Designing-in of Quality Through Axiomatic Design

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Summary & Conclusions — Decisions made during the design stage of product & process development profoundly affect product quality and process productivity. To aid in design decision making, a theoretical framework is advanced: the axiomatic approach to design. Axiomatic design consists of: 1) domains in the design world, 2) mapping between these domains, 3) characterization of a design by a vector in each domain, 4) decomposition of the characteristic vectors into hierarchies through a process of zigzagging between the domains, and 5) the design axioms, *viz*, Independence & Information Axioms. Statistical process control (SPC) and other methodologies to improve quality are valid only when they are consistent with the Independence & Information Axioms. This paper presents several criteria that govern the design & manufacture of quality products, To be able to control the quality of products, a design must satisfy the Independence Axiom. Based on this axiom and some theorems, several design criteria are derived & discussed. These criteria provide the bounds for the validity of some of the SPC techniques being used. When there is more than one acceptable design of a product or process, the Information Axiom must be used to select the best design(s).

1. INTRODUCTION[1]

Consumers, more than ever, expect high-quality products. To respond to this customer demand, many companies are concerned with improving the quality of their products. Currently, however, these companies cannot be certain of their product quality until the product is finally built & tested. This practice is expensive, and extends the lead time for new-product introduction. In some cases, companies ship products even when the product does not quite meet their specifications. The cause for this problem is their inability to make the right decisions at the design stage, requiring iterative steps to correct their mistakes. The redesign of products is not only costly, but takes years to complete. Wrong design decisions cannot be solved by simply fine-tuning the manufacturing process. Although SPC is popular in industry, SPC is useful only in sorting out the manufacturing problems associated with existing production systems, especially when many variables affect the outcome.

Much current design practice is *ad hoc* and empirical, although many design aids such as design-for-assembly and computer-aided design/engineering packages are used by many companies. However, these aids are not effective when the basic design concept is flawed; they cannot overcome mistakes made at higher levels of the design process. To provide a basis for correct design decisions, axiomatic design principles & theories have been advanced [1]. Design issues become easier to understand when they are analyzed using the framework of axiomatic design [1, 2].

What is quality? The word *quality* has many meanings & definitions, depending on the context. A product is defined to be a quality product when it satisfies the FR (or the design specifications) for the product, specified in terms of a nominal value and tolerance. These FR are derived from the perceived needs of customers.

There are many ways we can build quality into a product. The goal is to design a product that can easily be manufactured with functional requirements within their specified tolerances. The product must also be manufacturable within the specified cost. Therefore, to produce high quality products, we need good designs for both the product and the manufacturing process. Sometimes we use the term *robust design* to characterize those designs that ensure the manufacture of a quality product. Robustness is ultimately related to productivity, since the yield & productivity of the manufacturing operation are higher when the product-design is robust.

This paper discusses how quality can be built into the product at the design stage. This is done by axiomatic design, the essence of which is reviewed in section 2. Similar reviews of axiomatic design are in [3 - 5]. This review of axiomatic design is followed by discussion of quality issues based on axiomatic design.

Acronyms[2]

C	constraint
DP	design parameter
FR	functional requirement
PV	process variable
SPC	statistical process control.

Notation

A_{ij}, B_{ij}	element of [DM]
[DM]	design matrix
$\delta(x)$	random noise in x
$\Delta(x)$	derived tolerance on x
$\Omega(x)$	state change in x
DP	vector of DP
FR	vector of FR
PV	vector of PV
I	information content
p	probability

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[1]Acronyms, nomenclature, and notation are given at the end of the Introduction.

t time

η signal-to-noise ratio.

Other, standard notation is given in "Information for Readers & Authors" at the rear of each issue.

2. REVIEW OF AXIOMATIC DESIGN

The impetus for developing Axiomatic Design was the creation of the scientific basis for the field of design [1] — to make the field of design & manufacturing an academic discipline and thus make teaching & learning of the subject more systematic, generalizable, and rigorous. Since then, the basic ideas of axiomatic design have been applied in many fields, such as product, process, system, and organizational design. Design is a universal activity where synthesis must satisfy many functional requirements using a set of inputs [3-9]. There are two ways to deal with design: axiomatic and algorithmic.

- In algorithmic design, we try to identify or prescribe the design process, so in the end the process leads to a design embodiment that satisfies the design goals. Algorithmic methods can be divided into several categories: pattern recognition, associative memory, analogy, experientially based prescription, extrapolation, interpolation, selection based on probability, etc. Some of these techniques can be effective if the design has to satisfy only one functional requirement, but when many functional requirements must be satisfied at the same time, they are not very effective. Generally, the algorithmic approach is founded on the notion that the best way of advancing the design field is to understand the design process by studying current practice. An algorithmic approach is more useful at the final stages of detailed design than at the conceptual stage or at higher levels of design hierarchy.

- The axiomatic approach to any subject begins with a different premise: there are generalizable principles that govern the underlying behavior. Axioms are general principles or self-evident truths that cannot be derived or proven to be true except that there are no counter-examples or exceptions. Axiomatic approach has had a powerful impact in many fields of science & technology. Euclid's axioms for geometry are still the basis of geometric design, among other things; Newton's laws were axioms at the time Newton enunciated them; and the first & second laws of thermodynamics are axioms. Through these axioms, the concept of energy, entropy, and force have been defined. One of the main reasons for pursuing an axiomatic approach to design is the generality of axioms. ◄

The basic postulate of axiomatic design is: There are fundamental axioms that govern the design process. Two axioms were identified by examining common elements always present in good designs, be they product, process, or systems design.

1. *Independence Axiom*. The independence of *Functional Requirements* must always be maintained; *ie*, design decisions must always be made without violating the independence of each functional requirement from other functional requirements. The FR are defined as the *minimum number of independent requirements* that characterize the design goals.

2. *Information Axiom*. Minimize the information content; *ie*, among those designs that satisfy the Independence Axiom, the design that has the highest probability of success is the best design. ◄

Based on these design axioms, we can derive theorems & corollaries [1, 3, 4].

The world of axiomatic design has 4 *domains*:

- *customer domain*,
- *functional domain*,
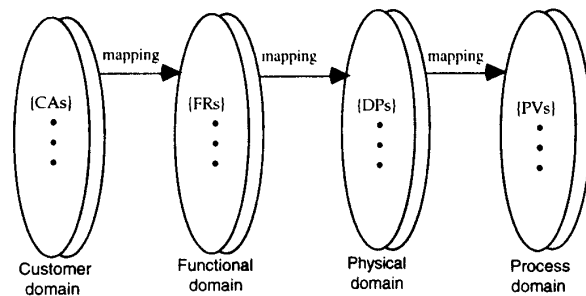- *physical domain*,
- *process domain*.

Figure 1. The 4 Domains of the Design World
$\{\cdot\}$ are the characteristic vectors of each domain

The domain structure is schematically illustrated in figure 1. The domain on the left relative to the domain on the right represents, what we want to achieve, whereas the domain on the right represents the design solution of how we propose to satisfy the requirements in the left domain. To go from *what* to *how* requires *mapping*. During this mapping process, the Independence Axiom must be satisfied.

Axiom 1. The Independence Axiom

Maintain the independence of the functional requirements. ◄

FR are defined as the *minimum set* of *independent* requirements that the design must satisfy. FR are the description of design goals, subject to constraints. Constraints provide the bounds on the acceptable designs and differ from FR in that they do not have to be independent.

In manufacturing, many disciplines and fields are involved, *eg*, mechanical, electrical, hardware, software. However, all designs can be represented using the 4 design domains, enabling us to generalize the design process. The design objectives can be different from one problem to another, but all designers go through the same thought process. Table 1 shows how all these seemingly different design tasks can be described in terms of the 4 design domains. For product design,

Character Vectors	Domains			
	Customer Domain **CA**	Functional Domain **FR**	Physical Domain **DP**	Process Domain **PV**
a. Manufacturing	Attributes which consumers desire	Functional requirements specified for the product	Physical variables which can satisfy the functional requirements	Process variables that can control design parameters (DP)
b. Materials	Desired performance	Required Properties	Micro-structure	Processes
c. Software	Attributes desired in the software	Output	Input Variables and Algorithms	Sub-routines
d. Organization	Customer satisfaction	Functions of the organization	Programs or Offices	People and other resources that can support the programs
(e) Systems	Attributes desired of the overall system	Functional requirements of the system	Machines or components, sub-components	Resources (human, financial, materials, etc.)

- the customer domain consists of the customer requirements or attributes which the customer is looking for in a product;
- the functional domain consists of functional requirements of the product (often called engineering specifications) and constraints;
- the physical domain is the domain in which the key DP are chosen to satisfy the FR;
- the process domain specifies the manufacturing methods that can produce the DP.

These first level FR, DP, PV can be further decomposed into hierarchies. To create these hierarchies, we must zigzag between the domains, because decomposition cannot be done by staying in a single domain. Product design is done through the mapping between the functional & physical domains, while process design is done through mapping between the physical & process domains. *Concurrent engineering* requires that these mapping & decomposition processes be simultaneously done by satisfying the design axioms.

The mapping process can be mathematically expressed in terms of the characteristic vectors that define the design goals and design solutions. At a given level of the design hierarchy, the set of functional requirements that define the specific design goals are a vector **FR** in the functional domain. Similarly, the set of design parameters in the physical domain (the *How's* for the FR) are a vector **DP**. The relationship between **FR** & **DP** is:

$$\mathbf{FR} = [DM] \times \mathbf{DP}. \tag{1}$$

$$FR_i = A_{ij} \cdot DP_j.$$

Eq (1) is a design equation for the design of a product. For processes, the design equation is:

$$\mathbf{DP} = [DM] \times \mathbf{PV}. \tag{2}$$

To satisfy the Independence Axiom, the matrix must be either diagonal or triangular. When [DM] is diagonal, each of the FR can be satisfied independently by means of one DP, *viz*, by an *uncoupled* design. When the matrix is triangular, the independence of FR can be guaranteed iff the DP are changed in a proper sequence, *viz*, a *decoupled* design. When there are many FR & DP, two quantitative measures, reangularity & semangularity, can be used to determine the independence of the functional requirements [1]. Although [DM] is a second order tensor, the usual coordinate transformation technique cannot be applied to (1) to create a diagonal or triangular matrix since [DM] typically involves physical things that are not amenable to coordinate transformation.

To satisfy the independence of a given set of FR,

'number of DP' ≥ 'number of FR'.

When,

'number of DP' < 'number of FR',

the design is always coupled [1: theorem 1]. Many other theorems & corollaries which can be used as design rules have been derived based on the axioms [1, 3].

The FR, DP, PV can be decomposed into a hierarchy. However, contrary to the conventional view of decomposition, they cannot be decomposed by remaining in one domain. One must zigzag between the domains to decompose them. For example, if one of the FR for a vehicle is *move forward*, we cannot decompose it without deciding in the physical domain *how we propose to go forward*. If we choose a horse & buggy as a means of moving forward, the next layer of FR are different from the case in which an automobile is chosen as the DP to satisfy the FR.

Even for the same task, defined by a set of FR, it is most likely that each designer will come up with different designs,

which are acceptable in terms of the Independence Axiom. However, one of these designs is likely to be superior to the others. The Information Axiom provides a quantitative means of measuring the merits of a given design, which can be used to select the best from among those that are acceptable. The Information Axiom is:

Axiom 2: The Information Axiom

Minimize the information content. ◄

Information is defined in terms of the information content that is related, in its simplest form, to the probability of satisfying a given FR. In the general case of n FR for an uncoupled design, information content is:

$$I = \sum_{i=1}^{n} -\log(p_i). \tag{3}$$

Notation

p_i Pr$\{DP_i$ satisfies FR$_i\}$

$\log()$ $\log_2()$ (with unit of bits) or $\log_e()$ (with unit of nats).

Since there are n FR, the total information content is the sum of all these probabilities. The Information Axiom states that the design that has the minimum I is the best design, since it requires the least amount of information to achieve the design goals. When $p_i \equiv 1$ (for all i), then $I=0$, and conversely, the information required is infinite when $p_i = 0$ for some i.

A design is *complex* when its probability of success is low. The quantitative measure for complexity is I. According to (3), complex systems require more information to make the system function. Thus, a large system is not necessarily complex. Even a small system can be complex if the probability of its success is low. Therefore, the notion of complexity is tied to the tolerances for the FR: the tighter the tolerances, the more difficult it is to choose a design solution or a system that can satisfy the FR. Many large systems tend to be more complex since there are more FR to be satisfied, and the tolerance of a large system is often tighter since many components must fit together to function as a system.

In the real world, the probability of success is governed by the intersection of the design range (defined by the designer to satisfy the FR) and the tolerance (the ability) of the system (defined as the System Range).

Example. Let the available tool (system) for cutting a rod be only a hacksaw.

A. Let the design specification for cutting the rod be 1000 ± 0.001 mm. The probability of success is extremely low. The information required to achieve the goal approaches infinity as long as the only system available to cut the rod is the hacksaw. Therefore, this is a complex design.

B. Let the design specification for cutting the rod be 1000 ± 10 mm. The probability of success is extremely high. The information required to achieve the goal approaches zero. Therefore this is a simple design. ◄
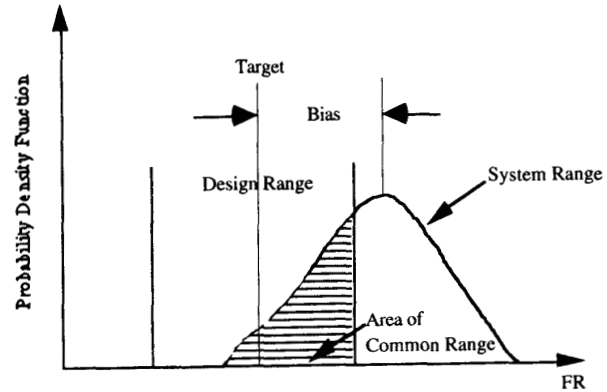

Figure 2. Design Range, System Range, and Common Range

[This is a plot of the pdf of a functional requirement. The functional requirement is satisfied only in the shaded region (common range). If the variation of the system range is larger than the tolerance of the design range, the functional requirements cannot be satisfied at all times. This type of design is *probabilistic* design, since the variation of the system range is so large that there is always a finite probability that the design cannot satisfy the functional requirement, even when the system range can be shifted horizontally. If the design is uncoupled, the bias and variation of the system range can be reduced.]

Notation

dr   design range
sr   system range
cr   common range
A_x area under x.

The probability of success can be computed by specifying the *design range* for the FR and by determining the *system range* that the proposed design can provide to satisfy the FR. Figure 2 illustrates these two ranges graphically. The vertical axis (the ordinate) is for the pdf, and the horizontal axis (abscissa) is for either FR or DP, depending on the mapping domains involved. When the mapping is between the functional domain and the physical domain, as in product design, the abscissa is for FR, whereas for the mapping between the physical domain and the process domain, as in process design, the abscissa is for DP. In figure 2, the System Range is plotted as a pdf *vs* specified FR. The overlap between design range and system range is the *common range*, and this is the only region where the design requirements are satisfied. Consequently, the area under the common range divided by the area under the system range is the design's probability of success (achieving the specification). Thus, the information content for this FR is [1]:

$$I = \log(A_{sr}/A_{cr}). \tag{4}$$

Since $A_{sr} = 1.0$ in most cases and there are n FR to satisfy, the information content is:

$$I = \sum_{i=1}^{n} -\log(A_{cr}). \tag{5}$$

When the system range is broad, as shown in figure 2, the system range can extend to regions outside the design range. Then there is a finite probability that the design cannot satisfy the design specification even for an uncoupled design. (Sometimes this type of design is known as *probabilistic*.) If the proposed solution is a coupled design, the system range cannot be shifted horizontally to put it inside the design range, since other FR are also affected by such a shift. At the same time, in a coupled design, the position of the system range shown in figure 2 is also affected whenever other DP are changed to satisfy other FR. When the design is uncoupled, the system range can be shifted horizontally without affecting other FR. Even then, unless the variation associated with the FR shown in figure 2 is reduced, there is always a non-zero probability that the FR cannot be satisfied. However, if the design satisfies the Independence Axiom, the variation can be reduced using the methods in section 3.
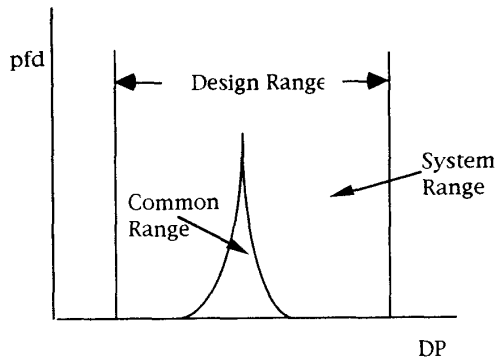

Figure 3. Preferred Distribution of the System Range

[When the variation of the system range is less than the tolerance of the design range and when the design is *uncoupled*, then the system range can be shifted horizontally and can be made to be within the design range. The case shown here is deterministic design, since the design always satisfies the design specification and the information required is zero.]

A deterministic case is shown in figure 3, where the common range is the same as the system range, and the system range is inside the design range. Thus the design specifications are always satisfied. Furthermore, if the FR is independent of other FR (the Independence Axiom is satisfied) or if there is only one FR, we can vary the DP to move the system range along the horizontal axis to place it within the design range. When the system range is contained within the design range, as shown in figure 3, the information content is always zero. Such a design is always deterministic.

In SPC, in addition to the design range, target values are given. The difference between the target value and the peak of the system range is the *bias* (see figure 2). Unfortunately, during the early stages of design, it is not always possible to state the target value precisely, although we know the range within which the functional requirement must be.

In an ideal uncoupled design, the number of FR, DP, PV are the same, and the information content is zero [1: theorem 4]. Such a design is deterministic. In this case, the design matrices are diagonal. In any uncoupled design, the variation associated with each FR can be reduced by eliminating the random variations of DP & PV. When the random noise associated with FR is less than the designer-specified tolerance on FR, the design can always be made deterministic. This point is further discussed in section 3.

Ultimately, quality control of the product and particularly productivity of the system are functions of many factors, such as human factors, product design, and manufacturing process design. Many manufacturing systems are large, and require considerations in addition to those covered thus far. For the design and operation of large systems, the **FR** are a large set and moreover, at any given time, only a subset of **FR** must be satisfied, and the subset changes as a function of time [3]. In contrast to the design issues discussed so far for simple systems — simple in that the number of FR at the highest level was small and that these FR did not change as a function of time — a large system is defined as [3]:

A system is *large* if: a) the total number of FR at the highest level that the system must satisfy during its lifetime is large, and b) at different times, the system is required to satisfy different subsets of FR.   ◄

Suppose that we have to design a system to satisfy n FR. We have to find a set of n DP that are acceptable according to the Independence Axiom. As we search for DP in the physical domain that enable us to satisfy the FR, we might find that there is more than one DP_i that can satisfy a given FR_i:

$$FR_1 \, \$ \, (DP_1^a, \, DP_1^b, \, ..., \, DP_1^m)$$

$$FR_2 \, \$ \, (DP_2^a, \, DP_2^b, \, ..., \, DP_2^q)$$

$$FR_3 \, \$ \, (DP_3^a, \, DP_3^b, \, ..., \, DP_3^w) \tag{6}$$
$$\vdots$$
$$FR_n \, \$ \, (DP_n^a, \, DP_n^b, \, ..., \, DP_n^s).$$

Eq (6) states:

FR_1 is satisfied (indicated by \$) by DP_1^a, DP_1^b, ..., DP_1^m.
$\vdots$
FR_n is satisfied by DP_n^a, ..., DP_n^s.

Eq (6) does not say which DP_3^j, for example, is the best solution for FR_3. Furthermore, since all or a subset of the FR must be satisfied at any given instant, one cannot say *a priori* which DP_3 is the best DP for FR_3 without considering its relationship to the other FR that must be satisfied at the same time. Hence, the choice of DP_3 can differ depending on the chosen subset of FR.

Eq (6) represents the *knowledge base* (*database*) for the large system. When additional DP are added to these equations, it is equivalent to expanding the knowledge base.

Now suppose the subsets of FR change as a function of time as follows:

For $t = 0$, $\quad \mathbf{FR}_0 = \{FR_1, FR_5, FR_7, FR_n\}$

For $t = T_1$, $\quad \mathbf{FR}_1 = \{FR_3, FR_5, FR_8, FR_m\}$ (7)

For $t = T_2$, $\quad \mathbf{FR}_2 = \{FR_3, FR_9, FR_{10}, FR_n\}$.

Eq (7) states that initially the system must satisfy the set $\{FR_1, FR_5, FR_7, FR_n\}$. Then, the FR set changes at $t = T_1$ and at $t = T_2$. To satisfy $\mathbf{FR}_0$, we must choose a DP set, *viz*, $\mathbf{DP}_0 = \{DP_1, DP_5, DP_7, DP_n\}$, which satisfies the independence of FR_1, FR_5, FR_7, FR_n. At $t = T_1$, a different subset of $\mathbf{FR}$ must be satisfied. This means that the system must *reconfigure* (*switch*) to satisfy $\{FR_3, FR_5, FR_8, FR_m\}$ independently. The switching mechanism to go from a given subset of DP to another must operate at an acceptable speed. This process is followed whenever the FR set changes.

For a given subset of $\mathbf{FR}$, there can be many different sets of $\mathbf{DP}$ that are acceptable from the functional point of view. The best solution can be chosen based on an evaluation of each of the proposed solutions by measuring the information content. We can evaluate the information content for each and all FR that comprise the subset, and then sum them to get the total information of that subset, using (5). Detailed discussion of the design of large systems, along with the necessary theorems, are in [3].

3. CRITERIA FOR QUALITY PRODUCTS

Several corollaries & theorems have been derived from the design axioms, many of which function as design *rules* that should be followed in producing quality products. Some of the specific, deriveable criteria that govern quality products are presented in this section. These criteria provide guidelines for *what to do* and *what not to do* based on these axioms, corollaries, and theorems [1, 5].

3.1 Criterion 1: Equal Number of FR, DP, PV

To manufacture quality products, we must develop designs that are uncoupled or, at least, decoupled. Coupled designs violate the Independence Axiom. The number of DP & PV must equal the number of FR for the Independence Axiom to be satisfied. When the number of DP or PV is less than the number of FR, the design becomes coupled. If the number of DP & PV is equal to the number of FR when the design matrix is diagonal, then the design is defined as *ideal* [1: theorem 4].

Criterion 1. To satisfy the Independence Axiom, all manufacturing processes must be designed so that:

'number of PV' $\geq$ 'number of DP' $\geq$ 'number of FR',

and the design matrices must be diagonal or triangular. The ideal design is when:

'number of PV' $=$ 'number of DP' $=$ 'number of FR',

and the design matrices are diagonal or triangular. ◄

3.2 Criterion 2: Robust Design

Assumptions

1. The design for a product is characterized by 3 design parameters $\{DP_1, DP_2, DP_3\}$.

2. We have designed an ideal uncoupled process such that 3 PV have been chosen to satisfy these DP, $\{PV_1, PV_2, PV_3\}$, as per criterion 1 and [1: theorem 4]. ◄

Then, the design equation for the process is:

$$\begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ 0 & B_{22} & 0 \\ 0 & 0 & B_{33} \end{bmatrix} \begin{Bmatrix} PV_1 \\ PV_2 \\ PV_3 \end{Bmatrix}$$ (8)

The term *robust design* means a design that produces DP within their required tolerances even when PV vary appreciably. For example, if the body shape of a stamped sheet metal part meets the design specification even when the dimensions of the die are not controlled tightly, it is a robust design. For this to be possible, the elements of the design matrix must be *small*, as shown in the remainder of this subsection.

Let —

$$DP_1 = (DP_1)_0 \pm \Delta(DP_1)$$ (9)

Notation

$(DP_1)_0$ target value

$\Delta(DP_1)$ allowable tolerance for DP_1 derived from the tolerance on functional requirements

$\Delta(PV_1)$ derived tolerance on PV_1.

$$PV_1 = (PV_1)_0 \pm \Delta(PV_1)$$ (10)

Given the desired $\Delta(FR)$, the allowable $\Delta(DP)$ is fixed, which in turn determines the allowable $\Delta(PV)$. Thus the requirement is: $\delta(PV_1) < \Delta(PV_1)$.

Since we want to make $\Delta(PV_1)$ as large as possible for the given tolerance of the product $\Delta(DP_1)$ — and ultimately for the specified tolerance for functional requirement $\Delta(FR_1)$ — it is better to let B_{11} be as small as possible, but much larger than the off-diagonal elements, which are zero in (8). The lower limit of B_{11} is obviously dictated by the DP_1 response required of the system when PV_1 is varied.

For the design in (8)

$$\delta(DP_1) \approx B_{11} \cdot \delta(PV_1) + \delta(B_{11}) \cdot PV_1.$$ (11)

The system noise might be due to changes in temperature, environmental conditions, humidity, *etc*. For the manufacturing process to be acceptable,

$$|\delta(DP_1)| \leq |\Delta(DP_1)|.$$

In production, the larger the tolerance on the process variables, the easier it is to manufacture a quality product. When $|B_{11}|$ is small, a large variation in PV_1 does not create large variations in DP_1, provided that $\delta(B_{11})$ is negligible. The variation in the coefficient is due either to the nonlinearity (discussed later in this subsection) or to the noise in the system.

In SPC, the notion of the signal-to-noise ratio, η, is extensively used in selecting PV (parameter design). For this example, η due to $\delta(DP_1)$ is:

$$\eta = 20\log_{10}[(DP_1)_0/\delta(DP_1)]$$

$$= 20\log_{10}[B_{11} \cdot (PV_1)_0/[B_{11} \cdot \delta(PV_1) + \delta(B_{11}) \cdot PV_1]].$$

(12)

The minimum acceptable η is:

$$\eta_{min} = 20\log_{10}[(DP_1)_0/\Delta(DP_1)].$$ (13)

To have a robust design,

$$\eta > \eta_{min}.$$

For this to be possible,

$$\delta(DP_1) < \Delta(DP_1),$$

which can be satisfied if $\delta(B_{11})$ is made negligible and the random variation in PV_1 is made smaller than the tolerance specified for PV_1. These arguments are valid iff the design is uncoupled.

Criterion 2. Robust design is an uncoupled design where the signal-to-noise ratio is greater than the minimum signal-to-noise ratio: $(DP_i)_0/\Delta(DP_i)$. The coefficient that relates DP_i and PV_i must be much larger than the off-diagonal elements, but as small as possible to make the allowable variation in process variable as large as possible while still satisfying the DP specification.

3.3 Criterion 3: Redundant Design

When,

'number of DP' > 'number of FR', or

'number of PV' > 'number of DP,

the design is defined as *redundant*. To illustrate the characteristics of a redundant design, consider the product design consisting of:

2 FR, 4 DP, 4 PV.

The design equation for the product is:

$$\begin{Bmatrix} FR_1 \\ FR_2 \end{Bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 & A_{14} \\ 0 & A_{22} & A_{33} & 0 \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \\ DP_4 \end{Bmatrix}$$ (14)

This type of situation can often exist when the product is designed *ad hoc* without the benefit of axiomatic design. In this case, the best strategy is to fix 2 DP: either (DP_1, DP_2), (DP_1, DP_3), (DP_2, DP_4), or (DP_3, DP_4). Then, the resulting system is an ideal design with a diagonal matrix.

If this strategy of fixing 2 DP is used, the question then is which DP should be fixed. Applying criterion 2, it is better to fix the ones associated with a smaller coefficient, provided the off-diagonal elements are sufficiently small to be negligible, and the signal-to-noise ratio is larger than the minimum value defined by (8). Another strategy is to use 2 DP, *eg*, DP_1 & DP_2, for coarse control and to use the other 2, DP_3 & DP_4, for fine control. If $A_{11} \gg A_{14}$, then DP_1 is a good candidate for coarse control, and DP_4 should be used as the fine control for FR_1.

Criterion 3. For a redundant design, select the PV that: a) appreciably affect the DP, and b) satisfy the Independence Axiom; freeze all other PV. When there are equivalent choices, choose the one with a smaller coefficient provided that the off-diagonal elements are negligible when compared to the on-diagonal elements, and that the signal-to-noise ratio is greater than a minimum value defined by the tolerance on FR, DP, or PV..   ◀

3.4 Criterion 4: Source of Variation & Errors

Section 3.2 considers a completely uncoupled ideal design where all off-diagonal elements are zero. In many cases, the off-diagonal elements are very small, but not zero. Suppose that we have to satisfy 3 FR: FR_1, FR_2, FR_3. To have an ideal design, we must choose 3 DP that yield a diagonal design matrix; these 3 DP in turn must be satisfied independently by 3 PV:

$$\begin{Bmatrix} FR_1 \\ FR_2 \\ FR_3 \end{Bmatrix} = \begin{bmatrix} A_{11} & a_{12} & a_{13} \\ a_{21} & A_{22} & a_{23} \\ a_{31} & a_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix}$$ (15a)

$$\begin{Bmatrix} DP_1 \\ DP_2 \\ DP_3 \end{Bmatrix} = \begin{bmatrix} B_{11} & b_{12} & b_{13} \\ b_{21} & B_{22} & b_{23} \\ b_{31} & b_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} PV_1 \\ PV_2 \\ PV_3 \end{Bmatrix}$$ (15b)

The off-diagonal elements, the a & b in (15), should be made much smaller than the on-diagonal elements, the A & B in (15), so that the design can be considered uncoupled through a proper design. To change the vector $\{FR_1, FR_2, FR_3\}$ from state A to state B, we have to change the vector $\{DP_1, DP_2, DP_3\}$ from state A to state B. We denote the state change in DP_1

from A to B by $\Omega(\mathrm{DP}_1)$. Now, all the errors & variations that can be introduced by changes in DP & PV must be less than the specified tolerances. That is, to develop an uncoupled design for a manufacturing process that can be controlled intelligently, the noise of the system must be smaller than the specified tolerances for FR, $\Delta(\mathrm{FR})$, and for DP, $\Delta(\mathrm{DP})$, by satisfying the conditions:

$$\Delta(\mathrm{FR}_1) > a_{12}\cdot\Omega(\mathrm{DP}_3) + a_{13}\cdot\Omega(\mathrm{DP}_2) + \delta(A_{11})\cdot\mathrm{DP}_1$$
$$+ A_{11}\cdot\delta(\mathrm{DP}_1)$$
$$\Delta(\mathrm{DP}_1) > b_{12}\cdot\Omega(\mathrm{PV}_3) + b_{13}\cdot\Omega(\mathrm{PV}_2) + \delta(B_{11})\cdot\mathrm{PV}_1$$
$$+ B_{11}\cdot\delta(\mathrm{PV}_1). \tag{16}$$

Or, more generally,

$$\Delta(\mathrm{FR}_i) > \sum_{j=1,j\neq i}^{n} a_{ij}\cdot\Omega(\mathrm{DP}_j) + \delta(A_{ii}\cdot\mathrm{DP}_i)$$

$$\Delta(\mathrm{DP}_i) > \sum_{j=1,j\neq i}^{n} b_{ij}\cdot\Omega(\mathrm{PV}_j) + \delta(B_{ii}\cdot\mathrm{PV}_i). \tag{17}$$

To be within the design specification when additional errors are present due to non-zero off-diagonal terms, it is required that:

$$\delta(\mathrm{DP}_i) \ll \Delta(\mathrm{DP}_i). \tag{18}$$

When (17) is satisfied, the Independence Axiom is satisfied because the effects of the off-diagonal elements are negligible, and the individual outputs such as FR & DP can be controlled using PV. This fact can be written as [1: theorem 8].

The signal-to-noise equations are:

$$\eta = 20\log_{10}[\mathrm{DP}_i/\delta(\mathrm{DP}_i)]$$
$$= 20\log_{10}\left[(\mathrm{DP}_i)_0\bigg/\left[\sum_{j=1,j\neq i}^{n} b_{ij}\cdot\Omega(\mathrm{PV}_j)\right] + \delta(B_{ii}\cdot\mathrm{PV}_i)\right]; \tag{19}$$

$$\eta_{\min} = 20\log_{10}[\mathrm{DP}_i/\Delta(\mathrm{DP}_i)]. \tag{20}$$

The criterion for robust design should be considered in selecting the design, *viz*,

$$\eta > \eta_{\min}.$$

Then, the criterion for being within the specified tolerance is:

When the design is nearly uncoupled but the off-diagonal terms are not negligible, the variation depends on the change of state of the characteristic vectors. And, the signal-to-noise ratio is a local property that has to be checked at every design point in the design space. ◄

3.5 Criterion 5: Control Sequence in Decoupled Designs

Let a decoupled design have the relationships:

$$\begin{Bmatrix} \mathrm{FR}_1 \\ \mathrm{FR}_2 \\ \mathrm{FR}_3 \end{Bmatrix} = \begin{bmatrix} A_{11} & 0 & 0 \\ a_{21} & A_{22} & 0 \\ a_{31} & a_{32} & A_{33} \end{bmatrix} \begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \\ \mathrm{DP}_3 \end{Bmatrix} \tag{21a}$$

$$\begin{Bmatrix} \mathrm{DP}_1 \\ \mathrm{DP}_2 \\ \mathrm{DP}_3 \end{Bmatrix} = \begin{bmatrix} B_{11} & 0 & 0 \\ b_{21} & B_{22} & 0 \\ b_{31} & b_{32} & B_{33} \end{bmatrix} \begin{Bmatrix} \mathrm{PV}_1 \\ \mathrm{PV}_2 \\ \mathrm{PV}_3 \end{Bmatrix} \tag{21b}$$

Eq (21) indicates that FR_1 is a function only of DP_1. If we monitor DP_1 during a manufacturing process and keep it within the specified tolerance by controlling PV_1, the process produces satisfactory (in terms of FR_1) parts. Once DP_1 is set, FR_2 can be controlled primarily by controlling DP_2, but the output is also affected by fluctuations of DP_1 because of the a_{21} term. Then, for this manufacturing process to work, it must satisfy the relationship:

$$\Delta(\mathrm{FR}_1) > \delta(A_{11}\cdot\mathrm{DP}_1)$$
$$\Delta(\mathrm{DP}_1) > \delta(B_{11}\cdot\mathrm{PV}_1) \tag{22}$$
$$\Delta(\mathrm{FR}_2) > \delta(A_{22}\cdot\mathrm{DP}_2) + \delta(a_{21}\cdot\mathrm{DP}_1)$$
$$\Delta(\mathrm{DP}_2) > \delta(B_{22}\cdot\mathrm{PV}_2) + \delta(b_{21}\cdot\mathrm{PV}_1).$$

In writing (22), the higher order terms are assumed to be negligible. Eq (22) indicates that for a decoupled design, it is important to minimize the random variations in the process variables and to make the off-diagonal elements as small as possible, but they are not affected by the state change of FR & DP — which was the case in (15).

Criterion 5. To make a quality product using a *decoupled* design, the random fluctuations of the process variables should be minimized and the off-diagonal elements (coupling elements) should be made close to zero, in addition to varying the DP & PV in the proper sequence. ◄

3.6 Criterion 6: Non-Linear Design

Many manufacturing processes are non-linear. Often the elements of the design matrix for the process are functions of the process variables. Then we have to search the design space to identify the design window where the process behaves either as an uncoupled design or as a decoupled design. The design window can be identified by:

- calculating numerical values of the elements of the design matrix at a given design point; or
- evaluating the *semangularity*, S, and *reangularity*, R [1]. R & S are defined so that when $R = S = 1$, the design is uncoupled.

Non-linearity can also introduce errors when a DP or PV is not at the exact target value, since any random error $\delta(DP)$ or $\delta(PV)$ changes the design elements, A_{ij}, which are functions of DP or PV. Consider a simple uncoupled nonlinear design at state A:

$$DP_1 = B_{11} \cdot PV_1$$

$$DP_2 = B_{22} \cdot PV_2. \tag{23}$$

$$\delta(DP_1) = \delta(B_{11} \cdot PV_1)$$

$$\delta(DP_2) = \delta(B_{22} \cdot PV_2). \tag{24}$$

In nonlinear design, DP_1 & DP_2 might be coupled by the changes in PV_1 & PV_2 if B_{11} & B_{22} are both functions of PV_1 & PV_2. However, if B_{11} is a function only of PV_1, and if B_{22} is a function only of PV_2, the design is always uncoupled.

Criterion 6. When the design of a manufacturing process is nonlinear, the operating window must be sought by identifying the design space where the system behaves as uncoupled or decoupled. If the design does not have an uncoupled or decoupled design window, the manufacturing process must be changed to satisfy the Independence Axiom before attempting to control the process. Non-linearity introduces errors when DP & PV vary randomly, since the elements of the design matrices are functions of DP or PV.   ◄

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