents is equal to the bidding quantity whose bidding prices do not exceed the market price. For markets with the LSE agents actively responding to the price of electricity, the dispatched consumption is equal to their bidding quantity corresponding to bidding price not less than the market price.

The crucial advantage of this agent-based approach is its ability to capture dynamic interactions among the agents that cannot be displayed by the traditional supply-demand and/or (static) oligopoly models. Furthermore, the agent-based model can be extended to analyze the effect of the factors influencing the agents' behaviors on the market outcomes. These factors include the effects of the different market structures and of the existence of active decision-making LSE agents on the bidding behavior of the power-producing agents. Because of the flexible nature of an agent-based approach, the agents can be modeled to represent market participants who have asymmetric characteristics, who make decisions with incomplete information, and who employ a learning algorithm in response to the other agents as well as to improve or to maintain outcomes. Nevertheless, the simulated outcomes depend highly on not only the agents' characteristics, but also on the learning algorithms that the agents use. These result in the difficulty in model verification and in the limitation of potential usages of this model for any existing-market analysis.

This thesis is organized as follows. Chapter 1 gives an overview of the existing electricity markets and highlights the literatures of related fields, including the research on electricity markets, game theory, agent-based modeling, and learning algorithms in multiagent systems. Chapter 2 provides a detailed analysis of electricity markets as repeated games played by market participants under different demand and supply characteristics, and explains the necessity of applying an agent-based model to replicate the markets. The by-product of this analysis is a proposed definition of market power based on aspects of game theory. Chapter 3 describes this new proposed modeling approach and learning schemes adopted by the market participants. Chapter 4 presents the hypothetical spot market models based on the approach described in Chapter 3. The simulation processes under different learning algorithms of the agents are outlined. Simulations show the market dynamics and provide insights into several important aspects of model characteristics and their influence on market price dynamics. Chapter 5 presents a study of the New England electricity market to support the validity of choosing the agent-based model. The study focuses on the bidding behavior of New England market participants under different demand conditions. Chapter 6 shows an application of the proposed model on analyzing the effect of market structures, including uniform and discriminatory-pricing rules, on market participants' behavior and price dynamics. The model is extended to show the preliminary effect of active load-serving entities on reducing price-markups. Suggestions for future research and conclusions are included in Chapter 7.

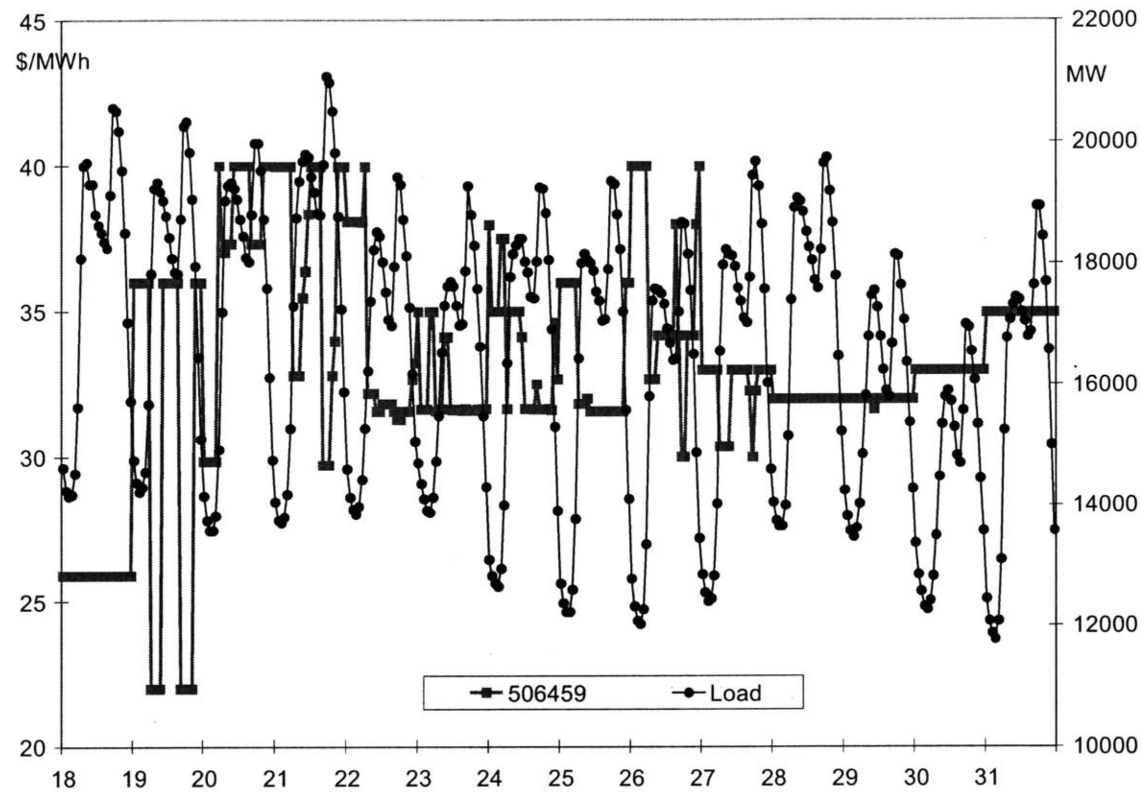

Figure 0-1: Forecast Demand and Bidding Prices for Bidding Quantity 2,000 MW of Lead Participant 506459 during January 18-31, 2000

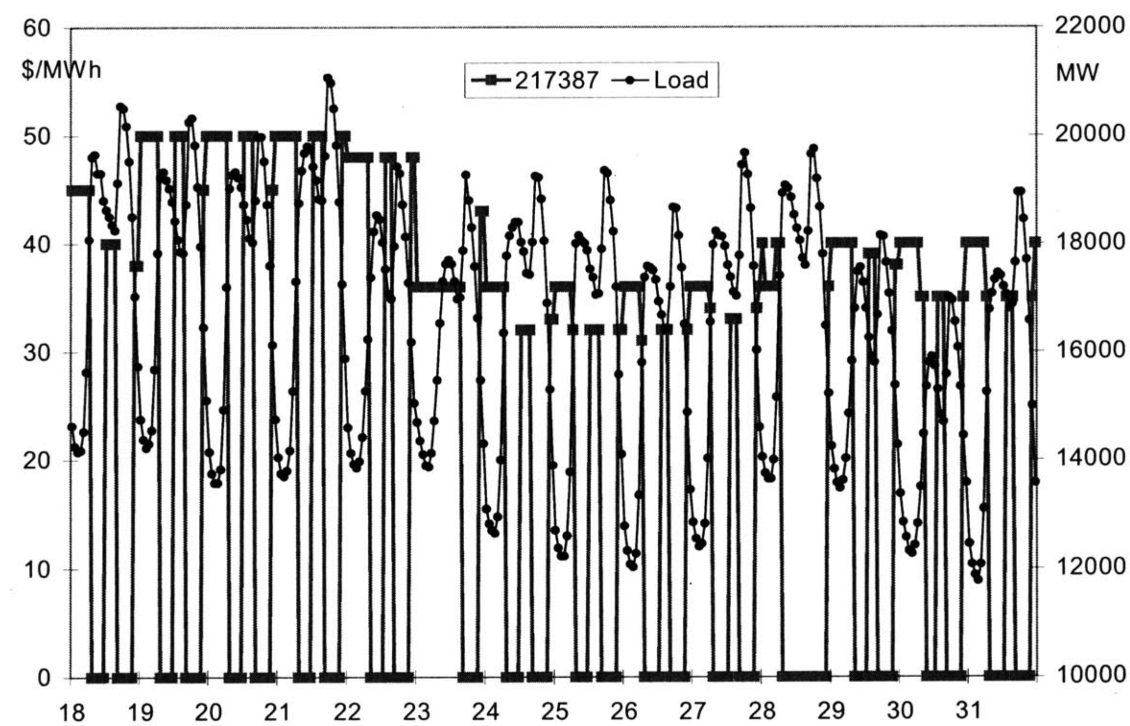

Figure 0-2: Forecast Demand and Bidding Prices for Bidding Quantity 1,000 MW of Lead Participant 218387 during January 18-31, 2000

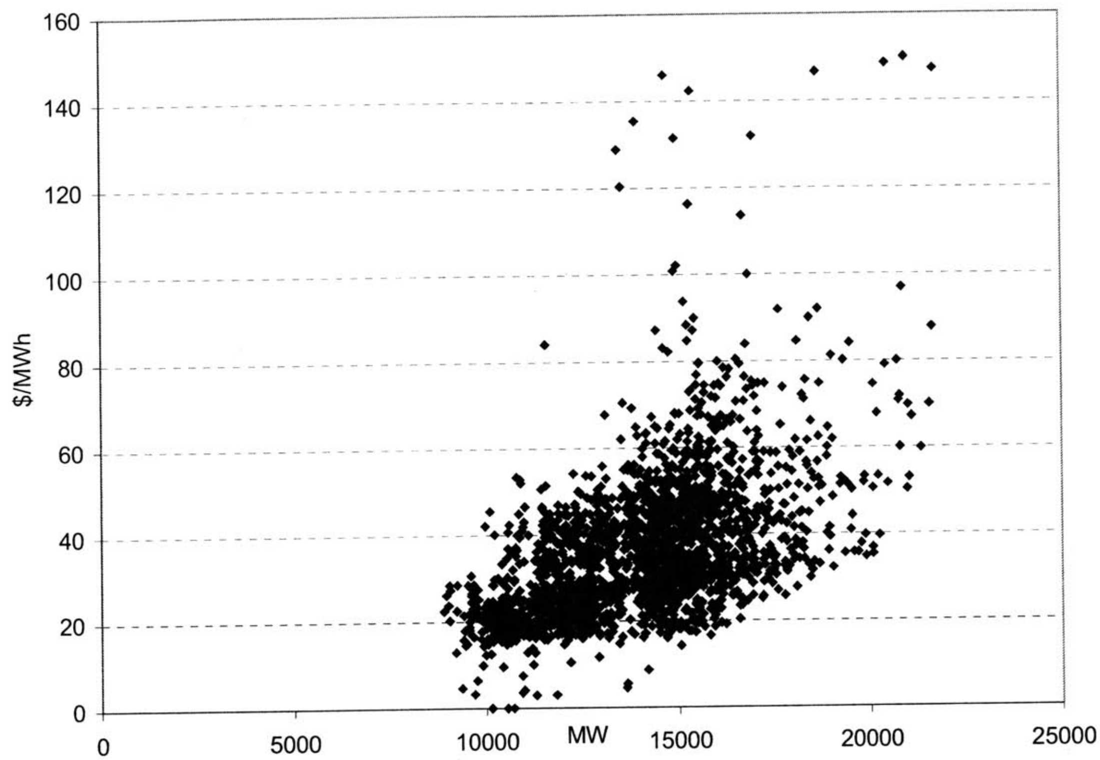

Figure 0-3: A Scatter Plot of Sampled Hourly Demands and Market Prices from May 1, 1999 to April 30, 2001

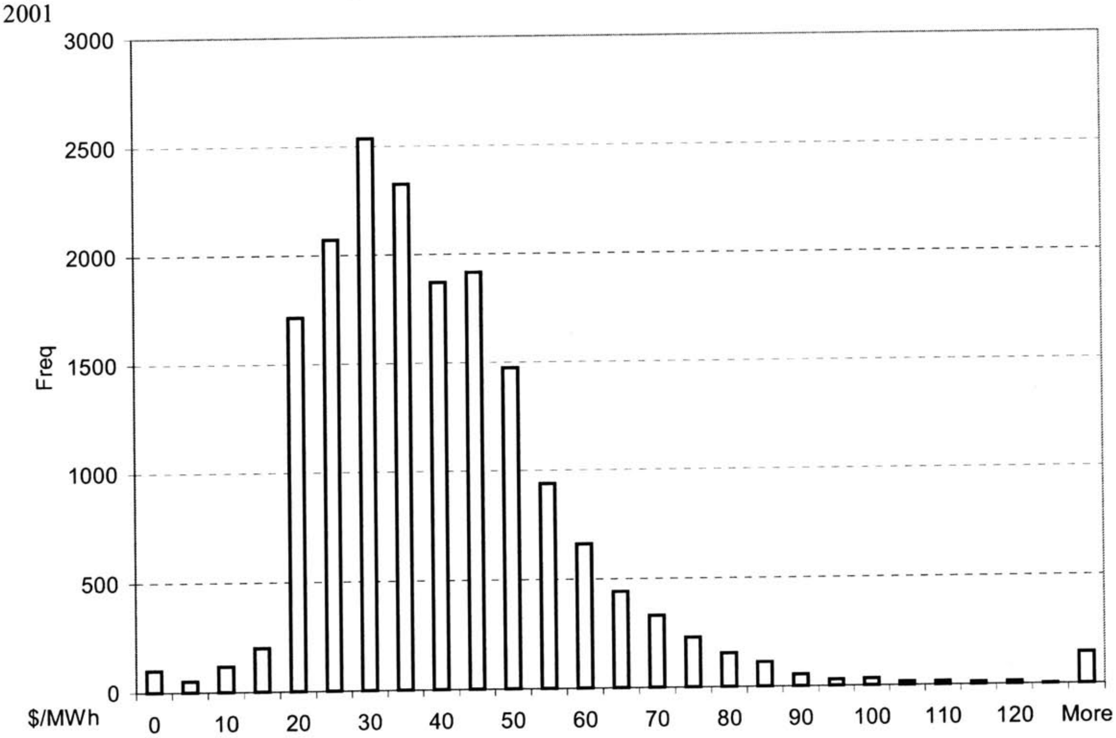

Figure 0-4: A Histogram of Hourly Market Prices from May 1, 1999 to April 30, 2001

Chapter 1

Reviews of Related Research and Studies

This chapter provides an overview of research in related fields that will be applied to the modeling of electricity spot markets. Section 1.1 gives an overview of several studies on electricity markets. Section 1.2 highlights some literature providing background information on game theory. Section 1.3 provides the basic concept of the agent-based modeling approach and introduces some of the research on electricity markets which use this approach. Section 1.4 describes some studies in reinforcement and multiagent learning as well as some learning algorithms used in this thesis. Finally, Section 1.5 summarizes the objective of the study and provides an overview of the modeling approach incorporating the research fields described above.

1.1 Electricity Markets

1.1.1 Overview

Electricity spot markets are the marketplaces where electricity is traded through auctions. An auction is a market institution with an explicit set of rules determining resource allocation and prices on the basis of bids from market participants (for a detailed overview of auction theory, see, for instance, McAfee and McMillan [32]). The type of auction usually used in these markets is a multiple-unit first-price sealed-bid auction for buying and selling electricity through a single system operator, such as in the New England electricity market.[1] Generally, the bidders are the power producers, because

[1] In the New England electricity market, as detailed in Market Rule 5-C of the New England system operator that can be found on the New England Independent System Operator's website [53], to determine a 5-minute real-time price from dispatch software which schedules the units to meet energy demand during the five-minute dispatch time frame, so that the system energy cost is minimized while meets the reliability requirement, when these conditions are present, the real-time market price (RTMP) is set equal to the price of the most expensive MW of all the desired dispatch that yields such solution and is eligible (according to the market rules) to set the RTMP.

the demand side still remains price-inelastic, and the system operator "buys" power on their behalf. In the California market, however, the auction is in a double multiple-unit sealed-bid first-price form. In this type of auction, both sellers (power producers) and buyers (load-serving entities (LSEs)) submit their bids[2] to the system operator simultaneously.

In markets which utilize the first-price sealed-bid auction, the power producers submit sealed bids indicating the amount of power they are willing to produce at specified prices to the operator, who schedules the units to meet the total demand on a price merit-order. On the other hand, in double auction markets, both power producers (sellers) and LSEs (or buyers on behalf of their customers) submit bids. The power producers indicate the price at which they are willing to sell their power (limited quantities), and the LSEs indicate how much they are willing to pay for the amount of power they want to consume. The sellers are ranked from the lowest to the highest bidding price, while the buyers are ranked from the highest to the lowest bidding price. The intersection of demand and supply gives a quantity (total demand) and an interval of prices, from which the market price is set according to predetermined rules. In both types of auctions, two market structures are employed: uniform and discriminatory-pricing structures. In uniform-pricing auctions, each successful bidder is paid an amount equal to the most expensive successful bidding price, multiplied by the scheduling quantity. In discriminatory auctions, each successful bidder is paid its bidding price, multiplied by the scheduled quantity.

The crucial inherent characteristic of using an auction mechanism to execute power trades relates to the asymmetric possession of information among the bidders and an operator;[3] the bidders have their private values for the power traded, i.e., the buyer does not know the true electricity production cost. Bidders also have asymmetric portfolios of generating units with differences in generating technology and capacity. Although the bidders know the system marginal cost function, they do not know their competitors' actual operating cost characteristics because each unit is different from the others in its operating constraints, as well as they may have different objective functions (values).

1.1.2 Previous Research

Most studies on electricity markets, and especially those on generation competition, focus mainly on the issue of market efficiency. These studies generally apply a static oligopoly model to the analysis of market equilibrium and also use it to perform an empirical study. For example, Green and Newbery [20] study the UK market by formulating the market as a single-shot game of two symmetric players and applying a supply function equilibrium model (SFE), based on the study of Klemperer and Meyer [28], to determine a Nash equilibrium of the market. Green and Newbery use the SFE to determine market prices under different levels of demand, assuming that the UK market was under the duopoly

[2] A bid in this thesis is referred to an offer to sell power of the bidders, who are power producers.

[3] Note that the system operator generally knows the operating costs of the generating units, however the customers generally do not know this information.

situation during 1988 and 1989. They find the existence of significant price-markups on marginal-cost prices. Further, they use their model to show the effect of entry in later years, and recommend that subdividing two players into five players would increase competition without the cost of excessive entry. Several studies apply the SFE concepts and extend the study to analyze characteristics of market equilibrium in further detail. Examples include Rudkevitch *et al.* [38], Baldick *et al.* [3], Baldick and Hogan [4].

Von der Fehr and Harbord [13] propose to model the market as a first-price sealed-bid multiple-unit auction (and use a Bertrand model to analyze market equilibrium). They show that pure-strategy equilibria do not always exist; instead, multiple equilibria are in fact more likely due to capacity-constrained price competition. Moreover, for a range of demand distributions no other pure-strategy combinations constitute an equilibrium. They believe this suggests inherent price instability in the present regulatory set up, which is confirmed by the evidence obtained from their empirical study of the UK market. In addition, they emphasize this finding by showing that the Bertrand outcome is unlikely and that the generating units with expensive operating cost may be sold at lower offering prices than the generating units with cheaper operating costs.

Borenstein and Bushnell [6] model the California electricity market as a static Cournot market with a competitive fringe. They argue that the quantity-setting Cournot paradigm seems to correspond to the electricity market much more closely than the price-setting Bertrand paradigm, because generally power producers have increasing marginal-cost functions and limited available capacities. The Cournot outcome can be used as a base-case analysis because the Cournot equilibrium represents a worst-case analysis of possible market power in static equilibria. By using historical cost data, they simulate benchmark competitive and Cournot equilibrium prices for several demand levels and for demand elasticity. From their model, significant price markups are found in high demand hours during several months of the period of study.

Several empirical studies confirm inefficiency in the existing electricity markets. For example, Wolak and Patrick [49] analyze the strategic behavior of market participants in the UK,[4] taking into account the market structure and its rules, from April 1, 1991 to March 31, 1995. They find that the majority of excess revenues, i.e., spot prices much higher than the average cost of supply during a given period, are due to the exercise of market power within a short period, i.e., a 3-hour window. The generators strategically bid by adjusting the maximum available capacity and the bidding prices of their generating units. However, they find that declaring the availability of each unit is a high-powered strategy that causes market prices to be substantially higher than average costs. This strategy can be implemented successfully because market rules require the units to submit the same capacity that is made available to the pool throughout the day, as well as to declare their availability on a half-hourly basis during the day at the discretion of generators.

[4]Note that during the study period, a bidder in the UK market is required to submit one bid for a one-day auction, which is comprised of 48 half-hour periods.

Wolfram [51] presents an empirical study of market power in the UK during 1992, 1993, and 1994 when two rivals owned substantial shares of generating units. Wolfram makes use of several approaches to construct this measurement and proposes a method to derive a system marginal cost accounting for strategic capacity withholding. For example, the short-run marginal cost of a fossil-fuel unit is a function of the type of fuel burned, the cost of the fuel, and its thermal efficiency. A constant marginal cost for a nuclear unit is assigned.[5] Pumped-storage capacity is assigned a cost based on the average pool price during the period of pumping water. A price-cost markup indicates the difference between market price and the marginal cost of producing power to meet demand. The study shows that capacity withholding has not generally resulted in markups as large as those predicted by conventional oligopoly models. Markups are higher for higher demand quantities. Moreover, there is evidence showing that the pool price is just below a potential entrant's long-run average cost. However, the study finds that although the power producers are charging prices significantly higher than the observed marginal-cost prices, the prices are not raised to the levels predicted by the oligopoly model.

Wolfram [50] also considers the characterization of bidder behavior and market outcomes in multi-unit auctions based on theoretical auction literature, and applies these findings to further evaluate the extent to which these predictions hold empirically in the daily electricity auction in the UK market during the years 1992 to 1994. This analysis shows that the strategic behavior of the power producers is to set the bidding price above marginal cost and to set a higher price for infra-marginal capacity. For example, the larger participant in the auction tends to bid more than its smaller competitor does for units with comparable costs. The bidders submit bids with a larger markup over marginal costs for generating units that are more likely to be used after a number of other units are already operating. Some power producers submit higher bids for given generating units during the periods when more of their other units are available to operate. Moreover, the incentive to set a high price for infra-marginal capacity is moderated by the incentive to ensure that a unit is not left out of the dispatch schedule.

Borenstein *et al.* [7] adopt a similar approach to Wolfram's [51], developing benchmark prices to analyze the efficiency of the California market from June to November 1998. The evidence indicates that market power in California's wholesale market was a significant factor in high-price power during the period of the study. They find that price markups are significantly larger during the higher demand months of July and August and during higher demand hours. Low markups are found during lower demand months and during off-peak hours. Borenstein *et al.* suggest that the causes of this phenomenon include the power producers' ability to take advantage of inelastic demand, the capacity limits of the opponents, and the lack of storage. Other research with a similar approach, i.e., to recreate a benchmark price that was introduced by Wolfram [50], can also be found in Joskow and Kahn [25]. Joskow and Kahn analyze the California market during the summer of 2000, accounting for the effect of NO_x emission allowance. They conclude that there is no substantial market power

[5] So that the nuclear units would not have been a marginal unit during the period of this study.

when the NO_x emission allowance is added to the operating cost.

In addition, several other studies in this field also focus on a top-down approach to model electricity spot and future prices. For example, Deng [12] proposes several mean-reversion jump-diffusion models to describe the dynamics of electricity prices. Skantze and Ilic [40] model spot price dynamics based on a principal component analysis. Both models provide potential benefits for physical and financial asset valuation, hedging, and speculation.

1.2 Game Theory

Game theory has been studied extensively. Two types of game characteristics are considered, including stage and dynamic games. Stage games have three elements: the set of players; the pure-strategy space; and payoff functions. These games are played only once. A game equilibrium strategy is determined using a Nash equilibrium concept, which describes a profile of strategies in which each player's strategy is an optimal response to the other players' strategies. Repeated games are stage games that are played repetitively. Dynamic games or multi-stage games are forms of modeling situations with dynamic structures. Players determine their actions depending on which stages they are in and the information available to them. After each stage game is played, a transition from the current game (stage) to the next one occurs. Stochastic games are one type of dynamic games, consisting of several stage-games and transition probabilities, in which each game represents one stage. The fundamental background of game theory can be found in any introductory text on this topic, for example, Fudenberg and Tirole [16] and Owen [37]. For general overviews and applications of game theory, one might find Gibbons [19] very useful as well. This thesis is concerned with a repeated game. Since the players in the game do not know their opponents' actions and their payoffs, this game is considered a game of incomplete and imperfect information. An explanation for viewing electricity spot markets as a repeated game is described later.

1.3 Agent-based Modeling

Tesfatsion [42] provides a complete overview of the agent-based computational economic (ACE) approach and of many studies that apply this method, with special focus on its importance in the study at market economies. As Tesfatsion [42] mentions, decentralized market economies are complex adaptive systems, consisting of large numbers of adaptive agents simultaneously involved in local interaction. Macro-economic regularities and behaviors emerge from these local interactions and then feedback into the determination of these interactions. The traditional model, such as the oligopoly model, lacks the means to model this feed back quantitatively and generally places the emphasis on extraneous agents and on imposed market equilibrium constraints. Interactions among decision-makers in this model typically play no role or appear in the form of tightly constrained game interactions. On the other hand, the agent-based model quantitatively formulates a wide variety of complex phenomena, such as inductive learning, imperfect competition, endogenous trade network formation, the on-going co-evolution of individual behaviors, and the overall system dynamics. In summary, the agent-based model consists of evolving systems of autonomous interacting agents. This model specifies the initial state of the system by setting the initial attributes of the agents. The system then evolves over time without further intervention from the modeler. All events that subsequently occur must arise from the historical time-line of agent-agent interactions.

In an electricity spot market, market participants, including power producers and sometimes load-serving entities are agents. Visudhiphan and Ilic [44] introduce a simple agent-based model of a electricity spot market, in which each agent has a constant marginal cost and a limited capacity. The agent performs myopic decision-making to determine a bid in the current period. The decisions to increase, to decrease, or to maintain their bidding prices are based on the observed market outcomes of the previous bidding period, and different assigned strategies. The simulated price dynamics show no trace of equilibrium under various demand conditions.

Bower and Bunn [8] apply an ACE approach to simulate the behavior of an oligopoly of bidders in a range of multi-unit, multi-period, auction settings. These researchers developed a detailed model of electricity trading in the UK market, and also used this model, which takes into account the discriminatory and uniform-pricing structures, to analyze the effects of different auction structures on bidding behavior. In the model, the agents have simple myopic internal decision rules. For example, the agents may raise or lower their bidding prices by a random percentage of the bids they submitted in the previous trading period. The agent is also continuously updating its profit objective, as the simulation progresses by using the previous trading day's profit as a benchmark against the current day's profits. In their model, the agents know everything about their own portfolio of plants, bids, output levels, and profits, but nothing about other agents or the state of the market. The simulations show that the settlement procedure from the uniform to the discriminatory-pricing structures, as well as changing the bidding procedure from daily to hourly bids, induces a rise in prices. When no bid prices or market prices are published, large agents gain an advantage over the small agents because of information asymmetry, especially in the discriminatory-pricing structure. The disparity in information between the uniform and discriminatory-pricing structures significantly alter market prices because the latter reduces the competitive pressure on large firms due to the increase in risk of overbidding, particularly by small firms. These effects are exacerbated when inelastic electricity demand approaches total bidding capacity. As mentioned, auction theory supports the view that increasing the amount of available information increases the efficiency of the auction but only at the expense of consumers, due to the difficulty of enforcing a collusive agreement.

Nicolaisen, *et al.* [35] propose an ACE model of a wholesale electricity market that can be used as a laboratory for systematic experimentation to investigate market power and efficiency in a double-auction pricing setup. The agents in this model also employ a learning algorithm. Their investigation is focused on variations in the relative market power of the buyers and sellers in response to changes in concentration and capacity. Also, the study developed a conceptual tool to understand the effects of the discriminatory-pricing rule on structural versus behavioral market power. Their experimental findings show that structural biases inherent in the discriminatory-pricing rule induce market power outcomes; however, the buyers and sellers with less market power are unlikely to improve through learning.

A similar concept of simulation-based agent-based modeling is an experimental economic approach. This approach also aims to mimic how the market (or economic system) works. Instead of using a computer simulation to obtain the outcomes of agents' interactions and associated dynamics, the experiments of interactions are performed in an economic lab. Several studies, such as those in Backerman *et al.* [2] and Schuler [39], use this experimental economic approach on electricity markets. Since this thesis does not focus on this approach, an in-depth overview on this matter is not included. Some related studies of this approach can be found in Schuler [39].

1.4 Learning Algorithm and Multiagent Learning

The learning algorithms are generally designed for either single-agent systems, in which an agent makes decisions against an uncertain environment, or multiagent systems, in which agents make decisions against one another. The fundamental background of reinforcement-learning for a single-agent system can be found in Sutton and Barto [41] and Bertsekas and Tsitsiklis [5], for instance, while Kaelbling *et al.* [26] provide a comprehensive survey of the field of reinforcement learning from a computer-science perspective. Learning algorithms in either single-agent systems or multiagent systems are generally in a form of myopic decision-making, because of the lack of knowledge of the systems.

Several learning algorithms in single agent-systems have been developed. A few of these algorithms are summarized below. Some studies are concerned with determining "optimal" decisions, while the others characterize the strategies/algorithms that guarantee near-optimal outcomes with efficient run-time. For example, Q-learning, the classic algorithm, which was first introduced by Watkins and Dayan [46], is a model-free reinforcement learning. The agent starts with an arbitrary initial value for all states. At each time, the agent chooses an action and observes its reward. In Q-learning, the agent updates its Q-values for each state using the previous Q-values, the current rewards, and a learning rate, which is a non-negative constant less than one.

McCallum [33] develops the nearest sequence memory algorithm for an agent to learn in a partially observable Markov decision process. This algorithm is a combination of instance-based methods, which are used in learning in continuous spaces and history sequences. The nearest sequence memory algorithm is different from the fixed-sized window techniques because it provides a variable memory-length, such as k-nearest neighbor. This algorithm improves performance compared to several algorithms, because recording raw experience is particularly advantageous when the agent is learning to partition the state spaces, especially when the agent is deciding the importance of history for uncovering a hidden state.[6]

In multiagent systems, the learning algorithm becomes more complicated when the agent is learning in an environment that changes due to other agents' actions and external uncertainties. Most studies characterize the learning algorithms under the framework of two-person repeated and/or stochastic games. The two-person games have one particularly beneficial feature in that they allow one player to apply a reinforcement learning algorithm by assuming that the other player faces an uncertain environment. Multiagent learning has a strong connection to game theory, where players select actions to maximize payoffs in the presence of other payoff-maximizing players. Learning is essential in the repeated or stochastic games of incomplete information, in which the players have no information of the opponents' strategies and payoffs. However, several learning algorithms yield outcomes that depend on assumptions about the opponents' policies, strategies, and learning algorithms. There is still demand for new techniques for developing learning algorithms for an agent in a multiagent environment that require the least amount of information and assumptions about the opponents' actions and payoffs. A few studies on learning algorithms in multiagent systems are summarized below.

Learning in repeated games in the economics community occurs in the form of fictitious play (more detail on this topic see Fudenberg and Kreps [18]). Fudenberg and Levine [16] study a variation of fictitious play, in which the probability of each action is the exponential function of that action's utility against the historical frequency of opponents' actions. This learning algorithm is the set of behavioral rules that map all the components from observations to actions without the internal thought process of the players, and can be implemented in an extensive form game in which opponents' strategies are not observed. Fudenburg and Levine show that this method yields approximately optimal outcomes and guarantees nearly the minmax outcome, regardless of opponents' behavior.

Several studies in the artificial intelligence community begin by developing a learning algorithm for an agent to make a decision against an uncertain nature, and continue by applying it to game-setups. A similar approach to that of Fudenberg and Levine is developed independently by Freund and Schapire [15]. Freund and Schapire [15] apply a weight-majority algorithm to an on-line prediction developed by Littlestone and Warmuth [30] to study the close connections between playing a repeated (zero-sum) game, on-line prediction, and boosting. This algorithm lets the learner maintain nonnegative weights on a set of actions (decision, hypothesis). The actions that yield satisfying outcomes will be chosen with a higher probability. Freund and Schapire's analysis of this algorithm yields a proof

[6] For more details on McCallum's nearest sequence memory algorithm as well as an overview on previous researches on this topic, see McCallum [33].

of von Neumann's minmax theorem, and applies this algorithm to find the approximate minmax or maxmin strategy of a zero-sum repeated game.

Auer *et al.* [1] introduce algorithms, which are partially based on a weight-majority algorithm of Freund and Shchapire [15], for an agent to play the non-stochastic multi-armed bandits. The objective is to develop learning algorithms to play multi-armed bandits, which yield the expected weak regrets within established bounds as a function of the number of actions, playing time, and probability of error, without any statistical assumption about the payoff generating process. The algorithms determine a probability distribution over the possible actions (the possible arms to be picked) that is a mixture of a uniform distribution and a distribution that is a function of weight-factors associated with each action, so that the algorithm tries out all actions and gets a good estimate of reward. In addition, Auer *et al.* [1] apply these algorithms to an agent playing a repeated game without knowledge of its opponents' actions and their associated payoffs. These algorithms require only the number of actions assigned to the agent and the maximum payoff that can be obtained. When the agent uses one proposed algorithm to play the game against its opponent, it is guaranteed to obtain payoffs which converge to the maximum payoff that can be obtained against the empirical distribution of plays by the opponents.

Bowling and Veloso [9] introduce a new concept of a variable learning rate, proving convergence in self-play[7] on a restricted class of repeated games. They define two properties of learning algorithms for the learner: rationality and convergence. If the other players' policies converge to stationary policies then the learning algorithm will converge on a policy that is a best response to the other players' policies, as well as the learner will necessarily converge on a stationary policy. Bowling and Veloso use gradient ascent as a technique for learning in simple two-player, two-action, general sum repeated matrix games, in which the players know the opponent's actions and associated payoffs. The utilization of this method, though rational, does not necessarily yield the convergent strategies. Bowling and Veloso introduce a variable learning rate, which contributes to the "Win or Learn Fast" principle, in which a learner should adapt quickly when it is doing worse than expected and be cautious when it is doing better than expected. The authors prove that the variable learning rate causes gradient ascents to converge. The concept of the variable learning rate is further applied to use with the policy hill-climbing algorithm, which is an extension of rational Q-learning.[8] These authors show by examples that when this algorithm is instructed to play several games the agents' plays converge to best-response policies.

Hart and Mas-Collel [21] demonstrate the characterization of an entire class of adaptive strategies, which are Hannan-consistent, for playing repeated games. Their strategies are a mapping direction that satisfies specific conditions for a target set, the approachable convex and closed set. Then the average payoff vector is guaranteed to approach the target set. The regret-based strategies can be

[7] All players use the same algorithm.

[8] Q-learning algorithm yields an optimal solution at each decision stage.

derived by replacing the payoff vector with the regret vector and by setting the target to the non-positive orthant. When an agent uses one of these regret-based strategies to play a repeated game, it needs to have full knowledge of its payoff matrix, but does not need to have knowledge of its opponents' actions and their associated payoffs.

Littman [31] applies a Q-learning-like algorithm to find optimal policies and demonstrates its application to two-person zero-sum stochastic games in which the optimal policy is probabilistic. The agents are trained against the opponent with different learning algorithms and are able to observe their opponents' actions. The trained agent with fixed policy using the "max" operator in the update step of a standard Q-learning algorithm is less successful than the update step using the "minmax" operator, because Q-learning is designed to find a deterministic policy while the minimax-Q is designed to find optimal probabilistic policies. Littman points out that the idea of probabilistic policies is also useful in the context of acting optimally when the agent's perception is incomplete. Random actions can be used against the agents' uncertainty of true states of environment, as well as the agents' uncertainty of the opponents' moves.

Based on Q-learning and on the Nash equilibrium concept, Hu and Wellman [23] present a multiagent Q-learning algorithm for the agents to play stochastic games. The agents maintain a Q-table containing the Q-values of each state. At each state, the agents determine a Nash equilibrium strategy of the Q-table associated with that state. Under particular conditions the Q-values converge to the Nash equilibrium Q-values. For an agent to play a stochastic game, given that the agent knows which state it is in, this algorithm requires full information of all the agents' actions and payoffs, as well as the state reached by the joint action at each stage of the game.

Brafman and Tennenholtz [10] introduce their R-MAX learning algorithm to play two-person general-sum stochastic games. This algorithm is proved to converge to a near-optimal average reward in polynomial time. Given that the agent knows which state it is in, not only does this algorithm require full information of all agents' actions and payoffs, as well as the state reached by the joint action at each stage of the game, but also requires the maximum possible reward and the desirable time required to learn the game. By using this algorithm, an agent optimizes its behavior with respect to a fictitious model associated with the real games. This optimal policy leads to either the exploration of new parameters or the exploitation of the current condition of the model. The near-optimal expected return that is achievable by the policy can be obtained with high probability in polynomial time.

Learning to act in multiagent systems poses a difficult problem since the normal definition of an optimal policy no longer applies because of a moving target, and the performance of the agents depends on the system environment. These agents face the changing environment due to the adaptive behavior of the other agents, which may have different goals, assumptions, strategies, and learning algorithms, as well as due to the dynamic changes of the system. The optimal course of action therefore is to change as all the other agents adapt. These external adapting agents violate the basic stationary assumption of traditional techniques for behavioral learning. In some typical situations, such as in an electricity market, in which information about the other agents is not available at the beginning, more information about the system and the other agents is revealed during the course of interaction. Identifying learning methods that require the least amount of knowledge of the systems and opponents become essential in this system.

1.5 Electricity Market Modeling

This thesis proposes to formulate a model that is able to closely mimic the day-ahead electricity markets in terms of market participants' bidding behavior and price dynamics. The repetitive auction of trading power is modeled as a repeated bidding game played by the bidders or the market participants. An agent-based approach is chosen to model and analyze the electricity market, in which an individual market participant who can influence the market outcomes is considered an active decision-maker or as an agent. The agents engage in a series of sealed-bid auctions or they play a repeated game. This proposed approach is different from the previous research on electricity market issues in several aspects. First, this is a dynamic model of agents' bidding behavior that is capable of capturing the adaptive behavior of the players in response to their opponents' actions. Second, in the real markets, information regarding the actions and payoffs of the others is confidential. In this model, though, the agents extract information about their competitors from available data over time with a learning algorithm and adjust their decisions (or actions) optimally. Third, the model consists of non-uniform agents, which conform to market participants in the existing markets who may have various objective functions, portfolio characteristics, and learning algorithms.

In summary, a broad spectrum of research fields is applied to this agent-based electricity market model. Game theory and learning in games become essential in characterizing bidding strategies, as well as in identifying the best and the most rational responses. A dynamic model that captures individual behavior and outcomes, such as an agent-based modeling approach, plays a crucial role. Moreover, because of incomplete information and repeated plays, the players need to learn about their opponents and respond in a profitable manner. A multiagent learning framework provides insight in developing the model as well as into the learning algorithm for the agents. Although this thesis develops an approach to state-of-the art modeling an existing complex system with interactions of multiple agents, it might not provide critical new theories in any of these fields.

This agent-based model is somewhat similar to the agent-based model of Bower and Bunn [8]. It offers a variety of learning algorithms and/or bidding strategies of the agents. Moreover, the agent may make its bidding decision based on its entire portfolio or its individual units with different learning algorithms. Depending on the bidding strategies (and/or learning algorithms) the price dynamics vary significantly; however, it is difficult to prove analytically and empirically, because the outcomes depend highly on characteristics and decision-making processes of the agents. The model is potentially useful for analyzing some factors that affect market outcomes, but which are impossible to describe with a few equations. In addition, the necessity to determine equilibrium and/or equilibrium strategy, and to prove convergence are not essential.

The next chapter first provides a characterization of electricity spot markets as repeated games of incomplete information. The reasons that the agent-based modeling approach are applied to formulate an agent-based electricity market model, a model in which only power producers are assumed to be active decision-makers or agents in the proposed model are outlined. Some learning algorithms that will be adopted in the model are highlighted.

Chapter 2

Electricity Spot Markets as a Repeated Bidding Game

This chapter characterizes electricity spot markets as repeated games, outlines the reasons behind choosing an agent-based modeling approach for these markets, and highlights the methods for developing market models. The electricity markets, especially day-ahead markets, are auction-style marketplaces where market participants bid to sell or buy power on a daily basis. In this thesis, the electricity market model is viewed as a series of bidding games, consisting of 24 hourly bidding games (or single-stage games). For each game, the players decide on a bid-supply function (bid) that will yield the best payoffs.

This chapter shows that in the single-stage bidding game, in which the bidders have finite choices of actions (bids) and know their opponents' actions and the associated payoffs, the bidders may have multiple equilibrium strategies. On the other hand, when the perfect information assumption is relaxed to better replicate the existing markets, the real bidding games played by the market participants become games of incomplete and imperfect information.

This chapter is organized as follows. Section 2.1 describes the methodology proposed to analyze electricity spot markets as repeated bidding games. It also explains why the repeated electricity day-ahead market can be thought of as a series of repeated bidding games of incomplete and imperfect information. Section 2.2 illustrates in detail the variation of equilibrium strategies in the bidding game under different demand conditions when the game is played by a finite set of symmetric and asymmetric players. A three-person bidding game with the assumption that each player has perfect information about the others is analyzed to show that the multiple equilibrium condition is possible and that the players' characteristics affect equilibrium strategies. Additional examples of a three-person repeated bidding game with different demand and supply characteristics are presented in the appendix to this chapter. Section 2.3 summarizes the contributing factors that formulate an agent-based electricity market model. Furthermore, the learning algorithms that are applicable to the model are highlighted.

2.1 A Bidding Game

The repeated games of electricity markets are not stochastic because the same stage games are played over time, and there is no change in the states of the games. An outline of possible outcomes when the agents have incomplete and imperfect information of their competitors, that is, when the agents make a decision individually without knowledge of the entire game and encounter different demand scenarios, is considered here.

- **Deterministic and price-inelastic demand**: The agents play a single-stage game with this demand by determining their "best" bid-supply functions or their set of bidding prices and quantities to maximize their profits. If this demand level is maintained, the same stage-game is played again. Since the agents do not know their opponents' bid-supply functions and payoffs, the agents adjust their bids according to some learning algorithms that determine their next bids based on the observed information, such as scheduling prices and quantities. For the same demand level, if the duration of the game is sufficiently long and the agents' portfolio characteristics remain unchanged, an equilibrium may be attained.

- **Deterministic and price-elastic demand**: The agents determine their "best-reply" bid-supply function in response to their opponents' actions by using learning algorithms to obtain their profit maximization objective. Like the previous scenario, the agents adjust their bids over time, and a game equilibrium may be reached.

- **Uncertain and price-inelastic**: The agents determine their "best-reply" bid-supply function in response to their opponents' actions to achieve their expected profit maximization objective. Von der Fehr and Harbord [13] show that no pure-strategy Nash equilibrium strategy exists in this type of stage game because one bidding price does not yield an equilibrium outcome for all possible demand levels. Although the agents play this game repetitively, they will not settle in any equilibrium bidding strategy. This issue is revisited in Section 2.2.2.

- **Uncertain and price-elastic**: When the game is played repetitively, the analysis is similar to the preceding scenario in which demand is uncertain and price-inelastic. No pure-strategy equilibrium exists in this game.

Even though demand may have different characteristics, when the demand level of any hour is similar to the other hours, the agents play the same stage game associated with that demand level. The bidding games are repeated games of an unknown game set-up. Moreover, depending on the demand level, the same bid-supply function may yield different payoffs; hence, the game changes its characteristics when the level of demand consumption changes. Since demands in the electricity markets vary over time, the markets can then be viewed as a demand-dependent series of unknown repeated bidding games.

Note that the concept of Nash equilibrium, which is a profile of strategies such that each player's strategy is an optimal response to the other players' strategies, is applied to bidding-game analyses presented in the following sections. The strategies mentioned in the following sections and chapters are concerned with both pure strategies, which are complete profiles of actions in response to any contingency of games, and mixed strategies, which are probability distributions over pure strategies.

2.2 A Three-person Bidding Game

This section presents a preliminary analysis of equilibrium strategies in a single-stage bidding game. This analysis shows the variations of the equilibrium strategies according to demand levels and the characteristics of the bidders. Both stage and repeated bidding games of small markets that consist of three power producers and deterministic demand are analyzed. The agents are allowed to have perfect information about the game and a finite number of bid-supply functions to choose from, i.e., bidding strategies and payoff functions. The effects of demand and power producers' characteristics on equilibrium strategies are considered. A three-person bidding game in strategic form has three elements: the set of players $i \in \{1, 2, 3\}$; the pure-strategy space S^i for each Player i; and the payoff functions $u^i(s)$ for each profile $s = (s^1, \ldots, s^N)$ of the strategies. All players except Player i are Player i's opponents and are denoted by $-i$.

In the following examples, the players participate in a uniform-pricing market. The players submit their bids to an operator, who schedules the players to meet the demand at the least cost. A bid contains a set of bidding price and quantity blocks. The operator sequences the bids according to a merit order, i.e., from the lowest bidding price to the highest one in order to create a market bid-supply function. The market price is equal to the bidding price where demand intersects the bid-supply function; in other words it is equal to the price on the bid-supply function with quantity equal to demand. The infra-marginal bidder, or the bidder whose bidding price is less than the market price, is scheduled to generate its bidding quantity. Residual demand is defined as total demand minus the sum of scheduled quantity of the infra-marginal bidders. The marginal bidder with its bidding price equal to the market price is scheduled to operate the residual demand. The successful bidders are paid the market price multiplied by their scheduled quantity. When more than one bidder is at the margin, this thesis assumes that these bidders are scheduled to operate a weighted-portion of the residual demand. For example, suppose that Players 1 and 2 are marginal bidders. The bidding quantities of Players 1 and 2 are x and y, respectively. Suppose residual demand is equal to L. In this scenario, Player 1 is scheduled to operate $\frac{x}{x+y} \cdot L$ and Player 2 is scheduled to operate $\frac{y}{x+y} \cdot L$. Note that the bids and the bid-supply functions are used interchangeably in this chapter.

2.2.1 A Single-stage Game with Deterministic Demand

Let us analyze the Nash equilibrium strategies in a bidding game. These analyses perform under different market characteristics, including market participants with asymmetric marginal-cost functions. The scenarios in which the agents have uniform marginal-cost functions or uniform capacity are shown in the appendix to this chapter. The first part of this section is focused on inelastic and deterministic demand. The second part analyzes the effect of demand uncertainties on the existence of a pure-strategy equilibrium of the games.

The general characteristics of the players are as follows: 1) each player owns one generating unit or one unit; 2) each unit has a limited generation capacity, q^i_{max}; 3) each unit generates power at a constant marginal cost,[1] mc^i; and 4) the units are ordered such that $mc^1 \leq mc^2 \leq mc^3$. The operating cost of producing q^i is therefore equal to $mc^i \cdot q^i$. Let B^i denote a bid of Player i (Agent i), which is a set of bidding prices and quantities, i.e., $B^i = \{b^i, q^i\}$, where b^i is a bidding price and q^i is a bidding quantity. The strategy space of each player is $b^i \in [0, P_{cap})$ and $q^i \in (0, q^i_{max}]$, where P_{cap} denotes a price cap, the maximum bidding price that is allowed to be bid on the market.

In the bidding game, which adopts a first-price mechanism, the winning bidders are paid the market price that is set equal to the most expensive bidding price of the winning bidders. (Note that another form of determining rewards for the winning bidders in which the second most expensive bidding prices is set as the market price for the winning bidders is not considered here. For more detail on the second-price auctions, see, for instance, Milgrom and Weber [34] and Vickery [43].) In the stage game or static game, when there is no demand uncertainty, each player maximizes its profit as follows:

$$\max_{b^i,\, q^i} (P - mc^i) \cdot \hat{q}^i$$

where P is market price, which is a function of bids and demand, $P = f(B^i, B^{-i})$. Let $\hat{q}^i$ be a scheduled quantity, in which $0 < \hat{q}^i \leq q^i_{max}$ when $b^i \leq P$, and $\hat{q}^i = 0$ when $b^i > P$.

Let us consider when the players have asymmetric marginal-cost functions (that is, no players have the same installed capacity and marginal-cost characteristics). Each Player i has maximum capacity q^i_{max} and marginal-cost mc^i. Suppose that $q^1_{max} > q^2_{max} > q^3_{max}$ and $mc^1 < mc^2 < mc^3$. Let Q^i denote $\sum_{j=1}^{i} q^j_{max}$. Let us consider when demand is equal to $L1$, $L2$, and $L3$, where $0 < L1 \leq Q^1$, $Q^1 < L2 \leq Q^2$, and $Q^2 < L3 \leq Q^3$. The bidding price is at least zero but not greater than a price

[1] A piece-wise linear marginal-cost function, a quadratic operating function, is another type of cost functions that is widely used to represent cost characteristics of generating units in several studies on electricity markets. Although this thesis does not use this form, the analysis presented here can be applied to analyze this form of cost functions with modifications.

cap P_{cap}, i.e., $b^i \in [0, P_{cap}]$, (also $P_{cap} > \max_i mc^i$). Suppose that each Player i chooses its bidding price from $[mc^i, P_{cap}]$ and its bidding quantity from $[0, q^i_{max}]$. Given the set of these bidding prices, the Nash equilibrium strategies in this game can be analyzed as in the following examples.

Case 1: $L = L1$

One Nash equilibrium occurs when Player 1 submits its bidding price equal to $mc^2 - \epsilon$, where $\epsilon > 0$ and $mc^2 - \epsilon > mc^1$, and the other players submit their marginal-cost bids.

$$\begin{cases} B^1 &= \{mc^2 - \epsilon, q^1_{max}\} \\ B^i &= \{mc^i, q^i_{max}\}, \ \ i \neq 1. \end{cases}$$

The market price is equal to $mc^2 - \epsilon$. Player 1 is scheduled to serve demand $L1$ and receives profit equal to $(mc^2 - mc^1 - \epsilon) \cdot L1$, while the others are not scheduled. Cooperation between Players 1 and 2 to raise the bidding price to be $mc^3 - \epsilon_1$ and $mc^3 - \epsilon_2$ is not possible, because both players will undercut their bidding prices until Player 2 is no longer making profits. However, there will be another Nash equilibrium, if Player 2 and Player 3's bidding prices are less expensive than Player 1's bidding price, resulting in Player 1's residual demand being greater than zero, i.e.,

$$L1 - q^2_{max} - q^3_{max} \ > \ 0.$$

Then, Player 1 submits its two-part bid in which the bidding price of the first part is equal to mc^1 and of the second part is equal to $mc^3 + \Delta$. When these conditions hold true, the market price is equal to $mc^3 + \Delta$[2] and the profit that Player 1 obtains from this two-part bid exceeds the profit from its marginal-cost bid, i.e.,

$$(mc^3 - mc^1 + \Delta) \cdot (L1 - q^2_{max} - q^3_{max}) > (mc^2 - mc^1 - \epsilon) \cdot L1. \tag{2.1}$$

Equation (2.1) always holds when $\Delta \gg 0$, e.g. $mc^3 + \Delta = P_{cap}$. When both conditions hold, the equilibrium strategy is as follows:

$$\begin{cases} B^1 &= \{(mc^1, q^{1,1}), (mc^3 + \Delta, q^{1,2})\} \\ B^i &= \{mc^i, q^i_{max}\}, \ \ i \neq 1 \end{cases}$$

where $q^{1,1} = L1 - q^2_{max} - q^3_{max} - \delta$, $\delta > 0$, and $q^{1,2} = q^1_{max} - q^{1,1} > 0$. The market price is equal to $mc^3 + \Delta$. Player 1 receives profit equal to $(mc^3 - mc^1 + \Delta) \cdot (L1 - q^2_{max} - q^3_{max})$, Player 2 receives

[2] Player 1 might consider a less aggressive strategy if $L1 - q^2_{max} > 0$. Player 1 sets the bidding price of the second part equal to $mc^3 - \epsilon$, where $\epsilon > 0$ and $mc^3 - \epsilon > mc^2$. The market price is equal to $mc^3 - \epsilon$ in which Player 2 is scheduled as an infra-marginal unit. This strategy is favorable when $(mc^3 - mc^1 - \epsilon) \cdot (L1 - q^2_{max}) > (mc^2 - mc^1 - \epsilon) \cdot L1$. In this case, Player 1 submits its bid, $B^1 = \{(mc^1, q^{1,1}), (mc^3 - \epsilon, q^{1,2})\}$, where $q^{1,1} = (L1 - q^2_{max} - \delta)$ and $q^{1,2} = q^1_{max} - q^{1,1}$.

profit equal to $(mc^3 - mc^2 + \Delta) \cdot q^2_{max}$, and Player 3 receives profit equal to $\Delta \cdot q^3_{max}$. In this thesis, this strategy is called a capacity withholding strategy, and it is an equilibrium because Player 1 can set $\Delta > 0$ such that Equation (2.1) holds, and Players 2 and 3 are not better off submitting bidding prices other than their marginal cost.

On the other hand, suppose that the players have a finite choice of bidding prices. Let Player 1 choose from $\{mc^1, mc^2 - \epsilon, mc^3 - \epsilon, mc^3 + \Delta\}$, where $\Delta > \epsilon > 0$, Player 2 choose from $\{mc^2, mc^3 - \epsilon, mc^3 + \Delta\}$, and Player 3 choose from $\{mc^3, mc^3 + \Delta\}$. Likewise, let each Player i choose a bidding quantity of either 0 or q^i_{max}. Without the capacity withholding strategy,[3] when demand is equal to $L1$, there are three equilibrium strategies. For the first equilibrium, the bidding prices of Players 1, 2, and 3 are $mc^2 - \epsilon$, mc^2, and mc^3, respectively. Only Player 1 is scheduled to serve the entire demand; the market price is equal to $mc^2 - \epsilon$. For the second equilibrium, the bidding prices of Players 1, 2, and 3 are $mc^3 - \epsilon$, $mc^3 - \epsilon$, mc^3, respectively. Players 1 and 2 are scheduled to generate half of the entire demand; the market price is equal to $mc^3 - \epsilon$. For the third equilibrium, the bidding prices of Players 1, 2, and 3 are $mc^3 + \Delta$, $mc^3 + \Delta$, and $mc^3 + \Delta$, respectively. Players 1, 2, and 3 are scheduled to generate one-third of the entire demand; the market price is equal to $mc^3 + \Delta$.

Case 2: $L = L2$

When demand is equal to $L2$, one Nash equilibrium is that Players 1 and 3 submit their marginal-cost bid, and Player 2 submits its bidding price equal to $mc^3 - \epsilon$, where $\epsilon > 0$ and $mc^3 - \epsilon > mc^2$, i.e.,

$$\begin{cases} B^2 &= \{mc^3 - \epsilon, q^2_{max}\} \\ B^i &= \{mc^i, q^i_{max}\}, \quad i \neq 2. \end{cases}$$

The market price is equal to $mc^3 - \epsilon$. Player 1 receives profit equal to $(mc^3 - mc^1 - \epsilon) \cdot q^1_{max}$, Player 2 receives profit equal to $(mc^3 - mc^2 - \epsilon) \cdot (L2 - q^1_{max})$, while Player 3 is not scheduled. Note that Player 1 is not better off submitting a bidding price greater than mc^1 and less than mc^2. Similarly, there are other equilibria in which Players 1 and 2 submit two-part bids. For Player 1, if the following conditions hold,

$$\begin{array}{lcl} L2 - q^2_{max} - q^3_{max} &>& 0, \\ (mc^3 - mc^1 + \Delta) \cdot (L1 - q^2_{max} - q^3_{max}) &>& (mc^3 - mc^1 - \epsilon) \cdot q^1_{max}, \end{array} \tag{2.2}$$

the players submit their bids such that,

$$\begin{cases} B^1 &= \{(mc^1, q^{1,1}), (mc^3 + \Delta, q^{1,2})\} \\ B^i &= \{mc^i, q^i_{max}\}, \quad i \neq 1, \end{cases} \tag{2.3}$$

[3] Since the players are allowed to choose a bidding quantity of either 0 or q^i_{max}.

where $q^{1,1} = L2 - q^2_{max} - q^3_{max} - \delta$, $\delta > 0$, and $q^{1,2} = q^1_{max} - q^{1,1}$. The market price is equal to $mc^3 + \Delta$. All players are scheduled. Player 1 receives profit equal to $(mc^3 - mc^1 + \Delta) \cdot (L1 - q^2_{max} - q^3_{max})$, Player 2 receives profit equal to $(mc^3 - mc^2 + \Delta) \cdot q^2_{max}$, and Player 3 receives profit equal to $\Delta \cdot q^3_{max}$. Suppose that all players know Δ ($\Delta + mc^3 = P_{cap}$). For Player 2, if the following conditions hold,

$$\begin{aligned} L2 - q^1_{max} - q^3_{max} &> 0, \\ (mc^3 - mc^2 + \Delta) \cdot (L1 - q^1_{max} - q^3_{max}) &> (mc^3 - mc^2 - \epsilon) \cdot (L2 - q^1_{max}), \end{aligned} \tag{2.4}$$

Player 2 submits its bid,

$$\begin{cases} B^2 = \{(mc^2, q^{2,1}), (mc^3 + \Delta, q^{2,2})\} \\ B^i = \{mc^i, q^i_{max}\}, \ i \neq 2, \end{cases} \tag{2.5}$$

where $q^{2,1} = L2 - q^1_{max} - q^3_{max} - \delta$, in which $\delta > 0$, and $q^{2,2} = q^2_{max} - q^{2,1}$. The market price is equal to $mc^3 + \Delta$. All players are scheduled to operate. Player 1 receives profit equal to $(mc^3 - mc^1 + \Delta) \cdot q^1_{max}$, Player 2 receives profit equal to $(mc^3 - mc^2 + \Delta) \cdot (L2 - q^1_{max} - q^3_{max})$, and Player 3 receives profit equal to $\Delta \cdot q^3_{max}$.

Note that since $q^1_{max} > q^2_{max}$, when Equation (2.4) holds, Equation (2.2) also holds, but not vice versa. If only Equation (2.2) holds, Player 1 will have a dominant strategy, which is to exercise the strategy in Equation (2.3). When both Equations (2.2) and (2.4) hold, there exists a multiple-equilibrium condition, in which Players 1 and 3 are better off submitting marginal-cost bids, while Player 2 applies its capacity withholding strategy as in Equation (2.5). Players 2 and 3 are better off submitting a marginal-cost bid, while Player 1 applies its capacity withholding strategy and submits a bid as in Equation (2.3). When the capacity withholding strategy is applicable to Players 1 and 2, by submitting a marginal-cost bid Player 3 benefits from being scheduled as an infra-marginal bidder. (Similarly, Players 1 and 2 are better off submitting a marginal-cost bid if Player 3 submits a bidding price equal to $mc^3 + \Delta$.)

Case 3: $L = L3$

There is one unique Nash equilibrium in which Players 1 and 2 submit their marginal-cost bids, and Player 3 submits a bidding price equal to $mc^3 + \Delta$, where $\Delta > 0$, i.e.,

$$\begin{cases} B^3 = \{mc^3 + \Delta, q_{max}\} \\ B^i = \{mc^i, q_{max}\}, \ i \neq 3. \end{cases}$$

The market price is equal to $mc^3 + \Delta$. Player 1 receives profit equal to $(mc^3 - mc^1 + \Delta) \cdot q_{max}$, Player 2 receives profit equal to $(mc^3 - mc^2 + \Delta) \cdot q_{max}$, and Player 3 receives profit equal to $\Delta \cdot (L3 - 2q_{max})$. Note that Players 1 and 2 are not better off submitting a bidding price greater than their marginal costs. Note also that they are not better off applying the capacity withholding strategy because by submitting a marginal-cost bid, they are scheduled to operate at maximum capacity. The strategy in which Players 1, 2 and 3 submit the same bidding price equal to $mc^3 + \Delta$ to be scheduled to operate equal to $(q^1_{max}/Q^3) \cdot L2$, $(q^2_{max}/Q^3) \cdot L2$, and $(q^3_{max}/Q^3) \cdot L2$, respectively, is not an equilibrium strategy because each player is better off undercutting its bidding price slightly. Note that when there is asymmetry in marginal-cost functions among the players, the equilibrium strategy is a function of both bidding price and bidding quantity. Market price always deviates from marginal-cost price. The undercutting is not an issue since the players with the less expensive units are always better off submitting marginal-cost bids.

2.2.2 A Single-stage Game with Uncertain Demand

Von der Fehr and Harbord [13] show that when the players face demand uncertainties that one supplier is unable to serve within the range of demand variation, no equilibrium in pure strategies exists. The following examples extend this finding. Let us use the demand model of von der Fehr and Harbord. When the market opens, demand (λ) is determined as a random variable independent of price; in particular, $\lambda \in [\underline{\lambda}, \bar{\lambda}] \subseteq [0, Q^3]$, according to a probability distribution $G(\lambda)$.

When the players have asymmetric marginal-cost functions, the pure-strategy equilibrium when demand equals $L1$ is that Player 1 submits a bidding price less than the next expensive marginal cost and submits a marginal-cost bid for other demand levels. When the capacity withholding strategy is implementable, the pure-strategy equilibrium for Player 1 is to exercise this strategy and to set the bidding price of withheld capacity to be higher than the most expensive marginal cost, such as $mc^3 + \Delta$. Similarly, Player 2 submits a bidding price less than the next expensive marginal cost when demand is equal to $L2$ and submits a marginal-cost bid for other demand levels. Like Player 1, when the capacity withholding strategy is implementable, Player 2 shall exercise this strategy and set the bidding price of withheld capacity higher than the most expensive marginal cost, such as $mc^3 + \Delta$, to obtain higher profits. When Player 2 is able to exercise the capacity withholding strategy, so is Player 1. In this case, a mixed equilibrium condition exists, such that either Player 1 or Player 2 can implement the capacity withholding strategy and the other benefits from the increased market price. Player 3 shall submit a bidding price higher than its marginal cost when demand is higher than Q^2 and submit a marginal-cost bid for other demand levels. Hence, by applying the proposition presented by von der Fehr and Harbord [13] when there is uncertainty of demand in the form described above, let us consider the following scenarios:

1. When $\bar{\lambda} - \underline{\lambda} \geq \min_i q^i_{max}$. The capacity withholding strategy is not applicable to Players 1 and 2 and, depending on the forecast demand, there could be a pure-strategy equilibrium as in the deterministic case. For example, if $\bar{\lambda} < q^1_{max}$, there will be a pure-strategy equilibrium in which all players will submit bids as if demand is equal to $L1$. All players should submit their pure-

strategy equilibrium as if they were scheduled to operate as a marginal unit in the deterministic case because, when demand varies, each player becomes a marginal unit. Therefore, the players apply the same pure-strategy equilibrium as when they are scheduled as a marginal unit.

2. $\bar{\lambda} - \underline{\lambda} < \min_i q^i_{max}$. When the capacity withholding strategy is not applicable to Players 1 and 2, the pure-strategy equilibrium is that the player (called Player m) who will be a marginal unit if the actual demand is equal to the forecast demand, submits its bid following the strategy in the deterministic case. If $\underline{\lambda} < Q^{m-1}$, where $m = \{2, 3\}$, the player whose marginal cost is the most expensive but less than that of Player m, also submits its bid to be a marginal unit in the deterministic case. If $\bar{\lambda} > Q^m$, Player m submits its bid to be a marginal unit in the deterministic case.

However, when the capacity withholding is implementable, there is no pure-strategy equilibrium for the players because the capacity withholding strategy would not yield maximum profits for any actual load levels.

2.2.3 Comment and Discussion

Two main conclusions from the previous examples are a) there exists a multiple-equilibrium condition under different demand conditions, and b) an asymmetry of portfolios creates an opportunity to apply a capacity withholding strategy. For any player, whenever the capacity withholding strategy is applicable in the deterministic demand environment, that player should choose to exercise this strategy because

1. If this strategy is only applicable to the player, it is dominant.

2. This strategy yields minmax values if this strategy is applicable to the player and its opponents. Although this player is better off submitting a marginal-cost bid when other players exercise their capacity withholding strategy, by exercising the capacity withholding strategy the player is guaranteed its minmax payoffs.

These examples also emphasize the effect of the asymmetric marginal-cost functions of the players on the equilibrium bidding strategies. Without the unit-commitment constraints of operating the units, the players who own the units with large capacity (large q^i_{max}) and which are economical to operate (small mc^i) are easily able to strategically submit a bid-supply function that causes the market price to deviate from the marginal-cost price. On the other hand, when the players are uniform in their capacity and operating cost characteristics, as shown in the appendix to this chapter, then it is unlikely that the players will non-cooperatively raise the bidding price above the marginal cost. As a result, when the players are uniform, the marginal-cost prices are enforceable. Therefore, to prevent the players from setting the market price higher than their marginal costs, the market should have more than one player to prevent monopoly conditions, and each player should have marginal-cost functions as uniform as possible. (That is, after divesting any power system, the power producers in that power market should have portfolio characteristics as similar as they can possibly be.)

In addition, when inelastic demand is relatively close to installed capacity, for example, when demand is equal to $L3$, the players are able to set the bidding price as high as possible, i.e., $mc^3 + \Delta = P_{cap}$. This thesis calls this condition an ***absolute market power*** condition of the power producers. When this condition exists, the customers pay the highest price for electricity. To prevent the bidders from exploiting an absolute market power condition, without considering any constraint in dispatching the units to serve demand, such as transmission constraints, reserve requirements, and unit-commitment constraints, the maximum capacity of the largest power producer should be less than the total installed capacity minus the maximum demand (during a specified period). For example, consider period T. Let L_k denote demand at time k (which belongs to period T), and let L_{max} denote the maximum demand during that period, i.e., $L_{max} = \max_{k,k \in T} L_k$. Let q^i_{max} denote the maximum installed or available capacity of power producer i. The absolute market condition occurs when

$$\max_i q^i_{max} > \sum_j q^j_{max} - L_{max}.$$

On the other hand, when the constraints, such as transmission constraints, unit-commitment constraints, and reserve requirements are accounted for, the largest capacity that one player can own ($\max_i q^i_{max}$) will be reduced. For example, when the power producers also sell ancillary services to the system operator, the total demand is the energy demand (L_k) plus the ancillary service requirements. Suppose the ancillary service requirement (L^r_k) at each time k is $\alpha\%$ of the energy demand, i.e., $L^r_k = (1 + \alpha/100) \cdot L_k$. The absolute market condition occurs when

$$\max_i q^i_{max} > \sum_j q^j_{max} - (1 - \alpha/100) \cdot L^{max}.$$

Any player has "market power" in a bidding game when its dominant strategy to submit a strategic bid exists under some demand conditions. When the game experiences different demand levels, the player who has market power at one demand level may not have market power at the other. This implies that the player with market power is able to maintain its strategic bid and reap profits because the same game is played over and over again, and that strategic behavior is likely to be implemented when portfolio-based decision-making is in place or when the players own multiple generating units.

2.3 Multiagent Market Model

This thesis provides a framework for formulating electricity spot markets using an agent-based model to analyze dynamic interactions of the power producers and the price dynamics as a result of those interactions. The agent-based modeling approach provides simulation-based analyses that can capture the dynamic interaction of the agents in the electricity markets and their effect on market price dynamics. The key motivations to the adoption of this approach are the number of the agents in the markets, the asymmetric characteristics and objective functions of these agents, and the repetitive auctions.

The classic oligopoly model was previously used by several researchers, such as Green and Newbury [20], to analyze the market in a static setup with only a few active power producers (i.e., $N \geq 3$) facing (almost) inelastic demand. This model is very sensitive to the price-elasticity of demand. Also, in this setup at least one player will have at least 33.3% of market shares, so this player can always set the market price different from the marginal-cost price. Market prices determined using this model deviate substantially from marginal-cost prices; therefore, the model becomes an unrealistic one for analyzing the ability of a power producer to influence the market price. Moreover, existing electricity markets in the US are not dominated by just a few players but rather by around 10 power producers, as seen in the California and New England markets; see [52] and [53]). On the other hand, with the oligopoly model for many agents with asymmetric marginal-cost functions, it is difficult to characterize market prices even statically.

Furthermore, one may argue that an HHI index (see the appendix to this chapter) to determine the level of competitiveness may be used instead. Rudkevitch *et al.* [38] use these indices to indicate that the New England markets are competitive; however, the price spikes (indicating very expensive market prices) in the New England market during low or high demand periods can be observed regularly.

Additionally, as shown in the examples in Section 2.2, the pure-strategy equilibrium in a single-stage bidding game for any given demand level depends on the players' characteristics. The equilibrium strategy of the bidding game generally varies whether the players own a single unit or a portfolio of units. For example, two portfolios with the same marginal-cost characteristics (including generation capacities and operating costs) may have the same optimal bidding strategy when bids are determined on a unit-by-unit basis, and different optimal bidding strategies may be obtained when the entire portfolio is considered. If the units are able to generate power at any output (less than its capacity), one large unit can generate power equal to two small units at the same costs of mc per unit of power. Consider when Player 1 owns one unit with capacity q^1_{max} and Player 2 owns two units with capacity $q^{2,1}_{max}$ and $q^{2,2}_{max}$, in which $q^1_{max} = q^{2,1}_{max} + q^{2,2}_{max}$. Suppose that the bids are determined based on a unit-by-unit basis and that capacity withholding is applicable for Player 1. Player 1 anticipates market price to be $m\acute{c}$ if capacity withholding strategy is applied. Player 1 submits its bid as $B^1 = \{(mc, q^{1,1}), (m\acute{c}, q^{1,2})\}$, where $mc < m\acute{c}$. Suppose $q^{1,2} > q^{2,2}_{max}$ and the capacity withholding strategy is not applicable to any one of Player 2's units. Player 2 is unable to set the market price equal to $m\acute{c}$ by simply submitting a bidding price as $B^2 = \{(mc, q_{max}^{2,1}), (m\acute{c}, q_{max}^{2,2})\}$. Note that different optimal bidding strategies may also be obtained when operating constraints, such as unit-commitment constraints, are accounted for in determining bid-supply functions.

In existing electricity markets, power producers participate in sealed-bid auctions to trade electricity daily. These auctions are modeled as a series of a repeated game of incomplete information. Without the information regarding the opponents' actions and payoffs, it is not feasible to determine an optimal bidding strategy. Fortunately, the auction occurs repeatedly, and market prices and total demand are public information after each auction finishes. The bidders can use this information, together with their bid-supply function and scheduled outcomes, to learn their opponents' strategies over time. The algorithms that are suitable for an agent to play a repeated game of incomplete information are designed so that no information about the opponents' actions and payoffs is required.

In the next chapter, the proposed electricity market model as a multiagent system and/or a bidding game is described in detail, and a state-of-the-art agent based modeling approach is introduced.

Appendix to Chapter 2

A. Equilibrium in a Three-person Bidding Game

In this section, let us consider when the players have uniform marginal-cost functions and when the players have the same installed capacity. General characteristics of the players are the same as where the players have asymmetric portfolios. The equilibrium strategies in a bidding stage-game under different demand conditions can be derived as follows.

I. Deterministic Demand

a) Uniform Marginal-cost Functions

First, let us analyze an equilibrium of price competition in a bidding game in which there are three identical players. Each generating unit has maximum capacity q_{max} and marginal cost mc. Let us consider when demand equal to $L1$, $L2$, and $L3$, where $0 < L1 \leq q_{max}$, $q_{max} < L2 \leq 2q_{max}$, and $2q_{max} < L3 \leq 3q_{max}$.

Suppose that the player can submit its bidding price such that $b^i \in [0, P_{cap}]$ and $P_{cap} > 0$. The only equilibrium in the game when demand is equal to $L1$ and $L2$ is that each agent submits a marginal-cost bid, i.e.,

$$B^i = \{mc, q_{max}\}, \ \ i \in \{1, 2, 3\}.$$

This is an equilibrium strategy because the agents cannot unilaterally benefit from submitting a bidding price higher than mc. If only one player submits a bidding price higher than mc and the others submit a marginal-cost bid the market price is still equal to mc. On the other hand, when demand is equal to $L3$, if only one player submits its bidding price higher than mc, it benefits from the market price equal to its bid, while the others become free-riders. Cooperation in which all players submit the same bidding price greater than mc is unlikely, because one player can benefit from undercutting the price slightly to be scheduled to operate at its full capacity q_{max}. To have one player submit a different bid seems to contradict the presupposition that all the players are identical. Therefore, the only equilibrium in this game is that the players submit the marginal-cost bid in the case of demand equal to $L1$ and $L2$ as well.

$$B^i = \{mc, q_{max}\}, \;\; i \in \{1,2,3\}.$$

Next, suppose that the players have finite choices of bidding price, i.e., $b^i \in \{mc, mc + \Delta\}$, where $\Delta > 0$. Let us consider the three scenarios of $L1$, $L2$, and $L3$. There are two pure strategy Nash equilibria for demand equal to $L1$, i.e., either all players submit a bidding price equal to mc or a bidding price equal to $mc + \Delta$.

$$B^i = \begin{cases} \{mc, q_{max}\} \\ \{mc + \Delta, q_{max}\} \end{cases}, \;\; i \in \{1,2,3\}.$$

In either one of these equilibria, the players are scheduled to generate $L1/3$. (Residual demand is equal to $L1$. Each player is scheduled to operate $\frac{q_{max}}{q_{max}+q_{max}+q_{max}} \cdot L1 = \frac{L1}{3}$.) While the former case yields zero profit, the later case yields profit equal to $(\Delta \cdot L1)/3$. When demand is equal to $L2$, there are four pure-strategy equilibria. These include that all players submitting a bidding price equal to mc and that one of the players submits a bidding price equal to mc and the others submit a bidding price equal to $mc + \Delta$. A marginal-cost bidder receives profit equal to $\Delta \cdot q_{max}$, while the others receive profit equal to $(\Delta \cdot (L2 - q_{max}))/2$. Cooperation strategy in which all players submit the bid equal to $mc + \Delta$ is not an equilibrium strategy, although it yields profits equal to $(\Delta \cdot L2)/3$. This is because one player can defect by submitting a marginal-cost bid to obtain $\Delta \cdot q_{max}$ instead. The four equilibria strategies for this scenario are:

$$B^i = \{mc, q_{max}\} \qquad\qquad i \in \{1,2,3\}$$

$$\begin{cases} B^i &= \{mc, q_{max}\} \\ B^{-i} &= \{mc + \Delta, q_{max}\} \end{cases} \qquad i \in \{1,2,3\}.$$

When demand is equal to $L3$, there are three pure-strategy equilibria, which are that two players submit a marginal-cost bid and the other player submits a bidding price equal to $mc + \Delta$. For two marginal-cost bidders, each player obtains profit equal to $\Delta \cdot q_{max}$, and the non marginal-cost bidder obtains the profit equal to $\Delta \cdot (L3 - 2q_{max})$. If all players cooperated to submit a bidding price equal to $mc+\Delta$, each player would receive profit equal to $(\Delta \cdot L3)/3$ which is better than $\Delta \cdot (L3 - 2q_{max})$ for a non marginal-cost bidder. This is not an equilibrium because one player may defect by submitting a marginal-cost bid to obtain more profit equal to $\Delta \cdot q_{max}$.

$$\begin{cases} B^i &= \{mc + \Delta, q_{max}\} \\ B^{-i} &= \{mc, q_{max}\} \end{cases} \quad i \in \{1,2,3\}.$$

Note that when there are multiple pure-strategy equilibria, a mixed-strategy equilibrium is also obtained.[4] When the player chooses its bidding price from an infinite set, the equilibrium that is not a marginal-cost bid is not possible because the player is unable to agree on Δ non-cooperatively. However, the equilibrium resulting in positive profits when the player faces an infinite set of bidding prices may be reached when the game is played sufficiently often. If the game is played often enough, the players will be able to agree on some Δ, and the strategy of cooperation to raise the price will be enforceable.

b) Uniform Capacity

Let us consider the bidding game played by three players called Player i, where $i \in \{1,2,3\}$. Each Player i has installed capacity q_{max} and marginal cost equal to mc^i. Suppose that $mc^1 < mc^2 < mc^3$. Three demand scenarios, i.e., $L1$, $L2$, and $L3$, where $0 < L1 \leq q_{max}$, $q_{max} < L2 \leq 2q_{max}$, and $2q_{max} < L3 \leq 3q_{max}$ are considered.

Suppose that the player can choose a bidding price that belongs to $b^i \in [0, P_{cap}]$, where $P_{cap} > 0$. In this game, when demand is equal to $L1$, one equilibrium is that Player 1 submits a bidding price equal to $mc^2 - \epsilon$, where $\epsilon > 0$ and $mc^2 - \epsilon > mc^1$. The other players submit their marginal-cost bid, i.e.,

$$\begin{cases} B^1 &= \{mc^2 - \epsilon, q_{max}\} \\ B^i &= \{mc^i, q_{max}\}, \quad i \neq 1. \end{cases}$$

The market price is equal to $mc^2 - \epsilon$. Only Player 1 is scheduled to serve demand and receives profit equal to $(mc^2 - mc^1 - \epsilon) \cdot L1$, while the other players are not scheduled.

When demand is equal to $L2$, one equilibrium is that Players 1 and 3 submit their marginal-cost bid and Player 2 submits its bidding price equal to $mc^3 - \epsilon$, where $\epsilon > 0$ and $mc^3 - \epsilon > mc^2$, i.e.,

$$\begin{cases} B^2 &= \{mc^3 - \epsilon, q_{max}\} \\ B^i &= \{mc^i, q_{max}\}, \quad i \neq 2. \end{cases}$$

The market price is equal to $mc^3 - \epsilon$. Players 1 and 2 are scheduled to operate, but not Player

[4] However, note that in this thesis the value of σ^i, a probability distribution over a set of pure-strategy equilibria, is not determined.

3. Player 1 receives profit equal to $(mc^3 - mc^1 - \epsilon) \cdot q_{max}$ and Player 2 receives profit equal to $(mc^3 - mc^2 - \epsilon) \cdot (L2 - q_{max})$. Note that Player 1 is not better off submitting a bidding price greater than mc^1 and less than mc^2.

Likewise, when demand is equal to $L3$, one equilibrium is that Players 1 and 2 submit their marginal-cost bids and Player 3 submits its bidding price equal to $mc^3 + \Delta$, where $\Delta > 0$, i.e.,

$$\begin{cases} B^3 &= \{mc^3 + \Delta, q_{max}\} \\ B^i &= \{mc^i, q_{max}\}, \quad i \neq 3. \end{cases}$$

The market price is equal to $mc^3 + \Delta$. Player 1 receives profit equal to $(mc^3 - mc^1 + \Delta) \cdot q_{max}$, Player 2 receives $(mc^3 - mc^2 + \Delta) \cdot q_{max}$, and Player 3 receives $\Delta \cdot (L3 - 2q_{max})$. From this example, one can observe that Players 1 and 2 are not better off submitting a bidding price greater than their marginal cost.[5]

From these examples, when there is asymmetry in marginal costs, the equilibrium strategy is no longer a marginal-cost bid. Price-undercutting does not play a role, since the players with the cheaper marginal-cost units are always better off submitting their marginal-cost bids.

Second, when demand is equal to $L2$, there is a unique equilibrium strategy where Player 2 chooses the bidding price equal to $mc^2 + \Delta$, whereas Players 1 and 3 are indifferent to either of their bidding prices. The market price equals $mc^2 + \Delta$. Players 1 and 2 are scheduled to operate, but not Player 3. Player 1 receives profit equal to $(mc^2 + \Delta - mc^1) \cdot q_{max}$ and Player 2 receives profit equal to $\Delta \cdot (L2 - q_{max})$. When demand is equal to $L3$, there is a unique equilibrium strategy where Player 3 chooses the bidding price equal to $mc^3 + \Delta$. Players 1 and 2 are indifferent to either of their bidding prices. The market price equals $mc^3 + \Delta$. All players are scheduled. Player 1 receives profit equal to $(mc^3 + \Delta - mc^1) \cdot q_{max}$, Player 2 receives profit equal to $(mc^3 - mc^2 + \Delta) \cdot q_{max}$, and Player 3 receives profit equal to $\Delta \cdot q_{max}$.

Furthermore, let us consider when Player 1 chooses its bidding price from $\{mc^i, mc^2 - \epsilon, mc^3 - \epsilon, mc^3 + \Delta\}$, where $\Delta > \epsilon > 0$, Player 2 chooses its bidding price from $\{mc^2, mc^3 - \epsilon, mc^3 + \Delta\}$, and Player 3 chooses the bidding price from $\{mc^3, mc^3 + \Delta\}$. When demand is equal to $L1$, one equilibrium is $b^1 = mc^2 - \epsilon$, $b^2 = mc^2$, and $b^3 = mc^3$. The market price is equal to $mc^2 - \epsilon$. Only Player 1 is scheduled to operate. Suppose that $(mc^3 - \epsilon - mc^1) \cdot (L1/2) > (mc^2 - \epsilon - mc^1) \cdot L1$. There is one equilibrium that yields a higher market price than that of $mc^2 - \epsilon$, that is, where Players 1 and 2 submit their bidding prices equal to $mc^3 - \epsilon$. The market price is then equal to $mc^3 - \epsilon$ and Player

[5] To submit a bidding price such as $mc^3 + \acute{\Delta}$ if $\acute{\Delta} = \Delta$, their profits are reduced to $(mc^3 - mc^1 + \Delta) \cdot \frac{L3}{3}$. On the other hand, if $\acute{\Delta} > \Delta$, their profits are $(mc^3 - mc^1 + \acute{\Delta}) \cdot \frac{(L3 - q_{max})}{2}$. By increasing $\acute{\Delta} \gg \Delta$, they are able to make more profits than if they were scheduled to operate as marginal units. However, both Players 1 and 2 have to choose the same $\acute{\Delta}$. If only one of these two players submits the bid with $\acute{\Delta} \gg 0$, its will be a marginal unit and may be more profitable than being scheduled to operate as an infra-marginal unit. Since Player 3 is able to apply a similar strategy and Players 1 and 2 know that bidding above marginal costs is Player 3's dominant strategy (over a marginal-cost bid), they are always better off if they bid ϵ lower than Player 3's bidding price, $\epsilon > 0$.

3 is indifferent to any of its bidding strategies. Players 1 and 2 are scheduled to generate $L1/2$, but not Player 3. Moreover, when $\Delta \gg 0$ such that $(mc^3 + \Delta - mc^1) \cdot (L1/3) > (mc^2 - \epsilon - mc^1) \cdot L1$, there is another equilibrium, that is, where all players submit their bidding prices equal to $mc^3 + \Delta$. In this case, each player is scheduled to generate $L1/3$. Due to the existence of multiple pure-strategy equilibria, there is also a mixed-strategy equilibrium. Likewise, with a analysis similar to that of demand equal to $L1$, multiple equilibrium conditions can also be found when demands are equal to $L2$ and $L3$. The finite sets of bidding strategies of the players can ensure the possibility that the market price is set to $mc^3 + \Delta$ in any demand $L1$, $L2$, or $L3$.

II. Uncertain Demand

a) Uniform Marginal-cost Functions

When the players have uniform portfolio characteristics, the pure-strategy equilibrium is that the players submit their marginal-cost bids, regardless of the demand levels. Therefore, demand uncertainty does not affect the pure-strategy equilibrium in which the players submit their marginal-cost bids.

b) Uniform Capacity

When the players have uniform capacity but not uniform marginal costs, the pure-strategy equilibrium of each player can be characterized as follows. Player 1 submits a bidding price to be less than the next expensive marginal cost when demand is equal to $L1$ and submits a marginal-cost bid for other demand levels. Player 2 submits a bidding price to be less than the next expensive marginal cost when demand is equal to $L2$ and submits a marginal-cost bid for the other demand levels. Likewise, Player 3 submits a bidding price to be less than the next expensive marginal cost when demand is equal to $L3$ and submits a marginal-cost bid for the other demand levels. When there is demand uncertainty in the forms described in Section 2.2 by von der Fehr and Harbord [13], the players should apply the same pure-strategy equilibrium, as if they were scheduled to operate as a marginal unit. That is, Player 1 submits a bidding price equal to $mc^2 - \epsilon > mc^1$, Player 2 submits a bidding price equal to $mc^3 - \epsilon > mc^2$, and Player 3 submits a bidding price equal to $mc^3 + \Delta$. If there is a sufficiently large demand deviation, the anticipated marginal unit may become an infra-marginal unit and the anticipated extra-marginal unit may become a marginal unit, and the players would be better off setting the bidding price as if they encounter the deterministic demands and they are scheduled to operate at the margin.

B. HHI Index

An HHI index is defined as

$$HHI = \sum_i S_i^2, \ \sum_i S_i = 100\% \tag{2.6}$$

where S_i is market shares of each firm in a market. From Equation (2.6), HHI ranges between 0 and 10,000. That $HHI = 10,000$ refers to a market with a monopoly, and that $HHI = 0$ refers to a competitive market. See, for example, Landes and Posner [29] and Ordover *et al.* [36], for more detail on this subject.

Chapter 3

Agent-based Modeling Approach

This chapter presents a detailed explanation of an agent-based electricity market model. In this model, the market has a uniform-pricing rule and active market participants are agents, who learn the bidding behavior of the other participants from available information and determine their bids in response to the others. There are only the power-producing agents facing inelastic demands. The market-clearing mechanism uses the price-merit order method. The agents know forecast demand, actual demand, and their scheduled quantity and market price, but not the others' bids and/or scheduled quantity.

This chapter is organized as follows. Section 3.1 lays out the general characteristics of the model and describes the details of the model and load-based decision schemes. Section 3.2 outlines the characteristics of the agents, available information, learning algorithms, as well as the actions or the bid-supply functions. This thesis applies three learning algorithms: 1) Auer *et al.'s* algorithms to play multi-armed bandits that is later applied to play unknown repeated games (see Auer *et al.* [1]); 2) the softmax action selection using a Boltzmann distribution, which shares a similarity with Auer *et al.'s* algorithms in term of the action selection (see Sutton and Barto [41]); and 3) a model-based learning algorithm that is designed specifically for this bidding game. These algorithms are presented in Sections 3.3, 3.4, and 3.5, respectively. The conclusion is outlined in Section 3.6.

3.1 Model Characteristics

The agent-based electricity market model represents the electricity spot markets with the following characteristics. Power-producers have portfolios of generating units with different marginal costs and capacities, and demand is inactive and inelastic (no load-serving entity). The power-producers participate in a sealed-bid first-price auction to sell electricity daily. They submit bid-supply functions to a system operator prior to demand being realized. The bid-supply functions are piece-wise and non-decreasing functions of quantities and prices, indicating the amount of power the power-producer is willing to generate at the specified price. An independent system operator clears the market by using a price-merit order method, matching supply to demand and setting the market price to be the bidding price of a marginal unit. The market has a uniform-pricing rule. After the market clears, the power-producers are informed of total demand and market prices, as well as their scheduled outcomes, such as scheduled prices and quantities. No bid-supply functions of the competitors are revealed. In addition, the agents know the aggregate (system) marginal-cost function. This function is determined by assuming that all the units are on at their full capacity. The agents do not know the marginal-cost function of individual opponents.[1]

In this model, the agents do not known when the game starts, since the bid outcomes depend on the bid-supply functions of all agents. The agents want to learn the game so that their bid-supply functions yield profits at least better than the profits from a marginal-cost bid, and as good as the profits from the previous periods. The agents can therefore adopt any learning algorithm that is suitable for available information, and they may use the information revealed through the interactions for learning about the others' joint actions. According to Fudenberg and Kreps [18], in a repeated game of incomplete information an agent's play may have a broad class of assessing actions played by its opponent given information observed through repetition of game playing, actions which were themselves dependent on the information available to the opponents.[2] Learning the opponents' bidding behaviors of the agents can also be viewed as on-line decision-making.

As mentioned in previous chapters, the equilibrium strategy of the agent in a single-stage bidding game varies when demand changes, because the change in demand affects payoffs obtained from the same actions and equilibrium strategies. This claim is supported by the observation of the New England market prices, which is shown in the appendix to this chapter. The histograms of market prices change their characteristics under different demand ranges. Generally, a typical characteristic of the system marginal-cost function is a piece-wise non-decreasing function. The portfolio of each power producer is a part of the system supply function. Different demand levels lead to different groups of power producers that influence the scheduled outcomes. Figure 3-1 shows an example of an aggregate marginal-cost function and the portfolios of power producers G1 - G6 as parts of this function. As shown in Figure 3-1, when demand is equal to $L1$, power producers G1, G2, G3, and G4 are competitors. That is, they have the units that can be bid as marginal units, setting market prices and obtaining positive profits. Similarly, when demand is equal to $L2$, power producers G1, G2, G3, G4, and G5 are competitors. Figure 3-2 shows the New England moving-average demand from May, 1999 to April, 2001. Electricity demand generally exhibits a seasonal pattern. For example, as shown in Figure 3-2, in the New England market there are two peak demand periods in the winter

[1] This is quite a strong assumption. In the existing markets, the units are operated under unit-commitment constraints. The aggregate marginal-cost function at each hour should account for these operating constraints; however, the unit-commitment constraints are discarded throughout this thesis.

[2] Fudenberg and Kreps suggest four examples of the assessment rules including: a) the opponent will play in the current period (t) whatever it plays in the previous period ($t-1$); b) the opponent will play the weighted average of the past plays; c) the opponent is equally likely to play any action that has been played at least 1% of the time with zero probability for all other actions; and d) the opponent will play the equally weighted actions played previously.

and summer months.

To play the game consecutively without considering demand variation might not capture the true behavior of the bidders. This thesis proposes to divide the learning and decision-making processes based on load levels and calls this approach a load-based modeling approach, meaning that the agents play multiple bidding games simultaneously (during a daily auction round). Each game is technically correlated because of constraints of operating the generating units, i.e., the unit-commitment constraints; however, these particular constraints are not considered here.

The concept of partitioning the bidding game into a set of bidding games depending on demand levels shares a similarity with the idea of adaptive resolution models, which is to partition the environment into regions of states that can be considered the same for the purposes of learning and generating actions, as mentioned in Kaelbling *et al.* [26]. Note that the bidding game has only one state. Kaelbling *et al.* also state that without detailed prior knowledge of the environment, it is very difficult to know what granularity or placement of partitions is appropriate. This problem is overcome in methods that use adaptive resolution and where during the course of learning a partition is constructed that is appropriate to the environment. Moreover, the only way to behave truly effectively in a wide-range of environments is to use memory of previous actions and observations to clarify the current state. This data collection is based on a finite-history window of memory concept.

In this model, a decision-making algorithm to determine the bids of the agents is formulated based on the levels of demand. A set of indices $\mathcal{L}^j$ representing a range of demand such that $\bar{L}^{j-1} < \mathcal{L}^j \leq \bar{L}^j$ is introduced. Learning and data-collecting over the period are based on these load indices. Continuous demand value is discretized into N^d load/demand indices ($\mathcal{L}$). Each index d, $\mathcal{L}^d$, represents demand in range d denoted by $[\bar{L}^{d-1}, \bar{L}^d)$, where

$$\mathcal{L}^d \equiv L \in [\bar{L}^d, \bar{L}^{d+1}), \;\; d \in N^d.$$

This range is set arbitrarily and may affect the behavior of the agents as well as price dynamics. Using these indices, load in each hour is mapped to one of these ranges, meaning that the continuous load is mapped to a set of discrete load indices, as follows:

$$\begin{aligned} 0 \leq L < \bar{L}^1 &\rightarrow \mathcal{L}^1 \\ \bar{L}^1 \leq L < \bar{L}^2 &\rightarrow \mathcal{L}^2 \\ &\vdots \\ \bar{L}^{N^d-1} \leq L < \bar{L}^{N^d} &\rightarrow \mathcal{L}^{N^d} \end{aligned}$$

where $\bar{L}^{N^d} \rightarrow \infty$. Figure 3-3 shows an example of the mapping from continuous demand to 16 indices, in which each index represents 500 MW of power. This figure shows also that demand in different hours can be mapped to the same index. Although the same demand index can occur in several hours a day, this thesis allows an information update to take place only daily for each demand index. Therefore, during several hours in each day demands are mapped with the same index, and an average of any collected information during those hours is used for any update.

In addition, the concept of storing the observed data is motivated by a real-world situation in which market participants analyze and assess market conditions to determine the best response to any market circumstance. The agents record market outcomes or create their database in a form of memory matrices. The matrices contain collected data for a window of Md periods and have each row representing one load index (or one load range), $\mathcal{L}^d$; that is, the data stored in any row is associated with the load index that is mapped to that row. Note that $Md \geq 1$. For any myopic decision, $Md = 1$. Generally, these data can be categorized by different references, such as time of day, load levels, or seasons.

3.2 Model of Power-producing Agents

The power-producing agents (agents) own a portfolio of generating units, which consists of at least one generating unit. These units have a constant marginal cost. The agents submit their bid-supply functions, pairs of bidding price and bidding quantity, to sell electricity to the market. Note that a bid and a bid-supply function are used interchangeably throughout this chapter. The agents submit 24 bid-supply functions each day.

Let $S^i = P \times Q^i$ be action (or bid) spaces of Agent i, where $P \in [0, P_{cap}]$ and $Q^i \in (0, q^i_{max})$, where P_{cap} is the maximum possible price and q^i_{max} is total available capacity or installed capacity of Agent i. In this market model, the agents choose their bid-supply function so that the undiscounted expected sum of profits ($\hat{R}$) are maximized. Suppose Agent i owns at least one generating unit j. Prior to determining the bid-supply function, Agent i calculates its expected profits of its N^i units over K periods. This expectation is taken over uncertain demand and over variations of market prices, which are a function of the agent and its opponents' bid-supply functions. The undiscounted expected profits are defined as follows:

$$\hat{R}^i = \max\ \mathcal{E}\left\{\sum_{k=0}^{K}\sum_{j\in N^i}(\hat{P}^{i,j}_k \cdot q^{i,j}_k - c^{i,j}(q^{i,j}_k) - \mathcal{U}^{i,j}_k)\right\}$$
$$\text{subject to } q^{i,j}_{k,min} \leq q^{i,j}_k \leq q^{i,j}_{max}$$

where $\hat{P}^{i,j}_k$ denotes a forecast price paid to Agent i for scheduled quantity $q^{i,j}_k$ of generating unit j. Let $c^{i,j}(q)$ denote an operating cost incurred due to producing q. Marginal cost defined by $(\frac{\partial c^{i,j}(q)}{\partial q} = mc^{i,j})$ is a constant for $q \in [0, q^{i,j}_{max}]$; therefore, $c^{i,j}(\cdot)$ is a linear function. Let $q^{i,j}_{max}$ denote the installed capacity of unit j. Let $q^{i,j}_{k,min}$ denote the minimum capacity that unit j needs to be operating at time k and let $U_k^{i,j}$ be the cost incurred each time k due to the unit-commitment constraints. The constraint on $q_{k,min}^{i,j}$ is imposed to capture the inflexibility due to the unit-commitment constraints. Since the unit-commitment constraints are not accounted for in this thesis, these constraints are discarded by setting $q_{k,min}^{i,j} = 0$ and $\mathcal{U}_k^{i,j} = 0$. As a result, no intertemporal relationships from period to period are considered and the agents determine their bid-supply functions for each period k based on that period only.

Let us further assume that demand is deterministic; that is, forecast demand is equal to actual demand. The objective function of the agents at time k is simplified such that the expectation is taken over the variation of market prices, and it is reduced to

$$\hat{R}^i = \max \ \mathcal{E} \left\{ \sum_{k=0}^{K} \sum_{j \in N^i} (\hat{P}_k^{i,j} \cdot q_k^{i,j} - c^{i,j}(q_k^{i,j})) \right\} \tag{3.1}$$
$$\text{subject to } 0 \leq q_k^{i,j} \leq q_{max}^{i,j}.$$

According to Equation (3.1), the agents determine their "best" bids from the available information and the assigned learning algorithm. The set of available information is summarized in Section 3.2.1, whereas learning algorithms employed in this model are presented in Section 3.2.2.

3.2.1 Available Information

The information that the agents know before and after each bidding rounds includes:

- *Forecast demand.* Prior to a bid submission, the agents are informed of total forecast demand for the next twenty-four hours. After the market clears in each bidding hour, total demand for that hour becomes publicly available.

- *System marginal-cost function.* The agents uniformly know the aggregate marginal-cost function of the market. This function is determined by assuming that all units are available and ready to operate at its full capacity. (Before deregulation began a few years ago, information regarding operating costs and operating constraints of the generating units in the market was publicly known. This information is currently confidential and this assumption may not be realistic.)

- *Scheduled prices and quantities.* Each agent is informed only of its scheduled price and quantity of each hour. Since the uniform-price rule is adopted here, the scheduled price in each hour is equal to the market price of that hour. In addition, no unavailable capacity due to outage and maintenance are considered, nor are unit-commitment constraints.

3.2.2 Learning in the Repeated Bidding Game

As shown in the previous chapter, this market model is considered a repeated bidding game of incomplete and imperfect information, which may also have multiple equilibria in some demand levels. The typical problems of the agents in playing the incomplete information game with multiple equilibria are to decide which actions to play based on the available information. A "good" learning algorithm requires the least information regarding the opponents' actions and the agent's payoff characteristics. The learning algorithms in general must also balance exploration of new actions and exploitation of current best actions. (See, for instance, Sutton and Barto [41], for more detail.) Besides, in the long run the learning algorithms may guarantee an average payoff as large as the best-reply payoff to the empirical distribution of play of the other agents; that is, the learning algorithm or strategy has Hannan-consistent properties.[3] This concept plays a significant role as a measure and a desired property of learning algorithms or strategies.

The agents participate in the bids daily without an opportunity to have off-line training to learn to bid. They face an on-line learning problem, and consequently the agents have to make their decisions myopically. That is, the agent determines its decision to maximize its objective function, using only current and past available information.[4] This action is called a myopic play.[5]

One key factor in determining a bid-supply function, besides forecast demand, is to know market prices. Market price at each time k denoted by P_k is a function of the bids of all N agents and demand, L_k, i.e., $f : S^1 \times \ldots \times S^N \times \Re^+ \rightarrow \Re$,

$$P_k = f(B_k^i, B_k^{-i}, L_k).$$

Note that one can think of demand as a mapping function from the bids to market price, i.e., $L : S^1 \times \ldots \times S^N \rightarrow \Re$, $P_k = L(B_k^i, B_k^{-i})$, where B_k^i is a bid function of Agent i at time k and B_k^{-i} is an aggregate bid-supply function of other agents except Agent i at time k.

Prior to determining the bids for the next period k, from Agent i's point of view, anticipated market price at time k is a function of its bid B_k^i and its anticipated opponents' bids, which is an assessment based on its past bids $\{B_{<k}^i\}$,[6] demand ($\{L_{<k}\}$), and price ($\{P_{<k}\}$), that is, $\hat{B}_k^{-i}(\{B_{<k}^i\}, \{L_{<k}\}, \{P_{<k}\})$. Hence,

$$\hat{P}_k^i = P^i(B^i(b_k^i, q_k^i), \hat{B}_k^{-i}).$$

The auctions occur daily with new revealed information. Although the decisions or actions of the

[3] The definition of Hannan consistency strategies is described in the appendix to this chapter.

[4] This assumption is not realistic when the unit-commitment constraints are accounted for. For simplicity of the model and model formulation, no unit-commitment constraints are accounted for; therefore, the agent makes a myopic decision.

[5] See the appendix to this chapter for a complete definition of a myopic play.

[6] The subscript on variable x, $(x)_{<k}$, refers to a string of variable x from time 0 up to time $k-1$.

opponents are not observable, the joint actions may be obtained. The agents may either a) learn about their opponents from their own actions and their payoffs obtained after each bidding round without considering their actions, or b) try to "anticipate" the market prices from past information; that is, $\hat{B}_k^{-i}$ is obtained from learning. This claim suggests that the agent with larger capacity and tendency to be scheduled is likely to better estimate the actions of the agents with less capacity. Note also that the asymmetry in the agents' portfolio characteristics can be found in Appendix A.

To determine a bid-supply function, making use of game theory perspectives, Agent i should play a Nash equilibrium strategy. Agent i is unable to do so, because it does not know its opponents' bid-supply functions and their associated scheduled outcomes; therefore, it either determines its bid-supply functions (a set of bidding prices and quantities) so that the anticipated profit, calculated from its anticipated price $\hat{P}_k^i$ at time k, are maximized by following some learning algorithms, or determines its mixed strategies according to some learning algorithms without attempting to determine anticipated prices. One may view a simplified version of Equation (3.1) as

$$\{b_k^{i,j,*}, q_k^{i,j,*}\} = \arg \max_{\{b_k^{i,j}, q_k^{i,j}\}} \left\{ \sum_{j \in N^i} (P^i(B^i(b_k^i, q_k^i), \hat{B}_k^{-i}) \cdot q_k^{i,j} - c^{i,j}(q_k^{i,j})) \right\}.$$

This equation tells us that Agent i plays the best response strategy to the joint actions it believes the opponents might play. Since the objective of this thesis is to construct a computer-based market model that closely mimics characteristics of the existing electricity markets, the equilibrium strategy or equilibrium dynamics is not a main focus. The learning algorithms may not necessarily yield the value of the bidding game or have Hannan-consistency properties. The simulated price dynamics depend highly on an ability of the algorithm to allow the agents to explore and exploit favorable actions. This thesis explores three different learning algorithms/bidding selection strategies:

- The algorithms select a mixed strategy for choosing a bid-supply function. These algorithms are similar to Auer *et al.'s* algorithms [1] which play multi-armed bandits and they are outlined in Section 3.3.

- The algorithm selects a mixed strategy for choosing a bid-supply function in which the selection method is updated based on the Boltzman distribution. This algorithm can be found in, for example, Sutton and Barto [41], and it is highlighted in Section 3.4.

- The model-based algorithm selects a pure strategy of bid-supply function. This algorithm is developed solely for this particular model, and it is presented in Section 3.5.

3.3 Modified Auer *et al.*'s Algorithms

Auer *et al.* [1] provide the algorithms for an agent to play multi-armed bandits so that expected regrets after playing for a given time T are within established bounds. Their algorithms are based on the weight-majority algorithm of Freund and Schapire [15]. In the multi-armed bandit problem, a gambler must decide which arm of K non-identical slot machines to play in a sequence of trials so as to maximize its reward. Auer *et al.* assume that each arm delivers rewards that are independently drawn from a fixed and unknown distribution; that is, no statistical assumptions are made about the generation of rewards. Each slot machine is initially assigned an arbitrary and unknown sequence of rewards, one for each time step, chosen from a bounded real interval. The "worst-case" regret is used to measure the gambler's performance, which is the difference between the return the gambler would have had by pulling arms $j_1, \ldots, j_T$ and the actual gambler's return, where both returns are determined by the initial assignment of rewards.

In this thesis, three algorithms developed by Auer *et al.*, including algorithms **Exp3**, **Exp3.1**, and **Exp3.P.1**, are implemented. Algorithms **Exp3** and **Exp3.1** are illustrated in detail in the appendix to this chapter, and only an overview of Algorithm **Exp3.P.1** is presented in this section. For those who are familiar with these algorithms, Section 3.3.1 can be skipped entirely. These algorithms are based on the assumption that the agent knows the number K of actions and, after each trial t, the agent knows the rewards $x_{i_1}(1), \ldots, x_{i_t}(t)$ of the previously chosen actions $i_1, \ldots, i_t$.

3.3.1 Auer *et al.*'s Algorithm Exp3.P.1

Algorithm Exp3.P.1 works as follows:

Initialization

1. Set real values of α, γ, and δ, where $\alpha > 0$, $\gamma \in (0, 1]$, and $\delta \in (0, 1)$.

2. Initialize T_r and δ_r. Determine r^*. For $r = 0, 1, \ldots$, let $T_r = 2^r$, $\delta_r = \frac{\delta}{(r+1)(r+2)}$, and

$$r^* = \min\{r \in \mathcal{N} : \delta_r \geq KT_r e^{-KT_r}\}. \tag{3.2}$$

Repeat For $r^*, r^* + 1, \ldots$, by letting $T = T_r$ and $\delta = \delta_r$

Initialization

1. Set $\gamma = \min\left\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K \ln K}{T}}\right\}$ and $\alpha = 2\sqrt{\ln(KT/\delta)}$.

2. Initialize $w_1(i)$, for $i = 1, \ldots, K$,

$$w_1(i) = \exp\left(\frac{\alpha\gamma}{3}\sqrt{\frac{T}{K}}\right).$$

Repeat For each $t = 1, 2, \ldots, T$

1. For $i = 1, \ldots, K$, set the mixed strategy $\bar{p}_t = \{p_t(1), \ldots, p_t(i), \ldots, p_t(K)\}$ as follows:

$$p_t(i) = (1-\gamma)\frac{w_t(i)}{\sum_{j=1}^{K} w_t(j)} + \frac{\gamma}{K}. \tag{3.3}$$

2. Choose i_t randomly according to the distribution $\{p_t(1), \ldots, p_t(K)\}$.

3. Receive rewards $x_t(i_t) \in [0, 1]$.

4. For $j = 1, \ldots, K$, set

$$\hat{x}_t(j) = \begin{cases} x_t(j)/p_t(j) & \text{if } j = i_t \\ 0 & \text{otherwise,} \end{cases}$$

$$w_{t+1}(j) = w_t(j) \cdot \exp\left(\frac{\gamma}{3K}\left(\hat{x}_t(j) + \frac{\alpha}{p_t(j)\sqrt{KT}}\right)\right). \tag{3.4}$$

In this algorithm, δ denotes the probability of error. If the agent desires to have a small probability of error so that the weak regret lies within the bound presented by Auer *et al.*,[7] the agents will have to suffer the larger bound. This bound is an increasing function of the number of arms (K), implying that the more arms to be tried, the bigger the guaranteed bound. Furthermore, as shown in Equation (3.3), this algorithm yields a mixed strategy or probability distribution $\bar{p}_t$ over possible K arms. This probability distribution is a mixture of a uniform distribution (γ/K) and a function of the weight factor $w_t(i)$ associated with each arm i. As a result, the agent has a chance to explore all K arms even with a small probability. The favorable arm, the arm that yields the large reward, is chosen with an increasing probability, as shown in Equation (3.4).

Furthermore, this algorithm requires only the number of possible actions K and the probability of error δ, while it requires no knowledge of the characteristics of the agent's reward or the time horizon that the algorithm is performed. These two features make this algorithm a suitable one for an agent playing a repeated game in an unknown game set-up, such as in the bidding game. Note also that if the reward $x_i(t)$ is in the range $[a, b]$, $a < b$, then the algorithm can be used after the rewards are translated and rescaled to the range $[0, 1]$.

[7] See the appendix to this chapter for more detail.

Application to Game Theory

The adversarial bandit problem can be easily related to the problem of playing repeated games. For an N-person finite game, let sets $\{S^1, \ldots, S^i, \ldots, S^N\}$ denote pure strategies for each Agent i. Let $u^1, \ldots, u^N$ denote sets of payoffs for each agent, where function $u^i : S^1 \times \ldots \times S^N \to \Re$ denotes agent i's payoff function. Let $S = S^1 \times \ldots \times S^N$ and let $S^{-i} = S^1 \times \ldots \times S^{i-1} \times S^{i+1} \times \ldots \times S^N$. Let $s \in S$ and $s^{-i} \in S^{-i}$. Then, given $s \in S$, let (j, s^{-i}) denote $(s^1, \ldots, s^{i-1}, j, s^{i+1}, \ldots, s^N)$, where $j \in S^i$.

Suppose that the game is played repeatedly over time. Each agent knows all payoff functions and, after each round t, the agent also knows the vector of pure strategies, $s(t) = (s_t^i, \ldots, s_t^N)$, chosen by the agents. The average regret of Agent i for the pure strategy j after T rounds is defined by:

$$R_T^i(j) = \frac{1}{T} \sum_{t=1}^{T} [u^i(j, s_t^{-i}) - u^i(s_t)].$$

A desirable property for an agent is Hannan-consistency, in which Agent i is Hannan-consistent if

$$\lim_{T \to \infty} \sup \max_{j \in S^i} R_T^i(j) = 0, \quad \text{with probability 1.}$$

Next, let us consider the unknown game setup, that happens when the payoffs obtained by the agent belong to a known bounded real interval. Let $x_{i_t}(t)$ be viewed as the payoff $u_i(i_t, s_{-i}(t))$ received by Agent i at round t of the game. This payoff $u^i(i_t, s_t^{-i})$ depends on the possibly randomized choices of all agents which are functions of their realized payoffs. By using this algorithm, the agents can obtain:

Auer et al.'s Theorem 9.1 *If Agent i has $K \geq 2$ pure strategies and plays in the unknown game setup with payoffs in $[0,1]$ using the mixed strategy* **Exp3.P.1***, then*

$$\max_{j \in S^i} R_T^i(j) \leq \frac{10}{\sqrt{2}-1} \sqrt{\frac{2K}{T}\left(\ln \frac{KT}{\delta} + c_T\right)} + \frac{10(1+\log_2(T))}{T}\left(\ln \frac{KT}{\delta} + c_T\right),$$

where $c_T = 2\ln(2 + \log_2 T)$, holds with probability at least $1 - \delta$, for all $0 < \delta < 1$ and for all $T = (K/\delta)^{\Omega(1/K)}$.

From this theorem, the bound of the expected weak regret obtained from this algorithm does not depend on the time horizon of learning and the maximum possible rewards up to that time, but depends on the number of actions (K) and the probability of error (δ). This theorem, along with Auer *et al.'s* Corollary 6.5,[8] implies that in a repeated game of incomplete information when the agent adopts algorithm **Exp3.P.1**, the agent's average regret is Hannan-consistent:

[8] See the appendix to this chapter for more detail.

Auer *et al.'s* **Corollary 9.2** *Agent's strategy* **Exp3.P.1** *is Hannan-consistent in the unknown game setup.*

Two examples to show the implementation of algorithm **Exp3.P.1** for players to play two repeated games are presented next.

Examples

Algorithm **Exp3.P.1** is applied to play two-player Prisoner's Dilemma and Battle-of-the-sexes games. Each player, Player R or C, adopts this algorithm as its strategy. These games are presented in Table 3.1.

Table 3.1: Prisoner's Dilemma and Battle-of-the-sexes

Prisoner's Dilemma

		Player C	
		Cooperate	*Defect*
Player R	*Cooperate*	(0.6, 0.6)	(0.2, 0.8)
	Defect	(0.8, 0.2)	(0.4, 0.4)

Battle-of-the-sexes

		Player C	
		Football	*Ballet*
Player R	*Football*	(0.4, 0.4)	(0,0)
	Ballet	(0, 0)	(0.4,0.4)

From Table 3.1, the non-cooperative Nash equilibrium strategy of Prisoner's Dilemma is that Players R and C choose *Defect.* The simulated average rewards and the mixed strategies ($\bar{p}_t$) of both players using algorithm **Exp3.P.1** with $\delta = 0.1$ are presented in Table 3.2. One can observe that this algorithm yields average rewards close to the Nash equilibrium rewards of the game. Likewise, three non-cooperative Nash equilibrium strategies of Battle-of-the-sexes are that Players R and C choose to go 1) to *Ballet,* 2) to *Football,* and 3) to *Ballet* and *Football* with an equal probability. Furthermore, the simulated average rewards and the mixed strategies ($\bar{p}_t$) of both players using algorithm **Exp3.P.1** with $\delta = 0.1$ are presented in Table 3.2. One can observe that this algorithm yields average rewards close to the mixed-strategy equilibrium rewards of the game.

Table 3.2: Simulated Mixed Strategies and Average Rewards

Prisoner's Dilemma

	Mixed Strategies		Reward
	Cooperate	*Defect*	
Player R	0.3489	0.6511	0.4788
Player C	0.3610	0.6390	0.4795

Battle-of-the-sexes

	Mixed Strategies		Reward
	Football	*Ballet*	
Player R	0.4954	0.5046	0.2017
Player C	0.4954	0.5046	0.2017

3.3.2 Playing Bidding Games Using Algorithm Exp3.P.1

Algorithm **Exp3.P.1** can be modified for the agents to play the bidding games. This learning algorithm lets the agents choose the bidding price and bidding quantity. Let $(*)^b$ denote any variable associated with the bidding price and let $(*)^q$ denote any variable associated with the bidding quantity. This revised algorithm is called Algorithm A3 from here on. Note that the revised Algorithms **Exp3** and **Exp3.P** for the agents to play the bidding game are called Algorithms A1 and A2, respectively. Algorithm A3 follows these steps.

Initialization Agent i has K^b choices of bidding prices, i.e., $\bar{B}^i = \{B^i(1), \ldots, B^i(K^b)\}$ and K^q choices of bidding quantities $\bar{Q}^i = \{Q^i(1), \ldots, Q^i(K^q)\}$. Agent i determines T^b_r, δ^b_r, $r^{b,*}$, T^q_r, δ^q_r, and $r^{q,*}$ using the formula as shown in Section 3.3.1.

Repeat For each day $t = 1, 2, \ldots,$

1. Agent i obtains the scheduled prices and quantity and calculates profits (Π^i_k) from the previous bids, i.e.,

$$\Pi^i_k = P_k \times \sum_j q^{i,j}_k - \sum_j c(q^{i,j}_k),$$

where P_k is the market price at hour k, $q^{i,j}_k$ is the scheduled unit associated with unit j, and $c(q^{i,j}_k)$ is the operating cost of producing $q^{i,j}_k$ of unit j.

2. Agent i determines the vectors of rewards associated with all possible bidding prices, $\bar{x}^b_t = \{x^b_t(1), \ldots, x^b_t(K^b)\}$, and bidding quantities, $\bar{x}^q_t = \{x^q_t(1), \ldots, x^q_t(K^q)\}$, as follows.

   (a) For all $k \in t$, let $\tilde{x}^b_k(m)$ be defined as

$$\tilde{x}^b_k(m) = \begin{cases} \Pi^i_k(i^b_k) & \text{if } m = i^b_k \\ 0 & \text{otherwise,} \end{cases}$$

   where i^b_k denotes the choice of bidding price chosen at hour k of day t and $\Pi^i_k(i^b_k)$ denotes the profit obtained from choosing the bidding price i^b_k.

   (b) Then, for $m \in K^b$, $x^b_t(m)$ is an average of profits associated with action m obtained in day t and is determined as follows:

$$x^b_t(m) = \frac{\sum_k \tilde{x}^b_k(m)}{\tilde{K}},$$

   where $\tilde{K}$ is the total number of auction rounds in each day t that action m is chosen.

   Then, for $n \in K^q$, $x^q_t(n)$ can be determined by using a similar method.

3. Agent i receives forecast demand $\hat{L}_{t+1}$ for the next bidding round.

4. Agent i checks whether $t \in T^b_r$; otherwise, it sets $r^{b,*} = r^{b,*} + 1$, sets $(r = r^{b,*})$, sets $T^b = T^b_r$, and sets $\delta^b = \delta^b_r$.

5. Agent i checks whether $t \in T_r^q$; otherwise, it sets $r^{q,*} = r^{q,*}+1$, sets $(r = r^{q,*})$, sets $T^q = T_r^q$, and sets $\delta^q = \delta_r^q$.

6. Agent i determines its bid for an anticipated marginal unit for hour k based on the load index associated with forecast demand $\hat{L}_k$. The bid consists of two parts: bidding price and bidding quantity. Agent i chooses its bidding price from K^b possible values as follows:

   (a) Agent i determines $\gamma^b = \min\left\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K^b \ln K^b}{T^b}}\right\}$ and $\alpha^b = 2\sqrt{\ln\frac{K^b T^b}{\delta^b}}$.

For $m = 1, \ldots, K^b$

   (b) Agent i calculates $\hat{x}_t^b(m)$ as follows:

$$\hat{x}_t^b(m) = x_t^b(m)/p_t^b(m).$$

   Note that $\hat{x}_t^b(m) = 0$ for action m that is not chosen in day t.

   (c) Agent i updates its weight associated with choice m of K^b possible bid prices, $w_{t+1}^b(m)$, using

$$w_{t+1}^b(m) = w_t^b(m) \cdot \exp\left(\frac{\gamma^b}{3K^b}\left(\hat{x}_t^b(m) + \frac{\alpha^b}{p_t^b(m)\sqrt{K^b T^b}}\right)\right),$$

   and updates its probability of selecting choice m, $p_{t+1}^b(m)$, using

$$p_{t+1}^b(m) = (1-\gamma^b)\frac{w_{t+1}^b(m)}{\sum_{h=1}^{K^b} w_t^b(h)} + \frac{\gamma^b}{K^b}.$$

   (d) Agent i chooses $i_{k\in t+1}^b$ randomly according to the distribution $\{p_{t+1}^b(1), \ldots, p_{t+1}^b(K^b)\}$ and sets

$$BM_k^i = B(i_k^b) \quad \text{for all } k \in t+1$$

   where $(B(\cdot) \in \bar{B}^i)$ is a choice of bidding price.

   Similarly, to determine a bid quantity, Agent i chooses its bidding quantity from K^q possible values as follows:

   (a) Agent i determines $\gamma^q = \min\left\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K^q \ln K^q}{T^q}}\right\}$ and $\alpha^q = 2\sqrt{\ln\frac{K^q T^q}{\delta^q}}$.

   (b) Agent i calculates $\hat{x}_t^q(n)$ as follows:

$$\hat{x}_t^q(n) = \begin{cases} x_t^q(n)/p_t^q(n) & \text{if } n = i_t^q \\ 0 & \text{otherwise.} \end{cases}$$

   (c) Agent i updates its weight associated with choice n of K^q possible bid quantities, $w_{t+1}^q(n)$, using

$$w_{t+1}^q(n) = w_t^q(n) \cdot \exp\left(\frac{\gamma^q}{3K^q}\left(\hat{x}_t^q(n) + \frac{\alpha^q}{p_t^q(n)\sqrt{K^q T^q}}\right)\right),$$

and updates its probability of selecting choice n using

$$p^q_{t+1}(n) = (1-\gamma^q)\frac{w^q_{t+1}(n)}{\sum_{h=1}^{K^q} w^q_{t+1}(h)} + \frac{\gamma^q}{K^q}.$$

(d) Agent i chooses $i^q_{k\in t+1}$ randomly according to the distribution $\{p^q_{t+1}(1),\ldots,p^q_{t+1}(K^q)\}$ and sets

$$q^i_k = Q(i^q_k) \text{ for all } k \in t+1$$

where $(Q(\cdot) \in \bar{Q}^i)$ is a choice of bidding quantity. Let $q_{k,WH}$ denote the withheld capacity and $q^i_{k,WH} = q^i_{max} - q^i_k$.

7. Agent i determines the bid-supply function for each hour k by using BM^i_k and q^i_k as follows.

(a) The bidding price of the withheld capacity (WH_k) is set to

$$WH_k = \min\{BM^i_k + C_2, P_{cap}\}$$

where C_2 is a positive constant and P_{cap} is a price cap, indicating the maximum market price allowed in the market. This bidding price is assigned to the capacity of the units with the lowest marginal costs summed to the withheld capacity.

(b) For any unit j with non-zero capacity that is not considered the withheld capacity, its bidding price b^j_k is set to

$$b^j_k = mc^{i,j}$$

where $mc^{i,j}$ is the marginal cost of unit j.

8. Agent i submits the bid-supply functions for day $t+1$ to the system operator.

9. The system operator clears the market for each hour k and informs the agents of market prices, total demand, and their scheduled quantities.

3.4 Softmax Action Selection Using a Boltzmann Distribution

This section presents another simple learning algorithm for an agent to learn the repeated bidding game. The concept of this algorithm is *softmax action selection*, adopted from reinforcement learning in a single-agent environment, and is explained in detail in Sutton and Barto [41]. This algorithm maintains estimates of the actions, as well as balances exploring new actions and exploiting current knowledge of the value of the actions. A probability distribution over all actions is a function of the rewards associated with the actions. The action with the most satisfactory reward, or the most greedy action, is given the highest selection probability, but all the others are ranked and weighted according to the reward estimates. As a result, this algorithm is improved from ϵ-greedy action selection, in which the action selection rule selects the action with the highest estimated reward and then selects uniformly at random an action that is independent of the reward estimates with small probability ϵ. The drawback of the uniform distribution of the ϵ-greedy algorithm is that the worst-appearing and the next-to-best actions are equally chosen.

This thesis chooses the most common softmax method that uses a Gibbs or Boltzmann distribution. Based on the Boltzmann distribution, action j is chosen on the t-th play with probability ($p_t(j)$) as follows:

$$p_t(j) = \frac{e^{R_t(j)/\tau}}{\sum_{h=1}^{K} e^{R_t(h)/\tau}}$$

where $R_t(j)$ is the value estimate of action j of K possible actions at time t. Let τ be a positive parameter called the temperature. High temperatures cause the actions to be selected nearly equally, while low temperatures cause a greater difference in selection probability for actions that differ in their value estimates. In the limit as $\tau \to 0$, softmax action selection becomes the same as greedy action selection; that is, all actions are selected almost uniformly. In the softmax action selection algorithm, the reward associated with each action is updated using the following formula:

$$R_{t+1}(j) = \begin{cases} (1-\alpha)R_t(j) + \alpha \cdot \Pi_t(j) & \text{if } j = i_t \\ R_t(j) & \text{otherwise.} \end{cases} \tag{3.5}$$

Note that α is a step-size parameter and ($0 < \alpha \leq 1$) is a constant. In addition, the estimate reward for each action j as shown in Equation (3.5) follows an incremental update rule for reinforcement learning in a nonstationary environment, in which a constant step-size parameter, α, is used (see, for example, Sutton and Barto [41]). This rule determines the next estimate reward ($R_{t+1}(j)$) by weighting the recent rewards more heavily than the past ones, and the next estimate reward is a weighted average of the past rewards and the initial estimate $R_0(j)$.[9] The learning algorithm with softmax action selection using a Boltzmann distribution works as follows:

Initialization Set input parameters $\alpha \in (0, 1)$ and $\tau > 0$. Set the reward associated with each action j to be $R_j(t) = c > 0$, for all $j = (1, \ldots, K)$, where c is a constant.

[9]From Sutton and Barto [41],

$$\begin{aligned} R_{t_j+1}(j) &= \alpha \cdot \Pi_{t_j}(j) + (1-\alpha)R_{t_j}(j) \\ &= \alpha \cdot \Pi_{t_j}(j) + (1-\alpha)(\alpha \cdot \Pi_{t_j-1}(j) + (1-\alpha)R_{t_j-1}(j)) \\ &= (1-\alpha)^{t_j} R_0(j) + \sum_{k=1}^{t_j} \alpha \cdot (1-\alpha)^{t_j-k} \Pi_k \end{aligned}$$

where t_j denotes the most recent period when action j is selected. Note that $\sum_{k=1}^{t_j} \alpha \cdot (1-\alpha)^{t_j-k} = 1$.

Repeat For $t = 1, \ldots$

1. Set the mixed strategy $\bar{p}_t = \{p_t(1), \ldots, p_t(m), \ldots, p_t(K)\}$ as follows:

$$p_t(j) = \frac{e^{R_t(j)/\tau}}{\sum_{h=1}^{K} e^{R_t(h)/\tau}}.$$

2. Choose i_t randomly according to the distribution $\{p_t(1), \ldots, p_t(K)\}$.

3. Receive rewards $x_t(i_t)$.

4. For $j = 1, \ldots, K$, set

$$R_t(j) = \begin{cases} (1-\alpha)R_t(j) + \alpha \cdot x_t(j) & \text{if } j = i_t \\ R_t(j) & \text{otherwise.} \end{cases}$$

3.4.1 Application to the Bidding Game

One can observe that this algorithm, like Auer *et al.'s* algorithms, provides a learning tool for an agent that requires no knowledge of its opponents' actions and their associated rewards (payoffs). Consequently, this algorithm can be applied by an agent to play a repeated game in an unknown game set-up. As for the bidding game, the softmax action selection algorithm can be modified as follows. Note that the modified algorithm is called Algorithm SAB from here on. Let K^b be all possible choices of bidding prices and let K^q be all possible choices of bidding quantities.

Initialization Agent i determines its input parameters $\alpha \in (0,1)$ and $\tau > 0$. Set the reward associated with each action m for all $m \in \{1, \ldots, K\}$ to be $R_t(m) = 0$. Agent i has K^b choices of bidding prices, i.e., $\bar{B}^i = \{B^i(1), \ldots, B^i(K^b)\}$ and K^q choices of bidding quantities $\bar{Q}^i = \{Q^i(1), \ldots, Q^i(K^q)\}$.

Repeat For each day $t = 1, 2, \ldots$

1. Agent i obtains the scheduled prices and quantity and calculates profits (Π_k^i) from the previous bids, i.e.,

$$\Pi_k^i = P_k \times \sum_j q_k^{i,j} - \sum_j c(q_k^{i,j}),$$

where P_k is market price at hour k, $q_k^{i,j}$ is scheduled unit associated with unit j, and $c(q_k^{i,j})$ is the operating cost of producing $q_k^{i,j}$ of unit j.

2. Agent i determines the vectors of rewards associated with all possible bidding prices, $\bar{x}_t^b = \{x_t^b(1), \ldots, x_t^b(K^b)\}$, and bidding quantities, $\bar{x}_t^q = \{x_t^q(1), \ldots, x_t^q(K^q), \}$ as follows.

(a) For all $k \in t$, let $\tilde{x}_k^b(m)$ be defined as

$$\tilde{x}_k^b(m) = \begin{cases} \Pi_k^i(i_k^b) & \text{if } m = i_k^b \\ 0 & \text{otherwise,} \end{cases}$$

where i_k^b denotes the choice of bidding price chosen at hour k of day t and $\Pi_k^i(i_k^b)$ denotes the profit obtained from choosing the bidding price i_k^b.

(b) Then, for $m \in K^b$, $x_t^b(m)$ is an average of profits associated with action m obtained in day t and is determined as follows:

$$x_t^b(m) = \frac{\sum_k \tilde{x}_k^b(m)}{K}$$

where K is the total number of auction rounds in each day t that action m is chosen. Let $I^b(m)$ be boolean, in which it is equal to 1 when $x_t^b(m) > 0$, and equal to 0 otherwise.

Then, for $n \in K^q$, $x_t^q(n)$ and $I^q(n)$ can be determined by using a similar method.

3. Agent i receives forecast demand $\hat{L}_{t+1}$ for the next bidding round.

4. Agent i determines its bid for an anticipated marginal unit for hour k based on the load index associated with forecast demand $\hat{L}_k$. The bid consists of two parts: bidding price and bidding quantity. To determine a bid price, Agent i follows these steps.

(a) Agent i determines $R_t^b(m)$ as follows:

$$R_{t+1}^b(m) = \begin{cases} (1-\alpha)R_t^b(m) + \alpha \cdot x_t^b(m) & \text{if } I^b(m) = 1 \\ R_t^b(m) & \text{otherwise.} \end{cases}$$

(b) Agent i updates its probability of choosing choice m using

$$p_t^b(m) = \frac{e^{R_t^b(m)/\tau}}{\sum_{h=1}^{K^b} e^{R_t^b(h)/\tau}}.$$

(c) Agent i chooses $i_{k \in t+1}^b$ randomly according to the distribution $p_{t+1}^b(1), \ldots, p_{t+1}^b(K^b)$ and sets

$$BM_k^i = B(i_k^b) \text{ for all } k \in t+1,$$

where $(B(\cdot) \in \bar{B}^i)$ is a choice of bidding price.

Similarly, to determine a bid quantity, Agent i follows these steps.

(a) Agent i determines $R_t^b(n)$ as follows:

$$R_{t+1}^q(n) = \begin{cases} (1-\alpha)R_t^q(n) + \alpha \cdot x_t^q(n) & \text{if } I^q(n) = 1 \\ R_t^q(n) & \text{otherwise.} \end{cases}$$

(b) Agent i updates its probability of choosing choice n using

$$p_t^q(n) = \frac{e^{R_t^q(n)/\tau}}{\sum_{j=h}^{K^q} e^{R_t^q(h)/\tau}}.$$

(c) Agent i chooses $i^q_{k\in t+1}$ randomly according to the distribution $\{p_{t+1}^q(1), \ldots, p_{t+1}^q(K^q)\}$ and sets

$$q_k^i = Q(i_k^q) \text{ for all } k \in t+1,$$

where $(Q(\cdot) \in \bar{Q}^i)$ is a choice of bidding quantity. Let $q^i_{k,WH}$ denote the withheld capacity and $q^i_{k,WH} = q^i_{max} - q^i_k$.

5. Agent i determines the bid-supply function for each hour k by using BM_k^i and q_k^i as follows:

(a) The bidding price of the withheld capacity is set to

$$WH_k = \min\{BM_k^i + C_2, P_{cap}\}$$

where C_2 is a positive constant and P_{cap} is a price cap.

(b) For any unit j with non-zero capacity that is not considered the withheld capacity, its bidding price b_k^j is set to

$$b_k^j = mc^{i,j}$$

where $mc^{i,j}$ is the marginal cost of unit j.

6. Agent i submits the bid-supply functions for day $t+1$ to the system operator.

7. The system operator clears the market for each hour k and informs the agents of market prices, total demand, and their scheduled quantities.

3.5 An Algorithm Based on Electricity Model Characteristics

This section presents a model-based learning algorithm designed for the agent-based electricity market model. This learning algorithm is based on the game theoretical concept to determine a "rational" action of an agent in response to anticipated actions of opponents. The agent chooses a pure strategy of the possible bid-supply functions to do better than its marginal-cost bid, especially when the agent anticipates being scheduled to operate as a marginal agent. The agent always bases its decision on a strategy that directs the agent to "cooperate" in raising its bidding price rather than to "undercut" the bidding prices of its opponents. This strategy unilaterally yields profit at least equal to a marginal-cost bid.

According to this algorithm an agent follows a two-step decision-making process. First, the agent determines its bidding capacity by applying a capacity withholding strategy. Then, the agent determines its bidding price for the anticipated marginal unit. The capacity withholding strategy is motivated also by the evidence from several previous empirical studies, indicating that the capacity withholding strategy is exercised by the market participants in the existing electricity markets, for instance, Wolak [49] and Wolfram [50]. In this model, the agent stores the finite data, such as its bids, its bidding outcomes, market prices, total forecast and actual demand, and observed opponents' joint actions, based on the load indices. The approach is most in line with the finite-history window approach, such as in McCallum [33].

Although this algorithm does not require knowledge of the opponents' actions and their associated rewards, it has a few disadvantages because the agents always select their pure-strategy actions. First, if the repeated bidding game has a mixed-strategy equilibrium, the agents will be unable to reach this outcome. Next, the agents might not get to explore all possible actionsbecause when all agents adopt this algorithm, they could possibly "reach" the equilibrium without ever trying out the actions that have never been played. This algorithm is called the model-based algorithm from here on, and it is described below.

3.5.1 Capacity Withholding Strategy

The capacity withholding (CW) strategy is motivated by the potential of an agent to unilaterally influence the market price in the three-agent bidding game shown in the previous chapter. One can observe that the CW strategy yields a minmax-strategy outcome. Its rewards are greater than the rewards when all agents submit their marginal-cost bids. An agent implements this strategy for its own profits regardless of how other agents would play. In this strategy, an agent determines its "optimal" withheld capacity by assuming that the opponents submit their marginal-cost bids (the system marginal-cost function subtracted by its marginal-cost function). When the demand is deterministic, the optimization becomes

$$W_k^{i,*} = \arg\max_{W_k^i} \left(\sum_{j=1}^{N^i} \hat{P}_k^i(W_k^{i,j}) \cdot (q_k^{i,j} - W_k^{i,j}) - c_k^{i,j}(q_k^{i,j} - W_k^{i,j}) \right).$$

The characterization of the CW strategy exploits the characteristics of the agents' piece-wise marginal-cost functions and also piece-wise system marginal-cost function. The agent searches for the minimum capacity that should be withheld so that the market price increases, resulting in profits that are greater than those obtained from a marginal-cost bid.

The concept of the minmax value of the game is applied to constructing a bidding strategy of the agents in the bidding games, and consequently, the agent employs the CW strategy when it is implementable. When the CW strategy is implemented successfully, the agent is able to "manipulate" market price. Note that when the competitors are also able to change the prices by imposing the CW strategy, the agent is better off not withholding its capacity. However, to guarantee a long-term increase in market price, the agent always chooses to withhold its capacity whenever possible to guarantee its minmax payoff in each game.

3.5.2 The Model-based Algorithm

The model-based algorithm is used in order to determine the bid daily, and the agents make use of the following information: historic market prices, past bidding prices of the (anticipated) marginal unit, past bidding prices of the units, scheduled outcomes, analyzed outcomes (these variables are discussed later in this chapter), system marginal-cost function, forecast demand, profits, anticipated profits, and marginal-cost function. Let AP denote anticipated profits (calculated from the previous bidding round), OP denote actual profits obtained from the previous bidding round, MP denote market price, BM denote the bidding price of an anticipated marginal unit, and O denote the analyzed outcome. The model-based algorithm works as follows:

Initialization Agent i submits its marginal-cost bid-supply functions to an operator. The operator schedules the agents and informs market prices, total demand, and scheduled quantities.

Repeat For each day $t \geq 1$, Agent i follows the scheme below. This scheme is called the *PORTFOLIO* scheme.

1. Agent i obtains the scheduled prices and quantity and calculates profits (Π_k^i) from the previous bids, i.e.,

$$\Pi_k^i = P_k \times \sum_j q_k^{i,j} - \sum_j c(q_k^{i,j}),$$

   where P_k is market price at hour k, $q_k^{i,j}$ is scheduled unit associated with unit j, and $c(q_k^{i,j})$ is operating cost of producing $q_k^{i,j}$ of unit j.

2. For each hour k, Agent i determines the bidding outcome (O) of its portfolio using the following scheme called the *OUTCOME* scheme:

   (a) $OP < AP$: This implies that the previous bid is not successful. Consider BM and MP.
   1) $BM \leq MP$: This means the agent under-estimates the BM; the other agents increase their BMs (from the previous period); or the agent overestimates the market prices so that

the agent would be scheduled to operate less than anticipated. Note that it is not possible to have $BM < MP$ and $OP < AP$. But $BM = MP$ is possible. (For example, when the agent anticipates being scheduled to operate more than it is actually scheduled.) Agent i then sets $O = 11$.

2) $BM > MP$: This implies that the agent over-estimates the market prices. For this reason, to increase the scheduled quantity and subsequently profits in the next bidding round, Agent i then sets $O = 10$. If, however, $AP = 0$, Agent i then sets $O = 00$.[10]

(b) $OP = AP$: This implies that the previous bid is successful. Consider BM and MP.

1) $BM < MP$: This implies that the agent underestimates its BM or the other agents increase their BMs from their previous values. Agent i then sets $O = 11$ when $OP = 0$, otherwise, Agent i then sets $O = 00$.[11]

2) $BM = MP$: This implies the agent is able to set the market price, or the agent is likely to set the high MP the next period, because the agent is a marginal agent at the current period. Therefore, the agent should increase its BM the next period. Agent i then sets $O = 11$.

3) $BM > MP$: This implies that the agent overestimates the market prices. It is unlikely to have $OP = AP$ when $BM > MP$; however, to complete the algorithm, Agent i then sets $O = 00$.

(c) $OP > AP$: This implies that the previous bid is overly successful, the opponents set the market prices, or the agent is scheduled to operate as a marginal agent and its scheduled quantity is more than the anticipated one. Since the outcome is satisfying, the agent does not change its bidding price the next period. Agent i then sets its $O = 00$. That $AP < OP < 0$ implies that its BM is too low and the agent operates at loss. Agent i then sets its $O = 11$.

Agent i updates its O and MP.

3. Agent i determines the bidding quantity through the CW strategy.

4. Agent i assesses whether each individual unit obtains its profit as anticipated. This scheme is trivial in the market with a uniform-pricing rule. The scheduled unit gets paid the market price; therefore, the outcome of each unit depends on BM. Also, the agent uses *OUTCOME* to determine the bidding outcome of each unit (O_u).

5. Agent i determines the load indices associated with each hourly forecast demand. Let BM for each hour of the next bidding round be calculated by the following scheme, called the *SETPRICE*

[10] To reduce BM means to submit a lower-than-marginal-cost bid.

[11] Note that when the agent receives $OP > 0$, it is better off not to adjust its bid because the increased BM might result in not being scheduled. The increased BM might allow the other agents with lower bidding prices to be scheduled.

scheme:

$$BM_k = Tar_k + \bar{c}_k$$

where Tar_k is the target price and $\bar{c}$ is a constant. There are several ways to determine Tar_k. These methods directly affect the bids of the agent and subsequently affect the price dynamics of the market. For example, when the length of the recorded memory is set to 1 ($Md = 1$), Tar_k can be set to

$$\text{Method M1: } Tar_k = BM_{k-1},$$

or

$$\text{Method M2: } Tar_k = MP_{k-1}.$$

In addition,

$$\bar{c} = \Delta, \text{ if } O = 11; \; \bar{c} = 0, \text{ if } O = 00; \text{ and } \bar{c} = -\Delta, \text{ if } O = 10,$$

where Δ is a positive constant. Note that BM_k, Tar_k and $\bar{c}_k$ are associated with the load indices.

6. Agent i determines the bidding prices of each unit (*BU*) from its O_u using the *SETPRICE* scheme.

7. Agent i determine the bid-supply function by using b^i_{t+1} and q^i_{t+1} as follows:

   (a) For unit j with BU^j less than or equal to BM, its bidding price b^j is set to

   $$b^j = \max\{mc^j, \min\{BU^j, BM\}\}$$

   where mc^j is the marginal cost of unit j. Note that if the $BM < mc^j$, BM is set to be mc^j.

   (b) For unit j with BU^j greater than BM, its bidding price b^j is set to

   $$b^j = \max\{mc^j, BU^j\}.$$

   (c) For the withheld capacity, the bidding price of the withheld capacity (WH) can be either

   $$\text{Method C1: } WH = \min\{c_1, P_{cap}\}$$

   where c_1 is a constant and $c_1 \gg \max_j mc^j$, or

   $$\text{Method C2: } WH = \min\{BM + c_2, P_{cap}\}$$

where c_2 is a positive constant and P_{cap} is a price cap.

The agent updates its recorded BM and BU of each unit.

8. Agent i calculates its AP. The anticipated profit is determined by assuming that BM is the market price. The bidding blocks with bidding prices of at least BM get scheduled and get paid at BM (for the market with the uniform-pricing rule). Similarly, the anticipated profit of each block is calculated as well (to be used in determining O_u). Then the agent records its new AP.

9. Agent i submits the bid-supply functions for day $t+1$ to the system operator.

10. The system operator clears the market for each hour k and informs the agents of market prices, total demand, and their scheduled quantities.

3.5.3 Algorithms with a Game Matrix

The previous algorithm can be modified by adding a memory, so that $Md > 1$. This change tends to make the decision scheme more conservative, meaning that the agents is less likely to raise the bidding price. This modified algorithm directs an agent to a three-step decision-making process. The first and last steps are similar to the previous algorithm in which the agent determines its bidding capacity by applying a CW strategy and then determines its bidding price for the anticipated marginal unit. The second step is added in order to have the agent estimate the potential joint behavior of the opponents from the available information. This algorithm works as follows:

Analysis of Opponents' Joint Actions

The agent can observe opponents' joint actions. Because the agent is assumed to know the aggregate supply function, it can differentiate strategic market prices from marginal-cost or "competitive" prices, $\hat{P}_k^{mc}$. When the agent knows demand in each hour with certainty, it is able to calculate the hourly marginal-cost price. Similarly, when the actual demand in any hour is realized, marginal-cost price of that hour can also be determined. By observing market price and comparing it with the anticipated marginal-cost price, the agent can considers three possibilities:

- A positive markup means a joint *strategic action* of the agents; that is, at least one agent successfully submits the bid-supply function to cause an increase in the market price. Note that if outages, maintenance, and unit-commitment constraints are accounted for, the positive markup may result from competitive behavior.

- A negative markup indicates that some agents do not submit a marginal-cost bid; instead they underbid so that they are scheduled to operate with more confidence, though, this condition does not exist in the model-based algorithm since the agents always submit as the bidding price at least their marginal cost.

- A zero markup means that the market price is actually competitive.

After the market clears, Agent i compares market prices for each hour with anticipated competitive prices and records the joint strategic behavior of the agents in a matrix. For each action in which each element contains either "0" or "1," let "0" represent competitive joint actions of the opponents or a no-game condition. Let "1" represent a joint strategic action or a game-condition. This matrix is called a GM matrix. One would expect a GM matrix to be sparse if the agents are marginal-cost bidders. An example of a GM matrix is shown below:

$$\begin{array}{c} \mathcal{L}^1 \rightarrow \\ \vdots \\ \mathcal{L}^d \rightarrow \\ \vdots \\ \mathcal{L}^{N^d} \rightarrow \end{array} \begin{bmatrix} 1 & 0 & \ldots & 1 \\ \vdots & & & \\ 0 & 0 & \ldots & 0 \\ \vdots & & & \\ 0 & 1 & \ldots & 1 \end{bmatrix} \Rightarrow \text{GM}^i = \begin{bmatrix} 1 & 0 & \ldots & 1 \\ \vdots & & & \\ 0 & 0 & \ldots & 0 \\ \vdots & & & \\ 0 & 1 & \ldots & 1 \end{bmatrix}.$$

For each hour, the agent obtains this information (0 or 1) and stores it in the row associated with the demand index of actual demand of that hour. When uncertainties due to outages, maintenance, and unit-commitment constraints are considered, the non-zero price markup does not generally imply the strategic behavior, but, when these factors are not accounted for, the non-zero price markup implies the strategic behavior. Let γ^i denote an error factor. The game condition is determined using the following scheme, which is called the *GAME* scheme. For given actual demand equal to $L_k \equiv \mathcal{L}^d$,

1. Determine $G_k^i(\mathcal{L}^d)$:

$$\begin{aligned} |\, P_k(\mathcal{L}^d) - \hat{P}_k^{i,mc}(\mathcal{L}^d)\,| &\leq \gamma^i, & G_k^i(\mathcal{L}^d) &= 0 \\ |\, P_k(\mathcal{L}^d) - \hat{P}_k^{i,mc}(\mathcal{L}^d)\,| &> \gamma^i, & G_k^i(\mathcal{L}^d) &= 1 \end{aligned}$$

   where $G_k^i(\mathcal{L}^d)$ denotes the game condition for hour k of load index $\mathcal{L}^d$. When a positive markup does not necessarily imply a strategic behavior, for all i, $\gamma^i > 0$, otherwise $\gamma^i = 0$. The information in any memory matrix is recorded in order of occurrence.

2. For any demand $L_k \equiv \mathcal{L}^d$, $d \in N^d$, where N^d is the number of demand indices, the GM matrix is updated as follows:

$$\left.\begin{aligned} GM_l^i(\mathcal{L}^h) &= GM_{l+1}^i(\mathcal{L}^h) \\ GM_{Md}^i(\mathcal{L}^h) &= G_k^i(\mathcal{L}^h) \end{aligned}\right\} \quad \forall h = d,\, 1 \leq l \leq Md - 1,$$

$$GM_l^i(\mathcal{L}^h) = GM_l^i(\mathcal{L}^h) \qquad \forall h \neq d,\, 1 \leq l \leq Md$$

   where $GM_l(\mathcal{L}^h)$ is an element in row h and column l of the GM matrix. Each row h is associated with load index h.

Updating Memory Matrices

After each bidding round, memory matrices are updated and in this thesis the new information ($x(h)$) is recorded at most once for each load index. That is, for a memory matrix M, which has its element, $M^{h,l}$, in row h and column l, is updated as follows:

$$\begin{aligned} M^{h,l} &= M^{h,l+1}, \quad 1 \leq l < Md \\ M^{h,l} &= x(h), \qquad l = Md \end{aligned}$$

The agents record $x(h)$ to represent information, such as market prices, associated with load index $\mathcal{L}^h$ occuring during that day. For example,

$$x(h) = \frac{\sum_k \tilde{x}_k(h)}{K}$$

where $\tilde{x}(h)$ is market price (MP) and K is the total number of hours in each day that have load index $\mathcal{L}^h$. Also,

$$x(h) = \max_{k \in K} \tilde{x}_k(h).$$

Similarly, the agents can also record the bidding price of their anticipated marginal units (BM) in a similar fashion as they record MP.

The Model-based Algorithm with a Game Matrix

When each agent has the portfolio of units with different minimum operating capacity constraints, this matrix becomes unique to that agent. This is because the agents may view the system marginal-cost differently. A GM matrix enters the *PORTFOLIO* scheme as follows:

1. Agent i obtains market prices and quantity and calculates profits (Π_k^i) from the previous bids.

2. Agent i determines O by using the *OUTCOME* scheme. Agent i updates its recorded O and MP.

3. Agent i determines the bidding quantity through the CW strategy.

4. Agent i assesses whether each individual unit obtains its profit as anticipated by using the *OUTCOME* scheme to determine O_u.

5. Agent i assesses the joint actions of the opponents by using the *GAME* scheme to determine GM.

6. Agent i determines the load indices associated with each hourly forecast demand ($\mathcal{L}^h$). Let BM for each hour of the next bidding round be calculated by the following scheme, which is called

the *SETPRICE-GAME* scheme:

$$\begin{aligned} BM_k &= Tar_k + \bar{c}_k && \text{if } \sum GM^i(\mathcal{L}^h) \geq \max\{1, Md/2\} \\ BM_k &= mc(\mathcal{L}^h) && \text{otherwise,} \end{aligned}$$

where Tar_k is the target price and $\bar{c}_k$ is a constant. Let Tar_k be set by either Method M1 or M2 and $\bar{c}$ be selected using the same method as in the *SETPRICE* scheme. Let $mc(\mathcal{L}^h)$ denote the marginal-cost price when demand is equal to $L \equiv \mathcal{L}^h$.

7. Agent i determines the bidding prices of each unit (BU) from its O_u by using the *SETPRICE* scheme.

8. Agent i sets the bidding price for each block of the bidding quantity as in Section 3.5.2. Agent i updates its recorded BM and BU of each unit.

9. Agent i calculates its AP.

10. Agent i submits the bid-supply functions for day $t+1$ to the system operator.

11. The system operator clears the market for each hour k and informs the agents of market prices, total demand, and their scheduled quantities.

Note that the withheld capacity (q_{WH}) obtained from the capacity withholding strategy is not affected by the *SETPRICE-GAME* scheme.

3.6 Conclusion

An immediate problem with stage-games of incomplete and imperfect information is that determining a Nash equilibrium strategy is no longer applicable because the agents have neither their own entire payoff functions nor their opponents' entire payoff functions. The concept of on-line learning to determine the agent's actions or bid-supply functions is implemented. Three learning algorithms in multi-agent systems are selected.

The output of Algorithms A1, A2, A3, and SAB is a mixed strategy distribution over the actions which are sets of price-quantity pairs, whereas the model-based algorithm yields a pure-strategy action. This algorithm lets the agent choose a bid function such that its anticipated profit is maximized given its belief about the others' actions and/or assuming that the other agents' behavior is based on a strategy. The simulations of the agent-based market model are presented in the next chapter.

Appendix to Chapter 3

A. Preliminary Empirical Study of the New England Electricity Market

I. Demand Levels and Price Characteristics

This appendix provides an empirical study, analyzing the New England electricity spot market during the period May 1999 to October 1999. From available data of hourly prices and demand, the histograms of (ex post) prices under several load conditions are presented. This analysis shows the different characteristics of market prices under different demand conditions to confirm the importance of deriving the load-based decision scheme of the agent in the proposed agent-based electricity spot market model.

Instead of the probability density functions (PDFs) of price given a load level, the probability mass functions (PMFs) of finite prices given a range of demand is derived from historic data of market prices and demand. This is because actual demand and market prices are continuous values. However, the observed data are limited (only 24 data points per day) and deriving a PDF of prices for each load is not possible. Instead, therefore, the PMFs or histograms of prices given a range of demand are determined. The procedures for constructing the PMFs of ex post hourly market prices given a range of demand are as follows:

1. *Recording the historic hourly market prices and actual demand.* This information is obtained from the ISO-NE website [53].

2. *Determining load index.* As mentioned, actual hourly load takes on a continuous value. To simplify and obtain sufficient data points to represent the PDF, actual demand is discretized into several ranges or indices, covering the maximum and minimum demand. By doing so, continuous demand is mapped to a set of discrete indices. The more indices used, the greater the accuracy of the mapping of actual load.

3. *Mapping ex post prices to load indices.* Historic hourly demand is mapped to a load index, and its associated market price is recorded based on the load index.

4. *Representing the probability of (ex post) price distribution given a load index.* For each index, a histogram of prices is plotted and normalized so that it can be used as a PMF given a load index. Note that to obtain a relatively smooth distribution function, sufficient data points are required.

Figures 3-4 and 3-5 show the histograms of market prices from May 1, 1999 to April 30, 2002 with demand in a 10,000-11,000 MW range and a 15,000 - 16,000 MW range, respectively. These histograms indicate the different characteristics of market prices under different load ranges. When demand is in a 15,000 - 16,000 MW range, market prices exhibit a larger variance than prices when demand is in a 10,000 - 11,000 MW range.

II. Observed Absolute Market Power Conditions

This analysis is performed on the New England electricity spot market during the months of May to October 1999[12] to show that when the supply margin is small, there is higher likelihood of price spikes where the market prices are substantially higher than (calculated) marginal-cost prices. The procedures for identifying absolute market power conditions are described as follows:

1. *Recording the hourly price and actual and forecast demand.*
2. *Calculating total available capacity for each day.* Total available capacity for day d (Q_d) is equal to total net claimed capacity (or installed capacity (Q_d^{max})) plus net imported power (Q_d^{Im}) from the neighboring areas such as the New York power pool[13] and Canada minus the planned maintenance capacity (Q_d^{Maint}), i.e.,

$$Q_d = Q_d^{max} + Q_d^{Im} - Q_d^{Maint}.$$

3. *Determining the marginal cost of operating each unit.*
4. *Calculating the hourly demand-to-supply index.* This index indicates the hourly ratio of forecast demand and total available capacity.
5. *Observing the relationship between hourly market clearing price and hourly demand-to-supply index.* Note that one advantage of the index is that it incorporates both demand and supply factors.

During the period of this study, no marginal-cost data for each unit is available. Therefore, to follow the third step, the average operating costs of each technology type are used. These data, as shown in Table 3.3, are obtained from the Department of Energy website [54]. Further, the demand-supply ratios are calculated from the forecasted demand and available generation capacity, obtained by subtracting the summer net claimed capacity (Table 3.4) from the generation scheduled for maintenance (Table 3.5) and the interchange. In this thesis, it is assumed that the interchange during the entire months of May, June, and July is equal to 2,400 MW. The best publicly available information is based on the (assumed) average of interchange equal to 2,400 MW (around 60% of (supposed) maximum transfer limits: New York Power Pool = 1,100 MW, New Brunswick = 700 MW, and Hydro Quebec = 2,200 MW).[14]

[12]This is the period of the first five months after the market started and prior to when the actual bid data were published.

[13]See http://www.nyiso.com. for more detail on the New York power pool.

[14]This analysis reflects information available to public during the specified period.

Table 3.3: Average Operating Expenses for Major Investor-owned Electric Utilities, 1993 - 1997 (Mills per kWh)

	Years				
Plant Type	1993	1994	1995	1996	1997
Nuclear	21.80	20.86	20.39	20.65	24.80
Fossil Steam	22.97	21.80	21.11	21.25	21.34
Hydroelectric	6.47	7.43	5.89	5.96	5.73
Gas Turbine and Small Scale	40.38	32.16	28.67	40.64	32.84

The types of generation technology in the New England market during the period of study are shown in Table 3.4. Note that unit categories include CC-Combined Cycle, D-Diesel, F-Fossil, G-Gas, GF-Gas or Oil, HD-Daily Hydro (Normally No Pondage), HW-Weekly Hydro (Pondage), J-Jet Engine, N-Nuclear, and PS-Pumped Storage.

Table 3.4: Summer Season Net Claimed Capacity during July 1999

Plant Types	HD	HW	PS	F	N	CC	D	G	GF	J
Capacity (MW)	652	882	1,685	9,039	4,343	2,542	106	684	2,525	837
% of Total Capacity	2.8	3.8	7.2	38.8	18.6	10.9	0.5	2.9	10.8	3.6

Table 3.5: Samples of Scheduled Maintenance during 1999

Dates	5/01-07	5/08-14	5/15-21	5/22-28	5/29-6/04	6/05-11	6/12-18
Cap. (MW)	5,100	5,700	4,800	4,300	3,300	3,400	3,600
Dates	6/19-25	6/26-7/02	7/03-09	7/10-16	7/17-23	7/24-30	7/31-8/06
Cap.(MW)	2,600	2,300	1,400	800	800	500	0

Figure 3-6 shows the relationship between the market clearing prices and the actual demand-supply and forecast demand-supply ratios, which are obtained from public data for the period from May 1, 1999 to October 31, 1999. The scatter plot in this figure shows that when the demand-supply ratio is not less than 0.8 and not greater than 1, the observed prices vary substantially and take on expensive values.

B. Regret and Hannan-consistency

Regret defines the difference between the payoffs of playing two actions; that is, the regret of action $a(h)$ defines the difference between playing action $a(j)$ instead of any action h (see Foster and Vohra [14], and Hart and Mas-Collel [21]). Let $R(S)$ denote the expected loss from using an algorithm (or a strategy) S over T periods, and be defined as

$$R(S) = \sum_{t=1}^{T} \sum_{a(j)\in\mathcal{A}} w_t(j) R_t(j),$$

where $\{w_t\}_{t\geq 0}$ is the probability weight implied by the algorithm, let $\mathcal{A}$ denote the action space, and let $R_t(j)$ represent the loss incurred from choosing action $a(j)$ at time t. Let $R_T(j|S)$ define the regret incurred by S from choosing decision $a(j)$ to be

$$R_T(j|S) = \sum_{i \in D} \max \left\{ 0, \left(\sum_{t=1}^{T} w_t(j)(R_t(j) - R_t(i)) \right) \right\}.$$

A learning algorithm should yield the payoff for an agent over a long period of play such that its average is as large as the maximum payoff that can be obtained against the empirical distribution of plays by the other agents. This condition is called *Hannan-consistent.* A learning algorithm (strategy) is Hannan-consistent if, given the play of the others, there is no regret in the long run for not having played (constantly) any particular action.

C. A Myopic Play

A myopic play can be defined as follows. Let h_t be a collection of pure strategy profile (s_t), i.e., $h_t = (s_1, \ldots, s_t)$. An assessment rule of Agent i (μ^i) is Agent i assessment of the possible pure-strategy profiles that its opponents will choose at time t, as a function of the past plays h_t. A behavior rule (ϕ^i) is a function of the past plays h_t. The myopic play is defined by, for example, Fudenburg and Levine [18], as follows:

Definition Given an assessment rule $\mu^i = (\mu_1^i, \mu_2^i, \ldots)$ for Agent i, the behavior rule, $\phi^i = (\phi_1^i, \phi_2^i, \ldots)$ for i is myopic relative to μ^i if, for every t and h_t, $\phi_t^i(h_t)$ maximizes i's immediate expected payoff, given assessment $\mu_t^i(h_t)$. That is, $a^i(\phi_t^i(h_t), \mu_t^i(h_t)) = \max_{s^i \in S^i} a^i(s^i, \mu_t^i(h_t))$.

D. Auer *et al.'s* Learning Algorithms

I. Multi-armed Bandit Model

Let K denote the number of possible actions and i denote each action taken by an agent, in which $i \in \{1, \ldots, K\}$. An infinite sequence $\mathbf{x}(1), \mathbf{x}(2), \ldots$ of vectors $\mathbf{x}_t = (x_t(1), \ldots, x_t(K))$ denotes an assignment of rewards where $x_t(i) \in [0, 1]$ denotes the reward obtained if action i is chosen at time t or trial t. The agent's algorithm, therefore, is a sequence $I_1, I_2, \ldots$, where each I_t is a mapping from the set $(\{1, \ldots, K\} \times [0 \times 1])^{t-1}$; that is, the action indices and previous rewards to the set of action indices. Let $G_T(A) \stackrel{def}{=} \sum_{t=1}^{T} x_t(i_t)$ denote the return at time horizon T ($T > 0$) of algorithm A choosing actions $i_1, i_2, \ldots$. Given any time horizon T and any sequence of actions $(j_1, \ldots, j_T)$, the (worst-case) regret of algorithm A for $(j_1, \ldots, j_T)$ is defined as $G_{(j_1,\ldots,j_T)} - G_T(A)$, where $(G_{(j_1,\ldots,j_T)} \stackrel{def}{=} \sum_{t=1}^{T} x_t(j_t))$ is the return at time T, obtained by choosing actions $(j_1, \ldots, j_T)$. Therefore, the weak regret is defined by

$$G_{T,max} - G_T(A)$$

where $G_{T,max} \stackrel{def}{=} \max_j \sum_{t=1}^{T} x_t(j_t)$.

Fixing an algorithm defines a probability distribution over the set of all sequences of actions. Let $\mathbf{P}\{\cdot\}$ and $\mathbf{E}[\cdot]$ denote the probabilities and the expectations with respect to this distribution.

Assumptions Auer *et al.'s* algorithms are based on the assumptions that the agent knows the number K of actions. In addition, after each trial t, the agent knows the rewards $x_1(i_1), \ldots, x_t(i_t)$ of the previously chosen actions $i_1, \ldots, i_t$.

Two bounds on the performance of the algorithms are considered. The first bound is on the expected regret, i.e., $G_{(j_1,\ldots,j_T)} - \mathbf{E}[G_A(T)]$ of A for an arbitrary sequence $(j_1, \ldots, j_T)$ of actions. The second bound is a confidence bound on the weak regret, having the form $\mathbf{P}\{G_{max}(T) > G_A(T)+\epsilon\} \leq \delta$. That is, the return of A up to time T is not much smaller than that of the globally best action. Next, let us consider Auer *et al.'s* algorithm **Exp3**.

II. Auer *et al.'s* Algorithm Exp3

Auer *et al.'s* Algorithm **Exp3** can be described by the following pseudo-codes:

Parameters: Select $\gamma \in (0, 1]$.

Initialization: Set $w_1(i) = 1$ for $i = 1, \ldots, K$.

Repeat: For each $t = 1, 2, \ldots, T$

1. Set
$$p_t(i) = (1-\gamma)\frac{w_t(i)}{\sum_{j=1}^{K} w_t(j)} + \frac{\gamma}{K} \quad i = 1, \ldots, K.$$

2. Draw i_t randomly accordingly to the probabilities $p_t(1), \ldots, p_t(K)$.

3. Receive reward $x_t(i_t) \in [0, 1]$.

4. For $j = 1, \ldots, K$ set
$$\begin{aligned} \hat{x}_t(j) &= \begin{cases} x_t(j)/p_t(j) & \text{if } j = i_t \\ 0 & \text{otherwise,} \end{cases} \\ w_j(t+1) &= w_t(j)\exp(\gamma \hat{x}_t(j)/K). \end{aligned}$$

Algorithm **Exp3** draws an action i_t according to the distribution $p_t(1), \ldots, p_t(K)$. This distribution is a mixture of the uniform distribution (γ/K) and a distribution which assigns to each action a probability mass exponential in the estimated cumulative reward for that action. Uniform distribution is added in to guarantee that the algorithm tries out all K actions and gets good estimates of the rewards for each. To compensate the reward of actions that are unlikely to be chosen, the estimated reward $\hat{x}_{i_t}(t)$ is set to x_{i_t}/p_{i_t}, yielding $\mathbf{E}[\hat{x}_t(j)|i_1,\ldots,i_{t-1}] = \mathbf{E}[p_t(j)\cdot\frac{x_t(j)}{p_t(j)} + (1-p_t(j))\cdot 0] = x_t(j)$. This algorithm yields the main results, including

Auer *et al.'s* **Theorem 3.1** *For any $K>0$ and for any $\gamma \in (0,1]$,*

$$G_{max} - \mathbf{E}[G_{\mathbf{Exp3}}] \leq (e-1)\gamma G_{max} + \frac{K\ln K}{\gamma}$$

holds for any assignment of rewards and for any $T>0$.

Note that **Exp3** yields an expected regret of $O(\sqrt{gK\ln K})$ whenever an upper bound g on the return G_{max} is known in advance. If the time horizon T is known, g can be set to T, since there is no payoff greater than 1. As $G_{max} = G_{max}(T) \leq T$, the bound is never worse than $O(\sqrt{TK\ln K})$. Note also that if the reward $x_t(i)$ is in the range $[a,b]$, $a<b$, then the algorithm can be used after the rewards are translated and rescaled to the range $[0,1]$.

This algorithm can be modified to yield expected weak regret to be $O(\sqrt{G_{max}K\ln K})$ uniformly over T. The modified algorithm, called algorithm **Exp3.1**, proceeds in epochs. Let $r=0,1,2,\ldots$ denote the indices of the epochs. On each epoch r, the algorithm guesses a bound g_r, i.e., $g_r = \frac{K\ln K}{e-1}\cdot 4^r$, and determines γ_r, where $\gamma_r = \min\left\{1, \sqrt{\frac{K\ln K}{(e-1)g_r}}\right\}$ before restarting **Exp3** at the beginning of each epoch. After finishing **Exp3** in each round, an estimate of the return of each action i, $\hat{G}_{t+1}(i)$ is updated as $\hat{G}_{t+1}(i) = \hat{G}_t(i) + \hat{x}_t(i)$. Once the actual gain of some action has advanced beyond the estimate $\hat{G}_t(i)$ of any action i, i.e., $\hat{G}_t(i) \leq g_r - K/\gamma_r$, the algorithm goes to the next epoch. Algorithm **Exp3.1** yields

Auer *et al.'s* **Theorem 4.1** *For any $K>0$,*

$$\begin{aligned} G_{max} - \mathbf{E}[G_{\mathbf{Exp3.1}}] &\leq 8\sqrt{e-1}\sqrt{G_{max}K\ln K} + 8(e-1)K + 2K\ln K \\ &\leq 10.5\sqrt{G_{max}K\ln K} + 13.8K + 2K\ln K \end{aligned}$$

holds for any assignment of rewards and for any $T>0$.

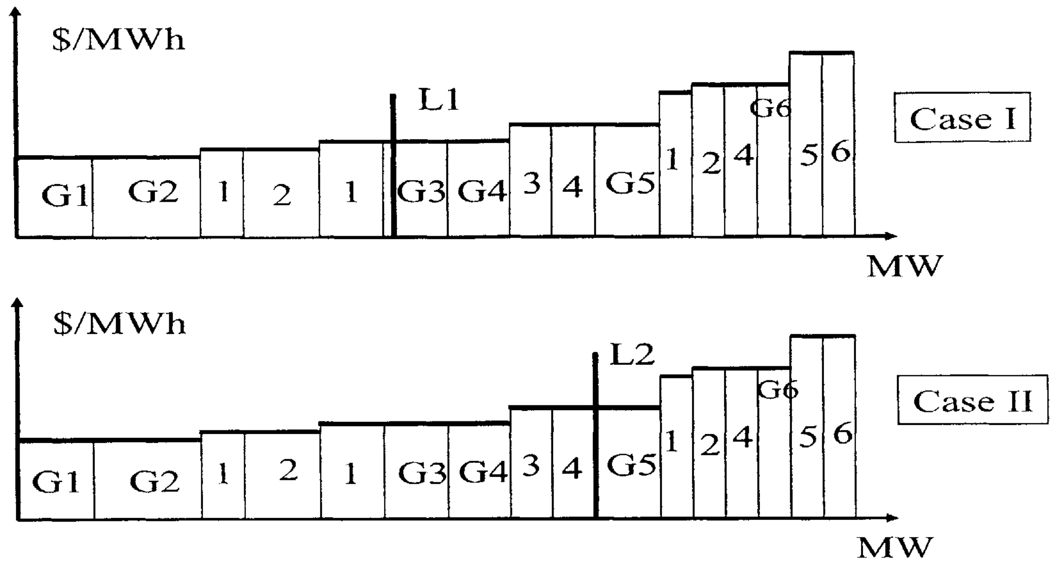

Figure 3-1: Examples of Different Power Producers Competing to Sell Electricity at Different Demand Levels

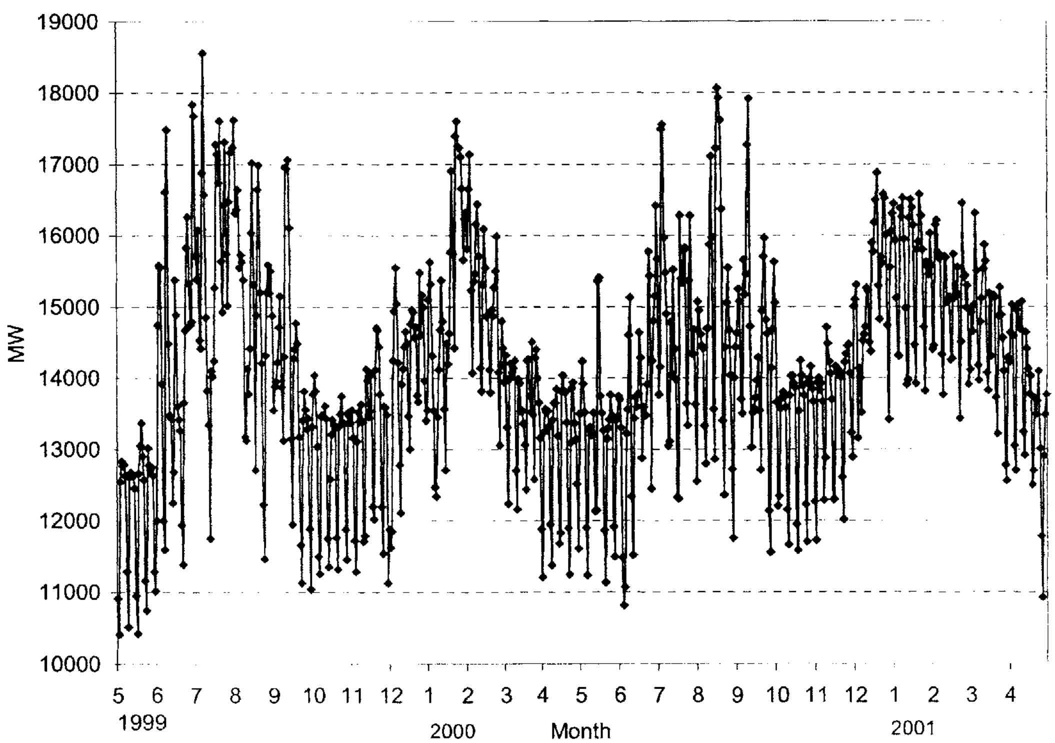

Figure 3-2: An Example of Yearly Demand Characteristics in the New England Electricity Market from May 1999 to April 2000

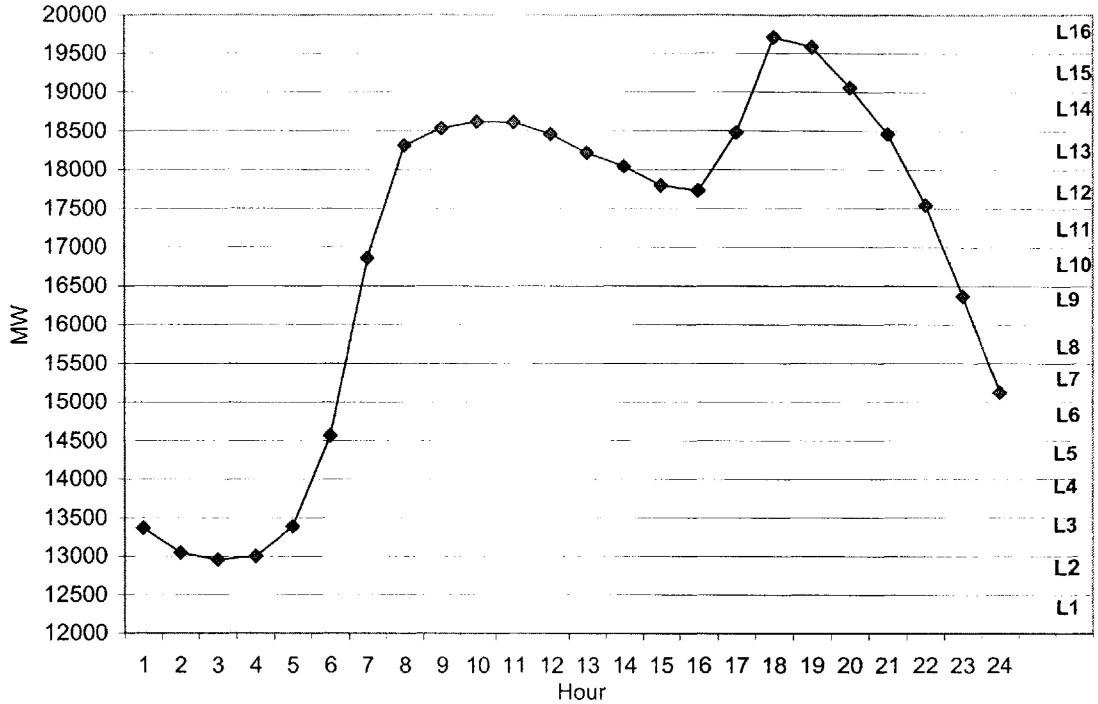

Figure 3-3: Mapping Demand to Load Indices

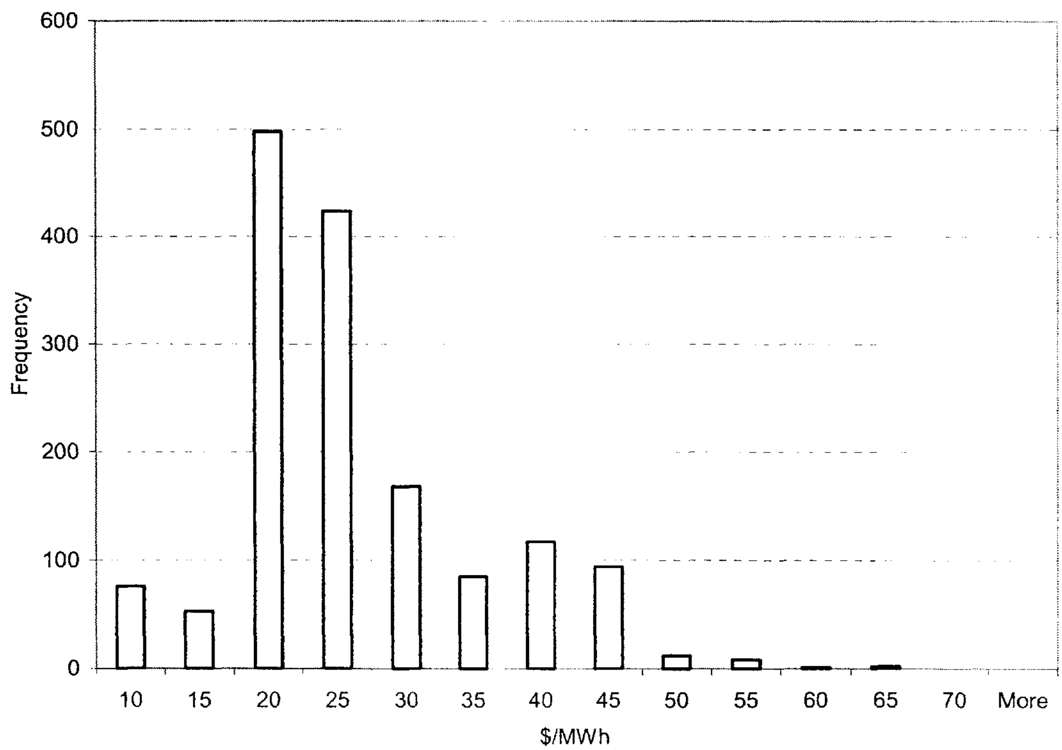

Figure 3-4: Histogram of Hourly Market Prices in New England When Demand is within a 10,000 – 11,000 MW Range from May 1, 1999 to April 30, 2001

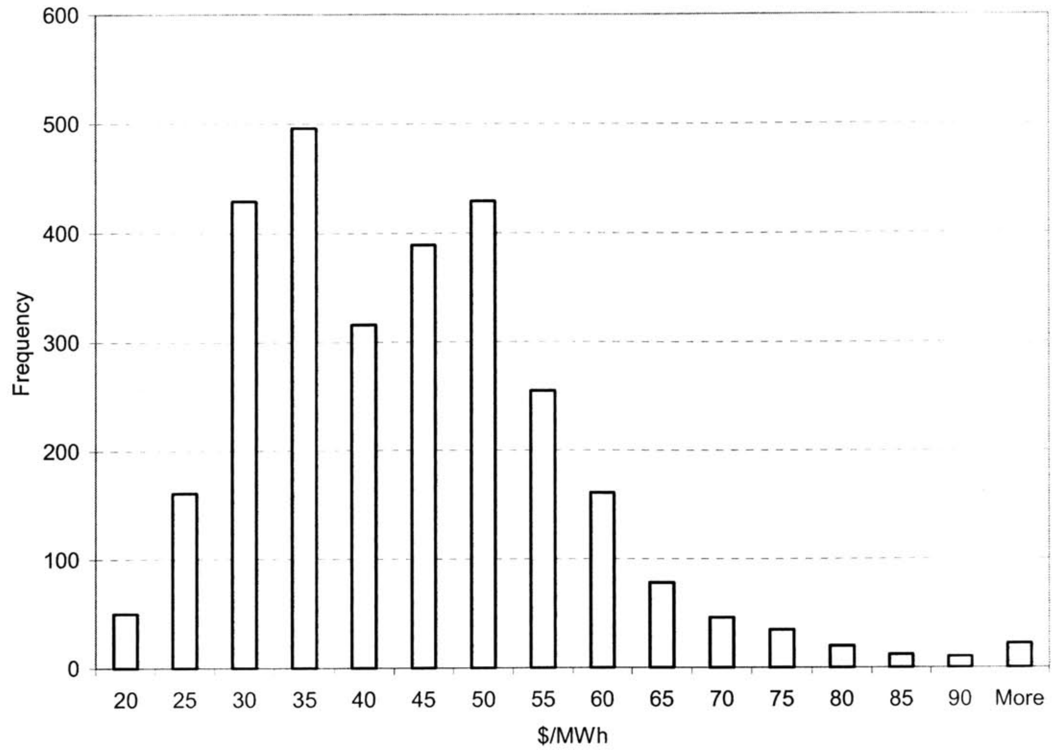

Figure 3-5: Histogram of Hourly Market Prices in New England When Demand is within a 15,000 – 16,000 MW Range from May 1, 1999 to April 30, 2001

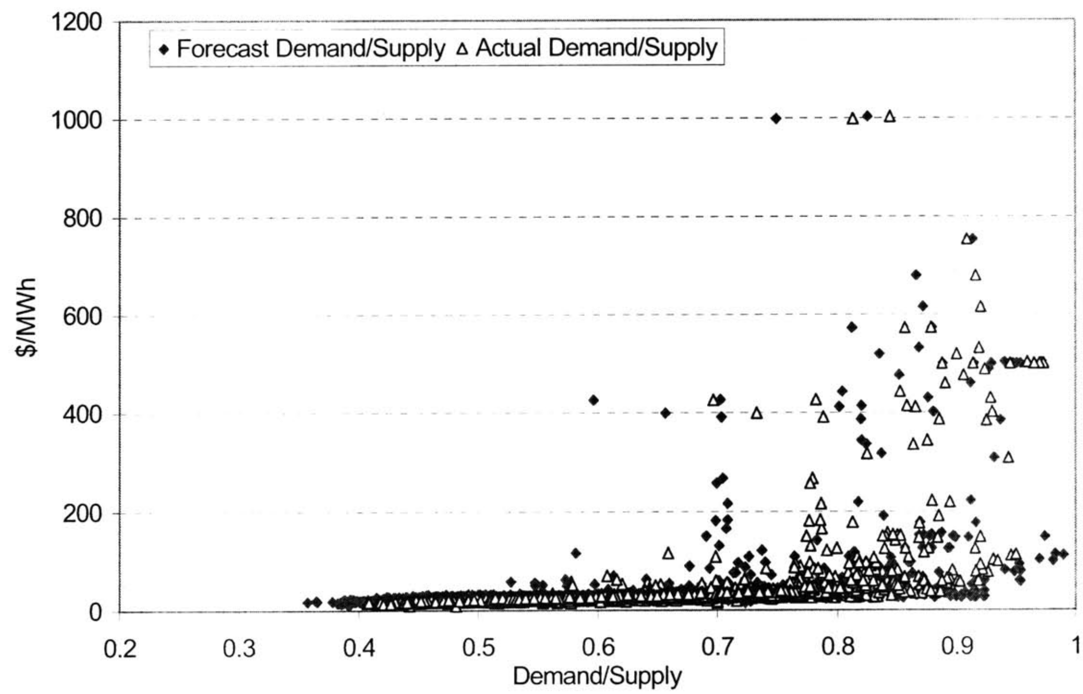

Figure 3-6: Demand-supply Ratio and Market-clearing Prices from May 1, 2000 to October 31, 2000

Chapter 4

Simulations and Analyses

This chapter presents the simulated outcomes of price dynamics from the agent-based market model when the power-producing agents use different learning algorithms. The simulations show the effect of different learning algorithms on market price dynamics and the agents' bidding behavior. The simulations are performed under the assumptions that demand is inelastic and deterministic and that the agents submit piece-wise bid-supply functions.

This chapter is organized as follows. Section 4.1 outlines the hypothetical agent-based electricity model. Section 4.2 presents simulations and analyses when the agents use Algorithms A1, A2, and A3. Section 4.3 shows simulations and analyses when the agents use Algorithm SAB. Section 4.4 presents simulations and analyses when the agents use the model-based algorithm. Section 4.5 presents an analysis on the effectiveness of Algorithm A3 and compares the simulated outcomes obtained when the agents use Algorithm A3 and when they use the model-based algorithm. Section 4.6 investigates the effects of the input parameters of these learning algorithms on the simulated price dynamics. Section 4.7 introduces two methods that can be applied to verify which learning algorithms provide the best match for the existing markets. Finally, the conclusion is provided in Section 4.8.

4.1 Market Model

The hypothetical market model in this chapter consists of power-producing agents, inelastic demand, and a system operator. The following sections describe the characteristics of the power-producing agents, the demand, and the market rules used in the model.

4.1.1 Characteristics of Agents

As shown in Table 4.1, there are 11 agents with non-uniform portfolio characteristics.[1] The aggregate marginal-cost function (or the system marginal-cost function) is shown in Figure 4-1. From Table 4.1, one can observe that the agents have different marginal-cost functions and that Agent 5 owns the largest capacity, equal to 21 MW.

4.1.2 Characteristics of Demand

For the simulations presented in this section, demand is assumed to be deterministic and inelastic; in addition demand is considered as an input of the market model. Daily demand has a repeated pattern, as shown in Figure 4-2. As mentioned previously, the agents play a series of repeated games, in which each stage game is defined by demand levels. The learning algorithm used is also based on these demand levels in the form of demand indices. Each index represents a demand range of 5 MW. The demand pattern has values between 30 to 100 MW. Thus, 15 demand indices are used to represent this demand pattern, as shown in Table 4.2.

Table 4.1: Characteristics of Power-producing Agents

Agent No.	Marginal Cost ($/MWh)													Total (MW)
	10	12	15	20	27	30	35	38	42	48	55	60	72	
	Capacity (MW)													
1	3	0	2	1	0	0	0	2	0	0	0	0	0	8
2	2	0	0	3	0	2	2	0	0	0	0	0	0	9
3	2	0	2	0	2	0	1	0	0	1	0	0	0	8
4	2	0	1	0	2	0	1	0	0	1	0	0	0	7
5	6	4	3	3	0	2	0	0	2	0	0	0	1	21
6	0	7	0	0	0	0	1	0	0	0	0	0	0	8
7	5	0	0	2	0	0	1	0	0	2	0	0	0	10
8	0	2	0	0	3	0	0	2	2	0	0	0	0	9
9	0	2	0	0	0	2	0	1	0	0	0	0	0	7
10	0	0	2	2	0	0	0	0	0	0	1	1	0	6
11	0	0	3	1	0	2	0	0	0	0	1	0	0	7

Table 4.2: Characteristics of Demand Indices

Indices	Demand (MW)	Indices	Demand (MW)	Indices	Demand (MW)
1	< 30	6	$50 - 55$	11	$75 - 80$
2	$30 - 35$	7	$55 - 60$	12	$80 - 85$
3	$35 - 40$	8	$60 - 65$	13	$85 - 90$
4	$40 - 45$	9	$65 - 70$	14	$90 - 95$
5	$45 - 50$	10	$70 - 75$	15	> 95

[1]This number of agents is chosen to closely represent the number of active market participants in existing markets, such as those in New England and California.

4.1.3 Market Rules

This agent-based electricity market model has a uniform-pricing payment rule, in which the agents are paid market prices for the scheduled quantities. Prior to making bidding decisions, the agents are assumed to know system marginal-cost function (as shown in Figure 4-1), scheduled quantities of previous periods, market price and total demand of previous periods, and forecast demand. In addition, the model uses the market clearing mechanism and adopts additional market rules as described below. The competitive outcomes when the agents submit their marginal-cost bids are shown as well.

Determining Market Clearing Prices

The agents have a piece-wise marginal-cost function and submit a piece-wise bid-supply function, or a set of bid-price and bid-quantity blocks (or bid-blocks). The operator uses a price merit order method to schedule the units to match demand. To determine the market prices from the bid-supply functions and demand, the bid blocks are sequenced from the block with the cheapest bidding price to the block with the highest bidding price. Market price in any hour is set to the bidding price of the most expensive bid-block that is scheduled to serve demand at that hour. Let L_k denote demand at time k, P_k denote market clearing price, $q_k^{i,j}$ denote agent i's bidding capacity of bid block j, and $y_k^{i,j}$ denote scheduled quantity, i.e., $0 \leq y_k^{i,j} \leq q_k^{i,j}$. Let Ψ^i be a set of units of Agent i scheduled to operate during period k. The system operator schedules the units to meet demand (L_k) such that total cost is minimized, where the total cost is the sum of market price multiplied by the scheduled capacity, $\sum_i \sum_j y_k^{i,j}$. The market price is defined as the maximum bidding price of the scheduled bid-blocks, i.e., $P_k = \max_i \max_{j \in \Psi_k^i} b_k^{i,j} \cdot I(y_k^{i,j} > 0)$. The bid blocks with the bidding prices most equal to the market price are dispatched.

Other Rules

When the agents submit their bid-supply functions such that more than one unit is scheduled to operate as a marginal unit and its scheduled quantity is a weighted-portion of residual demand, which is defined as the total demand subtracted by the total scheduled quantities of all infra-marginal units. In addition, since no outage and maintenance capacities are considered, a supply-deficiency condition, in which demand exceeds available capacity, is not possible; however, when the supply-deficiency condition occurs in any hour, the total demand of that hour is set to that hour's total available capacity. Let \$/MWh be a unit-price for 1 MWh of energy sold. In addition, the market price is set to the most expensive price, such as $P_k = \max_i \max_j(mc^{i,j}) + C$, where C is a constant that yields a market price to be higher than the most expensive marginal cost. For all simulations in this chapter, there is a price cap P_{cap}, the maximum possible price in the market and equal to \$150/MWh, and the market price when the supply-deficiency condition occurs can be set to P_{cap}, i.e., $\max_i \max_j(mc^{i,j}) + C = P_{cap}$.

Competitive Solution

Suppose the agents submit the marginal-cost bid-supply function in every bidding round. The market prices are then the competitive prices which are shown in Figure 4-2.

4.2 Agents with Algorithms A1, A2, and A3

The agents with Algorithms A1, A2, and A3 have rewards ranging from negative values to positive values that are greater than one. To use these algorithms, the range of the reward values is rescaled to be between zero and one, as follows:

$$\begin{aligned} R &= 1 - \exp(-c \cdot \Pi) \quad \text{for} \quad \Pi > 0 \\ R &= 0 \qquad\qquad\qquad\quad \text{for} \quad \Pi \leq 0 \end{aligned} \tag{4.1}$$

where Π represents the actual reward that the agents obtain, c is a positive constant, and R denotes the rescaled reward. Note that $R \in [0, 1]$. Equation (4.1) indicates that any loss from bidding is equally unfavorable, being assigned the rescaled reward equal to zero.

4.2.1 Algorithms A1 and A2

In the simulation in which the agents use Algorithm A1, γ is set to $\gamma = 0.1, 0.3, 0.5, 0.7$, or 0.9. For Algorithms A1 and A2, each agent (Agent i) selects the bidding price of the anticipated marginal unit from \$0/MWh to P_{cap}, with an increment of \$3/MWh. The total choices of the bidding prices (K^b) are equal to 51. Likewise, each agent selects its bidding quantity that has a bidding price no greater than the bidding price of the anticipated marginal unit, from 0.25 MW to its available capacity (q^i_{max}), with an increment of 0.25 MW. Therefore, the total choices of the bidding quantities (K^q) vary from agent to agent, depending on the available capacity.

Although these algorithms do not include the capacity withholding (CW) strategy as in the model-based algorithm, the withheld capacity can be defined as the difference between total available capacity subtracted by the bidding quantity selected through the learning algorithms ($q_{k,WH} = q^i_{max} - q^i_k$). The bidding price for this withheld capacity (WH_k) is set to $WH_k = \min\{BM + C, P_{cap}\}$, where C is equal to \$3/MWh.

The simulations run for 1,200 hours (that is, for 50 days). Figure 4-3 shows the samples of simulated price dynamics when all agents use Algorithm A1 with $\gamma = 0.1$, Figure 4-4 shows the samples of simulated price dynamics when all agents use Algorithm A1 with $\gamma = 0.9$, and Figure 4-5 shows the samples of simulated price dynamics when all agents use Algorithm A2.

4.2.2 Algorithm A3

In the simulations in which the agents use Algorithm A3, each agent selects the bidding price of the anticipated marginal unit from 0 to P_{cap} with an increment of \$3/MWh. The total choices of bidding prices (K^b) are equal to 51. Likewise, each agent selects its bidding quantity from 0.25 MW to its available capacity (q^i_{max}) with an increment of 0.25 MW. The bidding price for this withheld capacity (WH_k) is set to $WH_k = \min\{BM + C, P_{cap}\}$, where C is equal to \$3/MWh. The simulations run for 1,200 hours. Recall that the minimum number of stages (r) in the algorithm for the bidding price and bidding quantity ($r^{b,*}$ and $r^{q,*}$) is determined as follows:

$$r^* = \min\{r \in \mathcal{N} : \delta_r = \frac{\delta}{(r+1)(r+2)} \geq KT_r e^{-KT_r}\}.$$

Therefore, for $\delta = 0.9$, $r^{b,*} = 1$. Also, let α^b be set to $\alpha^b = 2\ln\frac{K^b T^b_r}{\delta}$ and α^q be set to $\alpha^q = 2\ln\frac{K^q T^q_r}{\delta}$.

Let us further extend the model in which the agents use Algorithm A3 to examine the several values of δ on the price dynamics; for example, $\delta = 0.1, 0.3, 0.5, 0.7$, or 0.9. The samples of simulated price dynamics when all agents use Algorithm A3 with $\delta = 0.1, 0.5$, and 0.9 are shown in Figures 4-6, 4-7, and 4-8, respectively.

Average Price Dynamics An average price dynamics of simulated outcomes when the agents employ Algorithm A3 with $\delta = 0.1$ are also presented. A total of 100 simulations are performed. Figure 4-16 shows that average price dynamics across 100 simulations for each hour.

4.2.3 Analyses

Algorithms A1, A2, and A3 yield a mixed strategy for the agents to choose bidding prices and bidding quantities. The analyses are described as follows.

Algorithms A1 and A2: From Figures 4-3 - 4-5, one can observe that the simulated price dynamics are similar when γ is set to different values. Recall from Chapter 3 that the probability distribution over all possible bidding prices (and bidding quantities) is a mixture of the uniform distribution γ/K^b (and γ/K^q) and a probability mass exponential in the estimated cumulative rewards, i.e., $w_t(m)$ for an action m. Since the initial conditions for $w_{t=0}(m)$ are set uniformly to one, at the beginning of the simulations each action is chosen almost uniformly, i.e., $p^b_t(m) = \frac{1}{K^b}$ (and $p^q_t(m) = \frac{1}{K^q}$). Additionally, γ is small compared to K^b (or K^q). When $\gamma = 0.1$, more weight is assigned to the probability mass exponential of the cumulative rewards. When the weight associated with action m is discounted by γ, i.e., $w_t(m) = \exp(\frac{\gamma \hat{x}_t(m)}{K})$, the estimated cumulative rewards do not grow quickly. Although the large value of γ contributes to the large weight of the uniform distribution, the small value of γ does not put substantial weight on the probability mass exponential. Therefore, the price dynamics exhibit similar patterns for small or for large values of γ.

Algorithm A3: The simulated price dynamics with different values of δ do not exhibit large differences when $\delta = 0.1, 0.3, 0.5, 0.7$, or 0.9. Let us consider the effect of δ on choosing a mixed-strategy action. Recall that α is a function of δ, i.e., $\alpha = 2\sqrt{\ln \frac{KT}{\delta}}$. For all possible actions K and the time horizon T, the weight $(w_t(m))$ associated with action m is a function of α, i.e.,

$$w_{t+1}(m) = w_t(m) \cdot \exp(\frac{\gamma}{3K}(\frac{x_t(m)}{p_t(m)} + \frac{\alpha}{p_t(m)\sqrt{KT}})), \tag{4.2}$$

where $x_t(m)$ is the reward associated with action m and the probability of selecting action m, $p_t(m)$, is defined as $p_t(m) = (1-\gamma)\frac{w_t(m)}{\sum_h^K w_t(h)} + \frac{\gamma}{K}$. Note that $\gamma = \min\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K \ln K}{T}}\}$. When K is large, i.e., $K^b = 51$ for the bidding price and $K^q = 24$ for Agent 10 (that is, an increment of 0.25 MW for the bidding quantities of 6 MW), and $T < K \ln K$, γ is always set to 3/5. One can rewrite Equation (4.2) as follows:

$$w_{t+1}(m) = w_t(m) \cdot \exp(\frac{\gamma}{3K}(\frac{x_t(m)\sqrt{KT} + \alpha}{p_t(m)\sqrt{KT}})).$$

One can also observe that when δ has the smallest value, α has the highest value. If $\alpha = 2\sqrt{\ln \frac{KT}{\delta}} \gg x_t(m)\sqrt{KT} \simeq \sqrt{KT}$, the weight is mainly determined by α. On the other hand, when $\sqrt{\ln \frac{KT}{\delta}} \ll \sqrt{KT}$, δ plays almost no role in determining the weight. Also, when δ is large, i.e., $\delta \geq 0.1$, δ for each trial yields at most $\sqrt{\ln \frac{KT}{\delta}} \simeq \sqrt{\ln 10KT}$; hence, $\sqrt{KT} >> \sqrt{\ln(10KT)}$ and δ plays no significant role. As a result, each action is selected almost uniformly. When δ is small, there is no significant difference in the price dynamics compared to when δ is large as one can observe from Figure 4-9, because, for large K and T, $\sqrt{KT} \gg \sqrt{C + \ln(KT)}$, where $C = -\ln \delta$. Figure 4-9 shows the simulated prices when the agents use the Algorithm A3 with $\delta = 0.1, 0.001$, or 0.00001.

In analyzing the simulations obtained from the model with Algorithms A1, A2, and A3, as shown in Figures 4-3 - 4-8, one can observe that the bidding price of the anticipated marginal unit (BM) does not affect the bid-supply function when the bidding quantity is equal to zero. This issue is investigated further in Section 4.6.1.

4.3 Agents with Algorithm SAB

This section presents the simulations when the agents use Algorithm SAB with different values of model parameters, including a temperature (τ) and a learning rate (α). Recall from Chapter 3 that the probability distribution over the possible actions at any period t, $\{p_t\}$, is defined as follows. For any action j,

$$p_t(j) = \frac{e^{R_t(j)/\tau}}{\sum_h e^{R_t(h)/\tau}}$$

and the cumulative reward associated with each action j is defined as

$$R_t(j) = \begin{cases} (1-\alpha)R_t(j) + \alpha \cdot x_t(j) & \text{if} \quad j = i_t, \\ R_t(j) & \text{otherwise.} \end{cases}$$

Like the model with Algorithms A1, A2, and A3, each agent selects the bidding price of the anticipated marginal unit with the value from 0 to P_{cap}, equal to \$150/MWh, with an increment of \$3/MWh. Likewise, each agent selects its bidding quantity from 0.25 MW to its available capacity with an increment of 0.25 MW. Note that the total choices of the bidding quantities (K^q) vary from agent to agent, depending on their available capacities. The withheld capacity can also be defined as the difference between the total available capacity and the bidding quantity selected through the learning algorithms ($q_{k,WH} = q^i_{max} - q^i_k$). The bidding price for this withheld capacity (WH_k) is set to $WH_k = \max\{BM + C, P_{cap}\}$, where C is equal to \$3/MWh. The simulations run for 1,200 hours, or 50 days.

4.3.1 Effects of τ on Price Dynamics

In this section the simulations explore the effects of τ for $\tau = 0.1, 1, 10$, or 100 on price dynamics when α is set to 0.9. The simulated market prices when 1) $\tau = 1$ are shown in Figure 4-10, 2) $\tau = 10$ are shown in Figure 4-11, and 3) $\tau = 100$ are shown in Figure 4-12.

4.3.2 Effects of α on Price Dynamics

In this section the simulations also explore the effects of α on price dynamics for $\alpha = 0.1, 0.3, 0.5, 0.7$, or 0.9 when τ is set to 10 or 100. The simulated market prices when 1) $\alpha = 0.1$ and $\tau = 100$ are shown in Figure 4-13, 2) $\alpha = 0.5$ and $\tau = 100$ are shown in Figure 4-14, and 3) $\alpha = 0.1$ and $\tau = 10$ are shown in Figure 4-15.

Average Price Dynamics An average price dynamics of simulated outcomes when the agents employ Algorithm SAB with $\alpha = 0.1$ and $\tau = 100$ are also presented. A total of 100 simulations are performed. Figure 4-17 shows the average price dynamics across 100 simulations for each hour.

4.3.3 Analyses

Algorithm SAB yields the mixed strategy for choosing the bidding prices and the bidding quantities. As shown in the simulations (Figures 4-10 - 4-14), one can observe that when α is held constant and τ is varied, the fluctuation of price dynamics increases as τ increases. Let us consider when $\tau = 0.1, 1, 10$, or 100, and when $\alpha = 0.1$ or $\alpha = 0.9$. As one might anticipate, the higher temperature, τ, causes all actions to be selected more equally, while the lower temperature causes the actions that yield the higher rewards to be selected with a higher probability. Furthermore, as shown in Figure 4-10 when $\tau = 1$, the price dynamics shift to a steady-state pattern.

When τ is set at the large value the agents increasingly explore the possible actions. For example, when $\tau = 100$, the simulated price dynamics are similar to the price dynamics obtained from the model, in which the agents use Algorithms A1, A2, or A3 (comparing Figures 4-8 and 4-12). When τ is held constant at $\tau = 100$ and α is varied, i.e., $\alpha = 0.1, 0.3, 0.5, 0.7$, or 0.9, the result as anticipated is that α does not play a significant role in exploring the good actions, and the simulated price dynamics are very similar under these values of α.

Additionally, one can observe that the average simulated prices when the agents use Algorithm A3 are higher than those when the agents use Algorithm SAB. This result may be caused by the difference in mixed-strategy action selection methods, in which in Algorithm A3 each action is selected with a probability of at least γ/K, while in Algorithm SAB each action is selected with a probability depending on the associated rewards. Algorithm SAB chooses actions that yield satisfying outcomes often and may not trial out other actions, such as the expensive bid-supply functions, as often as Algorithm A3 may do.

4.4 Agents with the Model-based Algorithm

This section presents simulations that examine the effects of several parameters of the algorithms, such as target price (Tar) and increment or decrement (Δ) in setting the bidding price for an anticipated marginal unit (BM), as well as the bidding price of withheld capacity (WH_k). In the model with the model-based learning algorithm, the agents can choose any bidding price greater than the lowest marginal cost and less than or equal to P_{cap}, equal to \$150/MWh, and the agents can determine the withheld capacity by using the CW strategy.

4.4.1 Choosing Tar by Methods M1 and M2

Recall from Chapter 3 that the bidding price of the anticipated marginal unit for the next period $(k+1)$, BM_{k+1}, is determined by $BM_{k+1} = Tar_k + \bar{c}$, $c \in \{-\Delta, 0, \Delta\}$. Methods M1 and M2 to select the target price, Tar_k, are examined. Method M1 sets Tar equal to the BM of the previous period, BM_k, that is, $Tar = BM_k$. Method M2 sets Tar equal to the market price of the previous period, MP_k, that is, $Tar = MP_k$. This section shows that both Methods M1 and M2 contribute to different price dynamics, given that other parameters, such as Δ and WH_k, are held constant. The difference can be substantial when the market price is unavailable to the agents, for instance when the market has a discriminatory-pricing structure. This issue is examined in Chapter 6. Let $\Delta = 2$ and the bidding price of the withheld capacity is set to P_{cap} (Method C2). The bidding price of the withheld capacity is set to P_{cap}. Figure 4-18 shows samples of simulated price dynamics when all agents set $Tar = BM_k$, using Method M1. Figure 4-19 shows samples of simulated price dynamics when all agents set $Tar = MP_k$, using Method M2.

4.4.2 Effects of Δ on Price Dynamics

This section explores the effect of the values of Δ for determining BM_k on the overall price dynamics. Recall that $BM_k = Tar_k + \bar{c}$, where Tar_k is determined by Method M1. Three values of Δ are analyzed, including setting Δ equal to \$1, \$2, or \$3/MWh. In the simulations in this section the bidding price of the withheld capacity is set to P_{cap}. Figure 4-20 shows samples of simulated price dynamics when all agents set $\Delta = 1$, Figure 4-18 shows samples of simulated price dynamics when all agents set $\Delta = 2$, and Figure 4-21 shows samples of simulated price dynamics when all agents set $\Delta = 3$.

4.4.3 Effects of WH_k on Price Dynamics

This section explores the effects of methods used to set the bidding price for the withheld capacity (WH_k) on overall price dynamics. Methods C1 and C2 to set this bidding price are analyzed. For Method C1, the agents set $WH_k = BM_k + C$, and the constant C is set to \$3/MWh. For Method C2, the agents set $WH_k = P_{cap}$. In the simulations in this section Δ is set to \$2/MWh and Tar is set by using Method M1 to $Tar = BM$. The samples of simulated price dynamics of Methods C1 and C2 are shown in Figure 4-25.

4.4.4 Analyses

This model-based algorithm yields a pure-strategy bid-supply function. The bid-supply function is a function of the values of Tar, Δ, and WH, in which the effect of Tar, Δ, and WH on simulated price dynamics can be explained as follows:

Methods to Set Tar: From the *PORTFOLIO* scheme, the agents determine the bid-supply functions based on each individual unit and the entire portfolio. The scheduled agents are paid the market price for any scheduled quantity; hence, the agents know the market prices and tend to obtain profits at least equal to the price that they anticipate. The incentive of the agent to increase the bidding price, especially for the infra-marginal units, is minimal. At any period k the agent increases/decreases its bidding price of the anticipated marginal unit for the next period (BM_{k+1}) based on the market price (MP_k), the anticipated profits (AP_k), the actual profits (OP_k), and the bidding price of the anticipated marginal unit of the current period (BM_k). Note that AP_k is calculated by assuming that $MP_k = BM_k$. According to the *OUTCOME* scheme, when $OP_k > AP_k$, the agents no longer need to adjust their BM_k, thus, by using Method M1 to set Tar, having $BM_{k+1} = BM_k + \bar{c}$, the agents no longer adjust their bidding prices and the market prices no longer change.

When Method M2 is used to set Tar, the agents determine their bidding price of the anticipated marginal unit for the next period as $BM_{k+1} = MP_k + \bar{c}$. Under the uniform-pricing structure, the agents are paid the market price for the scheduled quantities and the agents know this price with certainty. Consequently, Methods M1 and M2 tend to yield very similar simulated price dynamics. The simulated market prices can still diverge when Method M2, $BM_{k+1} = MP_k + \bar{c}$, is used, because even if the agents obtain positive profits, they may still change their BM_k. Even if $\Delta = 0$, if MP_k changes, BM_{k+1} also changes. This will either lead to over-bidding by the agents or an increase in the market price as a result of the cumulative effect of raising the bid-supply functions of the agents. Market prices increase due to the agents simultaneously increasing their bidding prices b^i_{k+1}, hence, the BM_{k+1} of all agents increases, and subsequently, the cumulative effect of increasing BM_{k+1} causes the prices to escalate.

On the other hand, when the market prices are not publicly available, such as in the discriminatory-pricing market,[2] market price estimation plays an important role in the determination of market prices from the scheduled prices and scheduled quantity. The agent may over-estimate or under-estimate the market prices, hence, BM may rise over time, causing the divergence of market prices over time.

Values of Δ: As shown in Figures 4-18, 4-20, and 4-21, one can observe that different values of Δ can create different price dynamics, which can be divergent. An increase or decrease in each agent's BM may result in not being scheduled or in being scheduled to operate at full capacity for the anticipated marginal units. Using Hour 8 of each day, let us consider market price divergence of this hour when the agents use $\Delta = 1$, and market price convergence, when the agents use $\Delta = 2$ or $\Delta = 3$. In Figures 4-22 - 4-24, let "M1 D1" denote the simulated outcomes when $\Delta = 1$ is used, "M1 D2" denote the simulated outcomes when $\Delta = 2$ is used, and "M1 D3" denote the simulated outcomes when $\Delta = 3$ is used. Figure 4-22 shows the market price during Hour 8 for a period of 50 days. Figures 4-23 and 4-24 show $(OP_k - AP_k)$ of agents 1 and 5, respectively. One can observe that when the agents obtain $OP_k - AP_k > 0$ and $OP_k > AP_k > 0$, the agents stop raising their BM_{k+1}, resulting in the convergence of the market prices. When the agents use $\Delta = 2$ and $\Delta = 3$, the convergence of prices is observed.

When the agents employ Method M1 to set Tar, the agents that are scheduled to operate at the margin raise the bidding price the next period. Having $BM_k = MP_k$ and $OP_k = AP_k$, from the $OUTCOME$ scheme, these agents set $O_k = 11$. Hence, the agents with at least one unit that has a marginal cost equal to the market price, which is defined as Group A, gradually raise their bidding price with an increment of Δ so that it becomes either higher than or equal to the agents with at least one unit having the next most expensive marginal cost. This second group is defined as Group B. At this point, there are several possibilities. First, if Group A's BM_k is higher than Group B's BM_k,

[2] This issue is examined in Chapter 6.

Group A no longer obtains profits as anticipated and decides to decrease its BM_{k+1}. Group B, on the other hand, obtains the same profits as it anticipated (for the agents in Group B, $MP_k = BM_k$). In the next period, Group A obtains the profits it anticipates after lowering BM_k and the bid-supply function; hence, Group A will increase the bidding price again. The same explanation is applied to Group B. As a result, one can observe that the market prices for the same load levels alternates between two values (odd and even values) over time. Second, Group A's BM_k at certain hours can be equal to Group B's BM_k (because of an increment of \$1/MWh). When BM_k of both Group A and Group B are the same (for the first time) and equal to MP_k, following the *OUTCOME* scheme, both Groups A and B determine $O_k = 11$. For Group A, $OP_k < AP_k$ and $BM_k = MP_k$. For Group B, $OP_k = AP_k$ and $BM_k = MP_k$. In the next period, the market price increases according to the bidding prices of the agents in both groups. Hence, they obtain less profit than they anticipate (since there are more agents (Groups A and B) scheduled to operate at the margin); i.e., $OP_{k+1} < AP_{k+1}$ and $BM_{k+1} = MP_{k+1}$, and then $O_{k+1} = 11$.

Consequently, depending on whether Δ is such that BM_k of Group A is equal to BM_k of Group B or such that BM_k of Group A is greater than BM_k of Group B, the market prices can diverge or shift to a steady-state pattern. In the simulations, $\Delta = 1$ leads to divergence of market price during Hour 8, while $\Delta = 2$ or $\Delta = 3$ leads to a steady-state pattern of market price during that same hour.

Values of WH_k: As shown on simulated outcomes in Figure 4-25, market prices during the lower demand hours can be higher than prices during the higher demand hours. This outcome results from the cumulative bidding behavior of the agents, because many more agents are able to withhold capacity during the low demand hours than during the high demand hours. Let us consider Hours 12 and 17 in Figure 4-25, where "C1" denotes the simulated prices when the agents use Method C1 and where "C2" when the agents use Method C2. At Hours 12 and 17, demand is equal to 75.7 and 74.2 MW, and marginal-cost prices are equal to \$35 and \$30/MWh, respectively. At Hour 12, to raise the price from \$35 to \$38/MWh by withholding capacity each agent needs 5 MW of capacity. Agents 1-7 and 11 are eligible to exercise the CW strategy (the total capacity with marginal cost less than \$30/MWh is greater than 5 MW), but they will lose 5 MW of scheduled capacity for only a \$3/MWh increase in prices, which is not profitable to any agent except Agent 5. The agents use the price-setting strategy only to set the market price at \$37/MWh in the first day. On the other hand, at Hour 17, to raise the price from \$30 to \$35/MWh by withholding capacity, each agent needs only 1 MW of capacity. All agents are able to set the market price at \$35/MWh. Moreover, if Agent 5 withholds an extra 6 MW, it could raise the bidding price from \$30 to \$38/MWh, meaning an increase in profit for Agent 5. In this scenario, total withheld capacity is 16 MW, of which 10 MW comes from each agent plus (additionally) 6 MW comes from Agent 5, resulting in a price equal to \$48/MW in Day 1.

On the other hand, when Method C2 is used, $WH_k = \min\{BM_k + C, P_{cap}\}$, the agents set their

WH_k to be $BM_k + 3$. The market price at Hour 17 is equal to \$35/MWh, compared to \$48/MWh when Method C1 is used. This example suggests that without explicitly setting the expensive bidding price for the withheld capacity, the agents are still able to raise the market prices higher than market prices when the CW strategy is not exercised.

4.4.5 Simulations with a Game Matrix

This section presents simulation and analysis when the agents employ the model-based algorithm and have their GM, historic price (HP), and historic bidding price (HB) matrices record the opponents' joint actions, MP, and BM, respectively. The *SETPRICE* scheme in the model-based algorithm used in the previous simulations is replaced by the *SETPRICE-GAME* scheme as shown in Chapter 3. The agents choose the bidding price equal to \$150/MWh for the withheld capacity and they set $\Delta = 2$. The agents use either Method M1 or M2 to set Tar when they determine the bidding price of the anticipated marginal unit for the next period (BM_{k+1}) when the game condition is observed, i.e., $BM_{k+1} = Tar + \bar{c}$, where $\bar{c} = \{-\Delta, 0, \Delta\}$.[3] For any load index $\mathcal{L}^h$, Method M1 sets

$$Tar = \max_j \; HB^{h,j},$$

where $HB^{h,j}$ is an element in row h and column j of an HB matrix. Method M2 sets

$$Tar = \max_j \; HP^{h,j},$$

where $HP^{h,j}$ is an element in row h and column j of an HP matrix. Note that these methods are the same as those in the model-based algorithm when the length of the HP or HB matrix (Md) is equal to one. Setting Tar to be the maximum recorded value allows the agents to use the most "optimistic"[4] action.

When no new bid-supply function is tried and subsequently no new market price is obtained, Method M1 produces no new value of BM. Recall from the previous chapter, the conditions in which the agents obtain 1) $OP < AP$ and $BM = MP$; 2) $OP = AP = 0$ and $BM < MP$; 3) $OP = AP$ and $BM = MP$; or 4) $AP < OP < 0$, together with the game condition is equal to one, lead to an increase in BM (when $O = 11$). Method M2 tends to produce a new value of BM, even if $O = 00$, due to variation of MP. Two sets of simulations with different initial conditions are considered, including the one with an initial condition of the GM matrix equal to one (i.e., $GM_0 = 1$) and the other with an initial condition of the GM matrix equal to zero (i.e., $GM_0 = 0$). Both sets have initial conditions of the HP and HB matrices equal to zero.

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[3] As shown in Chapter 3, the game condition for each load index occurs when the sum over the row of the GM matrix, which is associated with that load index, exceeds 0.5.

[4] One may consider the most "pessimistic" action by setting $BM_{k+1} = \min_j HP^{h,j}(\mathcal{L}^h) + \Delta$, instead.

Figure 4-26 shows the simulated price dynamics when the agents use Method M2 to set Tar and Md =1, 3, 5, or 10 during Hour 1 of each day. Demand is equal to 46.1 MW and the agents do not use the CW strategy. In this figure, "Mem1" denotes the price dynamics when the agents use $Md = 1$ and $GM_0 = 1$, and it represents the price dynamics when the agents use $Md = 1$, 3, 5, or 10 and $GM_0 = 0$. In addition, "Mem3," "Mem5," and "Mem10" denote the price dynamics when $GM_0 = 1$ and the agents use $Md = 3$, 5, and 10, respectively. Figure 4-27 shows the simulated price dynamics when the agents use Method M1 to set Tar and Md =1, 3, 5, or 10 during Hour 1 of each day and the agents exercise the CW strategy if possible. Let "Mem1" denote the price dynamics when the agents use $Md = 1$, "Mem5" denote the price dynamics when the agents use $Md = 5$, and "Mem10" denote the price dynamics when the agents use $Md = 10$. These price dynamics obtain from the simulations that have $GM_0 = 1$.

When the agents employ Method M2, $GM_0 = 0$, and no CW strategy is present, no significant difference between the outcomes with various values of Md are observed. According to the *OUTCOME* scheme, the agents that are scheduled to operate as marginal agents will increase their bidding price the next period if the game condition is equal to one. If $Md > 2$, no game condition occurs at bidding-round 2, yielding no increase in their bidding prices. Since there is no change in BM, there is no change in MP. Therefore, price dynamics shift to a steady-state pattern. On the other hand, when $GM_0 = 1$, the agents with $Md > 1$ experience the game condition and are able to raise the bidding prices for several periods. The simulated outcomes for Hour 1 when the agents use Method M1 to set Tar are similar to those when Method M2 is used.

Note that market price equal to $27/MWh exhibits the game condition since the marginal-cost price is equal to $15/MWh. From Figure 4-27, when the agents employ Method M1, $GM_0 = 0$, and the CW strategy is in place, the agents with $Md = 10$ take several hours than the agents with $Md < 10$ before deciding to increase their bidding prices, resulting in market prices remaining at $27/MWh for five hours. On the other hand, when $GM_0 = 1$, the agents are able to raise the bidding price at the first hour and continue adjusting their bidding price according to the bidding outcomes in the later hours.

With $GM_0 = 0$, when there is no demand uncertainty as in the simulations presented here, the length of the memory matrices plays an insignificant role in shaping price dynamics once the agents obtain $O = 00$. Demand uncertainty generally creates unsuccessful outcomes and different market prices for the same forecast demand. These factors cause changes in the HP and HB matrices which lead to changes in bid-supply functions and price dynamics. A large Md tends to lead to more conservative outcomes. For example, suppose the agents experience a price spike for one period after a long period of the no-game condition. When the agents use $Md = 1$, a strategic bid is expected for the next bidding round with the same demand index. When the agents use $Md = 3$, a competitive bid is expected, instead. One may observe higher market prices when the agents use Method M2 than prices when the agents use Method M1. This is because a change in BM may occur due to a change in MP.

4.5 Learning Algorithms and Bidding Behavior

This section presents further analysis of the effectiveness of Algorithm A3, especially when, by using Algorithm A3, the agents gain profits as high as those they would obtain using one best-response bid-supply function over time.[5] To observe long-term dynamics as a result of this learning algorithm, simulations in this section are performed over longer periods than those of the simulations in Sections 4.2 - 4.4, which are 1200/24 = 50 periods for each load index. This section also compares the simulated outcomes obtained when the agents use Algorithm A3 and the model-based algorithm.

4.5.1 Analyzing Algorithm A3

The effectiveness of Algorithm A3 is examined to show that the agents "learn" to improve their bidding behavior over time and reach the best-response profits according to Auer *et al.'s* Theorem 9.1 as shown in Section 3.3.1. Because in the bidding game the agents have incomplete information of the opponents' actions and the opponents can use any action, the best-response bid-supply (BRBS) function cannot be determined in a closed-form formulation; instead, it is obtained from simulations. The steps used in this analysis follow.

1. *Identifying the BRBS function for an agent in response to bidding behaviors of the other agents.*
   Each simulation assumes that the agent uses one bid-supply function obtained from one bidding price (BM) and one bidding quantity (q_k), whereas the other agents use Algorithm A3. A total of 50 particular bid-supply function simulations are performed, and average cumulative profits across these simulations are calculated. The BRBS function is a bid that yields the highest average cumulative profits.

2. *Comparing the bidding outcomes of the agent when using Algorithm A3 to those when using the BRBS function.*
   All agents employ Algorithm A3. A total of 100 simulations are performed, and average cumulative profits across these simulations are calculated. The average profits that Agent 1 obtains are compared to the profits that it obtains from the BRBS function.

Note that the BRBS function determined from a set of bid-supply functions in which BM is equal to 27, 33, 39, 45, 51, or 75 and q is equal to 0, 1, 2, 3, 4, 5, or 6 is explored. Note that all possible bid-supply functions include $BM \in \{0, P_{cap}\}$ with an increment of \$3/MWh and $q \in \{0.25, q_{max}\}$ with

[5] See the appendix to Chapter 3. Auer *et al.* define the weak regret over T periods (the deviation of rewards from learning algorithm A, $G_T(A)$, from the best outcome, $G_{T,max}$) as $G_{T,max} - G_T(A)$, where $G_{T,max} \stackrel{def}{=} \max_j \sum_{t=1}^{T} x_t(j_t)$ and when $x_t(j_t)$ is the reward the agent obtains from choosing action j_t at time t.

an increment of 0.25 MW that yield $K^b = 51$ and $K^q = 32$, respectively. Total simulated scenarios are shown in Table 4.3.

Table 4.3: Simulation Scenarios

Scenarios	BM	q	Scenarios	BM	q	Scenarios	BM	q	Scenarios	BM	q
1	27	0	9	33	1	17	39	2	25	51	0
2	27	1	10	33	2	18	39	3	26	51	2
3	27	2	11	33	3	19	39	4	27	51	4
4	27	3	12	33	4	20	39	5	28	75	0
5	27	4	13	33	5	21	39	6	29	75	2
6	27	5	14	33	6	22	45	0	30	75	4
7	27	6	15	39	0	23	45	2			
8	33	0	16	39	1	24	45	4			

Simulation and Analysis

Agent 1 is selected for this study because it has a relatively small installed capacity of 8 MW and most of its units have an inexpensive marginal cost. Let δ be set to 0.1. Demand is set to 66 MW for every hour. Let each hour be considered the beginning of each trading round. The duration of each simulation is equal to 500 hours. The simulated outcomes that show the cumulative average profits over 50 simulations of each scenario at Hour 500 are shown in Figure 4-28.

According to Auer *et al.'s* Theorem 9.1, Algorithm A3 should yield average profits to Agent 1, at least within a certain bound (BB) of the profits from the best-response action. The BB is calculated using the formulation from Auer *et al.'s* Theorem 9.1 as follows:

$$BB = \frac{10}{\sqrt{2}-1}\sqrt{\frac{2K}{T}\left(\ln\frac{KT}{\delta}+c_T\right)} + \frac{10(1+\log_2(T))}{T}\left(\ln\frac{KT}{\delta}+c_T\right),$$

where K is the total number of actions and $c_T = 2\ln(2+\log_2 T)$. One may calculate $r^*_{min} = 1$ and $T_r = [2, 4, 8, \ldots]$. For the simulation duration of 500 hours, the last epoch is $r^* = 8$ and $T = (K/\delta)^{\Omega(1/K)} = 2^8 = 256$. One can observe that the higher the simulation duration, T, the lower the bound and that the higher the possible actions, the higher the bound. In this agent-based model, $K = K^b \times K^q = 1,632$, where $K^b = 51$ and $K^q = 32$. Hence, when $T = 256$ hours and $\delta = 0.1$, $BB =$\$391. That is, the rescaled cumulative rewards over a T-period when the agent uses Algorithm A3 will be within \$391 of the rewards from the BRBS function within probability $1 - \delta = 0.9$.

From the simulated outcome shown in Figure 4-28, the BRBS function is identified as $BM =$ \$39/MWh, and $q = 6$ MW or $q_{WH} = 2$ MW (Scenario 21). This bid-supply function yields a cumulative average of \$303 for rescaled profits across 50 simulations during 500 hours. When Agent 1 employs Algorithm A3, it obtains a cumulative average of \$198 for rescaled profits across 100 simulations during 500 hours. The difference is \$105. This result indicates that Algorithm A3 effectively has Agent 1 choose the BRBS function over time. Although the simulations show that Algorithm A3 yields cumulative profits within the calculated bound, one must keep in mind that in the actual electricity markets the power producers are unable to identify their BRBS functions in near real-time by using the steps described above.

Similarly, when the agents employ Algorithm A3 and choose $q \in \{1, q_{max}\}$ with an increment of 1 MW, then $K^q = 8$ and $BB = \$191$. The simulation, when Agent 1 employs a strategy, $BM =$ \$39/MWh and $q = 6$ MW or $q_{WH} = 2$ MW (Scenario 21), yields Agent 1 a cumulative average of \$307 for rescaled-profits across 100 simulations during 500 hours. The simulation when Agent 1 employs Algorithm A3 yields Agent 1 an cumulative average of \$225 for rescaled-profits across 100 simulations during 500 hours. The difference is \$82, which is acceptable given BB =\$191. Figure 4-29 shows the cumulative average rescaled profits over 500 hours of these simulations.

4.5.2 Algorithm A3 and the Model-based Algorithm

This section analyzes and compares the simulated outcomes obtained when only Agent 1 or 5 employs Algorithm A3 or the model-based algorithm and when the other agents submit their marginal-cost bids. The comparison shows the effect of the mixed and pure-strategy action selections on price dynamics and bidding outcomes. Demand is equal to 66 MW in every hour and each hour is considered a bidding round. A total of 100 simulations are performed, and the average prices and profits that Agents 1 and 5 obtain are observed. Average prices across 100 simulations are shown in Figure 4-30. In this figure, let "Algorithm A3-Gen1" denote price dynamics when only Agent 1 employs Algorithm A3, and "Algorithm A3-Gen5" denote price dynamics when only Agent 5 employs Algorithm A3. Also, let "Model-based Gen1-M1" denote price dynamics when only Agent 1 employs the model-based algorithm with Method M1, and let "Model-based Gen5-M1" and "Model-based Gen5-M2" denote price dynamics when only Agent 5 employs the model-based algorithm with Methods M1 and M2, respectively.

According to the simulations in Figure 4-30, when only Agent 5 employs Algorithm A3, it is able to raise the market prices to be higher than when only Agent 1 does. Additionally, when Agent 5 employs Algorithm A3, hourly average prices are higher than those obtained when Agent 5 employs the model-based algorithm with Method M1 to set Tar. Similarly, Agent 1 using Algorithm A3 is able to raise the market prices to be higher than when it uses the model-based algorithm with Method M2. Moreover, when Agent 5 uses Method M2 to set Tar, the market prices increase to a higher level than the prices obtained from other scenarios. Agent 5 obtains the profits it anticipates, and its bidding price of the anticipated marginal unit (BM) is equal to the market prices (MP), i.e., $BM = MP$. The $OUTCOME$ scheme results in $O = 11$, causing an increase in BM until this condition no longer exists, resulting in the price dynamics shifting to a steady-state pattern. For example, the agent decreases its BM when profits are less than what it anticipates.

4.6 Exploring the Model

In this section, the agent-based electricity market models in which the agents use different learning algorithms are extended to examine the possible outcomes under different market scenarios. First, if agents submit the total capacity with the bidding prices in order of their marginal costs, they compete only in their bidding prices without consideration of the CW strategy. Second, let us consider the scenario when only the agent with the largest capacity (Agent 5) uses the learning algorithm, while the other agents submit their marginal-cost bid-supply functions. This analysis examines the effect of a dominant agent on the price dynamics. Finally, the agent-based model is applied to compare the simulated outcomes when the agents use the model-based algorithm to determine the bid-supply function based on the unit-by-unit and the portfolio-based decision schemes. These analyses are presented in the following sections.

4.6.1 Price-war

This section explores the market price dynamics when the agents use the learning algorithms to determine their bidding prices but not their bidding quantities. That is, the agents compete with each other by undercutting or raising their bidding prices of the anticipated marginal units. Algorithm A3, Algorithm SAB, and the model-based algorithms are analyzed.

Simulations

Like the previous simulations, in Algorithms A3 and SAB the agent selects the bidding price for the anticipated marginal unit no greater than P_{cap}. Let the agents using Algorithm A3 choose $\delta = 0.9$. The samples of simulated price dynamics are shown in Figure 4-31. Let the agents using Algorithm SAB choose $\alpha = 0.9$ and $\tau = 100$. The samples of simulated price dynamics are similar to the ones obtained from Algorithm A3 as shown in Figure 4-31.

When the agents use the model-based algorithm, the bidding price for the anticipated marginal unit is set to $BM_k = Tar + \bar{c}$, where Tar is determined by Methods M1 and M2 and $\bar{c} \in \{-\Delta, 0, \Delta\}$. Let $\Delta = 2$ and the bidding price of the withheld capacity be set to P_{cap}, which is equal to \$150/MWh. Figure 4-32 shows the samples of simulated price dynamics when the agents use Method M1 to set Tar. Let "M1 D2 C2" and "M1 D2 C2 noW" denote the simulated outcomes when the agents use the CW strategy and when the agents do not, respectively. Figure 4-33 shows the samples of simulated price dynamics when the agents use Method M2 to set Tar. Let "M2 D2 C2" and "M2 D2 C2 noW" denote the same things as in Figure 4-32.

Analyses

Algorithms A3 and SAB: From Figure 4-31, one can observe that when the agents use Algorithm A3, the market price dynamics shift to a steady-state pattern. Let us consider the method to set BM for Algorithm A3 (recall Chapter 3). When the agents set the withheld capacity equal to zero, the bid-supply function is always equal to the marginal-cost function. Since the bidding price b_k, that is, $b_k = BM_k$, is used for setting the bidding price of the withheld capacity WH_k, i.e., $WH_k = BM_k + c$, where c is a constant, with zero withheld capacity, BM_k plays no role in the bid-supply function. Consequently, the bid-supply function is equal to the marginal-cost function, and the market prices are equal to the marginal-cost prices. Similarly, when Algorithm SAB is used for learning and the withheld capacity is set to zero, the simulations yield the market prices equal to the marginal-cost prices. An explanation for this result is similar to that of the simulations when the agents use Algorithm A3. Since BM_k has no role in setting the bid-supply function, the agents submit the bid-supply function equal to the marginal-cost function.

The Model-based Algorithm: Let us consider Figure 4-32. When the agents use the model-based algorithm with zero withheld capacity, they are able to raise the market prices above the marginal-cost prices. Additionally, they are sometimes able to raise the market price higher than the prices obtained when the agents exercise the CW strategy, such as in Hour 4 of each day. From the $OUTCOME$ scheme, when the agents obtain $OP_k - AP_k > 0$ and $OP_k > 0$, if the agents are not scheduled to operate at the margin, they will not raise their prices (recall $O_k = 00$ for $OP_k - AP_k > 0$ and $BM_k < MP_k$). Conversely, if the agents are scheduled to operate at the margin, they will raise BM for the next period. However, suppose the CW strategy is in place and some agents obtain $OP_k > AP_k$. This outcome implies that at least one agent exercises the CW strategy. When this happens, these agents have $BM_k < MP_k$ and they obtain 1) an increase in the scheduled capacity, 2) an increase in the market price, or 3) an increase in the scheduled capacity and the market price. These agents have $BM_k < MP_k$. Hence, according to the $OUTCOME$ scheme, when $OP_k > AP_k$ and $BM_k < MP_k$, $O_k = 00$ and there is no change in BM_{k+1}.

Let us also consider, for example, when the agents use Method M2 to determine Tar during Hour 4 and the CW strategy is present. Let us consider Figure 4-34, which shows MP_k, $OP_k - AP_k$, and BM_k of all agents. At Day 4 ($k = 4$), Agent 1 obtains $OP_k > AP_k$ and $BM_k = MP_k$, as well as the $OUTCOME$ scheme yields $O_k = 00$, and, consequently, Agent 1 does not adjust its bid-supply function the next period. Similarly, Agent 5 and the others also have $OP_k > AP_k$ and $BM_k = MP_k$, as well as the $OUTCOME$ scheme yields $O_k = 00$. As a result, no agents adjust their BM_{k+1}. Note that only Agent 5 is able to profitably exercise the CW strategy. Moreover, at Day 5 ($k = 5$), Agents 1, 3, 4, 10, and 11 obtain $OP_k > AP_k$ and $BM_k = MP_k$ and they do not attempt to raise their BM_k. The other agents obtain $OP_k = AP_k$ and $BM_k = MP_k$, so they raise their BM_k. However, the other agents are able to set the market price and the market price remains the same as the price of Day 5.

On the other hand, let us consider Figure 4-35, which shows MP, $OP - AP$, and BM of all agents when the agents are not allowed to exercise the CW strategy. On Day 4 ($k = 4$), when Agent 5 does not receive the profit as anticipated, i.e., $OP_k < AP_k$ and $BM_k > MP_k$, the $OUTCOME$ scheme advises Agent 5 to lower its BM_{k+1} and also its bid-supply function. The other agents do not adjust their bid-supply function during this period. Further, Agent 5 is unable to change the market price at Day 5, but it obtains higher profits than the anticipated ones. This causes Agent 5 to maintain the same BM for Day 6 (that is, $BM_6 = BM_5$). The other agents obtain $OP_k = AP_k$ and $BM_k = MP_k$ at Day 5 ($k = 5$), so that they increase their BM_{k+1} at Day 6. The market price at Day 6 then is higher than that in the market when the agents are allowed to withhold their capacity. From this point on, by applying an analysis similar to one presented here, one can show that the agents raise the bid-supply function, resulting in the increase in market prices greater than the increase in market prices when the CW strategy is in place.

4.6.2 Dominant Agent

This section explores market price dynamics when only the dominant agent, Agent 5, who owns 21 MW of installed capacity (or 21% of the total installed capacity), uses Algorithm A3, Algorithm SAB, and the model-based algorithm.

Simulations

Like the previous simulations, when Agent 5 uses Algorithms A3 and SAB and selects its bidding price for the anticipated marginal unit to no greater than P_{cap}. Let Agent 5 choose $\delta = 0.9$. The bidding price for this withheld capacity (WH_k) is set to $WH_k = \max\{BM_k + C, P_{cap}\}$, where C is equal to \$3/MWh. The samples of simulated price dynamics are shown in Figure 4-36. When Agent 5 uses Algorithm SAB, let Agent 5 choose $\alpha = 0.9$ and $\tau = 100$; the samples of simulated price dynamics are shown in Figure 4-37. When Agent 5 uses the model-based algorithm, $BM_k = Tar + \bar{c}$, where Tar_k is determined by Method M1 on the overall price dynamics and $\bar{c} \in \{-\Delta, 0, \Delta\}$. Let Δ be set to $\Delta = 2$ and let the agent use Method C2 to set $WH_k = P_{cap}$. The samples of simulated price dynamics when Agent 5 uses the model-based algorithm are shown in Figure 4-38.

Analyses

From Figures 4-36 - 4-38, one can observe that Agent 5 is able to influence the price dynamics, causing the prices to be higher than the marginal-cost prices in many hours. When Agent 5 uses Algorithm A3, one can observe that market prices shift from the marginal-cost prices to more expensive prices. In addition, when demand is equal to 75.9 MW, Agent 5 may cause the market price to be as high as \$54/MWh compared to the marginal-cost price equal to \$35/MWh. Agent 5 obtains this price by determining the withheld capacity to be $q_{k,WH} = 18.25$ MW with $WH_k = \$54$/MWh, and then setting the bidding quantity, q_k^5, equal to 1.75 MW with $BM_k = \$51$/MWh. Note that Agent 5 can implement this bid successfully, because there is total capacity of 74 MW from other agents except Agent 5 with marginal cost less than \$55/MWh.

Moreover, Agent 5 could cause the market price to be at most \$55/MWh by having $q_{k,WH} \geq 18.25$ MW and $WH_k > \$54$/MWh, though, this agent may not be scheduled to operate $q_{k,WH}$ and does not receive the benefit from the high market price. When Agent 5 uses Algorithm SAB, one can observe that the simulated outcomes are similar to those obtained from the model with Algorithm A3; furthermore, when different values of δ are assigned to the model, i.e., $\delta = 0.1, 0.3, 0.5$, or 0.7, the simulated price dynamics yields outcomes similar to the model with $\delta = 0.9$.

In addition, when Agent 5 uses the model-based algorithm, one can observe that it can apply the CW strategy to raise the bidding prices during the lower demand hours daily (such as by withholding 6 MW at Hour 17) as the simulations in Section 4.4.3. Without other agents trying to raise or undercut the bidding prices, Agent 5 alone is unable to raise the market prices as high as those when all agents uniformly adopt the learning algorithms. When Agent 5 raises its bidding price and is not scheduled to operate as it anticipates, the *OUTCOME* scheme directs Agent 5 to stop raising the bidding price; consequently, the market price dynamics shift to a steady-state pattern.

Furthermore, when Agent 1, with only 8 MW of installed capacity, is the only agent who uses the learning algorithm to determine its bid-supply function, the simulations show that Agent 1 is unable to change the market price from the marginal-cost prices as much as Agent 5 is able to. The maximum deviation that Agent 1 can cause during the maximum demand hour by using Algorithm A3 is equal to \$38/MWh. That is, Agent 1 has the bidding quantity, q_k^1, at most equal to 0.75 MW and the withheld quantity equal to 5.25 MW (the unit with 2 MW of capacity and marginal cost equal to \$38/MWh plays no role), as well as $BM_k \geq \$36$/MWh.[6] Figure 4-39 shows samples of simulated price dynamics when Agent 1 uses the model-based algorithm with Method M1 to set Tar and $\Delta = 2$.

4.6.3 Unit-by-unit vs. Portfolio Decision Scheme

The simulations in this section show the price dynamics obtained from the unit-by-unit decision scheme, as well as the portfolio-based decision scheme with and without the CW strategy. The objective of this section is to demonstrate that market efficiency may occur when the agents have the least information about the entire market. Market efficiency is defined as the difference between market prices and competitive prices. The smaller the difference, the higher the price efficiency. These outcomes do not guarantee that the power-producing agents can profitably operate in the market and

[6] In this case, Agent 1 is scheduled to operate its q_k^1, plus the weight-portion of the residual demand, in which the residual demand is equal to total demand subtracted by the capacity of the other agents with marginal cost less than \$38/MWh and by q_k^1, i.e., $(75.9 - 75 - q_k^1)$. That is, the weight-portion of Agent 1 is $\frac{2}{5}$, which is the ratio of 2 MW and 5 MW from capacity of the units with marginal cost equal to \$38/MWh.

the issue regarding the profitability of power-producing agents is left for future research. In this section, the simulations are based on the assumption that each agent owns one generating unit. Each unit may be different in capacity and in its constant marginal cost. The agents use the model-based algorithm to determine a pure-strategy bid-supply function. The agents adopt a similar decision scheme as in the portfolio-based case except that no CW strategy is considered. This scenario is similar to the ones in Section 4.6.1, in which the agents use the learning algorithm to determine only the bidding price. Two methods to select Tar for determining the bidding price of the anticipated marginal unit BM are considered. In addition, two scenarios are considered. First, with Method U1 Agent i determines BM and sets the bidding price b^i such that $\tilde{b}^i = \max\{BU, mc^i\}$ and $b^i = \max\{BM, \tilde{b}^i\}$. Note that mc^i is the marginal cost of Agent i and BU is the bidding price for its unit based on the previous outcomes. Second, with Method U2 Agent i determines BM and and sets b^i such that $b^i = \max\{BM, BU\}$. Hence, four possible scenarios are considered, including the agents use 1) Methods M1 and U1, 2) Methods M2 and U1, 3) Methods M1 and U2, and 4) Methods M2 and U2.

Simulations

The agents in the following simulations, as well as the demand pattern, are the same as the previous simulations. Furthermore, the bidding price of the withheld capacity is set to P_{cap}, and all agents set $\Delta = 2$. Figure 4-40 shows the samples of simulated price dynamics when the agents use the unit-by-unit decision scheme with Methods M1 and U1 denoted by "U1 M1 D2", and with Methods M2 and U1 denoted by "U1 M2 D2". The simulated outcomes yield the same market prices from Methods U1 and U2; therefore, only the outcomes from Method U1 are presented. Figure 4-41 shows the prices obtained from the unit-by-unit decision schemes with Method U1 and from the portfolio-based scheme. In both cases, the agents use Method M1 to set Tar. In addition, Figure 4-42 shows the prices obtained from the unit-by-unit decision schemes with Method U1, and from the portfolio-based scheme when no CW strategy is present. Finally, Figure 4-43 shows the prices obtained from the unit-by-unit decision schemes with Method U1, and from the portfolio-based scheme when no CW strategy is present. To determine BM, the agents use Method M2 to set Tar. Note that in Figures 4-41 - 4-43 the simulated outcomes from the unit-by-unit decision scheme are denoted by "Unit-by-unit," and the outcomes from the portfolio-based scheme are denoted by "Portfolio."

Analyses

From Figures 4-40 - 4-43, one can observe that the price dynamics when the agents use Methods M1 and U1 are identical to the price dynamics when the agents use Methods M1 and U2; in addition, these price dynamics shift to a steady-state pattern. When the agents submit a marginal-cost bid for the first day, the agents will either increase or maintain the bidding price. (Recall that for any k the outcome $O_k = 10$ only when $0 \leq\ll OP_k < AP_k$ and $BM_k > MP_k$. When the agents submit the marginal-cost bid, the anticipated profit is equal to zero for some agents when the marginal cost is greater than the market price.) Note that when the agents determine the anticipated profit, they assume that the others submit marginal-cost bids. For the infra-marginal unit, the agents obtain satisfying outcomes, since the market price is higher than their bidding price and their BM_k; therefore, profits that they obtain (OP_k) are greater than anticipated profits (AP_k). Recall from the *OUTCOME* scheme that when $OP_k > AP_k$ and $BM_k < MP_k$, the agents have no incentive to adjust their bidding price.

Only the agents that are scheduled to operate at the margin raise the bidding price for the next period, because, for these agents, $BM_k = MP_k$ and $OP_k = AP_k$, resulting in $O_k = 11$. Hence, the agents with marginal cost equal to market prices, denoted by Group A, gradually raise their bidding price with an increment of \$2/MWh (that is, $\Delta = 2$) to be either higher than or equal to the agents with the next expensive marginal costs, denoted by Group B. At this point, with an argument similar to the one used in analyzing the effect of Δ on price dynamics, if Group A's bidding price is higher than Group B's bidding price, Group A no longer obtains profits as anticipated and decides to decrease their bidding prices. Group B, on the other hand, obtains the same profits that it anticipates (the market price is equal to the marginal cost of the agents in Group B). In the next period, Group A obtains the profits that it anticipates after lowering the bidding price; hence, Group A will increase the bidding price again. The same explanation is applied. As a result, one can observe that the market prices of the same load levels alternate between two values (odd and even values) over time.

In the unit-by-unit decision scheme, because the CW strategy is absent the agents are unable to raise the bidding price. In addition, if the $\Delta = 1$, the divergence of market prices can be observed when the agents use Method M1 together with U1 or U2. When the bidding prices of both Group A and Group B are the same (for the first time) and equal to market prices, following the *OUTCOME* scheme, both Groups A and B determine $O_k = 11$. This result occurs because, for Group A, $OP_k < AP_k$ and $BM_k = MP_k$, and for Group B, $OP_k = AP_k = 0$ and $BM_k = MP_k$. In the next period, the market price increases according to the bidding prices of the agents in both Groups. Hence, they both obtain $OP_k < AP_k$ (since there are more agents (Groups A and B) that are scheduled to operate at the margin) and $BM_k = MP_k$, and as a result $O_k = 11$.

Additionally, when the agents use Methods M2 and either U1 or U2 with $\Delta = 2$, the price dynamics of some hours, such as Hour 7, diverge. In this scenario, the infra-marginal agents raise their bidding price in the next period (since $BM_{k+1} = MP_k + \bar{c}$ and $MP_k > mc$, where mc denotes marginal cost). The marginal agents also raise their bidding price in the next period, because they obtain $OP_k = AP_k$ and $MP_k = BM_k$. The infra-marginal agents who obtain satisfying outcomes also raise the bidding price in the next bidding round. When these agents keep raising the bidding prices, their bidding prices may eventually be comparable to the more expensive units, as explained previously. As a result, several units with low marginal costs are scheduled to operate at the margin, whereas some units with expensive marginal costs are scheduled to operate as infra-marginal units. The profits that the anticipated marginal agents receive are less than they anticipate (since many units are scheduled to operate at the margin). As a result, the *OUTCOME* scheme yields $O_k = 11$; subsequently, the agents maintain increases in the bidding prices. As a result, divergence of the simulated market prices is common when the agents adopt Method M2 to determine the target price Tar in either the unit-by-unit or portfolio-based decision scheme.

Figures 4-44 and 4-45 illustrate the divergence of market price at Hour 7. Let us consider Agents 1 and 6 with marginal costs equal to \$10 and \$20/MWh, respectively. In Figure 4-44, "Price" denotes the simulated prices, "Comp" denotes the marginal-cost prices, "Agent 1" denotes the bidding prices of Agent 1, and "Agent 6" denotes the bidding prices of Agent 6. The competitive price at Hour 7 is \$20/MWh. Agent 1 is scheduled to operate as an infra-marginal unit, while Agent 6 is scheduled to operate at the margin on the first day. Even though the agents obtain satisfying outcomes, i.e., $OP_k > AP_k$, they could change the bidding price for the next period (BM_{k+1}), because when MP_k changes, BM_{k+1} changes regardless of the bidding outcomes.

Implication: Generating-unit Divestitures

From the results shown above, one may conclude that when the agents use the learning scheme, such as Method M2 and $\Delta = 2$, that yields the divergence of simulated price dynamics regardless of the decision schemes, no conclusive effect of the decision schemes on price dynamics is made. On the other hand, when the agents use the learning scheme, such as Method M1 and $\Delta = 2$, which yields non-increasing dynamics over time, the unit-by-unit decision scheme results in lower market prices than those obtained from the portfolio-based decision scheme. Recall that the agents do not exercise the CW strategy in the unit-by-unit scheme simulation, while they exercise the CW strategy in the portfolio scheme. The CW strategy is a critical factor in causing the expensive bid-supply functions and, consequently, in resulting in increases in price markups. As shown in Section 4.6.1, when the agents employ only the price-setting strategy, the simulated prices may be lower than those when the capacity withholding strategy is in place. The agents may be unable to withhold their capacity because they may own a number of generating units which have to operate at their full capacity or not operate at all, and because they may have less capacity than that required to implement the strategy successfully ($W_k^{i,*}$).

In summary, this outcome suggests that when the agents own small portfolios of generating units, which are portfolios consisting of small capacity units or a few small generating units, they are less likely to submit the strategic bid-supply functions that substantially deviate from the marginal-cost, bid-supply functions, and cause the high price markups. Consequently, to increase the possibility of achieving perfect competition as a result of power-system privatization, when a regulator divests the generating units, the largest portfolio should have as small a capacity as possible. In addition, as described in Chapter 2, the agents could not increase the bidding prices when they own portfolios with uniform capacity and/or the same level of marginal cost as when they own asymmetric portfolios. The regulator should be concerned that after divestiture there should be as many power producers as possible. To sum up, the largest portfolio in the markets should have as small a capacity as possible, and power producers should have as similar portfolios as possible.

4.7 Verification of Agent-based Market Model

This section introduces two methods to determine whether the agent-based model can create dynamics sufficiently close to those of the actual system that it represents. The outcomes from these methods indicate the relationship between the learning algorithms that the agents use and the actual bidding strategies that the power producers use in the actual system. The first method measures the average square deviation of the simulated outcomes from the actual ones. The other method applies the concept of Chernoff Bound to determine the "correctness" probability of the simulated outcomes relative to the actual outcomes. In addition, it is important to determine the degree to which the error between the simulated and the actual outcomes changes over time, for example whether error decreases as the simulation time proceeds. Unfortunately, numerical verification of the model cannot be presented because the required data are confidential and were not made available.

4.7.1 Average Square Error

This section presents the average square error method, which measures the error between the actual and simulated outcomes when the model has the same input, such as inelastic demand and agent characteristics, as the actual system. The smaller the error, the better the model. This method is described as follows. Let y be the actual outcome observed from the market and let $\tilde{y}(A)$ be the simulated outcome obtained when the agents use a learning algorithm A. The error for any period k ($e_k(A)$) is the difference between the actual and simulated outcomes, i.e., $e_k(A) = y_k - \tilde{y}_k(A)$. The average square error for a T-period interval of the algorithm A, $E_T(A)$, is equal to

$$E_T(A) = \frac{\sum_{k=1}^{T}(e_k(A))^2}{T} = \frac{\sum_{k=1}^{T}(y_k - \tilde{y}_k(A))^2}{T}.$$

The model that closely mimics the actual system over time T shall have this property

$$\lim_{T\to\infty} E_T(A) = 0.$$

That is, the difference between the actual and simulated outcomes will decrease, causing the average square error to converge to zero over time. Since modeling the behavior of decision-makers closely is difficult, the ideal model, which would yield $\lim_{T\to\infty} E_T(A) = 0$, may not exist. Therefore, instead of identifying the ideal model, one may consider a model that yields acceptable error, i.e., within a desirable threshold $\acute{E}$, or $E_T(A) \leq \acute{E}$ for some period T. Another way to interpret this measure is to say that after time T, if $E_T(A) \leq \acute{E}$, the model used in conjunction with learning algorithm A is sufficiently good to represent the actual system.

4.7.2 Probability of Correctness

Another measurement of error uses the concept of on-line prediction. The simulated outcomes from the agent-based model, in which agents use any learning algorithm, can be viewed as an on-line prediction. Also the agent-based model could also be considered as one player in a two-person general-sum game playing against nature with the payoff values equal to $\{0, 1\}$. In this two-person game, the player representing the agent-based model has a strategy which chooses actions based on the simulated outcome from the model, whereas the other player representing nature has a strategy which chooses actions based on actual outcomes. The payoffs of this game indicate how closely the model predicts the actual system.

This method is described as follows. Let us define a bi-matrix game with finite possible actions and let us define an acceptable error Δ. When the difference between the simulated outcomes and the actual ones is within this acceptable band, the player obtains the payoff equal to zero. On the other hand, when the difference is outside this band, the player obtains the payoff equal to one. Over time T, the sum of the payoffs that the player obtains is the accumulative error between the simulated outcomes and the actual ones. A payoff matrix of this game, in which the nature is a column player, is as follows:

$$\begin{bmatrix} 0 & \dots & 0 & 1 & \dots & & & \dots & 1 \\ \vdots & & & & & & & & \\ 1 & \dots & 1 & 0 & \dots & 0 & 1 & \dots & 1 \\ \vdots & & & & & & & & \\ 1 & \dots & & & \dots & 1 & 0 & \dots & 0 \end{bmatrix}.$$

The payoff matrix has diagonal elements equal to zero. The concept of Chernoff Bound is applied to determine the probability that the errors are within the acceptable band Δ is greater than a threshold $\acute{Z}$ over period T. Let p denote this probability and let X_k be an indicator of a random variable at any period k that is equal to 1 if the player receives a payoff equal to 0, and otherwise it is equal to 0.

According to the Chernoff Bound, $\text{Prob}\,(\sum_{k=1}^{T} X_k \geq \acute{Z}) \;\leq\; e^{-s\cdot\acute{Z}} \cdot \mathcal{E}(e^{s(\sum_{k=1}^{T} X_k)})$, where s is a positive constant and $\mathcal{E}(\cdot)$ denotes the expected value. Therefore, the probability that the error is less than the threshold $\acute{Z}$ is

$$\text{Prob}\,(\sum_{k=1}^{T} X_k < \acute{Z}) \;\geq\; 1 - e^{-s\cdot\acute{Z}} \cdot \mathcal{E}(e^{s(\sum_{k=1}^{T} X_k)}).$$

Note that if X_k is an independent random variable, this probability is then equal to

$$\text{Prob}\,(\sum_{k=1}^{T} X_k < \acute{Z}) \;\geq\; 1 - e^{-s\cdot\acute{Z}} \cdot (\mathcal{E}(e^{sX}))^T,$$

and $\mathcal{E}(e^{s(\sum_{k=1}^{T} X_k)}) = p \cdot e^{S\cdot 0)} + (1-p)\cdot e^S$.

Since the outcomes of an agent-based model depend on the learning algorithms/strategies of the agents, the cumulative error indicates how well the learning algorithms/strategies contribute to the dynamics of the actual decision-makers' behavior. One can notice that the method presented in this section is similar to the average square error method, which is to determine numerically the deviation of the simulated from the actual outcomes over time.

4.7.3 Insufficient Information

This thesis does not provide the numerical calculation of both methods to demonstrate the accuracy of the model and the actual market because of insufficient information required for constructing the model. This missing information includes marginal-cost functions, operating constraints and/or unit-commitment constraints, bilateral contract obligations, as well as outage and maintenance schedules of the power producers. Furthermore, in some markets, such as in New England, to schedule the generating units to meet demand, the system operator incorporates power system operating constraints, including transmission constraints, voltage support, and operating reserve requirement. The preliminary study by Visudhiphan *et al.* [45] on the dispatch process shows that, depending on whether the power system operating constraints as well as the generating units' operating constraints are accounted for, one may not be able to reproduce the market prices from the observed bid-supply functions and demands. These factors need to be considered before comparing the observed market prices to the simulated prices which do not account for them.

Moreover, in the learning algorithms used in this thesis, the choices of possible bid-supply functions are limited, i.e., the agents choose a bidding price from zero to a price cap with an increment of $3/MWh when Algorithms A1, A2, A3, and SAB are present. Conversely, in the actual market, the power producers can choose the bidding prices from any value less than a price cap. This actual market feature could yield different equilibrium strategies compared to the model with the limited choices of bidding prices (recall Chapter 2). In addition, the agents discretize the range of possible bidding prices (such as from zero to a price cap) using the same increment and also adjust the bidding price in the model-based algorithm using the same increment. This uniform behavior may not represent the actual behaviors of the power producers whose bidding strategies are confidential and possibly unique.

Consequently, one can conclude that the factors that hinder the proposed model verification include lack of system information, complexity of power system operations and their effects on market clearing outcomes, and infinite sets of possible actions of the market participants. However, to demonstrate that the agent-based modeling approach should be considered an appropriate tool in analyzing the electricity spot markets, despite these issues of the model verification, a study on the New England power-producer bidding behavior is presented in the next chapter. This study will show that the power producers exhibit time-varying bidding behavior that may result from learning, and also that the bidding behavior of each power producer is rather unique.

4.8 Conclusion

The agent-based approach to model electricity markets contains both advantages and disadvantages. This approach provides high flexibility in formulating the agents; that is, the agents can be modeled to have different marginal costs, capacities, objective functions, and/or learning algorithms. This chapter explores the agent-based model when the agents employ different learning algorithms and then draws a relationship between the simulated price dynamics and the learning algorithms. The simulations show that the simulated outcomes depend highly on not only the agents' characteristics, but also on the learning algorithms that the agents use. The disadvantages of this modeling approach lie in the difficulty in model verification. To use this model to analyze any actual market, the model should be tested to prove whether or not it closely represents the actual market. Unfortunately, without information relating to power-system operating constraints, generating-unit operating constraints and characteristics, and system conditions, model verification turns out to be nearly impossible.

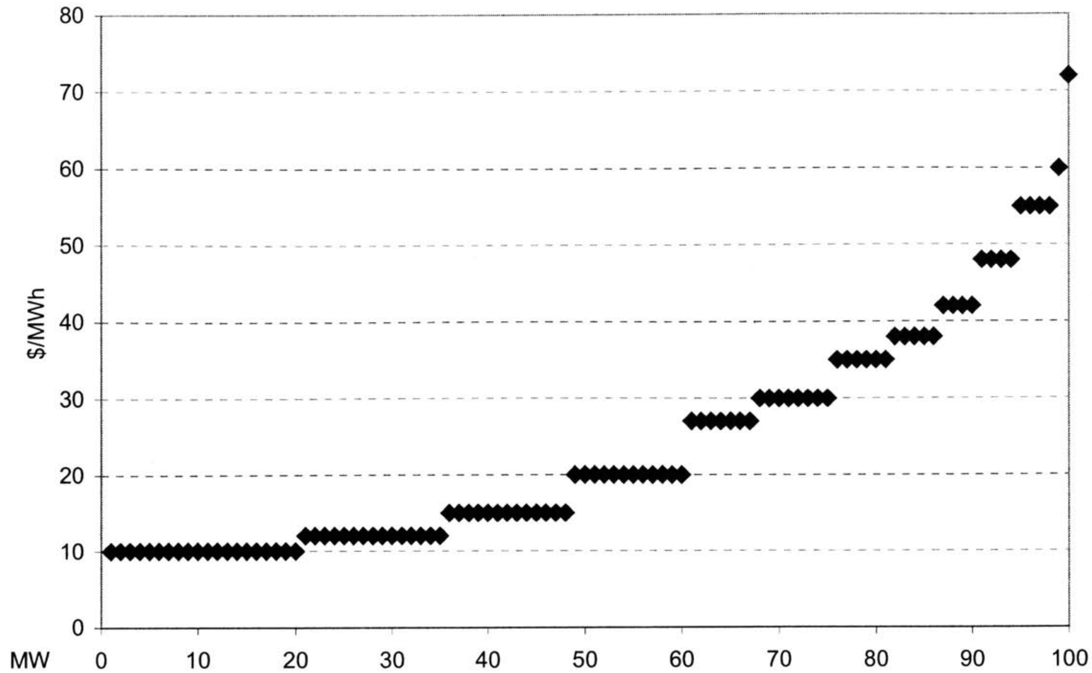

Figure 4-1: Aggregate Marginal-cost Function of the Hypothetical Market

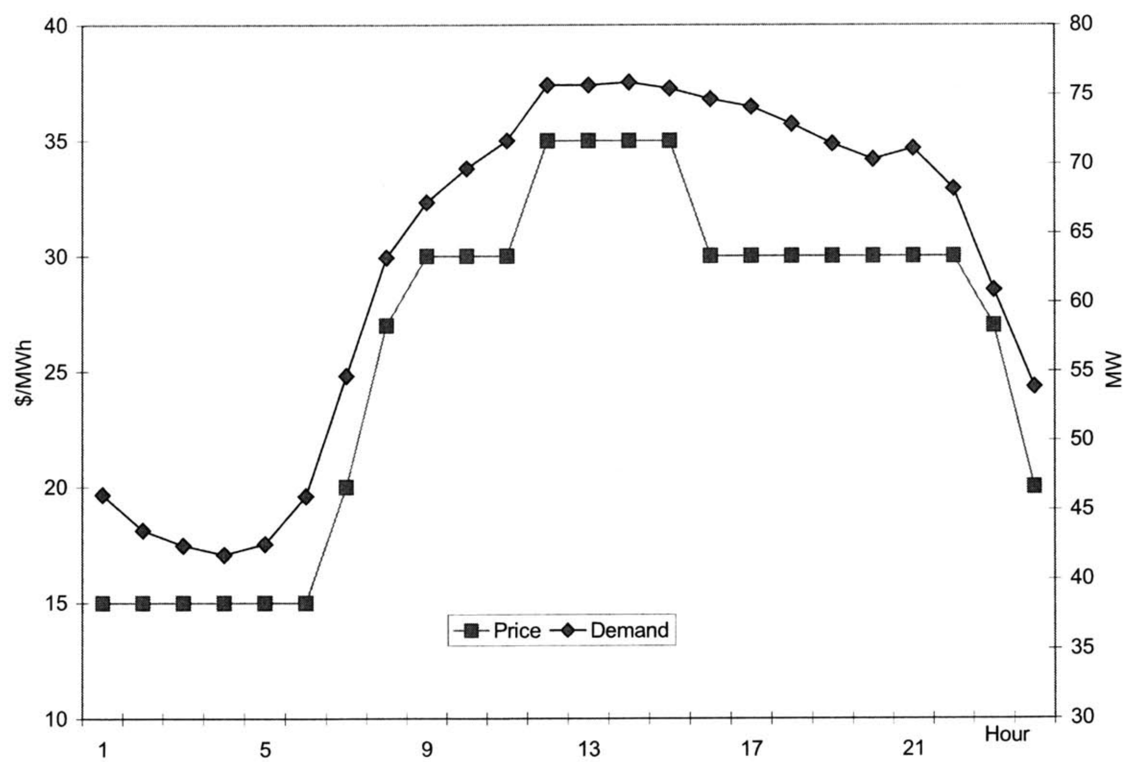

Figure 4-2: Hourly Inelastic and Deterministic Demands and Marginal-cost Prices

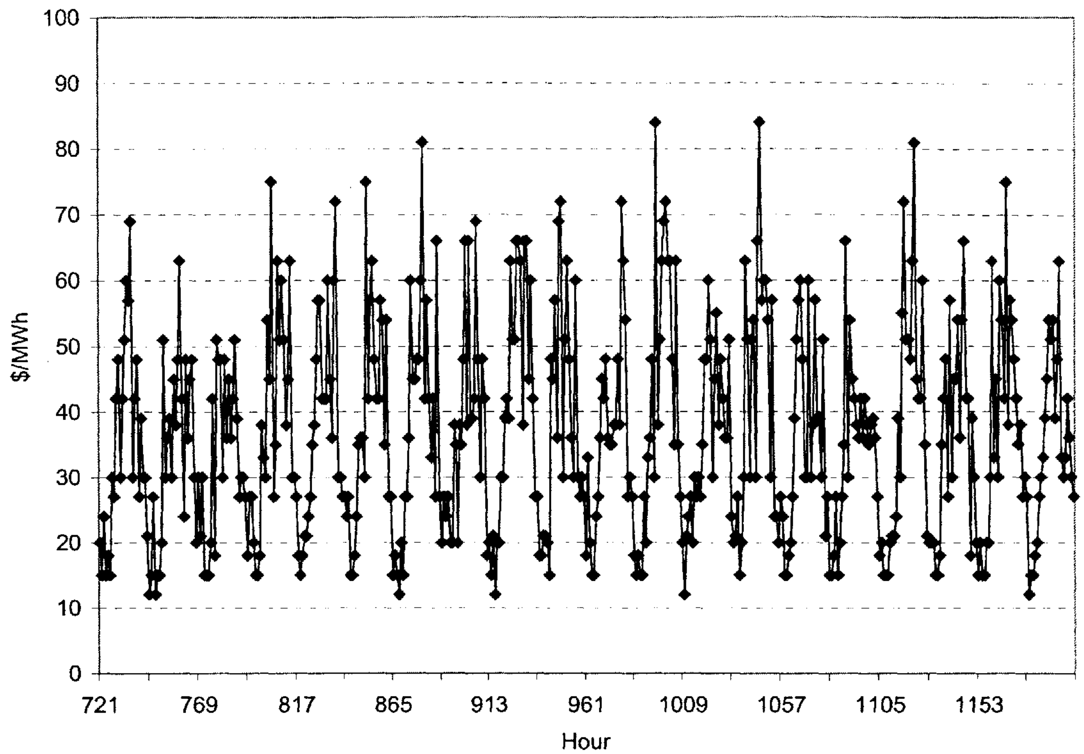

Figure 4-3: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A1 with $\gamma = 0.1$

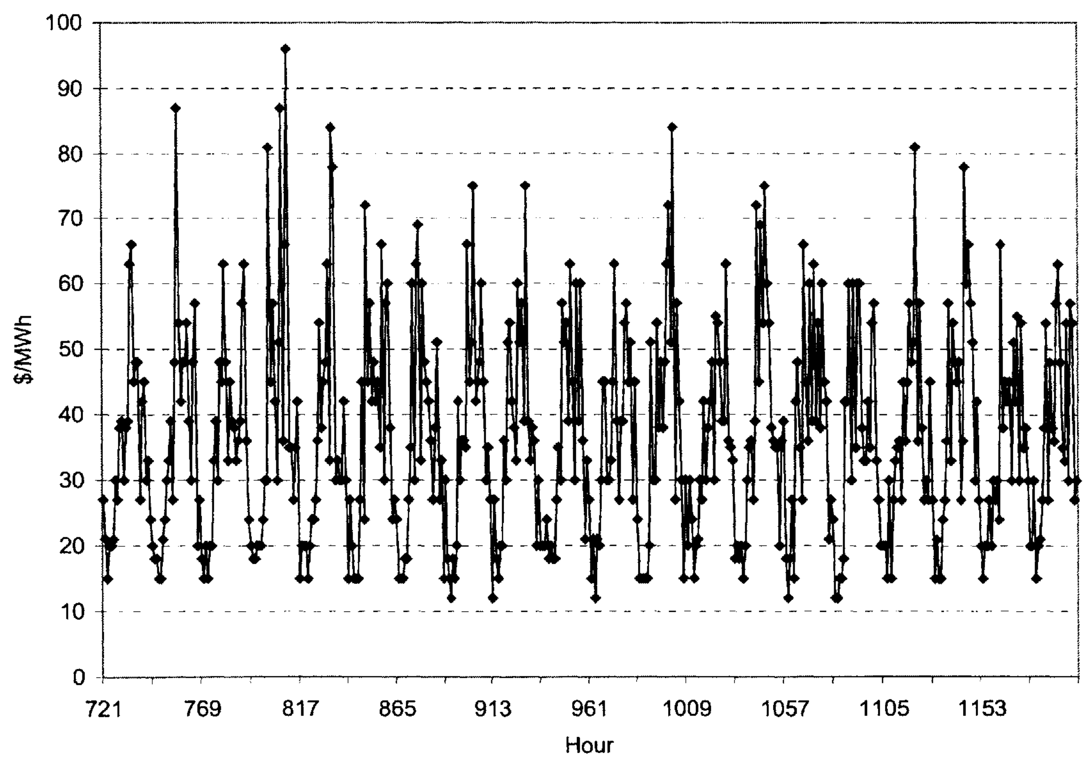

Figure 4-4: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A1 with $\gamma = 0.9$

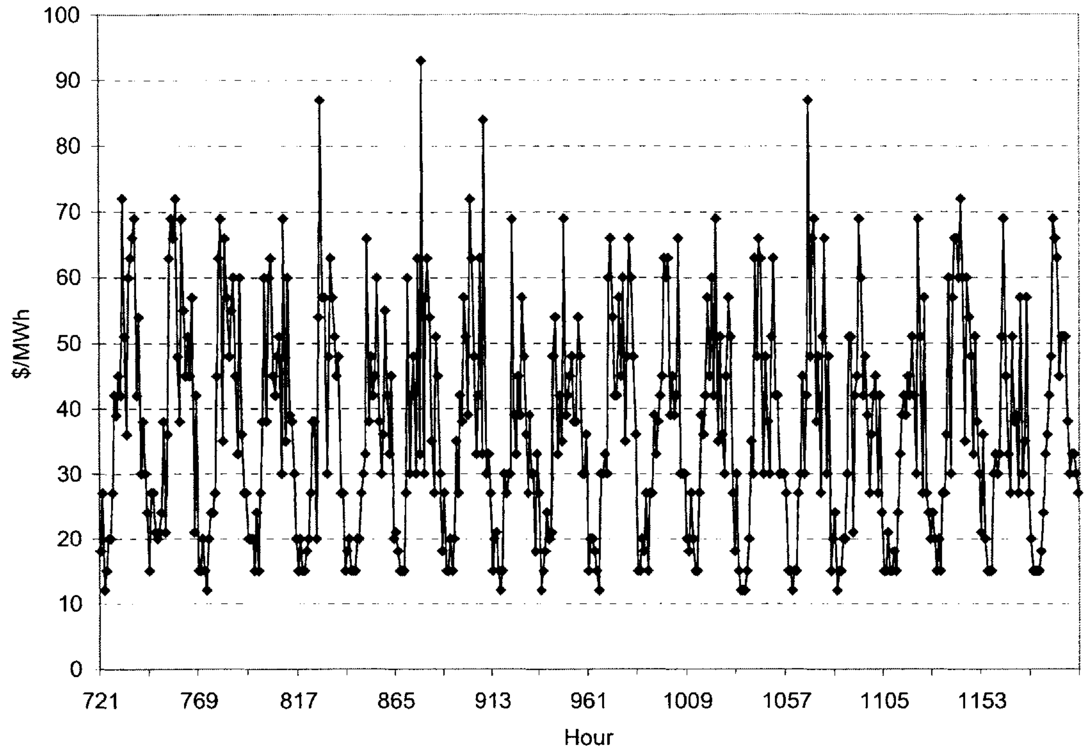

Figure 4-5: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A2

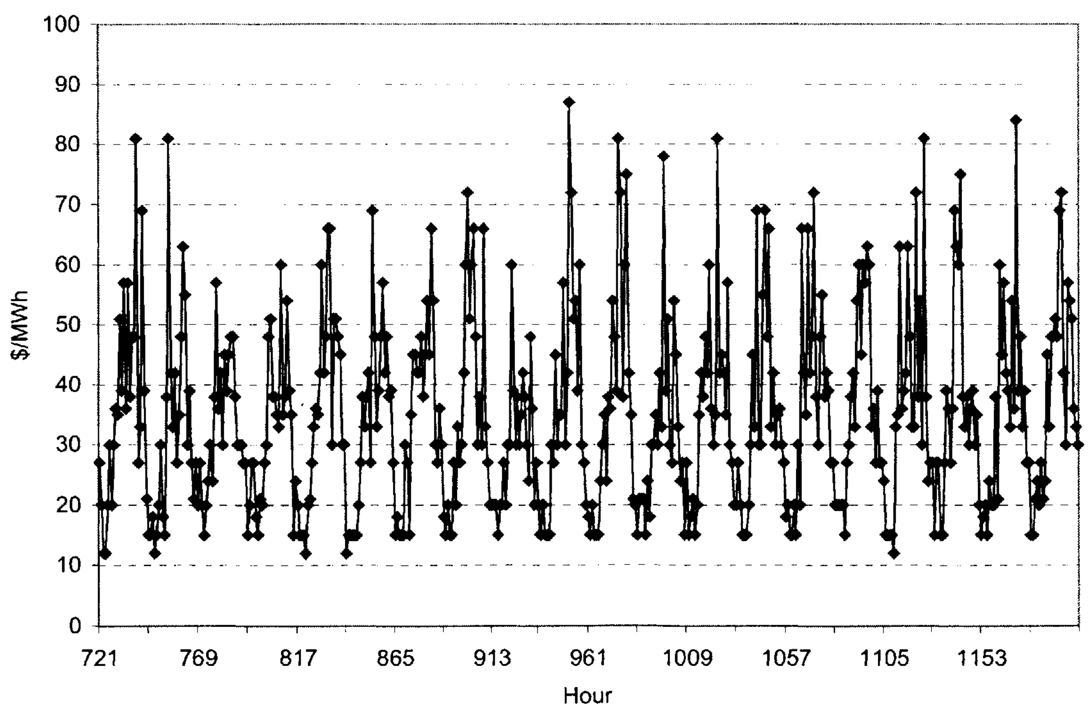

Figure 4-6: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A3 with $\delta = 0.1$

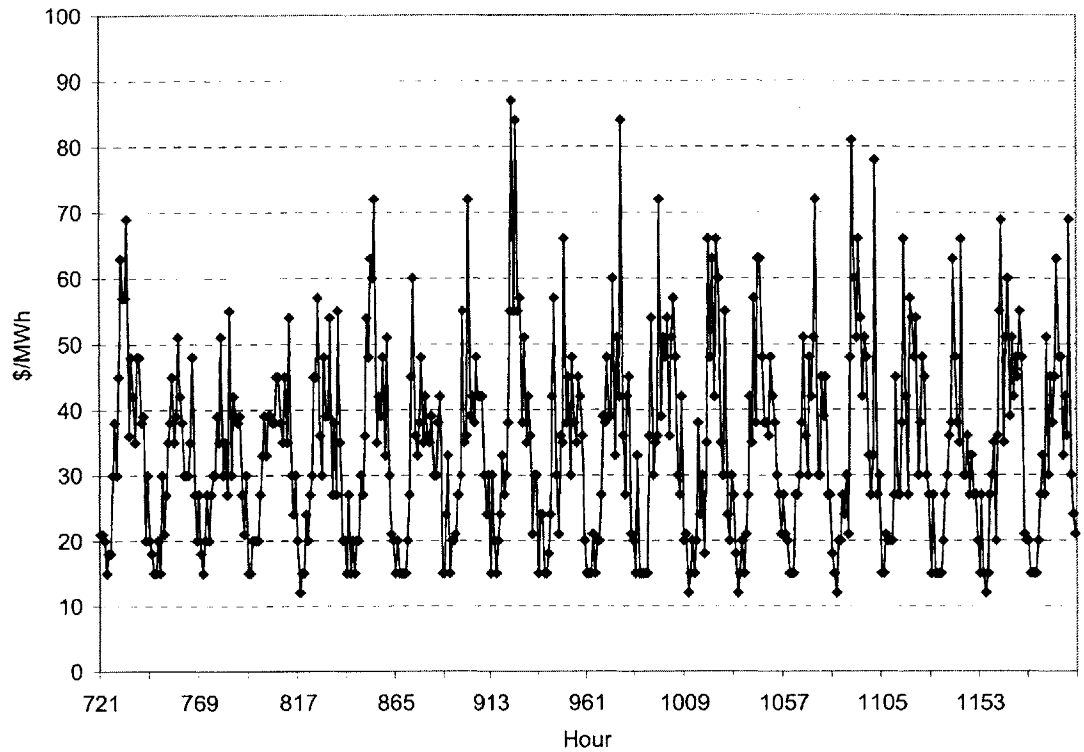

Figure 4-7: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A3 with $\delta = 0.5$

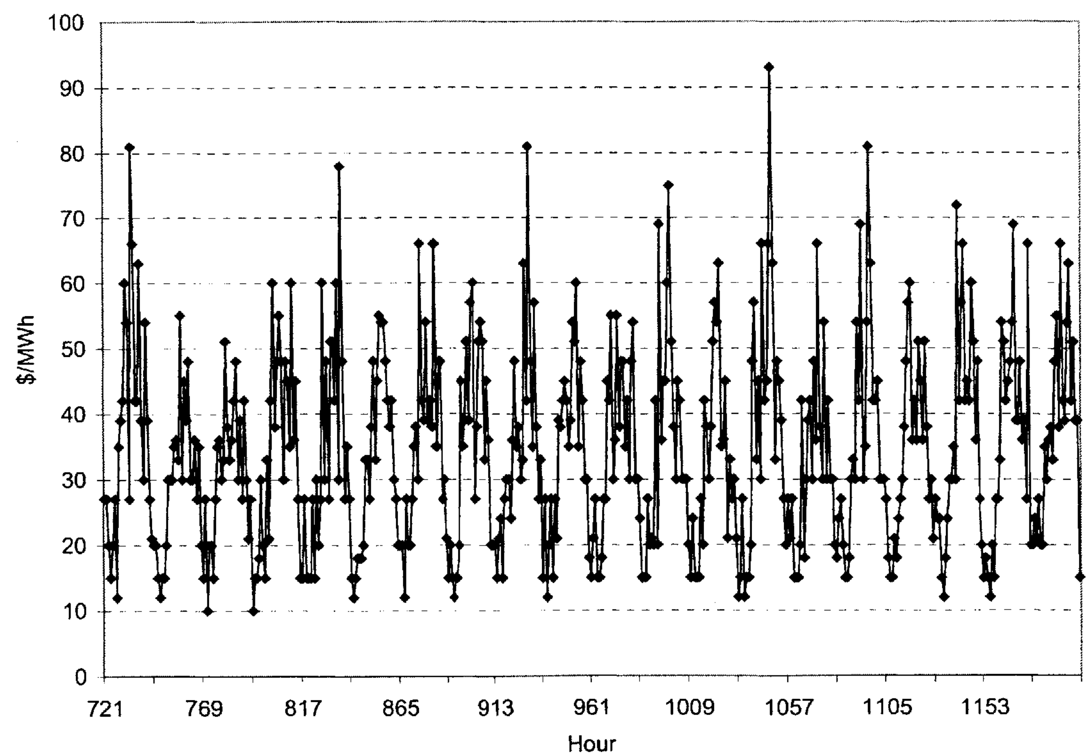

Figure 4-8: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm A3 with $\delta = 0.9$

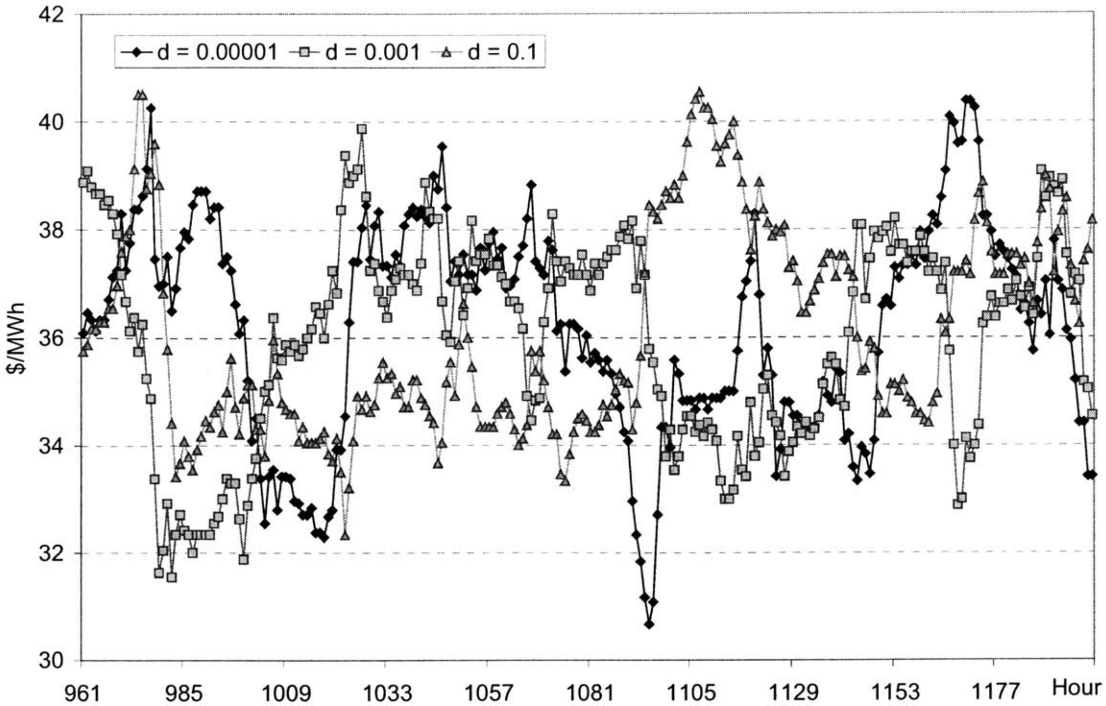

Figure 4-9: Moving-average Prices from Hours 961 to 1,200 When the Agents Employ Algorithm A3 with $\delta = 0.1$, 0.001, and 0.00001

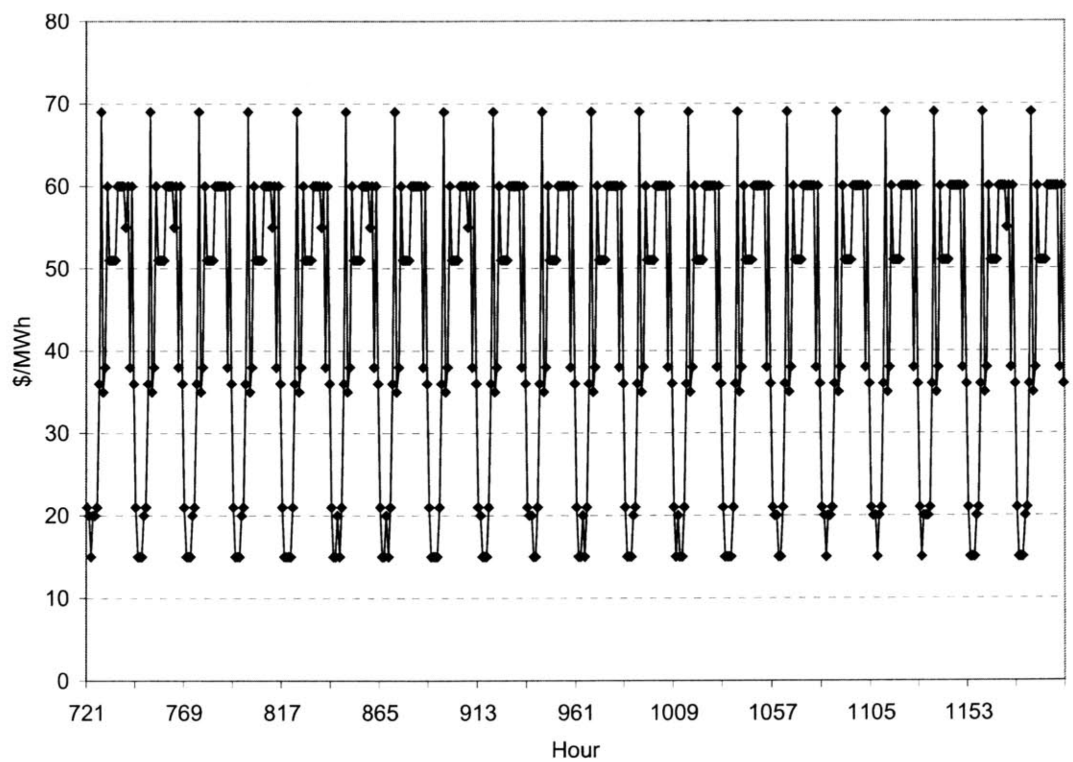

Figure 4-10: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm SAB with α = 0.9 and $\tau = 1$

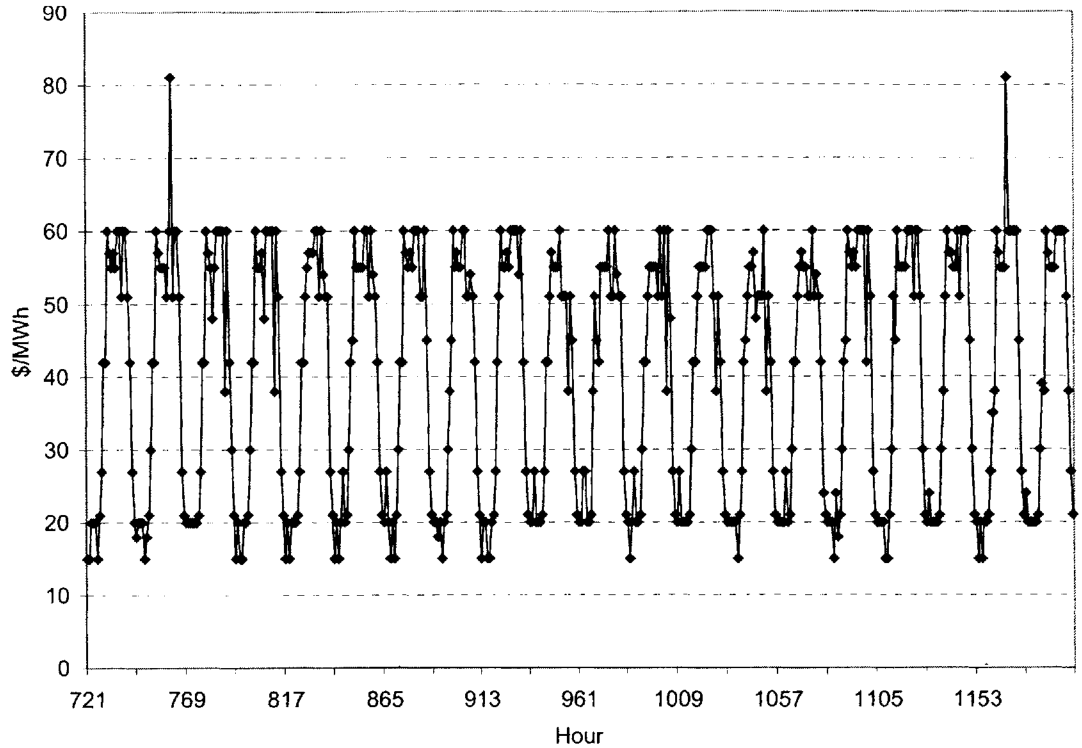

Figure 4-11: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm SAB with α = 0.9 and τ = 10

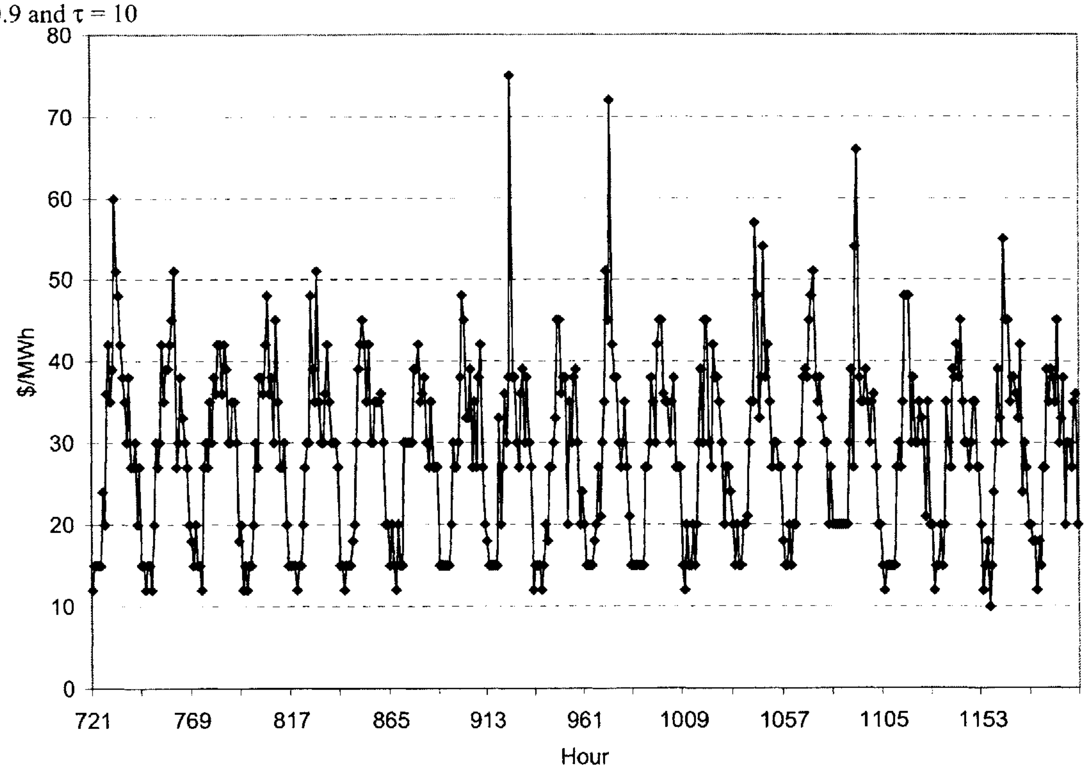

Figure 4-12: Price Dynamics during Hours 721- 1200 When the Agents Employ Algorithm SAB with α = 0.9 and τ = 100

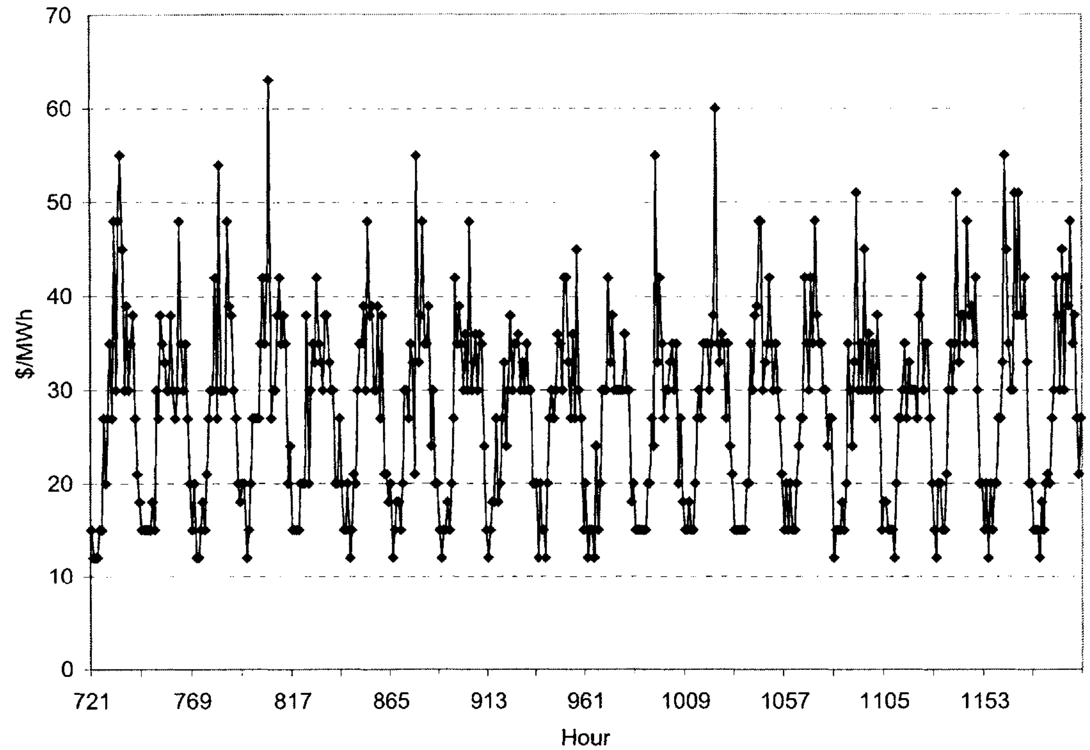

Figure 4-13: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm SAB with α = 0.1 and τ = 100

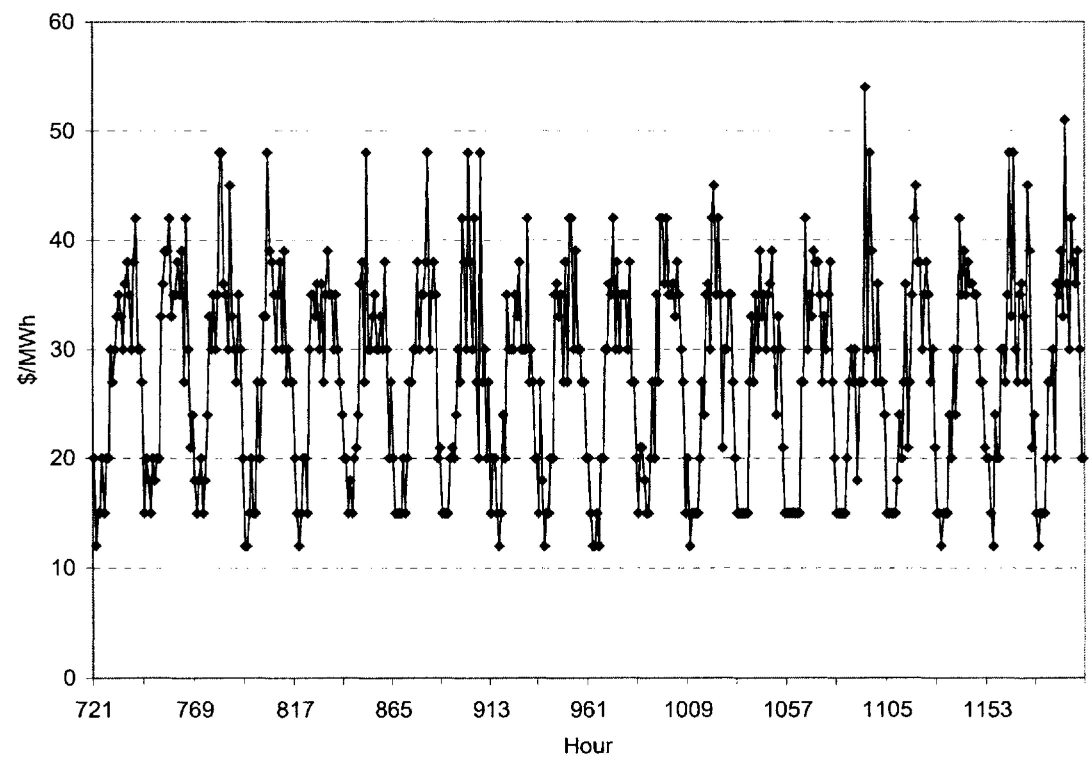

Figure 4-14: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm SAB with α = 0.5 and τ = 100

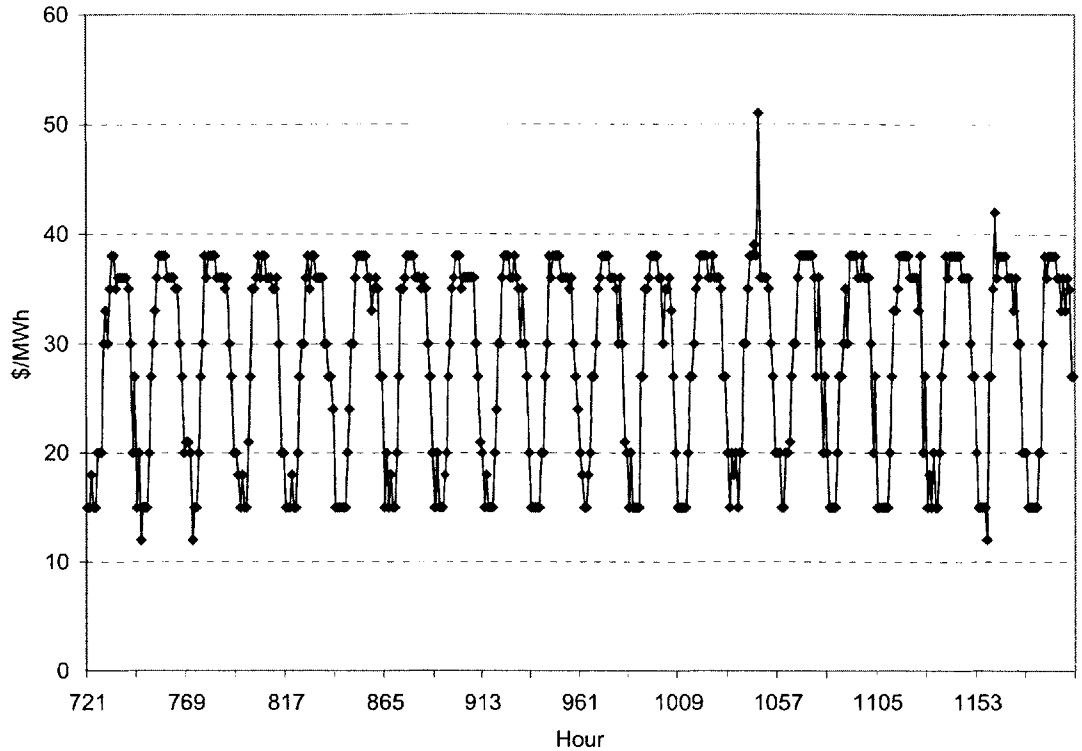

Figure 4-15: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithm SAB with α = 0.1 and $\tau = 10$.

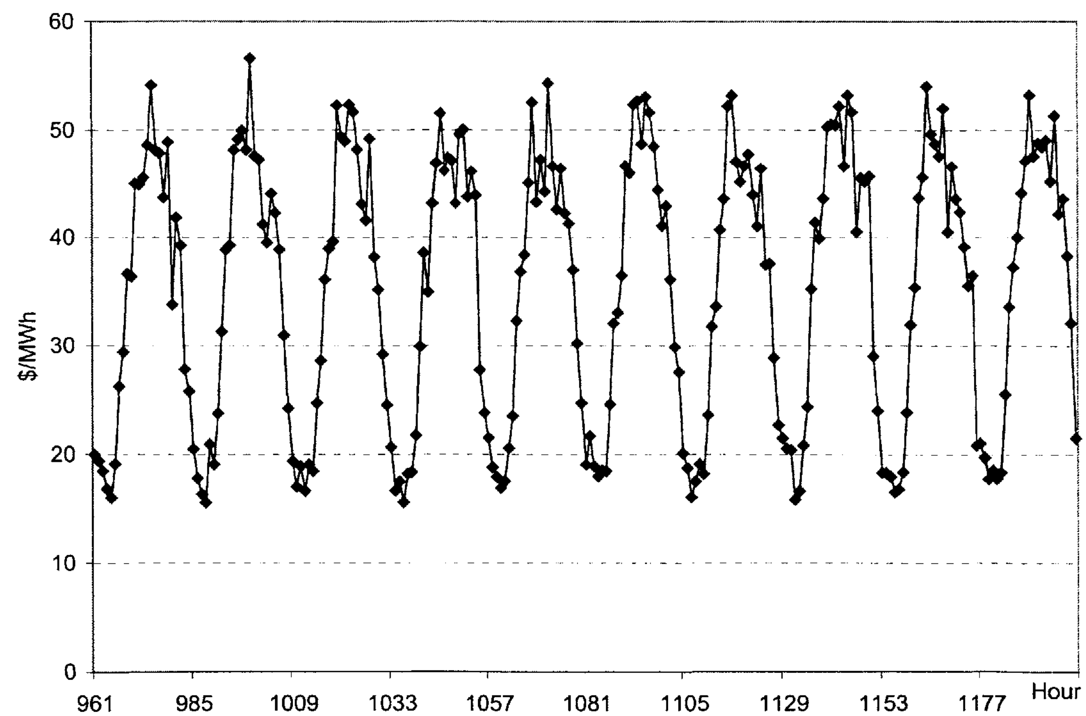

Figure 4-16: Average Price Dynamics across 100 Simulations from Hours 961 to 1,200 When the Agents Employ Algorithm A3 with $\delta = 0.1$.

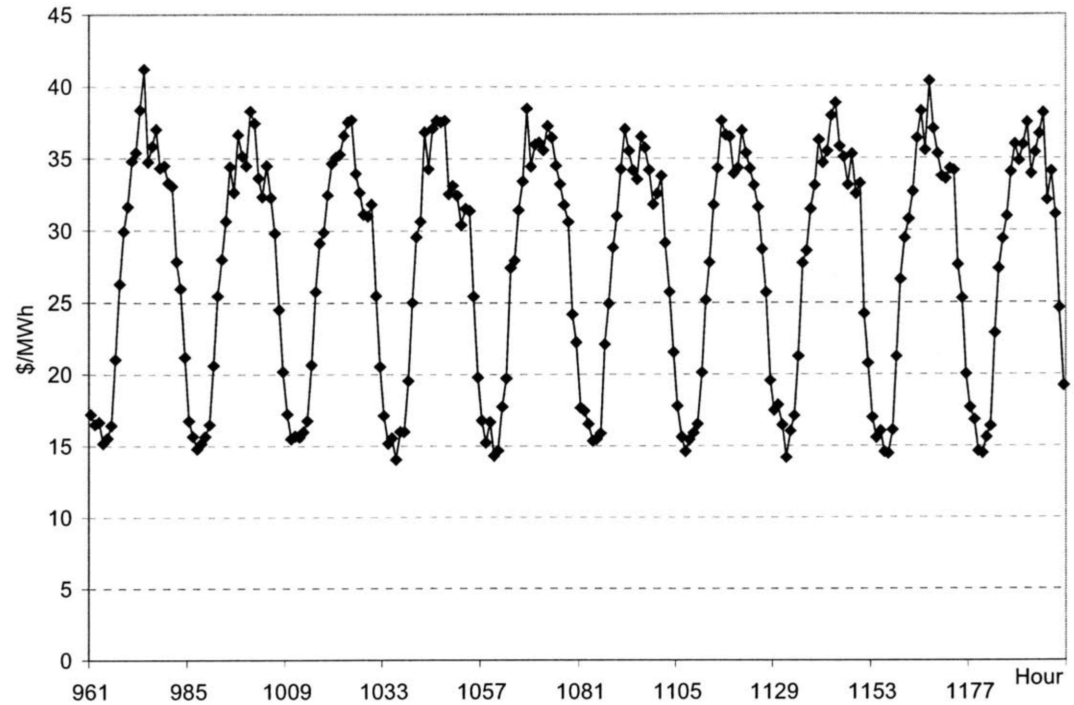

Figure 4-17: Average Price Dynamics across 100 Simulations from Hours 961 to 1,200 When the Agents Employ Algorithm SAB with $\alpha = 0.1$ and $\tau = 100$

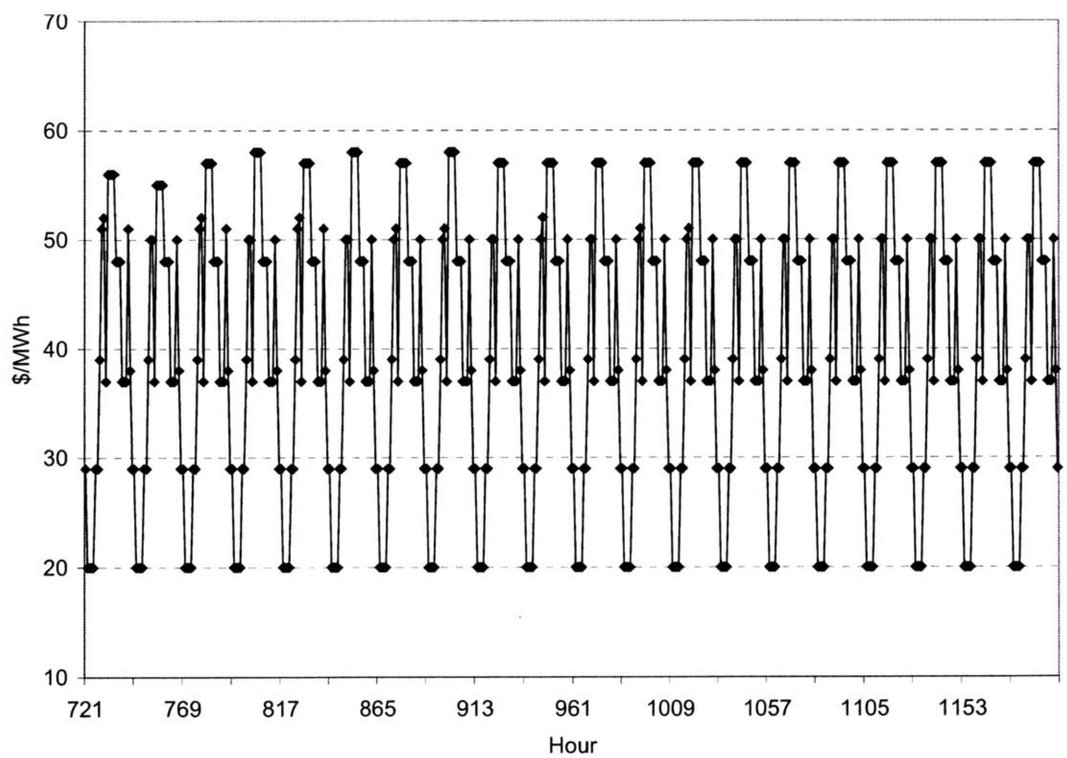

Figure 4-18: Price Dynamics from Hours 721 to 1,200 When the Agents Employ the Model-based Algorithm with Methods M1 and C2 and $\Delta = 2$

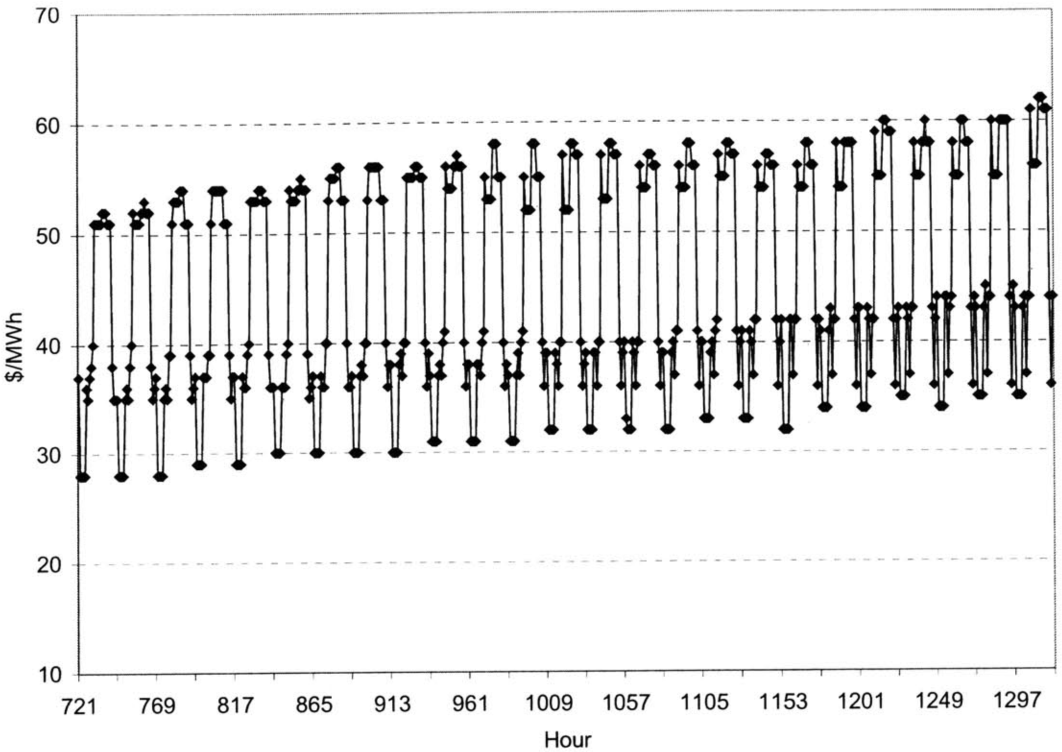

Figure 4-19: Price Dynamics from Hours 721 to 1,320 When the Agents Employ the Model-based Algorithm with Methods M2 and C2 and $\Delta = 2$

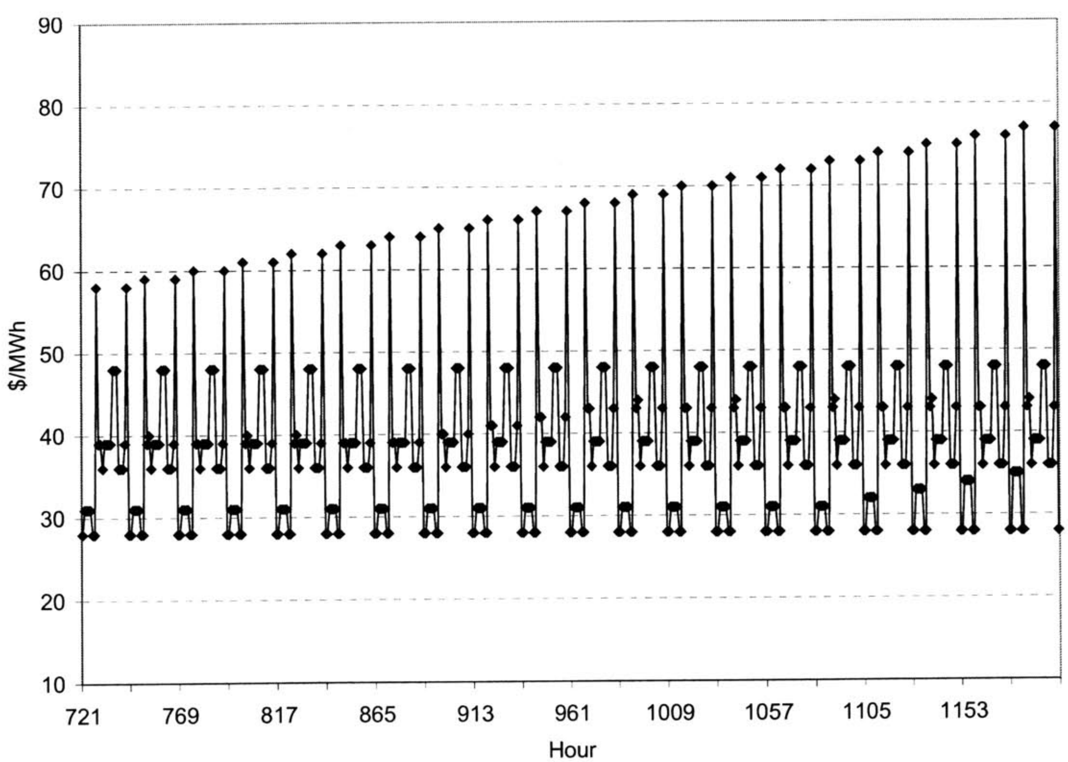

Figure 4-20: Price Dynamics from Hours 721 to 1,200 When the Agents Employ the Model-based Algorithm with Δ=1 and Methods M1 and C2

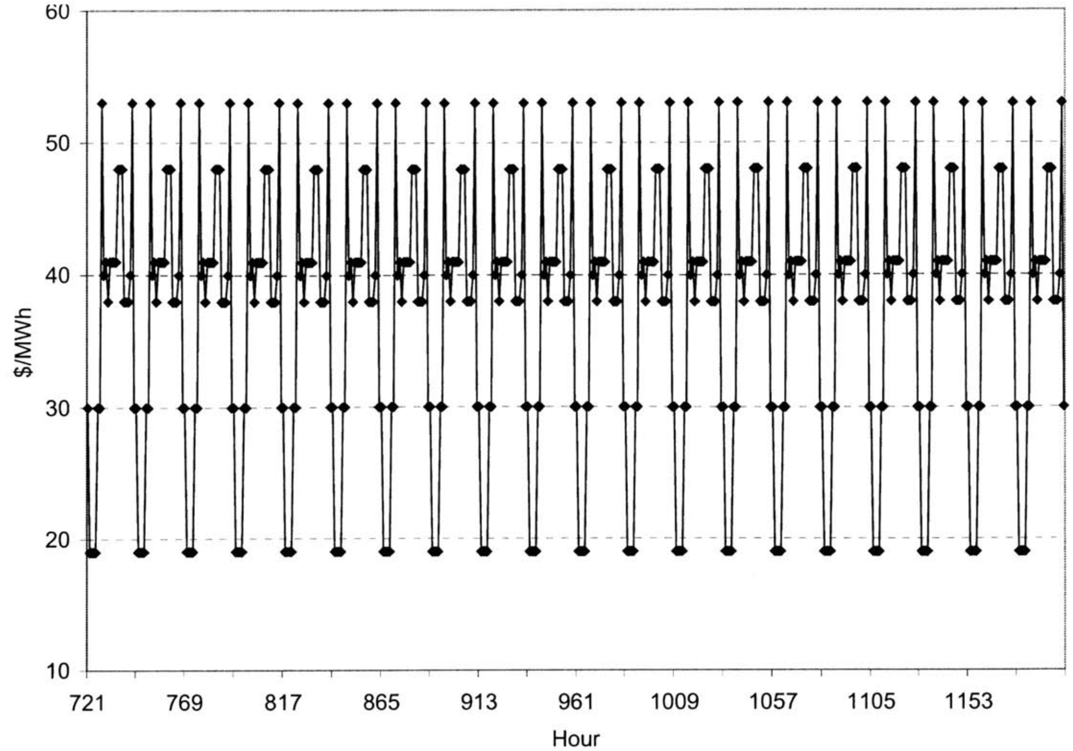

Figure 4-21: Price Dynamics from Hours 721 to 1,200 When the Agents Employ the Model-based Algorithm with Δ=3 and Methods M1 and C2

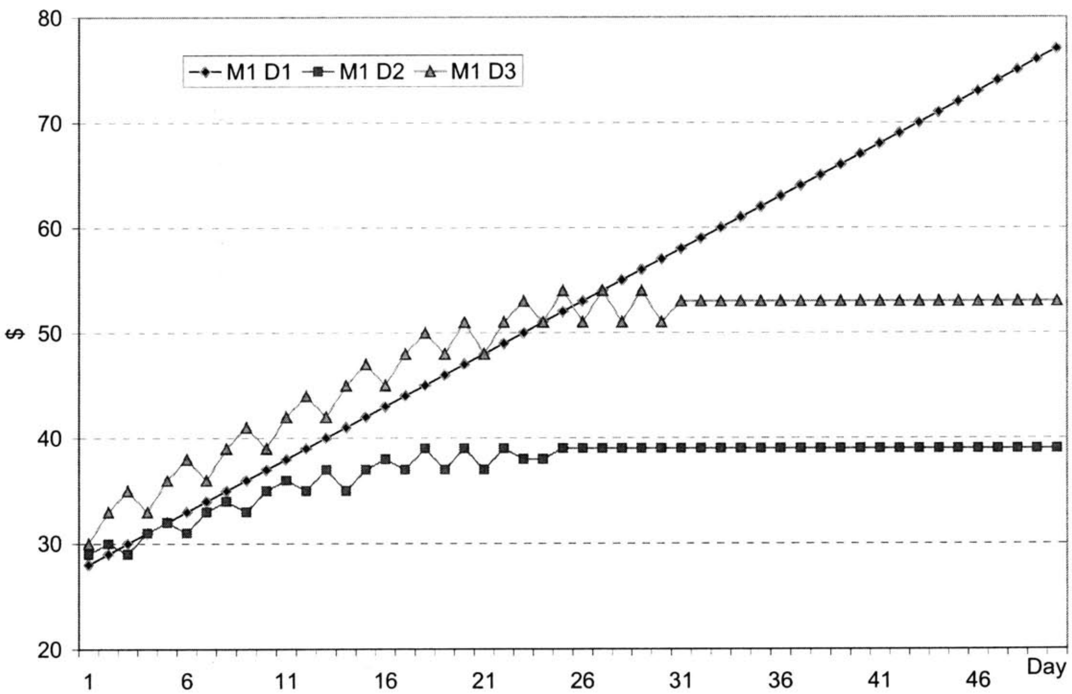

Figure 4-22: Daily Price Dynamics at Hour 8 When the Agents Employ the Model-based Algorithm with Δ=1, 2, or 3

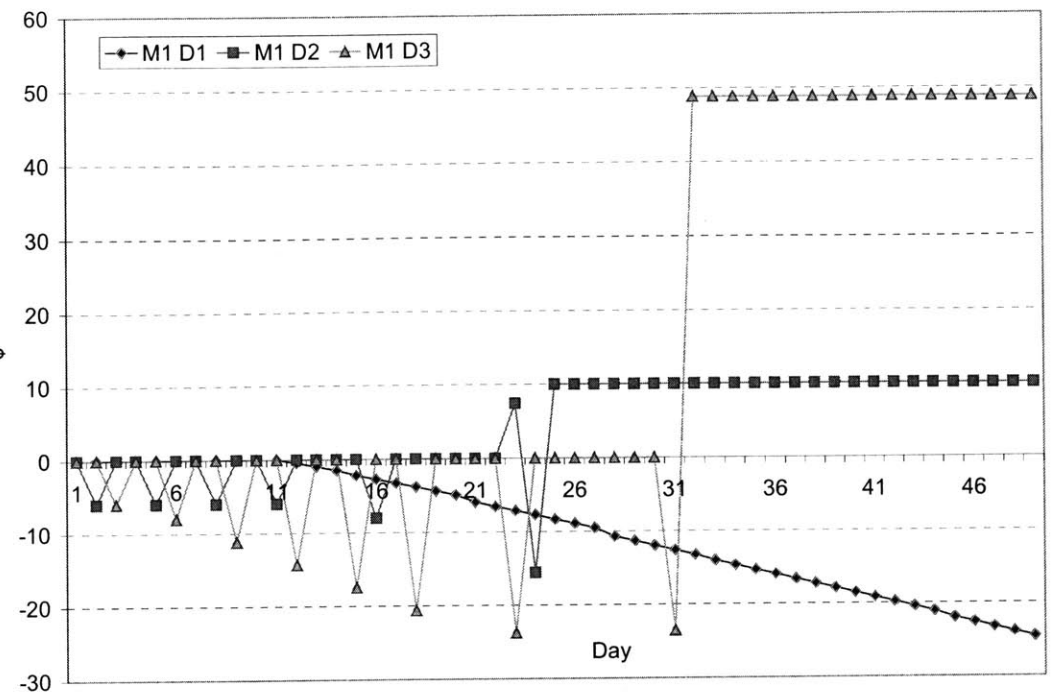

Figure 4-23: Daily Profits that Agent 1 Obtains at Hour 8 When the Agents Employ the Model-based Algorithm with Δ=1, 2, and 3.

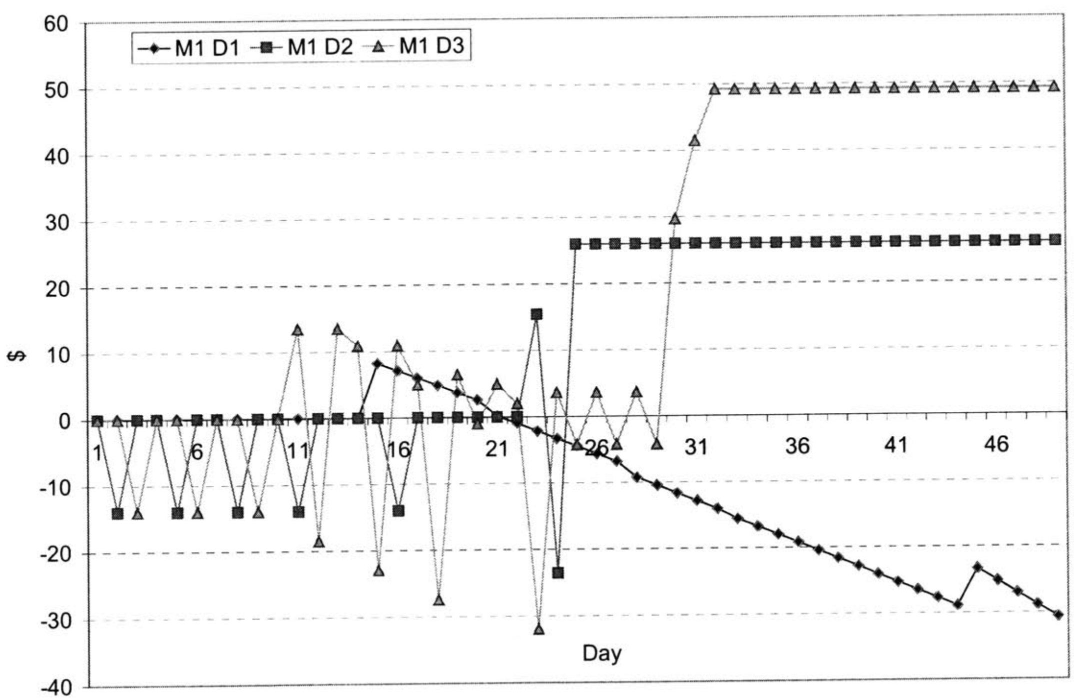

Figure 4-24: Daily Profits that Agent 5 Obtains at Hour 8 When the Agents Employ the Model-based Algorithm with Δ=1, 2, and 3.

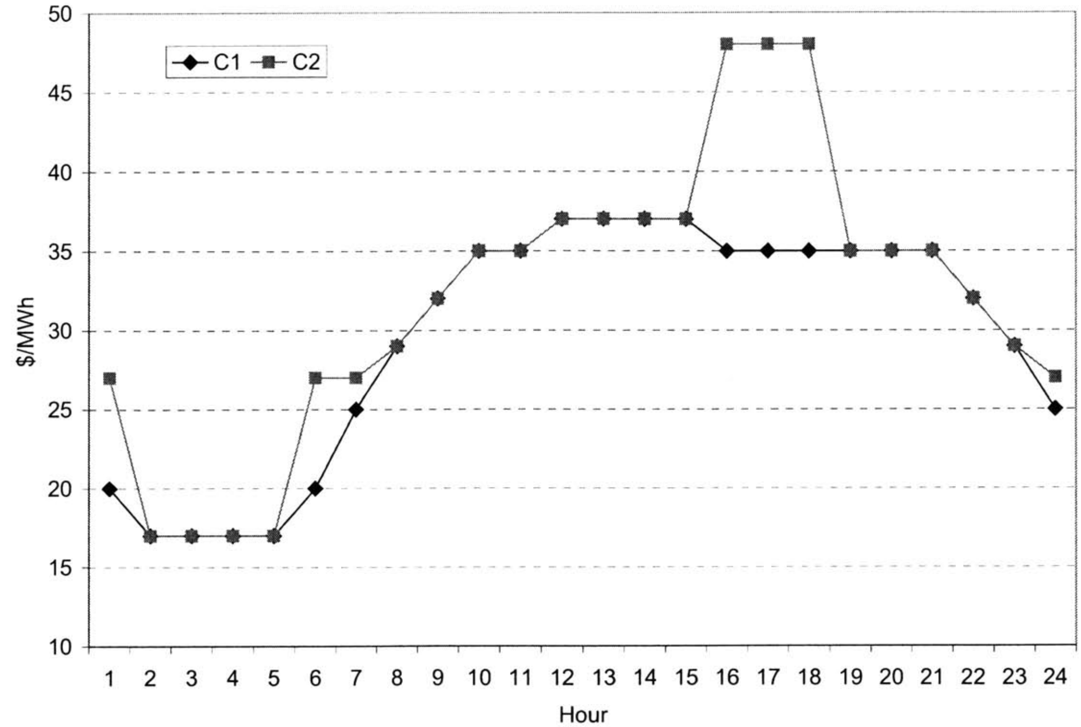

Figure 4-25: Samples of Simulated Prices When the Agents Employ the Model-based Algorithm with Method C1 or C2 to Determine WH.

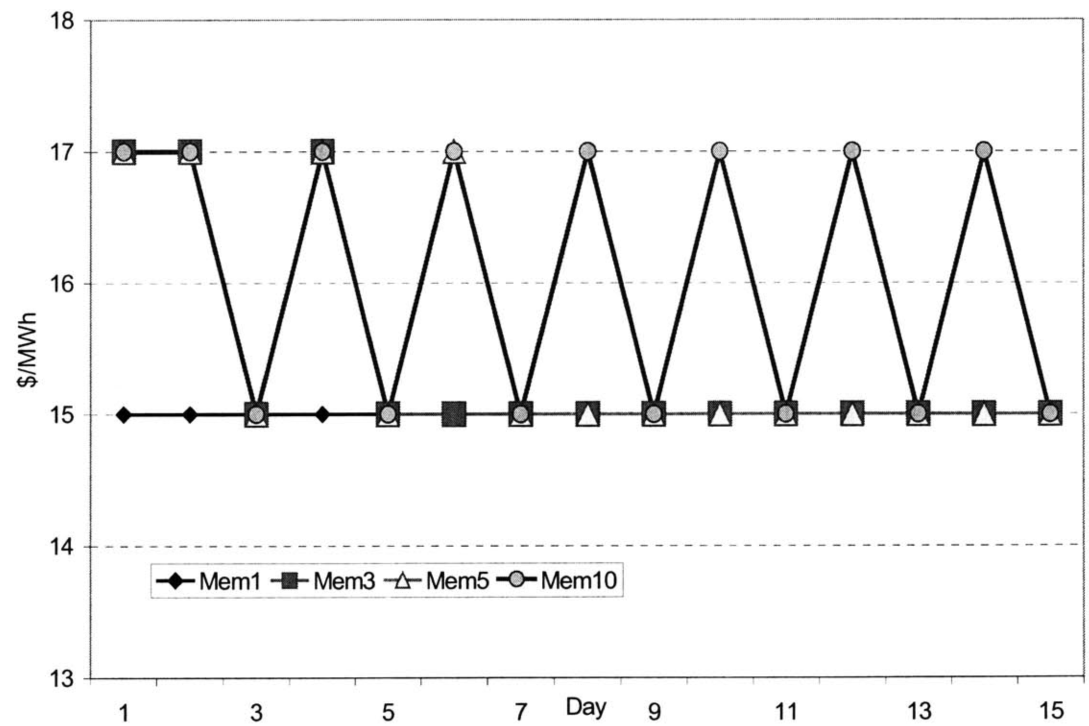

Figure 4-26: Price Dynamics When the Agents Employ the Model-based Algorithm with and Method M2 and a GM Matrix

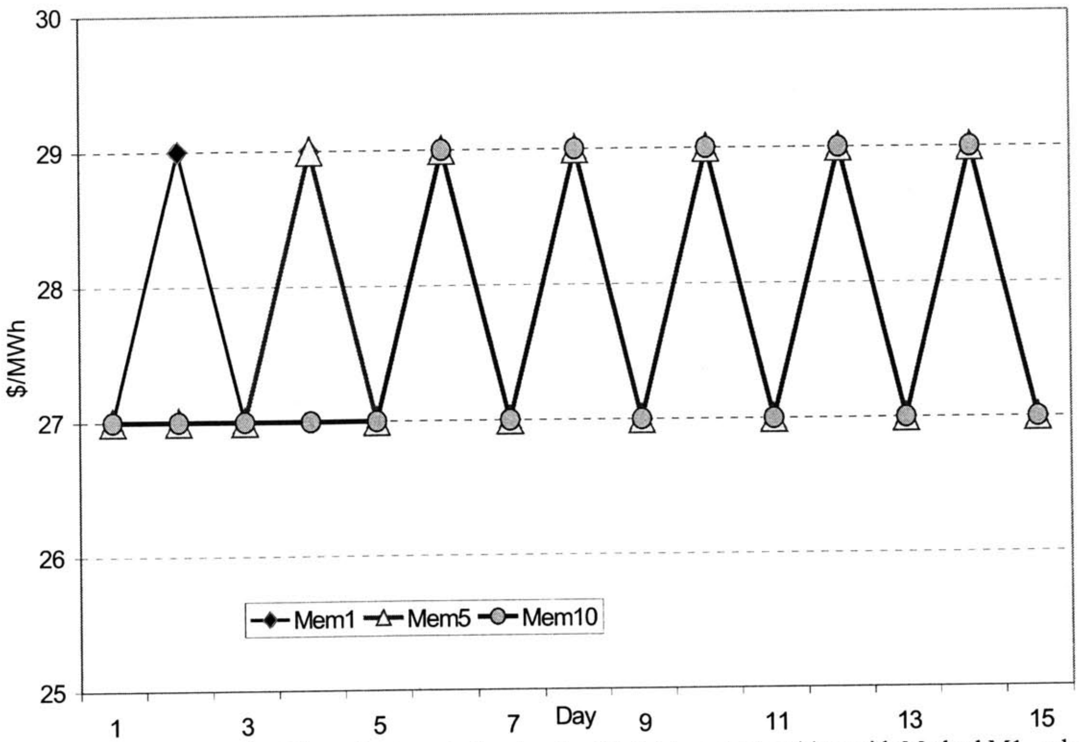

Figure 4-27: Price Dynamics When the Agents Employ the Model-based Algorithm with Method M1 and a GM Matrix

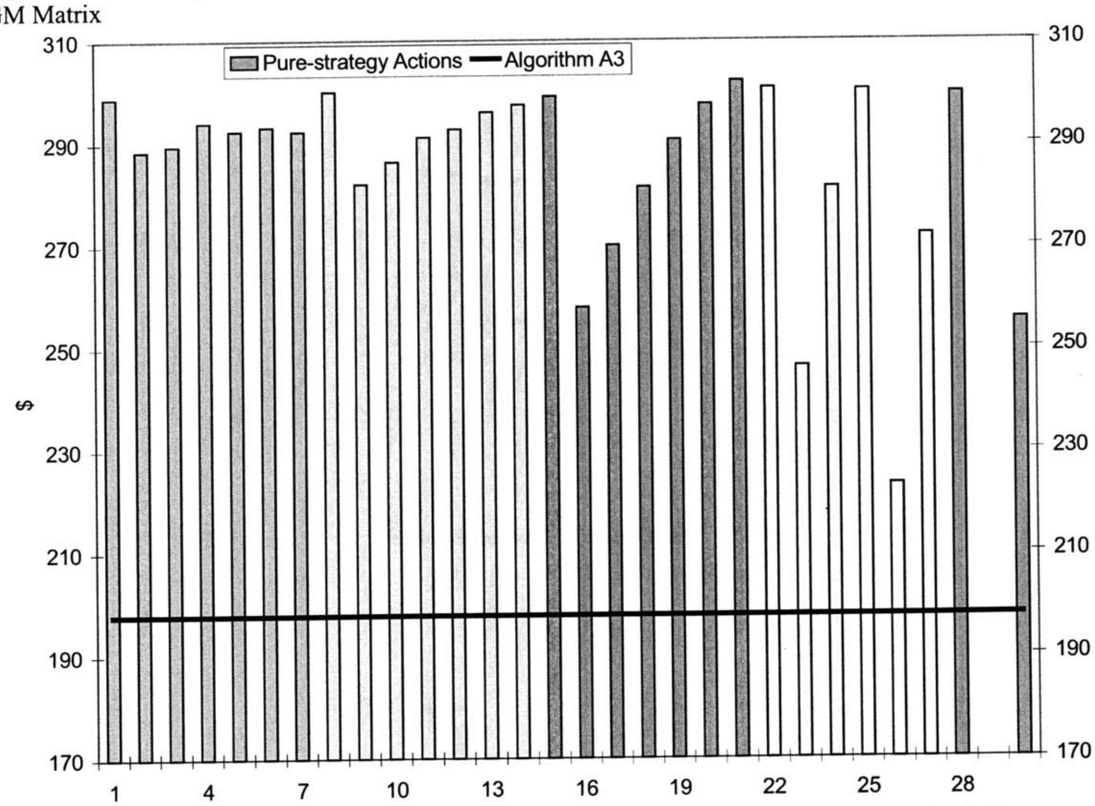

Figure 4-28: Cumulative Profits that Agent 1 Obtains When It Submits a Bid-supply Function in Response to the Opponents Employing Algorithm A3

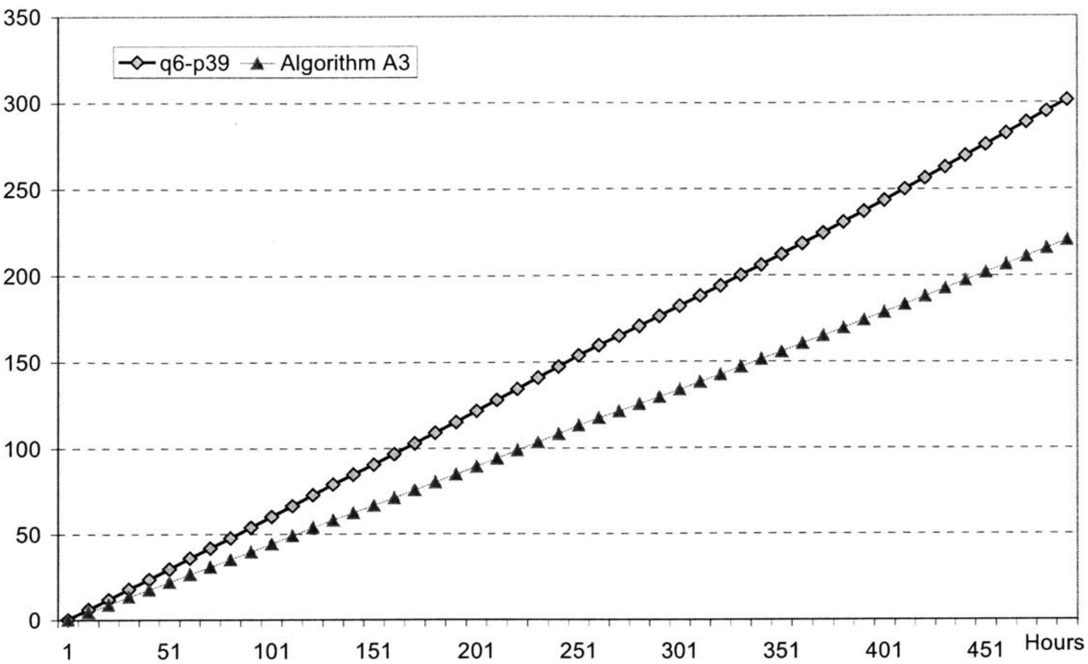

Figure 4-29: Cumulative Profits When Agent 1 Employs either Algorithm A3 or When It Submits a Bid-supply Function with q = 6 MW and BM = $39/MWh

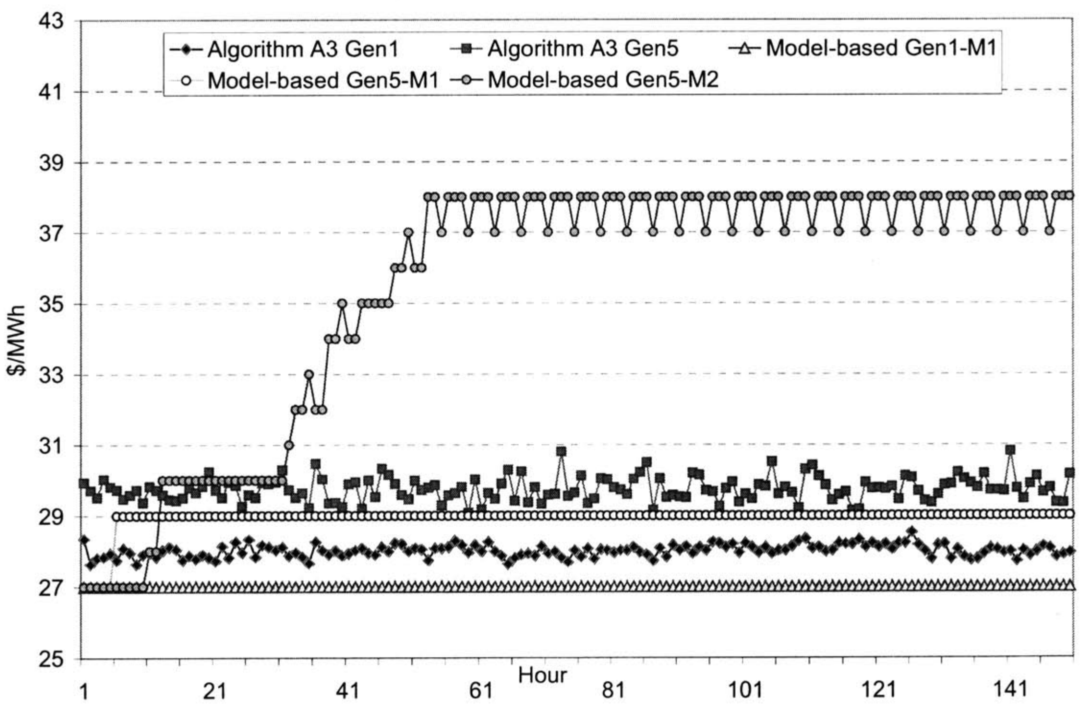

Figure 4-30: Price Dynamics Obtained When Demand is Equal to 66 MW, and either Agent 1 or 5 Employs a Learning Algorithm

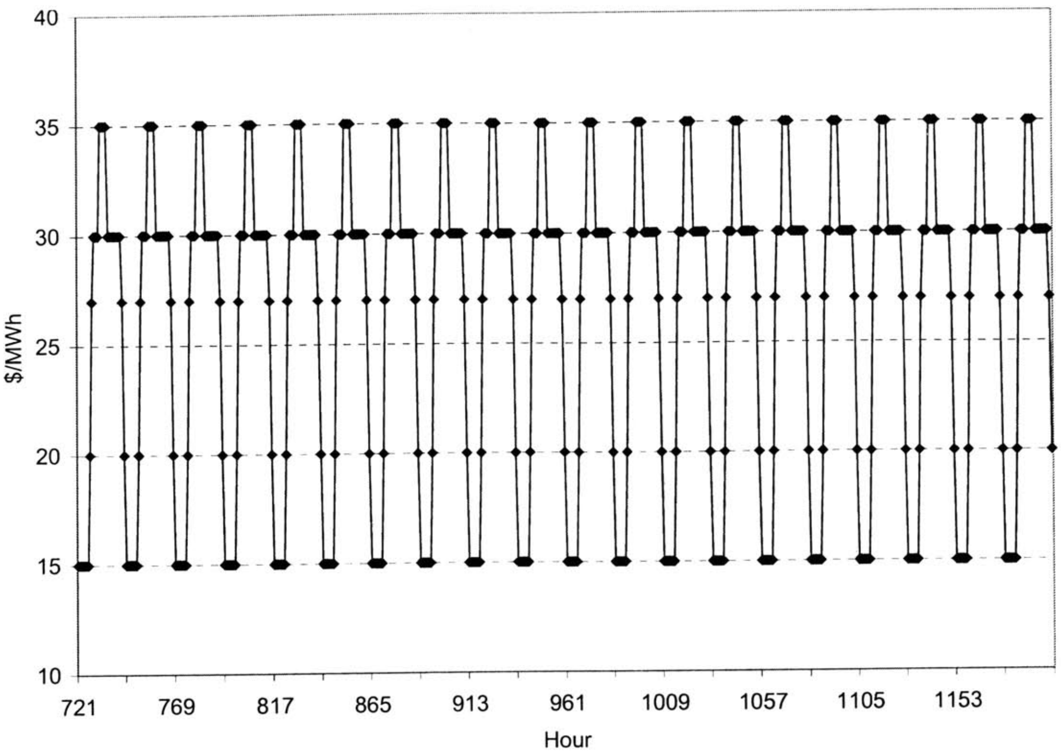

Figure 4-31: Price Dynamics from Hours 721 to 1,200 When the Agents Employ Algorithms A3 with δ = 0.9 or Algorithm SAB with α = 0.9 and τ = 100 to Determine Only *BM*

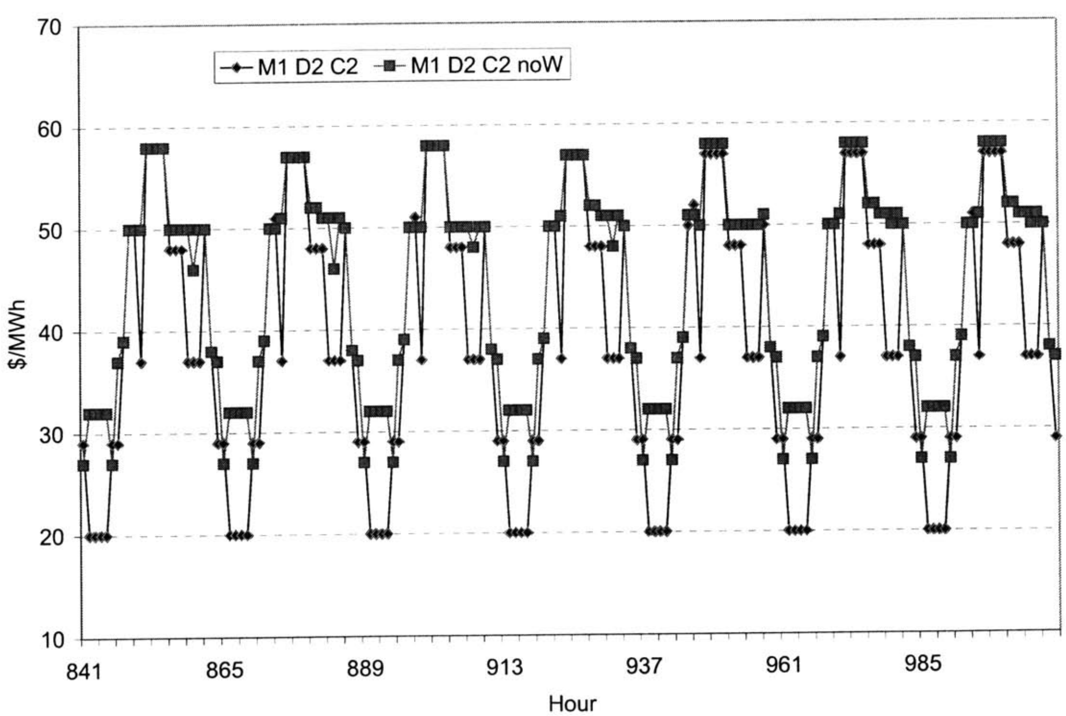

Figure 4-32: Price Dynamics from Hours 841 to 1008 When the Agents Employ the Model-based Algorithm with Method M1 and Δ=2 to Determine Only *BM*

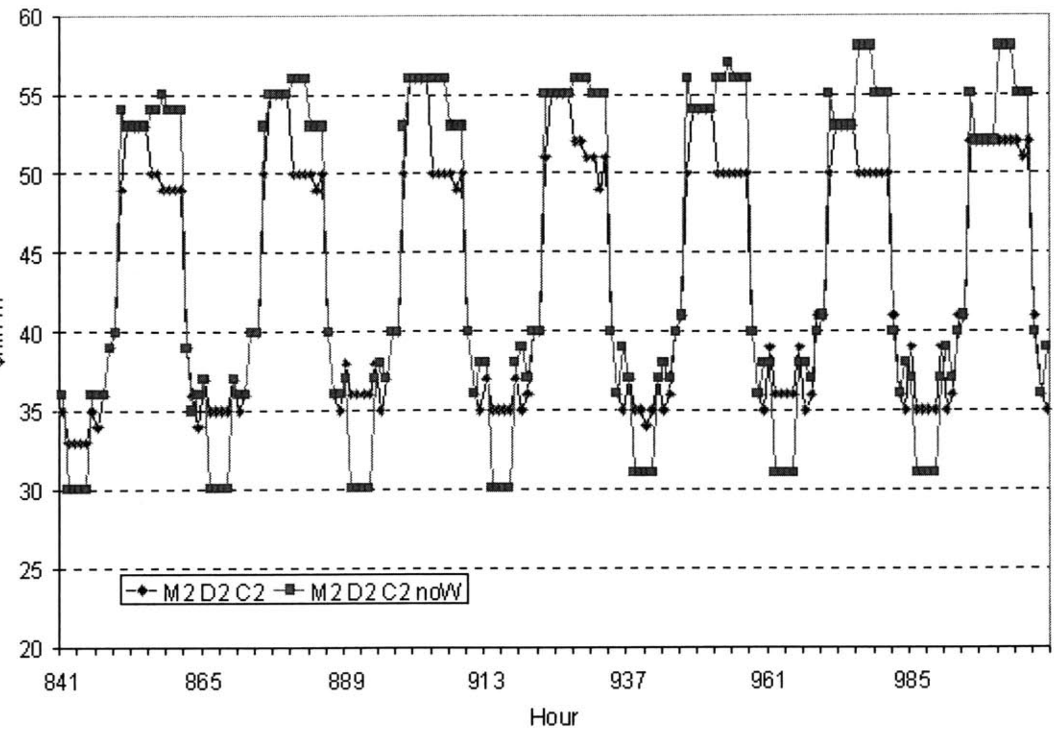

Figure 4-33: Price Dynamics from Hours 841 to 1008 of the Model-based with Method M2 and Δ=2 to Determine Only *BM*

Day	1		2		3		4		5		6	
MP		17		19		20		19		19		19
Agent #	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM
1	0.3	17	0.08	19	-7.6	21	9.22	18	2.05	19	2.05	19
2	0	17	0	19	-5	21	2	18	0	19	-4	21
3	0.3	17	0.08	19	-8.26	21	8.22	18	2.05	19	2.05	19
4	0.16	17	0.03	19	-5.13	21	5.11	18	1.02	19	1.02	19
5	0	17	0	19	-9	21	9	18	0	19	-18	21
6	0	17	0	19	-7	21	7	18	0	19	-14	21
7	0	17	0	19	-7	21	5	18	0	19	-10	21
8	0	17	0	19	-2	21	2	18	0	19	-4	21
9	0	17	0	19	-2	21	2	18	0	19	-4	21
10	0.3	17	0.08	19	-2.95	21	6.22	18	2.05	19	2.05	19
11	0.46	17	0	19	-4	21	9.34	18	3.08	19	3.08	19

Figure 4-34: Relationship between *MP*, *(OP-AP)*, and *BM* of the Agents at Hour 4 from Day 1 to 6 When the Agents Employ the Model-based Algorithm with Method M2

Day	1		2		3		4		5		6	
MP		17		19		20		19		19		21
Agent #	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM	OP-AP	BM
1	0	17	-3.28	19	-9.93	21	5.33	18	0	19	-5.32	21
2	0	17	0	19	-5	21	2	18	0	19	0	21
3	0	17	-3.28	19	-8.26	21	4.33	18	0	19	-5.32	21
4	0	17	-1.64	19	-5.13	21	3.16	18	0	19	-2.66	21
5	0	17	28.87	17	13	19	-13	20	26.26	17	31.7	19
6	0	17	0	19	-7	21	7	18	0	19	0	21
7	0	17	0	19	-7	21	5	18	0	19	0	21
8	0	17	0	19	-2	21	2	18	0	19	0	21
9	0	17	0	19	-2	21	2	18	0	19	0	21
10	0	17	-3.28	19	-7.61	21	2.33	18	0	19	-5.32	21
11	0	17	0	19	-4	21	3.49	18	0	19	0	21

Figure 4-35: Relationship between *MP*, *(OP-AP)*, and *BM* of the Agents at Hour 4 from Day 1 to 6 When the Agents Employ the Model-based Algorithm with Method M2 Without the CW Strategy

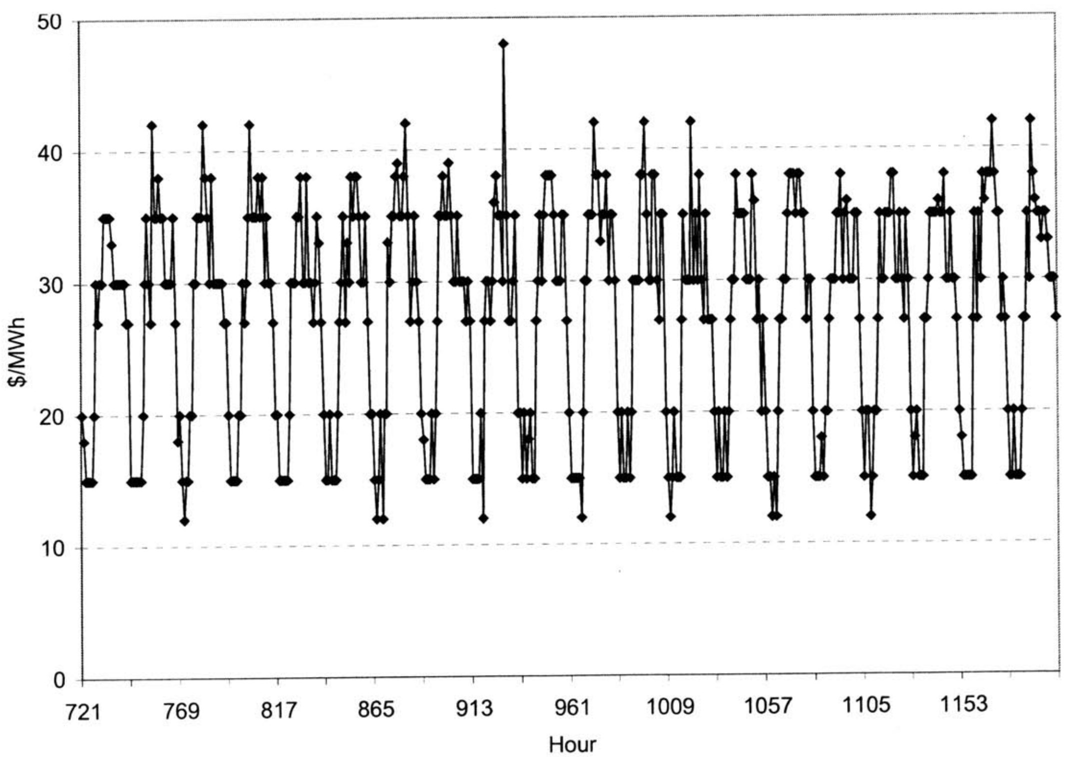

Figure 4-36: Price Dynamics from Hours 721 to 1,200 When Only Agent 5 Employs Algorithm A3 with $\delta = 0.9$, While the Other Agents Submit their Marginal-cost Bids

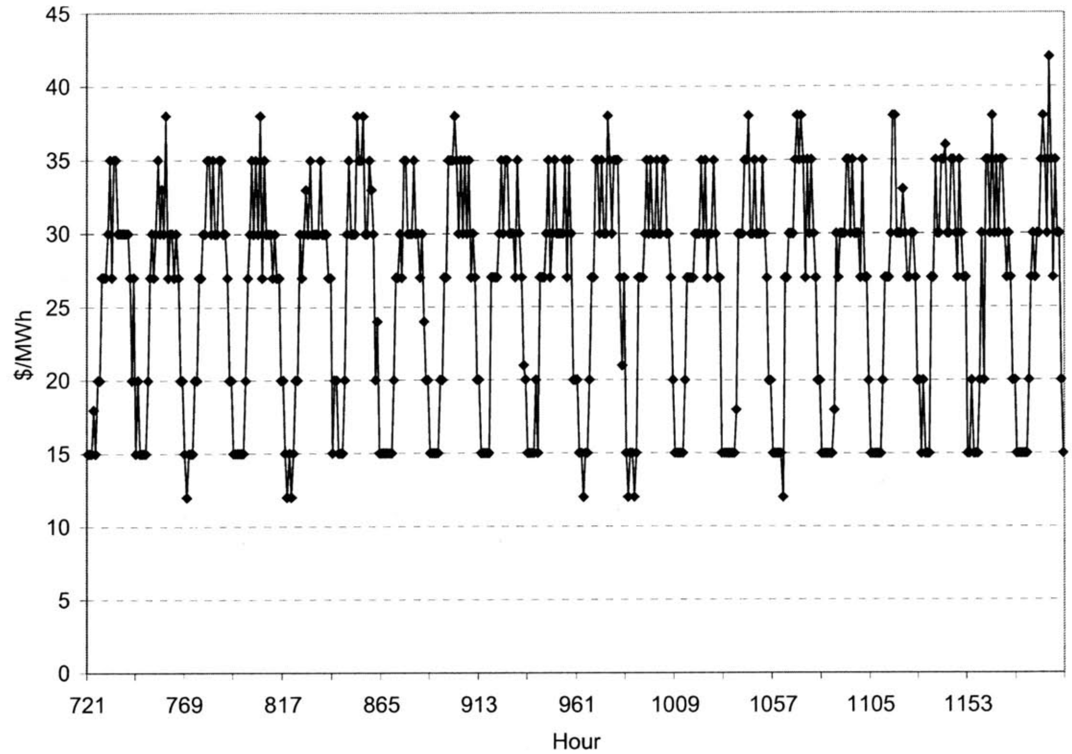

Figure 4-37: Price Dynamics from Hours 721 to 1,200 When Only Agent 5 Employs Algorithm SAB with $\alpha = 0.9$ and $\tau = 100$, While the Other Agents Submit their Marginal-cost Bids

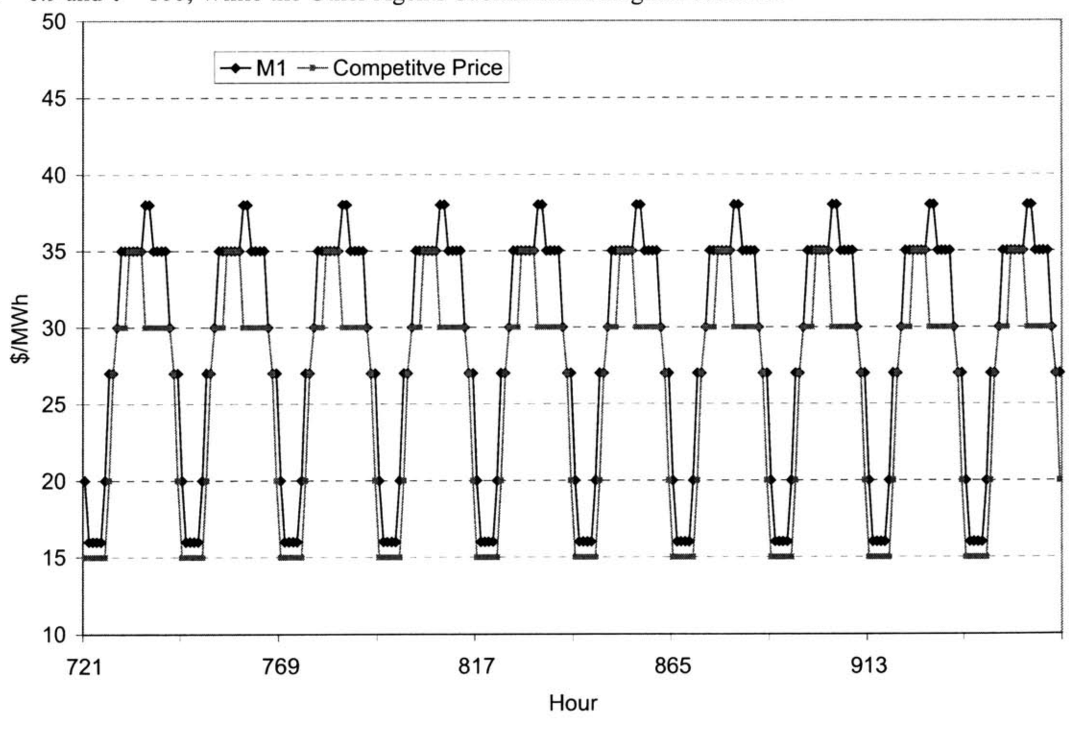

Figure 4-38: Price Dynamics from Hours 721 to 960 When Only Agent 5 Employs the Model-based Algorithm with Methods M1 and C1 and $\Delta = 2$, While the Other Agents Submit Their Marginal-cost Bids

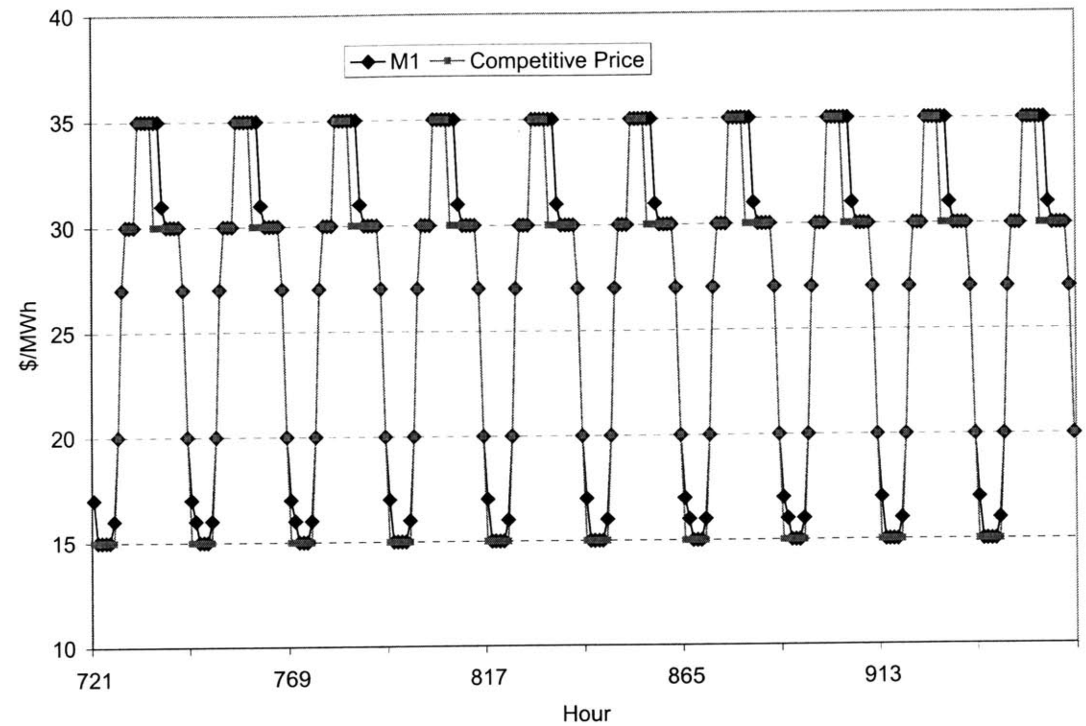

Figure 4-39: Price Dynamics from Hour 721to 960 When Only Agent 1 Employs the Model-based Algorithm Methods M1 and C1 and Δ= 2, While the Agents Submit Their Marginal-cost Bids

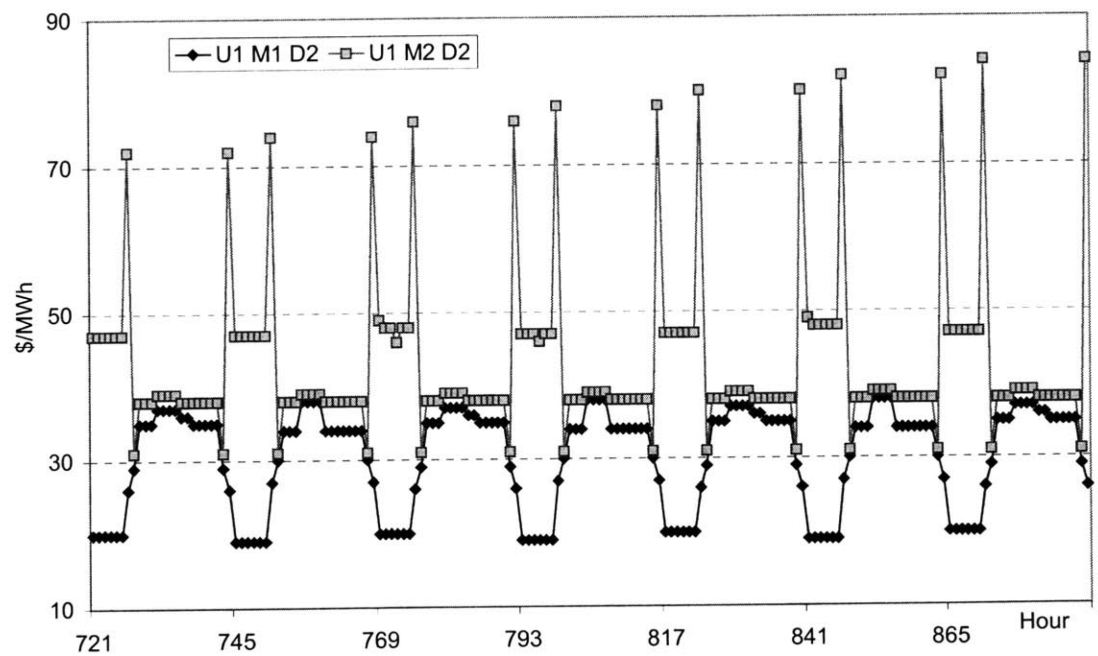

Figure 4-40: Price Dynamics from Hour 721 to 888 When the Agents Employ the Model-based Algorithm with a Unit-by-unit Decision Scheme

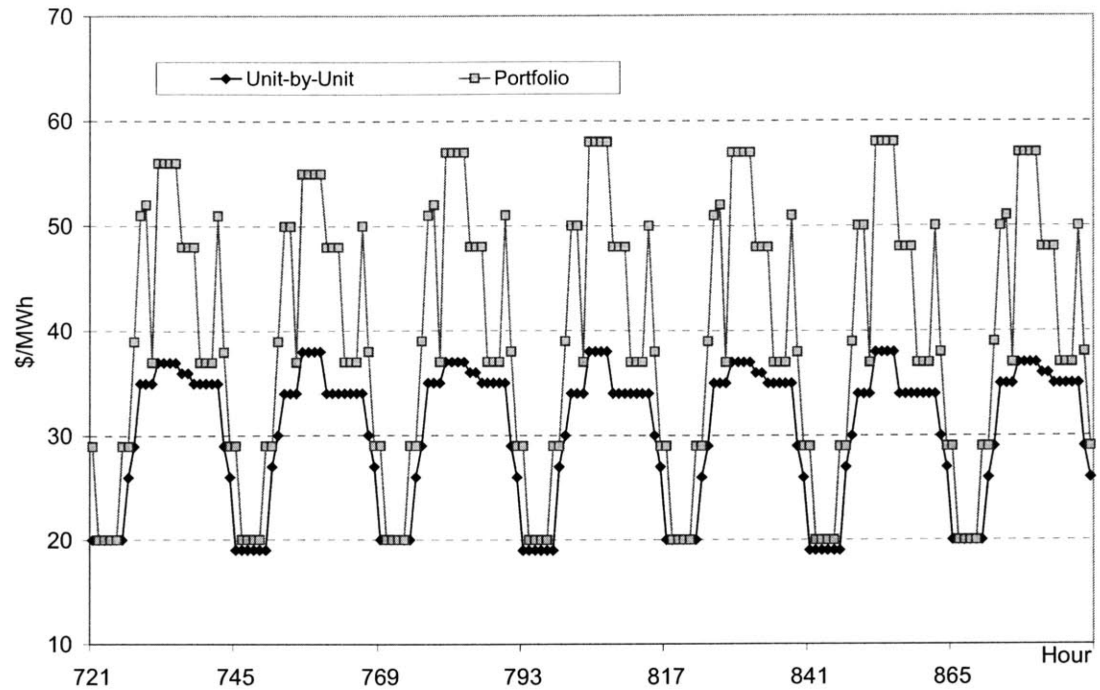

Figure 4-41: Price Dynamics from Hours 721 to 888 When the Agents Employ the Model-based Algorithm with Unit-by-unit and Portfolio-based Schemes with Methods M1 and C1 and Δ= 2

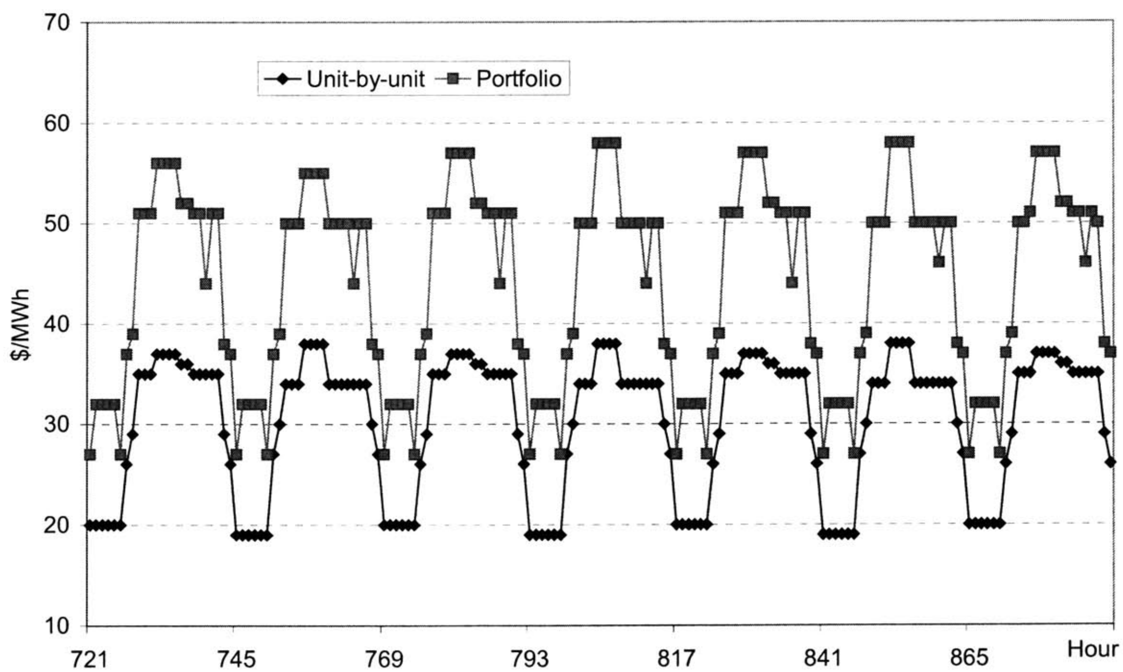

Figure 4-42: Price Dynamics from Hours 721 to 888 When the Agents Employs the Model-based Algorithm with the Unit-by-unit and Portfolio-based Schemes with Method M1 and Δ= 2 without the CW Strategy

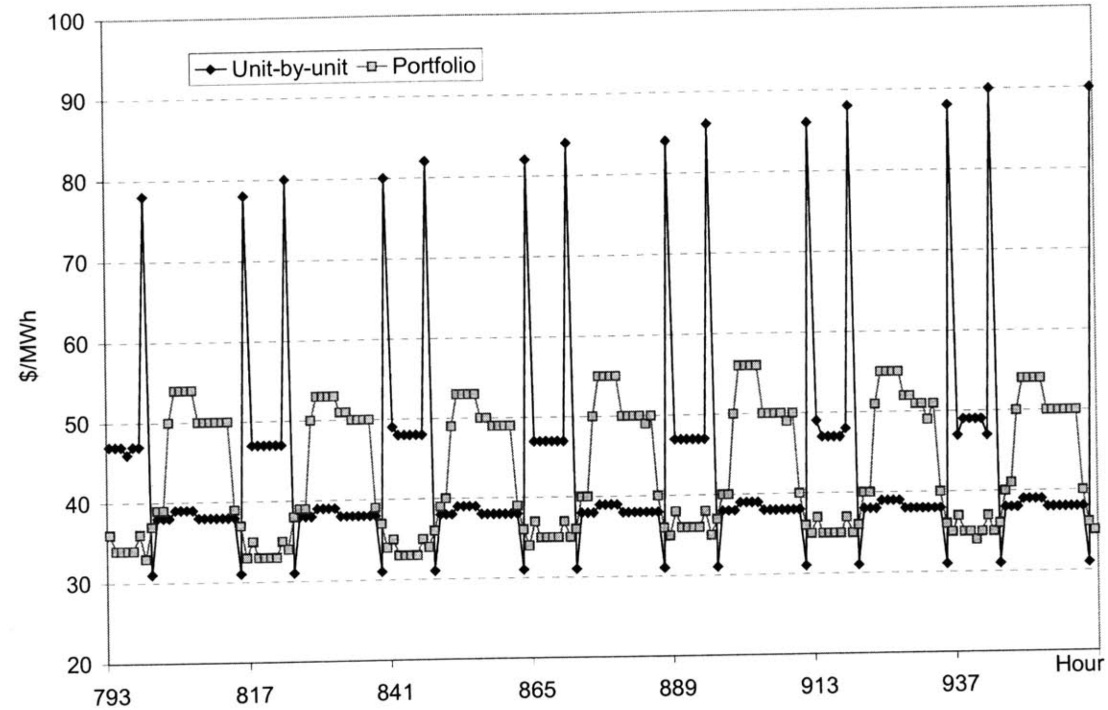

Figure 4-43: Price Dynamics during Hours 793-960 the Agents Uses the Model-based Algorithm with the Unit-by-unit and Portfolio-based Schemes with Method M2 and Δ= 2 without the CW Strategy

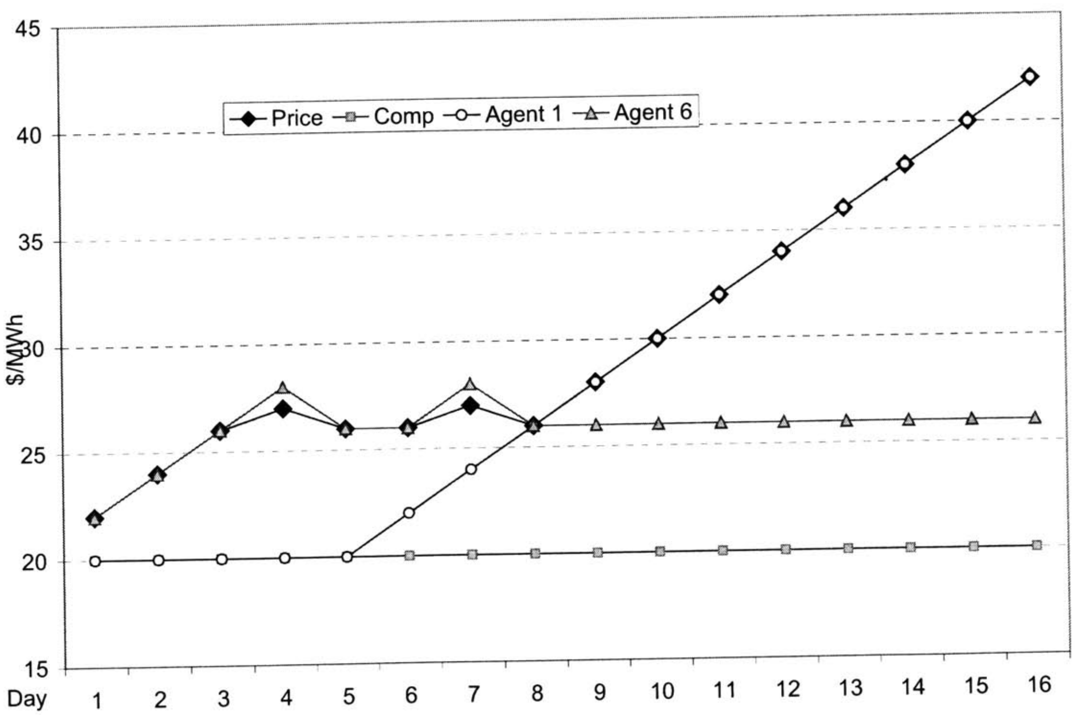

Figure 4-44: Prices and Bidding Prices of Agents 1 and 6 at Hour 7 Daily When the Agents Employ the Model-based Algorithm with a Unit-by-unit Decision Scheme, Methods U1 and M2, and Δ= 2

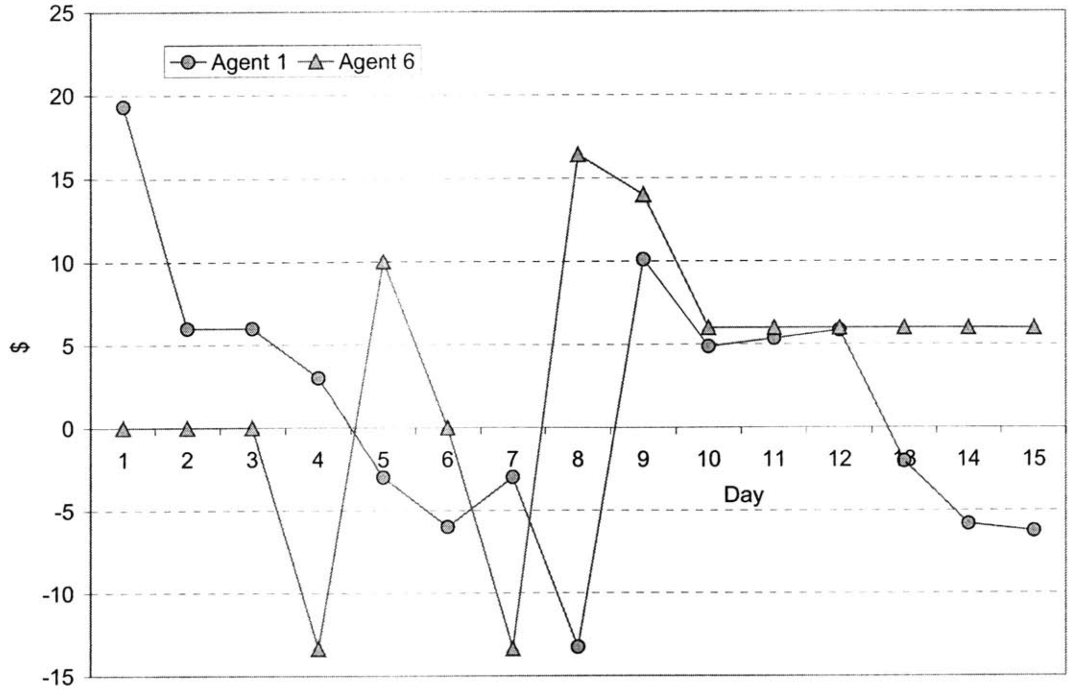

Figure 4-45: *(OP-AP)* of Agents 1 and 6 at Hour 7 Daily When the Agents Employ the Model-based Algorithm with a Unit-by-unit Decision Scheme, Methods U1 and M2, and Δ= 2

Chapter 5

Analyzing the New England Electricity Market

This chapter presents an empirical study on the New England electricity market to support the concept that an agent-based approach is an appropriate tool for modeling electricity spot markets. The agent-based approach is selected for this analysis because of the characteristics of electricity spot markets, which consist of several market participants with the potential to influence market outcomes. The model must be capable of reproducing price dynamics from market data. The crucial problem at hand is that these data are unfortunately not made available by the market operator and without knowledge of the market participants' bidding strategies/learning algorithms, it is difficult to reproduce the price dynamics.

This empirical study analyzes the bidding behavior of the market participants, also known as Lead Participants (LPs), and shows that agent-based behavior is a key component of modeling the dynamics of the existing markets. The available information currently includes historic bid data, LP names and portfolio characteristics, net claimed capacity of each generating unit, forecast and actual demand, total capacity used for alleviating network-constraints, net imported capacity, and market rules. The bidding behavior of the LPs is observed directly from the historic bid data, and the analysis of LP 506459's bid data shows a possible learning algorithm. In addition, the bidding strategies of the LPs depend on their portfolio characteristics, as well as on the types of generating units.

In summary, this empirical study analyses 1) whether the LP bidding behavior exhibits certain patterns, i.e., demand-dependent or daily patterns, 2) whether the LPs learn the markets and how they learn, 3) the learning algorithm that is likely to be used by the LPs, and 4) whether the LPs submit a portfolio bid or a unit-by-unit bid. The results from this empirical study show that the agent-based approach is an appropriate model for electricity spot markets, since by using this model the strategic behavior of the market participants can be captured, and that potential price dynamics due to the strategic behavior of the market participants can be simulated.

This chapter is organized as follows. Section 5.1 provides the method used to perform an analysis of LPs' bidding behavior. Section 5.2 presents the method used to identify possible marginal units. Section 5.3 presents the detail on bid characteristics of a few LPs. Section 5.5 investigates a possible bidding strategy of LP 506459. Section 5.4 examines the bid characteristics of different types of generating units with the same owner. The conclusion is presented in Section 5.6. For readers who are interested in the background of the New England electricity spot market, which is operated by the New England Independent System Operator (ISO-NE), an overview of this market and the available information is outlined in the appendix to this chapter.

5.1 Analyzing Bidder Behavior

This section presents the method used to determine the bidding characteristics of the LPs and the preliminary results of a few LPs' bidding behavior. The analyses indicate the non-uniform behavior of the LPs in the New England electricity spot markets. They also suggest that in order to understand the dynamics of the current electricity markets it is essential to model electricity markets in general by using an agent-based approach.

The method to determine the bidding characteristics of the LPs follows these steps:

1. *Matching LP IDs and their bidding capacity with the names of the Lead Participants and their portfolios to obtain the portfolios and generating units' characteristics.*

The bidder names and their portfolios, which contain the number of generating units, installed capacity, and types of generation technology are obtained from the posted net-claimed capacity. By comparing a total sum of high operating limits (HOLs) with the total net claimed capacity of each bidder's portfolio, the LPs can be matched with the bidder names and their portfolios. Some LPs have similar capacity and the units' characteristics and bidding constraint characteristics (such as self-scheduled capacity (SS)) are used to identify the LP and its portfolio. For example, pumped-storage units have limited available capacity in each day, and nuclear units are generally under a self-scheduled condition to avoid being turned off. The matching results are shown in Table 5.1. There are several benefits of identifying the LPs and their portfolio characteristics. First, to differentiate the causes of bid adjustments during the day, as to whether they come from strategic behavior or operation constraints, is essential. The hydropower units, for example, are limited energy sources (due to limited water flow) and generally are not operated during the off-peak demand hours, while other units, such as nuclear units, are able to operate all day. The unavailability of the hydropower units during certain hours may be caused from limited flow of the rivers/streams, and not because of the strategic behavior of the LPs who bid those units to the market.

Second, by matching the LPs and their types of units, one can identify which LPs tend to be scheduled to operate at the margin and thus set the market price. The crucial benefit of identifying which units are scheduled to operate as marginal units, and is it is possible that observe the relationship between the types of units and their bid-supply functions, as well as the portfolio characteristics and bidding strategies. Consequently, when the agent-based market model is used for analyzing the existing markets, proper bidding strategies (or learning algorithms) can be assigned to the agents.

Table 5.1: Some New England Market LPs during July 2000

LP ID	140603	184983	196063	206845	218387	331313
LP	SITHE	CLP	PPLEP	FPL	SEI	TMLP
LP ID	333704	353795	400693	405573	412080	465936
LP	NU	CPS	WSVST	UI	MMWEC	CMP
LP ID	483669	484516	505718	506459	515039	519412
LP	NU_NAESCO	BE	TPM	PGET	MPLP	UAELT
LP ID	529934	529988	532832	547596	607144	629513
LP	CES	NRGPM	SCEM	BELD	ENGC	CMEEC
LP ID	647399	649626	659984	674577	780847	854478
LP	BHE	CCT	DPA	DETM	ENGEN	INDCK
LP ID	902793	910093	934720			
LP	CEEI	VELCO	PEC			

Table 5.2 shows the portfolios of net claimed capacity (summer claimed capacity) of four LPs, including LPs 206845, 218387, 506845, and 529988 during July 2000.

Table 5.2: LPs' Net Claimed Capacities

	Types of Technology (% of Capacity)								total
LP ID	D	F	HD	HW	G	J	CC	PS	(MW)
206845	0	70.0	10.7	16.5	2.8	0	0	0	1,349
218387	0	7.0	9.0	6.0	0	12.0	0	66.0	1,645
506459	0.2	56.3	3.5	10.0	0	0	16.7	13.3	4,422
529988	0	83.0	0	0	7.0	10.0	0	0	2,313

2. *Reconstructing the bid-supply functions of the LPs.*

To reconstruct an hourly bid-supply function, bid-blocks MW of the units with positive HOL are stacked from the lowest bid-block \$ to the highest bid-block \$. For the units with positive SS capacity, the bid-blocks \$ of the bid-blocks MW added up to their SS capacity are set to zero. The minimum quantity that is scheduled to operate but not allowed to set the market price of each generating unit is the maximum between the SS capacity and low operating limits (LOLs). Some LPs do not necessarily set the bidding prices of the SS quantity block at zero. In this analysis, when the LPs do not set their bidding prices of the SS capacity at zero, their bidding prices are automatically set to zero. Figure 5-1 shows a set of LP 506459's hourly bid-supply functions. As observed in Figure 5-1, the hourly bid-supply functions cannot be represented by a simple function, such as $y = a \cdot x + b$ or $y = \exp(a \cdot x + b)$, where a and b are constants. Moreover, for each trading day, there are 24 bid-supply functions for each LP. To simplify the analysis, the bidding prices of sampled bidding quantity of each trading hour are used.

3. *Reconstructing the aggregate bid-supply function of each hour from the bid data to determine the market price and dispatched capacity of each LP.*

Similar to reconstructing an hourly aggregate bid-supply function, bid-blocks MW of the units with positive HOL are stacked based on a price-merit order. For units with positive self-scheduled capacity the bid-blocks $ from the lowest price of bid-blocks MW summed equal to the SS capacity are set to zero. Several LPs submit the non-zero (especially positive) bidding prices for the self-scheduled blocks, such as LP 218387. An example of the aggregate bid-supply functions in a typical day for the New England market is shown in Figure 5-2.

4. *Determining actual demand.*

The actual demand L_k^a in each hour is the demand served by merit-order generating units. This demand, the merit-order dispatch capacity, includes the import/export power flowing from and into the neighboring grids, including the New York power pool and the Canadian power system, such as New Brunswick and Hydro Quebec [53], though it does not account for the capacity dispatched out of merit order to transmission constraints. The actual demand in each hour is calculated by

$$L_k^a = L_k - Q_k^T + Q_k^{Im},$$

where L_k is the actual consumption for each hour k, Q_k^T is the total capacity used to compensate the system constraints due to transmitting power, and Q_k^{Im} is imported power (negative Q_k^T means the power is exported out of the New England market).

5. *Determining scheduling capacity and revenue.*

From the given bid data, total demand, and market prices, scheduled capacity for each hour is determined by using the price-merit order method, that is, by finding an intersection point of the aggregate bid-supply function and actual demand. The market price of that hour is defined as the value on the price axis of the intersection point, and is equal to the maximum bidding price of the bid-blocks scheduled to operate at the margin. For several reasons, this market price for the New England market is generally different from the actual price during that hour. First, an hourly calculated market price is the price where a market-wide bid-supply function intersects with the actual demand. The hourly price published by the ISO-NE is determined using ISO-NE's dispatching software, resulting in that demand to meet supply at the minimized total cost, which accounts for unit-commitment constraints. Therefore, by simply identifying an intersection point of the demand and supply functions, the unit-commitment factors are not captured.[1] Second, an hourly published price is an average of 12 five-minute prices; also hourly demand is average demand during that hour. Since the aggregate

[1]See the market rules for more details on dispatching and also Visudhiphan *et al.* [45] for analyzing the effect of the clearing mechanism on market prices.

bid-supply function is not linear, an average of prices (with different demand in each 5-minute interval) is not equal to the price of the average demand in a given hour. Third, the ISO-NE dispatches may alleviate transmission congestion in the network; nevertheless, real-time outage of the operating units may have occurred, and the addition dispatch might be needed. Hourly total capacity dispatched to alleviate transmission congestion is not factored in determining the market price in this study. The specific units, which are constrained on, turned on out-of-merit order, or constrained off, are not available. Visudhiphan *et al.* [45] show that one cannot reproduce market prices by simply determining an intersection of the aggregate bid-supply function and actual demand from the available bid data, total demand, imported/exported power, and capacity dispatched out-of-merit order, because the real dispatch accounts for the above factors.

The scheduled capacity of each LP is a total sum of the bid-blocks MW with bid-blocks $ at most equal to the calculated market price. When there is more than one bid-block MW with the same bid-block $ equal to the calculated market price, these bid-blocks MW are scheduled to operate based on the weighted portion of the residual demand. Also, when the scheduled capacity of each unit is less than its declared LOL, the unit is dispatched at zero, and the unscheduled bid-block MW of the units that are dispatched above their LOLs are scheduled to operate instead.

Revenue of each LP in each hour is simply the market price during that hour multiplied by the total scheduled quantity. Note that the scheduled quantity, as well as revenue presented later in this analysis, does not reflect the real revenue that the LPs receive from their actual electricity sale.

6.*Discretizing demand and categorizing hourly demand into sequences of load-index events.* To observe whether the load-based modeling approach is reasonably good at capturing market participant adaptive behavior, the bidding behavior of the market participants is analyzed in an order of load-index events. Hourly forecast demand is discretized and represented by a load index. The hours with the same load-index sequence are grouped together in their order of occurrence. The bid data relating to each load-index sequence is analyzed.

5.2 Identifying Marginal Units

The marginal units are the last units to be dispatched; that is, the market price is equal to their bidding prices. For the LPs to set the prices that yield the most desirable outcomes, bidding strategies and/or learning algorithms play a critical role for the units to be scheduled to operate as marginal units. Therefore, examining the bidding behavior of the potential marginal units may shed light on learning algorithms/bidding strategies of the LPs. Since details regarding units dispatched in each hour are not available, given the available data, this thesis identifies the marginal units used during each hour using the following steps:

1. *Determining the aggregate bid-supply function.* The same method described in Section 5.1 is used.

2. *Determining actual demand L_k^a.* Actual demand is defined as the demand served by merit-order generating units. Total demand L_k^a is simply the actual consumption (L_k) subtracted by the quantity needed for alleviating transmission constraints (Q_k^t) and to which the imported power (Q_k^{Im}) is added, i.e., $L_k^a = L_k - Q_k^T + Q_k^{Im}$.

3. *Determining the actual demand range, a 1600-MW band around the L_k^a, e.g., $L \in [L_k^a - 800, L_k^a + 800]$.* This range is chosen arbitrarily. Motivations behind establishing this demand range are that the actual consumption varies within the hour[2] and the dispatch incorporates unit-commitment constraints. This demand range allows us to capture the units that may potentially be dispatched as marginal units.

4. *Identifying the marginal units.* The marginal units are defined as the units on the aggregate bid-supply function that have bid-blocks MW in the specified range $[L_k^a - 800, L_k^a + 800]$. From Figure 5-3, the units that have their bid-blocks MW in band A are called marginal units.

5. *Mapping an Asset ID to a unit in the LP portfolio to identify its generation technology.* By comparing the HOL of the Asset ID and the installed capacity, one can identify some units and their generating technology without difficulty. Several units of the same portfolio have similar installed capacity and when this happens, bid data are used. The units with daily limited available energy are considered either hydropower or pumped-storage units. The units with no limited available energy (DEA = 0) and with low operating limit greater than zero (LOL > 0) are considered either combined-cycle or fossil-fueled units.

5.2.1 Results

This section presents the findings on potential marginal units in the New England market during the observed two-week periods in January, April, July, and October of 2000. Table 5.3 shows the LPs with the units that could be scheduled to operate as marginal units during January 18-31, 2000 and October 18-31, 2000. Demand in January is higher than in October in almost every hour (as shown in Figure 5-31). Since bid data of a two-week period of those months are considered, there are a total of 336 hours, meaning 14-hour groups of data for each Trading Hour (TH). Only the data associated with THs 4, 9, 12, 16, and 18 and the LPs with more than 7 hours scheduled to operate as a marginal unit in each TH are presented. The $*$ denotes that the LPs have at most 6 hours in the TH to be scheduled to operate as a marginal unit.

The lists of the marginal units are shown in Table 5.4. The results show that several units are consistently scheduled to operate at the margin, as shown in Table 5.3. These LPs are eligible to set

[2] The published total consumption is an average of 12 values of 5-minute demand.

Table 5.3: LPs with Marginal Units during January 18-31 and October 18-31, 2000

January 18-31, 2000						
	Trading Hours					
LP ID	4	9	12	16	18	22
	No. of hours					
218387	14	7	*	7	*	*
333704	11	*	7	*	9	*
400693	13	*	11	7	11	*
412080	*	7	7	*	*	*
465936	*	*	*	11	*	8
484516	10	*	*	*	*	*
505718	*	7	*	8	7	11
506459	14	14	14	14	13	14
529988	8	13	13	12	13	11
532832	*	9	8	11	9	11
787013	11	*	*	*	*	*
910093	7	*	*	10	*	9

October 18-31, 2000						
	Trading Hours					
LP ID	4	9	12	16	18	22
	No. of hours					
218387	13	13	9	10	7	10
400693	*	13	14	11	12	*
505718	8	11	13	11	14	14
506459	13	13	13	14	14	14
515039	*	*	10	8	7	*
529934	7	*	*	*	*	*
529988	11	7	*	*	7	*
532832	*	12	13	12	7	14
674577	*	9	9	8	8	11
787013	13	*	*	*	*	*
959445	10	*	*	*	*	*

the market prices. Different LPs influence market prices in different hours and in different months. For example, LP 218387 tends to be a marginal unit more often in October than in January, whereas LP 529988 tends to be a marginal unit more often in January than in October. LP 506459 always submits the bid-supply functions such that some of its units are scheduled to operate as a marginal unit almost every hour. Furthermore, one would expect to observe the variation of the bid-supply functions of the LPs within a day or from day to day. This, in fact, is true, especially for LP 506459. The bid-supply functions of LP 506459 over several days and the sample plots of bidding prices associated with a few bidding quantities are shown in Figures 5-14 - 5-19.

Since LP 506845 submits the bid-supply functions so that the units are likely to be scheduled to operate as marginal units, the extended analysis of its bid-supply function is performed to identify the types of generating units that are likely to be scheduled to operate at the margin. By observing the bid data, it turns out that LPs submit bid-supply functions such that most of its units (accounting for more than half of total HOL) can be scheduled to operate as marginal units. Table 5.4 shows the units may be scheduled to operate as a marginal unit during THs 9 and 18 in the periods of January 18-31 and October 18-31, 2000. The numbers in the columns "TH 9" and "TH 18" indicate a number of trading hours during the observed periods that the units are scheduled to operate at the margin (14 means that the unit is a marginal unit for the entire observed period). Let H denote a hydropower unit (either a HD or HW unit), F denote a fossil-fueled unit, PS denote a pumped-storage unit, and CC/F denote the unit that is either a combined-cycle unit or a fossil-fueled unit. Due to limited information on operating characteristics of the units, some units are not exclusively identified and let x denote that the type of generating unit cannot be identified.

From Table 5.4, it is clear that several hydropower, combined-cycle, and fossil-fueled units of LP

Table 5.4: Marginal Units of LP 506459 during January 18 - 31 and October 18 - 31, 2000

January				
Asset ID.	TH 9 (Hours)	TH 18 (Hours)	HOL (MW)	Tec.
26161	14	0	64	CC
29086	12	12	22	H
31965	7	0	14	H
38850	12	14	13	H
43414	11	10	435	F
45823	12	11	440	F
58508	7	0	7	H
71825	12	14	192	H
72183	12	13	145	CC/F
79606	12	14	164	H
81361	12	11	290	PS
81483	14	14	290	F
88818	14	14	48	H
89472	14	14	42	H
90417	14	13	290	F
92137	14	14	147	CC/F
93720	14	0	41	H
Total (MW)			2,644	

October				
Asset ID.	TH 9 (Hours)	TH 18 (Hours)	HOL (MW)	Tec.
16337	12	12	150	CC/F
26161	0	8	64	CC
29086	14	14	22	H
31965	0	12	14	H
34993	14	14	82	F
37274	12	11	150	CC/F
38850	14	14	13	H
42841	0	13	290	PS
43414	13	13	435	F
47946	13	12	10	H
58508	13	12	7	H
71825	14	14	192	H
79606	14	14	164	H
86967	0	12	5	x
88625	13	13	18	x
88818	14	14	48	H
89472	14	14	42	H
92137	12	11	147	CC/F
93270	13	13	41	H
Total (MW)			1,894	

506459 are generally scheduled to operate as marginal units. Moreover, marginal units vary over time. For example, Asset IDs 81361 and 81483 are likely to be scheduled to operate as marginal units in January but not in October. The generating units of LP 506459, which are likely to be scheduled to operate as marginal units, are analyzed in Section 5.5.

5.3 Lead Participant Bidding Behavior

The bidding behavior of four LPs, including LPs 206845, 218387, 506459, and 529988, is analyzed. These LPs are selected because the size of their installed capacity is more than 5% of total capacity in the market. In particular, LP 506459 is responsible for the bid submission of the largest capacity. Based on the bid data, LP 206845 has several units with parts of their capacity self-scheduled, LP 218387 owns the units with limited energy generation, and LP 529988 owns the units that can be dispatched without energy constraints and with no self-scheduled constraints. By comparing the bid data to the lists of LP generating units, one can conclude that LP 218387 owns a pumped-storage facility, LP 506459 owns 27% hydropower capacity in its portfolio, and LP 529988 owns fossil-fueled, gas-turbine, and jet-engine units, but no hydropower unit. The analyses were performed during the periods January 18-31, April 17-30, July 18-31, and October 18-31, 2000. These months cover demand during winter, spring, summer, and autumn, respectively. The demand patterns during January 18-31 are shown in Figure 5-4, during April 17-30 in Figure 5-5, during July 18-31 in Figure 5-6, and during October 18-31 in Figure 5-7.

5.3.1 Observing Bidding Behavior

The samples of time-series of the bidding prices given the bidding quantity of four LPs are shown in Figures 5-8 - 5-22 below. These plots reflect the true bidding prices at the specified bidding quantities. They are not adjusted by setting the bidding prices to zero for the self-scheduled capacity, as occurs when the market prices and scheduled quantity are determined. A few values of bidding quantities between the self-scheduled quantities and HOLs are chosen for presenting the plots of a time series of bidding prices and quantities sampled from the daily bid-supply functions. Each line represents one bidding quantity in which its value is specified on the plots.

To demonstrate that the LPs may not submit the same total HOLs daily and/or weekly, total self-scheduled (SS) capacity and total HOLs of TH 14 during January 18-31, 2000 are shown in Table 5.5.

Table 5.5: Self-scheduled Quantity and Bidding Prices during January 18-31, 2000: Trading Hour 14

Date	LP 206845		LP 218387		LP 506459		LP 529988	
	SS (MW)	HOL (MW)	SS (MW)	HOL (MW)	SS (MW)	HOL (MW)	SS (MW)	HOL (MW)
18	0	921	298	2,680	1,011	4,964	491	2,333
19	0	921	288	2,680	965	5,001	491	2,333
20	0	601	200	2,680	956	4,839	491	2,181
21	0	601	200	2,680	980	4,501	491	2,333
22	0	601	210	2,680	965	4,672	431	2,181
23	0	601	200	2,680	948	4,677	431	1,931
24	0	601	198	2,680	944	4,669	431	1,931
25	0	601	191	2,680	950	4,942	431	2,333
26	0	601	191	2,680	972	4,818	416	2,333
27	0	301	178	2,680	948	4,499	416	2,333
28	0	301	315	2,680	949	4,501	416	2,333
29	0	921	199	2,680	958	4,630	416	2,333
30	0	921	158	2,680	959	4,037	416	2,333
31	0	921	145	2,680	947	4,596	416	1,773

The examples show the bidding prices at the specified bidding quantities between January 18-24, and April 17-23, 2000 of LPs 206845, 218387, 506845, and 529988. Additional plots of bidding prices given quantities of LP 506459 during the last two weeks of January, April, July, and October of 2000 are also presented.

LP 206845

The bid-supply functions of LP 206845 are sampled. Examples of a time-series of observed bidding prices associated with sampled bidding quantities 150, 300, 450, 600, and 750 MW between January 18-24, 2000 are shown in Figure 5-8, and with sampled bidding quantities 450, 600, 750, and 900 MW between April 17-23, 2000 are shown in Figure 5-9. These plots, together with the analyses of historic bid data between the periods of study, indicate that LP 206845 adjusts its bidding prices seasonally. It intends to be scheduled to operate during the peak-demand hours, especially during the peak-demand months of January and July when one can observe the low bidding prices for the same amount of power. Some of its capacity is self-scheduled during low-demand months, especially in April, and the rest of the capacity is offered at expensive bidding prices. In other words, when total demand in the market is low, this LP is not scheduled to operate beyond its total self-scheduled quantity. Using the method presented previously, the scheduled quantities during the last two weeks of January, April, July, and October 2000 are shown in Figure 5-10. The SS capacity of trading TH 14 during these periods is shown in Figure 5-11.

LP 218387

Examples of observed bidding price time-series when the bidding quantities are equal to 100, 900, 1,300, 1,700, and 2,100 MW during January 18-24, 2000 are shown in Figure 5-12, and when the bidding quantities are equal to 100, 500, 900, 1,300, 1,700, and 2,100 MW during April 17-23, 2000 are shown in Figure 5-13. This LP is responsible for determining the bid-supply function for the second largest capacity, or 10.4% of the installed capacity in July 2000. It self-schedules parts of the capacity, especially during the off-peak hours. Table 5.6 shows the variation of maximum available capacity, or the total HOLs and SS capacity of this LP on January 19, 2000. During the morning hour of low demand, the SS capacity is set to the highest, or 44% of its maximum available capacity.

Table 5.6: Self-scheduled and and Maximum Available Capacity on January 19, 2000 of LP 218387

Hour	1	6	10	13	15	18	21	23
HOL (MW)	2,680	2,680	2,680	2,680	2,680	2,680	2,680	2,680
SS (MW)	1,179	1,179	388	288	246	948	367	200

Notice that the bidding prices in Figure 5-12 do not reflect the SS capacity since this LP submits positive bidding prices for its SS portion. From Figure 5-12, one can observe that LP 218387 submits lower bidding prices during high-demand hours than during the low-demand hours in the morning for the same bidding quantity, (i.e., less than half of its maximum available capacity) 100 and 900 MW. LP 218387 submits the same bid-supply functions during the other months as well. The maximum available capacity during April is lower than the other months. This LP is dispatched to at least its

SS capacity for every hour. Therefore, when combined with its bidding prices, this LP is scheduled to operate mostly during high-demand hours.

LP 506459

This LP is responsible for determining the bid-supply function for the largest capacity, or 17.8% of total installed capacity. Several generating units are dispatched as self-scheduled units or marginal units. The bid-supply functions of LP 506459 during the periods of interest are sampled. Examples of observed bidding time-series prices given bidding quantities ranging from 1,000 to 3,500 MW with an increment of 500 MW between January 18-31, April 17-23, July 18-31, and October 18-24, 2000 are shown in Figures 5-14-5-19. The bid-supply functions shift substantially within a one-day period and tend to move in the same direction as the levels of demand within a day. For example, LP 506459 offers a higher bidding quantity during peak-demand hours than during off-peak hours at the same bidding prices.

The portfolio of this LP contains the largest capacity and variety of units. This LP submits low bidding prices for the first 2,000 MW of its capacity so that it is scheduled to operate every hour, and at generally more than its SS capacity. A possible bidding strategy of this LP is described later in Section 5.4. In addition, the bid-supply functions of five units of LP 506459 are further examined in Section 5.5.

LP 529988

Examples of observed bidding price time-series when bidding quantities are equal to 450, 900, 1,450, and 1,800 MW between January 18-24, 2000 are shown in Figure 5-22, and when bidding quantities are equal to 450, 900, 1,450, and 1,650 MW between April 17-23, 2000 are shown in Figure 5-22. This LP owns 8.4% of total installed capacity. Its total available capacity, or total HOLs, are lower during the low demand months of April and October. The SS capacity varies within the day as well as over months. When SS capacity is non-zero, this capacity is lower during the low-demand hours than during the high-demand hours. Examples of the maximum available capacity and SS capacity of this LP on a Wednesday in January, April, July, and October 2000 are shown in Table 5.7. This LP increases its bidding prices (shifting the bid-supply functions) in April, July, and October compared to the bidding prices in January. During those months, it is basically scheduled to operate at its SS capacity.

5.3.2 Observation and Analyses

Based on the bid data, one can observe that:

- The bidding prices given bidding quantity (or the bidding prices for the same bidding quantity)

Table 5.7: Self-scheduled and Maximum Available Capacity of LP 529988

Date	Hour	1	6	10	13	15	18	21	23
1/19/00	HOL (MW)	2,333	2,333	2,333	2,333	2,333	2,333	2,333	2,333
	SS (MW)	235	282	516	491	491	627	669	359
4/19/00	HOL (MW)	1,682	1,682	1,573	1,573	1,573	1,573	1,682	1,682
	SS (MW)	0	0	0	0	0	0	0	0
7/19/00	HOL (MW)	2,209	2,209	2,209	2,209	2,209	2,209	2,209	2,209
	SS (MW)	85	85	85	85	85	85	85	85
10/18/00	HOL (MW)	1,950	1,950	1,950	1,950	1,950	1,950	1,891	1,786
	SS (MW)	535	535	850	725	725	935	935	790

do change over time on both daily and seasonal bases. (See, for example, Figures 5-14, 5-16, 5-17, and 5-18.)

- The LPs tend to submit the bid-supply functions that reflect the unit types as well as their entire portfolios. For example, one can observe from Figures 5-24, 5-28, and 5-27, the difference between the bid-supply functions of several units submitted by the same LP. In addition, the bid-supply functions of the entire portfolio change according to demand levels on a daily and weekly, as well as on a seasonal, basis.

- One possible strategy of some LPs (such as LP 218387) is that they tend to submit bids to fill their target scheduled capacity (similarly to a target utilization rate [8]). That is, the bidding prices corresponding to the target scheduled capacity are likely to be adjusted to match anticipated prices (or demand patterns).

- The LPs tend to submit high bidding prices for two possible reasons, 1) to avoid being scheduled to operate in low-demand (low-price) periods and 2) to set the market prices. As observed, the bidding prices during the off-peak hours for low bidding quantity are generally higher than the same portion of bidding quantity during peak hours. One must also keep in mind that some units might have limited energy generation capacity, i.e., hydropower units, so that their bid-supply function might reflect this constraint.

- Whether the LPs follow a certain learning algorithm is difficult to assess. One possible learning algorithm and/or bidding strategy of LP 506459 as observed from the bid data during the periods January 18-31 and October 18-31, 2000 is described in the next section. As observed from the bid data, it is not necessary that the LPs adopt only one strategy or learning algorithm over time.

- The LPs have different bidding strategies and/or learning algorithms. Some LPs submit the same bids over time without adjustment to demand levels, while some LPs submit time-varying bids, which do not necessarily depend on demand levels.

- Without marginal costs or operating costs such as fuel prices of each LP, it is difficult to differentiate whether the high bidding prices are a result of learning to bid strategically, of changing operating costs, or of implementing the capacity withholding strategy.[3] The total available capacity in each trading hour does change over time. This may reflect that the LPs are withholding their capacity or that the units may be unavailable due to operating constraints.

5.4 Load Indices and Bidding Behavior

Another important issue in the agent-based market model is the assumption that the agents behave strategically in the electricity market model based on demand levels. This assumption originates from the observed total demand and prices without knowledge of the LP bid-supply functions. To verify whether this assumption is reasonable, LP bidding behavior is observed based on the forecast demand level. After the demand indices associated with forecast demand are determined, the trading hours are rearranged and grouped such that the hours with the same forecast demand index are ordered consecutively based on their order of occurrence. For example, the total forecast demand in New England can be discretized into 15 indices, in which each index represents demand of a 1,000-MW range. The first index represents demand not more than 9,000 MW, while the last index represents demand more than 22,000 MW. Table 5.8 shows the number of hours associated with each index during year 2000.

Table 5.8: Examples of Discretized Demand in Year 2000

Range	< 9,000	9-10,000	10-11,000	11-12,000	12-13,000
Hours	34	465	862	763	785
Range	13-14,000	14-15,000	15-16,000	16-17,000	17-18,000
Hours	812	961	1,609	1,006	719
Range	18-19,000	19-20,000	20-21,000	21-22,000	> 22,000
Hours	431	201	66	35	10

There are, however, several possible ways to observe the load-based behavior of the LPs, methods such as referencing the observations on maximum-minimum daily demand and/or average of daily demand. From the historic bid data, the bidding patterns of the LP are unlikely to change hourly (for example, see Figures 5-8 - 5-23) if the hourly load indices are accounted for as in the proposed agent-based model. Instead, the observed bid data seem to change on a daily basis. Moreover, to determine the bid-supply function for the LPs by considering demand on an hourly basis is impractical because the generating units generally operate on at least a daily basis due to unit-commitment constraints. Instead of collecting the data in the memory matrices (as described in Chapter 3) based on hourly load-indices, one can base the data on indices of daily maximum-minimum and/or average peak demand,

[3] Moreover, the LPs may submit high bidding prices when they realize the potential benefits of out-of-merit scheduling to alleviate transmission constraints. However, this condition is not considered in this thesis.

which must be during specified hours such as between 7:00 a.m. and 11.00 p.m. when demand is relatively high (or the peak-demand hours). The information in a memory matrix is updated on a daily basis, reducing the necessary frequency of data updating to at most 24 times a day compared to updating hourly. The possible load-based bidding strategy/learning algorithm of LP 506459 as observed from the bid data is presented as follows. In this study, LP 506459 is chosen due to its capacity, and its units are likely to be scheduled to operate as marginal units because LP 506459 has a mixed-type of generation technology, which could provide flexibility in shaping a bid-supply function.

5.4.1 A Possible Bidding Strategy

The following examples and analyses show a method to analyze a bidding strategy and/or a learning algorithm of LP 506459. Bid data during two 2-week periods in January and July, 2000 are observed. The steps to analyze the possible learning (bidding strategies) are as follows:

1. *Discretizing demand between 7:00 a.m. and 11:00 p.m. of January 18-31, 2000 into four indices.* The criteria are based on the maximum and minimum forecast demand because prior to a bid submission the LP is informed of the forecast demand. The first index represents the maximum demand greater than 21,000 MW and the minimum demand greater than 17,000 MW, and the second index represents the maximum demand between 19,500 and 21,000 MW and the minimum demand between 16,000 and 17,000 MW. The third index represents the maximum demand between 18,000 and 19,500 MW and the minimum demand between 15,000 and 16,000 MW. The last index represents the maximum demand less than 18,000 MW and the minimum demand less than 15,000 MW. As shown in Figure 5-4, Day 21 is represented by the first index, Days 18, 19, 20, 27, and 28 are represented by the second index, Days 22, 23, 24, 25, 29, and 31 are represented by the third index, and Day 30 is represented by the forth index.

2. *Calculating total revenue of each day from scheduled quantity and prices.* This revenue is the total sum over 24 hours of scheduled quantities multiplied by scheduled prices (market prices), assuming that the scheduled quantity (power) remains constant throughout the hour. (These sets of information are shown in Table 5.9.) Figure 5-20 shows the scheduled quantities and market prices for January 18-31, 2000.

3. *Examining a possible learning pattern.* For each index, the first day of the observed period is considered an initial condition. The bid-supply function of this day is used as a reference bid. Let us consider the load series associated with only the second and third indices. Days 18 and 22 are the initial conditions for the second and third indices.

From Table 5.9, x M\$/h (or M for short) denotes x million dollars per hour. For the second index, on the 1st day (January 18) the revenue and the bidding quantity equal to an average of scheduled

Table 5.9: Average Scheduled Quantity, Calculated Market Prices, and Revenues of LP 506459

Date	January				July			
	Index	Q (MW)	P ($/MWh)	Revenue (M$/h)	Index	Q (MW)	P ($/MWh)	Revenue (M$/h)
18	2	3,100	38.24	2.02	1	2,830	42.61	2.09
19	2	2,302	37.60	1.49	2	2,505	35.39	1.53
20	2	2,078	37.49	1.33	3	2,355	33.94	1.39
21	1	2,467	39.78	1.67	3	2,753	36.16	1.73
22	3	2,592	49.34	1.74	4	2,500	34.12	1.48
23	3	2,154	35.73	1.32	4	2,197	28.32	1.12
24	3	2,207	35.64	1.35	2	2,553	35.16	1.55
25	3	2,806	37.17	1.78	2	2,547	34.04	1.49
26	3	2,407	36.77	1.52	2	2,539	33.51	1.46
27	2	2,672	39.10	1.81	2	2,638	34.52	1.56
28	2	2,700	40.11	1.85	1	2,621	34.20	1.54
29	3	2,535	35.52	1.55	3	2,202	28.42	1.09
30	4	2,024	25.49	0.90	4	2,158	27.10	1.05
31	3	2,242	32.39	1.25	2	2,456	36.78	1.54

quantity are $2.02M and 3,100 MW. The averaged scheduled price, or scheduled price (calculated market price), is equal to $38.24/MWh. The LP substantially increases its bidding prices for the bidding quantities 2,500 MW and 3,000 MW for the second day (January 19) and obtains scheduled quantity and revenue equal to 2,302 MW, and $1.49M, though the scheduled price does not change from the first day, remaining at $37.60/MWh. The LP reduces its bidding prices for bidding quantity 3,000 MW and increases its bidding prices for bidding quantity 2,500 MW (during the evening hours) for the third day (January 20). However, the scheduled quantity and revenue decrease further to 2,078 MW and $1.33 M, respectively. On the fourth day (January 27), LP 506459 decreases its bidding prices for bidding quantities 2,000, 2,500, and 3,000 MW. Its revenue and scheduled quantity increase to $1.81M and 2,672 MW, respectively, with the scheduled price equal to $39.10/MWh. On the fifth day (January 28), the LP continues decreasing the bidding prices of bidding quantity 2,000 and 3,000 MW for all hours, and 2,500 MW for the morning and evening peak-hour periods. Thus, the LP increases its revenue and scheduled quantity to 2,700 MW and $1.85 M, respectively.

For the third index, on the first day (January 22) the initial revenue and the bidding quantity are $1.74M and 2,592 MW, respectively, while the LP decreases the bidding prices for 2,500 and 2,800 MW for the 2nd day (January 23) during the evening peak-hours, and obtains the scheduled quantity and revenue equal to 2,154 MW and $1.32M with the scheduled price decreasing from $49.34 to $35.73/MWh. The LP maintains its bidding prices for 2,500 and 3,000 MW, (and, though not of interest to this thesis, increases the bidding prices for 2,000 MW during the morning off-peak hours) on the third day (January 24). The scheduled quantity and revenue increase slightly to 2,207 MW and $1.35M, respectively, without significant change of the scheduled price. On the fourth day (January 25), the LP decreases its bidding prices for 2,000 and 2,500 MW and increases its bidding prices for

3,000 MW. Revenue and scheduled quantity increase to \$1.78M and 2,806 MW, with an increase of the scheduled price to \$37.17/MWh. Then on the fifth day (January 26) the LP increases the bidding prices of 2,000, 2,500, and 3,000 MW, which reduces the revenue and scheduled quantity to 2,407 MW and \$1.52M, respectively. The LP decreases its bidding prices for 2,000, 2,500, and 3,000 MW for the evening peak-hour period on the sixth day (January 29), causing a revenue and scheduled quantity increase to \$1.55M and 2,535 MW, respectively. The LP increases its bidding prices for bidding quantity 2,000 and 2,500 MW the next day (January 31), which results in revenue and scheduled quantity reductions to \$1.25M and 2,242 MW.

Similarly, the other example is focused on the two-week period of July 18-31, 2000. Steps similar to those in the previous example are applied:

1. *Discretizing demand between 7:00 a.m. and 11:00 p.m. of July 18-31, 2000 into four indices.* The first index represents the maximum demand greater than 19,000 MW and the minimum demand between 13,500 and 14,000 MW, and the second index represents the maximum demand between 17,500 and 19,000 MW and the minimum demand between 13,000 and 13,500 MW. The third index represents the maximum demand between 16,000 and 17,500 MW and the minimum demand between 12,500 and 13,500 MW, whereas the last index represents the maximum demand less than 16,000 MW and the minimum demand less than 12,500 MW.[4] Therefore, as shown in Figure 5-6, Days 18 and 28 are represented by the first index, Days 19, 24, 25, 26, and 27 are represented by the second index, Days 20, 21, and 29 are represented by the third index, and Days 22, 23, and 30 is represented by the forth index.

2. *Calculating total revenue from scheduled quantity and prices.* Figure 5-21 shows the scheduled quantities and market prices between July 18 and 31, 2000. The total revenue, scheduled quantity, and scheduled prices during this period are also shown in Table 5.9.

3. *Examining a possible learning pattern.* Let us analyze the series associated with the first index. The first day, or January 19, 2000, is considered an initial condition. The bid-supply function of this day is used as a reference bid.

The revenue and the bidding prices on the first day (July 19), given that the bidding quantity is equal to an average over a day of the scheduled quantity, are \$1.53M and 2,505 MW, respectively. The LP decreases bidding prices for bidding quantity 2,500 MW and increases them for bidding quantity 3,000 MW on the second day (July 24), and obtains revenue \$1.55M and scheduled quantity 2,553 MW. The LP lowers then its bidding prices for 2,500-3,000 MW on the third day (July 25) and increases its scheduled quantity to 2,547 MW and revenue \$1.49M. On the fourth day (July 26), the LP continues lowering its bidding prices for 2,500-3,000 MW, and, its scheduled quantity and revenue decrease to

[4] Note that the indices may vary according to the demand levels. Some units might not be available during different months, and as shown in Figure 5-32 demand in January, 2000 is higher than in July, 2000 for almost every hour.

2,539 MW and \$1.46M. Again on the fifth day (July 29), the LP lowers its bidding price further and the scheduled quantity and revenue increase to 2,638 MW and \$1.56M. On the sixth day of this series (July 31st), the LP decreases its bidding prices for the peak demand hours further and increases the bidding prices for the off-peak hours. By doing so the LP obtains less scheduled quantity 2,456 MW, but the revenue remains similar at \$1.54M due to the scheduled price increases from \$34.52 to \$36.78/MWh.

5.4.2 Observation and Analysis

A set of the bidding prices with bidding quantities of LP 506459 is shown in Figures 5-14, 5-15, 5-18, and 5-19. Although these plots show that there is no significant evidence to indicate that load-based behaviors corresponding to finely discretized load levels exist, there are possible load-based behavior on a wider band of load levels. This strategy is referenced to the maximum-minimum load indices. From this observation, LP 506459 is likely to use a strategy in which the bidding prices of a target bidding quantity (the quantity anticipated to be scheduled) are increased if the LP obtains the anticipated revenue; likewise, the bidding prices are decreased if the LP does not obtain the anticipated scheduled quantity and revenue. In addition, this observation suggests possible modifications to improve the effectiveness of the agent-based market model.

5.5 Bidding Behavior of Generating Units

This section presents the analysis of the bidding behavior of generating units, which is focused on the units that are likely to be scheduled to operate at the margin (see Table 5.9). The objectives of this study are to observe the bidding characteristics of generating units that are likely to be marginal and to observe the characteristics of the bid-supply functions of the units with different generation technology and with different flexibility of bid-supply functions.

Five generating units of LP 506459, including Asset IDs 23789, 37274, 43414, 79606, and 81361 are selected for this analysis. Asset ID 23789 is chosen due to its 620 MW capacity, which could be eligible for the CW strategy, and this unit is generally bid as a base-load unit. Asset ID 37274 has a medium capacity of 150 MW and is occasionally scheduled to operate as a marginal unit during the observed periods. Asset ID 43414 is a large unit with 440 MW capacity and it is regularly scheduled to operate as a marginal unit. Asset ID 79606 is a medium-size unit with 160 MW capacity and limited available energy and it is frequently scheduled to operate as a marginal unit. Asset ID 81361 is a unit with 290 MW capacity and limited available energy.

5.5.1 Bidding Behavior

The following analyses focus on 1) whether the LP determines its bid-supply function of each unit according to its type of generating technology, 2) whether the bid-supply functions of the units bid by the same LP have similar patterns, 3) whether the bid-supply function depends on demand levels, 4) how the units can be bid to operate as marginal units, and 5) whether the bidding prices of the same set of units are ordered consistently over time (how the marginal cost (opportunity cost) of the units change).

Asset ID 23789

By matching the capacity of Asset ID 23789 to the unit with a similar capacity in LP 506459's portfolio, Asset ID 23789 is identified as a fossil-fueled generating unit with in the stalled capacity 620 MW. The plots of the bidding prices with the sampled bidding quantities equal to 300, 600, 610, and 630 MW during January 18-24, 2000 are shown in Figure 5-24. According to the bidding characteristics, LP 506459 is likely to bid this unit to be an infra-marginal unit through SS capacity of at least 350 MW daily (except during its unavailable period in April with a zero HOL bid). The bidding prices for the bidding quantity greater than 350 MW are similar during the observed two-week period but vary across the observed months. The unavailability occurs during the off-peak hours for a few days in July.

Asset ID 37274

By matching the capacity of Asset ID 37274 to the unit with a similar capacity in LP 506459's portfolio, Asset ID 37274 is identified as either a combined-cycle or fossil-fueled unit. The plots of the bidding prices with the sampled bidding quantities equal to 20, 80, 120, and 160 MW during January 18-24, 2000 are shown in Figure 5-25. These plots with zero bidding quantity show that this unit becomes unavailable for several days. When the unit is available (i.e., HOL > 0), the unit has bidding prices no less than $30/MWh except during the first observed days in January. For most of the observed days, this unit has one bidding price for its capacity and one bidding price for a one-day period. One can also observe that the unit adjusts its bidding prices in almost an identical pattern of daily demand during the observed period in July. The bidding prices for these periods are comparable to the calculated market prices during the same hours shown in Figure 5-25. This suggests that this unit is anticipated a marginal one.

Asset ID 43414

By matching the capacity of Asset ID 43414 to the unit with a similar capacity in LP 506459's portfolio, Asset ID 43414 is identified as a (fossil-fueled) generating unit with the installed capacity of 430 MW. The plots of the bidding prices with the sampled bidding quantities equal to 20, 320, and 440 MW during January 18-24, 2000 are shown in Figure 5-26. This unit becomes unavailable for a few days during the observed periods in July and October. As observed from the bid-supply functions, this unit submits its bidding price of no less than $40/MWh except in January, when its bidding prices are lower than $40/MWh. Moreover, when the unit is available, as observed from its bid-supply function, it is not intended to operate at full capacity since the last portion (near its full capacity) of its unit has a bidding price substantially higher than the calculated prices. This bidding characteristic may suggest the possibility of the capacity withholding strategy imposed by the LP, and that the LP would want to set a high market price if this unit were scheduled to operate at its full capacity. Being scheduled to operate at part of its capacity, this unit would be eligible to provide reserve capacity[5] and also to be dispatched at a short notice if other cheaper units could not to be turned on instantaneously to meet an abrupt demand change.

Asset ID 79606

By matching the capacity of Asset ID 79606 to the unit with a similar capacity in LP 506459's portfolio, Asset ID 79606 is identified as a hydropower unit with installed capacity 160 MW because it has limited daily energy available. The plots of the bidding prices with the sampled bidding quantities equal to 20, 80, and 120 MW during January 18-24, 2000 are shown in Figure 5-28, the sampled bidding quantities equal to 20, 80, 120, and 140 MW during April 17-23, 2000 are shown in Figure 5-29, while the sampled bidding quantities equal to 20, 40, 80, and 120 MW during July 18-24, 2000 are shown in Figure 5-30. During high-demand months (January and July), the hourly bid-supply functions change throughout the day. The LP appears to submit bidding prices so that this unit is scheduled to operate as a marginal unit during the peak-demand hours (compared with the bidding prices for forecast demand during the same periods). On the other hand, during the low-demand months (such as April and October), the hourly bid-supply functions are similar throughout the day and vary across different days. As shown in Figures 5-28 - 5-30, LP 506459 tends to lower the bidding prices for this unit during the high-demand period and raise the bidding prices during the low-demand period. This observation, together with an analysis of the bids of other hydropower units of LP 506459, indicate that the bid-supply functions of the hydropower units in general reflect the limited energy availability.

Asset ID 81361

By matching the capacity of Asset ID 81361 to the unit with a similar capacity in LP 506459's portfolio, Asset ID. 81361 is identified as a pumped-storage unit with an installed capacity of 290 MW. This is because it has limited daily energy available and its capacity matches the capacity of

[5]See the ISO-NE market rules.

the pumped-storage units in LP 506459's portfolio. The plots of the bidding prices with the sampled bidding quantities equal to 20, 200, 260, and 290 MW during January 18-24, 2000 are shown in Figure 5-27. The bidding prices of this unit is no less than \$40/MWh in most of observed periods except in April. During the two-week period in January the bidding prices for a given bidding quantity are higher during the low-demand hours than the bidding prices during the high-demand hours. Similar characteristics are observed in July, except that the bidding prices are set to be lower in the evening hours than the mid-day hours (peak-demand periods) and the morning hours. This unit is unlikely to be scheduled to operate in October due to the high bidding price.

5.5.2 Observation and Analysis

The plots of the bidding prices, given set bidding quantities, show the variation of the bidding prices of the units with different capacity and generation technologies. From the bid-supply functions, one can observe that

- The bid-supply functions of the generating units submitted by the same LP may change on a daily basis, as well as on a seasonal basis. This is especially true for a hydropower unit, such as Asset ID 79606, as shown in Figures 5-28 - 5-30. However, the change of bid-supply functions depends on the types of units. The bid-supply functions of some units do not change, for example, nuclear units are submitted as a self-scheduled unit daily.
- The bid-supply function for each unit has its own characteristics and does not move in the same fashion. For example, let us consider the bid-supply functions of units 79606 and 81361 in July. The bidding prices of unit 79606 increase on the fifth day in the second week while the bidding prices of unit 81361 decrease on the same day.
- Depending on the demand levels, the units that are dispatched as marginal units have different types of generation technology. However, these units must be highly flexible to be turned on or off. The marginal units of LP 506459 determined by the method presented in Section 5.2 during THs 9 and 18 of January 18-31 and October 18-31, 2000 are shown in Table 5.4.
- The characteristics of the bid-supply function of each generating unit depend also on the unit's installed capacity and generation technology; for example, when Asset ID 23789, a large fossil-fueled unit, is available, it tends to be scheduled to operate as a base-load unit. Its bidding prices are generally lower than the scheduled price. Asset IDs 37274 and 43414 are also fossil-fueled units but have different bidding characteristics due to the basic fact that the marginal costs and operating constraints vary among units with different installed capacity and generating technology. In addition, generating units that are similar in size and type of generation technology may be bid to the market with different strategies. These different strategies may be caused by the locational advantage of the units, though, this issue is not explored in this thesis.

- The hydropower and some fossil-fueled units have operating flexibility in terms of being turned on and off within a short period of time. From Table 5.4, the analysis of the bid data shows that these types of units are often scheduled to operate as marginal during high-demand periods. During the peak-demand months, such as January and July primarily, LP 506459 submits the bid-supply functions, especially of the hydropower and fossil-fueled units, following daily demand characteristics. Given the same bidding quantity, the bidding prices are lower during the hours of high demand than those during the hours of low demand. Therefore, these units are likely to be scheduled to operate as marginal units in most hours.

- The daily bid-supply function of the hydropower unit (not a pumped-storage unit) such as Asset ID 79606 tends to have negative correlations with hourly demand levels. For instance, at bidding quantity equal to 50 MW, the bidding price is higher during low demand than during high demand. Similarly, the bid-supply functions of the pumped-storage units such as Asset ID 81361 show that the units are intended to be scheduled to operate only during the peak-demand hours. That the hydropower unit can be turned on and off easily allows the unit to adjust its operating condition as often as every hour. As observed, these units are bid so that they are scheduled to operate during high-demand hours, meaning that the bidding prices during the peak-demand hours are lower than the bidding prices during the off-peak demand hours. This causes a significant shift during peak and off-peak hours of the bid-supply function of the LPs who own a large capacity of hydropower units. This strategy implies that the unit can operate at specified market prices whenever it is scheduled to operate in any hour. When demand exhibits a two-peak pattern (such as in January, as shown in Figure 5-32) for the hydropower units, increasing the bidding prices between two peak periods may result in a market price similar to the prices during the peak-demand periods. When this strategy is implemented by several hydropower units, the portfolio bid-supply function shifts toward the higher bid quantities when demand is large; that is, the bidding quantity for the same bidding price (drawing a line parallel to a quantity axis) during the peak-hour periods becomes lower than during the off peak period. This is somewhat consistent with the CW strategy.

- Large generating units, such as Asset ID 23789, tend to have low bidding prices for one part of their installed capacity, resulting in it being scheduled to operate as a base-load unit, and the rest of its capacity, which is generally not scheduled to operate, tends to have high bidding prices. There are several possible causes for such behavior. First, the cost of operating at the units near their capacity may be non-constant or non-linear. Second, LP 506459 may implement its strategic behavior, such as the CW strategy. Third, by operating less than the full capacity, the units could be dispatched to serve near real-time demand variation (or could be scheduled to provide reserve capacity) instead of units with the cheaper bidding prices that could not be turned on sufficiently quickly (note that these units could also set high market prices if the market rules allow the units with out-of-merit dispatch to set the market price).

- As observed from the bid data, some units, such as Asset IDs 37274 and 43414, are not available for a few days in a one-week period, i.e., their HOLs are equal to zero. These units may be unavailable because the LP might implement the CW strategy, or these units could be under planned maintenance. Without information of unit outages and/or operating constraints, the unavailability may not necessarily imply that the LP applies the CW strategy.

5.6 Conclusion

The New England historic bid data, market prices, and reports show that the LPs own non-uniform portfolios of generating units and are unlikely to share the same bidding strategy. The bidders with hydropower can adjust their bid-supply functions hourly, but the bidders with only nuclear units cannot adjust their bid-supply function within a day. Moreover, the bidding strategy influences market prices. These findings support the concept that an agent-based approach is essential in modeling and analyzing the electricity spot markets. However, to verify whether the proposed agent-based model is a valid model to represent the electricity spot market, information about the generating units, especially their operating constraints, is crucial. The operating constraints of each unit could play a key role in determining bid-supply functions, bidding outcomes, and market price dynamics. Another key factor in reproducing price dynamics using the agent-based model is to understand the bidding strategy of the market participants. This is a very difficult and tedious task because, as mentioned, massive amounts of generally confidential information on generating units is required, including:

- Unit-commitment constraints.
- Fuel (such as oil and/or gasoline) costs.
- Water levels of river-flow hydropower units.
- Environmental constraints, for example NO_x emission allowance.[6]
- Maintenance schedule.
- Possible load-obligation or bilateral contracts of the bidders, affecting the self-scheduled portion of the bid-supply function.
- Possible change in the portfolio characteristics due to the addition of new units or decommissioning of old units.

[6] See, for instance, the study of the effect of the NO_x emission allowance on the market price markups in the California market by Joskow and Kahn [25].

Note that fuel costs affect operating costs and subsequently cause a change in the bid-supply function. Therefore, without information regarding operating costs and constraints of generating units it might not be sufficient to suggest that the changes in the observed bid-supply function are the result of learning, and not of operating costs and constraints. Knowledge of the maintenance schedule is also important in differentiating between unavailability of the unit due to implementation of the CW strategy and that which is due to scheduled maintenance and outages.

If this information and the historic bid data of each bidder are available, one may differentiate the variation in the bid-supply functions that may be caused by operating constraints and operating costs, and may also be able to identify bidder learning algorithms and/or bidding strategies. Without this information, the most that one can conclude is that the LPs are likely to adopt different learning algorithms/bidding strategies or some forms of mixed strategies.

On the other hand, when only operating constraints and operating costs are available, without accounting for unit-commitment constraints, the agent-based model can be used to analyze the price dynamics. The real and simulated price dynamics might be different because the LPs apply different learning-algorithms from those used in the model. Furthermore, the market participants may have different objective functions. Additionally, prior to applying this model to analyze the existing market, some modifications of load-based behavior are needed, because based on the historic bid data there is no sufficiently explicit sign exhibiting hourly load-based decision-making. Instead, the LPs tend to respond to daily demand patterns or daily average demand. The agent-based market model should be used with caution since it does not fully take into account the factors that influence the decisions of the LPs that could in turn play a key role in determining bid functions. These factors are such as bilateral deals, transmission-related strategic behavior, and scheduling processes (accounting for ancillary products).

Appendix to Chapter 5

A. The New England Wholesale Electricity Market

The wholesale electricity market for the New England region opened on May 1, 1999. This market is administered by the Independent System Operator New England (ISO-NE, [53]). This is a "day-ahead - hourly" marketplace in which wholesale electricity suppliers and power producers or LPs bid their resources into the market the day before and submit separate bids for each resource for each hour of the day. The bid (or bid-supply function) of each LP is a set of bid-blocks in which each block indicates the quantity of power in MW and the associated price for that block in $/MWh. The bids are tabulated and stacked in dollar terms from the lowest to the highest, matching the expected hourly demand forecast for that hour and each hour in the next day. The least cost dispatch sequence for the next day which reflects the actual bids is determined. (The dispatch algorithm can be found on the

ISO-NE website [53].) The generating units are dispatched to match the actual (near real-time) load on the system. The highest bid that is dispatched to meet actual load sets the market clearing price (MCP) for all purchased electricity during that hour. Note that ISO-NE calculates a 5-minute-period MCP. A published hourly price is an average of all 5-minute-period MCPs during that hour. All the units dispatched to meet demand during that hour are paid the market price by buyers who purchase power from the market.[7] The New England market adopts a uniform-pricing market rule in which the LPs whose units are scheduled to produce power get paid the hourly MCP multiplied by the scheduled quantity.

I. Demand Characteristics

Electricity demand in general exhibits a seasonal consumption pattern. Figure 5-31 shows the daily average of forecasted demand in the New England Electricity market during year 2000. The peak-demand periods occur during the winter month of January and the summer month of August, respectively, and the low-demand period occurs during April. Average demand is higher during the weekdays than during the weekends. Within different seasons, the daily pattern varies considerably. For example, during the summer months peak consumption occurs once during the day, while during the winter months, two peak consumption periods occur daily, as shown in Figure 5-32. This figure shows a one-week period of forecast demand publicly posted on the ISO-NE website by the operator[8] for Monday to Sunday periods during the weeks of January 17-21, 2000 and July 31-August 4, 2000.

II. Bid Characteristics

In the New England market, the daily bids of all market participants are revealed to the public after a 6-month delay period, and these historic bid data are published on the ISO-NE website [53]. Examples of typical bids are shown in Table 5.10, where TH denotes trading hour end and LP denotes lead participant. The LP is a supplier responsible for bidding its generating unit, which is represented by a masked number, for instance LP ID 140603. The Asset ID Number is the particular asset being bid by a masked number, for instance Asset ID 65758. Self-scheduled (SS) capacity is the MW's (if any) that were scheduled to run. Daily energy available (DEA) is the entire amount of energy available in MW's for the specific date for a limited energy generating unit, such as a hydropower or a pumped-storage unit. The high operating limit of the unit, denoted by HOL, is the maximum available capacity that the unit offers at any hour, and the low operating limit, denoted by LOL, is the minimum operating capacity of the unit. Bid-block $ is the dollar figure the unit was bid at. Bid-block MW is the amount of MW's bid at a specific price for the unit. For each asset the LP can bid up to 10 pairs of $ and MW blocks.

[7]ISO-NE is a residual market. Residual means that to the extent that a power supplier produces electricity in excess of the demand of its customers, its can sell the excess into the wholesale market to other market participants.

[8]Forecast demand is used instead of actual demand to be consistent with the rest of the analyses.

Table 5.10: Typical Energy Bids Submitted by Electricity Suppliers

TH	LP	Asset	SS	DE	HOL	LOL	BB	BB	BB	BB	...	BB
	ID	ID	MW	MW	MW	MW	$	MW	$	MW	...	MW
1	140603	65758	0	0	380	160	72.9	160	72.9	190	...	0
...												
1	184983	39697	868	0	868	868	0	875	0	0	...	0
...												
8	140603	65758	0	0	380	160	131.4	160	131.4	190	...	0
...												
8	184983	39697	868	0	868	868	0	875	0	0	...	0
...												

III. Lead Participants

During July 2000, at least 42 LPs submitted their bid-supply functions to the ISO-NE. Among these LPs, the 12 largest own around 83% of installed capacity. The total HOLs of the twelve largest LPs as of July, 2000 is shown in Table 5.11. The largest LP owns around 18% of installed capacity, while the smallest owns less than 0.05% of installed capacity. Each LP owns at least one generating unit. These units are generally different in generation technology types and installed capacity. According to the market rules, the maximum capacity that the LPs can bid to the markets is dependent upon their the net claimed capacity. LPs must notify the ISO-NE of their net claimed capacity. The net claimed capacity of the generating units may change over the seasons (Seasonal Claimed Capacity (SCC)) due to their operating conditions.[9] The types of generation technology in the New England market are shown in Table 5.12. These data are obtained from the ISO-NE website [53]. Note that total installed capacity shown in Table 5.11 can be lower than total HOL from the bid data and that based on the ISO-NE's market rules, the units are not dispatched beyond their HOLs and the units, once eligible, are not dispatched below LOLs.

Table 5.11 shows the HOL of the 12 largest LPs between 2:00 p.m. and 3.00 p.m. on a weekday, July 28, 2000.

B. Absolute Market Power Conditions

Another interesting issue to consider is the possibility of an absolute market power condition, which would occur if the LP submitting the most expensive bidding prices was still being scheduled. This thesis chooses LP 506459 for the analysis, because it has the largest percentage of market installed capacity. In this analysis, the trading days that the market power condition may exist are first identified, and then the bidding behavior on those particular days is examined. To determine the

[9]SCC represents the Summer and Winter Claimed Capacity of a generating unit. A summer period runs from June 1 through September 30, and the winter period runs from October 1 through May 31. Claimed capacity is the maximum dependable load carrying ability, in megawatt, of units, excluding capacity required for station use. For example, the units may operate all day during the summer months and only a few hours during the spring months.

Table 5.11: Examples of Available Capacity of 12 Largest LPs of July, 2000

Lead Participant ID	Capacity (MW)	% of Installed Cap.
506459	4,423	17.8 %
218387	2,590	10.4 %
333704	2,382	9.6 %
529988	2,096	8.4 %
140603	1,789	7.2 %
532832	1,327	5.3 %
206845	1,257	5.1 %
483669	1,158	4.7 %
674577	1,141	4.6 %
400693	881	3.5 %
184983	867	3.5 %
910093	761	3.1 %
Others	4,162	16.8 %
Total	24,834	100 %

Table 5.12: Summer Seasonal Claimed Capacity of July, 2000

Unit Type	Capacity (MW)	% of Installed Cap.
Fossil (F)	11,580	47.3 %
Nuclear (N)	4,359	17.8 %
Combined Cycle (CC)	3,722	15.2 %
Gas Turbine (G)	663	2.7 %
Jet Engine (J)	774	3.2 %
Diesel (D)	126	0.5 %
Pumped Storage (PS)	1,650	6.7 %
Hydro-Conventional Daily (HD)	746	3.0 %
Hydro-Conventional Weekly (HW)	878	3.6 %
Total	24,499	100 %

market power condition the following steps are used:

1. *Calculating the market capacity surplus.* Forecast the total available capacity and peak demand of each day (d) which are available in the ISO-NE morning report [53]. Total available capacity Q_d is total installed capacity Q_d^{max} plus imported capacity Q_d^{Im}, subtracted by outages Q_d^{Out}, where the imported capacity is the maximum forecast imported capacity within that day and the outage capacity is the maximum forecast outage capacity within that day. Hence, $Q_d = Q_d^{max} + Q_d^{Im} - Q_d^{Out}$. The forecast peak demand does not include forecast peak-demand reserve requirements. The capacity surplus is simply the difference between forecast available capacity and forecast peak demand. Then, the percentage of capacity surplus over total available capacity (and/or total installed capacity) is calculated.

2. *Calculating the available capacity of the LP.* Some units of the LP may be unavailable due to outages or operating constraints. The information of available capacity can be determined from the hourly HOLs.

3. *Comparing whether the capacity surplus is less than the LP's available capacity.* If the available capacity is greater than the capacity surplus, the market power condition is possible. The LP available capacity $\bar{Q}$ is defined by the difference between total available capacity $\bar{H}$, which is the total sum of high-operating-limit capacity of each unit j (HOL^j), and total SS capacity $\bar{SS}$, which is the total sum of SS capacity of each unit j (SS^j); i.e.,

$$\bar{Q} = \bar{H} - \bar{SS} = \sum_j (HOL^j - SS^j).$$

Note that the LP may have a load-obligation, meaning that the LP also provides power. Therefore, the net available capacity may be less than HOLs.

Due to the market power mitigation scheme imposed by the ISO-NE, the LPs who submit substantially high bidding prices are subject to a legal investigation. Therefore, when the absolute market power condition occurs for any LP, meaning that capacity surplus is less than total available capacity of the LP, the LP may submit higher bidding prices than usual (but not at the substantial value). The available capacity of the LP, however, may not be available for imposing the strategic bidding strategy (such as the CW strategy) because some units might not be ready to operate due to operating constraints. The information regarding the operation constraints of the units is generally not available to the public. These factors limit the frequency of market power conditions.

To examine the possible abuses of absolute market power conditions by some LPs, let us consider the time-series of bidding prices given the bidding quantity of the largest LP, LP 506459, for four 2-week periods in January, April, July, and October as shown in Figures 5-14 - 5-19. These bidding prices are accompanied by the LP minimum daily capacity surplus $\bar{Q}_{LP}$ or the anticipated scheduled capacity if the LP were to withhold its capacity to take advantage of this condition. Let $\bar{Q}_{LP}$ be defined by $\bar{Q}$ minus the market capacity surplus. The higher the $\bar{Q}_{LP}$, the greater the possibility for the LP to abuse the market power condition.

Total available capacity and minimum available capacity of LP 506459 are shown in Table 5.13. This table contains 1) a set of 2-week period of daily total available capacity ("Total Avail. Cap.") which is the installed capacity (Q_d^{max}) plus imported capacity (Q_d^{Im}) minus outage (Q_d^{Out}), i.e., ($Q_d^{max} + Q_d^{Im} - Q_d^{Out}$), 2) capacity surplus, which is total available capacity minus the maximum forecast demand of that day when the reserve requirement is not accounted for ("Surplus I"), 3) capacity surplus that includes the reserve requirement ("Surplus II"), and 4) the lowest $\bar{Q}$ of LP 506459 ("Min. Avail. Cap."). By comparing "Surplus I" and "Min. Avail. Cap.," when "Min. Avail. Cap." of each day exceeds "Surplus I," it is possible to anticipate the possibility of LP 506459 having absolute market power. During these 56 days of observations, LP 506459 might choose January 18, 21, and 28, April 26, as well as October 30 to take advantage of its absolute market-power condition. Similarly, with reserve requirement or by comparing "Surplus II" and "Min. Avail. Cap.," when "Min. Avail.

Cap." of each day exceeds "Surplus II," one could anticipate an even greater possibility of LP 506459 having absolute market power. Note that when the reserve requirement is accounted for, one might anticipate that the LPs may exercise the absolute market power in both energy and reserve markets. During these 56 days of observations, besides those five days previously identified, LP 506459 might also choose several more days to raise the bidding price.

By comparing the bidding prices of these four two-week periods, the bidding prices during the days with positive $\bar{Q}_{LP}$ are likely to be much higher than during the days with negative $\bar{Q}_{LP}$. Without the true marginal costs in each period, it is not possible to conclude that the high bidding prices have high price markups by comparing the bidding prices. There is no significant evidence to indicate LP 506459's exploitation of market power during the days with large $\bar{Q}_{LP}$ to raise the bidding prices substantially higher than the prices during days with low $\bar{Q}_{LP}$.

Table 5-13: Available Capacity of LP 506459 during January, April, July, and October 2000

January					April				
Date	Total Avail Cap	Surplus I	Surplus II	Minimum Avail Cap.	Date	Total Avail Cap	Surplus I	Surplus II	Minimum Avail Cap.
1/18/00	22891	1941	197	2218	4/17/00	19624	4399	2119	2750
1/19/00	23147	2872	1132	2109	4/18/00	19898	3823	1543	2894
1/20/00	22744	2769	1029	2247	4/19/00	19142	3242	962	3037
1/21/00	22932	1857	75	2083	4/20/00	19238	3563	1283	3037
1/22/00	22739	3164	1424	2121	4/21/00	19382	4482	2202	3373
1/23/00	22790	3515	1775	2200	4/22/00	18267	4242	1785	3389
1/24/00	22844	3594	1814	2196	4/23/00	17455	4180	1905	3280
1/25/00	23288	3938	1858	2155	4/24/00	19533	3883	1603	3267
1/26/00	22954	4279	2199	2138	4/25/00	18686	3386	906	3276
1/27/00	23336	3661	1581	2132	4/26/00	18991	3091	1111	3301
1/28/00	21493	1743	1	2130	4/27/00	19597	3597	1617	3332
1/29/00	21656	3506	1237	2099	4/28/00	19617	3942	1962	3349
1/30/00	21628	4753	2673	1985	4/29/00	17131	3456	1512	3443
1/31/00	23021	4121	2041	2102	4/30/00	17149	3649	1650	3587

July					August				
Date	Total Avail Cap	Surplus I	Surplus II	Minimum Avail Cap.	Date	Total Avail Cap	Surplus I	Surplus II	Minimum Avail Cap.
7/18/00	22902	3102	1122	2102	10/18/00	20966	3791	2131	2213
7/19/00	21998	3573	1593	2159	10/19/00	21090	4390	2727	2193
7/20/00	21101	3826	1462	2191	10/20/00	20524	4199	2611	2223
7/21/00	20669	3619	1255	1892	10/21/00	17747	3547	1966	1903
7/22/00	18761	3036	1252	1912	10/22/00	18203	3703	2114	1904
7/23/00	19002	3727	1946	1871	10/23/00	21097	4372	2732	1846
7/24/00	21569	3734	1664	1721	10/24/00	21145	4720	3071	1918
7/25/00	20937	3187	1107	1755	10/25/00	20711	4386	2737	1693
7/26/00	21285	3110	1030	1798	10/26/00	20831	4506	2857	1747
7/27/00	21127	3602	1522	1792	10/27/00	20926	4776	3132	1885
7/28/00	21271	3046	966	1765	10/28/00	19166	4216	2627	1602
7/29/00	19753	3678	1889	1788	10/29/00	19447	3247	1658	2007
7/30/00	19467	3742	1961	1791	10/30/00	20362	1812	193	1997
7/31/00	21328	3028	948	2191	10/31/00	20245	2295	702	1651

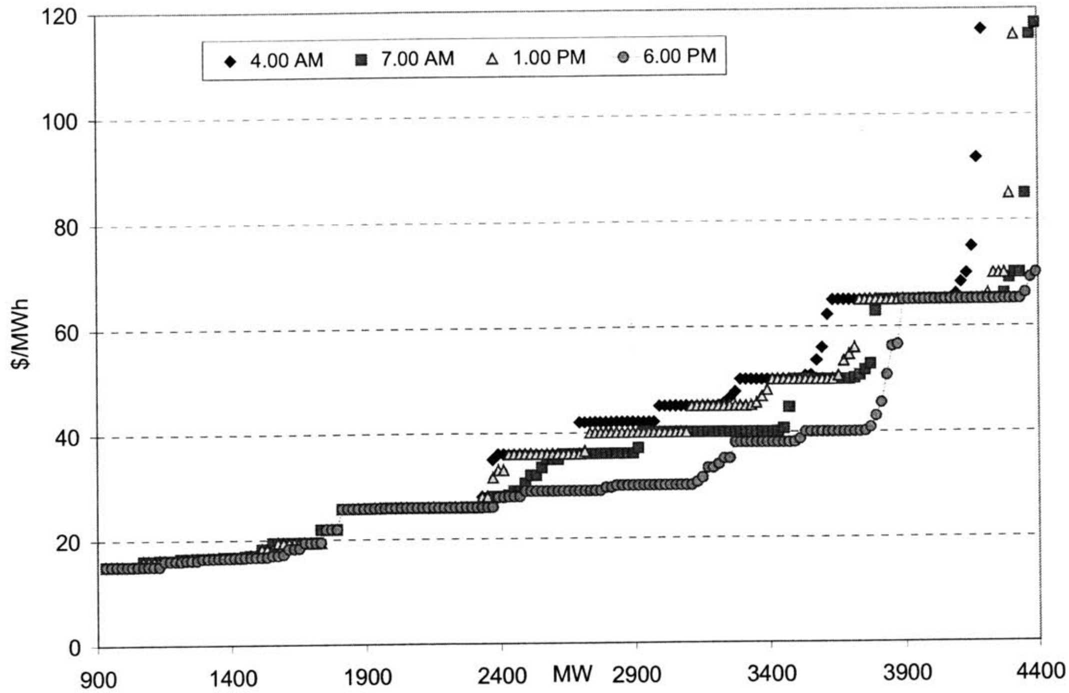

Figure 5-1: Examples of Hourly Bid-supply Functions of LP 506459 in January 2000

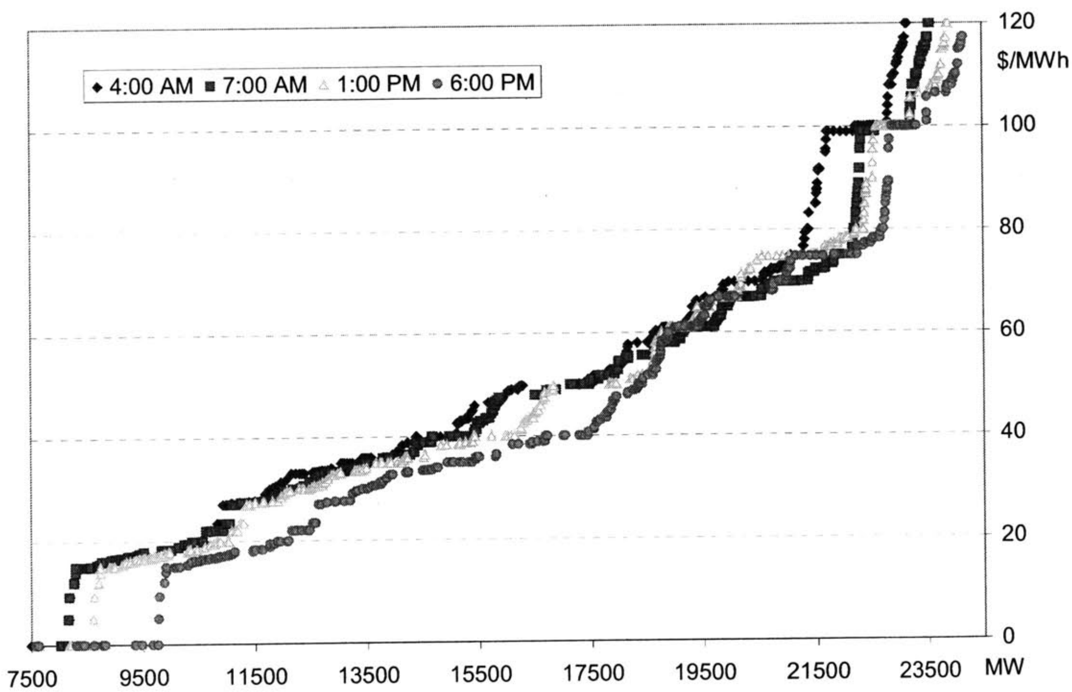

Figure 5-2: Examples of Hourly Aggregate Bid-supply Functions in January 2000

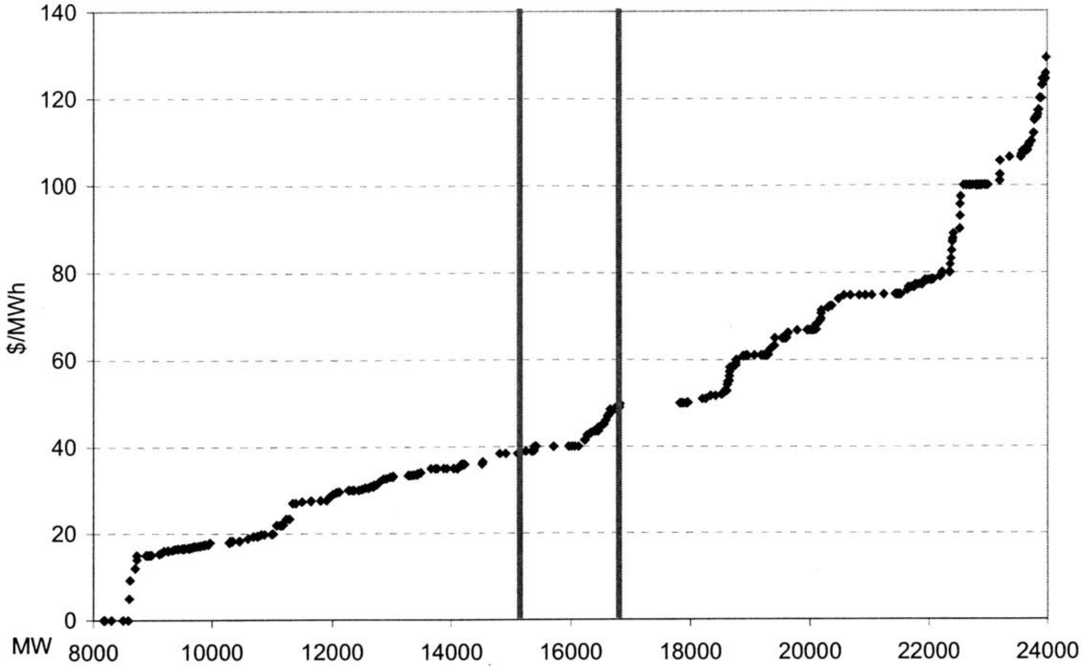

Figure 5-3: A Band of Marginal Units When Demand is Equal to 16,000 MW

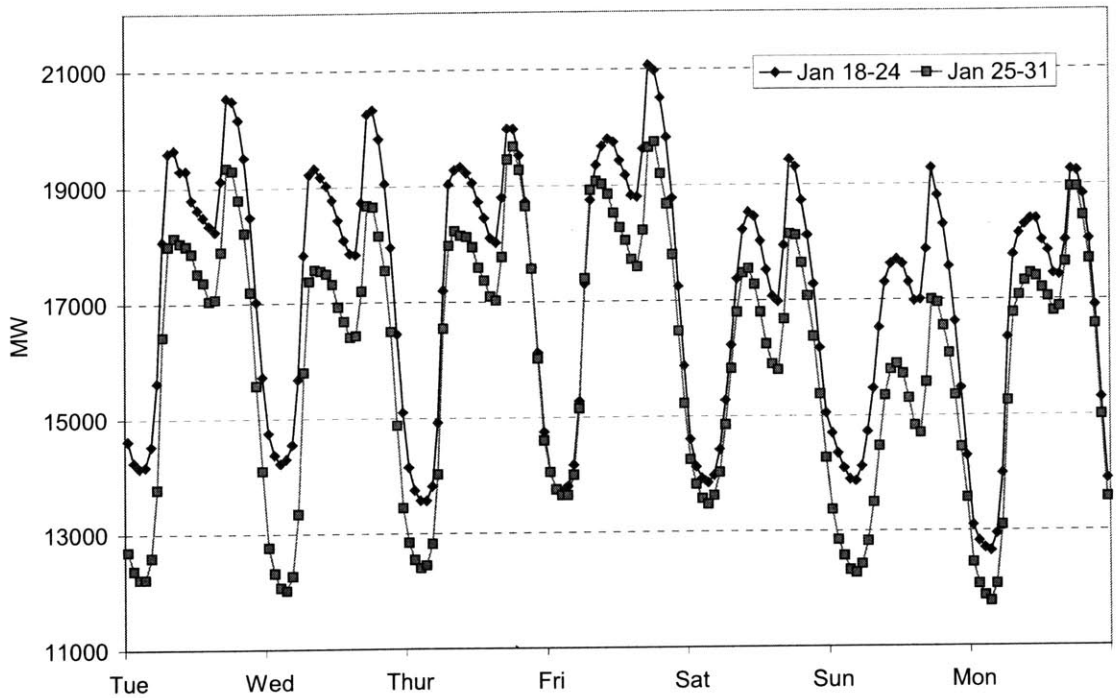

Figure 5-4: Daily Forecast Demand in New England during January 18-31, 2000

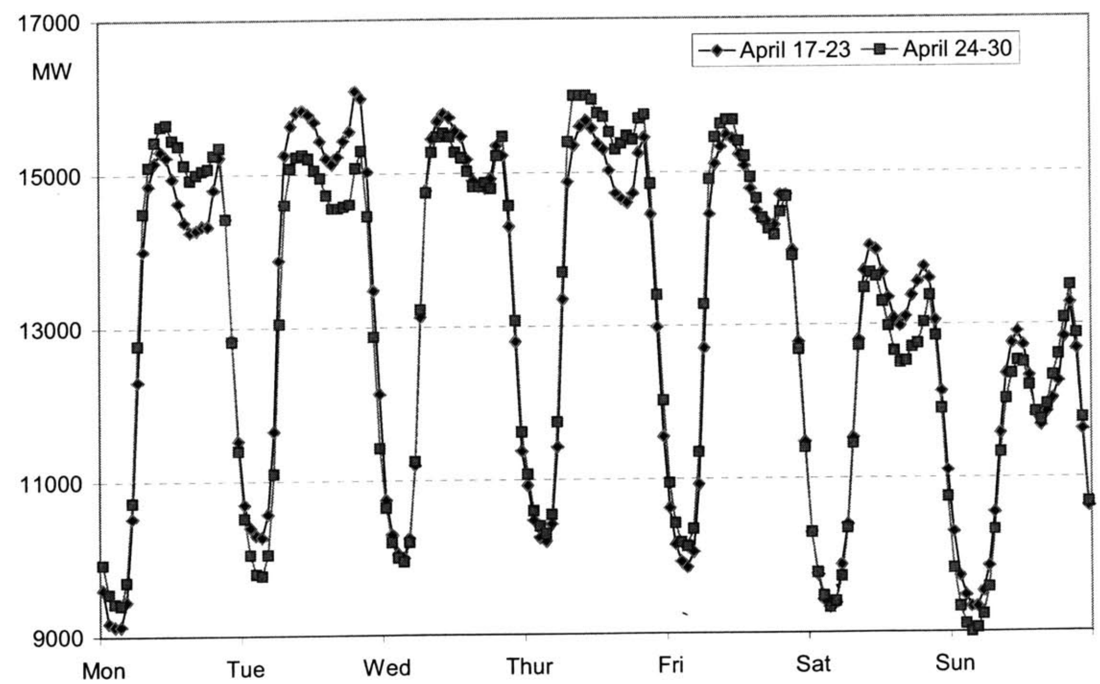

Figure 5-5: Daily Forecast Demand in New England during April 17-30, 2000

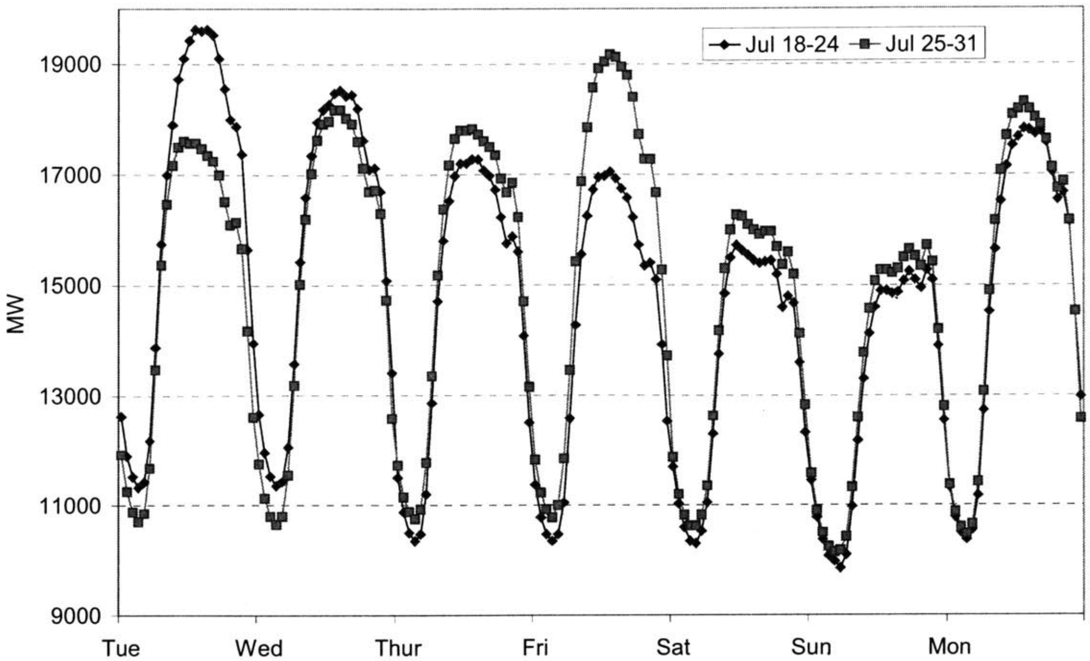

Figure 5-6: Daily Forecast Demand in New England during July 18-31, 2000

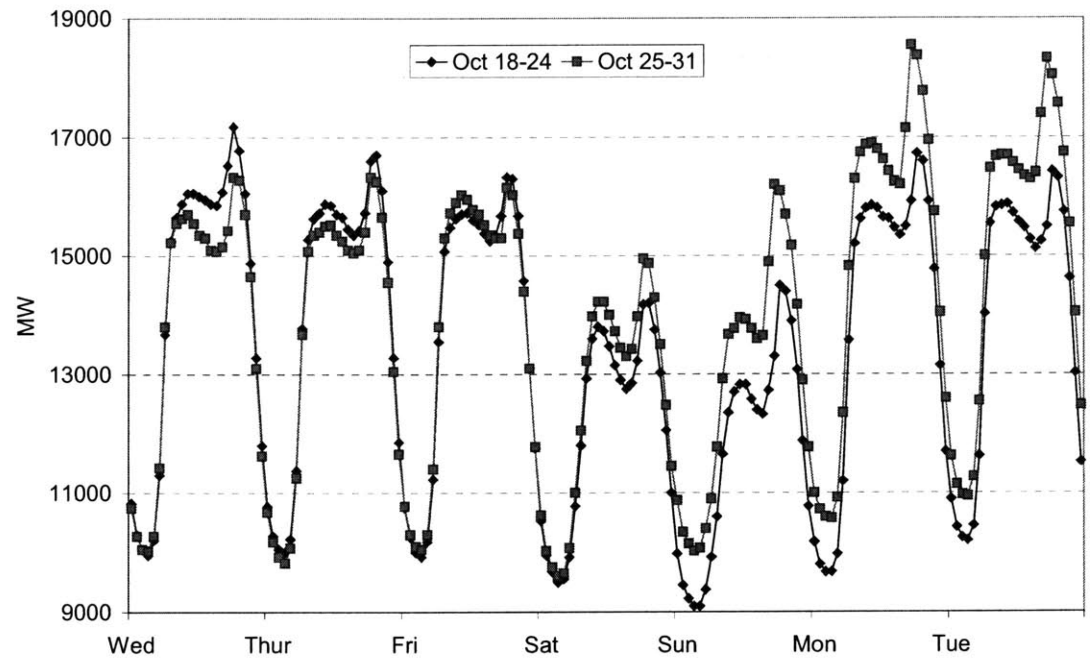

Figure 5-7: Daily Forecast Demand in New England during October 18-31, 2000

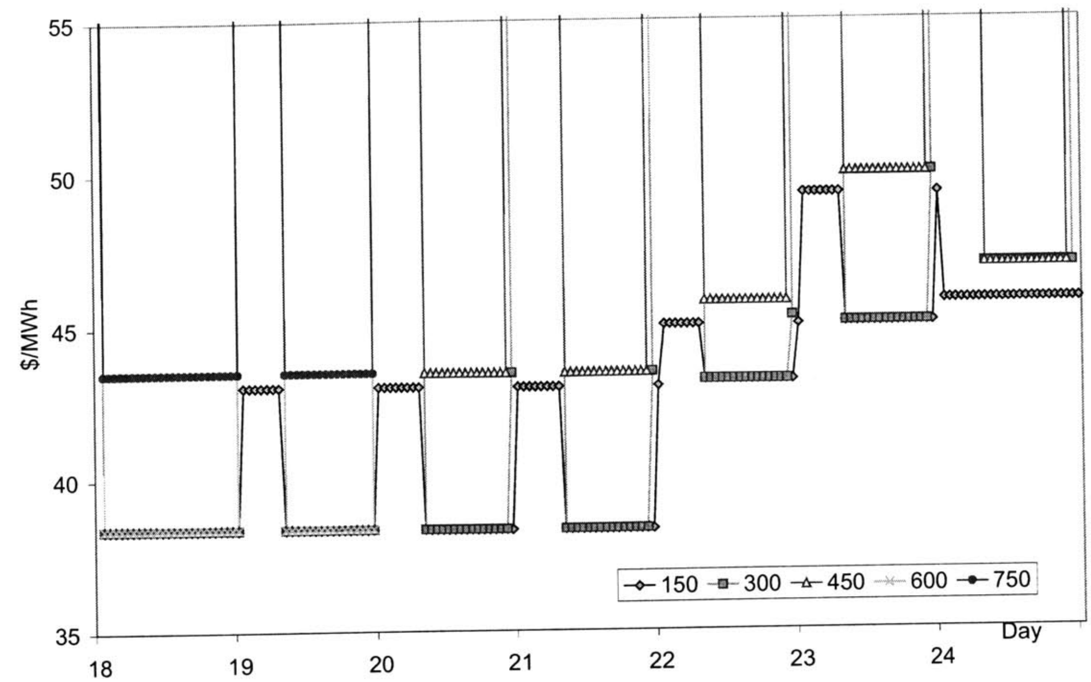

Figure 5-8: Sampled Bidding Prices for Some Bidding Quantities of LP 206845 during January 18 – 24, 2000

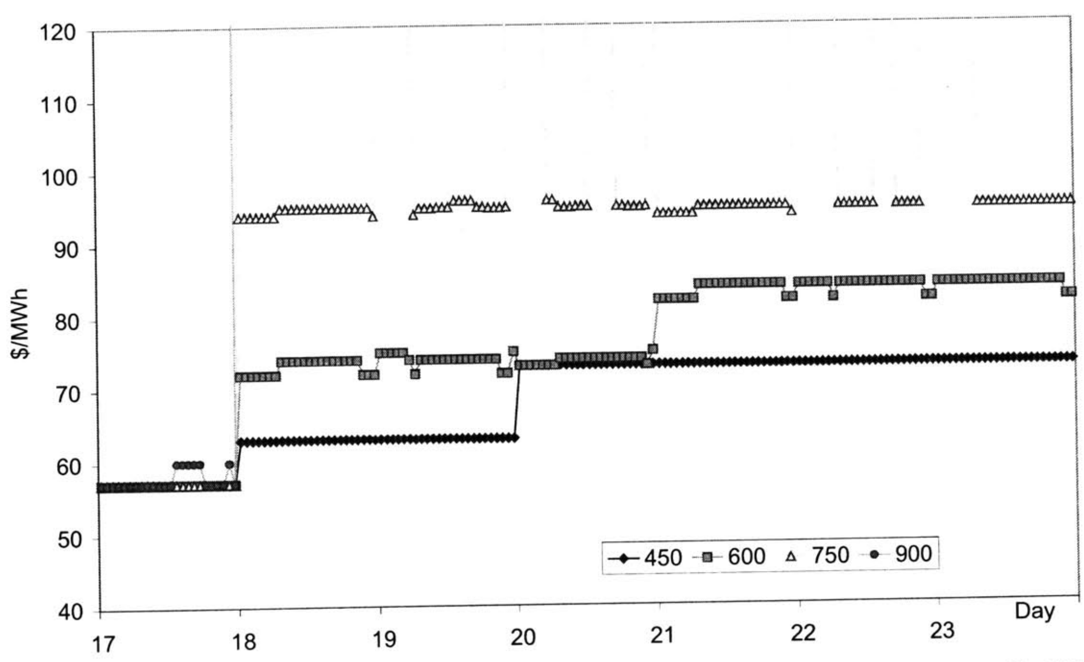

Figure 5-9: Sampled Bidding Prices for Some Bidding Quantities of LP 206845 during April 17 – 23, 2000

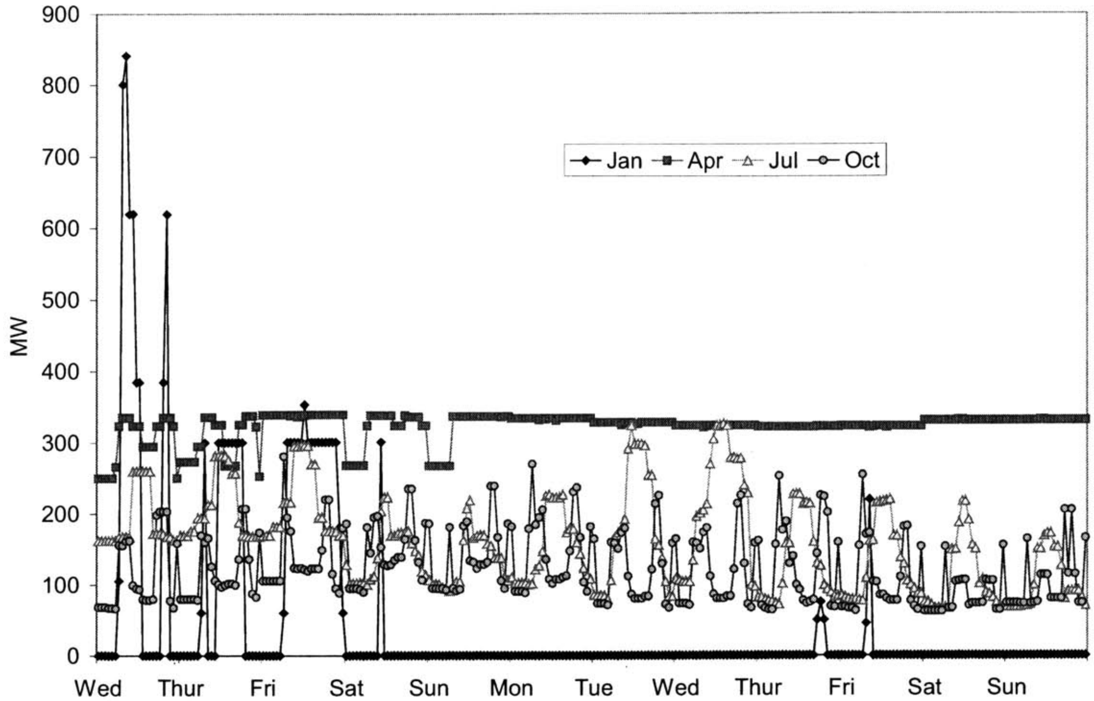

Figure 5-10: Scheduled Quantities of LP 206845 during Four Two-week Periods of Study

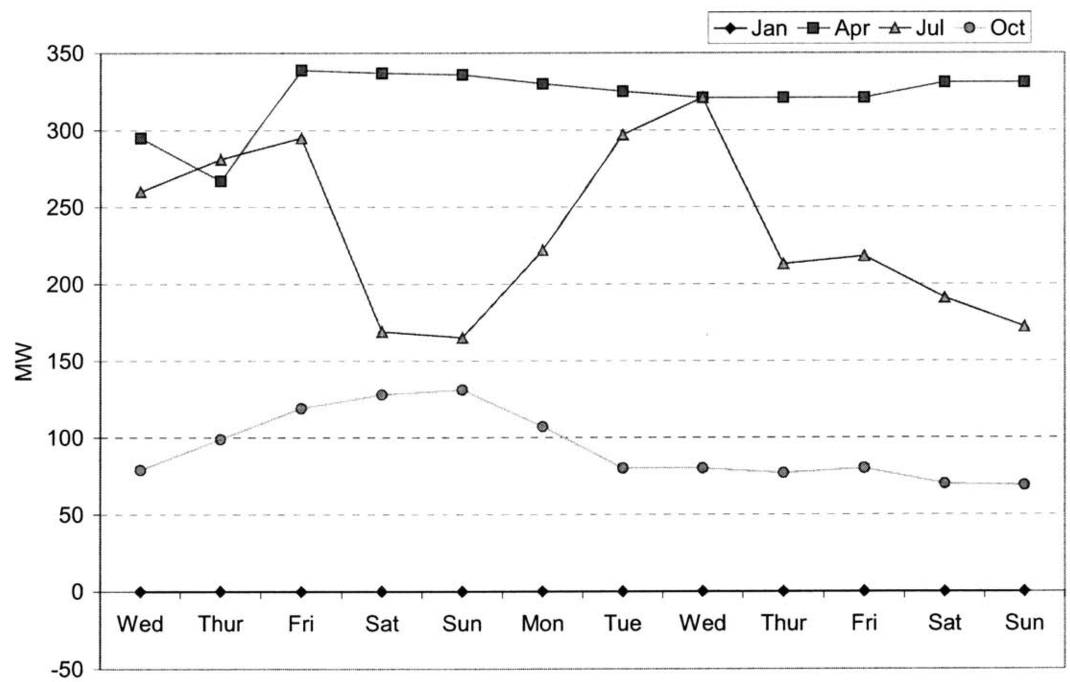

Figure 5-11: Self-scheduled Quantities of LP 206845 at Hour 14 during Four Two-week Periods of Study

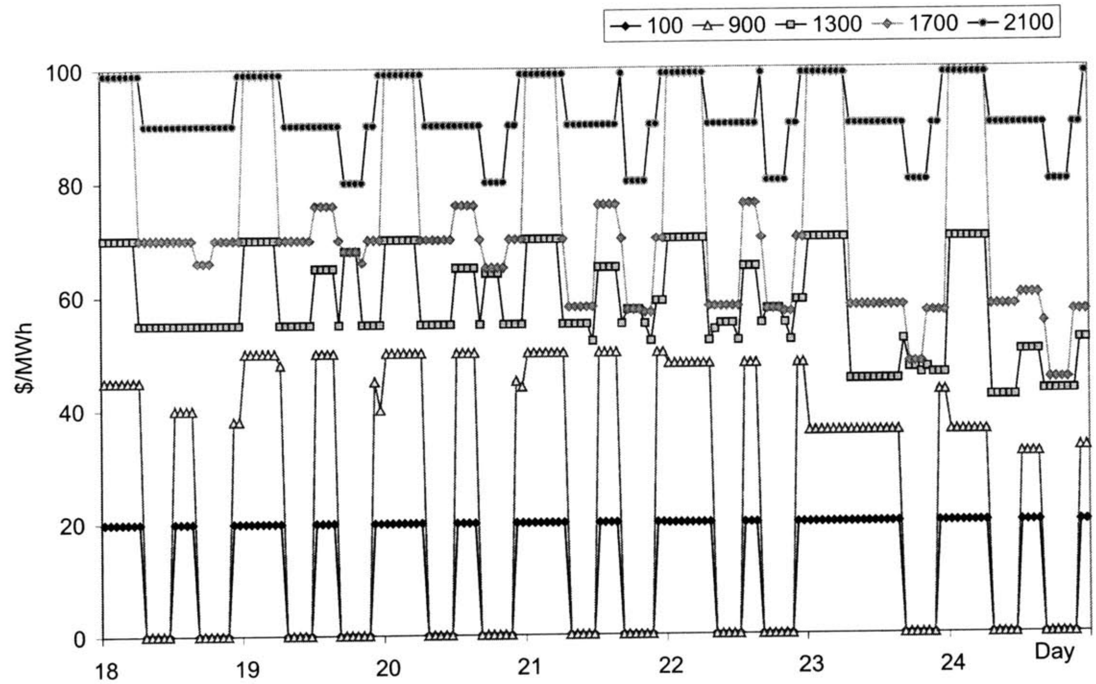

Figure 5-12: Sampled Bidding Prices for Some Bidding Quantities of LP 218387 during January 18 – 24, 2000

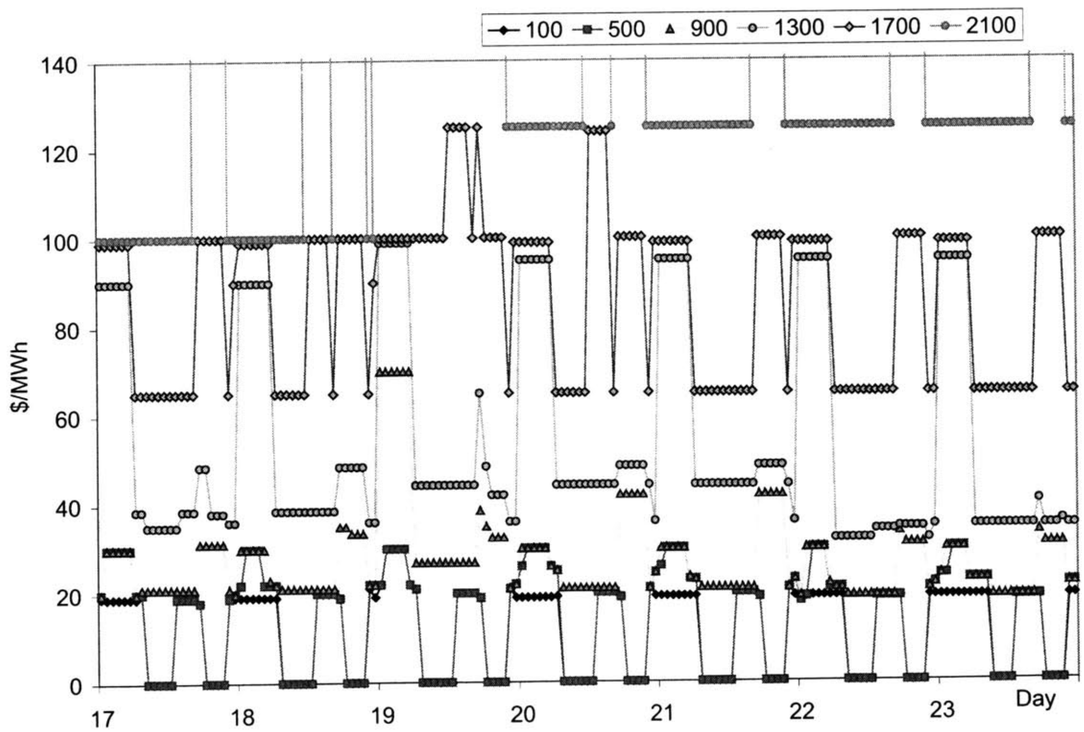

Figure 5-13: Sampled Bidding Prices for Some Bidding Quantities of LP 218387 during April 17– 23, 2000

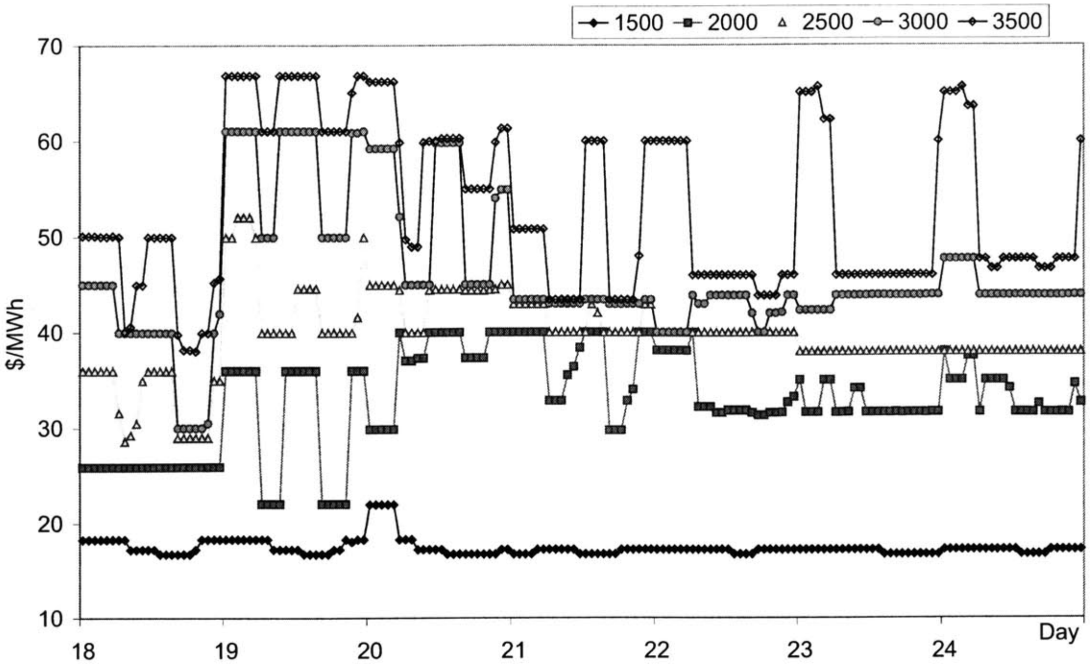

Figure 5-14: Sampled Bidding Prices for Some Bidding Quantities of LP 506459 during January 18 – 24, 2000

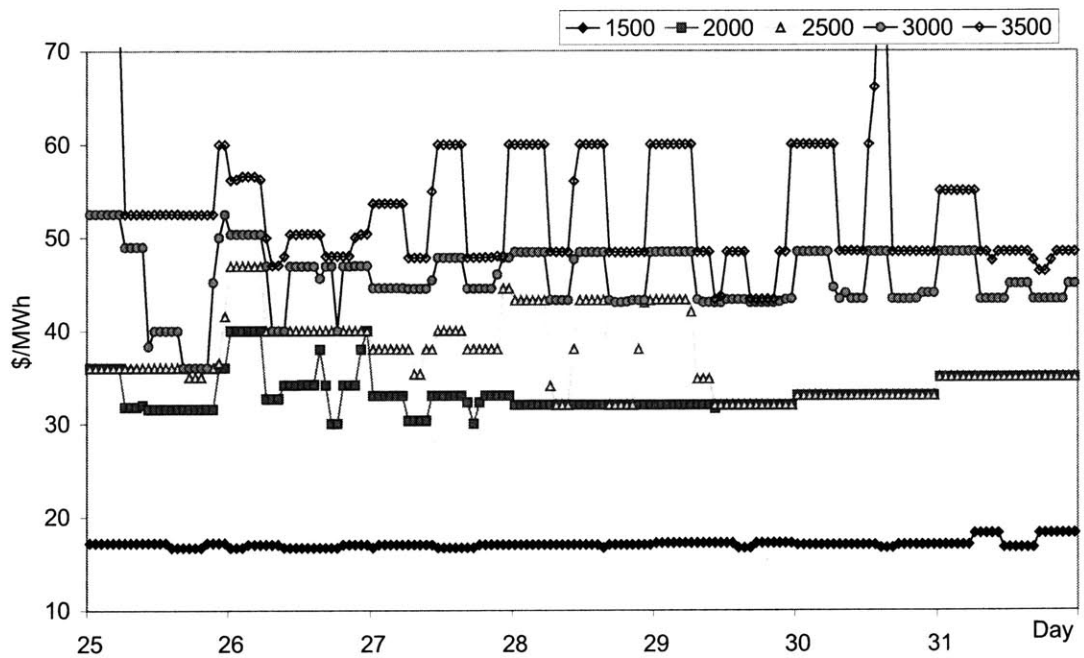

Figure 5-15: Sampled Bidding Prices for Some Bidding Quantities of LP 506459 during January 25 – 31, 2000

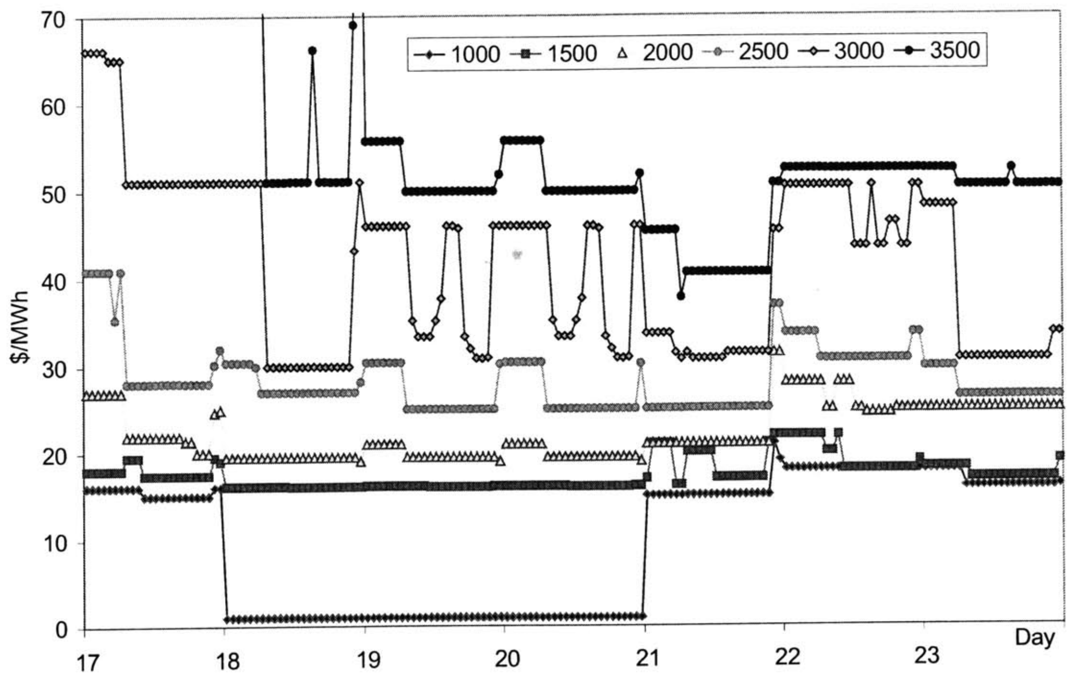

Figure 5-16: Sampled Bidding Prices for Some Bidding Quantities of LP 506459 during April 17 – 23, 2000

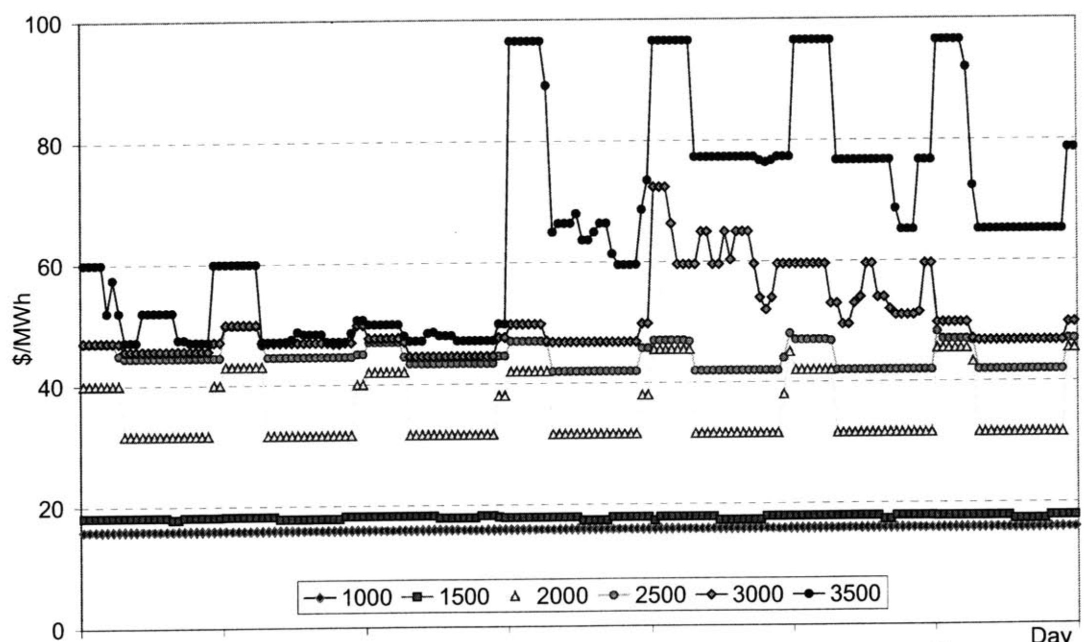

Figure 5-17: Sampled Bidding prices for Some Bidding Quantities of LP 506459 during October 18 – 24, 2000

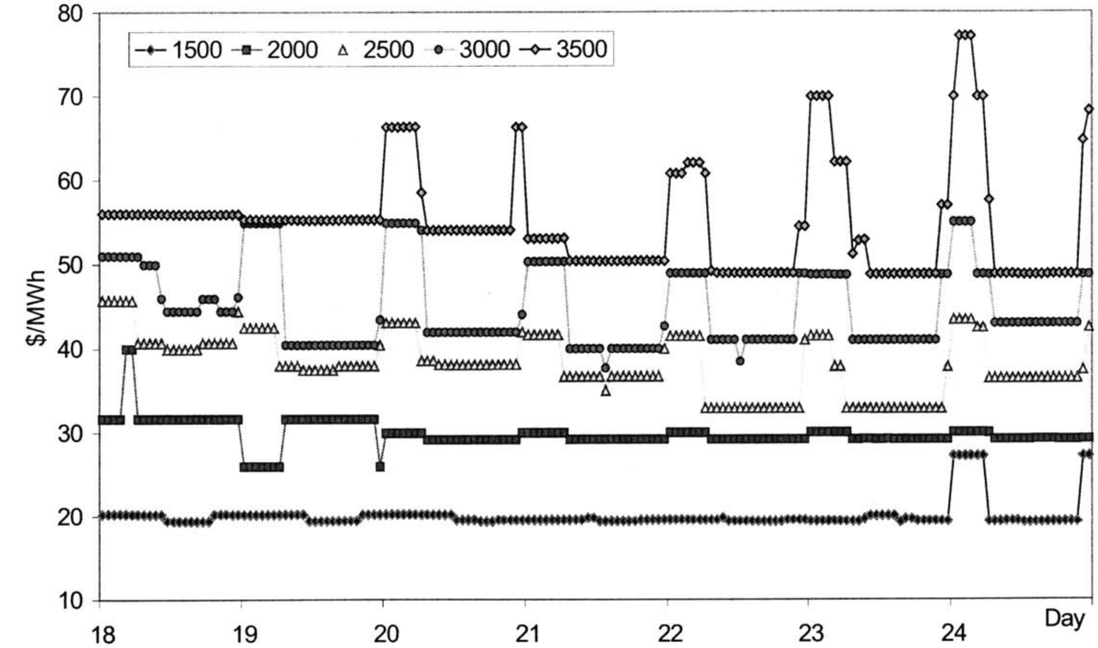

Figure 5-18: Sampled Bidding Prices for Some Bidding Quantities of LP 506459 during July 18 – 24, 2000

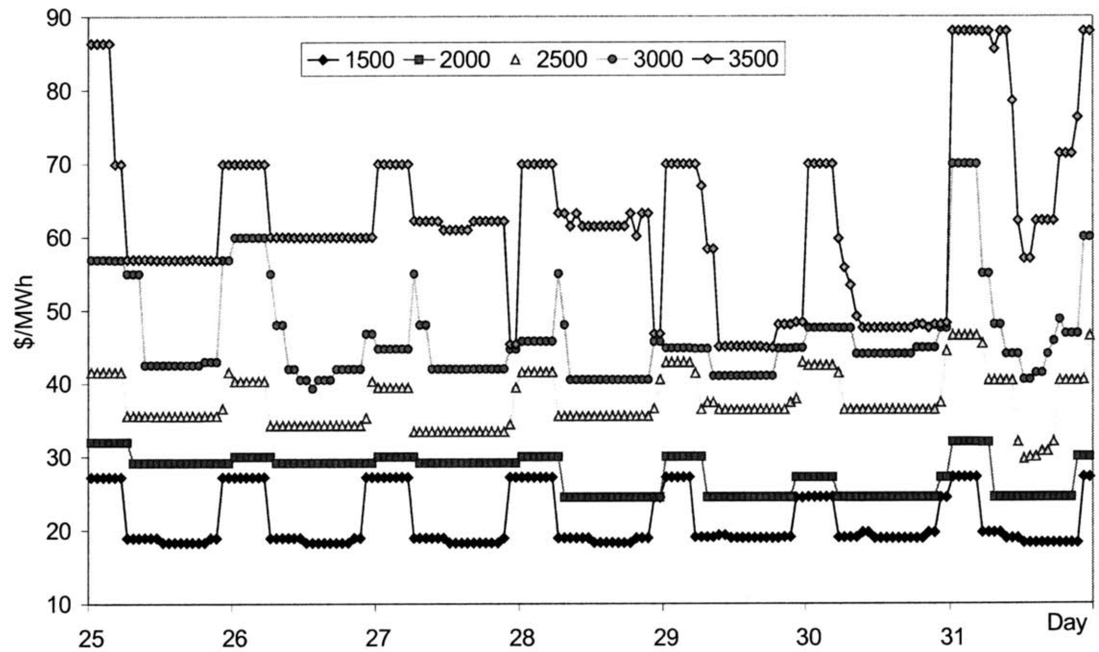

Figure 5-19: Sampled Bidding Prices for Some Bidding Quantities of LP 506459 during July 25 – 31, 2000

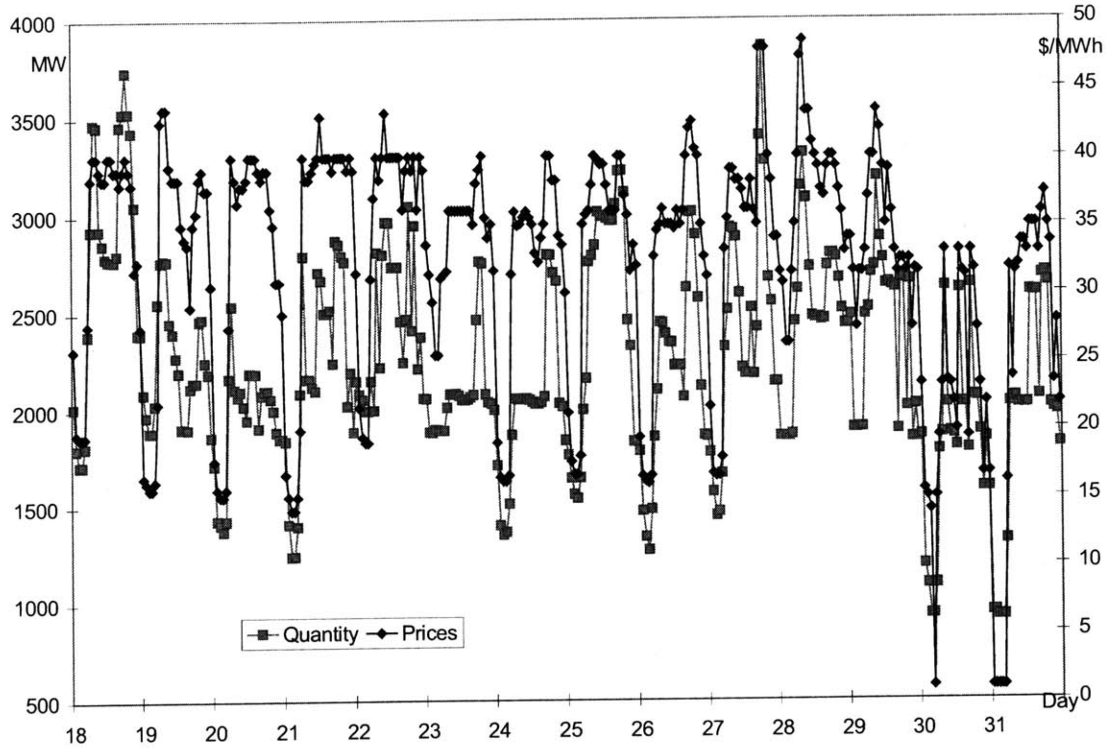

Figure 5-20: Scheduled Quantities of LP 506459 and Calculated Prices during January 18 – 31, 2000

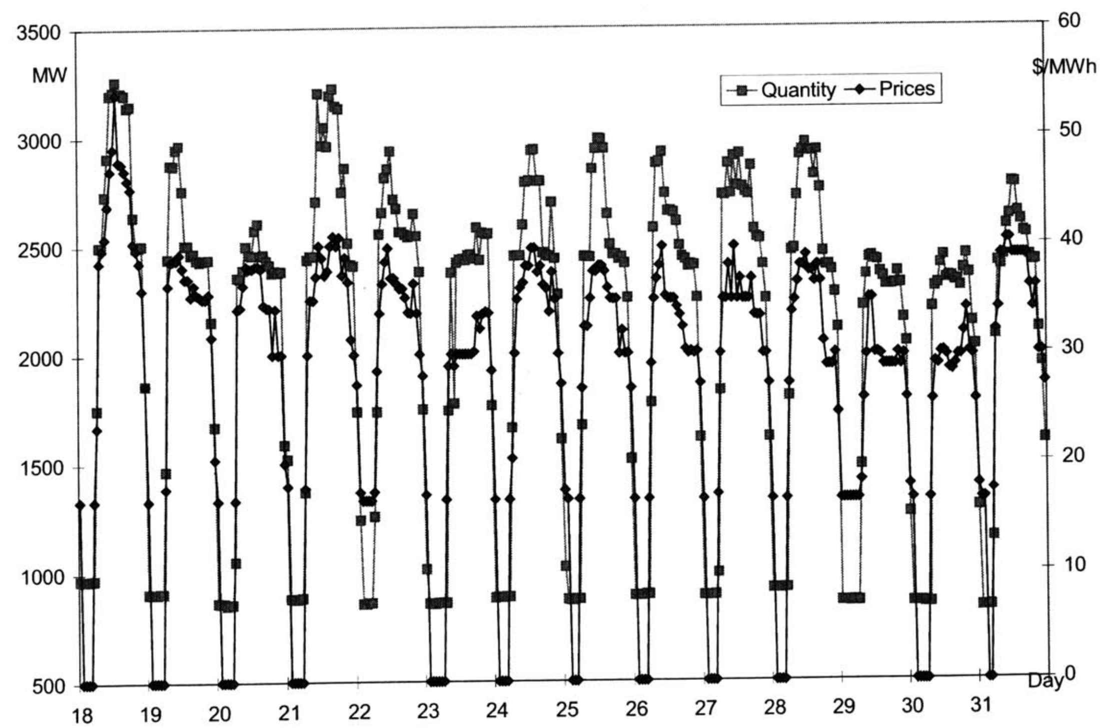

Figure 5-21: Scheduled Quantities of LP 506459 and Calculated Prices during July 18 – 31, 2000

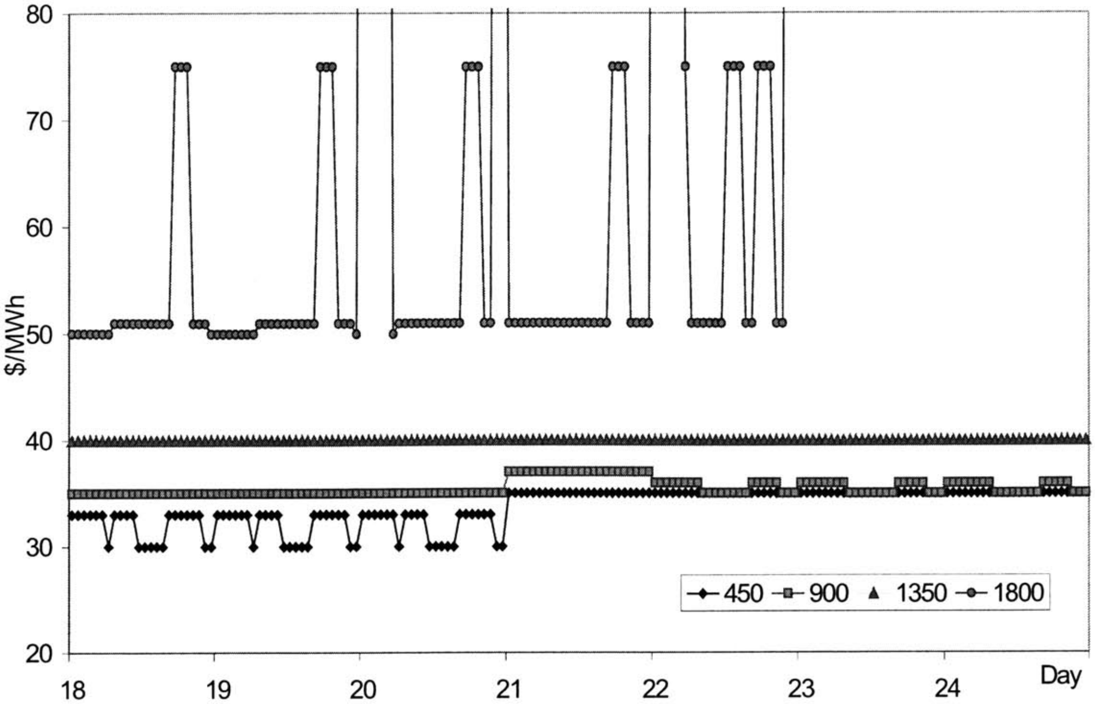

Figure 5-22: Sampled Bidding Prices for Some Bidding Quantities of LP 529988 during January 18 – 24, 2000

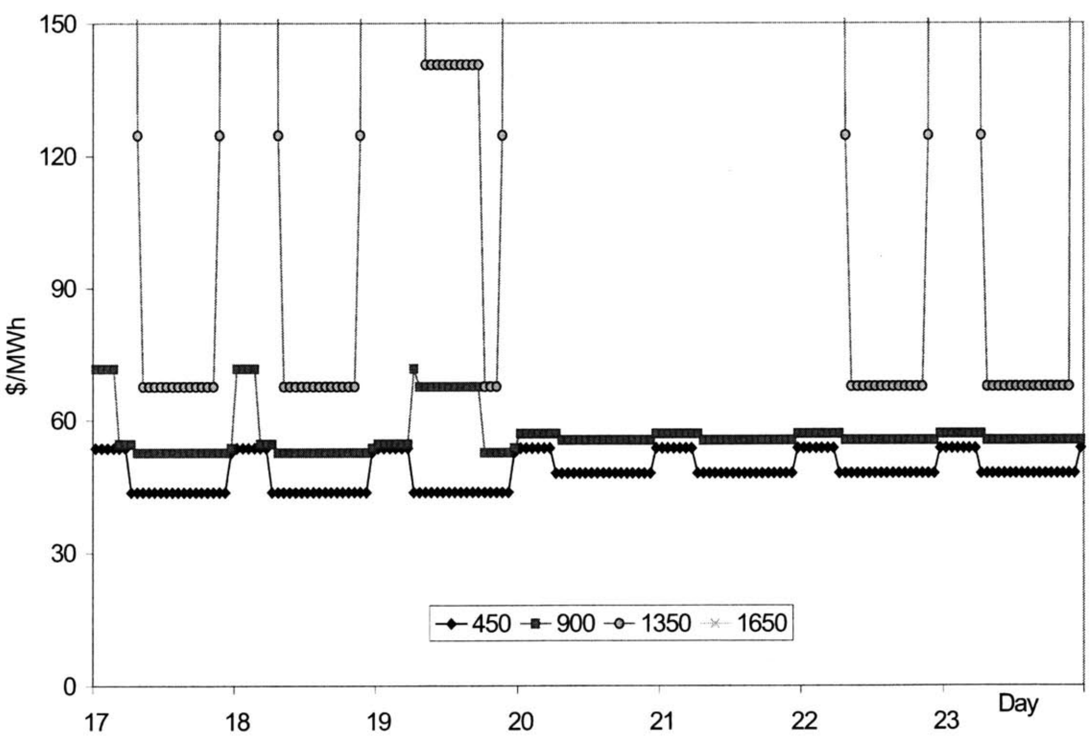

Figure 5-23: Sampled Bidding Prices for Some Bidding Quantities of LP 529988 during April 17 – 23, 2000

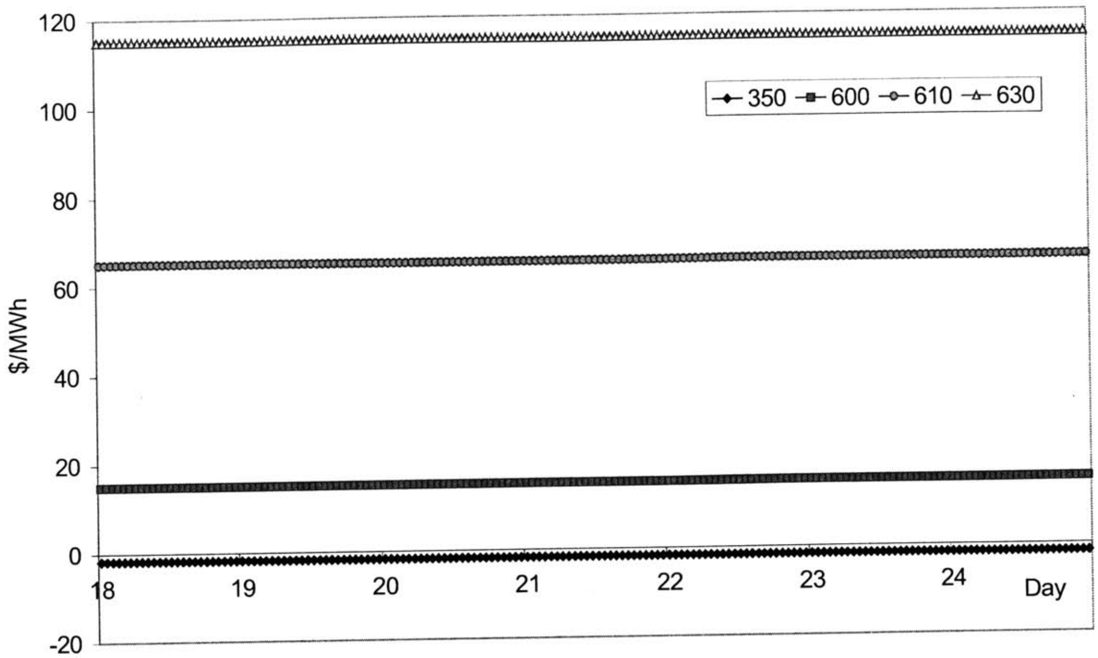

Figure 5-24: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 23789 during January 18 – 24, 2000

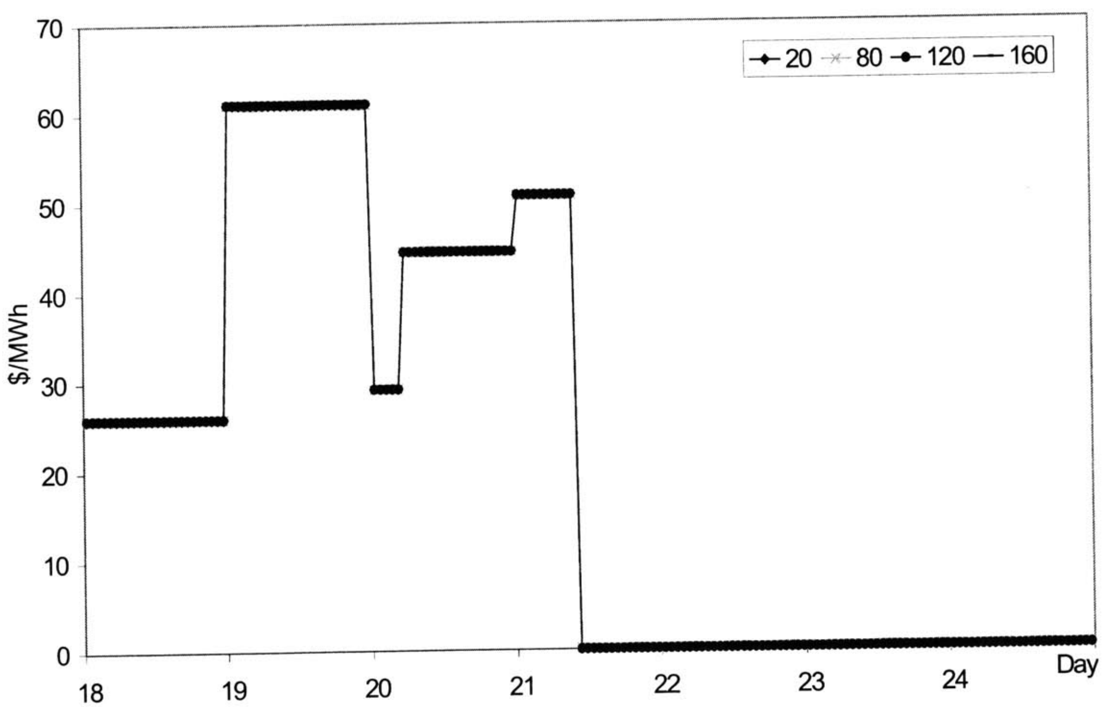

Figure 5-25: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 37274 during January 18 – 24, 2000

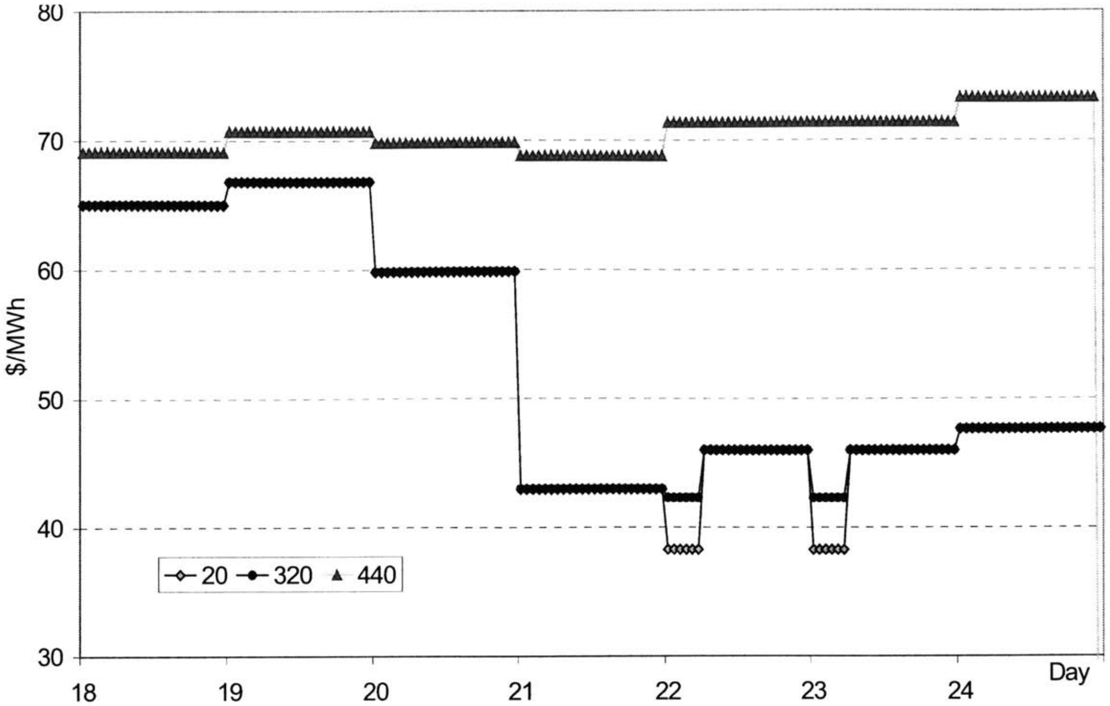

Figure 5-26: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 43414 during January 18 – 24, 2000

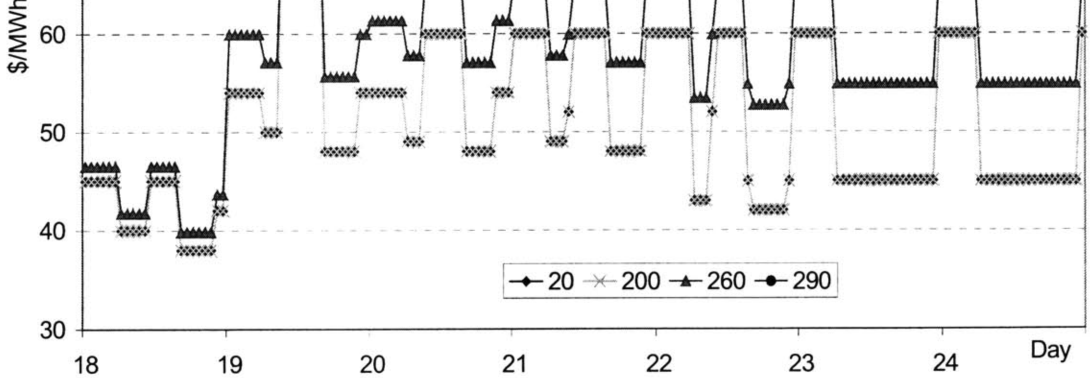

Figure 5-27: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 81361 during January 18 – 24, 2000

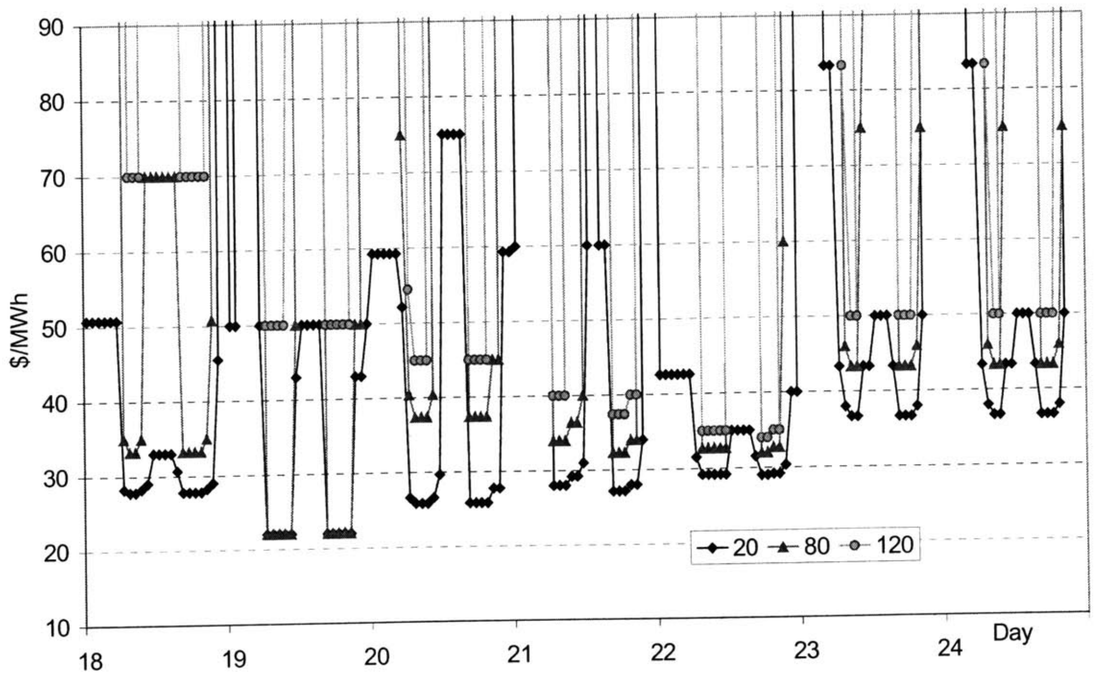

Figure 5-28: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 79606 during January 18 – 24, 2000

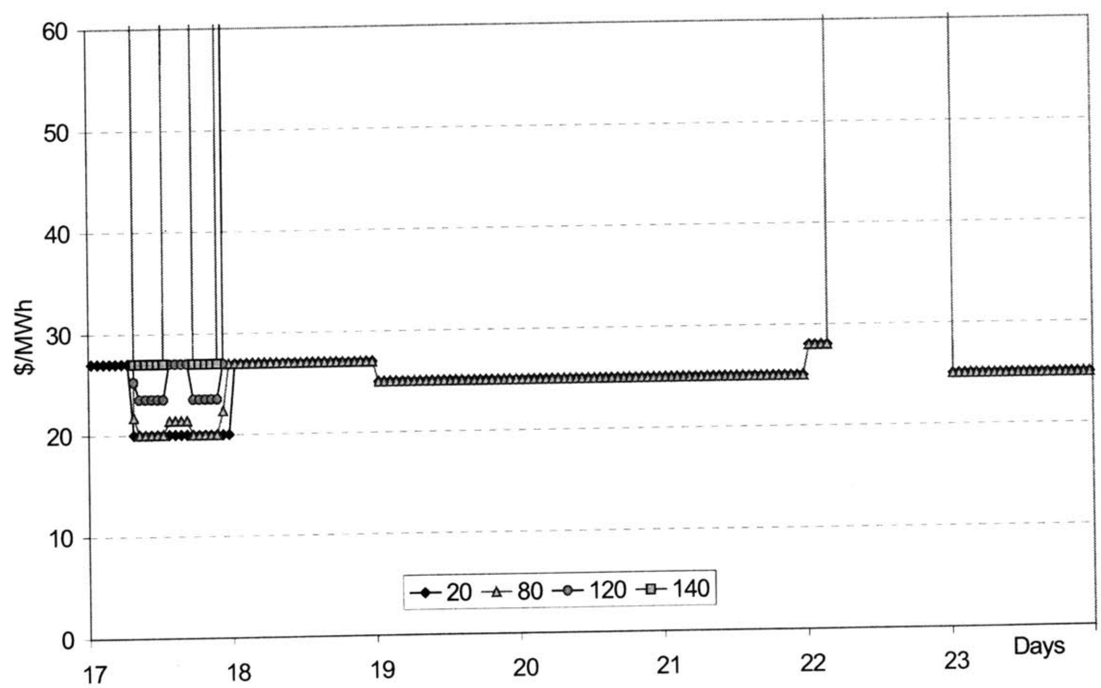

Figure 5-29: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 79606 during April 17 – 23, 2000

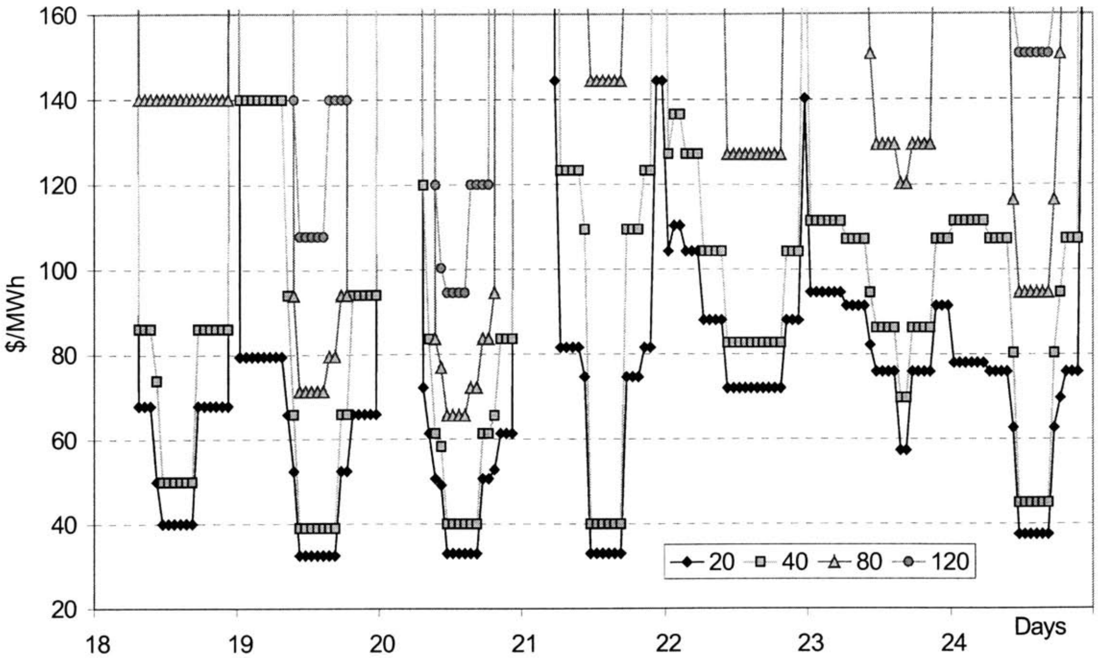

Figure 5-30: Sampled Bidding Prices for Some Bidding Quantities of Asset ID 79606 during July 18 – 24, 2000

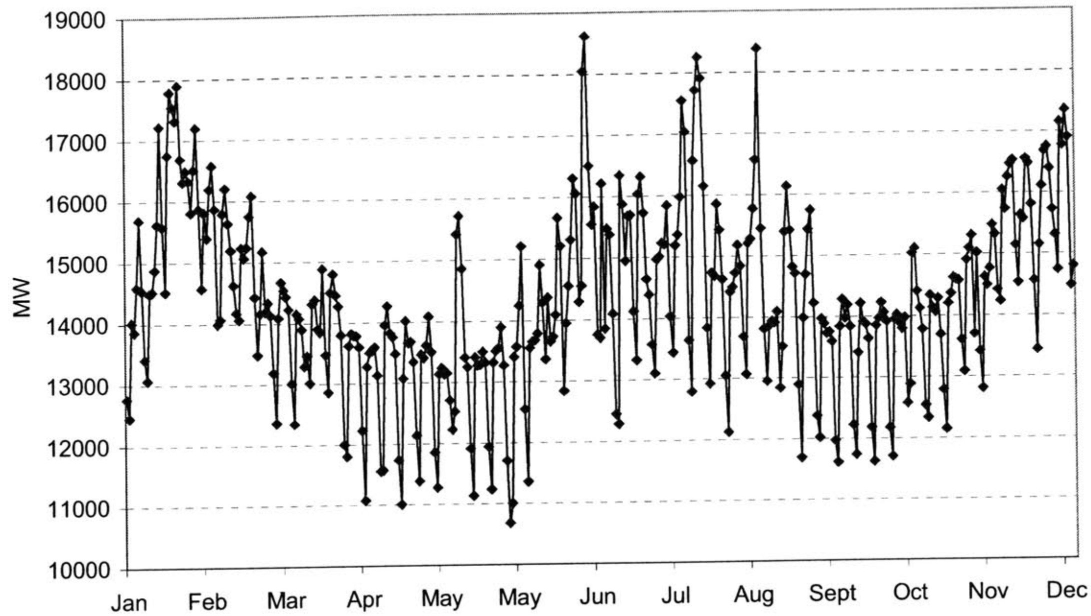

Figure 5-31: Daily Average Electricity Demand in New England during Year 2000

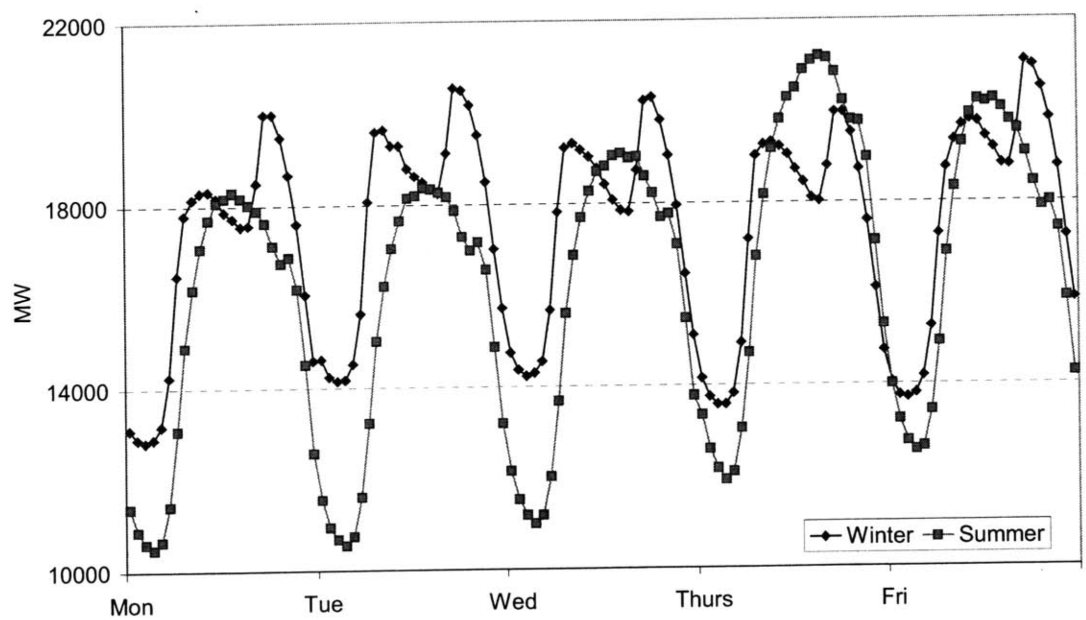

Figure 5-32: Examples of Daily Electricity Demand Characteristics in New England during Year 2000

Chapter 6

Applications of the Agent-based Market Model

An agent-based approach is an alternative tool for modeling a multiagent system to observe the dynamic outcomes that result from interactions among the agents or individual decision-makers. This chapter explores two factors that might affect the agents' bidding behavior, that is the market structures, as well as the role of active demand-side agents or load-serving entity agents. The uniform and discriminatory-pricing market structures are considered. In uniform-pricing markets, the agents are paid market prices for their scheduled bidding quantities. In discriminatory-pricing markets, the agents are paid bidding prices for their scheduled bidding quantities. The simulations and analyses when the market model adopts either of these market structures are presented in Section 6.1.

Generally, the power producers benefit from high market prices, while load-serving entities (LSEs) benefit from low market prices. However, the LSEs are not yet active players in the markets. For example, in the California electricity market, which operates under a sealed-bid double auction format, clear indicators of the LSE inactivity were seen in price-spikes and rolling-blackouts due to insufficient supply surplus during the summer of 2001.[1] The presence of active LSEs might diminish the ability of the power producers to successfully implement strategic behavior and might reduce the magnitude of market prices. Section 6.2 presents the agent-based market model with several power-producing agents and one LSE agent. The simulations and analyses are then outlined. Like the power-producing agents, the LSE agent determines its bid-demand functions by following some learning algorithms.

[1] If the active LSEs had been in place, the LSEs would have been able to reduce consumption and/or their willingness to pay.

6.1 Uniform and Discriminatory-pricing Markets

In the auction theory framework, both the uniform and the discriminatory-pricing auctions are used to sell multiple units of goods. In the uniform-pricing (UP) electricity market, the agents are paid the market prices for the power they produce, and in the discriminatory-pricing (DP) electricity market, the agents with scheduled quantities are paid the bidding price of those quantities. This thesis analyzes the effects of these payment rules on the bidders' behavior and price dynamics using the agent-based market model presented in the previous chapters. In this model, the payment rule is modified to fit a DP structure, while the learning algorithm/bidding strategy of the agents remains unchanged, except for the setting of the bidding prices of the anticipated infra-marginal units. The characteristics of the model and learning algorithms, as well as the simulations and analyses are presented next.

Note that, since the market price of each hourly auction is not publicly available to the agents, a market price estimation scheme of each agent is added to the learning algorithms and this price estimation is presented in the appendix to this chapter. A preliminary analysis of the bidding behavior of the agents in markets with both UP and DP structures is also presented in the appendix to this chapter.

6.1.1 Models

This thesis analyzes the impact of the DP structure on price dynamics by performing two sets of simulations. In the first set the agents use Algorithm A3 with a slight modification to the algorithm used in Chapter 4. The second set is when the agents use the model-based algorithm. The simulated price dynamics are compared to the ones obtained from the market model with the UP structure. The agent-based model used for simulations in this section shares the same characteristics as those of the model used in Chapter 4. That is, the power-producing agents have the same marginal-cost functions in which the aggregate marginal-cost function is shown in Figure 6-1, the daily demand pattern is shown in Figure 6-2, and the market-clearing mechanism adopts a price-merit order method. No intertemporal effects of unit-commitment constraints of operating the generating units are in place and the operator schedules the generating units independently to serve hourly demand through the hourly auction. The learning algorithms used by the agents in this analysis are described in the next section.

Algorithm A3

Algorithm A3 is used in this section. There is only one modification to replace the price-setting scheme of each unit in the portfolio (after the bidding price of the anticipated marginal unit (BM_k^i) and the bidding quantity (q_k^i)) scheme, as follows:

1. The bidding price of the withheld capacity (WH_k) is set to

$$WH_k = \min\{b^i_{k+1} + C_2, P_{cap}\}$$

where C_2 is a positive constant and P_{cap} is a price cap, indicating the maximum market price allowed in the market. This bidding price is assigned to the capacity of the units with the lowest marginal costs summed to the withheld capacity.

2. For any unit j with non-zero capacity that is not considered the withheld capacity, it determines $\tilde{b}^j_k = \max\{mc^{i,j}, BM^i_k\}$. Then its bidding price b^j_k is set to

$$\begin{aligned} b^j_k &= \tilde{b}^j_k - m \cdot \epsilon, \quad mc^j < BM^i_k \\ b^j_k &= \tilde{b}^j_k, \quad\quad\quad\;\; mc^j \geq BM^i_k \end{aligned} \tag{6.1}$$

where $mc^{i,j}$ is the marginal cost of unit j. Let $\epsilon > 0$ be a positive constant and be equal to an increment of the choice of the possible bidding prices. Let m denote an order of the unit such that the marginal cost is less than BM^i_k and the lower m is the more expensive marginal cost.

Note that this price-setting scheme is based on the analysis presented in the appendix to this chapter in which the anticipated marginal units have the bidding prices less than the anticipated marginal unit but higher than their marginal costs, as well as having bidding prices in order of their marginal costs.

The Model-based Algorithm

After the end of each bidding round the agents follow the price-estimation scheme, as shown in the appendix to this chapter, to estimate market price, denoted by $\hat{P}$, from their scheduled outcomes. The same price-setting scheme as that in Chapter 3 is used, except that market price is replaced by $\hat{P}$, i.e., $MP \equiv \hat{P}$. From the *PORTFOLIO* scheme, when the agent is in the market using the DP rule, the agent may calculate the anticipated profit by 1) assuming that BM is the payment it anticipates to receive, or 2) assuming that BM is the market price, and it receives the payment $b^{i,j}$ for each scheduled block, i.e., $b^{i,j} \leq BM$. When the agents cautiously anticipate their profits, the profits obtained after each bidding round are likely to be closer to the anticipated ones than when the agents overly estimated their profits. This in turn reduces the possibility that the agents increase the bidding prices to explore more profitable opportunities. Therefore, if the second method is chosen, one would anticipate the price dynamics of the markets with the UP and DP rules to be similar.

6.1.2 Simulations

This section presents simulations of price dynamics and profits of the agents in both the UP and the DP market structures. The agents use either Algorithm A3 or the model-based algorithm. The market price of each hour in the case of the DP structure refers to the maximum bidding price of the scheduled bid-blocks at that hour. In the simulations in which the agents use Algorithm A3, the agent selects BM from 0 to P_{cap}, which is equal to \$150/MWh, with an increment of \$3/MWh so that the total choices of the bidding prices (K^b) are equal to 51. Likewise, the agents select their bidding quantity (q^i_{max}) from 0.25 MW to their available capacity with an increment of 0.25 MW.

Let α^b be set to $\alpha^b = 2\ln\frac{K^bT^b_r}{\delta^b}$ and α^q be set to $\alpha^q = 2\ln\frac{K^qT^q_r}{\delta^q}$. Figure 6-3 shows the samples of the simulated price dynamics under the UP structure when all agents use $\delta = 0.9$. Figure 6-4 shows the samples of the simulated price dynamics under the DP structure when all agents use $\delta = 0.9$. Furthermore, the profits of the agent with the largest capacity (Agent 5) received in the two scenarios are shown in Figure 6-5. The moving-average sum of agent profits is shown in Figure 6-6.

When the agents use the model-based learning algorithm, $\Delta = 2$ and the bidding price of the withheld capacity is set to P_{cap}. Figure 6-8 shows the samples of simulated price dynamics when all agents use Method M1 to set Tar and set $\Delta = 2$, and Figure 6-9 shows the samples of simulated price dynamics when all agents use Method M2 to set Tar and set $\Delta = 2$.

6.1.3 Analyses

Bower and Bunn [8] use their agent-based model to show that in the DP structure, the agents with the lower cost units try to submit a higher bidding price, closer to the anticipated price. This causes the supply function to become flat in the region anticipated to be scheduled, the lower-cost capacity. This finding is consistent with the preliminary analysis presented in the appendix to this chapter. In addition, this behavior is incorporated into the agent-based model by having the agents set their bidding prices as shown in Equation (6.1) when they use Algorithm A3. The agent behavior of gradually raising the bidding prices of their infra-marginal units closer to the anticipated price is observed when the agents use the model-based algorithm with the *SETPRICE* scheme.Therefore, the simulated price dynamics when Equation (6.1) is used can be viewed as steady-state dynamics.

The difference between the price dynamics and bidding behavior of the agents when the market model has the UP or the DP structure is rather substantial. To understand how the payment rules may affect the agents' bidding behavior and, consequently, the price dynamics, let us begin by analyzing the impact of the learning algorithms on the price dynamics.

Comment on Learning Algorithms

The general characteristics of Algorithm A3 and the model-based algorithms that might affect the simulated price dynamics are explained as follows:

- As mentioned in Chapter 4, Algorithm A3 yields a mixed strategy action that allows the agents to explore all of their possible actions (K^b and K^q), whereas the model-based algorithm yields a pure-strategy action that allows the agents to choose the next action to be higher or lower than, or equal to, the current one. Using the model-based algorithm, the exploration takes a longer time and some actions may never be tried.

- Algorithm A3 selects the mixed-strategy action in which there is a uniform probability distribution assigned to every action regardless of the outcome (γ/K); therefore, when the agents use this algorithm, the market prices can take on any value from the available choices.

- When the model-based algorithm is implemented in the model, one can observe that in the market with the DP structure the price-estimation error of the agents may contribute to a divergence of market price dynamics (the market price may be bounded by P_{cap}). The overestimation of market prices when the agent has no units in the portfolio scheduled as marginal units and the use of Method M1 to determine BM cause the anticipated profits (AP) of the agent to be higher than the actual profits (OP) received from bidding. Recall that Method M1 sets $Tar = BM$, while Method M2 sets Tar equal to the market price (MP) of the previous period. Also recall that $AP > OP$, which implies $BM \leq \hat{P}$ in the previous auction round. From the *OUTCOME* scheme, when $AP > OP$ and $BM < \hat{P}$, $O = 11$. That is, the agent increases BM regularly, and submits increasing bidding prices over time through Method M1. Therefore, when all agents use the same decision scheme, they simultaneously raise their bidding prices for the anticipated marginal unit; consequently, the divergence of simulated market prices is unavoidable.

Simulated Outcome Analyses

This thesis proposes to analyze the simulated outcomes from the model with either the UP or DP structures by comparing 1) the simulated price dynamics with the same demand pattern, and 2) the profits that the agents receive over time. The difference in price dynamics and profits between the two structures is caused mainly by the accuracy of the agents' market price anticipation as well as by the bid-supply function which is a result of the anticipated price.[2]

Algorithm A3: From Figures 6-3 and 6-4, let "UP" and "DP" represent the simulated price dynamics from the UP and DP models, respectively. One can observe that the simulated prices from the

[2]This outcome is consistent with the analysis presented in the appendix to this chapter.

market model with Algorithm A3 under the DP structure are likely to be higher than the simulated prices from the model under the UP structure. Let us consider Equation (6.1). With the same BM, in the DP market model Equation (6.1) yields a flat bid-supply function compared to the price-setting scheme in the UP market model. For example, consider Agent 5 and suppose that its BM is equal to \$55/MWh and all bidding quantities are accounted for. The bid-supply functions obtained from Algorithm A3 under the UP and DP market models are shown in Figure 6-7. One can observe that the bid-supply function under the DP market model is likely to lie above the bid-supply function under the UP market model on the price axis; that is, the agent sells its power at a higher price on the DP market than on the UP market. The cumulative effect regarding the expensive bid-supply functions from every agent leads to high market prices in the DP market model.

Additionally, this result, in which the market prices on the DP market are generally higher than on the UP market, is true when the agents set δ to other values such as $\delta = 0.1, 0.3, 0.5$, or 0.7. The expensive simulated market prices on the DP market contribute to the substantial profits that the agents obtain. One can also observe from Figure 6-5 that the profits Agent 5 receives from the DP market are likely to be higher than those Agent 5 receives from the UP market. Similarly, as shown in Figure 6-6, the average profits that the agents obtain from the DP market are higher than the average profits the agents obtain from the UP market. Note that one key advantage of Algorithm A3 in the DP market is that the agents require a knowledge of market prices. Therefore, the price estimation scheme is not necessary.

The model-based Algorithm: From Figures 6-8 and 6-9, the simulated price dynamics in both the UP and DP markets depend on the methods to set Tar and on the values of Δ. The simulated prices from the market model under the DP structure over time can either be higher or lower than the simulated prices from the model under the UP structure. Let "UP-M1" and "UP-M2" in Figures 6-8 and 6-9 represent the price dynamics when the agents use Methods M1 and M2 to set Tar in the UP market, respectively, and let "DP-M1" and "DP-M2" represent the price dynamics when the agents use Methods M1 and M2 to set Tar in the DP market, respectively.

In addition, Figures 6-10 and 6-11 illustrate the relationship between AP, OP, BM, and MP during Hour 18 of each trading day. Recall from Chapter 3 that when $OP - AP \geq 0$, $BM < MP$, and $OP > 0$, the agents no longer increase their BM in the next period. One can observe that when the $OP - AP$ plot exceeds zero (crosses the zero-price axis), the BM plot no longer changes.

Recall the *PORTFOLIO* scheme in Chapter 3. The agents determine their bid-supply functions based on their individual units as well as their entire portfolio. In the market with the DP structure, the bidding prices of the anticipated infra-marginal units increase rapidly so that their bidding price converges closely to BM. In the market with the UP structure, the agents increase the bidding prices of their anticipated marginal units slowly, because the agents are paid the market price for their scheduled quantity. Hence, the agents tend to obtain the profits they anticipate and the incentive to increase the bidding price, especially for the infra-marginal units, is lower than in the DP market.

In the DP market when the agents use Method M1 they increase or decrease BM of the next period based on BM of the current period. Note that BM stops changing when the agents obtain profits at least equal to the profits they anticipate, i.e., when $OP \geq AP$. Since the agents use the same learning scheme, when the agents no longer adjust their bidding prices, the cumulative effect causes the market prices to shift to a steady-state pattern. In this case, the agents do not use the information about the estimated market prices (the *ANTPRICE* scheme). One can observe similar outcomes in the market with the UP structure when the agents use Method M1; that is, prices shift to a steady-state pattern when the actual profits exceed the anticipated ones.

When the agents in the market with the DP structure use Method M2 to set Tar, the market price dynamics tend to diverge. Unlike Method M1, the agents set BM of the next period based on the estimated market price of the current period, $\hat{P}$. This estimated price is obtained from the scheduled prices and scheduled quantity via the *ANTPRICE* scheme (see the appendix to this chapter), in which the market price over-estimation or under-estimation is possible. Although the bidding outcome is satisfying and the agent does not adjust the price, BM of the next period might change. Note that, for Method M2, $BM = MP + \bar{c}$, where MP is obtained from the *ANTPRICE* scheme and $\bar{c} = \{-\Delta, 0, \Delta\}$. Since MP depends on the agent's and the competitors' bid-supply functions, as well as on the positive estimation error, each agent's BM tends to rise over time. The cumulative effect of this outcomes contributes to the divergence of market prices.

Nonetheless, when the agents set $\Delta = 1$ and use either Method M1 or M2, as shown in Figures 6-12 and 6-13, the price competition of the marginal agents in the UP market to raise the bidding prices persists. The *OUTCOME* scheme, which tends to direct the agents to cooperate to raise the bidding price, encourages this behavior. Therefore, when all agents use the same strategy, the cumulative effect of this behavior creates a divergence of market prices. In summary, when all agents use the model-based learning algorithm, the divergence of simulated prices can be observed in both the UP and DP structures. Three factors contributing to market-price divergence include the usage of Method M2 to set Tar, the usage of the price-estimation scheme in the DP model, and the value of Δ .

6.1.4 Implications of the Simulations

The simulations demonstrate the effects of market structures and information asymmetry among the agents on the agents' bidding behavior. When the agents follow the model-based learning algorithm with the different parameter setting presented in this section, the agents may determine expensive bid-supply functions, causing high market prices in a market model with either the DP or the UP structure. On the other hand, when the agents follow Algorithm A3, the agents submit more expensive bid-supply functions in the market model with the DP structure than in the model with the UP structure. Consequently, this agent-based model with different learning algorithms suggests that the market prices and profits of the agents in the DP market are likely to be higher than those in the UP market; that is, the DP structure tends to deteriorate market efficiency more than the UP structure.

In addition, although the simulated outcomes tend to suggest tendencies towards higher market prices in the DP markets than in the UP markets, one should realize that the outcomes significantly depend on the learning algorithms that the agents employ. The finding from simulations may be substantiated if the model with different learning algorithms is properly tested against the actual market by using the method presented in Chapter 4.

6.2 The Role of Load-serving Entity

The existing electricity markets can be divided into two setups based on the activities of the demand side. The first setup is a sealed-bid auction-style market, a market without active LSEs, such as in New England.[3] The other setup is a sealed-bid double auction-style market, such as in California, with active LSEs who buy the power on the behalf of customers.

This section analyzes the double auction-style market, where power producers submit a bid-supply function, indicating the amount of power they want to sell at the bidding prices, and LSEs submit a bid-demand function, indicating the amount of power they are willing to pay for at the bidding prices. The aggregate bid-demand function is the LSEs' bid-demand functions stacked from the highest to the lowest bidding prices. The aggregate bid-supply function is the power producers' bid-blocks, stacked from the lowest to the highest bidding prices. The intersection of the bid-demand and bid-supply functions gives a quantity and an interval of prices. A specified rule chooses a price from the interval. Demand-side bidding is introduced to the market to promote efficiency outcomes when there is a lack of price-elastic demand. However, as mentioned in McAfee and McMillan [32], the choice of bids reflects individuals' strategic attempts to manipulate the market selling/buying price, so that the quantity and price interval reached are not necessarily those of the competitive equilibrium.[4]

The model used to analyze the effects of the LSE agents on the power-producing agents' behavior and on price dynamics in the double auction markets is modified from the model introduced in Chapter 4 to reflect the presence of another set of decision-makers, the LSE agents. The LSE agents use learning algorithms similar to those of the power-producing agents, though with slightly different bidding price adjustment strategies. This model shows that market efficiency, which is defined as the difference between the market prices and marginal-cost prices, is likely to improve once active LSEs are introduced to the market (for more detail on the effect of LSE agents on market outcomes, see, for instance, Watz [47]). Although the outcomes are as one might anticipate, the agent-based

[3] Currently, the load-response program has been implemented and the customers do not bid in the market.

[4] As quoted from [32], "Wilson [48] showed that, for the case of equal numbers of buyers and sellers with valuation distributed uniformly, the double auction satisfies the stronger criteria of ex ante efficiency: It maximizes the expected gain from trade."

model provides an alternative tool for verification. This section begins by introducing the double auction-style agent-based market model. Then simulations and analyses are presented.

6.2.1 Models

The market model presented in this section is assumed to have the UP structure. Like in the model in Chapter 4, prior to making the daily bidding decisions, both power-producing agents and LSE agents are informed of scheduled quantities of previous periods, market price and total demand of previous periods, and forecast demand. The forecast demand is defined as the quantity at which the aggregate bid-supply function intersects the aggregate bid-demand function. In addition, the power-producing agents also know the system marginal-cost function, while the LSE agents know the system marginal-utility function. Next, let us consider the characteristics of the agents.

Power-producing Agents' Characteristics

Two sets of power-producing agents are considered. Let Market-A and Market-B denote the first set and the second set of power-producing agents, respectively; and let both markets have the same total capacity. The aggregate marginal-cost functions of both sets are shown in Figure 6-14. Market-B represents markets with more expensive marginal-cost units, whereas Market-A represents markets with less expensive marginal-cost units, similar to that presented in Chapter 4. The objective of having Market-A and Market-B representing different marginal-cost units to observe the effect of the LSE agent on market outcomes due to different power-producing agent characteristics.

LSE Agent's Characteristics

Let this double-auction market model have only one active LSE agent. This LSE agent has a set of marginal-utility functions that vary hourly to exhibit peak and off-peak demand. The LSE agent maximizes its total profits by buying power from the market and selling it back to customers. The LSE agent anticipates its profits ($\hat{R}$) during K periods as follows:

$$\hat{R} = \mathcal{E}\left\{ \sum_{k=0}^{K} \sum_{j \in N} (-\hat{P}_k \cdot y_k^j + \mu^j(y_k^j) - \mathcal{U}_k^j) \right\}$$

$$\text{subject to } 0 \leq L_{k,min}^j \leq y_k^j \leq L_{max}^j$$

where $\hat{P}_k$ denotes a forecast price of the LSE agent at time k, y_k^j denotes the bidding demand quantity of bid-block j, and μ^j denotes a utility function of the LSE agent (or the obligation to serve its consumers under a specified contract) associated with bid-block j. Let $\mathcal{U}_k^j$ denote the compensation fee that the LSE agent has to pay when it curtails the consumption associated with bid-block g at time k when the market price is lower than the customers' willingness-to-pay prices. This thesis assumes that the consumers buy a power contract from the LSE agent. This contract indicates the maximum market price (or the willingness-to-pay) that the customers are willing to pay for the power they consume, as well as the minimum consumption $L^j_{k,min}$ to be delivered in each period.

Let $\mathcal{U}^j_k$ consist of two parts. The first part is equal to $\mathcal{U}^j_{k,1}$, and is associated with the cost that the LSE agent pays customers when it is unable to buy the power up to $L^j_{k,min}$. The other part is equal to $\mathcal{U}^j_{k,2}$, and is associated with the compensation that the LSE agent pays its customers when the LSE agent curtails its customers' consumption. For example, this cost incurs when the market price is at least equal to the customers' willingness to pay and the customers are not scheduled. Hence, $\mathcal{U}^j_k = \mathcal{U}^j_{k,1} + \mathcal{U}^j_{k,2}$. In summary, there are three additional constraints when an active LSE agent is added to the model, including:

- Minimum Load Obligation $L_{k,min}$. This is the minimum consumption of each period with the maximum willingness to pay equal to the maximum market prices or a price cap (P_{cap}). This portion of LSE $i's$ demand is price-inelastic.
- Minimum Load or $L^{i,j}_{k,min}$. This is the minimum load that the LSE agent has to serve customer j when the market price is greater than its willingness-to-pay; otherwise the LSE is subjected to pay $\mathcal{U}^j_{k,1}$. For simplicity, the minimum load is set to 0 ($L_{min} = 0$).
- Curtailable contracts allow the LSE agent to curtail consumers' actual consumption L^j_k from the contracted quantity $L^j_{k,max}$ for compensation fee mf^j multiplied by the curtailed quantity, i.e.,

$$U^j_{k,2} = \max\left(L^j_{k,max} - L^j_k, 0\right) \cdot mf^j \cdot \mathcal{I}(P_k < \mu^j_k).$$

  where $\mathcal{I}(Y)$ is boolean, equal to 1 if statement Y is true and equal to 0 otherwise.

Contracts with the customers of the LSE agent are pre-determined and have no intertemporal relation between hours. The LSE agent's marginal-utility functions of Hours 4, 12, and 18 are shown in Table 6.1 and in Figure 6-15.

The LSE agent has incomplete information about its competitors, the power-producing agents, and also encounters an on-line decision-making process and makes its bidding decision myopically, i.e., the LSE agent calculates its anticipated profits as follows:

$$\hat{R} = \sum_{k=0}^{K} \sum_{j \in N} \left(-\hat{P}_k \cdot y^j_k + \mu^j(y^j_k) - \mathcal{U}^j_k\right)$$
$$\text{subject to } 0 \leq y^j_k \leq L^j_{max}.$$

To determine its bid-demand function, this agent can either 1) determine $\hat{P}_k$ based on the observed past and current information, such as market prices and total demand, and then derive its bid-demand

Table 6.1: Samples of the LSE Agent's Marginal-utility Functions

	Marginal Utility ($/MWh)											
Hours	300	290	280	250	230	200	180	165	140	120	105	100
4	6	7	8	7	6	6	4	0	1	1	1	2
12	30	13	6	10	8	6	7	6	6	3	0	0
18	23	10	6	6	6	5	3	5	5	2	1	2
	Marginal Utility ($/MWh)											
Hours	90	80	75	65	60	50	40	35	20	10		
4	2	1	0	2	1	1	1	0	1	1		
12	0	2	0	0	1	1	1	1	0	1		
18	0	0	2	0	0	0	0	1	1	1		

function following some established criteria accordingly, such as those of the model-based algorithm, i.e.,

$$\{b_k^*, q_k^*\} = \max_{b_k, q_k} \left\{ \sum_{j \in N} (-\hat{P}_k \cdot L_k^j + \mu^j(L_k^j) - \mathcal{U}_k^j) \right\},$$

or 2) follow some learning algorithms, such as Algorithm A3 and the model-based algorithms, that allow the agent to derive its bid-demand function without estimating $\hat{P}_k$. Both methods are described in detail in Sections 6.2.1 and 6.2.1, respectively.

Market-Clearing Prices

The power-producing agents have piece-wise marginal-cost functions and submit piece-wise bid-supply functions. The aggregate bid-supply function (ABS) is a collection of the bid-supply functions of all the power-producing agents and is constructed by sequencing the bid-blocks from the cheapest to the highest bidding prices. The LSE agent has a set of piece-wise marginal-utility functions and submits piece-wise bid-demand functions. The aggregate bid-demand function (ABD) is a collection of bid-demand functions of the agents and is constructed by sequencing the bid-blocks from the highest to the lowest according to willingness to pay. Total demand is the quantity value at the intersection point of the ABS and ABD functions. In this thesis, the market price (P) is determined as the following method and this method is also illustrated in Figure 6-16.

1. Case 1: The ABS function intersects with the ABD function from below; that is, the bidding quantity of the intersected ABD block on the left of the intersection is positive and less than the ABD block-quantity. Suppose that the bidding price of this ABD block is $U2$. The market price is set to $P = U2$. Hence, only the quantities on the left of the intersection points are scheduled to operate, and the entire block on the ABD function with bidding price $U2$ is not scheduled to operate. If the market price is less than $U2$, this entire ABD block will be scheduled for purchasing. However, there is insufficient supply to serve this demand block at any price less than $U2$. Consequently, the market price is set to $U2$.

2. Case 2: The ABD function intersects the ABS function from above; that is, the bidding quantity of the intersected ABS block on the left of the intersection is positive and less than the ABS block-quantity. Suppose that the bidding price of this ABS block is $C2$. The market price is set to $P = C2$. Hence, only the quantities on the left of the intersection points are scheduled to operate/or to be purchased, and the entire block of the ABS function with bidding price $C2$ is not scheduled to operate. If the market price is more than $C2$, this ABS block will be scheduled to operate. However, there is insufficient demand to buy this power at a price higher than $C2$. Consequently, the market price is set to $C2$.

3. Case 3: The ABD function intersects the ABS function at the end of their blocks. Suppose that the minimum bidding price of the ABD block on the left-hand side of the intersection is equal to $U1$ and the maximum bidding price of the ABD block on the right-hand side of the intersection is equal to $U2$. Similarly, suppose that the maximum bidding price of the ABS block on the left-hand side of the intersection is equal to $C1$ and the minimum bidding price of the ABS block on the right-hand side of the intersection is equal to $C2$. Only the capacity on the left of the intersection is scheduled to operate or to be purchased; hence, the market price must be less than the bidding prices of the next most expensive ABS blocks that are not scheduled to operate, and must be higher than the bidding prices of the next most expensive ABD blocks that are not scheduled for purchasing. Consequently, the market price is set to $P = 0.5 \times (\min(U1, C2) + \max(U2, C1))$.

Modified Auer *et al.*'s Learning Algorithm

Like the power-producing agents, the LSE agent (Agent i) determines its bid-demand function by using a modified algorithm based on algorithm **Exp3.P.1** of Auer *et al.*. This modified algorithm is called Algorithm A3L. Let $(*)^b$ denote any variable associated with the bidding price and let $(*)^q$ denote any variable associated with the bidding quantity.

Initialization Agent i has K^b choices of bidding prices, i.e., $\bar{B} = \{B(1), \ldots, B(K^b)\}$, and K^q choices of bidding quantities, i.e., $\bar{Q} = \{Q(1), \ldots, Q(K^q)\}$. Agent i determines T_r^b, δ_r^b, $r^{b,*}$, T_r^q, δ_r^q, and $r^{q,*}$ using the formula as shown in Chapter 3.

Repeat For each day $t = 1, 2, \ldots$

1. Agent i obtains the scheduled prices and quantity and calculates the profits (Π_k) obtained from the previous bids, i.e.,

$$\Pi_k = -P_k \times \sum_j y_k^j + \sum_j (\mu^j(y_k^{i,j}) - \mathcal{U}_k^j),$$

where P_k is the market price at Hour k, L_k^j is the scheduled quantity associated with demand-block j, $\mu(q_k^j)$ is a contract price of consuming L_k^j of bid-block j, and $\mathcal{U}$ is the compensation fee if the agent cannot serve its customers as indicated in the contracts.

2. Agent i determines the vectors of rewards associated with all possible bidding prices, $\bar{x}_t^b = \{x_t^b(1), \ldots, x_t^b(K^b)\}$ and $\bar{x}_t^q = \{x_t^q(1), \ldots, x_t^q(K^q)\}$ as follows:

   (a) For all $k \in t$, let $\tilde{x}_k^b(m)$ be defined as

   $$\tilde{x}_k^b(m) = \begin{cases} \Pi_k(i_k^b) & \text{if } m = i_k^b \\ 0 & \text{otherwise,} \end{cases}$$

   where i_k^b denotes the choice of bidding price chosen at Hour k of day t and $\Pi_k(i_k^b)$ denotes the profit obtained from choosing bidding price i_k^b.

   (b) Then, for $m \in K^b$, $x_t^b(m)$ is an average of profits associated with action m obtained in day t and is determined as follows:

   $$x_t^b(m) = \frac{\sum_h \tilde{x}_h^b(m)}{\sum h}$$

   where h denotes the hour in day t that action m is chosen.

   Likewise, for $n \in K^q$, $x_t^q(n)$ can be determined by using a similar method.

3. Agent i receives forecast demand $\hat{L}_{t+1}$ for the next bidding round.

4. Agent i checks whether $t \in T_r^b$; otherwise, it sets $r^{b,*} = r^{b,*} + 1$, sets $(r = r^{b,*})$, sets $T^b = T_r^b$, and sets $\delta^b = \delta_r^b$.

5. Agent i checks whether $t \in T_r^q$; otherwise, it sets $r^{q,*} = r^{q,*} + 1$, sets $(r = r^{q,*})$, sets $T^q = T_r^q$, and sets $\delta^q = \delta_r^q$.

6. Agent i determines its bid-demand function for an anticipated marginal bid-block for Hour k based on the load index associated with forecast demand $\hat{L}_k$. The bid-demand function consists of two parts, bidding price and bidding quantity. Agent i chooses its bidding price from K^b possible values as follows:

   (a) Agent i determines $\gamma^b = \min\left\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K^b \ln K^b}{T^b}}\right\}$ and $\alpha^b = 2\sqrt{\ln \frac{K^b T^b}{\delta^b}}$.

For $m = 1, \ldots, K^b$

   (b) Agent i calculates $\hat{x}_t^b(m)$ as follows:

   $$\hat{x}_t^b(m) = \begin{cases} x_t^b(m)/p_t^b(m) & \text{if } m = i_t^b \\ 0 & \text{otherwise.} \end{cases}$$

(c) Agent i updates its weight $(w_{t+1}^{b}(m))$ associated with choice m of K^b possible bidding prices using

$$w_{t+1}^{b}(m) = w_t^b(m) \cdot \exp\left(\frac{\gamma^b}{3K^b}\left(\hat{x}_t^b(m) + \frac{\alpha^b}{p_t^b(m)\sqrt{K^bT}}\right)\right),$$

and updates its probability of selecting choice m using

$$p_{t+1}^{b}(m) \;=\; (1-\gamma^b)\frac{w_{t+1}^{b}(m)}{\sum_{h=1}^{K^b} w_t^b(h)} + \frac{\gamma^b}{K^b}.$$

(d) Agent i chooses $i_{k\in t+1}^{b}$ randomly according to the distribution $\{p_{t+1}^{b}(1),\ldots,p_{t+1}^{b}(K^b)\}$ and sets

$$BM_k = B(i_k^b) \;\; \text{for all } k \in t+1$$

where $(B(\cdot) \in \bar{B})$ is a choice of bidding price.

Similarly, to determine a bid quantity, Agent i chooses its bidding quantity from K^q possible values as follows:

(a) Agent i determines $\gamma^q \;=\; \min\left\{\frac{3}{5}, 2\sqrt{\frac{3}{5}\frac{K^q \ln K^q}{T^q}}\right\}$ and $\alpha^q \;=\; 2\sqrt{\ln \frac{K^qT^q}{\delta^q}}$.

(b) Agent i calculates $\hat{x}_t^q(n)$ as follows:

$$\hat{x}_t^q(n) = \begin{cases} x_t^q(n)/p_t^q(n) & \text{if } n \;=\; i_t^q \\ 0 & \text{otherwise.} \end{cases}$$

(c) Agent i updates its weight associated with choice n of K^q possible bid quantities, $w_{t+1}^{q}(n)$, using

$$w_{t+1}^{q}(n) = w_t^q(n) \cdot \exp\left(\frac{\gamma^q}{3K^q}\left(\hat{x}_t^q(n) + \frac{\alpha^q}{p_t^q(n)\sqrt{K^qT}}\right)\right),$$

and updates its probability of selecting choice n using

$$p_{t+1}^{q}(n) \;=\; (1-\gamma^q)\frac{w_{t+1}^{q}(n)}{\sum_{h=1}^{K^q} w_{t+1}^{q}(h)} + \frac{\gamma^q}{K^q}.$$

(d) Agent i chooses $i_{k\in t+1}^{q}$ randomly according to the distribution $\{\, p_{t+1}^{q}(1),\ldots,p_{t+1}^{q}(K^q)\}$ and sets,

$$y_k = Q(i_k^q) \;\; \text{for all } k \in t+1,$$

where $(Q(\cdot) \in \bar{Q})$ is a choice of bidding quantity. Let WH_k denote the withheld capacity and $WH_k = y_{max} - y_k$, where y_{max} is the maximum demand.

7. Agent i determines the bid-supply function for each Hour k by using BM_k and q_k as follows:

(a) The bidding price of the curtailed capacity (WH_k) is set to

$$WH_k = b_{k+1} - C_2,$$

where C_2 is a positive constant. In this model, $C_2 = 3$.

(b) For any block j with non-zero capacity that is not considered the withheld capacity, its bidding price b_k^j is set to

$$b_k^j = \mu^j,$$

where $\mu^{i,j}$ is the marginal utility of bid-block j.

8. Agent i submits the bid-demand functions for day $t+1$ to the system operator.

9. The system operator clears the market for each Hour k and informs the agents of market prices, total demand, and their scheduled quantities.

The Model-based Algorithm

Load Curtailment Like the capacity withholding (CW) strategy of the power-producing agents, the LSE agent has a strategy for determining the demand to be consumed at the anticipated price. By reducing some consumption and paying the customers through compensation fees, the LSE agent might make more profit, i.e.,

$$W_k^* = \arg\max_{W_k} \left\{ \sum_{j \in N} -\hat{P}_k(W_k^j) \cdot (L_k^j - W_k^j) + \mu_k^j (L_k^j - W_k^j) - \mathcal{U}_k^j(W_k^j) \right\}$$

where W_k^* denotes the optimal curtailed consumption at time k based on the assumption that the other agents submit their marginal-utility or marginal-cost bids, $\hat{P}_k$ denotes the anticipated market price of the LSE agent, L_k^j denotes the demand obligation associated with the willingness to pay μ_k^j, and $\mathcal{U}$ denotes the compensation payments when the agent is unable to serve its demand obligation.

The model-based algorithm is modified for the LSE agent to determine its bid-demand function as follows. This modified algorithm is called the model-based LSE algorithm. Let BM_k denote the bidding price of the anticipated marginal block, OP_k denote the actual profits obtained at time k, AP_k denote the anticipated profits, and MP_k denote the market price at time k.

Initialization Let an LSE agent submit its marginal-utility bid-demand functions to an operator. The operator schedules the agents to purchase based on both the ABS and ABD functions, and then informs them of market prices, total demand, and scheduled consumptions.

Repeat For each day $t \geq 1$ that Agent i follows the scheme which is called the *PORTFOLIO-LSE* scheme,

1. Agent i obtains the scheduled prices and quantity and calculates the profits (Π_k) obtained from the previous bids, i.e.,

$$\Pi_k = -P_k \times \sum_j L_k^j + \sum_j \mu^j(L_k^j) - \mathcal{U}^j,$$

where P_k is the market price at Hour k, L_k^j is the scheduled quantity associated with demand-block j, $\mu(L_k^j)$ is a contract price of consuming L_k^j of block j, and $\mathcal{U}$ is the compensation fee if the agent cannot serve its customers as indicated in the contracts.

2. Agent i determines the bidding outcome (O) from the following scheme, called the *OUTCOME-LSE* scheme:

   (a) $OP < AP$: This implies that the previous bid was not successful, and that the agent has submitted a lower BM. Let us consider the following cases.

   i. $OP \geq 0$. Consider two sub-cases a) when $BM \geq MP$, the agent sets $O = 10$, and b) when $BM < MP$, the agent sets $O = 00$.

   ii. $OP < 0$. This implies under-bidding or submitting a bid-demand function at which the agent is unable to buy power. Hence, the agent would increase the bidding prices to improve its willingness-to-pay for power it could buy. The agent could decrease the bidding prices in cases in which it over-pays for the consumed power. Consider three sub-cases a) when $BM > MP$, the agent sets $O = 10$, b) when $BM = MP$, the agent sets $O = 11$, and c) when $BM < MP$, the agent sets $O = 11$ as long as $AP > 0$, and $O = 00$, otherwise.[5]

   (b) $OP = AP$. This implies that the previous bid was successful and that the agent has no reason to change its bids. The agent sets $O = 00$, except 1) when $BM = MP$ and $OP > 0$, in which case the agent in this case sets $O = 10$,[6] and 2) when $BM > MP$ and $AP = OP = 0$, and the agent here sets $O = 10$.[7]

   (c) $OP > AP$. This implies that the previous bid was overly successful or that the agent could be a marginal consumer, being scheduled to buy power more than expected. (When there is more than one LSE agent, this may imply that the other LSEs set the market prices.)

[5] When the agent anticipates positive profits but does not receive them, the agent considers submitting a low bidding price. Likewise, when the agent anticipates non-positive profits, which may result when the compensation fee exceeds the gain from buying cheaper power, an increase in bidding prices implies that the agent would not underbid. To underbid could result in no scheduling and in losses incurred in compensating the customers.

[6] The agent is considered to be dispatched as a marginal consumer and the agent may be able to lower the market price the next round.

[7] The agent anticipates the lower market price for the next period, because the agent is able to buy the power as it anticipates. It may also buy power at lower than its willingness-to-pay by decreasing its bidding prices. Since $MP < BM$ in the current period and the agent can anticipate positive AP the next period, by decreasing the bidding price further, the agent could cause the MP to be lower and it might obtain more profits.

The agent sets the $O = 00$[8] except when 1) $AP, OP > 0$ and $BM = MP$, the agent sets $O = 10$,[9] and 2) $AP < 0$, $OP = 0$, and $BM < MP$, the agent sets $O = 11$ (so that its BM increases).[10]

Agent i updates its O and MP.

3. Agent i determines the bidding quantity through the load curtailment strategy. This bidding quantity is the maximum load obligation, in which the agent does not compensate to the customers for reducing their consumption. Overall profits from this lower price offset the losses from the compensation payment for the unserved obligation.

4. Agent i assesses whether each individual bid-block obtains its profit as anticipated. The agent also uses the *OUTCOME-LSE* scheme to determine the bidding outcome of each unit (O_u).

5. Agent i determines the hourly demand from the aggregate supply function and the aggregate demand function. The agent determines BM for each hour of the next bidding round by using the scheme, which is called *SETPRICE-LSE* scheme, as follows:

$$BM_k = Tar_k + \bar{c}_k$$

where Tar_k is the target price and $\bar{c}_k$ is a constant, which is $\bar{c} \in \{-\Delta, 0, \Delta\}$. Like the power-producing agents, Tar_k is set to

$$\begin{aligned} \text{Method M1:} \quad & Tar_k = BM_{k-1}, \\ \text{Method M2:} \quad & Tar_k = MP_{k-1}. \end{aligned}$$

In the market model with the DP structure, Method M2 is an estimation of the market price of each agent, i.e., $MP_{k-1} \equiv \hat{P}^i$. Let $\bar{c}$ be defined as follows:

$$\bar{c} = \Delta, \text{ if } O = 11; \; \bar{c} = 0, \text{ if } O = 00; and \; \bar{c} = -\Delta, \text{ if } O = 10$$

where Δ is a positive constant. Note that BM_k, Tar_k, and Δ_k are associated with the load indices.

6. Agent i determines the bidding prices of each unit (BU) from O_u using the *SETPRICE-LSE* scheme.

7. Agent i sets the bidding price for each block of the bidding quantity as follows.

[8] Since the outcome is satisfying, the agent does not change its bidding price for the next period.

[9] The agent is scheduled to purchase power as a marginal consumer and it may set the market price the next period: therefore, the agent shall submit a bid-demand function that may result in lowering the market price.

[10] During the current period, the agent decreases its bidding price to lower than its willingness-to-pay price (due to negative anticipated profits). However, the outcome shows that the agent has been scheduled to purchase power more than it anticipates and the agent keeps MP in the positive profit zone.

(a) For demand-block j with BU^j greater than BM, its bidding price b^j is set to

$$b^j = \min\{\mu^j, BU^j\}$$

where mu^j is the willingness-to-pay of block j.

(b) For demand-block j with BU^j equal to BM, its bidding price b^j is set to

$$b^j = \min\{\mu^j, BM\}.$$

Note that if the BM is less than μ^j, BM is set to μ^j.

(c) For bid-block j with BU^j less than BM, its bidding price b^j is set to

$$b^j = \min\{\mu^j, BU^j\}.$$

(d) The bidding price of the curtailed capacity can be set to either WH where $WH \ll \min_j \mu^j$, or $WH = \max\{BM - C, 0\}$, where C is a positive constant.

Agent i updates its recorded BM and BU of each demand block.

8. Agent i calculates its AP, by assuming that $BM = MP$. The bidding blocks with the bidding prices of at least BM are scheduled and paying BM. Similarly, the anticipated profit of each block is calculated as well (to be used in determining O_u). Then, Agent i records its new AP.

9. Agent i submits the bid-demand functions for day $t + 1$ to the system operator.

10. The system operator clears the market for each Hour k and informs the agents of market prices, total demand, and their scheduled quantities.

6.2.2 Preliminary Analysis

Suppose that the aggregate marginal-cost function of the power-producing agents is an increasing function and is denoted by $MC(x)$, where x is the quantity of power. That is, $MC(x)$ indicates the price of the associated quantity of power x that the power-producing agents are willing to produce. Suppose that the aggregate marginal-utility function of the power-producing agents is a decreasing function and it is denoted by $MU(x)$. That is, $MU(x)$ indicates the willingness to pay for the associated power quantity x that the LSE agents are willing to consume. This section provides a preliminary analysis to show that, given the aggregate bid-supply function of the power-producing agents, the LSE agent may not necessarily be better off submitting a strategic bid-demand function. Note that the strategic bid-demand function is defined as a bid-demand function that is not the marginal-utility function and is determined by the demand-curtailment or by the price-setting strategy.

The best response of the LSE agent depends on the characteristics of the aggregate bid-demand function. The simulations shown in the next section support this finding, especially when the agents use the model-based learning algorithm.

For simplicity, let us assume that the LSE agents have the aggregate marginal-cost function as follows.

$$MU(x) = \begin{cases} = & \hat{a} \cdot x + \hat{b} \quad \text{if} \quad x \geq x_L \\ = & Y_L \quad\quad\quad \text{if} \quad 0 \leq x < x_L \end{cases}$$

where $\hat{a}$ and $\hat{b}$ are constants, $\hat{a} < 0$, and $\hat{b} > 0$. Let x_L denote the minimum demand that the LSE agent needs to serve the customers before it has to pay the compensation fee. Let $Y_L > 0$. The power-producing agents have the aggregate marginal-cost function as follows:

$$MC(x) = \begin{cases} = & \hat{c} \cdot x + \hat{d} \quad \text{if} \quad x \geq x_S \\ = & Y_S \quad\quad\quad \text{if} \quad 0 \leq x < x_S \end{cases}$$

where $\hat{c}$ and $\hat{d}$ are constants; $\hat{c} > 0$ and $\hat{d} < 0$. Let x_S denote the minimum capacity at which the power-producing agents need to operate before they are subject to a penalty fee. Let $Y_S > 0$. Note that $x_S \geq 0$; however, in this agent-based model x_S is always set to zero. Figure 6-17 shows the samples of these aggregate marginal-cost and marginal-utility functions.

Suppose that the LSE agent knows the aggregate bid-supply function (y), i.e., $y = c \cdot x + d$. The LSE agent maximizes the anticipated profit (Π) by determining the slope (a) and the intercept value on a y-axis (b) of the bid-demand function as follows:

$$\max_{a,b} \Pi = \max_{a,b} \left(-P \cdot x + U - \mathcal{U}(\max(x_L - x, 0))\right)$$

where P denotes the market price and U denotes the contract payment that the LSE agent obtains from the customers, and where U is assumed to be a constant. Let $\mathcal{U}$ denote the compensation fee that the LSE agent has to pay the customers when demand of at least equal to x_L is not delivered. Let us consider a set of $MU(x)$ and $MC(x)$ in which $MU(x)$ intersects with $MC(x)$ at the quantity x^*, and x^* is such that $x^* > X_S, X_L$. At the intersection point, $c \cdot x + d = a \cdot x + b$ and, consequently, $x = \frac{d-b}{a-c}$ and $P = a \cdot x + b$. Therefore,

$$\begin{aligned} \max_{a,b} \Pi = \max_{a,b} (-P \cdot x + U) &= \max_{a,b} (-(a \cdot x + b) \cdot x + U) \\ &= \max_{a,b} \left(-(a \cdot \frac{d-b}{a-c} + b) \cdot \frac{d-b}{a-c} + U\right). \end{aligned}$$

The LSE agent determines the optimal values of a and b as follows:

$$\frac{\partial \Pi}{\partial a} = (\frac{d-b}{a-c})^2 - 2a \cdot \frac{(d-b)^2}{(a-c)^3} - b \cdot \frac{(d-b)}{(a-c)^2} = 0.$$

$$\frac{\partial \Pi}{\partial b} = -2\frac{d-b}{(a-c)^2} - \frac{d-2b}{a-c} = 0.$$

With some algebraic calculation, one can obtain

$$a = \frac{2bc}{d} - c. \tag{6.2}$$

Suppose that the LSE agent submits the bid-demand function with the same values for Y_L and x_L as in the marginal-cost function. At the point on the aggregate marginal-utility function where $x = x_L$,

$$Y_L = (\frac{2bc}{d} - c) \cdot x_L + b,$$

and

$$b = (Y_L + cx_L)/(\frac{2c}{d} \cdot x_L + 1), \quad \text{and} \quad a = c \cdot (2Y_L - d)/(2cx_L + d).$$

Note that $a < 0$ and $b > 0$. From Equation (6.2) for any given value $c > 0$ and $d < 0$, when the bid-supply function has $c \gg 1$ and $d \ll 0$, the best response of the LSE agent is to have $a \ll 0$. That is, when the bid-supply function has a steep slope for $x > x_S$, the best-response bid-demand function of the LSE agent will have a steep slope for $x > x_L$ as well.

Suppose that the LSE agent does not submit a bid-demand function that is more expensive than its marginal-utility function, i.e., $MU(x) \geq y$ for all x. This condition implies that $\hat{a} > a$ for $x \geq x_L$. Therefore, the slope of the bid-demand function (a) and its intercept value on a y-axis (b), which is the best response to the bid-supply function of the power-producing agents with slope (c) and intercept value on the y-axis (d), should be equal to

$$a = \min\{c \cdot (2Y_L - d)/(2cx_L + d), \hat{a}\} \text{ and } b = \max\{(Y_L + cx_L)/(\frac{2c}{d} \cdot x_L + 1), \hat{b}\}.$$

Figure 6-18 illustrates this relationship.

As a result, in the market that has an aggregate marginal-cost function with a steep slope, such as with a large value of $\hat{c}$ and a small value $\hat{d}$, and in which the power-producing agents submit the strategic bid-supply function, such that $0 < \hat{c} < c$ and $d < \hat{d} < 0$, the LSE agent may be better off submitting its strategic bid-demand function, especially when $(2cx_L + d < 0)$. On the other hand, when the power-producing agents do not have a steep slope of the aggregate marginal-cost function or of the aggregate bid-supply function, the LSE agent might be better off submitting its marginal-utility bid-demand function, especially when $(2cx_L + d > 0)$.

This analysis suggests that the LSE agent will be more likely to submit a strategic bid-demand function to buy power in Market-A than in Market-B. This observation further implies that market prices in Market-A should be lower than those in Market-B. Note that the best-response aggregate bid-supply function of the power-producing agents can be determined using the same method as the LSE agent as presented previously.

6.2.3 Simulations

This section presents the simulated price dynamics and simulated profit dynamics of both power-producing and LSE agents. Two sets of simulations are considered, including when the power-producing agents use Algorithm A3 and the LSE agent uses Algorithm A3L, and when the power-producing agents use the model-based algorithm and the LSE agent uses the model-based LSE algorithm.

Algorithms A3 and A3L

In the simulations in which the power-producing agents use Algorithm A3 and the LSE agent uses Algorithm A3L, the agents select the bidding price of the anticipated marginal unit from 0 to \$300/MWh, which is the maximum willingness-to-pay of the LSE agent, with an increment of \$3/MWh. Likewise, the power-producing agents select their bidding quantity from 0.25 MW to its available capacity with an increment of 0.25 MW, whereas the LSE agent selects its bidding quantity from 2.5 MW to its available capacity with an increment of 2.5 MW. Note that the increment in bidding quantity of the power-producing agents is set to be smaller than that of the LSE agent, because the capacity of the power-producing agents is relatively smaller than the LSE agent's total demand obligation. The simulations shown in this section have δ set to 0.1 for both the power-producing and LSE agents. Hence, for $r = 0, 1, \ldots$, for the power-producing agents $r^{b,*} = 0$, as well as for the LSE agent, $r^{b,*} = 0$. In addition, let $\alpha^b = 2 \ln \frac{K^b T_r^b}{\delta^b}$ and $\alpha^q = 2 \ln \frac{K^q T_r^q}{\delta^q}$.

The Model-based and Model-based LSE Algorithm

In the simulations in which the power-producing agents use the model-based algorithm and the LSE agent uses the model-based LSE algorithm, $\Delta = 2$ and the bidding price of the withheld capacity is set to the maximum willingness-to-pay of the LSE agent. The bidding price of the curtailed capacity of the LSE agent is set to \$0/MWh.

Market Scenarios

Let us define a strategic bid as a bid function (either bid-supply or bid-demand function) in which the bidding prices and bidding quantities are determined by the assigned algorithms. Several simulation scenarios are considered.

- **Scenario I:** The power-producing agents and the LSE agent submit strategic bid functions; that is, they apply the assigned learning algorithm to determine the bid-supply and the bid-demand functions, respectively.

- **Scenario II:** The LSE agent is assumed to have no active role; that is, the LSE agent submits its marginal-utility bid-demand function in every period. This case implies that the power-producing agents encounter price-elastic demand.

- **Scenario III:** The power-producing agents use the learning algorithm to determine only the bidding price of the anticipated marginal unit (BM) without the bidding quantity, and the LSE agent submits a strategic bid-demand function.

- **Scenario IV:** The LSE agent uses the learning algorithm to determine only the bidding price of the anticipated marginal block (or when the curtailed capacity is maintained to zero), and the power-producing agents submit a strategic bid-supply functions.

The hourly competitive market prices and the demands of Market-A and Market-B for a five-day period are shown in Figures 6-19 and 6-20, respectively. A 24-hour window of moving average is applied to all simulations. Figures 6-21 and 6-22 show the dynamics of moving-average prices and demand when the power-producing and LSE agents in Market-A use the Algorithm A3 and Algorithm A3L, respectively. In addition, Figures 6-25 and 6-26 show the dynamics of moving-average prices and demand when the power-producing and LSE agents in Market-B use the Algorithm A3 and Algorithm A3L, respectively. Note that in the plots shown in Figures 6-21, 6-22, 6-25, and 6-26, "d01" denotes the simulated outcomes under Scenario I, "d01noL" denotes the simulated outcomes under Scenario II, "d01noWG" denotes the simulated outcomes under Scenario III, and "d01noWL" denotes the simulated outcomes under Scenario IV. Also, "Comp" denotes the simulated competitive outcomes of prices and demand obtained from the assumption that the power-producing agents submit their marginal-cost bid-supply functions and the LSE agent submits its marginal-utility bid-demand function.

Figures 6-23 and 6-24 show the dynamics of moving-average prices and demand when the agents in Market-A use the model-based algorithm with Method M1 to set the target price when they determine the bidding price for their anticipated marginal unit with $\Delta = 2$. In addition, Figures 6-27 and 6-28 show the dynamics of moving-average prices and demand when the agents in Market-B also use the model-based algorithm with the same setting. Note that in the plots shown in Figures 6-23, 6-24, 6-27, and 6-28, "PWPW" denotes the simulated outcomes under Scenario I, "PWnoPnoW" denotes the simulated outcomes under Scenario II, "PnoWPW" denotes the simulated outcomes under Scenario III, and "PWPnoW" denotes the simulated outcomes under Scenario IV. Also, "Comp" denotes the simulated competitive outcomes.

The existence of the active LSE agent will decrease the ability of the power-producing agents to raise the bid-supply functions, causing the market prices to be reduced regardless of the learning algorithms implemented in the model. Typically, the marginal-utility function has a non-increasing characteristic in willingness-to-pay as the quantity increases. Then, given a marginal-utility function where the market price increases, actual consumption increases as well. Therefore, the accumulative moving-average profits of both the power-producing and LSE agents are used in addition to price dynamics as measures to identify the effect of the active decision-making of the LSE agent.

Figure 6-29 shows the moving-average profits that the LSE agent in Market-A obtains according to the four scenarios, whereas Figure 6-30 shows the moving-average profits that the power-producing agents obtain when the power-producing and LSE agents in Market-A employ the Algorithm A3 and Algorithm A3L, respectively. Figure 6-33 shows the moving-average profits that the LSE agent in Market-B obtains according to the four scenarios, whereas Figure 6-34 shows the moving-average profits of the power-producing agents.

When the power-producing agents use the model-based algorithm and the LSE agent uses the model-based LSE algorithm to determine their strategic bid functions, the moving-average profits that the LSE agent in Market-A obtains according to the four scenarios are shown in Figure 6-31. Figure 6-32 shows the moving-average profits that the power-producing agents obtain. Figure 6-35 shows the moving-average profits of the LSE agent in Market-B and Figure 6-36 shows the moving-average profits of the power-producing agents in Market-B accordingly.

6.2.4 Analyses

As shown earlier, Algorithm A3L yields mixed strategy actions and the model-based LSE algorithm yields pure-strategy action. Because the agents can choose any action at any time when they employ Algorithm A3 or A3L, the simulated outcomes exhibit more fluctuations than those when the agents employ the model-based algorithm. For simplicity in analyzing the simulated outcomes, the moving average with a 24-hour window of prices, demand, and profits are presented to capture the trend of the outcomes over time. The analyses are described as follows.

Algorithms A3 and A3L

As one can observe from Figures 6-21, 6-22, 6-25, and 6-26, in Scenario I, when the power-producing and LSE agents use Algorithms A3 and A3L, respectively, in either Market-A or Market-B, the simulated prices and demand exhibit large fluctuations compared with prices and demand when the algorithm is used by only the power-producing agents or the LSE agent. One would also anticipate similar outcomes from Scenarios II and IV. In Scenario II the LSE agent submits marginal-utility bids and in Scenario IV the LSE agent does not determine the bidding quantity. Recall from Section 4.6.1 that when the power-producing agents use Algorithm A3 and do not determine the bidding quantity, the bid functions of these agents are their marginal-cost bid-supply functions. The same argument is applied to the LSE agent; that is, the LSE agent submits a marginal-utility bid-demand function in Scenarios II and IV.

When the power-producing agents submit their marginal-cost bid-supply function in Scenario II, the LSE agent is able to submit a strategic bid-demand function that yields lower prices for a higher amount of power. One can observe that in both Market-A and Market-B, the LSE agent obtains the highest average profits over the simulation period of 1440 hours in Scenario III. In addition, the LSE agent obtains more profit in Scenario I than in Scenario II. Although Scenarios II and IV will have similar outcomes, in the simulated outcome presented here the profits that the LSE agent receives in Scenario II are higher than the profits received in Scenario IV. Note that each simulation with Algorithm A3 yields one possible simulated path, resulting from a series of random draws according to the probability distribution that is obtained from the algorithm.

As anticipated, the simulated outcomes from Market-A and Market-B do not exhibit a substantial difference in price dynamics. That is, the profits that the LSE agent in Market-A or Market-B obtains are highest in Scenario III and lowest in Scenario IV. These simulations show that the strategic bid-demand function is a better response to the power-producing agents than the marginal-utility bid-demand function. Using Algorithm A3 the power-producing agents tend to submit more expensive bid-supply functions. The mixed strategy selects the bidding prices and quantities from all possible actions. The large cumulative withheld capacity can easily lead to an expensive aggregate bid-supply function. Besides, as a result of a series of random draws described in the algorithm, each simulation represents one possible set of time-series dynamics. To obtain a better conclusion of the outcomes when the agents use Algorithm A3, several simulations will be performed and the expected outcomes of those simulations will be used, as follows.

Average Simulated Outcomes The moving-average simulated profits of the LSE agent in Market-A and Market-B from 100 simulations are shown in Figures 6-37 and 6-38, respectively. In each simulation, the power-producing and LSE agents employ Algorithms A3 and A3L, respectively, with $\delta = 0.1$. In each figure, the profits that the LSE agent obtains in Scenario I are denoted by "d01," in Scenario II they are denoted by "d01NoL," and in Scenario IV they are denoted by "d01NoWL." Note that Scenario III is not investigated because the LSE agent always obtains more profits in Scenario III than other scenarios.

From these figures, one can observe that the simulated moving-average profits of the LSE agent exhibit a somewhat periodic characteristic, especially in Scenario II; that is, when the LSE agent submits its marginal-utility bid. In Scenario II, the simulated profits decrease and then increase abruptly before decreasing again. This pattern may result from the cumulative effect of strategic bid-supply functions of the power-producing agents. These agents are likely to choose the bidding quantity resulting in a large withheld capacity to raise the prices over time until total consumption decreases, such that these strategic bids are no longer profitable. When this occurs each power-producing agent is more likely to choose a smaller withheld capacity, which yields the increase in profits of the LSE agent.[11]

Moreover, the LSE agent obtains more profits by submitting a strategic bid-demand function than by submitting a marginal-utility bid-demand function either in Market-A or Market-B. Figure 6-39 shows the plots of the difference between the profits of the LSE agent from strategic bid-demand functions and from the marginal-utility one in Market-A and in Market-B. The difference in Market-A is higher than that in Market-B. This result suggests that this agent in Market-A should always submit a strategic bid-demand function. Note that this result contradicts the preliminary analysis, which suggests that the LSE agent in Market-B should benefit more from the strategic bids than the LSE agent in Market-A because the slope of the marginal-utility function in Market-A is steeper than that in Market-B. This outcome may result from the action selection based on mixed strategies. That is, the chosen actions do not respond directly to the opponents' actions, but rather respond to the opponents' average actions in a form of a probability distribution over all actions.

The Model-based Algorithm

One can observe from Figure 6-23 that in Market-A the simulated prices in Scenario I are the highest, whereas the ones in Scenario II are lower than in Scenario I but higher than in Scenario IV. These scenarios yield simulated prices that are substantially higher than the competitive prices and the prices in Scenarios III, which themselves are lower than competitive prices. The simulated prices in Scenario III are the lowest, and the associated demand is similar to that of Scenario I. In Scenario IV, the demand decreases and converges closely to the demand in Scenarios I and III. From Figures 6-31 and 6-32, the LSE agent in Market-A receives the highest profits in Scenario III, the scenario in which the power-producing agents do not exercise the CW strategy. The LSE agent in Market-A receives the lowest profits in Scenario I, the scenario in which the LSE agent submits the strategic bid-demand functions. The power-producing agents receive the highest profits when they submit strategic bid-supply functions and receive the lowest profits when they do not exercise the CW strategy. Note that when the LSE agent uses the model-based LSE algorithm without implementing the demand curtailment strategy, it can have bid-demand functions that are not marginal-utility functions.

In Market-B the simulated prices in Scenario II are the highest and are higher than the competitive prices as shown in Figure 6-27, whereas the prices in Scenario IV are lower than in Scenario II but higher than in Scenarios I and III. The simulated prices in Scenario I and III are substantially lower than the competitive prices, while the simulated prices in Scenario IV decrease over time and become lower than the competitive prices, but still remain higher than those of Scenario I and III. The

[11] A somewhat periodic characteristic of the moving-average outcomes when the agents employ Algorithm A3 or A3L may result from a change in parameter settings after each epoch ($r^b orr^q$) ends.

associated demand of any scenario is lower than the demand in the competitive setup. Also, the associated demand of Scenario I is similar to that of Scenario III which itself is lower than the demand in Scenarios II and IV. Note that, like the simulated prices, the demand in Scenario IV decreases over time.

As seen in Figures 6-35 and 6-36, the LSE agent in Market-B receives the most profits in Scenario III. Although the LSE agent is most profitable in the situation when the power-producing agents do not exercise their CW strategy (or Scenario III), the LSE agent is more profitable when it submits a strategic bid-demand function than when it submits a marginal-utility bid-demand function. Moreover, the marginal-utility bid-demand function yields the least profits to the LSE agent when the power-producing agents submit strategic bid-supply functions. The LSE agent receives the lowest profits in Scenario II, in which the LSE agent submits only the marginal-utility bid-demand functions in response to the strategic bid-supply functions of the power-producing agents. The power-producing agents receive the highest profits when they submit strategic bid-supply functions in Scenario II, but not in Scenario I. The lowest profits that the power-producing agents receive occur when they do not exercise the CW strategy.

The simulated outcomes from Market-B are considerably different than those of Market-A. They demonstrate that the strategic bid-demand functions of the LSE agent may not necessary yield an increase in profits in response to the strategic bid-supply functions of the power-producing agents. Since Market-A has less expensive generating units than Market-B, Market-A can be considered to have an aggregate marginal-cost function with a flatter slope and Market-B can be considered to have an aggregate marginal-cost function with a steeper slope. From the preliminary analysis, the LSE agent in Market-A is probably better off submitting the marginal-utility bid-demand function compared to the LSE agent in Market-B, because the power-producing agents in Market-B can submit marginal-cost bid-supply functions that can easily result in an aggregate bid-supply function steeper than the aggregate bid-supply function of the agents in Market-A. When the agents use the model-based algorithm, the characteristics of the power-producing agents (the characteristics of the market-wide marginal-cost function) play a significant role in the simulation results.

6.2.5 Comments and Conclusions

The simulated outcomes suggest that the active LSE agent is likely to reduce an ability of the power-producing agents to set expensive market prices. The market prices do not reach the maximum willingness-to-pay of the LSE agent. As shown in the appendix to this chapter, when the competitive demand as shown in Figure 6-19 is used as an input to the model without the LSE agent, the market prices are substantially higher than any outcomes obtained in the model with the LSE agent. Although the simulated outcomes depend highly on the behavior of the power-producing agents, which is typical of agent-based models or multiagent systems, the LSE agent with the proper response strategy is able to improve its profits over time. Given the same LSE characteristics, the outcomes from the model that the agents use with the model-based learning algorithm further suggest that an LSE agent does not need to respond strategically to power-producing agents with a flatter aggregate marginal-cost function slope (such as Market-A) in order to obtain the highest profits, as it does if the power-producing agents have a steeper aggregate marginal-cost function slope (such as Market-B). That is, in Market-A the LSE agent can increase its profits by submitting its marginal-utility bid-demand function.

Appendix to Chapter 6

A. Preliminary Analyses

Let us consider electricity markets where demand is deterministic and inelastic and where the markets have uniform-pricing (UP) and discriminatory-pricing (DP) structures. Let us assume that no intertemporal effects of unit-commitment constraints of operating the generating units are accounted for and the operator schedules the generating units to serve hourly demand through the hourly auction independently. Let agents refer only to the power producers. As in Chapter 4, the agents submit piece-wise bid-supply functions or bid-blocks to the operator for scheduling. In markets with the UP structure, the revenue of Agent i for any scheduled bid-block j at Hour k is equal to the market price (P_k) multiplied by scheduled quantity ($y_k^{i,j}$), where the scheduled quantity is less than or equal to the bidding quantity ($q_k^{i,j}$), i.e., $0 \leq y_k^{i,j} \leq q_k^{i,j}$. The market price of each hour is equal to the maximum bidding price of the most expensive bid-block that is scheduled to meet the demand (L_k) at that hour. That is, the system operator schedules the bid-blocks to serve demand based on a merit order so that the total system cost, the sum of the market price (P_k) multiplied by the scheduled capacity $\sum_i \sum_j y_k^{i,j}$, is minimized. Therefore, at each Hour k, the system operator optimizes

$$\begin{aligned} \min \sum_i \sum_j & \quad (P_k \cdot y_k^{i,j}) \\ \text{st. } L_k &= \sum_i \sum_j y_k^{i,j} \\ 0 \leq y_k^{i,j} &\leq q_k^{i,j}, \quad \forall i, j. \end{aligned}$$

This optimization yields

$$P_k = \max_i \max_{j \in \Omega_k^i} b_k^{i,j} \cdot I(y_k^{i,j} > 0),$$

where Ω_k^i is a set of Agent i's bid-blocks scheduled to operate during Hour k.

On the other hand, in markets with the DP structure, any scheduled bid-block is paid the associated bidding-price ($b_k^{i,j}$) multiplied by that scheduled quantity. Similarly, the system operator schedules the bid-blocks to meet demand such that the total system cost, the sum of bidding prices multiplied by the scheduled capacity, is minimized. At each hour k, the system operator optimizes

$$\begin{aligned} \min \sum_i \sum_j & \ (b_k^{i,j} \cdot y_k^{i,j}) \\ \text{st. } L_k \ = & \ \sum_i \sum_j y_k^{i,j} \\ 0 \ \leq \ y_k^{i,j} \ \leq & \ q_k^{i,j}, \quad \forall i, j. \end{aligned}$$

Let P_k denote the market price of Hour k. As with the UP market structure, this market price refers to the maximum bidding price of the units that are scheduled to produce electricity to serve demand at that hour, which is defined as

$$P_k \ = \ \max_i \ \max_{j \in \Psi_k^i} \ b_k^{i,j} \cdot I(y_k^{i,j} > 0)$$

where Ω_k^i is a set of Agent i's bid-blocks scheduled to operate during Hour k. Note that it is assumed that this price not publicly available in the markets with the DP structure. Therefore, at each hour the agents know only their scheduled prices and quantities. Next let us consider the bidding behavior of the agents in the markets with the UP and DP structures.

I. Determining Bidding Prices

This section shows that in each hourly bidding round the units with marginal costs that are less than the market price of that hour are likely to submit a higher bidding price in the DP markets than in the UP markets. Suppose the agents submit a piece-wise bid-demand function or a set of bid-blocks of prices and quantities to the system operator for scheduling. Let one bid-block represent one unit and let one agent own one generating unit; therefore, one bid-block represents one agent. First, let us consider the markets with the UP structure and then those with the DP structure. In both cases, without the unit-commitment constraints, prior to a bid submission, Agent i anticipates market price $\hat{P}_k^i$ from the forecast demand and the system marginal-cost function. Let us assume the bidding price of any unit is equal to at least its marginal cost. Let demand be deterministic and inelastic. Suppose that the agents submit their full-capacity bids.

Let the marginal-cost price of any demand be defined as the price at which the aggregate marginal-cost function and a demand function (a straight line parallel to a price-axis) intersect. Given the forecast demand, this price is the minimum market price when there is no demand uncertainty and no system constraints.[12] The uncertainty of market prices for any given deterministic demand is caused

[12]When the system constraints are taken into consideration, the units may be scheduled to serve demand out-of-merit-order, resulting in a market price either higher or lower than, or equal to the marginal-cost price. However these factors are not considered in this thesis.

partly by the unavailability of the units due to strategic bidding or unplanned outage, as well as other operating constraints found in the power systems. As shown in Appendix A, when no system constraints are accounted for, the market price of any demand level tends to be higher than the price when the system constraints are not accounted for. Let $\hat{P}_{min}(L_k)$ denote the minimum market price when demand is equal to L_k, that is, the price when all units are able to operate at their full capacity. Let market prices be continuous values. Suppose that the market prices of any demand level can be described by probability distribution functions and that the market price is equal to at least $\hat{P}_{min}(L_k)$ and is bounded above by a price cap, P_{cap}. Let $\wp_{k|\hat{L}_k,b_k^i}(\hat{P}_k^i)$ denote a probability density function of the anticipated market price at Hour k viewed by Agent i.

UP Structure: In markets with the UP structure, Agent i determines its bidding prices such that its expected profit, given its assumption about market prices, is maximized as follows:

$$\begin{aligned} b_k^{i*} &= \arg\max_{b_k^i} \mathcal{E}_{\hat{P}_k} \left(\hat{P}_k^i \cdot q_k^i - c(q_k^i)\right) \\ \text{s.t.} \quad & q_k^i > 0, \quad \text{if} \quad b_k^i \leq \hat{P}_k^i \\ & q_k^i = 0, \quad \text{if} \quad b_k^i > \hat{P}_k^i. \end{aligned} \tag{6.3}$$

The expectation is taken over the anticipated price observed by Agent i. Without an explicit stochastic model of market prices, let us assume that the anticipated price at Hour k viewed by Agent i is a random variable conditioned on the forecast demand $\hat{L}_k$ and its bidding price b_k^i. Let us assume that all agents share the same knowledge of demand. Equation (6.3) is rewritten as follows:

$$b_k^{i*} = \arg\max_{b_k^i} \int_{b_k^{i,j}}^{\infty} \wp_{k|\hat{L}_k,b_k^i}(\hat{P}_k^i) \cdot (\hat{P}_k^i \cdot q_k^i - c^i \cdot q_k^i) \cdot d\hat{P}_k^i.$$

Let $\hat{P}_k^i \cdot q_k^i - c^i \cdot q_k^i = R_k^i$. An infra-marginal unit is a unit with a bidding price less than the market price, which is the highest bidding price of the scheduled units. With the uncertainty of market prices given demand L_k, an anticipated infra-marginal unit is defined as a unit with the probability of being scheduled to operate when it submits a marginal-cost bid equal to 1, i.e.,

$$\int_{mc_k^{i,j}}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i = \int_{P_{min}(L_k)}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i = 1.$$

Like an anticipated infra-marginal unit, an anticipated marginal unit is the unit with a probability of being scheduled to operate when it submits a marginal-cost bid equal to 1, and $mc^i = P_{min}(L_k)$. On the other hand, an extra-marginal unit is a unit with a bidding price greater than the market price or the highest bidding price of the scheduled units. With the uncertainty of market prices given demand L_k, an anticipated extra-marginal unit is defined as a unit with a probability of being scheduled to operate when it submits a marginal-cost bid less than 1, i.e.,

$$0 \leq \int_{mc_k^i}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i < \int_{P_{min}(L_k)}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i = 1.$$

Let us consider the following observations.

1. The higher the bidding price, the lower the chance of the agent being scheduled. Let us consider the two cases when $b_k^i = C_1$ and $b_k^i = C_2$. Given $\epsilon > 0$, suppose that $C_1 = mc^i < C_2 = \hat{P}_{min}(L_k) + \epsilon$. When the agent changes its bidding price b_k^i from C_1 to C_2, the agent may experience one of two following outcomes:

   (a) That $\wp_{k|\hat{L}_k, b_k^i}(\hat{P}_k^i)$ does not change, i.e.,

   $$\wp_{k|\hat{L}_k, b_k^i=C_1}(\hat{P}_k^i) \ = \ \wp_{k|\hat{L}_k, b_k^i=C_2}(\hat{P}_k^i).$$

   In this case, the anticipated profits when the bidding price is equal to C_1 are not less than the anticipated profits when the bidding price is equal to C_2, i.e.,

   $$\int_{b_k^i=C_1}^{\infty} \wp_{k|\hat{L}_k, b_k^i=C_1}(\hat{P}_k^i) \cdot R_{k|C_1}^i \cdot d\hat{P}_k^i \ \geq \ \int_{b_k^i=C_2}^{\infty} \wp_{k|\hat{L}_k, b_k^i=C_2}(\hat{P}_k^i) \cdot R_{k|C_2}^i \cdot d\hat{P}_k^i.$$

   (b) That $\wp_{k|\hat{L}_k, b_k^i}(\hat{P}_k^i)$ does change, i.e.,

   $$\wp_{k|\hat{L}_k, b_k^i=C_1}(\hat{P}_k^i) \ \neq \ \wp_{k|\hat{L}_k, b_k^i=C_2}(\hat{P}_k^i), \qquad \forall \hat{P}_k^i \neq \tilde{P}.$$

   This result does not determine whether submitting a bidding price equal to $C1$ is better than submitting one equal to $C2$. To understand this result, let us consider the difference between the expected profits when b_k^i is equal to C_1 and C_2 is equal to

   $$\int_{b_k^i=C_1}^{\infty} \wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i) \cdot R_{k|C_1}^i \cdot d\hat{P}_k^i \ - \ \int_{b_k^i=C_2}^{\infty} \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i) \cdot R_{k|C_2}^i \cdot d\hat{P}_k^i$$

   $$= \int_{C_1}^{\tilde{P}} (\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i)) \cdot R_k^i \cdot d\hat{P}_k^i + \int_{\tilde{P}}^{\infty} (\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i)) \cdot R_k^i \cdot d\hat{P}_k^i$$

   $$\leq \int_{C_1}^{\tilde{P}} (\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i))\, d\hat{P}_k^i \cdot R_{max}^{i,1} + \int_{\tilde{P}}^{\infty} (\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i))\, d\hat{P}_k^i \cdot R_{max}^{i,2}.$$

   Let $\tilde{P}$ denotes a point on an anticipated price axis where $\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i)$ crosses $\wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i))$ or where $\wp_{k|\hat{L}_k, C_1}(\hat{P}_k^i = \tilde{P}) = \wp_{k|\hat{L}_k, C_2}(\hat{P}_k^i = \tilde{P}))$. Let $R_{max}^{i,1}$ denote the bounds on the return for the anticipated price at most equal to $\tilde{P}$, and let $R^{i,2}_{max}$ denote the anticipated price greater than $\tilde{P}$.[13] Let $A1 = \int_{C_1}^{\tilde{P}} (\wp_{k|\hat{L}_k,C_1}(\hat{P}^i_k) - \wp_{k|\hat{L}_k,C_2}(\hat{P}^i_k))d\hat{P}^i_k$ and $A2 = \int_{\tilde{P}}^{\infty} (\wp_{k|\hat{L}_k,C_1}(\hat{P}^i_k) - \wp_{k|\hat{L}_k,C_2}(\hat{P}^i_k))d\hat{P}^i_k$. Since $C1 < C2$, the minimum market price $(\hat{P}_{min}(L_k))$ when $b^i_k = C1$ is lower than when $b^i_k = C2$. Hence, $A1 > 0$ and $A2 < 0$. Moreover, $R^{i,1}_{max} < R^{i,2}_{max}$. Therefore, the anticipated profits when the bidding price is equal to C_1 may be one of the following: higher than, lower than, or equal to the anticipated profits when the bidding price is equal to C_2.

Therefore, when the units are anticipated to be infra-marginal or marginal, and they are unable to influence the market price, they should submit bidding prices equal to their marginal costs. The bidding price (b^i_k) of any anticipated infra-marginal unit for any demand $\hat{L}_k$ should be equal to at least the marginal cost but not greater than the minimum anticipated price, i.e., $mc^i_k \leq b^i_k < \hat{P}_{min}(L_k)$. Submitting a bidding price less than the marginal cost results in a loss in profits when the unit is scheduled and its bidding price is paid. The agent is likely to lose profits if $b^i_k = \hat{P}_{min}(L_k)$, because when $P_k = \hat{P}_{min}(L_k) = b^i_k$, the unit may be scheduled to operate its bidding quantity. For example, if this unit submits $b^i_k < \hat{P}_{min}(L_k)$, it will be scheduled to operate q^i_k. Likewise, if this unit submits $b^i_k = \hat{P}_{min}(L_k)$, it will be scheduled to operate less than q^i_k, because there is at least one other unit submitting a marginal-cost bidding price equal to $\hat{P}_{min}$ or a unit with bidding price equal to $\hat{P}_{min}$. Therefore, each of the units with a bidding price equal to $\hat{P}_{min}$ is scheduled to operate $\tilde{q}^i_k \leq q^{i,j}_k$. Consequently,

$$\hat{P}_{min}(L_k) \cdot q^i_k \;\geq\; \hat{P}_{min}(L_k) \cdot \tilde{q}^i_k.$$

On the other hand, when the unit is able to influence the market price, submitting a bidding price higher than its marginal cost may be profitable.

2. The bidding price for an anticipated extra-marginal unit for any demand $\hat{L}_k$ is greater than its marginal cost, and is greater than $\hat{P}_{min}(\hat{L}_k)$. The agent should set the bidding price higher than the marginal cost, even if the marginal cost is higher than $\hat{P}_{min}(\hat{L}_k)$. Let us consider the two cases in which $b^i_k = C_1$ and $b^i_k = C_2$. Suppose that $\hat{P}_{min}(L_k) < C_1 < C_2$. Next, suppose that the anticipated demand is less than the total available capacity, $q_{max,k}$, i.e., $L_k \ll q_{max,k}$. By varying the bidding price b^i_k from C_1 to C_2, the agent may observe

   (a) That $\wp_{k|\hat{L}_k,b^i_k}(\hat{P}^i_k)$ does not change, i.e.,

   $$\wp_{k|\hat{L}_k,b^i_k=C_1}(\hat{P}^i_k) \;=\; \wp_{k|\hat{L}_k,b^i_k=C_2}(\hat{P}^i_k).$$

   Hence, setting the bidding price equal to C_1 yields higher anticipated profits than setting

[13] Note that $R^{i,2}_{max}$ can be less than $P_{cap} \cdot q^i_k - c^i \cdot q^i_k$.

the bidding price equal to C_2 (as in the previous case). In addition, the probability of the unit being scheduled to operate when the bidding price is equal to $C1$ is larger than when the bidding price is equal to $C2$, i.e.,

$$\int_{b_k^i=C_1}^{\infty} \wp_{k|\hat{L}_k,b_k^i=C_1}(\hat{P}_k^i) \cdot d\hat{P}_k^i \;\; \geq \;\; \int_{b_k^i=C_2}^{\infty} \wp_{k|\hat{L}_k,b_k^i=C_2}(\hat{P}_k^i) \cdot d\hat{P}_k^i.$$

(b) That $\wp_{k|\hat{L}_k,b_k^i}(\hat{P}_k^i)$ does change, i.e.,

$$\wp_{k|\hat{L}_k,b_k^i=C_1}(\hat{P}_k^i) \;\neq\; \wp_{k|\hat{L}_k,b_k^{i,j}=C_2}(\hat{P}_k^i), \qquad \forall\; \hat{P}_k^i \neq \tilde{P}.$$

Using an argument similar to that in the previous case, it is not clear whether setting the bidding price equal to C_1 yields anticipated profits that are higher than, lower than, or equal to the anticipated profits from setting the bidding price equal to C_2.

Then, let us consider when the anticipated demand is close to the total available capacity[14](or the supply scarcity could occur), i.e., $L_k \rightarrow q_{max,k}$, so that by varying the b_k^i of the agent from C_1 to C_2, $\wp_{k|\hat{L}_k,b_k^i}(\hat{P}_k^i)$ does change. Let us consider also when

$$\int_{b_k^i=C_1}^{\infty} \wp_{k|\hat{L}_k,C_1}(\hat{P}_k^i) \cdot R_{k|C_1}^i \cdot d\hat{P}_k^i \;-\; \int_{b_k^i=C_2}^{\infty} \wp_{k|\hat{L}_k,C_2}(\hat{P}_k^i) \cdot R_{k|C_2}^i \cdot d\hat{P}_k^i$$
$$\leq \int_{C_1}^{\tilde{P}} (\wp_{k|\hat{L}_k,C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k,C_2}(\hat{P}_k^i))\, d\hat{P}_k^i \cdot R_{max}^{i,1} + \int_{\tilde{P}}^{\infty} (\wp_{k|\hat{L}_k,C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k,C_2}(\hat{P}_k^i))\, d\hat{P}_k^i \cdot R_{max}^{i,2}.$$

When $R_{max}^{i,2} \gg R_{max}^{i,1}$, as well as when $A1 = \int_{C_1}^{\tilde{P}} (\wp_{k|\hat{L}_k,C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k,C_2}(\hat{P}_k^i))d\hat{P}_k^i < 0$ and $A2 = \int_{\tilde{P}}^{\infty} (\wp_{k|\hat{L}_k,C_1}(\hat{P}_k^i) - \wp_{k|\hat{L}_k,C_2}(\hat{P}_k^i))d\hat{P}_k^i > 0$, the bidding price b_k^i equal to $C2$ yields a higher expected profit than the bidding price equal to $C1$, i.e.,

$$\int_{b_k^i=C_1}^{\infty} \wp_{k|\hat{L}_k,b_k^i=C_1}(\hat{P}_k^i) \cdot R_k^i \cdot d\hat{P}_k^i \;\; \ll \;\; \int_{b_k^i=C_2}^{\infty} \wp_{k|\hat{L}_k,b_k^i=C_2}(\hat{P}_k^i) \cdot R_k^i \cdot d\hat{P}_k^i.$$

Consequently, given that Δ is a constant and $\Delta > 0$, the bidding price for an anticipated extra-marginal unit should be set to

$$b_k^{i,j} = mc^i + \Delta.$$

[14]Such as on extremely hot or cold days

DP Structure: Let us consider the markets with the DP structure when the agent determines its bidding price such that its expected profit is maximized as follows:

$$
\begin{aligned}
b_k^{i*} &= \arg\max_{b_k^{i,j}} \int_{b_k^i}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot (b_k^i \cdot q_k^i - c(q_k^i)) \cdot d\hat{P}_k^i \\
\text{s.t.} \quad & q_k^i > 0, \quad \text{if} \quad b_k^i \leq \hat{P}_k^i \\
& q_k^i = 0, \quad \text{if} \quad b_k^i > \hat{P}_k^i.
\end{aligned}
$$

Let us consider the following observations.

1. When $b_k^i \leq \hat{P}_{min}(L_k)$, the unit is scheduled to operate as a marginal unit or an infra-marginal unit in which the scheduled capacity is equal to q_k^i. Let us consider when

   (a) The bidding price does not affect market price. In this case, the bidding quantity q_k^i and profit $(b_k^i \cdot q_k^i - c(q_k^i))$ are not a function of prices. Since $c(q_k^i)$ is a constant, $c^i(q_k^i) = c^i \cdot q^i$,

   $$b_k^{i*} = \arg\max_{b_k^i} \int_{b_k^i}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i \cdot (b_k^i \cdot q_k^i - c^i \cdot q_k^i),$$

   and

   $$0 < \int_{b_k^i}^{\infty} \wp_{k|\hat{L}_k}(\hat{P}_k^i) \cdot d\hat{P}_k^i \leq 1, \quad \forall \; b_k^i.$$

   Also

   $$b_k^i \cdot q_k^i - c^i \cdot q_k^i > 0, \quad \forall \, b_k^i > c^i;$$

   hence,

   $$b_k^{i,1} \cdot q_k^i - c^i \cdot q_k^i > b_k^{i,2} \cdot q_k^i - c^i \cdot q_k^i, \quad \forall \, \hat{P}_{min}(L_k) \geq b_k^{i,1} > b_k^{i,2} \geq 0.$$

   Therefore, in order to maximize anticipated profits, Agent i should submit the highest bidding price for an anticipated infra-marginal unit, the bidding price is set to

   $$b_k^i = \max \{ \hat{P}_k^i - \epsilon, \, mc^i \} = \hat{P}_k^i - \epsilon.$$

   Similarly, for an anticipated marginal unit, the bidding price is set to

   $$b_k^i = \max \{ \hat{P}_k^i - \epsilon, \, mc^i \}.$$

   (b) The bidding price affects market price. Just as in the UP case, the agent should select a bidding price that yields the highest anticipated profit.

2. The bidding price of an anticipated extra-marginal unit is determined as follows. First, when

$L_k \ll q_{max,k}$, the agents submit bidding prices such that

(a) The bidding price does not affect the market price. The bidding price of an anticipated extra-marginal unit is equal to

$$b_k^{i,j} = \max \{ \hat{P}_k^i,\ mc^{i,j} \}.$$

(b) The bidding price affects the market price. The agent should select a bidding price that yields the highest anticipated profit.

On the other hand, when the anticipated demand is close to the total available capacity ($L_k \rightarrow q_{max,k}$), the agents are very likely to influence the market prices. Utilizing an argument similar to that when $L_k \rightarrow q_{max,k}$ in the markets with the UP structure, given $\Delta > 0$, the agents should submit the highest possible bidding price such that

$$b_k^{i,j} = mc^{i,j} + \Delta.$$

II. Determining a Bid-supply Function

The previous analysis can be extended to cover a scenario in which the agents own at least one unit. For any deterministic demand, let a marginal unit be m with the marginal cost equal to the marginal-cost price and denoted by $\hat{b}_k^{i,m} = \hat{P}_{min}^i(L_k)$. Should Agent i set the bidding prices of all of its anticipated infra-marginal units to $b_k^{i,j} = \hat{P}_k^i - \epsilon$, for $\epsilon > 0$? Let us assume that Agent i has N^i generating units which have $mc^{i,1} \leq mc^{i,2} \leq \ldots \leq mc^{i,j} \leq \ldots \leq mc^{i,N^i}$. Suppose that units 1 to j are anticipated infra-marginal units, i.e., $\hat{P}_{min}^i > mc^{i,j} \geq \ldots \geq mc^{i,2} \geq mc^{i,1}$, such that the bidding price $b_k^{i,j} < \hat{P}_{min}(L_k)$ does not affect the anticipated market price given demand L_k, and that Agent i sets the bidding prices such that $b_k^{i,1} = \ldots = b_k^{i,j} = b_k^{i,m} < \ldots < b_k^{i,N^i}$. Hence, the probability of Agent i's units being scheduled to operate are equal to

$$\int_{b_k^{i,1}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,1}}(\hat{P}_k^i) \cdot d\hat{P}_k^i \quad = \quad \ldots \quad = \quad \int_{b_k^{i,m}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,j}}(\hat{P}_k^i) \cdot d\hat{P}_k^i.$$

On the other hand, if Agent i submits a bidding price such that none of its anticipated infra-marginal units has the same bidding price as the anticipated marginal unit, i.e.,

$$b_k^{i,1} \leq \ldots \leq b_k^{i,j} < b_k^{i,m} < \ldots < b_k^{i,N^i},$$

the probability of each unit j, with the lower bidding price, being scheduled to operate will be higher than if the unit has a higher bidding price, i.e.,

$$\int_{b_k^{i,1}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,1}}(\hat{P}_k^i) \cdot d\hat{P}_k^i \;=\; \int_{b_k^{i,j}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,j}}(\hat{P}_k^i) \cdot d\hat{P}_k^i \;>\; \int_{b_k^{i,m}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,m}}(\hat{P}_k^i) \cdot d\hat{P}_k^i.$$

In addition, when demand uncertainty is taken into consideration, there is a possibility that one unit can be scheduled to operate either as an anticipated marginal unit or a marginal unit (recall Chapter 2). To set the bidding price of the units with the less expensive marginal costs to be lower than the bidding price of the units with the more expensive marginal costs increases the probability of the less expensive units being scheduled to operate, i.e.,

$$\int_{b_k^{i,1}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,1}}(\hat{P}_k^i) \cdot d\hat{P}_k^i \;>\; \int_{b_k^{i,j}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,j}}(\hat{P}_k^i) \cdot d\hat{P}_k^i \;>\; \int_{b_k^{i,m}}^{\infty} \wp_{k|\hat{L}_k, b_k^{i,m}}(\hat{P}_k^i) \cdot d\hat{P}_k^i.$$

Actual demand could be lower than the forecast demand, and as a result the anticipated infra-marginal units may be scheduled to operate as marginal units. To submit the bidding prices of the anticipated infra-marginal units in their marginal-cost order, the probability of the lower cost units being scheduled to operate will be higher than that of the higher cost units. This scheduled outcome implies the efficiency of utilizing the units.

B. A Method to Estimate Market Price

Let us consider the markets with the DP structure. Suppose that the agents in the DP structure are not informed of the market price (the maximum bidding price of the scheduled units). Since the agents require the market prices in some learning schemes, the agents have to estimate the market price of each auction round based on the available information obtained from the current and the past auctions; however, as mentioned previously, to have a well-defined stochastic model representing the market prices is not reasonable. The agents face the problem of information asymmetry, because they have non-uniform portfolio characteristics. Over-estimated market prices could lead to a divergence of market prices. The following analysis shows that the difference between the prices in markets with the DP and UP structures can be caused by the characteristics of the system marginal-cost function and the market participants' portfolio characteristics.

Suppose that there are M agents owning N generating units where $1 < M < N$, meaning that each of the agents owns at least one unit. Each unit has uniform capacity q and no single unit has the same marginal cost, i.e.,

$$0 < mc^1 < \ldots < mc^n < \ldots < mc^N.$$

Let us define $\Delta^n = mc^n - mc^{n-1} > 0$. Let P be an actual market price, $\hat{P}^i$ be an estimated price by agent i, and $b^{i,j}$ be the bidding price of unit j of Agent i. Suppose that the agents submit a marginal-cost bid for each of their units, i.e., $b^{i,j} = mc^{i,j}$ with a bidding quantity equal to q. The units with marginal cost of at most P are scheduled to operate capacity $x^{i,j}$, where $0 \leq x^{i,j} \leq q$. To estimate the market prices, Agent i considers whether it is scheduled to operate as one of the following:

1. An infra-marginal agent: An agent is an infra-marginal agent when none of its units has a scheduled quantity equal to zero, or all of its units have their schedule quantities equal to their bidding quantities, i.e., $x^{i,j} = q$. On the other hand, if this agent has a unit (unit g) such that its scheduled quantity is equal to zero ($x^{i,j} = 0$), then one of the agent's units may operate at the margin. Consequently, the agent may anticipate the market price to be at most equal to the marginal cost of unit g ($P \leq mc^{i,g}$). Likewise, if this agent has unit g, in which its scheduled quantity is equal to zero ($x^{i,g} = 0$), then none of the agent's units may operate at the margin. Consequently, the agent may anticipate the market price to be less than the marginal cost of unit g ($P < mc^{i,g}$). Similarly, when the agent has unit h as the most expensive unit that is scheduled and unit g as the least expensive unit that is not scheduled, the agent knows only the interval of the market prices, i.e., $mc^{i,h} < P < mc^{i,g}$.

2. A marginal agent: An agent is a marginal agent when one of its scheduled units (unit j) has a scheduled quantity less than its bidding quantity, i.e., $0 < x^{i,j} < q$. Therefore, the agent knows the market price, i.e., $P = mc^{i,j}$.

3. An extra-marginal agent: An agent is an extra-marginal agent when none of its units are scheduled, i.e., $x^{i,j} = 0, \forall\, j$. Therefore, this agent knows only that $P < mc^{i,q}$, where unit q has the least marginal cost in Agent i's portfolio.

Since demand also determines the number of units to be scheduled, demand also dictates whether the agents know the market price in each auction. At any demand level, the agent may be operating at the margin and know the market price, while, at the other demand level, the agent may not be scheduled and will not know the market price exactly.

When any Agent i requires a market price in the learning algorithm, such as the model-based algorithm, let it use a price estimation method, which is called the *ANTPRICE* scheme, as follows:

- **Case I:** Agent i is an infra-marginal agent that has at least one unit scheduled to operate as an infra-marginal unit and at least one unit not scheduled. The estimated market price is set to an average of the most expensive bidding price of the scheduled units and the lowest bidding price of the non-scheduled units, i.e., $\hat{P}^i = \frac{A^i + B^i}{2}$, where $A^i = \min_j (mc^{i,j} \cdot I(x^{i,j} = 0))$ and $B^i = \max_j (mc^{i,j} \cdot I(x^{i,j} = q))$.

- **Case II:** Agent i is a marginal agent. The estimated market price is set to the most expensive bidding price of the scheduled units, i.e., $\hat{P}^i = mc^{i,j}$. When one unit is scheduled to operate at the margin, the agent knows the market price exactly.

- **Case III:** Agent i has all units scheduled to operate as infra-marginal units. The estimated market price is set to the most expensive bidding price of the scheduled units plus a constant, i.e., $\hat{P}^i = \max_j \, mc^{i,j} + C$, where C is a non-negative constant.

- **Case IV:** Agent i has no unit scheduled to operate. The estimated market price is set to the lowest bidding price of the non-scheduled units minus a constant, i.e., $\hat{P}^i = \min_j \, mc^{i,j} - C$.

The agents can over-estimate or under-estimate the market price by using the *ANTPRICE* scheme. Let us consider the difference between the actual market price P and the estimated price of agent i, $\hat{P}^i$:

- For Case I, the difference between the estimated and the actual market prices is determined as follows:

$$\begin{aligned} |P - \hat{P}^i| &= |P - \frac{A^i + B^i}{2}| \\ &= |\frac{P - A^i}{2} + \frac{P - B^i}{2}| \\ &\leq \max\left(|\frac{P - A^i}{2}|, |\frac{P - B^i}{2}|\right). \end{aligned}$$

- For Case II, the difference between the estimated and the actual market prices is $|P - \hat{P}^i| = 0$.

- For Case III, the difference between the estimated and the actual market prices is $|P - \hat{P}^i| = C$.

- For Case IV, the difference between the estimated and the actual market prices is $|P - \hat{P}^i| = C$.

Hence,

$$0 \leq |P - \hat{P}^i| \leq \max\left(C, \max_i\left(\max\left(|\frac{P - A^i}{2}|, |\frac{P - B^i}{2}|\right)\right)\right).$$

The simulations show that when the agents use the model-based learning algorithm and this *ANTPRICE* scheme, a divergence of prices is possible. Although the deviation of this estimation is bounded,[15] an over-estimation of market prices by the agents, especially when Tar is set by Method M2, could lead to a divergence of market prices, which is the major drawback of modeling when agents use the same decision schemes to choose pure strategies.

C. Competitive Outcomes without an LSE agent

The simulated moving-average prices shown in Figure 6-40 obtain under the assumptions that the demand is equal to the competitive demand in Market-A as shown in Figure 6-19 and that there are

[15]The maximum anticipated price is equal to $\max_i \, max_j \, mc^{i,j} + C = mc^N + C$, and the minimum anticipated price is equal to $\min_i \, min_j \, mc^{i,j} - C = mc^1 - C$.

only the power-producing agents as active decision-makers using the model-based algorithm. Similarly, the moving-average prices shown in Figure 6-41 obtain under similar assumptions except that the demand is equal to the competitive demand in Market-B as shown in Figure 6-20. The agents use the model-based algorithm with Method M1 to determine the bidding price of the anticipated marginal units, Δ is set to 2, and the bidding price of the withheld capacity is set to \$150/MWh. This scenario is denoted by "M1 D2". Also, when the power-producing agents do not use the capacity withholding strategy as in Section 4.6.1, this scenario is denoted by "M1 D2 noW". In addition, the market price is not allowed to be higher than a price cap which is equal to \$150/MWh.

In Market-A, by comparing the "PWPW," "PWnoPnoW," and "PWPnoW" plots in Figure 6-23 to the "M1 D2" plot in Figure 6-40, one can observe that the moving-average prices as a result of the agents' strategic bid-supply functions ("M1 D2") are generally higher than the moving-average prices when these agents encounter the LSE agent. The similar outcomes are applied in Market-B by simply comparing the "PWPW," "PWnoPnoW," and "PWPnoW" plots in Figure 6-27 to "M1 D2" plot in Figure 6-41. Since demand is constant in each hour, the profits of the power-producing agents have a characteristic similar to that of the prices.

By comparing the "PnoWPW" plot in Figure 6-23 to the "M1 D2 noW" plot in Figure 6-40, and the "PnoWPW" plot in Figure 6-27 to the "M1 D2 noW" plot in Figure 6-41, when the power-producing agents do not use their capacity withholding strategy, the simulated moving average prices are also higher than the prices when there is an LSE agent in both Market-A and Market-B.

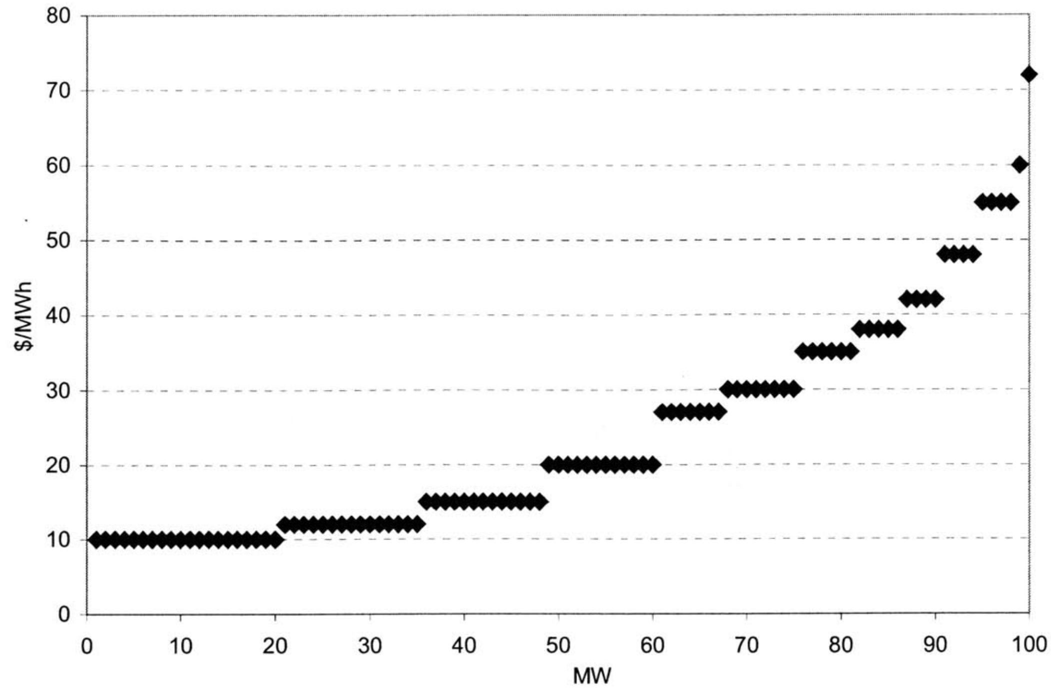

Figure 6-1: Aggregate Marginal-cost Function of the Hypothetical Market

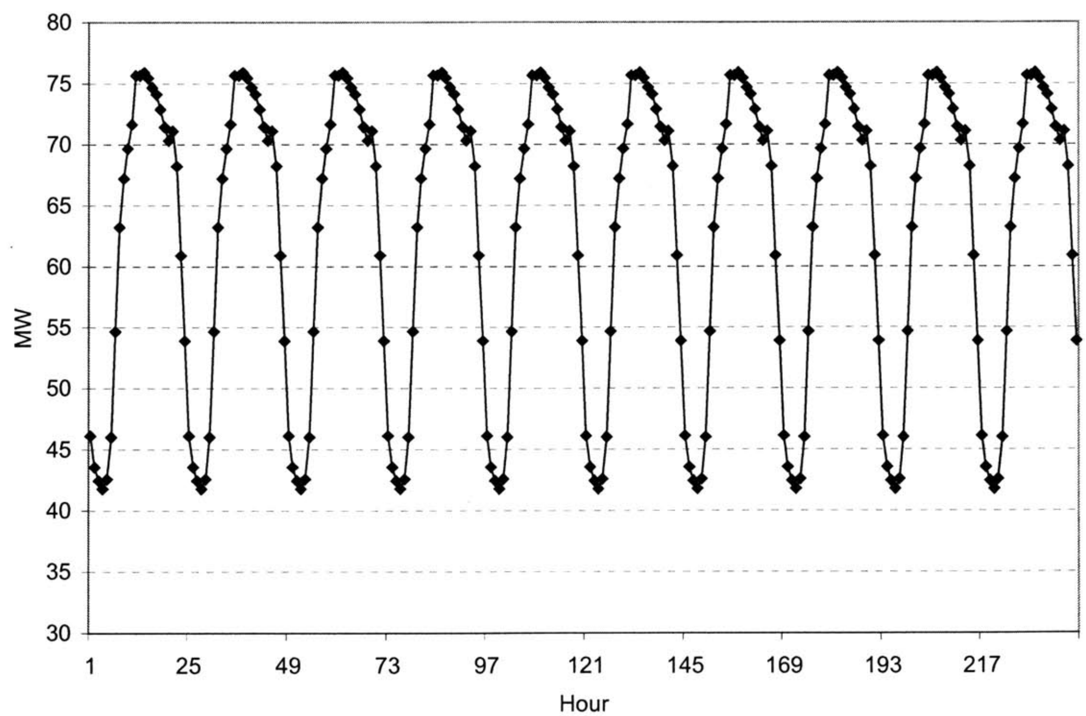

Figure 6-2: Daily Deterministic and Inelastic Demand Pattern

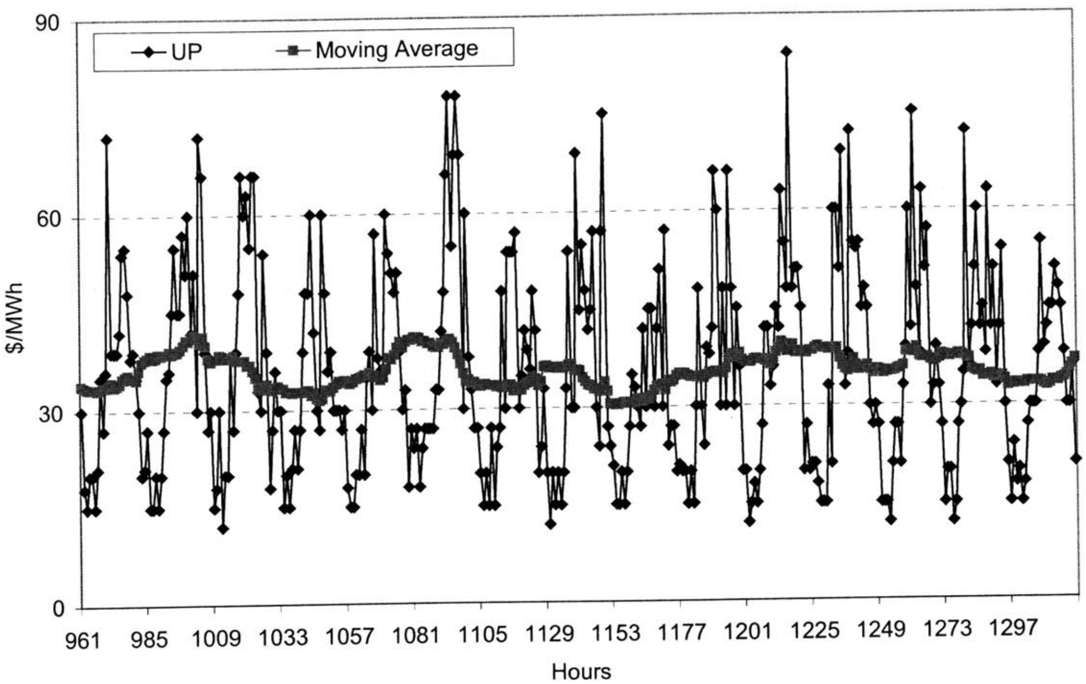

Figure 6-3: Price Dynamics and Their Moving-average from Hours 961 to 1,320 When the Agents Employ Algorithm A3 with $\delta = 0.9$ in the Market with a UP Structure

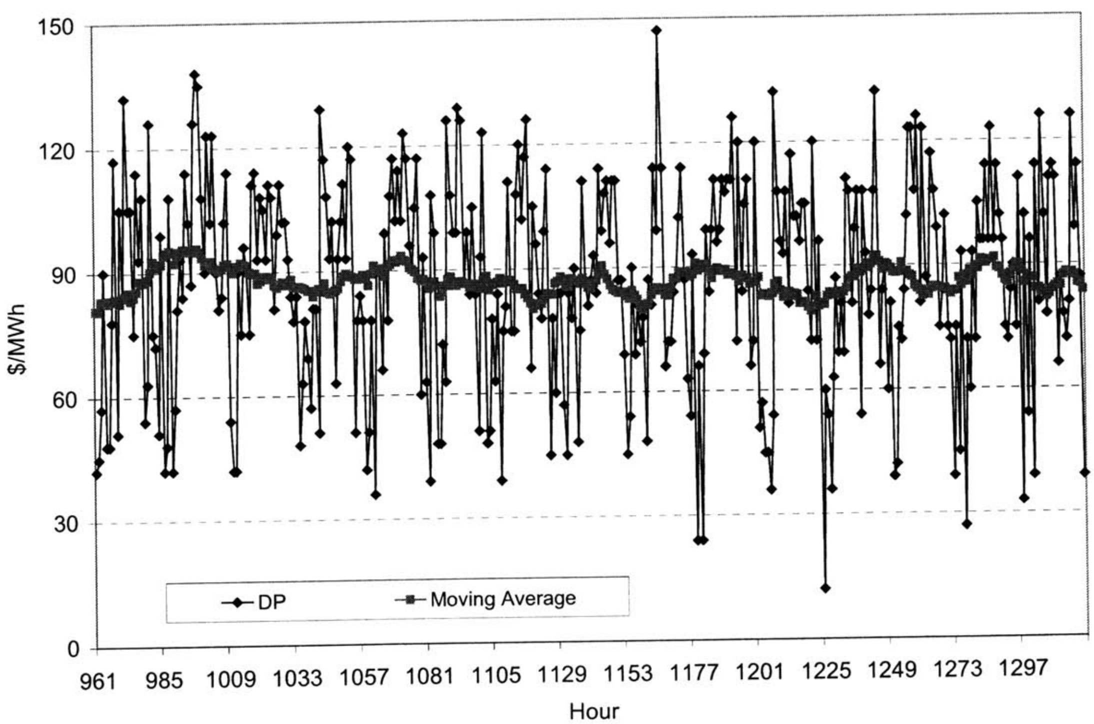

Figure 6-4: Price Dynamics and Their Moving -average from Hours 961 to 1,320 When the Agents Employ Algorithm A3 with $\delta = 0.9$ in the Market with a DP Structure

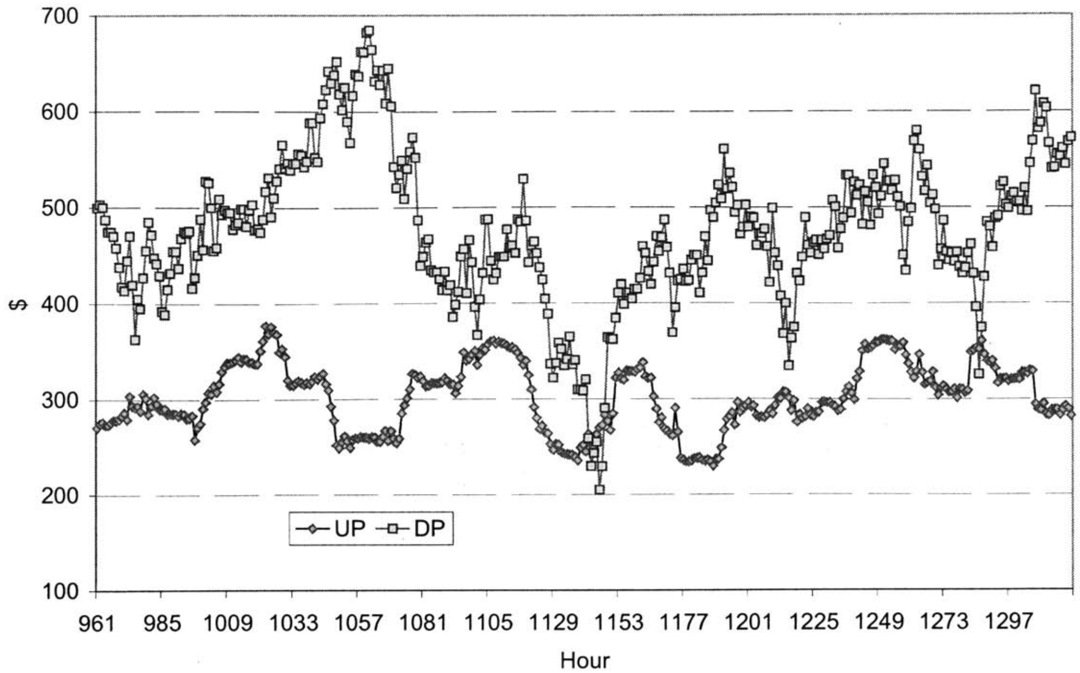

Figure 6-5: Moving-average Profits of Agent 5 from Hours 961 to 1,320 When the Agents Employ Algorithm A3 in the Markets with UP and DP Structures

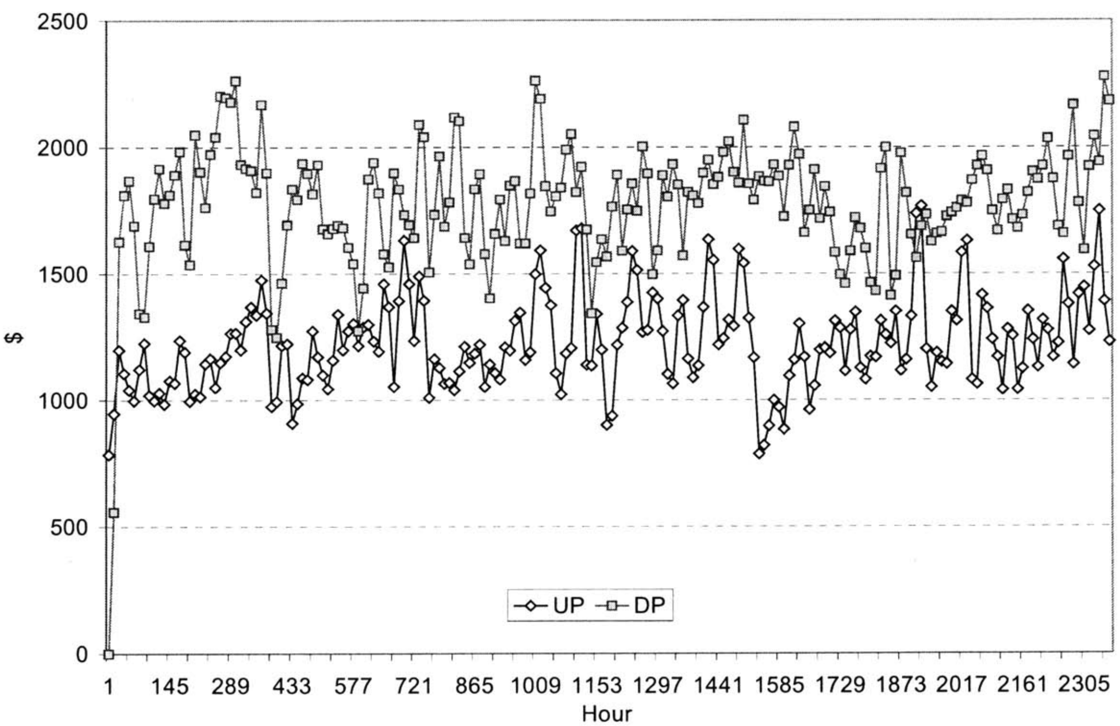

Figure 6-6: Moving-average Profits of all Agents When They Employ Algorithm A3 in the Markets with UP and DP Structures

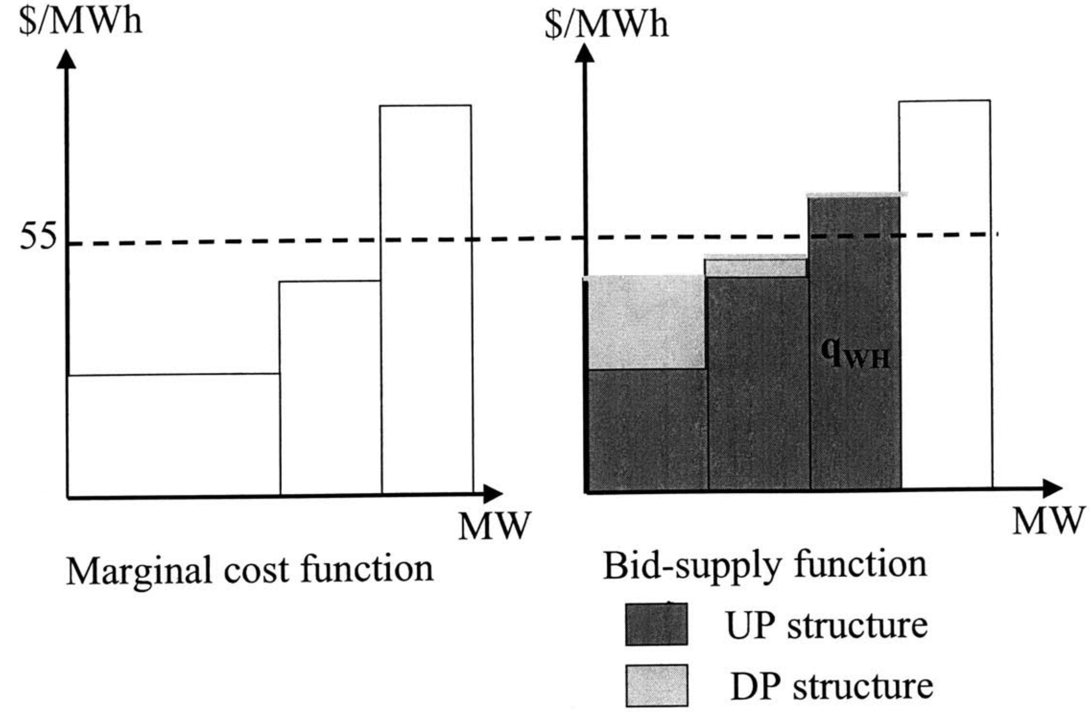

Figure 6-7: Examples of the Bid-supply Functions for the UP and DP Markets When the Agents Employ Algorithm A3 with $BM = 55$

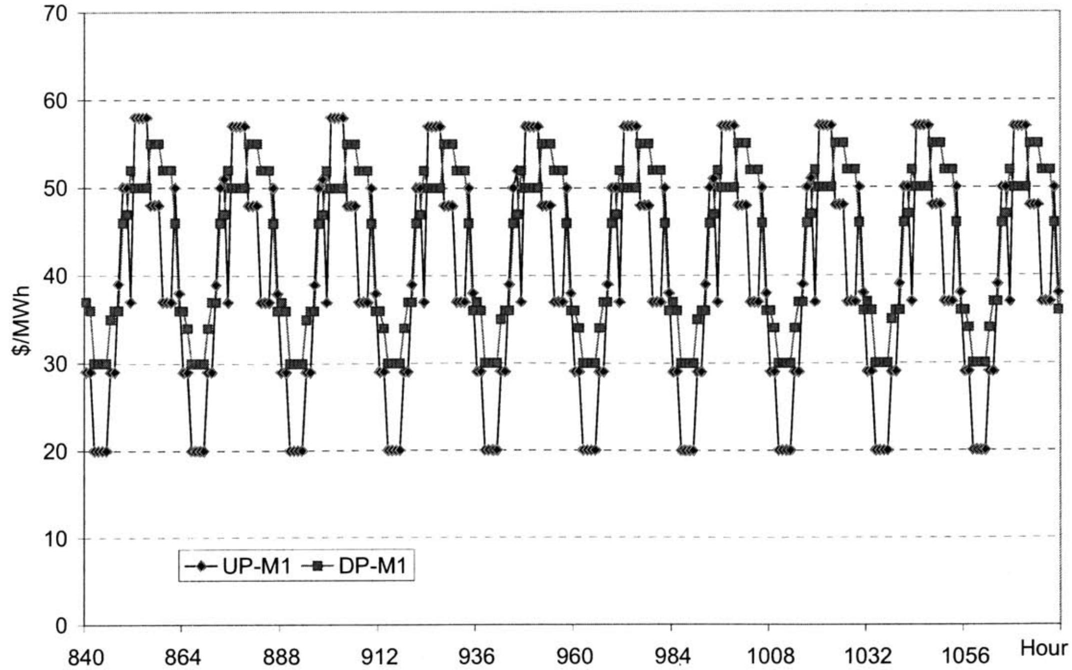

Figure 6-8: Price Dynamics from Hours 840 to 1,079 When the Agents Employ the Model-based Algorithm with Method M1 and Δ= 2 in the Markets with UP and DP Structures

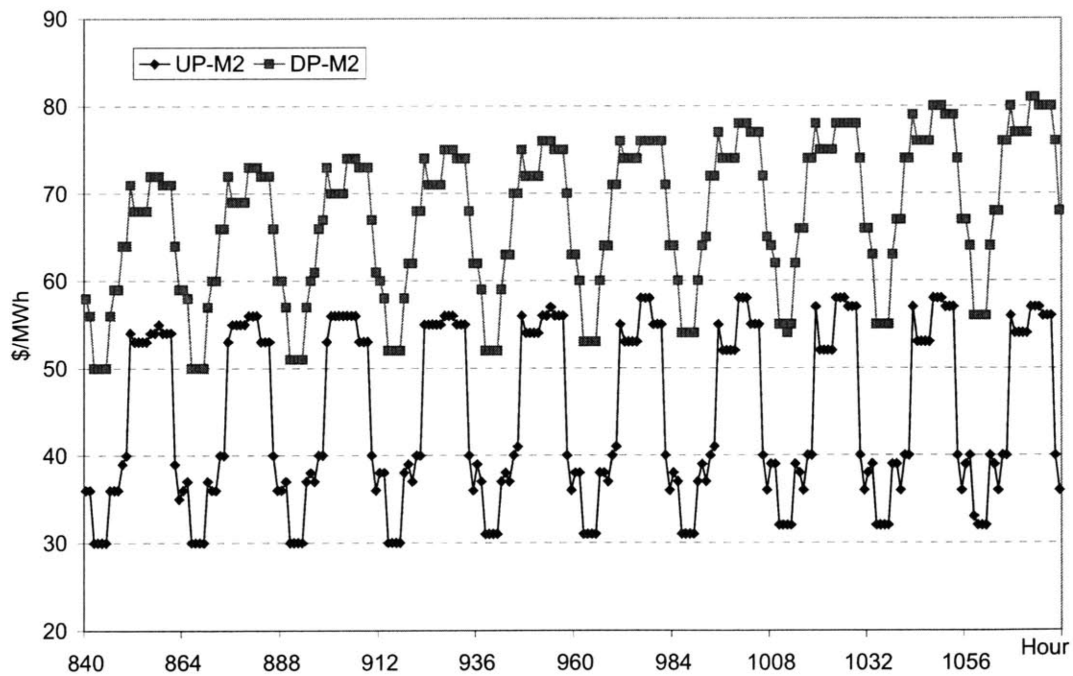

Figure 6-9: Price Dynamics from Hours 840 to 1,079 When the Agents Employ the Model-based Algorithm with Method M2 and Δ = 2 in the Markets with UP and DP Structures

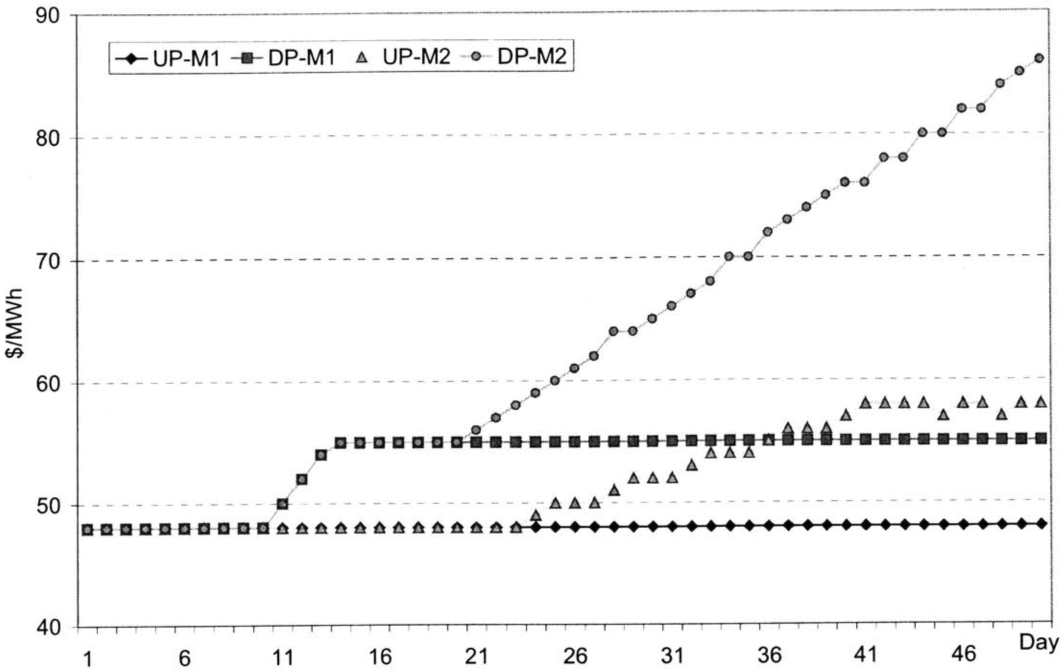

Figure 6-10: Price Dynamics at Hour 18 When the Agents Employ the Model-based Algorithm in the Markets with UP and DP Structures

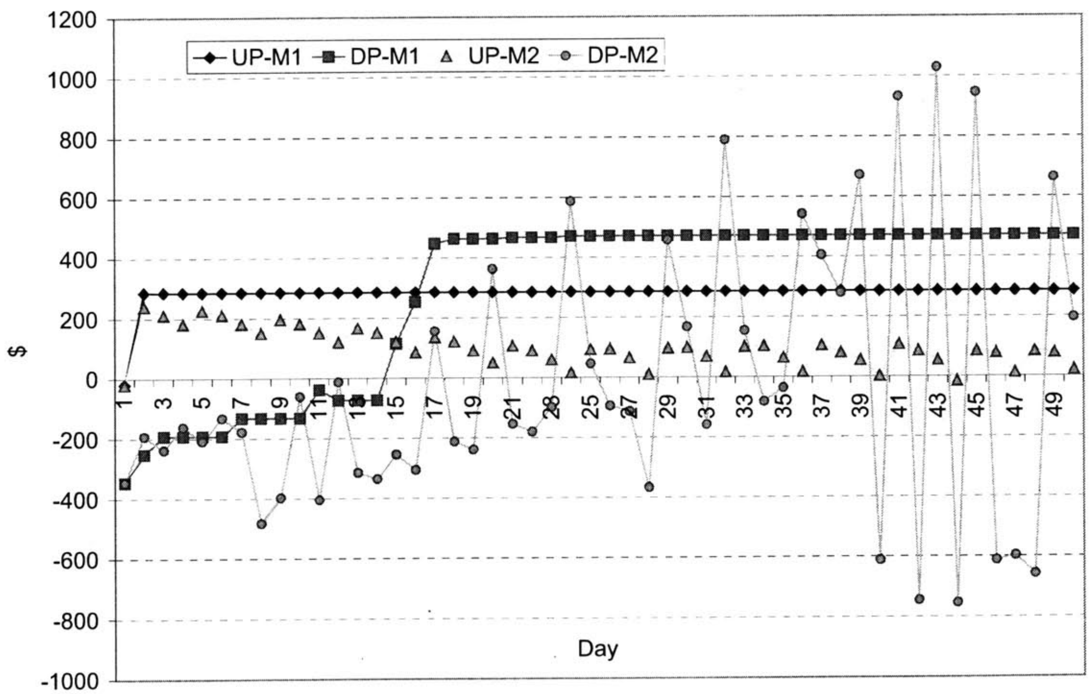

Figure 6-11: Dynamics of *(OP - AP)* of Agent 5 at Hour 18 When the Agents Employ the Model-based Algorithm in the Markets with UP and DP Structures

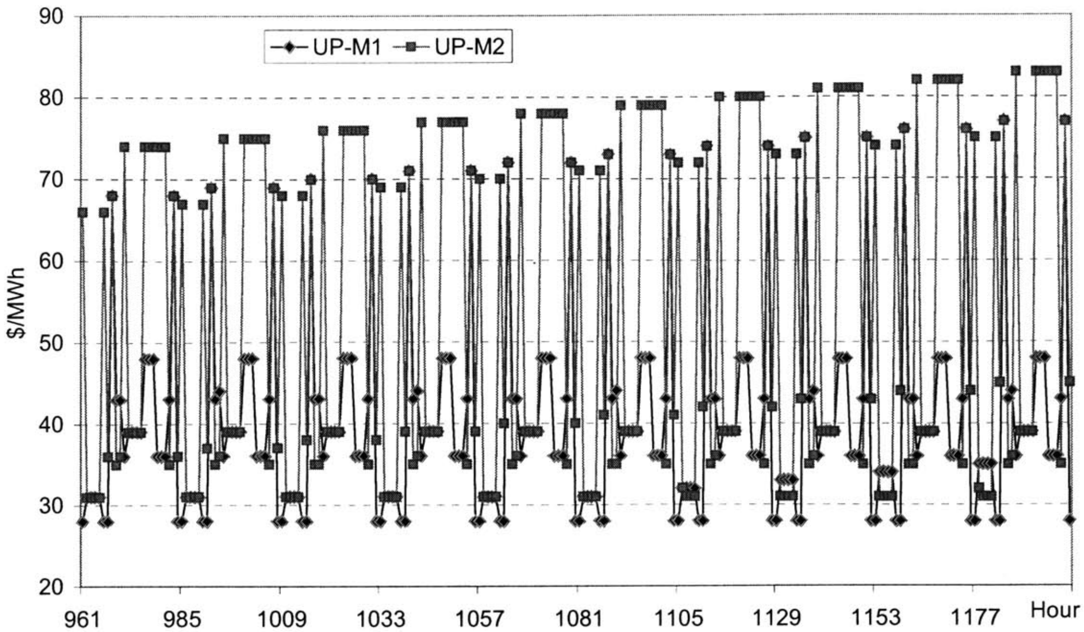

Figure 6-12: Price Dynamics from Hours 961 to 1,200 When the Agents Employ the Model-based Algorithm with Method M1 or M2 and $\Delta = 1$ in the Market with a UP Structure

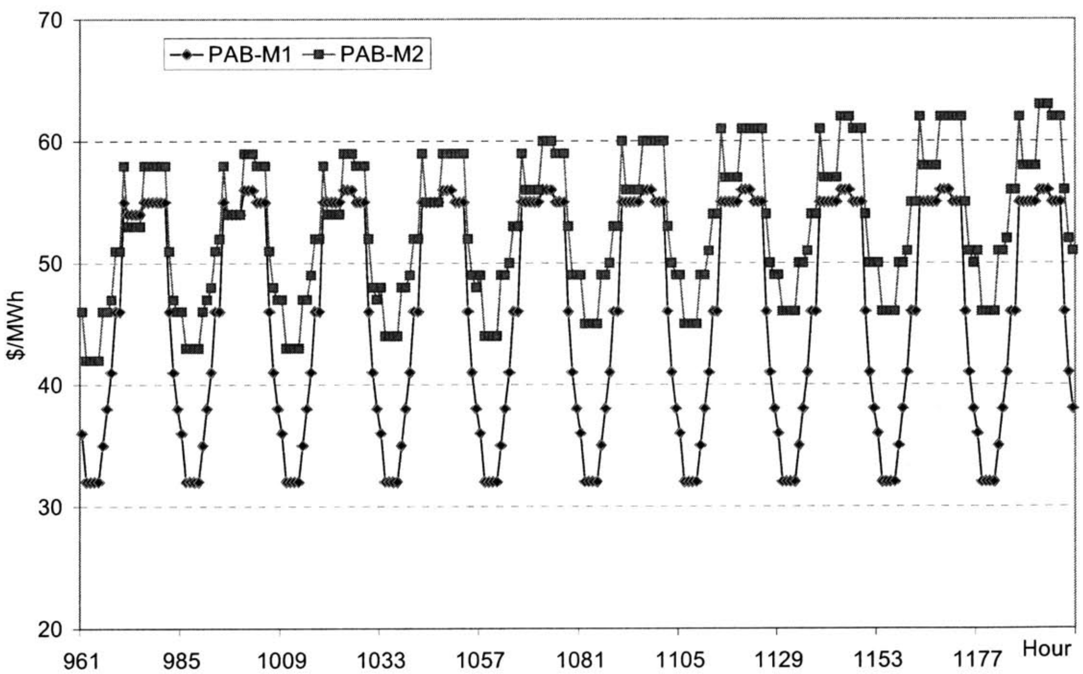

Figure 6-13: Price Dynamics from Hours 961 to 1,200 When the Agents Employ the Model-based Algorithm with Method M1 or M2 and $\Delta = 1$ in the Market a DP Structure

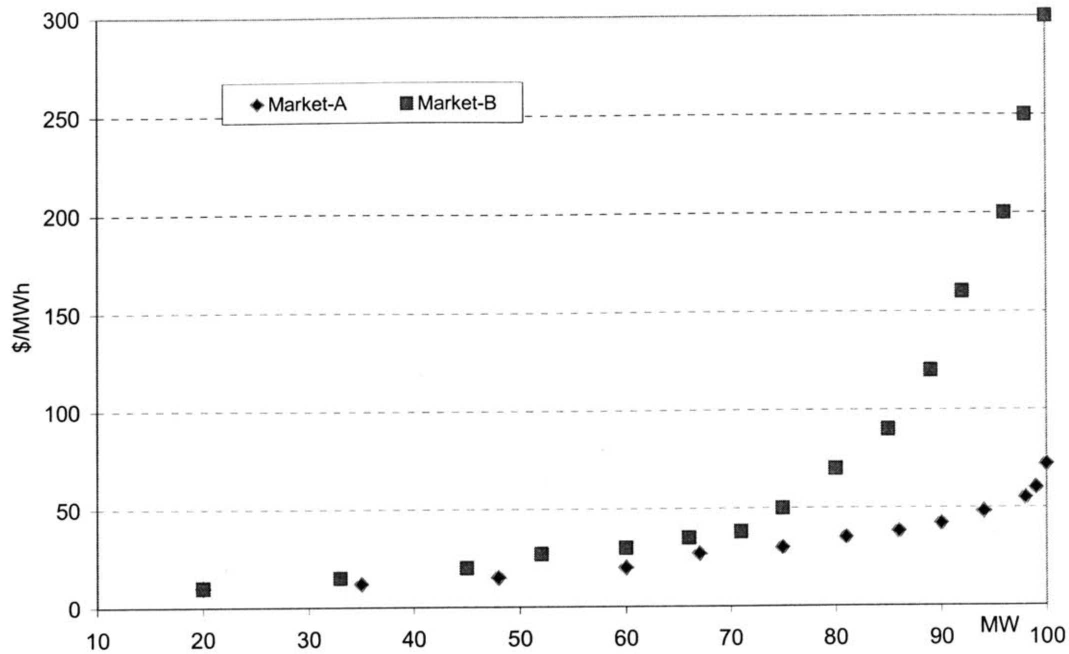

Figure 6-14: Aggregate Marginal-cost Characteristics of Market-A and Market-B

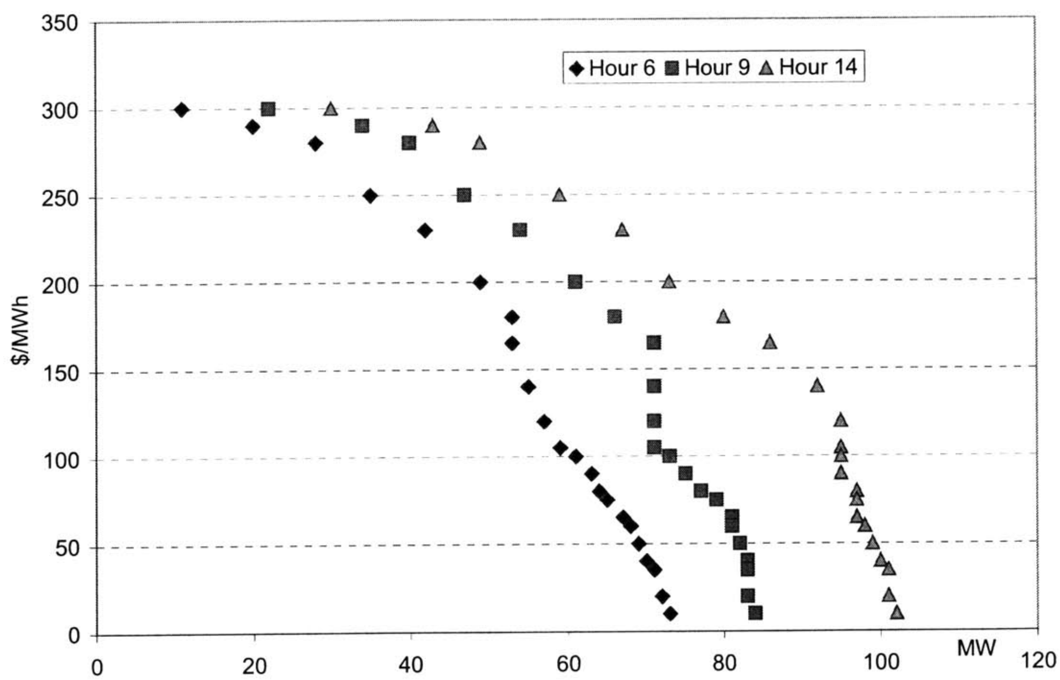

Figure 6-15: Samples of an LSE Agent's Marginal-utility Functions

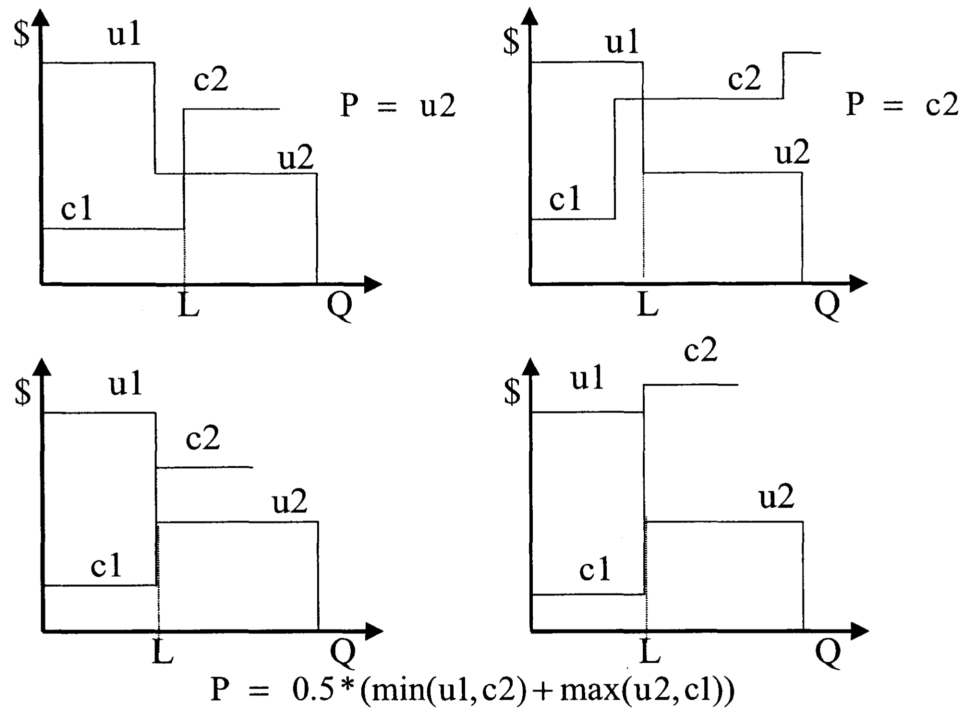

Figure 6-16: Price Setting Criteria in the Double-auction Agent-based Market Model

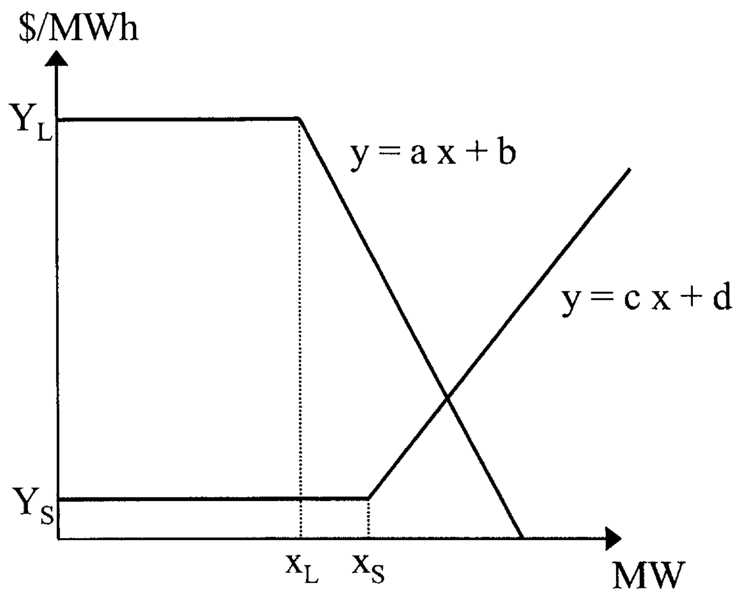

Figure 6-17: Examples of Piece-wise Marginal-cost and Marginal-utility Functions

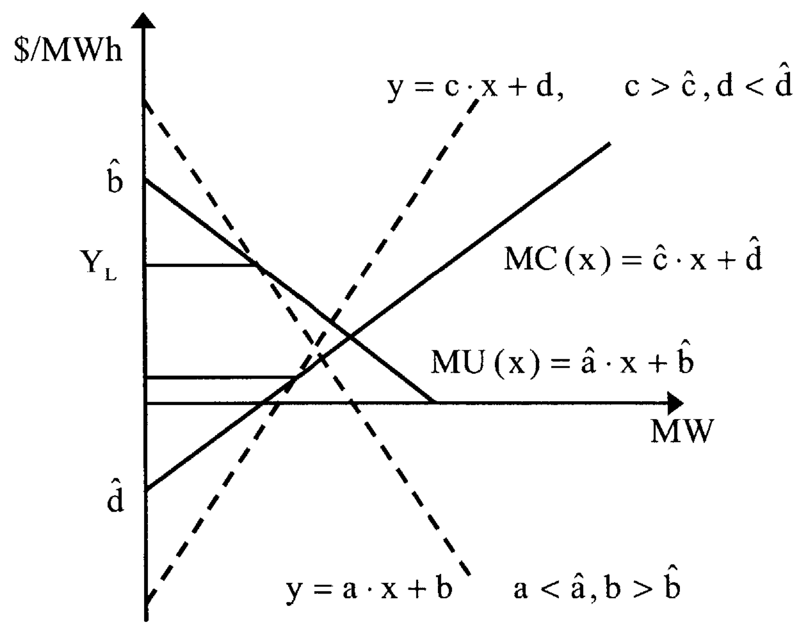

Figure 6-18: Examples of Piece-wise Bid-supply and Bid-demand Functions Relative to Marginal-cost and Marginal-utility Functions

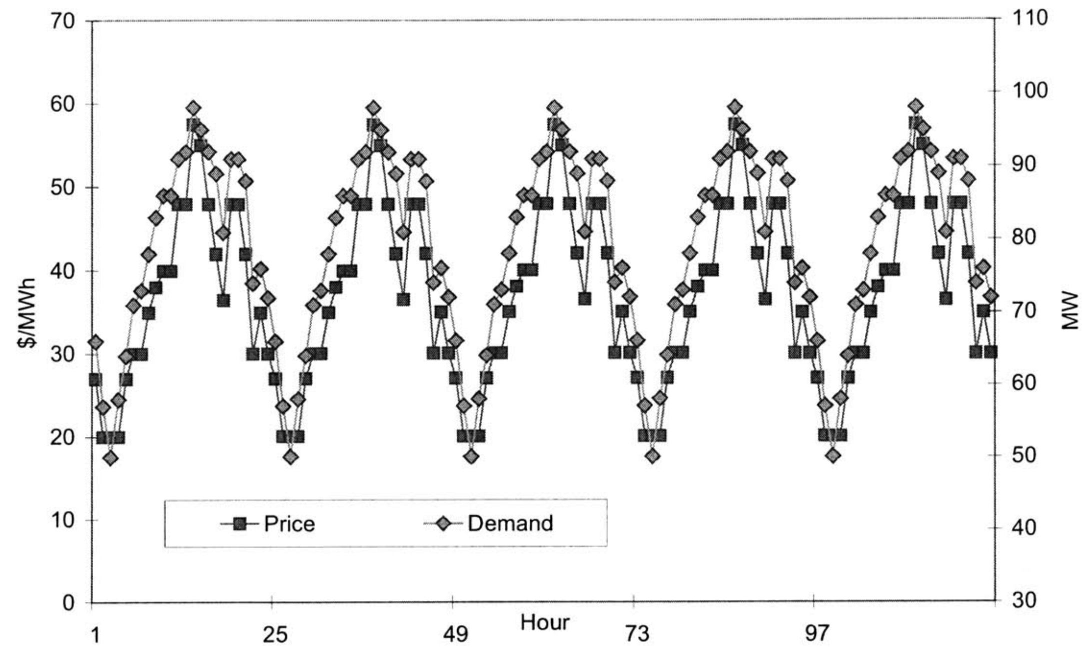

Figure 6-19: Daily Marginal-cost Prices and Demand in Market-A

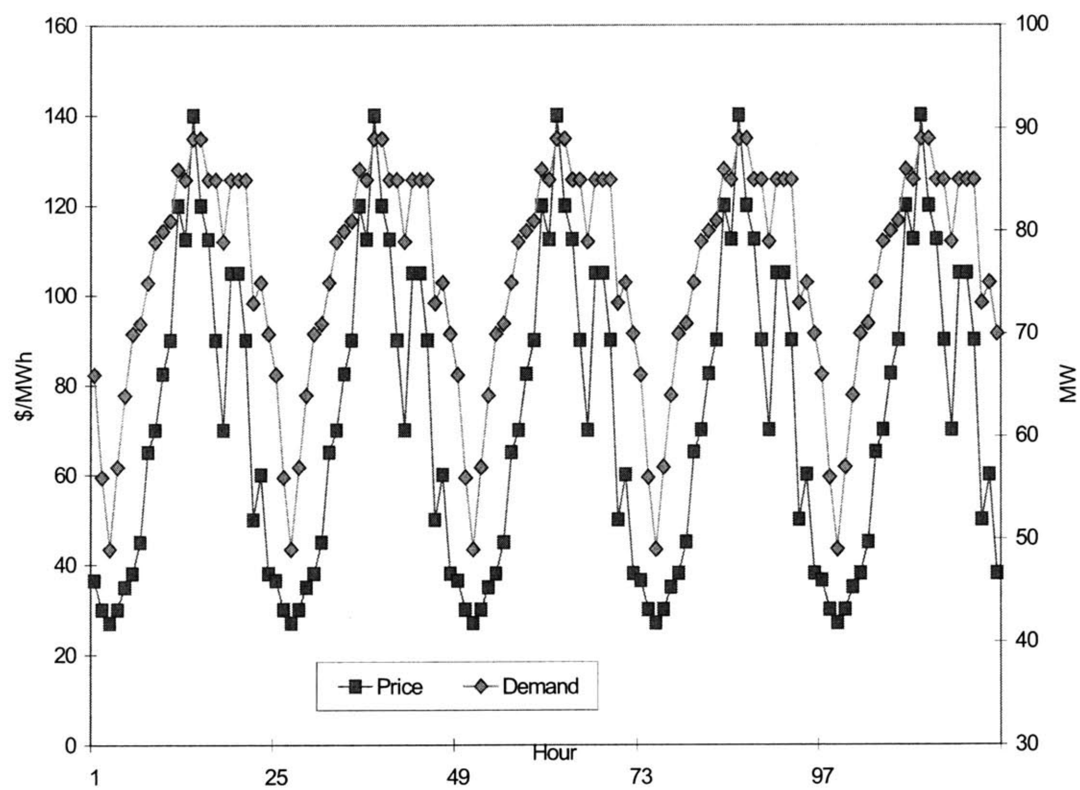

Figure 6-20: Daily Marginal-cost Prices and Demand in Market-B

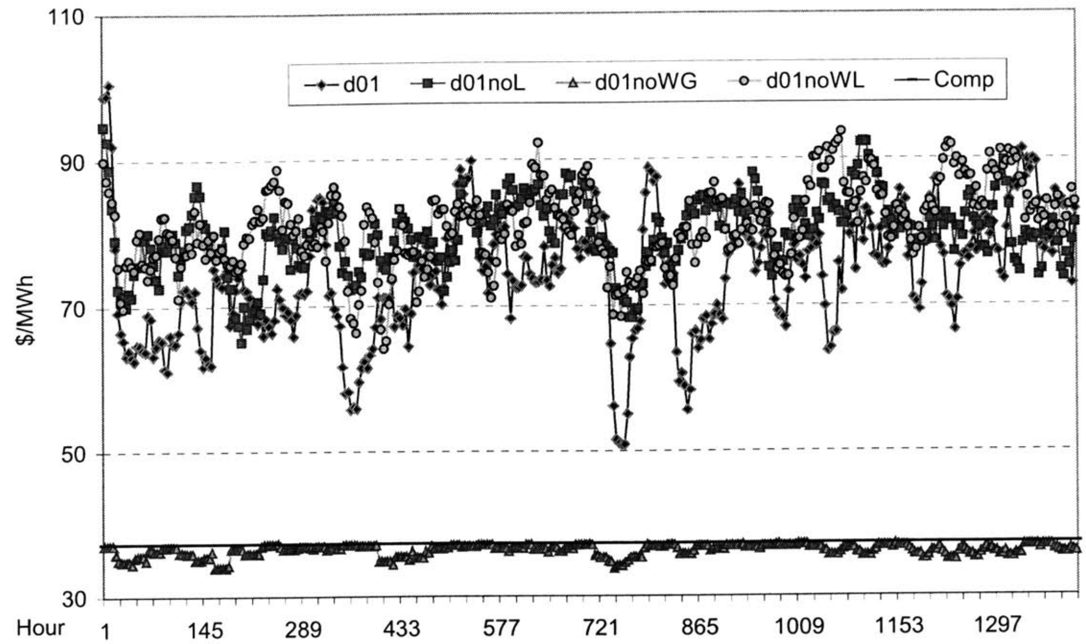

Figure 6-21: Moving-average Price Dynamics When the LSE Agent in Market-A Employs Algorithm A3L with $\delta = 0.1$

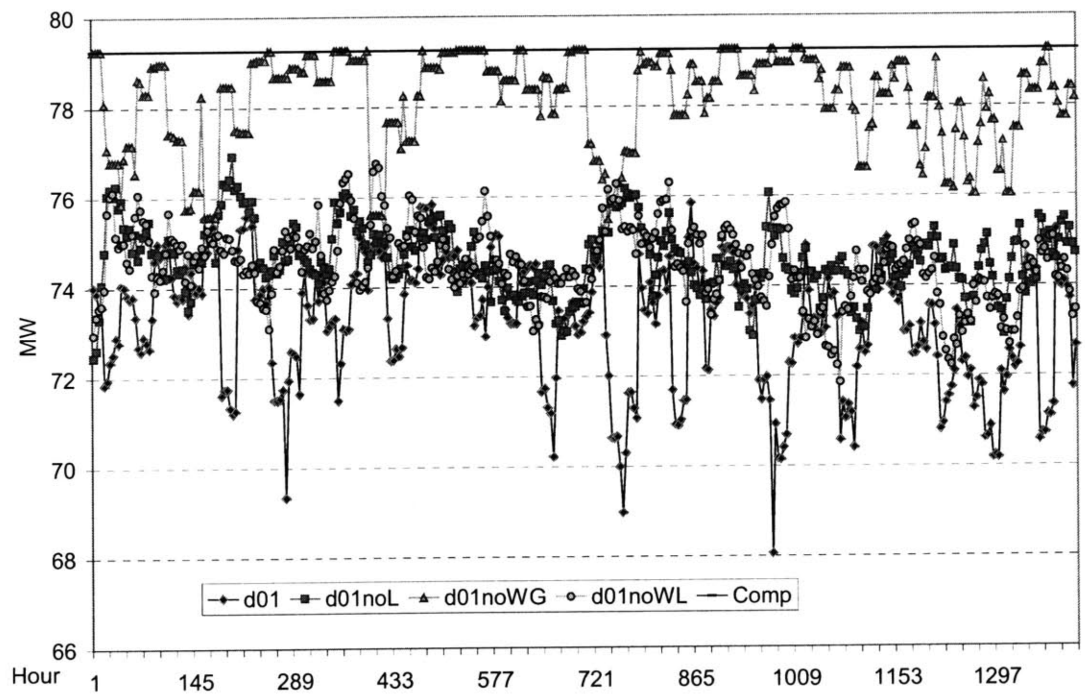

Figure 6-22: Moving-average Demand Dynamics When the LSE Agent in Market-A Employs Algorithm A3L with $\delta = 0.1$

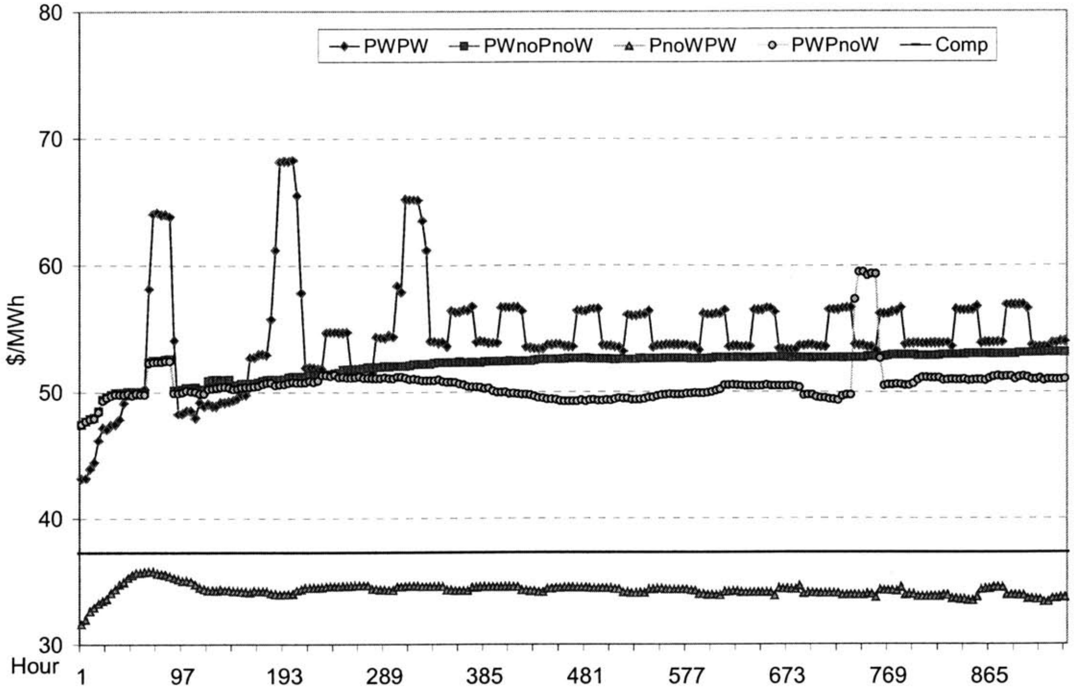

Figure 6-23: Moving-average Price Dynamics When the LSE Agent in Market-A Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

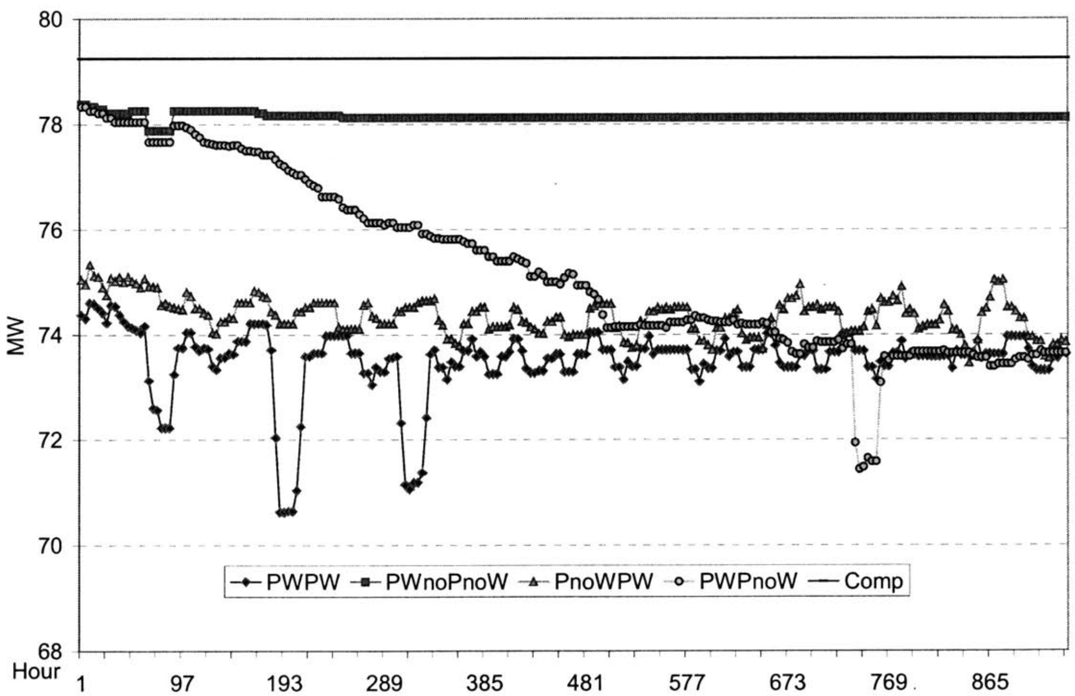

Figure 6-24: Moving-average Demand Dynamics When the LSE Agent in Market-A Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

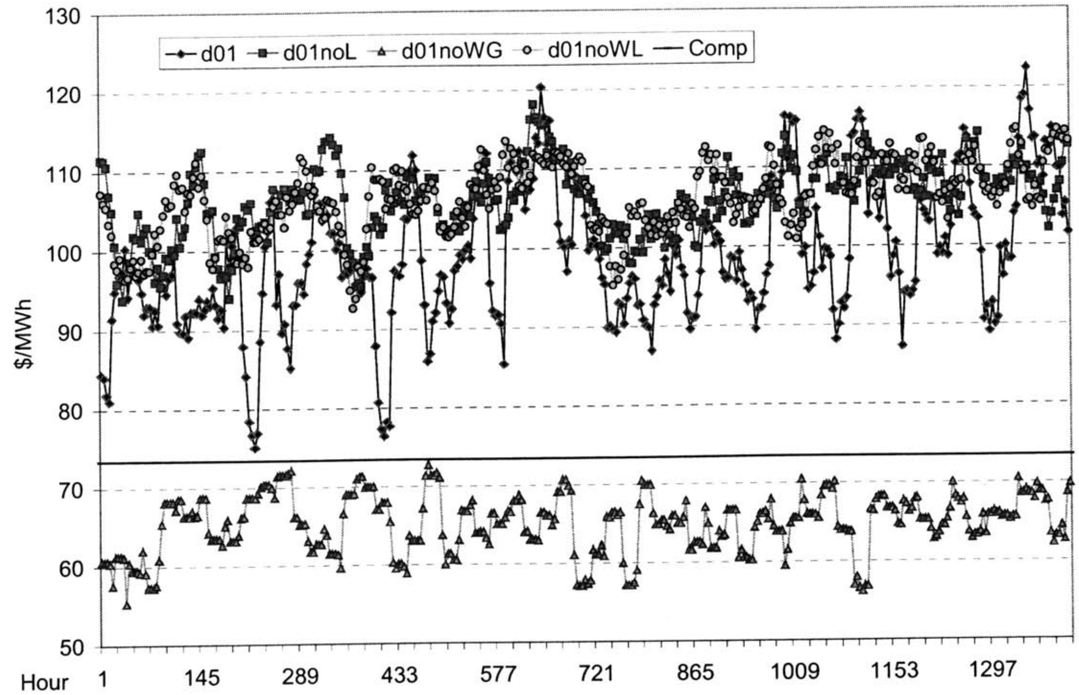

Figure 6-25: Moving-average Price Dynamics When the LSE Agent in Market-B Employs Algorithm A3L with $\delta = 0.1$

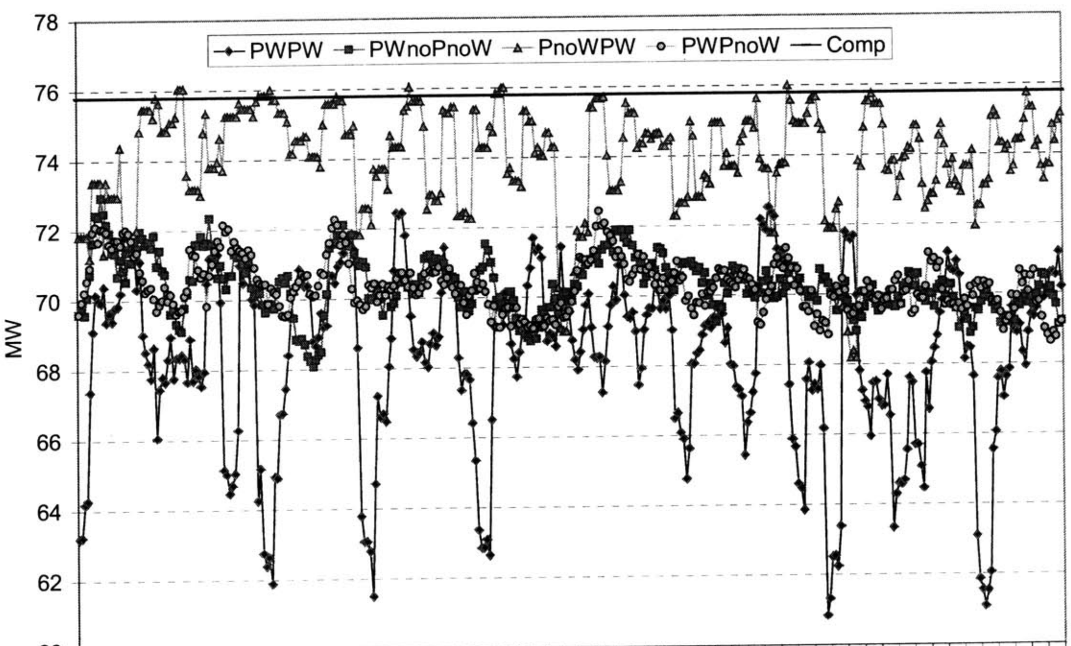

Figure 6-26: Moving-average Demand Dynamics When the LSE Agent in Market-B Employs Algorithm A3L with $\delta = 0.1$

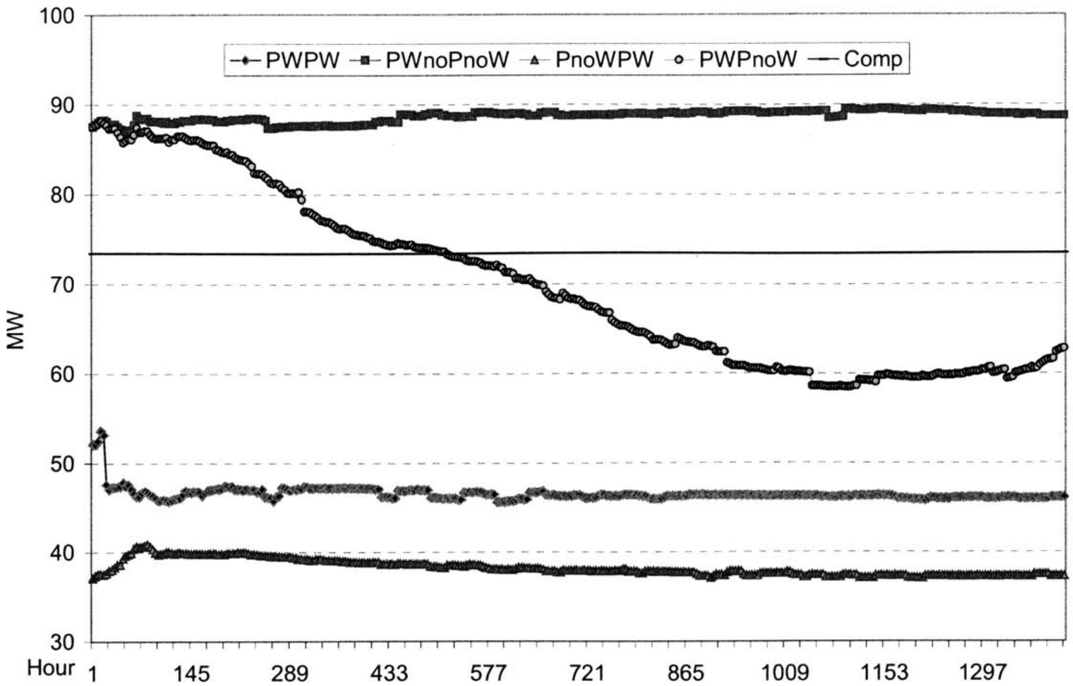

Figure 6-27: Moving-average Price Dynamics When the LSE Agent in Market-B Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

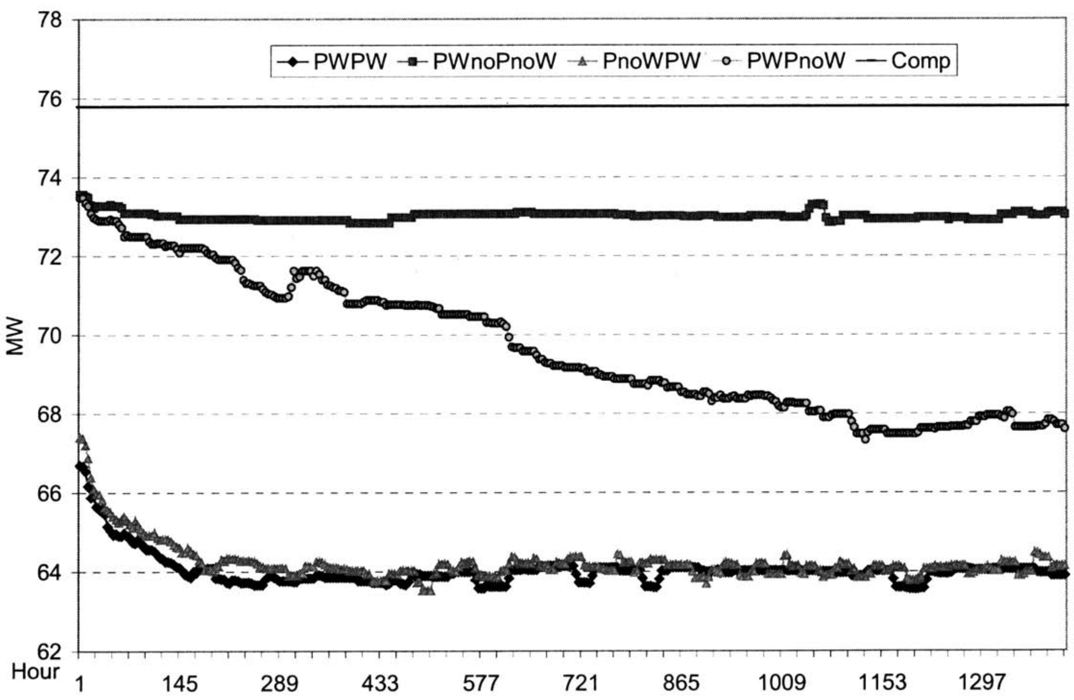

Figure 6-28: Moving-average Demand Dynamics When the LSE Agent in Market-B Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

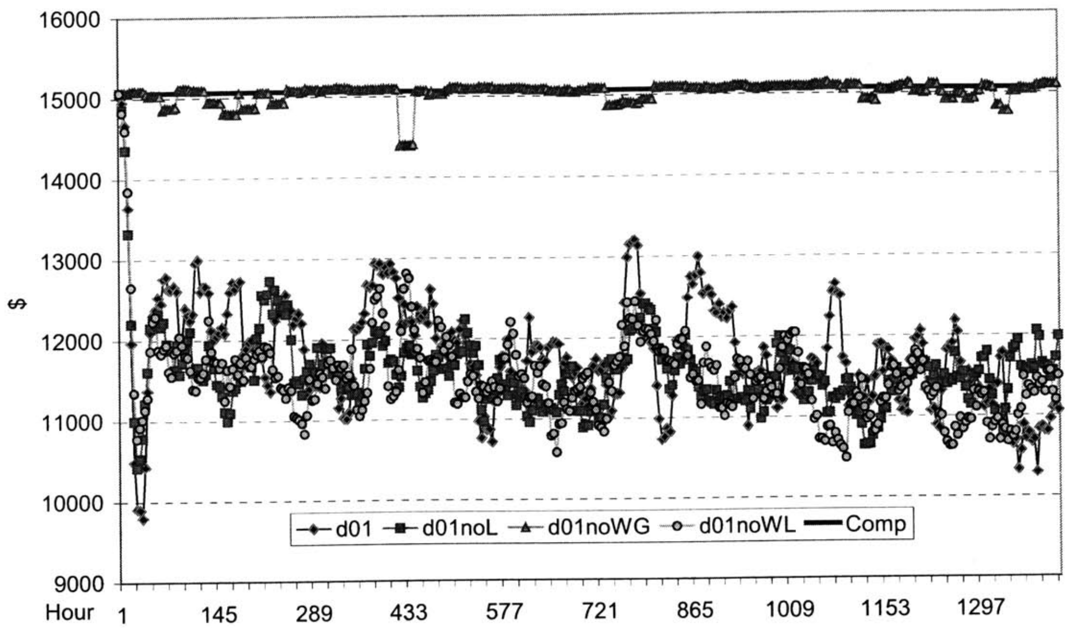

Figure 6-29: Moving-average Profit Dynamics That the LSE Agent Obtains in Market-A When It Employs Algorithm A3L with $\delta = 0.1$

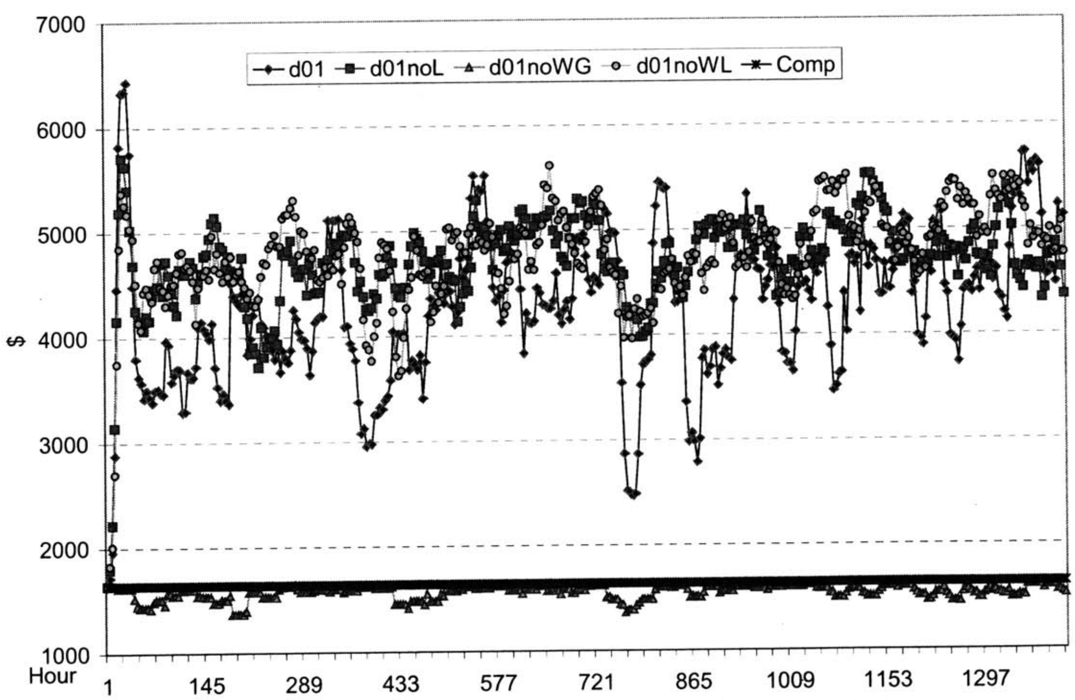

Figure 6-30: Moving-average Profit Dynamics That the Power-producing Agents Obtain in Market-A When the LSE Agent Employs Algorithm A3L with $\delta = 0.1$

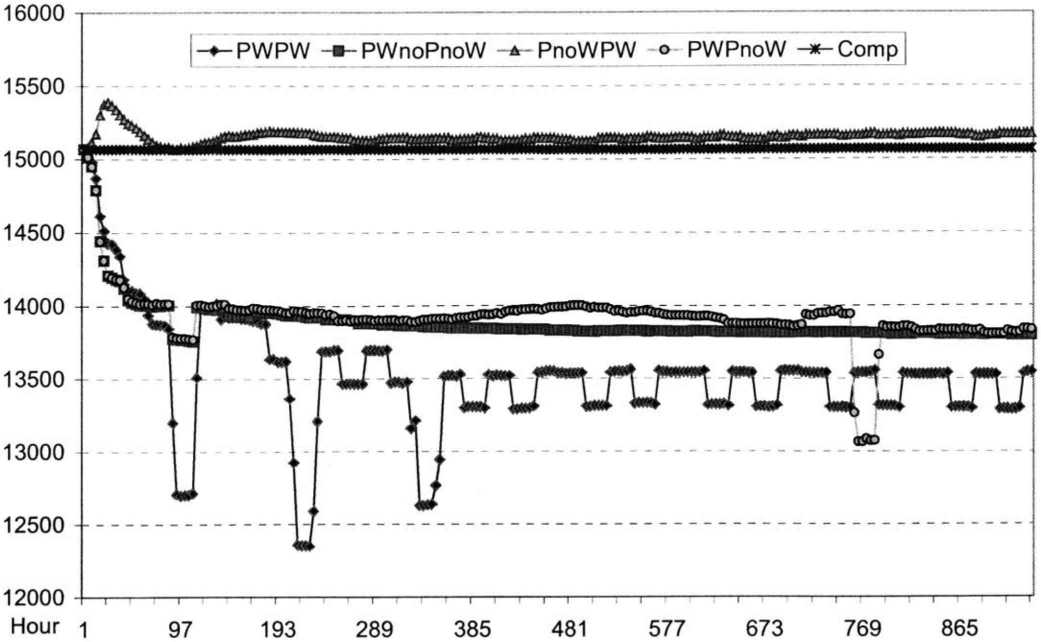

Figure 6-31: Moving-average Profit Dynamics That the LSE Agent Obtains in Market-A When It Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

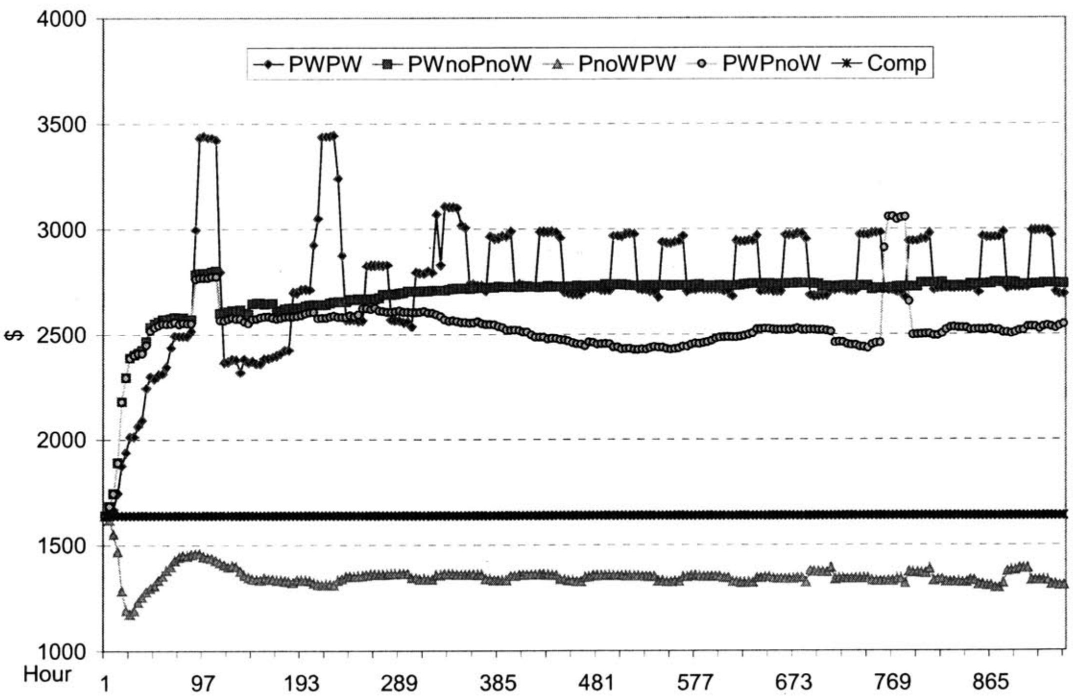

Figure 6-32: Moving-average Profit Dynamics That the Power-producing Agents Obtain in Market-A When the LSE Agent Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

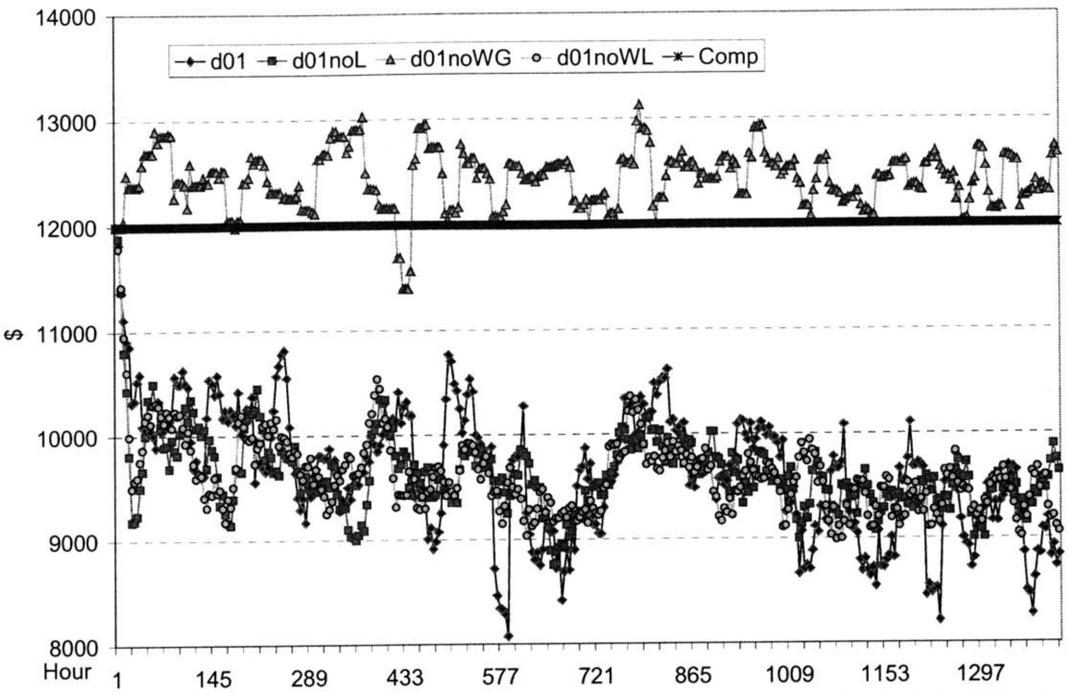

Figure 6-33: Moving-average Profit Dynamics That the LSE Agent Obtains in Market-B When It Employs Algorithm A3L with $\delta = 0.1$

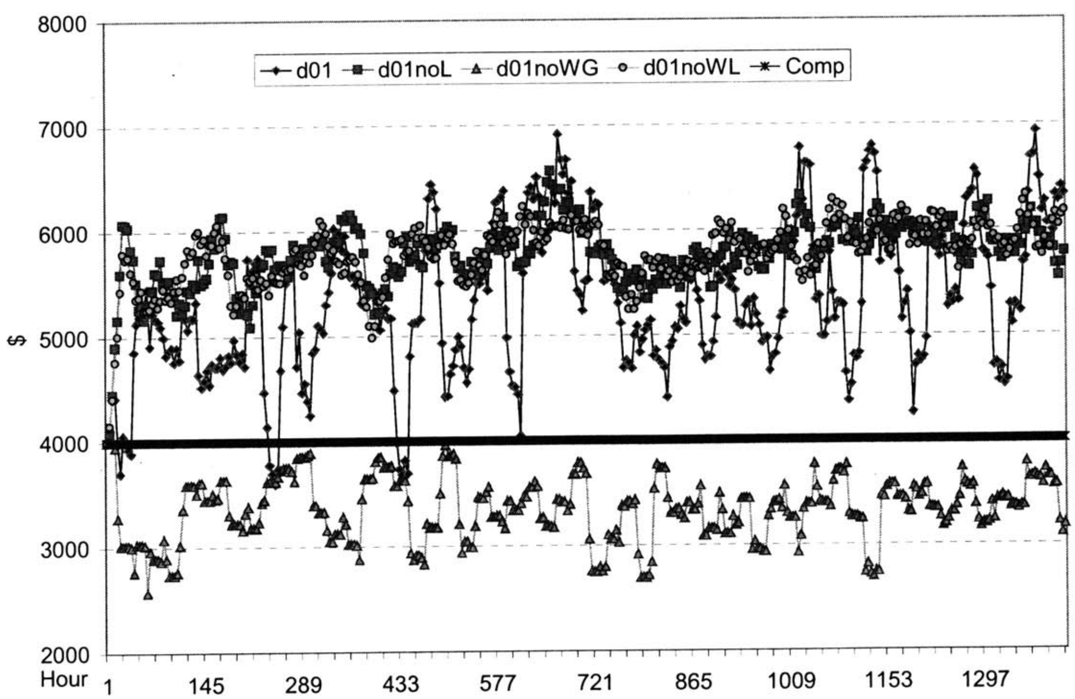

Figure 6-34: Moving-average Profit Dynamics That the Power-producing Agents Obtain in Market-B When the LSE Agent Employs Algorithm A3L with $\delta = 0.1$

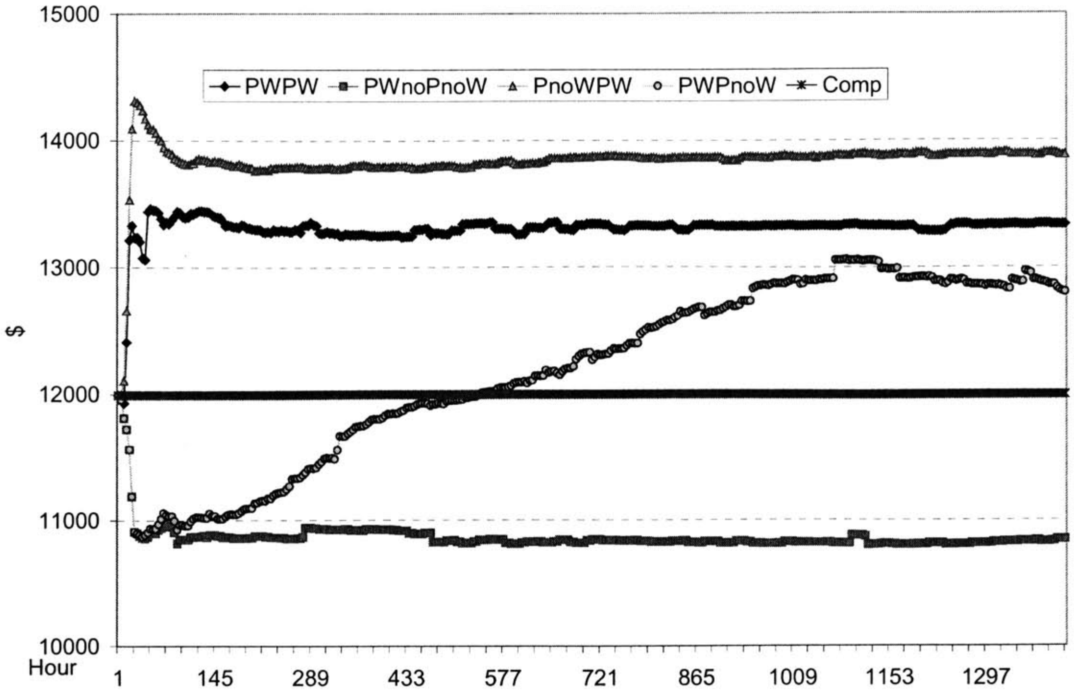

Figure 6-35: Moving-average Profit Dynamics That the LSE Agent Obtains in Market-B When It Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

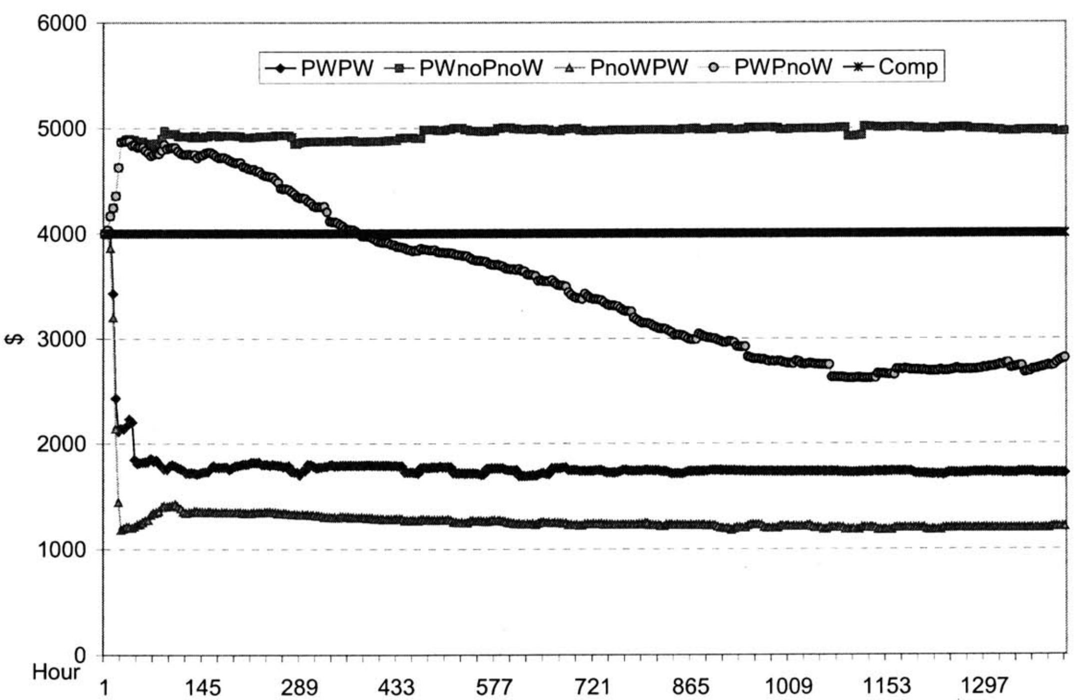

Figure 6-36: Moving-average Profit Dynamics That the Power-producing Agents Obtain in Market-B When the LSE Agent Employs the Model-based LSE Algorithm with Method M1 and $\Delta = 2$

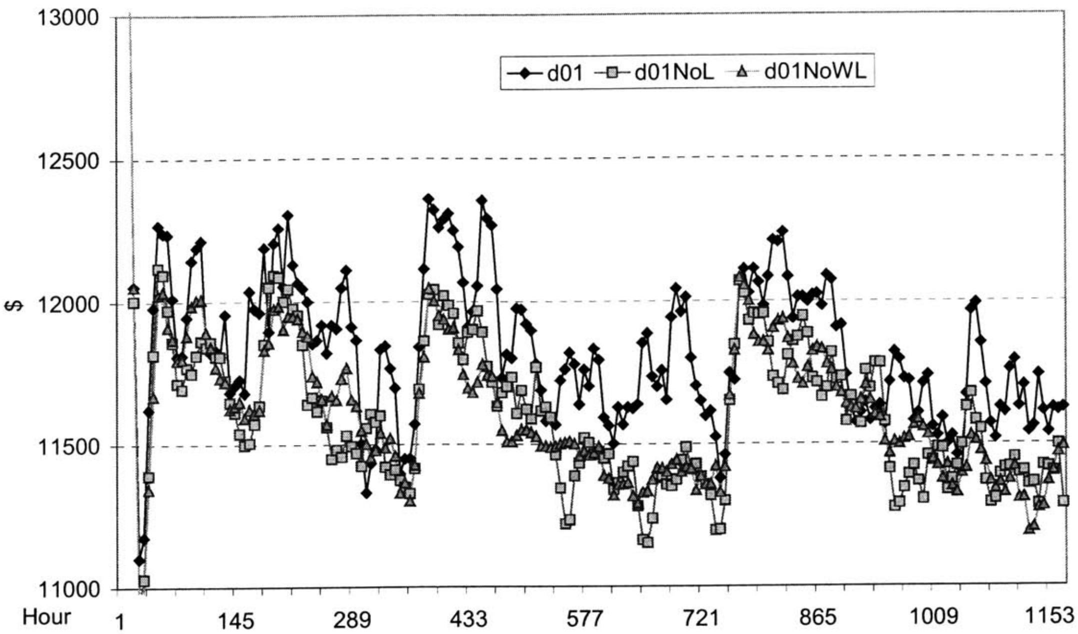

Figure 6-37: Moving-average Profits across 100 Simulations That LSE Agent in Market-A Obtains When It Employs Algorithm A3L with $\delta = 0.1$

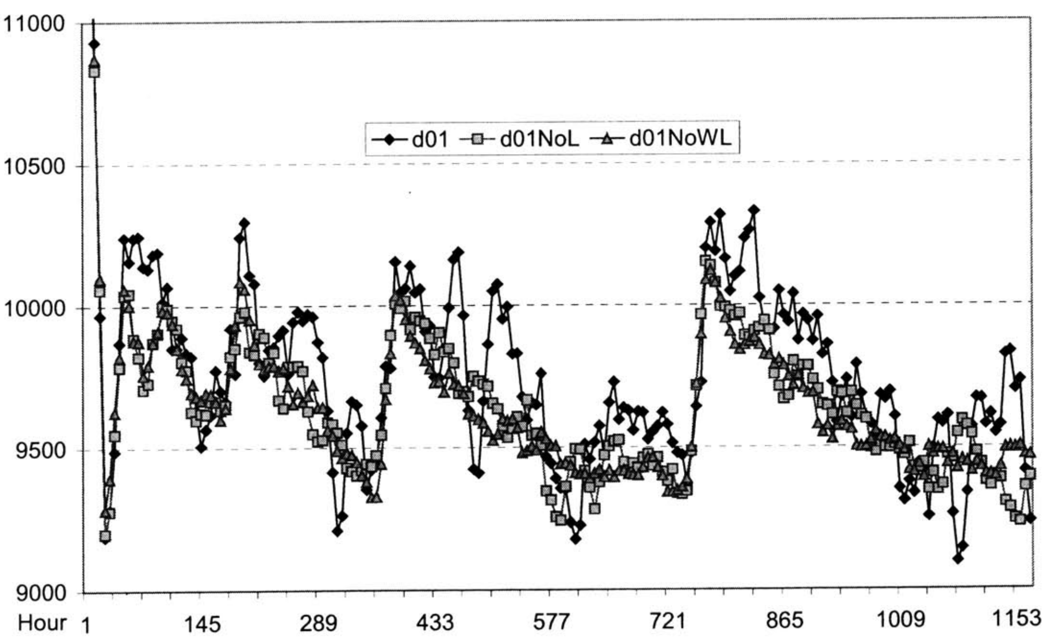

Figure 6-38 Moving-average Profits across 100 Simulations That LSE Agent in Market-B Obtains When It Employs Algorithm A3L with $\delta = 0.1$

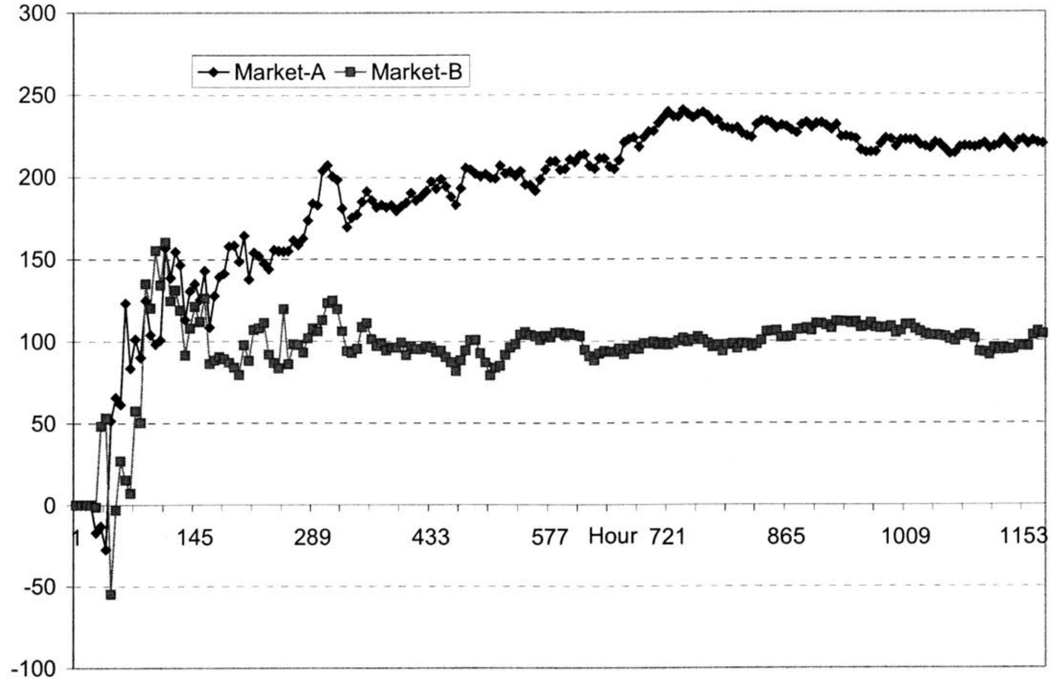

Figure 6-39: Time-dependent Average-profit Difference between the Profits from Scenarios I and II That the LSE Agent, Employing Algorithm A3L with $\delta = 0.1$, Obtains in Market-A and Market-B

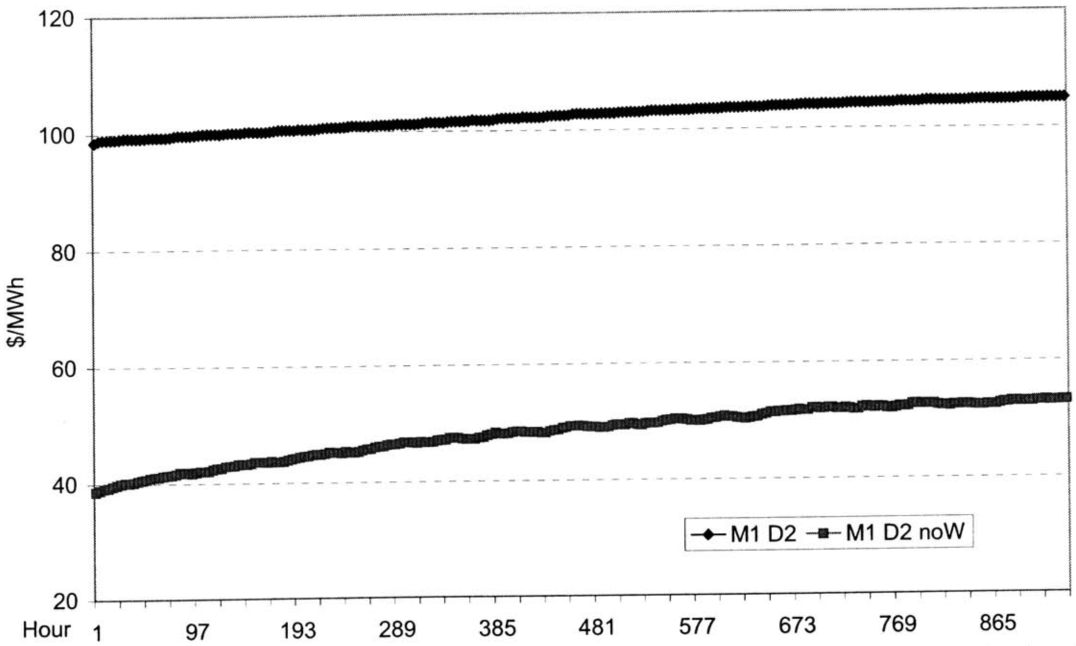

Figure 6-40: Moving-average Price Dynamics When the Power-producing Agents in Market-A Employ the Model-based Algorithm with Method M1 and $\Delta = 2$ and Demand is as Shown in Figure 6-19

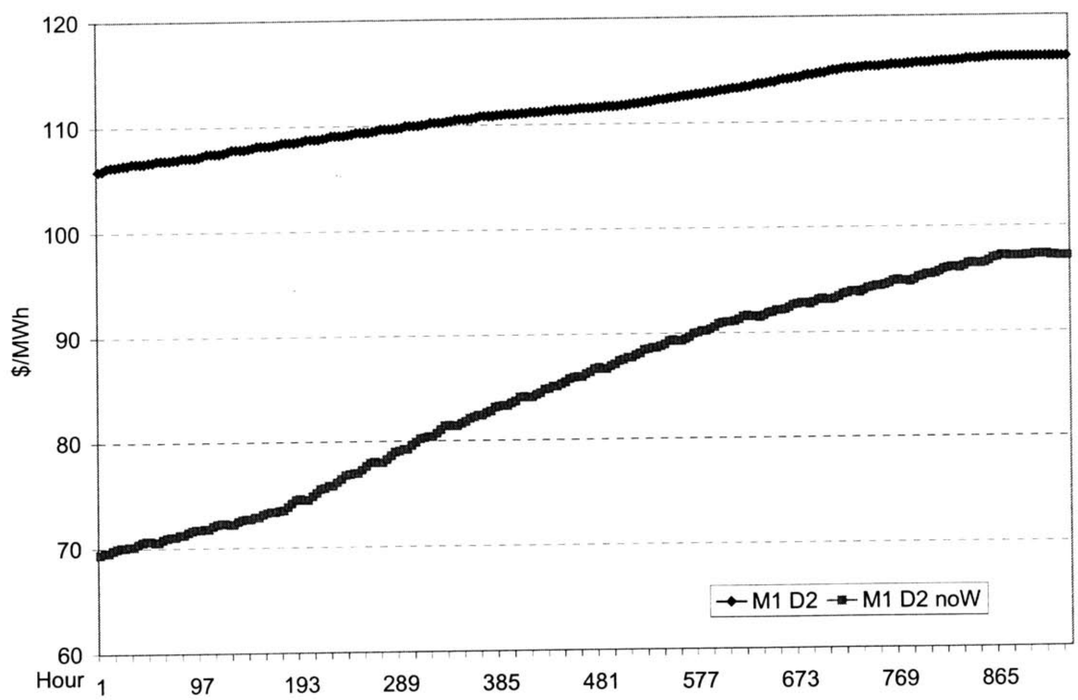

Figure 6-41: Moving-average Price Dynamics When the Power-producing Agents in Market-A Employ the Model-based Algorithm with Method M1 and $\Delta = 2$ and Demand is as Shown in Figure 6-20

Chapter 7

Possible Future Research and Conclusions

The agent-based model presented in this thesis demonstrates another possible modeling approach that can capture the dynamic interactions of decision-makers in dynamic systems. The electricity spot markets are examples of such dynamic systems, which consist of several active decision makers, such as the power producers and the load-serving entities, who can influence market outcomes. One potential benefit of the agent-based model for electricity markets is its ability to provide insight into the effects of decision-maker behavior on overall outcomes. In this thesis the agent-based electricity market model is used to analyze the effects of the market structures on market power-producer bidding behavior and market outcomes, and the role of active load-serving entities in the markets. In addition, the agent-based model creates simulated outcomes that incorporate the cumulative effects of bidding behaviors not generally captured when a top-down aggregate model is used.

The simulated outcomes from the agent-based model depend highly on not only the characteristics of the agents, but also on the learning algorithms that the agents employ. Model verification plays a key role in determining which learning algorithms yield the dynamics that most closely mimic the actual dynamics of the markets, though, verifying this agent-based market model is very difficult due to a lack of market information. This information has not been made available to the public by the system operator. Several aspects of the agent-based model shall be further investigated. These aspects, which are left for future research, are summarized in Section 7.2. The contributions of this thesis are summarized in the next section.

7.1 Contributions of the Agent-based Model

As mentioned previously, this thesis presents an alternative approach to model multiagent systems. Additionally, this thesis also provides potential benefits to a regulator, a system planner, and market-participants, as follows:

- For a regulator, the agent-based model can be used to investigate market conditions that could lead to higher prices by using scenario simulations on the condition that the model is verified and the well-matched learning algorithm is identified. The regulator may become aware of those market conditions and monitor the market participants cautiously. For example, cumulative effects, such as those from the capacity withholding strategy, may provide insight into the cause of price-spikes. These spikes may result from an insignificant action of an individual bidder.
- For a system planner, the agent-based model can be used to analyze market factors (such as new market rules) and their effects on market price dynamics as well as bidders' behaviors before any changes in market prices may take place. For example, the concept of unit-by-unit and portfolio-based decision schemes could provide a fundamental guideline in a generation-asset divestiture.
- For a market participant, if the model agent-based model is tested properly according to the method suggested in Chapter 4 of this thesis, it could potentially be extended to identify the "best" response action of one power producer (or one LSE) against its opponents' actions. That is, the power producer (or the LSE) may employ the model to simulate the possible market outcomes by using different learning algorithms in response to its opponents' actions, which are assumed to follow patterns observed in the actual markets. This power producer (or this LSE) then determines a learning algorithm that would yield the best profits based on the observed opponents' actions.

7.2 Future Work

The agent-based market model presented in this thesis could be improved so that it can represent the markets more closely. Several model modifications are suggested in Section 7.2.1. When the model is used to analyze the actual markets without information about the marginal-cost function of each unit in the markets, an equivalent marginal-cost function can be determined from historic bid data. The method to construct this function is presented in Section 7.2.2

7.2.1 Model Improvement

The modeling approach described in Chapter 3, is heavily dependent on somewhat limited assumptions about agents' actions and decision-making schemes. These assumptions may be made more complex in the following ways.

- The addition of non-uniform decision-making to the model. All previous simulations show the simulated price dynamics when the agents uniformly use one learning algorithm. In Chapter 5 the empirical analysis suggests that the actual market participants use different bidding strategies. The model, therefore, should be modified to examine the impact of non-uniform learning algorithms chosen by the agents on the market price dynamics.

- The improvement of the load-based decision scheme. In this model, the agents record the bidding outcomes and update their actions based on the hourly demand levels represented by load indices; however, when the intertemporal effects from generating-unit operating constraints are accounted for, the hourly load-based reference might not be valid. Another way to record and update agents' actions and bidding outcomes is by using daily demand average, in which a reference value to represent demand characteristic within a day has to be defined.

- The integration of more realistic system constraints. In the agent-based model presented in this thesis, no constraints of power system operation, such as unit-commitment constraints, ancillary services, and transmissions, are accounted for. The model can, for example, be modified to reflect the inflexibility of generating units due to unit-commitment constraints by imposing necessary constraints on the minimum operating capacity of the units. To simplify solutions of unit-commitment constraint problems of any market participant with a large portfolio, one may assume that the units plan their operations on a seasonal basis and that the agents cannot affect the average hourly prices as price-takers. Then the agents determine the average daily optimal operating schedule for that season, such as when to turn on or off the units in each day based on the average hourly prices. The daily operation is constrained by imposing the minimum operating capacity of each unit for each hour. One must keep in mind that when all units follow this method the actual market prices might change from the prices in the seasonal calculation. For example, several units go on maintenance during the low-demand period which may result in supply shortages and may subsequently contribute to an increase in market prices.

- The addition of demand uncertainty and development of strategies that capture uncertain payoffs that are caused by external factors such as demand variation. A few thoughts on this issue are summarized in Section 7.2.3

- The development of a long-term dynamic model. A long-term agent-based model can be developed using a similar methodology. Instead of making a bidding decision, the agents make an investment decision, such as adding new capacity, entering or exiting the market, or merging with other agents. A learning process with new decision and assessment rules is required.

7.2.2 A Simplified Method for Reproducing Market Prices

When the individual cost characteristics of each market participant, such as marginal costs and unit-commitment constraints, are not available for market-price reproducing as mentioned in Chapter 4, the simulated price dynamics based on the actual markets could be obtained by using information from historic bid data. In addition, from historic bid data, one could examine possible bidding strategies/learning algorithms of each market participant. The steps for reproducing price dynamics from these data are described as follows:

1. *Identifying possible unit-commitment constraints.* These constraints can be estimated based on the type of units: nuclear, coal, oil, and hydropower. This task is quite complicated since the same type unit of different sizes may have different constraints.

2. *Assigning the market participants their bidding strategies and/or learning algorithms.* The main problem of reconstructing price dynamics using historic bid data is to assign the right bidding strategy and/or learning algorithm to the market participants. Generally, a nuclear unit is inflexible in terms of feasibility of being turned on or off in a short period of time; therefore, it always operates as a base-load unit or employs a price-taker strategy.

3. *Determining the system marginal-cost function and market participants' marginal-cost functions from bid data.* When the actual marginal-cost function is not available, an "equivalent" marginal-cost function can be determined from the bid data. The minimum bidding prices at different bidding quantities over some period form an equivalent marginal-cost function which indicates the cheapest prices to buy power during that period, as shown in Figure 7-1. This equivalent function might be an optimistic estimation of the actual one because in some low-demand hours the market participants may "underbid" their units, so the units are scheduled for all hours to avoid being turned off. Moreover, some units may be bid under their self-scheduled capacity, and their bidding prices may not reflect the real marginal cost.

4. *Simulating market prices using the agent-based model and the available information.* The market-price simulation follows these steps.

   (a) Rearranging marginal-cost and operating-constraint data to have a format that fits with the model. For example, in existing markets, such as the New England electricity market, the market participants are required to submit a piece-wise bid-supply function in which the bid-supply function for each unit can be up to 10 bidding blocks (bid block MW and bid block $).

   (b) Simulating the price dynamics using the model with equivalent marginal-cost functions and demand obtained from the actual markets. If the bidding strategies cannot be extracted from the bid data, assign the bidders some rational bidding strategies as well as general objective functions, such as profit maximization.

(c) Modifying bidding strategies and observing the effects of bidding strategies on simulated prices and bidding behaviors of the agents in the model.

Note that although the marginal-cost functions of all units are available, determining the strategic behavior from their competitive behavior by comparing the bid functions to the marginal-cost functions alone might not be sufficient. This is because the generating units generally operate under unit-commitment constraints as well as contract obligations, and this information is generally confidential.

7.2.3 Effects of Demand Uncertainties

Although, in this agent-based model, demand is assumed to be deterministic. Forecast demand is generally accurate within a few percents of error.[1] This error, which is small in terms of percentage, may be greater than 1,000 MW in terms of capacity; this amount of power could require a few entire units to be turned on or off completely.[2] Without considering outages or the bidding prices that are lower than marginal costs, when actual demand is lower than forecast demand, the agents might not be scheduled to operate as anticipated. Similarly, when actual demand is higher than forecast demand, the agents might want to set higher prices. How should the agents take demand uncertainty into consideration as part of their bidding strategy?

This thesis suggests that in order to account for demand uncertainty, the agents must define the objective in their bidding-game participation, for example, whether

- the agents would like to be scheduled to operate with a high probability. (The agents may want to be scheduled to operate also when actual demand deviates within a bound of forecast demand with a high probability.)
- the agents would like to obtain the maximum expected profits.
- the agents would like to obtain the minimum-variance profits from each bidding round.
- the agents would like to raise market prices to be as high a level as possible regardless of demand levels.

Let us consider when each agent owns one generating unit, one bidding price is allowed for each unit (that is, withheld capacity is equal to zero), and no outage and strategic bids from the competitors are accounted for. An agent that wants to be scheduled to operate with a high probability determines

[1] From conversations with a ISO-NE staff and Dr. Robert Brammer, a thesis committee member.

[2] The ISO-NE provides information regarding hourly forecast and actual demand in the New England system.

a bidding price as follows:

$$b^* = \arg\max_{b \geq mc} \ \mathcal{E}_L\{q(L,b)\} \tag{7.1}$$

where b is a bidding price, mc is marginal cost, q is a scheduled quantity, P is a market price, and L is demand, which is characterized by a random variable. Let $\mathcal{E}_x(\cdot)$ refer to an expected value with respect to a random variable x. Let an hourly market price observed by each agent be characterized by a random variable as a function of the agent's bidding price and demand level. Suppose that when each agent is scheduled to operate ($b \leq P$), it operates at full capacity, q_{max}. Therefore, Equation (7.1) means

$$\arg\max_{b \geq mc} \int_b^\infty \wp(P|L,b) \cdot q(L,b) \cdot dP \equiv \arg\max_{b \geq mc} \ (\int_b^\infty \wp(P|L,b) \cdot dP) \cdot q_{max}$$

where $\wp$ is a probability distribution. The outcome of this optimization is similar to the outcome when the agent submits a bid that yields the highest market price, as follow:

$$b^* = \arg\max_{b \geq mc} \ \mathcal{E}_L\{P(L,b)\} \equiv \arg\max_{b \geq mc} \ (\int_b^\infty \wp(P|L,b) \cdot dP)$$

where $P(L,b)$ is market price when demand is equal to L and the agent's bidding price is equal to b.

An agent that wants to maximize its expected profits determines a bidding price as follows:

$$b^* = \arg\max_{b \geq mc} \ \mathcal{E}_L\{P(b,L) \cdot q - C(q)\} \tag{7.2}$$

where $P(b,L)$ is an anticipated market price when forecast demand is equal to L, assuming that the agent submits bidding price b and the other agents submit their marginal-cost bids. Let $C(q)$ be the operating cost of producing q and $C(q) = 0$ when $q = 0$. Note that $q = q_{max}$ when $b < P(b,L)$, $q = 0$ when $b > P(b,L)$, and $0 \leq q \leq q_{max}$ when $b = P(b,L)$. Hence, $(P(b,L) \cdot q - C(q)) \geq 0$. When the uncertainty due to outage and other agents' bids (B^{-i}) are accounted for, Equation (7.2) of Agent i is modified to

$$b^{i*} = \arg\max_{b^i \geq mc^i} \ \mathcal{E}_{L,B^{-i}}\{P(b^i, L, \bar{B}^{-i}) \cdot q - C^i(q)\}.$$

To simplify this analysis, let us consider only the demand uncertainty. Equation (7.2) can be rewritten as follows:

$$b^* = \arg\max_{b \geq mc} \int_0^\infty \wp(L) \cdot (P(b,L) \cdot q - C(q)) \cdot dL \tag{7.3}$$

where $\wp(L)$ is the probability density function of actual demand given forecast demand L.

Suppose that whenever $b \leq P(b, L)$, the agent is scheduled to operate q_{max} or the agent is paid $P \cdot q_{max}$ for any scheduled quantity. One can observe that for each bidding price b the integral in Equation (7.3) can be interpreted as calculating a value for a call option with a strike price equal to $C(q_{max})$ without considering a discounted factor.[3] The concept of this valuation remains unchanged when a price cap (P_{cap}) is accounted for and a total of installed capacity is exhausted. Hence, choosing the "best" bidding price in order to maximize the expected profits is similar to choosing the most expensive option.

In addition, instead of maximizing expected profits, the agent may try to minimize variances of expected profits obtained from bidding, i.e.,

$$b^* = \arg\min_{b \geq mc} \int_0^\infty \wp(L) \cdot (P(b, L) \cdot q - C(q) - \bar{\Pi}(b))^2 \cdot dL$$

where $\bar{\Pi}(b)$ is the expected profit, that is, $\mathcal{E}_L\{P(b, L) \cdot q - C(q)\}$. Therefore,

$$b^* = \arg\min_{b \geq mc} \{\int_0^\infty \wp(L) \cdot (P(b, L) \cdot q - C(q))^2 \cdot dL - \bar{\Pi}(b)^2\}.$$

By applying this objective, the agent chooses a bidding price that yields the lowest variance of expected profits, which might not result in the maximum profits as shown in Equation (7.2).

When the agents own more than one generating unit and/or the unit-commitment constraints are accounted for, the bid-supply function determination regardless of the objective function becomes more complicated due to the addition of inter-temporal factors. One might expect less flexibility in submitting an expensive bid-supply function to raise a market price, because of losses that occur when some units have to be turned off during their operations. Moreover, the complexity of the bid determination increases when the agents in the model may account for transmission constraints (and/or location-based prices). These issues are left for future research.

7.3 Conclusions

The agent-based electricity market as a bidding game belongs to a class of unknown game setups in which the agents do not know the actions and associated payoffs of their opponents before or after the bidding decisions are made. Determining a Nash-equilibrium strategy of the players playing this game is not applicable, because they have neither their own entire payoff functions nor their opponents' entire payoff functions. When multiple equilibria of the game exist, some of the typical problems of these players who have incomplete information about others' actions and associate payoffs may be expressed as follows:

- How do the players learn the game efficiently with the least information about the opponents?

[3] For more detail on this subject, see Hull [24], for instance.

- Among all possible equilibria (pure and mixed-strategy equilibria), which equilibrium should be chosen?

- If there is only a single Nash-equilibrium in the game, how long does it take before the equilibrium strategy is reached?

These issues play a key role in modeling and analyzing multiagent systems (or games) to observe dynamic outcomes from agent interactions. Although, as shown throughout this thesis, the potential benefits of the agent-based electricity market models are significant, analyses based on the model need to be performed with cautions until some of these issues are resolved.

In addition, the observations from the simulations and empirical studies indicate that to have a proper agent-based model for a multiagent system, the model must have these properties:

- The number of active agents that could affect the dynamics of the system of interest must be realistic. Each agent should have characteristics similar to those of actual decision-makers.

- Learning algorithms must have both exploration and exploitation. Without exploration in strategies, the agents do not get to experience other possible actions and the outcomes may converge to some dynamics. This may not be reasonable if such phenomena are not observed in the system. Note that the learning algorithm that the agents employ might not necessarily yield the convergence outcomes. This characteristic might not be critical as long as the simulated dynamics mimic the actual ones closely.

- The agents should have learning algorithms that are different from the others and are similar to the algorithms that may be adopted by the actual decision-makers. The empirical studies show that the LPs' bidding strategies depend on their portfolios' characteristics. When the agents adopt the same learning algorithm, such as the model-based algorithm, as shown in the simulations, there are not many adversarial outcomes. The decisions of the agents tend to move in tandem. The learning algorithm may result in a steady-state pattern of the price dynamics. These simulated results might not be realistic even under the deterministic demand condition.

- The significant characteristics of the decision-makers shall be captured. However, some constraints may be relaxed for model simplification. For example, in the agent-based electricity market model, unit-commitment constraints may be considered a long-term problem and may be calculated prior to determining the bids. The agents then take these constraints as inputs to their bidding decisions.

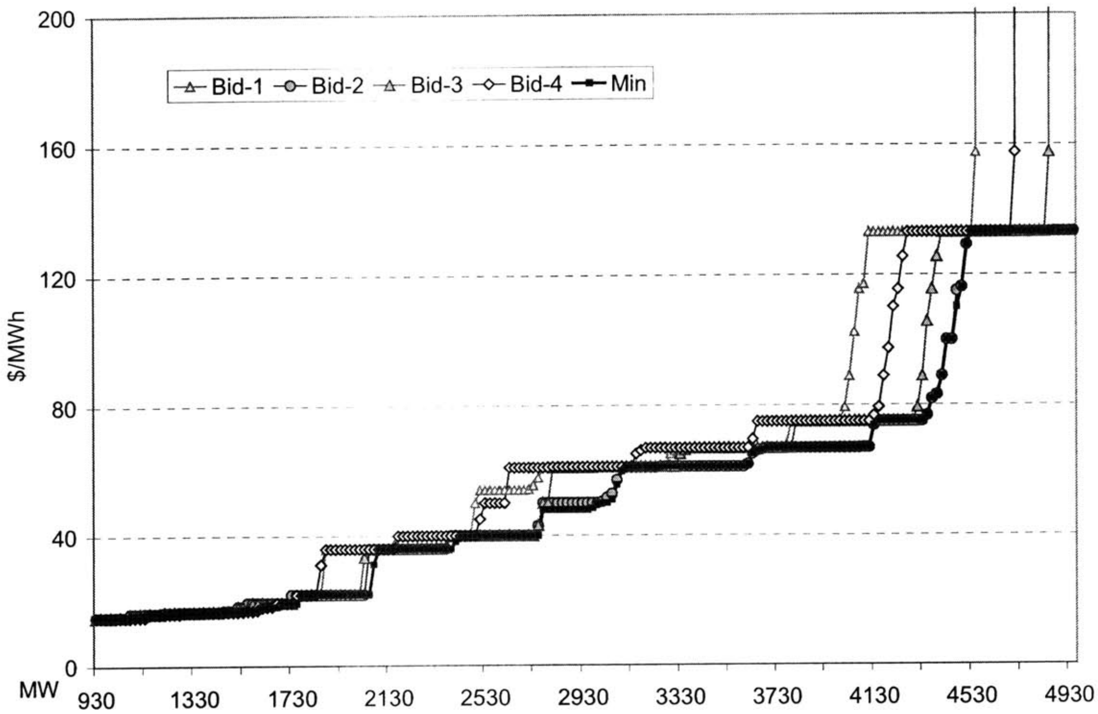

Figure 7-1: An Example of "Equivalent" Marginal-cost Function

Appendix A

Available Information and Spot Prices

This appendix shows how market participants or agents perceive forecast prices differently when they have asymmetric information. How does the information affect the forecast prices? To answer this question, the probability mass functions (PMFs) of market prices given a deterministic (inelastic) demand (or the PMFs of prices given demand, for short) and different sets of information are derived. The PMFs of prices given a specific demand level under various sets of additional information show that the agents can have different forecast prices if they possess different sets of information. This study provides an analytical understanding of the effect of information asymmetry and its influence on the agents' possible strategic behavior with different portfolio characteristics. To start, suppose that each agent, who is a power producer, holds a portfolio of generating units. During each round of auctions, prior to submitting its bid, the agent does not know the others' bids and the market prices. The market prices are a function of all agents' bids and total demand. After the market clears in each hour, only the market prices and total demand are publicly known. The bids of the agents may be revealed, if at all, after a long period of delay.[1]

Suppose that the agents know the system's marginal-cost function and the past bids (revealed bids) of the other agents. For the purpose of this study, planned outages are excluded. From the revealed bids, let us assume that the agent is able to determine the probability of the availability of each unit in each hour. This probability of the availability of a unit refers to the probability that a particular unit has a bidding price deviating from its marginal cost. Let us further assume that each unit has a constant marginal cost and submits a single bidding price for each unit. Moreover, the unavailability of each unit is independent. (This is a rather strong assumption because units with the same owner are likely to have correlated bidding prices.) The correlation between hours is assumed

[1] For example, a 6-month delay in the New England electricity market.

to be zero. Given the probability of the availability of each unit and the above assumptions, the PMF of price given demand can be derived. When the agent knows for certain the bidding prices of its units or whether a few units are not available due to planned maintenance, the PMFs (of prices given demand) seen by the agent become conditional on this added information.

This study shows that the agent with a larger portfolio perceives anticipated prices given demand with more accuracy (as reflected in the narrower width of the PMFs or the smaller variance of anticipated prices) than the agent with a smaller portfolio. Moreover, without demand elasticity, the larger the demand, the more variance of the anticipated market prices or the wider the PMFs. When demand elasticity is present, the width of the PMFs or the variance of the anticipated prices is reduced. Further, when the constraints associated with unit operation such as the unit-commitment constraints are accounted for, the variance of anticipated prices can increase.

This appendix is organized as follows. Section A.1 provides general background, assumptions, and framework. Section A.2 shows the derivation of the PMFs of market prices given demand in the market where all generators are uniform in size, have a constant marginal cost, and share the same probability of the availability. The system's marginal cost function is a piece-wise non-decreasing function. Section A.3 shows the derivation of the PMFs when the agents are asymmetric in terms of marginal cost, size, and the probabilities of the availability. Section A.4 presents the effect of demand elasticity on the PMFs of prices given demand. Section A.5 describes the potential effect of unit-commitment constraints on market prices. Section A.6 provides a possible extension of the PMFs.

A.1 General Background

Let us consider a market that consists of N independent agents. Let $\mathcal{N} = \{1, \ldots, N\}$. Each Agent i ($i \in \mathcal{N}$) owns one generating unit or one unit. An agent, a generating unit, or a unit is interchangeable. Each unit has a constant marginal cost, $mc^i > 0$. Let index i rank agents from the least expensive to the most expensive units, i.e., $mc^i \leq mc^{i+1}$. Let q^i_{max} denote the total capacity of generator i, Q_{max} denote the total installed capacity, where $Q_{max} = \sum_i^N q^i_{max}$, and X^i denote the cumulative capacity of generators 1 to i, i.e., $X^i = \sum_{g=1}^i q^g_{max}$.

Definition of Perfectly Competitive Markets

In a perfectly competitive market without unit-commitment constraints, an agent is a price taker and maximizes its profits (Π) as follows:

$$\max(\Pi) = \max_q \left(P \cdot q - C(q)\right)$$

where $C(q)$ is an operating cost of producing q units of power. The solution to the above optimization when the market price is not a function of q is

$$mc(q) = \frac{\partial C(q)}{\partial q} = P$$

where P is market price. The above equation means the agent is willing to generate its power when its marginal cost ($mc(q)$) is not greater than market price. Suppose each unit has the same capacity and has marginal cost such that $mc^i < mc^{i+1}, \ \forall i \in \mathcal{N}$. The merit-order (aggregate) supply function is a non-decreasing function and no single unit has the same marginal cost. Suppose that deterministic demand at a particular hour is equal to L, that the market is perfectly competitive, that together with that all units are available at their full capacity, and that the market clearing price is equal to the marginal cost of the marginal unit (denoted by index m). Let marginal unit m be the unit with the most expensive marginal cost to be scheduled to operate, so that total cost is minimized. Let P^0 denote the marginal-cost price when all units are available, i.e.,

$$P^0 = mc^m(L - \sum_{i=1}^{m-1} q^i_{max}), \quad \text{s.t.} \quad \sum_{j=1}^{m-1} q^j_{max} < L < \sum_{j=1}^{m} q^j_{max}.$$

Let each unit i have the probability of the availability at any time k equal to $\Psi^i_k = \Psi^i$ and probability of unavailability equal to $(1 - \Psi^j)$. The derivations of the PMFs presented in this thesis can be performed period by period, because the intertemporal factors associated with the units are not accounted for. For simplicity of notation, the index indicating time $*_k$ is omitted. Probability Ψ may be interpreted as the probability that the unit j submits a different bid from its marginal-cost bid. This could result from unit-commitment constraints, bilateral trades, and strategic bids. The bidding prices of unavailable units are assumed to be higher than the most expensive marginal cost.[2] Unavailability due to real-time outages and planned maintenance are not accounted for. Probability of outages of each unit is also independent.[3]

Next, let Ω^{-n} denote a set of n unavailable units. Let $\wp^{n|\Omega^{-n}}$ denote the probability of these n units becoming unavailable. The probability that one unit g is unavailable at any time k is equal to

$$\wp^{1|\Omega^{-1}} = \prod_{j=1, j \neq g}^{N} (\Psi^j) \cdot (1 - \Psi^g).$$

Similarly, the probability of n units becoming unavailable at time k is equal to

$$\wp^{n|\Omega^{-n}} = \sum_{i, i \in n} (\prod_{j \in \Omega^{-i}} (\Psi^j) \cdot \prod_{g \in \Omega^i} (1 - \Psi^g)).$$

[2]One can think of this as an economic withholding strategy.

[3]This assumption will not be true if Agent i owns more than one unit. Scheduled maintenance or planned outages will be correlated.

Note that $\Omega^{-i} \cup \Omega^{i} = \mathcal{N}$. The total events of n unavailable units, where $n \in \mathcal{N}$, denoted by $\aleph(\Omega^{-n})$, can be calculated as follows:

$$\aleph(\Omega^{-n}) = \begin{pmatrix} N \\ n \end{pmatrix} = \frac{N!}{n!(N-n)!}.$$

Binomial Distribution

The probability of any specific sequence of n units being unavailable in the market of N units is called the binomial density and is denoted by P^n. There are $\begin{pmatrix} N \\ n \end{pmatrix}$ such sequences that have n unavailable units. Thus,

$$P^n = \begin{pmatrix} N \\ n \end{pmatrix} \Psi^n \cdot (1-\Psi)^{N-n}.$$

This density is positive for $n = 0, \ldots, N$. The parameters of the density are N, which is a positive integer, and $0 \leq \Psi \leq 1$.

A.2 PMF of Prices Given Demand: Case I

This section presents a procedure for deriving PMFs of market prices given load when the probability of the availability of each unit is known and the units are uniform in capacity and in the probability of their availability at any period. Let each unit have the same maximum capacity $q^i_{max} = q_{max}$, $\forall i \in \mathcal{N}$. Let demand L at each period k be such that $X^{m-1} < L \leq X^m$. Let $x^m = L - X^{m-1}$ denote the residual demand served by unit m, and let $\wp_{P|L}(P < mc^m | L)$ denote the probability of price equal to mc^m given demand L. The market price, given demand L without any outage, denoted by P^0, is

$$P^0 = mc^m \cdot I(x = (L - \sum_{j=1}^{m-1} q^j_{max}) \geq 0).$$

Let us consider unit i in which $mc^i < mc^m_k(x^m) = mc^m$. When this unit i becomes unavailable during time k, the market price increases to $P > mc^m$. This event yields $\wp_{P|L}(P < mc^m | L) = 0$. Note that because of the uniform capacity of each unit, when one unit with marginal cost less than mc^m becomes unavailable, the market price increases to the marginal cost of the next more expensive unit. Exploring all possible scenarios of n unavailable units, where $n \in \mathcal{N}$, shows that the market price is equal to mc^{m+1} because one of the following is true:

- One unit with $mc^i < mc^{m+1}$ is unavailable, while the other units are available; or
- One unit with $mc^i < mc^{m+1}$ is unavailable, and other units with $mc^i > mc^{m+1}$ are unavailable.

Similarly, market price is equal to the marginal cost of unit $m+h$, i.e., $P = mc^{m+h}$ when

- h units with $mc^i < mc^{m+h}$ are unavailable, while the other units are available; or

- h units with $mc^i < mc^{m+h}$ are unavailable and at least another unit (unit i, where $i \geq h$), with a maximum of $(N-m-h)$ units with $mc^i > mc^{m+h}$, is unavailable.

As a result, the probability that market price equals mc^{m+h} is equal to the sum of all possible events of h unavailable units with marginal cost less than mc^{m+h} (defined by Ω^{-h}) and the sum of all possible events of at most $(h^* = N-m-h \geq 0)$ unavailable units with marginal cost greater than mc^{m+h} (denoted by Ω^{-h^*}), i.e.,

$$\wp_{P|L}(P = mc^{m+h}) = \prod_{g\in\Omega^{-h},g=1}^{h}(1-\Psi^g)\cdot\prod_{g\notin\Omega^{-h},g=1}^{h}\Psi^g\cdot\prod_{g\in\Omega^{-h^*},g=1}^{Z}(1-\Psi^h)\cdot\prod_{g\notin\Omega^{-h^*_1},g=1}^{Z}\Psi^h$$

where $Z = N-m-h$ and $\Omega^{-(h+h^*)} = \Omega^{-h}\cup\Omega^{h^*}$. When each unit has the same probability of the availability $\Psi^i = \Psi$, $\forall i \in \mathcal{N}$, the PMF of market prices given load L, $\wp_{P|L}(P = P)$, is reduced to

$$\wp_{P|L}(P = mc^{m+z}) = \binom{m+z-1}{z}\Psi^{m-z}(1-\Psi)^z\cdot\{\sum_{d=0}^{N_N-z}\binom{N_N-z}{d}\Psi^{N_N-z-d}(1-\Psi)^d\}$$

where $N_N = N-m$ and $0 \leq z \leq N_N$. Note that due to the uniform capacity and probability of the availability of the units, a closed-form formulation for an event of h unavailable units can be obtained, where $h \in \mathcal{N}$.

It is also possible that more than N_N units could become unavailable simultaneously. If this happens, supply scarcity would occur and the available units would drive the market price to be (literally) infinite.[4] Therefore, the probability of market prices higher than the most expensive marginal cost mc^N or mc^{m+N_N}, is equal to

$$\wp_{P|L}(P > mc^N) = 1-\sum_{z=0}^{N_N}[\binom{m+z-1}{z}\Psi^{m-z}(1-\Psi)^z\cdot\{\sum_{d=0}^{N_N-z}\binom{N_N-z}{d}\Psi^{N_N-z-d}(1-\Psi)^d\}].$$

Note that when $P > mc^{m+N_N}$, it could be considered $P \to \infty$.

[4]Demand is assumed to be inelastic.

A.2.1 Known Number of Unavailable Units

This section analyzes the effect of known unavailable capacity on the PMFs of prices given demand, observed by Agent i. How would Agent i perceive the market price differently, if the agent knows the total number of unavailable units N^{out}? Suppose that the agent knows which units are unavailable. One would expect that this information improves confidence in anticipated prices; that is, the width of the PMF of prices given demand is reduced. Let us consider a scenario in which total unavailable units are known. Suppose $X^{m-1} < L \leq X^m$. Let $N^N = N - m$. The following cases are considered:

1. For $N^{out} = 0$. All units are available at their marginal costs. Since there is no unavailable capacity, the supply function is the system's marginal-cost function, i.e., the market price defined by P^0 is equal to mc^m. Therefore, the agents know with certainty that the market price is going to be this value.

$$\wp_{P|L,N^{out}=0}(P = mc^m) = 1.$$

2. For $N^{out} = 1$, let $\bar{K} = \binom{N}{1} \cdot \Psi^{N-1}(1-\Psi)$. To determine a set of all possible prices, let us consider two cases:

   (a) An unavailable unit i has marginal cost less than P^0 or mc^{m+1}, i.e., $mc^i \leq mc^m$. Since an infra-marginal unit or a marginal unit is unavailable, the market clearing price increases from P^0 to $P = mc^{m+1}$. Hence, the probability of market price equal to mc^{m+1} is the probability that one unit with marginal cost of at most mc^m becomes unavailable, i.e.,

$$\wp_{P|L,N^{out}=1}(P = mc^{m+1}) = \frac{\binom{m}{1} \cdot \Psi^{m-1}(1-\Psi) \cdot \Psi^{N_N}}{\bar{K}} = \frac{m}{N}.$$

   (b) An unavailable unit i has its marginal cost greater than P^0, i.e., $mc^i > mc^m$. Since an extra-marginal unit becomes unavailable, the market clearing price is not affected by this unavailability; in addition, the market price remains the same as when there is no unavailable unit, i.e., $P = mc^m$. Hence, the probability of market price equal to mc^m is the probability that one unit with marginal cost greater than mc^m becomes unavailable, i.e.,

$$\wp_{P|L,N^{out}=1}(P = mc^m) = \frac{\Psi^m \cdot \binom{N_N}{1} \cdot \Psi^{N_N-1}(1-\Psi)}{\bar{K}} = \frac{N-m}{N}.$$

3. For $N^{out} = n$ and $1 < n < N$. Let $\bar{K} = \begin{pmatrix} N \\ n \end{pmatrix} \cdot \Psi^{N-n}(1-\Psi)^n$. When n units become unavailable, this yields seven basic scenarios, as follows:

   (a) When $n \leq m < N_N$. The unavailable units have marginal costs less than $P^0\,(P^0 = mc^m)$. Suppose that the total available capacity after subtracting the capacity of the unavailable units is sufficient to serve demand. Proceeding similarly to identify a set of all possible prices, let us consider cases when n_1 unavailable units have their marginal costs less than or equal to P_k^0 (i.e., $mc^i \leq mc^m$) and n_2 unavailable units have their marginal costs greater than P^0 (i.e., $mc^i > mc^m$). Since only n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases to $P = mc^{m+n_1}$. Hence, the probability of price given demand when price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most equal to P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P = mc^{m+n_1}) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}$$

   where $\forall n_1 \in \{0, 1, \ldots, m)\}$ and $\forall n_2 \in \{0, 1, \ldots, m)\}$, such that $n_1 + n_2 = n$.

   (b) When $n < N_N \leq m$. The unavailable units have marginal costs less than $P^0\,(P^0 = mc^m)$. Suppose that the total available capacity after subtracting the capacity of the unavailable units is sufficient to serve demand. Proceeding similarly, let us consider cases when n_1 unavailable units have their marginal costs less than or equal to P^0 (i.e., $mc^i \leq mc^m$) and n_2 unavailable units have their marginal costs greater than P^0 (i.e., $mc^i > mc^m$). Since only n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases to $P = mc^{m+n_1}$, which is less than the most expensive marginal cost or mc^N. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most equal to P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P < mc^{m+N_N} = mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}$$

   where $\forall n_1 \in \{0, 1, \ldots, m)\}$ and $\forall n_2 \in \{0, 1, \ldots, m)\}$, such that $n_1 + n_2 = n$.

   (c) When $N_N < n \leq m$. The unavailable units have marginal costs less than $P^0\,(P^0 =$

mc^m). Suppose that the total available capacity after subtracting the capacity of the unavailable units may not be sufficient to serve demand. As a result, the price could exceed the most expensive marginal cost. Proceeding similarly to identify a set of all possible prices, let us consider cases when there are n_1 unavailable units that have their marginal costs less than or equal to P^0 (i.e., $mc^i \leq mc^m$) and there are n_2 unavailable units that have their marginal costs greater than P^0 (i.e., $mc^i > mc^m$). Only n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, and the market price increases to $P = mc^{m+n_1}$. The price exceeds the most expensive marginal cost (mc^N) since $n_1 > N^N$. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P > mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

(d) When $n_1 \leq N_N$. Since n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases from P^0 to $P = mc^{m+n_1}$. The price does not exceed the most expensive marginal cost (mc^N) since $n_1 \leq N^N$. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P \leq mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

(e) When $n > m \geq N_N$. The unavailable units may have marginal costs greater than P^0 ($P^0 = mc^m$) and the total available capacity after subtracting the capacity of the unavailable units may not be sufficient to serve demand. The price could exceed the most expensive marginal cost. Proceeding similarly to identify a set of all possible prices, let us consider the following cases:

i. When $n_1 > N^N$. There are n_1 unavailable units that have their marginal costs not greater than P^0 (i.e., $mc^i \leq mc^m$) and there are n_2 unavailable units that have their marginal costs greater than P^0 (i.e., $mc^i > mc^m$). Only n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, and the market price increases from P^0 to $P = mc^{m+n_1}$. The price exceeds the most expensive marginal cost (mc^N) since $n_1 > N^N$. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P > mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

ii. When $n_1 \leq N_N$. Since n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases from P^0 to $P = mc^{m+n_1}$. The price does not exceed the most expensive marginal cost (mc^N) since $n_1 \leq N^N$. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P \leq mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

(f) When $n > N_N \geq m$. The unavailable units may have marginal costs greater than $P^0\,(P^0 = mc^m)$ and the total available capacity after subtracting capacity of the unavailable units may not be sufficient to serve demand. The price could exceed the most expensive marginal cost. Proceeding similarly to identify a set of all possible prices, let us consider the following cases:

i. When $n_1 > N^N$. There are n_1 unavailable units that have their marginal costs not greater than P^0 (i.e., $mc^i \leq mc^m$) and there are n_2 unavailable units that have their marginal costs greater than P^0 (i.e., $mc^i > mc^m$). Only n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal cost $mc^i > mc^m$, and the market price increases from P^0 to $P = mc^{m+n_1}$. Since $n_1 > N^N$, the price exceeds the most expensive marginal cost or mc^N. Hence, the probability of price given demand when price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P > mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}},$$

where $N_N = N - m$.

ii. When $n_1 \leq N_N$. Since n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases from P^0 to $P = mc^{m+n_1}$. The price does not exceed the most expensive marginal cost (mc^N) since $n_1 \leq N^N$. Hence, the probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P \leq mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

(g) When $N_N \geq n > m$. The unavailable units may have marginal costs greater than $P^0\,(P^0 = mc^m)$. The total available capacity after subtracting the capacity of the unavailable units is sufficient to serve demand. The price would be no greater than the most expensive marginal cost. Proceeding similarly to identify a set of all possible prices, let us consider a case when $n_1 \leq N_N$. Since n_1 unavailable infra-marginal and/or marginal units affect the price, but not those n_2 units with marginal costs $mc^i > mc^m$, the market price increases from P^0 to $P = mc^{m+n_1}$. Additionally, the price does not exceed the most expensive marginal cost or mc^N since $n_1 \leq N^N$. The probability of price given demand where price equals mc^{m+n_1} is equal to the probability that n_1 units with marginal costs at most P^0 and n_2 units with marginal costs greater than P^0 become unavailable, as follows:

$$\wp_{P|L,N^{out}=n_1}(P \leq mc^N) = \frac{\begin{pmatrix} m \\ n_1 \end{pmatrix} \cdot \Psi^{m-n_1}(1-\Psi)^{n_1} \cdot \begin{pmatrix} N_N \\ n_2 \end{pmatrix} \cdot \Psi^{N_N-n_2}(1-\Psi)^{n_2}}{\bar{K}}.$$

The more information obtained, the more accuracy the anticipated price. As observed, the uncertainty is minimal in one of the extreme cases, such as no unavailable units or no available units. The uncertainty increases when less information is available, as reflected in a wide distribution (or large variance).

A.2.2 Imperfect Competition

For each agent, the difficulty in deriving the PMF of (ex ante) prices is that the competitors' bids may be unknown and/or the aggregate supply function may not be available. The agents are able to choose any bidding price or quantity that complies with the constraints, such as a price cap or installed capacity. The optimal strategy for each agent in the market with N players is not easy to characterize. Importantly, the payoff function for all agents associated with their bids is also unknown. To understand the effect of a bid on a PMF of price given demand, let us follow these assumptions. Agent i assumes that other agents bid at their marginal costs and it bids strategically at b^i and $q^i_{max} = q_{max}$. When the unplanned outage is not accounted for, since Agent i is certain about its own availability, the uncertainty is reduced from N to $N-1$. Note that the uncertainty of price observed by Agent i will be reduced if this agent owns several units. Suppose that Agent i bids (b^i, q_{max}) and that L is such that $X^{m-1} < L \leq X^m$. Let P^0 denote market price when every agent submits a marginal-cost bid, there is no unavailable unit, and $P^0 = mc^m$. Let us consider the following scenarios:

1. If $b^i < P^0$ and $mc^i < P^0$

   If Agent i bids lower than the system's marginal-cost price and all agents are available, this bid will not affect the intersection point of the load and aggregate supply functions. Hence, bidding below the system marginal cost does not change the anticipated market price at that given load.

2. If $b^i < P^0$ and $mc^i \leq P^0$

   In this case, Agent i underbids. This will make the PMF of prices given demand shift to the left (relative to that of a marginal-cost bid), that is, toward the lower price.

3. If $b^i \geq P^0$ and $mc^i < P^0$

   In this case, Agent i overbids (or tries to bid strategically). This will make the PMF of prices given demand shift to the right, that is, toward the higher price.

4. If $b^i \geq P^0$ and $mc^i \geq P^0$

   In this case, Agent i overbids and its effect on the system supply curve is similar to submitting a marginal-cost bid.

From the above observations, in any period k when demand is inelastic and deterministic,

- The agents with marginal costs less than P^0 are very likely to strategically influence the market more easily than the agents with higher marginal costs. Since the bids with the bidding price higher than the marginal costs of the agents cause the PMF of market prices to shift to the right (toward the higher price zone), the expected price is likely to increase.

- The number of agents that can strategically bid tends to increase as the load increases.

- The variances of market prices at any load and/or in any period k are not constants. They depend on the system marginal-cost price (or on the operating cost function), as well as on the level of demand, the probability of the availability of each unit, and bidding prices (when the market is imperfect).

The PMF of market prices given demand as a function of b^j, the probability of the availability, and the marginal cost of each agent, can be determined as follows:

1. Agent i with $mc^i \leq mc^{m+z}$. Suppose that

   (a) Agent i's bidding price is equal to mc^h, where $0 < h \leq m+z$, and $0 \leq z \leq N_N - 1$:

$$\wp^i_{P,\, b^i}(P = mc^{m+z}) = \binom{m-1+z}{z} \Psi^{m-1-z}(1-\Psi)^z \cdot \{ \sum_{d=0}^{N_N - z} \binom{N_N - z}{d} \Psi^{N_N-1-z-d}(1-\Psi)^d \}.$$

   (b) Agent i's bidding price is equal to mc^h, where $m+z < h \leq N$, and $0 \leq z \leq N_N - 1$:

$$\wp^i_{P,\, b^i}(P = mc^{m+z+1}) = \binom{m+z}{z} \Psi^{m-z}(1-\Psi)^z \cdot \{ \sum_{d=0}^{N_N-1-z} \binom{N_N-1-z}{d} \Psi^{N_N-1-z-d}(1-\Psi)^d \}.$$

2. Agent i with $mc^i > mc^{m+z}$. Suppose that

   (a) Agent i's bidding price is equal to mc^h, where $0 < h \leq m+z$, and $0 \leq z \leq N_N - 1$:

$$\wp^i_{P,\, b^i}(P = mc^{m+z}) = \binom{m+z}{z} \Psi^{m-z}(1-\Psi)^z \cdot \{ \sum_{d=0}^{N_N-1-z} \binom{N_N-1-z}{d} \Psi^{N_N-1-z-d}(1-\Psi)^d \}.$$

   (b) Agent i's bidding price is equal to mc^h, where $m+z < h \leq N$, and $0 \leq z \leq N_N - 1L$

$$\wp^i_{P,\, b^i}(P = mc^{m+z}) = \binom{m+z-1}{z} \Psi^{m-z}(1-\Psi)^z \cdot \{ \sum_{d=0}^{N_N-1-z} \binom{N_N-1-z}{d} \Psi^{N_N-1-z-d}(1-\Psi)^d \}.$$

The PMFs of market price given demand observed by Agent i, presented above, have closed-form formulations when each unit has the probability of the availability equal to Ψ, the installed capacity equal to q_{max}, the bidding price set at $b^i = mc^i$, and $mc^i < \ldots < mc^N$.

A.2.3 Simulations

This section presents examples to show the variations of PMFs of prices given demand. These PMFs vary according to demand levels and the probability of the availability of each agent. Information asymmetry among the agents due to non-uniform marginal-cost functions may result in by different perceptions of the PMFs of prices given demand of by different agents.

PMFs of Prices Given Demand

The following examples show the effect of the probability of the availability on the PMF of prices given demand. The market has a system marginal-cost function as shown in Figure A-1. There are 30 agents and each agent owns one generating unit. Each unit has a unique constant marginal cost. Demand is equal to 26 units of power (or units, for short). The marginal cost price P^0 =\$66.7/unit-hour. When the supply scarcity condition exists, the market price is set at \$200/unit-hour. Two scenarios are considered. In the first scenario, each unit has a probability of availability (Ψ) equal to 0.85. In the second scenario, each unit has a probability of availability equal to 0.95. The PMFs of both scenarios are shown in Figure A-2.

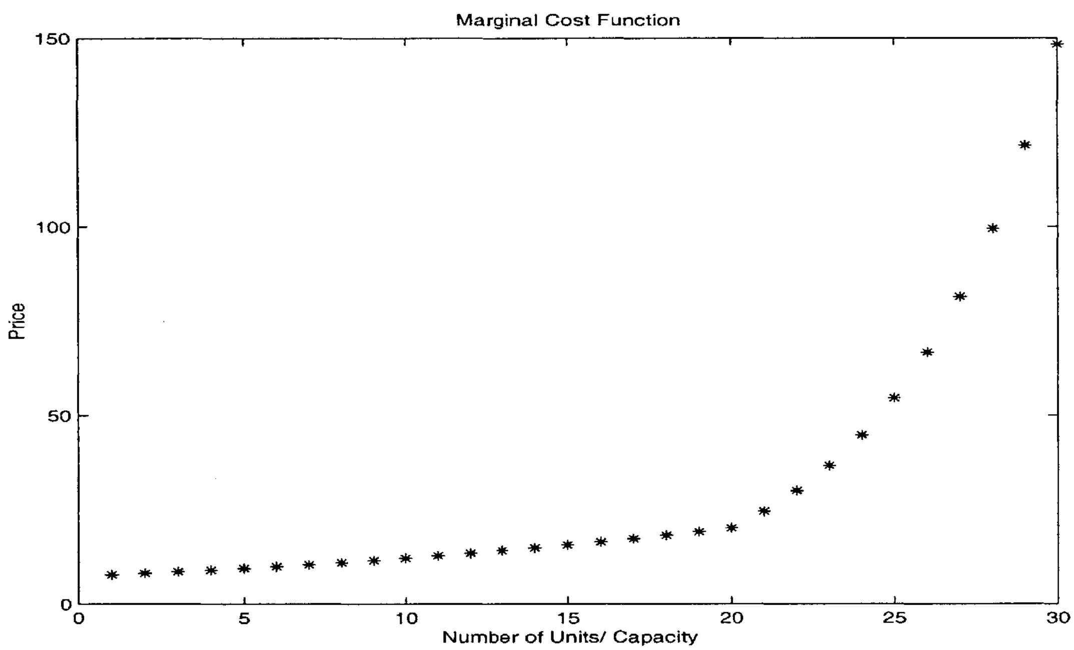

Figure A-1: Aggregated Marginal-cost Function of the Market with 30 Agents

Figure A-2 shows that when Ψ is large, the prices at any given demand tend to have a large variance; that is, they have a wider PMF. When demand is large relative to the total installed capacity and the units tend to be unavailable, the supply defficiency condition occurs with a high probability.

PMFs of Prices Given Demand Observed by Agent i

This section presents examples of PMFs of price given different demand levels observed by Agents A and B. This market has 125 generating units. Each unit has the same capacity. The total installed

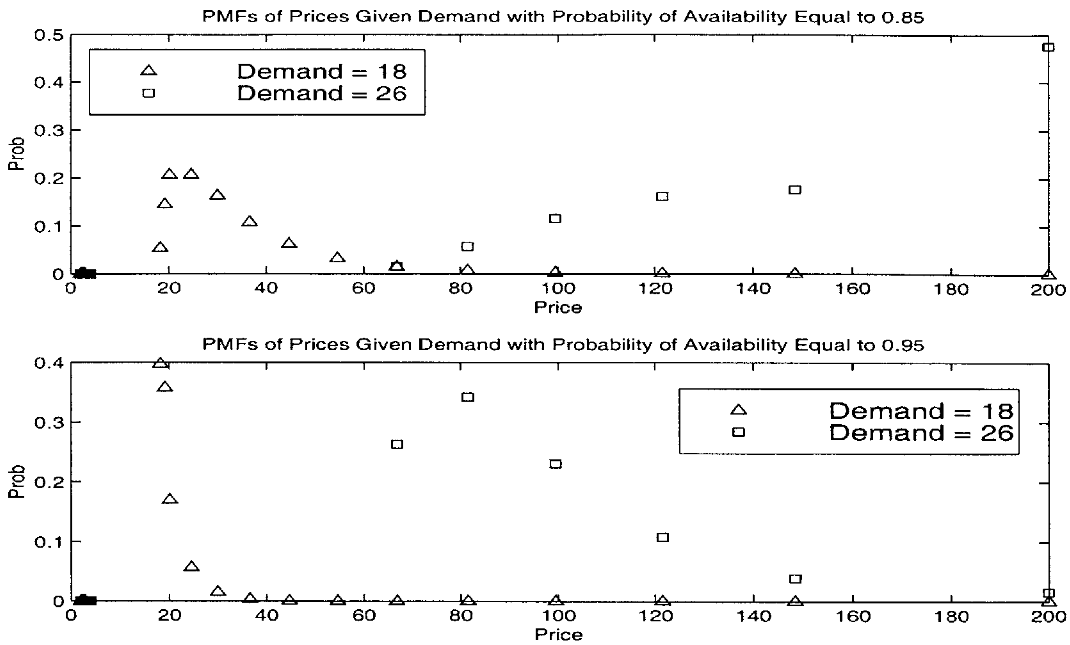

Figure A-2: PMFs of Prices Given Demand with Probability of Availability Equal to 0.85 and 0.95

capacity is equal to 125 units. Each unit has a constant marginal cost and no two marginal costs have the same values. This system's marginal-cost function is as shown in Figure A-3. Each unit has probability of availability equal to 0.9. When the supply scarcity condition exists, the market price is set at \$300/unit-hour. Two demand levels are considered. In the first scenario, demand is equal to 77 units and P^0 = \$29.4/unit-hour. This demand level is low compared to the total available capacity. In the second scenario, demand is equal to 103 units and P^0 = \$83.6/unit-hour. This demand level is high compared to the total available capacity. The agents are assumed to submit the same bidding price for all their units. Agents A and B own 6 units and their marginal-cost functions are shown in Table A.1. Agent A has fewer expensive units than Agent B has. In both cases under perfect competition, Agent A is scheduled to operate as an infra-marginal agent while Agent B is scheduled to operate as an extra-marginal agent in the first case, and as an infra-marginal agent in the second case.

Table A.1: Marginal Costs of Units Owned by Agents A and B

	Unit					
	1	2	3	4	5	6
Marginal Cost of Agent A	17.12	17.74	18.4	19.1	19.8	20.5
Marginal Cost of Agent B	49.40	51.42	53.51	55.7	57.98	60.34

As observed from Figures A-4 and A-5, the higher the demand, the wider the PMFs. When demand is large relative to total available capacity, the variance of price increases with a high probability that price will deviate from the marginal-cost price (P^0). Agent A with the cheaper marginal-cost units tends to have a greater impact on market price when demand is lower than when demand is higher. As observed from Figure A-4, when Agent A raises its bidding price higher than P^0 (or becomes unavailable), the market price increases. Agent B with the more expensive marginal-cost units tends to have a high impact on market price when demand is high and the least impact when demand is low. When the agent submits a bid that could set the market price, there is a high probability that the market price is equal to its bidding price. This corresponds to the fact that the peak of the PMF of prices given demand is equal to \$103/unit-hour as observed by Agents A and B when their bidding prices are equal to \$120/unit-hour. Moreover, when the agents with the same capacity are expected to be infra-marginal agents under a competitive condition, the perception of prices depends only on the bidding price. Both Agents A and B observe the same PMFs of prices given demand, when demand equals 103 units and their bidding prices equal \$20/unit-hour.

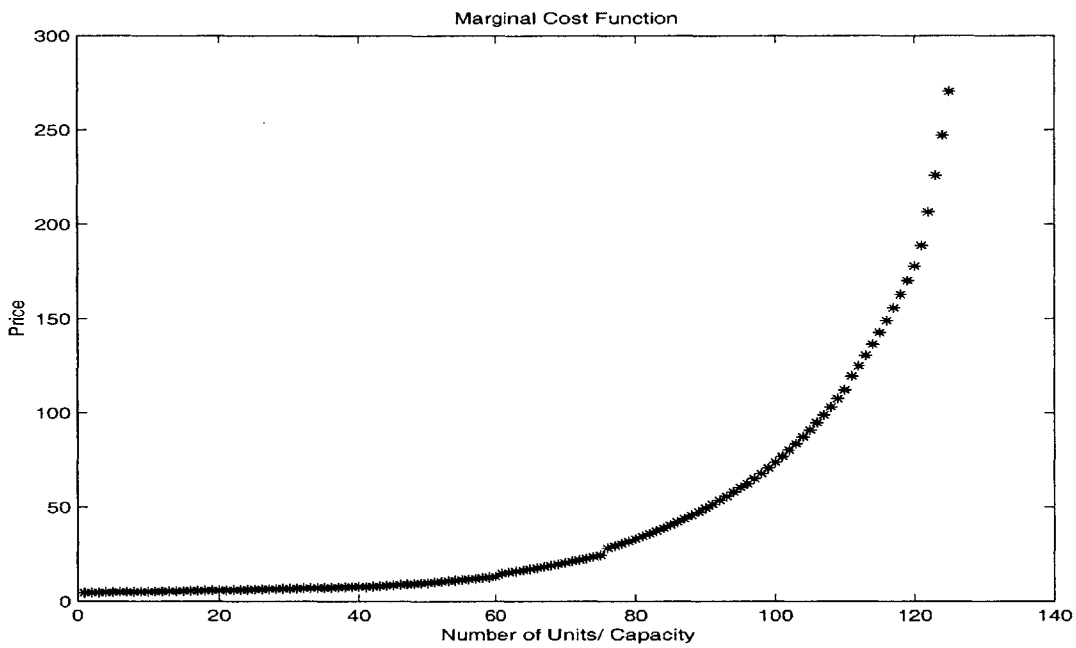

Figure A-3: Aggregated Marginal-cost Function of the Market with 125 Units

The next example in Figure A-6 shows the PMFs of prices given demand as observed by Agents 6 and 9 that have 6 and 9 generating units, respectively. Both agents' marginal-cost functions are shown in Table A.2. The same market setup as in the previous example is used (Figure A-3). Demand is assumed to be 77 units and the bidding prices of both agents are equal to \$20/unit-hour.

The agent who owns more units has relatively more information about the market compared to the agent owning fewer units. As shown in the simulated PMFs in Figure A-6, the PMF of price given demand of Agent 9 has a slightly narrower distribution compared to that of Agent 6. This emphasizes the findings presented by Bower and Bunn [8] that the size of agents could create

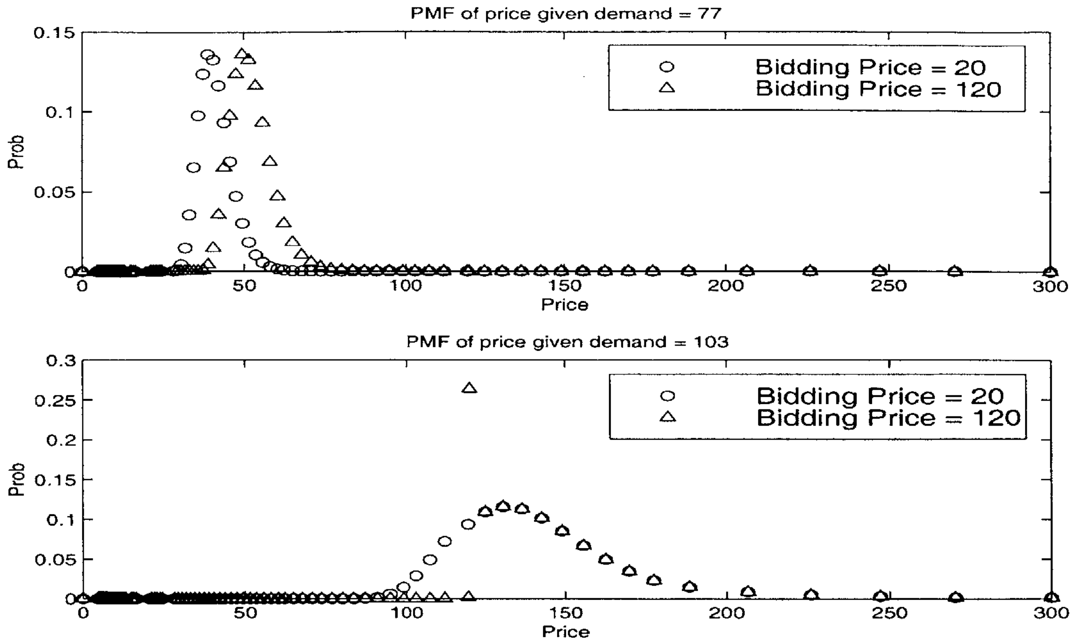

Figure A-4: PMFs of Prices Given Demand Equal to 77 and 103 Units as Observed by Agent A

Table A.2: Marginal Costs of Units

	Unit								
	1	2	3	4	5	6	7	8	9
Marginal Cost of Agent 6	17.12	17.74	18.4	19.1	19.8	20.5	-	-	-
Marginal Cost of Agent 9	17.12	17.74	18.4	19.1	19.8	20.5	21.2	22.0	22.8

information asymmetry and in turn influence their bidding behavior.

A.2.4 Observations

From the previous sections, one can observe that the larger the demand relative to the installed capacity, the wider the PMFs of prices given demand. When supply margin (i.e., the difference of the total installed capacity and demand) is small, there is a non-zero probability that supply scarcity occurs. This results in a high market price that could exceed the most expensive marginal cost. In the real market, the high price could be the price of the emergency contract, or the bidding prices of units that become unavailable at their (true) marginal costs.[5] When the unavailability due to unplanned outages is accounted for, the effect of supply scarcity and high prices would be substantiated. As observed, because the system's marginal-cost function is a convex and increasing function (which is a typical characteristic of the aggregate marginal cost functions in the existing markets), the PMFs of prices given demand are not normally distributed but rather are skewed to the right. The fat-tail

[5]Remember that the unavailability observed here is also due to strategic behavior.

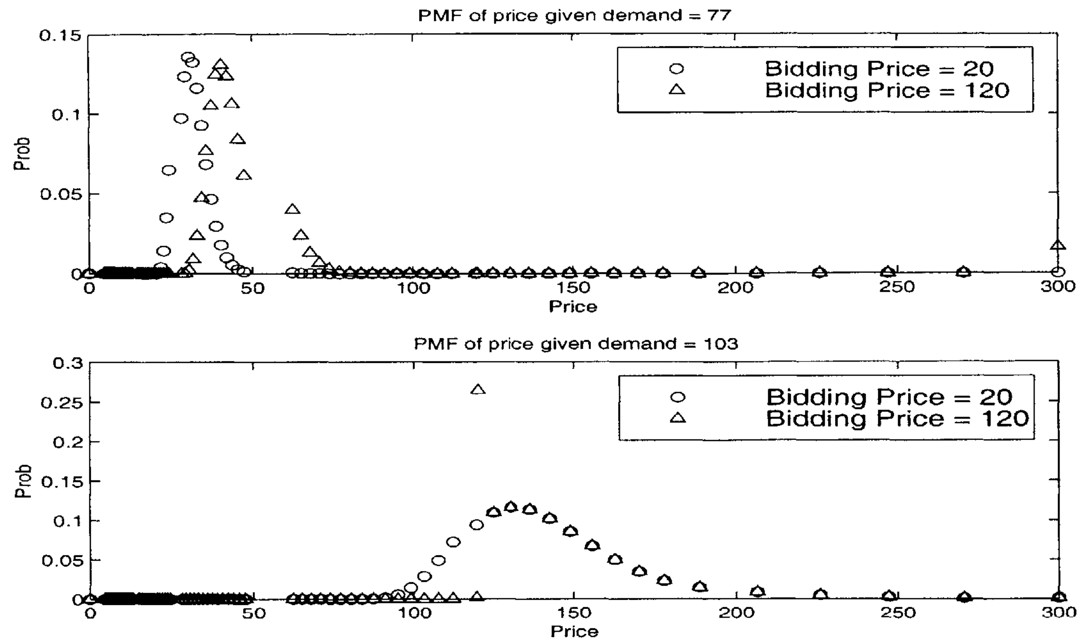

Figure A-5: PMFs of Prices Given Demand Equal to 77 and 103 Units as Observed by Agent B

phenomenon of the PMFs on the high price region exists, especially when supply margin is rather small.

A.3 PMFs of Prices Given Demand: Case II

This section presents generalized PMFs of prices given demand when units have non-uniform capacity, marginal costs, and probability of availability. No closed-form formulation for PMFs can be obtained.

A.3.1 PMFs of Prices Given Load

Suppose there are N units. Let $\mathcal{N}$ denote a set of generating units. Each unit i, $i \in \mathcal{N}$ has capacity q^i_{max} and marginal cost $mc^i(q^i)$. The probability of the availability of each unit i is denoted by Ψ^i. For any given load L, let P^0 denote the competitive price when all units are available at their marginal cost. Without unavailable units, let I_{In} denote a set of infra-marginal units, which are the units with marginal cost of at most P^0, i.e., $mc^g \leq P^0$, $\forall g \in I_{In}$.Without unavailability, extra-marginal units denoted by I_{Ex} are the units with marginal costs greater than P^0, i.e., $mc^h > P^0$, $\forall h \in I_{Ex}$. Note that $I_{In} \cup I_{Ex} = \mathcal{N}$.

Let U denote a set of unavailable units and P^u denote the marginal-cost price when u units are unavailable at time k and $u \in U$. Note that the index k representing time is omitted for simplicity of notation. To derive the PMF of prices given demand, let us follow a method similar to that used in the previous sections. To start, the following scenarios are considered.

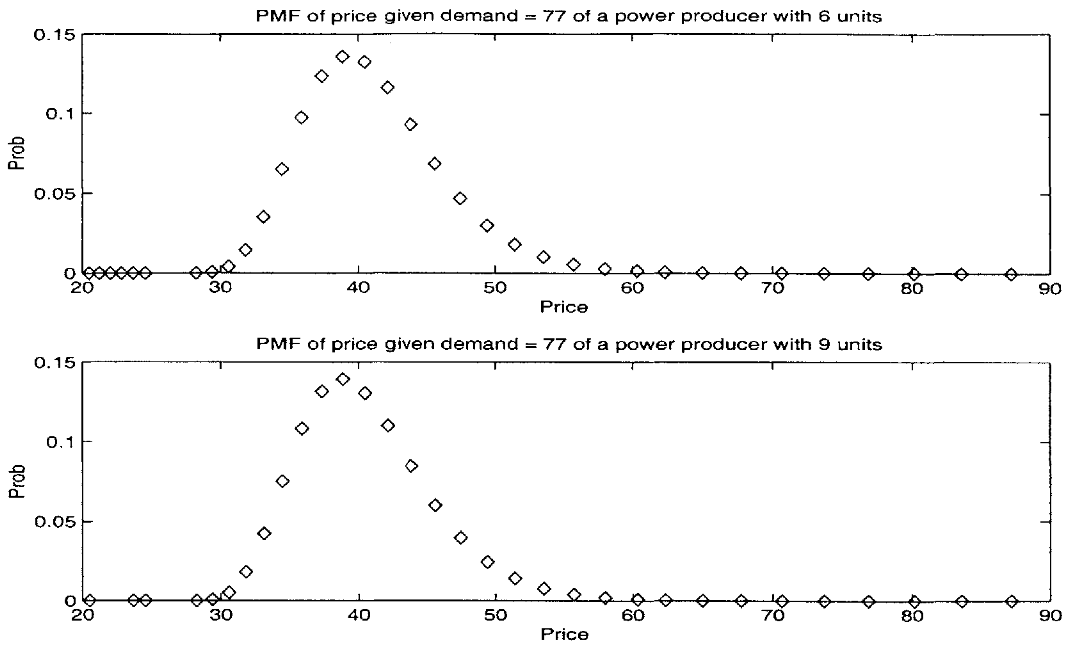

Figure A-6: PMFs of Prices Given Demand as Observed by Agents 6 and 9

1. One unavailable unit

Let P^1 denote the market price when one unit is unavailable. The anticipated price is at least equal to P^0, i.e., $P^1 \geq P^0$. When market price remains at P^0, the unavailable unit is small or not an infra-marginal unit. When the unavailable unit is sufficiently large, market price increases to be greater than P^0. Let us consider the following outcomes resulting from one unit's becoming unavailable.

1. When the unavailable unit belongs to set I_{Ex} ($u \in I_{Ex}$), the unavailability results in unchanged market price, i.e., $P^1 = P^0$, and the probability of prices equal to P^0 is equal to

$$\wp_{L|u \in I_{Ex}}(P^1 = P^0) = (1 - \Psi^u) \cdot \prod_{\substack{h \in I_{Ex} \\ h \neq u}} \Psi^h \cdot \prod_{g \in I_{In}} \Psi^g.$$

2. When the unavailable unit belongs to set I_{In} ($u \in I_{In}$), market price may increase, i.e., $P^1 \geq P^0$, depending on the capacity of the unavailable unit. When this unit is so small that its unavailability does not affect market price, $P^1 = P^0$. Otherwise, $P^1 = mc^j$, $j \in I_{Ex}$. When one unit is unavailable, the probability of prices equal to P^0 is equal to

$$\wp_{L|u \in I_{In}}(P^1 = P^0) = (1 - \Psi^u) \cdot \prod_{\substack{g \in I_{In} \\ g \neq u}} \Psi^g \cdot \prod_{h \in I_{Ex}} \Psi^h.$$

2. m unavailable units

When m units are unavailable, the anticipated price is at least equal to P^0, i.e., $P^m \geq P^0$, where P^m denotes the market price when m units are unavailable. These unavailable units may not change market prices if they are not infra-marginal units or the total capacity of unavailable units is small. However, if the unavailable units are sufficiently large to change the price, the market price will increase. Let us consider the following outcomes resulting from m units' becoming unavailable.

1. When the unavailable units belong to set I_{Ex} ($m \in I_{Ex}$), market price remains unchanged, i.e., $P^m = P^0$. The probability of prices equal to P^0 When m units are unavailable is equal to

$$\wp_{L|u \in I_{Ex}}(P^m = P^0) = \prod_{u \in U, I_{Ex}} (1 - \Psi^u) \cdot \prod_{\substack{h \in I_{Ex} \\ h \neq u}} \Psi^h \cdot \prod_{g \in I_{In}} \Psi^g.$$

2. When the unavailable units belong to set I_{In} ($m \in I_{In}$), depending on the capacity of the unavailable units, if the units are so small that the unavailability does not affect market price, $P^m = P^0$. Otherwise, market price increases to a marginal cost of a unit belonging to set I_{Ex}, i.e., $P^m = mc^j$, $j \in I_{Ex}$. The probability of prices equal to P^0 when m units are unavailable is equal to

$$\wp_{L|u \in I_{In}}(P^m = P^0) = \prod_{u \in U, I_{In}} (1 - \Psi^u) \cdot \prod_{\substack{g \in I_{In} \\ g \neq u}} \Psi^g \cdot \prod_{h \in I_{Ex}} \Psi^h.$$

3. when the unavailable units belong to set I_{In} and I_{Ex} ($u_1 \in I_{In}$ and $u_2 \in I_{Ex}$), market prices may be affected by the unavailability, i.e., $P^m \geq MC(L)$. Depending on the capacity of the unavailable unit, if this unit is small so that its unavailability does not affect market price, $P^m = P^0$. Otherwise, market price increases to a marginal cost of a unit in set I_{Ex}, i.e., $P^m = mc^j$, $j \in I_{Ex}$. The probability of prices equal to at least P^0 when m units are unavailable is equal to

$$\wp_{L|u \in I_{In}|I_{Ex}}(P^m \geq P^0) = \prod_{u_1 \in U, I_{In}} (1 - \Psi^{u_1}) \cdot \prod_{u_2 \in U, I_{Ex}} (1 - \Psi^{u_2}) \cdot \prod_{\substack{g \in I_{In} \\ g \neq u}} \Psi^g \cdot \prod_{h \in I_{Ex}} \Psi^h$$

where, $u_1 + u_2 = m$. The market price could exceed the maximum marginal cost ($\max_j mc^j$), i.e., $P^m > \max_j mc^j$ when demand is larger than supply margin (defined by $\sum_j^N q^j_{max} - L$), or when $\sum_{f \in I_{Ex}} q^f_{max} < \sum_{d \in I_{In}} q^d_{max}$ and $\sum_{u \in U} q^u_{max} > \sum_{f \in I_{Ex}} q^f_{max}$. When this condition holds true, the price spike is likely to occur.

3. N unavailable units

When N units are unavailable, the anticipated price exceeds the most expensive marginal cost, i.e., $P^N = P^0 > \max_j mc^j$. The probability of prices greater than P^0 when all units are unavailable is equal to

$$\wp_{L|u=N}(P > P^0) = \prod_{u \in U = N} (1 - \Psi^u).$$

Observations

From these scenarios, one can observe that unavailability does not necessarily cause market price to change. In addition, the PMFs of price given demand in this case have no closed-form formulation, and the complexity of the derivation can be analyzed as follows. Let $\mathcal{A}$ denote a set of possible scenarios in which there are u unavailable units ($u \in \mathcal{N}$). Following the previous derivation, after obtaining all possible prices and the associated probabilities of the availability, the PMFs of prices given demand are calculated by ordering possible prices from the smallest to the highest and adding the probability of all events contributing to that price. A total of possible scenarios including at least u units' becoming unavailable, where $0 \leq u \leq N$.

$$\mathcal{A} = \sum_{u=0}^{N} \begin{pmatrix} N \\ u \end{pmatrix}.$$

Several scenarios may create the same prices due to the non-uniform capacity of the units. Let a denote a scenario resulting in one market price P^a and $\cup a = \mathcal{A}$. The probability of prices equal to X given demand L is equal to

$$\wp_L(P = P^a) = \sum_{a,\ \mathrm{st.}\ P^a = X} \wp_{L|u \in a}(P^u = X).$$

In summary, the factors contributing to the complexity in deriving the PMF of prices given load from the set of non-uniform units include:

- The total possible outcomes of a unavailable units are equal to $\mathcal{A} = \sum_{u=0}^{N} \begin{pmatrix} N \\ u \end{pmatrix}$; therefore, the number of calculations increases as the number of generating units (N) increases.
- Non-uniform capacity and marginal costs make mapping from a market price to a scenario, in which there are u unavailable units, not unique. To derive the PMF of prices given demand, all possible scenarios and their associated market prices must be obtained before determining the PMF.
- The derivation of the probability of the availability of each unit (Ψ) and the PMF of prices

given demand is difficult without historic detailed information of each unit, including bidding prices, bidding quantities, marginal costs, maximum available (physical) capacity, maintenance schedules, and near real-time outages.

A.3.2 Simulations and Analyses

To show the effect of information available to an agent on its price perception, a set of simulations to determine the PMFs of prices given (inelastic) demand under different available information conditions is presented. Three market scenarios, Markets 1, 2, and 3, are selected. There are 10 units in Market 1, 13 units in Market 2, and 15 units in Market 3. The characteristics of the units in each market are summarized in Table A.3. When the generators in each market have a constant marginal cost. The system marginal-cost function is a step-wise function. The system marginal-cost function of each market is shown in A-7. The maximum installed capacity in the markets is set to 60 units and $P^0 =$ \$26/unit-hour. When supply deficiency occurs, market price is assumed to be \$100/unit-hour.

Cases of Interest

1. The simulations below are based on Market 2 with demand equal to 38 units and they show the effect of available information on the PMFs of prices given demand obtained by different observers.

   (a) Case I: The simulation shows the PMF of prices given demand observed by an outsider or an agent who does not own any unit in the market. This outsider knows demand and the probability of the availability of each unit. By assuming that the agents submit a marginal-cost bid, the PMF of prices given demand observed by this outsider is shown in Figure A-8 Case I.

   (b) Case II: The simulation shows the PMF of prices given demand observed by Agent i or unit 3. Agent i anticipates market price by assuming that the other agents submit their marginal-cost bids. The PMF of prices given demand observed by Agent i is shown in Figure A-8 Case II.

   (c) Case III: The simulation shows the PMF of prices given demand observed by an outsider or an agent who does not own any unit in the market. The outsider observes the market prices and knows which units are under maintenance. In this case, unit 2 is assumed to be under maintenance. By assuming that all agents submit their marginal-cost bids, the PMF of prices given demand and units under maintenance observed by this outsider is shown in Figure A-8 Case III.

   (d) Case IV: The simulation shows the PMF of prices given demand observed by Agent i or unit 3. Let Agent i set its bidding price at \$26.5/unit-hour (its marginal cost equal to \$20/unit-hour). By assuming that the other agents submit a marginal-cost bid and it submits a strategic bid (i.e., the bidding price higher than marginal cost), the PMF of prices given demand and its bid observed by Agent i is shown in Figure A-8 Case IV.

(e) Case V: The simulation shows the PMF of prices given demand observed by Agent i or unit 3. Let Agent i set its bidding price at \$26/unit-hour and unit 2 is under maintenance. By assuming that the other agents submit a marginal-cost bid and Agent i submits a strategic bid (i.e., the bidding price higher than marginal cost), the PMF of prices given demand, its bid, and the units under maintenance observed by Agent i is shown in Figure A-8 Case V.

Table A.3: Marginal-cost Functions of Markets 1, 2, and 3

	Market 1			Market 2			Market 3		
Gen	MC	Cap	Prob	MC	Cap	Prob	MC	Cap	Prob
1	12	15	0.93	12	5	0.95	12	5	0.92
2	20	10	0.98	12	10	0.97	12	5	0.93
3	23	10	0.92	20	10	0.95	12	5	0.98
4	26	5	0.96	23	3	0.92	20	5	0.98
5	27	4	0.95	23	3	0.93	20	5	0.98
6	29	4	0.97	23	4	0.97	23	3	0.95
7	30	4	0.93	26	5	0.98	23	3	0.96
8	35	3	0.97	27	4	0.96	23	4	0.93
9	45	3	0.95	29	4	0.98	26	5	0.93
10	60	2	0.98	30	4	0.98	27	4	0.97
11				35	3	0.93	29	4	0.97
12				45	3	0.96	30	4	0.96
13				60	2	0.93	35	3	0.91
14							45	3	0.91
15							60	2	0.95

These plots show that the PMFs of prices given demand and information of the observer change according to the observer's information set, for example, unit 3 clearly sees different anticipated prices from what an outsider sees (cases I and IV).

2. The simulation in Figure A-9 shows the PMFs of prices given demand of Markets 1, 2, and 3, consisting of 10, 13, and 15 units, respectively. Demand is equal to 38 units and P^0 = \$26/unit-hour. All units are assumed to submit their marginal-cost bids. This figure shows that when the number of infra-marginal units increases given that demand and total installed capacity remain the same, the PMFs of prices given demand become wider. The peak of the PMFs of prices given demand occurs at P = \$26/unit-hour and this peak of Market 3 is the smallest. The second peak occurs at P = \$27/unit-hour and this peak of Market 3 is the highest.

3. The simulation in Figure A-10 shows the PMFs of prices given demand of Market 1 when demands are equal to 18, 28, 38, and 48 units. All units are assumed to submit their marginal-

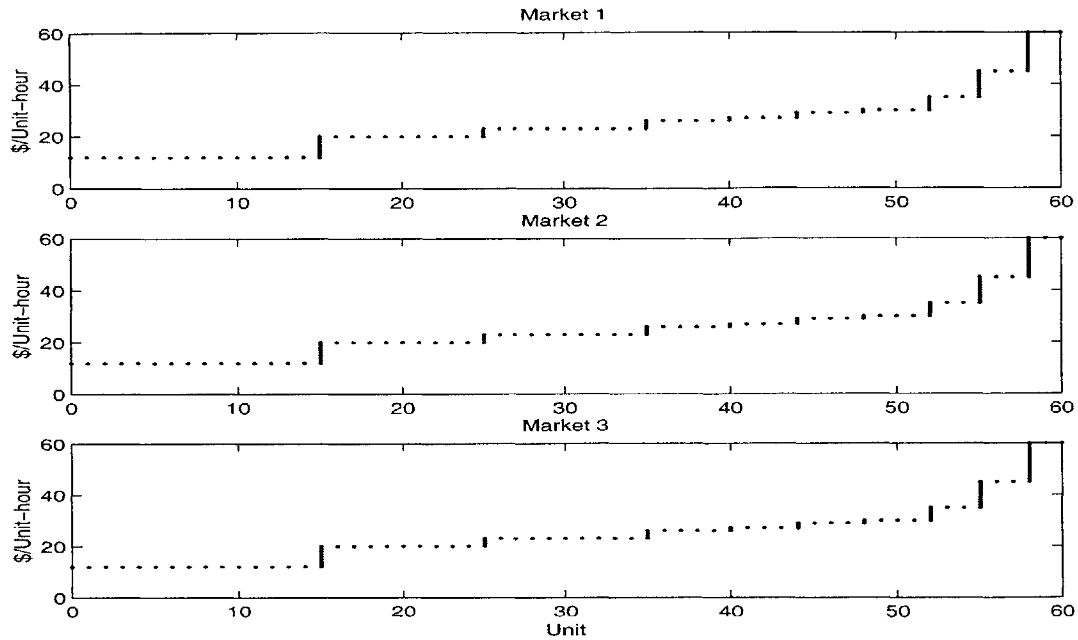

Figure A-7: Marginal-cost Characteristics of Markets 1, 2, and 3

cost bids. As one might anticipate, the higher the demand, the higher the market price. As in the previous section, the PMFs shift to the higher price zone when demand increases. Additionally, the higher the demand level, the wider the PMFs of prices given demand and the condition indicating demand scarcity (and, subsequently, price spikes) is observed with a high probability when demand increases or when the supply margin decreases.

A.3.3 Observation

Proposition: *Given two Markets A and B with the same total number of units $N^A = N^B = N$, in which each unit i has the same Ψ^i, total available capacity $Q^A_{max} = Q^B_{max} = Q_{max}$, and marginal-cost functions (when all units are available) such that $S^A(q) \geq S^B(q)$, $\forall 0 \leq q \leq Q_{max}$. The probability of u unavailable units with the same indices i in Markets A and B is equal to $\wp_{L,u}(P)$. The market price in Market A is no less than the market price in Market B, i.e., $P^{A,-u} \geq P^{B,-u}$.*

Proof: Let us consider each scenario s in which at least one unit i of Markets A and B is unavailable. Suppose this unit i has capacity q^i and marginal cost $mc^{A,i}$ and $mc^{B,i}$ in Markets A and B, respectively. Because $S^A(q) \geq S^B(q)$, $\forall 0 \leq q \leq Q_{max}$,

$$mc^{A,i} \geq mc^{B,i}.$$

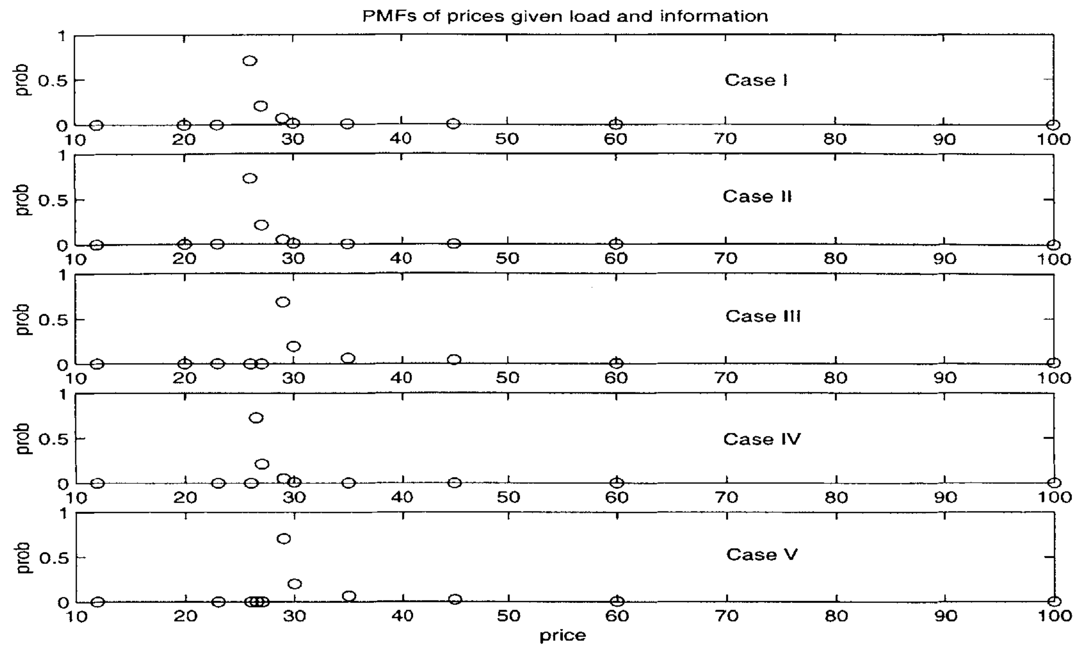

Figure A-8: PMFs of Prices Given Demand under Different Sets of Available Information

The supply functions without unavailable unit i, of Markets A and B, $S^{A,s}(q)$ and $S^{B,s}(q)$ are such that, i.e., for $s = 1$ and $Q^s = \sum_{j=1}^{i-1} q^j$,

$$
\begin{aligned}
&S^{A,s}(q) = S^A(q) && \forall\, 0 \le q \le Q^s, \\
&S^{B,s}(q) = S^B(q) && \forall\, 0 \le q \le Q^s, \\
&S^{A,s}(q) = S^A(q + q^i) && \forall\, Q^s \le q \le (Q_{max} - q^i), \\
&S^{B,s}(q) = S^B(q + q^i) && \forall\, Q^s \le q \le (Q_{max} - q^i), \\
&S^{A,s}(q) = S^A(q + q^i) \ge S^B(q + q^i) && \forall\, Q^s \le q \le (Q_{max} - q^i).
\end{aligned}
$$

Hence, for $s \ge 1$,

$$
\left.
\begin{aligned}
S^{A,s}(q) &\ge S^A(q) \\
S^{B,s}(q) &\ge S^B(q) \\
S^{A,s}(q) &\ge S^{B,s}(q)
\end{aligned}
\right\}
\quad \forall\, 0 \le q \le Q_{max} - \sum_{i \in s} q^i.
$$

A.4 Imperfect Competition with Elastic Demand

This section explains the effect of price elasticity on the PMFs of market prices given demand observed by Agent i. When demand is price-elastic, an increase in market price reduces the total consumption. Therefore, a narrower PMF of prices given an price-elastic demand function is anticipated compared to that given price-inelastic demand.

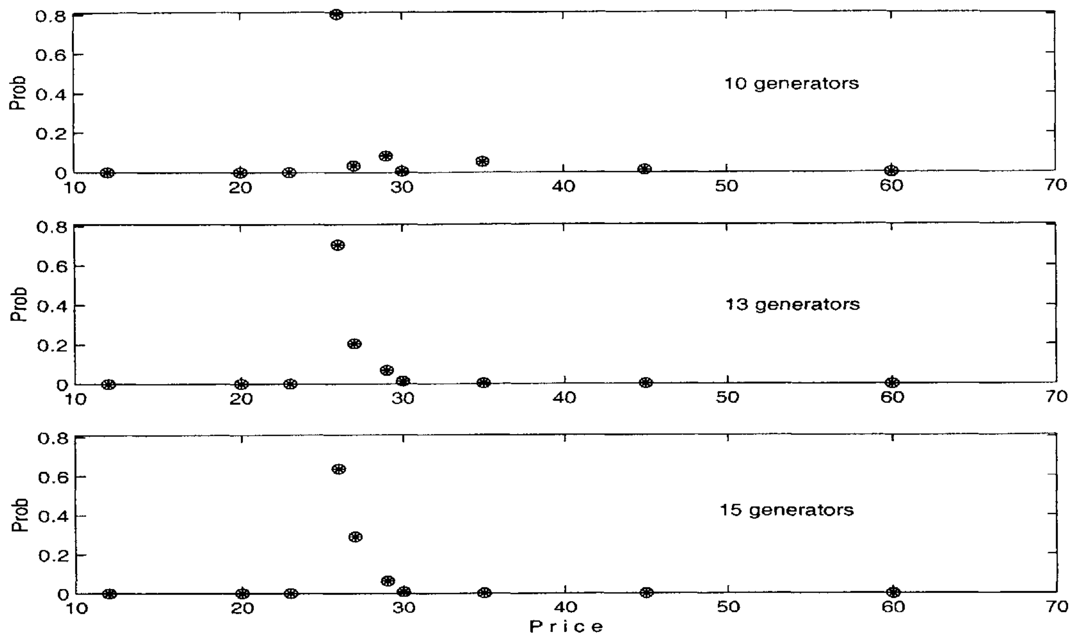

Figure A-9: PMFs of Prices Given Demand in the Markets 1, 2, and 3

Let L^{In} and L^E denote inelastic and elastic demand functions, respectively. Let $P_L = D(L^{In} = L)$ denote a willingness-to-pay function, $D(\cdot)$ be a non-increasing function of demand L, and $MC(\cdot)$ represent a system marginal-cost function and MC be an increasing function of bidding quantity. Let P^0 and L^0 denote market price and total demand when there is no unavailability and the agents submit their marginal-cost bids. Note that $P^0 = MC(q = L^{in} = L^0) = MC(q = L^E = L) = D(L^{In} = L))$.

Proposition: *Given that there are N units and each unit i has the total capacity q^i_{max} and the same Ψ^i, the anticipated market prices given elastic demand (L^E and $P^0 = D^{L^0}$) have lower mean and variance than the anticipated market prices given inelastic demand ($L^{in} = L^0$).*

Proof: Let elastic demand be a decreasing and continuous function and defined by

$$\begin{aligned} P_L = \;& D(L) \\ \text{s.t.} \;\; & \frac{\partial P_L}{\partial L} < 0 \\ P_L \le \;& D(0) < \infty, \;\; \forall\, L > 0. \end{aligned}$$

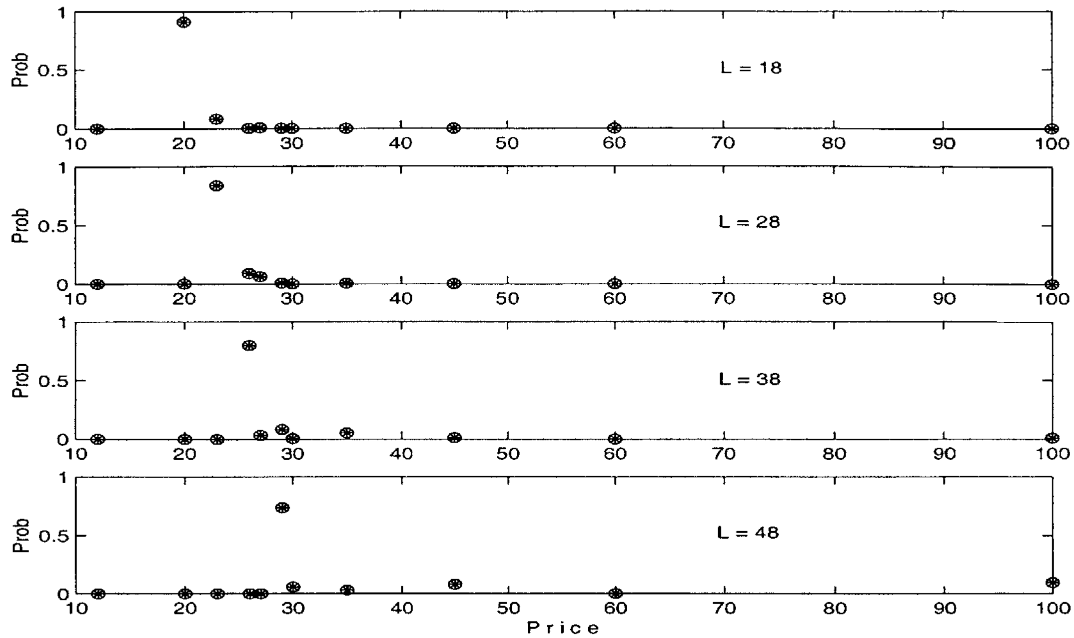

Figure A-10: PMFs of Prices Given Demand Equal to 18, 28, 38, or 48 Units in Market 1

Let a supply function[6] with no unavailable unit be a non-decreasing and continuous function and be defined by

$$P = S(q), \quad \forall\ 0 \leq q \leq q_{max}$$
$$\text{s.t.} \quad \frac{\partial P_P}{\partial q} \geq 0.$$

Market price P^0 is the price at which the demand function ($D(L)$) intersects with the supply function ($S(q)$) at demand equal to x^* such that

$$P^0 = S(x^*) = D(x^*),$$

where $x^* = L^0$. If there are u unavailable units such that the unavailable capacity is equal to q^{-u}, where $q^{-u} = \sum_{i \in u} q^i_{max}$, the supply function will change to $\acute{S}(q)$. Therefore,

$$\acute{S}(q) \geq S(q), \quad \forall\ 0 \leq q \leq (\sum_{i=1}^{N} q^i_{max} - q^{-u}). \tag{A.1}$$

Let us consider the case in which the above inequality strictly holds, i.e., $\acute{S}(q) > S(q)$. Let $\tilde{P} = S(\tilde{q})$. When a unit u with the cheapest marginal cost (at $q^u = 0^+$) equal to $\tilde{P}$ becomes unavailable, a part

[6] A supply function is a collection of all agents' marginal-cost functions ordered from the cheapest to the most expensive marginal cost. For example, let $mc^i(q)$ denote marginal-cost function of Agent i and $mc^i(q) = c^i \cdot q$, $0 \leq q \leq q^i_{max}$, where c^i is a constant. $S(q) = ((c^1)^{-1} + \ldots + (c^N)^{-N})^{-1} \cdot q$.

of the supply function, which has $0 < q < \tilde{q}$, does not change, i.e., $\acute{S}(q) = S(q)$. The other part of the supply function, in which $q > \tilde{q}$, is unable to serve demand $L > \tilde{q}$ at the same price due to q^{-u}_{max} being unavailable.[7] As a result, $\acute{S}(q) \geq S(q)$, where $\tilde{q} \leq q \leq (Q_{max} - q^{-u})$. On the other hand, when unit u with marginal cost at most $\tilde{P}$ becomes unavailable, to serve demand $L > \tilde{q}$ at the same price is not possible since unit u is not available. The part of the supply function in which $\tilde{q} \leq q \leq (Q_{max} - q^{-u})$ shifts to a high price zone resulting in $\acute{S}(q) \geq S(q)$, where $q \geq L^0$. A similar explanation holds for more than one unavailable unit and one can obtain $\acute{S}(q) \geq S(q)$, where $\acute{S}(q)$ is defined.

Let us consider the scenario in which unavailable units cause the supply function to change in the neighborhood of the demand function at $q = x^*$. Let ΔP denote a positive increment of prices at $q = x^*$ and be defined as follows:

$$\Delta P = \acute{S}(x^*) - S(x^*) > 0.$$

Since the demand function is a decreasing function,

$$D(x_1) > D(x_2), \quad \forall\, x_1 < x_2.$$

The supply function is an increasing function, therefore,

$$\acute{S}(x_1) < \acute{S}(x_2), \quad \forall\, x_1 < x_2.$$

For some demand $L = \tilde{x}$, in which $x^* < \tilde{x}$ and

$$D(\tilde{x}) = P^0 + \Delta P,$$

the willingness-to-pay of $L = \tilde{x}$ is

$$D(x^*) < D(\tilde{x}).$$

On the other hand, for some supply quantities, i.e., $q = \tilde{x}$, one can obtain

$$P^0 + \Delta P = \acute{S}(x^*) > \acute{S}(\tilde{x}).$$

This result means that an intersection between the supply function $\acute{S}$ and the demand function D exists, and there exists $\acute{x}$ such that

$$\tilde{x} < \acute{x} < x^*.$$

[7] Since unit u is part of the supply function.

Therefore,

$$\text{given} \qquad \tilde{x} < \acute{x} < x^*,$$
$$D(\tilde{x}) > D(\acute{x}) > D(x^*). \tag{A.2}$$

When at least one unit is unavailable, Equation (A.1) always holds. Equation (A.2) indicates that given two markets with the same supply function $S(q)$, $P^0 = S(L^0)$ and $L^E = L^{in} = L^0$, when some units become unavailable, the market price in the market with elastic demand is at least the market price in the market with inelastic demand.

A.4.1 PMFs of Prices Given Elastic Demand

Let us consider a market with N units and unavailability of each unit due to outages be independent and not accounted for. Note that the unavailability is, therefore, due to strategic bids. Each unit has the same probability Ψ and capacity q_{max}. Marginal cost of each unit is a constant and orders such that

$$mc^1 < \ldots < mc^i < \ldots < mc^N.$$

The supply function is defined by $P = S(q)$. Without unavailable units and with all units submitting their marginal cost bids, the market price is equal to P^0.

Demand Characteristics

Demand in each hour is

$$W = D(L)$$

where W is the willingness-to-pay to consume total demand L. The demand function $D(L)$ is defined by

$$W = \begin{cases} A^1, & 0 \leq L \leq L^2 \\ A^2 - B \cdot L, & L^2 < L \leq L^3 \\ 0, & L > L^3 \end{cases}$$

where A^1, A^2, and B are constant numbers and the maximum willingness-to-pay of the customers is A^1.

Procedures for Deriving PMFs

Suppose that there are u unavailable units ($0 \leq u \leq N$). This unavailability causes the supply function to shift to the left (because the supply function is a step-wise function when one step is subtracted from the supply function, it causes the function to the right of the step to move left.). The market price increases from P^0 to P^u, where P^u is the market price when u units become unavailable. The derivation of the PMFs of market prices given an elastic demand function is similar to that in the previous sections when demand is inelastic. The unavailability of infra-marginal units might not change the market prices due to the reduction of total demand, according to the demand function. The supply function may intersect the demand function into two regions including one which has the demand function has a negative slope and the other one which has a zero slope. Let us consider when

1. The demand function has a negative slope. When at least one unit (u) becomes unavailable during time k, the supply function from the position of unit u is shifted to the left (or costs of producing electricity increases) and market price increases. This price-increment is a function of capacity of unit u and the slope of the demand function ($\frac{B}{A}$). Let P^1 denote the market price when one unit with a marginal cost less than P^0 is unavailable and be equal to

$$P^1 = mc^{m+1}(q_{max}) - \frac{B}{A} \cdot q_{max}.$$

2. The demand function has a zero slope. When more than one unit (u) becomes unavailable during time k, the supply function shifts to the left and market price remains the same because demand has the constant willingness-to-pay ($\max(W)$) independent of the total consumption. This causes the decreasing consumption for any unavailable unit with a marginal cost less than $\max(W)$.

The PMFs of price given a demand function can be derived following the procedures in the previous section, by exploring all possible scenarios, determining the market price, and evaluating the probability associated with those scenarios and prices. The detailed procedure for deriving the PMFs of prices given a demand function are not presented here and only the simulated PMFs to compare the effects of price-elasticity on the PMFs are presented.

A.4.2 Simulations and Analyses

The simulations show the effect of price elasticity on the PMFs of prices given a demand function. Let us consider two markets with identical units, consisting of 125 units as shown in Figure A-3. Each unit has a constant marginal cost and the same capacity. The first market faces inelastic demand equal to L^0. The other market faces elastic demand with the willingness-to-pay function, $D(L)$. Without unavailable units and with all units submitting their marginal cost bids, the market price in both markets is equal to P^0. A demand function (W) shown in Figure A-11 is as follows:

$$W = \begin{cases} 280 & 0 \leq L \leq 45 \\ 280 - 4.308L & 45 < L \leq 110 \\ 0 & L > 110. \end{cases} \tag{A.3}$$

When there is no outage, the demand function intersects the supply function at demand approximately

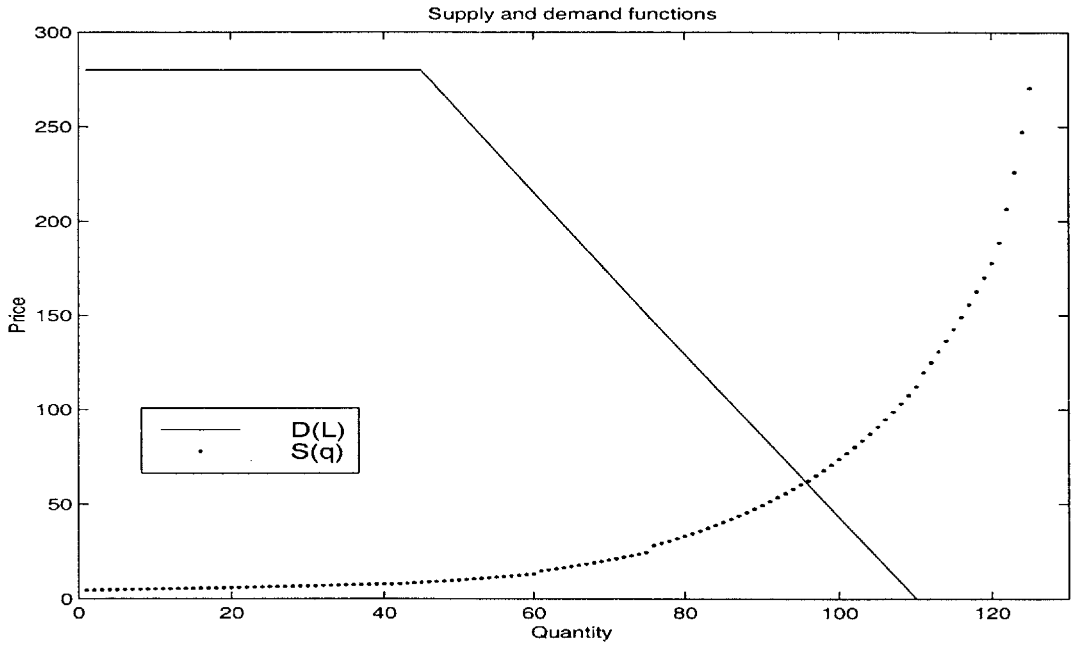

Figure A-11: Characteristics of Demand and Supply Functions

equal to 96 units and no-outage marginal-cost price equal to \$62.3/unit-hour. These PMFs of prices given a demand function are observed by Agent A with the marginal-cost function shown in Table A.1. Agent A submits the bidding price equal to \$20/unit-hour. As anticipated, the PMF of prices given inelastic demand equal to 96 units is wider than that of the PMF of prices given elastic demand according to Equation (A.3). This means the market prices in the market with inelastic demand has a higher variance than those in the market with elastic demand. These PMFs are shown in Figure A-12.

A.5 Effects of Unit-commitment Constraints on Prices

This section presents an introductory analysis of the effect of unit-commitment constraints on observed prices given demand of Agent i. Let us consider the PMF of prices given forecast demand at least one hour from now. Suppose that the unit-commitment constraints associated with each unit's operation is publicly known. This assumption implies that the variation of market prices at any given demand

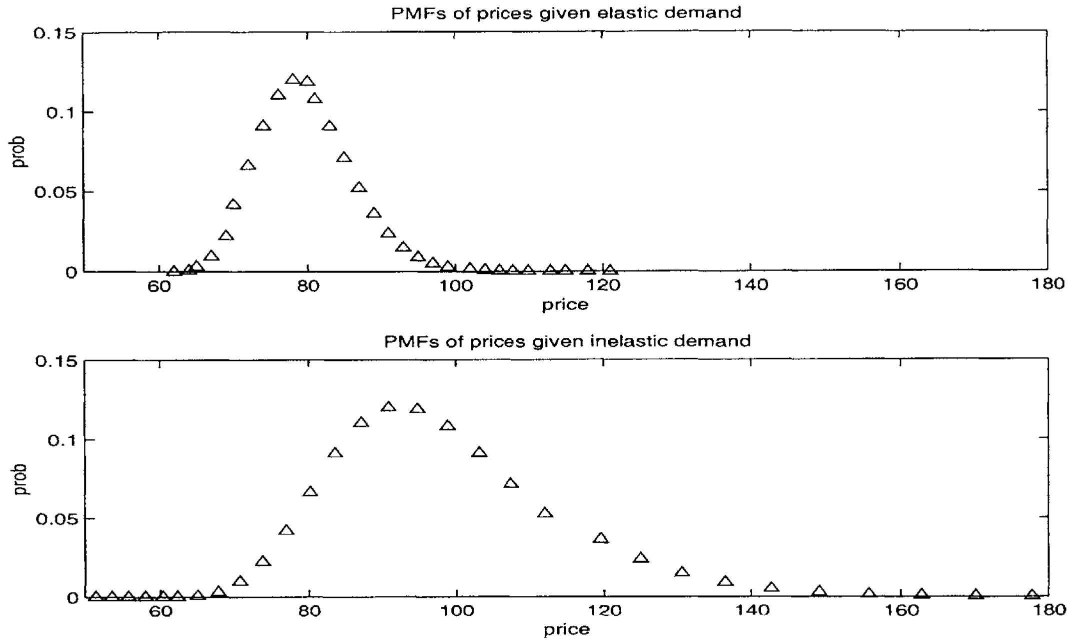

Figure A-12: PMFs of Prices Given Demand under Inelastic and Elastic Demand Conditions

is due to strategic bids. Let ψ_k^i denote whether a generating unit i is available at time k, i.e., $\psi_k^i = 1$ means unit i is available. Let the availability condition depend only on the previous period. When the unit-commitment constraints are considered, the availability of the units at time k depends on the availability at time $k-1$. Given the probability of the availability of Agent i at time $k-1$ denoted by Ψ_{k-1}^i and the transition probability of the availability from period $k-1$, ψ_{k-1}^i, to period k, ψ_k^i, denoted by $\Psi_{k-1,k|\psi_{k-1}^i}^i$, one may derive Ψ_k^i as follows:

$$\Psi_k^i = \Psi_{k-1,k|\psi_{k-1}^i=1}^i \cdot \Psi_{k-1}^i + \Psi_{k-1,k|\psi_{k-1}^i=0}^i \cdot (1 - \Psi_{k-1}^i).$$

When the unit is in the "off" state and needs more than one period to be in the "on" state. Therefore, the probability that the unit is going to be available the next period given that the unit is not available during the current period, i.e.,

$$\Psi_{k|\psi_{k-1}^i=0}^i = \Psi_{k-1,k|\psi_{k-1}^i=0}^i. \tag{A.4}$$

The transition from the "off" state in $k-1$ to the "on" state in k is not possible (due to unit-commitment constraints); hence,

$$\Psi_{k-1,k|\psi_{k-1}^i=0}^i = 0.$$

From Equation (A.4),

$$\Psi_k^i = 0.$$

This shows that once Agent i has more information about the availability of other units in the market (i.e., closer to the operation time), its ability to anticipate available capacity is improved. Let us consider the effect of this improved information on the PMFs of market prices given demand. Suppose that there are N_{Uc} units that are not in their "on" states at time $k-1$ and these units are unable to be turned on in one period. For each unit j, $j \in N_{Uc}$, its probability of the availability in period k is $\Psi_k^j = 0$ Suppose each unit has capacity q_{max}^i. The system's marginal cost function when all units are available, $S(q)$, changes to $\acute{S}(q)$ as in Equation (A.1), and

$$\acute{S}(q) \geq S(q), \quad \forall \; 0 \leq q \leq (Q_{max} - q^{-N_{Uc}}) \tag{A.5}$$

where $Q_{max} = \sum_{i=1}^{N} q_{max}^i$ and $q^{-N_{Uc}} = \sum_{i \in N_{Uc}} q_{max}^i$.

One could derive the PMF of prices given demand by applying the same procedure described in the previous sections, replacing total units N with $\acute{N} = N - N_{Uc}$ and $S(q)$ with $\acute{S}(q)$.

Proposition: *Given two Markets A and B with the same number of units $N^A = N^B = N$, in which each unit i has the same probability of the availability (Ψ^i), the total available capacity ($Q_{max}^A = Q_{max}^B = Q_{max}$), and the same marginal cost function (when all units are available), i.e., $S^A(q) = S^B(q)$, $\forall 0 \leq q \leq Q_{max}$. Suppose that Market A has a fewer flexible units (N_{Uc}^A) than Market B does. Given the same operating condition of two markets during period $k-1$, market prices at any given load for any period k of Market A have mean and variance higher than or equal to those of Market B with more flexible units, ($N_{Uc}^B < N_{Uc}^A$).*

Proof: Suppose at time $k-1$, there are the same number of units with the same marginal cost in the "on" states in both Markets A and B. At time k the supply functions of both markets are similar to those in Equation (A.5), in which

$$\acute{S}^A(q) \geq S^A(q), \quad \forall \; 0 \leq q \leq (Q_{max} - q^{-N_{Uc}^A})$$
$$\acute{S}^B(q) \geq S^B(q), \quad \forall \; 0 \leq q \leq (Q_{max} - q^{-N_{Uc}^B}).$$

Since $N_{k,Uc}^A > N_{k,Uc}^B$,

$$\acute{S}^A(q) \geq \acute{S}^B(q), \quad \forall \; 0 \leq q \leq (Q_{max} - q^{-N_{Uc}^A}).$$

The previous sections show that prices in Market A under different unavailable conditions are always higher than or equal to prices in Market B.

A.6 Possible Extension

Similar procedures can be adopted to derive the PMF of prices given the total available capacity observed by load-serving entities (LSEs). There is a similarity between the role of information on the "perception" of market prices by power producers and LSEs. Suppose that each LSE has a set of curtailable contracts in which curtailable quantity and associating prices are known (from the past). By observing whether the other LSEs bid at their willingness-to-pay, LSE m can obtain the PMFs of prices given a supply function. Applying a method similar to that is described in the previous sections and letting Ψ^i denote the probability of the availability of any LSE i, the PMFs of prices given supply function can be derived. When both demand and supply variations are considered, the PMFs of prices given supply and demand function can be obtained as well.

Appendix B

Samples of MATLAB Codes

```
%%%The following models are designed specifically for a 24-hour decision period.
%%%%%%%Agents with the Model-based Algorithm
clear all;
[marginalCostP, marginalCostQ, minUnitP, minUnitQ, PenaltyUnit] = UNITPRODUCER2(1,1);%
marginalCostP numGen*x marginalCostQ 1*x
numGen = length(marginalCostQ(:,1));
%minUnitP = 0*minUnitP; minUnitQ = 0*minUnitQ; PenaltyUnit = 0*PenaltyUnit; %No minimum
operating constraints
[systemCostP, systemCostQ, U1, U2, U3] = UNITPRODUCER2(0,1);
for i = 1:1:24
   systemCost([1:1:length(systemCostP(:,1))],2*(i-1)+1) = systemCostP;
   systemCost([1:1:length(systemCostP(:,1))],2*i) = systemCostQ;
end;
hEnd = length(marginalCostP(1,:));
fclose('all'); fid=fopen('C:\MatlabModel\ModelBased\loadmodl.txt','rt');
[dummy,count]= fscanf(fid, '%f',[2, 2400]); fclose('all');
actualLoad = dummy(1,:)';
forecast = dummy(2,:)';
totalperiod = length(actualLoad);
structure = 1; %Publicly Known Market Prices
UP = 1; % UP = 0 --> discriminatory pricing; UP = 1 -->uniform pricing
%%%%%INPUT%%%%%%%
mem = 1;%%Length of memory matrices
stChoice =   8*ones(1,numGen);
inc = 3*ones(1,numGen);
InL(1) = 1;
for m = 1:1:(numGen)
   index = [30:5:100];
   InL(m+1) = length(index);
   if m == 1
      indexG = index;
   else
      indexG = [indexG index];
   end;
   IndexLoadG(m,[1:1:length(index)]) = index;
end;
loadPrice = zeros(sum(InL)-1,mem);%Historic market prices
loadBidPrice = zeros(sum(InL)-1,mem);
loadBidUPrice = zeros(sum(InL)-1, mem*hEnd);
outcomeUnit = zeros(sum(InL)-1, mem*hEnd);
outcome = zeros(sum(InL)-1,mem);
lenIndex = length(index);
len1Index = lenIndex*numGen;
loadStat = eye(length(index), length(index));%For demand uncertainty
priceStat = zeros(length(index), 2);
agentPrice = zeros(sum(InL)-1, 2);
antProfit = zeros(numGen,24);
maxPrice =  zeros(numGen,24);
yL = forecast([1:1:24]);
for j = 1:1:numGen
   competitiveBidP([24*(j-1)+1:1:24*j],:) = ones(24,1)*marginalCostP(j,:);
   competitiveBidQ([24*(j-1)+1:1:24*j],:) = ones(24,1)*marginalCostQ(j,:);
   for m = 1:1:length(marginalCostP(j,:))
      loadBidUPrice([(j-1)*lenIndex+1:1:j*lenIndex],[mem*(m-1)+1:1:mem*m]) = marginalCostP(j,m);
   end;
```

```
end;
if UP == 0
   [yP, SchedulingGen, SchedulingPr]= CLEARPAB(competitiveBidP, competitiveBidQ, yL,0);
else
   [yP, SchedulingGen, SchedulingPr]= CLEARUP(competitiveBidP, competitiveBidQ, yL,0);
end;
[loadStat, probLoad, priceStat] = LOADSTATIC(yL, yL, index, loadStat, priceStat, yP);
[indexAA] = DISCRETIZELOAD(yL,index);
forecastLoad = yL;
for j = 1:1:numGen
   marginalCostM =[marginalCostP(j,:)' (marginalCostQ(j,:)')*ones(1,24)];%
   marginalCost = [marginalCostP(j,:)' marginalCostQ(j,:)'];
   ySchedule = SchedulingGen([j:numGen:(23*numGen+j)],:);
   ySchedulePr = SchedulingPr([j:numGen:(23*numGen+j)],:);
   minQgen  = minUnitQ([j:numGen:23*numGen+j],:);
   minPgen = minUnitP([j:numGen:23*numGen+j],:);
   penaltyG = PenaltyUnit([j:numGen:23*numGen+j],:);
   [antUnitProfit([24*(j-1)+1:1:24*j],:), antProfit(j,:), SS] = PROFITUNITGEN(yP, marginalCostM,
minQgen, minPgen, penaltyG, ySchedule, ySchedulePr, 1);
   yG= [marginalCostP(j,:)];
   for m = 1:1:24
      yG = [yG; marginalCostQ(j,:)];
   end;
   maxPrice(j,:) = yP;
   ymarginalCost([25*(j-1)+1:1:25*j],:)= yG;
   y = [sum(InL([1:1:j])):1:sum(InL([1:1:j+1]))-1];
   if UP == 1
      A14 = priceStat;
   else
      yBidPA = competitiveBidP([24*(j-1)+1:1:24*j],:);
      yBidQA = competitiveBidQ([24*(j-1)+1:1:24*j],:);
      [yP] = ANTPRICE(ySchedule, ySchedulePr, yBidPA, yBidQA, inc(j));
      A14 = agentPrice(y,[1:2]);
      [A14] = PRICESTATIC(yL, index, A14, yP);
   end;
   agentPrice(y,[1:2]) = A14;
   AA1 = loadBidPrice(y,:);
   BB1 = loadPrice(y,:);
   CC1 = loadBidUPrice(y,:);
   [AA, BB, CC] = INITIALLOAD1(indexAA, marginalCostP(j,:), yP, AA1, BB1, CC1);
   loadBidPrice(y,[1:1:length(AA(1,:))]) = AA;
   loadPrice(y,[1:1:length(BB(1,:))]) = BB;
   loadBidUPrice(y,[1:1:length(CC(1,:))]) = CC;
   bidPrice([24*(j-1)+1:1:24*j],:) = [competitiveBidP([24*(j-1)+1:1:24*j],:) zeros(24,1)];
   bidQuantity([24*(j-1)+1:1:24*j],:) =[competitiveBidQ([24*(j-1)+1:1:24*j],:) zeros(24,1)];
end;
yesPrice = 0;
for d = 1:1:(totalperiod/24)
   fLoad(d,:) = forecast([24*(d-1)+1:1:24*d],1)';
   marginalCostQM = marginalCostQ;
   fsystemCostM = systemCost;
   lenYes = length(yesPrice(1,:));
   for n = 1:1:numGen
      A1=0; A3=0;A7 =0;A8 = 0;A10 =0;A12 =0;A13 = 0;
      y = [sum(InL([1:1:n])):1:sum(InL([1:1:n+1]))-1];
      A0= antProfit(n,:);
      A3 = outcome(y,:);
      A13 = outcomeUnit(y,:);
      A7 = loadPrice(y,:);
      A8 = loadBidPrice(y,:);
      A12 = loadBidUPrice(y,:);
      A10 = antUnitProfit([24*(n-1)+1:1:24*n],:);
      if UP == 1
         A14 = priceStat;
      else
         A14 = agentPrice(y,:);
      end;
      ySchedule = SchedulingGen([n:numGen:(23*numGen+n)],:);
      ySchedulePr = SchedulingPr([n:numGen:(23*numGen+n)],:);
      yBidP = bidPrice([((n-1)*24+1):1:24*n],:);
      yBidQ = bidQuantity([((n-1)*24+1):1:24*n],:);
      minQgen  = minUnitQ([n:numGen:23*numGen+n],:);
```

```
      minPgen = minUnitP([n:numGen:23*numGen+n],:);
      penaltyG = PenaltyUnit([n:numGen:23*numGen+n],:);
      marginalCost = [marginalCostP(n,:)' marginalCostQM(n,:)'];
      for m = 2:1:24
         marginalCost =[marginalCost marginalCostP(n,:)' marginalCostQM(n,:)'];
      end;
      IndexLoad = indexG(1,[sum(InL([1:1:n])):1:sum(InL([1:1:n+1]))-1]);
      ymarginal = ymarginalCost([25*(n-1)+1:1:25*n],:)';
      [bidP,bidQ,A0,A3,A7,A8,todayProfit(n,:),yPrice, A13, A10,A12, maxPrice(n,:), A14] =
PORTFOLIONEWa(ymarginal,marginalCost,yP,yL,ySchedule,fLoad(d,:),fsystemCostM,
IndexLoad,A0,A3,A7,A8,stChoice(n), ySchedulePr, yBidP, yBidQ, structure, inc(n),
UP,loadStat,probLoad, A14, minQgen, minPgen, penaltyG, A12, A10,A13, maxPrice(n,:));
      bidPrice([((n-1)*24+1):1:24*n],:) = bidP;
      bidQuantity([((n-1)*24+1):1:24*n],:) = bidQ;
      ymarginalCost([25*(n-1)+1:1:25*n],:)= [marginalCostP(n,:);
ones(24,1)*marginalCostQM(n,:)];
      if structure == 2
         yesPrice0(n,:) = yPrice;
      else
         yesPrice0(n,:) = yP;
      end;
      loadPrice(y,:) = A7;
      antProfit(n,:) = A0;
      loadBidUPrice(y,:) = A12;
      antUnitProfit([24*(n-1)+1:1:24*n],[1:1:length(A10(1,:))]) = A10;
      outcome(y,:) = A3;
      outcomeUnit(y,:) = A13;
      agentPrice(y,:) = A14;
      loadBidPrice(y,:) = A8;
      lBid = length(bidP(1,:));
      bidAgentP([24*(d-1)+1:1:24*d],[lBid*(n-1)+1:1:n*lBid]) = bidP;
      bidAgentQ([24*(d-1)+1:1:24*d],[lBid*(n-1)+1:1:n*lBid]) = bidQ;
   end;
   if d == 1
      anticipatedProfit = antProfit;
      Profit = todayProfit;
      yesantPrice = yesPrice0;
      yesmaxPrice = maxPrice;
   else
      A = [anticipatedProfit antProfit];
      anticipatedProfit(:,[1:1:length(A(1,:))]) = A;
      G = [Profit todayProfit];
      Profit(:,[1:1:length(G(1,:))]) = G;
      APP = [yesantPrice yesPrice0];
      yesantPrice(:,[1:1:length(APP(1,:))]) =  APP;
      yPP = [yesmaxPrice maxPrice];
      yesmaxPrice(:,[1:1:length(yPP(1,:))]) = yPP;
   end;
   [dayLoad] = fLoad(d,:); %[dayLoad] = actualLoad([24*(d-1)+1:1:24*d],1)';
   [competitivePrice Sc BB1]= CLEARUP(competitiveBidP, competitiveBidQ, dayLoad, 0);
   if UP == 0
      [marketPrice, SchedulingGen, SchedulingPr]= CLEARPAB(bidPrice, bidQuantity, dayLoad, 0);
   else
      [marketPrice, SchedulingGen, SchedulingPr]= CLEARUP(bidPrice, bidQuantity, dayLoad, 0);
   end;
   [loadStat, probLoad, priceStat] = LOADSTATIC(fLoad(d,:), dayLoad, index, loadStat, priceStat,
Price);
   if d == 1
      Price = marketPrice;
      CPrice = competitivePrice;
      Load = dayLoad;
   else
      Price = [Price marketPrice];
      CPrice = [CPrice competitivePrice];
      Load = [Load dayLoad];
   end;
   yL = dayLoad;
   if structure == 1
      yP = marketPrice;
   else
      yP = 0;
   end;
```

```
end;
%%%%%% Functions %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [marketPrice, SchedulingGen, SchedulingPrice]= CLEARUP(BidPrice, BidQuantity,Load,flag)
total = length(Load);
NumGen = length(BidPrice(:,1))/total;
NumBid = length(BidPrice(1,:));
SchedulingGen = zeros(NumGen*total,1);
SchedulingPrice = zeros(NumGen*total,1);
for h = 1:1:total
   m = 0;       l = 1;
   tempPrice = 0;
   tempQuantity = 0;
   tempGenerator = 0;
   for j = 1:1:NumGen
      NumBid1 = length(BidPrice((j-1)*total+h,:));
      for i = 1:1:NumBid1
         if BidQuantity((j-1)*total+h,i) > 0
            tempPrice(l)= BidPrice((j-1)*total+h,i);
            tempQuantity(l)= BidQuantity((j-1)*total+h,i);
            tempGenerator(l)= j;
            l = l+1;
         end;
      end;
   end;
   [tempPriceSort IndexSort] = sort(tempPrice);
   accumulatedSupply = 0;
   acPrice = tempPriceSort(1);
   p =1;
   for g = 1:1:length(tempQuantity)
      if tempPriceSort(g) == acPrice(p)
         accumulatedSupply(p) = tempQuantity(IndexSort(g))+accumulatedSupply(p);
      else if tempPriceSort(g)> acPrice(p)
            p = p+1;
            acPrice(p) = tempPriceSort(g);
            accumulatedSupply(p) = tempQuantity(IndexSort(g))+accumulatedSupply(p-1);
         end;
      end;
   end;
   totalSup(h) = length(accumulatedSupply);
   b = 1;
   indexLoad(h) = 0;
   while b <= totalSup(h) %| indexLoad(h) == 0
      if b == 1
         if Load(h) <= accumulatedSupply(1)
            indexLoad(h) = 1;
         end;
      else
         if Load(h) <= accumulatedSupply(b) & Load(h) > accumulatedSupply(b-1)
            indexLoad(h) = b;
         end;
      end;
      b = b+1;
   end;
   if indexLoad(h) == 0
      indexLoad(h) = totalSup(h);
      marketPrice(h)= max(tempPriceSort)+100;
      Load(h) = accumulatedSupply(totalSup(h));
   else
      marketPrice(h) = acPrice(indexLoad(h));
      Load(h) = Load(h);
   end;
   marG = 0; e = 0; in =zeros(NumGen,1);
   %%%This part is for determining how many units submit the same price
   for p = 1:1:length(tempQuantity)
      if marketPrice(h) == tempPriceSort(p)
         marG = marG+1;
         marginalUnit(marG) = p;
         marginal(marG) = tempQuantity(IndexSort(p));
      else if marketPrice(h) < tempPriceSort(p)
            e=e+1;
            extramarginal(e) = tempQuantity(IndexSort(p));
         else
```

```
            %%%%%%%Scheduling infra marginal units%%%%%%%%%%
            k = 1;
            while k <= NumGen
               if tempGenerator(IndexSort(p)) == k
                  in(k)= in(k)+1;
                  SchedulingGen((h-1)*NumGen+k,in(k)) =  tempQuantity(IndexSort(p));
                  SchedulingPrice((h-1)*NumGen+k,in(k)) = marketPrice(h);
                  k = NumGen +1;
               else
                  k = k+1;
               end;
            end;
         end;
      end;
   end;
   %%%%Scheduling marginal units%%%%%%%%%%%%%%%%%%%
   if marG == 1 %one unit is a marginal unit
      F=tempGenerator(IndexSort(marginalUnit(marG)));
      in(F) = in(F)+1;
      if indexLoad(h) == 1
         marginalCap(h) = Load(h);
      else
         marginalCap(h) = Load(h)-accumulatedSupply(indexLoad(h)-1);
      end;
      SchedulingGen((h-1)*NumGen+F,in(F)) = marginalCap(h);
      SchedulingPrice((h-1)*NumGen+F,in(F)) = marketPrice(h);
   else if marG > 1
         marginalSum(h) = sum(marginal);
         if indexLoad(h) == 1
            marginalCap(h) = Load(h);
         else
            marginalCap(h) = Load(h)-accumulatedSupply(indexLoad(h)-1);
         end;
         for j = 1:1:marG
            FF = tempGenerator(IndexSort(marginalUnit(j)));
            in(FF)= in(FF)+1;
            SchedulingGen((h-
1)*NumGen+FF,in(FF))=tempQuantity(IndexSort(marginalUnit(j)))*marginalCap(h)/marginalSum(h);
            SchedulingPrice((h-1)*NumGen+FF,in(FF))=marketPrice(h);
         end;
      end;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [LoadIndex] = DISCRETIZELOAD(Load,Index)
total = length(Load);
LoadIndex = zeros(total,1);
total1 = length(Index);
for j = 1:1:total
   h = 1;
   while LoadIndex(j) == 0 %| h <= total1
      if h == 1
         if Load(j) <= Index(h)
            LoadIndex(j)= h;
         end;
      else if h <= total1
            if Load(j)> Index(h-1) & Load(j) <= Index(h)
               LoadIndex(j) = h;
            end;
         else
            LoadIndex(j) = total1;%+1;
         end;
      end;
      h = h+1;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [todayUnitProfit, todayProfit, AA] = PROFITUNITGEN(yPrice, marginalCostM, minQ, minP,
Penalty, yscheduling, yScPrice, flag)
total = length(yPrice);ySchQ = yscheduling;CC = marginalCostM;
for i = 1:1:total
   marginalCost = [marginalCostM(:,1) marginalCostM(:,i+1)];
```

```
todayUnitProfit(i,:) = zeros(1,length(marginalCost(:,1)));
addCost = 0;
schedule = 0;
scheduleP = 0;
sumSchedule(i) = sum(yscheduling(i,:));
if sumSchedule(i) == 0
   todayProfit(i) = 0;
   for h = 1:1:length(marginalCost(:,1))
      if marginalCost(h,2) > 0
         if minQ(i,h) == marginalCost(h,2)
            addCost = addCost+minP(i,h);
            todayUnitProfit(i,h) = todayUnitProfit(i,h) -minP(i,h);
         end;
      end;
   end;
   todayProfit(i) = todayProfit(i) - addCost;
else if sumSchedule(i) < min(minQ(i,:))
      schedule(1) = sumSchedule(i);
      scheduleP(1) = Penalty(i);
      sumSchedule(i) = -1;
      sumCost = -Penalty(i)*schedule(1)+ sum(yScPrice(i,:).*yscheduling(i,:));
      X = 0;
      for b1 = 1:1:hEnd
         if marginalCost(b1,2) > 0
            X = X+1;
            if minQ(i,b1) == marginalCost(b1,2)
               addCost = addCost + minP(i,b1);
               todayUnitProfit(i,b1) = todayUnitProfit(i,b1) - minP(i,b1);
            end;
         end;
      end;
      X1 = and(marginalCost(:,2), ones(hEnd,1));%%to find the unit with positive capacity
      todayUnitProfit(i,:) = todayUnitProfit(i,:) + (sumCost/X)*X1';
   else
      h = 1;
      hEnd = length(marginalCost(:,1));
      while sumSchedule(i) > 0.0001
         if sumSchedule(i)- marginalCost(h,2) >= 0
            if marginalCost(h,2) > 0
               sumSchedule(i)= sumSchedule(i) - marginalCost(h,2);
               scheduleP(h) = marginalCost(h,1);
               schedule(h) = marginalCost(h,2);
            else
               sumSchedule(i)= sumSchedule(i);
               scheduleP(h) = marginalCost(h,1);
               schedule(h) = 0;
            end;
            h =h+1;
         else %%to reschedule
            found = 0; g1 = h;gb = h;
            while found == 0
               if g1 <= hEnd
                  if sumSchedule(i) >= minQ(i,g1)
                     scheduleP(g1) = marginalCost(g1,1);
                     schedule(g1) = sumSchedule(i);
                     sumSchedule(i) = -1;
                     found = 1;
                     if g1+1 <= hEnd
                        for b1 = (g1+1):1:hEnd
                           if marginalCost(b1,2) > 0
                              if minQ(i,b1) == marginalCost(b1,2)
                                 addCost = addCost + minP(i,b1);
                                 todayUnitProfit(i,b1) = todayUnitProfit(i,b1) - minP(i,b1);
                              end;
                           end;
                        end;
                     end;
                     for b1 = 1:1:g1
                        if marginalCost(b1,2) > 0
                           if minQ(i,b1) == marginalCost(b1,2)
                              if schedule(b1) == 0
                                 addCost = addCost + minP(i,b1);
```

```
                  todayUnitProfit(i,b1) = todayUnitProfit(i,b1) - minP(i,b1);
               end;
            end;
         end;
      end;
   else
      restSch = minQ(i,g1) - sumSchedule(i);
      if g1 > 1
         rest = marginalCost([1:1:g1-1],2)'- minQ(i,[1:1:g1-1]); %rest = 1*(h-
1)

         if sum(schedule) > 0
            if sum(rest) >= restSch
               ff1 = 1; f = g1-1; e = g1-1;
               while ff1 == 1
                  if rest(f) > 0
                     if schedule(e) > 0
                        if rest(f) >  restSch
                           schedule(e) = schedule(e) - restSch;
                           ff1 = 0;
                        else
                           schedule(e) = schedule(e) - rest(f);
                           restSch = restSch - rest(f);
                        end;
                     end;
                  end;
                  f = f-1; e = e-1;
               end;
               schedule(g1) = minQ(i,g1);
               scheduleP(g1) = marginalCost(g1,1);
               found = 1;
               sumSchedule(i) = -1;
               if g1+1 <= hEnd
                  for b1 = (g1+1):1:hEnd
                     if marginalCost(b1,2) > 0
                        if minQ(i,b1) == marginalCost(b1,2)
                           addCost = addCost + minP(i,b1);
                           todayUnitProfit(i,b1) = todayUnitProfit(i,b1) -
minP(i,b1);

                        end;
                     end;
                  end;
               end;
               for b1 = 1:1:g1
                  if marginalCost(b1,2) > 0
                     if minQ(i,b1) == marginalCost(b1,2)
                        if schedule(b1) == 0
                           addCost = addCost + minP(i,b1);
                           todayUnitProfit(i,b1) = todayUnitProfit(i,b1) -
minP(i,b1);

                        end;
                     end;
                  end;
               end;
            else
               if gb == hEnd
                  schedule(g1) = sumSchedule(i);
                  scheduleP(g1) = Penalty(i);
                  addCost = minP(i,g1);
                  sumSchedule(i) = -1;
                  found = 1;
               else
                  schedule(g1) = 0;
                  scheduleP(g1) = 0;
                  g1 = g1+1;
               end;
            end;
         else
            schedule(g1) = 0;
            scheduleP(g1) = 0;
            g1 = g1+1;
         end;
      else
```

```
                                schedule(g1) = 0;
                                scheduleP(g1) = 0;
                                g1 = g1+1;
                             end;
                          end;
                       else
                          schedule(g2) = sumSchedule(i);
                          scheduleP(g2) = Penalty(i);
                          sumSchedule(i) = -1;
                          found = 1;
                          ff2 = 0;
                       end;
                    end;
                 end;
              end;
              d1 = 1;
              todayUnitProfit(i,[1:1:length(schedule)]) =
todayUnitProfit(i,[1:1:length(schedule)]) -scheduleP.*schedule;
              for s = 1:1:length(schedule)
                 if marginalCost(s,2) > 0
                    if schedule(s) > 0
                       fd1 = 0;
                       tempSch = schedule(s);
                       while tempSch > 0.0001
                          if ySchQ(i,d1) == tempSch
                             todayUnitProfit(i,s) = todayUnitProfit(i,s)+ yScPrice(i,d1)*tempSch;
                             tempSch = tempSch - ySchQ(i,d1);
                             ySchQ(i,d1) = ySchQ(i,d1) - tempSch;
                             d1 = d1+1;
                          else if ySchQ(i,d1) > tempSch
                                todayUnitProfit(i,s) = todayUnitProfit(i,s)+
yScPrice(i,d1)*tempSch;
                                tempSch = 0;
                                ySchQ(i,d1) = ySchQ(i,d1) - tempSch;
                             else
                                todayUnitProfit(i,s) = todayUnitProfit(i,s)+
yScPrice(i,d1)*ySchQ(i,d1);
                                tempSch = tempSch - ySchQ(i,d1);
                                ySchQ(i,d1) = ySchQ(i,d1) - ySchQ(i,d1);
                                d1 = d1+1;
                             end;
                          end;
                       end;
                    end;
                 end;
              end;
           end;
           todayProfit(i) = sum(yScPrice(i,:).*yscheduling(i,:))-sum(schedule.*scheduleP)-
addCost;
        end;
    end;
end;
if total == 1
    AA = schedule;
else
    AA = 0;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [A1, A2, A4, A5, A6, A7, A8, A9, A10, A11, A12, A13, A14] =
PORTFOLIONEWa(ymargin,marginalCost,yP,yLoad,ySchedule,fLoad,systemCostM, IndexLoad,
antProfit,outcome,loadPrice,loadBidPrice, Astrategy, yScPrice, yBidP, yBidQ, structure, inc, UP,
loadStat, probLoad, priceStat, minQ, minP, penalty,loadBidUPrice, antUnitProfit, outcomeUnit,
maxPrice)
total= length(yLoad);
mem = length(loadBidPrice(1,:));
if UP == 1
    yPrice = yP;
else
    [yPrice schedulingUnit] =  ANTPRICE(ySchedule, yScPrice, yBidP, yBidQ, inc);
end;
[todayUnitProfit, todayProfit, XX] = PROFITUNITGEN(yPrice, ymargin, minQ, minP, penalty,
ySchedule, yScPrice, 1);
```

```
[yloadIndexD] = DISCRETIZELOAD(yLoad,IndexLoad);
yesBidPrice = yBidP;
[GamePrice, outcome]= GAMEDETECT11(yLoad,yPrice,yloadIndexD, outcome,antProfit, todayProfit,
maxPrice);
if UP == 1
   [priceStat] = priceStat;
else
   [priceStat] = PRICESTATIC(yLoad, IndexLoad, priceStat, yPrice);
end;
for j = 1:1:length(marginalCost(:,1))
   yesUnitBid = yesBidPrice(:,j);
   [GPriceUnit, oUnit] = GAMEDETECT11(yLoad,yPrice,yloadIndexD, outcomeUnit(:,[mem*(j-
1)+1:1:j*mem]),antUnitProfit(:,j), todayUnitProfit(:,j), yesUnitBid);
   outcomeUnit(:,[mem*(j-1)+1:1:j*mem]) =oUnit;
end;
[loadPrice] = STATICUNIT(GamePrice, loadPrice);
[loadIndexD] = DISCRETIZELOAD(fLoad,IndexLoad);
flag0 = 0;
[fcompPrice fcompSchedule fcStack fprofit] = COMPETITIVE(systemCostM, fLoad,
marginalCost,flag0);
[bidQuantity absolutePower maxWithhold] = WITHHOLDINGFULL1(fLoad, systemCostM, marginalCost,
fcompPrice, fcStack, fprofit);
[newSystemCostM] = CAPWITHHELD(systemCostM, marginalCost,bidQuantity);
marginalCostB(:,1) = marginalCost(:,1);
marginalCostB(:,[2:1:total+1]) = bidQuantity';
flag1 = 1;
[ncompPrice ncompSchedule ncStack nprofit] = COMPETITIVE(newSystemCostM,
fLoad,marginalCostB,flag1);
[bidPrice, bidQuantity, profit, loadBidPrice, loadBidUPrice, todayUnitProfit, maxPrice] =
SETPRICEZaCAP(loadPrice, loadBidPrice, loadBidUPrice,  marginalCost, newSystemCostM,  ncStack,
ncompPrice, Astrategy,inc, fLoad,loadIndexD,outcome, outcomeUnit, UP,loadStat,probLoad,
priceStat,maxWithhold, bidQuantity, minP, minQ, penalty);
A1 = bidPrice;A2 = bidQuantity;A4 = profit;A5 = outcome;
A6 = loadPrice;A7 = loadBidPrice;A8 = todayProfit;A9 = yPrice;A10 = outcomeUnit;
A11 = todayUnitProfit;A12 = loadBidUPrice;A13 = maxPrice;A14 = priceStat;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [GamePrice, outcome]= GAMEDETECT11(yesLoad,marketPrice,indexLoad, outcome,antProfit,
todayProfit, yesBidPrice)
Md = length(outcome(1,:));
Last = length(outcome(1,:));
total = length(yesLoad);
GamePrice = zeros(total,3);
for j = 1:1:total
   if todayProfit(j) > 0
      if antProfit(j) > 0
         if todayProfit(j) < antProfit(j)
            if yesBidPrice(j) > marketPrice(j)
               outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
               outcome(indexLoad(j),Last) = 10;
              else if yesBidPrice(j) == marketPrice(j)
                  outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                  outcome(indexLoad(j),Last) = 11;
               else
                  outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                  outcome(indexLoad(j),Last) = 11;
               end;
            end;
         else if todayProfit(j) == antProfit(j)
               if yesBidPrice(j) > marketPrice(j)
                  outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                  outcome(indexLoad(j),Last) = 00;
               else if yesBidPrice(j) == marketPrice(j)
                     outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                     outcome(indexLoad(j),Last) = 11;
                    else
                     outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                     outcome(indexLoad(j),Last) = 00;
                  end;
               end;
            else
               if yesBidPrice(j) > marketPrice(j)
                  outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
```

```
                outcome(indexLoad(j),Last) = 00;
             else if yesBidPrice(j) == marketPrice(j)
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 00;
                else
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 00;
                end;
             end;
          end;
       end;
    else if antProfit(j) == 0
          if yesBidPrice(j) > marketPrice(j)
             outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
             outcome(indexLoad(j),Last) = 00;
          else if yesBidPrice(j) == marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             else
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             end;
          end;
       else
          if yesBidPrice(j) > marketPrice(j)
             outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
             outcome(indexLoad(j),Last) = 00;
          else if yesBidPrice(j) == marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             else
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             end;
          end;
       end;
    end;
else if todayProfit(j) == 0
       if antProfit(j) > 0
          if yesBidPrice(j) > marketPrice(j)
             outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
             outcome(indexLoad(j),Last) = 10;
          else if yesBidPrice(j) == marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 11;
             else
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 11;
             end;
         end;
      else if antProfit(j) == 0
            if yesBidPrice(j) > marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             else if yesBidPrice(j) == marketPrice(j)
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 11;
                else
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 11;
                end;
             end;
          else
             if yesBidPrice(j) > marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 00;
             else if yesBidPrice(j) == marketPrice(j)
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 00;
                else
                   outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                   outcome(indexLoad(j),Last) = 00;
```

```
                    end;
                end;
            end;
        end;
    else
        if antProfit(j) > 0
            if yesBidPrice(j) > marketPrice(j)
                outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                outcome(indexLoad(j),Last) = 10;
            else if yesBidPrice(j) == marketPrice(j)
                    outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                    outcome(indexLoad(j),Last) = 11;
                else
                    outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                    outcome(indexLoad(j),Last) = 11;
                end;
            end;
        else if antProfit(j) == 0
                if yesBidPrice(j) > marketPrice(j)
                    outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                    outcome(indexLoad(j),Last) = 00;
                else if yesBidPrice(j) == marketPrice(j)
                        outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                        outcome(indexLoad(j),Last) = 11;
                    else
                        outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                        outcome(indexLoad(j),Last) = 11;
                    end;
                end;
            else
                if todayProfit(j) < antProfit(j)
                    if yesBidPrice(j) > marketPrice(j)
                        outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                        outcome(indexLoad(j),Last) = 10;
                    else if yesBidPrice(j) == marketPrice(j)
                            outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                            outcome(indexLoad(j),Last) = 11;
                        else
                            outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                            outcome(indexLoad(j),Last) = 11;
                        end;
                    end;
                else if todayProfit(j) == antProfit(j)
                        if yesBidPrice(j) > marketPrice(j)
                            outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                            outcome(indexLoad(j),Last) = 00;
                        else if yesBidPrice(j) == marketPrice(j)
                                outcome(indexLoad(j),[1:1:Last-1]) =
outcome(indexLoad(j),[2:1:Last]);
                                outcome(indexLoad(j),Last) = 11;
                            else
                                outcome(indexLoad(j),[1:1:Last-1]) =
outcome(indexLoad(j),[2:1:Last]);
                                outcome(indexLoad(j),Last) = 00;
                            end;
                        end;
                    else
                        if yesBidPrice(j) > marketPrice(j)
                            outcome(indexLoad(j),[1:1:Last-1]) = outcome(indexLoad(j),[2:1:Last]);
                            outcome(indexLoad(j),Last) = 00;
                        else if yesBidPrice(j) == marketPrice(j)
                                outcome(indexLoad(j),[1:1:Last-1]) =
outcome(indexLoad(j),[2:1:Last]);
                                outcome(indexLoad(j),Last) = 11;
                            else
                                outcome(indexLoad(j),[1:1:Last-1]) =
outcome(indexLoad(j),[2:1:Last]);
                                outcome(indexLoad(j),Last) = 11;
                            end;
                        end;
                    end;
                end;
```

```
            end;
         end;
      end;
   end;
   GamePrice(j,1) = marketPrice(j);
   GamePrice(j,2) = indexLoad(j);
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [loadPrice] = STATICUNIT(GamePrice, loadPrice)
LIndex = length(loadPrice(:,1));
LoadIndex = GamePrice(:,2);
total = length(GamePrice(:,1));
total1 = length(loadPrice(1,:));
for h = 1:1:total
   if total1 > 1
      loadPrice(LoadIndex(h), [1:1:total1-1]) =     loadPrice(LoadIndex(h), [2:1:total1]);
      loadPrice(LoadIndex(h), total1) = GamePrice(h,1);
   else
      loadPrice(LoadIndex(h), total1) = GamePrice(h,1);
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [compPrice, schedule, cStack, profit] = COMPETITIVE(systemCostMB, Load,
marginalCostB,flagbid)
for h= 1:1:length(Load)
   if flagbid == 1
      marginalCost =[marginalCostB(:,1) marginalCostB(:,2)];
      systemCostM1 = [systemCostMB(:,2*(h-1)+1) systemCostMB(:,2*h)];
      [sys1 sys2] = ZEROOUT(systemCostM1(:,1), systemCostM1(:,2));
      systemCostM = [sys1 sys2];
   else
      marginalCost= marginalCostB;
      systemCostM1 = systemCostMB;
      [sys1 sys2] = ZEROOUT(systemCostM1(:,1), systemCostM1(:,2));
      systemCostM = [sys1 sys2];
   end;
   total = length(marginalCost(:,1));
   total1 = length(systemCostM(:,1));
   f = 1;
   cStack(h) = 0;
   while cStack(h) == 0
      if f ==1
         if Load(h) <= systemCostM(1,2)
            compPrice(h) = systemCostM(1,1);
            cStack(h) = 1;
         end;
      else if Load(h) > systemCostM(f-1,2) & Load(h) <= systemCostM(f,2)
            compPrice(h) = systemCostM(f,1);
            cStack(h) = f;
         else if Load(h) > systemCostM(total1,2)
               compPrice(h) = 200;
               cStack(h) = f;
               end;
         end;
      end;
      f = f+1;
   end;
   for d = 1:1:total
      if marginalCost(d,2) > 0
         if marginalCost(d,1) < compPrice(h)
            schedule(h,d) = marginalCost(d,2);
            cost(h,d) = marginalCost(d,1);
            scheduleP(h,d) = compPrice(h);
         else if marginalCost(d,1) == compPrice(h)
               if d == 1
                  schedule(h,d) = (marginalCost(d,2)*Load(h))/(systemCostM(cStack(h),2));
               else
                  schedule(h,d) = (marginalCost(d,2)*(Load(h)-systemCostM(cStack(h)-
1,2)))/(systemCostM(cStack(h),2)-systemCostM(cStack(h)-1,2));
               end;
               scheduleP(h,d) = compPrice(h);
               cost(h,d) = marginalCost(d,1);
```

```
            else
               schedule(h,d) = 0;
               cost(h,d) = marginalCost(d,1);
               scheduleP(h,d) = compPrice(h);
            end;
         end;
      else
         schedule(h,d) = 0;
         cost(h,d) = 0;
         scheduleP(h,d) = compPrice(h);
      end;
   end;
   operatingCost(h) = sum(schedule(h,:).*cost(h,:));
   income(h) = sum(schedule(h,:).*scheduleP(h,:));
   profit(h) = income(h) - operatingCost(h);
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [bid_Quantity, absolutePower, maxWithhold] = WITHHOLDINGFULL1(Load, systemCostM,
marginalCost, compPrice, cStack, nprofit)
noWprofit = nprofit;
total = length(compPrice);total1 = length(marginalCost(:,1));
total2 = length(systemCostM(:,1));
jM = 1;
minWithhold = zeros(total,(total2-min(cStack)+1));
withholdCap = zeros(total, jM);
withholdPrice = zeros(total,jM);
ord = zeros(total,1);
eps = 0.005; %eps is tolerance
inc = 0.05;
for i = 1:1:total
   for b = 1:1:(total2-cStack(i)+1)
      minWithhold(i,b) =  systemCostM(cStack(i)+b-1,2)- Load(i)+eps;
      if b < (total2-cStack(i)+1)
         newCompPrice(i,b) = systemCostM(cStack(i)+b,1);
      else
         newCompPrice(i,b) = 150;
      end;
   end;
   j = 1;l(i) =0;
   while j <= total1
      if marginalCost(j,1) <= compPrice(i)
         if marginalCost(j,2) > 0
            l(i) = l(i)+1;
            withholdCap(i,l(i)) = marginalCost(j,2);
            withholdPrice(i,l(i)) = marginalCost(j,1);
          end;
         j = j+1;
      else
         ord(i)= j;
         %ord(i) tells us the order of the next marginal cost that might have  positive capacity
         j = total1+1;
      end;
   end;
   if ord(i) == 0
      ord(i) = total1+1;
   end;
   sumWithhold(i) = sum(withholdCap(i,:));
   j = 1;
   while j <= (total2-cStack(i)+1)
      if sumWithhold(i) < minWithhold(i,j)
         withhold(i,j) = 0;
         withholdP(i,j) = 0;
         j = (total2-cStack(i)+1)+1;
         absolutePower(i) = 0;
      else
         if (ord(i) <= total1)
            if marginalCost(ord(i),2) > 0
               withholdCap(i,l(i)+1) = marginalCost(ord(i),2);
               withholdPrice(i,l(i)+1) = marginalCost(ord(i),1);
               sumWithhold(i) = sum(withholdCap(i,:));
            end;
            withhold(i,j) = minWithhold(i,j);
```

```
            withholdP(i,j) = newCompPrice(i,j);
            ord(i) = ord(i)+1;
            l(i) = l(i)+1;
            j= j+1;
         else
            withhold(i,j) = minWithhold(i,j);
            withholdP(i,j) = newCompPrice(i,j);
            j = (total2-cStack(i)+1)+1;
            absolutePower(i) = 1;
         end;
      end;
   end;
end;
withholdR = ceil(withhold);
[maxWithhold indexMax] = max(withholdR,[],2);
maxWithholdPrice = max(max(withholdP,[],2),compPrice');
ordNew = ord-1;
lnew = l+1;
for i = 1:1:total
   v = indexMax(i);
   while v > 0
      capWithhold = withholdR(i,v);
      j = ordNew(i);
      h = length(marginalCost(:,1));
      T=h;
      schedule(i,:) =zeros(1,h);
      marginQ(i,:) = zeros(1,h);
      scheduleP(i,:)=zeros(1,h);
      while h > 0
         if marginalCost(h,1) <= withholdP(i,v)
            if marginalCost(h,2) > 0
               if (capWithhold- marginalCost(h,2)) >= 0
                  capWithhold =(capWithhold- marginalCost(h,2));
                  schedule(i,T-h+1) = 0;
                  scheduleP(i,T-h+1) = marginalCost(h,1);
               else
                  schedule(i,T-h+1) = marginalCost(h,2) -capWithhold;
                  capWithhold = 0;
                  scheduleP(i,T-h+1) = marginalCost(h,1);
               end;
            end;
         end;
         h = h-1;
      end;
      operatingCost(i) = sum(schedule(i,:).*scheduleP(i,:));
      returnWithhold(i) = withholdP(i,v)*sum(schedule(i,:));
      fprofit(i) = returnWithhold(i) - operatingCost(i);
      if fprofit(i) > noWprofit(i)
         for b = 1:1:total1
            if sum(schedule(i,:)) > 0
               if marginalCost(b,1)> withholdP(i,v)
                  if marginalCost(b,2) > 0
                     marginQ(i,b) = marginalCost(b,2);
                  end;
               else
                  marginQ(i,b)= schedule(i,total1-b+1);
               end;
            else
               marginQ(i,b) = marginalCost(b,2);
            end;
         end;
         wProfit(i) = fprofit(i);
         v = 0;
         maxWithhold(i) = sum(marginalCost(:,2))-sum(marginQ(i,:));
      else
         if v > 1
            v = v-1;
         else % v == 1
            marginQ(i,[1:1:total1]) = marginalCost(:,2)';
            wProfit(i) = noWprofit(i);
            maxWithhold(i) = 0;
            v = v-1;
```

```
         end;
      end;
   end;
   marginP(i,:) = marginalCost(:,1)';
end;
bid_Quantity = marginQ;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [newSystemCost] = CAPWITHHELD(systemCostM, marginalCost,bidQuantity)
total = length(bidQuantity(:,1));
marginalCostP = marginalCost(:,1);
marginalCostQ = marginalCost(:,2);
for i = 1:1:total
   withholdCap = marginalCostQ - bidQuantity(i,:)';
   if sum(withholdCap) > 0 %for positive withheld capacity.
      withholding = [marginalCostP withholdCap];
      systemNew0 = SUPPLYMAINT(systemCostM, withholding);
      systemNew = SUPPLY(systemNew0);
   else
      systemNew = systemCostM;
   end;
   if i == 1
      newSystemCost = systemNew;
   else
      newSystemCost([1:1:length(systemNew(:,1))],2*(i-1)+1) = systemNew(:,1);
      newSystemCost([1:1:length(systemNew(:,2))],2*i) = systemNew(:,2);
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [A1, A2, A3, A4, A5, A6, A7] = SETPRICEZaCAP(loadPrice,loadBidPrice, loadBidUPrice,
marginalCostB, systemCostMM,  cStack, compPrice,strategyChoice,inc, Load,loadIndexD,outcome,
outcomeUnit, UP,loadStat,probLoad, priceStat,maxWithhold, bidQuantity, minP, minQ, penalty)
Nl = length(loadPrice(1,:));
total = length(compPrice);
mem = length(loadBidPrice(1,:)); hEnd = length(marginalCostB(:,1));
total3 = length(loadBidPrice(:,1)); count = zeros(total3, hEnd);countP = zeros(total3,hEnd);
count1 = zeros(total3,1);countP1 = zeros(total3,1);
todayUnitProfit = zeros(24,hEnd);
bidPrice = 0*bidQuantity;
for i =1:1:total
   systemCostM = [systemCostMM(:,2*i-1) systemCostMM(:,2*i)];
   marginalCost = [marginalCostB(:,2*i-1) marginalCostB(:,2*i)];
   addCost = 0;
   c = 1;b=1;findPrice = 0;
   while b <= length(loadStat(1,:))
      if loadStat(loadIndexD(i),b) > 0
         floadIndex = b;
         fPrice = STRATEGY(loadPrice,loadBidPrice,floadIndex,Nl,inc,
compPrice(i),strategyChoice,outcome, marginalCost(cStack(i),1));
      else
         fPrice = 0;
      end;
      findPrice(1,b) = fPrice;
      b = b+1;
   end;
   maxBidPrice(1,i) = min(150, sum(findPrice.*probLoad(loadIndexD(i),:)));
   antPrice(i) = min( sum(findPrice.*probLoad(loadIndexD(i),:)), 150);
   for j = 1:1:length(marginalCost(:,1))
      c = 1;b=1;findPrice1 = 0;
      while b <= length(loadStat(1,:))
         if loadStat(loadIndexD(i),b) > 0
            floadIndex = b;
            findPrice1(1,b) = STRATEGY(loadPrice,loadBidUPrice(:,[mem*(j-
1)+1:1:j*mem]),floadIndex,Nl,inc,compPrice(i),strategyChoice,outcomeUnit(:,[mem*(j-
1)+1:1:j*mem]), marginalCost(j,1));
         else
            findPrice1(1,b) = 0;
         end;
         b = b+1;
      end;
      bidUnitPrice(i,j) = min(150, sum(findPrice1.*probLoad(loadIndexD(i),:)));
   end;
   for r = 1:1:length(marginalCost(:,1))
```

```
      if bidUnitPrice(i,r) < antPrice(i)
         schedule(i,r) = bidQuantity(i,r);
         cost(i,r) = marginalCost(r,1);
         bidPrice(i,r) = max(marginalCost(r,1),bidUnitPrice(i,r));
         bidQuantity(i,r) = bidQuantity(i,r);
         scheduleP(i,r) = antPrice(i);
         addCost = addCost;
         todayUnitProfit(i,r) = todayUnitProfit(i,r)+(scheduleP(i,r)-cost(i,r))*schedule(i,r);
      else if bidUnitPrice(i,r) == antPrice(i)
            if cStack(i) == 1
               schedule(i,r) = bidQuantity(i,r)*(Load(i))/systemCostM(cStack(i),2);
            else
               schedule(i,r) = bidQuantity(i,r)*(Load(i)-systemCostM(cStack(i)-
1,2))/(systemCostM(cStack(i),2)-systemCostM(cStack(i)-1,2));
            end;
            cost(i,r) = marginalCost(r,1);
            scheduleP(i,r) = antPrice(i);
            bidPrice(i,r) = bidUnitPrice(i,r);
            bidQuantity(i,r) = bidQuantity(i,r);
            addCost = addCost;
            todayUnitProfit(i,r) = todayUnitProfit(i,r)+(scheduleP(i,r)-
cost(i,r))*schedule(i,r);
         else
            schedule(i,r) = 0;
            cost(i,r) = bidUnitPrice(i,r);
            bidPrice(i,r) = min(150, max(bidUnitPrice(i,r), marginalCost(r,1)));
            scheduleP(i,r) = bidPrice(i,r);
            bidQuantity(i,r) = bidQuantity(i,r);
            todayUnitProfit(i,r) = todayUnitProfit(i,r) -minP(i,r);
         end;
      end;
      count(loadIndexD(i),r) = count(loadIndexD(i),r)+1;
      if countP(loadIndexD(i),r) == 0
         countP(loadIndexD(i),r) = bidPrice(i,r);
      end;
   end;
   bidPrice(i,r+1) = 150;%max(150, antPrice(i)+inc);
   bidQuantity(i,r+1) = maxWithhold(i);%bidQuantity(i,r+1);
   income(i) = sum(antPrice(i)*schedule(i,:));
   operatingCost(i) = sum(schedule(i,:).*cost(i,:));
   profit(i) = income(i) - operatingCost(i)-addCost;
   count1(loadIndexD(i)) = count1(loadIndexD(i))+1;
   if countP1(loadIndexD(i)) == 0
      countP1(loadIndexD(i)) = maxBidPrice(1,i);
   end;
end;
for i = 1:1:total3
   for r = 1:1:hEnd
      if count(i,r) > 0
         loadBidUPrice(i,[mem*(r-1)+1:1:mem*r-1]) = loadBidUPrice(i,[mem*(r-1)+2:1:mem*r]);
         loadBidUPrice(i,mem*r) = countP(i,r);
      end;
   end;
   if count1(i) > 0
      loadBidPrice(i,[1:1:mem-1]) = loadBidPrice(i,[2:1:mem]);
      loadBidPrice(i,mem) = countP1(i);
   end;
end;
A1 = bidPrice;A2 = bidQuantity;A3 = profit;A4 = loadBidPrice;
A5 = loadBidUPrice;A6 = todayUnitProfit;A7 = maxBidPrice;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [maxBidPrice] =
STRATEGY(loadPrice,loadBidPrice,loadIndexD,N1,inc,compPrice,strategyChoice,outcome, margin)
total = length(loadIndexD);
Ng = N1 -1;
for i = 1:1:total
   if outcome(loadIndexD(i),N1) == 00
      inc1 = 0;
   else if outcome(loadIndexD(i),N1) == 10
         inc1 = -2;
      else
         inc1 = 2;
```

```
      end;
   end;
   target = 0;
   if loadBidPrice(loadIndexD(i), Nl) > 0
      target = loadBidPrice(loadIndexD(i), Nl);
   else
      x = loadIndexD(i)+1;
      while target == 0
         if x <= length(loadBidPrice(:,1))
            if loadBidPrice(x, Nl) > 0
               target = loadBidPrice(x, Nl);
            else
               x = x+1;
            end;
         else
            target = margin; %%submitting a marginal cost bid
         end;
      end;
   end;
   maxBidPrice(i) = target +incl;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [systemCostM] = SUPPLYMAINT(systemCost, outageMaintenance)
total = length(outageMaintenance(:,1));total1 = length(systemCost(:,1));
for j = 1:1:total
   if outageMaintenance(j,2) > 0
      systemCost([j:1:total1],2) = systemCost([j:1:total1],2)-outageMaintenance(j,2);
   end;
end;
systemCostM = systemCost;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [Newsystem] = SUPPLY(systemCostN1)
total2 = length(systemCostN1(:,1));
cap = systemCostN1([1:1:total2],2)-[0;systemCostN1([1:1:total2-1],2)];
j = 1;d = 1;g = 0;
while j< total2+1
   if cap(j)> 0
      systemCostNN(d,2) =  systemCostN1(j,2);
      systemCostNN(d,1) = systemCostN1(j,1);
      d = d+1;
   else
      g = g+1;
   end;
j = j+1;
end;
if g == 0
   systemCostNN(total2,2) = systemCostN1(total2,2);
   systemCostNN(total2,1) = systemCostN1(total2,1);
end;
Newsystem = [systemCostNN(:,1) systemCostNN(:,2)];
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [loadStat, probLoad, priceStat] = LOADSTATIC(fLoad, dayLoad, index, loadStat,
priceStat, price)
[fLoadIndex] = DISCRETIZELOAD(fLoad,index);
[dayLoadIndex] = DISCRETIZELOAD(dayLoad,index);
total = length(fLoad);
for i = 1:1:total
   loadStat(fLoadIndex(i), dayLoadIndex(i)) =    loadStat(fLoadIndex(i), dayLoadIndex(i)) + 1;
   priceStat(dayLoadIndex(i),1) = priceStat(dayLoadIndex(i),1)+ price(i);
   priceStat(dayLoadIndex(i),2) = priceStat(dayLoadIndex(i),2)+1;
end;
for j = 1:1:length(loadStat(:,1))
   sumProb = sum(loadStat(j,:));
   if sumProb > 0
      probLoad(j,[1:1:length(loadStat(j,:))]) = loadStat(j,:)/sumProb;
   else
      probLoad(j,[1:1:length(loadStat(j,:))]) = 0;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [priceStat] = PRICESTATIC(dayLoad, index, priceStat, price)
[dayLoadIndex] = DISCRETIZELOAD(dayLoad,index);
```

```
total = length(dayLoad);
for i = 1:1:total
   priceStat(dayLoadIndex(i),1) = priceStat(dayLoadIndex(i),1)+ price(i);
   priceStat(dayLoadIndex(i),2) = priceStat(dayLoadIndex(i),2)+1;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [yP, schedulingUnit] = ANTPRICE(ySchedule, ySchedulePr, yBidP, yBidQ, inc)
total = length(ySchedule(:,1));total1 = length(ySchedule(1,:));
total2 = length(yBidP(1,:));
schedulingUnit = zeros(total, total2);
for i = 1:1:total
   if sum(ySchedule(i,:)) == 0
      k = 0; bidP = 0;
      for j = 1:1:total2
         if yBidQ(i,j) > 0
            k = k+1;
            bidP(1,k) = yBidP(i,j);
         end;
      end;
      [bidPm bidPI] = sort(bidP);
      yP(i) = min(bidPm) - inc;
      schedulingUnit(i,[1:1:total2]) = 0;
   else if sum(ySchedule(i,:)) == sum(yBidQ(i,:))
         k = 0; schePQ = 0;
         for j = 1:1:total1
            if ySchedule(i,j) > 0
               k = k+1;
               schePQ(1,k) = ySchedulePr(i,j);
            end;
         end;
         for f = 1:1:total2
            if yBidQ(i,f) > 0
               schedulingUnit(i,f) = 1;
            else
               schedulingUnit(i,f) = 0;
            end;
         end;
         yP(1,i) = max(schePQ)+inc;
      else
         k = 0; schePQ = 0;
         for j = 1:1:total1
            if ySchedule(i,j) > 0
               k = k+1;
               schePQ(1,k) = ySchedulePr(i,j);
            end;
         end;
         maxPQ = max(schePQ);
         d = 0; bidP = 0;
         for m = 1:1:total2
            if yBidQ(i,m) > 0
               d = d+1;
               bidP(1,d) = yBidP(i,m);
               bidC(1,d) = m;
            end;
         end;
         [bidPm bidPI] = sort(bidP);
         found = 0;p = 1;
         while found == 0
            if p <= length(bidPm)
               if bidPm(p) <= maxPQ
                  found = 0;
                  schedulingUnit(i,bidC(bidPI(p))) = 1;
               else
                  found = 1;
                  schedulingUnit(i,bidC(bidPI(p))) = 0;
               end;
               p = p+1;
            else
               found = 1;
               yP(1,i) = maxPQ;
            end;
         end;
```

```
         if (p-1) <= length(bidPm)
            yP(1,i) = (maxPQ + bidPm(max(1,p-1)))/2;
         end;
      end;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [GeneratorP, GeneratorQ, minP, minQ, Penalty] = UNITPRODUCER2(gen, market)
numGdata = 13; %including the marginal cost.
fclose('all');
fid1=fopen('C:\MatlabModel\ModelBased\sampleGENA2.txt','rt');
[dummy,count]=fscanf(fid1, '%f',[numGdata, 720]);
fclose('all');
fid1a=fopen('C:\MatlabModel\ModelBased\sampleGENA2a.txt','rt');
[dummy1,count]=fscanf(fid1a, '%f',[numGdata, 720]);
fclose('all');
fid1b=fopen('C:\MatlabModel\ModelBased\sampleGENA2b.txt','rt');
[dummy2,count]=fscanf(fid1b, '%f',[numGdata, 720]);
fclose('all');
numGen = 11;
marginalCostP = dummy(:,[1:1:numGen])';
marginalCostQ = dummy(:,[numGen+1:1:2*numGen])';
penalty1 = dummy(:,[2*numGen+1:1:26*numGen])';
Penalty = penalty1(:,1);
minQ = dummy1(:,[1:1:numGen*24])';
minP = dummy2(:,[1:1:numGen*24])';
if gen == 0
   [GeneratorP, GeneratorQ] = SUPPLYFUNCTION(marginalCostP, marginalCostQ);
   minP = 0; minQ = 0; Penalty = 0;
else
   GeneratorP = marginalCostP;
   GeneratorQ = marginalCostQ;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [systemCostP, systemCostQ] = SUPPLYFUNCTION(marginalCostP, marginalCostQ)
[CapI, CapPI] = NUMBERGEN(marginalCostP, marginalCostQ);
total = length(CapPI);d = 1; systemCostQ(1,1) = 0;
[CapOrder, CapOrderI] = sort(CapPI);
systemCostP(1,1) = CapOrder(1,1);
for i = 1:1:total
      if systemCostP(d,1) == CapOrder(i,1)
         systemCostP(d,1) = CapOrder(i,1);
         systemCostQ(d,1) = systemCostQ(d,1)+CapI(CapOrderI(i),1);
      else if systemCostP(d,1) < CapOrder(i,1);
            systemCostP(d+1,1) = CapOrder(i,1);
            systemCostQ(d+1,1) = CapI(CapOrderI(i),1)+systemCostQ(d,1);
            d = d+1;
         end;
      end;
end;
totals = length(systemCostP(:,1));
totalm = length(marginalCostP(1,:));
if totals < totalm
   for i = (totals+1):1:totalm
      systemCostP(i,1) = 0;
      systemCostQ(i,1) = 0;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [ CapI, CapPI] = NUMBERGEN(marginalCostP, marginalCostQ)
CapMarg = marginalCostQ'; CapP = marginalCostP(1,:)';
d = 1;
for g = 1:1:length(CapMarg(:,1))
   for h = 1:1:length(CapMarg(1,:))
      if CapMarg(g,h) > 0
         CapI(d,1) = CapMarg(g,h);
         CapPI(d,1) = CapP(g);
         d = d+1;
      end;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
%%%Agents with Algorithm A3%%%%%%%%%%%%%%%%%%%%%%%%
clear all;
[marginalCostPX, marginalCostQ, minUnitP, minUnitQ, PenaltyUnit] = UNITPRODUCER2(1, 1);%
marginalCostP numGen*x marginalCostQ 1*x
marginalCostP = marginalCostPX(1,:);
numGen = length(marginalCostQ(:,1));
hEnd = length(marginalCostP(1,:));
fclose('all');
fid=fopen('C:\MatlabModel\Exp3p1UPPABNew\loadmod1.txt','rt');
[dummy,count]=fscanf(fid, '%f',[2, 1200]);
fclose('all');
actualLoad = dummy(1,:)';
forecast = dummy(2,:)';
totalperiod = length(actualLoad);
xx1 = 5;xx2 = 3;minP = 0;maxP = 150;
indexL = [30:xx1:100];
indexP = [minP:xx2:maxP];
indexP = [indexP];
lengthL = length(indexL); indexQ = zeros(lengthL, lengthL);
xx = 5;
for h = 1:1:lengthL
   inTQ = [0:xx:indexL(h)];
   indexQ(h,[1:1:length(inTQ)]) = inTQ;
end;
lengthP = length(indexP);
lengthQ = length(indexQ(1,:));
delta = 0.1;
yL =  actualLoad([1:1:24],1)';
for j = 1:1:numGen
   competitiveBidP([24*(j-1)+1:1:24*j],:) = ones(24,1)*marginalCostP(1,:);
   competitiveBidQ([24*(j-1)+1:1:24*j],:) = ones(24,1)*marginalCostQ(j,:);
end;
[yP, SchedulingGen, SchedulingPr]= CLEARUP(competitiveBidP, competitiveBidQ, yL, 0);
aLoad = DISCRETIZE(yL, indexL);
for j = 1:1:numGen
   marginalCost =[marginalCostP' marginalCostQ(j,:)'];
   ymarginalCost([2*(j-1)+1:1:2*j],:)= marginalCost';
   ySchedule = SchedulingGen([j:numGen:(23*numGen+j)],:);
   maxQ(j) = sum(marginalCostQ(j,:));
end;
max_maxQ = max(maxQ);
indexBidQuantity = zeros(numGen, max_maxQ);
for j = 1:1:numGen;
   XqBid = [0.25:0.25:maxQ(j)];
   Xq(j) = length(XqBid);
   indexBidQuantity(j, [1:1:Xq(j)]) = XqBid;
end;
lengthBidQ  = length(indexBidQuantity(1,:));
%%%%%%%%%%%%%%%%%%%%%%%%Input%%%%%%%%%%%%%%%%%%%%%%
GmaxPa = zeros(lengthL,lengthP); %W2
GmaxQa = zeros(lengthL,max(Xq)); %W2
for m = 2:1:numGen
   GmaxPa = cat(3, GmaxPa, zeros(lengthL, lengthP)); %W3
   GmaxQa = cat(3, GmaxQa, zeros(lengthL, max(Xq))); %W4
end;
Kp = lengthP;
%%%%%%%%%%%%%%%%%%%Find r_star%%%%%%%%%%%%%%%%%%%%%%
r_starP = 0; TrPstar = 2^(r_starP);
deltarPstar = delta/((r_starP+1)*(r_starP+2));
while deltarPstar < Kp*TrPstar*exp(-Kp*TrPstar)
   r_starP = r_starP + 1;
   TrPstar = 2^(r_starP);
   deltarPstar = delta/((r_starP+1)*(r_starP+2));
end;
for m = 1:1:numGen
   r_starQ(m) = 0; TrQstar(m) = 2^(r_starQ(m));
   deltarQstar(m) = delta/((r_starQ(m)+1)*(r_starQ(m)+2));
   while deltarQstar(m) < Xq(m)*TrQstar(m)*exp(-Xq(m)*TrQstar(m))
      r_starQ(m) = r_starQ(m) + 1;
      TrQstar(m) = 2^(r_starQ(m));
      deltarQstar(m) = delta/((r_starQ(m)+1)*(r_starQ(m)+2));
   end;
```

```
   TrQ(m) =  2^(r_starQ(m));
end;
rP = zeros(1,numGen)+r_starP;
rQ = r_starQ;
TrP = zeros(1,numGen)+ 2^(r_starP);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
deltarP(1) = delta/((rP(1)+1)*(rP(1)+2));
deltarQ(1) = delta/((rQ(1)+1)*(rQ(1)+2));
gamma_rP = min(3/5, 2*sqrt((3*Kp*log(Kp))/(5*TrP(1))));
gamma_rQ = min(3/5, 2*sqrt((3*Xq(1)*log(Xq(1)))/(5*TrQ(1))));
alphaP = 2*sqrt(log(Kp*TrP(1)/deltarP(1)));
alphaQ = 2*sqrt(log(Xq(1)*TrQ(1)/deltarQ(1)));
TrP(1) = 2^(rP(1));
TrQ(1) = 2^(rQ(1));
W3 = zeros(lengthL,lengthP)+ exp((alphaP*gamma_rP/3)*(sqrt(TrP(1)/Kp))); %W2
W4 = zeros(lengthL,max(Xq))+exp((alphaQ*gamma_rQ/3)*(sqrt(TrQ(1)/Xq(1)))); %W2
p3 = 0; p4 = 0;
for i = 1:1:lengthL
   for j = 1:1:lengthP
      p3(i,j) = (1- gamma_rP)*(W3(i,j)/sum(W3(i,:))) +  gamma_rP/lengthP;
   end;
   W4(i,[Xq(1)+1:1:max(Xq)]) = 0;
   for h = 1:1:Xq(1)
      p4(i,h) = (1- gamma_rQ)*(W4(i,h)/sum(W4(i,[1:1:Xq(1)]))) +  gamma_rQ/Xq(1);
   end;
   Mx = max(Xq)- Xq(1);
   if Mx > 0
      p4(i,[Xq(1)+1:1:max(Xq)]) = zeros(1,Mx);
   end;
end;
for m = 2:1:numGen
   deltarP(m) = delta/((rP(m)+1)*(rP(m)+2));
   deltarQ(m) = delta/((rQ(m)+1)*(rQ(m)+2));
   gamma_rP = min(3/5, 2*sqrt((3*Kp*log(Kp))/(5*TrP(m))));
   gamma_rQ = min(3/5, 2*sqrt((3*Xq(m)*log(Xq(m)))/(5*TrQ(m))));
   alphaP = 2*sqrt(log(Kp*TrP(m)/deltarP(m)));
   alphaQ = 2*sqrt(log(Xq(m)*TrQ(m)/deltarQ(m)));
   TrP(m) = 2^(rP(m));
   TrQ(m) = 2^(rQ(m));
   xW3 = zeros(lengthL, lengthP)+ exp((alphaP*gamma_rP/3)*(sqrt(TrP(m)/Kp)));
   xW4 = zeros(lengthL, max(Xq))+ exp((alphaQ*gamma_rQ/3)*(sqrt(TrQ(m)/Xq(m))));
   W3 = cat(3, W3, xW3); %W3
   W4 = cat(3, W4, xW4); %W4
   m3 = 0; m4 = 0;
   for i = 1:1:lengthL
      for j = 1:1:lengthP
         m3(i,j) = (1- gamma_rP)*(W3(i,j,m)/sum(W3(i,:,m))) +  gamma_rP/lengthP;
      end;
      W4(i,[Xq(m)+1:1:max(Xq)],m) = 0;
      for h = 1:1:Xq(m)
         m4(i,h) = (1- gamma_rQ)*(W4(i,h,m)/sum(W4(i,[1:1:Xq(m)],m))) +  gamma_rQ/Xq(m);
      end;
      Mx = max(Xq)- Xq(m);
      if Mx > 0
         m4(i,[Xq(m)+1:1:max(Xq)]) = zeros(1,Mx);
      end;
   end;
   p3 = cat(3, p3, m3); %W3
   p4 = cat(3, p4, m4); %W4
end;
for j = 1:1:numGen
   bidPric = 0; bidQuantity = 0;
   indexBidP = indexP;
   indexBidQ = indexBidQuantity(j,:);
   lengthBidP = lengthP;
   ranNum = rand(24,1);
   for i = 1:1:length(yL)
      chooseP(i) = MAPPING(p3(aLoad(i,:),:,j), ranNum(i), indexP, lengthP);
      bidPrice(i,1) = chooseP(i);%??
      chooseQ(i) = MAPPING(p4(aLoad(i,:),:,j), ranNum(i), indexBidQ, Xq(j));
      bidQuantity(i,1) = chooseQ(i);
   end;
```

```
   setbidP(j,:) = chooseP;
   setbidQ(j,:) = chooseQ;
end;
%%
newRP = TrP;
newRQ = TrQ;
for d = 1:1:(totalperiod/24)
   fLoad(d,:) = forecast([24*(d-1)+1:1:24*d],1)';
   marginalCostQM = marginalCostQ;
   for n = 1:1:numGen
      A0 = 0; A1 = 0; A2 = 0; A3 = 0; A4 = 0; A5 = 0; A6 = 0;
      A3 = W3(:,:,n);A4 = W4(:,:,n);pA3 = p3(:,:,n);pA4 = p4(:,:,n);
      ySchedule = SchedulingGen([n:numGen:(23*numGen+n)],:);
      ySchedulePr = SchedulingPr([n:numGen:(23*numGen+n)],:);
      marginalCost =[marginalCostP' marginalCostQM(n,:)'];
      ymarginal = (ymarginalCost([2*(n-1)+1:1:2*n],:))';
      indexBidQ = indexBidQuantity(n,:);
      lengthBidQ = Xq(n);
      setP = setbidP(n,:);
      setQ = setbidQ(n,:);
      Kp =length(A3(1,:));
      Kq =Xq(n);
      GmaxP = GmaxPa(:,:,n);
      GmaxQ = GmaxQa(:,:,n);
      deltarP(n) = delta/((rP(n)+1)*(rP(n)+2));
      deltarQ(n) = delta/((rQ(n)+1)*(rQ(n)+2));
      gamma_rP = min(3/5, 2*sqrt((3*Kp*log(Kp))/(5*TrP(n))));
      gamma_rQ = min(3/5, 2*sqrt((3*Kq*log(Kq))/(5*TrQ(n))));
      alphaP = 2*sqrt(log(Kp*TrP(n)/deltarP(n)));
      alphaQ = 2*sqrt(log(Kq*TrQ(n)/deltarQ(n)));
      if newRP(n) == 0
         rP(n) = rP(n)+1;
         TrP(n) = 2^(rP(n));
         newRP(n) = TrP(n);
         A3 = A3*0+ exp((alphaP*gamma_rP/3)*(sqrt(TrP(n)/Kp)));
      end;
      if newRQ(n) == 0
         rQ(n) = rQ(n)+1;
         TrQ(n) = 2^(rQ(n));
         newRQ(n) = TrQ(n);
         A4 = A4*0+ exp((alphaQ*gamma_rQ/3)*(sqrt(TrQ(n)/Kq)));
      end;
      alpha = [alphaP alphaQ]; Tr = [TrP(n) TrQ(n)];
      [bidP,bidQ,A3,A4,tProfit,setP, setQ, pA3, pA4, GmaxP, GmaxQ] =
NEWGENExp3_P_1(ymarginal,marginalCost,yP,yL,ySchedule,fLoad(d,:), indexL, indexP, indexBidQ,
A3,A4,  gamma_rP, gamma_rQ, pA3, pA4, setP, setQ,ySchedulePr, lengthBidQ,GmaxP, GmaxQ, alpha,
Tr);
      newRP(n) = newRP(n) - 1;
      newRQ(n) = newRQ(n) - 1;
      bidPrice([((n-1)*24+1):1:24*n],[1:1:length(bidP(1,:))]) = bidP;
      bidQuantity([((n-1)*24+1):1:24*n],[1:1:length(bidQ(1,:))]) = bidQ;
      ymarginalCost([2*(n-1)+1:1:2*n],:)= marginalCost';
      setbidP(n,:) = setP;
      setbidQ(n,:) = setQ;
      todayProfit(n,:) = tProfit;
      W3(:, :, n) = A3;
      W4(:, :, n) = A4;
      p3(:, :, n) = pA3;
      p4(:, :, n) = pA4;
      GmaxPa(:,:,n) = GmaxP;
      GmaxQa(:,:,n) = GmaxQ;
      lBid = length(bidP(1,:));
      bidAgentP([24*(d-1)+1:1:24*d],[lBid*(n-1)+1:1:n*lBid]) = bidP;
      bidAgentQ([24*(d-1)+1:1:24*d],[lBid*(n-1)+1:1:n*lBid]) = bidQ;
   end;
   if d == 1
      Profit = todayProfit;
   else
      G = [Profit todayProfit];
      Profit(:,[1:1:length(G(1,:))]) = G;
   end;
   [dayLoad] = fLoad(d,:);
```

```
   [competitivePrice Sc BB1]= CLEARUP(competitiveBidP, competitiveBidQ, dayLoad, 0);
   [marketPrice, SchedulingGen, SchedulingPr]= CLEARUP(bidPrice, bidQuantity, dayLoad, 0);
   if d == 1
      Price = marketPrice;
      CPrice = competitivePrice;
      Load = dayLoad;
   else
      Price = [Price marketPrice];
      CPrice = [CPrice competitivePrice];
      Load = [Load dayLoad];
   end;
   yL = dayLoad;
   yP = marketPrice;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [bidP,bidQ,W3,W4,todayProfit, setP, setQ, p3, p4, GmaxP, GmaxQ] =
NEWGENExp3_P_1(ymargin,marginalCost,yPrice,yLoad,ySchedule,fLoad,indexL, indexP, indexQ, W3, W4,
gammaP, gammaQ, p3, p4, setP, setQ,yScPrice, lengthBidQ,  GmaxP, GmaxQ, alpha, Tr)
total= length(fLoad);
alphaP = alpha(1); alphaQ = alpha(2); TrP = Tr(1); TrQ = Tr(2);
[BB, todayProfit, AA] = PROFITCAL(yPrice, marginalCost, ySchedule, yScPrice, 1);
scheduleQ = sum(ySchedule, 2);
[yloadIndexD] = DISCRETIZE(yLoad,indexL);
ranNum = rand(24,1);
[W3, p3, GmaxP] = NEWWExp3_P_1(setP, yloadIndexD, W3, todayProfit, p3, gammaP, length(W3(1,:)),
indexP, GmaxP, alphaP,TrP);
[priceIndex] = DISCRETIZE(yPrice, indexP);
[W4, p4, GmaxQ] = NEWWExp3_P_1(setQ, yloadIndexD, W4, todayProfit, p4, gammaQ, lengthBidQ,
indexQ, GmaxQ, alphaQ, TrQ);
[loadIndexD] = DISCRETIZE(fLoad,indexL);
[aLoad] = [loadIndexD];
[bidP, bidQ, setP, setQ] = SETBID(aLoad, marginalCost, indexQ, indexP, W3, W4, ranNum, p3, p4,
lengthBidQ);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [todayUnitProfit, todayProfit, AA] = PROFITCAL(yPrice, marginalCostM, yscheduling,
yScPrice, flag)
total = length(yPrice);
ySchQ = yscheduling;
CC = marginalCostM;
if flag == 1 %%for aggregate profits
   for i = 1:1:total
      marginalCost = [marginalCostM(:,1) marginalCostM(:,2)];
      todayUnitProfit(i,:) = zeros(1,length(marginalCost(:,1)));
      addCost = 0;
      schedule = 0;
      scheduleP = 0;
      sumSchedule(i) = sum(yscheduling(i,:));
      if sumSchedule(i) == 0
         todayProfit(i) = 0;
         todayProfit(i) = todayProfit(i) - addCost;
      else if sumSchedule(i) < 0
            schedule(1) = sumSchedule(i);
            scheduleP(1) = 0;
            %scheduleP(1) = Penalty(i);
            sumSchedule(i) = -1;
            %sumCost = -Penalty(i)*schedule(1)+ sum(yScPrice(i,:).*yscheduling(i,:));
            sumCost = sum(yScPrice(i,:).*yscheduling(i,:));
            X = 0;
            X1 = and(marginalCost(:,2), ones(hEnd,1));%%to find the unit with positive capacity
            todayUnitProfit(i,:) = todayUnitProfit(i,:) + (sumCost/X)*X1';
         else %%sumSchedule(i) >=min(minQ(i,:))
            h = 1;
            hEnd = length(marginalCost(:,1));
            while sumSchedule(i) > 0.001
               if sumSchedule(i)- marginalCost(h,2) >= 0
                  if marginalCost(h,2) > 0
                     sumSchedule(i)= sumSchedule(i) - marginalCost(h,2);
                     scheduleP(h) = marginalCost(h,1);
                     schedule(h) = marginalCost(h,2);
                  else
                     sumSchedule(i)= sumSchedule(i);
                     scheduleP(h) = marginalCost(h,1);
```

```
            schedule(h) = 0;
         end;
         h =h+1;
      else %%to reschedule
         found = 0; g1 = h;
         gb = h;
         while found == 0
            if g1 <= hEnd
               if sumSchedule(i) >= 0
                  %if sumSchedule(i) >= minQ(i,g1)
                  scheduleP(g1) = marginalCost(g1,1);
                  schedule(g1) = sumSchedule(i);
                  sumSchedule(i) = -1;
                  found = 1;
               else
                  restSch = minQ(i,g1) - sumSchedule(i);
                  if g1 > 1
                     rest = marginalCost([1:1:g1-1],2)'- zeros(1,[1:1:g1-1]);
                     if sum(schedule) > 0
                        if sum(rest) >= restSch
                           ff1 = 1; f = g1-1; e = g1-1;
                           while ff1 == 1
                              if rest(f) > 0
                                 if schedule(e) > 0
                                    if rest(f) >  restSch
                                       schedule(e) = schedule(e) - restSch;
                                       ff1 = 0;
                                    else
                                       schedule(e) = schedule(e) - rest(f);
                                       restSch = restSch - rest(f);
                                    end;
                                 end;
                              end;
                              f = f-1; e = e-1;
                           end;
                           schedule(g1) = 0;
                           %schedule(g1) = minQ(i,g1);
                           scheduleP(g1) = marginalCost(g1,1);
                           found = 1;
                           sumSchedule(i) = -1;
                        else % sum(rest) < restSch
                           if gb == hEnd
                              schedule(g1) = sumSchedule(i);
                              %scheduleP(g1) = Penalty(i);
                              scheduleP(g1) = 0;
                              %addCost = minP(i,g1);
                              addCost = 0;
                              sumSchedule(i) = -1;
                              found = 1;
                           else
                              schedule(g1) = 0;
                              scheduleP(g1) = 0;
                              g1 = g1+1;
                           end;
                        end;
                     else %sum(schedule) == 0
                        schedule(g1) = 0;
                        scheduleP(g1) = 0;
                        g1 = g1+1;
                     end;
                  else %%if g1 == 1
                     schedule(g1) = 0;
                     scheduleP(g1) = 0;
                     g1 = g1+1;
                  end;
               end;
            else
               schedule(g2) = sumSchedule(i);
               %scheduleP(g2) = Penalty(i);
               scheduleP(g2) = 0;
               sumSchedule(i) = -1;
               found = 1;
```

```
                        ff2 = 0;
                     end;
                  end;
               end;
            end;
            d1 = 1;
            todayUnitProfit(i,[1:1:length(schedule)]) =
todayUnitProfit(i,[1:1:length(schedule)]) -scheduleP.*schedule;
            for s = 1:1:length(schedule)
               if marginalCost(s,2) > 0
                  if schedule(s) > 0
                     fd1 = 0;
                     tempSch = schedule(s);
                     while tempSch > 0.0001
                        if ySchQ(i,d1) == tempSch
                           todayUnitProfit(i,s) = todayUnitProfit(i,s)+ yScPrice(i,d1)*tempSch;
                           tempSch = tempSch - ySchQ(i,d1);
                           ySchQ(i,d1) = ySchQ(i,d1) - tempSch;
                           d1 = d1+1;
                        else if ySchQ(i,d1) > tempSch
                              todayUnitProfit(i,s) = todayUnitProfit(i,s)+
yScPrice(i,d1)*tempSch;
                              tempSch = 0;
                              ySchQ(i,d1) = ySchQ(i,d1) - tempSch;
                           else
                              todayUnitProfit(i,s) = todayUnitProfit(i,s)+
yScPrice(i,d1)*ySchQ(i,d1);
                              tempSch = tempSch - ySchQ(i,d1);
                              ySchQ(i,d1) = ySchQ(i,d1) - ySchQ(i,d1);
                              d1 = d1+1;
                           end;
                        end;
                     end;
                  end;
               end;
            end;
         end;
         todayProfit(i) = sum(yScPrice(i,:).*yscheduling(i,:))-sum(schedule.*scheduleP);
      end;
   end;
end;
if total == 1
   AA = schedule;
else
   AA = 0;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [W3, p3, Gmax] = NEWW3Exp3_P_1(priceBid, loadIndexD, W3,todayProfit, p3, gammaz,
lengthBid, indexP, Gmax, alpha,T)
total = length(loadIndexD); K = lengthBid;
sumreward = zeros(length(W3(:,1)),lengthBid); count = zeros(length(W3(:,1)),lengthBid);
for i = 1:1:total
   if todayProfit(i) < 0
      rewardz(i) = 0;
   else
      rewardz(i) = 1 - exp(-0.005*todayProfit(i));
   end;
   for z = 1:1:lengthBid
      if priceBid(i) == indexP(z)
         sumreward(loadIndexD(i),z) = sumreward(loadIndexD(i), z)+ rewardz(i);
         count(loadIndexD(i),z) = count(loadIndexD(i),z)+1;
      end;
   end;
end;
for m = 1:1:length(W3(:,1))
   for j = 1:1:lengthBid
      if count(m,j) > 0
         reward(m,j) = sumreward(m,j)/count(m,j);
      else
         reward(m,j) = 0;
      end;
   end;
```

```
end;
for i = 1:1:total
   for j = 1:1:lengthBid
      xhat = reward(loadIndexD(i),j)/p3(loadIndexD(i),j);
      Gmax(loadIndexD(i),j) = Gmax(loadIndexD(i),j)+ xhat;
      W3(loadIndexD(i), j) = W3(loadIndexD(i),j)*exp((gammaz/(3*K))*(xhat+
alpha/(p3(loadIndexD(i), j)*sqrt(K*T))));
   end;
end;
countP = zeros(total,1);
for i = 1:1:length(p3(:,1))
   for j = 1:1:length(W3(1,:))
      p3(i,j) = (1-gammaz)*(W3(i,j)/sum(W3(i,:))) +  gammaz/K;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [LoadIndex] = DISCRETIZE(Load,Index)
total = length(Load);LoadIndex = zeros(total,1);total1 = length(Index);
for j = 1:1:total
   h = 1;
   while LoadIndex(j) == 0 %| h <= total1
      if h == 1
         if Load(j) <= Index(h)
            LoadIndex(j)= h;
         end;
      else if h <= total1
            if Load(j)> Index(h-1) & Load(j) <= Index(h)
               LoadIndex(j) = h;
            end;
         else%h > total
            LoadIndex(j) = total1;
         end;
      end;
      h = h+1;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [bidP, bidQ, setP, setQ] = SETBID(aLoad, marginalCost, indexQ, indexP, W3, W4, ranNum,
p3, p4, lengthBidQ)
total = length(aLoad);
for i = 1:1:total
   chooseP(i) = MAPPING(p3(aLoad(i,:),:), ranNum(i), indexP, length(W3(1,:)));
   bidPrice(i,1) = chooseP(i);
   setP(1,i) = bidPrice(i,1);
   chooseQ(i) = MAPPING(p4(aLoad(i,:),:), ranNum(i), indexQ, lengthBidQ);
   setQ(1,i) = chooseQ(i);
end;
withholdP = 150;
total1 = length(marginalCost(:,1));
for i = 1:1:total
   capWithhold = max(0,sum(marginalCost(:,2))-chooseQ(i));
   h = length(marginalCost(:,1));
   T = h;
   schedule(i,:) = zeros(1,h);
   marginQ(i,:) = zeros(1,h);
   scheduleP(i,:)= zeros(1,h);
   while h > 0
      if marginalCost(h,1) <= withholdP
         if marginalCost(h,2) > 0
            if (capWithhold- marginalCost(h,2)) >= 0
               capWithhold =(capWithhold- marginalCost(h,2));
               schedule(i,T-h+1) = 0;
               scheduleP(i,T-h+1) = marginalCost(h,1);
            else
               schedule(i,T-h+1) = marginalCost(h,2) -capWithhold;
               capWithhold = 0;
               scheduleP(i,T-h+1) = marginalCost(h,1);
            end;
         end;
      end;
      h = h-1;
   end;
```

```
   for b = 1:1:totall
      if sum(schedule(i,:)) > 0
         if marginalCost(b,1)> withholdP
            if marginalCost(b,2) > 0
               marginQ(i,b) = marginalCost(b,2);
            end;
         else
            marginQ(i,b)= schedule(i,totall-b+1);
         end;
      else
         marginQ(i,b) = marginalCost(b,2);
      end;
   end;
   maxWithhold(i) = sum(marginalCost(:,2))-sum(marginQ(i,:));
   marginP(i,:) = marginalCost(:,1)';
end;
bidQuantity = marginQ;
schedule = 0;
for i =1:1:total
   antPrice(i) = chooseP(i);
   for r = 1:1:length(marginalCost(:,1))
      if marginalCost(r,1) < antPrice(i)
         bidPrice(i,r) = marginalCost(r,1);
         bidQuantity(i,r) = bidQuantity(i,r);
      else if marginalCost(r,1) == antPrice(i)
            bidPrice(i,r) = antPrice(i);
            bidQuantity(i,r) = bidQuantity(i,r);
         else
            bidPrice(i,r) =marginalCost(r,1);
            bidQuantity(i,r) = bidQuantity(i,r);
         end;
      end;
   end;
   bidPrice(i,r+1) = min(antPrice(i)+ 3,150);%max(bidPrice(i,r+1), antPrice(i)+inc);
   bidQuantity(i,r+1) = maxWithhold(i);%bidQuantity(i,r+1);
end;
bidP = bidPrice;
bidQ = bidQuantity;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function [X] = MAPPING(W, ranNum, indexP, mx)
lengthP = mx;
Wnor = W;Wsum = Wnor(1);
for i = 2:1:lengthP
   Wsum(i) = Wsum(i-1)+ Wnor(i);
end;
found = 0; i = 1;
while found == 0
   if i == 1
      if ranNum < Wsum(1)
         X = indexP(i);
         found = 1;
      else
         i = i+1;
      end;
   else if i < lengthP
         if ranNum >= Wsum(i) & ranNum < Wsum(i+1)
            X = indexP(i);
            found = 1;
         else
            i = i+1;
         end;
      else
         X = indexP(i);
         found = 1;
      end;
   end;
end;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Samples of loadmodl.txt
%46.11  46.11
%44.2      42.56
%48     46
```

```
%74.86  75.67
%73.58  74.11
%72.64  72.89
%70.7       71.44
%68.96  70.33
%69.76  71.11
%69.24  68.22
%62.24  60.89
%54.72  53.89
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Samples of sampleGENA2.txt
%marginal cost 10  12  15  20  27  30  35  38  42  48  55  60  72
%                  10  12  15  20  27  30  35  38  42  48  55  60  72
%                  10  12  15  20  27  30  35  38  42  48  55  60  72
%                  10  12  15  20  27  30  35  38  42  48  55  60  72
%Capacity       3  0   2   1   0   0   0   2   0   0   0   0   0
%                  6   4   3   3   0   2   0   0   2   0   0   0   1
%                  0   7   0   0   0   0   1   0   0   0   0   0   0
%                  0   0   2   2   0   0   0   0   0   0   1   1   0
%Penalty        0  0   0   0   0   0   0   0   0   0   0   0   0
%               0  0   0   0   0   0   0   0   0   0   0   0   0
%               0  0   0   0   0   0   0   0   0   0   0   0   0
%               0  0   0   0   0   0   0   0   0   0   0   0   0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Samples of sampleGENA2a.txt
%Minimum Quantity  0  0   0   0   0   0   0   0   0   0   0   0   0
%                   0  0   0   0   0   0   0   0   0   0   0   0   0
%                   0  0   0   0   0   0   0   0   0   0   0   0   0
%                   0  0   0   0   0   0   0   0   0   0   0   0   0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Samples of sampleGENA2b.txt
%Minimum Price   0 0   0   0   0   0   0   0   0   0   0   0   0
%                0 0   0   0   0   0   0   0   0   0   0   0   0
%                0 0   0   0   0   0   0   0   0   0   0   0   0
%                0 0   0   0   0   0   0   0   0   0   0   0   0
```

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