VALUING THE FLEXIBILITY OF FLEXIBLE MANUFACTURING SYSTEMS

by

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Valuing the Flexibility of Flexible Manufacturing Systems*

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Abstract

This paper studies a topical issue: Flexible Manufacturing System (FMS) justification. We contend that current evaluation methods fall short of capturing a key advantage of an FMS: the value of flexibility. We identify various benefits of FMS that arise from the ability to switch between modes of production, and in particular, we model the value derived from the ability to better cope with uncertainty. A model to capture this value must solve for the value of flexibility together with the dynamic operating schedule of the production process. We present a stochastic dynamic programming model that captures the essential elements of this problem. A numerical example further demonstrates the optimal mode switching decision rules.

This research has several important managerial implications. It emphasizes the importance of ex ante economic justification of flexible manufacturing systems and proposes a way to modify existing capital budgeting techniques to incorporate the special features of flexibility. As the value of flexibility depends inherently on the design of the manufacturing system, the design and justification stages must be conducted simultaneously. We also show how to include other operating decisions in the valuation model. These can include the investment timing or the decision to temporarily shut down or to abandon the project entirely.

Introduction

Many managers and production engineers believe that conventional evaluation methods, such as net present value techniques, are not suited for analyzing investment in modern flexible manufacturing systems (FMS). Instead such investment decisions often are based on non-economic criteria. Recent surveys, for example, show that improvements in throughput, product quality, information flows, reliability, and other strategic advantages have been the primary motivations behind the decision to invest in FMS.[1]

At the same time, there is much evidence that despite large investments in modern manufacturing hardware, the productivity gap between the U.S. and Japan has been widening.[2] In particular, researchers have found that while flexibility is purported to be a prime advantage of modern manufacturing technologies, systems installed in the U.S. either are not very flexible or do not use the available flexibilities to best advantage[3] These observations have given rise to growing realization of the need to revamp the design, procurement, and management techniques used in modern manufacturing.

There is no question that engineering design and good operations management are very important to the success of FMS, but it is our contention that the main failure of FMS in the U.S. stems from the use of inappropriate *ex ante* criteria used in the justification of investments. Consequently, in

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1 See Graham and Rosenthal [1986].

2 For example see Adler [1985] and Jaikumar [1986].

3 See exhibit 1 in Jaikumar [1986], a comparison of functioning FMSs in Japan and the U.S. The author finds that in the U.S. relatively fewer parts are manufactured per machine and drastically fewer new parts are introduced.

some installations, tasks that are not suited for FMS have been assigned to the system. In others, inappropriately designed FMS are used in tasks that would have been profitably mechanized with a correctly designed FMS.

For FMS to succeed, it is essential to (a) include all effects arising from the introduction of the FMS in the capital budgeting decision, and (b) conduct the ex ante analysis under all feasible configurations in order to choose the optimum design configuration.

This paper takes a closer look at why conventional capital budgeting rules fail to capture the complexities of FMS projects. Once problems are identified we suggest ways to incorporate, in the justification stage, the special features of FMS. In particular, we develop a dynamic model that captures the option value of flexibility arising from the ability to better cope with uncertainty, and we implement this stochastic dynamic programming model using a stylized example.

The paper is organized as follows: In section 1, we describe issues that complicate the justification of flexible projects. Section 2 highlights the need for a stochastic dynamic model. Section 3 elucidates the model using an illustration. Section 4 presents results from a numerical simulation and interprets them. Section 5 discusses the feasibility of practical implementation of the approach, with some caveats.

1. Complexities of FMS Justification

A flexible manufacturing process adds value to the firm that can be attributed to changes in direct and indirect cash flows, operating flexibilities that enhance the firm's ability to cope with uncertainty, and non-pecuniary effects such as learning value. An evaluation of such an investment must weigh these effects against the incremental initial investment costs of installing an FMS. Although the problems associated with measuring costs and benefits are not trivial, we do not address them in this paper.[4]

The primary aim of this paper is to introduce a framework that incorporates the option value of flexibility. Flexible production processes give the firm an ability to modify, or in some cases reverse, decisions made in earlier periods. The option value stemming from such operating flexibilities has been noted by several researchers.[5]

Most previous efforts to model production flexibility fail to capture the essentially dynamic facets of operating flexibility. They have not acknowledged that the decision to operate a flexible system during the current period in a particular mode influences not only the payoffs in future periods but also the future operating decisions.

It is only recently that the operations research and management science community has focused on the value of flexibility. Fine and Freund [1986] have developed a static model that captures the value of product flexibility when firms face uncertain product demand.[6] In their model firms make a capacity investment decision before the resolution of future demand uncertainties. Flexible capacity that gives the firm the ability to respond to a variety of future demand outcomes is traded off against the increased investment cost. They model a firm that produces two products and chooses a portfolio of fixed and flexible capacity before receiving final demand information. Under strong assumptions about the form of the technology

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4 Kulatilaka [1985] gives a review of the pertinent literature and develops a cash flow forecasting model that includes both direct and indirect effects.

5 Jones and Ostroy [1984] first formalized this notion.

6 Discussions of definitions and classification of flexibility types can be found in Jaikumar [1985] and Graham and Rosenthal [1986]. Son and Park [1987] also discuss some static measures of flexibility.

(linear production functions), they solve a two-stage convex quadratic program and characterize the optimal profit function and the optimal investment policies.

Although the Fine and Freund model provides useful insights and takes a first crack at modeling the value of flexibility, it fails to capture the extremely important dynamic features. An example is switching between modes of production. When switching is costly, future switching decisions will be affected not only by revelations of demand change but also by the current production mode (and hence, past switching decisions).

In this paper we study the value of flexibility within a stochastic dynamic model. Monahan and Smunt [1984] took a similar approach in their study of the FMS investment decision. They develop a discrete time stochastic dynamic programming model where interest rates and "levels of technology" are assumed to be exogenous and evolving stochastically over time. The proportion of production using FMS is then chosen so that discounted costs are minimized.[7]

The approach we take here departs from that of Monahan and Smunt in two important aspects. First, we endogenize the utilization of the flexible technology, in that we model the choice of the optimal operating mode of the flexible system simultaneously with the valuation of the technology. Second, we explicitly derive the value of flexibility. In addition, we think that the

7 A review of FMS justification studies can be found in Suresh, N. C. and J. R. Meredith [1985].

exogenous variable with the most striking impact on value is the level of either input or output prices. Hence, we treat these as exogenous.[8]

The main insight leading to our formulation is that operating flexibilities can be viewed as a series of nested compound options similar to those encountered in complex financial securities. Financial options allow the exchange of one asset, whose value evolves stochastically over time, for another. A call option on a stock, for example, gives the holder the right to purchase the stock at a future date at a pre-specified exercise price. At any future date until expiration of the option, the holder of the option has the right to exchange the exercise price for the stock (i.e., purchase the stock at the exercise price).

Pursuing this analogy, consider a flexible technology with two modes of operation A and B (or the ability to produce two products). The firm is operating currently in mode A, and it can costlessly and instantaneously switch modes. At the next decision point, if mode B is more profitable than mode A the firm will switch modes. Otherwise it will continue to produce under mode A. If switching between modes is costly, though, the decision rule must take into account the effects of a current switch on all future production scenarios. The process describes a set of sequential options that are nested. We can value such options using results from compound option valuation.[9]

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8 The interest rate dynamics and dynamics of technology levels, which are treated as exogenous in the Monahan and Smunt model, are very difficult to estimate in practice. Furthermore, the conventional wisdom in the finance literature is that interest rate uncertainty is not a critical element in capital budgeting.

9 See Kulatilaka and Marcus [1986].

Unfortunately, the imperfect (not necessarily efficient) nature of production input and output markets makes it difficult to relate models of financial option valuation to the manufacturing setting. In particular, the arbitrage arguments used in financial option valuation do not apply when assets are not traded in efficient markets. Furthermore, the stochastic processes used to model stock prices are inappropriate in modeling sources of uncertainties that are typical in production systems.

The framework we propose resolves these problems and permits the inclusion of many realistic features of manufacturing by making explicit a simultaneous system of stochastic dynamic programs, which must be solved to obtain the optimal scheduling of the FMS and the ex ante value of flexibility. As a result of this generalization it is no longer possible to obtain closed form solutions. Nevertheless, numerical techniques used in solving such systems of dynamic programs are well known and easily implemented.

2. Valuing Operating Flexibilities: The Theoretical Framework

Here we outline a stochastic dynamic programming model that explicitly derives the value of flexibility stemming from the ability of an FMS to respond to changing conditions.[10] A flexible technology is stylized as one that embodies M alternative "modes" of operation. A mode of operation may refer to a routing within a production system, or to a particular set of inputs used in manufacturing the product. Such a definition describes process flexibility. A mode also may refer to the choice of output, in which case it will describe product flexibility. Finally, a mode can be used to characterize waiting to invest, shutting down, and abandoning of projects.

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10 This model is described in detail in Kulatilaka [1986].

Associated with each mode is a cash flow pattern (profit stream) that is determined by the realization of some uncertain variable(s), Θ (e.g., demand, price, exchange rate). The i^{th} mode's profit (or cost) function (over a unit of time), given that state of the world Θ_k occurs, is denoted by $\pi^j(\Theta_k)$. The FMS can switch between modes i and j incurring a cost δ_{ij}. Switching costs may come from a variety of sources such as retooling, retraining, lost time, inventory changes, and compensatory wages.

The evolution of Θ is characterized by a stochastic process.[11] Suppose that Θ_k can realize N different states. Define the transition probability that $\Theta(t+1)$ is state i given that $\Theta(t)$ was state j as p_{ij}: i.e., $p_{ij}=prob(\Theta_{t+1}=i/\Theta_t=j)$. In general p_{ij} can depend on time, values of Θ, or any other variables. In the numerical example we assume Θ to be stationary, and hence parameters are constant over time. The NxN transition probability matrix with elements p_{ij} is denoted by P.

Suppose the flexible system has a life of T periods and that it reaches the last period employing mode m $\{m\varepsilon(1,M)\}$. The chronology is defined such that the first period begins at time 0 so that the last (T^{th}) period begins at time T-1. At T-1 the firm will observe the realization of $\Theta(T-1)$, say state j, and realize a profit of $\pi^m(\Theta_{T-1})$.[12] One way to think of the time periods

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11 For example, we can represent Θ with a diffusion process:

$$d\Theta = \alpha(\Theta,t)\ dt + \sigma(\Theta,t)\ dZ$$

where dZ is a standard Wiener process. We can then discretize the process to fit this model.

12 The profit functions are the begining-of-period present values of the flow profits over a single period.

is as discrete approximations to the continuous decision process.[13]

Alternatively, we can think of the time periods as given exogenously by contracting arrangements or other organizational constraints, for example.

With just one period remaining in its life (i.e., at time T-1), the value of the system is known with certainty and is given by the maximum of the profits under the different modes:

$$(1) \qquad F(T-1,m) = \max_{i} [\pi^{i}(\Theta_{T-1}) - d_{i}].$$

where the indicator variable $d_i = 0$ if $i=m$ (remain in mode m)

$d_i = \delta_{mi}$ if $i \neq m$ (switch to mode i)

At periods prior to T-1, however, the future realizations of Θ are unknown to the firm. For example, consider the mode choice decision at time T-2. Suppose T-2 is reached with mode m in use. The value of the flexible system at T-2 is then the value over the next period using the mode that maximizes that period's net profits (net of switching costs) plus the expected value (at time T-2) of the value at T-1.

$$(2) \qquad F(T-2,m) = \operatorname{Max}_{i} \{ \pi^{i}(\Theta_{T-2}) - d_{i} + \phi\, E_{T-2}[F(T-1,i)] \} \qquad i=1,\ldots,M$$

where ϕ is the one-period present value factor, which is computed as $1/(1+r)$, and r is the risk-adjusted discount rate.

Several caveats regarding the discount rate bear noting. If the underlying asset follows an equilibrium growth rate, then we can take "risk

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13 By choosing arbitrarily small time periods we can approximate the continuous time case.

neutral expectations" as in Cox and Ross [1976] and use the risk-free rate in discounting cash flows. The problem is that production inputs and outputs are not necessarily traded in efficient markets, unlike financial markets. Hence, their growth rates need not equal the fair rate of return. In such cases we can use an insight due to McDonald and Siegel [1984]: replace the drift term of the diffusion process by the equilibrium rate and then follow risk-neutral discounting. Such an adjustment, however, requires the additional imposition of an asset pricing model.[14]

Expectations are taken over all possible realizations of Θ and can, in general, be computed as follows:

$$(3) \qquad E_t[F(t)] = \sum_{k=1}^{N} \{F(\Theta_{t+1},k)\, p_{mk}(t)\}.$$

Thus, the dynamic programming equations at time t can be written as

$$(4) \qquad F(t,m) = \max_i \{ \pi^i - d_i + \phi\, E_t[F(t+1)] \} \qquad i,m=1,\ldots,M \text{ and } t=0,\ldots,T-2.$$

14 See Kulatilaka and Marcus [1986] for details.

These M-simultaneous dynamic programs can be solved numerically to obtain the value functions F(t,i) for all t and i. Furthermore, argument maximum will yield the optimal mode choice (i.e., M_t/M_{t-1}=m is obtained by replacing Max with Argmax in equation 4).

3. A Stylized Example

A firm facing some exogenous uncertainty, say price (Θ), is considering an investment in a flexible manufacturing system. The price dynamics are modeled by a mean reverting stochastic process:

$$d\Theta = \mu(\Theta^{\circ}-\Theta)\ dt + \sigma_{\Theta}\ dZ_{\Theta}$$

$$= .05(.5-\Theta)\ dt + .4\ dZ_{\Theta}$$

The instantaneous drift term $\mu(\Theta^{\circ}-\Theta)$ acts as an elastic force that produces mean reversion. In the numerical solutions we discretize Θ within the range [0,1] with a grid size of .02 (i.e., Θ is allowed to take 51 discrete values from 0 through 1.0). The base case probability transition matrix appears in Table 1.

We characterize the firm before it makes the investment as "waiting to invest," which we define as mode 1. Once it makes the investment, the firm can produce in one of two modes (mode 2 or mode 3) and incur the investment cost, I. While operating in either production mode, the system has the ability to switch to the other production mode or to temporarily shut down (mode 4). Shutting down the plant will incur some shutdown costs, and while the firm remains in this mode it will continue to incur fixed costs. From the shutdown mode, the firm can start up in one of the two production modes or abandon the plant (mode 5) and receive the salvage value. We summarize the options in our notation as follows;

Mode	Description	Profit Flow per Period
1	waiting to invest	0
2	production mode A	$\pi^2(\Theta)$
3	production mode B	$\pi^3(\Theta)$
4	shut down	-F
5	abandon	0

For purposes of this example, profit functions for the two production modes are assumed to be linear in Θ. The feasible set of the switching possibilities, switching costs, and their base case values is summarized below:

$\delta_{12} = \delta_{13} = I = 1.0$ (initial investment[15])
$\delta_{23} = \delta_{32} = 0.005$ (cost of switching between production modes[16])
$\delta_{24} = \delta_{34} = 0.005$ (cost of shutting down the plant)
$\delta_{42} = \delta_{43} = 0.005$ (cost of startup (from the shutdown mode)
$\delta_{45} = -S = -0.5$ (receive the scrap value)

To simplify the technologically feasible set of outcomes and the optimal technology choice, we first analyze the value of the flexible system within a single period setting. Figure 1 plots the net profit functions for each of the five fixed modes against the possible values of Θ.

The net profit of the two production modes is given by the single-period profit (π^i, i=2,3) less the initial investment (δ_{1i}, i=2,3). We can extend these plots to study the optimal mode choice in a static framework and derive the value of the flexible system. Figures 2a,2b, and 2c present the profit scenarios when the initial modes are 1,2, and 4, respectively. These figures

15 Initial investment cost and scrap value easily can be made contingent on time and usage.

16 Switching costs need not be symmetric.

also can be interpreted as the value functions as of the beginning of the last period: i.e., F(T-1,i), i=1,2,4.

Consider Figure 2a. The solid line plots F(T-1,1), the value of the flexible system if the firm reaches time T-1 without having made the initial investment, against Θ. The switching decision rule requires only a simple comparison of net profits.

Condition	Decision
$\Theta_{T-1} \leq \Theta^*_{12}$	do not invest
$\Theta^*_{12} \leq \Theta_{T-1} \leq \Theta^*_{13}$	invest and operate in mode 2
$\Theta^*_{13} \leq \Theta_{T-1}$	invest and operate in mode 3

Figure 2b graphs F(T-1,2) and the decision rule is now:[17]

Condition	Decision
$\Theta_{T-1} \leq \Theta^*_{24}$	shut down the plant
$\Theta^*_{34} \leq \Theta_{T-1} \leq \Theta^*_{24}$	continue to operate with mode 2
$\Theta^*_{34} \leq \Theta_{T-1}$	switch to operating with mode 3

We must highlight the determination of the switch points Θ^*_{24} and Θ^*_{23}. Θ^*_{24}, the price at which it becomes optimal to shut down given that the plant is operating in mode 2, is the intersection of π^2 and the horizontal line, $-F-\delta_{24}$; Θ^*_{23}, which has a similar interpretation as above is the intersection of π^2 and $\pi^3-\delta_{23}$. We compute these values for comparison with the corresponding switch points for the dynamic case.

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17 A very similar decision rule can be derived by studying F(T-1,3).

Figure 2c shows the case when starting with the shutdown mode. The resulting decision rule is

Condition	Decision
$\Theta_{T-1} \le \Theta^*_{42} = \Theta^*_{45}$	abandon project
$\Theta^*_{42} \le \Theta_{T-1} \le \Theta^*_{43}$	start up with mode 2
$\Theta^*_{43} \le \Theta_{T-1}$	start up with mode 3

Notice that in this essentially static approach, the firm will never remain shut down: it will either abandon the project or start up. As there is no future switching opportunity, when prices are low enough it is best to abandon and receive the scrap value, or, when prices are high enough it is best to start up.

Our model builds richness by expanding this static framework to a dynamic one. We have included so much discussion in order to clarify why and how the decision rules and the optimal switching points are modified when the firm has future switching opportunities. For example, consider the situation depicted in figure 2c at some time prior to T-1 and when the price is below Θ^*_{45}. As it is possible that future values of Θ can be very high, thereby allowing the realization of high profits, it may be optimal to remain shut down and not abandon the project altogether.

4. Numerical Simulations

The numerical simulations use the following profit functions to represent cash flows from the two production modes (mode 2 and mode 3);

$$\pi^2(\Theta) = a_2 + b_2\Theta = -.6 + 2\Theta$$
$$\pi^3(\Theta) = a_3 + b_3\Theta = -.1 + \Theta$$

The parameter values are chosen such that for low prices ($\Theta<.5$) $\pi^3>\pi^2$ while for higher prices ($\Theta>.5$) $\pi^3<\pi^2$.

We study dynamic behavior by setting the life of the project equal to 50 periods (i.e., T=50), solving the dynamic program to obtain the optimal switching points and the value functions for each period. The results are summarized in Table 2 and Figure 3.

Figure 3 plots the time zero value of the flexible system (given that it was waiting to invest,) $F(t=0,M_0=1)$, against the possible outcomes of the uncertain variable, Θ. For comparison with investments in fixed systems we also plot the profit functions of the two fixed mode technologies, π^2 and π^3, in the same graph. The shaded area between the flexible and the maximum profit of the fixed systems gives the value of flexibility. At the mean value of Θ (i.e., if $\Theta_0=0.5$), the present value of the flexible system (at approximately 19) is about twice that of the fixed systems (at approximately 8.2). The value comes from the existence of the options (a) to wait to invest, (b) to operate in one of two production modes, (c) to shut down and startup, and (d) to abandon the project altogether. If the incremental investment required to install the flexible system is less than the value of flexibility, the firm should chose the flexible system over the fixed-mode systems.

Table 2 presents a summary of the operating modes for different prior operating modes and for different values of Θ. For example, column 1 gives the optimal mode of operation for the first period, given that the time zero mode was 1 (i.e., $M_{50}/M_{49}=1$). Looking down column 1, we note that if the initial period value of Θ is less than .16, it is optimal to wait to invest. For $.16\geq\Theta_0\geq.38$, the firm should produce with operating mode 3. For $\Theta_0\geq.4$, it should operate with mode 2.

The second column of Table 2 gives optimal modes for time 1 when starting in mode 2 (i.e. $M_1/M_0=2$). Now it is optimal to shut down the plant if $\Theta_0\leq.10$; switch modes and operate with mode 3 if $0.10\leq\Theta_0\leq0.38$; continue operating with mode 2 if $\Theta\geq.40$. Similarly, column 3 gives the operating mode when $M_0=3$. The only change between this and the previous case is that the firm should choose to operate in mode 3 until $\Theta_0\leq.40$ (compared with .38) because of the cost of switching between modes 2 and 3. Finally, column 4 reports the recommended modes when the plant is shut down during at the start. Because there are no shutdown or startup costs, these values are identical to those in column 2.

It should be noted that for the base case parameter values, with 50 periods of life remaining, the plant will not be abandoned for any realization of Θ_0. With shorter times to maturity (remaining life), however, sometimes it is optimal to abandon the project. These are depicted in Table 3, which reports the optimal mode choices for the last period.

5. Managerial Implications, Caveats, and Limitations of the Model

Our point has been that current evaluation methods fall short of capturing an essential feature of an FMS: the value of flexibility. An important benefit of flexibility is enhancement of the firm's ability to better cope with uncertainty. The model developed here aims to specify how much flexibility is worth.

This research has several important managerial implications. Recognizing the importance of ex ante economic justification of FMS, it proposes a way to modify existing capital budgeting techniques to incorporate the special features of flexibility. As the value of flexibility is inherently dependent on the design of the manufacturing system, the design and justification must be conducted simultaneously. We also show that other operating decisions, such as investment timing and decisions to temporarily shut down or abandon a project, can be included in the valuation model.

Appealing as the concept may be, admittedly its practical implementation requires overcoming many measurement and modeling hurdles. Although common to all capital budgeting exercises, a foremost problem is the measurement of flow profit functions over the life of the project. Typically engineering cost studies or economic factor demand studies are used to estimate such profit functions.[18] This model, however, requires ex ante measures of future cash flows and deal with new technologies for which there may not be historical cost data. Hence, profit function estimation must be derived by combining information from design engineers and manufacturing managers. The profit function coefficients will depend not only on the engineering specifications but also on the way the plant is configured and operated.[19]

Another major hurdle is the estimation of the stochastic dynamics of Θ. If Θ is the price of a traded input, then the stochastic process can be statistically estimated using historical time series data on its spot price.[20]

An especially relevant example is seen in the case of modern manufacturing plants that purchase electricity in spot markets. A firm may switch away from purchased electricity (either to another source of energy or to an inoperative mode by shutting down the plant) depending on relative fuel prices. The spot price is determined by random supply and demand conditions

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18 See Berndt and Wood [1979] for details on the estimation and interpretation of cost and profit functions.

19 For example, scheduling algorithms and pricing strategies can both influence the cash flows.

20 See Marsh and Rosenfeld [1983].

facing the power grid, so the spot price of electricity (and hence relative fuel prices) will evolve stochastically over time.

In some cases, the implied parameter values of the stochastic process may be inferred from market rationality. For example, if the source of exogenous uncertainty is oil prices, and the market for a particular oil-using machine is competitive, then the implied variance can be solved for by setting the market price equal to the value of the machine.

In many other instances, however, the Θ process may be much harder to estimate. In extreme cases the possible outcomes and the associated probabilities may require subjective judgment.

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Table 1

Transition Probability Matrix

$$d\Theta = .05(.5-\Theta)\,dt + .4\,dZ$$

	0.0	.01	.02	.03	.04	.05	.06	.07	.08	.09	1.0
0.0	0.525	0.431	0.340	0.258	0.187	0.130	0.087	0.055	0.033	0.019	0.010
0.1	0.098	0.099	0.095	0.087	0.074	0.060	0.046	0.034	0.023	0.015	0.009
0.2	0.090	0.098	0.099	0.096	0.087	0.075	0.061	0.047	0.034	0.024	0.015
0.3	0.079	0.090	0.097	0.099	0.096	0.088	0.076	0.062	0.048	0.035	0.024
0.4	0.064	0.078	0.089	0.097	0.099	0.096	0.088	0.077	0.063	0.049	0.036
0.5	0.049	0.064	0.077	0.089	0.097	0.099	0.097	0.089	0.077	0.064	0.049
0.6	0.036	0.049	0.063	0.077	0.088	0.096	0.099	0.097	0.089	0.078	0.064
0.7	0.024	0.035	0.048	0.062	0.076	0.088	0.096	0.099	0.097	0.090	0.079
0.8	0.015	0.024	0.034	0.047	0.061	0.075	0.087	0.096	0.099	0.098	0.090
0.9	0.009	0.015	0.023	0.034	0.046	0.060	0.074	0.087	0.095	0.099	0.098
1.0	0.010	0.019	0.033	0.055	0.087	0.130	0.187	0.258	0.340	0.431	0.525

Table 2
The Optimal Mode to Operate: t=1

Θ	$M_1/M_0=1$	$M_1/M_0=2$	$M_1/M_0=3$	$M_1/M_0=4$
0.00	1	4	4	4
0.02	1	4	4	4
0.04	1	4	4	4
0.06	1	4	4	4
0.08	1	4	4	4
0.10	1	4	4	4
0.12	1	3	3	3
0.14	1	3	3	3
0.16	3	3	3	3
0.18	3	3	3	3
0.20	3	3	3	3
0.22	3	3	3	3
0.24	3	3	3	3
0.26	3	3	3	3
0.28	3	3	3	3
0.30	3	3	3	3
0.32	3	3	3	3
0.34	3	3	3	3
0.36	3	3	3	3
0.38	3	3	3	3
0.40	2	2	3	2
0.42	2	2	2	2
0.44	2	2	2	2
0.46	2	2	2	2
0.48	2	2	2	2
0.50	2	2	2	2
0.52	2	2	2	2
0.54	2	2	2	2
0.56	2	2	2	2
0.58	2	2	2	2
0.60	2	2	2	2
0.62	2	2	2	2
0.64	2	2	2	2
0.66	2	2	2	2
0.68	2	2	2	2
0.70	2	2	2	2
0.72	2	2	2	2
0.74	2	2	2	2
0.76	2	2	2	2
0.78	2	2	2	2
0.80	2	2	2	2
0.82	2	2	2	2
0.84	2	2	2	2

Definition: $M_1/M_0=i$ is the optimal mode at time 1 (M_1) given that the mode in the previous period (M_0) was mode i.

Table 3
The Optimal Mode to Operate: t=50

Θ	M_{50}/M_{49}=1	M_{50}/M_{49}=2	M_{50}/M_{49}=3	M_{50}/M_{49}=4
0.00	1	4	4	5
0.02	1	4	4	5
0.04	1	4	4	5
0.06	1	4	4	5
0.08	1	4	4	5
0.10	1	4	4	5
0.12	1	3	3	5
0.14	1	3	3	5
0.16	1	3	3	5
0.18	1	3	3	5
0.20	1	3	3	5
0.22	1	3	3	5
0.24	1	3	3	5
0.26	1	3	3	5
0.28	1	3	3	5
0.30	1	3	3	5
0.32	1	3	3	3
0.34	1	3	3	3
0.36	1	3	3	3
0.38	1	3	3	3
0.40	1	2	3	3
0.42	1	2	2	2
0.44	1	2	2	2
0.46	1	2	2	2
0.48	1	2	2	2
0.50	2	2	2	2
0.52	2	2	2	2
0.54	2	2	2	2
0.56	2	2	2	2
0.58	2	2	2	2
0.60	2	2	2	2
0.62	2	2	2	2
0.64	2	2	2	2
0.66	2	2	2	2
0.68	2	2	2	2
0.70	2	2	2	2
0.72	2	2	2	2
0.74	2	2	2	2
0.76	2	2	2	2
0.78	2	2	2	2
0.80	2	2	2	2
0.82	2	2	2	2
0.84	2	2	2	2

Definition: M_{50}/M_{49}=i is the optimal mode at time 50 (M_{50}) given that the mode in the previous period (M_{49}) was mode i.

Figure 1

Profit Functions for the Fixed Modes against θ

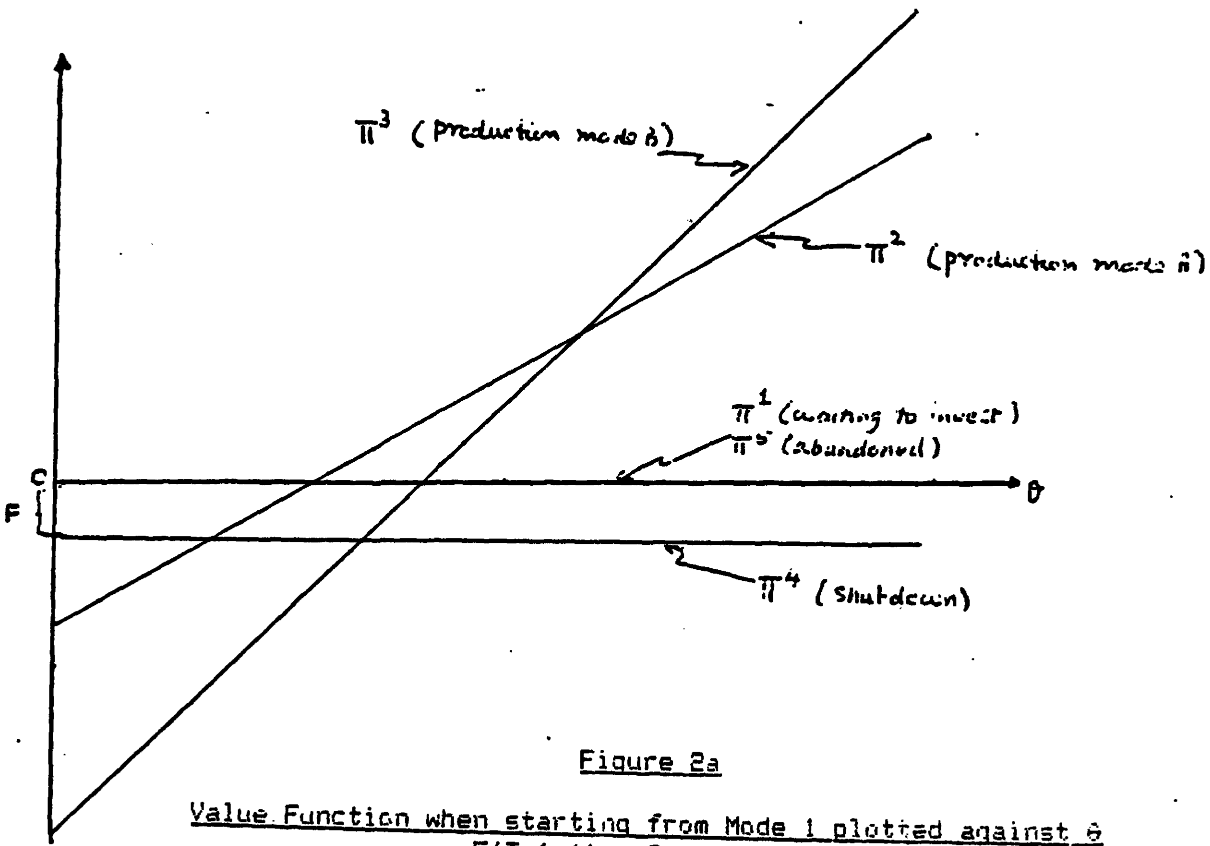

Figure 2a

Value Function when starting from Mode 1 plotted against θ

F(T-1,1) : Static Case

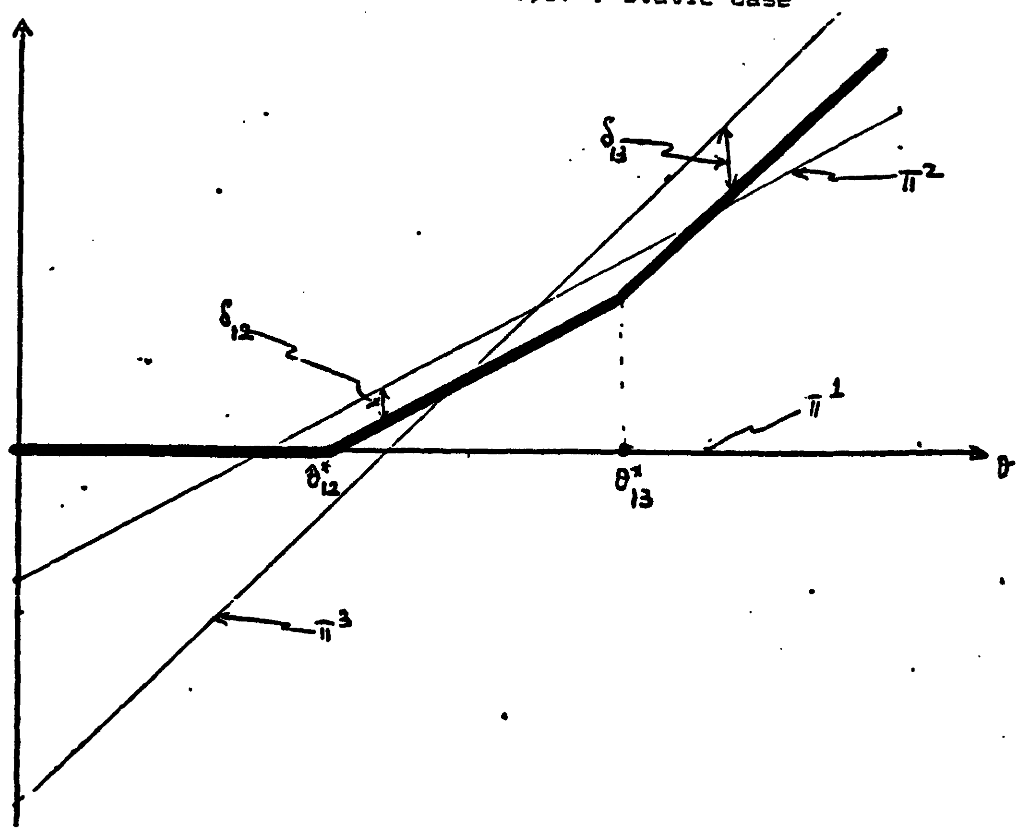

Figure 2b

Value Function when starting from Mode 2 plotted against θ

F(T-1,2) : Static Case

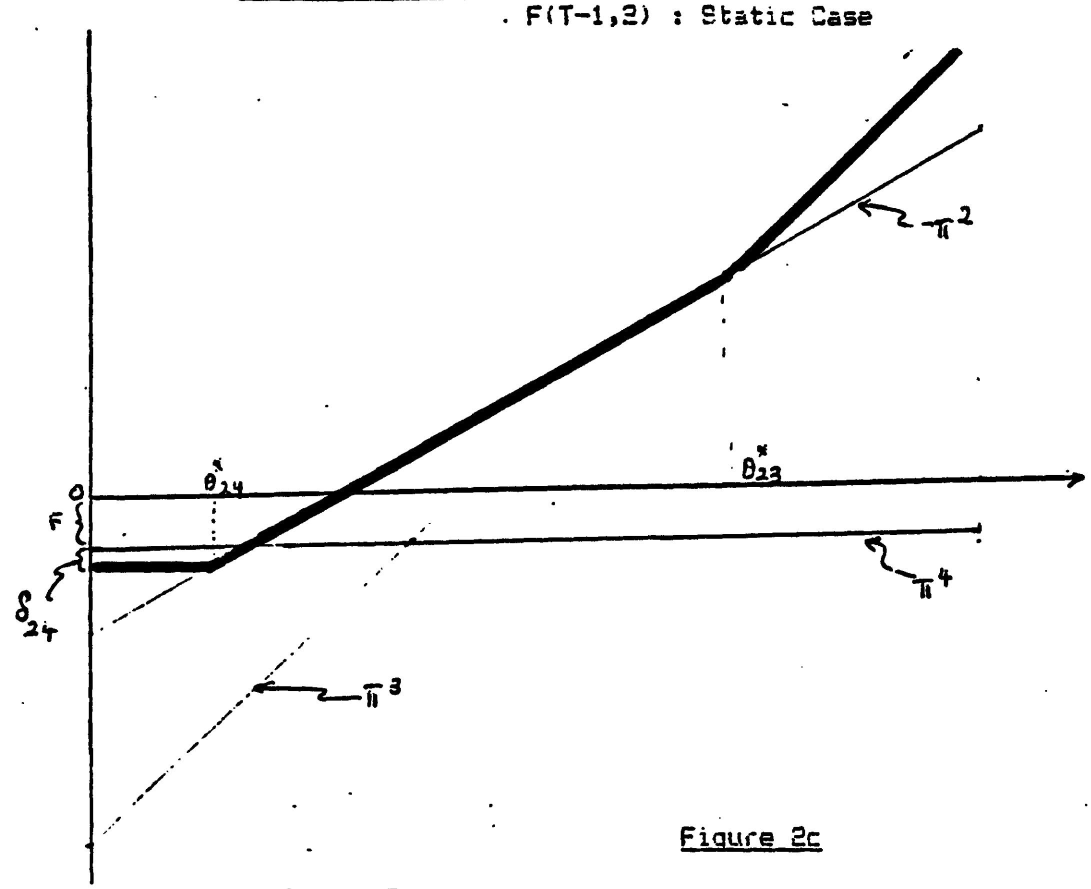

Figure 2c

Value Function when starting from Mode 4 plotted against θ

F(T-1,4) : Static Case

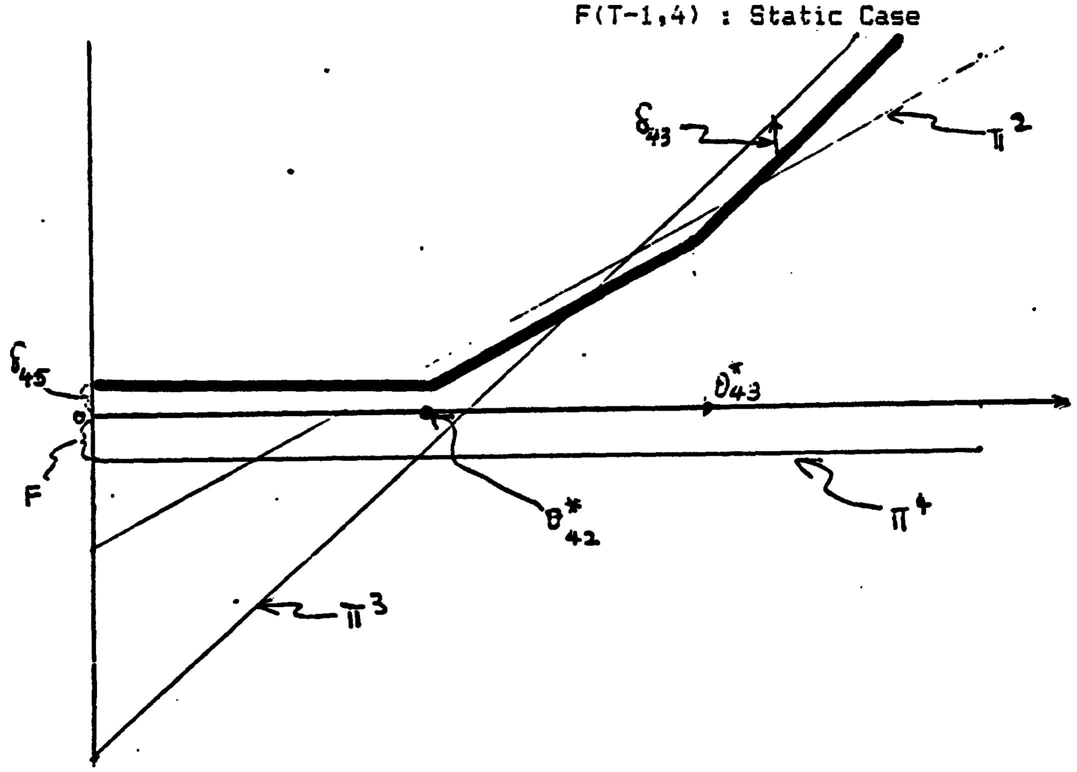

Figure 3

Value of Flexible System at t=0