Why Knot?

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Submitted to the Media Arts and Sciences Section, School of Architecture and Planning, in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the Massachusetts Institute of Technology

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Abstract

This dissertation examines *ways of knowing* through a study of the extent, range, and diversity of thinking about knots. The thesis focuses on epistemological and psychological aspects of this thinking: it describes in detail ways of representing intuitive knowledge about knots, strategies for thinking and communicating about knots, and understandings of the topological relationships inherent in individual knots and within groups of knots; as well as case studies that analyze in-depth and demonstrate the coherence in the thinking of particular individuals.

Piaget and Inhelder ([1948, 1956] 1967), Beth and Piaget (1966), and Papert (1980) describe the epistemological structures of topology (such as *proximity, continuity, connection,* and *separation),* order (such as *seriation),* and classification, which in combination contribute to the emergence of mathematical thinking. The thesis shows how these deep structures enter into thinking about knots and formulates a way of characterizing differences in terms of an implicit preference for one or another of the structures. This approach to psychological *style* draws also from the work of Winnicott (1971) and the *object-relations* school of psychoanalysis; Fairbairn's (1963) discussion of the formation of *internal objects* is compared to Papert's (1980) discussion of the formation of *intellectual structures*. The thesis considers the importance to each process of the role of *transitional objects*, which pertains to Papert's notion of *objects to think with,* and to working with string and thinking about knots. Thus both *cognition* and *affect* are considered; current discussions in cognitive science and cognitive psychology of *concrete* and *situated* thinking are also relevant.

Twenty-two children, aged ten to fourteen, participated in the study by learning to tie knots in a multicultural setting that grew in response to the children's interests and creations. The project began with the participants' exchanges with a video "pen pal," an older child who introduced knots and answered questions through videotaped "correspondence." The *thinking environment* that developed was explicitly concerned with both learning and research. Important themes included the learners' *construction* of knots and understandings about them; the learners' self-reflection on these constructions and their outcomes and implications; and the role of the social construction of knowledge. The richness of the data was supported by a qualitative approach involving the study's relatively long duration, the relatively high frequency of working sessions, and the researcher's roles as facilitator and participant observer.

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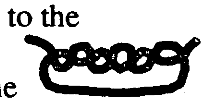
The ideas expressed here do not necessarily reflect the positions of the supporting agencies.

The child in me thanks the adult for attempting to recapture some of what wasn't quite lost.

This thesis is dedicated to the memories and actualities
of my brothers and sisters, related and "adopted,"
agencies together in a changing lattice.

Advice to the Reader

This thesis makes use of a manner of presentation that stems from a methodology combining techniques of learning environment design, participant observation, and clinical interview. Within an overall framework that arranges concerns of method, case studies, conclusions, etc., according to sections, observations and interpretations occur together throughout. The hurried reader may therefore find it difficult to navigate a path through the text that captures the knot-tyers' moments of discovery or breakthrough that led me to a given conclusion; any cursory reading, however, is likely to contain both salient points and the overall flavor of the work. With this caveat in mind, I can point out that an abbreviated reading including Sections I and II, the brief introductions preceding each of the other sections and sub-sections, and Section V, should convey a sense of the nature and contributions of the study.

The children's work is rich: a single example often lends itself to several explanations, emphasizing different characteristics of the thinking that produced the work. A "hypertext" medium would be best suited to the presentation, enabling the multiple links that a given passage might require. But in this book-like format, I must rely on other means of indicating connections between ideas and examples. I have settled on repeating certain passages liberally, with varying descriptions, in sections appropriate to the characteristic that is being emphasized. These repetitions are signalled by the  symbol, an illustration of the repetitive process of tying the so-called Stopper knot (see Sections 5.1 and 5.1.4). This symbol is accompanied by the section numbers of other places in the thesis where the passage occurs. One of these numbers appears in darker type; it is the number of the section that includes the description in its most complete form, or where the description is somehow most "at home."

Taking a cue from the Bourbaki mathematicians, I also make use of another symbol to "forewarn the reader against serious errors." (Bourbaki 1966, vi) Passages that become tricky or complicated and may require the reader's increased concentration are marked by a symbol resembling the Bourbaki's Z-shaped "dangerous bend" sign: . This shape represents a way of beginning the problematic cousin of the Square knot, the Thief knot (see Sections 5.1 and 7.1.1).

The reader is invited, even advised, to keep a piece of string handy and to tie the knots while reading about them. Words and pictures go a long way in describing the configurations, but knots are physical as well as conceptual objects, to be appreciated by fingers as well as eyes and minds. I include here some quotes from preliminary readers who have emphasized the importance of tying the knots as they read along:

> It certainly helped me – not only to tie the knots for myself, but to see how the kids were doing it.
>
> I take it for granted that a serious reader will want to do that.
>
> It's important if you really want to understand what these kids are thinking – how they're thinking about the knot, and how they view it. You just can't view it in the same way if you're looking at a picture of the knot on a page.
>
> Tying the knots myself made it much easier to follow the young people's trains of thought.

Throughout this thesis, pseudonyms have been substituted for the actual names of participants in the research. Pieces of the children's writing are included here with the original spelling and grammar; I have made editorial changes only in the few cases where the reader would likely have been confused. I have also made my best attempt to represent hand-drawn pictures so that they remain true to the children's expressions but are manageable within the form of the thesis.

Dialogues among the children, often including my own voice, are presented with illustrations and explanation. For the sake of accuracy, and in order to suggest vacillations and multiple influences in the thinking that occurs in a given episode, I have retained stuttering, partial utterances, and mumbling, where discernable. The resulting protocols form an important source information about the children's thinking. They also give a flavor of the atmosphere and culture of the research setting in a way that my words alone could never do.

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I Fair Leads

"Fair leads" is an expression that sailors and other knot-tyers use as a form of greeting or regards. A reference to the "leading end" of a piece of string as it moves into the form of a knot, the expression conveys best wishes for successful knotting or good sailing, metaphorical and otherwise.

1 Conceptual *Situs*[1]

This thesis describes an environment for studying children's learning and thinking about topological relationships. That environment, called the "Knot Lab," evolved over a period of five months and included a room, a group of people in communication with each other through various media, and a certain set of ideas. The thesis also offers interpretations of the natures of the participants' thinking – through annotated protocols, case studies, and other descriptions. These interpretations focus on the childrens' conceptions of fundamental aspects of certain knots. Such conceptions were expressed most distinctly through the detection of similarities and differences among the configurations, and through projects involving knots that the children constructed.

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[1] Perhaps I should explain this term. *Situs* is Latin for "situation." The original name of the mathematical discipline of topology was *analysis situs*, or the "study of place." Knot theory is a branch of topology.

The research is of an emerging tradition that enjoys some epistemological advantages: I assume that the process of *learning* occurs similarly in adults and children, but that younger peoples' thinking tends to be less affected by extenuating factors like acculturation and cumulative personal experience. I also assume that it is natural and important for a study to include both girls and boys, and children of different cultural and racial backgrounds. Finally, I assume that the children's process of constructing a meaningful context for their work, and my facilitation of this construction, combined with participant observation, constitute fruitful approaches to psychological and epistemological research. In keeping with the conventions of this methodology, I strive to develop a deep and comprehensive description of the many sorts of thinking that I encountered. Inevitably, I figure as a character in these descriptions. Much of what I learned about the children's thinking was enabled through the relationships that we developed.

Like the children in this project, topologists often find themselves concerned with reducing descriptions of a set of objects to a few fundamental objects or concepts. The mathematicians normally discover such translations through proofs that make use of axioms and specialized notations. The children used other, less formal ways of arriving at conclusions and representations. Their approaches are described here in detail. They are important in and of themselves, not only because they may contribute to our understanding of children's thinking, but because they are varied, and we dwell in an era of growing consciousness – even conviction – among many researchers that there can be no *one* way of knowing.

That awareness has to do with better understandings of ways in which different people think (children and adults, men and women, "neats" and "scruffies" (Minsky 1990), etc.), but also of different processes that can occur simultaneously, or evolve over time, within the mind of an individual. In children such processes tend to demonstrate their vibrancy to a greater extent than in adults. Those who have lived with a two-year-old, for example, may find it easy to imagine the distinct but "parallel" processes that become manifest in the child's daily life. The child is concurrently learning to run, experimenting with new words, asserting a sense of control by resisting (learning to say "no!"), refining various kinds of coordination, and so on. Marvin Minsky describes such processes in terms of societal "agencies" that are nearly separate in infants but gradually become simultaneous and interdependent as the person grows:

> Some readers may be horrified at picturing a baby's mind as made up of nearly separate agencies. But we'll never understand how human natures grow without some theories for how they start. One evidence for separateness is how suddenly infants switch from smiles of contentment to shrieks of hunger-rage. In contrast to the complex mixtures of expressions that adults show, young children seem usually to be in one or another well-defined state of activity – contentment, hunger, sleepiness, play, affection, or whatever. Older children show less sudden mood changes, and their expressions suggest that more different things are happening at once. Our minds may thus originate as sets of relatively simple, separate need machines. But soon enough each becomes enmeshed in all the rest of our growing machinery. (Minsky [1985]1986, 171)

In young children, such processes would be more easily distinguishable than in older children and adults. To a large extent, growth is a process of increasing the interdependence of such processes. Understanding how separate processes contribute to our thinking – how they come to be woven together, their interactions subtly orchestrated – constitutes an important epistemological and psychological enterprise.

In this thesis, I attempt to sort out some components of the thinking that happens in learning about knots. I premise my descriptions on the idea that "thinking about knots" is not just thinking about knots. It involves thinking about many things at the same time –

some of them involving the topology of knots, some relating to the string or material used to tie them, and some that are evoked in one way or another by working with string and knots. Many of these associations can be generalized and characterized, perhaps, as "topological thinking." But many of the associations are idiosyncratic and highly personal, relating to an individual's memories, goals, and so on.

Thus to focus only on knots would be to miss important parts of the picture. Yet to bring in too much about other aspects of the thinking that happens would risk diluting the picture of what "topological thinking" *is* involved. Here I attempt a balance: I bring into the description some parts of the thinking about knots, and some parts of the other "stuff" – stuff that the children themselves brought in. They volunteered information about personal kinds of involvement with the knots through direct reference in conversations with me or in conversations with each other that they knew I could hear, and through the ways in which they worked with the knots.

The incorporation of such personal information in any given experience of the Knot Lab would be reason enough to include the information in my description of the project. But the fact of its emergence raises questions about whether knots might somehow, by their very nature, tend to elicit certain kinds of thoughts in concert with the "topological" ones. This interplay of epistemological and psychological concerns is at the center of my work. It gives rise to a way of characterizing styles of thinking that can encompass both intellectual (or "cognitive") and emotional (or "affective") orientations, and both "objective" and "subjective" dimensions.

This approach is timely: we are nearing the end of a century in which researchers have made notable advances in studies of the brain, the mind, and the nature of human thought. Disciplines that have spawned or come of age in recent decades include psychoanalysis, ethology, neurology, cognitive science, artificial intelligence, developmental psychology, and linguistics. Each has developed its own methodologies and discourse for formulating questions and explaining findings about processes of thinking. Although such different disciplines have their particular interests, they often seek to answer similar questions – yet unfortunately, the assumptions and jargons of each can be so directed and refined as to defy appreciation of overlaps.

This thesis finds itself at a point when acknowledgment of some of the commonalities seems natural – indeed, inevitable – as we arrive at deeper understandings of the interplay of processes in what we call *thought.* Minsky ([1985] 1986), for example, has combined the perspectives of Freud, Piaget, Tinbergen, and Bowlby with his own theories; and Turkle (1988) has described common ground between artificial intelligence and psychoanalysis. The data from the study described here suggest certain means of analysis and interpretaton; my report, therefore, makes use of a convergence of specific views from Papert's theory of "constructionism" (Papert 1986; 1990 "Introduction," "Unified"; Harel and Papert (in press)), aspects of Piagetian theory (e.g., Gruber and Vonèche 1977; Piaget [1929] 1951, [1941, 1952] 1965; Piaget and Inhelder [1948, 1956] 1967), the "object-relations" school of psychoanalysis (e.g., Fairbairn [1952] 1963; Goethals 1973; Winnicott 1965, 1971, *Playing, Therapeutic*), and the "society of mind" theory of artificial intelligence researchers Minsky and Papert (Minsky [1985] 1986).

I have mentioned the emerging research tradition to which this thesis belongs. Though that tradition pre-dates Seymour Papert's (1980) publication of *Mindstorms,* much of it has been further developed since then. This tradition has its roots in Piagetian theory, the history and philosophy of science, artificial intelligence research, psychoanalytic theory, anthropology, and sociology. I explain relevant aspects of these precedents in appropriate parts of the thesis. In this introductory section, I describe some of the concepts that help to situate my work with knots.

1.1 Epistemological Pluralism

One of the central themes of this thesis concerns the diversity that emerges as people think about knots. They "see" them in different ways, and consider different things about them to be important enough to describe and to remember. Rather than attempting to de-emphasize this diversity by seeking general categories through which to describe the thinking, I have chosen to emphasize the differences through the way in which I have organized the report and through the tenor of my descriptions. This approach is consistent with several current veins of epistemological and psychological research. In my view, the emphasis is crucial in attempting to develop thorough understandings of how people think and, ultimately, of how they come to find their senses of themselves and their roles in society.

Although there are signs of a cultural shift toward acknowledging that people "think in different ways," there is a lag in the accompanying shift of values needed to mark change. The general and the "abstract" still tend to wield greater credibility and power than the specific and the "concrete" – and often, individuals' senses of self-worth and potentials to contribute hang in the balance. The traditional views of intellectual maturity as resulting from a progression from "concrete" to "abstract" stages, as well as of logical, scientific thinking as proceeding through a series of propositions, one of which implies the next, tend to exclude the thinking of individuals who ultimately comprise significant populations, often characterized in terms of gender or culture. By leaving out important information, we have formed problematically inaccurate views and expectations that find their niches in educational theory and practice, the workplace, research funding agencies, and other centers of influence.

Sherry Turkle and Seymour Papert (1990) point out that there is currently a shift in several intellectual domains toward questioning this value structure and attempting to better understand and appreciate orientations toward the particular and the "concrete." For these authors, "concrete thinking" has to do with a certain relationship to knowledge and ideas: it describes a way of thinking that stays close to its object. Particularity has to do with acknowledging multiple ways of knowing. In their explanation of "epistemological pluralism," Turkle and Papert describe "the hegemony of the abstract, formal, and logical as the privileged canon in scientific thought." (Ibid., 345) They enumerate three challenges to this hegemony – from feminist scholars, from social scientists, and from computer scientists.

In feminist scholarship, authors such as Carol Gilligan (1982, 1987; Brown 1987) and Evelyn Fox Keller (1983, 1985) describe alternative, feminine voices that differ from the canonical views of moral development as reaching a pinnacle in law-based reasoning, and of proper scientific thought as shaped by logic and objectivity. For these writers,

> ... the canonical style, abstract and rule-driven, is associated with power and elitism, and with the social construction of science and objectivity as male. (Turkle and Papert, 345)

Characterizing Gilligan's "different voice" and Nobel-prizewinner Barbara McClintock's way of doing science is a style of relating to objects through proximity rather than distance.

Turkle and Papert point out that social scientists such as Bruno Latour ([1976] 1986, 1987), Lucy Suchman (1987), Jean Lave (1988), and Sharon Traweek (1989)

> ... show us how within laboratories there is a great deal of thinking that does not respect the canon and how "ordinary" people in their kitchens and workplaces make very effective use of a down-to-earth mathematical thinking very different from the abstract and formal math they were taught at school. (Turkle and Papert, 345)

Again, the emphasis of this "down-to-earth" approach is on a proximal way of relating to things:

> ... closeness to objects tends to support a concrete style of reasoning, a preference for using objects to think with, and a bias against the abstract formulae that maintain reason at a distance from its objects. Conversely, a distanced relationship with objects supports an analytic, rule and plan oriented style. (Ibid., 350)

In the realm of computation, prevalent views of and approaches to programming mask the variable modes of thinking that are possible within its milieu:

> That the computer should be an ally in the revaluation of the concrete has a certain irony; in both the popular and technical cultures there has been a systematic construction of the computer as the ultimate embodiment of the abstract and formal. But the computer's intellectual personality has another side: computers provide a context for the development of concrete thinking. ... The *practice* of computing provides support for a pluralism that is denied by its social construction. (Ibid., 345-46)

Turkle and Papert describe styles of programming in terms of mastery. "Soft mastery" and "concrete thinking" are consistent with an approach that favors proximity and relatedness;

> Hard mastery is resonant with the logical and hierarchical elements of the traditional construction of "scientific method." Soft mastery has always had its place in the discourse of the arts and has always been glimpsed in the autobiographical writings of scientists. Only recently has it gained recognition as an integral element of scientific practice. ... Observation of the soft approach to programming calls into question deeply entrenched assumptions about the classification and value of different ways of knowing. It provides examples of the validity and power of concrete thinking in situations that are traditionally assumed to demand the abstract. It supports a perspective which encourages looking for psychological and intellectual development within rather than beyond the concrete and suggests the need for closer investigation

> of the diversity of ways in which the mind can use objects rather than the rules of logic to think with. (Ibid., 350)

Positioning themselves at the convergence of these approaches in feminism, social science, and computation, Turkle and Papert dub the trend a "revaluation of the concrete" and call for further research into ways of thinking that rely on experience and proximity to objects. My research is situated as one response to this call. The findings not only support the notion of pluralism, but provide a context for discussion of the role of "affect" in so-called "cognitive" thinking. Both of these discussions contrast a paradigm that has become dominant in cognitive science.

1.2 The Realm of Computation

> Gauss is said to have complained: "I have had my results for a long time; but I do not yet know how I am to arrive at them."
> – (Lakatos 1976, 9, n. 1)

In certain areas of artificial intelligence research, the propositional approach inherent in "plan-oriented" or "top-down" programming strategies can be seen as a manifestation of "the hegemony of the abstract, formal, and logical." (Turkle and Papert 1990, 345) This paradigm, the quintessential example of which is Newell and Simon's (1972) "General Problem Solver" (GPS), was itself part of a multi-disciplinary response to the behaviorist tradition that dominated psychological research until mid-century. As such, the effort involved an attempt to understand more deeply the nature of human thought, rather than limiting descriptions of human behavior to observable actions.

This new paradigm cast human thinking as problem-solving. The GPS computer was programmed so that by moving through various changes of state, the system progressed toward achieving a goal. Subproblems decreased the distance between the point of movement and the goal; specialized operators helped to achieve subgoals along the way. The pattern was one of determining "where you are," "where you want to be," getting the difference, and moving accordingly. The method assumed, of course, that you know "where you want to be" – that an end-state, or goal, is something that can be known and planned for. Although the researchers who created GPS were concerned at early stages of their work with "trying to understand how humans solve problems,"

> The present research – that is, the present version of GPS – is governed entirely by concern about how to attain generality. Therefore, we have labeled it artificial intelligence rather than psychology. There is no conflict, much less contradiction, between these two aims. Nevertheless, one will not find in this monograph any treatment of human data or any direct discussions of whether it is plausible for humans to behave or be structured in ways suggested by the present research. We do not consider such concerns to be irrelevant. After all, the history of GPS has been an alternation between questions of artificial and natural intelligence. However, it awaits additional positive research before anything of substance can be said. (Ernst and Newell 1969, 3)

This thesis differs from the GPS approach in three respects: the research described here is concerned with understanding aspects of natural intelligence, rather than simulating an artificial intelligence. It is also concerned with specificity more so than generality – with looking at how individuals solve problems that they define, as well as problems that others also attempt to solve and which are therefore more "general." However, I describe individual differences in approaches even to these problems. I present the work of three children in greatest detail, through case studies (Section 9), but also present the work of nineteen others. In many of these descriptions – and this is the third departure from a GPS-like approach – the emphasis is on how a child represents a problem at hand and develops ways to go about solving it. This emphasis is orthogonal to the top-down approach: an alternative to formulating a plan and achieving the goal is to ponder ways of representing and solving a problem through the course of thinking about it.

This alternative necessarily assumes that different representations – different approaches – may arise. Instead of a prior judgment about what representation is best for a given problem, there is the possibility that different kinds of solutions can emerge and different kinds of efficiency can come to be acknowledged. Different people come up with different representations, and they go about doing so in different ways. To formulate these processes as a function of a single mechanism or process is to leave out many important parts of the picture. Here, case studies describe three very different approaches to thinking about knots. Only one of these, the "step-by-step" approach, is similar to the goal-driven, propositional sort of process.

Allen Newell has continued the research agenda lauched by GPS, producing a new system, "Soar," which again is based on the model of problem solving as movement in a state space:

> ... the architecture of Soar is built around the problem space. Thus, the decisions that must be made are what problem space should be used (to attain a goal), what state should be used (within the problem space), and, finally, what operator should be used (to progress to the next state). These are the basic operations of the architecture. They are all made by a uniform decision process, called the decision cycle. ...
>
> ... A decision cycle takes place within the context of a stack of earlier decisions about goals, problem spaces, states, and operators ... (Newell 1990, 170-71)

My study challenges the notion of a "uniform decision process" in that it does not seem useful in describing the findings of the research. As the interpretation section of Jill's case study suggests (Section 9.3.4), the processes within an individual mind may have their own, separate origins and "agendas." Furthermore, these processes are not necessarily goal-driven, but may emerge through activity and thinking about something altogether different from the process that triggered them. But in Soar there is room only for causal reasoning. The system is governed by a belief in:

> ... the effectiveness, and apparent necessity, of a goal hierarchy, with goals, subgoals, and alternative subgoals, for controlling behavior to achieve intelligent performance. ...
>
> ... When Soar finds itself at an impasse it creates a subgoal to resolve the impasse. Thus, subgoals arise dynamically because Soar is unable to proceed. Furthermore, this is the only way subgoals can arise. If Soar knows what to do next, there is no need of a subgoal. ...
>
> ... Any change in a goal at some level – a change in problem space, state, or operator – completely preempts all changes about subgoals, because these latter were all created in response to the impasse at this higher level. (Ibid., 174-75)

In this system, the question of "how much" knowledge is available becomes more important than the nature of existing or new knowledge, or how knowledge is used in a given situation:

> If lots of knowledge is brought to bear Soar goes right to the goal, exhibiting routine behavior. If little knowledge is available, then Soar may do lots of undirected and combinatorial search.
>
> This uniform use of problem spaces as the task representation is called the problem space hypothesis ... It is only one of several aspects of Soar where a uniform structure or process is adopted. ...
>
> Memory for everything is handled by the same structure, the same writing processes, and the same reading (or access) processes – for search control, for how to implement an operation, for declarative knowledge, for historical (episodic) knowledge, for everything. This is another uniform assumption. It is arguably the most contentious feature of the Soar architecture. (Ibid., 164)

Newell goes further by offering Soar as a "unified theory of cognition," a candidate among many such theories that he hopes will soon be appearing. To posit a single process as being the basis of all the thinking that goes on in a mind would seem questionable enough, but to formulate that mechanism as the basis for all of human thought wildly generalizes an area that is becoming possible to understand in more particular terms. Newell acknowledges the difficulty of his stance:

> ... it is easy to name aspects that are simply missing in the version of Soar developed to date – emotion, dreams, imagery. These phenomena might require entirely new structures, different in kind from the current mechanisms of Soar. Insofar as that is not the case, the burden of proof rests on Soar to show how such phenomena are to be explained within the current architecture. This has not been done yet. (Ibid., 232)

Within the traditional research realm of "cognition," Soar seems to produce plausible results. Cryptarithmetic is one domain in which the problem-solving paradigm is fruitful. But the data reported in this thesis demonstrate that within the realm of "cognition," we can articulate varying styles and approaches as we examine the thinking of different individuals. And beyond this realization, is it possible (given the complexities and multiple influences of living in the world) that a purely cognitivist model can describe a "unified theory"? The concerns left out of Soar may very well turn out be areas that will demand "entirely new structures." By including this possibility in his discussion, Newell acknowledges that some combination of "cognition" and "affect" comprises intellectual activity – but for now, he has worked toward getting at only the "cognitive" side.[2] Perhaps he would agree with Turkle and Papert that getting also to the "affective" requires a special sort of research method and material. This brings us to a discussion of knots.

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[2] Indeed, some might argue that propositional reasoning constitutes just one aspect of "cognition." Also, many researchers would maintain (as I do) that "cognition" and "affect" cannot be separated; some use the terms "hot" and "cold" cognition in order to bring both concerns into the discourse of cognitive science.

1.3 Knots as Objects to Think With

Papert describes how properties of a particular object played a role in his development as a mathematician:

> Before I was two years old I had developed an intense involvement with automobiles. The names of car parts made up a very substantial portion of my vocabulary: I was particularly proud of knowing about the parts of the transmission system, the gearbox, and most especially the differential. It was, of course, many years later before I understood how gears work; but once I did, playing with gears became a favorite pastime. I loved rotating circular objects against one another in gearlike motions and, naturally, my first "erector set" project was a crude gear system.
>
> I became adept at turning wheels in my head and at making chains of cause and effect: "This one turns this way so that must turn that way so ..." I found particular pleasure in such systems as the differential gear, which does not follow a simple linear chain of causality since the motion of the transmission shaft can be distributed in many ways to the two wheels depending on what resistance they encounter. I remember quite vividly my excitement at discovering that a system could be lawful and completely comprehensible without being rigidly deterministic.
>
> I believe that working with differentials did more for my mathematical development than anything I was taught in elementary school. Gears, serving as models, carried many otherwise abstract ideas into my head. (Papert 1980, vi)

In Section IV, we shall attempt to unpack this idea of "carrying abstract ideas into the head." Here, we can say that by thinking with the object – using it as an image in his thought, maintaining in his thinking the properties that it would demonstrate in the physical world – Papert was able to sustain ideas that otherwise could have been impossibly confusing; it had become what he calls an "object to think with." The presence of the object in his everyday world was key to his understanding. He describes this important factor in the gradual understanding of principles of geometry:

> Jean Piaget's work on genetic epistemology teaches us that from the first days of life a child is engaged in an enterprise of extracting mathematical knowledge from the intersection of body with environment. The point is that, whether we intend it or not, the teaching of mathematics, as it is traditionally done in our schools, is a process by which we ask the child to forget the natural experience of mathematics in order to learn a new set of rules (Ibid., pp. 206-07).

In looking for a way to capture aspects of this natural experience, Papert later developed a computational environment that complements this "intersection of body with environment." The world of the Logo computer language includes "Turtle geometry," in which children can write programs to guide the movement of a graphic turtle on the computer screen. To do so, the children make use of the knowledge of their own movement through space. As they develop more precise and sophisticated specifications for the turtle, they are encouraged to think more deeply about their own movement, and this deeper understanding is, in turn, translated to the computer programs. As in Papert's experience with the gear, children can gain access to mathematical ideas by thinking with the turtle – even identifying with it, imagining themselves to *be* the turtle:

> The gear can be used to illustrate many powerful "advanced" mathematical ideas, such as groups or relative motion. But it does more than this. As well as connecting with the formal knowledge of mathematics, it also connects with the "body knowledge, " the sensorimotor schemata of a child. You can <u>be</u> the gear, you can understand how it turns by projecting yourself into its place and turning with it. It is this double relationship – both abstract and sensory – that gives the gear the power to carry powerful mathematics into the mind (Ibid., p. viii).

Papert uses the term "syntonic learning" to describe the nature of the correspondence between the child's experience and the turtle's behavior:

> This term is borrowed from clinical psychology and can be contrasted to the dissociated learning [often involved in "school math"]. Sometimes the term is used with qualifiers that refer to kinds of syntonicity. For example, the Turtle circle is *body syntonic* in that the circle is firmly related to children's sense and knowledge about their own bodies. Or it is *ego syntonic* in that it is coherent with children's sense of themselves as people with intentions, goals, desires, likes, and dislikes. A child who draws a Turtle circle wants to draw the circle; doing it produces pride and excitement. (Ibid., 63)

This sense of "ego syntonicity" is a key ingredient in the usefulness of gears, or computational circles, or other "objects to think with." It is not enough that the object itself contains specific physical properties and that it lends itself to analogy with the movement of one's own body – a sense of personal connection is what sustains the involvement with these other characteristics. Again, Papert describes how gears might or might not provide the key:

> A modern-day Montessori might propose, if convinced by my story, to create a gear set for children. Thus every child might have the experience I had. But to hope for this would be to miss the essence of the story. *I fell in love with the gears.* This is something that cannot be reduced to purely "cognitive" terms. Something very personal happened, and one cannot assume that it would be repeated for other children in exactly the same form. (Ibid., viii)

Papert describes still another kind of syntonicity that accounts for the learnability of mathematical ideas embodied by the Logo turtle:

> One of the most widespread representations of the idea of angle in the lives of contemporary Americans is in navigation. Many millions navigate boats or airplanes or read maps. For most there is a total dissociation between these *live* activities and the *dead* school math. We

> have stressed the fact that using the Turtle as metaphorical carrier for the idea of angle connects it firmly to body geometry. We have called this body syntonicity. Here we see a cultural syntonicity: The Turtle connects the idea of angle to navigation, actively, firmly and positively rooted in the extraschool culture of many children. (Ibid., 68)

In these passages, Papert has described processes of "thinking with objects," or "concrete thinking." The processes include both "cognitive" and "affective" criteria. An object describes certain physical properties that, in principle, anyone might recognize. But in order for a person to actively think with it, the object relates to individual experience through various syntonicities – of "body," "ego," and/or "culture."[3] Though many objects might embody potentially interesting physical characteristics, there is no guarantee that they will connect with more personal aspects of thought. Certain objects may be more or less likely to touch "affective" aspects of people's lives.

Knots constitute a class of objects that both embody mathematical principles and have a tendency to evoke a range of emotional, personal sorts of thoughts. There are a number of reasons for the latter. One is that knots are made of string, a common material that is part of people's everyday experience. We use string of different weights and textures for any number of odd tasks, from tying shoes to wrapping gifts to anchoring things that move. String embodies the potential for *connection,* a powerful notion in our senses of the social and physical worlds. By convoluting string into the forms of knots, we introduce complexities that have challenged practical knot-tyers, mathematicians, and psychologists.

In this thesis, I discuss how various people have worked with these challenges. I am especially concerned with the thinking of twenty children who participated in a study in

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[3] These combined "cognitive" and "affective" criteria form the basis of what Papert has described as "microworlds," a concept further developed in Section 3.1 of this thesis. The criteria also figure in the more recent discussion of "constructionism" (Papert 1988; 1990 "Introduction," "Unified"; Harel and Papert (in press)).

which they were learning about knots and looking at their own thinking as they did so. My interest was in studying the diverse nature of topological thought, considering both its "cognitive" and "affective" aspects.

Knots are a class of objects well suited to this focus. They have inspired a branch of formal mathematics, "knot theory," in which topologists seek algebraic means of identifying the seemingly infinite number of combinations of intertwinements that form what we call "knots." But they are also among the most common of everyday objects: pervasive through time and cultures, knots have become part of our arts, mythologies, and symbol systems, in addition to our mathematics, physics, and practical work. People become familiar with knots from a very young age and through a variety of means.

A property that makes knots especially interesting to study is that they do not necessitate a separation of the thinking about an object and the thinking about how it is produced. Additionally, knots offer a wide range of complexities. It is just as possible to think seriously about a simple Overhand knot as a more complicated sailor's knot or ornamental knot. This means that a project dedicated to learning about knots can offer an "in" for just about anyone. Knots are also well suited for such a study in that, as one knot can often be thought of as having been built up from another or from a combination of others, they can show clearly how a learner goes about combining skills and subskills. The range of complexity also allows for many possibilities in terms of uncovering the deep understandings of an individual as well as seeing what kinds of understandings emerge as people work together with these evocative objects. Importantly, knots are of epistemological and psychological interest for their way of eliciting great diversity in how people think about them.

In the following sections, I discuss in more detail such epistemological and psychological concerns, the ways in which working with knots surfaced both "cognitive" and "affective" thinking, and how the environment in which I conducted my study made use of these "objects to think with." The environment consisted both of knots and of a social substrate that encouraged the lively exchange of ideas about them. As the reader will discover, these ideas included detailed views of the topology of certain configurations, as well as deep connections between this thinking and children's senses of their bodies and their personal lives.

The discussion of one other conceptual area will help to situate my presentation of this study: that is, views of how mathematical thinking is structured.

1.4 Structures of Mathematical Thought

Theorists in two different disciplines have been concerned with articulating components of mathematical thought. The Bourbaki mathematicians set out to define an "architecture of mathematics":

> The Bourbaki have attempted to abstract the most basic structures under which all others can be subsumed. By retrospection (and not by any a priori process) they arrived at three 'mother structures' *(mère structures) [sic]*. First, there are algebraic structures (prototypically the group), which are characterized by the presence of operations. Second, there are structures of order, which have to do with relations, and whose prototype is the lattice, which assigns to each pair of its elements a successor and a predecessor of the pair. Third, and last, there are topological structures which are based on notions of neighbourhood, continuity, and limit. (Lane 1970, 23-24)

At about the same time, Piaget was making a similar, but separate, inquiry. He was concerned with the way in which mathematical thought develops in the individual, and based his conclusions on the observable behavior of children rather than on adult introspections. Thus, Piaget "emphasizes the part played by overt activities in building up the conceptual machinery of thought." (Beth and Piaget 1966, xvi) Upon discovering the findings of the Bourbaki, Piaget compared the structures they defined (which he sometimes calls the "parent structures") to his own characterization of the development of mathematical thought in children:

> Now when we study the intellectual development of the child, we find that the earliest cognitive operations, those which grow directly out of handling things, can be divided into precisely three large categories, according to whether reversibility takes the form of "inversion," of "reciprocity," or of "continuity" and "separation." Corresponding to the first – formally considered, algebraic structures – there are classificatory and number structures; corresponding to the second – formally considered, order structures – there are series and serial correspondences; corresponding to the last – formally considered, topological structures – there are operations that yield classes, not in terms of resemblances and differences, but in terms of "neighbourhoods," "continuity," and "boundaries." It is remarkable that, psychogenetically, topological structures antedate metric and projective structures, that psychogenesis inverts the historical development of geometry but matches the Bourbaki "genealogy"! (Piaget [1968] 1970, 26-27)

Piaget emphasizes that all other structures can be derived from the Bourbaki "matrix structures," and that the overlap of the structures constitutes an "effective architecture of mathematics":

> This regressive analysis has ... brought to light ... three fundamental structures which remain irreducible and are called "matrix structures", because all others known at present can be derived from them:
>
> (1) *Algebraic structures*, of which the prototype is the "group" ...
>
> (2) *Structures of order*, an important type of which is the network or lattice ...
>
> (3) *Topological structures* which deal with the concepts of neighbourhood, limit, and continuity.
>
> From these matrix structures we can ... derive all the others ...

> Finally we shall arrive back at the particular theories of classical mathematics by specifying the elements with which the differentiated or multiple structures deal. But they then cease to appear autonomous and take the form of an intersection of structures. ... [and] constitute ultimately the effective architecture of mathematics. (Beth and Piaget 1966, 165-66)

Of particular interest in this thesis is the character of the topological structures, a basis of what many cognitive psychologists call "spatial thinking":

> ... it is not an overstatement to suppose that besides the structures with reversible inversion or reciprocity, concerning which we shall enquire whether they foreshadow algebraic structures and structures of order, we must distinguish at all the elementary stages a third type of structure, the primary characteristics of which are essentially topological, and whose combinations with other sturctures give rise to more complex spatial structures (measurement etc.). (Ibid., 186)

Papert explains the "mother structures" in terms of how they make mathematics learnable and, therefore, how they can be useful in the development of specific types of learning environments that he calls "microworlds":

> Bourbaki is a pseudonym taken by a group of French mathematicians who set out to articulate a uniform theory for mathematics. Mathematics was to be one, not a collection of subdisciplines each with its own language and line of development. The school moved in this direction by recognizing a number of building blocks that it called "mother structures." These structures have something in common with our idea of microworlds. Imagine a microworld in which things can be ordered but have no other properties. The knowledge of how to work the world is, in terms of the Bourbaki school, the mother structure of order. A second microworld allows relations of proximity, and this is the mother structure of topology. A third has to do with combining entities to produce new entities; this is the algebraic microstructure. The Bourbaki school's unification of mathematics is achieved by seeing more complex structures, such as arithmetic, as combinations of

> simpler structures of which the most important are the three mother structures. This school had no intention of making a theory of learning. They intended their structural analysis to be a technical tool for mathematicians to use in their day-to-day work. But the theory of mother structures *is* a theory of learning. It is a theory of how number is learnable. ...
>
> Piaget observed that children develop coherent intellectual structures that seemed to correspond very closely to the Bourbaki mother structures. For example, recall the Bourbaki structure of order; indeed, from the earliest ages, children begin to develop expertise in ordering things. The topological and algebraic mother structures have similar developmental precursors. What makes them learnable? First of all, each represents a coherent activity in the child's life that could in principle be learned and made sense of independent of the others.
>
> Second, the knowledge structure of each has a kind of internal simplicity ... Third, although these mother structures are independent, the fact that they are learned in parallel and that they share a common formalism are clues that they are mutually supportive; *the learning of each facilitates the learning of the others* [my italics]. (Papert 1980, 160)

The microworld of Turtle geometry is based on properties of the turtle: its *position* and *heading* make it an object with which a child can explore the idea of *angle*. Similarly, knots are objects with which children (and others) can explore the ideas of *neighborhood* (or *surrounding*), *limit,* (or *boundaries*), and *continuity,* and the relationships of *proximity* to which they give rise.

2 Doing Epistemology and Psychology with Knots and String

> What is topology, that a child may know it, and a child, that she may know topology?
>
> – *apologies to Warren S. McCulloch* (1961; [1965] 1988, 1)

Papert explains a distinction between two kinds of research, which is useful in illustrating a similarity of my work to Piaget's:

> Piaget has described himself as an epistemologist. What does he mean by that? When he talks about the developing child, he is really talking as much about the development of knowledge. This statement leads us to a contrast between epistemological and psychological ways of understanding learning. In the psychological perspective, the focus is on the laws that govern the learner rather than on what is being learned. Behaviorists study reinforcement schedules, motivation theorists study drive, gestalt theorists study good form. For Piaget, the separation between the learner and the learning process is a mistake. To understand how a child learns number, we have to understand number. And we have to study number in a particular way: We have to study the structure of number, a mathematically serious undertaking. This is why it is not at all unusual to find Piaget referring in one and the same paragraph to the behavior of small children and to the concerns of theoretical mathematicians. (Papert 1980, 158)

Piaget's study of number led to his formulation of "mother" or "matrix structures," like those of the Bourbaki mathematicians, which he postulates as contributing to the development of mathematical understandings. He distinguishes epistemological concerns as those pertaining to the "object," and psychological concerns as those pertaining to the "subject":

> ... the aim of these analyses is epistemological and not logical, that is, on each of the points enumerated we intend not to raise questions of validity, but to contribute to the solution of two questions (a) to discover what is due to the subject and what belongs to the object and (b) the ontological nature of the latter. For example, as far as the "matrix structures" in Bourbaki's sense are concerned, the questions to which psychology can make some contribution are to determine whether these structures correspond to general mental structures in the operational mechanisms of the subject, or whether they are only due to a recent technical elaboration. If they are "natural" insofar as rooted more or less deeply in the subject's activity, we have to establish how they develop genetically as a function either of the internal conditions of this activity (we say of this activity as opposed to any introspective "experience"), or of diverse experiences (physical etc.) or of language etc. (Beth and Piaget 1966, 163)

In the case studies (Section 9), I demonstrate a connection between mathematical "mother structures" and aspects of three children's internal "activity." The "mother structures" provide a way of explaining specific representations that these children used in describing knots. In particular, their descriptions can be seen as reflecting the interdependent "mother structures" of number *(classification, seriation)* and topology *(proximity, separation, order, enclosure, continuity)*. Complementing the dramatic extent, range, and diversity of thinking about knots that is documented in this thesis, the work of each of these three children has an internal consistency and coherence that can be characterized in terms of a preference for one "mother structure" or another.

This view constitutes a novel approach to the notion of *style*. I am explicitly bringing together the epistemological and the psychological – not just through a particular kind of research, but for purposes of explanation. I am also bringing together two facets of psychology that are normally kept separate: "cognitive psychology" and what I call here, for purposes of comparison, "affective" psychology, which refers to the domains of clinical psychology and psychoanalysis.

Paradoxically, when I look at the psychological through the epistemological (or vice-versa), I am both in accord with Piaget's approach and at odds with it, partly because he is at times self-contradictory. In his introduction to the volume on *Mathematical Epistemology and Psychology,* W. Mays explains the importance to Piaget of actions – observable, overt activities – as the operational "glue" that holds together certain structured understandings. Mays points out – as Papert also has – that, contrary to much of the popular impression and much of Piaget's own writing, he was concerned with the study and emphasis of processes of "concrete" thought, and questioned the tendency to normalize formal, "abstract" thought:

> Piaget finds that intellectual behaviour consists at first of simple classificatory and relational activities in which the child compares, distinguishes and orders the objects around him, and that his later logical and mathematical activities, in which propositional or formal operations occur, develop out of these. Piaget uses the term 'operation' to refer to an action or system of bodily movements, which has become internalised in the form of thought activities. For Piaget mathematical and logical operations are real actions, whether they be actions performed by a child when he moves beads along an abacus or, at the adult level, manipulations performed upon symbols in accordance with specific rules of a calculus.
>
> Four main stages in the construction of such operations are distinguished. They are (1) sensory-motor: before language appears the sensory-motor activities of the young child can display some of the features of intelligence; (2) pre-operational thought, in which language, symbolic play and invention occur; (3) concrete operations: the activities involved in classifying, ordering and enumerating objects; (4) propositional or formal operations, i.e. verbal and formal logico-

> mathematical reasoning. As a result of neglecting the earlier, more concrete levels of logical thought philosophers have, in Piaget's view, tended to regard (4) as forming an independent normative realm of its own. (Ibid., xvi)

Turkle and Papert make clear that, for many older children and adults, the "more concrete levels" constitute a preferred way of thinking. These authors challenge

> ... the value-laden perspective on intellectual growth that has dominated Western philosophy. Piaget sees a progression from egocentric beginnings to a final, "formal stage" when propositional logic and the hypothetico-deductive method "liberate" intelligence from the need for concrete situations to mediate thinking. We disagree: for us, *formal reasoning is not a stage, but a style*. (Turkle and Papert 1990, 358)

My approach takes the same view. By discussing styles in terms of epistemological structures, I assume that while component structures of thought may be supplanted by increasingly "sophisticated" ones, this is not the only way in which thinking can develop. Certain structures may continue to be accessible as a person grows, to be called upon more or less frequently, and with greater or lesser clarity, as time, circumstance, or preference may dictate.[1]

In both the "cognitive" and "affective" domains of psychology, people are characterized according to categories of *style*. The implications of such categorization vary, but the politics of intervention suggest that any such characterization should be self-conscious and clear in its assertions about what it attempts to explain. Psychological researchers and practitioners use the notion of *style* in different ways and for different purposes. Two illustrative examples are the "cognitive styles" described by Kenneth M. Goldstein and Sheldon Blackman, and the "neurotic styles" described by David Shapiro.

Here is one defintion:

[1] See Section IV for a more detailed discussion of *style* with regard to the findings of my study.

> Cognitive style, emphasizing the structure rather than the content of thought, refers to the ways in which individuals conceptually organize their environments.
>
> ... A number of themes appear repeatedly in the various approaches to the study of cognitive style. These include the relationship of cognitive style to intelligence, rigidity and intolerance of ambiguity, and performance under stress. (Goldstein and Blackman 1978, vii)

Through particular tests and measurement techniques, the researchers whose work Goldstein and Blackman describe worked within categories designated as "authoritarianism," "dogmatism," "cognitive complexity," "integrative complexity," and considerations in the area of perception that led to designations such as "field dependence."

Shapiro clarifies his use of the term *style:*

> By "style," I mean a form or mode of functioning – the way or manner of a given area of behavior – that is identifiable, in an individual, through a range of his specific acts. By "neurotic styles," I mean those modes of functioning that seem characteristic, respectively, of the various neurotic conditions. I shall consider here, particularly, ways of thinking and perceiving, ways of experiencing emotion, modes of subjective experience in general, and modes of activity that are associated with various pathologies. It is not my aim to be exhaustive or even systematic, and it is clear that there are many interesting aspects of style that cannot even be touched on here – for example, body-movement styles. But I hope to provide an outline of at least four major neurotic styles: obsessive-compulsive, paranoid, hysterical, and impulsive. (Shapiro 1965, 1-2)

Through case analyses (often including specific tests), Shapiro refines descriptions of categories of thought and action which have become standard in the discourse of psychologists in his milieu.

Turkle explains Shapiro's interest in both "cognitive" and "affective" components of thought:

> Psychologist David Shapiro has used the idea of "neurotic styles" to capture what each of us knows intuitively about him- or herself: we are the same person whether we are solving an intellectual problem or sorting out a personal difficulty. And, indeed, the blocks we run into, the ways we achieve or avoid success in the cognitive and affective domains, often take us aback by their similarity. The use of clinical categories to describe these styles reflects the fact that when we look at human psychology there is a continuum between what we see as ill and what we see as normal. The underlying processes are the same for everyone; some simply suffer more from them than others. Thus we come to understand ourselves better by knowing what we would be like if the stresses of life led us to a breaking point. At that breaking point, our "neurotic style" would be transformed into a disabling symptom. At that point, the style "takes over," severely limiting our ability to cope with reality. Before that point, a neurotic style is simply a way of approaching the world and defending oneself against what is painful. (Turkle 1984, 107)

Although this psychological domain shares interests with the "cognitive" school, each domain has attempted certain kinds of rigor by isolating areas for research, analysis, and description. Each has its own agenda: cognitive psychologists may hope to describe categories capable of including "emotional" thoughts as well as "cognitive" ones, yet their training and discourse tend to sustain a certain skepticism about matters of "affect"; clinical psychologists and psychiatrists may seek to describe patterns of thought that span both "affective" and "cognitive realms," yet their descriptions are rooted in a milieu devoted to diagnosis and treatment, potential ills being an assumption of any given analysis.

The result, of course, is that we have available rich bodies of information that are not easily understood with regard to one another. My interest is in capturing how these habitually separate domains might inform each other. Therefore I used a method of inquiry that is strictly traditional in neither realm, but which combines elements of each and which yields information useful in an analysis from each perspective. The result is that my designations of *style*, in addition to being shaped by epistemological concerns, are intended to reflect aspects of individuals' ways of working and thinking which encompass both the "cognitive" and the "affective."

My hope is that I may have taken some steps toward developing a typology of mathematical thought, of the sort that Evert W. Beth calls for. His observations, and my own, go against the grain of efforts toward describing "unified theories of cognition":

> The mere fact that the result of original work in the mathematical field is called sometimes a *creation* or *invention*, and sometimes a *construction* or *discovery*, shows all the multiformity of mathematical experience.
>
> It seems to me that only a truly scientific typology of mathematical thought, established by well tried psychological methods, could give us a sufficiently expressive image of this experience in its divergent forms. As long as we lack such a typology it will remain very difficult to arrive at a more or less coherent interpretation of the introspective data, with which only mathematicians themselves can provide us on the subject of the true nature of mathematical thought. (Beth and Piaget 1966, 100)

In the following short sections, I describe the work of epistemologists and psychologists who have been concerned with knots and string, and with the thinking they evoke. Some of the researchers are concerned with epistemology and "cognitive" psychology, some are concerned with "cognitive" psychology, and some, with "affective" psychology. The studies are cited in chronological order, from those done earlier to later.[2]

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[2] I have not included the fascinating work of Kurt Lewin, *Principles of Topological Psychology*,in which he develops representations of the "psychological life space" based on constructs borrowed from the highly refined

2.1 Epistemology and "Cognitive" Psychology

Here I describe a study of knots by Piaget and Inhelder, and two pilot studies that I conducted prior to the research that spawned the Knot Lab.

2.1.1 Jean Piaget and Bärbel Inhelder

In Piaget's theory (1965), *classification* and *sequence* are considered as "mother structures" with which children construct the concept of number. Classifications are called a "cardinal" concern, involved with ways of arranging things that are similar. Sequence, or seriation, is an "ordinal" concern, involved with giving order to things that are dissimilar.

The concern with *order* appears again in the later work of Piaget and Inhelder as they discuss childrens' constructions of the concept of "space" (1967). The authors begin the volume with a differentiation of "perceptual space" and "representational space":

> ... the evolution of spatial relations takes place at two different levels. It is a process which takes place at the perceptual level and at the level of thought or imagination. (Piaget and Inhelder [1948, 1956] 1967, 3)

Sensori-motor constructs are not to be confused with "representational images and geometrical ideas," and they need to be studied in a way that acknowledges this differentiation. Complicating the distinction, however, is the interdependence of the two domains:

> Though in a sense profiting from the achievements of perception and motor activity (which at their own level provide experience of straight lines, angles, circles, squares, projective systems and so on),

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formal mathematics of topology. (Lewin 1936, 6) The work of the authors I have included here deals specifically with the use of string and/or knots.

> representational thought or imagination at first appears to ignore metric and perspective relationships, proportions, etc. Consequently, it is forced to reconstruct space from the most primitive notions such as the topological relationships of proximity, separation, order, enclosure, etc., applying them to the metric and projective figures yielded by perception at a higher level than that of these primitive relationships themselves. (Piaget and Inhelder [1948, 1956] 1967, 3-4)

Thus we should not consider geometric concepts as being based directly on sense data; rather, there are different "levels" of thought through which geometric understandings emerge. The perceptual level comes to recognize geometric relationships as experienced in the outside world; the representational level proceeds from understandings of topological relationships to understandings of geometric ones. But as the authors imply, these different sorts of geometric understandings do not grow in isolation; they are the result of an interaction between the level of perception and the level of thought or imagination:

> ... during the development of representational space, representational activity is, in a manner of speaking, reflected or projected back on to perceptual activity. (Piaget and Inhelder [1948, 1956] 1967, 4)

The distinction between perceptual and representational knowledge is crucial to these authors' choice of what to study and how to study it. *The Child's Conception of Space* is concerned with how children construct underlying topological concepts, as well as, ultimately, the geometric concepts involved in understanding projective and Euclidean space. This thesis shares their interest in the development of understandings of topological relationships.

Piaget and Inhelder consider the topological concepts of linear and circular order as leading to the "relationships of surrounding," one of which is the concept "between." In order to understand this relationship, the child must first be able to reverse a series. For example, understanding the series ABC involves realizing that B is simultaneously between

A & C and C & A, and part of this understanding involves recognizing the invariance of B's position.

"Between," say Piaget and Inhelder,[3] is "one particular instance of the more general relationships of 'surrounding'." They assign to these relationships even more importance than other elementary spatial relationships (such as *proximity, separation,* and *order*) because the relationships of surrounding lead the child "to differentiate and build up the three initial topological dimensions." (Piaget and Inhelder [1948, 1956] 1967, 104) As the authors define them, the general relationships of surrounding are: "between," which constitutes a one-dimensional surrounding (a line); a point lying inside or outside of a closed planar figure, which constitutes a two-dimensional surrounding (a surface); and a point lying inside or outside of a closed box, which constitutes a three-dimensional surrounding (a space).

The authors considered various situations within which to study these relationships, including containers and contained objects or rings placed around sticks. Ultimately they decided on knots, because the children with whom they were working would not yet have developed understandings of the perceptual relationships. As a research situation, knots would thus be more likely to reveal the precursor topological elements.

Piaget and Inhelder saw as an additional advantage the fact that knots had already been recognized as a branch of mathematical study focusing not on geometric properties such as distances, angles, or measures, but on elasticized forms. Of interest are the *homeomorphisms* that can be discovered as the forms are stretched or twisted: do the deformations retain the proximities, separations, or (for lines) relative order? At what point might children recognize these homeomorphisms?

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[3] Also credited for their collaboration in this work are Ascoli, Halperin-Goetschel, and Morf.

The authors mention still another advantage of knots as a domain for "psychogenetic investigation": children learn to tie them from an early age, so although much about given knots would not yet be understood, the domain itself would be familiar. Piaget and Inhelder characterize the gradual understanding of transformations in the string as happening in degrees. The children who participated in the Piagetian studies ranged in age from two to six years old. The researchers presented them with loosened and tightened transformations of the simplest kind of knot, the overhand knot, as well as with transformations of other string shapes, including a circle, a "figure of eight," a "pseudoknot," and mirror-image versions of the overhand. They considered the idea of surrounding to be embodied in the knot through the properties of enclosure or intertwinement. Their interviews addressed questions about homeomorphic correspondence or non-correspondence between the string shapes.

The research reported in this thesis posed similar questions, but sought children's responses with regard to more complex configurations, both in terms of the finished knots and processes of tying them. The children were older, ranging from ten to twelve years of age. There was a sufficient variety of knots that the children were able to perceive general similarities, articulations of which became the basis of a rudimentary "knot language." Whereas Piaget and Inhelder are concerned with homeomorphic transformations – gradations of looseness or tightness of a knot – the Knot Lab research included instances of configurations that were identical except for a key characteristic, which some of the children came to appreciate while others did not. By learning to tie and describe the knots, the children were grappling with understandings of topological relationships (relationships of *proximity*). By classifying the knots according to perceived similarities and differences, the children were working with other mathematical principles, such as the concept of set or group.

The study by Piaget and Inhelder of young children's thinking about simple knots becomes a basis of comparison with my approach. The study consisted of the classic interview style, and a series of questions that anticipated increased complexity of understanding with age. Stages of learning about properties of knots were carefully delineated (see Appendix A). In their interpretation, Piaget and Inhelder look broadly, considering the responses of many children and ordering them according to a developmental progression marked by increments of change that can be discerned in the thinking. I look broadly too, not in an effort to identify developmental stages, but to compare many different individuals' means of expression and ways of thinking about knots – their *styles*. This interest mandates two important shifts of focus from the Piagetian studies: it favors studying at length and in detail the work and thought of individuals, favoring a depth-first rather than a breadth-first approach; and concentrates on – indeed, "revalues" – so-called "concrete" aspects of thought.

2.1.2 Pilot Studies for the "Knot Lab" Research Effort

I conducted two initial experiments with groups of people working together to learn and communicate about knots. These sessions were short-term, and involved tying knots and giving others verbal and graphic instructions for how to tie them. The emphasis on notations (or "knotations," as we called them) was similar to Caron-Pargue's approach (Section 2.2.2) and one of the focuses of the analysis by van Sommers (Section 2.2.3). However, each of the pilot studies pointed to a need for more immersion, in order for participants to evolve both understandings of knots and means of representing the understandings.

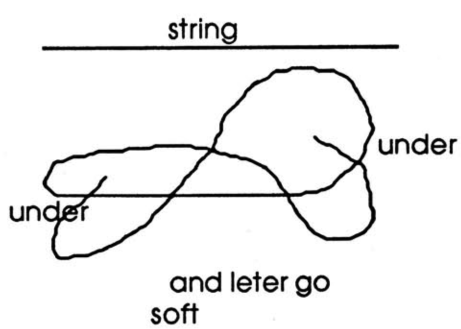
One of the groups consisted of four children, ten through twelve years of age. They worked together in two sessions. In the first, they each tied an Overhand knot and produced braids. Two of the children also tied Figure 8's, and two of them tied Bowlines. In the second session, the children worked with these same knots, in addition to the Square knot.


6.17

Ellen and Pete worked together on the Figure 8, and each of them developed instuctions for tying the knot. Ellen began with a procedure:

> <u>string</u>
> cross your hands
> take your right hand and your left hand
> leter go soft

Ellen also attempted a pictorial recipe that substituted specifications of "under" crossings for the references to hands, although it isn't entirely clear what strands go "under":



Pete tried to be verbally precise:

> put It in your ritt hand and hold one straight and the other one down the take the one that is hanging and Flip it over thes put it thru the hole then tight IT up soft and thats whow you do it.

While at this point emphases such as tightening the knot softly and the sense of the term "straight" as a horizontal appear as curiosities, a much broader study was needed to see both the regularity with which such concepts might emerge, and the individual conditions in which they might occur. The relevance of working with knots was also a consideration: there was a sense in which the children's expressions lacked a certain passion or sense of ownership, qualities which might have infused their responses with greater meaning and reliability, in terms of what was revealed about the thinking.

In various sessions with adults, one person tied a knot and then gave instructions to another for how to tie the same knot. Typically, the instructions were both verbal (spoken and written) and graphic. Again, while the particular focuses on crossings, maneuvers, etc., were revealing, the decontextualization of the exercises robbed them of a sense of purposiveness and therefore a certain kind of "meaning" that seemed related to reliability. Asking people to sit at a table for a few hours and work with knots in order to satisfy the curiosities of a researcher did not seem to evoke a satisfying range and depth of relevant thoughts.

One solution could have been to work for an extended period of time with people using knots in order to accomplish some activity in which they were engaged, such as sailing. But I decided to work with children for several reasons: Their thinking is affected by fewer years of cultural influences, and thus seems more "raw" and reliable as information about how people learn. In a sense, since children are relatively less inhibited, tending to be more spontaneous – and, perhaps, more straightforward – than adults, their thinking is also more accessible, if conditions are provided such that they feel secure and occupied. Exploring how to provide an environment that answered these needs while making knots a topic worthy of serious consideration was an interesting prospect in itself. The experimental design that I launched, and which the children appropriated and modified, was geared toward developing a setting in which learning and communicating about knots was the raison d'être.

Thus, the approach of the actual study extended the pilot studies in terms of constraints and setting. Longer duration and a wider variety of activities enabled the participants to develop richer understandings of knots, and enabled greater access to their thinking.

2.2 "Cognitive" Psychology

Knots form the basis of studies or theoretical discussion for three sets of researchers concerned with spatial thinking and representations. Forman discusses how the developmental perspective of Piaget and Inhelder differs from Olson's perspective in researching "spatial" knowledge. Caron-Pargue deals specifically with notations, discussing the variety, multiple meanings, and gradations of subtlety in subjects' representations of knots. Van Sommers used a variety pictures, including some string figures, to gauge subjects' memories of the complicated images.

2.2.1 George E. Forman on David R. Olson

Although Olson (1970) did not conduct specific research involving knots, Forman (Eliot and Salkind, 1975) uses knots as an example in illuminating theoretical differences between Olson's and Piaget and Inhelder's approaches to components of spatial representation. Olson identifies activity and instruction as means of bringing subjects into contact with alternatives in thinking through a problem (such as, Forman would have us imagine, that of producing a knot). Olson's mode of inquiry assumes a level of generality that differs from Piaget and Inhelder's: applied to the domain of knots, Olson's questions would pertain to remembering aspects of a particular configuration, whereas Piaget and Inhelder seek to extract principles that would apply in the domain of knots as well as in other well-chosen contexts, principles such as "order" and "surrounding." In Forman's words, Piaget and Inhelder are interested in how such principles are "constructed" rather than "contacted" for purposes of the immediate situation and then made available for use in other situations, as Olson would emphasize.

The research reported here, like Piaget and Inhelder's, was concerned with the *construction* of topological understandings, but also with the development of an environment that would facilitate that construction.

2.2.2 Josiane Caron-Pargue

Caron-Pargue (1982, 1983) elicited and analyzed adult subjects' graphic and verbal instructions for making knots. Of particular interest were the interactions between the subjects' mental representations of knots and their construction and use of notations. The evolution of these notations is interpreted as capturing specifics of the subjects' growth of understanding. It became clear that a given notation might correspond to several meanings, and a given meaning could be expressed through several notations. Caron-Pargue developed a detailed catalog of the notations and relevant contexts (including, for example, whether a certain part of the knot was elevated). She asserts that a specific process of coming to understand a knot may be hinged to the choice of a certain representative medium, and points to this methodological consideration as shaping directions for future research.

In the pilot studies preceding my research (see Section 2.1.2), participants also learned to tie certain knots and developed graphic and verbal instructions for tying them. Although these notations illuminate varying conceptions of the knots, it became clear that the depth of familiarity that I was hoping to study would require both a lengthy period of immersion in the subject and an environment conducive to a wide range of choices of knots, approaches to learning them, and media for expressing ideas about them. The specific representational choices of two of the participants (Sections 9.3 and 9.4) reflect – among other influences – the importance of the medium, consistent with Caron-Pargue's findings.

2.2.3 Peter van Sommers

Van Sommers (1984) developed a comparative measure of adult subjects' abilities to reproduce from memory graphic representations of "difficult" patterns. Among the fourteen designs were an overhand knot and a circle twisted to form a loop in the shape of an eight. (String representations of both of these shapes were included in Piaget and Inhelder's studies.) Beyond noting degrees of the drawings' resemblance to the original pictures, van Sommers catalogued styles and patterns of notation, developing terms such as "pathfinding" and "backtracking."

Although the research described here includes some analyses of graphic representations of knots, the scope is broader in that it includes various media as appropriate to individual participants' strategies for learning to tie and describe the knots.

2.3 "Affective" Psychology

These three psychoanalysts – Winnicott, Laing, and Lacan – saw in string and knots a powerful metaphor for the ideas with which they were working. Themes of their work include what we might call the "topology of social relations" and the complexities of human thought.

2.3.1 D. W. Winnicott

A well-known British pediatrician and child psychiatrist, Winnicott developed many concepts now in common parlance among psychoanalytic circles. Terms such as the "good-enough mother," the "holding environment," and the "transitional object" figure prominently in the object-relations tradition. Winnicott's theories were based on children's clinical case histories that he personally took – which by the time he died numbered more than 20,000. (Guido, Lamb, and Stevens 1990) He was known for his remarkable ability to relate to children; the cartoonist David Levine has caricatured him as a human teddy bear. Winnicott championed the idea of play as a natural form of communication.

Winnicott developed some original techniques that enabled him to get at the psychological material with which his young patients were struggling. One of these techniques became known as the "squiggle game," in which he made a scribble on a piece of paper, which the child would turn into some meaningful picture by adding more lines or scribbles. The child would then initiate a drawing by making a new scribble, which Winnicott would turn into a picture. The game would go back and forth in this manner, until some significant communication emerged through the series of drawings. (Winnicott 1971, *Therapeutic Consultations*)

In one such session with a seven-year-old boy,

> nearly everything I did with him was translated into something associated with string. Among his ten drawings there appeared the following: lasso whip crop a yo-yo string a string in a knot another crop another whip. (Winnicott 1965, 153-57; 1971, *Playing and Reality*, 15-20)

Upon questioning, the boy's parents described his preoccupation with string:

> ... whenever they went into a room they were liable to find that he had joined together chairs and tables; and they might find a cushion, for instance, with a string joining it to the fireplace. They said that the boy's preoccupation with string was gradually developing a new feature, one that had worried them instead of causing them ordinary concern. He had recently tied a string round his sister's neck (the sister whose birth provided the first separation of this boy from his mother). (*Playing and Reality*, 17)

Winnicott had learned not only of this separation, but of some others in the boy's early life, which apparently had been traumatic for him. Winnicott surmised that the boy was "dealing with a fear of separation, attempting to deny separation by his use of string." Although the boy's mother was skeptical about this interpretation, Winnicott encouraged her to discuss the matter with her son, should she find the inclination and opportunity. The suggestion was fruitful:

> ... one evening she had opened the subject with the boy and found him to be eager to talk about his relation to her and his fear of a lack of contact with her. She went all over the separations she could think of with him with his help, and she soon became convinced that what I had said was right, because of his responses. Moreover, from the moment that she had this conversation with him the string play ceased. She had had many other conversations with the boy about his feeling of separateness from her, and she made the very significant comment that she felt the most important separation to have been his loss of her when she was seriously depressed; it was not just her going away, she

said, but her lack of contact with him because of her complete preoccupation with other matters. (Ibid.)

Winnicott stayed in touch with this family as the boy grew. They described recurrences of his expressive play with string at times when the mother went into the hospital for an operation and when she experienced depression.

Summarizing this case, Winnicott comments:

> String can be looked upon as an extension of all other techniques of communication. String joins, just as it also helps in the wrapping up of objects and in the holding of unintegrated material. In this respect string has a symbolic meaning for everyone; an exaggeration of the use of string can easily belong to a sense of insecurity or the idea of a lack of communication. ... the function of the string is changing from communication into a *denial of separation*. (Ibid., 19)

The theme of denial of, or preoccupation with, separation is one that emerges here, and is discussed in the case studies of Jill and Tony (Sections 9.3 (especially 9.3.4) and 9.4 (especially 9.4.4)).

2.3.3 R. D. Laing

Laing, a psychiatrist and psychoanalyst associated with the Tavistock and Langham clinics in London, was intrigued by the complexities of interpersonal relationships. Influenced by the existentialist tradition, Laing made efforts to "convey above all that it was far more possible than is generally supposed to understand people diagnosed as psychotic" (Laing [1960, 1962] 1965, 11). In particular, he studied the phenomenon of schizophrenia and the dynamics of groups and families. Laing also participated in the development of a "notation for dyadic perspectives" (Laing [1961] 1969, 174-80), which enables the schematization of the explicit, the implied, the repetitive, and the contradictory in systems of dialog and relationship. He continued this interest in his book, *Knots,* which expresses in poetic form the intricacies and confusions that often arise between people. In a brief foreword, he explains:

> The patterns delineated here have not yet been classified by a Linnaeus of human bondage. They are all, perhaps, strangely familiar. In these pages I have confined myself to laying out only some of those I actually have seen. Words that come to mind to name them are: knots, tangles, fankles, *impasses,* disjunctions, whirligogs, binds. I could have remained closer to the 'raw' data in which these patterns appear. I could have distilled them further towards an abstract logico-mathematical calculus. I hope they are not so schematized that one may not refer back to the very specific experiences from which they derive; yet that they are sufficiently independent of 'content', for one to divine the final formal elegance in these webs of *maya.* (Laing 1970, iv)

This example from the book should give a sense of the qualities of overlapping and interconnectedness that Laing explores in the work:

> They are playing a game. They are playing at not playing a game. If I show them I see they are, I shall break the rules and they will punish me. I must play their game, of not seeing I see the game. (Ibid., 1)

2.3.4 Jacques Lacan

Lacan, a French psychoanalyst who became concerned with extending and interpreting Freud's work for popular understanding, made use of knots in his formulation of the *matheme.* Turkle describes this model in terms of knots composed of four components representing aspects of the psyche – the imaginary, the real, the symbolic, and the symptom:

> These, he explained, were Borromean knots made of interlocking circles. When one is cut, the whole chain of circles comes undone. (Turkle [1978] 1981, 235)

Lacan's use of knots represented an attempt to bring together things that he saw as importantly related, but which we often experience as separate:

> He speaks of how manipulating and perforating spheres in the "praxis of knots" is "the thing to which the spirit is most rebel." The circles that make up the knots are sections of spheres, "man's first representations of his own body and his first conceptions of science." The knots "so contradict our global sense of our bodies as enveloped and enveloping that to try oneself in the praxis of knots is to shatter inhibition," perhaps because it threatens our images of our bodies and our images of our science by reminding us of a connection between them.
>
> It is clear that for Lacan the role of mathematical theory is psychoanalytic. Doing the theory – working on the knots, practicing the manipulations – enters as an integral element, indeed, the critical element in the emergence of insight about the self, in the same sense that psychoanalytic insight grows out of the lived relationship with an analyst.
>
> The mathematical modeler often sees his enterprise as scientific and precise, as opposed to literary or poetic. Lacan refuses this dichotomy. He cuts across a line between poetry and science that has become axiomatic in the philosophy if not in the practice of Western science. (Ibid., 237)

Lacan's working with knots becomes a critique of the abstracted, disembodied practice that mathematicians have developed through formalized knot theory. Knots provide a way of gaining access to the "primitive roots" of topology "as a way of experiencing the body." (Ibid, 246) The discussion recalls Papert's concept of "body syntonicity" (Section 1.3).

3 Toward an Integrated Research Methodology

Studying people's thinking, particularly when both "cognitive" and "affective" aspects are of interest, requires a setting conducive to in-depth exploration of the subjects' dialogue and actions. This need has implications for the duration of the study and for the nature of the environment in which the inquiry is conducted. The researcher must be able to spend a good deal of time with the subjects, encourage or design problem scenarios that will tend to surface a certain kind of thinking, and develop a relationship with the subjects that allows for the exchange of relevant information.

Although I conducted one of the two pilot studies with adults, I decided to work with children in order to seek a generally more spontaneous kind of response, one which also has the characteristic of being less complicated by the broader range of adult experience. While other researchers have made use of string figures and knots to explore topological thinking, spatial reasoning, and emotional expression (Section 2), knots have not previously been studied in light of the evolving "constructionist" perspective rooted in Piaget's "constructivism" (Gruber and Vonèche 1977) and developed by Papert (1986; 1990, "Introduction," "Unified"; Harel and Papert (in press)). These discussions emphasize the importance of the culture and the context in which learning occurs:

> The work is both frequent enough ... and open-ended enough for differences in style to emerge. (Turkle and Papert 1990, 353)

In addition to the focus on design and description of the learning environment, the "constructionist" framework (and my study) have been influenced by methodologies that seek ways of making explicit the thinking of different individuals. Among these approaches are the Piagetian clinical interview ([1929] 1951, 1-32), clinical methods for research, as described by David N. Berg and Kenwyn K. Smith (1985), the interview and analysis techniques of Gilligan et al. (Brown 1987, 1-34), and the role of "participant observer" as described by Turkle (1984, 315-32). These techniques are briefly explained below. Related approaches are described in Donald Polkinghorne (1983, 1-13, 283-89), K. Anders Ericsson and Herbert A. Simon (1984, 1-52), and Evelyn Jacob (1987).

The means of data collection is a way in which my work both bears a similarity to and differs from that of the Piagetian school. The Piagetian clinical interview consists of a set of questions that are asked of many subjects in a more or less uniform manner. The interviews that I conducted at the end of the project included elements that were consistent from individual to individual (Section 4.3), but also responded to directions initiated by different individuals. These responses depended, of course, on the relationship that had developed between each of the children and myself during the course of the project. I combined the interviews with several other ways of capturing information (see Section 4.4), all of which were contained within an overall setting that "situated" the learning and thinking about knots. (c.f. Suchman 1987) The children themselves created many of the projects that were to become sources of data for this report.

Piaget describes the difficulty of accessing children's deeply rooted beliefs:

> The form and functioning of thought are manifested every time the child comes into contact with other children or with an adult and constitute a form of social behaviour, observable from without. The content, on the contrary, may or may not be apparent and varies with the child and the things of which it is speaking. It is a system of intimate beliefs and it requires a special technique to bring it to the light of day. Above all it is a system of mental tendencies and predilections of which the child himself has never been consciously aware and of which he never speaks.
>
> ... it is not merely useful but essential ... to examine the methods to be employed in studying these beliefs. To judge the logic of children it is often enough simply to talk with them or to observe

> them among themselves. To arrive at their beliefs requires a special method which, it must be confessed outright, is not only difficult and tedious, but demands also an outlook, the fruit of at least one or two full years' training. Mental specialists, trained in clinical practice, will immediately appreciate the reason. In order to assess a child's statement at its true worth the most minute precautions are necessary. (Piaget [1929] 1951, 2)

Piaget goes on to describe the method by which he and associated researchers went about ascertaining the unconscious systems of beliefs in the children they studied. He calls the method "clinical," and contrasts it with other ways in which researchers might attempt to study thought. One other way would be through tests:

> ... that is to say, the method of posing questions so arranged as to satisfy the two following requirements: first, that the question and the conditions in which it is submitted remain the same for each child, second that each answer be related to a scale or schedule which serves as a standard of comparison both qualitative and quantitative. ... But for our particular purpose the test method has two important defects. Firstly, it does not allow a sufficient analysis of the results. ... The essential failure of the test method in the researches with which we are concerned, is that it falsifies the natural mental inclination of the subject or at least risks so doing. (Ibid., 3)

Thus, tests could not allow for the depth of analysis appropriate for psychological and epistemological research; they ignore the natural context of the child's thinking; and they risk the phenomenon of "begging the question" – that is, suggesting through the form of a question some particular avenue of response. Piaget uses the analog of the clinician in psychology as a technique that is preferable in its emphasis on the subjects' own patterns of thought and in its respect for the context of that thinking:

> The skill of the practitioner consists not in making him answer questions but in making him talk freely and thus encouraging the flow of his spontaneous tendencies instead of diverting it into the artificial

> channels of set question and answer. It consists in placing every symptom in its mental context rather than in abstracting it from its context. (Ibid., 4)

Despite the tone of this passage, which might lead us to question the assumptions about how the interview setup comes to exist at all – (Is it really a matter of the subject's volition?) – the next concern reveals a spirit of following rather than leading the subject. Piaget begins by criticizing what might be seen as the antithesis of the test method, the method of pure observation. What is needed is something between the two, in terms of the degree of the researcher's intervention:

> Observation must be at once the starting point of all research dealing with child thought and also the final control on the experiments it has inspired. In the case of the present research it is the observation of the spontaneous questions of children which furnishes data of the highest importance. The detailed study of the contents of these questions reveals the interests of children at different ages and reveals to us those questions which the child is revolving in its own mind and which might never have occurred to us, or which we should never have framed in such terms. Further, a study of the exact form of the questions indicates the child's implicit solutions, for almost every question contains its solution in the manner in which it is asked. ... We may thus state the first rule of our method. When a particular group of explanations by children is to be investigated, the questions we shall ask them will be determined in matter and in form, by the spontaneous questions actually asked by children of the same age or younger. It is also important, before drawing conclusions from the results of an investigation, to seek corroboration in a study of the spontaneous questions of children. (Ibid., 4-5)

Piaget outlines what he sees as systematic defects in the method of pure observation: "it is, in fact, impossible to observe a large number of children under similar conditions," and unless you talk with children it is difficult to gain any insight into what they are thinking, or to distinguish their play from their beliefs. Piaget describes an alternative between this and the test method:

> It is therefore essential to go beyond the method of pure observation and without falling into the pitfalls of the test method, to take full advantage of what may be gained from experiment. With this in view we shall use a third method which claims to unite what is most expedient in the methods of test and of direct observation, whilst avoiding their respective disadvantages: this is the method of clinical examination, used by psychiatrists as a means of diagnosis. ... The clinical examination is thus experimental in the sense that the experimenter sets himself a problem, makes hypotheses, adapts the conditions to them and finally controls each hypothesis by testing it against the reactions he stimulates in conversation. But the clinical examination is also dependent on direct observation, in the sense that the good practitioner lets himself be led, though always in control, and takes account of the whole mental context, instead of being the victim of "systematic error" as so often happens to the pure experimenter. ... it is so hard to find the middle course between systematisation due to preconceived ideas and incoherence due to the absence of any directing hypotheses! (Ibid., 7-9)

Central to this formulation is the idea that making hypotheses is an ongoing and frequent process. Willingness to *change* a hypothesis is crucial, and the method relies on conscientiousness in interpreting the information it yields:

> The psychologist must in fact make up for the uncertainties in the method of interrogation by sharpening the subtleties of his interpretation. ... The greatest enemies of the clinical method are those who unduly simplify the results of an interrogatory, those who either accept every answer the child makes as pure gold or those on the other hand who class all as dross. (Ibid., 9)

More recently, "post-Piagetian" writers have elaborated the complementary need for well-designed environments in which learning and research of the learning can take place. (Ackermann 1987, 1989, 1990; Papert 1980, 1984, 1987, 1990 "Unified")

Berg and Smith describe their conception of the *clinical method*, emphasizing the importance of the relationship between researcher and researched. It becomes clear that in such a situation, these roles are likely to be reversed from time to time during the course of the interactions:

> In spite of the development of a definition that characterizes clinical as almost exclusively therapeutic and distinct from research, most of the social sciences use the term to refer to specific kinds of research methods ... an approach to research rather than an application to a therapeutic situation. In our experience as well, the term *clinical* is regularly used to refer to an approach to the study of social systems, a method with its own characteristics and its own demands.
>
> We argue that all social research has its clinical aspects. However, these are ignored more often than not ... we have chosen to use the term clinical for those aspects of research that have the following characteristics:
> (1) direct involvement with and/or observation of human beings or social systems;
> (2) commitment to a process of self-scrutiny by the researcher as he or she conducts the research;
> (3) willingness to change theory or method in response to the research experience *during* the research itself;
> (4) description of social systems that is dense and thick and favors depth over breadth in any single undertaking; and
> (5) participation of the social system being studied, under the assumption that much of the information of interest is only accessible to or reportable by its members. (Berg and Smith 1985, 24-25)

Turkle borrows from Geertz (1973) in describing a similar sense of involvement with her research:

> The research is systematic: one informant's account of how something is done is checked and rechecked against the accounts of others; careful note is taken of what people do so that this can be compared with what they say they do. But at the same time, the very process of research is interpretive. In writing about ethnography, Clifford Geertz has stressed this fundamental fact: "what we call our data are really our own constructions of what other people are up to. ... Right down at the factual base, the hard rock, insofar as there is any, of the whole enterprise, we are already explicating; and worse, explicating explications. (Turkle 1984, 315)

Gilligan et al. also make use of a style of clinical interview and an interpretive approach that demands self-awareness on the part of the researcher:

> Our open-ended clinical interview yields complex real-life narratives that, by their nature, demand attention to context – situational, personal, cultural – and therefore, to perspective – ours as well as the respondent's. We wish, therefore, to provide a theoretical framework and a way of reading that highlights the interpretive nature of our work, and so present a method that claims a theoretical stance and guides the reader through an understanding of the ways in which it is manifest in interview texts. (Brown 1987, 2)

These researchers have worked toward developing and clarifying qualitative techniques so that it will become possible to get in touch with the "deep structures" of their subjects' thinking. (e.g., Lane 1970, 15) The interest is in attempting to understand such aspects as personality, style, and epistemological preference rather than "surface structures" like attitudes and abilities. The techniques also make it possible to capture the natural diversity in people's thinking. The information that emerges through the research setting is a function of the relationship between researcher and subject, rather than the relatively limited and rigid bounds of tests and questionnaires. Duration of the study becomes a key concern; the relationships, like the subjects' thinking, need to time to grow.

Qualitative research necessitates an interpretive manner of report, as I provide through this thesis. Characteristic of my approach were a certain wariness about the potential influence of my interventions, willingness to change hypotheses or courses of action as the project developed, and dedication to the task of developing a "thick description" (Geertz 1973) of the children's work and thinking, which this thesis represents. The workshop, "lab"-like atmosphere of the room that was the center of the project, as well as the project's extended duration, are hallmarks of the research setting which enabled the gathering of a much richer set of data than would otherwise have been possible. The children had time to work with knots, to think a lot about them, and to find many ways of making explicit what they were thinking.

In making this report, my concerns are multiple: I look both broadly and deeply at the results of the study. For breadth, I look at the knot-tying culture that the participants formed, at commonalities that emerged in their thinking about knots, and at differences in their many ways of thinking. For depth, I consider in detail the work of three participants who became particularly immersed in the project and whose involvement came to serve important culture-binding functions. The work and thinking of these three children are described in the case studies (Section 9).

3.1 Knot Microworlds

Papert (1980) has developed the term "microworld" to describe Logo software environments inhabited by the "turtle." In the microworld of "Turtle geometry," this animal-shaped graphic has the properties of *position* and *heading,* which characterize it as an object with which children (and others) can construct *angles,* the basis of a particular sort of mathematics. By setting a position and a heading and then setting the turtle in motion, children can construct complicated geometric images that have "artistic" qualities as well as "mathematical" ones.

The aesthetics of making such images provide a kind of holding power for many of the children. Many others get involved because they can bring to the environment the everyday knowledge of how their own bodies move in the world. By transferring this knowledge to the movement of the turtle, the children connect the two worlds, and the computer microworld takes on meaning. Seeing the movement of the turtle as being similar to their own movement is one way in which these children "identify" with the turtle; this "identification" provides another kind of holding power. Papert terms it "body syntonicity."

In microworlds that simulate Newtonian motion, the turtle takes on properties of movement that children can control and use in constructing understanding of physics that are contrary to intuitive experience and therefore often difficult to assimilate otherwise. In each of these environments, the turtle also has another important property: it is likable. It is a brightly colored representation of a creature that moves according to the instructions of the child. Naming it, making stories about it, creating friends for it, and otherwise playing with it are ways in which the children sustain their work with the mathematical and physical properties that the objects embody. These kinds of play can become ways of gaining access to the mathematical and the physical through the personal, another kind of "identification." For this reason, Papert has characterized the microworlds as being "ego syntonic."

Learning research that makes use of these microworlds considers access to them through another important means as well: the social context in which the microworlds are made available to the children. This broader view includes not just the computational environment, but its situation among groups of children, teachers, and parents, in homes, schools, and communities. As the Logo users exchange stories, project ideas, and information about how to move the turtle, they build a culture dedicated to learning: depending on the circumstances, such a culture is often, like the computational microworld, centered around a set of ideas that pertain to certain mathematical, physical, and aesthetic principles. In this sense, the turtle microworlds can be syntonic with culture as well as with the body and the ego.

Microworlds of knots can make a similar offer. By twisting and turning with a piece of string, and by seeing and feeling the relationships of different parts of a knot, many children adopt a "body syntonic" approach that enables them to construct vivid understandings of what is crucial about those relationships (e.g., Sections 5.4.2, 6.22.7, and 10.1). By creating stories about the knots and imagining them to be like friends and family, children find ways of becoming personally involved with the world of knots (e.g., Sections 6.21, 7.2.11, and 9.1.4). And by asking one another questions about knots, writing letters to each other about them, and displaying constructions involving knots for others to see, the children developed a culture actively involved with both topological and social senses of proximity, separation, and connection.

In this thesis, I discuss the children's thinking about knots in terms of three "microworlds": the complex object that is each individual knot, the ways in which groups of knots can be seen as being related, and the environment that the children and I developed as a context for learning about knots. Each knot is, in a sense, its own universe, which invites contemplation of its topology both as it is being formed and as a completed object. Ways in which children explore this microworld are presented in various sections (including 5, 6, and 7.1). Additionally, different knots are often quite similar, so that understanding something fundamental about one can lead to an understanding of another. Ways in which children came to regard certain knots as belonging together are also discussed in various sections (especially 6.22.8, 7.2, and 9.1.4). This conceptual work was situated in a social and physical environment that developed during many weeks' time and which is discussed throughout this thesis. Particulars of that environment constituted important aspects of the methodology for the study, and are discussed in the next section.

II The Thinking Environment

Following a picture, a child carefully intertwines a piece of string and produces a knot. She checks it against the image in the book to see if the knot she holds is the one she attempted to tie. In so doing, she has set up for herself the fundamental problem of the knot theorist: When are two knots the same? For Maria and her knot-tying peers, there are many instances in which this classic question can be translated as: How can you come to recognize a new situation as a transformation of something you already know?

The Piagetian research tradition is known for its emphasis on the selection and design of settings and scenarios in which a certain line of exploration and questioning will yield information about children's understandings of a specific topic. More recently, "post-Piagetian" writers have acknowledged the importance of social and cultural factors involved in learning, by extending the discussion of design to entire environments that consist of people, projects, and places in which the activity happens. (Ackermann 1987, 1989, 1990; Papert 1980, 1984, 1987, 1990 "Unified") Such environments grow through extended periods of time, making it possible to examine cultural influences on the construction of ideas and the subjects' personal involvement with the ideas.

These environments have come to be called "learning environments" or, acknowledging the researcher's participation and the double purpose of the work, "environments for learning and research." (Ackermann 1987) In a further attempt to emphasize the participants' examination of their own learning, which constitutes what we might consider as another, special form of research, I have dubbed the Knot Lab a "thinking environment." In this environment, we looked at each others' active involvement with knots as well as ways of representing knowledge about knots. The following sections describe the setting in which the work occurred, the participants in the project, the frequency of meetings, and the nature of the work.

The idea of *relationship* is of particular importance in characterizing this "thinking environment." The idea emerges in several ways: My relationship to the children was not so much as an authority figure as someone who facilitated the project by introducing and furthering ideas, obtaining materials, and coordinating meetings and communication among the participants. Many of them came to understand that their relationship to me could be as a friend and fellow thinker. Secondly, the participants' relationships with each other grew and deepened through the course of the project. Some of the children already knew each other as the project began, but many of the participants were less familiar or were strangers. Therefore, much of the work had implicitly to do with getting to know other people through getting to know more about knots with them. Thirdly, the participants had different relationships to knots and the ideas they embody:[1] there was tremendous variance in degrees of excitement about the project and in what aspects of knots different people found interesting. Finally, "relationships" are what constitute knots. To understand a knot is to become familiar with how different parts of the string stand in relation to each other, in producing the knot and in regarding the finished object.

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[1] For a further, theoretical discussion of some of these relationships, see Section IV.

4 Welcome to the Knot Lab

The KNOT LAB THE WILD KNOT'S THE Funky cold KNOTS

– Marcos, José, and Julio, on their sign at the entrance to the Knot Lab

Imagine that you are in an inner-city elementary school – a gray building with graffiti on the walls, long hallways populated by lines of children, distorted sounds coming from the public-address system, people chattering, computers humming. Amid the noise is the clutter of busy classrooms and senses of urgency alternating with times of thoughtful quiet. One of the doors is decorated with a sign that boldly announces the room as "The KNOT LAB." You enter, and find children playing with string, writing letters, watching videos, climbing to tack knots on display boards, arguing over the use of the video camera, working together on stories about knots, and otherwise immersed in activities related to their study of knots. They are surrounded by their own constructions: three of the four walls boast large, colorful displays showing knots in various stages of formation, dangling from tree branches, tacked to accompanying pieces of writing, sewn on cardboard, and drawn on various pieces of paper. Books and other printed materials about knots are strewn about, and pieces of string are everywhere. You have entered a "thinking environment" dedicated to learning about knots and to reflecting on that learning.

What you do not see are aspects of the environment that are equally important, but reside elsewhere. Miles away, an older child waits for the end of his school day so he can meet with me in a video editing room where we will answer letters, in written and video form, which the younger knot-tyers have entrusted me to give to him. I am the courier; he is the "knot expert." A Boy Scout and now, to some, a TV star, he takes his responsibility seriously and tries to answer the detailed questions that the other children pose. Eventually they will all meet, when he comes to the "Knot Fair" that culminates the project.

Three of the boys captured both the serious and playful aspects of the environment in their construction of a “welcome” sign for the front door. It displays pieces of string that have been tied in the forms of various knots, as well as pictures of a chemist's flask and a party hat and balloons. The combination playground/workshop, "lab"-like atmosphere of the room that was the center of the project, and the project's relatively long duration, are hallmarks of the research setting that enabled the gathering of a much richer set of data than would otherwise have been possible. The children had time to think deeply about knots, and to find many ways of making explicit what they were thinking. What follows is a more detailed account of how the setting evolved.

4.1 Experimental Setting

I was concerned with developing conditions for both learning and research in two ways: through consideration of the overall environment in which the study was conducted, and through examination of particular work or inquiries in which the participants were engaged. These focuses evolved through my instigation of ideas, as well as ongoing adjustments based on the participants' responses and suggestions. Thus the research design is best characterized in terms of malleability, rather than the imposition of any pre-determined structure. The setting had to be flexible enough for the project to evolve in response to ideas and events that occurred during its course. I presented an initial context for learning about knots, which we gradually modified as the children became immersed in the project.

For each of the four groups of children involved, the project began with a discussion of knots and an invitation to join the working sessions in which they could learn more about knots. I made it clear that participation was voluntary and that they could stop at any time if they lost interest or didn't like the project. We also talked about the project as furthering *research* into how people think about knots. Participants would be researchers and well as learners – researchers into their own thinking, and that of others. I described the Boy Scout who wanted to be their "pen pal" through exchanges of videotapes. Soo Yong Chang would demonstrate how to tie various knots and respond to questions from the other participants as the project progressed.

Soo Yong was an older child who was himself in the process of learning to tie certain knots. I assumed the roles of facilitator and "courier," the person common to each end of the communication, who videotaped the sessions and arranged for the children and Soo Yong to see each others' video mail. Videotaping the exchanges between the children provided a means of recording visual and aural data, and also stimulated the children's excitement about the project. They enjoyed "seeing themselves on TV." The inevitable self-consciousness that resulted among the participants supported a theme of becoming aware of one's own thinking processes so that they can be made available to others – in this sense, the children were also researchers, and several came to think of themselves in this way. They were learning about knots, but about thinking as well – they were becoming epistemologists. (Ackermann 1987)

This "video correspondence" served a kick-off purpose but did not define the scope of the project, which quickly took on a life of its own. Most dramatic in its evolution were gradual shifts from an emphasis on video as an instruction and communication medium, to the children's initiation of their own activities and greater use of paper correspondence. The children worked individually or in teams, initially within four separate working groups. As the end of the school year approached, distinct boundaries between these groups relaxed and increasingly frequent but casual merging of the groups occurred. With this change came increases in the incidence of borrowing of ideas and of collaboration between members of initially different working groups. The children were building not only understandings of certain knots, but a culture dedicated to learning about knots and thinking.

Evidence of the children's appropriation of the project and the knots accumulated visibly during the three months. I responded to their suggestions by bringing to the environment references and materials with which they could further their ideas. Eventually they voted on a name for the room in which they met. It came to be known as the "Knot Lab," and was adorned by a wealth of printed and video information about knots, as well as bulletin board displays that the children constructed, showing knots in different stages of being tied, stories about the knots, a "family tree" of knots, and special words and graphic devices for describing knots.

Three important elements of the research occurred through the working sessions: they formed a period of culture-building and of immersion in thinking about the knots, so that discussions in the form of the "final interviews" (Section 4.3) fit within a context that all the children shared; the working sessions, in their own right, generated data on thinking about knots; and in the course of these sessions, many of the children built up a relationship with me that came to involve comfort and trust. Gradually throughout the project, and in the final interviews, many of the children were willing to show and explain to me what they thought. We had taken an approach that assumes that people will have different ways of thinking about aspects of knots, and which values these differences. Although many of the children seemed hesitant at first, they came to accept that this approach was genuine – that they weren't going to be told they were wrong or stupid if they risked articulating what they thought.

The Knot Lab, as a center of this approach, became a place where the children began having dialogues and debates about different ways to think about knots (and eventually, other issues in life, too). Many children engaged in "dialogues" with themselves – that is, they developed a form of critical thinking in which they would launch an interpretation of a knot and then retract or modify it as they continued the exploration. It was not unusual for a child to arrive at an understanding very different from the one she had started with, and to describe the initial interpretation as being "wrong." This form of self-critique was refreshing for its lack of the punitive overtones that can stem from incorporation of voices of authority who emphasize mistakes as being problematic.

4.2 Participants, Schedule, and Activities

Twenty-two children were involved in the study. Soo Yong, a 14-year-old Boy Scout, played a special role as the remote "knot expert." He and his family had come to the United States from Korea three years prior to the project. Four teachers in an elementary school associated with the M.I.T. Epistemology and Learning Group suggested children in their homeroom classes to participate in the project.[2] I had emphasized to the children that their participation was voluntary, and Patish, whose family was from India, decided to drop out after two sessions. The remaining twenty Knot Lab participants included ten girls

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[2] Their reasons for suggesting the children they did varied: Therese suggested children who knew each other well (many of them, since kindergarten) and worked well together; Ed suggested children he thought were doing well enough in their other work that they wouldn't be set back by missing class time to work on knots; and Kate suggested children who would comprise a varied group but wouldn't "kill each other." Al was absent on the day when the project was scheduled to begin for children in his class; the three who were finished their other work when I entered their classroom at the scheduled time were the ones who joined the knot project (see Section 9).

and ten boys. Seven of these children were Hispanic (three girls and four boys), seven were African-American (four girls and three boys), and six were Caucasian (three girls and three boys). The children comprised four working groups:[3]

Al's group	Ed's group	Kate's group	Therese's group
Alice 10;6	Celina 11;10	Althea 10;6	José 10;11
Jill 11;9	Curtis 10;1	Doreen 10;5	Juanita 13;10
Tony 11;11	Eugene 10;6	Jack 10;2	Julio 10;6
	Pablo (~11)	Leroy 11;7	Marcos 10;4
	Tiffany 10;11	Patish (~10)	Maria 10;6
		Patrick 11;10	Rosella 10;5
		Stacy 10;7	

The research project was conducted from February through July.[4] The most intensive activity in the Knot Lab occurred during the latter part of March, April, May, and the early part of June. Soo Yong was actively involved from mid-February through mid-June, during which time we had approximately seventeen meetings to practice knots, discuss the other children's work, and prepare video and paper correspondence; he also attended the field trip to the sailing dock and the Knot Fair. In mid-July, he assisted with knot demonstrations at an M.I.T. summer workshop for teachers.

The other children had regular weekly meetings in the Knot Lab from mid-March through mid-June. As the project neared its end, many of the children attended more frequently, either by casually joining another group for a session in the Knot Lab or by arranging additional times to meet with me there.[5] Aside from the one child who dropped out of the project after two sessions, the remaining twenty children participated in at least

[3] Pseudonyms have been substituted for the actual names of the participants.

[4] The pilot studies were conducted during the preceding year. Although many adults and children participated in this preliminary work, I include material from only two of the children here; for purposes of this report, I call them Ellen and Pete (see Sections 2.1.2 and 6.17).

[5] As we approached the end of the school year, some of the teachers were more relaxed about allowing children to miss certain classes in order to work in the Knot Lab. One of the teachers (Al) was retiring in June and became the least stringent about such matters. Also, several children traded their lunch and recess times for time in the Knot Lab.

one group work session for ten weeks, in an individual final interview averaging about fifteen minutes' duration,[6] and in the Knot Fair at the end of the project. Most of the children participated in more sessions: Al's group did twenty, Kate's did twelve, and Ed's did eleven plus the field trip to the sailing dock. Individual kids from each of the four groups came to the Knot Lab at additional times. So, for each of the twenty children who participated during the entire project, the number of working meetings ranged from ten to twenty within the three-month period. In addition, four children came to M.I.T. several weeks after the project had ended, to demonstrate some of what they had learned for teachers at a summer workshop.

A number of the children became so involved in the project and produced work of such noteworthiness that I considered developing detailed reports and analyses of their thinking about knots, in the form of case studies. However, in going through the information generated by the project, I decided to focus on case studies of the three children in Al's group – Alice, Jill, and Tony. The decision was based on their more frequent attendance, and the consequent facts that I had come to know them better, they had constructed more work, and, as they were in the same group, their work often dovetailed in ways that tended to highlight peculiar characteristics of the thinking of one child or another. The information about them tended to be richer and sharper, obviously an advantage for a report such as mine.[7]

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[6] One of these interviews lasted as little as five minutes, and some lasted more than half an hour. The duration was sometimes a function of the child's interest and desire to continue, and sometimes of available time. Most of the interviews, though, lasted about fifteen minutes.

[7] Although these children were in an "Advanced Work Class," the deciding factor in elaborating on their work here had to do with the circumstances that enabled them to spend a good deal of time in the Lab. As I was studying the development of topological understandings, seeing a good deal of an individual's work was tremendously beneficial – through regarding a relatively large volume of work, as well as through getting to know a child well, I was better able to understand how the work reflected changes in the individual's thinking. As noted, several of the children in the project had become interested and involved enough that, had they been able to spend more time in the lab, they would have been good candidates for case studies in this report.

My decision also presented a disadvantage, however: each of these children is Caucasian. Although they come from different socio-economic backgrounds and family situations, many of their cultural norms and expectations would likely differ from analogous ones for the African-American or Hispanic children. As a "self-conscious" researcher, I am obliged to point out (to the reader and to myself) that my racial and cultural similarity to these three children would likely have affected our interactions.

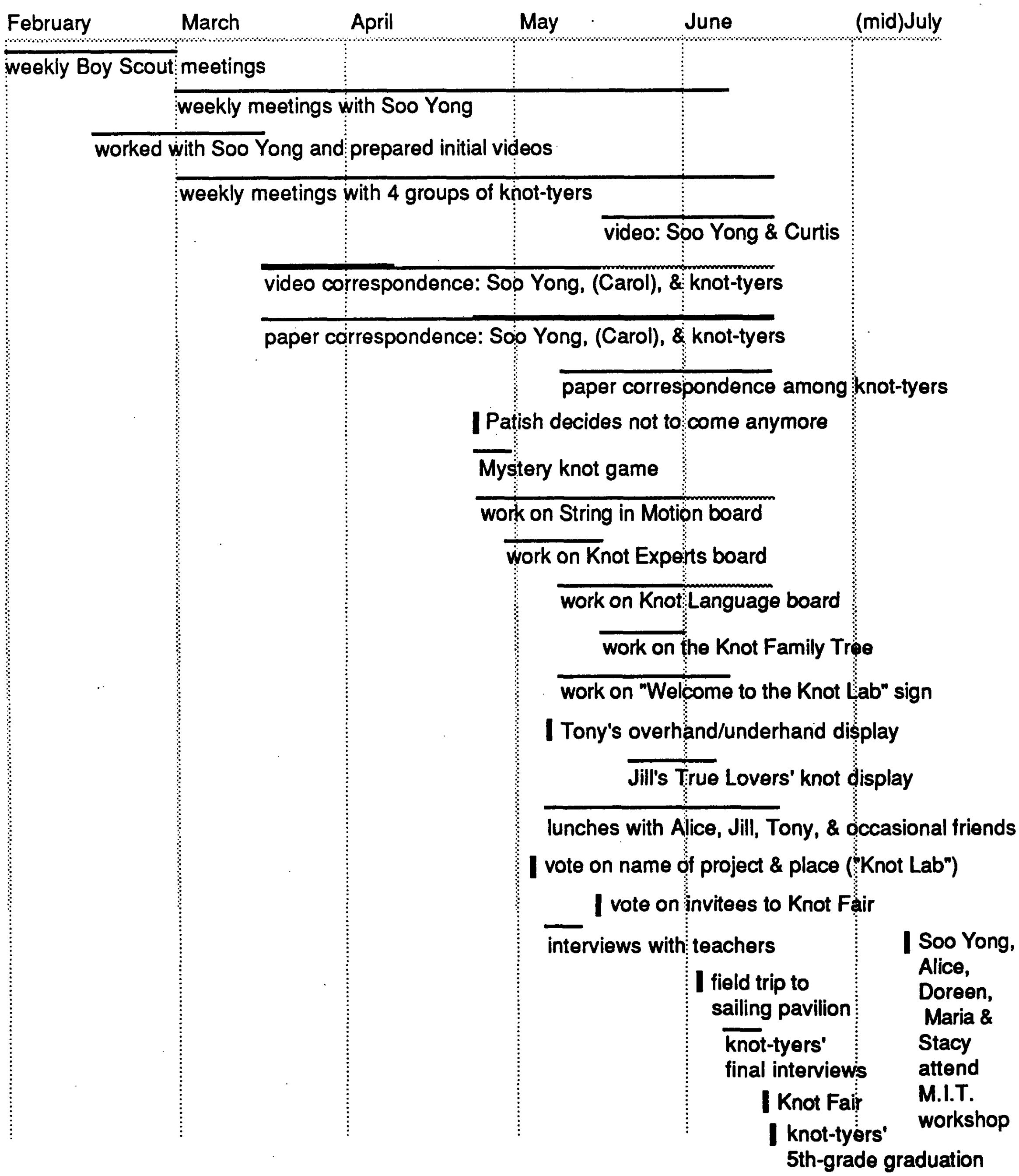
However, I do see the disadvantage mainly as a function of the larger setting in which the research project was conducted: within the confines of the schedules and customs of an inner-city elementary school, it is difficult to interweave a project such as the one I was organizing, loose-ended as it was in terms of time and commitment by the children, and unrelated as it seemed to anything in the curriculum. That some of the children were able to find extra time to spend in the lab was not only testament to their enjoyment and appropriation of the project, but a boon to the developing "knot culture" and environment. Striving for depth of individual analysis, the duration of the participants' involvement is directly related to the quality of the report. The more time the children could spend working in the Knot Lab, the more insightful and useful the results could be.

The schedule of meeting times is as follows:

2 February	initial meeting with Boy Scout troop
9 February	meeting with Boy Scout troop
16 February	videotaped knot-tying sessions at Boy Scout meeting (8mm)
23 February	videotaped knot-tying sessions at Boy Scout meeting (8mm)
3 March	videotaped Soo Yong practicing knots (8mm)
17 March	prepared video letter with Soo Yong (VHS here and subsequently)
20 March	Ed's, Therese's, and Al's groups
22 March	Kate's group
24, 25 March	Soo Yong
27 March	Ed's, Therese's, and Al's groups
31 March	Kate's group; Soo Yong

3 April	Ed's, Therese's, and Al's groups
7 April	Kate's group; Soo Yong
10 April	Ed's, Therese's, and Al's groups
14 April	Kate's group; Soo Yong
24 April	Ed's, Therese's, and Al's groups
28 April	Kate's group (Patish decided not to come any more); Soo Yong
30 April	Soo Yong
1 May	Ed's, Therese's, and Al's groups
2 May	interview Therese
3 May	interview Ed, Al, and Kate
5 May	Al's and Kate's groups; Soo Yong visited the school
8 May	Ed's, Therese's, and Al's groups
12 May	Kate's group; Soo Yong
15 May	Ed's, Therese's, and Al's groups
19 May	Kate's group; Soo Yong
22 May	Ed's, Therese's, and Al's groups (Stacy with Al's)
23 May	Al's group
24 May	Al's and Ed's groups
26 May	Kate's group (without Stacy); Soo Yong
31 May	Ed's group's field trip to the sailing pavilion
1 June	Al's and Kate's groups
2 June	Kate's, Al's, Ed's groups; Pablo's interview; Soo Yong
5 June	Ed's and Al's groups; José and Marcos
6 June	Al's and Kate's groups (partial)
12-14 June	final interviews
15 June	Knot Fair
16 June	(fifth-grade graduation)
mid-July	Soo Yong and Alice, Doreen, Maria, and Stacy attend an M.I.T. workshop

This chart shows the development of increased activity toward the end of the project:

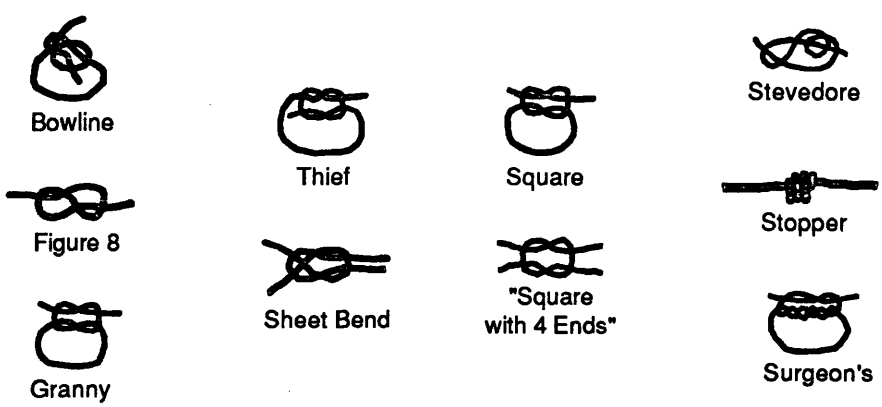


4.3 Final Interviews

At the end of the study, I had an individual meeting with each child. We conversed while the child tied various knots, compared two similar knots, and arranged a set of knots into groups, according to perceived similarities. These comparative techniques were useful in eliciting understandings of relationships among parts of the configurations.[8]

The last part of the project for each participant was a conversation about how to tie and think about certain knots. The immersion in Knot Lab projects that preceded the interviews had the effect of preparing the children for these detailed discussions. A stationary camera, positioned behind the child and myself, "looked over our shoulders" at the knot-tying activity while recording picture and sound.

The children worked with a set of knots that included a Granny, a Stopper, a Figure 8, a Stevedore, a Surgeon's, a Square and a "Square with 4 Ends", a Thief, a Sheet Bend, and a Bowline:



[8] I present analyses of these discussions primarily in Section 7.

Although this basic set varied slightly from interview to interview as different children tied new knots or changed existing ones in the course of our discussions, some version of each of these knots was included in each interview.

The interviews were geared toward three tacks of inquiry: tying at least one of the knots, grouping the knots into perceived "family resemblances," and comparing the Square and Thief knots. At some point in each interview, I explicitly asked the child to do and think about these things. However, I conducted the interviews in the form of a discussion with each child, so the sequence of these activities and the amount of time spent on each varied from child to child. Also, the varied and additional components of each interview sometimes formed a significant category of thinking about knots, which contributed in some way to this report: the comparisons of the Bowline and the Sheet Bend are an example.

In comparing the Square and Thief knots, several children acted on the suggestion that they imagine themselves to be a small ant crawling along the surface of the knot. This is a technique that Piaget and Inhelder had used in their studies of younger children working with simpler knots. It proved helpful in assisting these older children to imagine a change of scale (which was both spatial and temporal) in which portions of the knot could be considered separately, making more manageable the problem of finding one's way through the complicated configurations. Without this change of context, many of the children would have found difficulty in describing differences between the Square and the Thief.

It is important to note that without the children's prior three months of involvement in the knot project, the nature of the final interviews would have been quite different. By the time of the interviews, there was nothing strange about us sitting together and talking about knots that were strewn across the table. Playing with them, thinking about them, and talking about them had become familiar pastimes. Further, we were sitting in a room that by this time had been virtually taken over by knots: they were on various walls, pieces of furniture, and the floor; knots and string spilled out of various supply drawers. The knots that surrounded us were knots that the children had either tied or examined or written about, or with which they had been involved in some way. Further still, we had gotten to know each other. Of course the degree of comfort that the children felt with me, the knots, and the environment varied from individual to individual, but by this time the children had become accustomed to my questions and curiosities about knots, and the discussions that I have come to call the "final interviews" were not very different from discussions we had had all along.

In addition to tying, comparing, and grouping knots during these interviews, other topics tended to come up as the children were reminded of different things during the course of the conversation. For example, Juanita had not well understood the idea of the Family Tree, and by way of explanation we spent a while developing a genealogical chart of her family. Julio was worried that he was going to fail the fifth grade, and we talked about this concern for a while. Similar side-discussions were woven into the interviews with other children.

4.4 Collecting the Data

I collected data through note-taking, audiotaping, videotaping, and tangible projects that the children produced. Sources of information included conversations with the children (and with their teachers); the children's video and paper correspondence with Soo Yong; their written descriptions of knots, illustrated stories, instructions, etc.; and the bulletin-board displays that they constructed.

Video was a key element of the "Knot Lab" project. The idea of the "video pen pal" was one that the children accepted and played with immediately. They looked forward to Soo Yong's replies to their video mail: not only did they think of him as something of a celebrity, but his responses on video incorporated images of the children, so they could see both Soo Yong and themselves on TV. The most important role of the medium, in fact, must have been its effect of bringing the children into the project – immediately, and with enthusiasm.

We might well imagine a discussion of how helpful the videotapes were didactically – it is true, for example, that several of the children spent large amounts of time watching and listening to Soo Yong's tapes, repeating certain descriptions of knots over and over, slowing down certain parts so they could pay especially close attention, and so on. We might also imagine an elaboration of how useful the videotapes were in recording data or in providing a backup to data recorded in other ways, as in fact happened. But most important to the project was the medium's role in the design and evolution of the research situation.

The "video correspondence" launched the project and provided a way for the kids to get involved quickly. It also encouraged communication about knots, and a degree of self-consciousness not just about one's own appearance, but about how to describe knots and issues related to them. This aspect of self-reflection should not be underestimated. The video correspondence also established the presence of the camera as an everyday element of the research situation, so that its use as data-collecting device became relatively unobtrusive.

4.5 Analyzing and Reporting on the Data

A large part of the data analysis involved the development of illustrated protocols from transcripts of the videos and from my researcher's notes.[9] These videos and notes were concerned with both working sessions and final interviews. Excerpts from some of the transcripts are included in various parts of this thesis, which constitutes a report of the project. Data from the transcripts are coordinated with information from the children's writing and project work.

Several important questions emerged as I looked at the data: How did the children describe knots and tying maneuvers? What difficulties did they encounter? What strategies did they use for getting out of difficult situations? What knots did they perceive as being similar or related, and why? And finally, why was it that so many children became so interested in knots?!

These questions comprise the bases of my report. I look both broadly, at information from all twenty of the children who participated, and in depth, through case studies of the three children who became most immersed in the project. Sections 5, 6, 7, and 8 present the "epistemological" data and analyses, organizing them according to the children's ways of communicating about knots, comparing individual knots, clustering groups of knots, and noteworthy instances of ways of thinking about knots. In Section 9, case studies provide in-depth "psychological" descriptions and interpretations of the work and thought of three children. Section 10 uses a similar approach to include the work of children I did not come to know as well, but who were clearly involved with the project in personally meaningful ways.

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[9] Sally Araki and Mark Chakerian sorted through many of my scribbles and scrawls in assisting to develop these protocols.

III Epistemological Threads

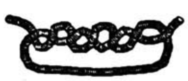
> Straight rational thinking is handy when dealing with planar surfaces, but knots are about relationships.
>
> – Brion Toss (author and producer of texts and videos about knot-tying), personal correspondence dated 20 March 1989

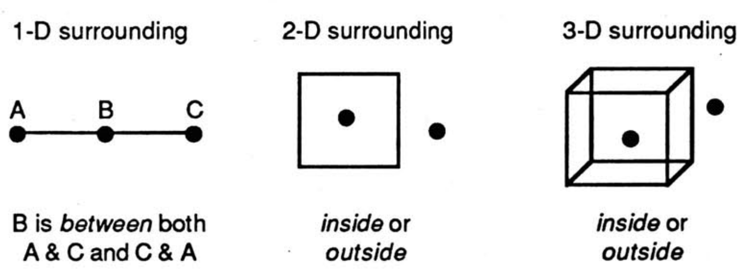
Here I present and analyze data from the study in two main sections, organized according to whether the interpretive approach has to do more with epistemological or psychological kinds of questions. Earlier we saw that Piaget distinguished these interests in terms of information pertaining to the "object" being considered or to the "subject" who is thinking about the object (Section 2). This distinction could be one way of looking at the organization of material here, as the "epistemological threads" have to do with knots and children's ways of thinking about them, and the "psychological threads" describe three particular children, partially through their thinking about knots. Papert offers a further explanation of the distinction:

> Piaget has described himself as an epistemologist. What does he mean by that? When he talks about the developing child, he is really talking as much about the development of knowledge. This statement leads us to a contrast between epistemological and psychological ways of understanding learning. In the psychological perspective, the focus is on the laws that govern the learner rather than on what is being learned. Behaviorists study reinforcement schedules, motivation theorists study drive, gestalt theorists study good form. For Piaget, the separation

between the learning process and what is being learned is a mistake. To understand how a child learns number, we have to study number. And we have to study number in a particular way: We have to study the structure of number, a mathematically serious undertaking. This is why it is not at all unusual to find Piaget referring in one and the same paragraph to the behavior of small children and to the concerns of theoretical mathematicians. (Ibid., 158)

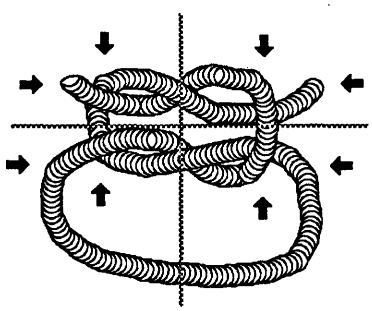
We have seen how this approach led to a formulation of the mathematical structures of *algebra, order,* and *topology* (Section 1.4). Elaborating on the structures of topology, Piaget and Inhelder include *proximity, separation, order, enclosure, continuity* (Piaget and Inhelder [1948, 1956] 1967). Consistent with Papert's mention of overlap among the structures of mathematics, they describe *order* as belonging to the realm of *number* (akin to the Bourbaki structure of *algebra*) as well, a realm that is also shaped by the structure of *classification* (Piaget [1941, 1952] 1965). But more particularly, the topological concepts of linear and circular *order*, they say, belong to the relationships of *surrounding*, of which the concept of *between* is an instance (Piaget and Inhelder [1948, 1956] 1967). The relationships of *inside* and *outside* are others:[1]


IV



[1] See the "Advice to the Reader" section for an explanation of this pictorial "repeat" symbol, which indicates that the illustration also occurs in Section IV.

Thinking about knots in terms of the relationships among their different parts is a key to understanding them:



Such relationships can be discerned in the completed object or through the historical process of its creation. A knot-tyer thinking about a knot may ask such questions as: "What is close to what?" "What is over what?" "Why is this part over that part, while what seems to be a comparable part is *under* something else?" Individual differences emerge through the implicit or explicit questions that different knot-tyers choose to ask, the topological relationships that strike them as being important, and how they may represent the thoughts that they settle on.

Diversity is a major theme of this section. It surfaced in many ways: through children's ways of discussing and describing knots, their ways of figuring out problems related to knots, and their perceptions of relationships in and among knots. Another theme concerns the children's reflections on their own learning – but of course, when one's own thinking becomes the object of epistemological inquiry, we begin to enter the realm of psychology.

5 Rudiments of a Language for Describing Knots

KNOT LANGUAGE
By: Patrick Gilmor

Knot Language is sometimes very hard to understand. To see if you talk the right way in Knot Language Try to explain to somebody how to tie a knot with their eyes closed. It probably won't come out right the First time because you said something Wrong.

Children in the Knot Lab struggled with making their ideas understandable to one another. In the process, they often disagreed or conversed at length about how to designate aspects of knots and tying processes. Their discussions and demonstrations relied on ways of describing the "anatomy" and spatial orientations of different knots, as well as on the usages of particular words, gestures, and pictures to convey ideas about knots and their processes of formation.

This externalized evidence of how the children were thinking can be considered as forms of "knowledge representation" in the sense that artificial-intelligence researchers Ronald J. Brachman and Hector J. Levesque explain it:

> The notion of the representation of knowledge is at heart an easy one to understand. It simply has to do with writing down, in some language or communicative medium, descriptions or pictures that correspond in some salient way to the world or a state of the world. In Artificial Intelligence (AI), we are concerned with writing down descriptions of the world in such a way that an intelligent machine can come to new conclusions about its environment by formally manipulating these descriptions.

> Despite this apparent simplicity in general goal, the research area of Knowledge Representation has a long, complex, and as yet non-convergent history. Despite the fact that just about every current AI program has what is called a "knowledge base" containing symbolic descriptions represented in some "representation scheme," there is still a vast amount to be understood before, for example, the knowledge in one system can be shared with another. There are tremendous subtleties in the notions of "representation," "knowledge," and their combination in AI. (Brachman and Levesque 1985, xiii)

Like Brachman and Levesque, I am concerned with the subtleties involved in making exterior certain ideas in order to communicate them and render them in some workable form. But I am also concerned with questions of what the natures of *internal* representations may be: how does someone *think* about a knot? And how do these internal representations get translated as the person tries to make the ideas undertandable to others? External representations can be considered as a kind of "snapshot" of given moments of the thinking process, in this case excerpted from a rich and ongoing "dialogue" between the knot and the thinker. In order to acknowledge these differences, Ackermann has suggested the term "knowledge in action" as an alternative to "knowledge representation."[1]

Such are the concerns I bring to my report of the children's "language" for representing their thinking about knots.

---

[1] Others, including Jeanne Bamberger, a musician and researcher who works with Logo, also tend to prefer this designation.

5.1 Knots Encountered in the Knot Lab

> Knots
>
> Knots are used for many different things. They are used for tying shoelaces, they are used for boats, and many other things. Different knots are used for different things. A regular knot is used to tie a jumprope or keep a door shut. Knots are important things that help hold different parts together. In the knot movie I'm going to make different kinds of knots, even regular knots. Knots are interesting things.
>
> -Pablo

Dozens of knots found their home in the Knot Lab. As Soo Yong showed new knots in the videos, he and I found and photocopied pictures of them for inclusion in our own book of knots. Known as "Soo Yong's Knots," it supplemented other books and visuals that could be found among the many materials in the lab.[1] The children worked from all of these pictures, as well as from the videos and the displays their peers had created.

The knots that were most popular among the children are pictured on the following pages. They include simple knots such as the Overhand, Figure 8, and Stopper; variations on a theme, as in the Bowlines, Hitches, and Heaving Line knot; Square knots and variants such as the Granny and Thief; the Stevedore; the Packer's; knots such as the Sheet Bend and Surgeon's, which effectively combine aspects of other knots; movable knots such as

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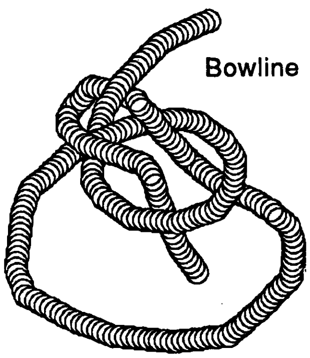
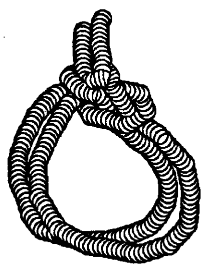
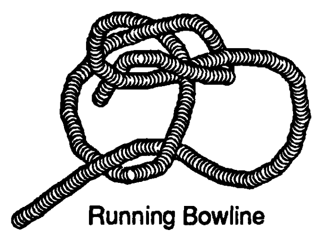
1 Most often used were "Knots and How to Tie Them" (Boy Scouts of America 1978), the ***Official Boy Scout Handbook*** (Hillcourt 1979), ***Sailing Knots*** (Altimiras 1984), "Ropefolk" (Grainger 1985), and the MIT "Sailing" booklet (Bisbee, Halloran, and Larkin 1981).

the Running Bowline, Trumpet, and True Lovers'; and more complicated knots such as the Monkey's Fist and Turk's Head.[2]

These are by no means the limit of what the children explored: additionally, several of them worked on macramé constructions such as jewelry and chains that they called the "Chinese Staircase" and "Zipper," braids of various kinds, the Hangman's Noose, variants of several of these knots, and even sewn patterns.


Bowline


Bowline on a Bight


Running Bowline

[2] In common parlance, there are a number of names for several of these knots. In this report, with one notable exception, I use the particular names by which the knots became known in the Knot Lab. The exception is the Thief knot, which the children actually referred to as the "Reef" knot. "Reef" is another common name for the Square knot. The Thief knot, though, was introduced in different ways at different times during the course of the project; it became, in some ways, more of a curiosity than the other knots. The children knew it originally through a game that challenged them to explain its differences from the Granny and Square knots. In this game, the knot was called the "Mystery" knot. Later, in a video, Soo Yong referred to it as the "Thief" knot – but some of the children must have misheard his pronunciation, and they were familiar with the term "Reef" from paging through different books. For whatever reasons, this knot became known as the "Reef" knot, a name that stuck through most of the project. Incidentally, there is an interesting story about how the knot was used on old seafaring vessels: Supposedly, ship captains would use it to close the lid of the box in which he kept the bread supply. For this reason, the knot is also sometimes called the "Bread" knot. Easily mistaken for a Square knot, it was a good indicator of whether someone had been pilfering the bread: thieves would most likely re-close the box with a Square knot, and the captain would thus know that someone had tampered with the box. Thus the knot is also called the "Thief" knot.

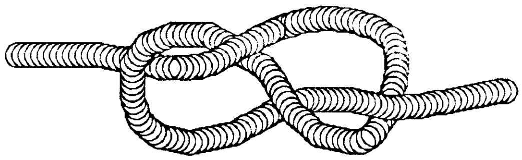
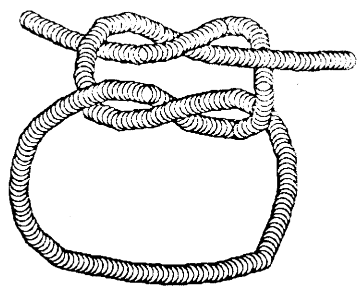
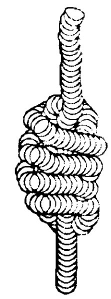


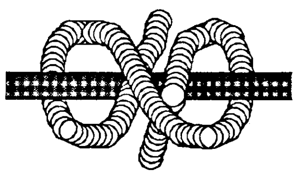
Figure 8



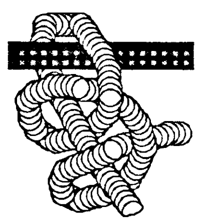
Granny



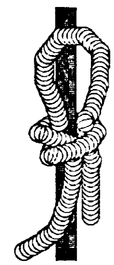
Heaving Line



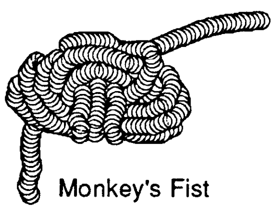
Clove Hitch



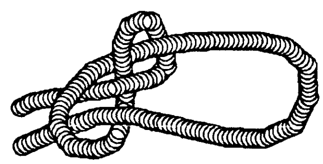
2 Half Hitches



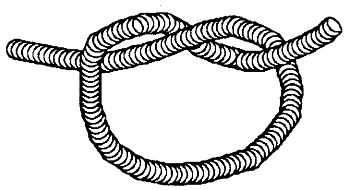
Slippery
Hitch



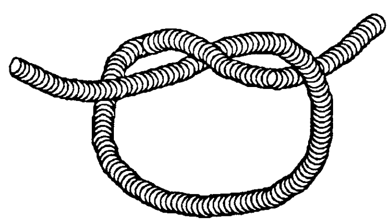
Monkey's Fist



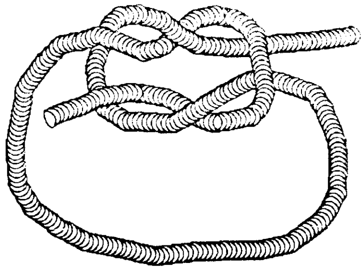
Packer's



"Pretzel" or Overhand

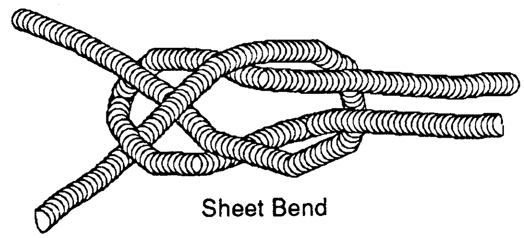


"Pretzel" or "Underhand"

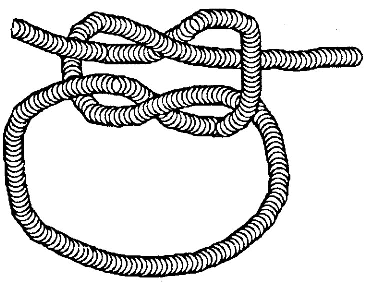


Thief

(called "Reef" knot and "Mystery" knot in the Knot Lab; actual names are "Bread" knot and Thief" knot)

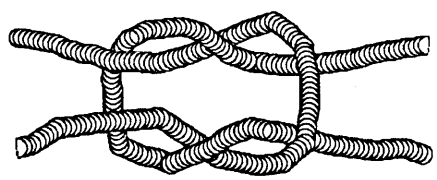


Sheet Bend

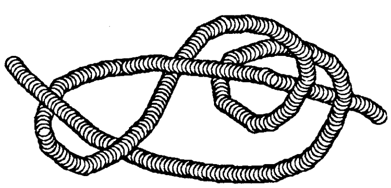


Square

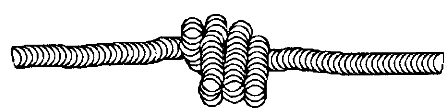
(another name in common parlance is "Reef" knot)



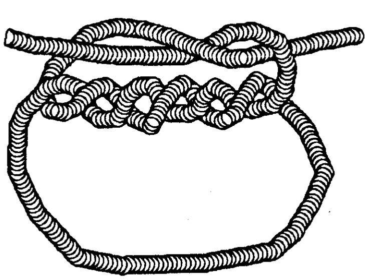
"Square with 4 ends"



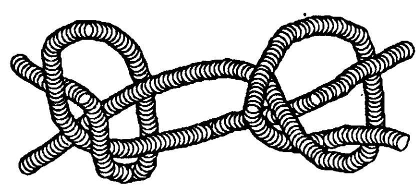
Stevedore



Stopper

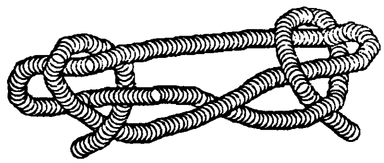


Surgeon's



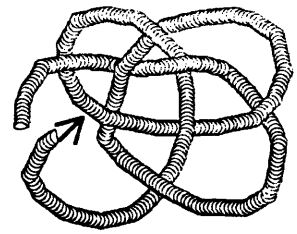
True Lovers'

(another name in common parlance is "Fisherman's" knot)



Trumpet

(similar to the"Sheepshank")



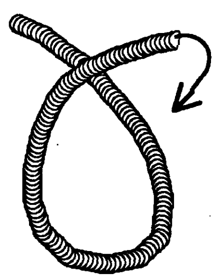
Turk's Head

(the pattern continues until there are three strands in each quadrant)

5.2 How Do You Tie a Square Knot?

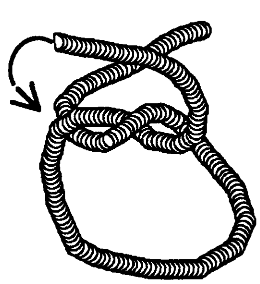
The Square knot is one of the simplest and most commonly used, yet both its configuration and its processes of formation present descriptive and procedural challenges. Here we look at ways in which people usually tie the Square knot, as well as ways in which children in the Knot Lab worked with it.

In order to remember how to tie a Square knot, people usually keep in mind some adage like, "Left over right, right over left." Notice that in so doing, they implicitly identify a moving end and a static (or "standing") end:

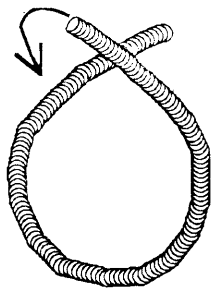


"left over right"

These roles remain consistent as the knot is completed – the end that moves initially is the one that continues to move, and the end that is passive at first remains the standing end. The apparent switch hinted by the wording ("left over right, right over left") has to do with the changing position of the moving end, not with the assumption of an opposite role:

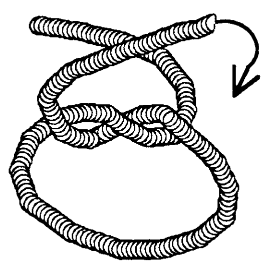


Preference may dictate that the knot is started the other way around:

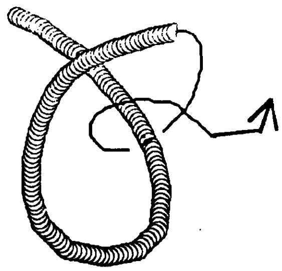


"right over left"

But the pattern is the same – one end moves throughout to form the knot, and the other end is passive:

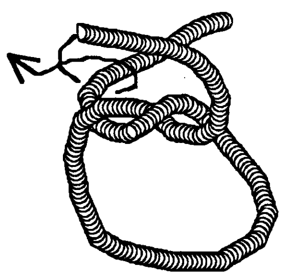


Notice that there is a caveat in this discussion, however, which requires considering a finer level of detail. It *is* possible, of course, to change the roles of the moving and standing ends, and still produce a correct Square knot. So far we have seen two ways of tying the knot: the "left over right, right over left" way and the "right over left, left over right" way. Each of these examples articulates two steps in the form, "X over Y, Y over X" – but each step encompasses *more than one move*. The first step in the first example, indicated by "left over right," includes the acts of crossing the left end over the right, wrapping the left end back and under the right, and continuing the wrapping motion so that the left end again points upward:

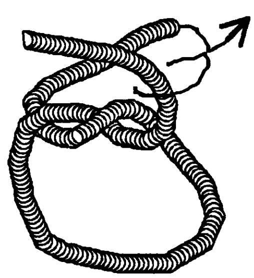


"left over right"

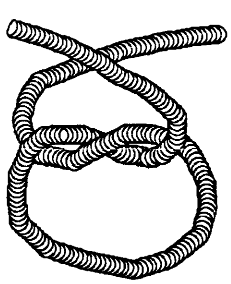
Likewise, the next step – the "right over left" step – includes both crossing and wrapping motions of the end that moved initially:

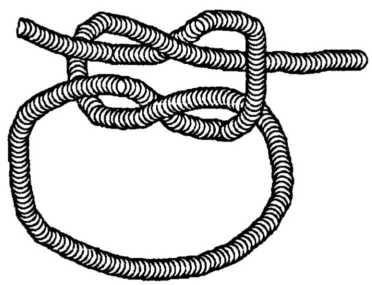


However, at this point, it would be also possible to move the *other* end. The end that had been moving could change roles, becoming passive, and the end that had been passive could move – forward, down and over the other end, back and upward.

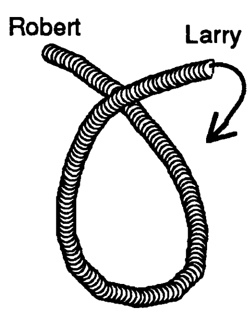


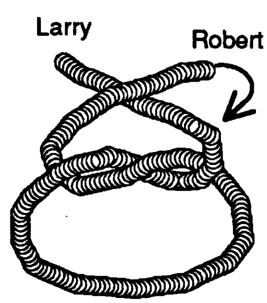
This possibility for change was a frequent source of confusion among Knot Lab participants. What does "right over left" really mean? Clearly, the end that is in the right position crosses over the left end, but then what is supposed to happen? Fortunately, a tyer who has gotten this far before becoming confused will produce the Square knot no matter which end she chooses to move:



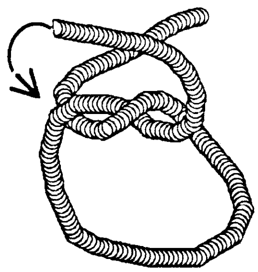


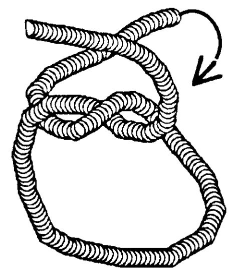
Confusion at an earlier point, though, can make the difference between producing the knot and not producing it (that is, between producing a Square knot or a Granny knot). If "left over right" is interpreted not as a designator of variable position, but as a phrase containing literally the name of each end, "left over right, right over left" would indicate a change of the moving end at a crucial point – a point that determines the final configuration. "Left" and "right" are less than optimal names, as they are primarily indicators of position, but people do sometimes make this nominal interpretation. Substituting more commonplace names can help to illustrate the point: suppose we call the two ends "Larry" and "Robert." Then the adage would be, "Larry over Robert, Robert over Larry":



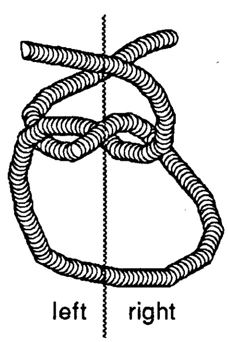


This adage would produce a Granny knot. In order to produce a Square knot, the second step intended by the "left over right, right over left" adage should actually look like either of these:



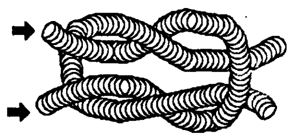


As an *instruction*, the "left over right, right over left" adage can be confusing, but as a *mnemonic* it can be useful. In its simplest enaction, the "left over right, right over left" saying works because the tyer employs a consistency that she might not even recognize – that is, moving only one of the ends, or maintaining an imaginary boundary that determines areas within which something can be called "left" or "right":

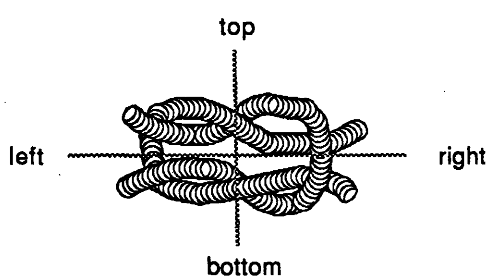


This boundary provides a kind of security in that it remains constant despite the potentially confusing motion of the end or ends. The boundary might be called an *invariant*.

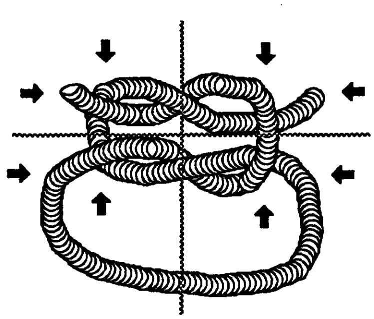
Deciding what to do in the second step could involve other strategies as well, such as "thinking ahead" in order to anticipate where an end will lie after it has been wrapped, and imagining its position relative to the result of the first step:



Here, the left/right invariant plays a role, but another invariant, involving a newly formed "top" and "bottom," becomes important:



Thus, there are many different ways to tie a Square knot, each of them being some combination of necessary and preferred moves. The key to tying the knot correctly is in accurately reproducing the relationships among its parts:



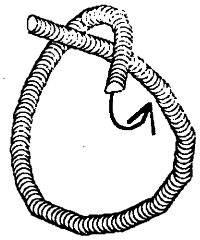
Despite their differences, the examples we have seen so far rely on a common view of how to hold the string in preparation for tying and through the process of tying. Here we refer to such examples as making use of an *upright* method of tying (see Section 5.4.1). Jill's diagram showing steps in producing a simple Overhand knot emphasizes the way in which the string is held between two hands, drooping to form the U-shaped loop characteristic of an upright method:

 5.4.3 6.16 6.17 **9.1.1**

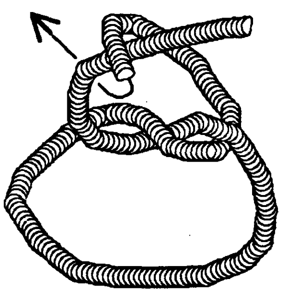
Also characteristic of this method is the tyer's consciousness of the potential for each of the two ends to move into the form of the knot.

Soo Yong's introductory videotape includes instructions for how to tie the Square knot, in which he also makes use of an upright method. Although the children subsequently used printed pictures and other ways of figuring out how to tie this knot, Soo Yong's demonstration clearly had some initial influence. His version of the "left over right" motif bears elucidation for its unique interpretation of timing in the procedure. He assigns the terms "left over right" to a slightly later phase than is often meant:

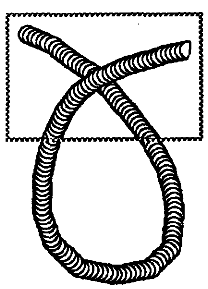
Soo Yong: Okay, the Square knot. Left over right, like this, and then, around the right part:



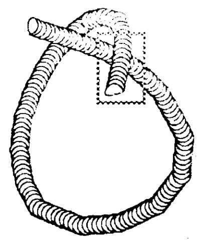
and then, right over left, like this, and then like that:



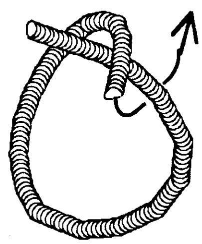
In a common interpretation, "left over right" refers to the initial act of crossing the two ends:



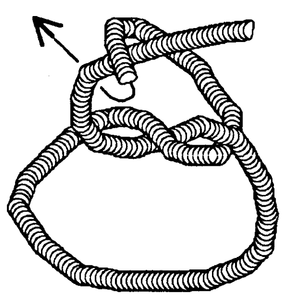
But Soo Yong thinks of a slightly later phase. The initial crossing has already been made (in what is described above as a "right over left" move – which, in deference to Soo Yong's style, we might here call "left under right," as he is also thinking of the left as the moving end). Then the left string is brought forward, down, and over the right; *this* is Soo Yong's "left over right" move:



He completes the motion by wrapping the moving end "around the right part":



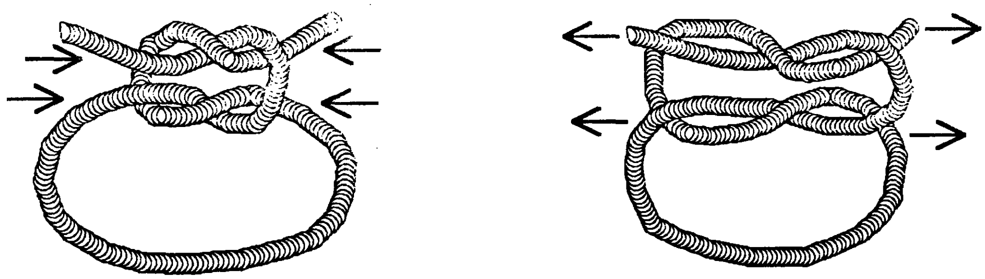
Soo Yong continues with the same moving end, wrapping it under, over, and around the other part as before:



Then he demonstrates a test for the completed Square knot, which was to become immensely popular in the Knot Lab:

 **6.8** 7.2.9 9.4.2

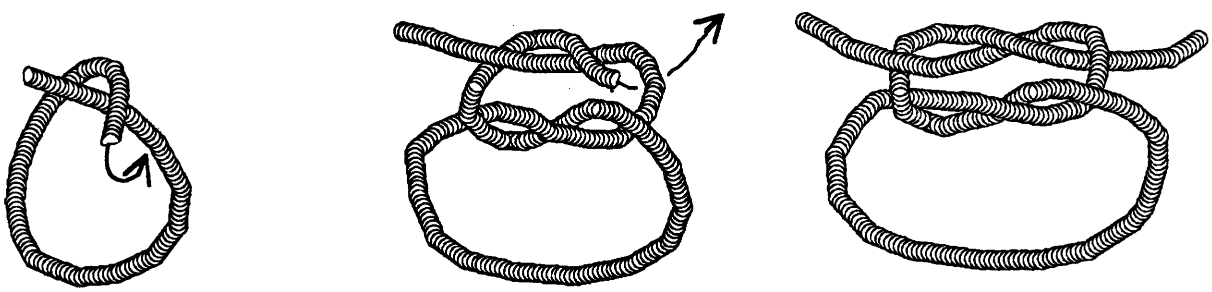
And sorta, you check if it's right by doing this. Just sorta loosen the knot and – and, sorta kinda do that.



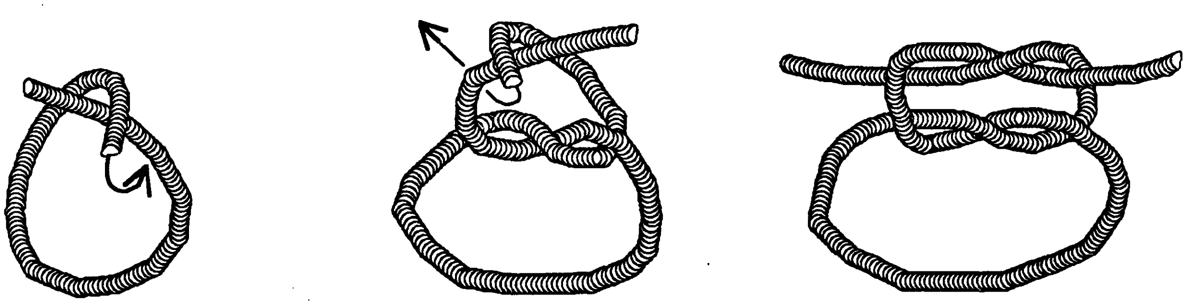
(He loosens and tightens the knot repeatedly by pushing the two horizontal loops so they move apart, and then pulling them closer together.)

Soo Yong explains how the procedure can go wrong, producing a Granny knot instead of a Square. He emphasizes the principle of doing the second step in the "opposite" way from the first:

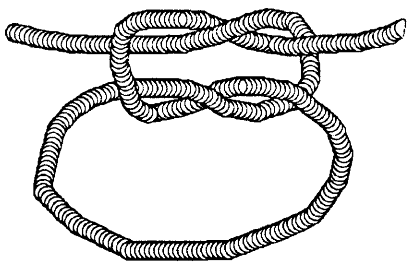
A lot of people are confused between the Granny knot and the Square knot because Granny knot – it would be like left over right and then left over right again, like this:

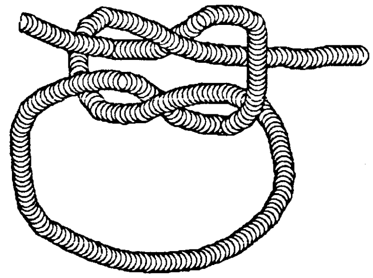


But a Square knot – left over right, and then, opposite way, which is right over left, like this:



Notice that Soo Yong's Square knot mirrors the one produced in earlier examples described above. Soo Yong's difference in identifying to what phase the "left over right" terminology should apply makes a crucial difference in the outcome of the procedure. In Section 5.5 we develop more reliable names for each of these results:





Children in the Knot Lab encountered different problems and demonstrated different approaches in learning to tie Square knots:

Patrick gets confused by the "left over right, right over left" wording. It seems to indicate an alternating pattern – yet he is not quite sure what to alternate. Should he switch the end that is moving? Or should he maintain the moving end and switch the direction in which it moves?

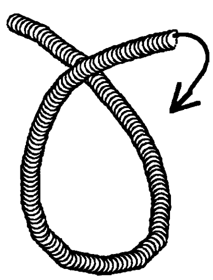
Maria realizes that the way she crosses the second step matters – but she can get confused about which way is really "opposite." Should the moving end should be crossed over or under the other one? Her difficulty is in remembering – actually, in remembering to remember – the way in which she crossed the moving end in the first step. And by the time she gets to the second step, the product of the first is unrecognizable. Its intertwining no longer resembles the way it looked when the crossing was made, and she is left without a model for the next step.

Alice uses some idiosyncratic wording to describe the sense of opposites contained by the knot. Her descriptions are often accompanied by a meaningful gesture:

Um – like – like two full – outerhand and an underhand *(does a twirly motion with her finger)* – and then do it again.

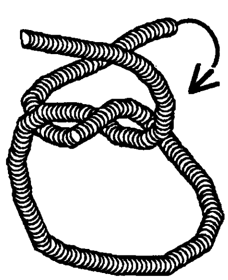
Her distinction acknowledges the plurality of moves often considered as a single step. She includes both the outward direction in which the string must move ("outerhand") and the wrapping that puts it under the other strand ("underhand").

Stacy finds various "left over right" methods confusing for their generality. She develops a way of describing her own way of tying the Square knot that is more detailed in its attention to particular movements. She begins with a familiar left-over-right move,



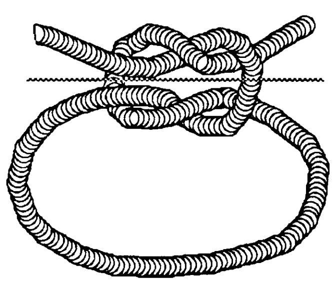
but finds a different way of encompassing the sense of "opposite":

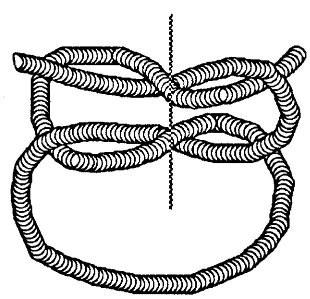
Take what is *now* left, and tuck it *under*.



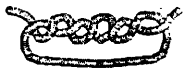
Stacy first acknowledges that the roles of the ends have switched – the left end no longer moves, and the right end does. The "now left" end becomes the moving end, and it goes not *over* the other end, but *under* it.

Tony dutifully learns the upright, two-ended approach described by Soo Yong and in various ways here, and indicated by many of the books in the Knot Lab. He becomes fascinated by the back-and-forth test for the finished knot, and sits for long whiles playing with the knot by moving it back and forth. He notices that the method he used to tie the knot builds it up vertically, emphasizing a first step that becomes the bottom of the knot and a second step that becomes the top. The test reveals a more horizontal character of the knot, though.

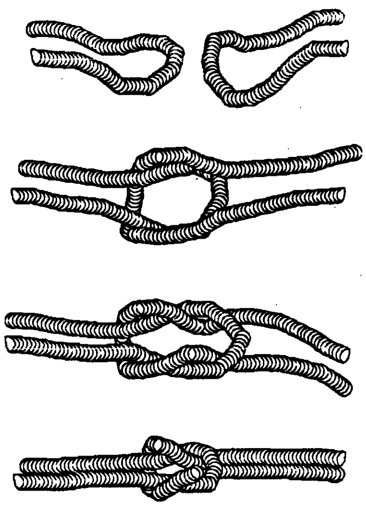




Tony gradually comes to see the knot as two interlocking, horizontal loops. He sets about trying various ways of putting two loops together in order to produce the knot:



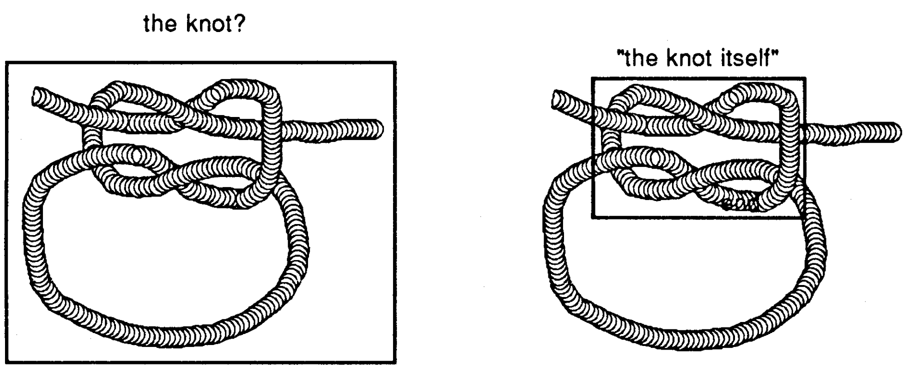
9.1.2 **9.4.2**



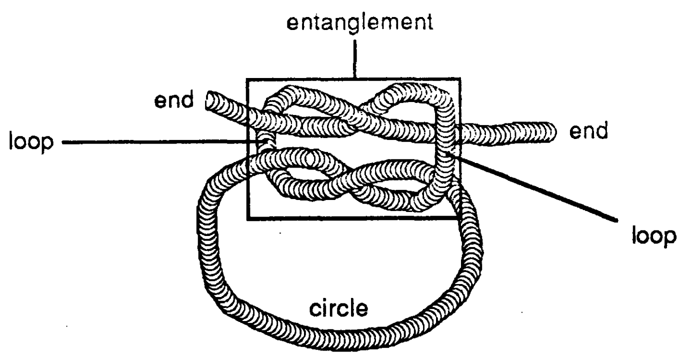
Although in this context, children usually thought of a Granny knot as a Square knot that had gone wrong, many were delighted to discover that in macramé, the same pattern can be desirable. A chain of Overhands tied in a pattern that produces a Granny for each doublet forms an attractive spiral. If the Overhands are tied in a pattern that produces a Square for each doublet, the resulting chain lays flat and seems relatively uninteresting. What is considered an error in one context may be desirable in another.

5.3 Anatomy of a Knot

Children in the Knot Lab were learning to tie knots as an exploration into learning and as an exploration into knots. Generally, their knots weren't really *doing* anything – they were simply objects that were produced and appreciated in and of themselves. An outcome of this approach was that the process of tying tended to yield a small, odd-looking object that spurred some confusion and debate: was the knot the entire object formed by the piece of string, or was it simply the part that resulted from a particular way of entangling the string?

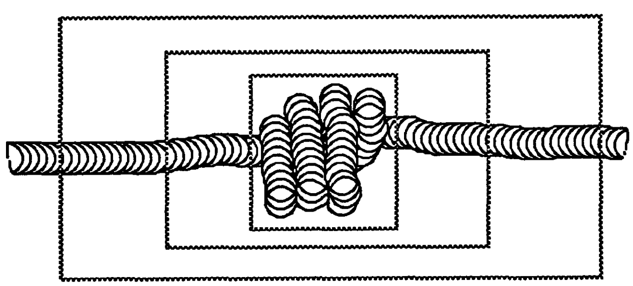


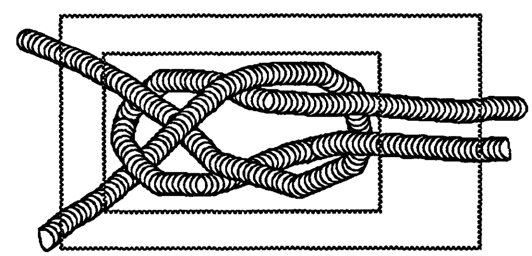
As they described the results of their work, most of the children included what is here shown as the lower, rounded part of the object (the "circle"). That is, they considered the entire object – the entire piece of convoluted string – as "the knot," and differentiated its constituents:



Some children used the term "loop" to mean what here is called the "circle." In such cases, what here are called "loops" were often referred to as "sides." In many texts on knots, "ends" are called "bights," but only a few children interested in the Bowline on a Bight came to use this term (see Sections 5.4.1 and 6.16). In their discussion of what constitutes a "loop," Alice and Jill called the moving end the "bight," distinguishing it from the "standing end"; for them, the term "bight" seemed to carry with it the notion of movement.[1]

Distinguishing the crucial part is not as problematic for knots whose configuration does not form a circle: the ends are simply the ends, and no matter what the length of string or how much of the ends are considered in the description, there seems to be no question about what is "the knot":





[1] Although the Bowline on the Bight is a knot that is stationary in its finished form, it bears a certain resemblance to knots such as the Running Bowline and the True Lovers' knot, whose completed states allow for moving the knots in certain ways. The children found such knots intriguing; perhaps such knots' capacity for movement influenced the children's sense of the term "bight."

5.4 Terminology

In discussing knots, the children used vocabularies that were often individual and spontaneous, and often the result of a synthesis of ideas and terms that developed among many participants through the course of the project. These terms included words, gestures, and pictorial symbols, some of which were displayed on the Knot Language board (see Section 9.1.3).

Supplementing the anatomical distinctions just described (Section 5.3), terminology relevant to identifying parts and orientations of knots is explained in the next few sections. The terms either echo or derive from the discourse of participants in the Knot Lab project.

5.4.1 Words

The following notable or persistent words and usages are described on the following pages:

across	*intersection*
around	*leading end, standing end*
between	*loop*
bight, end	*not-knot*
bridge, blockade	*over, under*
bridge, bridging	*overhand (outerhand), underhand*
coil	*through*
crooked, straight	*top, bottom*
cross, crossing	*track*
eye	*twist, wind, wrap*
in, out, over, under	*upright*

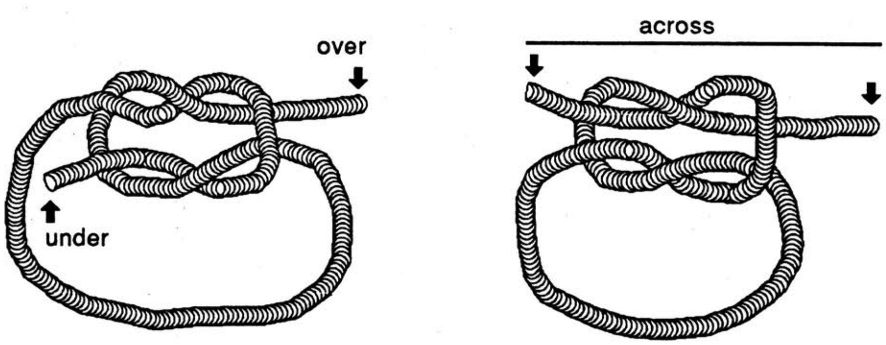
across

In a letter to Soo Yong, Tiffany explained the difference between the Thief knot and the Square knot. She implicitly chose the circle as the referent, describing the ends relative to the circle without pointing it out specifically:

7.1.1.12

> To Soo Yong I did the Mystery knot and Carol helped me. It's different from the square knot because the string goes under and over. But the string of the ... square knot go across.
>
> from Tiffany

Tiffany's interpretation can be pictured as follows:



For Tiffany, "over" and "under" allude not to specific crossings, but to the positions of the ends with regard to the circle. With her designation, "across," she considers both ends and notes that not only do both of them occur above the circle, but they define a neat line at the top of the knot.

Alice made use of the term in a similar way:

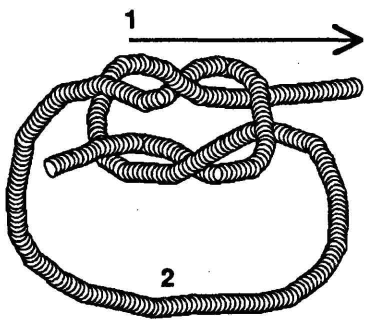
5.4.2 6.2 6.6 **7.1.1.13** 9.1

Carol: ... do these two knots look the same to you?

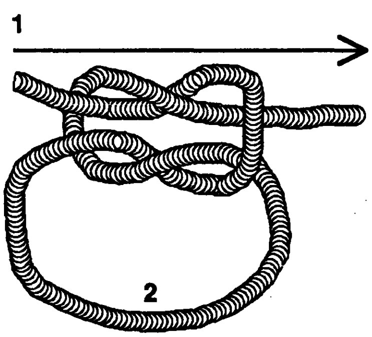
Alice: Um umm. *(She means "no.")*

What's different?

The string goes over this **(1)** and around like that **(2)**.



This one, the string goes straight across **(1)**, and then it goes like that **(2)**.



Alice based her comparison on a view of the top part of the knots.

around

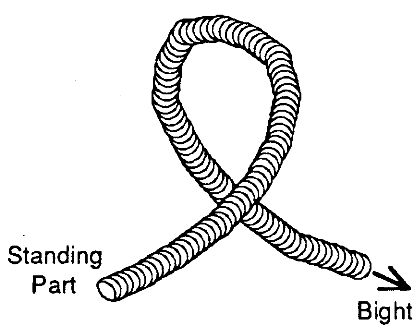
Alice and Stacy posted this term on the Knot Language board (see Section 9.1.3). Their usage was similar to that of *twist, wind,* and *wrap,* defined below.

between

"Between" was a designator often used in the obvious way.

bight, end

While most children referred to the ends of the string simply as "ends," some also used the sailor's term, "bight" (see Sections 5.3 and 6.16). The children tended to reserve this term for a moving end, effectively distinguishing it from the end that stay still in the course of tying (see *bridge, bridging* and *leading end, standing end* below).



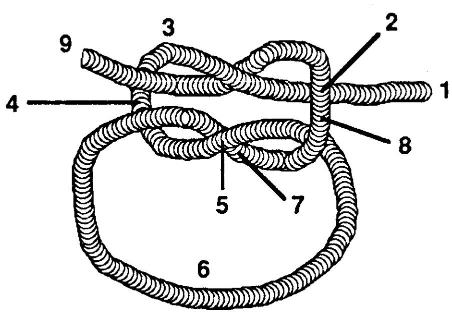
bridge, blockade

For Alice, a "bridge" was part of a knot that goes over or under another part. In following the path of the Square knot, she considered both sides – so from any one view, a bridge can appear to go either over or under. When she came to a crossing, she traversed the "bridge" and continued on her way, or imagined the knot turned over so that she could cross the bridge and continue. In the configuration of the Square knot, the coupling of "over" and "under" crossings makes for particularly pronounced "bridges":

5.4.2 6.2 6.6 **7.1.1.13**

Carol: ... if you were a tiny little ant, and you were walking all along that knot, how would it look? Can you show me with your finger what you would do?

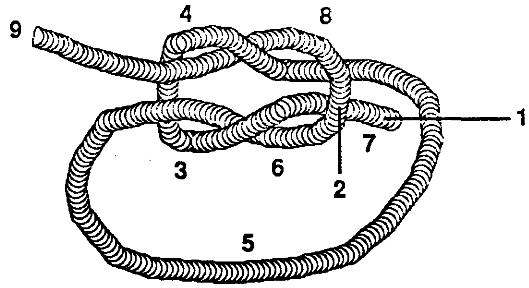
Alice: Um – *(does the Square)* – I'd start from here **(1)**, and then go on the bridge **(2)**, and then go over there **(3)**, and go on that other bridge **(4)** – *(she does motion with finger instead of turning knot around)* – and go to that one **(5)**, and go all the way around **(6)**, and go to that bridge **(7)**, and go over there **(8)**, and there **(9)**. *(Her finger points left, out of the knot.)*



When Patrick (as the ant) encountered certain crossings, however, he saw a "blockade" that made it more difficult to find his way through the knot. He was looking at only one side of the knot; it was only the overcrossings ("those things in it": (2) and (6)) that he found problematic. When the strand he was following went*under* another part, he seemed to make a break in his trail, jumping to the point at which the strand emerged again ((3) and (4)):

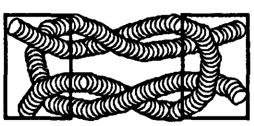
6.4 6.7 6.22.1 6.22.2 **7.1.1.15**

Patrick: Okay. I would start here **(1)** and then, I'd go um – hmm. Let's see. It's hard. It's a blockade **(2)**. I'd probably go there **(3)**, and then I'd switch to here **(4)**, come around **(5)**, go across over this **(6)**, go – oh no! **(7)** *(He realizes that he would leave the knot too early.)* Then come back here **(8)**, and go around there **(9)**. Okay, 'cause of those things in it, it's hard.

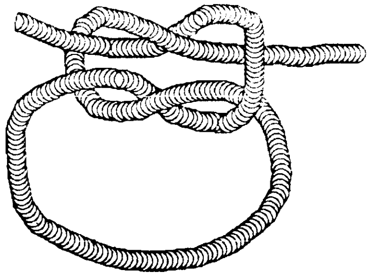


Alice and Patrick used their terms with regard to different ways of seeing the knots. Alice enjoyed both the advantage of her human-view, which enabled her to see that the strand she was following continued despite the interruption at a crossing, and her ant-view, in which she traversed the strand via the "bridge" formed by the perpendicular strand. Patrick allowed only his ant-view, so the perpendicular strand seemed like a "blockade" – there was no way to know that if he crawled up on it, he would find the continuation of the strand he was following.

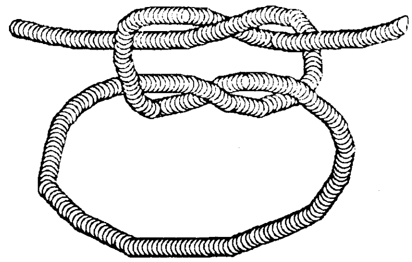
However, we can use aspects of their thinking in developing a way to identify versions of the Square and Thief knots. As Alice noticed, the left and right sides of these knots are characterized by two parallel strands that run over or under another strand, which runs perpendicular to them:



By taking this view of the knot, but using Patrick's choice of the situation in which the perpendicular strand runs *over* the other two, we find an identifier – designated by his term, "blockade":



blockade-right Square



blockade-left Square

(See Section 5.5 for further considerations in the use of this term.)

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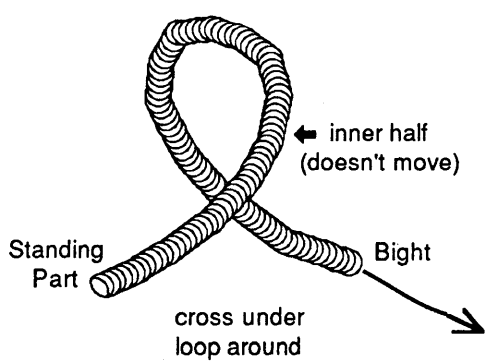
bridge, bridging

As a noun, this term was a designation for a crossing. As a verb, it identified the way in which some crossings are formed: Alice and Jill defined "bridging" as the process of making a loop. They designated a "standing part" of the string, which is a term that Soo Yong used, and a "bight," which they considered as the moving end (see Sections 5.3 and 6.16). They picked up the term "bight" from various books on knots that were available in the Knot Lab. Unlike these books, which generally use the term to refer to either end of the string, the girls used it in contrast to the "standing part," as a reference to the moving end:



They specified a formula for bridging: "cross under, loop around." In the result, part of the string lies over the other part.

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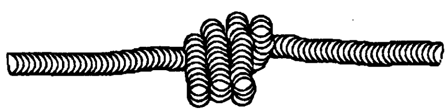
coil

9.1

A coil of a tightened knot is evidence of a *twist, wind,* or *wrap* in tying (see below). This version of a Stopper knot has, arguably, three and a half coils:



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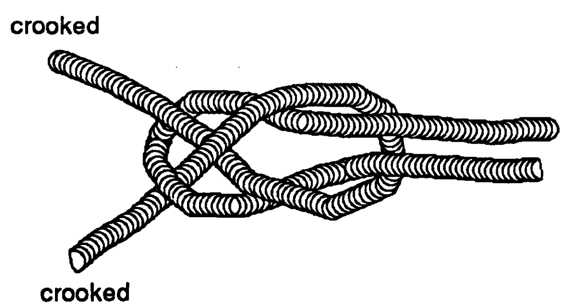
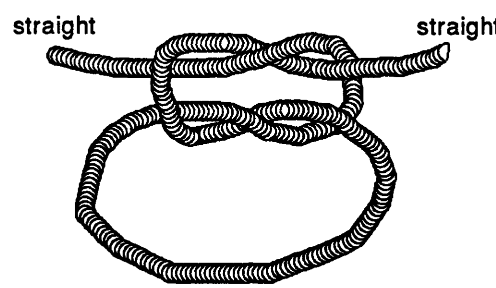
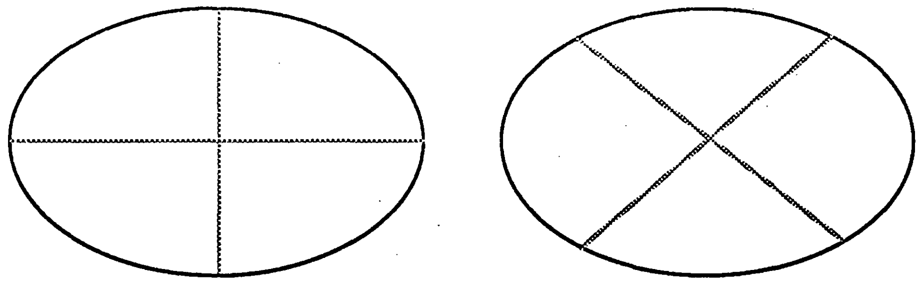
crooked, straight

Alice and Jill used these terms in describing the Sheet Bend relative to the Square knot. Their concern was with the way in which the ends exit the entanglement:





Their designations indicate a firm sense of "normal" orientation:



normal ("straight") unusual ("crooked")

Curtis used the same sense of the term *straight.* For him, the entanglement was the referent from which the ends of the Square knot "come out." But the circle became the referent in describing the Thief knot: one end goes "under" the left part of the circle that leads into the entanglement, and the other end goes "up," or above the circle.

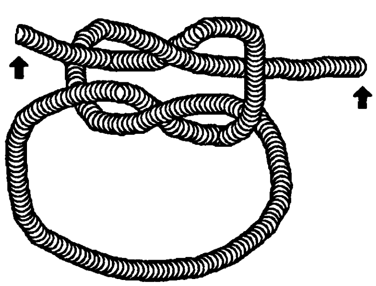
6.3 **7.1.1.4**

***Carol:** Okay. Take a look at – a closer look at these (the two "circular" knots, this time with the circle at the bottom). Are these really the same knot?*

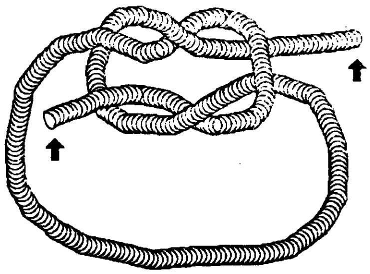
Curtis: No.

What's different?

These two strings come out straight,



and this one goes down under, like right there, and that line's straight up.



Curtis used the term "straight" in his description of each knot. His terms "out," "under," and "up" would seem sufficient to designate the position of the ends relative to a referent; "straight," therefore, seems to indicate an additional property of ends that exit the knot without pointing to a potential obstacle. Both of the ends of the Square knot are free in this sense (they "come out straight"), but only the right end of the Thief is "straight."

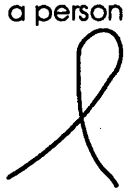
---

cross

Alice illustrated the term as the simple first step of the Overhand, or "Pretzel," knot:

9.1.1

> Take both ends and cross them so the strings look like like
> a person bending his knee like this

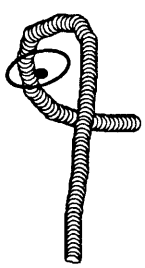


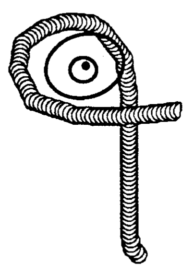
"Cross" is synonymous with her term, "bridge."

eye

 5.4.3 **9.1.3**

After hearing Soo Yong refer to the "eye" of a loop, Alice and Jill were confused about what he meant. It was clear that his reference was to the enclosed part of the loop, but did he mean an alphabetic "i," a personal "I," or a seeing "eye"? In a lengthy discussion, they decided that the negative space delineated by the string resembled the shape of a human eye. In order to clarify Soo Yong's term, they taped pieces of string on pages that were displayed on the Knot Language board, and drew representations of eyes in the loops:





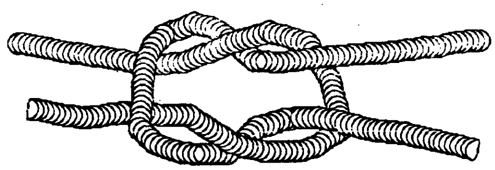
in, out, over, under

The children used these terms were arbitrarily and, often, synonymously. A preference for one of these terms often indicated a particular choice of referent.

Stacy explained the difference between the blockade-left "Square with 4 Ends" and the Sheet Bend in terms of the left ends of the knots. She noted how the ends are situated differently with regard to the left loop:

Carol: Okay – what if we did a Square knot without a circle?

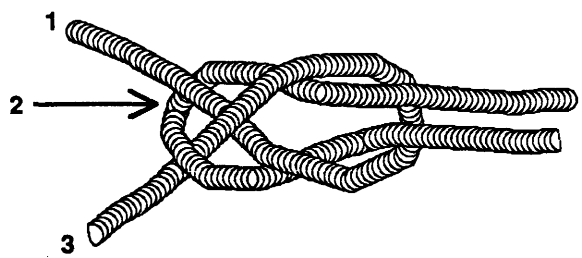
Stacy: Did a Square knot like this *(she ties it with two strings).*



It's almost exactly like these two *(the two strings that form the Sheet Bend).*

Okay – and what's the difference?

That this one goes out of the loop (1) – and if it went in – *(looks)* – I think so – yeah, if this one was on this side (2) *(putting* (1) *inside the loop)* and this one (3) was through here [it would be the same as the Square]. *((3) would stay where it is, becoming the partner of (1), newly placed in the same manner as the Square knot.)*



In this description, Stacy found it useful to designate "sides" of the left loop – the underside is the plane facing the table. Going "in" the loop, in another terminology, would be the same as going "under" the loop – away from her and toward the table. Going (or staying) "out" of the loop would mean being "over" the loop. In her usage, "in" the loop and "through" the loop imply the same movement (see *through,* below).

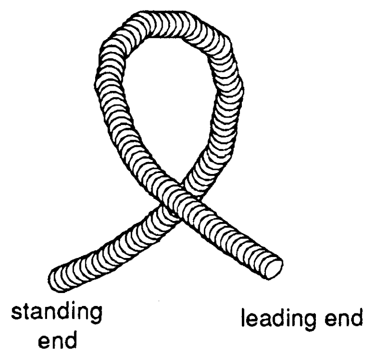
---

intersection

This was another term for *crossing* or *bridge* (see above).

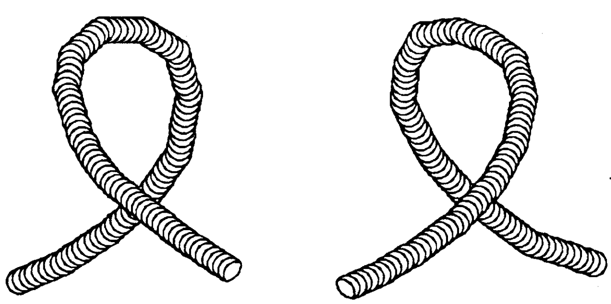
leading end, standing end

Soo Yong often referred to the "standing end" in his descriptions of how to tie various knots. Coupled with this designation was the need to name the end that moved around this stationary part – taking a cue from sailors and other knot-tyers, the children dubbed the end that moves the "leading end" (see *bight* and *bridge, bridging,* above).



loop

Children generally used this term for the most basic involvement, in which a single crossing completes a curve of the string:



Alice and Jill became involved in a debate about what constitutes a "loop," and Alice sought verification from Soo Yong:

9.1.3

Dear Soo Yong,
I was wondering what makes loops.

Jill's instructions were
A piece of string (or something) that has 2 ends & crosses
over one side.

And I said
The rope ends curl on its own.

which is right?
Alice

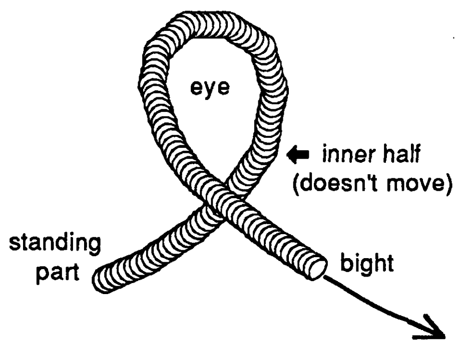
By "curl on its own," Alice referred to the apparent liveliness that can be generated by rolling the ends of a string in opposite directions – depending on the weight and texture of the string, a loop "curls" in somewhere along the string. Soo Yong accepted both methods:

Dear Alice,
You both are write.
As long as a rope makes a circle, it is a loop.
/\
complete

Sincerely,

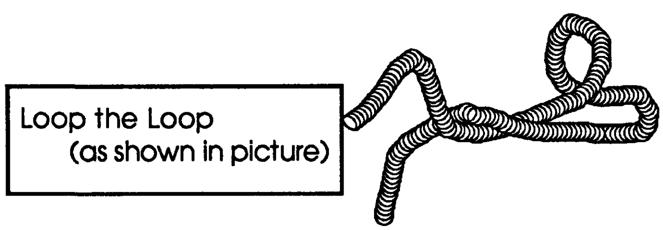
Soo Yong Chang

Alice and Jill designated parts of a loop, considering the end that moves, the end that doesn't move, and the negative space defined by the loop. They decided to name not only the origin of the end that doesn't move, but the portion of the string closest to it, the "inner half":

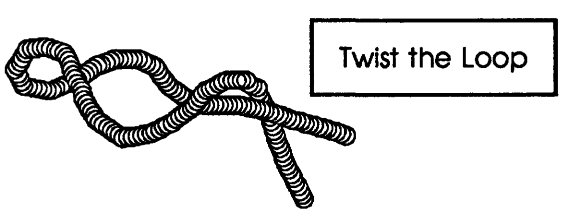



9.1.3

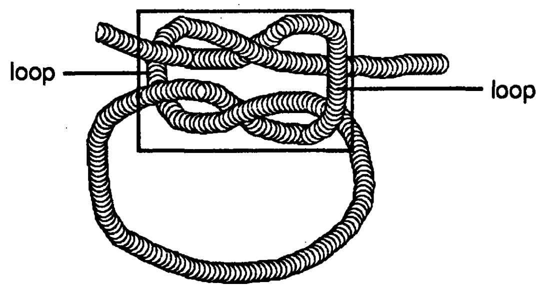
They continued the concern in showing steps for tying the Running Bowline. Jill chose to use the term "loop" for both the movement and its object:



Alice, however, chose to distinguish verb from noun:



The term "loop" also referred to a part of the Square knot (see Section 5.3). In the context of the knot, this loop may seem more complicated than those described above, but actually it is simpler:



A "loop" of the Square knot does not actually cross itself; it is the other "loop" in the knot that defines and completes the form.

---

not-knot

The concept that a complicated-looking tangle of string may appear to be a knot but might not actually be intertwined was very much present in the Knot Lab. Known as the "unknot" by mathematicians, the children dubbed this phenomenon the "not-knot." Although Jack didn't quite manage the spelling, his conception was accurate, and he wanted to share the fun with others:

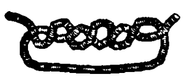
> First you wrap it around what your going to tie 3 times. And slip it at the top all the way through to the bottom. And voila! You have a Not Not!
>
> By Jack Benson

over, under

See *in, out,* above.

overhand (outerhand), underhand

Tony pointed out a similarity between the Figure 8 and the Stopper according to the ways in which the knots are formed:

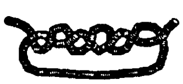

7.2.16

... the Stopper and the Figure 8 are both rather simple knots – which you get from using an overhand or an underhand – knot – using it that way – that's how you get those two –


Figure 8


Stopper

Alice used some idiosyncratic wording to describe the sense of opposites contained by the Square knot. She accompanied her descriptions with a meaningful gesture:


7.1.1.13

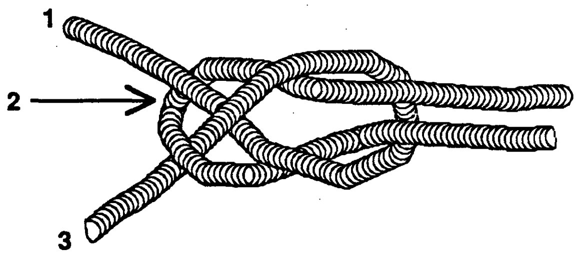
Um – like – like two full – outerhand and an underhand *(does a twirly motion with her finger)* – and then do it again.

Alice's distinction acknowledged the plurality of moves often considered as a single step. She included both the outward direction in which the string must move ("outerhand") and the wrapping that puts it under the other strand ("underhand").

through

For Stacy, "through" was an action that determined a relationship with a loop (see *in, out, over, under*, above). She described the end marked (3) as going through the left loop of the Sheet Bend:

... this one goes out of the loop (1) – and if it went in – *(looks)* – I think so – yeah, if this one was on this side (2) *(putting* (1) *inside the loop)* and this one (3) was through here [it would be the same as the Square]. *((3) would stay where it is, becoming the partner of* (1), *newly placed in the same manner as the Square knot.)*



9.2.2

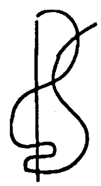
Alice used the term in a similar way in developing "formulas" for tying the Figure 8 and the Stevedore (see also *tuck* and *twist, wind, wrap,* below). For the Figure 8, she suggested:

make a loop
wrap it around the standing end
tuck it through

And for the Stevedore, she advised:

twist the string
take one of the strings, wrap it around the opposite,
put through the loop

Alice decided to illustrate the Stevedore formula. In her later drawings, she tended to clarify the relationships of the strands in the crossings.



first you make a loop and put the string under the straight line and wrap around string and put the string under and through the loop and Pull !

Alice

---

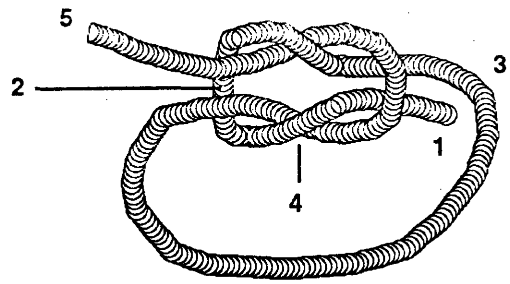
top, bottom

Marcos was concerned with both sides of the knot, turning it as needed in order to carefully follow the string – the ant's path – as it wound along. His use of the terms "top" and "bottom" fluctuated broadly. At first, "bottom" referred to the other face of the knot. Then, in identifying his starting points for tracing the path, "bottom" referred to the the circle of the Square knot, while "top" referred to the entanglement of the Thief. Then, in finally describing the difference between the two knots, he used both "top" and "bottom" with regard to the entanglements, as he situated the ends relative to the circles.


6.6 7.1.1.10 8.2

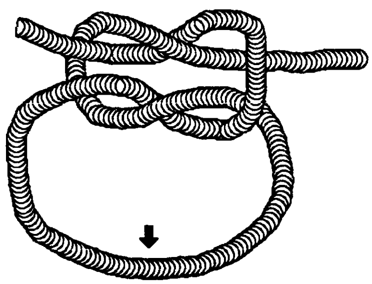
Carol: *Imagine you were a tiny little ant, and you were crawling on this string. And you wanted to crawl all over the whole string – you wanted to cover every little piece of it. Where would you start? Show with your finger how you would go around if you were a little ant.*

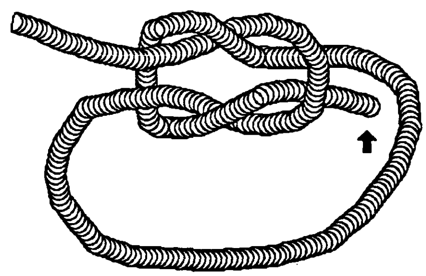
Marcos: *(He does the Square knot, then the Thief.)* This one – let me see. I'll go there **(1)** – there – yeah! Like this. I'll go like here – like – here, on the bottom **(2)**, and then I will go – let me see – what is it? Here – go around this **(3)**, then go here on the bottom **(4)**? *(He lifts the knot to point to the "bottom.")* Right? And I end up here **(5)**!



Okay. Is there anything different about how you went here (the Thief) and how you went here (the Square)?

Yeah. I start – I started in a different way. Here *(the Square)* I started – like, um – almost on the bottom. Here *(the Thief)*, I started on the top.



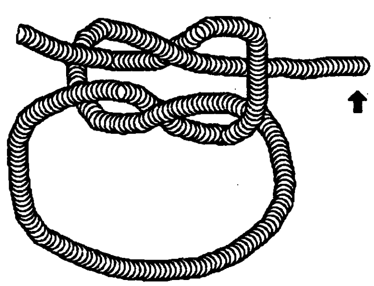


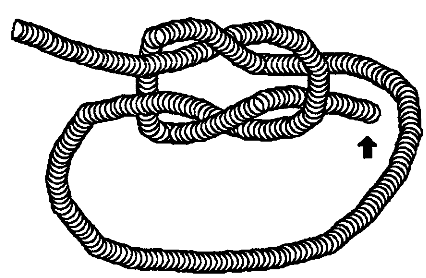
How come?

Because they are like different ways. See? *(He holds up both knots.)*

Oh, I see.

See – this one's on top *(the Square)*. This one's on the bottom *(the Thief)*.





I see.

So you have to – because they're different – they're the same knot, but they got – you gotta start them a different way, because they're not – almost the same. One is on the top, the other is on the bottom. *(He works with the Square knot.)* You have to go like this – go around – then go through there – you end here *(the left end of the Square)*. And here, you end right there *(the right end of the Thief)*.

track

Doreen's use of the term "track" recalls a transportation metaphor, but could also imply a way of thinking about the formation of a knot over time, as though the leading end of the string leaves a trace of where it has been, and that trace becomes the knot.

Doreen: Hmm – they look alike?

"Square with 4 Ends"

Sheet Bend

Carol: *Yeah, they do, don't they? What looks alike about them?*

The way – the way the track is *(she indicates the entanglement on each knot).*

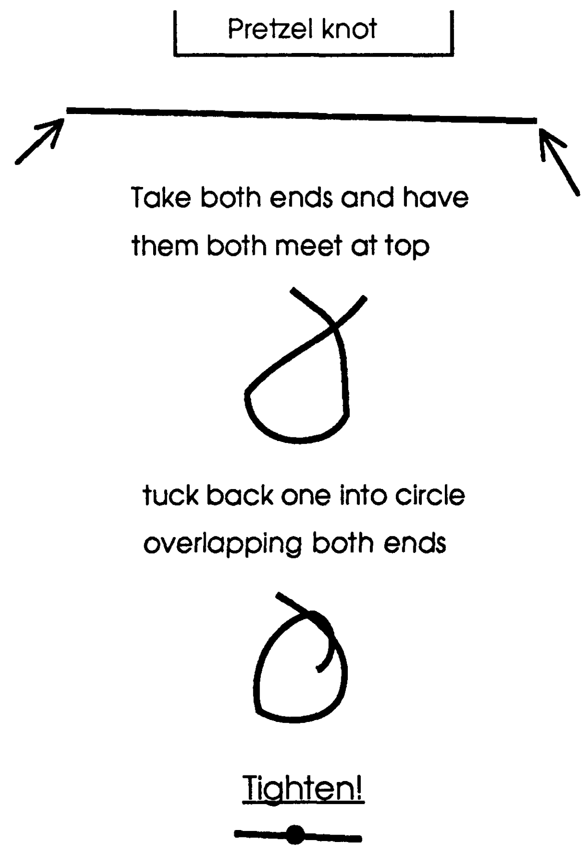
tuck

9.2.2

Alice's "formula" for tying the Figure 8 makes particular reference to the action that completes the knot (see also*through,* above, and *twist, wind, wrap,* below).

make a loop
wrap it around the standing end
tuck it through

Tony's use of the term, "tuck," evolves from designating a particular move to compacting three moves:



"Tuck" refers simply to the act of putting an end in the circle. But in his abbreviation of the procedure, the term comes to include the preparatory actions as well:

Pretzel 2

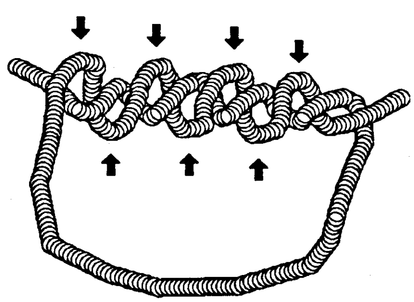
Do pretzel

Tuck left
through space
tighten

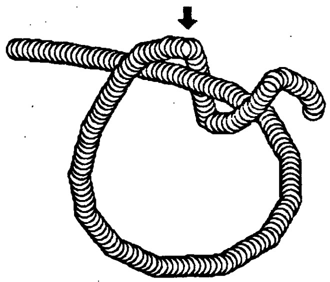
No longer referring to the two generic ends of the string, he adopts a distinction between right and left, and refers simply to "left" as the end demanding attention – it is the one doing the action. But he has not dispensed with the original first step altogether: "tuck" has now come to include what were before considered as three separate actions. Taking both ends, having them meet at the top, and tucking one back are all now implied by "tuck." What was a "circle" becomes a less rigid "space, " and "through" comes to mean involvement with that space as well as "overlapping both ends."

twist, wind, wrap

Many of the children used the terms "twist," "wind," and "wrap" to refer to the coils of knots such as the Stopper, or to the wrapping motion that produces the coils:

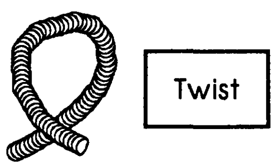


What they attempted to describe is the nature of the involvement of one part of the string with the other: in a "twist," the moving end makes a full revolution around the standing part.

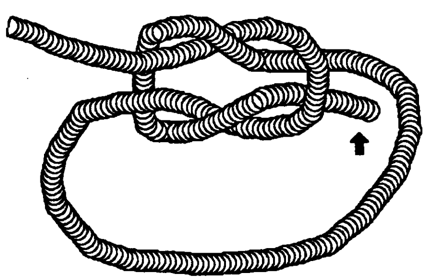


5.4.1

Alice used the term as the sole instruction for her first step for tying the Running Bowline, which is simply to form a loop:



Yet in discussing the Thief knot on the Family Tree, Jill used the term "twisted" to refer to the way in which one end lies within the circle:



9.1.4

Tony: The [Thief] is – if you notice, it's a lot like a Square knot, except for –

Jill: Almost exact.

Tony: Right. It's – the knot itself is exactly like a Square knot.

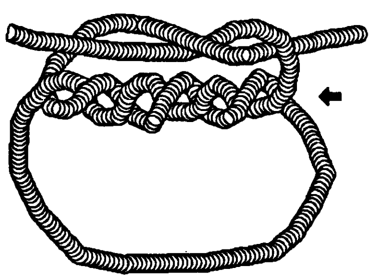
Jill: No –

Tony: Except – well – inverted.

Jill: Well, almost. It could be. It also looks a little bit like the Surgeon's, the way that sort of looks like it's twisted an extra time *(points to bottom of Thief).*

Tony: Right, and – and – but the difference is, this goes in *(exaggerates inward curl of right end).* On the Square knot it would be like that, without the twist *(holds right end above the part actually above it).* It would come up – fall that way naturally.

Jill's use of the term seemed to be more general, a reference to an unexpected – or comparatively complicated – involvement. Her reference pertained not only to position, but to a motion that is hinted by the right end of the Thief as though it anticipates the true "twist" of the Surgeon's:



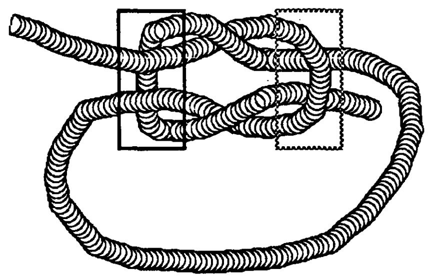
In the same discussion, Tony used yet another sense of the term "twisted" to describe the peculiarity of the Sheet Bend. His reference also seemed to be to a particular motion that results in a certain lay of the string:

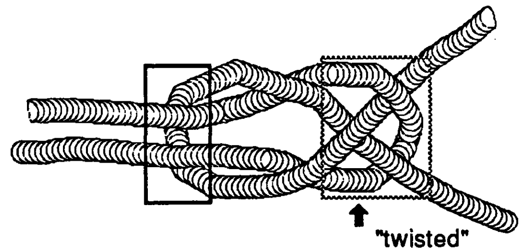
Jill: The Sheet Bend is very odd, and –

Tony: No, it isn't.

Jill: Yes, it is.

Tony: It's like a Square knot, except it's twisted. Well, not except if you see it that way. This is twisted *(shows front and back of Sheet Bend).* It's twisted that way – twisted – and it's twisted – I can't do it with this string. But it's twisted. And you notice *(climbs up to the tree),* I'm gonna use the [Thief] in this instance, since you know how similar the [Thief] is to the Square. If you notice that *(he puts his fingers on right side of both the Thief and the Sheet Bend: two threads go smoothly under one blockade of the Thief, but one crosses under the other and goes over the loop in the Sheet Bend),* you'll see how it's just twisted.

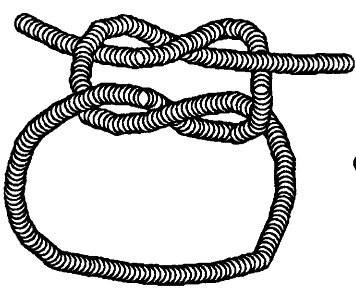




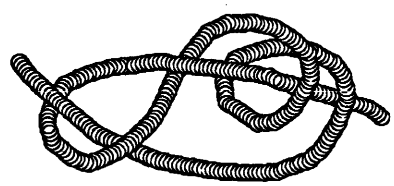
5.4.2 **7.2.7** 10.1

Doreen grouped the Granny, Stevedore, and Figure 8 together (see Section 7.2):

... because of the way they twist – they like, twist, like – into each other. *(As she speaks, she gestures broadly and symmetrically with her hands, pointer fingers extended to show curves going out together and then coming back in together.)*



Granny



Stevedore

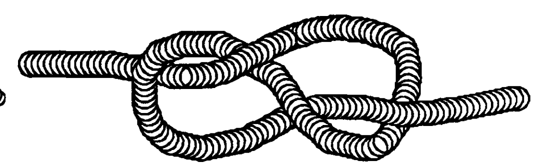


Figure 8

Bruckheimer translates Lietzmann's text on *Visual Topology* as defining a "twist" to mean the simplest involvement that produces a knot. Interestingly, he does not distinguish the Overhand from the Figure 8 in the way that a practical knot-tyer would; Alice, for example, would call his simple "twist," which produces an Overhand knot, a "loop." She would be more likely to concur with the term "twist" to describe the slightly more complicated ("firmer," as Lietzmann/Bruckheimer say) move that leads to the Figure 8. Lietzmann describes the Figure 8 as a more complicated "twist":

> If a thread has one – or two – accessible ends then a twist, which is commonly called a knot, can be more easily made. Since the mathematician only speaks of a knot when the thread is closed, we will here not speak of a knot when the thread is open, but of a twist – this is contrary to everyday usage. The existence of the free end permits one not only to make such twists, but also the opposite, to undo them again and to restore the untwisted thread.
>
> Fig. 10 shows the unusual simple twist, Fig. 11 a somewhat altered 'firmer' twist. The thumb knot, as the sailor calls the twist of Fig. 10, can be repeated many times. ... In both cases, in order to undo the twist, it is sufficient to withdraw the end of the thread from one loop. The simpler twist shows three, and the other four crossings. (Lietzmann [1955] 1969, 19)

Fig. 10

Fig. 11

9.2.2

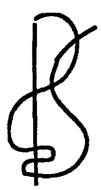
In working with Jill, Alice developed "formulas" for tying the Figure 8 and the Stevedore (see also*through* and*tuck,* above). While her use of the procedural form represented something of a concession, her choice of language reflected her preference for terms that imply a certain motion: "wrap," "tuck," "twist," (and, to a certain extent, even "loop" and "through") refer not to a relationship of one part of the strand to another, but to a movement that establishes the relationship. Her formula for the Figure 8 is:

make a loop
wrap it around the standing end
tuck it through

For the Stevedore, she advised:

twist the string
take one of the strings, wrap it around the opposite,
put through the loop

Alice decided to illustrate the Stevedore formula. In her later drawings, she tended to clarify the relationships of the strands in the crossings.

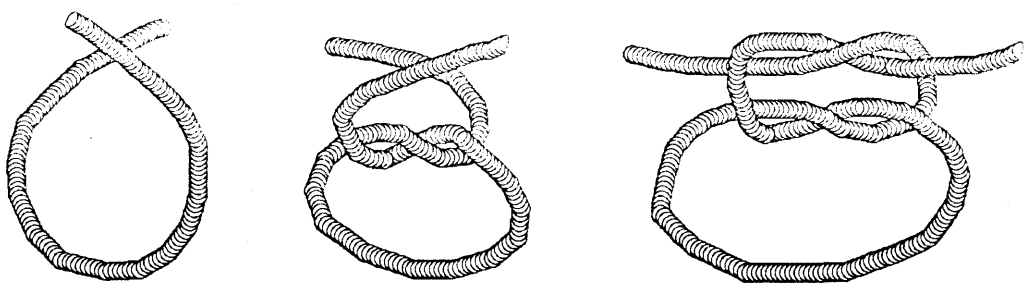


first you make a loop and put the string under the straight
line and wrap around string and put the string under and
through the loop and *Pull* !
Alice

---

upright

This general term categorizes ways of tying the Square and similar knots. The reference is to a way of holding the string, an end in each hand, with the length of the string drooping between the hands, ultimately forming the circle of the knot (see Section 5.2):



5.4.2 Gestures

The children represented ways of thinking about knots through spoken and written words, through pictures, and through gestures. With this brief catalog of some of the gestures that they used, I attempt to emphasize not only the importance of this mode of expression, but how it is often inseparable from other ways of representing thinking. Often, the tendency to gesticulate indicates a way of imagining oneself to be like a knot in certain ways – or even to *be* the knot. By moving parts of the body as the string would move, tyers can employ what we have called *body syntonicity* with the object of their thinking (see Sections 1.3 and 10.1).

---

point directly at an area or part

This is a gesture of assuredness; the particular indication is needed for the level of detail under discussion.

---

point in back-and-forth or circular motions

These indicate an approximation, a general area that is being discussed.

---

wave the hand back and forth

This gesture negates preceding actions in the discussion.

---

use fingers to represent characteristics of the knot

Eugene used his fingers to indicate the difference in positions of the ends of the Square knot. Not only did the "V" of his fingers point to the directions in which the ends extended, but he also created another mapping between finger position and the situation of the ends: the height of each finger was related to whether the end was "above" or "below" the circle.

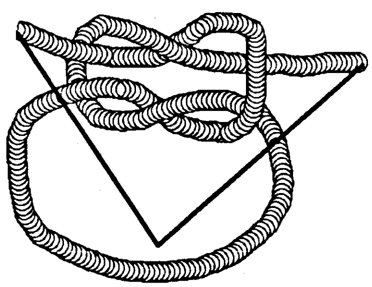
6.6 **7.1.1.5** 10.1

Carol: *How about these knots – these two knots. Do these look the same to you?*

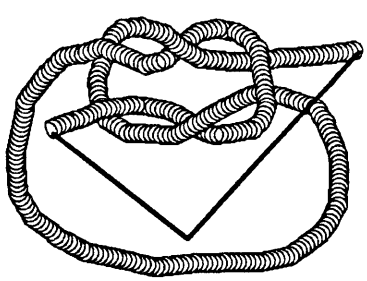
Eugene: Yeah.

What's the same about them?

They're the same – kind of. Because both of these go like that,



and both of these go like that.



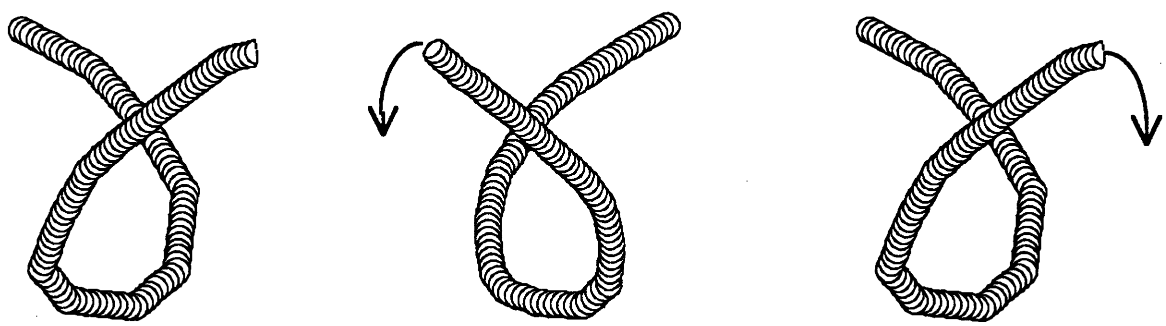
(He shows the two ends of each knot, using the index and middle fingers of his right hand to show the two ends at the same time. The index finger is below the middle finger for the Thief and they are in the same plane for the Square.)

Stacy tried to figure out how to tie a Thief knot by beginning with her knowledge of how to tie a Square. The upright approach didn't get her very far, though, which was confusing. After all, as she indicated with her fingers, the Thief and the Square look very much alike. The circle of the Thief, as of the Square, seemed to be a vestige of the curve of the string, formed in the first step of the upright approach:

 6.2 6.5 6.6 6.19 **7.1.1.7** 8.1 8.2 8.4

Carol: *... have you tied this one (the Thief) before?*

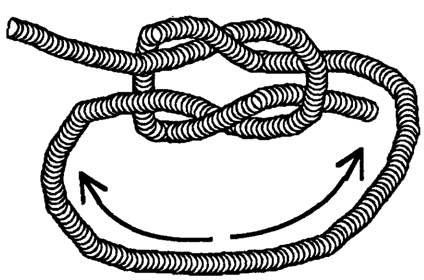
Stacy: Yeah, I think so. *(She tries a few approaches.)*



(She shakes her head, and partially undoes the loop.) Looks funny.

Can you think out loud as you try to figure it out? What's hard about it?

'Cause these look – because it looks like it goes first – like – *(motions with her fingers)*:

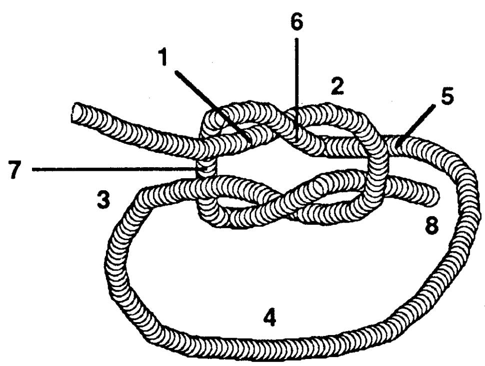


move a finger in the way the string seems to move

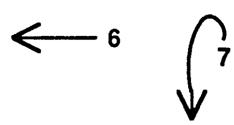
Leroy didn't just point, he practically danced through his discussion of the Thief knot. His gestures were important in conveying the motion that his sense of the knot contained:

6.22.1 **7.1.1.11** 8.2

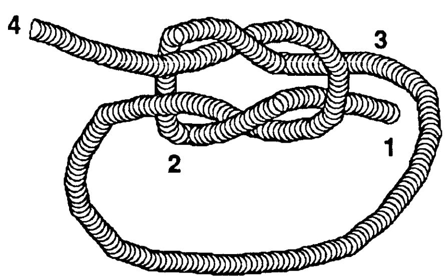
Um – well, if you was a tiny little ant, you would through here **(1)**, and go to – over there **(2)**, and out that side **(3)**, just go around **(4)**, go back in **(5)**, go through this side **(6)**, then go under **(7)**, and end up right here **(8)**.

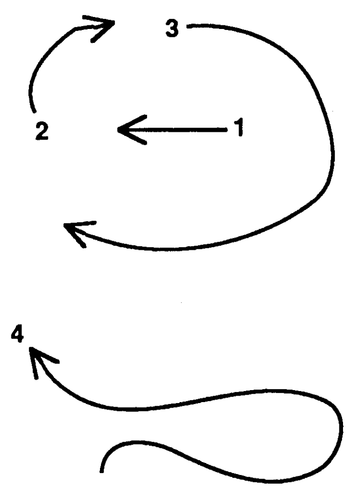


6, 7: He motions with his finger.



As his discussion proceeded, the gestures became broader and more inclusive:





Alice had a similar way of talking through the knots. Gesturing gave her a way of acknowledging both faces of the knot. Her use of the term "bridge" was another way of incorporating that concept of dimensionality, and of being precise in including every place that the ant must cover.

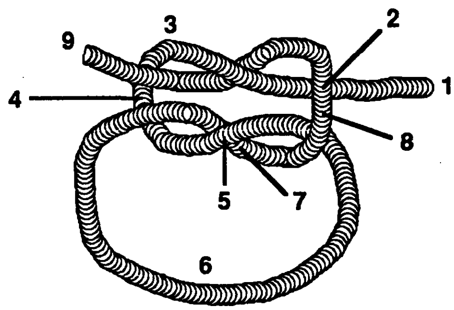
5.4.1 6.2 6.6 **7.1.1.13** 9.1

***Carol:** ... if you were a tiny little ant, and you were walking all along that knot, how would it look? Can you show me with your finger what you would do?*

Alice: Um – *(does the Square)* – I'd start from here (1), and then go on the bridge (2), and then go over there (3), and go on that other bridge (4) – *(she does motion with finger instead of turning knot around)* – and go to that one (5), and go all the way around (6), and go to that bridge (7), and go over there (8), and there (9). *(Her finger points left, out of the knot.)*

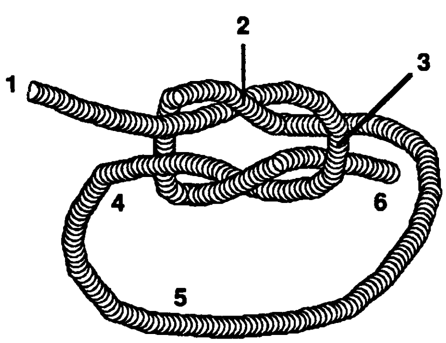
(4: Rather than turning the knot over to follow the string, she does a flip/curve sort of motion with her finger:)

(7 is in back of 5.)

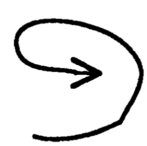


Okay, what about this knot (the Thief)?

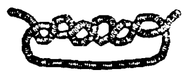
Start from here (1), and go under here (2), around there (3), there (4), there (5), and out here (6).



(5 is a finger motion showing the beginning of the final curve and the curve itself.)



Tony's "arch" and "twirl" gestures mimicked parts of the Bowline and the way in which he predicted the string would move into the form of the knot:



9.4.4 10.1

Tony: ... it seems to have that shape – *(He transfers the knot from both hands to his right hand. With his left hand he does a motion like an arch over the knot.)* – of a Bowline.

***Carol:** Which part is the shape of the Bowline?*

(He holds the knot with his left hand and with his right index finger follows the line along the loop and into the knot, doing a twirly motion with his finger as if to follow the curve of the string through the knot.) The loop with the – loop around it!

Tony's comparison of the Bowline and Packer's knots was even more animated:

Um – well – the shape and what's different about the Bowline is like an eye with things around it *(makes a circle with his right thumb and index finger around his left index finger)*, and this is more like an eye *(makes a circle with his left thumb and index finger; begins to put his right index finger into the circle, but instead grabs his left thumb with his right index finger and thumb)* kind of twisted *(tosses left hand as if to escape from the right hand's grip and dismiss the problem)*, with a thing around it, but higher up. It's strange. It's kind of like – it's kind of almost like a Bowline with a step added. The Bowline would go around, but this goes up and through *(makes corresponding motions around the Packer's knot)*.

move in the way the knot moves

Doreen used gestures to enact the salient features of certain knots, which became bases on which she arranged them into groups (see Section 7.2). The Granny, Stevedore, and Figure 8 go together:

5.4.1 **7.2.7** 10.1

... because of the way they twist – they like, twist, like – into each other. *(As she speaks, she gestures broadly and symmetrically with her hands, pointer fingers extended to show the curves going out together and then coming back in together.)*

The Surgeon's and Bowlines appeared more complicated; she saw the winds as their distinguishing feature:

These I put together because of the many different ways they go.

In many different – what do you mean?

Like ... they ... go around ... ***(She gestures broadly by crossing her two hands and then doing a circular motion with her right hand.)***

Her arms and hands became, for the moment, the ends of the string, as she herself seemed to become the knot in the process of being formed.

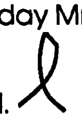
5.4.3 Pictorial Symbols

Knot-tyers everywhere seem to agree implicitly that, in order to communicate about knots, you need pictures. The children used images in books and videos as guides for tying and thinking about knots, and in their own descriptions of knots they used images involving a wide range of detail and abstraction. Many of their images can be found throughout this thesis; a few examples are included here:

6.21 **9.2.2**

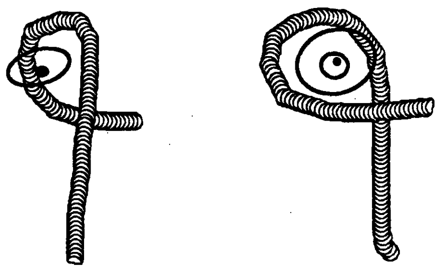
Alice interspersed diagrams with the words in her "Mr. Bird's Day" story:

<u>Mr. Bird's Day</u>

One day Mr. Bird decided to go take a walk around the city pond.  Then he decided to go swimming and he took off his pants and put on his shorts. Then he went in the pond . It started to get dark out and he got out like this . And went home.

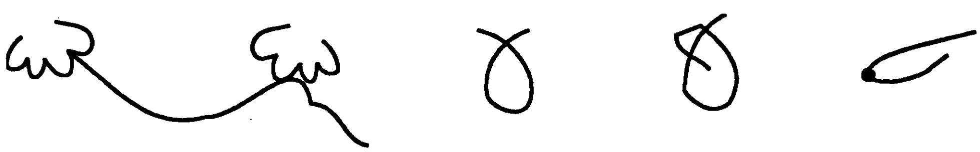
5.4.1 **9.1.3**

After hearing Soo Yong refer to the "eye" of a loop, Alice and Jill were confused about what he meant. It was clear that his reference was to the enclosed part of the loop, but did he mean an alphabetic "i," a personal "I," or a seeing "eye"? In a lengthy discussion, they decided that the negative space delineated by the string resembled the shape of a human eye. In order to clarify Soo Yong's term, they taped pieces of string on pages that were displayed on the Knot Language board, and drew representations of eyes in the loops:



5.2 6.16 6.17 9.1.1

As simply as they were rendered, there was no mistaking the hands in Jill's diagram of the sequence for tying an Overhand knot:



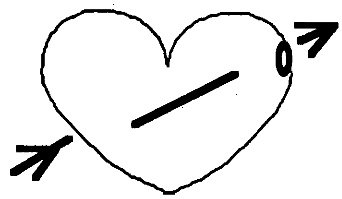
10.7

Rosella used a time-honored symbol in one of her letters to Soo Yong:

Dear Soo Yong Chang,

What do you mean by Monkey Fist. Well I don't care what you say but I still love you even if you have a girlfriend, and I hope you know that I like you because of your cheeks. Please be more specific about what you said about Monkey Fist, besides I like your handwriting it's not a mess.

P.S. Please
wright back.

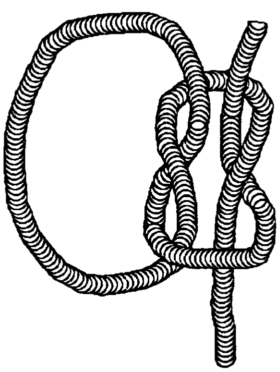


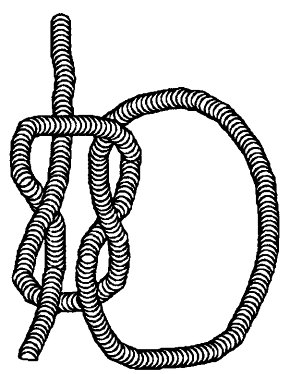
From: Rosella

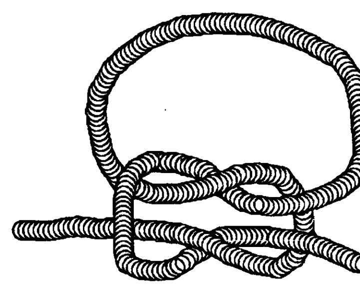
5.5 Orientations

The way in which we look at a knot is arbitrary to begin with. It may be a function of a preferred method of tying: the result is deemed "the knot." But as more such methods are learned, or as more examples of the knot are encountered, different orientations of the knot come to be appreciated, and accepted as being the same knot. And, as many children in the Knot Lab discovered, changing the orientation of a knot can be a useful strategy in figuring out a way to tie or describe it. Changes of orientation can include rotations in one plane, or turning the knot so that what was the "front" becomes the "back." Terms used to describe such changes are included here. Of course, each term can only be used relative to some arbitrary starting position.

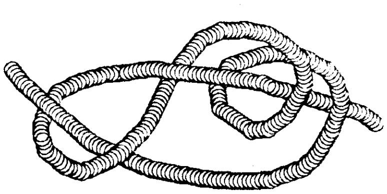
A knot can be rotated in the same plane, so that whatever side of the knot faces the viewer remains the facing side:


Square knot rotated right

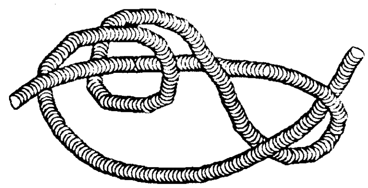

Square knot rotated left


"upside-down" Square knot

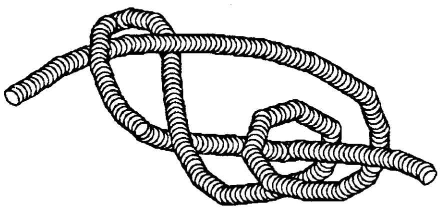
Or, a knot can be turned so that the front becomes the back. "Flipped" refers to a knot thus turned horizontally, and "inverted" refers to a knot turned vertically:



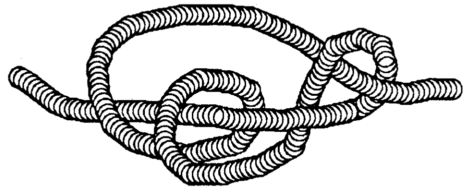
Stevedore



flipped Stevedore

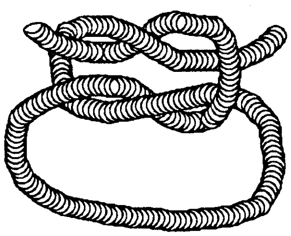


inverted Stevedore

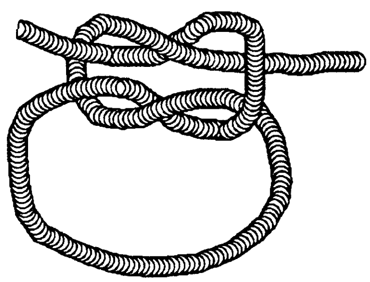


flipped & inverted Stevedore

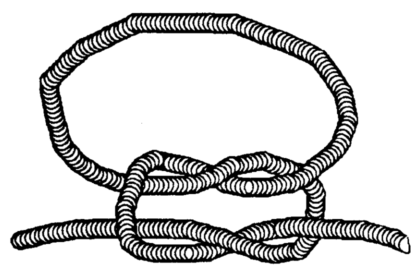
Notice that the Square knot seems to maintain its configuration when flipped, but when inverted its entanglement looks more like its mirror image. That is, a blockade-right Square flipped is a blockade-right Square (see Section 5.4.1, *blockade*), but a blockade-right Square inverted looks like a blockade-left Square!



blockade-right Square



blockade-right Square flipped



blockade-right Square inverted

Such phenomena caused various confusions and pleasures as the children discussed and played with different knots.

6 Strategies for Thinking and Communicating about Knots

When you get to the end of your rope, tie a knot and hang on.
– Franklin Delano Roosevelt

In *How to Solve It,* G. Polya develops a catalog of "methods of solution" of mathematical problems, called *heuristics*. He explains how this effort goes in a direction not always associated with the mathematical enterprise:

> Studying the methods of solving problems, we perceive another face of mathematics. Yes, mathematics has two faces; it is the rigorous science of Euclid but it is also something else. Mathematics presented in the Euclidean way appears as a systematic, deductive science; but mathematics in the making appears as an experimental, inductive science. Both aspects are as old as the science of mathematics itself. (Polya [1945, 1957] 1973, vii)

Here, I present a catalog that on the surface bears a marked similarity to Polya's: it is a listing of strategies that children developed to solve problems they encountered in thinking about knots and in making their ideas accessible to others. Like Polya, I strive to illustrate the diverse strategies that can emerge, this time in the particular domain of knots. But unlike Polya, I show how different individuals may conceive of these strategies, and under what varying conditions the approaches may emerge. In addition to outlining general heuristics, my interest is in describing how, when, and why particular children find them to be useful.

Papert suggests that "Turtle geometry" may be a "microworld" (see Section 3.1) in which the general ideas of heuristic problem-solving can find specific application, and thus become appropriable. Perhaps the same is true of knots.

> Mathematician George Polya has argued that general methods for solving problems should be taught. Some of the strategies used in Turtle geometry are special cases of Polya's suggestions. For example, Polya recommends that whenever we approach a problem we should run through a mental checklist of heuristic questions such as: Can this problem be subdivided into simpler problems? Can this problem be related to a problem I already know how to solve? Turtle geometry lends itself to this exercise. The key to finding out how to make a Turtle draw a circle is to refer to a problem whose solution is known very well indeed – the problem of walking in a circle. Turtle geometry provides excellent opportunities to practice the art of splitting difficulties. ... I believe that Turtle geometry lends itself so well to Polya's principles that the best way to explain Polya to students is to let them learn Turtle geometry. Thus, Turtle geometry serves as a carrier for the general ideas of a heuristic strategy. (Papert 1980, 64)

Many of the strategies described in this section can be seen as instances of more generally useful heuristics. Those knot-related instances include:

Assign a Name	*Vary the Context*
Isolate a Characteristic Part	*Anchor It*
Choose a Referent	*Do It in Reverse*
Consider More Detail	*Break It Down*
Loosen It	*One Bight or Two?*
Add Dimension	*Make It Tight!*
Turn It Around	*Stay with the Problem*
Give It a Test	*Use What You Have*
Look for a Pattern	*Make a Picture*
Use a Designator	*Tell a Story*
Debug It	*Find a Metaphor*

6.1 Assign a Name

Naming became an important theme in the Knot Lab: it was a way of distinguishing one knot from another, of engendering creative involvement with (and thus appropriation of) knots, of designating parts of string to facilitate instructions for tying, and of identifying parts of knots to facilitate discussion of the configurations.

In the initial meetings with the children who were to become knot-tyers in the Knot Lab, the paricipants talked about their conceptions of knots and experimented with tying a few knots. Some of their products resembled knots they were eventually to find documented in materials in the lab. A few examples are Eugene's "Disappearing Knot," which was an unknot, and knots that were either the Overhand or variations of it, including Rosella's "Pretzel," Juan's "Powers of Pain," Marcos's "Little Nikita," José's "Rosie Piggy," and Juanita's "Butterfly." Some of the invented knots were less distinguishable, though tthey often bore resemblances to the Figure 8 and Stopper knots: among these were the "Twisting Knot," "Knotty Knot," the "Bolt," "Roll On," "Barrel," Leopard," and "Thunder Blade." José's "Racadat Crew" was a form of braid. Chains of repeated knots included the "Cobra" and the "Zebra."

Alice, Jill, and Tony constructed a "family tree" on which they displayed knots according to similarities that they perceived in the formations (see Sections 9.1.4 and 10.1). To accompany the tree, Alice drew a portrait of each relative, forming a "family album" that included these personified knots:

"Pappy Pretzel"	(the Overhand knot)
"Great Grammy Stopper"	(the Stopper knot)
"Gram Granny"	(the Granny knot)

"Pop Square"	(the Square knot)
"Mammy Sheetbend"	(the Sheet Bend)
"Uncle Don Trumpet"	(the Trumpet knot)
"Cousin Surgina"	(the Surgeon's knot)
"Brother Love"	(the True Lovers' knot)
"Aunty Reefy"	(the Thief knot)[1]

Tying a knot involves knowing where and how to move the string. Thinking of the string in terms of different connected pieces can help in developing this knowledge. People often think about tying by imagining the string as being broken down into parts. By naming those parts, the tyer can keep track of them as they move. Similarly, appreciating differences between formed knots can involve comparing them piece by piece, and naming those parts is useful in producing descriptions of the knots.

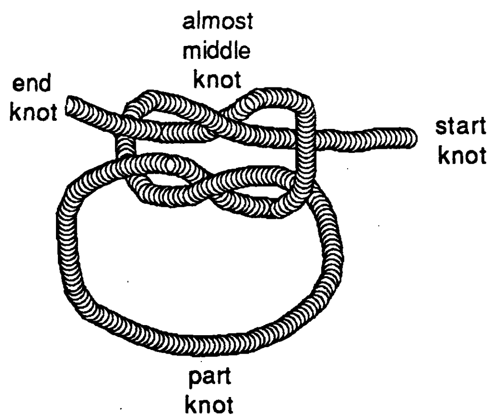
A macramé artist described his way of thinking of a piece of string as having a "head" and a "tail."[2] Tying a knot can be facilitated by such thoughts as, "the head goes under, and the tail goes around"; deciding which is the active part is often a key point.

After having some difficulty in following the imaginary ant's path along the Square and Thief knots, Celina gave names to different sections:

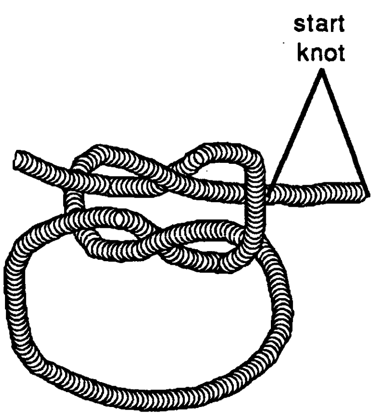
[1] Although "Reef" is actually another common name for the Square knot, the children used "Reef" to refer to the Thief, or Bread, knot (see Section 5.1). This variant of the Square knot was originally introduced as the "Mystery" knot and then, in a video by Soo Yong, as the "Thief" knot. But the name "Reef" is what became widely associated with it, probably both because the children misheard Soo Yong's pronunciation and because many of them confused the knot with a printed picture of the Reef – which looked, of course, very much like the Thief. Most of the children were able to distinguish these two knots by the end of the project.

[2] Chris Lombardi was one of five participants in a panel discussion of knots at M.I.T. during the "Science and Whole Learning" 1989 summer workshop. Other participants included a sailor, a practical knot tyer, a learning researcher, and a mathematician.

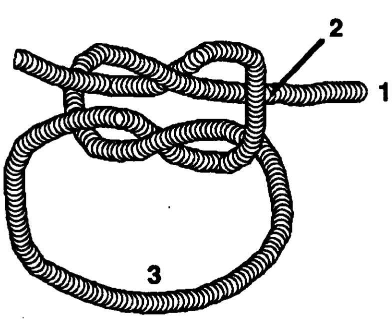
7.1.1.9



It's all of it, 'cause –



Well no, this part – 'cause if the ant crawled on the string, it'd start right here **(1)**, and then it goes down here **(2)**, and around to the middle part **(3)**.

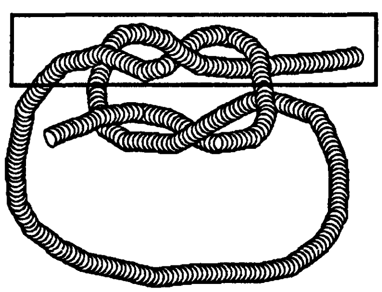


6.2 Isolate a Characteristic Part

When presented with the complicated problem of distinguishing the Square and Thief knots (see Section 7.1.1), two girls opted to begin by selecting a part representative of a key characteristic. The selections don't tell the whole story about each configuration, but are sufficient to make the distinctions they sought.

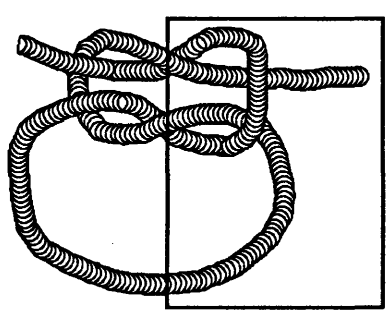
5.4.1 5.4.2 6.6 **7.1.1.13** 9.1

Alice focused on the top part of the knots:



5.4.2 6.5 6.6 6.19 **7.1.1.7** 8.1 8.2 8.4

And Stacy focused on the right side:



6.3 Choose a Referent

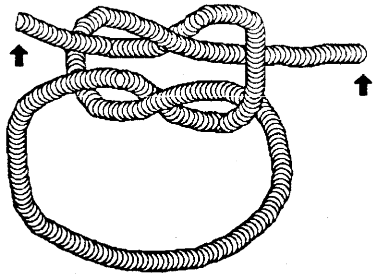
The interrelationships among parts of a knot can become so complicated that it sometimes helps to choose one part as the basis from which to describe others. The strategy can be useful in tying a knot as well. Naturally, different knot-tyers tend to choose different referents. Many choose the ends or the entanglement; Althea, in her discussion below, chose the circle. In a similar discussion, Curtis chose the entanglement of one knot, but described the ends relative to the circle of another knot. In order to tie a Turk's Head, Jill taped two wooden sticks to a piece of cardboard. She worked her way through the knot by rotating the cardboard and taping different points along the string as she wound it into the form of the knot. With the shape of the string changing and the board turning, the sticks provided a stable referent enabling her to keep track of where she had been and where she was going. In tying a Stopper, Tony went so far as to choose *himself* as the referent, by wondering whether he should wrap the string toward or away from himself.

Althea distinguished the Square and Thief knots based on where the ends were relative to a part of the knot that she had chosen implicitly as a referent, the circle:

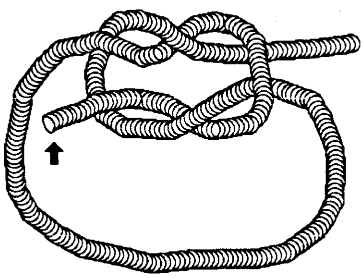
7.1.1.3

Carol: *...do these look the same?*

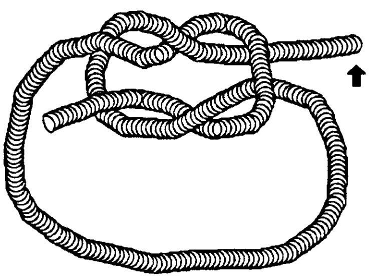
Althea: Not really, because...these two are up like this.



And one's under here,



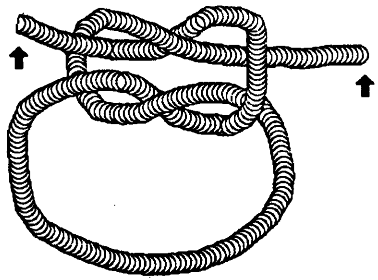
and this one's up.



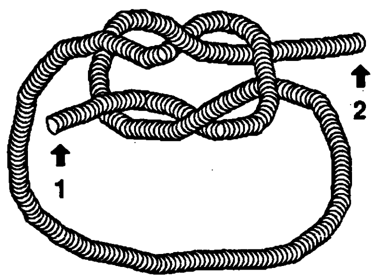
Curtis used the entanglement as the referent from which the ends of the Square knot "come out." But the circle became the referent in describing the Thief knot: one end went "under" the left part of the circle that leads into the entanglement, and the other end went "up," or above the circle:

5.4.1 **7.1.1.4**

These two strings come out straight,



and this one goes down under, like right there (1), and that line's straight up (2).

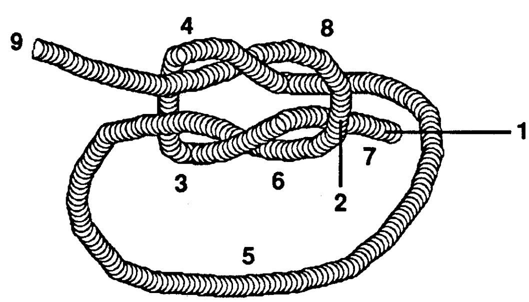


6.4 Consider More Detail

Crossings presented a difficulty for Patrick as he tried to trace the path of the Thief knot (see Section 7.1.1): at first, he broke the continuity of the path by going off on the wrong part of the string. To compensate, on the next try he looked more carefully, considering more detail as he traced the path. The effect was not only to help him keep track of exactly where the string went, but to slow him down somewhat so that no part of the string escaped his scrutiny.

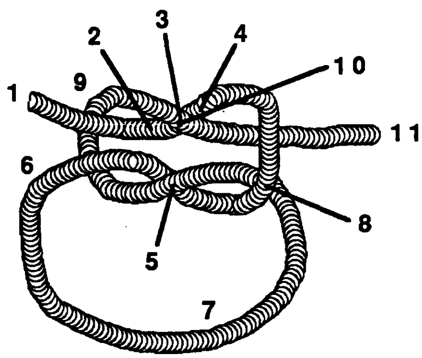
5.4.1 6.7 6.22.1 6.22.2 **7.1.1.15**

Patrick: Okay. I would start here **(1)** and then, I'd go – um – hmm. Let's see. It's hard. It's a blockade **(2)**. I'd probably go there **(3)**, and then I'd switch to here **(4)**, come around **(5)**, go cross over this **(6)**, go – oh no! **(7)** *(He realizes that he would leave the knot too early.)* Then come back here **(8)**, and go around there **(9)**. Okay, 'cause of those things in it, it's hard.



***Carol:** Uh huh. What about this one?*

This one. I start here (1), walk all the way to the end where this one (2) starts, and then I'd walk over this (3), and come back to where it would come in (4), walk onto the one that it was connected to (5) ('cause I couldn't go under it), and then I'd walk on this (6) and come over here (7), and then I'd walk on this (8), and come here (9), and walk over this (10) – like that (11).



6.5 Loosen It

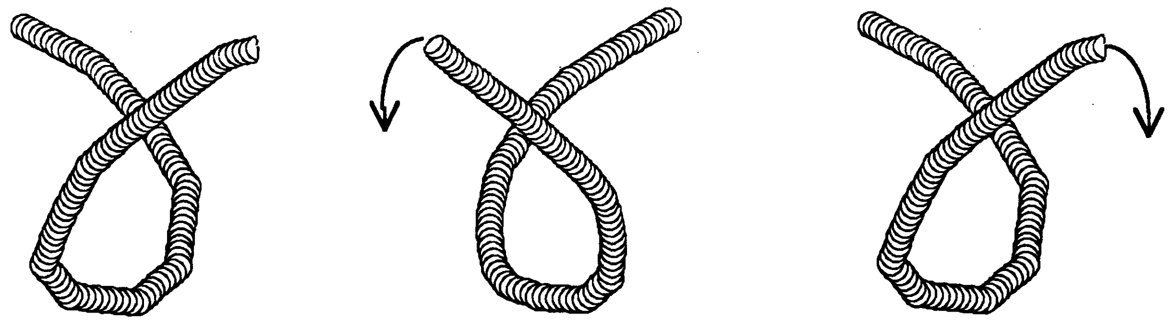
As they experimented with tying and describing various knots, children discovered that there was often an optimal degree of looseness or tightness that enabled them to appreciate the intricacies of a given configuration. A knot that was too loose had the effect of scattering the integrity of relationships among the strands; a knot that was too tight could obscure the crossings so they were not discernable at all. Jill was especially sensitive to the need for seeing varying degrees of looseness and tightness of any one knot in order to fully understand it; her concerns led to the development of the String in Motion board (Section 9.1.2). Many of the children used the strategy of loosening a tight knot that they were discussing in order to enable close examination of its features.

5.4.2 6.2 6.6 6.19 **7.1.1.7** 8.1 8.2 8.4

In trying to explain the difference between the Square and Thief knots (see Section 7.1.1), Stacy began by considering how the Square knot was formed. This approach led her to begin her description of the Thief with the circle – a reasonable starting point for a knot tied using the familiar upright, two-ended approach. But as soon as she got to the entanglement, she was in trouble. Then she had to switch the basis of her description to the confusing, asymmetric behavior of the ends. She could easily enough describe the ends separately relative to the circle: the left one was "up," and the right one was "down." But describing the way they came together was difficult; she could say only that they were "caught from in between."

Carol: ...have you tied this one (the Thief) before?

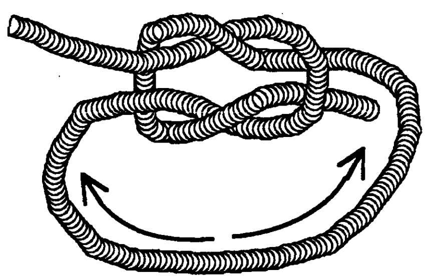
Stacy: Yeah, I think so. *(She tries a few approaches.)*



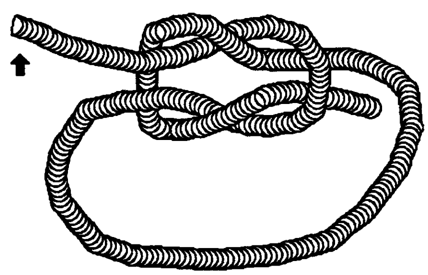
(She shakes her head, partially undoes the loop.) Looks funny.

Can you think out loud as you try to figure it out? What's hard about it?

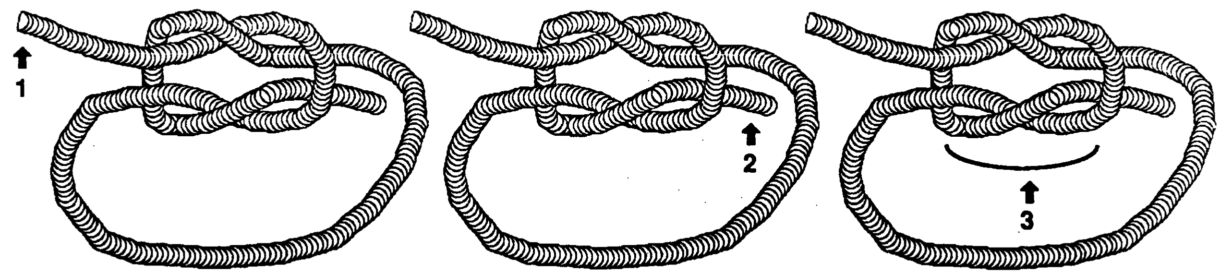
'Cause these look – because it looks like it goes first – like – *(motions with fingers)*



I'm following like, before the loop, and then how it – where the ends go – and like, they go up, and then – like, they go up –



– and then – like they go from being all the way up (1) and being all the way down (2), and then get caught from in between (3).



This approach hadn't gotten her very far, so Stacy reverted to a method that had been successful before: focus on the crossings. She loosened the Thief to get a better look at them. Recalling a Piagetian technique, I suggested a way of sorting through the tangle, by pretending that she was a small ant crawling along the knot and tracing her path as she goes. Immediately she shifted her starting point from the base of the circle to one of the ends. She gestured through a few broad tracings, gradually becoming more precise and finally lifting the knot so that she could turn it as she followed the ant's path. After tracing the path of the knot and then tying the knot correctly, she explained her success:

Carol: *...how come you could tie it that time but not before? What was different in the way you tied it?*

Stacy: Because it – it was, like, looser, so it was easier to see the path that it followed.

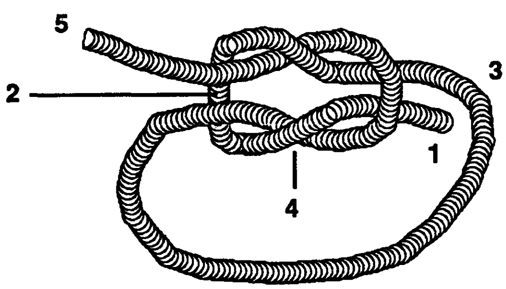
6.6 Add Dimension

In discussing various knots, many of the children concentrated solely on the side of the knot that faced them. Others found ways of bringing the reverse side into the discussion, or of otherwise acknowledging the knot's dimensionality.

5.4.1 7.1.1.10 8.2

Considering both faces of the Square and Thief knots helped Marcos to understand how the knots are different. He turned the knots over as he followed the path of the string, examining the crossings on the reverse side (the "bottom").

This one – let me see. I'll go there (1) – there – yeah! Like this. I'll go like here – like – here, on the bottom (2), and then I will go – let me see – what is it? Here – go around this (3), then go here on the bottom (4)? *(He lifts the knot to point to the "bottom.")* Right? And I end up here (5)!



5.4.2 6.2 6.5 6.19 7.1.1.7 8.1 8.2 8.4

Stacy wanted to keep careful track of the crossings as she traced the path of an imaginary ant crawling along the Thief knot. She gestured through a few broad tracings, gradually becoming more precise and finally lifting the knot so that she could turn it as she carefully followed the ant's path. (By regarding the knot in more than one plane, she could "do the unders," as she put it.) Stacy's move – from thinking about one surface of the knot to turning it and thinking in terms of more than one plane – is a strategy that few of the children employed. It helped her to understand the path well enough that she was able to imitate the pattern with a new piece of string, thereby producing a correctly tied Thief knot.

 5.4.1 5.4.2 6.2 **7.1.1.13** 9.1

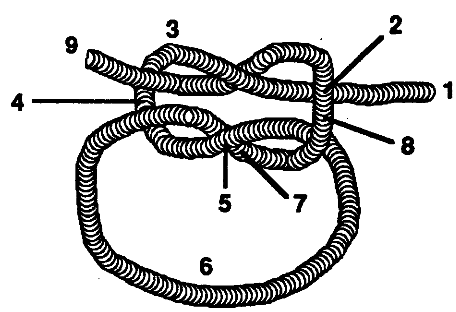
In following the path of the Square knot, Alice considered both sides – so that from any one view, the perpendicular strand at a crossing (what she called a "bridge") could be considered as going either over or under. When she came to a crossing, she traversed the "bridge" and continued on her way, or *imagined* the knot turned over so that she could cross the bridge and continue. Gesturing gave her a way of acknowledging both faces of the knot:

***Carol:** ... if you were a tiny little ant, and you were walking all along that knot, how would it look? Can you show me with your finger what you would do?*

Alice: Um – *(does the Square)* – I'd start from here (1), and then go on the bridge (2), and then go over there (3), and go on that other bridge (4) – *(she does motion with finger instead of turning knot around)* – and go to that one (5), and go all the way around (6), and go to that bridge (7), and go over there (8), and there (9). *(Her finger points left, out of the knot.)*

(4: Rather than turning the knot over to follow the string, she does a flip/curve sort of motion with her finger:)

(7 is in back of 5.)



 5.4.2 **7.1.1.5** 10.1

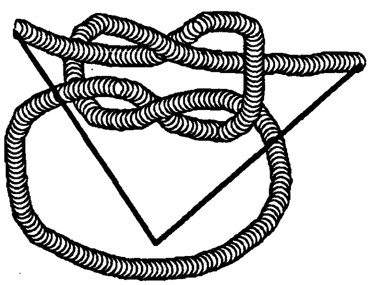
Eugene used his fingers to indicate the difference in positions of the ends. Not only did the "V" of his fingers point to the directions in which the ends extended, but he also created another mapping between finger position and the situation of the ends: the height of each finger was related to whether the end was "above" or "below" the circle:

Carol: *How about these knots – these two knots. Do these look the same to you?*

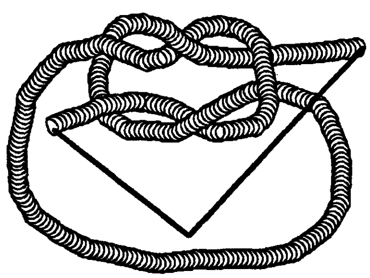
Eugene: Yeah.

What's the same about them?

They're the same – kind of. Because both of these go like that,



and both of these go like that.

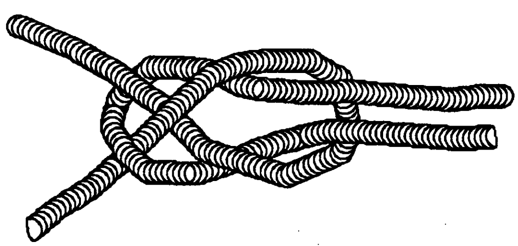


(He shows the two ends of each knot, using the index and middle fingers of his right hand to show the two ends at the same time. The index finger is below the middle finger for the Thief, and they are in the same plane for the Square.)

6.7 Turn It Around

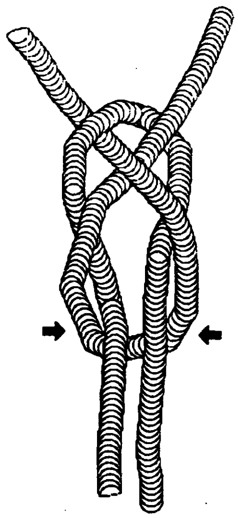
It is interesting to note the consistency with which different books and printed materials about knots tend to show knots in certain canonical positions, as though they were especially recognizable or understandable from those views.[1] However, for many children, varying the view of a knot was often the key to understanding it.

In working on the Sheet Bend with Tiffany, Celina looked at a picture showing the knot in its customary horizontal state:



She found the knot difficult at first, and turned it so that one of the loops better approximated the upright model with which she had become familiar in tying the Square knot:

[1] Indeed, the circumstance sometimes goes so far that certain knots have a particular name when seen from one view and another name when seen differently. The Lark's Head, for example, a macramé knot often used to anchor threads to a pole as a way of starting a piece such as a wall hanging, is an orientation of the Square knot. But the name, "Lark's Head," is given only to this specific orientation of the Square and, for that matter, only to a certain view of that orientation.



The new orientation helped the girls to understand and tie the knot. They wrote instructions for tying the Sheet Bend, which they decorated by taping two Sheet Bends in this vertical position to either side of the page. They made use of the idea of a "circle" in a manner similar to how they were accustomed to beginning the Square knot:

The Sheet Bend Knot

To make the sheet bend you need two pieces of string. Then you take one string and make it into a circle. Then you take the other piece of string and put it through the circle. It has 3 loops. First you the string under one part of the circle. Then on the next part of the circle. You put the same piece of string and put it over. The sheet bend is a simple and easy Knot. There are other knots like the sheetbend. To find out more about the sheetbend or other knots, look on the wall of knots or look in the book of knots. Sometimes people think that the sheetbend Knot is hard but it is not.

Tiffany Melinda Wilson

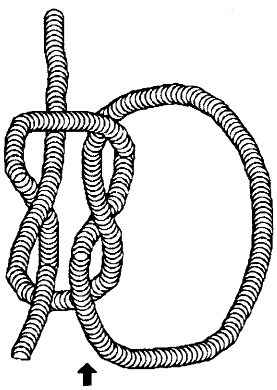
+

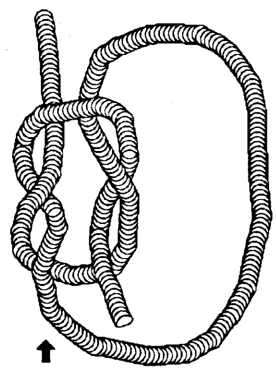
Celina Inabel Holmes

5.4.1 6.4 6.22.1 6.22.2 **7.1.1.15**

Patrick's preference for describing a part of the knot in terms of "left" and "right" led him to look at the Square and Thief knots rotated from the usual view. Turning the knots enabled him to see the difference between them, which previously had eluded him.

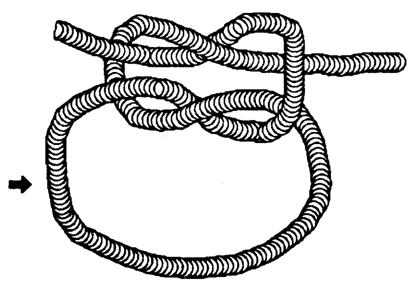
Patrick: They look the same – Oh! Wait a minute. *(He turns the knots sideways.)* I found it! Right here *(on the Square),* this Square – the one on the right is a loop, and the one on the right here *(the Thief)* is not. The one on the left is the loop.

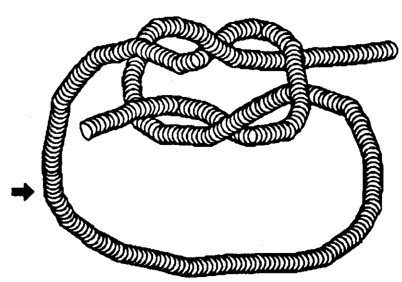




***Carol:** What made you suddenly see it?*

I was comparing *(he turns the knots upright again)* – to find out which one – and I was looking at this one, and this one *(he points to loops of knots)*, and I thought, "Wait a minute! Something's wrong here!"

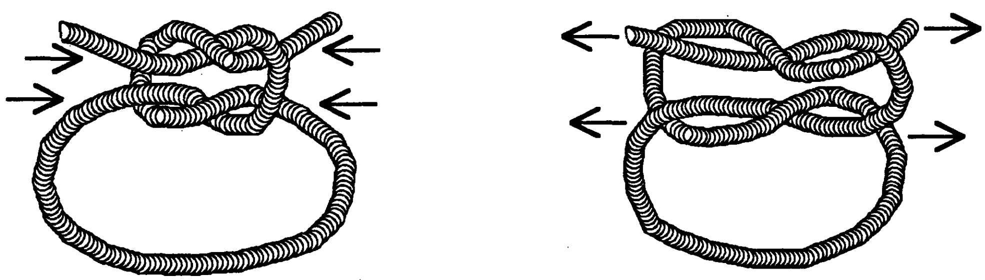




6.8 Give It a Test

The test for the Square knot was immensely popular. It was fun to move the knot back and forth, and to think of the knot as a movable object even in its finished state. Many children used the test as a way of distinguishing Granny from Square knots – although, as many persistent kids discovered, most knots will respond in some interesting way to this prodding. Leroy liked doing it to all of the knots. It was so important to him that he used it as a basis on which to group the knots in his final interview (see Sections 4.3 and 7.2).

5.2 7.2.9 9.4.2



(The "Square knot test" is a way of loosening and tightening the knot repeatedly, by pushing the two horizontal loops so they move apart, and then pulling them closer together.)

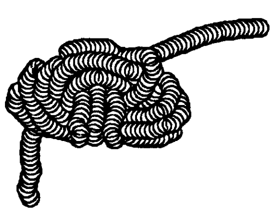
6.9 Look for a Pattern

A veteran craft-knot tyer, Althea had discovered that remembering a pattern can help in producing knots. She incorporated the strategy in her own invention:

> O.K Soo Yong Chang
> This is the million tie I made it up.
> You cross the string
> Put one string under and over and under till you want and then pull both strings and then you can do it again and again and again if you want.
>
> By Althea Moore
> Age 10

Althea's knot was essentially what the children knew as a Stopper, elaborated by repeating the pattern as many times as desired.

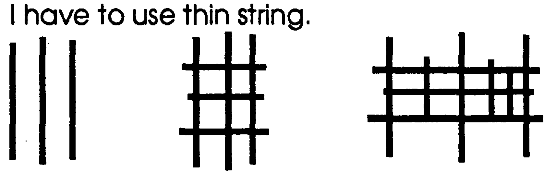
Understanding a complicated knot like the Monkey's Fist can be simplified if one notices the repeating pattern of three's that comprises the knot:



Leroy came to appreciate this pattern in comparing the knot to a volleyball. Jack's schematic drawing of the Monkey's Fist illustrated the knot's increasing complexity as he represented the sets of three strands layering over one another:

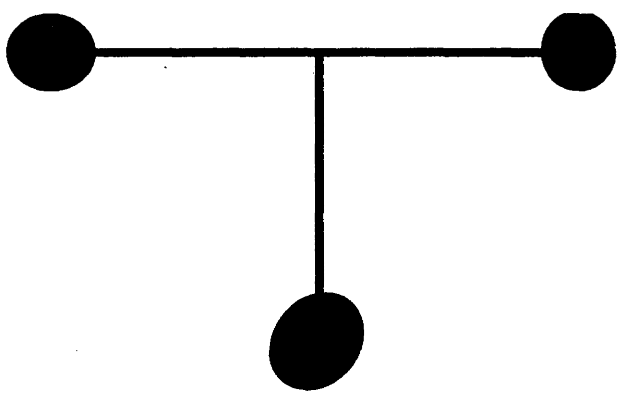
6.12

I'm an expert in the monkeys fist. But I found out that for me
I have to use thin string.



Jack Benson

The pattern seemed to appear again in Jack's drawing of a weapon that could be made of Monkey's Fists:



6.10 Use a Designator

Jill wanted to be clear about where things were and where they were going as she tied a knot. In constructing a display of the True Lovers' knot, she wanted an invariant name for the ends – something reliable, and more succinct than, for example, Stacy's designation of "the string that was left but is now right" in tying the Square knot. Jill used different-colored strings to help distinguish what was happening as the two knots slid together and apart:

9.3.2 9.3.4

<u>True Lovers Knot</u>

At the very top (on the black pipe) Notice the "square knot" to hold it in place. The knot holding on to the Lovers Knot (True Lovers knot) is the "Bowline". Notice the way the strings are two colors. It is that way so it is easyer for you to pull it.

To pull take the two strings with the black lego pieces Pull hard until the two pretzel knots meet. The pull hard the two string without anything on them. Repeat if wish.

Please Pull me

The strings had a tendency to get tangled during the sliding, and without the color differentiation it was difficult to keep track of which string was which. Yet seeing how the component knots moved with regard to each other was essential to understanding the configuration. Jill discovered that the different colors helped to keep important aspects of the configuration visible.

She also made use of another designator, the plastic LEGO pieces, which served both as handles to make pulling the two strings easier, and as markers of different ends of the strings. Each of the two pieces of string had one end with LEGO and one without, so the four ends involved in her True Lovers' knot were distinctly marked: each was either green or brown, and either had LEGO or did not.

6.11 Debug It

Bugs are problems in seeing or tying a knot. They can be thought of as strategies that don't work. An important aspect of learning to tie a knot is debugging – that is, identifying specific problems, figuring out how to overcome them, and incorporating the solution into the procedure for tying. Some of the bugs encountered by children in the Knot Lab were prevalent:

Cutting or selecting a cord too short for the chosen process or knot usually meant abandoning that effort and starting again with a better-chosen piece of string.

One of the most common bugs had to do with improperly tightening the knots (see Section 6.17). The prevalent temptation was to yank the ends of the string after the knot was formed – a tightening technique that works for many of the simpler knots but wreaks havoc with the more complex ones. As they worked on more and more knots, most of the children eventually found themselves engaged in a debugging process that eased them out of their original concept of what "tightening" means, to a more detailed, complicated, and refined concept that enabled them to construct the more difficult knots (see Section 12).

In macramé configurations involving several strands, many children tended to lose track of which strand should be moving at a given time. Even in knots with only two ends, confusion often occurred regarding which one was free to move and which was the standing end. The situation was complicated profoundly when these roles alternated.

In many knots, it helped to find ways of maintaining a distinct view or concept of the leading end the relatively static strand. Holding the strands too close together was one way in which children blundered this effort.

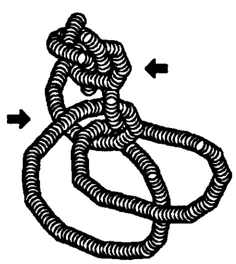
Related to this problem was a difficulty in remembering (or remembering to remember) which end had been designated implicitly as the moving end, in situations where maintaining this role was important. This error was complicated by the difficulty in discerning one strand from another as a knot became more complicated.

Similarly, children often had a tendency to get confused about whether one strand went over or under another one. This problem was more slippery than an either-or situation: it was not just a matter of A being over B, and B being under A. The use of the term "over" or "under" depended on which strand first had been chosen as the referent, and this choice could waver or be forgotten as the tying proceeded.

Jill and Stacy worked together on the Turk's Head and found mistakes in some of the over/under crossings in each of their knots. They realized that Jill's was an error of the eye – she had not traced a line correctly – while Stacy's was an error in interpreting a shaded part of the picture. Reading the dimensions of each crossing in the photocopied pictures was sometimes problematic. But in trying to trace a complicated path, Jill had mistakenly skipped to the alternate string at a crossing, which disrupted the flow of the knot. Her strategy was to anchor the starting point, and then key points along the way as she tied the knot; she could mark the picture accordingly.

In some of the more complicated knots, children found it difficult to match the number of twists that were represented in the pictures from which they were tying. Either they had difficulty interpreting the picture properly, or, even if they understood the picture, they tended to produce too many or too few of the twists in their own version of the knot. Compensating for these tendencies usually required feedback from others in the form of discussion or correspondence, as well as practice and perseverance.

Although it never found its way onto the Family Tree (Section 9.1.4), Jill wanted to tie a Bosun's Chair for the Bowline branch. Her first attempt had an extra wrap, and her next attempt had too few:



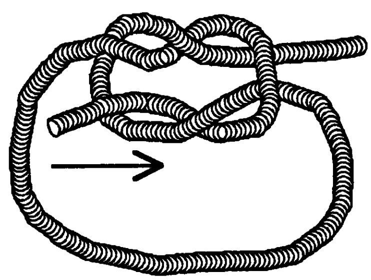
Another common error had to do with generating repetitions of movement that would result in a series of a particular form in the knot, such as in some of the macramé chain-type knots. Tying the patterns with sufficient regularity that the series would lay smoothly presented a significant challenge for many of the children.

Although keeping in mind an image of the finished form of a knot often helped in remembering or figuring out how to tie it, the technique also led to difficulty: children often preferred a certain orientation, usually some canonical position of the knot as shown in a printed source. But maintaining that orientation could severely limit possibilities in tying. Some, through experience or imagination, eventually seemed able to hold in mind more than one possible orientation for a knot.

After successfully following the Thief knot through two crossings, Maria fell prey to an extremely common bug: at the next crossing, she lost track of the path she had been following. Presented with a choice between continuing along the same strand or jumping to another strand of the knot, she got confused and took a "wrong turn":

***Carol:** Where would be a good starting point? If you wanted to start right at the end of the knot – or right at the beginning?*

Maria: *(points from the left rightward)*



Okay. So now you're at the beginning, and you're a little ant. Where do you go?

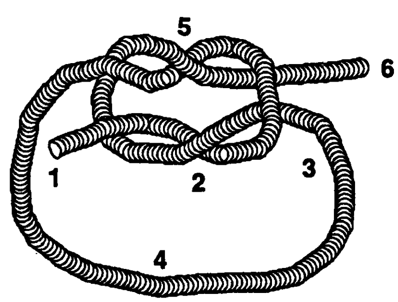
Along here (1) and here (2).

Okay. Then where do you go?

Down here (3)?

Okay.

And I go all around (4), and go like that (5, 6).

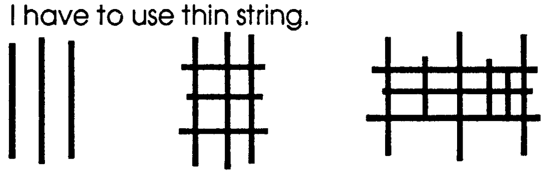


6.12 Vary the Context

In experiencing difficulty when tying a knot, many of the children found it helpful to vary the surface on which they were tying or the material that they were using. Maneuvering the string on a flat surface rather than holding it in an upright position often proved helpful, as did taping well-chosen parts of the knot to a table or card. A number of weights and textures of string were available, and children (like Jack) developed preferences for certain materials:


6.9

I'm an expert in the monkeys fist. But I found out that for me
I have to use thin string.



Jack Benson

(Jack's schematic representation of the Monkey's Fist captures the repeating-three's pattern that is the essence of producing the knot.)

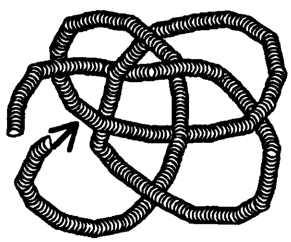
In working on the Family tree, Alice, Jill, and Tony got stuck on the Running Bowline, a relatively difficult knot to tie. Frustrated after several unsuccessful attempts, Tony decided to try a different approach: "I'll do it on a flat surface," he announced. Jill and Alice joined him in working through renditions of the knot on the Language Board (Section 9.1.3).

The choice of material for tying concerned many of the children. Several different kinds of rope and string were available. One of the kinds of string was composed of a soft, tissue-like material rolled and covered with the fibrous material that at first glance seemed to comprise the string. In cutting lengths for tying, the children discovered the stuffing and began pulling it out to reveal long rolls that could be unwound and played with. They noted the similarity of this thin, semi-transparent material to fabric-softener sheets and toilet tissue. Many of them enjoyed dissecting this string, and pulling and stretching the inner material. One of the boys wrapped himself up in it and pretended to be a mummy. There was also a large sailing rope, which Tony used to tie a Hangman's Noose and, later, a huge Stopper, arranged on the floor and pulled tight by several children. Leroy preferred this rope for tying the Monkey's Fist.

By entertaining such preferences, the children appropriated particular knot-tying efforts and came to better understandings of the knots with which they were involved. Varying the context of a problem in some way – by changing material, position, or however else – often freed the tyer to regard the situation differently and focus more clearly on the nature of the problem, rather than on some distractant.

6.13 Anchor It

The Turk's Head is a particularly involved knot, requiring many alternating over- and underpasses of each end:



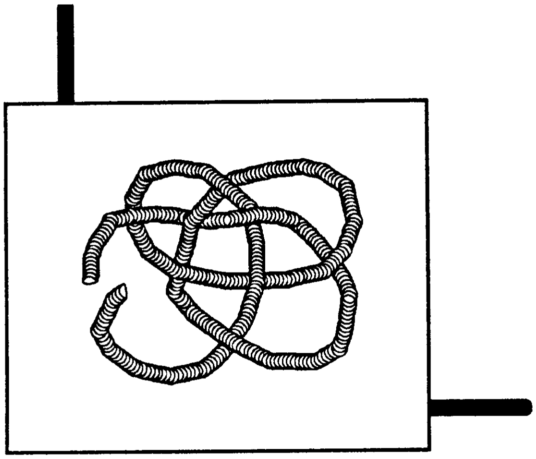
(the pattern continues until there are three strands in each quadrant)

Most children had no problem starting the knot, but quickly became confused as the circular, interwoven form began to emerge. It seems as though the string loses its singularity and becomes an indecipherable mass, like an image thrown out of focus. Doreen dramatized this perception by crumpling a long piece of string into a pile without crossings and calling it a Turk's Head, before asking for help.

A strategy for hanging onto the sense of the increasingly complicated blob emerged and quickly spread through the lab: as each new over- or underpass was accomplished, the child taped it to the table or a movable (and savable) cardboard surface. This technique served to define open areas within which the moving ends could be tucked and wound; otherwise, the blob of string tended to move around unmanageably as the tying proceeded.

9.3.2

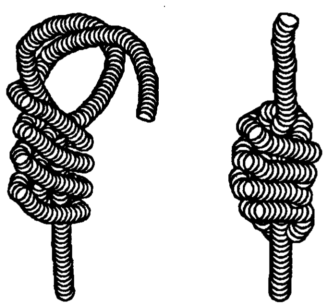
Jill elaborated the technique by taping two red sticks to one side of her piece of cardboard, as a device for keeping track of her tying. "It kept getting turned around," she explained. She had gotten mixed up and so sought an orientation device:



6.14 Do It in Reverse

Sometimes this approach meant simply looking at a knot differently, and sometimes its usefulness had as much to do with the encouragement provided by another's shared interest in a problem, but the strategy is crucial in managing some of the more complicated knots.

Leroy often became frustrated with the knots and with himself; he seemed to feel that he did everything backwards. He had some trouble in learning the Heaving Line knot, which looks much like a Stopper:



He tried following the pictorial directions in one of the books, but made two critical moves in exactly the opposite way from what the book suggested: the initial loop went under itself rather than over, and he wrapped the series of winds in a downward direction (away from the loop) rather than upward (toward the loop). Leroy recognized the discrepancy between his actions and those prescribed, but seemed to continue looking at things in this sort of "backwards" way. After all, with some effort, it had eventually been successful. He decided to let the style work for him rather than against him. When Jack began having problems with the Clove Hitch, Leroy suggested looking at the knot in a different way, "in reverse." In this instance, the suggestion meant considering the other end as the leading end – which Jack tried, ultimately producing the knot.

Jill finished her Turk's Head and worked on tightening it. At first she made it too tight and became confused – she had seen one that looked good before. We talked about how the tightening of this knot is like a reverse process rather than a completion step containing its own separate motion. Tightening the Turk's Head involves working back through the knot, smoothing slack in the string into an evenly distributed cloverleaf shape.

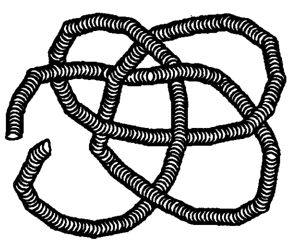
6.15 Break It Down

When the problem seems overwhelming, it often helps to break it down into smaller parts that somehow encapsule the situation.

Like many of the children, Stacy was fascinated by the decorative but complicated Turk's Head. Although she had managed to tie the Monkey's Fist, another difficult knot, the Turk's Head presented a greater challenge. She wrote to Soo Yong for advice, only to find that he had also had trouble with that knot. He did suggest a strategy, though:

> Dear Stacy,
> I tried the turk's head and it's very confusing. It's almost like tieing Monkey's Fist. Why don't you try to make it single line instead of double?
>
> Sincerely,
> Soo Yong Chang

Soo Yong's suggestion would produce a knot that looks something like this:



Normally, the Turk's Head is an elaboration of this essential pattern, with at least two – or, more commonly, three – rounds of string in each quadrant. By guessing that tying a less complicated version of the same configuration may be a step toward learning the Turk's Head, Soo Yong broke down the problem in a way that ultimately helped many of the knot-tyers.

6.16 One Bight or Two?

Miksch's Law: If a string has one end, then it has another end.

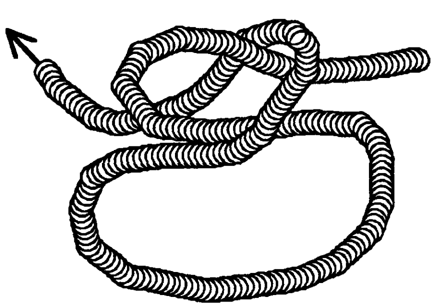
– *Macmillan's Book of Proverbs*

Two markedly different approaches to tying became prevalent in the Knot Lab. One approach involves holding a piece of string with both hands, an end of the string in each hand, and proceeding with some sequence of entwining the two ends, as Jill illustrated:

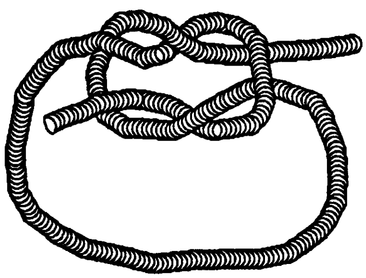
5.2 5.4.3 6.17 9.1.1

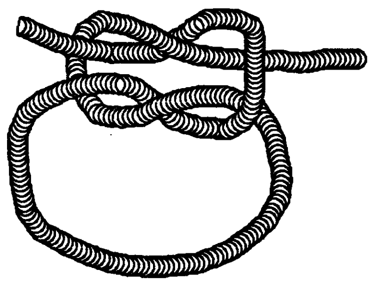
etc.

This approach usually involves some alternation of the ends, or at least thinking in terms of each end being potentially active. The other approach assumes only one active end: the string becomes like a snake or a worm, as some children thought of it, with one end leading the way through the knot:



Alice and Pablo pioneered this approach, but it gradually became more common as the participants' knot-tying repertoires increased. The Thief knot, in particular, encouraged adoption of this latter technique. The Thief looks so much like the Square knot that the impulse is often to attempt tying it in the two-ended, upright way. Yet a more fluid, snake-like approach produces the knot much more easily:



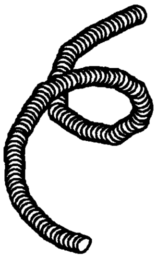


(Each of these knots can be easily produced by beginning with the string in an S-like shape and proceeding with the left end. But only the Square (shown at right) is easily produced with the upright approach.)

At the beginning of the project, Pablo had greater success than the other children in tying knots by following pictures in the *Boy Scout Handbook*. He was able to describe his process of identifying a starting point by choosing one of the ends as the active one, and then using his string to mimic the path of the knot pictured.

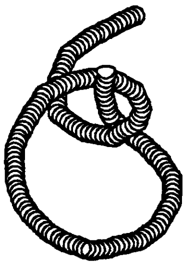
Tony used his own one-bight method for tying the Bowline, rather than the sequence reminded by the rabbit story:

6.21

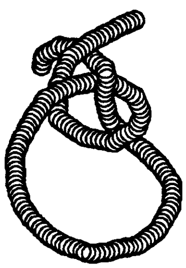


The Bowline Story

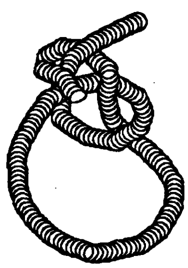
There's a hole in the ground.



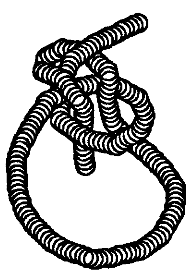
A rabbit comes out of the hole,



climbs up the tree,



goes around it,



and goes back down into the hole.

9.4.4 10.1

Carol: *How did you tie it?*

Tony: I – don't really know, I just – looked at the picture and I tried to follow my string, the string which I was currently using.

How did you follow around?

(He holds the picture so we can both see it, and points to the inner, leftmost end of the pictured Bowline.) I used – a mental finger. ***(He taps his head with his finger, then returns his finger to the picture and begins to follow the end as it curves around into the knot.)***

Uh huh.

I just – that's where it gets confusing –

Uh huh.

– 'cause I couldn't – 'cause the strings, they don't quite match.

Um hmm.

So, go around. ***(He continues following it around.)***

Um hmm. (He finishes.) Very nice.

When you – when you tie a knot from a picture, it's kind of hard to – you have to remember that, it doesn't matter where you start – on the – which string you start out with, that one or that one – ***(He indicates both ends on the picture.)*** – they both –

Um hmm.

– if it's one string, they both are gonna get – both to their places, eventually. ***(He sets aside the page of pictures).***

6.17 Make It Tight!

The concept of tightening is often part of children's very definition of a knot:

It's something with string that's tied together tight.

Like if you tie a rope or something that's hard to get out, just make a big knot.

When you tie your sneakers you loop it, then pull it real tight, it makes a knot.

So you can tie something real tight so it won't go apart, so you could have it real tight and it won't fall off, like your shoe; you tie it real hard.

Conversely, realizing that pulling the string *too* tight can render a knot unrecognizable, some children advise caution in tightening.


2.1.2

Ellen's set of directions for the Figure 8 knot was accompanied by a scribble-like diagram that specified where one part of the string went "under" the other:

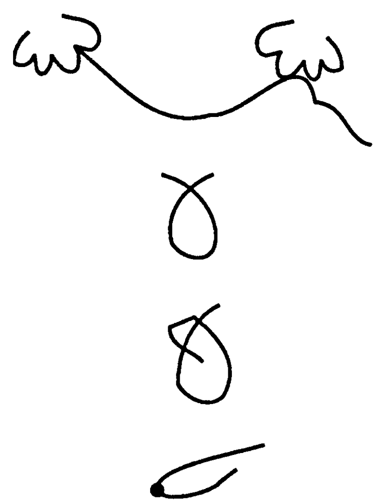
> string
> cross you hands
> take your right hand and your left hand
> leter go soft

Pete's instructions for the Figure 8 made a similar emphasis on keeping the knot slightly loose – if you don't pull it hard, you pull it "soft":

> put it in your ritt hand and hold one straight and the other
> one down the take the one that is hanging and flip it over
> thes put it thru the hole then tight it up soft and thats whow
> you do it.

5.2 5.4.3 6.16 **9.1.1**

Preference may dictate a certain technique for tightening the knot. Although Jill acknowledged that either end could move in the course of tying an Overhand knot, she chose to designate the left end as the moving end. Its role was consistent throughout the procedure: the "end that you put in the circle" is also the end that you pull to tighten the knot. This approach to tightening seems one-sided when compared to most others, in which the pleasure of pulling comes from a symmetrical final movement.



Over Hand Knots

Take both ends in your hands. Then take the left end and put it in back of the right end. Next take the lefl (either end) and tuck it into the circle the pull the end that you put in the circle and pull

Jill Bachman

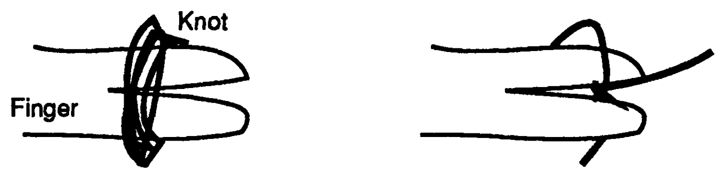
Patrick, dubbed the "Stopper expert" in the Knot Lab, expressed difficulties to Soo Yong, who offered the following advice in the return video:

You told me that you messed up when it's – 'cause you pulled it too fast and too hard. So when you're doing the Stopper knot, pull it slow and kinda straighten them out.

Maria included the emphasis in her instructions for a "knot":

M. Knot

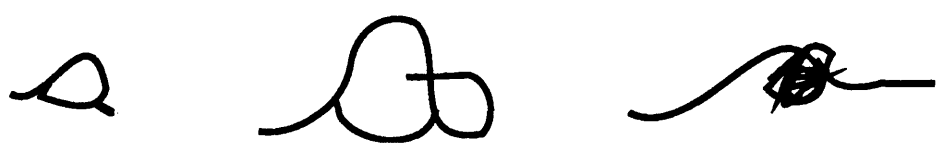
Roll it in two Finger then put it in the hole and pull it hard



Rosella made a similar emphasis:

Rosella

Make a circle. Then put the string in the circle. The pull it really Hard, And this is call the Pretzel.



The issue of whether a knot is loose or tight can play fundamentally in how the knot is understood. Althea, for example, was convinced that if you tightened the Granny, it would be a Square. Pablo, believing that the knots pictured in books like the *Boy Scout Manual* were to be taken literally as "the knots," insisted that the tightened version was not an Overhand knot, only the looser version was.[1]

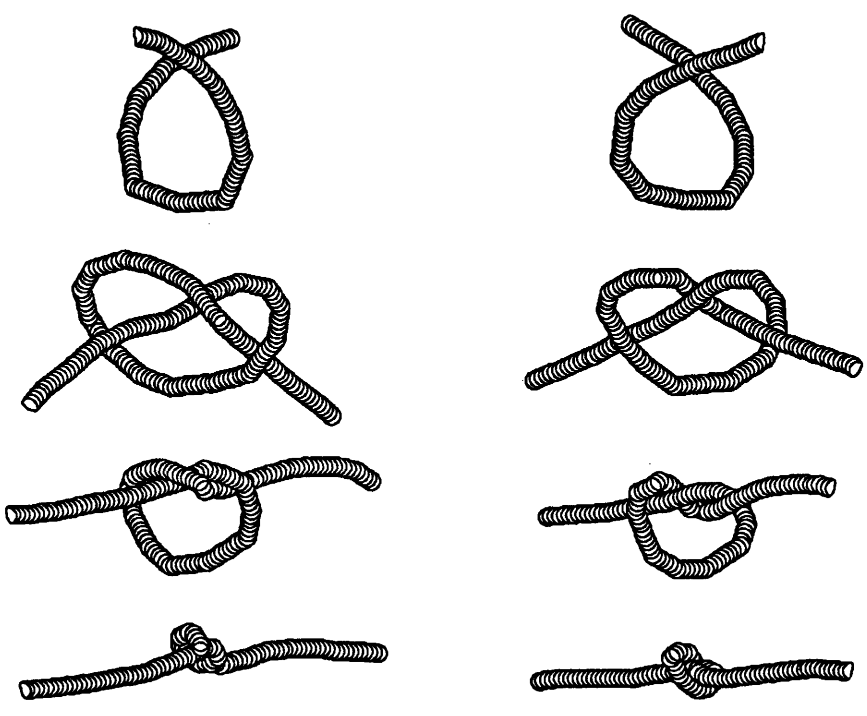
Jill generally did well in tying knots from pictures, but early in the project she complained about the distinct, disconcerting difference between her tightened, finished knot and the looser one pictured as a guide for tying. She was so bothered by the difficulty in

[1] See Section 2.1.1 and Appendix A for discussions by Piaget and Inhelder of such understandings.

recognizing variously tied and tightened stages of a knot as ultimately being the same knot, that she suggested expanding on the pictorial guides available in the Knot Lab by showing different stages and states of certain knots. Her suggestion led to one of the major displays in the Knot Lab, the String in Motion bulletin board (Section 9.1.2).

One aspect of this display highlighted particularly well a stylistic difference between Jill and Alice (see Section 9). Jill wanted to show different stages of each knot in formation; thus, these renditions of the Overhand knot were typical of the work she showed on the board:

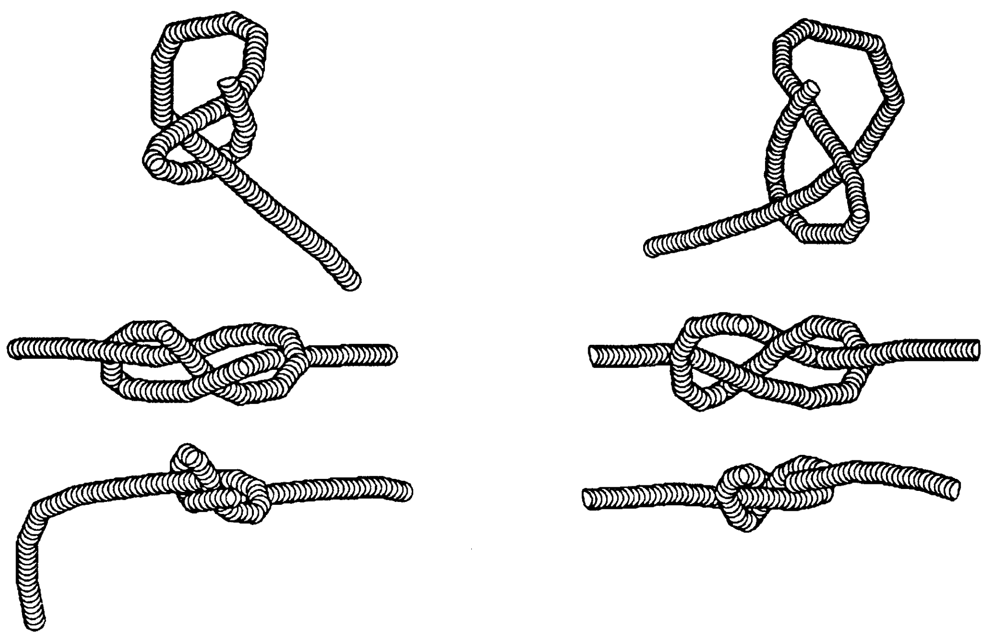
9.1.2 9.3.2



The knot progresses through both stages of formation and of tightening. Alice, however, considered the formation of a knot as a single, fluid process. Thus, her renditions of the

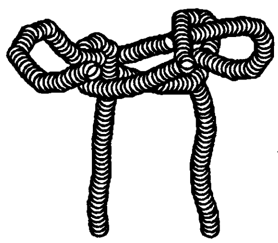
Figure 8 knot began with the knot virtually completely formed; what she illustrates in more detail is the process of tightening:

9.1.2 **9.2.2**



Tony, like many of the project's participants, encountered repeatedly a problem we might call the "pull-too-tight bug": For the simpler knots, tightening the knot is the final step, which many of the knot-tyers found satisfying and fun. As if to announce their glee at completing the knot, some would pull the two bights very hard, in an exaggerated expression of triumph at having won control over the knot. Tony often enjoyed this triumphant last step, but found as he got into the more complicated knots that exuberance was likely to lead to trouble.

The Trumpet knot, he discovered, can flip back on itself and become unrecognizable if tightened improperly. The process involves both entire hands, as the tyer must tug the two ends downward while pulling the inner loop through the two outer loops. Achieving the proper balance is important: pulling either the ends or the loops too much collapses the knot into an unusable shadow of itself. The finished knot is not meant to look like a rounded chunk in the string; rather, it is elongated and retains more open space than many other knots:



The Turk's Head presented another problem: if tightened insufficiently, it looks floppy, uneven, and unsatisfying; but if tightened too much, it becomes a blob without even a hint of its elegant cloverleaf shape. Like the Turk's Head, the Monkey's Fist requires an understanding of tightening as being the reverse process from tying, in that the end that finally leads out of the knot must be retraced and adjusted back through the knot. This conception requires such a modification of the children's original conception of what it means to "tighten," that a qualifying term or a different name probably would have helped to designate the critical new approach.

6.18 Stay with the Problem

> Quiet, calm deliberation disentangles every knot.
> – W. S. Gilbert (Budworth 1985, ii)

"Sticking with it" may be as important as any aspect of how the problem is approached. The emotional climate, individual and social, can be seen as a larger context that enables or prevents a child from staying with a problem.

10.3

Juanita was at first interested in the knot project, but became more and more frustrated as she experienced difficulty with some of the knots. Her reaction was to believe that she was somehow at fault, inherently "not good for this." After coming to this conclusion, she continued coming to the lab, but her work virtually stopped.

> To. Soo Yong Chang
> From. Juanita Lopez
> Dear Soo Yong Chang
> I will like to know who thought you how to do knots.
> answer back.

All of the kids in Juanita's group had asked him this, in addition to requesting new knots, so Soo Yong responded to all of them in the next video:

Rosella, Juanita, Maria, Julio, José, and Marcos: well, I learned all the knots in Boy Scouts...and, well, it's part of the skill awards that, uh, you earn for the things you get. And – I'll teach you how to do Surgeon's knot in a minute, but Trumpet knot, Monkey's Fist, and Turk's Head might take a little while so I'll teach you guys that later.

To. Soo Yong Chang
I am having a hard time lirning to do the monkeys Fist and I get very mad and ten I don't want to do enything.
From. Juanita L.

Dear Juanita,
Did you ever look in the book called "Sailor's Knot", or something like that?
If you didn't, look at the picture of monkey's fist in the book and read the instructions next to it. It should help you a little bit.

Sincerely,
Soo Yong Chang

Soo Yong Chang
thasnk but I think im not good for this I could only do some knots but it is difficult to do the monkeys fist.
From
Juanita
Lopez

10.5

Like Juanita, Pablo had done well in the beginning of the project, but reached a point at which he began to tie fewer knots, fumble with difficult ones, and generally lose interest. However, the friendship that he developed with Soo Yong perked his interest and encouraged him not to give up. He persevered and came to learn many more knots.

Dear Soo Yong,
On the sailing knot book I learn a knot I think. They are easy. If you want to send me another knots. I was fun doing them.

Sincerely,
Pablo Almirez

Dear Pablo,

So, do you like the book? I'm happy to hear that you learned many other knots.

I my guess is correct you are probably ready for the next knot. Would you like to tell me what you've learned?

Sincerely,
Soo Yong Chang

Dear Pablo
I haven't seen you doing anything lately why don't you try stevedore surgeon's knot and some other hitches. write back as soon as possible and tell me what you have done. If you have any questions just write it in the letter.

Sincerely,
Soo Yong Chang

Dear Soo Yong,
I've been working on the Packer's knot. Next time show me a nother knot and next week i'm going to work on the monkey's fist. Bye

Your friend,
Pablo

Dear Pablo,

This week I'm going to show you how to tie Sheet Bend. It is used to connect the two ropes together especially when their thickness is different.

Sincerely,
Soo Yong Chang

Dear Soo Yong,

I've tried to do the Monkey's Fist. It is not ease. I tried to learn by waching your tape over and over. I'm going to try until I got it.

Your friend,
Pablo

Dear Soo Yong,

I'm making a knot tree. And Eugene and I are trying too make the monkeys fist. So can you teach us. Thank you,

Pablo Almirez

6.19 Use What You Have

In the *New Mindstorms* videotape series, Seymour Papert (1986) demonstrates a process of learning to tie a bow tie as a combination of procedural and intuitive actions. Reading instructions from a book seems nonsensical at first – there is no starting point on which to base an interpretation of the author's instructions. Instead, Papert begins with a familiar bit of knowledge – pretending the necktie is a shoelace – which does not exactly match the problematic situation, but bears some resemblance to it. By working within this analogous context, he begins to formalize what he is doing, and finds similarities between his description and the one in the book. By going back and forth between these two realms – the "microworld" of tying and the formalized world of book instructions – Papert transfers knowledge from each realm into a newly understood one, that of tying the bow tie. He calls the approach "using what you have." Several instances of this strategy occurred during working sessions in the Knot Lab.

9.1.4

In developing the Knot Family Tree, Tony "used what he had" in bringing a familiar classification system to his thinking about knots. By working with the analogy in detail, his group introduced a basis for discussion about knots, which was to spread throughout the Knot Lab culture. Toward the end of the project, the group explained why they decided to build the Family Tree:

Carol: *How did you get the idea to do a family tree about knots?*

Jill: It was Tony's idea.

Tony: I had the idea, because a lot of knots are related, or they look alike, like the Square knot and the Granny. I mean, you know they're not the same, but if you put them in a family, like scientists do and things, you know they'd be in the same family and stuff. And like, things that would be in, would be the Sheet Bend and things like that.

Jill: Sort of, they have the same genes.

Why do scientists put things in families?

Tony: To help them know, like –

Alice: Organizing.

Tony: Yeah, to help them organize it, and to help them, like, 'cause if you wanna – kind of helpful, 'cause you can classify them that way.

Jill: Yeah.

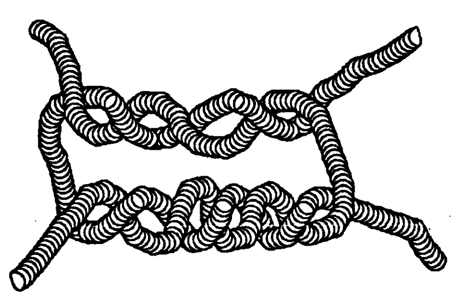
What other things do they put in families?

Tony: Animals.

Jill & Alice: Animals.

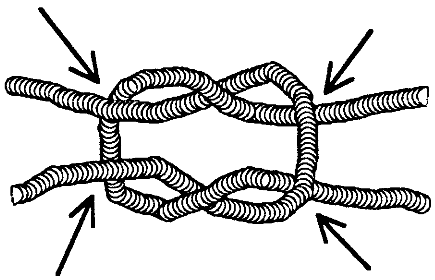
Tony had instigated a significant kind of cooperation among his working group: they were both imitating things they had encountered before (systems that classify animals and families of people) and inventing a way of using this approach to express the discovery that different knots have similar characteristics.

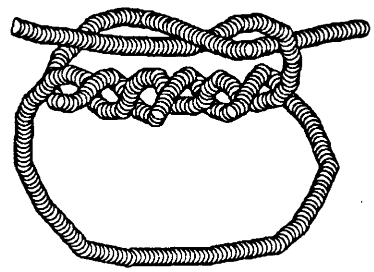
After mastering a few key elements of certain knots, Marcos came up with his own variation, which he taped to a sheet of paper and sent to Soo Yong as a letter. He assumed that he could "mix and match" elements from different knots, that the result would also be a knot, and that Soo Yong would know what it was:



what kind of knot is it?

Here Marcos was combining pieces of knowledge that he had learned from working with the Square and Surgeon's knots. From the Square, he made use of agents that might be called: "it's okay to use two strings instead of one," and "the over/under property of the top and bottom ends relative to the left and right loops should match."





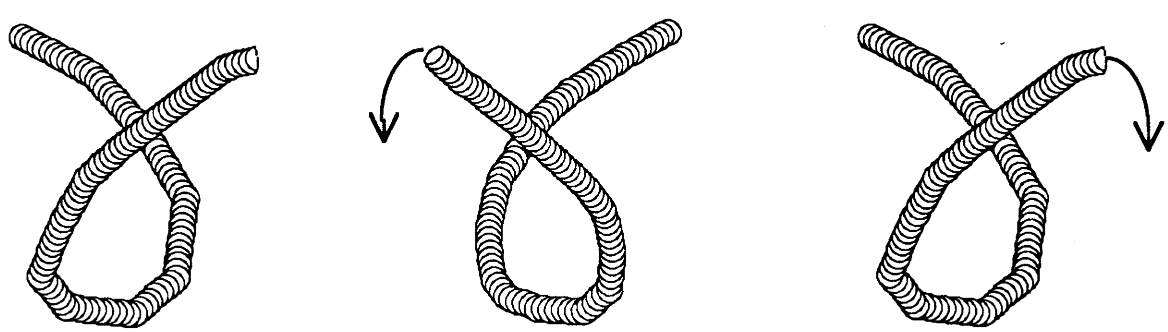
Noticing that the Surgeon's knot was pictured in the *Boy Scout Handbook* with multiple wraps at the bottom, Marcos made a knot that embodied the question, "Couldn't there be multiple wraps at the top, too?"

 5.4.2 6.2 6.5 6.6 **7.1.1.7** 8.1 8.2 8.4

Stacy knew how to tie a Square knot using an upright approach, but she ran into trouble when she tried to transfer the technique to the Thief knot:

Carol: *... have you tied this one (the Thief) before?*

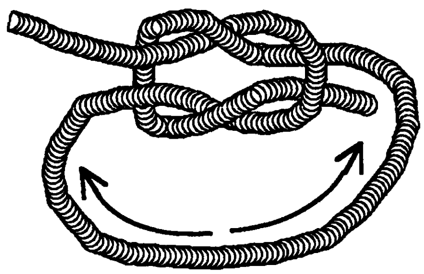
Stacy: Yeah, I think so. *(She tries a few approaches.)*



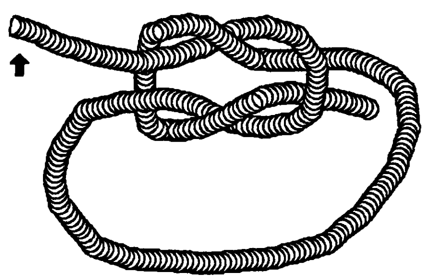
(She shakes her head, partially undoes the loop.) Looks funny.

Can you think out loud as you try to figure it out? What's hard about it?

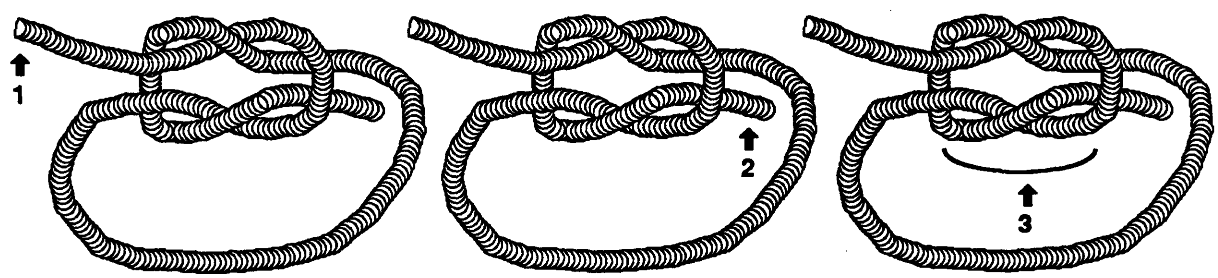
'Cause these look – because it looks like it goes first – like – *(motions with fingers)*



I'm following like, before the loop, and then how it – where the ends go – and like, they go up, and then – like, they go up –



– and then – like they go from being all the way up (1) and being all the way down (2), and then get caught from in between (3).



... you – were trying in a way kind of like the Square knot?

Yeah. I was trying – I was trying to think – and the Square knot, I just started like, you know how to ...
(She gestures with her hands, referring to the upright approach of tying the Square knot.)

6.20 Make a Picture

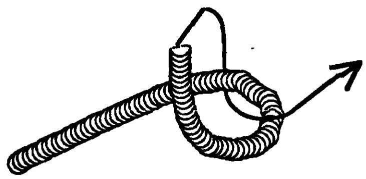
Patrick and Soo Yong came to the same conclusion as the authors of many of the books in the Knot Lab: if you want to explain how a knot is tied, use pictures. This strategy appears in Patrick's correspondence with Soo Yong:

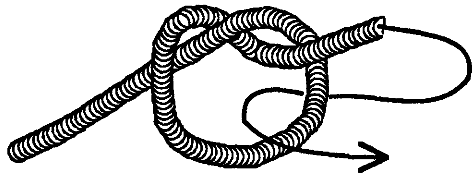
> Dear Soo Yong,
>
> I am working on some Events. Like the Knot Stories. I wrote The One on the Stopper. Hope to see you soon
>
> Sincerely,
> Patrick Gilmor

> Dear Patrick,
>
> I really would like to hear your stories. And YES, you will see me soon
>
> Sincerely,
> Soo Yong Chang

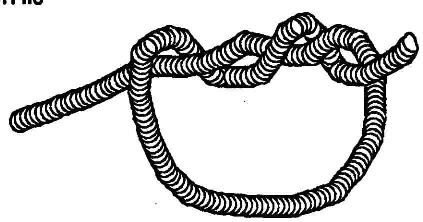
> Dear Soo Yong,
>
> I am writing to tell you that I am trying to be an Expert at the Stopper Knot Will you please send me directions on how to do the Stopper Knot
>
> Sincerely,
> Patrick Gilmor

> Dear Patrick,
>
> Stopper knot is one of the easiest not you've learned, and there should be a picture of stopper knot in the classroom. The way to tie it is almost like the overhand knot.

1. 

2. 

3. repeat step 2. 3 or 4 more times. THE END should look like this



Sincerely,
Soo Yong Chang

Dear Soo Yong,

Today I became an Expert. I Taught two people The Stopper. I know Many Knots. I know The Figure of eight, The Stopper and the Square Knot (All the easy ones!).

Sincerely,
Patrick Gilmor

(In showing the knot to his friends, Patrick had drawn pictures and commented that he liked learning knots from the "sailing book," a particularly well illustrated one, because the pictures help.)

Dear Patrick
Your Welcome very much.
If you have any more to talk about just write it in the letter.

Sincerely,
Soo Yong Chang

6.21 Tell a Story

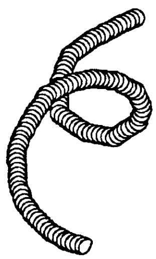
> A story is a little knot or complex of that species of connectedness called *relevance*.
>
> – Gregory Bateson ([1979] 1980, 13)

Creating a story with reminders of steps in tying a knot embedded in the plot is an old trick that many children in the Knot Lab adopted. Their stories appear throughout this report; I include a few here to illustrate the strategy.

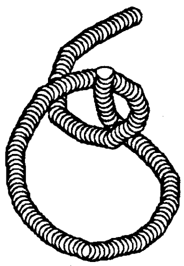
6.16

Most prospective knot-tyers have encountered some version of the Bowline story, which identifies key moments of the tying process:

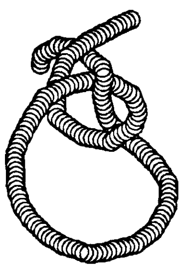
The Bowline Story



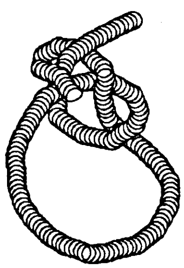
There's a hole in the ground.



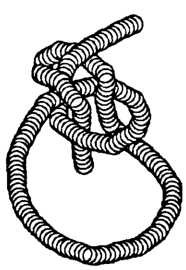
A rabbit comes out of the hole,



climbs up the tree,



goes around it,



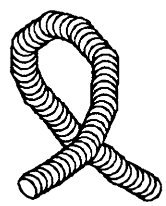
and goes back down into the hole.

Many of the children found this story helpful, amusing, and worth imitating. They developed their own stories, some of which relied on a metaphor such as a rabbit or snake, and others which were more blandly procedural.

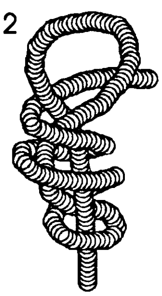

6.22.6 **9.3.2**

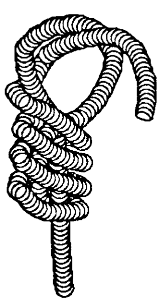
Jill created a story about the Heaving Line knot, but despite its fantasy content, she could not depart entirely from her intent to give instructions for producing the knot:

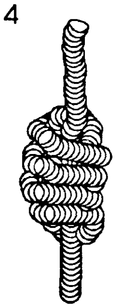
The Heaving Line Knot
(A story)

1.


There was once a faucet out in a yard. (The man who owned the house that the yard was in name was Mac.) Mac was gardening and watering his garden. When his was done he wrap the hose around the faucet (four times) Then he put the end through the faucet. Pull

2


3


4


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5.4.3 **9.2.2**

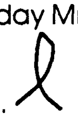
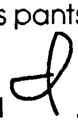
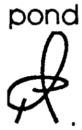
As she worked on the Packer and the Bowline, Alice wrote stories about them:

<u>Bowline</u>

One day a little birdy didn't know what to do. So he decided to take a walk around the pond. Then he got board again, so he went in the pond. He got sick and tired, so he went home and got lost. Then he went back to the pond and went swimming. But he got out on the other side and went home. His mommy was calling him home for dinner.

She added hand-drawn pictures in another Bowline story, in order to clarify the ways in which the plot paralleled the tying process:

<u>Mr. Bird's Day</u>

One day Mr. Bird decided to go take a walk around the city pond.  Then he decided to go swimming and he took off his pants and put on his shorts. Then he went in the pond . It started to get dark out and he got out like this . And went home.

6.22 Find a Metaphor

> Every thought is to some degree a metaphor. ... Our greatest ideas, like our evolutionary genes, need form only once, by accident, and then can spread from brain to brain.
>
> – Minsky ([1985] 1986, 299)

In trying to make sense of a knot, the children often made analogies to things that the knots seemed to resemble. In many instances, the analogies were so fitting that the knot seemed to *become* the object. Such identifications are a powerful strategy: they help to cast a problematic situation in more familiar terms. Metaphors have the effect of altering a conceptual frame in such a way that a child may become willing to spend more time or imagine a different level of spatial detail, which lead to a sense of "ownership" of the knot and its space. And often, simply by virtue of injecting some fun into the situation, the metaphors have a way of making the unfamiliar accessible.

In the following short sections, I describe uses of these metaphors:

ants	*volleyball*
highways	*garden hose*
snakes	*wrestling*
turtles	*family trees*

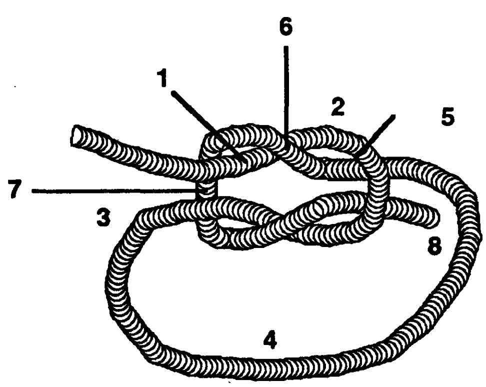
6.22.1 Ants

In comparing the Square and Thief knots during the final interviews (see Sections 4.3 and 7.1.1), several children followed my suggestion that they imagine themselves to be a small ant crawling along the surface of the knot. This technique, borrowed from the Piagetian school, encouraged spending time and focusing on details in thinking about and describing the object. Imagining oneself to be so small as to be surrounded by the knot can engender a "soft" style of thinking in which the child becomes very much a part of a system that includes both the knot and the child (see, for example, Turkle 1984). The knot is no longer an external object, but something that the child can "enter" and experience.

5.4.2 7.1.1.11 8.2

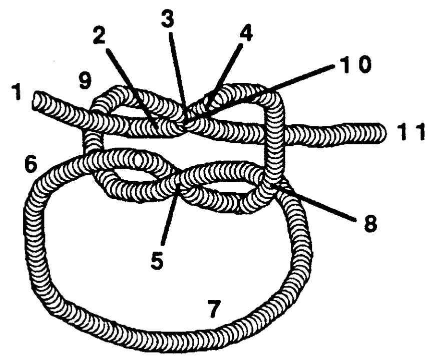
Carol: ... if you were a tiny little ant, and you wanted to crawl along those knots, how would you go? Can you show me with your finger?

Leroy: Um – well, if you was a tiny little ant, you would through here (1), and go to – over there (2), and out that side (3), just go around (4), go back in (5), go through this side (6), then go under (7), and end up right here (8).



 5.4.1 6.4 6.7 6.22.2 **7.1.1.15**

Patrick: I start here (1), walk all the way to the end where this one (2) starts, and then I'd walk over this (3), and come back to where it would come in (4), walk onto the one that it was connected to (5) ('cause I couldn't go under it), and then I'd walk on this (6) and come over here (7), and then I'd walk on this (8), and come here (9), and walk over this (10) – like that (11).



6.22.2 Highways

A recognized expert in practical knot-tying has explained his method of visualizing the final product in order to remember how to produce a desired knot.[1] He also thinks of knot-tying in terms of driving a car. Others involved with knots speak in similar terms, and, interestingly, the analogy to a highway came up for several children in the Knot Lab. One of them described it during his final interview, in which thinking this way about a knot clearly had helped him, after much difficulty, to distinguish the Thief from the Square. Another spent a good bit of time building imaginary roads out of LEGO bricks, in knot-like patterns.

Knot-Tying Expert:

I imagine myself starting here, sort of walking around, and driving a car pulling a rope behind me until ... I get to a certain point, and I turn right and make the loop. Eventually I come to a crossing, where I have to decide whether to go over or under. And so I just keep driving and after I go over or under, I come to something else and I go under that or over that, and eventually I go right or left, and just keep doing that until I go through the whole knot.

José:

... I figured, like, if I'm going on a trip with my parents and my family, we're going around on the highway. *(Here he begins tracing the path of the string in the Thief knot.)* So, this looks like a highway – go all the way, go up here, go around – looks like – you're always on the highway, and you turn around to be two places.

Eugene suggested that a road track could be like a knot, and others in his group got excited about the idea. During the following weeks he used LEGO bricks to build roads. Many of his constructions imitated pictures of highway designs in a civil architecture manual that I brought into the lab in response to his interest. Eugene called his creation

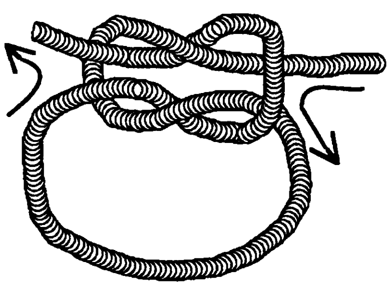
1 Frederick Browne is a founding member of the International Guild of Knot-Tyers. Conversations with him informed several aspects of this research.

"Highway B," and worked on building overpasses and underpasses, considering directional restrictions. Unfortunately the LEGO bricks were needed for projects by other groups in the school; Eugene got frustrated with repeatedly deconstructing and rebuilding his highway, and eventually abandoned the project. The idea lived on, though: Julio and Marcos busied themselves by constructing a network of over- and underpasses out of LEGO tracks. José watched them, and eventually made his own version, which he called "wayhi."

5.4.1 6.4 6.7 6.22.1 **7.1.1.15**

Patrick seemed to make a reference to driving in following the path around the Square knot:

So, I'd just actually make a U curve and make a U *(he traces out a curve with his fingers).*



He traced the path broadly; here, his concern was with the shape of the path rather than with the leaps from "underpass" to "overpass," and vice-versa, that he would have to make as the ant following the path.

In the following account of the development of techniques in calculating knot polynomials, "underpasses" and "overpasses," another apparent allusion to highways, are used as metaphors for crossings:

A mathematical label corresponding to any knot property that remains unchanged by deformations is called an *invariant*. One possible example of such an invariant is the minimum number of crossing points found in a drawing of a knot. This number often serves as the basis for organizing lists of knots. ...

Another approach is to use the arrangement of crossings in a knot diagram to produce an algebraic expression that serves as a label for the knot. In 1928, John W. Alexander discovered a systematic procedure for generating such a formula. ...

If two knots have different Alexander polynomials, then the knots are definitely not equivalent. ... But knots that have the same polynomial aren't necessarily equivalent. The procedure doesn't distinguish, for example, between left-handed and right-handed knots.

... The formula is the mathematical equivalent of systematically snipping the two strands of the knot at each crossing and refastening the ends so they are no longer twisted.

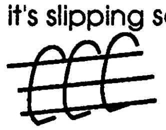
In 1963, mathematician John H. Conway, exploiting this new understanding, developed an easier method for computing the Alexander polynomial. Conway's method recognizes that a knot can be progressively unknotted by changing selected over and under crossings. Step by step, his unknotting game leads to diagrams with fewer crossings and finally to the Alexander polynomial. ... In 1984 ... knot theorists were suddenly and unexpectedly thrust into new mathematical territory overrun with novel invariants.

The mathematician who triggered the stampede was Vaughan F. R. Jones of the University of California at Berkeley. He found a completely new invariant, another polynomial that does a better job than the Alexander polynomial. Unlike the method for finding an Alexander polynomial, the Jones approach was based on the idea that overpasses and underpasses (or positive and negative crossings) play different roles. His discovery prompted a great deal of excitement in the mathematical community because his polynomial detects the difference between a knot and its mirror image. (Peterson 1988, 74-76)

6.22.3 Snakes

> ... a piece of knotted string with its ends spliced together so that it can't be untied is an excellent physical model for the abstract object with which mathematicians work. A mathematical knot can be thought of as a one-dimensional curve that snakes through three-dimensional space, then catches its own tail to form a loop. Like a circle, such a curve starts and ends at the same point and never intersects itself.
>
> – Peterson (1988, 70)

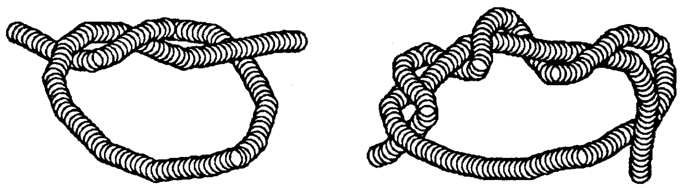
Jack used the metaphor of a snake and the form of a story to develop a procedure for tying the Monkey's Fist:

> A snake wraps around a branch 3  times. It feels like it's slipping so it wraps around 3 times around itself.
>  A bug lands on it's back and he chases it 3 times until it flies away.  A person finds him stuck and oulls him out coil by coil!
> The Monkeys Fist by
> *Jack Benson*

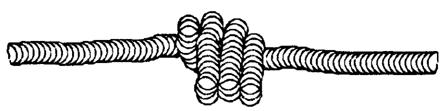
Patrick used the snake metaphor for his story about the Stopper knot. He illustrated the instructions with actual knots:

> Stopper
>
> There was asnake and one day he turned around and put his neck over it's body. Then It went under his body. He kept on going over and under For a while and

then somebody took the snake and pulled the head and tail. Then the person said, "That's a Stopper."



Patrick Gilmor



6.22.4 Turtles

One booklet in the Knot Lab took a craft-like approach to using knots as components of animal figures. Among them were a snail and a turtle, the latter being an animal that many of the children became interested in making. José, Marcos, and Julio decided to include it on their "Welcome to the Knot Lab" sign. As the children tried to construct the animal, their adventures with the Turk's Head, which formed the shell, became a theme through many weeks of work. Its interest probably had not only to do with the attractiveness of the difficulty and form of the ornamental knot, but with other associations to the turtle as an important animal in the children's school activities.

The children working in the Knot Lab had a good deal of experience with the Logo programming language, which relies on identification with a graphic object representing a turtle. The children enter Logo commands that instruct the turtle to do various things, such as moving about the screen while leaving a trail that can form complex geometric patterns. Thus their experience of interacting with a computer is really one of playing with a turtle.

Another association had to do with the school play, which the children were rehearsing during the time of the Knot Lab project. Several of the participating children had roles in the play, a spin-off from *Aesop's Fables*. Tony was the slow and steady tortoise, heckled by the hare but destined to win the race. His costume was a papier-mâché construction smoothed into shell-like sections and painted tortoise-like colors. One day he brought it into the Knot Lab and asked whether we could make shoulder straps and tie them onto it. Tony's arrival sent ripples of tortoise envy through the lab. For these computer programmers gone knot-tyers, the string tortoise had already resonated with their use of Logo's graphic turtle. Now, as we worked on Tony's costume, Alice, who had been making bracelets of knots and string for people, began making a tail to attach to his shell, and Tony admired the string turtle that Maria had made for the "Welcome" sign, commenting on how difficult it must have been to tie – "but it's cute." As a finishing touch, José and Marcos added thumb-tack eyes to the turtle to "make it Tony" (who always wore glasses).

6.22.5 Volleyball


6.9 8.1

In one of the videotaping sessions, Soo Yong had noted that the Monkey's Fist knot is similar to a volleyball in its repeating pattern of three elements. I brought a ball into the Knot Lab, and it became part of a display comparing it with the knot. Several children had discussed the similarity, but (like many details about knots) it did not seem to impress Leroy, who was often only mildly interested in the goings-on in the lab. He came because he was part of the group, and every now and then something would strike his fancy.

Although he had been occasionally interested in the Monkey's Fist as a potential accessory for martial arts, he hadn't been able to tie it successfully, and easily gave up trying. The more interesting part of the volleyball display was, for him, neither the knot nor the suggested comparison, but the ball itself. He eyed it for several weeks before garnering enough courage to ask whether he could borrow it and play with it outside during recess. He did borrow it, and respectfully returned it to its place when finished, apparently pleased that he had managed to do something out of the ordinary with respect to the lab.

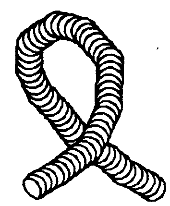
In his group's next session in the lab, something dramatic happened. In his usual half-hearted manner, Leroy casually began working with a few others who were trying to tie the Monkey's Fist. Suddenly he got up and ran across the room, grabbed the volleyball and looked at it. He stood with it for a few moments, studying it with excitement, and then began shouting about how it was the same as the Monkey's Fist. He hurriedly described the sets of three components, and ran back to the table with his piece of string, explaining the similarity to Stacy and working with her to finish tying the knot. With his help, she learned the knot too.

6.22.6 Garden Hose

Jill used the metaphor of a garden hose to help her reader imagine the coiling of the string in producing a Heaving Line knot:

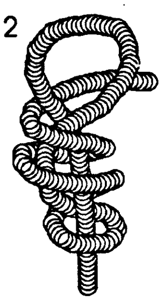
The Heaving Line Knot
(A story)

1.



There was once a faucet out in a yard. (The man who owned the house that the yard was in name was Mac.) Mac was gardening and watering his garden. When his was done he wrap the hose around the faucet (four times) Then he put the end through the faucet. Pull

2



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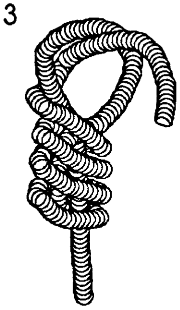

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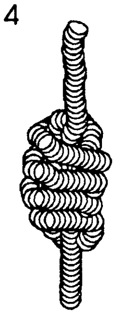

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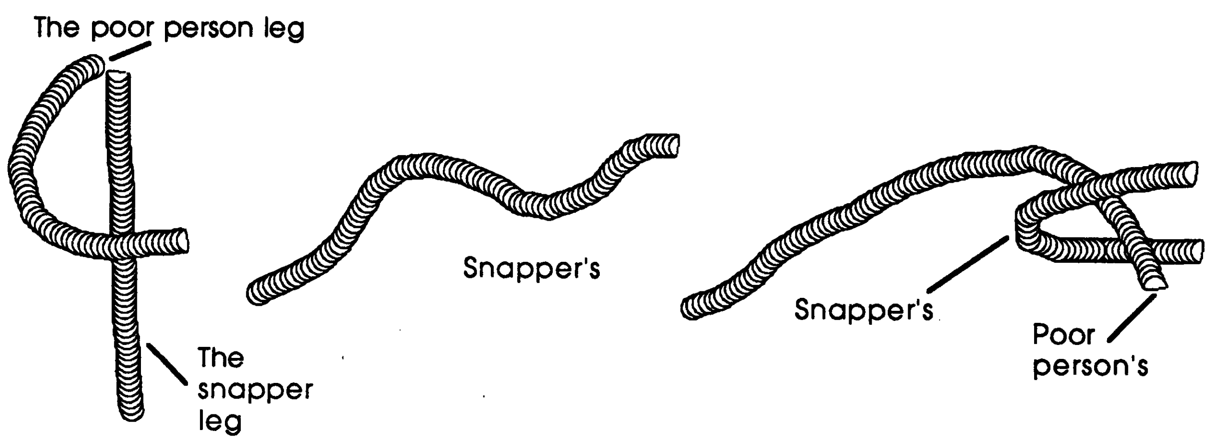
6.22.7 Wrestling

Leroy often preferred working by himself rather than with a group. Early on, he made knots that wrapped around his body or that he could swirl around like a martial arts champion. As the project progressed, he made several comments about how tying knots is like wrestling. One day he came into the lab with a black eye, explaining that he got it from "running into" his cousin. He began working on various knots with his peers, and then decided to make a pun: "OK, let's do the Figure 8. First you have to do a Figure 4." His reference was to a wrestling move known as the Figure 4, which he attempted to explain, first in an unsuccessful pencil drawing and then by using pieces of string to represent the legs of the two wrestlers. The move progresses through three steps:



(Leroy seems to have switched the labels in the final picture.)

6.22.8 Family Trees

Tony, Alice, and Jill were frequent visitors to the Knot Lab. They worked on individual projects, but often worked together, as a team. In noticing similarities between the Square knot, the Granny, and the Sheet Bend, Tony one day commented that the knots seemed related, as though they were in a family. A lively discussion followed, in which Alice and Jill picked up the analogy and the three decided to construct a "family tree" that would show how different knots are related. They proceeded to develop the display during the next several weeks. It consisted of twigs bound together to form a "tree" on which knots were glued or tied according to interpreted groupings, and a set of pictures that formed a kind of "family album" that personified each of the knots on the tree.[1]

As the display grew, other visitors to the Knot Lab came to like the idea, and discussions of family trees and relationships among knots became prevalent.

[1] This project is discussed in detail in Section 9.1.4. Related discussions appear in Sections 7.1.2.4 and 7.2.1.

7 Similarities and Differences between Knots

In the following sub-sections, ways in which knots can function as "microworlds" for exploring topological relationships come into particular focus. During the final interviews (see Section 4.3), children worked with knots by tying them, comparing similar knots, and arranging knots into groups according to perceived similarities. Many of the protocols included in the discussions that follow are from these interviews.

In several Knot Lab sessions, children had become involved in debates about the distinction between knots like the Overhand and "Underhand," and the Square and the Granny. Comparing knots was a useful way of seeing more clearly the elements of the configurations and coming to understand relationships among the parts. Likewise, consistencies from one knot to another formed a useful background against which differentiating features could become apparent.

As a way of further studying such comparisons, I asked participants in the final interviews to compare two other similar knots, the Square and the Thief. During the course of the conversations, some of the children got involved in comparing the Bowline and the Sheet Bend as well. Drawing from the lab-wide interest in relationships among knots – interest that the Family Tree had generated – I also asked each participant to arrange knots into groups according to which ones they thought should "go together." These activities proved useful in eliciting particular ways in which the children thought about the knots.

7.1 Relationships within Knots

Here, individual knots are highlighted as topological worlds that can sustain intricate thinking and dialogue about the relationships of *proximity* that they embody. In this sense, each knot is itself a *microworld* for exploring thinking about relationships of topology (see Sections 1.3 and 3.1).

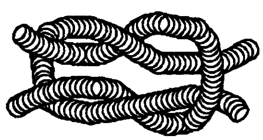
In coming to understand a knot, children often found it useful to place the knot next to another similar one and examine the ways in which the crossings, negative spaces, and other features corresponded. Such comparisons often triggered transformations of the perception of a knot as a relatively undifferentiated "blob" to a collection of particular and striking relationships. In comparing one knot side-by-side with another, each became like a magnifying glass through which the other knot could be better seen and understood.

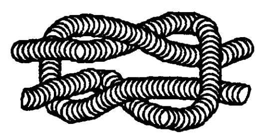
Fifteen of the discussions about the Square and Thief knots in the final interviews are included here. Three of the children also got involved in comparing the Bowline and the Sheet Bend, and Jill and Tony brought realizations from these conversations to their further explanation of the Family Tree (Section 9.1.4).

7.1.1 Comparisons of the Square and Thief Knots

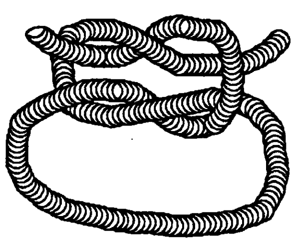
Many of the discussions in working sessions in the Knot Lab involved comparing two knots that are strikingly similar, the Square knot and a variant of it (the understanding of which can seem so elusive that even its name presents a problem: normally known as the "Thief knot" or "Bread knot," it was called the "Mystery knot" and the "Reef knot" in the Knot Lab).

What is especially deceiving is that in a very localized view, the two knots seem identical:

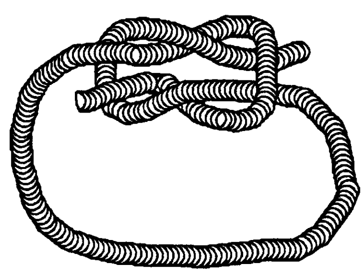




Yet, looking at the entire configurations, it becomes clear that they are different:



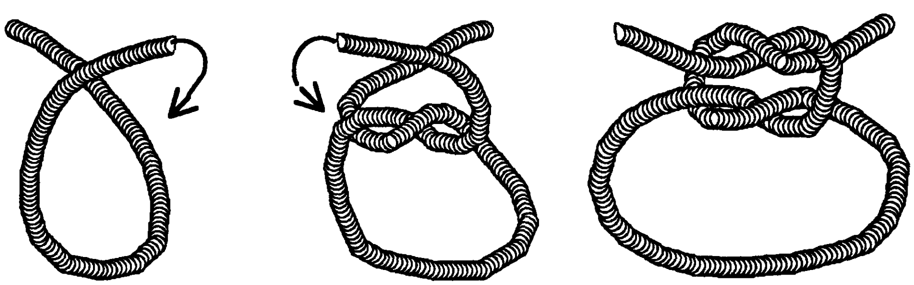
Square knot



Mystery or Thief knot

Articulating the nature of this difference posed a challenge for many of the children. Their explanations reveal a variety of ways of thinking about the knots. In some cases, confusion about the identical nature of the localized views persists: is the word "knot" to mean the only the entangled part, or the entire object? In most cases, the descriptions rely on some part of the knot having been (implicitly or explicitly) chosen as a referent. Frequently, the referent is the "circle," shown here as the lower part of the object (see Section 5.3).

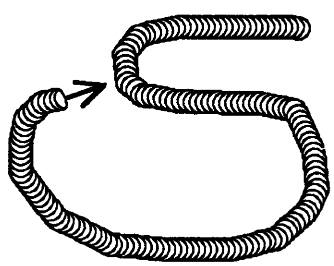
This orientation was the usual way of viewing the objects that were tied with the small pieces of string that the children normally used. No doubt this way of viewing the finished knots stemmed from the children's experience of producing them (see Section 5.2). The typical tying procedure leads to an object that we called a "knot," consisting of an "entanglement" situated above a "circle": by holding an end of the string in each hand, crossing them in some way, and proceeding with a specific set of winds or crossings, most of the knots that were of interest could be produced, as in this "upright" method of tying the Square knot:



Appreciating the difference between the Square and Thief knots is related to understanding how they are produced. For this reason, comparison of these particular knots shaped an important set of discussions: the comparison elicited conceptions of knot-tying as well as of the finished knots. The discussions tended to keep the childrens' thinking focused at a precarious sort of borderline. The choice of description in terms of process (tying) or product (the knot as a relatively static object) became revelatory of stylistic differences among the children in terms of how they thought about the knots.

The upright way of tying the Square and other knots had become customary, to the point of serving as a kind of "default" for many children as they began to learn a new knot. There is another fundamentally different approach to tying, though, which at first only a few of the children adopted (notably, Alice and Pablo). That approach involves thinking in terms of a single moving end that winds along a kind of pathway, wrapping around itself in certain places to produce the knot. The string becomes snake-like.

Interestingly, although both the Square knot and the Thief knot can be easily produced by this alternative method, some significant distortions of the "upright" method are required in order for it to produce a Thief knot. In fact, most children came to regard this effort as impossible: the Thief was tied in the snake-like manner or not at all. Many of them never managed to produce it, but those who did usually made some fundamental changes in their outlook and tying procedure. Such changes included laying the string flat on the table rather than holding it upright in two hands, and beginning the knot with some "S-like" pattern:



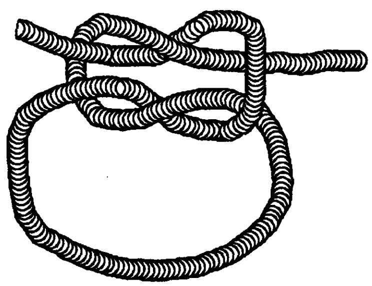
In the course of trying to figure out this approach, some children never quite reached the final solution, but struggled with the realization that the string had to be arranged differently. Their attempts at finding a workable arrangement sometimes took the form of an apparent expression of a sense of the movement embodied by the object. Typically, these forms didn't arrive at the completed knot that their originators were striving for, but seemed to capture something essential about conceptions of the configuration.

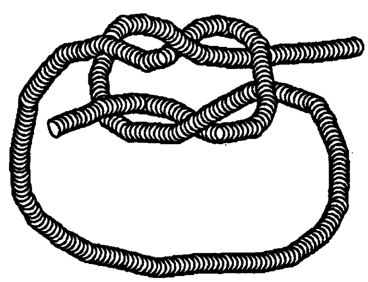
Instances of different childrens' approaches to understanding the relationships between the two knots are described on the following pages. Most of the descriptions are based on discussions that occurred during the final interviews.

7.1.1.1 Doreen's Square and Thief

Doreen sees no difference between the Square and the Thief knots. She includes the "Square with 4 Ends" with them, but differentiates the Sheet Bend. These decisions indicate that she is regarding the knot in a localized view that considers only the entanglement. The other parts of the object are not important to her discussion.

***Carol:** Okay, let's look in this group. Are these – what about these two knots? Are they the same knot?*

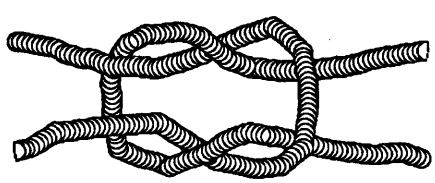




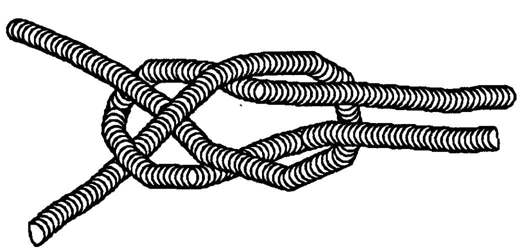
Doreen: I think they're the same.

Same?

(She picks them up and turns them over to look at them, and then looks at the Sheet Bend and the "Square with 4 ends.") These two are the same ***(Square and Thief)***. This one too ***("Square with 4 ends")***.



This one ***(Sheet Bend)*** I don't think is the same knot, 'cause of the way it ... ***(inaudible; she seems to indicate the crossed end)***.



7.1.1.2 Rosella's Square and Thief

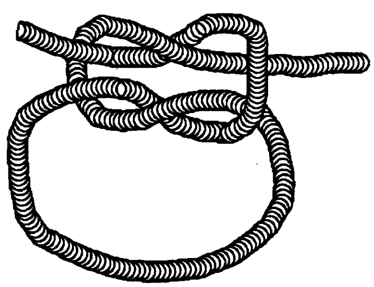
After noticing the superficial difference in size of the circles, Rosella differentiates the knots in terms of the ends relative to the circles. At first she is satisfied to characterize the ends simply as "out" or "in," but with prompting, she notes that this situation is not the same for each end of the Thief.

***Carol:** Is there anything different about these two?*

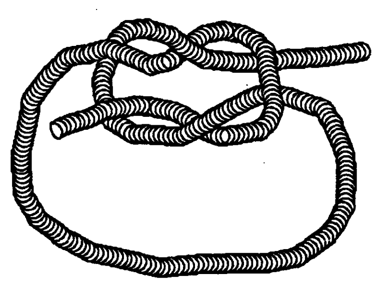
Rosella: No. The – oh yeah – the thing is bigger *(pulls on the circle of the Thief).*

It's bigger? What else?

Oh, this is – these are out *(touches the Square),*



and these are in *(touches the Thief).*



Both of them?

Yeah – not on both of them, one is in.

Oh, one is in.

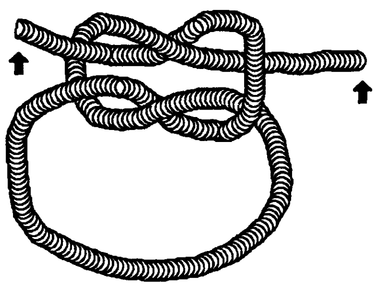
7.1.1.3 Althea's Square and Thief

Althea distinguishes the Square and Thief knots based on where the ends are relative to the circle. "Up" means above the circle, and "under" means within it – that is, under a part of the circle that leads into the entanglement.

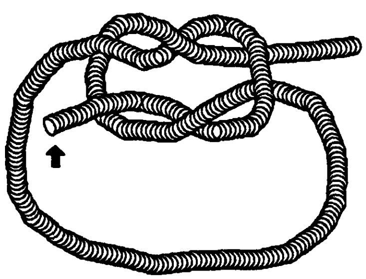
 6.3

***Carol:* ... do these look the same?**

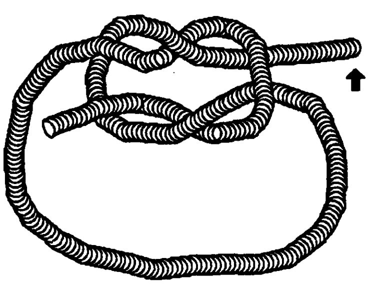
Althea: Not really, because ... these two are up like this.



And one's under here,



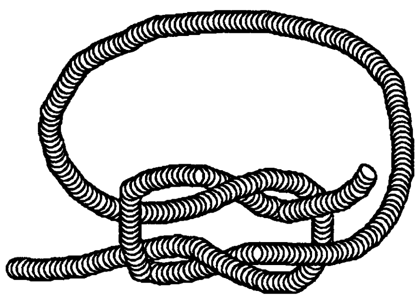
and this one's up.

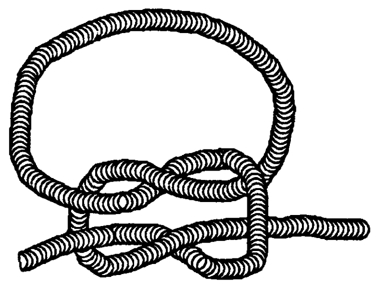


7.1.1.4 Curtis's Square and Thief

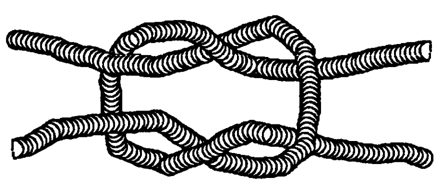
During the course of Curtis's interview, the knots have been strewn across the table, and he initially regards the Square and Thief knots oriented so that the circles are above the entanglements. At first he considers these to be the same knot – the Square knot – and brings in the "Square with 4 Ends" as another example. Thus he is concerned initially only with the entanglement.

Carol: *Okay – and what about these two knots?*





Curtis: Um – this is the – um – same as – ***(searches)*** this one! Wait a minute ***(gets the "Square with 4 Ends")*** –



– yeah, same as this one – the, um, Square knot.

Uh huh.

That one – this one, same thing! Square knot.

When asked to look again at the first two knots, inverted from their original position, he describes the difference in terms that indicate several simultaneous ways of thinking about the knots:

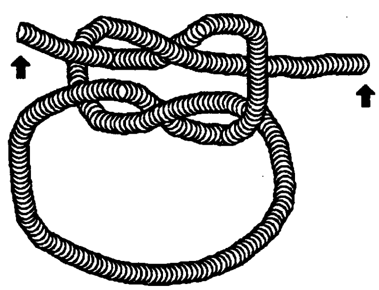

5.4.1 6.3

Carol: *Yeah? Okay. Take a look at – a closer look at these (the two "circular" knots, this time with the circle at the bottom). Are these really the same knot?*

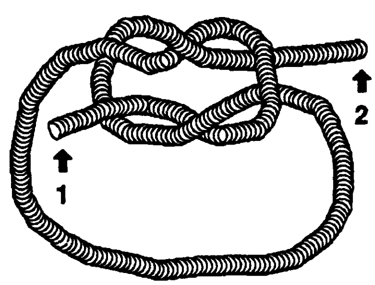
Curtis: No.

What's different?

These two strings come out straight,



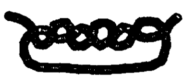
and this one goes down under, like right there (1), and that line's straight up (2).



Curtis uses the entanglement as the referent from which the ends of the Square knot "come out." But the circle becomes the referent in describing the Thief knot: one end goes "under" the left part of the circle that leads into the entanglement, and the other end goes "up," or above the circle. The term "straight" is present in his description of each knot. The terms "out" and "up" are sufficient to designate position of the ends relative to a referent; "straight," therefore, seems to indicate a property of the ends that exit the knot without pointing to a potential obstacle. Both of the ends of the Square knot are free in this sense (they "come out straight"), but only the right end of the Thief is "straight."

7.1.1.5 Eugene's Square and Thief

Eugene uses his fingers to indicate the difference in positions of the ends. Not only does the "V" of his fingers point to the directions in which the ends extend, but he also creates another mapping between finger position and the situation of the ends: the height of each finger is related to whether the end is "above" or "below" the circle. Thus he has implicitly chosen the circle as a referent.



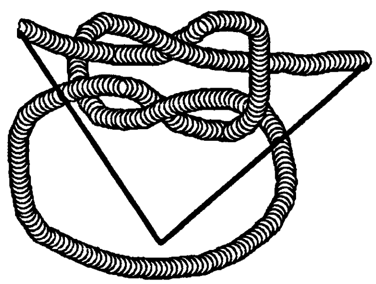
5.4.2 6.6 10.1

Carol: *How about these knots – these two knots. Do these look the same to you?*

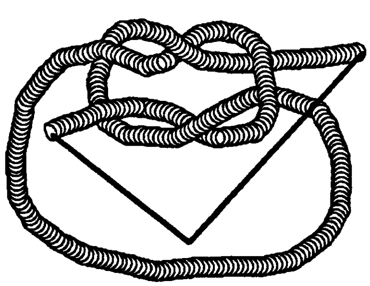
Eugene: Yeah.

What's the same about them?

They're the same – kind of. Because both of these go like that,



and both of these go like that.



(He shows the two ends of each knot, using the index and middle fingers of his right hand to show the two ends at the same time. The index finger is below the middle finger for the Thief, and they are in the same plane for the Square.)

Eugene's gesture is evidence of a way of thinking that relies on involvement of his body to gain an understanding of the knots – or, at least, to express that understanding. In a sense, part of Eugene has momentarily *become* the knot. His peculiar involvement is an example of the phenomenon of *body syntonicity* (see Sections 1.3 and 3.1).

7.1.1.6 Juanita's Square and Thief

Juanita's concern with the Square and Granny knots became a basis for a discussion about differences, which continues in her comparison of the Square and the Thief. At first, she describes the Square knot in terms of the ends relative to the entanglement: the ends at the right go "under" the right loop, and the ends at the left go "across" (or over) the left loop. The Granny, however, has one end "in the bottom" (under the left loop) and one "in the top" (over the right loop):

Carol: *How about if – let's take a look at these two (puts Square and Granny close together). You – you ... [described] these before, right?*

Juanita: Um hmm.

That's called what? (pointing to the Granny)

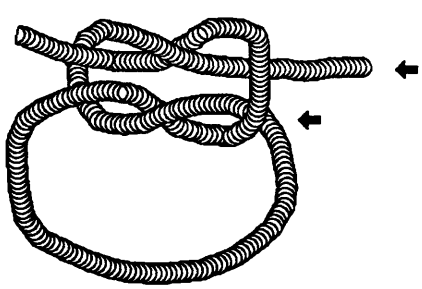
The Granny knot. *(Carol points to the knot above it, the Square.)* And the Square knot.

How can you tell them apart?

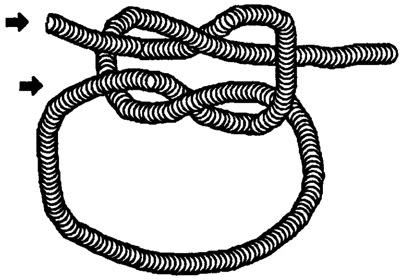
Because *(thinking)* – you want me to tell how I could do 'em apart – *(Carol nods her head "yes.")* – or you just want me to tell you about how I could know how it is?

Um, how do you know how it is.

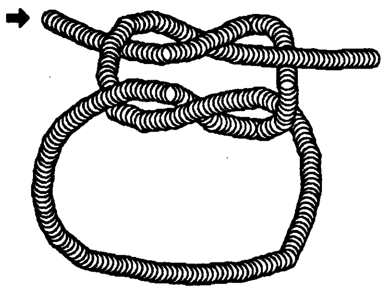
(She holds the Square knot and points to parts of it.) Because – um – the Square knot has these two under,



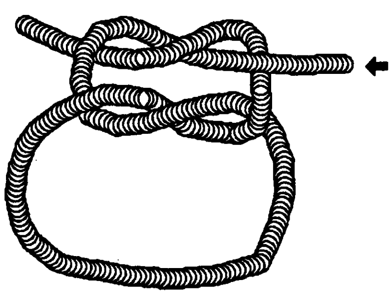
and these two across,



and this one doesn't. *(picks up the Granny)* It has one in the bottom *(strokes the left free end),*



and one in the top *(strokes the right free end).*



That's how I could tell which one is the Granny and which is the Square.

She sees the Thief as a Square knot until I ask her to continue her focus on the ends. Now she describes them not relative to the loops, but to the circle:

Carol: And which one is this one? (She picks up the Thief and looks at it.) You can make it looser if you want to.

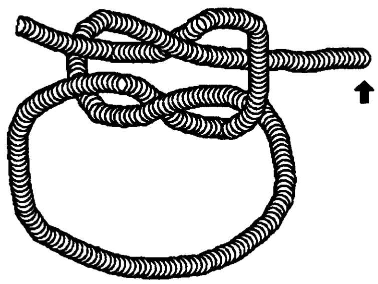
Juanita: The Square knot.

The Square knot? Now – now, what if we look at the ends of these two? (puts the Square and the Thief together.) I'm going to make it looser (loosens the Thief). Do they look the same? (She puts the Thief above the Square.)

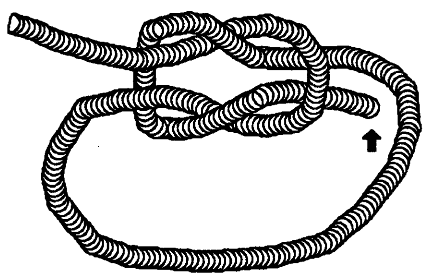
No.

What's different?

Because this one's on this side,



and this one's on this side.

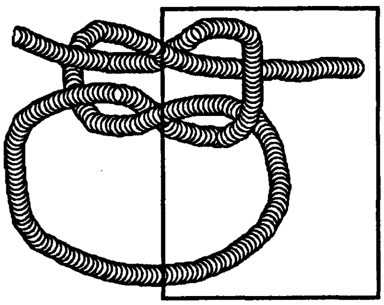


Juanita's description points to one of the reasons why the Thief is so difficult: there is an ambiguity between parts of the knot. Whereas in the Square knot the ends and the circle are distinct, in the Thief knot they have a way of running into each other.

7.1.1.7 Stacy's Square and Thief


5.4.2 6.2 6.5 6.6 6.19 8.1 8.2 8.4

Like Juanita, Stacy focuses on the right half of the knots in differentiating the Square and Thief knots:

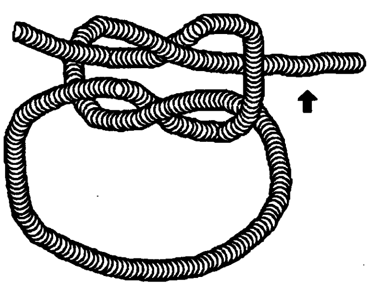


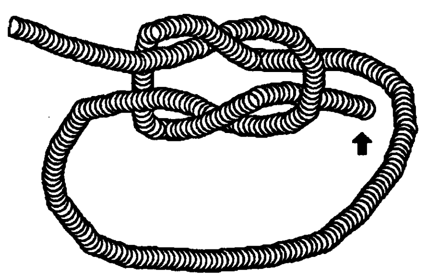
She then compares the different orientation of the right ends relative to the circles, using the terms "over" and "under."

Also like Juanita, Stacy includes the Granny knot in her discussion. She now changes her choice of referent, but maintains her use of the terms "over" and "under."

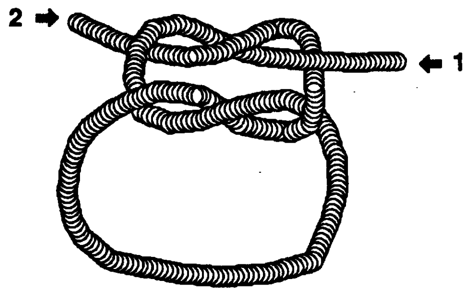
***Carol:** How about these two knots (Square and Thief)? Do they look the same to you or is there anything different about them?*

Stacy: Yes, this one – on this side, this one goes over *(above the circle on the Square),* and this one goes under *(inside the circle of the Thief).*

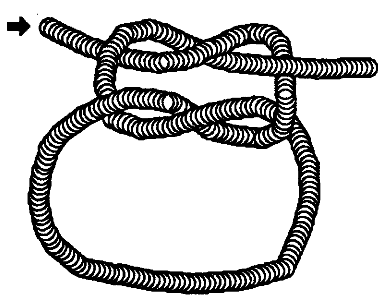




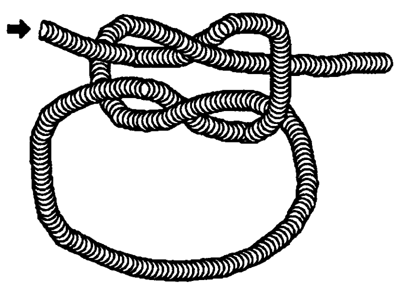
On this one *(the Granny knot),* this one goes over it (1), and this one goes over that (2).



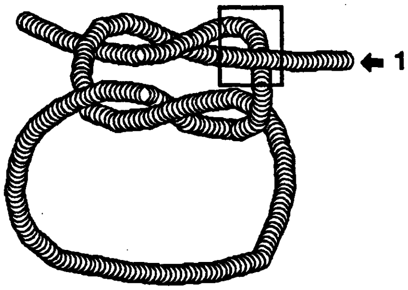
And it's suppose[d] to be going like – this line is suppose[d] to be going in ***(points to upper left, moves left to right with her finger to show how that crossing on the Granny knot should be like the same one on the Square knot).***

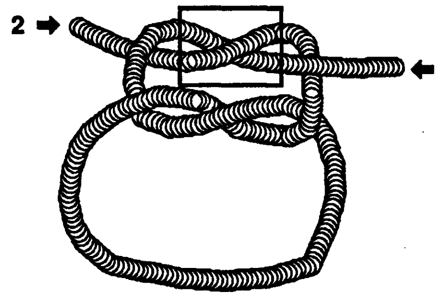


'Cause like right here, it goes in.

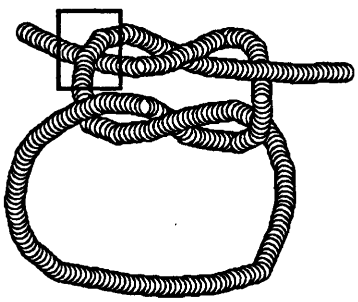


Stacy's two-part description of the Granny knot is unusual in its use of what we might call a *transitive referent* (see Section 8.4). She first describes the right end **(1)** as an active part – it goes over the upper part of the right loop. But then she abandons the loop as the referent. Now that designation switches to the end itself – no longer the actor, the right end becomes the new anchor with which Stacy can describe the action of the left end. The left end **(2)** goes over the right. Her focus is again at a crossing:





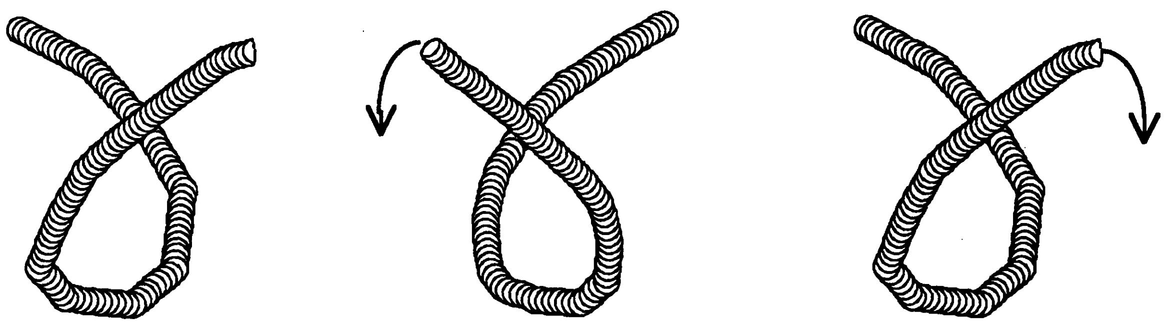
Stacy makes use of the Square knot in describing how the left end of the Granny should go "in" to the entanglement. Notice that her focus has now shifted fully to the left half of the knot:



The discussion continues, with some other knots being compared and tied. Then Stacy returns to the Square and Thief knots. She successfully ties the Square knot by experimenting with "left-over-right" sorts of approaches, holding the two ends of the string upright. But when she tries to tie the Thief knot in this manner, she runs into difficulty, and a new discussion of the knot ensues:

Carol: *... have you tied this one (the Thief) before?*

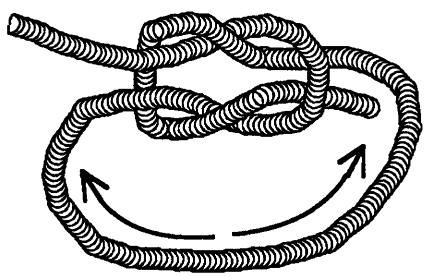
Stacy: Yeah, I think so. *(She tries a few approaches.)*



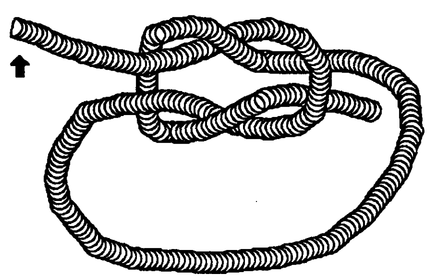
(She shakes her head, partially undoes the loop.) Looks funny.

Can you think out loud as you try to figure it out? What's hard about it?

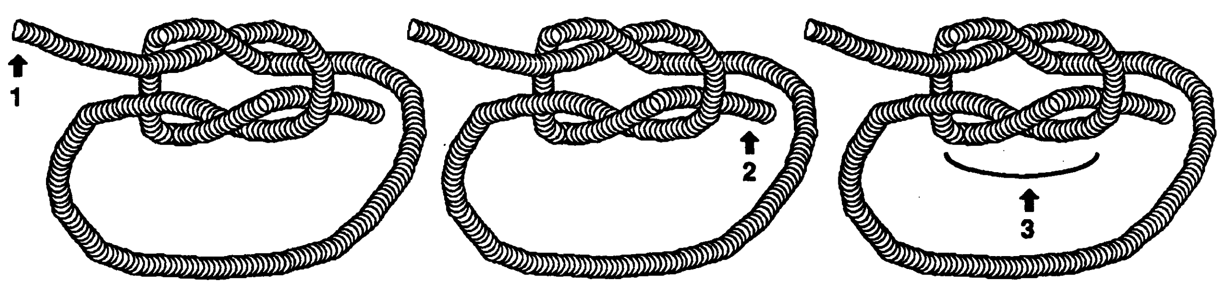
'Cause these look – because it looks like it goes first – like – *(motions with fingers)*



I'm following like, before the loop, and then how it – where the ends go – and like, they go up, and then – like, they go up –



– and then – like they go from being all the way up (1) and being all the way down (2), and then get caught from in between (3).



She begins her description of the Thief with the circle, which would be a reasonable starting point for a knot tied using the familiar "upright" approach. But as soon as she gets to the entanglement, she is in trouble. Then she must switch the basis of her description to the confusing, asymmetric behavior of the ends. She can easily enough describe the ends separately relative to the circle: the left one is "up," and the right one is "down." But describing the way they come together becomes difficult; she can say only that they are "caught from in between." Her remark may be taken symbolically as well as literally: not only do the two ends become entangled in a mysterious way, but that way seems to defy description – the ends, and her understanding, are somehow caught between being "up" and "down."

She decides to loosen the Thief knot to get a better look at the crossings. Recalling a Piagetian technique, I suggest a way of sorting through the tangle, by pretending that she is a small ant crawling along the knot and tracing her path as she goes. Immediately she shifts her starting point from the base of the circle to one of the ends. She gestures through a few broad tracings, gradually becoming more precise and finally lifting the knot so that she can turn it as she carefully follows the ant's path. (By regarding the knot in more than one plane, she can "do the unders," as she puts it: she continues her focus on crossings.) Stacy's move – from thinking about one surface of the knot to turning it and thinking in terms of more than one plane – is a strategy that only a few of the children employed. By examining the knot in this way, she comes to understand the path well enough that she is able to imitate it with a new piece of string, thereby producing a correctly tied Thief knot.

***Carol:** ... how come you could tie it that time but not before? What was different in the way you tied it?*

Stacy: Because it – it was, like, looser, so it was easier to see the path that it followed.

Right – and before you were thinking about the path, remember you – were trying in a way kind of like the Square knot?

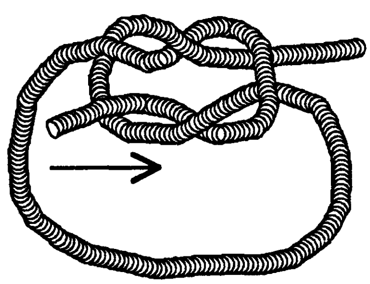
Yeah. I was trying – I was trying to think – and the Square knot, I just started like, you know how to *(she gestures with her hands, referring to the upright approach of tying the Square knot)* – so it wasn't that hard.

7.1.1.8 Maria's Square and Thief

For Maria, the Square and Thief knots seem the same at first, and tracing the imaginary ant's trail along the Thief knot is difficult. She loses track at the second crossing, effectively simplifying the path. She does discover, nevertheless, the idea that the string might form a continuous path, from one end of the knot to the other. And although her simplification is inaccurate, it captures the flavor of the curve – she does not manage the fully interlocked "S" shape, but the "G"-like shape that she sketches approximates the flavor of the Thief knot.

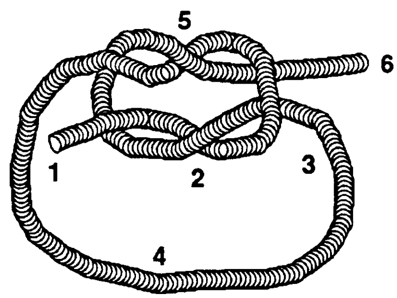
Carol: *Where would be a good starting point? If you wanted to start right at the end of the knot – or right at the beginning?*

Maria: *(points from the left rightward)*



Okay. So now you're at the beginning, and you're a little ant. Where do you go?

Along here (1) and here (2).



Okay. Then where do you go?

Down here (3)?

Okay.

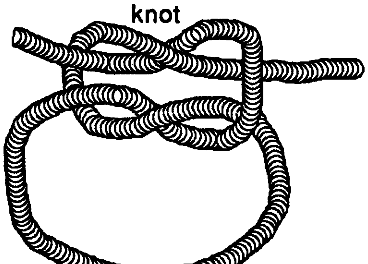
And I go all around (4), and go like that (5, 6).

7.1.1.9 Celina's Square and Thief

In trying to distinguish the Square and Thief knots, Celina ponders sections of the knots and tries naming them in order to describe the pathway. Her names indicate that she considers the entire object to be the knot, rather than just the entanglement. By focusing on the ends (in particular, what she calls the "start knot"), she is able to the describe the difference between the knots in terms of the right end relative to the circle. Although she comes to appreciate this fundamental difference, her discussion also includes more incidental ones, such as the shape of the circles and the way the string bends as it comes out of each knot. For many children, properties of an individual knot distracted from the understanding of its topology.

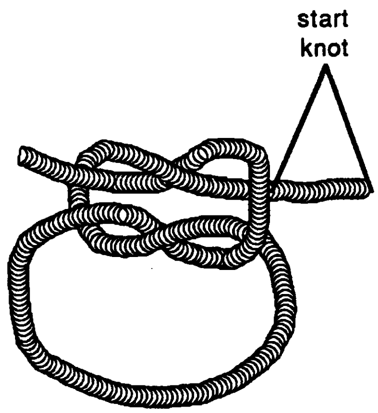

6.1

(Celina names parts of the knot in order to facilitate identification of the beginning of the ant's path along the Square and Thief knots.)

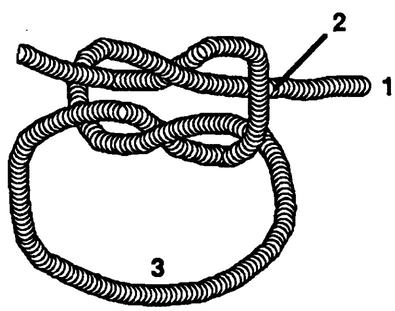


Carol: *Where is the "start knot" on this one?*

Celina: It's all of it *(she points to it)*, 'cause –

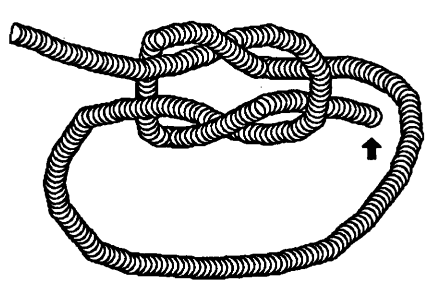


Well no, this part – 'cause if the ant crawled on the string, it'd start right here (1), and then it goes down here (2), and around to the middle part (3).

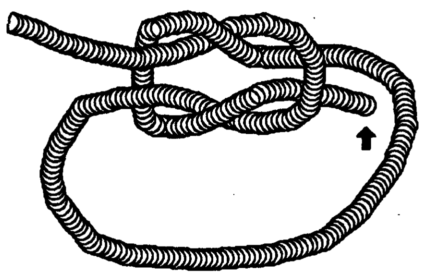
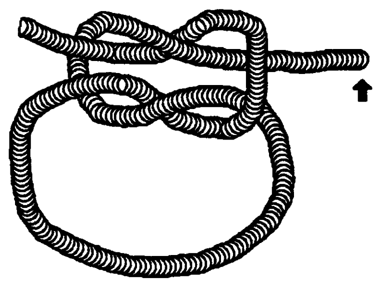


I urge her to make use of her naming system in looking at the Thief knot:

What about the "start knot" part on this one (the Thief)? Is there anything different?



This one's inside the circle *(the Thief)*, and this one's not *(the Square)*.

The end part is on the table *(for the Thief knot)*, and it's up here *(for the Square)*. This circle *(on the Thief)* is more oval-shaped.

7.1.1.10 Marcos's Square and Thief

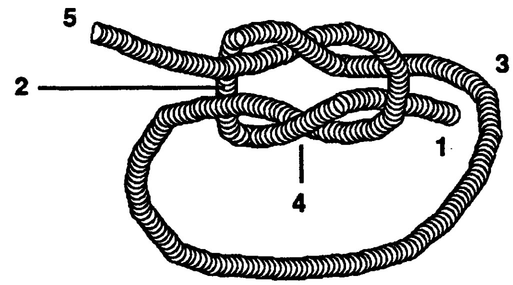
Marcos is concerned with both sides of the object, turning the knot as needed in order to carefully follow the string – the ant's path – as it winds around. His use of the terms "top" and "bottom" fluctuates broadly. At first, "bottom" refers to the other face of the knot. Then, in identifying his starting points for tracing the path, "bottom" refers to the the circle of the Square knot, while "top" refers to the entanglement of the Thief. Then, in finally describing the difference between the two knots, "top" and "bottom" are both terms used with regard to the entanglements, as he situates the ends relative to the circles.

Fluctuations characterize Marcos's thinking about these knots: fluctuations between which plane to consider, between focus on the entanglement or on the entire object, and between thinking about the tying process or the finished knot. His choice of starting point in tracing the Square knot reflects his recollection of the "upright," two-ended approach to tying it; his choice of starting point for the Thief points to his newfound understanding that this knot is most easily produced in a different way.

 5.4.1 6.6 8.2

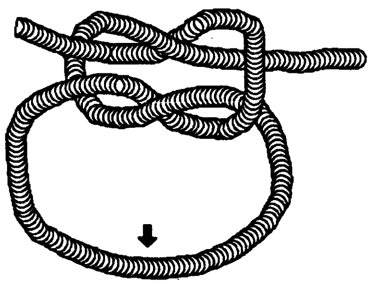
Carol: Imagine you were a tiny little ant, and you were crawling on this string. And you wanted to crawl all over the whole string – you wanted to cover every little piece of it. Where would you start? Show with your finger how you would go around if you were a little ant.

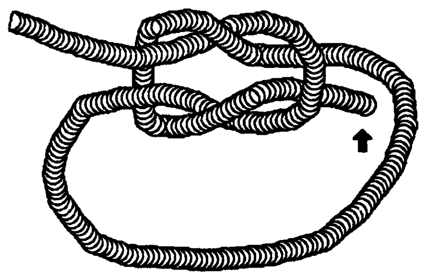
Marcos: *(He does the Square knot, then the Thief.)* This one – let me see. I'll go there (1) – there – yeah! Like this. I'll go like here – like – here, on the bottom (2), and then I will go – let me see – what is it? Here – go around this (3), then go here on the bottom (4)? *(He lifts the knot to point to the "bottom.")* Right? And I end up here (5)!



OK. Is there anything different about how you went here (the Thief) and how you went here (the Square)?

Yeah. I start – I started in a different way. Here *(the Square)* I started – like, um – almost on the bottom. Here *(the Thief)*, I started on the top.



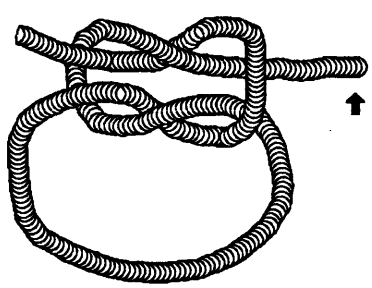


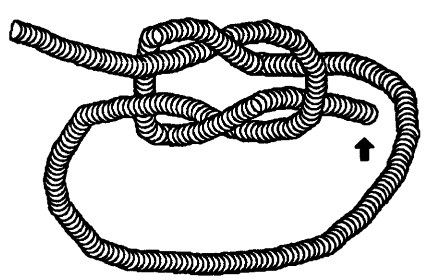
How come?

Because they are like different ways. See? *(He holds up both knots.)*

Oh, I see.

See – this one's on top *(the Square)*. This one's on the bottom *(the Thief)*.





I see.

So you have to – because they're different – they're the same knot, but they got – you gotta start them a different way, because they're not – almost the same. One is on the top, the other is on the bottom. *(He works with the Square knot.)* You have to go like this – go around – then go through there – you end here *(the left end of the Square)*. And here, you end right there *(the right end of the Thief)*.

Marcos's last explanation dramatizes his fluctuating views of the knots: in virtually the same breath, he says, "they're different," "they're the same knot," and (hesitating) "they're not – almost the same."

7.1.1.11 Leroy's Square and Thief

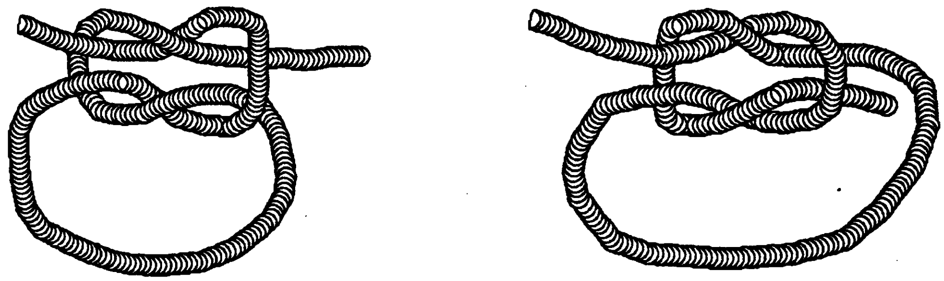
At first, Leroy insists that the Square and the Thief are the same knot. He discovers much evidence that they are different – but his belief that they really are seems fragile. He has difficulty explaining *why* the knots are different even though he has come close to demonstrating to himself that they are. He repeatedly falls back on old conceptions or strategies, even mistakes.

In tracing the knots, he is able to follow the pathway accurately, and explicitly considers both faces of the knot. However, he starts with different ends (the left for the Thief and the right for the Square) and then expresses surprise that he ends at a different place on each knot. In re-tracing, trying to keep the beginning point constant, he confuses crossings and loses track of the paths; nevertheless, he finally ends at the left of each knot – but declares that he has not ended at the same spot and continues to assert that the knots are the same. In beginning the tracings again, he identifies starting and ending points inconsistently with what he had done before, and re-traces each knot with different starting points: he seems unable to "anchor" his thinking, but decides the knots are different anyway. He has been through so many tracings that it seems possible he really has come to appreciate this difference, yet cannot explain it adequately. Instead he relies on more superficial attributes, such as size of the ends and shape of the circles, to explain it.

 5.4.2 6.22.1 8.2

***Carol:** Okay, let me ask you this. Let's see – see these two knots? Do they look like the same knot? (Square and Thief) Are they the same?*

Leroy: Yeah, kind of. *(He compares them side by side.)*

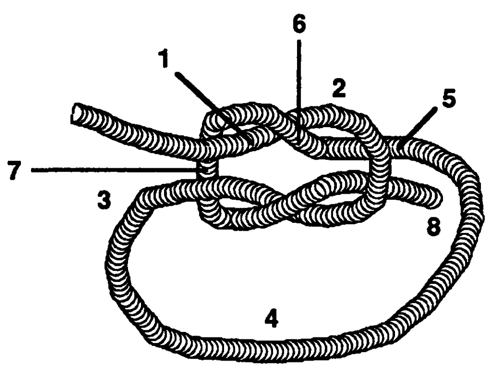


How are they the same?

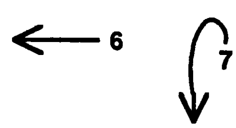
Like, like, this comes under right there, this comes over, this goes in, that goes in, that goes under – and so does this. It's the same one. They're identical. ***(He gestures quickly and broadly, pointing back and forth at the two knots as he speaks.)***

Okay. Now, if you put them on the table, can you – if you were a tiny little ant, and you wanted to crawl along those knots, how would you go? Can you show me with your finger?

Um – well, if you was a tiny little ant, you would go through here **(1)**, and go to – over there **(2)**, and out that side **(3)**, just go around **(4)**, go back in **(5)**, go through this side **(6)**, then go under **(7)**, and end up right here **(8)**.



6, 7: He motions with his finger.

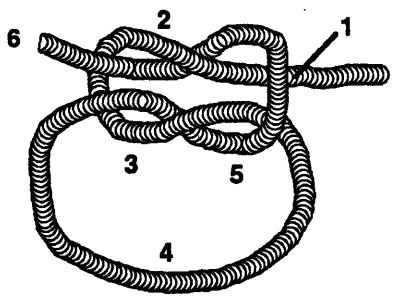


Uh huh. And how would it go on this one?

This one? Well, which – it depends on which end it starts with.

It would start at the same end you started with on this one.

(He chooses the other end.) It would go through here **(1)**, go through there **(2)**, go through here **(3)**, come through here now **(4)**, go though there **(5)**, and you come out here **(6)**. ***(He doesn't turn either knot.)***

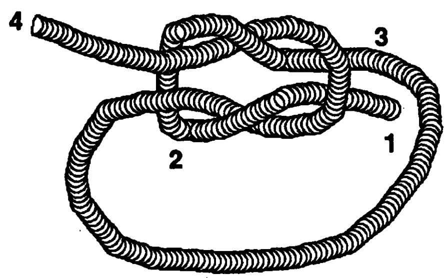


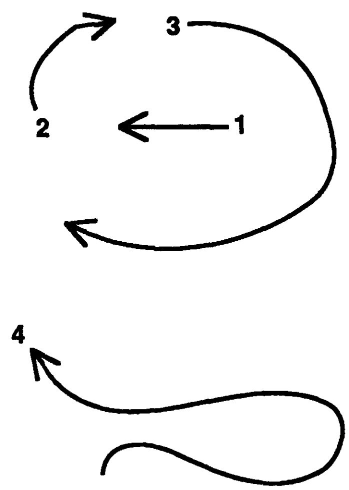
Do you end at the same spot?

(puzzled and surprised) No.

Oh, how come?

I don't know. *(He goes back to the Thief and traces it in four motions, while mumbling inaudibly.)*



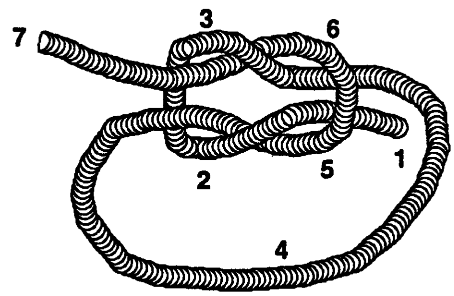


Yes – you – yeah, you do. (This re-tracing of the Thief matches his tracing of the Square, although that tracing had not matched the original tracing of the Thief.)

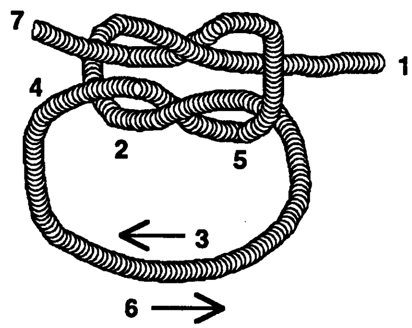
No you don't – *(He senses the inconsistency.)*

Oh, I see – you're doing – But do you start at the same spot? (I try to call his attention to the discrepancy between the original tracing of the Thief and the subsequent two tracings, but he moves on to another tracing of the Thief.)

Yeah. See. *(Now he is consistently beginning with the end at the right.)* You go through there (1), go through there (2), go under (3), come back up here (4), you go this way (5), and go that way (6), and you come here (7).



Then – and here *(on the Square)*, I started at that end *(the right)*. You go here (1), go through here (2), go through here (3), go through – here (4), go over here (5), go back around here *(He loses sight of both the path of the string and the direction of the movement.)* (6), and go over there (7). *(It's not clear how he gets directions and crossings confused. Even if his (4) and (5) are as shown here, it's hard to tell how he goes from (6), which is definite, to (7).)*

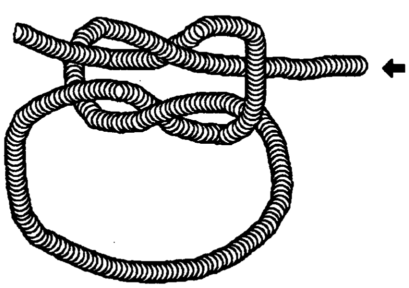


No! You don't.

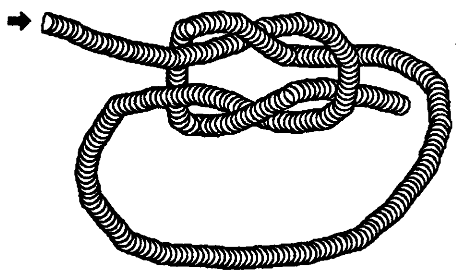
You don't end at the same spot? (Leroy shakes his head "no.") So does – is it the same knot, then?

Yeah, it's the same knot. I guess, but – it looks the same. *(He is confused, but engaged. He holds the knots next to each other to compare.)*

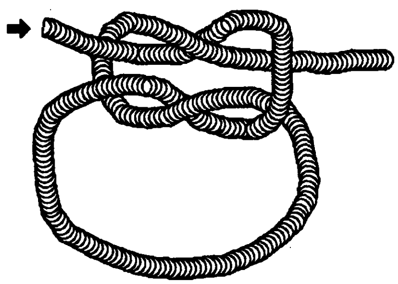
Where did you end on this one (the Square)? (He points.)



And where did you end on that one (the Thief)?

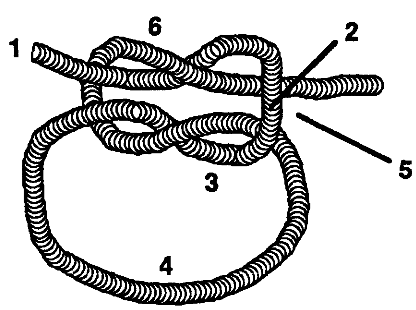


Where did you start on this one (Square)?

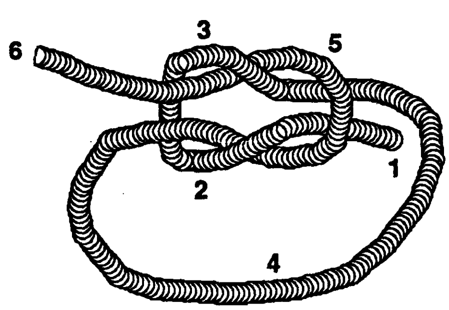


Let's see where you went again.

All right. *(Square:)* Go here (1), then go here (2), go through here (3), go over here (4), then I come over there (5), and go through (6). *((5) and (6) cover a lot of motion. He is pointing to the general areas but is perhaps too vague to enable his understanding.)*



(Thief:) Start here (1), go through there (2), go here (3), go around (4), go here (5), and go through (6).



So you end in different places? (Leroy nods: "yes.") So – are they the same knot, then?

Um – no! They just look the same.

How are they different?

(compares them) I think this one *(the Square)* has longer ends. *(He compares them again.)* And this one is bigger *(the Thief)*. This one opens more *(the Thief)*.

Okay.

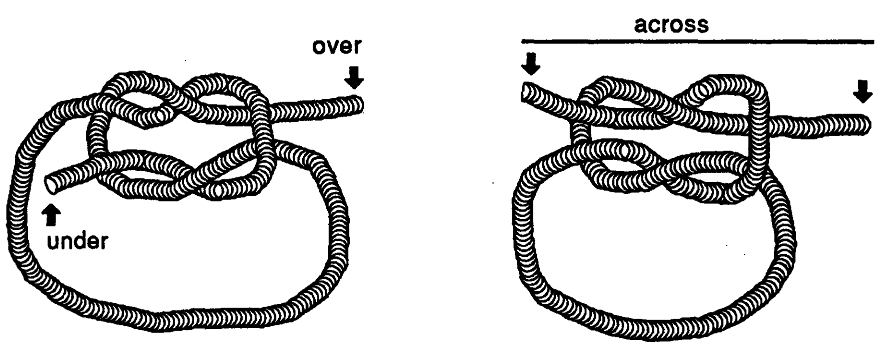
7.1.1.12 Tiffany's Square and Thief

In a letter to Soo Yong, Tiffany describes the difference between the Thief, or "Mystery knot," and the Square knot. She implicitly chooses the circle as the referent, describing the ends relative to the circle without pointing it out specifically:

> To Soo Yong I did the Mystery knot and Carol helped me. It's different from the square knot because the string goes under and over. But the string of the ... square knot go across.
>
> from Tiffany

Tiffany's interpretation can be pictured as follows:

5.4.1



In the Thief knot, her "over" and "under" designations refer not to the relationships of strands in the crossings, but to the position of the ends relative to the circle.

7.1.1.13 Alice's Square and Thief

In her initial explanation of the difference between the Square and the Thief, Alice identifies the top of the entanglement (which is composed differently in the two knots) and the circle as key parts. She does not use the circle as a referent, but indicates it as a noteworthy component of the knot. The top part of the entanglement is, for her, the distinguishing feature of the knots. In her consideration of this part, she looks not just at the ends, but beyond them to the lengths of string that become the ends as they extend out from the knots. Interestingly, though her focus on the top of the entanglement leads her to indicate one such length in the case of the Thief knot, it leads to considering two lengths as though they were a unit, in the case of the Square. Alice seems not to consider this a problem; her focus on the feature she has chosen to distinguish the knots remains stable despite its comparative unorthodoxy.

There is a subtle conflict in Alice's description, however. Her inclusion of the circles (that the string goes "around like that" or "like that") is accompanied by a gesture that indicates broadly the directional movement contained by that part of the knot. The circle is secondary to the entanglement, but too pronounced to exclude. It is connected through movement, rather than being considered as a component of the knot itself. The circle is a by-product of the process that produced the knot; it is irrelevant, but impossible to ignore, in regarding the finished object.

Alice is reluctant to dismiss completely the vestiges of the tying process: it is evident not just in her inclusion of the circles, but, subtly, in her selection of the feature that distinguishes the two knots. The Thief knot tends to be the product of a single strand wound around itself in a particular way; the Square knot is, in Alice's experience, produced through the involvement of the two ends of a piece of string. Aptly, the top part of the Thief is singular, and the top of the Square is comprised of two components.

Thus, her simple gestures speak to an understanding of the knots far deeper than her use of the criterion of *length of the top of the knot* would seem to indicate. Yet, that well-chosen criterion is sufficient in itself to answer the question of how the knots differ.

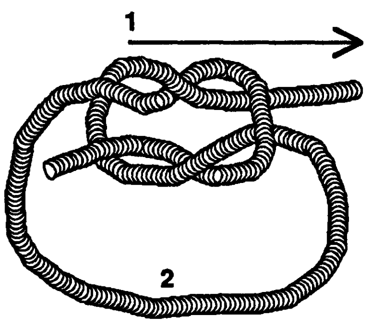
 5.4.1 5.4.2 6.2 6.6 9.1

Carol: *... do these two knots look the same to you?*

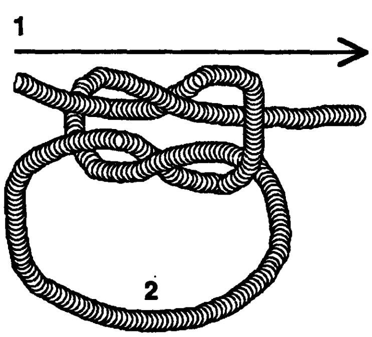
Alice: Um umm. *(She means "no.")*

What's different?

The string goes over this (1) and around like that (2).



This one, the string goes straight across (1), and then it goes like that (2).



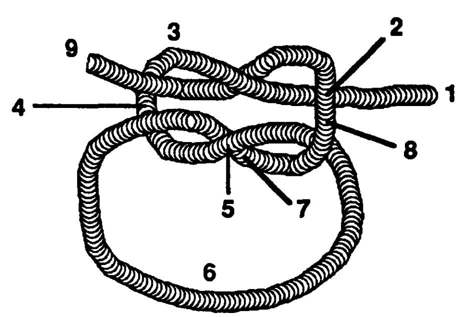
Alice's reliance on gestures to indicate motion contained by the knot is further apparent in her descriptions of the imaginary ant's paths. Gesturing gives her a way of acknowledging both faces of the knot. Her use of the term "bridge" is another way of incorporating that concept of dimensionality, and of being precise in including every place that the ant must cover. She indicates that she is considering both sides of the knot, which enables her to refer to both loops as constituting "bridges": one loop may seem to be a bridge while the other loop goes under its accompanying strands, but one need only flip the knot to see that what goes "under" from one view goes "over" from the other. Thus her ant would encounter two "bridges."

Carol: *... if you were a tiny little ant, and you were walking all along that knot, how would it look? Can you show me with your finger what you would do?*

Alice: Um – *(does the Square)* – I'd start from here (1), and then go on the bridge (2), and then go over there (3), and go on that other bridge (4) – *(she does motion with finger instead of turning knot around)* – and go to that one (5), and go all the way around (6), and go to that bridge (7), and go over there (8), and there (9). *(Her finger points left, out of the knot.)*

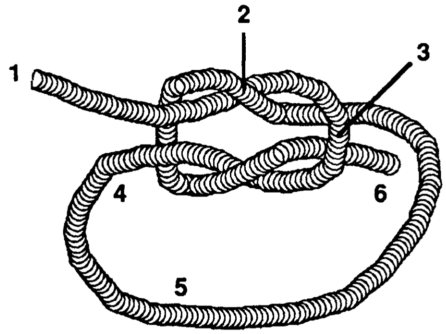
(4: Rather than turning the knot over to follow the string, she does a flip/curve sort of motion with her finger:)

(7 is in back of 5.)



Okay, what about this knot (the Thief)?

Start from here (1), and go under here (2), around there (3), there (4), there (5), and out here (6).



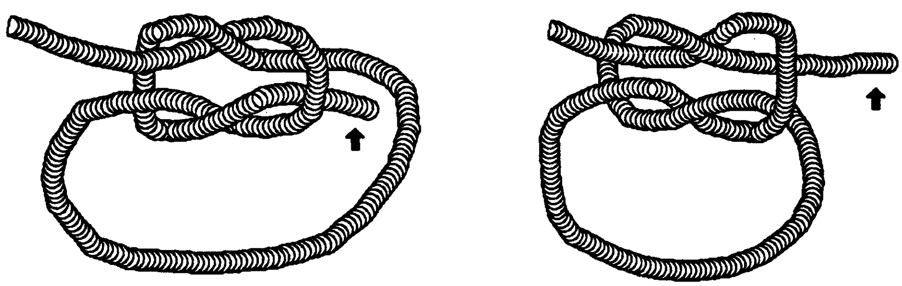
(5 is a finger motion showing the beginning of the final curve and the curve itself.)

7.1.1.14 Julio's Square and Thief

After distinguishing the Square and Thief knots by describing the right ends as "up" or "down" relative to the circles, Julio tries his hand at tying the knots.

Carol: *What's different?*

Julio: 'Cause this knot ***(the Thief)*** goes down and this one's up ***(the Square).***



He has a few encounters with the Granny knot when trying to produce the Square using the "upright," two-ended approach. Then he tries interlocking two loops, still without success.

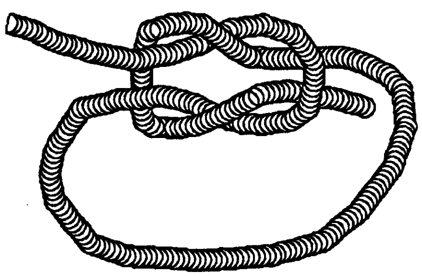
He tries both of these approaches in tying the Thief knot, but neither of them seem to work. He does manage to tie the Thief in the course of messing around with a complicated combination of the two approaches: he maintains the structure of two loops while moving each end around in a variety of ways, until the Thief appears almost by accident. Duplicating the effort is the challenge: he demonstrates a breakthrough in switching from the upright model to laying the string flat on the table and proceeding with one moving end. He vacillates for a while – beginning flat, returning to an upright position, and laying the string flat again, this time approximating an upside-down "G" shape to begin the Thief. Still he vacillates, crossing one of the ends in recall of the starting configuration of the upright model even though the string is still flat; then he returns to the flat curve, now not "G"-like but "S"-like, which leads him to the final knot. It is a triumphant moment.

Characterizing Julio's experimentation with the Thief are attempts to capture his sense of the movement contained by the knot. Unlike most of the other children, he manages to transform these "G" and "S"-shaped expressions into vehicles for embodying the exact configuration of the knot.

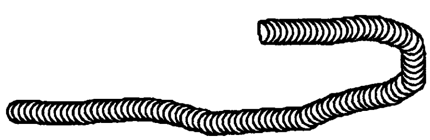

8.1

Carol: *... Could you tie it again?*

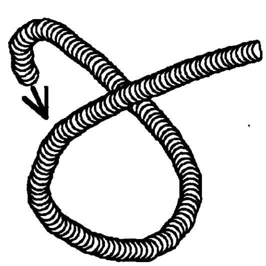
(He nods and begins. As he ties, he continually compares his work to the finished knot:)



(He holds the ends in two hands, thinks, and puts the string on the table.)

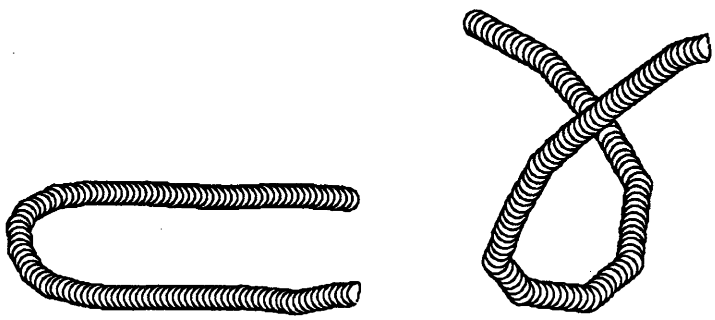


Julio: Um umn. *(He starts over.)* I used to tie it like this *(puts the right end under the left).*



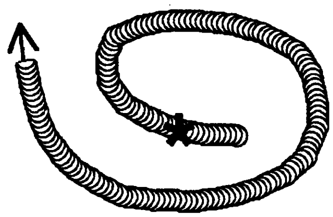
Yeah.

(He begins a completion of the Square knot by putting the "new right" end over the "new left" one. Then he stops and undoes the string.) Oooooh! 'Cause I used to go like – *(puts it on the table to see what his move would look like flat. He rearranges the string by uncrossing and moving the left end instead of the right.)* – somethin' like this –

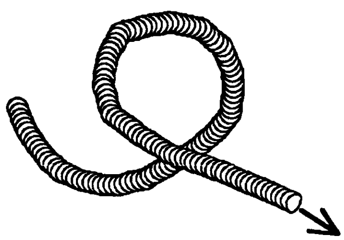


Um hmm.

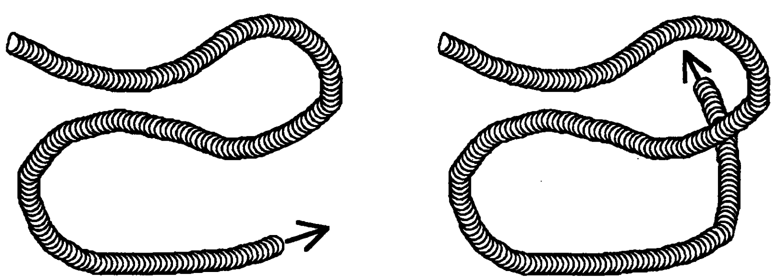
(He picks up the string and starts over.) Start – lemme start –



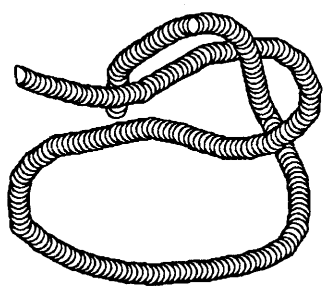
(He puts his finger at the "" to anchor the string, then pulls the other end around.)*

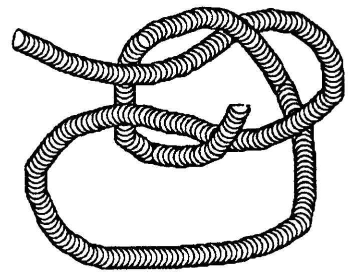


Lemme start – *(he undoes it)* – from this way:

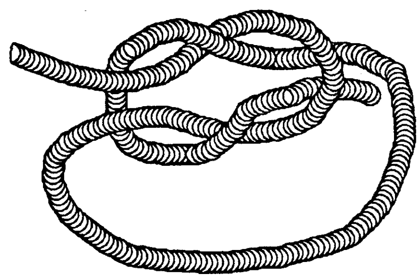


(Now he moves quickly; he knows what to do.)





(He tightens it, excited.)



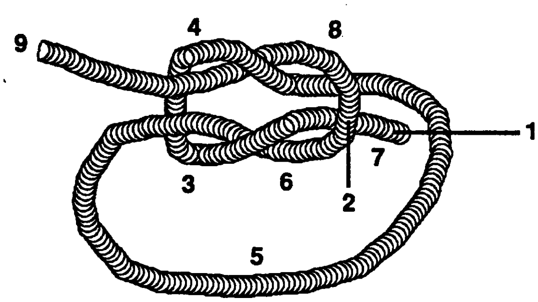
That's beautiful. Very nice! You developed a technique, I think. (He smiles.)

7.1.1.15 Patrick's Square and Thief

Patrick has some trouble following the imaginary ant's path at first. He complains about the "blockades" obstructing his way and loses track of the path as he gets confused at the crossings. He modifies his descriptions at subsequent crossings, including more detail in order to keep track of where he is.

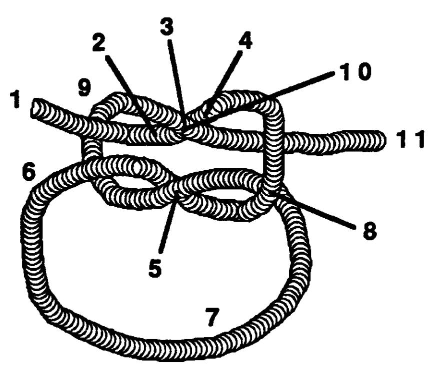
5.4.1 6.4 6.7 6.22.1 6.22.2

Patrick: Okay. I would start here (1) and then, I'd go um – hmm. Let's see. It's hard. It's a blockade (2). I'd probably go there (3), and then I'd switch to here (4), come around (5), go across over this (6), go – oh no! (7) *(He realizes that he would leave the knot too early.)* Then come back here (8), and go around there (9). Okay, 'cause of those things in it, it's hard.



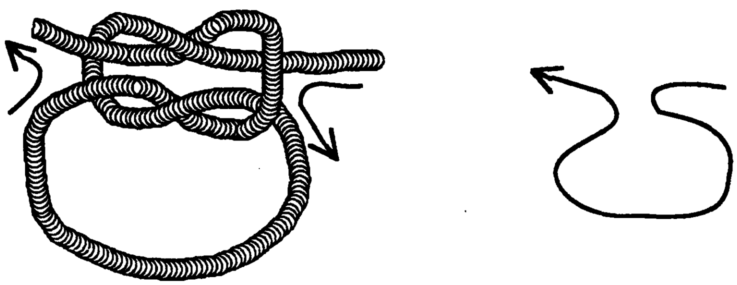
Carol: Uh huh. What about this one?

This one. I start here (1), walk all the way to the end where this one (2) starts, and then I'd walk over this (3), and come back to where it would come in (4), walk onto the one that it was connected to (5) ('cause I couldn't go under it), and then I'd walk on this (6) and come over here (7), and then I'd walk on this (8), and come here (9), and walk over this (10) – like that (11).



His "U-curve" concept breaks down the Square knot into two interlocking loops. He tries applying this view to tying the Thief knot, but without success:

Patrick: So, I'd just actually make a U curve and make a U *(he traces out a curve with his fingers).*



***Carol:** So, that's like one continuous pathway?*

Sort of.

Sort of. A U and another U. And are these two pathways the same (on the Square and on the Thief)?

Um – not really. I don't know why. I just thought of a better idea for this one.

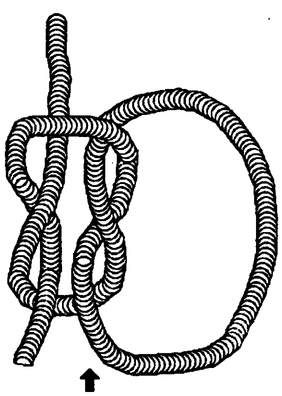
And how were they different?

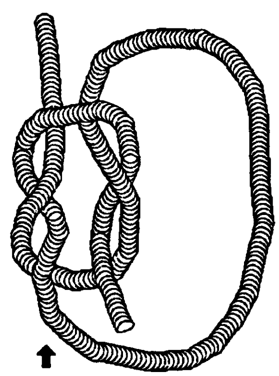
This one *(the Thief),* I just – I had this one all over the place *(motions his finger along the knot in a dancing motion)* – I just came from this to this one – I just switched. *(He moves the knot around a bit, ultimately flipping it.)*

Patrick inadvertently flips the Thief knot during the conversation, which may help to get him "unstuck" in his way of looking at the knot. The novel approach of turning the knots sideways enables him to distinguish the Square and Thief knots. In this view, the question is not whether Patrick chooses the entanglement or the circle as a referent – by changing the orientation in the way that he does, the two components no longer seem separate. He makes his distinction based on properties they share, which are made more obvious by taking a different view of the knot. In the Square knot, the lower part of the entanglement (or the right part, in Patrick's orientation) *is* part of the circle, and in the Thief knot, the *upper* part (his left part) is part of the circle. In the original orientation, the circle can become a distractant; Patrick finds a way to use it, instead, to his advantage. He considers it as a fundamental aspect of the knot.

Carol: *... you think they're the same – they're both a Square knot?*

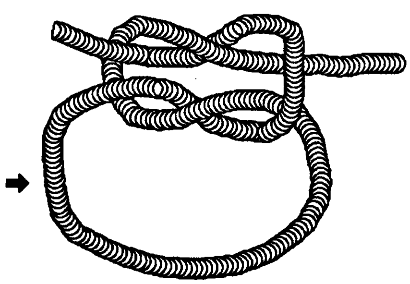
Patrick: They look the same – Oh! Wait a minute. *(He turns the knots sideways.)* I found it! Right here *(on the Square),* this Square – the one on the right is a loop, and the one on the right here *(the Thief)* is not. The one on the left is the loop. *(He refers to the "circle" as the "loop.")*

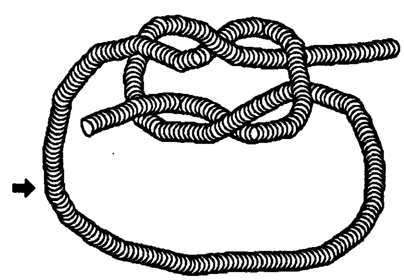




What made you suddenly see it?

I was comparing *(he turns the knots upright again)* – to find out which one – and I was looking at this one, and this one *(he points to the circles of the knots),* and I thought, "Wait a minute! Something's wrong here!"

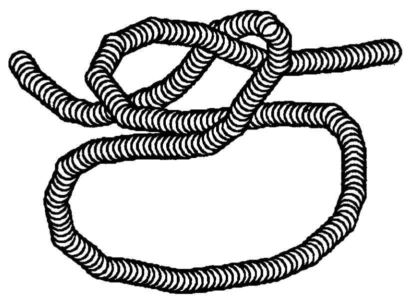




Later, in tying the Thief, Patrick produces a knot that is a flipped version of the one he has come to understand (though this flipped version matches the knot he started with).

He is confused at first, but it doesn't take him long to realize that the knot is the same either way.

... (He tries tying the Thief, manipulating the string while holding it in his two hands. He makes two loops, puts the left over the right (so the right is surrounded by the left), and tries tucking the left end in various ways over, through, etc.; then stops.)

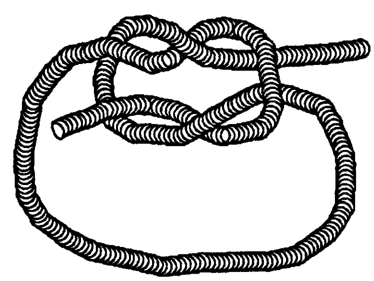


Patrick: Hmm – don't know. Well – I can't do this. How about if – well, I'll show what I do. I try to look at – like, make that loop, and then go around, and then make that one.

Carol: *Oh, okay.*

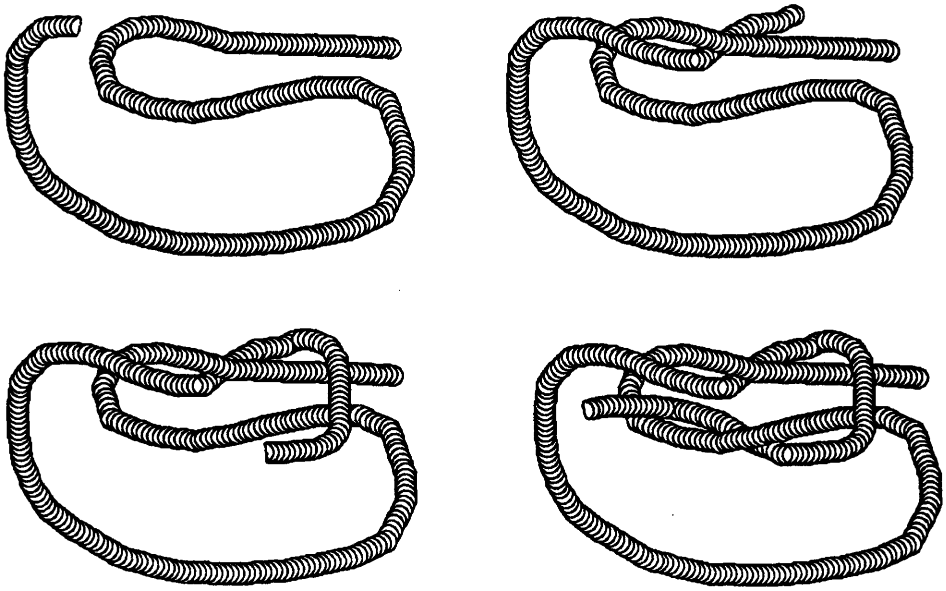
(He tries it: he makes a right loop, then a left) – this would go *(tries putting the left loop in the right, reversing his previous try)* – I was trying that.

Yeah, I saw you were trying that. How about if you put it flat on the table, and try to match what you see there? (He begins.) If you want, I can put my finger to hold it somewhere. (I made a similar offer to a number of the other children when I saw they were going in a direction that needed an additional set of fingers.)

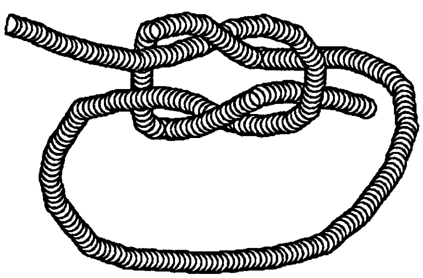


Let's see. I'm following it –

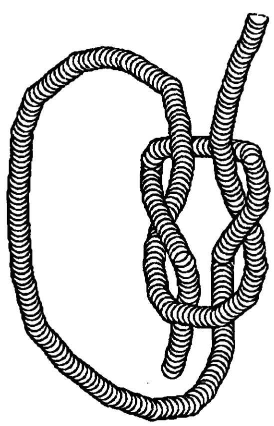
Very good! (He maneuvers more off the table than on, twisting hands and string awkwardly.)



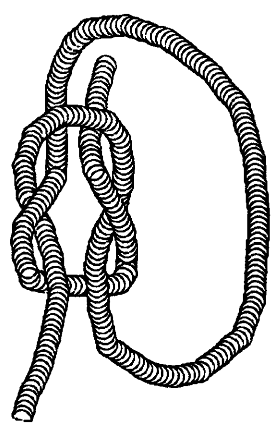
(Though he has started and proceeded in a way that would produce the Thief on the table, he has managed to turn it around in tightening it, so he gets an awkward-looking rendition of this confusing variation:)



Is that it? That's certainly close, if it's not it – (he rotates it one way) – that is <u>extremely</u> close! (He rotates the other way.)



the original Thief



Patrick's Thief

Close, but not it. *(He plays with it, tightening, straightening.)*

That might be it – let's see if we can –

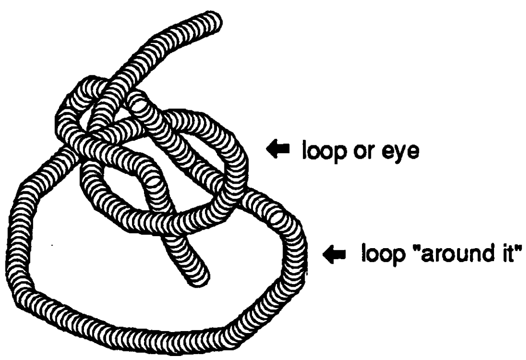
Yeah. *(He flattens the knot, and puts next to the Thief.)*

Very good – you tied it! You didn't think you could, but you did!

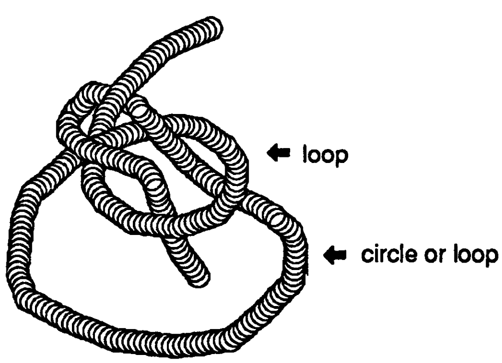
7.1.2 Comparisons of the Bowline and the Sheet Bend

Three of the children (Tony, Stacy, and Jill) discussed the relationship between the Bowline and the Sheet Bend in their final interviews (see Section 4.3). Although they had not paid careful attention to the similarity in developing the Family Tree of knots, Tony and Jill did discuss the relationship between the Bowline and the Sheet Bend when explaining the Family Tree in a session that occurred after the final interviews (see Section 9.1.4).

In the following descriptions, Tony uses the two "loops" as referents:



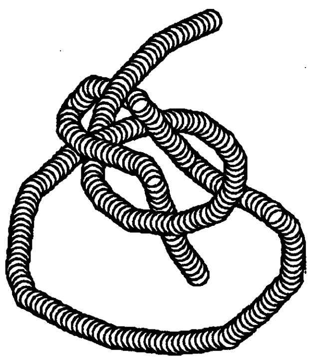
Stacy relies on these rounded parts as well, but has slightly different names for them:

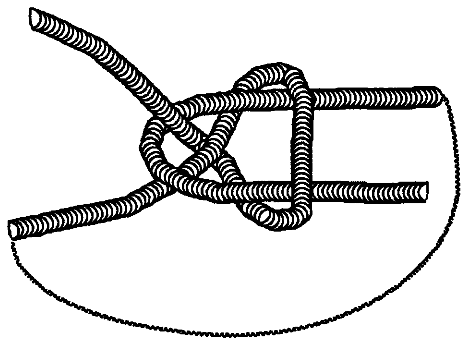


Stacy and Jill also rely on these parts as referents, but come to include the ends of the knots as focal points of their discussions.

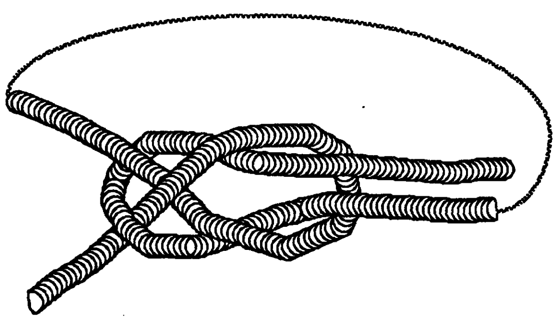
All of the children understand that a simple modification of one of the knots could result in a configuration matching that of the other knot. Tony and Jill describe this phenomenon in terms of clipping the larger loop of the Bowline; but both Jill and Stacy see the relationship in terms of joining two particular ends of the Sheet Bend.

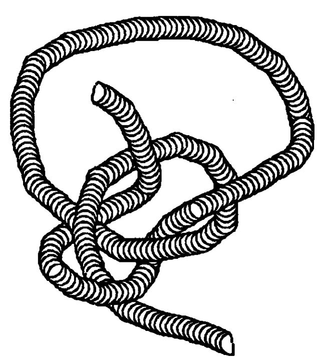
Tony and Jill get somewhat confused by the change in orientation needed to effect one-to-one comparison of the two knots, but Stacy makes use of her clear understanding that one knot is most like the inverted[1] version of the other:





the Bowline is like the inverted Sheet Bend,





and the Sheet Bend is like the inverted Bowline.

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[1] See Section 5.4.1 for an explanation of "inverted" and other terms.

7.1.2.1 Tony's Bowline and Sheet Bend

In looking through the set of knots during his final interview, Tony confuses both the "Square with 4 Ends" and the Sheet Bend with the Bowline. In trying to identify the Sheet Bend, he also considers the Figure 8 and a variation of the Bowline: "I *would* say the Running Bowline," he says, "but that moves." The Sheet Bend doesn't "move," so he knows that it can't be the Running Bowline. Finally, he identifies the Sheet Bend as the Bowline, which leaves him without a name for the Bowline itself. He considers the Running Bowline, the Bowline on the Bight, the Granny, and the Stevedore, and finally settles simply on the "Bowline family." His explanation relies heavily on a gesture that imitates what to him is a key part of the knot:

Tony: *(He holds the knot by the two ends and turns it slowly and carefully.)* Um – huh – first all, let's see what – it'd probably be in the Bowline family. *(He examines it closely.)*

Carol: Why do you say that?

Well, it seems to have that shape – *(He transfers the knot from both hands to his right hand. With his left hand he does a motion like an arch over the knot.)* – of a Bowline.

Which part is the shape of the Bowline?

(He holds the knot with his left hand and with his right index finger follows the line along the loop and into the knot, doing a twirly motion with his finger as if to follow the curve of the string through the knot.) The loop with the – loop around it!

The loop with a loop around it?

Yeah. Or an eye with a loop around it. *(He probably found the term "eye" in a book; see Section 5.4.1.)*

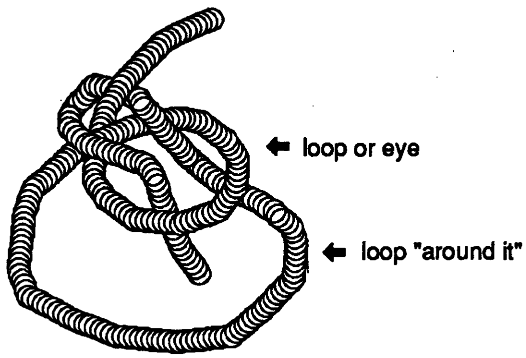
Well, you know what? That is the Bowline.

(He hesitates, examining and turning the knot.) *(To the camera:)* Haha! I knew it! *(To Carol:)* This is a Bowline?

Um hmm.

(He scratches his head and looks at the knot some more.) You sure?

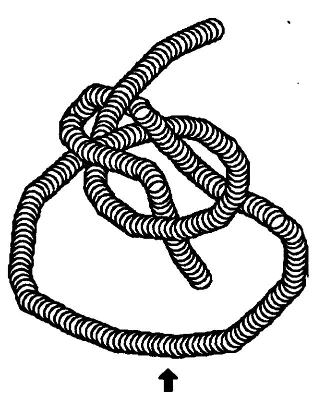
Tony focuses on the two "loops" as key parts of the Bowline:



His arch-like gesture mimics the larger loop.

Tony's uncertainty about whether he fully understands the knot is demonstrated by the contrast between the playful assuredness in his aside comment to the camera and his query to me: "You sure?" He returns to the other knots for clarification. Looking for the Sheet Bend, he examines the "Square with 4 Ends," the Figure 8, and the Granny, and finally realizes that the one he originally designated as the Bowline is actually the Sheet Bend. He offers a means of comparing these two knots:

Tony: ***(He takes the Bowline and compares it to the Sheet Bend that he holds, then shows them to Carol.)*** Take a look! ***(laughs)*** Well, there's that – ***(indicates the loop on the Bowline)*** – if you snipped that, it – if you snipped that, that would help.

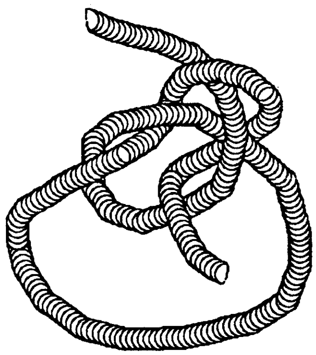


Carol: Um hmm.

It would end up looking – and, um – if you look in the back *(turns the Bowline around)* –

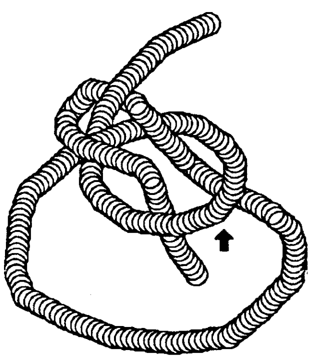
Um hmm – oh, yeah.

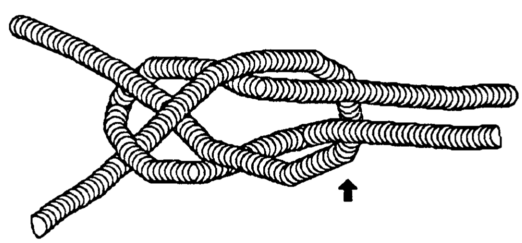
It's another one.



By flipping the Bowline, Tony's description here shifts from a primary focus on the larger "loop" to the "eye":

Tony: The only difference is that, in this *(the Bowline)*, it goes up, and in this *(the Sheet Bend)*, it goes down.

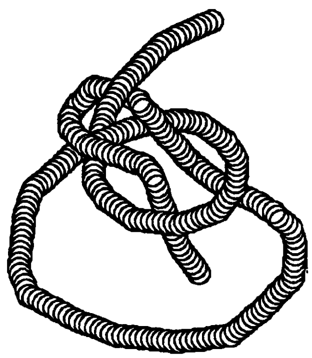


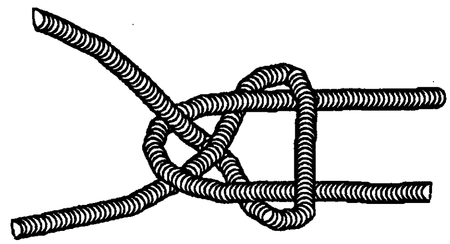


That's the only difference. So – um – *(continues looking at them)* – it looks an awful lot like the Bowline.

In the Bowline, two strands go under the curved part of the "eye," which he therefore describes as going "up." In the Sheet Bend, two strands go over the curved part of the "eye," which he therefore says goes "down." By flipping the Bowline, Tony demonstrates that he is on the way to understanding the relationship between the two knots

– but he doesn't quite understand the relative orientations. He flips the knot instead of inverting it, and then returns it to the position in which he is most accustomed to seeing it. For that orientation, holding the Sheet Bend in an inverted position could have provided a comparison making more obvious the similarities between the knots:





In these orientations, snipping the bottom of the Bowline would indeed produce the Sheet Bend. Although Tony has frequently used changes of orientation in developing quite sophisticated understandings of the Square knot, he does not take the technique far enough in this discussion of the Sheet Bend and the Bowline.

7.1.2.2 Jill's Bowline and Sheet Bend

In arranging the knots into groups (see Section 7.2.15), Jill sees a similarity between the Sheet Bend and the Bowline. Like Tony, she uses the term "loop" for both rounded parts of the Bowline. She begins to understand the need to change the orientation of one of the knots in order to enable a close comparison, but doesn't quite manage a clear description of this process.

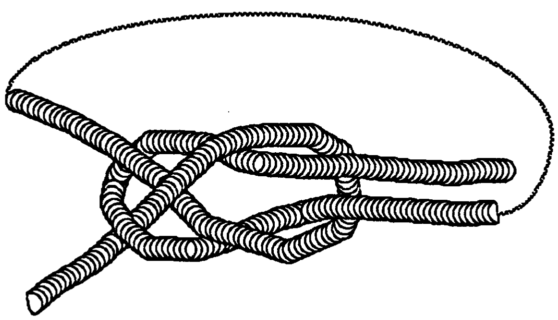
Jill: ... if it's the Sheet Bend, I would say that it goes with the Bowline.

Carol: Okay. Why?

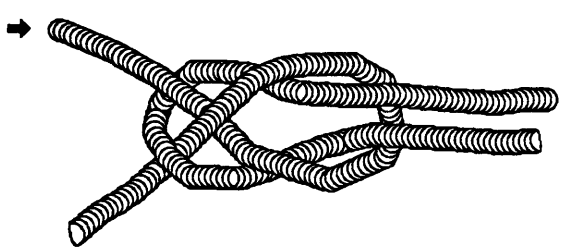
Well, because they both have the loop and a string coming out of the loop. *(She holds both.)*

Um hmm. They look very much alike, don't they?

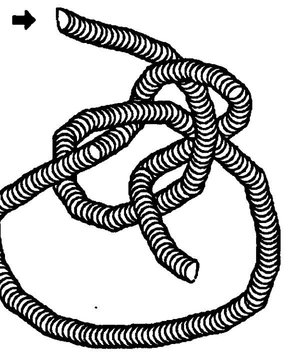
Yeah. The only difference is that, like – well, like the difference is that the Bowline – well that this *(the Sheet Bend)* – is – doesn't have a loop,



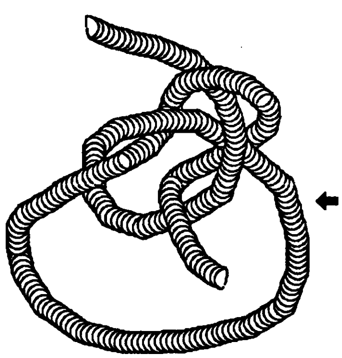
and the Bowline – this string comes out of the left *(of the Sheet Bend)*,



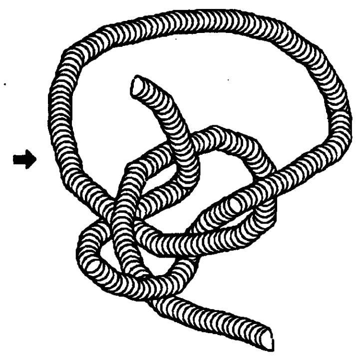
and this string *(on the Bowline)* comes out of the opposite:



Jill chooses, as the characteristic feature of the Sheet Bend, the end that crosses under itself in its idiosyncratic way. This focus leads her to look at the Bowline in an orientation that is flipped from the way it is normally seen (after tying it according to the story of the rabbit and the tree (see Section 6.21)); this way, both of the knots present the peculiar end at the upper part of the knot. She uses the term "opposite" in an attempt to indicate that, if the lower loop of the Bowline were clipped, the corresponding portion of the newly formed Sheet Bend would be inverted. However, she loses sight of the preferred end when performing this mental transformation. In the orientation of the Bowline that she chooses, the preferred end would be at the right:

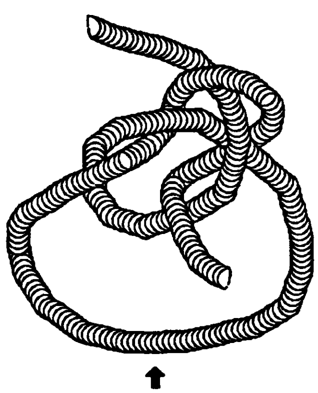


Had she inverted the Bowline, the crossing on which she wants to focus would present at the top of the knot, and the end of particular interest would be in the same position as on the Sheet Bend:

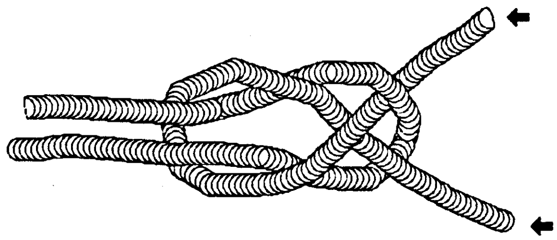


Perhaps to ameliorate her confusion, Jill resorts to a simpler description, which probably uses just a part of her thinking in the more complicated description above. Now she discusses the knots mainly in terms of the ends, making explicit the idea of cutting the larger loop of the Bowline. She makes this description without reference to the orientation of the knots, but seems sure that the result would be the same as the Sheet Bend. Her understanding includes an intuitive sense of the shift of orientation, illustrated here:

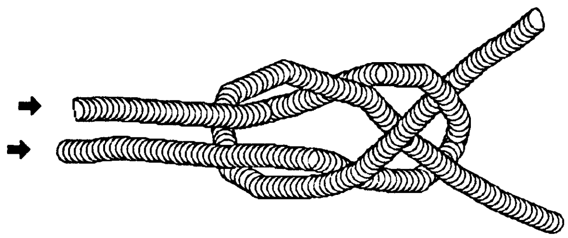
Jill: I guess they would be alike if you just cut this,



and then there would be two hanging,



and two just coming out.



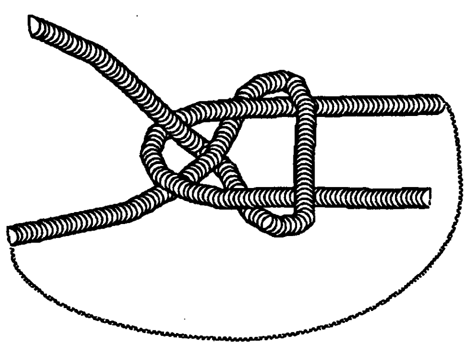
Umhmm.

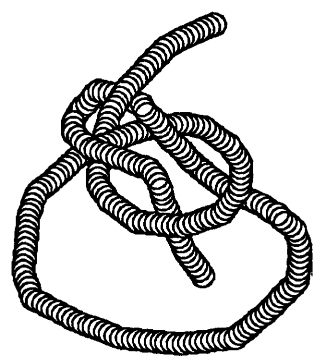
That would probably – they would definitely be alike.

7.1.2.3 Stacy's Bowline and Sheet Bend

Stacy originally groups the Sheet Bend with the Figure 8, and the Bowline with the Thief and the Square. Then she reconsiders the Sheet Bend, and suddenly sees the similarity between it and the Bowline. Stacy holds the Sheet Bend in an orientation that allows her to compare it directly to the familiar orientation of the Bowline. She suggests joining the appropriate ends of the Sheet Bend, so that it would become identical to the Bowline. Based on this similarity, Stacy groups the Sheet Bend with the Bowline, the Square, and the Thief (see Section 7.2.12).

Stacy: This one, maybe – ***(picks up Sheet Bend).*** I don't think this looks like it goes with anything – oh, this one – ***(compares with Bowline).*** Yeah – this way if – it if – 'cause there's a little thing that goes over – if this went through here – it would be really close – it would be like the circle. ***(She joins the ends of the Sheet Bend.)***

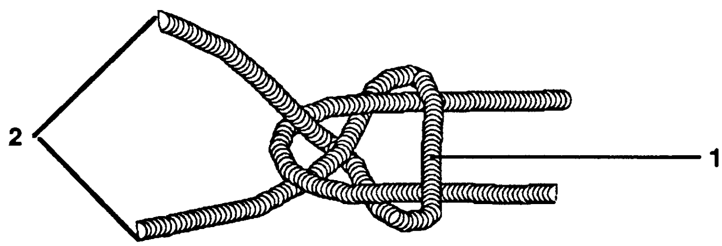




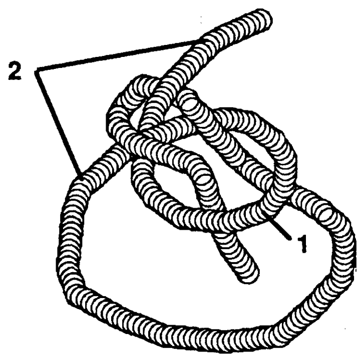
Okay, so those four – ***(Thief, Square, Bowline, Sheet Bend).***

In explaining why she has settled on this grouping, Stacy elaborates the difference she perceives between the Sheet Bend and the Bowline:

Stacy: ... this one – ***(Sheet Bend)*** with those, because, like I said, there's a loop right here (1) and then, right here (2). ***(She goes back and forth over this area with her finger, as if to join the ends as she described before.)***



And this one – *(the Bowline; she turns it around to match Sheet Bend)* –yeah, this way – because there's a loop right here *(like (1) on the Sheet Bend)* and another right here *(like (2) on the Sheet Bend).* There's a circle at the bottom, and one going through each end *(she holds it by the two ends).*



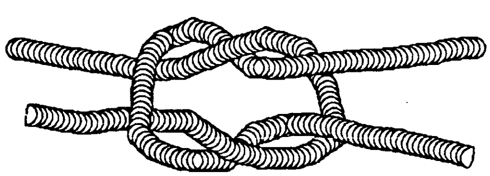
Here Stacy's discussion combines reliance on the ends of the knot as well as on the rounded parts of the Bowline (she refers to the smaller one as the "loop" and the larger one as the "circle"). By comparing the Sheet Bend to the "Square with 4 Ends," she swings the focus more to the ends of the knots:

Carol: *Okay – so what is that (the Sheet Bend) like?*

Stacy: These – but – um, the Square knot.

Okay – what if we did a Square knot without a circle?

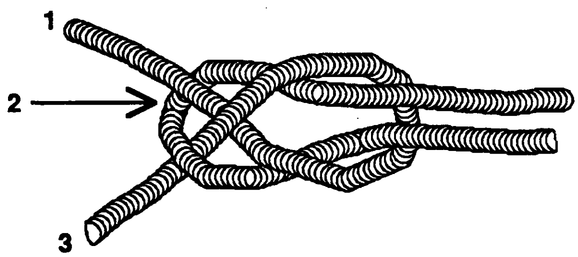
Did a Square knot like this – *(ties two strings):*



It's almost exactly like these two *(the two strings that form the Sheet Bend).*

Okay – and what's the difference?

That this one goes out of the loop **(1)** – and if it went in – ***(looks)*** – I think so – yeah, if this one was on this side **(2)** ***(putting (1) inside the loop (2))*** and this one was through here **(3)** ***[it would be the same as the Square]. ((3) would stay where it is, and (1) would become its partner in the same manner as for the Square knot.)***



7.1.2.4 The Bowline and Sheet Bend on the Family Tree

Although they constructed the Family Tree during several weeks prior to the final interviews (see Section 9.1.4), Jill, Tony, and Alice described what they had done in a session after the interviews. It is interesting to note that, although none of them were aware of the exact nature of the similarity between the two knots when making the tree, they had considered the Sheet Bend as a "cross with the Bowline and the Square." Also, although Tony demonstrates here that his understanding of the similarity has faltered, Jill has maintained the understanding she developed during the interview, insisting that the knots are, in fact, the same.

Jill: And next, we have the Sheet Bend, which is a cross – which is a cross with, um, with Bowline – *(points to the branch just left of the Square branch)*

Tony: *(corrects her pronunciation)* Bowline.

Jill: – and the Square family *(points to the Square branch).*

Carol: Ah.

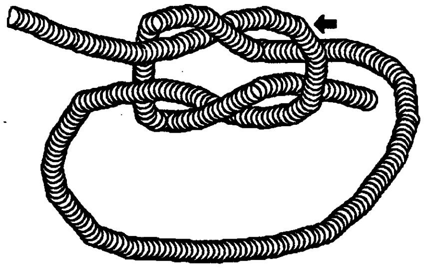
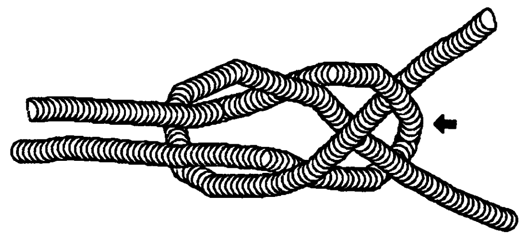
Tony: I can do the Sheet Bend.

Jill: The Sheet Bend is very odd, and –

Tony: No, it isn't.

Jill: Yes, it is.

Tony: It's like a Square knot, except it's twisted. Well, not except if you see it that way. This is twisted *(shows front and back of Sheet Bend).* It's twisted that way – twisted – and it's twisted – I can't do it with this string. But it's twisted. And you notice *(climbs up to the tree),* I'm gonna use the [Thief] in this instance, since you know how similar the [Thief] is to the Square. If you notice that *(puts his fingers on the right sides of both the Thief and the Sheet Bend: two threads go smoothly under the loop of the Thief, but one crosses under the other and goes over the loop in the Sheet Bend),* you'll see how it's just twisted.

Carol: *Oh.*

Jill: Okay.

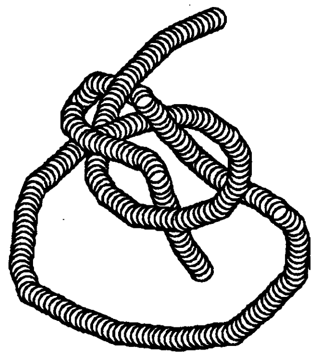
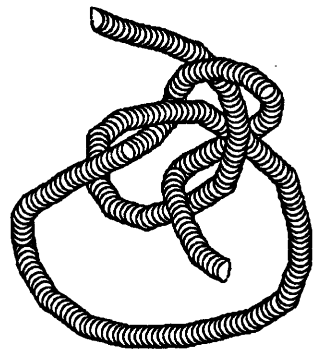
Tony: And that's the difference.

They proceed with an explanation of how the Bowline is like (and, in fact, is) the Sheet Bend. Tony focuses on the "twisted" strand, emphasizing the similarity of its configuration to that of the Sheet Bend, while Jill insists that they are identical. During her final interview, she explains, she noticed first that the Sheet Bend does not have a large loop as the Bowline does. Then she noticed that the "twisted" strand was the same, but on the "opposite" side, as she puts it. Although her use of this term in her interview was not precise enough to capture the exact orientation of the Sheet Bend relative to the Bowline, she now demonstrates a firm understanding of how the knots are related. Jill points out that Tony's accidental clipping of the loop of the Bowline has produced a correct Sheet Bend. (He inadvertently clipped the knot during their construction of the Family Tree.) Tony, responsive to Alice's chiding, seems unsure the mistake was fortuitous, and calls attention to a difference between the two knots, having to do with the practical purposes.

Tony: *(holding the Bowline)* This is the Bowline.

Carol: *Okay.*

Tony: You notice, if you look at it this way – ***(He turns it over to make visible the string crossing and going through the loop.)***

Carol:* *Uh huh.

Tony: The Bowline – see the similarity? It's almost exactly the same.

Jill: It <u>is</u> exactly the same! When I – when I was doing the interview with –

Tony: But for the Sheet Bend, there's two strings –

Jill: No – when I was doing the interview... my – the Sheet Bend – this – the Bowline had a loop, and this didn't. And I thought – at first, I thought they're different – um – they're like – there was that – *(she handles the Bowline)* –

Tony: Two versions.

Jill: No. It was like, the opposite side. *(She turns the knot over.)* And then I realized that if you cut the loop on the Bowline, it would be exactly the same as the Sheet Bend.

Tony: Right.

Jill: So this, here, is a Sheet Bend *(referring to the Bowline on the tree)*. Tony cut the loop, by mistake.

Tony: I cut the loop by mistake.

Alice: Again.

Jill: So he – he got the Sheet Bend.

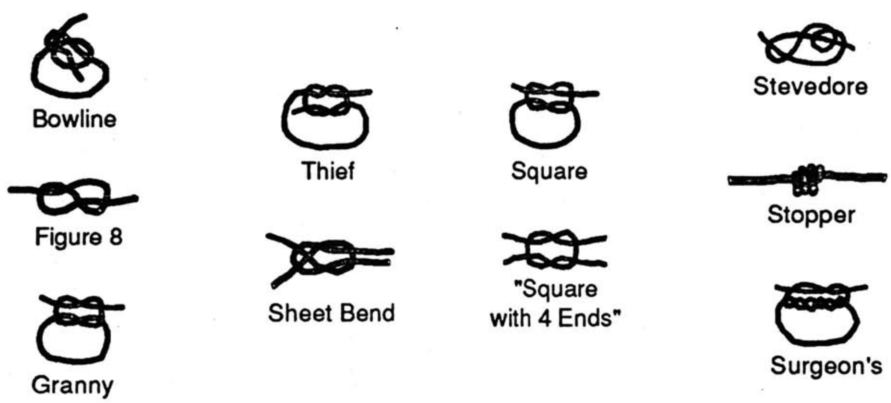
Carol:* *Okay.

Tony: Also, if – this is actually only using one string *(referring to the Bowline)*, and this is – the Sheet Bend is always using two. It's used in trying to put two strings of dif – di – eve – uneven sizes together. You know, like normal people just tie a Pretzel – well, sailors use – need something stronger: the Square – the Sheet Bend.

7.2 Relationships within Groups of Knots

Here, children's arrangements of knots into groups reveal the topological and combinatorial characteristics they considered to be somehow important.

One aspect of the final interviews involved grouping knots according to perceived similarities. This activity was inspired by Tony's, Jill's, and Alice's construction of the Family Tree, an idea that had caught on generally in the Knot Lab. In the final discussions, I gathered now-familiar knots into a pile and asked each child, "Which of these knots go together?"



The question needed little explication. The children set about arranging knots into groups, which are described here.

The groupings sometimes reflected a child's general knowledge of or feeling about certain knots, but often indicated a sense of fundamental aspects of the configurations. Despite the children's unique ways of arranging the knots, many of their groupings reflected a common perceived importance of certain properties. Some of the children explained their categories in more detail than others, but with or without verbal explanations, some consistencies of principle became apparent.

In the following portrayals of the children's groupings, the isolated properties are given descriptive names, some of which the children themselves used and some of which I developed. Many of these designations simply characterize the knots in broad, obvious ways, but others strive to express singular details that distinguish the objects. The designations, *more or less complicated versions of the same basic idea* and *part of the knots produced in a similar manner*, refer to similar properties, but the latter goes beyond treatment of the knot as a finished object, to considering how it came to be that object. The children's explanations of why they grouped certain knots together usually provided some indication of whether the process of formation was a criterion in their arrangement.

Not only are the groupings themselves revealing, but the position of an individual knot relative to a certain cluster often conveyed a perception of subtle relationships among objects in and outside of the group. (And sometimes, children considered whole clusters as related in their entirety.) Where relevant, designations for these borderline knots or positions are included as well.

Where more than one example of a knot was present, children sometimes grouped the same knot in different categories to acknowledge various characteristics of the configuration. A striking theme has to do with variations that emerged in descriptive levels of detail (a knot has two loops) and broad consideration (a knot seems complicated) and in degrees of specificity (a knot has longer ends than another) or generality (a knot, like some others, is formed by two strings).

In characterizing the bases of these arrangements of knots, I sometimes quote the children directly, sometimes derive a description more or less directly from what they say or do, and sometimes – when their intent is not so clearly communicated – I infer a description based on a sense of what they mean, other aspects of the discussion, their project work, etc. Such inferences are marked with a ~ symbol. The designations, which are listed here, are accompanied by textual description that elaborates on each child's discussion.

Distinguishing characteristics:

thought to be the same knot
distinguishing part: circle
distinguishing part: loops
distinguishing part: ends
roughly the same size
distinguishing characteristic: size
distinguishing characteristic: tightness
conspicuously horizontal
two horizontal loops
two loops
more familiar
less familiar
many winds
more complicated
less complicated
involved, or intertwined, in a complicated and/or unfamiliar way
made from one piece of string
made from two pieces of string
learned the knots at about the same time
similar expression or containment of movement
leftover or miscellaneous knots
different versions of the same basic idea
more or less complicated versions of the same basic idea
part of the knots tied in a similar manner
formed in a similar way
alternating over/under pattern

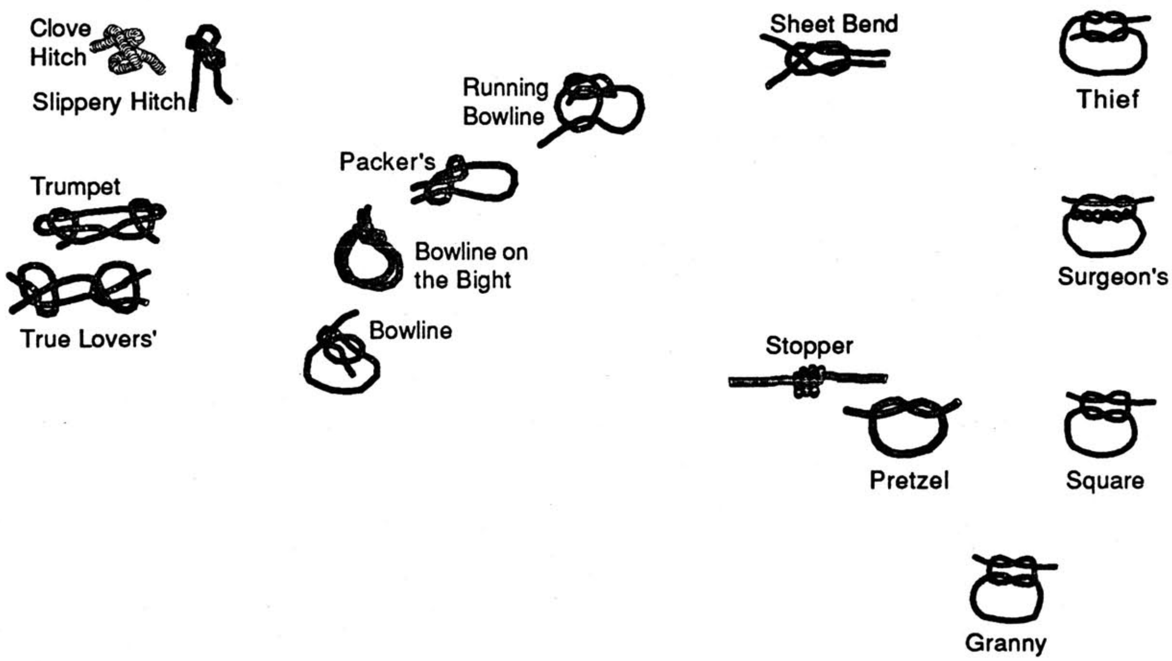
Borderline positions of knots:

alone but close to something similar
adjacent but not quite in the group
shared knot that indicates overlap of groups
a quintessential instance of a property that many have

7.2.1 Groups on the Family Tree

An early step toward understanding any sort of phenomenon is to learn what kinds of things there are in the set – to develop a taxonomy.

– Herbert A. Simon ([1968] 1981, 154)



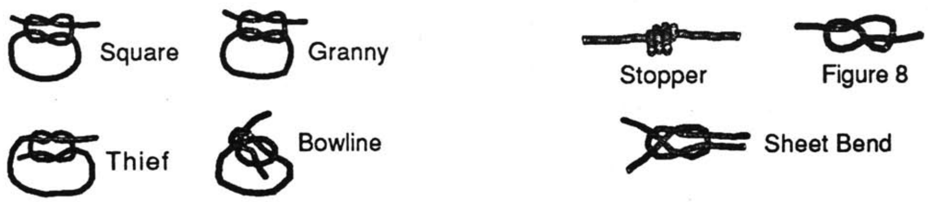
Group	Basis for Grouping
Slippery Hitch, Clove Hitch.	~ usually tied on a stick or other support
True Lovers', Trumpet.	True Lovers: "a version of two Pretzels" ~ conspicuously horizontal ~ symmetric quality ~ distinguishing characteristic: tightness
Bowline, Packer's, Bowline on the Bight, Running Bowline.	Packer's: "a Bowline with a step added" Packer's: "too manipulated – very interesting" Packer's, Bowline: similar containment of movement distinguishing parts: circles, loops
Sheet Bend.	~ similar to Bowline "like a Square knot, except it's twisted"
Square, Surgeon's, Thief.	more or less complicated versions of the same basic idea ~ distinguishing part: circle

	~ distinguishing part: ends
Granny.	"a bridge between the Pretzel and the Square"
Pretzel, Stopper.	"primitive" "most knots start with [the Pretzel]"

One of the bulletin board displays in the Knot Lab was a "family tree" that Alice, Jill, and Tony made together (see also Sections 6.22.8 and 9.1.4). The display consisted of twigs bound together to form a "tree" on which knots were glued or tied according to interpreted groupings. The Family Tree included knots that were not among the set used in the final interviews – the Hitches, Trumpet, and True Lovers' knots, as well as some of the more elaborate Bowlines, are not included in the following discussions of groupings, whereas the Stevedore and Figure 8 are.

The groupings of knots on the Family Tree do, however, provide an interesting comparison to the ways in which Alice, Jill, and Tony clustered knots during their final interviews. Alice put the Stopper with the Thief and the Square; and the Granny, with the Surgeon's (see Section 7.2.14). Jill also grouped the Granny with the Surgeon's, but maintained their proximity to the Square and the Thief. She also maintained relationships among the Square, Sheet Bend, and Bowline (see Section 7.2.15). Tony grouped the Sheet Bend and the Bowline together; the Granny, Surgeon's, and Thief went with the Square (see Section 7.2.16).

7.2.2 Juanita's Groups

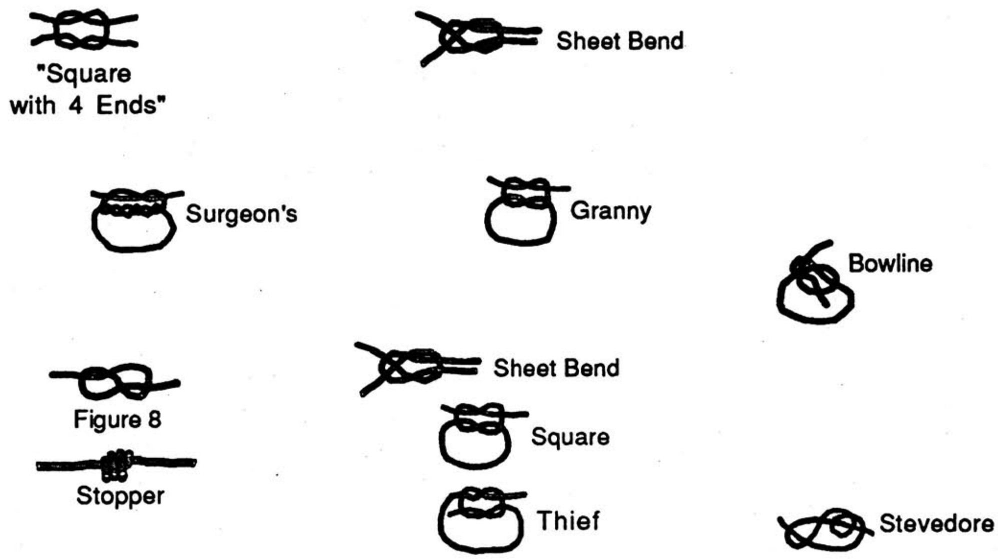


Group	Basis for Grouping
Square, Granny, Thief, Bowline.	~ distinguishing part: circles
Stopper, Figure 8, Sheet Bend.	~ conspicuously horizontal

Juanita's rationale seems straightforward: the knots with circles go together, and the horizontal knots go together.

She takes the "family" metaphor seriously, assigning relationships to the knots based on their sizes. In one family, the Sheet Bend is the father, the Figure 8 is the mother, and the Stopper is the baby. In the other family, she designates the Square as the father, the Thief as the mother, the Granny as the "big daughter," and the Bowline as the "little son."

7.2.3 Rosella's Groups



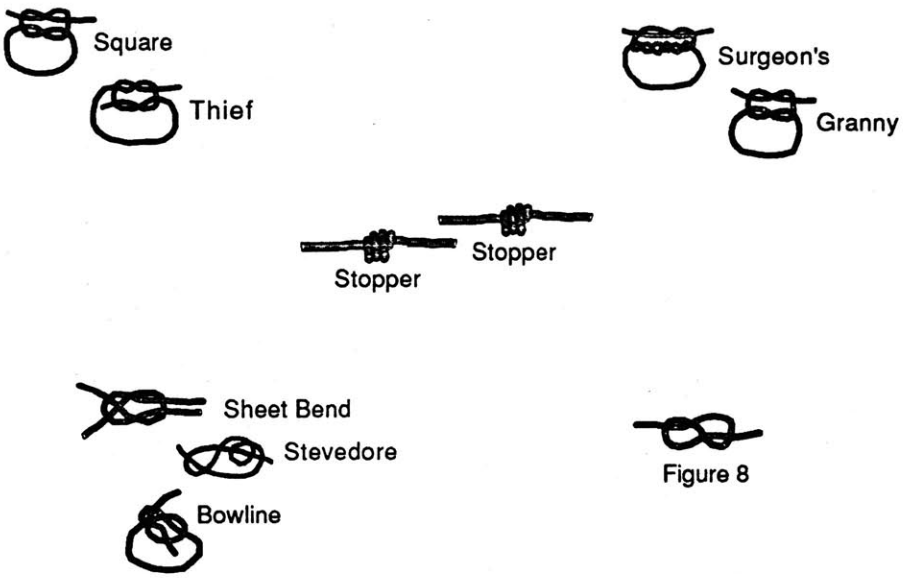
Group	Basis for Grouping
Figure 8, Stopper.	~ similar expression or containment of movement (winding) ~ (thought to be the same knot)
Sheet Bend, Square, Thief.	Square & Thief thought to be the same knot (Square) ~ learned the knots at about the same time ~ two horizontal loops
Surgeon's.	~ involved in a complicated and/or unfamiliar way
Granny.	~ involved in a complicated and/or unfamiliar way
Bowline.	~ involved in a complicated and/or unfamiliar way
Stevedore.	~ involved in a complicated and/or unfamiliar way
"Square with 4 Ends."	~ involved in a complicated and/or unfamiliar way
Sheet Bend.	~ involved in a complicated and/or unfamiliar way

Some misconceptions about the knots no doubt affected Rosella's decisions. For example, she calls the Thief the Square; she believes them to be the same knot, and so puts them together. She calls the grouped Sheet Bend the Figure 8; this may affect her arrangement because the Figure 8 and the Square were knots that she learned at about the same time.

She seems to think that the actual Figure 8 and the Stopper are versions of the same knot (if not loose and tight versions of the same knot); the relatively simplified winding of the Figure 8 recalls the process of winding that is so prominent in producing the Stopper knot.

The other knots seem so foreign to her that they each require their own group, separate from the groups of more familiar knots. Although she calls the Granny the Bowline, the misnomer does not affect her grouping of either knot. Interestingly, she does not notice that there are two Sheet Bends; one of them is kept apart in its own group. Similarly, she does not group the "Square with 4 Ends" with the other Square knot.

7.2.4 Celina's Groups



Group	Basis for Grouping
Square, Thief.	~ different versions of the same basic idea
Surgeon's, Granny.	more or less complicated versions of the same basic idea many winds formed in a similar way
Stoppers.	thought to be the same knot
Figure 8.	~ involved in a complicated and/or unfamiliar way alone but close to something similar
Sheet Bend, Stevedore, Bowline.	~ involved in a complicated and/or unfamiliar way

Celina groups the Square and Thief knots as though there is no question about their belonging together.

The Surgeon's and the Granny deserve an explanation:

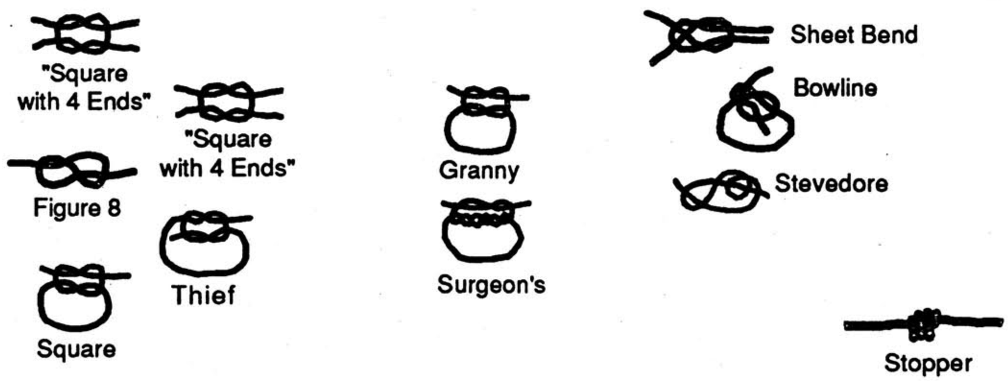
Celina: Well, these look the same, except this one *(the Surgeon's)* has more loops.

Her sense of the word "loops" refers to the winding motion that produces the lower part of the Surgeon's knot.

It's clear the Stoppers are the same knot, but the remaining knots are somewhat problematic.

After comparing the Stevedore and the Sheet Bend without satisfaction, Celina compares the Stevedore and the Figure 8. That doesn't look right either, so she settles on establishing groups for the problematic knots: the least complicated, the Figure 8, forms its own group, while the more complicated ones stay together. Interestingly, the group formed by the Figure 8 knot is thus best appreciated with regard to the threesome at its left, which developed simultaneously.

7.2.5 Marcos's Groups



<u>Group</u>	<u>Basis for Grouping</u>
"Square with 4 Ends," Figure 8,	thought to be the same knot "almost the same" distinguishing part: ends
Thief, Square.	made from one piece of string distinguishing part: circles roughly the same size distinguishing part: ends
Granny, Surgeon's.	more or less complicated versions of the same basic idea many winds
Sheet Bend, Bowline, Stevedore.	distinguishing part: ends ~ distinguishing part: circles ~ involved in a complicated and/or unfamiliar way many winds
Stopper.	~ many winds ~ distinguishing characteristic: tightness

The accumulation of knots in the leftmost group is key to understanding the relationships that Marcos deems important. The two "Square with 4 Ends" knots obviously go together. The Figure 8 is "almost the same," except for the fundamental difference about the way in which the ends are positioned at the left and right of each knot:

Marcos: ... this one's got these two *(the "Square with 4 Ends")*, and this one's got one *(the Figure 8)*.

Marcos thinks about several aspects of the Square and the Thief which he first groups together and then puts with the "Square with 4 Ends" and Figure 8:

Because they're – they got the same string – see? circle? Same – everything's almost the same size. And out the ends.

The Square and Thief are each tied with only one piece of string, which forms a circle. The proportions of these particular two knots are about the same, so they look a lot alike. And, in each knot, the ends provide a kind of "exit" from the knot.

He considers the Surgeon's to be basically the same as the Granny, except that it has more "winds."

The loops of the Sheet Bend, Bowline, and Stevedore make them look similar. They go together:

... 'cause they almost look the same. You see? They got two [loops] here – they look like that, like this.

Marcos turns the knots around, indicating the pairs of ends on each. He notes further that the Bowline and the Sheet Bend would be the same if two of the ends of the Sheet Bend were joined rather than separated:

This one and this one are almost the same. See – they go like that and that. See – why this one's separated. If this one was – like in the top, more like this – it would go – go – see? These two?

If the Sheet Bend were joined to form a circle like the Bowline's, they would be the same. The Stevedore is not so easy, though:

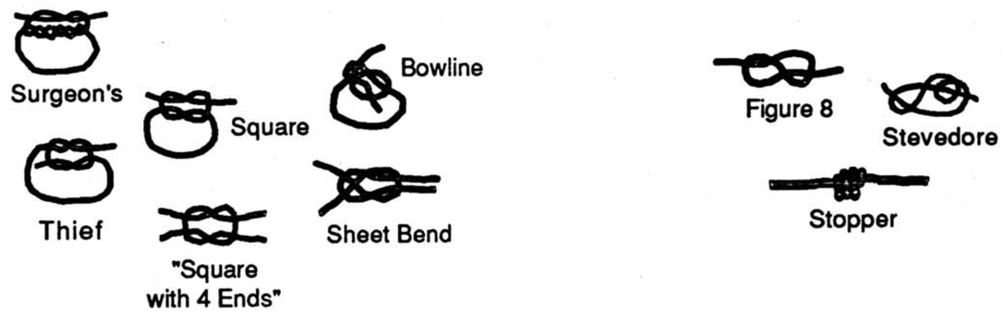
... this one, I'm not sure, because, this is like a – has like three wraps, you know?

"This one's weird," Marcos says about the Sheet Bend. "It looks like a man." It goes with the other strange-looking knots, like the Bowline and Stevedore – and possibly the Stopper:

This one and this one, they almost look alike. I think these four maybe go together – no – this one goes alone.

Unlike the others, the hallmark of the Stopper is the clump resulting from its tightened state. The Stopper also has many wraps, but Marcos chooses to categorize it according to its finished appearance rather than his knowledge of how it is produced.

7.2.6 Jack's Groups



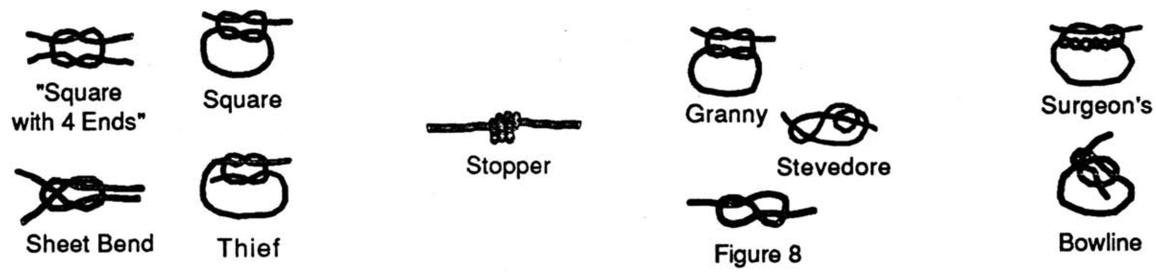
Group	Basis for Grouping
Thief, Square, "Square with 4 Ends,"	~ different versions of the same basic idea
Bowline,	"it's like the same design" different versions of the same basic idea
Sheet Bend,	"just looks like a bigger version" distinguishing characteristic: size
Surgeon's.	~ more or less complicated version of the same basic idea
Figure 8, Stevedore.	~ formed in a similar way ~ more or less complicated versions of the same basic idea
Stopper.	"completely different than all of them"

Although, at first, Jack compares the Bowline and Granny knots, the Granny does not appear in his final groupings. Instead, he puts the Bowline in the group containing the Squares:

Jack: This one *(the Bowline)* seems like it would sorta go into this one, 'cause it's like the same design.

He also puts the Sheet Bend in that group, but views it as related to the Bowline in particular – it "just looks like a bigger version" of the Bowline – the actual entanglement is larger in the Sheet Bend.

7.2.7 Doreen's Groups



Group	Basis for Grouping
"Square with 4 Ends," Sheet Bend,	~ conspicuously horizontal ~ two horizontal loops different versions of the same basic idea
Square and Thief.	thought to be the same knot
Granny, Stevedore, Figure 8.	"because of the way they twist" ~ alternating over/under pattern
Surgeon's, Bowline.	"because of the many ways they go" many winds similar expression or containment of movement ~ involved in a complicated and/or unfamiliar way
plus Stopper	distinguishing characteristic: tightness ~ many winds ~ similar expression or containment of movement

Doreen notices a similarity between the "Square with 4 Ends" and the Sheet Bend, based on "the way the track is." Calling attention to the form of the entanglements leads her to notice the similarity to the Square and Thief, which she thinks are the same knot. She adds them to the group. She notes that the "Square with 4 Ends" is also the same – her concern is with the entanglements in particular. She acknowledges the Sheet Bend as a slightly different knot, but does not specify why, though she pays some attention to the crossed ends at the left of the knot.

The Granny, Stevedore, and Figure 8 go together:

Doreen: ... because of the way they twist – they like, twist, like – into each other. *(As she speaks, she gestures broadly and symmetrically with her hands, pointer fingers extended to show the curves going out together and then coming back in together.)*

Interestingly, she makes no comment on the potential similarity between the Granny and the Squares. This group of three seems to be based on the alternating over/under pattern of the crossings in the knots, as opposed to the "matched pairs" of crossings formed by the loops of the Square knots.

5.4.1 5.4.2 10.1

The Surgeon's and Bowlines appear more complicated, and the winds are their distinguishing feature:

These I put together because of the many different ways they go.

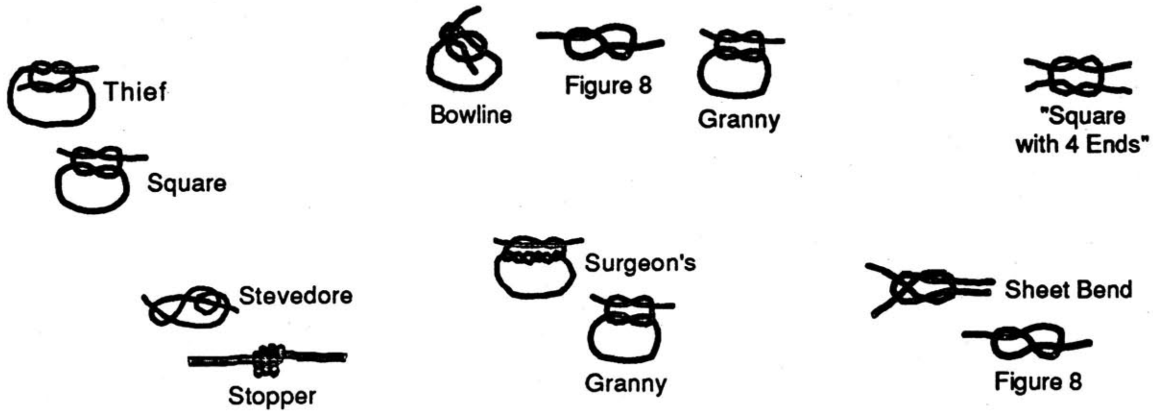
Carol: In many different – what do you mean?

Like ... they ... go around ... *(She gestures broadly by crossing her two hands and then doing a circular motion with her right hand.)*

Her arms and hands become, for the moment, the ends of the string, as she herself seems to become the knot in the process of being formed.

Although at first Doreen declares that the Stopper "should be alone," she later decides that it should go with the Surgeon's and the Bowline. The move indicates a change in her thinking about the knot as a finished object, to the process that produces it.

7.2.8 Althea's Groups



<u>Group</u>	<u>Basis for Grouping</u>
Sheet Bend, Figure 8.	"both kind of look like the Figure 8" ~ conspicuously horizontal ~ two loops distinguishing part: ends distinguishing characteristic: size
Surgeon's, Granny.	"both look like the Granny" different versions of the same basic idea
Bowline, Figure 8, Granny.	"it doesn't look like anything" (Bowline) leftover or miscellaneous knots (Figure 8 & Granny)
Stevedore, Stopper.	many winds ~ part of the knots tied in a similar manner distinguishing part: ends distinguishing part: loops
Thief, Square.	different versions of the same basic idea
"Square with 4 Ends."	"it doesn't look like anything" ~ made from two pieces of string ~ distinguishing part: ends ~ alone but close to something similar

What is most immediately striking about Althea's arrangement is her separation of the two Figure 8 knots and the two Granny knots. In explaining why the Figure 8 and the Sheet Bend go together, she moves from an explanation involving their general similarity, to an explanation of their difference, which relies on the crossed ends of the Sheet Bend and the sizes of the knots – differences insufficient, however, for her to place the knots in separate groups:

Carol: *Why do these go together (Figure 8 and Sheet Bend)?*

Althea: They look like – they both kind of look like the Figure 8 a little – like – this is the Figure 8, this kind of – if you take a good look –

Okay – what about it looks a little different from the Figure 8?

This – this is in here *(the crossed end through the loop of the Sheet Bend)*, and the Figure 8 is not like this. This is much bigger, right here *(the entanglement of the Sheet Bend)*, and *(shrugs)* it doesn't really look like the Figure 8.

Um hmm.

These go in the same group, though.

Like the Figure 8, the Granny is the more familiar knot against which another is compared. The Surgeon's goes with it because, simply, they look alike.

Okay. And how about these? Why do these go together (Granny and Surgeon's)?

They're kind of the same. They – they – they – they – both look like the Granny.

On the other hand, the presence of the Granny and the Figure 8 in the same group, with the Bowline, does not seem to imply a relationship. Having already grouped the Granny and Figure 8 meaningfully, the remaining two knots become extraneous, and are put in a category with a knot that is so mysterious (the Bowline) that it doesn't warrant serious consideration. Originally, Althea had it in a class by itself:

Because it *(the Bowline)* doesn't look like anything. I mean, it's a knot, but it doesn't look like any of those *(indicates the rest of the knots on the table)*.

But the Bowline thus becomes the basis for a kind of non-category, and Althea adds to it the extra Figure 8 and Granny.

In her general view of the Granny and Surgeon's, Althea calls attention to an overall similarity of appearance, but does not focus on what makes the knots different. Had she done so, she would likely have discussed the coils of the Surgeon's, a feature that comes up in her discussion of the Stopper and Stevedore knots. In fact, she might even have grouped the Surgeon's knot with these two, on the basis of that feature.

Okay. Why do these go together (Stopper and Stevedore)?

This is the Stopper. And this is like a Stopper, winding like this here, kind of *(she fingers the coils of the Stevedore).* This – if these two weren't there, these two both would be the Stopper.

That is, if an end of the Stevedore were not tucked into the left loop, the knot would better approximate the Stopper – or, for Althea, "both would be the Stopper."

A familiar knot, the Square, again becomes the basis for a group, as Althea puts the Thief knot with it:

These go together 'cause they look like the Square knot *(Square and Thief).*

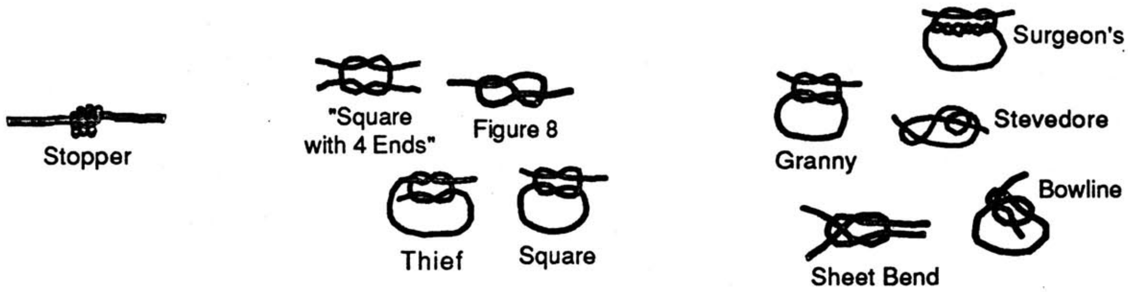
And, rather than noting the similarity of the Square and the "Square with 4 Ends," Althea seems to get stuck by the use of two strings to tie the "Square with 4 Ends." She momentarily compares it to the Sheet Bend, the only other knot with this feature, but abandons the tack because its peculiarly crossed end makes it look so different from the flat, relatively straightforward entanglement of the "Square with 4 Ends."

This *("Square with 4 Ends")* – I didn't really know where to put this one, because it doesn't look like anything – except for this one *(Sheet Bend)*, but this does not really look the same.

So, the "Square with 4 Ends" is in a class by itself. Its placement near the Sheet Bend and the Figure 8 may be a vestige of her comparison of the "Square with 4 Ends" and the Sheet Bend.

Althea seems to vacillate continually between taking a generalized view of the knots, which allows her to see their similarities, and a more detailed view, which enables her to examine differences between them.

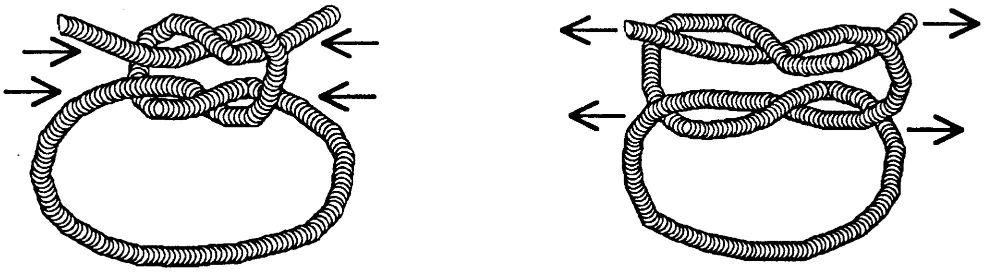
7.2.9 Leroy's Groups



Group	Basis for Grouping
"Square with 4 Ends," Figure 8, Thief, Square.	"these are just easy [to open]" ~ similar expression or containment of movement ~ two loops
Granny, Surgeon's, Stevedore, Sheet Bend, Bowline.	"they're hard to open" ~ similar expression or containment of movement ~ involved in a complicated and/or unfamiliar way ~ alternating over/under pattern
Stopper.	"this isn't like any of them" ~ distinguishing characteristic: tightness

5.2 **6.8** 9.4.2

Leroy adopts a unique basis for grouping the knots: he does the "Square knot test" to all of them. For the Square knot, this test is easy: it is a way of loosening and tightening the knot repeatedly, by pushing the two horizontal loops so they move apart, and then pulling them closer together.

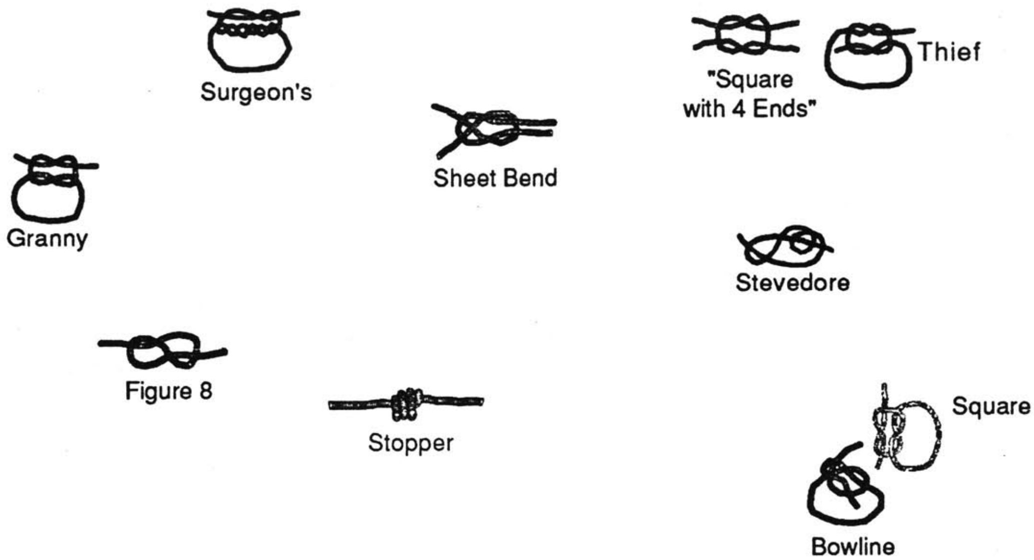


Other knots with two loops are also easily tested in this manner, but knots that are more convoluted, or whose loops are interrupted by an alternating over/under pattern, do not respond as easily to the test. Such knots may move back and forth, but have to be pushed and yanked in a more pronounced, less fluid way.

Leroy applies the test to each knot with care and thoughtfulness, painstakingly grouping the knots according to whether they respond by readily moving back and forth, or have a tendency to resist his prompting. The Stopper, of course, is completely tightened and won't move at all – so it has to be in a class by itself.

That Leroy would have chosen this basis for categorizing the knots is delightfully consistent. Throughout the project, his way of becoming involved with certain knots was to see in them an expression of some kind of movement or potential for physical activity, as in his description of the "Figure 4" wrestling hold (Section 6.22.7) and in his likening of a Monkey's Fist knot to a volleyball (Section 6.22.5).

7.2.10 Eugene's Groups



Group	Basis for Grouping
Figure 8.	~ conspicuously horizontal ~ alone but close to something similar
Sheet Bend.	~ conspicuously horizontal
Stevedore.	~ conspicuously horizontal
Granny.	distinguishing part: circle
Surgeon's.	~ more complicated distinguishing part: circle
"Square with 4 Ends," Thief.	~ distinguishing part: circle (Thief) ~ distinguishing part: loops
Square, Bowline.	~ distinguishing part: circle ~ more complicated (Bowline) ~ distinguishing part: loops
Stopper.	~ conspicuously horizontal ~ alone but close to something similar

Eugene leaves so many of the knots in their own groups that the order in which he works with them becomes as revealing as the classifications he does make. First he selects the Figure 8, the Sheet Bend, and the Stevedore, all knots with a conspicuously horizontal orientation. The Granny, Thief, and Square are next; later, he mentions the circle as an important part of knots like these. The Stopper is alone, but its position next to the Figure 8, also a predominantly horizontal knot, is suggestive, especially given his play with the orientation of the Square knot immediately thereafter (grouped with the Bowline). Then Eugene selects the nearly identical "Square with 4 Ends," although he groups it with the Thief, which he may consider to be the same knot. Interestingly, although he speaks about the circle part of various knots, Eugene's grouping of the "Square with 4 Ends" and the Thief suggests a focus on the entanglements. The Surgeon's and Bowline are more complicated, but still have a circle, which guides their classifications:

Eugene: These two are a group *(Square and Bowline).* They are two. ... 'Cause they're on the same branch.

Eugene is working very literally within the "family tree" metaphor. It enables him to describe the relationship between the Granny and the Surgeon's, although they are not as closely related as the Square and the Bowline:

Carol: *How about these two (Granny and Surgeon's)?*

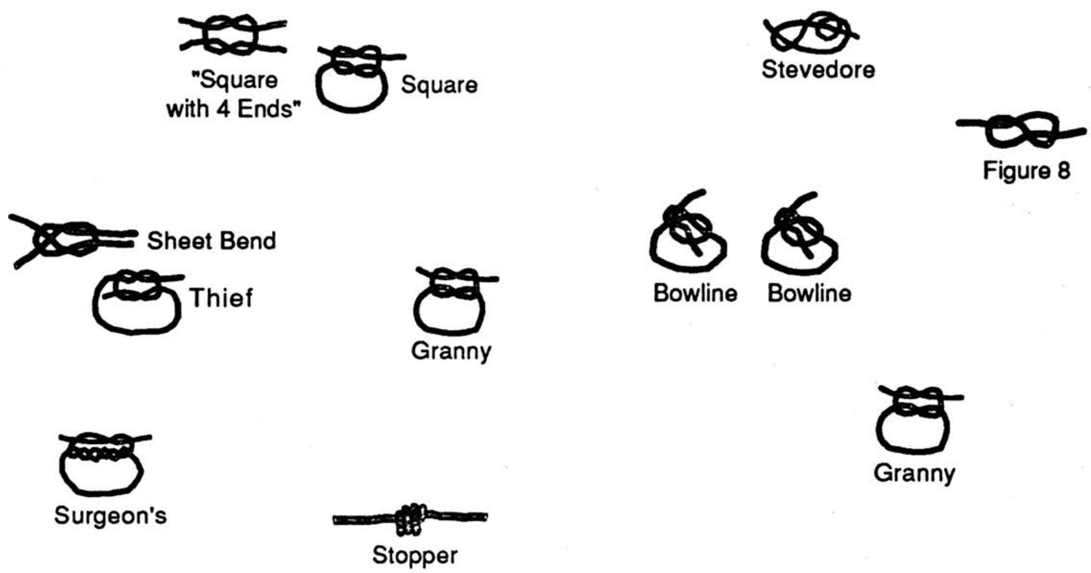
Eugene: 'Cause they all have circles, so I put them on the same branch.

These have circles (Square and Bowline).

Yeah, and these are on the same branch.

Beyond the circles, Eugene seems to see a similarity between the Square and the Bowline, which he tries to explore by rotating the Square. He does not articulate a discovery, but chooses to leave the Square knot in this orientation.

7.2.11 Curtis's Groups



Group	Basis for Grouping
Sheet Bend, Thief.	~ two loops ~ conspicuously horizontal (Sheet Bend) "family"
"Square with 4 Ends," Square.	~ two loops ~ conspicuously horizontal ("Square with 4 Ends") different versions of the same basic idea "half-cousins"
Surgeon's.	~ two loops ~ alone but close to something similar "friend"
Stopper.	~ conspicuously horizontal ~ distinguishing characteristic: tightness "friend"
Granny.	~ two loops ~ alone but close to something similar "friend"
Bowlines.	~ two loops thought to be the same knot "together by theirself" "family"
Stevedore.	~ conspicuously horizontal ~ alone but close to something similar

	"friend"
Figure 8.	~ conspicuously horizontal ~ alone but close to something similar "friend"
Granny.	~ leftover or miscellaneous knot "friend"

As Curtis explains his groupings, he is continually revising them. Originally, he develops a "Square group," which consists of the Thief, the Square, a Granny, a Bowline, the Surgeon's, and the other Granny, all notable for their way of embodying two facing loops. He is careful in his positioning of knots to match orientations or align corresponding parts of the knots. He compares the Figure 8 to the "Square with 4 Ends," but doesn't group them; he does put two other horizontally oriented knots together, the Stopper and the Sheet Bend. He puts the two Bowlines together and then, as he talks, moves and compares several of the horizontal knots: the "Square with 4 Ends," the Sheet Bend, the Figure 8, and the Stevedore. He dwells on these knots for a while, shifting them nearer and between one another. The Sheet Bend and the "Square with 4 Ends" were together; as he moves the Sheet Bend instead next to the Square knot, he explains, "I thought they looked the same for a minute." His perceptions of similarities are transient.

He declares the Granny and Thief to be "the same kind," yet does not put them close enough to be considered as the same group. Instead, they are on "two branches." The Square and Surgeon's, however, go on the "same branch." What the Granny and Thief have in common, perhaps, is that they are odd variations of the Square knot – they seem to have somehow gone awry, whereas the Surgeon's maintains the integrity of the Square knot while embellishing it.

The game of musical chairs continues as he comes around to the "Square with 4 Ends" again. He moves it closer to the Square knot, saying to it, "Let me get you in here."

The group of Squares is beginning to take shape. He adds the Thief saying to the knot, "Well, I'll put you there," and to me, "because it looks the same." The knots seem very much alive, mobile and part of our conversation.

Curtis moves the Stopper, speaking for it: "Put me at the bottom." To me, he explains, "Lonely." I ask:

10.1

***Carol:** ... what does it mean to be at the top, or to be at the bottom? What do they mean?*

Curtis: Uh – the bottom ones, they don't have no – like a friend to be with. The top ones, they have like, a partner. Like these two, the Bowline.

Oh, I see.

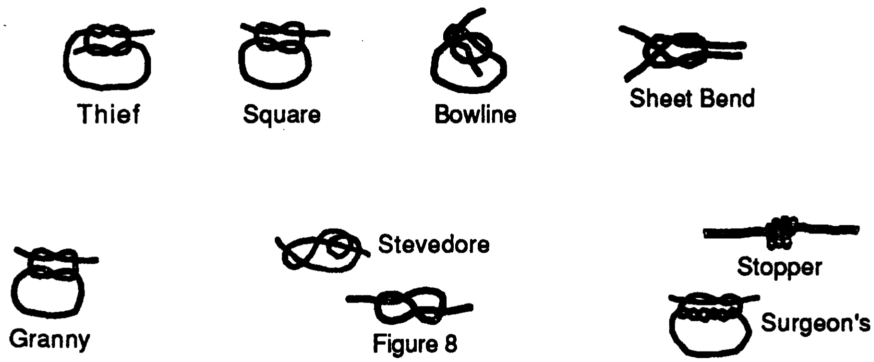
They would be together. Like they're family, and these are not family.

Uh huh. Which is the family part?

These two are family *(the Bowlines)*, these two *(the Square and the "Square with 4 Ends")* are cou – half-cousins. And, um, these two are family *(the Sheet Bend and the Thief)*. And this one, he's a friend of the family *(the Granny)*. And he's a friend *(the Surgeon's)*. He's a friend *(the Stopper)*. He's a friend *(the other Granny)*. He's a friend *(the Figure 8)*. He's a friend *(the Stevedore)*. *(Curtis laughs.)*

Curtis perceives the closest relationships among the knots he has come to know the best: the Bowline, the Squares, the Sheet Bend, and the Thief. His designation of the top part of the scheme as containing knots that have a "partner" implies that, while the Stevedore and the Figure 8 appear in their own groups, their position next to each other indicates some similarity.

7.2.12 Stacy's Groups

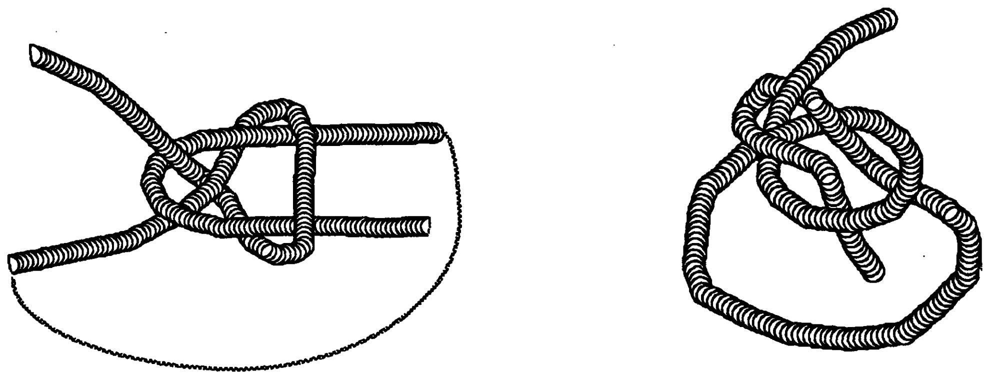


Group	Basis for Grouping
Thief, Square, Bowline, Sheet Bend.	different versions of the same basic idea (Thief, Square) "one's over and one's under" (Thief, Square, Bowline) distinguishing part: loops distinguishing part: ends (Bowline, SheetBend) distinguishing part: circle (Bowline, Sheet Bend) ~ alternating over/under pattern (Sheet Bend)
Stevedore, Figure 8.	conspicuously horizontal distinguishing part: circles distinguishing part: ends distinguishing part: loops ~ more or less complicated versions of the same basic idea
Stopper, Surgeon's.	part of the knots tied in a similar manner ~ many winds distinguishing part: ends
Granny.	~ leftover or miscellaneous knot

Stacy begins by grouping the Thief and Square knots, then adds the Bowline. At first she thinks the Sheet Bend doesn't look like it "goes with anything," but then her eye catches the Bowline and she compares the two. She decides the two knots are alike:

Stacy: ...'cause there's a little thing that goes over – if this went through here – it would be really close – it would be like the circle. *(She joins the ends of the Sheet Bend.)*

That is, the crossed end is the same on each knot, and the other two loose ends on the Sheet Bend could become the circle of the Bowline:



Stacy expresses clearly that the similarity she sees between the Stopper and the Surgeon's knots has to do with her knowledge of how they are formed:

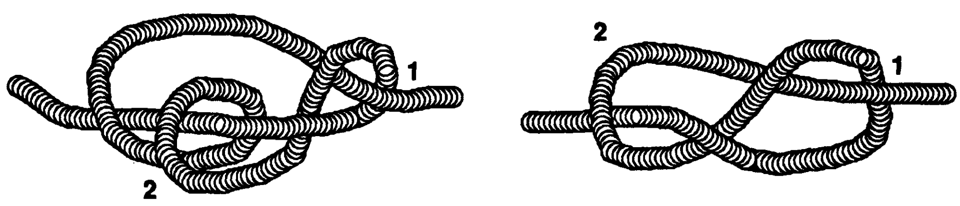
***Carol:** Okay - so why do these two go together? (Stopper and Surgeon's)*

Stacy: Because that – the process – this part here *(lower part of Surgeon's)* is the process of making that *(Stopper)*, so if you just took this *(upper crossing of Surgeon's)* and pulled it, it would be – *(the Stopper).*

In order to show the similarity between the Stevedore and the Figure 8, it helps to orient them so that the Stevedore's extra twist becomes apparent:

Okay – how about these two? (Figure 8 and Stevedore)

Um - because there's *(she reorients Figure 8 to match Stevedore)* – like a circle right here, and then an end that goes through the circle (1), and – even though there's two loops here *(Stevedore)* and one there *(Figure 8)* (2).



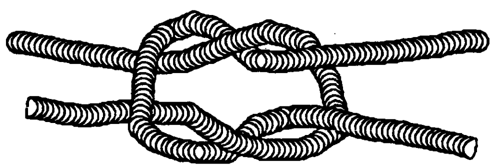
Stacy's comparison of the Sheet Bend and the Bowline is facilitated by a discussion of the Sheet Bend's similarity to the entanglement of the Square knot. Using the "Square with 4 Ends" helps – its two strings, matching those of the Sheet Bend, provide an easy start:

Okay – so what is that like?

These – but – um, the Square knot.

Okay – what if we did a Square knot without a circle?

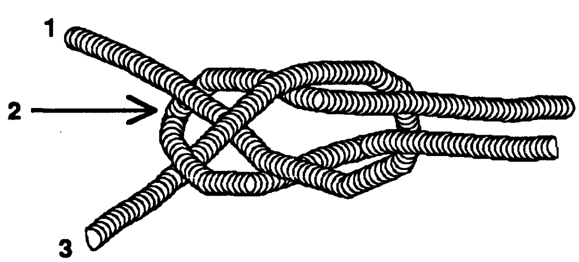
Did a Square knot like this *(ties two strings).*



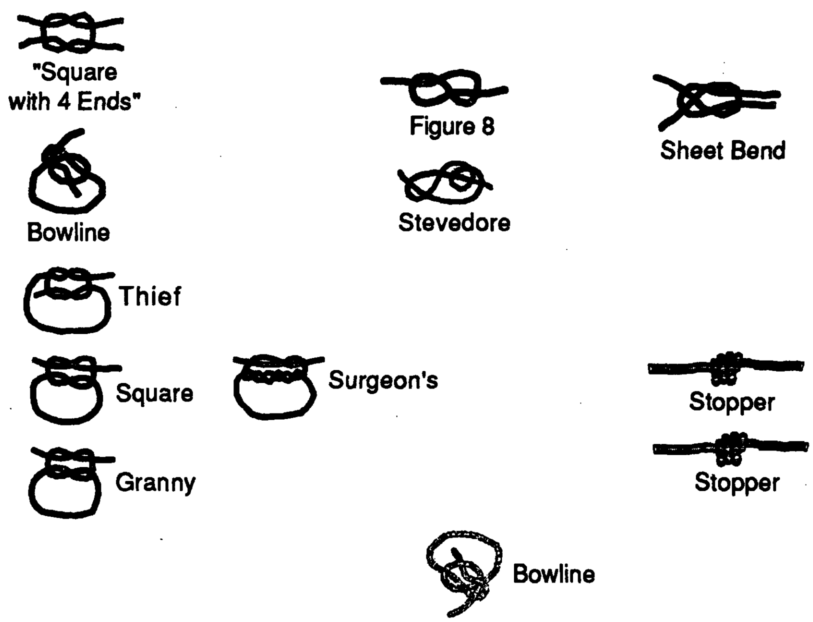
It's almost exactly like these two *(the two strings that form the Sheet Bend).*

Okay – and what's the difference?

That this one goes out of the loop (1) – and if it went in – *(looks)* – I think so – yeah, if this one was on this side (2) *(putting the strange bight inside the loop)* and this one was through here (3) *(it would be the same as the Square]. ((3) would stay where it is, becoming the partner of the newly placed* (1) *in the same manner as the Square knot.)*



7.2.13 Julio's Groups



Group	Basis for Grouping
"Square with 4 Ends," Bowline, Thief, Square, Granny.	~ more or less complicated versions of the same basic idea ~ distinguishing part: ends ("Square with 4 Ends," Bowline) "kind of the same" (Thief, Square, Granny) distinguishing characteristic: tightness (Square)
Surgeon's.	"looks the same as" the Square (but looser)
Figure 8, Stevedore.	~ conspicuously horizontal distinguishing part: winds (Stevedore)
Sheet Bend.	~ alone but close to something similar ~ conspicuously horizontal
Stoppers.	~ thought to be the same knot
Bowline.	~ leftover or miscellaneous knot

Julio first compares the "Square with 4 Ends" and the Bowline, carefully putting the entanglements side-by-side and letting the ends hang down next to each other. He arranges the Thief, Square, and Granny in a row, aligning the entanglements so he can get a good look at them. "These are kind of the same," he says. They stay together, joining the other two knots as a group.

He compares the Figure 8 and Stevedore side-by-side, noting that the Stevedore is different because it "has two of those things right there." The Sheet Bend goes near them, but is not similar enough to belong in the group.

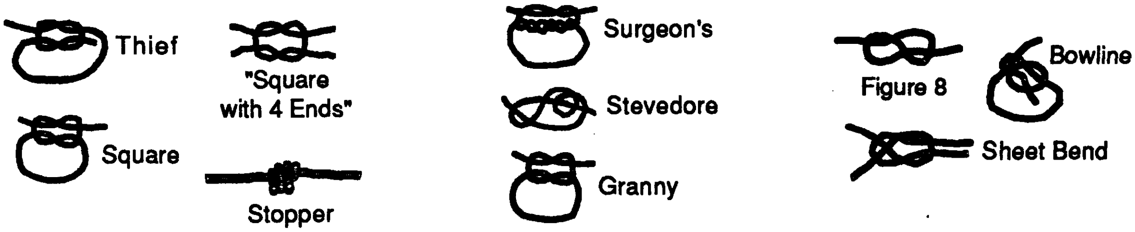
Interestingly, though the extra wind of the Stevedore is the feature that distinguishes it from the Figure 8, Julio seems to ignore this repeated feature of the Surgeon's knot. He compares it to the Square, saying:

Julio: This one *(the Surgeon's)* looks the same as this one *(the Square)*. Except this one's tighter *(the Square)*.

He overlooks the extra winds of the Surgeon's, apparently thinking that the string is just loose, so that if the knot were tightened, it would be a Square.

He leaves the second Bowline inverted and in a group by itself; perhaps he does not recognize it as the same as the knot he put with the Square-like knots.

7.2.14 Alice's Groups



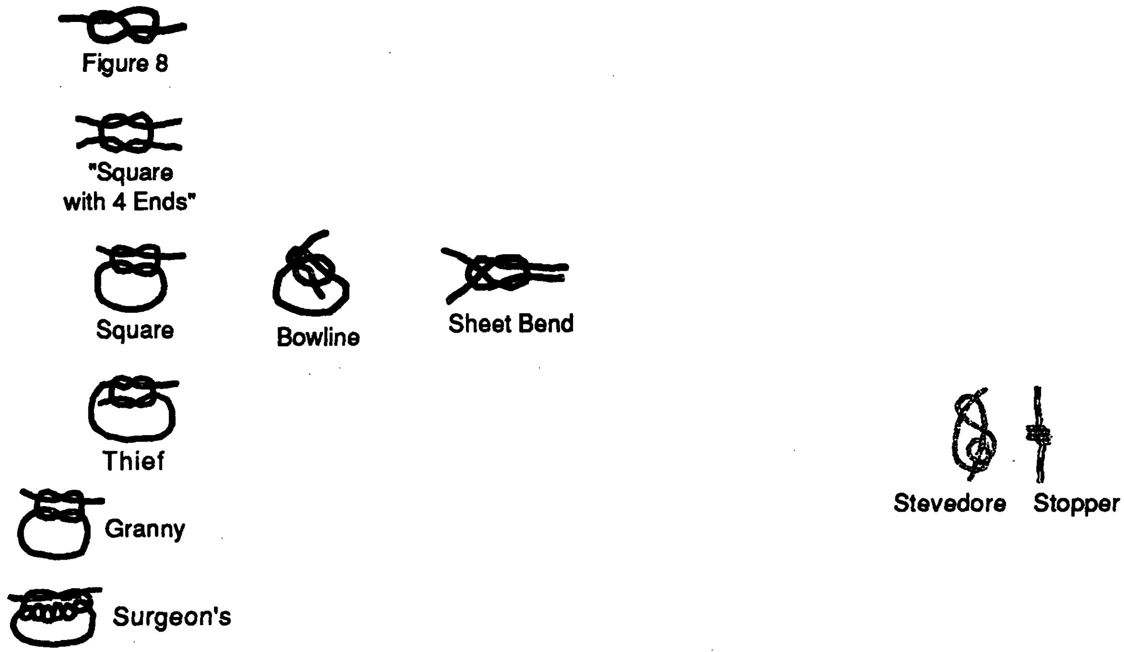
Group	Basis for Grouping
Thief, "Square with 4 Ends," Square, Stopper.	"look somewhat the same" (Thief, Square, "Square with 4 Ends") "this one, I guessed" (Stopper) ~ distinguishing characteristic: tightness (Stopper) ~ leftover or miscellaneous knot (Stopper)
Surgeon's, Stevedore, Granny.	"the shape" (Granny, Surgeon's) many winds (Surgeon's, Stevedore) ~ shared knot that indicates overlap (Surgeon's)
Figure 8, Bowline, Sheet Bend.	"mostly because of the knot" (Figure 8, Sheet Bend) ~ conspicuously horizontal (Figure 8, Sheet Bend) ~ two loops (Figure 8, Sheet Bend) ~ shared knot that indicates overlap (Sheet Bend) "these 2 look like that *(shows peculiar crossing)*" (S.Bend, Bowline)

Alice's Square group is based on the specific blockade-type pattern formed by the configuration of the Square knot; she doesn't include other "circle"-type knots, as other children do. Toward the end of the conversation she is hurried, though, and haphazardly tosses the Stopper into this group. She is not clear about whether or not she was thinking that, if the Stopper were loosened, it would reveal a configuration like that of the Square; she says only, "I guessed."

The two horizontal loops in the Figure 8 and the Sheet Bend seem to be her criterion for putting these knots together, though she says only that she grouped them "mostly because of the knot." She does not verbalize the similarity between the Sheet Bend and the Bowline, but indicates with her finger the peculiar crossed end and its situation relative to the facing loop, on each knot ("these two look like that"). Her arrangement places the Sheet Bend in an interesting conceptual position: it is related to each of the other knots, though these two are not notably related. Thus the Sheet Bend serves a kind of binding function, holding the group together.

The same function is served by the Surgeon's knot, which she relates both the Stevedore and the Granny, though she does not express a relationship between the latter two. The winds on the Surgeon's and the simpler Stevedore are the basis for their pairing, and "the shape" of the Surgeon's and Granny constitutes their similarity. They have circles and they are not Squares, so they must belong together.

7.2.15 Jill's Groups



Group	Basis for Grouping
Figure 8, "Square with 4 Ends," Square, Thief.	distinguishing part: loops distinguishing part: ends ~ two horizontal loops ~ alternating over/under pattern ~ shared knot that indicates overlap of groups (Square)
Bowline, Sheet Bend.	"the loop & a string coming out of the loop"
Granny, Surgeon's.	"one goes out the bottom, and one goes out the top"
Stevedore, Stopper.	~ many winds ~ formed in a similar way ~ more or less complicated versions of the same basic idea conspicuously vertical distinguishing part: ends

Jill's explanations for why she grouped the knots in the way that she did rely heavily on the position of the ends in the knot, and on distinctions of the "top" and "bottom" of each knot.

Her "Square family" begins with the Figure 8, which:

Jill: ... sort of looks like this one *("Square with 4 Ends")*, because, even though it doesn't have two strings, it has both loops with one coming out, each way.

"Coming out each way" refers to the fact that the ends of the Figure 8 are oriented differently as they exit the knot: one goes under a loop, and the other goes over. Likewise:

... the Thief is almost like the Square, because the one that comes out *(of the right end of the Thief)* is on the bottom, and this one's *(the left end)* on the top.

The ends of the Granny have a different relationship to the "blockade" formed by the loops of the Square and Thief knots. Jill calls attention to this difference and notes that the Surgeon's knot has the same characteristic. (In most of the other children's groupings, the Surgeon's knot has a more Square-like formation, which is how the knot is normally tied. But among the knots with which Jill is working, the Surgeon's must have been re-tied; it is more Granny-like.) To emphasize the difference in the blockades of the Granny and the Surgeon's, Jill sets these two knots slightly to the left, although they are still in the Square group.

And the Granny *(she touches it)* is a little bit – I guess it would be separate, because they don't – because one goes out on the bottom *(left end of Granny)* and one goes out on the top *(right end of Granny.)* And this *(the Surgeon's)* – one goes out on the bottom *(left end)* and one goes out the top *(right end)*, like there. So these are – the Granny, here *(moves these two to the left)*.

"The Bowline looks a bit like the Square," she says. Its similarity to the Square knot is pronounced enough that she places the two knots next to one another. In this way, the Square knot becomes an expression of overlap between two groups. Jill at first doesn't recognize the Sheet Bend and has it in a category by itself, but when she realizes its close relationship to the Bowline, she groups these two knots together:

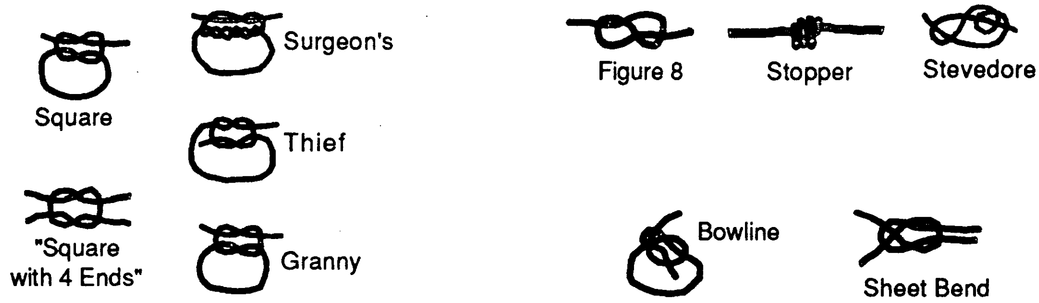
... they both have the loop and a string coming out of the loop.

Jill describes of the similarity and difference between the Bowline and the Sheet Bend in terms of the ways in which the ends exit the knots relative to the loops and circle.

Her comparison of the Stevedore and Stopper knots is much more general:

Well, I put these two together because, even though this is supposed to be a Stevedore, it looks really like a Stopper. If you compare it like that, it does look a lot like a Stopper. *(She holds both vertically with ends showing.)*

7.2.16 Tony's Groups



Group	Basis for Grouping
Square, Surgeon's, Thief, "Square with 4 Ends," Granny.	"note the resemblance" (Square, Surgeon's) ~ distinguishing part: loops ~ different versions of the same basic idea
Bowline, Sheet Bend.	~ different versions of the same basic idea ~ distinguishing part: loops ~ distinguishing part: ends ~ distinguishing part: circle (Bowline) "The Sheet Bend is a cross between a Square and a Bowline."
Figure 8, Stopper, Stevedore.	"their shape" (Figure 8, Stevedore) "rather simple knots" (Figure 8, Stopper) "you get from using an overhand or an underhand" (Fig. 8, Stopper) formed in a similar way (Figure 8, Stopper) ~ conspicuously horizontal ~ more or less complicated versions of the same basic idea (8, Steve.)

Tony begins with his favorite knot, the Square, compares it to the Surgeon's, and quickly adds the Thief and the "Square with 4 Ends." He does not add the Granny to the group by stumbling across it, but actively searches for it among the remaining knots. Unlike some of the children who considered the Granny more as a "Square knot gone wrong" than a legitimate knot, or who were bothered by its interruption of the Square knot's "blockades," Tony considers it to be just another version of the Square knot, well within the "family."

He becomes engrossed in two knots, one of which reminds him of the "Square with 4 Ends" because of its use of two strings:

Tony: **This is – the question is, what's the difference between the Bowline and the Sheet Bend?** ***(pause)*** **The Sheet Bend is tied with two strings! The Sheet Bend is a cross between a Square and a Bowline.**

Naturally the Sheet Bend and Bowline belong in the same group, though his positioning of the knots does not reflect the relationship to the Square that he has verbalized.


5.4.1 8.4

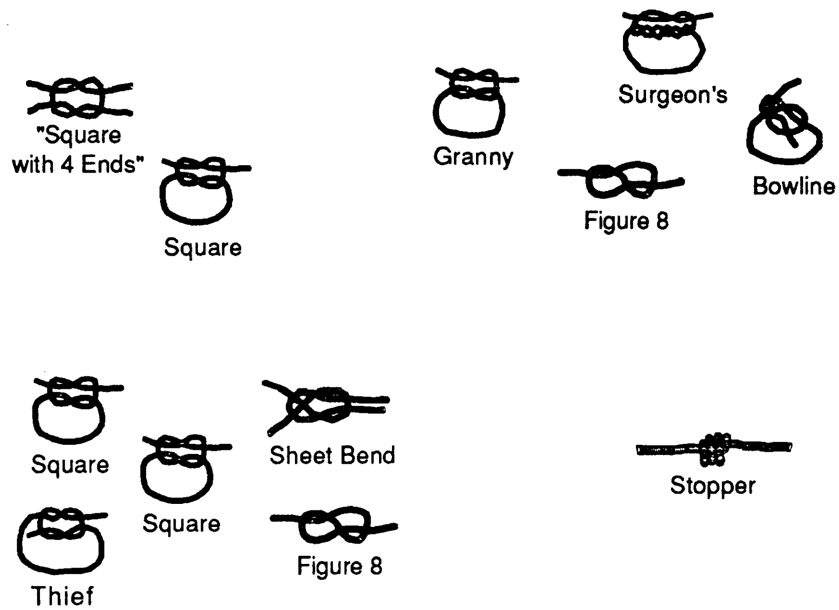
Tony's reason for grouping the Figure 8 and the Stopper has to do with they way in which the knots are formed:

... the Stopper and the Figure 8 are both rather simple knots – which you get from using an overhand or an underhand – knot – using it that way – that's how you get those two ...

The Stevedore belongs with them because of its resemblance to the Figure 8. Thus the Figure 8 is the "binding" knot in this group.

Though Tony has demonstrated an appreciation of fine details of the knots he has worked with during the course of the project, his tendency in this conversation is to describe them at a more general level.

7.2.17 Patrick's Groups



Group	Basis for Grouping
"Square with 4 Ends," Square.	~ different versions of the same basic idea
Granny, Surgeon's, Figure 8, Bowline.	"twist on top and twist on bottom" (Gran., Surg., Bow.) "I don't think that goes anywhere." (Bowline) ~ involved in a complicated and/or unfamiliar way (Bowline) "they go in the same places, sort of" (Fig.8, Surg.) part of the knots tied in a similar manner (Fig.8, Surg.) ~ more or less complicated versions of the same basic idea (Gran., 8) ~ alternating over/under pattern
Square, Thief, Square, Sheet Bend, Figure 8.	"look alike...sort of – like the same" (S.Bend, Fig.8) ~ more or less complicated versions of the same basic idea (Bend, 8) two horizontal loops ~ distinguishing part: ends (S.Bend, Fig.8)
Stopper	"It would be alone." ~ distinguishing characteristic: tightness

Patrick begins by selecting the knots he "knows": the Granny, Surgeon's, Square, and Figure 8. (By now there are several duplicates in the group, as he and other children have added newly tied knots.) He does the Square-knot test to the Square-like knots, at first believing the Thief to be a Square. Patrick comments on the proliferation of Square knots:

Patrick: I always get those – have a lot of Squares.

Regardless of what similar knot he tries to tie, his attempt often produces a Square knot.

He sets aside the Granny and the Sheet Bend; these are both very much like a Square knot, yet have a variation involving an alternation of the ends relative to the loops. He compares the Figure 8 and the Sheet Bend and keeps these two together. They are also very much like the Square, expressing the fundamental nature of its entanglement in different ways:

Okay. I think the Square should go in this group *(Sheet Bend and Figure 8)* 'cause this and this look a lot alike.

The Figure 8 seems to reduce the Sheet Bend to a simpler statement of the involvement of the ends with regard to the loops.

Patrick deals with the growing complexity of this group by splitting off the Granny, adding to it the Surgeon's and the Bowline. He explains their similarity:

These *(the Surgeon's, Granny, Bowline)* are like the twist on top and twist on bottom.

But on second thought, the Bowline does not seem so clear-cut:

Patrick: Except for this one here *(the Bowline.)*

***Carol:** What's – how does – why does that one go there?*

Well, I don't think that goes anywhere.

Oh, it's alone.

Yeah.

He says it is alone, but still leaves it in the group. The complicated involvement above the circle, at some level of generality, is similar to that of the Granny and Surgeon's.

An extra Figure 8 accompanies this group:

The Figure 8 should go here *(with the Surgeon's, Granny, Bowline.)*

How come?

Because they're like – the same – they go in the same places, sort of. *(He compares the Figure 8 and Surgeon's.)*

The presence of the two Figure 8 knots may keep Patrick from joining the group containing the Sheet Bend and the group containing the Surgeon's. With only one Figure 8, he would have to have used the knot as a shared member of each group, or chosen its relationship with one of the two knots as being stronger than the relationship with the other.

The "Square with 4 Ends" causes him no hesitation: it is simply another Square knot, and aspects of its two-stringedness or resulting horizontal orientation do not lead him to compare it with other knots.

The Stopper, his favorite knot, causes some problem in classification. At first he skips it, and then groups it singly. The fact that he had focused on the twists of the Surgeon's, Granny, and Bowline doesn't seem to help; the distinctive final result of the Stopper distracts from his knowledge of how it is formed. It seems to belong in a class by itself.

The Stopper – hmm, that's a hard one. ... Uh – there's the Stopper again. Hmm – I don't know where that would go. ... It would be alone.

8 Vignettes

Aside from the various terms and strategies that children developed for their work with knots, and aside from the comparisons and groupings that focused attention on particular characteristics of knots, there were several instances in which an aspect of epistemological understanding was so dramatic that it deserves special attention. Through the following selection of "vignettes," I offer explanations of moments such as when a child experienced a dramatic change in understanding by taking a different view of a knot, hesitated on the verge of abandoning one way of understanding for another, and discerned a fundamental difference between *doing* and *knowing*.

8.1 Breakthroughs

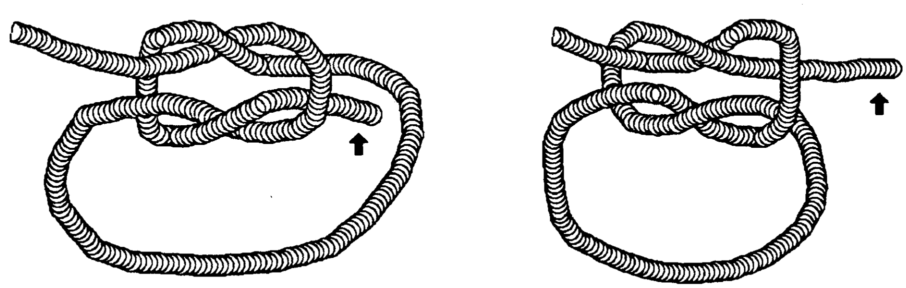
"Breakthroughs" occur as sudden changes of view, approach, or understanding – of mind. On several occasions in the Knot Lab, a child experienced such a change, which happened as a result of a suggestion from someone else that made the sense of a situation suddenly "click," through a gradual sort of simmering of the problem over a long period of time, or simply through perseverance.

7.1.1.14

After experimenting with several ways of beginning the Thief knot, some of them informed by a distracting upright approach (see Sections 5.2 and 5.4.1), Julio suddenly came to an understanding of how to tie the knot:

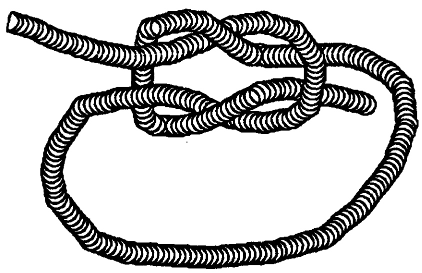
Carol: *What's different?*

Julio: 'Cause this knot *(the Thief)* goes down and this one's up *(the Square).*

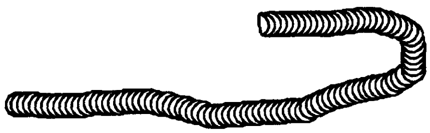


. . . Could you tie it again?

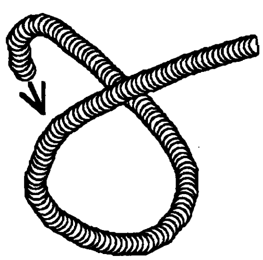
(He nods and begins. As he ties, he continually compares his work to the finished knot:)



(He holds the ends in two hands, thinks, and puts the string on the table.)

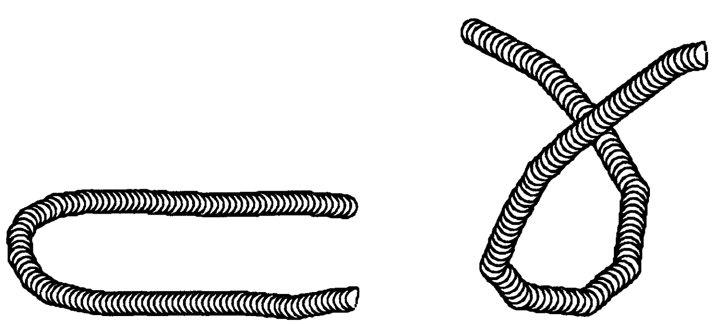


Um umn. *(He starts over.)* I used to tie it like this *(puts the right end under the left).*



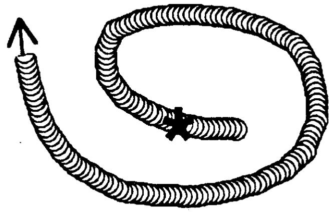
Yeah.

(He begins a completion of the Square knot by putting the "new right" end over the "new left" one. Then he stops and undoes the string.) Oooooh! 'Cause I used to go like – *(puts it on the table to see what his move would look like flat. He rearranges the string by uncrossing and moving the left end instead of the right.)* – somethin' like this –

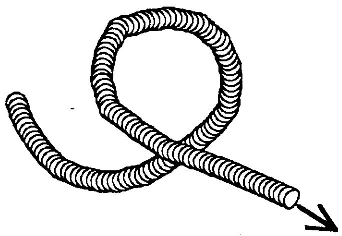


Um hmm.

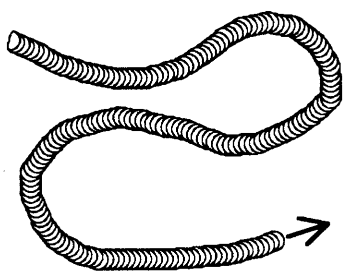
(He picks up the string and starts over.) Start – lemme start –

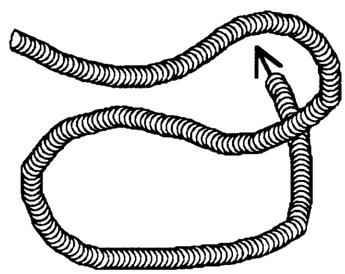


(He puts his finger at the "" to anchor the string, then pulls the other end across.)*

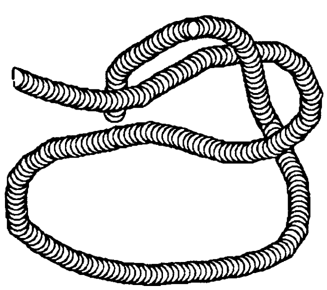


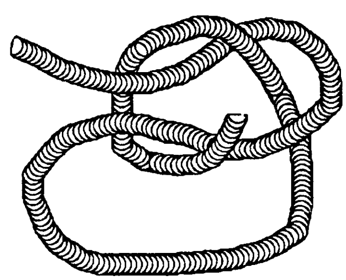
Lemme start – *(he undoes it)* – from this way:



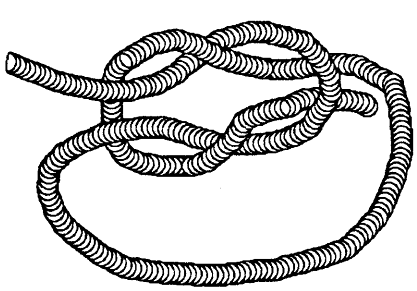


(Now he moves quickly; he knows what to do.)





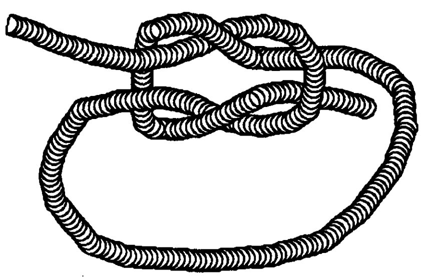
(He tightens it, excited.)



That's beautiful. Very nice! You developed a technique, I think. (He smiles.)

 5.4.2 6.2 6.5 6.6 6.19 **7.1.1.17** 8.4

Stacy began her description of the Thief knot with the circle, which would be a reasonable starting point for a knot tied using a familiar upright approach. But as soon as she got to the entanglement, she was in trouble. Then she had to switch the basis of her description to the confusing, asymmetric behavior of the ends. She could easily enough describe the ends separately relative to the circle: the left one was "up," and the right one was "down":



But describing the way they come together becomes difficult; she could say only that they were "caught from in between." Her remark may be taken symbolically as well as literally: not only do the two ends become entangled in a mysterious way, but that way seems to defy description – the ends, and her understanding, are somehow caught between being "up" and "down."

She decided to loosen the Thief knot to get a better look at the crossings. Recalling a Piagetian technique, I suggested a way of sorting through the tangle: she could pretend that she was a small ant crawling on the knot and tracing her path as she went along.

Immediately she shifted her starting point from the base of the circle to one of the ends. She gestured through a few broad tracings, gradually becoming more precise and finally lifting the knot so that she could turn it as she carefully followed the ant's path. (By regarding the knot in more than one plane, she could "do the unders," as she put it: she continued her focus on crossings.) Stacy's move – from thinking about one surface of the knot to turning it and thinking in terms of more than one plane – was a strategy that only a few of the children employed. By examining the knot in this way, she came to understand the path well enough that she is able to imitate it with a new piece of string, thereby producing a correctly tied Thief knot.

Carol: *... how come you could tie it that time but not before? What was different in the way you tied it?*

Stacy: Because it – it was, like, looser, so it was easier to see the path that it followed.

Right – and before you were thinking about the path, remember you – were trying in a way kind of like the Square knot?

Yeah. I was trying – I was trying to think – and the Square knot, I just started like, you know how to *(she gestures with her hands, referring to the upright approach of tying the Square knot)* – so it wasn't that hard.

6.9 **6.22.5**

In one of the videotaping sessions, Soo Yong had noted that the Monkey's Fist knot is similar to a volleyball in its repeating pattern of three elements. I brought a ball into the Knot Lab, and it became part of a display comparing the knot with the ball. Several children had noted and discussed the similarity, but (like many details about knots) it did not seem to impress Leroy, who was often only mildly interested in the goings-on in the lab, at best. He came because he was part of the group, and every now and then something would strike his fancy.

Although he had been occasionally interested in the Monkey's Fist as a potential accessory for martial arts, he hadn't been able to tie it successfully, and easily gave up trying. The more interesting part of the volleyball display was, for him, neither the knot nor the suggested comparison, but the ball itself. He eyed it for several weeks before garnering enough courage to ask whether he could borrow it and play with it outside during recess. He did borrow it, and respectfully returned it to its place when finished, apparently pleased that he had managed to do something out of the ordinary with respect to the lab.

In his group's next session in the lab, something dramatic happened. In his usual half-hearted manner, Leroy casually began working with a few others who were trying to tie the Monkey's Fist. Suddenly he got up and ran across the room, grabbed the volleyball and looked at it. He stood with it for a few moments, studying it with excitement, and then began shouting about how it was the same as the Monkey's Fist. He hurriedly described the sets of three components, and ran back to the table with his piece of string, explaining the similarity to Stacy and working with her to finish tying the knot. With his help, she learned the knot too.

8.2 Vacillations

As children in the Knot Lab worked toward new understandings, they sometimes got caught in struggles that caused them to falter in a description, to contradict themselves, and/or to retreat to an earlier conception.

Leroy's repeated attempts to distinguish the Square and Thief knots, by tracing the paths that they formed, seemed only to further confuse him, even though several of his tracings were correct. His ability to anchor beginning and end points, as well as points along the way, was so variable that he finally gave up discussion of relationships among parts of the knots and settled on less intrinsic properties, such as size and the length of the ends.

Marcos's discussion of the same two knots fluctuated wildly – between considering different sides of the knot, between focus on different parts of the knot, and between thinking about the knot and the process that produced it. He also fluctuated between thinking that the knots were the same and thinking that they weren't.

Stacy ran into trouble as she tried to think about the Thief knot in the same way that worked for the Square. She retreated to a manner of description that had worked for the Granny, and thus seemed like a safer starting point from which to reconstruct her thinking about the Thief. Her strategy paid off, as she conjured a number of models and finally ended up tying the difficult knot.

 5.4.2 6.22.1 7.1.1.11 8.2

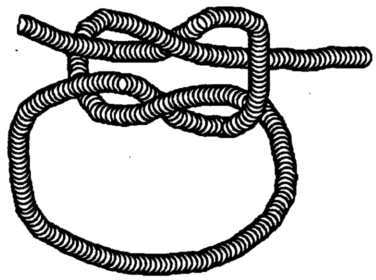
At first, Leroy insisted that the Square and the Thief were the same knot. He discovered much evidence that they were different – but his belief that they really were different seemed fragile. He had difficulty explaining *why* the knots were different even though he had come close to demonstrating to himself that they were. Leroy seemed to be on the verge of understanding something important about the knots, but didn't quite grasp the difference between them. He repeatedly fell back on old conceptions or strategies, even mistakes.

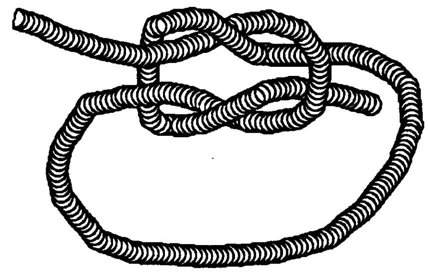
In tracing the knots, he was able to follow the pathway accurately, and explicitly considered both faces of the knot. However, he started with different ends (the left for the Thief and the right for the Square) and then expressed surprise that he ended at a different place on each knot. In re-tracing, trying to keep the beginning point constant, he confused crossings and lost track of the paths; nevertheless, he finally ended at the left of each knot – but declared that he had not ended at the same spot and continued to assert that the knots were the same. In beginning the tracings again, he identified starting and ending points inconsistently with what he had done before, and re-traced each knot with different starting points: he seemed unable to "anchor" his thinking, but decided the knots were different anyway. He had been through so many tracings that it seemed possible he really had come to appreciate this difference, yet could not explain it adequately. Instead he relied on more superficial attributes, such as size of the ends and shape of the circles, to explain it.[1]

Carol: Okay, let me ask you this. Let's see – see these two knots? Do they look like the same knot? (Square and Thief) Are they the same?

[1] I should note that Leroy was very much engaged in this discussion; the text may read as though I was pushing him inordinately, but he was clearly thinking hard about the problem, and my questions and suggestions flowed within the context of our discussion, following grimaces, gestures, and other indications that he was struggling but wanted to sustain our pursuit.

Leroy: Yeah, kind of. *(He compares them side by side.)*



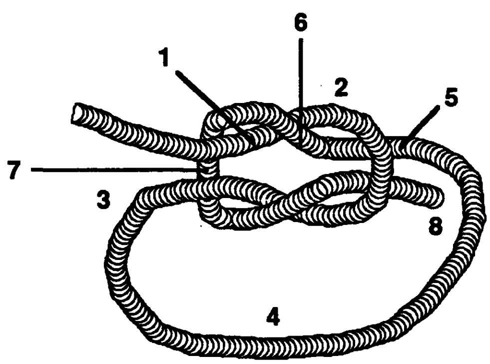


How are they the same?

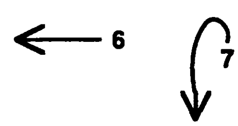
Like, like, this comes under right there, this comes over, this goes in, that goes in, that goes under – and so does this. It's the same one. They're identical. *(He gestures quickly and broadly, pointing back and forth at the two knots as he speaks.)*

Okay. Now, if you put them on the table, can you – if you were a tiny little ant, and you wanted to crawl along those knots, how would you go? Can you show me with your finger?

Um – well, if you was a tiny little ant, you would through here (1), and go to – over there (2), and out that side (3), just go around (4), go back in (5), go through this side (6), then go under (7), and end up right here (8).



6, 7: He motions with his finger.

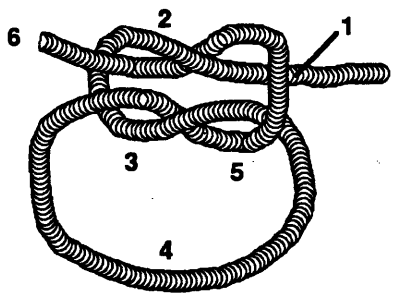


Uh huh. And how would it go on this one?

This one? Well, which – it depends on which end it starts with.

It would start at the same end you started with on this one.

(He chooses the other end.) It would go through here (1), go through there (2), go through here (3), come through here now (4), go though there (5), and you come out here (6). *(He doesn't turn either knot.)*

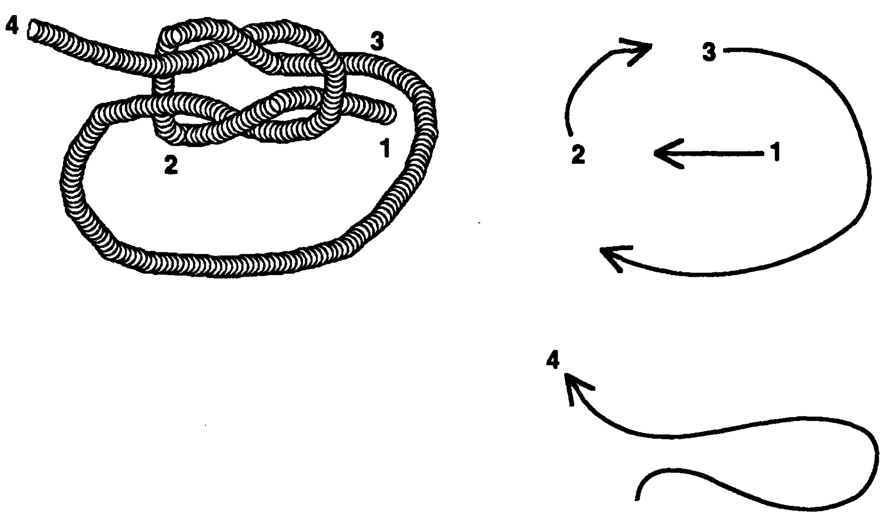


Do you end at the same spot?

(puzzled and surprised) No.

Oh, how come?

I don't know. *(He goes back to the Thief and traces it in four motions, while mumbling inaudibly.)*

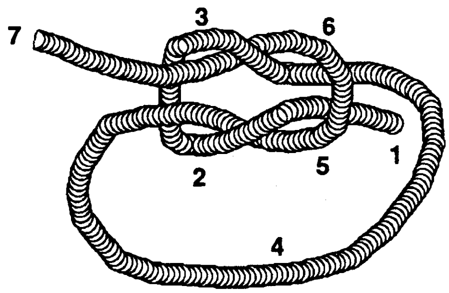


Yes – you – yeah, you do.

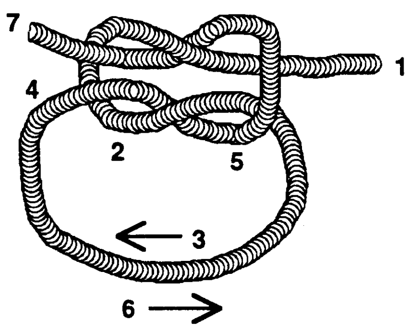
No you don't –

Oh, I see – you're doing – But do you start at the same spot?

Yeah. See. *(Now he is consistently beginning with the end at the right.)* You go through there (1), go through there (2), go under (3), come back up here (4), you go this way (5), and go that way (6), and you come here (7).



Then – and here, I started at that end *(the right)*. You go here (1), go through here (2), go through here (3), go through – here (4), go over here (5), go back around here *(He loses sight of both the path of the string and the direction of the movement.)* (6), and go over there (7). *(It's not clear how he gets directions and crossings confused. Even if his (4) and (5) are as shown here, it's hard to tell how he goes from (6), which is definite, to (7).)*

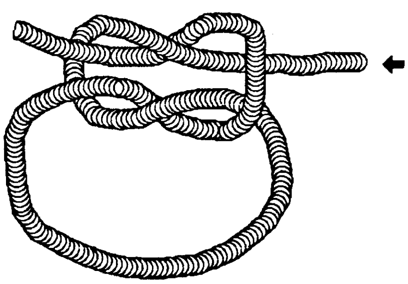


No! – you don't.

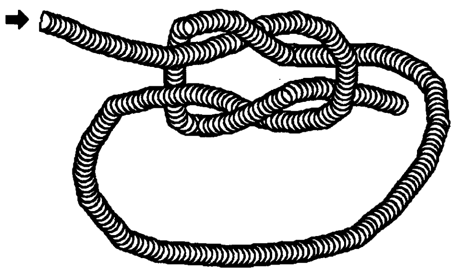
You don't end at the same spot? (Leroy shakes his head "no.") So does – is it the same knot, then?

Yeah, it's the same knot. I guess, but – it looks the same. *(He holds them next to each other to compare.)*

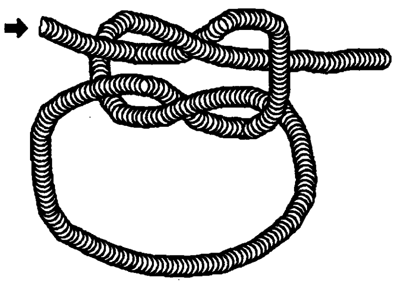
Where did you end on this one (the Square)? (He points.)



And where did you end on that one (the Thief)?

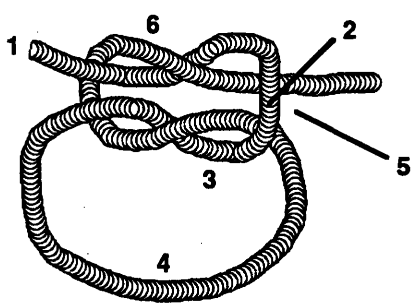


Where did you start on this one (Square)?

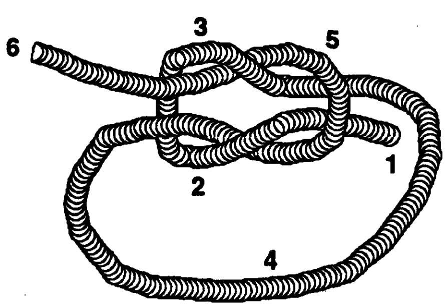


Let's see where you went again.

All right. *(Square:)* Go here (1), then go here (2), go through here (3), go over here (4), then I come over there (5), and go through (6). *((5) and (6) cover a lot of motion. He is pointing to the general areas but is perhaps too vague to enable his understanding.)*



(Thief:) Start here (1), go through there (2), go here (3), go around (4), go here (5), and go through (6).



So you end in different places? (Leroy nods: "yes.") So – are they the same knot, then?

Um – no! They just look the same.

How are they different?

(compares them) I think this one *(Square)* has longer ends. *(He compares them again.)* And this one is bigger *(Thief)*. This one opens more *(Thief)*.

Okay.

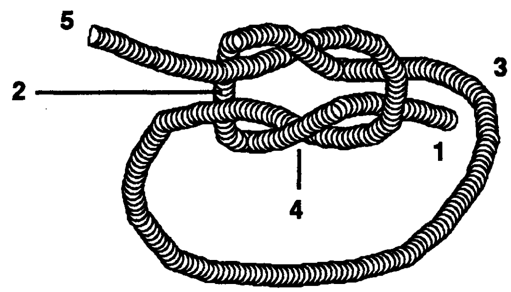
5.4.1 6.6 7.1.1.10 8.2

Marcos was concerned with both sides of the object, turning the knot as needed in order to carefully follow the string – the ant's path – as it wound around. His use of the terms "top" and "bottom" fluctuated broadly. At first, "bottom" referred to the other face of the knot. Then, in identifying his starting points for tracing the path, "bottom" referred to the the circle of the Square knot, while "top" referred to the entanglement of the Thief. Then, in finally describing the difference between the two knots, "top" and "bottom" were both terms used with regard to the entanglements, as he situated the ends relative to the circles.

Fluctuations characterized Marcos's thinking about these knots: fluctuations between which plane to consider, between focus on the entanglement or on the entire object, and between thinking about the tying process or the finished knot. His choice of the starting point in tracing the Square knot reflected his recollection of an upright two-ended approach to tying it; his choice of the starting point for the Thief pointed to his newfound understanding that this knot must be produced in a different way.

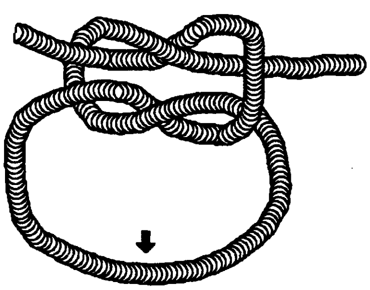
***Carol:** Imagine you were a tiny little ant, and you were crawling on this string. And you wanted to crawl all over the whole string – you wanted to cover every little piece of it. Where would you start? Show with your finger how you would go around if you were a little ant.*

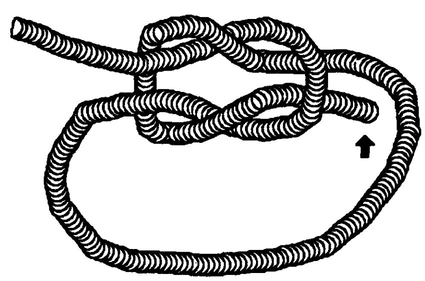
Marcos: *(He does the Square knot, then the Thief.)* This one – let me see. I'll go there (1) – there – yeah! Like this. I'll go like here – like – here, on the bottom (2), and then I will go – let me see – what is it? Here – go around this (3), then go here on the bottom (4)? *(He lifts the knot to point to the "bottom.")* Right? And I end up here (5)!



Okay. Is there anything different about how you went here (the Thief) and how you went here (the Square)?

Yeah. I start – I started in a different way. Here *(the Square)* I started – like, um – almost on the bottom. Here *(the Thief)*, I started on the top.



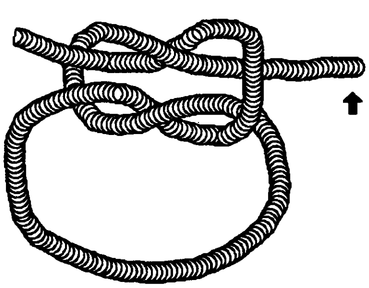


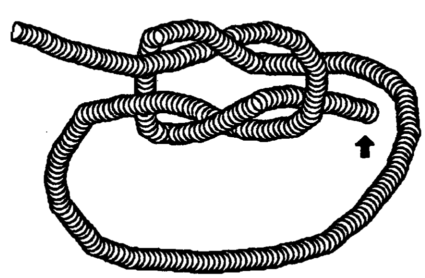
How come?

Because they are like different ways. See? *(He holds up both knots.)*

Oh, I see.

See – this one's on top *(the Square)*. This one's on the bottom *(the Thief)*.





I see.

So you have to – because they're different – they're the same knot, but they got – you gotta start them a different way, because they're not – almost the same. One is on the top, the other is on the bottom. *(He works with the Square knot.)* You have to go like this – go around – then go through there – you end here *(the left end of the Square)*. And here, you end right there *(the right end of the Thief)*.

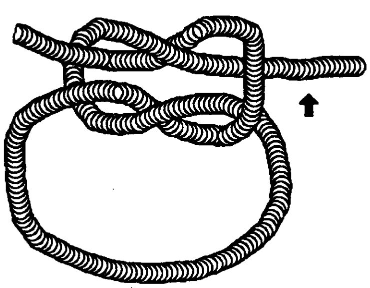
Marcos's last explanation dramatized his fluctuating views of the knots: in virtually the same breath, he said, "they're different," "they're the same knot," and "they're not – almost the same."

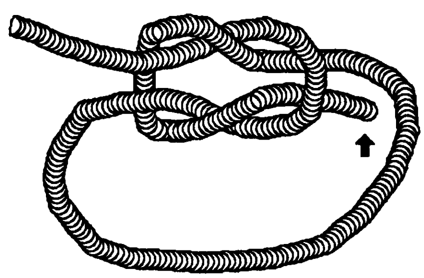
5.4.2 6.2 6.5 6.6 6.19 **7.1.1.7** 8.1 8.2 8.4

Stacy's comparison involved the Square, Thief, and Granny knots:

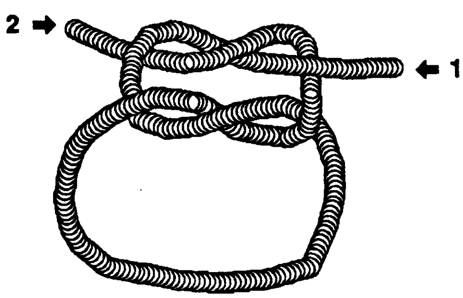
***Carol:** How about these two knots (Square and Thief)? Do they look the same to you or is there anything different about them?*

Stacy: Yes, this one – on this side, this one goes over *(above the circle on the Square)*, and this one goes under *(inside the circle of the Thief)*.

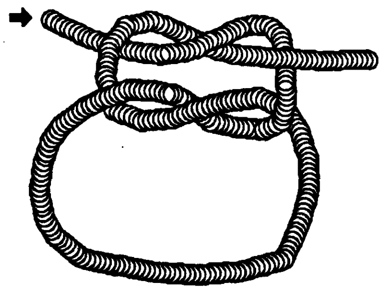




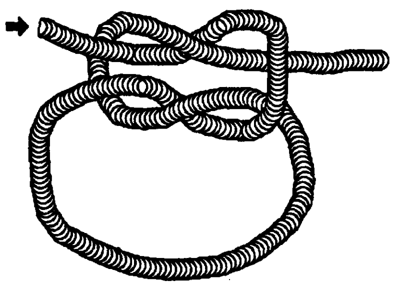
On this one *(the Granny knot)*, this one goes over it **(1)**, and this one goes over that **(2)**.



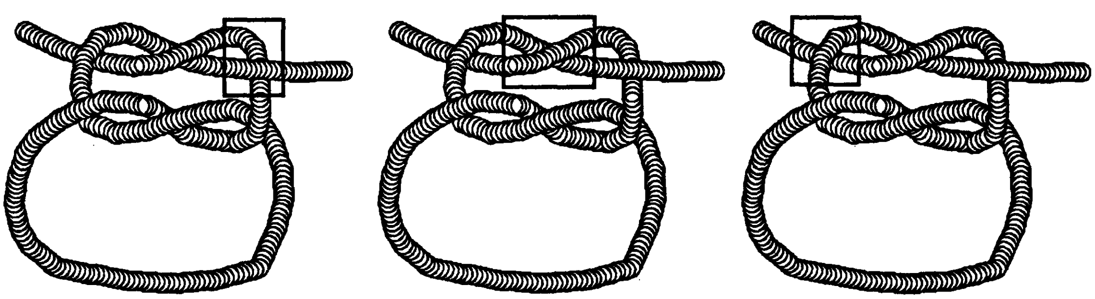
And it's suppose to be going like – this line is suppose to be going in *(points to upper left, moves left to right with her finger to show how that crossing on the Granny knot should be like the same one on the Square knot).*



'Cause like right here, it goes in.



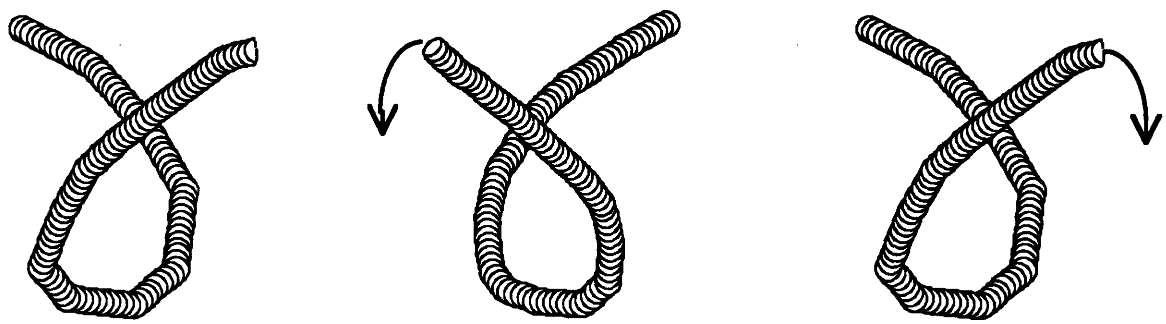
Stacy's description of the Granny knot depended on her focus, at any given time, on a particular crossing:



She was able to tie the Square knot successfully, and tried transferring the technique to tie the Thief knot. But here she ran into difficulty, and a new discussion of the knot ensued:

***Carol:** ... have you tied this one (the Thief) before?*

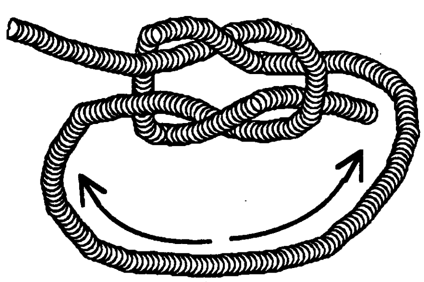
Stacy: Yeah, I think so. *(She tries a few approaches.)*



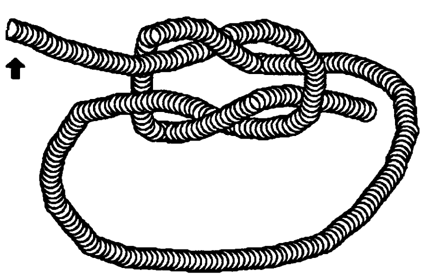
(She shakes her head, partially undoes the loop.) Looks funny.

Can you think out loud as you try to figure it out? What's hard about it?

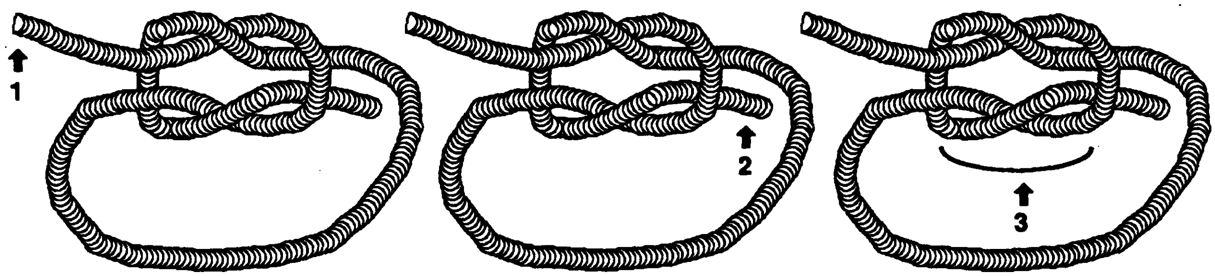
'Cause these look – because it looks like it goes first – like – *(motions with fingers)*



I'm following like, before the loop, and then how it – where the ends go – and like, they go up, and then – like, they go up –



– and then – like they go from being all the way up (1) and being all the way down (2), and then get caught from in between (3).



Abandoning thinking in terms of the process that produced the Square knot, Stacy returned to an earlier, tried-and-true technique: she loosened the Thief knot and looked closely at the crossings. This retreat alone did not solve the mystery of the knot: she combined it with tracing the path of the knot, considering both sides so she could "do the unders," as she put it. She continued her detailed examination of the crossings, and finally came to understand the knot – to the point of being able to tie it.

Carol: *...how come you could tie it that time but not before? What was different in the way you tied it?*

Stacy: Because it – it was, like, looser, so it was easier to see the path that it followed.

Right – and before you were thinking about the path, remember you – were trying in a way kind of like the Square knot?

Yeah. I was trying – I was trying to think – and the Square knot, I just started like, you know how to *(she gestures with her hands, referring to the upright approach of tying the Square knot)* – so it wasn't that hard.

8.3 Process and Product

[A knot is] as much an action as a finished structure.

– Brion Toss (producer of texts and videos about knots), in personal correspondence dated 20 March 1989

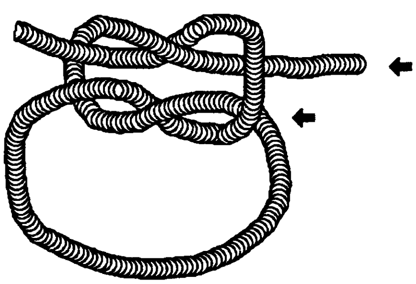
The children's descriptions of knots can be broadly distinguished in terms of whether they relied on knowledge of how a knot was formed, or on observations of the finished object. This distinction was often implicit, but Juanita posed it as a near-philosophical criterion on which to base her discussion. For her, *doing* and *knowing* were not necessarily the same:

***Carol:** How can you tell them apart?*

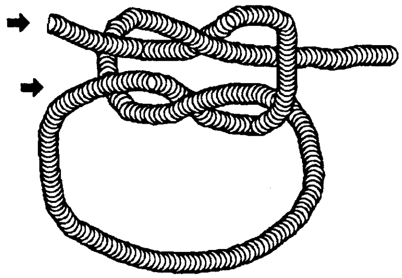
Juanita: Because *(hesitates)* – you want me to tell how I could do 'em apart – *(Carol shakes her head "yes.")* – or you just want me to tell you about how I could know how it is?

Um, how do you know how it is.

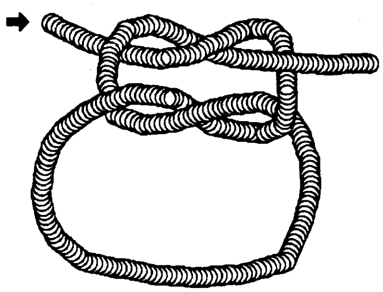
(She holds the Square knot and points to parts of it.) Because – um – the Square knot has these two under,



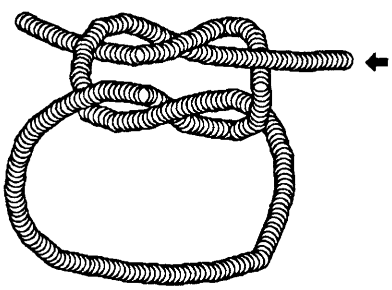
and these two across,



and this one doesn't. *(picks up the Granny)* It has one in the bottom *(strokes the left free end),*



and one in the top *(strokes the right free end).*



That's how I could tell which one is the Granny and which is the Square.

9.1.1

Soo Yong recommended thinking in terms of process in order to distinguish two similar knots:

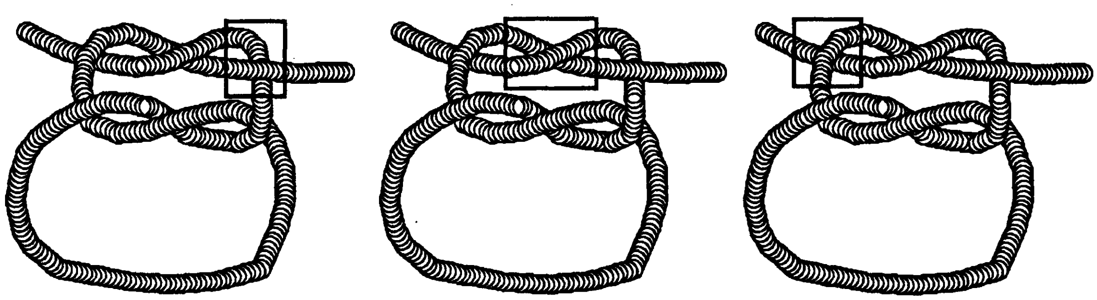
> Dear Soo Yong,
> Today we were talking about over hand and under.-hand.
> We couldn't figure out what is the differnce between them?
>
> Alice, **Tony**, *Jill*

> Dear Alice, Tony, and Jill,
> I know it is hard to figure out overhand and underhand knots. It's hard to find the difference when they are tightened but when you are tieing them you'll find the difference. I'm not going to tell you guys the difference because it will be too easy. So why don't you find out and tell me the difference.
>
> 
>
> Sincerely,
> Soo Yong Chang

8.4 Shifting Frames of Reference

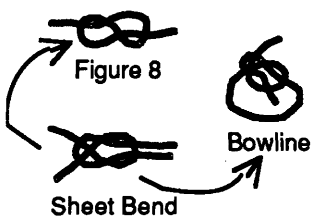
An interesting phenomenon occurred both in descriptions of individual knots and in groupings of many knots. The phenomenon took the form of a kind of shifting. A child would designate some property as being important to thinking about a knot, use the property to accomplish some momentary elucidation, and then move to another point in the discussion, carrying the concept of the property with her as a kind of tool, to be applied again as needed. Here I describe two aspects of such shifting.

One aspect involves a *shift of focus*, as in Stacy's implicit movement from one crossing to the next in her description of the Granny knot. Her use of the right end as a referent was meaningful only when put in the context of the upper right crossing. Similarly, she discussed the left end in terms of the right end, which necessarily meant that she looked at the upper middle crossing. And, in describing how the left end exits the knot, she considered the upper left crossing:



In her search for conceptual stability in the realm of movement embodied by the knot, Stacy was like a mountain climber who carefully achieves a foothold, places a stake at that spot and, thus secured, carefully reaches for the next foothold. As she made progress, a previously used stake, no longer needed where it was, could be lifted and placed at the next point of focus.

Another aspect of such a shifting characteristic of what are here termed as *transitive relationships*, has to do less with a way of keeping different points of focus separate than with keeping different components together. In arranging different knots into groups, several children used one knot as a kind of glue, holding other knots together. The binding knot was related to each of the other knots, but the other knots were not necessarily related to each other. Alice's grouping of the Figure 8, the Sheet Bend, and the Bowline is an example:



The similarity of form shared by the Figure 8 and the Sheet Bend held them together, and the peculiar crossing on both the Sheet Bend and the Bowline held them together. Therefore these three knots went together, even though Alice did not relate the Figure 8 and the Bowline. The Sheet Bend became a kind of anchor between them, holding together the entire group.

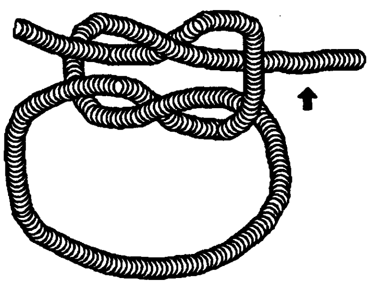
Included here are some detailed discussions of examples of such transitive relationships.

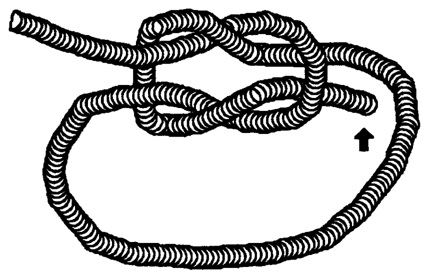
Stacy's use of a *transitive referent* grew through her discussion of the Square, Thief, and Granny knots. Her referent was a crossing, but it shifted from one crossing to another:

5.4.2 6.2 6.5 6.6 6.19 **7.1.1.7** 8.1 8.2

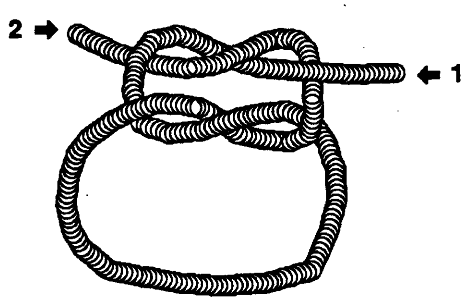
***Carol:** How about these two knots (Square and Thief)? Do they look the same to you or is there anything different about them?*

Stacy: Yes, this one – on this side, this one goes over *(above the circle on the Square),* and this one goes under *(inside the circle of the Thief).*

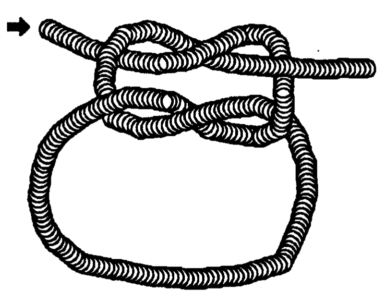




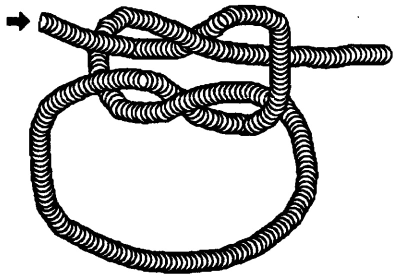
On this one *(the Granny knot)*, this one goes over it **(1)**, and this one goes over that **(2)**.



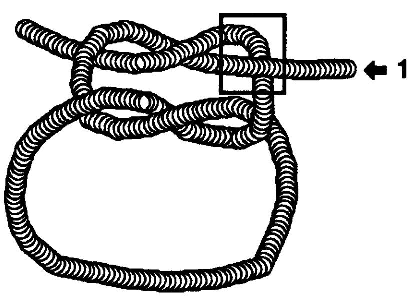
And it's suppose to be going like – this line is suppose to be going in *(points to upper left, moves left to right with her finger to show how that crossing on the Granny knot should be like the same one on the Square knot).*

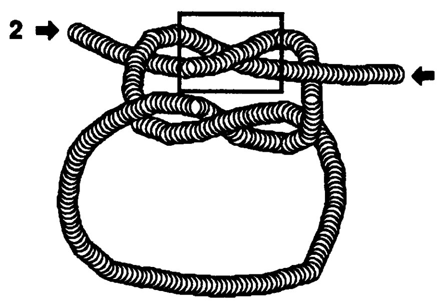


'Cause like right here, it goes in.

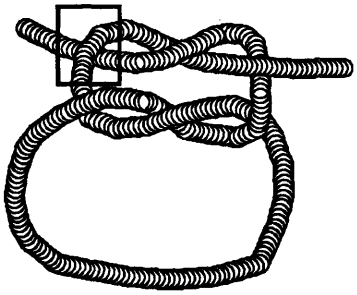


Stacy's two-part description of the Granny knot was unusual in its use of a transitive referent. She first described the right end as an active part – it went over the upper part of the right loop. But then she abandoned the loop as the referent. Her designation switched to the end itself – no longer the actor, the right end became the new anchor with which Stacy could describe the action of the left end. The left end went over the right. Her focus was again at a crossing:



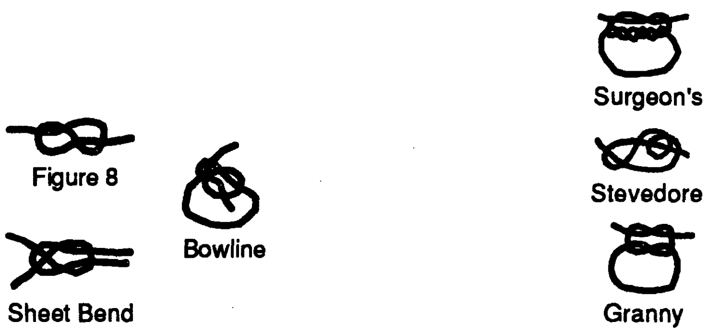


Stacy made use of the Square knot in describing how the left end of the Granny should go "in" to the entanglement. Notice that her focus had now shifted fully to the left half of the knot:

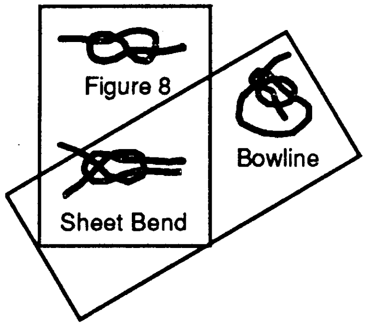


 7.2.4

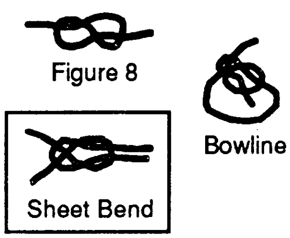
Looking at similarities among different knots, Alice produced two groups that relied on a sort of "glue" supplied by one of the member knots:



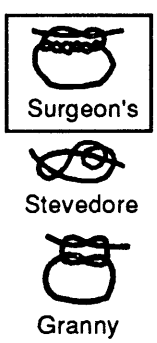
The two horizontal loops in the Figure 8 and the Sheet Bend seemed to be Alice's criterion for putting these knots together, though she said only that she grouped them "mostly because of the knot." She did not verbalize the similarity between the Sheet Bend and the Bowline, but indicated with her finger the peculiar crossed end and its situation relative to the facing loop, on each knot ("these two look like that").



Her arrangement placed the Sheet Bend in an interesting conceptual position: it is related to each of the other knots, though these two are not notably related. Thus the Sheet Bend served a kind of binding function, holding the group together.



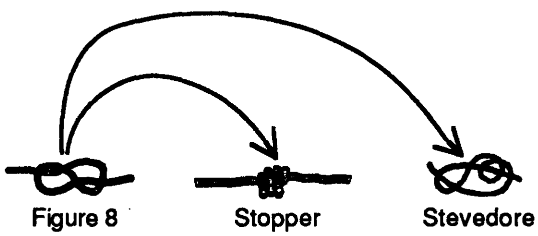
The Surgeon's knot served the same function. She related the knot to both the Stevedore and the Granny, though she did not express a relationship between the latter two.



The winds on the Surgeon's and the simpler Stevedore were the basis for their pairing. The "shape" of the Surgeon's and Granny constituted their similarity: they have circles and they are not Squares, so they must belong together.

7.2.16

Tony made use of a similar "binding" property:



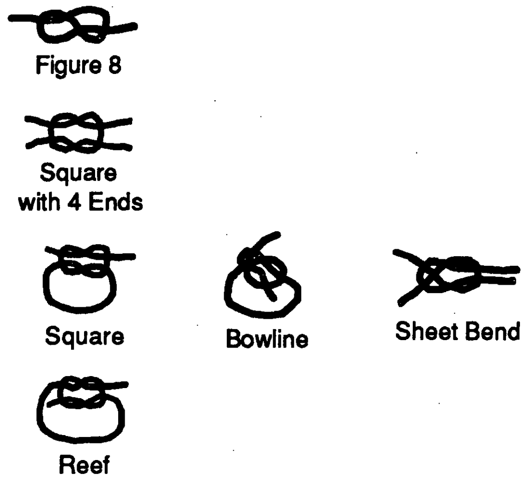
Tony's reason for grouping the Figure 8 and the Stopper had to do with they way in which the knots are formed:

> ...the Stopper and the Figure 8 are both rather simple knots – which you get from using an overhand or an underhand – knot – using it that way – that's how you get those two...

The Stevedore belonged with them because of its resemblance to the Figure 8. Thus the Figure 8 was the "binding" knot in this group; it was the knot that distributed the "glue" that held the group together.

7.2.15

Jill positioned two groups relative to each other in such a way that the Square knot was expressed as the "binding" knot:



The Figure 8, the "Square with 4 Ends," the Square, and the Thief have a similarity of form that was the basis for their grouping: two horizontal loops face each other in the entanglement. Yet there are some other knots that also have some form of this feature.

"The Bowline looks a bit like the Square," she said. The Bowline is, in a way, its own knot, yet its similarity to the Square knot is pronounced enough that she placed the two knots next to one another. In this way, the Square knot became an expression of overlap between two groups. Jill at first doesn't recognize the Sheet Bend and had it in a category by itself, but when she realized its close relationship to the Bowline, she grouped these two knots together:

> ...they both have the loop and a string coming out of the loop.

Jill described the similarity and difference between the Bowline and the Sheet Bend in terms of the ways in which the ends exit the knots relative to the loops and circle.

In their "sharing" of the Square knot, the two groups expressed an overlap of salient features of the knots.

IV Psychological Threads

> Grown-ups never understand anything by themselves, and it is tiresome for children to be always and forever explaining things to them.
>
> – Antoine de Saint-Exupéry, *The Little Prince*

In Section 1.3, we looked at how objects can assist with thinking about potentially complicated ideas. Characteristic of this "concrete" thinking can be a resonance of the object with one's own body experience or one's own likes and dislikes, which we referred to as "syntonicities" of body and ego. Recall this excerpt from Papert's discussion of the gear:

> Before I was two years old I had developed an intense involvement with automobiles. The names of car parts made up a very substantial portion of my vocabulary: I was particularly proud of knowing about the parts of the transmission system, the gearbox, and most especially the differential. It was, of course, many years later before I understood how gears work; but once I did, playing with gears became a favorite pastime. I loved rotating circular objects against one another in gearlike motions and, naturally, my first "erector set" project was a crude gear system.
>
> I became adept at turning wheels in my head and at making chains of cause and effect ...
>
> I believe that working with differentials did more for my mathematical development than anything I was taught in elementary

school. Gears, serving as models, carried many otherwise abstract ideas into my head. (Papert 1980, vi)

What does it mean for an object to "carry ideas into the head"? Psychological theorists have looked to the worlds of very young people in order to gain insight into how thought develops through interactions with the environment, and how objects can mediate the process.

Some of Sigmund Freud's writing established terms for a key psychological process that becomes relevant to our discussion of thinking with objects. In his model, our minds are in a particular state as we come into the world. They are like bundles of energy, instincts, drives – what he calls the "id." Gradually, through interactions with the environment, an infant begins to form a sense of itself and of others. The infant's mind is changing in the process: the id is giving rise to the "ego," and for several years the two will co-exist as the mind of the child. In Freud's theory, the mind undergoes another fundamental change at the time of the oedipal conflict. The ego now gives rise to the "super-ego," which will supervise the ego as it is influenced by the unchecked urges of the id. The child now has a "conscience," an internal model of the parents' values and castigations. The superego develops through a certain kind of interaction between child and parent, and mind and environment:

> The basis of the process is what is called an 'identification' – that is to say, the assimiliation of one ego to another one, as a result of which the first ego behaves like the second in certain respects, imitates it and in a sense *takes it up into itself* [italics mine]. Identification has been not unsuitably compared with the oral, cannibalistic incorporation of the other person. It is a very important form of attachment to someone else, probably the very first, and not the same thing as the choice of an object. The difference between the two can be expressed in some such way as this. If a boy identifies himself with his father, he wants to *be like* his father; if he makes him the object of his choice, he wants to

> *have* him, to possess him. In the first case his ego is altered on the model of his father; in the second case that is not necessary. (Freud [1933] 1965, 56)

Two aspects of this explanation later became important to a group of psychoanalysts in what has become known as the "object-relations" school: the concept of identification (also called "incorporation," "internalization," or "introjection"), and extensions of the idea of the so-called "split" that occurs in the ego as the superego develops.

The object-relations theory of human development is concerned with the importance of Freud's concepts of identification and object choice to the early relationship between infant and mother, and to later relationships in the life of an individual. The theory makes use of several senses of the word "object," which sometimes refers to an actual person (or part of that person), sometimes to a representation of a person (which exists in the mind and has developed through a process of identification), and sometimes to an external object that substitutes temporarily for a person (while the process of identification is occurring).

W.R.D. Fairbairn, one of the foundation thinkers for the direction of psychoanalytic theory toward object-relations, makes use of the concepts of identification and object choice, but explains a process of ego-splitting that differs from Freud's. Fairbairn does not rely on the same developmental progression from id to ego to superego. Rather, in Fairbairn's model, there is no id. An ego is present from birth, and the early interaction with the mother or caring figure plays a crucial role in how the infant's ego develops. Fairbairn's discussion of the "internalization of the object" is concerned with the infant's way of coming to understand the alternating presence and absence of the caring figure. The frequency and duration of the absences increase, of course, as the child grows.

When alone, the infant is thought to keep in mind some image or internal representation of the caregiver. The infant developed this representation during times when it was with the other person. At such times, their interaction includes for the infant a process of internalizing the person or part of the person – for example, the mother and her breast. The internal representation that results is called an "object." At first, this object is relatively simple. However, given that the external version of the object sometimes satisfies the infant but is not always available when desired (or is sometimes "empty"), the object is perceived as having a double nature. Both the external and internal versions of the object are said to be both "exciting" and "frustrating" (or "rejecting"). As these two aspects are acknowledged, the internalized object becomes more complicated: its "exciting" and "rejecting" aspects "split off from the main core of the object."[1] Then, because they are accompanied by powerful, problematic feelings with which the infant struggles, they are repressed by the ego. What remains of the internalized object – what is not repressed – "is described as the ideal object or ego-ideal." This ideal separates what the infant might like the object to be from the problematic capabilities it is known to have.

> Internalization of the object is a defensive measure originally adopted by the child to deal with his original object (the mother and her breast) in so far as it is unsatisfying. ... Internalization of the object is not just a product of a phantasy of incorporating the object orally, but is a distinct psychological process. (Fairbairn [1952, 1963] 1972, 224)

Thus Fairbairn sketches a scenario of progressive splitting of internal objects as the external separation is enacted. Given that every human being must go through some version of this process of separating from the mother, Fairbairn posits the tendency to form "splits," enabling development of internalized objects, as fundamental to the human psyche.

> The universal phenomenon of 'the super-ego' as described by Freud must also be interpreted as implying the presence of a split in the ego; for, in so far as the 'super-ego' is regarded as an ego-structure capable of

[1] See Appendix B for Fairbairn's summarial explanation of this process.

> distinction from 'the ego' as such, its very existence *ipso facto* provides evidence that a schizoid position has been established. (Ibid., 9)

"Schizoid" is a term meant to describe such a "split in the ego." In this sense, it is not intended to imply any malfunction or pathology; formation of this split is a developmental phenomenon symptomatic only of the human condition.

> [The] fundamental schizoid position is the presence of splits in the ego; and it would take a bold man to claim that his ego was so perfectly integrated as to be capable of revealing any evidence of splitting at the deepest levels, or that such evidence of splitting of the ego could in no circumstances declare itself at more superficial levels, even under conditions of extreme suffering or hardship or deprivation. ... The all-important factor here is the depth which requires to be plumbed before evidence of the splitting. In my opinion, at any rate, some measure of splitting of the ego is invariably present at the deepest mental level – or (to express the same thing in terms borrowed from Melanie Klein) *the basic position of the psyche is invariably a schizoid position.* (Ibid., 8)

Melanie Klein, another influential theorist of the object-relations school, posited

> ... views of the inner worlds of children being populated by relationships with highly emotional figures derived from their experience ... a structuring within the self from the start by the infant's relationships between its unitary ego and the caring family figures. (Sutherland 1989, 37)

That is, the infant's ego *relates* in some emotionally charged way with the caring figures. Experiences with the caregivers lead to the infant's formation of internal objects that represent the figures, as well as the natures of the relationships.

> At first, the objects in these relationships were 'part objects' ... with the distinction of external from internal being embryonic. ... A world of internal objects was fashioned from experience, with each of these structures embodying the intense quality of the primitive affects which had cohered in their formation. (Sutherland 1989, 37)

The importance of the idea of these internal "structures embodying the intense quality of the primitive affects which had cohered in their formation" cannot be understated.

Another writer, D. W. Winnicott (see Section 2.3.1), also describes the formation of emotionally charged inner objects. For Winnicott, the infant's process of separating from the mother occurs gradually and involves the progressive distinction of "me" and "not-me" objects. (Winnicott 1971, *Playing*) Here again, the "objects" are internal conceptions – of the mother and of the infant's own "self." The model is one in which the infant moves slowly from a sense of being merged with the mother (then understood as a "me" object), to a sense of autonomy as a separate individual (when the mother is understood as a "not-me" object). Winnicott sketches the dynamic using distinct *spaces* that are meant to represent concepts or states of mind – one for the "me," one for the "not-me." They are mediated by a space that lies between them, in which *play* occurs. Through play, the distinction of "me" and "not-me" comes to be understood – that is, the process of separation occurs.

"Transitional objects" assist in the process. The child substitutes something tangible (such as a blanket, teddy bear, or favorite toy) for the physical closeness to another person, which by necessity lessens as the child grows. The substituted object is a sign of a process of psychological incorporation of the person. This incorporation is what enables the child to tolerate realities of the loss of the external relationship. What develops in its stead is an *internal* relationship, with a representation of the loved one. In other words, not only does an internal object develop, but some quality of *relationship* with that object is internalized as well. Or, in the language of Fairbairn, Klein, and Sutherland, we form both "good objects" and "bad objects," so called because they embody "the intense quality of the primitive affects which had cohered in their formation."

We are familiar with the image of a child carrying around a soft, comforting object like a blanket or stuffed animal, and many of us have become accustomed to Winnicott's discussion of this phenomenon in terms of the "transitional object." But we hear less often stories like that of Marie Curie, a grown woman, saving and clinging to her husband's coat after suffering loss through his death. (E. Curie 1938, 243-59; Giroud [1981] 1986, 145-54) The example (like many that might be cited) points to the likelihood that the process of introjection, which we develop in infancy and childhood, sets up a pattern of interaction between our internal and external worlds that can be repeated, in different forms, throughout life.[2]

Implicit in the idea of defending against the perceived loss of a person's love is a process of "working through," often mediated by an actual physical object, which Winnicott describes as "transitional." The person involved in this gradual act both uses the object to substitute for the presence of the person – *introjects* – and infuses the external object with powerful senses of whom it represents – *projects*. In the course of developing the internal object, these senses are worked back in along with other attributes of the person (and the external object), projected out again, worked in again, and so on. The process is gradual and complex. It can be thought of as taking the form of a silent, though charged, dialogue.

When a physical object plays such an important role in such a highly charged process, it is likely to have been carefully chosen, based on inherent properties that resonate with some sense of the person for whom it is meant to compensate, or the situation in which it is used. The child's blanket can be soft, warm, and enveloping, like a mother's body and love; Pièrre's coat, although stained and damaged through the accident that caused his death, retained the shape of his body and evoked memories of times he had

[2] The continual nature of the process of introjection is widely accepted, as demonstrated through discussions in the "Psychoanalytic Approaches to Personality" seminar conducted by George Goethals (Harvard University, 1989-1990).

worn it. In these examples, the objects both *represent* and *embody,* or "carry with them," qualities of the lost relationship for which they compensate.

The way in which objects assist in a "working-through" can be echoed in other situations that do not necessarily involve a significant loss, but in which an object is an integral part of a process – as when a person opts for a favorite pencil to write drafts of an essay but a pen for writing a letter to a friend. The instrument mediates the author's sense of her messages and moods as the work develops. And along with the finished written piece, some "internal object" – some representation of the completed work – results, which the author can use in the course of other work. Recall again Papert's use of gears in his early thinking:

> I became adept at turning wheels in my head and at making chains of cause and effect ...
>
> I believe that working with differentials did more for my mathematical development than anything I was taught in elementary school. Gears, serving as models, carried many otherwise abstract ideas into my head. ...
>
> Slowly I began to formulate what I still consider the fundamental fact about learning: Anything is easy if you can assimilate it into your collection of models. If you can't, anything can be painfully difficult. Here ... I was developing a way of thinking that would be resonant with Piaget's. *The understanding of learning must be genetic.* It must refer to the genesis of knowledge. What an individual can learn, and how he learns it, depends on what models he has available. This raises, recursively, the question of how he learned those models. Thus the "laws of learning" must be about how intellectual structures grow out of one another and about how, in the process, they acquire both logical and emotional form. (Papert 1980, vi-vii)

In Papert's language, structures "grow out of one another"; in Freud's and Fairbairn's, one structure "splits off from" another. Papert's description is more constructive in tone, but the notions of derivation are strikingly similar. Klein and Fairbairn describe how internal objects become infused with affect, emotional charge, as they are formed; Papert describes how they "acquire both logical and emotional form." I raise the question of whether these writers need necessarily be describing processes of a very different nature – there is a logic, after all, in the child's original development of an internal object. The mother *has* left, for the time being. This realization, together with the anger, fear, loneliness, or whatever other senses the individual may experience, are slowly bundled into the complex internal object that comes to represent her.

Minsky expresses a complementary view:

> There is a popular view that emotions are inherently more complex and harder to understand than other aspects of human thought. I maintain that infantile emotions are comparatively simple in character and that the complexity of adult emotions results from accumulating networks of mutual exploitations. In adults, these networks eventually become indescribably complicated, but no more so than the networks of our adult "intellectual" structures. Beyond a certain point, to distinguish between the emotional and intellectual structures of an adult is merely to describe the same structures from different points of view. (Minsky [1985] 1986, 328)

And Papert, in this now-familiar quote, corroborates:

> ... I find myself frequently reminded of several aspects of my encounter with the differential gear. First, I remember that no one told me to learn about differential gears. Second, I remember that there was *feeling, love,* as well as understanding in my relationship with gears. ... A modern-day Montessori might propose, if convinced by my story, to create a gear set for children. Thus every child might have the experience I had. But to hope for this would be to miss the essence of the story. *I fell in love with the gears.* This is something that cannot be reduced to purely "cognitive" terms. Something very personal

> happened, and one cannot assume that it would be repeated for other children in the same form. (Papert 1980, viii)

Winnicott's account of the boy who obsessively played with string (see Section 2.3.1) is consistent with such a view: the case report includes lengthy descriptions of both the fear of loss and the denial of separation that the boy communicated through his play, as well as the boy's more rational and conscious acknowledgements that he had been feeling these ways. These discussions with his mother were followed by lapses in the boy's play with string. Given the opportunity to communicate about the ideas verbally, he no longer needed to express them through the use of string.

> String can be looked on as an extension of all other techniques of communication. String joins, just as it also helps in the wrapping up of objects and in the holding of unintegrated material. In this respect string has a symbolic meaning for everyone; an exaggeration of the use of string can easily belong to the beginnings of a sense of insecurity or the idea of a lack of communication. (Winnicott 1971, *Playing)*

String joins. As an "object to think with" (see Section 1.3), its potential to influence constructions of meaning is huge, and this facilitation occurs on more than one level. Beyond mere *symbolism*, and beyond the combined symbolism and *embodiment* that we find in many "transitional objects," string can be *manipulated* in order to take on different shapes and forms, and in order to join – physically, literally.[3] It is an object to think with and an object to *work* with (or to play with). Like the blanket and the coat, string can facilitate transition from one state of mind to another. Such transitions may be amplified by physical involvement with the material, which results in the formation of knots.

Papert's discussion emphasizes the naturalness and importance of such negotiation between mind, body, and the outside world. Winnicott's discussion of string is more concerned with communication and expression – with exteriorizing that which is in the

[3] Sherry Turkle stressed this point in a recent discussion.

mind – than with extracting knowledge from the environment. But in other descriptions, especially of transitional objects, his model includes both, as the concepts inevitably are related. The process of using an object to mediate between inner and outer worlds involves both a "taking in" and a "putting out"; the result is a change in the structures of thought that are involved (and, no doubt, in the external object as well). The nature of the object influences the interaction.

Another psychoanalytic writer, Janine Chasseguet-Smirgel, emphasizes the fundamental interaction of the physical and psychological. The physical expression of the mother's love (her "narcissistic cathexis") is the infant's initial way of encountering the world. This experience has both a unifying and confirming effect:

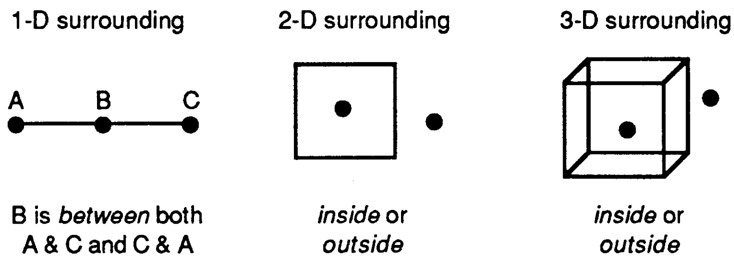
> The mother's narcissistic cathexis of her child as a 'whole' is linked to the physical care and caresses she lavishes on him, bringing together in this way his body ego and psychic ego and conferring value on his different functions. Amongst certain mammals, if one of a litter is not licked by the mother, it will die. (Chasseguet-Smirgel [1975] 1984, 31)

Such interaction becomes not only a fundamental need, but a model for gaining assurance of the internal through contact with the external.

In this section, we have looked at the growth of internal objects, or structures of thought, in terms of how the process can be mediated by external objects; at how the nature of the external object can influence the character of the internal structure; and how that character can be seen as infused with both "cognitive" (or "intellectual" or "logical") and affective (or "emotional") properties. These focuses should help to enrich the understanding of "mother structures" and their relevance to thinking about knots.

Our epistemological approach led to a description of structures of *number, order,* and *topology,* which includes *proximity, separation, order, enclosure, continuity.* While *order* also belongs to the structure of *number,* the topological concepts of *linear and circular* *order* are included among relationships of *surrounding*. As noted earlier, other instances of these relationships include the concepts of *between*, *inside*, and *outside:*


III



The Bourbaki and Piaget, who describe such "mother structures," are interested in the "architecture" of mathematical thought. Both acknowledge the interrelatedness of the basic structures of *order, topology,* and *number* (or *algebra*). Piaget goes further, by pointing to connections between these structures and the life of the developing child, who is engaged in noticing pairs of parents, playing with blocks, and other activities in which *order* and the other structures come into play. Chasseguet-Smirgel describes, in other terms, the emergence of these important concepts in the physical and psychological development of the child. Working with the Freudian developmental model of progression through "oral," "anal," and "genital" phases, she says:

> It is the anal phase that allows the child to emerge from the primary undifferentiated state, gives him an inside and an outside, situates him in time and space. It is the oedipal situation and the incest prohibition that consolidate the acquisition of the *third dimension*. Immediate gratification leaves us in immediate proximity to the object, immersed in it. Successive frustrations (which may, after all, *post facto*, acquire an oedipal significance) and the triangular situation allow us to

> maintain a distance from the object, affording us a certain perspective.
> (Chasseguet-Smirgel [1975] 1984, 33-34)

Thus, the original oral phase is when Winnicott's "me" and "not-me," and boundaries of "inside" and "outside," have yet to be established. The oedipal or genital phase is when the child discovers that only certain forms of love for a chosen parent figure are permissible. The "perspective" to which Chasseguet-Smirgel refers, which is involved with distinguishing properties of the "object," brings us again from the psychological to the epistemological.

9 Case Studies

In the following sections, I present the work and thinking of the three children who spent the greatest amount of time in the Knot Lab. While the varying approaches of these children continue the theme of diversity in ways of thinking about knots, the focus of this section is on the consistency of each individual's work and thought. An in-depth analysis, made possible by their lengthy involvement with the project and their relatively close relationships to me, to each other, and to the work, reveals a notable *coherence* within the thinking of each individual. As I report on the children's involvement, I am concerned with their approaches to knots and knot-tying, but also with their models of themselves and of their own learning (to the extent that these models became evident through their discussions and work in the Knot Lab).

Alice, Jill, and Tony were able to spend far more time working in the lab than the other participants. As a result, their involvement with knots became both deeper and more accessible. Another result is that we came to know each other fairly well. For these reasons, the three are ideal participants on whom to base these case studies of learning and thinking styles. Their work is presented here in four sections: a description of their work as a team is followed by a section for each child, including a profile, a description of important projects, a history of work, and my interpretation of key moments, factors, or processes in the child's learning.

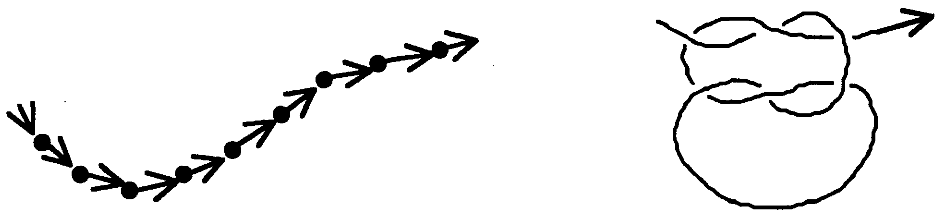
Alice, Jill, and Tony were all in Al's class, and were already friends when the project began. They often worked together, were the three who went to a different class (Kate's) for math, and often finished the work that Al assigned before the the other kids in the class did. It was at such a time, on a day when Al was away from school, that I first met them. They sat idly together, talking, while the substitute teacher tried to subdue the circus-like atmosphere that had taken over the class. Though I had previously discussed the knot project with Al, this teacher knew nothing about it and suggested that I "take the three" who had already finished their work. Alice, Jill, and Tony were curious, and delighted to join me in the then-barren room that was to become the Knot Lab.

During the weeks that followed, they worked as a team, in pairs, and individually, creating some of the major displays in the lab. They corresponded and, in some instances, collaborated with Soo Yong and other participants in the project (in particular, with members of Kate's group). As time went on, they found ways to spend every spare moment in the Knot Lab, hurrying to finish the work that Al assigned, occasionally skipping classes, and coming in during play and lunch times. It was especially during some lengthy mid-day sessions, enabled by combining their Knot Lab time with lunch and recess periods, that the four of us began to know each other well and the children became involved in some particularly detailed work with the knots. Even during our regularly scheduled meetings, the fact that their group was the smallest of any involved in the project meant that I was able to focus more on the work of each individual.

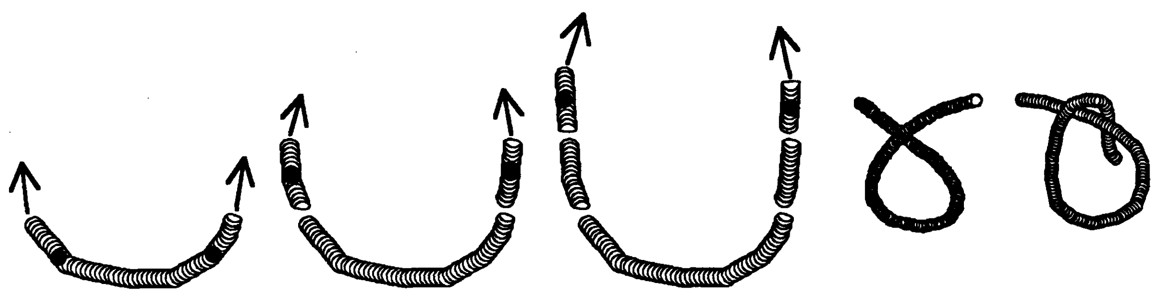
They were well aware of the fact that they were participating in a research project. As Tony once quipped to his colleagues, "She isn't studying the knots, she's studying *us*." Of course, *they* eventually began to study *me*, and to study their own thinking as well. Together, we tried to understand the circuitous pathways involved in learning about knots.

The style designations with which I have described Alice, Jill, and Tony reflect particular "mother structures" that seemed to prevail in their thinking (see Section 1.4). The structures were relevant both to regarding a knot as a finished object, and to the process of tying it.

With her "all at once" approach, Alice perceived in each knot a certain *continuity*: any given position of string continues smoothly from the previous one and to the next one. She formed a knot by leading one end of the string along a continuous pathway.

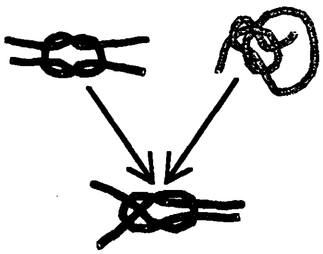


Jill was more concerned with interrupting the flow of movement; she found it confusing, unsettling. Where Alice resisted separation of points along the string and steps within the knot and the process of its formation, Jill insisted on it – her concerns were with *separation* and *seriation*. She wanted to know "where she was" at any given moment, and so fixed points along the knot in the form of well-defined steps.



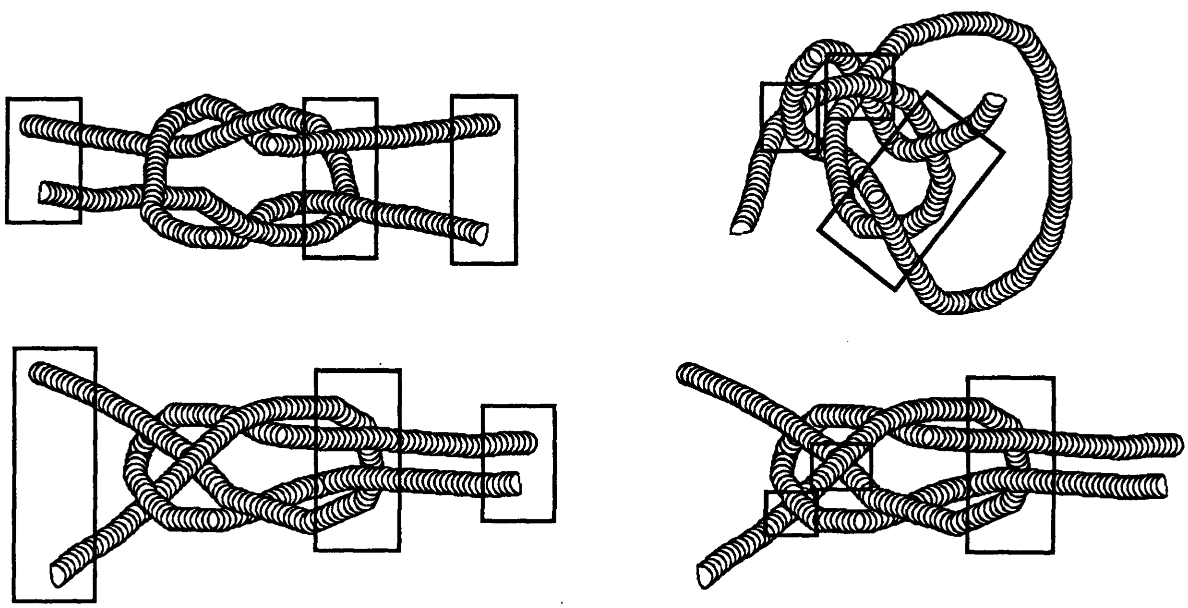
etc.

Tony used topological relationships as a basis on which to arrange knots into groups. Describing one knot in terms of another was one way in which he expressed his concern with what Piaget described as a "mother structure" of number, *classification*.



"The Sheet Bend is a cross between a Bowline and a Square."

He habitually looked for relationships among parts of a knot, and then noted similarities to other knots, as in these correspondences between the Square knot and the Sheet Bend, and the Bowline and the Sheet Bend:



Tony developed an especially poignant kind of involvement with knots, both in his sensitivity to the medium (demonstrated through such comments as "Knot-tying is really manipulating thread" and "... as you know, a knot can't be made ... without string – strings make knots") and in his expressive use of the medium. His concern with classifying knots according to perceived similarities and differences raised questions each time he produced a new knot: Does the knot belong with the others, or doesn't it? If it doesn't look like anything sanctioned or familiar, is it still acceptable as a knot? The resonance of these questions with ones he may have silently asked about himself was demonstrated by his development of a category of "mutations" and a "pseudo" knot. Did he, Tony, belong with the other kids, or not? He was from a different country. His speech was different. He wanted to be smarter than the others. He wanted to lose weight. Would people accept him if they perceived his differences?

By dubbing Alice's style an "all-at-once" approach, I am referring both to a way of doing knots and to a way of being in the world – that is, being generally with people and things, and in the places they inhabit. This is not to say that Alice might *not* adopt some other approach in other circumstances; I simply attempt to describe a way of thinking and doing that captures her tendencies and work – specifically, with knots, and more generally, with people and projects in the Knot Lab. The same is true in my descriptions of Jill and Tony, who also brought into their discussions and work many of the concerns in their lives outside of the lab.

Thus, I allow two senses of the term "style," to which Shapiro calls attention:[1]

> Psychological writers have a tendency to use "style" in two ways. One is descriptive of a formal consistency, presumably resulting from form-giving structures. Another use, however, casts style *as* a form-giving structure. The latter use, however, happens to be quite convenient. (Shapiro 1965, 191n)

For Shapiro's purposes, the latter causality is useful. The data from my study are insufficient to warrant interpretation of either causality, but I can allow either as a possibility. In the sense that Alice's way of working with knots is related to the topological

[1] An unpublished paper by Aaron Falbel discusses Shapiro's distinctions in the manner of a Wittgensteinian "language game." (Falbel 1985, Epistemology and Learning Group, M.I.T.)

"mother structure" of *continuity,* her style *results from* a form-giving structure; to the extent that her way of being in the world influences her implicit choice of this particular structure, her style can be seen *as* a form-giving structure. It is no surprising coincidence, nor need it be confusing, that Piaget refers to bases of topological thought in terms of "structures" and Shapiro refers to styles in terms of "structures." While different usages of the term may lead to appreciation of different kinds or states of thinking, the idea that thoughts stem from others, and the implied systemicity as the components interrelate, are consistent.

Jill's "step-by-step" style can be seen as *resulting from* the topological mother structure of *seriation.* Yet, the concerns with proximity and separation that characterized her world view may have influenced her focus on that topological structure; in this sense, her style *gave form* to her work with knots. Jill's True Lovers' knot is a poignant embodiment of her concerns, which can be seen as a statement of her own sense of being divided, of embodying separation and the potential of its reparation.

Similarly, Tony's invention of the Family Tree of knots and knot mutations reflect his concern with *classification;* characterizing his style with regard to this concern suggests that the style *results from* a form-giving structure. Yet, understanding the concerns expressed by his creations frames it differently, suggesting that his worry about whether or not he "belonged" *gave form* to the nature of the knots he tied.

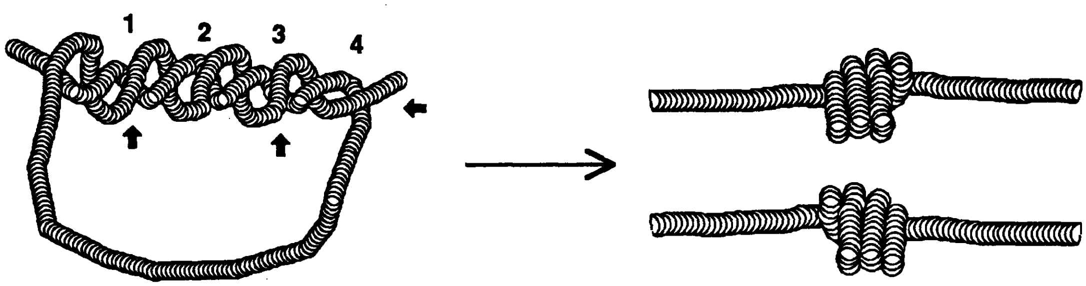
9.1 A Team of Three

Do you think you guys will know each other years from now?
(The three of them answer simultaneously and immediately:)
Alice and Jill: "Yes." Tony: "No." *(Jill and Alice seem surprised.)*

Alice, Jill, and Tony worked together so often, as a team or in pairs, that their work histories parallel one another closely. Their different styles of thinking about knots can be especially well appreciated by comparing their approaches to projects on which they worked together. The following four sub-sections describe their collaboration – on simple knots, and on the three major displays in the Knot Lab. Some short descriptions of their joint work and of the group profile are presented here:

Alice and Jill were puzzled by the discrepancy between the number of times they wrapped the moving end in producing a Stopper, and the number of coils in the final knot. Intuition suggested that by counting the number of revolutions in wrapping the moving end around the standing end, the girls could predict how many coils would result in the finished knot. Yet repeatedly, they seemed to come up with one short – or, at least, half a coil short. Their testing was further confused by the change in position of that "half-coil" as they turned the finished Stopper around. Finally, they decided to count only complete coils in the finished knot.

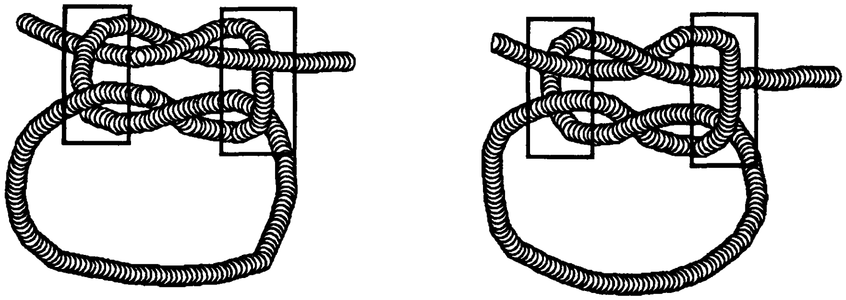
5.4.1



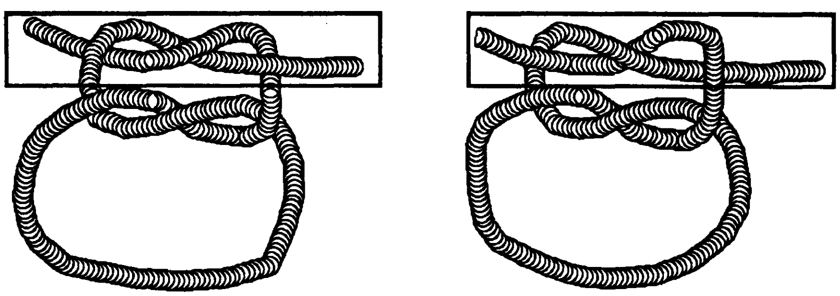
By clarifying that what they were seeking was the relationship between the number of *full* wraps and the number of *complete* coils, they came up with what might be called the "n minus one" formula: if you wrap four times, you'll come up with three; if you wrap three times, you'll come up with two, and so on.

One of the displays in the Knot Lab showed a Square knot and a Granny knot together, and asked, "How are these knots different?" Those who described the difference learned, in exchange, the secret of where the "Mystery knot" was: another display was hidden in the school library. It included identical Square and Granny knots, as well as a "Mystery knot" (the Thief knot).

Jill and Tony learned the clue right away. Alice was the last to get it; such contests did not really appeal to her. When she did describe the difference between the Square and Granny knots, her explanation bore a marked dissimilarity to Jill 's and Tony's. They had described the difference in terms of how the ends went over or under the loops of each entanglement. Their views tended to divide the knots vertically, so that they might imagine a right half and a left half:



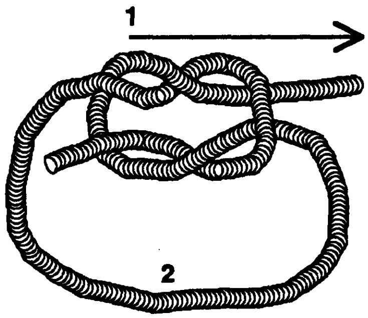
Presumably, this view stemmed from their use of the upright way of tying, producing the knots through some involvement of what they considered as right and left ends of the piece of string. But Alice's description relied on a view of the entanglements as being divided from top to bottom:



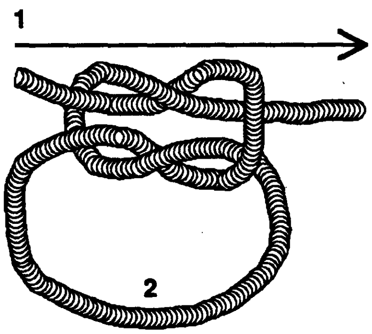
This view is consistent with the focus that she took weeks later in discussing the Square and Thief knots:

 5.4.1 5.4.2 6.2 6.6 **7.1.1.13**

Alice: The string goes over this (1) and around like that (2).



This one, the string goes straight across (1), and then it goes like that (2).



Toward the middle of the project, Alice, Jill , and Tony began finding extra time to work in the Knot Lab. Often they would finish their classwork early and ask to go into the lab. Eventually, at their request, we arranged for them to eat lunch in the lab rather than the cafeteria on certain days, and to spend the following period, which was their recess time, in the lab as well. Such extended working sessions enabled them to get deeply into their projects and into thinking about knots.

The sessions also enabled us to get to know each other better: when eating, our hands were busy with food, so we couldn't tie knots. Instead, we talked. They told me about their classwork, their teachers, and their families, and wanted to know about mine.

They also talked among themselves about ideas for knot projects and their feelings about school, their friends and families, and each other. Lunchtimes were occasions to be free, relax, and get silly if they wanted to. But our lunches also presented opportunities to talk seriously.

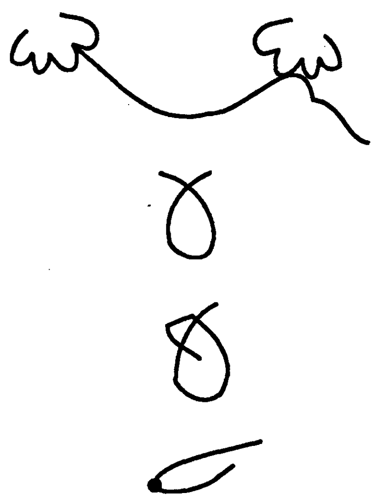
Several of these sessions are documented in the following sections. The reading sketches dynamics between two of the individuals, the entire group of three, and knot-tyers from other groups who occasionally collaborated with Alice, Jill, and/or Tony. Among the images are Alice and Jill discussing subtleties of the phenomenon of "tightening" (even if you tie a difficult knot like the Turk's Head or the Heaving Line knot correctly, careless or ungraceful tightening can ruin it); Jill and Tony engaged in heated debate with Alice about making sure her knots are "right" when put on a board; Alice, Jill, and Tony working calmly and industriously on the Family Tree, and suddenly beginning to sing together a song that they all knew from the school play; and the muddle of a dozen children pulling apart one of their favorite kinds of string, revealing the inside stuffing – a fabric-softener, tissue-like material – with lots of pulling, stretching, wrapping themselves in it, and so on. The picture I hope to leave with the reader is one of a community and sub-communities of thinkers immersed in the pleasure and the struggle of making their ideas apparent to themselves and to others.

9.1.1 Overhand or "Underhand"?

Most of the children began their work in the Knot Lab with the Overhand knot, and Jill, Alice, and Tony were no different.

Jill began by playing with words as well as string, experimenting with what the "Underhand" knot might be. This name is not common, but it stood to reason that if there was an Overhand knot, there must be an Underhand as well, and Jill set out to explore them both. She developed a procedure for tying "Over Hand Knots," which reflects an unusual concentration on the fact that *hands* are what produce the knot. She interprets "overhand" as two words, and her first step explicitly refers to the hands that hold the string:

 5.2 5.4.3 6.16 6.17



Over Hand Knots

Take both ends in your hands. Then take the left end and put it in back of the right end. Next take the lefl (either end) and tuck it into the circle the pull the end that you put in the circle and pull

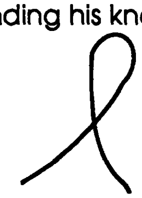
Jill Bachman

Jill vacillates a bit in attempting to describe which end goes into the loop. After all, it could be *either* end – or could it? Would it really be the same knot no matter which end you tucked into the circle? Or would they be different, necessitating different names like "Overhand" and "Underhand"? In her third step, not only does the designator "left" become ambiguous (the end that was originally on the left has now crossed to the other side), but anyway, she decides, "either end" can be moved in order to produce the knot she is after. Jill leaves this decision to the tyer's option, but is careful to specify that the "end that you put in the circle" is the one that you pull to tighten the knot. This approach to tightening seems one-sided when compared to most others, in which the pleasure of pulling comes from a symmetrical final movement.

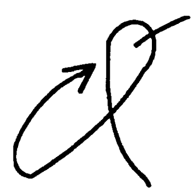
By referring to the "Pretzel" knot, Alice avoids the problem of considering whether it is an Overhand or "Underhand." She also wrote a procedure for tying it:

<u>The Pretsel KNot</u>

Take both ends and cross them so the strings look like like
a person bending his knee like this



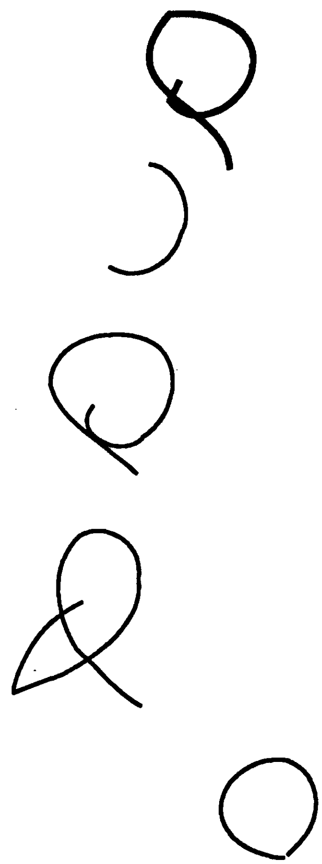
Then take the top string and put it under one of the sides of
the circle



then pull.
Alice

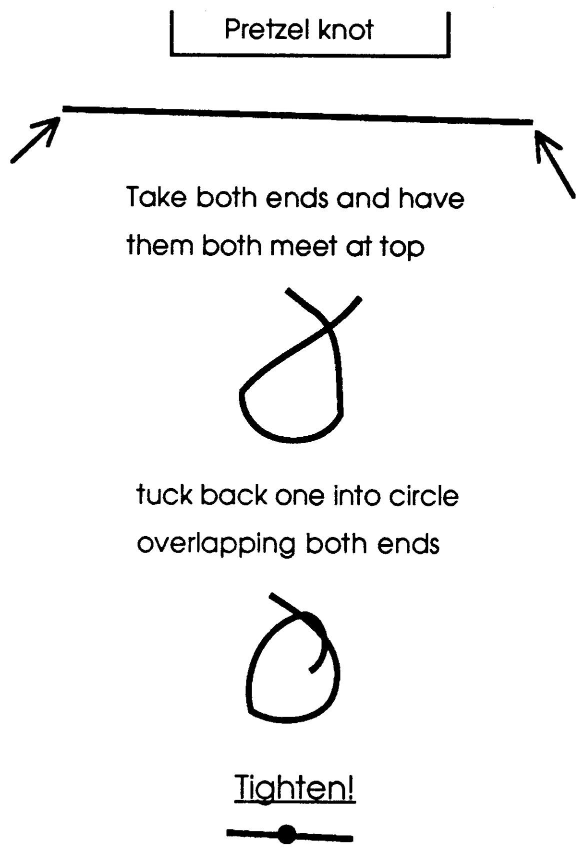
Alice doesn't call attention to hands as Jill does, but she does rely on a reference to the body in order to express the relation "cross." Her analogy to a person's leg recalls Leroy's answer to the Figure 8 knot, a string-based illustration of the "Figure 4" wrestling hold (Section 4.22.7), in which two pieces of string engage in convolutions analogous to two pairs of human legs.

Tony also called his Overhand knot the "Pretzel." Frustrated by his attempt to depict the process of tying the knot,



he resorted to a procedural description that included pictures and words:

5.4.1



Like Jill and Alice, he calls attention to the two ends of the string as the initial step. (He says the ends should meet, but he draws them as already having been crossed.) In rewriting the procedure, however, he is willing to abbreviate:

Pretzel 2

Do pretzel

Tuck left
through space
tighten

No longer referring to the two generic ends of the string, he adopts a distinction between right and left, and refers simply to "left" as the end demanding attention – it is the one doing the action. But he has not dispensed with the original first step altogether: "tuck" has now come to include what were before considered as three separate actions. Taking both ends, having them meet at the top, and tucking one back are all now implied by "tuck." What was a "circle" becomes a less rigid "space, " and "through" comes to mean involvement with that space as well as "overlapping both ends." The last instruction needs no condensing synonym. It is the fait accompli, always a separate, satisfying motion, expressed simply and perfectly by the word "tighten." It also involves some tension, for upon completion of this step, the procedure's success or failure comes into full view.

Building from this procedure, Tony developed a method of tying what he called the "Double Overhand." Having tied one Overhand knot, the wrapping motion is simply repeated to produce a "double" knot:

Pretzel 2

Do Pretzel

Tuck left behind knot
and pull through space
Tighten

Tony rewrote the Pretzel procedure, adding references to the knot already produced and to the term "tuck" that he had already defined. With a few more repetitions of the same wrapping motion, this knot would become what Tony soon learned as the Stopper.

After Tony and his working partners had spent several sessions working on the String in Motion board (Section 6.1.2), becoming well versed in their chosen knots, they tried to develop more exacting ways of describing the relationships and processes they had been illustrating. About the time of beginning work on the Knot Language board (Section 6.1.3), the three returned to the early theme. The discussion began with the Overhand knot, quickly acknowledging the complexities even of discerning an Overhand from an Underhand. Trying to pinpoint the difference produced dilemmas bringing into question issues of point of view, right and left, inner and outer, and the meaning of parallel lines.

8.3

Dear Soo Yong,
Today we were talking about over hand and under.-hand.
We couldn't figure out what is the differnce between
them?

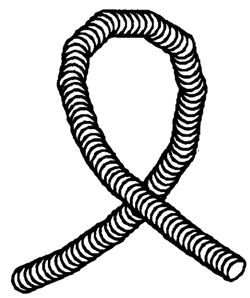
Alice, **Tony**, *Jill*

Dear Alice, Tony, and Jill,
I know it is hard to figure out overhand and underhand knots. It's hard to find the difference when they are tightened but when you are tieing them you'll find the difference. I'm not going to tell you guys the difference because it will be too easy. So why don't you find out and tell me the difference.

Sincerely,
Soo Yong Chang

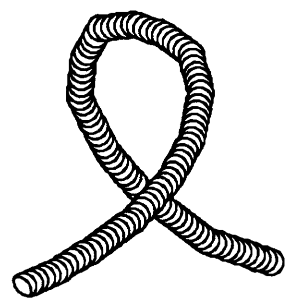
The quandary led to a discussion that prompted Tony to construct a display posing the crucial question:

Overhand?



Underhand?

What is the difference?



Tony's answer took on a tone of resignation: "All things are fundamentally the same."

The most exacting result of these inquiries took the form of clarifications of "loops," which they elaborated on the Knot Language board (see Section 6.1.3).

9.1.2 The String in Motion Board

Among the materials in the Knot Lab were copies of pages from a Boy Scout manual, which depicted a variety of knots. The pictures were hand-drawn, and showed the knots completely tied but just shy of being tightened. The children used these images as references for tying and as a source of ideas for new knots to learn.

Jill worked on the Stevedore from one of these sheets, but became frustrated when the different stages of its formation were nowhere to be found in the pictorial reference. How could she be sure she was intertwining the string correctly as she worked toward the final knot? Even her completed knot looked different from the one pictured, which was kept loose enough that the different strands of the configuration could be discerned.

Her concerns led to a solution in which not just one, but many stages of each knot could be shown. One of the bulletin boards in the room provided an inviting surface. Blank except for the word "motion," it gradually became an area in which pieces of string were curved and twisted into the forms of different knots. Jill, Alice, and Tony spent weeks stapling their serializations of knots to this display, which became known as the "string in motion" board. Their construction of it became an activity that fostered thinking through the knots, and the finished board became a reference for tying them.

After several weeks, their efforts resulted in a detailed assortment, as shown on the following page. (Closer views of these knots are shown in the "projects" section of each case study (Sections 6.2.2, 6.3.2, and 6.4.2).)

Jill began by constructing displays of "right-over-left" and "left-over-right" Pretzel knots *(top left)*, while Alice showed "loose and tight" Figure 8 knots *(top center)*. Tony put different versions of the Square knot on the board, along with variations such as the Thief and the Sheet Bend *(right side of the board)*. Jill also came to work on Stopper knots *(third column from the left, at bottom)*, Stevedores *(bottom left)*, and the Trumpet and True Lovers' knots *(bottom center)*; Alice did the Packer's *(center and bottom center)*, Bowline, Slippery Hitch, and Monkey's Fist *(single knots at center)*.

Jill wanted to identify all possible points of confusion that a knot-tyer might encounter, and to provide visual guides for questions that might arise. Through her work with the Overhand knot, she stumbled upon the idea that mirror images of a knot may look similar, and may have the same name, but may not be exactly the same.[1] She found the idea disconcerting, and it led her to specify two versions of several of the knots she worked on – often, by beginning with the left end, and then by making another version of the knot, beginning with the right end.

Alice went along with this method at first, but it soon became apparent that her preferred approach was quite different. She did her part of the project of putting knots on the board by putting knots on the board – she tied a knot, and put it up. Hers was a more fluid way of tying. She saw the knots as resulting from the movement of a single end, formed in just one step. In this view, serializations didn't make sense. It was in response to protests from Jill and Tony that Alice finally broke down the Packer's knot into a series of steps.

---

[1] "These two knots are completely symmetrical 'mirror images' of one another, and are topologically equivalent, but they are not congruent. The problem arises whether it is possible to deform one of these knots into the other in a continuous way. The answer is in the negative, but the proof of this fact requires considerably more knowledge of the technique of topology and group theory than can be presented here." (Courant and Robbins [1941, 1969] 1978, 256)

Tony demonstrated still another approach. At first, he was more reticent about committing a configuration to the board. While Jill used the surface as a means of anchoring different steps in working her way through a knot, and Alice regarded the board as a means of displaying a finished knot, Tony sat at a table and experimented. He wanted to understand a knot very well before relegating any aspect of it to the board. He was fascinated by the Square knot, and spent hours twisting pieces of string into different ways of forming it – or something like it. Often his explorations produced oddities that he quickly hid or untied, but eventually he developed an understanding of subtle differences between versions and variations of the Square knot, many of which found their way onto the board.

Alice often took on herself the role of correspondent for the group, initiating exchanges with Soo Yong that the three of them signed. She told him about their project, referring to herself in the objective as needed in order to preserve the voice of the group:

> Dear Soo Yong,
> We are making a bulleton board on knots. We made several. Could you teach us a couple more knots. Got to go.
>
> From: Alice *Jill* **Tony**

> Dear Alice, Jill, and Tony,
> I will teach you new knots every week. Practice the knots few times so you'll get the hang of it. I'm really looking forward to see your bulletin board.
>
> Sincerely,
> Soo Yong Chang

Dear Soo Yong,

Tony has been working on square knots and got some very interesting kinds.

Jill has been trying to make Stevedores. Now she's doing one backwards.

Alice has been working on some bowline knots. We have put them on our bullentin board. Theres a whole bunch of knots on the board.

From: *Jill* Alice **Tony**

The "motion" board triggered interest from other groups – Kate's in particular. Although the knot-tyers in this group contributed most significantly to the "Knot Experts" board, several of them corresponded with members of Al's group, and some collaborations emerged as the two groups worked together toward the end of the project. Most notable among these collaborations were the work shared by Alice and Stacy, and by Tony and Patrick. Perhaps understandably, given his self-imposed stipulation that a knot be thoroughly understood before becoming part of a Knot Lab display, Tony was concerned that some of the knot-tyers treated the term "expert" too lightly. Patrick seemed to share his sense of rigor:

Dear TONY,

I heard that you were in The Knot class. I know how to do all the Wimpy Knots. Like the Square knot and the Figure of Eight Knot. I see that you come up for Recess and Work on the Board And do all the Major Knots.

Sincerely,

Patrick Gilmor

It was impossible to ignore the development of "the Board." Large, colorful, and detailed, it gradually became the centerpiece of the lab. The other boards, interesting as they were, were clearly spin-offs. But its creators gradually shed a tendency toward pride and possessiveness as the other displays became more pronounced and the work of other participants in the project became more apparent. Some time after Patrick wrote this letter, Tony found a way to compliment him for his work on the Stopper knot: "There is an intimate relationship between Patrick and the Stopper," he announced.

The important role played by the "motion" board in shaping the atmosphere and projects in the Knot Lab is well expressed by a gesture of Alice's toward the end of the project. When she, Jill, and Tony first began working on the board, Alice had put up a sign with the number of their homeroom, as a declaration of both authorship and ownership. But one day several weeks later, amid the din and activity of knot-tyers from several groups working together in the room, she climbed up to that spot on the board and carefully removed the sign, acknowledging that the lab had become an open, shared workspace. There was no longer a need to claim it as their own; it belonged to everyone.

9.1.3 The Knot Language Board

Jill liked the idea of doing a bulletin board about language used in knot-tying; it appealed to her sense of detail and precision. A discussion of what constitutes loops set Alice, Jill, and Tony to work on the board. Alice sought clarification from Soo Yong:

5.1.4

Dear Soo Yong,
I was wondering what makes loops.

- -

Jill's instructions were
A piece of string (or something) that has 2 ends & crosses over one side.

- -

And I said
The rope ends curl on its own.

- -

which is right?
Alice

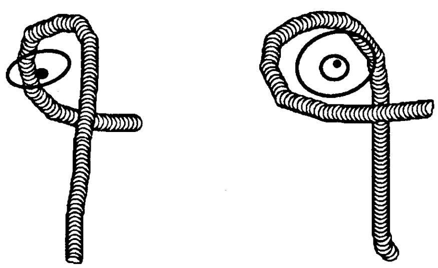
Dear Alice,
You both are write.
As long as a rope makes a circle, it is a loop.
^
complete

Sincerely,
Soo Yong Chang

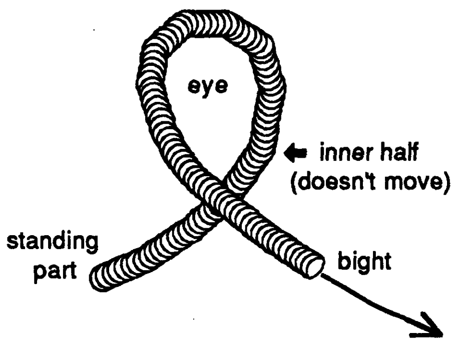
In this letter, Soo Yong used the same designator that Alice, Jill, and Tony had originally used in their instructions for the Overhand knot (Section 9.1.1): the term "circle" seems to be intuitively understood.

 5.4.1 5.4.3

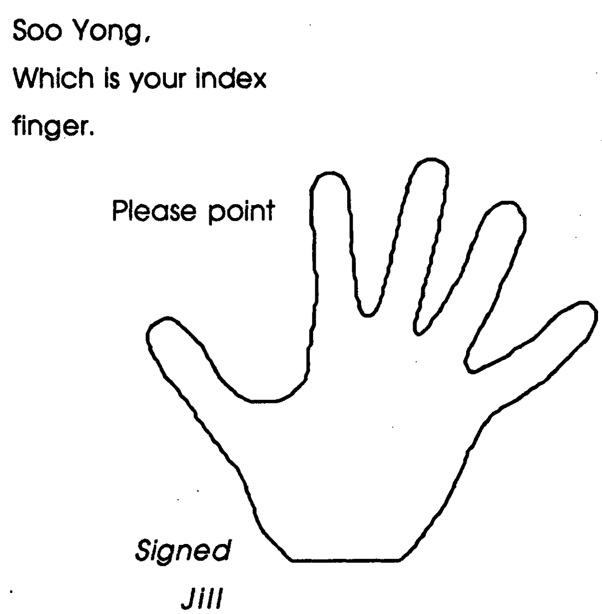
However, Soo Yong used a different designator for this open part of the knot in one of his videos. After hearing him refer to the "eye" of a loop, Alice and Jill were confused about what he meant. It was clear that his reference was to the enclosed part of the loop, but did he mean an alphabetic "i," a personal "I," or a seeing "eye"? In a lengthy discussion, they decided that the negative space delineated by the string resembled the shape of a human eye. In order to clarify Soo Yong's term, they taped pieces of string on pages that were displayed on the Knot Language board, and drew representations of eyes in the loops:



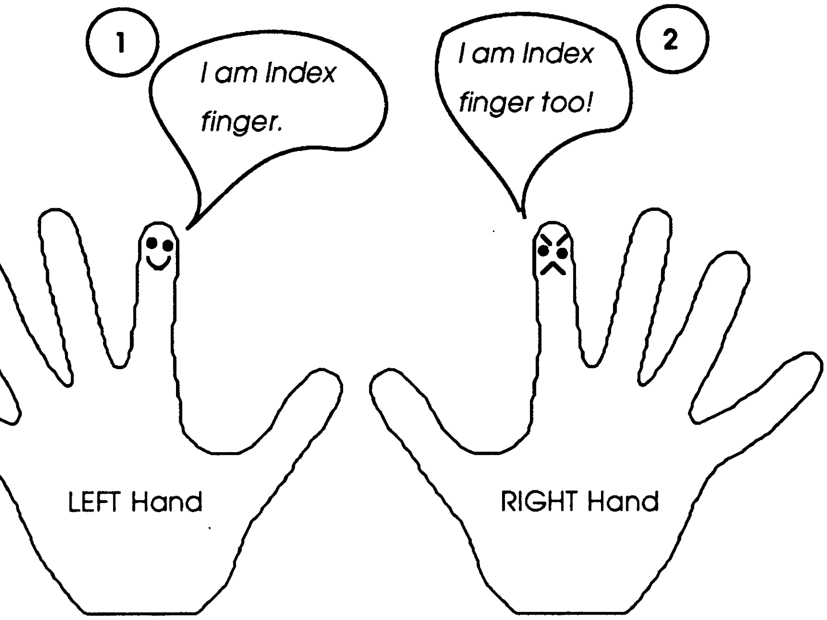
Alice and Jill also designated other parts of a loop, considering the end that moves, the end that doesn't move, and the negative space that their changed positions define. They decided to name not only the origin of the end that doesn't move (the "standing part," derived from Soo Yong's "standing end"), but the portion of the string closest to it, the "inner half." This designation echoes Alice's concern with "one of the sides of the circle" in her instructions for the Overhand: although what she calls the "inner half" doesn't move, it is still an important piece of the scenario because something moves *around* it. It is the stable referent relative to which the knot-tyer can decide what to do next:



One of Jill's early concerns was also repeated in developing the Knot Language board: again, she focused on the importance of *hands* in producing knots. If you want to talk about knots, she decided, you need to be able to talk about hands as well. She and Alice began to make diagrams of hands, carefully labelling each finger, but got stuck on an important term. Jill sought help from Soo Yong:

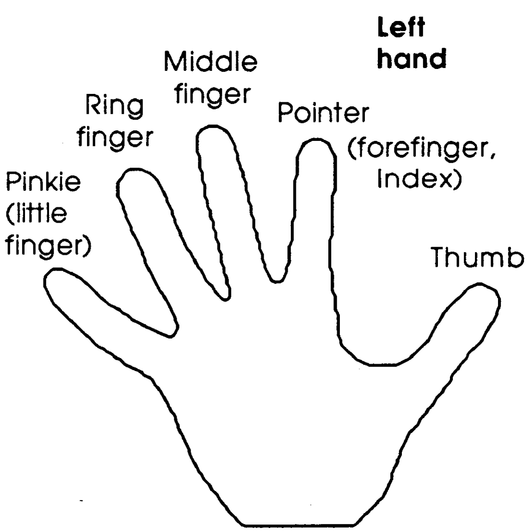


Dear Jill,

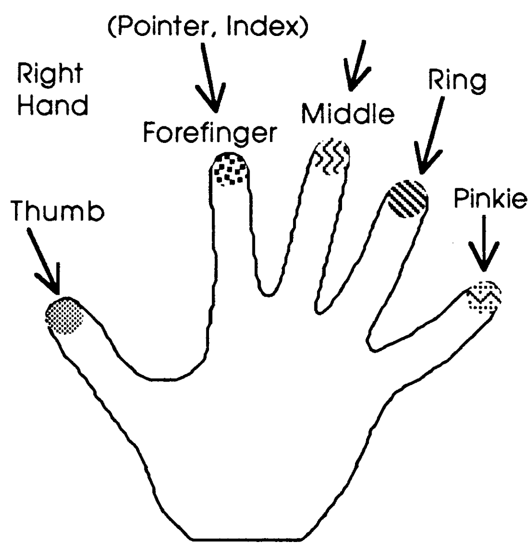


Sincerely,
Soo Yong's
index fingers

Jill and Alice added the clarification to their diagrams:

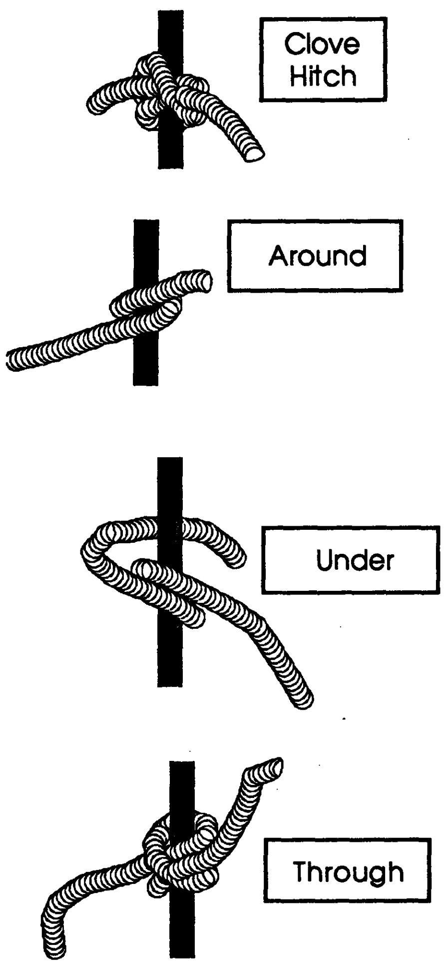


Alice's hand chart



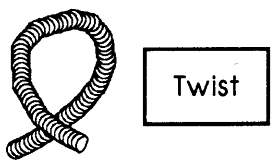
Jill's hand chart

Later, Alice and Stacy worked together and added some terms to the board. Though they didn't define them specifically, they decided that knot-tyers should be aware of these terms: *around*, *over*, *under*, *between*, *through*, and *intersection*. Alice added three of her own words to this list: *straight*, *twist*, and *cross*. (See Section 5.4.1 for various usages.) Alice illustrated how some of these terms could be used, in her instructions for tying the Clove Hitch:




5.4.1

Twist is a term that came up again when Alice, Jill, and Tony displayed on the board serializations of the Running Bowline (shown below). Alice used the term as the sole instruction for her first step:

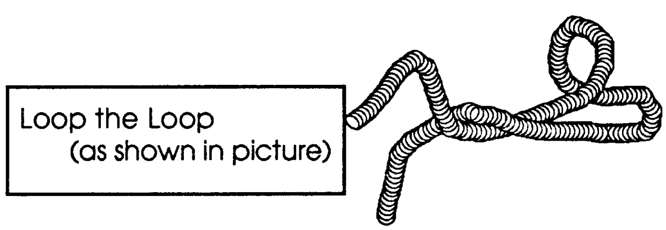


Alice compressed a lot into one word: "twist" is a specific motion that could be described by a series of steps – yet, characteristically, she expressed them as a single, continuous movement.

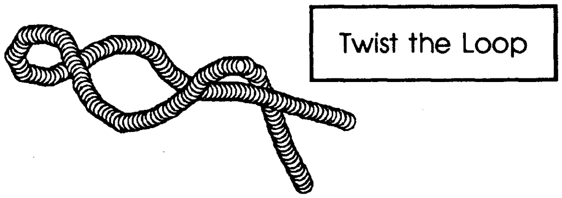
Alice tried to convince Jill of the power and significance of the term: referring to the loop in Jill's third step, Alice said, "You *twist* that."


5.4.1

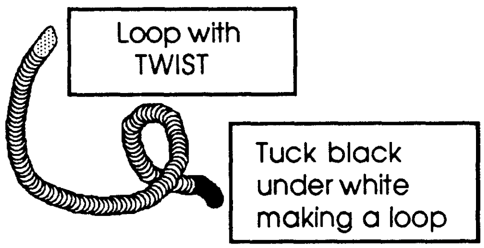
Jill's step was essentially the same as Alice's, but Jill chose to use the term "loop" for both the movement and its object:



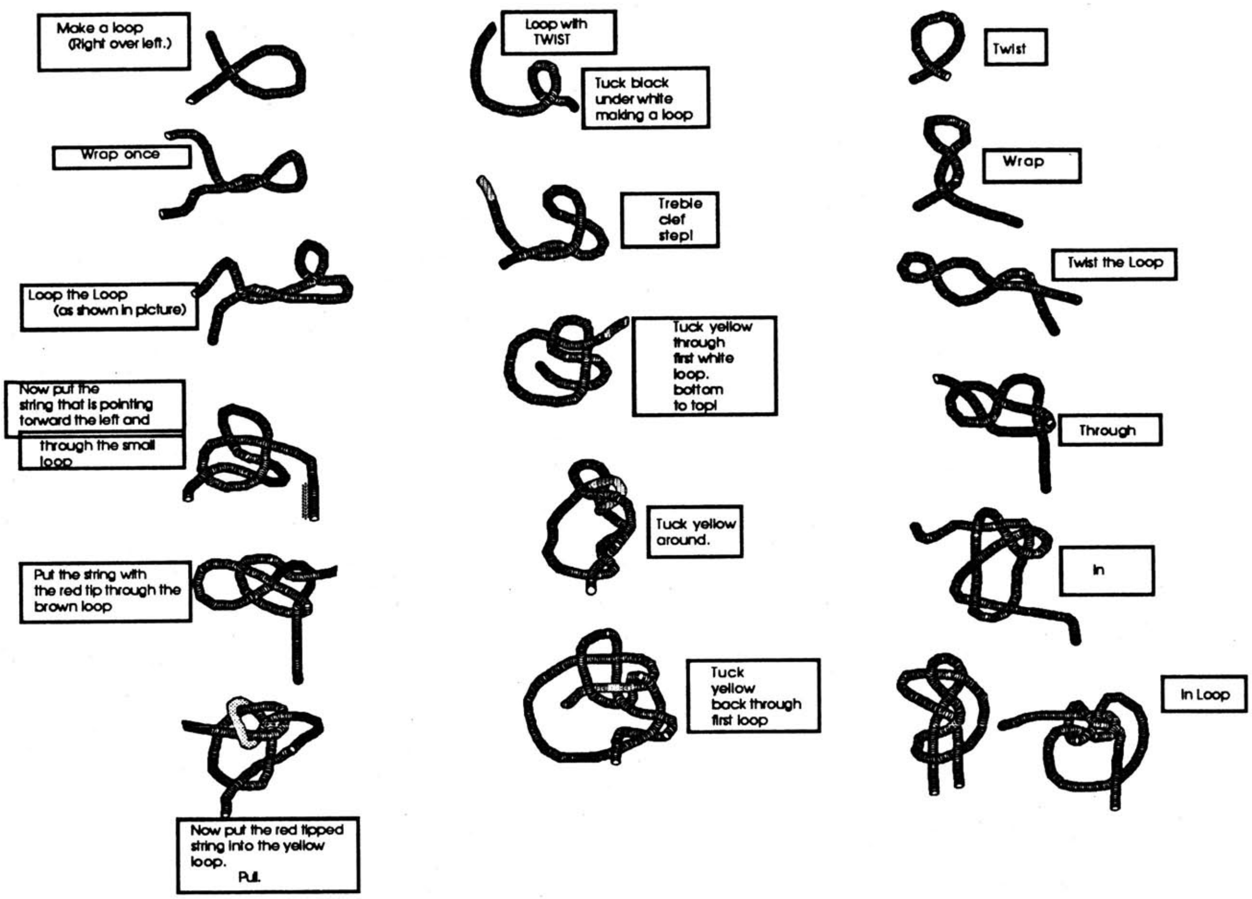
Alice, however, chose to distinguish verb from noun:



Tony also incorporated the term into his first step. He separates very clearly the name of the step from the instruction for how to produce it:



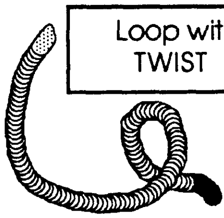
These illustrations were a way of sorting out a dilemma that arose as the three children worked on the Family Tree (Section 9.1.4): they had trouble tying the Running Bowline, and used the surface of the Knot Language board to serialize it:



Alice's steps were distinctive in their compression of instructions into a single word, whereas Jill and Tony used entire sentences to accompany each step. Jill and Tony also colored parts of their strings in order to facilitate their descriptions.

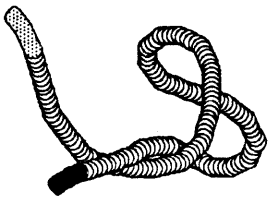
As a sidelight of their explanation of the Family Tree, the three clarified their instructions for the Running Bowline:

Tony: We had a lot of trouble with the Running Bowline, because we didn't agree. ... Um, mine – it starts out with a "loop with a twist." And sometimes people get confused. They think it might be the other way, but it really has to be under, and that's why I wrote there, "tuck black under white." Now, I labelled – I put these – I put specific colors on them so that it would help people. White is the center. This is – I call this the "treble clef step," because that's kinda – it looks kind of like a treble clef. And you bring it around and through so it'd go into there *(gestures to show the motion of the string)*. Then you tuck the yellow through the first – through the first white loop. Bring it up there, bottom to top – so you go up, you don't tuck it down. ... And then "tuck the yellow around." You can see – and then you tuck the yellow back through the first loop, then you tighten ... my instructions are really good, so I suggest you read them and look them over.

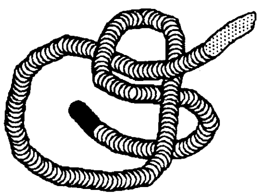


Loop with
TWIST

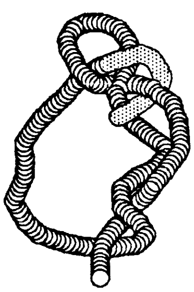
Tuck black
under white
making a loop



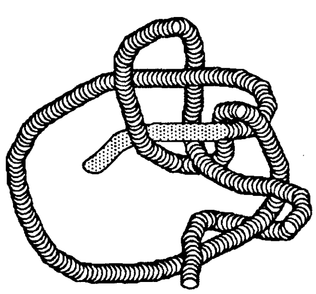
Treble
clef
step!



Tuck yellow
through
first white
loop.
bottom
to top!

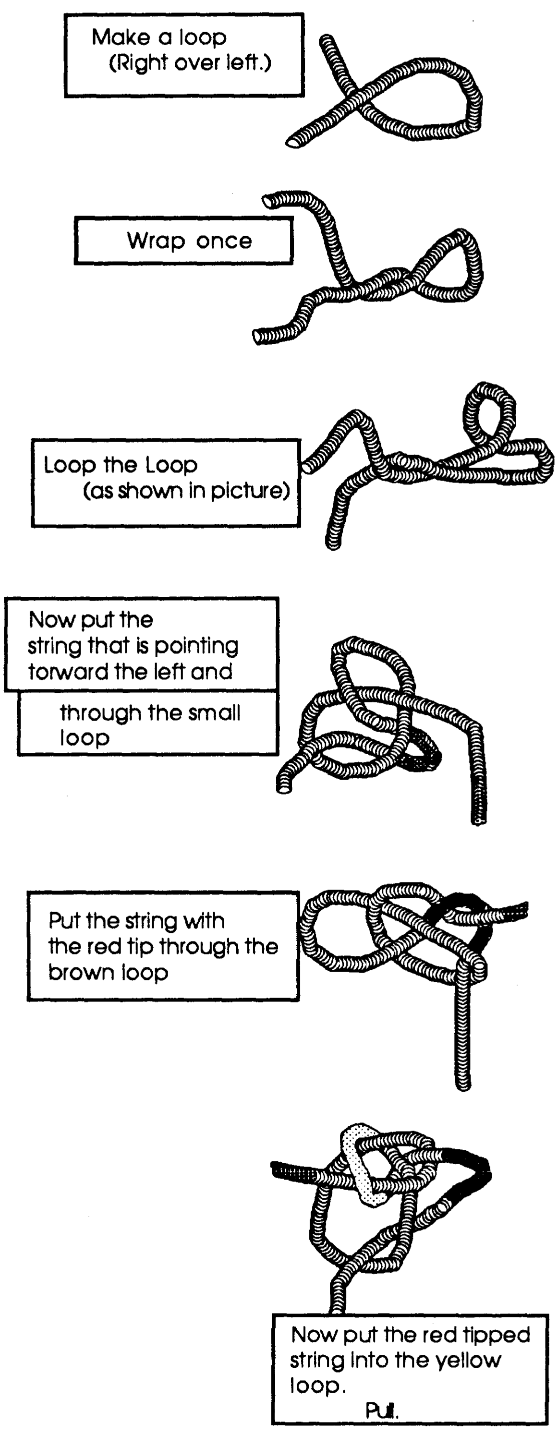


Tuck yellow
around.

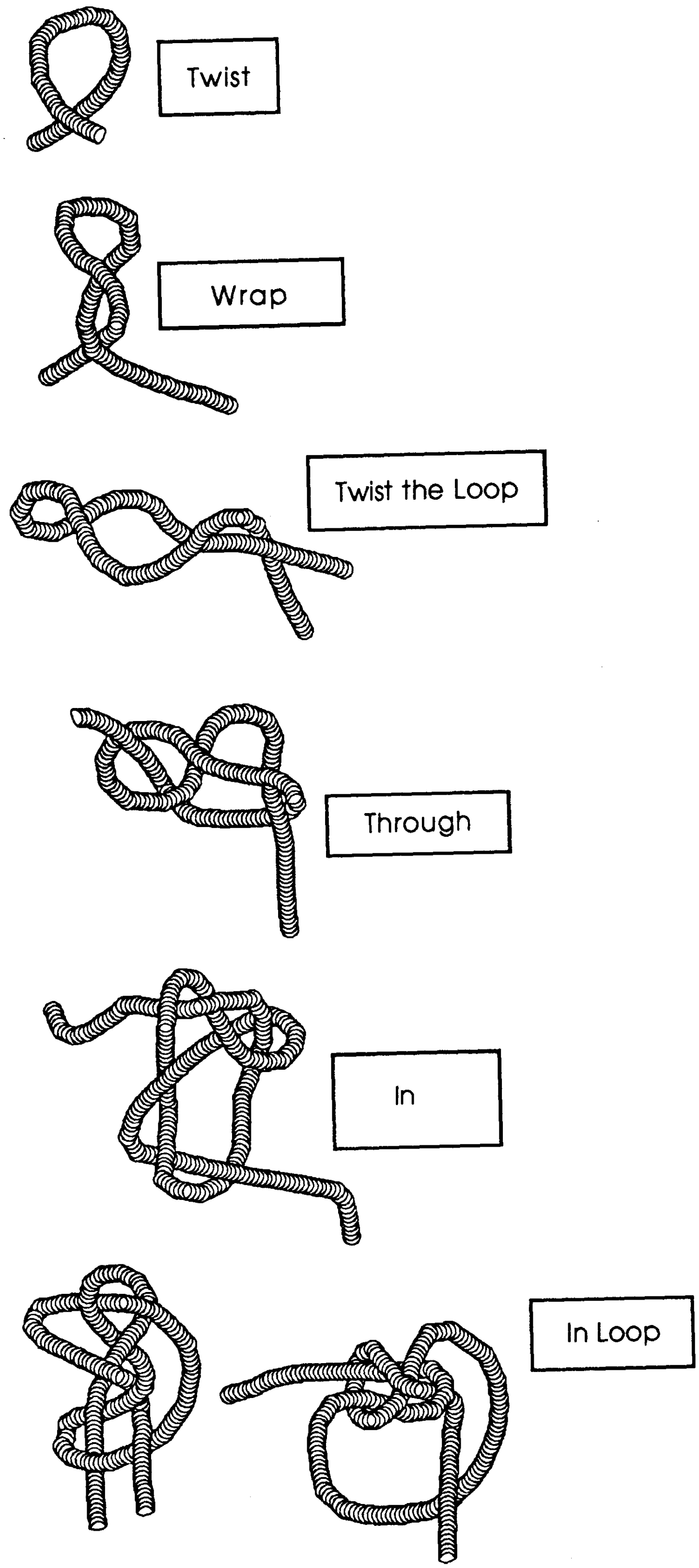


Tuck
yellow
back through
first loop

Jill: Well, mine are sort of – I sort of just did my own way. And Tony did his own way too, but – um – you just start with a regular loop, but you have to have it move a certain way, with the right over left – or the right – left over – under right. ... And so – um – and then, we have – you have to sort of make a loop – or, twist – and then you have to make a loop here *(she indicates her "loop the loop" step)*, but you have to have this string go over – over that one, or else it will – or else it will turn out wrong. And, um – then, you put this string – after you put it through the loop, you put it through the brown loop, and then – then after you put it through the bottom loop, you put it through the other loop, and you just pull and you've got it.



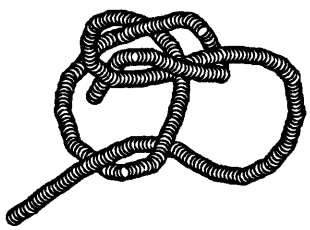
Alice: Um – this is mine. "Twist," "wrap," "twist the loop," "through," "in," and "in loop." Okay?


Twist
Wrap
Twist the Loop
Through
In
In Loop

(Al had sent a messenger to call the children back to their classroom, so Alice's explanation was abbreviated. It is likely that she would have given a more compact description than Jill or Tony in any case, though.)

Jill demonstrated her favorite property of the knot:

Jill: Here – here's the Running Bowline.



Tony: Watch this.

Jill: Now watch. You'll see how it moves. See? *(She does it.)* It can run. If you – but – you have to pull it right – the string.

Carol: *... Okay. How does it move?*

Jill: Well –

Tony: There's a loop at the bottom, which it goes through.

Jill: There's a loop at the bottom, which if you pull the bottom back, it will move forward, and if you move the bottom top – if you move the back top, it will move backwards. *(That is, if you move the bottom back, the knot will move forward, and if you move the top back, the knot will move backward.)*

9.1.4 The Family Tree of Knots

One of the bulletin board displays in the Knot Lab was a "family tree" that Alice, Jill, and Tony made together (see also Sections 6.22.8 and 7.2.1). The display consisted of twigs bound together to form a "tree" on which knots were glued or tied according to interpreted groupings. The children personified each knot, giving it a human name that reflected its place in the family. Alice drew portraits of each family member – "knot people," so to speak – the collection of which became a "family album" posted next to the tree (see also Sections 6.1 and 10.1). Each portrait embodied the shape of the knot in some way, carefully representing the "unders" and "overs" of the crossings. Jill and Tony tied most of the knots.

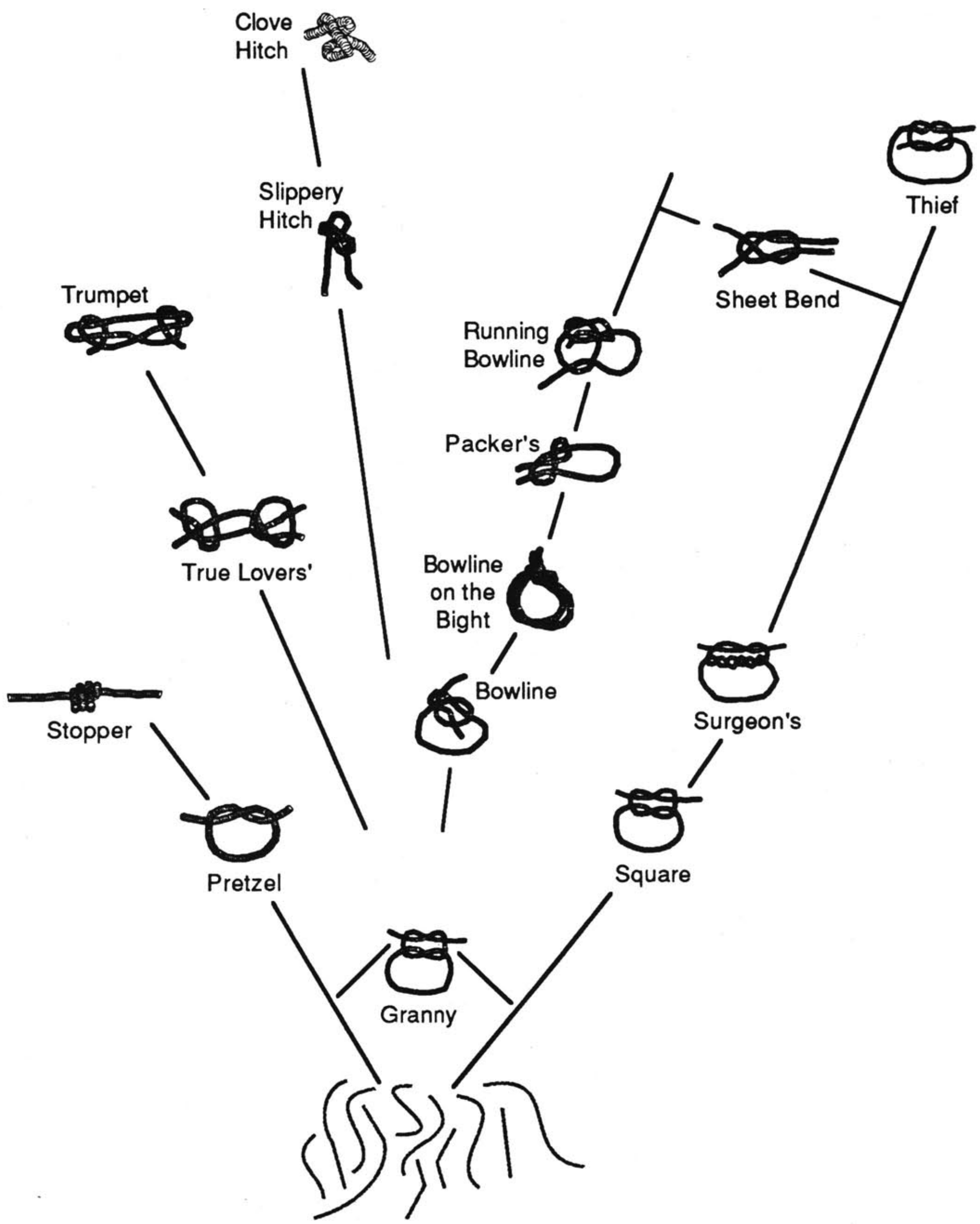
Before beginning the actual construction of the tree, the team thought about what families to represent and what knots belonged in each family. They drew up a chart, first grouping the knots, and then rearranging the families from simpler to more complex. Their order of families was: Pretzel, Square, Hitch, Bowline, and Trumpet. Each family came to occupy a branch of the tree, and the knots were arranged along the appropriate branch. In working through the construction of the actual tree, the co-workers made some modifications to their original plan, which are shown on the following page. Aside from a few omissions and the addition of "roots" in the final display, the most notable of these "before-and-after" results are the maintenance of the Sheet Bend's designation as *both* a Square and a Bowline, the shifting of the Granny from the Square family to being *between* the Square and the Pretzel families, and some changes of genealogical hierarchy within some of the families:

Plans and Actualizations of Groups on the Family Tree:

Planned family	**-and knots**	**Tree branch**	**-and knots**
		(represented as underground)	untied fragments of string: "knot roots"
Pretzel	Stopper Pretzel Double-Pretzel	Pretzel	Pretzel Stopper
		Pretzel/Square ("node between two branches")	Granny
Square	Square Granny Sheetbend Reef [1] Surgent	Square	Square Surgeon's Thief
		Square/Bowline	Sheet Bend
Hitch	Rolling Slippery Clove Stopper	Bowline	Bowline Bowline on the Bight Packer's Running Bowline
Bowline	Bowline Bowline on the end running bowline Packer's Sheetbend	Hitch	Slippery Clove
Trumpet	Trumpet True lover's knot	Trumpet	True Lovers' knot Trumpet

[1] "Reef" was their term for the Thief knot (see Section 5.1).

The Family Tree of Knots:



Alice's Illustrations for the Family Album:

After completing the Family Tree, Alice, Jill, and Tony explained why they decided to build the display:

Carol: *How did you get the idea to do a family tree about knots?*

Jill: It was Tony's idea.

Tony: I had the idea, because a lot of knots are related, or they look alike, like the Square knot and the Granny. I mean, you know they're not the same, but if you put them in a family, like scientists do and things, you know they'd be in the same family and stuff. And like, things that would be in, would be the Sheet Bend and things like that.

Jill: Sort of, they have the same genes.

Why do scientists put things in families?

Tony: To help them know, like –

Alice: Organizing.

Tony: Yeah, to help them organize it, and to help them, like, 'cause if you wanna – kind of helpful, 'cause you can classify them that way.

Jill: Yeah.

What other things do they put in families?

Tony: Animals.

Jill & Alice: Animals.

Before beginning work on the Family Tree, Jill had spent a good bit of time on her "True Lovers Knot" display (see Section 9.3.2), in which the use of colored strings helped to distinguish parts of the knot. Her use of a marker to color the strings seemed intriguing enough that Tony and Alice wanted to do it too, so they found a way to incorporate colored string in the Family Tree display:

Jill: We put the tree up, and then we did this, the roots.

Alice: Our "knot roots."

Jill: Which <u>must</u> be string.

Tony: Because as you know, a knot can't be made – used – made without string. Strings make knots.

They spent a long while selecting materials for the roots and carefully arranging them. The roots for the Family Tree consisted of a variety of colors and textures. Scattered nearby were other roots and blossoms not connected with the tree, as indications of possibilities for more knots. Knowing that the knots would be made from a certain kind of string, which was white, Tony was concerned with the implications of coloring the roots. "Should we have at least one white root?" he asked, "Can a red root have a white gene?" The team decided to add several strands of white roots.

The first knot to go on the tree was the Pretzel, the lower knot on the leftmost branch. They avoided a debate about whether it was an Overhand or an Underhand by simply calling it the Pretzel knot. Among their reasons for beginning with it were that "it's the most used knot," "it's the most common," "it's the easiest," and:

Tony: Most knots start with it –

Jill: Kind of the idea –

Tony: – and you know, if you – if you – somebod – asks you to tie a knot, you tie the Square. *(He has used the wrong name. He is talking about the Pretzel knot but mistakenly says "Square" instead, perhaps because it is the knot that he explored the most thoroughly.)* Also, because it's like the most primitive knot, really. I mean, when somebody has a piece of string, they generally cross it *(gestures this tying motion)* and tuck it under, and that's what you get *(the Pretzel).*

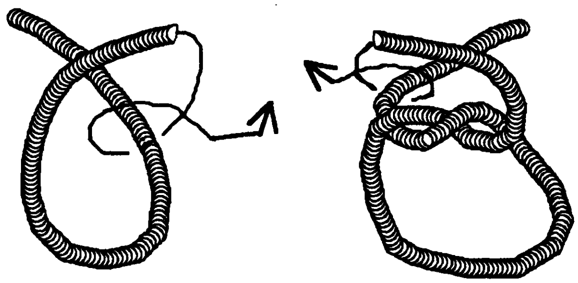
Their rule of thumb for placing the knots on each branch was to put "primitive" ones at the bottom and progressively harder ones toward the top. Here they used "primitive" in the senses of "easy," or "simple," but Tony had also described the Pretzel as the "Neanderthal, or Cro-Magnon, knot." He considered it to be primitive also in the sense that it is "basic" – both because people seem to tie it instinctively and because it is a fundamental unit of many knots. "Harder," they explained, means "you need more string, or you have to wrap it around more times." Thus they encompassed several meanings in their positioning of knots on the tree: they considered the ranges of simpler to more complex, basic to derivative, and primitive to further evolved.

A literal debate about whether the tree should represent species or race vs. generations never ensued, and the group settled into age and the extended family as the guiding metaphors.

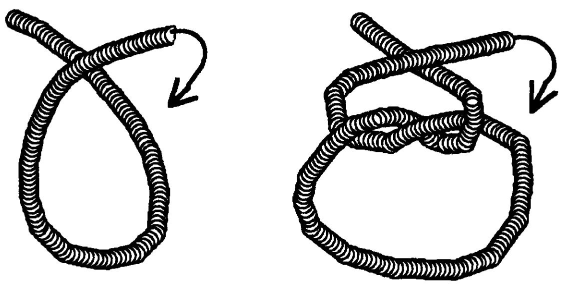
Jill complained that the names for the families should be different from the names of the knots, and Alice thought they should use "people names" (though they never changed the surnames). She named the Pretzel "Walter," and began drawing a picture of him. To her question of whether he should have a beard or a mustache, Tony replied, "Yes – he's old. A grandfather, in fact." Walter became known as "Pappy Pretzel" and "Great Pappy Pretzel." Above him, on the same branch, they put a Stopper. Then they moved to the Granny knot, or "Double Pretzel," as Tony liked to call it. They put "Gram Granny" on a node between two branches. Tony explains:

The Granny is a kind of bridge between the Pretzel and the Square...It's like a Pretzel, double, but it's not quite a Square, 'cause it's a Square if you keep on going instead of if you do it right. Some people say that a Square is you do it one way and then you do it the other way, but since the string – strings change places, it's really doing it twice...

Instead of thinking of the Square knot as resulting from the fragmented steps, "left over right, right over left," Tony has come to think in terms of a single active end that "keeps on going," repeating its fundamental motion:



...And the Granny's when you do it once one way and the other time the other way...But it looks like you're doing it the same way.



In other words, the moving end changes for the Granny. He continues:

And, um, Square knot family – this whole branch. This is the Square knot, which is the knot – which is a really good knot. It's good for – well, it's good for, when you tie it, it's like a Pretzel, like I said, it's – it's an exact Double Pretzel, is a Square.

Jill points out the contradiction of his earlier designation of the Granny as the Double Pretzel. To do so, she uses Tony's own reasoning in considering the Square knot as, basically, two loops:

Jill: No, not exactly. It's more like two loops with sides coming out, which is not exactly a Pretzel, Tony.

Tony: Yeah, it's a Double Pretzel, exactly.

Carol: Which is the Double Pretzel?

Tony: The Square. *(He points to it.)*

Oh, the Square.

Jill: But I thought you said that was a Double Pretzel *(points to the Granny at the node).* That's what you said.

(Tony shakes his head no.)

Jill: Yes, you did. You said that –

Tony: I said it can be called a Double Pretzel, but – in the way you tie it, it's a Double Pretzel – but in this *(he points to the Square knot),* the way it is is a Double Pretzel. I mean, it looks like you're tying it one way, and then the other way.

Jill: No, Tony –

Tony: But since the strings change, it's a Double Pretzel.

What Tony means is that the *appearance* of symmetrical movement in tying the Granny may lead to calling it a Double Pretzel, but that appearance is distracting. The sense of repeating what you have done before ("left over right, left over right") dulls the awareness that the knot can be understood in terms of the movement of a single end. Properly understood, the second step of the Granny represents an interruption rather than a continuation, since the motion of the first end is eclipsed by the movement of the second one. The second step of the Square, though, truly *is* a continuation, as the same end travels smoothly along the path that produces the knot. It is the change of position resulting from the first crossing that produces the apparent alternation of ends ("left over right, right over left"). Tony illustrates his point by tying the Square knot, and it becomes clear that he and Jill have in mind different criteria for the meaning of "double":

Jill: Well, I don't think that's double – Tony, once you tie it, and once it's tightened, it doesn't look like a double – it doesn't look like a Double Pretzel.

Tony: I didn't say it looked – I said it is. There's a difference, you know. There's such a thing as, "I look, therefore I am." It's – I mean – *(does back-and-forth motion as the check for the Square knot)* – see? It is a Square.

Jill bases her concept of "double" on appearance, and therefore refers to the finished Granny, in which the two crossings sit in nearly perpendicular planes, as the "Double Pretzel." Its "doubleness" is obvious in that a glance reveals two components, which look identical except for the shift in orientation. Tony bases his concept of "double" on the essence of how the knot was produced, and therefore chooses the Square. Never mind that a vertical view of the finished knot seems to reveal the second crossing as a mirrored image rather than a duplication of the first. Duplication may be more subtle in the finished Square knot, but the end appearance is not to be trusted as a sole indicator of what the object embodies and therefore what it *is*. Tony summarizes by pronouncing the Granny "a kind of warped Square." But they are clearly different:

Tony: The Square is so square, and the Granny is so granny.

(Jill laughs)

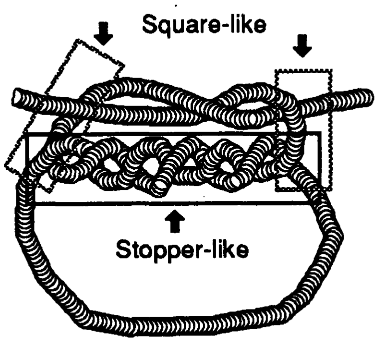
Tony: That's the only way to put it. I'm sorry, I'm sorry. *(He is tying.)*

Carol: *The Square is what?*

Tony: So square. *(He finishes the knot.)*

Alice designates the Square a "grand-pop," named "Pop Square." They discuss the Surgeon's, "Cousin Surgina," as being like a combination of a Stopper and a Square. The wrappings of the Stopper constitute the lower portion of the entanglement, and the blockades match those of the Square:[2]

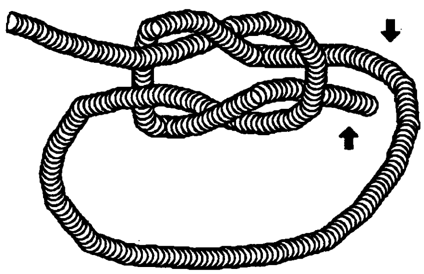
[2] See Section 5.4.1 for explanations of these this terminology.

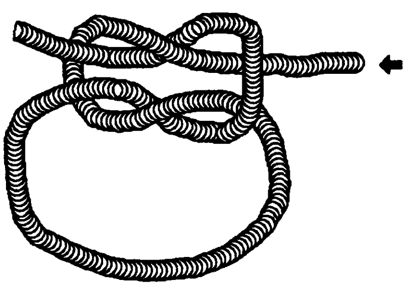


Cousin Surgina's mother is "Auntie Reefie," the last in their family of Squares:


5.4.1

Tony: ...the difference is, this goes in *(exaggerates inward curl of the right end of the Thief)*. On the Square knot it would be like that, without the twist *(holds the right end)*. It would come up – fall that way naturally.





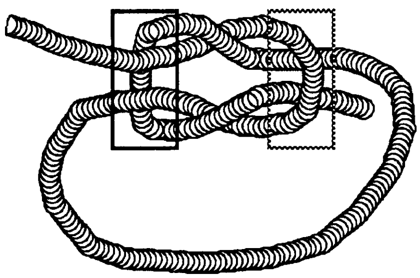
They describe "Mammy Sheet Bend" as a "cross with the Bowline...and the Square family." Tony describes its peculiarity:

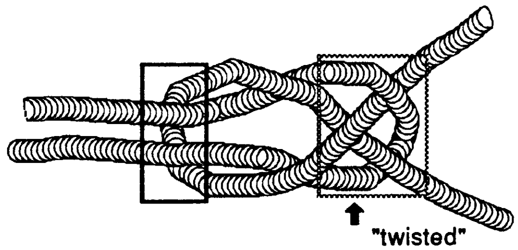
Jill: The Sheet Bend is very odd, and –

Tony: No, it isn't.

Jill: Yes, it is.

Tony: It's like a Square knot, except it's twisted. Well, not except if you see it that way. This is twisted *(shows front and back of Sheet Bend)*. It's twisted that way – twisted – and it's twisted – I can't do it with this string. But it's twisted. And you notice *(climbs up to tree)*, I'm gonna use the [Thief] in this instance, since you know how similar the [Thief] is to the Square. If you notice that *(puts fingers on right side of both the Thief and the Sheet Bend: two threads go smoothly under one side loop of the Thief, but one crosses under the other and goes over the loop in the Sheet Bend)*, you'll see how it's just twisted.





The next branch is named for the Trumpet knot, "Uncle Don Trump" (or "*Don* Trumpet," as they emphasized.) Its horizontal and symmetrical characteristics are similar to those of the True Lovers' knot, which Alice, Jill, and Tony interpret to be somewhat simpler. It tightens more neatly and therefore looks less complicated – and, after all, they are well familiar with the Overhand knots that comprise it:

Jill: And next is the True Lovers' knot *(handling the one on the tree)*, which is a version of two – which is a version of two Pretzels –

Um hmm.

Jill: – on top of each other's string. So one string is like a Pretzel which is over it. Like, one – one string has a Pretzel that has a Pretzel with – that's on it, and then the other one – the same string is over the part – is over the string that did the Pretzel on the other side.

Okay.

Jill: It's kind of hard to explain.

Jill gets stuck in attempting to describe the recursive nature of the True Lovers' knot, but Alice moves the discussion forward by focusing on the knot's property of movement. It takes the form of facial expressions that "Brother Love" can embody:

Carol: (Alice is at the picture.) And we have here Brother Love?

Alice: *(She is adding a brown and a green line at right.)* Yes. ... If you pulled his ears, they would go together like that *(shows her index fingers sliding together horizontally).*

Show me again?

Tony: That's pretty intimate. *(He throws a string onto Alice's head.)*

Show me again, Alice?

Alice: Ouch. If you pulled his ears *(she shows a mock outward pulling motion on the picture)*, um, his brain would go slap! *(She slaps her hands on her cheeks and closes her eyes.)*

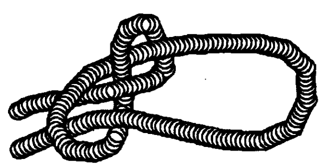
Jill: I'll do it! I'll do it right now, okay? *(Jill demonstrates the movement with the actual knot.)*

(Jill's construction of an exhibit centered around the True Lovers' knot is described in Sections 9.3.2 and 9.3.4.)

They saw the Bowline as a unit of several other knots. One of the knots on its branch was the Packer's, which Tony described:

Tony: Pack – this is the Packer's!

Carol: *Okay.*

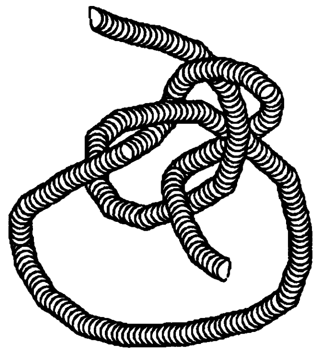


Tony: Definitely the Packer's. *(He holds it.)*

All right.

Tony: It's very similar to the Bowline *(touches the Bowline)*. No it isn't. I mean, yeah, it is.

Alice: Yes, it is.



Tony: Very similar. And it – it's – too manipulated – it's very interesting ***(does circular motions around the knot with his finger).***

How is it different from the Bowline?

Tony: The name.

Yeah...

Tony: And the shape.

What – what's different about the shape?

Tony: Um – well – the shape and what's different about the Bowline is like an eye with things around it ***(makes a circle with his right thumb and index finger around his left index finger)***, and this is more like an eye ***(makes a circle with his left thumb and index finger; begins to put his right index finger into the circle, but instead grabs his left thumb with his right index finger and thumb)*** kind of twisted ***(tosses left hand as if to escape from the right hand's grip and dismiss the problem)***, with a thing around it, but higher up. It's strange. It's kind of like – it's kind of almost like a Bowline with a step added. The Bowline would go around, but this goes up and through ***(makes corresponding motions around the Packer's knot).***

Also on the Bowline branch was the Running Bowline, a seldom-tied knot that became part of another display, on the Knot Language board (see Section 9.1.3). In attempting to tie the knot for the Family Tree, the children ran into difficulty and decided to examine more closely some different ways of producing the knot. They each developed a set of steps with accompanying instructions or keywords. "The Running Bowline gets really complicated-looking really fast," Tony explained. Though by this time they had abandoned the exercise of naming and making portraits of the knots as family members, Alice noted the correspondence between the character of the knot and its name: "it actually runs."

Both production and description of knots for the Family Tree dwindled at this point, although Tony made an attempt to continue discussion of the Bowline branch ("I get it – the Bowline on a Bight is a Double Bowline."). His terminology echoes usages he developed in labelling the Double Overhand and Double Pretzel (see Section 9.4.2). Jill considered adding the Bosun's Chair to the Bowline family, but abandoned it after two unsuccessful attempts. Instead she worked with Alice on the Clove Hitch and Slippery Hitch. Alice grew tired of making pictures, and the project reached its natural conclusion.

9.2 Alice: All at Once

"All at once" describes two important aspects of Alice's thinking:

The term designates a manner of knot-tying that involves leading one end of a piece of string along and around itself, in such a way that the trail made by the rest of the string forms the completed knot. Comparison to either a picture or a tied version of a finished knot is usually an important feature of the technique. I develop the premise that what lies behind Alice's preference for this method is an aesthetic or understanding of *continuity* and a kind of *simultaneity*. These qualities comprise views of both the finished object and the process of forming it, and involve a reluctance to separate the knot into bits or stages of production. Rather, the object is perceived and produced nearly "all at once."

The components of *continuity* and *simultaneity* are also apparent in Alice's way of working in the Knot Lab. In the following sub-sections I describe her way of going from project to project, working on several at once, and bringing discoveries and realizations from one activity to another.

9.2.1 Alice: Profile

Al characterized Alice as "the lowest" of the group of three; this was his sense of her abilities and potential.[1] "She is not as bright as the others, but she will produce," he said. "She gives you what she has. She is a good worker." Al explained that Alice's parents were separated, and her father had lost visitation rights. Al thought that Alice did "get attention," though, from her mother and from her grandmother, a school librarian who took an interest in her grandchildren. Alice has a younger sister who attended the same school.

Most striking about Alice's work with knots was her way of tying as though the configuration resulted from a single, smooth movement. She didn't like breaking down the procedure into steps as most of the other children did. "Flows" might be a better term than "ties" for describing her way of producing knots. Approaching a new knot, she would sit with one that had already been formed, or with a picture of it, study it for a while, and then lead one end of the string around so that it intertwined itself, ultimately forming the knot. Her frequency of accurately producing knots, and the speed with which she learned new knots, were, when compared with those characteristics of her peers' work, remarkable.

Alice didn't like to sit still. She loved taking off her shoes and running from one place to another in the Knot Lab, working on several projects at the same time. She liked to dance. She ate peanut-butter-and-"Fluff" (a marshmallow-based spread) sandwiches for lunch. Her fine blond hair curled wildly in all directions. "Her mother wants her to get a *perm!*" Jill exclaimed. Alice was the only person Jill knew for whom a permanent would be an attempt to *tame* the hair. "*Shhhhh!*" Alice told her. Alice felt very self-conscious about her hair, and on the day when she came into school with the new permanent, she

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[1] Alice was more than a year younger than Jill and Tony (see Section 4.2).

wore her jacket, with the hood up, all day long. At least she liked her new glasses better than the old ones.

When left alone, Alice was prolific, producing work of originality and quality. When pressed, she was likely to make mistakes or answer questions incorrectly. Alice liked working on things, though *completing* a task was for her neither a requirement nor the main pleasure of the work. She would spend a good deal of time on her projects, treating them with care and in detail. She was interested in thinking deeply and in varied ways about things, but wasn't prone to philosophizing in the way that Jill and Tony often were. Alice liked to *do* things.

A typical session in the Knot Lab would find Alice with her shoes off, working on several projects at once, going from one to the other, leaving some evidence of her involvement at various places in the room. She often liked to lie on the floor or on a table, with a book propped up on her knees so she could look at it as she tied. She liked to play, and would hide under a table if I left the room, in order to surprise me when I returned. One day, a wasp flew into the room through an open window and buzzed around the ceiling light. Alice began screaming and jumping up and down; finally, she hid under a table, and her friend Stacy hid with her.

Alice and Tony had lengthy conversations in mock baby-talk. Her enjoyment of this game may somehow have balanced her sense of responsibility as a "big sister." In one day, she both bantered with Tony at length in this manner and went to look after her younger sister, who was crying after having been punched in the stomach. The baby-talk conversations were for Alice and Tony only; they excluded Jill when she tried to play. "Jill, you can't be funny like us," Tony would say. Alice did try to bring Soo Yong into the game, though:

Hello.
Soo Yong Chang?

Bye-byes

Oh!
Where you GO?

I go Bye-byes

Alice seemed to have a crush on Tony, and often tried to get his attention. They sometimes worked together, though he could become irritated with what he seemed to regard as her cavalier attitude.

Although she could have tied any of the knots for the Family Tree, Alice preferred to make drawings of the people they represented. While producing the pictures, she was very involved in the project, asking the others what names to use, what knot to work on next, and so on. She was very much in tune to the overall goal, but wanted to establish her own approach to the project. She made a space for herself by tacking the paper to the board and placing a tied knot next to its picture so she could refer to it in drawing. The facial composition of each portrait carefully reflected the configuration of the represented knot. Although the drawings originally represented crossings as flat, not distinguishing which strand went over or under the other, Alice carefully indicated the "overs" and "unders" in explaining the pictures later.

Alice's participation in the construction of the Family Tree on one rainy day illustrated a dilemma between her desires to work individually and to have some social context for the work. "I can only do this when you're watching," she said to me. "It doesn't work when I do it alone." But when I stood with her and watched her work, she would invariably make mistakes. "It doesn't like me," she complained about the drawing, and stared dismally out of the window.

At another time, in a Knot Lab session toward the end of the project, when children from many of the different groups were working together, a simple but poignant gesture of Alice's illustrated her sense of the community that had developed: When she, Jill, and Tony had begun the String in Motion board, Alice had put on the board a sign with their homeroom number – a statement of authorship and ownership. Later, in the midst of the din and activity of collaboration, she quietly climbed up to the sign and carefully removed it. As the knot-tying community had expanded, the board, and the lab, had come to belong to everyone. Characteristically, Alice initiated an action that expressed a statement for the entire group.

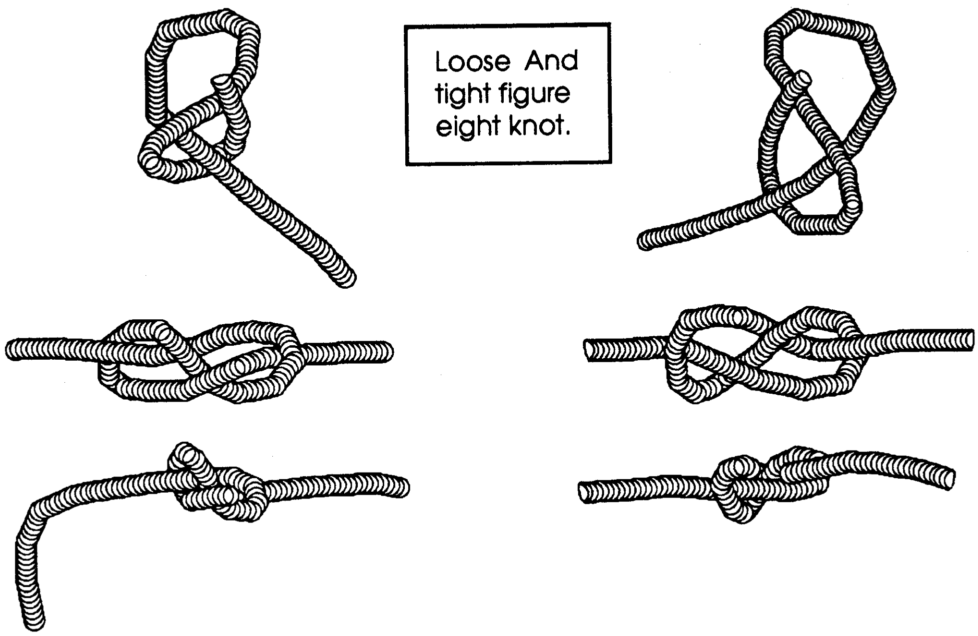
9.2.2 Alice: Projects

In addition to the projects on which she collaborated with Jill and Tony, Alice worked on a number of projects individually. Her "Ring knot" was an extension of her "Pretzel" knot, and a precursor to her understanding of the Stopper:

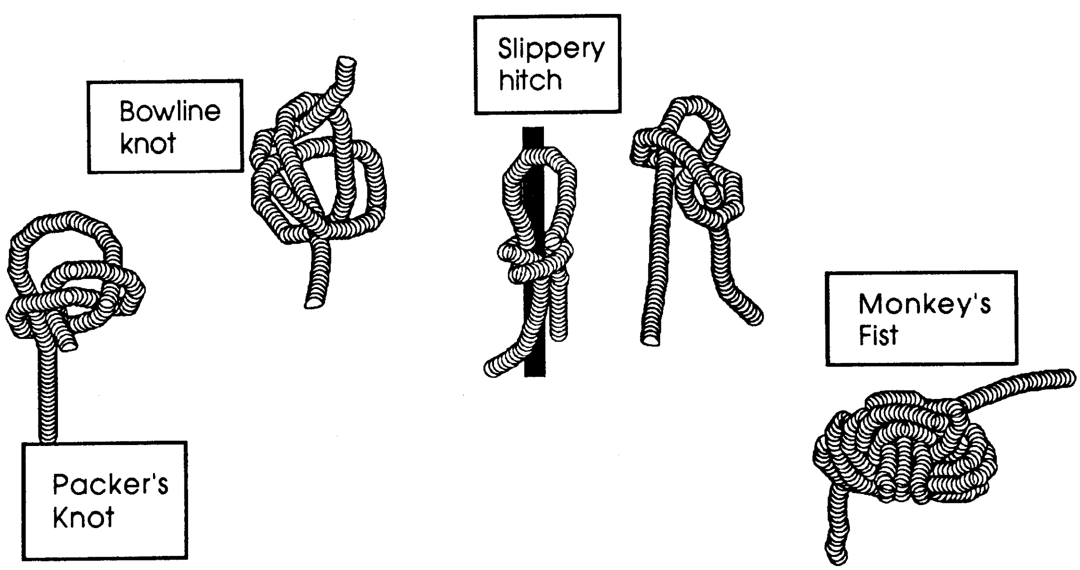
> <u>The Ring Knot</u>
>
> do Pretsel knot, then take the right string and put it behind the Pretsel and pull through hole. Then take left string put it in front of you, pull though hole. The pull both strings lightly.
>
> *Alice*

She quickly extended the "Pretzel" knot to produce the Stopper, and learned the Square knot using one of Tony's as a model. She loosened it, and then replicated the path of one of the ends in her usual manner. Whereas the other children often wanted to follow or develop some procedural instruction as they worked, Alice was especially good at simply tying from a picture of a knot. She liked working independently, moving at her own speed and inclination.

Alice mimicked Jill's method of showing two versions of the Figure 8 knot on the String in Motion board. There was an interesting difference in Alice's emphasis, however: consistent with her "all at once" way of thinking about knots, Alice interpreted not so much a need for serializing steps along the way of producing a knot, as providing different views of the completed knot. Her first "steps" show the knots almost entirely formed – this is the part of the display intended to show how to produce the knots. The subsequent "steps" simply show a change or orientation and a tightened version – should a knot-tyer encounter the Figure 8 in either of these forms, she should be assured that these, too, are the Figure 8:



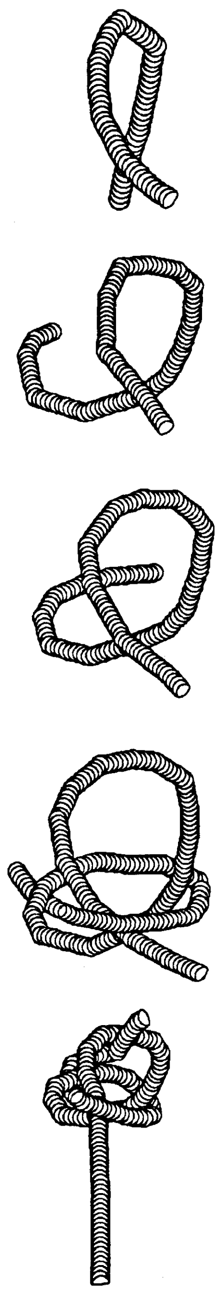
Alice then found a section of the board that became hers, and displayed the Packer's, Bowline, Slippery Hitch (which she showed with and without a supporting stick), and Monkey's Fist:



With her more fluid way of tying, Alice formed the knots in just one step. Serializations didn't seem natural. It was in response to protests from Jill (especially) and Tony that Alice finally broke down the Packer's knot into a series of steps. She did so reluctantly, however, and incorrectly at first. Again at Jill's and Tony's prodding, she began correcting it a few times, but never actually completed the project.

What she displayed is actually a Bowline:

Packer's



Alice was fascinated by both the Packer's and the Bowline, and spent a good deal of time working with each of these knots. As she worked, she wrote some stories:

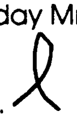
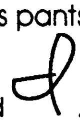
Bowline

One day a little birdy didn't know what to do. So he decided to take a walk around the pond. Then he got board again, so he went in the pond. He got sick and tired, so he went home and got lost. Then he went back to the pond and went swimming. But he got out on the other side and went home. His mommy was calling him home for dinner.

She added pictures in other Bowline stories:

 5.4.3 6.21

Mr. Bird's Day

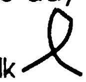
One day Mr. Bird decided to go take a walk around the city pond.  Then he decided to go swimming and he took off his pants and put on his shorts. Then he went in the pond . It started to get dark out and he got out like this . And went home.

"Mrs. Mouse" was a character who evolved through a series of drafts. In the final version, Alice respected Jill's call for specific steps in producing the Bowline:

Mrs. Mouse's day

This is Mrs. Mouses's walking les . This is what she did today. She

Mrs. Mouse's day

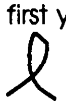
One day Mrs. Mouse was very depressed, so she took a walk .

Mrs. Mouse's Lesson #1

"Hello I am Mrs. Mouse and I want to show you how to walk when you are mad."

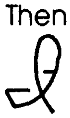
Step #1

first you walk in a kinda circle, like this

.

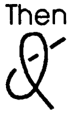
Step #2

Then you walk like this

.

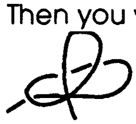
Step #3

Then you take the string and walk under the kinda circle

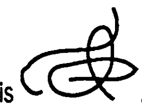
.

Step #4.

Then you walk like this.



Step # 5

Finally you walk like this  .

Thanks for walking with me. Remember follow my lessons and you will never be mad.

Signed

Mrs. Mouse

Mrs. Mouse

Alice sought clarification from Soo Yong on specifics about the Bowline. He commented on a version of the knot that she had sent to him:

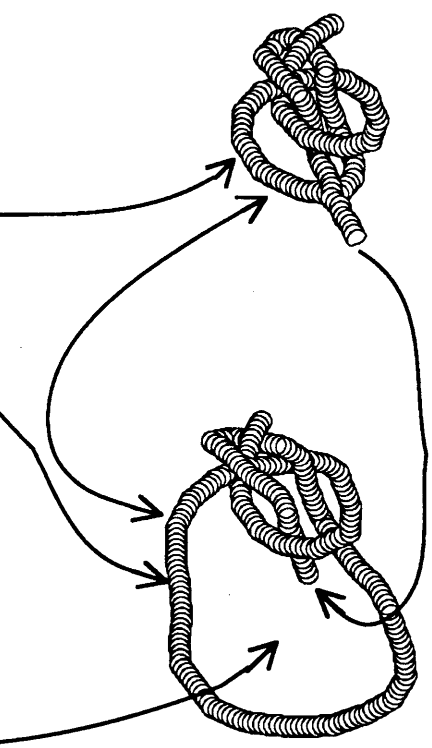
Dear Alice

on bowline this part

must be long

like this

because
this knot is
used for
rescueing people So
they can grab on
to the big loop
on the knot.



PS: the loop
should be
much bigger.

Soo Yong,

Bowline

Why is it used for rescuieng?

A l i c e

P.s. Thanks for telling me

Dear Alice,

Bowline is used for rescue because the big loop on the knot won't close up. so it won't get small or big.

P.S. your welcome

Sincerely,

Soo Yong Chang

5.4.1

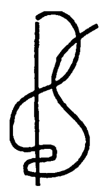
In working with Jill, Alice developed "formulas" for tying the Figure 8 and the Stevedore. While her use of the procedural form represented something of a concession, her choice of language reflected her preference for terms that imply a certain motion: "wrap," "tuck," "twist," (and, to a certain extent, even "loop" and "through") refer not to a relationship of one part of the strand to another, but to a movement that establishes the relationship. Her formula for the Figure 8 is:

make a loop
wrap it around the standing end
tuck it through

For the Stevedore, she advised:

twist the string
take one of the strings, wrap it around the opposite,
put through the loop

Alice decided to illustrate the Stevedore formula. In her later drawings, she tended to clarify the relationships of the strands in the crossings.



first you make a loop and put the string under the straight
line and wrap around string and put thestring under and
through the loop and *Pull* !
Alice

Interestingly, although many children saw the Figure 8 and the Stevedore as being related, grouping the knots together in their final interviews or describing the Stevedore as a "Figure 8 with an extra twist," Alice did not explicitly relate the two knots, nor did she group them together later. Her formulas demonstrate some similarity in her ways of thinking about the two knots, however. Although her Figure 8 formula and her Stopper formula differ slightly in the first step (which is where the greater complexity of the Stevedore is introduced), the remainder of the formulas are quite similar. And in the first step of her elaborated Stevedore formula, the term "loop" again captures the key concept.

Alice enjoyed exchanging ideas and information with others in the project, as these letters demonstrate:

Dear Althea,
I tried your zipper. Mine came out different. Could you
show me how to do it. Your picture didn't work.
Alice

Stacy,
I read your letter in your folder. It was pretty interesting.
Could you teach me it,
Alice

Alice,
Were you talking about the knot I made up, or the
Chinese Staircase? I'll teach you either one. Will you
teach me the Monkey's Fist?
Stacy

Alice and Stacy collaborated in several sessions when Stacy was able to find overlapping meeting times. They worked at tying knots together and developing terms for the Knot Language board.

Dear Soo Yong
Could you show me how to do the Turk's Head
Alice

Dear Alice,
It took me some time to do the Turk's Head. I told Stacy to
try with single line instead of double. Why don't you try it ?
^
too

Sincerely,
Soo Yong Chang

Like most of the knot-tyers, Alice discovered that tightening is a key consideration. Many knots seemed best completed by pulling them firmly and tightening them snugly. But some of the more complicated knots became ruined and unrecognizable when pulled too tight.

9.2.3 Alice: Work History

Salient aspects of Alice's work and thinking are here presented here according to projects that she worked on at about the same time. Projects shown in lighter gray occurred earlier in the study, and projects shown in darker gray occurred later. The presentation is intended to suggest not only her way of combining projects and working on several at the same time, but the range and interplay of factors involved in learning about knots: using understandings of simple knots to form more complex ones, communicating with friends, finding similar details in different knots, becoming frustrated with a project, and so on.

designates "loops" as an important criterion for the Thief knot

discusses the Square and Thief in terms of "two strings"

engages in a debate with Jill and Tony about the meanings of "over" and "under"

writes to Soo Yong asking "What makes a loop?"

includes 'twist' as a key concept in tying the Running Bowline

attends a workshop at M.I.T. with Doreen, Maria, Stacy, and Soo Yong

writes to Soo Yong telling him about progress on the String in Motion board and asking him to teach new knots

writes stories about the Packer's and Bowline knots

begins to debug the Packer's knot

makes hands chart with Jill

converses in baby talk with Tony

sings songs from the school play with Jill and Tony but engages in baby talk only with Tony

develops the "n-1" formula for the Stopper with Jill (see Section 9.1)

writes to Soo Yong asking about the Bowline

at lunch, agrees with Jill that she, Jill, and Tony will know each other years from now

her "Pretzel" and "Ring" knots lead to the Stopper

makes tying formulas for the Figure 8 and Stevedore knots

ties a Square using Tony's as a model

makes a turtle tail for Tony's school play costume

tightens the Turk's Head

Soo Yong writes a letter explaining that the Turk's Head is harder to learn than the Monkey's Fist because "it's complicated" and hard to hold

ties the Stevedore

does a macramé "Chinese Staircase" with Tony and Stacy

writes to Althea and Stacy

declares herself to be an "expert at Pretzel knot"

makes portraits for the Family Tree, talking in baby talk

misses Tony when he doesn't arrive and loses interest in doing pictures for the Family Tree

9.2.4 Alice: Interpretation

Almost from the start, Alice's way of thinking about how to tie knots incorporated a strategy that many children discovered much later, and some never seemed to appreciate. Rather than thinking of a knot as an intertwinement of the two ends of a piece of string, Alice preferred to think of it as a kind of trace of a single moving end. This sense of how to form the knot led her to think of the final product more as an embodiment of a single, fluid motion than as the result of a series of steps. She liked to tie the knot "all at once."

Alice seemed to approach a lot of things in this way. Making her sit still probably wouldn't have prevented the tendency from happening – even with her body sitting in one spot, her mind would still be jumping from idea to idea, elaborating some and leaving others unfurthered. Her language for describing knots was replete with a lively sense of movement: where Jill and Tony made detailed procedural statements, as in their instructions for the Running Bowline (see Section 9.1.3), Alice used single words that captured, for her, both the idea of a configuration and the movement that produced it.

The metaphor of "foraging" could capture Alice's presence in the Knot Lab – her way of going from one place to another, sampling this and that. And Papert's phrase, "logic on tap, not on top," might describe her relative flamboyance despite her ability to muster precise technique when a situation called for it. But I have settled on the "all at once" designation for its capability of describing both Alice's "here, there, everywhere" tendency and her peculiar manner of tying knots as though they resulted from a single gesture.

9.3 Jill: Step by Step

"Step by Step" is a designation of Jill's way of organizing work and ideas. In learning to tie a knot, she wanted to know where to start and where to go next, and she wanted verifications along the way that she was proceeding correctly. Thus it was in her interest to find or develop procedural instructions, and to declare points of interruption – checkpoints – for producing various knots. Her concern with the sequence of steps that produces a knot became the paradigm for displays on the String in Motion board, and although Alice and Tony did not strictly adhere to it, Jill was consistent and precise in her dedication to this approach. Her concern was with *seriation,* the order in which movements and processes should be executed (and displayed) in order to ensure that a knot produced would be the knot desired. Such ordering provided a way of knowing "where she was" at given moments within a complicated process. What became especially interesting to Jill were knots that seemed to resist being pinned down in this way – knots such as the Running Bowline and the True Lovers' knot, which in their completed form are capable of (and, in fact, are designed for) some kind of movement.

She dwelled in particular on the True Lovers' knot, making a special exhibit that allowed others to play with the "back-and-forth" movement of the finished knot. Jill's fascination with the dual nature of the knot, its way of embodying two states even after the steps of its formation were completed, is here discussed in terms of her sense of the transience – a certain *lack* of security and order – in her life situation.

9.3.1 Jill: Profile

Al described Jill as "good, and conscientious, but not as bright as Tony. She is not top – top – top, but gives you her potential." Jill's parents are divorced, and, as Al explained, she and her younger brother went "back and forth between them." Jill and her brother often arrived early at school, and on such days she would help Al in the classroom, preparing for the day.

Jill tended to be serious, and didn't play around as much in the Knot Lab as Alice and Tony did. She was neat and polite. She liked to sit close, touch, and talk at length about the knots she worked on. She also liked continual assurance and feedback about her work, which was careful and deliberate. It seemed more important to Jill than to any of the other knot-tyers to designate clear anchor points for the string as she learned to tie new knots. She accomplished this by literally stapling or taping down parts of the string along the way to producing a knot, and by developing clear labels for parts of knots and tying procedures.

Some of Jill's concerns are what led to development of the String in Motion board. She found looking at a finished picture of a knot insufficient as a guide for tying – it left too many questions unanswered in the course of arriving at some rendition that would be comparable to the picture. She wanted verification at several points along the way that she was "on the right track," and set out to provide for herself and others a reliable, step-by-step reference. The knots that she displayed on the board were precise and well thought through. She spent a good deal of time deciding which in-betweens to use and tended to opt for more and more particularity in the steps she illustrated. She was very articulate and liked explaining the steps and individual crossings in detail.

Jill didn't like leaving something unfinished. She stayed with a project until it was done, and convinced others to do the same. She also didn't like to skip steps; she wanted the sense of accuracy and assuredness that careful progression from one detail to the next would provide. Perhaps because of this contrast in their approaches, Jill sometimes became impatient with Alice.

Like Tony, Jill was concerned with "performance" – that is, with my surmise of who were the "best" knot-tyers. She often asked about who were the "quickest in getting" knots, her concerns relating both to her working trio and to all the participants in the project. But her sense of competitiveness was balanced by a certain pleasure in collaboration and in being a member of a productive group. At one point, she noted that the Running Bowline would make a good motion exhibit, as had the True Lovers' knot. Tony picked up the suggestion, saying that he could do it, but Jill then became concerned that Alice would be the only one who hadn't done such an exhibit. Someone would be left out of an important activity, and that wouldn't be fair.

In several of our lunchtime sessions, Jill told Alice, Tony, and myself about her family. She and her brother lived during half of the week with their mother and half of the week with their father. There was some tension in the communication between their parents, and Jill was troubled by it. One day she decided not to finish her sandwich after having taken just a few nibbles, and explained how she came to have that particular sandwich in her lunch. It was a scenario that was not uncommon, apparently, and one that I think illustrates some underlying dynamics that found their way into Jill's work with knots:

She had stayed with her father the previous night. In the morning they rushed so that he could take Jill and her brother to school and still be on time for work. Unlike their mother, who normally prepared appealing and balanced lunches for them, their father tended not to be so organized. Sometimes he would give them money to buy a school lunch, but Jill had learned from her mother that a homemade lunch, which typically included a sandwich, was preferable. She felt better when her father provided her with a sandwich. So often, as on this day, they would make a quick stop on the way to school – he would pull into the parking lot of a 24-hour convenience store, hand Jill some money, and she would run in and scout out the pre-packaged ingredients of lunch for her brother and herself. In addition to potato chips and some kind of dessert, she ended up with a vending-machine style sandwich, which not only lacked flavor, but contained chemicals such as preservatives that Jill would rather have avoided. Through conversations with her mother, she had become aware of issues regarding nutrition and processed foods. (In another lunchtime discussion, she announced that she had decided to stop eating sugar after June 16th, the date of her graduation from fifth grade.) Jill was upset about discarding the sandwich, because, as much as she didn't want to eat it, she felt that wasting food was wrong. (She was in a classic "double-bind," both in terms of being doomed to doing something wrong no matter *what* she chose to do, and of feeling torn between decisions that somehow represented one parent or the other. (Bateson 1972, 271-78)) Alice, Tony, and I talked about ways in which she could avoid such a situation in the future, by finding things in the store that would be tasty, healthy, *and* convenient – she could buy a zip-open can of tuna fish, for example, along with a bagel and a piece of fruit. As the suggestions continued and the knot-tyers shared their lunches with her, Jill came to feel better.

Another lunchtime conversation presented yet another dilemma involving Jill's family: Tony had a new wristwatch, which included a calculator and a small memory. He wanted to store Alice's and Jill's phone numbers. Alice's was straightforward, but Jill explained that he would not only have to store two phone numbers for her, one for her father's house and one for her mother's, but also the days on which he could reach her at either number. As she continued her explanation, it turned out that those days were not as predictable as might be conveniently encoded and stored in his wristwatch. Tony finally became so confused and frustrated that he abandoned the project.

Jill's preoccupation with her family situation is discussed further in connection with her work on the True Lovers' knot (see Sections 9.3.2 and 9.3.4).

9.3.2 Jill: Projects

In addition to the projects on which she collaborated with Alice and Tony, Jill worked on a number of projects individually. She began by exploring the "Underhand" knot. This name is not common, but it stood to reason that if there was an Overhand knot, there must be an Underhand as well, and Jill set out to explore them both (see Section 9.1.1). These ways of producing simple knots led her to an interest in *loops,* which she found elaborated by the Figure 8 and Stevedore knots (see Sections 5.4.1 and 9.1.3). She tied the Figure 8 with little problem, but the Stevedore presented some difficulty. It looked very similar, yet the slightly increased complexity caused by the extra loop was enough to render the knot problematically confusing.

Jill spent a long time on the Stevedore, working from the picture in the *Boy Scout Handbook.* She became uneasy and frustrated when different stages that she discerned in her own tying were nowhere to be found in the pictorial reference – only one picture of the completed knot, still loose, was included. How could she be sure she was intertwining the string correctly as she worked toward the final knot? Even her finished knot, which she carefully tightened, looked different from the loose one pictured. Jill's concerns were paralleled by general discussion among many of the children during the first few weeks of the project, about the themes of "looseness" and "tightness."

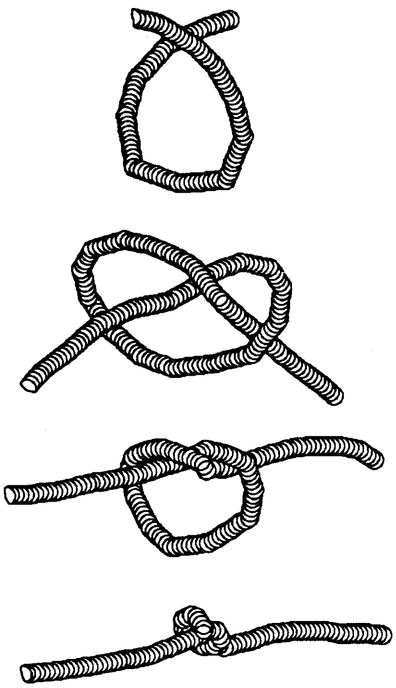
As Jill worked on the Square knot, her reservations about the pictures in the *Boy Scout Handbook* became even more pronounced. After vacillating between tries that produced a Square knot and tries that didn't, Jill had a revelation, which she formulated in a way that few children mustered. That is, she came to understand that you can start a Square knot *either* by doing "right over left" or "left over right," and the way in which you do the second crossing depends on the way in which you did the first. Jill had come to a realization that the invariance of the relationship between the two crossings is the essence of the knot.

In spending so much time with the Square knot, she had become well familiar with the "Square knot test" (see Section 5.2) as a way of checking to see whether the knot she had tied was correctly formed. While this act presented for Jill the same sort of pleasures that other knot-tyers enjoyed, it also presented yet another concern about predominant techniques in the Knot Lab. Jill was troubled by the fact that, if stiff enough cord is used, *many* knots, even a Granny, will respond in a satisfying way to the Square knot test. Thus the test wasn't sufficiently reliable for her sensibility.

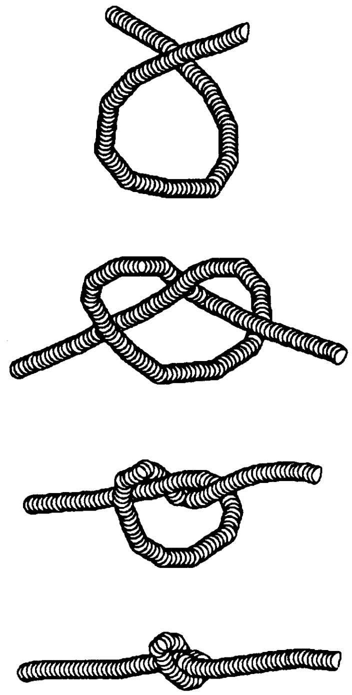
Jill's concerns led her to propose a solution that would provide more security as she tied. She wanted to show interim stages of a knot in formation, so that knot-tyers would have not just one, but several check-points – *reliable* gauges of whether they were "on the right track" in producing the knot. One of the bulletin boards in the room provided an inviting surface for displaying the different steps. Blank except for the word "motion," the board gradually became a domain in which pieces of string were curved and twisted into the forms of different knots. With Alice and Tony, Jill spent weeks stapling and taping serializations of knots on what became known as the String in Motion board.

She began with the "Pretzel" knot, acknowledging her sense that the mirror images, which she called "right" and "left" knots, are somehow different (see Section 9.1.2, footnote). Although she had often used the term "Underhand" along with "Overhand" in discussions of this knot (see Section 9.1.1), Jill settled here on the less ambiguous "right" and "left" designations, along with the customary term, "Overhand":

6.17 9.1.2

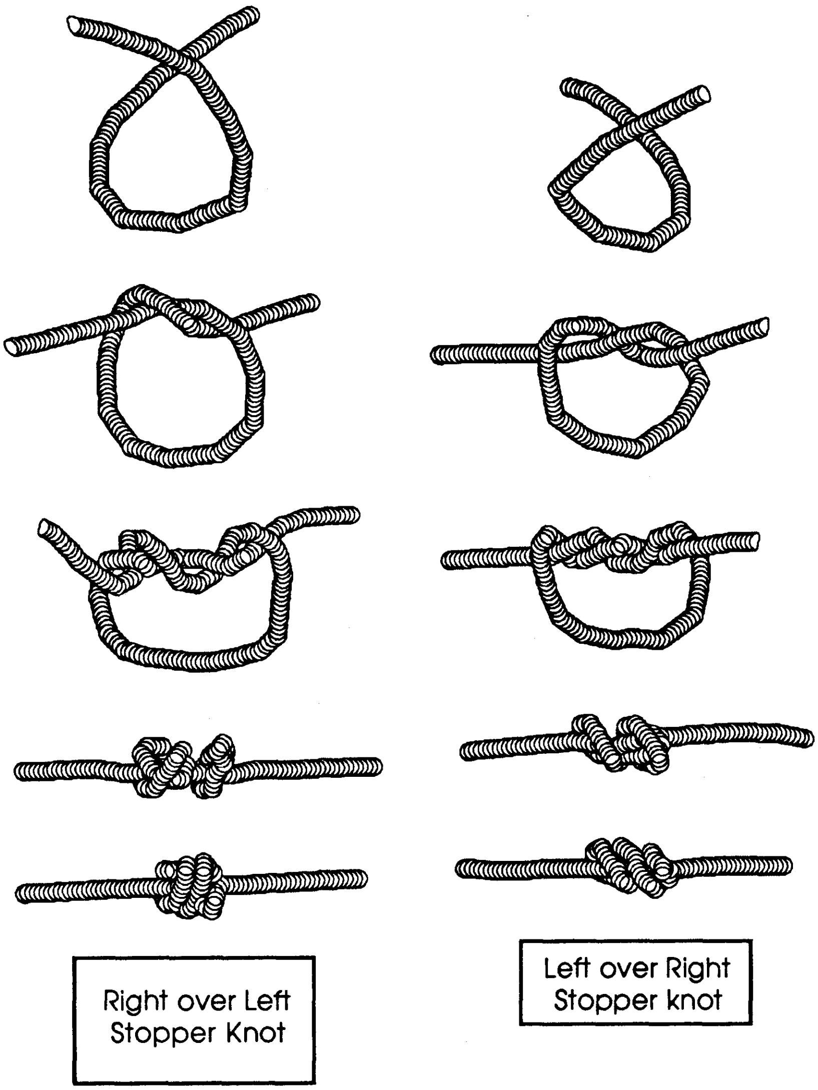


Right over left
Overhand (pretzel)

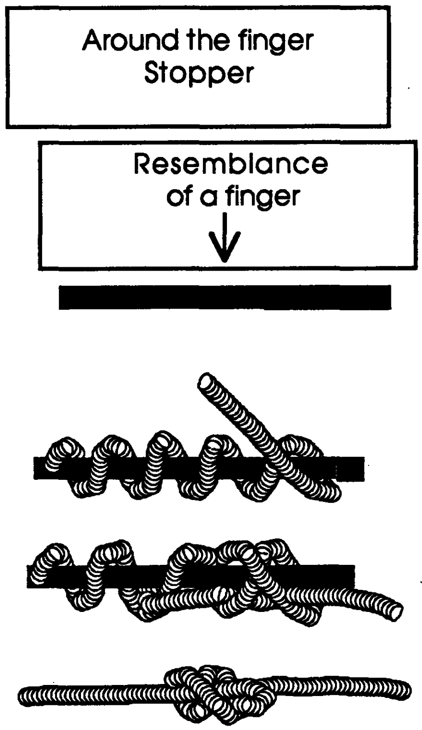


Left over Right
Overhand (pretzel)

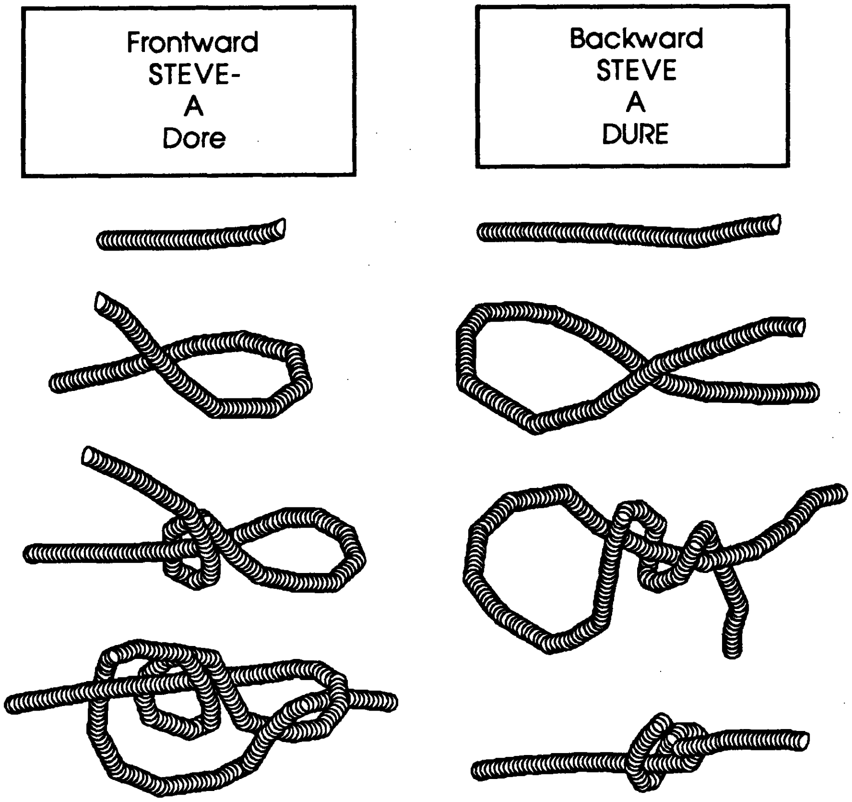
Her Stoppers were done in the same manner:



Later, she did another version of the Stopper:



Jill did an entire line of Stevedores before deciding they were wrong and taking them down. She worked carefully on getting correct renditions of the knot, using the window as a light table in order to see the reverse image of the knot through the flip side of the sheet on which it was printed. (The rendering of the knot in the *Boy Scout Handbook* is such that she was still able to discern details in the crossings.) The technique led to her serialization of the "backward" version of the knot:

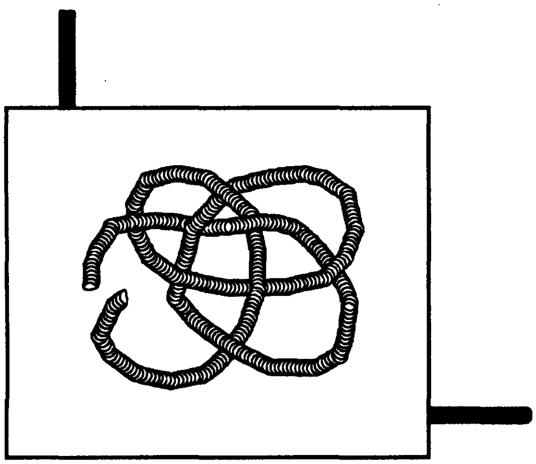


Jill had come to think in terms of the knots as being built up from components: the Stevedore, for example, can be seen as a Figure 8 with an extra loop. Her first attempt to tie the Bosun's Chair resulted, essentially, in a Figure 8 and a Bowline on the same string – this version had an "extra twist." In her attempt to compensate, the next version she produced had not enough twists. She described the Running Bowline in similar terms: "See, it's a Bowline with an extra loop." And, explaining a bug in one of Alice's knots, Jill said, "It wasn't a Bowline. It's an unidentified Figure 8."

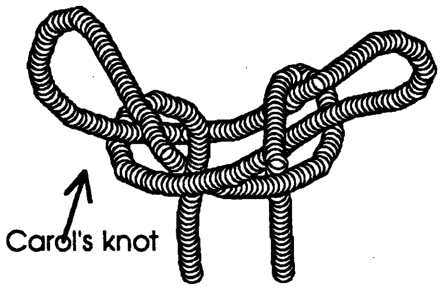
Jill managed to finish tying a Turk's Head, but had some trouble tightening it. She pulled the two ends hard as she would have with simpler knots, of course making it too tight and ruining its distinctive cloverleaf shape. After some discussion and experimentation, she came to understand that this knot requires working back along the path that formed the knot, using many careful, separate motions rather than one final tug.

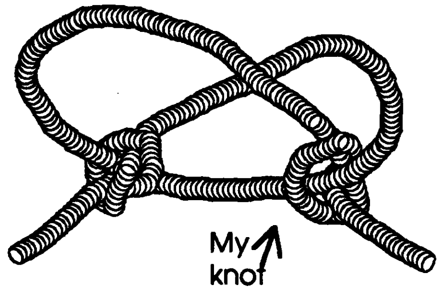
6.13

Jill used the unique orientation strategy of taping two sticks to a piece of cardboard, to which she also taped new parts of the knot as she formed them, rotating the board as she worked her way around in developing the curvilinear shape. "It kept getting turned around," she explained about the board. Consequently, her concept of the knot got turned around, too, but the sticks helped to fix her conception of where it began, regardless of the position of the board:



She carefully taped some other knots to a letter to Soo Yong:





Dear Soo Yong,

I'm tring to do the trumpet knot. Carol's knot is probably the right way, but I'm not sure.

Signed

Jill

P.S.

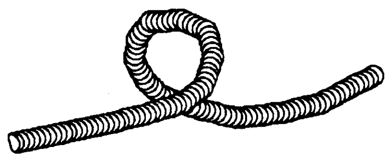
Will you please send us written letters untill we're done with the bulleten board.

Jill's misconception had to do with thinking in terms of the "Pretzel" knot that she and her peers had talked so much about. She saw the Trumpet knot, roughly, as two Overhand ("Pretzel") knots at the base of two loops. In trying to re-create that configuration, she developed yet another pretzel shape, construed as the main body of the knot. Soo Yong's instructions and hand-drawn illustrations take a completely different view of the knot:

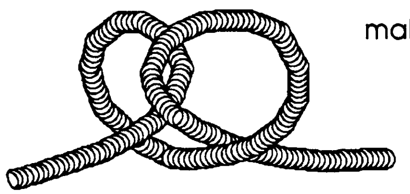
Dear, Jill

I saw both of your and Carol's trumpet knot and Carol's knot is right. So since you didn't get it I'll show you how to tie it step by step to make you an expert on trumpet knot as someone says.

step 1. make a loop



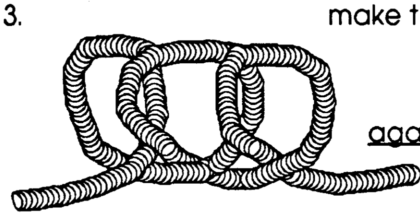
2.



make second loop

watch the crossing.

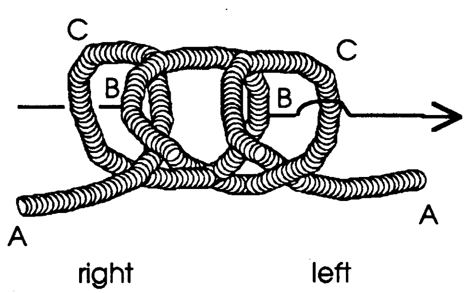
3.



make third loop

again watch the crossing.

4.



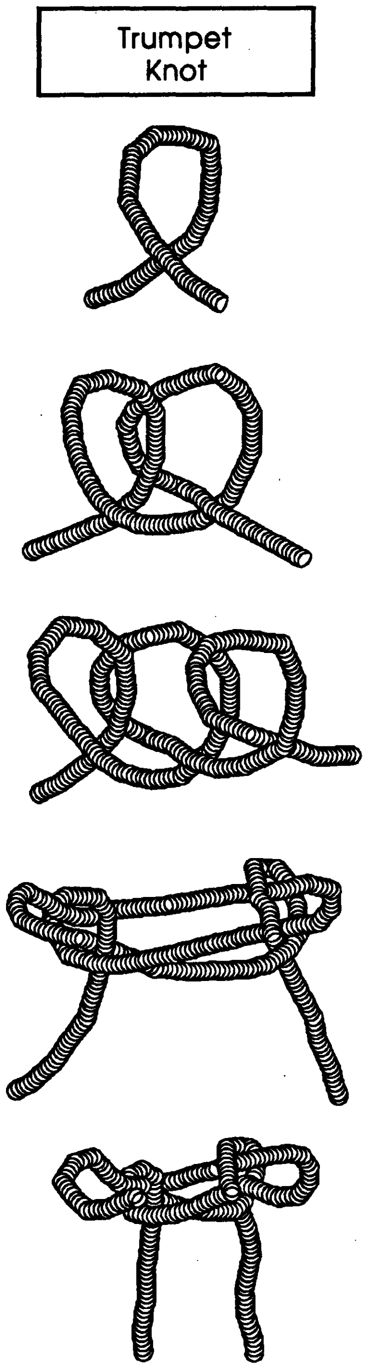
hold A with your 2 pinkies. pull Left B front of Left C and right B behind right C with thumb and index finger

tighten both As with your pinkies and don't let go on Bs.

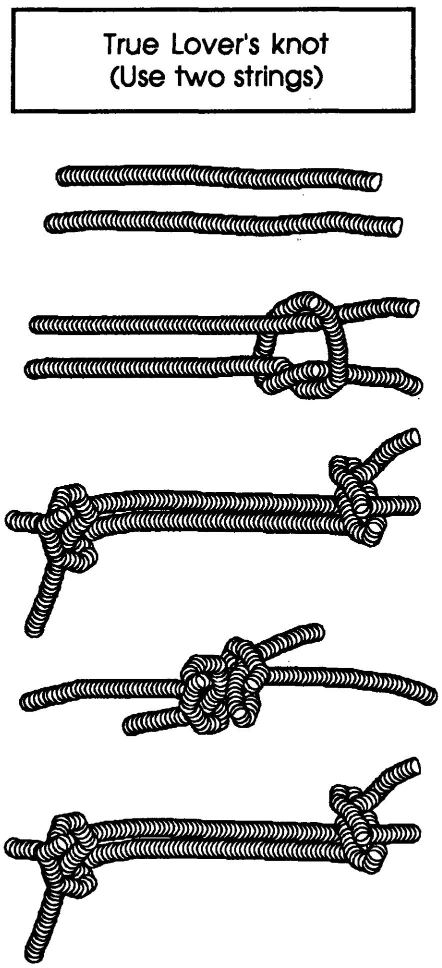
Sincerely

Soo Yong Chang

Jill did use Soo Yong's instructions and successfully produced a Trumpet knot, which she displayed on the String in Motion board:



She then moved to another double-component knot, which actually *was* comprised of two Overhands: the True Lovers' knot. This knot became very important to her. She used it to decorate her folder of correspondence and work, and made a display for the String in Motion board:



Jill also made a museum-style exhibit in which the knot was suspended from a pipe on the ceiling, in such a way that passers-by could pull it, playing with the knot's movable property. At first Jill hung the knot very high and centered above the String in Motion board; she had climbed up to assemble it. It partially obscured the view of the board, though, and was too high to reach easily, so Jill and I decided to hang it on a ceiling pipe to the left of the board, using a long suspension string to make the display low enough that people could maneuver it. She used a Square knot to anchor the support string to the pipe; a Bowline at the end of this string held one of the two strings comprising the True Lovers' knot. Jill was excited about all three knots, and wanted to make a "museum label" that would call attention to each of them. Making this sign contributed to her construction of the True Lovers' knot: she noted the problem of how to identify what one should do in order to maneuver it. After a lengthy conversation, she decided to make the strings different colors, and spent a long while with a marker, meticulously coloring the absorbent string. The frayed edges of the string were another problem to be taken seriously. Jill dabbed glue on the ends, and held them in twists until they dried. Then she touched them up with color again. It was a tedious process that she repeated several times, until she felt she had it right.

Jill's involvement with the material in these ways is important: her careful modification of the string and her climbing to position the knots are integral parts of her construction of the display, demonstrating the earnestness with which she approached the project. She thought it was important for others to have access to this knot – but obviously, it was important for her too.

Having distinguished the two strings by making them different colors, Jill was still faced with the problem that there were four ends that needed to be included in her descriptions of the knot, yet for any given move, two of the ends were involved. She needed to differentiate sets of two ends, and decided to use two X-shaped, plastic LEGO pieces that she found in the room – "helicopter things," as she called them. They became not only designators, but convenient handles as well.

As Jill continued developing the label for the exhibit, she commented, "You know what I like about this project? We're *writing* as well as learning about knots." She recalled a label that she had seen at an art museum, near a chair provided for weary gallery visitors: "You know how at the Museum of Fine Arts they have a chair with a label that says, 'Please sit in me.'?" Jill decided to imitate this idea, and toyed with different phrasings of her final instruction: "Please play with me." "Please pull me." She settled on the latter, and her label was complete:

 6.10 9.3.4

True Lovers Knot

At the very top (on the black pipe) Notice the "square knot" to hold it in place. The knot holding on to the Lovers Knot (True Lovers knot) is the "Bowline". Notice the way the strings are two colors. It is that way so it is easyer for you to pull it.

To pull take the two strings with the black lego pieces Pull hard until the two pretzel knots meet. The pull hard the two string without anything on them. Repeat if wish.

Please Pull me

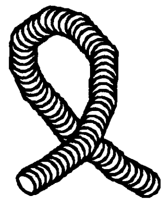
Jill created a story about the Heaving Line knot, a knot that no one else in the Knot Lab had attempted. Her dedication to developing step-by-step procedures led her to specify parenthetically that, in fact, her instructions take the form of a story:

6.21 6.22.6

The Heaving Line Knot

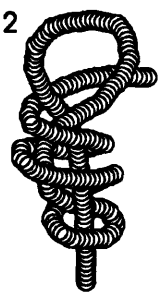
(A story)

1.



There was once a faucet out in a yard. (The man who owned the house that the yard was in name was Mac.) Mac was gardening and watering his garden. When his was done he wrap the hose around the faucet (four times) Then he put the end through the faucet. Pull

2



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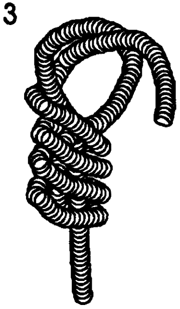

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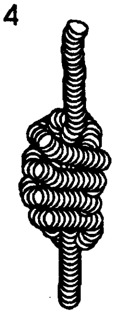

By

Jill Bachman

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But story or not, there was to be no getting away from the reality that Jill's purpose in developing such a document was to assist her reader in tying a knot – and, in Jill's view, one must have instructions to tie knots, or at least know what steps are necessary. As Mac finishes with the hose, Jill abandons the metaphor and instructs the reader/tyer to "pull." The end. Interestingly, she does not qualify any special way in which one should "pull," though she did have some trouble tightening the Heaving Line knot. An over-zealous tug can render this knot as an unrecognizable blob.

Once again, Jill made her illustrations in string and glued them to the paper containing the story. Her numbering of the illustrations gives the tyer some choices in producing the knot: she can rely on the verbal sequence, perhaps assisted by the illustrations, or simply follow the illustrations as examples.

9.3.3 Jill: Work History

Salient aspects of Jill's work and thinking are here placed in a "step-by-step," roughly chronological progression. The progression is intended to suggest the range and interplay of factors involved in learning about knots: combining understandings of simple knots to form more complex ones, communicating with friends, finding similar details in different knots, becoming frustrated with a project, and so on.

invents the "Underhand" knot
examines the loops in the "Underhand," Figure 8, and Surgeon's
understands that the Square knot can start with either end, and that the second crossing depends on the first
becomes concerned with degrees of looseness and tightness
ties the Figure 8 easily but has trouble with the Stevedore
develops the "n-1" formula for the Stopper with Alice (see Section 9.1)
decides her line of Stevedores is wrong and removes them from the String in Motion board
makes "frontward" and "backward" Stevedores, using the window as a light table
likes explaining crossings
ties by carefully anchoring a starting point, and then key points along the way
complains that is is sometimes hard to distinguish "overs" and "unders" in drawings
explains the difference between the Square and the Granny, and then the Thief
finishes the Stevedore and likes the idea of doing a language board as a new project
writes to Soo Yong about the Trumpet knot
engages in a debate with Alice and Tony about the meanings of "over" and "under"
makes hands chart with Alice

makes the True Lovers' knot and suspends it from the ceiling as a display
decides she won't eat sugar anymore after graduation
makes a sign for the True Lovers' display and modifies the knot accordingly
wants to know who is quickest at "getting knots"
creates the Heaving Line knot story
sings songs from the school play with Alice and Tony, but when she tries to talk in baby talk with them, Tony says, "Jill, you can't be funny like us."
works on tightening the Heaving Line knot
wonders if a Figure 8 and a Bowline together would form a Bosun's Chair
the twists on the Bosun's Chair are confusing: she has too many or not enough
at lunch, agrees with Alice that she, Alice, and Tony will know each other years from now
thinks of the Running Bowline as a Bowline with an extra loop
confused about whether to give her mother's or father's phone number to Tony
declares that Alice's mysterious knot is "not a Bowline. It's an unidentified Figure 8."
imagines that the Running Bowline would be a good motion exhibit
makes her Turk's Head too tight and has trouble tying it; develops an orientation device for the Turk's Head

6.3.4 Jill: Interpretation

Jill's involvement with knots was marked by her insistence on articulating as much as possible about the ones she worked on, in as much detail as she could muster and as others would tolerate, and finding ways to demonstrate or illustrate what she deemed as important aspects of the knots. Of prime importance was keeping track of the steps involved in tying a knot. She tended to think in terms of the involvement of two moving ends, and wanted to anchor their intermediary configurations on the way to producing the final knot, both as a way of coming to understand the knot and as a record for later reference. Even for knots that were most naturally tied by moving only one of the ends, Jill sought this anchoring of intermediary steps.

"Step by step" describes not only Jill's approach to tying individual knots, but her way of working in the Knot Lab. She wanted to work on a project carefully and diligently until it was done – a project at a time, step by step along the way to developing both a showcase and an environment conducive to working with knots.

Given her urge to fix the positions of several intermediary steps for any given knot, it is curious that the knot Jill seemed to find the most fascinating was one whose state of completion involves motion, resistance to being fixed, a changing of places of the two components of the knot. The True Lovers' knot must have held many attractions for Jill. ("What a curious name for a knot!" she may have wondered. "What does 'true love' mean? If lovers part, do they always come back together again?") The label that Jill made to accompany her display of the True Lovers' knot signals possible reasons for her fascination:

6.10 **9.3.2**

<u>True Lovers Knot</u>

At the very top (on the black pipe) Notice the "square knot" to hold it in place. The knot holding on to the Lovers Knot (True Lovers knot) is the "Bowline". Notice the way the strings are two colors. It is that way so it is easyer for you to pull it.

To pull take the two strings with the black lego pieces Pull hard until the two pretzel knots meet. The pull hard the two string without anything on them. Repeat if wish.

Please Pull me

Jill's final phrase is poignant. It signals her involvement with the display, and her identification with the knot. It is like an echo of another voice in Jill's mind, which wants to say:

> *Notice how I am suspended by two knots, one that anchors me and one that holds me. Notice how I <u>am</u> two knots, waiting to be pulled this way and that. I understand being pulled; it is something that I know. Allowing others to pull me is a purpose that I serve.*

This is a voice that enjoys a different time range, in a given moment apparently stifled by the deliberateness and focus of attention on a task at hand. Yet, over time, the voice has a way of poking through the presence of everyday events. It makes itself known through discourse and action. Hearing this voice requires a kind of listening that is patient, and combines watching, acting, *and* listening. It is not on the frequency of the hurried.

In many ways, throughout the course of the project, Jill had been saying something like this. Her stories of concern for her brother, confusion about her parents, and dissatisfaction with outcomes in her own life were manifested in her search for security in the checkpoints that her step-by-step approach to knot-tying proffered. Pinning down different steps can be a metaphor for attempting to answer the question, "Where am I?," a question Jill had reason to ask herself frequently. Jill's knot steps can be seen as a form of commitment to a situation – a compensation, perhaps, for the breach[1] she experienced through her parents' separation. Jill, like the knot, embodied two positions. She felt suspended by her parents' whim and pulled in different directions – literally, as she moved from home to home each week, and spiritually, as she attempted to understand the discord of two people she loved.

In constructing the True Lovers' knot, it seems that Jill had found a not only way of expressing her "pulled this way and that" situation, but of putting herself in the position, finally, of one who can also do the pulling.

[1] Webster's (1975) gives a tertiary definition of "breach" as "a temporary gap in continuity." The contrast with Alice's metaphorical insistence on continuity is compelling.

9.4 Tony: Relatives and Mutations

"Relatives and mutations" describes concerns that emerged as Tony learned about knots. In the course of lengthy sessions in which he experimented with string, he wiggled his way to an understanding of several knots (most notably, the Square knot) by tying scores of renditions of it, observing and commenting on how each version echoed or departed from the chosen model. Tony did not adhere to any particular knot-tying technique – he was willing to try anything. The result was not only a collection of well-formed knots, but many knots that in some sense *should* have been related, as they stemmed from the same conceptual or physical starting points, but which didn't quite "make it" as recognizable finished objects.

Tony's concern was thus with *classification*, wondering how the relationships he perceived within a given knot were manifested also in other knots. As he worked on the Square knot, he deemed as "relatives" those products for which he could find some documented counterpart, and "mutations" those products that looked particularly odd or were solely the result of his "mucking around," not found to be corroborated anywhere. The Family Tree of knots was an extension of his "relatives" idea to larger groups of configurations.

"Relatives and mutations," as a designator of Tony's style, captures both a way of thinking about knots and a way of thinking about himself. Along with his work with knots, his uncertain sense of "belonging" to aspects of his world are described in the following sub-sections.

9.4.1 Tony: Profile

> You know, Carol, I'm glad we're doing this project. It's really increased my awareness of knots.
>
> – Tony Hayes

Al considered Tony to be "very talented, very bright ... a typical 'advanced work child.'" Tony has a younger brother and two older sisters. According to Al, and consistent with Tony's own stories, Tony didn't "get too much attention at home." His parents were busy professionals: the father was, according to Al, a "leading brain surgeon," and the mother was ordained as an Episcopal priest during the course of the knot project. The family was originally from Canada and had moved to this country within Tony's memory. He became eleven years old during the course of the project.

Tony was outspoken, good-natured, and self-conscious. He liked to fantasize, hum, or otherwise play as he worked. Teasing his two partners and making puns based on the project ("most definitely not") were par in any given session. Though he formed solid friendships in school and with his siblings at home, Tony thought of himself as an oddball. He was overweight and reluctantly attended sessions with the school's speech therapist. He referred to himself as a nerd and a weirdo, someone whom other people would just as soon do without. He asked on several occasions whether I wasn't "getting tired of these kids." Tony seemed to see as his only saving grace the fact that he was "smart," a belief for which he was constantly seeking reassurance.

Tony was well aware of the lab's dual function as both a research and learning environment. He seemed to hone his self-observational skills, which were already acute, while learning about knots. He also had an unusual awareness of my roles as researcher and facilitator. To his queries about my "studying" the children, I replied that I was studying the knots, too, and myself, too, and hoped that they were studying themselves as well. He seemed intrigued by this idea and spent an unusual amount of time experimenting with "loud thinking."

Tony liked to think about the knots as he went along, figuring out how one part should relate to another in the course of tying. Compared with some of the other children's ways of tying, his approach was haphazard and likely to generate strange configurations as easily as desired ones. Messing about with string was the process by which he strengthened his understanding of different knots, often generating new ones along the way. At one point, when two working groups came together, several people attempted some of the more complicated, decorative knots. After some concerted effort, Tony produced one of them and showed it to a friend. "Look," he said, "I did the Carrick Bend." "How?" Patrick asked. "I played," came the reply.

For Tony, taking a knot apart was as frequent an activity as tying. Undoing a knot was an explorative process, as it helped him to see how the parts of the string related in configuration. But it was also a process of erasure: "I like to destroy," he declared as he undid a knot that had turned out differently from what he wanted. It seemed better to claim control over a desired process – destruction – than to admit that he had gotten the knot wrong. Tony tended to wish that his rate of correctly producing knots was better.

But although he – even more so than Jill – was very concerned with doing well, Tony was also able to forget such concerns and enjoy exploring new ways of intertwining string to produce knots that he liked. He was selective about his medium, choosing particular kinds of string for different tying projects. "Knot-tying is really manipulating thread," he observed.

Tony was willing to take risks and often enjoyed working independently. At times, though, some simple reassurance helped him to continue a strategy he had already begun.

Serializing the knots for the String in Motion board, for example, was fraught with distractions, as the knot-tyers scrambled between steps to find more string, staples, and so on. Tony sometimes found it difficult, upon completion of a certain step, to regain his position in the knot so that he could visualize the next step. He would get frustrated, but would calm down and respond well to my simple suggestions – like, "Start with the same step as was already up, and carry it a little further." Knot-tying has also to do with manipulating time, and that takes some getting used to.

At one point Tony expressed worry that he hadn't learned as many knots as his two partners; I pointed out that, instead, he had gone deeply into one configuration, producing some interesting and pleasing results. Tony knew the word "topology," and used it in describing his exploration of the Square knot. By producing different knots (deliberately and accidentally) and varying familiar knots, he saw similarities that led him to think of certain knots as being related. His observations spawned the Family Tree display (Section 9.1.4), which greatly influenced others' thinking and activity in the Knot Lab.

In spite of Tony's depth of understanding of the Square knot and relationships among different versions of it, he felt inadequate. He was was not alone in this sense of his accomplishments, though, as this letter from Patrick indicates:

> Dear TONY,
>
> I heard that you were in The Knot class. I know how to do all the Wimpy Knots. Like the Square knot and the Figure of Eight Knot. I see that you come up for Recess and Work on the Board And do all the Major Knots.
>
> Sincerely,
> Patrick Gilmor

Tony was slightly embarrassed by the letter, especially since the only knot he felt he had truly mastered was the Square knot, which Patrick had characterized as "wimpy." Tony did not respond in writing, but collaborated with Patrick in a session when their two working groups came together. Patrick industriously rolled dozens of Stoppers to use as modules in building up an animal. Tony decided to help him, and at one point put the large rope on the floor and made a huge Stopper, which everyone pulled to tighten. Tony asked, "Patrick, do you agree that the Stopper is in the Pretzel family?" Patrick allowed that it is. Tony declared that Patrick knew the Stopper "intimately." "There is an ongoing relationship between Patrick and the Stopper," he said. By the end of this session, there were Stopper knots all over the room. Tony had made a Stopper choker that he wore around his neck, and Alice sported a Stopper ring that he had given her.

In a previous session, Tony had made a necklace for his mother. It consisted of a Turk's Head, which he thought resembled a cross. On this occasion as well as others, he spoke about his parents. His mother had put his father through medical school, and he had put her through theology school. On Tony's birthday, there was a party – but it wasn't for him. It was to celebrate his mother's ordination, a landmark event within their family and their community. Tony felt sad and forgotten, although he also seemed to understand the importance of the event and felt proud of his mother.

Toward the end of the school year, there was a lot of excitement among these fifth-graders about their upcoming graduation. It turned out that Tony's father, who traveled frequently, would be away and would not be able to attend the graduation. Tony told the story in a downtrodden manner, mumbling, "Thanks a lot, Dad. I really appreciate it." His father's absence also meant that Tony was unable to ask him about the Surgeon's knot before the end of the study – another disappointment.

9.4.2 Tony: Projects

In addition to the displays on which he collaborated with Jill and Alice, Tony worked on a number of projects individually. Early in the study, he had described the "Double Overhand" and "Double Pretzel" knots, using a designation that appeared again later in his declaration, "I get it – the Bowline on a Bight is a Double Bowline." Although what he meant by the term "double" varied, the tendency that gave rise to his use of these names was his way of seeking some essential property in understanding a knot. "Double" in each case signified a repetition of some fundamental aspect of the knot: his "Double Overhand" was formed by repeating a wrapping motion (see Section 9.1.1); his "Double Pretzel" repeated both crossing and wrapping motions – at first he said it was a Granny;[1] and his "Double Bowline" referred to the doubling of the line that enables production of the knot.

Tony's involvement with the Square knot was so intense and prolonged that it warrants a detailed description:

Like most of the project's participants, Tony quickly learned to tie the Granny knot, but took a while learning to distinguish it from the Square knot and to tie either at will. Even after tying the Square knot correctly in several successive meetings, he would get confused, tying it but then losing it in the next attempt. Or he would correctly identify one that someone else had tied, explain the difference between it and a Granny, and then falter anyway as he again attempted to tie a Square knot.

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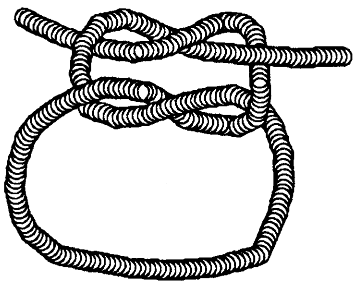
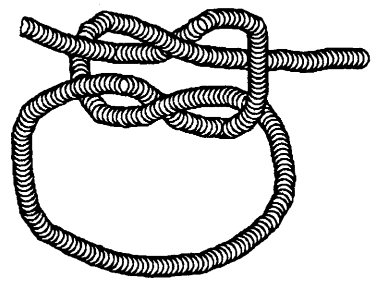
[1] But later, he described the Square knot as the "Double Pretzel" (see Section 9.1.4).

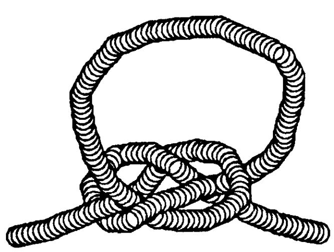


Granny



Square

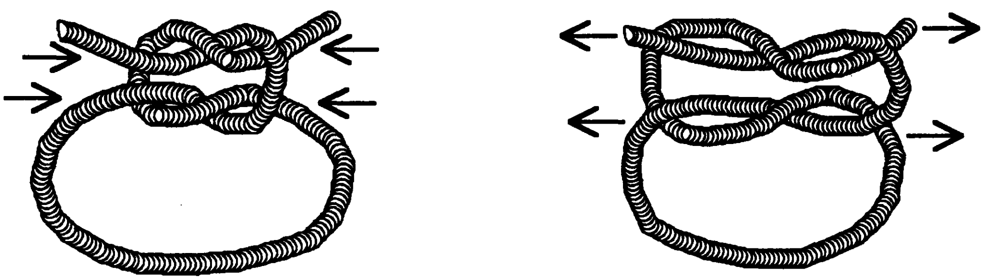
At first, the slightest change from a neatly tied Square knot was enough to prevent him from recognizing it again. In particular, changing the size of the circle often distracted him. For a while, Tony was unable to recognize the Square if its circle was very small relative to the knot. Similarly, if any part of it the actual knot was stretched out of the usual proportion, he had trouble identifying the knot. But gradually, as he saw it in more and more situations, understanding that the knot remained the same even if it was somehow twisted, he began to play at creating his own deformations. He especially loved flipping over the circle to make the knot look different:



As he tried to tie the knot, he would often successfully produce it, but then continue to manipulate the string so that the knot, still intact, would become deformed. He would fear that he had lost it until I intervened, holding his knot but not changing it in any way: "Is this the same knot as the one you tied?" "Hmm...I don't know. I guess so. Yes."

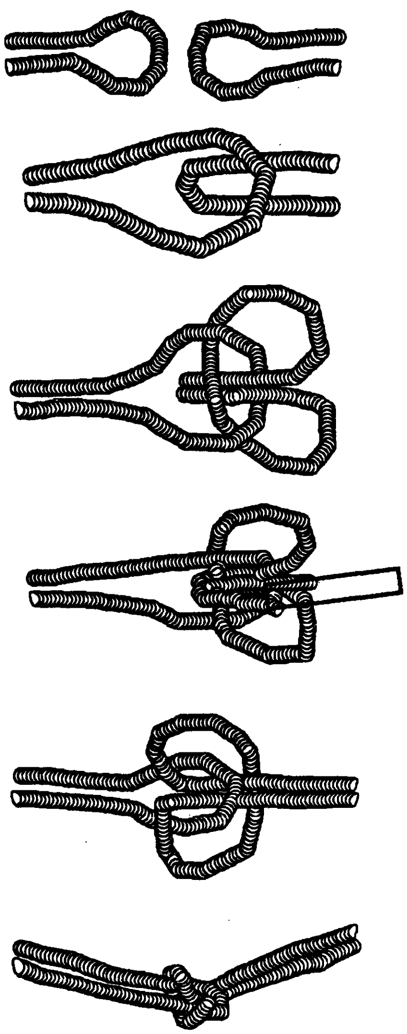
His sense of satisfaction in finally coming to a firm understanding of the Square knot was compounded by his delight in doing the "Square knot test":

5.2 6.8 7.2.9



Beyond the sweet-and-sour thrill of tightening the knot – (Is it a correctly tied Square, or not?) – the test was a final step that, for Tony, expressed and even celebrated the essence of the knot: what appears to be a complicated set of turns and crossings is revealed by the test to be, simply, two interlocking loops. Manipulating the knot through this "test" gave Tony a new view of the knot: whereas he had learned from Soo Yong a way of tying the Square that built up the knot vertically, he now began looking at the finished knot more horizontally. This new view was his own discovery. It omitted the circle, which he had found so distracting, and enabled Tony to develop a different way of tying the knot, which he called the "Easy Square Knot" and displayed on the String in Motion board:

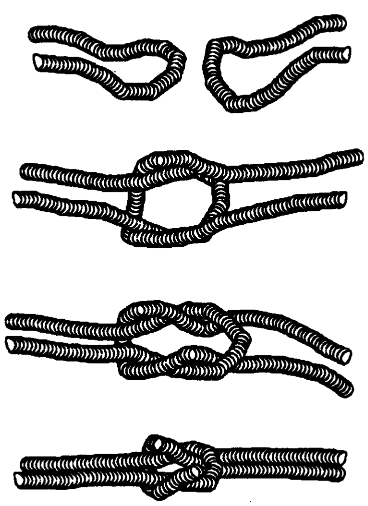
Easy Square Knot



He expressed the knot as two loops, with which he formed the Square knot in six steps. First, the tips of the two loops face each other. The right loop slides under the left, and then the two ends of the right loop move leftward, going over the left loop and under their own right loop. These ends then reverse direction. By pulling them hard to the right, the tip of the right loop flops to the right, so it is no longer facing the left loop. Tony found it difficult to represent the movement that produces this result. The configuration seems to change so markedly as a result of that single pull, that showing the resulting configuration as the next step would seem to be a mistake. It is not obvious how to get from the third step to what he has shown as the fifth. So he settled on an in-between step that demonstrates the motion of pulling: the two ends are held in mid-air by a piece of clear tape as they begin their rightward movement. Getting from the fifth to the final step is a proof left to the tyer, so to speak. It takes some fumbling, but an inadvertent leftward slip of the right loop produces the finished Square knot.

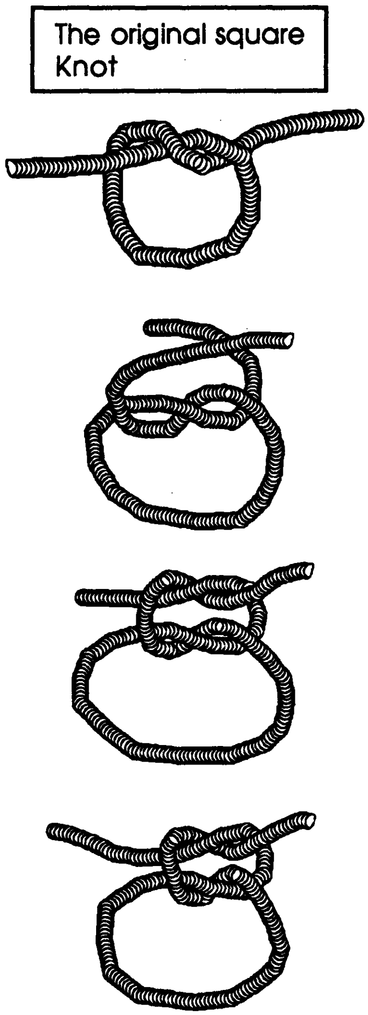
It was probably the process of articulating the "Easy Square" knot that led Tony to realize what he called the "Super-Easy Square" knot:

Super-Easy Square Knots



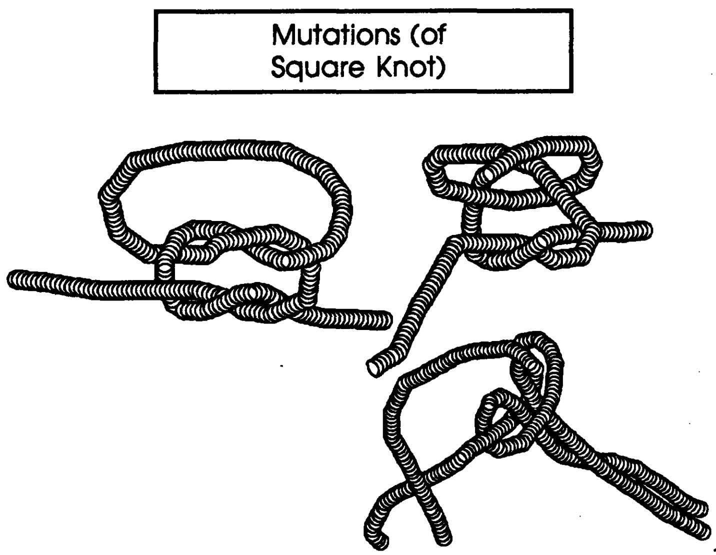
The tips of two loops again face each other, but the right loop slides into the left and the two ends of the right loop are tucked through the left loop. A Square knot results!

Discussion of whether he could produce the Square knot in this way if he were using only one string instead of two led to his display of the "Original Square" knot:



When reminded of this way of building the knot vertically, he said, "Yichkkk!" He preferred the two-loop method that he had developed, liking both the aesthetic of his alternative and, probably, that it *was* an alternative, one that he discovered.

The kind of fumbling that produced Tony's "Easy Square" knot was typical of his way of exploring different configurations. Other participants were less adventurous, choosing to follow pictures or other guides as they attempted new knots. Though Tony's method did produce some innovative results for which he received admiration, it more often resulted in unrecognizable topologies of which he seemed ashamed – they were mistakes, accidents to be untied, hidden, or discarded. Yet these deviations usually bridged his understanding to a desired result. We discussed the value of these strange forms, and Tony finally decided to put some of them on the board as well:



One of his "Mutations" (top left) is a Granny knot, upside-down relative to the customary view. The knot next to it is also an upside-down Granny, with the top loop re-formed to take on the shape of a "Pretzel." The lower knot is a tighter version of the fifth step of his "Easy Square" knot, flipped so that is seen from the other side.

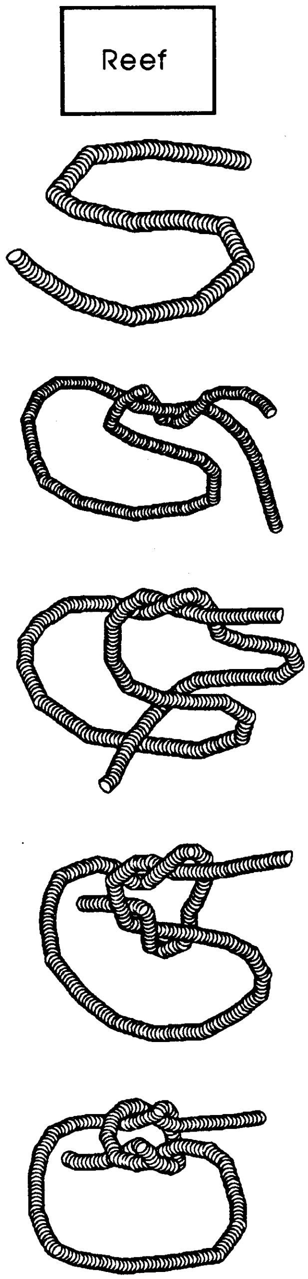
By this time, Tony had become so immersed in the topology of the Square knot that we had dubbed him an "expert" on it. He wanted to continue with it, and decided to try the Thief knot (which he called the "Reef") that seen earlier as the project's "Mystery" knot.[2]

---

[2] This name is unfortunate; "Reef" is actually another common name for the Square knot. The best names for the new knot would have been "Thief knot" or "Bread knot," in reference to a story about how proverbial ship

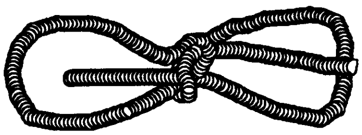
It was a challenge for Tony, as he had not yet attempted to tie it. With discussion and some coaching, from myself and his teammates, he developed a way of winding around a single string, which is distinctly different from his previous approaches to Square-like knots. His "Easy" and "Super-Easy Square" knots had made use of two separate strings, and even though the "Original Square" knot was tied with one string, he thought of tying it as the intertwinement of two active ends.

captains used the knot to secure the lid on the ship's bread box. Easily mistaken for a Square knot, the Bread knot was a good indicator of whether someone had been pilfering the bread: thieves would most likely re-close the box with a Square knot. But the Thief knot was originally introduced to participants in our project as the "Mystery" knot, and later, the name "Reef" became associated with it.



The single active end that could produce a Thief knot was revolutionary for Tony, and as was his wont, he continued exploring the idea. One of the outcomes was his "Square Knot with a Line through It," which looks like a bow tie or the knot in a shoelace, but is actually a variation of a Figure 8 knot:

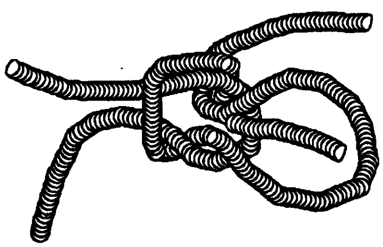
Square Knot
with a line
through it.



Tony's name indicates a return to his preferred conception of Square knots as being comprised of loops – the ends form a line that adorns but is kept separate from the knot itself, which sits comfortably between two loops.

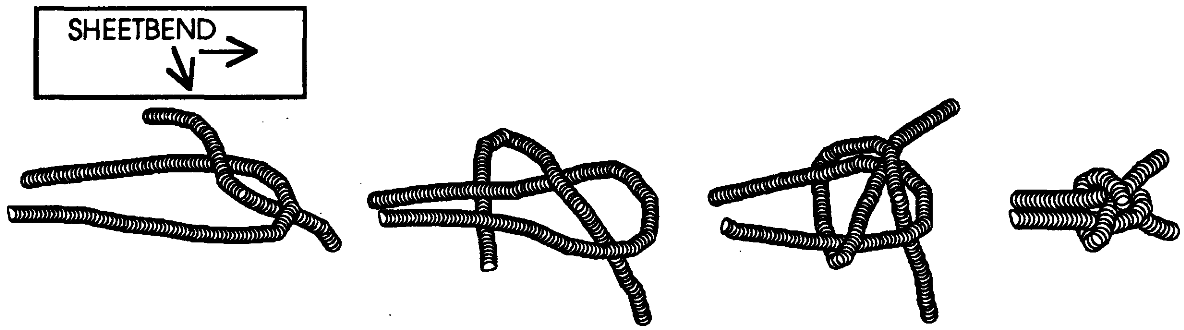
Returning to loops as his starting point, Tony embodied an arrangement similar to the "Square Knot with a Line through It": his "Pseudo-Square" knot also has a loop which seems to be an integral part of the knot, yet minor manipulation reveals the configuration to be nothing more than an Overhand tied around another string:

Pseudo-Square
Knot
Don't be deceived!



Tony accompanied this knot by a cautionary note: "Don't be deceived!" He was no longer fooled or startled by such oddities, and wanted to prevent others from being confused as well.

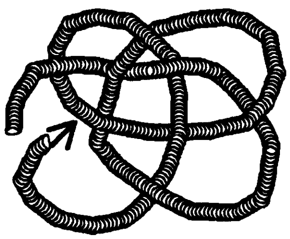
Tony discovered another variation of the Square knot in the Sheet Bend. It begins as he liked, with two strings, but represents an assortment of techniques that he had developed so far: one of the strings takes the form of a loop – but a passive one – and the other string winds around it in order to produce the knot:



Tony had had some difficulty in coming to understand this knot, but eventually finished it by tucking the active string under itself, rather than simply exiting the loop as in a Square knot. He declared, "The Sheet Bend is a Square knot crossed."

As Tony began working with the Bowline, he noticed a similarity to the Sheet Bend, but could not say exactly why they seemed alike. He came close by mentioning the distinction between one- and two-line knots, offering his "Square Knot with a Line through It" as an example of a knot similar to a Bowline in its use of just one string. He furthered the comparison in a discussion with Jill and Alice about the relationship between these knots on the Family Tree (see Section 9.1.4).

Many of the participants in the project had become intrigued by the Turk's Head, a complicated, decorative knot. Part of its fascination had to do with a booklet that gave instructions for constructing animals from different knots. (Grainger 1987) It showed the Turk's Head as the shell of a tortoise, an animal several of the participants, including Tony, wanted to make. Starting with a loop, the knot is produced by a long series of over and under moves, in which keeping track of the current location is imperative. The result resembles a cloverleaf:

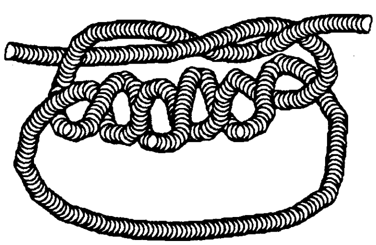


To make the tortoise shell, the instructions required not one, but two or even three complete rounds of such intertwining. Even for those who correctly finished the first round, figuring out where next to put the end in order to begin the second round was a common stumbling point, one which caused Tony some trouble.

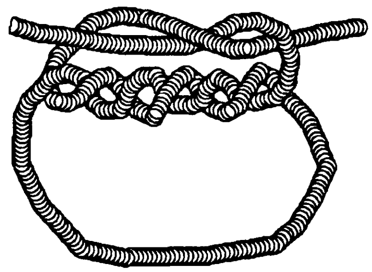
Another common difficulty with the Turk's Head was tightening it, after any number of rounds. This process becomes a matter of rectifying the uneven lengths of string that inevitably result as one creates the loops of the cloverleaf. Pulling too far on one loop can easily damage the shape of a neighboring loop. Achieving a pleasing balance requires a lot of patience. What becomes clear after many attempts is the reverse nature of the tightening process: ideally, the knot is tightened by retracing the path along which the knot was tied. And for this decorative knot, the most pleasing result is in leaving the knot slightly, but uniformly, loose. After thinking of "tightening" merely as giving two ends a satisfying tug, the multi-dimensional process required for this knot becomes difficult indeed.

Tony stayed with this knot and gradually managed to produce a "single-round" one. The ends were long enough that he was able to tie it on his wrist, and he proudly wore it as a bracelet. His mother, newly ordained as a priest, had complimented him on this creation, noting the similarity of the knot's shape to that of a cross. The following week, he decided to give the bracelet to his friend Alice, and to make a bigger Turk's Head as a necklace for his mother. This required several more attempts, stumbling through difficulties in tying and tightening the knot.

Tony next tried the Surgeon's knot, but had some trouble with it until coming to see it as a combination of a Stopper and a Square, both of which he had tied. He could tell when he produced a Granny-style Surgeon's instead of a conventional Square-style:



Granny-like Surgeon's



Square-like Surgeon's

He pointed out that his father was a surgeon, and decided to ask him about the knot.

Tony had complained several times that members of one of the other working groups declared themselves to be "Knot Experts" too easily. After constructing even one display of a single knot, several of them announced their expertise. Tony took the term "expert" far more seriously. Even after his intense involvement with the Square knot, work for which others were readily willing to designate him an expert, Tony wondered whether he was deserving of the title. He still had many questions and uncertainties about the knots he had worked on, and he thought the superlative should be reserved until he had a comfortably thorough understanding of the knot and its variations. His hesitation was evident in a letter that he wrote to Soo Yong:

Soo Yong,
Today we have done more on our board and Alice wrote 2 stories about bowlines. I made a pseudo square knot. I am an expert in square knots (I hope!).

Tony Hayes

Tony's relentless exploration of a given configuration led him to appreciate that a knot remains the same regardless of how it is twisted, folded, or contorted. He gave himself the task of viewing a knot he wanted to know in as many situations as possible. He used the term "intimacy" quite a bit. Gradually, it seemed, the domain of knots had become one in which he felt he could experiment with the concept.

9.4.3 Tony: Work History

Salient aspects of Tony's work and thinking are here presented according to the knots he worked on, his ways of working with others, and his expressions of ways of thinking about himself during the course of the project. The presentation is intended to suggest not only Tony's concern with classification, but the range and interplay of factors involved in learning about knots: combining understandings of simple knots to form more complex ones, communicating with friends, finding similar details in different knots, becoming frustrated with a project, and so on.

"Rather Simple Knots"

works on the "Pretzel" and extends ideas from it to the "Double Overhand" & Stopper

works from pictures of the Figure 8 and Stevedore

does a Stopper, but stumbles through the Surgeon's

doesn't want to do Hitches for the Family Tree

helps Patrick with many Stoppers; makes himself a Stopper choker

Squares & Relatives

falters over Square even though has done it well many times; gets the knot but then deforms it: "Is it the same? – no – yes."

calls odd knots "mutations"

uses tape to make a 3-D effect on the String in Motion board

displays mutations & the "Pseudo-Square" knot

displays the Thief knot

has trouble with Sheet Bend

develops a formula for the Sheet Bend

snips the Bowline on the Family Tree: "Now it's a Sheet Bend ... glue it."

renders a Square knot on the String in Motion board as two interlocked black plastic loops

More Complex Knots

wonders whether he should wrap the string toward or away from himself (as opposed to some other referent)

loops with "over" and demonstrates an interest in the rope as material

encounters "pull-too-tight" bug: the Trumpet flips back on itself; the Turk's Head loses form

makes a successful Turk's Head and uses it as a necklace for his mom

does a Carrick Bend and shows it to Leroy, saying, "I played."

"Knot-tying is really manipulating thread."

"I get it – the Bowline on a Bight is a 'Double Bowline.'"

compares his Running Bowline to the one in the sailing book (Altimiras 1984)

tries the Runing Bowline on a flat surface; finishes it as I help to staple earlier steps to the board; about one, he chants in baby talk, "To some it looks retarded ... "

Collaborations

engages in a debate with Alice and Jill about the meanings of "over" and "under"

makes a display of

"under" crossings & the philosophical question, "What is the difference?"

does a macramé "Chinese Staircase" with Alice and Stacy

wants to know if he, Alice, and Jill are the smartest in their class, and the best knot-tyers

notes that he hasn't done as many knots as Alice and Jill, and wants to know if that is okay

sings songs from the school play with Alice and Jill, but when she tries to talk in baby talk with them, he says, "Jill, you can't be funny like us."

plays with dissecting one kind of string, comparing its inner material to fabric softener and toilet tissue; with friends, does a lot of pulling and stretching of the material

Classifying Himself

begins carefully finding & using specific terms as a way to remember the knot

talks of himself as a "weirdo"; calls himself a "nerd"

says he "likes to destroy"

wants to know, "Was my Turk's Head the best?"

talks about his father traveling and that he will be away when graduation happens: "Thanks a lot, Dad."

talks about his parents putting each other through school

stops bringing in sandwiches; eats very little for lunch and announces that he is on a diet learns the Packer's from Alice's

"The Running Bowline gets really complicated-looking really fast."

puts the big rope on the floor and makes a huge Stopper, which several children pull to tighten

talks with Patrick about the Stopper: "There is an ongoing relationship between Patrick and the Stopper."

at lunch, disagrees that he, Alice, and Jill will know each other years from now

wants to store Alice's and Jill's telephone numbers in his watch

we make shoulder straps for his turtle costume for the school play

Marcos and Julio put thumb-tack eyes on the string turtle on their "Welcome to the Knot Lab" sign, to "make it Tony"

9.4.4 Tony: Interpretation

> But freaks should not be played down! They are the *real* refutations: they cannot be fitted into a pattern of continuous 'generalizations,' and may actually force us to revolutionise our theoretical framework ...
>
> – (Lakatos 1976, 96)

As Tony came to be familiar with more and more knots, he had a way of seeing them as being composed of one another. Commonality of anatomy or tying process became the basis on which he perceived certain knots as being "related." He expressed the Surgeon's as a combination of a Stopper and a Square knot, for example, and declared that "the Sheet Bend is a Square knot crossed," and "the Bowline on a Bight is a double Bowline." He also described the Packer's knot as being "like a Bowline with a step added":


5.4.2 10.1

Tony: Um – well – the shape and what's different about the Bowline is like an eye with things around it ***(makes a circle with his right thumb and index finger around his left index finger)***, and this is more like an eye ***(makes a circle with his left thumb and index finger; begins to put his right index finger into the circle, but instead grabs his left thumb with his right index finger and thumb)*** kind of twisted ***(tosses left hand as if to escape from the right hand's grip and dismiss the problem)***, with a thing around it, but higher up. It's strange. It's kind of like – it's kind of almost like a Bowline with a step added. The Bowline would go around, but this goes up and through ***(makes corresponding motions around the Packer's knot)***.

Tony's typically detailed gesticulations added to his explanation of the knots and demonstrate a way of identifying with them: both mind and body were actively involved in thinking about the knot. His fingers assumed the roles and movements of different parts of the knots – encircling one another, interrupting or going through one another, in various ways acting out the intertwinements and finding ways to exteriorize the topological relationships his mind was trying to apprehend. Even when Tony was not so demonstrative, a similar process was often at work, as when he explained that he had come to understand the Thief knot by using a "mental finger" to follow the path of the knot in his head.

6.16 10.1

***Carol:** How did you tie it?*

Tony: I – don't really know, I just – looked at the picture and I tried to follow my string, the string which I was currently using.

How did you follow around?

(He holds the picture so we can both see it, and points to the inner, leftmost end of the pictured Bowline.) **I used – a mental finger.** ***(He taps his head with his finger, then returns his finger to the picture and begins to follow the end as it curves around into the knot.)***

Tony's way of identifying with knots becomes poignant in light of expressions about his deflated view of himself, and the time and attention he devoted to the deformations, variations, and so-called "mutations" that he produced while experimenting with tying different knots.

Tony was concerned that his long and thorough struggle to understand the Square knot detracted from time he might have spent learning other knots, and that what he considered to be his resultant lack of productivity would be construed as some kind of deficiency in performance. He made continual queries about whether he, Jill, and Alice were the best knot-tyers, and I stressed the idea that people do things differently – some are faster, some take their time and get to know a knot really well. ("Like me!" Tony chimed in. Interestingly, his part in the school play was that of the legendary tortoise, slow but steady, who won the race.) At another time, as Tony finished a serialization and I helped to staple it to the board, he chanted in his baby talk, "To some it looks retarded ..."

("... but it's mine and it is what it is," he seemed to imply.)

Tony's game of baby talk with Alice demonstrated a desire to have a special closeness with her, but also to experiment with ways of speaking. His work with the school's speech therapist had made him aware of being able to control different ways of speaking, but also contributed to his sense of self-consciousness by setting him apart from most of the other children – the message was that he couldn't talk as well as they could. Again, he interpreted that he was somehow not up to par.

Such feelings of insufficiency may have been heightened during the time of the Knot Lab project, as events at home were particularly distracting for Tony's parents (especially his mother). Tony's stories indicated that his parents were not available as often as he would have liked, to provide a certain security or reflection of himself that he needed. His account of the celebration of his mother's ordination, and its distraction from his birthday, indicated a profound sense of having been forgotten. It was as though his sense of worthlessness had been inadvertently verified. In carefully making the Turk's Head ("cross") necklace for his mother, he seemed both to want to show her that he understood the importance of what she had done, and to find a way to present himself as someone worthy of her attention. As he tied the knot, he relived in detail a conversation with her, in which she had complimented him on the Turk's Head bracelet that he had made, and commented on its resemblance to a cross as well as its "summery" look.

When Tony had first begun generating knots that defied categorization in familiar terms, he seemed to channel his descriptions of himself as a "weirdo" and a "nerd," angrily, into rejection or destruction of the oddities. They were mistakes, perhaps signs of his own dullness or inadequacy. But gradually, he came to accept them as being useful in leading to new creations, as well as to deeper understandings of the knots he had set out to tie. An interesting validation of the "mutations" occurred as Tony made a place for them on the String in Motion board, near their recognizable counterparts, sanctioned versions of the Square knot. Having assumed an important role in his learning about Square knots, the anomalies deserved recognition too.

Whether this validation extended to some change in Tony's view of himself remains a question. Suffice it to say that the Knot Lab project had clearly become important to him – according to his own pronouncement, it had "really increased [his] awareness of knots," and it had provided an environment in which he could safely express many kinds of thoughts. He certainly didn't spare himself in his parting shots as we concluded his final interview:

Tony: I think I did pretty well.

***Carol:** I think you did, too, and I hope you can come this summer [to the M.I.T. workshop].*

Bye.

See ya. (He waves to the camera as he leaves, making a funny face. Carol stands and their paths collide.) Oh, good, Carol! *(laughter)* That's so nice of you! *(more noise)*

Well –

(to the camera:) Can you believe she just tripped me?!

I'm running – (more noise)

Bye, Carol.

See ya.

10 Vignettes

Aside from Alice, Jill, and Tony, who spent the most time in the Knot Lab, there were other participants whose involvement with the project suggested particular "affective" influences. If these children had been able to spend more time in the lab, we could have come to know each other better and insights into their thinking may have provided the basis for case studies. As it is, only these short "vignettes" are possible; I hope they provide a further glimpse of the range of involvements that emerged as the children worked with knots.

10.1 Becoming One with the Object

Others have described an approach to thinking about an object in such a way that it the thinker becomes, effectively, part of the object (Gilligan 1982; Keller 1983; Motherwell 1988; Papert 1984, 1987, 1988; Turkle 1984). Using such a "soft" or "relational" style, the thinker becomes involved with the object specifically moreso than generally, and emotionally as well as cognitively. A process akin to Freud's "introjection" may occur, so that as the thinker imagines herself to be part of the object, she also "brings in" the object to her mind – it becomes, effectively, part of her (see Section IV, as well as Sections 1.3 and 5.4.2).

Several incidents in the Knot Lab suggested particularly strongly this kind of involvement:

Jack was interested in tying knots, but he was also especially interested in the nature of the string that he used. He liked looking for different thicknesses and textures of string, and he often played by adorning himself with it. One day he put a loop around his ear and called it a hearing aid. Another time he put a noose around his neck and hands, calling himself a "prisoner." Althea let Jack tie her hands behind her back, and wrap a long piece of string all around her. Later, he sat on the floor, surrounded by coils of a large rope that he used to tie a Turk's Head. Jack was among a group of children who dissected a certain kind of string that had a woven, cloth-like wrapping around a tissue-like stuffing. Using the stuffing, he wrapped himself up like a mummy. Later, as he worked on tying knots, he wore a headband in the style of a native American. At the Knot Fair, as others rolled bread dough into string-like shapes with which to tie knots that could be baked and eaten, Jack made a similar "roll" with real string. It became sticky and dirty, but when Stacy said, "Eat it," he did.

Marcos often worked alone in the Knot Lab, and one day became particularly busy tying knots around himself and around various objects in the room.

Tony held in mind representations of both the knot and his finger, with which he managed to tie the challenging Thief knot:

6.16 **9.4.4**

Carol: *How did you tie it?*

Tony: I – don't really know, I just – looked at the picture and I tried to follow my string, the string which I was currently using.

How did you follow around?

(He holds the picture so we can both see it, and points to the inner, leftmost end of the pictured Bowline.) I used – a mental finger. *(He taps his head with his finger, then returns his finger to the picture and begins to follow the end as it curves around into the knot.)*

In order to draw the pictures for the family album of knots, Alice tacked the pieces of paper to the board next to the Family Tree, climbed up to them, and became so engrossed that she seemed to imagine herself as each of the characters she was creating. She caricatured each face in the shape of a certain knot and carefully rendered the "over" and "under" aspects of each crossing (see Sections 6.1 and 9.1.4).

Doreen's gestures were an important part of her explanations of why she grouped certain knots in the way that she did. The Granny, Stevedore, and Figure 8 went together:

5.4.1 5.4.2 **7.2.7**

... because of the way they twist – they like, twist, like – into each other. *(As she speaks, she gestures broadly and symmetrically with her hands, pointer fingers extended to show the curves going out together and then coming back in together.)*

The Surgeon's and Bowlines appeared more complicated, and the winds were their distinguishing feature:

These I put together because of the many different ways they go.

In many different – what do you mean?

Like ... they ... go around ... *(She gestures broadly by crossing her two hands and then doing a circular motion with her right hand.)*

Her arms and hands became, for the moment, the ends of the string, as she herself seemed to become the knot in the process of being formed.

Leroy had dialogs on several levels as he explained why he grouped certain knots together (see Section 7.2.9). He talked to me, to himself, to the knots, and for the knots. As he moved the "Square with 4 Ends" closer to the Square knot, he said to it, "Let me get you in here." He added the Thief, saying to the knot, "Well, I'll put you there," and to me, "because it looks the same."

Curtis moved the Stopper, speaking for it: "Put me at the bottom." To me, he explained, "Lonely." I asked:

7.2.11

... what does it mean to be at the top, or to be at the bottom? What do they mean?

Uh – the bottom ones, they don't have no – like a friend to be with. The top ones, they have like, a partner. Like these two, the Bowline.

Oh, I see.

They would be together. Like they're family, and these are not family.

Uh huh. Which is the family part?

These two are family *(the Bowlines)*, these two *(the Square and the "Square with 4 Ends")* are cou – half-cousins. And, um, these two are family *(the Sheet Bend and the Thief)*. And this one, he's a friend of the family *(the Granny)*. And he's a friend *(the Surgeon's)*. He's a friend *(the Stopper)*. He's a friend *(the other Granny)*. He's a friend *(the Figure 8)*. He's a friend *(the Stevedore)*. *(Curtis laughs.)*

Curtis perceived the closest relationships among the knots he had come to know the best: the Bowline, the Squares, the Sheet Bend, and the Thief. By anthropomorphizing the knots, he charged them with a lifelike quality that sustained his interest. They were like his friends and family, and he was willing to spend quite a bit of time with them. Sometimes he actually *became* one of the knots as he momentarily adopted its voice.

Gestures used in describing knots often revealed a way of thinking that seemed to equate one's own body with the knot (see Sections 1.3 and 5.4.2). Tony's "arch" and "twirl" gestures mimicked parts of the Bowline and the way in which the string would move into the form of the knot:


5.4.2 **9.4.4**

Tony: ... it seems to have that shape – *(He transfers the knot from both hands to his right hand. With his left hand he does a motion like an arch over the knot.)* – of a Bowline.

***Carol:** Which part is the shape of the Bowline?*

(He holds the knot with his left hand and with his right index finger follows the line along the loop and into the knot, doing a twirly motion with his finger as if to follow the curve of the string through the knot.) The loop with the – loop around it!

Tony's comparison of the Bowline and Packer's knots was even more animated:

Um – well – the shape and what's different about the Bowline is like an eye with things around it *(makes a circle with his right thumb and index finger around his left index finger)*, and this is more like an eye *(makes a circle with his left thumb and index finger; begins to put his right index finger into the circle, but instead grabs his left thumb with his right index finger and thumb)* kind of twisted *(tosses left hand as if to escape from the right hand's grip and dismiss the problem)*, with a thing around it, but higher up. It's strange. It's kind of like – it's kind of almost like a Bowline with a step added. The Bowline would go around, but this goes up and through *(makes corresponding motions around the Packer's knot)*.

Eugene used his fingers to indicate the difference in positions of the ends of the Square knot. Not only did the "V" of his fingers point to the directions in which the ends extended, but he also created another mapping between finger position and the situation of the ends: the height of each finger was related to whether the end was "above" or "below" the circle.

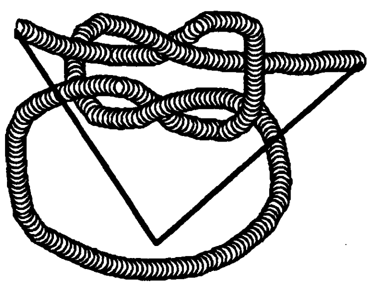
5.4.2 6.6 7.1.1.5

Carol: How about these knots – these two knots. Do these look the same to you?

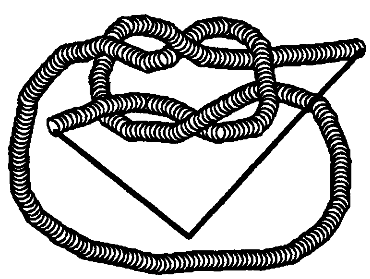
Eugene: Yeah.

What's the same about them?

They're the same – kind of. Because both of these go like that,



and both of these go like that.



(He shows the two ends of each knot, using the index and middle fingers of his right hand to show the two ends at the same time. The index finger is below the middle finger for the Thief, and they are in the same plane for the Square.)

10.2 Loving It

The Knot Lab became a place where kids could talk about a variety things while working on knots. Many of them discussed their teachers, parents, siblings, and other aspects of their lives. Often a group would discuss some issue before working, as if to "get it off their chests" so they could concentrate on knots. The environment was one in which many of the children developed a certain kind of trust, which enabled them to say what they were thinking without fear of censorship or repercussion. This trust was important in establishing a place where the children felt they were accepted on many levels, so that articulating their thinking about knots – or whatever came to mind in association with knots – could happen freely. In many cases it seemed that the resulting sense of security became part of a useful "feedback loop" in terms of learning knots: those who seemed to feel the most "at home" in the lab also seemed to learn more knots or to develop relatively deep understandings of the knots they worked on.

Kate's group came into the Knot Lab with many complaints about the substitute teacher who was working with them one day. They gathered around and blurted out story after story of how awful the substitute was. He was being very strict, the kids would be loud and disobedient, he would punish them, their behavior would become worse – the situation apparently had become progressively worse during the course of the day, and the kids were very upset. "The problem with substitutes is," one of them summarized, "they don't trust the kids." The children wanted some respect. After their clamorous account, they went to the letter boxes and folders, found their work, and settled into a productive session.

10.3 Hating What?

When the children lost interest in knots or had some difficulty with the project, it was impossible to ignore the hints of reasons that they seemed to offer through their words and actions. To interpret any of the following examples would require more familiarity with each child, developed through spending far more time together than we were able to manage.

6.18

Juanita was at first interested in the knot project, but became more and more frustrated as she experienced difficulty with some of the knots. Her reaction was to believe that she was somehow at fault, inherently "not good for this." After arriving at this conclusion, she continued coming to the lab, but her work virtually stopped.

> To. Soo Yong Chang
> From. Juanita Lopez
> Dear Soo Yong Chang
> I will like to know who thought you how to do knots.
> answer back.

All of the children in Juanita's group had asked him this, in addition to requesting new knots, so Soo Yong responded to all of them in the next video:

Rosella, Juanita, Maria, Julio, José, and Marcos: well, I learned all the knots in Boy Scouts...and, well, it's part of the skill awards that, uh, you earn for the things you get. And – I'll teach you how to do Surgeon's knot in a minute, but Trumpet knot, Monkey's Fist, and Turk's Head might take a little while so I'll teach you guys that later.

To. Soo Yong Chang
I am having a hard time lirning to do the monkeys Fist and I get very mad and ten I don't want to do enything.
From. Juanita L.

Dear Juanita,
Did you ever look in the book called "Sailor's Knot", or something like that?
If you didn't, look at the picture of monkey's fist in the book and read the instructions next to it. It should help you a little bit.

Sincerely,
Soo Yong Chang

Soo Yong Chang
thasnk but I think im not good for this I could only do some knots but it is difficult to do the monkeys fist.
From
Juanita
Lopez

At one point, Maria had made some clump-like knots out of thin pieces of flexible plastic. She arranged them on a piece of cardboard to form a face, with a knot for each of the eyes, the nose, and the mouth. She displayed her portrait next to some other cardboard assemblages of knots, which demonstrated more pedantic purposes in their use of labels for the knots, etc. For several weeks her work stood next to these displays, propped carefully on a piano in a corner the room.

Toward the end of the project, Maria was working with some others in her group on tying some difficult knots, including the Turk's Head. She suddenly became frustrated, ran for the face picture, and started tearing the knots off the card.

Eugene fluctuated between absorption in projects like LEGO highways, and apathy. His teacher said that he has "an identity problem," and gets "depressed." Ed attributed these troubles to the fact that Eugene's mother worked the 3-11 shift and was not around when he got home, so he spent a lot of time alone. Eugene seemed to find ways to express his alienation: the first knot he created was "the disappearing knot," and as the Family Tree that Alice, Jill, and Tony were making continued to be embellished, Eugene got angry. He wanted to "rip it down," complaining that "it's taking up too much room."[1]

[1] Eventually he and Pablo began work on their own Family Tree.

10.4 Teams

Within and across the four groups involved in the Knot Lab, partnerships emerged as children worked together on certain knots or corresponded with each other. The most notable working group was the threesome formed by Alice, Jill, and Tony, on whom the case studies are based (see Section 9). But there were many other collaborations as well, which produced significant understandings and contributed to the culture of the Knot Lab. Sometimes the collaborations were formalized through declarations of friendship and shared interests; at other times, the children wandered from table to table in the Knot Lab, trying what others were doing for a while, and then moving on to another knot or another group of people.

Patrick shows Stacy and Jack how to tie the Stopper, and Stacy makes one into a friendship bracelet. Patrick decides to make an animal out of Stoppers. Although he has tied many of these knots, he still often encounters trouble when he tries to tighten it. Jack sees the finished turtle on the "Welcome" sign and wants to make one. He works on the Turk's Head, which becomes the shell, commenting, "Patrick, you really like Stoppers." "Yeah," Patrick replies, "they look good." Stacy also works on the Turk's Head for the turtle. Jack and Doreen work on the Turk's Head together, but are having trouble with it. Doreen shows it as piled up without crossings, and finally asks for help. The children break for lunch, eating together in the Knot Lab as they talk about the knots they are working on. Patrick soon stops eating and returns to his Stopper construction site. He wraps the string quickly and pulls each knot tight, in a fast production mode. Tony puts a big rope on the floor and makes a huge Stopper. All of the children join in, pulling the rope to tighten it. Then Jack sits on the floor with a big string to do the Turk's Head.

Patrick goes from tape to bad string to good string as joining material for his Stoppers. He is arranging them into one conglomerate animal. There is more discussion about the inner material of one of the types of string as being like fabric-softener material. Jack puts it on and pretends to be a mummy. Tony asks, "Patrick, do you agree that the Stopper is in the Pretzel family?" Patrick says yes, he does think the Stopper goes with the Pretzels. About Patrick and the Stopper knot, I ask, "Does he know it really well?" "Yes," Tony says, "intimately. There is an ongoing relationship between Patrick and the Stopper." Jack decides that he no longer wants to work on the turtle, but will help Patrick. He seems to have taken a cue from Tony, who has already announced that he will help Patrick and is busily tying a Stopper. After a while, Jack has a bandage around his head like a western headband and is again on the floor trying to tie a Turk's Head. Tony makes a choker necklace out of a Stopper and wears it around his neck. Alice makes a Stopper ring. Jill carefully alternates the leading end in her Turk's Head, but still produces something that comes close, though it isn't quite the knot she was hoping for. Jack anchors his Turk's Head to the table with tape, but still it doesn't match the picture closely enough for that technique to be useful – so in tying, he confuses parts of the knot and thus confuses the "overs" and "unders."

Curtis loved watching the videos from Soo Yong and decided that he wanted to make a movie as well. He walked around the lab with the video camera, interviewing his fellow knot-tyers in the style of a news reporter. He also spent a good deal of time learning and perfecting the Bowline so that he could feature it in his movie. Placing the camera on a table, he stood in front of it and imitated Soo Yong's style of explaining the formation of the knot. When his group joined Soo Yong on a field trip to the M.I.T. Sailing Pavilion (see Section 4), Curtis made use of the extra hands to record the event so that it, too, could be included in the movie. He teamed up with Eugene, who held the microphone to record good-quality sound when Curtis held the camera; they switched off from time to time so each could have a turn with each piece of equipment. Pablo and Celina also formed a camera crew, taking turns recording images and sound. The sailing master showed them various knots made of large rope on a wall display.[1] The children tried tying a few knots with this relatively thick and heavy material: Pablo paid special attention to the Bowline, and everyone worked on the Figure 8 and the Fisherman's knot (which was known as the True Lovers' knot in the Knot Lab). They watched as people used knots in preparing their dinghies for sailing and in unhurling the rigs before returning the boats to the shelter. Later, Curtis and Soo Yong made a video that combined scenes of their trip to the sailing pavilion with Curtis's interviews and takes on "the fabulous Bowline." They used Eugene's suggested name for the video: *The Knot Team*.

[1] Hatch Brown also participated in panel discussion of knots during the summer workshop at M.I.T. (see Section 4.2)

10.5 Friends

Many of the children became involved in lengthy correspondences with Soo Yong. Pablo, however, seemed to take the exchange more seriously than most. Initially he was reticent, but the encouragement that he got from Soo Yong's letters prompted him to study the knots carefully, and to report on his progress to the stranger he had come to accept as his teacher. Once, after reading a particularly encouraging note from Soo Yong, Pablo quietly folded it and put it in his pocket, perhaps to re-read later in the day. It was, for him, something worth saving. A few of their letters demonstrate the mutual respect that developed through the exchange:

 6.18

Dear Soo Yong,
On the sailing knot book I learn a knot I think. They are easy.
If you want to send me another knots. I was fun doing
them.

Sincerely,
Pablo Almirez

Dear Pablo,
So, do you like the book? I'm happy to hear that
you learned many other knots.
I my guess is correct you are probably ready for
the next knot. Would you like to tell me what you've
learned?

Sincerely,
Soo Yong Chang

Dear Pablo

I haven't seen you doing anything lately   why don't you try stevedore surgeon's knot and some other hitches. write back as soon as possible and tell me what you have done. If you have any questions just write it in the letter.

Sincerely,
Soo Yong Chang

Dear Soo Yong,

I've been working on the Packer's knot. Next time show me a nother knot and next week i'm going to work on the monkey's fist. Bye

Your friend,
Pablo

Dear Pablo,

This week I'm going to show you how to tie Sheet Bend. It is used to connect the two ropes together especially when their thickness is different.

Sincerely,
Soo Yong Chang

Dear Soo Yong,

I've tried to do the Monkey's Fist. It is not ease. I tried to learn by waching your tape over and over. I'm going to try until I got it.

Your friend,
Pablo

Dear Soo Yong,

I'm making a knot tree. And Eugene and I are trying too make the monkeys fist. So can you teach us. Thank you,

Pablo Almirez

10.6 Enemies!

As strong as were some of the friendships that grew during the course of the project, clashes between Knot Lab participants affected the work with knots. Often, affinities and conflicts rooted in other times, places, or circumstances found their way into the lab.

Pablo and Curtis often argue about whose turn it is to use the VCR or video camera. There is a tension between them that often comes up in the Knot Lab, in the form of consistent disagreements, occasional outbursts, and teasing. Curtis describes what for him are the "Big 3" knots: "the Bowline, the Sheep Shank,[1] and the Figure 8, with Pablo's big head on it." Their teacher, Ed, tells of the boys' continual fighting in the classroom. Once, an argument culminated in Curtis's throwing a piece of chalk at Pablo, who "went after" Curtis. Ed says Pablo "wanted to kill." Interestingly, these two boys were among the children who developed the strongest relationships with Soo Yong: Pablo, in the form of written correspondence, and Curtis, in the form of a video, which he shot and Soo Yong edited, and about which they exchanged many ideas (see Section 10.4).

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[1] (He means the Sheet Bend.)

10.7 Making Amends

The context of the Knot Lab offered the potential for participants to work together, to polarize, and to express or "work through" the situations.

Rosella is fighting with the boys who are making the "Welcome" sign. They tell her that she can't help them, so she starts on her own project. But they all need the same tools, which offers them a chance to demonstrate their hostility. They argue over who should have priority with the scissors and tape. This kind of scene is common among Rosella and the boys in her group. They tease her terribly. Early in the project, José had named his invented knot "Rosie Piggy" in order to taunt her. They sometimes hurl names and insults back and forth in ways that are clearly painful to Rosella, but she has learned to fight and hold her own. Her teacher says she harbors a fear that she will be unusually short, like her mother. Rosella wants no one to see her mom. Already, her classmates have grown far taller than she. She has known most of them since kindergarten. Rosella has become an actress, her teacher says. She invents stories and lives in a fantasy world.

When the boys tell her she can't help them (though they do allow Juanita and Maria to help), she returns defiantly to her own project. She calms down as she stitches knots on a piece of cardboard. Rosella's act of sewing seems symbolic of a need to counterbalance their rejection; it is what Winnicott might call a *reparative act* (e.g., Phillips 1988, 27). Through it, she finds a way to achieve the connection that is hard to come by in other ways. She demonstrates her desire for acceptance in her correspondence with Soo Yong:

5.4.3

Dear Soo Yong Chang

I think you're really cute and really intelligent I like how you did the square knot its really neat.

P.S. I hope you wright back please, and I hope I learn alot from you.

(Soo Yong chose not to reply.)

Dear Soo Yong Chang,

Please right back and I did another knot. I love you

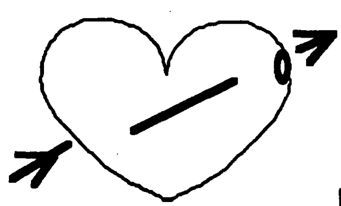
Bye, Bye

(Soo Yong's reply was kept confidential.)

Dear Soo Yong Chang,

What do you mean by Monkey Fist. Well I don't care what you say but I still love you even if you have a girlfriend, and I hope you know that I like you because of your cheeks. Please be more specific about what you said about Monkey Fist, besides I like your handwriting it's not a mess.

P.S. Please wright back.



From: Rosella

Dear Rosella

Monkey's fist is used when you need to throw the rope to someone else and I have no idea why it's called monkey's fist. I'll be more specific about Monkey's fist later.

Soo Yong Chang

Dear, Soo Yong Chang

I'm glad you are coming to the Knot Fair. And I saw you last Friday But you didn't recognized me, I had a Pony Tala in my hair And I am going to get your otorgraft. Bye.

Dear Rosella.

I would like to know what kind of knots you have been working on.

Sincerely,

Soo Yong Chang

Dear Soo Yong Chang,

I did a story with knots

I hope you like it. I voted for you so you coul'd go to the Knot Fair

Bye.

Rosella soon becomes tired of sewing knots. She finds it difficult to remember how to do the ones she wants. She switches to story-writing: perhaps by writing a metaphorical story about a knot-tying procedure, she can remind herself of how to form the knots. She starts writing about one knot, but in the middle of the story she switches to another knot, and furiously scribbles through the rejected writing.

V Denouement

> ***le dénouement*** **1:** untying, undoing of knots, etc. **2:** issue upshot, result, or outcome of an event, solution of a difficulty, or the ending of a plot or story
>
> – (Mansion [1967] 1978)

In this concluding section, I recount the contributions of the study and this report. I also look to directions for future work suggested by some of the findings.

11 Contributions

This thesis contributes to the growing body of literature on the diversity of thinking styles. It also offers an example of a research environment designed so that this diversity could emerge. And finally, it formulates a theoretical lens through which different styles of mathematical thinking, and the personal means through which it becomes mobilized, can be regarded.

11.1 Toward a Theory of the Particular

Gilligan (1982, 1987) and Turkle (1984) are among researchers whose recent work leads to better understanding of the different ways in which people think. In these researchers' writing, as well as that of others, a recurrent theme has to do with the distinction of two fundamental approaches. One of these is characterized by contextualization and a preference for thinking in a way that stays close to its object. The other favors abstraction, a certain removal from the context or distance from the object of thought. Often, such distance is achieved through generalization or an attempt to separate "cognitive," or rational, from "affective," or emotional, aspects of thought. This style has for generations enjoyed greater credibility in the Western tradition – so much so that, in response to this dominance, a priority on many research agendas has become the exploration, documentation, and understanding of so-called "concrete" or "situated" thinking. My study and report represent one such effort. Supplementing other research in psychology and the cognitive sciences, I strive toward a certain balance and, ultimately, greater acceptance of multiple ways of knowing.

The research is within the domain of mathematical thinking and is specifically concerned with how understandings of topology develop. The study required both methods and materials conducive to surfacing such understandings: through participant observation of people working with knots, I was able to appreciate certain deep, "concrete," and "affective" aspects of topological thinking.

Knots are well suited to such a study: they are manipulable objects that embody the process of their formation. They are also composed of a particularly evocative medium, string. These qualities have previously attracted the interest of researchers in the domains of cognitive psychology and psychoanalysis. Thinking about knots tends to elicit a wide range of diversity; the methods and materials used in this study enabled understanding of particular ways in which general patterns of thought may become mobilized within the mind of an individual.

11.2 Design of a Thinking Environment

In order for diversity in thinking to become apparent, the research must allow for work that is both frequent and open-ended. Individuals need time and freedom to get involved with their work in whatever ways best suit them. My study had the further need to sustain the interest of children, whose relatively unencumbered thinking was more likely to enable me to see patterns and changes as their conceptions grew.

The environment that evolved was concerned with focuses on changes in understandings, on the participants' awareness of their own processes of learning, and on developing a supportive and rich social context for the learning. Certain ingredients of the environment's design lent themselves to these emphases: the extended period of time, which allowed each participant to spend many hours working – alone or in teams – and within which special events could be scheduled; the use of video as way of heightening self-awareness and as a medium for corresponding about knots; and the presence of an intriguing figure in the form of an older child considered to be both an "expert" and, sometimes, a TV personality. The ready acceptance of this use of video, and the sustained interest in written correspondence as the video equipment came to be seen as less novel and more cumbersome, may point to the potential for telecommunications and shared knowledge bases in facilitating children's work.

The environment became particularly supportive of conducting a study through participant observation. There was so much going on that my presence could not help but blend in with the activity. Of course, as an older person whose involvement with the project extended beyond the Knot Lab, my role was different from that of most of the participants. Still, they understood and shared with me an interest in looking at *learning* as well as at knots, and our conversations were formed by exchanges of ideas about knots, school, our personal lives, practical considerations in the use of various media for communicating about knots, and so on.

11.3 Styles and Structures

The term "microworld" (Papert 1980) refers to learning environments based on fundamental concepts that give rise to other, more complicated ideas. In mathematics, such fundamental concepts have been described as structures of *topology, order,* and *classification.* (Piaget [1941, 1952] 1965, Piaget and Inhelder [1948, 1956] 1967, Beth and Piaget 1966, Bourbaki 1966) These structures are believed not only to comprise a kind of "architecture" of mathematics, but to be constituent concepts that form various mathematical understandings; they are relevant both epistemologically and psychologically.

Knots can be seen as a "microworld" in which all of these structures can be explored. They are pertinent especially to topological properties such as *proximity, continuity,* and *surrounding,* but can pertain to the other realms as well: in thinking of a knot as formed through a series of steps, the tyer engages structures of *order,* and in thinking of simpler knots as comprising more complex ones, as well as thinking of different knots as being related based on common characteristics, the tyer makes use of structures of combination and *classification.*

What becomes striking in a detailed and in-depth analysis is that as different as one individual's work may be from another's, it sustains a consistency of approach that can be characterized in terms of a preference for one or another of these epistemological structures. This observation addresses a previously stated concern:

> A valuable feature of Beth's account is his examination of the typology of mathematicians. He stresses the diversity of mathematical experience – that mathematicians differ in their accounts of the way they come to make their discoveries. In order to obtain a scientific typology of mathematical thought we need, he says, a sufficiently varied image of this experience in its diverse forms, and this can only be achieved by adequate psychological methods. Only thus will we be able to give a coherent interpretation of the introspective data furnished by mathematicians. (Beth and Piaget 1966, xv)

The result is even more striking when we see indications of a link between the implicitly chosen structure and the tyer's way of thinking about herself in relation to interpersonal aspects of her world. In these cases, an interdependence of "cognitive" and "affective" aspects of thought cannot be ignored. In Section IV, we saw ways in which the precursor topological principles may develop early in life, as a young child forms proximal relationships with caregiving figures. In fact, some version of the process of *introjection,* originally the means through which we defend against loss in our early relationships with other people, may be the means through which we are able, later, to relate to and think about people and objects. But beyond considering such origins of "cognitive" and "affective" aspects of thought, my emphasis is that they simply exist together, and in cases in this study, found avenue for expression through the medium of knots.

The interdependence of these two aspects of thought, and the idea that each can be characterized in terms of the same epistemological structure, underscores the consistency that can be found in an individual's thinking. Such consistency may elude appreciation, as a person typically engages in many activities and ways of thinking as may be appropriate to different tasks. Yet through prolonged study of an individual's choices for involvement with a specific set of ideas, we find a consonance that can be both startling and poetic.

12 Further Work

Several of the concerns and contributions of the work reported here lead to possibilities for extension and further work. Here I would like to suggest continuations that have to do both with the addition of another theoretical framework with which to view the findings, and with the emphasis on "cognitive" and "affective" aspects of thought as being intertwined.

Aspects of Minsky's "society of mind" theory lend themselves to discussion of the splitting and growth of internal components of thought, which in Section IV we referred to in terms of Fairbairn's "objects" and Papert's "structures."[1] Minsky expresses his theory in terms of "many little parts, each mindless by itself." (Minsky [1985] 1986, 17) He calls these parts "agents." Thinking emerges from their interactions. Each agent is skilled at performing some particular function, but is unaware of the skills – or even the existence – of the others. Agents become associated with other agents based on useful functions that result from their combined efforts. These associations comprise larger "agencies," which grow, organize, and re-organize in certain ways. In time, entire "societies" develop, with their member agencies assuming various roles – of finding a match to some scene or event,

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[1] While it is clear that these entities represent different things – Fairbairn's "objects" represent *people*, and relationships with them – my discussion (like Papert's) is concerned with how the formation of the conceptual entities is mediated by interaction with things (and people) in the external world.

performing a calculation, sending a message to another agency, deciding what information to ignore, what to prefer, and so on.

Minsky's explication of "Papert's Principle" forms a model that can be useful in describing the changing nature of children's thinking as they learn about knots. This is:

> The hypothesis that many steps in mental growth are based less on the acquisition of new skills than on building new administrative systems for managing already established abilities. (Minsky, 1986, p. 330)

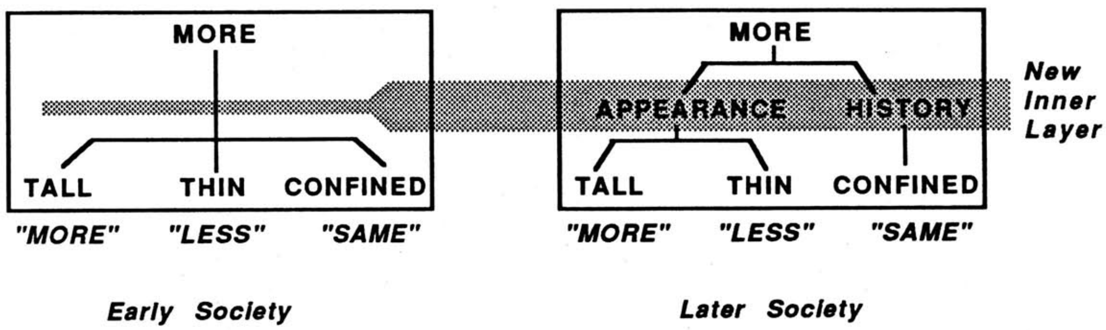
The emphasis is on reorganization of existing knowledge rather than acquisition of new knowledge. Minsky develops an explanation of the phenomenon of conservation, originally documented by Piaget and later corroborated in countless cross-cultural situations:

> Most previous theories had tried to explain Piaget's experiments by suggesting that children develop different kinds of reasoning as time goes by. That is certainly true, but the importance of Papert's conception is in emphasizing not merely the ingredients of reasoning, but how they're organized: a mind cannot really grow very much merely by accumulating knowledge. It must also develop ways to use what it already knows. That principle deserves a name. *Papert's Principle: Some of the most crucial steps in mental growth are based not simply on acquiring new skills, but on acquiring new administrative ways to use what one already knows.* ... what decides which groups to form? Papert's principle suggests that the processes which assemble agents into groups must somehow exploit relationships among the skills of those agents. (Minsky [1985] 1986, 102)

Minsky's diagram of the "Society-of-More" represents the reorganization of the conservationist child's thinking as "new administrative ways" develop to resolve a dilemma about quantities.[2] The dilemma can be raised through scenarios that vary the amount of

[2] Note that these elements, while "new" to the particular agency in question, are not new to the thinking mind to which the agency belongs. They are "around," and lend an organizational hand, so to speak, in administering other elements in the agency.

space that some fixed quantity of material occupies, such as the well-known experiment of pouring equal amounts of liquid in transparent containers of different shapes and sizes.



In Minsky's shorthand, MORE is the name of the agency before and after it undergoes the organizational change; "MORE" is what the child responds when seeing the liquid in a TALL container; "LESS" is the response when the container is THIN; and "SAME" is the response when the situation is CONFINED – that is, when the child cannot see the containers and is therefore not distracted by them, but knows that there has been no variation in the amount of liquid. A dramatic change occurs when the agents APPEARANCE and HISTORY appear, so that the child can distinguish the distracting appearance of the situation from what she knows to have occurred (or not to have occurred). In becoming concerned with the layer associated with APPEARANCE and HISTORY rather than with the TALL/THIN/CONFINED layer, the child's thinking becomes both more efficient and more accurate.

Some of the observable changes in children's thinking about knots, while not always suggesting a general phenomenon, lend themselves to representation in terms of such a reorganization of existing knowledge. One child's transfer of knowledge about the Square knot to learning about a very similar knot, the Sheet Bend, is an example of how adjusting a few familiar topological properties can lead to outward results that seem new and different. Many of the children experienced consternation as encounters with new knots led them to re-evaluate their concept of what it means to "tighten" a knot, a concept most consider to be just as important as the process of tying (see, for example, Sections 6.17, 9.3.2, and 9.4.2).

A diagram for what might be called the "Society-of-Tighten" would become complicated quickly. Imagine a progression that includes four knots that present notable pleasures or difficulties in tightening: the Overhand gets simply a hard, satisfying tug; the Square can, but also begins to suggest other considerations involved in using more fingers and paying attention to components of the knot other than the ends; the Trumpet has a particular symmetric form that must be preserved in its tightening in order to avoid turning the knot into an unrecognizable blob; and the Turk's Head requires a score of delicate movements – pushing, tugging, rolling, nudging, even loosening – aimed at preserving the balance of the cloverleaf shape and the uniformity of the tension on the string.

Modifications to the agency that is originally concerned with tightening the Overhand knot would occur as an agent concerned with SYMMETRY overrides the agency concerned with PULL[ing] HARD, for example, and as an agent concerned with the material used for tying modulates the idea of how much pressure is needed in order to achieve the desired tension in shaping the ornamental Turk's Head knot.

This sort of scheme would seem useful in representing the growth of knowledge structures related to tightening various knots. Through its generalized approach, however, it may run the risk of presenting an overly "clean," abstracted view of the process. The participants in my study present a number of different progressions through realizations related to their understandings of what it means to tighten a knot. And their ways of thinking about the key ideas may be significantly different. At the very least, their "Societies-of-Tighten" may respond to what we might call different "supporting agencies" that influence what agencies become involved in the reorganization, and when.

Tony's experience in learning to tighten the Turk's Head is an example: while it culminated a progression from simpler knots like the Overhand and Figure 8 through a number of Square-like knots, the Turk's Head presented new problems and new ways of thinking about skills he had already developed. He gave up several times, had tried several different approaches, and had worked with several different people in an effort to find helpful contexts and solutions. Finally, as he settled in with the Turk's Head that was to become a gift for his mother, he mustered the patience, perseverance, and concentration that enabled him to resolve difficulties or retrace his steps as needed in order to produce a satisfying version of the knot. As he worked on it, he chatted continually about his mother – about how she had complimented him on another Turk's Head that she had seen him wearing as a bracelet, and about how she had commented that its shape resembled that of a cross and that it had a "summery look." He wanted to present her with a version of this object, which she had found interesting. Tony's "Society-of-Tighten" would have to include representations of concepts related to such desires to please his mother, for example, and how a concept such as CROSS-LIKE, which probably would not be present in other children's societies, influenced SYMMETRY and other agents associated with MAINTAIN[ing the] SHAPE of Tony's knot.

Another aspect of this method of representation could also be enriched by taking a more personalized view of the processes of forming and reorganizing agencies: as agents needed for a certain task are originally "bundled" into agencies or reorganized as the agency grows, some agents that might have been possibilities for inclusion sometimes must be left out or discarded. In learning to tighten various knots, for example, the tyer must at some point relinquish the PULL HARD idea. Related to this relinquishing may be some sense of *loss* or *separation* – like *connection,* a fundamental factor in the psychological position of any individual.

Fairbairn, mentioned earlier as having re-interpreted Freud's concept of identification as a key factor in the early formation of internal "objects," saw individual ways of dealing with loss as shaping an attitude pervasive through the life of an individual. Fairbairn sees separation anxiety, in fact, as the root of *all* anxiety:[3]

> The early experience of complete and utter helplessness (what Fairbairn calls "infantile dependence") coincident with one's attempts to relate with objects leaves, according to Fairbairn, a potentiality for all subsequent attempts at object-relationships to be tinged with anxiety. (Goethals 1973, 92)

The anxiety is related to a fear of loss, loss being the condition against which the original process of introjection defends. This process, as we have seen, is continual and relevant not only to developing later relationships with people, but to working with objects as a way of assisting the construction of ideas. Individuals' ways of dealing with loss and separation have deeply rooted effects on how they think and on their attitudes to learning. Such considerations demand in-depth study and point to directions for further work.

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[3] Aspects of this idea come from discussions in the "Psychoanalytic Approaches to Personality" seminar conducted by George Goethals, Harvard University, 1989-1990. See Appendix B for a more detailed explanation of Fairbairn's theory of object-relations.

References

Ackermann, E. 1987. "Helping Children Become Epistemologists." Epistemology and Learning Group, M.I.T.

Altimiras, J. 1984. *Sailing Knots*. Trans. T. Willis. New York: Arco Publ., Inc.

Barthes, R. [1970] 1982. *Empire of Signs*. Trans. R. Howard. New York: Hill and Wang.

Bateson, G. 1972. *Steps to an Ecology of Mind*. New York: Ballantine Books.

Bateson, G. [1979] 1980. *Mind and Nature: A Necessary Unity*. New York: E. P. Dutton.

Berg, D. N., and K. K. Smith. 1985. *Exploring Clinical Methods for Social Research*. Beverly Hills: Sage Publ.

Beth, E. W. [1959, 1964], 1966. *The Foundations of Mathematics: A Study in the Philosophy of Science*. New York: Harper & Row.

Beth, E. W. 1965. *Mathematical Thought: An Introduction to the Philosophy of Mathematics*. Dordrecht, Holland: D. Reidel.

Beth, E. W., and J. Piaget. 1966. *Mathematical Epistemology and Psychology*. Trans. W. Mays. Dordrecht-Holland: D. Reidel.

Bisbee, J., P. Halloran, and N. Larkin. 1981. "Sailing and the Tech Dinghy." Cambridge, MA: MITNA Sailing Pavilion, M.I.T.

Bourbaki, N. 1966. *Elements of Mathematics: General Topology*. 2 vols. Reading, MA: Addison-Wesley.

Brachman, R. J., and H. J. Levesque, eds. 1985. *Readings in Knowledge Representation*. Los Altos, CA: Morgan Kaufmann.

Brown, L. M., ed. 1987. "A Guide to Reading Narratives of Moral Conflict and Choice for Self and Moral Orientation." GEHD Study Center, Harvard Univ. Graduate School of Education.

Caron-Pargue, J. 1983. "Codage verbal et codage graphique de noeuds." In proceedings of La Pensée Naturelle: Actes du Colloque de Rouen. Paris: Press Universitaire de France (Publications de l'Université de Rouen).

La Pensée Naturelle: Actes du Colloque de Rouen. Paris: P.U.F. (Publications de l'Université de Rouen).

Chasseguet-Smirgel, J. [1975] 1984. *The Ego Ideal: A Psychoanalytic Essay on the Malady of the Ideal.* Trans. Paul Barrows. New York: W. W. Norton.

Courant, R., and H. Robbins. [1941, 1969] 1978. *What is Mathematics? An Elementary Approach to Ideas and Methods.* New York: Oxford Univ. Press.

Dawson, Michael. 1985. "W.R.D. Fairbairn: Relating his Work to his Life." Cambridge, MA: Unpublished Master's thesis, Master of Liberal Arts in Extension Studies, Harvard Univ.

de Saint-Exupéry, A. 1971. *The Little Prince.* New York: Harcourt Brace Jovanovich.

Douglas, M. [1966] 1984. *Purity and Danger: An Analysis of the Concepts of Pollution and Taboo.* London: Ark Paperbacks.

Ernst, G. W., and A. Newell. 1969. *GPS: A Case Study in Generality and Problem Solving.* New York: Academic Press.

Fairbairn, W.R.D. [1952, 1963] 1972. *Psycho-Analytic Studies of the Personality.* London: Routledge & Kegan Paul.

Fairbairn, W.R.D. 1963. "Synopsis of an Object-Relations Theory of the Personality." *International Journal of Psycho-Analysis* 44:224-25.

Freud, S. [1933] 1965. "Lecture XXXI: The Dissection of the Psychical Personality," *New Introductory Lectures on Psychoanalysis.* Trans. and ed. by James Strachey. New York: Norton.

Geertz, Clifford. 1973. *The Interpretation of Cultures.* New York: Basic Books.

Gilligan, C. 1982. *In a Different Voice: Psychological Theory and Women's Development.* Cambridge, MA: Harvard Univ. Press.

Gilligan, C., et al. 1987. "Remapping the Moral Domain: The Contribution of Women's Thinking to Psychological Theory and Education." GEHD Study Center, Harvard Univ. Graduate School of Education.

Goethals, G. W. 1973. "Symbiosis and the Life Cycle." *British Journal of Medical Psychology* 46:91-96.

Goldstein, K. M. and S. Blackman. 1978. *Cognitive Style: Five Approaches and Relevant Research.* New York: John Wiley & Sons.

Grainger, Stuart. 1985. "Ropefolk." London: International Guild of Knot Tyers.

Gruber, H. E., and J. J. Vonèche, eds. 1977. *The Essential Piaget.* New York: Basic Books.

Guido, J., W. Lamb, and J. Stephens. 1990. "Donald Woods Winnicott: Pediatrician and Psychoanalyst." Unpublished paper, "Psychoanalytic Approaches to Personality" seminar with George Goethals, Harvard Univ.

Harel, I., and S. Papert, eds. (in press). *Constructionism.* Norwood, NJ: Ablex.

Hillcourt, W. 1979. *Official Boy Scout Handbook.* Irving, TX: Boy Scouts of America.

Jacob, E. "Qualitative Research Traditions: A Review." *Review of Educational Research* 57, 1(1987): 1-50.

Keller, E. F. 1983. *A Feeling for the Organism: The Life and Work of Barbara McClintock.* San Francisco: W. H. Freeman.

Keller, E. F. 1983. *A Feeling for the Organism: The Life and Work of Barbara McClintock.* San Francisco: W. H. Freeman.

Keller, E. F. 1985. *Reflections on Gender and Science.* New Haven: Yale Univ. Press.

Laing, R. D. [1960, 1962] 1965. *The Divided Self: An Existential Study in Sanity and Madness.* London: Penguin Books.

Laing, R. D. [1961], 1969. *Self and Others.* London: Penguin Books.

Laing, R. D. 1970. *Knots.* New York: Random House.

Lakatos, I. 1976. *Proofs and Refutations: The Logic of Mathematical Discovery.* Ed. J. Worral and E. Zahar. Cambridge: Cambridge University Press.

Latour, B., and S. Woolgar. [1979] 1986. *Labotatory Life: The Construction of Scientific Facts.* Princeton: Princeton Univ. Press.

Latour, B. 1987. *Science in Action.* Cambridge, MA: Harvard Univ. Press.

Lave, J. 1988. *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life.* Cambridge: Cambridge Univ. Press.

Lewin, K. 1936. *Principles of Topological Psychology.* Trans. F. Heider and G. M. Heider. New York: McGraw-Hill.

Lietzmann, W. [1955] 1969. *Visual Topology.* Trans. M. Bruckheimer. New York: American Elsevier.

Mansion, J. E. [1967] 1978. *Harrap's New Collegiate French and English Dictionary.* Skokie, IL: National Textbook Co.

McCulloch, W. S. 1961. "What is a Number, that a Man May Know It, and a Man, that He May Know a Number?" *General Semantics Bulletin* 26/27: 7-18.

McCulloch, W. S. ([1965] 1988). *Embodiments of Mind.* Cambridge, MA: M.I.T. Press.

Minsky, M. [1985] 1986. *The Society of Mind.* New York: Simon and Schuster.

Minsky, Marvin. 1990. "Logical *vs.* Analogical *or* Symbolic *vs.* Connectionist *or* Neat *vs.* Scruffy." Japan Prize address. Cambridge, MA: Music & Cognition Group, Massachusetts Inst. of Technology.

Newell, A. 1990. *Unified Theories of Cognition*. Cambridge, MA: Harvard Univ. Press.

Newell, A. and H. A. Simon. 1972. *Human Problem Solving*. Englewood Cliffs, NJ: Prentice-Hall.

Papert, S. 1984. "New Theories for New Learnings." *School Psychology Review*.

Papert, S. 1986. *On Logo*. "Thinking": Tape 3, New Mindstorms videotape series. St. Louis: MMi Media Microworlds.

Papert, S. 1987. "About Project Headlight." Cambridge, MA: Epistemology and Learning Group, M.I.T.

Papert, S. 1988. "The Conservation of Piaget: The Computer as Grist for the Constructivist Mill." *Constructivism in the Computer Age*. Ed. by G. Forman and P. B. Pufall. Hillsdale, New Jersey: Lawrence Erlbaum Assoc.

Papert, S. 1990. "Introduction." *Constructionist Learning*. Cambridge, MA: Epistemology and Learning Group, M.I.T.

Papert, S. 1990. "A Unified Computer Environment for Schools: A Cultural/Constructionist Approach." Proposal to National Science Foundation. Cambridge, MA: Epistemology and Learning Group, M.I.T.

Peterson, I. 1988. *The Mathematical Tourist: Snapshots of Modern Mathematics*. New York: W. H. Freeman.

Piaget, J. [1929] 1951. *The Child's Conception of the World*. Trans. by J. and A. Tomlinson. New York: Humanities Press.

Piaget, J. [1941, 1952] 1965.*The Child's Conception of Number*. New York: W. W. Norton.

Piaget, J., and B. Inhelder. [1948, 1956] 1967. *The Child's Conception of Space*. Trans. F. J. Langdon and J. L. Lunzer. New York: W. W. Norton.

Polkinghorne, D. 1983. *Methodology for the Human Sciences: Systems of Inquiry*. Albany: State Univ. of New York Press.

Shapiro, David. 1965. *Neurotic Styles*. New York: Basic Books.

Suchman, L. 1987. *Plans and Situated Actions*. Cambridge, UK: Cambridge Univ. Press.

Sutherland, J. D. 1989. *Fairbairn's Journey into the Interior*. London: Free Association Books.

Traweek, S. 1989. *Beamtimes and Lifetimes: The World of High-Energy Physicists.* Cambridge, MA: Harvard Univ. Press.

Turkle, Sherry. [1978] 1981. *Psychoanalytic Politics: Freud's French Revolution.* Cambridge, MA: MIT Press.

Turkle, S. 1984. *The Second Self: Computers and the Human Spirit.* New York: Simon and Schuster.

Turkle, S. 1988. "Artificial Intelligence and Psychoanalysis: A New Alliance." *Daedalus: Journal of the American Academy of Arts and Sciences* 117,1:241-68.

Turkle, S. 1990. "Romantic Reactions: Paradoxical Responses to the Computer Presence." *Boundaries of Humanity: Humans, Animals, Machines.* Mort Sousna & James Sheean, eds. Berkeley: Univ. of California Press.

Turkle, S., and S. Papert. 1990. "Epistemological Pluralism: Styles and Voices Within the Computer Culture." *Constructionist Learning.* Epistemology and Learning Group, M.I.T. See also *Signs* 16, 1(1990), Chicago University Press.

van Sommers, P. 1984. *Drawing and Cognition: Descriptive and Experimental Studies of Graphic Production Processes.* Cambridge, UK: Cambridge Univ. Press.

Winnicott, D. W. 1965. *The Maturational Processes and the Holding Environment: Studies in the Theory of Emotional Development.* Madison: International Univ. Press.

Winnicott, D. W. 1971. *Playing and Reality.* London: Tavistock.

Winnicott, D. W. 1971. *Therapeutic Consultations in Child Psychiatry.* London: Hogarth Press.

Bibliography

Ackermann, Edith. "Helping Children Become Epistemologists." Epistemology and Learning Group, Massachusetts Institute of Technology, 1987.

Ackermann, Edith. "Circular Reactions and Sensori-Motor Intelligence: A Discussion on Piaget's Theory of Early Cognitive Growth." Epistemology and Learning Group, Massachusetts Institute of Technology, 1989.

Ackermann, Edith. "From Decontextualized to Situated Knowledge: Revisiting Piaget's Water-Level Experiment." *Constructionist Learning*. Epistemology and Learning Group, Massachusetts Institute of Technology, 1990.

Agre, Philip E. "Computation and Situated Activity." Department of Computer Science, University of Chicago, 1990.

Altimiras, J. *Sailing Knots*. Translated by Tom Willis. New York: Arco Publishing, Inc., 1984.

Anderson, John R. Cognitive Psychology and Its Implications. New York: W. H. Freeman and Co., [1980] 1985.

Asher, Harry. *A New System of Knotting*. Warwickshire, UK: Quinton House, 1967.

Ashley, Clifford W. *The Ashley Book of Knots*. Garden City, New York: Doubleday & Co., 1944.

Austin, Howard. "A Computational Theory of Physical Skill." Doctoral dissertation, Massachusetts Institute of Technology, 1976.

Austin, Howard. "A Computational View of the Skill of Juggling." Artificial Intelligence Memo no. 330, Logo Memo no. 17. Cambridge, MA: A. I. Laboratory, Massachusetts Institute of Technology, 1974.

Barthes, Roland. *Empire of Signs*. Translated by Richard Howard. New York: Hill and Wang, [1970] 1982.

Bateson, Gregory. *Mind and Nature: A Necessary Unity*. New York: E. P. Dutton, [1979] 1980.

Bateson, Gregory. *Steps to an Ecology of Mind*. New York: Ballantine Books, 1972.

Berg, David N., and Kenwyn K. Smith. *Exploring Clinical Methods for Social Research*. Beverly Hills: Sage Publications, 1985.

Beth, Evert W. *The Foundations of Mathematics: A Study in the Philosophy of Science*. New York: Harper & Row, [1959, 1964], 1966.

Beth, Evert W. *Mathematical Thought: An Introduction to the Philosophy of Mathematics*. Dordrecht, Holland: D. Reidel Publ. Co., 1965.

Beth, Evert W., and Jean Piaget. *Mathematical Epistemology and Psychology*. Translated by W. Mays. Dordrecht-Holland: D. Reidel Publ. Co., 1966.

Bisbee, J., P. Halloran, and N. Larkin. "Sailing and the Tech Dinghy." Cambridge, MA: MITNA Sailing Pavilion, Massachusetts Institute of Technology, 1981.

Bourbaki, Nicolas. *Elements of Mathematics: General Topology*. 2 vols. Reading, MA: Addison-Wesley Publ. Co., 1966.

Boy Scouts of America. "Knots and How to Tie Them." Irving, TX: 1978.

Brachman, Ronald J., and Hector J. Levesque, eds. *Readings in Knowledge Representation*. Los Altos, CA: Morgan Kaufmann Publishers, Inc., 1985.

Bress, Helene. *The Macramé Book*. New York: Charles Scribner's Sons, 1972.

Breyan, Debra, Joel Kurtzberg, and Carol Strohecker. "W.R.D. Fairbairn [3 parts]: Schizoid Factors in the Personality, An Object-Relations theory of Personality, Observations on Fairbairn's Life and Work." Unpublished paper, "Psychoanalytic Approaches to Personality" seminar with George Goethals, Harvard University, 1990.

Brown, Lyn Mikel, ed. "A Guide to Reading Narratives of Moral Conflict and Choice for Self and Moral Orientation." GEHD Study Center, Harvard University Graduate School of Education, 1987.

Browne, Frederick D. "A Numerical Way to Classify Knots." *Knotting Matters* 7(1984). Edited by Robert Jackson. London: International Guild of Knot Tyers.

Budworth, Geoffrey. *The Knot Book*. Surrey, UK: Elliot Right Way Books, 1983.

Budworth, Geoffrey. *Knots & Crime*. UK: Police Review Publishing Co. Ltd., 1985.

Budworth, Geoffrey. *The Identification of Knots*. London, UK: Metropolitan Police Forensic Science Laboratory, 1982.

Burde, Gerhard, and Heiner Zieschag. *Knots*. Studies in Mathematics, vol. 5. De Gruyter, 1985.

Bussis, Anne M., and Edward A. Chittenden. "Inquiry into Meaning: An Investigation of Learning to Read." Hillsdale, NJ: Lawrence Erlbaum Assoc., Inc., 1985.

Caron-Pargue, Josiane. "Analyse d'instructions graphiques pour faire un noeud: La filiation des significations dans des dessins d'enfants." Laboratoire de Psychologie, Université de Rouen, 1985.

Caron-Pargue, Josiane. "Codage verbal et codage graphique de noeuds." In proceedings of La Pensée Naturelle: Actes du Colloque de Rouen. Paris: Press Universitaire de France (Publications de l'Université de Rouen), 1983.

Cassidy, John. *The Klutz Book of Knots*. Palo Alto: Klutz Press, 1985.

Chabaud, Jean-François. *Le noeud dit du fantasme: Topologie de Jacques Lacan*. Paris: Le Trente Mars, 1984.

Chasseguet-Smirgel, Janine. *The Ego Ideal: A Psychoanalytic Essay on the Malady of the Ideal*. Translated by Paul Barrows. New York: W. W. Norton & Co., [1975] 1984.

Chisnall, Robert. "An Investigation of Handedness and Knot Tying Habits: Implications for Experiential Education and Forensic Science." Queen's University, Kingston, Ontario, 1989.

Chisnall, Robert, ed. *The O.R.C.A. Rock Climbing Safety Manual*. Toronto, Ontario: Ministry of Tourism and Recreation, 1985.

Chisnall, Robert. "Teaching Mathematics with Knot and Rope." Queen's University, Kingston, Ontario, 1987.

Chisnall, Robert. "A Training Program for Knot Investigators." Queen's University, Kingston, Ontario, 1987.

R. S. Cohen, Feyerabend, P. K., and W. M. Wartofsky. *Essays in Memory of Imre Lakatos*. Boston Studies in the Philosophy of Science, vol. 34. Dordrecht, Holland: D. Reidel Publishing Co., 1976.

Courant, Richard, and Herbert Robbins. *What is Mathematics?* Oxford: Oxford University Press, [1941, 1969] 1978.

Crossley, David A. "Surgical Threads." *Knotting Matters* 15: 11-14, 1986.

Curie, Eve. *Madame Curie: A Biography by Eve Curie*. Translated by Vincent Sheean. New York: Doubleday, Doran & Co., Inc., 1938.

Damarin, Suzanne K. "On the Necessity of Finding New Ways to Address the Women and Math Problem." Unpublished draft, Ohio State University.

Davis, Philip J., and Reuben Hersh. *The Mathematical Experience*. Boston: Houghton Mifflin Co., 1981.

Dawson, Michael. "W.R.D. Fairbairn: Relating his Work to his Life." Cambridge, MA: Unpublished Master's thesis, Master of Liberal Arts in Extension Studies, Harvard University, 1985.

Day, Cyrus Lawrence. *Quipus and Witches' Knots: The Role of the Knot in Primitive and Ancient Cultures*. Lawrence: University of Kansas Press, 1967.

de Saint-Exupéry, Antoine. *The Little Prince.* New York: Harcourt Brace Jovanovich, 1971.

Douglas, Mary. *Purity and Danger: An Analysis of the Concepts of Pollution and Taboo.* London: Ark Paperbacks, [1966] 1984.

Dreyfus, Hubert L. and Stuart E. Dreyfus. *Mind over Machine: The Power of Human Intuition and Expertise in the Era of the Computer.* New York: Macmillan, Inc., 1986.

Dreyfus, Hubert L. with Harrison Hall, eds. *Husserl, Intentionality, and Cognitive Science.* Cambridge, MA: MIT Press, 1982.

Duckworth, Eleanor. *The Having of Wonderful Ideas: Essays on Teaching and Learning.* New York: Teachers College Press, Columbia University, 1987.

Eisenstein, Sergei. *Nonindifferent Nature.* Cambridge: Cambridge University Press, 1988.

Eliot, John, and Neil J. Salkind, eds. *Children's Spatial Development.* Springfield, Illinois: Charles C. Thomas, 1975.

Ellenberger, Henri F. *The Discovery of the Unconscious: The History and Evolution of Dynamic Psychiatry.* New York: Basic Books, Inc., 1970.

Ericsson, K. Anders, and Herbert A. Simon. *Protocol Analysis: Verbal Reports as Data.* Cambridge, Massachusetts: MIT Press, 1984.

Erikson, Erik H. *Childhood and Society.* New York: W.W. Norton & Co., [1950] 1963.

Ernst, George W., and Allen Newell. *GPS: A Case Study in Generality and Problem Solving.* New York: Academic Press, 1969.

Fairbairn, W. Ronald D. *Psycho-Analytic Studies of the Personality.* London: Routledge & Kegan Paul, [1952, 1963] 1972.

Fairbairn, W. Ronald D. "Synopsis of an Object-Relations Theory of the Personality." *International Journal of Psycho-Analysis* 44 (1963): 224-25.

Falbel, Aaron. "Friskolen 70: An Ethnographically Informed Inquiry Into the Social Context of Learning." Ph.D. diss., Massachusetts Institute of Technology, 1989.

Ferenczi, Sándor. "Confusion of Tongues between the Adult and the Child." *International Journal of Psycho-analysis* 30:225-30, 1949.

Feyerabend, Paul K. *Realism, Rationalism and Scientific Method.* Cambridge, UK: Cambridge University Press, 1981.

Fisher, Joan. *The Art of Macramé: Modern Design in Knotting.* London: The Hamlyn Publishing Group Ltd., 1972.

Flinn, R. M. "Knotting in Medicine and Surgery." *Knotting Matters* 12: 8-14, 1985.

Freud, S. "Lecture XXXI: The Dissection of the Psychical Personality," *New Introductory Lectures on Psychoanalysis*. Translated and edited by James Strachey. New York: W.W.Norton & Company, [1933] 1965.

Freyd, P., D. Yetter, J. Hoste, W.B.R. Lickorish, K. Millett, and A. Ocneanu. "A New Polynomial Invariant of Knots and Links." *Bulletin (New Series) of the American Mathematical Society* 12, 2 (1985): 239-46.

Gardner, Howard. *The Mind's New Science: A History of the Cognitive Revolution*. New York: Basic Books, Inc., [1985] 1987.

Geertz, Clifford. *The Interpretation of Cultures*. New York: Basic Books, Inc., 1973.

Gentner, Dedre, and Albert L. Stevens, eds. *Mental Models*. Hillsdale, New Jersey: Lawrence Erlbaum Associates, 1983.

Gilligan, Carol. *In a Different Voice: Psychological Theory and Women's Development*. Cambridge, MA: Harvard University Press, 1982.

Gilligan, Carol, et al. "Remapping the Moral Domain: The Contribution of Women's Thinking to Psychological Theory and Education." GEHD Study Center, Harvard University Graduate School of Education, 1987.

Giroud, Françoise. *Marie Curie: A Life*. Translated by Lydia Davis. New York: Holmes & Meier, [1981] 1986.

Goethals, George W. "The Evolution of Sexual and Genital Intimacy: A Comparison of the Views of Erik H. Erikson and Harry Stack Sullivan." *Journal of the American Academy of Psychoanalysis* 4(4):529-44.

Goethals, George W. "Symbiosis and the Life Cycle." *British Journal of Medical Psychology* 46:91-96, 1973.

Goldstein, Kenneth M. and Sheldon Blackman. *Cognitive Style: Five Approaches and Relevant Research*. New York: John Wiley & Sons, Inc., 1978.

Grainger, Stuart. "Ropefolk." London: International Guild of Knot Tyers, 1985.

Graumont, Raoul, and John Hensel. *Encyclopedia of Knots and Fancy Rope Work*. Cambridge, MD: Cornell Maritime Press, Inc., 1952.

Gruber, Howard E., and J. Jacques Vonèche, eds. *The Essential Piaget: An Interpretive Reference and Guide*. New York: Basic Books, Inc., 1977.

Guido, Joseph, William Lamb, and Jennifer Stephens. "Donald Woods Winnicott: Pediatrician and Psychoanalyst." Unpublished paper, "Psychoanalytic Approaches to Personality" seminar with George Goethals, Harvard University, 1990.

Guntrip, Harry. "My Experience of Analysis with Fairbairn and Winnicott: (How Complete a Result Does Psycho-Analytic Therapy Achieve?)" *International Review of Psycho-Analysis* 2 (1975):145.

Hadamard, Jacques. *The Psychology of Invention in the Mathematical Field.* New York: Dover Publications, Inc., 1945.

Harel, Idit. "Software Design for Learning: Children's Construction of Meaning for Fractions and Logo Programming." Ph.D. diss., Massachusetts Institute of Technology, 1988.

Harel, Idit, editor. *Constructionist Learning*. Epistemology and Learning Group, Massachusetts Institute of Technology, 1990.

Harel, Idit, and Seymour Papert, editors. *Constructionism.* Norwood, NJ: Ablex Publishing Co., in press.

Hebb, D. O. *The Organization of Behavior: A Neuropsychological Theory.* New York: John Wiley & Sons, Inc., 1949.

Hensel, John. *The Book of Ornamental Knots.* New York: Charles Scribner's Sons, 1973.

Hillcourt, William. *Official Boy Scout Handbook.* Irving, TX: Boy Scouts of America, 1979.

Higginson, W. "Language, Logic and Mathematics: Reflections on Aspects of Education and Research." *Revue de Phonétique Appliquée* 46-47:101-30, 1978.

Holden, Alan. *Orderly Tangles: Cloverleafs, Gordion Knots, and Regular Polylinks.* New York: Columbia University Press, 1983.

Hughes, Judith M. *Reshaping the Psycho-Analytic Domain: The Work of Melanie Klein, W.R.D. Fairbairn, and D. W. Winnicott.* Berkeley: University of California Press, 1989.

Husserl, Edmund. *Phenomenology and the Crisis of Philosophy: Philosophy as Rigorous Science and Philosophy and the Crisis of European Man.* Translated by Quentin Lauer. New York: Harper & Row, 1965.

Jacob, Evelyn. "Qualitative Research Traditions: A Review." *Review of Educational Research* 57, 1(1987): 1-50.

Jacobs, Harold R. *Mathematics: A Human Endeavor.* San Francisco: W. H. Freeman, 1970.

Jones, Vaughan F. R. "Knot Theory and Statistical Mechanics." *Scientific American* 98-103, November 1990.

Karmiloff-Smith, Annette. "From Meta-Processes to Conscious Access: Evidence from Children's Metalinguistic and Repair Data." *Cognition* 23, 2 (1986): 95-147.

Kauffman, Louis H. "New Invariants in the Theory of Knots." *Astérique* 163-164 (1988): 137-219.

Kauffman, Louis H. *On Knots*. Princeton: Princeton University Press, 1987.

Kauffman, Louis H. "State Models and the Jones Polynomial." *Topology* 26, 3 (1987): 395-407.

Keller, Evelyn Fox. *A Feeling for the Organism: The Life and Work of Barbara McClintock*. San Francisco: W. H. Freeman and Co., 1983.

Keller, Evelyn Fox. *Reflections on Gender and Science*. New Haven: Yale University Press, 1985.

Kliman, Marlene. "Breaking the Law: Competing Perspectives in Children's Thinking about the Balance Scale." Master of Science thesis, Massachusetts Institute of Technology, 1987.

Kliman, Marlene. "Children's Learning about the Balance Scale." *Instructional Science* 15 (1987): 307-40.

Kliman, Marlene. "Learning and Intervention: A Case Study." Epistemology and Learning Group, Massachusetts Institute of Technology, 1986.

Kliman, Marlene. "Learning in the Face of Conflict: An Analysis of a Style of Coping." Epistemology and Learning Group, Massachusetts Institute of Technology, 1985.

Korzybski, Alfred. *Science and Sanity*. Lakeville, Connecticut: International Non-Aristotelian Library Publishing Co., 1958.

Kuhn, Thomas S. *The Structure of Scientific Revolutions*. Chicago: University of Chicago Press, 1962, 1970.

Kuperstein, Michael. "Neural Model of Adaptive Hand-Eye Coordination for Single Postures." *Science* 239 (1988): 1308-11.

Lacan, Jacques. *Écrits: A Selection*. Translated by Alan Sheridan. New York: W. W. Norton & Co., 1977.

Lacan, Jacques. *The Four Fundamental Concepts of Psycho-Analysis*. Edited by Jacques-Alain Miller. Translated by Alan Sheridan. New York: W. W. Norton & Co., 1973, 1977.

Laing, R. D. *The Divided Self: An Existential Study in Sanity and Madness*. London: Penguin Books, [1960, 1962] 1965.

Laing, R. D. *Knots*. New York: Random House, 1970.

Laing, R. D. *Self and Others*. London: Penguin Books, [1961], 1969.

Lakatos, Imre. *Proofs and Refutations: The Logic of Mathematical Discovery*. Edited by John Worral and Elie Zahar. Cambridge: Cambridge University Press, 1976.

Lakatos, Imre, and Alan Musgrave, editors. *Problems in the Philosophy of Science*. Proceedings of the International Colloquium in the Philosophy of Science, London, 1965, vol. 3. Amsterdam: North-Holland Publishing Co., 1968.

Lampert, Magdalene. "The Teacher's Role in Reinventing the Meaning of Mathematical Knowing in the Classroom." Research Series no. 186. East Lansing: The Institute for Research on Teaching, College of Education, Michigan State University, 1988.

Landis, Bernard. "Discussions with Harry Guntrip." *Contemporary Psychoanalysis* 17, no.1 (1981): 103-112.

Lane, Michael, ed. *Introduction to Structuralism*. New York: Basic Books, Inc., 1970.

Latour, Bruno. *Science in Action*. Cambridge, MA: Harvard University Press, 1987.

Latour, Bruno, and Steve Woolgar. *Labotatory Life: The Construction of Scientific Facts*. Princeton: Princeton University Press, [1979] 1986.

Lave, Jean. *Cognition in Practice: Mind, Mathematics, and Culture in Everyday Life*. Cambridge: Cambridge University Press, 1988.

Leiber, Justin. *Structuralism: Skepticism and Mind in the Psychological Sciences*. Boston: Twayne Publishers Division of G. K. Hall & Co., 1978.

Levine, J. "A Characterization of Knot Polynomials." *Topology* 4 (1965): 135-41.

Lévi-Strauss, Claude. *The Savage Mind*. Chicago: University of Chicago Press, [1962] 1966.

Lewin, Kurt. *Principles of Topological Psychology*. Translated by Fritz Heider and Grace M. Heider. New York: McGraw-Hill Book Company, 1936.

Lickorish, W.B.R., and K. C. Millet. "The New Polynomial Invariants of Knots and Links." *Mathematics Magazine* 61, 1 (1988): 3-23.

Lietzmann, W. *Visual Topology*. Translated by M. Bruckheimer. New York: American Elsevier Publishing Co., [1955] 1969.

Lipkin, Richard. "Unraveling More than Just Knots." *Science* (October 24, 1988): 54-55.

Mansion, J. E. *Harrap's New Collegiate French and English Dictionary*. Skokie, IL: National Textbook Co., [1967] 1978.

McCulloch, Warren S. *Embodiments of Mind*. Cambridge, MA: MIT Press, ([1965] 1988).

McCulloch, Warren S. "What is a Number, that a Man May Know It, and a Man, that He May Know a Number?" Alfred Korzybski Memorial Lecture 1960. *General Semantics Bulletin* 26/27 (1961): 7-18.

McCulloch, Warren S., Seymour A. Papert, Manuel Blum, Jose L. S. Da Fonseca, and Roberto Moreno-Diaz. "After-Dinner Address: The Fun of Failures." *Annals of the New York Academy of Sciences* 156:963-68, April 1969.

Mendez, Anita M., Harold J. Fine, with Harry Guntrip. "A Short History of the British School of Object Relations and Ego Psychology." *Bulletin of the Menninger Clinic* 40,4:357-82, 1976.

Minsky, Marvin. "A Framework for Representing Knowledge." Artificial Intelligence Memo no. 306. Cambridge, MA: A. I. Laboratory, Massachusetts Institute of Technology, 1974.

Minsky, Marvin. "Logical *vs.* Analogical *or* Symbolic *vs.* Connectionist *or* Neat *vs.* Scruffy." Japan Prize address. Cambridge, MA: Music & Cognition Group, Massachusetts Institute of Technology, 1990.

Minsky, Marvin. "Nature Abhors an Empty Vacuum." Massachusetts Institute of Technology Artificial Intelligence Laboratory Memo, no. 647 (August 8, 1981).

Minsky, Marvin. *The Society of Mind.* New York: Simon and Schuster, [1985] 1986.

Minsky, Marvin and Seymour Papert. *Artificial Intelligence.* Condon Lectures. Eugene: Oregon State System of Higher Education, 1973.

Motherwell, Lise. "Gender and Style Differences in a Logo-Based Environment." Ph.D. diss., Massachusetts Institute of Technology, 1988.

Neuwirth, Lee. "The Theory of Knots." *Scientific American* 240, 6(June 1979): 110-24.

Newell, Allen. *Unified Theories of Cognition.* Cambridge, MA: Harvard University Press, 1990.

Newell, Allen and Herbert A. Simon. *Human Problem Solving.* Englewood Cliffs, NJ: Prentice-Hall, Inc., 1972.

Noam, Gil G., Regina O'Connell Higgins, and George W. Goethals. "Psychoanalytic Approaches to Developmental Psychology." *Handbook of Developmental Psychology.* New York: Prentice-Hall, 1982.

Olson, David R. *Cognitive Development: The Child's Acquisition of Diagonality.* New York: Academic Press, 1970.

Papert, Seymour. "About Project Headlight." Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1987.

Papert, Seymour. "Beyond the Cognitive: The Other Face of Mathematics." Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1986.

Papert, Seymour. "The Conservation of Piaget: The Computer as Grist for the Constructivist Mill." *Constructivism in the Computer Age*. Edited by George Forman and Peter B. Pufall. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1988.

Papert, Seymour. "A Critique of Technocentrism in Thinking about the School of the Future." Paper delivered at the "Children in an Information Age: Opportunities for Creativity, Innovation, and New Activities" Conference, Sofia, Bulgaria, May 1987.

Papert, Seymour. "Introduction." *Constructionist Learning*. Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1990.

Papert, Seymour. "Microworlds: Transforming Education." *The Computer Culture*. Edited by Susan H. Evans and Peter Clarke. Indianapolis: White River Press, Inc., 1984.

Papert, Seymour. *Mindstorms: Children, Computers, and Powerful Ideas*. New York: Basic Books, Inc., 1980.

Papert, Seymour. "New Theories for New Learnings." *School Psychology Review* October 1984.

Papert, Seymour. "Objects-to-Think-With." Unpublished course syllabus, Epistemology and Learning Group, Massachusetts Institute of Technology, 1988.

Papert, Seymour. *On Logo*. "Thinking": Tape 3, New Mindstorms videotape series. St. Louis: MMi Media Microworlds, Inc., 1986.

Papert, Seymour. "One AI or Many?" *Daedalus: Journal of the American Academy of Arts and Sciences* 117,1:1-14, 1988.

Papert, Seymour. "Redefining Childhood: The Computer Presence as an Experiment in Developmental Psychology." North-Holland Publishing Company, 1980.

Papert, Seymour. "Teaching Children Thinking." Artificial Intelligence Memo no. 247, Logo Memo no. 2. Cambridge, MA: A. I. Laboratory, Massachusetts Institute of Technology, 1971.

Papert, Seymour. "Teaching Children to be Mathematicians vs. Teaching About Mathematics." Artificial Intelligence Memo no. 249, Logo Memo no. 4. Cambridge, MA: A. I. Laboratory, Massachusetts Institute of Technology, 1971.

Papert, Seymour. "A Unified Computer Environment for Schools: A Cultural/Constructionist Approach." Proposal to National Science Foundation.

Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1990.

Papert, Seymour. "Uses of Technology to Enhance Education." Artificial Intelligence Memo no. 298, Logo Memo no. 8. Cambridge, MA: A. I. Laboratory, Massachusetts Institute of Technology, 1973.

Papert, Seymour. "Using Computers to Combat Illiteracy: Towards a Constructionist Theory of Creative Learning. "Proposal to the MacArthur Foundation. Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1986.

Peterson, Ivars. *The Mathematical Tourist: Snapshots of Modern Mathematics*. New York: W. H. Freeman and Co., 1988.

Peterson, Ivars. "Shareware, Mathematics Style." *Science News* (Jan. 2, 1988): 12-13.

Phillips, Adam. *Winnicott*. Cambridge, MA: Harvard University Press, 1988.

Piaget, Jean. *The Child's Conception of Movement and Speed*. Translated by G.E.T. Holloway and M. J. Mackenzie. New York: Basic Books, Inc., [1946] 1970.

Piaget, Jean. *The Child's Conception of Number*. New York: W. W. Norton & Co., Inc., [1941, 1952] 1965.

Piaget, Jean. *The Child's Conception of the World*. Translated by Joan and Andrew Tomlinson. New York: The Humanities Press, Inc., [1929] 1951.

Piaget, Jean. *The Equilibration of Cognitive Structures: The Central Problem of Intellectual Development*. Translated by Terrance Brown and Kishore Julian Thampy. Chicago: The University of Chicago Press, [1975] 1985.

Piaget, Jean. *Genetic Epistemology*. Translated by Eleanor Duckworth. New York: Columbia University Press, 1970.

Piaget, Jean. *The Grasp of Consciousness: Action and Concept in the Young Child*. Translated by Susan Wedgwood. Cambridge, MA: Harvard University Press, [1974] 1976.

Piaget, Jean. *Play, Dreams, and Imitation in Childhood*. Translated by C. Gattegno and F. M. Hodgson. New York: W. W. Norton & Co., Inc., 1962.

Piaget, Jean. *Psychology and Epistemology*. Translated by Arnold Rosin. New York: Grossman Publishers, [1970] 1971.

Piaget, Jean. *Structuralism*. Translated and edited by Chaninah Maschler. New York: Basic Books, Inc., [1968] 1970.

Piaget, Jean, and Bärbel Inhelder. *The Child's Conception of Space.* Translated by F. J. Langdon and J. L. Lunzer. New York: W. W. Norton & Co., Inc., [1948, 1956] 1967.

Polkinghorne, Donald. *Methodology for the Human Sciences: Systems of Inquiry.* Albany: State University of New York Press, 1983.

Polya, G. *How to Solve It: A New Aspect of Mathematical Method.* Princeton: Princeton University Press, [1945, 1957] 1973.

Reidemeister, K. *Knot Theory.* Moscow, Idaho: BCS Associates, [1932, 1974] 1983.

Robey, David, editor. *Structuralism: An Introduction.* Wolfson College Lectures 1972. Oxford: Clarendon Press, 1973.

Rolfsen, Dale. *Knots and Links.* Berkeley: Publish or Perish, Inc., 1976.

Rolfsen, Dale, editor. *Knot Theory and Manifolds.* Lecture Notes in Mathematics 1144, vol. 163. Springer-Verlag, 1985.

Schafer, Roy. "Narration in the Psychoanalytic Dialogue." *Critical Inquiry* (Autumn 1980): 29-53.

Shapiro, David. *Neurotic Styles.* New York: Basic Books, Inc., 1965.

Shepard, Roger N., and Lynn A. Cooper. *Mental Images and Their Transformations.* Cambridge, MA: MIT Press, 1982.

Simon, Herbert A. Sciences of the Artificial. Cambridge, MA: MIT Press, [1969] 1981.

Singer, Erwin. *Key Concepts in Psychotherapy.* New York: Basic Books, Inc., 1965, 1970.

Skemp, Richard R. *The Psychology of Learning Mathematics.* Harmondsworth: Penguin Books, 1971.

Smith, Daniel I. *Handbook of Cave Rescue Operations.* Petaluma, CA: National Cave Rescue Commission, 1978.

Steinhaus, H. *Mathematical Snapshots.* New York: Oxford University Press, 1960.

Stewart, Ian. *The Problems of Mathematics.* New York: Oxford University Press, 1987.

Strohecker, Carol. "Elucidating Styles of Thinking through Thinking about Knots." *Constructionism.* Edited by Idit Harel and Seymour Papert. Norwood, NJ: Ablex Publishing Corp., in press.

Strohecker, Carol. "Knot-Tying: A Constructionist Environment for Topology." *Constructionist Learning.* I. Harel, ed. Epistemology and Learning Group, Massachusetts Institute of Technology, 1990.

Strohecker, Carol. "Electronic Collage: The Videodisc and Interactive Narrative." Unpublished Master's thesis, Massachusetts Institute of Technology, 1986.

Sturrock, John. *Structuralism.* London: Paladin Grafton Books, 1986.

Suchman, Lucy. *Plans and Situated Actions.* Cambridge, UK: Cambridge University Press, 1987.

Sumners, D. W. "The Knot Theory of Molecules." *Journal of Mathematical Chemistry* 1(1987): 1-14.

Sumners, D. W. "The Role of Knot Theory in DNA Research." Department of Mathematics, Florida State University, Tallahassee.

Sutherland, John D. "The British Object Relations Theorists: Balint, Winnicott, Fairbairn, Guntrip." *Journal of the American Psychoanalytic Association* 28:829-60, 1980.

Sutherland, John D. *Fairbairn's Journey into the Interior.* London: Free Association Books, 1989.

Toss, Brion. *The Complete Rigger: Wire and Rope.* UK: Stanford Maritime Ltd., 1985.

Traweek, Sharon. *Beamtimes and Lifetimes: The World of High-Energy Physicists.* Cambridge, MA: Harvard University Press, 1989.

Turkle, Sherry. "Artificial Intelligence and Psychoanalysis: A New Alliance." *Daedalus: Journal of the American Academy of Arts and Sciences* 117,1:241-68, 1988.

Turkle, Sherry. *Psychoanalytic Politics: Freud's French Revolution.* Cambridge, Massachusetts: MIT Press, [1978] 1981.

Turkle, Sherry. "Romantic Reactions: Paradoxical Responses to the Computer Presence." *Boundaries of Humanity: Humans, Animals, Machines.* Edited by Mort Sousna & James Sheean. Berkeley: University of California Press, 1990.

Turkle, Sherry. *The Second Self: Computers and the Human Spirit.* New York: Simon and Schuster, 1984.

Turkle, Sherry, and Seymour Papert. "Epistemological Pluralism: Styles and Voices Within the Computer Culture." *Constructionist Learning.* Cambridge, MA: Epistemology and Learning Group, Massachusetts Institute of Technology, 1990. See also *Signs* 16(1), Chicago University Press, 1990.

van Sommers, Peter. *Drawing and Cognition: Descriptive and Experimental Studies of Graphic Production Processes.* Cambridge, UK: Cambridge University Press, 1984.

Von Foerster, Heinz. "To Know and to Let Know: An Applied Theory of Knowledge." *Canadian Library Journal* 39, 5 (1982).

Walker, Jearl. "The Amateur Scientist: In Which Simple Equations Show Whether a Knot Will Hold or Slip." *Scientific American* (August 1983): 122-27.

Wasserman, Steven A., Jan M. Dungan, and Nicholas R. Cozzarelli. "Discovery of a Predicted DNA Knot Substantiates a Model for Site-Specific Recombination." *Science* 229,4709 (July 12, 1985): 171-74.

Watzlawick, Paul, John Weakland, and Richard Fisch. *Change: Principles of Problem Formation and Problem Resolution*. New York: W. W. Norton & Co., 1974.

Webster's New Collegiate Dictionary. Springfield, MA: G & C Merriam Company, 1975.

Whyburn, G. T. "Dynamic Topology." *The American Mathematical Monthly* 77, 6 (1970): 556-70.

Winch, Quniton. *Nets and Knots*. Warwickshire, UK: Quinton House, 1987.

Winnicott, Donald W. *The Child, the Family, and the Outside World*. Reading: Addison-Wesley Publishing Co., Inc., [1957, 1964] 1987.

Winnicott, Donald W. *The Family and Individual Development*. London: Tavistock Publications, 1965.

Winnicott, Donald W. *The Maturational Processes and the Holding Environment: Studies in the Theory of Emotional Development*. Madison: International Universities Press, Inc., 1965.

Winnicott, Donald W. *Playing and Reality*. London: Tavistock Publications, 1971.

Winnicott, Donald W. *Therapeutic Consultations in Child Psychiatry*. London: Hogarth Press, 1971.

Wittgenstein, Ludwig. *The Blue and Brown Books*. New York: Harper & Row, 1958.

Wittgenstein, Ludwig. *Philosophical Investigations*. Translated by G. E. M. Anscombe. New York: Macmillan Publishing Co., Inc., [1953] 1968.

Wittgenstein, Ludwig. *Remarks on the Foundations of Mathematics*. G. H. von Wright, R. Rhees, and G.E.M. Anscombe, eds. Translated by G.E.M. Anscombe. Cambridge, Massachusetts: MIT Press, [1956] 1983.

Appendix A: Experimental Method and Stages in the Piaget and Inhelder Study

Piaget and Inhelder (Piaget and Inhelder [1948, 1956] 1967, 104-24) asked children to work with transformations of string shapes, including a circle, a "figure of eight," a "pseudoknot," and mirror-image versions of the overhand. The interviews followed this procedure, with each new step assuming successful completion of the previous one:

1. The child was asked to make an overhand knot by looking at one that had been formed and copying it. If she couldn't, she was asked to form the knot around a stick, and her way of doing this was observed. If this option was too difficult, the researcher formed the knot while the child imitated the procedure. If this option was too difficult, the researcher used a two-colored string and used the colors in a description of what to do while forming the knot.

2. When the child could form the knot, she was shown a loose version of the overhand and asked to produce it. Especially of interest was whether the child used the same technique as for #1.

3. With #2 accomplished, similar questions were used in asking the child to produce an even looser version of the overhand (the "clover" shape).

#1, #2, and #3 were intended as ways of exploring the perceptual continuity between shapes.

4. The child was asked whether the left and right "clover" were the same, and made the judgement by eye or by tracing the knot with a finger. If these methods didn't work, the researcher would put a bead on the string or ask the child imagine it as a pipe through which an ant crawled. If the child still did not perceive the distinction between the knots, she could move the bead along the string or try to draw the knots.

5. Similarly, they compared the true and false overhand knots.

6. They compared the true and false figure of eight knots.

7. They compared the two circles and the two interlocking loops.

The researchers came to categorize the responses according to three stages of development of certain understandings:

Stage I includes children younger than about four years of age. It involves learning simple overhand knots, and is divided into two substages.

At Substage IA, children could not copy the knot – that is, they could not grasp the principle of intertwinement. They would wind an end of the string without inserting it into the loop, or they would insert an end into what seemed to be a loop but actually wasn't. An understanding of the necessary relationship of "surrounding" had not yet evolved.

At Substage IB, children could copy the knot, but were confused when it was loosened, even if they tried following sections of the slack knot with a finger. They also could not distinguish true and false overhand knots.

Stage II includes children who can reproduce the overhand knot, indicating that they understand the principle of entwinement and can distinguish complete from incomplete loops.

At Substage IIA, children understood that two tight overhands were the same knot and that two loosened overhands were the same knot, but could not perceive the sameness of two overhands if one was tight overhand and the other was loose. A very loosened overhand, resembling a clover shape, was similarly confusing.

At Substage IIB, children could see the sameness of two overhands when one was tight and the other loose, but the very loose clover-shaped knot still was not identifiable as an overhand. The children still could not distinguish false from true overhand knots.

In a transitional phase, children tended to get correct answers by trial and error, but their rate of success was inconsistent enough to indicate that the understandings had not fully formed.

Stage III includes children who understood the correspondences between versions of the overhand knot, and for whom the understandings were reversible (and therefore "operational"). These children could recognize such correspondences between loose and tight versions the other knots as well, and could distinguish true from false knots. They also understood the difference between right and left overhands, whether loose or tight. These children were about seven to seven and a half years of age.

Appendix B:
Fairbairn's Synopsis of Object-Relations Theory

This text is quoted directly from W.R.D. Fairbairn's (1963) succinct "Synopsis of an Object-Relations Theory of the Personality":

> (1) An ego is present from birth.
>
> (2) Libido is a function of the ego.
>
> (3) There is no death instinct; and aggression is a reaction to frustration or deprivation.
>
> (4) Since libido is a function of the ego and aggression is a reaction to frustration or deprivation, there is no such thing as an "id."
>
> (5) The ego, and therefore libido, is fundamentally object-seeking.
>
> (6) The earliest and original form of anxiety, as experienced by the child, is separation-anxiety.
>
> (7) Internalization of the object is a defensive measure originally adopted by the child to deal with his original object (the mother and her breast) insofar as it is unsatisfying.
>
> (8) Internalization of the object is not just a product of a phantasy of incorporating the object orally, but is a distinct psychological process.
>
> (9) Two aspects of the internalized object, viz. its exciting and frustrating aspects, are split off from the main core of the object and repressed by the ego.
>
> (10) Thus there come to be constituted two repressed internal objects, viz. the exciting (or libidinal) object and the rejecting (or anti-libidinal) object.
>
> (11) The main core of the internalized object, which is not repressed, is described as the ideal object or ego-ideal.
>
> (12) Owing to the fact that the exciting (libidinal) and rejecting (anti-libidinal) objects are both cathected by the original ego, these objects carry into repression with them parts of the ego by which they are

cathected, leaving the central core of the ego (central ego) unrepressed, but acting as the agent of repression.

(13) The resulting internal situation is one in which the original ego is split into three egos – a central (conscious) ego attached to the ideal object (ego-ideal), a repressed libidinal ego attached to the exciting (or libidinal) object, and a repressed antilibidinal ego attached to the rejecting (or antilibidinal) object.

(14) This internal situation represents a basic schizoid position which is more fundamental than the depressive position described by Melanie Klein.

(15) The antilibidinal ego, in virtue of its attachment to the rejecting (antilibidinal) object, adopts an uncompromisingly hostile attitude to the libidinal ego, and thus has the effect of powerfully reinforcing the repression of the libidinal ego by the central ego.

(16) What Freud described as the "superego" is really a complex structure comprising (a) the ideal object or ego-ideal, (b) the antilibidinal ego, and (c) the rejecting (or antilibidinal) object.

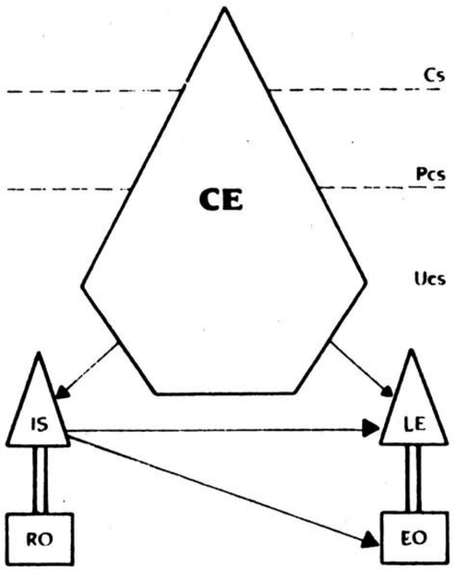
(17) These considerations form the basis of a theory of the personality conceived in terms of object-relations, in contrast to one conceived in terms of instincts and their vicissitudes.

The following page shows a pictorial comparison of Fairbairn's and Freud's "endopsychic structures" (Dawson 1985, 52).

-554-

Endopsychic Structures
Fairbairn and Freud

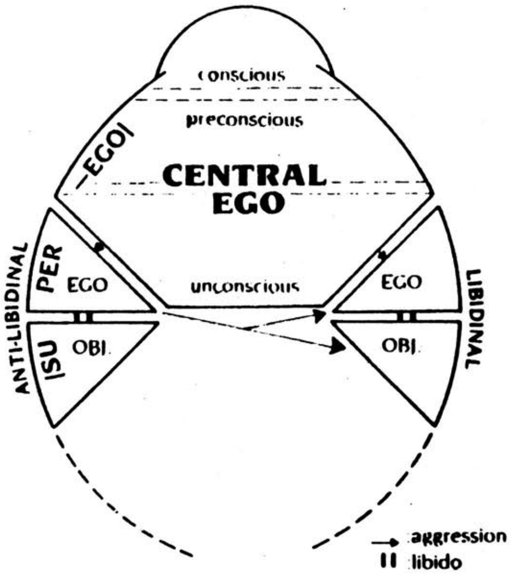
Fairbairn (1944)



CE Central Ego. IS Internal Saboteur LE Libidinal Ego RO, Rejecting Object EO Exciting Object, Cs. Conscious Pcs Preconscious Ucs. Unconscious ⟶ Aggression Libido II

Fairbairn Psychoanalytic Studies of the Personality

Fairbairn
Modified by Padel(1973)



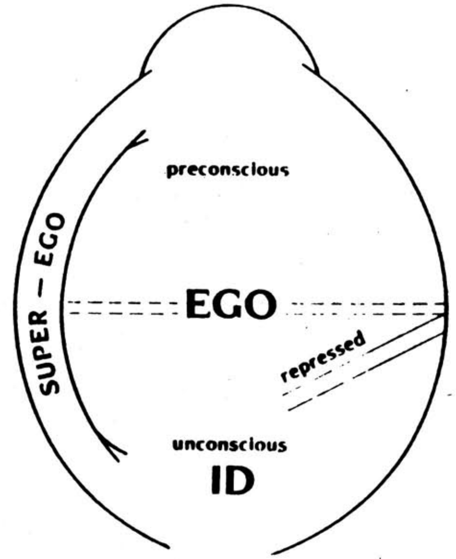
Modified;
(a) Changed Shape of outlines

(b) Fairbairn's changes in terminology and (SUPER-EGO) written in

(c) (dotted line)
Ideal object
(accepted object)

Padel The Contribution of W.R.D. Fairbairn (1889-1965) to Psychoanalytic Theory and Practice

Freud (1933)



Freud; New Introductory Lectures (S.E. Vol. 22 p. 78)