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How the Use of Curriculum Resources Explains Inter-Teacher Differences in Lesson Planning

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Abstract: This study concerned teacher education in mathematics—specifically, curriculum resources (textbooks and other media that teachers use when planning a lesson or delivering classroom instruction). Four levels of curriculum resources (media) were identified, and the concept of "translation" was adopted to describe the process in which a medium at a higher level is replaced by a medium at a lower level. A survey was conducted among 15 trainee teachers in the 2019 academic year. The purpose of the survey was to examine inter-teacher differences in lesson planning; particularly, the media the teachers used in their translations, the order in which they used the media, and how each medium shaped the translation process. The results revealed that (1) the trainee teachers simulated the actions of the textbook authors or students and that (2) the functions of the media changed after a translation was performed. These findings imply that differences in lesson planning can be explained by differences in the type of media translated, the order in which they are used, and the way the media are used.

1. Introduction

This study concerns teacher education in mathematics. Teachers make several decisions during classroom instruction; however, they are unlikely to make purely intuitional decisions (i.e., drawn exclusively from expertise-based heuristics), and their expertise and intuition are themselves not developed in a vacuum. If it is the case that the phenomena in mathematics lessons derive from the decisions made by teachers, then clarifying the grounds for these decisions should yield more practical insights into teacher education. Thus, it should help move the research on from its tendency to focus only on the static knowledge and skills that teachers have or ought to have. This study focused on the curriculum resources that teachers use as referential media to guide their decisions during classroom instruction. "Curriculum resources" can refer to Japan's national curriculum guidelines, textbooks, or any other media that teachers use to plan lessons or manage the class. The purpose of this study was to determine (1) how teachers interpret and use curriculum resources when planning in their classes, (2) what teachers evaluate a previous lesson and decide the trajectory for subsequent lessons.

In an examination of how classroom instruction is shaped by the teacher's interpretations of curriculum resources, a useful perspective is "curriculum enactment." "Enactment" translates directly into Japanese as *gutaika* or *joen. Gutaika* back-translates into English as "embodying" or "instantiating," while *joen* back-translates as "acting something out." As used in the former sense, enacting a curriculum could mean going from a curriculum resource, such as the national curriculum guidelines, to another curriculum resource, such as a learning unit plan or teaching plan. In the latter sense, it could mean applying the teaching plan in classroom instruction. If "curriculum" has a broad meaning, then so does curriculum enactment. Accordingly, curriculum enactment has garnered interest among researchers of mathematics teacher education. For example, the 46th issue of ZDM - Mathematics Education contained an editorial titled "Researching the enacted mathematics curriculum: learning from various perspectives on enactment." This editorial identified four levels of curriculum enactment (Thompson & Huntley, 2014):

- Level 1: Enactment that occurs at a national level "as educational goals are enacted into a set of national objectives or standards." In the case of Japan, this would include the national curriculum guidelines and the official annotations for these, as well as official public documents related to education.
- Level 2: Enactment that occurs "as goals or standards are embedded into written curriculum materials or textbooks." Examples include textbooks, teacher guides, learning materials, and local or school-level educational objectives.
- Level 3: Enactment that occurs "as teachers make decisions about how to use their written curriculum materials." Examples include a learning unit plan, teaching plan, learning materials, and learning tools.
- Level 4: Enactment that occurs "as teachers and students engage and interact with written materials during classroom instruction." Examples include the intentive questioning used

Suppose, for example, that a teacher is planning a lesson. If the teacher views a textbook to identify the content to teach and to envisage the lesson's flow, this would be an example of curriculum enactment at the third level. If, during classroom instruction, the teacher evaluates their instruction in real time and recalibrates the lesson accordingly, then this would be an example of level 4 enactment. In the former example, the teacher "translates" (see Note 1) the content of the textbook into a teaching plan; in the latter example, the teacher "translates" the teaching plan into classroom instruction. In each case, a translation is performed. Specifically, each case involves curriculum resources ("media"); one medium is created from an existing medium, and this process must necessarily involve translation. Translation, therefore, is key to curriculum resources) and translation (curriculum enactment). Figure 1 shows our conceptual framework concerning media and translations across the four levels. More fundamental (upstream) medium ↓ Translation i Medium I (e.g., national curriculum guidelines) ↓ Translation ii Medium II (e.g., textbook) ↓ Translation iii Medium III (e.g., learning unit plan, teaching plan) ↓ Translation iv Medium IV (classroom instruction)

Figure 1: Four levels of media and translation

Across the four levels of curriculum enactment, translations i to iv are paired with the media (curriculum resources) I to IV. This conceptual framework is designed to convey the multilayered nature of translation. For an example of this multilayered nature, consider a scenario involving translation iv (translation of lesson plan). Although it could be performed by either the classroom instructor or the students, the teacher, unlike the students, could potentially perform the translation at another level. That is, if the teacher is highly experienced, they already anticipate the content of translation iv based on their previous teaching experience and knowledge of the students. The translation the teacher would actually perform, then, would be translation iii (translation of learning unit plan or teaching plan). Teachers' translations are also multilayered, in the sense that they simulate the role of students while retaining that of the teacher.

Translations iii and iv are performed by the teacher and students during the lesson, while translations i and ii are performed by mathematicians and researchers of mathematics educations. Due to the limitations inherent in media, the national curriculum guidelines and textbooks are unlikely to give the reader a complete and perfect understanding of the author's intentions. In other words, teachers are unlikely to gain a perfectly accurate understanding of translations i and ii. Does this mean, then, that teachers will completely ignore these translations when planning their lessons? The answer must be no. Since teachers must perform some kind of translation in order to incorporate the medium into the lesson, they may simulate the actions of those who performed translation at other levels. Let us suppose, for example, that a teacher is planning a lesson on functions for square proportionality (y=ax^2) in the third year of junior high school. The teacher knows that the textbook uses the analogy of a vehicle's stopping distance. Referring to this content, the teacher starts envisaging how they could apply this analogy in the lesson. While doing so, the teacher may have a simple question: what did the authors of the textbook have in mind when they chose stopping distance as an example? The teacher could then find a clue by reading the relevant passage of the national curriculum guidelines. The explanation states the reason as "to use ideas that can help make better predictions." From this annotation, the teacher will understand that the authors cited the phenomenon of stopping distance not only to illustrate how stopping distance is not proportional to the vehicle's velocity but, crucially, because they felt that stopping distance

would provide a more relatable example. In summary, it is possible that a teacher would surmise the intentions of those who performed translation ii and incorporate these into translation iii. If, as the above scenario illustrates, frontline teachers perform translation at multiple levels, then it is entirely possible that a given medium can be translated in markedly different ways. Accordingly, it is necessary to carefully examine the multilayered nature of the translations.

We believe that the conceptual framework shown in Figure 1 can describe the translations teachers perform for a given class, including which level of translation they perform, whether they perform multiple levels of translation, and different levels of translation interact with each other. More specifically, it can tell us which media were used for each of the teacher's translations, what order the teacher translated them in, and how each medium shaped the translations. Little is known about how the translations at different levels interact, as the research has tended to focus more on teachers' intuitive judgments. To obtain more insights into how teachers plan and conduct lessons, the present study aimed to describe how media are interpreted/translated at each of the four levels.

In our research, we have an opportunity to observe trainee teachers. Additionally, our institution has several affiliated schools and preschools, meaning that we can implement a given lesson plan across different schools and compare the in-lesson teaching actions of multiple teachers with different backgrounds (e.g., varied teaching experience) to gain broad insights into teachers' practical knowledge. The present report focuses on how the trainee teachers performed translations during their classroom instruction.

2. Situating the Present Study within the Literature

This section summarizes the literature on theory-based curriculum development, curriculum enactment (creating a narrative from the itemized curriculum guidelines), and teachers' self-evaluation.

We have adopted the premise that curriculum enactment occurs at the four levels described in the previous section. Thompson and Huntley (2014) set forth this framework as follows. First, they mentioned that Travers (1992) and other researchers in the Second International Mathematics Study (SIMS) identified three levels of curriculum. The first level is the intended curriculum, represented in official curriculum documents and related textbooks for use; the second level is the implemented curriculum, which is the content that teachers teach; the third level is the attained curriculum, which refers to actual student achievement. They then mentioned that other TIMMS researchers added a fourth curriculum level between the intended and implemented levels: the potentially implemented curriculum, which is the textbook. This level emphasizes how the textbook mediates between intentions and implementation. The researchers emphasized the importance of understanding that the curriculum is replaced at each of the four stages.

Described below are the articles on curriculum enactment that were cited in the ZDM editorial.

Confrey, Maloney, and Corley (2014) presented a conceptual framework showing how curriculum (or learning trajectories) are associated with educational objectives articulated at a national level (in the U.S., in their case).

In a U.S. study, Remillard, Harris, and Agodini (2014) analyzed the effects of textbook design upon student test results. They analyzed four different textbooks and concluded that the design differences may explain differences in student testing. Similarly, Huntley and Terrell (2014) analyzed design differences among five prominent U.S. textbooks or units in integrated textbook related to linear equations. They reported that these differences affected student learning. Sears and Chávez (2014) compared two different textbooks in terms of how they encourage students to engage with geometric proof and investigated how these differences affected the way teachers enact proof tasks during lessons. Thompson and Senk (2014) reported that the same textbook can have different effects in the classroom depending on the teacher. Taking a different approach, Hunsader et al. (2014) focused on the assessment instructions accompanying prominent elementary and secondary-school curricula in the U.S., noting inconsistencies among these instructions. Focusing on classroom instruction, Otten and Soria (2014) compared how three teachers addressed algebra tasks in the classroom. They observed differences between the teachers in their expected acts in the lesson, in the verbal discourse expected of students, and in the after-class evaluation. These studies demonstrate that differences in the medium used—and in the way teachers translate it during classroom instruction-affects the lesson, evaluation, and student performance.

Remillard and Heck (2014) presented a conceptual model of curriculum enactment, indicating where the above articles fit within the model. As the editorial noted, their model provides a useful means for researchers from different countries to understand how curriculum terms are used in different countries. In a commentary that compared Singapore's educational system with that of the U.S., Kaur (2014) identified the types of curriculum research in Singapore that could fit into Remillard and Heck's model.

Thus, whereas the literature once consisted of disparate research at different curriculum levels, there is now a focus on integrating the perspectives and showing how the levels link up in a sequential flow. Additionally, it is now easier to share theoretical perspectives on curriculum across national boundaries and language barriers.

3. Survey and Analysis

In this section, we report our survey of trainee teachers' classroom instruction and our analysis of the observations. The purpose of the analysis was to clarify the media that teachers used in their translations, the order in which they used these media, and how differences in media affected the translations. In order to illustrate the multilayered nature of translation, we traced the teachers' thoughts and ideas that inspired them to shift from one medium to another.

(1) Method

The survey consisted of a questionnaire conducted among 15 trainee teachers in the 2019 academic year. At the time of the survey, they were undergoing a one-year internship at a junior high school affiliated with our institution (13 were assigned to Shinonome Junior High School and two to Mihara Junior High School). In the questionnaire, these trainee teachers reported on a lesson they taught as part of the internship.

(2) Questionnaire

The questionnaire consisted of four questions shown below. The questions correspond, primarily, to the corresponding translation (question 1 corresponds to translation i, question 2 to translation ii, and so on). We explained to the participants beforehand that "curriculum resources" could refer to the national curriculum guidelines (and the annotations for such), textbooks, teaching instructions, problem sets, journals (such as Meijito's Sugaku Kyoiku), and online resources. We also ensured that these resources were always accessible to the participants. The survey was conducted on the final day of the internship. Only 11 out of the 15 participants gave a response for the fourth question as we added this after the four respondents in question had completed the first period of their internship.

Please recall one of the lessons you taught and answer the following four questions concerning this lesson.
Question 1: What problem did you set for the class?
Question 2: What question did you link with this topic?
Question 3: When deciding on the learning task and question, which curriculum resource(s) did you refer to and how did you refer to them?
Question 4: How did the first lesson shape the second lesson?

(3) Results

Table 1 shows the results for the first three questions.

Type of curriculum resource	No.	What the resource was used for
	respondents	(no. respondents)
National curriculum guidelines and	8	Problems/questions (3), Lesson objectives (5),
annotations for such (Medium I)		Evaluation (2),Purpose (1)
Textbook (Medium II)	14	Problems/questions (14), Lesson objectives
		(1), Comprehension of subject matter (1),
		Nomenclature (1), Inter-lesson connection (2)
Teaching instructions (Medium II)	3	Problems/questions (1), Lesson objective (1),
		Key points for lesson (1), Lesson flow (1)
Ideas for teaching plans in online	6	Problems/questions (6), Lesson objectives (1),
resources and literature (Medium		Evaluation (1), Time allocation (1)
II)		
Other (previous classes at high	3	Problems/questions (3)
school or university)		

Table 1: Tvi	pes of curriculu	m resources	used and	what they	were used	for
Table 1. Ly	pes of curriculu.	III I COULCES	uscu anu	what they	were used	101

Textbooks were used the most, followed by the national curriculum guidelines (or the annotations for such). In terms of what the respondents used the resources for, all 14 of the respondents who

used textbooks said that they utilized them for tasks and questions. The national curriculum guidelines were most likely to be used for setting the lesson objectives or for evaluation. For example, one of the respondents (Yam) gave the following responses:

Question 1: What problem did you set for the class?

Under what conditions will a straight line drawn to a plane be perpendicular to the plane? Question 2: What question did you link with this topic?

How many set squares would you need in order to place Rod L perpendicular to Plane P? The figure on the right [omitted here] shows a square piece of paper that has been folded in two, placed on the surface of the desk (Plane P), and labelled. Explain how Line EF is perpendicular to Plane P.

Question 3: When deciding on the learning task and question, which curriculum resource(s) did you refer to and how did you refer to the resource(s)?

First, I checked the section in the textbook for my class to get a rough idea of how the lesson would flow. Next, to decide on the question to set, I looked online for examples of past lessons on this topic. I got some ideas for stimulus questions: "Could you do it with just one set square?" "What about placing two set squares along the same line?" When working out how to evaluate the learning, I checked the national curriculum guidelines to confirm what the lesson objective should be. Finally, I checked a teacher guide in the textbook to confirm the key points for the lesson.

According to the above responses, Yam began by viewing the textbook to decide a topic for the lesson content and work out the lesson flow. Yam then used an online resource to pick a question related to the content. However, at this point, Yam was yet to situate/contextualize their classroom instruction within this flow. Yam finally did so when they integrated the two different media (textbook and online resource) to derive a question that they would pose during classroom instruction. Next, Yam started thinking about evaluation. For this step, they reconsidered the lesson objective and consulted the national curriculum guidelines. However, Yam failed to derive from this medium any specific pointers on evaluation. Like Yam, seven other respondents referred to the national curriculum guidelines when deciding on lesson objective and evaluation.

Another respondent, Dai, managed to derive a question from the annotations to the national curriculum guidelines:

Question 1: What problem did you set for the class?

Using the phrase "rate of change," explain that a graph of $y = x^2$ is nonlinear.

Question 2: What question did you link with this topic?

Why is the graph of $y = x^2$ nonlinear when the rate of change is nonconstant?

Question 3: When deciding on the learning task and question, which curriculum resource(s) did you refer to and how did you refer to the resource(s)?

I looked at a number of textbooks to plan out the gist of the lesson. Each textbook contained the

conclusion: "The rate of change of $y = ax^2$ is nonconstant." I felt that this lacked something, so I checked out the national curriculum guidelines. The guidelines included the same statement, but they also added the following: "Students will understand that the graph of $y = ax^2$ is curved." I reckoned that junior high-school students would actually struggle to understand that the graph is curved. I therefore decided to focus on what we mean by rate of change in the case of a linear function. Although it may be unrealistic to expect the students to visualize the rate of change as a curve, they should at least be able to understand that it is not straight. I drafted the question accordingly.

Dai compared the conclusion derived in textbooks with that stated in the national curriculum guidelines, finding the latter to be more elaborated than the former. They then set a question that would lead to that more precise conclusion. During this process, Dai surmised the reasons for the difference between the two media and attempted to reconcile the difference. Table 2 clarifies the order of Dai's translations and what these translations consisted of. The table arranges the translations into a time series and shows, for each translation, (1) the main medium Dai referred to, (2) Dai's attitudes and behavior regarding the medium, and (3) the idea (derived from the medium) that prompted Dai to move to the next medium or translation.

Ordinal	Translation	Medium		Dai's attitudes and behavior	Idea that led Dai to the next
position	level			regarding the medium.	medium.
1	iii	П	Textbook	Dai decided to use several	The medium contained the
				textbooks.	conclusion: "The rate of
					change of y = ax^2 is
					nonconstant." Dai felt that
					this was inadequate.
2	ш	Ι	National	Dai focused on the part	Dai wondered why this point
			curriculum	reading, "Students will	was missing from the
			guidelines	understand that the graph of	textbooks.
				y = ax^2 is curved."	
3	ii	П	Textbook		Dai felt it would be
					unrealistic to expect the
					students to visualize the
					rate of change as a curve.
4	ш	Ш	Teaching	Dai set the question on the	
			plan	assumption that students	
				would understand that the	
				function is nonlinear.	

Table 2: Order of Dai's translations and the ideas that led to the next translation

Dai began by viewing several textbooks. They felt that the content in these textbooks was insufficient, and this reaction inspired them to view the national curriculum guidelines. Dai then noted that this medium contained a statement that was absent in the textbooks ("Students will understand that the graph of $y = ax^2$ is curved"). It is likely that Dai would have wondered why the textbook authors had omitted this point. If so, this would have meant that Dai had extended their focus to translation ii. Ultimately, Dai developed the topic and question for the class using content from both media (textbooks and national curriculum guidelines). In this way, Dai derived their lesson plan from multiple media and thus performed translation at multiple levels. This case also suggests that the function of a medium can change after a translation is performed. Initially, the textbooks functioned as a means for Dai to work out the content for the lesson; however, after Dai performed translation ii, they assumed a more practical function for classroom instruction in that they offered Dai a phrase to use in class ("Why is the graph of $y = x^2$ nonlinear when the rate of change is non-constant?").

(4) Results for Question 4

The trainee teachers at both schools delivered the same lesson twice, each time to a different class. This meant that, for their second lesson, they had an opportunity to adjust the lesson plan based on student responses in the first lesson. Any adjustments they made would have corresponded to translation iv. Accordingly, we asked question 4 ("How did the first lesson shape the second lesson?") in an attempt to obtain descriptive data on the respondents' experience in performing translation iv. Table 3 shows the responses of the 11 respondents who answered this question.

Change	No. respondents
Changed time allocation	6
Elaborated further on the topic	5
Elaborated further on the question	4
Added a question	2
Used additional tool	1
Shared discourse of other students	1
Gave students idea of next lesson	1
Encouraged students to share ideas	1
Encouraged students to explain in their own words	1

Table 3: Change between lesson 1 and 2 (n = 11)

In Table 3, "changed time allocation" denotes that the respondent felt that they did not have enough time in the first lesson to accomplish the key activities. "Elaborated further on the topic/question" denotes that the students struggled to grasp the problem/question as it was not stated clearly enough. In their descriptive responses, the respondents mentioned that these adjustments helped them observe what the students were thinking and how they were expressing these thoughts. To give an example of the responses, Nis elaborated on time allocation:

I divided the lesson into three parts: (1) recap of previous lesson, (2) development of current lesson, (3) recap of current lesson. In the first lesson, the first part took too long, leaving insufficient time for the third part. For the second lesson, I skipped the first part to leave more time for the third part, which enabled the students to reflect on what they had learned in the lesson. Consequently, more students accomplished the learning objective in the second lesson compared to those in the first.

Before starting on the main topic of the lesson, I spent plenty of time on a stimulus question: is it true that an angle [bisector of a triangle] divides [the opposite side into segments that are] proportional [to the other two sides of the triangle]? I spent more time in lesson 2 (compared to lesson 1) asking questions that students could answer by using what they had already learned, and this extra time resulted in an increased motivation in the students. I also increased the number of references I made (in my speech) to "angle," which prompted many of the students to recall the properties of a congruent triangle.

In the second lesson, Nis devoted more time to stimulating the students' interest and reflecting on the lesson, and this change encouraged the students to think critically. Although it was not made in real-time during class (in response to feedback, the change was still a product of translation iv.

Another respondent, Nak, added a question after considering student responses. This addition markedly changed the flow of the lesson.

[In a topic in which students must draw an additional line to determine the measure of an angle], the students seemed more interested in how many additional lines they could draw than they were in the properties of the additional line. I therefore decided to switch the focus to the number of additional lines. I got the students to compete with each other in thinking up solutions for this new challenge. Since the students had been unable to express the reason for drawing an additional line, I asked them to draw a number of lines and then divide them into groups. This approach helped the students verbalize their intuition about where to draw a line (as "just because").

Noting that the students were interested in how many additional lines they could find, Nak switched to a different question to encourage them to engage freely and generate several ideas. Nak then used the outcomes of this new questioning as a means to achieve the lesson objectives. Nak performed translation iv in that, rather than using a textbook, they focused on student responses during the lesson and reconsidered the question for that lesson accordingly. Nak's responses to the first three questions were as follows:

Question 1: What problem did you set for the class?

Find the measure of angle $\angle x$ when 1 and m are parallel (the figure shows parallel lines 1 and m intersecting a polyline).

Question 2: What question did you link with this topic?

"How many additional lines can you draw?" "Can you express in words how you intuited that the line should go there?"

Question 3: When deciding on the learning task and question, which curriculum resource(s) did you refer to and how did you refer to the resource(s)?

The textbook contains a figure showing an additional line to illustrate how you work out where the line goes. [...] According to the textbook, drawing an additional line makes it easier to use the properties of a 2D shape and thus helps in solving the problem. On reading this, I wondered why an additional line can be drawn. After considering how students' responses might drive the lesson forward, I concluded that the reason for drawing the line there is intuitional ("because it just seems to go there"). Since I myself had wondered why the stated answer features an additional line in a place where I wouldn't have thought to draw one, I assumed that students would also wonder about this. Accordingly, I decided to verbalize the idea that the line goes there "just because." Given the way students had responded in the first lesson, I predicted that students in the second lesson would be less interested in the properties of the additional line and more interested in the number of additional lines. Accordingly, I decided to ask them to see how many lines they could draw.

In the lesson planning process, Nak decided on the question based on the students' responses in the first lesson. In this sense, Nak arguably performed translation iv. As Table 2 did for Dai, Table 4 shows the order of Nak's translations and what these translations consisted of.

From the lesson planning stage onward, Nak placed importance on student responses and performed translation iv to get ideas for a teaching plan on multiple occasions. On the other hand, Nak gave no consideration to the actions of the parties at higher levels (the authors of the textbook). By contrast, Dai had wondered about the intentions of such parties but gave no consideration to student responses. Nak and Dai were polar opposites in this respect. While we are unsure whether a different learning unit would entail a different approach to lesson planning, the results in Tables 2 and 4 show that the differences between Nak and Dai in the way they planned their lessons can be explained by differences in the types of translation they performed, the order in which they performed the translations, and the way they used the media.

Ordinal	Translation	Medium		Dai's attitudes and behavior	Idea that led Dai to the next
position	level			regarding the medium.	ineurum.
1	ш	Π	Textbook	Nak read the relevant page,	Nak was interested in the
				examined the figure and	question of why an additional
				problems, and selected a	line could be drawn.
				problem.	
2	iv	IV	Lesson	Nak predicted student	Nak decided to verbalize the idea
				responses from his/her	that the line can be drawn "just
				experience.	because."
3	iii	Ш	Teaching	Nak decided to set the	
			plan	question: "Can you express	
				in words how you intuited	
				that the line should go	
				there?"	
4	iv	IV	Lesson	Nak felt that the students	Nak predicted that the students
				were more interested in how	would be more interested in the
				many additional lines could	number lines than they would be
				be drawn.	in the properties of the lines.
5	iv	IV	Lesson	Nak found that the students	Nak got the students to separate
				failed to verbalize their	the lines into groups to help
				intuition for the additional	them verbalize.
				line.	
6	iii	Ш	Teaching	Nak set the question: "How	
			plan	many additional lines can	
				you draw?"	

Table 4: Order of Nak's translations and the ideas that led to the next translation

4. Conclusion

Summarized below are our findings:

- 1. When planning their lessons, teachers perform multiple iterations of translation.
- 2. Teachers may simulate the actions of parties at different levels.
- 3. Translation can potentially change a medium's function.
- 4. Differences in the way teachers plan lessons can be explained by differences in the types of translation they performed, the order in which they performed the translations, and the way they used the media.

Outlined below are the ongoing research tasks:

- To confirm the first two findings, we must obtain more statistical data.
- To confirm the third finding, we must investigate how a teacher's attitudes toward a given medium changes and how a translation that the teacher performed affects the lesson.

- To confirm the fourth finding, we must compare the lessons delivered by teachers who used the same medium to determine whether differences in the lessons can be explained by differences in the translation of the medium.
- To develop the fourth finding, we must determine whether the order in which a teacher translates a medium affects the method of translation.

Note 1

"Translation" can mean simply translating text from one language to another, but it can also imply a process of interpretation. As used in this manuscript, "translation" implies the latter meaning.

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