Diversity with Practical Channel Estimation in Arbitrary Fading Environments

by

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Abstract

This thesis presents a framework for evaluating the bit error probability of N_{d}-branch diversity combining in the presence of non-ideal channel estimates. The estimator structure is based on the maximum likelihood (ML) estimate and arises naturally as the sample mean of N_{p} pilot symbols. The framework presented requires only the evaluation of a single integral involving the moment generating function of the norm square of the channel gain vector, and is applicable to channels with arbitrary distribution, including correlated fading. Analytical results show that the practical ML channel estimator preserves the diversity order of an N_{d}-branch diversity system, contrary to conclusions in the literature based upon a model that assumes a fixed correlation between the channel and its estimate. Finally, the asymptotic signal-to-noise ratio (SNR) penalty due to estimation error is investigated. This investigation reveals that the penalty has surprisingly little dependence on the number of diversity branches.

Thesis Supervisor: Moe Z. Win
Title: Charles Stark Draper Assistant Professor

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Chapter 1

Introduction

Diversity techniques can significantly improve the performance of wireless communication systems [1–4]. Among the various forms of diversity techniques, perfect coherent maximal-ratio combining (MRC) plays an important role as it provides the maximum instantaneous signal-to-noise ratio (SNR) at the combiner output. The performance of MRC over flat fading channels has been extensively investigated in the literature. For example, multipath diversity using Rake reception with MRC has played an increasingly important role in spread spectrum multiple-access systems [5–7] and more recently in third generation wireless systems [8–10], as well as in ultra-wide bandwidth (UWB) systems [11–14]. These results assume perfect channel knowledge; however, practical receivers must estimate the channel, thereby incurring estimation error which needs to be accounted for in the performance analysis.

The problem of weighting error in what is essentially a maximal-ratio combiner was examined in [15, 16]. The system was assumed to be operating in independent identically distributed (i.i.d.) Rayleigh fading channels, and estimates of the channel were derived from a pilot tone. The pilot tone was transmitted at a frequency offset from the data channels and used to provide appropriate weighting for combining. Expressions for the distribution of the instantaneous SNR[1] as well as the error probability of both non-coherent and coherent binary orthogonal signaling schemes were developed in [15, 16]. Similarly, [17, 18] analyzed the distribution of the SNR in the presence of complex Gaussian weighting errors for MRC. In these studies, the weighting errors were characterized by a correlation coefficient between

[1] Throughout this paper, we use the term SNR to refer to instantaneous SNR. The term average SNR is explicitly used to describe the SNR averaged over the fading ensemble.

the channel gain and its estimate.

While the SNR is a meaningful measure for analog systems, it does not completely describe the performance of a digital system. A more meaningful measure for digital systems is the bit error probability (BEP). In [19–21], the BEP was derived for MRC systems by averaging the conditional BEP, conditioned on the SNR. Note however, that these results can be misleading as they do not truly reflect the actual BEP [15, 22]. The averaging in [19–21] was performed over a distribution developed from the SNR distribution given in [15–18]. The studies in [19–21] considered a model where the correlation coefficient between the channel estimate and the true channel is *independent* of the SNR, that is, the BEP was parameterized by fixed values of correlation.

Regardless of the choice of the model, one expects the accuracy of the estimator to improve as the SNR increases. Along these lines, [23–25] considered a different model for analyzing error probability in digital transmission systems using pilot signals in which the correlation coefficient between the channel estimate and the true channel is *dependent* on the SNR. This model reflects the fact that as the SNR increases the estimator is capable of achieving a higher level of accuracy. Pilot symbol assisted modulation for single antenna systems in time varying Rayleigh fading channels has been analyzed assuming frequency-flat and frequency-selective channels in [26] and [27], respectively. Note that the work in [15–21, 23, 24] was applicable only to i.i.d. Rayleigh fading environments.

This thesis develops an analytical framework that enables the evaluation of the performance of N_{d}-branch diversity systems with practical channel estimation. This framework is applicable to any environment, provided that its fading can be characterized by a moment generating function (m.g.f.). Our methodology, requiring only the evaluation of a single integral with finite limits, is valid for channels with arbitrary distribution, including correlated fading. To illustrate the proposed methodology, we consider Nakagami-m fading channels that have been shown to accurately model the amplitude distribution of the UWB indoor channel [28].[2] We also examine the case of Ricean fading, as it is appropriate for channels with line of sight components, such as satellite communication channels [3,29]. We consider a channel estimator structure in which the correlation between the estimate and the true channel is a function that is *dependent* on the SNR. The SNR penalty, arising from degradation due to practical channel estimation, is quantified and we reveal a surprisingly

[2] Note that the special case of $m = 1$ reduces to the classical Rayleigh fading channel.

small dependence on N_{d}.

This thesis is organized as follows. In the next chapter, the models for both the system and estimator are presented. Chapter 3 examines the mean and normalized standard deviation (NSD) of the decision variable to provide initial insights into the behavior of diversity systems with practical channel estimation. In Chapter 4 the BEP of N_{d}-branch diversity for channels with arbitrary fading distributions is evaluated and some special cases are discussed. The BEP expressions are applied to a few common independent fading channel models and asymptotic expressions are developed in Chapter 5. Chapter 6 investigates performance in correlated fading channels. Chapter 7 discusses important aspects of practical diversity systems including the correlation coefficient between the true channel gain and its estimate and the SNR penalty due to practical channel estimation. Numerical results are given for the BEP of systems operating in Nakagami and Ricean fading environments, including correlated fading. Finally, Chapter 8 presents concluding remarks.

Chapter 2

Model

We consider an N_d-branch diversity system utilizing a binary phase-shift keying (BPSK) signaling scheme. In the interval $(0, T)$ we transmit signals of the form[1]

$$s_m(t) = \Re\Big\{a_m\, g(t)\, \mathrm{e}^{j2\pi f_c t}\Big\}, \quad m = 0,\, 1, \tag{2.1}$$

where a_m denotes the data symbols taking on the values ± 1 with equal probability. Here the signal pulse shape, $g(t)$, is a real-valued waveform that has energy $E_\mathrm{s} = \frac{1}{2}\int_0^T |g(t)|^2\, dt$ and support $(0, T)$. The received signal on the k^th branch is then modeled as

$$r_k(t) = h_k s_m(t) + n_k(t), \quad 0 \le t \le T, \quad 1 \le k \le N_\mathrm{d}. \tag{2.2}$$

Such a diversity system is depicted in Fig. 2-1.

The receiver demodulates $r_k(t)$ using the matched filter with impulse response $\frac{1}{\sqrt{2E_\mathrm{s}}} g^*(T - t)$.[2] Sampling the output yields

$$r_k = h_k s_m + n_k, \tag{2.3}$$

where $s_m \in \{+\sqrt{2E_\mathrm{s}}, -\sqrt{2E_\mathrm{s}}\}$ represents the message symbol, h_k is a complex, multiplicative gain introduced by fading in the channel, and n_k represents a sample of the additive noise on the k^th branch. The additive noise is modeled as a complex Gaussian random variable (r.v.) with zero mean and variance N_0 per dimension and is assumed to be independent among the diversity branches. We consider slowly fading channels, so $\mathbf{h} = [h_1,\, h_2,\, \ldots,\, h_{N_\mathrm{d}}]$

[1] $\Re\{\cdot\}$ is used to denote the real part.
[2] The complex conjugate is denoted by $(\cdot)^*$.

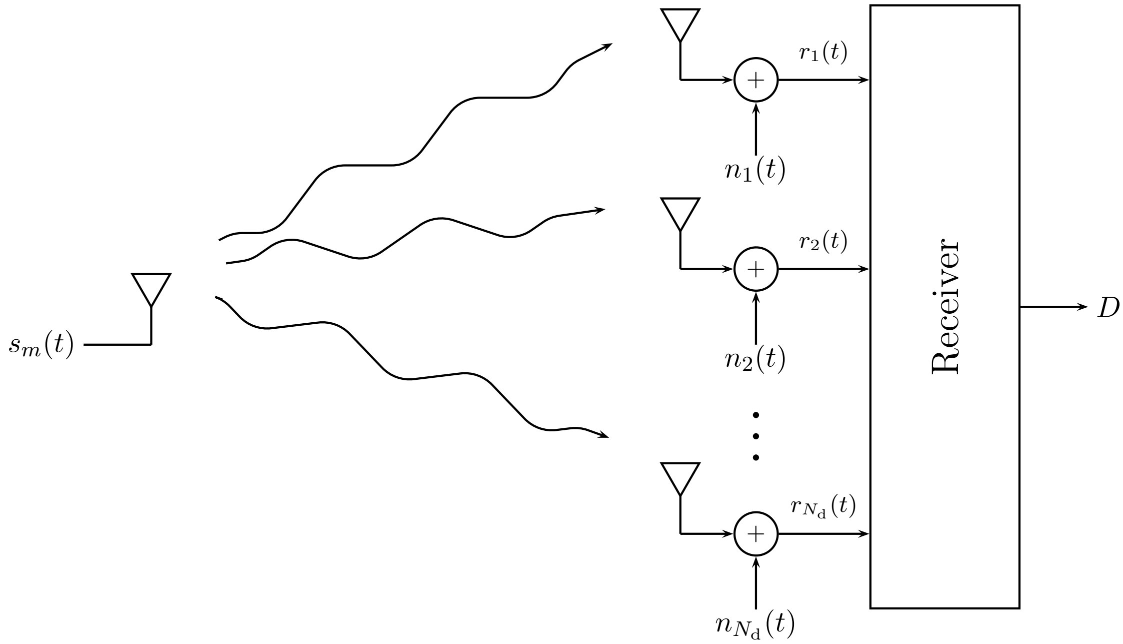

Figure 2-1: A diversity system employing multiple antennas.

is effectively constant over a block of symbols, without making any assumptions about the distribution of $\mathbf{h}$. Note also that there are no restrictions placed on the correlation between individual branch fading gains, h_k, that is, our analysis is valid for channels with arbitrary correlation matrix.

If $\mathbf{h}$ were known at the receiver, the optimal combiner that maximizes the output SNR is well known to be MRC,

$$r = \sum_{k=1}^{N_\mathrm{d}} h_k^* r_k.$$

In practice, however, $\mathbf{h}$ must be estimated; thus the combiner output is

$$r = \sum_{k=1}^{N_\mathrm{d}} \hat{h}_k^* r_k, \tag{2.4}$$

where $\hat{h}_k$ is an estimate of the multiplicative gain, h_k, on the k^{th} branch. Clearly, the performance of this combining scheme greatly depends on the quality of the estimate $\hat{h}_k$.[3]

[3]This receiver structure is the same as studied in [23–25] and is referred to as "fixed-reference coherent detection" in [15].

As in [23–25], information can be derived from a pilot transmitted in previous signaling intervals to form an estimate of the channel. Without loss of generality, all pilot symbols are considered to be +1. The received pilot, after demodulation, matched filtering and sampling can be represented by

$$p_{k,i} = \sqrt{2E_{\mathrm{p}}}h_k + n_{k,i}\,, \tag{2.5}$$

where $p_{k,i}$ and $n_{k,i}$ denote the pilot symbol and noise samples, respectively, received on the k^{th} branch during the i^{th} previous signaling interval and E_{p} is the energy of the pilot symbol. Then, the linear estimate based on the previous N_{p} pilot transmissions is given by

$$\hat{h}_k = \frac{\sum_{i=1}^{N_{\mathrm{p}}} c_i p_{k,i}}{\sqrt{2E_{\mathrm{p}}}\sum_{i=1}^{N_{\mathrm{p}}} c_i} = h_k + \frac{\sum_{i=1}^{N_{\mathrm{p}}} c_i n_{k,i}}{\sqrt{2E_{\mathrm{p}}}\sum_{i=1}^{N_{\mathrm{p}}} c_i}, \tag{2.6}$$

where c_i is an estimator weighting coefficient [30, 31]. The maximum likelihood estimate arises if we let $c_i = 1,\ \forall i$, which gives[4]

$$\hat{h}_k = h_k + \frac{\sum_{i=1}^{N_{\mathrm{p}}} n_{k,i}}{\sqrt{2E_{\mathrm{p}}}N_{\mathrm{p}}}.$$

Note that this particular estimator structure is the sample average of N_{p} pilot transmissions. Furthermore, this estimator is both unbiased and efficient, with $\mathbb{E}\left\{\hat{h}_k\right\} = h_k$ and variance $\mathbb{E}\left\{\left|\hat{h}_k - h_k\right|^2\right\} = \frac{N_0}{E_{\mathrm{p}}N_{\mathrm{p}}}$, achieving the Cramér-Rao lower bound with equality. It is also important to realize that both the pilot energy and the number of pilot symbols play a critical role in the performance of this estimator. As the pilot energy and/or the number of pilot symbols increase, the estimate becomes more accurate. That is, the estimate, and hence its correlation with the true channel gain, depends on both the average pilot SNR and the number of pilots, N_{p}, used to form the estimate [32]. Figure 2-2 shows the diversity combining system utilizing practical channel estimation in detail.

[4] In reality, knowledge of E_{p} is not needed since scaling $\hat{h}_k$ by any positive scalar does not affect the performance of the decision process in (3.1).

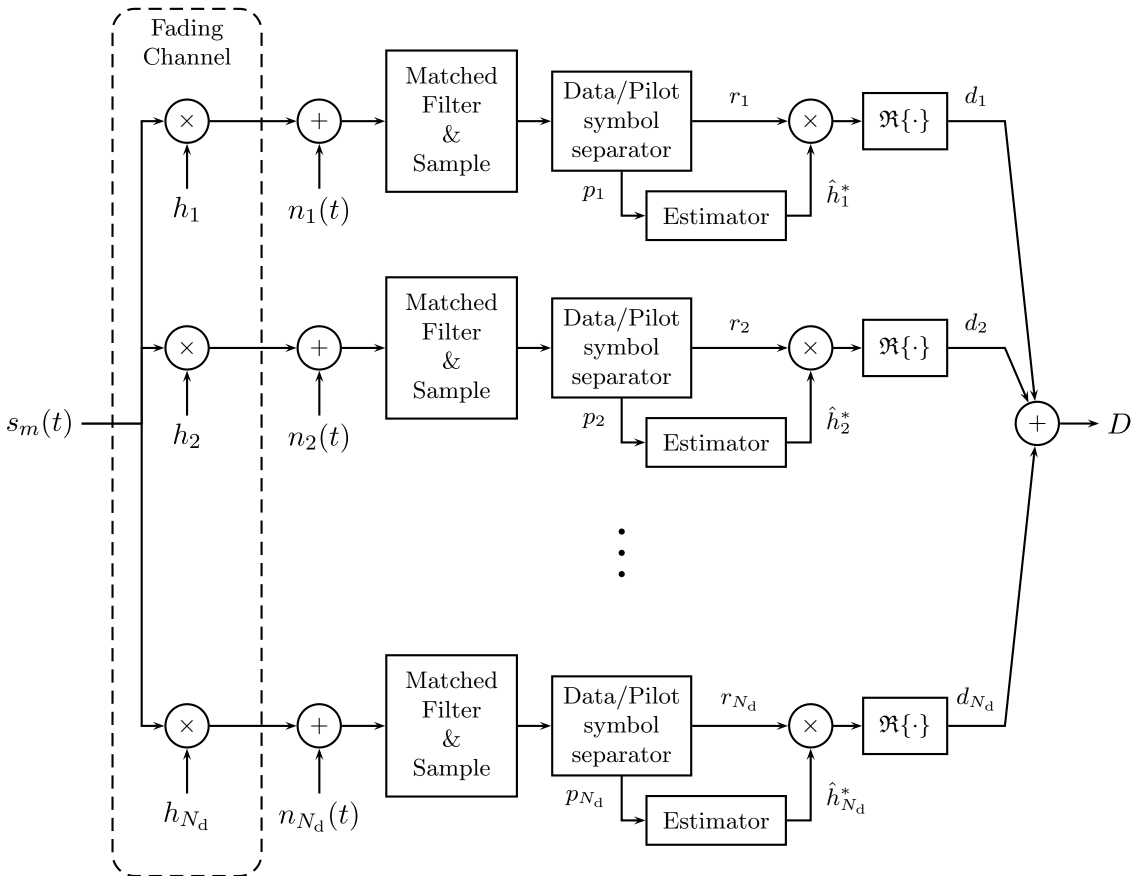

Figure 2-2: A diversity system utilizing practical channel estimation.

Chapter 3

Analysis of the Mean and Variance of the Decision Variable

The performance of diversity systems in fading channels can be characterized by various performance measures. These measures include average output SNR, symbol error probability (SEP), symbol error outage (SEO), outage probability, and outage capacity. Depending on the nature of the traffic (type of data), operating environments, etc., some measures are more meaningful than others, with a varying degree of difficulty in their evaluation. Among them, SEP is one of the most commonly used measures, since it provides insights into the performance of digital communication systems.

Occasionally one encounters diversity combining systems operating in environments that do not lend themselves to tractable SEP analysis. In such cases, one may resort to other performance measures, such as the average output SNR, averaged over the fast fading [33, 34]. The notion of the normalized standard deviation (NSD) of the instantaneous output SNR was introduced in [35] to assess the effectiveness of diversity systems in the presence of fading.

In the case of diversity systems with practical channel estimation, where knowledge of the channel is derived from imperfect estimates, the mean and NSD of the decision variable can be used to provide insights into the behavior of the system. Using the decision variable's characteristic function (c.f.), we first derive the mean and variance of the decision variable for arbitrary fading channels. The NSD is then computed using these quantities and, along with the mean, is used to examine the effectiveness of diversity systems with practical channel estimation.

3.1 Characteristic Function of the Decision Variable

The decision variable, on which the receiver bases its decision, is given by $D = \Re\{r\}$. Using (2.4), we can rewrite D as

$$D = \sum_{k=1}^{N_\mathrm{d}} d_k ,$$

where

$$d_k = \Re\Big\{\hat{h}_k^* r_k\Big\} = \frac{1}{2}\left(h_k^* + e_k^*\right)\left(h_k s_m + n_k\right) + \frac{1}{2}\left(h_k + e_k\right)\left(h_k^* s_m + n_k^*\right) , \tag{3.1}$$

and $e_k = \frac{\sum_{i=1}^{N_\mathrm{p}} n_{k,i}}{\sqrt{2E_\mathrm{p}}N_\mathrm{p}}$ is the complex Gaussian estimation error.

In general, if the diversity branches are correlated, the variables, d_k, in (3.1) will not be independent. However, conditioned on the channel gain vector, $\mathbf{h}$, the branches are conditionally independent and (3.1) can be viewed as a Hermitian quadratic form involving complex normal random variables [36, 37]. Conditioned on h_k and given that $a_1 = +1$ was transmitted,[1] we can write[2]

$$d_k = v_k^\dagger Q v_k ,$$

where

$$v_k = \begin{pmatrix} h_k + e_k \\ \sqrt{2E_\mathrm{s}}h_k + n_k \end{pmatrix} , \qquad \mathbb{E}\{v_k|\, h_k\} = \begin{pmatrix} h_k \\ \sqrt{2E_\mathrm{s}}h_k \end{pmatrix} , \qquad \text{and} \qquad Q = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix} . \tag{3.2}$$

Using the result of [37], the c.f., conditioned on h_k, is

$$\begin{aligned} \phi_{d_k|\, h_k}(t) &= \mathbb{E}\Big\{e^{jtd_k}\Big|\, h_k\Big\} \\ &= |I - jtLQ|^{-1} \exp\Big(-\mathbb{E}\{v_k|\, h_k\}^\dagger L^{-1}\left[I - (I - jtLQ)^{-1}\right]\mathbb{E}\{v_k|\, h_k\}\Big) , \end{aligned} \tag{3.3}$$

[1] Unless otherwise stated, we assume that $a_1 = +1$ was transmitted. Of course, since we are using BPSK with equiprobable symbols, the analyses are symmetric.

[2] $(\cdot)^\dagger$ denotes Hermitian transpose.

where

$$L = \mathbb{E}\Big\{(v_k - \mathbb{E}\{v_k|\, h_k\})(v_k - \mathbb{E}\{v_k|\, h_k\})^\dagger \Big|\, h_k\Big\} = \begin{pmatrix} \frac{N_0}{E_\mathrm{s} N_\mathrm{p}\,\varepsilon} & 0 \\ 0 & 2N_0 \end{pmatrix}$$

is the covariance matrix, and $\varepsilon \triangleq \frac{E_\mathrm{p}}{E_\mathrm{s}}$ is the ratio of pilot energy to data energy. After some simplification, (3.3) can be written as

$$\phi_{d_k|\, h_k}(t) = \frac{2E_\mathrm{s} N_\mathrm{p}\,\varepsilon}{N_0^2 t^2 + 2E_\mathrm{s} N_\mathrm{p}\varepsilon} \exp\left(\frac{E_\mathrm{s}\, |h_k|^2 \left(2j\sqrt{2E_\mathrm{s}} N_\mathrm{p}\,\varepsilon\, t - N_0\,(N_\mathrm{p}\,\varepsilon + 1)\, t^2\right)}{N_0^2 t^2 + 2E_\mathrm{s} N_\mathrm{p}\,\varepsilon} \right).$$

Since each d_k is conditionally independent, the conditional c.f. of D is given by the product of the individual c.f.'s

$$\begin{aligned} \phi_{D|\,\mathbf{h}}(t) &= \prod_{k=1}^{N_\mathrm{d}} \phi_{d_k|\, h_k}(t) \\ &= \left[\frac{2E_\mathrm{s} N_\mathrm{p}\,\varepsilon}{N_0^2 t^2 + 2E_\mathrm{s} N_\mathrm{p}\varepsilon}\right]^{N_\mathrm{d}} \exp\left(\frac{E_\mathrm{s}\, \|\mathbf{h}\|^2 \left(2j\sqrt{2E_\mathrm{s}} N_\mathrm{p}\,\varepsilon\, t - N_0\,(N_\mathrm{p}\,\varepsilon + 1)\, t^2\right)}{N_0^2 t^2 + 2E_\mathrm{s} N_\mathrm{p}\,\varepsilon} \right), \end{aligned} \quad (3.4)$$

where $\|\mathbf{h}\|^2 = \sum_{k=1}^{N_\mathrm{d}} |h_k|^2$ is the norm square of the channel gain vector.

3.2 Statistics of the Decision Variable

Using (3.4) and properties of c.f.'s one can evaluate the first and second moments of D when conditioned on $\mathbf{h}$:

$$\mathbb{E}\{D|\,\mathbf{h}\} = \frac{1}{j}\frac{d}{dt}\phi_{D|\,\mathbf{h}}(t)\Big|_{t=0} = \sqrt{2E_\mathrm{s}}\, \|\mathbf{h}\|^2 \quad (3.5)$$

$$\mathbb{E}\big\{D^2|\,\mathbf{h}\big\} = \left(\frac{1}{j}\right)^2 \frac{d^2}{dt^2}\phi_{D|\,\mathbf{h}}(t)\Big|_{t=0} = 2E_\mathrm{s}\, \|\mathbf{h}\|^4 + \frac{N_\mathrm{d}\, N_0^2 + E_\mathrm{s}\, \|\mathbf{h}\|^2\, N_0(N_\mathrm{p}\,\varepsilon + 1)}{E_\mathrm{s} N_\mathrm{p}\,\varepsilon}. \quad (3.6)$$

To find $\mathbb{E}\{D\}$ we simply average $\mathbb{E}\{D|\,\mathbf{h}\}$ over $\mathbf{h}$

$$\mathbb{E}\{D\} = \mathbb{E}_\mathbf{h}\big\{\mathbb{E}\{D|\,\mathbf{h}\}\big\} = \sqrt{2E_\mathrm{s}}\, \mathbb{E}\Big\{\|\mathbf{h}\|^2\Big\}\ . \quad (3.7)$$

Using the law of total variance [38], the variance of the decision variable is

$$\begin{aligned}\mathrm{Var}\{D\} &= \mathbb{E}\{\mathrm{Var}\{D|\,\mathbf{h}\}\} + \mathrm{Var}\{\mathbb{E}\{D|\,\mathbf{h}\}\} \\ &= 2E_{\mathrm{s}}\left(\mathbb{E}\left\{\|\mathbf{h}\|^4\right\} - \mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}\right) + \frac{N_{\mathrm{d}}\,N_0^2 + E_{\mathrm{s}}\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}N_0(N_{\mathrm{p}}\,\varepsilon + 1)}{E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}. \end{aligned} \tag{3.8}$$

Using (3.7) and (3.8) we define the NSD of the decision variable as

$$\begin{aligned}\check{\sigma}_D &\triangleq 10\log_{10}\left[\frac{\sqrt{\mathrm{Var}\{D\}}}{\mathbb{E}\{D\}}\right] \\ &= 10\log_{10}\left[\sqrt{\frac{\mathrm{Var}\left\{\|\mathbf{h}\|^2\right\}}{\mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}} + \frac{N_{\mathrm{d}}}{2N_{\mathrm{p}}\,\varepsilon}\frac{N_0^2}{E_{\mathrm{s}}^2\mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}} + \frac{N_{\mathrm{p}}\,\varepsilon+1}{2N_{\mathrm{p}}\,\varepsilon}\frac{N_0}{E_{\mathrm{s}}\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}}\right] \\ &= 10\log_{10}\left[\sqrt{\frac{\mathrm{Var}\left\{\|\mathbf{h}\|^2\right\}}{\mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}} + \frac{N_{\mathrm{d}}}{2N_{\mathrm{p}}\,\varepsilon}\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)^2 + \frac{N_{\mathrm{p}}\,\varepsilon+1}{2N_{\mathrm{p}}\,\varepsilon}\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)}\right], \end{aligned} \tag{3.9}$$

where we have defined $\Gamma_{\mathrm{tot}} \triangleq \mathbb{E}\left\{\|\mathbf{h}\|^2\right\}\frac{E_{\mathrm{s}}}{N_0}$ as the average total SNR. Note that this expression makes no assumption about the distribution of the fading. In Chapter 7 the NSD of the decision variable is evaluated for Nakagami fading channels.

Chapter 4

Analysis of the Bit Error Probability

In this chapter we determine the BEP via a m.g.f. approach. We develop a methodology that requires evaluation of a single integral with finite limits and is applicable to channels with arbitrary distribution, including correlated fading.

4.1 Bit Error Probability Conditioned on h

In Chapter 3 the c.f. of the decision variable conditioned on the channel gain vector was derived as

$$\phi_{D|\,\mathbf{h}}(t) = \left[\frac{2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}{N_0^2t^2 + 2E_{\mathrm{s}}N_{\mathrm{p}}\varepsilon}\right]^{N_{\mathrm{d}}} \exp\left(\frac{E_{\mathrm{s}}\left\|\mathbf{h}\right\|^2\left(2j\sqrt{2E_{\mathrm{s}}}N_{\mathrm{p}}\,\varepsilon\,t - N_0\left(N_{\mathrm{p}}\,\varepsilon + 1\right)t^2\right)}{N_0^2t^2 + 2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}\right),$$

given that $a_1 = +1$ was transmitted. In this case, a bit error will occur if $D < 0$. Thus, to evaluate the BEP, we need to determine

$$\Pr\{e\,|\,\mathbf{h}\} = \Pr\{D < 0\,|\,\mathbf{h}\} = \int_{-\infty}^{0}\frac{1}{2\pi}\int_{-\infty}^{\infty}\phi_{D|\,\mathbf{h}}(t)e^{-jtD}dt\,dD\,,$$

where the inner integral is the inversion of the conditional c.f. The order of integration can be exchanged, and the integration over D performed, provided that a small imaginary constant, $j\epsilon$, is added to avoid the singularity at $t=0$

$$
\begin{aligned}
\Pr\{e\,|\,\mathbf{h}\} &= -\frac{1}{2\pi j}\int_{-\infty+j\epsilon}^{\infty+j\epsilon}\frac{\phi_{D|\mathbf{h}}(t)}{t}dt \\
&= -\frac{1}{2\pi j}\int_{-\infty+j\epsilon}^{\infty+j\epsilon}\left[\frac{\frac{2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}{N_0^2}}{t\left(t^2+\frac{2E_{\mathrm{s}}N_{\mathrm{p}}\varepsilon}{N_0^2}\right)}\right]^{N_{\mathrm{d}}} \\
&\qquad\times\exp\left(\frac{\frac{2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}{N_0^2}\left(j\sqrt{2E_{\mathrm{s}}}\,\|\mathbf{h}\|^2\,t-\frac{N_0}{2}\frac{N_{\mathrm{p}}\,\varepsilon+1}{N_{\mathrm{p}}\,\varepsilon}\,\|\mathbf{h}\|^2\,t^2\right)}{t^2+\frac{2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}{N_0^2}}\right)dt\,. \qquad (4.1)
\end{aligned}
$$

Note that the integral in (4.1) is of the form

$$
-\frac{(v_1v_2)^L}{2\pi j}\int_{-\infty+j\epsilon}^{\infty+j\epsilon}\frac{1}{t(t+jv_1)^L(t-jv_2)^L}\exp\left(\frac{v_1v_2(jt\alpha_2-t^2\alpha_1)}{(t+jv_1)(t-jv_2)}\right)dt\,, \qquad (4.2)
$$

with

$$
\begin{aligned}
L &= N_{\mathrm{d}} \\
v_1 = v_2 &= \frac{\sqrt{2E_{\mathrm{s}}N_{\mathrm{p}}\,\varepsilon}}{N_0} \\
\alpha_1 &= \frac{N_0}{2}\frac{N_{\mathrm{p}}\,\varepsilon+1}{N_{\mathrm{p}}\,\varepsilon}\,\|\mathbf{h}\|^2 \\
\alpha_2 &= \sqrt{2E_{\mathrm{s}}}\,\|\mathbf{h}\|^2\,.
\end{aligned}
$$

Such an integral was evaluated in [23, 25] as

$$
\begin{aligned}
&Q_1(a,b)-I_0(ab)\exp\left[-\frac{1}{2}(a^2+b^2)\right] \\
&\quad+\frac{I_0(ab)\exp\left[-\frac{1}{2}(a^2+b^2)\right]}{(1+v_2/v_1)^{(2L-1)}}\sum_{k=0}^{L-1}\binom{2L-1}{k}\left(\frac{v_2}{v_1}\right)^k \\
&\quad+\frac{\exp\left[-\frac{1}{2}(a^2+b^2)\right]}{(1+v_2/v_1)^{(2L-1)}}\sum_{n=1}^{L-1}I_n(ab)\sum_{k=0}^{L-1-n}\binom{2L-1}{k} \\
&\quad\times\left[\left(\frac{b}{a}\right)^n\left(\frac{v_2}{v_1}\right)^k-\left(\frac{a}{b}\right)^n\left(\frac{v_2}{v_1}\right)^{2L-1-k}\right], \qquad (4.3)
\end{aligned}
$$

where $Q_1(\cdot,\cdot)$ is the Marcum Q-function, $I_n(\cdot)$ is the modified Bessel function of the n^{th} order, and

$$a = \left[\frac{2v_1^2 v_2(\alpha_1 v_2 - \alpha_2)}{(v_1+v_2)^2}\right]^{1/2}, \quad b = \left[\frac{2v_2^2 v_1(\alpha_1 v_1 + \alpha_2)}{(v_1+v_2)^2}\right]^{1/2}. \tag{4.4}$$

Using (4.3) in (4.1), in conjunction with the fact that $\frac{v_2}{v_1} = 1$, we obtain the conditional error probability, conditioned on the channel vector $\mathbf{h}$, as

$$\begin{aligned}\Pr\{e \,|\, \mathbf{h}\} = {} & Q_1(\zeta b, b) - \frac{1}{2} I_0(\zeta b^2) \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\ & + \frac{1}{2^{(2N_\mathrm{d}-1)}} \sum_{n=1}^{N_\mathrm{d}-1} I_n(\zeta b^2) \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\ & \times \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k} \left[\zeta^{-n} - \zeta^{n}\right],\end{aligned} \tag{4.5}$$

where, as in [39], we define $\zeta \triangleq \frac{a}{b}$, $0 < \zeta \leq 1$, and

$$a = \frac{\sqrt{E_\mathrm{s}}\,\|\mathbf{h}\|\,\left|\sqrt{N_\mathrm{p}\,\varepsilon} - 1\right|}{\sqrt{2N_0}} \tag{4.6}$$

$$b = \frac{\sqrt{E_\mathrm{s}}\,\|\mathbf{h}\|\,(\sqrt{N_\mathrm{p}\,\varepsilon} + 1)}{\sqrt{2N_0}}. \tag{4.7}$$

Now we make use of the following expressions for $Q_1(\zeta b, b)$ and $I_n(z)$ [39, 40],

$$\begin{aligned}Q_1(\zeta b, b) = {} & \frac{1}{4\pi}\int_{-\pi}^{\pi} \left\{\exp\left(-\frac{b^2}{2}(1 + 2\zeta\sin\theta + \zeta^2)\right)\right. \\ & \left. + \exp\left(-\frac{b^2}{2}\left[\frac{(1-\zeta^2)^2}{1+2\zeta\sin\theta+\zeta^2}\right]\right)\right\} d\theta\end{aligned} \tag{4.8}$$

$$I_n(z) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos\left(n\left(\theta + \frac{\pi}{2}\right)\right) \mathrm{e}^{-z\sin\theta} d\theta. \tag{4.9}$$

Application of (4.8) and (4.9) to (4.5) and further simplification yields

$$\Pr\{e \,|\, \mathbf{h}\} = \frac{1}{4\pi}\int_{-\pi}^{\pi} \left\{\exp\left(-\frac{b^2}{2}\left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) + f(\theta;\zeta)\exp\left(-\frac{b^2}{2}g(\theta;\zeta)\right)\right\} d\theta, \tag{4.10}$$

where

$$f(\theta;\zeta) = \frac{1}{2^{(2N_\mathrm{d}-2)}} \sum_{n=1}^{N_\mathrm{d}-1} \cos\left(n\left(\theta+\frac{\pi}{2}\right)\right)\left[\zeta^{-n}-\zeta^{n}\right] \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k} \tag{4.11}$$

$$g(\theta;\zeta) = 1 + 2\zeta\sin\theta + \zeta^2\,. \tag{4.12}$$

Now, we note that

$$b^2 = \frac{E_\mathrm{s}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{2N_0} = \frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{2\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} \tag{4.13}$$

$$\zeta = \frac{\left|\sqrt{N_\mathrm{p}\,\varepsilon}-1\right|}{\sqrt{N_\mathrm{p}\,\varepsilon}+1}\,. \tag{4.14}$$

Substitution of (4.13) and (4.14) into (4.10) yields the simplified expression for the BEP when conditioned on the channel

$$\begin{aligned}\Pr\{e\,|\,\mathbf{h}\} = \frac{1}{4\pi}\int_{-\pi}^{\pi} \Bigg\{ &\exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}\left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) \\ &+ f(\theta;\zeta)\exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} g(\theta;\zeta)\right)\Bigg\}\,d\theta\,.\end{aligned} \tag{4.15}$$

The advantage of (4.15), compared to the original equation (4.5), is now apparent in that averaging over $\mathbf{h}$ is a simple process because it lies only in the exponents. In addition, (4.15) only depends on $\|\mathbf{h}\|^2$, that is, it is sufficient to only condition on a single r.v., namely the norm square of the channel gain vector, as opposed to conditioning on the entire vector, involving N_d r.v.'s.

4.2 Bit Error Probability for Arbitrary Fading Channels

We now determine the BEP of our practical diversity system in arbitrary fading channels by averaging (4.15) over the channel ensemble:

$$P_e = \mathbb{E}_\mathbf{h}\big\{\Pr\{e\,|\,\mathbf{h}\}\big\}.$$

In [19–21] the conditional BEP, conditioned on the SNR, is averaged over the distribution of the SNR. Our derivation shows that one must average $\Pr\{e \mid \mathbf{h}\}$ in (4.15) over the distribution of the fading ensemble to get the *exact* BEP. This is in agreement with the observation made recently in [22]. A similar observation was also made more than four decades ago in [15], "Since we do not have exact coherent detection one can not average over the nonfading coherent detection error probability ... to obtain the error probability of fixed-reference coherent detection."

Since the N_{d} terms that we are averaging over appear only as $\|\mathbf{h}\|^2$ in the exponents of $\Pr\{e \mid \mathbf{h}\}$ in (4.15), we obtain the exact BEP expression as

$$P_e(\Gamma_{\mathrm{tot}}) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{ M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_{\mathrm{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right) + f(\theta;\zeta) M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_{\mathrm{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} g(\theta;\zeta)\right)\right\} d\theta, \tag{4.16}$$

where $M_{\|\mathbf{h}\|^2}(s) \triangleq \mathbb{E}_{\mathbf{h}}\left\{\mathrm{e}^{+s\|\mathbf{h}\|^2}\right\}$ is the m.g.f. of the norm square of the channel gain vector. Thus, we have an exact BEP expression for practical diversity systems in the presence of channel estimation error, for arbitrary channels. All we require is the evaluation of a single integral with finite limits and an integrand involving only the m.g.f. of the norm square of the channel gain vector.

4.3 Special Cases

In this section we consider some special cases of the BEP expression. From (4.16) we see that the BEP depends on N_{p} and ε through the quantity $N_{\mathrm{p}}\,\varepsilon$. Here, we investigate the cases where $N_{\mathrm{p}}\,\varepsilon$ is large, $N_{\mathrm{p}}\,\varepsilon \to 1$, and $N_{\mathrm{p}}\,\varepsilon = 0$.

4.3.1 Large $N_{\mathbf{p}}\,\varepsilon$

Since N_{p} and ε represent the number of pilot symbols used to form the estimate of the channel and the ratio of pilot energy to data energy, respectively, with increasing $N_{\mathrm{p}}\,\varepsilon$ we expect to see performance approach that of perfect channel knowledge. For a particular value of $\theta \in (-\pi, \pi)$ we consider the limit of the integrand in (4.15),

$$\lim_{N_\mathrm{p}\,\varepsilon\to\infty} \left\{ \exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} \left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) + f(\theta;\zeta)\exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} g(\theta;\zeta)\right)\right\}. \tag{4.17}$$

From (4.14), as $N_\mathrm{p}\,\varepsilon \to \infty$, we note that $\zeta \to 1$. This causes $f(\theta;\zeta)$ in (4.11) to go to zero. Furthermore, for large $N_\mathrm{p}\,\varepsilon$ the exponent in the second term of (4.17) tends to $-\infty$, hence the limit of the second term is zero. Thus, we need only consider the limit of the first term

$$\begin{aligned}
&\lim_{N_\mathrm{p}\,\varepsilon\to\infty} \exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2\,(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} \left[\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]\right) \\
&= \lim_{N_\mathrm{p}\,\varepsilon\to\infty} \exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} \left[\frac{\left((\sqrt{N_\mathrm{p}\,\varepsilon}+1) - \frac{(\sqrt{N_\mathrm{p}\,\varepsilon}-1)^2}{\sqrt{N_\mathrm{p}\,\varepsilon}+1}\right)^2}{g(\theta;\zeta)}\right]\right) \\
&= \lim_{N_\mathrm{p}\,\varepsilon\to\infty} \exp\left(-\frac{\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2}{4\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}} \left[\frac{\left(\frac{4\sqrt{N_\mathrm{p}\,\varepsilon}}{\sqrt{N_\mathrm{p}\,\varepsilon}+1}\right)^2}{g(\theta;\zeta)}\right]\right) = \exp\left(-\frac{2\,\Gamma_\mathrm{tot}\,\|\mathbf{h}\|^2}{\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}(1+\sin\theta)}\right)
\end{aligned} \tag{4.18}$$

Using (4.18) in (4.15), averaging over the fading ensemble, and simplifying gives

$$P_e(\Gamma_\mathrm{tot}) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} M_{\|\mathbf{h}\|^2}\left(-\frac{\Gamma_\mathrm{tot}}{\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}\sin^2\theta}\right) d\theta, \tag{4.19}$$

as $N_\mathrm{p}\,\varepsilon \to \infty$. We recognize (4.19) as the BEP for BPSK with perfect channel knowledge [39, p. 268].

4.3.2 $N_\mathbf{p}\,\varepsilon \to 1$

This case is of interest as it includes the simplest estimator, namely the case where $N_\mathrm{p} = 1$ and $\varepsilon \to 1$. From (4.14), when $N_\mathrm{p}\,\varepsilon \to 1$, $\zeta \to 0$. In order to evaluate the BEP performance in this case, we begin with (4.5) and apply the small argument form of the modified Bessel function of the n^{th} order [39, p. 84],

$$I_n(z) \approx \frac{\left(\frac{z}{2}\right)^n}{n!}, \quad z \text{ small},$$

where we have assumed that n is a non-negative integer. Assuming that ζ is small, we have

$$\begin{aligned}
\Pr\{e \,|\, \mathbf{h}\} &= Q_1(\zeta b, b) - \frac{1}{2}\exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\
&+ \frac{1}{2^{(2N_\mathrm{d}-1)}} \sum_{n=1}^{N_\mathrm{d}-1} \frac{1}{n!}\left(\frac{\zeta b^2}{2}\right)^n \left[\zeta^{-n} - \zeta^n\right] \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\
&\times \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k}.
\end{aligned}$$

Simplifying gives

$$\begin{aligned}
\Pr\{e \,|\, \mathbf{h}\} &= Q_1(\zeta b, b) - \frac{1}{2}\exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\
&+ \frac{1}{2^{(2N_\mathrm{d}-1)}} \sum_{n=1}^{N_\mathrm{d}-1} \frac{1}{n!}\left[\left(\frac{b^2}{2}\right)^n - \left(\frac{\zeta^2 b^2}{2}\right)^n\right] \exp\left(-\frac{b^2}{2}(1+\zeta^2)\right) \\
&\times \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k}.
\end{aligned}$$

Now, we take the limit as $\zeta \to 0$. Noting that $Q_1(0, b) = \exp\left(-\frac{b^2}{2}\right)$, after careful simplification we have

$$\Pr\{e \,|\, \mathbf{h}\} = \frac{1}{2^{2N_\mathrm{d}-1}} \sum_{n=0}^{N_\mathrm{d}-1} \frac{1}{n!}\left(\frac{b^2}{2}\right)^n \exp\left(-\frac{b^2}{2}\right) \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k},$$

where, from (4.13), $\frac{b^2}{2} = \frac{\Gamma_\mathrm{tot}\|\mathbf{h}\|^2}{\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}$. Using the fact that for a random variable X,

$$\mathbb{E}\left\{X^n e^{sX}\right\} = \frac{d^n}{ds^n} M_X(s),$$

we obtain the unconditional BEP expression as

$$P_e(\Gamma_\mathrm{tot}) = \frac{1}{2^{2N_\mathrm{d}-1}} \sum_{n=0}^{N_\mathrm{d}-1} \frac{1}{n!}\left(\frac{\Gamma_\mathrm{tot}}{\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}\right)^n \frac{d^n}{ds^n} M_{\|\mathbf{h}\|^2}(s)\bigg|_{s=-\frac{\Gamma_\mathrm{tot}}{\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}} \sum_{k=0}^{N_\mathrm{d}-1-n} \binom{2N_\mathrm{d}-1}{k}.$$

4.3.3 $N_{\mathbf{p}}\,\varepsilon = 0$

In this case, no channel estimation is performed, so we expect performance to degrade completely. From (4.14), when $N_{\mathrm{p}}\,\varepsilon = 0$ we have $\zeta = 1$. This causes $f(\theta;\zeta)$ to equal zero for all θ, hence the second term in (4.16) does not contribute to the integral. Also, note that the argument of the m.g.f. in the first term of (4.16) is zero. Using the fact that $M_{\|\mathbf{h}\|^2}(0) = 1$, we have

$$P_e(\Gamma_{\mathrm{tot}}) = \frac{1}{4\pi} \int_{-\pi}^{\pi} d\theta = \frac{1}{2}\,.$$

As expected, without performing any estimation the receiver achieves the worst possible performance.

Chapter 5

BEP for Independent Fading

Using the analytical framework developed in the previous chapter, which is valid for arbitrary fading distributions, this chapter evaluates the BEP for some common independent non-identically distributed (i.n.i.d.) channel models.[1] First, Nakagami-m distributed channels with arbitrary m parameters are considered. Next, the case of Rayleigh fading is presented. The case of Ricean distributed channels is also considered. Asymptotic results for the special case of i.i.d. fading are obtained to determine the diversity order of systems with practical channel estimation operating in these channels.

5.1 Nakagami and Rayleigh Fading Environments

Nakagami-m fading channels have received considerable attention in the study of various aspects of wireless systems [41, 42]. In particular, it was shown recently that the amplitude distribution of the resolved multipath components in ultra-wide bandwidth (UWB) indoor channels can be well-modeled by the Nakagami-m distribution [28]. The Nakagami-m family of distributions, also known as the "m-distribution," contains Rayleigh fading ($m = 1$) as a special case; along with cases of fading that are more severe than Rayleigh ($1/2 \leq m < 1$) as well as cases less severe than Rayleigh ($m > 1$).

In a Nakagami fading environment, the probability density function (p.d.f.) of each $|h_k|$ is given by the Nakagami distribution

$$f_{|h_k|}(x) = \frac{2}{\Gamma(m_k)} \left(\frac{m_k}{\Omega_k}\right)^{m_k} x^{2m_k-1} e^{-m_k x^2/\Omega_k}\,, \qquad x \geq 0\,, \tag{5.1}$$

[1]This generalizes the case of i.i.d. fading; that is, it includes i.i.d. fading as a special case.

where $\Omega_k = \mathbb{E}\Big\{|h_k|^2\Big\}$. The n^{th} moment of $|h_k|$ can be expressed as

$$\mathbb{E}\{|h_k|^n\} = \frac{\Gamma\left(m_k + \frac{1}{2}n\right)}{\Gamma\left(m_k\right)} \left(\frac{\Omega_k}{m_k}\right)^{\frac{n}{2}} , \tag{5.2}$$

where $\Gamma(\cdot)$ is the gamma function,[2,3]

$$\begin{aligned}
\Gamma(p) &= \int_0^\infty t^{p-1} \mathrm{e}^{-t} dt \,, & p &> 0 \\
\Gamma(p) &= (p-1)! \,, & p &\in \mathbb{Z}^+ \\
\Gamma\left(\frac{1}{2}\right) &= \sqrt{\pi} & \Gamma\left(\frac{3}{2}\right) &= \frac{1}{2}\sqrt{\pi} \,.
\end{aligned}$$

Here, we are interested in the squared-magnitude of the fading gains, $|h_k|^2$, whose p.d.f. is given by the chi-square distribution with $2m_k$ degrees of freedom

$$f_{|h_k|^2}(y) = \frac{1}{\Gamma(m_k)} \left(\frac{m_k}{\Omega_k}\right)^{m_k} y^{m_k - 1} e^{-m_k y/\Omega_k} \,, \qquad y \geq 0 \,. \tag{5.3}$$

The p.d.f. of $\|\mathbf{h}\|^2$ is given by the $(N_\mathrm{d}-1)$-fold convolution of $f_{|h_k|^2}(\cdot)$

$$f_{\|\mathbf{h}\|^2}(z) = f_{|h_1|^2}(z) * f_{|h_2|^2}(z) * \cdots * f_{\left|h_{N_\mathrm{d}}\right|^2}(z) \,, \qquad z \geq 0 \,. \tag{5.4}$$

The expression for the BEP of diversity systems with practical channel estimation, (4.16), relies on the m.g.f. of $\|\mathbf{h}\|^2$. In an i.n.i.d. Nakagami fading environment, this m.g.f. is given by

$$M_{\|\mathbf{h}\|^2}(s) = \prod_{k=1}^{N_\mathrm{d}} \left[\frac{1}{1 - s\frac{\Omega_k}{m_k}}\right]^{m_k} . \tag{5.5}$$

The m.g.f. for a Rayleigh fading environment is obtained by setting $m_k = 1, \ \forall k$ in the Nakagami-m model above.

Using (5.5) in (4.16), the BEP of diversity systems with practical channel estimation

[2]Note that $\Gamma(\cdot)$ is used to denote the gamma *function*, while Γ_{tot} denotes the average total SNR.
[3]$\mathbb{Z}^+$ denotes the set of positive integers.

operating in an i.n.i.d. Nakagami fading environment is given by

$$P_e(\Gamma_{\text{tot}}) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left\{\prod_{k=1}^{N_\text{d}}\left[1+\frac{\Gamma_{\text{tot}}(\sqrt{N_\text{p}\,\varepsilon}+1)^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}\frac{\Omega_k}{m_k}\frac{(1-\zeta^2)^2}{g(\theta;\zeta)}\right]^{-m_k}\right.$$
$$\left.+\,f(\theta;\zeta)\prod_{k=1}^{N_\text{d}}\left[1+\frac{\Gamma_{\text{tot}}(\sqrt{N_\text{p}\,\varepsilon}+1)^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}\frac{\Omega_k}{m_k}g(\theta;\zeta)\right]^{-m_k}\right\}d\theta\,. \tag{5.6}$$

5.2 Ricean Fading Environment

The Rice distribution is appropriate for modeling communication environments where there are line of sight components, such as satellite channels [3, 29]. In a Ricean fading environment environment, each h_k has a complex Gaussian distribution with nonzero mean. Correspondingly, the p.d.f. of each $|h_k|^2$ is given by the non-central chi-square distribution

$$f_{|h_k|^2}(y) = \frac{\kappa_k}{|\mu_k|^2}\exp\left(-\frac{(|\mu_k|^2+y)\kappa_k}{|\mu_k|^2}\right)I_0\left(2\sqrt{y}\frac{\kappa_k}{|\mu_k|}\right), \qquad y\geq 0\,, \tag{5.7}$$

where $\mu_k = \mathbb{E}\{h_k\}$ and $\kappa_k \triangleq \frac{|\mu_k|^2}{\mathbb{E}\left\{|h_k|^2\right\}-|\mu_k|^2}$ is the Rice factor. Note that μ_k is complex in general, but the p.d.f. of $|h_k|^2$ does not depend on the phase of μ_k. The p.d.f. of $\|\mathbf{h}\|^2$ is given by the $(N_\text{d}-1)$-fold convolution of $f_{|h_k|^2}(\cdot)$

$$f_{\|\mathbf{h}\|^2}(z) = f_{|h_1|^2}(z) * f_{|h_2|^2}(z) * \cdots * f_{|h_{N_\text{d}}|^2}(z)\,, \qquad z\geq 0\,. \tag{5.8}$$

The m.g.f. of $\|\mathbf{h}\|^2$ is given by

$$M_{\|\mathbf{h}\|^2}(s) = \prod_{k=1}^{N_\text{d}}\left[\frac{\kappa_k}{\kappa_k - s\,|\mu_k|^2}\right]\exp\left(\frac{s\kappa_k\,|\mu_k|^2}{\kappa_k - s\,|\mu_k|^2}\right). \tag{5.9}$$

Using (5.9) in (4.16), the BEP for diversity systems with practical channel estimation operating in i.n.i.d. Ricean channels is given by

$$
\begin{aligned}
P_e(\Gamma_{\text{tot}}) = \frac{1}{4\pi}\int_{-\pi}^{\pi} \Bigg\{ \prod_{k=1}^{N_{\mathrm{d}}} \Bigg[& \frac{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} g(\theta;\zeta)\,\kappa_k}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} g(\theta;\zeta)\,\kappa_k + \Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2(1-\zeta^2)^2\,|\mu_k|^2} \\
& \times \exp\left(\frac{-\Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2(1-\zeta^2)^2\,\kappa_k\,|\mu_k|^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} g(\theta;\zeta)\,\kappa_k + \Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2(1-\zeta^2)^2\,|\mu_k|^2} \right) \Bigg] \\
+\, f(\theta;\zeta) \prod_{k=1}^{N_{\mathrm{d}}} \Bigg[& \frac{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}\,\kappa_k}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}\,\kappa_k + \Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2 g(\theta;\zeta)\,|\mu_k|^2} \\
& \times \exp\left(\frac{-\Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2 g(\theta;\zeta)\kappa_k\,|\mu_k|^2}{4\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}\,\kappa_k + \Gamma_{\text{tot}}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2 g(\theta;\zeta)\,|\mu_k|^2} \right) \Bigg] \Bigg\}\, d\theta\,.
\end{aligned}
\tag{5.10}
$$

5.3 Asymptotic Results

5.3.1 Nakagami Fading

We now consider the behavior of the expressions in (4.16) and (4.19) as the SNR increases asymptotically for the case of i.i.d. Nakagami fading channels with $\Omega = 1$. In this case, the m.g.f. becomes

$$
M_{\|\mathbf{h}\|^2}(s) = \left(\frac{1}{1-\frac{s}{m}}\right)^{mN_{\mathrm{d}}} \approx \left(-\frac{m}{s}\right)^{mN_{\mathrm{d}}}, \tag{5.11}
$$

where the approximation is for large s. Using (5.11) in (4.16), one can obtain the asymptotic behavior for the case of imperfect channel knowledge as $\Gamma_{\text{tot}} \to \infty$,

$$
\begin{aligned}
P_{e,\substack{\text{Asym-I}\\ \text{Nakagami}}}(\Gamma_{\text{tot}}) &= \frac{1}{4\pi}\int_{-\pi}^{\pi} \Bigg\{ \left[\frac{m\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} g(\theta;\zeta)}{\frac{\Gamma_{\text{tot}}}{4}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2(1-\zeta^2)^2} \right]^{mN_{\mathrm{d}}} \\
&\qquad + f(\theta;\zeta) \left[\frac{m\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\}}{\frac{\Gamma_{\text{tot}}}{4}(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2 g(\theta;\zeta)} \right]^{mN_{\mathrm{d}}} \Bigg\}\, d\theta \\
&= K_{\text{I, Nakagami}}(m,\, N_{\mathrm{d}},\, N_{\mathrm{p}},\, \varepsilon) \left(\frac{1}{\Gamma_{\text{tot}}}\right)^{mN_{\mathrm{d}}},
\end{aligned}
\tag{5.12}
$$

where we have defined

$$K_{\mathrm{I,Nakagami}}\,(m,\,N_{\mathrm{d}},\,N_{\mathrm{p}},\,\varepsilon) \triangleq \frac{1}{4\pi}\left[\frac{4mN_{\mathrm{d}}}{(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2}\right]^{mN_{\mathrm{d}}} \times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{mN_{\mathrm{d}}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{mN_{\mathrm{d}}}}\right\} d\theta\,. \tag{5.13}$$

In (5.13) we have used the fact that $\mathbb{E}\Big\{\|\mathbf{h}\|^2\Big\} = N_{\mathrm{d}}\,\Omega = N_{\mathrm{d}}$. The subscript Asym-I is used to denote the *asymptotic behavior with imperfect channel knowledge.*

Using (5.11) in (4.19), one can similarly derive the asymptotic behavior for the case of perfect channel knowledge as

$$P_{e,\substack{\text{Asym-P}\\ \text{Nakagami}}}\,(\Gamma_{\text{tot}}) = K_{\mathrm{P,\,Nakagami}}\,(m,\,N_{\mathrm{d}})\left(\frac{1}{\Gamma_{\text{tot}}}\right)^{mN_{\mathrm{d}}}, \tag{5.14}$$

where

$$K_{\mathrm{P,\,Nakagami}}\,(m,\,N_{\mathrm{d}}) \triangleq \frac{(mN_{\mathrm{d}})^{mN_{\mathrm{d}}}\,\Gamma(\frac{1}{2}+mN_{\mathrm{d}})}{2\sqrt{\pi}\,\Gamma(1+mN_{\mathrm{d}})}. \tag{5.15}$$

The subscript Asym-P is used to denote the *asymptotic behavior with perfect channel knowledge.*

5.3.2 Rayleigh Fading

For the special case of Rayleigh fading, the asymptotic results can be derived by setting $m = 1$ in (5.12) and (5.14). In doing this we have,

$$P_{e,\substack{\text{Asym-I}\\ \text{Rayleigh}}}\,(\Gamma_{\text{tot}}) = K_{\mathrm{I,\,Rayleigh}}\,(N_{\mathrm{d}},\,N_{\mathrm{p}},\,\varepsilon)\left(\frac{1}{\Gamma_{\text{tot}}}\right)^{N_{\mathrm{d}}}, \tag{5.16}$$

where $K_{\mathrm{I,Rayleigh}}\,(N_{\mathrm{d}},\,N_{\mathrm{p}},\,\varepsilon) \triangleq K_{\mathrm{I,Nakagami}}\,(1,\,N_{\mathrm{d}},\,N_{\mathrm{p}},\,\varepsilon)$. Similarly,

$$P_{e,\substack{\text{Asym-P}\\ \text{Rayleigh}}}\,(\Gamma_{\text{tot}}) = K_{\mathrm{P,\,Rayleigh}}\,(N_{\mathrm{d}})\left(\frac{1}{\Gamma_{\text{tot}}}\right)^{N_{\mathrm{d}}}, \tag{5.17}$$

where $K_{\mathrm{P,\,Rayleigh}}\,(N_{\mathrm{d}}) \triangleq K_{\mathrm{P,\,Nakagami}}\,(1,\,N_{\mathrm{d}})$.

5.3.3 Ricean Fading

Similar to the case above, we consider the asymptotic behavior of (4.16) and (4.19) in i.i.d. Ricean fading with $\mathbb{E}\left\{|h|^2\right\} = 1$. In this case the m.g.f. becomes

$$M_{\|\mathbf{h}\|^2}(s) = \left[\frac{1+\kappa}{1+\kappa-s}\right]^{N_\mathrm{d}} \exp\left(\frac{sN_\mathrm{d}\,\kappa}{1+\kappa-s}\right) \tag{5.18}$$

$$\approx \left[-\frac{1+\kappa}{s}\right]^{N_\mathrm{d}} \exp\left(-N_\mathrm{d}\,\kappa\right), \tag{5.19}$$

where κ is the Rice factor and the approximation is valid for large s. Using (5.19) in (4.16), one can obtain the asymptotic behavior for the case of imperfect channel knowledge in Ricean fading as $\Gamma_{\mathrm{tot}} \to \infty$,

$$\begin{aligned} P_{e,\,\substack{\text{Asym-I}\\ \text{Ricean}}}(\Gamma_{\mathrm{tot}}) &= \left[\frac{N_\mathrm{d}(1+\kappa)\mathrm{e}^{-\kappa}}{\Gamma_{\mathrm{tot}}}\right]^{N_\mathrm{d}} \frac{1}{4\pi}\left[\frac{4}{(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}\right]^{N_\mathrm{d}} \\ &\quad\times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_\mathrm{d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{N_\mathrm{d}}}\right\} d\theta \\ &= K_{\mathrm{I,\,Ricean}}(\kappa,\, N_\mathrm{d},\, N_\mathrm{p},\, \varepsilon)\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)^{N_\mathrm{d}}, \end{aligned} \tag{5.20}$$

where we have defined

$$\begin{aligned} K_{\mathrm{I,\,Ricean}}(\kappa,\, N_\mathrm{d},\, N_\mathrm{p},\, \varepsilon) &\triangleq \frac{1}{4\pi}\left[\frac{4N_\mathrm{d}(1+\kappa)\,\mathrm{e}^{-\kappa}}{(\sqrt{N_\mathrm{p}\,\varepsilon}+1)^2}\right]^{N_\mathrm{d}} \\ &\quad\times \int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{N_\mathrm{d}} + \frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{N_\mathrm{d}}}\right\} d\theta\,. \end{aligned} \tag{5.21}$$

A similar calculation using (5.19) in (4.19) yields the asymptotic behavior for perfect channel knowledge in Ricean fading,

$$\begin{aligned} P_{e,\,\substack{\text{Asym-P}\\ \text{Ricean}}}(\Gamma_{\mathrm{tot}}) &= \left[\frac{N_\mathrm{d}(1+\kappa)\,\mathrm{e}^{-\kappa}}{\Gamma_{\mathrm{tot}}}\right]^{N_\mathrm{d}} \frac{\Gamma(\frac{1}{2}+N_\mathrm{d})}{2\sqrt{\pi}\,\Gamma(1+N_\mathrm{d})} \\ &= K_{\mathrm{P,\,Ricean}}(\kappa,\, N_\mathrm{d})\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)^{N_\mathrm{d}}, \end{aligned} \tag{5.22}$$

where

$$K_{\mathrm{P,\,Ricean}}\,(\kappa,\ N_{\mathrm{d}}) \triangleq \frac{\left[N_{\mathrm{d}}(1+\kappa)\,\mathrm{e}^{-\kappa}\right]^{N_{\mathrm{d}}}\,\Gamma(\frac{1}{2}+N_{\mathrm{d}})}{2\sqrt{\pi}\,\Gamma(1+N_{\mathrm{d}})}. \tag{5.23}$$

Note that the results in (5.20) and (5.22) differ from their counterparts in Rayleigh fading, (5.16) and (5.17), by only the multiplicative factor $\left[(1+\kappa)\,\mathrm{e}^{-\kappa}\right]^{N_{\mathrm{d}}}$.

For the case of Nakagami fading, it is clear from (5.12) and (5.14) that regardless of the number of pilot symbols used in the formation of an estimate of the channel, a diversity order of mN_{d} is still maintained as in the case of ideal MRC. Similarly, for the case of Ricean fading, (5.20) and (5.22) show that a diversity order of N_{d} is preserved. This behavior, arising purely from the analytical asymptotic expressions given in this chapter, is also evident from numerical results as will be shown in Chapter 7. These results are in contrast to the analytical results presented in [19–21] which showed that, even with the estimate arbitrarily close to the ideal one, the asymptotic BEP is proportional to $\frac{1}{\Gamma_{\mathrm{tot}}}$. That is, even with an arbitrarily good estimate, diversity order is that of a single branch system. For example, the expression [19, eq. (20)] shows that the diversity order is equal to that of a single branch system.

Chapter 6

BEP for Correlated Fading

Using the analytical framework developed in Chapter 4, which is valid for arbitrary fading distributions, the BEP is evaluated for some common correlated fading models. We consider the case of Nakagami-m distributed channels with arbitrary m parameters. Then, the case of Rayleigh fading is presented. The case of Ricean distributed channels is also considered.

6.1 Correlated Nakagami Fading Environment

Correlated Nakagami-m fading channels have received considerable attention in the study of various aspects of wireless systems [43, 44]. To investigate this environment, we consider a correlated fading channel with N_d diversity branches, whose squared fading gains are specified by $[|h_1|^2, |h_2|^2, \ldots, |h_{N_\mathrm{d}}|^2]$. Each $|h_i|$ has a Nakagami distribution with parameter $m_i \in \mathbb{Z}^+$. As in [45], it is assumed that the m_i's are integers, noting that the measurement accuracy of the channel is typically only of integer order. In a correlated Nakagami fading environment, the marginal p.d.f. of each $|h_i|^2$ is given by the chi-square distribution with $2m_i$ degrees of freedom

$$f_{|h_i|^2}(x) = \frac{1}{\Gamma(m_i)} \left(\frac{m_i}{\Omega_i}\right)^{m_i} x^{m_i-1} e^{-m_i x/\Omega_i}\,, \qquad x \geq 0\,. \tag{6.1}$$

where $\Omega_i = \mathbb{E}\left\{|h_i|^2\right\}$. In general the joint p.d.f. of $|h_1|^2, |h_2|^2, \ldots, |h_{N_\mathrm{d}}|^2$,

$$f_{|h_1|^2, |h_2|^2, \ldots, |h_{N_\mathrm{d}}|^2}\left(\{x_i\}_{i=1}^{N_\mathrm{d}}\right) \neq \prod_{i=1}^{N_\mathrm{d}} f_{|h_i|^2}(x_i)\,, \qquad \{x_i\}_{i=1}^{N_\mathrm{d}} > 0\,. \tag{6.2}$$

That is, the joint p.d.f. is not equal to the product of the marginals because the $|h_i|^2$'s are correlated. However, one can transform the dependent physical branch variables into a new set of independent *virtual branches* and express the norm square of the channel gain vector as a linear function of the independent virtual branch channel gains.

Let X_i be the $1 \times 2m_i$ vector defined by

$$X_i \triangleq \begin{bmatrix} X_{i,1} & X_{i,2} & \cdots & X_{i,2m_i} \end{bmatrix}, \qquad i = 1, 2, \ldots, N_{\mathrm{d}}\,, \tag{6.3}$$

where the elements of X_i, $X_{i,k}$'s, are i.i.d. Gaussian random variables with zero mean and variance $\mathbb{E}\Big\{X_{i,k}^2\Big\} = \frac{\Omega_i}{2m_i}$. It can be shown that each $|h_i|^2$ is infinitely divisible [46–48]. The infinite divisibility has implications on the *statistical* representation of $|h_i|^2$ as[1]

$$|h_i|^2 \overset{\mathrm{L}}{=} X_i X_i^t\,, \tag{6.4}$$

where the notation "$\overset{\mathrm{L}}{=}$" denotes "equal in their respective distributions" (or "equal in their respective Laws").[2] Therefore,

$$\|\mathbf{h}\|^2 \overset{\mathrm{L}}{=} \sum_{i=1}^{N_{\mathrm{d}}} X_i X_i^t = X X^t\,, \tag{6.5}$$

where X is the $1 \times D_T$ vector defined by

$$X \triangleq \begin{bmatrix} X_1 & X_2 & \cdots & X_{N_{\mathrm{d}}} \end{bmatrix}\,, \tag{6.6}$$

and $D_T = \sum_{i=1}^{N} 2m_i$ denotes twice the sum of the Nakagami parameters.

The statistical dependence among the N_{d} correlated branches can be related to the statistical dependence among the elements of X, by carefully constructing X. When there is only second-order dependence, it suffices to construct the covariance matrix of X given by $K_X = \mathbb{E}\big\{X^t X\big\}$. Without loss of generality, one can assume that the $|h_i|$'s are indexed in increasing order of their Nakagami parameters, i.e., $m_1 \leq m_2 \leq \ldots \leq m_{N_{\mathrm{d}}}$. We construct

---

[1] $(\cdot)^t$ denotes transpose.

[2] It is important to stress that, in general $|h_i|^2 \neq X_i X_i^t$ and the notation "$\overset{\mathrm{L}}{=}$" is used to merely indicate that only the respective distributions (or Laws) are equal [46–48]. One can view $X_i X_i^t$ as *a statistical* representation of $|h_i|^2$, and both forms can be used interchangeably in performing statistical analyses.

the covariance among the elements X such that

$$\mathbb{E}\{X_{i,k}X_{j,l}\} = \begin{cases} \frac{\Omega_i}{2m_i}, & \text{if } i=j \text{ and } k=l \\ \rho_{i,j}\sqrt{\frac{\Omega_i\,\Omega_j}{2m_i\,2m_j}}, & \text{if } i\neq j \text{ and } k=l=1,\,2,\ldots,\,2\min\{m_i,m_j\} \\ 0, & \text{otherwise} \end{cases} . \tag{6.7}$$

This construction implies that the k^{th} entries of X_i and X_j, with $i \neq j$, are correlated for $k = 1, 2, \ldots, 2\min\{m_i, m_j\}$. However, all the entries of X_i are mutually independent, and all other entries are independent. As shown in Appendix A.2, the coefficient $\rho_{i,j}$ is related to the branch correlation, $\rho_{|h_i|^2,|h_j|^2}$, in the following way

$$\rho_{|h_i|^2,|h_j|^2} \triangleq \sqrt{\frac{\min\{m_i,m_j\}}{\max\{m_i,m_j\}}}\,\rho_{i,j}^2\,.$$

The lower and upper bounds for the correlation between the two Nakagami branches are given by $0 \leq \rho_{|h_i|^2,|h_j|^2} \leq \sqrt{\frac{\min\{m_i,m_j\}}{\max\{m_i,m_j\}}}$.[3] Note in passing that a given correlation model of diversity branches does not *uniquely* determine K_X.

Then, as described in Appendix B, the m.g.f. of $\|\mathbf{h}\|^2$ is given by

$$M_{\|\mathbf{h}\|^2}(s) = \mathbb{E}\left\{e^{s\|\mathbf{h}\|^2}\right\} = \prod_{l=1}^{L}\left[\frac{1}{1-s2\lambda_l}\right]^{\frac{\nu_l}{2}} . \tag{6.8}$$

Here $\{\lambda_l\}$ is the set of L *distinct* eigenvalues of K_X where each λ_l has algebraic multiplicity ν_l such that $\sum_{l=1}^{L}\nu_l = D_T = \sum_{i=1}^{N_\text{d}} 2m_i$.

6.2 Correlated Rayleigh Fading Environment

To evaluate the performance in a Rayleigh fading environment, we need to characterize the m.g.f. of $\|\mathbf{h}\|^2$ and evaluate (4.16). The m.g.f. for Rayleigh fading is given by the Nakagami-m model above when we set $m_i = 1,\ \forall i$. In this case $K_X = \mathbb{E}\{X^tX\}$ is a $2N_\text{d} \times 2N_\text{d}$ matrix,

[3] The fact that two Nakagami branches with *different* fading parameters m_i and m_j can not be completely correlated (i.e., $\rho_{|h_i|^2,|h_j|^2} < 1$) is not a drawback in our statistical representation; it is just a manifestation of the basic fact that two r.v.'s with different distributions can not be completely correlated.

whose elements are given by

$$
\mathbb{E}\{X_{i,k}X_{j,l}\} = \begin{cases} \frac{\Omega_i}{2}, & \text{if } i = j \text{ and } k = l \\ \rho_{i,j}\sqrt{\frac{\Omega_i\,\Omega_j}{4}}, & \text{if } i \neq j \text{ and } k = l \\ 0, & \text{otherwise} \end{cases}, \tag{6.9}
$$

where

$$
\rho_{|h_i|^2,|h_j|^2} \triangleq \rho_{i,j}^2 .
$$

The m.g.f. for the norm square of the channel gain vector is then given by (6.8) where $\{\lambda_l\}$ is the set of L distinct eigenvalues of the matrix $\mathbf{K_X}$, as determined by (6.9), where each λ_l has algebraic multiplicity ν_l such that $\sum_{l=1}^{L}\nu_l = D_T = 2N_{\mathrm{d}}$.

6.3 Correlated Ricean Fading Environment

The Rice distribution is appropriate for modeling communication environments where there are line of sight components, such as satellite channels [3, 29]. Using a procedure similar to [49] we can derive the m.g.f. of the norm square of the channel gain vector in a Ricean fading environment. In such an environment, each h_k has a complex Gaussian distribution with nonzero mean. The m.g.f. of the norm square of the channel gains, $\|\mathbf{h}\|^2$, is given by

$$
M_{\|\mathbf{h}\|^2}(s) = [\det(\mathbf{I} - s\mathbf{K})]^{-1} \exp\left\{-\boldsymbol{\mu}\left(\mathbf{K} - \frac{\mathbf{I}}{s}\right)^{-1}\boldsymbol{\mu}^{\dagger}\right\}, \tag{6.10}
$$

where $\mathbf{K} = \mathbb{E}\{(\mathbf{h}-\boldsymbol{\mu})^{\dagger}(\mathbf{h}-\boldsymbol{\mu})\}$ is the covariance matrix and $\boldsymbol{\mu} = [\mu_1,\, \mu_2,\, \ldots,\, \mu_{N_{\mathrm{d}}}] = \mathbb{E}\{\mathbf{h}\}$ is the vector of (complex) means. For a Ricean environment, the Rice factor is given by $\kappa_i \triangleq \frac{|\mu_i|^2}{\mathbb{E}\{|h_i|^2\} - |\mu_i|^2}$. In Appendix A.3 it is shown that the elements of the correlation matrix, $\mathbf{K}$, can be expressed as

$$
\mathrm{Cov}\{h_i,\, h_j\} = -\Re\{\mu_i\mu_j^*\} \pm \sqrt{\Re^2\left\{\mu_i\mu_j^*\right\} + \sqrt{\mathrm{Var}\left\{|h_i|^2\right\}\mathrm{Var}\left\{|h_j|^2\right\}}\,\rho_{|h_i|^2,\,|h_j|^2}} . \tag{6.11}
$$

Note that, as in the case for correlated Nakagami fading, a given correlation model of diversity branches does not *uniquely* determine $\mathbf{K}$.

Chapter 7

Discussion and Numerical Results

In this chapter we discuss aspects of the correlation coefficient between the true channel gain and its estimate, including the relation of this correlation coefficient to the SNR and the number of pilot symbols. The mean and NSD of the decision variable are first examined, as they provide initial insights into the performance of diversity systems with practical channel estimation. Then, performance in terms of BEP is analyzed for the case of independent, as well as correlated fading channels. We also examine the SNR penalty due to channel estimation error and give some numerical results.

7.1 Relationship Between Estimate Correlation, SNR, and Number of Pilot Symbols, $N_{\mathbf{p}}$

The correlation coefficient of the channel gain estimate with the true channel gain plays a crucial role in the performance of diversity systems with practical channel estimation. Here we have used an estimator structure that employs pilot symbol transmission. The correlation coefficient that arises from such an estimator is given by

$$\rho_k = \frac{\mathbb{E}\left\{h_k \hat{h}_k^*\right\} - \mathbb{E}\{h_k\}\,\mathbb{E}\left\{\hat{h}_k^*\right\}}{\sqrt{\mathbb{E}\left\{|h_k - \mathbb{E}\{h_k\}|^2\right\}\mathbb{E}\left\{\left|\hat{h}_k - \mathbb{E}\left\{\hat{h}_k\right\}\right|^2\right\}}}$$
$$= \begin{cases} \frac{\sqrt{N_\mathrm{p}\,\varepsilon}}{\sqrt{N_\mathrm{p}\,\varepsilon + \frac{1}{\Gamma_k}}}, & \text{Nakagami-}m\text{ fading} \\ \frac{\sqrt{N_\mathrm{p}\,\varepsilon}}{\sqrt{N_\mathrm{p}\,\varepsilon + \frac{1+\kappa_k}{\Gamma_k}}}, & \text{Ricean fading} \end{cases}$$

where $\Gamma_k \triangleq \mathbb{E}\left\{|h_k|^2\right\}\frac{E_\mathrm{s}}{N_0}$ is the average SNR on the k^{th} diversity branch. It is important to note here that ρ_k is a function of the average branch SNR, Γ_k, as well as the number of pilot symbols, N_p. As Γ_k tends toward the high SNR regime, the correlation approaches one. This fact makes intuitive sense, if a system is operating under high SNR, it should be able to achieve better accuracy in its estimate. This model is significantly different from other correlation models [17–21], where the correlation coefficient is explicitly set to a particular value, irrespective of the branch SNR. Similarly, as the number of pilot symbols used to form the estimate increases, the correlation approaches one. Naturally, as the number of channel measurements increases we expect our knowledge of the channel to become more accurate.

Choosing the number of pilot symbols to use in the channel estimation is an important aspect of system design. Clearly, the number of pilot symbols cannot be arbitrarily large. The choice is governed foremost by the coherence time of the channel, and then by the requirements of the communication system in terms of bit rates and transmission power. Throughout, we have considered slowly fading channels in which a block of symbols experiences the same fading condition. Provided that the data symbols and the corresponding pilot symbols used to form an estimate are within the coherence time, the performance should follow what we have given above.

7.2 Normalized Standard Deviation

In this section the mean and NSD of the decision variable for diversity with practical channel estimation are examined for the case of i.i.d. Nakagami fading channels. Recall that the mean of the decision variable is given by (3.7) as

$$\mathbb{E}\{D\} = \sqrt{2E_{\mathrm{s}}}\,\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} = \sqrt{2E_{\mathrm{s}}}N_{\mathrm{d}}\Omega\,,$$

where we have used the fact that $\mathbb{E}\left\{\|\mathbf{h}\|^2\right\} = N_{\mathrm{d}}\Omega$ for i.i.d. Nakagami channels. This indicates that the mean of the decision variable increases with signal energy and the number of diversity branches. Thus, as the SNR's and/or number of diversity branches increases, we expect receiver performance to improve, because the mean of the distribution of the decision variable is moving away from the decision boundary (i.e. $D = 0$).

Recall that the expression for the NSD in (3.9) depends on the quantity $\frac{\mathrm{Var}\left\{\|\mathbf{h}\|^2\right\}}{\mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}}$. For i.i.d. Nakagami fading we have

$$\begin{aligned}
\mathrm{Var}\left\{\|\mathbf{h}\|^2\right\} &= N_{\mathrm{d}}\mathrm{Var}\left\{|h|^2\right\} \\
&= N_{\mathrm{d}}\left(\mathbb{E}\left\{|h|^4\right\} - \mathbb{E}^2\left\{|h|^2\right\}\right) \\
&= N_{\mathrm{d}}\Omega^2\left[\frac{(m+1)m}{m^2} - 1\right] = \frac{N_{\mathrm{d}}\Omega^2}{m}\,,
\end{aligned}$$

where we have made use of (5.2). Thus,

$$\frac{\mathrm{Var}\left\{\|\mathbf{h}\|^2\right\}}{\mathbb{E}^2\left\{\|\mathbf{h}\|^2\right\}} = \frac{N_{\mathrm{d}}\Omega^2}{{N_{\mathrm{d}}}^2\Omega^2}\left(\frac{1}{m}\right) = \frac{1}{m\,N_{\mathrm{d}}}\,. \tag{7.1}$$

Using (3.9) and (7.1), the NSD for Nakagami fading is given by

$$\breve{\sigma}_D = 10\log_{10}\left[\sqrt{\frac{1}{m\,N_{\mathrm{d}}} + \frac{N_{\mathrm{d}}}{2N_{\mathrm{p}}\,\varepsilon}\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)^2 + \frac{N_{\mathrm{p}}\,\varepsilon + 1}{2N_{\mathrm{p}}\,\varepsilon}\left(\frac{1}{\Gamma_{\mathrm{tot}}}\right)}\,\right]\,.$$

Figures 7-1 – 7-3 evaluate the NSD of the decision variable for Nakagami fading channels with $m = 0.5$, $m = 1$, and $m = 4$, respectively, for varying N_{d} and N_{p}. In each case, note that the NSD is reduced as the number of diversity branches, N_{d}, increases. This is expected because the receiver can better mitigate the effects of fading and improve its performance using a larger number of diversity branches. Increasing the number of branches effectively increases the diversity order. For Nakagami fading environments, the diversity order is given by $m\,N_{\mathrm{d}}$. Compared to Fig. 7-1, Figs. 7-2 and 7-3 exhibit lower NSD as the diversity

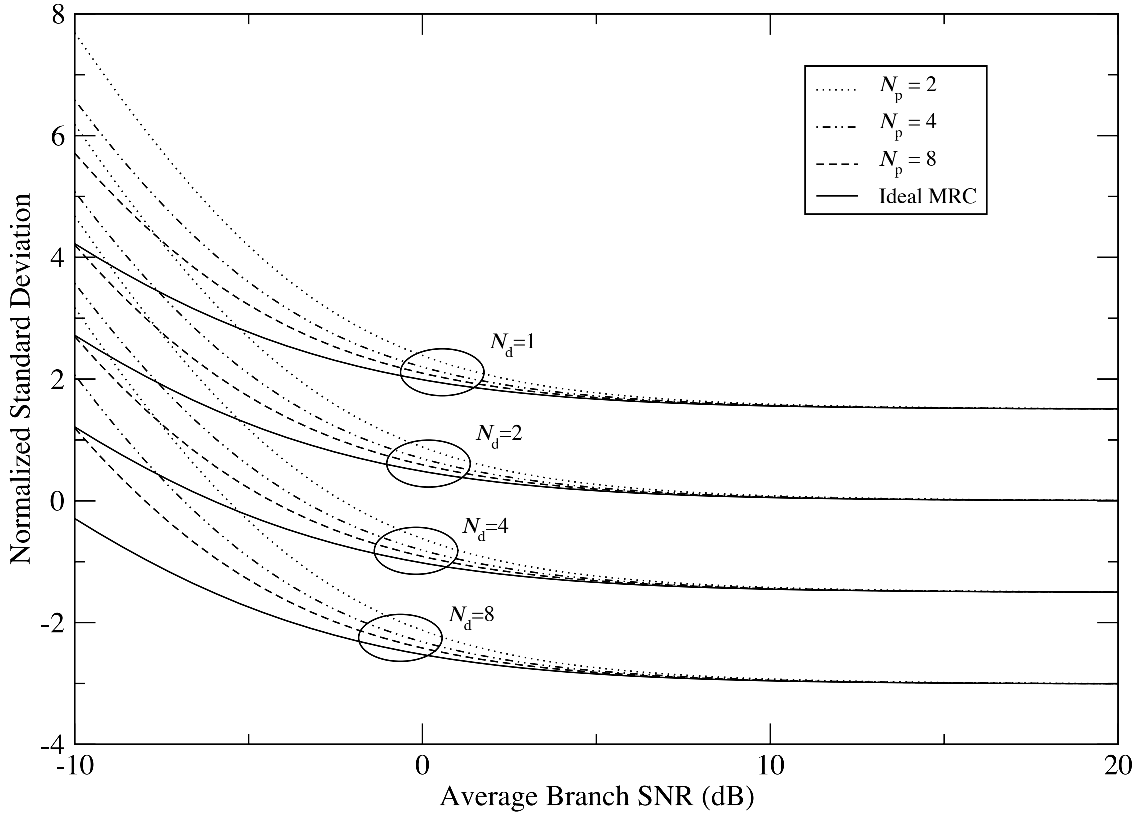

Figure 7-1: Normalized standard deviation of the decision variable for BPSK in i.i.d. Nakagami fading with $m = 0.5$, for various N_d, N_p.

order is increased from $0.5N_\mathrm{d}$ (Fig. 7-1) to N_d (Fig. 7-2) and $4N_\mathrm{d}$ (Fig. 7-3). A lower NSD corresponds to less uncertainty in the decision variable, indicating that the distribution of the decision variable is becoming more concentrated at its mean. The reduced uncertainty present in the decision variable translates directly to better receiver performance; that is, the receiver is less likely to make an error in the decision process.

It is also interesting to note that some combinations of N_d and N_p outperform systems with larger N_d with smaller N_p for certain ranges of SNR. For example, in Fig. 7-2, the curve corresponding to $N_\mathrm{d} = 2$, $N_\mathrm{p} = 8$ performs better than the case where $N_\mathrm{d} = 4$, $N_\mathrm{p} = 2$ for average branch SNR's less than about -6 (dB).

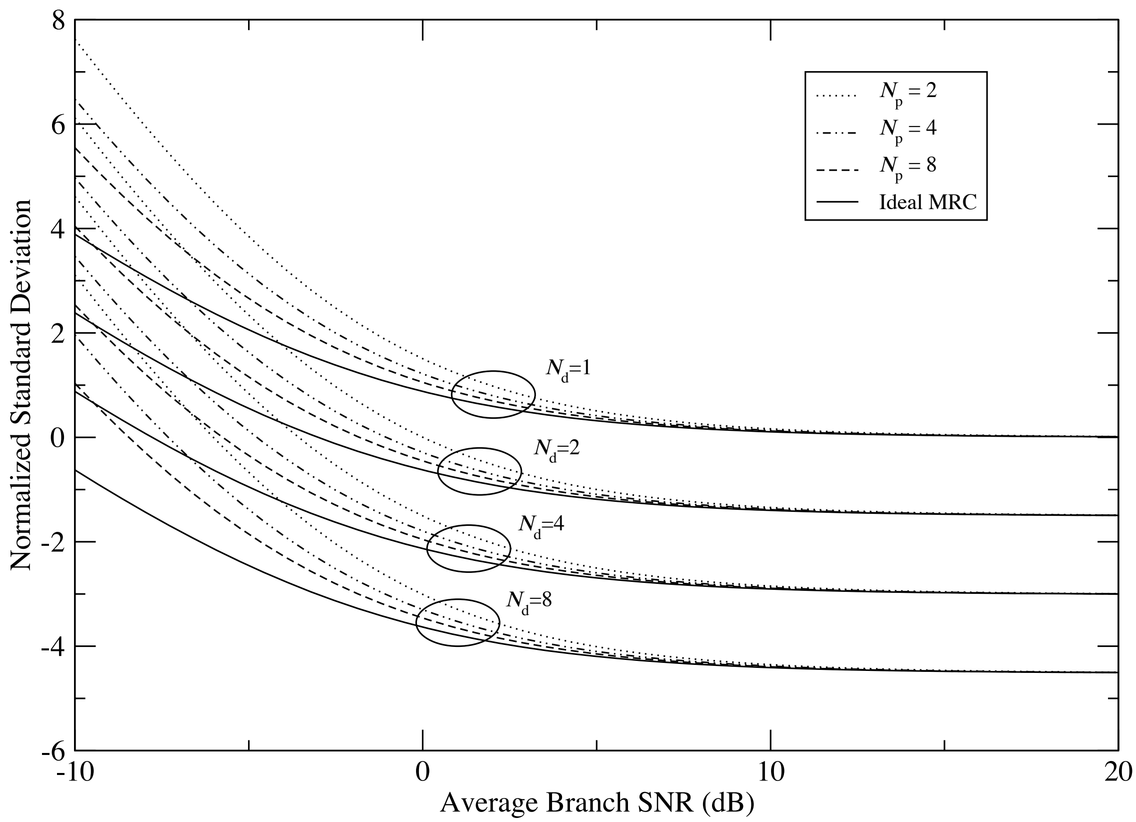

Figure 7-2: Normalized standard deviation of the decision variable for BPSK in i.i.d. Nakagami fading with $m = 1$ (Rayleigh fading), for various N_{d}, N_{p}.

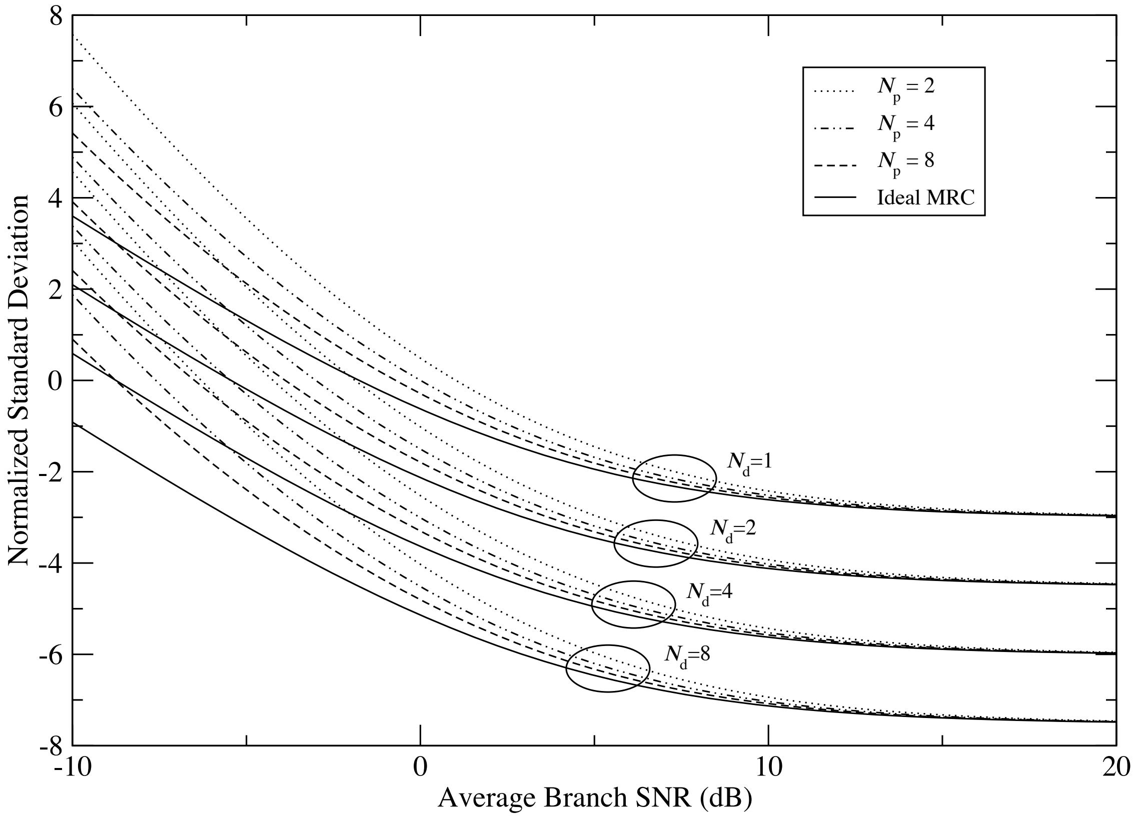

Figure 7-3: Normalized standard deviation of the decision variable for BPSK in i.i.d. Nakagami fading with $m = 4$, for various N_d, N_p.

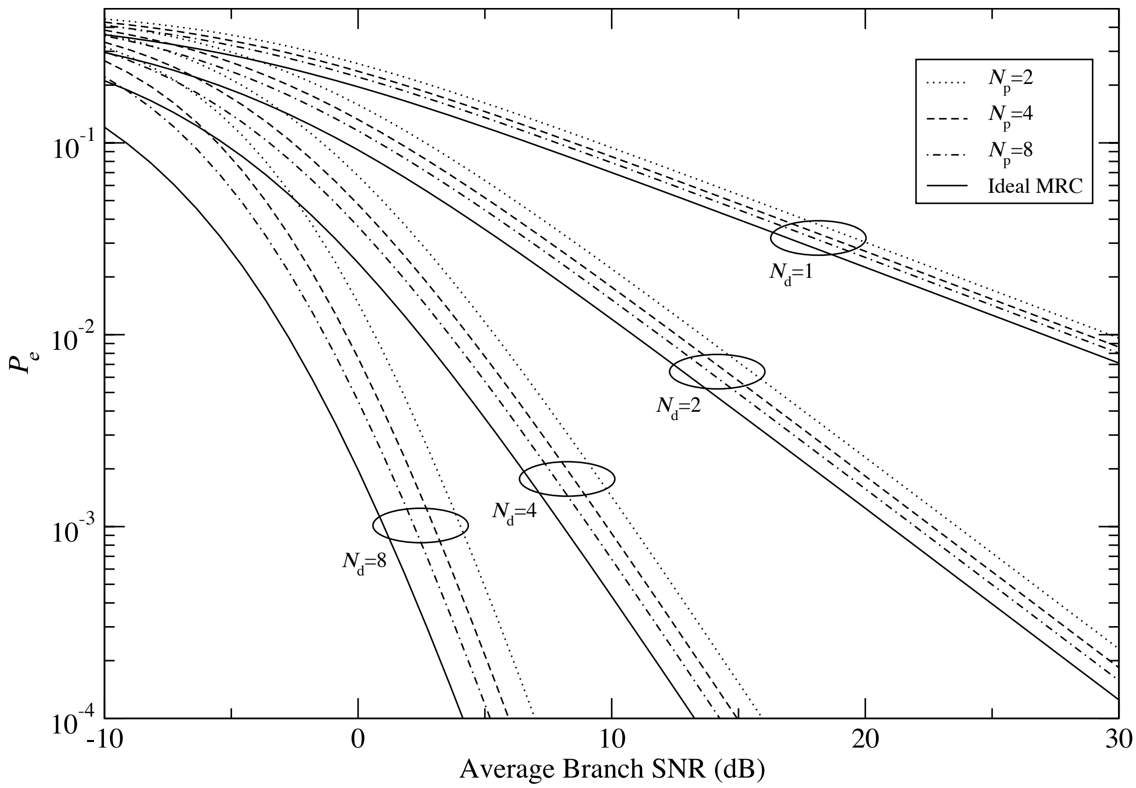

Figure 7-4: Performance of BPSK in i.i.d. Nakagami fading with $m = 0.5$, for various N_d, N_p.

7.3 Performance in Independent Fading

Figures 7-4 – 7-6, show the BEP for Nakagami fading for the cases where $m = 0.5$, $m = 1$, and $m = 4$, respectively, and $\varepsilon = 1$. In each case, note that the diversity order is preserved, regardless of the number of pilot symbols used in the estimation process. Also, note that as N_p increases, performance approaches that of perfect channel knowledge. These results are in agreement with our asymptotic analytical results in (5.12), (5.16), and (5.20), respectively. Previous numerical results in [15, 16, 19–21] were only valid for i.i.d. Rayleigh fading environments, and showed that the diversity order was not preserved. For example, in [19] numerical results with $\rho = 0.9$, 0.99, 0.999 ($\rho = 1$ corresponds to an ideal estimate) all display asymptotic behavior of a single branch system. Similar behavior can also be found in [21, Fig. 6].

Results for a Ricean fading environment are shown in Figs. 7-7 and 7-8, with $\kappa = 5$ dB and $\kappa = 10$ dB, respectively, and $\varepsilon = 1$. Note that the results for Ricean fading, exhibit a "skirt" where the curve becomes less steep and begins to follow the slope of the diversity

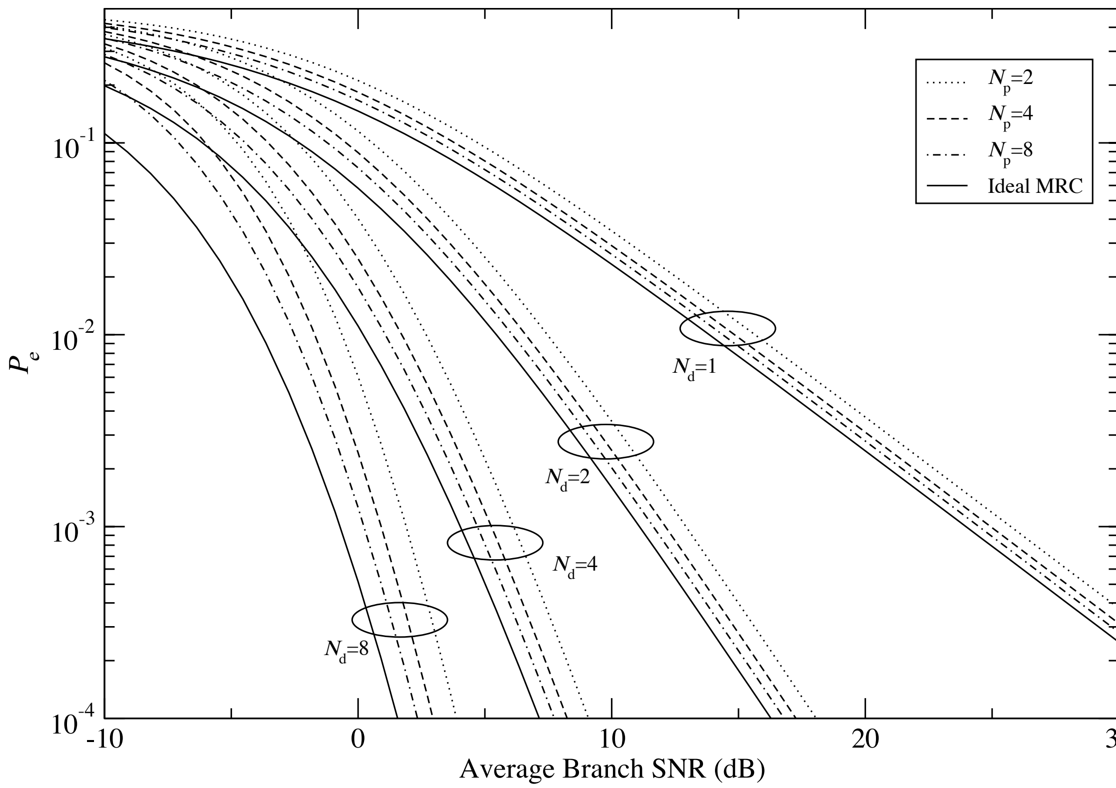

Figure 7-5: Performance of BPSK in i.i.d. Nakagami fading with $m = 1$ (Rayleigh fading), for various N_d, N_p.

order. This behavior is caused by the line of sight component, or mean, present in Ricean fading environments. Also note that performance in Ricean is better than that of Rayleigh fading because of the line of sight component. This is confirmed by comparing Figs. 7-7 and 7-8 with Fig. 7-5.

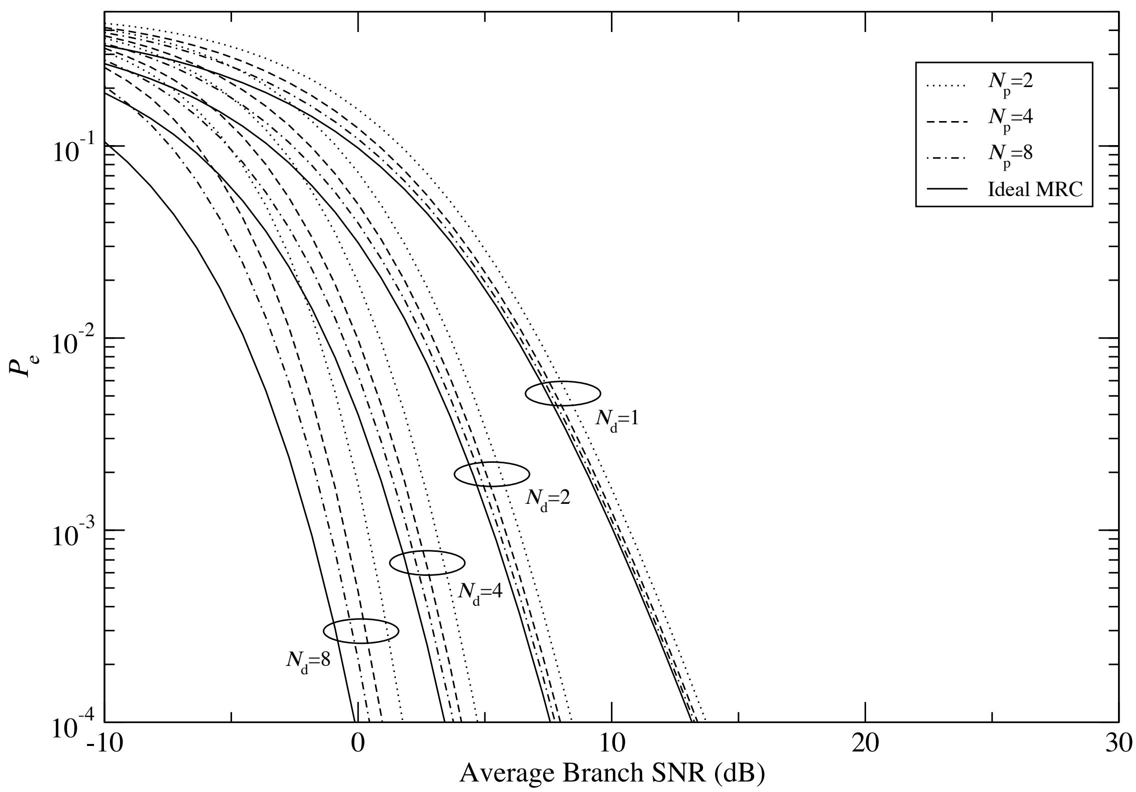

Figure 7-6: Performance of BPSK in i.i.d. Nakagami fading with $m = 4$, for various $N_{\rm d}$, $N_{\rm p}$.

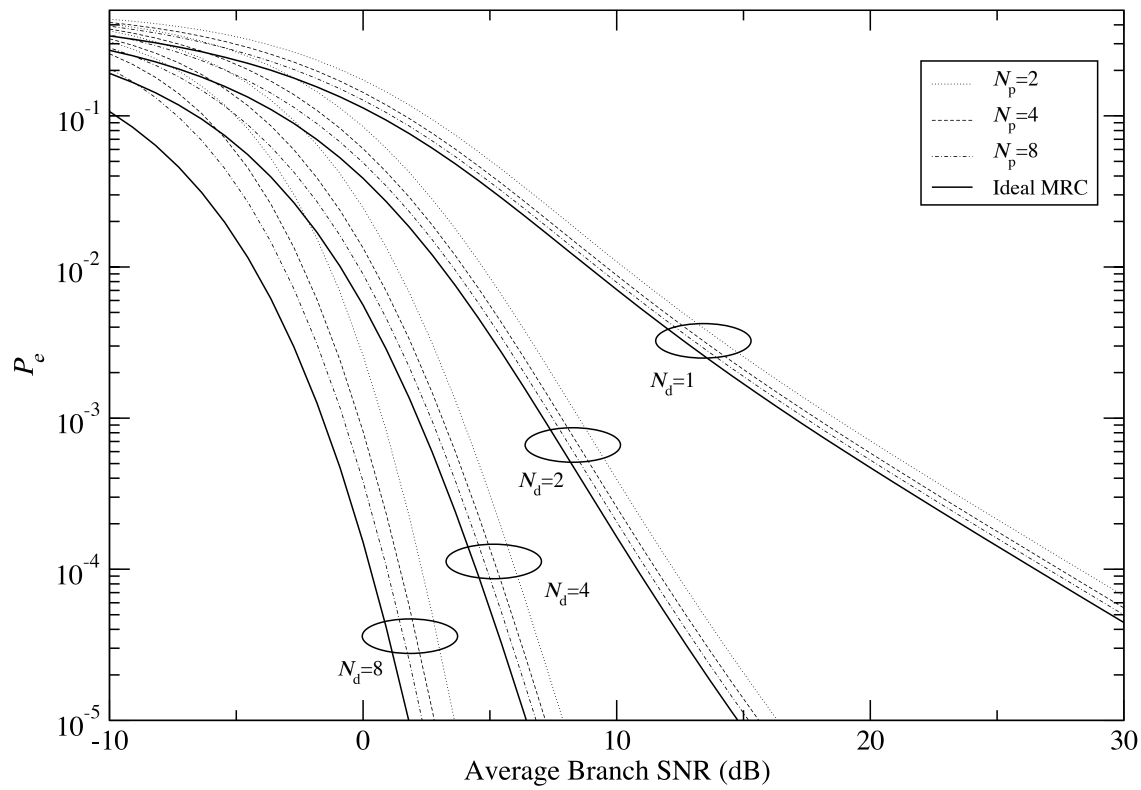

Figure 7-7: Performance of BPSK in i.i.d. Ricean fading with $\kappa = 5$ dB, for various N_{d}, N_{p}.

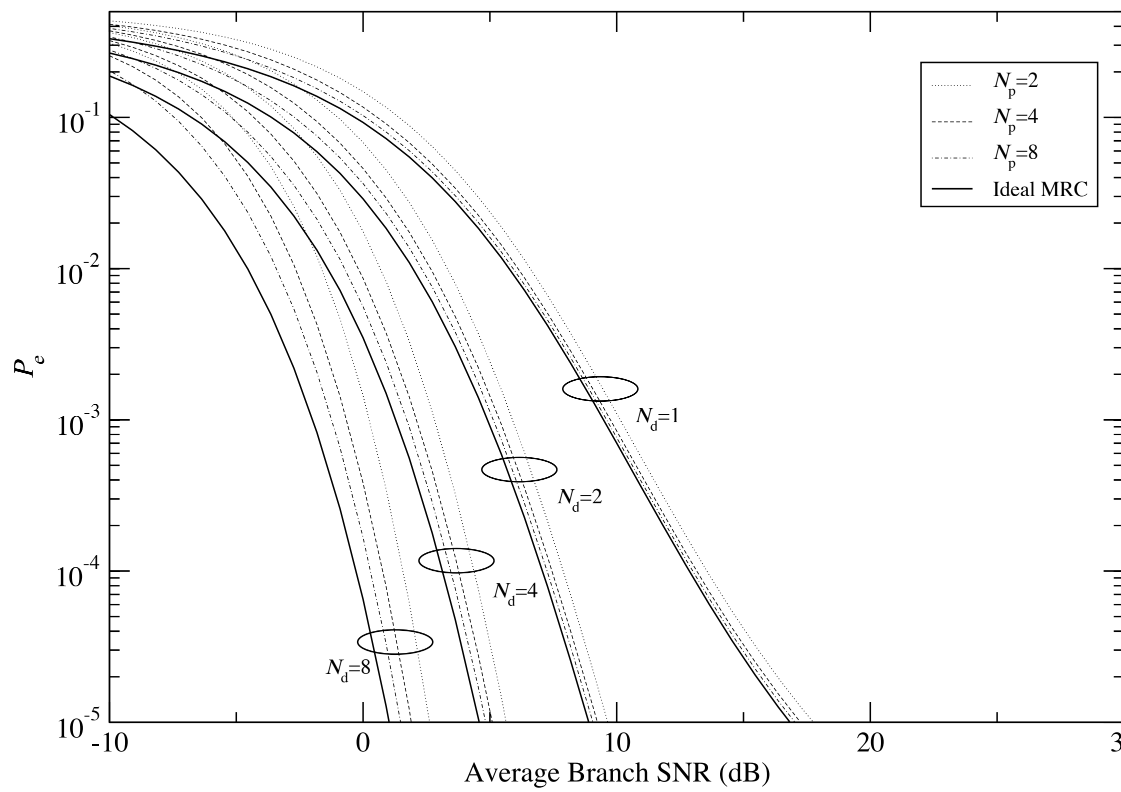

Figure 7-8: Performance of BPSK in i.i.d. Ricean fading with $\kappa = 10$ dB, for various N_{d}, N_{p}.

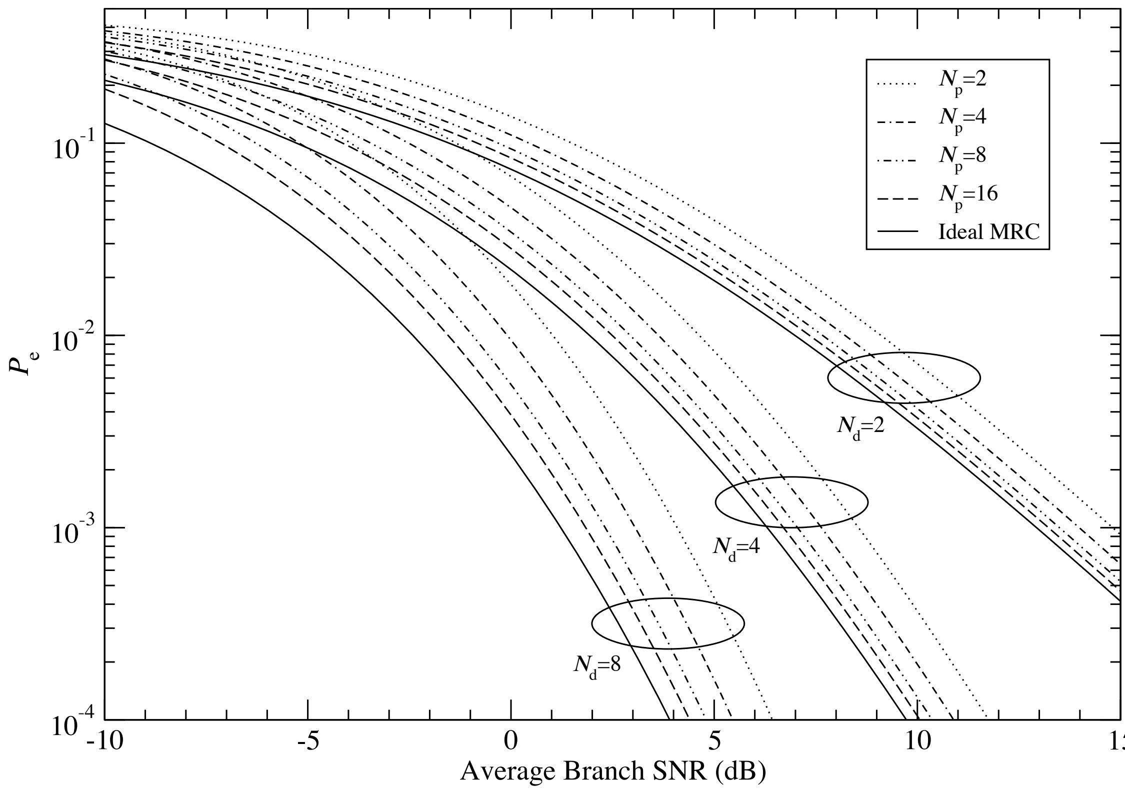

Figure 7-9: Performance of BPSK in correlated Nakagami fading with $m = 1$ (Rayleigh fading), $\eta = 0.6$, for various N_d and N_p.

7.4 Performance in Correlated Fading

Using the analytical framework developed in the previous chapters, which is valid for arbitrary fading distributions, we can evaluate the BEP for some common correlated fading models. We consider Nakagami-m and Ricean channels under an exponential correlation model, where the correlation between diversity branches is given by

$$\rho_{|h_i|^2,|h_j|^2} = \eta^{|i-j|}.$$

Using this correlation model with (6.8) we can analyze the BEP given by (4.16) for correlated Nakagami fading. Figures 7-9 and 7-10 show the BEP for the cases where $\eta = 0.6$ and $\eta = 0.9$, respectively for Rayleigh fading ($m = 1$) and $\varepsilon = 1$. Figures 7-11 and 7-12 show the BEP for the cases where $\eta = 0.6$ and $\eta = 0.9$, respectively for Nakagami fading with $m = 2$ and $\varepsilon = 1$. Similarly, results for the case of Ricean fading under this correlation model are shown in Figs. 7-13–7-16.

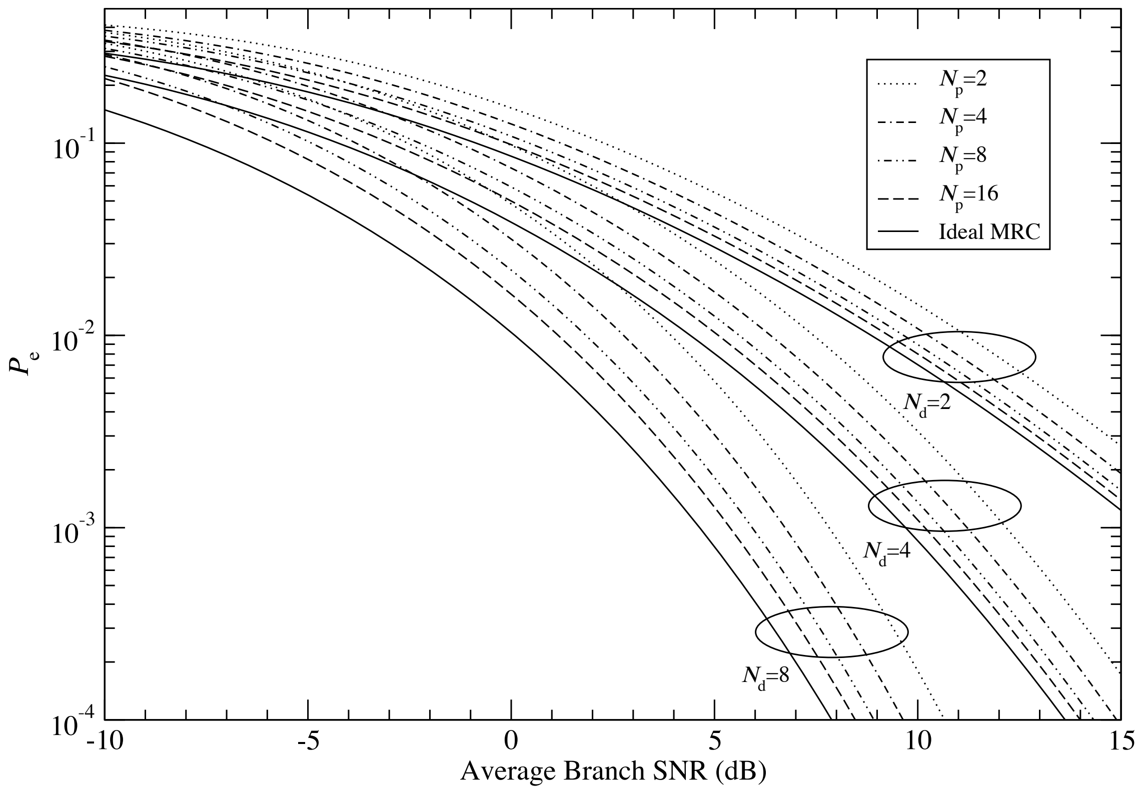

Figure 7-10: Performance of BPSK in correlated Nakagami fading with $m = 1$ (Rayleigh fading), $\eta = 0.9$, for various N_d and N_p.

In each case, note that the diversity order of the system with practical channel estimation matches that of an ideal system operating in the same correlated fading environment, regardless of the number of pilot symbols used in the estimation process. That is, the slope of the performance curves for practical channel estimation matches the slope of the curves for perfect channel knowledge (ideal MRC). Also, note that as N_p increases, performance approaches that of perfect channel knowledge.

In comparison to Fig. 7-5, Figs. 7-9 and 7-10 show that diversity systems with practical channel estimation perform worse in correlated fading environments. This result is expected, as increased branch correlation reduces the effective diversity order, thereby limiting the benefits of diversity reception. This is further verified by the fact that a diversity system performs worse in an environment with $\eta = 0.9$ than when $\eta = 0.6$, as indicated by Figs. 7-9 and 7-10. Similar results can be observed by comparing Figs. 7-11 and 7-12.

Figs. 7-13–7-16 show that correlation has a similar effect on the performance in Ricean fading environments. Also note that performance in correlated Ricean fading is better

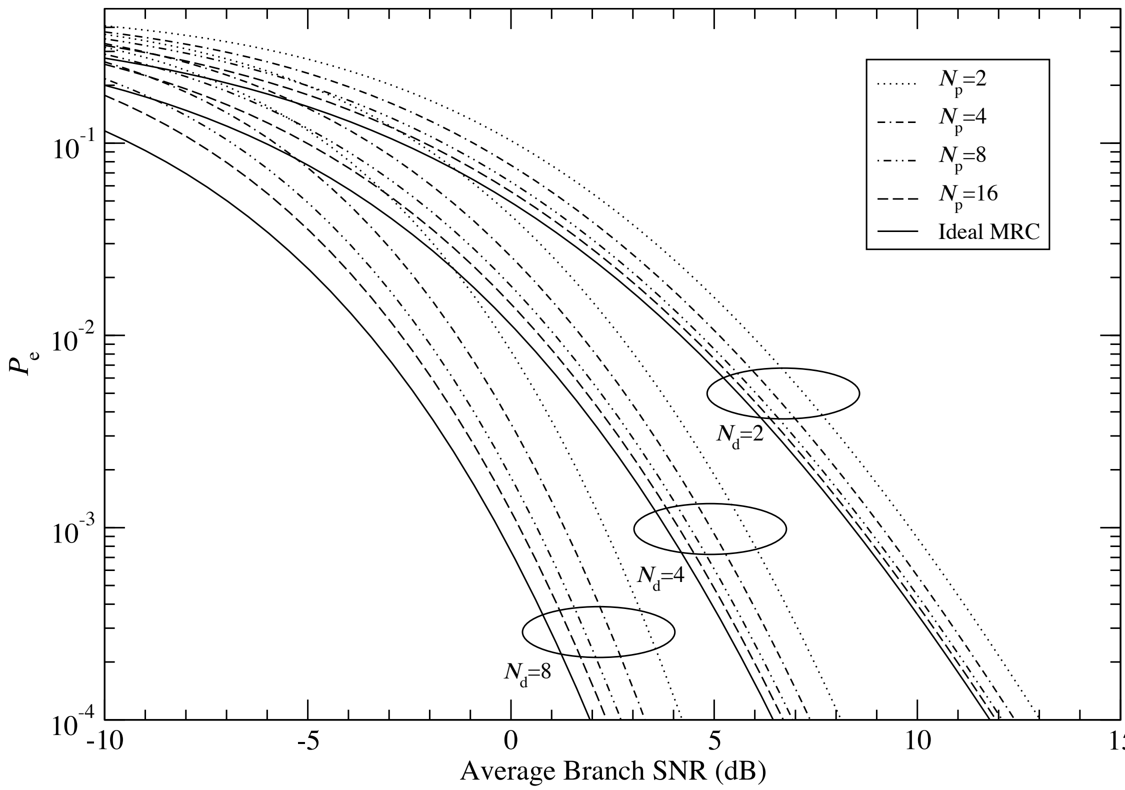

Figure 7-11: Performance of BPSK in correlated Nakagami fading with $m = 2$, $\eta = 0.6$, for various N_d and N_p.

than in correlated Rayleigh fading because of the line of sight component present in Ricean channels. This can be seen by comparing Figs. 7-13 and 7-15 with Fig. 7-9, and Figs. 7-14 and 7-16 with Fig. 7-10.

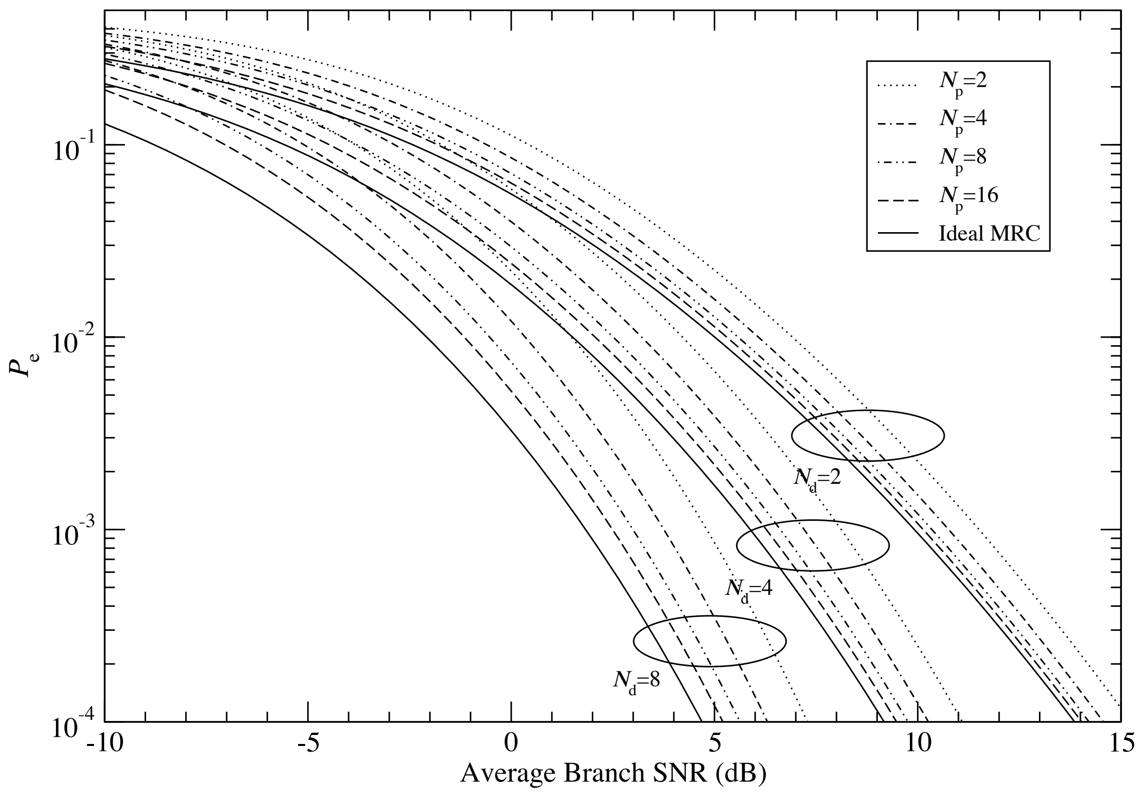

Figure 7-12: Performance of BPSK in correlated Nakagami fading with $m = 2$, $\eta = 0.9$, for various N_d and N_p.

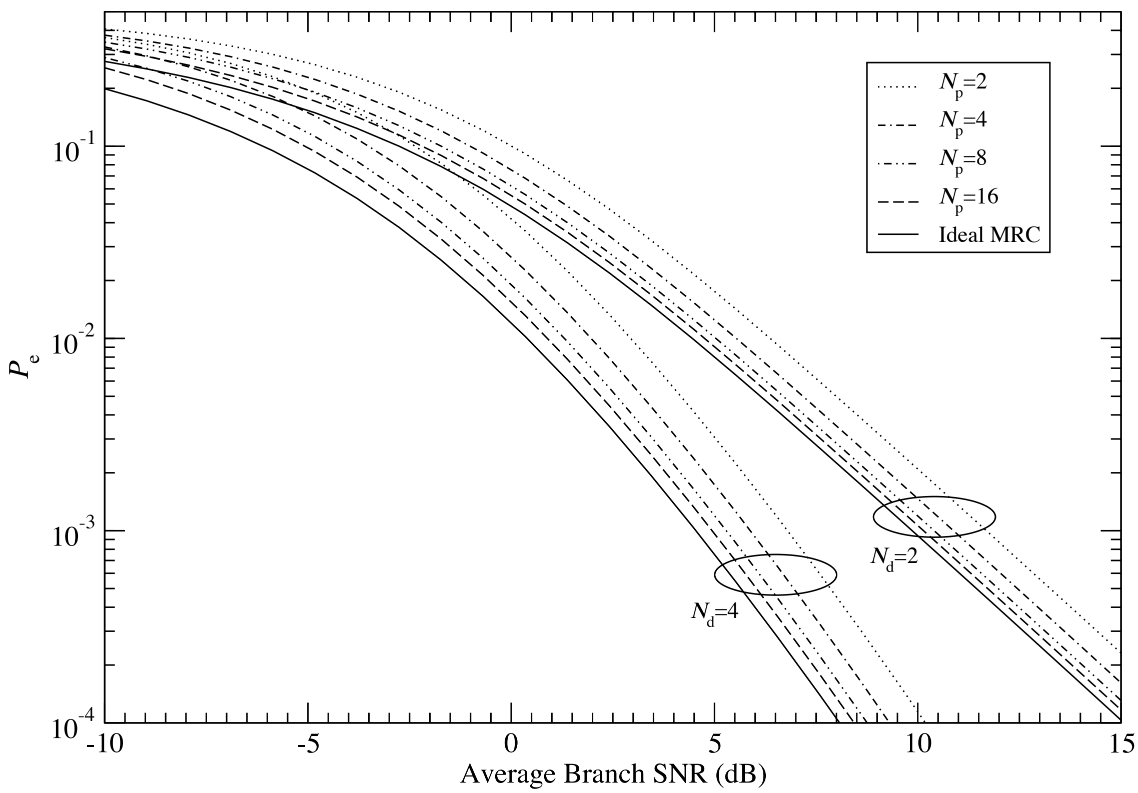

Figure 7-13: Performance of BPSK in correlated Ricean fading with $\kappa = 5$ dB, $\eta = 0.6$, for various N_d and N_p.

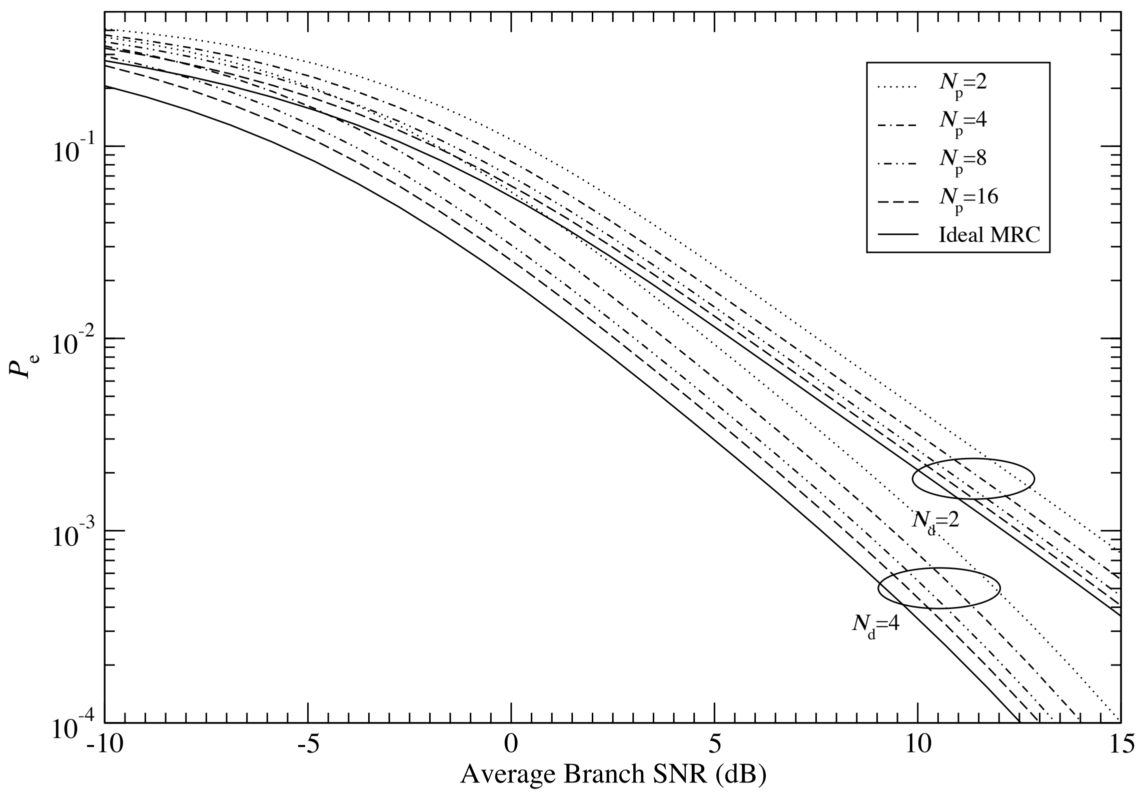

Figure 7-14: Performance of BPSK in correlated Ricean fading with $\kappa = 5$ dB, $\eta = 0.9$, for various N_d and N_p.

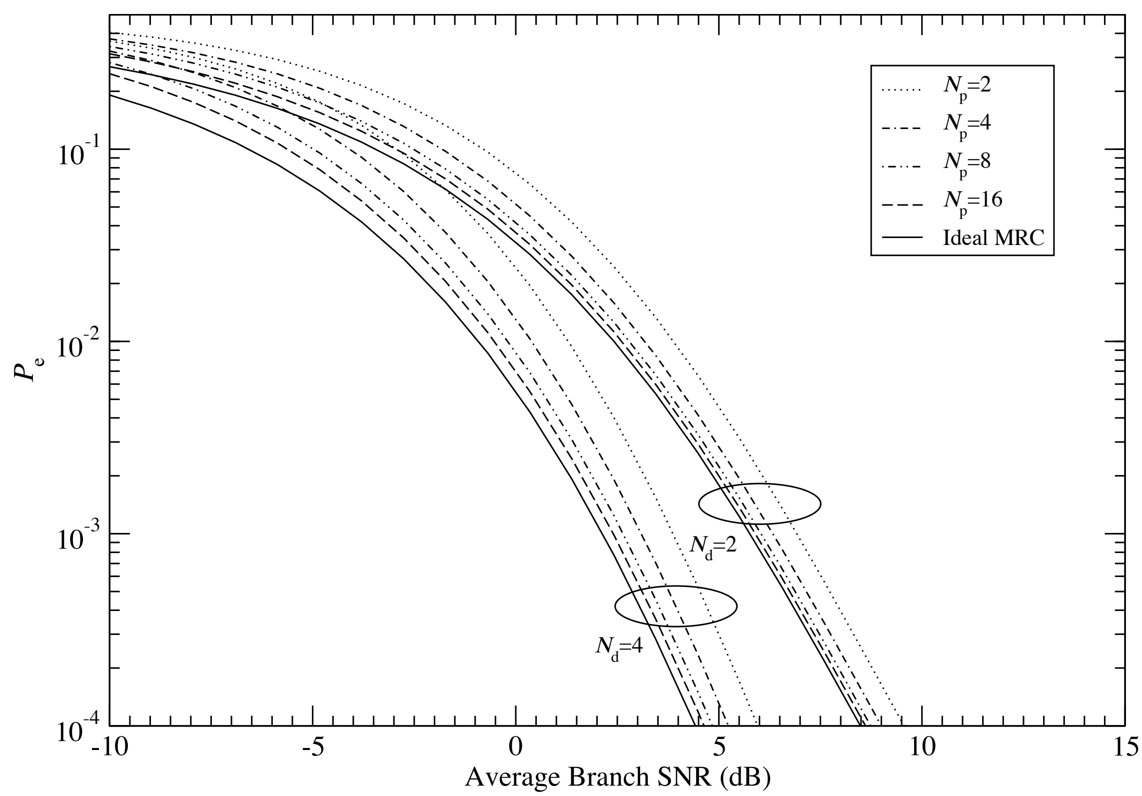

Figure 7-15: Performance of BPSK in correlated Ricean fading with $\kappa = 10$ dB, $\eta = 0.6$, for various N_d and N_p.

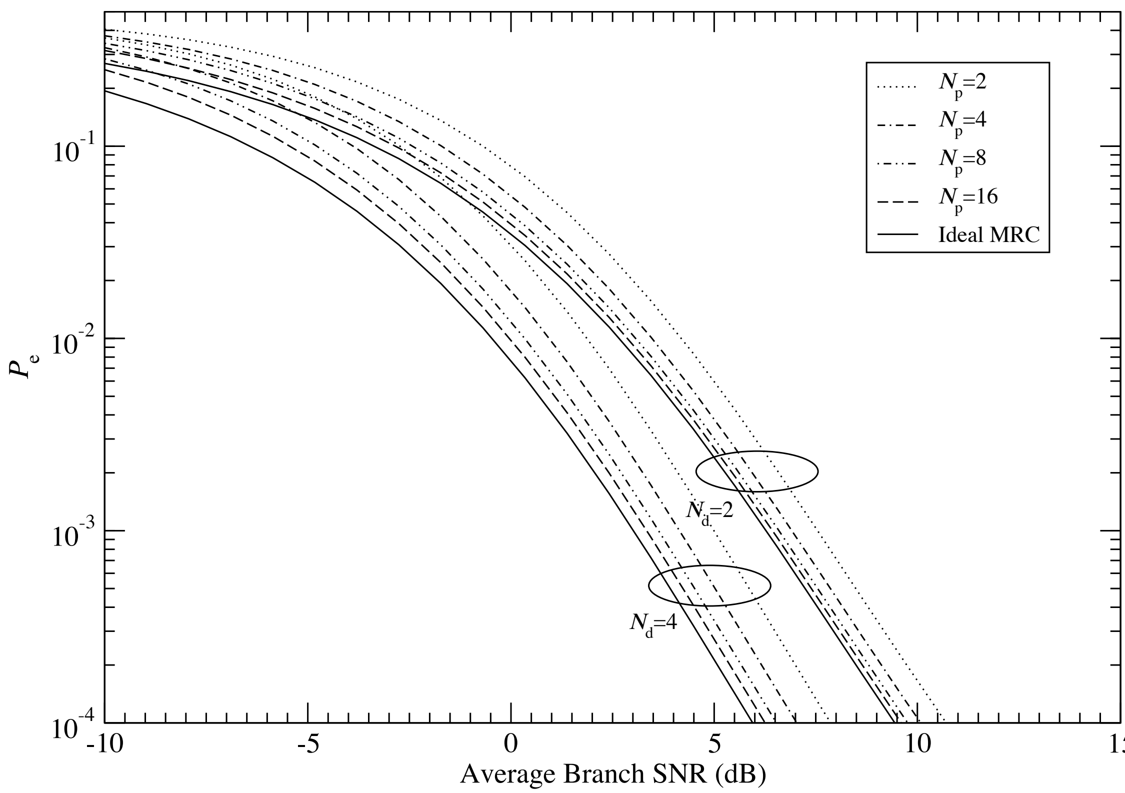

Figure 7-16: Performance of BPSK in correlated Ricean fading with $\kappa = 10$ dB, $\eta = 0.9$, for various $N_{\rm d}$ and $N_{\rm p}$.

7.5 SNR Penalty

In comparison to ideal MRC, diversity with practical channel estimation will incur a loss in SNR, due to the fact that completely coherent combining is not possible. For analog systems, the SNR penalty is defined in terms of the degradation in the SNR. Instead, as in [50], we consider a measure that is more suitable for digital systems; the SNR penalty required to maintain a target BEP.

For a digital communication system, we define the SNR penalty, β, as the increase in SNR required for a diversity system with practical channel estimation to achieve the same target BEP as ideal MRC. Implicitly, we have

$$P_{e,\mathrm{I}}(\beta\Gamma_{\mathrm{tot}}) = P_{e,\mathrm{MRC}}(\Gamma_{\mathrm{tot}}),$$

where $P_{e,\mathrm{I}}(\cdot)$, $P_{e,\mathrm{MRC}}(\cdot)$, β, and Γ_{tot} are the BEP for diversity combining with imperfect channel knowledge, the BEP for ideal MRC, the SNR penalty, and the total average SNR, respectively.

Note that the SNR penalty is a function of the target BEP, and therefore a function of the average SNR; that is, $\beta = \beta(\Gamma_{\mathrm{tot}})$. A closed form expression for β is difficult to obtain, if at all possible. However, using (5.12) - (5.15) we can derive the asymptotic SNR penalty, β_A, for large SNR, such that

$$P_{e,\text{Asym-I}}(\beta_A\Gamma_{\mathrm{tot}}) = P_{e,\text{Asym-P}}(\Gamma_{\mathrm{tot}}).$$

Solving this relation for the specific case of Nakagami fading channels gives

$$\begin{aligned}\beta_A &= \left[\frac{K_{\mathrm{I,Nakagami}}\,(m,\,N_{\mathrm{d}},\,N_{\mathrm{p}},\,\varepsilon)}{K_{\mathrm{P,\,Nakagami}}\,(m,\,N_{\mathrm{d}})}\right]^{\frac{1}{mN_{\mathrm{d}}}}\\ &= \frac{4}{(\sqrt{N_{\mathrm{p}}\,\varepsilon}+1)^2}\left\{\frac{\Gamma(1+mN_{\mathrm{d}})}{2\sqrt{\pi}\,\Gamma(\frac{1}{2}+mN_{\mathrm{d}})}\int_{-\pi}^{\pi}\left\{\left[\frac{g(\theta;\zeta)}{(1-\zeta^2)^2}\right]^{mN_{\mathrm{d}}}+\frac{f(\theta;\zeta)}{[g(\theta;\zeta)]^{mN_{\mathrm{d}}}}\right\}d\theta\right\}^{\frac{1}{mN_{\mathrm{d}}}}.\end{aligned} \tag{7.2}$$

Clearly, the asymptotic SNR penalty for Rayleigh fading is given by (7.2) when $m = 1$. A similar expression for Ricean fading can be derived using (5.20) – (5.23). Since (5.21) and (5.23) only differ by a multiplicative constant from (5.13) and (5.15) when $m = 1$, the

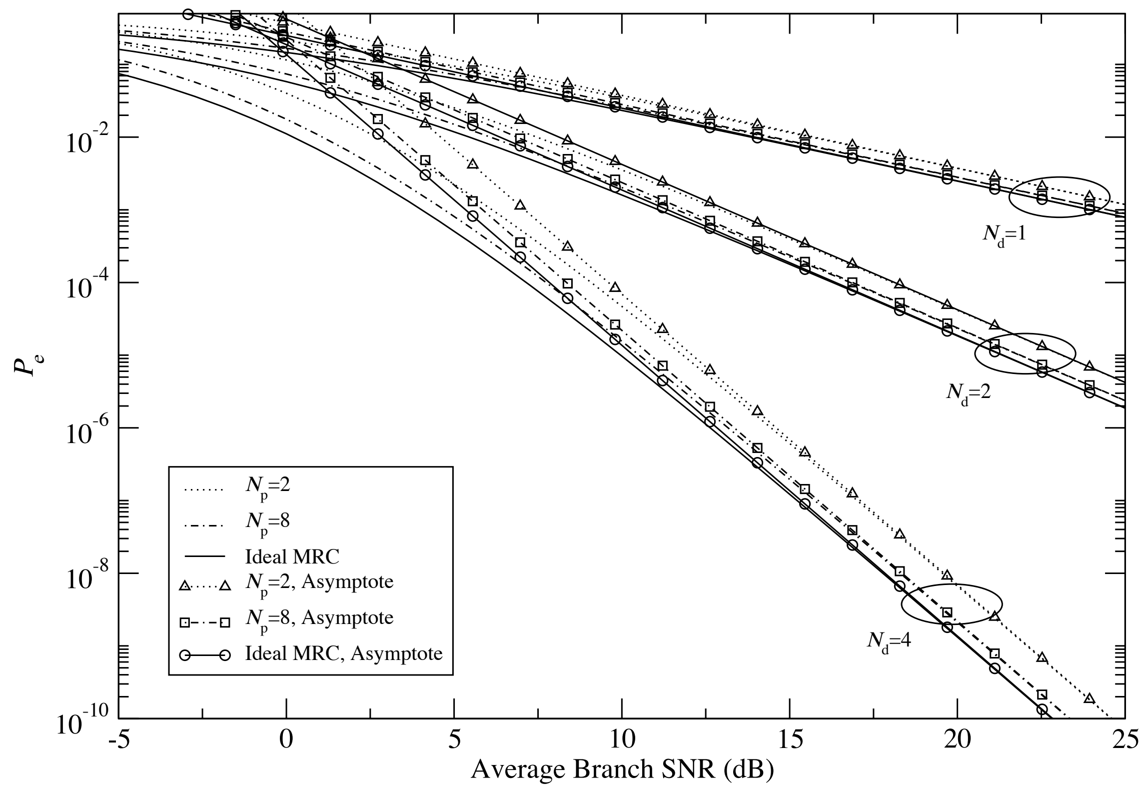

Figure 7-17: Comparison of exact performance with asymptotic performance of BPSK in i.i.d. Nakagami fading with $m = 1$ (Rayleigh fading), for various N_d, N_p.

asymptotic SNR penalty in Ricean fading is given by (7.2) with $m = 1$.

Figures 7-17 and 7-18 show[1] the asymptotic BEP given by (5.12) and (5.20). These figures provide further confirmation that the practical channel estimation scheme preserves the diversity order. From these figures we see that the performance given by the asymptotic expressions quickly approaches the exact error probability, indicating the efficiency of the asymptotic BEP expressions.

Figure 7-19 shows the asymptotic SNR penalty β_A as a function of $N_\mathrm{p}\,\varepsilon$ in Nakagami-m fading for several values of m and N_d. Several important observations can be made from looking at these graphs. First, note that curves are clustered according to m parameter, with better performance (lower penalty) occurring for more benign environments, $m > 1$. More importantly there is surprising lack of dependence on N_d, if any. In particular, for $m > 1$ increasing N_d slightly increases the SNR penalty. However, for the case where

[1]Figs. 7-17 and 7-18 show the BEP for error rates as low as 10^{-10} only to illustrate the asymptotic behavior and to further provide numerical confirmation that the practical channel estimation scheme preserves the diversity order; these extremely low BEP's are not practical, especially for wireless mobile communications.

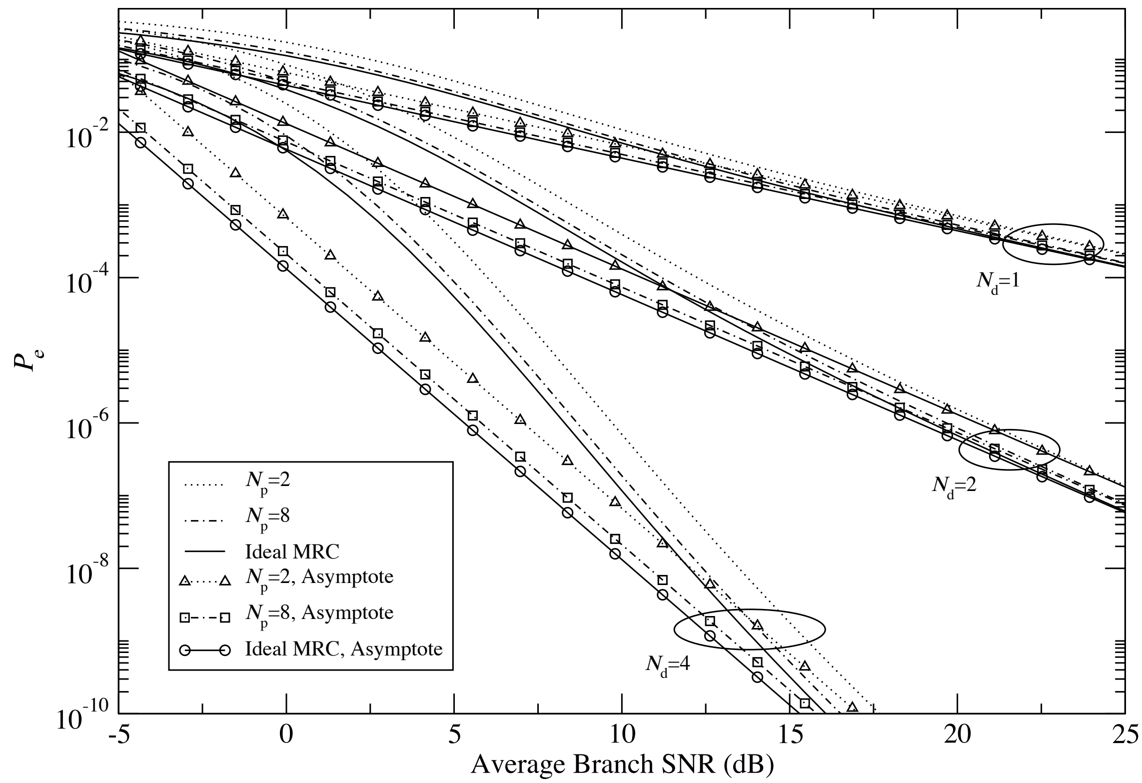

Figure 7-18: Comparison of exact performance with asymptotic performance of BPSK in i.i.d. Ricean fading with $\kappa = 5$ dB, for various N_d, N_p.

$\frac{1}{2} \leq m < 1$, the effect is reversed; increasing N_d decreases the penalty. In all the cases investigated, the difference in SNR penalties between $N_\mathrm{d} = 1$ and $N_\mathrm{d} = 8$ does not exceed 0.2 dB. For the case of Rayleigh or Ricean fading, where $m = 1$, changes in N_d have no effect on the SNR penalty. This can be seen from the $m = 1$ curve in figure 7-19, where the curves line up for all N_d. These results are surprising because one could expect that, as the number of diversity branches increases, the error due to practical channel estimation would also increase, thereby incurring a larger SNR penalty. These results show that this is not the case.

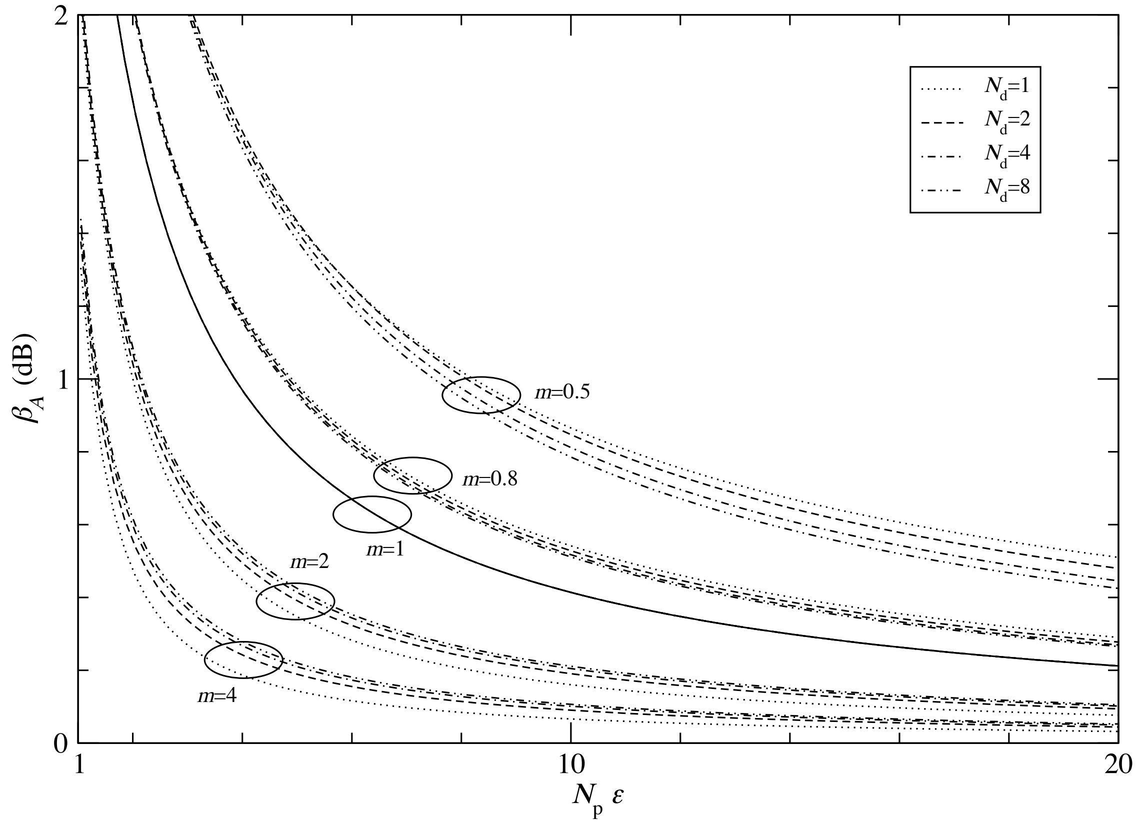

Figure 7-19: Asymptotic SNR penalty as a function of $N_\mathrm{p}\,\varepsilon$, for various N_d and m.

Chapter 8

Conclusion

This thesis develops a general framework for evaluating the exact BEP of BPSK in N_{d}-branch diversity systems utilizing practical channel estimation. The methodology, requiring only the evaluation of a single integral with finite limits, is applicable to channels with arbitrary distribution, including correlated fading, provided that the norm square of the channel gain vector can be characterized by a m.g.f. The results of this thesis show that the pilot symbol estimation technique, appropriate for digital communication systems, preserves the diversity order of an N_{d}-branch diversity system. This is in contrast to the results of [15,16,19–21], where the BEP was analyzed for fixed values of correlation. The asymptotic SNR penalty, arising from practical channel estimation, was quantified. The results show that the penalty has little dependence on the the diversity order of the system.

Appendix A

Covariance Relationships

In this appendix we derive the relationships between $\rho_{|h_i|^2,\,|h_j|^2}$ and the covariance of the underlying complex Gaussian r.v.'s for the case of correlated Nakagami and Ricean fading.

A.1 Preliminaries

We begin by reviewing and deriving some basic relations.

Lemma: For two Gaussian r.v.'s, X and Y,

$$\mathrm{Var}\big\{X^2\big\} = 4\mathbb{E}^2\{X\}\,\mathrm{Var}\{X\} + 2\mathrm{Var}\{X\}^2 \tag{A.1}$$

and

$$\begin{aligned}\mathrm{Cov}\big\{X^2,\,Y^2\big\} &= \mathbb{E}\big\{X^2Y^2\big\} - \mathbb{E}\big\{X^2\big\}\,\mathbb{E}\big\{Y^2\big\} \\ &= 2\mathrm{Cov}\{X,\,Y\}\,\Big(2\mathbb{E}\{X\}\,\mathbb{E}\{Y\} + \mathrm{Cov}\{X,\,Y\}\Big)\,.\end{aligned} \tag{A.2}$$

Proof: The joint c.f. of the bivariate Gaussian distribution [36] is given by

$$\phi(t,u) = \exp\left(j\left(\mathbb{E}\{X\}\,t + \mathbb{E}\{Y\}\,u\right) - \frac{1}{2}\left(\mu_{20}t^2 + 2\mu_{11}tu + \mu_{02}u^2\right)\right)\,, \tag{A.3}$$

where $\mu_{ij} = \mathbb{E}\Big\{(X - \mathbb{E}\{X\})^i\,(Y - \mathbb{E}\{Y\})^j\Big\}$. It can be shown that

$$\mathbb{E}\{X^m Y^n\} = \frac{1}{j^{m+n}} \frac{\partial^{m+n}}{\partial t^m \partial u^n} \phi(t,u)\bigg|_{t=0,\,u=0} . \tag{A.4}$$

Applying the above relation, the moments of interest are easily computed as

$$\begin{aligned}
\mathbb{E}\big\{X^2\big\} &= \mathbb{E}^2\{X\} + \mu_{20} \\
\mathbb{E}\big\{Y^2\big\} &= \mathbb{E}^2\{Y\} + \mu_{02} \\
\mathbb{E}\big\{X^4\big\} &= \mathbb{E}^4\{X\} + 6\mathbb{E}^2\{X\}\,\mu_{20} + 3\mu_{20}^2 \\
\mathbb{E}\big\{X^2Y^2\big\} &= \mathbb{E}^2\{X\}\,\mathbb{E}^2\{Y\} + \mathbb{E}^2\{X\}\,\mu_{02} + \mathbb{E}^2\{Y\}\,\mu_{20} + \mu_{02}\mu_{20} + 4\mathbb{E}\{X\}\,\mathbb{E}\{Y\}\,\mu_{11} + 2\mu_{11}^2 .
\end{aligned}$$

Thus,

$$\begin{aligned}
\mathrm{Var}\big\{X^2\big\} = \mathbb{E}\big\{X^4\big\} - \mathbb{E}^2\big\{X^2\big\} \quad &= 4\mathbb{E}^2\{X\}\,\mu_{20} + 2\mu_{20}^2 \\
&= 4\mathbb{E}^2\{X\}\,\mathrm{Var}\{X\} + 2\mathrm{Var}\{X\}^2 \\
\mathrm{Cov}\big\{X^2,\,Y^2\big\} = \mathbb{E}\big\{X^2Y^2\big\} - \mathbb{E}\big\{X^2\big\}\,\mathbb{E}\big\{Y^2\big\} \quad &= 4\mathbb{E}\{X\}\,\mathbb{E}\{Y\}\,\mu_{11} + 2\mu_{11}^2 \\
&= 2\mathrm{Cov}\{X,\,Y\}\,\Big(2\mathbb{E}\{X\}\,\mathbb{E}\{Y\} + \mathrm{Cov}\{X,\,Y\}\Big) .
\end{aligned}$$

$\square$

For the special case where $\mathbb{E}\{X\} = \mathbb{E}\{Y\} = 0$, (A.1) and (A.2) simplify to

$$\mathrm{Var}\big\{X^2\big\} = 2\mathrm{Var}\{X\}^2 = 2\mathbb{E}^2\big\{X^2\big\} \tag{A.5}$$

$$\mathrm{Cov}\big\{X^2,\,Y^2\big\} = 2\mathrm{Cov}\{X,\,Y\}^2 = 2\mathbb{E}^2\{XY\} , \tag{A.6}$$

which is in agreement with [51, (7.37)].

A.2 The Nakagami Case

In Section 7.4, our numerical results are parameterized by the correlation coefficient between the squared magnitude of the channel gains on each diversity branch. In Section 6.1, each squared fading gain, $|h_i|^2$, is represented by the sum of $2m_i$ i.i.d. r.v.'s, $X_{i,k}^2$. In this appendix we derive the relation between the correlation of $|h_i|^2$ and $|h_j|^2$ and the correlation of the elements of X for $i \neq j$, as follows

$$\begin{aligned}
\rho_{|h_i|^2,|h_j|^2} &\triangleq \frac{\mathbb{E}\left\{\left(|h_i|^2 - \Omega_i\right)\left(|h_j|^2 - \Omega_j\right)\right\}}{\sqrt{\mathrm{Var}\left\{|h_i|^2\right\}\mathrm{Var}\left\{|h_j|^2\right\}}} \\
&= \frac{\mathbb{E}\left\{\sum_{k=1}^{2m_i}\left(X_{i,k}^2 - \frac{\Omega_i}{2m_i}\right)\sum_{l=1}^{2m_j}\left(X_{j,l}^2 - \frac{\Omega_j}{2m_j}\right)\right\}}{\sqrt{\frac{\Omega_i^2}{m_i}\frac{\Omega_j^2}{m_j}}} \\
&= \frac{\sum_{k=1}^{2\min\{m_i,m_j\}}\mathbb{E}\left\{\left(X_{i,k}^2 - \frac{\Omega_i}{2m_i}\right)\left(X_{j,k}^2 - \frac{\Omega_j}{2m_j}\right)\right\}}{\sqrt{\frac{\Omega_i^2}{m_i}\frac{\Omega_j^2}{m_j}}}\,. \qquad \text{(A.7)}
\end{aligned}$$

The second equality follows from the representation of each $|h_i|^2$ as a sum of $2m_i$ i.i.d. r.v.'s. The fact that $X_{i,k}$ and $X_{j,k}$ are only correlated for $k = l = 1, \ldots, 2\min\{m_i, m_j\}$, gives rise to the third equality.

For zero mean Gaussian r.v.'s $X_{i,k}$ and $X_{j,k}$, (A.7) can be further reduced, using (A.6), to

$$\begin{aligned}
\rho_{|h_i|^2,|h_j|^2} &= \frac{\sum_{k=1}^{2\min\{m_i,m_j\}} 2\mathbb{E}^2\{X_{i,k}X_{j,k}\}}{\sqrt{\frac{\Omega_i^2}{m_i}\frac{\Omega_j^2}{m_j}}} \\
&= \frac{4\min\{m_i,m_j\}\rho_{i,j}^2\frac{\Omega_i}{2m_i}\frac{\Omega_j}{2m_j}}{\sqrt{\frac{\Omega_i^2}{m_i}\frac{\Omega_j^2}{m_j}}} \\
&= \sqrt{\frac{\min\{m_i,m_j\}}{\max\{m_i,m_j\}}}\,\rho_{i,j}^2\,. \qquad \text{(A.8)}
\end{aligned}$$

A.3 The Ricean Case

In Section 7.4, our numerical results are parameterized by the correlation coefficient between the squared magnitude of the channel gains on each diversity branch. On the other hand, in Section 6.3, the expression for the m.g.f. of correlated Ricean fading is written in terms of the covariance matrix of $\mathbf{h}$. Therefore, we need to determine the relationship between the two quantities $\mathrm{Cov}\{h_i,\, h_j\}$ and $\rho_{|h_i|^2,\,|h_j|^2}$.

We consider four Gaussian r.v.'s X, Y, V, W all with non-zero means and define two complex fading gains of a Ricean channel as $h_i = X + jY$ and $h_j = V + jW$. The correlation coefficient is then given by

$$\rho_{|h_i|^2,\,|h_j|^2} = \frac{\mathrm{Cov}\Big\{|h_i|^2,\,|h_j|^2\Big\}}{\sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}}$$
$$= \frac{\mathrm{Cov}\{X^2,\,V^2\} + \mathrm{Cov}\{X^2,\,W^2\} + \mathrm{Cov}\{Y^2,\,V^2\} + \mathrm{Cov}\{Y^2,\,W^2\}}{\sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}}\,. \tag{A.9}$$

We seek to relate the quantity in (A.9) to the covariance between the underlying complex Gaussian r.v.'s. We assume that there is correlation between the in-phase components of h_i and h_j. Similarly, we assume that the quadrature components of h_i and h_j are correlated, but there is no correlation between the in-phase and quadrature components. Furthermore, h_i and h_j are circularly symmetric complex Gaussian r.v.'s, with means $\mu_i = \mu_{i,R} + j\mu_{i,I}$ and $\mu_j = \mu_{j,R} + j\mu_{j,I}$, respectively. Mathematically,

$$\begin{aligned}
\mathrm{Cov}\{X,\,Y\} &= \mathrm{Cov}\{V,\,W\} = 0 & \mathrm{Cov}\{X,\,W\} &= \mathrm{Cov}\{Y,\,V\} = 0\\
\mathrm{Cov}\{X,\,V\} &= \mathrm{Cov}\{Y,\,W\} = \xi & &\\
\mathbb{E}\{X\} &= \mu_{i,R} & \mathbb{E}\{V\} &= \mu_{j,R}\\
\mathbb{E}\{Y\} &= \mu_{i,I} & \mathbb{E}\{W\} &= \mu_{j,I}\\
\mathrm{Var}\{X\} &= \mathrm{Var}\{Y\} = \sigma_i^2 & \mathrm{Var}\{V\} &= \mathrm{Var}\{W\} = \sigma_j^2\,.
\end{aligned}$$

Using the relations in (A.1) and (A.2), (A.9) reduces to

$$\rho_{|h_i|^2,\,|h_j|^2} = \frac{\mathrm{Cov}\{X^2,\,V^2\} + \mathrm{Cov}\{Y^2,\,W^2\}}{\sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}}$$
$$= 4\frac{\xi\left(\mu_{i,R}\mu_{j,R} + \mu_{i,I}\mu_{j,I}\right) + \xi^2}{\sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}}\,. \tag{A.10}$$

Solving for ξ gives

$$\xi = -\frac{\mu_{i,R}\mu_{j,R} + \mu_{i,I}\mu_{j,I}}{2}$$
$$\pm\frac{1}{2}\sqrt{\left(\mu_{i,R}\mu_{j,R} + \mu_{i,I}\mu_{j,I}\right)^2 + \sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}\rho_{|h_i|^2,\,|h_j|^2}}\,. \tag{A.11}$$

Thus,

$$\begin{aligned}
\mathrm{Cov}\{h_i,\, h_j\} &= \mathrm{Cov}\{X,\, V\} + \mathrm{Cov}\{Y,\, W\} = 2\xi \\
&= -\left(\mu_{i,R}\mu_{j,R} + \mu_{i,I}\mu_{j,I}\right) \\
&\quad \pm \sqrt{\left(\mu_{i,R}\mu_{j,R} + \mu_{i,I}\mu_{j,I}\right)^2 + \sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}\rho_{|h_i|^2,\,|h_j|^2}} \\
&= -\Re\big\{\mu_i\mu_j^*\big\} \pm \sqrt{\Re^2\Big\{\mu_i\mu_j^*\Big\} + \sqrt{\mathrm{Var}\Big\{|h_i|^2\Big\}\,\mathrm{Var}\Big\{|h_j|^2\Big\}}\rho_{|h_i|^2,\,|h_j|^2}}\,. \qquad \text{(A.12)}
\end{aligned}$$

If h_i and h_j are identically distributed, (A.12) reduces to

$$\mathrm{Cov}\{h_i,\, h_j\} = -\left|\mu_i\right|^2 \pm \sqrt{\left|\mu_i\right|^4 + \mathrm{Var}\Big\{|h_i|^2\Big\}\,\rho_{|h_i|^2,\,|h_j|^2}}\,. \qquad \text{(A.13)}$$

Using (A.1), we can express the variance of $|h_i|^2$ as

$$\begin{aligned}
\mathrm{Var}\Big\{|h_i|^2\Big\} &= \mathrm{Var}\big\{X^2 + Y^2\big\} = \mathrm{Var}\big\{X^2\big\} + \mathrm{Var}\big\{Y^2\big\} + 2\mathrm{Cov}\big\{X^2,\, Y^2\big\} \\
&= 4\mathbb{E}^2\{X\}\,\mathrm{Var}\{X\} + 2\mathrm{Var}\{X\}^2 + 4\mathbb{E}^2\{Y\}\,\mathrm{Var}\{Y\} + 2\mathrm{Var}\{Y\}^2 \\
&= 4\left|\mu_i\right|^2\sigma_i^2 + 4\sigma_i^4\,. \qquad \text{(A.14)}
\end{aligned}$$

Appendix B

Virtual Branch Transformation

In this appendix, Karhunen-Loève (KL) expansion is used to represent the vector X, of Section 6.1, in terms of a linear combination of independent, zero mean, unit variance Gaussian r.v.'s. This allows the expression of the norm square of the channel gain vector, $\|\mathbf{h}\|^2$, as a linear function of the *independent* virtual branch channel gains and facilitates characterization by its m.g.f.

Let $\{\lambda_l\}$ be the set of L *distinct* eigenvalues of $K_X = \mathbb{E}\big\{X^t X\big\}$ where each λ_l has algebraic multiplicity ν_l such that $\sum_{l=1}^{L} \nu_l = D_T = \sum_{i=1}^{N} 2m_i$. The corresponding *orthonormal* eigenvectors are denoted by $\{\phi_{l,k}\}$. Then the KL expansion of the vector X is [52]

$$X = \sum_{l=1}^{L} \sqrt{\lambda_l} \sum_{k=1}^{\nu_l} W_{l,k} \phi_{l,k} \, , \tag{B.1}$$

where the $\{W_{l,k}\}$'s are *independent*, zero mean, unit variance Gaussian r.v.'s. A similar technique employing a frequency domain KL expansion was used in [53] to study diversity combining in a frequency-selective Rayleigh fading channel. Another technique similar to KL expansion was also used in [54] to study the reception of noncoherent orthogonal signals in Rician and Rayleigh fading channels.

Recall that the norm square of the channel gain vector is given by

$$\|\mathbf{h}\|^2 \overset{\mathrm{L}}{=} X^t X \, . \tag{B.2}$$

Using (B.1), $\|\mathbf{h}\|^2$ can be described in a statistically equivalent representation as

$$
\begin{aligned}
\|\mathbf{h}\|^2 &\overset{\mathrm{L}}{=} \left(\sum_{l=1}^{L}\sqrt{\lambda_l}\sum_{k=1}^{\nu_l} W_{l,k}\phi_{l,k}\right)\left(\sum_{n=1}^{L}\sqrt{\lambda_n}\sum_{m=1}^{\nu_n} W_{n,m}\phi_{n,m}^t\right) \\
&= \sum_{l=1}^{L}\sum_{n=1}^{L}\sqrt{\lambda_l}\sqrt{\lambda_n}\sum_{k=1}^{\nu_l}\sum_{m=1}^{\nu_n} W_{l,k}W_{n,m}\phi_{l,k}\phi_{n,m}^t \\
&= \sum_{l=1}^{L}\lambda_l\sum_{k=1}^{\nu_l} W_{l,k}^2 \\
&= \sum_{l=1}^{L}\lambda_l V_l\,,
\end{aligned}
\tag{B.3}
$$

where the third equality follows from the orthonormality of $\{\phi_{l,k}\}$ and the virtual branch variables, V_l's, are defined by

$$
V_l \triangleq \sum_{k=1}^{\nu_l} W_{l,k}^2\,. \tag{B.4}
$$

Exploiting the fact that the $\{W_{l,k}\}$'s in the KL expansion are independent, zero mean, unit variance Gaussian r.v.'s, it can be shown that the V_l's are *independent* chi-square r.v.'s with ν_l degrees of freedom. The m.g.f. of V_l is given by

$$
M_{V_l}(s) \triangleq \mathbb{E}\big\{e^{+sV_l}\big\} = \left[\frac{1}{1-2s}\right]^{\frac{\nu_l}{2}}\,. \tag{B.5}
$$

Therefore the m.g.f. of $\|\mathbf{h}\|^2$ is

$$
\begin{aligned}
M_{\|\mathbf{h}\|^2}(s) &= \mathbb{E}\Big\{e^{+s\|\mathbf{h}\|^2}\Big\} \\
&= \mathbb{E}\Big\{e^{+s\sum_{l=1}^{L}\lambda_l V_l}\Big\} \\
&= \prod_{l=1}^{L} M_{V_l}(s\lambda_l) \\
&= \prod_{l=1}^{L}\left[\frac{1}{1-s2\lambda_l}\right]^{\frac{\nu_l}{2}}\,.
\end{aligned}
\tag{B.6}
$$

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