Intentional Fragmentation for Material Storage

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Abstract

A novel technique (location-relaxed storage) of mixing products within warehouse storage bins is presented and evaluated. Analyses of warehouse operations, storage space efficiency, error sensitivity, and placement policies are presented and compared to traditional warehousing techniques. The major factors that drive the performance differences between traditional, highly organized storage and location-relaxed storage are shown to include the number of unique stock keeping units (SKUs) served by the warehouse and the picking lot size characteristic of demand. The analyses demonstrate traditional storage techniques have greater difficulty dealing with a large SKU base. Furthermore, location-relaxed storage is shown to have a lower sensitivity to operation errors and a greater opportunity for cost savings through optimization opportunities. Finally, a new placement strategy especially suited for location-relaxed storage is presented.

As the popularity of Radio Frequency Identification (RFID) increases and the technical issues of widespread RFID implementation are addressed, new applications of RFID technology will change the way the world operates. An ongoing, industry-wide effort to implement RF-tags throughout the material goods supply chain has the support of manufacturers, retailers, and technology companies. RFID in the supply chain represents an enabling technology that will allow warehouse operations to break away from traditional methodologies and adopt revolutionary techniques, such as location-relaxed storage.

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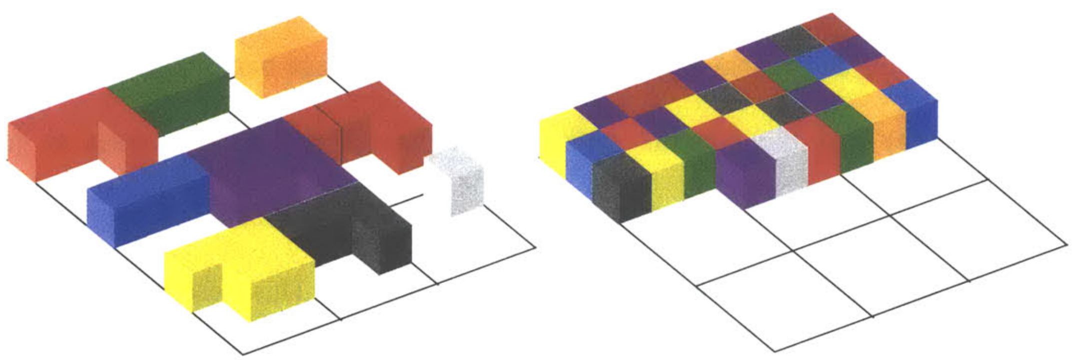
Chapter 1: Introduction

Currently, typical storage warehouses operate under an aggregated and highly organized system that stores identical items in the same storage bin. By directing all copies of the same product into a single storage bin, the warehouse system creates an organizational structure that is simple to understand and maintain. Order-picking needs for specific items map directly to storage bins with a simple one-to-one correspondence. Encoding identity through location has been vital for maintaining an accurate database of materials stored in the warehouse and preventing lost items. This tight association between storage location and item identity has been the dominant paradigm for warehouse operations for centuries. Figure 1.1a illustrates traditional storage which we call strict organization.



a) Strict Organization b) Location-relaxed Storage

Figure 1.1: Strict organization and location-relaxed storage

A new approach to warehouse operations offers to revolutionize the primary philosophy that binds location and identity in traditional warehouse methodology. Location-relaxed storage removes the restriction that all copies of the same product must be directed to the same storage bin. Furthermore, the new storage method permits items of different types to reside in the same bin at the same time. Figure 1.1b illustrates the new storage method with the same products as in Figure 1.1a. This mixing of products within the warehouse fundamentally changes how the warehouse operates. The new model compromises the simplicity found in traditional organization schemes; however, the benefits of relaxed storage include higher space utilization and increased productivity in picking and placing items under certain circumstances.

Amazon.com employs this new approach to warehouse storage for its warehouse fulfillment needs.[1] They have seen that mixed storage is appropriate and beneficial for the specific warehousing characteristics of online, direct-to-consumer business. This thesis takes the view that radio-frequency identification is an enabling technology for location-relaxed storage because RFID technology addresses many important disadvantages with location-relaxed storage. However, Amazon.com's use and success with mixed storage via bar-coded labeling shows that much of location-relaxed storage benefits do not rely on the new technology and the benefits sufficiently outweigh the disadvantages to justify location-relaxed storage. Furthermore, new automatic identification technologies will offer even greater benefits for location-relaxed storage. This research explores the ramifications of location-relaxed storage on warehouse operations including storage capacity, error sensitivity, as well as picking effort, and it demonstrates how location-relaxed storage is especially suited for warehouse operations with a high number of unique products and a low picking lot size.

The most robust evaluation of location-relaxed storage would compare its optimal performance to the optimal performance of traditional methods. Unfortunately, optimal techniques for warehouse operations are not well understood. Even for traditional

1. Personal conversation with Louis Usarzewicz, director of fulfillment engineering at Amazon.com.

warehouse methods, where existing research is much more abundant, optimal solutions tend to focus on one aspect of warehouse operations rather than the entire system. Both traditional and location-relaxed warehouse operations are complex optimization problems. Optimal policies for one may not be optimal for the other. Furthermore, optimal policies are likely to be problem dependent and different warehousing situations will have different optimal policies. The complexity of the optimization task and problem dependency make direct comparison of optimal solutions unwise and limited in scope. Demonstrating superiority of one storage method over the other for a single warehouse case will not necessarily prove anything beyond that specific case. This research demonstrates the potential of location-relaxed storage by presenting the optimization opportunities for warehouse operations without offering an optimal solution then analyzing specific characteristics of location-relaxed storage and comparing them to traditional methods.

1.1 Comparing Apples to Oranges

The two storage systems, while similar in many aspects, have differing features that make direct comparison both difficult and limited in overall significance. As stated before, optimization of these systems is very difficult. A very large number of factors influence warehouse operations. Warehouse size, warehouse shape, item characteristics, and ordering characteristics are just a few examples of parameters that have potentially significant influence on warehouse performance. Furthermore, the influences of these factors may differ between the two systems. Any specific comparison between location-relaxed storage and strict organization would only be valid for the specific case explored.

Despite the difficultly, some type of comparison must be made. This thesis explores location-relaxed storage as a possible alternative to traditional strict organization methods. To investigate this new approach, several simplified analyses explore the strengths and weaknesses of location-relaxed storage. In making these specific comparisons between location-relaxed storage and strict organization, many assumption about the warehouse operations are necessary, both to establish a system to analyze and to simplify the model so that an analysis is possible and reasonable. Analysis of carefully selected specific comparisons yield insights to the trends and influences on the two warehousing systems. The temptation is to dismiss such analyses with so many assumptions as flawed. However, the alternative is to do nothing and ignore the potential of location-relaxed storage for improving material storage techniques. The analyses make a strong case for further investigation into the effects of warehouse characteristics on location-relaxed storage and the complex optimization problems imbedded within warehouse routing and order-fulfillment.

1.2 Outline

Chapter 2 provides background on warehousing and introduces the new approach to warehouse operations, location-relaxed storage. Chapter 3 discusses location-relaxed storage in greater detail, including an analysis of the effect of multiple disparate locations on the picking process and simple heuristics for the warehouse routing problems. Chapter 4 discusses warehouse operations including placement and picking issues under location-relaxed storage. The first of four analyses compares location-relaxed storage to strict organization storage and scrutinizes the effect arrival and departure characteristics on warehouse operations. In Chapter 5, the second analytical model quantifies the storage capacity utilization of strict organization storage to illustrate the benefits of greater efficiency with location-relaxed storage. The error model of Chapter 6 analyzes the sensitivity of Auto-ID enabled strict organization storage and location-relaxed storage to warehouse operations mistakes such as misplacement or mis-picking. Chapter 7 investigates demand-based storage policies and their applicability to location-relaxed storage with a simple model that compares a demand-based placement policy to randomized placement. Chapter 8 concludes with a discussion of location-relaxed storage findings and opportunities for modern warehouse operations.

1.3 Summary

This research provides clear evidence that the two main characteristics that influence the appropriateness of location-relaxed storage for warehouse operations are the number of unique products served by the warehouse and the picking characteristics of the warehouse fulfillment needs. The benefits of location-relaxed storage increase as the number of unique products involved in warehouse operations increase. Furthermore, benefits are more substantial when the picking characteristics are such that a small quantity of a particular product are picked at a time (small picking lot size). The relative efforts involved between traditional techniques and location-relaxed storage are specific to each warehouse operations system. However, the strong potential benefits demonstrated compel the evaluation of each system to determine the appropriateness of location-relaxed storage in each individual case.

Chapter 2: Background

2.1 Warehousing

The main purpose of a warehouse is to buffer the disparity between supply and demand. Whether buffering the need for raw materials against source availability or buffering the supply of finished products against consumer demand, warehouses accept and store material goods for some time, then retrieve the item when needed. Three main phases describe the warehousing process: placement, storage, and retrieval. Placement is the process of taking arriving products, determining where each product should be stored, and physically bringing the product to the designated storage location. Storage is a passive phase where items reside in storage locations until needed. Finally, during the retrieval stage, the product is removed from the storage location and shipped out of the warehouse.

When addressing warehouse optimization, there are many metrics that are desirable to optimize as well as many features that influence operational efficiency. Warehouse metrics include the total utilization of warehouse space, which measures how much of the warehouse space is used to store items and how much space is left vacant. Another metric that is indicative of an effective warehouse is the average walk distance to pick and place an item. An efficient warehouse must keep walking distances to a minimum.

Other metrics of warehouse operations include *stockout frequency* and *turnover rates*, which describe the rate at which the warehouse is unable to meet the needs of an order and the length of time a particular item resides before being used to fill an order, respectively. These last two metrics relate to the *inventory management*, which determines how much of each *stock keeping unit* (SKU) should be stored in the warehouse. If the warehouse stocks too little of an SKU, then the warehouse may not be able to meet demand. This situation is called a stockout and is undesirable. Furthermore, if a warehouse stocks too much of an SKU, the amount of time that an SKU resides in the warehouse will be high, which is also undesirable because holding too much of an item wastes capital and occupies additional space in the warehouse. Inventory management and the associated stockout rate and turnover rate are separate issues from inventory storage and are not considered in this research. Instead, how warehouses handle incoming supplies and outgoing demand is the focus of this thesis. Metrics such as storage capacity utilization and average walking distance to place and pick an item are most relevant to the research.

2.2 Warehouse Characteristics

Many factors contribute to warehouse efficiency including physical characteristics, the number of unique SKUs served by the warehouse, SKU demand level, storage policy, placement policy, and picking policy. Physical characteristics include the area of the warehouse footprint, number of aisles, number of cross aisles, size of each storage bin, and location of pick up and drop off locations. The number of unique SKUs refers to how many different products the warehouse will handle. Storage policies refer to rules that impose in which storage bins a particular item may or may not be stored. Placement policies indicate where an item is to be stored while following the restrictions of the storage policy. Picking policies select which item should be picked to fill an order when multiple copies of the same SKU make a choice necessary.

2.2.1 Warehouse Design

The physical layout of the warehouse determines the storage capacity and affects the overall efficiency of operations. Physical layout includes the overall storage capacity, number of aisles, length of the aisles, number of storage bins in each aisle, and the number of cross aisles. Petersen [1997] investigated the effect of aspect ratio on warehouse performance when the storage capacity is fixed and reports that performance improves with deeper warehouses (warehouses with fewer but longer aisles). Petersen also investigates the effect of pick-up/drop-off (p/d) location by comparing simulations of warehouse operations with different p/d locations and concludes that a middle location p/d point results in shorter walking distances than locating the p/d at a corner. The significance of p/d location increases with wider (more aisles) warehouse configurations and smaller pick list sizes. Vaughan and Petersen [1999] report that the number of cross aisles in the warehouse configuration also has an effect on performance, in general, additional cross aisles provide more efficient warehouse operations.

2.2.2 Storage Policy

In addition to physical considerations, operational rules and policies have a strong influence on warehouse performance. The warehouse *storage policy* describes the rules that determine where an item may and may not be placed. Traditionally, warehouses fall into two main categories of storage policies, dedicated storage and shared storage. Under dedicated storage, every storage bin within the warehouse is assigned to a particular SKU permanently. Only items with a matching SKU may be placed in a particular storage bin. This static assignment means that when an item arrives at the warehouse for placement, there is a set number of possible locations in which to place the newly arrived item. Furthermore, this set of possible locations does not change over time. In contrast to dedicated storage, shared storage assigns a particular SKU to a storage bin when needed. Empty locations within a shared storage warehouse are not assigned to any particular SKU. If needed, these empty bins may be assigned to a particular SKU and accept newly arriving items of a matching SKU. When the last item in a storage bin is removed, leaving the bin empty, the SKU assignment that previously existed for the bin is then erased, and the bin may be reassigned to any SKU as needed in the future. In the relevant literature, this storage policy is also referred to as random storage.

The new method for storage, called **location-relaxed storage**, permits even greater flexibility in storage. Although dedicated storage and shared storage enforce a one SKU per storage bin rule, location-relaxed storage removes this restriction. For example, compact discs of different titles could be stored in the same bin under location-relaxed storage, but such mixing is not permitted by dedicated storage or shared storage.

Each of these three policies has its advantages and disadvantages. On the surface, dedicated storage is the simplest methodology but is also the least flexible. Shared storage adds greater flexibility and benefits at the cost of a more complex implementation. Finally, the newest method, location-relaxed storage, is the most flexible and complex. The extent to which flexibility adds to overall effectiveness will depend on other aspects of warehouse operations. Furthermore, the cost of complexity will be influenced by the warehouse operations and level of technology implementation.

For simplicity, we refer to dedicated storage and shared storage as strict organization storage policies to contrast them collectively with location-relaxed storage. Strict organization storage is homogeneous as all the items in a single storage bin must have the same SKU number. Location-relaxed storage is heterogeneous since a single storage bin may contain multiple SKU numbers.

2.2.3 Placement Policy

Related to the storage policy is the *placement policy* which determines how incoming items are assigned to a storage bin and the method by which incoming items arrive at the assigned bin. While the storage policy determines the rules that govern which SKU may be assigned to which storage bin, the placement policy determines the actual assignment. The placement policy arises from the fact that not all storage bins are the same. Even if all storage bins have the same size and dimensions, the location of the bin effects the ease with which one can access it. For example, Heskett [1963] offered the cube-per-order index (COI) placement policy to determine product assignments for the dedicated storage policy. The COI equals the total required space divided by the turnover frequency. Having calculated the COI, Heskett locates items with a low COI closer to the dock with the understanding that locations closer to the dock are easier to access. As a result, the COI placement policy creates an assignment that matches storage bins close to the dock with high turnover items. In other words, COI matches popular items to favorable locations.

Another component of the placement policy is how the item is handled in order to reach the assigned storage bin. In essence, putting items away is the simple task of carrying the item to the storage bin and storing it there. However, how warehouse operations physically handle the items arriving as a group will affect overall efficiency as well and is also consider part of the placement policy. One example of this placement policy component is batch placement. Batching is a common method for putting items away that involves sorting the items into groups based on warehouse geography. The sorted groups can then be placed into the warehouse more easily since all items in the group need to go to the same general area within the warehouse.

2.2.4 Picking Policy

The opposite of placement policy is the *picking policy*, which determines which copy of an SKU should fill a particular need. Having stored the items in the warehouse according to the selected storage policy and placement policy, the final action of the warehouse requires the retrieval of items for order-fulfillment. At a minimum, the picking policy must determine which storage bins to visit and the order in which to visit them. For cases where there is only one storage bin for each SKU, bin selection is trivial; however, determining the path to follow in order to visit the required bins is more complex. Extensive research has been devoted to the path planning component of the warehouse order-picking policy. Sections 2.3 and 2.4 discuss order-picking and path planning in greater detail, including a discussion on the additional complexity involved when the number of storage bins for an SKU exceeds one.

Similar to the placement policy, the picking policy also goes beyond determining which item should be used to fill a need. The policy also entails the method by which picking a group of items should be handled. Such methodologies include zone picking, which is similar to batch placement. In zone picking, items within a predefined area are picked in the same group, then all picked groups are sorted into individual orders and shipments. Another methodology is order batching, which aggregates several orders into one picking batch to be picked during one pass through the warehouse.

The picking policy also can take additional information into account while determining which copy of an SKU should meet the needs of an order. For example, imposing a first-in-first-out (FIFO) restriction uses the SKU copy that has been in the warehouse for the longest time. FIFO restrictions enforce stock rotation, which is important for items that expire or degrade over time.

A large portion of research in warehouse operations focuses on order-picking, the process by which items are retrieved from the warehouse storage bins to fill the desired orders. Frazelle [2002] explains the importance of order-picking efficiency by noting that improvements to warehouse efficiency often target order-picking as the area capable of providing the greatest productivity improvements. Frazelle cites studies that show typically 63% of all warehouse operation costs are from order-picking. He also notes that new operating paradigms such as just-in-time and quick response as well as new marketing trends such as micromarketing and megabranding have introduced added difficulties to order-picking management. These changes have led to a greater number of smaller order deliveries and a higher number of SKUs in the warehouse system, which further complicates the order-picking process.

2.3 Traditional Storage

The prominence of warehouse order-picking in overall costs makes it a primary area of research. Much research focuses on the travel distance involved with order-fulfillment. In traditional warehousing, maintaining order within the warehouse shelves is essential for smooth operations. The first step in organizing the warehouse typically takes the form of storing items of the same SKU in the same bin (strict organization storage policy). In other words, *location encodes identity.*[1] Figure 2.1 illustrates typical strict organization used in warehousing. Identical colors indicate identical SKUs. Both dedicated storage and shared storage discussed in Section 2.1 are examples of strict organization. The order created and

1. "Location encodes identity" is a concept of Dr. David Brock of the MIT Auto-ID center.

maintained with strict organization is crucial for finding inventory and as well as preventing lost merchandise. These needs are critical to warehouse operations and have mandated the use of strict organization in warehouse operations for centuries.

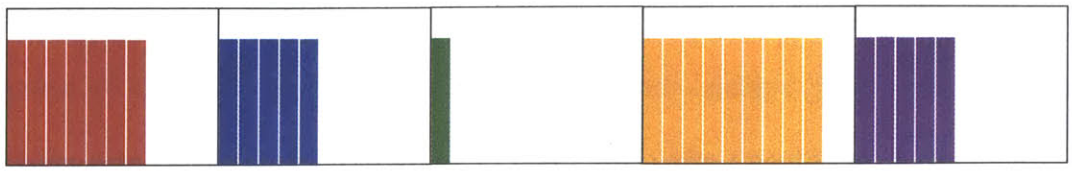


Figure 2.1: Strict organization for traditional warehouse storage

Strict organization storage relies heavily on order to meet the needs of warehouse operations. Deviations from order degrade the ability of the warehouse to operate properly. For example, the location encodes identity paradigm relies on maintaining a mapping between SKUs and storage bins. Given the task of finding a specific SKU, the mapping indicates where in the warehouse the desired SKU is stored. Placing a product within the warehouse that is inconsistent with the mapping prevents future retrieval as it creates a disconnection between where the item physically resides and where the mapping believes the item resides.

Inconsistency between the mapping and physical inventory adds inefficiency to warehouse operations because irretrievable items lead to the inability to fill orders. Occasional counting of products in the warehouse helps maintain map accuracy but costs additional effort. Maintenance is usually achieved through continuous cycle-counting and annual full physical inventory of the warehouse contents.

2.3.1 Strict Organization order-picking

Under a strict organization storage policy, there is generally a one-to-one correspondence between SKUs and storage bins. In the case where a particular SKU requires more storage space than a single bin allows, secondary storage bins handle the overflow but order-picking continues to use the primary bin exclusively. As a result, the mapping of an SKU to a storage bin remains one-to-one. For a given order, retrieval of items requires visiting specific storage bins. The decision that remains is what sequence should the order picker visit the required storage bins. A common optimization criterion for determining the picking route is the minimization of walking distance to visit the list of storage locations. The task of creating a minimum distance path that visits all the needed storage bins is a variation of the traveling salesman problem.

2.3.2 Traveling Salesman Problem

The traveling salesman problem (TSP) seeks to find the minimum cost tour for visiting n cities exactly once, where the cost of traveling from city i to city j is c_{ij}. This seemingly simple problem is very difficult to solve and belongs to the set of NP-hard problems [Karp 1972]. Consequently, no known polynomial time algorithm exists for solving the traveling salesman problem and computation times increase exponentially with the size of the problem.

In the case of warehouse order-fulfillment, the need for items translates directly to a need to visit a set of storage bins. By setting the cost between locations equal to the distance between locations the order-fulfillment problem becomes an instance of the TSP. Thus, the order-fulfillment problem must address the traveling salesman problem to obtain an optimal solution.

2.3.3 Warehouse-TSP

In the case of minimizing walking distance, the cost associated with traveling from location to location is simply the distance between locations. When the city-to-city transit cost equals the distance between the cities, the problem is called the Euclidean traveling salesman problem (Euclidean-TSP) and is also NP-hard [Papadimitriou 1977].

Unlike the general Euclidean traveling salesman problem, the warehouse traveling salesman problem (warehouse-TSP) has additional constraints. The locations to be visited must lie in the regular pattern found in the warehouse structure and walking is restricted to the aisles only. The rectilinear-TSP (also called the Manhattan-TSP) is a special case of the Euclidean-TSP where the distances between locations is the sum of travel along the two perpendicular axes rather than the euclidean distance. The rectilinear-TSP is also NP-hard [Garey 1976].

Typical warehouses are rectilinear, but the warehouse-TSP has one additional constraint. Not only is movement restricted along the x and y axes, the arrangement of the warehouse into rows of storage bins restricts movement along the aisles and movement from one aisle to the next may only occur at the cross aisles. Figure 2.2 illustrates the different forms of the traveling salesman problem. Unlike the Euclidean and rectilinear TSPs, the restricted movement found in the warehouse version of the TSP permits a polynomial time solution.

Ratliff and Rosenthal [1997] provide a solution to the warehouse-TSP using dynamic programming. Their presentation includes a detailed solution of the warehouse with no center cross aisles (Figure 2.3), where movement from one aisle to the next is restricted to the ends of the aisles; however, this technique can also extend to include additional cross

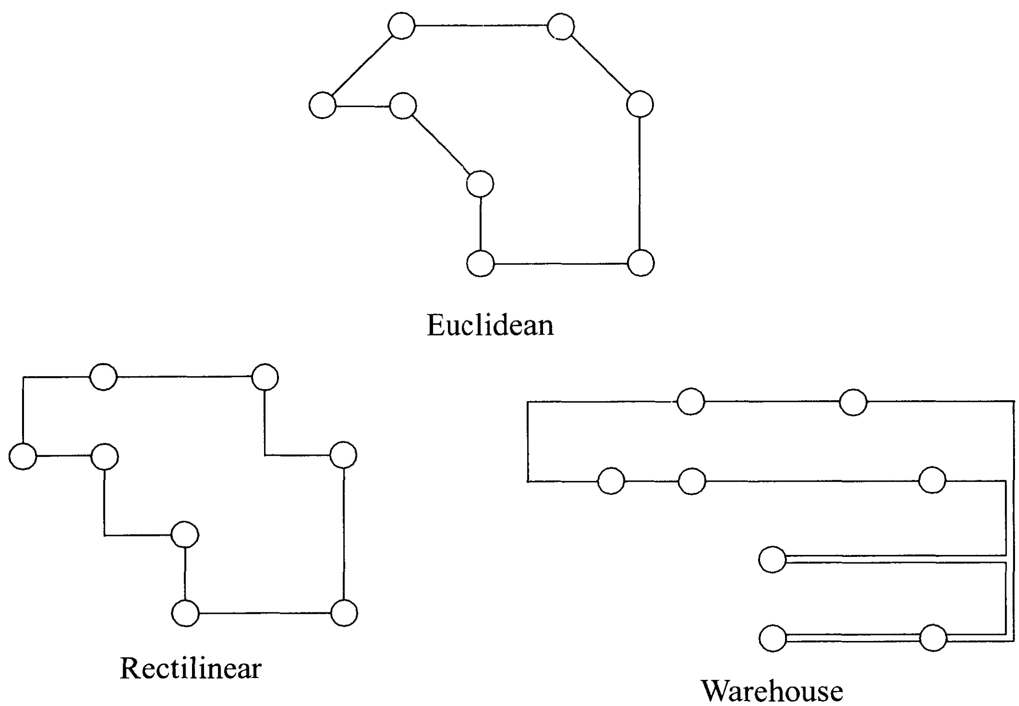


Figure 2.2: Traveling salesman problem tours

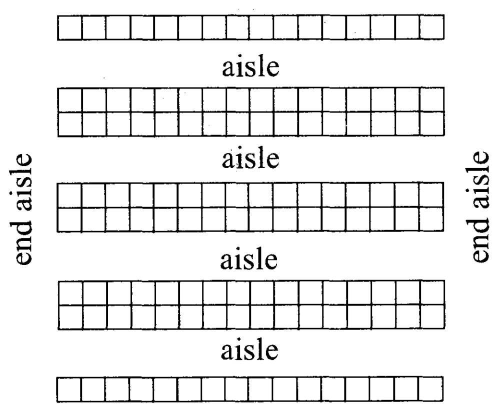


Figure 2.3: Warehouse layout with no cross aisles

aisles. When row crossing occurs only at the ends of the aisles, the configuration is also referred to as a single block. Figure 2.4 shows a warehouse configuration with one additional cross aisle, which permits movement between aisles at the midpoint of an aisle in addition to the two ends. With one additional cross aisle, the configuration is referred to as having two blocks. Roodbergen and DeKoster [2001a] show the solution for a mid-cross aisle warehouse and note that additional cross aisles (*i.e.* additional blocks) greatly complicate the solution. These techniques provide the means for establishing an optimal tour for warehouse order-picking when SKU to storage bins have a one-to-one correspondence.

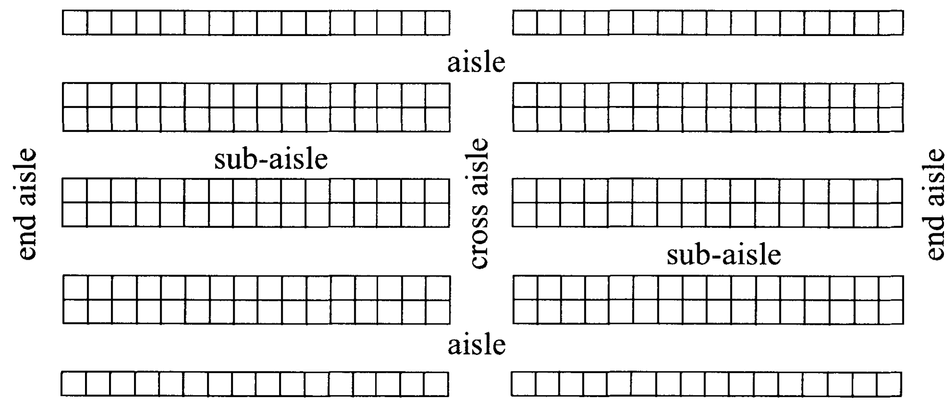


Figure 2.4: Warehouse with one cross aisle

2.3.4 Heuristics

Despite the existence of methods to compute optimal walking paths for warehouse locations, actual warehouse operations do not use the optimal solution and instead rely on routing heuristics. Petersen [1995] cites the non-intuitiveness and lack of consistency in optimal paths, which lead to confusion for human order pickers, as the reason heuristics are preferred over the optimal solution.

Petersen [1997] describes six routing strategies, including five heuristics and the optimal solution for a warehouse with no cross aisles (Figure 2.3). Figure 2.5 illustrates four of the heuristics investigated by Petersen. The heuristics include simple routes such as the transversal strategy where the picker traverses the entire length of all aisles that contain at least one pick as well as the return strategy where the picker always enters and exits an aisle from the same side. More complex heuristics include the midpoint strategy where the picker enters and leaves an aisle from the same side, but may also enter (and subsequently exit) both sides of the same aisle. The largest-gap strategy is similar to the midpoint strategy but allocates the break between sides by the greatest distance between two targeted locations rather than the midpoint. Finally, Petersen [1995] includes a fifth heuristic called the composite strategy that combines elements of the transversal and return strategies so that route construction has the option of either traversing the entire aisle or entering and exiting the aisle from the same side. Petersen reports that the largest-gap heuristic performs well when the number of picks is small; however with larger pick numbers, the composite heuristic yields the best performance of the heuristic solutions.

Roodbergen and De Koster address the situation where the warehouse has a single cross aisle [2001a] and multiple cross aisles [2001b] by providing heuristics and comparing performance against optimal solutions. Several of the heuristics are similar to the heuristics for a single block (no middle cross aisles), but are adapted to accommodate the additional cross aisles. The S-shape heuristic parallels the transverse strategy by having the picker walk through an entire sub-aisle that requires at least one pick. The largest-gap heuristic contains features that resemble the largest-gap strategy for the single block case, where typically the picker enters and exits a sub-aisle from the same side. The aisle-by-aisle heuristic, originally presented by Vaughan [1999], visits every aisle once and determines which cross aisle is best to access the next aisle. Finally, Roodbergen and De Koster present a combined heuristic that visits a sub-aisle containing at least one item exactly once; either the route will traverse the entire sub-aisle (transversal / S-shape) or the route will enter and exit the sub-aisle from the same side (return strategy).

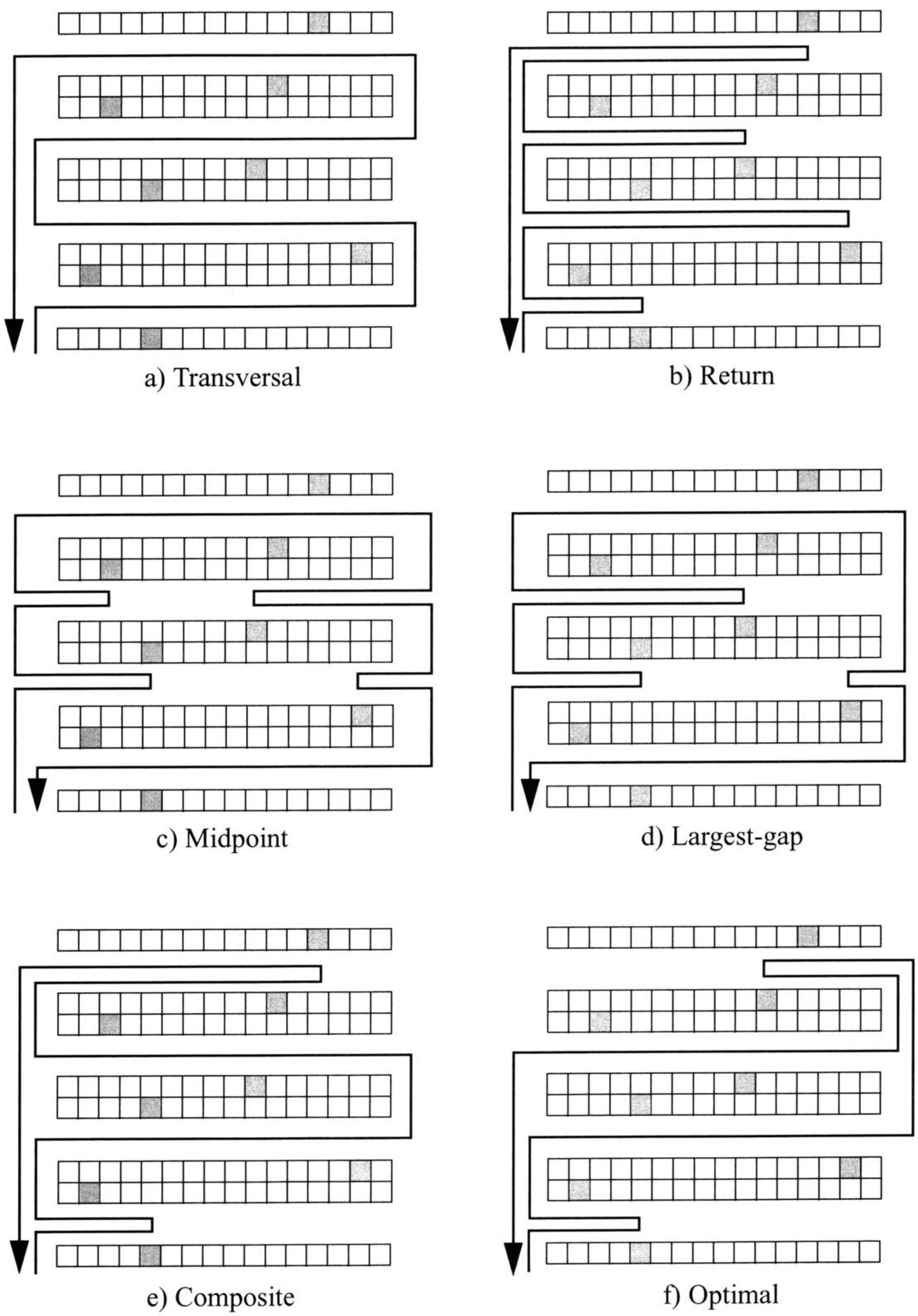


Figure 2.5: Order-picking routing strategies for warehouses with no cross aisles

Roodbergen and De Koster's evaluation of the heuristics with multiple cross aisles showed that the S-Shape heuristic never yields the best performance of the heuristic methods. The authors note that the combined heuristic considers the S-shape solution in its search; therefore at best, the S-shape may only equal the combined heuristic solution and never surpass it. They also note that the aisle-by-aisle heuristic and the combined heuristic produce the same results when the number of cross aisles is limited to the two end cross aisles (single block). For warehouses with three or more cross aisles, the combined heuristic gave the best results.

2.4 Location-Relaxed Storage

Traditional warehouse techniques also include an exception for low demand items called library shelving. Since allocating an entire storage bin for a low demand item can be very wasteful of storage space, library shelving assigns all low demand items into the same storage bin or bins. This mixing is similar to libraries where books of different titles are placed one after the other. The result saves considerable space at the cost of making the retrieval of the items more difficult since the picker must sift through an array of items rather than a bin of matching items to find the correct one. In other words, the task of finding a specific item in a mixed bin is harder than simply finding the bin. However, since library shelving is a special case for low demand items, the added difficulty encountered in picking is small and easily compensated by savings in storage space.

Extending library shelving to the entire warehouse is not traditionally done in warehouse operations. We designate the storage policy where all SKUs are subject to library shelving as location-relaxed storage. Unlike designated storage and shared storage, location-relaxed storage does not match a storage bin to a particular SKU. Under location-relaxed storage, *any item* may be placed in *any location* at *any time*. Furthermore, the correspondence between SKU and storage bins is no longer one-to-one. Figure 2.6 illustrates location-relaxed storage.

Figure 2.6: Location-relaxed storage

In addition to storage space advantages, location-relaxed storage offers a simplified storage placement process. Rather than having to seek out a specific storage bin corresponding to an SKU, location-relaxed storage permits storage in any bin that is not full. As a result, this simplified storage process leaves SKUs of the same type in many different storage bins as well as SKUs of different types within the same bin.

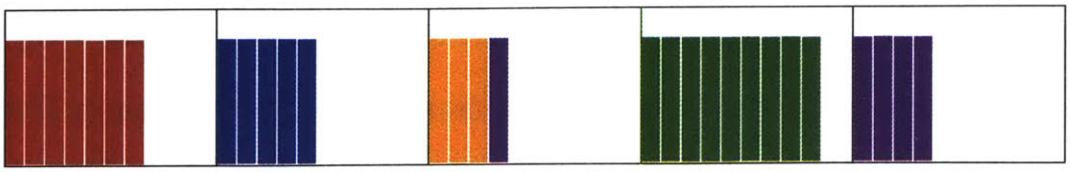
Despite the obvious advantage in storage space utilization and the possible advantages in storage and retrieval efforts, location-relaxed storage is not generally used by warehouses.[1] Location-relaxed storage, as described so far in this section, has serious disadvantages that outweigh the possible advantages. The main disadvantage is sensitivity to errors. If warehouse operations were 100% accurate, then location-relaxed storage would function smoothly and all the advantages of location-relaxed storage could be realized. However, operations are never fully error free; mistakes in placement are a common occurrence in warehouse operations. The warehouse database that tracks all items in the warehouse assumes that all items are placed correctly, and that item location

1. The vast majority of warehouse operations use strict organization storage. Few exceptions exist such as Amazon.com's implementation of a barcode based location-relaxed storage system.

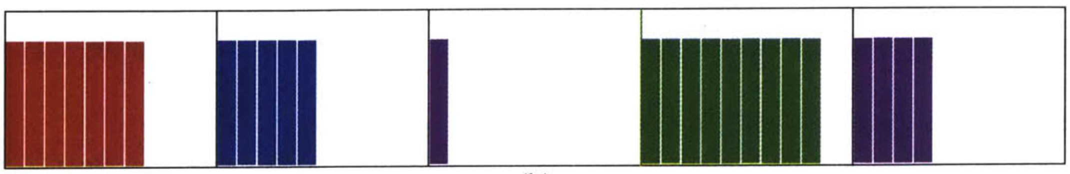
remains static while stored in the warehouse. These two assumptions are fundamentally unrealistic. Items placed into the warehouse are often placed in the wrong storage bin, placement can sometimes fail to occur, and items may unintentionally migrate from one location to another. As a result of these errors, the database that maps SKU to storage bins becomes out of synchronization and inconsistent with where the items actually are. Regardless of whether the storage policy is strict organization or location-relaxed, a misplaced item is not useful for order-fulfillment.

The difference in how errors impact the two storage policies, strict organization and location-relaxed, lies in the ease with which operations can deal with errors. In the strict organization case, a misplaced item will likely be visually detectable. Figure 2.7 illustrates two possible cases of a misplaced item in a strict warehouse. Figure 2.7a shows a mix of items in the third storage bin. Warehouse workers can identify a problem with the items when noticing a commingling of items. Figure 2.7b shows a case where detection of an error is not as easy where the item has been misplaced into a previously empty bin. Visually, there appears to be nothing amiss. The error will not be detected until the warehouse system directs a different SKU into the third storage bin, or an audit of the warehouse contents finds the mistake. With location-relaxed storage, visual detection of misplaced items is not possible. Since all storage bins may contain a mix of SKUs, a misplaced item will not stand out as incorrect. More direct means are required to detect mistakes in location-relaxed storage.

To maintain order, warehouses rely on cycle-counting and occasionally taking a full physical inventory of the warehouse. Furthermore, these item-by-item checks maintain synchronicity between the warehouse SKU-to-storage bin mapping and the warehouse



(a)



(b)

Figure 2.7: Misplacement in a strict organization setting

Incorrect placement (a) of the purple item into the center storage bin is visually detectable due to the conflict with the orange items. Incorrect placement (b) of the purple item into the center storage bin is not noticeable since there are no other items in the bin.

physical inventory. When strictly organized, cycle-counting an item involves visiting the storage bin(s) assigned to a particular SKU and counting the number of items in each bin. The cycle count can then be compared to the database and appropriate adjustments then maintain correspondence. Under location-relaxed storage, database maintenance becomes more arduous. Two approaches are possible, neither of which is simple. If one tried to count the number of copies of each SKU in the warehouse, the counter would have to visit many storage bins, and search each bin to verify the existence of the SKU in that bin. If one tried to count bin-by-bin, then the counter must determine the number of each SKU in the bin, of which there will be many. Furthermore, reconciliation of discrepancies between the physical inventory and the database requires greater detail in the location-relaxed storage case than in the strict organization case. In strict organization, the database assigns an SKU to each location and tracks the number of copies of the SKU in each location. Reconciliation between database and physical count involves simply changing the number of copies to match the physical count. Under location-relaxed storage, the database must track a list of SKUs in each location and the number of copies of each SKU found in the storage bin. This listing alone is more complex than the analogous list for strict organization. Furthermore, the location-relaxed listing requires an elaborate reconciliation process that must verify all copy counts as well as add and delete SKUs found/not found during the counting.

As a result of the high maintenance costs involved with keeping order with location-relaxed storage, the benefits of greater storage space utilization and distributed storage are severely diminished. In the case of library shelving, the application of mixed storage to a small subset of the SKUs rather than the entire SKU population mitigates the issues that make mixed storage undesirable. Furthermore, limiting library shelving to low demand items further diminishes the negative aspects involved. Although the problems with location-relaxed storage have prevented warehouses from applying location-relaxed storage in the past, Section 2.5 describes a technological advancement that makes location-relaxed storage a feasible option for the entire warehouse operation.

2.4.1 Traveling Purchaser Problem

All previous literature on routing for order-picking focuses on the warehouse version of the traveling salesman problem in which the required warehouse locations to be visited are determined in advance. With the introduction of location-relaxed storage, a slight change in the problem statement leads to significant changes to the solution and solution technique. Instead of listing the required locations definitively, the option of several storage bins provides additional choices that must be addressed in pick routing and optimization. This change greatly alters the solution process for both optimal and heuristic route planning.

The optimization problem,

> Given a set of markets, $M = \{1, \ldots, m\}$, a set of commodities $N = \{1, \ldots, n\}$, the cost of travel between market i and market j, c_{ij}, and the cost of product k at market j, d_{kj}; find a tour of markets that purchases one of each n products with minimum total travel and purchasing cost.

was labelled the Traveling Purchaser Problem (TPP) by Ramesh [1981]. The problem is similar to the Traveling Salesman Problem but contains additional decisions. In the TSP, route selection requires determining the visiting sequence of the markets (cities). For the TPP, in addition to sequencing the market locations, the route planner must also determine which market of many should be visited to satisfy the commodity needs. The TPP reduces to the TSP when each commodity is sold exclusively at one market.

Table 2.1 shows the effect of multiple locations on the number of feasible solutions to the TPP with the assumption that all n items are unique. The traveling salesman problem with n cities has $n!$ possible routes. In other words, there are $n!$ possible orderings of n items. Expanding the problem to the traveling purchaser problem, the number of possible routes increases. If there are two locations that satisfy each of the n products then the number of possible routes becomes $2^n \cdot n!$. 2^n describes the selection of locations; since each item appears in two distinct locations and there are n items, then 2^n combinations of locations are possible. In general, when there are n items to be picked and each item appears in k unique locations, the number of routes to consider is $k^n \cdot n!$.

Table 2.1: Possible combinations for the traveling purchaser problem

	TSP	Traveling Purchaser Problem		
	1 Location per SKU	**2 Locations per SKU**	**3 Locations per SKU**	**k Locations per SKU**
1 pick	1	2	3	k
2 picks	2	$2^2 \cdot 2$	$3^2 \cdot 2$	$k^2 \cdot 2$
3 picks	3!	$2^3 \cdot 3!$	$3^3 \cdot 3!$	$k^3 \cdot 3!$
***n* picks**	$n!$	$2^n \cdot n!$	$3^n \cdot n!$	$k^n \cdot n!$

Shortly after Ramesh formalized the traveling purchaser problem, Ong [1982] presented four computationally fast approximation solutions for the Traveling Purchaser Problem. Singh and van Oudheusden [1997] provide an optimal solution via a branch and bound algorithm. Improved approximation solutions are presented by Pearn and Chien [1998].

2.4.2 Warehouse-TPP

The general TPP describes the location-relaxed warehouse retrieval problem when the costs of travel, c_{ij}, equals the walking distance between storage bins; the purchase costs, d_{kj}, are zero when the product is available in the storage bin j; and the purchase cost is very large (infinite) when the product is not available in the storage bin.

$$
\begin{aligned}
c_{ij} &= \text{walking distance from storage bin } i \text{ to storage bin } j \\
d_{kj} &= \begin{cases} 0, \text{ when item } k \text{ appears in storage bin } j \\ \infty, \text{ otherwise} \end{cases}
\end{aligned}
\tag{2.1}
$$

General TPP heuristics, discussed in Section 2.4.1, could be used for the Warehouse-TPP, since the Warehouse-TPP has a more specific definition than the general TPP.

Unfortunately warehouse-TSP heuristics that address the warehouse structure in route construction, such as those discussed in Section 2.3.4, do not extend directly to the TPP case. Even with a heuristic for determining visitation order, the number of choices for which location to visit remains very large. Short of randomly choosing locations and then applying warehouse-TSP heuristics, the heuristics outlined in Section 2.3.4 offer little help in establishing a Warehouse-TPP heuristic. A method could be constructed by combining the warehouse-TSP heuristic with a separate location selection heuristic. However, the problem then shifts to finding a proper selection method for the locations to choose. Chapter 3 discusses the Warehouse-TPP in greater detail, including simple heuristics for Warehouse-TPP solutions for comparison to strict organization (warehouse-TSP) solutions.

2.5 RFID Technology

Due to the critical need for maintaining order within a warehouse, strict organization is the logical choice over location-relaxed storage. By aggregating and segregating SKUs the subsequent database is a simple mapping between SKU and storage bin. This "location encodes identity" paradigm provides the necessary structure with very simple maintenance.

New technologies, however, enable an escape from the "location encodes identity" philosophy of strict organization. Instead of inferring an object's identity based on the storage bin, new identification methods permit identity to be ascertained automatically and independent of location. Furthermore, these new technologies simplify database management through automated rather than manual maintenance. As a result, warehouses no longer need a well organized structure for the sake of maintenance. The new-found insignificance of location-to-SKU correspondence, which previously had been a guiding principle in warehouse design, changes the rules under which warehouses must function.

Radio frequency tags (RF-tags) are one example of an automatic identification enabling technology. An ongoing, industry-wide effort to place RF-tags on consumer products supports the idea of using RF-tags for identification in warehouse operations. Unlike traditional bar-code labeling, RF-tags do not require line-of-sight scanning and can be read automatically without being re-oriented and without manual assistance. This significant advantage over bar-codes eliminates the requirement for either human manipulation to get the scanner close to the marking, or elaborate automation via conveyor-systems to move objects past a scanner in a very controlled and predictable manner. RF-tags have other advantages as well; each RF-tag has sufficient memory to provide each individual item with a unique identity code.[1] For this reason, the technology is referred to as Radio Frequency Identification (RFID).

Implementation of RFID technology in warehouse operations is expected to provide several benefits. First, RFID will directly alleviate the problem of database inconsistency. In traditional warehousing, information updates occur when the system directs an item to be moved. Between initial placement and ultimate removal, the system assumes that the item remains static. Furthermore, traditional methods assume placement is always correct. These two fundamental assumptions, however, are incorrect. Misplacement and intermediate movements, while undesirable, occur regularly in warehouse operations and create inconsistencies between where the system believes items are stored and where

1. Universal product codes, which label items of the same type with the same barcode identifier, are not able to distinguish one copy of an SKU from another of the same SKU. RF-tags have a much greater number of unique labels and can assign a unique identifier to each item. Thus mass serialization of all products enables the system to distinguish one copy of an SKU from another.

items actually reside. Active observation of RF-tags unifies what is believed to be true with what is actually true. RF-tag readers continuously note each item's presence, identity, and location and detect all movements whether intended or not. The database remains up-to-date at all times and the system immediately 'finds' any 'lost' item.

Automatic database consistency opens new opportunities in warehouse operations. The main motivation for using strict organization over location-relaxed storage was the need for database integrity, the sensitivity of location-relaxed storage to placement errors, and the difficulty of maintenance with location-relaxed storage. RFID technology, however, addresses these issues and permits access to all of the advantages of location-relaxed storage.

Under strict organization, the requirement of putting items into specific storage locations increases the storage effort since additional walking is necessary to visit *specific* locations. Organization requires effort. Greater flexibility for item placement under location-relaxed storage leads to a reduction in the effort required to place items in the warehouse, since placement into any location is easier than placement into a specific location. Although placing items in a seemingly random manner will lead to items stored at unpredictable locations, the RF-tag on each product compensates for the lack of organization.

On the order-fulfillment aspect of warehouse operations, location-relaxed storage changes the dynamics of order-picking. Under strict organization the warehouse operates under a strongly organized and strongly aggregated mode. With location-relaxed storage aggregation is ignored. As a result, warehouse storage becomes distributed with a particular SKU appearing in many storage bins rather than just one or two. Holding many copies of the same SKU in the same storage bin is a benefit when picking many items of the same type at the same time. When copies of the same SKU reside in many disparate locations, picking many copies of the same SKU requires one to visit many different storage bins instead of just one.

Conversely, storing copies of the same SKU in many different locations provides opportunities ideal when picking a single copy of an SKU at a time. As the number of copies of the same SKU picked increases, the benefit of aggregating items of the same type into the same location increases and the benefit of distributed storage decreases. Above some picking number, traditional storage techniques will perform better than location-relaxed storage. Determining the criteria that delineate which storage method offers the best performance is one of our research goals.

Additionally, the location-relaxed method increases the operational capacity of the warehouse. Strict organization dedicates a particular shelf of the warehouse to a specific item type. Once an item occupies a particular shelf, only items of the same type may be stored there. This restriction, which is necessary under traditional circumstances to maintain database consistency, wastes space that could be used by other products. With RF-tag technology, the relaxed storage method removes the restriction and permits items of different types to reside on the same shelf. This reclaimed space permits more usable storage space than a traditional warehouse of the same size.

New technology combined with location-relaxed storage has the potential to greatly reduce warehouse operation labor costs. The greatest impact will likely occur in situations where the picking lot sizes are small such as in e-Commerce and other direct to consumer situations. In these situations the warehouse operates with a large 'step-down ratio,' where items enter the warehouse in large groups of the same product and exit in much smaller groups.

Tables 2.2 and 2.3 show the characteristics of strict organization and location-relaxed storage with and without the benefits of RFID technology. The major problems with location-relaxed storage without RFID technology are shown in red in Table 2.2. A complex accounting system for tracking materials in the warehouse, high sensitivity to inevitable placement errors, and a difficult process for inventory counting make location-relaxed storage without RFID technology highly undesirable. Table 2.3 lists the operation characteristics with the benefit of RFID automated monitoring, which ameliorates each of the major problems with location-relaxed storage.

Table 2.2: Storage methods without RFID technology

Strict Organization	Location-relaxed Storage
Simple database	Complex database
Location encodes identity	Identity independent of location
Specific location for each item	Place any item in any location at any time
Misplacement is bad, but visually detectable	Misplacement is extremely bad, and not visually noticeable
Limits free space due to identity restrictions	Liberated use of free space
Well ordered	No order
Simple physical inventory and cycle-counting	Very difficult for physical count

2.6 Complexity

As mentioned in sections 2.3 and 2.4, the traveling salesman problem and the traveling purchaser problem are NP-Hard. To show how warehouse graph-related optimization problems compare to other similar optimization problems, Table 2.4 delineates several graph related combinatorial optimization problems and indicates the complexity of each.

Table 2.3: Storage methods with RFID technology

Strict Organization	Location-relaxed Storage
Automatic simple database	Automatic complex database
Location encodes identity	Identity independent of Location
Specific location for each item	Place any item in any location at any time
Misplacement is bad, but automatically detected	'Misplacement' has no effect
Limits free space due to identity restrictions	Liberated use of free space
Well ordered	No Order, Organization enforced by tags
Simple physical count Very simple electronic count	Very difficult physical count Very simple electronic count

Each column corresponds to a different optimization problem on a graph $G = (V, E)$, where V is a set of vertices, E is a set of edges connecting two vertices, and c_{ij} is the cost associated with the edge connecting vertex i to vertex j. The first column, minimum spanning tree (MST), refers to the optimization problem of building a tree that connects all vertices V with a subset of the edges E such that the total cost of the edges is minimized. The second column, Steiner tree, is similar to the minimum spanning tree problem except the set of vertices to be connected is a subset S of the vertices V. The steiner tree may or may not include vertices that do not belong to S. The third column, traveling salesman, refers to the optimization problem described in greater detail in Section 2.3.2. The traveling salesman problem must construct a tour of the vertices such that the sum of the connecting edges' cost is minimized. Finally the fourth column, traveling purchaser problem, is described in greater detail in Section 2.4.1.

Each row corresponds to a variation on the optimization problem for each column. General refers to the most general form of the problems as described in the previous paragraph. Euclidean restricts the edge cost to the euclidean distance between two vertices. k or prize collecting refers to the case where k of the vertices must be connected but it does not matter *which* vertices are connected. Rectilinear restricts the edge cost to the distance between the two vertices along two perpendicular axes. And warehouse refers to edge costs that are rectilinear and restricted by warehouse barriers so that travel distance must follow the aisles and cross aisles of a warehouse. k-rectilinear and k-warehouse combine the attributes of prize collecting with the added constraints of rectilinear path lengths and warehouse barriers.

Table 2.4: Combinatorial Optimization Problems

	Minimum Spanning Tree (MST)	**Steiner Tree**	**Traveling Salesman Problem (TSP)**	**Traveling Purchaser Problem (TPP)**
General	Polynomial Time	NP-Hard	NP-Hard (Karp 1972)	NP-Hard
Euclidean	Polynomial Time	NP-Hard	NP-Hard (Papadimitriou 1977)	NP-Hard
k (prize collecting)	NP-Hard (Ravi 1994)		NP-Hard (Balas 1989)	
rectilinear	Polynomial Time	NP-Hard (Garey 1976)	NP-Hard (Garey 1976)	NP-Hard
warehouse	Polynomial Time		Polynomial Time (Ratliff 1983)	*Location-relaxed fulfillment problem*
k-rectilinear			NP-Hard	
k-warehouse			*Location-relaxed placement problem*	

For example, the general-MST is solvable by a polynomial time algorithm.[1] Furthermore, because the general-MST has a polynomial time algorithm, the Euclidean-MST, rectilinear-MST, and warehouse-MST also have a polynomial time algorithm since the general-MST solves the restricted cases as well. However, the k-MST problem asks for

1. Kruskal's algorithm and Prim's algorithm solve the MST problem in polynomial time [Cormen 1999].

the construction of a minimum cost tree that connects any k vertices of V. Furthermore, the k-MST problem is NP-Hard as shown by Ravi *et al.* [1994].

Under strict organization storage, warehouse fulfillment and item placement relate to the warehouse-TSP which was shown to have a polynomial time solution by Ratliff and Rosenthal [1983]. Under Location-relaxed storage, warehouse fulfillment is an instance of the Warehouse-TPP, and warehouse item placement is an instance of the k-warehouse-TSP. Chapter 4 discusses location-relaxed storage, the Warehouse-TPP, and the k-warehouse-TSP in greater detail.

2.7 Comparisons to Disk Drive Fragmentation

The topics of storage and fragmentation naturally bring forth comparisons to disk drive operations. Since both warehouses and hard disks place, store, then retrieve as their basic operations, the analogy between warehouses and hard disks would seem quite appropriate. Furthermore, the fragmentation involved with location-relaxed storage naturally leads to additional comparisons with hard disk fragmentation. In fact, hard disk data storage is a type of location-relaxed storage since any data can be stored on any part of the disk. However as similar as the basic operations are between warehousing and hard disk operations, similarities between the operations' execution and fragmentation issues are more tenuous.

2.7.1 Hard Disk Drives

Hard disks store information by writing file data to the hard disk surface. The disk is divided into blocks and files typically occupy more than one block. In order to write data to the blocks, a servo must position the write-head above the target block. This process is similar to warehouse placement, however the motion of the write head is not constrained while warehouse motions are constrained to the aisles.

Hard disk fragmentation and warehouse fragmentation are fundamentally different. Ideally, the hard drive writes a file to the disk along adjacent blocks. When there is no series of empty blocks large enough to store the file, hard drive fragmentation breaks the data set (file) into two or more pieces to fit the data into the available, but smaller, spaces on the hard disk. In a sense, hard disk fragmentation improves the storage space utilization, which is similar to the result of location-relaxed storage in warehousing. While these two methods achieve similar results, the circumstances and methodology are significantly different.

Location-relaxed warehouse fragmentation results from the fact that any item may be placed in any location, which results in the copies of an SKU residing in disparate storage bins across the warehouse. This fragmentation of the SKU copies contrasts the non-fragmented and aggregated result of strict organization storage.

Hard drive fragmentation breaks up the stored data, which is significantly different from warehouse storage where *whole* items are stored and breaking an individual SKU is not permitted. Furthermore, hard drive storage does not encounter multiple copies of the same data. Even if a file is duplicated, separate copies are treated as unique and no association between the copies exists.[1] Therefore, fragmentation refers to totally different processes in warehouse operations and hard disk operations.

---

1. With location-relaxed storage, retrieval of an SKU may be met with any copy of the needed SKU. With hard disk data, even if a copy of a file exists, the call for one of the copies refers to that specific copy. Substituting another copy is not permitted.

Fragmentation achieves the similar goal of opening up more capacity for warehouse and hard disk operations, but the methods and structure of the problems are so different that direct comparison of the two is not appropriate. Furthermore, fragmentation in hard disks is undesirable, while several analyses in Chapters 4 and 7 will show evidence that warehouse fragmentation can be an asset.

2.7.2 Redundant Arrays of Inexpensive Disks

Another disk storage application also has some similarities to warehouse operations. Redundant Disk Arrays or Redundant Arrays of Inexpensive Disks (RAID) use several disks to increase fault tolerance and improve performance for data storage [Gibson 1992]. The aspect of redundant disk arrays that draws comparisons to location-relaxed warehouse operations includes data striping, which is the process of interleaving data over the many disks in the array to improve access performance. Data striping, which is also a type of intentional fragmentation, breaks a file into pieces so that some parts of the file are on each of the disks in the disk array.

Breaking up data across multiple disks can improve performance of the disk array by enabling and encouraging parallel access to the data. However, the analogous process in warehouse operations is not the fragmentation of warehouse SKUs, but implementing multiple pickers. The key to greater throughput is parallelism. In the case of redundant arrays, fragmentation is the means of enabling parallel data access. In warehousing, parallel picking simply requires additional workers.

In warehouse picking, a process similar to load balancing is a concern for planning. Directing multiple pickers to the same aisle at the same time, while not forbidden, is undesirable. The congestion of multiple pickers in the same area can slow down efficiency. Similarly, in redundant disk arrays, multiple reads from the same disk is not physically possible. Fragmentation via data striping assists in load balancing.

Furthermore, redundancy in disk arrays is not the same type of redundancy found when warehouses hold multiple copies of the same SKU. The redundancy comes from a parity bit among the disk array. Storing redundant SKUs in different locations permits the picker to choose from several locations to fill the order needs. The redundancy found in disk arrays does not permit this type of choice. The parity bit enables the reconstruction of lost data. By reading the corresponding bits on the intact drives and the parity bit, the system can reconstruct the data on the malfunctioning drive. Therefore, the redundancy is only useful for protection from disk failure and not to increase productivity.

While similar to physical material storage issues, hard drives are not directly equivalent. Increased productivity with disk arrays does not come from storing multiple copies of the same data. Rather, disk arrays exploit the parallelism of multiple drives to read more data at a time. The analogous warehouse employment would be multiple workers, where increased rates of picking and placement are achieved by having more people working. Furthermore, in the case of redundant disk arrays, the increase in data reading rates from parallel operations comes at a cost of slower writing rates since redundancy requires multiple writing operations. Despite the similarities in vocabulary, drawing analogies between material storage and disk drive operations is not appropriate.

2.8 Related Optimization Opportunities

While much research falls into the category of route planning for order-picking, other issues also influence warehouse fulfillment efficiency as well. This section outlines additional optimization problems that affect warehouse performance. The order batching problem is related to the more common vehicle routing problem and describes the optimization opportunity of wisely pairing orders together for picking. The bin packing problem also has relevance to warehouse operations since storage bin capacities and worker carrying capacities are finite.

2.8.1 Order Batching and Vehicle Routing

The vehicle routing problem (VRP) optimizes the set of routes that collectively visit all target locations. With the TSP, one traveller must visit each city in a single tour. In the VRP, several travellers work together to visit all the cities via several separate tours. For additional information on the vehicle routing problem refer to Toth [2002].

Aspects of the vehicle routing problem arise in warehouse fulfillment when designating many pick routes to retrieve items for order-fulfillment. Typically the load of many orders are broken into smaller batches to accommodate the maximum capacity of the order picker, which is similar to the Capacitated Vehicle Routing Problem (CVRP). order-picking is similar to vehicle routing since both boil down to a division of labor; however, the differences between vehicle routing and order-picking can be significant depending on the order-picking policy.

When the order-picking policy uses a pick and sort technique, where all items are treated individually regardless of the order to which they belong, the order-picking process is equivalent to the capacitated vehicle routing problem. The collection of orders translates to a list of items which corresponds to a list of locations that must be visited by one of the many pickers. After picking, the system sorts the mass of items into individual orders.

When the order-picking policy uses batch picking, where members of an orders must be picked together and not separated, the optimization problem takes a step away from the traditional vehicle routing problem. Maintaining whole orders within each batch requires additional constraints not usually found in vehicle routing. These constraints allow fewer feasible solutions for batching than with pick and sort.

This thesis does not address order batching optimization. All simulations, whether using strict organization storage or location-relaxed storage, batch pick by randomly grouping orders into composite batches. See Chapter 4 for further discussion of order batching and its role in warehouse fulfillment optimization as well as the significance of ignoring order batching optimization.

2.8.2 Bin Packing Problem

The bin packing problem is another optimization problem that appears in warehouse operations: given a set of n objects with sizes $l_1, l_2, l_3, \ldots, l_n$ where $0 < l_i < 1$; find the minimum number of unit size bins needed to pack all the objects. The bin packing problem is also NP-Hard, but Martello and Toth [1990] provide effective solutions for problems with hundreds of items.

Typically the bin packing problem applies to warehouse operations as part of the capacitated vehicle routing problem and order-picking. With variable sized objects and a finite capacity for each order picker, designating items to a particular picker depends not only on the relative locations of the items (TSP or TPP) but also the sizes of the items and maximum capacity of the order pickers.

The optimal solution to the capacitated vehicle routing problem does not necessarily use the minimum number of bins (vehicles). The main concern for the CVRP regarding bin packing is that the number of vehicles at least equal the minimum necessary for bin packing in order to maintain a feasible solution. Otherwise, bin packing is not explicitly addressed in the CVRP.

In the location-relaxed storage case, multiple SKUs in a single storage bin establishes another instance of the bin packing problem. When all items in a storage bin have the same SKU, as is the case with strict organization, selection of bins is restricted by type and no bin packing choices are possible. Furthermore, all the items in a strictly organized storage bin have the same size. With location-relaxed storage, items may be placed in any location and a mixing of items makes the bin packing problem relevant. As a result, the relative sizes of items in a storage bin affects how many items may be placed there. This thesis avoids the bin packing problem by considering only the case where all SKUs have equal size.

2.9 Thesis Overview

The advances in RFID technology and growing interest by manufacturers and retailers to use RFID technology for a variety of applications from supply chain management to automated checkout to theft detection and avoidance makes the ubiquitous use of RFID technology a matter of time. RFID technology and the systems developed at the MIT Auto-ID Center will enable a wide variety of applications to increase productivity and decrease waste during a product's lifetime. This research focuses on the impact of RFID technology on warehouse operations; specifically centering on RFID as an enabling technology for location-relaxed storage. The research makes the case for location-relaxed storage as a valuable alternative to traditional storage techniques, especially for direct to consumer warehouse operations as found in eCommerce.

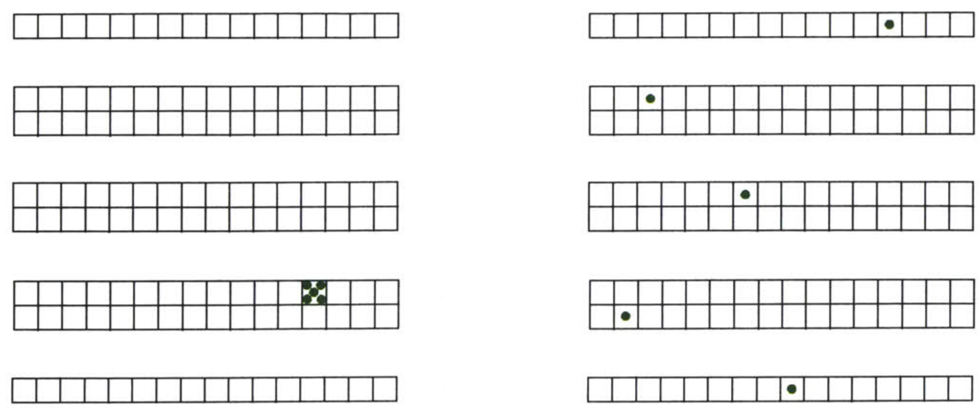
Chapter 3: Location-relaxed Storage

In the previous discussion of location-relaxed storage, one of the proposed benefits is a reduction in the route walking distance because of the availability of multiple locations for satisfying an SKU need. This chapter quantifies the benefit of multiple locations associated with location-relaxed storage by first analyzing the impact of multiple locations on the retrieval of a single item from the warehouse. The analysis then extends to routes that involve the retrieval of more than one item, which necessitates a closer look at the Warehouse-TPP. The findings for both, single picks and routes with more than one pick, show that multiple locations reduce the route walking distance. Furthermore, the most significant reduction occurs between having one location and two locations for filling the SKU needs.

3.1 Multiple Locations: Single Pick Analysis

A strict organization storage policy tends to minimize the number of storage bins that possess a particular SKU. Typically in strict organization, all copies of a particular SKU are found in a single storage bin. Under a location-relaxed storage policy, the freedom to place any item in any location tends to create multiple sites for a particular SKU. Figure 3.1 illustrates an example for a particular SKU under strict organization and location-relaxed storage. Multiple locations provide more routing choices when picking items. These additional choices may translate to a more efficient order-picking because path planning for order-picking can take advantage of the multiple locations in the search for the shortest route.

With a few assumptions, an analytical comparison of the cost of picking a single item when found in one location versus two locations reveals the potential for order-picking



a) Five copies of an item in a single location b) Five copies of an item in five locations

Figure 3.1: Comparison of multiple SKU copy distribution

savings in warehouse operations. In the single pick analysis, the cost associated with a particular location is the round trip walking distance of the location. Unlike the multiple pick case,[1] where the route length depends on other items in the picking list, the cost of picking a single item is easily computed and is constant for a particular location.

3.1.1 Continuous Approximation

Consider the case where a product appears in a single location of the warehouse and the distribution of walking distance costs for all locations of the warehouse varies between some minimum value a and some maximum value b. Furthermore, assume that random selection of a location will yield a corresponding walking distance that has an equal likelihood of being any value between a and b. In other words, approximate the corresponding walking distances as a constant probability distribution between minimum distance a and maximum distance b. Even though the model uses a continuous distribution to represent what should be a discrete distribution, the analysis will provide useful

1. Multiple item picking routes are also referred to multi-address picking.

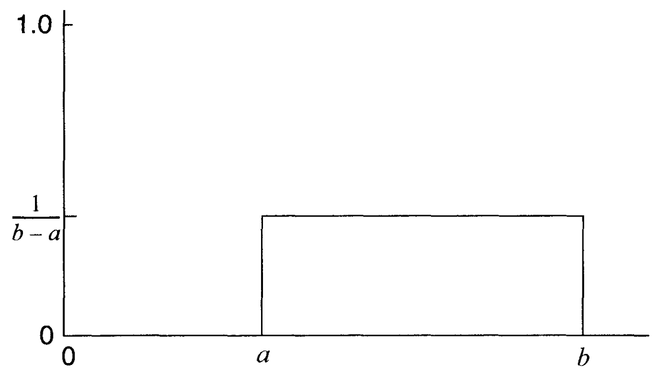


Figure 3.2: Single location walking distance probability density function

The distribution shown indicates all path lengths are between a and b units long. Furthermore, the probably of a path length is equal for all values between a and b.

information regarding the effect of multiple locations on expected walking distances. Figure 3.2 illustrates the probability density function of walking distances for the model when picking a single item with a single choice of storage location. In this case, the expected value of the walking distance when choosing one location at random is the average of the maximum and minimum walking distances:

$$E_1 = \frac{a+b}{2} \tag{3.1}$$

If multiple locations are to be considered for path planning, then the minimum walking distance of the possible choices fills the need of the single pick and yields the shortest route. When only one location is available to fulfill the need, the path length is equal to the round trip distance associated with the location. When multiple locations can fulfill the need, the path length is the *minimum* distance of the possible location choices.

When two locations meet the single pick need, the model chooses the location that has the shorter walking distance. Although the distribution of location distances is the same, the probability density function when considering the minimum of two randomly selected locations is not constant. Figure 3.3 shows the probability density function when

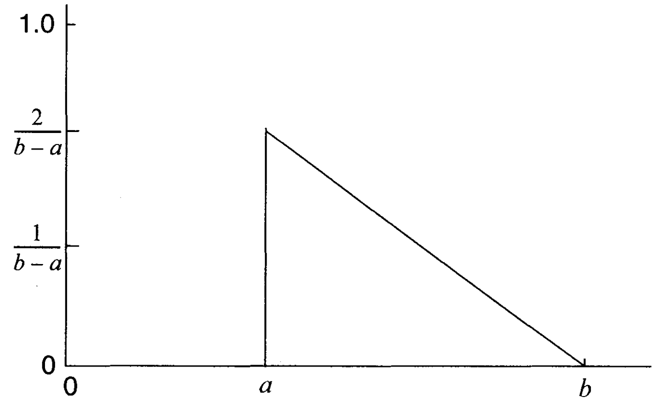


Figure 3.3: Walking distance probability for visiting one of two locations

Choosing the minimum of two randomly selected locations that have the probability density function shown in Figure 3.2 creates a probability density function that favors lower length paths.

considering the minimum of two locations. In this case the expected value of the walking distance is

$$E_2 = \int_{-\infty}^{\infty} x \cdot P_2(l = x)dx = \int_a^b x\left(\frac{b-x}{b-a}\right)\left(\frac{2}{b-a}\right)dx \tag{3.2}$$

where $P_2(l = x)$ is the probability that the minimum distance of two randomly selected locations is equal to l. The expression in equation 3.2 evaluates to become

$$E_2 = \frac{2a+b}{3}. \tag{3.3}$$

Therefore, the expected improvement when choosing from two locations instead of one is

$$\frac{E_2}{E_1} = \frac{2(2a+b)}{3(a+b)}, \tag{3.4}$$

where by definition, $a \le b$. Figure 3.4 illustrates the range of improvements depending on the relative values of a and b. The maximum improvement of 33% shorter expected distance occurs when $a = 0$. This best case scenario corresponds to a large difference between minimum and maximum walking distances. This large difference magnifies the

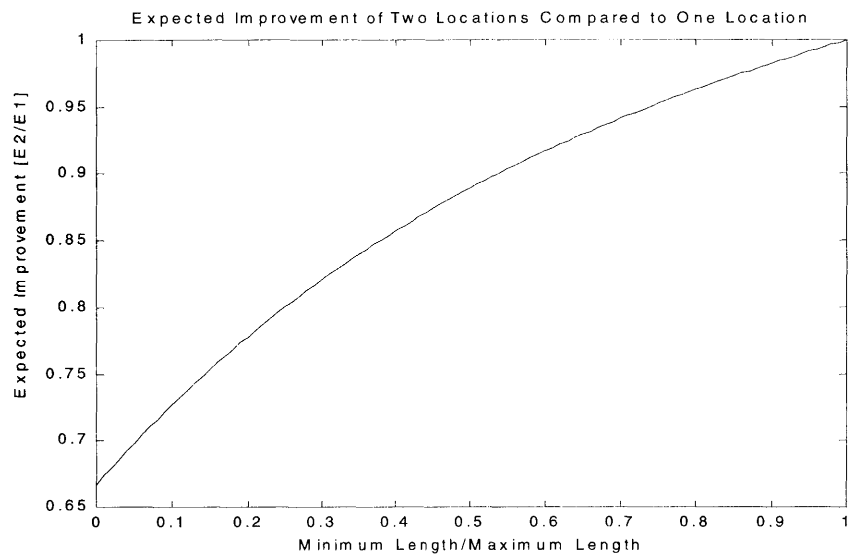


Figure 3.4: Expected improvement when considering two locations

The amount of expected improvement when considering two locations instead of just one location varies depending on the relative values of the path length minimum and path length maximum.

significance of the walking distance cost savings potential when considering two randomly chosen locations rather than just one.

Minimum improvement occurs when $a = b$, which shows no improvement because all locations have the same walking distance and considering a second location does not offer any potential for walking cost savings. The amount of expected improvement is directly related to the maximum and minimum walking distance difference.

$$E_1 - E_2 = \frac{b-a}{6} \tag{3.5}$$

In other words, the absolute walking distance savings is 1/6 the difference between maximum and minimum. Figure 3.5 illustrates the cost savings of two locations.

3.1.2 Discrete Analysis

The single pick analysis extends to cover discrete distributions that are not necessarily equiprobable. While not as intuitive as the simple continuous example, the expected

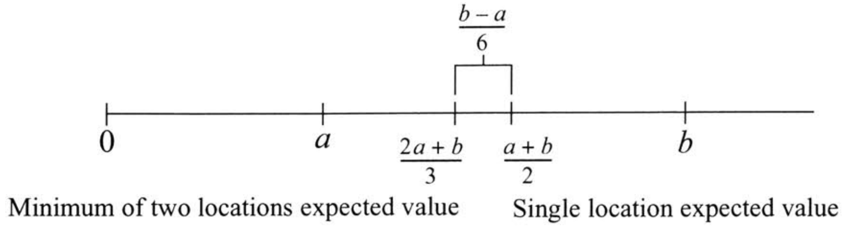


Figure 3.5: One location versus two locations

When considering only one randomly selected location, the expected path length is $\frac{a+b}{2}$. When considering the minimum of two locations, the expected path length is $\frac{2a+b}{3}$.

walking distance value for one location and the expected distance value for two locations can be calculated directly given the probability distribution. Figure 3.6 shows the probability distribution for the warehouse layout shown in Figure 3.7. The distances for each warehouse location extend from the start point to the warehouse location to the finish

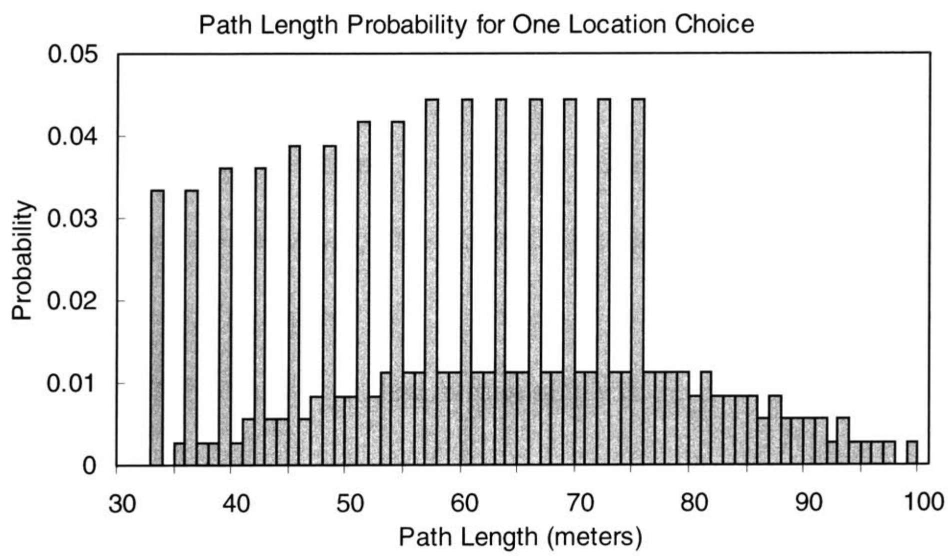


Figure 3.6: Trip distance probability distribution when visiting one location

point. By calculating the trip distance for each location, the probability of randomly picking a storage location that has a trip distance of 44.5 meters is the number of locations that have 44.5 meters as the trip distance divided by the total number of locations. For example, the probability distribution of figure 3.6 indicates that randomly choosing one location has a 0.0389 probability of selecting a round trip path length of 44.5 meters. In other words, 3.89% of the locations require 44.5 meters to walk from the starting location, to the selected location, and to the finish location. The fifteen peaks that appear in figure 3.6 correspond to the fifteen row aisles in the warehouse layout. Many storage locations in the same row aisle will have the same trip distance.

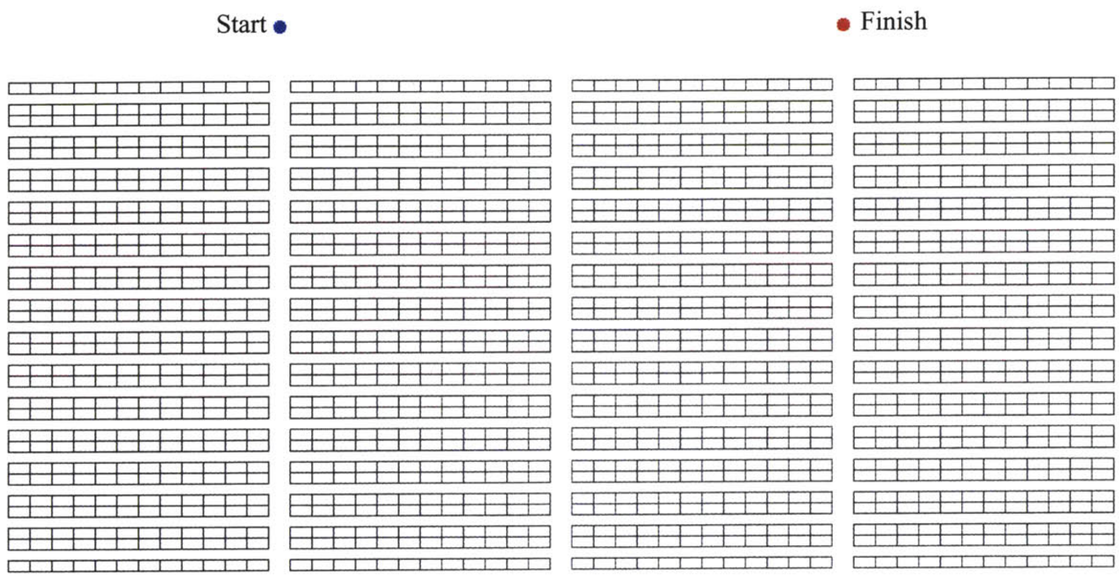


Figure 3.7: Warehouse layout

The expected value of the walking distance for this probability distribution is

$$E_1 = \sum_{l=a}^{b} P(L = l) \cdot l = 60 \,, \tag{3.6}$$

where a is the minimum length, b is the maximum length, and $P(L = l)$ is the probability that a randomly selected location has a path length of l.

When two locations are considered, then the walking distance probability distribution becomes

$$
\begin{aligned}
P_2(L = l) &= P(L_1 = l, L_1 \le L_2) + P(L_2 = l, L_2 \le L_1) - P(L_1 = l, L_2 = l) \\
&= 2P(L = l) \cdot P(L \ge l) - P(L_1 = l) \cdot P(L_2 = l)
\end{aligned}, \tag{3.7}
$$

which indicates that the path length probability distribution for two randomly chosen locations is two times the probability that one location equals l times the probability that the other location is greater than or equal to l, minus the probability that both locations have path lengths equal to l. Figure 3.8 shows the probabilities when choosing between two locations. Just as with figure 3.3, the shape of the distribution shifts so that lower trip distances have greater weight than higher trip distances, which makes sense because the distribution considers the *minimum* of two possible trip distances. The expected value of the two location probability distribution is

$$
E_2 = \sum_{l = a}^{b} P_2(L = l) \cdot l = 51.2 . \tag{3.8}
$$

The expected value when considering two randomly chosen locations is 14.6% shorter than when considering only one location (Equation 3.6).

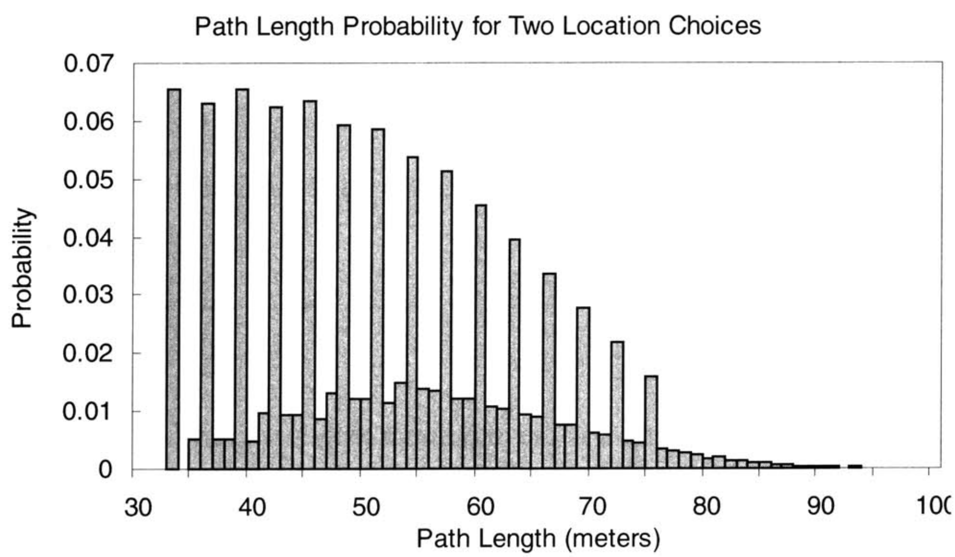


Figure 3.8: Trip distance probability distribution for visiting one of two locations

The single pick analysis shows the potential for walking distance savings when two locations meet the needs for a single pick. As the number of locations that meet the pick requirement increases, the expected value for the walking distance will continue to decrease. In the single pick case, computing the walking distance cost associated with a particular location over another is simple and choosing the location with the minimum cost is straightforward. When considering multi-address picking (the retrieval of many items in a single route through the warehouse), the analysis becomes more complicated due to the relationship of the minimum walking path distance to NP-hard problems like the Traveling Salesman Problem (Section 2.3.2) and the Traveling Purchaser Problem (Section 2.4.1). In the multi-address picking case, the walking distance associated with a set of storage bins is not easily computed and the minimum walking distance depends heavily and non-trivially on the other items in the pick list.

3.2 Heuristics for Route Construction

Evaluation of location-relaxed storage and its comparison to strict organization policies in multi-pick situations require reasonable solutions to the location-relaxed storage picking problem and the inherent Warehouse-TPP. Section 2.3.4 discussed routing heuristics for the warehouse traveling salesman problem. The options for building a Warehouse-TPP heuristic from warehouse-TSP heuristics are not promising. Possible methods for constructing a derivative heuristic include selecting locations based on a separate selection heuristic and then applying the warehouse-TSP heuristic. However, this approach merely shifts the burden to finding an appropriate selection method.

Another possibility is to construct the selection process using the warehouse-TSP heuristic as a metric for identifying a good selection. For example, a hill climbing search would use a warehouse-TSP heuristic to evaluate the routing cost for a particular set of location choices. After randomly changing the selections for a few of the items, if the new cost determined by the warehouse-TSP heuristic is less than the previous cost, then the changes are kept; otherwise the changes are discarded and a new set of changes considered. Repetition of random changes in this manner constitutes a hill climbing optimization technique that will find a local optimum, but computation time will tend to be long.

This section establishes simple heuristics for the Warehouse-TPP appropriate for further investigation of multiple picking analysis. The heuristics used are greedy algorithms for selecting storage bins and constructing routes. Section 3.3 establishes the utility of these heuristics by noting that heuristic results and optimal solution results respond similarly when given access to multiple locations.

3.2.1 A Greedy Heuristic for Route Construction

A greedy heuristic makes decisions based on what seems best at the time of the decision. Greedy heuristics for the Warehouse-TPP begin with an empty route that travels from the starting point to the ending point. Each step of the greedy heuristic selects a new location to be visited that satisfies one of the SKUs on the pick list. Augmenting the route to incorporate the additional location completes the step. Repeating the process for each SKU on the pick list satisfies the Warehouse-TPP.

The first greedy heuristic chooses the next SKU to be satisfied randomly. Then for each storage bin that contains the chosen item, the heuristic computes the extra walking path distances for adding the storage bin location to the route in each position of the existing route. For example, if there were two locations, A and B, already in the route in the order AB, then when considering storage bin location C, the added walking path distances considered include routes CAB, ACB, and ABC. Finally, the heuristic selects the storage bin and order number in the route that adds minimum length to the route. Figure 3.9 illustrates the first greedy heuristic.

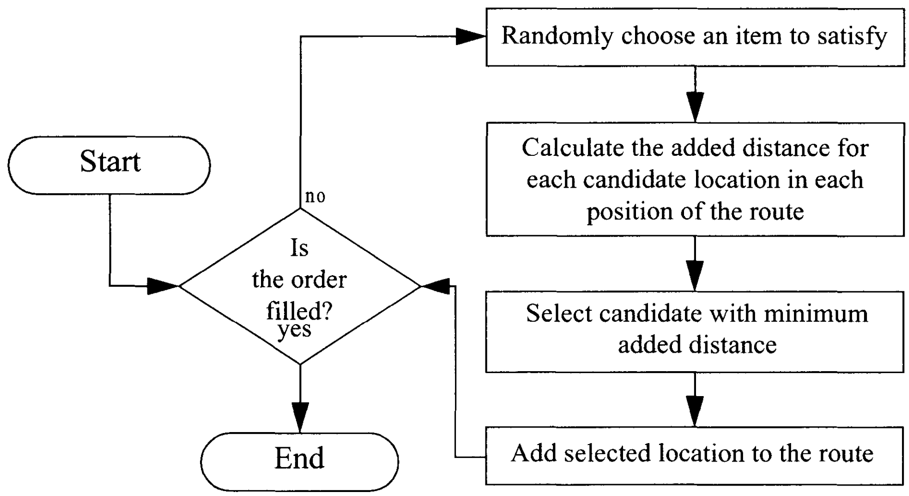


Figure 3.9: Random greedy heuristic

3.2.2 Greedy Heuristic Variations

The random greedy heuristic discussed in section 3.2.1 randomly chooses the next item to be added to the route. Variations on this greedy heuristic include using specific criteria to select the next item to satisfy. For example, instead of randomly choosing an item from the pick list, an alternate greedy heuristic considers *all* unsatisfied items and chooses the item, storage bin, and route order position with the smallest additional walk distance for the route. Figure 3.10 illustrates this *minimum* greedy heuristic.

The average performances of these two heuristics are compared by solving a series of warehouse-TPPs with each method. The problem instances correspond to the warehouse configuration shown in Figure 3.7 and take the form of 'pick one copy of n different

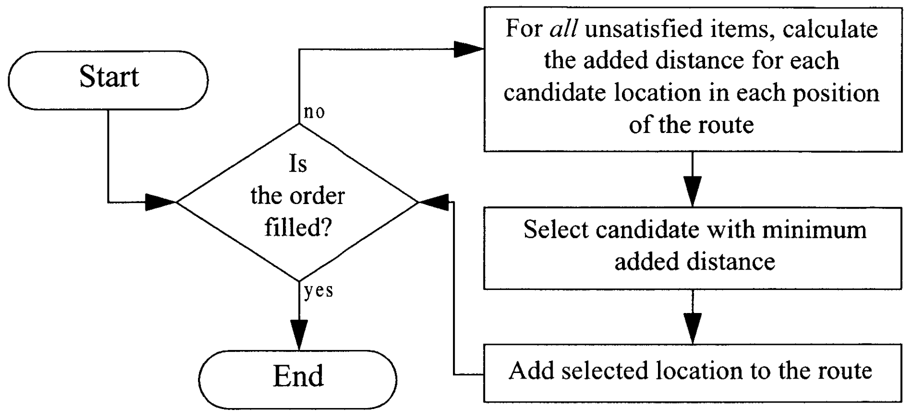


Figure 3.10: Minimum greedy heuristic

products, each of which is available in 5 different locations.'[1] Appendix A provides additional information on problem construction and heuristic comparison.

Figure 3.11 shows the results of the comparison and reports that on average, the minimum greedy heuristic performs worse than the random greedy heuristic. This surprising result illustrates the weakness of greedy heuristics with respect to optimal solutions for the Warehouse-TPP problem. The minimum greedy heuristic is 'too greedy' in selecting the absolute shortest possible route addition at each step. These myopic choices at each step lead to a poor overall result.

Since choosing the shortest addition of all possible items proved to decrease performance, the third greedy heuristic attempts to do the opposite by selecting the SKU from the pick list that increases the route length the *most*. Like the minimum greedy heuristic, this *maximum* greedy heuristic considers all storage bins of each unsatisfied item, and computes the additional travel distance for each storage bin. However, instead of taking the overall minimum, the heuristic determines the minimum *for each item*, then

1. In Chapter 3 all problems consider picking one copy of a particular product. Chapter 4 will expand the discussion to include picking multiple copies of the same product (lot size).

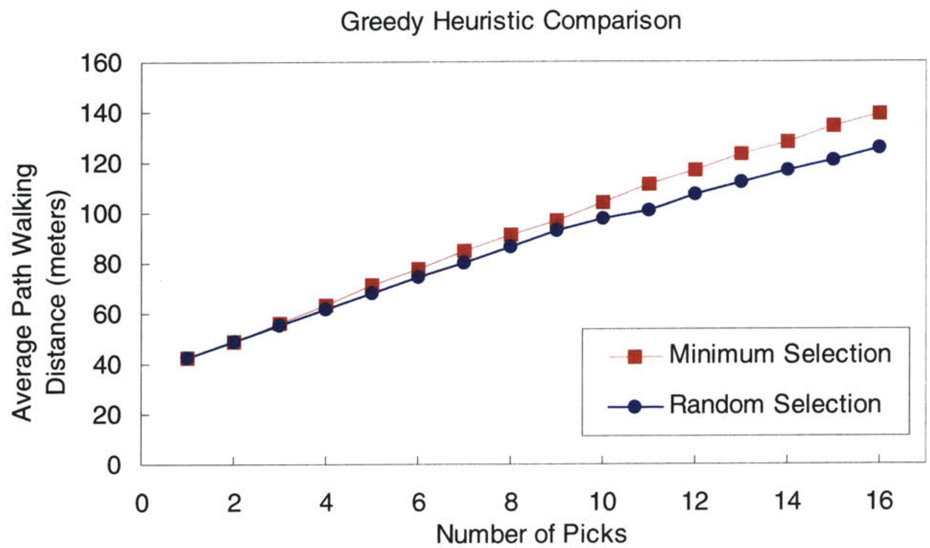


Figure 3.11: Greedy heuristic comparison (5 locations per item)

selects the item which increases the walking distance the most. Figure 3.12 illustrates the selection process for the maximum greedy heuristic. While counter-intuitive, the third greedy heuristic achieves, on average, better results than the random and minimum greedy heuristics. The 'un-greedy' step of selecting the item with the maximum additional walking distance assists in determining a short overall route by addressing the hard-to-

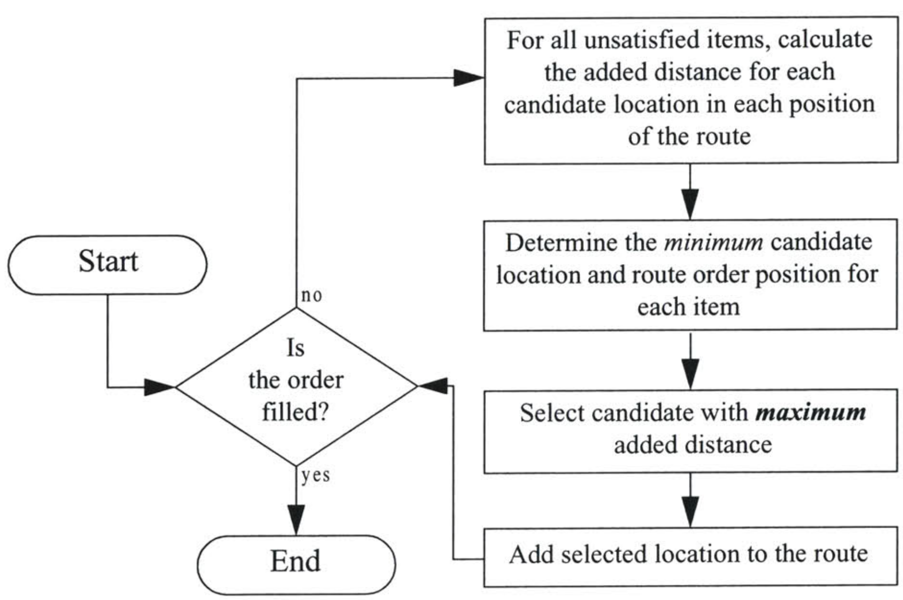
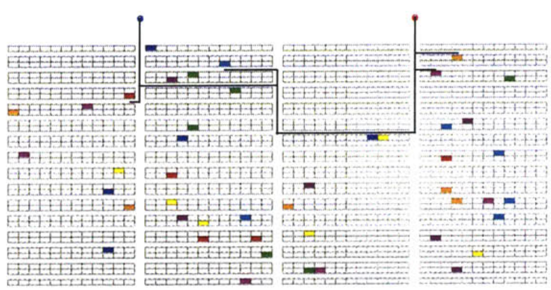


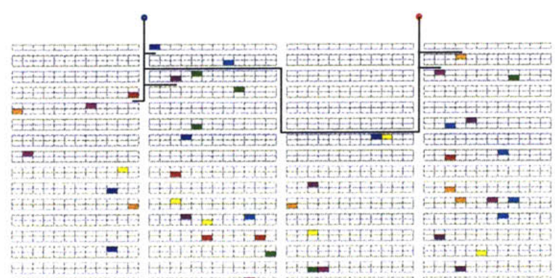
Figure 3.12: Maximum greedy heuristic

reach items first rather than last as with the minimum greedy heuristic. Selecting the hard-to-reach item first encourages better decisions for the other items.

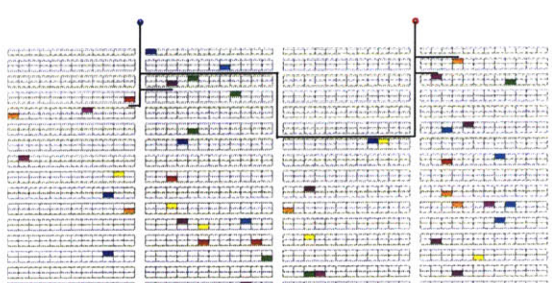
Figure 3.13 illustrates these three greedy heuristic solutions to the same Warehouse-TPP problem along with an optimal solution. This instance of the Warehouse-TPP requires the collection of eight items found at six locations each. In other words, pick eight different items, each of which is found in six different locations. Locations carrying the same SKU bear the same color. The problem was generated by randomly selecting six locations for each of the eight items as described further in Appendix A. The same problem was then solved by each technique. For this example, the maximum greedy heuristic performs closest to the optimal solution of the three heuristics, and the random greedy heuristic performs the worst.



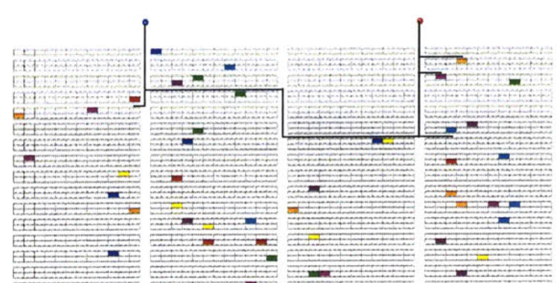
Random Selection: 77.5 meters



Minimum Selection: 75.5 meters



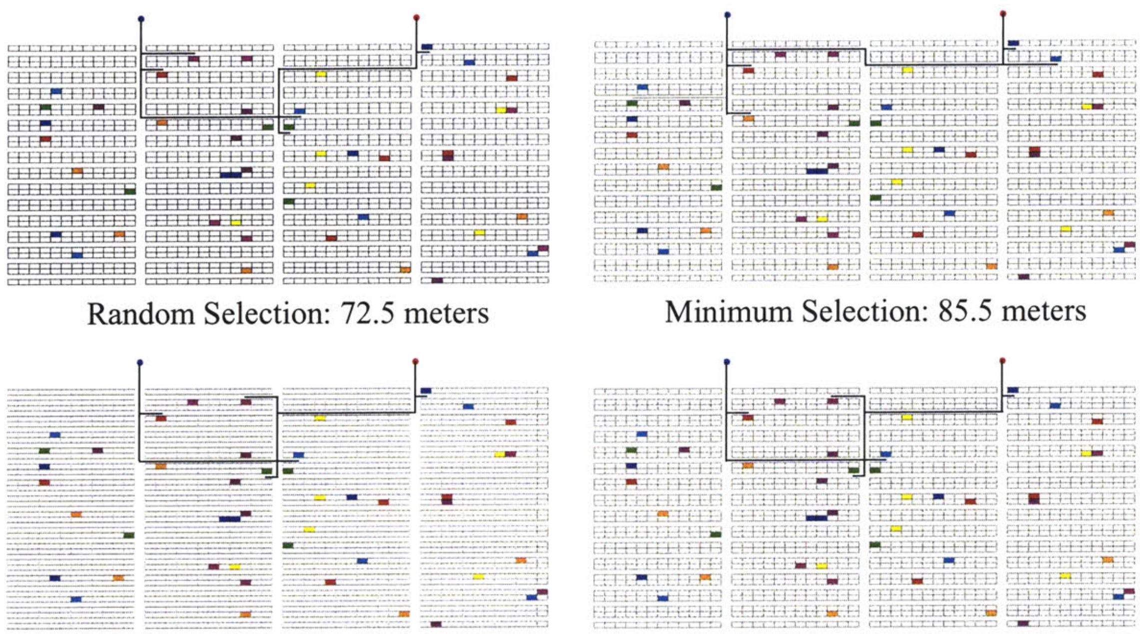
Maximum Selection: 73.5 meters



Optimal: 70.5 meters

Figure 3.13: Heuristic solutions, example one

Figure 3.14 shows solutions to a second example Warehouse-TPP. In this example, the best heuristic (maximum) matches the optimal solution. Furthermore, while the minimum greedy heuristic performs better than the random heuristic in example one, in this example, the random greedy heuristic performs better than the minimum greedy heuristic.



Random Selection: 72.5 meters

Minimum Selection: 85.5 meters

Maximum Selection: 68.5 meters

Optimal: 68.5 meters

Figure 3.14: Heuristic solutions, example two

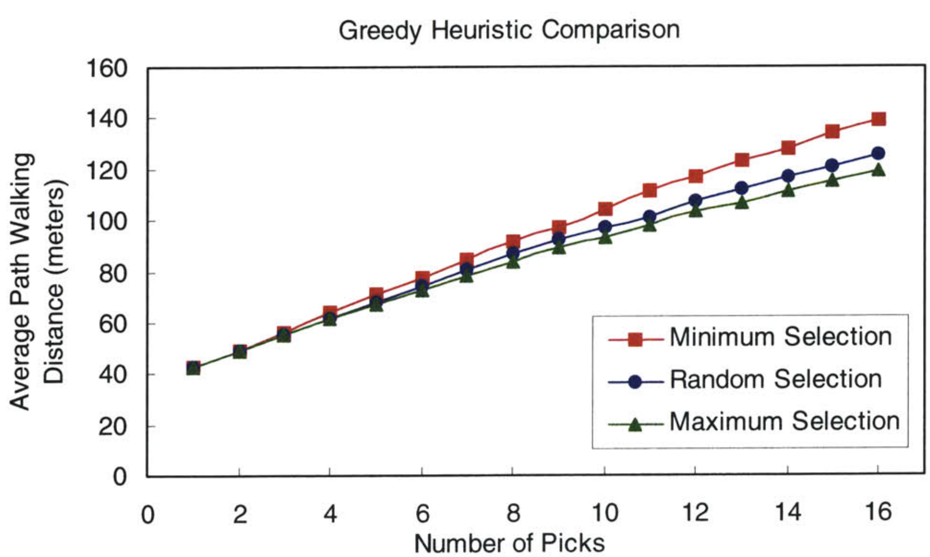


Figure 3.15: Heuristic comparison (5 locations per item)

Figure 3.15 shows the average performance for all three greedy heuristics with five storage bin locations for each SKU. For a randomly selected problem any of the three heuristic could have the best solution. However, the maximum greedy heuristic, which satisfies the farthest item first, typically performs the best.

3.2.3 Greedy Heuristics Versus Optimal Solutions

Figure 3.16 shows the performance of the three greedy heuristics relative to the optimal solution for the same problem. With only one pick, all four methods select the same solution. As the number of picks increases, the difference between heuristic solutions and the optimal solution increases as well. Figure 3.17 estimates the performance of the best greedy heuristic compared to the optimal solution. The linear trend line implies that heuristic routes with 20 picks will be approximately 15% longer than optimal routes. Since the size of Warehouse-TPP grows exponentially with larger numbers of picks, a linear trend line may be optimistic but illustrates the sub-optimality of the greedy heuristics and serves as a lower bound for the expected deviation from optimality.

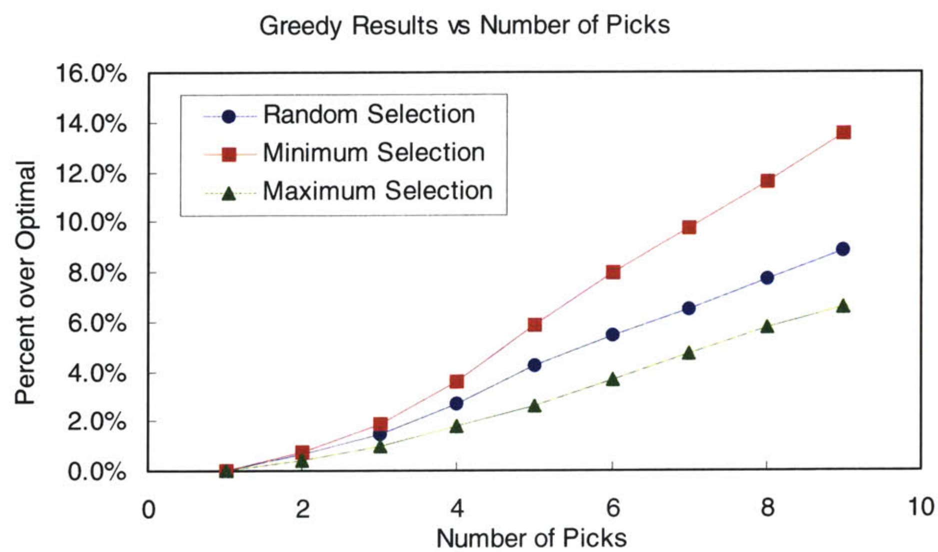


Figure 3.16: Comparison of greedy heuristics to optimal solution

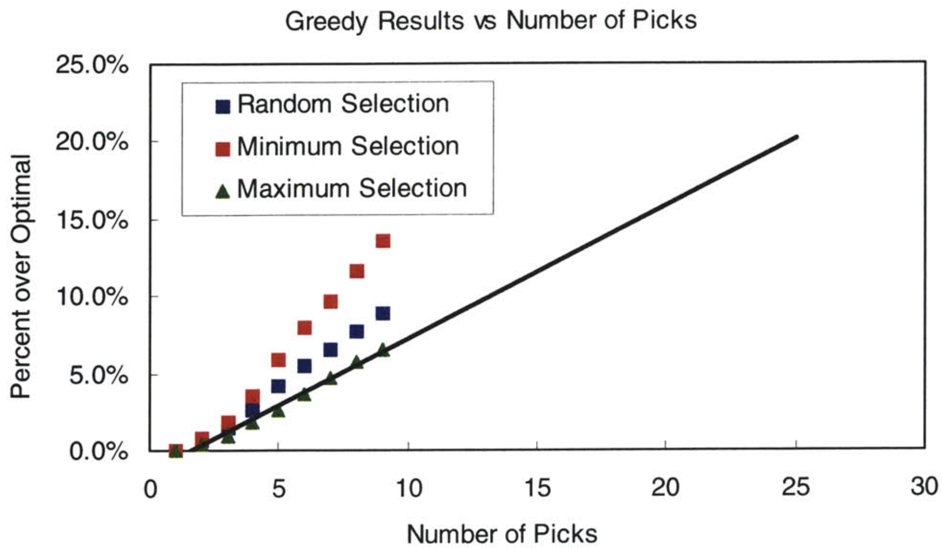


Figure 3.17: Linear projection of heuristic deviation from optimality

3.3 Multiple Pick Analysis

Section 3.1 analyzed the effect of multiple storage bins on the walking path length for a single pick. Multiple picks in a route necessitate more complex analyses because of the NP-hard nature of the routing problem and the large number of possible routes to consider. The single pick analysis determined the expected walking path distance by considering all route possibilities and their probabilities. The very large number of possible routes in the multi-pick case, however, makes enumeration of routes and probabilities infeasible. To estimate a solution, the following analysis for multiple picks estimates the expected route walking distance by randomly generating Warehouse-TPP instances and recording the solution's route distance through heuristics described in section 3.2.

The picking problems that the analysis experiments upon are generated by randomly selecting storage bin locations for each SKU needed. For example, if the problem is to pick eight items and each item is found in three different locations, then the problem is generated by randomly selecting three storage bin locations and designating those locations as containing SKU #1. Then randomly selecting three storage locations for each remaining SKU in the pick list completes the problem parameters. To obtain a better estimate of the expected route length for the given problem size, several different picking problems are generated and solved, and the average route length computed.

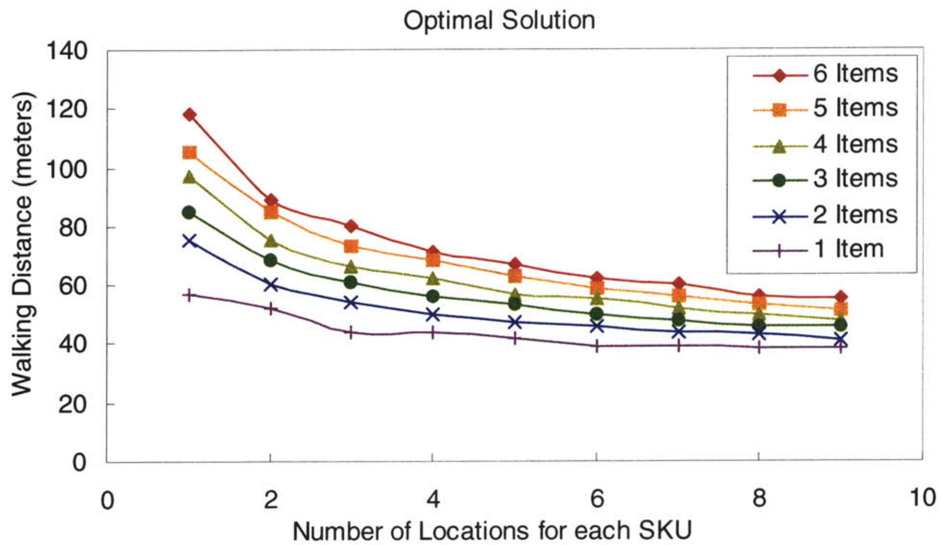


Figure 3.18: Effect of multiple picking locations on optimal routes

Figure 3.18 illustrates the average picking route length for optimal solutions. The different curves correspond to different numbers of items picked along the route. The red curve shows how picking routes that collect six items are affected by multiple locations from which to access the items. The *x*-axis corresponds to the number of locations in which each SKU may be found. When the number of locations equals one, the problem corresponds to the traveling salesman routing problem. When the number of locations is two or larger, the problem is a travelling purchaser problem. As the number of locations for each SKU increases, the route length decreases. The most significant decline in route length occurs between having one location for each SKU and having two locations for each SKU. These results are consistent with the single pick analysis.

Figure 3.19 illustrates the average pick-route length for routes obtained with a greedy heuristic. In this case it is the maximum selection greedy heuristic. The greedy heuristic results behave similar to optimal routing; as the number of storage bins available to meet a picking need increase, the overall route length decreases. The decrease is especially strong

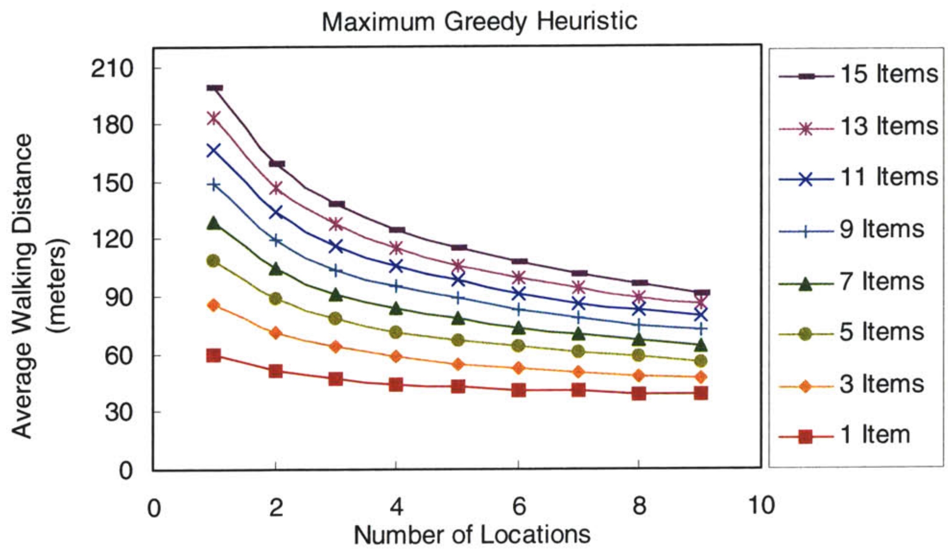


Figure 3.19: Maximum selection heuristic

between one available location and two locations, just as with the optimal route case. Furthermore, both optimal routes and greedy heuristic routes show diminishing advantages as the number of available storage locations increases. Figure 3.20 shows the same results from Figures 3.18 and 3.19 on the same graph for the specific case where there are five items to be picked. The data confirm that the greedy solutions, while consistently higher in value as the optimal solution, behave similarly to an increasing number of location from which to pick the needed items.

A thorough analysis of warehouse operations in Chapter 4 requires the solution to a large series of warehouse travelling purchaser problems. For the sake of simplicity and

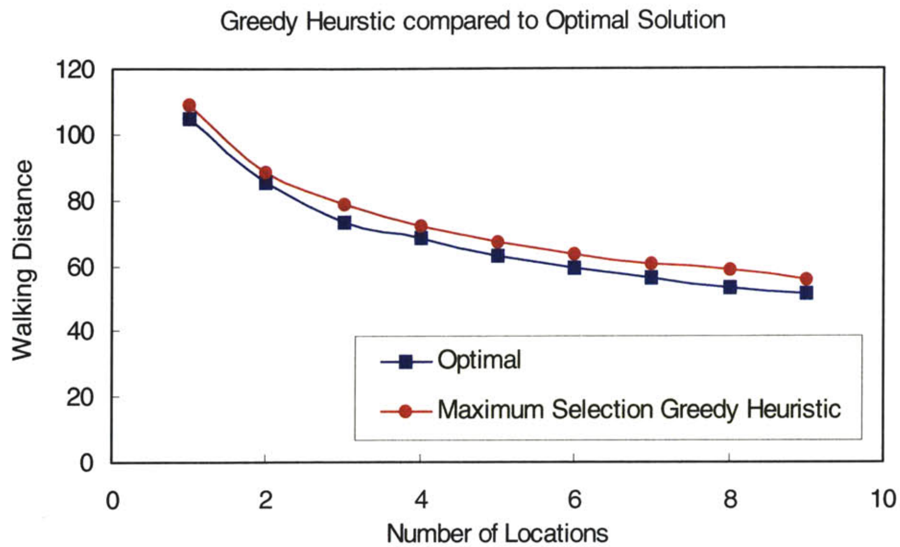


Figure 3.20: Greedy heuristic compared to the optimal solution

The number of items picked is five.

computation time, solutions to the Warehouse-TPP are obtained with the best greedy heuristic, the maximum greedy heuristic. As demonstrated by comparison to the optimal solutions, this heuristic, while sub-optimal, behaves similarly to changes in the number of storage bins for each SKU as the optimal solution.

Chapter 4: Warehouse Picking and Placement

The major factor of warehouse operations effort is order-picking, which makes efficiency in order-picking a top priority in warehouse management [Frazelle 2002]. The order-picking with location-relaxed storage is especially well positioned for optimization. The previous chapter outlines at a qualitative level how we expect location-relaxed storage to behave compared to traditional methods. This chapter focuses on routing for picking and placement operations and provides a quantitative comparison between location-relaxed storage and strict organization storage. The first section of this chapter discusses the warehouse order-fulfillment problem in greater detail and contrasts the location-relaxed fulfillment problem with traditional strict organization problems. While opportunities for optimization are numerous, the complexity of the problem makes optimization difficult. The second section covers warehouse item placement and discusses how location-relaxed item placement differs from strict organization placement. The last section outlines a warehouse simulation of placement and picking operations that quantifies the differences between location-relaxed storage and strict organization storage and reveals that location-relaxed storage functions best with small picking lot sizes.

4.1 Location-relaxed Warehouse Order-fulfillment

As mentioned in Section 2.2.4, order-picking has been identified as the major component of warehouse cost. Reduction in the effort required for order-picking has typically focused on establishing efficient picking routes. This section explains the warehouse order-fulfillment problem and details additional areas where optimization may yield better efficiency. The warehouse order-fulfillment problem as a whole reaches into several simultaneous optimization problems, which makes a globally optimal solution difficult to find and manage.

4.1.1 Interdependence of Simultaneous Problems

Chapter 3 treats the Warehouse-TPP as an independent problem, that is, given needed items *X*, *Y*, and *Z*, find the shortest route by selecting the storage bins and visitation sequence that meet the demand with the least cost. The problem as presented is self-contained and does not consider the influences of other order needs. However, in real warehouse picking, several concurrent traveling purchaser problems will arise simultaneously. These simultaneous problems are not independent. The solution of one problem influences the solution of the others because satisfying one order consumes resources which are then no longer available for the others.

For example, consider three traveling purchaser problems where TPP_1 requires items *X*, *Y*, and *Z*; TPP_2 requires items *A*, *B*, and *C*; and TPP_3 requires items *B*, *X*, and *Z*. TPP solutions meet the needs of TPP_1 without any consideration for TPP_2 or TPP_3. In reality, the overall goal is to minimize the *total* cost for all three problems. The items used to satisfy *X* and *Z* for TPP_1 may provide significantly better solutions for TPP_3, however, this possibility is ignored and the overall solution may suffer higher costs than necessary. A globally optimal solution must consider all problems simultaneously.

This type of interdependence does not occur with TSPs and strict organization. The one-to-one correspondence between SKUs and storage bins insulate concurrent warehouse-TSPs from affecting each other. Since access to a particular SKU is restricted to a single location in a strict organization warehouse, the solution of one warehouse-TSP never[1] denies another problem of a resource or opportunity.

---

1. Resource denial might occur if the item is the last copy of the SKU and meeting one problem's need necessarily denies the item to another. However, this is an issue of stockout, not optimization.

4.1.2 Order Batching

In addition to the major complication of interdependent problems, order batching adds another layer of optimization to the order-fulfillment problem. As discussed in section 2.8.1, order batching aggregates separate orders into a picking list for a single route through the warehouse.[1] After picking, the operator divides the batch into individual orders. Order batching relates to the vehicle routing problem and is a complex optimization problem. Table 4.1 shows how the number of possible groupings grows rapidly with the number of groups (batches) and the number of orders in each group.

Table 4.1: Number of order batching possibilities[a]

	Number of Orders in each Group				
Number of Groups	**1**	**2**	**3**	**4**	***n***
1	1	1	1	1	1
2	1	3	10	35	$\frac{\binom{2n}{n}}{2}$
3	1	45	840	17,325	$\frac{\binom{3n}{n}\binom{2n}{n}}{2}$
4	1	1260	184,800	31,531,500	$\frac{\binom{4n}{n}\binom{3n}{n}\binom{2n}{n}}{2}$
k	1	$\frac{\binom{2k}{2}\binom{(k-1)2}{2}\cdots\binom{4}{2}}{2}$	$\frac{\prod_{i=2}^{k}\binom{3i}{3}}{2}$	$\frac{\prod_{i=2}^{k}\binom{4i}{4}}{2}$	$\frac{\prod_{i=2}^{k}\binom{in}{n}}{2}$

a. The total number of orders is $k \cdot n$

In the location-relaxed case, the difficulties of order batching are compounded by the interdependence of the subsequent traveling purchaser problems. Each different batching

1. Creating larger picking lists increases efficiency by filling many orders with a single route through the warehouse. Furthermore, the amount of material picked is limited by the picker capacity and not by the size of the order.

possibility leads to a different set of interdependent traveling purchaser problems, which in turn are not easily evaluated. Furthermore, the number of combinations will be extremely high even for moderately sized problems. With four groupings and four orders in each group, the number of combinations is over 31 million. With five groupings and four orders in each group, the number of combinations is over 150 billion.

Figure 4.1 illustrates the warehouse picking process with order batching. Strict organization order-picking with batching results in k independent warehouse-TSPs and individually optimized solutions lead to a globally optimal solution for the entire set. Location-relaxed storage order-picking with batching results in k interdependent Warehouse-TPPs, and individual optimization of each Warehouse-TPP will not necessarily lead to the optimal solution for the entire set of problems. A fully optimal solution for location-relaxed storage must optimize both the batching process and the subsequent set of TPPs simultaneously.

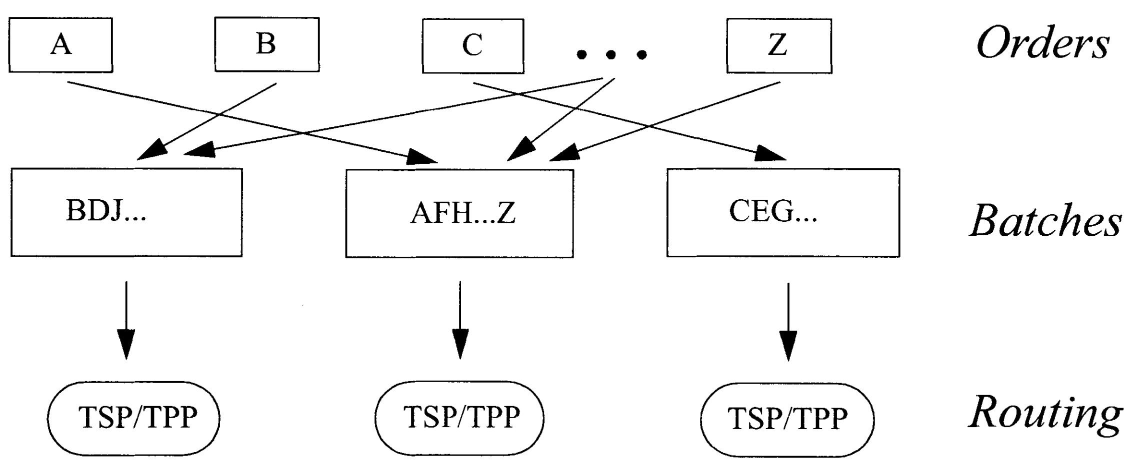


Figure 4.1: Location-relaxed warehouse picking

4.1.3 Impact of Turnover on Optimization

Order batching introduces a potentially large problem for optimization purposes, especially as the number of batches increases. If warehouse operations filled orders day-by-day,[1] then the opportunity for optimization would be enormous because a large volume of orders provide ample order pairings to reduce route walking distances. In other words, there is sufficient time to accumulate orders with favorable matchings. Finding the optimal would be difficult, but the opportunity for cost savings is high and even sub-optimal solutions will benefit from a moderate amount of optimization. However, warehouse operations typically fill orders as soon as possible and cannot wait to collect an entire day's orders.

Filling orders as soon as possible or hour-by-hour rather than day-by-day keeps the size of the optimization problem smaller, but also reduces the potential for optimization. Interdependence will still remain and complicate the problem, but the number of batching combinations and the number of simultaneous problems are kept small by the need for rapid turnover.

4.1.4 Changing the Optimization Criterion

Interdependence and the constant flow of new orders into the warehouse reveal a benefit to considering optimization mindful of future possibilities rather than solely focusing on current needs. In other words, solving a current problem has an impact on future tasks; therefore, a wise optimization method would take the effects on the future into account. Adding this type of foresight to the problem solving process requires a deviation from the simple and greedy optimization criterion of shortest walking distance.

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1. Fill yesterday's orders today

For example, consider the likely scenario of two pick route solutions to the same TPP problem where one is *slightly* longer than the other. Optimizing purely based on walking distance would myopically choose the shorter route. Now suppose that the shorter route uses more favorable locations and the slightly longer route uses more unfavorable locations. Choosing the slightly longer route leaves items in favorable locations for an upcoming and as yet unknown pick list. For the relatively small cost of a slightly longer walking distance, the favorably located items would be available for future picking.

The advantage of this trade-off between current cost of walking distance and future benefits is not completely clear. The decision would depend on the added walking distance involved, the difference in favorability of the two locations, and the expectation of needing the same item in the future. Furthermore, because different picking routes frequently have the same walking distance this type of decision making is mandatory.

Additional complications to such routing decisions arise when considering the combined effects of order-picking and inventory placement. Typically, route construction does not take into account the fact that the items picked leave 'holes' where future incoming items may be placed. In the strict organization storage case, ignoring placement effects is appropriate because the one-to-one correspondence between SKU and storage bin remains fairly static. With location-relaxed storage, every open spot is a viable storage spot for incoming material. Therefore, opening up favorable locations during the picking phase of warehouse operations leaves favorable locations for the placement phase, which is also advantageous. When confronted with a choice between two routes of similar walking distance, one that uses favorable locations and the other using less favorable locations, deciding which one to use is a choice between making future placement tasks easier or making future picking tasks easier. Using the route with more favorable locations deprives upcoming picking routes from using the more favorable items but opens up favorable space for placement. Using the route with less favorably located items leaves the more favorable items for future picking but also leaves fewer favorable 'holes' for incoming material.

The best choice will be time dependent. Early in a picking session, when it is known that many more picking operations will follow before new placement operations, choosing the less favorable route has a greater chance of being advantageous since there is more opportunity to use the favorably located items in the upcoming picking routes. Later in the picking session, when upcoming placement operations are known to be closer, using the more favorable route makes more sense since there are fewer upcoming picking events to take advantage of the favorably located items and using the favorably located items opens advantageous space for upcoming placement operations.

4.1.5 Implications of Interdependence and Order Batching on Analysis

The potential for lower walking distances stems from the larger search space of feasible solutions available with the TPP over an equal sized TSP. When considering the global optimization of many warehouse order-picking operations, however, interdependence found within a set of Warehouse-TPPs limits the set of solutions in a way that is not found in a set of warehouse-TSPs. The significance of Warehouse-TPP interdependence on optimization is not fully understood. While concurrent TPPs put some restrictions on the possible solutions, the feasible solutions for the set of TPP remains considerably large.

Furthermore, when considering global optimization of the entire warehouse operations, interdependence effects cannot be separated from item placement operations.

TSP analyses generally do not consider the relationship between picking and placement since aggregate storage tends to minimize the link and leaves each placement operation and picking operation independent of the others. With location-relaxed storage, current decisions have a strong effect on future tasks. With strict organization storage, current decisions have little effect on future tasks.

The warehouse operations simulation to be discussed in section 4.3 compares location-relaxed storage with strict organization storage but does not directly address all of the warehouse optimization issues discussed in this chapter. In our approach, several optimization decisions are abandoned in favor of a randomized approach to focus on the *potential* of location-relaxed storage against traditional strict organization storage. Section 4.3.2 discusses the model simplifications employed and the limitations of the simulation.

4.2 Location-relaxed Warehouse Replenishment (Item Placement)

Item placement in a strict organization warehouse is simply the reverse process of picking and also relates to the warehouse-TSP. Under location-relaxed storage, however, greater flexibility in placing items changes the nature of the route optimization problem to a version of the prize collecting traveling salesman problem. This section explains location-relaxed item placement and discusses its differences from strict organization.

4.2.1 Prize Collecting Traveling Salesman Problem (*k*-TSP)

Since all open spaces are available for placement, the problem of replenishment into a location-relaxed storage warehouse relates to the prize collecting traveling salesman problem[1] [Balas 1989]. The regular traveling salesman problem requires the visitation of a

1. The prize collecting traveling salesman problem can also be referred to as a *k*-TSP.

specific list of cities. In the prize collecting TSP, the route must visit k cities of a list of n cities provided.[1] In other words, given n cities construct a minimum cost route that visits any k cities. Just as with the regular TSP, a list of cities is part of the problem specification, but with the prize collecting case only some of the cities on the list need to be visited. In the location-relaxed storage warehouse, a replenishment routing problem has the set of all open spaces as the list of 'city' options, and the number of 'cities' that must be visited is determined by the number of items to be stored.

The location-relaxed warehouse replenishment problem is also a special case of the traveling purchaser problem where there is one commodity needed in great quantity and is available in many different cities. Location-relaxed warehouse item placement can be considered as a Warehouse-TPP where storage bins that have free space comprise the set of available cities and the amount of free space in each corresponds to the amount of the commodity available. In a sense, the commodity is free space within the warehouse and the Warehouse-TPP must find the route that acquires enough free space to meet the storage requirements.

4.2.2 Interdependence

Just as with location-relaxed picking, multiple instances of concurrent placement problems influence each other and the solution of one affects the solution of the others. The interdependence effect is even stronger with the item placement process than with picking because there is only one commodity involved. The links between interdependent Warehouse-TPPs in picking are limited to repeated SKUs across the series of TPPs. As a result, the interdependence is not strong because each decision does not necessarily affect

1. The prize collecting TSP described by Balas is more general than the problem statement provided here.

the other route optimization problems. Placement k-TSPs on the other hand, have a very strong interdependence as every storage task links to the other concurrent k-TSPs since the commodity involved is generic 'free space.' In placement, every decision affects all other placement problems.

4.3 Warehouse Placement and Picking Simulation

This section introduces a simulation of warehouse operations that compares the efficiency of location-relaxed storage against traditional strict organization and evaluates the potential of location-relaxed storage for warehouse operations. By running the simulation with a variety of input characteristics, the results reveal how these input characteristics affect the relative merits of location-relaxed storage and strict organization storage under different loading conditions. This simulation analysis focuses on the effect placement lot size and picking lot size have on each warehouse system. Chapter 2 speculated that location-relaxed storage is especially suited for warehouse characteristics with small picking lot sizes. The simulation results reported later in this chapter support this hypothesis.

4.3.1 Lot Size

The *order size* is the number of units in a group that are collated together at the warehouse and then shipped out of the warehouse to a single destination. Order size makes no reference to the identity of the items involved; the order could be 12 units of the same SKU or one copy each of 12 different SKUs and the order size would be 12 in both cases.

The *lot size* is the number of units with the same SKU in a group or order. An order with three copies of a SKU, has a lot size of 3. An order comprised of several different SKU has a different lot size for each one. *Placement lot size* refers to the lot size of products brought into the warehouse. In other words, SKUs arriving in groups of four have an placement lot size of 4. Similarly, *picking lot size* refers to the lot size of SKUs in orders leaving the warehouse.

Location-relaxed storage should have its strongest advantages over traditional strict organization storage when the picking lot size is small. Small lot sizes are better capable of taking advantage of location-relaxed storage's distributed placement of SKU items because only a small number of bins need to be visited and the relevant bins are numerous and widespread. Larger lot sizes are better able to take advantage of aggregated storage found in strict organization storage because obtaining several copies of an SKU at the same time is easier with aggregation since a single bin visit yields several copies of the needed SKU.

4.3.2 Model Simplification

The best method to compare the relative merits of location-relaxed storage to traditional storage techniques would calculate the optimal policies for location-relaxed storage and the optimal policies for strict organization storage and compare the warehouse operations work effort for each. Unfortunately, such optimal policies are not computationally feasible. As described in sections 4.1 and 4.2, the full warehouse order-fulfillment problem and warehouse placement problem for location-relaxed storage contain many complications and interdependent subproblems. Even with strict organization, where prior research is significantly more abundant, optimal techniques are not well understood. Furthermore, any optimal policy is likely to depend on a variety of warehouse characteristics, and the optimal policy itself will be problem dependent. In other words, different techniques may be necessary for different warehouse sizes and input characteristics to reach an optimal solution. For the purpose of evaluating location-relaxed storage, many simplifications are employed in the following simulation to make comparisons feasible.

4.3.2.1 Equal Sized SKUs

Setting all SKUs to the same size simplifies the analysis of warehouse operations by removing the significance of the bin packing problem. Capacity is still an issue, both for storage bins and worker carrying capacity; however, since all SKUs are the same size, size-based decisions to maximize bin storage utilization are not an issue. Each bin will be able to fit a fixed number of items, regardless of the identity of those items.

4.3.2.2 Interdependence

Interdependence is a complicating factor that cannot be simplified or removed from the warehouse problem. The simulation deals with interdependence by myopically solving each problem one-by-one. After a problem is solved, the resources used by that problem are no longer available for the remaining problems. This technique makes no attempt to optimize the order of solving the set of problems, but the constraints of interdependence are always met. As a result, the solutions are valid, but make no attempt to be efficient.

Interdependence is a larger issue for location-relaxed storage than with strict organization storage. For the most part, strict organization storage routing problems (both placement and picking) are independent of other routing problems. The consistent correlation between SKU and storage bin makes strict organization routing insensitive to previous problem decisions. Therefore, the decision to treat problems one-by-one has little effect on optimization in the strict organization case. This simplification ignores an opportunity for added optimization that is available in the location-relaxed storage case, and this cost savings opportunity is not available with strict organization storage. In other words, this simplification is more of a detriment to location-relaxed warehouses than strict organization warehouses.

4.3.2.3 Order Batching

Many options for order-picking technique are available including, pick by order, zone picking, and batch picking. The warehouse simulation arbitrarily selects batch picking for order-fulfillment. Order batching, as described in section 4.1.2, offers a rich opportunity for optimization through a large number of possible combinations of organizing orders into pick batches. Similar to interdependence, the simplification for this difficult optimization problem ignores this opportunity for cost savings in favor of a straightforward solution. The simulation randomly creates batches from a set of orders without regard to SKU identity or routing possibilities. This lack of vision applies to both location-relaxed storage routing and strict organization routing; however, the relative impact that random assignment has in each case is not simple to evaluate. In other words, how order batching optimization will affect location-relaxed storage operations relative to optimized strict organization order batching is not known.

4.3.2.4 Greedy Routing Heuristic

As shown in section 3.2, greedy heuristics for the TPP and TSP behave similarly to optimal routes. For computational simplicity, greedy solutions are used for route construction for both TPP (location-relaxed) and TSP (strict organization). Using the optimal solution is not possible for the simulation because of computation time constraints; however, comparisons of a few optimal solutions and greedy heuristic solutions show that although greedy heuristic solutions have an overall longer path length, both behave similarly to changes in the number of SKU locations.

4.3.2.5 Demand Characteristics

All SKUs have an equal demand rate in the warehouse simulation. Assigning equal demand rates to all the SKUs involved in the warehouse operations simulation nullifies the role of demand-based placement policies on warehouse performance. Chapter 7 discusses demand-based placement strategies further and its relative merits in location-relaxed and strict organization storage warehousing. By setting all SKUs to the same demand rate all items would be treated equally by a demand based storage policy.[1] Furthermore, the equivalence of the SKUs' size and demand rate simplifies the analysis of warehouse results since changes in behavior can be attributed more easily to changes in other warehouse characteristics rather than to the demand or size distribution.

4.3.2.6 No Stockout

For the warehouse simulation, the data set that governs which items are to be placed and removed each cycle is constructed such that no stockout occurs at any point in the simulation. Incoming and outgoing items are selected randomly from the equiprobable demand distribution for the SKUs. However, if an outgoing item selected is not in stock, then that selection is discarded and a new one selected. Stockout is an issue for inventory control, which governs how much of an item should be stored in the warehouse. The comparison of location-relaxed storage and strict organization storage is not concerned with how much of an item is stored, but *where* it is stored. Furthermore, the data used in the simulations are the same for both warehouses so even if stockout were to occur, it would occur equally in both warehouses. Therefore, this simplification will have no effect on the results.

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1. Item size is sometimes a factor in demand based placement policies, but in the warehouse simulation all SKUs have the same size as well.

4.3.2.7 Warehouse Dimensions

As stated before, a high number of factors influence warehouse operations and efficiency including the size and shape of the warehouse. Due to computation time constraints the research focuses on the effect of lot size on warehouse operations but leaves other potential factors, such as warehouse size and shape, for future investigation. The simulation analysis arbitrarily sets the warehouse dimensions to 4 columns and 30 rows (15 aisles, 5 cross aisles), 12 units per section, 8 bins per unit, and a capacity of 18 items in each bin. Whether this arbitrary selection favors one method over the other is unknown and left for future exploration.

4.3.2.8 Constant Order Size, Constant Lot Size

The simulation varies the placement and picking lot sizes during different runs of the simulation to determine the effect of lot size on warehouse operations. For simplicity, the order sizes are held constant throughout all simulations and the lot size is constant within a single simulation. Keeping order size and lot size fixed simplifies the analysis and helps keep the focus on the varying input parameters.

Since both order size and lot size are constant in each simulation, occasionally an order must have an off-lot size of an SKU to complete the order in the correct size. In other words, when order size is not a multiple of lot size one SKU in the order will have a lot size smaller than the designated lot size for the simulation. For example, when the order size is 20 and the desired lot size is 3, the order will have 6 SKUs with three copies (lot size 3) and one SKU with two copies (lot size 2). This consequence is unavoidable when maintaining a constant order size across all simulations.

4.3.3 Picking and Placement Simulation Procedures

The warehouse operations simulation begins with an empty warehouse. As mentioned in section 4.3.2.7, the warehouse consists of 4 columns and 30 rows; each section has 12 units with 8 storage bins in each unit. The size of the storage bins are all equal, and hold no more than 18 items. The maximum capacity of the warehouse is 207,360 items. These dimensions are held constant throughout the analysis.

The simulation initializes the warehouse by generating an initial inventory for the warehouse and placing the initial inventory into the warehouse. The simulation then cycles through picking phases and replenishment phases while keeping track of the route walking distances for each operation. The initial inventory and subsequent cycles are generated once and the same data is used for both location-relaxed and strict organization simulations.

Simulation Procedure
1. Begin with an empty warehouse
2. Generate the initial inventory and place items into the warehouse based on the placement lot size and placement policy
3. Generate orders for picking according to the picking lot size
4. Pick items to meet the needs of the orders and track the walking distance
5. Generate incoming stock for warehouse inventory replenishment according to placement lot size
6. Place items and track the walking distance
7. Repeat steps 2 through 6, 30 times.

The test warehouse has 11520 storage bins, and each storage bin has the capacity to contain 18 items, and the warehouse services 2000 unique SKUs. The operating inventory for the simulations is 160,000 items, which is 77.2% of the maximum capacity. For the entire analysis order size is 20 units, both for incoming products and outgoing orders. Each cycle has 100 orders of 20 units leaving the warehouse and 100 boxes of 20 units entering the warehouse for a total of 2,000 units each cycle. Placement and picking lot sizes vary from one SKU to 20 SKUs with each run of the simulation to gather information about the influence of lot size on performance. For a more detailed explanation of the warehouse picking and placement simulation, refer to Appendix B.

4.3.4 Strict Organization Simulation Results[1]

Figure 4.2 shows the effect of lot size on the total walking distance for both picking and placement in a strict organization warehouse. The graph shows strict organization warehousing is very sensitive to placement lot size. This result is not surprising because strict organization must aggregate matching SKUs into the same storage bin. Strictly organized warehouses expend less effort with larger placement lot sizes because each lot is already aggregated. Smaller lot sizes need additional aggregation to fill a storage bin while larger lot sizes will fill the storage bin outright. The data show significantly less walking with a large placement lot size than with small placement lot size. The graph also reveals the total walking distance decreases as the picking lot size increases. Larger picking lot sizes take better advantage of the aggregation that occurs with strict organization. Regardless of placement lot size, after item placement the SKU items are highly aggregated. This aggregation favors larger picking lot sizes since fewer location visits are necessary when the picking lot size is large.

4.3.5 Location-relaxed Storage Simulation Results

Figure 4.3 shows the results for the location-relaxed storage. Unlike the strict organization case, where a placement lot size of 18 always outperforms a placement lot size of one; with location-relaxed storage the small placement lot sizes outperform large placement lot sizes when the picking lot size is also small. When the picking lot size is large, then larger placement lot sizes outperform smaller placement lot sizes. For the specific example shown where the storage capacity of each bin is 18, picking lot sizes of nine or less prefer a unit sized placement lot rather than a placement lot size of 18. Picking lot sizes of ten or larger prefer a placement lot size of 18 rather than the unit sized lot. This result is not

1. For the discussion of lot size effects on warehouse picking and placement route lengths, solid points represent strict organization data and hollow points represent location-relaxed data.

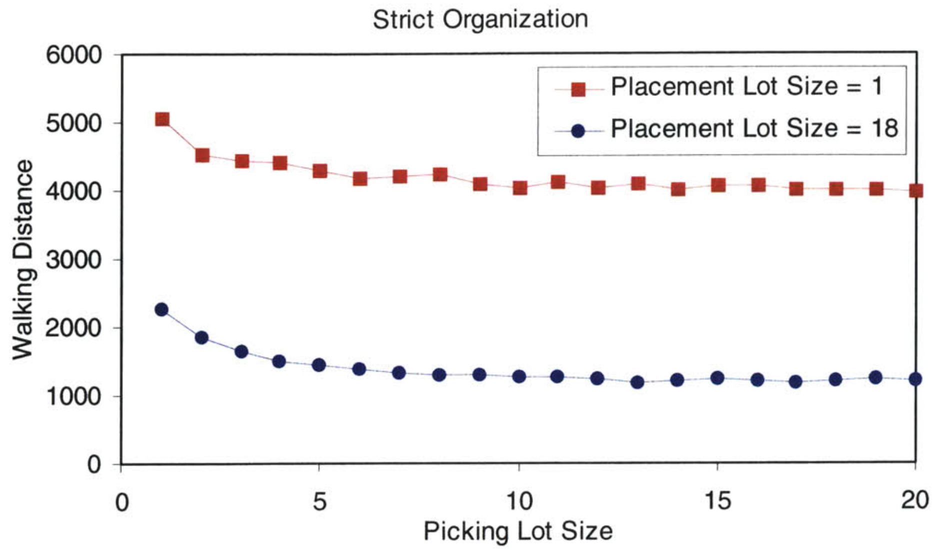


Figure 4.2: Lot size effect on strict organization route walking distance

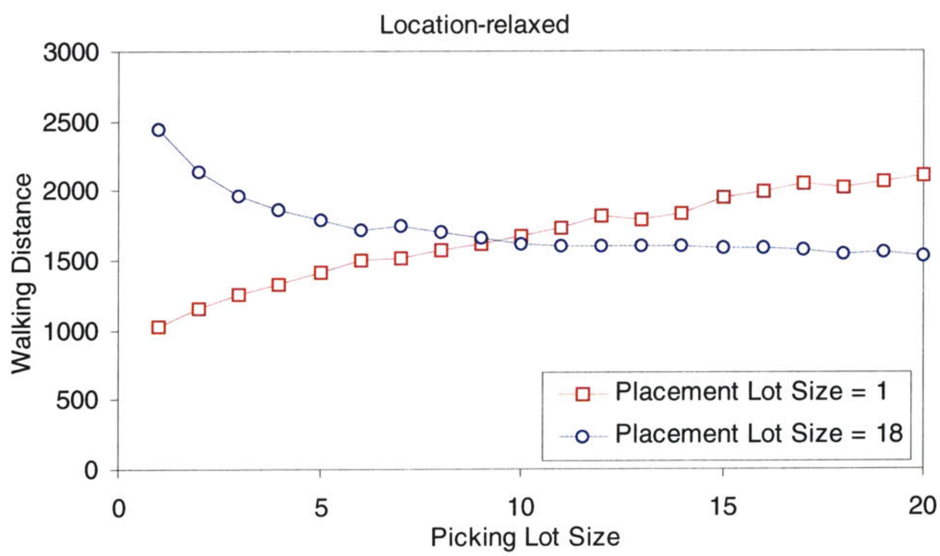


Figure 4.3: Lot size effect on location-relaxed route walking distance

surprising since no additional aggregation occurs in location-relaxed placement. Items arriving pre-aggregated stay semi-aggregated during location-relaxed placement. As a result, items arriving together tend to be placed in adjacent storage bins or bins within the same sub-aisle. This pre-aggregation has stronger benefits with large picking lot sizes. Small picking lot sizes benefit little from aggregation. Furthermore, small placement lot sizes tend to distribute the products across more storage bins in the warehouse. The graph indicates that this type of distributed storage provides advantages that outweigh the benefits of aggregation. The advantage stems from the proliferation of many disparate storage bins holding the same SKU. In other words, smaller placement lot sizes tend to create more choices for the Warehouse-TPP routing problem. The advantage of these multiple locations is not limited to when the picking lot size is one or two, but applies to picking lot sizes as large as eight or nine. This indicates that having to visit eight locations to obtain eight copies of the same SKU does not impose a heavy burden on routing.

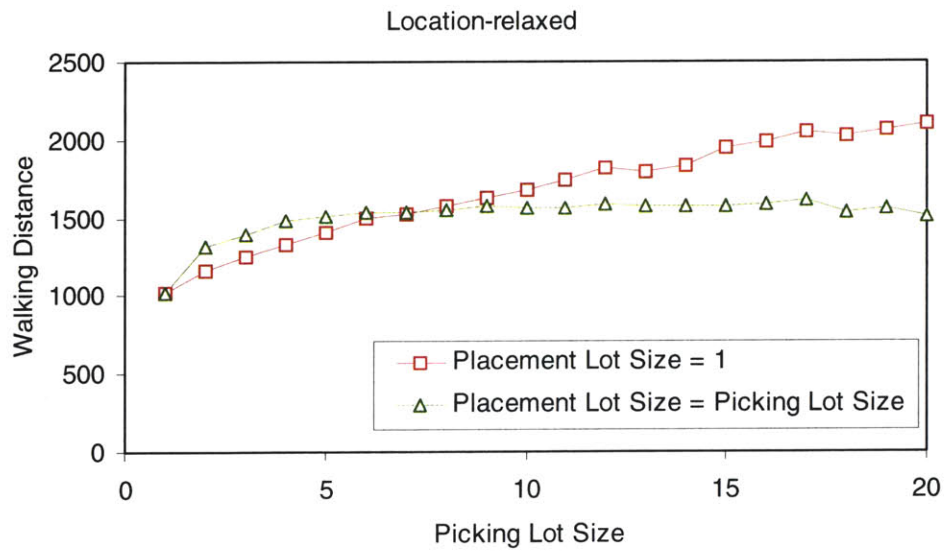
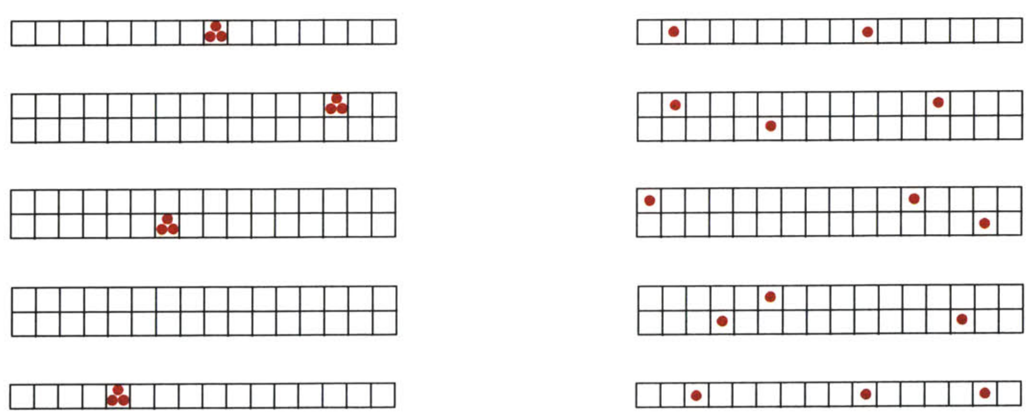


Figure 4.4: Effect of lot size on location-relaxed route walking distance

The benefits of highly distributed storage are further demonstrated by the data in Figure 4.4. Common sense would indicate that if the picking lot size matched the placement lot size, then the amount of pre-aggregation would be sufficient to obtain the benefits of aggregation while still providing for some distribution of the materials across the warehouse. The results show that this intuition is only true when the picking lot size is large. Figure 4.3 already demonstrated that large picking lot sizes prefer large placement lot sizes. The counter-intuitive result is small picking lot sizes prefer the unit lot size for placement, even compared to an equal sized placement lot size. In other words, placing items as singles is better than placing items in threes even when items are known to be picked in lots of three. This surprising result illustrates the strength of multiple locations for storage over aggregation.

For example, Figure 4.5 shows the choice between having four locations that have three copies of the needed SKU and 12 locations with one copy of the SKU. Because the 12 singletons are randomly distributed in the warehouse, almost any path though the warehouse with 12 singletons are likely to pass by three copies of the needed SKU. Furthermore, warehouse picking address multiple SKUs at a time, creating a situation where the set of items benefits from being next to as many other SKUs as possible. In other words, many singleton locations are likely to be next to other relevant locations for the pick route than a few locations with many copies in each. Visiting three separate locations but having more locations from which to choose is easier than visiting one location but having few choices.



A) Aggregated B) Distributed

Figure 4.5: Aggregation versus distribution

4.3.6 Location-relaxed vs. Strict Organization

Section 4.3.4 and Figure 4.2 show the very poor performance of strict organization when the placement lot size is small. Clearly, strict organization with small placement lot size is not a viable option. Fortunately for traditional warehouse operations, material arriving for storage in the warehouse typically arrive in large lots. Warehouses serve as a "step-down buffer" for materials as items arrives in large lots and depart in smaller lots. The step-down ratio, which describes the relative value of incoming lot size and outgoing lot size, should have a significant impact on warehouse performance. For example, a warehouse with products arriving in groups of 100 and departing in groups of 10 will operate very differently than a warehouse with products arriving in groups of 100 and departing in groups of one. Figure 4.6 compares location-relaxed storage and strict organization storage route walking distances for picking and placement when the placement lot size is 18. The graph shows that strict organization yields lower walking distances in every case.

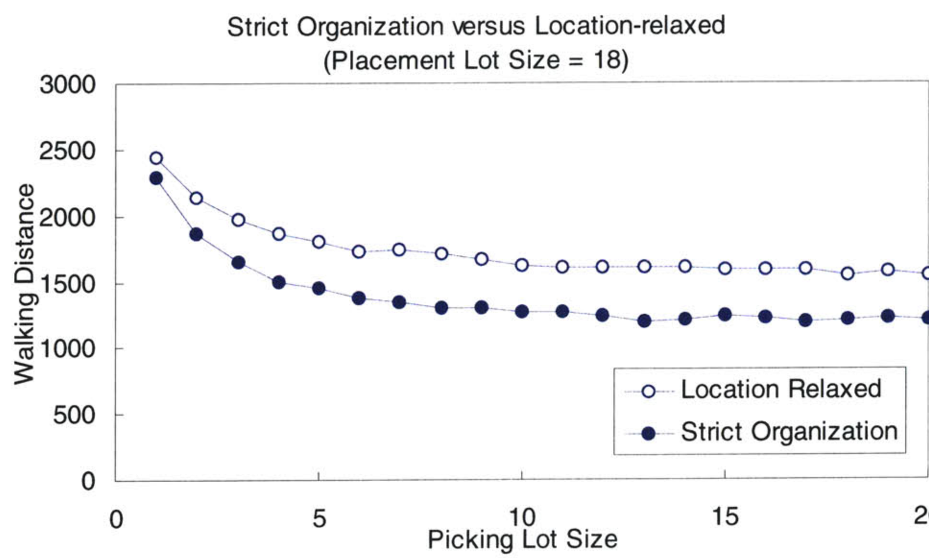


Figure 4.6: Lot size comparison between strict and location-relaxed

While at first, this seems to imply that strict organization is always better than location relaxed storage, location-relaxed storage offers an additional option that strict organization does not. While the data in Figure 4.2 show the inappropriateness of small placement lot sizes for strict organization, Figure 4.3 shows that small lot sizes are not only appropriate with location-relaxed storage, but often the better choice.

Figure 4.7 compares strict organization with a placement lot size of 18 against location-relaxed storage with a placement lot size of one. This seemingly incongruent comparison is valid since material arriving pre-aggregated does not *have* to be placed into the warehouse in large lots. Aggregation requires significant effort, but mixing requires little to none. Through forced randomization, the location-relaxed warehouse can easily create small placement lot sizes even when material arrives in large lots. The result is significantly shorter route walking distances when the picking lot size is also small. The results show that when the picking lot size is five or less, forcing unit sized placement lots with location-relaxed storage has shorter routes than strict organization with placement lot sizes of 18.

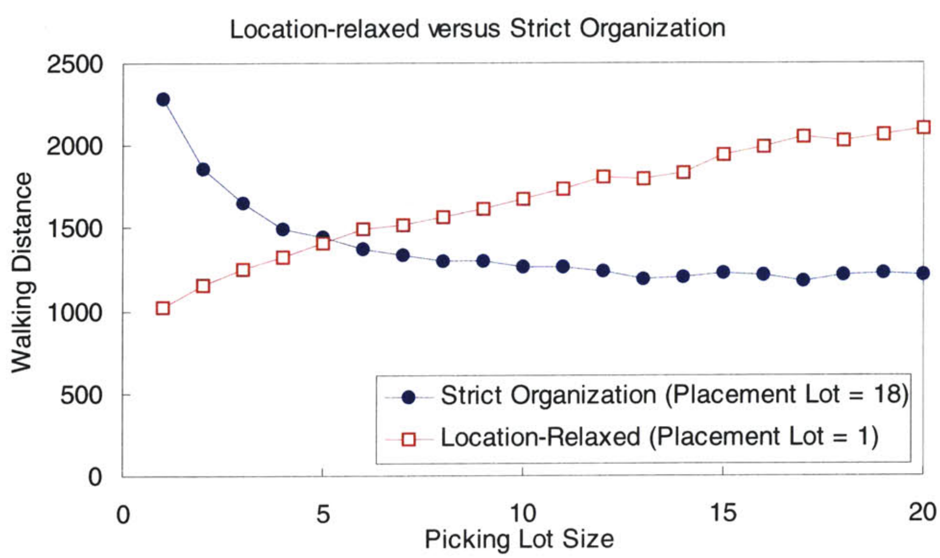


Figure 4.7: Comparison between Strict Organization and Location-relaxed

4.3.7 Discussion

As speculated, location-relaxed storage is most appropriate with small picking lot sizes. Furthermore, location-relaxed storage with small picking lot sizes operates best when the placement lot size is one. Picking lot size is a function of the demand characteristics placed on the warehouse and cannot be changed easily. However, regardless of the characteristics of incoming materials, warehouse operations can enforce unit sized placement lots by mixing the materials before placement. Therefore the critical characteristic for choosing between location-relaxed storage and strict organization is picking lot size.

Warehouses that fill orders directly to consumers are more likely to have unit lot sizes or very close to unit sized lots. One copy of consumer products is usually sufficient for a consumer. This fact makes location-relaxed storage a strong candidate for mail order catalog, electronic commerce, and other direct-to-consumer warehouse applications.

While typical supply chain warehouses that distribute products to retail outlets are not likely to use unit sized picking lots with great frequency, small picking lot sizes around three for average volume items are common in retail stores; therefore, location-relaxed storage cannot be discounted as a possible means for operations improvement. However, many items in retail stores will require high picking lot sizes as well. Mixed lot sizes are not addressed in the current analysis. The applicability of location-relaxed storage to more complex warehouse situations such as mixed lot sizes will be problem dependent and no simple generalization is appropriate. However, the benefits of location-relaxed storage with picking lot sizes as high as five for the small case simulation demonstrates a worthwhile opportunity for further investigation in any warehouse configuration.

The key insights provided by the analyses include:

- Location-relaxed storage offers the more benefits when the picking lot size is small.
- Location-relaxed storage functions best when the placement lot size is one.
- Intentional mixing of products during placement can improve performance of location-relaxed storage

Chapter 5: Efficiency of Strict Warehouses

This chapter quantifies the storage capabilities of a strict storage warehouse under special conditions to estimate the effect of strict organization on the effective storage space capacity of the warehouse. Due to strict constraints on where particular items may be placed, the warehouse may meet the situation where an incoming item has no viable storage bin in which to be stored. This type of overflow may even occur when the warehouse is not completely full because all storage bins with additional space may be assigned to non-matching SKUs.

Under location-relaxed storage, every location is available for use regardless of the identity of an incoming item. Therefore, 100% of the storage space capacity may be used in the location-relaxed case. The analyses described in this chapter quantify strict organization storage space utilization, which unlike location-relaxed storage will vary depending on the number of unique SKUs serviced by the warehouse. For the analyses, all items are assumed to have the same size, and all storage bins have the same capacity. The first analysis (section 5.1) is a worst case scenario for strict organization storage which demonstrates how many items the warehouse may safely contain with no possibility of overflow. The second analysis (section 5.2) generates the most probable distribution of items and determines the maximum number of items whose most probable distribution fits into the warehouse. The third analysis (section 5.3) uses a probabilistic approach to determine how many randomly selected items may fit into the warehouse with probability p of no overflow. Section 5.4 extends the discussion by changing the size of the storage bin capacities to see how bin capacity affects the effective storage space capacity.

5.1 Worst Case Analysis

The worst case scenario finds the minimum number of items such that all the items cannot be placed in the warehouse and maintain strict organization. For example, if there is only one SKU stored in the warehouse, then all locations must be filled before the arrival of another item causes a problem. In this case, the worst case maximum utilization equals 100%. When the warehouse services more than one SKU, the worst case maximum utilization will decrease. Consider a warehouse where:

> U equals the number of SKUs
> S equals the number of storage bins
> M equals the maximum number of items stored in each bin

Since the number of storage bins is S, and the maximum number of items that can be stored in a bin is M, the maximum number of items that may be stored is $M \cdot S$. If the warehouse services two SKUs, A and B; $S-1$ bins are completely full (M items); and the last storage bin has one A item, then any attempt to add a B item to the warehouse will fail since there is no place to store an additional B item without violating strict organization. The total number of items in such a warehouse is $M(S-1)+1$ which is equivalent to $MS-(M-1)$. $MS-(M-1)$ represents the number of items *guaranteed* to fit into the warehouse when the number of unique SKUs is two. Table 5.1 extends the guaranteed fit number to a general number of SKUs.

Between two SKUs and ($S+1$) SKUs, the guaranteed fit decreases by ($M-1$) for each additional SKU. For i SKUs, generate a worst case scenario by filling $S-(i-1)$ storage bins to capacity, then choose one copy of each of the first ($i-1$) SKU items and put one in each of the remaining ($i-1$) bins. Any attempt to place the i^{th} SKU into the warehouse will fail. The maximum number of items for this worst case is $M(S-(i-1))+(i-1)$, which equals $MS-(i-1)(M-1)$.

Table 5.1: Worst case scenario, guaranteed fit

# SKU	Guaranteed Fit
1	$M \cdot S$
2	$M \cdot S - (M-1)$
3	$M \cdot S - 2(M-1)$
4	$M \cdot S - 3(M-1)$
i	$M \cdot S - (i-1)(M-1)$
S	$S + M - 1$
S+1	S
>S+1	S

When the number of SKUs is greater than or equal to $(S+1)$, the guaranteed fit is a constant S, which is the total number of storage bins. This worst case scenario corresponds with the situation where each storage bin contains one SKU that is unique in the warehouse. An additional SKU of a different type can not fit into the warehouse. When the number of SKUs exceeds the number of locations, the worst case capacity is equal to the number of locations. Figure 5.1 shows the guaranteed fit. Utilization at or below the guaranteed fit will always fit into the warehouse regardless of the distribution.

5.2 Expected Distribution Analysis

The expected distribution analysis assumes equal probability for all SKUs. In other words, the probability that one SKU should be added to the distribution is equal to all other SKUs. This does *not* imply that the expected distribution has an equal number of all SKUs. With equal probabilities for all SKUs, the expected distribution becomes binomially distributed.

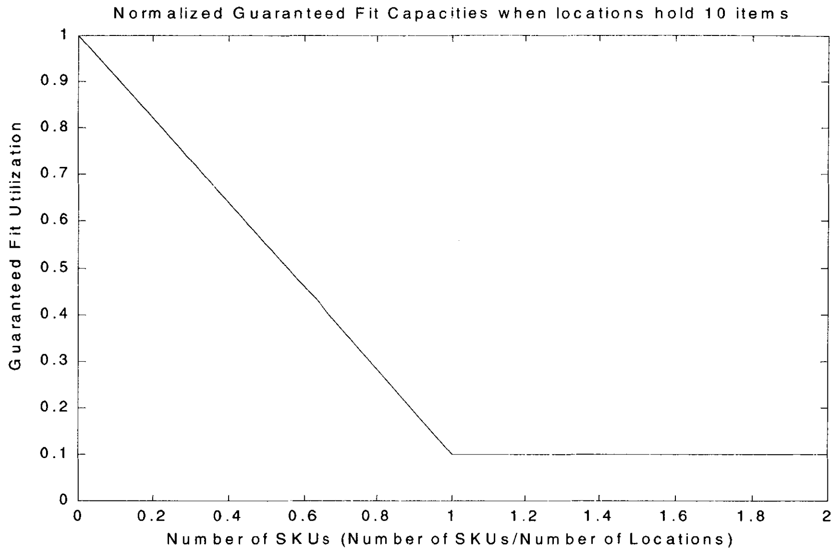


Figure 5.1: Worst case limitation on utilization

For a given warehouse size, the number of unique SKUs serviced by the warehouse, and a target number of items, N, the analysis determines whether or not the expected distribution of size N will fit into the warehouse. Repeating the procedure for several values of N determines the largest number of items whose expected distribution will fit into the warehouse.

5.2.1 Number of Copies Distribution

The analysis begins by determining the probability distribution for how many copies of a particular SKU are in a randomly created distribution. If there are N items in the distribution, then the SKU could have any number of copies from zero to N; however, the probability of all N items being the same SKU becomes extremely small as N gets larger. The probability distribution of numbers of copies of a particular title is a binomial distribution.

Let U equal the number of unique SKUs serviced by the warehouse.
Let N equal the number of items in the warehouse.

Then the probability that there are n copies of a particular SKU in the warehouse is:

$$P(n, N) = \binom{N}{n}\left(\frac{1}{U}\right)^n\left(1 - \frac{1}{U}\right)^{N-n} \tag{5.1}$$

which is binomially distributed with a $\frac{1}{U}$ probability of success and N trials.

Figure 5.2 illustrates the binomial probability distribution corresponding to 2,000 unique SKUs and a total item population of 12,000. While the average number of copies is 6, the probability of any one SKU having exactly 6 copies is approximately 0.16.

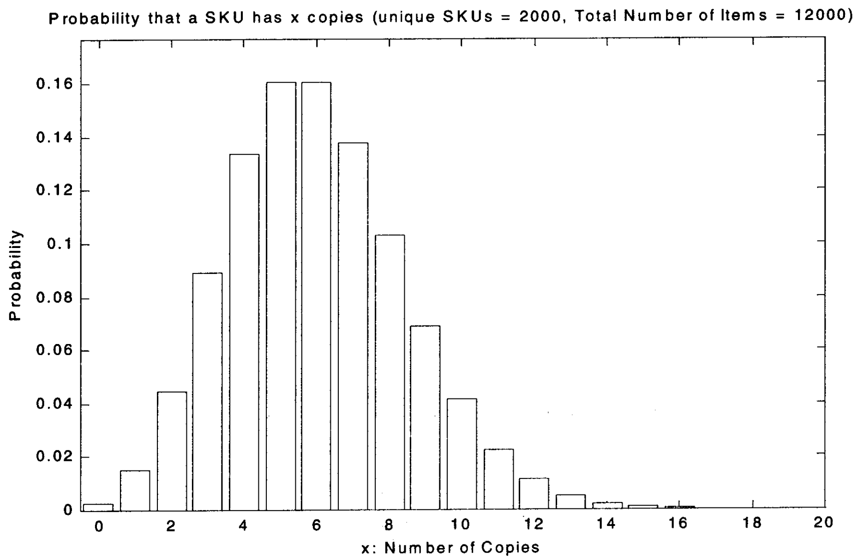


Figure 5.2: Binomial distribution

The binomial distribution for 12,000 trials with a probability of success of 1 in 2000 corresponds to the probability distribution that a particular SKU has x copies in a 12,000 item warehouse of 2000 unique SKUs.

5.2.2 Expected SKU Distribution

Having determined the probability distribution for the number of copies of a particular SKU, the next step establishes the expected SKU distribution for all N items. $P(i, N)$ equals the probability that an SKU had i copies in the distribution with N total items. $U \cdot P(i, N)$

equals the expected number of SKUs that have i copies in a randomly generated distribution containing N total items. Furthermore,[1]

$$\sum_{i=0}^{N} i \cdot U \cdot P(i, N) = N \tag{5.2}$$

In other words, the *expected* distribution has $U \cdot P(0, N)$ SKUs with zero copies in the distribution; $U \cdot P(1, N)$ SKUs with one copy in the distribution; $U \cdot P(2, N)$ SKUs with two copies in the distribution; *etc.* This expected distribution is illustrated in Figure 5.3, which shows the expected distribution of 12000 items, pulled with equal probability from 2000 SKUs. The graph shows that the expected distribution has 4.95 SKUs with zero copies, and 29.7 SKUs with one copy. Although a real distribution must have a non-negative integer number of SKUs corresponding to each number of copies, the expected distribution has fractional numbers representing the number of SKUs with a particular number of copies in the distribution. The expected distribution is not an actual distribution but the average of all possible distributions.

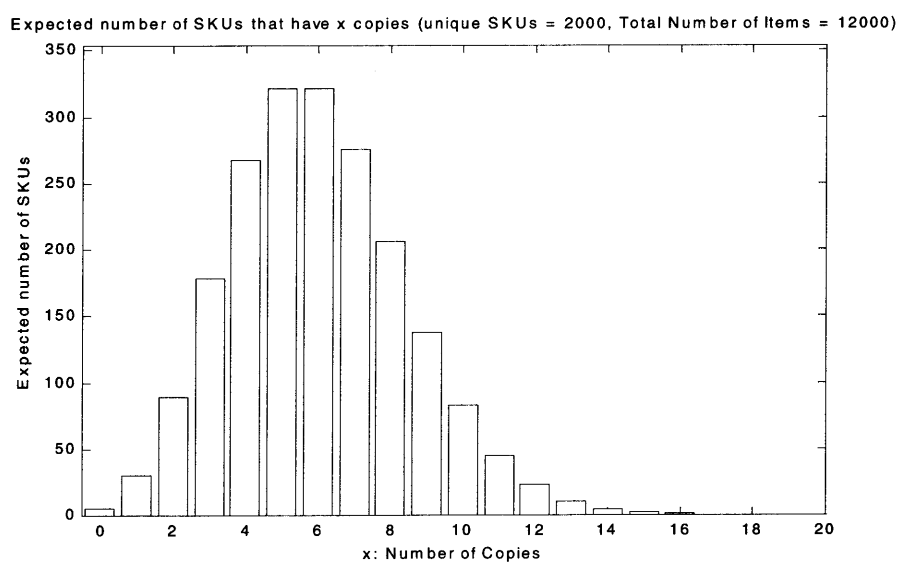


Figure 5.3: Expected number of copies

1. See Appendix C for a proof of equation 5.2.

5.2.3 Storage Limitation

Using this expected distribution of SKUs and the number of items that fit into each location, determining the number of storage bins needed to house the distribution is a matter of simple accounting. The analysis assumes all SKUs are the same size and all storage bins have the same capacity. In other words, given the number of SKUs that fit in a particular location, M, and the total number of storage locations, S, determine the maximum expected distribution that will fit in the space limitation, and the total number of SKUs corresponding to that distribution.

Given:
U equals the number of unique SKUs
S equals the number of storage locations
M equals the capacity of each storage location
Find:
N^*, the maximum number of items corresponding to the expected distribution that fits into a warehouse with S storage locations with a maximum of M items in each location.

Since the expected distribution assigns a rational number of SKUs for each quantity of items, a method of dealing with a fractional expected number of SKUs must be employed in the accounting. Counting the number of storage bins needed for the expected distribution takes a conservative approach by shifting the fractional portion of a quantity and re-assigning that fraction to the next higher quantity. For example, if the expected number of SKUs with zero copies is 4.9 and the expected number of SKUs with one copy is 7.3, the analysis uses an adjusted distribution with 4 SKUs having zero copies. The adjusted distribution would then take the remaining 0.9 SKU and add it to the number of SKUs expected to have one copy, which yields 8.1 SKUs. Truncating the result leaves the adjusted distribution with 8 SKUs having one copy and 0.1 SKU to be added to the next quantity. Because the adjusted distribution assigns a higher number of copies to a portion of the SKUs, the resulting total number of items represented by the adjusted distribution is larger than the original expected distribution. Therefore, if the adjusted distribution fits into the warehouse under strict conditions, then stating that the expected distribution 'fits' into the warehouse is justified.

Figure 5.4 illustrates how the number of unique SKUs serviced by a warehouse affects the effective storage capacity. For this example, the number of storage bins is 1,000 and each storage bin holds 10 items. As the number of unique SKUs increases the expected number of items that fit into the warehouse decreases. As should be the case, the expected distribution results are always greater than the worst case scenario. With the exception of a very small number of SKUs (near zero) and very many SKUs (many multiples of the number of storage bins), the expected distribution analysis shows that the worst case scenario is not likely to occur. Therefore, the utilization found for the expected distribution can be much larger than the worst case result. For example, with 801 SKUs in a warehouse with 1,000 storage bins with the capacity for 10 items, the worst case analysis yields a utilization of 0.28. However, the expected distribution analysis predicts a much larger utilization of 0.7.

The utilization drops sharply when the number of unique SKUs equals the number of locations. As the number of SKUs increases above the total number of storage locations, the expected distribution analysis approaches the worst case result. In other words, as the number of unique SKUs becomes very large, the possibility of randomly selecting 1,000 different SKUs for storage in the warehouse becomes more and more probable.

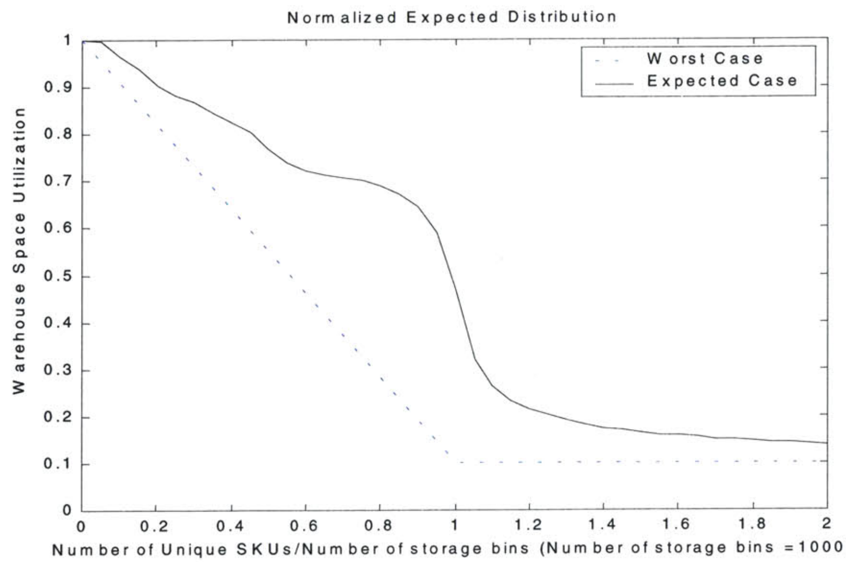


Figure 5.4: Utilization versus normalized number of unique SKUs

5.3 Probabilistic Analysis

The expected distribution illustrates how the number of unique SKUs serviced by the warehouse affects its ability to use all of its warehouse space with the most likely item distribution. Unfortunately, the expected distribution fails to illustrate a metric of reliability. Under the worst case scenario, the guaranteed fit shows the maximum number of items that can occupy the warehouse under strict conditions and *never* cause an overflow. In other words, the worst case analysis shows the storage capacity utilization for 100% reliability.

The expected distribution analysis qualitatively reveals how the storage capacity utilization will react to an increase in number of SKUs serviced; however, the expected distribution analysis gives no indication of the reliability of the result. In other words, we would like to know how often the warehouse would encounter an overflow state when using the storage capacity utilization dictated by the expected distribution analysis.

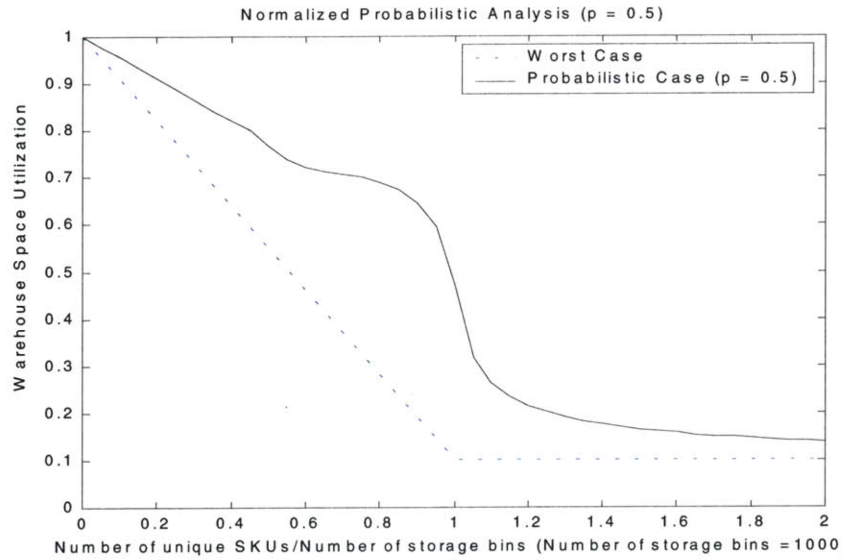


Figure 5.5: Probability analysis for 50% success

The final strict organization warehouse storage capacity analysis quantifies the probability of successful placement for a particular number of items by randomly generating a distribution of items, and calculating whether or not this distribution fits under strict organization. Repeating the procedure several times and tracking the ratio of successful placements to number of attempts yields the probability of success for the given number of items. Furthermore, such a probabilistic analysis can determine the number of items that correspond to a particular probability of success. Figure 5.5 shows the utilization levels that correspond to a 50% probability of successful placement. The solid curve represents the fraction of the warehouse space that can be filled with items and have a 50% chance of successful placement under strict organization. In other words, 50% of randomly generated distributions will successfully fit into the warehouse, and 50% will not fit.

Figure 5.6 compares the results of the probabilistic analysis with the expected distribution analysis. Warehouse utilization corresponding to a 50% success rate in

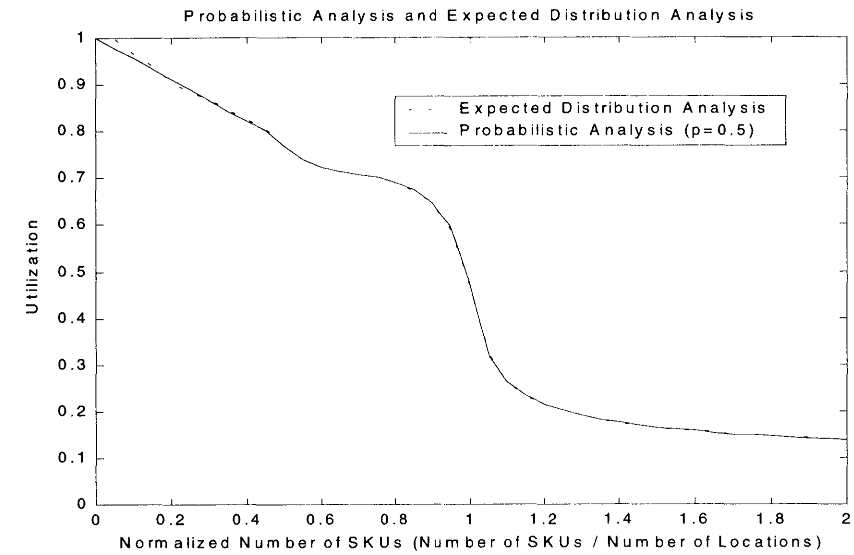


Figure 5.6: Expected distribution and 50% success rate

placing all items while maintaining strict organization closely follows the warehouse utilization for success with the expected distribution. The close correlation between the expected distribution analysis and the 50% success rate analysis seems to imply that the expected distribution corresponds to a 50% success rate. Closer inspection shows that small changes in utilization change the success rate rapidly near the 50% success rate.

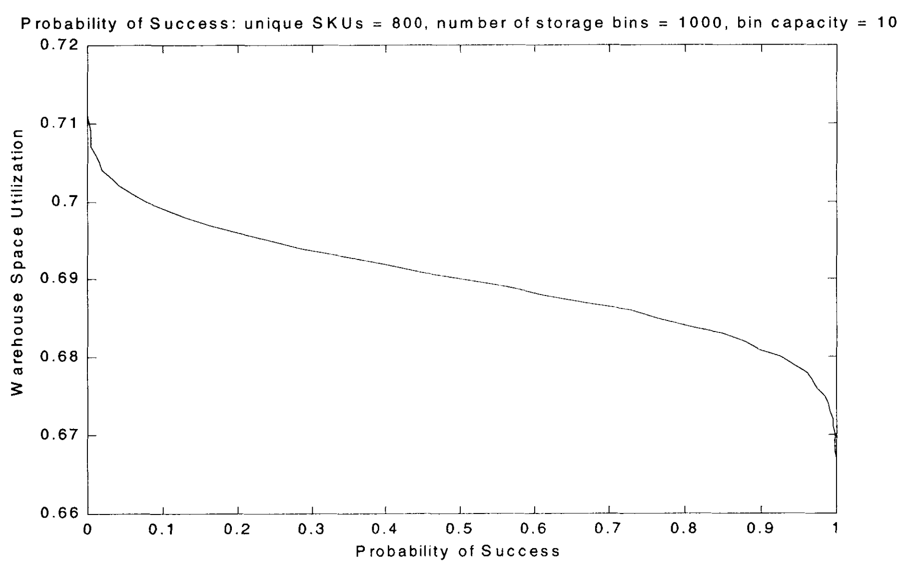


Figure 5.7: Rapid success rate change

Figure 5.7 shows the rapid change from a very high success rate to a very low success rate over a span of less than 5% of the warehouse space. Furthermore, Figure 5.8 shows the boundaries for 1% and 99% success rates. The very narrow band between 1% and 99% success rates further illustrate the rapid switch from low to high success rates. The expected distribution curve, which falls within this narrow band, is a very good indicator of this region of rapid success rate change. Therefore, as long as the warehouse utilization is slightly below the amount described by the expected distribution, the success rate will be very near 100%.

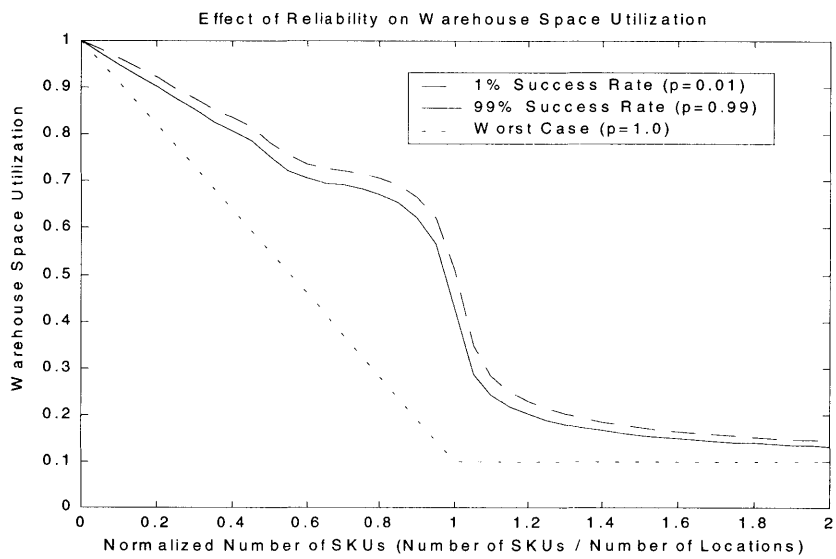


Figure 5.8: Warehouse utilizations for different success rates

5.4 Effect of Location Size

For the previous space utilization analyses, all storage bins were considered to have a maximum capacity of ten items each. This section explores the effect of different storage bin sizes on the strict organization warehouse space utilization curves. The following results consider bin storage capacities of two and 20.

5.4.1 Small Capacity

Figure 5.9 shows the result of the storage capacity utilization analysis with a smaller storage bin size. With only the capacity to house two items, the shape of the curve is dramatically different, but the overall result is very similar. As with the previous analysis, the difference between a 1% success rate, and 99% success rate is very small. The major difference is the greater smoothness of the utilization curve with the smaller capacity storage bins. While the storage capacity utilization curve decreases at a fluctuating rate when there are 10 units in each storage bin, the curve decreases at a fairly constant and well behaved rate when the storage size is reduced to 2. The smoothness is attributable to the fact that unused storage space is limited to no more than one unit per storage bin.

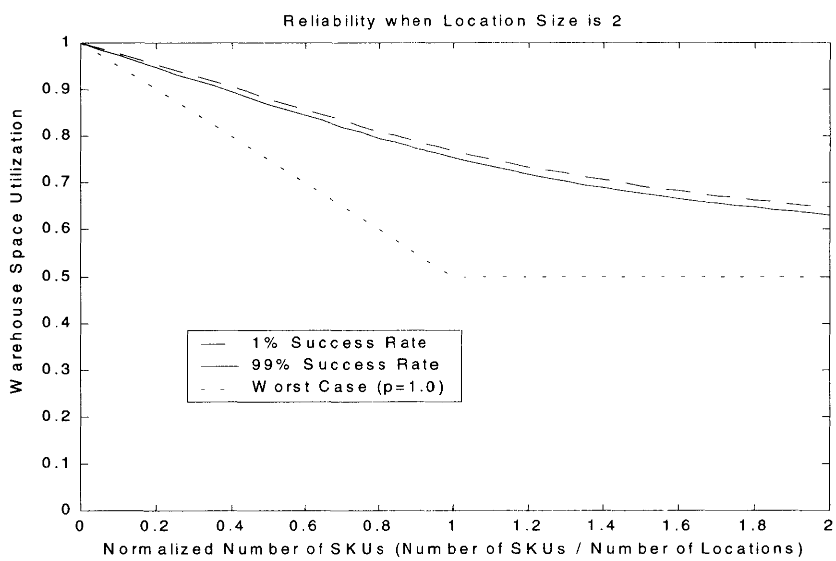


Figure 5.9: Smaller capacity results

5.4.2 Large Capacity

Figure 5.10 shows the effect of a larger storage bin on the utilization curve. Again, the small difference between 1% and 99% success rate is similar to smaller storage bin capacity results. While smaller storage bin capacities increase the smoothness of the utilization curve, increasing the storage bin capacity continues the trend by decreasing the smoothness. Fluctuations in the curve are much more pronounced in the 20 units per storage bin utilization curves to the point that the curves are no longer monotonically decreasing with increasing SKU service level.

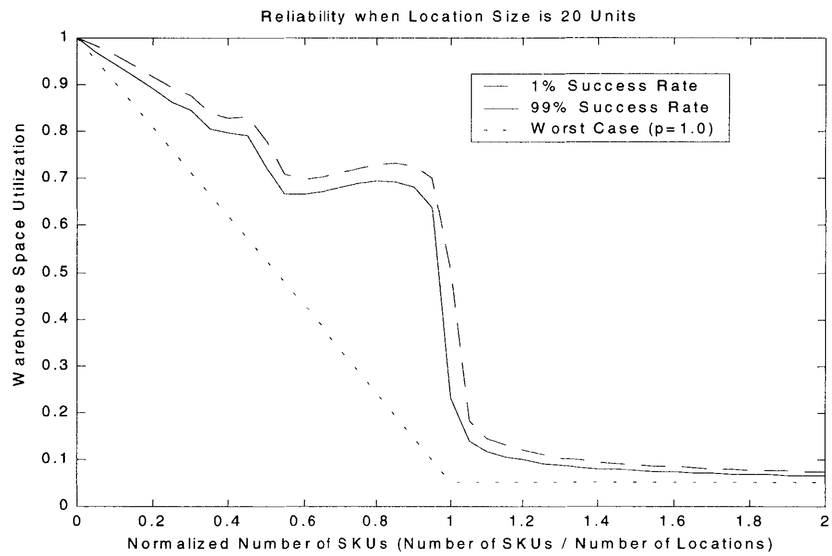


Figure 5.10: Larger capacity results

Over the span of normalized number of SKUs between 0.6 and 0.8, the warehouse space utilization *increases*. Over all other spans, the warehouse space utilization *decreases* with increasing number of SKUs serviced. Strict organization, and the fixed size of each storage location explains the region that runs counter to the normal trend of decreased utilization with increasing number of unique SKUs. The region of increased utilization is due to the fact that when the storage capacity is 20, SKUs that have 20 copies fit into a single storage bin, but increasing the number of copies by a only one unit to 21 requires two storage bins.

Figure 5.11 illustrates the average number of copies for all SKUs. The average number of copies is determined by dividing the total number of items by the total number of SKUs serviced. The average number of copies crosses 20 units when the normalized number of SKUs is approximately 0.65, which falls within the region of increasing utilization. In other words, when the average number of copies starts to fall below the storage space size, warehouse space utilization increases slightly. The average number of copies corresponds to the most frequent number of copies found amongst the SKUs. Figure 5.3 in Section 5.2.2 illustrates the relationship between average number of copies and the expected distribution. When many SKUs have slightly more than 20 copies, each of those SKUs must occupy two locations under strict organization rules. As the number of unique SKUs increases, the average number of copies decreases and more SKUs have 20 copies or less, which only require one location.

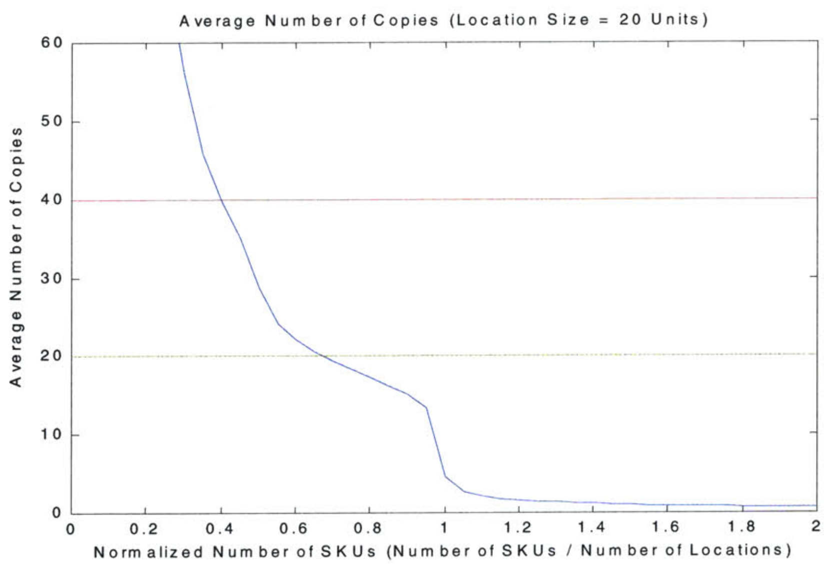


Figure 5.11: Average number of copies

The average number of copies graph shows a correlation between portions of the graph where the average number of copies is close to a multiple of the location size and portions of Figure 5.10 where the utilization opposes the trend of decreasing with increasing number of unique SKUs.

Furthermore, the normalized number of SKUs corresponding to an average of 40 copies is approximately 0.4. In Figure 5.10, 0.4 normalized SKUs corresponds to a region on the graph where the rate of utilization decline changes suddenly and decreases at a slower rate. While less pronounced, the reason for this change in slope is the same as for the increase in utilization around 0.65 normalized SKUs. When filling the warehouse, SKUs with slightly more than 40 copies require three locations, whereas SKUs with 40 copies or less need only two locations. This effect explains both why the curves are less smooth with larger storage bin sizes and why the curves sometimes have regions increased storage capacity utilization with increasing number of SKUs.

5.5 Discussion

The various analyses of strict organization warehouse space utilization show that the worst case scenario yields very poor space utilization levels but is very unlikely to occur. By accepting a very slight chance of overflow, the acceptable utilization level increases dramatically. Up to a certain point, increases in space utilization levels result in miniscule decreases in success rates. However beyond that point, increases in space utilization levels lead to a very large and rapid drop in success rate. Finding this drop-off point for the given warehouse capacities and number of SKUs is the key to determining the best space utilization level for strict organization. However, this operation point will always be less than the potential utilization level for location-relaxed storage because full utilization of the warehouse space is always possible with location-relaxed storage.

Chapter 6: Effect of Errors on Warehouse Operations

Idealized views of warehouse operations do not address the real issue of errors in day-to-day operations. This chapter looks at the role of unintended errors on location-relaxed and strict organization operations by modeling warehouse operations as a series of error probabilities and operation costs.

During operations the item confronts four action stages within the warehouse. Upon arrival, the item is placed within a storage location (stock); when filling an order, the item is picked (pick); the picked items are then sorted into individual orders (sort); and finally the order is packaged into the outgoing order (pack). At any of these stages an error can occur. The ramifications of these errors depends on the type of storage system implemented.

6.1 Action Costs

The analysis of errors depends directly on the relative effort costs for each step and the effort cost of corrective measures. The variable c_{stock} is the cost associated with taking an incoming item and placing it into the warehouse. The variable c_{pick} is the cost associated with locating and picking an item. The cost associated with dividing the picked items into groups that fill orders is represented by c_{sort}. Finally, c_{pack} is the cost associated with placing the sorted groups into bundles for shipment. The effort for each stage will be different between the strict organization method and the location-relaxed storage method.

For this error analysis, the effort associated with walking distances is included in the cost to stock and cost to pick values. While each item will encounter a different walking distance cost, this error analysis considers the average walking cost. In other words, c_{stock} is the average cost to stock, and c_{pick} is the average cost to pick.

6.2 Probability of Errors

Under ideal conditions where no errors occur, each item goes through each stage of the warehouse exactly once for a total cost:

$$\text{ideal cost} = c_{stock} + c_{pick} + c_{sort} + c_{pack} \tag{6.1}$$

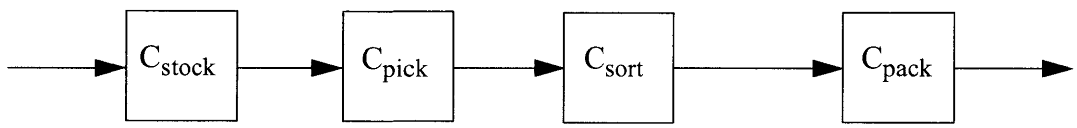


Figure 6.1: Ideal operations

At each stage of the warehousing process, the possibility of error will increase the overall effort above the ideal. The variable p_{stock} is the probability of placing an item in the incorrect location. The probability of picking the incorrect item is p_{pick}. The variable p_{sort} is the probability of sorting the item into the incorrect group, and p_{pack} is the probability of incorrectly packing an item. Figure 6.1 shows the ideal operation steps. The probability of error at each stage will be different between the strict organization method and the location-relaxed storage method.

The error model begins with the ideal operations and then investigates how the ideal total cost changes when we introduce the possibility of errors to the warehousing process. Furthermore, by assigning appropriate values for costs for each method we can compare the effect of errors between the strict organization and location-relaxed methods.

6.2.1 Strict Organization

In a strict organization warehouse, the misplacement of an incoming item must be revisited (extra pick operation needed) and restocked (extra stock operation needed). Incorrectly picked items must be restocked, then the correct item picked. A mistake in sorting the picked items requires the operation to be repeated. And a mis-packed item will need to be repacked.

Table 6.1: Strict organization storage action list

Action	Cost	Probability of Error	Remedy
Stock	c_{stock}	p_{stock}	Pick the mis-stocked item, then stock the item again
Pick	c_{pick}	p_{pick}	Restock the mis-picked item, then pick the correct item
Sort	c_{sort}	p_{sort}	Re-sort the item
Pack	c_{pack}	p_{pack}	Repack the item

An accurate account for the impact of errors on the warehouse must also consider the possibility of error during the remedy. The remedy for a picking error, sorting error, and packing error merely require repeating steps. For example, Figure 6.2 shows the result of a sorting error returning the item to the sort process for re-sorting, then continuing again as usual. The cost of the sorting operation is c_{sort}, and the probability of a sorting error is p_{sort}. The arrow labeled with p_{sort} represents a sorting error and the arrow labeled 1-p_{sort} represents a successful sorting action. Analyzing the flow diagram in Figure 6.2 requires

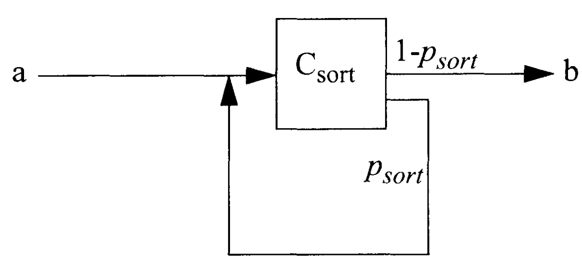


Figure 6.2: Sorting error remedy

calculating the cost associated with moving from state a to state b. The cost of going from state a to state b is the cost of a sorting operation plus the probability of a stocking error times the cost of going from state a to state b (Equation 6.2). In other words, the effective cost of a stocking operation is the idealized cost of a stocking operation plus the added cost associated with stocking errors.

$$\begin{aligned}(\text{Cost from state } a \text{ to state } b) &= c_{stock} + p_{stock}(\text{Cost from state } a \text{ to state } b) \\ (\text{Cost from state } a \text{ to state } b) &= \frac{c_{stock}}{1 - p_{stock}}\end{aligned} \tag{6.2}$$

The remedy for a stocking error is slightly more complicated since it requires an intermediate step before returning to the normal flow of operations. The possibility of error during the intermediate step complicates the analysis. Figure 6.3, shows that the remedy for a stocking error requires an intermediate step, an additional pick step, before returning the item for re-stocking; however, the diagram in Figure 6.3 does not consider the possibility of errors during the intermediate picking step. Figure 6.4 shows the remedy for a stocking error that does account for intermediate errors. The stacking of errors are increasingly unlikely to occur deeper in the model shown in Figure 6.4. Furthermore, the model greatly complicates the error model for what is an unlikely event. For simplicity, the truncated model shown in Figure 6.3 represents a stocking error for the error analysis.

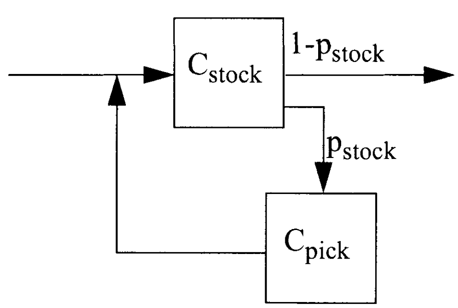


Figure 6.3: Stocking error remedy

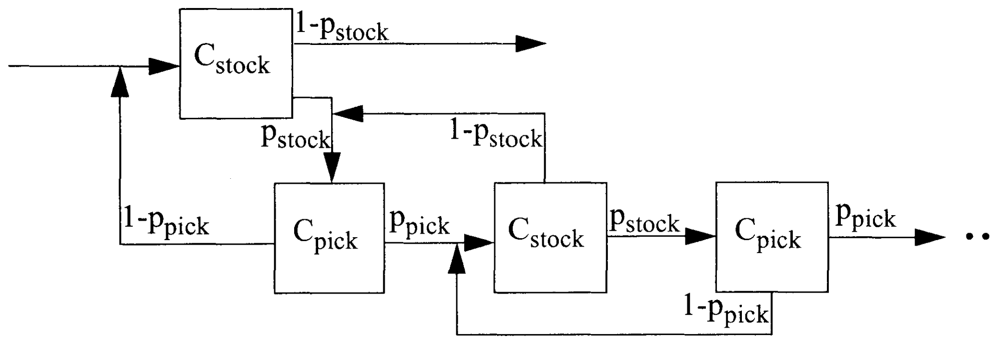


Figure 6.4: Stocking error remedy with intermediate errors

Comparing the location-relaxed storage error model to the strict organization error model without intermediate errors is a conservative comparison. The actual strict organization result with intermediate errors is slightly larger than the simplified analysis shown here with the model illustrated in Figure 6.5.

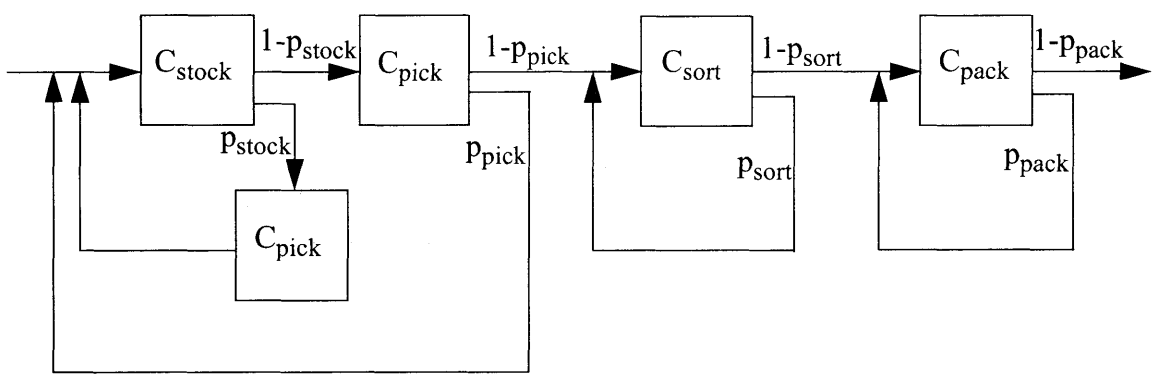


Figure 6.5: Effect of errors on strict organization

Neglecting the possibility of error during the corrective step for a stocking error yields a total cost for strict organization operations with errors:

$$\text{Strict Organization Cost} = \frac{c_{stock} + c_{pick}}{(1 - p_{stock})(1 - p_{pick})} + \frac{c_{sort}}{1 - p_{sort}} + \frac{c_{pack}}{1 - p_{pack}} \tag{6.3}$$

The cost difference between ideal operations and an error prone strict organization warehouse is:

$$\text{Cost with errors - Ideal} = \frac{1-(1-p_{stock})(1-p_{pick})}{(1-p_{stock})(1-p_{pick})}(c_{stock}+c_{pick}) + \frac{p_{sort}}{1-p_{sort}}c_{sort} + \frac{p_{pack}}{1-p_{pack}}c_{pack} \quad (6.4)$$

6.2.2 Location-Relaxed Storage

The process for location-relaxed storage with errors is very similar to the strict organization model. The only difference in error analysis format is the remedy for a stocking error. However, the actual values for each step's effort and each step's probability for error are potentially different. The action, *update*, which indicates the corrective step of updating the computer database to reflect the change in position, is shown mainly for illustration. Furthermore, the update action is considered error-free because mistakes in the database are not considered in either the strict organization or location-relaxed error analyses. In practice, the update action would require very little effort, and would be performed continuously after every action for constant verification. RFID technology makes this kind of management very easy and is one of the main benefits of RFID technology for material handling. The update action is included for a stocking error in the location-relaxed storage error model and can be removed by setting the update cost to zero.

Table 6.2: Location-relaxed Storage Action List

Action	Cost	Probability of Error	Remedy
Stock	c_{stock}	p_{stock}	Update the database to reflect the error
Update	c_{update}	0	--
Pick	c_{pick}	p_{pick}	Stock the item, then pick the correct item
Sort	c_{sort}	p_{sort}	Re-sort the item
Pack	c_{pack}	p_{pack}	Repack the item

Both strict organization and location-relaxed storage treat picking, sorting and packing errors the same. Stocking errors, however, require picking of the mis-stocked item, and restocking in strict organization operations; but only an update of the database in location-relaxed storage and no physical intervention. Figure 6.6 illustrates the location-relaxed storage error model.

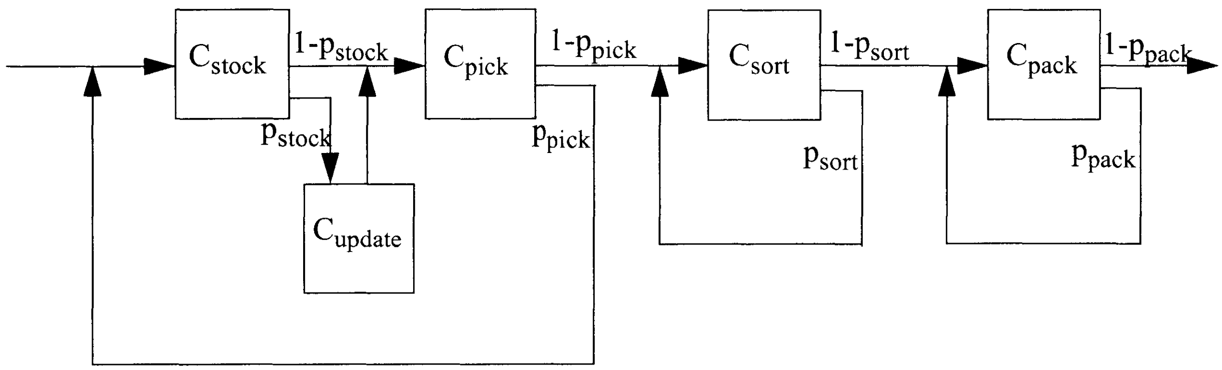


Figure 6.6: Effect of errors on location-relaxed storage

The location-relaxed operation cost with errors is:

$$\text{Location-relaxed Cost} = \frac{c_{stock} + c_{pick} + p_{stock}c_{update}}{(1 - p_{pick})} + \frac{c_{sort}}{1 - p_{sort}} + \frac{c_{pack}}{1 - p_{pack}} \tag{6.5}$$

The cost difference between ideal operations and an error prone location-relaxed warehouse is:

$$\text{with Errors - Ideal} = \frac{p_{pick}}{(1 - p_{pick})}(c_{stock} + c_{pick}) + \frac{p_{stock}c_{update}}{1 - p_{pick}} + \frac{p_{sort}}{1 - p_{sort}}c_{sort} + \frac{p_{pack}}{1 - p_{pack}}c_{pack} \tag{6.6}$$

6.3 Comparison of Strict Organization to Location-relaxed Errors

Two error comparisons show how location-relaxed storage tends to be less sensitive to operation errors than strict organization. In the first comparison, all costs and probabilities are considered equal between the strict organization and location-relaxed methods. The second comparison method takes into account the expected differences between the two models and treats the costs and error probabilities accordingly.

6.3.1 Equal Costs, Equal Probabilities

When the costs for each action are the same between corresponding actions of the strict organization warehouse and the location-relaxed warehouse, the difference in total cost is:

$$\text{Strict Organization Cost - Location-relaxed Cost} = \frac{p_{stock}(c_{stock} + c_{pick})}{(1 - p_{stock})(1 - p_{pick})} - \frac{p_{stock}c_{update}}{1 - p_{pick}} \tag{6.7}$$

As stated before, the update action shown in the location-relaxed method emphasizes the difference between the strict organization and location-relaxed cases. In reality an update occurs after every action, whether in strict warehousing or location-relaxed warehousing. For this reason, the value of c_{update} is neglected. Therefore,

$$\text{Strict Organization Cost - Location-relaxed Cost} = \frac{p_{stock}(c_{stock} + c_{pick})}{(1 - p_{stock})(1 - p_{pick})} \tag{6.8}$$

Since probabilities p_{stock} and p_{pick} must have values between zero and one, and all costs are positive, the expression in Equation 6.8 must be positive. Therefore, in the case where the probabilities and action costs are the same between the two error models, the location-relaxed method will always have a lower total cost.

6.3.2 Unequal Costs, Unequal Probabilities

Different costs and probabilities between the two models complicate the relationship between the total cost of the strict warehouse relative to the location-relaxed warehouse.

Designating difference values for cost and probabilities makes the difference in total cost equal:

$$\frac{c_{so,\,stock} + c_{so,\,pick}}{(1 - p_{so,\,stock})(1 - p_{so,\,pick})} - \frac{c_{lr,\,stock} + c_{lr,\,pick}}{1 - p_{lr,\,pick}} + \frac{c_{so,\,sort}}{1 - p_{so,\,sort}} - \frac{c_{lr,\,sort}}{1 - p_{lr,\,sort}} + \frac{c_{so,\,pick}}{1 - p_{so,\,pick}} - \frac{c_{lr,\,pick}}{1 - p_{lr,\,pick}} \tag{6.9}$$

6.3.2.1 Probability of Stocking Error

Due to the aggregation of items in the strict organization warehouse, the occurrence of errors during a manual stocking action is likely to be less frequent in a strict organization warehouse than in a location-relaxed warehouse. Under strict organization, items of the same type reside in the same location. When a new item is added to the warehouse, the item will either be placed in an empty location or a location in which matching items have already been placed. This visual clue assists in stocking and prevents errors.

If all values of cost and probability are the same between the two models except for the probability of stocking error, the total cost difference (strict - location-relaxed) becomes

$$\frac{p_{so,\,stock}(c_{stock} + c_{pick})}{(1 - p_{so,\,stock})(1 - p_{pick})}. \tag{6.10}$$

According to Equation 6.10, no matter what the relative values of stocking error probabilities, the cost for strict organization will always be greater than or equal to the cost of the location-relaxed warehouse. The two costs are equal only when strict organization stocking actions operate error free ($p_{so,\,stock} = 0$).

Since the aggregation of same type items under strict organization provides visual clues for proper placement procedures, the probability of stocking error is expected to be less for strict organization operations than in location-relaxed warehousing. Despite this disparity in favor of strict organization, the error model shows that the influence of stocking error will tend to favor location-relaxed storage, regardless of the fact that strict organization stocking errors are less likely.

6.3.2.2 Stocking Cost

If all values of cost and probability are the same between the two models except for the cost of stocking, the total cost difference (strict - location-relaxed) becomes

$$\frac{c_{so,\,stock} - (1 - p_{stock})c_{lr,\,stock} + p_{stock}c_{pick}}{(1 - p_{stock})(1 - p_{pick})} \tag{6.11}$$

The location-relaxed warehouse operation cost is less than the strict organization warehouse operation cost when

$$c_{so,\,stock} > (1 - p_{stock})c_{lr,\,stock} - p_{stock}c_{pick} \tag{6.12}$$

When $p_{stock} = 0$, Equation 6.12 reduces to $c_{so,\,stock} > c_{lr,\,stock}$, which indicates that the location-relaxed warehouse has lower overall costs when the cost of a stocking operation in a strict organization warehouse exceeds that of a location-relaxed warehouse.

When p_{stock} is non-zero, for a strict organization warehouse to have lower cost than the location-relaxed warehouse, the cost of a stocking action for the strict warehouse must be less than the cost of a stocking action in the location-relaxed method. If $c_{pick} = 0$, the location-relaxed stocking cost must exceed the strict stocking cost by a factor of $\frac{1}{1 - p_{stock}}$ for the strict organization total cost to be less than location-relaxed total cost. If c_{pick} is non-zero, then the location-relaxed stocking cost must be even larger for the strict organization total cost to be less than location-relaxed total cost. Again, the influence of the stocking error on strict organization storage drives the difference between the two models.

6.3.2.3 Picking Cost

When all values of cost and probability are the same between the two models except for the cost of picking, the total cost difference (strict - location-relaxed) becomes

$$\frac{p_{stock}c_{stock} + c_{so,pick} - (1 - p_{stock})c_{lr,pick}}{(1 - p_{stock})(1 - p_{pick})}. \tag{6.13}$$

The strict organization total cost is greater than the location-relaxed total cost when

$$c_{so,pick} > (1 - p_{stock})c_{lr,pick} - p_{stock}c_{stock}. \tag{6.14}$$

When $p_{stock} = 0$, the strict organization total cost is greater than the location-relaxed total cost when the cost to pick in the strict warehouse is greater than the cost to pick in the location-relaxed warehouse.

When p_{stock} is non-zero and $c_{stock} = 0$, the location-relaxed picking cost, $c_{lr,pick}$, must be larger than the strict organization picking cost, $c_{so,pick}$ by a factor of $\frac{1}{1 - p_{stock}}$, in order for the strict organization cost to be less than the location-relaxed total cost. When p_{stock} is non-zero and c_{stock} is non-zero, then the difference in picking costs must be even wider for the strict total cost to be less than the location-relaxed total cost.

Again, the error model shows a greater sensitivity in the strict organization case since $c_{lr,pick}$ must exceed $c_{so,pick}$ by at least $\frac{1}{1 - p_{stock}}$ for the total costs to be equal, and the greater sensitivity can be attributed to the probability of stocking error.

6.3.2.4 Probability of Picking Error

When all values of cost and probability are the same between the two models except for the probability of picking errors, the total cost difference (strict - location-relaxed) becomes

$$\frac{(p_{stock} + p_{so,pick} - p_{lr,pick} - p_{stock}p_{so,pick})(c_{stock} + c_{pick})}{(1 - p_{stock})(1 - p_{lr,pick})(1 - p_{so,pick})}. \tag{6.15}$$

The strict organization total cost is greater than the location-relaxed total cost when

$$p_{so,pick} > \frac{p_{lr,pick} - p_{stock}}{1 - p_{stock}}. \quad (6.16)$$

When $p_{stock} = 0$, the strict organization total cost is greater than the location-relaxed total cost when the probability of a picking error in the strict warehouse is greater than the probability of a picking error in the location-relaxed warehouse.

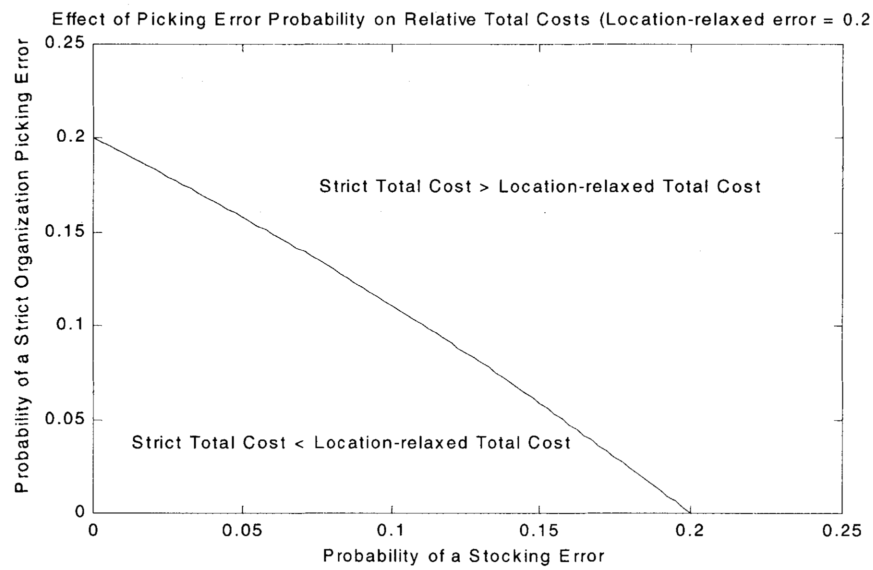


Figure 6.7: Effect of picking errors on relative costs

When p_{stock} is non-zero, the relative values of picking costs determine whether the strict total cost is larger than the location-relaxed total cost. Figure 6.7 shows the effect of various error probabilities on the relative values of strict and location-relaxed total cost. In this example, the probability of a location-relaxed picking error is fixed at 0.2. The x-axis shows varying probabilities of a stocking error, which is equal for both location-relaxed storage and strict organization storage. The y-axis shows varying error probabilities for strict organization picking. The region in the upper right corresponds to values that yield a strict total cost that is greater than the location-relaxed total cost. The region in the lower left corresponds to values that yield a strict organization total cost that is less than the location-relaxed total cost. In all instances, the probability of a strict picking error must be less than the probability of a location-relaxed picking error for the strict total cost to be less than the location-relaxed total cost. Aggregation in the strict organization warehouse will tend to make the probability of a picking error less likely in the strict case. Therefore, lower total cost for the strict organization warehouse is possible. However, the relative values of the total costs according to the error model will also depend on the probability of a *stocking* error. In order for strict organization to have a lower total cost than location-relaxed total cost, the stocking error probability must be significantly lower than the picking error probability. Stocking error again drives the difference between the two models.

6.4 Discussion

Without an automatic monitoring system, such as the RFID tag based Auto-ID system, warehouses are very sensitive to operation errors, especially location-relaxed storage warehousing. With an effective monitoring system, errors are not eliminated, but the impact of these errors are greatly reduced. The most prominent example of this error insensitivity is with location-relaxed storage and storage errors. Location-relaxed storage with automated monitoring 'virtually' removes storage errors. The errors still occur; however, the actions required to rectify the error are reduced to an automated and effortless computer database update. This insensitivity to stocking mistakes drives all other error comparisons between location-relaxed storage and strict organization storage.

Chapter 7: Demand-based Storage Policies

Traditional warehouse operations often use demand information about each SKU to make placement decisions. These choices attempt to increase picking efficiency by placing higher demand items in favorable warehouse locations. The cube-per-order index (COI) proposed by Heskett [1963] is a strict organization placement policy for dedicated storage that assigns high turnover SKUs closer to the dock. This type of assignment increases efficiency by increasing the utilization of easy to access storage bins. High demand SKUs will need to be picked more often; therefore, visiting the corresponding storage bin more often and increasing the use of favorable storage locations.

Under ideal strict organization conditions, there is a one-to-one correspondence between each SKU and storage location. Strict organization demands that each SKU be assigned to at least one storage bin; *which* storage bin is not a concern for satisfying the requirements of strict organization. Demand-based policies, however, give guidance in making good choices in assigning SKUs to storage bins.

With location-relaxed storage, the role of demand-based storage policies is not as straight-forward as with strict organization. Many different SKUs may occupy the same storage bin at the same time and *a priori* assignment of SKUs to storage bins makes little sense. Demand-based policies are more compatible with strict organization because they supplement the already existing need to assign SKUs (or more precisely, the corresponding storage bin) to a particular location. In location-relaxed storage this assignment does not exist. Furthermore, one of the main advantages of location-relaxed storage is the *laissez-faire* approach to item placement, and demand-based storage runs counter to the undirected, easy location-relaxed storage scheme.

This chapter investigates the effect of demand-based placement policies on location-relaxed storage. Section 7.1 discusses the structure of a rectangular warehouse and three metrics that may be used to evaluate the attractiveness of a particular storage location. Section 7.2 describes the results of a simplified simulation that compares demand-based placement to random placement. Section 7.3 explains the effect of configuration on warehouse performance and how these effects apply to demand-based storage.

7.1 Favorable Locations

The basic premise of demand-based storage hinges on the idea that certain storage locations are more easily accessed than others. Typically, favorable locations include storage bins close to the pickup/drop off location or close to the cross aisles. This section investigates several metrics for quantifying favorable conditions for warehouse picking operations. The three qualities considered include the distance of the storage bin to the pickup/drop-off location, the cost to add the storage location to a pre-existing picking path, and the cost to add some other location to a picking route containing the storage bin.

7.1.1 Base Distance

The most direct and simple metric for measuring the favorability of a storage bin is the walking distance between the pickup/drop-off location and the storage location. This simple-to-calculate base metric provides a loose lower bound on all walking routes that access the storage location. Figure 7.1 illustrates the base distance metric measured as distance from a pickup/drop-off location. Locations that are closer to the pickup/drop-off location are considered more favorable because accessing these locations require less walking from the starting point.

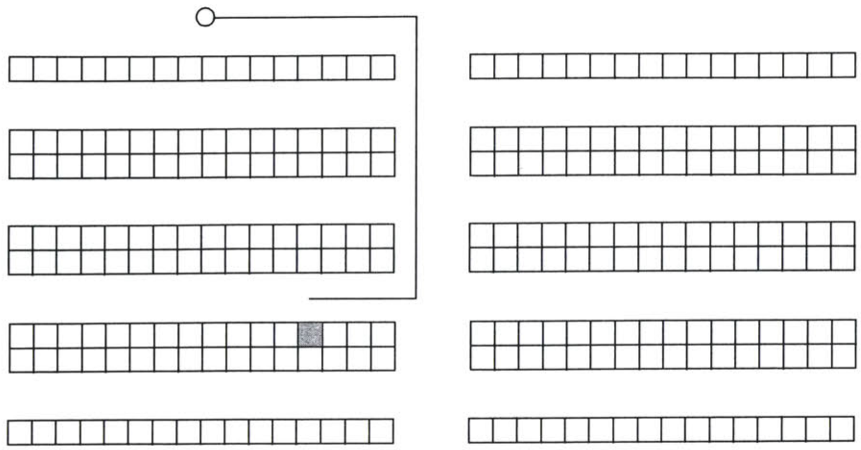


Figure 7.1: Base metric from pickup location to storage location

Figure 7.2 compares base distances for all storage locations in a rectangular warehouse. The distribution of favorable base distances is intuitively obvious, as distance to the starting location is visually identifiable.

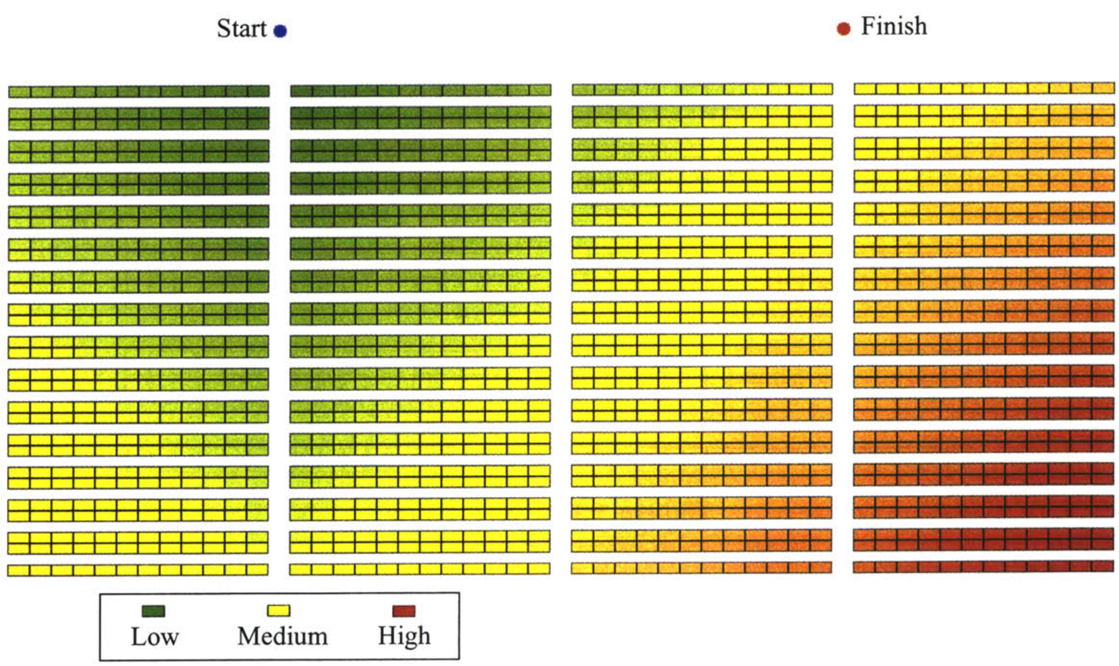


Figure 7.2: Relative base distances for a rectangular warehouse

The simplicity of the base metric makes for easy calculation; however, the base metric overlooks the interdependent aspects of multi-address picking routes. Under single-address picking, warehouse operations satisfy all orders by traveling from the start point to one storage bin and then to the finish point. However, multi-address picking begins at the start point, then visits many storage locations before traveling to the finish point, and the overall picking route depends on many storage locations. The base metric does not fully describe the favorability of the storage bin locations in this case.

7.1.2 Cover Metric

To better address the interdependence of storage locations when visiting more than one location, the cover metric considers the relationship between disparate storage locations and path building for multi-address pick routes. The cover metric quantifies the expected additional walking distance when adding another location to a pick route. If we construct the picking route for visiting the location marked black in Figure 7.3, then the cover

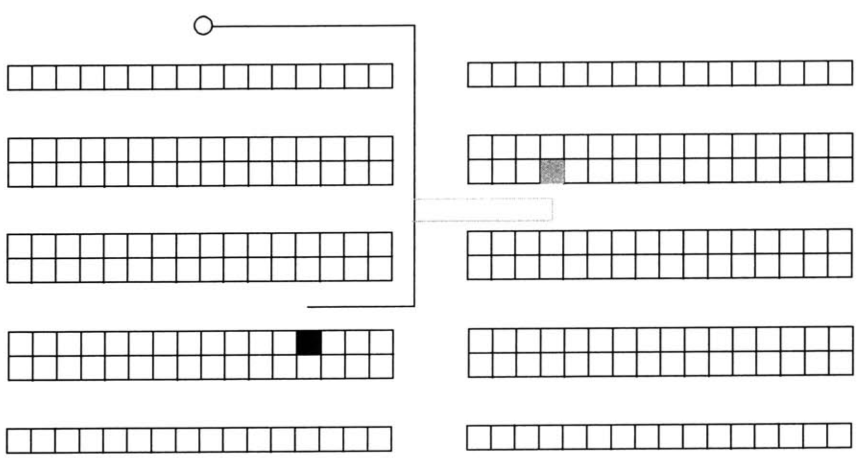


Figure 7.3: Cover metric

metric for the black location is the expected additional walking path distance for visiting another randomly selected storage bin. For example, if we add the grey location in Figure 7.3, then the picking route increases by the path indicated in grey. The summation

$$\text{cover metric} = \frac{1}{N} \cdot \sum_{j \in S} \text{cost to add location } j, \tag{7.1}$$

calculates the cover metric by considering the additional path length for every storage location and the probability of picking each storage location, where N is the number of storage locations in the warehouse and S is the set of all storage locations.

Figure 7.4 shows the relative values of the cover metric for the same rectangular warehouse. Unlike the base metric, locations far from the starting location have lower

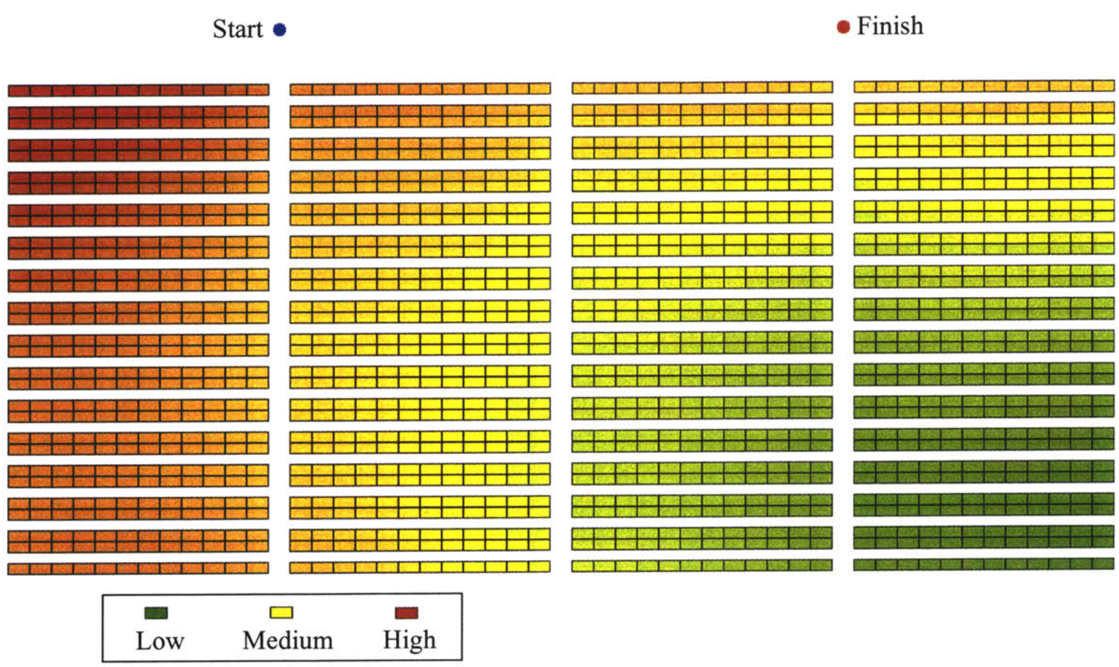


Figure 7.4: Relative values for the cover metric

cover values than locations close to the starting location. This reversal is due to the fact that more locations lie along the path to far away locations (high base metric). When considering the cover metric, locations that lie along the original base path can be added to the picking route without much additional walking.

7.1.3 Add Metric

The cover metric quantifies how easily other locations might be added to the picking path containing the described storage location. The add metric considers similar properties but describes how easily a particular location might be added to a pre-existing picking route. In other words, the cover metric addresses how easily other locations might be added, and the add metric describes how easily a bin might be added to other routes.

Like the cover metric, the add metric is an expected value: the expected increase in walking path distance if the location is added to a picking route containing some other randomly selected location. Figure 7.5 shows the relative values of the add metric for the rectangular warehouse.

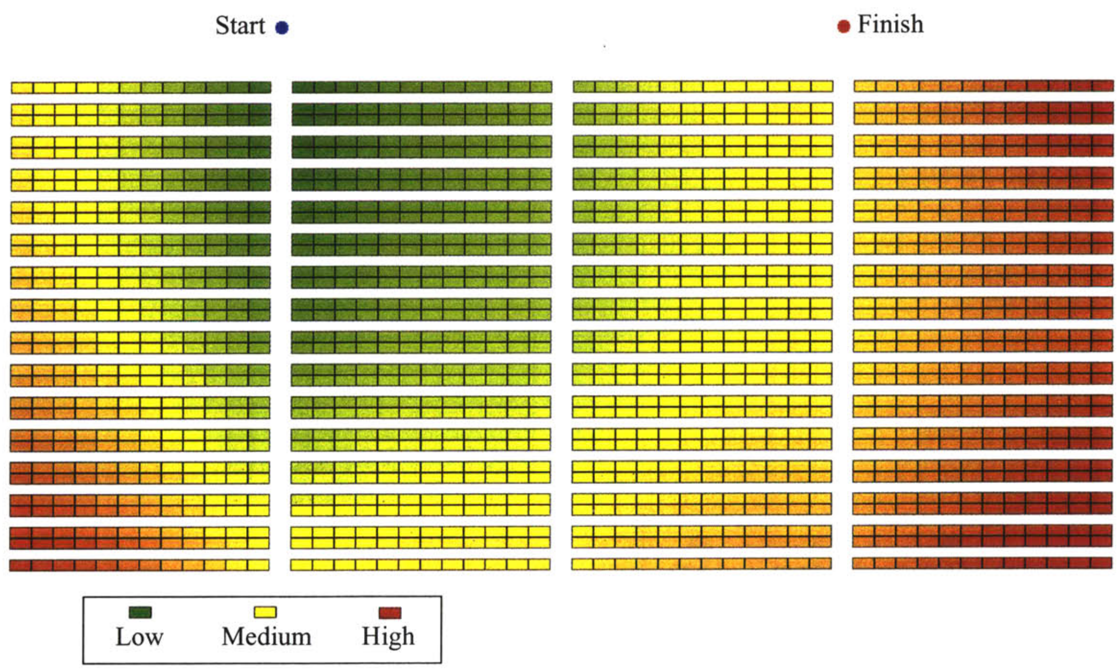


Figure 7.5: Relative values for the add metric

Both the add metric and cover metric are somewhat limited in their description. While both consider the effects of adding a location to a picking route, they limited the initial pick route to contain exactly one location. Considering longer picking routes is not feasible because the number of location combinations increases quickly and the calculation needs to solve larger traveling salesman problems. However, the simple add metric and cover metric reveal the trend that locations that appear to be unfavorable due to a high base metric may have other merits due to low cover and add metrics.

7.2 Simplified Analysis of Demand-based, Location-relaxed Storage

Understanding the differences between storage locations is significant when considering targeted placement schemes that try to maximize the use of favorable locations. Typically, such placement schemes rely on the expected demand for each SKU to determine to which storage location the SKU should be assigned and how favorable that location should be.

Demand-based placement runs against the general simplicity found in location-relaxed storage. Location-relaxed storage operates by delaying the expenditure of effort until it is clearly needed by placing items in any location (the most convenient) during the storage phase of warehouse operations. However, demand-based placement necessitates planning and targeted placement during storage,[1] which requires additional effort costs.

7.2.1 Demand-based Simulation

A simplified simulation of warehouse operations considers the possible benefits of instituting a demand-based placement policy with location-relaxed storage. To simplify the analysis and remove the complicating factors of interdependence as well as the influence of add and cover metrics, the simplified simulation considers cost of retrieval as a function of the storage location only, and not dependent on the other storage locations in the pick route. In other words, a single location will have some constant cost associated

1. Under strict organization storage, planning and targeted placement are already the status quo and a necessary part of storage operations. The addition of demand-based placement does not require additional effort on the part of a strict organization warehouse.

with it that is the cost of using that location to store and retrieve an item, regardless of what other locations are used and/or accessed.

The abstract warehouse model is of fixed size with all SKUs considered equal in size as well. The number of storage bins and the number of items that fit in each storage bin characterize the warehouse size. Warehouse shape is not an issue with this abstract warehouse model, as access costs are assigned to each storage bin without regard to physical location or physical constraints.

The analysis compares demand-based placement to random placement, by filling the warehouse in one of two methods. For demand-based placement, incoming items are paired with storage bins based on SKU demand such that higher demand items are placed in storage bins with the lowest cost. Under random placement, incoming items are randomly paired with storage bins.

The number of storage bins and the number of items that fit into each storage bin are held constant throughout the analyses. The costs associated with using a particular storage bin follow a linear progression with the lowest cost storage bin having a cost of 1 and the highest cost storage bin having a cost equal to the number of storage bins. The analyses vary the number of SKUs serviced by the warehouse, the percent of total capacity removed each cycle, and the relative SKU demand distribution.

The simulation is based on the following steps:

1. Fill the warehouse to full capacity using the demand distribution to determine the identity of the items placed into the warehouse and using the placement policy (demand-based or random) to determine the placement location.
2. Based on the percent removed each cycle and the current inventory, identify a subset of the warehouse contents for removal then select the proper copies of each SKU to meet the removal needs while minimizing access cost.
3. Generate a random replacement population for the items removed in the previous step based on the demand distribution.
4. Place the items into the warehouse based on the placement policy.
5. Repeat steps 2 through 4 while collecting the total access cost.

After repeating the removal and replacement of items many times, the access cost for demand-based placement can be compared to the access cost for random placement

7.2.2 Results

Repeating the simulation with different running conditions indicates when demand-based placement policies have a positive effect on location-relaxed storage. For the following simulations, the warehouse capacity was 1,000 units from 100 storage bins with a 10 unit capacity each.

Figure 7.6 shows the results of a simulation with 250 SKUs serviced by the warehouse and a demand distribution with half of the SKUs having twice the demand of the other half. The average cost is calculated by determining the expected access cost if storage bins were visited randomly. With 100 storage bins, the average cost per pick is 50.5. The

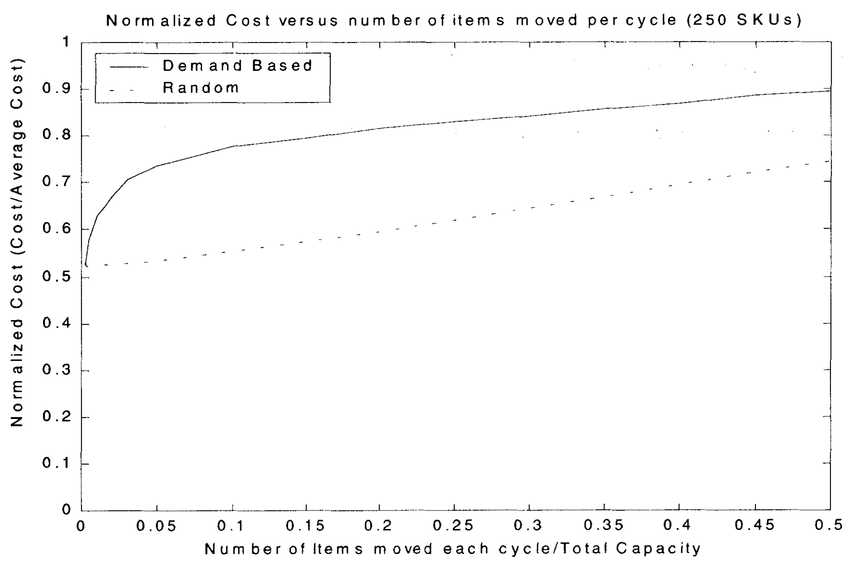


Figure 7.6: Random versus demand-based, 2:1 high-low demand distribution

normalized cost is equal to the access cost divided by the average cost. The simulation varies the number of items removed each cycle to show the effect of volume removal on the policies. The result shows that random placement always performs as well or better than a demand-based placement policy.

Figure 7.7 shows the results for the same operating conditions, but varying number of SKUs serviced by the warehouse and fixing the number of items removed each cycle to 100 (10% of total capacity). Again, random placement outperforms demand-based

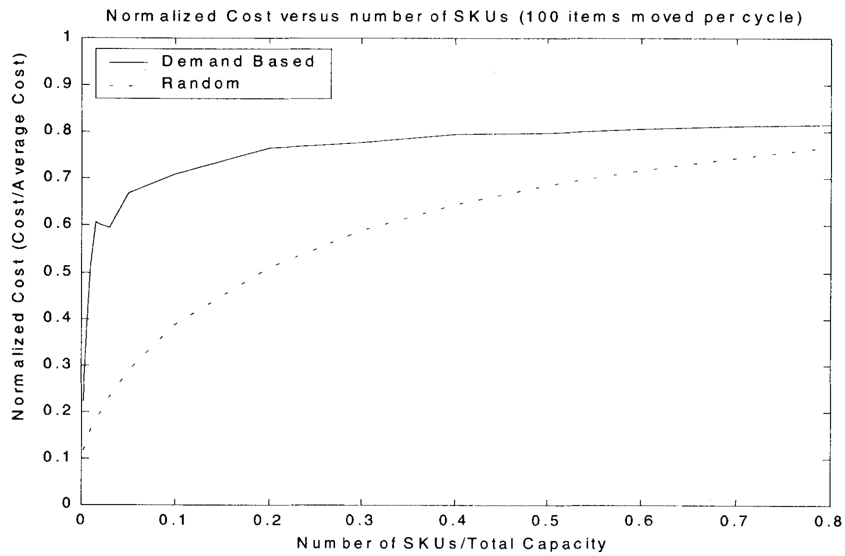


Figure 7.7: Random versus demand-based, 2:1 high-low demand distribution

placement in all cases presented; however, when the number of SKUs becomes large the cost difference diminishes.

Figures 7.8 and 7.9 show the same simulation with a demand distribution such that half of the SKUs have ten times the demand as the other half. All curves follow the same general upward trend with increasing numbers of SKUs and with increased removal percentage. However, with the demand of half the SKUs significantly higher (ten times higher) than the other items, the demand-based placement policy competes more closely with random placement than in the 2:1 case. The number of items moved each cycle has

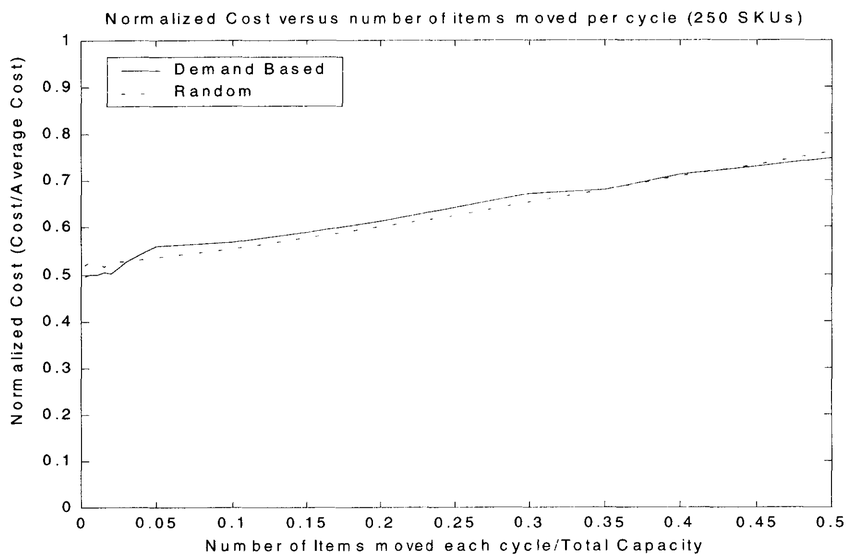


Figure 7.8: Random versus demand based, 10:1 high-low demand distribution

little effect on the relative values between demand-based placement and random placement. However, the number of SKUs serviced by the warehouse has a significant effect on relative access costs. With a higher number of SKUs serviced by the warehouse, demand-based placement outperforms random placement. In this case, when the number of SKUs serviced by the warehouse exceeds 30% of the total warehouse capacity,

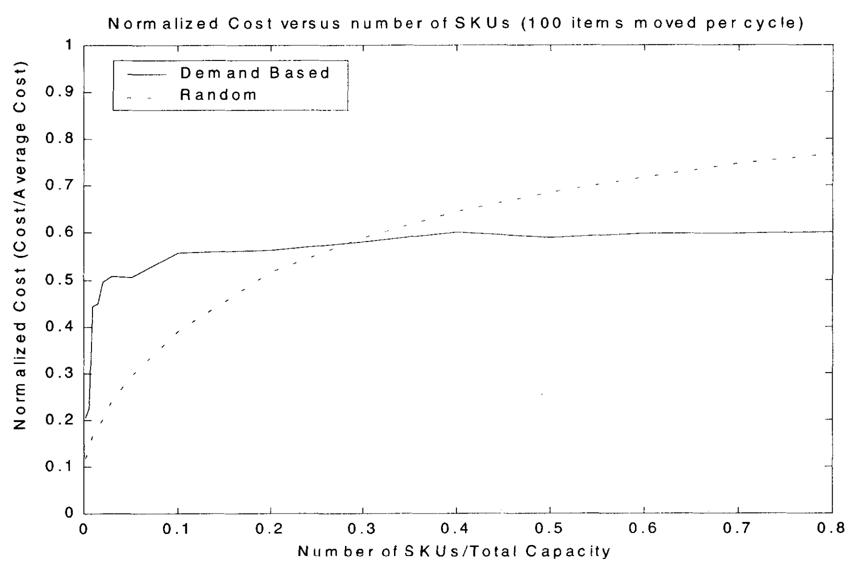


Figure 7.9: Random versus demand-based, 10:1 high-low demand distribution

demand-based placement performs better than random placement. However when the number of SKUs is smaller, random placement performs better than demand-based storage.

Figure 7.10 show the results when the demand distribution is linearly spaced. The SKU with the highest demand has two times the demand of the SKU with the lowest demand. The remaining SKUs fall evenly spaced between the highest and lowest demand. The result is very similar to the High-Low demand distribution and random placement always performs better than demand-based placement.

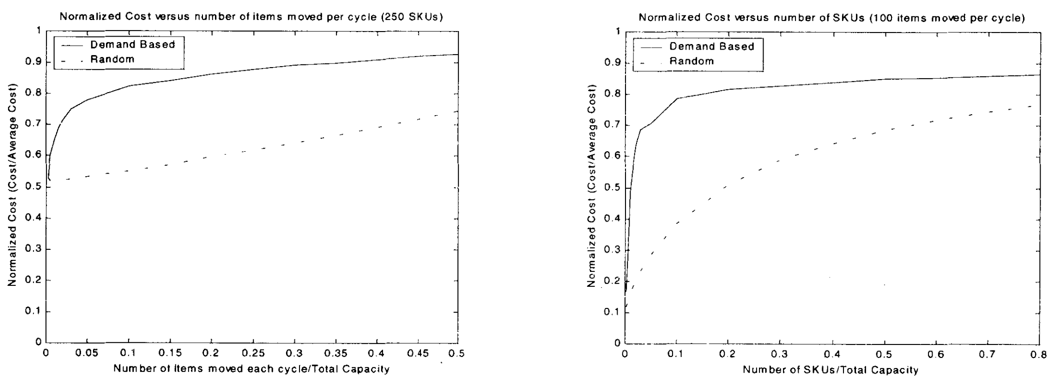


Figure 7.10: 2:1, Linear demand distribution

Figure 7.11 shows the results for a linearly spaced demand distribution with the highest demand SKU having ten times the demand of the lowest demand SKU. The stronger separation between the high demand items and low demand items diminishes the clear advantage of random placement over demand-based placement. Furthermore, when the number of SKUs serviced by the warehouse exceeds 58% of the warehouse total capacity, demand-based placement performs better than random placement.

Figure 7.12 illustrates the continued trend of an increased disparity between high and low demand items, in this case showing the same simulation with a linear demand

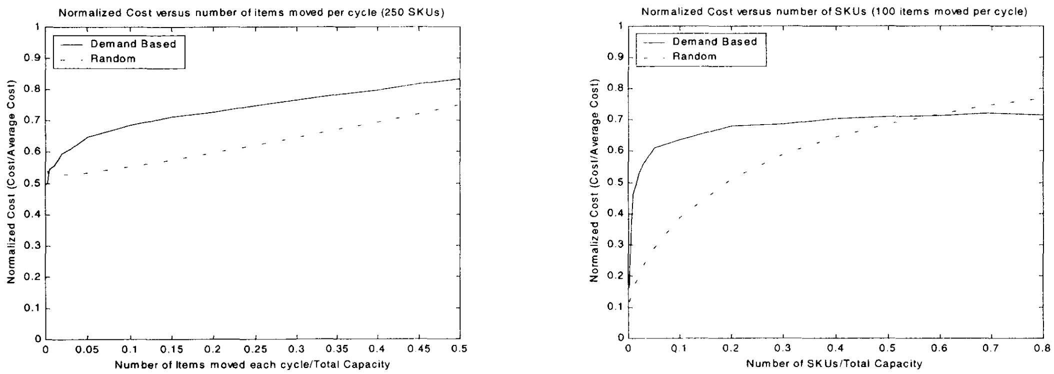


Figure 7.11: Random versus demand-based, 10:1 linear demand distribution

distribution with the highest demand SKU having twenty times the demand of the lowest SKU. Again, a stronger disparity in demand rates diminishes the superiority of random placement over demand-based placement. In this scenario, demand-based placement outperforms random placement when the number of SKUs serviced by the warehouse exceeds 48% of the warehouse total capacity.

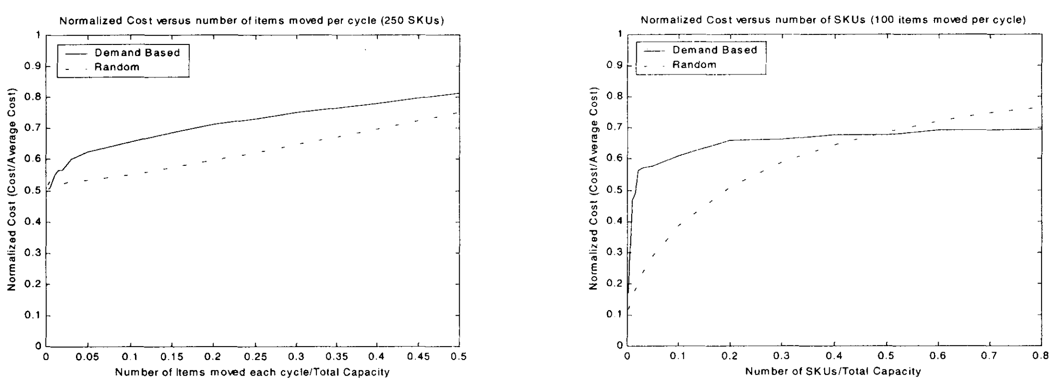


Figure 7.12: Random versus demand-based, 20:1 linear demand distribution

7.2.3 Discussion of Location-relaxed Demand Based Placement

The simulation results show that demand-based placement performs better when the number of SKUs serviced by the warehouse is very large. Demand-based placement works better with more SKUs because the number of copies of each item decrease as the number of unique SKUs increases. The number of copies of each SKU will vary depending on the demand distribution, however in general, a greater number of SKUs corresponds to a smaller number of copies for all SKUs serviced by the warehouse. With a smaller number of copies, the redundancy of storing multiple copies of the same SKU in the same storage bin, or very close to the same storage bin, is more likely to be advantageous and less likely to be troublesome. For example, if there are many copies of a particular SKU in the warehouse, then placing all of them in favorable locations is redundant since only a fraction of them will likely be needed during the next cycle. Smaller number of copies of each SKU reduces the chance of redundancy. In the extreme case where the number of SKUs serviced is very high such that most SKUs have a single copy in the warehouse and there is no chance of redundancy, demand-based placement is clearly superior.

Demand-based placement operates on the balance between greater utilization of favorable storage bins and the risk of redundant use of these favorable storage bins. Localized redundancy tends to undo the beneficial factors developed by location-relaxed storage. In the strict organization case, where storage bins are assigned exclusively to specific SKUs, localized redundancy is unavoidable as it is a fundamental part of strict organization. In location-relaxed storage, the de-aggregation and distribution of storage is part of what contributes to its success. Demand-based placement in a location-relaxed storage environment tends to over use favorable locations for high demand items.

The results also show that demand-based placement performs better when the differences in demand rates are very large, which should not be surprising since demand-based placement is driven by differences in demand rates from one SKU to another. If the differences in demand rates are small, then there is little reason to enforce a demand-based placement policy since SKUs with similar demand rates should not be treated differently.

The results also indicate that the high-low distribution is more sensitive to differences in relative demand values than linear distribution. This sensitivity is due to the intermediate demand SKUs between the extreme low and extreme high that make differences in demand rates less sharp in the linear distribution case. In other words, a high-low distribution tends to have greater differences between demand rates than the linear distribution.

Demand-based placement tends to group similar SKUs together. While not specifically aggregating items into the same storage bin as with strict organization storage, demand-based placement favors certain storage bins over others thereby influencing placement such that like SKUs end up in the same region (*i.e.* local redundancy).

Furthermore, demand-based placement requires additional effort in the storage phase of warehouse operations to direct items to specific storage bins and locations based on demand rates. These added efforts further diminish any benefit gleaned from demand-based placement.

7.2.4 Model Simplification Revisited

The demand-based analysis for location-relaxed storage makes a significant simplifying assumption that access costs are independent. In real warehouse picking operations multi-address picking is generally the mode of operation and access costs are dependent not only on which storage bin a particular SKUs resides, but the storage bins of all the SKUs on the pick list. The simplifying assumption makes the base metric, discussed in section 7.1.1, the most important characteristic of a storage location. The add metric and cover metric have no weight in this analysis because the access costs are assumed to be independent of other locations.

This simplifying assumption offers several benefits to the simulation and analysis. Because access costs are constant and independent of pick list or pick order, favorable storage bins are easily identifiable for the simulation. With constant and independent access costs, comparing the access costs of different locations directly indicates whether one location is more favorable than another. Without the simplifying assumption, one location may have a lower base metric while another has more favorable add and cover metrics. Evaluating which location is more favorable than the other in this case is not straight forward. The simplified model ignores the benefits measured by the add and cover metrics. In other words, the real-world benefit of picking two items that are close to each other does not appear in the simplified model, thereby allowing the formulation of a simple and clear demand-based strategy.

Removing the simplification from the analysis creates a more complicated interaction between storage bin and picking cost (access cost). It is likely that the benefits of picking items from storage locations that are close to each other, rather than focusing on picking from storage locations that are merely close to the pickup/drop-off location, will reduce the benefit of demand-based placement since locations that have favorable base metrics tend to have less favorable add and cover metrics.

7.2.5 Alternative Placement Strategy

Even though the simplified model emphasizes the benefits associated with demand-based placement, the analysis does not demonstrate a clear advantage over random placement. The pitfall that weakens demand-based placement is the overuse of favorable locations for high demand items. For example, if the warehouse capacity was 1,000 units and on-average product Z accounts for 10% of all demand, then on average, the warehouse would have 100 units of product Z stored in the warehouse. If 50% of the warehouse were removed each cycle, then on average 50 units of product Z would be needed each cycle. Placing all 100 units of product Z in favorable locations is inefficient since we only expect about 50 units to be needed each cycle. The extra 50 units occupy favorable space that could better be used servicing some other product.

Demand-based placement does not offer the same cost savings potential as with the strict organization storage case. However, the notion that placing a sufficient number of items in favorable locations while leaving the rest in less favorable locations leads to other possible placement strategies specifically designed for location-relaxed storage.

7.3 Configuration Analysis

Consider the case where there are three types of items (three SKUs) with a demand ratio of 3:2:1. Let the cost of using a particular storage location be independent of the other locations used just as with the simulation of section 7.2. Also, assign a constant cost of using a location equal to the location numbers 1 to n. Section 7.2, simulated the placement and removal of items according to demand-based placement and random placement schemes. This configuration analysis considers a fixed placement pattern rather than a replenishment policy.

The configuration pattern that corresponds to pure demand-based placement has the highest demand SKU occupying the most favorable storage bins, followed by the next highest demand SKU. For example, three SKUs with a 3:2:1 demand rate ratio would be arranged AAAAAAAAAAAAAAABBBBBBBBBBCCCCC. This configuration has no mixing of the SKUs and would not perform well since the first 15 locations contain all the same SKU. Strong demand-based placement aggregates SKUs in a way that is similar to strict organization, which is why demand-based placement works well with strict organization storage and less well with location-relaxed storage.

When removing *m* items from the configuration, the minimum access cost occurs when all *m* items appear in the most favorable locations. With no mixing in the configuration, it is highly unlikely that all copies of the highest demand SKU will be needed before any other SKU. In fact, the best configuration should have a 3:2:1 distribution in the first *m* locations.

Consider the case where product *Z* occupies 10% of all demand. With a 50% removal of the warehouse capacity, the smallest possible cost occurs when all picked items lie in the first half of the warehouse. An optimal configuration would have 10% of the first half of the warehouse containing product *Z*. If all of product *Z* had been placed in the first of the warehouse, then the first half of the warehouse would have product *Z* as 20% of the items. Half of which would be useful during the picking process, and the other half would be an obstruction. Furthermore, if 10% of the items were removed, then the optimal configuration would have product *Z* make up 10% of the first one-tenth of the warehouse. If the amount of product removed were unknown and random, then the best configuration would have product *Z* evenly distributed throughout the warehouse storage bins so that no matter what fraction of the warehouse was to be removed, the fraction would contain as close to 10% of product *Z* as possible.

A better configuration is AAABBCAAABBCAAABBCAAABBCAAABBC, which has a moderate amount of mixing and distributes the SKU's more evenly such that choosing $m = 6$, $m = 12$, $m = 18$, $m = 24$, or $m = 30$ results in the first m locations having a distribution matching 3:2:1. An even better assignment is ABACABABACABABACABABACABABACAB, which has a high amount of mixing and spreads repeated SKUs most evenly throughout the configuration.

Figure 7.13 compares various configurations' performance when removing 30% of the warehouse contents based on a linear demand distribution. The upper bound describes the worst case scenario where *every* pick comes from the least favorable storage locations. The lower bound describes the minimum cost possible for picking the items and corresponds to all picked items residing in the most favorable storage locations. The curve labelled "No Mixing" corresponds to the configuration where all copies of the highest demand reside in the most favorable storage bins (*e.g.* AAAAAAAAAAAAAAABBBBBBBBBBCCCCC). The curve labelled "Strong Mixing" corresponds to a configuration that maximizes the separation between identical SKUs (*e.g.* ABACABABACABABACABABACABABACAB). The curve labelled "Moderate Mixing" has some characteristics of both aggregation and distribution (*e.g.* AAABBCAAABBCAAABBCAAABBCAAABBC). Finally, the curve labelled random, corresponds to a randomly generated configuration. All four configurations contain the same inventory; the only difference is the location of the inventory.

Figure 7.13 shows the strength of mixing for obtaining a configuration that minimizes the cost to retrieve items. When the demand rates of the SKUs are known, evenly distributing each SKU across the warehouse performs better than placing higher demand SKUs in more favorable storage bins. This analysis confirms that demand-based storage, while helpful in strict organization storage, is counter-productive to location-relaxed

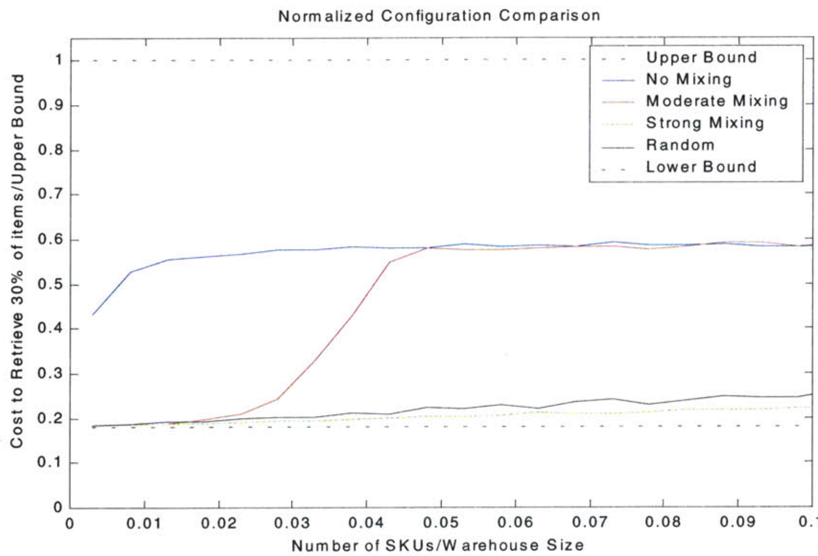


Figure 7.13: Configuration performance

storage operations. Furthermore, the configuration analysis confirms the positive benefits of distributed storage for order-picking and emphasizes the importance of *evenly* distributed storage in location-relaxed environments.

Demand-based placement policies only have a place in location-relaxed storage when the demand differences are extremely large between SKUs. In other words, demand-based placement is appropriate only for outliers far beyond the demand rates of typical items. While demand-based placement policies do not offer any contributions to location-relaxed storage, another placement policy based on the strong mixing configuration may offer cost savings benefits. For example, a simple placement policy emulating the results of strong mixing could choose a storage location for an incoming item based on the current locations of all other matching items already stored. The policy would maximize the distribution of the item's population by selecting a location that is as far away from the other items as possible. Such a placement policy may succeed in providing cost savings where demand-based placement fails.

The configuration analysis results also indicate a very good performance by a randomly generated configuration. Random placement of items is easier than specifying a location that maximizes distribution. The cost savings of a configuration based placement policy would have to obtain sufficient picking cost savings to justify the additional effort required to specify the target locations for placement.

Chapter 8: Conclusions

Location-relaxed storage holds great potential for revolutionizing warehouse operations. The initial analyses in this thesis have demonstrated the benefits of location-relaxed storage in reducing routing walk distances by distributing products across the warehouse. Furthermore, location-relaxed storage offers superior efficiency in storage space utilization. The research has also uncovered many aspects of location-relaxed warehouse operations where further optimization and research are warranted. Full optimization of location-relaxed warehouse fulfillment takes an already complex optimization problem with strict organization and superimposes additional layers of complexity. This added complexity also offers additional opportunity for optimization.

In the future, depending on the characteristics of a particular warehouse operation, some systems will find that strict organization is still the best option, others will determine that location-relaxed storage offers significant benefits over the old methods. Amazon.com already benefits from this new approach to warehousing. As our understanding of this tantalizing and complex warehouse problem improves, its benefits and application to warehouse operations will change the fundamentals of warehouse storage. Rather than settling for a configuration based on the perceived need for organization, the more worthwhile criterion of less costly operations will become the driving force in warehouse design.

8.1 Review

Analysis of the traveling purchaser problem demonstrates the benefits of having a choice of many storage locations for order-picking. Simulation of warehouse operations confirm the hypothesis that location-relaxed storage is especially beneficial with small outgoing order lot sizes. The surprising result from the warehouse simulation analysis indicates that there is not only compelling reason to use location-relaxed storage when the outgoing lot size is small, but deliberate mixing of incoming materials yields benefits as well. Even though incoming product may be pre-aggregated from the manufacturer, maintaining this aggregation is not only unnecessary in some cases, it is undesirable.

Location-relaxed storage allows all of the warehouse capacity to be available for use. Several analyses demonstrate that strict organization storage can not devote 100% of space to storage without risk of blocking; some space must always be left idle. Furthermore, strict organization storage capacity generally decreases as the number of SKUs serviced by the warehouse increases. For warehouses addressing a very large number of unique SKUs, location-relaxed storage becomes more desirable. In fact above a certain level, approximately equal to the number of storage bins, location-relaxed storage is the only viable storage method.

Without technological improvements location-relaxed storage suffers a very high sensitivity to operation errors. Traditional strict organization techniques also suffer from operational mistakes, but the impact of these mistakes are less severe than with location-relaxed storage. In fact, the order enforced with strict organization is the main point of recourse in response to operation errors and is the one of the main reasons strict organization methods dominate warehouse operations today.

With new technologies such as Radio Frequency Identification (RFID) tagging and automatic identification systems (Auto-ID), the sensitivity of location-relaxed storage techniques to operation errors is greatly reduced. Furthermore, the analysis of the expected effect of operation errors on location-relaxed storage with Auto-ID technology shows location-relaxed storage is less sensitive to errors than similarly equipped strict organization storage.

Demand-based placement policy analysis uncovers the surprising result that demand-based storage, a staple in traditional warehousing operations, offers little benefit to location-relaxed storage methods. In fact, the evidence shows demand-based placement policies to be counter-productive. Additional investigation indicates an alternative placement policy based on establishing a widespread distribution of each SKU across the warehouse storage bins might offer cost savings in lieu of demand-based placement. Deliberately placing copies of the same SKU far apart maximizes the benefits of multiple locations.

8.2 Discussion

Location-relaxed storage is an example of using entropy within a system as an advantage rather than a disadvantage. Strict organization approaches entropy in an adversarial manner as the design and operation of the warehouse focuses on maintaining order and fights inevitable disorder with high aggregation and periodic checks for discrepancies from the ordered ideal. On the other hand, location-relaxed storage accepts entropy and uses randomness as an asset. This novel approach to disorder illustrates that deriving order from randomness can be easier than maintaining order in a system that tends to be disordered.

Location-relaxed storage shows strong potential as a replacement to traditional storage techniques. The introduction of new technologies to automatically track and monitor materials within a warehouse address the major problems with location-relaxed storage that had previously made such an approach unfeasible. RFID technology removes the serious difficulty location-relaxed storage has with placement errors.

The case for location-relaxed storage is especially compelling when the warehouse operations characteristics include small outgoing order lot sizes and a high number of SKUs to be serviced by the warehouse. The proliferation of product lines tends to increase the number of SKUs served by a warehouse, and the emergence of direct-to-consumer warehouse operations tend to decrease the order size and lot size of outgoing product. These characteristics are consistent with the strengths of location-relaxed storage.

Unlike the traditional product distribution paradigm where the warehouse services several retail outlets, website-based stores tend to serve the individual customers directly. This direct-to-consumer model reduces the lot size of outgoing product from dozens in the 'to retail store' case down to singles for the direct-to-consumer case. Furthermore, website based stores tend to offer as much product variety as possible which leads to a high number of SKUs involved in the warehouse operations. Hence, location-relaxed storage is especially suited for eCommerce ventures that cater directly to the consumer.

8.3 Further Study

Location-relaxed warehouse storage offers a wide variety of optimization opportunities for maximum cost savings. The analyses included in this research focused on proving the strengths and potential of location-relaxed storage. Fully optimal techniques for location-relaxed warehouse storage are discussed but not pursued.

The analyses included in this thesis cover only a few of the many warehouse characteristics important to evaluating a warehouse policy. The warehouse simulation of placement and picking operations focuses on incoming and outgoing lot size to demonstrate the areas of strength location-relaxed storage has over traditional methods. Additional parameters of relevance that were not pursued include warehouse size, number of SKUs serviced by the warehouse, warehouse dimensions, and other picking policies such as zone picking. The areas of investigation for further insight and understanding of location-relaxed storage warehouse operations are numerous.

Beyond testing how different sizes, dimensions, and policies effect location-relaxed warehouse storage, optimization opportunities emerge in the location-relaxed warehouse operations. The warehouse traveling purchaser problem presents research opportunities in Warehouse-TPP optimal solutions, and warehouse specific heuristics. Adaptations of the warehouse-TSP heuristics discussed by Petersen, Roodbergen and DeKoster would be of great benefit for understanding the Warehouse-TPP problem better.

Optimality in order-picking offers a rich and complicated combinatorial optimization problem. Not only is the order batching problem itself difficult, it is intimately linked to the Warehouse-TPP problems which are interdependent.

Finally, the emergence of configuration-based placement policies for location-relaxed storage deserves a closer investigation and cost-benefit analysis. Demand-based placement strategies have been a staple for traditional warehousing, but do not offer much benefit to location-relaxed storage. Whether configuration-based placement offers the same level of benefit to location-relaxed storage as demand-based placement does with strict organization storage remains to be seen.

Although fully understanding the role of RFID technologies and location-relaxed storage on warehouse operations will require much additional investigation, the analyses presented showcase the potential of this new storage method. Furthermore, the investigation offers a strong motivation for undertaking additional research into location-relaxed storage and its associated optimization opportunities.

Appendix A : Warehouse Traveling Purchaser Problem

This appendix describes how examples of the warehouse traveling purchaser problem are generated for evaluation in Chapter 3 including the warehouse layout and dimensions and the step-by-step procedure for generating the problem instance.

A.1 Warehouse layout

Figure A.1 shows the warehouse layout used throughout Chapters 3 and 4 as well as parts of Chapter 7 (Section 7.1: Favorable Locations). The warehouse consists of 30 rows and 4 columns of storage locations. Within each column there are 12 stacks of storage locations. Each stack has 8 storage locations within it (not shown in the diagram) for a total of 11,520 locations. The diagram also shows the key dimensions. Each storage bin has a width of 1 meter and depth of half a meter. Row aisles are 0.5 meters, and column cross-aisles are 1 meter wide.

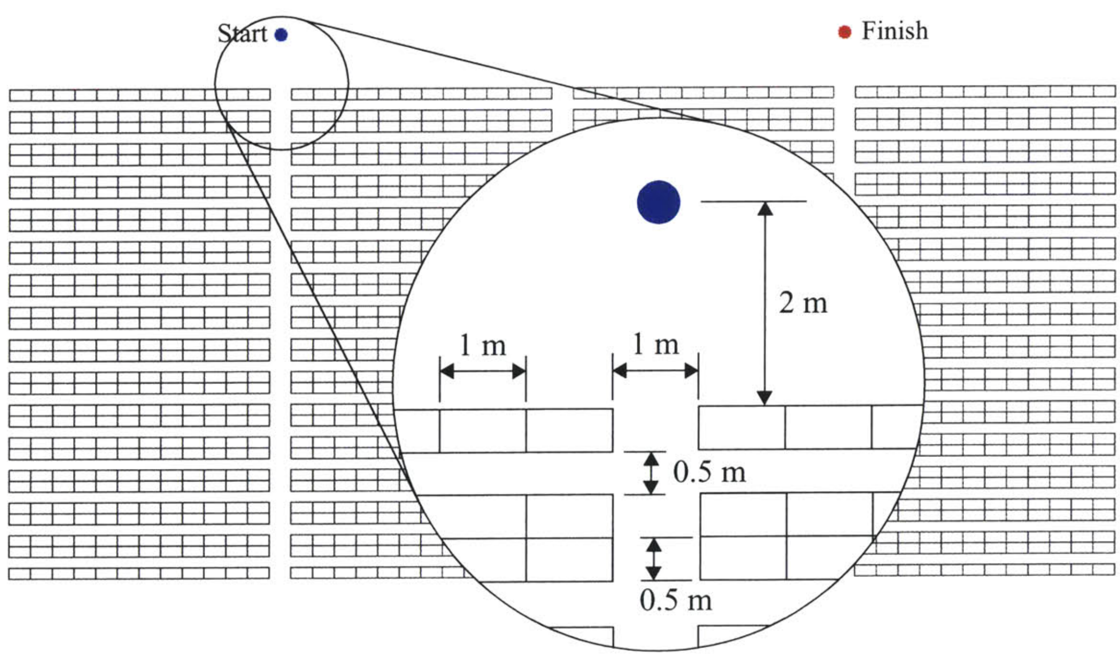


Figure A.1: Warehouse layout

A.2 Generating a traveling purchaser problem example

Chapter 3 evaluates warehouse-TPPs with varying number of items in the pick list and a varying number of locations in which each item resides. To generate these problems to be solved by the greedy heuristics (or optimal search), for each of the n items in the pick list, k locations are randomly selected from all locations with equal probability (repeated locations are permitted). The result is a list of n unique items each found in k locations. In other words, the generated problem is specifically for picking n **different** products. For evaluating optimal and heuristic solutions, a list of unique items is sufficient. Further investigation into the effect of picking lists with multiple copies of the same product (and therefore identical locations for matching products) is discussed in Section 4.3 and Appendix B.

Appendix B : Warehouse Picking and Placement

This appendix covers the process of generating sets of incoming products and sets of outgoing orders for the warehouse picking and placement simulation and the process by which the sets of products are placed into the warehouse. The warehouse layout and dimensions are the same as explained in Appendix A. The warehouse picking and placement simulation operates in alternating intervals of placing newly acquired items into the warehouse and removing items from the warehouse to fill orders.

B.1 Incoming products

Products entering the warehouse arrive in boxes of size N. The box size is fixed at 20 units per box for all simulations in Chapter 4. The product distribution within a box depends on the placement lot size, which varies between simulations but is constant within a particular simulation run. Demand is equal for all products; therefore each product has an equal chance of being in a particular incoming box.

Having set the box size to 20 and selected a placement lot size for the simulation run, incoming boxes of products are created by randomly selecting a product from the list of products served by the warehouse. Copies of this selected product are added to the box until either the box contains copies of the product equal to the placement lot size for the simulation run, or the box is full (20 items). After generating the box or boxes, the items are placed into the warehouse according to the placement policy. For the simulations described in Chapter 4, strict organization warehouses use a zone placement policy where items within incoming boxes are aggregated based on their assigned location within the warehouse. These zone-based, aggregated groups are limited in size and for the simulations in Chapter 4, the number of items in a single group cannot exceed 100 items.

Placement under location-relaxed storage follows a box-by-box methodology (*i.e.* no mixing from one box to another and no aggregation).

Permitting a grouping operation with strict organization storage but not location-relaxed storage gives a slight advantage to strict organization storage which indicates an even stronger superiority for cases where location-relaxed storage performs better than strict organization storage. Furthermore, the cost associated with the grouping process is not counted against strict organization storage in the simulation tabulation of operation costs.

B.2 Outgoing products

For the picking and placement simulations of Chapter 4, both strict organization and location-relaxed storage warehouses use batch picking to retrieve items from the warehouse to fill order demand. Orders are generated in a method similar to incoming product generation. The only difference is items are checked against the warehouse inventory to before being put into in order to eliminate the possibility of stockout. Order sizes are 20 for all simulations in Chapter 4; picking lot sizes very with each simulation. Orders are then combined into picking batches to reduce the number of passes through the warehouse necessary to retrieve the needed products. The maximum batch size is the same for both versions of the warehouse and is set to 150 items.

B.3 Simulation procedure

Simulations begin with an empty warehouse that is filled with a standard incoming product phase to initialize the warehouse. For all simulations of Chapter 4, the number of unique products is 2000 and the number of items in the warehouse after initialization is 160,000. Each simulation run consists of adding and removing 60,000 items divided into 30 alternating picking and placement phases.

Appendix C : Storage Efficiency Derivation

This appendix provides a proof for equation 5.2:

$$\sum_{i=0}^{N} i \cdot U \cdot P(i, N) = N$$

Replacing $P(i, N)$ with equation 5.1 yields

$$\sum_{i=0}^{N} i \cdot U \cdot \binom{N}{i}\left(\frac{1}{U}\right)^{i}\left(1-\frac{1}{U}\right)^{N-i} = N \tag{C.1}$$

$$\sum_{i=0}^{N} i \cdot \binom{N}{i}\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i} = N \tag{C.2}$$

$$0 \cdot \binom{N}{0}\left(\frac{1}{U}\right)^{-1}\left(1-\frac{1}{U}\right)^{N} + \sum_{i=1}^{N} i \cdot \binom{N}{i}\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i} = N \tag{C.3}$$

$$N\sum_{i=1}^{N} \frac{i}{N} \cdot \binom{N}{i}\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i} = N \tag{C.4}$$

$$N\left[\sum_{i=1}^{N-1}\left(\frac{i}{N} \cdot \binom{N}{i}\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i}\right) + \frac{N}{N} \cdot \binom{N}{N}\left(\frac{1}{U}\right)^{N-1}\left(1-\frac{1}{U}\right)^{0}\right] = N \tag{C.5}$$

$$N\left[\sum_{i=1}^{N-1}\left(\frac{i}{N} \cdot \binom{N-1}{i}\left[\frac{N}{N-i}\right]\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i}\right) + \left(\frac{1}{U}\right)^{N-1}\right] = N \tag{C.6}$$

$$N\left[\sum_{i=1}^{N-1}\left(\binom{N-1}{i}\left[\frac{i}{N-i}\right]\left(\frac{1}{U}\right)^{i-1}\left(1-\frac{1}{U}\right)^{N-i}\right) + \left(\frac{1}{U}\right)^{N-1}\right] = N \tag{C.7}$$

Let $M = N-1$ and $p = \frac{1}{U}$:

$$N\left[\sum_{i=1}^{M}\left(\binom{M}{i}\left[\frac{i}{M+1-i}\right](p)^{i-1}(1-p)^{M+1-i}\right) + (p)^{M}\right] = N \tag{C.8}$$

$$N\left[\sum_{i=1}^{M}\left(\binom{M}{i}\left[\frac{i}{M+1-i}\right]p^{i-1}(1-p)^{M+1-i}\right)+p^{M}\right] = N \qquad \text{(C.9)}$$

$$N\left[\sum_{i=1}^{M}\left(\binom{M}{i-1}p^{i-1}(1-p)^{M+1-i}\right)+\binom{M}{M}p^{M}\right] = N \qquad \text{(C.10)}$$

Let $j = i-1$

$$N\left[\sum_{j=0}^{M-1}\left(\binom{M}{j}p^{j}(1-p)^{M-j}\right)+\binom{M}{M}p^{M}\right] = N \qquad \text{(C.11)}$$

$$N\sum_{j=0}^{M}\binom{M}{j}p^{j}(1-p)^{M-j} = N \qquad \text{(C.12)}$$

The quantity $\sum_{j=0}^{M}\binom{M}{j}p^{j}(1-p)^{M-j}$ is the summation of a binomial distribution with M trials and $\frac{1}{U}$ rate of success. The summation of any probability distribution is unity. Therefore,

$$N = N \qquad \text{(C.13)}$$

And equation 5.2 is shown to be valid.

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