THE EFFECT OF THE PRINCIPLES OF DYNAMIC SYMMETRY ON MODERN ART AND SCIENCE

by

ERIC BEGLEITER

B.A., Hampshire College
June 1980

SUBMITTED TO THE DEPARTMENT OF ARCHITECTURE
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
OF THE DEGREE OF

MASTER OF SCIENCE IN VISUAL STUDIES

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
June, 1984

Signature of Author.......................................
Eric Begleiter, Department of Architecture
MAY 11, 1984

Certified by..
Otto Piene, Professor of Visual Design, Director
C.A.V.S., Thesis Supervisor

Accepted by...
Professor Nicholas Negroponte, Chairman
Departmental Committee for Graduate
Students

THE EFFECT OF THE PRINCIPLES OF DYNAMIC SYMMETRY ON MODERN ART AND SCIENCE

by

ERIC BEGLEITER

Submitted to the Department of Architecture in partial fulfillment of the requirements for the Degree of Master of Science in Visual Studies

May 11, 1984

ABSTRACT

A selection of ancient concepts of ratio, form, proportion and harmony is used as the basis for the motivation and design of geometric art forms. Through a comparison of both contemporary concepts of dimensional space and ancient structural forms, new relationships are inferred. These relationships are then applied in the creation of original sculpture by the artist.

Thesis Supervisor: Otto Piene

Title: Professor of Visual Design,
Director C.A.V.S.

TABLE OF CONTENTS

CHAPTER I INTRODUCTION

The intention of this thesis is to explore the factors which have influenced the design of a series of original geometric sculptures constructed by the author. These works explore the fundamental elements of space and the externalization of dimension as revealed in ancient and modern thought. It is also the purpose of this thesis to show the implications that visually ambiguous projections of polyspace have had with respect to the concept of self as revealed in art and science.

Starting from the somewhat vague qualitative notion of dynamic symmetry as a harmony of proportions the mythic importance different cultures have placed on the visual fundamentals of point, line, plane and volume will be described. Having developed an appreciation of these basic elements, this thesis will briefly deal with how the internal structuring of space has been revealed through an exact qualitative analysis of naturally occurring forms. This will in turn provide a basic model governing diverse visual forms of communication based on the natural language of organic patterns. It will then be shown how an art form based on these patterns is both constrained by the natural laws of space and as such is also symbolic of it's self-determined nature. These principles of space will then be viewed in the contemporary context of non-euclidean geometry providing insight into the implications a multi-dimensional crystalline structure has on the artistic expression of the world around us.

One concept utilized throughout this thesis is that of multi or poly-dimensional symmetry. Though both a complete qualitative and quantitative analysis of this notion is contained within the body of this work, it is nevertheless appropriate at this point to define its meaning in the context of a three-dimensional sculpture. This mathematical projection of space can most easily be understood through a form of analogy. A point is extended and so draws out a line, a line which moves perpendicular to its length traces out a plane. (The two end-points of this line segment now become extended and so make the edges of a new two-dimensional shape). If this plane moves perpendicular to its two directions then it spans a volume. By utilizing the correct perspective cues this process can be repeated from dimension to dimension. The fact that this image is a projection into hyperspace does not preclude our representing it on a two-dimensional canvas. With the correct perspectival clues this process can be visually repeated in the third dimension with respect to the fourth. Just as the geometry of two and three dimensional figures can reveal the structure of space and its influence on the physical forces expressed in art and science, so too the geometry governing the structure of hyperspace can also be utilized in a similar manner.

One of the major obstacles preventing the artist, before the renaissance, from creating realistic illusions of space was the very way they thought about the world around them. Linear point perspectives as well as parallel, orthogonal and oblique projections were developed in Europe and the Orient as people's observation and conception of space and art changed. In Europe this change paralleled a movement away from the canvas as a recorder of idealized factual information to the artist as the preserver of visual moment of time. The use of perspective not only incorporated a different concept of time, but also showed that though the canvas might be flat and two-dimensional there was no logical reason why an extra dimension could not be accurately portrayed within its physical limits. It should be pointed out that affirming this realization in a positive light, as a forward development of its art, is not to neglect the work of many modern artists. These artists as a reaction to the photographic image, and in order to address the issues of artistic medium, moved back to asserting the frontality of the two-dimensional canvas. It is however an attempt to show how, whether asserting reality, illusion or the psychological spaces in between, one's information base determines the possible visual realizations accessible to the artist. In the same way that it was a change of the notion of space and time which allowed artists to visually incorporate one additional dimension into the two-dimensional canvas, so too is it the change of the notion of space and time in geometry which now allows the artist to project the structure of a fourth spacial dimension into the physical limits of a three-dimensional sculpture. Moreover the type of visual imagery which this process lends itself to, namely the volumizing and externalizing of an idealized geometrical space, seems to entail certain perspectives on the human condition. These perspectives can be seen to resonate with many ancient cosmologies. Thus a method is initiated in which the artist strives to find a form appropriate to the expression of space as intricate and nuanced as the interconnections of physical and psychological space itself.

CHAPTER II THE EMERGENCE OF THE MATHEMATICAL ARTS

The development of an art based on science and geometry, as a metaphor for the internal structuring of space, has its roots in ancient philosophical and mathematical thought.. Man's first truly mathematical appreciation of the world around him was one of qualitative ratios. These primary relationships were seen mainly in terms of larger and smaller, more and less. This type of mathematical analysis was simply the more of one quantity, the more of some other quantity. The bigger the rock the heavier it is, the larger the field the more food it gives. The earliest recorded pre-Mesopotanian writing which can be said to be a form of mathematical logic was of this type describing such phenomenon as "The Wider the Circle is Across the Longer it is Around".[1]

Later during the rise of the first agricultural revolution between 4000 and 3000 B.C. a basic form of qualitative analysis was developed called the methods of rearrangement. In Afro-Asia, Mesopotania, and later in the Nile Valley, adverse conditions created the impetus to change and grow an interlocked society. Then with increased production and surplus food there was more time to develop such specialization as administrators, priests, timekeepers and astronomers. Later, surveyors, accountants, tax collectors and navigators became an integral part of a large complex society. It has been conjectured that because of the pre-Mesopotanian transition into agriculture, Babylonian mathematics and geometrical art also advanced. The agricultural revolution and the rise of mathematics also followed each other in Central and South America. The Aztecs of the Yucatan peninsula in Western Honduras, as well as the Chibcha people of upper Mexico domesticated corn and potato crops in 1180 B.C. and developed a complex infrastructure within their respective societies. In 1000 B.C. many of these cultures concept of astronomy was advanced enough to create a remarkably accurate calendar based on the intermeshing periods of the sun, moon and Venus.[2] During this period many cultures understood and appreciated the importance of mathematics and geometry in their models of the world around them. These models were able to blend the art and science of the day with the deeply ancient knowledge of the symbols of the deep stratas of the mind. Any preliminary study of Egyptian, Greek, Islamic, Mayan and American Indian cultures shows the great importance of these people placed on a sacred art based on geometry. These people saw a great mystery which was revealed to them in their understanding of the structure of space as a metaphor for the mind and the universe. The fundamental elements of circle , square, triangle and spiral as well as the regular solids of the tetra, octa, hexa, icosa and dodecahedron were in the ancient world conceived of not just as shapes, but as motifs. These basic organizational devices of space lent themselves to the composition of an art based on geometrical rhythms and order.

The ancient goal of charging space and form with deep feelings and in so doing articulating the aspirations of humanity is the object of much ancient artistic perception and creation. Systematic investigation of this notion through the annals of recorded history leads one deeper into the activity of creation itself. This movement often mythically culminates in the ancient conception of art as a perfect harmony of form and idea, and of art and science. This conception has manifested itself in many cultures which have developed a philosophy of mathematics akin to the Pythagorean-Platonic traditions of antiquity. In these disciplines, geometry was viewed as an integral part of the universal structure. This structure supported many insights which have characterized the artistic expressions of ancient man. This can be found in the great Chinese and Japanese paintings of Southern Sung and later in Zen art. The Yantras and Mandalas of Hindu, Tibetan and Buddhist art as well as Egyptian, Greek, Islamic and Mayan art all display a conception of art based on a mystic interpretation of geometry.[3]

Cultures time and time again sought to penetrate the mystery of existence by descending into the archetypical world of pure crystalline space. The archetype of number was appreciated by many ancient cultures as the major symbol governing the structure of space and their art. Thus their visual literacy appealed both to the intellect as well as the eye. Many philosophers and artisans from ancient Greece and the Islamic traditions used number as a motif of the generating and volumizing of the universe.

Greek philosophers such as Thales and Pythagoras felt it was number which was the first principle of being itself. Which is to say that the universe was not just represented in the structure of number, nor that universal structure is revealed in the order of number, but that the universe was itself "number". Thus we experience number directly as form and shape. This is a conception of number which transcends the notion of one or two things but which is a form of experience abstracted as the pure elements of unity and multiplicity. Beyond number is ratio, proportion and the natural vibrations of each "shape number". Therefore it can be said that four points determine the number square and eight the number cube. This concept of number is pure form and can be used to impose shape upon matter. The conception of number was considered a penetration of the mind into the ultimate construction of reality. The first century Pythagorean Nicomachus said that "number is the true the good, the pursuit of knowing order and harmony."[4] The Greek conception of cosmos as opposed to chaos presupposes an ordered universe be revealed in its cosmology. Pure number was thus the logic of the law inherent in that ordered universe. One major goal of the ancient disciplines of art, science, religion and philosophy was to find the unity of existence revealed in both the senses and the intellect. This was a form which reflected both outer and inner reality. The resolution of this dichotomy determines a movement not just backward to the fundamental elements of shape and number but also inward to the locus of consciousness itself.

Plato wrote his dialogue "Timaeus" at the end of his life and it subsequently became a popular creation myth of the ancient world. The main speaker of the book, Timaeus, was actually a distinguished Pythagorean well known in Plato's time. The dialogue begins with the story of Atlantis, the floods and earthquakes which destroyed it, and of the transmission of ancient Atlantian knowledge through Egypt. The stories Timaeus tells can be seen as expressing the battle between order and chaos. The Pythagorean knowledge as communicated by Plato's Timaeus is a description in which ordered space itself is created from the chaos of the everlasting by the introduction of the geometric ordering principle of number. Plato says in Timaeus "all kinds of things thus established receive their shape from ordering one, through the action of idea and pure number". Plato conceived of this, "ordering one" as the interrelations of a harmonious cosmos based on the preexisting categories of space and number. The Neoplatonic view of the value of art conceived of the artist as creating his work according to a preexisting system of natural order. A composition ruled by the structure of space realized in the inherent ratios of basic natural forms.[5]

This concept of objective beauty, that something can have admirable qualities of composition and proportion, seems at first to be based on a type of intellectual mediation. While on the other hand the conception of subjective beauty at first seems to be the opposite, as the enjoyment of the immediacy in beholding that which is pleasurable and good. Their approaches seem different, one advocating a cultivation of the eye just as the mind must be cultivated to think logically about difficult subjects; the other advocating that one must not come to art with logical ideas but be opened to the pleasure and freedom that it creates. These two oscillate between one and another as do matter and spirit or form and content, sometimes emphasizing one aspect, sometimes the other. Yet one of art's most distinctive attributes, and the one which was of supreme importance to the ancients, was its ability to express the most profound ideas in a purely sensuous form thereby bringing them into the human sphere of feeling. Art then functions as a reconciliation between that which is external and that which is internal, opening out between the mediation of the visual cortex and in the case of visual art, the retinal immediacy of the eye itself. This ancient dialogue has not ceased in modern times, it has appropriated new forms of expression based on the work of past artists and philosophers. The knowledge of the ancient Plato-Pythagorean and Islamic traditions speak to us about the ultimate questions of perception and consciousness in a compelling way where the inner and outer worlds become two aspects of the same thing.

Pythagoras originally migrated from Leonia to West Magna Graecia and became an exponent of Ionian enlightenment. Later he studied with Thales, one of the seven wise men of Miletos. This was thought to be the original home of Greek philosophy in Asia Minor opposite Samos. It is told that Thales traveled to Egypt where he gained knowledge of the carefully guarded secrets of the Egyptian priesthood which had been handed down through generations for thousands of years. Yet it is not known how much Greek geometry actually came from Egypt.[6] Pythagoras later founded the religious order of the Brotherhood of Purity in Croton. Because of the lack of tolerance of the day and the antagonism brought on by their own overt secrecy, their religion was continually persecuted for their esoteric beliefs. Through religious devotion reinforced by mathematical study, its followers tried to conform their lives to the alternating rhythms of nature in action, thought and being. Their desire to find shelter in the order and peace of mathematical and philosophical discipline can be seen to be both a social experiment as well as an anti-social reaction to the lack of tolerance and pluralism of the ancient world. Pythagoras wrote almost nothing for the public at large for fear that his sacred vision might be tainted by the profanity of an uncaring mob.[7]

According to Diogenes and Laertius, Pythagoras did write three treaties which were kept secret until the time of Philolaos who sold them to Plato for 100 minos and which became the basis for the text "Timaeus". In the "Timaeus" Plato writes "God discovered and bestowed sight upon man in order that we might observe the orbits of nature and make use of them for the revolution and turning of the mind; and that learning them and acquiring natural truth of reason we might extend the divine movement and bring into order within us that which is astray. Having been bestowed by the muses on Him who with reason seeks their help, not for senseless pleasure but as an ally against the discord of our souls and our times".[8] The Brotherhood of Purity's cloistered devotion combined with an intense mistrust of the uninitiated proved to be too much for the people of the time. The Brotherhood of Purity was supposedly attacked by an angry mob who set fires to their temples killing many disciples. The religion never totally recovered from this catastrophe. It did continue in some form for 300 years after.

CHAPTER II

FOOTNOTES

1. Beekmann, P., The History of Pi, p. 96.
2. Kuller, H., Epitome of Astronomy, p. 180.
3. Vanderbroeck, Andre, Philosophical Geometry, pp. 74-88.
4. Heath, T., A History of Greek Mathematics, p. 204.
5. Cornford, F., Plato's Timaeus, p. 14.
6. Heath, T., A History of Greek Mathematics, p. 94.
7. Brunes, T., The Secrets of Ancient Geometry and Its Use, pp. 109-117.
8. Cornford, F., Plato's Timaeus, p. 40.

CHAPTER III THE INFORMING OF THE TRIANGLE

One of Pythagoras' greatest achievements was the quantitative analysis of musical intervals. He found that the basic ratios of the lengths of a vibrating string corresponded to chords that were pleasurable to hear. The fascination of this discovery was a major foundation of Pythagorean philosophy. Encouraged by such discoveries Pythagoras concluded that the principle underlying existence and beauty was based on mathematical laws and ratios. This inclusion of philosophical questioning with mathematics shows how the idea of rhythmic progression and ratio was universally applied by the Pythagoreans in an attempt to reveal the hidden relationships of the universe. This ancient concept of ratio is wider in dramatic scope than our purely arthimetic concept. Ratio was perceived of as a functional relationship of values between different objects of knowledge. Thus the conception of ratio was one of judgment as well as measure. Proportion was then introduced into this comparison of values as an ordering principle, an invariant criteria which was transmitted from one ratio to another. Plato says in the Timaeus, "it is not possible to combine satisfactorily two things without a third. We must have between them a correlative link".[1] This basic triadic relationship permeates much of ancient thought in the form of a thesis, an antithesis and a synthesis of elements. These three fundamental tendencies of the universe derive from the mythic archetypes; descent from unity, subsequent expansion, final assent and return to the one.[2] They parallel the spontaneous invention of structure believed to be implicit in the first three dimensions of space. As Joseph Campbell has written "the generative splitting of the one passes to the manifold of earthly existence through the transforming medium of space and form. This is the frame of space and time. The lure moving the self born absolute to the act of its own creation".[3] In Aristotle's theory of dramatic unity any one action of process is conceived of as a whole onto itself. He says, "the beginning implies no necessary antecedent but that which is itself the natural antecedent of that which will come".[4] Indivisible and fundamental, making no additions, reality makes itself manifest by applying the "system" to itself.

This image of a watertight dramatic process has captured the soul of much art, science and philosophy. As Johannes Kepler wrote in his "Mysterium Cosmographicum", "Great thinkers, scientists and artists start with a deep contemplation of their subject matter, developing higher powers of concentration and observation. They gain the ability to marshal their "power" towards a particular goal. Eventually this flow of effort results in the illumination of the essential structure".[5] Aristotle continued, "the middle needs no other elements beyond itself to propel itself forward and the end must naturally follow from the elements upon which it rests".[6] This is an interesting image of the desire for an internally consistent structure, self-contained and self-propelling which the artist strives for. There is here a compelling formulation, perhaps unattainable, that drove past civilizations to create rigorous art forms based on the regenerative principles of the universe externalized in space and number.

In the 1900's the scholar of Islamic geometry, S.H. Nasr, commented upon that culture's interest in geometry saying "through the revelation of geometric form one sees the different sacred traditions which are inwardly united to the center, which transcends all cultural form. They are a bridge from unity to multiplicity, they are the correlative link, a sacred rediscovery of the creation of order itself".[7] The question now becomes what transcultural form can possibly hold this procession of historical thought. What image can serve as a link between thought and form, between old and new. Self-contained and self-caused, closing onto itself out into infinity. Expressed in a phrase Einstein used to refer to the underlying structure of space/time; finite yet unbounding. For years the major vision quest of artists in divergent cultures has been to produce an image which revealed the structure, harmony and regenerative properties of the universe.

Length, breath, expanse and volume all can become a metaphor for the materialization and volumizing of spirit and the creation of external form. In the ancient creation myths of Egypt communicated by Heliopolis, the cosmic ocean, the one having no form. The formless form ... gives itself over to all form. This act is performed by the Goddess Nun, who exists prior to all existence as pure undifferentiated spirit/ space. Nun is pure potentiality, the seed-will of God implicit in all form. In Nun undifferentiated space is compelled to contract and to define itself, bounded by itself as pure volume. This process evolves Atum, the creative exernalization of the internal spirit of Nun. Atum is distinguished and so creates himself from the undefinable Nun by the volumizing of space from the first dimension to the second and from the second to the third.[8]

In ancient Greece, the five essential volumetric forms (all of which individually have equal angles and edges) bore Plato's name. These "platonic solids" were frequently employed by Plato in the "Tiameus" to explain natural phenomena. In the many dialogues of "Timaeus", Plato outlined a cosmology based on planar and solid geometries. Plato also related the five solids with the four elements, earth, air, fire and water and the fifth solid which was a type of ether or prama out of which the other four are generated. Much later, Kepler attempted to show why there are specific number of planets, and what causes their planetary intervals, by application of platonic solids to astronomy. He wrote, "so there existed only five perfect platonic solids and five intervals between the six planets. It is I believe, not by chance that this is so, but by divine arrangement. For the planets have been spaced in such a manner that the solids can be exactly fitted into the intervals, as an invisible frame".[9] This mystery of the universe was thought to be solved by young Kepler, teacher at Rotestant School in Gratz.

This model did not comply with the experimental observations of the time and to Kepler's credit as an astronomical experimenter, he modified and ultimately dismissed this notion as being untenable replacing it with one that described the true elliptical orbit of planets. Yet through it we can see the great inexorable pull, and perhaps the folly, of attributing a pre-conceived geometrical structure to nature. Nature does perhaps have a mathematical structure, but she often gives up her secrets exceedingly slowly. The poetic archetype of the mathematical arrangement of nature remains unchanged while the actual theories come and go. The science of space and the traditions of ancient art began through a use of the most fundamental elements of visual literacy all based on the archetypes of point, line, plane and solid.

Pythagoras has written that the Monad, the one, is represented as one point. The one which is not number but the principle of number, the point which has no magnitude but is itself the principle of magnitude. The Monad in relation to the other gains substance. Two points have no substance within themselves, yet by their relation and projection into each other they define a line which does. The Monad is the principle of all things. It is the unity, the first cause, the active intellect. The volumizing of one dimension out of another is part of the traditional interpretation of the four fold polarization of the universe; hot-cold, moist and dry, fire-water, earth and sky. The repetition of the one within itself is the diad, and therefore the multiplicity of the world around us. The triad is the propelling principle between the one and the two and thus from one and the two the three is formed and from the first and second dimensions the third is created. Thus the triad is the extended theme of the logic of cosmos, from point springs line, from line extends plane, from plane is realized solid and from solid is born the world and the body.

This concept of space relates to the notion that context and creativity is the place in which things happen. Space, not as a thing, but as the space of things. Entering the space of an object we become aware of its psychic pull.

Space so defined, as context, is form giving. This is similar to the extraordinary psychological space in which one summons the freedom to create an image of self and express that image into the context of form. Nietzsche speaks of an irrational surge of self-created free will, which when inverted is propelled into the rational world of form.

Number was viewed in the past as the ultimate construction of reality. It was thought that number existed independent of space as pure form,independent and non-physical an entity in the realm of pure logic. It can be said that this pure crystalline view of number as the actualizing of spirit searched for its own expression in the dreams and aspirations of artists for thousands of years. These artists often felt that the rudiments of geometry and the theories of proportion held the key to the expression of the universe.

The Islamic philosopher Scharwardi describes his view of the transmission of the universal wisdom through Greek and Persian lines. He says that the Greek line proceeds from Thales, Pythagoras, Plato and Nicomachus. The Persian line from Al-Misri, Dhulnun and the Persian priest kings. The connection of the two traditions is called "Ishraq" by Moslem historians and was supposedly initiated by the followers of Seth, who according to Islamic sources, were the original founders of the craft guilds in Islam.[10] Al-Misri, the Islamic teacher of sacred goemetry wrote, "the point is the moment of emergence. From this focus of conscious awareness the point externalizes itself, becoming two extremes and the relation between them. The center is passive, the projection is active and so the arc is initiated and closes on itself. The center is the perfect pivot, the timeless dimensionless zero point encompassing all space. The outer circumference is the boundary and the mean is the field or domain. The circle departs from itself and nested in space it becomes the "in-forming" of the triangle, the flowering of the perfect circle into the structure of space. The triangle inverted and overlayed on itself becomes the six-pointed star. The "sixth" is the interception of the three triadic spheres expressing the days of creation in both the Bible and the Koran. The seventh intersection is the sublated moment perserved and exalted, the day of rest and the day of worship".[11]

CHAPTER III

FOOTNOTES

1. Cornford, F., Plato's Timaeus, p. 56.
2. Campbell, J., The Hero with a Thousand Faces, p. 69.
3. Ibid., p. 79.
4. Cassirer, E., The Philosophy of Symbolic Form, pp. 194-196.
5. Taylor, H., Ancient Ideals, p. 134.
6. Cassirer, E., The Philosophy of Symbolic Form, pp. 194-196.
7. Nasr, S., An Introduction to Islamic Cosmological Doctrines, p. 86.
8. Critchlow, K., Order in Space, p. 64.
9. Hilbert, D., Geometry and the Imagination, p. 105.
10. Rescher, N., Studies in Arabic Philosophy, p. 76.
11. Sirag, A., The Book of Certainty, p. 165.

CHAPTER IV ANCIENT FUNDAMENTALS OF SPACE, AN ABSTRACTION OF ZERO

Unlike Euclidean and modern non-Euclidean geometries, the starting point of ancient geometries was not a group of formal postulates. Instead they were built on a general conception of the order of the cosmos as symbolically visualized in the images of unity and multiplicity. Geometry was an attempt to make manifest the pure order of the oneness of space. The numeral "one" was considered the origin of "two" and the source of all natural geometries.

All known cultures had a conception of nothing, but the use of the number zero was a relatively modern occurrence dating from 700 A.D. It first appeared in India paralleling the Buddhist increased emphasis on the imperishable void of "Sunya" as being generative of the universe. The ancient concept of the one, as unity and source, was upset when zero became the first numeral.[1] Many historians believed that before this, in the Babylonian and Mayan civilizations, a notational procedure was used to denote an empty column, but it is in India that the zero was first treated as a number. Between the eighth and tenth centuries, the Indian concept of zero was passed to Arabia and was integrated into the arabic numerals we use today. Between the eleventh and twelveth century the Arab mathematician, Algorisma (whose name is the basis for modern term Algorithm) popularized the Arabic number system in Spain and initiated its introduction into western scientific thought.[2] By the fifteenth and sixteenth century the intervention of the zero in Renaissance mathematics, with its ability to facilitate multiplication and division, soon allowed a complex numerical logic to be developed. The zero permitted numbers to represent ideas which have no apparent physical form. Today the zero is an indispensable part of the mathematics on which we build our science and technology. The zero has become a crucial part of our theologies and philosophies of self and the environment around us.

The expert on Islamic geometry and architecture, Seyyed Hossien Nasr commented that, "we must appreciate the profound esoteric way in which what appears to the initiated as merely a decorative two-dimensional pattern, is in fact, the very law of space in the manifest realm. Both the contemplation and creative skill in making these patterns lead to an appreciation of the everlasting perfection of nature. Moreover, it is our very perception of these forms which activate the space of their existence, and so reveal ourself within them".[3] Islamic concentration on geometric space, dimension and symmetry is partly meant to draw attention away from the representational world to one of pure form and the pure action of the creative spirit. Many of these ancient art forms which express the volumizing of space, addressed themselves to a basic duality of spirit and form. They say in part that it is only by moving past the appearance of the material, to the ideal and spiritual, can the truth of world and our being become perceivable. It was thought that in order to avoid making graven images, art objects needed to turn away from complete material preoccupation with representational idolic form, to one of pure spiritual significance. This in turn allowed one to see that which was not immediately apparent on the surface, but rather that which was hidden within. It was a conscious decision to express spirit, not in its evolved social sphere, but in its fundamental creation of space.

The Roman mathematician, Vitruvius, who was both a philosopher and a writer on architecture defined dynamic symmetry as follows, "symmetry resides in the correlation by measurement between the various elements of a plan and between each of these elements and the whole, achieving a total consonance of form". The renissance masters who rewrote Timaeus' ten books of architecture over and over again, were intrigued by the fact that he found universal laws of proportion in the human form. Vitruvius decreed that these symmetries and proportions be employed as a measure for all perfect structures, especially for the temples of the Gods".[4]

The Italian mathematician Luca Pacioli in his "Divina Proportione" of 1509 wrote, "from the human form all measures derive and in it is to be found all and every ratio and proportion by which God reveals the innermost secrets of nature".[5] Vitruvius has written "symmetry results from proportion. Proportion is the interrelating of various constituent elements with the whole".[6] Andrea Palladio, an architect of the Italian renaissance once wrote "beauty will result from form and correspondence of the whole with respect to the several parts, of the parts with regard to each other, and of these again to the whole. Thus the structure may appear as an entire and a complete body wherein each member agrees with the others".[7]

This concept of the interrelation of parts and whole is easily related to the structural elements of any visually unstable image such as the hypercrystal. Its visual movement allows ones attention to dwell on the build up of small patterns. These are then reflected in the general overall structure. Moreover the build up of these structural elements can be based on the basic ratios and proportions which are utilized by nature.

CHAPTER IV

FOOTNOTES

1. Boyer, C., "Zero: the concept, the number", National Mathemetical Magazine, Jan. 1973, p. 28.

2. Ibid.

3. Nasr, S., An Introduction to Islamic Cosmological Doctrines, p. 107.

4. Ghyka, M., Geometrical Composition and Design, p. 17.

5. Scholfield, P., The Theory of Proportion in Architecture, p. 81.

6. Ibid, p. 88.

7. Palladio, A., The Four Books on Architecture, p. 72.

CHAPTER V NATURALLY OCCURRING PATTERNS

Pythagoras' analysis of natural dynamic symmetry began with the conception of musical harmony. The intervals between notes were measured by the length of the strings of the Lyna. This is of course different, but proportional, to the actual cycles per second of the frequencies they produced. They found that pleasing chords were produced by a system of proportions in which the ratios are compatible. The first harmonic fundamental of a string is its simplest mode of harmonic or regular motion. Its first overtone is the next frequency above the fundamental that is resonant with the string. Two notes often sound well together when their frequencies are in ratios of whole numbers such as the unison (one to one), the octave (one to two) or the fifth (two to three). The harmonic series is an additive progression of frequency ratios, such as one to two, three to two, four to three, and five to four.

Pythagoras evolved out of the concept of harmony and aesthetic proportion in time, that of a system of aesthetics in space. Thus the Greeks, through a generalized concept of repeated proportions, introduced the notion of symmetry as a recurrent phenomena in all structures composed of harmoniously ordered ratios. Ratio here is to be considered as the analogy of comparison; proportion as the permanent quality which is transmitted from one similar ratio to another. These result in a symmetry unlike the modern concept of equal distribution, but instead in a notion in which symmetry is to space as rhythm is to time. This system of symmetry is a type of pure pattern which is applied to the forms of nature. One of the most harmonious combinations of notes is the major triad with its four to five to six ratio. An explanation for this curious observation was advanced by H.L.F. von Helmolts who lived from 1821 to 1894. He believed that the presence and absence of positive and negative interference beats between the overtones of certain chords determined the pleasing quality of their sound. According to Helmolts "a stream of sound impulses from two or more notes produces consonents and aesthetic pleasure if the sound is free of beats. A sound emitted by notes separately by only a semitone is dissident, such an interval being rich in beats between interfering harmonies".[1]

It was thought that if the visual cortex of the brain is effected in an analogous manner, the musical intervals which give the most satisfying consonance and which produce no negative interference (such as the unison, octave and the major sixth). Then they could be effectively used in spatial art forms. These ratios convert visually into the square (one to one, as unison) the double square (one to two, as the octave) and a special rectangle divided by a ratio used in nature, the golden rectangle or the "major spatial six". Francis Warrain wrote "rhythm is a succession of phenomena which are produced by intervals, either constant or variable, which are regulated by a specific law. This notion of periocity and proportion can be used for succession in space as well as time. It is also apparent that in space combinations of proportion can cause the periodical reappearance of shape".[2]

In Huntley's book, "The Divine Proportion", he says, "the experience of glimpsing beauty in mathematics is as difficult to interpret to oneself, as it is to communicate. It must be caught not taught. The aesthetic joy of numerical relating is mediated through the intellect but originates from the lowest stratas of the mind".[3] Thus it was thought that there is a constrained structure of equilibrium utilized by nature from which the mind receives pleasure upon taking it in. Proportion and the triadic form is linked to this symmetry of design between its parts and the whole. The modern concept of rhythm is a notion of time, yet the Greek concept of eurhythmy was more broadly applied. Plato in particular wrote much about this concept of eurhythmy as a general structure dominating all aesthetics in which rhythm, symmetry and number are one.[4]

There are of course some deficiences in any theory which attempts to correlate space and spatial art with time and such things as music only in terms of a one dimensional mathematical congruence. It is perhaps true that space and time are intimately connected and that all of consciousness is a weaving of both space and time. It can thus be seen that any expression of that consciousness can emphasize either space or time. Thus music can be seen in the listener's mind as visual events or painting can be felt in terms of a temporal flow. However, time and music are more than just intervals, they also have the quality of a dense fabric of tensions and resolutions. Ultimately we are time, it is our being. This pure temporality is part of our own deepest consciousness. Our experience of the unfolding of time and music is an inward realization of the outward structuring of events. Many mathematical conversions from music to painting disallow an intuitive grasp of this inward dynamic of our experience of time's passage. Art can build its realities and its illusions in this experience of time and space. However to reduce this original analysis of space time to merely the mathematics of a wave form, or the spatial frequency of an image, disallows the intuitive notion that time and music are a poetic weaving of our memories. And yet perhaps it is not the mathematical correlation of space and time which weakens our understanding of this poetic "space of time", but merely the literal and sequential image of mathematics itself which is the problem.

One basic proportion called the golden or divine proportion is utilized by nature in such diverse things as beehives, seashells, plants and the human body. It was allegedly used by the Greeks and renaissance culture as a basic motif for art and architecture. One of the attempts of the late nineteenth century and twentieth century scholars (Zeising, Brickoff, Hambritge and Ghyka) was to discover recurring patterns by which Egyptian, Greek and medieval buildings were constructed thereby extending the principles of the renaissance back into history. However much of the employment of the golden proportion by the ancients appears to be marked with conjecture. There is little concrete evidence of its conscious use before popularization by such Greeks as Pythagoras, Plato, Euclid, and Latertius. This proportion represents the rhythm or symmetry that exists between the parts and the whole such that the whole is to the largest as the largest is to the smallest of any three triadic terms. This is one of the simplest proportions and the one most utilized by nature.

The smallest number of terms needed to express a proportion is the triad. Yet in order to obtain greater simplicity one can reduce this by making the third element (the whole) equal to the other two (the parts). This divine proportion does, in fact, operate in the laws of Phyllotaxis which governs the arrangement of leaves around the stem. Similar spiral patterns occur in the florets of the sunflower, the daisy, in the pine cone and the surface patterning of a pineapple. As in the geometry of platonic solids and the helix, these interlocking patterns embody incremental relations in the Fibonacci series. This series 1,1,2,3,5,8,13,21 ..., in which each number is the sum of the two preceding numbers, very quickly develops a divine proportion 1.6180 between successive terms.[5] Using this formulation of nature as the basis for artistic expression, the artistic structure functions not just as a stimulus to the eye, but also as a complex categorizing phenomena accessible to the mind. Thus when discernable features of the spatial environment is used as a basis for going beyond itself, that structure itself becomes a symbol of what is beyond, of that which no person can fully apprehend or comprehend.

H. Poincare in "Science and Method" wrote, "it may seem surprising that aesthetic sensibility should be introduced in connection with mathematics which it would seem can only interest the intellect. This impression is not the full truth. We must bear in mind sensuous feeling of mathematical beauty. For the harmony of numerical structuring, which reveals the physical forces of space and the geometric elegance of a proof are only in need of an appropriate vehicle to propel them into the forefront of human artistic expression".[6] Given the existence of three quantities or elements in a basic triadic proportion, there is given the first difference between elements A and B, a second difference between elements B and C, and the relations of these to each other. The relationship of AB to BC can be seen in terms of a larger triadic form. This is expressed as A is to C, A is to B, or A is to itself. We have then the difference of A and B as to the difference of B and C as A is to A or A is to B or A is to C. These three relationships in turn create three different series. The arithmetic series has a additive or subtractive progression (1,2,3, ...). The geometric series which is multiplative or divisional (4,8,16 ...) and the harmonic series which is a combination of both the additive and geometric series. The harmonic progression (A:B = B:C) takes its two extremes A and C finds its product and then divides this by the arithmetic average of the two. These relationships are employed by nature and have been used in the construction of a series of original art works by the author. These proportions have been applied to the successive buildup of dimensions in hyperspace. It is the goal of these works to be constructed in a way that each section is related to the whole as the whole is related to some of its parts.

CHAPTER V

FOOTNOTES

1. Huff, W., Symmetry Offprints, Vol.6, p. 13.
2. Atkins, K., Symmetry and Conservation Laws in Physics, pp. 76-81.
3. Huntley, H., The Divine Proportion, p. 64.
4. Cornford, F., Plato's Timaeus, p. 59.
5. Tyng, A., Simultaneous Randomness and Order, p. 28.
6. Rosenblum, R., Cubisum and Twentieth Century Art, pp. 37.

CHAPTER VI DIMENSIONAL SYMMETRY

The experience of symmetry is a common occurrence in every day life appearing in three dimensional objects, natural patterns and man made designs. The distinctive quality of the symmetrical object is its ability to be divided into two or more equal parts. Thus through rotation, reflection and translation one part of the pattern can be brought into congruence with another. The process of growth both in inanimate crystals and living matter shows a diverse application of the principles of symmetry. The world is pervaded with symmetrical shapes which we come into contact with everyday. Thus to some degree symmetry, or the conscious avoiding of symmetry, has played a predominant role in creation of much art. From the simplest handmade pots to Gothic cathedrals there has been an abiding interest in the balance of equilibrium reflected in symmetrical patterns. In Arabic design, symmetry has been explored by artists forbidden by their religion to express human, animal and worldly forms. In many other cultures' painting and sculpture, perfect symmetries are more often avoided than pursued. They are often replaced with the sense of balance and equilibrium of pictorial forces. Imperfections of symmetry in deference to the gods are known to have existed with oriental rug makers, among American Indian weavers and the artisans of many other cultures. Greek

Goddesses were often characterized as being envious of mortals who possessed too great a beauty, perhaps as a warning against pride and vanity.[1] Throughout history this position in aethetics has been asserted first and foremost as a cosmological principle alluding to man's fall from grace and secondarily as a visual source of dynamic tension.[2] Cultures with a highly developed appreciation of asymmetrical tension explored it in both the spatial and temporal aspects of visual art. This of course leads to subtler levels of visual tension and equilibrium not immediately apparent to the uninitiated eye. In the past, many,often dismissed these images as mere decorative repetition, when in fact they were an attempt to give sensuous form to the laws which govern the structure of space itself. Consequently it can be seen that mathematical symmetry is of interest to both the artist and the scientist. It underlies much scientific thought, playing an important role in chemistry, physics, crystalography as well as architecture, graphics and sculpture design.

There are three primary forms of symmetry; mirror reflection, rotation and translation (or rotation at infinity). Mirror reflection creates pairs of right and left. The bilateral symmetries of right and left are of the simplest forms of balance and equilibrium. The extended image of this bilateral equilibrium of forces is typified by a fulcrum and scale in which both factors of distance and weight determine the center of gravity and so the point of balance.

The mathematical philosophy of attributing right and left handedness to phenomena can be seen as a reflection of the primordial dichotomy eliciting from the structure of both space and language. God and Christ, when depicted as everlasting symbols are often shown in a frontal bilateral view and not in profile. When iconic images are in profile, this often has a bilateral significance such as Michaelangelo's creation of Adam in the Sistine Chapel. In this work God's right hand touches Adam's left reflecting unity of seeing one's own arms outstretched.[3] Many ancient cultures believed man to be created in the image of God and as such saw the universe replicated in themselves. For these cultures the repetition of symmetries in nature and the human body intermingled with the cultures' aesthetic values and artistic products. Thus the cultures' artifacts were often viewed as being both in harmony with and a model of the universe revealing its innermost structure.

The symmetrical volumizing of a fertilized egg and the unfolding of a germinated seed have long been pre-conscious images of the dualistic split of unity into the multiplicity of material existence. In the reiteration of the myth told by Aristophanes in Plato's "Symposium" it says, "original man was spherical and in perfect harmony, a model of his universe ... his back and sides forming a circle and he had four hands and four feet, one head with two faces looking opposite ways. He could walk upright or he could roll over and over at great pace turning on his four hands and four feet, eight in all like tumblers going over and over with their legs in the air. Terrible was the might and the strength of these men ... and then they made an attack on the Gods ... and the Gods then suffered their insolence. At last Zeus discovered a way to repay man for his insolence, he said "methinks I have a plan which will humble their pride and improve their manners. Man shall continue to exist but I shall cut them in two and they shall be diminished in strength and will increase in number, this will have the advantage of making them more profitable to us. They shall walk upright and if they continue insolent and will not be quiet I shall split them again and they shall hop about on a single leg".[4] Creation myths infused with the primordial split by which unity volumizes into multiplicity are often the basis for much art pertaining to the structure of space and geometry.

Mirror symmetries also exist in the conservation laws of physics. This is initially revealed by the fact that if a clock is constructed totally backwards it would still function. Conservation of momentum exists in mirror frames of reference.

Particles and anti-particles are mirror images of each other analogous to right and left-handed screws moving through time. The basic laws of physics are also symmetrical under the operations of translation given a homogeneous universe. This can be seen in the symmetries of the general theory of relativity. Basic physical laws hold if a system is rotated through a fixed angle this leads to the laws of the conservation of angular momentum. Through these theories one can see that the symmetrical breakdown of force and pattern is a powerful form of analysis. It allows one to find structures which organize space in such a dynamic way that they are repeatedly utilized by nature. Moreover these organizational devices of space can be expanded revealing minimum surface structures which exists in multidimensional space.

Rotational symmetry is the movement through an angle around a pivotal point or rotocenter. A 180 degree turn creates a two-fold rotocenter, and is the simplest form two dimensional rotation revealed in such things as letters, playing cards and propellers. Translation is the repetition of an image along a line and can be seen as rotation with the rotocenter at infinity. Mirror reflection and translational repetition can combine to create a type of sliding projection called glide symmetry. The right and left sides are then displaced by a half turn interval creating a type of sine wave.

This can be extended to a combination of rotation and translation to form the twist, torgue and helix forms of nature. It can be seen that nature has a complicated mathematical method of its own which artistically explores the mathematics of space and through which minimal surface structures in polydimensions (hyper-dimensional geodetic domes) can be effectively visualized. This in turn leads to geometric sculpture based on the design control which spacetime exerts on the forces of nature. This network of relationships is a sensuous form (versus its purely mathematical description), and allows for an art which embodies the underlining principles of space,concretely and explicitly.

Many qualities of these principles of space can be inferred through analogy. This approach of acquiring "hyper-visualization" by contemplating the relationship between the first three dimensions and extrapolating them into poly-dimensional space has often been a powerful form of analysis. This approach was freely used by Edwin Abbott in his book "Flatland". However this rather intuitive process has led some people to attribute mystical qualities to the fourth dimension such as the ability of spirits to walk through walls. Other qualities of multi-dimensional space can be deduced by modifying the five basic postulates of Euclidean geometry. The extrapolation through analogy goes something like this, a point is moved and so traces out a line segment. This segment is then translated into a perpendicular direction of equal length and so traces out a square plane. This plane can be projected perpendicular to its axis so as to produce a cubic volume. And the cube in turn can be projected to a cube of the next dimension, the hypercube. The general procedure for projecting a figure in multi dimensional space can also be described as a form of mirror reflection. Given a space X and the opposing space Y the spanning space between them XY is implied. Moreover we may project from X to Y by drawing through each point or verticy of X into its image point in Y through this newly constructed space XY. This type of orthogonal projection can be extended on infinitely. As Buckminster Fuller has said, "it appears as though going into infinity in an XY coordinate system perpendicular to sub-divisions of that coordinate system reveals the fact that nature is not doing what she is doing radially or convergently but expanding multi-dimensionally".[5]

The mapping of nets is another form of translation from one to another dimension. A cube can be unfolded into two dimensional space to form a kind of cross consisting of a square with four "side squares" attached to each of its sides (which will become the bottom and four walls of the cube) and a sixth side (the top) beyond one of the four "side squares".

The cube is folded so that the two extreme ends come into contact. This is possible because the square can turn through three dimensional space. When a hyper-hexahedron (hypercube) is unfolded into a three dimensional space, it creates a net which looks like a double cross containing a central cube with six others attached to one of each of its sides, and an eighth connected beyond these. In a three dimensional space the highest and lowest cubes are separated but in four dimensional space, where extra-dimensional rotation can take place, the two upper and lower cubes can be folded into contact. The column of cubes is now converted into a type of ring. This ring has within it a second reciprocal ring which interlocks with the first. In the same way that the two two-dimensional squares can be projected through perpendicular space and then connected at their vertices to form a three dimensional cube. Two three-dimensional cubes can also be projected and then connected at their vertices creating six new cubes between them, one corresponding to each surface projection. This combined with the two original cube projections makes eight cube facets in the four dimensional polycube, just as the three dimensional cube has six faces pointing in three different directions. Each of the eight cubes in the four dimensional polycubes is made up of six faces however there are not forty-eight separate faces in the hypercube. This is because many faces are shared in the same way that the edges are shared by different faces in any three-dimensional structure. This sharing of planes by the cube aggregates of hyperspace, projected into three-dimensional space, causes a visual ambiquity. This is like the inversion of a necker cube in which one cannot tell if one is looking down into or up through the cube. It has been suggested that the visual ambiquity caused by the sharing of planes in contradictory perpendicular cubes of hyperspace is the vacillation of hyperspace itself as it "pokes" through our limited three-dimensional perception. It should be pointed out that in this context, time is not to be considered the fourth dimension. The fourth dimension is a purely abstract spatial reality and as such is anything intercepting the third dimension as a series of points. This can be related to time's passage as it is in Einstein's general theory of relativity in which time fluctuates with space in its three-dimensional warpage as gravity. However the actual non-euclidean projections of space utilized by the author were primarily spatial in nature.

In many books like Hinton's "Era of a New Thought" and Ouspensky's "Tertium Organum", there is a direct philosophical relation between dimension and consciousness.[6] These books also supply a visual and intellectual motivation governing the construction of polydimensional space. In these works the triad of sensation, perception and conception are directly related to the first, second and third dimension. This correlation is due to the idea that dimension is not an empirically existing fact, but is a categorizing phenomena of the observer. This means that a part of our experience of binocular fusion and perspective is the mind's cohering of a thing's attributes together so they become one thing. The back, sides and front of an object are not lost when out of view but incorporated into a conception of the thing as a three dimensional object. It can also be said that we are doing this through time thereby cohering the moments of the thing into "one thing" which is itself thought to change through time. If this ability to cohere cluster attributes is part of the third dimension, then what would it be like if an observer had a less developed categorizing ability, and so a poorer sense of spatial coherence. It is true that if this observer were perceived by a being which could cohere the world three-dimensionally the observer in that frame of reference, would seem three dimensional. The contention is, the alien observer would not be able to hold the visual images (front, back and sides) as an unseen part of the object as we do. To this observer we might say that the world is all perception. As the angles of an object would change so too it would seem that the object was changing. To turn something would be to expand or to contract it; it's size actually seeming to change.

This movement itself would be a type of inadequate spatial awareness of phenomena. But now motion, (which is time to us) would be the third dimension for this two dimensionally coherent consciousness. All of the dimensions would have moved back, as it were, with time or motion taking on the role of the mind's inability to cohere space. To the one dimensionally coherent consciousness the world would be a line of pure sensation. Almost nothing would be cohered, everything not immediately experienced as a yes/no, pleasure/pain reality, would be "time-like". One might say then that a tissue or vegetable is a form of one dimensional consciousness in which all "two dimensional" events are reduced to then and now reactions of stimuli. Yet for us the very abstraction of this "now" is itself a temporal cohering. Thus the first, second and third dimensions are not static discreet perpendicular directions, but a continuum of partial dimensions in which time is more or less cohered.

From the concept of partial dimension involved in the conscious cohering of event and process, one comes to the notion of partial dimensions interacting so as to produce diminishing patterns in nature. Much of this type of analysis was done by Benoit Mandelbrot in his book on fractional or fractal sets. The first person responsible for the concept of partial dimensions and an irregular continuous functions was the mathematician Weierstrass who wrote about it in 1875. Later in 1883 the cantor set was used to describe cascade analysis as the formation of point events in a linear progression, later this was seen as an intersection of points and lines (the zero and first dimensions).[7] In 1919 the physicist Jean Perrins described Browning motion as an interference of fragmented dimensions in space. In Euclidean space sets in which topological dimensions coincide there are always real numbers (zero, one, two, three ...). In fractal theory every set of points or events is assigned a number and cohered as a dimensional set. This leads to the concept of a discordant set in a fractional dimension. In using a "triadic cantor set" to analyze error and static, the event of error free transmission and static is seen as a series of points existing on a line of time. The ability to see error is a function of the magnitude of possible observation. Rough analysis will see longer periods of error free transmission. As one discriminates and so coheres more information, more errors appear. At one level (called rank "zero") we have a specific error to non-error pattern. If we increase the accuracy of our analysis we might find what we had thought was error free was made up of many errors. At a certain scale of enlargement or reduction we will pass a theoretical threshold, like that between biology and chemistry or chemistry and physics, and one will then find themselves no longer analyzing the same type of phenomena. If we think of a closed interval from the smallest level of sub-division to the largest amount of cohering of any one phenomena (from point to line) we would see a pattern begin to form. This pattern would be indicative of the specific type of phenomena we are examining, whether it be the shape of an organic or inorganic form or the structure of an event. In the example of the triadic cantor set the spacing of any of the cross sections of this "pattern" entails the removing of the center element and the leaving of the two side elements. This relationship of event to baseline quickly resolves to a general form as one approaches infinite sub-divisions. This geometric series between baseline and infinite sub-divisions theoretically represents the intersection of two partial dimensions. All phenomena in theory has this type of "gamut of coherent". Thus these partial dimensions are derived through the intersection of any two dimensional events and the objects we use to measure them (the lowest level of sub-division).[8]

CHAPTER VI

FOOTNOTES

1. Frey, D., "On the Problem of Symmetry in Art", pp. 276-279.

2. Ouspensky, P., Tertium Organum, p. 14.

3. Hilbert, D., Geometry and the Imagination, p. 52.

4. Jowett, B., Plato's Symposium, Vol. I, Section 190, pp. 316-317.

5. Fuller, B., Comprehensive Design Strategy, p. 245.

6. Dunne, J., An experiment with Time, p. 21.

7. Mandelbrot, B., Fractals, p. 29.

8. Ibid., p. 109.

CHAPTER VII A NEW ORDER OF SPACE

The historical development of non-euclidean hyperspace geometry has its basis in the modification of Euclid's five postulates. One of the first analytical uses of geometry as a science of measurement was recorded in Egypt as early as 2000 B.C. This form of geometry was later abstracted by Greek architects in 700 B.C. The Greek mathematician Thales who lived from 640 to 506 B.C. abstracted positions and angles into points, lines and vertices. Much work in the understanding of the precise relationships of space was later done by the disciples of Phythagoras. Around 300 B.C. Euclid wrote his book "The Elements" (one of the most widely reproduced books in the world). This work of Euclid's greatly improved the mathematician's knowledge of geometry. Euclid's basic assumption, according to the oldest editions of "The Elements" consisted of the five following postulates:

1. A straight line may be drawn from any one point to any other point.
2. The finite straight line may be produced to any length in a straight line.
3. A circle's circumference may be described with any center at any equal distance from that circumference.

4. All right angles are equal.
5. If a straight line meets two other straight lines, so as to make the two interior angles on one side together less than two right angles, the other straight lines will meet.

This was also stated as given a line and a point not on that line, there is one and only one line that can be drawn through that point and be parallel to the original line.[1] Unlike the first four postulates the fifth is not immediately evident. In 430 A.D., Proclus reduced this postulate to read "if a line intercepts one of two parallel lines, it also intercepts the other". A still more modern version used today is, two lines which are parallel and exist in the same plane will not intercept.[2]

All of Euclid's analysis of space in "The Elements" was based on these five postulates. In the 19th Century it was discovered that in a theoretical space where the fifth postulate was invalidated, the other four postulates could still be extrapolated into a complete geometry. In fact it was found that if the fifth postulate was replaced with its opposite, then on this strange foundation one could build all types of wonderful, but perplexing new non-euclidean structures of space. These geometries are as free of logical contradictions as Euclidean geometry. The fact that we, in our three dimensional space, cannot "see" the physical correlations of space in the usual way means little. Before we ask which of these geometries describes our perceptual space it is important to recognize that this question has little practical value until one determines what correlation we assign between the basic concepts of geometry and their possible physical equivalence. Many actual concepts in geometry are abstractions and are only approximated in real life. A point which is defined as a position without magnitude doesn't really exist in space except as a mental abstraction. Nor can it be said that true lines exist, a light beam might seem like a straight line but according to developments in physics a warping of both three dimensional space and the straight path of light rays are due to matter and physical accelerations. Two rays of light perpendicular to a single plane might ultimately divert as in hyperbolic geometry, or converge as an elliptical geometry. Therefore analysis of space and geometry is also more or less accurate depending upon the scale which one is examining.

The mathematician Gauss who lived from 1777 to 1855 was one of the first to believe that the fifth postulate could be removed and not create contradictions in a new form of geometry. Gauss did not widely express his ideas at first thinking that they were too revolutionary to be more than just an analogy of structure with no actual significance to physical reality. This led him to first try to find a way to prove the fifth postulate. However he was not able to do so, having worked on this attempt for almost twenty years. Finally he overcame his prejudice and developed the first truly non-Euclidean geometry.[3]

Nikolai Leonovich Lobatschewsky, who lived from 1793 to 1856, was a Russian mathematician who independent of Gauss developed what he called pandimensional or the pangeometrical method. Lobatschewsky pointed out that while a geometry not based on Euclid's five postulates might seem odd when compared to the "real" world, its validity as a logical structure is not effected, but only depends on its internal consistency. This idea of internal consistency became one of the fundamental criterias in the creation of new theories of space and science. This is particularly interesting in respect to the ancient concepts of a self-referential structure as one basis for aesthetic unity. This is also a powerful reason to undertake a slow and thorough development of a visual language based on the constraints of hyperspace.

One interesting researcher in the field of non-Euclidean and pandimensional geometry was Bolyai who lived 1820 to 1860. He worked on what he called "the absolute science of space", or absolute geometry. This was a geometry consisting of propositions not pertaining to the fifth postulate at all and was applicable to both Euclidean and pandimensional geometries. In 1923 he wrote to his father saying, "I have resolved to publish, yet the goal is not quite reached. I have made such wonderful discoveries that I have been so overwhelmed by them ... it is as though I have created a new universe from nothing". In his reply Bolyai's father wrote back "many things have an epoch in which there are found at the same time in several places, just as the violets bloom on all sides in the spring ... get your material in order and publish soon, my son".[4]

George Frederick Reimann lived from 1826 to 1866 and modified the first, second and fifth postulates so as to read:

1. Any two points describe a line which is not necessarily unique to those two points.
2. A line is unbounding (for a line to be both finite and unbounding it must be a re-entrant, like a circle).
5. Any two lines in a plane will meet.

In Riemann's own words of 1854, "the infinite unboundedness of space possesses a greater empirical certainty than any other experience, and if we ascribe to space a consistent curvature it too must be finite". Albert Einstein later predicted in his general theory of relativity that space does in fact have a local curvature whenever there is matter and so space is finite in the fourth dimension and unbounding in the third.

During this time Schlafi, who lived from 1840 to 1895, began development on a precise analytical geometry of N-dimensional space. One disturbing quality of non-Euclidean geometry was that two geodesic lines of a sphere (defined as the largest circle possible whose radius is equal to the spheres radius) have not just one but two points in common; a point and its exact anti-point. The mathematician Klien later found a way to remove this blemish from spherical geometry. He realized that complex theories of space are clarified by changing the meaning of the word point so as to call each pair (the point and its anti-point) a single "point phenomena". In this way the point phenomena and line phenomena of Klien's elliptical geometry of N-space can be represented by lines and planes in our own Euclidean perceptual space. This in turn allows for the construction of a projective plane which corresponds to an arbitrary frame of reference passing through the network of a higher N-space. Allowing then, two basic generalizations to present themselves. One, that we may take four elemental references of three dimensional space (the point, the line, the plane and the volume) and use them to describe an N-plus one space and so obtain a N-dimensional perspective.

It is directly from this work that the modern artist and architect can receive new insights into the structure of his most basic medium, namely space itself. In so doing one may create forms that not only reflect and objectify modern scientific thought but also resonate with the basic structural relationships we share, accessible to all being on all planets. As the artist mathematician, B.L. Chilton has said, "we now believe that many geometric theories of space are truly universal, such that a being native to a distant planet working out the same geometric problems would end up with some similar results which could be directly related to our own".[5] This leaves open the possibility of creating a truly universal space-art in the future.

In a modern context from the 1840's to the present many individuals and mathematicians have worked on developing an understanding of the natural patterns of space and dimension. These men and women have had diverse backgrounds but shared an abiding interest in revealing the hidden relationships which make up the structure of space. Their concept of space was not just a place in which things happen, but was itself a guiding force governing the physical structure of nature and the laws of physics.

CHAPTER VII

FOOTNOTES

1. Alexandroff, P., Elementary Concepts of Topology, p. 53.
2. Lyusternik, L., Convex Figures and the Polyhedron, p. 67.
3. Coxeter, H., Non-Euclidean Geometry, p. 148.
4. Ibid., p. 173.
5. Chilton, B., "Shadows of Four-Dimensional Polytopes", Mathematics Magazine, November 1971, p. 44.

CHAPTER VIII PURSUING THE SHAPE OF SPACE

Much of the work based on natural forms and poly-dimensional space, dates back to the mid 1800s and the works of H.G. Grassmann. In 1844 Grassmann was one of the first mathematicians to develop a primary analytical geometry including the concept of more than three dimensions in his work on the theory of expansion.[1]

In 1852 the mathematician P. Mobius published an article in which he discussed the implications of just such geometry. It has even been suggested that as early as 1827 Mobius realized that a four dimensional rotation would bring two enantiomorphic solids (mirror opposites) into coincidence.[2] This idea was later used by Abbott in his book "Flatland" (1884) as well as by H.G. Wells in "The Platter Story". In these works a simple notion is extended from an ordinary context to one that is extraordinary. This idea is that a righthanded glove can only be visually reversed by lifting it off a flat plane (like a table) turning it, and placing it down again. Through the process of analogy it is contended that if a person could be lifted out of three dimensional space and rotated he too could be reversed and inverted.

Following this analogy of looking at a two dimensional plane, we see that rotating a two dimensional figure through a plane creates the intersection of a line within that plane. Thus when four dimensionally rotating something through our three dimensional world it too would become a mere "cross-section" of itself. Looking down upon a two dimensional plane from three dimensional space it is possible for us to look inside of something such as a triangle. If this analogy holds true then in a theoretical fourth spatial dimension one could do such things as remove the yolk from an egg without opening it, or pass "around" walls "dimensionally". It would also enable four dimensionally coherent beings to see all sections of an N-dimensional figure simultaneously, holding fast in its thoughts the visual unstability that we see as the temporal flow of the necker or hypercube inversion.

In 1853 Ludwig Schafli published a series of definitive articles on the subject of polydimensional space and time. Schafli was born in Grasswyl, Switzerland in 1814 and studied both theology and science at Berne. After completing his studies he later taught at a small primary school in Thun where he improvised much of his mathematical lessons to stimulate his students to think abstractly about space.[3] In 1847 he began his work on hyperspace. However very little of Schafli's work was ever published. Two small fragments were accepted in England and America in 1855 and 1858 but they did not attract much attention at the time. Twenty years later in 1880 much of Schafli's work was rediscovered by the American mathematician Stringham.[4] The result was that many people attributed the first truly mathematical description of hyperspace to him. Between 1881 and 1900 many mathematicians such as Rudel, Forchhammer, Gosset and Curjel published much work on hypersolids. One of these men, Theodore Gosset, was born 1869 and went to college in Cambridge in 1888. Gosset studied law and got his degree but for many years had trouble getting clients. He wrote that he passed much of his time after law school trying to figure out what regular figures might fit into N-dimensional space. In 1900 Gosset published his paper on regular and semi-regular figures in space of N-dimensions.[5] This work became an important reference for later mathematicians such as Micowski and Hilderbrant who used poly-dimensional space to describe the physical phenomena of gravity.

In 1932 the mathematician Room considered two five-dimensional configurations which was directly related to the polytopes of Gosset. Later J.A. Todd made use of Room's configurations in his geometric proofs concerning N-dimensional topology. John Petrie, born 1907(son of Sir William Petri, the great Egyptologist)showed remarkable promise in mathematical and spatial thought while in school. It has been recorded that in periods of intense concentration he would answer questions about very complicated four dimensional figures simply by visualizing them. Later John Petrie went on to develop an analytical analysis of the expanential increase of edges and planes in any number dimension.[6]

One story conveyed by the mathematician H.S. Coxeter in his book "Regular Polytopes" concerned Alicia Boole Stott who lived from 1860 to 1930. She was one of five daughters whose father was a professor of Algebra. At 13 Alicia lived in a run-down tenement in London due to her father's sudden death. She was subsequently forced to work in a factory making wooden toys for children. Alicia's mother was friends with Mr. James Hinton who had a son, Howard. Howard was fascinated by a set of wooden blocks Alicia had made in the factory. Howard would play with the blocks, studying all the parts and this Alicia later said inspired her at the age of 17 to begin having vivid dreams in which she had extraordinary glimpses of multi-dimensional cubes. Howard later went on to write books on hyperspace with a considerable amount of mystical interpretation.[7] In 1890 Alicia married the lawyer, Walter Stott. In 1891 Alicia read a paper written by the mathematician R. Schoute in which he described figures that were perpendicular to the three principle directions of space. Alicia wrote to him relaying her and Howard Hinton's experience with hyperspace. In his reply he asked if he might come to visit them which he did. He arranged for the publication of her work in 1900 in a book entitled "On Certain Series of

Sections of Regular Four-Dimensional Hypersolids".[8] Together this group came to know Paul Dunchian who felt that hyperspace geometry was the hidden structure implicit in the forces of nature. Paul S. Dunchian, who was born in 1894, was of American and Armenian descent. His great grandfather was a jeweler in the Courts of the Sultan of Turkey, and many of his ancestors were also experienced jewelers. Dunchian studied high school geometry and science but then went to work in a rug business with his father. He inherited his father's business but at the age of 30 began having vivid dreams about his ancestors and their work precisely cutting precious stones. This continued for months and he reported suddenly being able to think about space in a new and strange way. He began to experience a number of startling dreams of multi-dimensional space. The author Jerry Dunne describes this phenomena in his book "An Experiment with Time". In this work Dunne relates the visual unstability of perceiving hypercubes with the workings of the unconscious mind.[9]

Today there are a few artists who have begun using the work of mathematicians in the creation of drawings and sculptures which depict abstract multispatial realities. David Brisson, Associate Professor of Design at the Rhode Island School of Design has been working for many years in the area of geometry and perception. He has generated many stereoscopic diagrams depicting four dimensional hyperspace. Harriet Brisson is a sculptor working at Rhode Island College who is also interested in hyper-geometry as a basis for art work. Scott Kim and Thomas Banchoff have written articles concerning the geometry of curves and surfaces in three and four dimensional space as well as producing computer graphics in related research on geometry. J.M. Yeurralde, who was a fellow at the Center for Advance Visual Studies at M.I.T., has also created many intricate paintings based on hyper-spatial geometry.

CHAPTER VIII

FOOTNOTES

1. Sommerville, D., An Introduction to the Geometry of N-Dimensions, p. 35.
2. Coxeter, H., Regular Polytopes, p. 207.
3. Schlafli, L., An Attempt to Determine Twenty-Seven Lines upon a Surface of N-Dimensional Space, pp. 110-120.
4. Stringham, L., Regular Figures in N-Dimensional Space, pp. 1-14.
5. Gosset, T., On the Figures of N-Space, pp. 43-48.
6. Coxeter, H., Regular Polytopes, p. 241.
7. Hinton, H., A new Era of Thought, p. 31.
8. Stott, A., On Certain Series of Sections of the Regular Four Dimensional Hypersolid, pp. 21-27.
9. Dunne, J., An Experiment with Time, p. 193.

CHAPTER IX SYNTHESIS

The enumeration of a series of ambigious perspectival views of hyperspace leads to a natural consideration of the inherent structure of space in general, and to the laws underlying the visual language of naturally occurring forms. This pure spatial and temporal environment can be exemplified by volume, dimension and motion which can itself become the medium of artistic expression. LeCorbusier wrote that nature is ruled by mathematics, and the physical forces of which mathematics is a language, describing basic physical relationships. Moreover it is the true masterpieces of art which are in consonance with nature; expressing the laws of nature which they themselves proceed from, and by which they are constrained.[1] Thus to the degree that hypercrystalline form explicitly concerns one's perception of space and time, it too must necessarily adhere to the natural system of organization inherent in the medium of space itself. There is an obvious formal visual language implicit in these relationships which can become the basis of a rigorous form of art. The aim of this mode of inquiry into the perceptual ambiguity of hyperspace is to verify basic spatial phenomena so as to recognize new modes of visual communication. This in turn gives one a means of explicitly expressing in concrete visual terms, the many levels of complexity, dimension and order implicit in the structure of space itself. These patterns can then be organized in accordance with a series of simple repeated structures, varying by degree, so as to produce new structural units. These new units can in turn be reconstructed in the production of further interrelated patterns.

The filling of perceptual space with a series of structural units having progressive variation of dimension and proportion becomes a metaphor for the processing of nature in which one form germinates from another. Moreover given the ambiquity of hyperspatial forms, the visual movement they produce while enfolding in the mind of the observer speaks particularly well to some issues addressed by modern science, namely how observation is itself categorizing phenomena of the viewer.

This crystalline image of inner space also permeates many dreams and ancient myths of the self. In these mythic images a path of redemption out of material brokenness is formed. We pass through the changing pattern of the crystal's symmetry with a renewed spirit. The mathematically precise arrangement of this mythic crystal often invokes within one the intuitive feeling that there is a spiritually ordering principle at work revealing itself within the structure of inorganic matter. Carl Jung has written, "the image of a crystal often symbolically stands for the union of extreme opposites of matter and spirit".[2] This symbol implies something more than just the obvious and immediate meanings that come to mind. It has a wider pre-conscious aspect which is not, and can never, be fully explained. For its ability to open out onto itself is a magic ring of fire, opening it closes a circle with itself. This ring of the multi-dimensional hypercrystal has embedded within it a second reciprocal ring, interlocked they weave through each other. They appear as a solid structure, yet on closer inspection they can be seen to dissolve, eluding one's attempt to fix their manifold relationships. The choice of this infusion of mathematics with perceptual space is both symbolic of eternal permanence and a mythic image of continual flux. Immobility scintillant it reveals a network of hidden relationships to those who are willing to engage it in observation. This form of the hypercrystal is a visual realization of a process by which we focus depth and articulate space, as the mind realizes new planes through the transition of old ones. Ultimately this cognitive transmission of the structure of space is the motif of a transcendent image of the self. Therefore the construction of spatial relationships can be orchestrated in such a way that the organized space is a dynamic temporal flow and so reveals the processing of the mind and its interplay with the world. This is an ancient dialectical inversion in which one seeks the inner immediacy of experience in the objective structure of the world. A revealing of the hidden aspects of self, which has been alienated as objective reality. It is foremost a study of the objective effects of line, form, volume and space with the goal of finding at its center the principle of our own creation. Picking up the system of number and geometry seriously as a basis for artistic expression is a movement from simple interspection of inner sensation, to a revealing of the constructive patterning of objective reality itself.

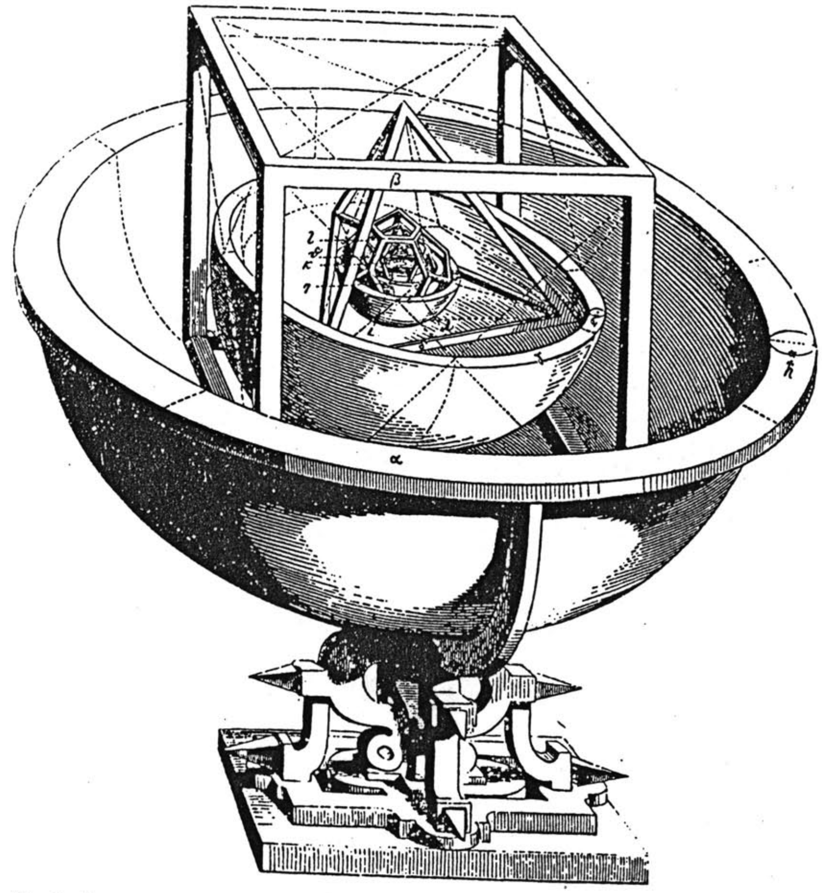
Artistic inquiry into the structure of space has often been based on geometry and science. This process has allowed art to proceed in both a systematic and patient method without disallowing a purely intuitive form of visual discovery. This is not to say that the translation of the laws governing physical forces and geometry into an artistic formulation, has the same methodology as the physical sciences. It does not utilize experimental controls to verify or falsify the validity of specific claims. Nor can it empirically prove its results in an artistic context. Therefore, though art can base its content on the structure of space revealed in geometry it is not to be equated with geometry itself. Both attempt to penetrate the mystery of the world around us, but they do so in different ways. As one shifts attention from measurement to meaning one begins to enter the realm of art. For it is art that can communicate the laws underlying the structure of space in a purely sensuous form, thereby bringing them into the human sphere of feeling.

Visually conceptual graphics based on geometry has had a long historical development. It has been an attempt to reveal in a tangible form the unifying basis of the physical forces which manifest themselves in the underlying structure of space and time. The geometric sculptures and drawings created by the author are visual projections of the mathematics of six and seven dimensional space. These works are meant to stand on their own, visualizing the statements contained within this text. These sculptures are executed through the use of lines made of neon, illuminated plexiglass and string. These lines are then reflected between mirrors, creating a series of floating cells where the actual lines and implied lines develop varying spatial relationships. Each of these cells corresponds to a specific fixed ratio of length to height and width to depth. The issues these works address are primarily concerned with the design control which space exerts on form through the successive buildup of dimension. Transcendent to any form, pure and self-volumizing, this absolute relating of space is the intended goal of expression and embodies ancient cosmologies in a context of modern science.

CHAPTER IX

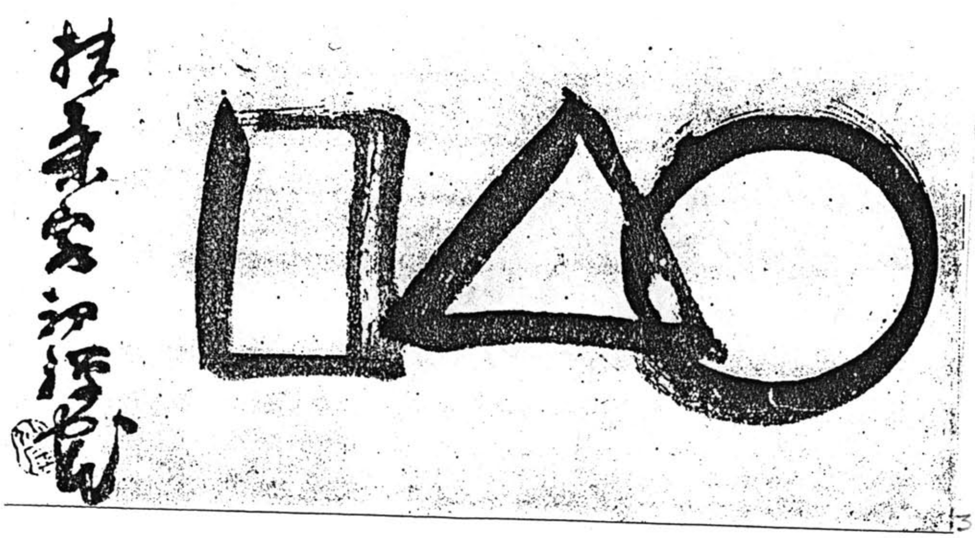
FOOTNOTES

1. Hilbert, D., Geometry and the Imagination, pp. 31-48.
2. Jung, C., Man and his Symbols, p. 221.

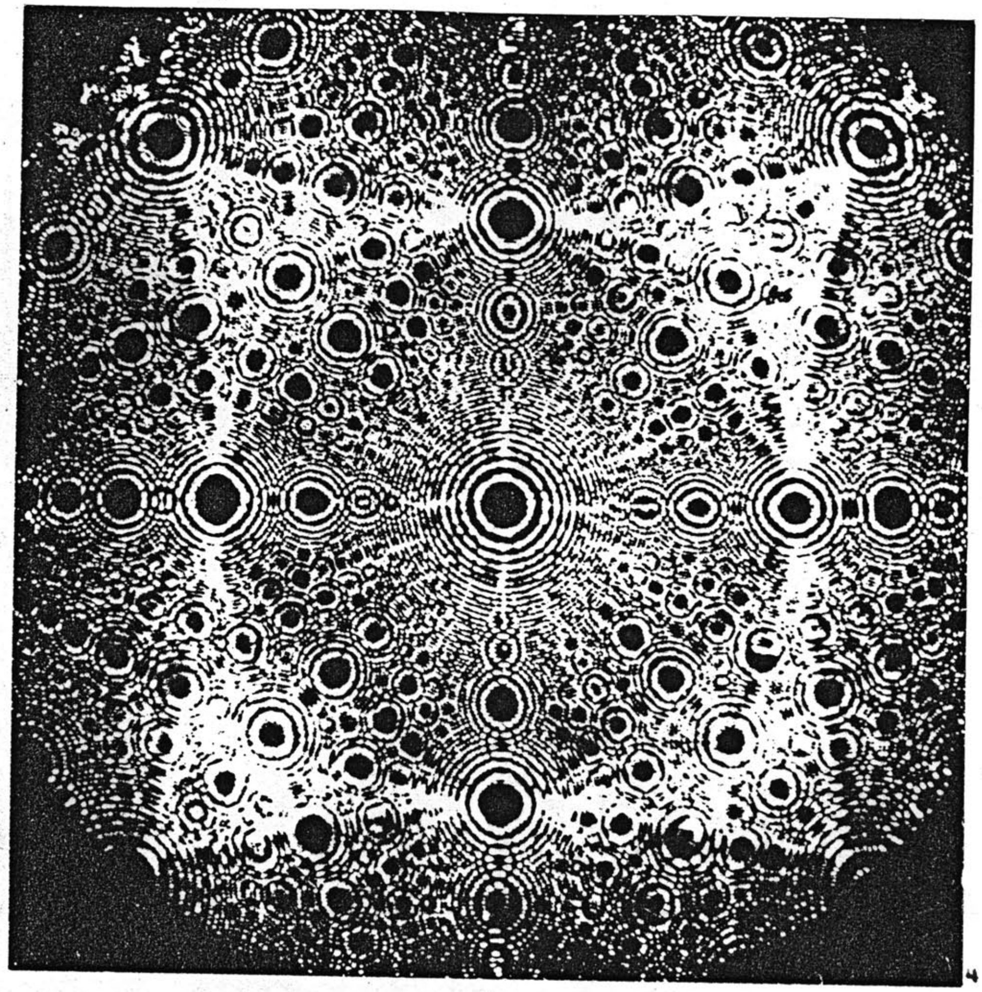


Kepler's version of the solar system was as one Platonic solid within another, the radii of the intervening concentric spheres relating to the orbits of the planets.

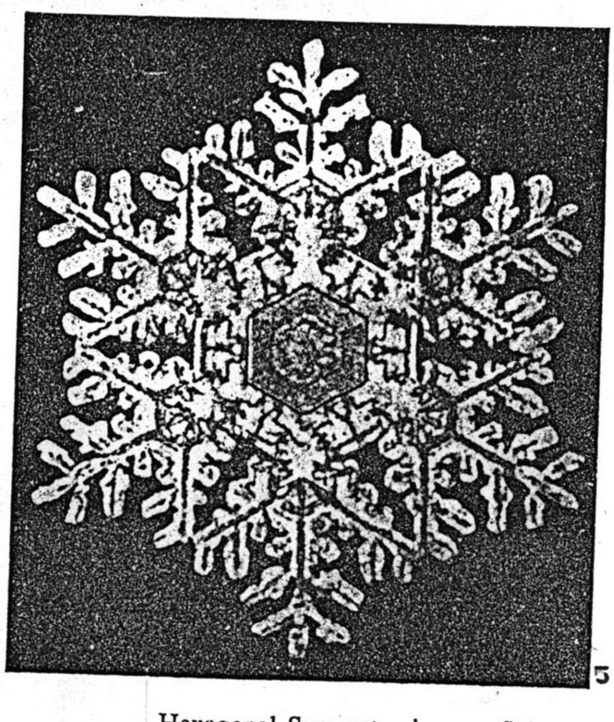
2



3

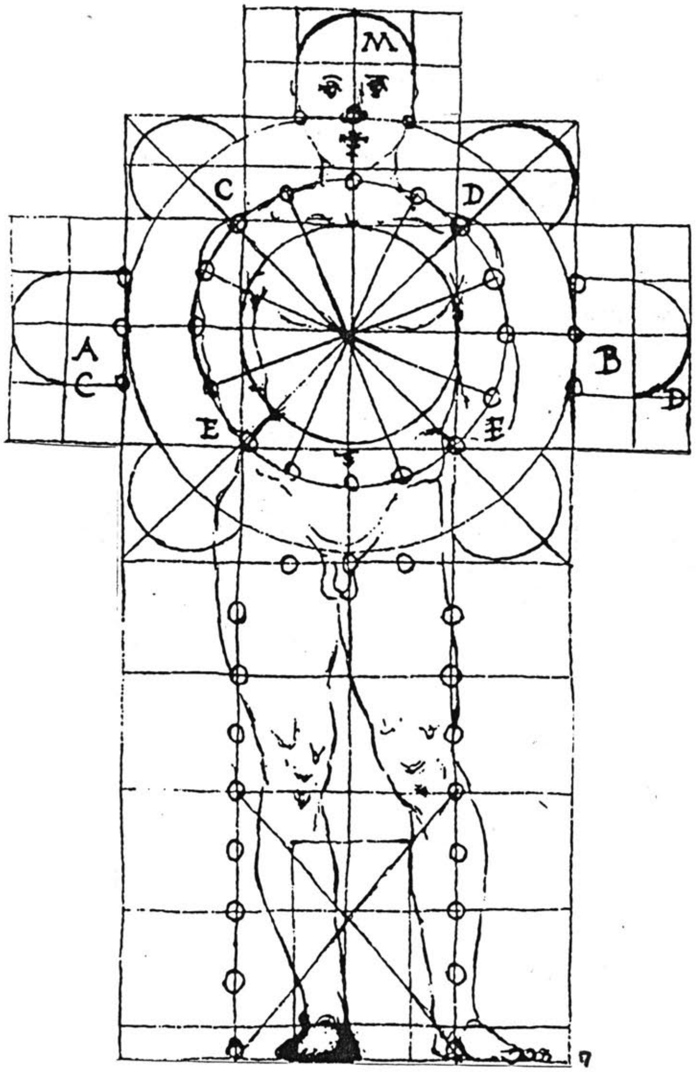


4



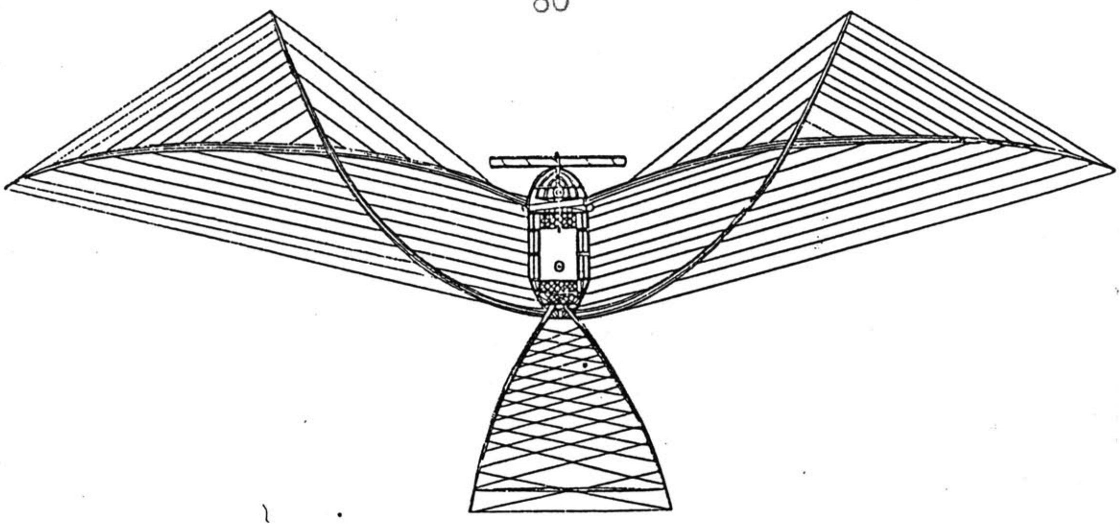
5

Hexagonal Symmetry in snowflake.

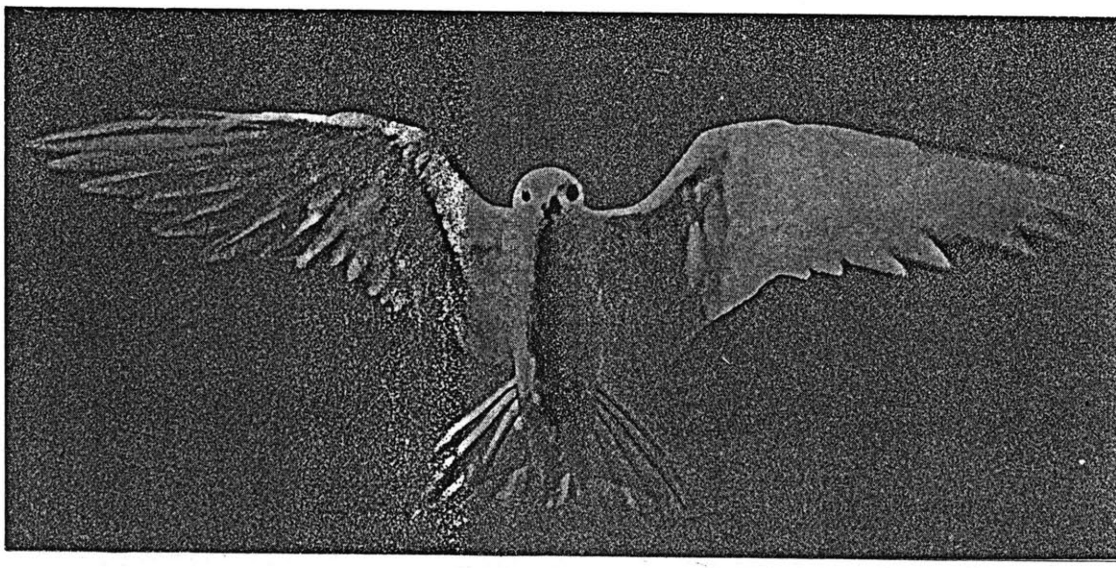


7

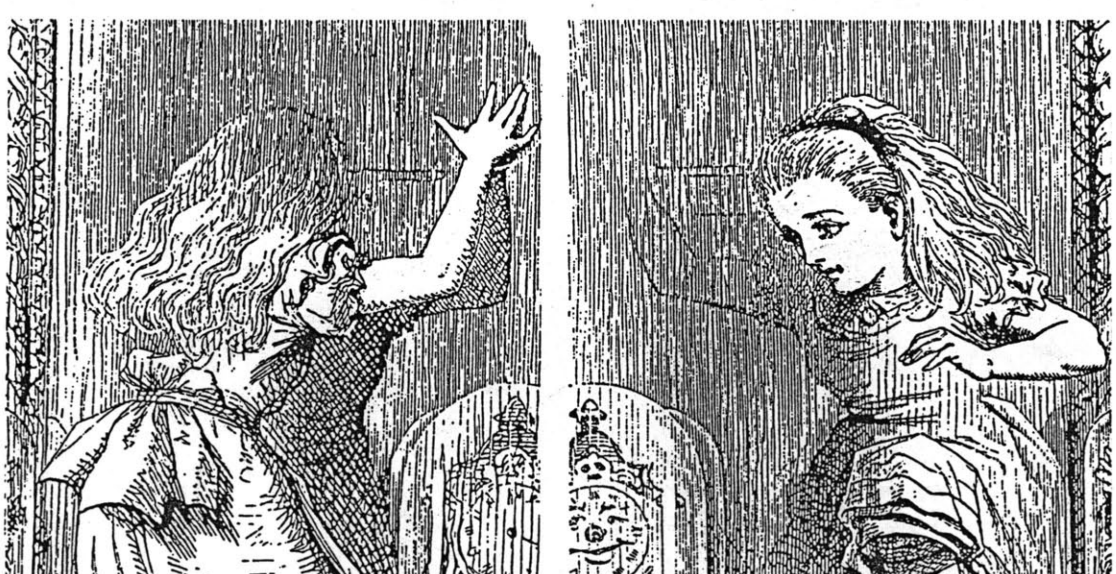
8



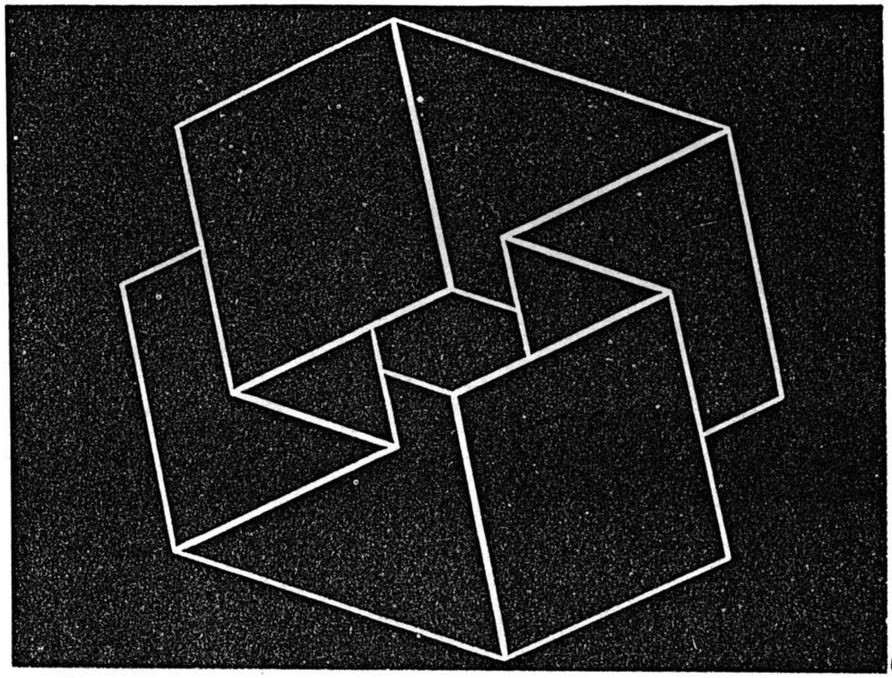
9



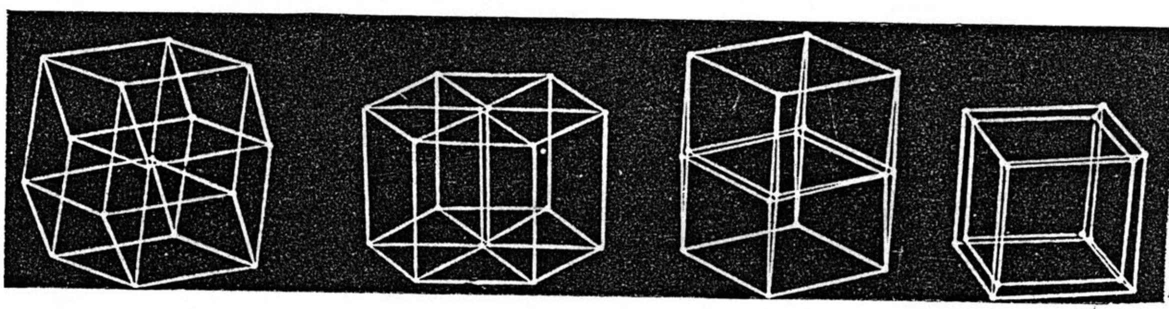
10



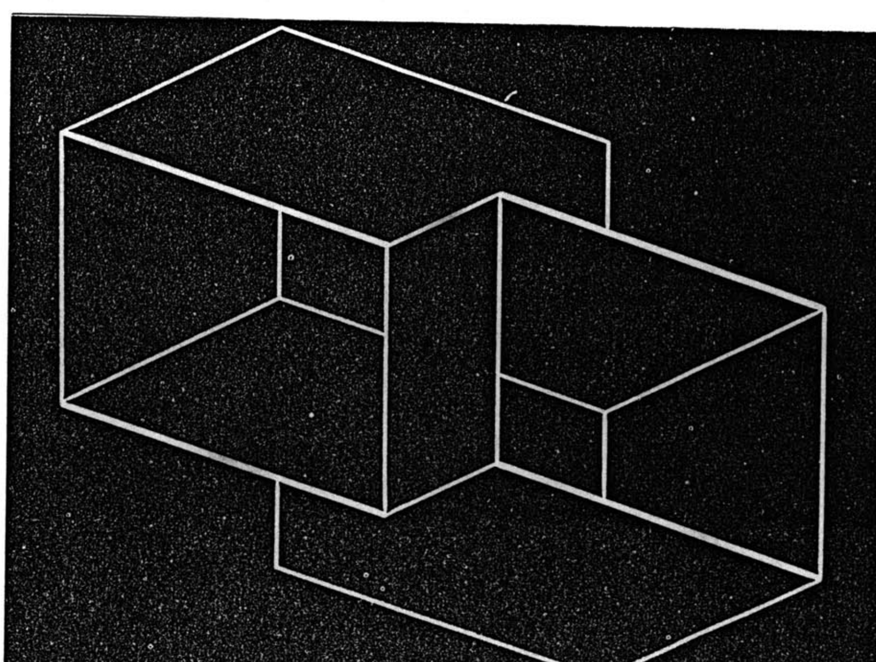
11

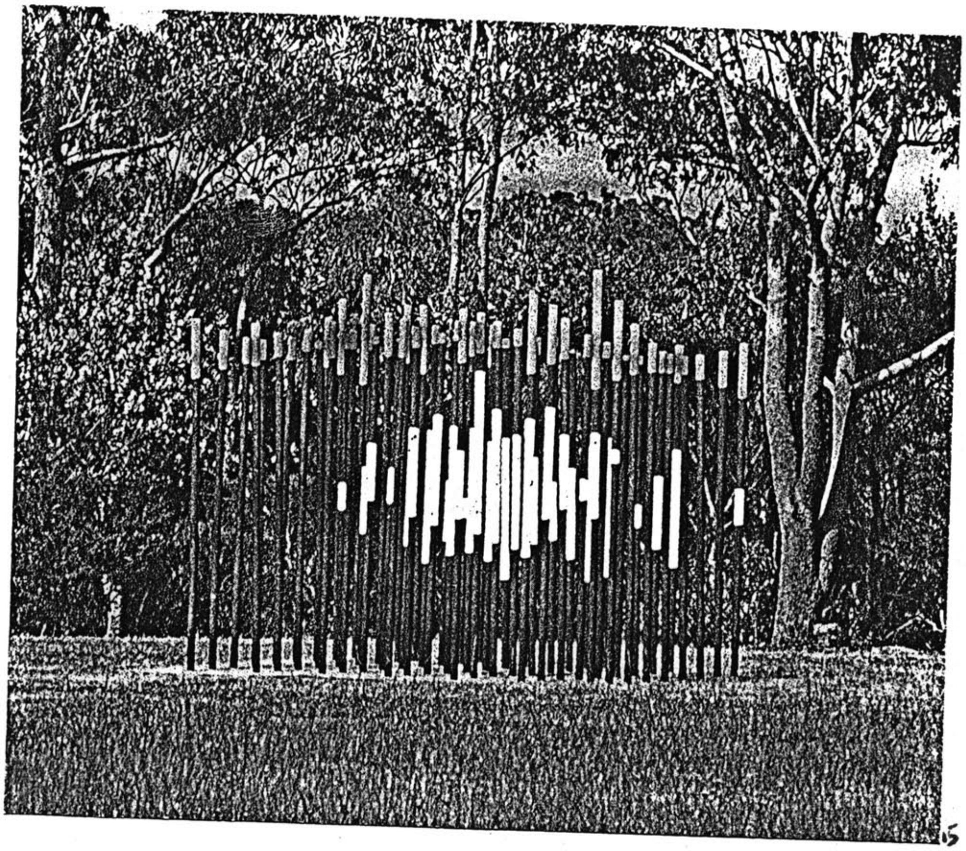


12

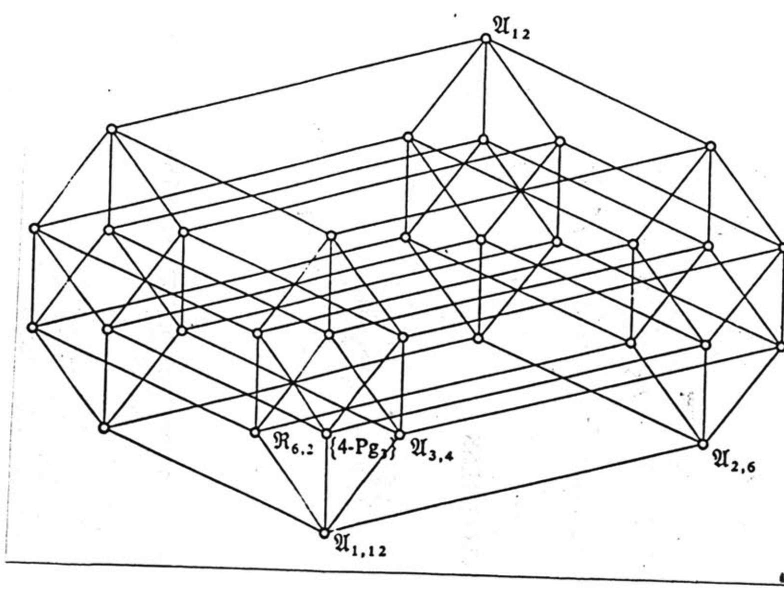


13

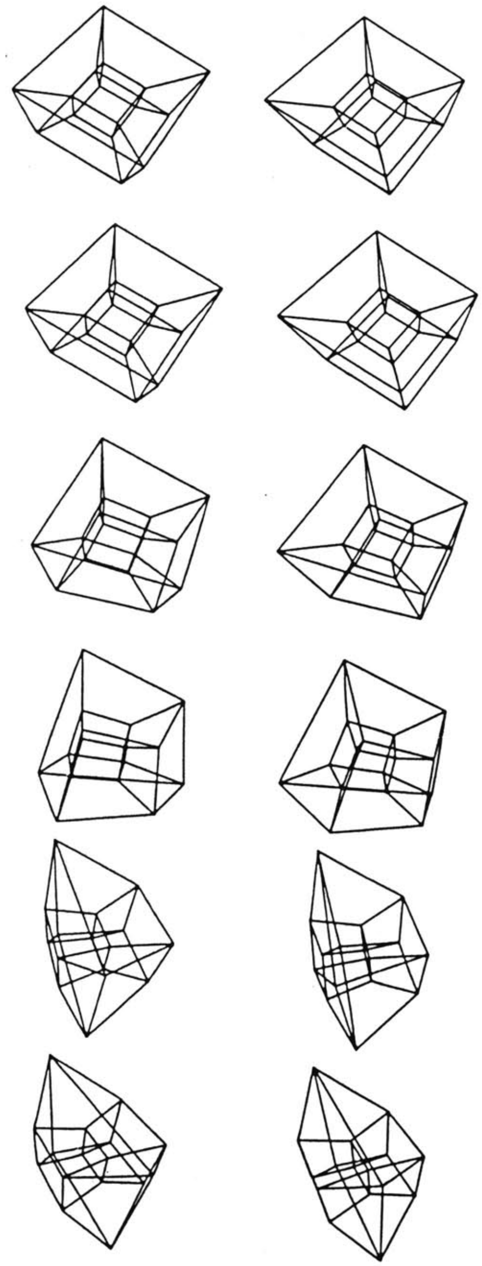




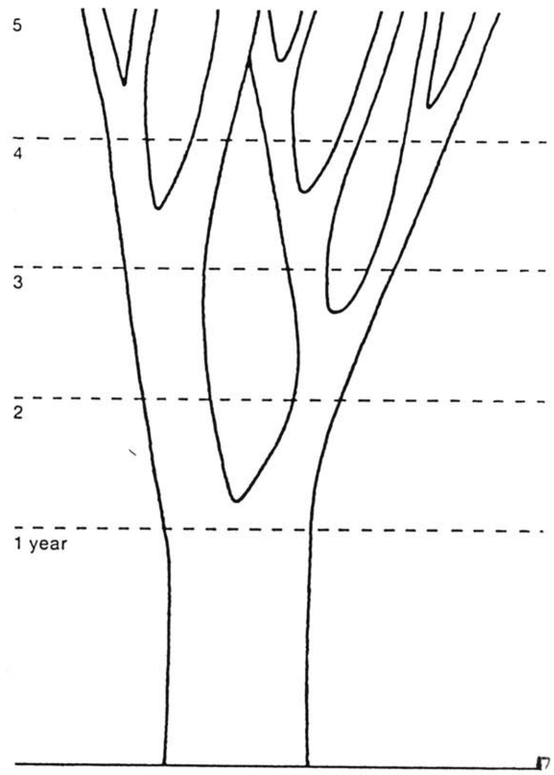
15


$\mathfrak{A}_{12}$
$\mathfrak{R}_{6,2}$
{4-Pg$_1$}
$\mathfrak{A}_{3,4}$
$\mathfrak{A}_{2,6}$
$\mathfrak{A}_{1,12}$

14

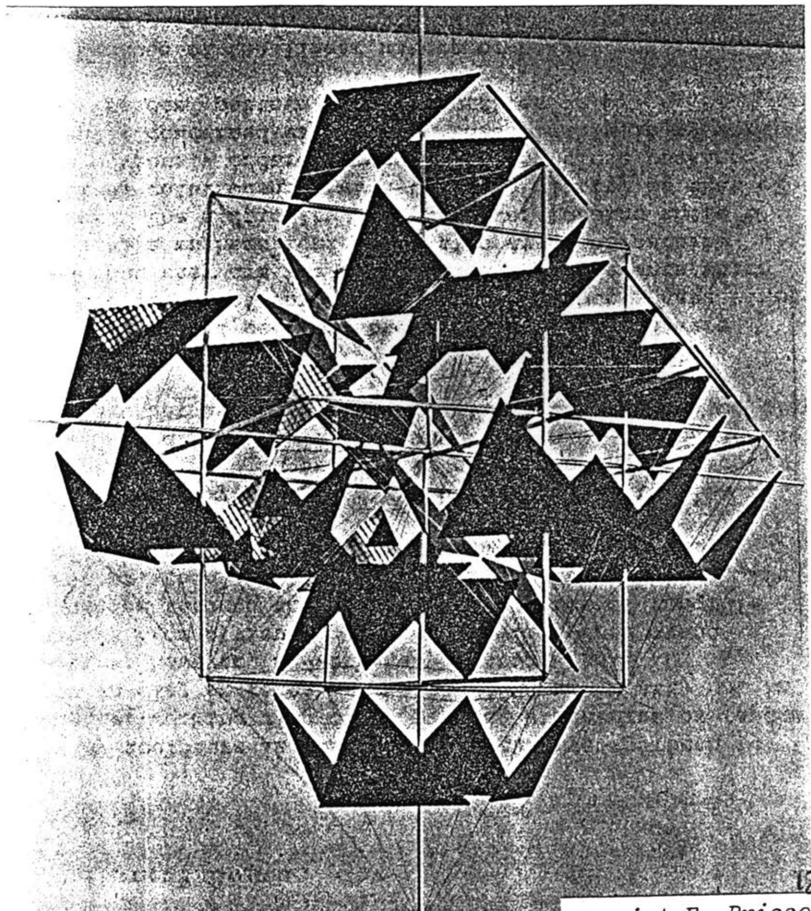


16

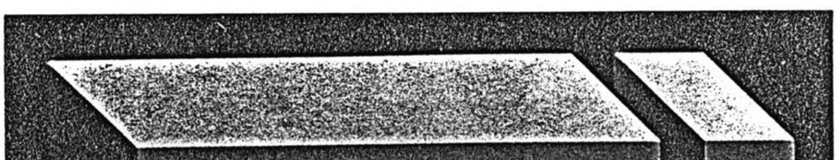


17

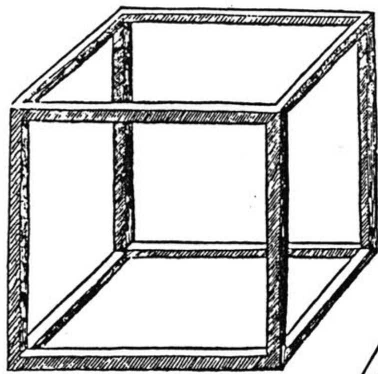
1+1=2, 1+2=3, 2+3=5, 3+5=8, 5+8=13, ...



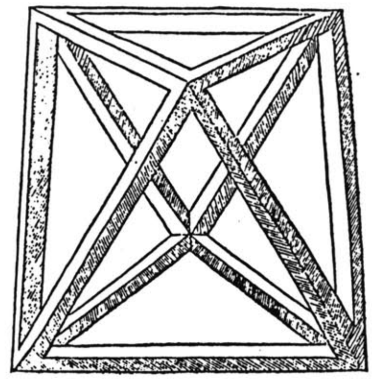
18 Harriet E. Brisson



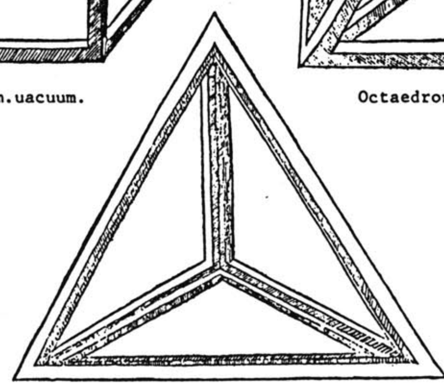
19 J. M. Yturralde



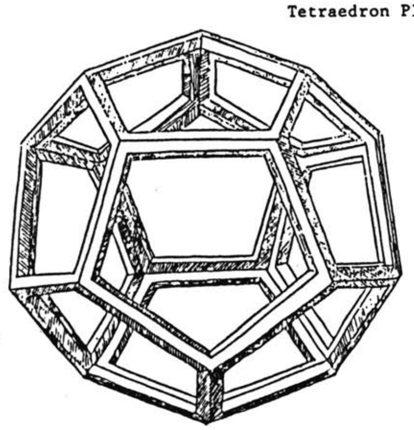
Hexaedron.Planum.uacuum.



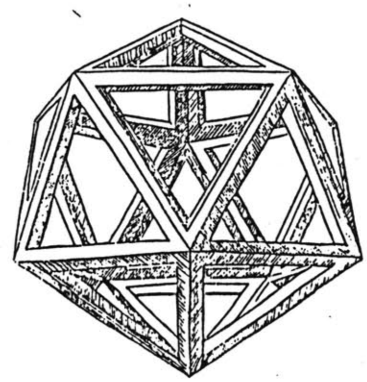
Octaedron Planum Vacuum



Tetraedron Planum Vacuum

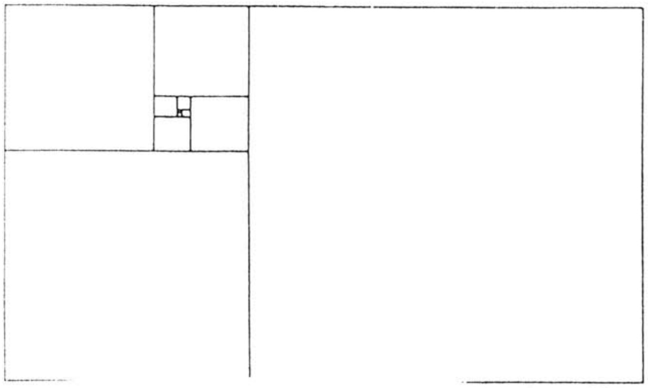


Dodecaedron Planum Vacuum

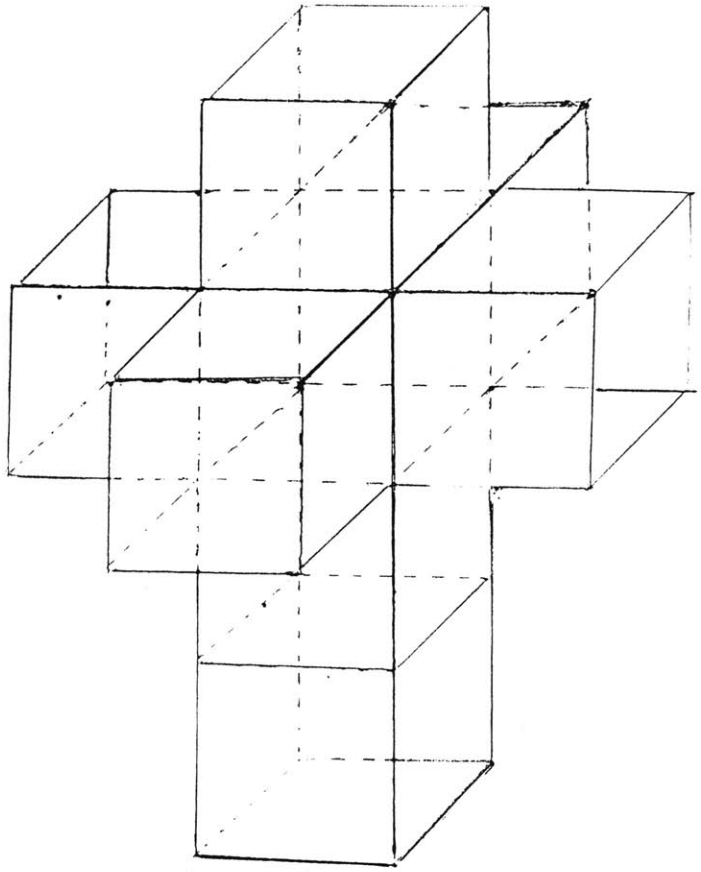


Icoſaedron Planum Vacuum

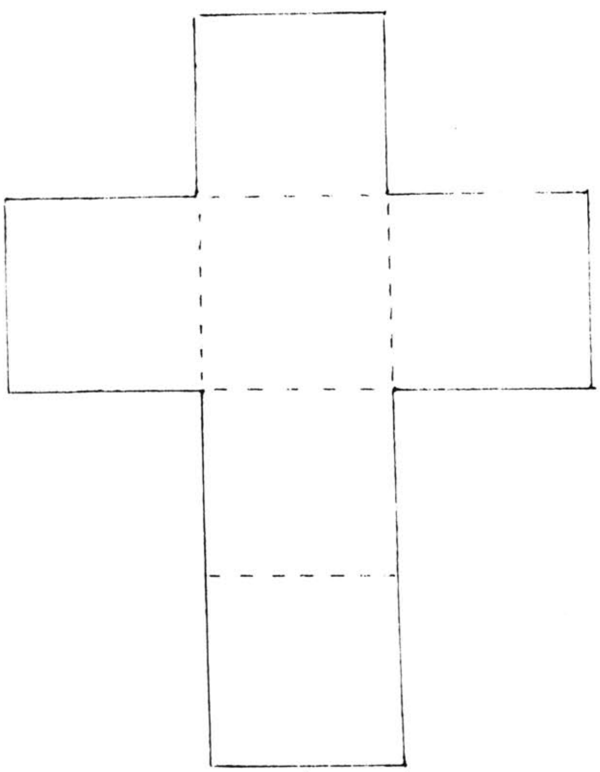
The Five Platonic Solids drawn by Leonardo da Vinci from Pacioli's De Divina Proportione [20]



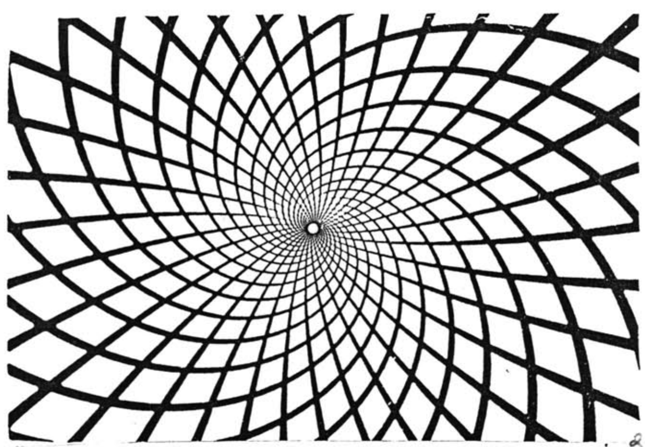
The golden rectangle, showing the recursive pattern of repeatedly subtracting the square which fits the shorter side to leave another, smaller, golden rectangle.



hyper-hexahedron net

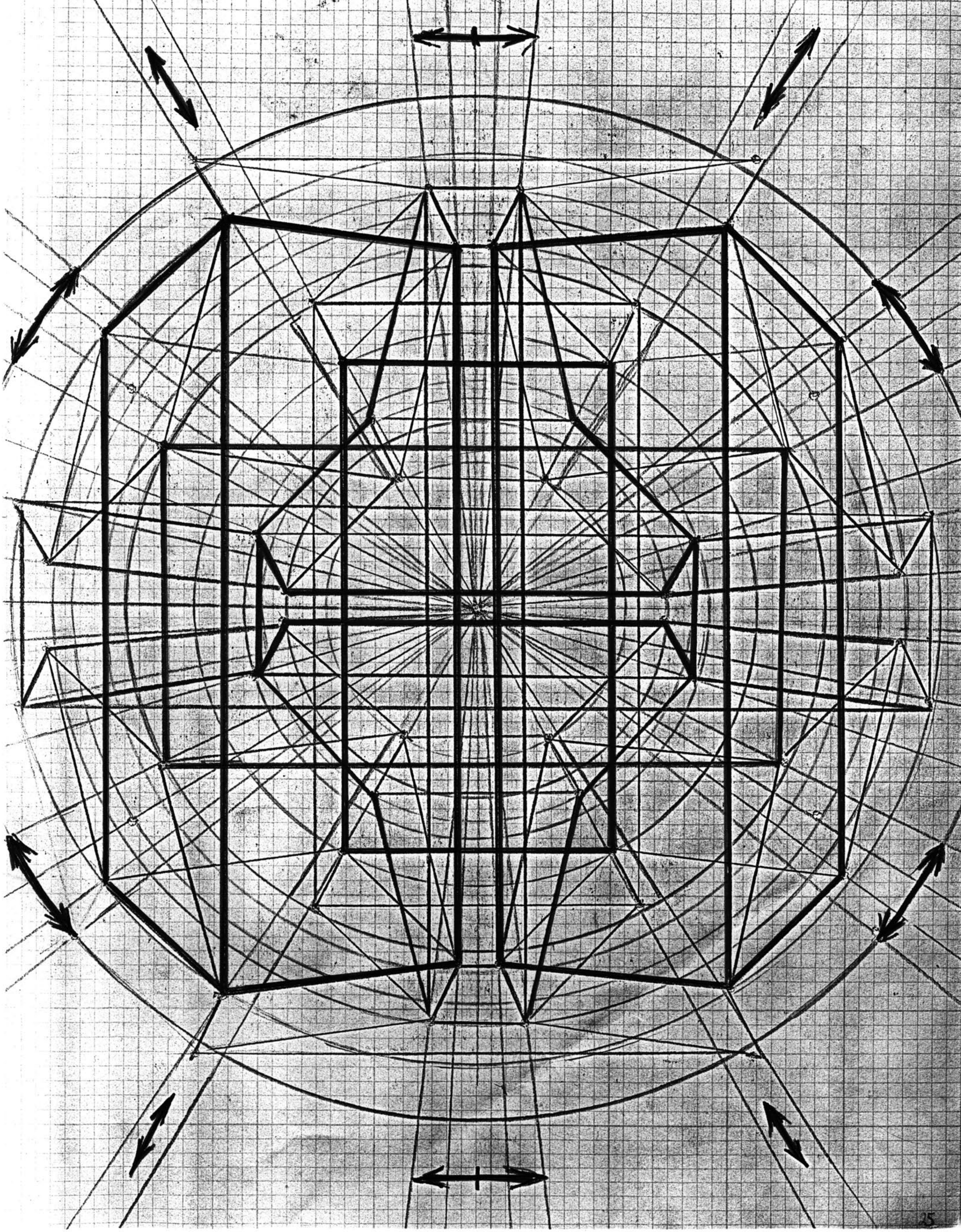


The cube net



a logarithmic spiral.

86.2



25

Polygon # sides 3=Δ 4=□ - 2D

angle # of vertices - 3D {3,4}

- Vertices = $\frac{4p}{4-(p-2)(q-2)}$

- edges = $\frac{2pq}{4-(p-2)(q-2)}$

- Faces = $\frac{4q}{4-(p-2)(q-2)}$

$p = (1-\frac{2}{p})\pi$

86.3

EULER'S Formula - $0_D - 1_D = 0$ $V-E=0$

in hyperdimensions Vert - edges

$0_D - 1_D + 2_d = 2$ $V-E+f=2$

$0_D + 1_D + 2_D - 3_D = 0$

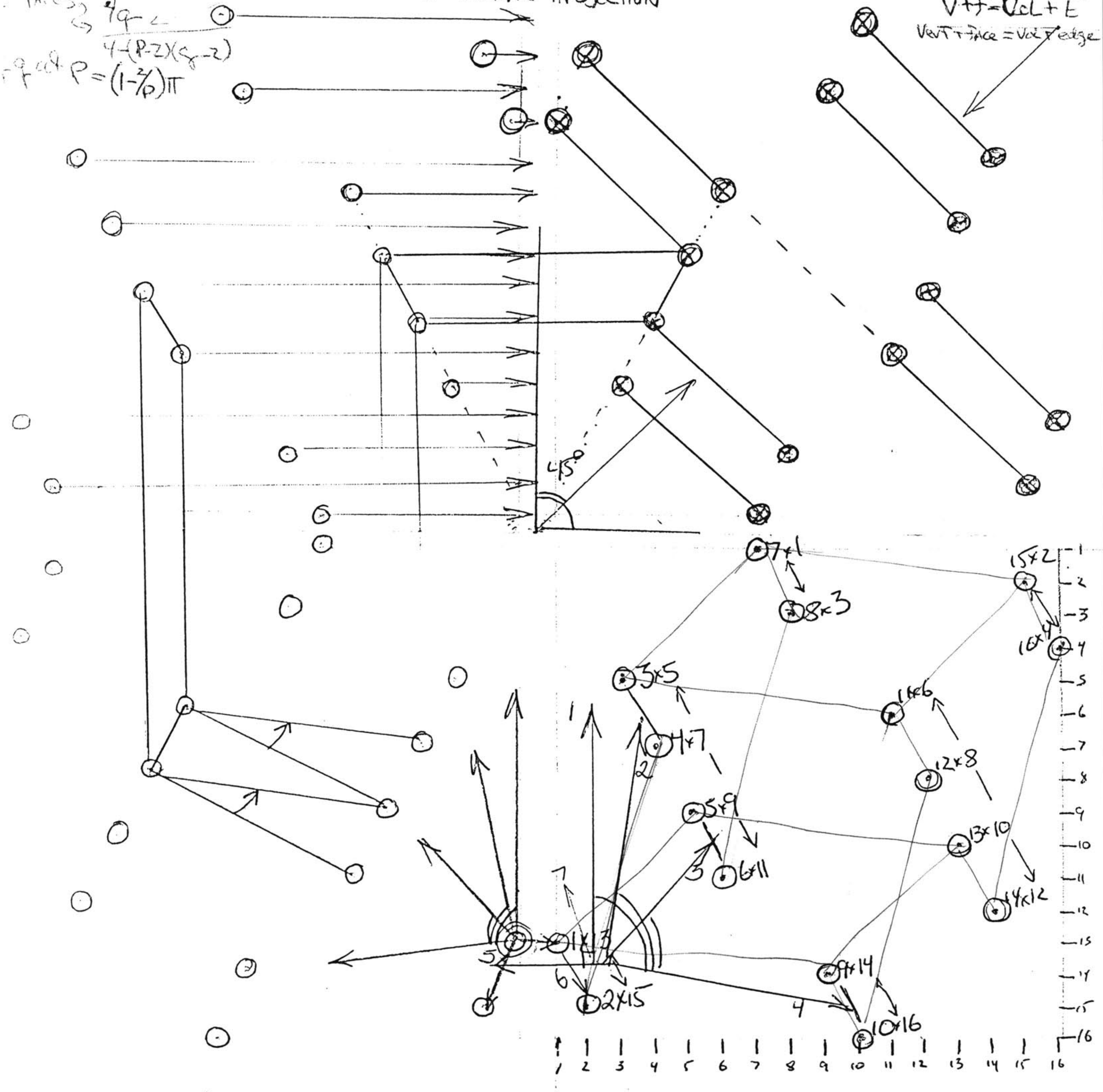
$V-E+f-V_{cl}=0$

$V+f=V_{cl}+E$

Vert + face = Vol + edge

LAYOUT FOR 8-D HYPERCRYSTAL

VERTICES, TOP LAYER OF VERTUAL PROJECTION



χ = NUMBER OF SIDES OF 2-D CELL (3=Δ, 4=□)

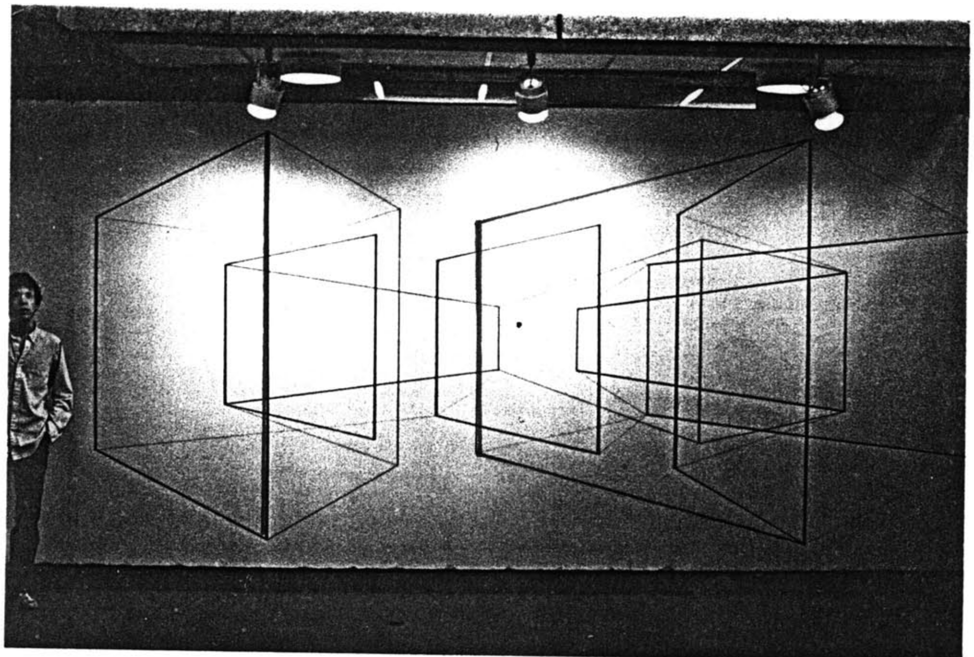
y = NUMBER OF SOLID ∢'S AT VERTEX (3=)

of ∢ at $\chi = (1-\frac{2}{\chi})\pi$ SUCH THAT $y < 2\pi$ (OR would lie FLAT)

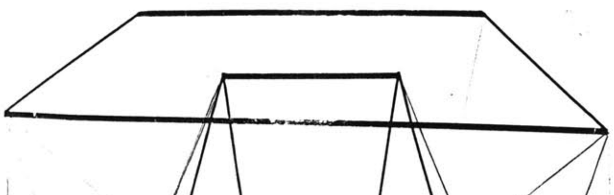
2D 3D 4D 5D 6D
$\{X, Y_1, Y_2, Y_3, Y_4\}$

$\{3,3,3,4,4\}$ ⟷ $\{4,3,3,3\}$ 8D

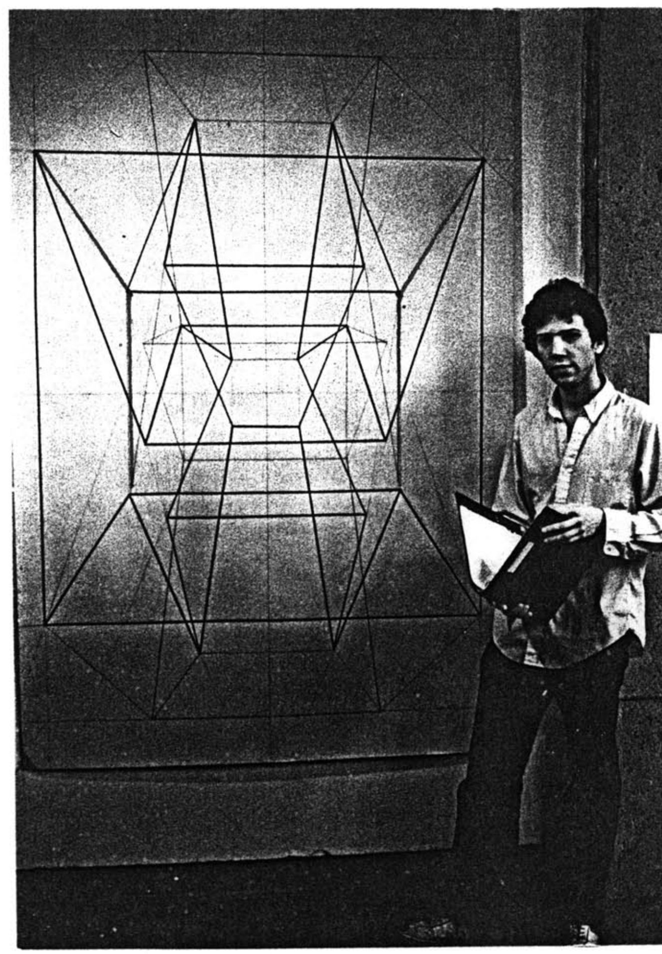
(vertual projection) (5D Hyper cube)



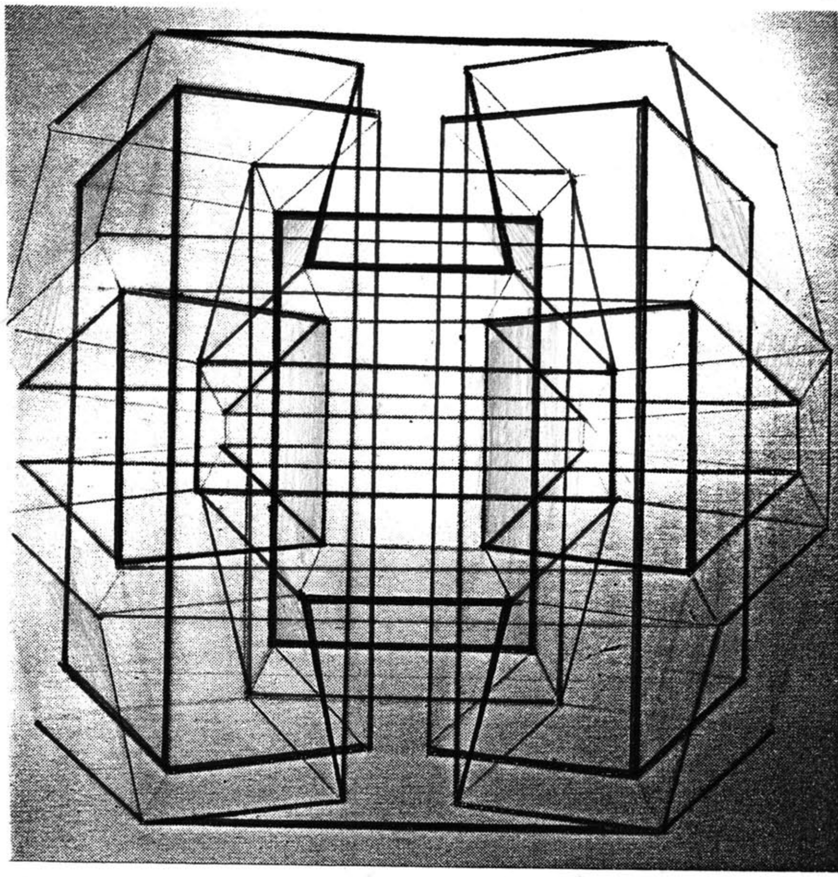
27



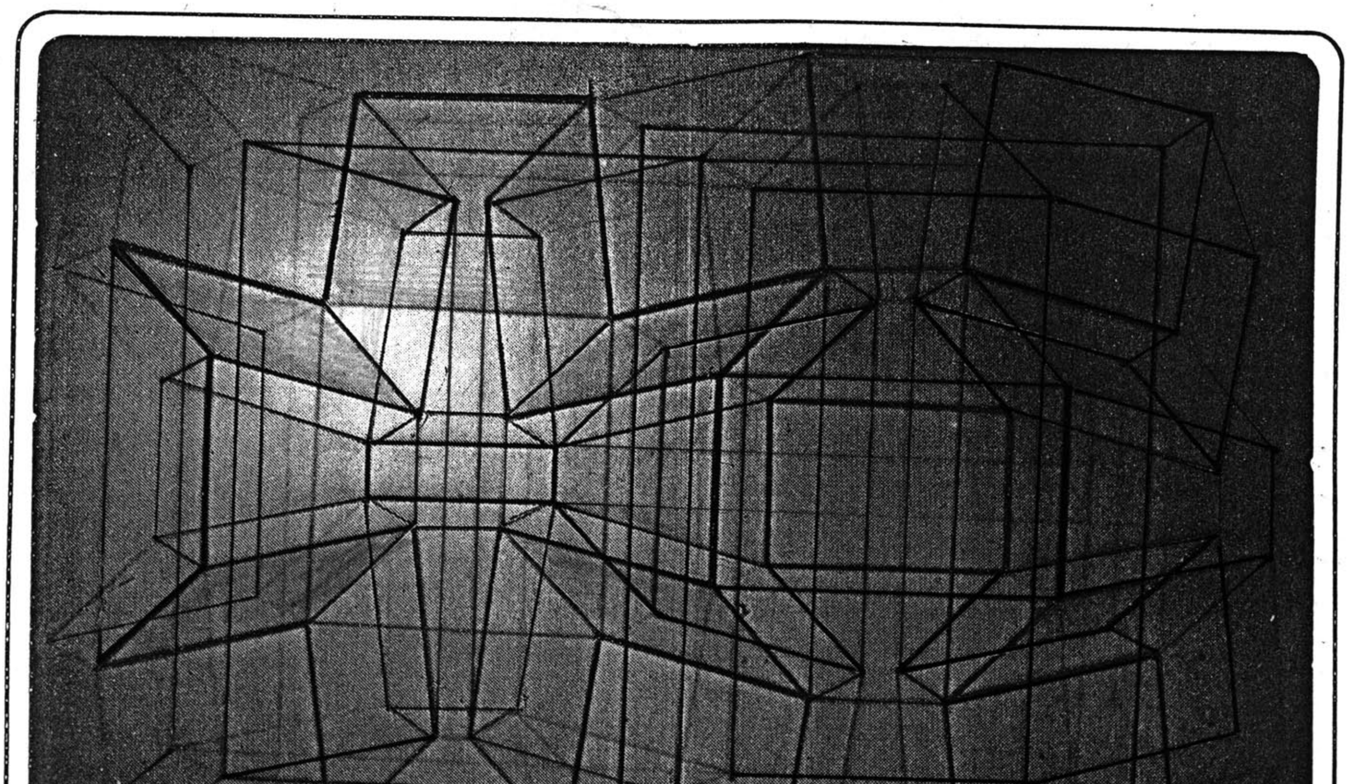
28



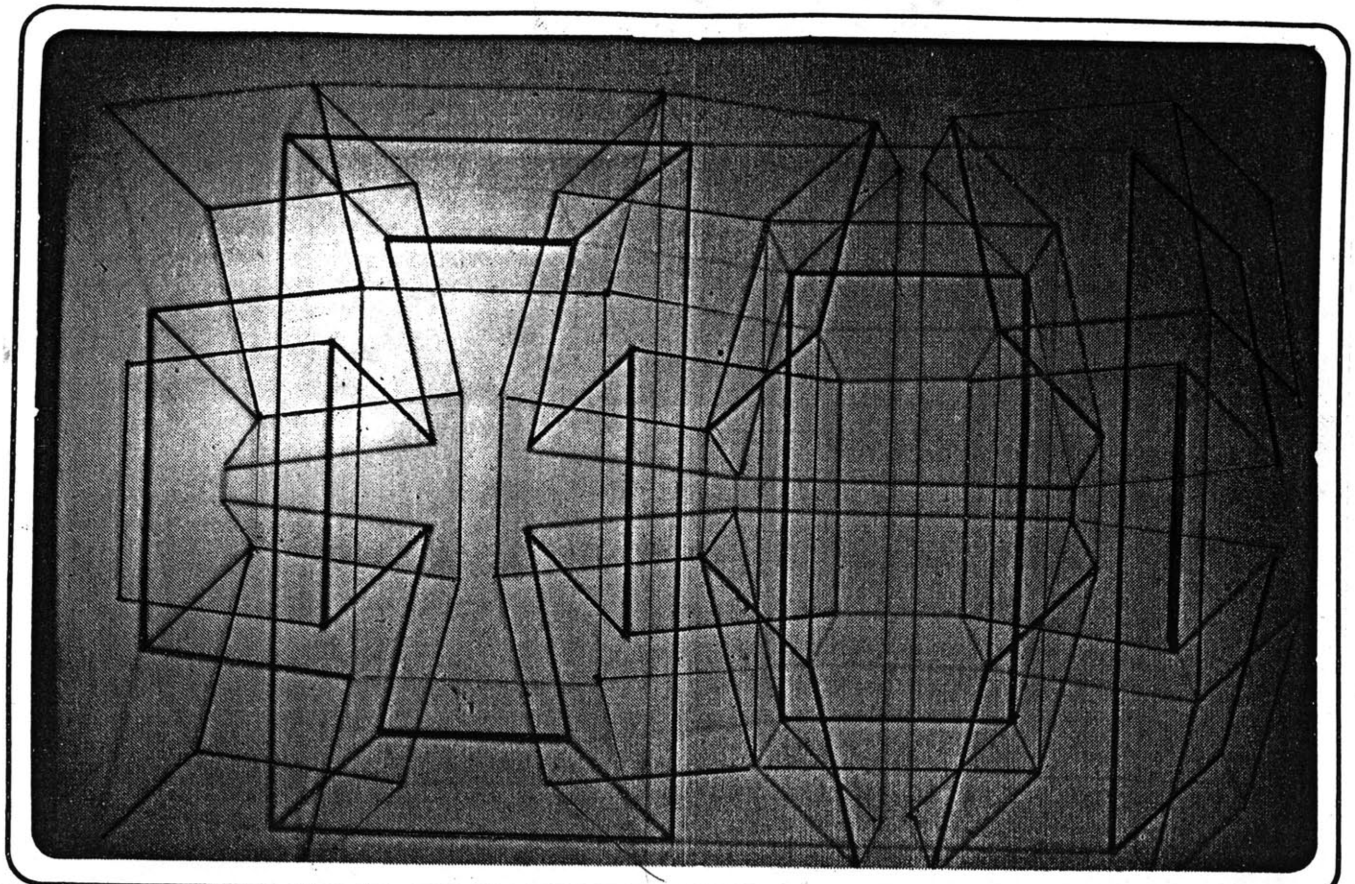
29



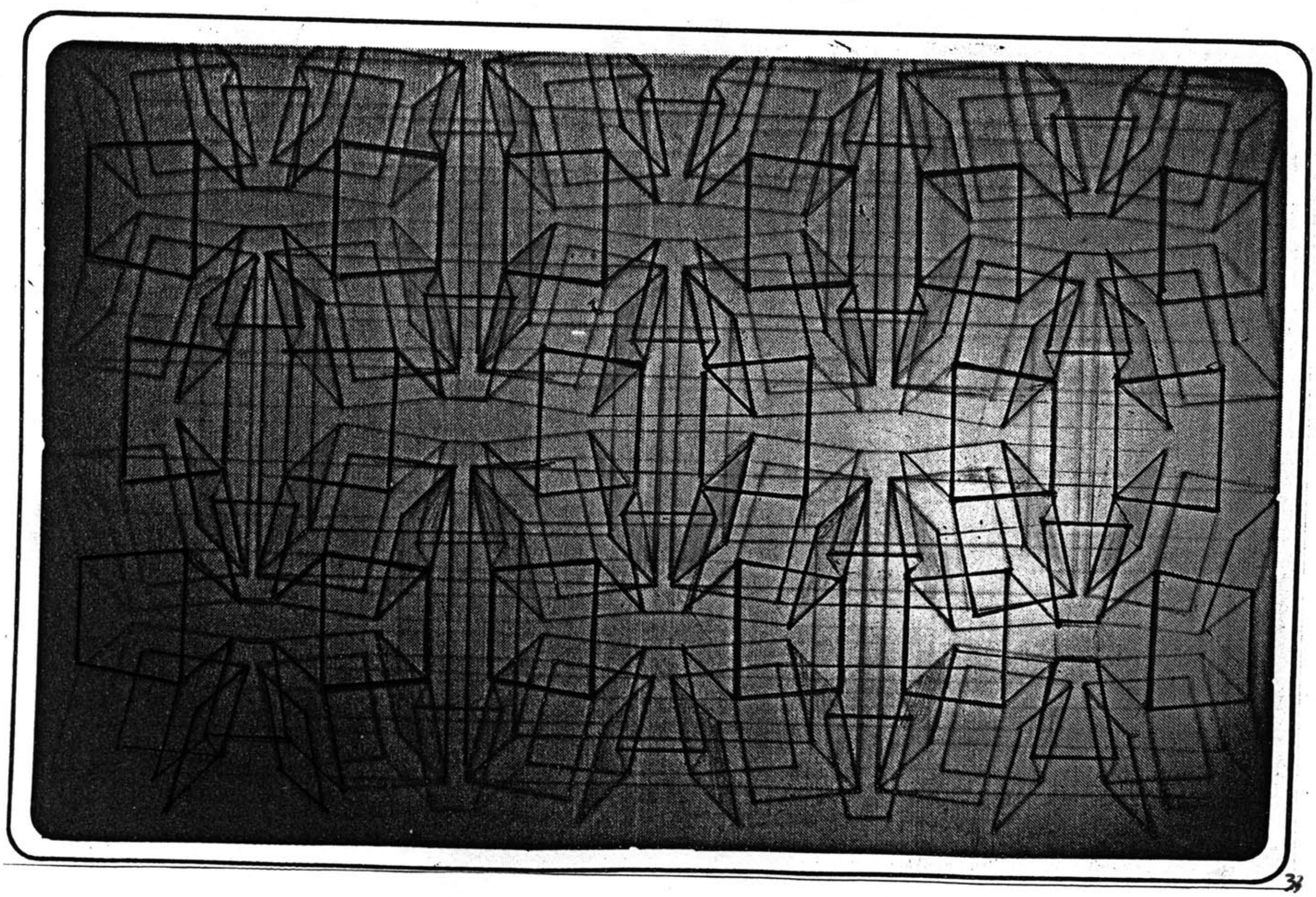
30



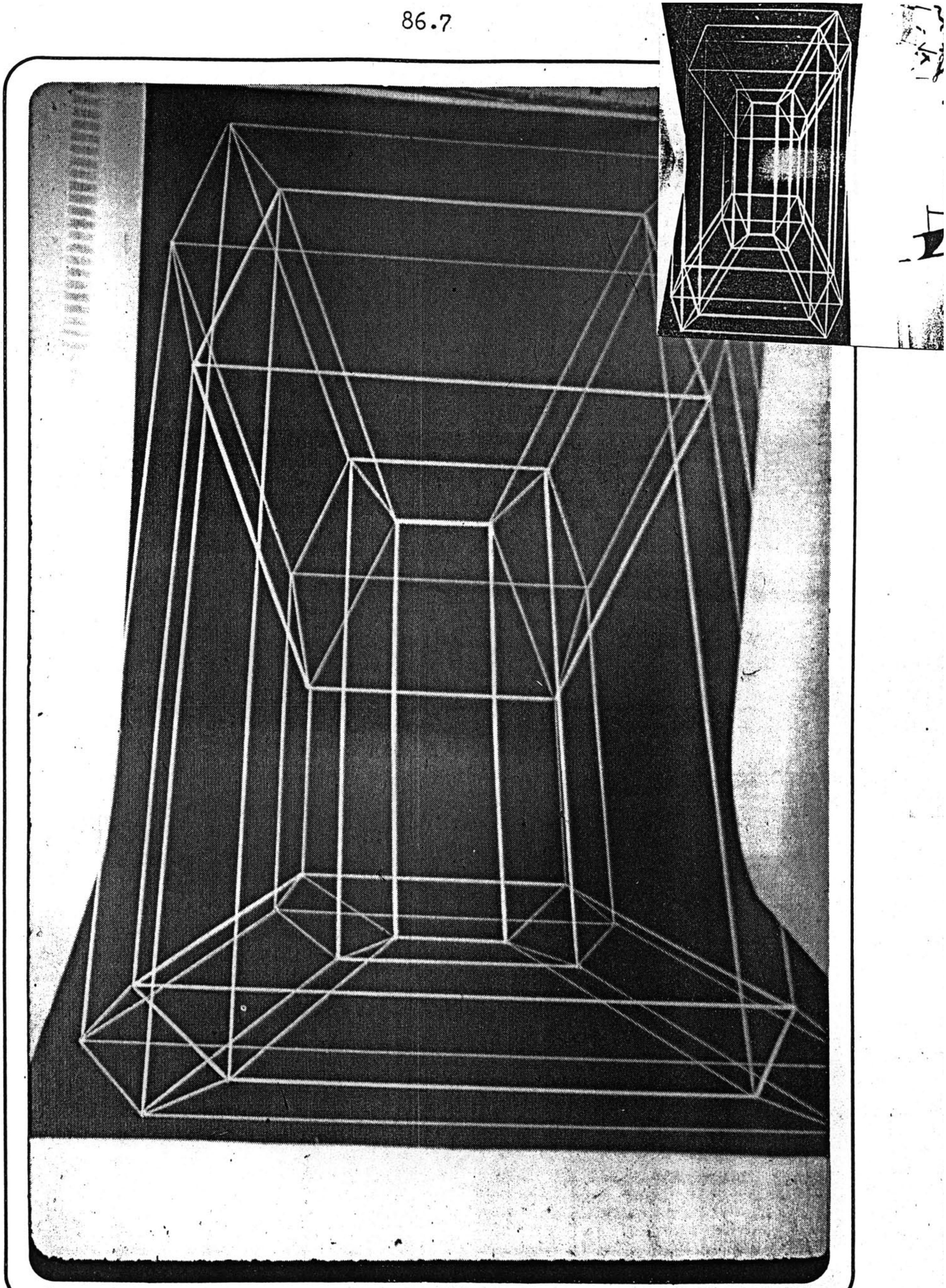
31

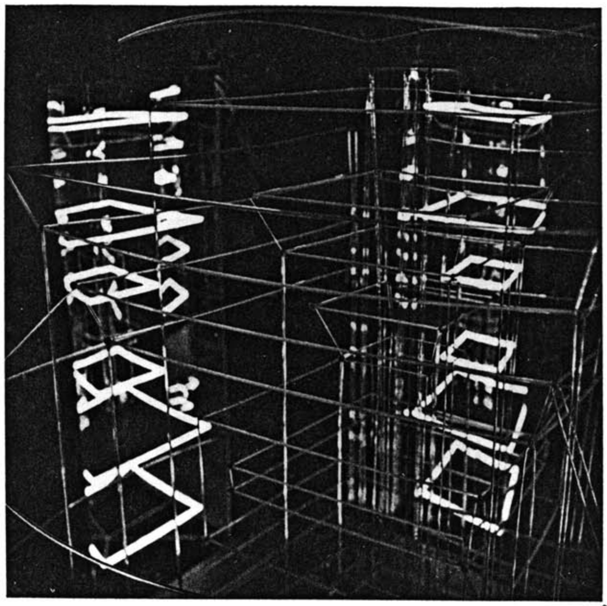


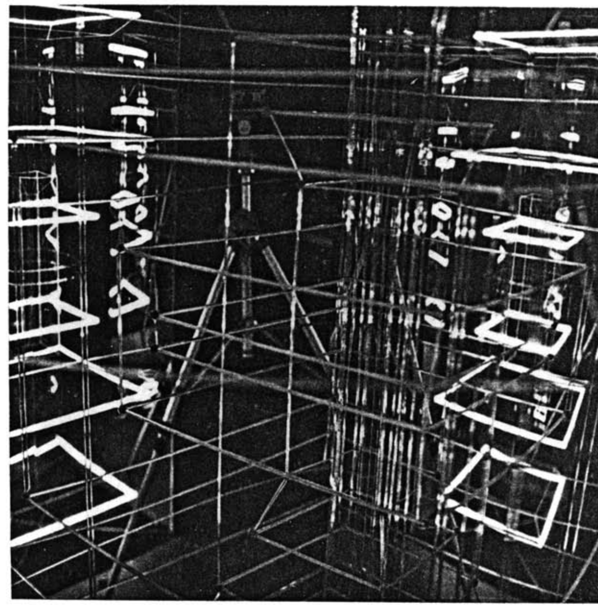
32

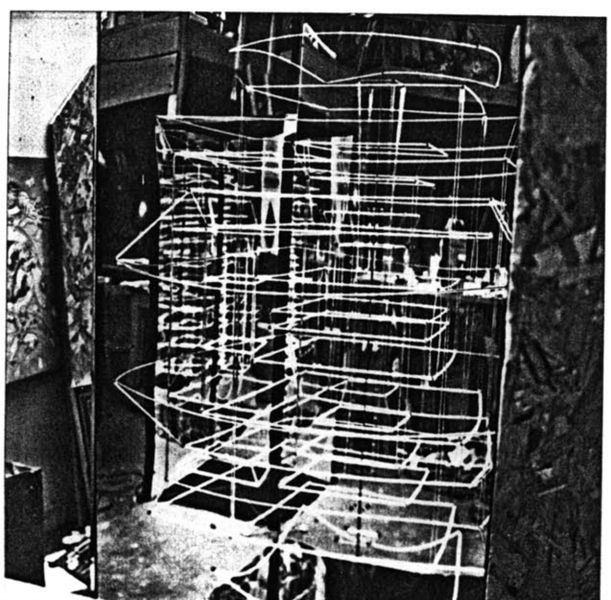


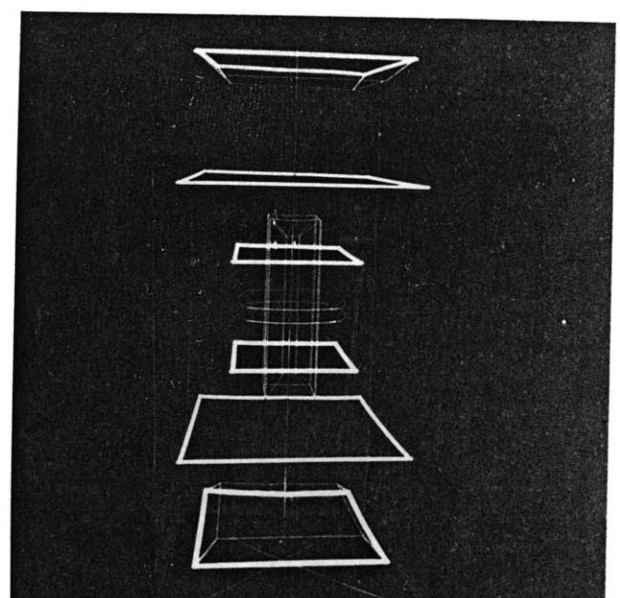
33











ILLUSTRATIONS

1. Armillary sphere I.M.S.S. Florence
Beckmann, P., The History of Pi, p. 98.

2. Brunes, T., The secrets of ancient geometry and its use, p. 106.

3. Japanese Zen calligraphic shows creation through progression from circle of unity, to triangle and manifestation of fourfold square. Symbolizing creation triadic tendency and manifest realm. Brush painting by Singai, Japan C 1830, Mitsu Art Gallery, Tokyo Arts Council of Great Britain.
Marie E., The Function of Esthetic Research, p. 84.

4. "Dancing atoms" refraction photo of atomic substrate. Dr. Edwin Muller, Pennsylvania State University
Lawlor, R., Sacred Geometry, p. 76.

5. Hexagonal symmetry in Snow Flake.
Ghyka, M., Geometrical Composition and Design, p. 69.

6. Fra Luca Pacioli and pupil C 1440 by Jide Barbari, Capodimonte Museum, Naples
Richter, I., Rhythmic Form in Art, p. 117.

7. Vitruvius' notion of man, "Vitruvian Man" drawing by Leonardo da Vinci, C 1490, Academy of Venic,
Huff, W., Symmetry Offprints, Vol. 6, p. 6.

8. Complete set of Scottish neolithic 'platonic solids' Ashmolean Museum at Oxford, from 1,000 years before Plato c 1500 B.C.,
Chitchlow, K., Time Stands Still, p.109.

9 &
10. Bilateral Symmetry.
Huff, W., Symmetry Offpints, Vol. 5, p. 10.

11. Carrol, L., Through the Looking Glass and What Alice Found There, p. 54.

12. Albers, J., "Structural Constellations", C 1954, Despite Straight Lines, p. 63 & 79.

13. Paul Donchian's models for the century of progress exposition in Chicago C.1930.
Coxeter, H., Regular Polytopes, p. 128.

14. (4-Pg.4) Class of counted hyper-parallelogram of five dimensional space.
Bachman, F., N-Gons, p. 197.

15. "6th Movement", The fourth dimension as temporal flow in space.
Alexander, R., Leonardo, vol. 16, no. 2, p. 85.

16. Michael, A., Bell Telephone Labs Stereoscopic Image of Hypercube Rotating on Four perpendicular Axis.

17. Fib. Tree growth in which each number of branches in the series is the sum of the two preceding numbers.

18. Harriet E Brisson, Hypergraphics, p.171

19. J.M.Yturralde, Hypergraphics, p.184

20. Five platonic solids drawn by Leonardo da Vinci form Paciolis De Divina Proportione, p.17

21. Logarithmic spiral, Leonardo, vol.16, no.2, p.92.

22-34. Selected drawings by author.

35. Streched rope hypercube.

36-39. neon and plexi hypercubes.

BIBLIOGRAPHY

1. Abbott, Edwin A., Flatland (A romance of many dimensions), Boston, Mass., Little, Brown & Company, 1941.

2. Alexandroff, Peter. Elementary Concepts of Topology, NY, Dover Books, 1953.

3. Arquelles,Jose, The Transformative Vision. Berkeley, CA. Random House, 1975.

4. Arnheim, Rudolf, Art and Visual Perception, Los Angeles, University of Cal., 1971.

5. Atkins, K.R., Symmetry and Conservation Laws in Physics, NY, John Wiley and Sons Inc., 1965.

6. Attneave, Fred, "Multistability in Perception", Sci. Am., Vol 225, Dec 1971, pp. 63-69.

7. Beckmann, Petr, The History of Pi, Boulder, Colorado, The Golem Press, 1970.

8. Boyer, Carl, "Zero: The symbol, The concept, The number", National Mathematical Magazine, Jan 1974.

9. Bragdon, Claude, 4D Vistas (Explorations into Four Dimensional Space, Lakemont, Georgia, CSA Press 1972.

9a. Bragdon, Claude, A Primer of Hyperspace, Tucson, Arizona, Omen Press, 1970.

10. Brauer, D., "Generalized Theorems of Polyotopic Projection", Transactions of the Royal Society of Canada, Vol 34 (1940) Section III, pp. 29-34.

11. Brisson, David W., Hypergraphics, Boulder, Colo., Westview Press, 1978.

12. Brisson, David W., "Visually Scanning Four-Dimensional Objects", Leonardo, Vol 13, (310), 1980.

13. Brunes, Tons, The Secrets of Ancient Geometry and its Use, NY, Copenhagen Press, 1967.

14. Bruno, Ernst, The Magic Mirror of McEscher, NY Random House Press, 1976.

15. Campbell, Joseph, The Hero With a Thousand Faces (Departure, initiation and return in myth and dream), NY, Princeton University Press, 1973.

16. Carrol, Lewis, Through the Looking Glass and What Alice Found There, Boston, Lee and Shephard Press, 1954.

17. Casey, Edward, Imaging as a Phenomenological Study, Bloomington Indiana, University Press, 1976.

18. Cassirer, Ernst, The Philosophy of Symbolic Form, NY, Yale University Press, 1967.

19. Ching, Francis, Architecture: Form, space and order, NY, Van Nostrand Reinhold Co., 1979.

20. Chilton, B.L., "Shadows of Four-Dimensional Polytopes", Mathematics Magazine, 44 (No. 5, Nov 1971).

21. Corbusier, L., The Modular, Cambridge, Mass., Harvard Press, 1954.

22. Coxeter, H.S.M., "Regular Polytopes in More Than Four-Dimensions", Journal of Mathematics and Physics, Vol 12, 1933, pp. 334-345.

23. Coxeter, H.S.M., Non-euclidean geometry, Canada, University of Toronto Press, 1942.

24. Coxeter, H.S.M., Regular Polytopes, NY, Macmillan Company, 1948.

25. Coxeter, H.S.M., Regular Complex Polytopic Projection, Cambridge University Press, 1974.

26. Critchlow, Keith, Islamic Patterns (an analytical and cosmological approach), NY, Schocken Books, 1976.

27. Critchlow, Keith, Order in Space, London, Thames and Hudson Limited, 1969.

28. Critchlow, Keith, Time Stands Still, London, Gordon Fraser Inter-scholarly Book Service, 1980.

29. Curjel, P., "Notes on the regular hypersolids", messenger of mathematics, Vol 28, 1899, pp. 190-193.

30. Dewey, John, Art As Experience, NY, Capricorn Books, 1981.

31. Dunne, L., An Experiment with Time, NY, Dover Publ. Inc.,1921.

32. Eckhart, Lugwig, Four-Dimensional Space (From Der Vierdimensionale Raum, 1929, trans by Arthur Bigelow, 1968), Bloomington, Indiana, Indiana University Press, 1968.

33. Escher, M.C., Antisymmetrical Arrangements in the Plane and Regular Three-dimensional Bodies as a Source of Inspiration to the Artist (Abs.)", Acta Crystall., Vol. 13, 1960.

34. Frey, D., "On the Problem of Symmetry in Art", Studium Generale, Vol. 34, 1975, pp. 276-279.

35. Fuller, R. Buckminster, Comprehensive design strategy, Illinois, Southern Illinois University Press, 1963.

36. Ghyka, Matila, Geometrical Composition and Design, London Alec Tiranti Ltd., 1952.

37. Ghyka, Matila, The Geometry of Art and Life, London, Alec Tiranti Ltd., 1954.

38. Greenberg, Clement, Art and Culture, Boston, Dover Books, 1961.

39. Goethe, Johann Wolfgang, The Theory of Colours and Perception, (Translated from German by C. Eastlake 1840), Cambridge, Mass., M.I.T. Press, 1970.

40. Gombrich, E.H., The Sense of Order, Ithica, Cornell University Press, 1977.

41. Gosset, T., "On Regular and Semi-Regular Figures in Space of N-dimensions", Messenger of Mathematics, Vol 29, 1900, pp. 43-48.

42. Gregory, R.L., The Intelligent Eye, NY, McGraw-Hill Publ., 1970.

43. Hambridge, Jay, Zeysing's Cannons of Polycletes (Ca.300 BC), NY, Dutton and Company Inc., 1919.

44. Hambridge, Jay, The Elements of Dynamic Symmetry, NY, Rover Press, 1967.

45. Health, T., A History of Greek Mathematics, Vol 2, Oxford Press, 1921.

46. Hilbert, D., Geometry and the Imagination, NY, Chelsa Publishing Co., 1952.

47. Henton, A New Era of Thought, London, Kristall Press, 1921.

48. Holden, Alan, Shapes, Space and Symmetry, NY, Columbia University Press, 1971.

49. Huff, William S., Symmetry Offprints Vol. 6 (Man's Aesthetic Response to his Contemplation of Self), Pittsburgh, Pa., 1970.

50. Huntley, H.E., The Divine Proportion, NY, Dover Publ. Inc., 1970.

51. Ihde, Don, Experimental Phenomenology, NY, Paragon Books, 1977.

52. Ivins, William M. Jr., Art and Geometry (A study in spacial intuition), NY, Dover Publ. Inc., 1946.

53. Kandinsky, W. , Concerning the Spiritual in Art , NY, Wassily Press, 1949.

54. Kepes, Gyorgy, Structure in Art and Science, NY, George Braziller Publ., 1965.

55. Klee, Paul, On Modern Art, NY, Faber and Faber, 1967.

56. Klee, Paul, Notebook Vol 2 (On the nature of nature), from Unendlich Naturgeschichte Trans. Heinz Norden, NY, George Wittenbern Inc., 1973.

57. Kubler, George, The Shape of Time, Yale University Press, 1962.

58. Kuller, H. Epitome at Astronomy, Book IV, Great books at the Western world, Chicago, Britannica Publ. 1952.

59. Lawlor, R., Sacred Geometry, NY, Crossroad Publ., 1975.

60. Lockwood, E.H., Geometric Symmetry, London, Cambridge University Press, 1978.

61. Lyusternik, L.A., Convex Figures and the Polyhedron, (Trans. from Russian by T.J. Smith), NY, Dover Books, 1963.

62. Mari, Enzo, The Function of Esthetic Research, Milano, Endizioni D: Comunita, 1970.

63. Mandelbrot, Benoit, B., Fractals (Form, Change and Dimension), San Francisco, W.H. Freeman & Co., 1977.

64. Nasr, S.H., An Introduction to Islamic Cosmological Doctrines, Cambridge, Mass., Tehran University Press, 1964.

65. Palladio, Andra, The Four Books on Architecture, Book 1 Chapter 1 (in Francis Ching's "Architecture: form, space and order), NY, Van Nostrand Reinhold Co., 1979.

66. Pacroli, Fra Luca,"<u>De Divina Proportione</u>"(illustrated by Leonardo de Vinci, 1509) reprinted in <u>Architectural Principles in the Age of Humanism</u>, London, University of London Press, 1949.

67. Penrose, L.S., "Impossible Objects: A special type of illusion", <u>British Journal of Psychology</u>, Vol. 49 (31), 1958.

68. Plato's <u>Timaeus</u> (translation by Francis M. Cornford), NY, Babba-Merrill Company Inc., 1959.

69. Plato's <u>Symposium</u> The diaglogues of Plato (translation by B. Jowell), NY, Random House, 1957, Vol. 1.

70. Rescher, N., <u>Studies in Arabic Philosophy</u> (Treatise on platonic solids and islamic philosophy, Al-Kind:, ABS.), NY, Columbia Press, 1966.

71. Richter, Irma A., <u>Rhythmic Form in Art</u>, London, Richard Clay and Sons, Ltd., 1932.

72. Rosenblum, R., <u>Cubisum and Twentieth Century Art</u>, NY Abrams & Sons, 1976.

73. Rucker, Rudolk V.B., <u>Geometry, Relativity and the Fourth Dimension</u>, NY, Dover Press, 1977.

74. Schlafli, L., "An Attempt to Determine the Twenty-Seven Lines Upon a Surface of N-Dimensional Space", Quarterly Journal of pure and applied mathematics, Vol. 2, 1858, pp. 110-120.

75. Scholfield, P.H., The Theory of Proportion in Architecture (from The"Geometry of the Environment"),Cambridge, Mass., M.I.T. Press, 1974.

76. Shubnikov, A.V., Color Symmetry (translated from Russian by Jack Itzkoff) Publications from the Academy of Sciences of U.S.S.R. Moscow 1951, NY, Macmillan Company, 1964.

77. Simkhovitch, V.G., "Approaches to History", Political Science Quarterly, Vol. 29, 1934, pp. 44-46.

78. Sirag, Abu Bakr, The Book of Certainty, NY, Scherken Books, 1952.

79. Sommerville, D.H., An Introduction to the Geometry of N-Dimensions, NY, Dover Press, 1941.

80. Stott, A.B., On Certain Series of Sections of the Regular Four-Dimensional Hypersolid, "Verhandelingen der Koninklyke", Vol.7.3, 1900, pp. 21.

81. Stringham, "Regular Figures in N-Dimensional Space", American Journal of Mathematics, Vol. 3, 1880, pp. 1-14.

82. Taylor, H.O., Ancient Ideals, NY, Macmillan Company, 1923.

83. Thomas, Brian, Geometry in Pictorial Composition, London, Oriel Press, 1971.

84. Tyng, AG, Simultaneous Randomness and Order: The fibonacci divine proportion as a universal forming principle, Ph.D. diss. 1974, Oxford Un., University microfilms international, Harvard.

85. Vanderbroeck, Andre, Philosophical Geometry, NY, Sandana Press, 1972.

86. Vitruvius, The Ten Books on Architecture, Book 2 (from "De Architectira" Ca. 27BC) translated by Morris Morgan, Cambridge, Harvard Press, 1926.

87. Jung, Carl, Man and His Symbols, N.Y., Dell Publ. Co., 1979.