Journal of Engineering Science and Technology Review 8 (2) (2015) 1-6
Special Issue on Synchronization and Control of Chaos: Theory, Methods and Applications

JOURNAL OF
Engineering Science and Technology Review

www.jestr.org

Research Article

An Eight-Term Novel Four-Scroll Chaotic System with Cubic Nonlinearity and its Circuit Simulation

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Received 29 September 2014; Revised 1 November 2014; Accepted 14 November 2014

Abstract

This research work proposes an eight-term novel four-scroll chaotic system with cubic nonlinearity and analyses its fundamental properties such as dissipativity, equilibria, symmetry and invariance, Lyapunov exponents and Kaplan-Yorke dimension. The phase portraits of the novel chaotic system, which are obtained in this work by using MATLAB, depict the four-scroll attractor of the system. For the parameter values and initial conditions chosen in this work, the Lyapunov exponents of the novel four-scroll chaotic system are obtained as $L_1 = 0.75335$, $L_2 = 0$ and $L_3 = -22.43304$. Also, the Kaplan-Yorke dimension of the novel four-scroll chaotic system is obtained as $D_{KY} = 2.0336$. Finally, an electronic circuit realization of the novel four-scroll chaotic system is presented by using SPICE to confirm the feasibility of the theoretical model.

Keywords: Chaos, chaotic systems, four-scroll system, Lyapunov exponents, Kaplan-Yorke dimension, circuit simulation.

1. Introduction

Chaotic systems are nonlinear dynamical systems which are sensitive to initial conditions, topologically mixing and also with dense periodic orbits. Especially, sensitivity to initial conditions means that an arbitrarily small perturbation of the current trajectory of the dynamical system may lead to significantly different future behavior.

Nonlinear dynamics occurs widely in engineering, physics, biology and many other scientific disciplines. There is great interest in chaos literature in discovering of chaos in nature and physical systems. Poincaré was the first, in 1880, to observe the possibility of *chaos*, in which a deterministic system exhibits aperiodic behavior that depends on the initial conditions, thereby rendering long-term prediction impossible [1]. In 1898 Jacques Hadamard published an influential study of the chaotic motion of a free particle gliding frictionlessly on a surface of constant negative curvature [2].

Much of the earlier theory was developed almost entirely by mathematicians, under the name of ergodic theory. Later studies, also in the topic of nonlinear differential equations, were carried out by Birkhoff (1927), Kolmogorov (1941), Cartwright and Littlewood (1945) and Stephen Smale (1960) [3-6].

However, the revolution on this field was made by Lorenz in 1963 [7] when he found a 3-D chaotic system while studying weather patterns.

The discovery of Lorenz chaotic system was followed by the discovery of many classical paradigms of 3-D chaotic systems such as Rössler system [8], Rabinovich system [9], Arneodo system [10], Sprott systems [11], Chen system [12], Lü system [13], Shaw system [14], Cai system [15], Tigan system [16], Colpitt's oscillator [17], Zhou system [18], etc.

Recently, many chaotic systems have been discovered such as Li system [19], Sundarapandian system [20], Sundarapandian-Pehlivan system [21], Zhu system [22], Vaidyanathan systems [23-27], Vaidyanathan-Madhavan system [28], Pehlivan-Moroz-Vaidyanathan system [29], Jafari system [30], Pham system [31], etc.

Also, the last decades many famous chaotic systems, which are exhibited n-scroll chaotic attractors, such as double-scroll attractors (like the Lorenz system [7], Chen system [12], Lü system [13], Tigan system [16]), three-scroll attractors (like Wang system [32], Dadras system [33], Pan system [34]), and four-scroll chaotic attractors (like Lü-Chen-Cheng system [35], Liu-Chen system [36], Pehlivan system [37], Liu system [38]), have been discovered.

Nowadays, chaos theory has applications in several fields such as oscillators [39-41], lasers [42,43], robotics [44-47], chemical reactors [48,49], biology [50,51], ecology [52,53], neural networks [54,56], secure communications [57-60], cryptosystems [61-64], economics [65-67], etc.

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ISSN: 1791-2377

This research work describes a novel four-scroll chaotic system and analyses its fundamental qualitative properties. The Lyapunov exponents and Kaplan-Yorke dimension of the novel chaotic system have been described. Finally, a circuit realization of the novel chaotic system has been made using SPICE simulations.

2. A Novel Four-Scroll Chaotic System

We consider the nonlinear system described by

$$\begin{cases} \dfrac{dx_1}{dt} = a(x_2 - x_1) + bx_2x_3 \\ \dfrac{dx_2}{dt} = -10x_2^3 - x_2 + 4x_1x_3 \\ \dfrac{dx_3}{dt} = cx_3 - x_1x_2 \end{cases} \tag{1}$$

where x_1, x_2, x_3 are state variables and a, b, c are positive constant parameters.

We note that the system (1) is an eight-term polynomial system with 3 quadratic nonlinearities and a cubic nonlinearity.

The system (1) exhibits a four-scroll chaotic attractor when the parameter values are taken as:

$$a = 3,\ b = 14,\ c = 3.9 \tag{2}$$

For numerical simulations, we have used the classical fourth order Runge-Kutta method in MATLAB with step size $h = 10^{-6}$ for solving the system (1) with parameter values as in (2) and initial conditions as:

$$x_1(0) = 0.2,\ \ x_2(0) = 0.4,\ x_3(0) = 0.2 \tag{3}$$

Figure 1 shows the 3-D view of the four-scroll attractor of the novel chaotic system (1), while Figs. 2-4 show the 2-D views (projections) of the four-scroll attractor on the three coordinate planes.

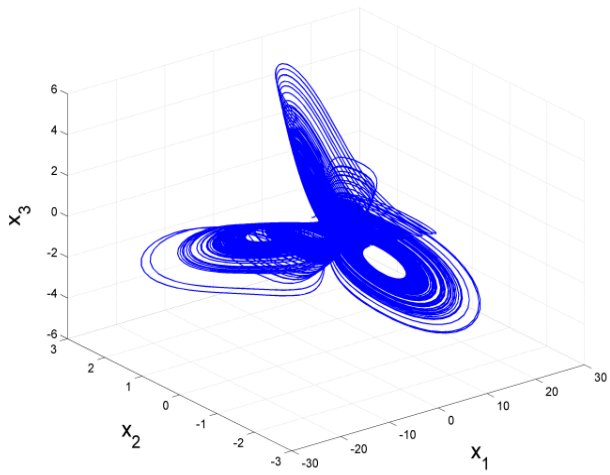

Fig. 1. The four-scroll attractor of the novel chaotic system.

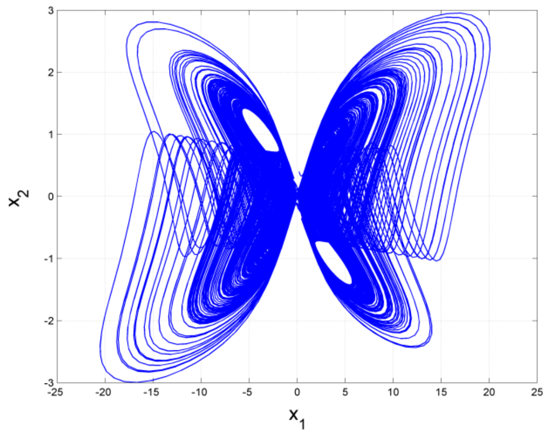

Fig. 2. 2-D projection of the novel chaotic system in (x_1, x_2) - plane.

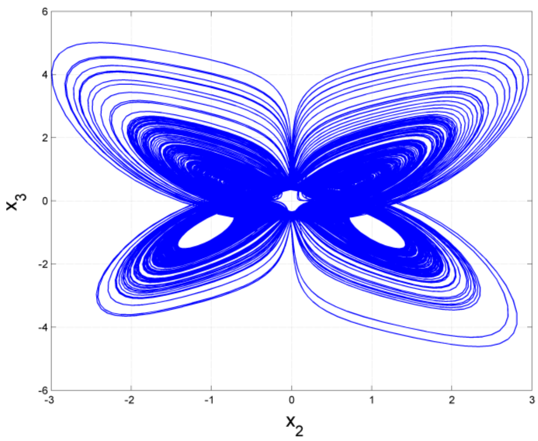

Fig. 3. 2-D projection of the novel chaotic system in (x_2, x_3) - plane.

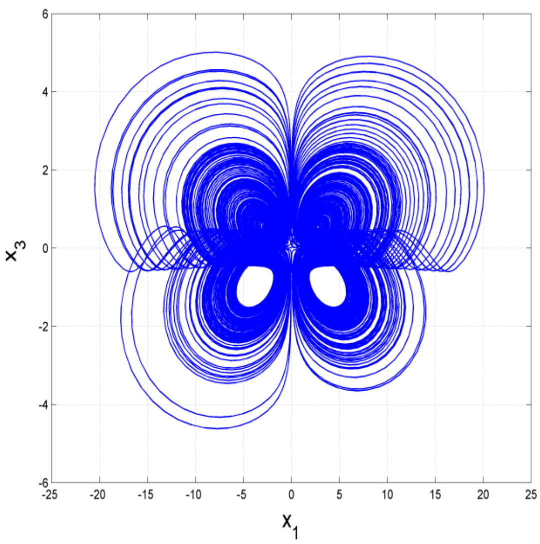

Fig. 4. 2-D projection of the novel chaotic system in (x_1, x_3) - plane.

3. Properties of the Novel Four-Scroll Chaotic System

In this section, we analyse the novel four-scroll chaotic system (1) and detail its fundamental properties like dissipativity, symmetry and invariance, equilibria, Lyapunov exponents and Kaplan-Yorke dimension.

3.1. Dissipativity

In vector notation, we may express the system (1) as:

$$\frac{dx}{dt} = f(x) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix} \tag{4}$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = a(x_2 - x_1) + b x_2 x_3 \\ f_2(x_1, x_2, x_3) = -10x_2^3 - x_2 + 4x_1 x_3 \\ f_3(x_1, x_2, x_3) = c x_3 - x_1 x_2 \end{cases} \tag{5}$$

We take the parameter values as in the chaotic case, *viz.* $a = 3$, $b = 14$ and $c = 3.9$.

Let Ω be any region in $\boldsymbol{R}^3$ with a smooth boundary and also $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f. Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we have

$$\frac{dV}{dt} = \int_{\Omega(t)} (\nabla \cdot f)\, dx_1 dx_2 dx_3 \tag{6}$$

The divergence of the novel system (1) is easily found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -a - 1 + c = -0.1 \tag{7}$$

Substituting (7) into (6), we obtain the first order ODE

$$\frac{dV}{dt} = -0.1\, V(t) \tag{8}$$

Integrating (8), we obtain the unique solution as:

$$V(t) = \exp(-0.1t) V(0) \tag{9}$$

It is evident from equation (9) that $V(t) \to 0$ exponentially as $t \to \infty$. This shows that the novel chaotic system (1) is dissipative.

Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel chaotic system (1) settles onto a strange attractor of the system.

3.2. Symmetry and Invariance

The novel chaotic system (1) is invariant under the change of coordinates

$$(x_1, x_2, x_3) \to (-x_1, -x_2, x_3) \tag{10}$$

The transformation (10) persists for all values of the system parameters. Thus, the novel chaotic system (1) has a rotation symmetry about the x_3-axis. Hence, it follows that any nontrivial trajectory of the novel chaotic system must have a twin trajectory.

3.3. Equilibrium Points

The equilibrium points of the novel chaotic system (1) are obtained by solving the following system of equations (with $a = 3$, $b = 14$, $c = 3.9$.

$$\begin{cases} a(x_2 - x_1) + b x_2 x_3 = 0 \\ -10x_2^3 - x_2 + 4x_1 x_3 = 0 \\ c x_3 - x_1 x_2 = 0 \end{cases} \tag{11}$$

A simple calculation yields the five equilibrium points

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.2595 \\ 0.7670 \\ 0.5095 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.2595 \\ -0.7670 \\ 0.5095 \end{bmatrix},$$

$$E_3 = \begin{bmatrix} 3.4089 \\ -1.0449 \\ -0.9134 \end{bmatrix}, \quad E_4 = \begin{bmatrix} -3.4089 \\ 1.0449 \\ -0.9134 \end{bmatrix} \tag{12}$$

The Jacobian matrix of the system (1) at x is given by

$$J(x) = \begin{bmatrix} -a & a + b x_3 & b x_2 \\ 4x_3 & -30x_2^2 - 1 & 4x_1 \\ -x_2 & -x_1 & c \end{bmatrix} \tag{13}$$

The Jacobian matrix at the equilibrium E_0 is obtained as:

$$J_0 = J(E_0) = \begin{bmatrix} -3 & 3 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3.9 \end{bmatrix} \tag{14}$$

Since J_0 is a triangular matrix, its eigenvalues are given by its diagonal entries, i.e.

$$\lambda_1 = -3,\ \lambda_2 = -1,\ \lambda_3 = 3.9 \tag{15}$$

Thus, the equilibrium E_0 is a *saddle* point.

Next, the Jacobian matrix at the equilibrium point E_1 is obtained as:

$$J_1 = J(E_1) = \begin{bmatrix} -3.0000 & 10.1330 & 10.7380 \\ 2.0380 & -18.6487 & 10.3620 \\ -0.7670 & -2.5905 & 3.9000 \end{bmatrix} \tag{16}$$

The eigenvalues of the matrix J_1 are obtained as:

$$\lambda_1 = -19.1228,\ \lambda_{2,3} = 0.6871 \pm 3.4276i \tag{17}$$

Thus, the equilibrium E_1 is a *saddle-focus* point.

Using a similar calculation, we can easily show that the other equilibrium points E_2, E_3, E_4 are also *saddle-foci*.

Hence, all the five equilibrium points of the novel chaotic system (1) are unstable.

3.4. Lyapunov Exponents and Kaplan-Yorke Dimension

For the chosen parameter values (2) and initial conditions (3), the Lyapunov exponents of the novel chaotic system (1) are obtained using MATLAB as:

$$L_1 = 0.75535,\ L_2 = 0,\ L_3 = -22.43304 \tag{18}$$

The maximal Lyapunov exponent (MLE) of the novel chaotic system (1) is L_1 = 0.75535. Since the sum of the Lyapunov exponents is negative, it follows that the novel chaotic system (1) is dissipative.

Also, the Kaplan-Yorke dimension of the novel chaotic system (1) is calculated as:

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.0336 \tag{19}$$

which is fractional. Figure 5 depicts the dynamics of the Lyapunov exponents of the novel chaotic system (1).

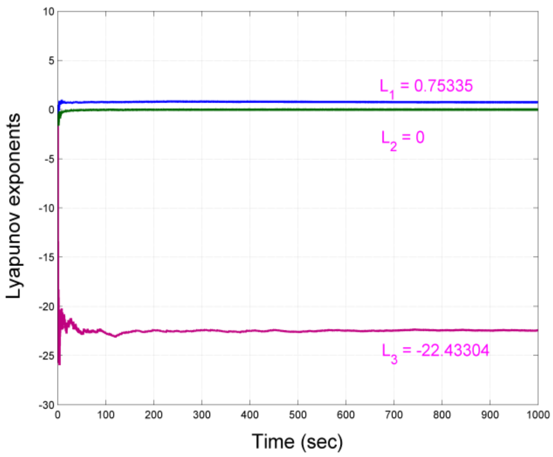

Fig. 5. Dynamics of the Lyapunov Exponents of the Novel System.

4. Circuit Realization of the Novel Chaotic System

An analogue circuit has been designed to realize the novel four-scroll chaotic system. It is noted that in order to obtain chaotic attractors in the dynamical range of operational amplifiers, the state variable x_3 of system (1) is scaled down. As a result, the novel chaotic system (1) can be represented as:

$$\begin{cases} \dfrac{dX_1}{dt} = \dfrac{a}{5}X_2 - aX_1 + \dfrac{b}{5}X_2X_3 \\ \dfrac{dX_2}{dt} = -10X_2^3 - X_2 + 20X_1X_3 \\ \dfrac{dX_3}{dt} = cX_3 - 5X_1X_2 \end{cases} \tag{20}$$

in which $X_1 = {}^{x_1}\!/_5$, $X_2 = x_2$ and $X_3 = x_3$. The schematic of the proposed circuit is shown in Fig. 6.

By applying Kirchhoff's laws to this circuit, its dynamics are described by the following circuital equations:

$$\begin{cases} \dfrac{dv_{C_1}}{dt} = \dfrac{1}{R_1C_1}v_{C_2} - \dfrac{1}{R_2C_1}v_{C_1} + \dfrac{1}{10R_3C_1}v_{C_2}v_{C_3} \\ \dfrac{dv_{C_2}}{dt} = -\dfrac{1}{100R_4C_2}v_{C_2}^3 - \dfrac{1}{R_5C_2}v_{C_2} + \dfrac{1}{10R_6C_2}v_{C_1}v_{C_3} \\ \dfrac{dv_{C_3}}{dt} = \dfrac{1}{R_7C_3}v_{C_3} - \dfrac{1}{10R_8C_3}v_{C_1}v_{C_2} \end{cases} \tag{21}$$

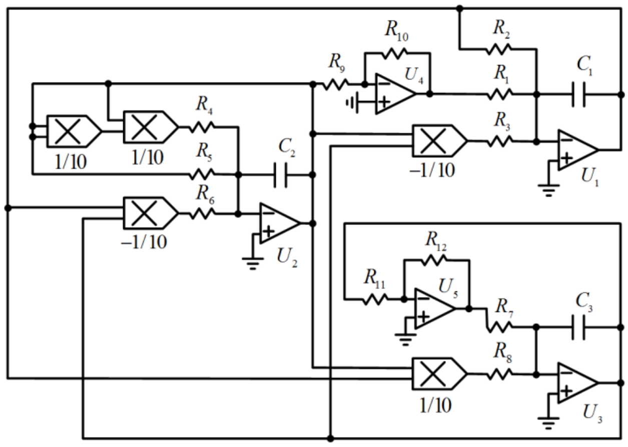

Fig. 6. Circuit realization of the novel four-scroll chaotic system (20).

where v_{C_1}, v_{C_2}, v_{C_3} are the voltages across the capacitors C_1, C_2 and C_3, respectively. Here each variable of system (20), i.e. X_1, X_2, X_3 is implemented as the voltage across the corresponding capacitors C_1, C_2 and C_3, respectively.

The values of the electronic components in Fig. 6 are selected in order to match known parameters of system (20): R_1 = 500 kΩ, R_2 = 100 kΩ, R_3 = 10.714 kΩ, R_4 = 0.3 kΩ, R_5 = 300 kΩ, R_6 = 1.5 kΩ, R_7 = 76.923 kΩ, R_8 = 6 Ω, $R_9 = R_{10} = R_{11} = R_{12}$ = 300 kΩ and $C_1 = C_2 = C_3$ = 1 nF. The power supplies of all active devices are ±15 Volts.

The designed circuit has been implemented by using the electronic simulation package Multisim. The obtained phase portraits are shown in Figs. 7-9. Obviously, these SPICE phase portraits are similar to the theoretical ones (see Figs. 2-4).

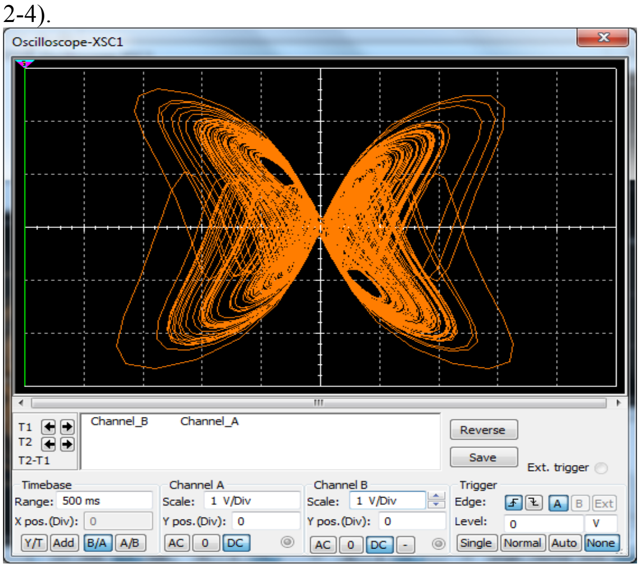

Fig. 7. Chaotic attractor exhibited by the circuit in Fig. 6 in $v_{C_1} - v_{C_2}$ phase portrait.

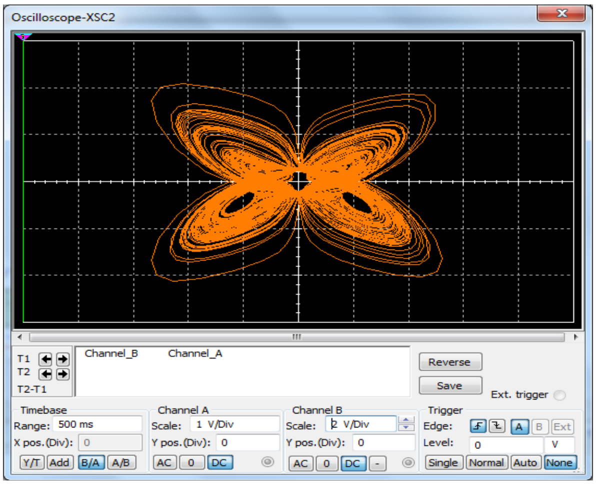

Fig. 8. Chaotic attractor exhibited by the proposed circuit, in $v_{C_2} - v_{C_3}$ phase portrait.

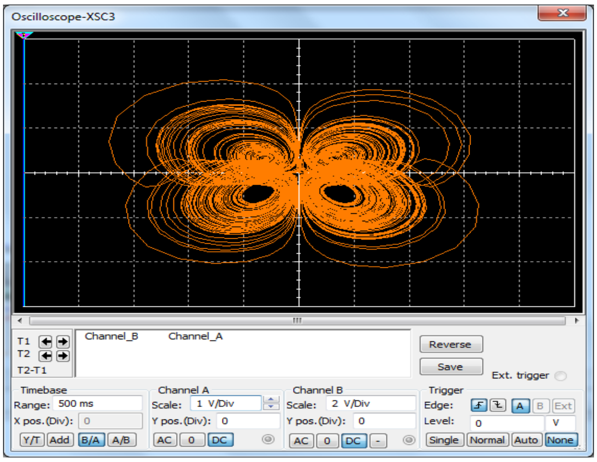

Fig. 9. Chaotic attractor exhibited by the proposed circuit, in $v_{C_1} - v_{C_3}$ phase portrait.

5. Conclusion

In this paper, an eight-term novel four-scroll chaotic system with a cubic nonlinearity has been introduced. Its complex dynamics characteristics such as dissipativity, symmetry, invariance, equilibrium points, Lyapunov exponents and Kaplan-Yorke fractional dimension are analysed. Furthermore, the feasibility of the theoretical model is also confirmed by an electronic circuitry.

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