The Single Airport Static Stochastic Ground Holding Problem

by

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B.S., Massachusetts Institute of Technology (1994)

Submitted to the Department of Electrical Engineering and Computer Science
in partial fulfillment of the requirements for the degree of

Master of Science in Operations Research

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

June 1998

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Abstract

We discuss methods for formulating and solving the single airport static stochastic ground holding problem in air traffic flow management. We begin by exploring a seminal model of Richetta and Odoni. We define a new model, the Static Stochastic model, that represents a substantial simplification of the Richetta model. We introduce a new model, the Maximum Air Delay model, which solves a closely related problem. We prove that the linear programming relaxations of both the Static Stochastic and Maximum Air Delay models are guaranteed to yield integer solution to their respective problems. We also consider an extension of the Static Stochastic model that explicitly penalizes wasted capacity.

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Acknowledgments

I would like to acknowledge Professor Amedeo Odoni of MIT and Dr. Stephen Kolitz of Draper Laboratories, whose guidance and support made this thesis possible.

I would like to acknowledge Bill Hall, for helping me find a power cord when I needed it most.

Contents

Chapter 1

Introduction

In air traffic flow management (ATFM), large amounts of costs and congestion are incurred due to uncertainty of future landing capacity over a several hour time horizon. Ground holding is one of the basic methods of lowering these costs. The idea is simple: it is preferable to have a flight wait on the ground at its point of origin than to have it circle the airport at its destination, unable to land. Therefore, if it is known with certainty, or at least with high probability, that a flight will be unable to land due to lack of capacity, it may be advantageous to hold the flight on the ground at its point of origin. Ground holding saves fuel costs and increases safety margins by relieving airborne congestion.

The FAA introduced a ground holding strategy in the early 1980's. For each possibly capacitated airport, the FAA generated an estimate, or forecast, of capacity over the next few hours. The FAA then treated this forecast as a deterministic profile of future landing capacity, and groundheld exactly enough airplanes such that if capacity materialized as planned, there would be no air holds (planes forced to wait in the air at their destination due to lack of landing capacity). The software used to implement this functionality for the FAA is known as *Grover Jack.* Throughout this thesis, this policy will be known as the *deterministic* ground holding policy.

The essential problems with the deterministic ground holding policy are that a forecast of future capacity is generated in an ad hoc manner, and that this forecast is then treated as exactly correct. In other words, once the forecast is made, the stochastic nature of future capacity is ignored.

An additional problem with the deterministic policy is that if the FAA's forecast of capacity is not equal to the *expected* values of future capacity, the deterministic policy will introduce a systematic bias in ground holding. This appears to be the case in practice: the FAA's capacity forecasts seem to be overly conservative, corresponding more closely to worst-case scenarios than expected-case scenarios, leading to a large number of ground holds which are, from the airlines' point of view, unnecessary, and therefore to a large amount of wasted capacity.

In this thesis, we look at *static stochastic* models for the single-airport ground holding problem: stochastic, in that they explicitly take into account the stochastic nature of future capacity, and static, in that they require all decisions over a given time horizon to be made in advance. In Chapter 2, we look at previous work on this problem. In Chapter 3, we define the problem more precisely, and in Chapter 4, we examine a particular model, developed by Richetta and Odoni. In Chapter 5, we introduce the Static Stochastic model, which represents a substantial simplification of the Richetta and Odoni model. In Chapter 6, we prove that the constraint matrix associated with the Static Stochastic Model is totally unimodular, indicating that applying the ellipsoid method to this model yields a polynomial-time algorithm for this problem. In Chapter 7, we explore two simple extensions of the Static Stochastic model, involving new types of constraints. In Chapter 8, we see how adding one of these new types of constraints and deleting a portion of the Static Stochastic Model yields a model for a closely related, and possibly more applicable, model. In Chapter 9, we explore a more complicated extension, involving a new type of objective function that explicitly penalizes "wasted" capacity. In Chapter 10, we present several examples and attempt to gain insight into the workings of the models. Finally, in Chapter 11, we draw conclusions and discuss directions for future work.

Chapter 2

Literature Review

The seminal paper on the static stochastic ground holding problem is "Solving Optimally the Static Ground-Holding Policy Problem in Air Traffic Control," by Richetta and Odoni [RO93b]. This paper deals with stochasticity by assuming that a probabilistic distribution of "scenarios", or possible realizations of capacity, is known. By treating arrivals as flows rather than as individual flights, and making use of the fact that empirically, the linear programming relaxation always yields integer solutions, the model defined in this paper is practically solvable for reasonable problem sizes.

An earlier paper which assumes that the airport is capacitated for only a single time period is [ARJ87]. Mina Sheel, in her Masters' Thesis at the MIT Operations Research Center [She94], performs an empirical analysis of the model developed by Richetta and Odoni. In [RO93a], Richetta and Odoni introduce a dynamic version of the problem, in which additional decisions may be made as partial information on the capacity realization becomes available.

A large body of work exists on deterministic versions of the ground-holding problem. Good examples include the PhD theses of Terrab [Ter90] and Vranas [Vra92b], papers by Vranas [Vra92a], Vranas and Bertsimas [BV95], Vranas, Bertsimas and Odoni [VBO94], and Bertsimas and Stock [BSar]; this list is by no means exhaustive.

Chapter 3

Description of the Problem

The static stochastic single-airport ground holding problem assumes that a single capacitated airport exists, and that travel times are deterministic and known in advance; the only element of uncertainty is the arrival capacity at the destination airport. An instance of the static stochastic single-airport ground holding problem (SSGHP) with $T \in \mathcal{Z}_+$ time periods, $Q \in \mathcal{Z}_+$ scenarios and $F \in \mathcal{Z}_+$ flights consists of a vector $S \in |T|^{|F|}$ of scheduled arrival times, Q arrival capacity scenarios $M_{q,t}$, $1 \leq q \leq Q$, $1 \leq t \leq T$, a probability vector p_q over the scenarios, c_g, the cost of ground holding a single plane for one time period, and $c_a > c_g$, the cost of one period of air delay for a single plane. (If $c_g \leq c_a$, it is always optimal to allow all planes to take off as scheduled.) The objective is to generate a sequence $S^* \in |T+1|^{|F|}$ of actual arrival times that minimizes the sum of the expected air and ground holding costs, subject to the constraint that no airplane may arrive early: each airplane's actual arrival time may not be before its originally scheduled arrival time. Note that the arrival time of a flight is defined to be *the time it arrives in the airspace of the arrival airport*, not necessarily the time it actually lands. We also assume that the landing capacity at time $T+1$ is $|F|$, i.e., that all planes will be able to land by time $T+1$.

Chapter 4

The Richetta Model

Richetta made some important simplifications to the problem that allowed him to solve it to optimality. The crucial idea was the attempt to treat planes arriving at the capacitated airport as a *flow* rather than as individual planes. More specifically, we can aggregate the originally scheduled arrivals at each time period t into a vector of *demands* D_t, $1 \leq t \leq T$, where $D_t = |f \in F : F_f = t|$. Once we have done this, we can formulate the problem using decision variables $X_{i,j}$, $i \leq j$, where $X_{i,j}$ is the number of planes originally scheduled to arrive at time i that we reschedule to arrive at time j. We define variables $W_{q,t}$, $1 \leq q \leq Q$, $1 \leq t \leq T$, where $W_{q,t}$ is the number of planes experiencing airborne delay at time period t under scenario q.

In his original formulation, Richetta used slightly super-linear ground holding costs (i.e., it is slightly more than twice as expensive to ground hold planes for two periods than for one), in order to avoid solutions where some planes are ground held for a very long time while other planes land immediately. We observe that this is not necessary if we agree that for $i < j$, all planes originally scheduled to land at time i must land before all planes originally scheduled to land at time j. We therefore use a linear ground holding cost function, with the understanding that we can recover a solution that satisfies the above constraint by post-processing.

This yields the following integer programming model:

$$\min \sum_{i=1}^{T} \sum_{j=i+1}^{T+1} c_g(j-i)X_{i,j} + c_a(\sum_{q=1}^{Q} p_q \sum_{i=1}^{T} W_{q,i}) \tag{4.1}$$

$$\sum_{j=i}^{T+1} X_{i,j} = D_i \qquad i = 1, \ldots, T \tag{4.2}$$

$$W_{q,i} - W_{q,i-1} - \sum_{j=1}^{i} X_{j,i} - S_{q,i} = -M_{q,i} \qquad t = 1, \ldots, T+1, \quad q = 1, \ldots, Q \quad (W_{q,0} = W_{q,T+1} = 0) \tag{4.3}$$

$$\sum_{i=1}^{T+1} S_{q,i} = \sum_{i=1}^{T} M_{q,i} \qquad q = 1, \ldots, Q \tag{4.4}$$

$$X_{i,j} \in \mathcal{Z}_+, W_{q,t} \in \mathcal{Z}_+, S_{q,t} \in \mathcal{Z}_+ \tag{4.5}$$

In the above integer program, the objective function (1) is the sum of the (fixed) ground costs and the (expected) air delay costs. We can define the problem in terms of a single cost parameter r because multiplying both c_a and c_g by a positive scalar does not affect the optimality of any given solution; we express the model in terms of two parameters, c_a and c_g, for clarity. Constraint set (2) stipulates that all planes scheduled to arrive at time i must arrive between time i and time $T+1$, inclusive. Constraint set (3) sets the air delays $W_{q,t}$ for each scenario — in words, this constraint set states that during a given time period, under a given scenario, the number of planes being air held is at least as large as the number of planes being air held from the previous time period, plus any new planes that arrive during that time period, minus the capacity at that time period under that scenario. Constraint set (4) is an artifact of the way in which Richetta posed the deterministic version of this problem as a network flow problem; in this formulation, it is redundant.

The key feature of this model is that by treating arrivals to the airport as a flow, we are able to avoid associating variables with individual flights. This greatly reduces the size of the problem, making solution to optimality plausible for reasonable problem sizes. Additionally, empirically, Richetta observed that the linear programming relaxation of the model always yielded integer solutions. He conjectured that that the constraint matrix of his model was unimodular, but did not prove or disprove his conjecture.

Performing several simple experiments, Richetta compared his model to both the deterministic policy, and a passive policy of allowing all planes to take off and arrive in the destination airport's airspace as scheduled. Note that the passive policy has the property that it minimizes the total *amount* of delay. Richetta found that in his experiments, his static stochastic model performed extremely well, in some cases generating solutions with only slighly more total delay than the passive model, and far lower costs.

Chapter 5

The Static Stochastic Model

Once we observe that under a first-come first-served (FCFS) discipline, we can recover large amounts of structural information via post-processing, we see that it is possible to simplify the problem substantially. We derive a new model, with decision variables A_t, $1 \leq t \leq T$, where A_t is simply the number of planes we allow to arrive in the airspace of the capacitated airport at time t. If we can derive an optimal solution for this model, we can recover arrival times for the individual flights by post-processing, using the assumption of an FCFS discipline. This assumption is quite reasonable and natural, as it corresponds precisely to current FAA policy, as well as to our intuition about fairness. We now have the following model:

$$\min \sum_{t=1}^{T} c_g G_t + \sum_{q=1}^{Q} \sum_{t=1}^{T} c_a p_q W_{q,t} \tag{5.1}$$

$$\sum_{t=1}^{j} A_t \leq \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{5.2}$$

$$\sum_{t=1}^{T+1} A_t = \sum_{t=1}^{T+1} D_t \tag{5.3}$$

$$W_{q,t} - W_{q,t-1} - A_t \geq -M_{q,t} \qquad t = 1, \ldots, T; \quad q = 1, \ldots, Q; \quad (W_{q,0} = 0) \tag{5.4}$$

$$G_j + \sum_{t=1}^{j} A_t = \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{5.5}$$

$$A_t \in \mathcal{Z}_+, W_{q,t} \in \mathcal{Z}_+, G_t \in \mathcal{Z}_+ \tag{5.6}$$

Here, the objective function (1) is again the sum of the (fixed) ground costs and the (expected) air delay costs; instead of presenting the ground delay costs explicitly in terms of the decision variables, we choose to define auxiliary variables G_t in constraint set (5). This constraint set is redundant — we could have substituted the A_t into the ground delay portion of the objective function, eliminating the G_t variables from the model, but we include them for clarity. Constraint set (2) stipulates that no plane may arrive before it is originally scheduled to do so, while constraint set (3) stipulates that all planes must arrive by time $T+1$. Constraint set (4) sets the air delays $W_{q,t}$ for each scenario.

This model achieves solutions equivalent to those found by the Richetta model, but represents a substantial improvement in terms of model size and simplicity. The following table compares the number of rows, columns, and non-zero elements in the constraint matrices of the two models, given Q scenarios and T time periods, after both models have been rephrased entirely in terms of equality constraints:

	Richetta Model	Static Stochastic Model
Rows	$QT + 2Q + T$	$QT + 2T + 1$
Columns	$\frac{(T+1)(T+2)}{2} + Q(2T+1)$	$2QT + 3T - Q + 1$
Non-Zero Elements	$(T+1)((Q+1)(\frac{T+2}{2}) + 4Q) - 2Q$	$T^2 + 4T + 4QT - Q + 1$

For example, if we have ten scenarios and thirty-two time periods, corresponding to an eight hour time horizon, the Richetta model has 372 rows, 1178 columns, and 7411 non-zero elements, while the Static Stochastic model has 385 rows, 727 columns, and only 2423 non-zero elements. The number of non-zero elements is probably the most important measure of the size of a model, because the models are solved as linear programming relaxations, and CPLEX stores a sparse representation of the model matrices. Indeed, we observe empirically that unless the problem is so small that CPLEX startup times dominate the running time, the Static Stochastic model does solve problems of this form approximately three times faster than the Richetta model. More importantly, the Static Stochastic model, by using a simpler form of decision variables, makes the model simpler and easier to understand.

Chapter 6

Unimodularity of the Static Stochastic Model

In this chapter, we prove that the constraint matrix associated with the Static Stochastic model is totally unimodular. All general results about totally unimodular matrices used in this chapter can be found in Nemhauser and Wolsey [NW88]. We will use the following characterization of totally unimodular matrices:

Theorem 1 *An $m \times n$ matrix is totally unimodular if and only if for every subset of the rows $S \subseteq N$ there exists a partition of S into two subsets S_1 and S_2 such that*

$$\left| \sum_{i \in S_1} a_{ij} - \sum_{i \in S_2} a_{ij} \right| \leq 1, \ \ j = 1, \ldots, N$$

We will show that such a partition exists for the constraint matrix of the Static Stochastic model. Additionally, we will need the following result:

Definition 2 *An $m \times n$ $0-1$ matrix A is an interval matrix if the 1's in each row appear consecutively.*

Theorem 3 *Interval matrices are totally unimodular.*

In order to see concretely the form of the constraint matrix, we include the matrix for the case $t = 4, q = 3$:

A_t					$W_{q,t}$												G_t			
+																				
+	+																			
+	+	+																		
+	+	+	+																	
+	+	+	+	+																
−					+															
	−				−	+														
		−				−	+													
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Looking back to our model, we first have $T+1$ constraints involving only the A_t; these are the constraints stipulating that no plane may arrive early, and that all planes must arrive by time $T+1$. We then have Q sets of T constraints each relating the A_t and the $W_{q,t}$. We call these constraints the *air-holding* constraints. These are the constraints that assign the $W_{q,t}$, the air delays; in our model, they are greater than or equal to constraints. We could rewrite them as less than or equal to constraints without affecting the total unimodularity of the matrix by multiplying these constraints by -1; it will be clear that this does not affect the proof. The final T constraints are the redundant constraints that set the ground holding variables, G_t.

We now show, given a subset of the rows $S \subseteq N$ that we can partition S into S_1 and S_2 in a manner that satisfies Theorem 1. We first note that since each G_t appears only once, those columns of the matrix will never violate the constraints of Theorem 1, and may consider only the A_t and the $W_{q,t}$ columns.

1, and may consider only the A_t and the $W_{q,t}$ columns.

We now examine the $W_{q,t}$ columns. Consider a single set of T air-holding constraints, corresponding to a particular scenario $q \in Q$. If our set S contains two adjacent rows from this set, either they must both be placed in S_1 or both be placed in S_2; otherwise we will generate a $+2$ or a -2 in some column corresponding to a $W_{q,t}$. This argument extends obviously to more than two adjacent air-holding constraints corresponding to the same scenario. This prompts the following definition:

Definition 4 *Given a subset S of the rows of A, we define a maxset M to be a maximal set of* adjacent *rows of air-holding constraints corresponding to the same scenario $q \in Q$. Additionally, we define functions $t_{start}(M)$ and $t_{end}(M)$, which, given a maxset M, return, respectively, the start and end times that that maxset covers.*

The maxset is a useful concept because maxsets are precisely the groupings of the air-holding constraints that need to be placed in the same subset of our partition of S in order to ensure that the sum in every $W_{q,t}$ column is $0, 1$, or -1. Any partition which respects maxsets will satisfy the requirements of Theorem 1 as far as the $W_{q,t}$ are concerned.

We complete our proof with the following construction. Given a subset S of the rows of A, we construct a new auxiliary matrix A' with $T+1$ columns. For each row in S which does not correspond to an air-holding constraint, we put the portion of that row corresponding to the A_t directly into A'. Additionally, for each maxset $M \subseteq S$ of the air-holding constraints, A' contains a single row with a 1 in the i'th column if $t_{start}(M) \leq i \leq t_{end}(M)$, and a 0 in the i'th column otherwise. By construction, A' is an interval matrix, and by Theorem 3 it is totally unimodular. Therefore, there exists a partition of the rows of A' into two subsets satisfying Theorem 1: let S'_1 and S'_2 be such a partition. Given such a partition, we can construct a partion of S into S_1 and S_2 that also satisfies Theorem 1. For a row of A' that corresponds to a row of S, place that row in S_1 if the corresponding row of A' is in S'_1, otherwise place it in S_2. For a row of A' corresponding to a maxset M in S, place all rows of M in S_1 if the corresponding row of A' is in S'_2, otherwise place all rows of M in

S_2; the change of sign accounts for the fact that the A_t variables are negated in the air-holding constraints.

Each A_t variable is added and subtracted the same number of times in S_1 and S_2 as it is in S_1' and S_2', therefore, S_1 and S_2 satisfy Theorem 1 as far as the A_t are concerned. Furthermore, this partitioning respects maxsets, so clearly none of the $W_{q,t}$ can cause Theorem 1 to be violated. We conclude that the constraint matrix A is totally unimodular.

Because the constraint matrix is totally unimodular, linear programming relaxations of this problem are guaranteed to yield integral solutions. Because there are only a polynomial number of constraints, the ellipsoid algorithm (or any polynomial time linear programming algorithm) yields a polynomial time algorithm for the single-airport static stochastic ground holding problem.

We note in passing that the Richetta model constraint matrix is not totally unimodular as written, but if the redundant constraint set is omitted, the matrix becomes totally unimodular; this can be proven by a method similar to the proof of total unimodularity for the Static Stochastic model constraint matrix above, with the addition of a step in which the columns are reordered.

Chapter 7

Simple Extensions of the Static Stochastic Model

We discuss two simple extensions to the Static Stochastic Model that increase its expressive power. These extensions were made to the Richetta model in [RO93b].

Suppose, due to safety considerations, that we wish to limit the amount of airborne delay at the capacitated destination airport. For instance, imagine we wish to insist that all planes that have arrived by time t must be able to land by time $t + 3$. This is easily modeled, thanks to our FCFS assumption. We simply add constraints of the form:

$$W_{q,t} \leq \sum_{i=t+1}^{t+3} M_{q,i}$$

Now, assume that we wish to limit the amount of ground delay any plane experiences to k time periods. Because of our FCFS assumption, this amounts to insisting that *all* planes originally scheduled to arrive by time t must be allowed to arrive by time $t + k$. This can be achieved by adding constraints of the form:

$$\sum_{i=1}^{t+k} A_i \geq \sum_{i=1}^{t} D_i$$

We observe that adding either or both of these forms of constraints do not affect the total unimodularity of the Static Stochastic model. In the first case, we are adding a row with a single non-zero entry, in the second case, we are adding an "interval" row; a row containing only 0's and 1's, whose 1's all occur consecutively. Therefore, the proof of total unimodularity given in the previous chapter extends directly to handle both these cases. We also observe that we can include arbitrarily strong forms of either type of constraint independently without making the problem infeasible: in the first instance, a schedule that ground holds all arrivals until time $T+1$ will always be feasible, and in the second case, a schedule that allows no ground holds will always be feasible. However, if we include constraints of both forms simultaneously, we may make the problem infeasible.

Chapter 8

The Maximum Air Delay Model

By adding the first types of "extension" constraint given above, and deleting the portion of the objective function corresponding to the air delay costs, we generate a new model with several interesting properties, the Maximum Air Delay model:

$$\min \textstyle\sum_{t=1}^{T} G_t \tag{8.1}$$

$$\textstyle\sum_{t=1}^{j} A_t \leq \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{8.2}$$

$$\textstyle\sum_{t=1}^{T+1} A_t = \sum_{t=1}^{T+1} D_t \tag{8.3}$$

$$W_{q,t} - W_{q,t-1} - A_t \geq -M_{q,t} \qquad t = 1, \ldots, T \quad q = 1, \ldots, Q \quad (W_{q,0} = 0) \tag{8.4}$$

$$G_j + \textstyle\sum_{t=1}^{j} A_t = \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{8.5}$$

$$W_{q,t} \leq L_{q,t} \qquad t = 1, \ldots, T \quad q = 1, \ldots, Q \tag{8.6}$$

$$A_t \in \mathcal{Z}_+, W_{q,t} \in \mathcal{Z}_+, G_t \in \mathcal{Z}_+ \tag{8.7}$$

This model contains all the constraints of the Static Stochastic model, so every feasible solution of the Maximum Air Delay model is also feasible in the Static Stochastic model. The additional constraints, constraint set (6), stipulate that no more than

$L_{q,t}$ planes can be held in the air at period t under scenario q. By setting $L_{q,t}$ to be the $\sum_{t'=t+1}^{t+k} M_{q,t'}$, we guarantee that all planes airheld at time t under scenario q will be able to land by time $t+k$. In other words, we can easily use constraint set (6) to make guarantees on the amount of time any given plane can be airheld.

When compared to the Static Stochastic model, the Maximum Air Delay model has several interesting features. Most importantly, it solves a different, but closely related problem. Whereas the Static Stochastic model finds the schedule with the lowest expected costs, the Maximum Air Delay model essentially seeks to minimize the amount of ground delay subject to safety constraints.

The Maximum Air Delay model is essentially a worst-case model, as opposed to the Static Stochastic model which is more of an expected case model. In fact, given two scenarios, one of which has higher capacity at every time period, and the assumption that we use constraint set (6) in such a way as to enforce an identical maximum air hold length for any given plane, it is easily seen that the scenario with the higher capacity can be deleted from the model.

We next note that the scenario probabilities do not appear in the Maximum Air Delay model. This is a consequence of the model being a worst case rather than an expected case model, and should be considered simultaneously a strength and a weakness of the model. On the positive side, this model is not sensitive to errors in scenario probability measurement, since these probabilities need not be measured. In other words, while the performance of the Static Stochastic model can be degraded substantially by inaccurate probabilistic measurements, the Maximum Air Delay model, by virtue of its simplicity, is immune to this implementation difficulty. On the negative side, perhaps the relative probabilities of the scenarios are of use in determining the correct schedule, and by throwing these probabilities away, we are impeding our ability to determine optimal schedules.

Finally, we see that because we have deleted the second term from our objective function, this model does not require c_g and c_a, the relative costs of ground and airborne delay. In the context of air traffic control, this should be considered a crucial advantage. One of the primary arguments against the Static Stochastic model is that the relative costs of air and ground holding are different for different airlines, and that the FAA has no right to set arrival rates based on a single such value. The Maximum Air Delay model avoids this pitfall. In fact, the Maximum Air Delay model was designed as an attempt to addres the perceived (political) problems with the Static Stochastic model. Under this model, the FAA restricts arrival rates based only on its operational and safety concerns (the $L_{q,t}$), without having to assign relative costs to air and ground delays. This model could easily be used to set *maximum* allowable arrival rates, with the individual airlines deciding the actual arrival rates amongst themselves in a collaborative decision making framework.

We note in passing that by the arguments in the previous chapter, the constraint matrix associated with the Maximum Air Delay model is totally unimodular, and that the linear programming relaxation of this model is therefore guaranteed to yield integer solutions.

Chapter 9

The Static Stochastic Model with Waste Penalties

In this chapter, we develop a more sophisticated extension to the Static Stochastic model. One problem that airlines have had with the current deterministic ground holding policy has been a feeling that the policy is overly conservative: that a large number of delays were being taken on the ground, when planes could have been arriving and successfully landing. We consider a model in which we penalize this waste explicitly.

We introduce new *penalty* variables, $P_{q,t}$, the "waste" penalty at time t under scenario q. We also introduce a new model parameter, c_w, the cost associated with "wasting" one unit of capacity. We wish to penalize only situations in which we are ground holding planes that *could be landing.* If we are ground holding planes, but those planes would not have been able to land, we assess no penalty; similarly, if there is excess capacity, but all planes have arrived, we assess no penalty. For this reason, we agree that $P_{q,t}$ should be the *minimum* of the number of planes being ground held at time t, and the excess, or slack, capacity at time t under scenario q. Introducing slack variables to turn our air holding assignment inequalities into equalities, we derive the following model:

$$\min \sum_{t=1}^{T} c_g G_t + \sum_{q=1}^{Q} \sum_{t=1}^{T} p_q (c_a W_{q,t} + c_w P_{q,t}) \tag{9.1}$$

$$\sum_{t=1}^{j} A_t \le \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{9.2}$$

$$\sum_{t=1}^{T+1} A_t = \sum_{t=1}^{T+1} D_t \tag{9.3}$$

$$W_{q,t} - W_{q,t-1} - A_t - S_{q,t} = -M_{q,t} \qquad t = 1, \ldots, T \quad q = 1, \ldots, Q \quad (W_{q,0} = W_{q,t} = 0) \tag{9.4}$$

$$G_j + \sum_{t=1}^{j} A_t = \sum_{t=1}^{j} D_t \qquad j = 1, \ldots, T \tag{9.5}$$

$$P_{q,t} = \min(G_t, S_{q,t}) \qquad q = 1, \ldots, Q \quad t = 1, \ldots, T \tag{9.6}$$

$$W_{q,t} = 0 \ OR \ S_{q,t} = 0 \qquad q = 1, \ldots, Q \quad t = 1, \ldots, T \tag{9.7}$$

$$A_t \in \mathcal{Z}_+, W_{q,t} \in \mathcal{Z}_+, G_t \in \mathcal{Z}_+, S_{q,t} \in \mathcal{Z}_+ \tag{9.8}$$

We see that we have added the penalty term to our objective function. Constraint sets (2) and (3) remain unchanged, and the only change to constraint set (4) is that we explicitly represent the slack variables. Constraint set (5) remains unchanged. Constraint set (6) is new, and represents the assignment of the penalty values. Note that constraint set (6) is shorthand; strictly speaking, minimums are not part of the language of integer programming. Each statement of the form (5) can be written as four constraints, and requires the introduction of 0-1 variables:

$$P_{q,t} - G_t \le 0$$

$$P_{q,t} - S_{q,t} \le 0$$

$$P_{q,t} - G_t + \omega x_{q,t} \qquad \ge 0$$

$$P_{q,t} - S_{q,t} - \omega x_{q,t} \ge -\omega$$

$$x_{q,t} \in 0, 1$$

where ω is large in terms of the problem data; in our implementation, we set $\omega = \sum_{t=1}^{T} D_t$, which is clearly an upper bound on both $S_{q,t}$ and G_t. Constraint set (7) is also shorthand, and needs to be expanded into two disjunctive constraints, each involving a new 0-1 variable. Constraint set (7) is necessary in order to avoid situations where, under extremely high penalty costs, the system "decides" to force arriving planes to take air holds under certain scenarios, even though capacity exists to allow those planes to land immediately, in order to avoid waste penalties at later times. Without waste penalties, under the assumption that air holds were costlier than groundholds, the system always automatically minimized the $S_{q,t}$, the number of air holds taken. With waste penalties this is not necessarily the case.

Looked at from a different perspective, constraint set (7) gives us further insight into both the original and extended models. We notice that the Static Stochastic (and Richetta) models, as specified, model situations where once the decisions (the A_t or the $X_{i,j}$, respectively) are taken, the scenario is realized, and the future is known at that time. This is unrealistic, but is not a problem in the original Static Stochastic model; we are *already* acting in the manner that is most advantageous once the scenario is realized, because the ability to force planes to wait in the air when there is capacity available to land that plane is of no value. However, this ability does exist, and under the extended model without constraint set (7), it could be of value. Constraint set (7) eliminates this option, forcing us to land as many planes as possible.

The constraint matrix for the extended model is clearly *not* totally unimodular; the expansion of the minimization terms involves large numbers ω. Indeed, the linear programming relaxation of this model does not yield integer solutions on test problems, and for even moderate sized instances, the problem is intractable under the current formulation.

In the next chapter, we provide some examples and begin an analysis of the model with waste penalties.

Chapter 10

Examples and Analysis

In this chapter we present several examples and analyses that will give us further insight into the static stochastic algorithm, both with and without waste penalties. We begin by presenting an elementary result that will greatly simply further analyses.

Definition 5 *Given an instance S of SSGHP, the upper envelope of S is the sequence $U \in \mathcal{Z}_+^T$ satisfying $U_t = \max_{q \in Q} M_{q,t}$, $1 \leq t \leq Q$, and the lower envelope is the sequence $L \in \mathcal{Z}_+^T$ satisfying $L_i = \min_{q \in Q} M_{q,i}$, $1 \leq t \leq T$.*

Lemma 6 *Given an instance of SSGHP, and an optimal schedule A^*, $A_t^* \leq U_t$ for all $1 \leq t \leq T$.*

PROOF:Assume A_t^* is strictly greater than U_t for some t. Defining $d = A_t^* - U_t$, consider the new schedule A' constructed by setting $A_t' = U_t$, $A_{t+1}' = A_{t+1}^* + d$, and $A_{t'}' = A_{t'}^*$ for all other t'. Under all possible scenarios, this schedule will have d additional ground holds and d fewer air holds at period t, and the same number of air and ground holds for all $t' \neq t$. Since $c_a > c_g$, $C(A') = C(A^*) - d * (c_a - c_g) < C(A^*)$, contradicting our assumption that A^* was optimal. □

The intuition behind this proof is straightforward. During any period for which $A_i > U_i$, some number of planes are *guaranteed* to experience air delay. By ground holding these planes for one period, we obtain a new schedule which is guaranteed to be of lower cost.

This lemma is simple and intuitive, but extremely useful. It's value lies in allowing us to easily bound the number of "plausible" schedules we need to consider when looking at a given problem. In particular, we need only consider those schedules for which $A_t \leq U_t$ for $1 \leq t \leq T$.

10.1 Static Stochastic vs. Deterministic

In this chapter, we give some simple examples to illustrate the weaknesses in the deterministic algorithm currently in use, and how the static stochastic algorithm corrects these weaknesses. Recall from Chapter 1 that the deterministic algorithm operates by picking the most probable scenario, and ground holding enough planes such that capacity limits will exactly be met if that scenario occurs. There are two closely intertwined, but conceptually different, errors in this approach. The first is that the distribution of future capacity is ignored. In some sense, the deterministic algorithm focuses on the *mode* of the distribution rather than the distribution itself. The second is that the relative costs of air holding and ground holding are ignored. The static stochastic algorithm fixes both these problems; indeed, it is difficult to see how to address one of these concerns without addressing the other.

This approach is admittedly conceptually ingenuous. It is not the case that the FAA receives a probabilistic capacity forecast, broken up into scenarios, and then chooses to use only the most probable scenario. The FAA receives only a single forecast. However, it does seem at least plausible to assume that the forecast the FAA receives corresponds to the most likely scenario. In practice, there is also some evidence that the FAA is biased towards overly conservative forecasts; we ignore that issue in this analysis.

We begin by considering an example consisting of two scenarios and two time periods. (When we say that an example consists of T time periods, we mean there are T possibly capacitated time periods, and that any planes that are either ground held or air held at time T will land at time $T + 1$.) Under the first scenario, which occurs with probability .6, the capacity is 0 at times 1 and 2. Under the second scenario, which occurs with probability .4, the capacity is 1 at times 1 and 2. The demand is 1 during both time periods. By Lemma 6, $U_t = 1$ for $1 \leq t \leq 2$, so we need only consider the four schedules that have zero or one arrival during times one and two. The following table lists these four schedules, as well as the number of ground holds and the *expected* number of air holds incurred under each schedule:

Schedule Number	Arrivals t=1	Arrivals t=2	Ground Holds	Expected Air Holds
1	0	0	3	0.0
2	0	1	2	0.6
3	1	0	1	1.2
4	1	1	0	1.8

For this problem, the first scenario, that of no capacity at times one and two, has probability .6, so the deterministic algorithm will choose schedule 1. Note that the deterministic algorithm is not dependent on the cost parameters c_g and c_w, since it does not use them in any way. The static stochastic algorithm, which *is* dependent on these cost parameters, will make different decisions depending on the parameter values. If $c_a > 1.6c_g$, then schedule 1 is optimal, and the static stochastic and deterministic algorithms give the same result. If, however, $c_a < 1.6c_g$, then schedule 4 becomes optimal; the static stochastic algorithm finds this optimal schedule, but the deterministic one does not. In particular, if $c_g = 100$ and $c_a = 150$, the deterministic algorithm will suggest schedule 1, with an expected cost of 300, and the static stochastic algorithm will suggest schedule 4, with an expected cost of 270. (If $c_a = 1.6c_g$, then all four schedules are optimal; in this case, the deterministic algorithm *does* produce an optimal schedule.) This example illustrates the basic problems with the deterministic algorithm: ignoring the *distribution* of capacity, concentrating only on the most likely scenario, and ignoring the cost information in the problem.

It is not even necessary to have two time periods. Consider a single time period, three scenario problem. The three possible capacities are 0, 1, and 2, with respective probabilities .4, .3, and .3. The demand is 2. Under the deterministic algorithm, the most likely outcome is a capacity of 0, and both planes are ground held until time 1, incurring two ground holds and no air holds. This schedule will be optimal only if $c_a \geq \frac{5}{2}c_g$. Now consider schedules where we allow one or two planes to land at time 0. If we allow one plane to land, we incur one ground hold, and expect to incur .4 air holds; this schedule is optimal whenever $\frac{10}{7}c_g \leq c_a \leq \frac{5}{2}c_g$. If we allow both planes to land, we incur no ground holds, and expect to incur 1.1 air holds; this schedule is optimal if $c_a \leq \frac{10}{7}c_g$. In all cases, the static stochastic algorithm finds the optimal solution, since it explicitly takes into account the distribution over the scenarios. This example again shows that the deterministic algorithm can easily produce non-optimal results.

In each of the above two examples, the schedule selected by the deterministic algorithm *is* optimal for *some* choice of the cost parameters c_g and c_a. This is not necessarily the case in general; it is easy to construct example problems such that the schedule selected by the deterministic algorithm is *never* optimal. We briefly discuss some properties of optimal schedules that will allow us to demonstrate the above result.

Perhaps the easiest method of showing that a given schedule is *never* optimal (i.e., is optimal for no choice of the cost parameters c_g and c_a) is to enumerate all feasible schedules, and to explicitly determine which schedule is optimal for each value of the ratio $r = \frac{c_a}{c_g}$ (see Chapter 4 for a discussion of why we need consider only this ratio, rather than all possible values of *both* parameters). This is exactly the method we have used for the last two examples. However, for large problems, this is completely impractical: for a problem with thirty-two time periods and up to forty planes landing each time period, we would have to individually consider as many as 40^{32} schedules. Fortunately, this isn't necessary.

We begin by fixing $c_g = 1$, so that by considering c_a, we are considering r. Assume we have a schedule A, a vector representing the number of arrivals at each time period 1 through T. If, for each value of the ratio r, we are able to produce a schedule A^r such that A^r has cost lower than A for cost ratio r, then schedule A can never be optimal. In particular, if any set of schedules satisfies this requirement, the optimal schedules will satisfy it. Put differently, an alternate method for testing whether a schedule can ever be optimal is as follows: solve the SSGHP to optimality for all possible values of the cost parameter r. Because the SSGHP can be solved as an LP, sensitivity analysis allows us to do this reasonably effectively. By doing so, we obtain a function $f(r)$, representing the cost of the optimal solution as a function of the cost ratio r. If $f(r)$ lies below $f_A(r)$, the cost of schedule A as a function of r, for all r, then A can never be optimal.

It is worth taking a moment to investigate the shapes $f(r)$ and $f_A(r)$. $f_A(r)$ is simply a straight line. Its slope is the number of air holds under schedule A, and its x-intercept is the number of ground holds under schedule A. (Since we stipulate that $c_a > c_g$, we only consider cost ratios of 1 or higher; $f_A(r)$'s value at 1 is the number of ground holds plus the number of air holds under schedule A.) $f(r)$, the optimal cost as a function of r is a piecewise linear convex nondecreasing function over r. This can be seen intuitively in several ways, and is not difficult to prove formally. One way of looking at it is that every feasible schedule A' has an associated function $f_{A'}(r)$; for any given r, the optimal schedule is that A' for which $f_{A'}(r)$ is minimal. Each $f_{A'}(r)$ is linear and nondecreasing, and there are a finite number of feasible schedules. The result follows.

We are now ready to construct a problem with the property that the schedule selected by the deterministic algorithm is not optimal for any choice of the cost ratio r. The problem has four scenarios and seven time periods. Demand at each time period is one. The following table lists the four scenarios and their probabilities:

Scenario Number	Scenario Probability	Capacity						
		t=1	t=2	t=3	t=4	t=5	t=6	t=7
1	.4	0	0	0	0	1	1	1
2	.2	1	0	0	0	0	1	1
3	.2	1	1	0	0	0	0	1
4	.2	1	1	1	0	0	0	0

These four scenarios have a regular structure. In each scenario, capacity is zero during a block of four consecutive time periods, and is one during the remaining three periods. This corresponds to an "uncertainty in time" sort of scenario: we might know that capacity is going to drop, for how long, and by how much, but not know when that drop is going to begin. In this particular case, since the first scenario has the highest probability, .4, the deterministic algorithm will select as optimal a schedule that matches it exactly, with no planes landing during the first four time periods, and one plane each landing during the final three periods. However, this schedule is *never* optimal, as we now show. The schedule in question incurs 22 ground holds, and expects to incur 2.8 air holds.

The following table lists the schedules which are optimal over some *range* of values of the ratio r, the ranges over which they are optimal, the number of ground holds and air holds incurred by each, and $f(r)$ over the ranges, in both symbolic and numeric form, assuming that $c_g = 1$:

Optimal Range	Arrivals							Ground Holds	Expected Air Holds	$f(r)$	
	1	2	3	4	5	6	7			Symbolic	Numeric
$1 \leq r \leq \frac{4}{3}$	1	1	1	0	0	0	0	10	7.2	$10 + 7.2r$	$17.2 - 19.6$
$\frac{4}{3} \leq r \leq \frac{5}{2}$	1	1	0	0	0	0	1	14	4.2	$10 + 4.2r$	$19.6 - 24.5$
$\frac{5}{2} \leq r \leq \frac{35}{8}$	1	0	0	0	0	0	1	20	1.8	$20 + 1.8r$	$24.5 - 27.875$
$\frac{35}{8} \leq r \leq 5$	0	0	0	0	0	0	1	27	.2	$27 + .2r$	$27.875 - 28$
$r > 5$	0	0	0	0	0	0	0	28	0	28	28

The study of this example yields many insights into the structure of optimal solutions to SSGHP. First, note that as the cost ratio r increases, air holding becomes more expensive relative to ground holding, and the number of air holds in the optimal solution *decreases.* As a result of this, $f(r)$ is nondecreasing, and its slope is nonincreasing. Eventually, for $r > 5$, air holds become so expensive that it is cheaper simply to ground hold enough planes so that no air holds are incurred. For $r \geq 5$, $f(r)$ has slope 0. These properties are true for *every* instance of SSGHP: $f(r)$ is nondecreasing, its slope is nonincreasing, and there exists an r' such that $f(r) = f(r')$ for $r \geq r'$. In rough terms, raising the cost ratio r by one unit generally makes the problem more expensive, each additional unit by which we raise the cost ratio generally has less effect than previous unit raises, and eventually, additional cost ratio raises have no effect, because we choose not to air hold any planes.

Note that we may not have found *all* optimal solutions to the problem. In particular, at the points where two line segments meet ($r \in \{\frac{4}{3}, \frac{5}{2}, \frac{35}{8}, 5\}$), there may be additional solutions that are optimal only for that particular value of r. However, we have found $f(r)$ for all r, and for each r, we do have at least one schedule A for which $f_A(r) = f(r)$ (two at each of the boundary points), so we may ignore these extra optimal schedules.

By looking at the chart, we can easily see that the schedule selected by the deterministic method, with 22 ground holds and 2.8 expected air holds, is never optimal; it lies above $f(r)$ everywhere. In particular, the schedule with 20 ground holds and 1.8 expected air holds *dominates* the deterministic schedule: because it has fewer air holds *and* fewer ground holds, we always prefer it to the deterministic schedule regardless of r. In fact, in this case, we could have produced that particular schedule as a quick proof that the deterministic schedule was optimal. It *is* possible for a given schedule to be nonoptimal without another schedule that dominates it in the above sense existing. If we *can* produce a dominating schedule, we do not even need to show that the dominating schedule is itself optimal for any value of r, it immediately acts as a proof that the schedule in question is *never* optimal. However, we have not been able to find a systematic way to produce a dominating schedule in general. On the other hand, the method of computing $f(r)$ over the entire range of r is guaranteed to decide whether or not a given schedule can ever be optimal.

10.2 Static Stochastic With and Without Penalties

In this chapter, we begin to explore the properties of the static stochastic algorithm with waste penalties, and compare its behavior to the algorithm without waste penalties. The model with waste penalties is much more complicated, and therefore this is really the beginnings of an analysis; more work remains to be done. Nevertheless, this simple exploration gives us several insights into the workings of the model.

We note in passing that Lemma 6 applies to the model with waste penalties: if $A_t > U_t$ for some t, then, by delaying a plane from time t to $t+1$, we trade one ground hold for one air hold, and we do not affect the number of waste penalties. We still have at least U_t planes arriving at time t, so no waste penalties can occur at that time period, and, since the excess plane was guaranteed to be air held until time $t+1$, we still have just as many planes trying to land at time $t+1$.

We begin with an example consisting of two time periods and two scenarios of equal probability. The demand is 1 at each time period. Under the first scenario, there is capacity 1 at each time period, and under the second scenario, there is capacity 0 at each time period. For reasons discussed above, there are only four plausible schedules for this problem; for each schedule, we list the expected number of ground holds, air holds, and waste penalties incurred by each schedule, and the expected cost of each schedule if $c_g = 1, c_a = 2.5$ and $c_w = .75$. In all cases, any excess planes arrive and land at time $t = 3$:

Schedule Number	Arrivals t=1	Arrivals t=2	Ground Holds	Expected Air Holds	Expected Waste Penalties	Cost
1	0	0	3	0.0	1.0	3.750
2	0	1	2	0.5	0.5	3.625
3	1	0	1	1.0	0.5	3.875
4	1	1	0	1.5	0	3.750

If we are solving the problem using the original model, there is no cost associated with the waste penalties; i.e., $w = 0$. In this case, Schedule 4 is optimal if $c_a < 2c_g$, and Schedule 1 is optimal if $c_a > 2c_g$. If $c_a = 2c_g$, all four schedules are optimal. If, instead, we solve this problem under the model with waste penalty extensions with the cost parameters given above, then schedule 2 is the unique optimum; indeed, if $c_g = 1$, and $c_a = 2.5$, then schedule 2 is the unique optimum whenever $.5 < c_w < 1$. In this simple example, we see that it is possible for a schedule which is only optimal for a single cost value under the original model, and even then is only one of several optima, to become the unique optimum over a range of values under the model with waste penalties.

A natural question to ask about the waste penalty extension of the Static Stochastic model is whether it ever produces schedules that are qualitatively different than those produced by the original model. In particular, does there exist an instance of SSGHP, and an assignment of the cost parameters, c_a, c_g, and c_w, such that the extended model produces a schedule A_t with the property that there is *no* assignment of the cost parameters c_a and c_g that will cause the original model to produce this same schedule? We answer this question in the affirmative, with an example.

We consider an example consisting of four time periods and two scenarios of unequal probability. The demand is 1 at each time period. Under the first scenario, capacity is 0 during the first three time periods and 1 during the fourth period (as a vector, the scenario is $\{0, 1, 1, 1\}$); this scenario occurs with probability .75. Under the second scenario, which occurs with probability .25, capacity is 1 during the first period and 0 during the other three periods $\{1, 0, 0, 0\}$.

We need only consider the 16 schedules that have 0 or 1 arrival each during times one through four, with all remaining arrivals at time five; no other schedule can possibly be optimal. In the following table, we list the 16 schedules, the expected number of ground holds, air holds, and waste penalties incurred by each schedule, and the expected cost of each schedule if $c_g = 1, c_a = 18.9$ and $c_w = 10$. In all cases, any excess planes arrive and land at time $t = 5$:

Schedule Number	Arrivals t=1	t=2	t=3	t=4	Ground Holds	Expected Air Holds	Expected Waste Penalties	Cost
1	0	0	0	0	10	0.00	2.50	35.000
2	0	0	0	1	9	0.25	1.75	31.225
3	0	0	1	0	8	0.50	1.75	34.950
4	0	0	1	1	7	0.75	1.00	31.175
5	0	1	0	0	7	0.75	1.75	38.675
6	0	1	0	1	6	1.00	1.00	34.900
7	0	1	1	0	5	1.25	1.00	38.625
8	0	1	1	1	4	1.50	0.25	34.850
9	1	0	0	0	6	0.75	1.50	35.175
10	1	0	0	1	5	1.00	0.75	31.400
11	1	0	1	0	4	1.25	0.75	35.125
12	1	0	1	1	3	1.50	0.00	31.350
13	1	1	0	0	3	2.25	0.75	53.025
14	1	1	0	1	2	2.50	0.00	49.250
15	1	1	1	0	1	3.50	0.00	67.150
16	1	1	1	1	0	4.50	0.00	85.050

Using the cost parameters specified above, the optimal schedule is schedule 4, with arrival vector $0, 0, 1, 1$ and an optimal cost of 31.175. However, this schedule can *never* be optimal for this instance of SSGHP if waste penalties are not used. If we are not using waste penalties, then the only costs considered are ground holding and air holding costs. Comparing schedule 4 with schedule 9 ($\{1, 0, 0, 0\}$), we see that these two schedules have the same number of expected air holds, but that schedule 9 has fewer ground holds; schedule 9 dominates schedule 4. Therefore, if we consider an identical instance of SSGHP without waste penalties, schedule 9 will *always* have lower expected costs than schedule 4.

The above example indicates that it is theoretically *possible* for the model with waste penalties to yield optimal schedules that are never yielded by the original model. In other words, the model with waste penalties *is* a stricly more expressive model, since this model with $c_w = 0$ yields the original model, but there exist examples, such as the one above, such that no setting of the parameters c_a and c_g will cause the original model to yield the same solution as the model with waste penalties. Whether this difference is of practical importance is, however, an open question. The above example was difficult to find, and should be considered unrealistic because of the example was difficult to find, and should be considered unrealistic because of the large value of $\frac{c_a}{c_g}$, 18.9. The value of $\frac{c_a}{c_g}$ is unknown in practice, and likely varies with the particular example being considered, but is almost certainly no higher than 5. Additionally, the cost difference involved was very small.

We have looked at several more complicated examples that were derived from real world data. In all of these examples, the optimal schedules generated by the model with waste penalties were always generated by the model without waste penalties for at least a single value of the cost parameters. These "real world" examples therefore correspond much more closely to the first example above rather than the second.

It is an interesting thought exercise to consider what happens when we increase either c_a or c_w, while holding the other two parameters constant. As we increase c_a, air holding becomes more expensive, and we begin to favor schedules with more ground holds and fewer air holds. If we increase c_a far enough, air holds will become so expensive that we will choose a schedule with no air holds whatsoever. If we increase c_w, then waste penalties become more expensive, and we attempt to avoid them. Waste penalties are caused by too few planes arriving, so we will attempt to avoid them by ground holding fewer planes, increasing the number of air holds. Specifically, the expected number of air holds is an increasing function of the number of planes arriving at each time period, and the number of ground holds and the expected number of waste penalties is a decreasing function of the number of planes arriving at each time period. However, although the number of ground holds and the number of waste penalties incurred by a schedule are highly correlated, the problem has a complicated enough structure that it is in some cases possible to take a schedule and add arrivals to some time periods and remove arrivals from others such that the total number of ground holds is increased but the expected number of waste penalties is decreased. It is this property that allows examples like the one above to exist.

We have demonstrated that the model with waste penalties can generate optimal solutions that are never generated by the model without waste penalties. In order to get it to generate truly different solutions, it appears that specially constructed examples are needed. On the other hand, regardless of whether the solutions generated are truly different, the model may be of value if it paints a more realistic and accurate picture of the costs involved. Currently, the model without waste penalties seems more useful. It is easier to understand, and is computationally tractable. The model with waste penalties is more powerful, but it is unclear that this extra power is useful in practice, and the model is intractable for even moderate problem sizes.

10.3 The Maximum Air Delay Model

The Maximum Air Delay model, as described in Chapter 8, takes a very different approach to the ground holding problem. Under this model, we attempt to minimize the number of ground holds subject to constraints on how long any given plane may be delayed in the air. In this chapter, we analyze this model under the reasonable assumption that these maximum delay constraints will be identical for all planes under all scenarios. More specifically, we consider the model as attempting to minimize the total number of plane-periods of ground delay, subject to the constraint that no plane will be held in the air for more than L time periods under any scenario. We accomplish this by setting $L_{q,t} = \sum_{t'=t+1}^{t+L} M_{q,t'}$ for all q and t (letting $M_{q,t}$ be arbitrarily large for $t > T$).

We begin by examining the behavior of the Maximum Air Delay model under "extreme" settings of the parameter L. We first consider $L = 0$. This amounts to a strict requirement that *no* plane experience air delay under any scenario. Therefore, the "optimal" schedule will correspond to an optimal "most conservative" one, in which just enough planes are ground held to guarantee no air delays. At the other end of the spectrum, as L gets very large, the system will be able to tolerate larger and larger air delays for any given plane. In particular, we may consider a schedule in which no plane is ground delayed. Define L^* to be the maximum number of periods of air delay any plane experiences under this schedule. By setting $L_{q,t} \geq \sum_{t'=t+1}^{t+L^*} M_{q,t'}$ for all q and t, we guarantee that this schedule will be feasible, and, since its cost in the Maximum Air Delay model is zero, it is clearly optimal.

Although the Maximum Air Delay model may be viewed as having q times t

control parameters (the $L_{q,t}$), it is more intuitive, and more relevant to its possible use, to view it as having a *single* parameter L, and agreeing that the $L_{q,t}$ will be used to implement this parameter, as described above. Under this discipline, we note that the Maximum Air Delay model experiences the same sorts of extreme effects as the Static Stochastic model. A Maximum Air Delay of 0 ($L = 0$) corresponds to infinitely high air delay costs under the Static Stochastic model, and in both cases, the schedule with no air holds and the fewest possible ground holds given no air holds is produced. What is the structure of this schedule A^*? Necessarily, $A_t^* \leq M_{q,t}$ for all t and all q; otherwise, there exists a scenario under which air delays would occur, and the schedule would be infeasible. But clearly, this condition is sufficient as well; the condition ensures that all planes will be able to land during the period they arrive, under any scenario. We conclude that the optimal schedule for the Maximum Air Delay model with $L = 0$, or the Static Stochastic model with arbitrarily large costs, is the schedule where $A_t = \min(\min_{q \in Q} M_{q,t}, D_t + G_{t-1})$ for all t; at each time period, the number of planes arriving is the minimum of the capacity at that time period under any scenario and the "demand" at that time period, where demand is adjusted upward by the number of planes ground held from the previous period.

Similarly, an infinite maximum air delay corresponds to a cost ratio of 1 or smaller under the Static Stochastic model. In both cases, a schedule with no ground delays is optimal. We see that the "extreme" behavior of the two models is identical. However, the behavior of the two models in intermediate cases is quite different, as we show below.

We note in passing that Lemma 6 does *not* apply to the Maximum Air Delay model. The proof of this lemma rested on the idea that air delays are always more expensive than ground delays, and that planes that were guaranteed to be delayed in the air were better off being delayed on the ground. The Maximum Air Delay model penalizes only ground delays, not air delays. Indeed, it is easy to make up examples in which the optimal schedule under the Maximum Air Delay model in which $A_t^* > U_t$ for some t; a trivial such example is a one scenario, one period example in which capacity is zero, demand is one, and $L_{1,1} = 1$. The optimal schedule will be $A_1^* = 1$, with an associated cost of zero. As a result of the inapplicability of this lemma, the approach pursued above, where we enumerate all "plausible" scenarios in order to exhaustively show the optimality of a given schedule under given cost parameters, is inappropriate. Instead, we take a higher-level approach, where we specify an example problem, *solve* the Maximum Air Delay model, and observe the results.

Unfortunately, there is some flexibility in the specification of the Maximum Air Delay model. This flexibility relates to the need to "project" capacity for $t > T$, in order to determine whether ground holds are necessary at some time $t' > T - L$, where L is the maximum allowable air delay. The Static Stochastic model assumes that the capacity at time $T + 1$ is large enough to allow all planes to land. This is the approach we take with the Maximum Delay Model as well, for the purposes of this analysis. In practice, it may be more reasonable to project, for each scenario, the capacity at time $t - 1$ into the future; this may in turn lead to some planes not arriving at all. This issue is not dealt with in this analysis, but is an important avenue of further investigation.

We now revisit the first example we studied for the Static Stochastic model. This example consists of two time periods and two scenarios. The first scenario occurs with probability .6, and has capacity zero at both time periods. The second scenario has probability .4, with capacity one at both time periods. Demand is one during each time period. Of course, under the Maximum Air Delay model, the probabilities themselves are irrelevant; only the scenarios matter. The following table lists the optimal schedule as a function of L, the maximum air delay:

Maximum Air Delay	Arrivals		Ground Holds	Expected Air Holds
	t=1	t=2		
0	0	0	3	0
1	0	2	1	1.2
2	1	1	0	1.8

We immediately note that even for such a simple example, if the maximum air delay, L, is one, we obtain an optimal schedule that is never optimal under the Static

Stochastic model. This is an effect of our decision to allow all planes to land at time $T+1$. Under such a model, all planes will arrive by time $T-L$, as they cannot possibly be delayed for more than L periods. It is for this reason that we may, in practice, choose to formulate the model in an alternative way (see discussion above).

We at this point note another important feature of the Maximum Air Delay model. Suppose there exist two different scenarios, q_1 and q_2, such that $M_{q_1,t} < M_{q_2,t}$ for all t. Then we remove scenario q_2 from the model. Clearly, if no planes are air held for more than L periods under scenario q_1, no planes can be air held under scenario q_2, where capacity is larger *for every time period.* Therefore, we may remove all such "dominated" schedules from our model. In the example above, the schedule with capacity one in both time periods is such a schedule. A different interpretation of this phenomenon is that the Maximum Air Delay model is essentially a "worst case" model (see Chapter 8).

We turn now to the second example from Chapter 10.1, where there are four scenarios and seven time periods, a block of four consecutive periods of capacity zero, and capacity one during the remaining periods. We note in passing that no scenario dominates any other here, so no scenario may be removed from the model. The following table lists the optimal schedule as a function of L, the maximum air delay:

Maximum	Arrivals							Ground
Air Delay	t=1	t=2	t=3	t=4	t=5	t=6	t=7	Holds
0	0	0	0	0	0	0	0	28
1	0	0	0	0	0	0	7	21
2	0	0	0	0	0	6	1	15
3	0	0	0	0	5	1	1	10
4	1	1	1	1	1	1	1	0

The "staircase" phenomenon at the right-hand side of the table is again an artifact of the fact that if all planes are allowed to land at time $T+1$, no planes will be ground held after time $T-L$. Ignoring that effect, we see that increasing L from zero to three does not allow planes to arrive during the early time periods, but suddenly, when we increase L to four, all planes may land as scheduled. Intuitively, this is because there are only four periods of zero arrivals; if L is four, any given plane may, no matter when in this zero capacity block it happens to arrive, "wait out" the zero capacity periods without violating the maximum air delay constraints.

This analysis of the Maximum Air Delay model is highly preliminary; additional examples should yield further insights into the workings of the model.

Chapter 11

Conclusions and Future Work

In this thesis, we have defined a model which represents a substantial simplification of Richetta's model. We have proved that the matrix induced by this model is totally unimodular, guaranteeing that the linear programming relaxation of this model will yield integer solutions. We have also defined an extension to this model that explicitly penalizes wasted capacity, and argued that this extension does add power to the model.

On the complexity theoretic front, the total unimodularity of the constraint matrix indicates that the static stochastic ground holding problem is solvable in polynomial time. It may be worthwhile to search for an explicit polynomial time algorithm, rather than relying on polynomial time LP solvers; this may yield faster algorithms, and, more importantly, deeper insights into the problem structure. Additionally, it is unclear how difficult the problem becomes when we allow waste penalties. Possibly, the problem is still polynomial, and a better model would reveal this; possibly the problem becomes NP-complete. It would be worthwhile, both from a theoretical and a practical standpoint, to explore this question.

More importantly, the work to date all assumes that a completely accurate distribution of probability scenarios is available. This assumption is very optimistic. Currently, no probabilistic forecasts whatsoever are available; even if some probabilistic information were given, it would be unrealistic to assume that it was entirely accurate. Ideally, the models should be extended to perform as well as possible using only partially specified probabilistic information — obviously, the model's performance would depend on the accuracy of the information available. At the very least, it should be possible to derive bounds about how well the current models perform if the forecasts they are given are only partially accurate.

Additionally, the models presented in this thesis require an unrealistically high level of aggregation. If the models could be extended to include different cost structures at different times and for different planes without too much increase in solution times, this would represent a substantial improvement; this is a promising direction for future investigations.

Finally, a deeper investigation is required into the practical, operational concerns of both the FAA and the airlines, in an attempt to find a model that will improve capacity while being politically satisfactory to all parties. The Static Stochastic model has the advantage that it takes advantage of the information provided by relative scenario probabilities; assuming it is acceptable to specify c_g and c_a, the model finds optimal solutions. On the other hand, this dependency on relative costs may be a politically fatal flaw, necessitating the use of a model that avoids these constraints such as the Maximum Air Delay model. It may be possible to develop compromise models — a simple example would be a variant of the Maximum Air Delay model that threw away all scenarios with probabilities below a certain threshold, thereby stopping the performance of the Maximum Air Delay model from being dominated by extremely low probability events. More generally, looking for ways to combine improving capacity through stochastic optimization techniques with modern collaborative decision making support tools will be an important area of future research.

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