Electromagnetic Scattering and Induction Models for Spheroidal Geometries

by

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B.S., Brigham Young University, Provo (1999)
M.S., Brigham Young University, Provo (1999)

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Abstract

Electromagnetic scattering from a medium containing randomly distributed discrete dielectric spheroidal inclusions is studied. Also, the broadband magnetoquasistatic solution for the induced magnetic field from a conducting and permeable spheroid under time harmonic excitation is demonstrated. Analytical electromagnetic solutions for spheroidal geometries are desirable because of their versatility in modeling manmade and natural shapes including solid and hollow needles, spheres, and disks, while at the same time possessing analytic solutions.

Coherent scattering from a collection of small dielectric spheroids populating a dense medium is compared to scattering from a homogeneous sphere. A Method of Moments (MoM) solution is adopted which accounts for spheroid-spheroid interactions directly. Coherent scattering results from these collections are compared to Mie scattering and the effective permittivity of the dense medium is obtained. Results are in good agreement to the classical mixing formula and this lends credibility to both models.

In order to reduce memory requirements and computational complexity, the Sparse Matrix/Canonical Grid (SMCG) method is applied to 3-D dense media scattering. By approximating the dyadic Green's function about a canonical rectilinear grid, weak interaction between spheroid far apart may be quickly approximated. Strong interactions between dielectric spheroid in close proximity are still calculated directly. Weak interactions are quickly evaluated using a novel multilevel block-Toeplitz matrix vector multiply based on the Fast Fourier Transform.

Electromagnetic induction (EMI) models of conducting and permeable spheroids under time harmonic excitation are refined to produce the broadband response with high dependability. A hybrid method is constructed consisting of three different approaches: 1) at low frequencies, the formally exact (but truncated) solution is applied, 2) at moderate frequencies, asymptotic expansions of the spheroidal wave functions (SWFs) are employed, 3) and at high, but still magnetoquasistatic, frequencies, a Small Penetration Approximation is borrowed. The combined EMI response is accurate except near a switchover point where there is typically less than a two percent error. Results are compared to data from a set of 17 steel and aluminum machined spheroids taken by the GEM-3 instrument and found to be in excellent agreement.

Asymptotic expressions of the SWFs are found to depend on branch points and associated characteristic eigenvalues of the spheroidal wave equation. These branch points are found using polynomial estimation techniques and a quadruple precision Newton-Rhapson

search method. Branch points for $(n-m) \leq 100$ are found in greater accuracy than previously available and many are tabulated in this thesis.

The solution for the induced magnetic field from multiple permeable and conducting spheroidal objects in close proximity under time harmonic excitation is presented. Interactions between spheroids is accounted for by a interspheroidal modal interaction matrix. This multibody solution may provide the basis for a forward model used by inversion routines designed to detect and discriminate UXO. Specifically, this multibody solution may help to isolate non-UXO clutter from actual UXO in the field.

Thesis Supervisor: Jin Au Kong
Title: Professor of Electrical Engineering and Computer Science

Acknowledgments

This thesis and this man are a culmination of years of effort on the part of many people. It is imperative to me to recognize and acknowledge key individuals and thank them for their help, support, encouragement, and examples.

I am deeply indebted to my parents and ancestors who deserve my utmost gratitude. In this academic context, it is appropriate to note that my father especially has had a profound influence on my academic career. He has played a crucial role in my intellectual development and is an exemplary scholar and a scientist. My mother has been a bastion of strength in my life. She has maintained a belief in my character and ability, even though there were times when no one else did. It seems impossible to appropriately exhaust one's gratitude to good parents. Thank you.

A turning point in my academic career occurred as an undergraduate when I decided to take a graduate class in remote sensing from Professor David G. Long at Brigham Young University (BYU). The excitement of discovery and the mutual respect encountered in that class was of significant importance in my immediate family's decision to further my education. Additionally, the example of character I perceived in Professor Long and the other Faculty at BYU still inspire me. I am very grateful for these experiences, these examples, and for the encouragement and advice given me while at BYU.

I have come to recognize a rare greatness in Professor Jin Au Kong, my Ph.D. advisor at the Massachusetts Institute of Technology (MIT). The specific aspect which I will notice here is his ability to elicit greatness in others, especially in his students, who often never recognize their own potential. Some people are hindered in discovering their ultimate best simply because there was no spark or catalyst to ignite them. Through his broad, extensive knowledge and skill, Professor Kong helped me uncover my latent abilities and then insist on their development. Though the abilities are mine in the end, the necessary spark for their ignition has been provided in many cases by Professor Kong. This merits gratitude, and I thank him.

Also deserving of special thanks is Kevin O'Neill and Chi Ao. Kevin has in many respects been a second advisor for this thesis specifically with reference to the conducting

and permeable spheroid chapters. Kevin has accomplished the difficult task of being a skilled mentor without being a micromanager. His help, patience, and respect were invaluable. Chi's work in spheroid scattering and induction models have had an important impact on the content of this thesis. I am indebted to him for his early tutoring and subsequent guidance as well as for his friendship.

I have had a chance to associate with many outstanding students in the course of my formal education. I make mention of two students in particular from the Microwave Earth Remote Sensing (MERS) Laboratory at BYU: Quinn Remund, whose integrity and character I admire, and James Bates who I hope will indeed be a friend for life. Also from BYU, Tom Stevens has been a genuine and straightforward friend. Thank you.

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A very personal recognition of influence is due. I am inescapably and profoundly thankful for the teachings and example of Jesus Christ. I believe that the theories and pronouncements on human existence from Jesus represent an interpersonally unprovable, yet intrapersonally self consistent, observable, and repeatable reality. His precepts have played

a periodic and central role in my inner dialogue, moral structure, and behavior decision tree. I would be remiss if I did not recognize this source of fulfillment, opportunity, and joy in my life.

Finally, I wish to express my deepest love, respect, and gratitude to my wife Masako. She has willingly sacrificed much comfort and convenience in order to uproot and be a family together with me in a country foreign to her. We have found that our love for each other and for our family is perhaps the only force powerful enough to enable these two disparate people to discard stubborn weaknesses and improve together. Our relationship has led us to be more honest, patient, loving, understanding, believing, excited, hopeful, empathetic, obedient, and the list continues. This thesis and this man would be much less were it not for my Masako. Thank you.

For my parents,

And especially for 雅子 ***(Masako)***

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Chapter 1

Introduction

Most human sensation of and interaction with the physical world has its basis in electromagnetic phenomena. Macroscopic electromagnetic phenomena are in turn completely described by Maxwell's equations combined with the constitutive relations. The ability to understand, explain, and predict physical phenomena is the first step in mastering and controlling the physical world rather than being powerless bystanders of it. Therefore, analytical and numerical solutions to Maxwell's equations are indispensable tools that enable man to direct the physical world to our advantage.

This thesis extends certain electromagnetic analytic and numerical solutions to Maxwell's equations enabling the accommodation of spheroidal geometries. Spheroidal geometries are desirable because of their versatility in modeling manmade and natural shapes including solid and hollow needles, spheres, and disks, while also possessing analytic solutions.

Beginning with Chapter 2, I study the electromagnetic wave propagation in three-dimensional (3-D) dense random discrete media containing dielectric spheroidal scatterers. I employ a Monte Carlo method in conjunction with the Method of Moments to solve the volume integral equation for the electric field. I calculate the effective permittivity of the random medium through a coherent-field approach and compare my results with a classical mixing formula. A parametric study on the dependence of the effective permittivity on particle elongation and fractional volume is included.

In Chapter 3, the Sparse Matrix/Canonical Grid (SMCG) method, which has been shown to be an efficient method for calculating the scattering from 1-D and 2-D random

rough surfaces, is extended to 3-D dense media scattering. In particular, I study the scattering properties of media containing randomly positioned and oriented dielectric spheroids. Mutual interactions between scatterers are formulated using a Method of Moments (MoM) solution of the volume integral equation. Iterative solvers for the resulting system matrix normally require $\mathcal{O}(N^2)$ operations for each matrix-vector multiply. The SMCG method reduces this complexity to $\mathcal{O}(N \log N)$ by defining a neighborhood distance, r_d, by which particle interactions are decomposed into "strong" and "weak". Strong interaction terms are calculated directly requiring $\mathcal{O}(N)$ operations for each iteration. Weak interaction terms are approximated by a multivariate Taylor series expansion of the 3-D background dyadic Green's function between any given pair of particles. Greater accuracy may be achieved by increasing r_d, using a higher order Taylor expansion, and/or increasing mesh density at the cost of more interaction terms, more FFTs, and longer FFTs, respectively. Scattering results, computation times, and accuracy for large-scale problems with r_d up to 2 grid points, 14×14×14 canonical grid size, fifth order Taylor expansion, and 15000 discrete scatterers are presented and compared against full solutions.

The efficient calculations in Chapter 3 are accomplished using a new $O(N \log N)$ FFT-based method to expedite matrix-vector multiplies involving multilevel block-Toeplitz (MBT) matrices. This new method is presented as Appendix A. The method is also a minimal memory method with $O(N)$ memory requirements because only nonredundant entries of the MBT matrix are stored. The accuracy and convergence of the method are illustrated in the calculation of the scattered field and the effective permittivity of a sphere with size parameter ka up to 5 using a volume integral formulation akin to the discrete dipole approximation.

The detection and discrimination of subsurface unexploded ordnance (UXO) is of critical and timely importance to human populations and governments. Electromagnetic induction (EMI) models of canonical shapes are promising in this effort to detect and discriminate UXO. I develope a hybrid model including asymptotic expressions of the spheroidal wave functions (SWFs) to obtain a reliable, broadband solution for the electromagnetic induction (EMI) response from a conducting and permeable spheroid in Chapter 5. I obtain this broadband response, valid in the magnetoquasistatic regime from 0 Hz to 100's of KHz, by

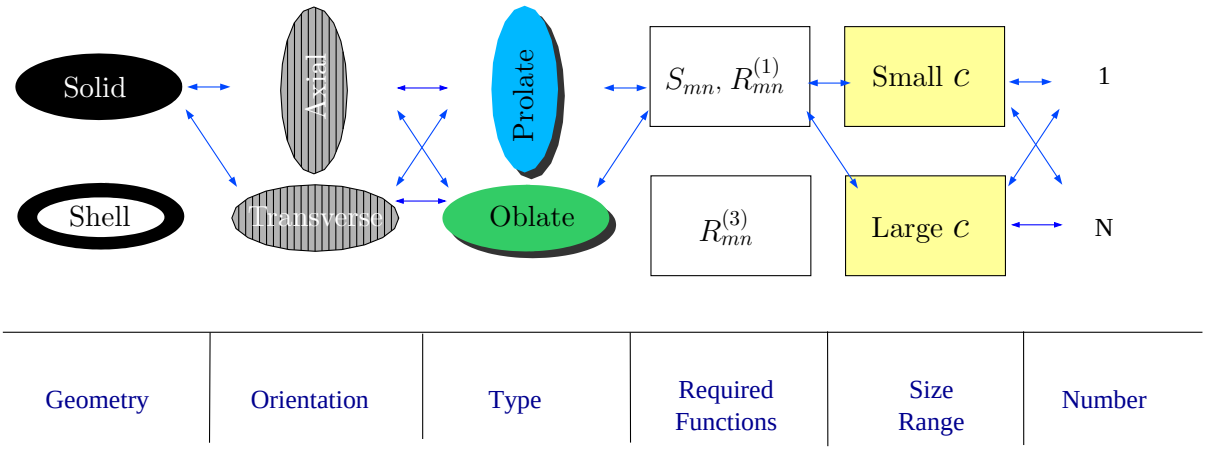

Figure 1-1: Electromagnetic induction (EMI) model flowchart.

combining three different techniques each applicable over a different frequency range. At low frequencies, the exact analytical solution is used. At mid-range frequencies, asymptotic expressions for the angular and radial SWFs, $S_{mn}(c,\eta)$ and $R_{mn}(c,\xi)$, are implemented to maintain a stable solution for the induced magnetic field. At higher frequencies, a Small Penetration Approximation (SPA) solution is used when the SPA solution approaches the asymptotic assisted solution to within some predefined tolerance. Validation of this combined technique is accomplished by comparison of the induced magnetic field predicted by my model to both a finite element method/boundary integral (FEM/BI) numerical solution and experimental data from various spheroids taken by an ultra-wideband EMI instrument. An outline of the various possibilities for spheroidal induction models si presented in Fig. 1-1.

This broadband EMI response is obtained through the use of asymptotic expansion of the SWFs for complex size parameters. In Chapter 4, I provide a rapid and accurate method for calculating the prolate and oblate spheroidal wave functions (PSWFs and OSWFs), $S_{mn}(c,\eta)$, and their eigenvalues, λ_{mn}, for arbitrary complex size parameter c in the asymptotic regime of large $|c|$, m and n fixed. The ability to calculate these SWFs for large and complex size parameters is important for many applications in mathematics, engineering, and physics. For arbitrary $\arg(c)$, the PSWFs and their eigenvalues are accurately

expressed by established *prolate*-type or *oblate*-type asymptotic expansions. However, determining the proper expansion type is dependent upon finding spheroidal branch points, $c_{\circ;r}^{mn}$, in the complex c-plane where the PSWF alternates expansion type due to analytic continuation. I implement a numerical search method for tabulating these branch points as a function of spheroidal parameters m, n, and $\arg(c)$. The resulting table allows rapid determination of the appropriate asymptotic expansion type of the SWFs. Normalizations, which are dependent on c, are derived for both the *prolate* and *oblate*-type asymptotic expansions and for both $(n-m)$ even and odd. The ordering for these expansions is different from the original ordering of the SWFs and is dictated by the location of $c_{\circ;r}^{mn}$. I document this ordering for the specific case of $\arg(c) = \pi/4$ which occurs for the diffusion equation in spheroidal coordinates. Some representative values of λ_{mn} and $S_{mn}(c,\eta)$ for large, complex c are also given while Appendix D contains a more complete listing of the branch points of the spheroidal wave equation, $c_{\circ;r}^{mn}$, along with their associated eigenvalues, $\lambda_{\circ;r}^{mn}$. Results are tabulated for a greater range of m and n than previously available as well as with greater accuracy. Also included in Section D.2 are tables containing the angular SWF for large and complex size parameter, c.

UXO discrimination is often hampered by multiple targets in close proximity corrupting individual induced field detection and processing. Exact calculation of the induced field produced by multiple spheroidal targets in close proximity may facilitate discrimination during inversion. Thus, I present the solution for the induced electromagnetic induction (EMI) response from multiple permeable and conducting spheroidal objects in close proximity under time harmonic excitation in Chapter 6. The exact formulation for the induced EMI response is based on magnetic field boundary conditions. The internal magnetic field of each spheroid is represented in terms of an infinite sum of the vector spheroidal wave functions. The external magnetic field is composed of the primary field and the induced secondary field from each spheroid, each expressed as an infinite sum of solutions of the first and second kinds of the Laplace equation in the spheroidal coordinate system. The total external secondary field summation coefficients are solved for simultaneously thus including all multiple body interactions.

In Appendix B, I present the exact formulation for the induced magnetic field from

a conducting and permeable spheroidal shell under time harmonic excitation. Some low frequency results are also presented.

Chapter 2

Monte Carlo Simulation of Electromagnetic Wave Propagation in Dense Random Media with Dielectric Spheroids

2.1 Introduction

In applications related to the remote sensing of the environment, the characterization of the electromagnetic wave interaction with natural media is of great importance. Natural media (e.g., snow, ice, and soil) often consist of a large number of densely packed, electrically small discrete scatterers that are randomly distributed in some background host medium.

The "classical" approach to study random discrete scatterers involves hypotheses such as the consideration of tenuous and/or sparse media. In sparse media, discrete scatterers occupy only a small volume fraction (typically less than 5%). In tenuous media, the constitutive parameters of discrete scatterers differ only slightly from that of the background medium. Such hypotheses allow for the solution of the problem through the independent scattering assumption, which neglects coherent interactions between the particles. However, in dense nontenuous media, the independent scattering assumption is no longer

valid [8,9,10]; hence the effects of multiple scattering and coherent wave mutual interactions must be taken into account. This is also true for media in which the scatterers occupy a low overall fractional volume but can be locally dense due to clustering properties (e.g., branches and leaves in vegetation canopies).

For such dense media, analytical wave theory and approximations such as Foldy's approximation [11], the quasicrystalline approximation (QCA) [12,13,14], and the QCA with coherent potential (QCA-CP) [12,8] are frequently employed. In QCA and QCA-CP, the pair distribution function, which constitutes a second-order spatial correlation among the scatterers, must be specified. Common approximations for the pair distribution function are the hole correction and the Percus-Yevick (PY) pair distribution function [8,15]. The use of such analytical techniques is adequate in the study of problems presenting configurational symmetries, such as media composed only of spherical particles. In many practical cases of interest, however, the adherence to spherical geometries is not able to capture the essential physics of the problem [16,17]. The need to incorporate nonspherical statistics into the analytical models makes such analytical treatments rather involved.

The alternative to deal with more complex problems in a systematic manner is to resort to numerical methods. In [16], the scattering problem involving dense dielectric scatterers is formulated using a volume integral equation, which may then be solved by the Method of Moments (MoM) [18]. This gives the complete solution for a given configuration of scatterers. For a random medium problem, the positions and properties of the scatterers are determined for different realizations according to some prescribed statistics using the Monte Carlo method. The MoM solution is sought for each realization so that statistical averages of the quantities of interest (e.g., effective permittivity, absorption rates, extinction rates, and phase functions of co-polarization and cross-polarization) can be determined.

In this work, we use the Monte Carlo method to characterize the effective permittivity of dense discrete random media composed of three-dimensional spheroidal dielectric scatterers as a function of the fractional volume of scatterers, particle elongation, the electromagnetic size of the particle (ka), and particle permittivity (media contrast). This extends previous studies (e.g. [19,20,21,15,22]) on the numerical calculation of effective permittivity which were limited to spherical particles and/or two-dimensional scattering.

2.2 Formulation

In this section, we briefly describe the volume integral formulation and the Monte Carlo methods used in our work. More detailed expositions of this MoM solution based on the volume integral forumation can be found in Chapter 3 and in [16].

Consider an incident electric field $\mathbf{E}_{inc}(\mathbf{r})$ impinging on N randomly positioned and oriented dielectric prolate spheroids (see Fig. 2-1). The total electric field $\mathbf{E}(\mathbf{r})$ can be expressed in terms of a volume integral equation as

$$\begin{aligned}\mathbf{E}(\mathbf{r}) = \mathbf{E}_{inc}(\mathbf{r}) + k^2 \sum_{j=1}^{N} \int_{V_j} d\mathbf{r}' g(\mathbf{r},\mathbf{r}')[\chi(\mathbf{r}')\mathbf{E}(\mathbf{r}')] \\ - \sum_{j=1}^{N} \nabla \int_{V_j} d\mathbf{r}' \nabla' g(\mathbf{r},\mathbf{r}') \cdot [\chi(\mathbf{r}')\mathbf{E}(\mathbf{r}')] \end{aligned} \tag{2.1}$$

where $k = \omega\sqrt{\mu\epsilon}$ is the background wavenumber and V_j is the volume of the spheroid j. The scalar Green's function $g(\mathbf{r},\mathbf{r}')$ and electric susceptibility $\chi(\mathbf{r}')$ are given by

$$g(\mathbf{r},\mathbf{r}') = \frac{\exp\left(ik|\mathbf{r}-\mathbf{r}'|\right)}{4\pi|\mathbf{r}-\mathbf{r}'|}$$
$$\chi(\mathbf{r}') = \frac{\epsilon_p(\mathbf{r}')}{\epsilon} - 1.$$

with ϵ_p being the permittivity inside the spheroid.

To solve Eq. (2.1) using MoM, we assume that the spheroids are electrically small and choose the electrostatic solution of the Laplace equation to form a set of N_b basis functions. In this work, only the dipole basis functions are considered. The matrix equation that results for the internal field $\mathbf{E}(\mathbf{r})$ is solved iteratively using the biconjugate gradient stabilized method (Bi-CGSTAB). It should be pointed out that permittivities of the background medium and the spheroid must satisfy the conditions $ka \ll 1$ and $k_p a \ll 1$, where $k_p = \omega\sqrt{\mu\epsilon_p}$, in order for the small scatterer assumption to be valid.

To obtain the average fields, a Monte Carlo simulation is performed by creating multiple realizations of media consisting of thousands of discrete spheroids with random positions and orientations contained within some test volume. The Metropolis shuffling process [23, 24]

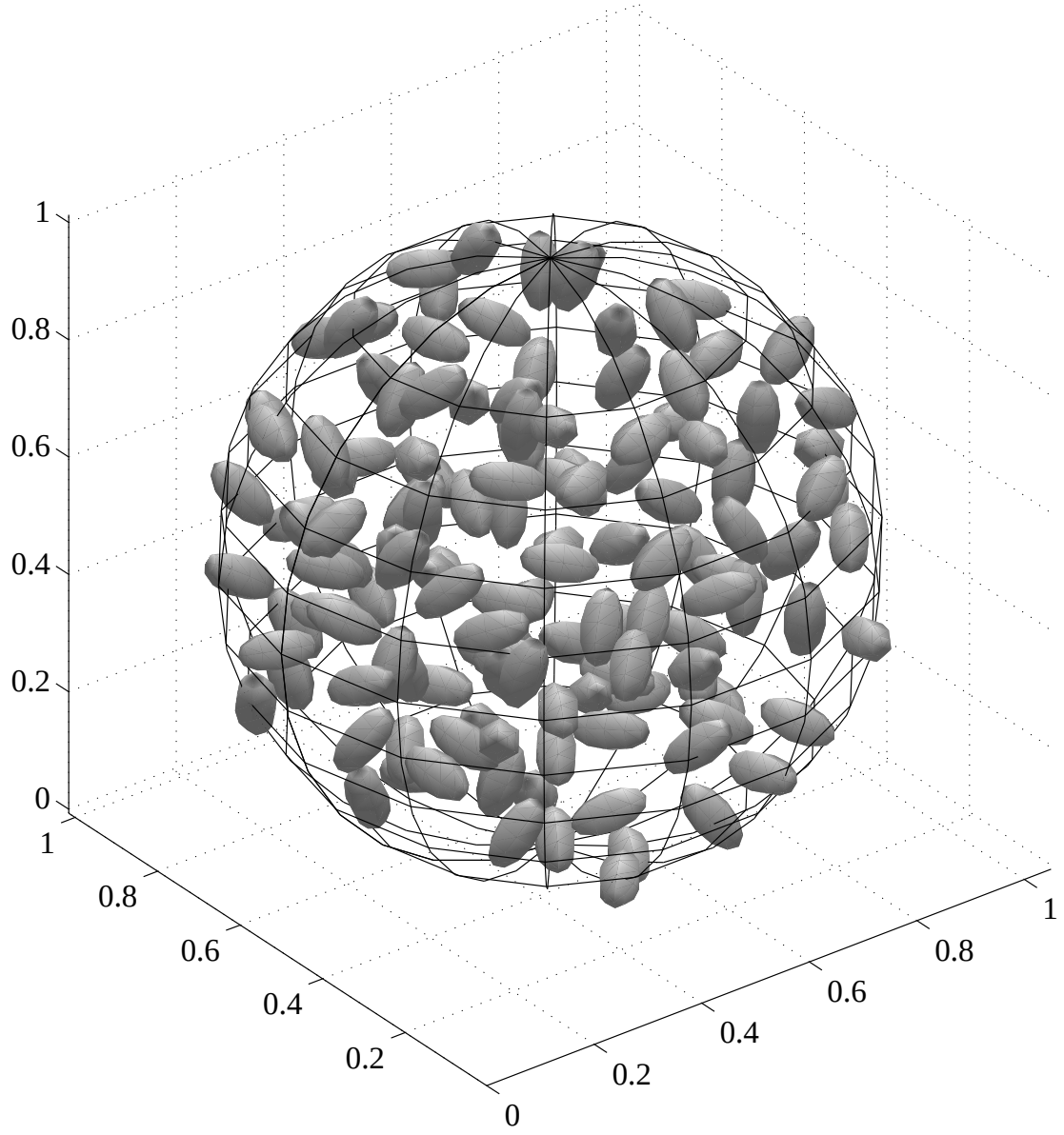

Figure 2-1: N non-overlapping small dielectric spheroids in a spherical test volume.

is used to generate the positions and orientations of a system of densely packed spheroids. Periodic boundary conditions [16] are employed for the test volume in order to minimize edge effects.

2.3 Characterization of Random Media

Generally speaking, the characterization of a random medium containing discrete scatterers can be studied from two different perspectives: the coherent and incoherent fields. The coherent scattered field is obtained by averaging the electric field solution over N_r realizations:

$$<\mathbf{E}_s>=\frac{1}{N_r}\sum_{\sigma=1}^{N_r}\mathbf{E}_s^{\sigma} \tag{2.2}$$

The coherent field is closely related to the propagation characteristics of the random medium. On the other hand, the scattering of the coherent wave away from its forward propagation direction is related to the incoherent field, which is defined as

$$\mathcal{E}_s^{\sigma}=\mathbf{E}_s^{\sigma}-<\mathbf{E}_s> \tag{2.3}$$

where $\sigma = 1, 2, \ldots, N_r$ is the realization index. The incoherent intensity is then given by

$$\mathcal{I}_s=\frac{1}{N_r}\sum_{\sigma=1}^{N_r}|\mathcal{E}_s^{\sigma}|^2 \tag{2.4}$$

The incoherent intensity can be used to obtain the extinction coefficient and phase functions that are used in radiative transfer (RT) theory [16]. In contrast to "conventional" RT theory, the extinction coefficient and phase functions obtained through the Monte Carlo simulations outlined in Section 2.2 take into account of coherent multiple interactions among the scatterers.

In this work, we focus on the characterization of the propagation properties of the random medium. Thus the coherent field will be used to determine the effective propagation constant of the random medium (or equivalently the effective permittivity for a non-magnetic medium). In this "coherent-field approach," the effective permittivity is obtained as the result of an inverse scattering problem. The coherent scattered fields from a collection of particles in a given finite test volume are compared to fields scattered from a homogeneous volume of the same size and shape. The permittivity of the homogeneous medium which yields scattered fields that best match the averaged scattered fields produced

by the random medium is considered to be the effective permittivity. This approach has the advantages of taking the size and shape of the test volume into account. It has been used for spheres contained in a cubic volume using the Born approximation for scattering from the cube [21], for 2-D rectangular regions [20], and for spheres contained in spherical volumes using Mie theory for scattering from the sphere [21, 25]. In the following, we apply this method to obtain the effective permittivity (ϵ_{eff}) of a dense discrete random medium containing many spheroids by comparing the coherent scattered fields from collections of spheroids contained in a spherical test volume (Fig. 2-1) to Mie scattering from the same size homogeneous sphere with permittivity equal to ϵ_{eff}. The bistatic radar cross section (RCS) will be used for the comparisons.

Let the incident electric field $\mathbf{E}_{inc}(\mathbf{r})$ be a plane wave

$$\mathbf{E}_{inc}(\mathbf{r}) = \mathbf{y} \exp(ikz) \tag{2.5}$$

The radar cross section (RCS) for the random spheroid medium (σ_{ran}) with the electromagnetic wavelength set to unity is then

$$\sigma_{ran} = 4\pi r^2 \left| < \mathbf{E}_s > \right|^2 . \tag{2.6}$$

The Mie scattering in the far field is computed using Wiscombe's code [26]. According to the notation in [26], the RCS of a Mie sphere σ_{mie} in the scattering plane is given by

$$\sigma_{mie} = \frac{4\pi}{k^2} \left| S_1(\theta) \right|^2 \tag{2.7}$$

where S_1 is the complex scattering amplitude, and θ is the scattering angle. The effective permittivity is calculated using a multistart Nelder-Mead simplex search method [27]. Only the real part of the effective permittivity is reported here. The error term used in this search method was chosen to be the absolute value of the cumulative difference between the spheroid medium RCS, σ_{ran}, and the Mie sphere RCS, σ_{mie}:

$$\text{error} = \delta_\sigma = \sum_{j=1}^{N_a} \left| (\sigma_{mie,j}) - (\sigma_{ran,j}) \right| . \tag{2.8}$$

where $\sigma_{mie,j}$ and $\sigma_{ran,j}$ denote the RCS for the Mie sphere and the random spheroid medium respectively at the j^{th} angle. N_a represents the number of scattering angles used in the minimization process (N_a=39 for the results presented in this work).

A cumulative logarithmic error was also considered. However, the resulting ϵ_{eff} was slightly unstable. Using a logarithmic fit, even changes in scattering amplitude on the order of 10^{-4} would alter the resulting fit due to the emphasis placed on all scattering point. When using the linear error, the strong forward scattered field is fit best, and the behavior in the tails is accounted for automatically. In this work, only the real part of ϵ_{eff} is considered.

2.4 Numerical Results

The effective permittivity, ϵ_{eff} of dense random media containing discrete prolate spheroids was calculated for many combinations of f_v (fractional volume) and e (elongation, $c = ae$). For each ϵ_{eff} calculation, N_r=50 realizations were performed, and the results averaged according to Eq. (2.2) to yield the coherent scattered electric field $<\mathbf{E}_s>$. The spherical test volume was first generated as a cubic test volume in order to employ periodic boundary conditions. After placement, the center of each spheroid was used to determine whether or not that spheroid would remain in the spherical test volume. N=3000 spheroids were placed in the cubic test volume, and therefore an average of $(N\pi/6) \approx 1571$ spheroids remained in the spherical test volume. The radius of the equivalent homogeneous sphere (R) used for Mie scattering was determined by

$$R = \left(\frac{N}{f_v} \left(\frac{\pi}{6} \right) a^3 e \right)^{\frac{1}{3}}. \tag{2.9}$$

The parameters for the simulations reported here were $f_v \Rightarrow$ [.05, .10, .15, .20, .25, .30, .35, .40], $e \Rightarrow$ [1.0, 1.8, 2.6], $\epsilon_p \Rightarrow$ 3.2, and $ka \Rightarrow$ 0.2. Both the random spheroid media scattering and the Mie sphere scattering were copolarized scattering with $\phi = 0°$ and $\theta = 0°$ to 180°. Typical scattering results for the case of f_v=0.20, e=1.0 (sphere case) and 2.6, ka=0.2, and ϵ_p=3.2+0.0i are shown in Fig. 2-2. As with previous results for spheres [21] and 2-D infinitely long cylinders [20], the coherent scattering from a homogeneous volume

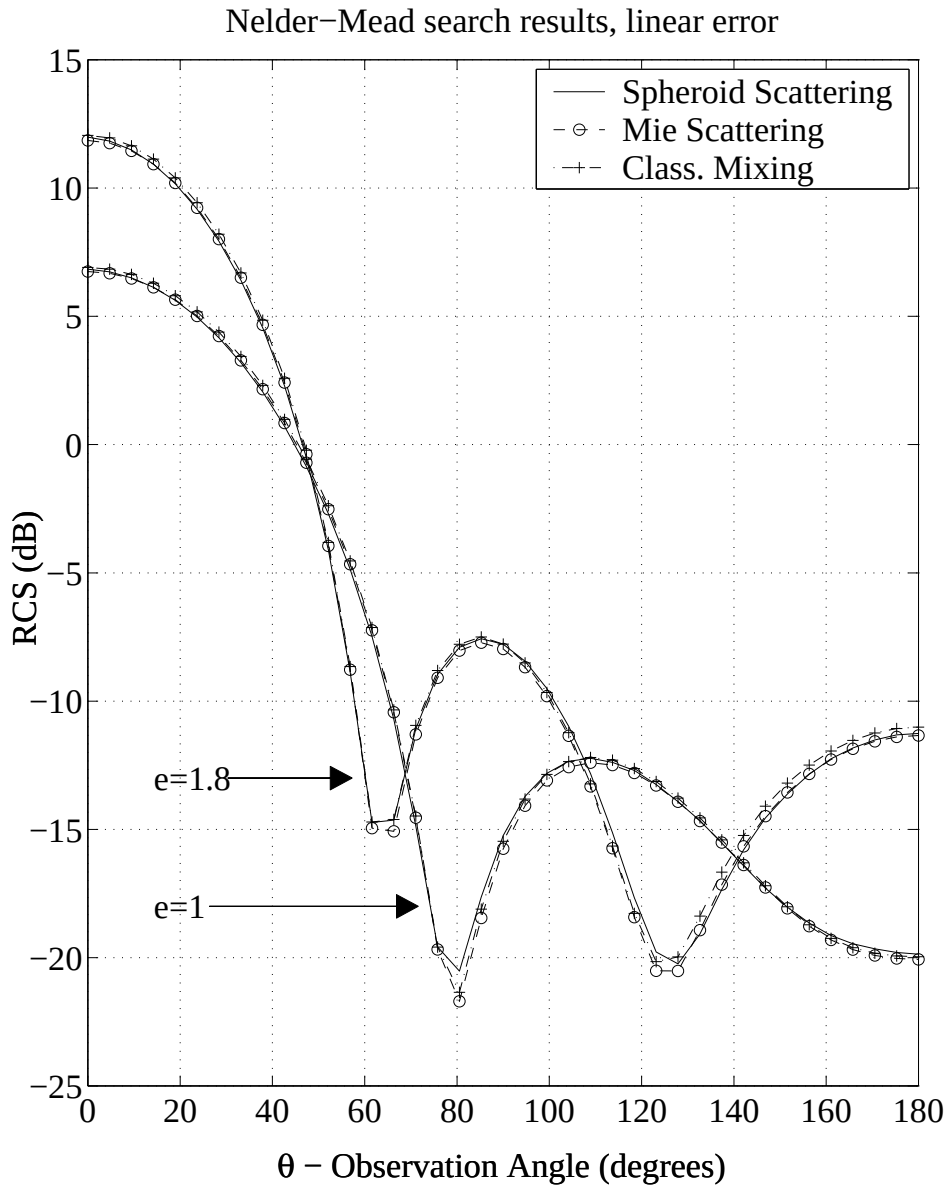

Figure 2-2: Scattering from the spheroid test volume (solid line) compared to scattering from a homogeneous dielectric sphere with ϵ_{eff} obtained from using Eq. (2.8) as the search method error, and with ϵ_{eff} a classical mixing formula [1]. On average, 1571 particles were used in N_r=50 realizations.

with permittivity ϵ_{eff} adequately model the coherent scattering from dense random media consisting of discrete spheroids.

The variation of ϵ_{eff} with f_v and e is depicted in Fig. 2-3. Also shown is the ϵ_{eff} predicted

by the classical mixing theory [1] for spheres (solid line) and for spheroids with e=2.6 (dash-dot line). This classical mixing formula for randomly oriented ellipsoids is expressed as

$$\epsilon_{\text{eff}} = \epsilon + \left(1 - \frac{f_v}{3}\sum_{i=1}^{3} N_i\beta_i\right)^{-1} \frac{f_v}{3}\sum_{i=1}^{3}\epsilon\beta_i \tag{2.10}$$

where

$$\beta_i = \frac{(\epsilon_p - \epsilon)}{\epsilon + N_i(\epsilon_p - \epsilon)}. \tag{2.11}$$

In the prolate spheroid case considered in this work, the depolarization factors N_i are given by [28]

$$N_z = \frac{1-\varepsilon^2}{2\varepsilon^3}\left(\ln\frac{1+\varepsilon}{1-\varepsilon} - 2\varepsilon\right) \tag{2.12}$$

$$N_x = N_y = \frac{1}{2}(1 - N_z) \tag{2.13}$$

with the eccentricity $\varepsilon = \sqrt{1-(1/e^2)}$.

For the case of a dense medium consisting of spheres (e=1), the results for effective permittivity are in excellent agreement the classical mixing formula except at higher fractional volumes where the results are slightly lower. For the case of a dense medium consisting of randomly oriented prolate spheroids (e=2.6), the results for effective permittivity are lower than that predicted by the classical mixing formula at f_v larger than about 0.2. These results are consistent with [21] for the spherical case. Note that as the fractional volume increases, the close proximity of the spheroids might necessitate the use of quadrupole terms to accurately model multiple scattering.

When interpreting numerical simulation results, accuracy and convergence of the computational method are important. A convergence criteria of residual error $\leq 10^{-6}$ was used in the iterative solver (Bi-CGSTAB) for Eq. (2.1). To ensure that the medium sample set was sufficiently random, many realizations were performed. Fig. 2-4 shows the calculated ϵ_{eff} and the final error term Eq. (2.8) resulting from the Nelder Mead simplex search as a function of the number of realizations performed. As the number of realizations increases the residual error decreases and ϵ_{eff} converges to its final value.

Table 2.1 lists the real part of ϵ_{eff} calculated from the method described in this work.

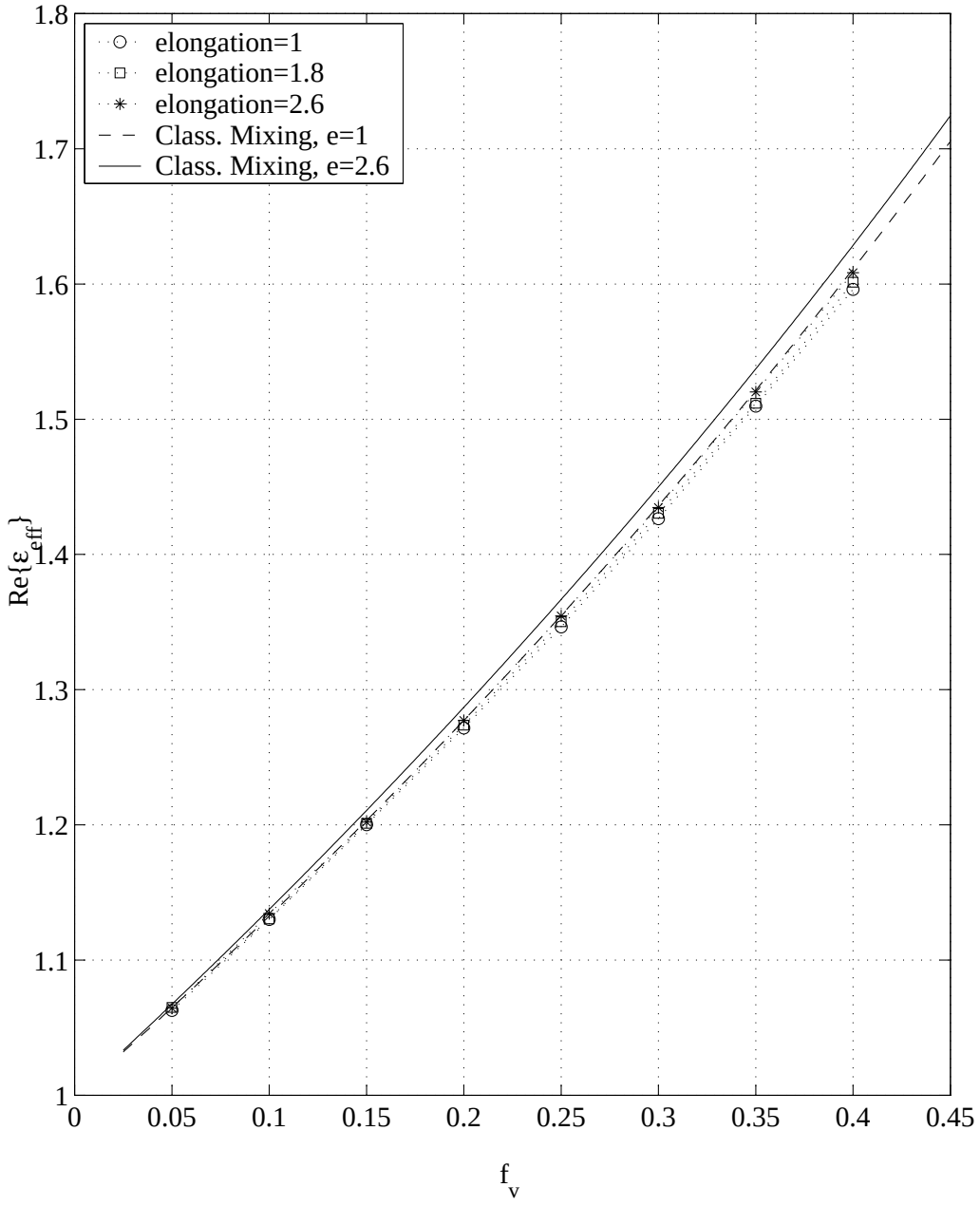

Figure 2-3: ϵ_{eff} as a function of fractional volume and elongation as compared to the classical mixing formula for e=1 and e=2.6.

These results provide numerical validation for the classical mixing formula for fractional volumes in the range of 0.05 to 0.4 for spheroids with elongations up to 2.6.

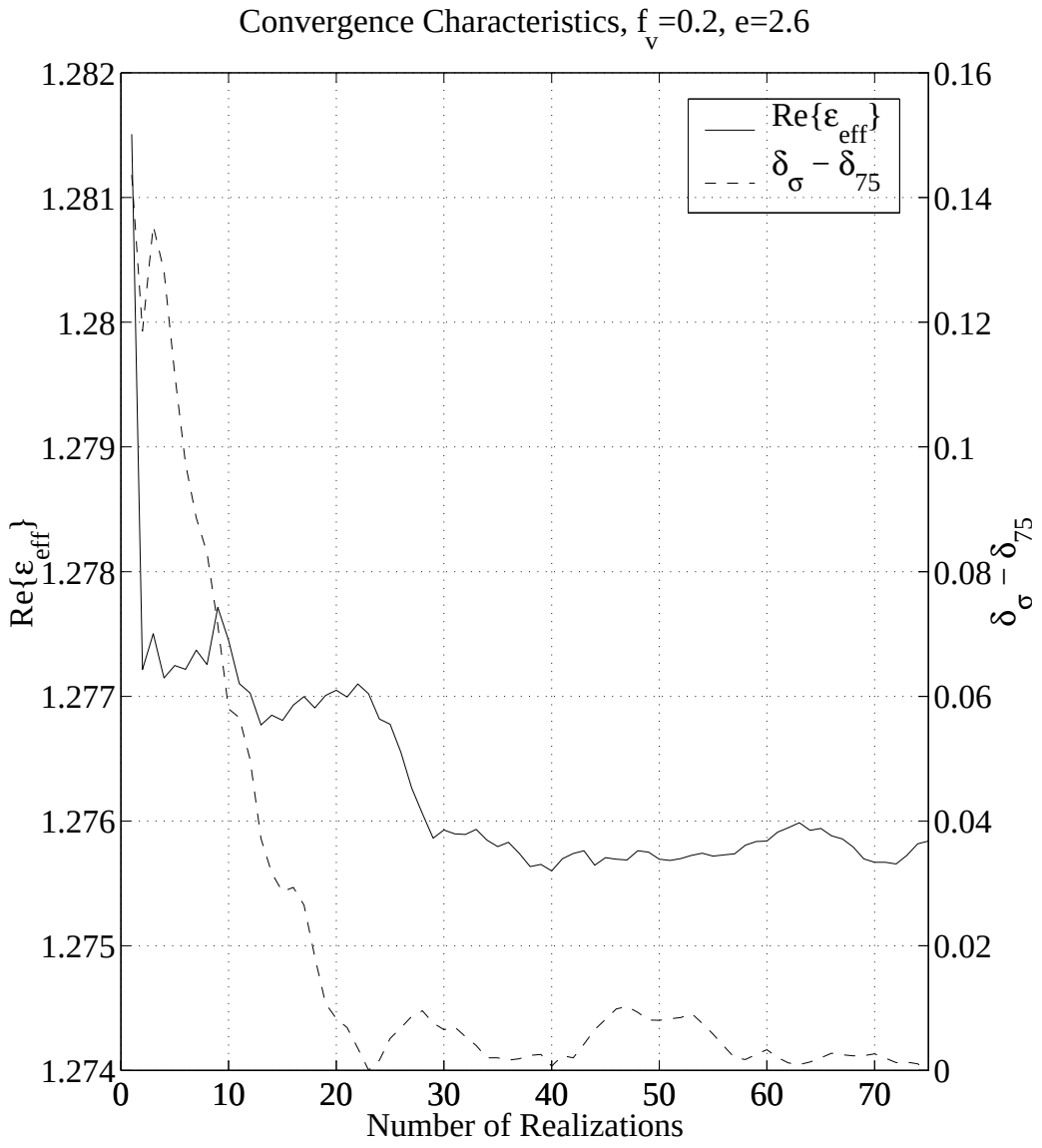

Figure 2-4: $\Re\{\epsilon_{\text{eff}}\}$ and the error term δ_σ (subtracted from the final error ($\delta_{75} = \delta_\sigma(N_r = 75)$) and normalized for N_a=39) resulting from the Nelder Mead simplex search according to realization number. This is for the case of f_v=0.2, e=2.6, ϵ_p=3.2, ka=0.2, and N_r=1...75 realizations.

2.5 Conclusions

In this work, the effective permittivity (ϵ_{eff}) of dense random media was studied. The coherent scattered fields from a collection of particles in a given test volume were compared to fields scattered from a homogeneous volume of the same size and shape. The permittivity

	e=1	1.8	2.6
f_v=0.05	1.0628	1.0650	1.0645
0.10	1.1301	1.1304	1.1345
0.15	1.2001	1.2012	1.2021
0.20	1.2717	1.2738	1.2772
0.25	1.3465	1.3502	1.3546
0.30	1.4265	1.4304	1.4347
0.35	1.5097	1.5118	1.5202
0.40	1.5961	1.6014	1.6084

Table 2.1: Summary of ϵ_{eff} results. For all cases presented, ka=0.2 and ϵ_p=3.2.

of the homogeneous medium which yielded scattered fields that best matched the averaged scattered fields produced by many configurations of particles was considered to be equal to ϵ_{eff}.

The effective permittivity of dense random media containing discrete spheroids was characterized through a Monte Carlo simulation employing the Method of Moments on the volume integral equation. The basis functions were chosen to be the electrostatic dipole solutions of a spheroid. Simulations with N=3000 particles, N_r=50 realizations and different fractional volumes (f_v = 0.05–0.40) and particle elongations (e = 1.0–2.6) were performed in a spherical test volume. The scattering from this volume was then compared to Mie scattering from a homogeneous sphere of the same size. Results indicate that ϵ_{eff} for a dense random medium containing randomly oriented discrete prolate spheroids agrees well with results from the classical mixing formula. This is consistent with previous studies considering spherical particles only.

Chapter 3

Sparse Matrix/Canonical Grid Method Applied to 3-D Dense Medium Simulations

3.1 Introduction

Electromagnetic scattering from and wave propagation in three-dimensional (3-D) discrete random media has been a topic of continued research due to its broad range of applications. For instance, in applications related to the remote sensing of the environment, the characterization of the electromagnetic wave interaction with natural media is of great importance. Natural media (e.g., snow, ice, and soil) often consist of a large number of densely packed, electrically small discrete scatterers that are randomly distributed in some background host medium (discrete random media). Moreover, many artificial materials (e.g., particulate composites) can also be characterized as being composed of randomly distributed scatterers in a host medium. Even some materials which can in principle be conceptualized and realized by regular, periodic arrangements, are often subject to random perturbations, both in terms of the positions, sizes and/or constitutive properties of the discrete scatterers.

For dense random media, several analytical methods based on wave theory, such as the quasi-crystalline approximation (QCA [12, 13]), the quasi-crystalline approximation with

coherent potential (QCA-CP [29]), or other approximations such as independent scattering and Foldy's approximation [11], are frequently employed. These analytical methods rely on basic simplifying assumptions about the media, such as tenuous media (low permittivity and permeability contrast), sparse media (low fractional volume of scatterers, typically less than 5%), or particle shape (*e.g.* spherical). These assumptions facilitate tractable analytical solutions but are not able to capture the essential physics of many real world problems. Moreover, it is often difficult to assess the range of validity of such analytical techniques *a priori*.

The alternative approach to deal with more complex media problems in a systematic manner is to resort to numerical techniques. Numerical methods such as the Method of Moments (MoM [18]), the finite difference time domain (FDTD [30, 31]), and related ones have allowed a partial relaxation of some of the limiting assumptions concerning the medium (Chapter 2 and [16]). In particular, the MoM provides a self-consistent solution which includes multiple interactions between discrete particles through an interaction (impedance) matrix $\overline{\overline{Z}}$. The main disadvantages of the MoM are the computation time required for solving the often large and dense resulting system of equations and the computer memory requirements for the storage of $\overline{\overline{Z}}$.

To help alleviate these bottlenecks, several so-called fast numerical methods have been developed in recent years for both 2-D and 3-D electromagnetic problems (e.g. [32, 33, 34, 35, 36, 37, 29, 38]). One of these fast methods, the Sparse Matrix/Canonical Grid (SMCG) method [39, 36, 40, 35, 41, 42], speeds up the solution of certain electromagnetic problems by decomposing the interaction matrix $\overline{\overline{Z}}$ into two separate matrices: a strong interaction matrix $\overline{\overline{Z}}^{\mathrm{s}}$ which contains the interactions of nearby elements calculated exactly, and a weak interaction matrix $\overline{\overline{Z}}^{\mathrm{w}}$ which contains an approximation of the weaker interaction terms from elements located farther apart. In contrast to Fast Multipole Methods (FMM), this procedure does not group interactions together, but rather approximates each individual weak interaction based on a canonical grid. The strong interaction matrix tends to be sparse while the vast majority of the interactions between elements are included in the weak interaction matrix. This latter matrix can be derived by a Taylor series expansion around a canonical grid [36, 42] or through the use of translation addition theorems [39, 29].

The advantage in doing this is that during the solution of $\overline{\overline{Z}}$ using iterative solvers (as is frequently the case), the original full matrix-vector multiply $\overline{\overline{Z}} \cdot \overline{x}$ ($\mathcal{O}(N^2)$ complexity) may be replaced by a sparse matrix multiply, $\overline{\overline{Z}}^{\mathrm{s}} \cdot \overline{x}$ ($\mathcal{O}(N)$) plus a Fast Fourier Transform (FFT) assisted multiply $\overline{\overline{Z}}^{\mathrm{w}} \cdot \overline{x}$ ($\mathcal{O}(N \log N)$). This FFT assisted matrix-vector multiply is possible because of the multilevel block Toeplitz (MBT) structure inherent in $\overline{\overline{Z}}^{\mathrm{w}}$ as a result of the canonical grid expansion of the weak interactions.

In this work, we extend the SMCG method to 3-D case and consider its application to the scattering from complex media consisting of a collection of randomly positioned and oriented dielectric spheroids in a homogeneous host medium. In Section 3.2, we describe the MoM formulation of $\overline{\overline{Z}}$ derived from a many-body volume integral equation. In Section 3.3, we describe the 3-D SMCG method in detail, including a discussion on the various tradeoffs associated with choosing the method parameters as well as the Taylor series expansion of the background dyadic Green's function around the canonical grid grid points. In Section 3.4, scattering results from these collections of spheroids calculated by the SMCG method are presented and compared to results obtained from the full MoM solution. Computation times, memory requirements, and accuracy for the approximated and full MoM solutions for cases of 500–15000 particles, varying neighborhood distances, Taylor series expansion orders of 0-5, and grids of 6×6×6 to 14×14×14 are also considered in Section 3.4. This is followed by conclusions in Section 3.5.

3.2 Formulation

The electric field $\overline{E}(\overline{r})$ in a random medium consisting of N arbitrary dielectric objects (discrete scatterers) with permittivity $\epsilon(\overline{r}'_j)$ excited by an incident electric field $\overline{E}_{inc}(\overline{r})$ can be described by the volume integral equation (VIE) [43] through a summation over each scatterer, i.e.,

$$\overline{E}(\overline{r}) = \overline{E}_{inc}(\overline{r}) + \frac{k^2}{\epsilon} \sum_{j=1}^{N} \int_{V'_j} dV'_j \, (\epsilon(\overline{r}'_j) - \epsilon) \, \overline{\overline{G}}(\overline{r}, \overline{r}'_j) \cdot \overline{E}(\overline{r}'_j), \tag{3.1}$$

where V_j is the volume of scatterer j, k is the wavenumber of the homogeneous background medium, ϵ is the permittivity of the background medium, $\epsilon(\overline{r}'_j)$ is the permittivity inside the j^{th} scatterer. $\overline{\overline{G}}(\overline{r},\overline{r}'_j)$ is the background generalized dyadic Green's function whose principal value is given by

$$\overline{\overline{G}}(\overline{r},\overline{r}'_j) = \left[\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right] g(\overline{r},\overline{r}'_j), \tag{3.2}$$

which, to account for the singularity encountered when $\overline{r} = \overline{r}'_j$, should be written as [44, 29, 45]

$$\overline{\overline{G}}(\overline{r},\overline{r}'_j) = P.V.\overline{\overline{G}}(\overline{r},\overline{r}'_j) - \frac{\overline{\overline{L}}\delta(\overline{r}-\overline{r}'_j)}{k^2}. \tag{3.3}$$

where $P.V.$ stands for Principal Value. The exclusion volume dependent term in Eq. (3.3) becomes important when calculating the self interaction terms (see below). Finally, $g(\overline{r},\overline{r}'_j)$ is the scalar Green's function,

$$g(\overline{r},\overline{r}'_j) = \frac{e^{ik|\overline{r}-\overline{r}'_j|}}{4\pi|\overline{r}-\overline{r}'_j|}. \tag{3.4}$$

Primed variables indicate those variables pertaining to the scatterers while unprimed variables indicate quantities positioned outside the scatterers (background or host medium). The induced polarization inside each object due to the electric field at $\overline{r}'_j$ is given by $\overline{P}(\overline{r}'_j) = (\epsilon(\overline{r}'_j) - \epsilon)\overline{E}(\overline{r}'_j)$ [from Eq. (3.1)].

To solve Eq. (3.1) using the Method of Moments (MoM), the electric field inside the each particle is expanded in a set of N_b basis functions, *e.g.* for the j^{th} particle,

$$\overline{E}(\overline{r}'_j) = \sum_{\alpha=1}^{N_b} c_{j\alpha}\,\overline{f}_{j\alpha}(\overline{r}'_j). \tag{3.5}$$

The basis functions $\overline{f}_{j\alpha}(\overline{r}_j)$ for each particle can be chosen based on particle size and geometry as detailed later on. For our purposes, we will also assume that these basis functions are orthonormal:

$$\int_{V'_j} dV'_j\ \overline{f}_{j\alpha}(\overline{r}'_j)\cdot\overline{f}_{j\beta}(\overline{r}'_j) = \delta_{\alpha\beta}, \tag{3.6}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta.

To find the field inside each object, we substitute Eq. (3.5) into Eq. (3.1) to arrive at

$$\left[\sum_{\beta=1}^{N_b} c_{i\beta}\,\overline{f}_{i\beta}(\overline{r}_i')\right] = \overline{E}_{inc}(\overline{r}_i') + \frac{k^2}{\epsilon}\sum_{j=1}^{N}\int_{V_j'} dV_j'\ (\epsilon(\overline{r}_j')-\epsilon)\,\overline{\overline{G}}(\overline{r}_i',\overline{r}_j')\ \cdot\ \left[\sum_{\alpha=1}^{N_b} c_{j\alpha}\,\overline{f}_{j\alpha}(\overline{r}_j')\right]. \tag{3.7}$$

By invoking the orthonormality of the basis functions [Eq. (3.6)] and applying Galerkin's technique, Eq. (3.7) can be written as:

$$c_{i\beta} = \int_{V_i'} dV_i'\ \overline{f}_{i\beta}(\overline{r}_i')\cdot\overline{E}_{inc}(\overline{r}_i') + \sum_{j=1}^{N}\sum_{\alpha=1}^{N_b} c_{j\alpha} z_{ij,\alpha\beta}, \tag{3.8}$$

where

$$z_{ij,\alpha\beta} = B_j \int_{V_i'} dV_i' \int_{V_j'} dV_j'\ \overline{f}_{i\beta}(\overline{r}_i')\cdot\overline{\overline{G}}(\overline{r}_i',\overline{r}_j')\cdot\overline{f}_{j\alpha}(\overline{r}_j'), \tag{3.9}$$

with

$$B_j = \frac{k^2}{\epsilon}(\epsilon(\overline{r}_j')-\epsilon). \tag{3.10}$$

The first term on the right hand side of Eq. (3.7) is determined by the incident field and is known for a given configuration while the other two terms describe the interaction matrix $\overline{\overline{Z}}$ with $c_{j\alpha}$ forming the unknown excitation vector $\overline{x}$. Suppose that the self term (that is, $i=j$, $\alpha=\beta$ term) in Eq. (3.8) is [16]

$$z_{ij,\alpha\beta}|_{i=j,\alpha=\beta} = C_{i\beta}, \tag{3.11}$$

then Eq. (3.8) can be recast into the matrix equation

$$\overline{\overline{Z}}\cdot\overline{x} = \overline{b}, \tag{3.12}$$

where

$$\overline{b} = \begin{bmatrix} \begin{bmatrix} \int_{V_1'} dV_1' \; \overline{f}_{11}(\overline{r}_1') \cdot \overline{E}_{inc}(\overline{r}_1') \\ \int_{V_1'} dV_1' \; \overline{f}_{12}(\overline{r}_1') \cdot \overline{E}_{inc}(\overline{r}_1') \\ \vdots \\ \int_{V_1'} dV_1' \; \overline{f}_{1N_b}(\overline{r}_1') \cdot \overline{E}_{inc}(\overline{r}_1') \end{bmatrix} \\ \vdots \\ \begin{bmatrix} \int_{V_N'} dV_N' \; \overline{f}_{N1}(\overline{r}_N') \cdot \overline{E}_{inc}(\overline{r}_N') \\ \int_{V_N'} dV_N' \; \overline{f}_{N2}(\overline{r}_N') \cdot \overline{E}_{inc}(\overline{r}_N') \\ \vdots \\ \int_{V_N'} dV_N' \; \overline{f}_{NN_b}(\overline{r}_N') \cdot \overline{E}_{inc}(\overline{r}_N') \end{bmatrix} \end{bmatrix}. \tag{3.13}$$

Similarly

$$\overline{x} = \left[\left[c_{11}, c_{12}, \ldots, c_{1N_b} \right], \ldots, \left[c_{N1}, c_{N2}, \ldots, c_{NN_b} \right] \right]^T, \tag{3.14}$$

and the $N_b \times N_b$ block of $\overline{\overline{Z}}$ which describes the interaction between particles i and j is given by

$$\overline{\overline{Z}}_{ij} = \begin{cases} -\begin{bmatrix} z_{ij,11} & z_{ij,12} & \cdots & z_{ij,1N_b} \\ z_{ij,21} & z_{ij,22} & & \\ \vdots & & \ddots & \vdots \\ z_{ij,N_b1} & & \cdots & z_{ij,N_bN_b} \end{bmatrix} & i \neq j, \\ \begin{bmatrix} (1-C_{i1}) & 0 & \cdots & 0 \\ 0 & (1-C_{i2}) & & \\ \vdots & & \ddots & \vdots \\ 0 & & \cdots & (1-C_{iN_b}) \end{bmatrix} & i = j, \end{cases} \tag{3.15}$$

where $\overline{\overline{Z}}$ is then understood to be

$$\overline{\overline{Z}} = \begin{bmatrix} \overline{\overline{Z}}_{11} & \overline{\overline{Z}}_{12} & \cdots \\ \vdots & \ddots & \vdots \\ \overline{\overline{Z}}_{N_b1} & \cdots & \overline{\overline{Z}}_{N_bN_b} \end{bmatrix}. \tag{3.16}$$

The size of $\overline{\overline{Z}}$ is $N_bN \times N_bN$, and the lengths of vectors $\overline{x}$ and $\overline{b}$ are N_bN. The calculation

of the self terms $C_{i\beta}$ are based on the electrostatic solution of spheroids and are described in detail in [16] Appendix A therein.

In this work, we consider identical prolate spheroidal particles at arbitrary locations and with arbitrary orientations. Assuming electrically small particles, we choose the basis functions to be the electrostatic solutions to the Laplace equation for a prolate spheroid. The interaction equation, Eq. (3.12), was derived above to show the generality of the method: for particles of different shapes, one need only choose appropriate basis functions (as many as needed) in order to calculate scattering from the collection. Also note that each individual particle need not be identical for the method described in Section 3.3 to be valid. Each particle may differ in shape, size, permittivity, *etc.* as long as the particle remains small enough compared to the wavelength (size parameter $ka \ll 1$) so that its response to $\overline{E}_{inc}(\overline{r})$ may be accurately modeled by a point response. For reference, the first three basis functions for the spheroid corresponding to the dipole solutions of the Laplace equation are given as

$$\overline{f}_{j1} = \hat{z}_{bj}\frac{1}{\sqrt{v_j}}, \tag{3.17a}$$

$$\overline{f}_{j2} = \hat{x}_{bj}\frac{1}{\sqrt{v_j}}, \tag{3.17b}$$

$$\overline{f}_{j3} = \hat{y}_{bj}\frac{1}{\sqrt{v_j}}. \tag{3.17c}$$

Here, $\hat{x}_{bj}$, $\hat{y}_{bj}$, and $\hat{z}_{bj}$ are the principal axes of spheroid j. For the expressions of the higher order basis functions and further details concerning this MoM solution for the case of small prolate spheroids, the reader is referred to [16]. An example of one realization of a random medium filled with 30 spheroids with elongation $e = 1.8$, and fractional volume $f_v = 0.3$, contained a unit cube test volume is illustrated in Fig. 3-1.

3.3 3-D SMCG Method

Consider a medium filled with many discrete scatterers (*e.g.* spheroids, as in Fig. 3-1) at random positions and with random orientations contained in a cubic test volume. The

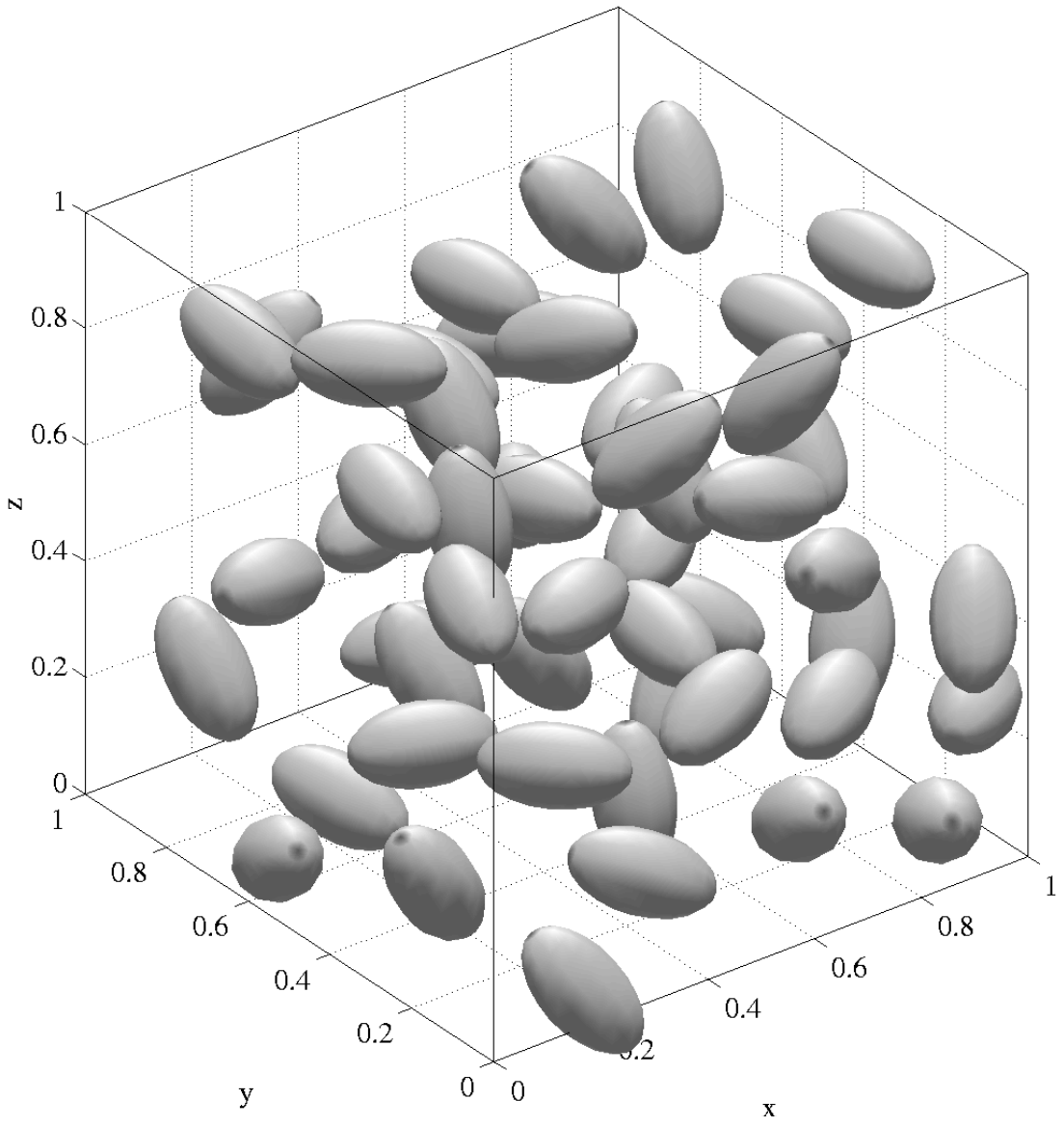

Figure 3-1: Densely packed medium consisting of 50 dielectric spheroids. Elongation=1.8, f_v=0.1.

dielectric spheroids have a statistical distribution realized by a shuffling process described in [16]. After a suitable distribution of N_{cube} particles is generated, the collection may be truncated to any shape. In our case we will truncate to a spherical test volume consisting of N_{sphere} particles.

3.3.1 SMCG Model Parameters

The Sparse Matrix/Canonical Grid (SMCG) method achieves reduced complexity and reduced memory requirements by assuming that scatterers are small enough so that accuracy is maintained though only point interactions are considered, and by decomposing the interaction matrix $\overline{\overline{Z}}$ into two separate matrices: the "strong" and "weak" interaction matrices. This decomposition is governed by a neighborhood distance, r_d, beyond which radius interactions are considered weak. The choice of the neighborhood distance is discussed in more detail in Section 3.3.2.

If the particles are small compared to the wavelength of the incident radiation, the induced electric field inside the particle can be considered constant and may therefore be accurately modeled by a dipole response located at the center of the particle. In this case, the integrals in Eqs. (3.7) and (3.13) may be replaced by the volume of that particle as:

$$\int_{V_i'} dV_i' \;\rightsquigarrow v_i = \frac{4}{3}\pi a_i^2 c_i, \tag{3.18}$$

where a_j is the length of the semiminor axis and $c_j = a_j e_j$ with e_j being the elongation or aspect ratio. The assumptions stated above allow the separation of $\overline{\overline{Z}}$ into three distinct matrices: 1) a pre-multiplying basis function matrix $\overline{\overline{f}}_i$ dependent on particle i, 2) the Green's function kernel $\overline{\overline{G}}(\overline{r}_i', \overline{r}_j')$ (which now only depends on the distance separating the centers of particles i and j, R_{ij} [see Eq. (3.23)]), and 3) a post-multiplying basis function matrix $\overline{\overline{f}}_j$ dependent on j. In rectangular coordinates this becomes

$$\overline{\overline{Z}}_{ij} = \overline{\overline{f}}_i \cdot \overline{\overline{G}}(\overline{r}_i', \overline{r}_j') \cdot \overline{\overline{f}}_j \tag{3.19}$$

where

$$\overline{\overline{f}}_i = \begin{bmatrix} f_{i1,x} & f_{i1,y} & f_{i1,z} \\ \vdots & \vdots & \vdots \\ f_{iN_b,x} & f_{iN_b,y} & f_{iN_b,z} \end{bmatrix} \tag{3.20}$$

and

$$\overline{\overline{G}}(R_{ij}) = \begin{bmatrix} G_{xx}(R_{ij})\, G_{xy}(R_{ij})\, G_{xz}(R_{ij}) \\ G_{yx}(R_{ij})\, G_{yy}(R_{ij})\, G_{yz}(R_{ij}) \\ G_{zx}(R_{ij})\, G_{zy}(R_{ij})\, G_{zz}(R_{ij}) \end{bmatrix}. \tag{3.21}$$

Note that the pre-multiplying $\overline{\overline{f}}_j$ is the transpose of the post-multiplying $\overline{\overline{f}}_i$ if i=j. If ξ denotes x, y, or z, then the terms of Eq. (3.21) are

$$G_{\xi_1\xi_2}(R_{ij}) = g(R_{ij})\delta_{\xi_1\xi_2} + \frac{1}{k^2}\frac{\partial^2}{\partial\xi_1\partial\xi_2} g(R_{ij}), \tag{3.22}$$

where $g(R_{ij})$ is the scalar Green's function [Eq. (3.4)], again $\delta_{\xi_1\xi_2}$ is the Kronecker delta, and

$$R_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}. \tag{3.23}$$

If $\overline{\overline{f}}$ is constructed by inserting $\overline{\overline{f}}_i$ into the appropriate columns (determined by grid point index) and rows (determined by particle index), then Eq. (3.19) can be expanded to include all particle interactions as

$$\overline{\overline{Z}} = \overline{\overline{f}} \cdot \overline{\overline{G}} \cdot \overline{\overline{f}}^{\mathrm{T}}, \tag{3.24}$$

where $\overline{\overline{f}}^{\mathrm{T}}$ denotes the transpose of $\overline{\overline{f}}$. Note that $\overline{\overline{f}}$ is sparse, while $\overline{\overline{G}}$ is a dense, $(NN_b) \times (NN_b)$ matrix containing the exact interactions between every pair of particles.

The next step in the SMCG method involves approximating the weak interaction between particles i and j in the interaction matrix $\overline{\overline{Z}}$ by a multivariate Taylor series expansion of $\overline{\overline{G}}(R_{ij})$ about the grid points nearest to the particles i and j, respectively. Let this approximated matrix be $\overline{\overline{Z}}^{\mathrm{w}}$ defined according to the following. Let a cubic lattice be superimposed onto the test volume with $N_{g,x}$, $N_{g,y}$, and $N_{g,z}$ lattice points, or grid points, in the x, y,

and z dimensions, respectively. For all of the simulations considered in this work,

$$N_{g,x} = N_{g,y} = N_{g,z} = N_g \tag{3.25}$$

with

$$N_{g,i} = \frac{(i-1)+\frac{1}{2}}{N_g} \tag{3.26}$$

for grid points located in a unit cube. The spacing between grid points is then $\Delta r_g = 1/N_g$. All those interactions located further apart than the predetermined neighborhood distance are considered "weak" interactions. Thus, decomposing $\overline{\overline{Z}}$ through the relation

$$\overline{\overline{G}} = \overline{\overline{G}}(\mathcal{R}_s) + \overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w), \tag{3.27}$$

for all $i, j = 1 \ldots N$ where $\mathcal{R}_s$ denotes the set of distances between pairs of particles whose interactions are considered strong, and $\mathcal{R}_w$ is a set containing all other pair interactions (considered weak), Eq. (3.19) can be written as

$$\begin{aligned} \overline{\overline{Z}} &= \overline{\overline{Z}}^{\mathrm{s}} + \overline{\overline{Z}}^{\mathrm{w}} \\ &= \underbrace{\overline{\overline{f}} \cdot \overline{\overline{G}}(\mathcal{R}_s) \cdot \overline{\overline{f}}^{\mathrm{T}}}_{\overline{\overline{Z}}^{\mathrm{s}}} + \underbrace{\overline{\overline{f}} \cdot \overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w) \cdot \overline{\overline{f}}^{\mathrm{T}}}_{\overline{\overline{Z}}^{\mathrm{w}}} \end{aligned} \tag{3.28}$$

where the superscript "cg" denotes an approximation based on the distance between the particle and its associated canonical grid point located at $\overline{r}_g$. $\mathcal{R}_s$ and $\mathcal{R}_w$ are mutually exclusive sets with $\mathcal{R}_s \bigcup \mathcal{R}_w = R$. $\overline{\overline{G}}(\mathcal{R}_s)$ contains the exact Green's function between particles only in close proximity (closer than r_d) while all other entries are set to 0. $\overline{\overline{G}}(\mathcal{R}_s)$ is the same size as $\overline{\overline{G}}$ from Eq. (3.24), but is sparse. $\overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w)$ is also the same size as $\overline{\overline{G}}$ and contains an approximation of the remaining weak interactions. $\overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w)$ is actually a sum of as many matrices as is required for the expansion (see Table 3.2). The derivation of $\overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w)$ as well as fast methods for the subsequent matrix-vector multiply will be discussed below.

3.3.2 Parameter Considerations

A natural question at this point is: beyond what distance should interactions be considered "weak"? The distance at which the separation occurs is called the neighborhood distance (r_d) and is defined in terms of the number of grid points away from the grid point of interest. It is clear that the choice for r_d influences both the accuracy of the results using the SMCG method and the computation time required. On one hand, for small r_d (< 1 inter-grid point space Δr_g), approximation errors tend to accumulate and cause the iterative solver of Eq. (3.28) either to not converge or converge to incorrect values. On the other hand, a large r_d causes a greater number of interactions to be classified as "strong" and the SMCG method reverts to a full iterative approach. Reasonable r_d values from simulations are between 1 and 2 grid points.

The neighborhood distance must be defined in terms of grid points because for weak interactions, particles interact through the Green's function matrix $\overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w)$ which is defined only in terms of the number of grid points and their respective locations (see Section 3.3.3). Thus, all particles associated with grid points closer than r_d (measured with respect to the inter-grid point distance) will have strong interactions with the particle under consideration, while those particles associated with grid points which are located further away than r_d are classified as weak interactions. In this way, r_d defines the strong interaction cloud (or molecule) which is illustrated in Fig. 3-2 for three spheroids S_1, S_2, and S_3 with $N_g = 3$. Table 3.1 lists the number of grid points included in the strong interaction cloud (g_{r_d}) for the first few ranges of r_d.

Range of r_d (# of Δr_g)	g_{r_d}
$0<r_d\leq 1$	1
$1<r_d\leq\sqrt{2}$	7
$\sqrt{2}<r_d\leq 2$	19
$2<r_d\leq\sqrt{5}$	33
$\vdots$	

Table 3.1: Number of surrounding gridpoints, g_{r_d}, included in the strong interaction cloud as a function of neighborhood distance r_d.

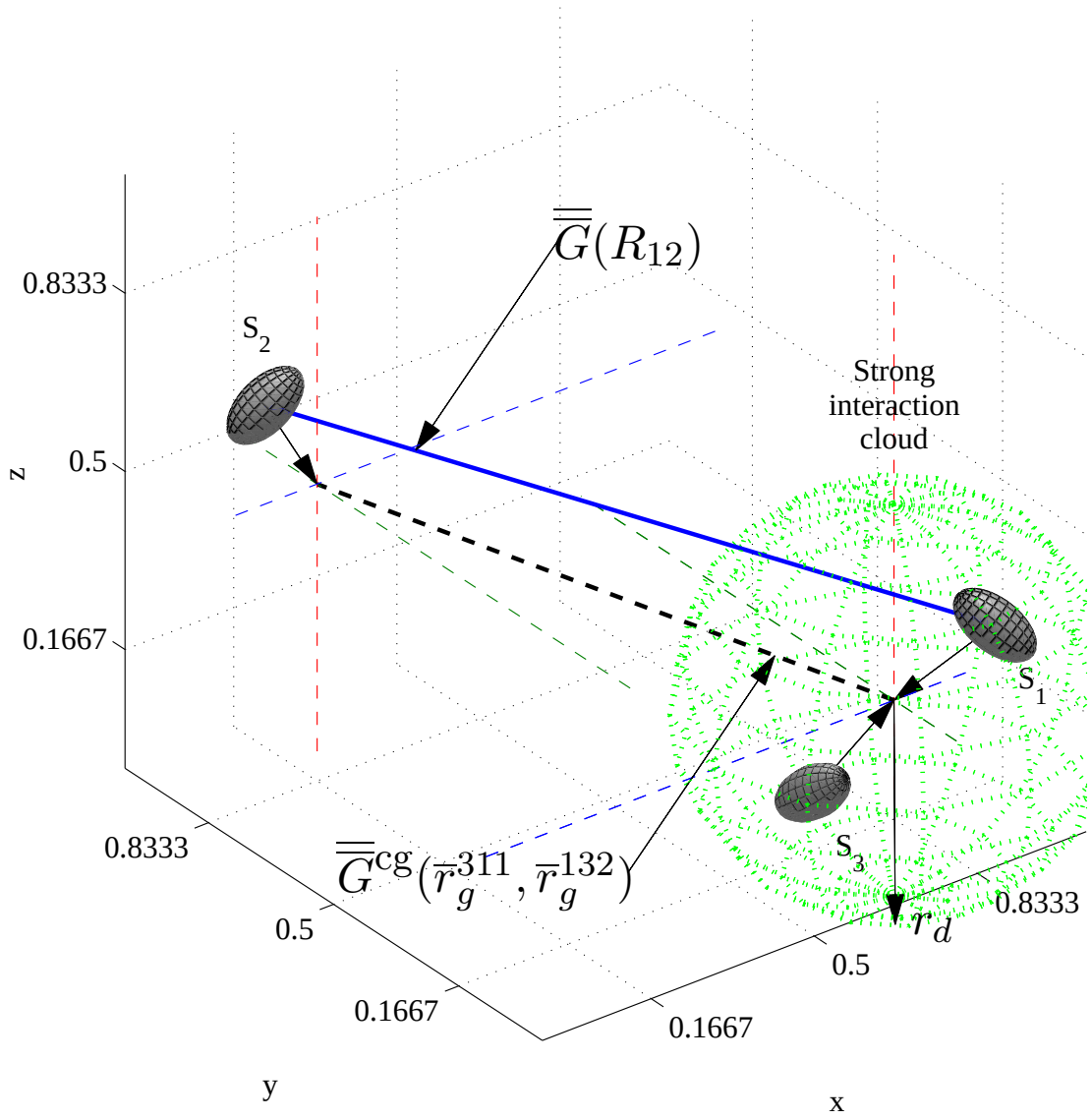

Figure 3-2: Strong interaction cloud with r_d=1.1. Spheroids S_1 and S_3 are closest to the same grid point and thus their interaction is included in $\overline{\overline{Z}}^{s}$. Spheroid S_2 is beyond r_d and thus its exact interactions with both S_1 ($\overline{\overline{G}}(R_{12})$ shown as a thick solid line between S_1 and S_2) and S_3 are approximated as $\overline{\overline{G}}^{cg}(\overline{r}_g^{311}, \overline{r}_g^{132})$ and included in $\overline{\overline{Z}}^{w}$.

One related question is, how fine or coarse of a grid is necessary or optimum for a given collection of scatterers? At the very least, there must be as many or fewer undulations (local extrema) in $\overline{\overline{G}}$ between gridpoints as the order of the Taylor series expansion (γ) for it to be possible for $\overline{\overline{G}}^{\mathrm{cg}}$ to approximate $\overline{\overline{G}}$ adequately. In terms of computation time, the number of multiplies M^s required during each iteration of the iterative solver for $\overline{\overline{Z}}^{\mathrm{s}} \cdot \overline{x}$ from Eq. (3.28) is $\mathcal{O}(N)$. M^s can be approximated as the average number of interactions with each particle from other particles associated with the same gridpoint, multiplied by the number of gridpoints in the strong interaction cloud, and multiplied again by the number of spheroids, *i.e.*

$$M^s = N \left(\frac{N}{N_g^3} \right) g_{r_d}. \tag{3.29}$$

Because $1/N_g^3 \ll 1$, M^s is $\mathcal{O}(N)$ unless N_g is small and/or g_{r_d} is large.

The number of multiplies M^w required during each iteration of the iterative solver for $\overline{\overline{Z}}^{\mathrm{w}} \cdot \overline{x}$ in Eq. (3.28) depends on the FFT length ($\approx 8N_g^3$ for $\overline{\overline{G}}^{\mathrm{cg}}(N_g)$, see Eq. (3.43)) and the number of expansion matrices (N_T, see Section 3.3.3 Table 3.2). Thus, if we estimate the number of multiplies required for an N-length FFT as $N \log_2 N$, M^w is

$$M^w = 8N_g^3 \left(\log_2(8N_g^3) \right) N_T. \tag{3.30}$$

Therefore, the total number of multiplies M for one iteration can be estimated by $M = M^w + M^s$. The N_g which minimizes this function can be found by solving the following transcendental equation for N_g given g_{r_d} and N_T,

$$\frac{-3N^2 g_{r_d}}{N_g^4} + 24 N_g^2 N_T \left(\log_2(8N_g^3) + \frac{1}{\ln 2} \right) = 0. \tag{3.31}$$

Equation (3.31) provides an estimate of the number of grid points which minimizes computation time as a function of the neighborhood distance and the expansion order.

The approximation error due to the SMCG method decreases if a larger r_d is chosen because more terms are being counted in the strong interaction cloud and computed exactly. This error will also decrease if N_g is chosen larger because a finer mesh implies that the Taylor series will estimate functions over smaller regions. In other words, the Taylor

approximation will have less error because it is fitted to a smoother function. The effect on computation time and accuracy of the expansion order, r_d, and N_g can be seen in the results listed in Table 3.3.

3.3.3 3-D Dyadic Green's Function Approximation

After the parameters of the 3-D SMCG method have been chosen, weak particle interactions are approximated by expanding the background dyadic Green's function about distance between the grid points nearest to each pair of particles. Let each particle located at $\overline{r}_i$ be associated with its nearest grid point $g^{l_i m_i n_i}$ located at $\overline{r}_g^{l_i m_i n_i}$ where l_i, m_i, and n_i are grid point indices in the x, y, and z dimensions respectively and are integers in the range

$$1 \leq \{l_i, m_i, n_i\} \leq N_g, \quad \forall \ i = 1, \ldots, N. \tag{3.32}$$

If particles i and j are associated with grid points $g^{l_i m_i n_i}$ and $g^{l_j m_j n_j}$ located at $\overline{r}_g^{l_i m_i n_i}$ and $\overline{r}_g^{l_j m_j n_j}$ respectively, $\overline{\overline{G}}(R_{ij})$, defined in rectangular coordinates by Eq. (3.21), can be approximated by a γ^{th} order Taylor series expansion $\overline{\overline{G}}^{\mathrm{cg}}$ as

$$\overline{\overline{G}}^{\mathrm{cg}}_{ij}(\mathcal{R}_w) = \sum_{\gamma=0}^{\infty} \left(\sum_{\xi=x,y,z} \Delta\xi_{ij} \frac{\partial}{\partial \xi} \right)^{\gamma} \frac{\overline{\overline{G}}(\overline{r}_g^{l_i m_i n_i}, \overline{r}_g^{l_j m_j n_j})}{\gamma!}, \tag{3.33}$$

or via the trinomial theorem as

$$\begin{aligned} \overline{\overline{G}}^{\mathrm{cg}}_{ij}(\mathcal{R}_w) = & \sum_{\gamma=0}^{\infty} \sum_{\gamma_x+\gamma_y+\gamma_z=\gamma} \left(\frac{1}{\gamma_x!\gamma_y!\gamma_z!} \right) \times \\ & \left\{ \prod_{\xi=x,y,z} (\Delta\xi_{ij})^{\gamma_\xi} \right\} \frac{\partial^{\gamma_x}}{\partial x^{\gamma_x}} \frac{\partial^{\gamma_y}}{\partial y^{\gamma_y}} \frac{\partial^{\gamma_z}}{\partial z^{\gamma_z}} \overline{\overline{G}}(\overline{r}_g^{l_i m_i n_i}, \overline{r}_g^{l_j m_j n_j}) \end{aligned} \tag{3.34}$$

where

$$\begin{aligned} \Delta\xi_{ij} &= \Delta\xi_i - \Delta\xi_j \\ &= (\overline{r}_{i,\xi} - \overline{r}_{g,\xi}^{l_i m_i n_i}) - (\overline{r}_{j,\xi} - \overline{r}_{g,\xi}^{l_j m_j n_j}), \end{aligned} \tag{3.35}$$

and again ξ is a placeholder for x, y, or z. $\overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w)$ from Eq. (3.28) is realized by Eq. (3.33) over all values of i and j and depends only on grid point locations $\overline{r}_g$ and the *relative* distance of each particle from its associated grid point.

Note that the relative distances $\Delta\xi_i$ in Eq. (3.33), are the only quantities which are particle specific and that they are scalars. Accordingly, we move these particle dependent quantities into $\overline{\overline{f}}$ by multiplying $\overline{\overline{f}}_i$ by $\Delta\xi_i$ over $\overline{\overline{f}}$ [*c.f.* Eqs. (3.19)–(3.24)], *i.e.*

$$\overline{\overline{f}}_{\gamma_x\gamma_y\gamma_z} \equiv \left(\prod_{\xi=x,y,z} (\Delta\xi_i)^{\gamma_\xi} \right) \overline{\overline{f}}_i \,, \quad \forall \;\; i = 1..N. \tag{3.36}$$

The remaining terms in Eq. (3.34) are dependent only upon the grid points, we define

$$\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z} \equiv \left(\frac{1}{\gamma_x!\gamma_y!\gamma_z!} \right) \frac{\partial^{\gamma_x}}{\partial x^{\gamma_x}} \frac{\partial^{\gamma_y}}{\partial y^{\gamma_y}} \frac{\partial^{\gamma_z}}{\partial z^{\gamma_z}} \overline{\overline{G}}(\overline{r}^{\Delta l \Delta m \Delta n}) \tag{3.37}$$

where $\overline{\overline{G}}(\overline{r}^{\Delta l \Delta m \Delta n})$ is an $3N \times 3N$ matrix defined by Eq. (3.21) with R_{ij} defined by Eq. (3.23) being the distance between grid points located at $\overline{r}_g^{l_i m_i n_i}$ and $\overline{r}_g^{l_j m_j n_j}$. Because of the translational invariance of the dyadic Green's function, the entries of $\overline{\overline{G}}(\overline{r}^{\Delta l \Delta m \Delta n})$ depend only on the differential number of grid points Δl, Δm, and Δn if the ordering of l, m, and n are sequential along the axes.

The i subscript in Eq. (3.36) corresponds to a pre-multiplication of $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ and can be thought of as a "source" particle from which radiation is being emitted, while the similar post-multiplication matrix corresponding to the j subscript would be $\overline{\overline{f}}^{\mathrm{T}}_{\gamma_x\gamma_y\gamma_z}$ and is analogous to a "sink" particle with an induced electric field caused (in part) by particle i.

Using the definitions above, we can now expand the weak interaction matrix $\overline{\overline{Z}}^{\mathrm{w}}$ as

$$\begin{aligned} \overline{\overline{Z}}^{\mathrm{w}} &= \overline{\overline{f}} \cdot \overline{\overline{G}}^{\mathrm{cg}}(\mathcal{R}_w) \cdot \overline{\overline{f}}^{\mathrm{T}} \\ &= \sum_{\gamma=0}^{\infty} \sum_{\gamma_x+\gamma_y+\gamma_z=\gamma} \overline{\overline{Z}}^{\mathrm{w}}_{\gamma_x\gamma_y\gamma_z} \end{aligned} \tag{3.38}$$

where

$$\overline{\overline{Z}}^{\mathrm{w}}_{\gamma_x\gamma_y\gamma_z} = \sum_{\begin{bmatrix}\gamma_{x_1}+\gamma_{x_2}=\gamma_x\\ \gamma_{y_1}+\gamma_{y_2}=\gamma_y\\ \gamma_{z_1}+\gamma_{z_2}=\gamma_z\end{bmatrix}} \sum \sum \Lambda\, \overline{\overline{f}}_{\gamma_{x_1}\gamma_{y_1}\gamma_{z_1}} \overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z} \overline{\overline{f}}^{\mathrm{T}}_{\gamma_{x_2}\gamma_{y_2}\gamma_{z_2}} \tag{3.39}$$

and where

$$\Lambda = (-1)^{\gamma_{x_2}+\gamma_{y_2}+\gamma_{z_2}} \binom{\gamma_x}{\gamma_{x_2}} \binom{\gamma_y}{\gamma_{y_2}} \binom{\gamma_z}{\gamma_{z_2}}. \tag{3.40}$$

The γ_{ξ_1} and γ_{ξ_2} in Eq. (3.39) are integers which take on all combinations in the interval $[0, \gamma_\xi]$ which satisfy $\gamma_{\xi_1} + \gamma_{\xi_2} = \gamma_\xi$. For example, in the second order (γ=2) Taylor series expansion, $\overline{\overline{Z}}^{\mathrm{w}}$ would be approximated by 6 terms as

$$\overline{\overline{Z}}^{\mathrm{w}} \approx \overline{\overline{Z}}^{\mathrm{w}}_{200} + \overline{\overline{Z}}^{\mathrm{w}}_{110} + \overline{\overline{Z}}^{\mathrm{w}}_{101} + \overline{\overline{Z}}^{\mathrm{w}}_{020} + \overline{\overline{Z}}^{\mathrm{w}}_{011} + \overline{\overline{Z}}^{\mathrm{w}}_{002}, \tag{3.41}$$

and, for example, the γ_x=2, γ_y=0, γ_z=0 ($\overline{\overline{Z}}^{\mathrm{w}}_{200}$) term would be [from Eq. (3.39)]

$$\overline{\overline{Z}}^{\mathrm{w}}_{200} = \overline{\overline{f}}_{200} \overline{\overline{G}}_{200} \overline{\overline{f}}^{\mathrm{T}}_{000} - 2\overline{\overline{f}}_{100} \overline{\overline{G}}_{200} \overline{\overline{f}}^{\mathrm{T}}_{100} + \overline{\overline{f}}_{000} \overline{\overline{G}}_{200} \overline{\overline{f}}^{\mathrm{T}}_{200} \tag{3.42}$$

with $\overline{\overline{f}}_{000} = \overline{\overline{f}}$ from Eq. (3.36). Lastly, if $(\epsilon(\overline{r}'_j) - \epsilon)$ is particle dependent, $\frac{k^2}{\epsilon}(\epsilon(\overline{r}'_j) - \epsilon)$ from Eq. (3.10) should also be multiplied into either the pre- or post- $\overline{\overline{f}}$ multiplication in a fashion similar to that of Eq. (3.36).

The reduction in computational complexity and memory requirements is realized by exploiting the structure inherent to $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$. As mentioned earlier, the Green's function matrix is highly redundant if the grid is ordered sequentially. In fact, $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ can be classified as a multilevel block Toeplitz (MBT) matrix with $M = 3$ levels, of size $N_{g,x}$, $N_{g,x}$, $N_{g,x}$ respectively, with the final level being a dense 3×3 block (Appendix A). Matrix-vector multiplies of MBT matrices can be achieved in $\mathcal{O}(N_{FFT} \log N_{FFT})$ operations, instead of $\mathcal{O}(N^2)$, where N_{FFT} is the FFT length defined as

$$N_{FFT} = 9 \left(\prod_{\xi=x,y,z} (2N_{g,\xi} - 1) \right). \tag{3.43}$$

For each unique vector, the matrix-vector multiply involving an MBT matrix requires another FFT with length given by Eq. (3.43). In terms of Eq. (3.39), because $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ remains unchanged for the same N_g and background wavenumber k, the FFT of the expansion matrices $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ may be performed once and stored. Then for each step in the iterative solution, one FFT must be performed for each unique combination of $\overline{\overline{f}}^{\mathrm{T}}_{\gamma_{x_2}\gamma_{y_2}\gamma_{z_2}}$ and $\overline{x}$, and likewise an inverse FFT must be calculated for each unique $\overline{\overline{f}}_{\gamma_{x_1}\gamma_{y_1}\gamma_{z_1}}$. Table 3.2 lists the total number of transforms performed for a single $\overline{\overline{Z}}^{\mathrm{w}} \cdot \overline{x}$ multiply using the canonical grid expansion (N_T).

Expansion order (γ)	Number of expansion terms ($\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$)	N_T
0	1	2
1	3	8
2	6	20
3	10	40
4	15	70
5	21	112

Table 3.2: Number of expansion terms and total number of FFTs (both forward and inverse) for expansion order γ.

Utilizing this 3-D SMCG method, the total complexity of solving Eq. (3.12) is reduced from $\mathcal{O}(KN^2)$ to $\mathcal{O}(KN_TN\log N)$ where K is the number of steps required for the iterative solver to converge. In Section 3.4, the 3-D SMCG method is shown to indeed realize results which require much less computation time while maintaining a desired tolerance.

3.4 Results

In this section, we apply the 3-D SMCG method to finding the scattering from a large collection of randomly distributed and oriented spheroids contained in a spherical test volume. Each of the N_{cube}=500–15000 spheroids considered are identical with respect to size, shape, and permittivity ($\epsilon(\overline{r}'_j)$). The size parameter ka for each spheroid is 0.2. Likewise, the fractional volume of the spheroids in the test volume is 20%. Only the dipole basis functions of Eq. (3.17) are employed, i.e., $N_b = 3$. Thus, there are a maximum of 45000 unknown coefficients $c_{j\alpha}$ to solve for. For a given N_{cube}, the same randomly generated configuration

was used for all combinations of γ, r_d, and N_g. All results were computed on a Compaq Alphaserver DS20E with 4GB of RAM.

Four parameters dictate the resulting accuracy and computation time of the SMCG method as compared to the full MoM method. While the computation time of the full method depends only on the number of spheroids in the test volume N_{sphere}, the SMCG method additionally depends on the neighborhood distance r_d, the Taylor series expansion order γ, and the number of grid points N_g in each dimension. The relationship between expansion order and computation time and accuracy is illustrated in Fig. 3-3 where expansion orders from γ=0 to γ=5 were considered. Note that γ=0 corresponds physically to approximating all of the spheroids as actually being located at their respective (nearest) grid points.

As a measure of the "distance" of the approximated scattering results from those given by the full method, the Kullback-Leibler distance [46] is used, *i.e.*,

$$p(\sigma_T||\sigma) = \sum_{n=1}^{N_\theta} \left| \sigma_T(n) \log \frac{\sigma_T(n)}{\sigma(n)} \right| / N_\theta \tag{3.44}$$

where σ is the radar cross section (RCS) resulting from the full solution and σ_T is the approximated RCS calculated using the SMCG method. The RCSs are computed from the dipole excitation strengths [16] at N_θ=200 distinct angles. An example can be seen in Fig. 3-6. As expected, for increasing expansion order and grid size, computation time increases dramatically until it approaches the computation time of the full solution. However, for γ=2, most grid sizes maintain a small Kullback-Leibler distance, while realizing an substantial reduction in computation time.

The effect of varying the neighborhood distance r_d is shown in Fig. 3-4. As the neighborhood distance increases from $0.1\Delta r_g$ to $2.1\Delta r_g$, the marginal speed of the SMCG method approaches the marginal speed of the full method (*i.e.* the slope of the curves are similar) due to many strong interactions being included in $\overline{\overline{Z}}^{\mathrm{s}}$ instead of being approximated in $\overline{\overline{Z}}^{\mathrm{w}}$. As a general trend, the Kullback-Leibler distance in Fig. 3-4 is smaller for larger r_d for the same reason.

Figure 3-5 shows computation times and Kullback-Leibler distances for the same set of

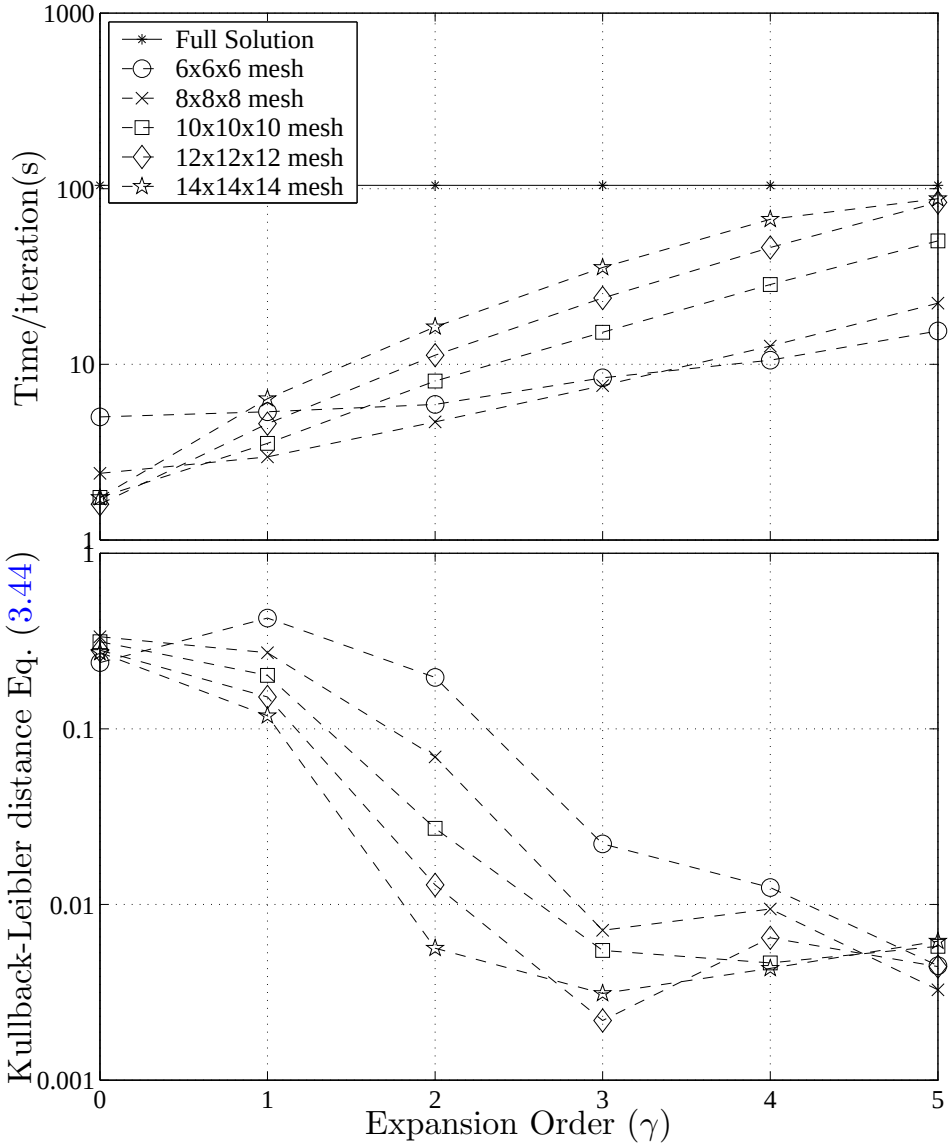

Figure 3-3: Computation times and Kullback-Leibler distances for SMCG method for γ=0-5 with r_d=1.1Δr_g and N_{cube}=10000.

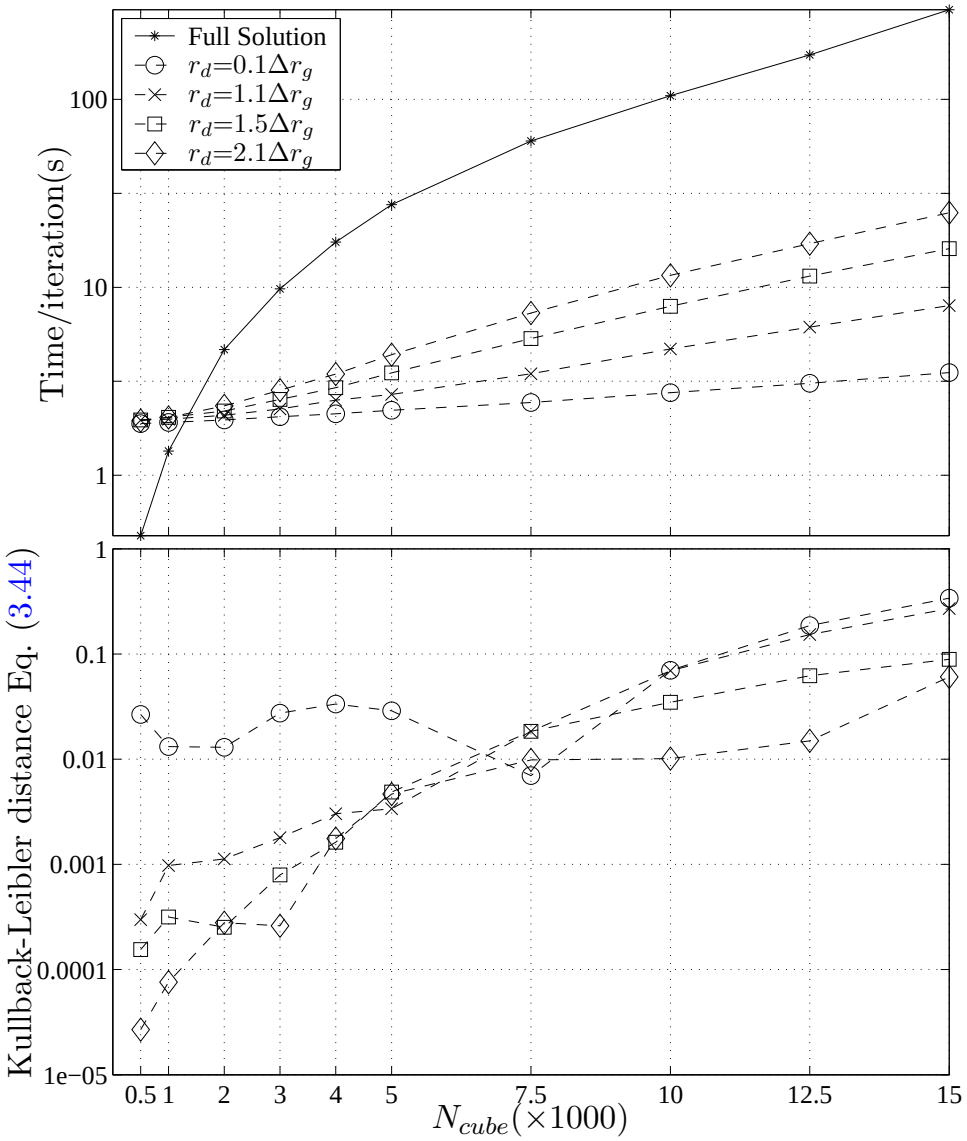

Figure 3-4: Computation times and Kullback-Leibler distances for SMCG method for r_d=0.1–2.1Δr_g with γ=2 and N_g=8.

data as Fig. 3-4 except that r_d is kept at r_d=1.1Δr_g, while the number of grid points is varied from $N_g = 6$ to $N_g = 14$. Note that for finer meshes, the FFT computation time dominates the overall computation time, which results in very flat curves with respect to N_{cube}. This also illustrates the tradeoff between grid size and the neighborhood distance discussed in more detail in Section 3.3.2. The final figure, Fig. 3-6, shows the convergence of σ_T to σ for successively accurate model parameters, and Table 3.3 records pertinent computation times and Kullback-Leibler distances for a number of cases.

Memory requirements can be estimated by the number of expansion matrices $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ stored (times 2 including $\overline{x}$) multiplied by the length of each of those matrices given by Eq. (3.43). The memory required for the SMCG method is thus seen to be $\mathcal{O}(N)$ as compared to $\mathcal{O}(N^2)$ for the full method. As a final note, this SMCG algorithm is easily adapted to other types of particles such as spheres, crystals, dust grains, etc., for which the basis functions are known. This method may be used in a way which imitates the Discrete Dipole Approximation (DDA) (Appendix A and [47, 48]), and the author has used this method to find the effective permittivity of collections of randomly distributed spheroids (Chapter 2).

3.5 Conclusions

The sparse matrix canonical grid (SMCG) method has been extended to 3-D and illustrated by finding the scattering from random media filled with dielectric spheroids at random positions and orientations. The 3-D SMCG method achieves $\mathcal{O}(N \log N)$ complexity instead of $\mathcal{O}(N^2)$ for the matrix-vector multiply when using an iterative solver by decomposing the interaction matrix $\overline{\overline{Z}}$, generated by an MoM solution to the many-body volume integral equation, into strong and weak interaction matrices $\overline{\overline{Z}}^{\mathrm{s}}$ and $\overline{\overline{Z}}^{\mathrm{w}}$, respectively. The matrix $\overline{\overline{Z}}^{\mathrm{s}}$ contains only those interactions which are between particles closer than the neighborhood distance r_d. Therefore, $\overline{\overline{Z}}^{\mathrm{s}}$ is very sparse and the matrix-vector multiply $\overline{\overline{Z}}^{\mathrm{s}} \cdot \overline{x}$ is accomplished in $\mathcal{O}(N)$ complexity. The matrix $\overline{\overline{Z}}^{\mathrm{w}}$ is formed by expanding the dyadic Green's function between each pair of particles whose associated grid points are located further apart than r_d in a γ^{th} order multivariate Taylor series expansion around a canonical grid superim-

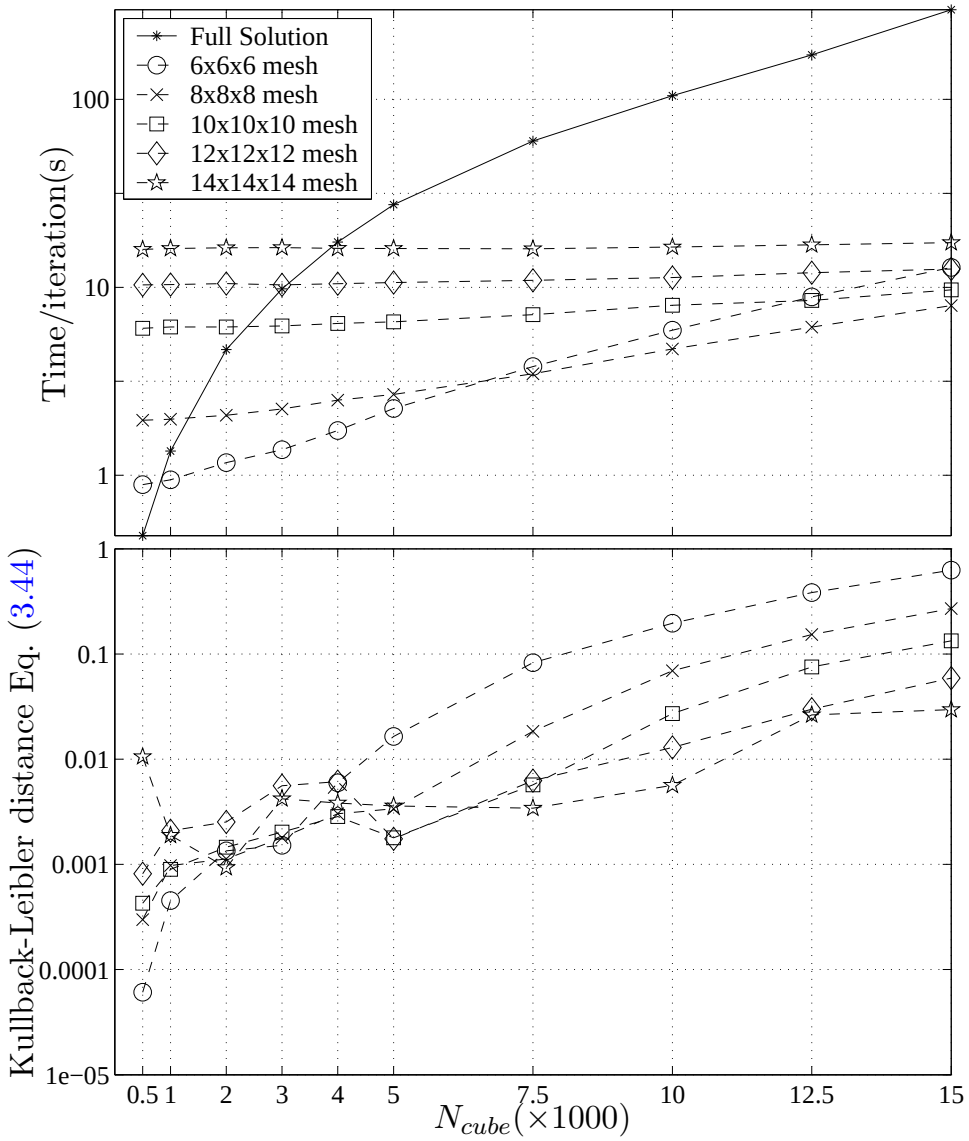

Figure 3-5: Computation times and Kullback-Leibler distances for SMCG method for N_g=6–14 with γ=2 and r_d=1.1Δr_g.

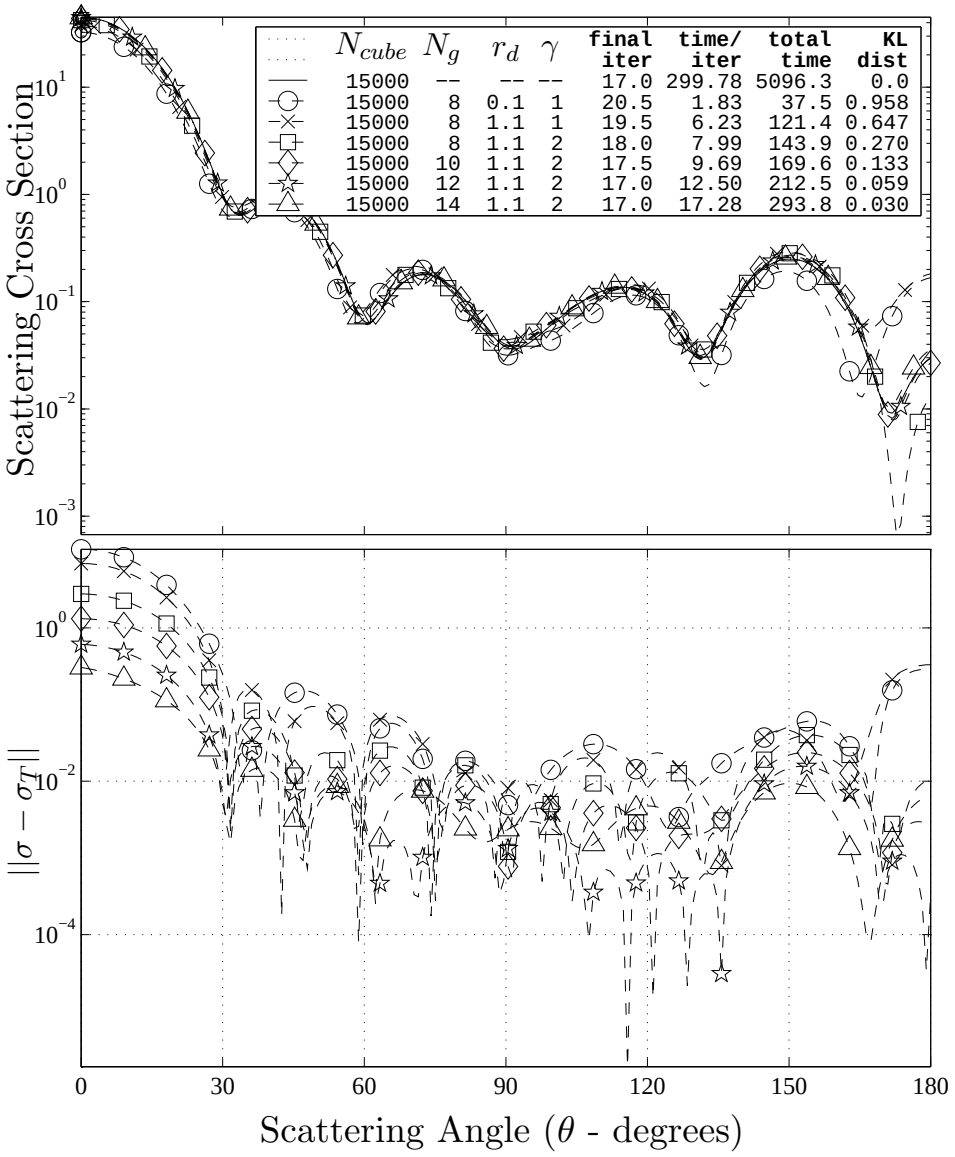

Figure 3-6: Radar Cross Section (σ) for increasing accurate approximations.

		N_g	N_{FFT}	total iters	time/ iter	Eq. (3.44)	speed up
Neighborhood dist. $r_d = 0.1\Delta r_g$	γ=1	6	11979	28.0	2.4	2.485	124.45
		8	30375	20.5	1.8	0.958	163.88
		10	61731	19.5	2.9	0.708	103.04
		12	109503	18.0	4.3	0.507	70.40
		14	177147	17.5	6.2	0.380	48.38
	γ=2	6	11979	28.0	3.4	0.800	87.12
		8	30375	20.5	3.5	0.339	85.24
		10	61731	18.5	7.1	0.139	41.99
		12	109503	18.0	11.0	0.044	27.31
		14	177147	18.0	16.4	0.046	18.31
	γ=3	6	11979	20.0	7.3	0.080	41.15
		8	30375	18.0	6.8	0.085	44.09
		10	61731	19.5	15.1	0.116	19.83
		12	109503	20.5	23.8	0.141	12.59
		14	177147	25.0	35.1	0.150	8.53
Neighborhood dist. $r_d = 1.1\Delta r_g$	γ=1	6	11979	24.0	11.8	1.926	25.49
		8	30375	19.5	6.2	0.647	48.15
		10	61731	18.5	5.4	0.519	55.96
		12	109503	18.0	5.7	0.390	52.42
		14	177147	18.0	7.2	0.308	41.37
	γ=2	6	11979	20.5	12.8	0.628	23.50
		8	30375	18.0	8.0	0.270	37.51
		10	61731	17.5	9.7	0.133	30.92
		12	109503	17.0	12.5	0.059	23.98
		14	177147	17.0	17.3	0.030	17.35
	γ=3	6	11979	17.5	21.1	0.122	14.19
		8	30375	17.0	11.5	0.022	25.97
		10	61731	16.0	17.4	0.018	17.25
		12	109503	16.0	26.5	0.015	11.31
		14	177147	15.0	36.4	0.009	8.23
Neighborhood dist. $r_d = 1.5\Delta r_g$	γ=1	6	11979	19.5	29.4	1.177	10.21
		8	30375	20.5	14.9	0.467	20.14
		10	61731	18.5	9.8	0.372	30.44
		12	109503	17.5	8.5	0.296	35.40
		14	177147	17.5	9.0	0.247	33.33
	γ=2	6	11979	20.0	29.7	0.289	10.09
		8	30375	17.5	16.0	0.089	18.68
		10	61731	16.0	14.3	0.085	20.90
		12	109503	16.0	15.3	0.055	19.58
		14	177147	17.0	19.2	0.033	15.60
	γ=3	6	11979	17.5	31.5	0.155	9.52
		8	30375	16.0	20.1	0.018	14.94
		10	61731	16.0	21.9	0.016	13.68
		12	109503	15.0	28.1	0.009	10.65
		14	177147	16.0	38.6	0.007	7.76

Table 3.3: Computation times and relative speedup for the SMCG method compared to the full method. For all cases N_{cube}=15000

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posed onto the test volume. The prolate spheroidal particles considered in this work were electrically small so that their response to electromagnetic excitation could be adequately approximated with a point response. Therefore, the Green's function interaction expansion

matrices $\overline{\overline{G}}_{\gamma_x\gamma_y\gamma_z}$ exhibit a multilevel block Toeplitz (MBT) structure. MBT matrices can be multiplied with an arbitrary vector using the only one forward and one inverse FFT. Thus $\overline{\overline{Z}}^{\mathrm{w}} \cdot \overline{x}$ can be accomplished in $\mathcal{O}(N_T N \log N)$ complexity where N_T is the number of FFTs and depends on the expansion order γ. The 3-D SMCG method was demonstrated to indeed realize the predicted reduction in complexity and memory requirements through large-scale examples including up to 15000 discrete scatterers.

Chapter 4

On the Asymptotic Expansion of the Spheroidal Wave Function and its Eigenvalues for Complex Size Parameter

4.1 Introduction

Spheroidal wave functions (SWFs) have become increasingly important in the solution of a variety of different physical problems due to the flexibility afforded by the spheroid's nonspherical yet canonical geometry. The use of SWFs can be found in such diverse areas such as light scattering [49, 50, 51], nuclear modeling [52], signal processing and communication theory [53, 54, 55, 56], electromagnetic modeling [57, 58, 59, 60, 61], and in finding the electromagnetic induction (EMI) response of canonical objects at magnetoquasistatic frequencies [62,63]. Three classic works on spheroidal wave functions are those by Stratton [64], Meixner [65], and Flammer [66], with tables of function values provided in [64, 66, 67].

Separation of variables of the wave equation in spheroidal coordinates results in a differential equation that is satisfied by both the angular and radial spheroidal wave functions

$$\frac{d}{d\zeta}\left[(1-\zeta^2)\frac{d\chi_{mn}}{d\zeta}\right]+\left[\lambda_{mn}-c^2\zeta^2-\frac{m^2}{1-\zeta^2}\right]\chi_{mn}=0, \tag{4.1}$$

where c is the size parameter, m and n are integers with $m \geq 0$, $n \geq m$, and λ_{mn} are the separation, or characteristic, eigenvalues at which values the solution to Eq. (4.1) is bounded. Table 4.1 classifies the solutions of Eq. (4.1), χ_{mn}, into four cases according to the ranges of ζ and c (assumed purely real).

$\chi_{mn}(c,\zeta)$	c	ζ	range	name
$S_{mn}(c,\eta)$	c	η	$\lvert\zeta\rvert<1$	angular prolate
$R^{(i)}_{mn}(c,\xi)$	c	ξ	$\lvert\zeta\rvert\geq 1$	radial prolate
$S_{mn}(-\mathrm{i}c,\eta)$	$-\mathrm{i}c$	η	$\lvert\zeta\rvert<1$	angular oblate
$R^{(i)}_{mn}(-\mathrm{i}c,\xi)$	$-\mathrm{i}c$	$\mathrm{i}\xi$	$\lvert\zeta\rvert\geq 1$	radial oblate

Table 4.1: Solutions to Eq. (4.1): angular and radial, prolate and oblate spheroidal wave functions.

Attempts to find rapid and accurate asymptotic expansions of the prolate angular and radial spheroidal wave functions (PASWF and PRSWF respectively) for large size parameter c (assumed real) have been ongoing. Flammer summarizes the work (up to 1957 [66]) and documents the asymptotic expansions of Eq. (4.1), for cases 1, 3, and 4 (see Table 4.1). The asymptotic expansion of Eq. (4.1), case 2, however, was unknown at the time of Flammer's work. Since that time, asymptotic approximations for this case have been developed by Slepian [68], for example, who built upon the asymptotic expansion of Eq. (4.1), case 1 found in Flammer (originally found by Meixner [65]) and extended the solution to five overlapping regions of validity. Miles [3, 69] provided compact, Bessel and Airy function expansions (used in part in Section 4.5) for the RSWFs in both the prolate and oblate cases. Streifer [70] (for the prolate case), and Cloizeaux and Mehta [71], also made strides in providing uniform asymptotic expansions for Eq. (4.1), and Sink and Eu [72] provided a WKB approximation for the angular SFW. More recently, asymptotic expansions of λ_{mn} for large c or n, $m = 0$ have been proposed in [73], expansions of Eq. (4.1) as they relate to the quantum shell model for both large n and large c have been proposed by [74, 75], and the work of Do-Nhat [76, 77] summarizes and provides more details of Flammer's expansions for Eq. (4.1) cases 1 and 3.

Evaluation of the SWFs for complex size parameter $c = c_r + c_i\mathrm{i}$, where $c_r = \Re\{c\}$ and $c_i = \Im\{c\}$, has received less attention for both the standard and asymptotic regimes. Applications utilizing complex c include light scattering from spheroidal particles [49, 78] and spheroidal antennas enveloped in a plasma medium [79]. Schäfke and Groh [80] were the first to investigate the branch cuts associated with the SWFs (emanating radially outward from the branch points, $c_{\circ;r}^{mn}$ in the complex c-plane) although their results for $c_{\circ;r}^{mn}$ were not reported until [81]. Hunter and Guerrieri [82] provided one method for estimating then numerically solving for $c_{\circ;r}^{mn}$ while the first few were earlier recorded by Oguchi [83]. Existing routines for the computation of λ_{mn} and $S_{mn}(c,\eta)$ for complex c can be found in [84, 85], but these Mathematica routines are too slow to be used for anything but reference. In addition, it appears that these codes do not properly determine the expansion type for $\arg(c) \neq [0, \pi/4, \pi/2]$.

In this work, we implement a numerical method for first tabulating the branch points $c_{\circ;r}^{mn}$, then rapidly and accurately computing the asymptotic expansions of λ_{mn} and $S_{mn}(c,\eta)$ for arbitrary complex size parameter c in the asymptotic regime of $|c|$ large with n and m fixed. This regime corresponds to, for example, case (i) in [3] and the regime discussed in part I of [68]. To accomplish this, the branch points $c_{\circ;r}^{mn}$ are incorporated into a lookup table used to determine which asymptotic expansion is appropriate for a given c. Specifically, we elaborate on the solution for the specific case of $c_r = c_i$, and Table 4.3 lists which expansion type applies for a given m and n under this condition. This special case is encountered in the solution of the diffusion equation for spheroidal systems [63]. We also provide normalizations for the standard *prolate*-type and *oblate*-type asymptotic expansions for both $(n-m)$ even and odd. Expansions for the oblate SWFs are also discussed as a simple extension of the current investigation. All calculations in this work were performed in double precision except for the Newton-Raphson method, which was implemented in quadruple precision. The routines presented are rapid enough to be of practical use in applications requiring the calculation of many SWFs of varying orders, degrees, and arguments.

The structure of this work is as follows. Section 4.2 reviews the basic geometry and the definitions of the prolate SWFs. We describe the most common asymptotic expansions of the spheroidal eigenvalues λ_{mn} for purely real or imaginary c in Sections 4.3.1 and 4.3.2

respectively. The asymptotic expansion of λ_{mn} in the case of complex c depends on finding the branch, or double, points and our numerical method for finding these points is described in Section 4.3.3. The ordering of the eigenvalues λ_{mn} (and the SWFs) for the case of complex c is not as straightforward as the case of purely real (PSWFs) or purely imaginary c (Oblate SWFs) but instead depends on $\arg(c_{\circ;r}^{mn})$ as compared to $\arg(c)$. The first few of these branch points are listed in Table 4.2. Section 4.3.5 addresses the expansion ordering and describes how to incorporate this ordering into existing asymptotic expansions. In Section 4.4 the overall asymptotic expansion of the PASWF for complex c is shown to be comprised of the established expansions outlined in Section 4.3, where the choice of which type of asymptotic expansion to use is determined by m, n, and $\arg(c)$ via the same lookup table of branch points used in determining the correct expansion type for λ_{mn}. Proper normalizations for these asymptotic expansions are derived in Section 4.4.1. Section 4.5 contains a discussion of the expansions and patterns under consideration as well as some examples in graphical form of the real and imaginary parts of the complex PASWFs, $S_{mn}(c,\eta)$ and $S'_{mn}(c,\eta)$, as compared to the original Legendre expansion solutions. At the end of Section 4.5 we discuss briefly one asymptotic expansion of the PRSWFs $R_{mn}^{(i)}(c,\xi)$ for complex c, in light of the current discussion of the PASWFs. This is followed by a conclusion (Section 4.6) and information on how to obtain the computer programs implemented for this work.

4.2 Standard Formulation of the Prolate Angular Spheroidal Wave Function

The prolate and oblate spheroidal coordinate systems are two of the eleven coordinate systems in which the scalar wave equation

$$(\nabla^2 + k^2)\psi = 0 \tag{4.2}$$

is separable. The solution to Eq. (4.2) in prolate spheroidal coordinates is

$$\psi_{pmn}^{(i)} = S_{mn}(c,\eta)\, R_{mn}^{(i)}(c,\xi)\, T_{pm}(\phi), \tag{4.3}$$

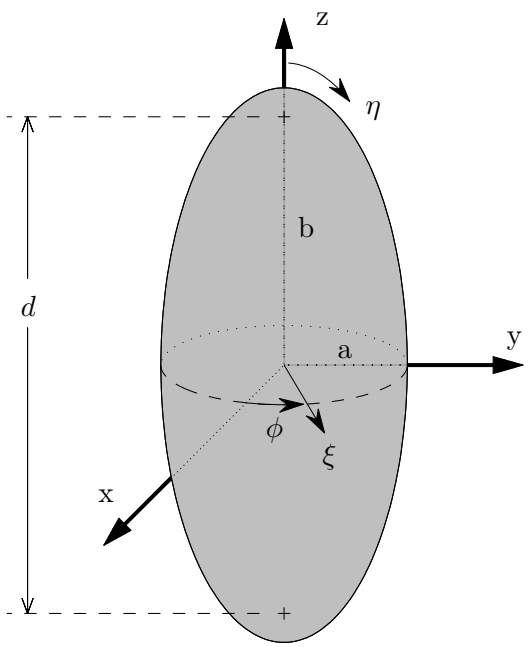

Figure 4-1: Prolate Spheroidal Geometry: $1 \leq \xi < \infty$, $-1 \leq \eta \leq 1$, $0 \leq \phi \leq 2\pi$, $x = \frac{d}{2}[(1-\eta^2)(\xi^2-1)]^{\frac{1}{2}}\cos(\phi)$, $y = \frac{d}{2}[(1-\eta^2)(\xi^2-1)]^{\frac{1}{2}}\sin(\phi)$, $z = \frac{d}{2}\eta\xi$, $e = \frac{b}{a}$, $\xi = (1-e^{-2})^{-\frac{1}{2}}$, $d = 2(b^2-a^2)^{\frac{1}{2}}$.

where c is the size parameter ($c = kd/2$, where d is the interfocal distance as shown in Fig. 4-1), $S_{mn}(c,\eta)$ is the PASWF, $R^{(i)}_{mn}(c,\xi)$ is a the PRSWF of the i^{th} kind, and $T_{pm}(\phi)$ is the azimuthal function given by

$$T_{pm}(\phi) = \begin{cases} \cos(m\phi), p = 0 \\ \sin(m\phi), p = 1. \end{cases} \tag{4.4}$$

Flammer's notation [66] is used throughout except for the notation used for the harmonic functions [Eq. (4.4)] and some minor notational (but not formal) differences in the asymptotic expansions of Eqs. (4.7)–(4.12) and Eqs. (4.13)–(4.19) below. Expressions such as $n = 0(1)6$ indicate n equals [1..6] with an increment of 1.

Expressions for the prolate angular and radial functions have been studied for quite some time beginning with Lamé [86] and then Niven [87]. The most common form of the prolate

angular function is expressed as an expansion in terms of associated Legendre functions

$$S_{mn}(c,\eta) = \sum_{r=0,1}^{\infty\prime} d_r^{mn}\, P_{m+r}^{m}(\eta) \tag{4.5}$$

where the prime indicates summation from $\{{}^{0}_{1}\}$ over $\{{}^{\text{even}}_{\text{odd}}\}$ indices when $(n-m)$ is $\{{}^{\text{even}}_{\text{odd}}\}$. The radial function can be expressed as an expansion of spherical Bessel functions [88] as

$$R_{mn}^{(i)}(c,\xi) = \frac{\left(\frac{\xi^2-1}{\xi^2}\right)^{m/2}}{\sum_{r=0,1}^{\infty\prime} d_r^{mn}\frac{(2m+r)!}{r!}} \sum_{r=0,1}^{\infty\prime} d_r^{mn}\frac{(2m+r)!}{r!}\, \mathrm{i}^{r+m-n} z_{m+r}^{(i)}(c\xi), \tag{4.6}$$

where $z_{m+r}^{(i)}(c\xi)$ represents the spherical Bessel functions $j_{m+r}(c\xi)$, $y_{m+r}(c\xi)$, $h_{m+r}^{(1)}(c\xi)$, and $h_{m+r}^{(2)}(c\xi)$, for $i = 1,2,3,4$ respectively [88]. The expansion coefficients d_r^{mn} are determined by solving a three term recurrence relation obtained by substituting Eq. (4.5) into Eq. (4.1). These expansions are accurate for small size parameters up to $|c| \lesssim 30$, but encounter numerical difficulties leading to inaccuracy for larger $|c|$.

4.3 Asymptotic Expansion of the Spheroidal Eigenvalues, λ_{mn}

Eigenvalues of the spheroidal differential equation, λ_{mn}, are special values for which the equation's solutions are bounded at $\zeta = \pm 1$. We first review the two most common asymptotic expansions of the spheroidal eigenvalues and angular wavefunctions including both the *prolate*-type (c purely real) and *oblate*-type (c purely imaginary) before applying these expansions to the case of complex size parameter c.

A number of methods can be used to determine the eigenvalues, including the truncation and solution of the (transcendental) infinite continued fraction equation directly [66, 85]; successive approximations [66]; a power series representation for small $|c|$ [66, 88] followed by Bouwkamp's method of refinement [89]; the relaxation method [90, 91]; and an eigenvalue matrix approach [2] (see also [92, 93]). We prefer to use Hodge's [2] method because it is straightforward and rapidly computes the full set of λ_{mn}. When c is in the asymptotic regime considered here, the λ_{mn} produced by Hodge's method require a larger truncation size of the

eigenvalue matrix in order to maintain accuracy, at the cost of increased computation time. In this case, increased accuracy can be achieved instead by applying either Bouwkamp's method or using an asymptotic representation for λ_{mn} (see below). It should be noted that Bouwkamp's method converges very slowly near the branch points $c_{\circ;r}^{mn}$.

We use the Matlab codes found at [94], modified to accept complex c to calculate the spheroidal wave functions. These programs are based on those given in [95] and the results for the PSWF were compared to published tables [64, 66, 67] and found to be in agreement for purely real and purely imaginary values of small c. Results also matched values for the PSWFs and eigenvalues for complex c given in [84, 85].

4.3.1 *Prolate*-Type Asymptotic Expansion

The asymptotic expansion typically applied to the PASWF, $S_{mn}(c,\eta)$, and its characteristic eigenvalue, λ_{mn}, for the case of purely real c [Eq. (4.1) case 1] consists of an expansion of $S_{mn}(c,\eta)$ in terms of a series of parabolic cylinder, or Weber functions [66,76]. We designate all asymptotic expansions typically applied to the PSWF for purely real c as *prolate*-type although as will be seen in Section 4.3.3, this expansion can also be applied to the OSWF in the case of arbitrary, complex c. We shall denote the asymptotic expansion by a superscript (a). The differential equation under these conditions becomes

$$\frac{d}{d\zeta}\left[(1-\eta^2)\frac{dS_{mn}}{d\eta}\right]+\left[\lambda_{mn}-c^2\eta^2-\frac{m^2}{1-\eta^2}\right]S_{mn}=0. \tag{4.7}$$

If the following substitutions are made:

$$S_{mn}(c,\eta)\approx S_{mn}^{(a)}(c,\eta)=(1-\eta^2)^{\frac{m}{2}}u_{mn}(c,\eta), \tag{4.8a}$$

$$\eta\Rightarrow\frac{x}{\sqrt{2c}} \tag{4.8b}$$

then in the limit as $c\uparrow\infty$, x fixed, Eq. (4.7) reduces to

$$\frac{d^2u_{mn}}{dx^2}+\left(\frac{\lambda_{mn}}{2c}-\frac{x^2}{4}\right)=0 \tag{4.9}$$

which is in the form of the parabolic cylinder differential equation. $u_{mn}(x)$ is therefore expanded in terms of an infinite series of parabolic cylinder functions

$$u_{mn}(x) = \sum_{r=-\infty}^{\infty}{}' C_r D_{n-m+r}(x) \tag{4.10}$$

where $D_{n-m+r}(x)$ is the parabolic cylinder function of order $n-m+r$ [88], the prime indicates summation over even r, and C_r are the expansion coefficients (unnormalized, see Section 4.4.1). Following this method, C_r and the eigenvalues λ_{mn} are further expanded as

$$C_r = \sum_{s=0}^{\infty} C_{rs} c^{-s} \tag{4.11}$$

and

$$\lambda_{mn} = \sum_{k=0}^{\infty} \Gamma_k c^{-k+1}. \tag{4.12}$$

Detailed description of the expansion coefficients, C_r and Γ_k, are given by Do-Nhat [76].

Slepian [68] states that the region of validity for this *prolate*-type expansion (Equation (1.6) in [68]) is $|\eta| \leq c^{-\frac{1}{2}}$. However, as noted by [70], if the infinite summations in equations Eqs. (4.10) and (4.11) are appropriately truncated, the approximation is remarkably accurate for a much larger range. Figures 4-9 and 4-10 show some examples of *prolate*-type $S_{mn}(c,\eta)$ and $S'_{mn}(c,\eta)$ as compared to $S^{(a)}_{mn}(c,\eta)$ and $S^{(a)\prime}_{mn}(c,\eta)$ respectively.

4.3.2 *Oblate*-Type Asymptotic Expansion

The asymptotic expansion typically applied to the oblate angular spheroidal wave function (OASWF) [case 3 of Eq. (4.1)] and its characteristic eigenvalue for the case of purely imaginary c, or $c \Rightarrow -ic$, consists of a similar method to that described by Eqs. (4.8)–(4.12) for the PSWF [66, 77]. Similar to the case above, we designate all asymptotic expansions typically applied to the OSWF for purely imaginary c as *oblate*-type although as will be seen in Section 4.3.3, this expansion can also be applied to the PSWF in the case of arbitrary, complex c. The original differential equation for the OASWF is

$$\frac{d}{d\zeta}\left[(1-\eta^2)\frac{dS_{mn}}{d\eta}\right] + \left[\lambda_{mn} + c^2\eta^2 - \frac{m^2}{1-\eta^2}\right] S_{mn} = 0, \tag{4.13}$$

which, under the following substitutions

$$S_{mn}(-\mathrm{i}c,\eta) \approx S_{mn}^{(a,\pm)}(-\mathrm{i}c,\eta) = (1-\eta^2)^{\frac{m}{2}} e^{-c(1\mp\eta)} u_{mn}(-\mathrm{i}c,\eta) \tag{4.14a}$$

$$\eta \Rightarrow \pm\left(\frac{x}{2c}-1\right), \tag{4.14b}$$

and in the limit as $c\uparrow\infty$, x fixed, becomes

$$\left[x\frac{d^2}{dx^2} + (m+1-x)\frac{d}{dx} + \nu\right] u_{mn}(x) = 0, \tag{4.15}$$

where $\nu = \left\{\begin{smallmatrix}(n-m)/2\\(n-m-1)/2\end{smallmatrix}\right\}$ when $(n-m)$ is $\left\{\begin{smallmatrix}\text{even}\\\text{odd}\end{smallmatrix}\right\}$. Equation (4.15) is satisfied by the associated Laguerre functions of degree m and order $\nu+r$, $L_{\nu+r}^{(m)}(x)$ [96, 88]. Accordingly, $u_{mn}(-\mathrm{i}c,\eta)$ is expanded as

$$u_{mn}(-\mathrm{i}c,\eta) = \sum_{r=-\nu}^{\infty} A_r \left\{L_{\nu+r}^{(m)}\left[2c(1\mp\eta)\right] + (-1)^{n-m} L_{\nu+r}^{(m)}\left[-\mathrm{i}2c(1+\eta)\right]\right\}, \tag{4.16}$$

so that $S_{mn}^{(a)}(-\mathrm{i}c,\eta)$ becomes

$$S_{mn}^{(a)}(-\mathrm{i}c,\eta) = (1-\eta^2)^{\frac{m}{2}} \sum_{r=-\nu}^{\infty} A_r \left\{e^{-c(1-\eta)} L_{\nu+r}^{(m)}\left[2c(1-\eta)\right] + (-1)^{n-m} e^{-c(1+\eta)} L_{\nu+r}^{(m)}\left[2c(1+\eta)\right]\right\}. \tag{4.17}$$

In a process similar to that for the prolate case, A_r and λ_{mn} are expanded in power series in c as

$$A_r = \sum_{s=0}^{\infty} A_{rs}(-\mathrm{i}c)^{-s} \tag{4.18}$$

and

$$\lambda_{mn} = \sum_{k=0}^{\infty} \Gamma_k(-\mathrm{i}c)^{-k+2}. \tag{4.19}$$

Detailed descriptions of the expansion coefficients, A_r and Γ_k, are given by Do-Nhat [77]. This *oblate*-type asymptotic expansion is found to be valid for all $|\eta| \leq 1$ and $|c| \gtrsim (n-m)+20$. Figures 4-11 and 4-12 show some examples of *oblate*-type $S_{mn}(c,\eta)$ and $S'_{mn}(c,\eta)$ as compared to $S_{mn}^{(a)}(c,\eta)$ and $S_{mn}^{(a)\prime}(c,\eta)$ respectively.

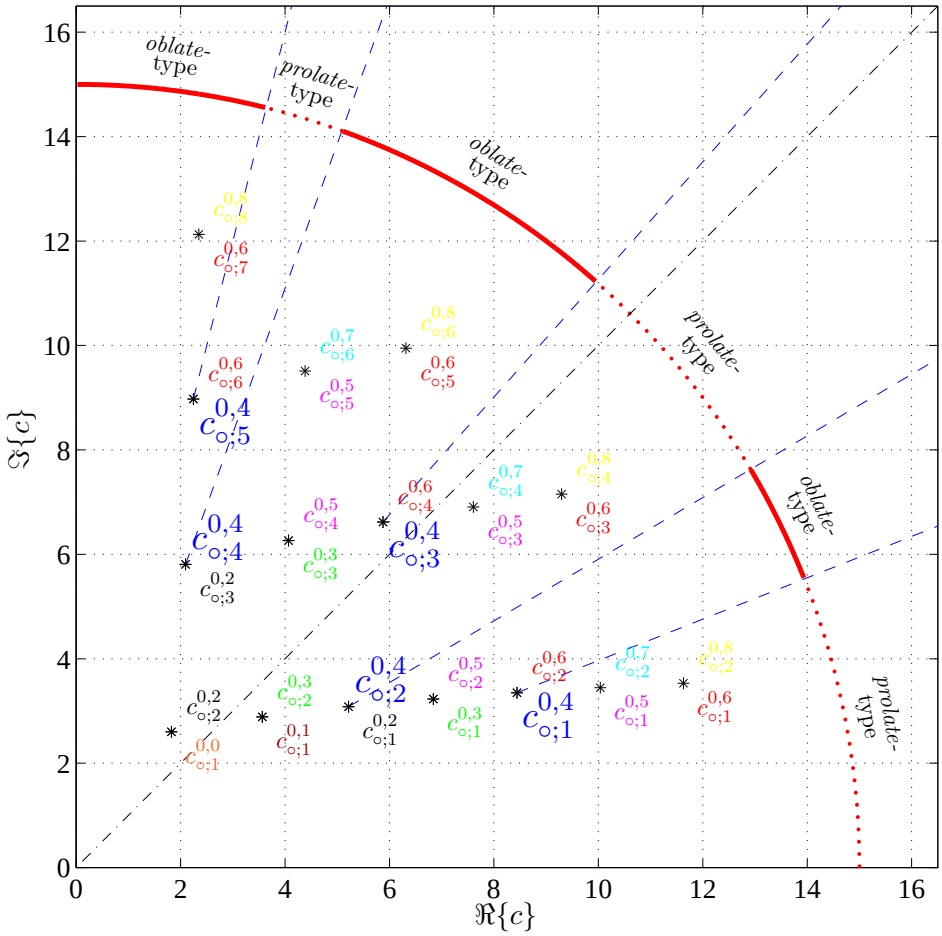

Figure 4-2: Branch points, $c_{\circ;r}^{mn}$ for $m = 0$ and $n = 0(1)6$. Branch cuts associated with $n = 4$ are indicated by the dashed lines with the appropriate expansion type indicated for each resulting sector. The dash-dot line indicates $\arg(c) = \pi/4$.

4.3.3 Asymptotic Expansion of the Eigenvalues for Complex c

When the size parameter c is complex, no new asymptotic expansions are necessary to determine the eigenvalues (and spheroidal wave functions) of the spheroidal wave equation. Instead, the expansions of λ_{mn} and $S_{mn}(c, \eta)$ for complex c are governed by a pattern, alternating between the aforementioned *prolate*-type (Section 4.3.1) and *oblate*-type (Section 4.3.2) expansions. This pattern is dictated solely by $\arg(c)$ for a given order m. The purpose of this Section is to provide a generally applicable numerical method for finding this asymptotic expansion pattern for the case of general complex c via a lookup table

consisting of the branch points, $c_{\circ;r}^{mn}$, for $0 \leq \arg(c) \leq \pi/2$. We also tabulate the pattern of correct asymptotic expansion types for the special case of $\arg(c) = \pi/4$ or $c = (1+\mathrm{i})\alpha$, where $\alpha \in \Re^+$. We restrict our attention to c^2 in the upper half plane because the branch points are symmetric about the real axis [82]. Asymptotic expansions for the OSWFs follow the same pattern as that of the PSWFs with $c \Rightarrow -\mathrm{i}c$ for the *prolate*-type expansions, and $c \not\Rightarrow -\mathrm{i}c$ but remains unchanged for the *oblate*-type expansions.

The set of $S_{mn}(c,\eta)$, with their accompanying eigenvalues, is complete in the sense that an arbitrary square-integrable function $y(\eta)$ can be expanded in terms of an infinite linear combination

$$y(\eta) = \sum_{n=m}^{\infty} a_n S_{mn}(c,\eta). \tag{4.20}$$

Direct solutions of Eq. (4.1), either using the series defined by Eq. (4.5) or by numerical methods such as a Runge-Kutta method, are unambiguous and theoretically valid for any m, n, and c. However, the spheroidal eigenvalues develop square root branch cuts where the solution to the infinite continued fraction equation, or equivalently the characteristic eigenvalue equation (e.g from Hodge's method), becomes degenerate and possesses double roots [97].

At these branch points, $c_{\circ;r}^{mn}$, two spheroidal eigenvalues merge and become analytic continuations of each other [83,82]. One consequence of this is that, as the eigenvalues exchange identities by level crossing onto each other's Riemann sheet, their respective asymptotic expansions are also interchanged. Thus λ_{mn} and $S_{mn}(c,\eta)$ are described alternately by a *prolate*-type asymptotic expansion then an *oblate*-type asymptotic expansion, as $\arg(c)$ (starting at $\arg(c) = 0$) crosses the branch cuts (emanating radially outward from $c_{\circ;r}^{mn}$) associated with a particular m and n. Given c, m, n, and $c_{\circ;r}^{mn}$ therefore, a simple lookup table determines which asymptotic expansion is appropriate by comparing $\arg(c)$ to $\arg(c_{\circ;r}^{mn})$. These transitions are depicted in Fig. 4-2, which shows the branch cuts and their appropriate asymptotic expansion regions for $m = 0$ and $n = 4$. As $\arg(c)$ progresses from 0 to $\pi/2$, each corresponding λ_{mn} crosses branch cuts (roughly proportional in number to $n+1$) and alternates expansion type with each crossing. At $\arg(c) = 0$, all λ_{mn} are *prolate*-type, at $\arg(c) = \pi/2$, all λ_{mn} are *oblate*-type, while at $\arg(c) = \pi/4$, the λ_{mn} are divided evenly

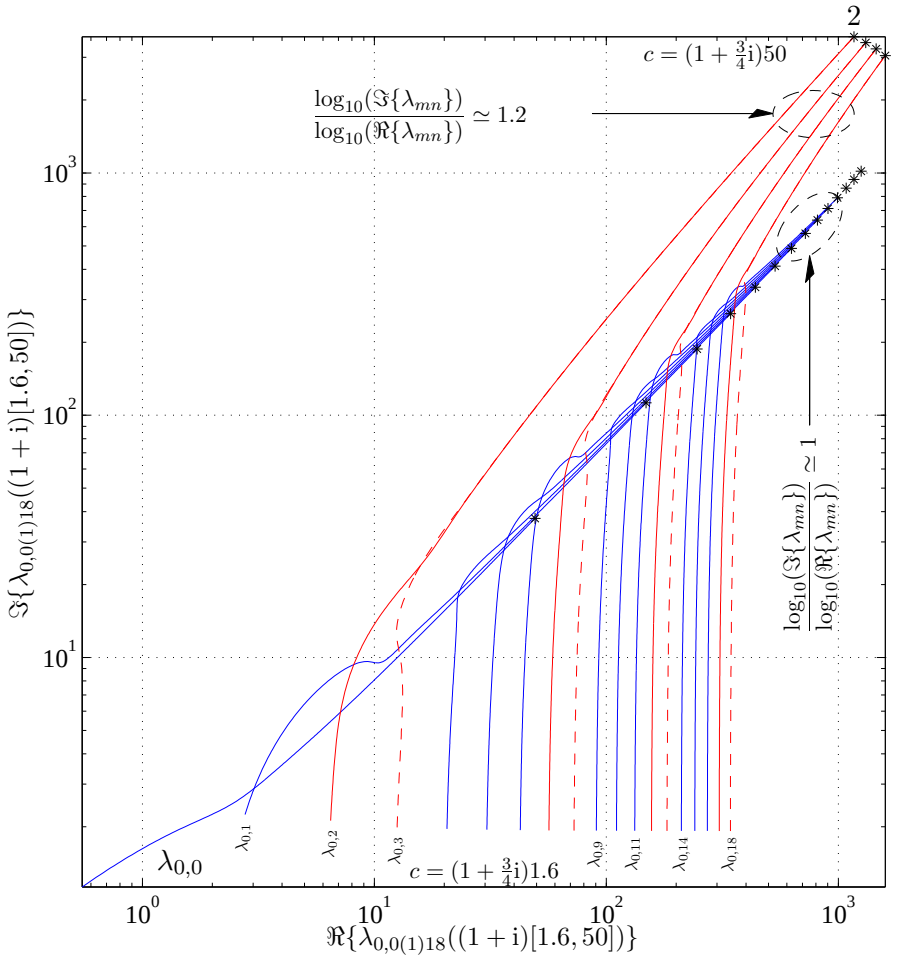

Figure 4-3: λ_{mn}: Eigenvalues of the spheroidal wave equation. $m = 0$ and $n = 0(1)18$. Each curve tracks an eigenvalue as c ranges from $c = (1 + \frac{3}{4}\mathrm{i})1.6$ (lower end of each curve) to $c = (1 + \frac{3}{4}\mathrm{i})50$ (upper end of each curve). Dashed curves indicate coalescing oblate eigenvalue pairs. The "2" indicates $\lambda_{0,2}((1 + \frac{3}{4}\mathrm{i})50)$.

between *prolate* and *oblate* types (see Fig. 4-4).

Further insight into the behavior of λ_{mn} for complex c can be obtained from Fig. 4-3 which shows λ_{mn} calculated by Hodge's method for $m = 0$ and $n = 0(1)20$ with $c = (1 + \frac{3}{4}\mathrm{i})\,\alpha$, as $\alpha \approx [1, 50]$. A similar figure except with $c = (1 + \mathrm{i})\,\alpha$ is shown in Fig. 4-4. The eigenvalues in Figs. 4-3–4-4 display two distinct, alternating behaviors in the asymptotic regime, depending on $(n - m)$ for these $\arg(c)$. The first behavior is that of a linear relationship between $\Re\{\lambda_{mn}\}$ and $\Im\{\lambda_{mn}\}$ indicated by the lines with a slope of one for

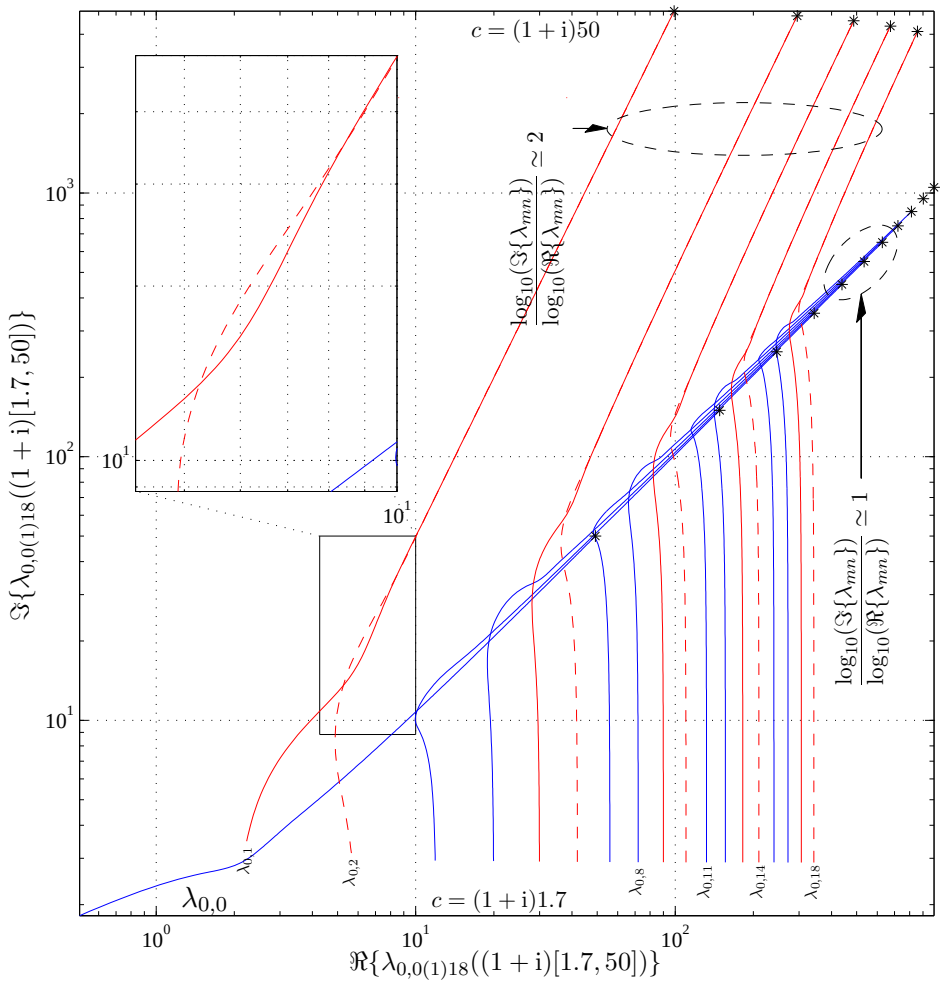

Figure 4-4: λ_{mn}: Eigenvalues of the spheroidal wave equation. $m = 0$ and $n = 0(1)18$. Each curve tracks an eigenvalue as c ranges from $c = (1+\mathrm{i})1.7$ (lower end of each curve) to $c = (1+\mathrm{i})50$ (upper end of each curve). Dashed curves indicate coalescing oblate eigenvalue pairs.

larger c (solid curves). This relationship can be seen more clearly in Fig. 4-7, in which the real and imaginary parts of $\lambda_{0,0}$ are shown explicitly as functions of c. This linear relationship is a consequence of the linear order term in the expansion of λ_{mn} in Eq. (4.12). The second behavior is a power law relationship between $\Re\{\lambda_{mn}\}$ and $\Im\{\lambda_{mn}\}$. Those eigenvalues that evince this type of behavior coalesce into pairs (dashed curves) and result from the *oblate*-type asymptotic expansion of λ_{mn} in Eq. (4.19). In the case of Fig. 4-3 with

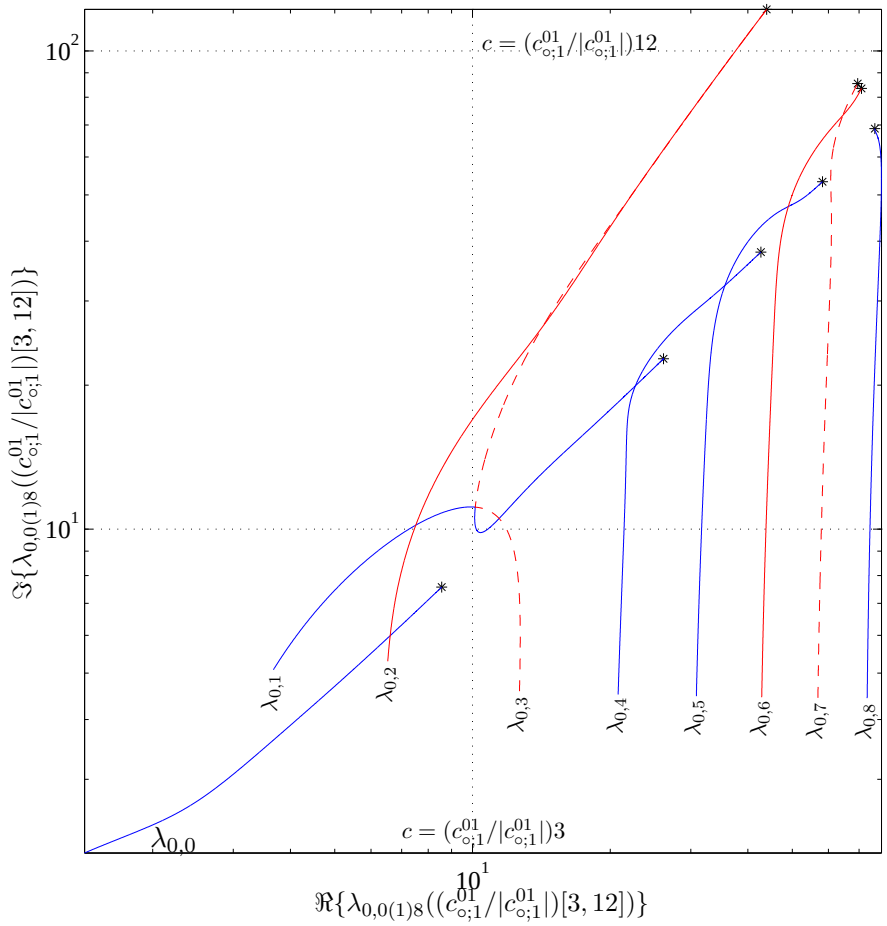

Figure 4-5: Branch point $c^{0,1}_{\circ;1} = c^{0,3}_{\circ;2} = 3.563644553545 + 2.887165344337\mathrm{i}$ where $\lambda_{0,1}$ and $\lambda_{0,3}$ merge at $\lambda^{0,1}_{\circ;1} = \lambda^{0,3}_{\circ;2} = 10.1408387872326 + 11.1215866842249\mathrm{i}$. Dashed curves indicate coalescing oblate eigenvalue pairs.

$c = (1+\frac{3}{4}\mathrm{i})\,\alpha$, this pairing occurs after the first two unpaired eigenvalues, and thereafter after every 3 unpaired eigenvalues. When $c = (1+\mathrm{i})\,\alpha$, oblate eigenvalue pairs occur after every two unpaired eigenvalues. This asymptotic behavior alternates with each level crossing, which occurs at the branch cuts associated with the branch points $c^{mn}_{\circ;r}$. At such points, two eigenvalues merge and become indistinguishable as shown for example in Fig. 4-5 where $\lambda_{0,1}$ and $\lambda_{0,3}$ merge at $c^{01}_{\circ;1} = 3.563644553545 + 2.887165344337\mathrm{i}$.

4.3.4 Computation of the Branch Points $c_{\circ;r}^{mn}$

The computation of the branch points can be decomposed into two stages: finding a sufficiently accurate initial estimate for $c_{\circ;r}^{mn}$, then using a numerical root finding technique such as the Newton-Raphson (NR) method [98] to refine the estimate to the desired accuracy. Estimates for the first few branch points $c_{\circ;r}^{m,m}$ to $c_{\circ;r}^{m,m+3}$ for each m were found by a simple bisection technique utilizing the process described in 4.3.5 to determine the correct ordering and expansion type at a given c. The bisection technique tested successively narrower sectors of $\arg(c)$ in order to specify the approximate angle at which the asymptotic expansions for $\lambda_{m,n}$ and $\lambda_{m,n+2}$ interchanged. Once the initial guess for $\arg(c_{\circ;r}^{mn})$ was found to the desired accuracy, $|c_{\circ;r}^{mn}|$ was searched for between 0 and $n+15$ (as an approximate upper bound) using a simplex search method [27] based on finding the minimum $|\lambda_{m,n} - \lambda_{m,n+2}|$ for the merging pair. After accurate $c_{\circ;r}^{m,m..m+1}$ were found in this manner, initial guesses for $\arg(c_{\circ;r}^{m,m+2..m+3})$ were obtained also via bisection by noting that $\arg(c_{\circ;r,\text{odd}}^{m,n+2})$ are interspersed with $\arg(c_{\circ;r,\text{even}}^{m,n})$, thus providing upper and lower bounds in $\arg(c)$ for the bisection method. An estimate for $|c_{\circ;r}^{m,m+2..m+3}|$ was obtained by noting that

$$\min\left(\left|c_{\circ;r,\text{odd}}^{m,n+2}\right|\right) > \max\left(|c_{\circ;r,\text{even}}^{m,n+2}| = \left|c_{\circ;r,\text{odd}}^{m,n}\right|\right) \tag{4.21}$$

and by observing the pattern which soon becomes apparent (see Fig. 4-2). Initial estimates for the remaining $c_{\circ;r}^{m,m+4...}$ were based on simple polynomial estimation techniques.

These initial guesses for $c_{\circ;r}^{mn}$ were then used as a starting point for a quadruple precision multivariate (in λ and c^2) NR root finding technique [82]. In most cases, these initial guesses were sufficiently accurate for the convergence of the NR method. Those that were not of sufficient accuracy were first refined using a simplex search method, which has a larger radius of convergence, prior to invoking the NR method. In all cases, this combined numerical procedure correctly and exhaustively identified the branch points for $m \le n \le 59$ as well as $m = 0(1)10$, $n \le 100$. The accuracy as defined by the smallest correction of the NR method was less than 1×10^{-15} in the vast majority of cases. However, as n increased, the accuracy of the NR method decreased until, for example, of $c_{\circ;r}^{0,100} \approx 1 \times 10^{-8}$. This numerical procedure for finding initial guesses is an alternative to the WKBJ, or phase-

integral, method outlined in [82]. A brief table of these results is given as Table 4.2 while a more complete listing is provided in D. The programs used in this calculation may be obtained by contacting the author.

m	n	r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
0	0	1	1.824770749208805+ 2.601670692890318i	1.705180091412460+ 4.220186348356370i
0	1	1	3.563644553545243+ 2.887165344336900i	10.140838787232552+11.121586684224873i
0	2	1	5.217093042404772+ 3.081362886557631i	23.915829574409656+18.743327216266508i
0	2	3	2.094267182395557+ 5.807965828216133i	1.998518598679208+ 8.578716573946885i
0	3	1	6.838151743509966+ 3.228540820485014i	42.839460829787278+26.892790566183677i
0	3	3	4.067274712533398+ 6.264358978587767i	11.780925838292136+22.541393207957562i
0	4	1	8.443063517943598+ 3.346896434708849i	66.833326753192964+35.456773864933439i
0	4	3	5.874514148328888+ 6.617314890038058i	27.516445359838094+37.764984283250755i
0	4	5	2.244329796261236+ 8.973752190228394i	2.156124860667898+12.810922196899876i
0	5	1	10.038665064092994+ 3.445770032753475i	95.856405565575329+44.361464108811766i
0	5	3	7.606334073445308+ 6.906465157219409i	48.806652414112627+54.011989822762004i
0	5	5	4.383443826328900+ 9.509669337618593i	12.778186437528630+33.481761709843944i
0	6	1	11.628375737753831+ 3.530602623592430i	129.884250145370260+53.555053907976102i
0	6	3	9.295569705184056+ 7.151702724612575i	75.462208083813508+71.112658178667672i
0	6	5	6.316233767329015+ 9.949229739353585i	29.846039092711770+55.689990108075811i
0	6	7	2.347893464857109+12.128185061133886i	2.263189447556893+16.978431740148483i
1	1	1	1.998555442181652+ 4.097453662365392i	2.915318658097470+ 6.133951063223160i
1	2	1	3.862833529248772+ 4.492300074953849i	12.201095993517072+16.244079086339937i
1	3	1	5.594491061915554+ 4.781582485576267i	27.134073085223168+27.423761751764438i
1	3	3	2.184204069300826+ 7.326156812534641i	3.102506602797785+10.539212581987126i
1	4	1	7.270040170458184+ 5.010809182556227i	47.410985357646396+39.426178765353157i
1	4	3	4.246372923512624+ 7.831360958311476i	13.386757375380991+27.641212328450003i
1	5	1	8.915451995089990+ 5.200792891998006i	72.896560506953961+52.094733920833313i
1	5	3	6.119087892218941+ 8.234638882858787i	29.880624051048471+46.181278766950726i
1	5	5	2.304084540373247+10.504790051881750i	3.223431021669091+14.784565087404808i
1	6	1	10.542501674387614+ 5.363017364859081i	103.515790242771686+65.322653293710630i
1	6	3	7.901814171761899+ 8.572100476598123i	52.112553092568930+65.921011346611721i
1	6	5	4.510843794687041+11.068777156965684i	14.199823500075389+38.585841097079758i
2	2	1	2.136987377094029+ 5.449457313914277i	6.102540356949614+ 7.684763813617406i
2	3	1	4.105156484215650+ 5.922750658440496i	16.136866268804905+20.406230631490939i
2	4	1	5.907125751703487+ 6.283464345814330i	32.087586148736847+34.487534677680138i
2	4	3	2.265822172027141+ 8.748077684520867i	6.207742694273197+12.250160879220145i
2	5	1	7.634658702877481+ 6.576768820638829i	53.561371101706115+49.638515941219318i
2	5	3	4.405746719793623+ 9.294389714166075i	16.974441589636069+32.109641041936442i
2	6	1	9.320494635222795+ 6.824432371374058i	80.376039435949536+65.668970414405067i
2	6	3	6.337223309080594+ 9.739915760209533i	34.182667033756140+53.573264399341902i
2	6	5	2.361167355756876+11.962429681198497i	6.294802569497238+16.568043718788960i
3	3	1	2.254441944326160+ 6.731940814252908i	11.273739025431583+ 9.046369132359118i
3	4	1	4.312789375877335+ 7.267942971679416i	21.989944786045928+24.064251342820775i
3	5	1	6.178462421804212+ 7.686866389128842i	38.868087480677623+40.699856835215551i
3	5	3	2.340324936128145+10.109946235283390i	11.310516001008940+13.800986115329859i
3	6	1	7.954733051349589+ 8.033437876318695i	61.436143112767034+58.624668698520892i
3	6	3	4.550052309842578+10.692087653052024i	22.538458584763827+36.168232895835459i
4	4	1	2.357663561227191+ 7.971913317957412i	18.433063143522340+10.285516884986537i
4	5	1	4.496463440238013+ 8.560827522001995i	29.782817773819936+27.394857362811521i
4	6	1	6.420513778117898+ 9.029346203504799i	47.523240440325630+46.358165420600599i
4	6	3	2.408979269086267+11.429969157729531i	18.410055957253928+15.237266195961114i
5	5	1	2.450444507315414+ 9.182664291501323i	27.583139404518864+11.436479619199886i
5	6	1	4.662342487692823+ 9.817718575239088i	39.529181751692839+30.489219805169245i
6	6	1	2.535162563188484+10.371846133322299i	38.725741591981532+12.519713968726423i

Table 4.2: Branch points and eigenvalues for $m = 0(1)6$ and $n = 0(1)[6-m]$. Accuracy was determined by the last correction of the Newton-Raphson method (in all cases $< 1 \times 10^{-15}$).

4.3.5 Eigenvalue Ordering for Complex c

For purely real, purely imaginary, or sufficiently small complex c, the ordering (indexed by n) is determined by the absolute magnitude of λ_{mn} [2]. However, for larger complex c, the eigenvalues may no longer be sorted merely by $|\lambda_{mn}|$. The power law nature of the imaginary part of those coalescing eigenvalues, when compared to the real part, especially in the asymptotic regime (large c), soon dominates and disrupts ordering by $|\lambda_{mn}|$ (see Fig. 4-6). A better choice for deciding the ordering of λ_{mn} seems to be $\Re\{\lambda_{mn}\}$ due to its linear increase in the asymptotic regime, for both types of behaviors noted above. Yet this choice of ordering is also not correct as coalescing oblate pairs of eigenvalues often reverse order, according to this measure, before becoming indistinguishable (see Fig 4-4 and inset therein). Additionally, ordering by $\Re\{\lambda_{mn}\}$ fails for cases of general c such as $c = (1 + \frac{3}{4}\mathrm{i})\,\alpha$, as can be seen by comparing Fig. 4-3 with Fig. 4-6. To illustrate the difficulty associated with eigenvalue ordering, note the eigenvalue corresponding to the asterisk labeled "$\lambda_{0,2}$" in Figs. 4-3 and 4-6. This eigenvalue is assigned order $n = 7$ by an eigenvalue solver in Hodge's method, $n = 12$ when arranged according to increasing $|\Re\{\lambda_{mn}\}|$, $n = 52$ when arranged according to $|\lambda_{mn}|$, while inspection of Fig. 4-3 reveals that this eigenvalue should in fact correspond to order $n = 2$.

At least it can be said that the eigenvalue ordering and expansion patterns for general, complex c should follow the ordering for λ_{mn} when $c = 0$. At that point, the eigenvalues reduce to

$$\lambda_{mn}(c = 0) = n(n + 1), \quad n \geq m, \tag{4.22}$$

and the spheroidal wave equation (4.1) reduces to the equation satisfied by the associated Legendre functions of the first kind of integral order and degree, $P_n^m(\eta)$. This fact is sufficient to suggest a brute force method for finding λ_{mn} ordering and expansion type for large, complex values of c. The method consists of simply computing λ_{mn} in sufficient density in the range

$$c = \frac{(c_r + c_i)\alpha}{|c|}, \quad \alpha \in [0, \text{large}) \tag{4.23}$$

so that the ordering may be determined by tracing each eigenvalue back to its corresponding value (and order) at $c = 0$ ((c.f. Figs 4-3–4-5). "Large" in this case is to be interpreted as

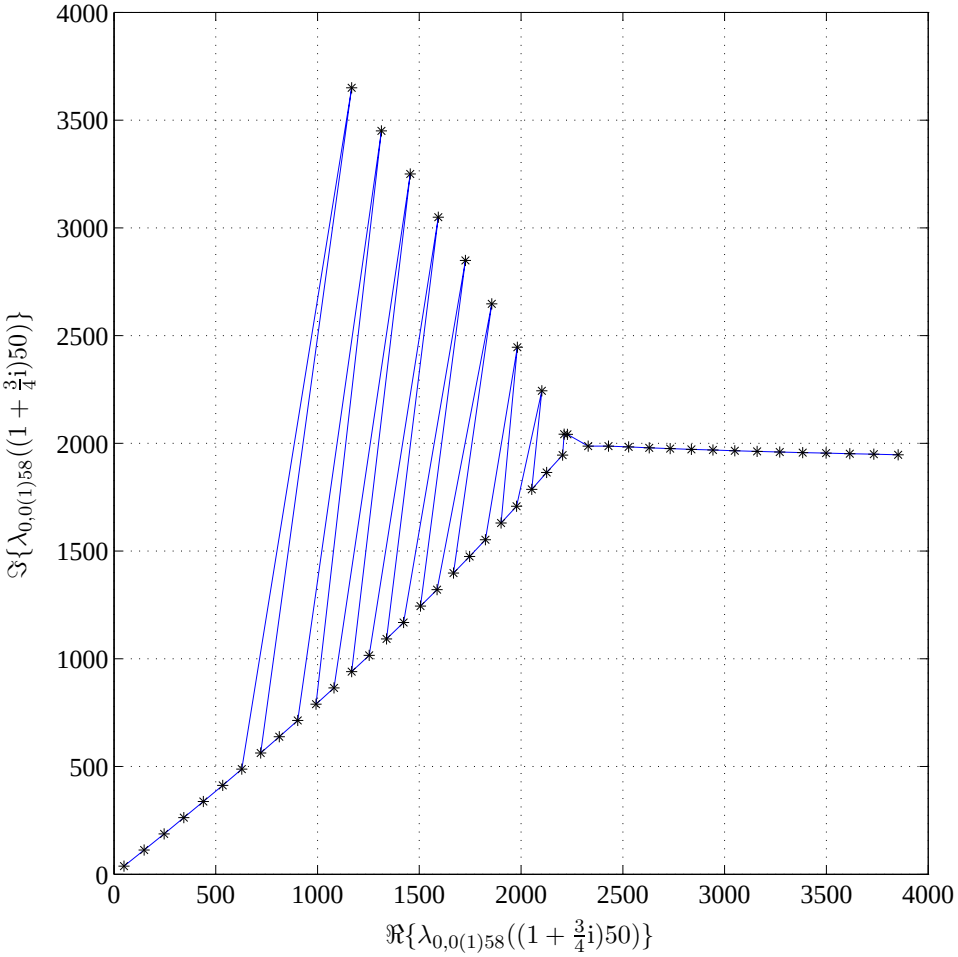

Figure 4-6: λ_{mn}: Eigenvalues of the spheroidal wave equation. $m = 0$ and $n = 0(1)58$ and $c = (1 + \frac{3}{4}\mathrm{i})50$. The solid line indicates the (incorrect) ordering returned from calculating λ_{mn} using Hodge's [2] method, while the correct ordering is labeled by $\lambda_{m,n}$.

any α large enough to satisfy the assumptions that $|c|$ is large compared to fixed m and n. As a rule of thumb, $|c| \gtrsim (n-m)+20$, although this has not been investigated thoroughly.

In practical application, a set of λ_{mn} is often obtained simultaneously using a method such as Hodge's method. Ordering for the entire set of eigenvalues can be determined uniquely by first making use of the fact that the *prolate* and *oblate*-type eigenvalues can be sorted as *separate* sets according to $\Re\{\lambda_{mn}\}$ and then by consulting the lookup table to determine which $n-m$ correspond to each type of expansion. Eigenvalues described by an

oblate-type asymptotic expansion can be identified according to $\Im\{\lambda_{mn}\} > c^2/2$ for c in the asymptotic regime.

The ordering of the *prolate*-type and *oblate*-type asymptotic expansions of λ_{mn} (denoted $\lambda_{mn}^{(a)}$) for general, complex c, is different from the ordering, according to n, of the eigenvalues themselves. While the n indicates λ_{mn} ordering for each m, the $\lambda_{mn}^{(a)}$ are ordered according to m, but independently, corresponding to the sequential ordering for each type of expansion separately. Let n_p be the number of *prolate*-type expansions of λ_{mn} necessary up to but excluding n, and n_o be the number of *oblate*-type expansions of λ_{mn} necessary up to but excluding n. λ_{mn} can then be approximated by *prolate*-type $\lambda_{mn}^{(a)}$ ordered according to n_p beginning at $n_p = m$, and *oblate*-type $\lambda_{mn}^{(a)}$ ordered according to n_o beginning at $n_o = m$, interspersed in a pattern determined by $\arg(c)$ via the lookup table described in Section 4.3.3.

As an illustration, for the case of c of the form $c = (1 + \mathrm{i})\,\alpha$, $\lambda_{0,0}$ would correspond to the zeroth ($n_p = 0$) *prolate*-type asymptotic expansion. On the other hand, the second eigenvalue for $m = 0$, $\lambda_{0,1}$, coalesces with the third eigenvalue, $\lambda_{0,2}$ (see Fig. 4-4), and these correspond to the zeroth ($n_o = 0$) and first ($n_o = 1$) *oblate*-type asymptotic expansions respectively. To be specific, we explicitly show the dependence of Γ_k on m and n, and express the *prolate*-type expansion of $\lambda_{0,0}^{(a)}$ from Eq. (4.12) as

$$\lambda_{0,0} \approx \lambda_{0,0}^{(a)} = \sum_{k=0}^{\infty} \Gamma_k(m = 0, n \Rightarrow n_p = 0) c^{-k+1}. \tag{4.24}$$

For completeness, we also show the *oblate*-type expansion of $\lambda_{0,1}^{(a)}$ and $\lambda_{0,2}^{(a)}$ from Eq. (4.19) as

$$\lambda_{0,1} \approx \lambda_{0,1}^{(a)} = \sum_{k=0}^{\infty} \Gamma_k(m = 0, n \Rightarrow n_o = 0)(-ic)^{-k+2}. \tag{4.25a}$$

$$\lambda_{0,2} \approx \lambda_{0,2}^{(a)} = \sum_{k=0}^{\infty} \Gamma_k(m = 0, n \Rightarrow n_o = 1)(-ic)^{-k+2}. \tag{4.25b}$$

In turn the fourth eigenvalue, $\lambda_{0,3}$, corresponds to the first ($n_p = 1$) *prolate*-type asymptotic expansion, and so on (c.f. Fig. 4-4). The oblate spheroidal eigenvalues are also expressed in terms of either the *prolate*-type and *oblate*-type asymptotic expansions, however, $c \Rightarrow (-\mathrm{i}c)$

	m=0				m=1				m=2			
$(n-m)$	type	n_p	n_o	norm.	type	n_p	n_o	norm.	type	n_p	n_o	norm.
0	Pro.	0	-	C_{Pe}	Pro.	1	-	C_{Pe}	Pro.	2	-	C_{Pe}
1	Obl.	-	0	C_{Oo}	Pro.	2	-	C_{Po}	Pro.	3	-	C_{Po}
2	Obl.	-	1	C_{Oe}	Obl.	-	1	C_{Oe}	Pro.	4	-	C_{Pe}
3	Pro.	1	-	C_{Po}	Obl.	-	2	C_{Oo}	Obl.	-	2	C_{Oo}
4	Pro.	2	-	C_{Pe}	Pro.	3	-	C_{Pe}	Obl.	-	3	C_{Oe}
5	Obl.	-	2	C_{Oo}	Pro.	4	-	C_{Po}	Pro.	5	-	C_{Po}
6	Obl.	-	3	C_{Oe}	Obl.	-	3	C_{Oe}	Pro.	6	-	C_{Pe}
7	Pro.	3	-	C_{Po}	Obl.	-	4	C_{Oo}	Obl.	-	4	C_{Oo}
8	Pro.	4	-	C_{Pe}	Pro.	5	-	C_{Pe}	Obl.	-	5	C_{Pe}
⋮		⋮				⋮				⋮		

Table 4.3: Pattern governing n_p, n_o and which type of asymptotic expansion, either the *prolate*-type (Pro. above) or the *oblate*-type (Obl. above), for different combinations of m and n. c is of the form $c = (1+\mathrm{i})\,\alpha$. Normalizations for each case are also given (see Section 4.4.1).

in Eq. (4.24) while $(-\mathrm{i}c) \Rightarrow c$ in Eq. (4.25a). The ordering of the *prolate*-type and *oblate*-type asymptotic expansions in Section 4.3 as applied to the OSWFs also adhere to this alternating pattern according to n_p and n_o.

Table 4.3 details which type of asymptotic expansion of the prolate spheroidal eigenvalue λ_{mn} is appropriate for a given m and n and indicates the ordering aspects of these expansions for the specific cases of $m = 0(1)2$, $(n-m) = 0(1)8$, and c of the form $c = (1+\mathrm{i})\,\alpha$. For this case, the first $m+1$ expansions of λ_{mn} are *prolate*-type expansions after which alternating pairs of *oblate*-type and *prolate*-type asymptotic expansions develop. The relationship between m, n, n_p, and n_o, again only for c of the form $c = (1+\mathrm{i})\,\alpha$, may be expressed compactly as

$$n_p = \begin{cases} n, & (n-m) \leq m \\ \left\lfloor \frac{(n-m)-m}{2} \right\rfloor + m + m, & (n-m) > m \end{cases} \tag{4.26}$$

and

$$n_o = \left\lfloor \frac{(n-m)-m}{2} \right\rfloor + m, \tag{4.27}$$

where $\lfloor\ \rfloor$ denotes the floor function.

The infinite summations for the asymptotic expansion of the eigenvalues and PASWFs,

either the *prolate*-type Eq. (4.12) or the *oblate*-type Eq. (4.19), must be truncated at some point or they will eventually diverge. The index, k_T, at which the summation is truncated is decided in the usual manner [97] by finding the minimum value in the series, truncating the summation at this index, and assuming the remaining error is less than this term. This truncation index is approximately proportional to $|c|$ as shown in Figs. 4-7b and 4-8b. We calculated the expansion coefficients C_r and A_r from Eqs. (4.11) and (4.18), respectively, for $r \leq 120$, $m = 0(1)5$, and $n = 0(1)100$. Thereafter, the eigenvalues themselves were calculated for both types of asymptotic expansions. These asymptotic values for λ_{mn} were compared to those eigenvalues obtained using other methods and found to be in excellent agreement in overlapping regions of high accuracy. Table 4.5 contains a few values for $\lambda_{mn}^{(a)}$ in the asymptotic regime, calculated using the methods described here.

4.4 Asymptotic Expansion of $S_{mn}(c,\eta)$

We now turn our attention to the asymptotic expansion of the PASWF, $S_{mn}(c,\eta)$, for complex argument c. The pattern governing which asymptotic expansion type is appropriate for obtaining the prolate spheroidal eigenvalues, λ_{mn}, for a given c, m, and n, can be applied directly to finding an appropriate asymptotic expansion of the PSWFs. This statement not only applies to the expansions summarized in Section 4.3, but also applies to all other known expansions in the literature that are typically applied to the PSWFs (*prolate*-type expansions) or the OSWFs (*oblate*-type expansions). This seems natural due to the fact that each of the respective asymptotic expansion types from Secs. 4.3 applies to its accompanying angular function as well as to the eigenvalue itself. Extending the example of $c = (1+\mathrm{i})\alpha$ and following the pattern given in Section 4.3.5, the asymptotic expansion of $S_{0,0}(c,\eta)$ is of the *prolate*-type given by Eq. (4.8), with $n_p = 0$, while $S_{0,1}(-\mathrm{i}c,\eta)$ and $S_{0,2}(-\mathrm{i}c,\eta)$ are of the *oblate*-type given by Eq. (4.14), i.e. $S^{(a)}_{m,n_o=1}(-\mathrm{i}c,\eta)$ and $S^{(a)}_{m,n_o=2}(-\mathrm{i}c,\eta)$, and so on. The same accounting for $(-\mathrm{i})$ must be done when finding the *oblate*-type asymptotic expansion for the PASWF as in the case of the *oblate*-type asymptotic expansion for the prolate eigenvalue (see Section 4.3.3). For the OSWFs this accounting is reversed, as noted in the previous Section.

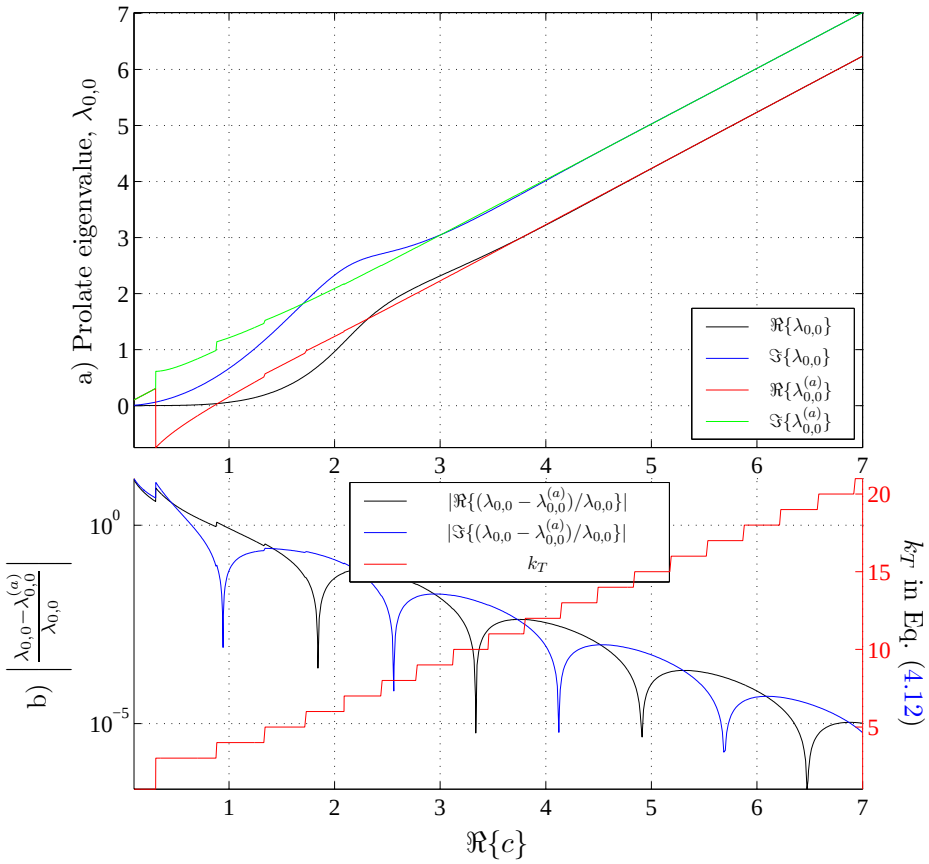

Figure 4-7: a) Spheroidal eigenvalues $\lambda_{0,0}$ for $c = (1+\mathrm{i})\,\alpha$, $\alpha = [0, 7]$. b) Relative error between the prolate eigenvalue from Hodge's method [2], $\lambda_{0,0}$, and its *prolate*-type asymptotic expansion [Eq. (4.12)], $\lambda^{(a)}_{0,0}$.

4.4.1 Normalizations of the Asymptotic Expansions

Both the *prolate*-type and *oblate*-type asymptotic expansions for the prolate angular functions are correct to within a constant. This discrepancy is due to the fact that a suitable normalization must be adopted in order for these asymptotic expansions to match with the Legendre expansion of Eq. (4.5). This normalization is dependent on the parameters m, n, and c as well as n_p or n_o. To find suitable normalizations for both the $(n-m) \Rightarrow even$ and $(n-m) \Rightarrow odd$ cases, we follow a procedure similar to that of Stratton [64] and Flammer [66].

For the *prolate*-type asymptotic expansion and in the $(n-m) \Rightarrow even$ case we constrain the value of the asymptotic expansion at $\eta = 0$ to be equal to the associated Legendre

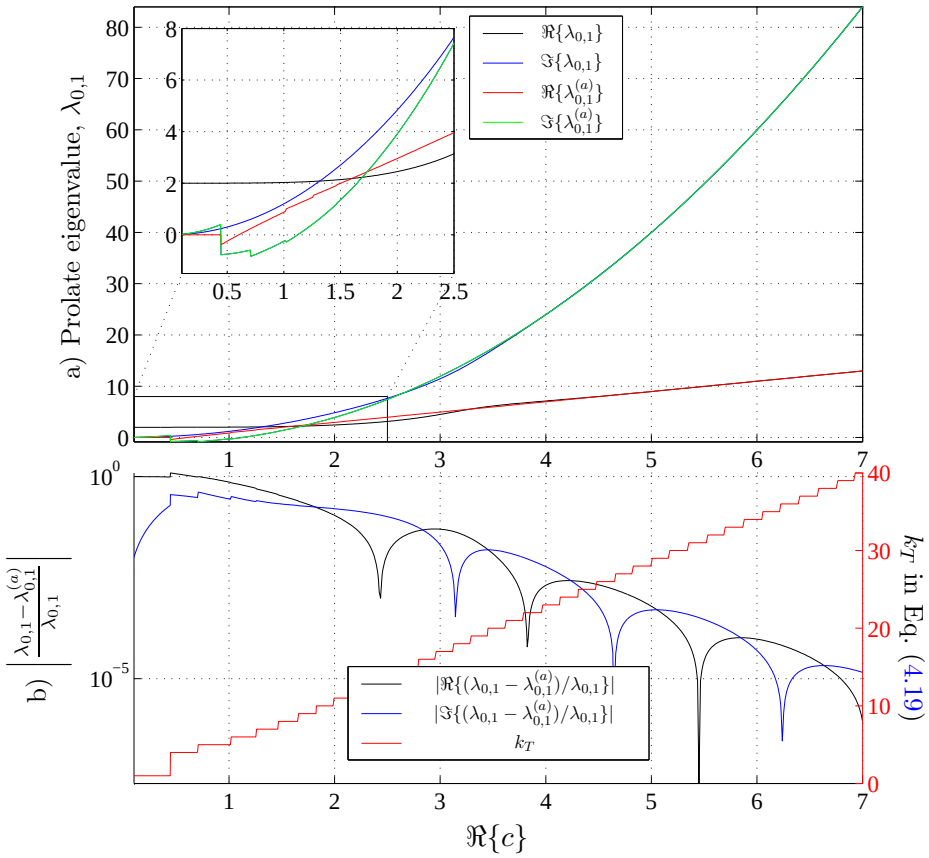

Figure 4-8: a) Spheroidal eigenvalues $\lambda_{0,1}$ for $c = (1+\mathrm{i})\,\alpha$, $\alpha = [0,7]$. b) Relative error between the prolate eigenvalue from Hogde's method [2], $\lambda_{0,1}$, and its *oblate*-type asymptotic expansion [Eq. (4.19)], $\lambda_{0,1}^{(a)}$.

function of degree n and order m,

$$S_{mn}(c,0) \approx C_{Pe}\left[\sum_{r=-\infty}^{\infty}{}' C_r D_{n_p-m+r}(0)\right] = P_n^m(0) \tag{4.28}$$

where we have extracted the normalizing constant C_{Pe}. Note that n_p has been inserted into the parabolic cylinder expansion reflecting the fact that this is only for *prolate*-type asymptotic expansions and therefore ordering must be according to n_p instead of n. The associated Legendre function $P_n^m(\eta)$ retains n as the order because this index is specified by the n of $S_{mn}(c,\eta)$. Based on the expressions for the Legendre and parabolic cylinder

functions of zero argument [88], C_{Pe} becomes

$$C_{Pe} = \frac{\left[(-1)^{\frac{n-m}{2}}(n+m)!\right] \Big/ \left[2^n \left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!\right]}{\sum_{r=-\infty}^{\infty}{}' C_r \frac{\sqrt{\pi}}{2^{\frac{-(n_p-m+r)}{2}} \Gamma\left(\frac{1-(n_p-m+r)}{2}\right)}} \tag{4.29}$$

where Γ refers to the Gamma function [88].

When $(n-m) \Rightarrow odd$, we constrain the value of the first derivative of the asymptotic expansion at $\eta = 0$ to be equal to the derivative of the associated Legendre function of degree n and order m,

$$S'_{mn}(c,0) \approx C_{Po} \frac{\partial}{\partial \eta} \left[(1-\eta^2)^{\frac{m}{2}} \sum_{r=-\infty}^{\infty}{}' C_r D_{n_p-m+r}\left(\eta\sqrt{2c}\right) \right] \Bigg|_{\eta=0} = P_n^{m\prime}(0) \tag{4.30}$$

having extracted the normalizing constant C_{Po}. The derivative of the left side of Eq. (4.30) can be expressed as

$$(1-\eta^2)^{\frac{m}{2}} \left[\sum_{r=-\infty}^{\infty}{}' C_r \left[D'_{n_p-m+r}\left(\eta\sqrt{2c}\right) \right] \sqrt{2c} \right] + \left[\sum_{r=-\infty}^{\infty}{}' C_r D_{n_p-m+r}\left(\eta\sqrt{2c}\right) \right] \frac{m}{2} (1-\eta^2)^{\frac{m}{2}-1}(-2\eta). \tag{4.31}$$

When $\eta = 0$ in Eq. (4.31), the normalization constant for the *prolate*-type asymptotic expansion for $(n-m) \Rightarrow odd$ becomes

$$C_{Po} = \frac{\left[(-1)^{\frac{n-m-1}{2}}(n+m+1)!\right] \Big/ \left[2^n \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)!\right]}{-\sum_{r=-\infty}^{\infty}{}' C_r \frac{\sqrt{2c\pi}}{2^{\frac{-(n_p-m+r+1)}{2}} \Gamma\left(\frac{-(n_p-m+r)}{2}\right)}}. \tag{4.32}$$

The *oblate*-type asymptotic expansions of the PASWF require similar normalizations. When $(n-m) \Rightarrow even$, we constrain the value of the *oblate*-type asymptotic expansion at

$\eta = 0$ to be equal to the associated Legendre function of degree n and order m,

$$S_{mn}(-\mathrm{i}c,0) \approx C_{Oe}\left[\sum_{r=-\nu}^{\infty} A_r\left\{e^{\mathrm{i}c}L_{\nu+r}^{(m)}\left(-\mathrm{i}2c\right)+(-1)^{n-m}e^{\mathrm{i}c}L_{\nu+r}^{(m)}\left(-\mathrm{i}2c\right)\right\}\right] = P_n^m(0). \quad (4.33)$$

where ν is now defined as

$$\nu = \begin{cases} \frac{n_o-m}{2}, & (n-m) \text{ even} \\ \frac{n_o-m-1}{2}, & (n-m) \text{ odd.} \end{cases} \quad (4.34)$$

It is important to note that the n in the factor $(-1)^{n-m}$ in Eqs. (4.33) and (4.37) remains the n of the prolate angular function, $S_{mn}(-ic,\eta)$ (as opposed to being replaced by n_o), because it is introduced to assure the symmetry or antisymmetry of the prolate angular functions. Therefore it remains associated with them rather than with the *oblate*-type asymptotic expansion. In this case, since $(n-m) \Rightarrow even$, Eq. (4.33) reduces to

$$S_{mn}(-\mathrm{i}c,\eta) \approx C_{Oe}\left[\sum_{r=-\nu}^{\infty} A_r 2\; e^{\mathrm{i}c}L_{\nu+r}^{(m)}(-\mathrm{i}2c)\right] = P_n^m(0). \quad (4.35)$$

Thus, the normalization constant, C_{Oe}, for the *oblate*-type asymptotic expansion, $(n-m) \Rightarrow even$, is

$$C_{Oe} = \frac{\left[(-1)^{\frac{n-m}{2}}(n+m)!\right] \Big/ \left[2^n\left(\frac{n-m}{2}\right)!\left(\frac{n+m}{2}\right)!\right]}{2e^{ic}\sum_{r=-\nu}^{\infty} A_r L_{\nu+r}^{(m)}(-i2c)} \quad (4.36)$$

Finally, the normalization constant for the *oblate*-type asymptotic expansion when $(n-m) \Rightarrow odd$ can be found by constraining the value of the first derivative of the asymptotic expansion at $\eta = 0$ to be equal to the derivative of the associated Legendre function of degree n and order m,

$$S'_{mn}(-\mathrm{i}c,0) \approx$$
$$C_{Oo}\frac{\partial}{\partial\eta}\left[(1-\eta^2)^{\frac{m}{2}}\sum_{r=-\nu}^{\infty} A_r\left\{\frac{L_{\nu+r}^{(m)}\left[-\mathrm{i}2c(1-\eta)\right]}{e^{-\mathrm{i}c(1-\eta)}}+(-1)^{n-m}\frac{L_{\nu+r}^{(m)}\left[-\mathrm{i}2c(1+\eta)\right]}{e^{-\mathrm{i}c(1+\eta)}}\right\}\right]\Bigg|_{\eta=0}$$
$$= P_n^{m\prime}(0). \quad (4.37)$$

Expansion Type	$(n-m)$	Normalization Constant
Prolate	even	$C_{Pe} = \dfrac{\left[(-1)^{\frac{n-m}{2}}(n+m)!\right] \Big/ \left[2^n \left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!\right]}{\sum_{r=-\infty}^{\infty}{}' C_r \dfrac{\sqrt{\pi}}{2^{\frac{-(n_p-m+r)}{2}} \Gamma\left(\frac{1-(n_p-m+r)}{2}\right)}}$
	odd	$C_{Po} = \dfrac{\left[(-1)^{\frac{n-m-1}{2}}(n+m+1)!\right] \Big/ \left[2^n \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)!\right]}{-\sum_{r=-\infty}^{\infty}{}' C_r \dfrac{\sqrt{2c\pi}}{2^{\frac{-(n_p-m+r+1)}{2}} \Gamma\left(\frac{-(n_p-m+r)}{2}\right)}}$
Oblate	even	$C_{Oe} = \dfrac{\left[(-1)^{\frac{n-m}{2}}(n+m)!\right] \Big/ \left[2^n \left(\frac{n-m}{2}\right)! \left(\frac{n+m}{2}\right)!\right]}{2e^{ic} \sum_{r=-\nu}^{\infty} A_r L_{\nu+r}^{(m)}(-i2c)}$
	odd	$C_{Oo} = \dfrac{\left[(-1)^{\frac{n-m-1}{2}}(n+m+1)!\right] \Big/ \left[2^n \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)!\right]}{-i2c\, e^{ic} \sum_{r=-\nu}^{\infty} A_r \left\{L_{\nu+r}^{(m)}(-i2c) - 2L_{\nu+r}^{(m)\prime}(-i2c)\right\}}$

Table 4.4: Normalization table for both *prolate*-type or *oblate*-type asymptotic expansions with $(n-m)$ both *even* and *odd*.

After taking the derivative of the left side of Eq. (4.37) and letting $\eta = 0$, one obtains the normalization constant for the *oblate*-type asymptotic expansion for $(n-m) \Rightarrow$ *odd* as

$$C_{Oo} = \frac{\left[(-1)^{\frac{n-m-1}{2}}(n+m+1)!\right] \Big/ \left[2^n \left(\frac{n-m-1}{2}\right)! \left(\frac{n+m+1}{2}\right)!\right]}{-\mathrm{i}2c\, e^{\mathrm{i}c} \sum_{r=-\nu}^{\infty} A_r \left\{L_{\nu+r}^{(m)}(-\mathrm{i}2c) - 2L_{\nu+r}^{(m)\prime}(-\mathrm{i}2c)\right\}}. \tag{4.38}$$

Table 4.4 summarizes the normalization constants required in order to calculate the asymptotic expansion of the prolate angular function for any combination of n and m and either the *prolate*-type or *oblate*-type asymptotic expansion.

4.5 Discussion

$S_{0,0}(c,\eta)$, $S_{0,3}(c,\eta)$, and $S_{0,4}(c,\eta)$ for $c = (1+\mathrm{i})20$ and $\eta = [0,1]$ with their asymptotic expansions (all *prolate*-type) are shown in Fig. 4-9. Their first derivatives for the same c

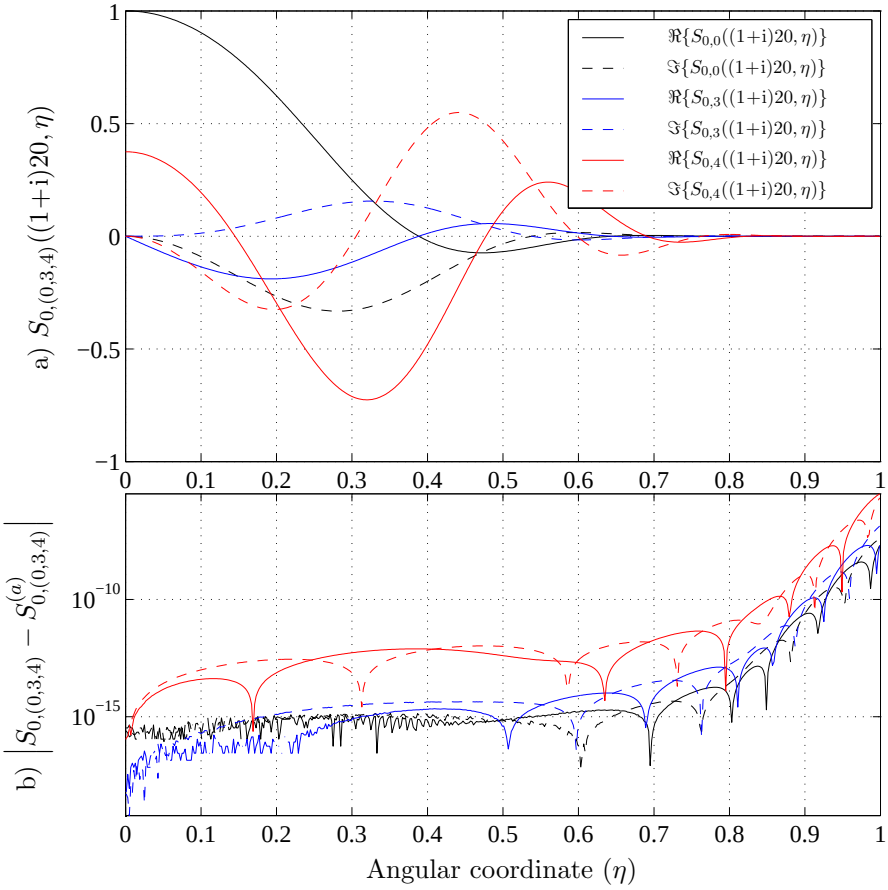

Figure 4-9: a) $S_{mn}(c,\eta)$ for $c = (1+\mathrm{i})20$, $m = 0$, $n = (0,3,4)$ (or $[n_p = 0,1,2]$), and $0 \leq \eta < 1$. b) Absolute error between Legendre expansion, $S_{mn}(c,\eta)$ from Eq. (4.5), and *prolate*-type asymptotic expansion, $S^{(a)}_{mn}(c,\eta)$ from Eqs. (4.7) and (4.11).

are shown in Fig. 4-10. For this size parameter c and range of n, both the original Legendre expansion, Eq. (4.5), and its *prolate*-type asymptotic expansion, Eqs. (4.7) and (4.11), are accurate expansions to at least 5 or 6 decimal places. This is not always the case. In fact, the *prolate*-type expansion of the PASWF described in Section 4.3.1 merely extends the region of accuracy for $S_{mn}(c,\eta)$ further toward $|\eta| \Rightarrow 1$ (see Fig. 4-13). This has been noticed by Slepian [68] and others, who provide other expansions in an effort to extend the region of applicability of the *prolate*-type asymptotic expansion to all values of η (and ξ). We do not implement or elaborate on these expansions here. However, it should be noted that all other *prolate*-type and *oblate*-type asymptotic expansions found in the literature

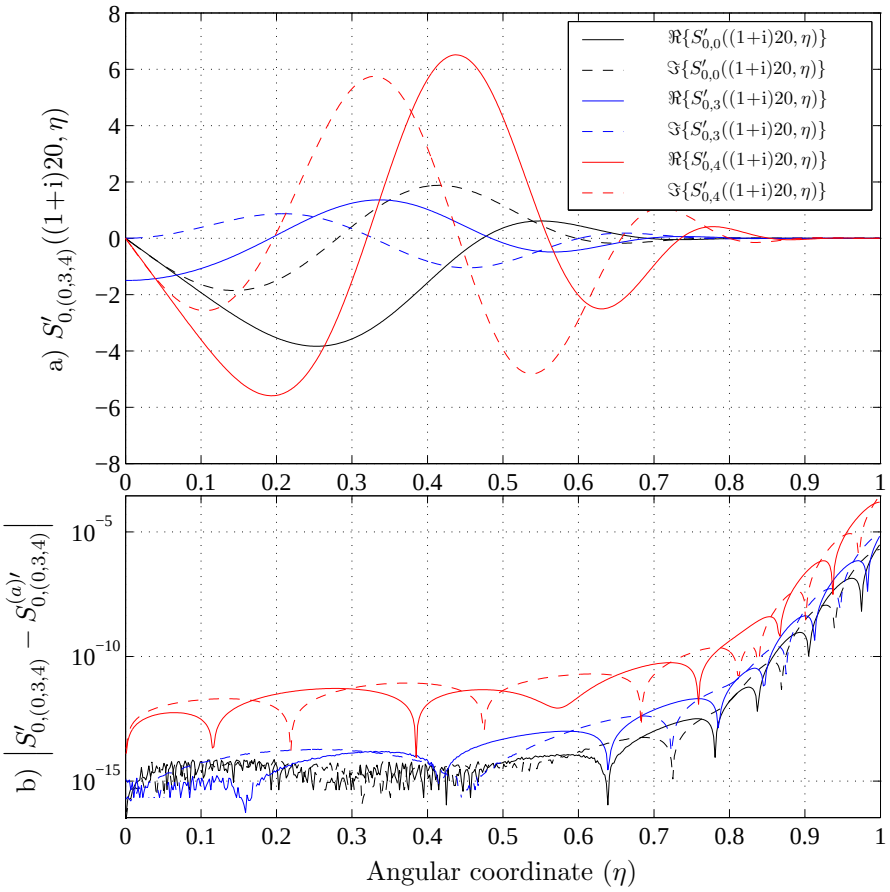

Figure 4-10: a) $S'_{mn}(c,\eta)$ for $c = (1+\mathrm{i})20$, $m = 0$, $n = (0,3,4)$ (or $[n_p = 0,1,2]$), and $0 \leq \eta < 1$. b) Absolute error between derivative of Legendre expansion, $S'_{mn}(c,\eta)$ from Eq. (4.5), and derivative of the *prolate*-type asymptotic expansion, $S'^{(a)}_{mn}(c,\eta)$ Eqs. (4.7) and (4.11).

adhere to the pattern dictated by the branch points, $c^{mn}_{\circ;r}$.

Some remarks about the behavior of these asymptotic expansions of the PASWFs for complex c are in order. To begin with, the magnitudes of the *oblate*-type functions increase with increasing $|\eta|$, whereas those functions approximated by *prolate* expansions [Eqs. (4.7) and (4.11)] decrease for increasing $|\eta|$. $S_{0,1}(c,\eta)$, $S_{0,2}(c,\eta)$, and $S_{0,5}(c,\eta)$ for $c = 20 + 20\mathrm{i}$ and $\eta = [0,1]$ with their asymptotic expansions (all *oblate*-type) are shown in Fig. 4-11. Their first derivatives for the same c are shown in Fig. 4-12. Furthermore, important differences are evident in the applicability of the expansions in Section 4.3 to the case of

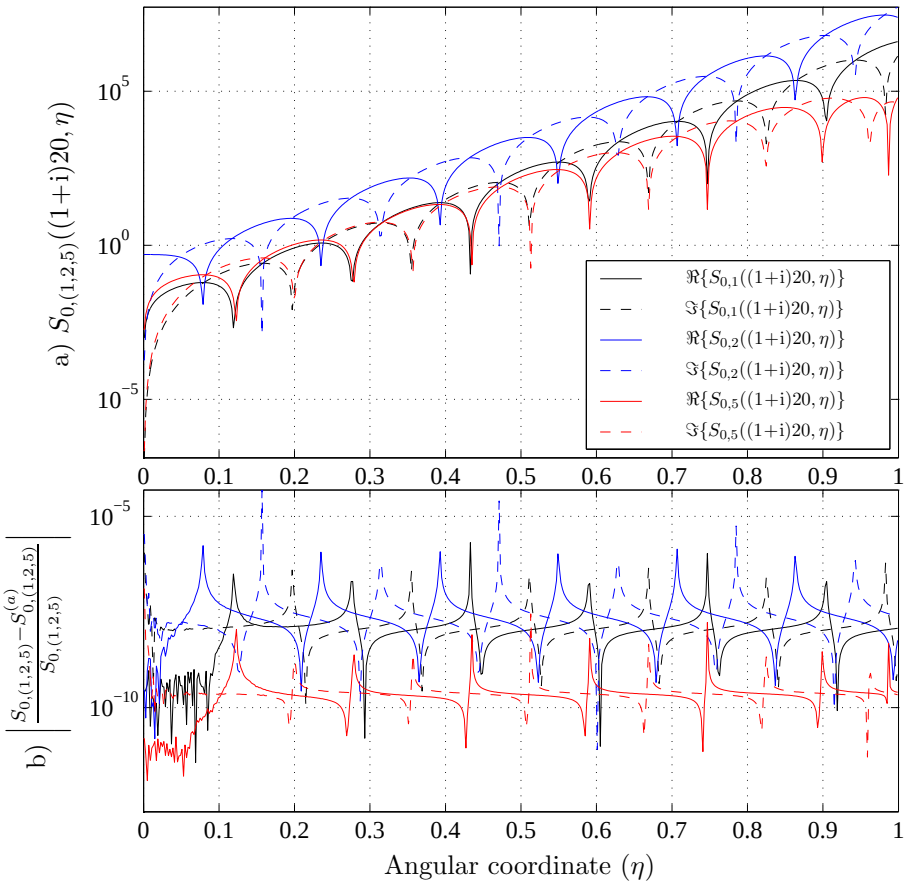

Figure 4-11: a) $S_{mn}(c,\eta)$ for $c = (1+\mathrm{i})20$, $m = 0$, $n = (1,2,5)$ (or $[n_o = 0,1,2]$), and $0 \le \eta \le 1$. b) Relative error between Legendre expansion, $S_{mn}(c,\eta)$ from Eq. (4.5) and *oblate*-type asymptotic expansion, $S^{(a)}_{mn}(c,\eta)$ from Eqs. (4.13) and (4.18).

$\eta > 1$, or in other words, to the PRSWF. For the case of the *prolate*-type expansions, the continued presence of singularities at $\eta = \pm 1$, even under transformation [Eq. (4.8)], causes increasing error as η approaches ± 1. Letting η become greater than 1 in Eq. (4.1) case 1 does not lead to Eq. (4.9), but leads instead to

$$\frac{d^2 u_{mn}}{dx^2} + \left(-\left((n-m) + \frac{1}{2} \right) - \frac{x^2}{4} \right) = 0, \tag{4.39}$$

for which the solutions are functionally different from those for Eq. (4.9).

The *oblate*-type asymptotic expansion, however, has no such difficulty and in fact, the resulting differential equation Eq. (4.15) remains unchanged as η becomes > 1. Letting η

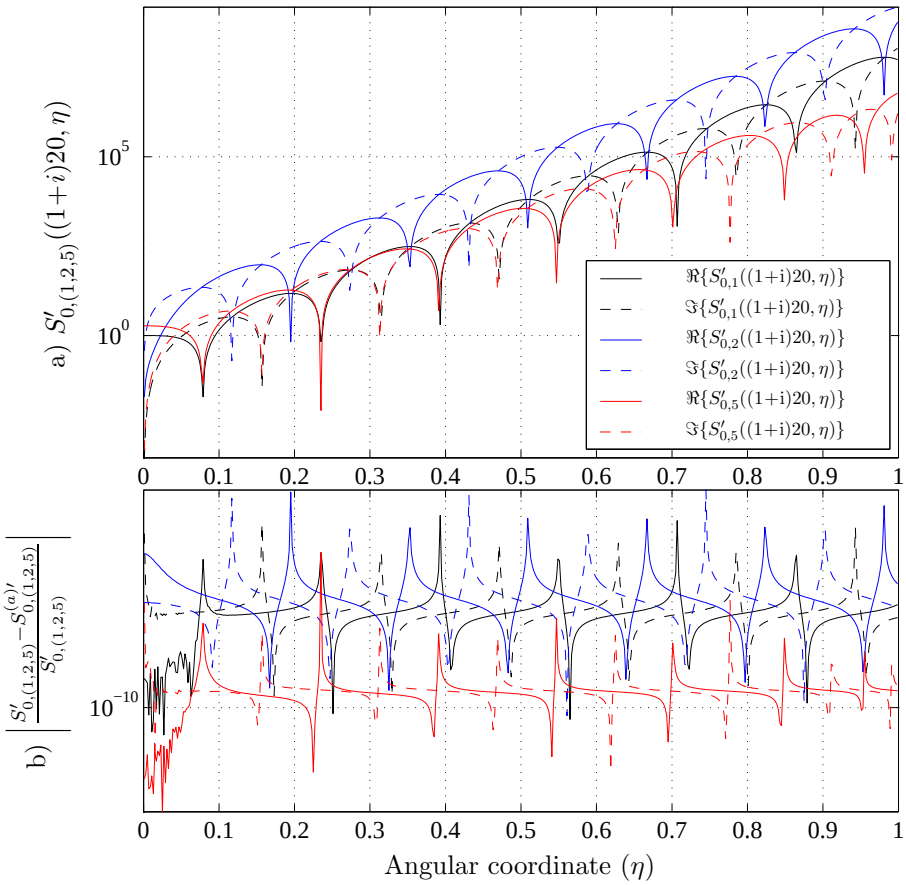

Figure 4-12: a) $S'_{mn}(c,\eta)$ for $c = (1+\mathrm{i})20$, $m = 0$, $n = (1,2,5)$ (or $[n_o = 0,1,2]$), and $0 \leq \eta \leq 1$. b) Relative error between derivative of Legendre expansion, $S'_{mn}(c,\eta)$ from Eq. (4.5)] and derivative of the *oblate*-type asymptotic expansion $S^{(a)\prime}_{mn}(c,\eta)$ from Eqs. (4.13) and (4.18).

become greater than 1 is equivalent to transforming Eq. (4.1) case 1 to case 2 (or similarly case 3 to case 4) and assuming $\eta \Rightarrow \xi$ (or $\eta \Rightarrow i\xi$). In this case, the associated Laguerre functions of the second kind are useful for finding asymptotic solutions of Eq. (4.1) case 4, the solution to which is the ORSWFs [66, 77]. Both types of expansions for the radial functions obey the same pattern as described in Section 4.3.5 and summarized for c of the form $c = (1+\mathrm{i})\,\alpha$ in Table 4.3 for the PASWFs. Normalizations for the prolate radial functions for complex c need to properly account for the altered ordering based on n_p and n_o as determined via the lookup table.

Figure 4-13 shows the limits of the original Legendre function expansions as compared

to the asymptotic expansions. The *prolate*-type functions are incorrect for large $|\eta|$ while the asymptotic expansion is accurate for a larger range. We can obtain accurate $S_{mn}(c,\eta)$ over the entire domain $(-1 \le \eta \le 1)$ by using a number of different expansions, for example the expansion Equation (3.11) in [3]. This expansion becomes more accurate for smaller ξ (or η) as $c\uparrow\infty$, and applies to η beyond the region where the expansion in Eq. (4.10) fails. To use this expansion the proportionality constant between $S_{mn}(c,\eta)$ and $R_{mn}(c,\xi)$, i.e. the joining factor κ_{mn}, must be calculated using values for the angular and radial spheroidal wave functions in the overlapping region of accuracy. Note also that for this *prolate*-type asymptotic expansion $R^{(a)}_{mn}(c,\xi)$, n should be replaced by n_p, i.e. $R_{mn}(c,\xi) \approx R^{(a)}_{mn_p}(c,\xi)$. *Oblate*-type expansions for the PRSWF would naturally be approximated by $R_{mn}(c,\xi) \approx R^{(a)}_{mn_o}(c,\xi)$. As with the case of the ASWFs, the expansion pattern governing the RSWFs can be extracted from the lookup table of branch points obtained in Section 4.3.4.

Some representative values for the PSWFs, based on the expansion of Section 4.3 and in the asymptotic regime considered here, are given in Table 4.5. More extensive tables cataloging the spheroidal eigenvalues, λ_{mn}, and the prolate spheroidal wave functions and their derivatives, $S_{mn}(c,\eta)$, $S'_{mn}(c,\eta)$, are given in D.

m	n	c or $\lvert c\rvert$	$\arg(c)$	exp. type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta)$	$S^{(a)\prime}_{mn}(c,\eta)$
0	0	(1+i) 20		Pro	1.92453281e+01 + 2.00049941e+01i	−6.83836700e−02 − 3.39951350e−02i	3.63351070e−01 + 1.16571397e+00i
0	1	(1+i) 20		Obl	3.89937623e+01 + 7.59993426e+02i	−2.57938462e+02 − 6.09345190e+01i	−6.20443272e+03 + 3.98256577e+03i
0	2	(1+i) 20		Obl	3.89937623e+01 + 7.59993426e+02i	3.05857460e+03 − 2.00264676e+03i	1.90507143e+04 − 9.98963377e+04i
0	3	(1+i) 20		Pro	5.82267144e+01 + 6.00256155e+01i	5.50084037e−02 + 2.73108705e−02i	−1.65474256e−01 − 8.74325313e−01i
0	0	(1+3/4i) 20		Pro	2.18720402e+01 + 1.69748407e+01i	−3.47455676e−02 − 4.09615124e−02i	3.53508024e−02 + 8.55512159e−01i
0	1	(1+3/4i) 20		Pro	6.61052450e+01 + 5.09337263e+01i	−1.86395360e−02 − 2.19543826e−02i	−2.39785921e−02 + 4.07982154e−01i
0	2	(1+3/4i) 20		Obl	2.56935925e+02 + 7.22737790e+02i	−2.55445410e+02 − 7.62763344e+02i	−2.14280028e+04 − 6.65075161e+03i
0	3	(1+3/4i) 20		Obl	2.56935925e+02 + 7.22737790e+02i	5.15777934e+01 − 7.02010195e+01i	−7.48251071e+02 − 2.31164090e+03i
1	1	$20\sqrt{2}$	82.5°	Obl	−6.62608483e+02 + 1.92282723e+02i	−6.38183239e+04 − 2.24521930e+05i	−2.49033999e+06 − 5.60757007e+06i
1	2	$200\sqrt{2}$	75°	Obl	−6.81912154e+04 + 3.97071788e+04i	9.09767490e+55 + 4.24988282e+56i	5.57846942e+58 + 1.08598335e+59i
1	3	$20\sqrt{2}$	67.5°	Pro	1.10776528e+01 + 2.61206172e+01i	3.66399689e−01 − 1.36935639e−01i	−4.17673071e+00 − 4.73542913e+00i
1	4	$200\sqrt{2}$	60°	Pro	4.23515050e+02 + 7.34845178e+02i	−4.46622017e−09 + 2.60371439e−08i	4.03436760e−06 − 1.42147750e−06i
1	5	$20\sqrt{2}$	52.5°	Pro	8.33329846e+01 + 1.12201499e+02i	1.37884345e+00 + 2.88997048e+00i	3.17195438e+01 − 3.17033885e+01i
3	3	$200\sqrt{2}$	45°	Pro	2.08260781e+02 + 1.99989137e+02i	−3.44330629e−12 − 3.82844150e−11i	−4.02443549e−09 + 4.79990764e−09i
3	4	$200\sqrt{2}$	37.5°	Pro	6.80467972e+02 + 5.16523773e+02i	−2.60817965e−12 + 4.83013584e−12i	8.10758985e−10 − 3.53034286e−10i
3	5	$200\sqrt{2}$	30°	Pro	1.23005566e+03 + 7.07071083e+02i	4.77547059e−11 + 2.20057916e−11i	−4.71197888e−09 − 6.89895126e−09i
3	3	$20\sqrt{2}$	45°	Pro	2.83577369e+01 + 1.98839189e+01i	−9.98471665e−01 − 5.19994344e−01i	5.16758633e+00 + 1.72331507e+01i
3	4	$20\sqrt{2}$	37.5°	Pro	7.49297681e+01 + 5.13430059e+01i	−2.00381999e+00 − 2.28550597e+00i	−1.99261253e+00 + 4.31871548e+01i
3	5	$20\sqrt{2}$	30°	Pro	1.28377385e+02 + 7.02592909e+01i	8.18626749e+00 − 3.13154480e+01i	−3.31331032e+02 + 2.10442556e+02i
4	4	$200\sqrt{2}$	22.5°	Pro	2.76588216e+02 + 1.08228546e+02i	−2.57734140e−14 − 6.79600600e−14i	−3.70059261e−13 + 1.18316534e−11i
4	5	$200\sqrt{2}$	15°	Pro	8.33944637e+02 + 2.19593771e+02i	−6.61149570e−14 + 2.65401397e−14i	1.13666158e−11 − 1.31862709e−12i
4	6	$200\sqrt{2}$	7.5°	Pro	1.41449695e+03 + 1.84574257e+02i	4.11004645e−13 + 4.04943129e−12i	2.18766576e−11 − 6.43661714e−10i
4	5	$20\sqrt{2}$	7.5°	Pro	9.92587462e+01 + 1.09491032e+01i	1.11029103e+01 − 5.89455918e+00i	−1.63601043e+02 + 5.69861065e+01i

Table 4.5: Some values for the PASWF and its derivative. $\eta = .5$ for all cases listed.

The computer codes used for evaluating the PSWFs in this work are available for educational and research use. Interested readers may contact the author or go to [99].

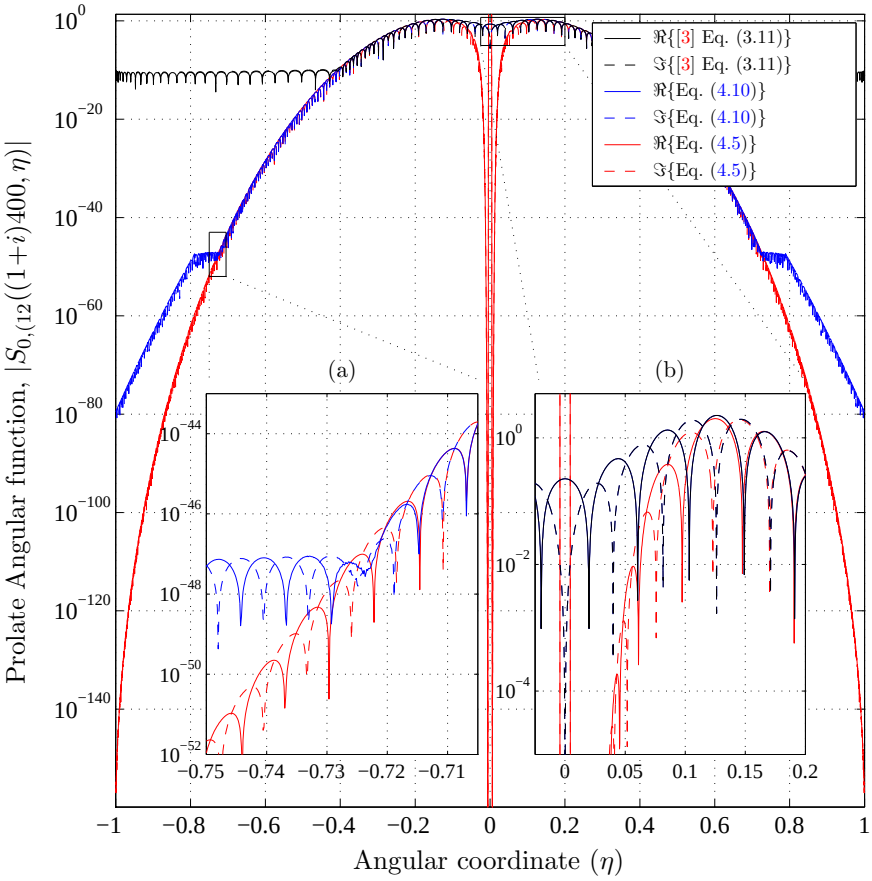

Figure 4-13: $\Re\{S_{0,12}((1+i)400,\eta)\}$ and $\Im\{S_{0,12}((1+i)400,\eta)\}$ ($n_p = 6$), for $-1 \leq \eta \leq 1$. Legendre expansion [Eq. (4.5), black curves], *prolate*-type asymptotic expansion [Eqs. (4.7) and (4.11), blue curves], and Miles [3] Bessel-type asymptotic approximation (Equation (3.11) therein, red curves). $\lambda^{(a)}_{0,12} = 5178.20430071674 + 5200.04621007177i$.

4.6 Summary and Conclusion

The asymptotic expansion of the spheroidal wave function (SWF) and its eigenvalues have been investigated for complex size parameter c. No fundamentally new asymptotic expansions are required, but instead, established *prolate*-type and *oblate*-type asymptotic expansions are directly applicable to finding the asymptotic expansion of the SWF for complex size parameter c. A rapid and accurate method for calculating the SWFs for complex c in the asymptotic regime has been presented based on a lookup table consisting of branch

points, $c_{\circ;r}^{mn}$. Through a brute force numerical method, this lookup table, based on the number of branch cuts (emanating from $c_{\circ;r}^{mn}$) traversed according to $\arg(c)$, is calculated once and recorded. This allows the assignment of any SWF to either a *prolate*-type or *oblate*-type expansion (ordered by n_p and n_o respectively) for any general complex size parameter c. The ordering for these expansions was found to be different than the original ordering of the spheroidal wave functions, with Table 4.3 documenting the ordering for the special case when $\arg(c) = \pi/4$. Normalizations, dependent on c, were derived for both the *prolate* and *oblate*-type asymptotic expansions and for both $(n - m)$ even and odd. These normalized asymptotic expansions for the PASWFs $S_{mn}(c, \eta)$ and $S'_{mn}(c, \eta)$, together with established asymptotic expansions for the prolate radial spheroidal wave functions $R_{mn}(c, \eta)$ and $R'_{mn}(c, \eta)$, allow the rapid calculation of the vector spheroidal wave functions for significantly larger (complex) size parameter c than previously possible.

Chapter 5

Broadband Analytical Magnetoquasistatic Electromagnetic Induction Solution for a Conducting and Permeable Spheroid

5.1 Introduction

Electromagnetic induction (EMI) techniques have long been important in geophysical surveying [100,101,102]. In the past ten years or so, intense interest has focused on small ultra-wide band (UWB) electromagnetic induction (EMI) (10's of Hz up to 100's of kHz) sensors used for shallow surveying to detect and discriminate metallic objects with one principal application being the detection and discrimination of unexploded ordnance (UXO) [103, 104, 105, 106, 107]. UXO contaminate wide regions of the globe and render those regions dangerous at best and uninhabitable at worst [108,109,110]. Typical UXO are nonspherical, however, thus complicating the detection and discrimination process. Some examples of UXO are pictured in Fig. 5-2.

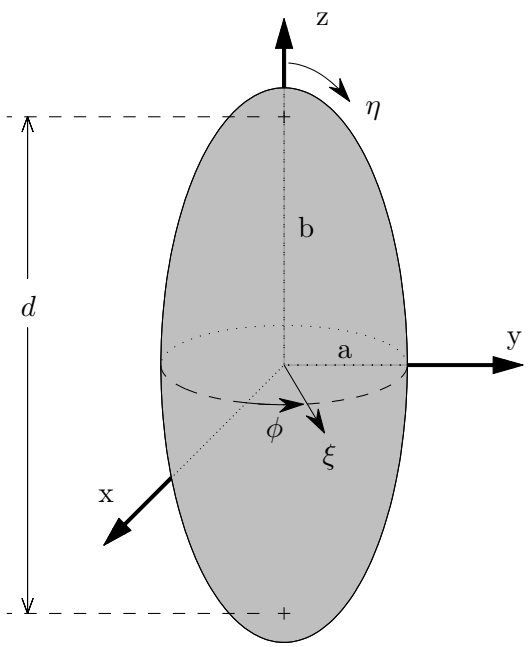

Figure 5-1: Spheroidal Geometry: $1 \leq \xi < \infty$ (Oblate case: $0 \leq \xi < \infty$), $-1 \leq \eta \leq 1$, $0 \leq \phi \leq 2\pi$, $x = \frac{d}{2}[(1 \mp \eta^2)(\xi^2 \mp 1)]^{\frac{1}{2}} \cos(\phi)$, $y = \frac{d}{2}[(1 \mp \eta^2)(\xi^2 \mp 1)]^{\frac{1}{2}} \sin(\phi)$, $z = \frac{d}{2}\eta\xi$, $e = \frac{b}{a}$, $\xi = \left(\pm(1 - e^{-2})\right)^{-\frac{1}{2}}$, $d = 2\left(\pm(b^2 - a^2)\right)^{\frac{1}{2}}$. Upper sign $\rightarrow$ prolate, lower sign $\rightarrow$ oblate.

While the solution for the induced magnetic field and potential within and around a conducting and permeable sphere under time harmonic excitation were first proposed by Wait [111, 112] over fifty years ago, lack of an analytical magnetoquasistatic solution for nonspherical shapes has hampered development of appropriate signal processing, inversion, and even instrument calibration. Recently some important progress has been made in formulating and evaluating spheroidal EMI solutions [63,62]. While special approximations resolved evaluation problems at high EMI frequencies, and the full analytical formulation worked well at low EMI frequencies, stability problems persisted in the mid-frequency range, depending on the geometrical and material properties of the spheroid. To remedy this, we construct a combined solution for the induced magnetic field from metallic spheroids that applicable over the entire EMI band, for both oblate and prolate spheroidal shapes.

Our combined system incorporates asymptotic approximations of the angular and radial spheroidal wave functions (ASWFs and RSFWs) (see Chapter 4) into these existing low and

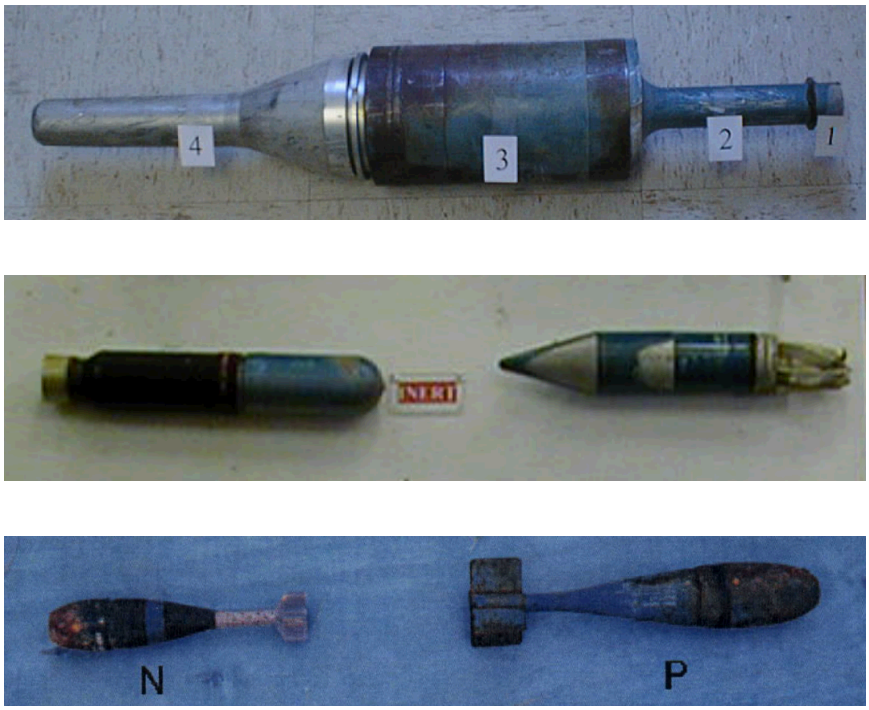

Figure 5-2: Some examples of unexploded ordnance (UXO).

high frequency solutions [63] for the induced magnetic field. This extends the frequency range over which the induced magnetic field external to the spheroid may be obtained to cover the problematical mid-frequency range. To be more specific, "low" frequency generally refers to frequencies ranging from static to frequencies below the resonant peak of the phase quadrature response of the induced magnetic field. "High" EMI frequencies are those at which the skin depth of the metallic object is very small relative to its dimensions. As frequency increases further into this "high" frequency range, the EMI response asymptotically approaches the high frequency limit of vanishing internal magnetic field and the object acts as a perfect (EMI) reflector.

Section 5.2 contains a brief review of the formulation for the exact solution for the magnetic field within and around a conducting and permeable spheroid under time harmonic excitation, valid at small size parameters (or frequencies). Section 5.3.1 presents an asymptotic-assisted analytical solution based on known asymptotic expansions (Chapter 4) of the spheroidal wave functions (SWFs). One other approach proposed in the literature to bridge this frequency gap is to implement a rational function approximation [62] which is based on the singular expansion method (SEM) [113]. However, this solution is neither rigorous nor exact, and can yield incorrect results for underdetermined systems. Our asymptotic-assisted solution realizes all higher order terms for mid-frequencies until it too encounters numerical difficulties for higher frequencies. A small penetration approximation (SPA) is then used to find the solution for the external induced magnetic field at high frequencies for which the exact and asymptotic-assisted solutions do not converge. This SPA solution is summarized in Section 5.3.2.

Results for the induced magnetic field from a conducting and permeable spheroid are compared to both Wait's solution (in the particular case of the sphere) as well as to a finite element/boundary integral (FE-BI) numerical solution [4] in the case of a spheroid, in Section 5.4.1. In Section 5.4.2, the induced fields predicted by our combined method are compared to ultrawide band (UWB) data obtained with the Geophex GEM-3 [114] instrument, for prolate and oblate, permeable (magnetic) and non-permeable machined spheroids. Model and data are found it to be in excellent agreement if the permeability and conductivity of the spheroids are allowed to vary within acceptable physical ranges. Results

illustrate that solutions are distinct based on spheroid characteristics and orientation. On this basis, our method could become the forward problem component on which inversion schemes may be based.

5.2 Exact Formulation

The formulation for the exact analytical solution for the induced magnetic field from a conducting and permeable spheroid under time harmonic excitation in the magnetoquasistatic regime is given in [63] and will only be summarized here.

Let a spheroid with high conductivity σ and relative permeability μ_r (as compared to the background medium) be excited by a time harmonic primary field $\overline{H}_\circ(\overline{r})e^{-\mathrm{i}\omega t}$ (time dependence expression $e^{-\mathrm{i}\omega t}$ is suppressed below). We assume the background medium is homogeneous, only weakly magnetic, and poorly conducting so that the wavenumber, $k_\circ$, of the host medium may be approximated by $k_\circ \approx 0$. As a result of this magnetoquasistatic approximation [102], the primary magnetic field, $\overline{H}_\circ(\overline{r})$, and the induced (or secondary) external magnetic field, $\overline{H}_\mathrm{s}(\overline{r})$, are irrotational and can be described by scalar potentials, $U_\circ(\overline{r})$ and $U_\mathrm{s}(\overline{r})$ respectively which both satisfy the Laplace equation in spheroidal coordinates. The known primary field potential can be expanded in terms of solutions of the first kind of the Laplace equation in spheroidal coordinates

$$U_\circ(\overline{r}) = H_\circ \frac{d}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} b_{pmn} P_n^m(\eta) P_n^m(\xi) T_{pm}(\phi), \tag{5.1}$$

where $P_n^m(\beta)$ represents the associated Legendre function of the first kind of degree n and order m [88]. Similarly, the secondary field potential can be expanded in terms of Laplace solutions of the second kind as

$$U_\mathrm{s}(\overline{r}) = H_\circ \frac{d}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} B_{pmn} P_n^m(\eta) Q_n^m(\xi) T_{pm}(\phi), \tag{5.2}$$

where $Q_n^m(\beta)$ represents the associated Legendre function of the second kind of degree n and order m [88] and B_{pmn} are unknown secondary induced field expansion coefficients.

The total external magnetic field is the sum of the primary and secondary fields

$$\overline{H}_2 = \overline{H}_\circ + \overline{H}_\mathrm{s} = -\nabla U_\circ - \nabla U_\mathrm{s}. \tag{5.3}$$

Within the highly conducting object, the internal magnetic field of the spheroid, $\overline{H}_1(\overline{r})$, can be found by solving the vector wave equation in spheroidal coordinates

$$\nabla \times \nabla \times \overline{H}_1(\overline{r}) - k_1^2 \overline{H}_1(\overline{r}) = 0 \tag{5.4}$$

where $k_1^2 = \mathrm{i}\omega\sigma_1\mu_1$. Under these assumptions, the Helmholtz equation above becomes a diffusion equation. $\overline{H}_1(\overline{r})$ can be expressed as an infinite series of vector spheroidal wave functions (VSWFs) $\overline{M}$ and $\overline{N}$ as

$$\overline{H}_1 = H_o \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} \left[A_{pmn}^{(M)} \overline{M}_{pmn}^{r(1)}(c_1;\eta,\xi,\phi) + A_{pmn}^{(N)} \overline{N}_{pmn}^{r(1)}(c_1;\eta,\xi,\phi) \right] \tag{5.5}$$

where the size parameter, c, of a spheroid is defined as

$$c = \frac{k_1 d}{2}, \tag{5.6}$$

k_1 is the wavenumber inside the spheroid, and d is the interfocal distance.

$\overline{M}_{pmn}^{r(1)}(c_1;\eta,\xi,\phi)$ and $\overline{N}_{pmn}^{r(1)}(c_1;\eta,\xi,\phi)$ are in turn composed of the angular and radial SWFs, $S_{mn}(c,\eta)$ and $R_{mn}(c,\xi)$ [66] as well as the harmonic functions $T_{pm}(\phi)$ defined as

$$T_{pm}(\phi) = \begin{cases} \cos(m\phi) & \text{p=0,} \\ \sin(m\phi) & \text{p=1.} \end{cases} \tag{5.7}$$

The exact solution for $\overline{H}_\mathrm{s}$ can be obtained by matching the tangential magnetic fields, $\overline{H}_\eta$ and $\overline{H}_\phi$, and the normal component of the magnetic flux density, $\overline{B}_\xi$, at the surface of

the spheroid, i.e.

$$H_{1\eta} = H_{2\eta} \tag{5.8a}$$

$$\mu_r H_{1\xi} = H_{2\xi} \tag{5.8b}$$

$$H_{1\phi} = H_{2\phi} \tag{5.8c}$$

where $\mu_r = \mu_1/\mu_2$ is the relative permeability of the spheroid compared to the surrounding medium. Substituting Eqs. (5.1)–(5.6) into Eq. (5.8) results in an infinite set of equations which must be massaged, truncated, and then cast into matrix form in order to be solved [63].

Even though this solution for the induced magnetic field from a conducting and permeable spheroid is theoretically applicable for any frequency satisfying the magnetoquasistatic assumptions, two problems limit the practical applicability of this solution. The first is that the basis functions used to express the magnetic field internal to the spheroid, in this case the angular (ASWF) and radial (RSWF) spheroidal wave functions, $S_{mn}(c,\eta)$ and $R_{mn}(c,\xi)$ respectively, are in general not orthogonal to each other for different m, n, and c as are the spherical wave functions. Because of this, the complexity of the solution is increased due to the infinite sets of equations arise when matching the boundary conditions at the spheroidal surface.

The second, more challenging difficulty is that the angular and radial SWFs, which must be evaluated as part of $\overline{M}^{r(1)}_{pmn}(c_1;\eta,\xi,\phi)$ and $\overline{N}^{r(1)}_{pmn}(c_1;\eta,\xi,\phi)$, become unstable at moderate size parameters (or frequencies) on the order of $|c| \approx 30$ [66, 63]. This tendency to diverge is shown qualitatively in Section 5.4.1 Fig. 5-4 where the frequencies to the left of the first dashed vertical line indicate the limited region of stability of the exact method. This numerical difficulty is typically encountered at frequencies lower than that where the resonant peak of the response is fully developed. Some other method is therefore desirable, to extend the range of frequencies over which solutions for the induced magnetic field are tractable . In the next section, we implement asymptotic expansions of the SWFs in order to extend the solution's range of stability to higher frequencies. However, the size parameter c for the wave (in this case, diffusion) equation is complex (see Section 5.2), and asymptotic

expansions of the SWFs for the case of complex size parameter have not been treated extensively in the literature (for a summary, see [115]).

5.3 High Frequency Approximations

5.3.1 Asymptotic Expressions for the Spheroidal Wave Functions

We use asymptotic expansions of the spheroidal wave function for complex size parameter c [115,82] to extend the range over which the full analytical solution is stable to larger c than previously possible. Through the use of these asymptotic expansions, the maximum c for which the full analytic solutions is stable is increased closer to the size parameter for which the small penetration approximation (SPA [63], see Section 5.3.2) is valid. Accordingly, the accuracy of the induced field solution for this mid-frequency range is on the same order as the accuracy of the asymptotic expansions. Unfortunately, some aspect of this asymptotic-assisted solution also becomes numerically unstable at a frequency that is a function of the relative permeability, μ_r, the overall dimensions a and b of the spheroid, and the aspect ratio $e = b/a$.

The original Legendre and Bessel function expansions for the ASWF and RSWF respectively become unstable in double precision for size parameters larger than $|c| \approx 30$ [63, 66]. Consequently, the full analytic solution outlined in Section 5.2 fails to converge for spheroidal size parameters beyond this range. For these larger c, asymptotic expansions of the SWFs do exist which remain stable. However, these established asymptotic expansions traditionally treat only the cases of purely real (for the case of the prolate SWFs) or purely imaginary (oblate SWFs) size parameter [66, 76, 77, 115].

When c is complex, asymptotic representations of the prolate SWFs (for prolate spheroids) and the oblate SWFs (for oblate spheroids) consist of either the *prolate*-type or *oblate*-type asymptotic expansions (Chapter 4). The choice of which type of asymptotic expansion is appropriate is determined by the spheroidal parameters m, n, and the size parameter c through a comparison of $\arg(c)$ to a look up table containing the branch points, $c_{\circ;r}^{mn}$ of the spheroidal wave equation. These branch points are found once through polynomial estimation techniques followed by a Newton-Raphson multivariable root finding method and

	exp.	m=0		exp.	m=1		exp.	m=2	
$(n-m)$	type	n_p	n_o	type	n_p	n_o	type	n_p	n_o
0	Pro.	0	-	Pro.	1	-	Pro.	2	-
1	Obl.	-	0	Pro.	2	-	Pro.	3	-
2	Obl.	-	1	Obl.	-	1	Pro.	4	-
3	Pro.	1	-	Obl.	-	2	Obl.	-	2
4	Pro.	2	-	Pro.	3	-	Obl.	-	3
5	Obl.	-	2	Pro.	4	-	Pro.	5	-
6	Obl.	-	3	Obl.	-	3	Pro.	6	-
7	Pro.	3	-	Obl.	-	4	Obl.	-	4
8	Pro.	4	-	Pro.	5	-	Obl.	-	5
⋮		⋮			⋮			⋮	

Table 5.1: Pattern governing n_p, n_o and which type of asymptotic expansion, either the *prolate*-type (Pro. above) or the *oblate*-type (Obl. above), for different combinations of m and n. c is of the form $c = (1+\mathrm{i})\,\alpha$.

stored for reference (see Chapter 4). $c_{\circ;r}^{mn}$ for $0 \le m \le n \le 110$ have been found, but for the purposes of this work, only $m \le 1$, $n \le 60$ are used. The diffusion equation, Eq. (5.4), dictates $\arg(c) = \pi/4$, for which a simple pattern, alternating between the *prolate* and *oblate* expansion types soon develops. Beginning at $n = m$, this pattern consists of $m+1$ *prolate*-type expansions followed by alternating pairs of *oblate* then *prolate*-type expansions.

When computing the *prolate*-type and *oblate*-type asymptotic expansions of the spheroidal eigenvalues, λ_{mn}, (and $S_{mn}(c,\eta)$, denoted $\lambda_{mn}^{(a)}$ and $S_{mn}^{(a)}(c,\eta)$ respectively) for complex c, the ordering is different from the ordering, according to n, of the eigenvalues themselves. While the n indicates λ_{mn} ordering for each m, the $\lambda_{mn}^{(a)}$ are ordered according to m, but independently, corresponding to the sequential ordering for each type of expansion separately. Let n_p be the number of *prolate*-type expansions of λ_{mn} necessary, up to but excluding n, and n_o be the number of *oblate*-type expansions of λ_{mn} necessary, up to but excluding n. λ_{mn} can then be approximated by *prolate*-type $\lambda_{mn}^{(a)}$ ordered according to n_p beginning at $n_p = m$, and *oblate*-type $\lambda_{mn}^{(a)}$ ordered according to n_o beginning at $n_o = m$. This behavior can clearly be seen in Fig. 5-3, is summarized in Table 5.1, and can be succinctly written as (Chapter 4)

$$n_p = \begin{cases} n, & (n-m) \le m \\ \left\lfloor \frac{(n-m)-m}{2} \right\rfloor + m + m, & (n-m) > m \end{cases} \tag{5.9}$$

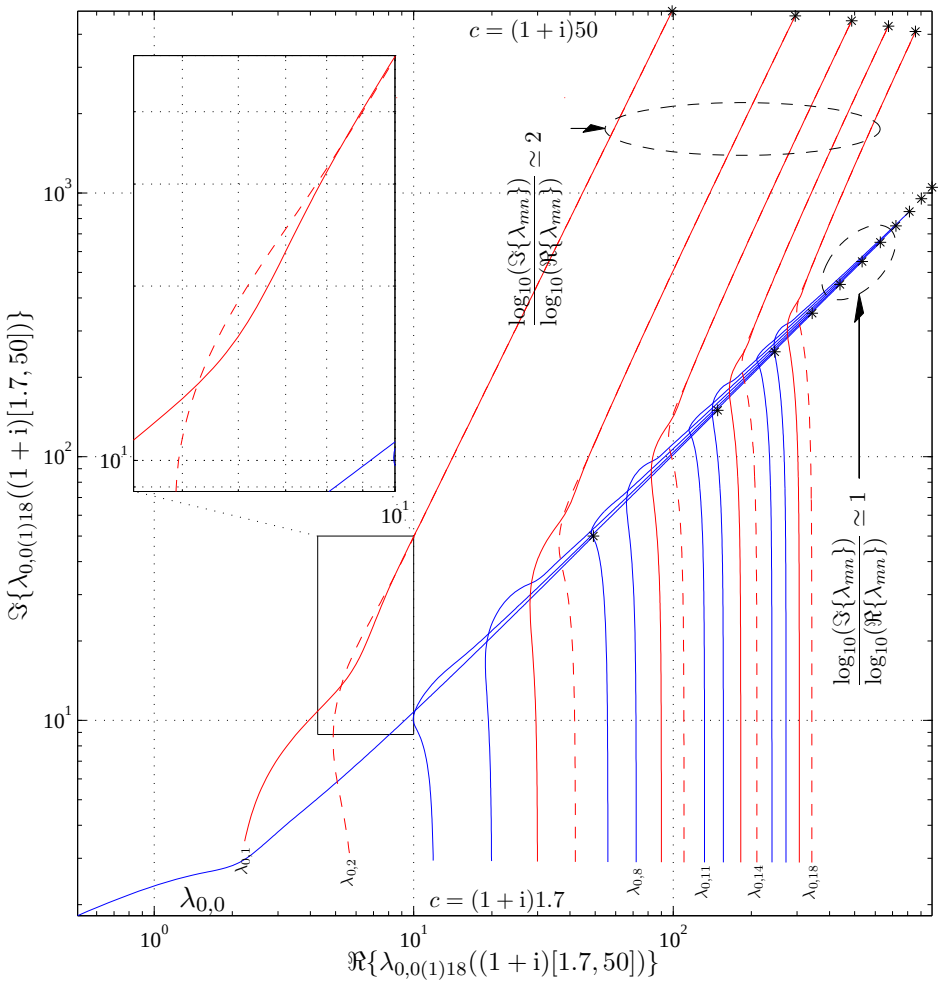

Figure 5-3: λ_{mn}: Eigenvalues of the spheroidal wave equation. $m = 0$ and $n = 0(1)18$. Each curve tracks an eigenvalue as c ranges from $c = (1+\mathrm{i})1.7$ (lower end of each curve) to $c = (1+\mathrm{i})50$ (upper end of each curve). Dashed curves indicate coalescing oblate eigenvalue pairs.

and

$$n_o = \left\lfloor \frac{(n-m)-m}{2} \right\rfloor + m, \tag{5.10}$$

where $\lfloor\ \rfloor$ denotes the floor function.

The *prolate*-type asymptotic expansion of the angular SWF, $S^{(a)}_{mn}(c,\eta)$, used in this work

is found by expanding $S_{mn}(c,\eta)$ in terms of an infinite series of the form,

$$S_{mn}(c,\eta) \approx S_{mn}^{(a)}(c,\eta) = (1-\eta^2)^{\frac{m}{2}} \sum_{r=-\infty}^{\infty}{}' C_r D_{n_p-m+r}(x) \tag{5.11}$$

where $D_{n_p-m+r}(x)$ is the parabolic cylinder function of order (n_p-m+r) [88] (note the dependence on n_p), the prime indicates summation over even r, and C_r are expansion coefficients (Chapter 4 and [66,76]). The eigenvalues in this case are expanded in an inverse power series as

$$\lambda_{mn} \approx \lambda_{mn}^{(a)} = \sum_{k=0}^{\infty} \Gamma_k(m,n_p) c^{-k+1} \tag{5.12}$$

where the functional dependence of the expansion coefficients Γ_k has been made explicit. C_r and $\Gamma_k(m,n_p)$ are found by substituting Eqs. (5.11) and (5.12) into the spheroidal wave equation [76].

If a given combination of m, n, and $\arg(c)$ requires an *oblate*-type asymptotic expansion for $S_{mn}(c,\eta)$, it is expressed as an infinite series of associated Laguerre polynomials, $L_{\nu+r}^{(m)}(x)$ [96,88],

$$S_{mn}^{(a)}(-\mathrm{i}c,\eta) = (1-\eta^2)^{\frac{m}{2}} \sum_{r=-\nu}^{\infty} A_r \left\{ e^{-c(1-\eta)} L_{\nu+r}^{(m)}[2c(1-\eta)] \right.$$
$$\left. +(-1)^{n-m} e^{-c(1+\eta)} L_{\nu+r}^{(m)}[2c(1+\eta)] \right\}, \tag{5.13}$$

where A_r are expansion coefficients and $\nu = \left\{ \begin{smallmatrix} (n_o-m)/2 \\ (n_o-m-1)/2 \end{smallmatrix} \right\}$ when (n_o-m) is $\left\{ \begin{smallmatrix} \text{even} \\ \text{odd} \end{smallmatrix} \right\}$ (Chapter 4 and [66,77]). The inverse power series representation for λ_{mn} now becomes

$$\lambda_{mn} = \sum_{k=0}^{\infty} \Gamma_k(m,n_o)(-\mathrm{i}c)^{-k+2}. \tag{5.14}$$

where $\Gamma_k(m,n_o)$ are expansion coefficients. In a manner similar to that used in the case of the *prolate*-type asymptotic expansion, A_r and $\Gamma_k(m,n_o)$ for the *oblate*-type asymptotic expansion can be found by substituting Eqs. (5.13) and (5.14) into the spheroidal wave equation [77].

The original expansion for radial SWFs in terms of an infinite series of spherical Bessel

functions also becomes inaccurate for large c [66]. Fortunately, the RSWFs adhere to the same pattern of asymptotic expansion types as in the case of the ASWFs. Additionally, $S_{mn}(c,\eta)$ and $R_{mn}(c,\xi)$ are proportional for a given c, i.e.

$$S_{mn}^{(i)}(c,\eta) = \kappa_{mn}^{(i)}(c) R_{mn}^{(i)}(c,\eta) \tag{5.15}$$

where the superscript (i) indicates the *kind* of the SWF [66]. For the *prolate*-type asymptotic expansion of $R_{mn}(c,\xi)$, we use Equation (3.11) in Miles [3]. For the *oblate*-type asymptotic expansion of the RSWF, we use Eq. (5.15) because $S_{mn}^{(a)}(-\mathrm{i}c,\eta)$ is much better behaved and suffers from fewer numerical problems for $|\eta| > 1$.

5.3.2 Small Penetration Approximation

For much higher frequencies which approach the limit of vanishing internal magnetic field, a small penetration approximation (SPA) for the internal magnetic field, $\overline{H}_1(\overline{r})$, has been developed [63, 7]. According to the SPA, the ξ-dependence of the azimuthal component of the internal magnetic field, $H_{1\phi}$, is approximated as a decaying exponential, $e^{-\mathrm{i}k_1 h_\xi \xi}$, leading to a self-consistent equation relating the internal fields of the spheroid at the boundary. Using the boundary conditions of Eq. (5.8), $\overline{H}_1$ can then be expressed in terms of the components of $\overline{H}_2(\overline{r})$, thus avoiding any reference to the SWFs and the accompanying computational difficulty. This approach yields stable solutions for very large c, up into the frequency range in which the magnetic field inside the spheroid becomes negligible as it approaches the high frequency limit of zero field. In this frequency range, and the induced field can be well modeled as a dipole response from a perfect EMI reflector.

This SPA solution applies at very high frequencies, but has also been found to be moderately accurate for lower frequencies, especially for highly permeable objects. In anticipation of the non-convergence of the asymptotic-assisted solution, both solutions are calculated over the entire frequency range. If the asymptotic-assisted solution remains convergent for frequencies at which the SPA solution is sufficiently accurate (arbitrarily set to some predefined tolerance), a seamless switchover to the SPA solution occurs. If instead the asymptotic-assisted solution fails to converge before some suitable switchover criterion is

satisfied, the asymptotic-assisted solution is abandoned and the SPA solution is adopted regardless. We choose the switchover frequency to be the frequency at which the dipole term of the asymptotic-assisted solution of the induced field is within some small error, B^{δ}_{0m1}, of the dipole term of the corresponding SPA solution in the asymptotic regime,

$$B^{\delta}_{0m1} = \left| \frac{B_{0m1} - B^{spa}_{0m1}}{B^{spa}_{0m1}} \right| \tag{5.16}$$

where $m = 0$ for axial excitation, and $m = 1$ for transverse excitation. For spheroids with $1 \leq \mu_r \leq \infty$ and $\frac{1}{8} \leq \mu_r \leq 8$, the asymptotic-assisted solution switches over to the SPA solution when B^{δ}_{0m1} is less than 5%. In the vast majority of cases, B^{δ}_{0m1} is less than 2% with only more extreme aspect ratios and slightly magnetic objects posing the most difficulty.

5.4 Validation and Results

5.4.1 Comparison with Simulation

The induced magnetic field potential and hence the magnetic field can be calculated anywhere external to the spheroid through Eq. (5.2) after the induced potential expansion coefficients B_{pmn} are found from the combined solution outlined in the previous sections. In this section, we compare this solution with results from both Wait's solution for a conducting and permeable sphere [111] and to results obtained from a hybrid solution based on a finite element/boundary integral method (FE-BI) combined with a thin skin approximation [4]. We compare both the axial ($m = 0$) and transverse ($m = 1$) excitations of prolate and oblate spheroids. For axial excitation, the primary field is aligned with the laboratory z-axis $\overline{H}_{\circ}(\overline{r}) = \hat{z}$, the axis of rotation of the spheroid is aligned with the z-axis, and $b_{001} = -1$ in Eq. (5.1) with all other $b_{00n} = 0$. In the case of transverse excitation, we choose to rotate the spheroid (as opposed to the laboratory reference frame) so that the spheroid is aligned with the x-axis while the primary magnetic field is still $\overline{H}_{\circ}(\overline{r}) = \hat{z}$ mandating $b_{011} = 1$ with all other $b_{01n} = 0$. Following this convention, the Cartesian coordinates refer to the laboratory reference frame. The spheroid's location is then expressed in terms of this Cartesian system, while its orientation relative to the laboratory is referenced

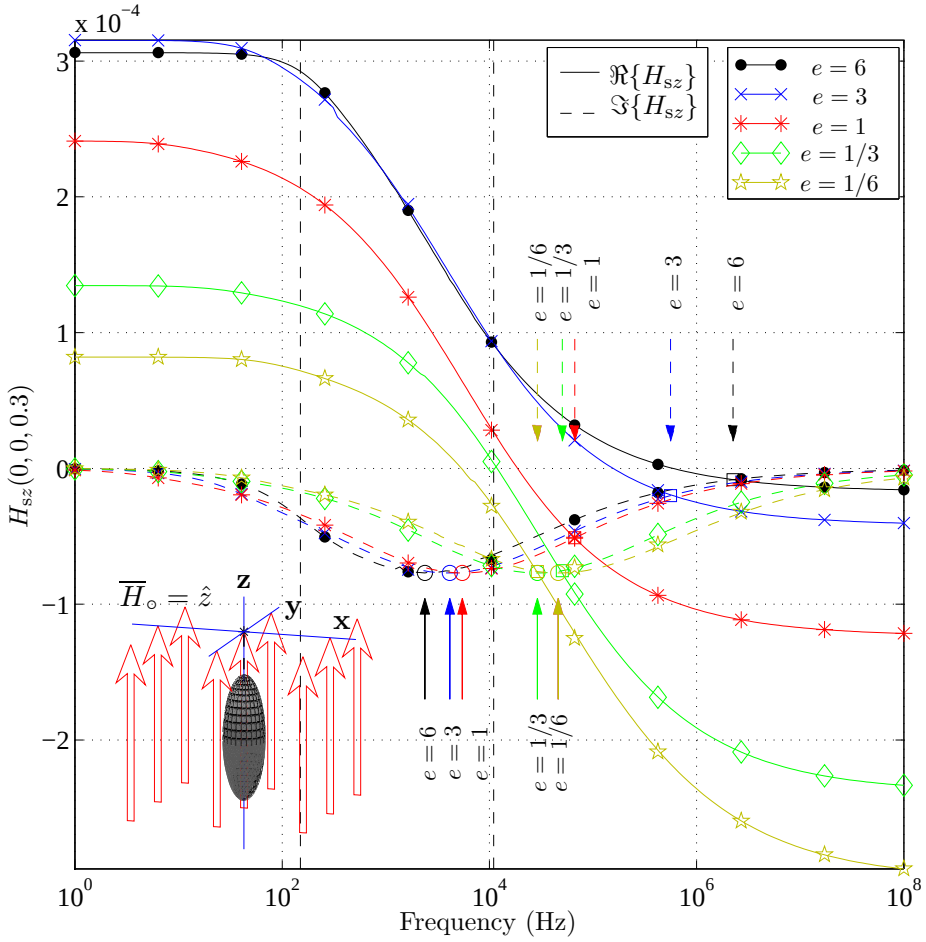

Figure 5-4: z-component of the induced secondary field, H_{sz}, from five different spheroids, aspect ratio $e = [6, 3, 1, 1/3, 1/6]$, with their longest dimension equal to 3cm, and axial excitation calculated by the combined method described in Sections 5.2 and 5.3. All spheroids were 30cm below the measurement point and under uniform excitation $\overline{H}_\circ = \hat{z}$ with $\sigma = 10^7$ and $\mu_r = 100$. Solid curves $\Rightarrow \Re\{H_{sz}\}$ (inphase component), while dashed curves $\Rightarrow \Im\{H_{sz}\}$ (quadrature component). Vertical dashed lines give a rough indication of the regions of validity for the exact, asymptotic-assisted, and SPA solutions, progressing from low to high frequency. Circles indicate the resonant peak of $\Im\{H_{sz}\}$. Squares indicate the frequency at which $|\Re\{H_{sz}\}| = |\Im\{H_{sz}\}|$. Dots indicate results from the corresponding numerical solution [4]. Note that H_{sz} for the sphere has been scaled by 1/3.

using the three Euler angles $\{\alpha, \beta, \gamma\}$ [43].

In all cases considered here, even at the "high" frequencies, we assume that electromagnetic phenomena are magnetoquasistatic in the sense that displacement currents are negligi-

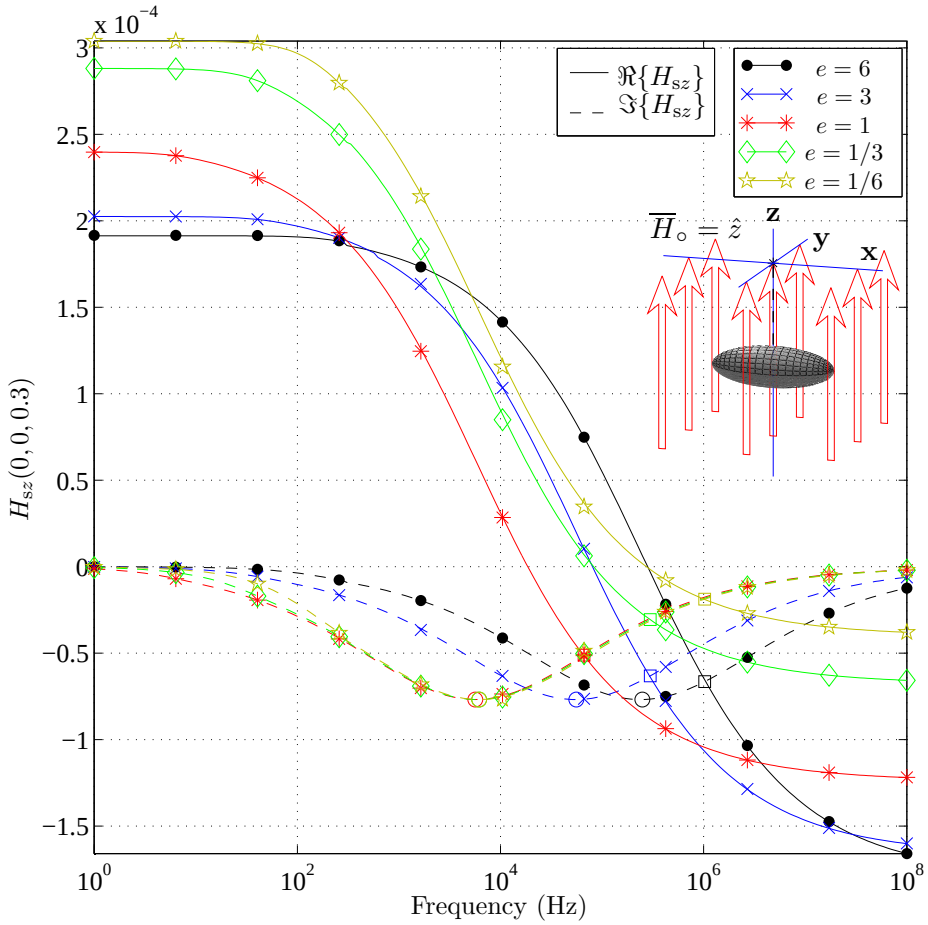

Figure 5-5: z-component of the induced secondary field, H_{sz}, from the same five spheroids as in Fig. 5-4 under transverse excitation. $\overline{H}_\circ = \hat{z}$ with spheroids' axes of revolution aligned along the x-axis. All spheroids were 30cm below the measurement point and under uniform excitation $\overline{H}_\circ = \hat{z}$ with $\sigma = 10^7$ and $\mu_r = 100$. Circles indicate the resonant peak of $\Im\{H_{sz}\}$. Squares indicate the frequency at which $|\Re\{H_{sz}\}| = |\Im\{H_{sz}\}|$. Dots indicate results from the corresponding numerical solution [4]. Again, the response for the sphere has been scaled by 1/3.

ble. Within the metallic object, the ratio of conduction to displacement currents is $\sigma/(\omega\varepsilon)$. Even at a frequency of 100 MHz (as shown in Figs. 5-4–5-6), this is $\approx 10^7/(10^8 10^{-11})$, so that conduction currents dominate by orders of magnitude [116, 117]. Accordingly, even at EMI frequencies above 1 MHZ, there is essentially no phase difference between one observation point and another within the length scale of interest (meters). External fields appear

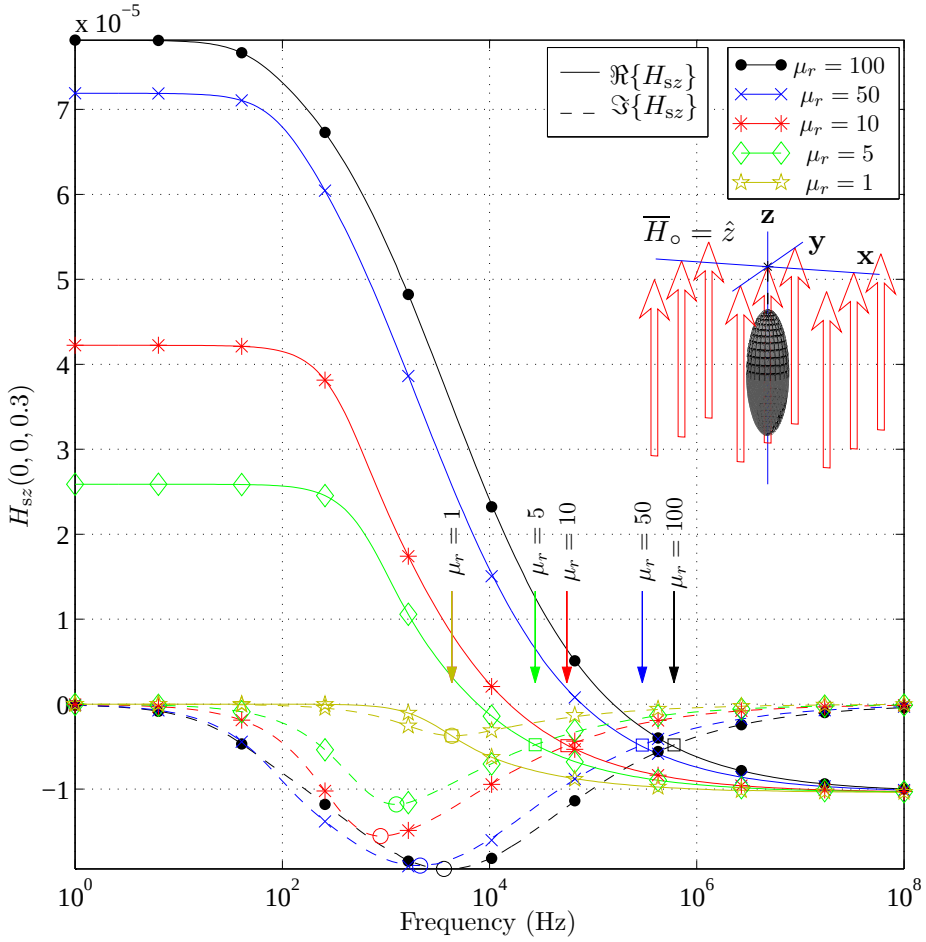

Figure 5-6: z-component of the induced secondary field, H_{sz}, from five spheroids under axial excitation with a similar configuration to that of Fig. 5-4, but with $\mu_r = [1, 5, 10, 50, 100]$ and aspect ratio $e = 3$ and $2a = 3$cm. All spheroids were 30cm below the measurement point and under uniform excitation $\overline{H}_\circ = \hat{z}$ with $\sigma = 10^7$. Circles indicate the resonant peak of $\Im\{H_{sz}\}$. Squares indicate the frequency at which $|\Re\{H_{sz}\}| = |\Im\{H_{sz}\}|$. Dots indicate results from the corresponding numerical solution [4]. $\overline{H}_\circ = \hat{z}$

.

static, being time dependent only as they follow imposed boundary conditions.

When the asymptotic representations of the SWFs outlined in Section 5.3 are utilized to find $\overline{H}_2(\overline{r})$, the solution is indeed stable for higher frequencies, as can be seen in Fig. 5-4, which shows combined results calculated by all three techniques outlined in this paper, viz. original exact solution, asymptotic-assisted solution, and the SPA solution. This

combined solution is capable of finding the induced secondary magnetic field over a broad EMI frequency range from static to very high frequencies.

Important characteristics of the broadband EMI response are illustrated in Figs. 5-4–5-6. For all the results reported in this work, our choice of sign convention follows the convention common in the geophysics field in that the conjugate of the magnetic field is shown. In Fig. 5-4, as the aspect ratio decreases from $e = 6$ (prolate) to $e = 1/6$ (oblate), the resonant peak of the induced field shifts to higher frequencies under axial excitation. Simultaneously, the frequency at which $|\Re\{H_{sz}\}| = |\Im\{H_{sz}\}|$ shifts from higher to lower frequencies.

Figure 5-5 shows $\overline{H}_2(\overline{r})$ for the same five spheroids considered in Fig. 5-4 except that they are under transverse excitation, i.e. the spheroids' axes of revolution are aligned with the x-axis, with the same primary field, $\overline{H}_\circ(\overline{r}) = \hat{z}$. Both Wait's solution ($e = 1$) and the numerical solution are included for reference. Figure 5-6 compares the combined analytical solution to solutions from the same numerical method for spheroids with aspect ratio $e = 3$ and $\mu_r = [1, 5, 10, 50, 100]$. Note that the high frequency limits of all the spheroids in Fig. 5-6 are identical because the spheroids' dimensions are equal, whereas the static limit and the peak and crossover points are characteristic of each spheroid. The distinct progression of the crossover point to lower frequencies as the permeability decreases is noteworthy.

For the spherical cases shown, the aspect ratio used for the combined method is slightly different from 1.0 (in this case 1.005) because $R_{mn}(c, \xi)$ possesses a singularity at $e = 1$ ($\xi = \infty$). Wait's solution is obscured by our analytical model. Results from our combined model, Wait's solution, and the FE-BI numerical method are essentially equal.

5.4.2 Comparison with Measurements

In this section we compare our results from the combined broadband method to measurements taken in the frequency domain using the UWB GEM-3 instrument [114]. The GEM-3 instrument is a monostatic, broadband, electromagnetic sensor used for subsurface geomagnetic sensing at magnetoquasistatic frequencies. The instrument is constructed as two large coils of 32cm and approximately 16cm diameters. The currents in these two concentric loops are in opposing directions thus creating a "magnetic cavity" region of near zero

primary field magnitude at the common center where the pickup coil(s) are located. The induced secondary response from metallic objects at 10 or more programmable frequencies are simultaneously recorded for later analysis. The frequency range of the GEM-3 is roughly 30Hz–100KHz (but with decreasing accuracy at the upper frequency range, see page 142) as in Figs. 5-20–5-23. Examples of the GEM-3 instrument are shown in Fig. 5-7 [5].

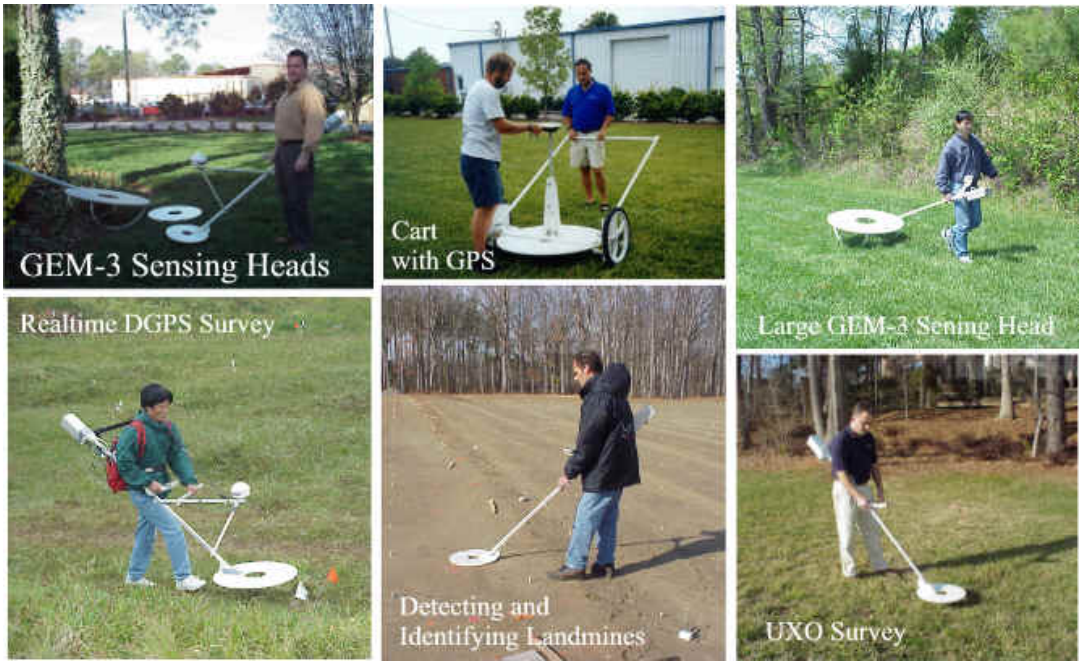

Figure 5-7: Geophex ultra-wideband GEM-3 instrument in various configurations [5].

The experimental setup used is pictured in Fig. 5-19. To minimize variance due to sensor position, the GEM-3's position and orientation was fixed relative to the laboratory. Test objects (see Table 5.2 and Fig. 5-18) were placed upon a supporting wooden platform built around the instrument. These objects were placed in various locations (e.g., a grid) relative to the instrument and their induced field responses were measured. The sensor head and grid are pictured in more detail in Fig. 5-8, while a qualitative representation of the magnetic field produced by the instrument is shown in Fig. 5-9 [7]. All of the measurements reported in this thesis (c.f. Figs. 5-20–5-23) were taken by my colleague Dr. Kevin O'Neill at Hanover, NH in October, 2003 [6].

For nonuniform primary fields such as those produced by the GEM-3 that still satisfy the

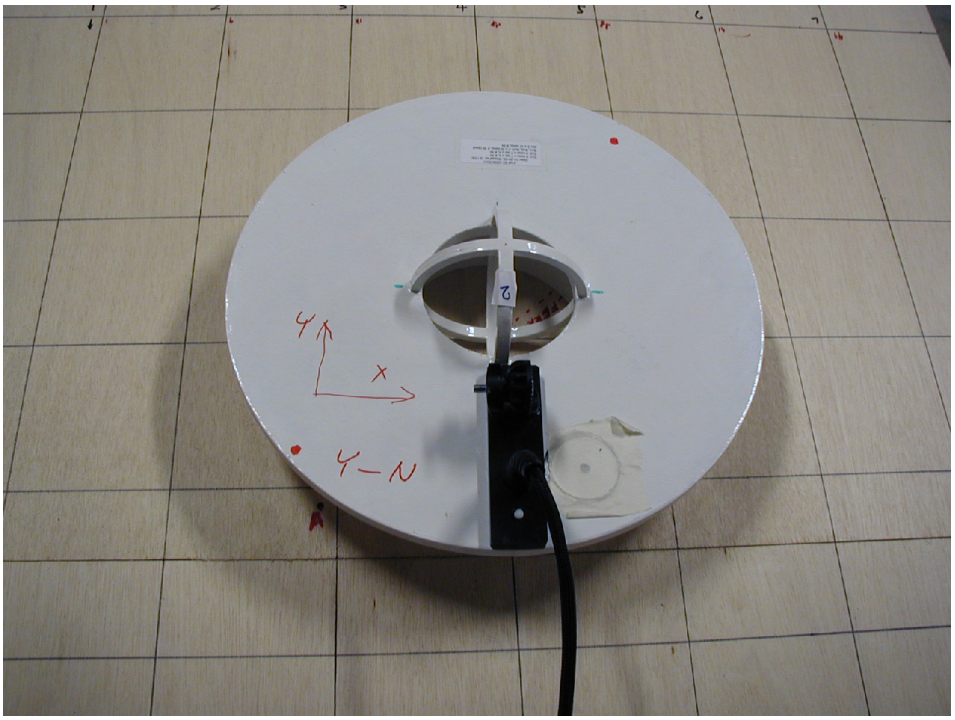

Figure 5-8: Geophex ultra-wideband GEM-3 instrument sensor head and measurement grid [6].

magnetoquasistatic assumptions, $b_{0mn} \neq 0$ for $n \neq 1$. In order to find the induced external magnetic field from Eqs. (5.2) and (5.3), the primary field from the GEM-3 must be expressed in terms of excitation coefficients b_{0mn}. The b_{0mn} are dependent upon the particular spheroidal coordinate system, (η, ξ, ϕ, d), and its orientation with respect to $\overline{H}_\circ(\overline{r})$.

One way to find these excitation coefficients is to utilize the orthogonality of the associated Legendre and harmonic functions in Eq. (5.1) in order to find an expression for b_{pmn} [88], i.e.

$$b_{pmn} = \frac{\int_0^{2\pi} T_{pm}(\phi) \int_{-1}^{1} P_n^m(\eta)\ U_\circ(\overline{r})\ d\eta\, d\phi}{\ell \pi H_\circ \frac{d}{2} \frac{P_n^m(\xi)}{(n+\frac{1}{2})} \frac{(n+m)!}{(n-m)!}}, \tag{5.17}$$

where

$$\ell = \begin{cases} 2\pi\widetilde{p}\,\mathrm{m}{=}0, \\ \pi \quad \text{otherwise} \end{cases} \tag{5.18}$$

with $\widetilde{p} = 1$ for $p = 0$ and vice versa. This assumes that the potential for the primary field

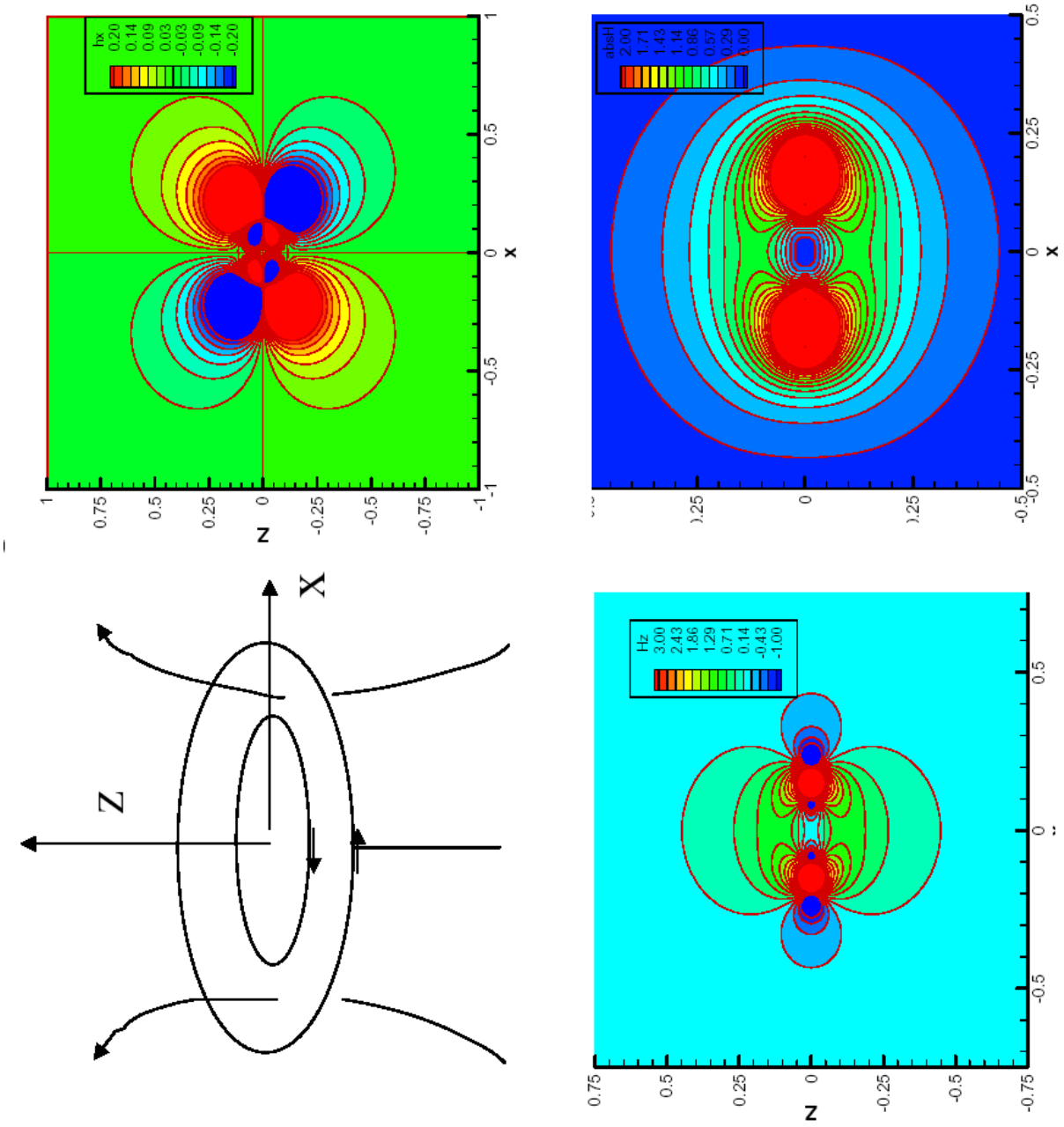

Figure 5-9: Magnetic field produced by the Geophex ultra-wideband GEM-3 instrument [7].

is known at the surface of the spheroid. If $U_\circ(\overline{r})$ is unknown, $\overline{H}_\circ(\overline{r})$ may be used instead by taking $-\nabla$ of both sides of (5.1) and performing the same procedure as shown in (5.17). These expressions involve multiple integrals which must be performed numerically unless some convenient analytical representation of $U_\circ(\overline{r})$ or $\overline{H}_\circ(\overline{r})$ is known.

A simpler method, which avoids time consuming numerical integrations, for obtaining b_{pmn} is to use point matching in (5.1) at $N_p > 2(\max(m)+1)(\max(n)+1)$ points on the surface of the spheroid defined by $\xi = \xi_\circ$. The resulting system of equations resolves up to $N \leq N_p$ expansion coefficients again assuming some convenient expression for $U_\circ(\overline{r})$ or $\overline{H}_\circ(\overline{r})$ is known. This system of equations can be expressed as

$$\begin{bmatrix} U_\circ(\overline{r}_1) \\ U_\circ(\overline{r}_2) \\ \vdots \\ U_\circ(\overline{r}_{N_p}) \end{bmatrix} = \begin{bmatrix} \Phi_1(\overline{r}_1) & \Phi_2(\overline{r}_1) & \cdots & \Phi_N(\overline{r}_1) \\ \Phi_1(\overline{r}_2) & \Phi_2(\overline{r}_2) & \cdots & \Phi_N(\overline{r}_2) \\ \vdots & \ddots & \ddots & \vdots \\ \Phi_1(\overline{r}_{N_p}) & \Phi_2(\overline{r}_{N_p}) & \cdots & \Phi_N(\overline{r}_{N_p}) \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_N \end{bmatrix} \tag{5.19}$$

where the potential at the i^{th} point on the surface is

$$\Phi_j(\overline{r}_i) = P_n^m(\eta_i) P_n^m(\xi_\circ) T_{pm}(\phi_i) \tag{5.20}$$

given some suitable mapping of $[p, m, n] \rightarrow j$. Equation (5.19) can easily be solved via (pseudo-) inversion or conjugate gradient techniques, yielding b_{pmn} in terms of the object's coordinates system. We adopt this approach, using an approximate representation of the GEM-3 primary field modeled as rings of magnetic sources [7]. Figure 5-11 depicts the primary and secondary induced potentials in the x–z plane which includes both the center of the GEM-3 (assigned to be the origin) and a spheroid located at $\overline{r} = [0.0, 0.0, -0.25]$ meters. The spheroid's axis of symmetry is aligned with the z-axis in this case. Figures 5-11 and 5-16 illustrate the dramatic falloff in magnitude of the induced field as compared to the primary field. For these realistic cases, the induced field at the sensor is five orders of magnitude smaller than the primary field even though the object is at a relatively shallow depth.

For high but still magnetoquasistatic frequencies (see Fig. 5-11), the induced magnetic

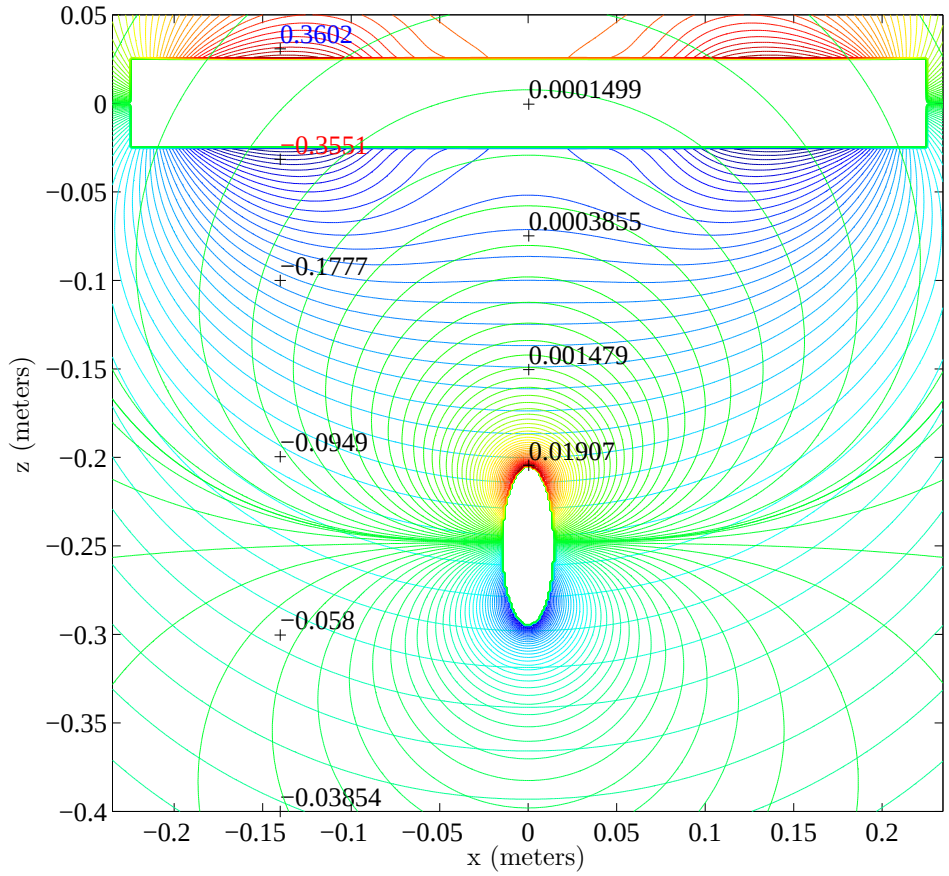

Figure 5-10: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at low frequency, under axial excitation, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

field will be out of phase with the primary field due to Lenz's law. Figure 5-16 shows $U_\circ(\overline{r})$ and $U_\mathrm{s}(\overline{r})$ for a low (near static) frequency case with the spheroid removed from the z-axis. In this case, the induced response in phase with the primary field.

We now compare the theoretical induced external magnetic field predicted by our combined broadband technique with the induced field measured by the GEM-3 instrument. The primary field and potential, $\overline{H}_\circ(\overline{r})$ and $U_\circ(\overline{r})$ respectively, at the surface of the spheroid are described by 12 rings of dipoles whose location and magnitudes are given by the method found in [7]. The primary field potential expansion coefficients, b_{pmn}, are then found via

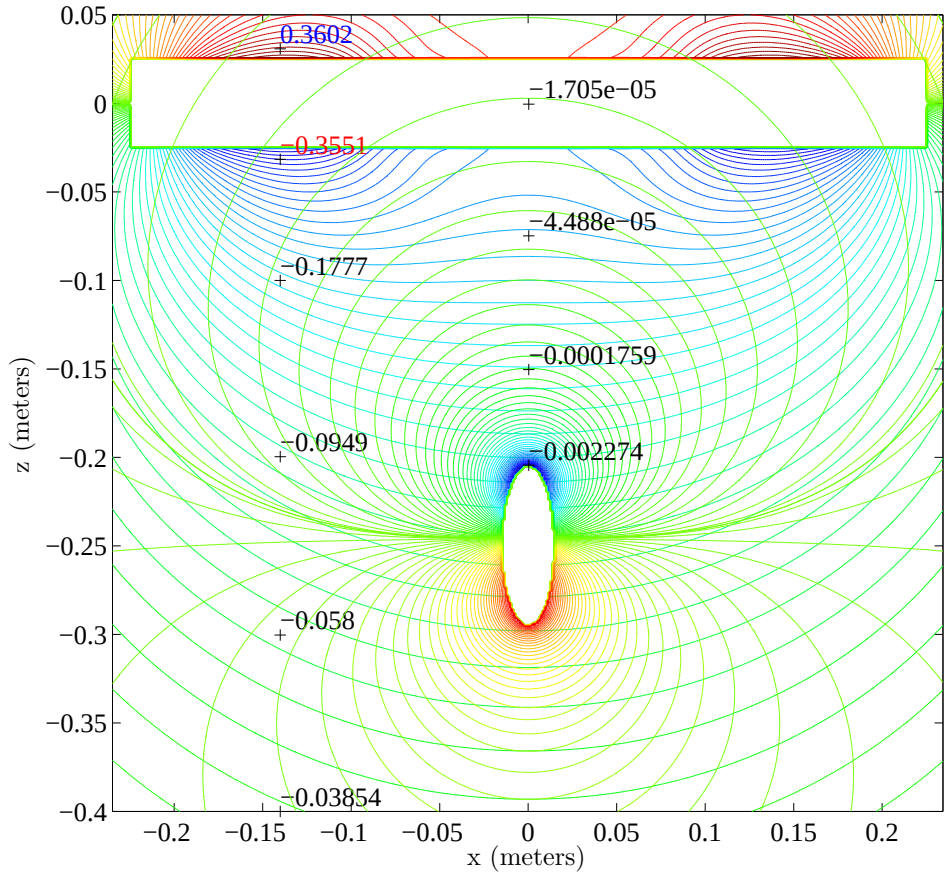

Figure 5-11: Primary field potential from the GEM-3, $U_{\circ}(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_{\mathrm{s}}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at high frequency, under axial excitation, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

Eq. (5.19).

I machined a set of 17 spheroids for test objects: three spheres, six prolate spheroids, and eight oblate spheroids machined from aluminum and mild steel. Table 5.2 summarizes the spheroids' designations and characteristics, and they are pictured in Fig. 5-18. The induced magnetic fields for prolate steel spheroid S2 for both axial and transverse orientations are shown in Fig. 5-20. Figure 5-21 shows similar results for the aluminum spheroid A1 and Figs. 5-22 and 5-23 show similar comparisons for oblate spheroids S8 and A7 respectively. The measurements were taken from a position directly above each object.

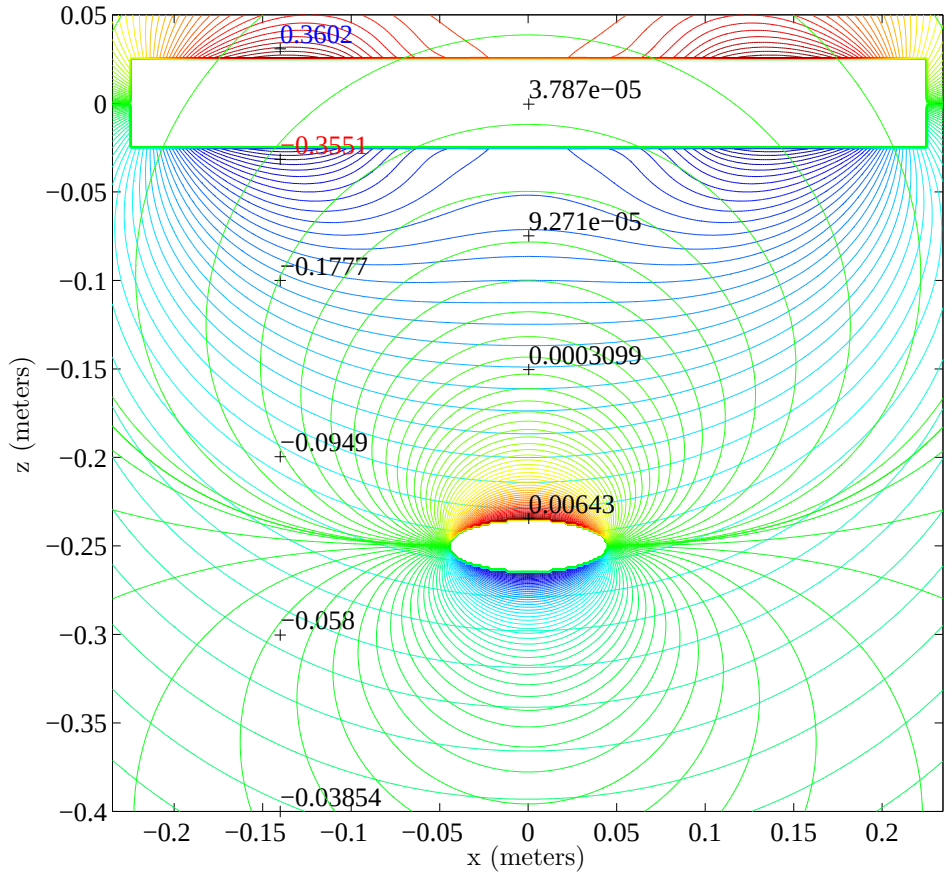

Figure 5-12: Primary field potential from the GEM-3, $U_{\circ}(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_{\mathrm{s}}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at low frequency, under transverse excitation, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

The distance from the center of the receiver to the closest edge of each object was 25cm in the case of the steel objects, and 15cm in the case of the aluminum objects. The experimental setup is shown in Fig. 5-19.

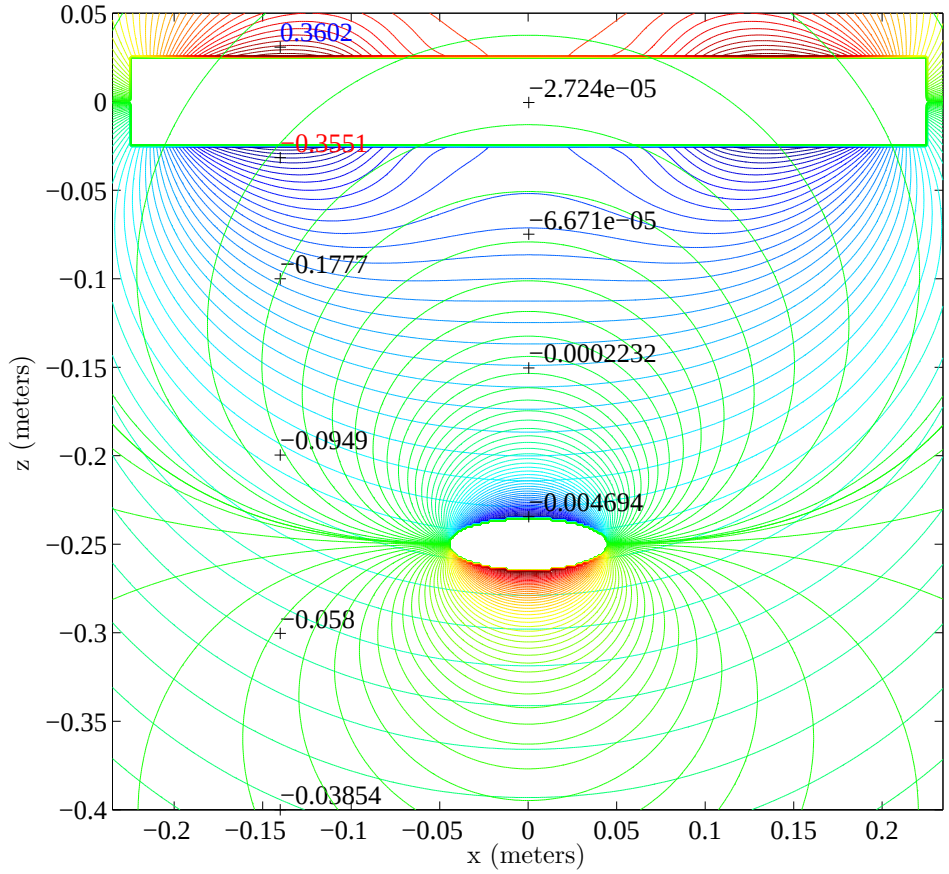

Figure 5-13: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at high frequency, under transverse excitation, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

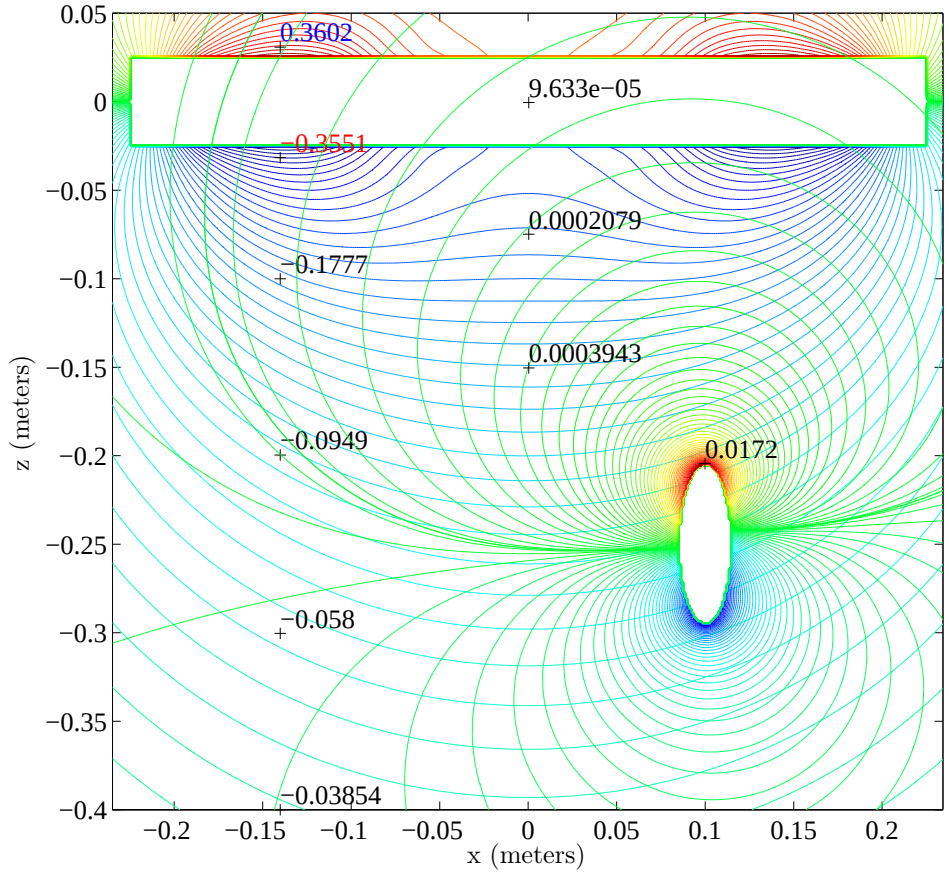

Figure 5-14: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at low frequency, under axial excitation, offcenter, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

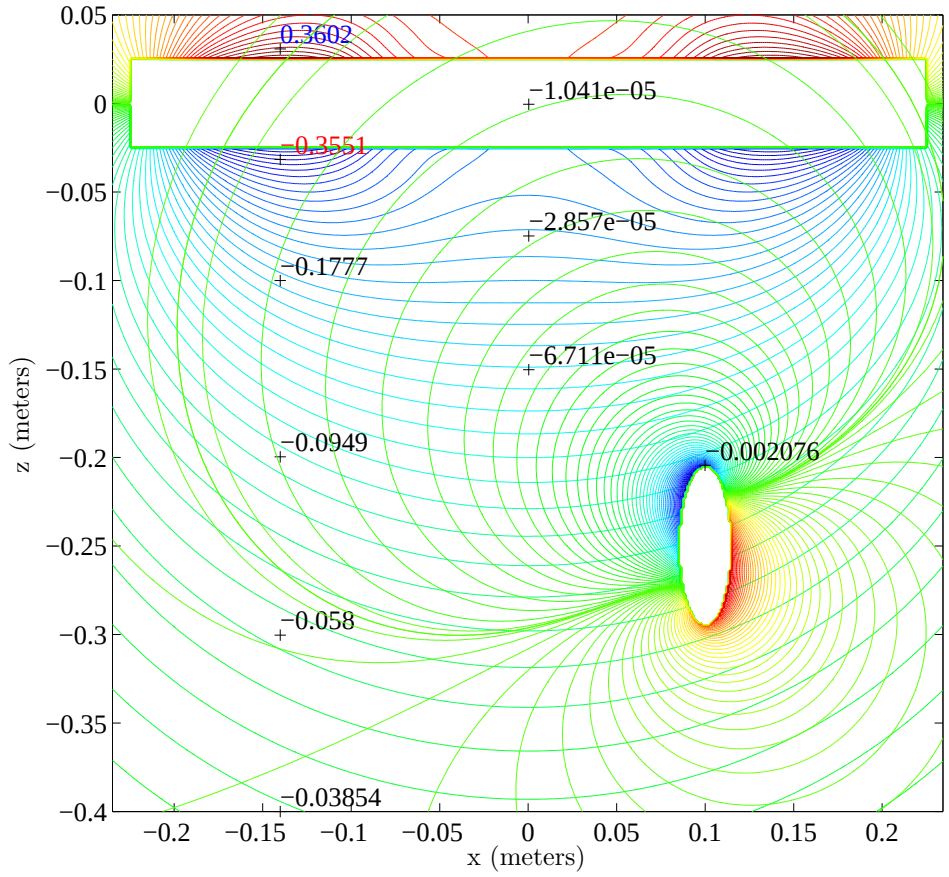

Figure 5-15: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at high frequency, under axial excitation, offcenter, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

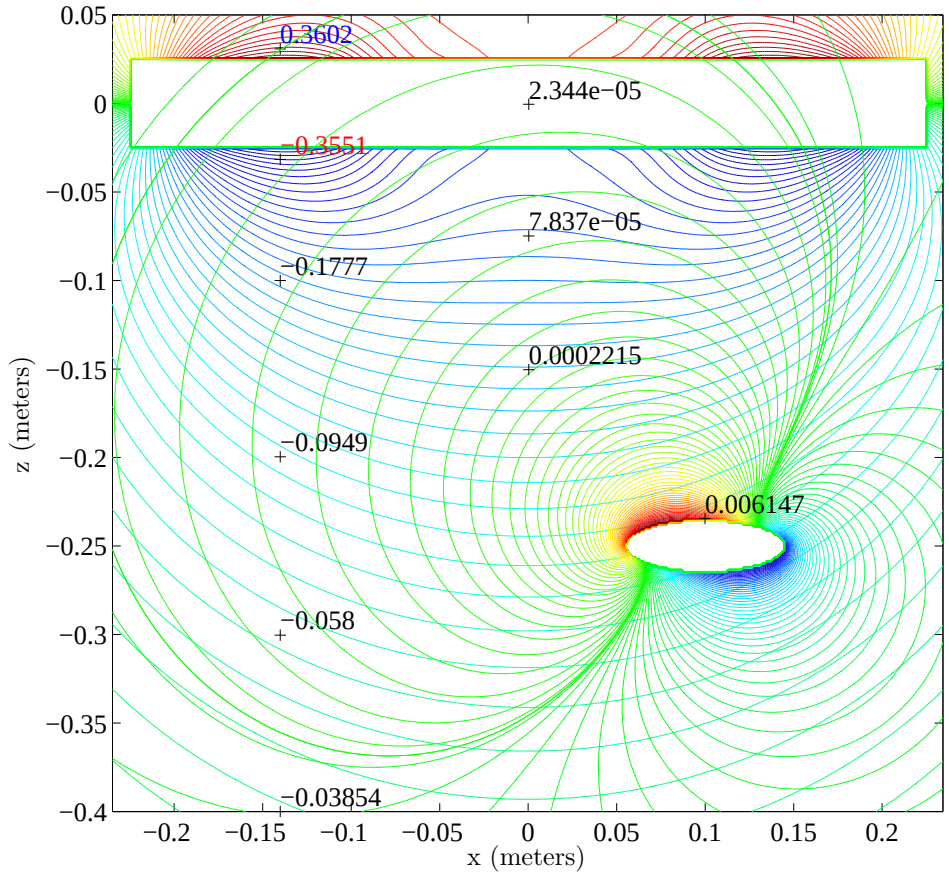

Figure 5-16: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at low frequency, under transverse excitation, offcenter, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

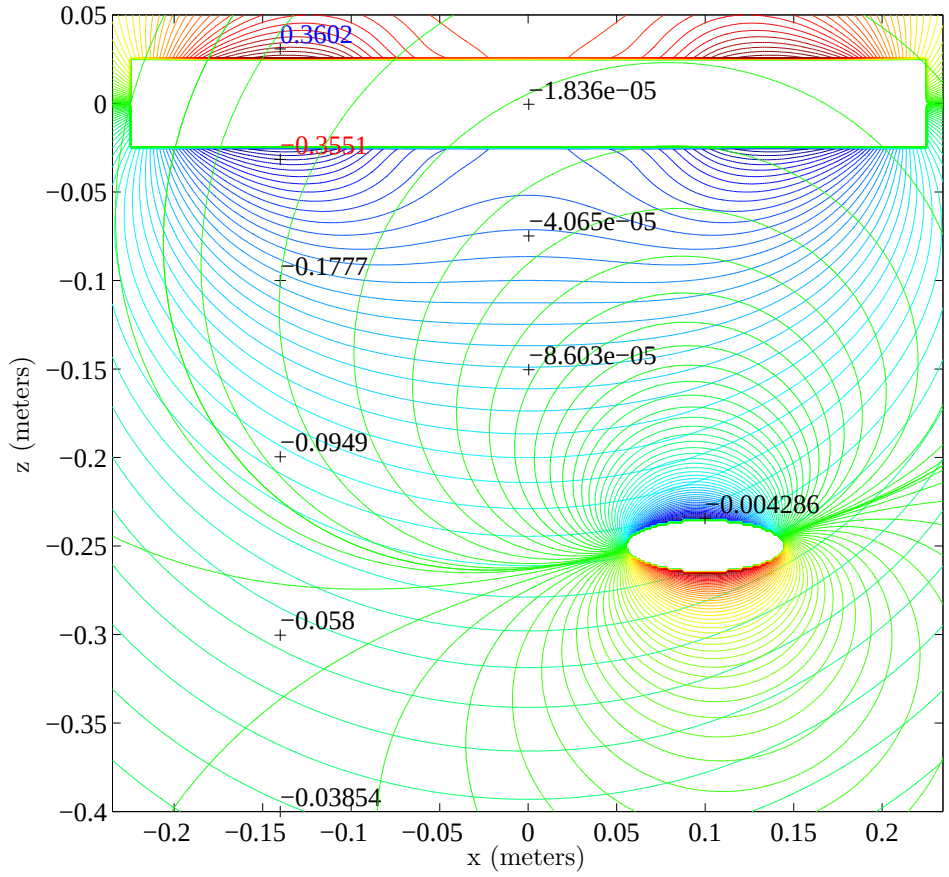

Figure 5-17: Primary field potential from the GEM-3, $U_\circ(\overline{r})$ (magnitudes marked on left), and the secondary potential induced by the spheroid, $U_\mathrm{s}(\overline{r})$ (magnitudes marked in center) in the $x-z$ plane for the S2 spheroid (see Table 5.2), at high frequency, under transverse excitation, offcenter, and choosing $\mu_r = 100$ and $\sigma = 10^7$. Color of induced response has been adjusted for clarity. Axes are in meters. The white rectangle encompasses the GEM-3 instrument.

Figure 5-18: Spheroid collection used for testing. Specifications are listed in Table 5.2.

Figure 5-19: GEM-3 experimental setup.

Therefore, the distance to the center of S2 under axial orientation was $z = -34$cm, while the depth to the center under transverse orientation was $z = -26.5$cm, etc.

	comp.	type	semiaxis (a)	semiaxis (b)	nominal $e = b/a$
S1	Iron	sphere	90.62mm	90.62mm	1
S2	Steel	PS	30.02mm	182.19mm	6
S3	Steel	PS	29.94mm	90.28mm	3
S4	Steel	PS	14.97mm	90.76mm	6
S5	Steel	OS	29.32mm	4.56mm	1/6
S6	Steel	OS	29.59mm	9.65mm	1/3
S7	Steel	sphere	29.87mm	29.87mm	1
S8	Steel	OS	89.85mm	28.39mm	1/3
S9	Steel	OS	89.95mm	15.32mm	1/6
A1	Al	PS	30.17mm	180.23mm	6
A2	Al	PS	29.9mm	91.29mm	3
A3	Al	PS	15.04mm	91.14mm	6
A4	Al	OS	29.36mm	4.34mm	1/6
A5	Al	OS	29.36mm	8.88mm	1/3
A6	Al	sphere	29.91mm	29.91mm	1
A7	Al	OS	89.92mm	30.38mm	1/3
A8	Al	OS	89.98mm	15.94mm	1/6

Table 5.2: Physical dimensions of steel and aluminum spheroids. PS $\Rightarrow$ Prolate Spheroid. OS $\Rightarrow$ Oblate Spheroid.

Our implementation truncates the infinite sets of equations resulting from Eq. (5.8) at $m = 1$. This is partially due to the fact that at remote points in the "far field", or farther than one or two characteristic lengths (i.e. longest dimension of the object), the lower order, or $m = 0, 1$, terms of the induced magnetic field tend to dominate. Consequently, our solution includes effects for $m = 0$ and $m = 1$ only, which correspond to the axial and transverse excitations respectively [63]. Even with this limitation, the primary and secondary fields may be spatially nonuniform, however, because all n in the range $n \leq 60$ are considered. With reference to the GEM-3 primary field (see Section 5.4.2), objects closer than one or two characteristic lengths from the instrument may have important interaction with $m \geq 2$ orders of the primary field. For objects this close, our model may not accurately predict the induced field response unless the response due to higher order terms are included.

The exact permeability (for the steel) and conductivity of the spheroids were not known

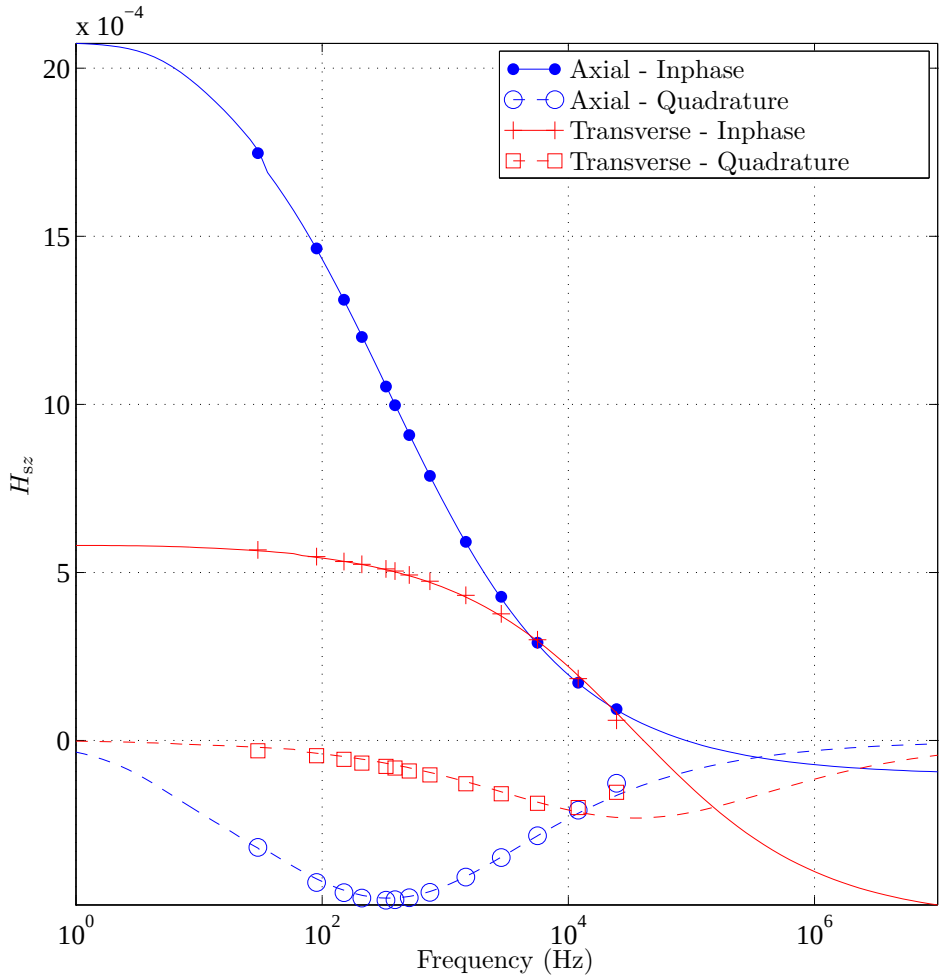

Figure 5-20: Comparison of GEM-3 induced magnetic field measurements, $H_{sz,d}$, from prolate steel spheroid S2 with combined model outlined in Sections 5.2 and 5.3. Both axial and transverse cases are shown. Orientations are similar to those shown in plots of $U_s(\overline{r})$ such as Fig. 5-11. Optimized permeability and conductivity were $\mu_r = 234.1$ and $\sigma = 4.315 \times 10^6 \mho/m$, respectively.

or measured beforehand. We sidestepped this lack of information by obtaining these parameters via inversion of the measurements. In other words, we extracted σ and μ_r using a simple search routine by fitting out model to the axial measurements first. We then used these extracted parameters for the transverse measurements from the same object and found them to deliver reasonable results. The resulting conductivity and permeability are within acceptable ranges for these metals.

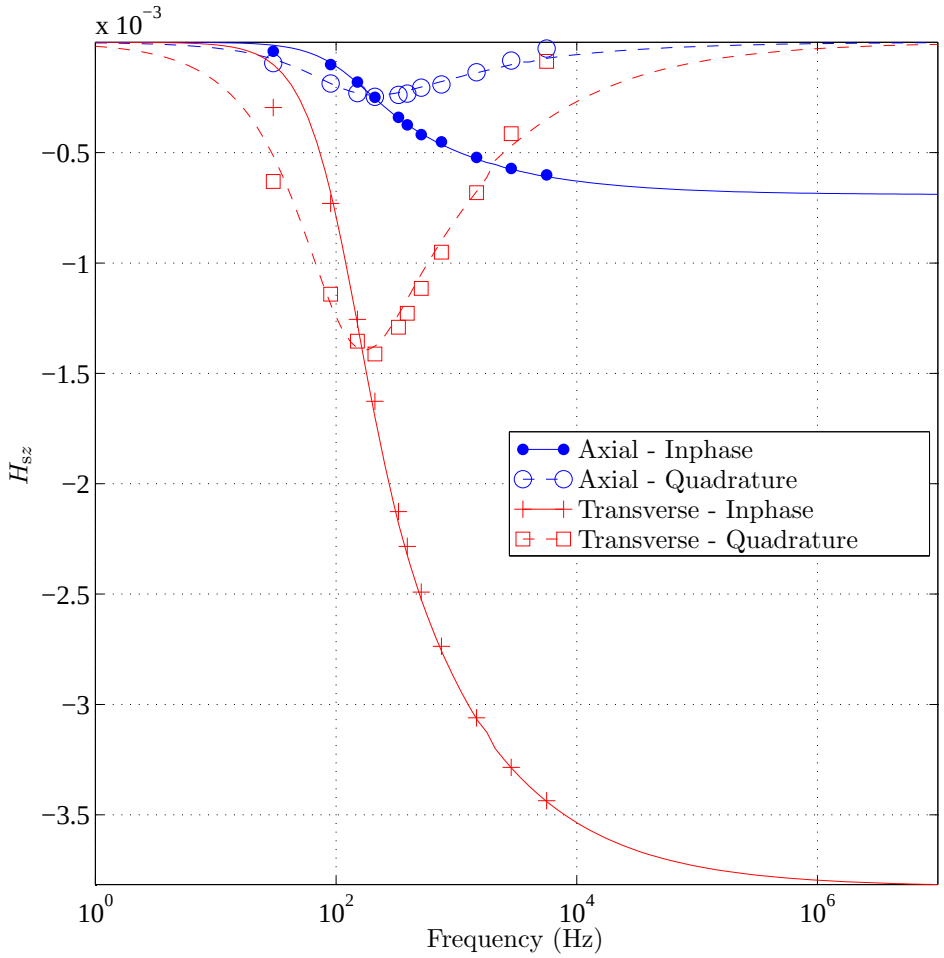

Figure 5-21: Comparison of $H_{sz,d}$ from prolate aluminum spheroid A1 with combined model outlined in Sections 5.2 and 5.3. Both axial and transverse cases. Optimized conductivity was $\sigma = 2.103 \times 10^7 \mho/m$.

The experimental GEM-3 we used for these measurements records reliable data at frequencies up to about 12kHz [118]. Measurements are not absolutely calibrated with respect to the primary field. Therefore, the following procedure to normalize and remove any possible constant offset was adopted. Let $H_{sz,d}$ be the z-component of the induced magnetic field measured with the GEM-3 and let H_z be the z-component of the induced magnetic field predicted by our model. In order to compare $H_{sz,d}$ and H_{sz} (as in Figs. 5-20–5-23), we

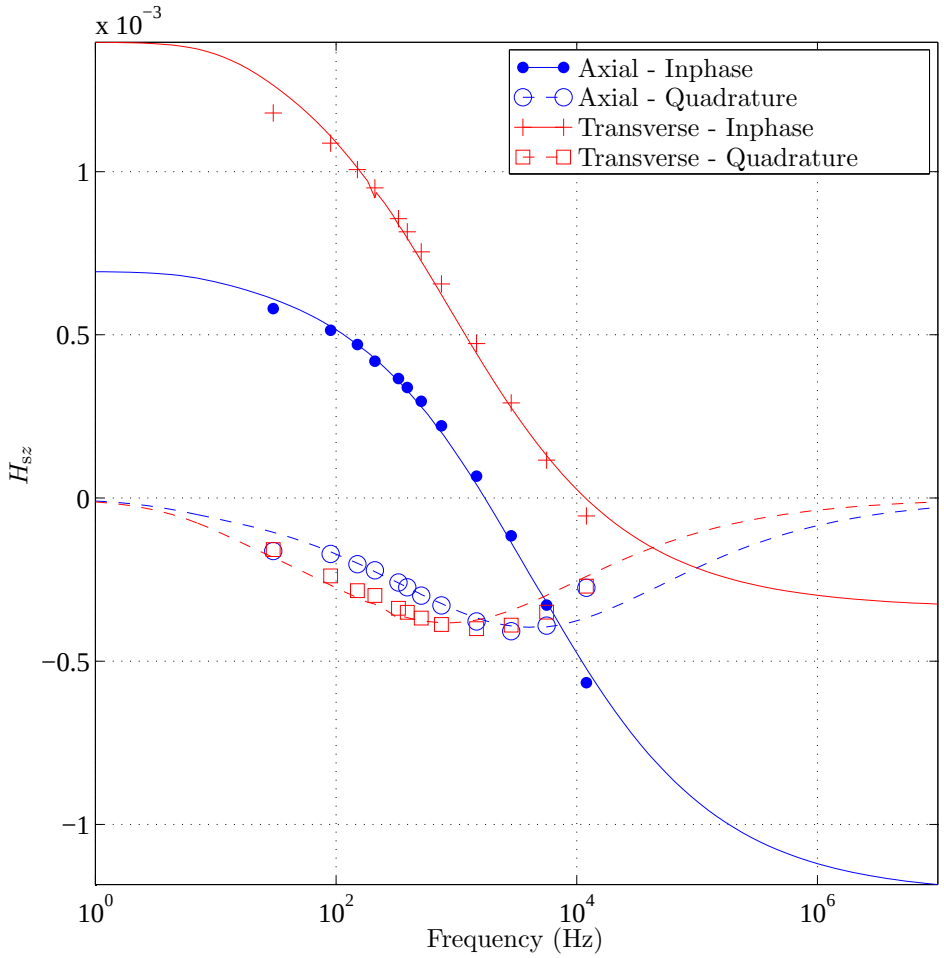

Figure 5-22: Comparison of $H_{sz,d}$ from oblate steel spheroid S8 with combined model outlined in Sections 5.2 and 5.3. Both axial and transverse cases. Optimized permeability and conductivity were $\mu_r = 268.0$ and $\sigma = 1.939 \times 10^7 \mho/m$, respectively.

set

$$H_{sz,d} = \left[\left(H_{sz,d} - \overline{H_{sz,d}} \right) \frac{\sum_f \left(\left| H_{sz} - \overline{H_{sz}} \right| \right)}{\sum_f \left(\left| H_{sz,dat} - \overline{H_{sz,dat}} \right| \right)} \right] + \overline{H_{sz}} \tag{5.21}$$

where the overline implies an average over frequency, and the summation is over all data frequencies used.

Data and model predictions are seen to agree quite well under all conditions except possibly for high frequencies where the accuracy of the data may be suspect [118]. The agreement is not as good for the cases comparing oblate spheroids. This may be due to not

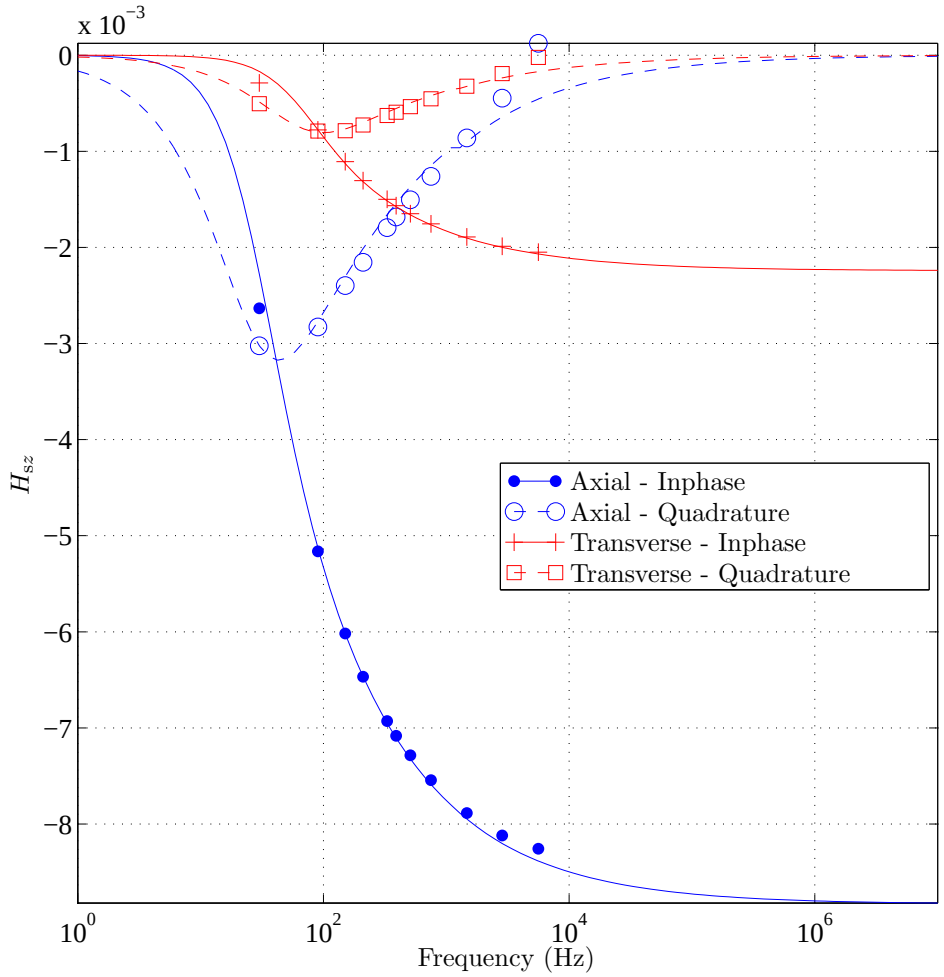

Figure 5-23: Comparison of $H_{sz,d}$ from oblate aluminum spheroid A7 with combined model outlined in Sections 5.2 and 5.3. Both axial and transverse cases. Optimized conductivity was $\sigma = 3.147 \times 10^7 \mho/m$.

having the globally optimal σ and/or μ_r for the these spheroids or due to our truncation of Eqs. (5.1) and (5.6) at $m = 1$.

5.5 Conclusion

We have constructed a broadband solution for the induced magnetic field from conducting and permeable prolate and oblate spheroids under time harmonic excitation in the magnetoquasistatic regime. Our combined method consists of the exact analytical solution,

an asymptotic-assisted analytical solution, and a small penetration approximation (SPA). This combined solution is accurate to within 5% error for most spheroids with aspect ratio $\frac{1}{8} < e < 8$. Results produced by this combined method were compared to results from a finite element/boundary integral numerical method and found to be in excellent agreement. Results were also compared to induced magnetic field measurements taken by the UWB GEM-3 instrument from metal prolate and oblate spheroids. If the permeability and conductivity of the spheroids is allowed to vary within accepted physical ranges, results were seen to match very well with measurements. Using this model, one can calculate the broadband induced magnetic field response from a flexible set of canonical shapes such as spheres, spheroids, plates, and needles is now possible. The results indicate that solutions from our model display distinct, systematic response patterns based on the spheroidal characteristics and orientation. On this basis, our method could become a forward model upon which inversions schemes may be based.

Chapter 6

Simultaneous Analytical Solution for the Broadband Magnetoquasistatic Electromagnetic Induction (EMI) Response from Multiple Conducting and Permeable Spheroids

6.1 Introduction

Electromagnetic induction (EMI) response techniques at low frequencies (0-100's kHz) comprise a promising method for the detection and discrimination of unexploded ordnance (UXO). The broadband analytical magnetoquasistatic solution for the induced EMI response from a conducting and permeable spheroid under time harmonic excitation has recently been theoretically and experimentally demonstrated [63, 119] and Chapter 5. This

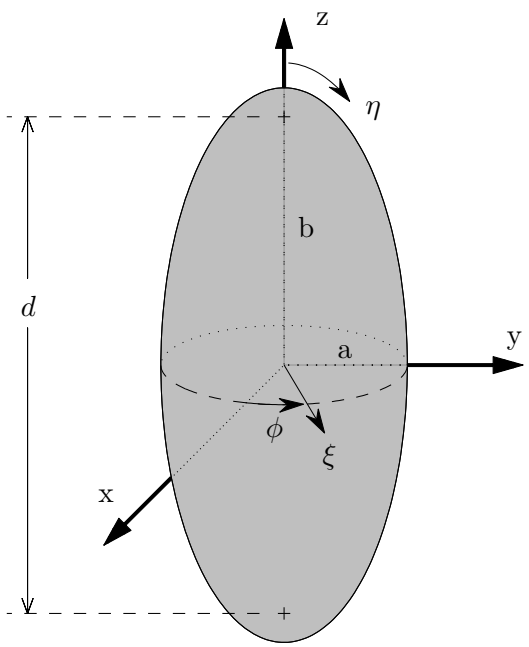

Figure 6-1: Spheroidal Geometry: $1 \leq \xi < \infty$ (Oblate case: $0 \leq \xi < \infty$), $-1 \leq \eta \leq 1$, $0 \leq \phi \leq 2\pi$, $x = \frac{d}{2}[(1 \mp \eta^2)(\xi^2 \mp 1)]^{\frac{1}{2}} \cos(\phi)$, $y = \frac{d}{2}[(1 \mp \eta^2)(\xi^2 \mp 1)]^{\frac{1}{2}} \sin(\phi)$, $z = \frac{d}{2}\eta\xi$, $e = \frac{b}{a}$, $\xi = \left(\pm(1 - e^{-2})\right)^{-\frac{1}{2}}$, $d = 2\left(\pm(b^2 - a^2)\right)^{\frac{1}{2}}$. Upper sign $\rightarrow$ prolate, lower sign $\rightarrow$ oblate.

model gives the induced secondary magnetic field from multiple spheroidal objects in close proximity and is based on a boundary condition formulation similar to the formulation for spherical objects given by the Wait [111, 112].

EMI models of canonical shapes are promising in the detection and discrimination of metallic UXO. As a practical matter, however, many field sites are littered with copious amounts of metallic debris (see Fig. 6-2). This clutter can be anything from other UXO, spent shell casings, old cutlery, coins, or any other highly conductive remnant of civilization. Distinguishing these non-hazardous targets from actual UXO, and along the same lines, detecting UXOs amid these minutiae has been an ongoing topic of research [120, 121, 122, 123, 110]. Previously, we have studied the case of a single, larger spheroidal target surrounded by a cloud of smaller spheroids [119]. However, this treatment did not account for the interactions between spheroids that do occur when multiple targets are in close proximity. This paper rigorously treats the case of multiple metallic spheroidal targets in close proximity

Figure 6-2: Site cluttered with multiple potential targets in close proximity.

and in so doing includes all interactions that occur between objects.

When multiple conducting and permeable spheroidal targets are in close proximity (e.g. Fig. 6-3), their respective induced secondary magnetic fields interact and form a nonlinear system. The separate responses may not be simply superposed to find the total induced field from the collection. Instead, the induced response of any individual spheroid is a result of both the primary magnetic field and the secondary field emanating from all the other spheroids. Our formulation accounts for these multiple interaction terms through the construction of an interaction matrix which couples excitation modes of one spheroid to those of each of the other spheroids. This interaction matrix scales in both memory and complexity as L^2 as compared to the case of a single isolated spheroid where L in the number of spheroids present.

We present the formulation of our model, including the use of asymptotic representations of the spheroidal wave functions in Section 6.2.

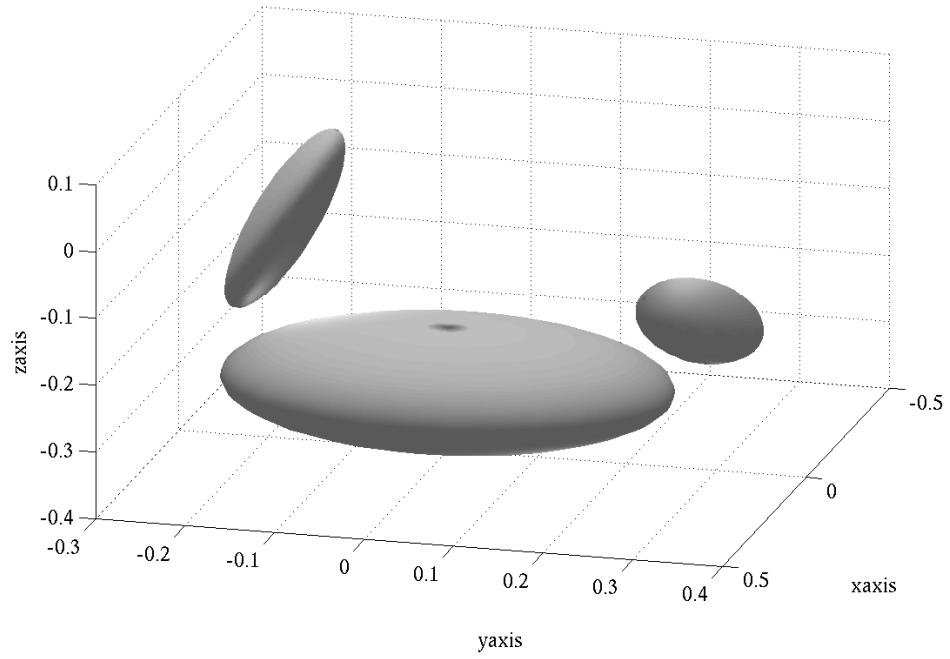

Figure 6-3: Example collection of 3 spheroids in close proximity.

6.2 Formulation

This formulation for the induced secondary magnetic field external to multiple spheroidal targets in close proximity has many similarities with the solution for a single isolated spheroid (Chapter 5 and [63]. In turn, these solutions follow a similar method based on boundary conditions used by Wait [112, 124] in the solution for the induced field from a spherical target. These solutions are exact except for the truncation of the infinite sets of resulting equations.

Let a time harmonic primary magnetic field, $\overline{H}_\circ(\overline{r},t) = \overline{H}_\circ(\overline{r})e^{-\mathrm{i}\omega t}$ ($e^{-\mathrm{i}\omega t}$ suppressed below), excite a collection of permeable and conducting spheroidal targets in close proximity. Spheroid ℓ is defined by its permeability relative to the host medium, $\mu_r^{(\ell)}$, conductivity, $\sigma^{(\ell)}$, semi-axis of revolution $a^{(\ell)}$, interfocal distance $d^{(\ell)}$, wavenumber

$$\left(k^{(\ell)}\right)^2 = \mathrm{i}\omega\sigma^{(\ell)}\mu_r^{(\ell)} \tag{6.1}$$

and size parameter

$$c^{(\ell)} = \frac{k^{(\ell)} d^{(\ell)}}{2} \tag{6.2}$$

(see Fig. 6-1). Each spheroid possesses its own local coordinate system defined by $\left(d^{(\ell)}, \eta, \xi, \phi\right)$ with

$$d^{(\ell)} = 2\sqrt{\pm\left(\left(b^{(\ell)}\right)^2 - \left(a^{(\ell)}\right)^2\right)} \tag{6.3}$$

where the upper sign applies to prolate spheroids and the lower sign applies to oblate spheroids. The orientation of each spheroid with reference to some laboratory Cartesian system is expressed by the three Euler angles, $\{\alpha^{(\ell)}, \beta^{(\ell)}, \gamma^{(\ell)}\}$ [43].

For frequencies low enough to satisfy the magnetoquasistatic assumptions, the magnetic field inside spheroid ℓ, $\overline{H}^{(\ell)}(\overline{r})$, satisfies the vector wave equation (in this case, the diffusion equation)

$$\nabla \times \nabla \times \overline{H}^{(\ell)} - \left(k^{(\ell)}\right)^2 \overline{H}^{(\ell)} = 0 \tag{6.4}$$

$H^{(\ell)}$ can naturally be expressed in terms of an infinite sum of r-type vector spheroidal wave functions of the first kind, $\overline{M}_{pmn}(c;\eta,\xi,\phi)$ and $\overline{N}_{pmn}(c;\eta,\xi,\phi)$ [66], as

$$\begin{aligned}\overline{H}^{(\ell)} = H_\circ \sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\sum_{p=0}^{1}\Big[&A_{pmn}^{(M)(\ell)}\overline{M}_{pmn}(c^{(\ell)};\beta_{\ell\ell}) \\ &+A_{pmn}^{(N)(\ell)}\overline{N}_{pmn}(c^{(\ell)};\beta_{\ell\ell})\Big]\end{aligned} \tag{6.5}$$

where $A_{pmn}^{(M)(\ell)}$ and $A_{pmn}^{(N)(\ell)}$ are expansion coefficients and $H_\circ$ is the relative magnitude of the primary field. Subscripts such as in $\beta_{\ell q} \equiv [\eta_{\ell q}, \xi_{\ell q}, \phi_{\ell q}]$ are interpreted as indicating locations in the coordinate system of spheroid ℓ expressed in the coordinate system of spheroid q. The azimuthal dependence of $\overline{M}_{pmn}(c^{(\ell)};\beta_{\ell\ell})$ and $\overline{N}_{pmn}(c^{(\ell)};\beta_{\ell\ell})$ can be compactly expressed as

$$T_{pm}(\phi) = \begin{cases}\cos(m\phi) & \text{p=0,} \\ \sin(m\phi) & \text{p=1.}\end{cases} \tag{6.6}$$

The background host medium is assumed to be homogeneous, only weakly magnetic and poorly conducting so that the wavenumber in the background medium, $k_\circ$, can be approximated as 0, or $k_{\text{sec}}^2 \approx 0$. In this case, the primary magnetic field becomes irrotational

and can be therefore represented by a scalar potential which satisfies the Laplace equation. Let $U_\circ$ be the primary magnetic field potential with

$$\overline{H}_\circ = -\nabla U_\circ. \tag{6.7}$$

At spheroid ℓ, $U_\circ$ can be decomposed into an infinite summation of solutions of the first kind of the Laplace equation in spheroidal coordinates which are regular at the origin,

$$U_\circ(\overline{r}) = H_\circ \frac{d^{(\ell)}}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} b_{pmn}^{(\ell)} P_n^m(\eta) P_n^m(\xi) T_{pm}(\phi). \tag{6.8}$$

The primary field expansion coefficients, $b_{pmn}^{(\ell)}$, are specific to each spheroid and are assumed to be known *a priori*.

The induced secondary magnetic field from spheroid ℓ may also be represented by a potential, $U_{\mathrm{s}}^{(\ell)}$, and can be expressed as

$$U_{\mathrm{s}}^{(\ell)}(\overline{r}) = H_\circ \frac{d^{(\ell)}}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} B_{pmn}^{(\ell)} P_n^m(\eta) Q_n^m(\xi) T_{pm}(\phi) \tag{6.9}$$

which consists of Laplace solutions of the second kind in spheroidal coordinates. The secondary field expansion coefficients, $B_{pmn}^{(\ell)}$, contain all multiple interaction effects and must be solved for. Once the $B_{pmn}^{(\ell)}$ are known, the total induced magnetic field potential can be found.

The total magnetic field on the surface of spheroid ℓ is a combination of the primary field, the self induced secondary field, and the contribution from the induced fields emanating from all the other spheroids. Thus, the total magnetic field in terms of spheroid ℓ's coordinate

system, $\overline{H}_{\rm tot}^{(\ell)}$, is

$$\begin{aligned}\overline{H}_{\rm tot}^{(\ell)} = \overline{H}_\circ + \overline{H}_{\rm s} &= -\nabla U_\circ - \sum_{\ell=1}^{L} \nabla U_{\rm s}^{(\ell)} \\ &= -H_\circ \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} \frac{d^{(\ell)}}{2} b_{pmn}^{(\ell)} \nabla \Psi_{pmn}(\beta_{\ell\ell}) \\ &\quad - H_\circ \sum_{m'=0}^{\infty} \sum_{n'=m'}^{\infty} \sum_{p'=0}^{1} \sum_{q=1}^{L} \frac{d^{(q)}}{2} B_{p'm'n'}^{(q)} \overline{\overline{R}}_{\ell q} \cdot \nabla \Phi_{p'm'n'}(\beta_{\ell q}) \end{aligned} \quad (6.10)$$

where $\overline{\overline{R}}_{\ell q}$ is a rotation matrix based on the three Euler angles that rotates the vector magnetic field from the coordinate system of spheroid q to that of spheroid ℓ. Note that when $q = \ell$, $\overline{\overline{R}}_{\ell q} = \overline{\overline{I}}$, or the identity dyad, which case corresponds to the self induced magnetic field of spheroid ℓ.

The total induced magnetic field, $\overline{H}_{\rm s}$, may be solved for by utilizing the boundary conditions at the surface of each of the L spheroids. At the surface of every spheroid, the normal component of the magnetic flux density and the tangential components of the magnetic field must be equal. Thus there are $3L$ sets of boundary conditions we use to solve for the unknowns $A_{pmn}^{(M)(\ell)}$, $A_{pmn}^{(N)(\ell)}$, and $B_{pmn}^{(\ell)}$. For spheroid ℓ, these boundary conditions are written

$$H_\eta^{(\ell)} = H_{{\rm tot};\eta}^{(\ell)} \quad (6.11a)$$

$$\mu_r^{(\ell)} H_\xi^{(\ell)} = H_{{\rm tot};\xi}^{(\ell)} \quad (6.11b)$$

$$H_\phi^{(\ell)} = H_{{\rm tot};\phi}^{(\ell)}. \quad (6.11c)$$

Considering Eqs. (6.5) and (6.10), Eq. (6.11b), for example, can now be written as

$$\begin{aligned}\mu_r^{(\ell)} H_\circ \sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\sum_{p=0}^{1}\Big[&A_{pmn}^{(M)(\ell)}\overline{M}_{pmn;\xi}(c^{(\ell)};\beta_{\ell\ell}) \\ &+A_{pmn}^{(N)(\ell)}\overline{N}_{pmn;\xi}(c^{(\ell)};\beta_{\ell\ell})\Big] \\ &= -H_\circ \sum_{m=0}^{\infty}\sum_{n=m}^{\infty}\sum_{p=0}^{1}\frac{d^{(\ell)}}{2}b_{pmn}^{(\ell)}\left[\nabla\Psi_{pmn}(\beta_{\ell\ell})\right]_\xi \\ &- H_\circ \sum_{m'=0}^{\infty}\sum_{n'=m'}^{\infty}\sum_{p'=0}^{1}\sum_{q=1}^{L}\frac{d^{(q)}}{2}B_{p'm'n'}^{(q)}\left[\overline{\overline{R}}_{\ell q}\cdot\nabla\Phi_{p'm'n'}(\beta_{\ell q})\right]_\xi \end{aligned} \tag{6.12}$$

In order to reduce the complexity of Eq. (6.12) and its counterparts from Eqs. (6.11a) and (6.11c), we use a similar procedure as used in [63] and exploit the orthogonality among the azimuthal and Associated Legendre functions between different p, m, and n. We first define an inner product for the azimuthal functions:

$$\langle T_{pm}(\phi),(\cdot)\rangle = \int_{-\pi}^{\pi} T_{pm}(\phi),(\cdot)d\phi. \tag{6.13}$$

Taking $\langle T_{pm}(\phi_{\ell\ell}),(\text{Eq. (6.12)})\rangle$ isolates m and p from the summations on the internal and primary magnetic field terms due to the orthogonality of the azimuthal functions and gives

$$\begin{aligned}\mu_r^{(\ell)} \sum_{n=m}^{\infty}\Big[&A_{pmn}^{(M)(\ell)}\left\langle T_{pm}(\phi_{\ell\ell}),\overline{M}_{pmn;\xi}(c^{(\ell)};\beta_{\ell\ell})\right\rangle \\ &+A_{pmn}^{(N)(\ell)}\left\langle T_{pm}(\phi_{\ell\ell}),\overline{N}_{pmn;\xi}(c^{(\ell)};\beta_{\ell\ell})\right\rangle\Big] \\ &= -\sum_{n=m}^{\infty}\frac{d^{(\ell)}}{2}b_{pmn}^{(\ell)}\left\langle T_{pm}(\phi_{\ell\ell}),\left[\nabla\Psi_{pmn}(\beta_{\ell\ell})\right]_\xi\right\rangle \\ &\quad -\sum_{m'=0}^{\infty}\sum_{n'=m'}^{\infty}\sum_{p'=0}^{1}\sum_{q=1}^{L}\frac{d^{(q)}}{2}B_{p'm'n'}^{(q)} \\ &\quad \left\langle T_{pm}(\phi_{\ell\ell}),\left[\overline{\overline{R}}_{\ell q}\cdot\nabla\Phi_{p'm'n'}(\beta_{\ell q})\right]_\xi\right\rangle. \end{aligned} \tag{6.14}$$

The summations over m' and p' must be retained for the terms involving $\ell \neq q$ spheroids because

$$\left\langle T_{pm}(\phi_{\ell\ell}),\left[\overline{\overline{R}}_{\ell q}\cdot\nabla\Phi_{p'm'n'}(\beta_{\ell q})\right]_\xi\right\rangle \neq 0 \tag{6.15}$$

in general due to coordinate systems of spheroids ℓ and q being non-colocated.

We define another inner product of the associated Legendre functions:

$$\langle P_n^m(\eta), (\cdot)\rangle = \int_{-1}^{1} P_n^m(\eta), (\cdot) d\eta. \tag{6.16}$$

$\left\langle P_{n''}^m(\eta_{\ell\ell}), (\text{Eq. } (6.14))\right\rangle$ results in

$$\begin{aligned}
&\mu_r^{(\ell)} \sum_{n=m}^{\infty} \left[A_{pmn}^{(M)(\ell)} I_{pmnn'';\xi}^{(\ell)} + A_{pmn}^{(N)(\ell)} J_{pmnn'';\xi}^{(\ell)} \right] \\
&= -\sum_{n=m}^{\infty} \frac{d^{(\ell)}}{2} b_{pmn}^{(\ell)} F_{pmnn'';\xi}^{(\ell)} \\
&- \sum_{m'=0}^{\infty} \sum_{n'=m'}^{\infty} \sum_{p'=0}^{1} \sum_{q=1}^{L} \frac{d^{(q)}}{2} B_{p'm'n'}^{(q)} G_{p'm'n'n'';\xi}^{(\ell)(q)}.
\end{aligned} \tag{6.17}$$

where

$$I_{pmnn'';\xi}^{(\ell)} \equiv \left\langle P_{n''}^m(\eta_{\ell\ell}), \left\langle T_{pm}(\phi_{\ell\ell}), \overline{M}_{pmn;\xi}(c^{(\ell)}; \beta_{\ell\ell}) \right\rangle \right\rangle \tag{6.18}$$

$$J_{pmnn'';\xi}^{(\ell)} \equiv \left\langle P_{n''}^m(\eta_{\ell\ell}), \left\langle T_{pm}(\phi_{\ell\ell}), \overline{N}_{pmn;\xi}(c^{(\ell)}; \beta_{\ell\ell}) \right\rangle \right\rangle \tag{6.19}$$

$$F_{pmnn'';\xi}^{(\ell)} \equiv \left\langle P_{n''}^m(\eta_{\ell\ell}), \left\langle T_{pm}(\phi_{\ell\ell}), [\nabla \Psi_{pmn}(\beta_{\ell\ell})]_{\xi} \right\rangle \right\rangle \tag{6.20}$$

and

$$G_{p'm'n'n'';\xi}^{(\ell)(q)} \equiv \left\langle P_{n''}^m(\eta_{\ell\ell}), \left\langle T_{pm}(\phi_{\ell\ell}), \left[\overline{\overline{R}}_{\ell q} \cdot \nabla \Phi_{p'm'n'}(\beta_{\ell q}) \right]_{\xi} \right\rangle \right\rangle. \tag{6.21}$$

The equations are very similar for the boundary condition on the η and ϕ components of the magnetic field [Eqs. (6.11a) and (6.11c), respectively].

Because the functional dependence on η of $[\nabla \Psi_{pmn}(\beta_{\ell\ell})]_{\xi \text{ or } \phi}$ is $P_{n''}^m(\eta_{\ell\ell})$, the only nonzero $F_{pmnn'';\xi \text{ or } \phi}^{(\ell)}$ occurs when $n = n''$. The η dependence of $[\nabla \Psi_{pmn}(\beta_{\ell\ell})]_{\eta}$ is $dP_{n''}^m(\eta_{\ell\ell})/d\eta_{\ell\ell}$ resulting in two nonzero modes, $n - 1 = n''$ and $n + 1 = n''$ [63, 88].

When $\ell = q$ in Eq. (6.17), the primed summation indices are equivalent to their unprimed counterparts. In this case, $B_{pmn}^{(\ell)}$ are the expansion coefficients for the self induced magnetic field potential, and the inner products in $G_{p'm'n'n''}^{(\ell)(q)}$ are again over an orthogonal set and

reduce to one or two terms in the same way as described above for the $F^{(\ell)}_{pmnn''}$.

The solution to this system, i.e. finding the expansion coefficients $A^{(M)(\ell)}_{pmn}$, $A^{(M)(\ell)}_{pmn}$, and $B^{(\ell)}_{pmn}$, requires the truncation of these infinite sets of equations. The total number of boundary conditions, N_{BC}, is

$$N_{BC} = 3L2\,(m_{\max}+1)\,(n_{\max,\circ}+1) \tag{6.22}$$

where $m_{\max}$ indicates the truncation order for the m index, and $n_{\max,\circ}$ indicates the truncation order for the n index when $m = 0$. These boundary conditions, one for each distinct mode on each spheroid, from Eq. (6.11) may then be cast into 3 matrix equations which we write as

$$\overline{\overline{Z}}^{M}_{\eta}\,\overline{A}^{M} + \overline{\overline{Z}}^{N}_{\eta}\,\overline{A}^{N} = -\,\overline{\overline{W}}_{\eta}\,\overline{b} \;-\; \overline{\overline{Y}}_{\eta}\,\overline{B} \tag{6.23a}$$

$$\overline{\overline{Z}}^{M}_{\xi}\,\overline{A}^{M} + \overline{\overline{Z}}^{N}_{\xi}\,\overline{A}^{N} = -\,\overline{\overline{W}}_{\xi}\,\overline{b} \;-\; \overline{\overline{Y}}_{\xi}\,\overline{B} \tag{6.23b}$$

$$\overline{\overline{Z}}^{M}_{\phi}\,\overline{A}^{M} + \overline{\overline{Z}}^{N}_{\phi}\,\overline{A}^{N} = -\,\overline{\overline{W}}_{\phi}\,\overline{b} \;-\; \overline{\overline{Y}}_{\phi}\,\overline{B}. \tag{6.23c}$$

The definitions and dimensions for $\overline{\overline{Z}}^{M}_{\zeta}$, $\overline{\overline{Z}}^{N}_{\zeta}$, $\overline{\overline{W}}_{\zeta}$, $\overline{\overline{Y}}_{\zeta}$, $\overline{b}$, and $\overline{B}$ where $\zeta = \eta$, ξ, or ϕ are found in Section 6.3.

Equation (6.23) includes all multiple body interaction effects and provides a simultaneous solution for the internal and induced, secondary external magnetic fields. In particular, the secondary induced magnetic field, $\overline{H}_{\rm s}$, may be calculated from the expansion coefficients $\overline{B}$ and compared to frequency domain measurements made with instruments such as the ultrawide band GEM-3 [114] instrument

The formulation for the induced electromagnetic induction (EMI) response from multiple permeable and conducting spheroidal objects in close proximity under time harmonic excitation presented here is formally exact except for the truncation of the infinite sets of equations. However, as the size parameters, $c^{(\ell)}$, become larger, the vector spheroidal wave functions $\overline{M}_{pmn}(c^{(\ell)};\beta_{\ell\ell})$ and $\overline{N}_{pmn}(c^{(\ell)};\beta_{\ell\ell})$ become numerically unstable preventing calculation of the expansion coefficients $A^{(M)(\ell)}_{pmn}$, $A^{(N)(\ell)}_{pmn}$, and $B^{(\ell)}_{pmn}$. To extend the range of c over which solutions can be calculated, we can use asymptotic forms of the angular

and radial spheroidal wave functions in the internal modal field matrices $\overline{\overline{Z}}_{\zeta}^{M}$, and $\overline{\overline{Z}}_{\zeta}^{N}$ (see Chapters 4 and 5) . As these representations also become inaccurate for even larger size parameters, it is expected that the small penetration approximation (SPA) [63, 7] in the spheroidal modal interaction matrix $\overline{\overline{Y}}_{\zeta}$ can be used at these frequencies.

This multibody solution may provide the basis for a forward model used by inversion routines designed to detect and discriminate UXO. Specifically, this multibody solution may help to isolate non-UXO clutter from actual UXO in the field.

6.3 Matrix Definitions and Dimensions

The matrices $\overline{\overline{Z}}_{\zeta}^{M}$, $\overline{\overline{Z}}_{\zeta}^{N}$, and $\overline{\overline{W}}_{\zeta}$, where $\zeta = \eta$, ξ, or ϕ, are coefficient matrices describing the relative magnitude of the various spheroidal modes internal and external to the spheroids. The size of these matrices are prescribed by the truncation values used. Given m_{max} and n_{max} as described in Section 6.2, then the size of $\overline{\overline{Z}}_{\zeta}^{M}$, $\overline{\overline{Z}}_{\zeta}^{N}$, and $\overline{\overline{W}}_{\zeta}$ are

$$2L(m_{\max}+1)(n_{\max,\circ}+1) \tag{6.24}$$

where the 2 accounts for the p index.

The entries of $\overline{\overline{Z}}_{\zeta}^{M}$, $\overline{\overline{Z}}_{\zeta}^{N}$, and $\overline{\overline{W}}_{\zeta}$ are arranged in blocks according to, first, spheroid index ℓ, then m, then p, and finally n (or n'). Defining matrix indices

$$\begin{aligned} i = {} & 1 + (\ell-1)(m_{\max}+1)2(n_{\max,\circ}+1) \\ & + m2(n_{\max,\circ}+1) \\ & + p(n_{\max,\circ}+1) \\ & + (n''-m) \end{aligned} \tag{6.25}$$

$$\begin{aligned} j = {} & 1 + (\ell-1)(m_{\max}+1)2(n_{\max,\circ}+1) \\ & + m2(n_{\max,\circ}+1) \\ & + p(n_{\max,\circ}+1) \\ & + (n-m), \end{aligned} \tag{6.26}$$

then

$$\overline{\overline{Z}}_{\zeta}^{M}(i,j) = I_{pmnn'';\zeta}^{(\ell)}, \tag{6.27}$$

$$\overline{\overline{Z}}_{\zeta}^{N}(i,j) = J_{pmnn'';\zeta}^{(\ell)}, \tag{6.28}$$

and

$$\overline{\overline{W}}_{\zeta}(i,j) = F_{pmnn''}^{(\ell)}. \tag{6.29}$$

Using index j, we also define the order of the spheroidal expansion coefficients $\overline{b}$ and $\overline{B}$ as,

$$b_{pmn}^{(\ell)} = \overline{b}(j) \tag{6.30}$$

and

$$B_{pmn}^{(\ell)} = \overline{B}(j). \tag{6.31}$$

The spheroidal modal interaction matrix $\overline{\overline{Y}}_{\zeta}$ has the same size as $\overline{\overline{Z}}_{\zeta}^{M}$, $\overline{\overline{Z}}_{\zeta}^{N}$, and $\overline{\overline{W}}_{\zeta}$ (assuming $m'_{\max} = m_{\max}$ and $n'_{\max,\circ} = n_{\max,\circ}$), but has nonzero terms which account for the interspheroidal interactions in addition to the self induced terms. We define different matrix indices such that

$$\begin{aligned} i' = & 1 + (\ell - 1)(m_{\max} + 1)2(n_{\max,\circ} + 1) \\ & + m2(n_{\max,\circ} + 1) \\ & + p(n_{\max,\circ} + 1) \\ & + (n'' - m) \end{aligned} \tag{6.32}$$

$$\begin{aligned} j' = & 1 + (q - 1)(m_{\max} + 1)2(n_{\max,\circ} + 1) \\ & + m'2(n_{\max,\circ} + 1) \\ & + p'(n_{\max,\circ} + 1) \\ & + (n' - m'). \end{aligned} \tag{6.33}$$

Now, the entries of $\overline{\overline{Y}}_\zeta$ can be specified as

$$\overline{\overline{Y}}_\zeta(i',j') = G^{(\ell)(q)}_{p'm'n'n'';\zeta}. \tag{6.34}$$

6.4 Conclusion

We have constructed a solution for the induced magnetic field from multiple conducting and permeable prolate and oblate spheroids in close proximity and under time harmonic excitation in the magnetoqausistatic regime. This solution solves for the induced field expansion coefficients for all spheroids in the collection simultaneously and includes all intertarget interactions and effects.

Chapter 7

Summary

In this thesis, I have studied electromagnetic scattering and induction models with regard to spheroidal geometries. Principal results recorded in this thesis include advances in finding the asymptotic expressions of the spheroidal wave functions, obtaining a broadband electromagnetic induction (EMI) response for realistic conducting and permeable spheroids and showing agreement of my model with data, formulating and implementing a simultaneous solution for the induced magnetic field from multiple permeable and conducting spheroidal objects in close proximity under time harmonic excitation, and the first application of the Sparse Matrix/Canonical Grid (SMCG) method to three dimensions as applied to scattering from dense random discrete media containing dielectric spheroidal scatterers.

These models advance our ability to calculate the scattered fields and the induced secondary magnetic fields from more realistic canonical shapes such as spheroids, plates, and needles.

First in Chapter 2, the effective permittivity (ϵ_{eff}) of dense random media was studied. The coherent scattered fields from a collection of particles in a given test volume were compared to fields scattered from a homogeneous volume of the same size and shape. The permittivity of the homogeneous medium which yielded scattered fields that best matched the averaged scattered fields produced by many configurations of particles was considered to be equal to ϵ_{eff}. The effective permittivity of dense random media containing discrete spheroids was characterized through a Monte Carlo simulation employing the Method of Moments on the volume integral equation. The basis functions were chosen to be the

electrostatic dipole solutions of a spheroid. Simulations with N=3000 particles, N_r=50 realizations and different fractional volumes ($f_v = 0.05$–0.40) and particle elongations ($e =$ 1.0–2.6) were performed in a spherical test volume. The scattering from this volume was then compared to Mie scattering from a homogeneous sphere of the same size. Results indicate that ϵ_{eff} for a dense random media containing randomly oriented discrete prolate spheroids agrees well with results from the classical mixing formula. This is consistent with previous studies considering spherical particles only.

The sparse matrix canonical grid (SMCG) method has been extended to 3-D and illustrated by finding the scattering from random media filled with dielectric spheroids at random positions and orientations in Chapter 3. The 3-D SMCG method achieves $\mathcal{O}(N \log N)$ complexity instead of $\mathcal{O}(N^2)$ for the matrix-vector multiply when using an iterative solver by decomposing the interaction matrix $\overline{\overline{Z}}$, generated by an MoM solution to the many-body volume integral equation, into strong and weak interaction matrices $\overline{\overline{Z}}^{\text{s}}$ and $\overline{\overline{Z}}^{\text{w}}$, respectively. The matrix $\overline{\overline{Z}}^{\text{s}}$ contains only those interactions which are between particles closer than the neighborhood distance r_d. Therefore, $\overline{\overline{Z}}^{\text{s}}$ is very sparse and the matrix-vector multiply $\overline{\overline{Z}}^{\text{s}} \cdot \overline{x}$ is accomplished in $\mathcal{O}(N)$ complexity. The matrix $\overline{\overline{Z}}^{\text{w}}$ is formed by expanding the dyadic Green's function between each pair of particles whose associated grid points are located further apart than r_d in a γ^{th} order multivariate Taylor series expansion around a canonical grid superimposed onto the test volume. The prolate spheroidal particles considered in this paper were electrically small so that their response to electromagnetic excitation could be adequately approximated with a point response. Therefore, the Green's function interaction expansion matrices $\overline{\overline{G}}_{\gamma_x \gamma_y \gamma_z}$ exhibit a multilevel block Toeplitz (MBT) structure. MBT matrices can be multiplied with an arbitrary vector using the only one forward and one inverse FFT. Thus $\overline{\overline{Z}}^{\text{w}} \cdot \overline{x}$ can be accomplished in $\mathcal{O}(N_T N \log N)$ complexity where N_T is the number of FFTs and depends on the expansion order γ. The 3-D SMCG method was demonstrated to indeed realize the predicted reduction in complexity and memory requirements through large-scale examples including up to 15000 discrete scatterers.

The procedure for expediting a matrix-vector multiply involving $\mathbf{T}_f^M$ multilevel block-Toeplitz (MBT) matrices outlined in Appendix A is efficient both in terms of memory and computational complexity. It can be easily applied to 3-D scattering from arbitrary shaped

targets using a discretized integral formulation. This was illustrated in Section A.3, where a comparison between the scattering from a spherical volume containing many small dielectric spheres and the scattering from a homogeneous sphere of the same size and shape of the spherical volume was presented. The effective permittivity ϵ_{eff} of the medium composed by the small particles was calculated and shown to converge to the permittivity of the Mie sphere for sufficiently fine grid spacings. In contrast to previously proposed algorithms, this fast matrix-vector multiply may be accurately described as a minimal memory method due to the fact that only the unique information from a $\mathbf{T}_f^M$ matrix is stored in memory. Also, the reduced complexity is achieved in a fashion similar, but not identical to those described in [125] and [126] with a speed improvement due to the 1-D FFT length of $\prod_{i=1}^{M}(2n_i - 1)$ in the present method.

The asymptotic expansion of the spheroidal wave function (SWF) and its eigenvalues have been investigated for complex size parameter c in Chapter 4. No fundamentally new asymptotic expansions are required, but instead, established *prolate*-type and *oblate*-type asymptotic expansions are directly applicable to finding the asymptotic expansion of the SWF for complex size parameter c. A rapid and accurate method for calculating the SWFs for complex c in the asymptotic regime has been presented based on a lookup table consisting of branch points, $c_{\circ;r}^{mn}$. Through a brute force numerical method, this lookup table, based on the number of branch cuts (emanating from $c_{\circ;r}^{mn}$) traversed according to $\arg(c)$, is calculated once and recorded. This allows the assignment of any SWF to either a *prolate*-type or *oblate*-type expansion (ordered by n_p and n_o respectively) for any general complex size parameter c. The ordering for these expansions was found to be different than the original ordering of the spheroidal wave functions, with Table 4.3 documenting the ordering for the special case when $\arg(c) = \pi/4$. Normalizations, dependent on c, were derived for both the *prolate* and *oblate*-type asymptotic expansions and for both $(n - m)$ even and odd. These normalized asymptotic expansions for the PASWFs $S_{mn}(c, \eta)$ and $S'_{mn}(c, \eta)$, together with established asymptotic expansions for the prolate radial spheroidal wave functions $R_{mn}(c, \eta)$ and $R'_{mn}(c, \eta)$, allow the rapid calculation of the vector spheroidal wave functions for significantly larger (complex) size parameter c than previously possible. In Appendix D, I provided some of the branch points of the spheroidal wave equation,

$c_{\circ;r}^{mn}$, with their associated eigenvalues, $\lambda_{\circ;r}^{mn}$ for a greater range of m and n than previously available as well as with greater accuracy. Also included are tables containing the angular SWF for large and complex size parameter, c.

Chapter 5 discusses my broadband solution for the induced magnetic field from conducting and permeable prolate and oblate spheroids under time harmonic excitation in the magnetoquasistatic regime. My combined method consists of the exact analytical solution, asymptotic assisted analytical solution, and a small penetration approximation (SPA). This combined solution is accurate to within 2% error for most spheroids with aspect ratio $\frac{1}{8} < e < 8$. Results produced by this combined method were compared to results from a method of moments/boundary integral method and found to be in excellent agreement. Results were also compared to induced magnetic field measurements taken by the GEM-3 instrument from metal prolate and oblate spheroids. If the permeability and conductivity of the spheroids is allowed to vary within accepted physical ranges, results were seen to match exceptionally well with measurements. Using this model, calculation of the broadband induced magnetic field response from more realistic canonical shapes such as spheroids, plates, and needles is now possible. The results indicate that solutions from my model are indeed unique for a given spheroid and orientation. On this basis, my method could become a forward model upon which inversions schemes may be based.

Finally in Chapter 6, I have constructed a broadband solution for the induced magnetic field from multiple conducting and permeable prolate and oblate spheroids in close proximity and under time harmonic excitation in the magnetoquasistatic regime. This solution solves for the induced field expansion coefficients for all spheroids in the collection simultaneously and includes all intertarget interactions and effects. My combined method consists of the exact analytical solution at low frequencies, asymptotic assisted analytical solution at mid-range frequencies, and a small penetration approximation (SPA) for high frequencies in the MQS regime.

Appendix A

Fast Algorithm for Matrix-Vector Multiply of Asymmetric Multilevel Block-Toeplitz Matrices in 3-D Scattering

A.1 Introduction

Multilevel block-Toeplitz (MBT) matrices often arise in the solution of electromagnetic scattering problems due to the translational invariance and convolutional nature of the free-space Green's function. For example, a three-level MBT matrix arises in connection with the solution of the scattering from arbitrarily shaped targets using either the Discrete Dipole Approximation (DDA) [47, 127, 128] or, similarly, using a (volumetric) discretized integral formulation [48, 129, 16]. In other contexts, MBT matrices are also generated as the result of the autocorrelation of a two dimensional (2-D) discrete random process [130], and in many problems involving symmetries based on cubic meshes [131, 132].

In this communication, we describe a new fast Fourier transform (FFT)-based method to expedite matrix-vector multiplies involving MBT matrices with minimal memory requirements. This method applies to MBT matrices with blocks and subblocks which are

themselves MBT and in general asymmetric, and provides for the last, M^{th}, level subblock to be a square, possibly asymmetric, arbitrary matrix (*i.e.*, not necessarily Toeplitz). The method has a similar purpose to the FFT-based method of [125], but it is more general in implementation due to its ability to multiply arbitrary MBT matrices. Moreover, it is based on 1-D FFTs implementations directly (as opposed to 2-D and 3-D FFTs [42]), and can be classified as a truly minimal memory method because it stores only nonredundant matrix entries. The computational complexity is $O(N \log N)$ and the memory requirement is $O(2^M N) \approx O(N)$. The convergence and accuracy of the method are assessed in the calculation of the scattering and effective permittivity of a dielectric sphere.

A.2 Formulation

A.2.1 Definitions

Let $\mathbf{A}$ be a square matrix in the matrix equation $\mathbf{A} \cdot \mathbf{x} = \mathbf{b}$ which consists of block-Toeplitz (BT) matrices each containing BT subblocks continuing for $M-1$ levels with the last M^{th} level containing a possibly asymmetric, square matrix, *i.e.*,

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}^1_0 & \mathbf{a}^1_1 \cdots \mathbf{a}^1_{n_1} \\ \mathbf{a}^1_{-1} & \mathbf{a}^1_0 \\ \vdots & \ddots \ \vdots \\ \mathbf{a}^1_{-n_1} & \cdots \ \mathbf{a}^1_0 \end{bmatrix}, \tag{A.1}$$

where

$$\mathbf{a}^1_{-n_1}, \ldots, \mathbf{a}^1_{n_1} = \begin{bmatrix} \mathbf{a}^2_0 & \mathbf{a}^2_1 \cdots \mathbf{a}^2_{n_2} \\ \mathbf{a}^2_{-1} & \mathbf{a}^2_0 \\ \vdots & \ddots \ \vdots \\ \mathbf{a}^2_{-n_2} & \cdots \ \mathbf{a}^2_0 \end{bmatrix}, \tag{A.2}$$

continuing until the $(M-1)^{\text{st}}$ level

$$\mathbf{a}^{M-1}_{-n_{(M-1)}}, \ldots, \mathbf{a}^{M-1}_{n_{(M-1)}} = \begin{bmatrix} \mathbf{a}^M_0 & \mathbf{a}^M_1 \cdots \mathbf{a}^M_{n_M} \\ \mathbf{a}^M_{-1} & \mathbf{a}^M_0 \\ \vdots & \ddots \quad \vdots \\ \mathbf{a}^M_{-n_M} & \cdots \mathbf{a}^M_0 \end{bmatrix}, \tag{A.3}$$

with $\mathbf{a}^M_{-n_M}, \ldots, \mathbf{a}^M_{n_M}$ being arbitrary $f \times f$ matrices. Furthermore, let $\mathbf{a}^{1\ldots(M-1)}$ be asymmetric and let $\mathbf{A}$ be referred to as $\mathbf{T}^M_f(n_1, n_2, \ldots, n_M)$ or simply $\mathbf{T}^M_f$ where M denotes the number of BT levels, f denotes the size of the final dense matrix, and $n_1, n_2, \ldots, n_M$ denote the number of BT blocks at each level. As an example, a typical $\mathbf{T}^3_1(2,2,2)$ matrix is shown in Fig. A-1. A $\mathbf{T}^3_3(2,2,2)$ matrix (the type produced in the solution to the example in Section A.3) would be similar except that each unique integer depicted in Fig. A-1 is replaced by a corresponding 3×3 dense matrix.

A.2.2 Recursive Algorithms

In order to illustrate the fast matrix-vector multiply method, we divided the method into parts 1–4 which are briefly described below. For brevity and ease of use, each step is also separately cast in terms of Matlab codes in the Appendix, and can be easily ported into other programming languages.

In part 1, full $\mathbf{T}^M_f$ matrices, which will be generically referred to as $\mathbf{A}$, are defined recursively. The definition is described in Algorithm 1, which steps through each of the first level of subblocks $(\mathbf{a}^1_{-n_1}, \ldots, \mathbf{a}^1_{n_1})$ recursively defining each level and then assigning that subblock along its diagonal when completed.

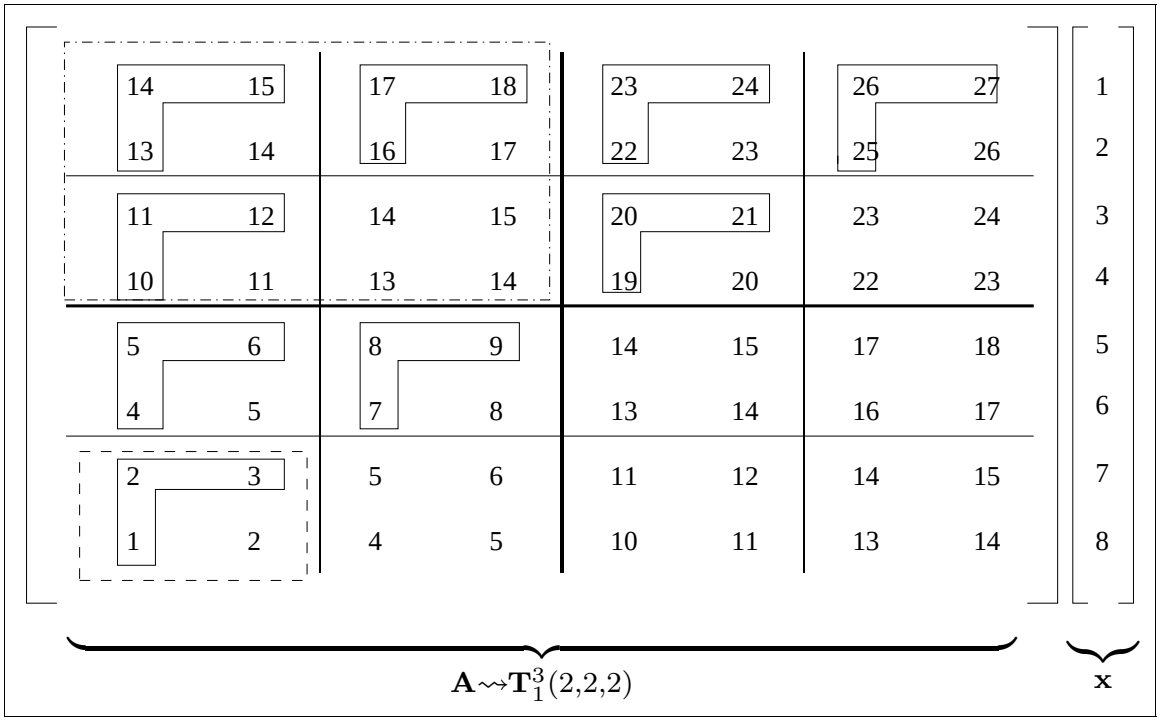

Figure A-1: Example of a $\mathbf{A} = \mathbf{T}_1^3(2,2,2)$ asymmetric MBT matrix. Unique entries are boxed with a solid line. In the upper left hand corner, $\mathbf{a}_0^1$ is enclosed by a dash-dot box, while at the lower left hand corner $\mathbf{a}_{-1}^1|\mathbf{a}_{-1}^2$ in delineated by a dashed box. Thin solid lines delineate interaction between different rows of a 2x2x2 cubic mesh, while bold solid line delineate between planes.

Algorithm 1: Generation of full $\mathbf{T}_f^M$ MBT matrices.

```
1  function [A]=BT_full(n,f,n_ind,level)
2  % [A]=BT_full(n,f,n_ind,level)
3  %    Recursive multilevel block Toeplitz (MBT) matrix generator.
4  %    n=[n1 n2 ... nM] is the number of BT blocks at each level.
5  %    f is the size of the final dense fxf block.
6  %    n_ind is size 2xn and indicates the current block index.
7  %       Row 1 => down, Row 2 => across,
8  %    level indicates which level is current.
9  %    BT_full is initially called with only n and f as arguments.
10 if nargin<3, n_ind=ones(2,length(n)); level=1; end
11 if level==length(n)+1
12  A=application_function(n,f,n_ind,level); %fxf block assignment.
13 else
14  this_n=n(level);
15  for i=1:this_n %Lower triangle and diagonal assignment.
16   b_edge=prod(n(level+1:length(n)))*f;
17   n_ind(1,level)=i;
18   blk=BT_full(n,f,n_ind,level+1);
19   for j=1:(this_n-i+1)
20    A(b_edge*(i-1)+[b_edge*(j-1)+1:b_edge*j],...
21                   [b_edge*(j-1)+1:b_edge*j])=blk;
22   end
23  end
24  for i=2:this_n %Upper triangle assignment.
25   n_ind(2,level)=i;
26   blk=BT_full(n,f,n_ind,level+1);
27   for j=1:this_n-i+1
28    A([b_edge*(j-1)+1:b_edge*j],...
29       b_edge*(i-1)+[b_edge*(j-1)+1:b_edge*j])=blk;
30   end
31  end
32 end
```

The application function in line 12 of Algorithm 1 assigns the final $f \times f$ block and should be replaced by appropriate statements involving the physical parameters and relationships of the system as a function of the current indices $n_1, n_2, \ldots, n_M$ (represented by n_ind).

Storing the entire matrix $\mathbf{A}$ in computer memory would be inefficient due to the many repeated entries in MBT matrices. Indeed, for many large scale problems, storing $\mathbf{A}$ is impractical if not impossible. Part 2 is designed to circumvent the matrix filling process by assigning only the nonredundant information from $\mathbf{A}$ to an associated *vector* $\mathbf{A}_u$. We call this a MBT projection, denoted by

$$\mathbf{A}_u = \Pi_{bt}(\mathbf{A}). \tag{A.4}$$

This projection is performed by Algorithm 2.

Algorithm 2: Π_{bt}: Direct assignment of $\mathbf{A}_u$.

```
1  function [Au]=BT_fft(n,f,n_ind,level)
2  if nargin<3, n_ind=ones(2,length(n));level=1; end
3  if level==(length(n)+1)
4   Au=flipud(application_function(f)).'; Au=Au(:).';
5  else
6   this_n=n(level);
7   for i=this_n:-1:1
8    b_edge=f^2*prod(2*n(level+1:length(n))-1);
9    n_ind(1,level)=i;
10   blk=BT_fft(n,f,n_ind,level+1);
11   Au(1+b_edge*(this_n-i):b_edge*(this_n-i+1))=blk;
12  end
13  for i=2:(this_n)
14   n_ind(2,level)=i;
15   blk=BT_fft(n,f,n_ind,level+1);
16   Au(1+b_edge*(i+this_n-2):b_edge*(i+this_n-1))=blk;
17  end
18 end
```

Algorithm 2 proceeds in the same block order convention as Algorithm 1 assigning only the left column and top row of each subblock to $\mathbf{A}_u$. The final $f \times f$ block is assigned in line 4 of Algorithm 2 from left to right, bottom to top, i.e.,

$$\begin{bmatrix} f_1 & f_2 & f_3 \\ f_4 & f_5 & f_6 \\ f_7 & f_8 & f_9 \end{bmatrix} \quad \rightarrow \quad [f_7, f_8, f_9, f_4, f_5, f_6, f_1, f_2, f_3]. \tag{A.5}$$

For example, applying Algorithm 2 to achieve a MBT projection from the matrix $\mathbf{A}$ of Fig. A-1 would result in $\Pi_{bt}(\mathbf{A}) = \mathbf{A}_u = [1{:}27]$. The memory requirement for storing MBT matrices in this way is reduced substantially from

$$f^2 \left(\prod_{i=1}^{M} n_i \right)^2 = f^2 N^2 \tag{A.6}$$

stored elements for the full MBT matrix $\mathbf{A}$ to

$$f^2 \prod_{i=1}^{M} (2n_i - 1) \approx f^2 2^M N \tag{A.7}$$

stored elements for $\mathbf{A}_u$. If $\mathbf{T}_f^M$ possessed any symmetry (as in the case of the DDA approximation, see [125]), this requirement could be reduced even further.

In part 3, zeros are inserted at appropriate locations into $\mathbf{x}$ according to the BT structure, in order to obtain an auxiliary vector denoted by $\mathbf{x}_z = \zeta_p(\mathbf{x})$, suitable for the direct

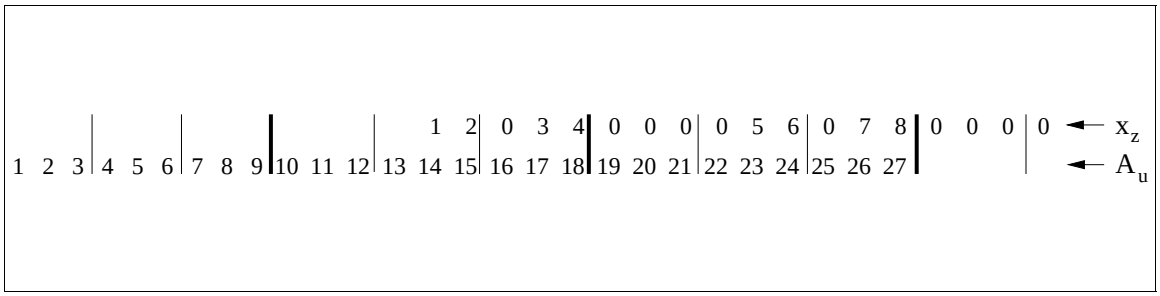

Figure A-2: $\mathbf{A}_u * \mathbf{x}_z$ step which yields the first term of $\mathbf{b}_z$. Note the insertion of zeros into $\mathbf{x}$ yielding $\mathbf{x}_z$ which causes proper alignment. Each entry in $\mathbf{A}_u$ corresponds to an $\mathbf{a}_i^M$ final level subblock where i is between $-n_M$ and n_M; the thin vertical lines mark the separation between $\mathbf{a}_{-1}^2 \ldots \mathbf{a}_1^2$ subblocks in $\mathbf{a}_{-1}^1 \ldots \mathbf{a}_1^1$, and thicker vertical lines indicate the boundary between $\mathbf{a}_{-1}^1 \ldots \mathbf{a}_1^1$ blocks.

convolution with $\Pi_{bt}(\mathbf{A})$.

Algorithm 3: ζ_p: Generation of $\mathbf{x}_z$ by inserting zeros into $\mathbf{x}$.

```
1  function xz=BT_pad(n,f,x)
2  xz=[];
3  if isempty(n)
4   xz=[x zeros(1,f^2-f)];
5  else
6   b_edge=prod(n(2:length(n)))*f;
7   this_n=n(1);
8   for i=1:this_n
9    if length(n)>1
10    xz=[xz BT_pad(n(2:length(n)),f,x(1+(i-1)*b_edge:i*b_edge))...
11        zeros(1,(n(2)-1)*prod(2*(n(3:length(n)))-1)*f^2)]
12    else
13     xz=[xz BT_pad(n(2:length(n)),f,x(1+(i-1)*b_edge:i*b_edge))]
14   end
15  end
16 end
```

These projections are done because the full matrix-vector multiply $\mathbf{A} \cdot \mathbf{x}$, which requires $O(N^2)$ complexity, may be replaced with a convolution, or outer product, in the Fourier domain. Rather than multiplying the full MBT matrix $\mathbf{A}$ with $\mathbf{x}$, the vector $\Pi_{bt}(\mathbf{A})$ is convolved with $\zeta_p(\mathbf{x})$. Algorithm 3 in Appendix performs the ζ_p operation which depends on $n_1, n_2, \ldots, n_M$. This is also depicted graphically in Fig. A-2. Note that $\mathbf{x}_z$ must be zero-padded to the length of $\mathbf{A}_u$ before the FFT and subsequent multiplication in the Fourier domain (see example in next Section).

The convolution is easily accomplished in $O(N \log N)$ complexity by multiplying $\widetilde{\mathbf{A}}_u$ and $\widetilde{\mathbf{x}}_z$ (the tildes denote Fourier transformed quantities: $\mathbf{A}_u \overset{\mathcal{F}}{\Leftrightarrow} \widetilde{\mathbf{A}}_u$) in the Fourier domain

to obtain $\widetilde{\mathbf{b}}_z$. In the final step, the vector $\mathbf{b}$ is recovered after the multiplication in the Fourier domain. This is done in a process similar to Algorithm 3, through the suppression of extra terms in $\mathbf{b}_z$, where

$$\mathbf{b}_z = \mathcal{F}^{-1}(\widetilde{\mathbf{A}}_u \cdot \widetilde{\mathbf{x}}_z), \tag{A.8}$$

to reconstruct $\mathbf{b}$. For completeness, this is illustrated in the last algorithm, Algorithm 4, which performs this reconstruction of $\mathbf{b}$ from the IFFT of $\widetilde{\mathbf{b}}_z$.

Algorithm 4: ζ_a^{-1}: Reconstruction of $\mathbf{b}$ from $\mathbf{b}_z$.

```
function b=BT_reconstruct(n,f,bz)
b=[];
if isempty(n)
 b=bz(f:f:f^2);
else
 b_edge=prod(2*n(2:length(n))-1)*f^2;
 this_n=n(1);
 for i=this_n:2*this_n-1
  b=[b BT_reconstruct(n(2:length(n)),f,...
       bz(1+(i-1)*b_edge:i*b_edge))];
 end
end
```

If we denote this last operation by ζ_a^{-1}, then the fast matrix-vector multiply can be concisely written as

$$\mathbf{b} = \zeta_a^{-1}\left[\mathcal{F}^{-1}\left(\widetilde{\Pi_{bt}(\mathbf{A})} \cdot \widetilde{\zeta_p(\mathbf{x})}\right)\right] \tag{A.9}$$

A.2.3 $\mathbf{T}_f^M$ Fast Multiply Example

As an example, consider the $\mathbf{T}_1^3(2,2,2)$ matrix shown in Fig. A-1. Here, $\Pi_{bf}(\mathbf{A}) = \mathbf{A}_u$= [1:27] and if $\mathbf{x}$=[1:8] as in Fig. A-1, The operator ζ_p implemented by algorithm 3 in the Appendix yields $\zeta_p(\mathbf{x}) = \mathbf{x}_z$ =[1 2 0 3 4 0 0 0 0 0 5 6 0 7 8 0 0 0 0]. A graphical representation of the step in the convolution $\mathbf{A}_u * \mathbf{x}_z$ that yields the first term of $\mathbf{b}_z$ (and $\mathbf{b}$) is provided as Fig. A-2.

The following Matlab commands perform the FFT on $\mathbf{A}_u$ and $\mathbf{x}_z$ to yield $\widetilde{\mathbf{A}}_u$ and $\widetilde{\mathbf{x}}_z$ respectively, the multiplication of $\widetilde{\mathbf{A}}_u$ and $\widetilde{\mathbf{x}}_z$ which yields $\widetilde{\mathbf{b}}_z$, and the reconstruction of $\mathbf{b}$ from the IFFT of $\widetilde{\mathbf{b}}_z$ using Algorithm 4. Here, A_u = [1:27], x = [1:8], n=[2 2 2], f=1, and fft_length=27.

```
x_z=BT_pad(n,f,x);
A_u_tilde=fft(fliplr(A_u),fft_length);
x_z_tilde=fft(x_z,fft_length);
```

```
b_z_tilde=A_u_tilde.*x_z_tilde;
b_z=ifft(b_z_tilde);
b=BT_reconstruct(n,f,b_z);
```

A.3 Numerical Implementation and Example: 3-D Scattering

We study the electromagnetic scattering from a arbitrarily shaped dielectric target inscribed in a cube which has a mesh having N_x, N_y, and N_z grid points in the x, y, and z directions respectively superimposed upon it. At each grid point, a small (electric field assumed constant or $ka \ll 1$) sphere with permittivity ϵ_p and volume $V = d_x d_y d_z$ (where d_x, d_y, and d_z are the lattice spacings) is placed and allowed to exhibit only an electric dipole response (see Fig. A-3). This method can be shown to be equivalent to the DDA ([48], [129]) and in fact the fast method presented in this work may also be applied to matrix-vector multiplications used for iterative solutions in the DDA.

The scattered electric field from a small dielectric scatterer is [43]

$$\overline{E}_s(\overline{r}) = \frac{-k^2}{\epsilon}\,\overline{\overline{G}}(\overline{r},\overline{r}')\,\overline{p}, \tag{A.10}$$

where

$$\overline{\overline{G}}(\overline{r},\overline{r}') = \left[\overline{\overline{I}} + \frac{\nabla\nabla}{k^2}\right] g(\overline{r},\overline{r}'), \tag{A.11}$$

$$\overline{p} = v_0(\epsilon_p - \epsilon)\overline{E}(\overline{r}'), \tag{A.12}$$

$g(\overline{r},\overline{r}')$ is the scalar Green's function, v_0 is the volume of the particle, ϵ_p is the permittivity of the particle, k is the background wavenumber, and ϵ is the permittivity of the host medium. Accordingly, the scattering from a collection of small scatterers excited by an incident electric field $\overline{E}_{inc}$ leads to the discretized volume integral equation (VIE), *i.e.*

$$\overline{E}(\overline{r}) = \overline{E}_{inc}(\overline{r}) + \frac{k^2}{\epsilon}\sum_{j=1}^{N} v_j(\epsilon_j - \epsilon)\,\overline{\overline{G}}(\overline{r},\overline{r}_j)\,\overline{E}(\overline{r}_j). \tag{A.13}$$

Thus the electric field at each scatterer caused by both the incident electric field and the

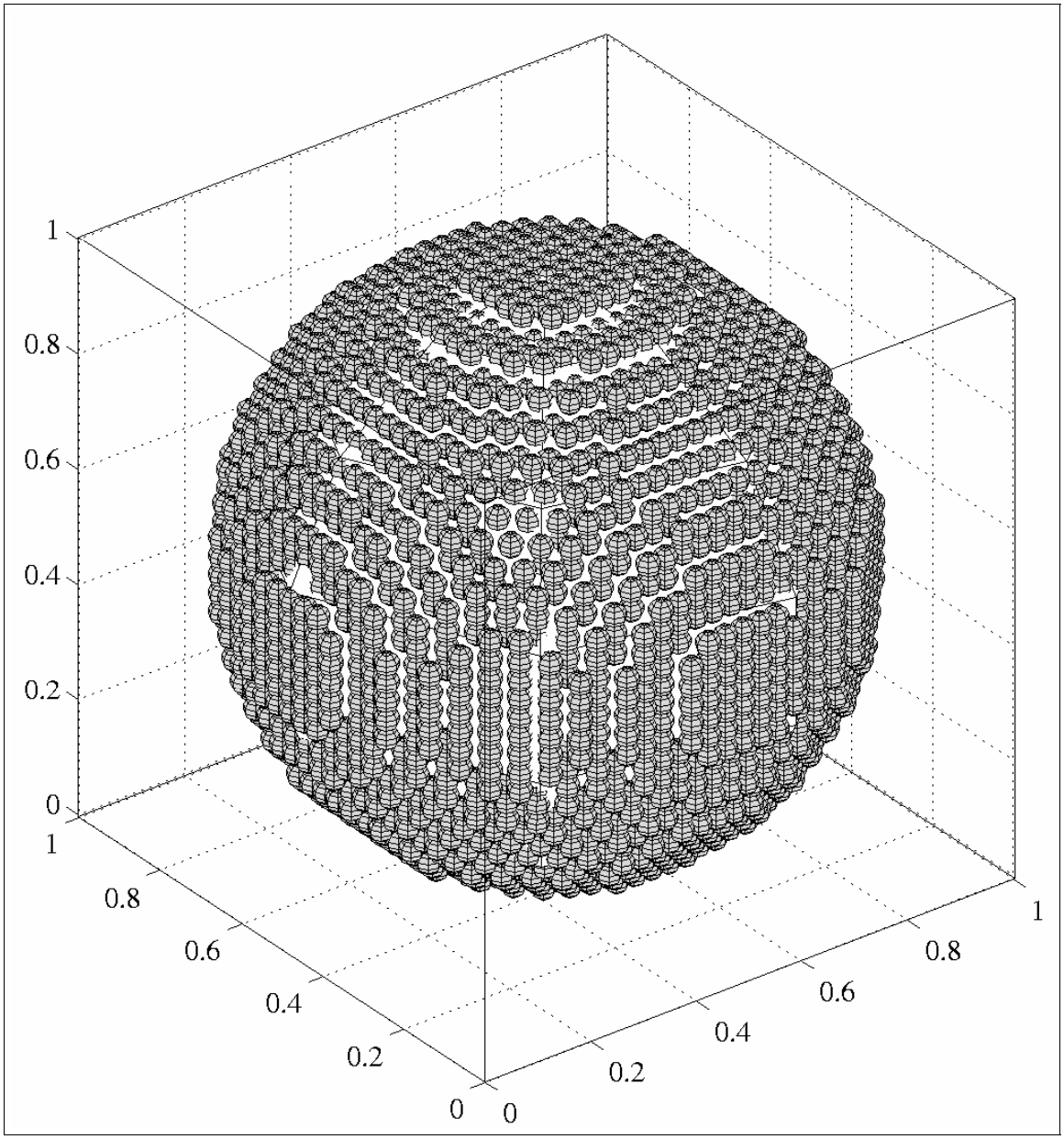

Figure A-3: Discrete Dipole Approximation of a homogeneous sphere using a 31x31x31 grid.

field radiated from each of the other dipoles can be expressed as

$$\overline{E}(\overline{r}_i) = \overline{E}_{inc}(\overline{r}_i) + \frac{k^2}{\epsilon} \sum_{j=1}^{N} v_j(\epsilon_j - \epsilon)\, \overline{\overline{G}}(\overline{r}_i, \overline{r}_j)\, \overline{E}(\overline{r}_j). \tag{A.14}$$

For otherwise similar spheres on lattice points, the interaction between particles i and j can be seen to depend only on $\overline{\overline{G}}(\overline{r}_i, \overline{r}_j)$ which is translationally invariant.

Using a method of moments (MoM) procedure [16], induced dipole strengths are computed using the electrostatic solution to the Laplace equation for a sphere as the basis functions. This model incorporates multiple scattering effects into an interaction (impedance) matrix $\mathbf{Z}$ based on Eq. A.14. The entries of $\mathbf{Z}$ constitute a MBT ($\mathbf{T}_3^3(N_x, N_y, N_z)$) due to the translationally invariant Green's function kernel. The scattered electric field in the far-field can be approximated using these induced dipole moments. The resulting system of equations is often ill-conditioned in the case of large scatterers. Therefore, the solution is usually sought through iterative techniques requiring (fast) matrix-vector multiplies.

The inverse problem of finding the effective permittivity (ϵ_{eff}) of the target – defined as the permittivity of a homogeneous solid of the same size and shape as the test volume which reacts to electromagnetic excitation in the most similar manner as the test volume – may be calculated through comparison of the scattered field to Mie scattering (Chapter 2 and [16, 21]) in the case that the target is a sphere, as illustrated in Fig. A-3. As the discretization size is decreased, or equivalently, as the number of grid points in each dimension increases, the result is expected to converge to the scattering from a homogeneous sphere with uniform permittivity ϵ_{eff}. Figures. A-4 and A-5 illustrate results of this comparison for two different sizes and contrasts with increasingly fine grid spacings. The %error column included on the table accompanying each plot is calculated according to

$$\%\text{error} = \text{rms}\left(\frac{\sigma_{vie}}{\sigma_{Mie}} - 1\right) \times 100, \tag{A.15}$$

where σ_{Mie} is the radar cross section (RCS) of a homogeneous dielectric sphere according Mie scattering, and σ_{VIE} is the RCS computed using the discrete volume integral equation (Eq. A.14). Table A.1 shows the total number of iterations used by the iterative solver (in

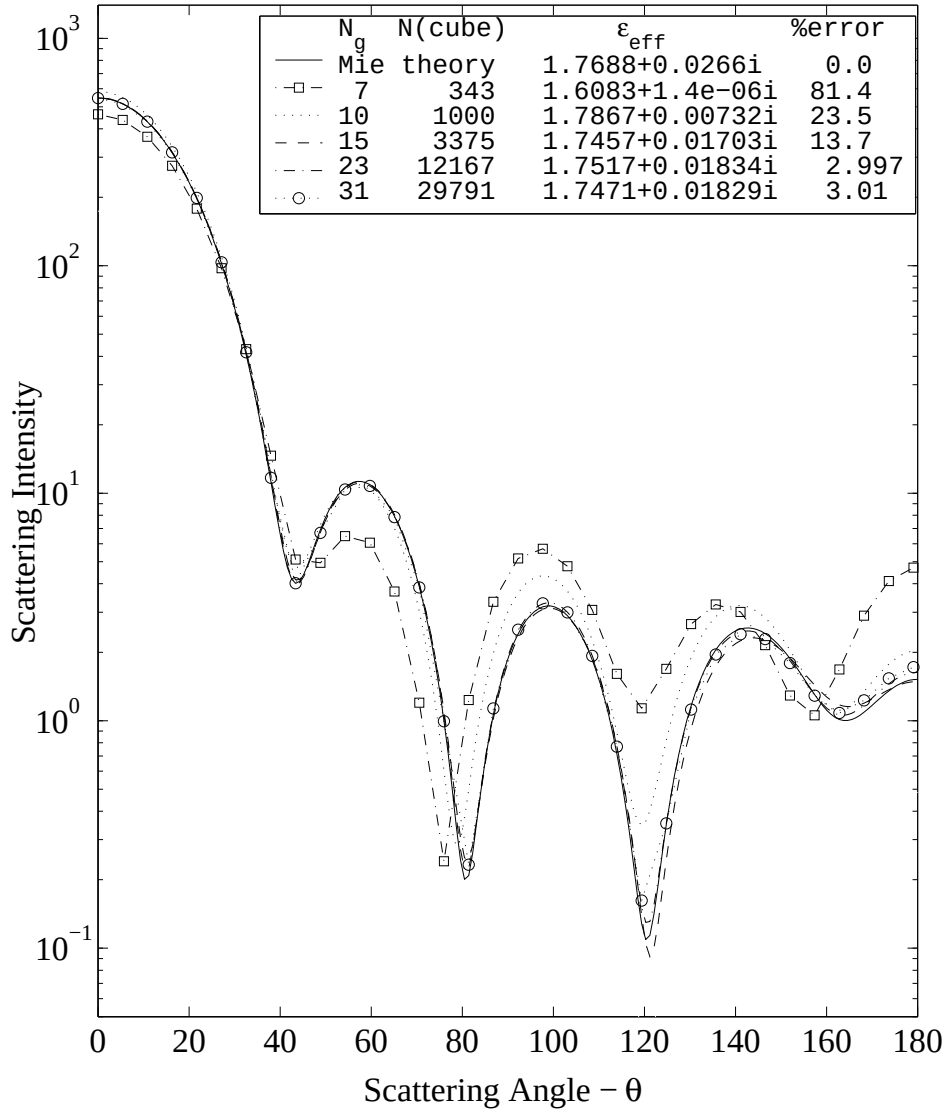

Figure A-4: Effective permittivity (ϵ_{eff}) results for a spherical test volume having meshes with 7-31 grid points per axis, $\epsilon_p = 1.7688 + 0.0266i$, and $ka = 5$ for the test volume.

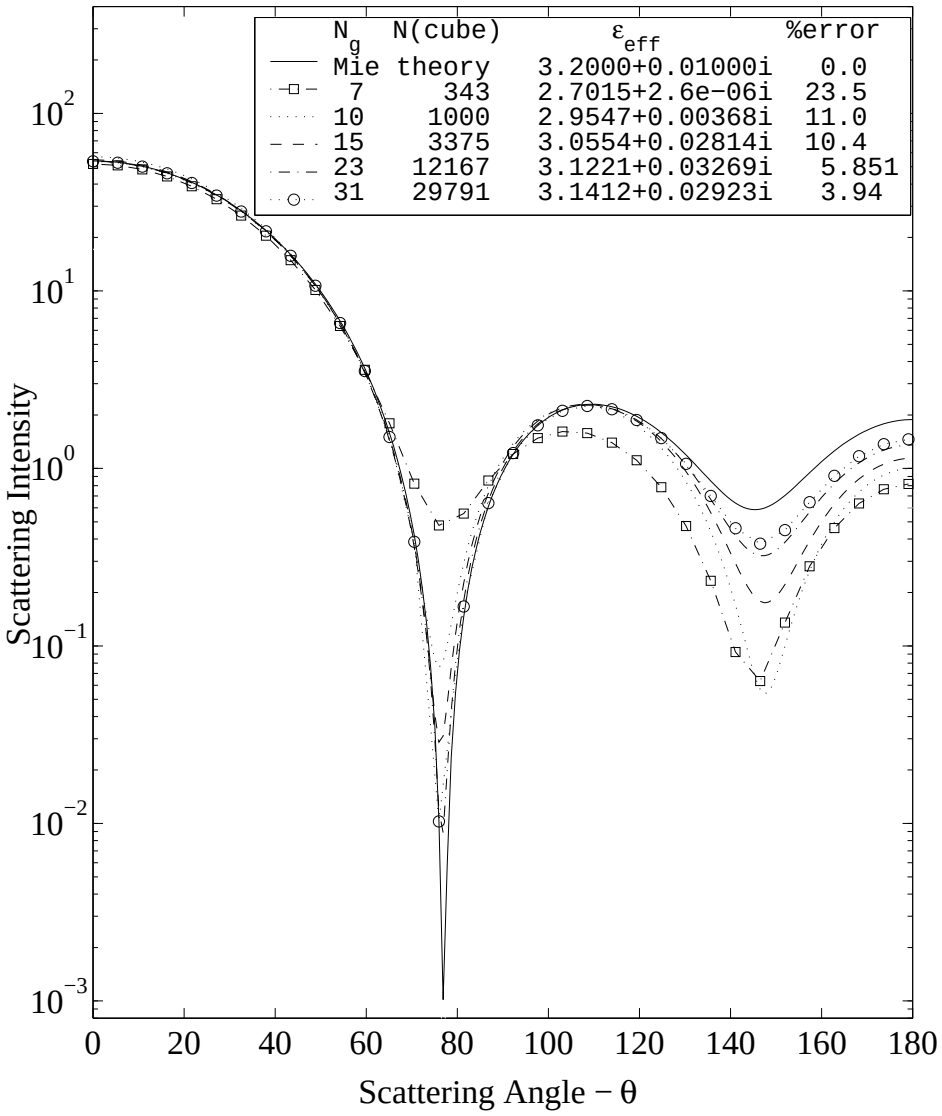

Figure A-5: Effective permittivity (ϵ_{eff}) results for $\epsilon_p = 3.2 + 0.01i$, and $ka = 2.5$.

this case the biconjugate gradient stabilized solver) and the total computational time for each solution on a Compaq Alphaserver DS20E with 4GB of RAM.

The far field scattering results for the collection of spheres contained in a spherical boundary approaches the Mie scattering results given a sufficiently fine grid spacing. As ka and ϵ_p increase the grid needs to be more densely spaced in order to maintain the validity of the previously mentioned assumptions.

N_g	length of $\mathbf{A}_u$ (FFT length) (Eq. A.7)	MBT Fast Method Time (s)	Full Method Time (s)	%error (Eq A.15)
7	19773	1.33	5.85	124.0
10	61731	5.98	25.5	37.2
15	219501	19.25	190.7	9.41
23	820125	87.9	2544.0	2.904
31	2042829	363.3	14295.3	3.06

Table A.1: Comparison of MBT fast method computation times with the full method for the case of Fig. A-4.

A.4 Discussion and Conclusion

The procedure for expediting a matrix-vector multiply involving $\mathbf{T}_f^M$ multilevel block-Toeplitz (MBT) matrices outlined in this communication is efficient both in terms of memory and computational complexity. It can be easily applied to 3-D scattering from arbitrary shaped targets using a discretized integral formulation. This was illustrated in Section A.3, where a comparison between the scattering from a spherical volume containing many small dielectric spheres and the scattering from a homogeneous sphere of the same size and shape of the spherical volume was presented. The effective permittivity ϵ_{eff} of the medium composed by the small particles was calculated and shown to converge to the permittivity of the Mie sphere for sufficiently fine grid spacings. In contrast to previously proposed algorithms, this fast matrix-vector multiply may be accurately described as a minimal memory method due to the fact that only the unique information from a $\mathbf{T}_f^M$ matrix is stored in memory. Also, the reduced complexity is achieved in a fashion similar, but not identical to those described in [125] and [126] with a speed improvement due to the 1-D FFT length of $\prod_{i=1}^{M}(2n_i - 1)$ in the present method.

Appendix B

Magnetoquasistatic Solution for Conducting and Permeable Spheroidal Shells

B.1 Exact Formulation

A conducting and permeable prolate spheroidal shell of uniform thickness is placed in a nonpermeable and weakly conducting homogeneous background and excited by a time-harmonic primary magnetic field $\overline{H}_o(\overline{r})e^{-i\omega t}$ (Fig. B-1). The time dependence expression of $e^{-i\omega t}$ is suppressed below. In the quasi-magnetostatic regime, we assume that the displacement current can be neglected in comparison to the conduction current. The magnetic fields in the spheroidal shell and in the core satisfy the vector wave equation [134]

$$\nabla \times \nabla \times \overline{H}_1 - k^2 \overline{H}_1 = 0 \tag{B.1}$$

with the wavenumber k being either $k_1^2 = i\omega\sigma_1\mu_1$ for the shell layer, or $k_3^2 = i\omega\sigma_3\mu_3$ for the core. Thus the magnetic field inside the shell layer ($\xi = \xi_0, \;\; d_\beta < d < d_\alpha$ [region 1]) can be expanded using the (divergence-free) vector spheroidal wavefunctions of first kind (which

[1]This Appendix, as well as Appendix C, borrows heavily from [133], Chapter 2 therein.

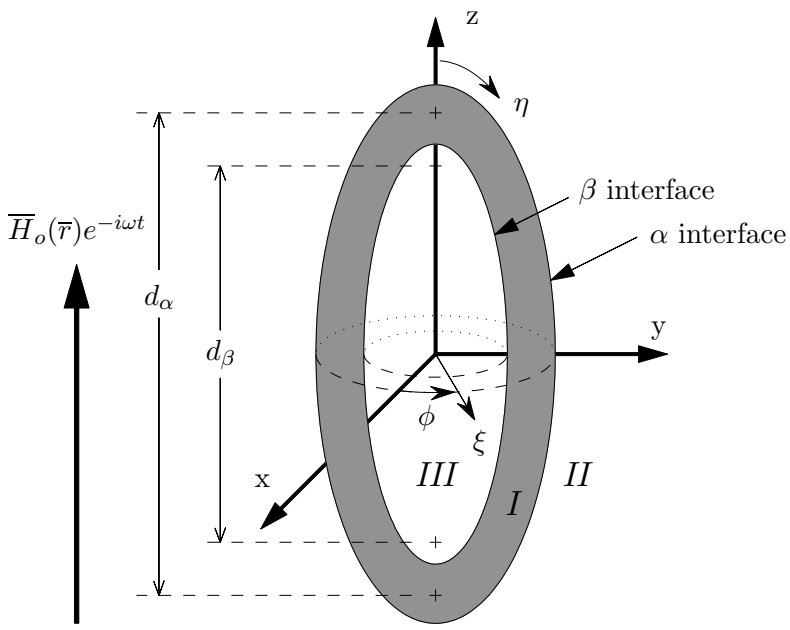

Figure B-1: A conducting and permeable prolate spheroidal shell is excited by a time-harmonic primary field $\overline{H}_o(\overline{r})e^{-i\omega t}$. The prolate spheroidal coordinate system is specified by (ξ, η, ϕ) with $-1 \leq \eta \leq 1$, $1 \leq \xi < \infty$, and $0 \leq \phi < 2\pi$. The outer (α) interface of the shell is given by $\xi = \xi_0 = 1/\sqrt{1-(1/e)^2}$ and d_α with $e_\alpha = b_\alpha/a_\alpha = b_\beta/a_\beta$ is the elongation ratio. The inner (β) interface is specified by ξ_0 and d_β.

are regular at the origin) and third kind (which are regular at infinity) [66]:

$$\overline{H}_1 = H_o \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} \Big[A_{pmn}^{(M)} \overline{M}_{pmn}^{r(1)}(c;\xi,\eta,\phi) + A_{pmn}^{(N)} \overline{N}_{pmn}^{r(1)}(c;\xi,\eta,\phi) + C_{pmn}^{(M)} \overline{M}_{pmn}^{r(3)}(c;\xi,\eta,\phi) + C_{pmn}^{(N)} \overline{N}_{pmn}^{r(3)}(c;\xi,\eta,\phi) \Big] \tag{B.2}$$

while the field inside the core of the spheroid (region 3), is expanded in terms of spheroidal wavefunctions of the first kind only:

$$\overline{H}_3 = H_o \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} \Big[D_{pmn}^{(M)} \overline{M}_{pmn}^{r(1)}(c;\xi,\eta,\phi) + D_{pmn}^{(N)} \overline{N}_{pmn}^{r(1)}(c;\xi,\eta,\phi) \Big] \tag{B.3}$$

In Eqs. (B.2) and (B.3), H_o is a constant and the dimensionless spheroidal frequency parameter c is defined as

$$c = k\frac{d}{2} \tag{B.4}$$

with $d = 2\sqrt{b^2 - a^2}$ being the interfocal distance for either the inner or outer layer.

The vector spheroidal wavefunctions of the third kind must be included in the description of the field in the shell layer due to the presence of the inner boundary of the shell layer. Using an analogy to microwave scattering, the spheroidal wavefunctions of the first kind represent the "ingoing" wave, while the spheroidal wavefunctions of the third kind represent the "outgoing" wave. This corresponds to the case of a layered sphere under the excitation of a standing wave [44]. The angular function is an infinite sum of associated Legendre functions, radial function of the first kind is an infinite sum of spherical Bessel functions, while the radial function of the third kind is an infinite sum of spherical Hankel functions of the first kind.

The vector wavefunctions can be generated from the scalar wavefunction as follows [66]:

$$\overline{M}_{pmn}^{r(i)} = \nabla\psi_{pmn}^{(i)} \times \overline{r} \tag{B.5}$$

$$\overline{N}_{pmn}^{r(i)} = \frac{1}{k_1}\nabla \times \overline{M}_{pmn}^{r(i)} \tag{B.6}$$

with i=1, 2, 3, or 4 and

$$\psi_{pmn}^{(i)} = S_{mn}(c_1, \eta)R_{mn}^{(i)}(c_1, \xi)T_{pm}(\phi) \tag{B.7}$$

where $S_{mn}(c_1, \eta)$ and $R_{mn}^{(i)}(c_1, \xi)$ are the spheroidal angle and radial functions, respectively, while the azimuthal function $T_{pm}(\phi)$ is simply

$$T_{pm}(\phi) = \begin{cases} \cos m\phi, & \text{p=0} \\ \sin m\phi, & \text{p=1} \end{cases} \tag{B.8}$$

Note that other choices for generating the vector wavefunctions are possible besides (B.5) and (B.6), (cf. [50, 66]). The r-vector wavefunctions used here represent a common choice

for scattering problems [49,59]. To simplify notation, we will dispense with the superscript r and the explicit dependence on xi_0 of the spheroidal wavefunctions.

Outside the spheroid (region 2) the medium is assumed to be poorly conducting and weakly magnetic. Thus $k_2 \to 0$ in the quasi-static limit, and the magnetic fields can be obtained from their scalar potentials, which satisfy the Laplace equation. We define a scalar potential $U_2(\overline{r})$ such that the external magnetic field is $\overline{H}_2 = -\nabla U_2$. The total external magnetic field can be written as a sum of the primary and secondary fields, i.e.,

$$\overline{H}_2 = \overline{H}_o + \overline{H}_{\rm s} = -\nabla U_o - \nabla U_{\rm s} \tag{B.9}$$

The primary field potential $U_o(\overline{r})$ is expanded as

$$U_o(\overline{r}) = \frac{H_o d_\alpha}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} b_{pmn} \Phi^{(1)}_{pmn}(\xi, \eta, \phi) \tag{B.10}$$

where

$$\Phi^{(1)}_{pmn}(\xi, \eta, \phi) = P_n^m(\xi) P_n^m(\eta) T_{pm}(\phi) \tag{B.11}$$

is the spheroidal Laplace solution of first kind which is regular at the origin ($\xi \to 1$). The source coefficients b_{pmn} are determined from the primary field. The functions $P_n^m(\eta)$ and $P_n^m(\xi)$ are the associated Legendre functions of first kind. The associated Legendre functions used here follow the convention of Abramowitz and Stegun [88] (Eqs. 8.6.6 and 8.6.7), which is slightly different from that of Flammer [66].

Similarly, the secondary magnetic field potential $U_{\rm s}(\overline{r})$ can be expanded as

$$U_{\rm s}(\overline{r}) = \frac{H_o d}{2} \sum_{m=0}^{\infty} \sum_{n=m}^{\infty} \sum_{p=0}^{1} B_{pmn} \Phi_{pmn}(\xi, \eta, \phi) \tag{B.12}$$

where

$$\Phi_{pmn}(\xi, \eta, \phi) = Q_n^m(\xi) P_n^m(\eta) T_{pm}(\phi) \tag{B.13}$$

is the spheroidal Laplace solution of second kind which is regular at infinity ($\xi \to \infty$). The function $Q_n^m(\xi)$ is the associated Legendre function of second kind.

The boundary conditions are specified by the continuity of tangential components of $\overline{H}$ and normal component of $\overline{B}$ [134] at the outer surface of the spheroidal shell (ξ_0, d_α)

$$H_{1\eta} = H_{2\eta} \tag{B.14a}$$

$$H_{1\phi} = H_{2\phi} \tag{B.14b}$$

$$\mu_{r1} H_{1\xi} = H_{2\xi} \tag{B.14c}$$

and at the inner surface (ξ_0, d_β)

$$H_{1\eta} = H_{3\eta} \tag{B.15a}$$

$$H_{1\phi} = H_{3\phi} \tag{B.15b}$$

$$\mu_{r1} H_{1\xi} = \mu_{r3} H_{3\xi} \tag{B.15c}$$

where $\mu_{r1} = \mu_1/\mu_0$ and $\mu_{r3} = \mu_3/\mu_0$ are the relative permeabilities of the spheroidal shell and core with respect to their surrounding respectively.

The vector spheroidal wavefunctions in component forms are given in Appendix C.1. In matching the boundary conditions, first note that from (B.8),

$$\frac{dT_{pmn}}{d\phi} = (-1)^{\tilde{p}} m T_{\tilde{p}mn}(\phi) \tag{B.16}$$

where $\tilde{p} = 0$ for $p = 1$ and vice versa. Now, matching the angular component H_η according

to (B.14a) gives

$$\sum_{n=m}^{\infty}\left\{\left[(-1)^p m A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\eta;mn}^{(1)}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}A_{pmn}^{(N)}\widetilde{N}_{\eta;mn}^{(1)}(c_{1\alpha},\eta)\right]+\left[(-1)^p m C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\eta;mn}^{(3)}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}C_{pmn}^{(N)}\widetilde{N}_{\eta;mn}^{(3)}(c_{1\alpha},\eta)\right]\right\}$$
$$=-\sum_{n=m}^{\infty}\left[b_{pmn}P_n^m(\xi_0)+B_{pmn}Q_n^m(\xi_0)\right]\frac{dP_n^m}{d\eta} \tag{B.17}$$

where

$$c_{1\alpha}=k_1\frac{d_\alpha}{2} \tag{B.18}$$

and we have made use of the orthogonality property of $\cos m\phi$ and $\sin m\phi$ to eliminate the sum over the indices m and p and However, the sum over n must be retained because no such orthogonality condition exists between the angular functions $S_{mn}(c,\eta)$ and $P_n^m(\eta)=S_{mn}(c=0,\eta)$. In general, $S_{mn}(c,\eta)$ and $S_{mn}(c',\eta)$ are orthogonal only when $c=c'$. This accounts for the complexity of the multi-media spheroidal boundary value problem. $\widetilde{M}_{mn}^{(i)}$ and $\widetilde{N}_{mn}^{(i)}$ are "reduced" vector spheroidal wavefunctions whose definitions can be found in Appendix C.

Similarly, matching H_ϕ and H_ξ according to Eqs. (B.14c)–(B.14b) yields

$$\mu_{r1}\sum_{n=m}^{\infty}\left\{\left[(-1)^p m A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\xi;mn}^{(1)}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}A_{pmn}^{(N)}\widetilde{N}_{\xi;mn}^{(1)}(c_{1\alpha},\eta)\right]+\left[(-1)^p m C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\xi;mn}^{(3)}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}C_{pmn}^{(N)}\widetilde{N}_{\xi;mn}^{(3)}(c_{1\alpha},\eta)\right]\right\}$$
$$=-\sum_{n=m}^{\infty}\left[b_{pmn}\frac{dP_n^m}{d\xi_0}+B_{pmn}\frac{dQ_n^m}{d\xi_0}\right]P_n^m(\eta). \tag{B.19}$$

$$\sum_{n=m}^{\infty}\left\{\left[(-1)^{p+1} A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\phi;mn}^{(1)}(c_{1\alpha},\eta)+\frac{m}{c_{1\alpha}}A_{pmn}^{(N)}\widetilde{N}_{\phi;mn}^{(1)}(c_{1\alpha},\eta)\right]+\left[(-1)^{p+1} C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\phi;mn}^{(3)}(c_{1\alpha},\eta)+\frac{m}{c_{1\alpha}}C_{pmn}^{(N)}\widetilde{N}_{\phi;mn}^{(3)}(c_{1\alpha},\eta)\right]\right\}$$
$$=-\sum_{n=m}^{\infty}m\left[b_{pmn}P_n^m(\xi_0)+B_{pmn}Q_n^m(\xi_0)\right]P_n^m(\eta) \tag{B.20}$$

For the inner boundary, we have respectively for the H_η, H_ξ, and H_ϕ boundary conditions:

$$\begin{aligned}
&\sum_{n=m}^{\infty}\left\{\left[(-1)^p m A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\eta;mn}^{(1)}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}A_{pmn}^{(N)}\widetilde{N}_{\eta;mn}^{(1)}(c_{1\beta},\eta)\right]+\right.\\
&\qquad\left.\left[(-1)^p m C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\eta;mn}^{(3)}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}C_{pmn}^{(N)}\widetilde{N}_{\eta;mn}^{(3)}(c_{1\beta},\eta)\right]\right\}\\
&=\sum_{n=m}^{\infty}\left\{\left[(-1)^p m D_{\tilde{p}mn}^{(M)}\widetilde{M}_{\eta;mn}^{(1)}(c_{3\beta},\eta)+\frac{1}{c_{3\beta}}D_{pmn}^{(N)}\widetilde{N}_{\eta;mn}^{(1)}(c_{3\beta},\eta)\right]\right\}
\end{aligned}\tag{B.21}$$

$$\begin{aligned}
&\mu_{r1}\sum_{n=m}^{\infty}\left\{\left[(-1)^p m A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\xi;mn}^{(1)}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}A_{pmn}^{(N)}\widetilde{N}_{\xi;mn}^{(1)}(c_{1\beta},\eta)\right]+\right.\\
&\qquad\left.\left[(-1)^p m C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\xi;mn}^{(3)}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}C_{pmn}^{(N)}\widetilde{N}_{\xi;mn}^{(3)}(c_{1\beta},\eta)\right]\right\}\\
&=\mu_{r3}\sum_{n=m}^{\infty}\left\{\left[(-1)^p m D_{\tilde{p}mn}^{(M)}\widetilde{M}_{\xi;mn}^{(1)}(c_{3\beta},\eta)+\frac{1}{c_{3\beta}}D_{pmn}^{(N)}\widetilde{N}_{\xi;mn}^{(1)}(c_{3\beta},\eta)\right]\right\}
\end{aligned}\tag{B.22}$$

$$\begin{aligned}
&\sum_{n=m}^{\infty}\left\{\left[(-1)^{p+1} A_{\tilde{p}mn}^{(M)}\widetilde{M}_{\phi;mn}^{(1)}(c_{1\beta},\eta)+\frac{m}{c_{1\beta}}A_{pmn}^{(N)}\widetilde{N}_{\phi;mn}^{(1)}(c_{1\beta},\eta)\right]+\right.\\
&\qquad\left.\left[(-1)^{p+1} C_{\tilde{p}mn}^{(M)}\widetilde{M}_{\phi;mn}^{(3)}(c_{1\beta},\eta)+\frac{m}{c_{1\beta}}C_{pmn}^{(N)}\widetilde{N}_{\phi;mn}^{(3)}(c_{1\beta},\eta)\right]\right\}\\
&=\sum_{n=m}^{\infty}\left\{\left[(-1)^{p+1} D_{\tilde{p}mn}^{(M)}\widetilde{M}_{\phi;mn}^{(1)}(c_{3\beta},\eta)+\frac{m}{c_{3\beta}}D_{pmn}^{(N)}\widetilde{N}_{\phi;mn}^{(1)}(c_{3\beta},\eta)\right]\right\}
\end{aligned}\tag{B.23}$$

where

$$c_{1\beta}=k_1\frac{d_\beta}{2}\qquad\text{and}\qquad c_{3\beta}=k_3\frac{d_\beta}{2}.\tag{B.24}$$

Equations (B.14) and (B.15) are used to determine the field expansion coefficients $A_{pmn}^{(M)}$, $A_{pmn}^{(N)}$, $C_{pmn}^{(M)}$, $C_{pmn}^{(N)}$, $D_{pmn}^{(M)}$, $D_{pmn}^{(N)}$, and B_{pmn}. Further simplifications can be made by considering the parity of the η dependence on the left- and right-hand sides of these equations. Take (B.19), for example. Under the parity transformation $\eta\to-\eta$, the nth term of its right-hand side, due to the functional form of $P_n^m(\eta)$, gives a factor of $(-1)^{n-m}$. On the other hand, $\widetilde{M}_{\xi;mn}^{r(i)}(\xi_0,\eta)$ and $\widetilde{N}_{\xi;mn}^{r(i)}(\xi_0,\eta)$ give $(-1)^{n-m+1}$ and $(-1)^{n-m}$, respectively. These can be deduced from their expressions in Appendix C and the fact that $S_{mn}(\eta)$ has

the same parity property as $P_n^m(\eta)$ [66]. These considerations imply that excitations b_{pmn} with $n-m$ even and $n-m$ odd are decoupled and can be treated separately. If $b_{pmn} \neq 0$ for $n-m$ even, then $B_{pmn'}$, $A^{(N)}_{pmn'}$, $A^{(M)}_{pm(n'+1)}$, $C^{(N)}_{pmn'}$, $C^{(M)}_{pm(n'+1)}$, $D^{(N)}_{pmn'}$, and $D^{(M)}_{pm(n'+1)}$ can be nonzero only for $n' = m, m+2, \ldots$. This will be referred to as the case of *even* excitation. If $b_{pmn} \neq 0$ for $n-m$ odd, then $B_{pmn'}$, $A^{(N)}_{pmn'}$, $A^{(M)}_{pm(n'-1)}$, $C^{(N)}_{pmn'}$, $C^{(M)}_{pm(n'-1)}$, $D^{(N)}_{pmn'}$, and $D^{(M)}_{pm(n'-1)}$ can be nonzero only for $n' = m+1, m+3, \ldots$. This will be referred to as the case of *odd* excitation.

Note that Eqs. (B.17)-(B.23) must hold for all η on the surface of the spheroid. A common procedure to solve such equations is to expand the left-hand sides, which are regular functions of η, in terms of an infinite series of $P_n^m(\eta)$. This can be accomplished by multiplying both sides of the equations by $P_{n'}^m(\eta)$, $n' = m, m+1, \ldots$, and integrating over η from -1 to 1. Applying this "testing" procedure to (B.19), (B.20), (B.22), (B.23), and (B.21) yield respectively,

$$\begin{aligned}
&\mu_{r1} \sum_{n=m}^{\infty} \left\{ \left[(-1)^p m A^{(M)}_{\tilde{p}mn} I^{(1)}_{\xi;n'n}(c_{1\alpha},\eta) + \frac{1}{c_{1\alpha}} A^{(N)}_{pmn} J^{(1)}_{\xi;n'n}(c_{1\alpha},\eta) \right] + \right. \\
&\qquad \left. \left[(-1)^p m C^{(M)}_{\tilde{p}mn} I^{(3)}_{\xi;n'n}(c_{1\alpha},\eta) + \frac{1}{c_{1\alpha}} C^{(N)}_{pmn} J^{(3)}_{\xi;n'n}(c_{1\alpha},\eta) \right] \right\} \\
&\quad = -\left[b_{pmn'} \frac{dP^m_{n'}}{d\xi_0} + B_{pmn'} \frac{dQ^m_{n'}}{d\xi_0} \right]
\end{aligned} \tag{B.25}$$

$$\begin{aligned}
&\sum_{n=m}^{\infty} \left\{ \left[(-1)^{p+1} A^{(M)}_{\tilde{p}mn} I^{(1)}_{\phi;n'n}(c_{1\alpha},\eta) + \frac{m}{c_{1\alpha}} A^{(N)}_{pmn} J^{(1)}_{\phi;n'n}(c_{1\alpha},\eta) \right] + \right. \\
&\qquad \left. \left[(-1)^{p+1} C^{(M)}_{\tilde{p}mn} I^{(3)}_{\phi;n'n}(c_{1\alpha},\eta) + \frac{m}{c_{1\alpha}} C^{(N)}_{pmn} J^{(3)}_{\phi;n'n}(c_{1\alpha},\eta) \right] \right\} \\
&\quad = -m \left[b_{pmn'} P^m_{n'}(\xi_0) + B_{pmn'} Q^m_{n'}(\xi_0) \right]
\end{aligned} \tag{B.26}$$

$$\begin{aligned}
&\mu_{r1} \sum_{n=m}^{\infty} \left\{ \left[(-1)^p m A^{(M)}_{\tilde{p}mn} I^{(1)}_{\xi;n'n}(c_{1\beta},\eta) + \frac{1}{c_{1\beta}} A^{(N)}_{pmn} J^{(1)}_{\xi;n'n}(c_{1\beta},\eta) \right] + \right. \\
&\qquad \left. \left[(-1)^p m C^{(M)}_{\tilde{p}mn} I^{(3)}_{\xi;n'n}(c_{1\beta},\eta) + \frac{1}{c_{1\beta}} C^{(N)}_{pmn} J^{(3)}_{\xi;n'n}(c_{1\beta},\eta) \right] \right\} \\
&= \mu_{r3} \sum_{n=m}^{\infty} \left[(-1)^p m D^{(M)}_{\tilde{p}mn} I^{(1)}_{\xi;n'n}(c_{3\beta},\eta) + \frac{1}{c_{3\beta}} D^{(N)}_{pmn} J^{(1)}_{\xi;n'n}(c_{3\beta},\eta) \right]
\end{aligned} \tag{B.27}$$

$$\sum_{n=m}^{\infty}\left\{\left[(-1)^{p+1}A^{(M)}_{\tilde{p}mn}I^{(1)}_{\phi;n'n}(c_{1\beta},\eta)+\frac{m}{c_{1\beta}}A^{(N)}_{pmn}J^{(1)}_{\phi;n'n}(c_{1\beta},\eta)\right]+\left[(-1)^{p+1}C^{(M)}_{\tilde{p}mn}I^{(3)}_{\phi;n'n}(c_{1\beta},\eta)+\frac{m}{c_{1\beta}}C^{(N)}_{pmn}J^{(3)}_{\phi;n'n}(c_{1\beta},\eta)\right]\right\}$$
$$=\sum_{n=m}^{\infty}\left[(-1)^{p+1}D^{(M)}_{\tilde{p}mn}I^{(1)}_{\phi;n'n}(c_{3\beta},\eta)+\frac{m}{c_{3\beta}}D^{(N)}_{pmn}J^{(1)}_{\phi;n'n}(c_{3\beta},\eta)\right] \tag{B.28}$$

$$\sum_{n=m}^{\infty}\left\{\left[(-1)^{p}mA^{(M)}_{\tilde{p}mn}I^{(1)}_{\eta;n'n}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}A^{(N)}_{pmn}J^{(1)}_{\eta;n'n}(c_{1\beta},\eta)\right]+\left[(-1)^{p}mC^{(M)}_{\tilde{p}mn}I^{(3)}_{\eta;n'n}(c_{1\beta},\eta)+\frac{1}{c_{1\beta}}C^{(N)}_{pmn}J^{(3)}_{\eta;n'n}(c_{1\beta},\eta)\right]\right\}$$
$$=\sum_{n=m}^{\infty}\left[(-1)^{p}mD^{(M)}_{\tilde{p}mn}I^{(1)}_{\eta;n'n}(c_{3\beta},\eta)+\frac{1}{c_{3\beta}}D^{(N)}_{pmn}J^{(1)}_{\eta;n'n}(c_{3\beta},\eta)\right]. \tag{B.29}$$

The integrals $I^{(i)}_{\gamma;n'n}(c,\eta)$ and $J^{(i)}_{\gamma;n'n}(c,\eta)$ for $i=1,3$ and $(\beta=\xi,\eta,\phi)$ are matrices that couple different angular modes as a result of the non-orthogonality of the angular wave-functions. They are defined in Appendix C.2.

For the angular component at the outer (α) layer, the angular dependence of the right-hand side of Eq. (B.17) is $dP^m_n/d\eta$. Multiplying with $P^m_{n'}(\eta)$ and integrating would result in somewhat complicated expressions on the right-hand side. We prefer therefore to multiply both sides of Eq. (B.17) by the factor $(1-\eta^2)$ before applying the testing procedure. Since

$$(1-\eta^2)\frac{dP^m_n}{d\eta}=\gamma_{1m}(n)P^m_{n-1}(\eta)-\gamma_{2m}(n)P^m_{n+1}(\eta) \tag{B.30}$$

with

$$\gamma_{1m}(n)=\frac{(n+1)(n+m)}{2n+1},\qquad \gamma_{2m}(n)=\frac{n(n-m+1)}{2n+1} \tag{B.31}$$

we get, from (B.17),

$$\begin{aligned}\sum_{n=m}^{\infty}\Bigg\{&\left[(-1)^p m A^{(M)}_{\tilde{p}mn} I^{(1)}_{\eta;n'n}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}A^{(N)}_{pmn}J^{(1)}_{\eta;n'n}(c_{1\alpha},\eta)\right]+\\&\left[(-1)^p m C^{(M)}_{\tilde{p}mn} I^{(3)}_{\eta;n'n}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}C^{(N)}_{pmn}J^{(3)}_{\eta;n'n}(c_{1\alpha},\eta)\right]\Bigg\}\\&=-\gamma_{1m}(n'+1)\left[b_{pm(n'+1)}P^m_{n'+1}(\xi_0)+B_{pm(n'+1)}Q^m_{n'+1}(\xi_0)\right]\\&\quad+\gamma_{2m}(n'-1)\left[b_{pm(n'-1)}P^m_{n'-1}(\xi_0)+B_{pm(n'-1)}Q^m_{n'-1}(\xi_0)\right]\end{aligned}\tag{B.32}$$

Equations (B.25)–(B.28), (B.29), and (B.32), are the master equations with which the unknown expansion coefficients can be solved. Using (B.25), we can solve for $B_{pmn'}$ in terms of $A^{(M)}_{\tilde{p}mn}$, $A^{(N)}_{pmn}$, $C^{(M)}_{\tilde{p}mn}$ and $C^{(N)}_{pmn}$:

$$\begin{aligned}B_{pmn'}=-\left[\frac{dQ^m_{n'}}{d\xi_0}\right]^{-1}\Bigg\{&b_{pmn'}\frac{dP^m_{n'}}{d\xi_0}\\&+\mu_{r1}\sum_{n=m}^{\infty}\Bigg\{\left[(-1)^p m A^{(M)}_{\tilde{p}mn} I^{(1)}_{\xi;n'n}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}A^{(N)}_{pmn}J^{(1)}_{\xi;n'n}(c_{1\alpha},\eta)\right]\\&\quad+\left[(-1)^p m C^{(M)}_{\tilde{p}mn} I^{(3)}_{\xi;n'n}(c_{1\alpha},\eta)+\frac{1}{c_{1\alpha}}C^{(N)}_{pmn}J^{(3)}_{\xi;n'n}(c_{1\alpha},\eta)\right]\Bigg\}\Bigg\}\end{aligned}\tag{B.33}$$

Incidentally, Eq. (B.33) can be used to obtain the high-frequency limit for $B_{pmn'}$. In this limit, only surface currents exist; therefore, the internal field must vanish. Thus setting $A^{(M)}_{pmn}=A^{(N)}_{pmn}=C^{(M)}_{pmn}=C^{(N)}_{pmn}=0$ in (B.33) gives

$$B_{pmn'}\to-\left[\frac{dQ^m_{n'}}{d\xi_0}\right]^{-1}\frac{dP^m_{n'}}{d\xi_0}\,b_{pmn'}\qquad\text{as }|c_{1\alpha}|\to\infty\tag{B.34}$$

which is only a function of ξ_0 (or equivalently elongation ratio $e=b/a$) and the source vector $b_{pmn'}$.

To obtain numerical results, the infinite system of equations must be truncated. Let N_T be the maximum order of coefficients considered, i.e.,

$$A^{(M)}_{\tilde{p}m(n+1)}=A^{(N)}_{pmn}=C^{(M)}_{\tilde{p}m(n+1)}=C^{(N)}_{pmn}=D^{(M)}_{\tilde{p}m(n+1)}=D^{(N)}_{pmn}=B_{pmn}=0\tag{B.35}$$

(even excitation) or

$$A^{(M)}_{\tilde{p}mn} = A^{(N)}_{pm(n+1)} = C^{(M)}_{\tilde{p}mn} = C^{(N)}_{pm(n+1)} = D^{(M)}_{\tilde{p}mn} = D^{(N)}_{pm(n+1)} = B_{pm(n+1)} = 0 \quad \text{(B.36)}$$

(odd excitation) for $n > N_T$. We choose N_T such that $N_T - m$ is even. Then the total number for each set of coefficients is $L_T = (N_T - m)/2 + 1$.

To cast the system of equations in matrix form, we let, for even excitation,

$$\overline{B} = \left[B_{pmm}, B_{pm(m+2)}, \cdots, B_{pmN_T}\right]^t \quad \text{(B.37a)}$$

$$\overline{b} = \left[b_{pmm}, b_{pm(m+2)}, \cdots, b_{pmN_T}\right]^t \quad \text{(B.37b)}$$

$$\overline{A} = \Bigg[A^{(N)}_{pmm}, A^{(N)}_{pm(m+2)}, \cdots, A^{(N)}_{pmN_T}, (-1)^p A^{(M)}_{\tilde{p}m(m+1)}, (-1)^p A^{(M)}_{\tilde{p}m(m+3)}, \cdots, (-1)^p A^{(M)}_{\tilde{p}m(N_T+1)}\Bigg]^t \quad \text{(B.37c)}$$

$$\overline{C} = \Bigg[C^{(N)}_{pmm}, C^{(N)}_{pm(m+2)}, \cdots, C^{(N)}_{pmN_T}, (-1)^p C^{(M)}_{\tilde{p}m(m+1)}, (-1)^p C^{(M)}_{\tilde{p}m(m+3)}, \cdots, (-1)^p C^{(M)}_{\tilde{p}m(N_T+1)}\Bigg]^t \quad \text{(B.37d)}$$

$$\overline{D} = \Bigg[D^{(N)}_{pmm}, D^{(N)}_{pm(m+2)}, \cdots, D^{(N)}_{pmN_T}, (-1)^p D^{(M)}_{\tilde{p}m(m+1)}, (-1)^p D^{(M)}_{\tilde{p}m(m+3)}, \cdots, (-1)^p D^{(M)}_{\tilde{p}m(N_T+1)}\Bigg]^t \quad \text{(B.37e)}$$

where b and B are column vectors with length L_T and A, C, and D are column vectors with length $2L_T$. The superscript t denotes matrix transpose. Similarly, for odd excitation,

$$\overline{B} = \left[B_{pm(m+1)}, B_{pm(m+3)}, \cdots, B_{pm(N_T+1)}\right]^t \tag{B.38a}$$

$$\overline{D} = \left[D_{pm(m+1)}, D_{pm(m+3)}, \cdots, D_{pm(N_T+1)}\right]^t \tag{B.38b}$$

$$\overline{b} = \left[b_{pm(m+1)}, b_{pm(m+3)}, \cdots, b_{pm(N_T+1)}\right]^t \tag{B.38c}$$

$$\overline{A} = \Bigg[A^{(N)}_{pm(m+1)}, A^{(N)}_{pm(m+3)}, \cdots, A^{(N)}_{pm(N_T+1)}, (-1)^p A^{(M)}_{\tilde{p}mm}, (-1)^p A^{(M)}_{\tilde{p}m(m+2)}, \cdots, (-1)^p A^{(M)}_{\tilde{p}mN_T}\Bigg]^t \tag{B.38d}$$

$$\overline{C} = \Bigg[C^{(N)}_{pm(m+1)}, C^{(N)}_{pm(m+3)}, \cdots, C^{(N)}_{pm(N_T+1)}, (-1)^p C^{(M)}_{\tilde{p}mm}, (-1)^p C^{(M)}_{\tilde{p}m(m+2)}, \cdots, (-1)^p C^{(M)}_{\tilde{p}mN_T}\Bigg]^t \tag{B.38e}$$

$$\overline{D} = \Bigg[D^{(N)}_{pm(m+1)}, D^{(N)}_{pm(m+3)}, \cdots, D^{(N)}_{pm(N_T+1)}, (-1)^p D^{(M)}_{\tilde{p}mm}, (-1)^p D^{(M)}_{\tilde{p}m(m+2)}, \cdots, (-1)^p D^{(M)}_{\tilde{p}mN_T}\Bigg]^t \tag{B.38f}$$

(B.38g)

Then (B.32), (B.26), (B.33), (B.29), (B.28), and (B.27) can be rewritten in matrix notation a

$$\overline{\overline{Z}}^{(1)}_{\xi}(c_{1\alpha},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\xi}(c_{1\alpha},\eta)\cdot\overline{C} = \overline{\overline{W}}^{(p)}_{\xi}\cdot\overline{b} + \overline{\overline{W}}^{(q)}_{\xi}\cdot\overline{B} \tag{B.39a}$$

$$\overline{\overline{Z}}^{(1)}_{\phi}(c_{1\alpha},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\phi}(c_{1\alpha},\eta)\cdot\overline{C} = \overline{\overline{W}}^{(p)}_{\phi}\cdot\overline{b} + \overline{\overline{W}}^{(q)}_{\phi}\cdot\overline{B} \tag{B.39b}$$

$$\overline{\overline{Z}}^{(1)}_{\eta}(c_{1\alpha},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\eta}(c_{1\alpha},\eta)\cdot\overline{C} = \overline{\overline{W}}^{(p)}_{\eta}\cdot\overline{b} + \overline{\overline{W}}^{(q)}_{\eta}\cdot\overline{B} \tag{B.39c}$$

$$\overline{\overline{Z}}^{(1)}_{\xi}(c_{1\beta},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\xi}(c_{1\beta},\eta)\cdot\overline{C} = \overline{\overline{Z}}^{(1)}_{\xi}(c_{3\beta},\eta)\cdot\overline{D} \tag{B.39d}$$

$$\overline{\overline{Z}}^{(1)}_{\phi}(c_{1\beta},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\phi}(c_{1\beta},\eta)\cdot\overline{C} = \overline{\overline{Z}}^{(1)}_{\phi}(c_{3\beta},\eta)\cdot\overline{D} \tag{B.39e}$$

$$\overline{\overline{Z}}^{(1)}_{\eta}(c_{1\beta},\eta)\cdot\overline{A} + \overline{\overline{Z}}^{(3)}_{\eta}(c_{1\beta},\eta)\cdot\overline{C} = \overline{\overline{Z}}^{(1)}_{\eta}(c_{3\beta},\eta)\cdot\overline{D} \tag{B.39f}$$

where $\overline{\overline{Z}}^{(i)}_{\gamma}(c,\eta)$ are $L_T \times 2L_T$ matrices and $\overline{\overline{W}}^{(p,q)}_{\gamma}$ are $L_T \times L_T$ matrices, respectively ($\gamma = \xi, \eta, \phi$). Their expressions can be found in Appendix C. Equations (B.39), can be

applied independently for even and odd excitations doe to decoupling. Thus, Eqs. (B.39) provide $12L_T$ linear equations for the $2L_T$ unknowns each in $\overline{A}$, $\overline{C}$, and $\overline{D}$, and the L_T unknowns in $\overline{B}$. Once $\overline{B}$ is determined, it can then be used to obtain the secondary magnetic field $\overline{H}_{\rm s}$.

It should be remarked that for the special axisymmetric case of $m = 0$ ($p = 0$), it follows from (B.26) and (B.28) that

$$\begin{aligned} A_{10n}^{(M)} &= 0, \quad \text{for } n = 0, 1, 2, \ldots \\ C_{10n}^{(M)} &= 0, \quad \text{for } n = 0, 1, 2, \ldots \\ D_{10n}^{(M)} &= 0, \quad \text{for } n = 0, 1, 2, \ldots \end{aligned} \tag{B.40}$$

Thus we only need to solve for $A_{00n}^{(N)}$, $C_{00n}^{(N)}$, $D_{00n}^{(N)}$, and B_{00n}, using (B.39a), (B.39c), (B.39d), and (B.39f). The size of the truncated system of equations in this case is reduced to $4L_T \times 4L_T$ (for either even or odd excitation).

B.2 Results

We show some preliminary low frequency results in Figures B-2–B-10.

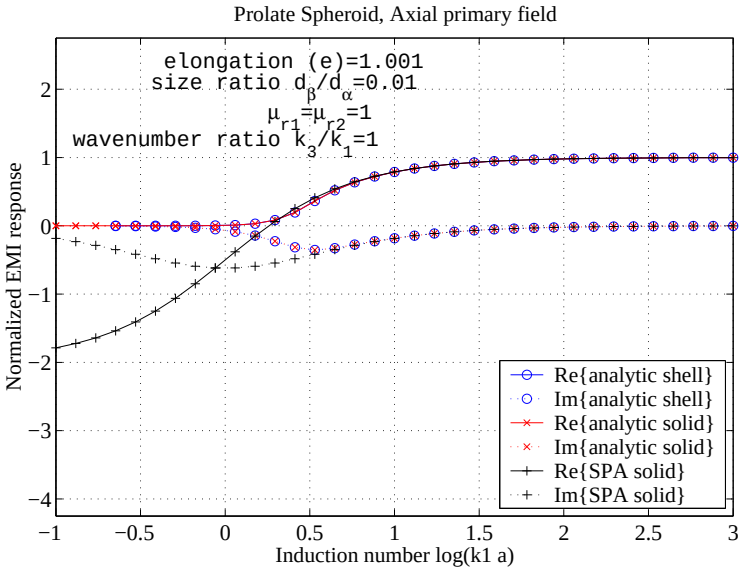

Figure B-2: Normalized dipole moment, m_z, from a nonmagnetic spherical shell with vanishing core size.

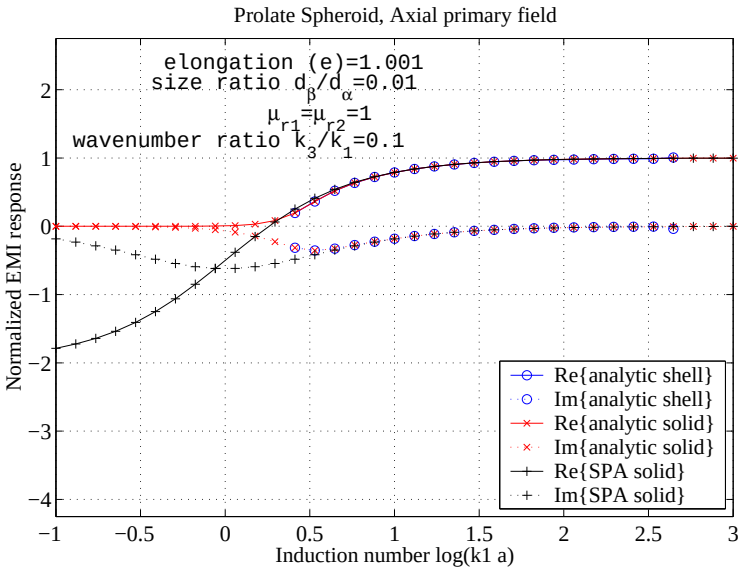

Figure B-3: Normalized dipole moment, m_z, from a spherical shell with vanishing core size but high core contrast.

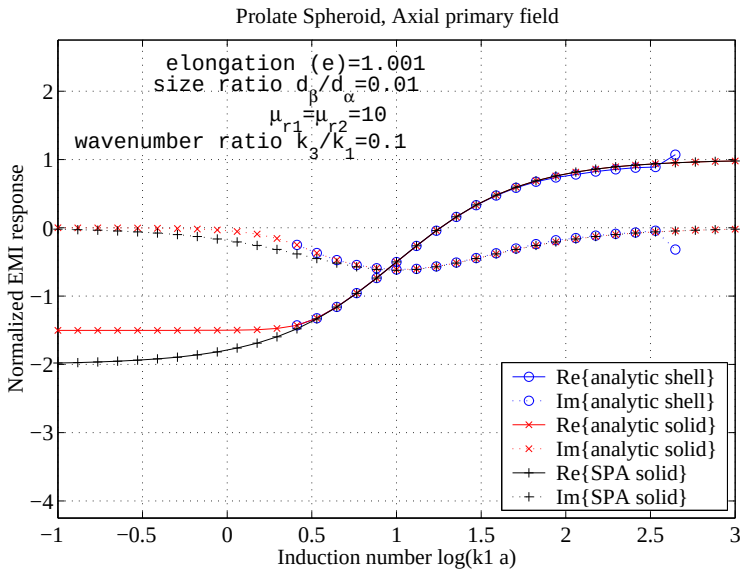

Figure B-4: Normalized dipole moment, m_z, from a magnetic spherical shell with vanishing core size and high core contrast.

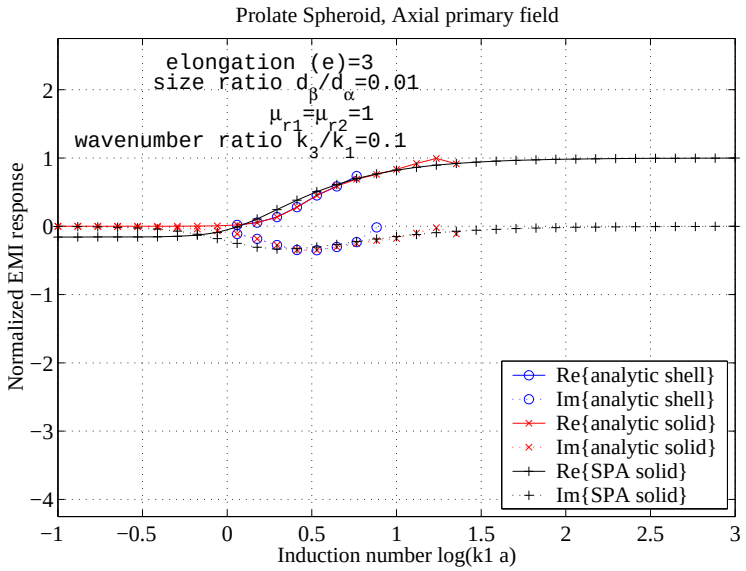

Figure B-5: Normalized dipole moment, m_z, from a nonmagnetic spheroidal shell with vanishing core size but high core contrast.

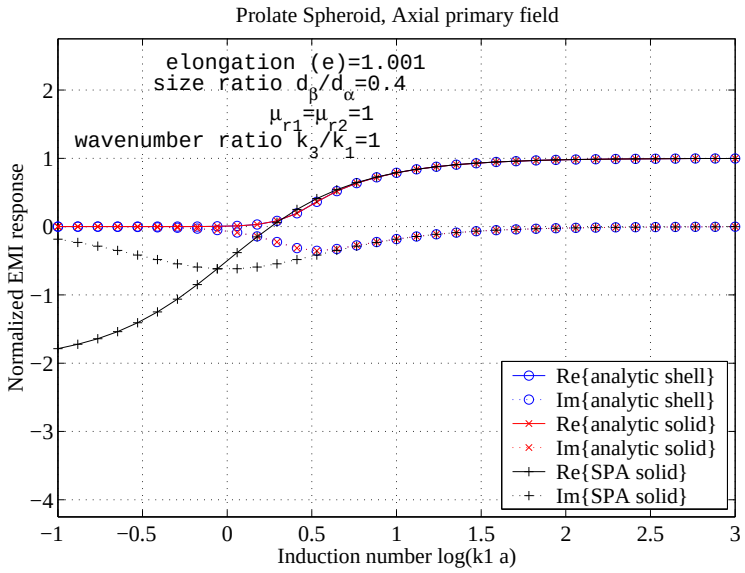

Figure B-6: Normalized dipole moment, m_z, from a nonmagnetic spherical shell with large core.

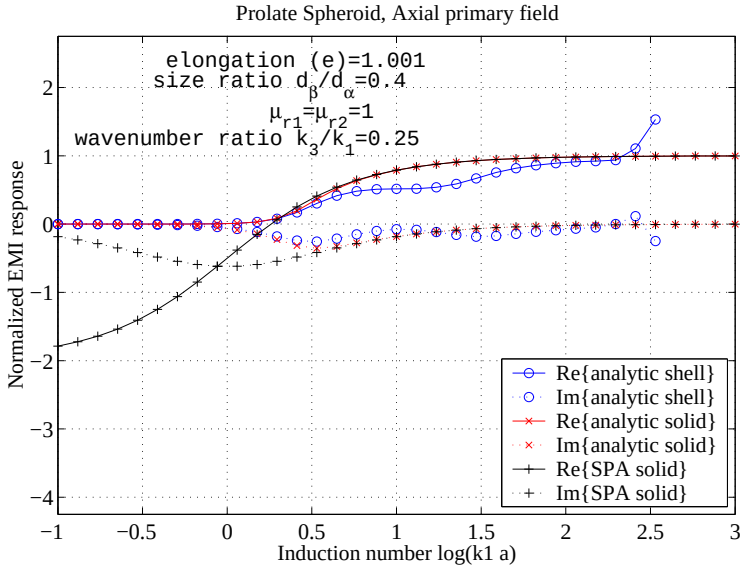

Figure B-7: Normalized dipole moment, m_z, from a nonmagnetic spherical shell with moderate core size and contrast.

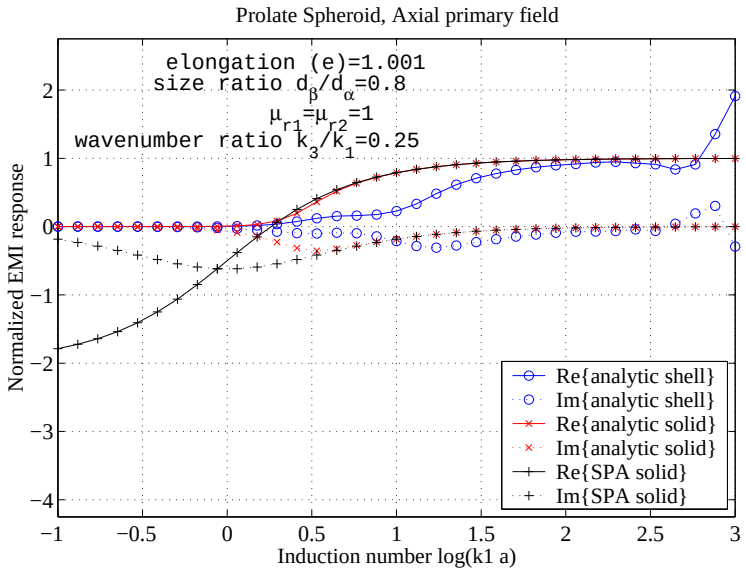

Figure B-8: Normalized dipole moment, m_z, from a nonmagnetic spherical shell with large core size and contrast.

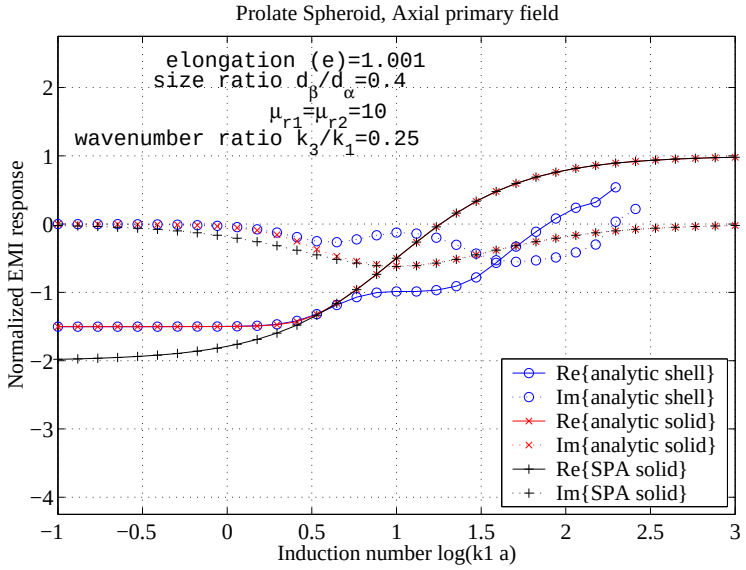

Figure B-9: Normalized dipole moment, m_z, from a magnetic spherical shell with moderate core size and contrast.

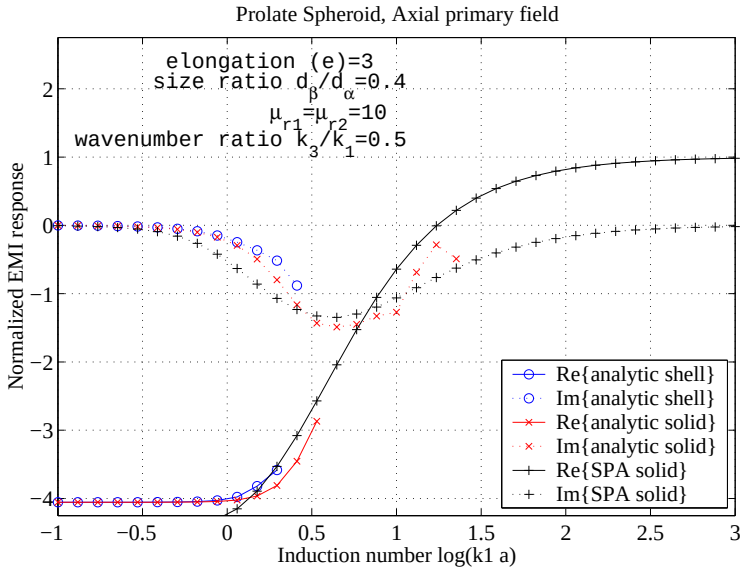

Figure B-10: Normalized dipole moment, m_z, from a magnetic spheroidal shell with moderate core size and contrast.

Appendix C

Computational Elements in Vector Spheroidal Wavefunction Expansions

C.1 Vector Spheroidal Wavefunctions

For computational purposes, it is convenient to express the vector spheroidal wavefunctions of (B.5) and (B.6) in partially separable forms. Explicit expressions of $\overline{M}$ and $\overline{N}$ in component forms can be found in Flammer's monograph on spheroidal wavefunctions [66]. They can be manipulated to yield

$$M^{(i)}_{\gamma;pmn}(c,\eta) = \frac{d}{2h_\gamma}\frac{dT_{pmn}}{d\phi}\widetilde{M}^{(i)}_{\gamma;mn}(c,\eta) \tag{C.1a}$$

$$M^{(i)}_{\phi;pmn}(c,\eta) = \frac{d}{2h_\phi}T_{pmn}(\phi)\widetilde{M}^{(i)}_{\phi;mn}(c,\eta) \tag{C.1b}$$

$$N^{(i)}_{\gamma;pmn}(c,\eta) = \frac{1}{c}\frac{d}{2h_\gamma}T_{pmn}(\phi)\widetilde{N}^{(i)}_{\gamma;mn}(c,\eta) \tag{C.1c}$$

$$N^{(i)}_{\phi;pmn}(c,\eta) = \frac{1}{c}\frac{d}{2h_\phi}\frac{dT_{pmn}}{d\phi}\widetilde{N}^{(i)}_{\phi;mn}(c,\eta) \tag{C.1d}$$

[2]This Appendix, as well as Appendix B, borrows heavily from [133], Appendix B therein.

where $\gamma = \eta, \xi$ and h_γ, h_ϕ are the metric coefficients for the prolate spheroidal coordinates:

$$h_\eta = \frac{d}{2}\sqrt{\frac{\xi^2-\eta^2}{1-\eta^2}}, \quad h_\xi = \frac{d}{2}\sqrt{\frac{\xi^2-\eta^2}{\xi^2-1}}, \quad h_\phi = \frac{d}{2}\sqrt{(1-\eta^2)(\xi^2-1)} \tag{C.2}$$

It is important to note that d in Eqs. (C.1) and (C.2) is implicitly decided by the choice of c through Eq. (B.18) or (B.24). The modified vector wavefunctions in (C.1) can be written as:

$$\widetilde{M}^{(i)}_{\gamma;mn}(c,\eta) = F^{(M)(1)(i)}_{\gamma;mn}(\xi) G^{(M)(1)}_{\gamma;mn}(\eta) \tag{C.3}$$

$$\widetilde{M}^{(i)}_{\phi;mn}(c,\eta) = \frac{1}{(\xi^2-\eta^2)} \sum_{t=1}^{2} F^{(M)(t)(i)}_{\phi;mn}(\xi) G^{(M)(t)}_{\phi;mn}(\eta) \tag{C.4}$$

$$\widetilde{N}^{(i)}_{\gamma;mn}(c,\eta) = \frac{1}{(\xi^2-\eta^2)^2} \sum_{t=1}^{5} F^{(N)(t)(i)}_{\gamma;mn}(c,\xi) G^{(N)(t)}_{\gamma;mn}(\eta) \tag{C.5}$$

$$\widetilde{N}^{(i)}_{\phi;mn}(c,\eta) = \frac{1}{(\xi^2-\eta^2)} \sum_{t=1}^{2} F^{(N)(t)(i)}_{\phi;mn}(c,\xi) G^{(N)(t)}_{\phi;mn}(\eta) \tag{C.6}$$

The functions $F^{(t)(i)}_{mn}(c,\xi)$ and $G^{(t)}_{mn}(\eta)$ represent the separable parts of the vector spheroidal wavefunctions. They are listed below.

For the $\overline{M}$ wavefunctions:

$$F^{(M)(1)(i)}_{\eta;mn}(\xi) = -\xi R^{(i)}_{mn}(\xi) \tag{C.7a}$$

$$G^{(M)(1)}_{\eta;mn}(\eta) = \frac{1}{(1-\eta^2)} S_{mn}(\eta) \tag{C.7b}$$

$$F^{(M)(1)(i)}_{\xi;mn}(\xi) = \frac{1}{(\xi^2-1)} R^{(i)}_{mn}(\xi) \tag{C.7c}$$

$$G^{(M)(1)}_{\xi;mn}(\eta) = \eta S_{mn}(\eta) \tag{C.7d}$$

$$F^{(M)(1)(i)}_{\phi;mn}(\xi) = (\xi^2-1)\xi R^{(i)}_{mn}(\xi) \tag{C.7e}$$

$$G^{(M)(1)}_{\phi;mn}(\eta) = (1-\eta^2)\frac{dS_{mn}}{d\eta} \tag{C.7f}$$

$$F^{(M)(2)(i)}_{\phi;mn}(\xi) = -(\xi^2-1)\frac{dR^{(i)}_{mn}}{d\xi} \tag{C.7g}$$

$$G^{(M)(2)}_{\phi;mn}(\eta) = (1-\eta^2)\eta S_{mn}(\eta) \tag{C.7h}$$

For the $\overline{N}$ wavefunctions:

$$F^{(N)(1)(i)}_{\eta;mn}(c,\xi) = 2\xi(\xi^2-1)\frac{dR^{(i)}_{mn}}{d\xi} - (\xi^2-1)(\lambda_{mn}-c^2\xi^2)R^{(i)}_{mn}(\xi) + m^2R^{(i)}_{mn}(\xi) \tag{C.8a}$$

$$G^{(N)(1)}_{\eta;mn}(\eta) = \eta S_{mn}(\eta) \tag{C.8b}$$

$$F^{(N)(2)(i)}_{\eta;mn}(c,\xi) = -(\lambda_{mn}-c^2\xi^2)R^{(i)}_{mn}(\xi) \tag{C.8c}$$

$$G^{(N)(2)}_{\eta;mn}(\eta) = (1-\eta^2)\eta S_{mn}(\eta) \tag{C.8d}$$

$$F^{(N)(3)(i)}_{\eta;mn}(\xi) = m^2(\xi^2-1)R^{(i)}_{mn}(\xi) \tag{C.8e}$$

$$G^{(N)(3)}_{\eta;mn}(\eta) = \frac{1}{(1-\eta^2)}\eta S_{mn}(\eta) \tag{C.8f}$$

$$F^{(N)(4)(i)}_{\eta;mn}(\xi) = (\xi^2-1)^2\left[\xi\frac{dR^{(i)}_{mn}}{d\xi} + R^{(i)}_{mn}(\xi)\right] \tag{C.8g}$$

$$G^{(N)(4)}_{\eta;mn}(\eta) = \frac{dS_{mn}}{d\eta} \tag{C.8h}$$

$$F^{(N)(5)(i)}_{\eta;mn}(\xi) = \xi(\xi^2-1)\frac{dR^{(i)}_{mn}}{d\xi} + (3\xi^2-1)R^{(i)}_{mn}(\xi) \tag{C.8i}$$

$$G^{(N)(5)}_{\eta;mn}(\eta) = (1-\eta^2)\frac{dS_{mn}}{d\eta} \tag{C.8j}$$

$$F^{(N)(1)(i)}_{\xi;mn}(c,\xi) = (\xi^2-1)\left[-2\frac{dR^{(i)}_{mn}}{d\xi} + \left(\lambda_{mn} - c^2 + \frac{m^2}{\xi^2-1}\right)\xi R^{(i)}_{mn}(\xi)\right] \tag{C.8k}$$

$$G^{(N)(1)}_{\xi;mn}(\eta) = S_{mn}(\eta) \tag{C.8l}$$

$$F^{(N)(2)(i)}_{\xi;mn}(c,\xi) = 3(\xi^2-1)\frac{dR^{(i)}_{mn}}{d\xi} + \left[(\xi^2-2)c^2 + \lambda_{mn} + \frac{m^2}{\xi^2-1}\right]\xi R^{(i)}_{mn}(\xi) \tag{C.8m}$$

$$G^{(N)(2)}_{\xi;mn}(\eta) = (1-\eta^2)S_{mn}(\eta) \tag{C.8n}$$

$$F^{(N)(3)(i)}_{\xi;mn}(c,\xi) = \frac{dR^{(i)}_{mn}}{d\xi} + c^2\xi R^{(i)}_{mn}(\xi) \tag{C.8o}$$

$$G^{(N)(3)}_{\xi;mn}(\eta) = (1-\eta^2)^2 S_{mn}(\eta) \tag{C.8p}$$

$$F^{(N)(4)(i)}_{\xi;mn}(\xi) = (\xi^2-1)\frac{dR^{(i)}_{mn}}{d\xi} - 2\xi R^{(i)}_{mn}(\xi) \tag{C.8q}$$

$$G^{(N)(4)}_{\xi;mn}(\eta) = (1-\eta^2)\eta\frac{dS_{mn}}{d\eta} \tag{C.8r}$$

$$F^{(N)(5)(i)}_{\xi;mn}(\xi) = \frac{dR^{(i)}_{mn}}{d\xi} \tag{C.8s}$$

$$G^{(N)(5)}_{\xi;mn}(\eta) = (1-\eta^2)^2\eta\frac{dS_{mn}}{d\eta} \tag{C.8t}$$

$$F^{(N)(1)(i)}_{\phi;mn}(\xi) = (\xi^2-1)\left[\xi\frac{dR^{(i)}_{mn}}{d\xi} + R^{(i)}_{mn}(\xi)\right] \tag{C.8u}$$

$$G^{(N)(1)}_{\phi;mn}(\eta) = S_{mn}(\eta) \tag{C.8v}$$

$$F^{(N)(2)(i)}_{\phi;mn}(\xi) = R^{(i)}_{mn}(\xi) \tag{C.8w}$$

$$G^{(N)(2)}_{\phi;mn}(\eta) = (1-\eta^2)\left[S_{mn}(\eta) - \eta\frac{dS_{mn}}{d\eta}\right] \tag{C.8x}$$

C.2 Coupling Matrices and System Matrices

The definitions for the coupling matrices are given below:

$$I^{(i)}_{\xi;n',n}(c,\eta) = \frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)\widetilde{M}^{(i)}_{\xi;mn}(c,\eta) \tag{C.9a}$$

$$I^{(i)}_{\eta;n',n}(c,\eta) = \frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)(1-\eta^2)\widetilde{M}^{(i)}_{\eta;mn}(c,\eta) \tag{C.9b}$$

$$I^{(i)}_{\phi;n',n}(c,\eta) = \frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)\widetilde{M}^{(i)}_{\phi;mn}(c,\eta) \tag{C.9c}$$

$$J^{(i)}_{\xi;n',n}(c,\eta) = \frac{1}{c}\frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)\widetilde{N}^{(i)}_{\xi;mn}(c,\eta) \tag{C.10a}$$

$$J^{(i)}_{\eta;n',n}(c,\eta) = \frac{1}{c}\frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)(1-\eta^2)\widetilde{N}^{(i)}_{\eta;mn}(c,\eta) \tag{C.10b}$$

$$J^{(i)}_{\phi;n',n}(c,\eta) = \frac{1}{c}\frac{2n'+1}{2}\frac{(n'-m)!}{(n'+m)!}\int_{-1}^{1} d\eta P^m_{n'}(\eta)\widetilde{N}^{(i)}_{\phi;mn}(c,\eta) \tag{C.10c}$$

The explicit expressions of $\widetilde{M}^{(i)}_{\gamma;mn}(c,\eta)$ and $\widetilde{N}^{(i)}_{\gamma;mn}(c,\eta)$ for $\gamma = \eta,\xi,\phi$ are given in Appendix C.1. Once they are determined, the integrals can be easily evaluated numerically by Gaussian quadrature.

The system matrices $\overline{\overline{Z}}^{(i)}_{\gamma}(c,\eta)$ and $\overline{\overline{W}}^{(p,q)}_{\gamma}$ are given as follows. We distinguish the cases of even and odd excitations by defining

$$r_j = \begin{cases} 2j+m-2 & \text{for even excitation} \\ 2j+m-1 & \text{for odd excitation} \end{cases} \tag{C.11}$$

The matrices $\overline{\overline{Z}}_{\gamma}^{(i)}(c,\eta)$ are of size $L_T \times 2L_T$. For $j,k = 1,...,L_T$,

$$Z^{(i)}_{\xi;j,k}(c,\eta) = \mu_r J^{(i)}_{\xi;r_j,r_k}(c,\eta) \tag{C.12a}$$

$$Z^{(i)}_{\xi;j,k+L_T}(c,\eta) = \mu_r m I^{(i)}_{\xi;r_j,r_k\pm1}(c,\eta) \tag{C.12b}$$

$$Z^{(i)}_{\eta;j,k}(c,\eta) = J^{(i)}_{\eta;r_j+1,r_k}(c,\eta) \tag{C.12c}$$

$$Z^{(i)}_{\eta;j,k+L_T}(c,\eta) = m I^{(i)}_{\eta;r_j+1,r_k\pm1}(c,\eta) \tag{C.12d}$$

$$Z^{(i)}_{\phi;j,k}(c,\eta) = m J^{(i)}_{\phi;r_j,r_k}(c,\eta) \tag{C.12e}$$

$$Z^{(i)}_{\phi;j,k+L_T}(c,\eta) = -I^{(i)}_{\phi;r_j,r_k\pm1}(c,\eta) \tag{C.12f}$$

In (C.12b), (C.12d), and (C.12f), the upper sign is for even excitation and the lower sign is for odd excitation.

The matrices $\overline{\overline{W}}_{\gamma}$ are of size $L_T \times L_T$. For $j,k = 1,...,L_T$,

$$W^{(p)}_{\xi;j,k} = -\frac{dP^m_{r_j}}{d\xi}\delta_{jk} \tag{C.13a}$$

$$W^{(q)}_{\xi;j,k} = -\frac{dQ^m_{r_j}}{d\xi}\delta_{jk} \tag{C.13b}$$

$$W^{(p)}_{\eta;j,k} = -\gamma_{1m}(r_j)P^m_{r_j}(\xi)\delta_{(j+1)k} + \gamma_{2m}(r_j+2)P^m_{r_j+2}(\xi)\delta_{(j-1)k} \tag{C.13c}$$

$$W^{(q)}_{\eta;j,k} = -\gamma_{1m}(r_j)Q^m_{r_j}(\xi)\delta_{(j+1)k} + \gamma_{2m}(r_j+2)Q^m_{r_j+2}(\xi)\delta_{(j-1)k} \tag{C.13d}$$

$$W^{(p)}_{\phi;j,k} = -mP^m_{r_j}(\xi)\delta_{jk} \tag{C.13e}$$

$$W^{(q)}_{\phi;j,k} = -mQ^m_{r_j}(\xi)\delta_{jk} \tag{C.13f}$$

where δ_{jk} is the Kronecker delta.

Appendix D

Reference Tables for the Spheroidal Wave Equation for Complex c

D.1 Branch points, $c_{\circ;r}^{m,n}$, and Associated Eigenvalues $\lambda_{\circ;r}^{m,n}$ of the SWE

These branch points and associated eigenvalues for a greater range of m and n can be reproduced by software written by the author. Tables in this section are included because no other source has published tables of branch points for this range of parameter and at this level of accuracy. Contact the author for access to these programs.

D.1.1 $m = 0$, $n = [m..80]$

Accuracy was determined by the last correction of the Newton-Raphson method.

Table D.1: Branch points and associated eigenvalues for $m = 0$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
0, 0 ,1	1.824770749208805 + 2.601670692890318 i	1.705180091412460 + 4.220186348356370 i	0, 1 ,1	3.563644553545243 + 2.887165344336900 i	10.140838787232552 + 11.121586684224873 i
0, 2 ,1	5.217093042404772 + 3.081362886557631 i	23.915829574409656 + 18.743327216266508 i	0, 2 ,3	2.094267182395557 + 5.807965828216133 i	1.998518598679721 + 8.578716573946881 i
0, 3 ,1	6.838151743509966 + 3.228540820485014 i	42.839460829787278 + 26.892790566183677 i	0, 3 ,3	4.067274712533398 + 6.264358978587767 i	11.780925838292136 + 22.541393207957562 i
0, 4 ,1	8.443063517943598 + 3.346896434708849 i	66.833326753192964 + 35.456773864933439 i	0, 4 ,3	5.874514148328888 + 6.617314890038058 i	27.516445359838094 + 37.764984283250755 i
0, 4 ,5	2.244329796261236 + 8.973752190228394 i	2.156124860667898 + 12.810922196899876 i	0, 5 ,1	10.038665064092994 + 3.445770032753475 i	95.856405565575329 + 44.361464108811766 i
0, 5 ,3	7.606334073445308 + 6.906465157219409 i	48.806652414112627 + 54.011898227620041 i	0, 5 ,5	4.383443826328900 + 9.509669337618593 i	12.778186437528630 + 33.481761709843944 i
0, 6 ,1	11.628375737753831 + 3.530602623592430 i	129.884250145370260 + 53.555053907976102 i	0, 6 ,3	9.295569705184056 + 7.151702724612575 i	75.462208083813508 + 71.112658178667672 i
0, 6 ,5	6.316233767329015 + 9.949229739353585 i	29.846039092711770 + 55.689990108075811 i	0, 6 ,7	2.347893464857109 + 12.128185061133886 i	2.263189447556893 + 16.978431740148483 i
0, 7 ,1	13.214108021867665 + 3.604842114031867 i	168.901012550649369 + 62.999226446425368 i	0, 7 ,3	10.957870542891056 + 7.364707014831354 i	107.373656814110845 + 88.944615504739886 i
0, 7 ,5	8.147467516661960 + 10.323899301379097 i	52.831395893010438 + 79.222018475547969 i	0, 7 ,7	4.613983308387243 + 12.710935561838975 i	13.496424361795992 + 44.232604764077692 i
0, 8 ,1	14.797019945736288 + 3.670810179656952 i	212.895788730994781 + 72.664518377383786 i	0, 8 ,3	12.601853656860371 + 7.552989106884000 i	144.470630704203188 + 107.415994842966938 i
0, 8 ,5	9.917892969755632 + 10.651089858500113 i	81.465390714716307 + 103.897214349459944 i	0, 8 ,7	6.649722635876492 + 13.206057605249066 i	31.579166145618920 + 73.187499640071934 i
0, 8 ,9	2.426755266724711 + 15.277799057286881 i	2.343980783624102 + 21.103815837875132 i	0, 9 ,1	16.377855800081942 + 3.730143922754754 i	261.860733447250368 + 82.527583895986282 i
0, 9 ,3	14.232725426889811 + 7.721687504996004 i	186.704669933275369 + 126.455376010690060 i	0, 9 ,5	11.648031656826387 + 10.941780781838141 i	115.584894994604014 + 129.575872480192004 i
0, 9 ,7	8.566807794592391 + 13.638874465481070 i	55.895670794459470 + 103.670385974408049 i	0, 9 ,9	4.795233775898897 + 15.891693671884648 i	14.057386209297746 + 54.879985036694741 i
0,10,1	17.957116787021921 + 3.784040426698697 i	315.790001887496999 + 92.569477952059643 i	0,10,3	15.853845319218598 + 7.874480991123857 i	234.040731693069716 + 146.005654344901444 i
0,10,5	13.349712997597559 + 11.203435277353591 i	155.081025165167944 + 156.148368174552417 i	0,10,7	10.410251940624978 + 14.024247058616169 i	86.118562824385393 + 135.505690619167808 i
0,10,9	6.917582942451463 + 16.425839365768446 i	32.960499851131885 + 90.459854620068782 i	0,10,11	2.490327262437059 + 18.424904659115747 i	2.408721597834776 + 25.198509979662479 i
0,11,1	19.535153320240660 + 3.833401137717104 i	374.679115756393969 + 102.774525241977997 i	0,11,3	17.467490992090958 + 8.014097371160032 i	286.452526568988958 + 166.020127896453602 i
0,11,5	15.030367143915981 + 11.441392537803358 i	199.876462460007843 + 183.526690015259589 i	0,11,7	12.203452288695585 + 14.371989749736837 i	122.042803329388377 + 168.549078518222814 i
0,11,9	8.909332180700922 + 16.901125867449363 i	58.375129164312511 + 127.715473425074194 i	0,11,11	4.944445543948445 + 19.061129762281226 i	14.517281031107508 + 65.460946543513145 i
0,12,1	21.112218078841369 + 3.878921577406139 i	438.524563710064440 + 113.129545165301150 i	0,12,3	19.075267730064859 + 8.142616883703791 i	343.919755416526698 + 186.459890248030973 i
0,12,5	16.694951390324288 + 11.659619367791977 i	249.913830458837651 + 211.638677650511283 i	0,12,7	13.960273952682158 + 14.689027894000139 i	163.528082012527875 + 202.683151467007804 i
0,12,9	10.818019546945614 + 17.330295405580152 i	89.929406582396609 + 166.482831755741131 i	0,12,11	7.141319713525860 + 19.624346718372887 i	34.108834335464920 + 107.593796180195483 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,12,13	2.543518354464380 + 21.570532475755499 i	2.462661333970665 + 29.269331709786822 i	0,13,1	22.688498049603837 + 3.921149467358161 i	507.323538837447018 + 123.623303140077070 i
0,13,3	20.678342962879118 + 8.261662901396956 i	406.426375302075428 + 207.292027788687903 i	0,13,5	18.346926056713983 + 11.861150849530645 i	305.149114822285128 + 240.424095084199280 i
0,13,7	15.689639674099469 + 14.980471804601065 i	210.472702709300791 + 237.811237983354744 i	0,13,9	12.668855389338326 + 17.722040124724117 i	127.384959274218318 + 206.619067233139759 i
0,13,11	9.198821610189677 + 20.132183581617266 i	60.458570491739088 + 151.513723605687204 i	0,13,13	5.071172125506672 + 22.223615003784481 i	14.906757379113769 + 75.994883405387441 i
0,14,1	24.264134691617055 + 3.960523726252934 i	581.073760141115940 + 134.246111395481620 i	0,14,3	22.277588001718076 + 8.372526946548424 i	473.959460730851788 + 228.488333590482171 i
0,14,5	19.988793373711303 + 12.048364785089129 i	365.547660901260656 + 269.831891436271064 i	0,14,7	17.397631778005625 + 15.250219110445345 i	262.799877149209749 + 273.852515964114730 i
0,14,9	14.477056541232377 + 18.082662369767373 i	170.575620876289207 + 248.004249403177710 i	0,14,11	11.166092176046561 + 20.595685174222506 i	93.159816359840647 + 197.070982744438567 i
0,14,13	7.333346163556755 + 22.809411118445475 i	35.091269269676218 + 124.634010860624684 i	0,14,15	2.589208330027252 + 24.715213695732540 i	2.508846525344480 + 33.320768350770322 i
0,15,1	25.839237075106841 + 3.997401420530968 i	659.773347558225055 + 144.989531669067475 i	0,15,3	23.873667519270032 + 8.476253538174426 i	546.508427744023834 + 250.024365140936794 i
0,15,5	21.622415056470839 + 12.223160795185768 i	431.081603102226666 + 299.818239433499798 i	0,15,7	19.088577942045678 + 15.501320025877984 i	320.449805972399361 + 310.738445006410586 i
0,15,9	16.252580109066024 + 18.416921291354679 i	219.379095965306874 + 290.537461986475648 i	0,15,11	13.069546704662997 + 21.022566779321046 i	131.947185453629743 + 244.127319564698610 i
0,15,13	9.449454955467191 + 23.343083266824951 i	62.255361539978637 + 175.146497972910993 i	0,15,15	5.181256710274163 + 25.381508876983606 i	15.244374747239075 + 86.493287543852858 i
0,16,1	27.413890701408814 + 4.032076870317803 i	743.420732257636814 + 155.846149297666642 i	0,16,3	25.467098124858225 + 8.573699477928775 i	624.064488845936012 + 271.878738120684091 i
0,16,5	23.249209268514452 + 12.387081666095698 i	501.728141365956503 + 330.345095961266168 i	0,16,7	20.765661872654380 + 15.736211711873958 i	383.374776834370323 + 348.410163375674585 i
0,16,9	18.002316611748597 + 18.728519577424038 i	273.701829967417382 + 334.132938535347478 i	0,16,11	14.925336858080875 + 21.418538844477492 i	176.63297605090676 + 292.56323805124379 i
0,16,13	11.469713646256640 + 23.834316390184501 i	95.964480947342238 + 227.397851475079875 i	0,16,15	7.501485538308003 + 25.985397103523361 i	35.949517815902333 + 141.606383354313834 i
0,16,17	2.629229023914731 + 27.859250742325603 i	2.549199691551395 + 37.355977677037764 i	0,17,1	28.988163578803029 + 4.064795497308687 i	832.014590851180401 + 166.809398521764962 i
0,17,3	27.058287902721471 + 8.665576318167293 i	706.620260071880693 + 294.032586594042471 i	0,17,5	24.870277941496550 + 12.541398149835944 i	577.468344610413851 + 361.379122299353241 i
0,17,7	22.431289423011670 + 15.956875308241884 i	451.535967022288844 + 386.816563021371394 i	0,17,9	19.731236328969889 + 19.020405593842671 i	333.470135498840648 + 378.716981300094972 i
0,17,11	16.744159632532558 + 21.787991446405044 i	227.077355396894490 + 342.276228880271333 i	0,17,13	13.421372718790622 + 24.289998476364175 i	135.930978871095022 + 281.256066144586157 i
0,17,15	9.670386647074043 + 26.540128126986414 i	63.834826079006852 + 198.661095035834336 i	0,17,17	5.278532220583037 + 28.536202482996774 i	15.5422233402859 + 96.96357839497779 i
0,18,1	30.562110503873850 + 4.095764058658622 i	925.55379622292423 + 177.87342529414965 i	0,18,3	28.647563812232441 + 8.752481443875894 i	794.169471390297304 + 316.469143271784503 i
0,18,5	26.486491946276669 + 12.687169786815033 i	658.286295642360301 + 392.890858315802177 i	0,18,7	24.087319336433872 + 16.164944864433181 i	524.901269909030589 + 425.912841354072441 i
0,18,9	21.443041605601852 + 19.294971456530501 i	398.624626816785792 + 424.225601473129927 i	0,18,11	18.533500520083226 + 22.134394120425185 i	283.172234691800554 + 393.177908360563094 i
0,18,13	15.321252573503804 + 24.715310219911824 i	181.948999084427413 + 336.603824397728431 i	0,18,15	11.738927010961380 + 27.054286562226665 i	98.442966283480459 + 257.538358053774516 i
0,18,17	7.650987671431293 + 29.154943011019043 i	36.711310703932888 + 158.527194480114844 i	0,18,19	2.664816878709987 + 31.002828834912670 i	2.58501051852747 + 41.377291773390873 i
0,19,1	32.135776139382322 + 4.125158341302432 i	1024.037380160035354 + 189.032978058511873 i	0,19,3	30.235191115661074 + 8.834921252801625 i	886.706749124546150 + 339.173408186434585 i
0,19,5	28.098549740839150 + 12.825289512134425 i	744.168464983908393 + 424.854080000930651 i	0,19,7	25.735214652043172 + 16.361785090412404 i	603.443766992919109 + 465.659392781788085 i
0,19,9	23.140562356598121 + 19.554188375071380 i	469.116547007323845 + 470.602716390520698 i	0,19,11	20.298814770192543 + 22.460548141295689 i	344.831702556976836 + 445.191454214688690 i
0,19,13	17.180594246186434 + 25.114291664814562 i	233.863648859081735 + 393.338686355437005 i	0,19,15	13.734956252616433 + 27.534063404369181 i	139.467671656242800 + 318.113250963450241 i
0,19,17	9.867876144454309 + 29.727204051505666 i	65.243769753912702 + 222.087248107604751 i	0,19,19	5.365647708002061 + 31.688571566956107 i	15.80861649262197 1 + 107.410857249501746 i
0,20,1	33.709197266738975 + 4.153129036331815 i	1127.464504527516056 + 200.283319728128987 i	0,20,3	31.821387432513127 + 8.913328731780346 i	984.227449707470441 + 362.131883644956986 i
0,20,5	29.707018768146455 + 12.956517005815272 i	835.103242636504547 + 457.245292024577736 i	0,20,7	27.376145415483265 + 16.548548200062402 i	687.140623444867970 + 506.020946515592868 i
0,20,9	24.826008033206616 + 19.799702227323017 i	544.905245064517885 + 517.798760188386268 i	0,20,11	22.044204751457375 + 22.768754538278269 i	411.985962766349132 + 498.249528116270426 i
0,20,13	19.007331246666652 + 25.490176512503776 i	291.55416477539052 + 451.37117371657791 i	0,20,15	15.675786845461573 + 27.984175116239726 i	186.687610789158896 + 380.271674730833070 i
0,20,17	11.980709955732012 + 30.260668715409807 i	100.663339548141224 + 287.539886048796177 i	0,20,19	7.785546275625205 + 32.319743542605025 i	37.396027055158733 + 175.407332846476208 i
0,20,21	2.696845249085518 + 34.146067942636392 i	2.617185498821204 + 45.386497045126646 i	0,21,1	35.282404460969559 + 4.179806286062766 i	1235.834438735374306 + 211.620155967940036 i
0,21,3	33.406333089020556 + 8.988076982149829 i	1086.727530751469430 + 385.332359662755493 i	0,21,5	31.312365312871716 + 13.081504051984696 i	931.080581535314764 + 490.043321520726238 i
0,21,7	29.011060336733646 + 16.726216330711974 i	775.972270781619045 + 546.965885994580503 i	0,21,9	26.501134660589873 + 20.032902894574875 i	625.956385513759187 + 565.769600089412279 i
0,21,11	23.772833822285659 + 23.060930618801489 i	484.577296446548587 + 552.292632411857198 i	0,21,13	20.807291664691540 + 25.845605454424341 i	354.923835420306887 + 510.622711897100828 i
0,21,15	17.573069399848119 + 28.408342621754489 i	239.934848841559301 + 443.912326951781836 i	0,21,17	14.017781014966555 + 30.760939041461381 i	142.647981218995227 + 354.767653280011984 i
0,21,19	10.046394130763435 + 32.906847680518773 i	66.515318096116346 + 245.444772653359678 i	0,21,21	5.444509801028503 + 34.839196163536293 i	16.049514207421382 + 117.838804164295439 i
0,22,1	36.855423354307291 + 4.205303247692535 i	1349.146541911980194 + 223.039576178994139 i	0,22,3	34.990178861281862 + 9.059489766451819 i	1194.20344972158250 + 408.76373839895456 i
0,22,5	32.914976444378460 + 13.200814111864016 i	1032.091721700790458 + 523.228988983039926 i	0,22,7	30.640737995998396 + 16.895633766890700 i	869.921789708117672 + 588.465704478259454 i
0,22,9	28.167359263681654 + 20.254975710933277 i	712.240643087344210 + 614.475680290168839 i	0,22,11	25.487191998865512 + 23.338693399521148 i	562.557265176491342 + 607.267812328717923 i
0,22,13	22.584891074027819 + 26.182769900936069 i	423.893562608378204 + 571.023842328343562 i	0,22,15	19.435081566464913 + 28.809575554265564 i	299.077498693126813 + 508.945658978569611 i
0,22,17	15.996772607392785 + 31.232327313459745 i	190.963039308574736 + 423.660285501501107 i	0,22,19	12.200111226177563 + 33.456819099118228 i	102.674274915682673 + 317.434131791885591 i
0,22,21	7.907858414555943 + 35.480939955424908 i	38.017744867266913 + 192.254436273466837 i	0,22,23	2.725953601051139 + 37.289049295215321 i	2.646385691020677 + 49.385001666075205 i
0,23,1	38.428275601970007 + 4.229718917881131 i	1467.400248646491718 + 234.538004497178463 i	0,23,3	36.573051851566419 + 9.127849831076814 i	1306.652083347261623 + 432.415889111993351 i
0,23,5	34.515176414861998 + 13.314937633241000 i	1138.128973801862685 + 556.784838747986555 i	0,23,7	32.265824202277926 + 17.057531803608889 i	968.974435889803772 + 630.494564302654453 i
0,23,9	29.825840440764527 + 20.466940384611849 i	803.732729068565959 + 663.881337289422959 i	0,23,11	27.189273375631196 + 23.603420913441219 i	645.884711069532045 + 663.127624633439950 i
0,23,13	24.343558185771894 + 26.503512797010746 i	498.397540402630682 + 632.512752248061474 i	0,23,15	21.267942948701371 + 29.190355034184268 i	364.009261738566806 + 575.292297348234115 i
0,23,17	17.929649992901023 + 31.678268306563950 i	245.429194979654227 + 494.118311749288921 i	0,23,19	14.275323989376487 + 33.974759115377502 i	145.537346079887328 + 391.265544005659422 i
0,23,21	10.209245358474858 + 36.080790471755748 i	67.673789668053800 + 268.747489068408981 i	0,23,23	5.516536046921436 + 37.988476355071157 i	16.269333873986248 + 128.250176160856341 i
0,24,1	40.000979629140737 + 4.253140394643984 i	1590.595057472352210 + 246.112158772221420 i	0,24,3	38.155060005667060 + 9.193405545233968 i	1424.070662834859604 + 456.279527277005343 i
0,24,5	36.113239097491018 + 13.424304169092089 i	1249.185547194198534 + 590.694916122950985 i	0,24,7	33.886859749067234 + 17.212548197007749 i	1073.117269929450003 + 673.028936170342945 i
0,24,9	31.477536448562489 + 20.669680938005140 i	900.410650681757602 + 713.954245988977050 i	0,24,11	28.880698529753349 + 23.856298246180991 i	734.524288530731383 + 719.829310643641634 i
0,24,13	26.086009569450432 + 26.809401593151783 i	578.380241414240459 + 695.034083202256511 i	0,24,15	23.076317883365675 + 29.552757851983610 i	434.642700406174015 + 642.881538483955637 i
0,24,17	19.824959809019955 + 32.101564664915095 i	305.904737566547055 + 566.052813916831724 i	0,24,19	16.290000898753064 + 34.464628168876544 i	194.858300530472405 + 466.832932494332113 i
0,24,21	12.400903236194793 + 36.645054771282027 i	104.511819635895463 + 347.243244125334002 i	0,24,23	8.019953209992481 + 38.639330508476178 i	38.587013521318859 + 209.074070513529875 i
0,24,25	2.752624006459813 + 40.431829910692677 i	2.673107766574164 + 53.373941562088135 i	0,25,1	41.573551215035437 + 4.275644705751269 i	1718.730521480293191 + 257.759015968767585 i
0,25,3	39.736295626432316 + 9.256376249265770 i	1546.456721271660626 + 480.346113033558197 i	0,25,5	37.709397547753042 + 13.529292074511069 i	1365.25541175030048 + 624.94458145729936 i
0,25,7	35.504301369347928 + 17.361242569210450 i	1182.338864715545469 + 716.047301028232482 i	0,25,9	33.123247934678353 + 20.863969078828017 i	1002.255138104712387 + 764.664966803342622 i
0,25,11	30.562801303851295 + 24.098352748268095 i	828.445362068904046 + 777.334126610538419 i	0,25,13	27.814434112853309 + 27.101782419049222 i	663.794248256111928 + 758.537966069434106 i
0,25,15	24.863848816055288 + 29.898544327544680 i	510.904695351524310 + 711.650052928726950 i	0,25,17	21.689053040788444 + 32.504545470671886 i	372.274816086866622 + 639.384295047102682 i
0,25,19	18.256353213437610 + 34.929606117293034 i	250.447810406203018 + 544.039108476281854 i	0,25,21	14.511716822122365 + 37.178405374895647 i	148.184591024034972 + 427.639395033619394 i
0,25,23	10.358941741432238 + 39.250257485726337 i	68.737570277856719 + 292.005395126791598 i	0,25,25	5.582808761183214 + 41.136697413456140 i	16.471439688323436 + 138.647102550101067 i
0,26,1	43.146003954169771 + 4.297300299527417 i	1851.806240603241804 + 269.475782782258307 i	0,26,3	41.316838135874171 + 9.316956602220074 i	1673.808050545249898 + 504.607765247718532 i
0,26,5	39.303851442633267 + 13.630236342319357 i	1486.333185709100007 + 659.520353784905410 i	0,26,7	37.118537784105847 + 17.504108744657110 i	1296.629071155011161 + 759.529901466121601 i
0,26,9	34.763649938426610 + 21.050482681607800 i	1109.249194521064055 + 815.986571729512434 i	0,26,11	32.236691779003074 + 24.330481400873040 i	927.62116295401142 5 + 835.606796260196120 i
0,26,13	29.530620960227139 + 27.381821157776955 i	754.598652266143631 + 822.979235759920584 i	0,26,15	26.633437119458460 + 30.229222446707414 i	592.733258797016447 + 781.540805185514841 i
0,26,17	23.526794201953422 + 32.889174629255777 i	444.44448801983486 + 714.04144248273280 i	0,26,19	20.183141046104392 + 35.372306928335078 i	312.155438467989484 + 622.796268350182686 i
0,26,21	16.559877850426034 + 37.684495578706816 i	198.435734571380436 + 509.834238027905030 i	0,26,23	12.585979468530960 + 39.827031187064641 i	106.203431715036871 + 376.983252842285879 i
0,26,25	8.123395363985731 + 41.795491214071902 i	39.111941998853673 + 225.870419908704378 i	0,26,27	2.777228767640468 + 43.574451026085150 i	2.697734203683512 + 57.354250601086527 i
0,27,1	44.718349624010074 + 4.318168270098028 i	1989.821855226624166 + 281.259870524495568 i	0,27,3	42.896756267908607 + 9.375320145311635 i	1806.122665767680701 + 529.057188298312440 i
0,27,5	40.896772932463222 + 13.727434991542390 i	1612.414043803179084 + 694.409778368148864 i	0,27,7	38.729902151931107 + 17.641584728471258 i	1415.97882860231994 + 803.45853271855015 i
0,27,9	36.399316233564029 + 21.229820572600321 i	1221.377738112996212 + 867.894332839537014 i	0,27,11	33.903302895571557 + 24.553472382068165 i	1032.028133153588669 + 894.615059055493475 i
0,27,13	31.236050689332238 + 27.650535148576214 i	850.757845018634725 + 888.316787644395390 i	0,27,15	28.387432409114300 + 30.546095870273046 i	680.075229319513824 + 852.502161440724876 i
0,27,17	25.341999859489764 + 33.257128052354716 i	522.334001181355120 + 789.960001725675170 i	0,27,19	22.076901506231987 + 35.794917839676245 i	379.858667864524307 + 703.025276180822516 i
0,27,21	18.557792875661690 + 38.166278505752089 i	255.067063670971351 + 593.733155245732178 i	0,27,23	14.730154351528814 + 40.373954187489531 i	150.6271473825740 9 + 463.9127808168289 6 i
0,27,25	10.497437136059792 + 42.416141496026881 i	69.720900371660022 + 315.225943863035013 i	0,27,27	5.644172841687634 + 44.284068455214495 i	16.658452347765675 + 149.031269421171061 i
0,28,1	46.290598480510511 + 4.338303371825231 i	2132.777040858797591 + 293.108873531878828 i	0,28,3	44.476109824840492 + 9.431622245073289 i	1943.398775675821526 + 553.687609316528210 i
0,28,5	42.488311290438503 + 13.821154318914992 i	1743.493641362652397 + 729.601313732855033 i	0,28,7	40.338681839295326 + 17.774060851298376 i	1540.380009951946704 + 847.816365650354101 i
0,28,9	38.030738105521714 + 21.402514481152359 i	1338.627313848900485 + 920.365460641588584 i	0,28,11	35.563425577119979 + 24.768022273493454 i	1141.645407494628898 + 954.329294196842397 i
0,28,13	32.931961806980922 + 27.908818035354056 i	952.240589057844659 + 954.513046036486344 i	0,28,15	30.127764380448095 + 30.850300622527286 i	772.884561267637082 + 924.487152457114121 i
0,28,17	27.137723623291890 + 33.609850377477905 i	605.875468031804189 + 867.081807335040480 i	0,28,19	23.942662095442927 + 36.199294877326935 i	453.455775799871958 + 784.654678913922567 i
0,28,21	20.514393723257268 + 38.626197186751241 i	317.920346289170766 + 679.249792371343460 i	0,28,23	16.809835621463414 + 40.894410545111022 i	201.743403864666817 + 552.696726236503650 i
0,28,25	12.757608065622629 + 43.003965041995563 i	107.770516640334563 + 406.666099036124535 i	0,28,27	8.219416673896072 + 44.949848691250992 i	39.59889741987568 + 242.64671310090060 i
0,28,29	2.800061319121512 + 46.716943225898177 i	2.72056571156404 + 61.32670883240976 i	0,29,1	47.862759497562045 + 4.357754864956531 i	2280.671503656463756 + 305.020550500649051 i
0,29,3	46.054951096135255 + 9.486002541404654 i	2085.634757832861396 + 578.492724077727871 i	0,29,5	44.078596640891433 + 13.911633248473880 i	1879.568051133261406 + 765.084234727766443 i
0,29,7	41.945126169188541 + 17.901886472993727 i	1669.825293929312693 + 892.587795805610654 i	0,29,9	39.658339017673356 + 21.569038791880043 i	1460.98585900636249 + 973.37888266853361 i
0,29,11	37.217735603749219 + 24.974749940524759 i	1256.454399645273497 +1014.722205127574853 i	0,29,13	34.619400218314418 + 28.157459505922269 i	1059.019291652291713 +1021.533521777658052 i
0,29,15	31.856037140603792 + 31.142833622359369 i	871.121027876083417 + 997.452862223035368 i	0,29,17	28.916448860327993 + 33.948597707035944 i	695.010448977184524 + 945.353965258565495 i
0,29,19	25.784382713620655 + 36.587031206734061 i	532.860975695227694 + 867.619742074423471 i	0,29,21	22.436369014498524 + 39.066307966815089 i	386.866162895230048 + 766.305724638113134 i
0,29,23	18.837586982347894 + 41.391151521467727 i	259.346126152821057 + 643.243221042088294 i	0,29,25	14.933158501058710 + 43.562942931631682 i	152.89436035902426 + 500.10330086026960 i
0,29,27	10.626281163938568 + 45.579109452883344 i	70.635035227460222 + 338.414832890186290 i	0,29,29	5.701300364887454 + 47.430746407897360 i	16.832452784470689 + 159.404039978382571 i
0,30,1	49.434840562425414 + 4.376567225061337 i	2433.504976644922863 + 316.992808270242278 i	0,30,3	47.633326013268963 + 9.538586997193054 i	2232.829137718576931 + 603.466650104426776 i
0,30,5	45.667742974206739 + 13.999086959340081 i	2020.633710305394743 + 800.848548858159575 i	0,30,7	43.549452625665033 + 18.025375542380456 i	1804.308058950663735 + 937.758313871986843 i
0,30,9	41.282486196472192 + 21.729818570723079 i	1588.44251059278849 +1026.91505480959313 i	0,30,11	38.866814456569287 + 25.174207840254468 i	1376.438467291244706 +1075.768552819562728 i
0,30,13	36.299256672870662 + 28.397161159370384 i	1171.069429085361890 +1089.346441186834682 i	0,30,15	33.573598275714858 + 31.424575205578993 i	974.749219783542117 +1071.359918017079053 i
0,30,17	30.680223315465845 + 34.274470481396904 i	789.688151462124210 +1024.728166797878430 i	0,30,19	27.605243539087393 + 36.959507623013657 i	618.000978945373276 + 951.861588565187390 i
0,30,21	24.328880372162818 + 39.488365448711079 i	461.796641824925189 + 854.829707402302461 i	0,30,23	20.822480038250639 + 41.866465629157368 i	323.270084125978883 + 735.467590404934867 i
0,30,25	17.042600735432796 + 44.096222093893637 i	204.819175327396181 + 595.444755692518811 i	0,30,27	12.917599980689861 + 46.176772060903900 i	109.23008886735667 + 436.30089608324937 i
0,30,29	8.309004093964743 + 48.102725989013997 i	40.052970281825239 + 259.405496680883743 i	0,30,31	2.821356984681268 + 49.859329687536238 i	2.741842927275834 + 65.291976679602556 i
0,31,1	51.006848636333331 + 4.394780741730171 i	2591.277216507062803 + 329.023687666790579 i	0,31,3	49.211275096503066 + 9.589489624848701 i	2384.980570993262518 + 628.603885818173126 i
0,31,5	47.255850603965435 + 14.083709931087176 i	2166.687375812915889 + 836.884923692098027 i	0,31,7	45.151851865859513 + 18.144811240775997 i	1943.822294253947575 + 983.314393876260056 i
0,31,9	42.903499876754402 + 21.885236222264400 i	1720.987445809191286 +1080.955799515559875 i	0,31,11	40.511165685391930 + 25.366891316675162 i	1501.582638563080309 +1137.444928729207504 i
0,31,13	37.972295545380476 + 28.628549396367287 i	1288.369084501992120 +1157.922432612270768 i	0,31,15	35.281590431517422 + 31.696307137255946 i	1083.737759240616697 +1146.172062366718592 i
0,31,17	32.430755736227269 + 34.588439203019391 i	889.864059202801968 +1105.160112690487040 i	0,31,19	29.407840149553991 + 37.317930798446945 i	708.812486716563512 +1037.326454952800304 i
0,31,21	26.196006806639371 + 39.893883492754348 i	542.620489239857648 + 944.756864683633125 i	0,31,23	22.771328066191149 + 42.322289098379251 i	393.379615988595219 + 829.292374409096055 i
0,31,25	19.098626452712420 + 44.606373658490227 i	263.331823073613066 + 692.601448703855112 i	0,31,27	15.122755122928888 + 46.746536275731607 i	155.009671487686290 + 536.224405451134885 i
0,31,29	10.746723441731165 + 48.739670236669099 i	71.489025236748006 + 361.576512093265535 i	0,31,31	5.754734633123008 + 50.576851428290482 i	16.995120643065150 + 169.766535926941486 i
0,32,1	52.578789887315338 + 4.412432026626102 i	2753.988000840980476 + 341.111351090231608 i	0,32,3	50.788834236266013 + 9.638813948980971 i	2542.087828369972158 + 653.899274795860606 i
0,32,5	48.843008184037075 + 14.165678515595987 i	2317.726086381931964 + 873.184623562784168 i	0,32,7	46.752491801559486 + 18.260449885541110 i	2088.362524982400373 +1029.243396164669775 i
0,32,9	44.521660748840652 + 22.035637051530085 i	1858.611748845708917 +1135.484166230664641 i	0,32,11	42.151227903967346 + 25.553246303731175 i	1631.87338642737359 +1199.72956027912392 i
0,32,13	39.639177258129727 + 28.852185990668644 i	1410.898572729779062 +1227.234259840677851 i	0,32,15	36.980990504472267 + 31.958727183558718 i	1198.058674880526951 +1221.855791421648064 i
0,32,17	34.169486839137249 + 34.891364865032109 i	995.498869735766448 +1186.609027973724551 i	0,32,19	31.194320152858445 + 37.663362856907980 i	805.240314000816397 +1123.965057182783767 i
0,32,21	28.041034619448311 + 40.284180456953678 i	629.259459163147994 +1036.027931241081433 i	0,32,23	24.689404115023237 + 42.760282046132680 i	469.561253478234619 + 924.646797989257266 i
0,32,25	21.110422260122977 + 45.095546473510225 i	328.260760931492314 + 791.490540347089336 i	0,32,27	17.260375607734932 + 47.291339151563413 i	207.693431206485712 + 638.096989672787799 i
0,32,29	13.067424111285044 + 49.346155555771162 i	110.595898217950804 + 465.894744275871574 i	0,32,31	8.392960502665726 + 51.254372438363262 i	40.478294093222374 + 276.148817428079894 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,32,33	2.841307336485150 + 53.001628305193471 i	2.761761375338454 + 69.250619818521216 i	0,33,1	54.150669800708876 + 4.429554446854491 i	2921.637125806150380 + 353.254071585042311 i
0,33,3	52.366035342062837 + 9.686654252151243 i	2704.149782644520656 + 679.347974357617659 i	0,33,5	50.429294375594772 + 14.245153121211846 i	2473.747130128763274 + 909.739454122537836 i
0,33,7	48.351520947982380 + 18.372524231393260 i	2237.923748632450497 +1075.533482792390032 i	0,33,9	46.137216008581781 + 22.181333941404365 i	2001.307298860138644 +1190.484310339473268 i
0,33,11	43.787385211503697 + 25.733675756587843 i	1767.298441060822370 +1262.602143211665179 i	0,33,13	41.300475962513083 + 29.068576833550139 i	1538.640132626225522 +1297.256593913482675 i
0,33,15	38.672639874089221 + 32.212461021264296 i	1317.686898060017029 +1298.380047495359577 i	0,33,17	35.897642471868622 + 35.184015382997913 i	1106.557657897944182 +1269.037251377507346 i
0,33,19	32.966481495760554 + 37.996744637368217 i	907.235956807119464 +1211.732051446822879 i	0,33,21	29.866654245231750 + 40.660413612536956 i	721.645769741988829 +1128.588580706497851 i
0,33,23	26.580887032015550 + 43.181883313449219 i	551.718661065013521 +1021.466103070270719 i	0,33,25	23.084904980655931 + 45.565566820706849 i	399.464659252703029 + 892.035718901835594 i
0,33,27	19.343258468368628 + 47.813590593436949 i	267.061872486308744 + 741.832538298119402 i	0,33,29	15.300596211194629 + 49.925633951632392 i	156.992108174357838 + 572.286581121243785 i
0,33,31	10.859786253238449 + 51.898219256493540 i	72.290256615439745 + 384.714521532494700 i	0,33,33	5.804921043485394 + 53.722477139082216 i	17.14783106412779 + 180.11969423503649 i
0,34,1	55.722493271622980 + 4.446178496432807 i	3094.224404093641624 + 365.450223179729221 i	0,34,3	53.942906884522486 + 9.733096641178330 i	2871.165397521455816 + 704.945427849707698 i
0,34,5	52.014779233467742 + 14.322280077375634 i	2634.748016750584156 + 946.541713565642226 i	0,34,7	49.949071190401504 + 18.481246278083312 i	2392.501380827697176 +1122.173543391808835 i
0,34,9	47.750384311144167 + 22.322611310445804 i	2149.066675145944828 +1245.941387639702953 i	0,34,11	45.419975626800287 + 25.908545057879596 i	1907.846632610706820 +1326.043696301949012 i
0,34,13	42.956693635009010 + 29.278179223516140 i	1671.57767249619701 +1367.96581665404574 i	0,34,15	40.357268039951222 + 32.458072060907632 i	1442.599852639529672 +1375.715956064696002 i
0,34,17	37.616274128543452 + 35.467078960959952 i	1223.009208068124963 +1352.409885952394234 i	0,34,19	34.725844846485572 + 38.318914262069541 i	1014.756478018656935 +1300.585575503081145 i
0,34,21	31.675098347080009 + 41.023605870571302 i	819.720164646861235 +1222.388842397369217 i	0,34,23	28.449152793730558 + 43.588351258384897 i	639.769030774971498 +1119.690888649128510 i
0,34,25	25.027441037620207 + 46.018007236489410 i	476.82493819395495 + 994.16687495585450 i	0,34,27	21.380685842372174 + 48.315313064747741 i	332.937733551338056 + 847.350149306213098 i
0,34,29	17.464965043988915 + 50.480855822146751 i	210.390929851376086 + 680.668007967836502 i	0,34,31	13.208288311939858 + 52.512665233357332 i	111.879216037816562 + 495.453274885302676 i
0,34,33	8.471947732638530 + 54.404983693166272 i	40.878270881817954 + 292.878347405213617 i	0,34,35	2.860070383211779 + 56.143853120583998 i	2.780482051095884 + 73.203127688576075 i
0,35,1	57.294264682711528 + 4.462332116163098 i	3271.749663168061943 + 377.698272316455814 i	0,35,3	55.519474350664510 + 9.778219964131301 i	3043.133717943467218 + 730.687340094923684 i
0,35,5	53.599525365853601 + 14.397193234290750 i	2800.726453540848524 + 983.584149545218906 i	0,35,7	51.545260084570266 + 18.586809670761252 i	2552.091208799864944 +1169.153129939979408 i
0,35,9	49.361359856653308 + 22.459728480897070 i	2301.883076360785253 +1301.841461915118771 i	0,35,11	47.049297971221641 + 26.078186591007615 i	2053.507758491501590 +1390.036434799964582 i
0,35,13	44.608271426186953 + 29.481407986727618 i	1809.696557680669230 +1439.339850558883882 i	0,35,15	42.035511325506867 + 32.696069616360816 i	1572.777117549233708 +1453.836599493433141 i
0,35,17	39.326290296562696 + 35.741175072530197 i	1344.825474599713743 +1436.694500178300814 i	0,35,19	36.473707952497065 + 38.630622145061317 i	1127.763627493047807 +1390.486857256434405 i
0,35,21	33.468241373085725 + 41.374666896465499 i	923.430427930438100 +1317.382599148548479 i	0,35,23	30.296972588043531 + 43.980794892205957 i	733.640435759041338 +1219.266507890584080 i
0,35,25	26.942293618428170 + 46.454236144014970 i	560.240633007648285 +1097.819568166027011 i	0,35,27	23.379661310393423 + 48.798232274020990 i	405.174518996067150 + 954.574790587910911 i
0,35,29	19.573415205672312 + 51.014086288028494 i	270.56711959500041 + 790.95583351294363 i	0,35,31	15.468046603329761 + 53.100942866978471 i	158.857329613568766 + 608.298145837026141 i
0,35,33	10.966316478492800 + 55.055068668545132 i	73.044835815278546 + 407.831723017778131 i	0,35,35	5.852229239533764 + 56.867697613131639 i	17.291723975193040 + 190.464307783776917 i
0,36,1	58.865987969916461 + 4.478040970264924 i	3454.212743738545214 + 389.996770221500981 i	0,36,3	57.095760628250126 + 9.822096602402240 i	3220.053861686115852 + 756.569655570812643 i
0,36,5	55.183588909775779 + 14.470015341603515 i	2971.680324608593310 +1020.859920975711020 i	0,36,7	53.140192781005055 + 18.689391762176523 i	2716.689351280137998 +1216.462399123938212 i
0,36,9	50.970315782509687 + 22.592922559985151 i	2459.750251341149578 +1358.171423625593661 i	0,36,11	48.675617529059863 + 26.242903631426259 i	2204.272470663032891 +1454.563659658669167 i
0,36,13	46.255598880553777 + 29.678640649422956 i	1952.983431993337717 +1511.358010751312349 i	0,36,15	43.707927838968047 + 32.926915750452828 i	1708.20014673852961 +1532.71682128529778 i
0,36,17	41.028481036394226 + 36.006863563332935 i	1471.981141083561624 +1521.860870879443382 i	0,36,19	38.211187169312787 + 38.932543258142225 i	1246.223137652607647 +1481.399879375836235 i
0,36,21	35.247673003856647 + 41.714410034971735 i	1032.730225884195761 +1413.527155730100503 i	0,36,23	32.126651984645598 + 44.360198144330909 i	833.269839126569877 +1320.142534600177214 i
0,36,25	28.832915017896013 + 46.875454857297882 i	649.62472682877603 +1202.93445990407781 i	0,36,27	25.345630159252195 + 49.263839786061702 i	483.648965088947705 +1063.437165485497644 i
0,36,29	21.635308879025409 + 51.527233231102770 i	337.338213130203542 + 903.071156637576223 i	0,36,31	17.657866648024971 + 53.665635767352150 i	212.93211910720407 + 723.16939277103154 i
0,36,33	13.341197753977694 + 55.676737356590614 i	113.089397705935355 + 524.981024008927307 i	0,36,35	8.546517731335694 + 57.554715539288637 i	41.255733994121478 + 309.595472106405964 i
0,36,37	2.877777959916811 + 59.286015311318302 i	2.798139077520676 + 77.149927536983327 i	0,37,1	60.437666678303380 + 4.493328686590048 i	3641.613498423733290 + 402.344346091100135 i
0,37,3	58.671786331839940 + 9.864793157726673 i	3401.925012022885312 + 782.588538947309644 i	0,37,5	56.767020355795403 + 14.540859241776856 i	3147.607672796185398 +1058.362564046693024 i
0,37,7	54.733963643671984 + 18.789155392492489 i	2886.292223754685892 +1264.092061225184352 i	0,37,9	52.577406997637667 + 22.722410917048364 i	2622.662439529478434 +1414.918918082532628 i
0,37,11	50.299170735521557 + 26.402973675584356 i	2360.132179308961440 +1519.609660145088810 i	0,37,13	47.899021488499514 + 29.870221835612522 i	2101.426066573828393 +1584.000875511898585 i
0,37,15	45.375009557844031 + 33.151031050715304 i	1848.852034860141885 +1612.333055874227966 i	0,37,17	42.723537486714044 + 36.264652257084116 i	1604.453256819465196 +1607.880760941527797 i
0,37,19	39.939249669999647 + 39.225287253776763 i	1370.104152754243387 +1573.291090624946037 i	0,37,21	37.014753421394133 + 42.043566047792204 i	1147.578189909566163 +1510.782867496632207 i
0,37,23	33.940131422833211 + 44.727439100938419 i	938.601564474443421 +1422.272297465675138 i	0,37,25	30.702145618726110 + 47.282725899149639 i	744.901451941315372 +1309.456776060862012 i
0,37,27	27.282927336599837 + 49.713438224585055 i	568.255844500344438 +1173.873305721022916 i	0,37,29	23.657722908530843 + 52.021930323625277 i	410.552990269740974 +1016.940317536059638 i
0,37,31	19.790706461497155 + 54.208878583996011 i	273.873120955023239 + 839.986732983622232 i	0,37,33	15.626246895566061 + 56.273026691263631 i	160.618379276780615 + 644.265797459245050 i
0,37,35	11.067023502129928 + 58.210468370929185 i	73.757868943603725 + 430.930462889367618 i	0,37,37	5.896969339982721 + 60.012572253500032 i	17.427754752006525 + 200.801055155446306 i
0,38,1	62.009304009693693 + 4.508217066017950 i	3833.951790581837486 + 414.739700986748176 i	0,38,3	60.247570080666890 + 9.906371050429060 i	3588.746411299916872 + 808.740357673490507 i
0,38,5	58.349865248658681 + 14.609828907284534 i	3328.506683881469144 +1096.085961882557967 i	0,38,7	56.326657618773432 + 18.886250432012286 i	3060.896508234326575 +1312.033334623898554 i
0,38,9	54.182772564261278 + 22.848393323049983 i	2790.614319427000737 +1472.072281758310965 i	0,38,11	51.920169084904053 + 26.558651303170105 i	2521.078970073862820 +1585.159627859205557 i
0,38,13	49.538846918937850 + 30.056467027013579 i	2255.013231134959824 +1657.250172541796474 i	0,38,15	47.037192177270505 + 33.368799533275521 i	1994.717319766077026 +1692.663179903239552 i
0,38,17	44.412067509603901 + 36.515003357828540 i	1742.22093432581119 +1694.72772615759095 i	0,38,19	41.658738762225575 + 39.509406892609540 i	1499.378761498489439 +1666.129156237853294 i
0,38,21	38.770655619518699 + 42.362794386489867 i	1267.937180419961578 +1609.112819983304917 i	0,38,23	35.739060465528297 + 45.083305481866830 i	1049.586100147787647 +1525.612475551933130 i
0,38,25	32.552353855863778 + 47.676995132386047 i	846.004254310393662 +1417.335821303045805 i	0,38,27	29.195070514564126 + 50.148175028579509 i	658.90424763986937 +1285.82409126269931 i
0,38,29	25.646169666714766 + 52.499594346421539 i	490.083882043957203 +1132.494942170048716 i	0,38,31	21.875995826893412 + 54.732475081814428 i	341.493120501029125 + 958.673301253518275 i
0,38,33	17.840336862599678 + 56.846370578266288 i	215.334085733912218 + 765.610480549822228 i	0,38,35	13.466997836899040 + 58.838722846913626 i	114.234293887979376 + 554.481695868453812 i
0,38,37	8.617135574160638 + 60.703693629045496 i	41.613068009872215 + 326.301354010196121 i	0,38,39	2.894541193035209 + 62.428123888975506 i	2.814845354621380 + 81.091395256983745 i
0,39,1	63.580902863469767 + 4.522726265652941 i	4031.227493281765874 + 427.181602350117089 i	0,39,3	61.823128736470821 + 9.946887042303235 i	3780.517355287391638 + 835.021666350852570 i
0,39,5	59.932164785266870 + 14.677020345542555 i	3514.375672723123444 +1134.024317371478446 i	0,39,7	57.918351397931850 + 18.980815123831000 i	3240.499126843070371 +1360.277905168842153 i
0,39,9	55.786537710326407 + 22.971053806520775 i	2963.600963787104774 +1529.620485604000805 i	0,39,11	53.538802408560848 + 26.710170649691577 i	2687.105532584868342 +1651.199580526749969 i
0,39,13	51.175350199040125 + 30.237665794419588 i	2413.734583643069982 +1731.088678623315218 i	0,39,15	48.694863201398761 + 33.580572830769590 i	2145.781814901550206 +1773.686381687784205 i
0,39,17	46.094608362016011 + 36.758338874875143 i	1885.265095624045443 +1782.376946586289023 i	0,39,19	43.370394026017770 + 39.785405114849539 i	1634.021610576960256 +1759.884741054456072 i
0,39,21	40.516398285886304 + 42.672692532487616 i	1393.773688985964554 +1708.482551476443859 i	0,39,23	37.524853767181739 + 45.428507247688295 i	1166.179150051710849 +1630.122747316135928 i
0,39,25	34.385537403346362 + 48.059109358736187 i	952.874226408728759 +1526.524548721360361 i	0,39,27	31.084959723570602 + 50.569068336655306 i	755.514840077448753 +1399.234775467983582 i
0,39,29	27.605045559163436 + 52.961466594512281 i	575.821725513957176 +1249.671653546006610 i	0,39,31	23.920873901635456 + 55.237974615871806 i	415.636561827251398 +1079.156873666190222 i
0,39,33	19.996487724465016 + 57.398785686164892 i	277.001294415644338 + 888.937671101786691 i	0,39,35	15.776160007375038 + 59.442340769499125 i	162.286237815136445 + 680.195001262189635 i
0,39,37	11.162507410997140 + 61.364620914329407 i	74.433668847179717 + 454.012689010734164 i	0,39,39	5.939404047053480 + 63.157149275510704 i	17.556731950484210 + 211.130522906587146 i
0,40,1	65.152465871671225 + 4.536874959655768 i	4233.440488395316606 + 439.668879061044834 i	0,40,3	63.398477607898442 + 9.986393695233287 i	3977.237188196789248 + 861.429192669895315 i
0,40,5	61.513956327220512 + 14.742522391323789 i	3705.213071066521934 +1172.172128759987118 i	0,40,7	59.509114411246486 + 19.072977256838712 i	3425.097218653430900 +1408.817889777483970 i
0,40,9	57.388815538505170 + 23.090562270089624 i	3141.617800501144302 +1587.553084432371406 i	0,40,11	55.155241639905128 + 26.857747551869277 i	2858.205098423805794 +1717.716294206305065 i
0,40,13	52.808778046738482 + 30.414084588248734 i	2577.580575279339428 +1805.500130747556796 i	0,40,15	50.348368641032685 + 33.786673788095342 i	2302.032466182766711 +1855.383046150494010 i
0,40,17	47.771637049638493 + 36.995045247781512 i	2033.568257880055626 +1870.805078647928667 i	0,40,19	45.074867503905338 + 40.053741016139668 i	1774.00958236292490 +1854.53032026181745 i
0,40,21	42.252871699506322 + 42.973803802330274 i	1525.057347601890342 +1808.859811850739788 i	0,40,23	39.298733990917860 + 45.763686981204287 i	1288.340870071479003 +1735.765485660351032 i
0,40,25	36.203398042649333 + 48.429830519534647 i	1065.458879237704878 +1636.979181955636250 i	0,40,27	32.955017905242990 + 50.977027262159822 i	858.017789578368593 +1514.054545028122675 i
0,40,29	29.537924601021629 + 53.408643832476095 i	667.67200222048712 +1368.41167370642734 i	0,40,31	25.930910499824741 + 55.726724901410563 i	496.171907251919663 +1201.370080107397598 i
0,40,33	22.104186766525775 + 57.931980936791831 i	345.428437930046869 +1014.172553998032413 i	0,40,35	18.013440139310248 + 60.023621101490129 i	217.611256048935758 + 807.998895126850925 i
0,40,37	13.586406314591137 + 61.998907372088567 i	115.320556932770714 + 583.958352297863598 i	0,40,39	8.684196758216407 + 63.852020482716334 i	41.952298648235164 + 342.996979335184790 i
0,40,41	2.910454611814311 + 65.570186201903766 i	2.830696800829548 + 85.027863878666295 i	0,41,1	66.723995429295613 + 4.550680480904525 i	4440.590665793658445 + 452.200416973109270 i
0,41,3	64.973630626851659 + 10.024939774797588 i	4178.905298271363790 + 887.959824719415792 i	0,41,5	63.095273841936567 + 14.806417403043241 i	3901.017416774585399 +1210.524167669756025 i
0,41,7	61.099009678818376 + 19.162855194220448 i	3614.688119294967237 +1457.645803728232067 i	0,41,9	58.989708484465389 + 23.207075903901668 i	3324.660578316402280 +1645.860171572074705 i
0,41,11	56.769641159099322 + 27.001581416845966 i	3034.371387059854442 +1784.697242773018615 i	0,41,13	54.439352516296836 + 30.585969159299978 i	2746.542368152783183 +1880.469147104605554 i
0,41,15	51.998018595805597 + 33.987399565190323 i	2463.457229077709144 +1937.734652986228639 i	0,41,17	49.443578855460878 + 37.225477311603235 i	2187.114351086645001 +1959.990124855338308 i
0,41,19	46.772736843585086 + 40.314834930516838 i	1919.321523977272136 +1950.040013505871002 i	0,41,21	43.980858371108454 + 43.266623920920559 i	1661.760522044285835 +1910.214352080289018 i
0,41,23	41.061765218131349 + 46.089428518335971 i	1416.035246541508968 +1842.505492315276797 i	0,41,25	38.007398169976121 + 48.789847298434871 i	1183.711165441634876 +1748.658884481587393 i
0,41,27	34.807292626349557 + 51.372868055356356 i	966.351166740052463 +1630.236123760163991 i	0,41,29	31.447758526636093 + 53.842102066758329 i	765.552058681607832 +1488.660407876067438 i
0,41,31	27.910554979408012 + 56.199910141711108 i	582.986350781884425 +1325.250226738262427 i	0,41,33	24.170626493391993 + 58.447428321720182 i	420.455969343121922 +1141.244382795623324 i
0,41,35	20.191911134588224 + 60.584473462219520 i	279.969770997089597 + 937.818815747561075 i	0,41,37	15.918606248667846 + 62.609257220909647 i	163.870236874452360 + 716.090269582400765 i
0,41,39	11.253280320232314 + 64.517692299297792 i	75.075911239102084 + 477.080036518994348 i	0,41,41	5.979757828283767 + 66.301468238043611 i	17.679345874306129 + 221.453222520897526 i
0,42,1	68.295493720553040 + 4.564158946160088 i	4652.677922633985872 + 464.775154870546601 i	0,42,3	66.548600501195722 + 10.062570606584677 i	4385.521113871946909 + 914.610599505144251 i
0,42,5	64.676148283761904 + 14.868781876580778 i	4101.787344286494772 +1249.075459243144905 i	0,42,7	62.688094543934369 + 19.250558778344512 i	3809.269342942016465 +1506.754531186984877 i
0,42,9	60.589309566784664 + 23.320740425930662 i	3512.725336675854123 +1704.532338121021439 i	0,42,11	58.382140791198793 + 27.141856857110284 i	3215.598558524007785 +1852.130543721849790 i
0,42,13	56.067274082977498 + 30.753546667607029 i	2920.611763719584815 +1955.981156595002403 i	0,42,15	53.644091936320272 + 34.183024326903997 i	2630.044962474864860 +2020.723686195662594 i
0,42,17	51.110814416914756 + 37.449961714263772 i	2345.888562044565333 +2049.911318615295386 i	0,42,19	48.464516060712839 + 40.569072779676517 i	2069.938017901986314 +2046.389438880165471 i
0,42,21	45.701049669870436 + 43.551606595983920 i	1803.85797201795685 +2012.51773977632865 i	0,42,23	42.814879305764173 + 46.406264184819300 i	1549.229585016755891 +1950.309765753409920 i
0,42,25	39.798804187003718 + 49.139784704739043 i	1307.588691823819090 +1861.525470246055875 i	0,42,27	36.643531417252738 + 51.757327177879446 i	1080.459689271205434 +1747.735420647393312 i
0,42,29	33.337017779834270 + 54.262715199713071 i	869.389001560331621 +1610.367047919179186 i	0,42,31	29.863430183017957 + 56.658579553159363 i	675.98202063803853 +1450.73897533650165 i
0,42,33	26.201426199615241 + 58.946411734535261 i	501.948682457193968 +1270.086850547288805 i	0,42,35	22.321109514634603 + 61.126520155628590 i	349.166213896227362 +1069.581997080861811 i
0,42,37	18.178086168355755 + 63.197847297745582 i	219.775922420228596 + 850.340933558232905 i	0,42,39	13.700037734756824 + 65.157525978341951 i	116.353872984046717 + 613.413551739872446 i
0,42,41	8.748040400705664 + 66.999780615311749 i	42.275161246117307 + 359.683193061082648 i	0,42,43	2.925599289108548 + 68.712208303765351 i	2.845775587013278 + 88.959630310432715 i
0,43,1	69.866962741693186 + 4.577325366986033 i	4869.702162724435766 + 477.392080798027109 i	0,43,3	68.123398847460166 + 10.099328391712467 i	4597.084099992335723 + 941.378692537126312 i
0,43,5	66.256607924453945 + 14.929686988092293 i	4307.521576138460659 +1287.821264165770117 i	0,43,7	64.276421306825753 + 19.336190129481718 i	4008.838566351034842 +1556.137298576859621 i
0,43,9	62.187703462816629 + 23.431691174121575 i	3705.808379089821756 +1763.560636227622581 i	0,43,11	59.992867516904546 + 27.278745125747491 i	3401.881171822210035 +1920.004909481209324 i
0,43,13	57.692724266693020 + 30.917027526874314 i	3099.781140242905167 +2032.022335735358638 i	0,43,15	55.286840254581179 + 34.373801585066381 i	2801.785336593018656 +2104.333553432298686 i
0,43,17	52.773685638687468 + 37.668799876016237 i	2509.877200091208124 +2140.549021970137801 i	0,43,19	50.150664423599224 + 40.816809814885886 i	2225.841186489950815 +2143.555583898692475 i
0,43,21	47.414059346519316 + 43.829168275668160 i	1951.326564906959902 +2115.743196890834952 i	0,43,23	44.558896928276724 + 46.714680910300814 i	1687.894085808906766 +2059.147297698019429 i
0,43,25	41.578720326610572 + 49.480212064464176 i	1437.053077093902857 +1975.543150124412023 i	0,43,27	38.465238697363134 + 52.131072012896730 i	1200.293721102461177 +1866.511219044814879 i
0,43,29	35.207793626511453 + 54.671269929032654 i	979.118070718019339 +1733.484208876015600 i	0,43,31	31.792532893892837 + 57.103669295219376 i	775.073168525217739 +1577.781983232129733 i
0,43,33	28.201081120783009 + 59.430062250206255 i	589.790390819216327 +1400.638016970800063 i	0,43,35	24.408273731339726 + 61.611516206375687 i	425.037353467495336 +1203.219195024090595 i
0,43,37	20.377964260406177 + 63.766489771797183 i	282.794037297470254 + 986.638574894443536 i	0,43,39	16.054290139325243 + 65.774083316418299 i	165.378373898426645 + 751.955367424558062 i
0,43,41	11.339782743439430 + 67.669819931304218 i	75.687754216562169 + 500.133891783411457 i	0,43,43	6.018223980848056 + 69.445561914589717 i	17.796190523722718 + 231.769603518533188 i
0,44,1	71.438404320917016 + 4.590193748323186 i	5091.663295957127048 + 490.050228721339806 i	0,44,3	69.698036306539038 + 10.135252487026383 i	4813.593755148521268 + 968.261408364419140 i
0,44,5	67.836678640755252 + 14.989199075446038 i	4518.218915406474480 +1326.757062349287025 i	0,44,7	65.864037774514628 + 19.419844352986349 i	4213.393614670545503 +1605.787650455520634 i
0,44,9	63.784967438449158 + 23.540054071189527 i	3903.906249547046173 +1822.936545912010843 i	0,44,11	61.601936943676083 + 27.412405380768888 i	3593.214148256548469 +1988.309603549686699 i
0,44,13	59.315867874415176 + 31.076607023676058 i	3284.043397927280694 +2108.579552006141057 i	0,44,15	56.926491215395600 + 34.559966246039878 i	2978.668752705775205 +2188.548513857167563 i
0,44,17	54.432500663921445 + 37.882270563917722 i	2679.067580938502033 +2231.884634500518587 i	0,44,19	51.831593841549832 + 41.058373852702580 i	2387.014524352589433 +2241.516691046371307 i
0,44,21	49.120434628661435 + 44.099692232676318 i	2104.145032962678215 +2219.865456372934204 i	0,44,23	46.294544556053992 + 47.015125429630018 i	1832.00148856735973 +2168.98889406683838 i
0,44,25	43.348115376223539 + 49.811649740143778 i	1572.069422599820655 +2090.678309248089136 i	0,44,27	40.273719537316325 + 52.494709734258770 i	1325.808464051496912 +1986.524902782536174 i
0,44,29	37.061873854061687 + 55.068477826158031 i	1094.681359543052622 +1857.967602043318038 i	0,44,31	33.700375151380712 + 57.536019702660475 i	880.184037612393809 +1706.328548110550400 i
0,44,33	30.173256631779555 + 59.899384529165360 i	683.88010002177339 +1532.84006073728415 i	0,44,35	26.459065938801245 + 62.159624045538642 i	507.444583733380114 +1338.665207899720599 i
0,44,37	22.527819461163475 + 64.316727635741586 i	352.725323197384796 +1124.912464452157337 i	0,44,39	18.335058492179826 + 66.369430206296229 i	221.838645270084839 + 892.641850620899390 i
0,44,41	13.808422298311754 + 68.314773929081142 i	117.339140446558417 + 642.849452952797947 i	0,44,43	8.808959452481483 + 70.147044351225361 i	42.583153656757219 + 376.360725584959766 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,44,45	2.940045273854695 + 71.854195227952673 i	2.860152636889170 + 92.886960756068788 i	0,45,1	73.009820135799856 + 4.602777176329987 i	5318.561237800710842 + 502.748675482570434 i
0,45,3	71.272522644889605 + 10.170379654613747 i	5035.049608594330493 + 995.256171951851684 i	0,45,5	69.416384165462659 + 15.047380066428175 i	4733.878238952233005 +1365.878538086971730 i
0,45,7	67.450987739535790 + 19.501610167265394 i	4422.932448789832051 +1655.699427611083593 i	0,45,9	65.381172154636332 + 23.645946479550556 i	4107.015711551975073 +1882.651945008882649 i
0,45,11	63.209454576009101 + 27.542985802524363 i	3789.592738962910516 +2057.034400869435103 i	0,45,13	60.936854925496938 + 31.232466743929763 i	3473.391910600640131 +2185.640312833812004 i
0,45,15	58.563251414164426 + 34.741736407642207 i	3160.686272863542399 +2273.353613399413007 i	0,45,17	56.087538078189930 + 38.090632140834771 i	2853.447925687305542 +2323.900511896232729 i
0,45,19	53.507675050498541 + 41.294068086153509 i	2553.442753846012693 +2340.252155896321256 i	0,45,21	50.820665397033487 + 44.363532089083186 i	2262.293765997557784 +2324.860635096667920 i
0,45,23	48.022468287251527 + 47.308008735376639 i	1981.526772380713510 +2279.807016852437300 i	0,45,25	45.107844042315151 + 50.134574824407345 i	1712.605872095295808 +2206.899310890739343 i
0,45,27	42.070113814454338 + 52.848794721218191 i	1456.963297387881084 +2107.740214737715178 i	0,45,29	38.900800022219627 + 55.454985252210470 i	1216.026793196966537 +1983.775743579036543 i
0,45,31	35.589086665488878 + 57.956389071346337 i	991.247211867552892 +1836.331275839584578 i	0,45,33	32.120988953842712 + 60.355277132085149 i	784.129170455664507 +1666.638972109877841 i
0,45,35	28.478021325032213 + 62.652987725046856 i	596.268574208761038 +1475.858902443824036 i	0,45,37	24.634930016797238 + 64.849896284960522 i	429.403138368986106 +1265.094872615020222 i
0,45,39	20.555500037438883 + 66.945289824980975 i	285.487427615878744 +1035.403971139534860 i	0,45,41	16.183821206059946 + 68.937075157009176 i	166.817555109727266 + 787.793466161596143 i
0,45,43	11.422396327194678 + 70.821118573301234 i	76.271931012421106 + 523.175440862600340 i	0,45,45	6.054970138122156 + 72.589457697168825 i	17.907780683062207 + 242.080063737768484 i
0,46,1	74.581211728582247 + 4.615087896859908 i	5550.395908845064696 + 515.486538018056535 i	0,46,3	72.846866843316079 + 10.204744284576993 i	5261.451217822953367 +1022.360520807319972 i
0,46,5	70.995746307318115 + 15.104287860627505 i	4954.498491371006821 +1405.181566519535409 i	0,46,7	69.037311398127244 + 19.581570462965075 i	4637.453154028072277 +1705.866747126731525 i
0,46,9	66.976382369543558 + 23.749477961113914 i	4315.133729439924537 +1942.699081871778617 i	0,46,11	64.815516916531735 + 27.670624584335915 i	3991.012496086230385 +2126.169551933736784 i
0,46,13	62.555822311845070 + 31.384775833762731 i	3667.820483008677911 +2263.192719518725426 i	0,46,15	60.197308825456780 + 34.919314941797275 i	3347.829558119787180 +2358.734626488635058 i
0,46,17	57.739050483988130 + 38.294124537878439 i	3033.007272636556536 +2416.579892935986663 i	0,46,19	55.179242824123776 + 41.524173537782588 i	2725.111699827140455 +2439.742436105903835 i
0,46,21	52.515191830220516 + 44.621014873512578 i	2425.754633333740003 +2430.706120814615133 i	0,46,23	49.743245214875202 + 47.593709911214162 i	2136.446900948499206 +2391.575643999899057 i
0,46,25	46.858664213475642 + 50.449425996984495 i	1858.63324248305412 +2324.17632321383144 i	0,46,27	43.855423228977394 + 53.193834809552840 i	1593.721232649572357 +2230.123043716917437 i
0,46,29	40.725911517660982 + 55.831381587796486 i	1343.107302042443280 +2110.869695796729957 i	0,46,31	37.460490794128773 + 58.365464853395707 i	1108.202315644036844 +1967.745777626992322 i
0,46,33	34.046826711541073 + 60.798548499683974 i	890.459162010737828 +1801.984237030737859 i	0,46,35	30.468842490338158 + 63.132216034129073 i	691.40541666605327 +1614.74303492810532 i
0,46,37	26.704995396710313 + 65.367165902749392 i	512.685718438869912 +1407.121731702286979 i	0,46,39	22.725230455847260 + 67.503132357893023 i	356.12205095163171 +1180.17301648687453 i
0,46,41	18.485036793331130 + 69.538688363291485 i	223.808564425050150 + 934.906071998455445 i	0,46,43	13.912020600975170 + 71.470814855531174 i	118.280609042732351 + 672.267893575748531 i
0,46,45	8.867208701246192 + 73.293870703865423 i	42.877577635708079 + 393.030213305948621 i	0,46,47	2.953853498459750 + 74.996151194836116 i	2.873889584632835 + 96.810095114865035 i
0,47,1	76.152580519627804 + 4.627137385747260 i	5787.167234391789862 + 528.262970811411947 i	0,47,3	74.421077175087333 + 10.238378594429955 i	5492.798166319610573 +1049.572097779767091 i
0,47,5	72.574785144173617 + 15.159976670884697 i	5180.078679554933842 +1444.662201269688467 i	0,47,7	70.623045716704638 + 19.659802802233230 i	4856.953929995077488 +1756.283984197287282 i
0,47,9	68.570657551895209 + 23.850750954403914 i	4528.257451675163793 +2003.070550527562546 i	0,47,11	66.420212424031732 + 27.795450813323374 i	4197.469247106422699 +2195.705750196487770 i
0,47,13	64.172895235250522 + 31.533692117504621 i	3867.323312941095992 +2341.225425519305190 i	0,47,15	61.828834910403771 + 35.092890893197222 i	3540.090814027054421 +2444.678003463493042 i
0,47,17	59.387267555406808 + 38.492970990687354 i	3217.735399939265790 +2509.906833809363434 i	0,47,19	56.846600389435345 + 41.748951208953478 i	2902.008180588064533 +2539.968969868129534 i
0,47,21	54.204410816379315 + 44.872443684903331 i	2594.510830033782440 +2537.380471249874518 i	0,47,23	51.457392842832910 + 47.872579449638977 i	2296.740604657187305 +2504.270144808750501 i
0,47,25	48.601251043994601 + 50.756607693710059 i	2010.124711727579552 +2442.481165720843364 i	0,47,27	45.630532928614258 + 53.530296601389288 i	1736.048459428406431 +2353.641235921600583 i
0,47,29	42.538379971810684 + 56.198206128717786 i	1475.880145214091272 +2239.212837739664792 i	0,47,31	39.316162047839704 + 58.763872865002867 i	1230.994973738842646 +2100.530397402283143 i
0,47,33	35.952933576047968 + 61.229929752830280 i	1002.800162948155048 +1938.828534761570381 i	0,47,35	32.434600335288373 + 63.598172104143813 i	792.763765285080353 +1755.263993736366729 i
0,47,37	28.742585747127563 + 65.869572056934658 i	602.450802388890793 +1550.932755455558436 i	0,47,39	24.851562636522772 + 68.044232772022340 i	433.572708522333301 +1326.882773986378879 i
0,47,41	20.725260185181742 + 70.121255211141943 i	288.061506264884542 +1084.120923074463008 i	0,47,43	16.307730310767983 + 72.098448005287651 i	168.193785510460344 + 823.607260157446945 i
0,47,45	11.501453877260168 + 73.971684862594643 i	76.830822866916066 + 546.205706739194852 i	0,47,47	6.090142612980125 + 75.733178664124111 i	18.014565383294745 + 252.384957502274517 i
0,48,1	77.723927819300116 + 4.638936411905378 i	6028.875144085088323 + 541.077163557342715 i	0,48,3	75.995161274860848 + 10.271312807998076 i	5729.090061535469431 +1076.888644457976852 i
0,48,5	74.153519193166858 + 15.214497329336780 i	5410.617867797489453 +1484.316663121928286 i	0,48,7	72.208224753988219 + 19.736379865610946 i	5081.433081479330212 +1806.945755509061200 i
0,48,9	70.164052418485937 + 23.949861379607242 i	4746.384195881077176 +2063.759268011174299 i	0,48,11	68.023622349853824 + 27.917585255786324 i	4408.959071903526819 +2265.634102410308060 i
0,48,13	65.788188456539515 + 31.679363091951494 i	4071.894957535060257 +2419.727598585848227 i	0,48,15	63.457986438118354 + 35.262640719328779 i	3737.462742376183996 +2531.170822976182535 i
0,48,17	61.032398660958002 + 38.687379573158637 i	3407.622757501835622 +2603.866148873214570 i	0,48,19	58.510023188443533 + 41.968643970234865 i	3084.119912453228608 +2640.914102612855913 i
0,48,21	55.888681364287216 + 45.118100023413845 i	2768.546743422930376 +2644.863323733561629 i	0,48,23	53.165376942796406 + 48.144942137341772 i	2462.388193113968100 +2617.867168773722824 i
0,48,25	50.336208544008194 + 51.056493705206528 i	2167.055553252264417 +2561.787172744482632 i	0,48,27	47.396229003113909 + 53.858610006017962 i	1883.91396378646732 +2478.26442773376311 i
0,48,29	44.339236512112556 + 56.555953912593033 i	1614.306351361512043 +2368.770661616830239 i	0,48,31	41.157470375302282 + 59.152184943174930 i	1359.575968828047735 +2234.645967917846974 i
0,48,33	37.841164456406005 + 61.650085102161832 i	1121.089471468824513 +2077.127458121182372 i	0,48,35	34.377875796280975 + 64.051634137827293 i	900.262661352226473 +1897.371520638642778 i
0,48,37	30.751436393561100 + 66.358038600095455 i	698.59175766253611 +1696.47132528747534 i	0,48,39	26.940228691917561 + 68.569710539785163 i	517.694695069790441 +1475.470328209239369 i
0,48,41	22.914139060482924 + 70.686178881840775 i	359.370547533338424 +1235.371297473129289 i	0,48,43	18.628614465606617 + 72.705890252245538 i	225.693643351712865 + 977.137356388903981 i
0,48,45	14.011235290803148 + 74.625786969868159 i	119.181989470131143 + 701.670450471167669 i	0,48,47	8.923011114244714 + 76.440309578069019 i	43.159571594409343 + 409.692214730453031 i
0,48,49	2.967077291357940 + 78.138079770313041 i	2.887040325044996 + 100.729250590823057 i	0,49,1	79.295254838471919 + 4.650495094100623 i	6275.519571578297473 + 553.928339014921562 i
0,49,3	77.569126199656424 + 10.303575316297744 i	5970.326533057012966 +1104.307995108861178 i	0,49,5	75.731965561056313 + 15.267897562372038 i	5646.115173375183986 +1524.141329639377545 i
0,49,7	73.792879944967069 + 19.811369853017212 i	5310.889010239505296 +1857.846904018217629 i	0,49,9	71.756617406681130 + 24.046899180585527 i	4969.511435388264545 +2124.758453645773443 i
0,49,11	69.625821470562713 + 28.037141059349793 i	4625.478282212819067 +2335.946101569756138 i	0,49,13	67.401807385101037 + 31.821926813096940 i	4281.530303204054690 +2498.688886306773838 i
0,49,15	65.084907066214853 + 35.428729393192626 i	3939.938498320454983 +2618.200748808527806 i	0,49,17	62.674635125610870 + 38.877544556707420 i	3602.660406798580880 +2698.443357066727003 i
0,49,19	60.169762098851365 + 42.183478230039533 i	3271.435425978606872 +2742.561020932345855 i	0,49,21	57.568329196202441 + 45.358245838198044 i	2947.847836674940936 +2753.135314038279830 i
0,49,23	54.867618154068339 + 48.411099575866139 i	2633.371393001384149 +2732.344546100706793 i	0,49,25	52.064079193628011 + 51.349430299367441 i	2329.402908458444472 +2682.069071696286755 i
0,49,27	49.153212773939828 + 54.179172146892888 i	2037.28920526428601 +2603.96389700069312 i	0,49,29	46.129393597386212 + 56.905080679118747 i	1758.350252011082830 +2499.510591920712159 i
0,49,31	42.985615790496325 + 59.530925373791348 i	1493.900550934984039 +2370.055593107260393 i	0,49,33	39.713123202673941 + 62.059620423102203 i	1245.270539873717325 +2216.839258964087549 i
0,49,35	36.300862661748695 + 64.493307333755467 i	1013.829844423666032 +2041.018436864645082 i	0,49,37	32.734648857672042 + 66.833395301576317 i	801.014700980321436 +1843.684280102879256 i
0,49,39	28.995829491990364 + 69.080558296659618 i	608.363012384115905 +1625.876269158624154 i	0,49,41	25.059016611175366 + 71.234678469428573 i	437.56293723375518 +1388.59249453693383 i
0,49,43	20.887893739219280 + 73.294708381011745 i	290.526368327267676 +1132.794459733850772 i	0,49,45	16.426482615542966 + 75.258384193379058 i	169.512319207593066 + 859.399056454063270 i
0,49,47	11.577247339192224 + 77.121600782019087 i	77.366516923019518 + 569.225578299411268 i	0,49,49	6.123869860541932 + 78.876744402343462 i	18.116938624249908 + 262.684602183942502 i
0,50,1	80.866562697847115 + 4.661822952147077 i	6527.100454232028824 + 566.815751031523519 i	0,50,3	79.142978482933913 + 10.335192822528306 i	6216.507230948506731 +1131.828071101275782 i
0,50,5	77.310140077378321 + 15.320222238209077 i	5886.569762551267559 +1564.132725622664566 i	0,50,7	75.377040351912868 + 19.884836844377098 i	5545.320207592908446 +1908.982484983349423 i
0,50,9	73.348399090986092 + 24.141948811593064 i	5197.636787117028689 +2186.061610063219177 i	0,50,11	71.226878731745629 + 28.154224382288437 i	4847.023403172206599 +2406.633602178281762 i
0,50,13	69.013849032436980 + 31.961512689087829 i	4496.224538724863123 +2578.099384688743612 i	0,50,15	66.709728717440015 + 35.591311386740081 i	4147.511652163385406 +2705.755990596002448 i
0,50,17	64.314152189809690 + 39.063647618641184 i	3802.839967495128803 +2793.624633319520399 i	0,50,19	61.826046204517837 + 42.393665412512782 i	3463.943992053560123 +2844.893692855372592 i
0,50,21	59.243650667120903 + 45.593125333162440 i	3132.400546830878284 +2862.178003259619345 i	0,50,23	56.564497561895450 + 48.671332392867804 i	2809.673207124048986 +2847.681198401578058 i
0,50,25	53.785351976061399 + 51.635738943599250 i	2497.145590754666500 +2803.302874149787385 i	0,50,27	50.902112568263647 + 54.492350742073363 i	2196.147841600120501 +2730.712430395360116 i
0,50,29	47.909662701167044 + 57.246007121257840 i	1907.97908898020682 +2631.40182437421072 i	0,50,31	44.801655813383896 + 59.900576333757726 i	1633.927865701430619 +2506.724453973212803 i
0,50,33	41.570207074172146 + 62.459090398040992 i	1375.292117939881109 +2357.924618636366631 i	0,50,35	38.205443928232469 + 64.923833595147556 i	1133.400418197636554 +2186.160385406419664 i
0,50,37	34.694832723998985 + 67.296391735107250 i	909.636305797637760 +1992.521672902114688 i	0,50,39	31.022129250130334 + 69.577663284867057 i	705.46847597297858 +1778.04464023899322 i
0,50,41	27.165653632057449 + 71.767826340782676 i	522.491225820005866 +1543.722761129218952 i	0,50,43	23.095243810742691 + 73.866243759436571 i	362.483187381230437 +1290.513808691526492 i
0,50,45	18.766312618805493 + 75.871263869017554 i	227.500862963660495 +1019.338920408816648 i	0,50,47	14.106420384733919 + 77.779807860205509 i	120.046540821028216 + 731.058486652853162 i
0,50,49	8.976562917139679 + 79.586403474735192 i	43.430136794657876 + 426.347223223206583 i	0,50,51	2.979763588818704 + 81.279983988533189 i	2.899652253854387 + 104.644624679573539 i
0,51,1	82.437852436249997 + 4.672928953166523 i	6783.617732840343706 + 579.738682720818929 i	0,51,3	80.716724182672124 + 10.366190473021463 i	6467.631824248180237 +1159.446875767751180 i
0,51,5	78.888057412686734 + 15.371513590306201 i	6131.980846953561922 +1604.287514327179224 i	0,51,7	76.960732886852185 + 19.956841124675410 i	5784.725247708745883 +1960.347753125487088 i
0,51,9	74.939440551331543 + 24.235089675350050 i	5430.758000636370525 +2247.662505785319354 i	0,51,11	72.826857815427630 + 28.268934958947671 i	5073.591156707177106 +2477.688797593427353 i
0,51,13	70.624402849411581 + 32.098242191137615 i	4715.973131085302157 +2657.949609441637222 i	0,51,15	68.332572782973969 + 35.750531550325164 i	4360.176155201153961 +2793.825268023862009 i
0,51,17	65.951110712731648 + 39.245858919553463 i	4008.153569954162322 +2889.396764379443539 i	0,51,19	63.479085189115146 + 42.599403270651941 i	3661.635556495633409 +2947.896813718861722 i
0,51,21	60.914916125904277 + 45.822966564887949 i	3322.192195092329712 +2971.973811765385108 i	0,51,23	58.256361439383859 + 48.925902189522787 i	2991.27779130253066 +2963.85705829401149 i
0,51,25	55.500469134653621 + 51.915718688619116 i	2670.263915819879912 +2925.465778115738431 i	0,51,27	52.643493497344238 + 54.798487044049182 i	2360.465492632902624 +2858.484204776390925 i
0,51,29	49.680768770786052 + 57.579122550924183 i	2063.162681612354845 +2764.415182197380545 i	0,51,31	46.606527476442650 + 60.261582533067454 i	1779.620476688039389 +2644.619635336599913 i
0,51,33	43.413641534121020 + 62.849004522521369 i	1511.107549925218109 +2500.346441829262403 i	0,51,35	40.093249658698134 + 65.343799537922067 i	1258.915780741893514 +2332.755593886259248 i
0,51,37	36.634208586390955 + 67.747708422519750 i	1024.382098096409663 +2142.936523895757091 i	0,51,39	33.022256630278306 + 70.061823865126513 i	808.914819283091674 +1931.922772869994787 i
0,51,41	29.238678106681768 + 72.286575068722172 i	614.027853452927388 +1700.703584180078451 i	0,51,43	25.258034496719571 + 74.421663717327093 i	441.38860446068782 +1450.23220411285160 i
0,51,45	21.043971874180922 + 76.465923809780406 i	292.890878949277806 +1181.428886174036734 i	0,51,47	16.540487983045178 + 78.417039249072616 i	170.777779620445699 + 895.170844589958847 i
0,51,49	11.650034214301195 + 80.270936358490587 i	77.880852693192423 + 592.235833151974930 i	0,51,51	6.156265266090407 + 82.020171647554037 i	18.215247999199484 + 272.979283531859380 i
0,52,1	84.009125018012767 + 4.683821553473130 i	7045.071351381919158 + 592.696444780119464 i	0,52,3	82.290368924214320 + 10.396591975747739 i	6723.699999601198215 +1187.162489662010785 i
0,52,5	80.465731183799704 + 15.421811419377587 i	6382.347680284824492 +1644.602489364817984 i	0,52,7	78.543982509240564 + 20.027439477568787 i	6029.102781526439685 +2011.938150803877761 i
0,52,9	76.529781699541076 + 24.326396518204302 i	5668.872948263226135 +2309.555159208141049 i	0,52,11	74.425817641589234 + 28.381366608925656 i	5305.178446534515388 +2549.104199235389842 i
0,52,13	72.233551463649349 + 32.232229492441334 i	4940.771803753318636 +2738.230469681534032 i	0,52,15	69.953551177759635 + 35.906525901208731 i	4577.926309103008862 +2882.397778116962400 i
0,52,17	67.585658658663021 + 39.424338066609302 i	4218.593812843627347 +2985.747108564369682 i	0,52,19	65.129071412772191 + 42.800877056543257 i	3864.500681962138060 +3051.555756990637747 i
0,52,21	62.582372811193075 + 46.047982861298351 i	3517.210907606432556 +3082.505959372263987 i	0,52,23	59.943525299889565 + 49.175053261829007 i	3178.170346384511504 +3080.852996817984604 i
0,52,25	57.209831891653351 + 52.189648264459912 i	2848.739553853732559 +3048.536080108988699 i	0,52,27	54.377865636639648 + 55.097898408430552 i	2530.219536637673173 +2987.254680770357027 i
0,52,29	51.443362153433583 + 57.904788077327702 i	2223.873142789051144 +2898.522987510030362 i	0,52,31	48.401065162622828 + 60.614355201038478 i	1930.943962950497280 +2783.709970979344234 i
0,52,33	45.244507843320463 + 63.229832198961830 i	1652.674191763733461 +2644.069671746816311 i	0,52,35	41.965701595567367 + 65.753743169903501 i	1390.322755480063734 +2480.764658875695659 i
0,52,37	38.554684530743224 + 68.187965915334658 i	1145.185108546486617 +2294.884583887150256 i	0,52,39	34.998846708935005 + 70.533762526333447 i	918.616486250687444 +2087.461076636023336 i
0,52,41	31.281879847324202 + 72.791777577747922 i	712.061240921750709 +1859.479697269159942 i	0,52,43	27.382051898305793 + 74.961996795814500 i	527.092603940051958 +1611.889058744382737 i
0,52,45	23.269160673790076 + 77.043648224990591 i	365.470855167654463 +1345.606119705057608 i	0,52,47	18.898591352518590 + 79.035004157198799 i	229.236377026162728 +1061.513535990285845 i
0,52,49	14.197888786376293 + 80.932978232973042 i	120.877140967905291 + 760.433188198648054 i	0,52,51	9.028037699475048 + 82.732188824619470 i	43.690158498801921 + 442.995677212750763 i
0,52,53	2.991953914950271 + 84.421866447921559 i	2.911767269542211 + 108.556397660481352 i	0,53,1	85.580381339574330 + 4.694508736569679 i	7311.461256793471875 + 605.688373934020774 i
0,53,3	83.863917938536034 + 10.426419707771220 i	6984.711460014687873 +1214.973066174925862 i	0,53,5	82.043174047699097 + 15.471153276427563 i	6637.669555329354807 +1685.074567224808789 i
0,53,7	80.126812402027667 + 20.096685451138995 i	6278.451531229639841 +2063.749297108891824 i	0,53,9	78.119459569467594 + 24.415939787339312 i	5911.979616083935980 +2371.733823850465797 i
0,53,11	76.023812812646014 + 28.491607696624033 i	5541.782344596181247 +2620.872617470313344 i	0,53,13	73.841371330893210 + 32.363582043721344 i	5170.616517076538003 +2818.933243800779110 i
0,53,15	71.572767269007116 + 36.059422332267559 i	4800.756738389569364 +2971.463165293574093 i	0,53,17	69.217932398652053 + 39.599234977085096 i	4434.153725187810778 +3082.663559007944059 i
0,53,19	66.776181720693359 + 42.998260567238233 i	4072.530496192027385 +3155.856529484804014 i	0,53,21	64.246247357385073 + 46.268374085097676 i	3717.445545261708048 +3193.758411026938575 i
0,53,23	61.626277377325650 + 49.419014127264070 i	3370.337023129976842 +3198.650757735939351 i	0,53,25	58.913805315896724 + 52.457787930483313 i	3032.55540036860748 +3172.49309580892259 i
0,53,27	56.105690914625242 + 55.390880548136359 i	2705.388933698734945 +3117.000507053790898 i	0,53,29	53.198028511217387 + 58.223339376747340 i	2390.084635052887734 +3033.698945979958353 i
0,53,31	50.186015204084846 + 60.959275530162607 i	2087.866577500251879 +2923.965904965396930 i	0,53,33	47.063765206141987 + 63.602007089035183 i	1799.952923498350401 +2789.061124397563162 i
0,53,35	43.824048089569949 + 66.154159515835630 i	1527.572877157333323 +2630.150350909604640 i	0,53,37	40.457914114583289 + 68.617732293474717 i	1271.984790491847889 +2448.324113816319368 i
0,53,39	36.954144019007423 + 70.994136330617110 i	1034.496913712968990 +2244.612818137297836 i	0,53,41	33.298411663655841 + 73.284202655196395 i	816.492879579352802 +2019.998957771028472 i
0,53,43	29.471948018942850 + 75.488158139046291 i	619.465224183488544 +1775.426770216723298 i	0,53,45	25.449272324416828 + 77.605556392425711 i	445.062730776703688 +1511.808908641716926 i
0,53,47	21.193999875280589 + 79.635136702020205 i	295.162865696989456 +1230.027912832712445 i	0,53,49	16.650109397945730 + 81.574546694079146 i	171.994256541125822 + 930.924351559102774 i
0,53,51	11.720042764997061 + 83.419751783191373 i	78.375459701802910 + 615.237155788530458 i	0,53,53	6.187429409936332 + 85.163474788117426 i	18.309801695356029 + 283.269260040844244 i
0,54,1	87.151622235387052 + 4.704998047680212 i	7582.787398763106467 + 618.713831492730719 i	0,54,3	85.437376096498213 + 10.455694812865673 i	7250.665923721867330 +1242.876827475698292 i
0,54,5	83.620397785492187 + 15.519574628903102 i	6897.945801224146635 +1725.700780355574352 i	0,54,7	81.709244128839643 + 20.164629598902224 i	6532.770285215458898 +2115.776977784725659 i
0,54,9	79.708508575471967 + 24.503785954329228 i	6160.076095795423498 +2434.192974743727063 i	0,54,11	77.620894008400938 + 28.599741546883308 i	5783.400078760226279 +2692.987144002231162 i
0,54,13	75.447933311967859 + 32.492401092842869 i	5405.503450561893260 +2900.049557285000901 i	0,54,15	73.190316695586901 + 36.209341250484513 i	5028.662365632539149 +3061.011493895030071 i
0,54,17	70.848057854173661 + 39.770690654437409 i	4654.826732300938602 +3180.134510024903193 i	0,54,19	68.420579024119419 + 43.191717082012815 i	4285.716645746491849 +3260.785730485586555 i
0,54,21	65.906747972068231 + 46.484327762264400 i	3922.885641257345469 +3305.715827367393558 i	0,54,23	63.304881628993563 + 49.657998883169640 i	3567.764838115326711 +3317.232897913901979 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,54,25	60.612722488047488 + 52.720381114115874 i	3221.69546270778346 +3297.31708828540741 i	0,54,27	57.827388949903550 + 55.677709519587026 i	2885.954071772715452 +3247.699434028510950 i
0,54,29	54.945297124276308 + 58.535089117508271 i	2561.773159972394751 +3169.918043085021054 i	0,54,31	51.962047934453658 + 61.296697667774474 i	2250.358955469820557 +3065.359367182602909 i
0,54,33	48.872268735432677 + 63.965930857623249 i	1952.907738043977815 +2935.289339904029930 i	0,54,35	45.669391817252453 + 66.545505394979031 i	1670.621799179376467 +2780.877438713124775 i
0,54,37	42.345341094855193 + 69.037529426968320 i	1404.726139993549850 +2603.215682430880406 i	0,54,39	38.890079129513040 + 71.443545431793723 i	1156.487171561716423 +2403.333904493762020 i
0,54,41	35.290931392819722 + 73.764547174704134 i	927.235110966208822 +2182.212168887235293 i	0,54,43	31.531535437171907 + 76.000968478026863 i	718.392634973179952 +1940.790783407851222 i
0,54,45	27.590115341213718 + 78.152636013559359 i	531.514086056368228 +1679.977830145258167 i	0,54,47	23.436435570755737 + 80.218668119584279 i	368.343176357229424 +1400.653033750271334 i
0,54,49	19.025858914412588 + 82.197278850560380 i	230.905637959465508 +1103.663607161081700 i	0,54,51	14.285918406183521 + 84.085384863983933 i	121.676343750278178 + 789.795593588808401 i
0,54,53	9.077589762188355 + 85.877697042439976 i	43.940423197544945 + 459.637968441032285 i	0,54,55	3.003685181095779 + 87.563729387054238 i	2.923422589611942 + 112.464734691477844 i
0,55,1	88.722848483214932 + 4.715296625190796 i	7859.049729541505258 + 631.772202014721529 i	0,55,3	87.010747939570649 + 10.484437290352730 i	7521.563123143951998 +1270.872060748688000 i
0,55,5	85.197413377648374 + 15.567109011798436 i	7163.175780966783350 +1766.478270755995027 i	0,55,7	83.291297774616325 + 20.231319698790077 i	6792.057893506069377 +2168.017135904357929 i
0,55,9	81.296960743237292 + 24.589997808782083 i	6413.160577276766162 +2496.927295854798103 i	0,55,11	79.217108337860012 + 28.705846821662110 i	6030.029021647183981 +2765.441135627045696 i
0,55,13	77.053303185206246 + 32.618782154933093 i	5645.428986819153579 +2981.571362283386861 i	0,55,15	74.806288093216054 + 36.356396153471039 i	5261.638389052572165 +3151.033222938580366 i
0,55,17	72.476151505139569 + 39.938837887430914 i	4880.606625125223218 +3278.148826269013625 i	0,55,19	70.062413687071967 + 43.381400204446244 i	4504.051254543970572 +3366.330514358480741 i
0,55,21	67.564066335274049 + 46.696020092814955 i	4133.521345406684304 +3418.363519623529100 i	0,55,23	64.979580337681483 + 49.892208419077967 i	3770.441599113109987 +3436.582733090632701 i
0,55,25	62.306888082363436 + 52.977655867307845 i	3416.144759973002238 +3422.989202905353977 i	0,55,27	59.543342025358996 + 55.958643479354834 i	3071.89663190985857 +3379.33023576695450 i
0,55,29	56.685647890443398 + 58.840329092227208 i	2738.916375253180377 +3307.156450581244371 i	0,55,31	53.729767717971093 + 61.626951328002889 i	2418.393863200310989 +3207.863661391838377 i
0,55,33	50.670784166726555 + 64.321976412922922 i	2111.505391757473262 +3082.724448890855456 i	0,55,35	47.502712040839896 + 66.928203509594780 i	1819.428797225878725 +2932.912530899042849 i
0,55,37	44.218234464426359 + 69.447838255927934 i	1543.358971997047092 +2759.521982098279750 i	0,55,39	40.808326719750610 + 71.882540115228892 i	1284.524869948731293 +2563.582678860439501 i
0,55,41	37.261705015401262 + 74.233445913846325 i	1044.209397543984323 +2346.072883000338607 i	0,55,43	33.563988515540899 + 76.501168744308799 i	823.774215109184070 +2107.929508038591393 i
0,55,45	29.696363083603654 + 78.685767221092974 i	624.692703447370718 +1850.056200782487394 i	0,55,47	25.633312551387252 + 80.786672995891948 i	448.596846993658971 +1573.328655316330924 i
0,55,49	21.338426893005710 + 82.802549851791568 i	297.349274636822031 +1278.594757682232967 i	0,55,51	16.755669844759169 + 84.731021840229403 i	173.165385203827384 + 966.661085515137302 i
0,55,53	11.787476273518159 + 86.568099093590405 i	78.851788240360150 + 638.230152178652816 i	0,55,55	6.217451922644292 + 88.306666265207767 i	18.400874223698477 + 293.554766564019815 i
0,56,1	90.294060808895424 + 4.725411229324424 i	8140.248203769134307 + 644.862892064452581 i	0,56,3	88.584037707444836 + 10.512666076091808 i	7797.402803939823571 +1298.957114699475142 i
0,56,5	86.774231071556500 + 15.613788165317455 i	7433.358889135653044 +1807.404284030042845 i	0,56,7	84.872992071714165 + 20.296800952475969 i	7056.313263556318816 +2220.465863227784666 i
0,56,9	82.884845916342329 + 24.674634725335071 i	6671.231341813093422 +2559.931668445225569 i	0,56,11	80.812499653368263 + 28.809997862075885 i	6281.666680458738483 +2838.228199218770442 i
0,56,13	78.657542102715027 + 32.742815438585545 i	5890.389696981038469 +3063.490918761320700 i	0,56,15	76.420763738028427 + 36.500694151155251 i	5499.680262236695853 +3241.519182871325484 i
0,56,17	74.102321280995270 + 40.103801881391192 i	5111.487532563814057 +3376.695814397197864 i	0,56,19	71.701824746726402 + 43.567454620832962 i	4727.526886590430877 +3472.478556281148030 i
0,56,21	69.218379262108769 + 46.903616858502851 i	4349.343374297676746 +3531.687408386871539 i	0,56,23	66.650596375684643 + 50.121831501786041 i	3978.355838656313153 +3556.684288434629252 i
0,56,25	63.996581460923785 + 53.229826165072339 i	3615.889234443166060 +3549.491408155840872 i	0,56,27	61.253899349355663 + 56.233924243266500 i	3263.19946974381537 +3511.87263925929665 i
0,56,29	58.419517262388894 + 59.139332100147946 i	2921.493435161113212 +3445.391441961780856 i	0,56,31	55.489721355853348 + 61.950344082747044 i	2591.945981282106459 +3351.453364290511672 i
0,56,33	52.460000015510431 + 64.670490725924566 i	2275.715105520794168 +3231.338052208174304 i	0,56,35	49.324882677611953 + 67.302645967297735 i	1973.956350104910371 +3086.223933368038161 i
0,56,37	46.077716269955587 + 69.849103280925846 i	1687.837318882871614 +2917.207661839766843 i	0,56,39	42.710350397965883 + 72.311626682147988 i	1418.553269641428187 +2725.319730933546907 i
0,56,41	39.212683308906975 + 74.691479565393522 i	1167.344952306971891 +2511.537204380347248 i	0,56,43	35.571985478094405 + 76.989431586706601 i	935.520320794115491 +2276.794171040117362 i
0,56,45	31.771848445778307 + 79.205740518183347 i	724.482633384310816 +2021.990192086367188 i	0,56,47	27.790459256423574 + 81.340100914471762 i	535.769201415831958 +1747.996513677750272 i
0,56,49	23.597554609521865 + 83.391541733247024 i	371.108704683889130 +1455.658718577595891 i	0,56,51	19.148479202368510 + 85.358233109294815 i	232.513499570310245 +1145.791231179334318 i
0,56,53	14.370757184527511 + 87.237102947631030 i	122.446425826546999 + 819.146617248174493 i	0,56,55	9.125356868997381 + 89.022955367643121 i	44.181632756623209 + 476.274448693118927 i
0,56,57	3.014990343104489 + 90.705574745163034 i	2.934651421025816 + 116.369787582087241 i	0,57,1	91.865259890627200 + 4.735348268336706 i	8426.382778317858538 + 657.985329056873525 i
0,57,3	90.157249362898852 + 10.540399116438644 i	8078.184724134674980 +1327.130396306518833 i	0,57,5	88.350860442306612 + 15.659642160503164 i	7708.494549800971072 +1848.476163863656211 i
0,57,7	86.454344513209506 + 20.361116167127449 i	7325.535356416637114 +2273.119392182006777 i	0,57,9	84.472191941538554 + 24.757752906856791 i	6934.286755903420271 +2623.201160281120792 i
0,57,11	82.407108831740572 + 28.912264999560893 i	6538.310687701079587 +2911.342177832889774 i	0,57,13	80.260706997621540 + 32.864586233006875 i	6140.382327437097956 +3145.800777084190941 i
0,57,15	78.033820119207689 + 36.642336438821786 i	5742.783675734330245 +3332.460554129370848 i	0,57,17	75.726667350699387 + 40.265700829414328 i	5347.463896456511065 +3475.765196988197658 i
0,57,19	73.338940990708807 + 43.750016784828809 i	4956.136512393037265 +3579.218020772326781 i	0,57,21	70.869850163236379 + 47.107274240000642 i	4570.342966566448013 +3645.673985839313445 i
0,57,23	68.318135182211975 + 50.347045749168238 i	4191.496754701480313 +3677.522253367472786 i	0,57,25	65.682059358214445 + 53.477093066667841 i	3820.915672886360426 +3676.806441722018917 i
0,57,27	62.959380724137851 + 56.503778674735898 i	3459.846510869454050 +3645.307260125593984 i	0,57,29	60.147303312184462 + 59.432353615030557 i	3109.48485065156638 +3584.60131585639283 i
0,57,31	57.242405179471781 + 62.267163379693031 i	2770.991715967251821 +3496.104234292743513 i	0,57,33	54.240537701884321 + 65.011797296170769 i	2445.508307456958846 +3381.103112444036014 i
0,57,35	51.136687109822340 + 67.669197334073829 i	2134.169783215182179 +3240.781520732401987 i	0,57,37	47.924783964884412 + 70.241736410824473 i	1838.118926183439044 +3076.239176263030004 i
0,57,39	44.597437824217593 + 72.731272416667380 i	1558.520549475266989 +2888.507723184127371 i	0,57,41	41.145559995437708 + 75.139181362659443 i	1296.577636471322194 +2678.563599985401652 i
0,57,43	37.557811578936750 + 77.466370623551910 i	1053.550546148962667 +2447.338643251315261 i	0,57,45	33.819765090060820 + 79.713270583971635 i	830.781261215514746 +2195.728786687880074 i
0,57,47	29.912568097106757 + 81.879799215522510 i	629.725899589692290 +1924.600848120096771 i	0,57,49	25.810674671717397 + 83.965287436788671 i	452.001213830404708 +1634.796695199592023 i
0,57,51	21.477653956437337 + 85.968339101459094 i	299.456298071576555 +1327.132227694673929 i	0,57,53	16.857457969574828 + 87.886564820789971 i	174.294411225398761 +1002.382370852178838 i
0,57,55	11.852516553252110 + 89.716023519842693 i	79.311134694055497 + 661.215361605648582 i	0,57,57	6.246413016358469 + 91.449756893484505 i	18.488711144999744 + 303.836017324267232 i
0,58,1	93.436446362838367 + 4.745113822484928 i	8717.453412145539914 + 671.138960181271045 i	0,58,3	91.730386614229744 + 10.567653435891874 i	8363.908653319813311 +1355.390367797228691 i
0,58,5	89.927310447480480 + 15.704699514074093 i	7988.582214607600690 +1889.691346887032296 i	0,58,7	88.035371454903085 + 20.424305921412110 i	7599.723183215349309 +2325.974088407788258 i
0,58,9	86.059024835270407 + 24.839405606360057 i	7202.325265592321557 +2686.731015617216599 i	0,58,11	84.000974026405089 + 29.012714839458646 i	6799.958792708787769 +2984.777137824408783 i
0,58,13	81.862850948414319 + 32.984175260344493 i	6395.403787739397558 +3228.493761898286721 i	0,58,15	79.645528449774531 + 36.781418726885001 i	5990.944540322833745 +3423.848847329035834 i
0,58,17	77.349282824951544 + 40.424646430321168 i	5588.530448894322944 +3575.347088494025229 i	0,58,19	74.973881910895244 + 43.929215536877017 i	5189.873478618174886 +3686.537532736264893 i
0,58,21	72.518630331955535 + 47.307139554340687 i	4796.511842650288600 +3760.310281082156507 i	0,58,23	69.982386496691888 + 50.568018506382366 i	4409.854157473122541 +3799.081940198935172 i
0,58,25	67.363558219732028 + 53.719645756866328 i	4031.211636426953646 +3804.917761244855683 i	0,58,27	64.660079718409094 + 56.768419924860403 i	3661.822658135303755 +3779.615544067820338 i
0,58,29	61.869370074335450 + 59.719633268150133 i	3302.87236625091509 +3724.76532646195892 i	0,58,31	58.988271072297046 + 62.577678326915326 i	2955.509034445290126 +3641.793128898954365 i
0,58,33	56.012960043817557 + 65.346198318086081 i	2620.858410225157513 +3531.993855865067417 i	0,58,35	52.938830429565741 + 68.028197295028320 i	2300.036963227391880 +3396.556620221565026 i
0,58,37	49.760328564949987 + 70.626120282309927 i	1994.164826328309346 +3236.584648943578941 i	0,58,39	46.470728609638407 + 73.141909814779069 i	1704.379197053646521 +3053.111232427218965 i
0,58,41	43.061816632188169 + 75.577042621842850 i	1431.849061237005799 +2847.112722290759848 i	0,58,43	39.523435811318819 + 77.932547995720100 i	1177.792343547405380 +2619.519260453570496 i
0,58,45	35.842809556135165 + 80.209007152658188 i	943.497065790569309 +2371.223659583730296 i	0,58,47	32.003490154149290 + 82.406529992138445 i	730.348993479441901 +2103.088567468602378 i
0,58,49	27.983633289864191 + 84.524700911998579 i	539.870004287122811 +1815.951574164183967 i	0,58,51	23.752952513216982 + 86.562476061082592 i	373.775075889973891 +1510.626810965999312 i
0,58,53	19.266777957444475 + 88.517993232576757 i	234.064301617420114 +1187.898247659475146 i	0,58,55	14.452627245415805 + 90.388197983354502 i	123.189425354590199 + 848.487068604481806 i
0,58,57	9.171462524582353 + 92.167987540732611 i	44.414416120441913 + 492.905435333466130 i	0,58,59	3.025898945729321 + 93.847404210791240 i	2.945483514959016 + 120.271696303634897 i
0,59,1	95.007620819682202 + 4.754713665993594 i	9013.460066162360818 + 684.323251397772879 i	0,59,3	93.303452935527602 + 10.594445199062706 i	8654.574371916729433 +1383.735543829414382 i
0,59,5	91.503589476632300 + 15.748987293561024 i	8273.621361012876150 +1931.047357889301111 i	0,59,7	89.616088207327351 + 20.486408717366796 i	7878.875801928692454 +2379.026443823946465 i
0,59,9	87.645368933635638 + 24.919643329826364 i	7475.345391272519919 +2750.516645886717924 i	0,59,11	85.594130894022499 + 29.111410519911406 i	7066.608853886563338 +3058.527356889273960 i
0,59,13	83.464023505624013 + 33.101658996903723 i	6655.451139555892041 +3311.562957189561075 i	0,59,15	81.255955123294740 + 36.918031632090134 i	6244.158971760563873 +3515.675884936646526 i
0,59,17	78.970254382024862 + 40.580744359257537 i	5834.682191609667825 +3675.431973028302309 i	0,59,19	76.606758550237728 + 44.105172665813541 i	5428.731480615821056 +3794.426150773957488 i
0,59,21	74.164860081947936 + 47.503351921903025 i	5027.842168478133317 +3875.583828250708393 i	0,59,23	71.643525883283658 + 50.784907636176747 i	4633.418421710386610 +3921.349246176391262 i
0,59,25	69.041296246948889 + 53.957662482168502 i	4246.765397841086269 +3933.809499257682546 i	0,59,27	66.356266422898585 + 57.028048543339082 i	3869.113709205112627 +3914.779713431619257 i
0,59,29	63.586051288366143 + 60.001396171182222 i	3501.638851254380370 +3865.863620214767707 i	0,59,31	60.727731611450977 + 62.882141276857233 i	3145.47732031256282 +3788.49792967519761 i
0,59,33	57.777778429442129 + 65.673976592300050 i	2801.740617240342090 +3683.985683592841724 i	0,59,35	54.731949643584556 + 68.379962984863894 i	2471.528035021747201 +3553.521906681473411 i
0,59,37	51.585149537588215 + 71.002611141062943 i	2155.938975693994053 +3398.213748819670400 i	0,59,39	48.331236745676840 + 73.543940212437164 i	1856.085497381604000 +3219.096605796793483 i
0,59,41	44.962757803531126 + 76.005517424617736 i	1573.105847473783797 +3017.147245562276112 i	0,59,43	41.470569112105863 + 78.388480616477793 i	1308.179912886761031 +2793.294648795365447 i
0,59,45	37.843284467100418 + 80.693543754579437 i	1062.547871014616248 +2548.429027090948694 i	0,59,47	34.066436423935052 + 82.920983165332984 i	837.533984697061896 +2283.409242997581714 i
0,59,49	30.121139925590910 + 85.070598744759636 i	634.578735904061091 +1999.068517678424996 i	0,59,51	25.981823978583211 + 87.141638055531303 i	455.285002382407583 +1696.217613325375851 i
0,59,53	21.612040605668291 + 89.132657724285338 i	301.489479906292786 +1375.642784397545938 i	0,59,55	16.955732774877333 + 91.041263031756714 i	175.384244344575393 +1038.089376607639906 i
0,59,57	11.915326864622733 + 92.863564571489007 i	79.754662550035036 + 684.193266345091615 i	0,59,59	6.274384757748132 + 94.592756120151051 i	18.573532995783747 + 314.113208443186238 i
0,60,1	96.578783818201941 + 4.764153287213904 i	9314.402703107745765 + 697.537686500487553 i	0,60,3	94.876451585030708 + 10.620789767529450 i	8950.181670499305255 +1412.164488860982601 i
0,60,5	93.079705396053456 + 15.792531213710356 i	8563.611490664485245 +1972.541805355889664 i	0,60,7	91.196509118889011 + 20.547461119550821 i	8162.992314410492327 +2432.273070164982073 i
0,60,9	89.231247027691481 + 24.998514021881469 i	7753.345722911652047 +2814.553621035718152 i	0,60,11	87.186612798575538 + 29.208411948610948 i	7338.258831596036543 +3132.587312947585815 i
0,60,13	85.064270985363351 + 33.217109966508218 i	6920.521586562613265 +3395.001692414025001 i	0,60,15	82.865162123166428 + 37.052261034249632 i	6502.423276868565154 +3607.933784280512100 i
0,60,17	80.589662826885217 + 40.734094696086295 i	6085.914377213553053 +3776.010683817948120 i	0,60,19	78.237674256621105 + 44.278003419079688 i	5672.704537483634340 +3902.873342541589409 i
0,60,21	75.808669755959755 + 47.696042870984094 i	5264.326522633075911 +3991.482637137501115 i	0,60,23	73.301716076064523 + 50.997862233373688 i	4862.180443651690439 +4044.310618600029102 i
0,60,25	70.715475191920234 + 54.191311394681918 i	4467.565885327308933 +4063.466421863658070 i	0,60,27	68.048189853376087 + 57.282853476408214 i	4081.706283008784794 +4050.782718328595365 i
0,60,29	65.297653638750333 + 60.277854099858821 i	3705.768203218757662 +4007.877178018091399 i	0,60,31	62.461164480719319 + 63.180789237059699 i	3340.877246225713861 +3936.197473989881018 i
0,60,33	59.535458911075487 + 65.995397218589204 i	2988.131753245564141 +3837.055090972982725 i	0,60,35	56.516622239845411 + 68.724791038432699 i	2648.615193756261760 +3711.651307432373414 i
0,60,37	53.399967120029906 + 71.371541356194285 i	2323.407943489991339 +3561.097578249148683 i	0,60,39	50.179868830726811 + 73.937736919281221 i	2013.599100210231882 +3386.431829974878383 i
0,60,41	46.849539086285617 + 76.425026608809759 i	1720.299009394961786 +3188.631715678237015 i	0,60,43	43.400709308862012 + 78.834645403194571 i	1444.653295711104420 +2968.625559442966733 i
0,60,45	39.823175279908718 + 81.167424853991506 i	1187.859458000659060 +2727.301485382760802 i	0,60,47	36.104119947617100 + 83.423786785816901 i	951.187574529352560 +2465.515017268821339 i
0,60,49	32.227061988924262 + 85.603716298920460 i	736.007573381524935 +2184.095178629463589 i	0,60,51	28.170130464168782 + 87.706705670761409 i	543.827281673840730 +1883.848660950122849 i
0,60,53	23.903019613666956 + 89.731651838002307 i	376.349134781747466 +1565.560500955253701 i	0,60,55	19.381047912635431 + 91.676669656443082 i	235.561939939369040 +1229.986278383869376 i
0,60,57	14.531728355289772 + 93.538727302808041 i	123.907174153910233 + 877.817667643124082 i	0,60,59	9.216017873910932 + 95.312814351626486 i	44.639339061777981 + 509.531215895992716 i
0,60,61	3.036437576616156 + 96.989219261227944 i	2.955945628998174 + 124.170590283180204 i	0,61,1	98.149935881200477 + 4.773437907152312 i	9620.281287436895582 + 710.781766241845389 i
0,61,3	96.449385621771484 + 10.646701752074929 i	9250.730349168681641 +1440.675814692440781 i	0,61,5	94.655665589340401 + 15.835355725010000 i	8858.552127905191810 +2014.172377301830011 i
0,61,7	92.776647651134056 + 20.607497882739988 i	8452.071863656594360 +2485.710692952315640 i	0,61,9	90.816680485763840 + 25.076063236035285 i	8036.324915661858540 +2878.837661447231312 i
0,61,11	88.778450995524864 + 29.303776019641539 i	7614.906781623457391 +3206.951673795531406 i	0,61,13	86.663636733629502 + 33.330597008865411 i	7190.612465178990533 +3478.803529604886535 i
0,61,15	84.473207390212707 + 37.184188402117215 i	6765.733940801714198 +3700.614941783629092 i	0,61,17	82.207583591883292 + 40.884792316070772 i	6342.222492080407392 +3877.074384164121057 i
0,61,19	79.866725355664570 + 44.447816967142955 i	5921.786969387569115 +4011.868961961807599 i	0,61,21	77.450180622538880 + 47.885336886917734 i	5505.957866585781630 +4107.995166079790579 i
0,61,23	74.957108169607253 + 51.207023272232014 i	5096.131602188464058 +4167.953022697385677 i	0,61,25	72.386281937513431 + 54.420751314585317 i	4693.602631942189873 +4193.873890770334583 i
0,61,27	69.736080054160738 + 57.533012965618013 i	4299.587753919860006 +4187.608191839211031 i	0,61,29	67.004459571717149 + 60.549206556087007 i	3915.245262062675920 +4150.787762426441986 i
0,61,31	64.188916276828877 + 63.473845130614855 i	3541.690661255285704 +4084.871492765105359 i	0,61,33	61.286427412500032 + 66.310709100845628 i	3180.010115511932781 +3991.179594225387518 i
0,61,35	58.293373426921185 + 69.062959402588859 i	2831.27248634869284 +3870.91991589256577 i	0,61,37	55.205432591506110 + 71.733221625591597 i	2496.540643448579885 +3725.208571489742098 i
0,61,39	52.017439025644698 + 74.323647941858823 i	2176.882651292451101 +3555.086412469291645 i	0,61,41	48.723189541419956 + 76.835961195534480 i	1873.383436826955176 +3361.532412939431197 i
0,61,43	45.315176367308894 + 79.271483700843675 i	1587.157627962923925 +3145.474714514212337 i	0,61,45	41.784208533353187 + 81.631151757831759 i	1319.364446878022591 +2907.799849904196435 i
0,61,47	38.118859042986088 + 83.915515311420634 i	1071.225907614855714 +2649.360474328827877 i	0,61,49	34.304626088849922 + 86.124720128630742 i	844.050235853805702 +2370.981728654932340 i
0,61,51	30.322596736594065 + 88.258466612092448 i	639.263685931795635 +2073.466036297732899 i	0,61,53	26.147178851680071 + 90.315933288671431 i	458.456443528542081 +1757.595433712197064 i
0,61,55	21.741910419546272 + 92.295639972010974 i	303.453803195607918 +1424.128597068216322 i	0,61,57	17.050727539032859 + 94.195193113564386 i	176.437503202592836 +1073.783139642592459 i
0,61,59	11.976054352659938 + 96.010756921304903 i	80.183419941578038 + 707.164299640665490 i	0,61,61	6.301432133142784 + 97.735672235823245 i	18.655538571303595 + 324.386520079403454 i
0,62,1	99.721077499846785 + 4.782572496523834 i	9931.095785216022705 + 724.055007513231999 i	0,62,3	98.022257920696504 + 10.672195060749155 i	9556.220216975938456 +1469.268178168254053 i
0,62,5	96.231476994219989 + 15.877484095097367 i	9158.442818392551999 +2055.936837376891617 i	0,62,7	94.356516446997887 + 20.666552069275667 i	8746.113631281936250 +2539.336145863340789 i
0,62,9	92.401689364204827 + 25.152334291002447 i	8324.281685815509263 +2943.364630405433218 i	0,62,11	90.369674798288273 + 29.397556812397244 i	7896.550849171469054 +3281.615287460496347 i
0,62,13	88.262161364749844 + 33.442185525459813 i	7465.721236061932359 +3562.962251371499860 i	0,62,15	86.080145153519652 + 37.313891091567839 i	7034.087615387162259 +3793.712018308665847 i
0,62,17	83.824087186132886 + 41.032927246795062 i	6603.602240706336488 +3978.614549775237720 i	0,62,19	81.494001752641068 + 44.614716827020509 i	6175.973376892663509 +4121.403228115311322 i
0,62,21	79.089505675357117 + 48.071351911817260 i	5752.729517651752758 +4225.110296895189094 i	0,62,23	76.609842675769144 + 51.412524194230400 i	5335.26372369900582 +4292.26391198699275 i
0,62,25	74.053889893320729 + 54.646132420597539 i	4924.865730009269100 +4325.017828343985457 i	0,62,27	71.420149946023798 + 57.778695359299782 i	4522.746192674710983 +4325.240408875720277 i
0,62,29	68.706729754030420 + 60.815641723588890 i	4130.055733358165526 +4294.577869263212961 i	0,62,31	65.911305807421328 + 63.761518925738450 i	3747.900490875951164 +4234.500553594079065 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,62,33	63.031074201969830 + 66.620146289480303 i	3377.355342608868341 +4146.337663578462525 i	0,62,35	60.062682289425275 + 69.394728943362040 i	3019.47563774594391 +4031.30391300570636 i
0,62,37	57.002136901821508 + 72.087942919013273 i	2675.308101132093270 +3890.520402486102284 i	0,62,39	53.844681433165277 + 74.701998363148007 i	2345.901476015281787 +3725.031273767408948 i
0,62,41	50.584629999448381 + 77.238685352490961 i	2032.317456781427381 +3535.817226996376576 i	0,62,43	47.215140410076906 + 79.699405055007432 i	1735.642517357695851 +3323.806664866420761 i
0,62,45	43.727896839495727 + 82.085187564575961 i	1457.001384112179721 +3089.885001359003581 i	0,62,47	40.112654085056846 + 84.396696367484921 i	1197.573176864782909 +2834.902494966011545 i
0,62,49	36.356560173321597 + 86.634217755707752 i	958.611739419502555 +2559.680793321475903 i	0,62,51	32.443104909092618 + 88.797631068635894 i	741.472595104331845 +2265.018139915188385 i
0,62,53	28.350394702702232 + 90.886351376817345 i	547.650725552457402 +1951.692735664965767 i	0,62,55	24.048107693664338 + 92.899227623403050 i	378.837040987719035 +1620.462600291514036 i
0,62,57	19.491553101845067 + 94.834359393396696 i	237.009925008669711 +1272.056759819884064 i	0,62,59	14.608240821835885 + 96.688741315456909 i	124.601324624679464 + 907.139057698856504 i
0,62,61	9.259123295823777 + 98.457454087526216 i	44.856912356142089 + 526.152051917266931 i	0,62,63	3.046630247305028 +100.131021194702143 i	2.966061914695278 + 128.066589518220297 i
0,63,1	101.292209136046978 + 4.791561791468704 i	10246.846164025519101 + 737.356942577466953 i	0,63,3	99.595071186422444 + 10.697282943150984 i	9866.651091388232089 +1497.940279024440088 i
0,63,5	97.807146136031093 + 15.918938483725002 i	9463.283127823127870 +2097.833021220731098 i	0,63,7	95.936127392796934 + 20.724655157058596 i	9045.116835190527127 +2593.146365465991039 i
0,63,9	93.986292507861563 + 25.227368414448687 i	8617.214807074451528 +3008.130527055591756 i	0,63,11	91.960311729010229 + 29.489805774326960 i	8183.189263324630701 +3356.573173200416932 i
0,63,13	89.859882976908594 + 33.551937705203947 i	7745.845476284125652 +3647.471849713940173 i	0,63,15	87.686026228794248 + 37.441442618876351 i	7307.481108422498437 +3887.217925518210450 i
0,63,17	85.439239599703470 + 41.178584994789489 i	6870.049531388035575 +4080.622952349695879 i	0,63,19	83.119587472231473 + 44.778801249194089 i	6435.258622089057098 +4231.466705659317995 i
0,63,21	80.726750347493365 + 48.254199800237075 i	6004.635124371245183 +4342.817311673752556 i	0,63,23	78.260050464307383 + 51.614491442829248 i	5579.569050140494255 +4417.231200892222660 i
0,63,25	75.718460232491211 + 54.867596876601006 i	5161.345789908526058 +4456.884685386403362 i	0,63,27	73.100596955010857 + 58.020059846919693 i	4751.170313195573726 +4463.664248335596312 i
0,63,29	70.404705227350632 + 61.077337329937258 i	4350.186119618780140 +4439.230683213106204 i	0,63,31	67.628626960823468 + 64.044008650914179 i	3959.490647869344230 +4385.066008657174280 i
0,63,33	64.769757753304461 + 66.923929182616845 i	3580.148298218609398 +4302.508662189703500 i	0,63,35	61.824987051537391 + 69.720344876724127 i	3213.20189820820588 +4192.78049562526758 i
0,63,37	58.790617967751459 + 72.435978197306213 i	2859.683251087244116 +4057.007900966251327 i	0,63,39	55.662260426887720 + 75.073092432156599 i	2520.623306317507286 +3896.238649268649624 i
0,63,41	52.434688071003947 + 77.633538971878195 i	2197.062459468047109 +3711.455542904293452 i	0,63,43	49.101644259565354 + 80.118790454960461 i	1890.061519925923903 +3503.587659526400785 i
0,63,45	45.655574175285658 + 82.529961351226888 i	1600.713989053872865 +3273.519740931023534 i	0,63,47	42.087245787010033 + 84.867816344968517 i	1330.160498643202573 +3022.100109514961787 i
0,63,49	38.385196777924570 + 87.132765525210715 i	1079.606633879407127 +2750.147342618521634 i	0,63,51	34.534895690326579 + 89.324843522407363 i	850.346039470412393 +2458.455752183883760 i
0,63,53	30.517405710527637 + 91.443666651536503 i	643.791968865389663 +2147.799404549661176 i	0,63,55	26.307116858565102 + 93.488356274371966 i	461.522952499792495 +1818.933704817435000 i
0,63,57	21.867555652527038 + 95.457403968452255 i	305.353763376966128 +1472.591586231877500 i	0,63,59	17.142653109145588 + 97.348422571574901 i	177.456552900501265 +1109.464583700640560 i
0,63,61	12.034832097357800 + 99.157631129315192 i	80.598354391538365 + 730.128852324238437 i	0,63,63	6.327613945250795 +100.878512547382016 i	18.734907687806533 + 334.656118248039434 i
0,64,1	102.863331224604266 + 4.800410308056041 i	10567.532392870336480 + 750.687118349095954 i	0,64,3	101.167827965769519 + 10.721978031279891 i	10182.022797794499638 +1526.690857870961281 i
0,64,5	99.382679158213207 + 15.959740011885778 i	9773.072640751739527 +2139.858833048323049 i	0,64,7	97.515491674625068 + 20.781837139068390 i	9349.080727420743642 +2647.138386289619120 i
0,64,9	95.570507641362966 + 25.301204875358732 i	8915.123107103738221 +3073.131479819526703 i	0,64,11	93.550387655340870 + 29.580571889063137 i	8474.820331943956262 +3431.820513094288799 i
0,64,13	91.456837347312472 + 33.659912731821329 i	8030.982872130518444 +3732.326515584745721 i	0,64,15	89.290898287951379 + 37.566912911562760 i	7585.911373838693180 +3981.125813163310340 i
0,64,17	87.053102667920797 + 41.321846844917815 i	7141.560463088080724 +4183.091644298860956 i	0,64,19	84.743561143608389 + 44.940163571684721 i	6699.637811325096663 +4342.050286636075725 i
0,64,21	82.362013151258068 + 48.433986735393148 i	6261.668644049538671 +4461.105871256102546 i	0,64,23	79.907853602283694 + 51.813044950929999 i	5829.04021003379876 +4542.84323939362002 i
0,64,25	77.380142990761826 + 55.085279401495818 i	5403.033902736236996 +4589.461411371471513 i	0,64,27	74.777604452726607 + 58.257257125126245 i	4984.849424601464307 +4602.865158219919067 i
0,64,29	72.098609302578737 + 61.334461426039589 i	4575.623658571443229 +4584.730036986400592 i	0,64,31	69.341151213219575 + 64.321501309634598 i	4176.445952691144157 +4536.549946939051551 i
0,64,33	66.502808094002802 + 67.222265604124090 i	3788.370967894809837 +4459.672790241123948 i	0,64,35	63.580689601561119 + 70.040038046595782 i	3412.429908508884182 +4355.327811112238123 i
0,64,37	60.571366880911398 + 72.777583939439552 i	3049.640759173543302 +4224.646975961763019 i	0,64,39	57.470779332368600 + 75.437215407305217 i	2701.018043654788016 +4068.681999991328667 i
0,64,41	54.274110577670598 + 78.020839925437485 i	2367.582577157336345 +3888.418137301995557 i	0,64,43	50.975621771917503 + 80.529995140431254 i	2050.371696371843882 +3684.785526155670141 i
0,64,45	47.568423944568124 + 82.965871747115912 i	1750.450525656688797 +3458.668655752325776 i	0,64,47	44.044160226112936 + 85.329325091119728 i	1468.924994016510027 +3210.914354712689601 i
0,64,49	40.392549827805539 + 87.620875021863242 i	1206.957601253480561 +2942.338562203864058 i	0,64,51	36.600710528589801 + 89.840692047801056 i	965.787435591306803 +2653.731993699717805 i
0,64,53	32.652107262777328 + 91.988565565582221 i	746.756863338865969 +2345.864589570178850 i	0,64,55	28.524827142235957 + 94.063845845337681 i	551.349076696284556 +2019.488176331982459 i
0,64,57	24.188534898413490 + 96.065343139971858 i	381.244357575435458 +1675.335598469175011 i	0,64,59	19.598532488254943 + 97.991148032033621 i	238.411431133473911 +1314.110969872833493 i
0,64,61	14.682327938140459 + 99.838284531632752 i	125.273372417028099 + 936.451816049586000 i	0,64,63	9.300869748056487 +101.601922901218416 i	45.067598676517683 + 542.768182159192065 i
0,64,65	3.056498714871942 +103.272811156844710 i	2.975854244473976 + 131.959805541890319 i	0,65,1	104.434444175189796 + 4.809122355685059 i	10893.154442096933053 + 764.045095718837160 i
0,65,3	102.740530659198186 + 10.746292377271525 i	10502.335169047246382 +1555.518694298534911 i	0,65,5	100.958081850111270 + 15.999908825634966 i	10087.810959497348449 +2182.012242447775861 i
0,65,7	99.094619829739216 + 20.838126615194437 i	9658.004592150215103 +2701.309336205746604 i	0,65,9	97.154351452196536 + 25.373881106090625 i	9218.005464343901622 +3138.363740230261101 i
0,65,11	95.139926914729841 + 29.669901831317937 i	8771.442436950646879 +3507.352644175933165 i	0,65,13	93.053058109229440 + 33.766166974714245 i	8321.131212454371052 +3817.520629136165098 i
0,65,15	90.894806103157450 + 37.690368538989588 i	7869.375502643824802 +4075.429057222161191 i	0,65,17	88.665734400363831 + 41.462790135226385 i	7418.131313368220617 +4286.012944512273862 i
0,65,19	86.365996437313342 + 45.098892544606471 i	6969.106279382454886 +4453.145173549147330 i	0,65,21	83.995386252636067 + 48.610813610015612 i	6523.824322228475467 +4579.965995245736849 i
0,65,23	81.553366105017474 + 52.008298586034599 i	6083.670192023632808 +4669.088789531587281 i	0,65,25	79.039078045708109 + 55.299307788447550 i	5649.921606394622358 +4722.735426908840964 i
0,65,27	76.451343033744024 + 58.490430003124672 i	5223.773387792720314 +4742.829123429180981 i	0,65,29	73.788649231149620 + 61.587173092596387 i	4806.356267549535005 +4731.060373700137461 i
0,65,31	71.049129827286748 + 64.594173706745451 i	4398.752062079447569 +4688.935150309507662 i	0,65,33	68.230529720926157 + 67.515351773818182 i	4002.006367008990765 +4617.811033672606754 i
0,65,35	65.330159400302165 + 70.354026069988805 i	3617.139580487732474 +4518.924897491957381 i	0,65,37	62.344833219517440 + 73.113001503710763 i	3245.156866252033069 +4393.414545967661979 i
0,65,39	59.270787772064388 + 75.794635189424525 i	2887.057552196255529 +4242.335931146774783 i	0,65,41	56.103573928874987 + 78.400886046944692 i	2543.844406753067688 +4066.677083750663769 i
0,65,43	52.837912893472883 + 80.933351045696909 i	2216.533233334972920 +3867.369561765186063 i	0,65,45	49.467501562805900 + 83.393290008120957 i	1906.163273856333490 +3645.297994734576150 i
0,65,47	45.984744170390584 + 85.781639880092385 i	1613.809029744934378 +3401.308137845028796 i	0,65,49	42.380372943146611 + 88.099018384487351 i	1340.594325062668304 +3136.213719025641240 i
0,65,51	38.642894870318266 + 90.345715782214810 i	1087.709816923811786 +2850.802235182145978 i	0,65,53	34.757752833659403 + 92.521671733599675 i	856.435837108820238 +2545.839685449957415 i
0,65,55	30.705989523549931 + 94.626431317816781 i	648.173712947917238 +2222.073921043777773 i	0,65,57	26.461979894585127 + 96.659068624390457 i	464.491233421411380 +1880.235569599051132 i
0,65,59	21.989241147624838 + 98.618054086418852 i	307.193429913919488 +1521.033459485999856 i	0,65,61	17.231700683744627 +100.501011110268365 i	178.443536688314964 +1145.134535186578887 i
0,65,63	12.091780848312181 +102.304214239839794 i	81.000325274802805 + 753.087278347537222 i	0,65,65	6.352983572414578 +104.021283520596910 i	18.811803520074481 + 344.922156377997567 i
0,66,1	106.005548374143117 + 4.817702049482779 i	11223.712283316217508 + 777.430448918848242 i	0,66,3	104.313181531259815 + 10.770237488297763 i	10827.588045037267875 +1584.422605100426381 i
0,66,5	102.533359672369656 + 16.039464155089735 i	10407.497703127988643 +2224.291281374277332 i	0,66,7	100.673521793449268 + 20.893550877080024 i	9971.887743846158628 +2755.656432094698175 i
0,66,9	98.737839666430020 + 25.445432815066805 i	9525.860805058591723 +3203.823677152992332 i	0,66,11	96.728952427556692 + 29.757840109779547 i	9073.054029963488574 +3583.165051067710010 i
0,66,13	94.648576912855972 + 33.870754164875081 i	8616.288382540718885 +3903.048750597294656 i	0,66,15	92.497791768145945 + 37.811872924651290 i	8157.870714573077748 +4170.121248818611093 i
0,66,17	90.277189278544441 + 41.601488509646003 i	7699.758527286087883 +4389.379425077022461 i	0,66,19	87.986962459520058 + 45.255072628130208 i	7243.659574947928377 +4564.742863597983160 i
0,66,21	85.626955988160887 + 48.784776375418907 i	6791.096673888542682 +4699.388043419917267 i	0,66,23	83.196694609506665 + 52.200360557493141 i	6343.45232073736315 +4795.95700359191233 i
0,66,25	80.695395991536643 + 55.509803378913269 i	5902.000854729393723 +4856.694598228075847 i	0,66,27	78.121971651795292 + 58.719713954016804 i	5467.932576080096624 +4883.542635981952117 i
0,66,29	75.475017684211480 + 61.835623081774237 i	5042.372493269573170 +4878.206712162846088 i	0,66,31	72.752795787095209 + 64.862193196716703 i	4626.395404866791978 +4842.205053082794620 i
0,66,33	69.953204149559923 + 67.803373182918207 i	4221.038458408208498 +4776.905117080907985 i	0,66,35	67.073736872589961 + 70.662514366116753 i	3827.311990824934128 +4683.551628594504109 i
0,66,37	64.111429617332831 + 73.442458345187219 i	3446.209250099259407 +4563.288475049105728 i	0,66,39	61.062787918057033 + 76.145603773970947 i	3078.715477539462881 +4417.176117768041877 i
0,66,41	57.923692847540337 + 78.773956883061331 i	2725.816768792395123 +4246.205666334441048 i	0,66,43	54.689276134752419 + 81.329168939674290 i	2388.509117917399635 +4051.310432834951371 i
0,66,45	51.353752831201206 + 83.812562679452554 i	2067.808081213665446 +3833.375554406035917 i	0,66,47	47.910193175519211 + 86.225148804089116 i	1764.759584911565071 +3593.246109613333374 i
0,66,49	44.350204491638934 + 88.567632760446244 i	1480.452566152411691 +3331.734028601320006 i	0,66,51	40.663474976445961 + 90.840411028108292 i	1216.034426655323614 +3049.623984020668559 i
0,66,53	36.837096127374977 + 93.043554794774451 i	972.730786742174814 +2747.678316960148095 i	0,66,55	32.854511404835542 + 95.176776740018838 i	751.871948647429804 +2426.640835679891552 i
0,66,57	28.693791461440213 + 97.239372711658447 i	554.930245552901624 +2087.238862838082241 i	0,66,59	24.324589888761604 + 99.230122021867757 i	383.576125763761411 +1730.181708941667011 i
0,66,61	19.702203037139611 +101.147111387062111 i	239.769338045659822 +1356.150050046136130 i	0,66,63	14.754138055112550 +102.987396408291843 i	125.924675631022211 + 965.756462747723731 i
0,66,65	9.341339908782633 +104.746235115883877 i	45.271818441419882 + 559.379825335442774 i	0,66,67	3.066062754962825 +106.414590162577483 i	2.985342488918528 + 135.850342262694085 i
0,67,1	107.576644186119651 + 4.826153321787002 i	11559.205889331991784 + 790.842764925783854 i	0,67,3	105.885782720159412 + 10.793824358882969 i	11157.78127229850543 +1613.40144260063380 i
0,67,5	104.108517780157086 + 16.078424369036583 i	10732.132506517922593 +2266.694041325391481 i	0,67,7	102.252206941967103 + 20.948135986608044 i	10290.729525548540551 +2810.176975776331346 i
0,67,9	100.320987117832701 + 25.515894090951484 i	9838.688100596786171 +3269.507771362524636 i	0,67,11	98.317485800261025 + 29.844429199109388 i	9379.653628258138269 +3659.253359074969012 i
0,67,13	96.243423571728840 + 33.973725557230395 i	8916.452358430753520 +3988.905611730832788 i	0,67,15	94.099894899267795 + 37.931486541880815 i	8451.394350378810486 +4265.196183857055075 i
0,67,17	91.887518525743999 + 41.738012150668034 i	7986.438707162677019 +4493.183898872071040 i	0,67,19	89.606524108535680 + 45.408784266451512 i	7523.293447253739032 +4676.835133972489530 i
0,67,21	87.256803330967358 + 48.955966361953649 i	7063.480466205193807 +4819.362698417748106 i	0,67,23	84.837938979694613 + 52.389333789699087 i	6608.380234700567598 +4923.437403824608737 i
0,67,25	82.349218922496902 + 55.716881496163779 i	6159.263989383128319 +4991.327213499394020 i	0,67,27	79.789638633100253 + 58.945237617897881 i	5717.317839401069250 +5024.992667430445181 i
0,67,29	77.157894065701981 + 62.079954401247065 i	5283.661466358047619 +5026.154614785414196 i	0,67,31	74.452365506600771 + 65.125718362749083 i	4859.363124111443540 +4996.343704715239255 i
0,67,33	71.671092151276767 + 68.086505385971620 i	4445.452078536031877 +4936.937460367983476 i	0,67,35	68.811736362845906 + 70.965697083787262 i	4042.929286251624490 +4849.188663671417089 i
0,67,37	65.871535712917122 + 73.766169107962824 i	3652.776902952070486 +4734.247514282691554 i	0,67,39	62.847239845257867 + 76.490358547097003 i	3275.967087094446470 +4593.179236668703197 i
0,67,41	59.735027756430433 + 79.140315246092598 i	2913.470497018840888 +4426.978300701290209 i	0,67,43	56.530398987814571 + 81.717740309239701 i	2566.264856294595575 +4236.580084000341230 i
0,67,45	53.228029031388758 + 84.224013916353144 i	2235.343979864714129 +4022.870574160730939 i	0,67,47	49.821574203744959 + 86.660213692382627 i	1921.727717640825404 +3786.694546148688914 i
0,67,49	46.303402942794122 + 89.027124077885219 i	1626.473398696128243 +3528.862527738893277 i	0,67,51	42.664216245695911 + 91.325236307744362 i	1350.689581727428504 +3250.156762174903633 i
0,67,53	38.892494341145380 + 93.554737531180407 i	1095.553302434063426 +2951.336267423674144 i	0,67,55	34.973657845631699 + 95.715485980456492 i	862.332690234252027 +2633.140932584865368 i
0,67,57	30.888731830039283 + 97.806966281113233 i	652.418092988428157 +2296.294284607962709 i	0,67,59	26.612078537850966 + 99.828213534666631 i	467.367367583216378 +1941.503823359002809 i
0,67,61	22.107207657012513 +101.777682913110695 i	308.976498486394121 +1569.455741202200898 i	0,67,63	17.318044177269414 +103.653011738743743 i	179.400402853327506 +1180.793736313767567 i
0,67,65	12.147010500196128 +105.450530276911437 i	81.390114410380463 + 776.039899433365463 i	0,67,67	6.377589614919197 +107.163990898531310 i	18.886374590211371 + 355.184776652165510 i
0,68,1	109.147731955599767 + 4.834479932794269 i	11899.635234074397886 + 804.281642898873315 i	0,68,3	107.458336246517746 + 10.817063500862453 i	11492.914703640486550 +1642.454093080629491 i
0,68,5	105.683561044436559 + 16.116807025533067 i	11061.715019470930201 +2309.218670684229437 i	0,68,7	103.830684131618185 + 21.001906848591545 i	10614.529307274657185 +2864.868350185038707 i
0,68,9	101.903807811062535 + 25.585297499075171 i	10156.486364850954487 +3335.412610450067859 i	0,68,11	99.905547419497566 + 29.929709662024404 i	9691.239811019877379 +3735.613327705666961 i
0,68,13	97.837626196190428 + 34.075130080658184 i	9221.621201665564513 +4075.086107823948169 i	0,68,15	95.701152818435389 + 38.049267094511208 i	8749.943864701697748 +4360.647853316417240 i
0,68,17	93.496770352041892 + 41.872427993881473 i	8278.168603138341496 +4597.419407966146537 i	0,68,19	91.224742397762469 + 45.560104140063821 i	7808.003833772934740 +4789.414028119255818 i
0,68,21	88.885004311865956 + 49.124470573638916 i	7340.970702704086762 +4939.880949596584287 i	0,68,23	86.477192851662878 + 52.575316264635227 i	6878.447866097214501 +5051.519863562312821 i
0,68,25	84.000661136114815 + 55.920651842447164 i	6421.703714074739764 +5126.621960826937539 i	0,68,27	81.454482582452144 + 59.167123261778016 i	5971.920471726586584 +5167.166643272012152 i
0,68,29	78.837445681289637 + 62.320302846835773 i	5530.212860082379848 +5174.890157872022428 i	0,68,31	76.148040342986107 + 65.384899634434504 i	5097.643024790943855 +5151.335735340084284 i
0,68,33	73.384435723728231 + 68.364914718904501 i	4675.232870957180239 +5097.891138770940415 i	0,68,35	70.544448721027891 + 71.263757939305293 i	4263.974598695265740 +5015.817401041183075 i
0,68,37	67.625501578803281 + 74.084336607962513 i	3864.840022522888376 +4906.271247991438031 i	0,68,39	64.624566138648149 + 76.829123446212591 i	3478.789128970473030 +4770.322904087556708 i
0,68,41	61.538091069311825 + 79.500208596381512 i	3106.778253780491923 +4608.970461800819066 i	0,68,43	58.361906689765618 + 82.099339024177240 i	2749.768229090684144 +4423.151654507665626 i
0,68,45	55.091099435659110 + 84.627947517929670 i	2408.732857101670106 +4213.753640225495474 i	0,68,47	51.719844031154970 + 87.087172641474709 i	2084.668112469954394 +3981.621240051689711 i
0,68,49	48.241174992852493 + 89.477870269061043 i	1778.602572239855817 +3727.563954527132864 i	0,68,51	44.646668287198139 + 91.800616600990438 i	1491.609743801272998 +3452.361983450886783 i
0,68,53	40.925984990569347 + 94.055701681769250 i	1224.823255792634427 +3156.771379918621733 i	0,68,55	37.066193701637218 + 96.243111190145370 i	979.456388265193482 +2841.528347421894523 i
0,68,57	33.050719303328336 + 98.362492210561314 i	756.828341832300566 +2507.352476282613225 i	0,68,59	28.857618404063814 +100.413094893542009 i	558.401414448562036 +2154.948247585942227 i
0,68,61	24.456535372562747 +102.393674091469421 i	385.836928274579350 +1785.002907476903829 i	0,68,63	19.802762332161439 +104.302316869383645 i	241.086266247845856 +1398.175023913544464 i
0,68,65	14.823806347979840 +106.136112052471901 i	126.556471165502302 + 995.053468027676672 i	0,68,67	9.380609150506933 +107.890403478870979 i	45.469954801220155 + 575.987182431604197 i
0,68,69	3.075340394767381 +109.556359114327549 i	2.994544753205965 + 139.738296698332761 i	0,69,1	110.718812008273176 + 4.842685480444582 i	12245.000292537990390 + 817.746693650492375 i
0,69,3	109.030844021410104 + 10.839964971186415 i	11832.988197806056633 +1671.579475297510953 i	0,69,5	107.258494071470594 + 16.154628918851088 i	11396.244905904160987 +2351.863372218226687 i
0,69,7	105.408961734772703 + 21.054887278175055 i	10943.286484534888586 +2919.728015770955608 i	0,69,9	103.486314979509757 + 25.653674170789060 i	10479.254651894363633 +3401.534884034731476 i
0,69,11	101.493156538222735 + 30.013720262346514 i	10007.811215864270707 +3812.240844582835962 i	0,69,13	99.431211315237448 + 34.175014476788007 i	9531.793054412069978 +4161.585290171982706 i
0,69,15	97.301600719857646 + 38.165269683865048 i	9053.516819470292830 +4456.470434151356130 i	0,69,17	95.104990177193841 + 42.004799926052058 i	8574.945104444128447 +4702.079212754695618 i
0,69,19	92.841674748803200 + 45.709105398382455 i	8097.786848869336609 +4902.471842899817602 i	0,69,21	90.511630400514989 + 49.290371959456628 i	7623.562608678073957 +5060.934077958926537 i
0,69,23	88.114544125734795 + 52.758401336778569 i	7153.649422187957498 +5180.194589618362443 i	0,69,25	85.649829765877982 + 56.121218863445300 i	6689.313071051537008 +5262.567907767945144 i
0,69,27	83.116633195403651 + 59.385487200775437 i	6231.732181312970170 +5310.052419177344746 i	0,69,29	80.513828781813189 + 62.556797489189705 i	5782.016852813579135 +5324.399904072302888 i
0,69,31	77.840007941148457 + 65.639879850678923 i	5341.223526407339705 +5307.166323872530484 i	0,69,33	75.093459831983594 + 68.638758951539998 i	4910.367226381944420 +5259.749845948125767 i
0,69,35	72.272143573663683 + 71.556870975382537 i	4490.431969089919221 +5183.419935367786820 i	0,69,37	69.373650711783270 + 74.397152719685522 i	4082.37991467612119 +5079.34004429504057 i
0,69,39	66.395155877922036 + 77.162110002427653 i	3687.159706743308561 +4948.585618447329580 i	0,69,41	63.333352579825899 + 79.853870277425997 i	3305.714367373518144 +4792.158617651227360 i
0,69,43	60.184369644460681 + 82.474222815252091 i	2938.989077630024894 +4610.999401698473775 i	0,69,45	56.943661758520840 + 85.024648718574852 i	2587.939170367474162 +4405.996597625729919 i

Table D.1: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
0,69,47	53.605864374235104 + 87.506342221864898 i	2253.538693381222402 +4177.995400046031136 i	0,69,49	50.164598211562975 + 89.920224046479291 i	1936.789974206899387 +3927.804636316712731 i
0,69,51	46.612200296398797 + 92.266946935297653 i	1638.734750777100999 +3656.202834460571921 i	0,69,53	42.939344245377335 + 94.546892758789710 i	1360.467659167143211 +3363.943446947570465 i
0,69,55	39.134486888192491 + 96.760159597694866 i	1103.153257813906521 +3051.759285508595440 i	0,69,57	35.183029478965935 + 98.906535666278700 i	868.048451581060704 +2720.366069173322558 i
0,69,59	31.065981924774793 +100.985454229079679 i	656.533446784120883 +2370.464678856181308 i	0,69,61	26.757695758998842 +102.995918366924997 i	470.156888393883492 +2002.740961809925921 i
0,69,63	22.221674674736892 +104.936372885267389 i	310.706335418094682 +1617.859797305553229 i	0,69,65	17.401842239545022 +106.804471692209091 i	180.328927654970727 +1216.442856125207982 i
0,69,67	12.200621354158539 +108.596600657768704 i	81.768435110026218 + 798.987009009475514 i	0,69,69	6.401476447919890 +110.306639800409783 i	18.958756468262102 + 365.444111166697951 i
0,70,1	112.289884652310434 + 4.850773409607537 i	12595.301040724010818 + 831.237539146916447 i	0,70,3	110.603307853750891 + 10.862538397752552 i	12178.001619152328203 +1700.776539087021092 i
0,70,5	108.833321220731349 + 16.191906123074457 i	11735.721843087594607 +2394.626400722287599 i	0,70,7	106.987047672814299 + 21.107100063401838 i	11277.000476950383018 +2974.753507110890496 i
0,70,9	105.068521138320321 + 25.721053886362643 i	10806.992053782476432 +3467.871379257227090 i	0,70,11	103.080331354514612 + 30.096498069810696 i	10329.366535602799559 +3889.131919720502992 i
0,70,13	101.024203987928090 + 34.273423428578361 i	9846.966134937787501 +4248.398359017509392 i	0,70,15	98.901271822237149 + 38.279546963300163 i	9362.110877781920863 +4552.658280753688814 i
0,70,17	96.712220833693905 + 42.135188968244456 i	8876.765231323170156 +4807.156781777590368 i	0,70,19	94.457375257932256 + 45.855857874542060 i	8392.638773312693957 +5016.001116569103033 i
0,70,21	92.136748851112173 + 49.453749663512966 i	7911.251617744498617 +5182.513642061713654 i	0,70,23	89.750075411545595 + 52.938678023029446 i	7433.979368222107041 +5309.452105856853450 i
0,70,25	87.296825351677242 + 56.318682083248461 i	6962.085419491385437 +5399.154482246414773 i	0,70,27	84.776211987991999 + 59.600440184494879 i	6496.745063497443880 +5453.638258875278552 i
0,70,29	82.187189497086308 + 62.789561119279874 i	6039.064093810449776 +5474.670876799706093 i	0,70,31	79.528443431558415 + 65.890794773739017 i	5590.093619942816076 +5463.821168447580931 i
0,70,33	76.798373951934295 + 68.908187881821746 i	5150.842228416112448 +5422.497859832652466 i	0,70,35	73.995071325362090 + 71.845201250097361 i	4722.286278772212427 +5351.979018221912156 i
0,70,37	71.116282650884983 + 74.704799178339584 i	4305.378906242111952 +5253.435009551811163 i	0,70,39	68.159368099389539 + 77.489518279613975 i	3901.05816791806046 +5127.9467071977326 i
0,70,41	65.121244101821802 + 80.201520623067609 i	3510.254688137364610 +4976.520168537783320 i	0,70,43	61.998309746649106 + 82.842634591449567 i	3133.899116287801462 +4800.098630845181106 i
0,70,45	58.786350952402792 + 85.414385773119790 i	2772.929699313342098 +4599.572469447563890 i	0,70,47	55.480414430528526 + 87.918019413550240 i	2428.300290786366077 +4375.787558693779829 i
0,70,49	52.074639476781627 + 90.354515310690033 i	2100.989173394698355 +4129.552385793472240 i	0,70,51	48.562029202334323 + 92.724595454556052 i	1792.009174702534438 +3861.644161468960647 i
0,70,53	44.934132017018051 + 95.028724086460471 i	1502.419732256585803 +3572.814097307171323 i	0,70,55	41.180585210396451 + 97.267099058652661 i	1233.341861759274934 +3263.791937074591715 i
0,70,57	37.288437381186405 + 99.439632197092891 i	985.977495697512154 +2935.289714500010177 i	0,70,59	33.241097315254017 +101.545914393351865 i	761.635584783477952 +2588.004498893163145 i
0,70,61	29.016609638787269 +103.585157469465585 i	561.769124489185401 +2222.619414273258371 i	0,70,63	24.584611122722688 +105.556097257014741 i	388.030943339709893 +1839.800964188511216 i
0,70,65	19.900390814173431 +107.456824630411177 i	242.364607211324198 +1440.186812608553510 i	0,70,67	14.891456329465608 +109.284462809738301 i	127.169888709136714 +1024.343258554921249 i
0,70,69	9.418746374990937 +111.034439374166595 i	45.662357910661832 + 592.590438690402152 i	0,70,71	3.084348112778115 +112.698118817268607 i	3.003477579690521 + 143.623759619669926 i
0,71,1	113.860950179532054 + 4.858747020628222 i	12950.537455586552824 + 844.753812037136868 i	0,71,3	112.175729457085396 + 10.884793003432529 i	12527.954837352979666 +1730.044264045454383 i
0,71,5	110.408046621366083 + 16.228654032632598 i	12080.145520934569504 +2437.506060796050861 i	0,71,7	108.564949446429011 + 21.158567023357087 i	11615.670726964281130 +3029.942429713919864 i
0,71,9	106.650438133065151 + 25.787465151976761 i	11139.69769850475677 +3534.41897653527394 i	0,71,11	104.667089083838363 + 30.178078557343433 i	10655.904515232808990 +3966.282680136261661 i
0,71,13	102.616627905387048 + 34.370399679564215 i	10167.138733404466620 +4335.520656910639445 i	0,71,15	100.500197507904502 + 38.392149281412635 i	9675.723798222710684 +4649.205916931320644 i
0,71,17	98.318502752073300 + 42.263653445330355 i	9183.626127543848270 +4912.645782164711818 i	0,71,19	96.071894938757922 + 46.000428283993962 i	8692.556044579458103 +5129.994617509087220 i
0,71,21	93.760423016467470 + 49.614679256030229 i	8204.033359435035891 +5304.611464828521093 i	0,71,23	91.383864431344222 + 53.116231270031790 i	7719.432411697921452 +5439.283237837769775 i
0,71,25	88.941742355190073 + 56.513136412703162 i	7240.014415659181395 +5536.371454743335562 i	0,71,27	86.433332953844527 + 59.812087752043588 i	6766.951575773270633 +5597.912813550807186 i
0,71,29	83.857664673631817 + 63.018710656895792 i	6301.345671968085298 +5625.690536441901713 i	0,71,31	81.213510500352243 + 66.137773559491265 i	5844.24282867977308 +5621.28645897918432 i
0,71,33	78.499373442496989 + 69.173343878040569 i	5396.645604370462934 +5586.120010996718520 i	0,71,35	75.713464928657061 + 72.128905463729666 i	4959.523187541777588 +5521.478021613814235 i
0,71,37	72.853675277984834 + 75.007448307203688 i	4533.82026667915034 +5428.53794631673645 i	0,71,39	69.917534816535976 + 77.811537722639329 i	4120.46500425642262 +5308.38628094652086 i
0,71,41	66.902163484701759 + 80.543367953067076 i	3720.37646065524359 +5162.03339110943489 i	0,71,43	63.804205801589660 + 83.204803618068567 i	3334.471767033227025 +4990.425630724103030 i
0,71,45	60.619746659583740 + 85.797411365365363 i	2963.673329002634546 +4794.455382732771795 i	0,71,47	57.344201359126252 + 88.322483312209641 i	2608.916353216289735 +4574.969487565742384 i
0,71,49	53.972170127558222 + 90.781053251720081 i	2271.157029442366820 +4332.776404904127048 i	0,71,51	50.497242337024161 + 93.173906063452890 i	1951.381780462103052 +4068.652364130618025 i
0,71,53	46.911727351233203 + 95.501580223142739 i	1650.618129592297464 +3783.346684963981261 i	0,71,55	43.206274707060487 + 97.764362652694757 i	1369.947965913773032 +3477.586378221019913 i
0,71,57	39.369320721668302 + 99.962274232071664 i	1110.524377753076351 +3152.080046848394886 i	0,71,59	35.386249779419174 +102.095042822804785 i	873.593910571475817 +2807.520957709696177 i
0,71,61	31.238058728114776 +104.162058032553645 i	660.527374232195029 +2444.588842625513735 i	0,71,63	26.899090097254174 +106.162296804075595 i	472.864845365327369 +2063.949221260136255 i
0,71,65	22.332842865124604 +108.094197657417453 i	312.386015688326154 +1666.246856067834415 i	0,71,67	17.483239988919756 +109.955433204398702 i	181.230734984000861 +1252.082499784529318 i
0,71,69	12.252705201343067 +111.742444539866668 i	82.135939945542091 + 821.928875554062984 i	0,71,71	6.424684696712908 +113.449234804619095 i	19.029073233320990 + 375.700282938605483 i
0,72,1	115.432008866485049 + 4.866609477285679 i	13310.709514982316250 + 858.295155207798871 i	0,72,3	113.748110455843886 + 10.906737628441698 i	12882.847727120197305 +1759.381658284961077 i
0,72,5	111.982674187353268 + 16.264887400025117 i	12429.515641339297872 +2480.500704745858002 i	0,72,7	110.142674163466580 + 21.209309062257223 i	11959.296698638971066 +3085.292457007859412 i
0,72,9	108.232077184469233 + 25.852935271309928 i	11477.370748074610674 +3601.174645561952275 i	0,72,11	106.253446025389508 + 30.258495691452318 i	10987.423949133122733 +4043.689364776048023 i
0,72,13	104.208505484330814 + 34.465984144577462 i	10492.309207953627265 +4422.947662459501771 i	0,72,15	102.098407450194586 + 38.503124814885396 i	9994.353429588853032 +4746.108028365885730 i
0,72,17	99.923874130247313 + 42.390249143085846 i	9495.525053452487555 +5018.540070661442769 i	0,72,19	97.685281943559005 + 46.142880408358479 i	8997.535247868154329 +5244.445333658812160 i
0,72,21	95.382712633838835 + 49.773232946921247 i	8501.903647722218011 +5427.219621192850354 i	0,72,23	93.015984386118390 + 53.291142200990997 i	8010.003487842434879 +5569.679098449797493 i
0,72,25	90.584669626440572 + 56.704672433666758 i	7523.093994644895247 +5674.208921657384053 i	0,72,27	88.088103157158258 + 60.020530558738493 i	7042.344514912588238 +5742.865102627675697 i
0,72,29	85.525382627982353 + 63.244357525839654 i	6568.853087218472865 +5777.446758207327548 i	0,72,31	82.895362347730753 + 66.380939188425344 i	6103.66117246308931 +5779.54885165187261 i
0,72,33	80.196640769051129 + 69.434362374564444 i	5647.76568055488224 +5750.60165329814026 i	0,72,35	77.427541453917186 + 72.408132530265121 i	5202.12907759645350 +5691.90090422144294 i
0,72,37	74.586086846655235 + 75.305263678705558 i	4767.688137486994492 +5604.631314482125163 i	0,72,39	71.669963666020806 + 78.128347925519066 i	4345.36176242530746 +5489.88518351537732 i
0,72,41	68.676478103813608 + 80.879609470901570 i	3936.058209853442349 +5348.677386897949873 i	0,72,43	65.602498194023624 + 83.560946573853670 i	3540.68201294807750 +5181.95761437273813 i
0,72,45	62.444379564675081 + 86.173963864508892 i	3160.140859458257182 +4990.620500391584756 i	0,72,47	59.197869103762947 + 88.719996640178621 i	2795.352696338513852 +4775.514119254765319 i
0,72,49	55.857978530347786 + 91.200128193465815 i	2447.253356533301940 +4537.447196216899101 i	0,72,51	52.418815892542305 + 93.615200722196050 i	2116.805065754853331 +4277.195296428704751 i
0,72,53	48.873356593045450 + 95.965819887728372 i	1805.006294708728774 +3995.506182501234434 i	0,72,55	45.213124749083207 + 98.252352538863590 i	1512.903601719029439 +3693.104467000576278 i
0,72,57	41.427736731110073 +100.474916386145892 i	1241.606410706520592 +3370.695612918036659 i	0,72,59	37.504223632777247 +102.633359293049025 i	992.306184804072245 +3028.969225012565857 i
0,72,61	33.425980275369291 +104.727223945250245 i	766.302381999563181 +2668.601363488382958 i	0,72,63	29.171041069312043 +106.755690085567196 i	565.039349836465135 +2290.255127088007612 i
0,72,65	24.709036568973939 +108.717479103250554 i	390.161990972695946 +1894.577470439535773 i	0,72,67	19.995253706043322 +110.610688523090374 i	243.606549300110316 +1482.186247884887962 i
0,72,69	14.957201151998069 +112.432476759068763 i	127.765962770230317 +1053.626222726406013 i	0,72,71	9.455814732330177 +114.178353001227080 i	45.849348606702527 + 609.189765319117782 i
0,72,73	3.093101010853956 +115.839869992141715 i	3.012156122286707 + 147.506816118058509 i	0,73,1	117.003060975434991 + 4.874363814211643 i	13675.817197623613538 + 871.861221362478091 i
0,73,3	115.320452391107239 + 10.928380751187103 i	13242.680167944698042 +1788.787757257218118 i	0,73,5	113.557207631469424 + 16.300620370967433 i	12783.831917557648922 +2523.608730602818923 i
0,73,7	111.720228564599850 + 21.259346219819598 i	12307.877876532456867 +3140.801327493992630 i	0,73,9	109.813448929570740 + 25.917490412167410 i	11820.010396746314655 +3668.135441529848777 i
0,73,11	107.839417623078532 + 30.337782016304836 i	11323.923678448478313 +4121.348319728776914 i	0,73,13	105.799857952932115 + 34.560216012663503 i	10822.475981059693368 +4510.674984442559435 i
0,73,15	103.695929730221067 + 38.612519691871505 i	10317.997705975019926 +4843.359455513697867 i	0,73,17	101.528371088510781 + 42.515029453958412 i	9812.459379518766582 +5124.833685190439610 i
0,73,19	99.297581765483713 + 46.283275265835670 i	9307.573107765814711 +5359.346462587357564 i	0,73,21	97.003674085499256 + 49.929479783512562 i	8804.858470397190104 +5550.330426507467564 i
0,73,23	94.646504288556528 + 53.463488343867986 i	8305.687746196443186 +5700.631074451895984 i	0,73,25	92.225690826962790 + 56.893376661422053 i	7811.318353530345121 +5812.657289740487613 i
0,73,27	89.740623268908692 + 60.225864677211732 i	7322.916995932704594 +5888.484495819903714 i	0,73,29	87.190463825566212 + 63.466607999081170 i	6841.57822430963915 +5929.92781146808102 i
0,73,31	84.574142548413860 + 66.620408861313990 i	6368.33913503653275 +5938.59544517596623 i	0,73,33	81.890346597163941 + 69.691372325889077 i	5904.19134155893516 +5915.92863660504736 i
0,73,35	79.137503486046157 + 72.683024099488563 i	5450.09100266255609 +5863.23218006753905 i	0,73,37	76.313757777293915 + 75.598400716562892 i	5006.967468435209412 +5781.698195304174078 i
0,73,39	73.416940234243626 + 78.440119328723100 i	4575.730963660097586 +5672.424955835992478 i	0,73,41	70.444527907453477 + 81.210432075576321 i	4157.27963914905376 +5536.43203483570414 i
0,73,43	67.393592930193520 + 83.911268502506786 i	3752.50626805306183 +5374.67266453469438 i	0,73,45	64.260736840767805 + 86.544268449889856 i	3362.30483762430140 +5188.04395857074724 i
0,73,47	61.042005870612144 + 89.110807090190974 i	2987.577283361506034 +4977.395475641185840 i	0,73,49	57.732780583967696 + 91.612013220738305 i	2629.240632061018005 +4743.536481217119217 i
0,73,51	54.327630096156966 + 94.048781450418502 i	2288.234867314424264 +4487.242174740062183 i	0,73,53	50.820116074408496 + 96.421778482248769 i	1965.531915374136815 +4209.259080404922315 i
0,73,55	47.202523444662006 + 98.731443222927794 i	1662.146290088648357 +3910.309739902562342 i	0,73,57	43.465480507995032 +100.977979855171256 i	1379.148168270277893 +3591.096783321447674 i
0,73,59	39.597405558394215 +103.161342142249907 i	1117.680059027488596 +3252.306370119984422 i	0,73,61	35.583668259251773 +105.281205820574527 i	878.978917389056733 +2894.610843970490805 i
0,73,63	31.405254209023671 +107.336923396828553 i	664.406822142619376 +2518.670128992448554 i	0,73,65	27.036498392429895 +109.327450659860318 i	475.495859003524515 +2125.130612400239443 i
0,73,67	22.440896154207920 +111.251223252994279 i	314.018355610005017 +1714.618025651435346 i	0,73,69	17.562370506419402 +113.105934158445805 i	182.107313291935839 +1287.713216450149730 i
0,73,71	12.303346257782735 +114.888079113598778 i	82.493227446397327 + 844.865745455618026 i	0,73,73	6.447251647070871 +116.591780018745141 i	19.097438734469893 + 385.953406785431639 i
0,74,1	118.574106755281576 + 4.882012943812880 i	14045.860483034453864 + 885.451672623655895 i	0,74,3	116.892756725928109 + 10.949730507716296 i	13607.452043852481438 +1818.261622640837913 i
0,74,5	115.131650478174976 + 16.335866517166057 i	13143.094073627844409 +2566.828580249048628 i	0,74,7	113.297619046983272 + 21.308697718215402 i	12661.413764647668359 +3196.466842058468956 i
0,74,9	111.394563459640267 + 25.981155668559492 i	12167.615869348805063 +3735.298501565275274 i	0,74,11	109.425018521659013 + 30.415968732018424 i	11665.402588647573793 +4199.255993710385155 i
0,74,13	107.390705429753197 + 34.653132842846368 i	11157.637536128742795 +4598.698356256832994 i	0,74,15	105.292790944076273 + 38.720378106712957 i	10646.654642199238879 +4940.955186917774881 i
0,74,17	103.132027811599727 + 42.638045512475330 i	10134.426580331746663 +5231.520836909959144 i	0,74,19	100.908837423542352 + 46.421671269344699 i	9622.666480508567474 +5474.691402161248334 i
0,74,21	98.623360636646993 + 50.083485833819473 i	9112.893979222386406 +5673.936425660816894 i	0,74,23	96.275489266364687 + 53.633343842628193 i	8606.480538202800744 +5832.130813852192659 i
0,74,25	93.864884814302556 + 57.079331787260145 i	8104.681935849206639 +5951.707261521936744 i	0,74,27	91.390988052674629 + 60.428181875320234 i	7608.662432725532199 +6034.760696348060264 i
0,74,29	88.853021493831889 + 63.685563516763011 i	7119.51332872284911 +6083.12234047354741 i	0,74,31	86.249985825953630 + 66.856294363046345 i	6638.26763413146818 +6098.41375865490500 i
0,74,33	83.580650772847150 + 69.944496623241420 i	6165.91199308574414 +6082.08728141626580 i	0,74,35	80.843540370584961 + 72.953715034841636 i	5703.39664173299116 +6035.45688942591551 i
0,74,37	78.036912250441802 + 75.887007245375912 i	5251.64395980283462 +5959.72225805106336 i	0,74,39	75.158730110245926 + 78.747013853613964 i	4811.55603133054501 +5855.98779509786073 i
0,74,41	72.206628087418196 + 81.536013097551205 i	4384.02153908598575 +5725.27794739420915 i	0,74,43	69.177865153074706 + 84.255963671593236 i	3969.92226121664589 +5568.54968334734622 i
0,74,45	66.069266843225108 + 86.908538121183909 i	3570.139408504090170 +5386.702808966011617 i	0,74,47	62.877150505138310 + 89.495148525023964 i	3185.56003200112764 +5180.58860185494177 i
0,74,49	59.597228566011658 + 92.016965621223406 i	2817.083742857913421 +4951.017125011098869 i	0,74,51	56.224481797978157 + 94.474932087053361 i	2465.630021613451845 +4698.763492736439730 i
0,74,53	52.752990589695763 + 96.869770281945762 i	2132.146460367584041 +4424.573292703849802 i	0,74,55	49.175705818159138 + 99.201984352129770 i	1817.618392197209005 +4129.167311652185163 i
0,74,57	45.484130124380165 +101.471856101827157 i	1523.080544086312102 +3813.245657512523849 i	0,74,59	41.667861437376935 +103.679433351768765 i	1249.631651375993442 +3477.491303672900358 i
0,74,61	37.713915501574853 +105.824508436783375 i	998.453488301884136 +3122.572973729349087 i	0,74,63	33.605675013610195 +107.906582651568954 i	770.836696124539571 +2749.147072110428326 i
0,74,65	29.321165686159361 +109.924808979565711 i	568.217561504440482 +2357.857872113735993 i	0,74,67	24.830013033669228 +111.877898231867164 i	392.233572800034096 +1949.333861561380445 i
0,74,69	20.087502674884409 +113.763956913162616 i	244.814100127389395 +1524.174083186516782 i	0,74,71	15.021144733269566 +115.580179131157607 i	128.345643066140298 +1082.902715189236460 i
0,74,73	9.491872242938022 +117.322153526244520 i	46.031221588258582 + 625.785320965170968 i	0,74,75	3.101612963067426 +118.981613286093548 i	3.020594297229168 + 151.387546107000048 i
0,75,1	120.145146442404297 + 4.889559662736415 i	14420.839351509386688 + 899.066180155885377 i	0,75,3	118.465024850247431 + 10.970794709878035 i	13977.163243176964897 +1847.802341288261459 i
0,75,5	116.706006075515802 + 16.370638866912714 i	13507.301843827965058 +2610.158737644801022 i	0,75,7	114.874851686091802 + 21.357382005880027 i	13019.903885449026347 +3252.286861429732653 i
0,75,9	112.975430355156206 + 26.043955118597026 i	12520.186419727195243 +3802.661041358131115 i	0,75,11	111.010262618448536 + 30.493085766134138 i	12011.859607240879996 +4277.408933798544240 i
0,75,13	108.981066996397999 + 34.744770653331834 i	11497.792414323052071 +4687.013630678353366 i	0,75,15	106.889016301370063 + 38.826742426717530 i	10980.322329536500547 +5038.890352901457845 i
0,75,17	104.734876679077132 + 42.759346321168621 i	10461.424229008634939 +5338.595902732532522 i	0,75,19	102.519086321032321 + 46.558124373444159 i	9942.812346785241971 +5590.473741762129066 i
0,75,21	100.241822652962426 + 50.235314356632003 i	9426.006480790782007 +5798.030382846941393 i	0,75,23	97.903000839029502 + 53.800779653048508 i	8912.377405707229627 +5964.170214059455247 i
0,75,25	95.502325992001502 + 57.262616903024814 i	8403.179417219800598 +6091.349821642980714 i	0,75,27	93.039286805642092 + 60.627569873001598 i	7899.57452019049560 +6181.68372522630671 i
0,75,29	90.513162177109251 + 63.901320979622469 i	7402.65098451594895 +6237.01934632165921 i	0,75,31	87.923018751074196 + 67.088702397709682 i	6913.437994025610351 +6258.991710928713474 i
0,75,33	85.267703203248914 + 70.193852477464475 i	6432.91752796135006 +6249.06435521384628 i	0,75,35	82.545829328426350 + 73.220363351582794 i	5962.03425690406675 +6208.56057176038576 i
0,75,37	79.755759625173354 + 76.171223993242378 i	5501.70400993735802 +6138.68772903660374 i	0,75,39	76.895580703150287 + 79.049185480929012 i	5052.82122545885159 +6040.55651982791642 i
0,75,41	73.963071428380530 + 81.856520967472974 i	4616.26570513716251 +5915.19643000623455 i	0,75,43	70.955662212510731 + 84.595216349942731 i	4192.90893227904871 +5763.56834587400499 i
0,75,45	67.870383174905285 + 87.266974608501187 i	3783.620182788692546 +5586.574965617616726 i	0,75,47	64.703797961165279 + 89.873242052324244 i	3389.27264406010909 +5385.06950542049253 i
0,75,49	61.451918632789734 + 92.415228168818857 i	3010.74976302536470 +5159.86306691658683 i	0,75,51	58.110094996600068 + 94.893919843821095 i	2648.952070759547496 +4911.730942723105727 i
0,75,53	54.672868589131355 + 97.310090348748574 i	2304.804782736923698 +4641.418069042269053 i	0,75,55	51.133776512389119 + 99.664301251169731 i	1979.266561571023203 +4349.643779846973302 i
0,75,57	47.485082040452951 +101.956909979867206 i	1673.339972282682766 +4037.105967588569001 i	0,75,59	43.717394699765059 +104.188042754451146 i	1388.084394563397836 +3704.484699850290326 i
0,75,61	39.819116916662836 +106.357596707166721 i	1124.632549934944109 +3352.445260733493797 i	0,75,63	35.775605492726754 +108.465092048482454 i	884.212489378920964 +2981.640437927462244 i
0,75,65	31.567836338646153 +110.510181094216151 i	668.178157166851065 +2592.711554995256392 i	0,75,67	27.170138145388581 +112.491471406197320 i	478.054168117172367 +2186.286948861481505 i
0,75,69	22.546003538768051 +114.407509037707669 i	315.605941048371335 +1762.974308987843642 i	0,75,71	17.639356129382609 +116.256008638057537 i	182.960030233350409 +1323.335505982096493 i
0,75,73	12.352621974464645 +118.033519850424838 i	82.840847899626780 + 867.797845469555455 i	0,75,75	6.469211600998566 +119.734279138955230 i	19.163957683552304 + 396.203590096493087 i
0,76,1	121.716180261444507 + 4.897066657913369 i	14800.753784074970099 + 912.704423808758179 i	0,76,3	120.037258085441621 + 10.991580862295807 i	14351.813658345392469 +1877.409024228214093 i
0,76,5	118.280277606127569 + 16.404949933669013 i	13876.454972167517553 +2653.597727149806360 i	0,76,7	116.451932255902008 + 21.405416798429904 i	13383.347778941137221 +3308.259303772205385 i
0,76,9	114.556058718100985 + 26.105911878538063 i	12877.721329283665909 +3870.220351974166078 i	0,76,11	112.595163111043860 + 30.569161849201777 i	12363.293701645730835 +4355.803781400859407 i

Table D.1: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
0,76,13	110.570960764464317 + 34.835164004681978 i	11842.939211593400614 +4775.616774913107292 i	0,76,15	108.484629715922196 + 38.931653291646690 i	11318.998931736054146 +5137.160219616655013 i
0,76,17	106.336948385161492 + 42.878978867806495 i	10793.449991982086431 +5446.053418270895236 i	0,76,19	104.128376956406996 + 46.692688210980890 i	10268.007805038047081 +5706.687254014436803 i
0,76,21	101.859107799847393 + 50.385025959543619 i	9744.192428030457449 +5922.605271940165039 i	0,76,23	99.529097170749722 + 53.965863724430477 i	9223.374070290399686 +6096.741410748364615 i
0,76,25	97.138084628698223 + 57.443307709223561 i	8706.80569204336643 +6231.57622403116693 i	0,76,27	94.685603759646142 + 60.824112579988238 i	8195.64721772925441 +6329.24390653521823 i
0,76,29	92.170986239723732 + 64.113973020112198 i	7690.98409390558481 +6391.60817008579488 i	0,76,31	89.593360372826623 + 67.317734897654248 i	7193.84192032356259 +6420.31760127120015 i
0,76,33	86.951644649723477 + 70.439551771476815 i	6705.1982949889643 +6416.8470504009192 i	0,76,35	84.244536455528618 + 73.483003119240564 i	6225.9926548679941 +6382.5292405202299 i
0,76,37	81.470495705314136 + 76.451185052270461 i	5757.1346675347349 +6318.5793628283564 i	0,76,39	78.627722856185613 + 79.346780779320724 i	5299.5115833741065 +6226.1145369951828 i
0,76,41	75.714130381990657 + 82.172115825221653 i	4853.99486354723012 +6106.16944347044409 i	0,76,43	72.727306357312500 + 84.929201513058302 i	4421.4463388161248 +5959.7090571242952 i
0,76,45	69.664468223079467 + 87.619769194658318 i	4002.72411873397368 +5787.63915577317130 i	0,76,47	66.522404016201534 + 90.245296990785548 i	3598.68845436486254 +5590.81510011404862 i
0,76,49	63.297397217152444 + 92.807030270233327 i	3210.207758521507003 +5370.049256440766840 i	0,76,51	59.985129705364436 + 95.305996682918050 i	2838.165006163267208 +5126.117342969164383 i
0,76,53	56.580554784394494 + 97.743016212643198 i	2483.464777760953893 +4859.763913016472543 i	0,76,55	53.077728279130305 +100.118706384999797 i	2147.041277950699168 +4571.707135137029582 i
0,76,57	49.469579294110204 +102.433482407005755 i	1829.867775198596519 +4262.642984205478569 i	0,76,59	45.747591435014392 +104.687551563875132 i	1532.968091888664503 +3933.248909803924562 i
0,76,61	41.901346338784528 +106.880900552537000 i	1257.431077276043879 +3584.186985444068796 i	0,76,63	37.917846270368614 +109.013273404143135 i	1004.429513223178219 +3216.106436276577824 i
0,76,65	33.780463394525356 +111.084135861040352 i	775.245830184680131 +2829.645227567681559 i	0,76,67	29.467216034348873 +113.092618691439853 i	571.308782407006788 +2425.429892361996735 i
0,76,69	24.947725668713289 +115.037425396742222 i	394.248906500075350 +2004.071436136187231 i	0,76,71	20.177277273527707 +116.916673367345069 i	245.989105918021068 +1566.151003076724010 i
0,76,73	15.083382733124797 +118.727592663675736 i	128.909803533536973 +1112.173060711457538 i	0,76,75	9.526972337743878 +120.465849210698551 i	46.208248176618731 + 642.377252997156688 i
0,76,77	3.109896744992037 +122.123349281881389 i	3.028804913943933 + 155.266024767286126 i	0,77,1	123.287208426029281 + 4.904356512213901 i	15185.603762453609306 + 926.366091778389432 i
0,77,3	121.609457688532871 + 11.012096178246058 i	14731.403185678418595 +1907.080805720114085 i	0,77,5	119.854468097421162 + 16.438811742797967 i	14250.553211910468235 +2697.144111932632313 i
0,77,7	118.028866247561353 + 21.452819116912046 i	13751.74500180490941 +3364.38214240720845 i	0,77,9	116.136457201816555 + 26.167048153288615 i	13240.219905610145361 +3937.973796837452483 i
0,77,11	114.179732541390820 + 30.644224563364691 i	12719.703877187257604 +4434.437268440752632 i	0,77,13	112.160403937315962 + 34.924346077444589 i	12193.07657590285271 +4864.50386591856113 i
0,77,15	110.079653889335461 + 39.035149706505564 i	11662.682681299711476 +5235.760183422031332 i	0,77,17	107.938272048994421 + 42.996988234644817 i	11130.501624135204111 +5553.888071180925181 i
0,77,19	105.736735955445880 + 46.825414220322585 i	10598.250065218880991 +5823.325886986322075 i	0,77,21	103.475261225150149 + 50.532678745942015 i	10067.448412299423580 +6047.654267430129948 i
0,77,23	101.153833301939059 + 54.128661168431336 i	9539.46642335855904 +6229.83676738511349 i	0,77,25	98.772227149542829 + 57.621476708143959 i	9015.555861172037112 +6372.377979850260090 i
0,77,27	96.330018446504852 + 61.017890316089513 i	8496.874733976239440 +6477.431853603445234 i	0,77,29	93.826588323042998 + 64.323608253255216 i	7984.50585842240980 +6546.87847700514612 i
0,77,31	91.261122790178177 + 67.543489308972681 i	7479.47147674020289 +6582.38009133054675 i	0,77,33	88.632607443572510 + 70.681701385223619 i	6982.7450703579689 +6585.4229636927939 i
0,77,35	85.939817621661348 + 73.741839831880000 i	6495.2611516744491 +6557.3493596341486 i	0,77,37	83.181303872826717 + 76.727018301268444 i	6017.9235881214063 +6499.3824154411677 i
0,77,39	80.355372284312153 + 79.639939389205423 i	5551.6128658000061 +6412.6458114365660 i	0,77,41	77.460058904030944 + 82.482950075784672 i	5097.1926042530222 +6298.1795690693853 i
0,77,43	74.493097104709307 + 85.258085484723665 i	4655.51557220746236 +6156.95291224383072 i	0,77,45	71.451876249830789 + 87.967103460093256 i	4227.42941640402114 +5989.87487444436647 i
0,77,47	68.333389356872999 + 90.611511741329210 i	3813.7822963575954 +5797.8031541051923 i	0,77,49	65.134166517378233 + 93.192588993081912 i	3415.428614540512172 +5581.551594178292362 i
0,77,51	61.850189471264628 + 95.711400543992809 i	3033.23504399875219 +5341.89657057357181 i	0,77,53	58.476780691736217 + 98.168809356710980 i	2668.087086305444700 +5079.582506472598652 i
0,77,55	55.008457187679475 +100.565482448863477 i	2320.896447844156683 +4795.326677175430632 i	0,77,57	51.438734214870756 +102.901892670389500 i	1992.609612621137103 +4489.823426372412541 i
0,77,59	47.759855813210280 +105.178315366055145 i	1684.218134821469221 +4163.747872499801815 i	0,77,61	43.962414873056069 +107.394818936563979 i	1396.771410063808844 +3817.759133048543845 i
0,77,63	40.034799826514053 +109.551247523809295 i	1131.393078687277921 +3452.503016371606464 i	0,77,65	35.962356227340983 +111.647192726533376 i	889.302902726750176 +3068.613982052708252 i
0,77,67	31.726051649737656 +113.681948853663300 i	671.847228793019667 +2666.715843744535960 i	0,77,69	27.300209566030141 +115.654441468636449 i	480.543670769741539 +2247.419871475755372 i
0,77,71	22.648320657542016 +117.563108546563939 i	317.151151891888503 +1811.316616456772408 i	0,77,73	17.714309576009619 +119.405687396447505 i	183.790145382086763 +1358.949824681524433 i
0,77,75	12.400603742032470 +121.178780714165342 i	83.179308392565531 + 890.725384838178456 i	0,77,77	6.490596186389138 +122.876735500575691 i	19.228726605977364 + 406.450933511732273 i
0,78,1	124.858231139442253 + 4.911611709742893 i	15575.389269029617935 + 940.050880286237771 i	0,78,3	123.181624856091133 + 11.032347594524786 i	15115.931725201933659 +1936.816842357093037 i
0,78,5	121.428580431019839 + 16.472235856584511 i	14629.596325127458840 +2740.796492462385231 i	0,78,7	119.605658886676821 + 21.499605323592213 i	14125.095126586826154 +3420.653403652830093 i
0,78,9	117.716634038632805 + 26.227385283633190 i	13607.681481205889213 +4005.918808871847432 i	0,78,11	115.763982836532193 + 30.718300416796904 i	13081.089175224844439 +4513.306213746510366 i
0,78,13	113.749412867140506 + 35.012348744675982 i	12548.203204627163359 +4953.671085977467555 i	0,78,15	111.674110388100118 + 39.137269128170495 i	12011.371876000572229 +5334.685765568476199 i
0,78,17	109.538875316236556 + 43.113417700344748 i	11472.576964256270003 +5662.094694873885601 i	0,78,19	107.344201313406259 + 46.956351763946664 i	10933.536442963897571 +5940.383756830206039 i
0,78,21	105.090325726976232 + 50.678328451881249 i	10395.771156021139177 +6173.170735876653453 i	0,78,23	102.777261361426497 + 54.289234416103604 i	9860.650516927436911 +6363.448865364664925 i
0,78,25	100.404816402742441 + 57.797193383267846 i	9329.42522046079830 +6513.74684616736249 i	0,78,27	97.972606031380508 + 61.208980015573751 i	8803.25151265267414 +6626.23845602842084 i
0,78,29	95.480057761419559 + 64.530311509053334 i	8283.20976149197486 +6702.82024165514122 i	0,78,31	92.926411670662077 + 67.766058855559308 i	7770.3190636933459 +6745.1681882136409 i
0,78,33	90.310716133360216 + 70.920403495708271 i	7265.5490313523205 +6754.7800768422876 i	0,78,35	87.631819280242851 + 73.996955749294770 i	6769.8295404226137 +6733.0078215605399 i
0,78,37	84.88835610487118 + 76.99884579437864 i	6284.058994286435 +6681.082619346391 i	0,78,39	82.078730858319531 + 79.928794466518411 i	5809.1115077676950 +6600.1348374013523 i
0,78,41	79.201094086787208 + 82.789168897629153 i	5345.8433200549252 +6491.2099761598365 i	0,78,43	76.253313333162112 + 85.582026521889787 i	4895.0986818708279 +6355.2816595875765 i
0,78,45	73.232936103088079 + 88.309149959399946 i	4457.7154226922739 +6193.2623423219447 i	0,78,47	70.137143134796816 + 90.972074574877254 i	4034.53038208726593 +6006.01224199539411 i
0,78,49	66.962689231197402 + 93.572109990777690 i	3626.38488241213645 +5794.34687719728936 i	0,78,51	63.705827790826682 + 96.110356441155787 i	3234.13042759719565 +5559.04349942137287 i
0,78,53	60.362213515132837 + 98.587716534902768 i	2858.634836137140155 +5300.846639397027502 i	0,78,55	56.926775245467468 +101.004902716120867 i	2500.789059977829311 +5020.472936285444121 i
0,78,57	53.393546926565129 +103.362440429583259 i	2161.515012990811556 +4718.615376016100527 i	0,78,59	49.755438282746539 +105.660666679317160 i	1841.774847947739545 +4395.947026523183013 i
0,78,61	46.003916008431965 +107.899723249819644 i	1542.582305934026635 +4053.124315078990094 i	0,78,63	42.128547320029405 +110.079543244913381 i	1265.017066141016130 +3690.789832997951180 i
0,78,65	38.116322630172412 +112.199828607770812 i	1010.243542135095026 +3309.574547637498199 i	0,78,67	33.950604954749871 +114.260014549413484 i	779.536498696077388 +2910.099083196861102 i
0,78,69	29.609406357472288 +116.259213516402369 i	574.317635061457167 +2492.973217559555451 i	0,78,71	25.062345140332695 +118.196124469945715 i	396.210955703710624 +2058.791372440640771 i
0,78,73	20.264706195495851 +120.068877239786090 i	247.133268345882072 +1608.117631306761723 i	0,78,75	15.144003404721998 +121.874737904346929 i	129.459250173467098 +1141.437557512888588 i
0,78,77	9.561164329171296 +123.609447521089152 i	46.380678721141386 + 658.965698621760680 i	0,78,79	3.117964146432328 +125.265078505725981 i	3.036799789095093 + 159.142322943291617 i
0,79,1	126.429248595245738 + 4.918774640803376 i	15970.110286817345695 + 953.758493274165858 i	0,79,3	124.753760727854129 + 11.052341785378806 i	15505.399180470185456 +1966.616312214551499 i
0,79,5	123.002617351511631 + 16.505233397674743 i	15013.584082275023320 +2784.553505077491081 i	0,79,7	121.182315149341349 + 21.545791155469256 i	14503.397740937462004 +3477.071164775073612 i
0,79,9	119.296597065461810 + 26.286943790445964 i	13980.105412273613183 +4074.052887791101966 i	0,79,11	117.347925346324459 + 30.791414891813311 i	13447.44867139463531 +4592.40751963012008 i
0,79,13	115.338003107711316 + 35.099202639755973 i	12908.317842118318367 +5043.114718507106772 i	0,79,15	113.268019714812539 + 39.238047546340397 i	12365.064875623525950 +5433.932607171057498 i
0,79,17	111.138784452786894 + 43.228308835141853 i	11819.673930787927020 +5770.668262572571621 i	0,79,19	108.950805960742983 + 47.085548239084716 i	11273.864354152497981 +6057.855140832227335 i
0,79,21	106.704341908008558 + 50.822028573665847 i	10729.157505816023331 +6299.148227847381349 i	0,79,23	104.399430761238378 + 54.447643364126669 i	10186.92255504068453 +6497.57049471523351 i
0,79,25	102.035911903726600 + 57.970524366146257 i	9648.40925012666230 +6655.67481528416101 i	0,79,27	99.613437617455105 + 61.397455417025292 i	9114.77221944373014 +6775.65486747268278 i
0,79,29	97.131478961379045 + 64.734164048077020 i	8587.08955231020082 +6859.42373402228168 i	0,79,31	94.589326721866470 + 67.985532783682743 i	8066.3773985340567 +6908.6712286239917 i
0,79,33	91.986088071524691 + 71.155755854408454 i	7553.6017321330291 +6924.9067385918161 i	0,79,35	89.320679199669172 + 74.248457711884171 i	7049.688061586436 +6909.491926766066 i
0,79,37	86.59181388868075 + 77.26678411898112 i	6555.529639269110 +6863.666160144223 i	0,79,39	83.797987754847398 + 80.213473090415562 i	6071.9945738257384 +6788.5666120327114 i
0,79,41	80.93745761375659 + 83.09091070852695 i	5599.932151326180 +6685.244392019485 i	0,79,43	78.008215136966228 + 85.901175348987522 i	5140.1786066892500 +6554.6776664208764 i
0,79,45	75.007953603770844 + 88.646072837215101 i	4693.5625457726692 +6397.7824667475988 i	0,79,47	71.934026074753291 + 91.327164346611497 i	4260.9101947041245 +6215.4217004033981 i
0,79,49	68.783392658844249 + 93.945788337136008 i	3843.0506433054475 +6008.4127485173958 i	0,79,51	65.552553616143925 + 96.503077447307831 i	3440.82125279214415 +5777.53394280682642 i
0,79,53	62.237463681429716 + 98.999970946381708 i	3055.07341520130331 +5523.53014500570862 i	0,79,55	58.833420955918058 +101.437223089782222 i	2686.678886321102254 +5247.117600633250731 i
0,79,57	55.334920568342298 +103.815407468004963 i	2336.536971481585169 +4948.988197681934253 i	0,79,59	51.735458293354789 +106.134917162767266 i	2005.582935609099650 +4629.813224846916455 i
0,79,61	48.027261043381358 +108.395960178487698 i	1694.798155315820395 +4290.246687029667555 i	0,79,63	44.200906937925630 +110.598549129911888 i	1405.222767527172209 +3930.928188391172625 i
0,79,65	40.244772048712235 +112.742483408984370 i	1137.971964239651015 +3552.485316244459227 i	0,79,67	36.144192087185260 +114.827320774094161 i	894.257771806209689 +3155.535309275917371 i
0,79,69	31.880127463508924 +116.852332967814647 i	675.419423993294572 +2740.685460261991011 i	0,79,71	27.426897357186537 +118.816435330250243 i	482.967959874939368 +2308.530868981374624 i
0,79,73	22.747991161149134 +120.718070189718688 i	318.656183357667317 +1859.645776740412202 i	0,79,75	17.787334926656023 +122.554998257061044 i	184.598821317627085 +1394.556590224357251 i
0,79,77	12.447357506143709 +124.323874341711445 i	83.509077213961945 + 913.648557128066727 i	0,79,79	6.511434627563156 +126.019152121355702 i	19.291834671110578 + 416.695531521482735 i
0,80,1	128.000260977858375 + 4.925847606552410 i	16369.766799431199615 + 967.488642114728691 i	0,80,3	126.325866390088819 + 11.072085175571363 i	15899.805458399448980 +1996.478414041397400 i
0,80,5	124.576581474574724 + 16.537815071052183 i	15402.516261799924905 +2828.413820626696634 i	0,80,7	122.758839777004852 + 21.591391755686757 i	14886.652446895655885 +3533.633552043215332 i
0,80,9	120.876353747531638 + 26.345743416111375 i	14357.491077588456392 +4142.373597528069695 i	0,80,11	118.931570878385102 + 30.863592498445165 i	13818.781473959475989 +4671.738168643565587 i
0,80,13	116.926189463229662 + 35.184937219857538 i	13273.419277418932325 +5132.831144088489054 i	0,80,15	114.861401374031090 + 39.337519559253707 i	12723.760098910503075 +5533.496464448323422 i
0,80,17	112.738024431337124 + 43.341701589798227 i	12171.790517847815863 +5879.603881688029105 i	0,80,19	110.556581185840457 + 47.213049181056263 i	11619.231309820190290 +6175.734470842355222 i
0,80,21	108.317348317604456 + 50.963830486898431 i	11067.604426088608307 +6425.580470304324990 i	0,80,23	106.020388375636387 + 54.603945511118674 i	10518.27888577001431 +6632.19464532943948 i
0,80,25	103.665570059136186 + 58.141533591783222 i	9972.50360484602606 +6798.15410468383743 i	0,80,27	101.252580524819265 + 61.583387239913371 i	9431.43172980899726 +6925.67249417813309 i
0,80,29	98.780931747875030 + 64.935243761697762 i	8896.13923089648051 +7016.67950641472271 i	0,80,31	96.249962120816008 + 68.201996588801236 i	8367.6394972621711 +7072.8788639715312 i
0,80,33	93.65883394700505 + 71.38785204413132 i	7846.895081395343 +7095.791647755095 i	0,80,35	91.006527125133417 + 74.496447931667177 i	7334.8273757112847 +7086.7893645169352 i
0,80,37	88.291829046410584 + 77.530944725811224 i	6832.3247735562454 +7047.1196547617228 i	0,80,39	85.51332048893370 + 80.49409663848098 i	6340.249717087327 +6977.926610622844 i
0,80,41	82.669357059662858 + 83.388307593175043 i	5859.444935642999 +6880.267073755922 i	0,80,43	79.758045474899802 + 86.215675647021129 i	5390.739112794008 +6755.123887018415 i
0,80,45	76.77721365514060 + 88.97802839014646 i	4934.952177832057 +6603.416805472102 i	0,80,47	73.72437320237889 + 91.67695114520448 i	4492.900391857352 +6426.011586781854 i
0,80,49	70.596672275518017 + 94.313809281661406 i	4065.4013864675389 +6223.7276503191451 i	0,80,51	67.390836105150271 + 96.889765580291197 i	3653.2793129319198 +5997.3446003278850 i
0,80,53	64.103091271772286 + 99.405793286140707 i	3257.37027198904707 +5747.60783963793529 i	0,80,55	60.729068215962243 +101.862685238318221 i	2878.52822123429041 +5475.23344861075111 i
0,80,57	57.263673927798379 +104.261059233278033 i	2517.63160315044161 +5180.91246342847990 i	0,80,59	53.700922808585553 +106.601359535519492 i	2175.591010683287550 +4865.314653628774977 i
0,80,61	50.033707289510211 +108.883854159250049 i	1853.358323852648937 +4529.091865836385296 i	0,80,63	46.253479009104723 +111.108627151876547 i	1551.937936925854046 +4172.880961734277662 i
0,80,65	42.349792398545581 +113.275564521728128 i	1272.401000414095051 +3797.306320299786421 i	0,80,67	38.309627438683982 +115.384331492466103 i	1015.904120823260996 +3402.981768819613080 i
0,80,69	34.116339200774469 +117.434337076657883 i	783.714889409369334 +2990.511585252344503 i	0,80,71	29.747934467825786 +119.424678744666380 i	577.248383096051725 +2560.489689593779076 i
0,80,73	25.174029100082180 +121.354053267400346 i	398.122456056943690 +2113.494742532237979 i	0,80,75	20.349908371692827 +123.220604174004265 i	248.248159231320955 +1650.074537750645277 i
0,80,77	15.203088338885337 +125.021633470606531 i	129.994727907430729 +1170.696480143635199 i	0,80,79	9.594493823266610 +126.752955222999603 i	46.548744703529984 + 675.550785861526833 i
0,80,81	3.125826070075052 +128.406801434038528 i	3.044589846334491 + 163.016507496881616 i			

Table D.1: Branch points and associated eigenvalues for $m = 0$.

D.1.2 $m = 1$, $n = [m..80]$

Accuracy was determined by the last correction of the Newton-Raphson method.

Table D.2: Branch points and associated eigenvalues for $m = 1$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
1, 1 ,1	1.998555442181652 + 4.097453662365392 i	2.915318658097470 + 6.133951063223160 i	1, 2 ,1	3.862833529248772 + 4.492300074953849 i	12.201095993517072 + 16.244079086339937 i

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
1, 3 ,1	5.594491061915554 + 4.781582485576267 i	27.134073085223168 + 27.423761751764438 i	1, 3 ,3	2.184204069300826 + 7.326156812534641 i	3.102506602797785 + 10.539212581987126 i
1, 4 ,1	7.270040170458184 + 5.010809182556227 i	47.410985357646396 + 39.426178765353157 i	1, 4 ,3	4.246372923512624 + 7.831360958311476 i	13.386757375380991 + 27.641212328450003 i
1, 5 ,1	8.915451995089990 + 5.200792891998006 i	72.896560506953961 + 52.094733920833313 i	1, 5 ,3	6.119087892218941 + 8.234638882858787 i	29.880624051048471 + 46.181278766950726 i
1, 5 ,5	2.304084540373247 + 10.504790051881750 i	3.22343102166909 + 14.78456508740481 i	1, 6 ,1	10.542501674387614 + 5.363017364859081 i	103.515790242771686 + 65.322653293710630 i
1, 6 ,3	7.901814171761899 + 8.572100476598123 i	52.112553092568930 + 65.921011346611721 i	1, 6 ,5	4.510843794687041 + 11.068777156965684 i	14.199823500075389 + 38.585841097079758 i
1, 7 ,1	12.157425518717748 + 5.504539533489089 i	139.221923002142887 + 79.032571331034433 i	1, 7 ,3	9.632074824342478 + 8.862803308220407 i	79.850651953842572 + 86.678174452737196 i
1, 7 ,5	6.497378520948285 + 11.540048903871105 i	31.833744549876009 + 64.029483325071382 i	1, 7 ,7	2.392325650572686 + 13.667125470943098 i	3.312353362945665 + 18.955959364310853 i
1, 8 ,1	13.763874712911164 + 5.630020430847810 i	179.983540743798642 + 93.165854343630244 i	1, 8 ,3	11.328299694262133 + 9.118350208514050 i	112.957518697341982 + 108.317344985330479 i
1, 8 ,5	8.372857881069370 + 11.947103162705321 i	55.547115833727254 + 90.915865684544613 i	1, 8 ,7	4.712585908410366 + 14.268247694128767 i	14.818382803267312 + 49.342071017855510 i
1, 9 ,1	15.364144468012046 + 5.742704317885289 i	225.778374983608074 + 107.676529019960242 i	1, 9 ,3	13.000893209948700 + 9.346416392190919 i	151.343076053641511 + 130.734820960417352 i
1, 9 ,5	10.180563799695939 + 12.306198709234776 i	85.039318060244113 + 119.062939528610698 i	1, 9 ,7	6.793437465093069 + 14.785372845485306 i	33.352512036522448 + 81.495894335999708 i
1, 9 ,9	2.461987524647070 + 16.822045374260245 i	3.382457695547487 + 23.081756627203863 i	1,10,1	16.959756886923902 + 5.844941740557042 i	276.589998642283888 + 122.527580467633641 i
1,10,3	14.656275969602667 + 9.552373783292623 i	194.944276890409270 + 153.848777963964721 i	1,10,5	11.942663645404307 + 12.627831816288797 i	120.125066813581512 + 148.326057038260927 i
1,10,7	8.749027039897156 + 15.241610918325712 i	58.263319271138862 + 115.263410679381295 i	1,10,9	4.875518793261963 + 17.448960842493992 i	15.317075805947175 + 59.993484725671252 i
1,11,1	18.551764580589989 + 5.938490783133454 i	332.405909055150289 + 137.688566688487924 i	1,11,3	16.298666990732507 + 9.740141886793172 i	243.714894944414766 + 177.593016007214658 i
1,11,5	13.672124078288958 + 12.919273290738930 i	160.679139122507962 + 178.589852701828477 i	1,11,7	10.625872897155935 + 15.650803076593942 i	89.199129948819362 + 150.472401258715792 i
1,11,9	7.036579565300370 + 17.999351990490915 i	34.595204714277656 + 98.753300589341109 i	1,11,11	2.519449898084327 + 19.972949349382343 i	3.440205760332667 + 27.175368910312638 i
1,12,1	20.140920533075978 + 6.024700938586217 i	393.216347726428978 + 153.134011701354439 i	1,12,3	17.930972400309869 + 9.912674099521714 i	297.619831913763051 + 201.912863512018902 i
1,12,5	15.377194798621701 + 13.185806599602564 i	206.611687570367508 + 209.760618523395408 i	1,12,7	12.448411897296973 + 16.022243129736466 i	125.937152863100550 + 186.977556888093574 i
1,12,9	9.062207249995353 + 18.492458163273412 i	60.511998736761100 + 139.253284151275210 i	1,12,11	5.012074674571113 + 20.618916807083224 i	15.734493842282545 + 70.577154354558488 i
1,13,1	21.727778657047793 + 6.104630658373134 i	459.013538130049369 + 168.842283302435760 i	1,13,3	19.555269207745162 + 10.072254073433612 i	356.631699139381510 + 226.762395892185083 i
1,13,5	17.063442736915551 + 13.431409424175248 i	257.855444145423007 + 241.760821147452134 i	1,13,7	14.231265434422278 + 16.362579957173196 i	168.323528906047642 + 224.658956929240901 i
1,13,9	11.000899587022626 + 18.940183946179314 i	92.676523819663501 + 181.334967339539531 i	1,13,11	7.242780446626386 + 21.194664094900219 i	35.646540549253452 + 115.879172795495478 i
1,13,13	2.568300292315292 + 23.121427531129132 i	3.489236429700662 + 31.244427740543561 i	1,14,1	23.312756117318234 + 6.179125414724703 i	529.791174036403959 + 184.794785403064139 i
1,14,3	21.173086729718918 + 10.220685524099080 i	420.728646512791499 + 252.102483501750015 i	1,14,5	18.734793289598922 + 13.659160057042378 i	314.358404406757359 + 274.525255742814352 i
1,14,7	15.983925161063713 + 16.676777265841398 i	216.245888823473905 + 263.416853843769957 i	1,14,9	12.878559412361513 + 19.350750224397650 i	130.835962532610978 + 224.856618282961762 i
1,14,11	9.330439063738341 + 21.716563585027686 i	62.430947663665691 + 163.018146826324909 i	1,14,13	5.129545548043640 + 23.782091876369972 i	16.093204598373720 + 81.112789163039864 i
1,15,1	24.896173461914415 + 6.248871216060073 i	605.544064799664056 + 200.975361854261308 i	1,15,3	22.785579087509738 + 10.359418740290204 i	489.892921131206833 + 277.899381738515331 i
1,15,5	20.394110919425131 + 13.871494281619029 i	376.079339618917572 + 307.998311781125551 i	1,15,7	17.712912286300956 + 16.968658931674522 i	269.618829229189828 + 303.167209544021603 i
1,15,9	14.710927293177342 + 19.730180452681697 i	174.812812633243453 + 269.697631069652743 i	1,15,11	11.324837990235599 + 22.194980627995026 i	95.665747817545551 + 211.850864778631973 i
1,15,13	7.421727356808509 + 24.377872406012361 i	36.557351660897702 + 132.914607081186091 i	1,15,15	2.610753334031841 + 26.268325538818395 i	3.531796951255998 + 35.293796264901921 i
1,16,1	26.478281311492612 + 6.314432300988091 i	686.267882690492570 + 217.369846746712739 i	1,16,3	24.393635515902606 + 10.489637718922772 i	564.109875442570228 + 304.123687194813499 i
1,16,5	22.043544438355521 + 14.070375221642998 i	442.98489237048545 + 342.13198784612200 i	1,16,7	19.422899982600057 + 17.241243675287226 i	328.375460209661810 + 343.838311569744064 i
1,16,9	16.508367692187985 + 20.083064524460742 i	224.475648587231092 + 315.755596992282221 i	1,16,11	13.252873887252669 + 22.637223811871610 i	135.073623088408510 + 262.241359902422914 i
1,16,13	9.564962959124538 + 24.922954240540911 i	64.104505264001375 + 186.629252102566767 i	1,16,15	5.232573449523292 + 26.940700137094073 i	16.407545397042004 + 91.612158619545042 i
1,17,1	28.059278608318817 + 6.376278315290989 i	771.958978682370798 + 233.965718784811031 i	1,17,3	25.997953428561853 + 10.612321859332116 i	643.367264083170539 + 330.749548102702363 i
1,17,5	23.684742981234820 + 14.257409775041946 i	515.047615903029737 + 376.884418914903108 i	1,17,7	21.117348550987550 + 17.496962135827737 i	392.462139930672890 + 385.368252211809931 i
1,17,9	18.278094539468061 + 20.413001762180922 i	279.723410724933501 + 362.942886837999538 i	1,17,11	15.131073682250650 + 23.048747207889637 i	180.457288788781767 + 314.070569223294854 i
1,17,13	11.609922651340975 + 25.426468705545723 i	98.287608917575938 + 242.129166988291274 i	1,17,15	7.579741774656437 + 27.552747191916115 i	37.360581462396652 + 149.883900044679876 i
1,17,17	2.648270598403898 + 29.414135460952782 i	3.569370217334297 + 39.326818867030951 i	1,18,1	29.639325398202242 + 6.434804309050247 i	862.614245409236105 + 250.751831265781334 i
1,18,3	27.599088273906524 + 10.728290686258543 i	727.654732696992937 + 357.754055909515557 i	1,18,5	25.318996723261769 + 14.433930840993449 i	592.244604366285216 + 412.218761752423859 i
1,18,7	22.798888474488290 + 17.737803753475124 i	461.835020722791057 + 427.703037525625746 i	1,18,9	20.025339146902052 + 20.722878266565978 i	340.476139115135936 + 411.183791154516996 i
1,18,11	16.970434203549889 + 23.433774245657908 i	231.669047780753971 + 367.235494303315363 i	1,18,13	13.584203421303766 + 25.894957524550929 i	138.809022327069584 + 299.285272830412623 i
1,18,15	9.773267447529681 + 28.116939469525981 i	65.588201018509196 + 210.128999929566760 i	1,18,17	5.324294883357478 + 30.096077778718858 i	16.687188868372196 + 102.082887897707252 i
1,19,1	31.218551974371358 + 6.490345716684749 i	958.231013397346942 + 267.718198060157306 i	1,19,3	29.197488431952713 + 10.838236937944201 i	816.963438237263290 + 385.116768730981505 i
1,19,5	26.947331712313886 + 14.601056861366935 i	674.556508302782959 + 448.102335996481941 i	1,19,7	24.469563198879342 + 17.965419568931296 i	536.457689198115986 + 470.795149422146778 i
1,19,9	21.754017478204943 + 21.015050213026598 i	406.669132698773581 + 460.412276004441253 i	1,19,11	18.778684655430968 + 23.795664217635931 i	288.594245362918230 + 421.646779032665052 i
1,19,13	15.504902346862567 + 26.333373436083104 i	185.454706083693736 + 357.981366112505782 i	1,19,15	11.864449656630457 + 28.641406116031519 i	100.622681379384531 + 272.235427477838016 i
1,19,17	7.721177736803752 + 30.721620828533432 i	38.078811229389373 + 166.802562404032329 i	1,19,19	2.681866490794754 + 32.559163519807171 i	3.602984975659500 + 43.345924445798872 i
1,20,1	32.797065542238727 + 6.543189758456037 i	1058.806971254923837 + 284.855821791378673 i	1,20,3	30.793520186823034 + 10.942751486845985 i	911.285761619416576 + 412.819332696167294 i
1,20,5	28.570575623009113 + 14.759735867943657 i	761.966809826925441 + 484.505951538866782 i	1,20,7	26.130989292577755 + 18.181196193361103 i	616.299499618637356 + 514.602435740765031 i
1,20,9	23.467136357863374 + 21.291470044919034 i	478.249077451358232 + 510.570241759189514 i	1,20,11	20.561481100110765 + 24.137144940629142 i	351.141409914567248 + 477.226374851620051 i
1,20,13	17.383486461537057 + 26.745601579745127 i	238.062942889999221 + 418.115171928915345 i	1,20,15	13.881394817881253 + 29.132043647836738 i	142.149285290210429 + 336.081530339525045 i
1,20,17	9.960595420570877 + 31.301866869030466 i	66.920591342091171 + 233.544519683540443 i	1,20,19	5.406927663811995 + 33.249081378715942 i	16.938966187853950 + 112.530225148535308 i
1,21,1	34.374955158875181 + 6.593584242497837 i	1164.340103427789018 + 302.156554303854421 i	1,21,3	32.387485947471077 + 11.042342407056635 i	1010.615086679172919 + 440.845177212603573 i
1,21,5	30.189404472633456 + 14.910778677397268 i	854.461277910156468 + 521.403374181271261 i	1,21,7	27.784465649979300 + 18.386310288351190 i	701.334364087998551 + 559.087240861878286 i
1,21,9	25.167053276792124 + 21.553776208465788 i	555.171375944207512 + 561.606166370418691 i	1,21,11	22.323093277987649 + 24.460468643606411 i	419.235979786427379 + 533.905591106962561 i
1,21,13	19.228076629232223 + 27.134768194969091 i	296.507194138681712 + 479.596843440543694 i	1,21,15	15.841624389897802 + 29.593368303979499 i	189.939627645692411 + 401.554745001292190 i
1,21,17	12.094310185816422 + 31.844120122499337 i	102.727513153353740 + 302.211879607458968 i	1,21,19	7.849161506738880 + 33.886019058164173 i	38.728197092598990 + 183.681083384993656 i
1,21,21	2.712273336393179 + 35.703610446734814 i	3.633383461812271 + 47.352950610116103 i	1,22,1	35.952295449019111 + 6.641744447453395 i	1274.828641044915230 + 319.612982177181664 i
1,22,3	33.979637772417711 + 11.137449769349606 i	1114.945627660412356 + 469.179266909140381 i	1,22,5	31.804376519162474 + 15.054884317674874 i	952.027551655028446 + 558.770895660717542 i
1,22,7	29.431050047411777 + 18.581769498158692 i	791.539853738461716 + 604.215715367652365 i	1,22,9	26.855649285223180 + 21.803358670707023 i	637.398245276626881 + 613.474038117180839 i
1,22,11	24.066825440560706 + 24.767520711219291 i	492.816109692215434 + 591.623525834834709 i	1,22,13	21.044658042052507 + 27.503438447873204 i	360.685801190698044 + 542.347128131435170 i
1,22,15	17.756962852999742 + 30.028960502616634 i	243.819808373225584 + 468.554367555879651 i	1,22,17	14.150809003296544 + 32.353731223635137 i	145.170278219439865 + 372.690624184407625 i
1,22,19	10.130761581939517 + 34.479961032393035 i	68.129558745714945 + 256.894143258396809 i	1,22,21	5.482097314068398 + 36.400286560333214 i	17.167879114868018 + 122.957954869077284 i
1,23,1	37.529149438699356 + 6.687858566860297 i	1390.271022661357847 + 337.218331910078632 i	1,23,3	35.570187563797248 + 11.228457266476999 i	1224.272292833019264 + 497.807897635043958 i
1,23,5	33.415957317447507 + 15.192659785865144 i	1054.654816466806551 + 596.586983656654070 i	1,23,7	31.071614100175825 + 18.768443738500906 i	886.896517558557889 + 649.957260970461334 i
1,23,9	28.534447865999642 + 22.041407825531476 i	724.897325618866603 + 666.132506737599215 i	1,23,11	25.795287252247405 + 25.059897974180057 i	571.829756138576272 + 650.325811844869122 i
1,23,13	22.837779275356027 + 27.853750312271664 i	430.515383409715184 + 606.295709738103028 i	1,23,15	19.635834433468990 + 30.441729136753914 i	303.652832227639180 + 536.990898596003717 i
1,23,17	16.147936320024574 + 32.834837157703326 i	194.008098590786346 + 444.872283060190455 i	1,23,19	12.303840789938272 + 35.037523758442617 i	104.643368046454299 + 332.087191070341476 i
1,23,21	7.966013303899047 + 37.046984645166198 i	39.320696966634131 + 200.526882928837892 i	1,23,23	2.740036829016563 + 38.847613408206669 i	3.661118380447543 + 51.349332394995706 i
1,24,1	39.105570742667588 + 6.732092061345869 i	1510.665862583708304 + 354.966390725554902 i	1,24,3	37.159314857426764 + 11.315701452096667 i	1338.590575420897494 + 526.718527129600943 i
1,24,5	35.024538545162478 + 15.324635599368614 i	1162.333548915623624 + 634.831993936331855 i	1,24,7	32.706883532482891 + 18.947089478397739 i	987.387357181783955 + 696.284079086359611 i
1,24,9	30.204698936271694 + 22.268951694591394 i	817.640639060174408 + 719.544200687134776 i	1,24,11	27.510575212866730 + 25.338966537291210 i	656.232586969964814 + 709.963611563178119 i
1,24,13	24.610982581046891 + 28.187508900290517 i	505.926394146923371 + 671.379816546492975 i	1,24,15	21.484483510469300 + 30.834082011465984 i	369.327980016545439 + 606.784677746390457 i
1,24,17	18.097793559626471 + 33.290746599049896 i	249.056405652329516 + 518.658273109354582 i	1,24,19	14.397176903207539 + 35.563592659625101 i	147.927791184479446 + 409.153860348497346 i
1,24,21	10.286628300595449 + 37.652762646303024 i	69.235927520005404 + 280.190798668082380 i	1,24,23	5.551030417665980 + 39.550095087737589 i	17.377698579913339 + 133.368908886715019 i
1,25,1	40.681605271995529 + 6.774591171299413 i	1636.011925074386909 + 372.851439909178794 i	1,25,3	38.747172850298092 + 11.399479159193781 i	1457.896465456906753 + 555.899633282568516 i
1,25,5	36.630452355315249 + 15.451278177755773 i	1275.055313231181344 + 673.487931378754411 i	1,25,7	34.337468219937662 + 19.118368838231206 i	1092.997415761591128 + 743.170799875308603 i
1,25,9	31.867439609652777 + 22.486884690167656 i	915.603795354855720 + 773.675172114498878 i	1,25,11	29.214398166763765 + 25.605905414093947 i	745.986442325101279 + 770.492805527048972 i
1,25,13	26.367082178014211 + 28.506255038476770 i	586.860012498804281 + 737.543078922368181 i	1,25,15	23.307681497079422 + 31.208042023409071 i	440.753953524162455 + 677.864504622879281 i
1,25,17	20.009043812990136 + 33.724169506300882 i	310.168945945851135 + 593.960018075347648 i	1,25,19	16.428864936428972 + 36.061974549940260 i	197.731223752675703 + 487.990021742763361 i
1,25,21	12.496327218807485 + 38.223656827541340 i	106.401302722700763 + 361.881633679988568 i	1,25,23	8.073502243827479 + 40.205255917073707 i	39.865408814474819 + 217.345403229149923 i
1,25,25	2.765574317923565 + 41.991269378962144 i	3.686612143001886 + 55.336219130137493 i	1,26,1	42.257292580820810 + 6.815485777033378 i	1766.30810316380757 + 390.86819829693724 i
1,26,3	40.333893117274663 + 11.480053512915696 i	1582.186377872244748 + 585.340594575776322 i	1,26,5	38.233982462368452 + 15.572999804303235 i	1392.812597180665307 + 712.538249882936839 i
1,26,7	35.963884960219026 + 19.282864791219936 i	1203.713451822756497 + 790.594174516732210 i	1,26,9	33.523539043727354 + 22.695990174118833 i	1018.765375762311464 + 828.494441128517337 i
1,26,11	30.908166524621922 + 25.861740053807129 i	841.058176284926276 + 831.873331627964717 i	1,26,13	28.108351499308970 + 28.811316392651879 i	673.265900917226304 + 804.734595566111466 i
1,26,15	25.109163373274360 + 31.565329641367793 i	517.854234735155160 + 750.166421703906281 i	1,26,17	21.888138509444715 + 34.137365769656299 i	377.226901259754072 + 670.698016997005766 i
1,26,19	18.411224453770643 + 36.535736560351140 i	253.859688378072207 + 568.499566955271575 i	1,26,21	14.624114447425482 + 38.764153087025029 i	150.464094913495671 + 445.500580273700564 i
1,26,23	10.430398280541020 + 40.821375037188474 i	70.255664163766340 + 303.443922876383454 i	1,26,25	5.614675083396651 + 42.698796259250791 i	17.571337164675139 + 143.765270663422257 i
1,27,1	43.832666938548734 + 6.854891746592612 i	1901.553401110442110 + 409.011774053096588 i	1,27,3	41.919589340305798 + 11.557658846227115 i	1711.457093321709408 + 615.031588523226446 i
1,27,5	39.835372807096022 + 15.690166716420769 i	1515.598678384166078 + 751.967683529235046 i	1,27,7	37.586574977432491 + 19.441093393956471 i	1319.523677427402617 + 838.532817737311234 i
1,27,9	35.173732129604488 + 22.896958376252041 i	1127.106447963404207 + 883.973618109988593 i	1,27,11	32.593057174390388 + 26.107368508018187 i	941.418768407038897 + 894.068642350539221 i
1,27,13	29.836653334358093 + 29.103846385427115 i	765.100545423660719 + 872.908168317317404 i	1,27,15	26.891910983220022 + 31.907423325233705 i	600.563822683492731 + 823.632684880049851 i
1,27,17	23.740029392106759 + 34.532247054218722 i	450.131836670914254 + 748.800727837690260 i	1,27,19	20.353109357200626 + 36.987408158337523 i	316.158686796923860 + 650.595240440859811 i
1,27,21	16.688282647434750 + 39.277769683602770 i	201.163193179315158 + 530.947966933063185 i	1,27,23	12.674318952845693 + 41.403994022040500 i	108.025288229541530 + 391.610023617676973 i
1,27,25	8.173007882887974 + 43.361371135159260 i	40.369414793903552 + 234.140756443580443 i	1,27,27	2.789212023978915 + 45.134648892737857 i	3.710194690947565 + 59.314550474765369 i
1,28,1	45.407758190928689 + 6.892912877469706 i	2041.746919775451261 + 427.277623272774633 i	1,28,3	43.504360284899242 + 11.632504750489220 i	1845.70570910275683 + 644.96350484067875 i
1,28,5	41.434834402024251 + 15.803105733371236 i	1643.407514434846235 + 791.762103097296290 i	1,28,7	39.205917549489065 + 19.593513722657327 i	1440.417545717198664 + 886.966990685674546 i
1,28,9	36.818645195891627 + 23.090400788263427 i	1240.610179168522109 + 940.086588016949008 i	1,28,11	34.270061657512990 + 26.343582134130628 i	1047.042630995244053 + 957.045255092307798 i
1,28,13	31.553532767794714 + 29.384854348042026 i	862.326000168116821 + 942.021671546927109 i	1,28,15	28.658344324172340 + 32.235604906293254 i	688.826867654334819 + 898.210904689954305 i
1,28,17	25.568609230734744 + 34.910449514403666 i	528.800903783163221 + 828.203517838413177 i	1,28,19	22.261138591165530 + 37.419112240559315 i	384.502126619167541 + 734.198108909093435 i
1,28,21	18.701325708748769 + 39.767359020317826 i	258.296268844908298 + 618.130382798691130 i	1,28,23	14.834448306076364 + 41.957253045605810 i	152.812006445052219 + 481.752301679533218 i
1,28,25	10.563802739556564 + 43.986610506321924 i	71.201280682900801 + 326.660603479235704 i	1,28,27	5.673779171284822 + 45.846603904052081 i	17.751090728588967 + 154.148765930314141 i
1,29,1	46.982592457162511 + 6.929642513641266 i	2186.88784434577337 + 445.66151424402523 i	1,29,3	45.088292196549389 + 11.704779437642360 i	1984.929598149584990 + 675.127870764340173 i
1,29,5	43.032550792684063 + 15.912109728867810 i	1776.233651838231481 + 831.908393334753441 i	1,29,7	40.822240739048198 + 19.740536017095049 i	1566.385576851733049 + 935.878416508416421 i
1,29,9	38.458815892793879 + 23.276861844379411 i	1359.261523943044040 + 996.809244422829124 i	1,29,11	35.940022552779290 + 26.571082175372627 i	1157.907061159498653 +1020.772376545369298 i
1,29,13	33.260285096941452 + 29.655229237753993 i	964.908920052506119 +1012.036528924018512 i	1,29,15	30.410454971263441 + 32.550994381252828 i	782.594912001736361 + 973.853331741085981 i
1,29,17	27.376998560529344 + 35.273387375710008 i	613.163452732293990 + 908.847744830538204 i	1,29,19	24.140411480554452 + 37.832655261794372 i	458.785114732738293 + 819.236690309049322 i
1,29,21	20.672251912044192 + 40.235287404114949 i	321.701441789900002 + 706.962166961421758 i	1,29,23	16.929237200262449 + 42.484416996144930 i	204.346294150603200 + 573.775575789010759 i
1,29,25	12.839834032266133 + 44.579630085573456 i	109.534218766299276 + 421.283489378627735 i	1,29,27	8.265626632445951 + 46.515732458595920 i	40.838335318363100 + 250.916128086183903 i
1,29,29	2.811209699940480 + 48.277804500440077 i	3.732128554013248 + 63.285107915790299 i	1,30,1	48.557192698498596 + 6.965164900919760 i	2336.975433964438707 + 464.159496434998232 i
1,30,3	46.671460745722790 + 11.774652549900424 i	2129.126374543070597 + 705.516786463153267 i	1,30,5	44.628682453827295 + 16.017442182912681 i	1914.07214997619485 + 872.39434734267843 i
1,30,7	42.435829931658311 + 19.882528409585515 i	1697.419214152124596 + 985.250122662137528 i	1,30,9	40.094708765621668 + 23.456828487548439 i	1483.046969687172805 +1054.119263822366293 i
1,30,11	37.603661378878037 + 26.790493182457613 i	1273.991801855396488 +1085.221586524868371 i	1,30,13	34.958006458842959 + 29.915758542173986 i	1072.819803232510139 +1082.917276170756168 i
1,30,15	32.149901698911933 + 32.854577055769148 i	881.825553834814968 +1050.516260667823417 i	1,30,17	29.167739812824230 + 35.622293420665414 i	703.158565493792707 + 990.679973707381464 i
1,30,19	25.994951278080368 + 38.229591855836581 i	538.919035863412887 + 905.646062746591497 i	1,30,21	22.607817273486255 + 40.683551818454220 i	391.246056806332092 + 797.365249051013279 i
1,30,23	18.971320628721987 + 42.988147022251049 i	262.418366354232887 + 667.589265513830583 i	1,30,25	15.030430415721426 + 45.144268709923608 i	154.997525279070544 + 517.925226107244498 i
1,30,27	10.688226927368174 + 47.149081208374781 i	72.082764565822117 + 349.846292574015820 i	1,30,29	5.728942879151541 + 48.993679621276108 i	17.91880069558484 + 164.52078722453280 i
1,31,1	50.131579185239232 + 6.999556329808445 i	2492.009012921105295 + 482.767873450889169 i	1,31,3	48.253932618240569 + 11.842277522519568 i	2278.29386432040792 + 736.12286889340737 i

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
1,31,5	46.223370357642040 + 16.119340993366411 i	2056.918517174814497 + 913.208575184005554 i	1,31,7	44.046934693673855 + 20.019822526188229 i	1833.510704257142152 +1035.066305260469790 i
1,31,9	41.726727589026396 + 23.630738069617358 i	1611.954327165064115 +1111.995912823424533 i	1,31,11	39.261600286133088 + 27.002373984223333 i	1395.278686219151496 +1150.366569922967756 i
1,31,13	36.647632226559097 + 30.167143523983402 i	1186.032389539840096 +1154.631193086189114 i	1,31,15	33.878080604175103 + 33.147225036247498 i	986.481413220400896 +1128.159530652653757 i
1,31,17	30.942933094354700 + 35.958250213595640 i	798.733214407374476 +1073.651313974916775 i	1,31,19	27.827994295700570 + 38.611272672196421 i	624.828063256485734 + 993.367052304543677 i
1,31,21	24.513243126533201 + 41.113860387924746 i	466.819354457870645 + 889.268506718791286 i	1,31,23	20.969833198098954 + 43.470662062440525 i	326.859700830496138 + 763.109491235266660 i
1,31,25	17.154171028598004 + 45.683567680547071 i	207.314181862033308 + 616.495100540242788 i	1,31,27	12.994497195715912 + 47.751396066529658 i	110.943253628873236 + 450.910583988607186 i
1,31,29	8.352244178319348 + 49.668646717550445 i	41.276705018487256 + 267.674038559648068 i	1,31,31	2.831777491394996 + 51.420776171659845 i	3.752625978264946 + 67.248550831520021 i
1,32,1	51.705769882808021 + 7.032886104592763 i	2651.987963127629428 + 501.483179345719634 i	1,32,3	49.835766824736922 + 11.907793582373499 i	2432.430080627133520 + 766.939202758698002 i
1,32,5	47.816738892081837 + 16.218021687619327 i	2204.768656604224816 + 954.340424390794510 i	1,32,7	45.655774328484320 + 20.152718181058049 i	1974.652996552512150 +1085.312211716884349 i
1,32,9	43.355225235039363 + 23.798984925617319 i	1745.972556634998909 +1170.419882411936669 i	1,32,11	40.914379126659185 + 27.207226733697624 i	1521.751346445781337 +1216.182887943245760 i
1,32,13	38.329966575621398 + 30.410011642753862 i	1304.523176734874824 +1227.147991751571453 i	1,32,15	35.596177558729600 + 33.429714467582144 i	1096.529319659226303 +1206.746104520703284 i
1,32,17	32.704333767001415 + 36.282214599755072 i	899.840858156785771 +1157.716860366940182 i	1,32,19	29.642186055004576 + 38.978880693859637 i	716.446608993422615 +1082.345523111411239 i
1,32,21	26.392661609477965 + 41.527690244761160 i	548.327449223358713 + 982.607023643195362 i	1,32,23	22.931644228338278 + 43.933843796117991 i	397.531696327410373 + 860.259259019735737 i
1,32,25	19.223806384617891 + 46.200017315137302 i	266.267710494649066 + 716.905433893941904 i	1,32,27	15.213884083813641 + 48.326251436530441 i	157.041606408684657 + 554.031826122122197 i
1,32,29	10.804796476345398 + 50.309257759511311 i	72.908215216858153 + 373.005270724616025 i	1,32,31	5.780655148152847 + 52.140147897991248 i	18.075966317373087 + 174.882478167502825 i
1,33,1	53.279780772848881 + 7.065217369411299 i	2816.911717658768794 + 520.302157784818860 i	1,33,3	51.417015785790475 + 11.971327447717945 i	2591.533202451582838 + 797.959297484825015 i
1,33,5	49.408898264596608 + 16.313680144531748 i	2357.61882022633836 + 995.77991048885701 i	1,33,7	47.262542414106854 + 20.281487335670928 i	2120.839658205836713 +1135.974038683418939 i
1,33,9	44.980511641128487 + 23.961925883152567 i	1885.091622404789405 +1229.373144671110140 i	1,33,11	42.562469035856964 + 27.405504426853327 i	1653.394973299385811 +1282.647781645665418 i
1,33,13	40.005705560429419 + 30.644926771322439 i	1428.271027161584698 +1300.439550627016160 i	1,33,15	37.305208123926796 + 33.702739516618330 i	1211.939664556275147 +1286.241711943627706 i
1,33,17	34.453424206936539 + 36.595037211020561 i	1006.440351812107565 +1242.835219417052940 i	1,33,19	31.439718772570529 + 39.333459382457079 i	813.717414019463263 +1172.531766848656616 i
1,33,21	28.249406474982095 + 41.926330515890804 i	635.689757403219801 +1077.321378656976549 i	1,33,23	24.862076844192533 + 44.379309057973714 i	474.318245513383715 + 958.968030850801256 i
1,33,25	21.248575515498128 + 46.695703349318855 i	331.683585002614564 + 819.074252078999280 i	1,33,27	17.365071961994488 + 48.876490327590489 i	210.094092025758727 + 659.123716962730896 i
1,33,29	13.139635833155891 + 50.919935019829531 i	112.26474218658551 + 480.49801147872000 i	1,33,31	8.433586153197796 + 52.820352283290902 i	41.688235170662345 + 284.416518574374948 i
1,33,33	2.851087779043231 + 54.563594856476847 i	3.771860951637443 + 71.205442467728957 i	1,34,1	54.853626121814060 + 7.096607815888505 i	2986.779755180981283 + 539.221743642087063 i
1,34,3	52.997726236592840 + 12.032994781219310 i	2755.601556333710050 + 829.177049314256124 i	1,34,5	50.999946495050331 + 16.406494913835772 i	2515.465569376896838 +1037.517656005356685 i
1,34,7	48.867410536227879 + 20.406377457646986 i	2272.064801939884546 +1187.038842859299621 i	1,34,9	46.602860298610366 + 24.119884909302964 i	2029.302370296946947 +1288.838828253447218 i
1,34,11	44.206283346209943 + 27.597617197580494 i	1790.196115786930477 +1349.740002263782344 i	1,34,13	41.675455343716216 + 30.872397669325153 i	1557.256844703324532 +1374.479686399759657 i
1,34,15	39.006048388417696 + 33.966923830615656 i	1332.685878813728550 +1366.614545193177491 i	1,34,17	36.191467629107073 + 36.897478194093665 i	1118.495087366605276 +1328.968107194108825 i
1,34,19	33.222430383194322 + 39.675934866681757 i	916.590087172084054 +1263.879980652776567 i	1,34,21	30.086210688196701 + 42.310915082443799 i	728.836378780420091 +1173.357006734441484 i
1,34,23	26.765354799477798 + 44.808462024972094 i	557.120631672673198 +1059.171122367434236 i	1,34,25	23.235442257151696 + 47.172402018493464 i	403.417795632418461 + 922.925791167644775 i
1,34,27	19.460906890958405 + 49.404449965475365 i	269.878195012068090 + 766.101507754861586 i	1,34,29	15.386306354614870 + 51.504019386187295 i	158.961389931493784 + 590.081887561143276 i
1,34,31	10.914438396801355 + 53.467508262118926 i	73.684291575219987 + 396.140958532991021 i	1,34,33	5.829319511984671 + 55.286106243810138 i	18.223824329042792 + 185.234792241934286 i
1,35,1	56.427318706833162 + 7.127110292121849 i	3161.591595125851200 + 558.239046686838151 i	1,35,3	54.577939985433702 + 12.092901438025926 i	2924.633600558177477 + 860.586707780247366 i
1,35,5	52.589971078181108 + 16.496629202551098 i	2678.305740852693361 +1079.544836699419875 i	1,35,7	50.470531380327706 + 20.527614385784076 i	2428.32302428156527 +1238.49446269280179 i
1,35,9	48.222513577323390 + 24.273157053555167 i	2178.596423737754776 +1348.801109615638552 i	1,35,11	45.846186437139039 + 27.783937623417810 i	1932.142513014877295 +1417.439663857814594 i
1,35,13	43.339746754240096 + 31.092885065200214 i	1691.463307074177692 +1449.243957101939941 i	1,35,15	40.699459114307651 + 34.222830011836088 i	1458.744006737246309 +1447.834998206668843 i
1,35,17	37.919549159137333 + 37.190220034580399 i	1235.972306694635790 +1416.080005888589540 i	1,35,19	34.991877523208878 + 40.007133690938531 i	1025.019967538130686 +1356.347820786079183 i
1,35,21	31.905345158666705 + 42.682448067032063 i	827.706129347875162 +1270.663639277110633 i	1,35,23	28.644894471102965 + 45.222533086449786 i	645.853888573357153 +1160.809087763535899 i
1,35,25	25.189815363070888 + 47.631645703523816 i	481.349343175334639 +1028.390265989735326 i	1,35,27	21.510715016246163 + 49.912095488193280 i	336.213939029324933 + 874.885209091020897 i
1,35,29	17.563579644358523 + 52.064177152771329 i	212.708383436509138 + 701.674929074614283 i	1,35,31	13.276348341823823 + 54.085752851633679 i	113.508878648938861 + 510.051116715273452 i
1,35,33	8.510254452720735 + 55.971037289443338 i	42.07600139436587 + 301.14523028606965 i	1,35,35	2.869283692164964 + 57.706284900124842 i	3.789977845553834 + 75.156269103059543 i
1,36,1	58.000870006616267 + 7.156773329063768 i	3341.346793491121844 + 577.351337072102410 i	1,36,3	56.157694553000752 + 12.151144542628783 i	3098.627911433963163 + 892.182845945543590 i
1,36,5	54.179050378353914 + 16.584232584302995 i	2846.136417595094372 +1121.853133974482716 i	1,36,7	52.072041309180626 + 20.645404786349047 i	2589.609352485463660 +1290.329449354615008 i
1,36,9	49.839687126671372 + 24.422011812020472 i	2332.96609510637245 +1409.24511758307426 i	1,36,11	47.482500970917179 + 27.964805225153693 i	2079.222952072605040 +1485.728114722566033 i
1,36,13	44.999047027015173 + 31.306807616725838 i	1830.874642145730604 +1524.709491299811816 i	1,36,15	42.386104871579676 + 34.470967515859336 i	1590.092355073178851 +1529.875441560223635 i
1,36,17	39.638607650470831 + 37.473878116741311 i	1358.842545662226485 +1504.137869168320094 i	1,36,19	36.749390396135929 + 40.327797191876009 i	1138.967224797446761 +1449.896020254260293 i
1,36,21	33.708718836118308 + 43.041824006509970 i	932.245032218976235 +1369.194819016134716 i	1,36,23	30.503503501665275 + 45.622608554262406 i	740.444125026074516 +1263.827148493429604 i
1,36,25	27.115996253090710 + 48.074770305311247 i	565.375069166214416 +1135.403398716718129 i	1,36,27	23.521539671296104 + 50.401106702407674 i	408.952376642130901 + 985.400396153860584 i
1,36,29	19.684381522490028 + 52.602607871006171 i	273.277736542771549 + 815.195315895845056 i	1,36,31	15.548941494431858 + 54.678219519738320 i	160.771076298459576 + 626.083215515966799 i
1,36,33	11.017925158071442 + 56.624125013797901 i	74.416534443053024 + 419.256131828852176 i	1,36,35	5.875272859140388 + 58.431632163970143 i	18.363406729708220 + 195.578534863085849 i
1,37,1	59.574290363571855 + 7.185641597377804 i	3526.044939173310468 + 596.556032382633475 i	1,37,3	57.737023713913359 + 12.207813421945817 i	3277.583171335625593 + 923.960333891483060 i
1,37,5	55.767254806002313 + 16.669442476758402 i	3018.954903233067853 +1164.434692606643466 i	1,37,7	53.672062524084069 + 20.759938268978292 i	2755.919198686658092 +1342.533005643465458 i
1,37,9	51.454573537437653 + 24.566696012742916 i	2492.40430967928478 +1470.15684924988705 i	1,37,11	49.115513854164725 + 28.140530304610110 i	2231.427147142668218 +1554.587824638529810 i
1,37,13	46.653769356357479 + 31.514546960004512 i	1975.476439666959777 +1600.854839156961589 i	1,37,15	44.066569365131926 + 34.711799284798623 i	1726.711201357770506 +1612.710027388236313 i
1,37,17	41.349460663609463 + 37.749009495696917 i	1487.079179484195038 +1593.110868099131267 i	1,37,19	38.496114732958020 + 40.638593271928855 i	1258.396138088666476 +1544.488060078842636 i
1,37,21	35.497952617005254 + 43.389844051951883 i	1042.405139334895466 +1468.907481017974760 i	1,37,23	32.343520448108151 + 46.009653851019223 i	840.826509450228173 +1368.174682581922298 i
1,37,25	29.017471846800344 + 48.502950598591482 i	655.405959504893644 +1243.905894013817033 i	1,37,27	25.498862185376776 + 50.872937986928022 i	487.968019544960441 +1097.578162701016709 i
1,37,29	21.758111411247643 + 53.121167256964370 i	340.484505989178160 + 930.565089648824255 i	1,37,31	17.751062461313531 + 55.247416340184131 i	215.175641283287348 + 744.159524886455415 i
1,37,33	13.405553900818996 + 57.249253389920980 i	114.684176314253449 + 539.574224795114446 i	1,37,35	8.582753812679279 + 59.120852298861443 i	42.442580777963016 + 317.861553089545112 i
1,37,37	2.886485355080955 + 60.848865720080532 i	3.807097752589699 + 79.101454486174262 i	1,38,1	61.147589122098694 + 7.213756305495686 i	3715.685650752680431 + 615.850686038758340 i
1,38,3	59.315957957599132 + 12.262990417066504 i	3461.498158239266559 + 955.914315025425481 i	1,38,5	57.354647814934296 + 16.752385423997236 i	3196.758699884460839 +1207.282083065170127 i
1,38,7	55.270704887055260 + 20.871389217504813 i	2927.248320114430044 +1395.094931711292929 i	1,38,9	53.067345407346309 + 24.707436302234171 i	2656.904540041229211 +1531.523095569618590 i
1,38,11	50.745481183966760 + 28.311397235337825 i	2388.74563605822686 +1624.00228558241679 i	1,38,13	48.304280732981873 + 31.716452011700053 i	2125.255491690355939 +1677.659841959026608 i
1,38,15	45.741367832148370 + 34.945747355849043 i	1868.582549598473861 +1696.314519423302954 i	1,38,17	43.052824312171687 + 38.016120242790407 i	1620.658047276995603 +1682.970170959707502 i
1,38,19	40.233044369842091 + 40.940126135109807 i	1383.274511030622307 +1640.089885546167579 i	1,38,21	37.274435005844673 + 43.727229141687367 i	1158.143597268604253 +1569.761590716864475 i
1,38,23	34.166915679660946 + 46.384531923403458 i	946.943719958549082 +1473.804775675338078 i	1,38,25	30.897113963024349 + 48.917227316639554 i	751.364421281559430 +1353.842932057347980 i
1,38,27	27.447050272443981 + 51.328861558849610 i	573.153614140673540 +1211.354772237509223 i	1,38,29	23.791880013731589 + 53.621446856471820 i	414.175220986048998 +1047.711240454995277 i
1,38,31	19.895704203300724 + 55.795418617426535 i	276.489609685260120 + 864.201131306005436 i	1,38,33	15.702834868588836 + 57.849370648509534 i	162.482563467581372 + 662.042125448222805 i
1,38,35	11.115907188721787 + 59.779343229427930 i	75.109603408344000 + 442.353073960301913 i	1,38,37	5.918799316980325 + 61.576788067752744 i	18.495583515794788 + 205.914394182411257 i
1,39,1	62.720774747055010 + 7.241155547720874 i	3910.268573664834548 + 635.232976883351512 i	1,39,3	60.894524882268527 + 12.316751592110768 i	3650.371736532597424 + 988.040184841600762 i
1,39,5	58.941286751790024 + 16.833178213977895 i	3379.545488721816128 +1250.388267816731286 i	1,39,7	56.868067465126622 + 20.979918380772450 i	3103.592784415691312 +1448.005576678186571 i
1,39,9	54.678157922215185 + 24.844441298043066 i	2826.460749248337834 +1593.331375271274510 i	1,39,11	52.372632378288486 + 28.477667298167944 i	2551.16969130424377 +1693.95592393283255 i
1,39,13	49.950908421780177 + 31.912842655117561 i	2280.199656475226675 +1755.105517307373248 i	1,39,15	47.410957158956535 + 35.173197632974556 i	2015.689924099463724 +1780.666144227853010 i
1,39,17	44.749329200387741 + 38.275671640663425 i	1759.557139295424122 +1773.688751516395087 i	1,39,19	41.961046880837060 + 41.232944407696252 i	1513.573192119997884 +1736.669660220690730 i
1,39,21	39.039364762525302 + 44.054630830153975 i	1279.421896580150587 +1671.719830563433561 i	1,39,23	35.975365985129500 + 46.748018078287885 i	1058.744731281379472 +1580.673829093080258 i
1,39,25	32.757319645954425 + 49.318528344262816 i	853.182226681532939 +1465.163704811493972 i	1,39,27	29.369651460215383 + 51.769999805328155 i	664.416398414081868 +1326.671218109585197 i
1,39,29	25.791230595210706 + 54.104829036086414 i	494.220261312867819 +1166.565834684903621 i	1,39,31	21.992328246604711 + 56.323982542526643 i	344.523495196894089 + 986.132129806065677 i
1,39,33	17.928674293956107 + 58.426842575058188 i	217.511483878489997 + 786.586243920344600 i	1,39,35	13.528029417305904 + 60.410763141400516 i	115.797818007343523 + 569.070881840059997 i
1,39,37	8.651511620493167 + 62.269919303137740 i	42.790153944746756 + 334.566645741275408 i	1,39,39	2.902794553552179 + 63.991352995992244 i	3.823323218806158 + 83.041370909948839 i
1,40,1	64.293854925655225 + 7.267874609710465 i	4109.793377704131672 + 654.700699804941792 i	1,40,3	62.472749533113152 + 12.369167355461951 i	3844.202848915816958 +1020.333571825773902 i
1,40,5	60.527223582804844 + 16.911928855942381 i	3567.313112894938513 +1293.746571099365838 i	1,40,7	58.464239845476804 + 21.085674260368016 i	3284.948939308796071 +1501.255795355956479 i
1,40,9	56.287151040915184 + 24.977903460122619 i	3001.067341352935273 +1655.569875961085017 i	1,40,11	53.997173646069641 + 28.639581135856492 i	2718.691243055779523 +1764.434022542552839 i
1,40,13	51.593945353192922 + 32.104012914484322 i	2440.29774173912256 +1833.17395768212668 i	1,40,15	49.075744203917047 + 35.394503969890479 i	2168.018194303309883 +1865.743460400356753 i
1,40,17	46.439533346266415 + 38.528085442638314 i	1903.756334316286484 +1865.241221315643088 i	1,40,19	43.680883987335029 + 41.517547964660587 i	1649.265677755171509 +1834.197551764163336 i
1,40,21	40.793784074601369 + 44.372640275502754 i	1406.205261491174952 +1774.747327928675759 i	1,40,23	37.770310846130862 + 47.100812086676235 i	1176.183850788804875 +1688.741218333693269 i
1,40,25	34.600112666058820 + 49.707685592177910 i	960.798928292254345 +1577.821002040579060 i	1,40,27	31.269593039414243 + 52.197350096456077 i	761.675338665786740 +1443.472777411530615 i
1,40,29	27.760587814284758 + 54.572526716284472 i	580.508587778730202 +1287.065781147463213 i	1,40,31	24.048102779370382 + 56.834618764095154 i	419.119669918407681 +1109.880976198758617 i
1,40,33	20.096122090319515 + 58.983631954138289 i	279.533426994747401 + 913.130537093672046 i	1,40,35	15.848873179459737 + 61.017894010609091 i	164.105919896110123 + 697.963792824695474 i
1,40,37	11.208937264544796 + 62.933354443030296 i	75.767454167322640 + 465.433685900638579 i	1,40,39	5.960140703271037 + 64.721624793841471 i	18.621094832596995 + 216.242964094982483 i
1,41,1	65.866836655447628 + 7.293946237442635 i	4314.259754813015206 + 674.251757271335009 i	1,41,3	64.050654693778611 + 12.420303006056209 i	4042.990509240269148 +1052.790320239872472 i
1,41,5	62.112505518224552 + 16.988737438338724 i	3760.059562471563822 +1337.350651729957235 i	1,41,7	60.059303260518803 + 21.188794325712976 i	3471.313385925255261 +1554.836909420772372 i
1,41,9	57.894451354247572 + 25.108000724220702 i	3180.719118154273474 +1718.227401453407083 i	1,41,11	55.619290918973157 + 28.797360887178030 i	2891.302812309833371 +1835.422651319357556 i
1,41,13	53.233654639163092 + 32.290233701482073 i	2605.53940396759253 +1911.84824046950303 i	1,41,15	50.736092695458851 + 35.609991682282072 i	2325.553425060522841 +1951.526243106474340 i
1,41,17	48.123932753097954 + 38.773748364727247 i	2053.237177530185363 +1957.603682346944652 i	1,41,19	45.393227983024403 + 41.794393708705471 i	1790.327780810710010 +1932.645544644780330 i
1,41,21	42.538604705844008 + 44.681795762973280 i	1538.462148402517641 +1878.811417990733162 i	1,41,23	39.552995618969383 + 47.443548166236411 i	1299.219942222860709 +1797.968995368693641 i
1,41,25	36.427218662013644 + 50.085448631000709 i	1074.160616157895220 +1691.770845783908953 i	1,41,27	33.149322892108948 + 52.611804239896216 i	864.859038941854692 +1561.708592040447002 i
1,41,29	29.703551574079896 + 55.025613089971372 i	672.943960048135636 +1409.152471123376699 i	1,41,31	26.068624256286849 + 57.328643143268756 i	500.144699675920720 +1235.380778222416438 i
1,41,33	22.214692782991595 + 59.521404219775619 i	348.354737074547756 +1041.601158230613009 i	1,41,35	18.097397144994311 + 61.602973127860281 i	219.72916397158988 + 828.96225412378624 i
1,41,37	13.644437419368352 + 63.570549127108109 i	116.855919627175069 + 598.544029421432356 i	1,41,39	8.716892926637041 + 65.418338240950320 i	43.120582289752079 + 351.261492230672900 i
1,41,41	2.918298277084937 + 67.133759530111746 i	3.838741834404301 + 86.976347842524220 i	1,42,1	67.439726320550264 + 7.319400874780286 i	4523.667417118616868 + 693.884151665962008 i
1,42,3	65.628261138514873 + 12.470219215319393 i	4246.733796156463541 +1085.406474560771585 i	1,42,5	63.697175550928669 + 17.063696884389820 i	3957.782961113424335 +1381.194478573307606 i
1,42,7	61.653331554550107 + 21.289406081743309 i	3662.682955306702752 +1608.740672475290467 i	1,42,9	59.500173673864040 + 25.234897932915924 i	3365.411241241693460 +1781.293324523757292 i
1,42,11	57.239152341322615 + 28.951212049857432 i	3068.997452529803013 +1906.908605176774472 i	1,42,13	54.870273378328356 + 32.471755202218013 i	2775.915061139360660 +1991.112347866259142 i
1,42,15	52.392328988404742 + 35.819960583929642 i	2488.28274790348905 +2037.99538174058648 i	1,42,17	49.802970126937964 + 39.013015943593487 i	2207.982691445899945 +2050.753597067764076 i
1,42,19	47.098675080255781 + 42.063900494126592 i	1936.737351768257895 +2031.987275675564433 i	1,42,21	44.274628854467771 + 44.982589051021442 i	1676.163829940687947 +1983.881436326962330 i
1,42,23	41.324505174456604 + 47.776803293051394 i	1427.815793224216804 +1908.321628592463867 i	1,42,25	38.240121886953936 + 50.452495845778216 i	1193.218928038948206 +1806.972169079409014 i
1,42,27	35.010911447526567 + 53.014164005322996 i	973.904126603022860 +1681.331288297571064 i	1,42,29	31.623097370834007 + 55.465044459370453 i	771.442139551225182 +1532.771389835227410 i
1,42,31	28.058366533721511 + 57.807213427742205 i	587.484037018213371 +1362.569169859726571 i	1,42,33	24.291603920471541 + 60.041595745987941 i	423.813954215734725 +1171.928060591706526 i
1,42,35	20.286699878094762 + 62.167861033225833 i	282.425872076491316 + 961.993047262333221 i	1,42,37	15.987815013858219 + 64.184135422362331 i	165.649741765596303 + 733.852507131126799 i
1,42,39	11.297489098297428 + 66.086316281268736 i	76.393473384006469 + 488.499567377324240 i	1,42,41	5.999504514795179 + 67.866184188113934 i	18.740575537326247 + 226.564761729658471 i
1,43,1	69.012529757940527 + 7.344266873926575 i	4738.016095183988000 + 713.595978334115784 i	1,43,3	67.205587851090414 + 12.518972453636275 i	4455.431847463241866 +1118.178265379227696 i
1,43,5	65.281272922806878 + 17.136893619635654 i	4160.481554250554836 +1425.272308354413781 i	1,43,7	63.246392017652013 + 21.387628010192778 i	3859.054687613123406 +1662.959238523364547 i
1,43,9	61.104422396459448 + 25.358748093815777 i	3555.139198558299086 +1844.757544399852122 i	1,43,11	58.856910395208708 + 29.101325113000108 i	3251.768698507906265 +1978.879348395086481 i
1,43,13	56.504015880073503 + 32.648808960326996 i	2951.415816727356287 +2070.951095331727629 i	1,43,15	54.044746898005208 + 36.024687623316289 i	2656.19424979217229 +2125.13278889011599 i
1,43,17	51.477042118345032 + 39.246215867018691 i	2367.977213917679364 +2144.669673296526526 i	1,43,19	48.797756350187790 + 42.326453346801365 i	2088.474042391581861 +2132.197889018676051 i
1,43,21	46.002565969571386 + 45.275470761622543 i	1819.284046998241593 +2089.928536795480795 i	1,43,23	43.085790455989937 + 48.101104181020531 i	1561.937594620098935 +2019.765775038456695 i
1,43,25	40.040108808430304 + 50.809443661413759 i	1317.930250831509966 +1923.386533883086940 i	1,43,27	36.856127233524937 + 53.405153696855699 i	1088.753983641286368 +1802.296636959442594 i
1,43,29	33.521717495004687 + 55.891678169722901 i	875.928769566355527 +1657.871740069245334 i	1,43,31	30.020975018650493 + 58.271356696901492 i	681.038206274162462 +1491.388026662008542 i
1,43,33	26.332497900429019 + 60.545460922077851 i	505.774114376477257 +1304.045442729662909 i	1,43,35	22.426341420673161 + 62.714140181960971 i	351.998549840481473 +1096.984375721189281 i
1,43,37	18.258073665078676 + 64.776234125447218 i	221.840024593409225 + 871.293499454793846 i	1,43,39	13.755347434169439 + 66.728831965544146 i	117.863731468480694 + 627.996133252807681 i

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
1,43,41	8.779211986348976 + 68.566191801102065 i	43.435467296673210 + 367.946936246669452 i	1,43,43	2.933071446771074 + 70.276095880104194 i	3.853428997480489 + 90.906678749916992 i
1,44,1	70.585252315285857 + 7.368570682398195 i	4957.305536446239785 + 733.385419258819411 i	1,44,3	68.782652215498558 + 12.566615368838027 i	4669.083855066493925 +1151.102096591638656 i
1,44,5	66.864833530028889 + 17.208408163486308 i	4368.153698553978757 +1469.578665540272368 i	1,44,7	64.838546107834347 + 21.483570402069631 i	4060.425813670378830 +1717.485133449171371 i
1,44,9	62.707292679917501 + 25.479693489503205 i	3749.898774843266438 +1908.610448408045158 i	1,44,11	60.472703722588165 + 29.247876992685299 i	3439.610521378659541 +2051.322964579875588 i
1,44,13	58.135076410312102 + 32.821609701979355 i	3132.033393231094578 +2151.350067467994450 i	1,44,15	55.693611783315113 + 36.224429183100085 i	2829.276876487239861 +2212.921319072673214 i
1,44,17	53.146505379106095 + 39.473650862735340 i	2533.20625862330007 +2239.33176189606320 i	1,44,19	50.490946762435698 + 42.582407100557219 i	2245.519104143559161 +2233.253907860193522 i
1,44,21	47.723046439984714 + 45.560854987656548 i	1967.798715397626665 +2196.925531017210233 i	1,44,23	44.837689697169502 + 48.416933187053992 i	1701.554507703160425 +2132.270080228493953 i
1,44,25	41.828302117715694 + 51.156854245450077 i	1448.255068689756172 +2040.977883195509548 i	1,44,27	38.686493990030371 + 53.785430463680377 i	1209.357734584993523 +1924.563251757991793 i
1,44,29	35.401523331033907 + 56.306286945642597 i	986.337735269287691 +1784.406093907820605 i	1,44,31	31.959467306582450 + 58.721990471199462 i	780.719977807657301 +1621.783131615335833 i
1,44,33	28.341891311724027 + 61.034106160358526 i	594.117411113543881 +1437.891724699211181 i	1,44,35	24.523581951466777 + 63.243178624982875 i	428.282195100143497 +1233.867707531630685 i
1,44,37	20.468353766182936 + 65.348612566475012 i	285.181257186801361 +1010.796560547229547 i	1,44,39	16.120314374225746 + 67.348381610793510 i	167.121426928675561 + 769.711859759860317 i
1,44,41	11.381971709956511 + 69.238359710137786 i	76.990582734842221 + 511.552077667011474 i	1,44,43	6.037070117628403 + 71.010501019071768 i	18.854574224127713 + 236.880240955766880 i
1,45,1	72.157898901553366 + 7.392337009589151 i	5181.535503817996869 + 753.250737296713737 i	1,45,3	70.359470182629707 + 12.613197123041070 i	4887.689060469554533 +1184.174533739807885 i
1,45,5	68.447890276415208 + 17.278315654943260 i	4580.797852537613835 +1514.108324055084722 i	1,45,7	66.429850078679991 + 21.577336096116984 i	4266.793738543937252 +1772.311229149985593 i
1,45,9	64.308871461332672 + 25.597866659811164 i	3949.686025416226585 +1972.842877276846139 i	1,45,11	62.086658694355357 + 29.391032298744157 i	3632.517288898983224 +2124.228111526612338 i
1,45,13	59.763631541250625 + 32.990356940638868 i	3317.760073808553443 +2232.295560381083760 i	1,45,15	57.339164015338639 + 36.419423093605417 i	3007.520348243004264 +2301.344695958760440 i
1,45,17	54.811681659683238 + 39.695601215332822 i	2703.65639425393283 +2334.72076550826387 i	1,45,19	52.178672709241646 + 42.832089546351114 i	2407.855215202122054 +2335.133120422667616 i
1,45,21	49.436632834864170 + 45.839123253862518 i	2121.685676856533973 +2304.846746374369559 i	1,45,23	46.580945554280717 + 48.724733341668212 i	1846.638301591267236 +2245.805001716220886 i
1,45,25	43.605687614601941 + 51.495241993871353 i	1584.157425372164425 +2159.712322830874200 i	1,45,27	40.503334597367193 + 54.155592848157710 i	1335.669428433215671 +2048.092322710070221 i
1,45,29	37.264321223409112 + 56.709570523781096 i	1102.609872799649111 +1912.330077453269951 i	1,45,31	33.876374659163311 + 59.159939309381414 i	886.452223023817282 +1753.703839163175189 i
1,45,33	30.323469291484546 + 61.508515352149388 i	688.741353021247619 +1573.409334388275738 i	1,45,35	26.584103764340412 + 63.756173865688019 i	511.136568649690275 +1372.578366477116106 i
1,45,37	22.628254678742419 + 65.902777135520253 i	355.472400698175363 +1152.291926752470545 i	1,45,39	18.411432325539586 + 67.946980009489536 i	223.853849845959672 + 913.584957711322886 i
1,45,41	13.861252596174843 + 69.885795625503917 i	118.825794171362162 + 657.429279711433992 i	1,45,43	8.838741250923450 + 71.713549016608027 i	43.736196700745772 + 384.623707482534542 i
1,45,45	2.947179043242463 + 73.418370831605031 i	3.867450068974474 + 94.832626557532180 i	1,46,1	73.730474031432905 + 7.415588975538987 i	5410.705774431969076 + 773.190270913378185 i
1,46,3	71.936056416388780 + 12.658763693216276 i	5111.246750728870211 +1217.392293372933182 i	1,46,5	70.030473382555115 + 17.346686321095035 i	4798.412568145585283 +1558.856290624393978 i
1,46,7	68.020355526740857 + 21.669021135747002 i	4478.156026874042254 +1827.430720020016906 i	1,46,9	65.909238341455691 + 25.713391273730998 i	4154.497252852946986 +2037.446093669570473 i
1,46,11	63.698890767044126 + 29.530944456164672 i	3830.483730257718889 +2197.583980400387190 i	1,46,13	61.389842171367562 + 33.155236393032560 i	3508.588650824968681 +2313.774529718066788 i
1,46,15	58.981621937659199 + 36.609890402540834 i	3190.915085934792387 +2390.387446991758225 i	1,46,17	56.472862124293393 + 39.912326968229273 i	2879.315139399338932 +2430.818556876980892 i
1,46,19	53.861318310553109 + 43.075804172220273 i	2575.46633119262378 +2437.81447836071357 i	1,46,21	51.143829207648665 + 46.110627940156327 i	2280.924486196211546 +2413.667899941733140 i
1,46,23	48.316219073671512 + 49.024912661211751 i	1997.163046987826874 +2360.342652992968397 i	1,46,25	45.373135714206285 + 51.825079033422469 i	1725.604476671915791 +2279.557928801395065 i
1,46,27	42.307805378820426 + 54.516187940375950 i	1467.647369734248969 +2172.847380369060375 i	1,46,29	39.111669876040757 + 57.102165212031956 i	1224.691937711434548 +2041.602088474155153 i
1,46,31	35.773844564164961 + 59.585948095453347 i	998.166252442786572 +1887.102757591326281 i	1,46,33	32.280286371471298 + 61.969569459252256 i	789.555913614064593 +1710.544424803829543 i
1,46,35	28.612460748986493 + 64.254184529359463 i	600.440829781230718 +1513.055604560964412 i	1,46,37	24.745073596855999 + 66.440032910120749 i	432.545169894723074 +1295.712587206977105 i
1,46,39	20.641877861056656 + 68.526309253413217 i	287.811953661750351 +1059.547698009130045 i	1,46,41	16.246939043856710 + 70.510872446296972 i	168.527387881283033 + 805.544885229651868 i
1,46,43	11.462740676508879 + 72.389594488770939 i	77.561320219903010 + 534.592381848527111 i	1,46,45	6.072993603571454 + 74.154604421850777 i	18.963568143399534 + 247.189802961654294 i
1,47,1	75.302981864441463 + 7.438348244137253 i	5644.816138511327154 + 793.202429365194348 i	1,47,3	73.512424422171250 + 12.703358140074148 i	5339.756254818057641 +1250.752233322230495 i
1,47,5	71.612610657049544 + 17.413585895707470 i	5020.996483202273339 +1603.817789570493687 i	1,47,7	69.610109870513526 + 21.758715355040053 i	4694.510389747504632 +1882.837101523331285 i
1,47,9	67.508466355806419 + 25.826382905571602 i	4364.328986171835822 +2102.411753577142917 i	1,47,11	65.309505660427575 + 29.667756700827599 i	4033.504904796843675 +2271.380258723713723 i
1,47,13	63.013855269290382 + 33.316421232623433 i	3704.512380336162096 +2395.774543692760744 i	1,47,15	60.621184405875418 + 36.796036935968672 i	3379.452146070143954 +2480.034844480936044 i
1,47,17	58.130311023473396 + 40.124069857608660 i	3060.170870676449795 +2527.607905522940655 i	1,47,19	55.539230730494275 + 43.313832557503680 i	2748.33755572204564 +2541.27800589464778 i
1,47,21	52.845088853375408 + 46.375695254804043 i	2445.496228436515139 +2523.365986179872834 i	1,47,23	50.044101178374234 + 49.317847865334940 i	2153.104855763415799 +2475.856664942505631 i
1,47,25	47.131418835766681 + 52.146799921300214 i	1872.566114714725927 +2400.484576829294838 i	1,47,27	44.100923247190863 + 54.867717416834950 i	1605.253565970885347 +2298.794087203868457 i
1,47,29	40.944924539384118 + 57.484651831122591 i	1352.535769846272387 +2172.183044798710853 i	1,47,31	37.653716843570749 + 60.000692837005495 i	1115.800505471594533 +2021.935457403101282 i
1,47,33	34.214908160149726 + 62.418061931686367 i	896.481364829280210 +1849.246576414260744 i	1,47,35	30.612372891424535 + 64.738154069104482 i	696.089670351295240 +1655.242455417573183 i
1,47,37	26.824527631321509 + 66.961524678937536 i	516.256279525321474 +1440.995013596523222 i	1,47,39	22.821284493081215 + 69.087805826525866 i	358.791417653422343 +1207.532325767624116 i
1,47,41	18.558107148847252 + 71.115508171923977 i	225.779138493125686 + 955.840834810595084 i	1,47,43	13.962582708098800 + 73.041594802331602 i	119.746061050890646 + 686.845249304820641 i
1,47,45	8.895718455841768 + 74.860467991804043 i	44.023980847727998 + 401.292441996106220 i	1,47,47	2.960677786193669 + 76.560591756009259 i	3.880862072698671 + 98.754428071888768 i
1,48,1	76.875426239440230 + 7.460635142676366 i	5883.866398350965937 + 813.285688281439093 i	1,48,3	75.088586660146177 + 12.747020849189271 i	5573.216940351450830 +1284.251343792400348 i
1,48,5	73.194327735221336 + 17.479075994158087 i	5248.548314619990379 +1648.988248904241118 i	1,48,7	71.199156770856192 + 21.846502902824582 i	4915.854672914416369 +1938.524150628248663 i
1,48,9	69.106622649231923 + 25.936949727766585 i	4579.177962208709687 +2167.731880251776147 i	1,48,11	66.918600383386817 + 29.801602966210123 i	4241.576174124250429 +2345.607096735592677 i
1,48,13	64.635805385350267 + 33.474073202664307 i	3905.524941685834165 +2478.283740510752523 i	1,48,15	62.258032975501486 + 36.978054679749341 i	3573.123163400748126 +2570.272852379069718 i
1,48,17	59.784268835575766 + 40.331055017150192 i	3246.212742094314763 +2625.072411721325352 i	1,48,19	57.212724686149521 + 43.546436473414381 i	2926.455027551226067 +2645.504718291888821 i
1,48,21	54.540820819962384 + 46.634627828384161 i	2615.38336071692311 +2633.91917656040050 i	1,48,23	51.765122189875804 + 49.603887598471985 i	2314.441658085600920 +2592.322062467784690 i
1,48,25	48.881225597547107 + 52.460805684024166 i	2025.014650188015366 +2522.463790994875581 i	1,48,27	45.883587445272568 + 55.210642675612775 i	1748.453266813443406 +2425.900052645771211 i
1,48,29	42.765271546576564 + 57.857562375502553 i	1486.097609404295326 +2304.036160514013773 i	1,48,31	39.517581276101666 + 60.404789553882225 i	1239.299501875937494 +2158.160207465441090 i
1,48,33	36.129514099337328 + 62.854711064353552 i	1009.446596116296746 +1989.468508728707548 i	1,48,35	32.586927269060517 + 65.208929017090497 i	797.990443857536889 +1799.085500245384992 i
1,48,37	28.871203381188025 + 67.468266548517022 i	606.482057812857192 +1588.079323798025825 i	1,48,39	24.956981715622202 + 69.632716646265010 i	436.620906204183598 +1357.473349202479994 i
1,48,41	20.807964952331474 + 71.701306714423112 i	290.328728592384380 +1108.252058071455849 i	1,48,43	16.368185090590707 + 73.671810234900903 i	169.873219379206063 + 841.354168866978739 i
1,48,45	11.540107040248119 + 75.540113329559716 i	78.107904452069718 + 557.621486476872860 i	1,48,47	6.107411643111367 + 77.298519001205918 i	19.067975033687091 + 257.493804645854539 i
1,49,1	78.447810705182192 + 7.482468769396589 i	6127.856367396768292 + 833.438585605936851 i	1,49,3	76.664554645412650 + 12.789789747732089 i	5811.628210625010070 +1317.886739185971464 i
1,49,5	74.775648289786687 + 17.543214450068895 i	5481.066852273946097 +1694.363287577844631 i	1,49,7	72.787536501105066 + 21.932462712541458 i	5142.186846185704781 +1994.485907904816713 i
1,49,9	70.703769067880827 + 26.045193130885853 i	4799.041108905191322 +2233.398840404249768 i	1,49,11	68.526264130685391 + 29.932608675163664 i	4454.693177178103724 +2420.255076744102553 i
1,49,13	66.255815966725521 + 33.628343607458362 i	4111.620401527906324 +2561.290789686554945 i	1,49,15	63.892333794912680 + 37.156123005965412 i	3771.920300067238259 +2661.088078069338735 i
1,49,17	61.434954966668009 + 40.533492485876799 i	3437.430614001865251 +2723.196446887523052 i	1,49,19	58.882086291902453 + 43.773859732950910 i	3109.805821828968874 +2750.476548516205639 i
1,49,21	56.231395407527422 + 46.887706986476474 i	2790.56957497922713 +2745.30672957497291 i	1,49,23	53.479759778714048 + 49.883355234956994 i	2481.153010560997700 +2709.715154278370392 i
1,49,25	50.623172505781710 + 52.767467307903303 i	2182.924541708151082 +2645.468608966466945 i	1,49,27	47.656597143366554 + 55.545389232695072 i	1897.214576772638566 +2554.134668740339293 i
1,49,29	44.573755670458780 + 58.221385647205771 i	1625.337531087832303 +2437.126746893423388 i	1,49,31	41.366821988812660 + 60.798801675809720 i	1368.612991676101501 +2295.737738035880284 i
1,49,33	38.025973758522369 + 63.280170052338221 i	1128.387787427364628 +2131.165811304952058 i	1,49,35	34.538719883071735 + 65.667273356872997 i	906.060980439536138 +1944.534606495170692 i
1,49,37	30.888851530858737 + 67.961164101416188 i	703.114560666878333 +1736.909198782923340 i	1,49,39	27.054717163602941 + 70.162137578321776 i	521.154294770055799 +1509.308401372010394 i
1,49,41	23.006175777428894 + 72.269640152174077 i	361.968790896718872 +1262.712780019894126 i	1,49,43	18.698653351731981 + 74.282070115957737 i	227.623319931527845 + 998.064713261966745 i
1,49,45	14.059714626465958 + 76.196360570202643 i	120.627995150795385 + 716.245573338263966 i	1,49,47	8.950352269203423 + 78.006997997892441 i	44.299881644440951 + 417.953698170986684 i
1,49,49	2.973617474155157 + 79.702764884786092 i	3.893715050843075 + 102.672297597292669 i	1,50,1	80.020138547415144 + 7.503867090440656 i	6376.785869411716703 + 853.659717862456546 i
1,50,3	78.240339036779389 + 12.831700499708907 i	6054.989501938120156 +1351.655650586695174 i	1,50,5	76.356594217281412 + 17.606055618240671 i	5718.550953466595274 +1739.938703779853540 i
1,50,7	74.375286273833979 + 22.016668924498347 i	5373.504993870570615 +2050.716661111877329 i	1,50,9	72.299962680280387 + 26.151208279874162 i	5023.915530275997298 +2299.405322423212510 i
1,50,11	70.132579069462935 + 30.060891448775230 i	4672.851807868943979 +2495.315185144737370 i	1,50,13	67.874000505790505 + 33.779374197608377 i	4322.793181692704820 +2644.784856814684645 i
1,50,15	65.524239249575501 + 37.330409764972771 i	3975.836200383285814 +2752.467728580734274 i	1,50,17	63.082570081168143 + 40.731578546179541 i	3633.814990247869900 +2821.965099605391515 i
1,50,19	60.547576353655160 + 43.996329825843951 i	3298.377863275856726 +2856.176281039309742 i	1,50,21	57.917148839715253 + 47.135194748726242 i	2971.039678121795532 +2857.508909813270293 i
1,50,23	55.188445647661737 + 50.156551332481939 i	2653.219930177757586 +2828.013434132464681 i	1,50,25	52.357813656680648 + 53.056273377097138 i	2346.272163872827605 +2769.473461634957857 i
1,50,27	49.420665820787015 + 55.872350508771937 i	2051.508127104936193 +2683.468963735000216 i	1,50,29	46.371302053540390 + 58.576572056809788 i	1770.218971635800699 +2571.422035123736350 i
1,50,31	43.202652179789041 + 61.183246258331991 i	1503.695257972018226 +2434.631028645966126 i	1,50,33	39.905904631722485 + 63.695035279722617 i	1253.247345831405255 +2274.296697689449957 i
1,50,35	36.469958563205253 + 66.113880052261749 i	1020.227889517056155 +2091.542667232529766 i	1,50,37	32.880587949593249 + 68.441031458494905 i	806.058682937517005 +1887.431694629212188 i
1,50,39	29.119106241905879 + 70.677041213953984 i	612.265264745594891 +1662.978490406121409 i	1,50,41	25.160097433672636 + 72.821701579107184 i	440.525148993918378 +1419.159019580231416 i
1,50,43	20.967223045409803 + 74.873906449606793 i	292.741011312819012 +1156.914411882189597 i	1,50,45	16.484488444354678 + 76.831366876616097 i	171.163831833214232 + 877.141929987970343 i
1,50,47	11.614344436774671 + 78.689995110481419 i	78.632286028387156 + 580.640267441557512 i	1,50,49	6.140444573656403 + 80.442265684129055 i	19.168162601674183 + 267.792565350211362 i
1,51,1	81.592412812986311 + 7.524847027444252 i	6630.654737791140940 + 873.947736712236519 i	1,51,3	79.815949715654824 + 12.872786682216418 i	6303.300281164634725 +1385.555418836924218 i
1,51,5	77.937185803457595 + 17.667650647858419 i	5960.999537913936365 +1785.710464167880673 i	1,51,7	75.962440530118656 + 22.099191266197614 i	5609.807306132301164 +2107.210930121570982 i
1,51,9	73.895256237332404 + 26.255084614256120 i	5253.798492853718017 +2365.744316405335212 i	1,51,11	71.737621031156863 + 30.186561742575488 i	4896.048194980605331 +2570.778786819072138 i
1,51,13	69.490463546222330 + 33.927297962681905 i	4539.038030404396522 +2728.755571414702899 i	1,51,15	67.153889395568754 + 37.501072260554217 i	4184.863950507471600 +2844.399570730623054 i
1,51,17	64.727298122074160 + 40.925496914776915 i	3835.356962410455708 +2921.364126640078666 i	1,51,19	62.209433204852758 + 44.214059368344095 i	3492.159849547798045 +2962.587491954500820 i
1,51,21	59.598387252882063 + 47.377335593720105 i	3156.779486942251879 +2970.506914989254255 i	1,51,23	56.891571184277005 + 50.423755786876470 i	2830.624749855175196 +2947.195492082209512 i
1,51,25	54.085648847887903 + 53.360109689966057 i	2515.035607310950127 +2894.454065296551562 i	1,51,27	51.176433121826250 + 56.191891109161013 i	2211.306795998854341 +2813.875471294901672 i
1,51,29	48.158733976079183 + 58.923537742167404 i	1920.708332074958207 +2706.891018168306800 i	1,51,31	45.026141644890863 + 61.558599250014616 i	1644.504539341691043 +2574.805118432145264 i
1,51,33	41.770716690906660 + 64.099853228398928 i	1383.973042229314160 +2418.821782596313824 i	1,51,35	38.382539405208838 + 66.549380438814495 i	1140.425218144632709 +2240.065353768863133 i
1,51,37	34.849035633336165 + 68.908604720804291 i	915.229957446731191 +2039.597015699996064 i	1,51,39	31.153926785246881 + 71.178297652342508 i	709.843400752215871 +1818.427976629591058 i
1,51,41	27.275504431246716 + 73.358541385390239 i	525.849026174831351 +1577.529577435223246 i	1,51,43	23.183583639393056 + 75.448613833776503 i	365.016091287297684 +1317.839437438532059 i
1,51,45	18.833559877519402 + 77.446880074469917 i	229.392926547961395 +1040.259667088609604 i	1,51,47	14.152980603447267 + 79.350204765164065 i	121.474647043132975 + 745.631578155931606 i
1,51,49	9.002826837436922 + 81.153181101816671 i	44.564835840383424 + 434.607969375142432 i	1,51,51	2.986042063688152 + 82.844895522217271 i	3.906053155347410 + 106.586429921589556 i
1,52,1	83.164636331330101 + 7.545424536826118 i	6889.462814514653473 + 894.301345775648429 i	1,52,3	81.391395856315825 + 12.913079944882648 i	6556.560043545649023 +1419.583488151455640 i
1,52,5	79.517441870384758 + 17.728047729402476 i	6208.411583195237664 +1831.674693946923071 i	1,52,7	77.549031196272523 + 22.180095395643583 i	5851.092071157337159 +2163.963453047451367 i
1,52,9	75.489698579507774 + 26.356906298968944 i	5488.687413437625764 +2432.409095811184216 i	1,52,11	73.341460122026490 + 30.309723418896471 i	5124.278684056765997 +2646.637601664430349 i
1,52,13	71.105301567317625 + 34.072398427617711 i	4760.349996430423744 +2813.192997519624896 i	1,52,15	68.781413213804015 + 37.668251283029020 i	4398.997042367852373 +2936.871894760129180 i
1,52,17	66.369308068972117 + 41.115419805842251 i	4042.048160138575895 +3021.379908367872758 i	1,52,19	63.867875158741647 + 44.427247392868487 i	3691.141183521327548 +3069.694496543504989 i
1,52,21	61.275390119692645 + 47.614358022506472 i	3347.775735665354205 +3084.282809956886922 i	1,52,23	58.589492269120889 + 50.685229731884199 i	3013.350992205741932 +3067.240934483806541 i
1,52,25	55.807130405509149 + 53.646707638933748 i	2689.19450538959700 +3020.38732479888721 i	1,52,27	52.924474707454799 + 56.504349679175398 i	2376.585468438390308 +2945.328113360857060 i
1,52,29	49.936787391158397 + 59.262668123341172 i	2076.774640365907089 +2843.504309400460897 i	1,52,31	46.838238861842754 + 61.925299989250931 i	1791.002546933325675 +2716.226936481814846 i
1,52,33	43.621647258004799 + 64.495126292386075 i	1520.517302461619693 +2564.703881155223371 i	1,52,35	40.278104703870952 + 66.974351974590149 i	1266.593374589747100 +2390.060886675409620 i
1,52,37	36.796427720491558 + 69.364552759151209 i	1030.552843329392772 +2193.358280440267663 i	1,52,39	33.162320229300192 + 71.666690503401853 i	813.791286575372396 +1975.604428283540301 i
1,52,41	29.357037644485487 + 73.881093403771430 i	617.811624506957060 +1737.766368700582461 i	1,52,43	25.355117872898457 + 76.007389452512896 i	444.271732028449151 +1480.777306404120736 i
1,52,45	21.120188651344474 + 78.044365878489302 i	295.057106611623055 +1205.538854851510905 i	1,52,47	16.596234229687365 + 79.989689385264967 i	172.403558548531009 + 912.910086917922058 i
1,52,49	11.685694853031006 + 81.839307383382376 i	79.136188977678088 + 603.649491969204860 i	1,52,51	6.172198898085409 + 83.585862386165488 i	19.264456189306145 + 278.086372319420832 i
1,53,1	84.736811733665874 + 7.565614681703327 i	7153.209950239383943 + 914.719297693168642 i	1,53,3	82.966685988639981 + 12.952610154381881 i	6814.768310679954084 +1453.737400216886272 i
1,53,5	81.097379907589115 + 17.787292318245207 i	6460.786120615242908 +1877.827667711809227 i	1,53,7	79.135087912277797 + 22.259443211876768 i	6097.357668046414801 +2220.969173458171099 i
1,53,9	77.083334998249427 + 26.456752631575387 i	5728.579847996684293 +2499.393200584017450 i	1,53,11	74.944161263375094 + 30.430474262955542 i	5357.539821038290938 +2722.883683036086040 i
1,53,13	72.718603763635215 + 34.214317368714227 i	4986.724405804568050 +2898.087606719117048 i	1,53,15	70.406929711033797 + 37.832106092994501 i	4618.229341300106171 +3029.873481084713148 i
1,53,17	68.008755473496549 + 41.301508882624972 i	4253.887060798772735 +3121.999408131771816 i	1,53,19	65.523102637514910 + 44.636080498607654 i	3895.311912725592720 +3177.482290429562454 i
1,53,21	62.948413200969604 + 47.844675948246116 i	3544.015994235337075 +3198.819466891969114 i	1,53,23	60.282533387498248 + 50.941217220168149 i	3201.383258727823886 +3188.130311712176990 i
1,53,25	57.522668965535161 + 53.927200192369469 i	2868.729883276883356 +3147.251246294755219 i	1,53,27	54.665310501139125 + 56.810041401540509 i	2547.320828927774073 +3077.802094942201165 i
1,53,29	51.706122919002333 + 59.594320987875669 i	2238.389263326705548 +2981.234015933771843 i	1,53,31	48.639788898126241 + 62.283576059169082 i	1943.154057408337621 +2858.865149904928878 i
1,53,33	45.459788668835785 + 64.881317712633958 i	1662.836619267061906 +2711.907828442956998 i	1,53,35	42.158087634607753 + 67.389324708736879 i	1398.678253421181807 +2541.489826605063172 i
1,53,37	38.724684038987625 + 69.809486010651270 i	1151.959371588398881 +2348.671294647707327 i	1,53,39	35.146933631045648 + 72.142929498200900 i	924.022290370830319 +2134.458427460147050 i

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
1,53,41	31.408499010159463 + 74.390194463312213 i	716.300207648542823 +1899.814457286124707 i	1,53,43	27.487624013343805 + 76.551187356957826 i	530.356674508675837 +1645.667987675451059 i
1,53,45	23.354087263621690 + 78.625081818661954 i	367.94352575162304 +1372.91757929838718 i	1,53,47	18.963259532149753 + 80.610121741083702 i	231.093732643240344 +1082.428351815221731 i
1,53,49	14.242675049204225 + 82.503223421036637 i	122.288717784696544 + 775.004420063312978 i	1,53,51	9.053305476743979 + 84.299053445793561 i	44.819673921516120 + 451.255694107249383 i
1,53,53	2.997990546471317 + 85.986988212123194 i	3.917915534491625 + 110.497002801979619 i	1,54,1	86.308941470186355 + 7.585431697239098 i	7421.896003008035223 + 935.200391403573008 i
1,54,3	84.541828054232695 + 12.991405525668490 i	7077.924628691157523 +1488.014788731219596 i	1,54,5	82.677016189228297 + 17.845427337526349 i	6718.122231434360401 +1924.165800981558050 i
1,54,7	80.720638235515679 + 22.337293136423959 i	6348.602560348399493 +2278.223228572362586 i	1,54,9	78.676207557540920 + 26.554698410839421 i	5973.473481596825877 +2566.690421587965830 i
1,54,11	76.545784670851305 + 30.548906449208314 i	5595.828337448722777 +2799.509397909991549 i	1,54,13	74.330452734350899 + 34.353641239636133 i	5218.156840816202021 +2983.430253404791074 i
1,54,15	72.030548889422050 + 37.992746726558906 i	4842.555056939633687 +3123.393569830603155 i	1,54,17	69.645783805085102 + 41.483916111480831 i	4470.847179744549976 +3223.210135095984697 i
1,54,19	67.175300028074048 + 44.840733880972671 i	4104.662675852091525 +3285.936523455739007 i	1,54,21	64.617691101870349 + 48.073889935120974 i	3745.488595859051657 +3314.100510930599285 i
1,54,23	61.970991163009060 + 51.191946715780091 i	3394.707132153209841 +3309.845052669424149 i	1,54,25	59.232638397740210 + 54.201846728717733 i	3053.624025869117304 +3275.024858440684966 i
1,54,27	56.399411636650875 + 57.109260190192018 i	2723.49118162794321 +3211.27380937713951 i	1,54,29	53.467335823419809 + 59.918829181986510 i	2405.525659082250058 +3120.053624844882052 i
1,54,31	50.431548072830331 + 62.634341679579052 i	2100.926567268438248 +3002.690027551380354 i	1,54,33	47.286110482867713 + 65.258855796907113 i	1810.891060051824070 +2860.400317330194866 i
1,54,35	44.023747245129670 + 67.794786732670644 i	1536.630516228684655 +2694.314884549813087 i	1,54,37	40.635469145544320 + 70.243963745510740 i	1279.388057207827160 +2505.494338844188405 i
1,54,39	37.110022766932374 + 72.607660615877478 i	1040.458840546282090 +2294.943361055640707 i	1,54,41	33.433052745888894 + 74.886599444016923 i	821.215183582860163 +2063.621953145269799 i
1,54,43	29.585765538652161 + 77.080923150643400 i	623.139794139903302 +1812.454313948886465 i	1,54,45	25.542660490774047 + 79.190124707579528 i	447.872876360599946 +1542.334837707780935 i
1,54,47	21.267337569319178 + 81.212906205339991 i	297.284366992637786 +1254.128925541683884 i	1,54,49	16.703764356218233 + 83.146904356124693 i	173.596242737290140 + 948.660308364463276 i
1,54,51	11.754373319473608 + 84.988108352631428 i	79.621144498891468 + 626.649836182332479 i	1,54,53	6.202769323653254 + 86.729324537381771 i	19.357145028346842 + 288.375485170403067 i
1,55,1	87.881027825478881 + 7.604889050129353 i	7695.520838086076765 + 955.743469619670691 i	1,55,3	86.116829456750139 + 13.029492741366980 i	7346.028566553017299 +1522.413374343435180 i
1,55,5	84.256365879017054 + 17.902493362569622 i	6980.419043427900760 +1970.685642361469945 i	1,55,7	82.305707822881303 + 22.413700368876604 i	6604.825290167517778 +2335.720938342179124 i
1,55,9	80.268355380720607 + 26.650814270993855 i	6223.366119239523869 +2634.294786238691813 i	1,55,11	78.146386280792925 + 30.665106963638433 i	5839.141136952520355 +2876.507408596823552 i
1,55,13	75.940925094466337 + 34.490315844782302 i	5454.643121000954125 +3069.212151995673594 i	1,55,15	73.652372602933639 + 38.150303031402132 i	5071.968716974197378 +3217.421832868883939 i
1,55,17	71.280525634352244 + 41.662784530218921 i	4692.940574627984461 +3325.000110227394089 i	1,55,19	68.824637305411628 + 45.041372255099063 i	4319.184654541887539 +3395.043439660122658 i
1,55,21	66.283439494466705 + 48.296788306056577 i	3952.182572534452447 +3430.110270650710390 i	1,55,23	63.655137408247484 + 51.437632423462354 i	3593.309090057256071 +3432.367405302212319 i
1,55,25	60.937380021458239 + 54.470890030123584 i	3243.860361727103282 +3403.688141040568098 i	1,55,27	58.127206348237131 + 57.402280625108112 i	2905.076293510193864 +3345.720752797868954 i
1,55,29	55.220964370210837 + 60.236502968900012 i	2578.159163178989729 +3259.937900724437441 i	1,55,31	52.214196065915921 + 62.977410512625262 i	2264.289997322544423 +3147.673317521581339 i
1,55,33	49.101477506010049 + 65.628137552613012 i	1964.643851487818210 +3010.149752704634011 i	1,55,35	45.876196230436754 + 68.191188813143924 i	1680.404994283307587 +2848.500750331341351 i
1,55,37	42.530237108736266 + 70.668500138220267 i	1412.783013961565075 +2663.787972220218307 i	1,55,39	39.053528763486376 + 73.061474345267669 i	1163.031060094715713 +2457.015213734899589 i
1,55,41	35.433367425574303 + 75.370993141331326 i	932.467877822542391 +2229.139849352067813 i	1,55,43	31.653365856530346 + 77.597402441109310 i	722.506171171243409 +1981.082070602350313 i
1,55,45	27.691727679797690 + 79.740463113853465 i	534.691571670175563 +1713.731763343733292 i	1,55,47	23.518201208922477 + 81.799249237878257 i	370.760144075294988 +1427.951771675116106 i
1,55,49	19.088137253780911 + 83.771953582057662 i	232.730867428244466 +1124.573075679739077 i	1,55,51	14.329060060431797 + 85.655499489779245 i	123.072610304699268 + 804.365113173698433 i
1,55,53	9.101933695815250 + 87.444646262198972 i	45.065135571926270 + 467.897264208837953 i	1,55,55	3.009497668007619 + 89.129046869797662 i	3.929337059076385 + 114.404179050672440 i
1,56,1	89.453072932388622 + 7.623999492847208 i	7974.084327411068443 + 976.347416483991879 i	1,56,3	87.691697107110201 + 13.066897060577400 i	7619.079714556753970 +1556.930959957078130 i
1,56,5	85.835443124381399 + 17.958528788819955 i	7247.675727740171169 +2017.383866275803257 i	1,56,7	83.890320593933168 + 22.488717119402970 i	6866.024472783366036 +2393.457795343255839 i
1,56,9	81.859814906888303 + 26.745166985471649 i	6478.255677513145201 +2702.200545214054273 i	1,56,11	79.746018130395953 + 30.779157986908899 i	6087.475283133699122 +2953.870655857814199 i
1,56,13	77.550092018160314 + 34.624439736300275 i	5696.179285903962409 +3155.424855948376262 i	1,56,15	75.272495315916515 + 38.304891041719436 i	5306.465143419013657 +3311.948348094346784 i
1,56,17	72.913103676925928 + 41.838248940993331 i	4920.154273258086505 +3427.357835078767948 i	1,56,19	70.471271457608196 + 45.238150686409817 i	4538.869530292488889 +3504.789849081297689 i
1,56,21	67.945857057685643 + 48.515348135867612 i	4164.087597500933953 +3546.833732864358353 i	1,56,23	65.335221770022656 + 51.678475476176175 i	3797.176428160934393 +3555.680382449420449 i
1,56,25	62.637206232870525 + 54.734557705505303 i	3439.423360670999955 +3533.221960272020169 i	1,56,27	59.849084992588026 + 57.689359665126993 i	3092.057256955358753 +3481.121446712030775 i
1,56,29	56.967496876711685 + 60.547632104365327 i	2756.26680271619807 +3400.86279320160838 i	1,56,31	53.988345998920103 + 63.313288301795119 i	2433.216438456338892 +3293.788136838594255 i
1,56,33	50.906664557390485 + 65.989531833197688 i	2124.061026353333091 +3161.126120259290474 i	1,56,35	47.716423245245579 + 68.578948359775922 i	1829.960187967567435 +3004.013937790542059 i
1,56,37	44.410266596595207 + 71.083569389749528 i	1552.093225904219025 +2823.514853527178730 i	1,56,39	40.979136219194267 + 73.504912515148902 i	1291.675668600241806 +2620.632370958996034 i
1,56,41	37.411719137912300 + 75.843999797024068 i	1049.978798404972849 +2396.321837006851638 i	1,56,43	33.693609552169939 + 78.101353042479857 i	828.354162010306140 +2151.499932467459985 i
1,56,45	29.805972439016873 + 80.276960732646373 i	628.266299930622949 +1887.052112145441242 i	1,56,47	25.723274781342187 + 82.370204485294693 i	451.339432492086871 +1603.837348910833953 i
1,56,49	21.409093709377824 + 84.379718644121141 i	299.429333098530321 +1302.687699916881456 i	1,56,51	16.807383743286522 + 86.303121353293179 i	174.745308719961201 + 984.394054411175262 i
1,56,53	11.820571764560569 + 88.136448450708755 i	80.088518648787726 + 649.641899245942227 i	1,56,55	6.232240439041958 + 89.872665501398345 i	19.446487383278285 + 298.660139583269824 i
1,57,1	91.025078784505041 + 7.642775113189662 i	8257.586349153694755 + 997.011155388691918 i	1,57,3	89.266437464189750 + 13.103642418201826 i	7897.077682906046903 +1591.565426365730445 i
1,57,5	87.414261141117521 + 18.013569985037364 i	7519.891496003338943 +2064.257266220012752 i	1,57,7	85.474498877359792 + 22.562392820650327 i	7132.198791730453195 +2451.429455397847050 i
1,57,9	83.450620120635804 + 26.837819743396363 i	6738.140176970874563 +2770.402160144826667 i	1,57,11	81.344728697507023 + 30.891137242666481 i	6340.827988362141696 +3031.592343289885321 i
1,57,13	79.158019723032950 + 34.756106058266170 i	5942.761579417317989 +3242.060238379350267 i	1,57,15	76.891004776875846 + 38.456620339020780 i	5546.039431122757378 +3406.963575726488216 i
1,57,17	74.543631716931401 + 42.010436536555737 i	5152.482014581203657 +3530.272263088510954 i	1,57,19	72.115347740766310 + 45.431215339391372 i	4763.709445678396150 +3615.163091340108167 i
1,57,21	69.605127176682487 + 48.729736143994792 i	4381.193933713307160 +3664.256501257400032 i	1,57,23	67.011474032303852 + 51.914664998985188 i	4006.297192008793445 +3679.767712431405926 i
1,57,25	64.332403640315988 + 54.993063460500188 i	3640.298443086278439 +3663.608009749646499 i	1,57,27	61.565404353196499 + 57.970738169520800 i	3284.416368874472028 +3617.455367760224817 i
1,57,29	58.707377694008827 + 60.852487670509767 i	2939.827134125582688 +3542.805353269434818 i	1,57,31	55.754552887983920 + 63.642280050775078 i	2607.679931688544457 +3441.008871853194705 i
1,57,33	52.702368674376551 + 66.343382078207455 i	2289.111121268144416 +3313.300868201837602 i	1,57,35	49.545311021404764 + 68.958452845582627 i	1985.257843879835036 +3160.822645065748475 i
1,57,37	46.276688741617754 + 71.489610086286504 i	1697.271941733883978 +2984.639578260649614 i	1,57,39	42.888318067794245 + 73.938474005830045 i	1426.335087938345168 +2785.755435532680167 i
1,57,41	39.370068180150540 + 76.306190878341155 i	1173.676260363423808 +2565.124116992235486 i	1,57,43	35.709184500998759 + 78.593436168277478 i	940.593161980785567 +2323.659330551051880 i
1,57,45	31.889237366040156 + 80.800393844485015 i	728.480084127306895 +2062.242403652297980 i	1,57,47	27.888396402982558 + 82.926703638416953 i	538.866458776015861 +1781.727947571254617 i
1,57,49	23.676384677534912 + 84.971358514496643 i	373.474007819508927 +1482.945985727774996 i	1,57,51	19.208536913916237 + 86.932513068318599 i	234.308907699289762 +1166.695856534844324 i
1,57,53	14.412369975630769 + 88.807105015138205 i	123.828471684922860 + 833.714551819675876 i	1,57,55	9.148841690006988 + 90.589986685758859 i	45.301882424824754 + 484.533031570716048 i
1,57,57	3.020594521035294 + 92.271074887347268 i	3.940348921752210 + 118.308108297275197 i	1,58,1	92.597047247429103 + 7.661227379605995 i	8546.026787314591274 +1017.733646945552550 i
1,58,3	90.841056571528270 + 13.139751515765246 i	8180.022100427060650 +1626.314728191571021 i	1,58,5	88.992832289660882 + 18.067651433273756 i	7797.065597694539974 +2111.302748486918972 i
1,58,7	87.058263542735148 + 22.634774321193213 i	7403.346994290461225 +2509.631728865745572 i	1,58,9	85.040802758314030 + 26.928832401718491 i	7003.017735159634867 +2838.894292196470815 i
1,58,11	82.942563205011794 + 31.001118314749167 i	6599.196603631633479 +3109.665922862416664 i	1,58,13	80.764769902711720 + 34.885402936809285 i	6194.386435519128099 +3329.110474146357319 i
1,58,15	78.507982618474486 + 38.605594524840043 i	5790.686928252143844 +3502.458336436207901 i	1,58,17	76.172215426302756 + 42.179467467494106 i	5389.917868418035141 +3633.732773146124600 i
1,58,19	73.757000787593029 + 45.620704154185745 i	4993.696969384900513 +3726.151003908743860 i	1,58,21	71.261419436566584 + 48.940109498967090 i	4603.492387577266527 +3782.364758471968798 i
1,58,23	68.684106128829981 + 52.146379064725004 i	4220.660115915653478 +3804.613793868669745 i	1,58,25	66.023235786023207 + 55.246608234098886 i	3846.471899317800762 +3794.828756778163552 i
1,58,27	63.276491347790873 + 58.246642254100905 i	3482.137023940881591 +3754.702883835400371 i	1,58,29	60.441012312290582 + 61.151323702157441 i	3128.820100894934058 +3685.743657394667935 i
1,58,31	57.513320779335743 + 63.964670998354734 i	2787.65627689220173 +3589.31108813540050 i	1,58,33	54.489219280650715 + 66.690008711437088 i	2459.764916396212357 +3466.646800416423503 i
1,58,35	51.363651221706938 + 69.330062773814291 i	2146.262594926862675 +3318.896628400993905 i	1,58,37	48.130509537179741 + 71.887028936527443 i	1848.276166749789809 +3147.128531007385391 i
1,58,39	44.782370742763149 + 74.362619572685006 i	1566.956705551376899 +2952.347058748065592 i	1,58,41	41.310117243912380 + 76.758091513391889 i	1303.495278802507073 +2735.505217978034580 i
1,58,43	37.702385070534802 + 79.074255424365859 i	1059.141860782855701 +2497.514289765025296 i	1,58,45	33.944725311494516 + 81.311464749180203 i	835.229343410271099 +2239.251895456228340 i
1,58,47	30.018267668916568 + 83.469578614677872 i	633.205851766125761 +1961.568248535836574 i	1,58,49	25.897451900793666 + 85.547886583444978 i	454.681078551017663 +1665.289831922940493 i
1,58,51	21.545836368215983 + 87.544969391436751 i	301.497849122795628 +1351.217866774325103 i	1,58,53	16.907365463375701 + 89.458435741751558 i	175.853820645206724 +1020.112609527231029 i
1,58,55	11.884462204725198 + 91.284371603761599 i	80.539535235667543 + 672.626214865598172 i	1,58,57	6.260688103919955 + 93.015896912089389 i	19.532714811282460 + 308.940550374102884 i
1,59,1	94.168980068958447 + 7.679367182731239 i	8839.405531353515471 +1038.513887093395169 i	1,59,3	92.415560090489393 + 13.175245904589589 i	8467.912613382302879 +1661.176890101179652 i
1,59,5	90.571168143925519 + 18.120805856977704 i	8079.197317707749789 +2158.517326325905287 i	1,59,7	88.641634119271600 + 22.705906062424891 i	7679.467887355908715 +2568.060572544482511 i
1,59,9	86.630392493611524 + 27.018261715532759 i	7272.886560233515411 +2907.671791462512829 i	1,59,11	84.539563894097228 + 31.109170936582284 i	6862.578609268001856 +3188.085081500895740 i
1,59,13	82.370400114216679 + 35.012413835508653 i	6451.050465263592741 +3416.568023253468709 i	1,59,15	80.123504893161297 + 38.751911650627100 i	6040.403218536002896 +3598.423791124352647 i
1,59,17	77.798953093624846 + 42.345455357073902 i	5632.456211641676418 +3737.729145202142263 i	1,59,19	75.396355589571627 + 45.806747459295636 i	5228.825064602490784 +3837.741892896795889 i
1,59,21	72.914890939610814 + 49.146616545021246 i	4830.974267296809558 +3901.145231277409039 i	1,59,23	70.353313908311193 + 52.373785554648762 i	4440.254568245881273 +3930.203654282857769 i
1,59,25	67.709945518362872 + 55.495381218789291 i	4057.930817791935169 +3926.867393232387712 i	1,59,27	64.982646236272117 + 58.517284503562394 i	3685.203619934894050 +3892.845195857256385 i
1,59,29	62.168771740848193 + 61.444378634067945 i	3323.226908273443314 +3829.656738530039092 i	1,59,31	59.265108809627961 + 64.280728357450926 i	2973.122865638040366 +3738.671448766092453 i
1,59,33	56.267786718966207 + 67.029711249457776 i	2635.995210022293122 +3621.137979778565295 i	1,59,35	53.172156724860628 + 69.694114265405176 i	2312.941651969913437 +3478.207088036422192 i
1,59,37	49.972628033002188 + 72.276203997839175 i	2005.066233215589136 +3310.949751654153260 i	1,59,39	46.662442115293260 + 74.777775955045740 i	1713.492261392871569 +3120.371785979972628 i
1,59,41	43.233356341739814 + 77.200185876529858 i	1439.376666511126359 +2907.425825060923216 i	1,59,43	39.675188861307987 + 79.544364151596426 i	1183.926858277045312 +2673.021258618293814 i
1,59,45	35.975141515653021 + 81.810812351633700 i	948.421646736250523 +2418.032478180064572 i	1,59,47	32.116748380966413 + 83.999578119030062 i	734.238693111435509 +2143.305513949336728 i
1,59,49	28.078150263158715 + 86.110200042542573 i	542.892714133338472 +1849.662675884901546 i	1,59,51	23.829049178820128 + 88.141605056428631 i	376.092329418223017 +1537.903694125442371 i
1,59,53	19.324766952340749 + 90.091920076852460 i	235.831954432056762 +1208.798467721747556 i	1,59,55	14.492815155644818 + 91.958102884200585 i	124.558228203297176 + 863.053528745781136 i
1,59,57	9.194146414055307 + 93.735098408916102 i	45.530508653458021 + 501.163313655202671 i	1,59,59	3.031309038942188 + 95.413075218494356 i	3.950979135150458 + 122.208928487803263 i
1,60,1	95.740878888310206 + 7.697204873499409 i	9137.722475847767782 +1059.350905331484228 i	1,60,3	93.989953330275242 + 13.210146062080192 i	8760.748884378372168 +1696.150003275337212 i
1,60,5	92.149279553549633 + 18.173064338415099 i	8366.285974119644379 +2205.898114498399991 i	1,60,7	90.224628903053897 + 22.775830240565103 i	7960.560333628701301 +2626.712082126360656 i
1,60,9	88.219417104847238 + 27.106161548808139 i	7547.744945093116257 +2976.729687098454633 i	1,60,11	86.135770270079476 + 31.215361255649196 i	7130.971606418104784 +3266.843728623974584 i
1,60,13	83.974964125581550 + 35.137217879725938 i	6712.750444890365543 +3504.425615458848824 i	1,60,15	81.737642552465928 + 38.895664609428096 i	6295.184105078006723 +3694.851422197870306 i
1,60,17	79.423936274155025 + 42.508507769440264 i	5880.091706523271569 +3842.251537726911920 i	1,60,19	77.033528369563655 + 45.989468527594759 i	5469.087060391936575 +3949.924506107207435 i
1,60,21	74.565687470361993 + 49.349397459879505 i	5063.631345277713081 +4020.585158518446860 i	1,60,23	72.019278687891344 + 52.597042935368265 i	4665.070502229970771 +4056.522912088781140 i
1,60,25	69.392757067346267 + 55.739560778555322 i	4274.663020721059183 +4059.707790572823797 i	1,60,27	66.684145408364586 + 58.782865058442596 i	3893.601473533274202 +4031.864284586383292 i
1,60,29	63.890996284220662 + 61.731876592968000 i	3523.029912493944266 +3974.524523261968170 i	1,60,31	61.010336382378277 + 64.590702859843375 i	3164.058534460026294 +3889.067640082008438 i
1,60,33	58.038589459410595 + 67.362770163331135 i	2817.776622313260759 +3776.749640479493792 i	1,60,35	54.971471866950324 + 70.050921326828174 i	2485.264538022084707 +3638.726564866505214 i
1,60,37	51.803851261286148 + 72.657487478140297 i	2167.605434359769788 +3476.072814137050500 i	1,60,39	48.529553959466178 + 75.184339403130252 i	1865.897333177532573 +3289.795915987347598 i
1,60,41	45.141098033108804 + 77.632921740170175 i	1581.266289910468231 +3080.848620775582731 i	1,60,43	41.629314941813860 + 80.004271503051527 i	1314.881549652377544 +2850.138941547933428 i
1,60,45	37.982796840484454 + 82.299020667680239 i	1067.973959958234218 +2598.538534976747542 i	1,60,47	34.187057817786958 + 84.517380381870936 i	841.859570106187789 +2326.889597661790958 i
1,60,49	30.223197480790891 + 86.659100989022235 i	637.971601249908076 +2036.010121946834261 i	1,60,51	26.065632645091089 + 88.723395843742310 i	457.906483773742366 +1726.696654872177305 i
1,60,53	21.677906272724488 + 90.708803633636663 i	303.495158383499700 +1399.721788645361585 i	1,60,55	17.003955022126057 + 92.612930930699505 i	176.924531266613229 +1055.817109371884953 i
1,60,57	11.946199402646304 + 94.431916256325124 i	80.975294881539909 + 695.603260709137999 i	1,60,59	6.288180608143869 + 96.159028946276095 i	19.616035714992098 + 319.216914072151667 i
1,61,1	97.312745244486450 + 7.714750298168702 i	9440.977520177113547 +1080.243763068617454 i	1,61,3	95.564241275137903 + 13.244471461797486 i	9058.530591358769016 +1731.232222111876808 i
1,61,5	93.727176700276843 + 18.224456426457287 i	8658.330916130973492 +2253.442324196594200 i	1,61,7	91.807265054046724 + 22.844586955263075 i	8246.623248121200959 +2685.582485165206435 i
1,61,9	89.807902626064504 + 27.192583067499875 i	7827.591261999006747 +3046.063178132508710 i	1,61,11	87.731219323992462 + 31.319752075577881 i	7404.373309241033894 +3345.935984550337253 i
1,61,13	85.578512228469052 + 35.259890153093664 i	6979.483304937560206 +3592.676235977626220 i	1,61,15	83.350461876856556 + 39.036941493387459 i	6555.025595571337362 +3791.733016206424054 i
1,61,17	81.047250370965116 + 42.668726636207921 i	6132.819281007522477 +3947.290466844774983 i	1,61,19	78.668627359122965 + 46.168984081897555 i	5714.476625682929807 +4062.688008143772095 i
1,61,21	76.213944529211545 + 49.548584851478488 i	5301.455824109464629 +4140.672261566471207 i	1,61,23	73.682168624658516 + 52.816300961841812 i	4895.098411639931328 +4183.557741633905607 i
1,61,25	71.071877866986242 + 55.979315277015452 i	4496.657006420670768 +4193.334458566930152 i	1,61,27	68.381243815792331 + 59.043572592321354 i	4107.316745144622473 +4171.742861941132105 i
1,61,29	65.607998812636666 + 62.014028554512542 i	3728.212522468237239 +4120.327774425320058 i	1,61,31	62.749387612730430 + 64.894830133003836 i	3360.443435507886988 +4040.478304048448081 i
1,61,33	59.802100226217036 + 67.689448528819511 i	3005.085423662517769 +3933.458108356263892 i	1,61,35	56.762181041525999 + 70.400777846920818 i	2663.202857837235115 +3800.428846688981594 i
1,61,37	53.624906588563590 + 73.031208182676934 i	2335.859710506217198 +3642.468716468260482 i	1,61,39	50.384620212170319 + 75.582678726056514 i	2024.130900670757228 +3460.587372908725683 i
1,61,41	47.034505421528863 + 78.056714355713581 i	1729.114449353892041 +3255.738138814606828 i	1,61,43	43.566268824704082 + 80.454447532778886 i	1451.945215844344375 +3028.828139154346900 i
1,61,45	39.969684545201368 + 82.776625770631469 i	1193.811253940454890 +2780.726788349896196 i	1,61,47	36.231916885224514 + 85.023603194410981 i	955.974321359391979 +2512.272867179523018 i
1,61,49	32.336468824433240 + 87.195312171567394 i	739.796986966318286 +2224.280179819714704 i	1,61,51	28.261456676924105 + 89.291206620950220 i	546.780541607764803 +1917.541321387702965 i
1,61,53	23.976564910326122 + 91.310159854389084 i	378.621587575402543 +1592.827949019612333 i	1,61,55	19.437105077920052 + 93.250279646726014 i	237.303696534609713 +1250.882475358906277 i
1,61,57	14.570585142308913 + 95.108548249913810 i	125.263614578147710 + 892.382749994230153 i	1,61,59	9.237953315935640 + 96.880002214720562 i	45.751549826372518 + 517.788398078210207 i
1,61,61	3.041666408722055 + 98.555050447373659 i	3.961252948620980 + 126.106767167418027 i	1,62,1	98.884580583873955 + 7.732012830553890 i	9749.170568232797450 +1101.191552078595578 i

Table D.2: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
1,62,3	97.138428609090241 + 13.278240637911344 i	9361.257426673853843 +1766.421761142674541 i	1,62,5	95.304869149111710 + 18.275010235669360 i	8955.331522167007279 +2301.147258295587562 i
1,62,7	93.389558684003731 + 22.912214346105849 i	8537.655594931080486 +2744.668134510451182 i	1,62,9	91.395873483743003 + 27.277574916785117 i	8112.423957613434141 +3115.667624895939298 i
1,62,11	89.325945732709869 + 31.422403078086226 i	7682.781537732100333 +3425.356169700467490 i	1,62,13	87.181091519876844 + 35.380501968993045 i	7251.246120256674658 +3681.313112183597241 i
1,62,15	84.962024862096612 + 39.175825920613576 i	6819.923888769199039 +3889.060647717187749 i	1,62,17	82.668975155715600 + 42.826208645838918 i	6390.634110710767345 +4052.836786989497796 i
1,62,19	80.301753492679055 + 46.345404755543299 i	5964.987745613836523 +4176.021957376725368 i	1,62,21	77.859788251809974 + 49.744304300418349 i	5544.44030571375788 +4261.39471703184972 i
1,62,23	75.342139930196765 + 53.031701314838479 i	5130.329290742195553 +4311.294840982909591 i	1,62,25	72.747500160795525 + 56.214803826263356 i	4723.901897414463747 +4327.732507338794676 i
1,62,27	70.074177102335497 + 59.299585192613016 i	4326.336371610987953 +4312.464326347281713 i	1,62,29	67.320067606602251 + 62.291033382246574 i	3938.759112248663314 +4267.048038601464214 i
1,62,31	64.482615162659457 + 65.193331931179912 i	3562.258922079973672 +4192.882976538648109 i	1,62,33	61.558751234963083 + 68.009993494422517 i	3197.89938386641961 +4091.24072834774643 i
1,62,35	58.544815969644731 + 70.743959362467436 i	2846.730097143729836 +3963.288882998725967 i	1,62,37	55.436452018916164 + 73.397673661376317 i	2509.797378268871398 +3810.109780871490329 i
1,62,39	52.228461959414609 + 75.973137942060433 i	2188.154974180850786 +3632.715589853521578 i	1,62,41	48.914614676017045 + 78.471949787974310 i	1882.875358345704399 +3432.060630070744537 i
1,62,43	45.487377741912574 + 80.895327499866198 i	1595.062272935216470 +3209.051598700912564 i	1,62,45	41.937538558569649 + 83.244121542795128 i	1325.865382710856693 +2964.556146190327127 i
1,62,47	38.253651368417145 + 85.518811972052859 i	1076.498276519523870 +2699.410083949726868 i	1,62,49	34.421198388852808 + 87.719489095816826 i	848.261724081354714 +2414.423309464610156 i
1,62,51	30.421253481568783 + 89.845811465304379 i	642.575355299625357 +2110.384217420756158 i	1,62,53	26.228214107569503 + 91.896929321805928 i	461.023444359765620 +1788.061659068436484 i
1,62,55	21.805610633665065 + 93.871348801453649 i	305.425983022583466 +1448.201551364693159 i	1,62,57	17.097373943173150 + 95.766680207753652 i	177.959922731672265 +1091.508562730007725 i
1,62,59	12.005923096667887 + 97.579116207246415 i	81.396790997043595 + 718.573466185449547 i	1,62,61	6.314779644922599 + 99.302070546211780 i	19.696638324248660 + 329.489411096277365 i
1,63,1	100.456386267159317 + 7.749001401728700 i	10062.301528148418583 +1122.193393053647242 i	1,63,3	98.712519738381502 + 13.311471244569516 i	9668.929096220963402 +1801.716892147689123 i
1,63,5	96.882365894809084 + 18.324752537527630 i	9257.287198122432528 +2349.010306912054602 i	1,63,7	94.971524936265709 + 22.978748718194907 i	8833.656384264391818 +2803.965502170297441 i
1,63,9	92.983352620714811 + 27.361183383965685 i	8402.241548429550676 +3185.538541021507172 i	1,63,11	90.919982040013352 + 31.523371026770207 i	7966.194211118152452 +3505.098794526052188 i
1,63,13	88.782746156485643 + 35.499121119522940 i	7528.036100839733990 +3770.329701223523898 i	1,63,15	86.572389567225329 + 39.312397334526132 i	7089.875362082849279 +3986.826664318602980 i
1,63,17	84.289185236375516 + 42.981045599670210 i	6653.531602460280737 +4158.881672943630292 i	1,63,19	81.933001028977415 + 46.518835512766159 i	6220.614699958034180 +4289.916284594612989 i
1,63,21	79.503336229084212 + 49.936674854030080 i	5792.577763303068423 +4382.741131522921933 i	1,63,23	76.999337949245103 + 53.243378180191300 i	5370.754598028092914 +4439.721402180393852 i
1,63,25	74.419802420743210 + 56.446176965525900 i	4956.387393623070238 +4462.887612415256626 i	1,63,27	71.763163475938512 + 59.551071156402294 i	4550.648005776200989 +4454.012721708159916 i
1,63,29	69.027468841014411 + 62.563078763108564 i	4154.654942822694466 +4414.667597988542184 i	1,63,31	66.210343565339357 + 65.486417239451811 i	3769.487446953375638 +4346.262030889103698 i
1,63,33	63.308938692908647 + 68.324637591760379 i	3396.197639052824798 +4250.075798305232638 i	1,63,35	60.319861883373470 + 71.080724625380896 i	3035.821446871008447 +4127.282707401706830 i
1,63,37	57.239084851044943 + 73.757172102028164 i	2689.388895594973746 +3978.969562964774468 i	1,63,39	54.061819844208884 + 76.356038595534855 i	2357.934275686365709 +3806.151403000265418 i
1,63,41	50.782353344268763 + 78.878987799701889 i	2042.506702349620127 +3609.783940262043416 i	1,63,43	47.393818689592287 + 81.327315540959404 i	1744.181646492455457 +3390.773875669838162 i
1,63,45	43.887878493816459 + 83.701964493786519 i	1464.074167465792016 +3149.987553127788942 i	1,63,47	40.254268732121133 + 86.003526340455437 i	1203.354857605712141 +2888.258266550228655 i
1,63,49	36.480121249315815 + 88.232229558954387 i	963.270009808686154 +2606.392368828400322 i	1,63,51	32.548912276667401 + 90.387908666927231 i	745.168435826937412 +2305.174102353766102 i
1,63,53	28.438737278991592 + 92.469946571921511 i	550.539127402134454 +1985.368612509784498 i	1,63,55	24.119266105747474 + 94.477173224749208 i	381.067623660945458 +1647.721445598810533 i
1,63,57	19.545802215149205 + 96.407684229023673 i	238.727464261901190 +1292.949268884197181 i	1,63,59	14.645851314001243 + 98.258489694817243 i	125.946198527902453 + 921.702847175014426 i
1,63,61	9.280357796192298 +100.024716413230564 i	45.965490357298876 + 534.408546438355188 i	1,63,63	3.051689418701653 +101.697002844744034 i	3.971193199007340 + 130.001742584277650 i
1,64,1	102.028163575630472 + 7.765724527432633 i	10380.370312050745270 +1143.248434248172998 i	1,64,3	100.286518811970979 + 13.344180110653941 i	9981.545318648342800 +1837.115941450558012 i
1,64,5	98.459675404189397 + 18.373708844504620 i	9564.197375737563561 +2397.028943245082246 i	1,64,7	96.553178058313591 + 23.044224657825588 i	9134.624669684000764 +2863.471173569454095 i
1,64,9	94.570361608672201 + 27.443452548408949 i	8697.042616551925676 +3255.671585963592406 i	1,64,11	92.513358820712355 + 31.622709954493924 i	8254.609341769695675 +3585.158550106490566 i
1,64,13	90.383517584737561 + 35.615812104173180 i	7809.850583379276031 +3859.719678466627101 i	1,64,15	88.181610428599910 + 39.446731278436779 i	7364.876560194070407 +4085.023672655290738 i
1,64,17	85.907950478191509 + 43.133324737989639 i	6921.507379215740912 +4265.416603139170547 i	1,64,19	83.562458108683202 + 46.689376033036424 i	6481.352043414632135 +4404.361273189460007 i
1,64,21	81.144698240436142 + 50.125809477215441 i	6045.86151583974242 +4504.70051826141753 i	1,64,23	78.653898120294173 + 53.451458776202891 i	5616.366223290287053 +4568.825083755554260 i
1,64,25	76.088950605133363 + 56.673577277532381 i	5194.103730032479689 +4598.785982475620585 i	1,64,27	73.448405359636254 + 59.798189711186410 i	4780.239962070155343 +4596.372699632495824 i
1,64,29	70.730448762658668 + 62.830342051916176 i	4375.886092030909822 +4563.169426195423512 i	1,64,31	67.932872119027024 + 65.774283266651409 i	3982.112471770089542 +4500.596626178038605 i
1,64,33	65.053026685084717 + 68.633599908924452 i	3599.960573816023953 +4409.942508481086406 i	1,64,35	62.087762815031276 + 71.411316998788777 i	3230.453648573771261 +4292.387366833122542 i
1,64,37	59.033349000857228 + 74.109974006381194 i	2874.606656860959447 +4149.022769037710532 i	1,64,39	55.885364421842169 + 76.731681792913648 i	2533.435963478876602 +3980.866955170437450 i
1,64,41	52.638555389090854 + 79.278164362215378 i	2207.969262293422616 +3788.877398251842806 i	1,64,43	49.286640995661863 + 81.750787828243560 i	1899.256245355725696 +3573.961206326312094 i
1,64,45	45.822044962798536 + 84.150577838223299 i	1608.380640109069873 +3336.983850149140835 i	1,64,47	42.235516421004256 + 86.478225590028188 i	1336.474450923607264 +3078.776549842076292 i
1,64,49	38.515576725923310 + 88.734081309952245 i	1084.735619008446520 +2800.142513997160222 i	1,64,51	34.647680554097896 + 90.918132257742698 i	854.450991585300812 +2501.862049077352822 i
1,64,53	30.612879693633413 + 93.029959319378591 i	647.027754180684497 +2184.696246432125008 i	1,64,55	26.385555270612961 + 95.068660504333963 i	464.038997137586932 +1849.388238150980214 i
1,64,57	21.929227397954719 + 97.032717233046824 i	307.294590898482227 +1496.659004719759878 i	1,64,59	17.187822789011094 + 98.919748243307680 i	178.962240903662064 +1127.187869593140249 i
1,64,61	12.063759881743238 +100.726001296344677 i	81.804923250062373 + 741.537218912053049 i	1,64,63	6.340541132630155 +102.445029602352236 i	19.774693213460601 + 339.758207604708673 i
1,65,1	103.599913716927460 + 7.782190333391690 i	10703.376835828699768 +1164.355850205871775 i	1,65,3	101.860429740205845 + 13.376383290348018 i	10299.105824617285180 +1872.617287381708366 i
1,65,5	100.036805654713078 + 18.421903487680250 i	9876.061511094190791 +2445.200719677121469 i	1,65,7	98.134531467837860 + 23.108675139191142 i	9440.559545563066422 +2923.181842170181426 i
1,65,9	96.156920750484460 + 27.524424419746850 i	8996.825805796048371 +3326.062557998055127 i	1,65,11	94.106104829656203 + 31.720471335944215 i	8548.025029581356648 +3665.530299357956210 i
1,65,13	91.983444749336286 + 35.730636340164196 i	8096.687023490896536 +3949.476926719656149 i	1,65,15	89.789738543853119 + 39.578899647796874 i	7644.924184582250746 +4183.644525406376488 i
1,65,17	87.525336383356318 + 43.283129039158183 i	7194.557266232955044 +4372.433344109826976 i	1,65,19	85.190207255809824 + 46.857121063053704 i	6747.194587568997122 +4519.347540738743191 i
1,65,21	82.783976910881748 + 50.311815464576426 i	6304.285204725911171 +4627.262275385018256 i	1,65,23	80.305946833274177 + 53.656063834758086 i	5867.156457671306271 +4698.593985259277360 i
1,65,25	77.755099276887947 + 56.897139949454562 i	5437.041638322191830 +4735.414329545928013 i	1,65,27	75.130090852053286 + 60.041091669048541 i	5015.101167381766572 +4739.529484600085198 i
1,65,29	72.429235605710858 + 63.092991035522132 i	4602.439391582471217 +4712.537147564747102 i	1,65,31	69.650477416294336 + 66.057116340696595 i	4200.118386015885335 +4655.868659741557167 i
1,65,33	66.791350545291792 + 68.937087144231782 i	3809.169716440650973 +4570.820886101320866 i	1,65,35	63.848926145328797 + 71.735965704871390 i	3430.604857935144992 +4458.580856860658059 i
1,65,37	60.819741233415428 + 74.456333679790873 i	3065.424813325761079 +4320.245180573314428 i	1,65,39	57.699704865253445 + 77.100349999688987 i	2714.629393058229653 +4156.835607926092052 i
1,65,41	54.483973643275597 + 79.669793852701829 i	2379.226591151605589 +3969.311714138890238 i	1,65,43	51.166784676978281 + 82.166095303711487 i	2060.242515491653194 +3758.581390808439210 i
1,65,45	47.741227659551242 + 84.590354977350870 i	1758.732412232727711 +3525.509643704409882 i	1,65,47	44.198926904471946 + 86.943353247130844 i	1475.793485343200928 +3270.926046978846443 i
1,65,49	40.529585200984194 + 89.225548306653977 i	1212.580499505879970 +2995.631078042384615 i	1,65,51	36.720306236896612 + 91.437061089042288 i	970.325660932711799 +2700.401416412041272 i
1,65,53	32.754543176461674 + 93.577642781810141 i	750.365190631027417 +2385.994069654532723 i	1,65,55	28.610373713916580 + 95.646616672678661 i	554.176771422486354 +2053.148729258672120 i
1,65,57	24.257455544374050 + 97.642777878502628 i	383.435722749036074 +1702.586574275429939 i	1,65,59	19.651085837165549 + 99.564215536501536 i	240.106274242396552 +1335.000086248942807 i
1,65,61	14.718769131240125 +101.407970189539554 i	126.607401518745661 + 951.014387649686455 i	1,65,63	9.321446443261159 +103.169257201281170 i	46.172769808891211 + 551.023997538074127 i
1,65,65	3.061398753010214 +104.838934414225136 i	3.980820607585773 + 133.893964644422709 i	1,66,1	105.171637830298010 + 7.798406578740542 i	11031.321018918937625 +1185.514840563948383 i
1,66,3	103.434256211885113 + 13.408096109893295 i	10621.610356117393167 +1908.219357896213978 i	1,66,5	101.613764169696594 + 18.469359688468103 i	10192.879083220519533 +2493.523264115026450 i
1,66,7	99.715597812994901 + 23.172131622934693 i	9751.460144725268037 +2983.094304428339001 i	1,66,9	97.743049173393501 + 27.604139065420977 i	9301.589818077964082 +3396.707387663887403 i
1,66,11	95.698247137253020 + 31.816704246741029 i	8846.439456777492524 +3746.209068805738298 i	1,66,13	93.582564282521673 + 35.843652356196436 i	8388.542988535857148 +4039.595526144802989 i
1,66,15	91.396821929121785 + 39.708970922269948 i	7930.015083877680809 +4282.682309128345878 i	1,66,17	89.141404434102469 + 43.430537494429267 i	7472.677278346473940 +4479.923935996484943 i
1,66,19	86.816325829584940 + 47.022160739636945 i	7018.137384352273330 +4634.866021862092566 i	1,66,21	84.421268301366027 + 50.494794817803715 i	6567.842772488552328 +4750.416165786095007 i
1,66,23	81.955602197261996 + 53.857308041006519 i	6123.117966359136517 +4829.016623647369670 i	1,66,25	79.418392600433222 + 57.116993284396926 i	5685.192312003806364 +4872.759841408139437 i
1,66,27	76.808395023326284 + 60.279920021657482 i	5255.221116596336287 +4883.468841782436357 i	1,66,29	74.124041282407035 + 63.351184625858501 i	4834.302370294109096 +4862.754999677083106 i
1,66,31	71.363415562870401 + 66.335092717951952 i	4423.490434355700927 +4812.060723500166205 i	1,66,33	68.524219792759439 + 69.235294555202913 i	4023.807645443242109 +4732.691744499509696 i
1,66,35	65.603726533390414 + 72.054886943684394 i	3636.254523769025582 +4625.842062439187430 i	1,66,37	62.598716496848290 + 74.796490559905266 i	3261.819115108201913 +4492.613585259668071 i
1,66,39	59.505396332389296 + 77.462308633236077 i	2901.485908442347409 +4334.031861326378930 i	1,66,41	56.319290208819574 + 80.054170986932462 i	2556.244734441098444 +4151.058886215982966 i
1,66,43	53.035095517529918 + 82.573566061181296 i	2227.100058975956927 +3944.603686053069396 i	1,66,45	49.646487965487857 + 85.021662495013217 i	1915.081144004514499 +3715.531184572609618 i
1,66,47	46.145853021694684 + 87.399320944458353 i	1621.253670582126233 +3464.669717785017838 i	1,66,49	42.523906439825126 + 89.707095898716545 i	1346.733638452994001 +3192.817442732202380 i
1,66,51	38.769140949830366 + 91.945226123353336 i	1092.704739685514141 +2900.747768277329669 i	1,66,53	34.866987363193033 + 94.113610681443717 i	860.441084254921179 +2589.213772287351276 i
1,66,55	30.798478508867380 + 96.211764609495816 i	651.338420886659833 +2258.951261655554845 i	1,66,57	26.537981730116481 + 98.238742773751980 i	466.959515266193534 +1910.679403166458542 i
1,66,59	22.049008847424208 +100.193008367984092 i	309.104852068472098 +1545.095796018325018 i	1,66,61	17.275483721072376 +102.072192332581764 i	179.933524409507157 +1162.855836163930235 i
1,66,63	12.119824805243502 +103.872597972125988 i	82.200508985338274 + 764.494870127842546 i	1,66,65	6.365515912640349 +105.587913104465841 i	19.850355438656067 + 350.023457076485329 i
1,67,1	106.743336991407176 + 7.814380677714311 i	11364.202784106606487 +1206.724628928657921 i	1,67,3	105.008001709870726 + 13.439333210874853 i	10949.058665830418249 +1943.920628334788717 i
1,67,5	103.190558050508173 + 18.516099624984015 i	10514.649592795793069 +2541.994276552984047 i	1,67,7	101.296389027442601 + 23.234624147285427 i	10067.325636255434802 +3043.205455058726329 i
1,67,9	99.328764914030899 + 27.682634728545594 i	9611.333410068207741 +3467.602131612268295 i	1,67,11	97.289811252913339 + 31.911455510344407 i	9149.850883104378227 +3827.190040875294926 i
1,67,13	95.180910676179522 + 35.954915971161583 i	8685.416150988176014 +4130.069744823826113 i	1,67,15	93.002905752476039 + 39.837010379545710 i	8220.146244962621495 +4382.130332891380021 i
1,67,17	90.756212403330593 + 43.575625360814556 i	7755.863608262003254 +4587.880679017473085 i	1,67,19	88.440886432477413 + 47.184580886379472 i	7294.175710849443931 +4750.907952243321233 i
1,67,21	86.056662440207617 + 50.674844591821085 i	6836.528443253128899 +4874.152298351707941 i	1,67,23	83.602974729276454 + 54.055300435884135 i	6384.243763646505613 +4960.081911343848333 i
1,67,25	81.078965232670257 + 57.333259168665187 i	5938.547374680480061 +5010.810156020109389 i	1,67,27	78.483481068273989 + 60.514810482529235 i	5500.589832259435752 +5028.177047267721719 i
1,67,29	75.815062879991871 + 63.605073489849019 i	5071.463202804852699 +5013.807798728604212 i	1,67,31	73.071924131075448 + 66.608379316603816 i	4652.214651276311997 +4969.156063718784026 i
1,67,33	70.251920700657962 + 69.528406815417753 i	4243.857905941506942 +4895.536636351658672 i	1,67,35	67.352509328160139 + 72.368284899244287 i	3847.38328037420206 +4794.15070355815988 i
1,67,37	64.370692511686173 + 75.130670405731678 i	3463.766771536257238 +4666.105713822279540 i	1,67,39	61.302946239626763 + 77.817807480100299 i	3093.978659159192375 +4512.431280564472218 i
1,67,41	58.145125203296445 + 80.431572527033083 i	2738.991987326834533 +4334.092115948899846 i	1,67,43	54.892337562754896 + 82.973507432976021 i	2399.791306442396490 +4131.998707787687636 i
1,67,45	51.538777342908503 + 85.444842753628762 i	2077.382093539012658 +3907.016256247420188 i	1,67,47	48.077496096209906 + 87.846511727471906 i	1772.801361363762226 +3659.972245132250009 i
1,67,49	44.500084658440755 + 90.179155172466622 i	1487.130135483579124 +3391.662910760376690 i	1,67,51	40.796216847753399 + 92.443116390476135 i	1221.508771098280022 +3102.858767526574866 i
1,67,53	36.952971853494581 + 94.638423904331376 i	977.156591264228723 +2794.309221823464668 i	1,67,55	32.953782904905502 + 96.764757743658123 i	755.398250734011185 +2466.746091447628260 i
1,67,57	28.776712538977041 + 98.821391127478179 i	557.700998037826366 +2120.885382464275153 i	1,67,59	24.391408375113844 +100.807091454719412 i	385.730682150747327 +1757.425463833217009 i
1,67,61	19.753162796226956 +102.719946074102396 i	241.442867650670109 +1377.036034837871512 i	1,67,63	14.789480045956740 +104.557027887269555 i	127.248516388800553 + 980.317883037962815 i
1,67,65	9.361298084789276 +106.313638960925942 i	46.373788255458194 + 567.634970110065183 i	1,67,67	3.070813242277758 +107.980846930567623 i	3.990154032773514 + 137.783535741832935 i
1,68,1	108.315012216744591 + 7.830119719761012 i	11702.022057340012907 +1227.984461816951125 i	1,68,3	106.581669525388918 + 13.470108590339846 i	11281.450516538554439 +1979.719619317325623 i
1,68,5	104.767194006042985 + 18.562144493530027 i	10841.372560946088925 +2590.611525840767172 i	1,68,7	102.876916380678736 + 23.296181412438891 i	10388.155223465890231 +3103.512282586344099 i
1,68,9	100.914084996088349 + 27.759947936958458 i	9926.055390086898115 +3538.742966831797730 i	1,68,11	98.880821237667689 + 32.004769833866959 i	9458.257641374495506 +3908.468546661626533 i
1,68,13	96.778516438596256 + 36.064480459200233 i	8987.304282296559904 +4220.894029917166336 i	1,68,15	94.608032546113037 + 39.963080292601987 i	8515.314784750398758 +4481.982117645053222 i
1,68,17	92.369814636355628 + 43.718464394082645 i	8044.112615761966481 +4696.296120824044920 i	1,68,19	90.063957279354739 + 47.344463286609347 i	7575.305055323179658 +4867.464853719532584 i
1,68,21	87.690243802544529 + 50.852057212774461 i	7110.336704824277149 +4998.461110483996890 i	1,68,23	85.248167973672423 + 54.250144785231356 i	6650.527190105959562 +5091.779135835967281 i
1,68,25	82.736943121247890 + 57.546053499397409 i	6197.098851088328956 +5149.553337764547905 i	1,68,27	80.155501335318462 + 60.745891982169205 i	5751.197827902129575 +5173.640860467351558 i
1,68,29	77.502483990111273 + 63.854800623121506 i	5313.910663124094754 +5165.680907509730787 i	1,68,31	74.776223885333607 + 66.877134382676445 i	4886.277802141643406 +5127.138543867028602 i
1,68,33	71.974718551047317 + 69.816598790103896 i	4469.304934591706115 +5059.337810605626146 i	1,68,35	69.095593542255344 + 72.676352646726215 i	4063.972851442436877 +4963.487285288429121 i
1,68,37	66.136053738890425 + 75.459086365063769 i	3671.246327258683777 +4840.700182084687185 i	1,68,39	63.092819634092884 + 78.167081961599635 i	3292.082439054673159 +4692.010428788257741 i
1,68,41	59.962044165418412 + 80.802258796816659 i	2927.438682463381610 +4518.385730194159805 i	1,68,43	56.739203555883456 + 83.366207827086626 i	2578.281233795551088 +4320.738340809362853 i
1,68,45	53.418952447986072 + 85.860216157544940 i	2245.593731773270065 +4099.934072339934573 i	1,68,47	49.994928571823912 + 88.285282901355174 i	1930.387021878618725 +3856.799919888225759 i
1,68,49	46.459483892716882 + 90.642126631030749 i	1633.710436276039218 +3592.130582539913121 i	1,68,51	42.803304950984057 + 92.931183921540409 i	1356.665407270339983 +3306.694059690821177 i
1,68,53	39.014859483213947 + 95.152597454666790 i	1100.422599001146637 +3001.236400529340244 i	1,68,55	35.079557571248571 + 97.306190571063723 i	866.244425107260213 +2676.485528147768946 i
1,68,57	30.978415738159633 + 99.391422394364994 i	655.516087252695911 +2333.153751576942341 i	1,68,59	26.685789709460582 +101.407312273617151 i	469.790789284499397 +1971.937836442256412 i
1,68,61	22.165184661015648 +103.352310566381789 i	310.860286748936915 +1593.513397988495853 i	1,68,63	17.360522680840710 +105.224063427477077 i	180.875629357417210 +1198.513187385917718 i
1,68,65	12.174222727939711 +107.018929764010323 i	82.584292957932632 + 787.446739250854876 i	1,68,67	6.389750344952977 +108.730727267319779 i	19.923766361090657 + 360.285301670993192 i
1,69,1	109.886664467668467 + 7.845630488210544 i	12044.778767558023901 +1249.293607659438976 i	1,69,3	108.155262771148756 + 13.500435638023056 i	11618.785680573511854 +2015.614894759329218 i
1,69,5	106.343678379743551 + 18.607514565618612 i	11173.047528123615848 +2639.372846642200329 i	1,69,7	104.457190524146782 + 23.356830858773748 i	10713.948142005250702 +3164.011865162292452 i
1,69,9	102.499025501377801 + 27.836113604239284 i	10245.754615219177140 +3610.126185221530704 i	1,69,11	100.471299807060419 + 32.096689933784589 i	9771.658133331988211 +3990.040059140307221 i
1,69,13	98.375412237451357 + 36.172396702343441 i	9294.205247196863638 +4312.062999371654769 i	1,69,15	96.212242399564957 + 40.087240111936957 i	8815.517942580361705 +4582.231386255425605 i
1,69,17	93.982262306903067 + 43.859123063747973 i	8337.420817738340702 +4805.163044669290684 i	1,69,19	91.685602532096979 + 47.501885934910746 i	7861.521104335963173 +4984.528520348902930 i
1,69,21	89.322091743735868 + 51.026520770599852 i	7389.262292213550609 +5123.333351792516623 i	1,69,23	86.891279060332508 + 54.441939918824048 i	6921.961891663289862 +5224.097940668211777 i

Table D.2: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
1,69,25	84.392444221465993 + 57.755486576588744 i	6460.839140623971616 +5288.977855364651077 i	1,69,27	81.824598246909915 + 60.973287121015325 i	6007.036074623828426 +5319.847498505910153 i
1,69,29	79.186475892756064 + 64.100501873561740 i	5561.634082459020647 +5318.360205742723338 i	1,69,31	76.476520311164307 + 67.141508096166390 i	5125.667329894647082 +5285.992610284594775 i
1,69,33	73.692859621536272 + 70.100036239806258 i	4700.133992024152576 +5224.078172745675147 i	1,69,35	70.833274454743105 + 72.979272972612492 i	4286.005964007156763 +5133.833051792578772 i
1,69,37	67.895154826341965 + 75.781939935500262 i	3884.237551945368523 +5016.376437732603335 i	1,69,39	64.875443818566680 + 78.510354267727749 i	3495.773543735105704 +4872.746805483492608 i
1,69,41	61.770564364019293 + 81.166475031552409 i	3121.557003808305126 +4703.915109944846336 i	1,69,43	58.576323719155674 + 83.751938352073338 i	2762.537115849997463 +4510.795656800484721 i
1,69,45	55.287787658203278 + 86.268083137416141 i	2419.677411151033539 +4294.255182375028198 i	1,69,47	51.899112436088465 + 88.715968499508790 i	2093.964771091548755 +4055.120534427706843 i
1,69,49	48.403316137877496 + 91.096383339665607 i	1786.419692363705963 +3794.185239577332140 i	1,69,51	44.791960230054144 + 93.409847886063858 i	1498.108477710808302 +3512.215155011223942 i
1,69,53	41.054693158978054 + 95.656606134621427 i	1230.158311169367380 +3209.953313643543879 i	1,69,55	37.178572743356590 + 97.836606929853431 i	983.776689471429222 +2888.123953495812657 i
1,69,57	33.147014953881872 + 99.949469404508250 i	760.277605589771952 +2547.435510184359828 i	1,69,59	28.938069397286405 +101.994424755656880 i	561.118650008122131 +2188.581879461638437 i
1,69,61	24.521375376498810 +103.970218625138187 i	387.956869692035923 +1812.240017330004775 i	1,69,63	19.852221739907804 +105.874940413123355 i	242.739742741927586 +1419.058108947491519 i
1,69,65	14.858113133098756 +107.705696786183680 i	127.870722396537175 +1009.613796366145721 i	1,69,67	9.399984686865736 +109.457874508452960 i	46.568910868249425 + 584.241665138384406 i
1,69,69	3.079950078135794 +111.122741971515453 i	3.999210686272515 + 141.670551483265257 i	1,70,1	111.458294654120692 + 7.860919477622712 i	12392.472846529184608 +1270.651355860300328 i
1,70,3	109.728784393390953 + 13.530327170926032 i	11961.063939302996005 +2051.605060002409573 i	1,70,5	107.920017174402162 + 18.652229240919688 i	11509.674053062639359 +2688.276136570053950 i
1,70,7	106.037221533520977 + 23.416598739437536 i	11044.703658097776497 +3224.701366624909951 i	1,70,9	104.083601634933331 + 27.911165123396316 i	10570.429988632276945 +3681.748188485956234 i
1,70,11	102.061268425294557 + 32.187256652435906 i	10090.050825811551476 +4071.900186786852828 i	1,70,13	99.971627030452368 + 36.278713331846333 i	9606.116998435330970 +4403.571434134708397 i
1,70,15	97.815573135899470 + 40.209546634131954 i	9120.753073173171288 +4682.872054161050073 i	1,70,17	95.593603650100263 + 43.997666751706184 i	8635.784878976339314 +4914.474458325607884 i
1,70,19	93.305882603443877 + 47.656923269213053 i	8152.819730866266582 +5102.091005377453257 i	1,70,21	90.952280891894631 + 51.198319288596480 i	7673.300172472862869 +5248.760068860965475 i
1,70,23	88.532399208746412 + 54.630780042250642 i	7198.541800378050539 +5357.028307716142081 i	1,70,25	86.045579141550377 + 57.961663463063303 i	6729.760993099285770 +5429.072561321169815 i
1,70,27	83.490905124915443 + 61.197112584502712 i	6268.095970581292931 +5466.784612417523022 i	1,70,29	80.867198613512542 + 64.342306419990749 i	5814.623310841921011 +5471.832062562783904 i
1,70,31	78.173004974077699 + 67.401643123799019 i	5370.371306747117160 +5445.703260389585921 i	1,70,33	75.406572941401109 + 70.378876460221335 i	4936.331101863795084 +5389.741248073629322 i
1,70,35	72.565825897683325 + 73.277219117956662 i	4513.466271152409718 +5305.169943912755116 i	1,70,37	69.648323615097212 + 76.099421832024234 i	4102.721341755296635 +5193.114711316743524 i
1,70,39	66.651212352447658 + 78.847834376205739 i	3705.029644008844116 +5054.618789866406587 i	1,70,41	63.571160203451484 + 81.524452584550659 i	3321.320822512627274 +4890.656624963236936 i
1,70,43	60.404273192023226 + 84.130954260263863 i	2952.528310991460785 +4702.144838978896587 i	1,70,45	57.145985540078186 + 86.668725898789148 i	2599.597078880523895 +4489.951385318212488 i
1,70,47	53.790914362257524 + 89.138881437277263 i	2263.491993883678788 +4254.903284379999604 i	1,70,49	50.332664009339389 + 91.542273635410865 i	1945.207221592026826 +3997.793235289617769 i
1,70,51	46.763556993694500 + 93.879498097109959 i	1645.777218655080560 +3719.385313317317468 i	1,70,53	43.074254203399072 + 96.150889249738583 i	1366.290104812647087 +3420.419884657749662 i
1,70,55	39.253201493354553 + 98.356506644593679 i	1107.904588534682262 +3101.617776333211623 i	1,70,57	35.285790902259173 +100.496108436257913 i	871.872305934433371 +2763.683587628756868 i
1,70,59	31.153024916755992 +102.569106233379046 i	659.568701037646747 +2407.307719007196283 i	1,70,61	26.829249487599995 +104.574490293170854 i	472.538096127724032 +2033.165936466793482 i
1,70,63	22.277964533655712 +106.510702626289131 i	312.564106253297211 +1641.913132109110848 i	1,70,65	17.443091257353558 +108.375406998797800 i	181.790250476671588 +1234.160577468519250 i
1,70,67	12.227049490411837 +110.165017677370471 i	82.956955670587249 + 810.393117738200885 i	1,70,69	6.413286819050765 +111.873477635810488 i	19.995055210957016 + 370.543873402771396 i
1,71,1	113.029903638043990 + 7.875992909924759 i	12745.104228701580723 +1292.057015910134169 i	1,71,3	111.302237182967872 + 13.559795465472268 i	12308.285082651580524 +2087.688760050760720 i
1,71,5	109.496216074955839 + 18.696307096475131 i	11851.251711805742161 +2737.319353484751446 i	1,71,7	107.617018947537730 + 23.475510187781065 i	11380.421066902488747 +3285.578032788416294 i
1,71,9	105.667827784734527 + 27.985134453850989 i	10900.080457077345272 +3753.605483328354239 i	1,71,11	103.650747391491493 + 32.276509066110187 i	10413.434247165740089 +4154.044667574072264 i
1,71,13	101.567188184854345 + 36.383476859207676 i	9923.037571866983853 +4495.414270836738069 i	1,71,15	99.418060472655910 + 40.330054157963232 i	9431.017640096739342 +4783.898220600262903 i
1,71,17	97.203884174874233 + 44.134157935996136 i	8939.201603620749665 +5024.223583692013563 i	1,71,19	94.924854433380204 + 47.809646385236874 i	8449.196983326111877 +5220.144609032809967 i
1,71,21	92.580881504061978 + 51.367532972162628 i	7962.445530708446313 +5374.732591000405591 i	1,71,23	90.171614184127733 + 54.816755024236599 i	7480.261116762444544 +5490.560540633039636 i
1,71,25	87.696451724704261 + 58.164684315533954 i	7003.857486496037382 +5569.826672740726281 i	1,71,27	85.154546932516041 + 61.417479524054578 i	6534.369313076748767 +5614.440264990453215 i
1,71,29	82.544801870097686 + 64.580337210597150 i	6072.868682140967394 +5626.083310950546547 i	1,71,31	79.865856730713205 + 67.657675124083255 i	5620.378390289044546 +5606.256013196567437 i
1,71,33	77.116071848800047 + 70.653268865211587 i	5177.88299555349658 +5556.3111477237987 i	1,71,35	74.293502271945485 + 73.570355453527199 i	4746.338282393682675 +5477.480559993745374 i
1,71,37	71.395863772110474 + 76.411712772239298 i	4326.679631038151456 +5370.895971082045435 i	1,71,39	68.420488528757858 + 79.179720970964354 i	3919.829673130441734 +5237.605588793802781 i
1,71,41	65.364267879243940 + 81.876410009323465 i	3526.705551681663110 +5078.587573724546019 i	1,71,43	62.223578370662288 + 84.503496234517144 i	3148.226072068173380 +4894.761112935955680 i
1,71,45	58.994185660018594 + 87.062409970004893 i	2785.319027399302740 +4686.995650175484116 i	1,71,47	55.671118264528403 + 89.554315401606672 i	2438.929006303681035 +4456.118678127849307 i
1,71,49	52.248499190853011 + 91.980123477883524 i	2110.026049737272388 +4202.922393471120813 i	1,71,51	48.719317048020230 + 94.340498008190806 i	1799.615628949703250 +3928.169434136167183 i
1,71,53	45.075107452301694 + 96.635854554792076 i	1508.750594760758304 +3632.597846757876141 i	1,71,55	41.305496567178508 + 98.866350952071969 i	1238.546047248681589 +3316.925354580566818 i
1,71,57	37.397523526912849 +101.031867137973364 i	990.198589558746562 +2981.852883592336639 i	1,71,59	33.334589340156050 +103.131970042243111 i	765.012355191608208 +2628.067093347990976 i
1,71,61	29.094732587904478 +105.165855649062607 i	564.435968569486704 +2256.241178873476201 i	1,71,63	24.647585749690460 +107.132252852814261 i	390.118273528290786 +1867.031942151170369 i
1,71,65	19.948435183694983 +109.029256258033769 i	243.999182719729077 +1461.067204473596803 i	1,71,67	14.924786491626218 +110.854007284854760 i	128.475098132231949 +1038.902548108129395 i
1,71,69	9.437572126805010 +112.601975303300918 i	46.758471854641542 + 600.844267845175864 i	1,71,71	3.088824997609218 +114.264620944479688 i	4.008006318809692 + 145.555101323915835 i
1,72,1	114.601492236528856 + 7.890856749438312 i	13102.672851062614427 +1313.509916548078991 i	1,72,3	112.875623785544761 + 13.588852287439336 i	12660.448908653212129 +2123.864677906228280 i
1,72,5	111.072280469462015 + 18.739765932492027 i	12197.780096794800556 +2786.500512945358878 i	1,72,7	109.196591803700016 + 23.533589280074086 i	11721.099690982322500 +3346.639187942869285 i
1,72,9	107.251717576566421 + 28.058052202288483 i	11234.705008560802526 +3825.694676921007613 i	1,72,11	105.239755918832628 + 32.364484585443783 i	10741.806983938256963 +4236.469363319741205 i
1,72,13	103.162121586963636 + 36.486731797768613 i	10244.965081897124037 +4587.586594906951177 i	1,72,15	101.019738169057590 + 40.448814629155358 i	9746.309209698285486 +4885.304160366511496 i
1,72,17	98.813146857878962 + 44.268656361013591 i	9247.667927263964884 +5134.403847038054664 i	1,72,19	96.542571740959218 + 47.960123234895320 i	8750.649075397452179 +5338.681867079849326 i
1,72,21	94.207959789964079 + 51.534238438616832 i	8256.693757164166527 +5501.242516909654114 i	1,72,23	91.809004710918259 + 54.999950661727766 i	7767.114293489414194 +5624.685249372954786 i
1,72,25	89.345159575210531 + 58.364644689532575 i	7283.122006520255127 +5711.229753438426087 i	1,72,27	86.815640942546977 + 61.634493907358383 i	6805.848272976162662 +5762.802910118163709 i
1,72,29	84.219425921841633 + 64.814711365197397 i	6336.36098209157626 +5781.10122394419068 i	1,72,31	81.555242811186289 + 67.909733209632691 i	5875.67778352515597 +5767.63688193635062 i
1,72,33	78.821555375721573 + 70.923355519108554 i	5424.77706230684544 +5723.77253715985898 i	1,72,35	76.016540330261336 + 73.858838094416939 i	4984.607300801178098 +5650.748119637218224 i
1,72,37	73.138057104472765 + 76.718984188798643 i	4556.09531297082412 +5549.70188125719505 i	1,72,39	70.183608408674615 + 79.506202272226773 i	4140.153726005785757 +5421.687188753098781 i
1,72,41	67.150289407663294 + 82.222554033143936 i	3737.68801734828185 +5267.68612815645156 i	1,72,43	64.034722341896000 + 84.869791539601110 i	3349.603379593223508 +5088.620683073949294 i
1,72,45	60.832972050437753 + 87.449385578216649 i	2976.811676115629780 +4885.362043038679985 i	1,72,47	57.540435792304159 + 89.962546517437730 i	2620.238765666327708 +4658.738453504330209 i
1,72,49	54.151697600552744 + 92.410238501582228 i	2280.832518249357690 +4409.541914252324204 i	1,72,51	50.660332377571841 + 94.793187296185380 i	1959.571926881999389 +4138.533953167750951 i
1,72,53	47.058636657036097 + 97.111881597693781 i	1657.477859120479025 +3846.450635471352143 i	1,72,55	43.337248735845243 + 99.366566996116376 i	1375.626223824074032 +3534.006901589775225 i
1,72,57	39.484595264993210 +101.557230886533247 i	1115.164720047448100 +3201.900239768560368 i	1,72,59	35.486052551389243 +103.683575182951415 i	877.335021249543502 +2850.813550620569004 i
1,72,61	31.322610993535065 +105.744971108782750 i	663.503517333906984 +2481.416746641311420 i	1,72,63	26.968608341065945 +107.740385263277560 i	475.206258197892964 +2094.365855499174813 i
1,72,65	22.387540426662778 +109.668255056482650 i	314.219248107761587 +1690.296188230013058 i	1,72,67	17.523328293670072 +111.526263760663667 i	182.678939283632474 +1269.798598776902736 i
1,72,69	12.278392917315381 +113.310880525768894 i	83.319120548464255 + 833.334272372667783 i	1,72,71	6.436164194052857 +115.016169173366933 i	20.064340434373346 + 380.799295161601378 i
1,73,1	116.173061224714758 + 7.905516716886138 i	13465.178653007906178 +1335.009404969830257 i	1,73,3	114.448946711002876 + 13.617508919849776 i	13017.555223032892172 +2160.131532995167163 i
1,73,5	112.648215468422961 + 18.782622814995996 i	12549.258816021501843 +2835.817685802262531 i	1,73,7	110.775948671146594 + 23.590859093892981 i	12066.738878874486545 +3407.882231550587221 i
1,73,9	108.835283924475647 + 28.129947697886397 i	11574.302670171442514 +3898.012472632575736 i	1,73,11	106.828312207264958 + 32.451219048773673 i	11075.167677762852691 +4319.170254359324645 i
1,73,13	104.756451742601953 + 36.588520775694342 i	10571.897717237097822 +4680.083634090769010 i	1,73,15	102.620638160862953 + 40.565877774758150 i	10066.625445462472271 +4987.084316052337272 i
1,73,17	100.421432320825957 + 44.401219195365179 i	9561.180909601103849 +5245.008869835971382 i	1,73,19	98.159085254114373 + 48.108418810081020 i	9057.172376613649249 +5457.695540079193051 i
1,73,21	95.833578206804447 + 51.698508929821386 i	8556.040435275303025 +5628.281702171077995 i	1,73,23	93.444646848346110 + 55.180448924791278 i	8059.096020357701491 +5759.393335703158300 i
1,73,25	90.991794534900066 + 58.561635820681488 i	7567.548227779708213 +5853.271697209468584 i	1,73,27	88.474297340859934 + 61.848256840900014 i	7082.525371221350724 +5911.861373529544835 i
1,73,29	85.891202333842983 + 65.045540544915127 i	6605.09141903119144 +5936.87349247734437 i	1,73,31	83.241319788808681 + 68.157940371115913 i	6136.25919841074028 +5929.83234859166441 i
1,73,33	80.523209483197761 + 71.189271623642711 i	5677.00130360914045 +5892.11060692965748 i	1,73,35	77.735160759342193 + 74.142815460708363 i	5228.25936606856885 +5824.95643011648099 i
1,73,37	74.875165600862104 + 77.021398877256559 i	4790.95216802409050 +5729.51476347891821 i	1,73,39	71.940883480469637 + 79.827456788612338 i	4365.982968805981727 +5606.844311542939067 i
1,73,41	68.929596129297778 + 82.563080435425405 i	3954.24634344179640 +5457.93128271388377 i	1,73,43	65.838149565485779 + 85.230055055834910 i	3556.63479402919666 +5283.70067411060245 i
1,73,45	62.662879575506516 + 87.829888877341617 i	3174.045379610953205 +5085.025659990936219 i	1,73,47	59.399515166267278 + 90.363834824003931 i	2807.386618107288996 +4862.735501118103457 i
1,73,49	56.043051969526410 + 92.832905817736389 i	2457.585946368865734 +4617.622287196685647 i	1,73,51	52.587583613694449 + 95.237884103291151 i	2125.598088446596648 +4350.446745540601114 i
1,73,53	49.026072659194568 + 97.579324579384803 i	1812.414791611319288 +4061.943285257999833 i	1,73,55	45.350031901428167 + 99.857551597144024 i	1519.076570474446498 +3752.824386620360201 i
1,73,57	41.549067884011500 +102.072647978909217 i	1246.687400523112046 +3423.784375072087641 i	1,73,59	37.610203373292322 +104.224433917022054 i	996.433818415466362 +3075.502510556657398 i
1,73,61	33.516826393758379 +106.312431538685459 i	769.610813000784901 +2708.645110614444093 i	1,73,63	29.246966134701349 +108.335807399900716 i	567.658662062062149 +2323.865936568658071 i
1,73,65	24.770249522560164 +110.293277869816094 i	392.218544931207646 +1921.802775307946831 i	1,73,67	20.041961296984489 +112.182945344477147 i	245.223279718260784 +1503.064131320572869 i
1,73,69	14.989608452914288 +114.001986650371364 i	129.062632646489988 +1068.184521317634562 i	1,73,71	9.474120860220994 +115.745951624237335 i	46.942777857402234 + 617.442949399862528 i
1,73,73	3.097452442326910 +117.406485108984938 i	4.016555380458016 + 149.437269127112927 i	1,74,1	117.744611338468061 + 7.919978302460648 i	13832.621576218600239 +1356.554846078462788 i
1,74,3	116.022208342116713 + 13.645776188985382 i	13379.603838815206473 +2196.488079680865667 i	1,74,5	114.224025922609471 + 18.824894115599310 i	12905.687492232791556 +2885.268995921840997 i
1,74,7	112.355097680947040 + 23.647341762534019 i	12417.338003754046440 +3469.304635125440655 i	1,74,9	110.418539077232808 + 28.200849062384655 i	11918.872506050816810 +3970.555665994615538 i
1,74,11	108.416433510378710 + 32.536746809034064 i	11413.515022469544419 +4402.143434520653500 i	1,74,13	106.350201869398930 + 36.688884641063538 i	10903.833736948266960 +4772.900752339887731 i
1,74,15	104.220790684064525 + 40.681291228031114 i	10391.964102759626257 +5089.233290746143211 i	1,74,17	102.028778992874692 + 44.531901178430495 i	9879.737727603222083 +5356.032460137109410 i
1,74,19	99.774442919707070 + 48.254595313121854 i	9368.763403620081590 +5577.178603294965797 i	1,74,21	97.457795728123969 + 51.860414509144128 i	8860.481330604587129 +5755.842247516811767 i
1,74,23	95.078612332119960 + 55.358328183171260 i	8356.201210396395254 +5894.675979627397282 i	1,74,25	92.636443115484283 + 58.755744884505056 i	7857.130096428150864 +5995.942712174855842 i
1,74,27	90.130619772137194 + 62.058864867393318 i	7364.39345722773760 +6061.60483478380229 i	1,74,29	87.560254665894931 + 65.272931292454317 i	6879.05159705941423 +6093.38820470471728 i
1,74,31	84.924234451013461 + 68.402413866676554 i	6402.11282251361536 +6092.82934018230389 i	1,74,33	82.221208165920757 + 71.451145964177584 i	5934.54429176260874 +6061.31104547682844 i
1,74,35	79.449569587873711 + 74.422428789940327 i	5477.281202841058075 +6000.089855210409951 i	1,74,37	76.607433238339809 + 77.319111586472545 i	5031.23479931545171 +5910.31756106109060 i
1,74,39	73.692602998066960 + 80.143654000223592 i	4597.29955767502361 +5793.05837329751284 i	1,74,41	70.702531768306173 + 82.898174842363645 i	4176.35984889294105 +5649.30280737778594 i
1,74,43	67.634269928547710 + 85.584490209889253 i	3769.296324477433245 +5479.979077171406061 i	1,74,45	64.484399389704336 + 88.204143047789245 i	3376.99225836307960 +5285.96256533375345 i
1,74,47	61.248948668383932 + 90.758425588243426 i	3000.340077641189964 +5068.083793750615769 i	1,74,49	57.923282358512779 + 93.248395604350321 i	2640.248338333544780 +4827.135211086086201 i
1,74,51	54.501955220587163 + 95.674886995362641 i	2297.649451989894260 +4563.877035848837295 i	1,74,53	50.978516087881559 + 98.038514819570409 i	1973.508555104392826 +4279.042331440470662 i
1,74,55	47.345238510076207 +100.339674447163532 i	1668.834055456013175 +3973.341434955118984 i	1,74,57	43.592740836188362 +102.578533932630620 i	1384.690623109363059 +3647.465802624052685 i
1,74,59	39.709432827353545 +104.755017861511973 i	1122.215786691669109 +3302.091252311659900 i	1,74,61	35.680677054850669 +106.868779538168042 i	882.641982876749580 +2937.880435473725356 i
1,74,63	31.487453507235230 +108.919155879080591 i	667.327176989349709 +2555.484052117224110 i	1,74,65	27.104093080373083 +110.905094436190495 i	477.799694164089715 +2155.539531260626063 i
1,74,67	22.494088510197450 +112.825031149727280 i	315.828406312074435 +1738.663641160372435 i	1,74,69	17.601361274878350 +114.676670282265448 i	183.543119762292918 +1305.427789379832575 i
1,74,71	12.328333685803164 +116.456535211785422 i	83.671360140438139 + 856.270448075370723 i	1,74,73	6.458418179582465 +118.158806336660291 i	20.131730858690556 + 391.051681602271742 i
1,75,1	119.316143276856295 + 7.934246778028510 i	14205.001564546460941 +1378.145621775201334 i	1,75,3	117.595410942570226 + 13.673664488674316 i	13746.594575957718916 +2232.933105855787744 i
1,75,5	115.799716439518818 + 18.866595548614814 i	13267.065762186970460 +2934.852618034200532 i	1,75,7	113.934046554056067 + 23.703058525772498 i	12772.896462183434778 +3530.903939282520241 i
1,75,9	112.001494661167882 + 28.270783275415656 i	12268.413615495581325 +4043.321140890710467 i	1,75,11	110.004136196999553 + 32.621100814725381 i	11756.847761381464807 +4485.385106379387253 i
1,75,13	107.943393981685858 + 36.787862559719848 i	11240.771466750720720 +4866.033444048462115 i	1,75,15	105.820224388509985 + 40.795100644632114 i	10722.323023951586947 +5191.745841148993350 i
1,75,17	103.635223259551339 + 44.660754756593541 i	10203.335669164556748 +5467.468604452525142 i	1,75,19	101.388690095796264 + 48.398712315057573 i	9685.418812054742375 +5697.124237203113807 i
1,75,21	99.080668089398159 + 52.020022244135390 i	9170.012380581705656 +5883.916487806411169 i	1,75,23	96.710968885837829 + 55.533663416142289 i	8658.42498700434226 +6030.52462664875929 i
1,75,25	94.279186891551674 + 58.947055236742358 i	8151.861814137681904 +6139.233306115408595 i	1,75,27	91.784705834613177 + 62.266410240450078 i	7651.44568898384568 +6212.02281042678624 i
1,75,29	89.226699094907218 + 65.496985345779436 i	7158.23349137899277 +6250.63382669132716 i	1,75,31	86.604124583363117 + 68.643265580213978 i	6673.22928847586263 +6256.61520665047919 i
1,75,33	83.915714442583976 + 71.709101319360059 i	6197.39513203760998 +6231.36001383039002 i	1,75,35	81.159959443077724 + 74.697812606400802 i	5731.66017370972440 +6176.13328624095629 i
1,75,37	78.335087586478210 + 77.612269557768016 i	5276.9285740381338 +6092.0938058500324 i	1,75,39	75.439036045066103 + 80.454954980457515 i	4834.08656541194614 +5980.31144654532818 i
1,75,41	72.469415114344528 + 83.228013447646703 i	4404.00895530661455 +5841.78120421048698 i	1,75,43	69.423462272326645 + 85.933289815383361 i	3987.56531153747665 +5677.43470003929360 i
1,75,45	66.297983643201931 + 88.572359284610641 i	3585.62604874220779 +5488.14973464478589 i	1,75,47	63.089279030226756 + 91.146550477826352 i	3199.06863188731040 +5274.75832129778792 i
1,75,49	59.793045015439205 + 93.656962515855071 i	2828.784128262380818 +5038.053519908576163 i	1,75,51	56.404248093423021 + 96.104476682255637 i	2475.684377462203884 +4778.795314741295442 i
1,75,53	52.916955844631197 + 98.489762897859322 i	2140.710116838561135 +4497.715718085106346 i	1,75,55	49.324107738792776 +100.813280842956516 i	1824.840512190084382 +4195.523230730040268 i
1,75,57	45.617196370172472 +103.075275092409314 i	1529.104726342951835 +3872.906738910625791 i	1,75,59	41.785810968032521 +105.275762964142643 i	1254.596460730371746 +3530.538862395981141 i
1,75,61	37.816959936222013 +107.414512733493126 i	1002.492922167449706 +3169.078661765348443 i	1,75,63	33.694020024390554 +109.491008051589375 i	774.080594345254553 +2789.173398722562524 i
1,75,65	29.395012436213911 +111.504390978487649 i	570.791965018437622 +2391.458544429096037 i	1,75,67	24.889559628093256 +113.453368925301334 i	394.261035206897475 +1976.553904845094848 i
1,75,69	20.132945448743591 +115.336054198983959 i	246.413955531909693 +1545.049623939423554 i	1,75,71	15.052678627464498 +117.149659414832939 i	129.634235083260734 +1097.460066010158698 i

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
1,75,73	9.509686499269559 +118.889812718652294 i	47.122110900405879 + 634.037868396604495 i	1,75,75	3.105845696563446 +120.548335595650613 i	4.024871159595857 + 153.317133658978690 i
1,76,1	120.887657704436720 + 7.948327208538953 i	14582.318563906148484 +1399.781130287518181 i	1,76,3	119.168556664370143 + 13.701183802986730 i	14118.527261007284324 +2269.465431608365634 i
1,76,5	117.375291398588203 + 18.907742205725263 i	13633.393275956605066 +2984.566775695765045 i	1,76,7	115.512802627136978 + 23.758029777258269 i	13133.413672941303957 +3592.677750946708329 i
1,76,9	113.584161719707097 + 28.339776235474549 i	12622.925131181535107 +4116.305865953045213 i	1,76,11	111.591435807980716 + 32.704312685435212 i	12105.164684787352599 +4568.891576775884459 i
1,76,13	109.536048968680859 + 36.885492106477081 i	11582.709295575392389 +4959.477328611144912 i	1,76,15	107.418966442401555 + 40.907349810829864 i	11057.700133825863304 +5294.616871081514546 i
1,76,17	105.240799599499383 + 44.787830210009908 i	10531.972127184124474 +5579.311460101209377 i	1,76,19	103.001869727887879 + 48.540826902774313 i	10007.135388995726316 +5817.525818555883234 i
1,76,21	100.702248012726798 + 52.177396376156665 i	9484.629684975681812 +6012.496981662350663 i	1,76,23	98.341780505256921 + 55.706526407132628 i	8965.762672031063630 +6166.930975807586037 i
1,76,25	95.920102858430354 + 59.135646635913680 i	8451.73782327590743 +6283.13427271650653 i	1,76,27	93.436647529327558 + 62.470981178580168 i	7943.675514689683041 +6363.105138215572879 i
1,76,29	90.890644978386092 + 65.717799927701719 i	7442.62942560212014 +6408.59918435345753 i	1,76,31	88.281119676875079 + 68.880602351525724 i	6949.59964598898478 +6421.17770021218712 i
1,76,33	85.606881245920789 + 71.963254837811348 i	6465.5434280473928 +6402.2441220109922 i	1,76,35	82.866510675122512 + 74.969095151654486 i	5991.38423635782601 +6353.07211512123831 i
1,76,37	80.058341235775998 + 77.901013018245862 i	5528.0195702684005 +6274.8275874348310 i	1,76,39	77.180433362503251 + 80.761512963252926 i	5076.3279151704810 +6168.5862250262744 i
1,76,41	74.230542382511999 + 83.552763667671115 i	4637.17510387301081 +6035.34766713948738 i	1,76,43	71.206077473080228 + 86.276636834060213 i	4211.42032246130839 +5876.04712117554664 i
1,76,45	68.104049557911310 + 88.934737688117892 i	3799.92196958497016 +5691.56500222149407 i	1,76,47	64.921004912809749 + 91.528428612546918 i	3403.54357050411318 +5482.73503076532415 i
1,76,49	61.652939885877757 + 94.058846938132191 i	3023.15995674517808 +5250.35111455941933 i	1,76,51	58.295190094002564 + 96.526917536870513 i	2659.663950973809278 +4995.173261711945997 i
1,76,53	54.842284306011670 + 98.933360524919806 i	2313.973850237299757 +4717.932711985782589 i	1,76,55	51.287748205644476 +101.278694043273276 i	1987.042089812879794 +4419.336424123414872 i
1,76,57	47.623834951808306 +103.563231687367022 i	1679.865614095506544 +4100.071011081356119 i	1,76,59	43.841145237831334 +105.787073623989571 i	1393.498716108422514 +3760.806161881993830 i
1,76,61	39.928073942460884 +107.950089628363841 i	1129.069501188425647 +3402.197509291143433 i	1,76,63	35.869971631336412 +110.051890933239903 i	887.801818464471353 +3024.888754352540673 i
1,76,65	31.647809333064053 +112.091785349826651 i	671.045774202304983 +2629.512534506839074 i	1,76,67	27.235912246174006 +114.068705306465887 i	480.322462846964413 +2216.688713777808971 i
1,76,69	22.597770846978275 +115.981087892274743 i	317.394057529816678 +1787.016464764051534 i	1,76,71	17.677307532330925 +117.826659506982935 i	184.384101956582356 +1341.048639489474681 i
1,76,73	12.376946079557204 +119.601996965518424 i	84.014201501160755 + 879.201870324325341 i	1,76,75	6.480081666672810 +121.301393139039092 i	20.197326704726017 + 401.301139923975143 i
1,77,1	122.459155253375428 + 7.962224462697272 i	14964.572522174139522 +1421.460785532134423 i	1,77,3	120.741647554709203 + 13.728343727462326 i	14495.401726777614385 +2306.083907959469343 i
1,77,5	118.950754965272495 + 18.948348588398691 i	14004.669696274728267 +3034.409739359170544 i	1,77,7	117.091372876442449 + 23.812275108811626 i	13498.889075924704230 +3654.623740709587764 i
1,77,9	115.166550749906264 + 28.407852816874414 i	12982.406217499992636 +4189.506891152475873 i	1,77,11	113.178347108629652 + 32.786412782345927 i	12458.464627575920531 +4652.659252575656865 i
1,77,13	111.128186666581982 + 36.981809350214135 i	11929.645672340331657 +5053.228145280749231 i	1,77,15	109.017042628526880 + 41.018080744390979 i	11398.093435331607907 +5397.841425353508384 i
1,77,17	106.845540710221812 + 44.913175770692085 i	10865.644594045857957 +5691.555347992522911 i	1,77,19	104.614022510723373 + 48.680993815931750 i	10333.910045927803367 +5938.376911962021040 i
1,77,21	102.322585412698473 + 52.332598478073137 i	9804.329497036809698 +6141.576501714558617 i	1,77,23	99.971107718196166 + 55.876985924438486 i	9278.20977471567130 +6303.88696843579328 i
1,77,25	97.559263758603521 + 59.321595449709811 i	8756.75279317806417 +6427.63667865317166 i	1,77,27	95.086531668823611 + 62.672662100387882 i	8241.07665578946035 +6514.84196232699833 i
1,77,29	92.552195365546524 + 65.935468013596292 i	7732.23205083153698 +6567.27344655042907 i	1,77,31	89.955341567514068 + 69.114526280974232 i	7231.21533603056105 +6586.50495605376636 i
1,77,33	87.294852224446004 + 72.213718385061668 i	6738.97925001165640 +6573.95040700932896 i	1,77,35	84.569392365833536 + 75.236398780184231 i	6256.44190441141291 +6530.89220924117581 i
1,77,37	81.777393073181159 + 78.185475633009148 i	5784.4945285476970 +6458.5035245058061 i	1,77,39	78.917028972463896 + 81.063473861615662 i	5324.0083203545473 +6357.8659910200531 i
1,77,41	75.986189297281769 + 83.872584738557563 i	4875.8406803849321 +6229.9840446761154 i	1,77,43	72.982441144191142 + 86.614705066782733 i	4440.84105701985482 +6075.79664716583102 i
1,77,45	69.902982974516988 + 89.291468068899547 i	4019.856603096552590 +5896.187012508904445 i	1,77,47	66.744585632012061 + 91.904267509810040 i	3613.73783306817131 +5691.99077086618672 i
1,77,49	63.503517020794277 + 94.454276110216526 i	3223.344474284607713 +5464.002899783038629 i	1,77,51	60.175444926555535 + 96.942458942710800 i	2849.55172722539874 +5212.98367368438176 i
1,77,53	56.755309938100531 + 99.369582187071217 i	2493.257191539187716 +4939.663822639779028 i	1,77,55	53.237156468118755 +101.736217316450649 i	2155.388785612517950 +4644.749043702809104 i
1,77,57	49.613903460115232 +104.042740446819678 i	1836.914101783068872 +4328.923965786080771 i	1,77,59	45.877025582337808 +106.289326147246115 i	1538.851823736549704 +3992.855623911641032 i
1,77,61	42.016096753472866 +108.475934378581726 i	1262.286136198073109 +3637.196437283449086 i	1,77,63	38.018112757120406 +110.602288218902714 i	1008.385574904868463 +3262.586579999198420 i
1,77,65	33.866440550014133 +112.667838272926048 i	778.428692692362006 +2869.655416304498431 i	1,77,67	29.539094563389842 +114.671706323923090 i	573.840689264993216 +2459.021163218421407 i
1,77,69	25.005693708675295 +116.612593844804010 i	396.248827686764287 +2031.286588045985809 i	1,77,71	20.221521550618693 +118.488624784746719 i	247.572979607771003 +1587.024350319756422 i
1,77,73	15.114088815194492 +120.297047712742355 i	130.190743052134167 +1126.729502921952871 i	1,77,75	9.544320315876709 +122.033566929651272 i	47.296730951480576 + 650.629172137203341 i
1,77,77	3.114017007398344 +123.690173422316605 i	4.032965903826777 + 157.194769027169059 i	1,78,1	124.030636525411481 + 7.975943222959544 i	15351.763389093825026 +1443.184016510691208 i
1,78,3	122.314685562326119 + 13.755153488982296 i	14877.217812046499603 +2342.787415664063701 i	1,78,5	120.526111104084805 + 18.988428638223336 i	14380.894697921121406 +3084.379824543486393 i
1,78,7	118.669763939922120 + 23.865813351859671 i	13869.322131119066398 +3716.739640324964512 i	1,78,9	116.748671736245129 + 28.475036922999664 i	13346.856068997993134 +4262.921344569254870 i
1,78,11	114.764884137158973 + 32.867430274123578 i	12816.746467019720512 +4736.684636657060764 i	1,78,13	112.719825925120503 + 37.076848933345424 i	12281.579102933616014 +5147.281748305099427 i
1,78,15	110.614477432983335 + 41.127333788730894 i	11743.501005593379887 +5501.414683971152954 i	1,78,17	108.449477623848836 + 45.036837732620548 i	11204.350656464952408 +5804.194745812283145 i
1,78,19	106.225187036993503 + 48.819265574526526 i	10665.739812184987386 +6059.671261945723018 i	1,78,21	103.941727585275601 + 52.485687601012508 i	10129.108215251271758 +6271.148025409529509 i
1,78,23	101.599007822519326 + 56.045107889217348 i	9595.761981408986685 +6441.384777574232430 i	1,78,25	99.196738379921840 + 59.504974846612171 i	9066.90160741594991 +6572.73185145163097 i
1,78,27	96.734440249596332 + 62.871533842634321 i	8543.64309127000706 +6667.22371947375905 i	1,78,29	94.211447461788538 + 66.150078579235100 i	8027.03432634759702 +6726.64610923590953 i
1,78,31	91.626905015524883 + 69.345135011045983 i	7518.0681671401726 +6752.5854742646316 i	1,78,33	88.979762466250932 + 72.460598863567043 i	7017.6931056151043 +6746.4663122064949 i
1,78,35	86.268763235795191 + 75.499840323569245 i	6526.8222116034440 +6709.5798880352077 i	1,78,37	83.492429424121767 + 78.465784920438466 i	6046.3408077159056 +6643.1067381755611 i
1,78,39	80.64904162461988 + 81.36097674252326 i	5577.113230001249 +6548.134584964844 i	1,78,41	77.736612938801841 + 84.187628261295089 i	5119.9889473926878 +6425.6728053063025 i
1,78,43	74.752856014949430 + 86.947659782310552 i	4675.8082627396088 +6276.6642732810415 i	1,78,45	71.695141452322687 + 89.642730677410398 i	4245.4077881915700 +6101.9951751495255 i
1,78,47	68.560445253477738 + 92.274263937492137 i	3829.62587368763116 +5902.50324080814426 i	1,78,49	65.345282074790319 + 94.843465130431866 i	3429.30816761604046 +5678.98472591030077 i
1,78,51	62.045619665708479 + 97.351336494407278 i	3045.313503824529562 +5432.200398965417662 i	1,78,53	58.656767848137257 + 99.798686599223302 i	2678.52034192234623 +5162.88072795863172 i
1,78,55	55.173232239381392 +102.186135730561801 i	2329.834044262381667 +4871.730414226731227 i	1,78,57	51.588517907696776 +104.514116856770656 i	2000.195364197527624 +4559.432381433772207 i
1,78,59	47.894859876329278 +106.782871676729840 i	1690.590665960698516 +4226.651287906399375 i	1,78,61	44.082843178425584 +108.992440746098126 i	1402.064632806551117 +3874.036581447415301 i
1,78,63	40.140849558096150 +111.142645906602837 i	1135.736614838405785 +3502.225038076021065 i	1,78,65	36.054219079509828 +113.233061943671188 i	892.822456567366658 +3111.842576987516622 i
1,78,67	31.803915067682166 +115.262972030190753 i	674.664915040929372 +2703.504813772143279 i	1,78,69	27.364258018532578 +117.231296818705061 i	482.778301289503247 +2277.814988230826202 i
1,78,71	22.698736857541977 +119.136476738190908 i	318.918483848618223 +1835.355543993365245 i	1,78,73	17.751275292538459 +120.976261194905078 i	185.203093799492365 +1376.661596981224193 i
1,78,75	12.424298646035590 +122.747279548048724 i	84.348130880357701 + 902.128747243556973 i	1,78,77	6.501185016369303 +124.443933204631648 i	20.261220469414461 + 411.547770555764544 i
1,79,1	125.602102093678923 + 7.989487994899689 i	15743.891116186310683 +1464.950266736022286 i	1,79,3	123.887672543405330 + 13.781621964388112 i	15263.975361271275688 +2379.574864073908884 i
1,79,5	122.101363590688166 + 19.027995765320199 i	14762.067967145734656 +3134.475390098330536 i	1,79,7	120.247982137709641 + 23.918662616231771 i	14244.712317631010592 +3779.023240334027378 i
1,79,9	118.330534181921323 + 28.541351536142841 i	13716.273908914583444 +4336.546429332602202 i	1,79,11	116.351060249512301 + 32.947393198547104 i	13180.009120697142862 +4820.964324111147107 i
1,79,13	114.310984669069583 + 37.170644146108408 i	12638.508147386959536 +5241.634102324316700 i	1,79,15	112.211294127077949 + 41.235147700863280 i	12093.920992180926987 +5605.331956658777017 i
1,79,17	110.052639813853901 + 45.158860554521262 i	11548.087990671101579 +5917.224281584483833 i	1,79,19	107.835399934211878 + 48.955692597855148 i	11002.621828830227969 +6181.402785450518422 i
1,79,21	105.559719381338340 + 52.636720411094657 i	10458.962375657136363 +6401.204726343377843 i	1,79,23	103.225535104365093 + 56.210955531830777 i	9918.41514601165363 +6579.41679800445763 i
1,79,25	100.832591828472658 + 59.685854974008926 i	9382.17935197589213 +6718.41136806924987 i	1,79,27	98.380450792070206 + 63.067673862662751 i	8851.36904311021135 +6820.24112585876082 i
1,79,29	95.868493051548370 + 66.361716830510304 i	8327.0295017513381 +6886.7069805861183 i	1,79,31	93.295918231281462 + 69.572521986913330 i	7810.1502935380131 +6919.4081029076924 i
1,79,33	90.661739153805300 + 72.703998508329178 i	7301.675913202263 +6919.779668118899 i	1,79,35	87.964772461956542 + 75.759531426228136 i	6802.5146789090768 +6889.1219010921914 i
1,79,37	85.203625077605182 + 78.742062634190930 i	6313.5463445396244 +6828.6228270959773 i	1,79,39	82.376676088618510 + 81.654154262679015 i	5835.6287790385340 +6739.3763771678496 i
1,79,41	79.482053383782088 + 84.498038700525655 i	5369.6039826644992 +6622.3970053179619 i	1,79,43	76.517604032065961 + 87.275658290740893 i	4916.3036583656622 +6478.6316468746736 i
1,79,45	73.480856989111587 + 89.988696866864871 i	4476.5545246786132 +6308.9696233296572 i	1,79,47	70.368976157362113 + 92.638604685452592 i	4051.18354008670258 +6114.25094289714798 i
1,79,49	67.178701048934883 + 95.226617861856056 i	3641.02320562631439 +5895.27333497719519 i	1,79,51	63.906271180959273 + 97.753773071425925 i	3246.91712262011060 +5652.79827614774422 i
1,79,53	60.547328675857017 +100.220917994523717 i	2869.726007678370479 +5387.556205381904874 i	1,79,55	57.096791016400168 +102.628717727508842 i	2510.334412049901402 +5100.251079902731362 i
1,79,57	53.548681949940772 +104.977657120634447 i	2169.658465408588199 +4791.564384851485556 i	1,79,59	49.895902126590208 +107.268038698703961 i	1848.655080127605061 +4462.158674189810881 i
1,79,61	46.129910272177234 +109.499975414810478 i	1548.333237675306918 +4112.680679503358078 i	1,79,63	42.240266742633892 +111.673376886114582 i	1269.768283179487753 +3743.763964389470402 i
1,79,65	38.213956219904333 +113.787926785093504 i	1014.120672671692205 +3356.030996668922853 i	1,79,67	34.034337156329336 +115.843047346937112 i	782.661545745984427 +2950.094290463616744 i
1,79,69	29.679418259961196 +117.837843697978116 i	576.809268300818417 +2526.555750584497218 i	1,79,71	25.118815688536074 +119.771013933430822 i	398.184765560776157 +2086.001966986437310 i
1,79,73	20.307813228517436 +121.640695053007889 i	248.701984723751593 +1628.988919697389065 i	1,79,75	15.173923800087445 +123.444171569368990 i	130.732929932801710 +1155.993126748943723 i
1,79,77	9.578069681326697 +125.177221804718684 i	47.466878160548106 + 667.216997749807888 i	1,79,79	3.121977689699523 +126.831999507802834 i	4.040850925595024 + 161.070245071319135 i
1,80,1	127.173552504398998 + 8.002863115995529 i	16140.955656666523282 +1486.758993687120437 i	1,80,3	125.460610267055074 + 13.807757697940756 i	15655.674224321208385 +2416.445190057463151 i
1,80,5	123.676516023118452 + 19.067062874977509 i	15148.189201126568150 +3184.694836555937854 i	1,80,7	121.826033491127035 + 23.970840326513560 i	14625.059132779389984 +3841.472387811864337 i
1,80,9	119.912147137858469 + 28.606818764184663 i	14090.658987806067671 +4410.379420718197252 i	1,80,11	117.936888160879832 + 33.026328520204366 i	13548.251544542230477 +4905.494998639813275 i
1,80,13	115.901679955153369 + 37.263226996069328 i	13000.431417225589030 +5336.281278011269933 i	1,80,15	113.807514843014488 + 41.341559733633432 i	12449.351609615203415 +5709.588677674011706 i
1,80,17	111.655055293543711 + 45.279286955891017 i	11896.854357902109768 +6030.638727582968386 i	1,80,19	109.444695990849326 + 49.090323315567154 i	11344.553342937015259 +6303.565564757142965 i
1,80,21	107.176603366345233 + 52.785751316950062 i	10793.888644675545947 +6531.739966081722741 i	1,80,23	104.850741038545124 + 56.374589537502118 i	10246.165281067880642 +6717.975636850540468 i
1,80,25	102.466885778633937 + 59.864303123946286 i	9702.58130426700154 +6864.66704413991829 i	1,80,27	100.024636651434378 + 63.261156426531578 i	9164.2489627789764 +6973.8851649047092 i
1,80,29	97.523418883916960 + 66.570464416638060 i	8632.2111004300677 +7047.4461670299443 i	1,80,31	94.962483353475363 + 69.796776697884312 i	8107.4541949119048 +7086.9620221382465 i
1,80,33	92.340902157535197 + 72.944015160362554 i	7590.918977080236 +7093.878674361678 i	1,80,35	89.657560416188815 + 76.015578855412897 i	7083.5092843514867 +7069.5054076813576 i
1,80,37	86.911144208574953 + 79.014425115156627 i	6586.0996167370304 +7015.0378442211622 i	1,80,39	84.100124311822384 + 81.943133069044308 i	6099.5417428852879 +6931.5762414295414 i
1,80,41	81.22273516813651 + 84.80395384178652 i	5624.670623234695 +6820.140258853611 i	1,80,43	78.276948222390743 + 87.598850467590509 i	5162.3098645722193 +6681.6810333669291 i
1,80,45	75.26043841654001 + 90.32952969695508 i	4713.276886934979 +6517.091175129052 i	1,80,47	72.170542154817866 + 92.997467265322797 i	4278.3879652595851 +6327.2131386176525 i
1,80,49	69.004204402103483 + 95.603927749693185 i	3858.4633021484478 +6112.8463108441301 i	1,80,51	65.757911650845955 + 98.149979801806651 i	3454.33229406652345 +5874.75307755915946 i
1,80,53	62.427606136448624 +100.636507273965719 i	3066.83917243353972 +5613.66406802982965 i	1,80,55	59.008574645078369 +103.064216514343883 i	2696.84923676778772 +5330.28273293410621 i
1,80,57	55.495302112481596 +105.433639874010467 i	2345.25595563402658 +5025.28937277799832 i	1,80,59	51.881275200241717 +107.745135205783413 i	2012.989303941089702 +4699.344701134781644 i
1,80,61	48.158712764750327 +109.998880801448365 i	1701.025852063969523 +4353.092992051417241 i	1,80,63	44.318185922500241 +112.194864738239616 i	1410.401359414923036 +3987.164814653836856 i
1,80,65	40.348064809656066 +114.332866860928902 i	1142.227020489089455 +3602.179281296254430 i	1,80,67	36.233680300653923 +116.412430363910161 i	897.711200446041062 +3198.745584882177809 i
1,80,69	31.955989108301722 +118.432817629980676 i	678.189768315416245 +2777.463264411056571 i	1,80,71	27.489307883161203 +120.392940398768033 i	485.170657916076550 +2338.919794490140248 i
1,80,73	22.797124600195488 +122.291244271512170 i	320.403792635996865 +1883.681685208668569 i	1,80,75	17.823364594150636 +124.125502301682815 i	186.001211446531414 +1412.267072145653628 i
1,80,77	12.470454771458845 +125.892395425750578 i	84.673597823619502 + 925.051271415331485 i	1,80,79	6.521756312351332 +127.586429814683626 i	20.323497698268412 + 421.791667761669316 i
1,81,1	128.744988278452922 + 8.016072763876551 i	16542.956965364250209 +1508.609668291023581 i	1,81,3	127.033500420398425 + 13.833568917705936 i	16052.314256225612553 +2453.397356973447131 i
1,81,5	125.251571832210828 + 19.105642392638963 i	15539.258107459547318 +3235.036604565678772 i	1,81,7	123.403923740330725 + 24.022363256141514 i	15010.362091240349400 +3904.084984227672976 i
1,81,9	121.493519229622450 + 28.671459884354295 i	14470.010582253764369 +4484.417663393537623 i	1,81,11	119.522379984188035 + 33.104262184552930 i	13921.472731012861914 +4990.273429139445398 i
1,81,13	117.491928024756405 + 37.354628273212782 i	13367.347572981117992 +5431.219447939282873 i	1,81,15	115.403160643916607 + 41.446605712677176 i	12809.791136092590023 +5814.180400896862011 i
1,81,17	113.256750707061911 + 45.398158006798312 i	12250.647600183732720 +6144.432994569440780 i	1,81,19	111.053108272713516 + 49.223204271435307 i	11691.531702241049061 +6426.153840786983892 i
1,81,21	108.792419967409941 + 52.932832588767397 i	11133.88381241531351 +6662.74728643239177 i	1,81,23	106.474674472817512 + 56.536068182161848 i	10579.00854946031177 +6857.05410471035611 i
1,81,25	104.099678702551614 + 60.040383887542440 i	10028.10292288896744 +7011.49092383662628 i	1,81,27	101.667067302271491 + 63.452052784064819 i	9482.2775186899453 +7128.1470757015541 i
1,81,29	99.176307024885404 + 66.776399628269388 i	8942.5729042263029 +7208.8540601128552 i	1,81,31	96.626696884737981 + 70.017984901427809 i	8409.9726577178026 +7255.2367292905228 i
1,81,33	94.017364574938654 + 73.180742520011449 i	7885.4139647272987 +7268.7518827346294 i	1,81,35	91.34725933383690 + 76.26808478784167 i	7369.796435213337 +7250.717957580417 i
1,81,37	88.61514120972829 + 79.28298361623742 i	6863.989609050155 +7202.338275081202 i	1,81,39	85.81956645905063 + 82.22803416761269 i	6368.839495930165 +7124.719530421244 i
1,81,41	82.958868594396407 + 85.105505211434917 i	5885.1744144183904 +7018.8867099982635 i	1,81,43	80.03113434913604 + 87.91737923372779 i	5413.810341079172 +6885.795284591114 i
1,81,45	77.03417351719968 + 90.66538448490151 i	4955.555945917673 +6726.341297610319 i	1,81,47	73.965481223478605 + 93.351020546832203 i	4511.2174700876358 +6541.3698078836824 i
1,81,49	70.822190631231464 + 95.975578561235935 i	4081.6035933569024 +6331.6820329678012 i	1,81,51	67.601013320908962 + 98.540156930067610 i	3667.5304413259878 +6098.0414568801971 i
1,81,53	64.298163459430086 +101.045673035180286 i	3269.8268965364609 +5841.1791055245149 i	1,81,55	60.909260224783729 +103.492871298848414 i	2889.34040581070258 +5561.79814709468519 i
1,81,57	57.429200430674918 +105.882327693079745 i	2526.94352326590661 +5260.57793830133505 i	1,81,59	53.851989342604455 +108.214450576797688 i	2183.541503049076709 +4938.177605985721129 i
1,81,61	50.170511269039814 +110.489477461084363 i	1860.081372993257219 +4595.239222768887885 i	1,81,63	46.376210729123116 +112.707466920494724 i	1557.563106855237265 +4232.390597772010551 i
1,81,65	42.458636045701049 +114.868284291165168 i	1277.053817844950117 +3850.247646001600060 i	1,81,67	38.404762126741922 +116.971578850733067 i	1019.706415063248869 +3449.416194131802513 i
1,81,69	34.197940043567478 +119.016748506781283 i	786.785092960715815 +3030.492856529460823 i	1,81,71	29.816173690499998 +121.002884842613014 i	579.701795990652158 +2594.064085024819178 i
1,81,73	25.229077149207800 +122.928684749316957 i	400.071476185858955 +2140.701081881933078 i	1,81,75	20.391934848286851 +124.792299415854259 i	249.802480700655963 +1670.943889189392394 i
1,81,77	15.232262046382077 +126.591049148232955 i	131.261511270996579 +1185.251208949301599 i	1,81,79	9.610978451623060 +128.320784189006787 i	47.632774821169875 + 683.801473168358029 i
1,81,81	3.129738218171835 +129.973814683698436 i	4.048536694760592 + 164.943627711611640 i			

Table D.2: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$

Table D.2: Branch points and associated eigenvalues for $m = 1$.

D.1.3 $m = [2..8]$, $n = [m..25 + m]$

Accuracy was determined by the last correction of the Newton-Raphson method.

Table D.3: Branch points and associated eigenvalues for $m = 2$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
2, 2 ,1	2.136987377094029 + 5.449457313914277 i	6.102540356949614 + 7.684763813617406 i	2, 3 ,1	4.105156484215650 + 5.922750658440496 i	16.136866268804905 + 20.406230631490939 i
2, 4 ,1	5.907125751703487 + 6.283464345814330 i	32.087586148736847 + 34.487534677680138 i	2, 4 ,3	2.265822172027141 + 8.748077684520867 i	6.207742694273197 + 12.250160879220145 i
2, 5 ,1	7.634658702877481 + 6.576768822063829 i	53.56137110170611 + 49.63851594121932 i	2, 5 ,3	4.405746719793623 + 9.294389714166075 i	16.974441589636069 + 32.109641041936442 i
2, 6 ,1	9.320494635222795 + 6.824432371374058 i	80.376039435949536 + 65.668970414405067 i	2, 6 ,3	6.337223309080594 + 9.739915760209533 i	34.182667033756140 + 53.573264399341902 i
2, 6 ,5	2.361167355756876 + 11.962429681198497 i	6.294802569497238 + 16.568043718788960 i	2, 7 ,1	10.980074056517031 + 7.038930557723234 i	112.427552975076566 + 82.446601921716592 i
2, 7 ,3	8.166812656483337 + 10.118357017967426 i	57.296877513680847 + 76.397746513311105 i	2, 7 ,5	4.629138586623777 + 12.552028695435473 i	17.624494826117377 + 43.212361461328676 i
2, 8 ,1	12.621844293125854 + 7.228162350072844 i	149.649461249734259 + 99.874080057743143 i	2, 8 ,3	9.935661817902666 + 10.448088781586954 i	86.048143140681574 +100.388658834670480 i
2, 8 ,5	6.664848432177823 + 13.051574944386537 i	35.80867087930098 + 71.60441777267100 i	2, 8 ,7	2.435870538004652 + 15.146579362179107 i	6.365563315843792 + 20.779945737626058 i
2, 9 ,1	14.250887964172007 + 7.397474007183994 i	191.996169854608070 +117.876711993110590 i	2, 9 ,3	11.664384566585689 + 10.740571478359586 i	120.27458870532745 +125.39875008739831 i
2, 9 ,5	8.581338084250451 + 13.487440074589898 i	60.223109253853508 +101.553325975282092 i	2, 9 ,7	4.806341649488908 + 15.765290336173946 i	18.148719317722193 + 54.065736247491166 i
2,10,1	15.870475565427192 + 7.550663615220111 i	239.434777209378126 +136.395310008916113 i	2,10,3	13.364818667036719 + 11.003533250197448 i	159.868707821270419 +151.313851318047568 i
2,10,5	10.424021320580719 + 13.875011481040593 i	90.538349153954456 +132.874097735796937 i	2,10,7	6.929144863851361 + 16.302794991673473 i	37.12940338946358 + 89.18995533841284 i
2,10,9	2.497017203916184 + 18.316234296095967 i	6.424492117200728 + 24.931009799783553 i	2,11,1	17.482820216559340 + 7.690531263611104 i	291.940629275122205 +155.381783797859043 i
2,11,3	15.044379042978385 + 11.242469999610398 i	204.754412204447533 +178.043025829063822 i	2,11,5	12.216439004059271 + 14.224396063855474 i	126.549069521458065 +165.416818274500457 i
2,11,7	8.920785770675087 + 16.780532331378613 i	62.621777629190639 +126.012736403774980 i	2,11,9	4.952996950513199 + 18.956019363165186 i	18.586208692183735 + 64.782757499057425 i
2,12,1	19.089479654866846 + 7.819202888793248 i	349.494701189557759 +174.796267512402835 i	2,12,3	16.708002697381598 + 11.461446441695077 i	254.875366212035516 +205.512134223820254 i
2,12,5	13.972510630890151 + 14.542693936779409 i	168.115251243772320 +199.060519216368476 i	2,12,7	10.829135406399161 + 17.211552398281750 i	94.251392476224979 +164.363422298875349 i
2,12,9	7.150507780896148 + 19.521857266786085 i	38.239390537003771 +106.531574363686829 i	2,12,11	2.548659439229380 + 21.477767913975164 i	6.474738482854820 + 29.041510886983069 i
2,13,1	20.691585686045180 + 7.938331768083366 i	412.081960779012888 +194.605164903426299 i	2,13,3	18.359130440845256 + 11.663561552198811 i	310.188422307607652 +233.659548750286035 i
2,13,5	15.701178302395029 + 14.835125737280784 i	215.135658793313837 +233.706117861674386 i	2,13,7	12.679542571894926 + 17.604732464963025 i	131.778991340134723 +204.094971867492944 i
2,13,9	9.208143451841947 + 20.031688168607726 i	64.652708557647529 +150.085857101528632 i	2,13,11	5.077990345047075 + 22.133583788759058 i	18.960884199883164 + 75.413517986014199 i
2,14,1	22.289982489857405 + 8.049229559312671 i	479.690296984367535 +214.779771560522448 i	2,14,3	20.000247427631397 + 11.851236335072683 i	370.659646591993067 +262.433208823767586 i
2,14,5	17.408529095014853 + 15.105661852017079 i	267.533989127114694 +269.271095216917047 i	2,14,7	14.487287283714512 + 17.966492473822285 i	175.038190915817722 +245.084654099085697 i
2,14,9	11.175311986424436 + 20.496747280603678 i	97.416923171568186 +195.291009988918546 i	2,14,11	7.340857167634913 + 22.721475484741436 i	39.195750124767187 +123.720194541035852 i
2,14,13	2.593295191572294 + 24.634279640612309 i	6.518419656878613 + 33.122403236260418 i	2,15,1	23.885313757013027 + 8.152954665783501 i	552.309791260345150 +235.295275101296085 i
2,15,3	21.633200948080297 + 12.026400498991594 i	436.261775123132452 +291.788535616370496 i	2,15,5	19.098889267867595 + 15.357402215168843 i	325.250900429849651 +305.685677027258748 i
2,15,7	16.262356702661364 + 18.301669596034067 i	223.906756961229718 +287.229589458028670 i	2,15,9	13.078549421581902 + 20.924871737688786 i	136.265549570726307 +242.005589620582981 i
2,15,11	9.457222581493030 + 23.256794015108071 i	66.413011463144557 +173.915327357443658 i	2,15,13	5.186838833094567 + 25.302736926452226 i	19.288155307047617 + 85.984438150548669 i
2,16,1	25.478079640729842 + 8.250373657055686 i	629.932206493277931 +256.130011536037273 i	2,16,3	23.259397146308093 + 12.190618316559769 i	506.972513163473025 +321.686914445172135 i
2,16,5	20.775438637452137 + 15.592818983725058 i	388.239093559495302 +342.890080119437926 i	2,16,7	18.011655622457983 + 18.614020149983386 i	278.291310745460123 +330.442586678974749 i
2,16,9	14.934067739510542 + 21.321853274712858 i	181.010429440296633 +290.107882068960180 i	2,16,11	11.477515746283125 + 23.749347597815337 i	100.175735219966938 +225.860782084914035 i
2,16,13	7.507760458050305 + 25.908335831939194 i	40.035340141879793 +140.804156042334512 i	2,16,15	2.632563453759253 + 27.787464128401766 i	6.556991404490268 + 37.180347347487618 i
2,17,1	27.068675151997031 + 8.342205075296862 i	712.550618960869770 +277.264900405007495 i	2,17,3	24.879927978056429 + 12.345176217180304 i	582.773356993585139 +352.094563866495832 i
2,17,5	22.440578367037478 + 15.813918553585728 i	456.460112093179248 +380.832492814872523 i	2,17,7	19.740160697747100 + 18.906530469171756 i	338.118382199705025 +374.648884700020346 i
2,17,9	16.752595845619528 + 21.692138104400204 i	231.511577313622723 +339.493775251115608 i	2,17,11	13.429089420409893 + 24.206107234761365 i	140.195130734985867 +279.422072786895001 i
2,17,13	9.676978909500871 + 26.464445600938188 i	67.965894503122811 +197.578086971132933 i	2,17,15	5.283198240060583 + 28.466167412298066 i	19.578457891342154 + 96.511110782040049 i
2,18,1	28.657416742145347 + 8.429051380073169 i	800.159147280022808 +298.683007833401575 i	2,18,3	26.495656027364426 + 12.491145395898663 i	663.648753407619665 +382.981675362405838 i
2,18,5	24.096162466421934 + 16.022353596302086 i	529.882171632539894 +419.467564971586000 i	2,18,7	21.451576580321262 + 19.181620226892097 i	403.328810645227009 +419.783682645883459 i
2,18,9	18.541636645536229 + 22.039235671833413 i	287.660901109607380 +390.073692372527091 i	2,18,11	15.328818106568360 + 24.632311283602839 i	186.264779613987372 +334.480103933908481 i
2,18,13	11.745634254458322 + 26.979733717995217 i	102.620337447477070 +256.184364276767781 i	2,18,15	7.656321482386069 + 29.086329167762205 i	40.78325891802104 +157.81204342157821 i
2,18,17	2.667594133265277 + 30.938328187998465 i	6.591486695834693 + 41.219740160482836 i	2,19,1	30.244561136672218 + 8.511422698606747 i	892.752748863423449 +320.369203123892248 i
2,19,3	28.107272846015569 + 12.629427595044637 i	749.585485074002463 +414.321746969141941 i	2,19,5	25.743649525911700 + 16.219502810535545 i	608.478636503399571 +458.755257920667930 i
2,19,7	23.148733380937525 + 19.441281066110459 i	473.874056554331275 +465.790265459013881 i	2,19,9	20.306654983013285 + 22.365976334436031 i	349.37255981879997 +441.7699013298832 8 i
2,19,11	17.187973267507665 + 25.032040962377401 i	238.229522577729284 +390.931117809265459 i	2,19,13	13.741662187396186 + 27.460449428527856 i	143.691292748163789 +316.496103025235925 i
2,19,15	9.873554192349513 + 29.659759766441287 i	69.354772390061385 +221.120074654822702 i	2,19,17	5.369613902680779 + 31.625517822966660 i	19.839161797276631 +107.003501607675062 i
2,20,1	31.830318951862001 + 8.589754787005729 i	990.327064446802638 +342.309885059353974 i	2,20,3	29.715340095510605 + 12.760789247250267 i	840.57221144029984 +446.09105889049380 i
2,20,5	27.384205551434096 + 16.406529091282753 i	692.226920259451163 +498.659953463359216 i	2,20,7	24.833839670198302 + 19.687174299142011 i	549.71367200992472 +512.61856651143898 i
2,20,9	22.051758435489088 + 22.674682768499999 i	416.576862296596687 +494.514355864247420 i	2,20,11	19.014506206902894 + 25.408560085346512 i	295.968487952470866 +448.684633771639426 i
2,20,13	15.682422592110980 + 27.911351066716190 i	190.956830739760932 +378.397761257735851 i	2,20,15	11.986550406007758 + 30.194199783185820 i	104.81481957161567 +286.32914831346426 i
2,20,17	7.790144799671900 + 32.257888325878952 i	41.457343250162744 +174.762081067643976 i	2,20,19	2.699197587412236 + 34.087508434670355 i	6.622660778486124 + 45.243665057417239 i
2,21,1	33.414864717304951 + 8.664422820531701 i	1092.878297534960438 +364.492761037425339 i	2,21,3	31.320319196060503 + 12.885887381191276 i	936.599119467148284 +478.268255030069838 i
2,21,5	29.018775649821521 + 16.584422850111050 i	781.107673002660704 +539.149752869206623 i	2,21,7	26.508650075765502 + 19.920701656652056 i	630.813508029264312 +560.224050710742404 i
2,21,9	23.780113044453564 + 22.967288693389101 i	489.216208889894745 +548.246997941316636 i	2,21,11	20.814255672441618 + 25.764531829980253 i	359.384943084959730 +507.661298634796481 i
2,21,13	17.579593877665875 + 28.336190318324590 i	244.248692477114446 +441.786943771779875 i	2,21,15	14.023674629524615 + 30.695293177549608 i	146.840317194329799 +353.320230633482709 i
2,21,17	10.051344875368903 + 32.845995988134341 i	70.610744479453047 +244.570758681878743 i	2,21,19	5.447928098181404 + 34.781851472317292 i	20.075654138909769 +117.468372014673861 i
2,22,1	34.998344371709223 + 8.735752125670283 i	1200.403119664148562 +386.906666847595488 i	2,22,3	32.922593102441141 + 13.005289578751245 i	1037.657653613165166 +510.834004868857221 i
2,22,5	30.648135217563816 + 16.754034864320161 i	875.10416835456977 +580.19591768321561 i	2,22,7	28.174580014207887 + 20.143057701318192 i	717.144409053116192 +608.566834906707186 i
2,22,9	25.494210031108761 + 23.245423771156730 i	567.242283546820886 +602.914422574541732 i	2,22,11	22.591643694402119 + 26.102164705323439 i	428.399816666101685 +567.791038200935304 i
2,22,13	19.441470687109053 + 28.737999867581074 i	303.434821699592760 +506.573255201658924 i	2,22,15	16.002651368801434 + 31.167383272739769 i	195.196011911951302 +421.981872490488342 i
2,22,17	12.205251590050134 + 33.396816825864903 i	106.805443728634941 +316.338725360269109 i	2,22,19	7.911870171306743 + 35.424619016998186 i	42.070712782205682 +191.666564273592456 i
2,22,21	2.727973980747985 + 37.235427472452784 i	6.651080650482439 + 49.254386206458484 i	2,23,1	36.580880949929757 + 8.804026634499937 i	1312.898595053473400 +409.541418134793105 i
2,23,3	34.522482568048005 + 13.119489559257850 i	1143.740304104418556 +543.770727205546336 i	2,23,5	32.272927263534960 + 16.916101582965748 i	974.201832802925878 +621.772418084044148 i
2,23,7	29.832786431478748 + 20.355269421290107 i	808.681240101964022 +657.610986247054029 i	2,23,9	27.196043825281276 + 23.510475918146792 i	650.614048080557609 +658.468820105225063 i
2,23,11	24.350102889148815 + 26.423314867815645 i	502.947355763591872 +629.011545468516488 i	2,23,13	21.274183310727086 + 29.119278414166889 i	368.408841230081919 +572.676653717949762 i
2,23,15	17.935470810419616 + 31.613928218759558 i	249.70223568151613 +492.21368884634148 i	2,23,17	14.280552956595594 + 33.915482989048606 i	149.704921708714664 +389.954827466898621 i
2,23,19	10.213606639826674 + 36.025339360927894 i	71.756848386450613 +267.950087083547089 i	2,23,21	5.519516573030510 + 37.935888578978414 i	20.291986793584499 +127.910527892229481 i
2,24,1	38.162578953898780 + 8.869495618553518 i	1430.362120024957449 +432.387686866515594 i	2,24,3	36.120258475371124 + 13.228919497710129 i	1254.840438796108629 +577.062362171293216 i
2,24,5	33.893690122916738 + 17.071264896413211 i	1078.387878733068419 +663.855564010763146 i	2,24,7	31.484225977126453 + 20.558226647969210 i	905.402147405511869 +707.323955779161111 i
2,24,9	28.887234831532787 + 23.763638043388465 i	739.296270964399696 +714.867130190247167 i	2,24,11	26.092352146445332 + 26.729560121410611 i	582.972099388457991 +691.267059902293454 i
2,24,13	23.082402934669670 + 29.482116366176012 i	439.083288892970756 +640.025899794997372 i	2,24,15	19.830695320726122 + 32.037748550337049 i	310.217074766654775 +563.926041834886519 i
2,24,17	16.295253258453592 + 34.405978350132855 i	199.062368708404279 +465.312004690632989 i	2,24,19	12.405468719946246 + 36.590349319313013 i	108.62671184293218 +346.24278612694201 i
2,24,21	8.023488359500291 + 38.587626936492427 i	42.633293130551067 +208.534163498157028 i	2,24,23	2.754379779892783 + 40.382377029014961 i	6.677181867692932 + 53.253627403508155 i
2,25,1	39.743527751745233 + 8.932379106339301 i	1552.791373817049362 +455.436897746587590 i	2,25,3	37.716151310645380 + 13.333959868776615 i	1370.952168130274231 +610.694181373394940 i
2,25,5	35.510878370635439 + 17.220087774496449 i	1187.651014371027941 +706.423700836880130 i	2,25,7	33.129697781829016 + 20.752705772570970 i	1007.287986789105162 +757.676116201652803 i
2,25,9	30.569116430438172 + 24.005943761224579 i	833.258423780268799 +772.070358199690304 i	2,25,11	27.820581673460286 + 27.022254812532335 i	668.426702469459997 +754.507381434956187 i
2,25,13	24.869776442043218 + 29.828284740169650 i	515.38505311386405 +708.55722832841911 i	2,25,15	21.694686114020652 + 32.441187247855595 i	376.625564137111041 +637.038858263818838 i
2,25,17	18.261585931424289 + 34.871500802214634 i	254.688101472695223 +542.312297231543425 i	2,25,19	14.516393804770425 + 37.124344522631148 i	152.332169487965587 +426.441291892859226 i
2,25,21	10.362817740437222 + 39.199314655417901 i	72.810623358827073 +291.272143902911353 i	2,25,23	5.585433454396442 + 41.088135100427131 i	20.49128118557920 4 +138.333514980374019 i
2,26,1	41.323804249663738 + 8.992872281952907 i	1680.184278289890472 +478.681140700286790 i	2,26,3	39.310358532945315 + 13.434947392169473 i	1492.070235560949186 +644.652628486269919 i
2,26,5	37.124878831311563 + 17.363066775960569 i	1301.981211715713926 +749.456955992636836 i	2,26,7	34.769875478316997 + 20.939388481300682 i	1114.321875087469380 +808.640380823619466 i
2,26,9	32.242798085133742 + 24.238295119781743 i	932.473837409858220 +830.043017302139901 i	2,26,11	29.536581295379960 + 27.302571419488096 i	759.270330165495693 +818.687069746422026 i
2,26,13	26.639208088959943 + 30.159300029622841 i	597.252173158884943 +778.213245461993779 i	2,26,15	23.532314502785436 + 32.826219159952522 i	448.832702784909827 +711.480357611955924 i
2,26,17	20.188325426434542 + 35.314678377900073 i	316.431462535997582 +620.867285705576364 i	2,26,19	16.564604803045764 + 37.630996244424892 i	202.616301886104594 +508.443076036521745 i
2,26,21	12.590066227141444 + 39.776747698408684 i	110.305048313075645 +376.062386502278343 i	2,26,23	8.126537517612817 + 41.747699990162161 i	43.152771690738952 +225.371214982447896 i
2,26,25	2.778769995521829 + 43.528565072193786 i	6.701305733801660 + 57.242739666767172 i	2,27,1	42.903475012862138 + 9.051149086563797 i	1812.538964643470763 +502.113096426633490 i
2,27,3	40.903050369118532 + 13.532181504010449 i	1618.189927815996953 +678.925184399340424 i	2,27,5	38.736022994329531 + 17.500642156925320 i	1421.369518832839958 +792.937026246099094 i
2,27,7	36.405331550080348 + 21.118876729215565 i	1226.488833416028228 +860.191886582241636 i	2,27,9	33.909212044100258 + 24.461484434314897 i	1036.919046002915820 +888.752668790465123 i
2,27,11	31.241831862030242 + 27.571532629594760 i	855.467446797348316 +883.764789184076221 i	2,27,13	28.393049353365186 + 30.476472738167320 i	684.631525810901849 +848.942019475698430 i
2,27,15	25.347401642087053 + 33.194528920330434 i	526.75871523800140 +787.18590084398556 i	2,27,17	22.082018414362519 + 35.737709421657989 i	384.169799738417737 +700.897334965673394 i
2,27,19	18.562527951970051 + 38.113271716179923 i	259.280571834336911 +592.152865168996186 i	2,27,21	14.734367076141275 + 40.324246248524382 i	154.758302217697690 +462.809140578023687 i
2,27,23	10.500908236634240 + 42.369020940572582 i	73.785729855555971 +314.547216434970096 i	2,27,25	5.646504028418240 + 44.238957328922830 i	20.675991289143486 +148.740030396329132 i

Table D.3: Branch points and associated eigenvalues for $m = 2$.

Table D.4: Branch points and associated eigenvalues for $m = 3$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
3, 3 ,1	2.254441944326160 + 6.731940814252908 i	11.273739025431583 + 9.046369132359118 i	3, 4 ,1	4.312789375877335 + 7.267942971679416 i	21.989944786045928 + 24.064251342820775 i
3, 5 ,1	6.178462421804212 + 7.686866389128842 i	38.868087480677623 + 40.699885683521551 i	3, 5 ,3	2.340324936128145 + 10.109946235283390 i	11.310516001008940 + 13.800986115329859 i
3, 6 ,1	7.954733051349589 + 8.033437876318695 i	61.436143112767034 + 58.624668698520892 i	3, 6 ,3	4.550052309842578 + 10.692087653052024 i	22.538458584763827 + 36.168232895835459 i
3, 7 ,1	9.679537838481016 + 8.329895457322275 i	89.470760015046380 + 77.619303900712296 i	3, 7 ,3	6.535247979983682 + 11.174335659270628 i	40.423739462325223 + 60.296458384158385 i
3, 7 ,5	2.415254551817370 + 13.368587097992547 i	11.366977844813620 + 18.216098734544630 i	3, 8 ,1	11.371207393612224 + 8.589281166145321 i	122.842143451250735 + 97.529614678553401 i
3, 8 ,3	8.408413761373746 + 11.588609602980835 i	64.372250401422662 + 85.935813365476434 i	3, 8 ,5	4.739546680924512 + 13.981710325186279 i	23.043511212415940 + 47.495489570279560 i
3, 9 ,1	13.039980886135291 + 8.820008939982213 i	161.466125749877762 +118.241208155844362 i	3, 9 ,3	10.213687567267053 + 11.952730900126889 i	94.081402785878936 +112.880009409366878 i
3, 9 ,5	6.820862640440326 + 14.506857833692713 i	41.760918770557318 + 78.626454612100531 i	3, 9 ,7	2.478038039562548 + 16.581311278054450 i	11.420466992754628 + 22.487384813472879 i
3,10,1	14.692131660511157 + 9.027864573426063 i	205.284176825622268 +139.665659403400156 i	3,10,3	11.973491905599939 + 12.278009686548042 i	129.367081505428217 +140.970754477275023 i
3,10,5	8.775744701011597 + 14.968769827964529 i	66.851325907066851 +111.423995118616020 i	3,10,7	4.895514170500202 + 17.216754405204874 i	23.480106559030069 + 58.494306828288252 i
3,11,1	16.331761264312494 + 9.217015015377619 i	254.253552696996650 +161.732450878841320 i	3,11,3	13.700863054665351 + 12.572180215435449 i	170.105890786047212 +170.084277723405791 i
3,11,5	10.651470589674066 + 15.382142966081950 i	97.958368861775313 +145.699374966281624 i	3,11,7	7.057698940565480 + 17.773240832154730 i	42.899673720710318 + 96.411770612096433 i
3,11,9	2.531408842471157 + 19.771007607492592 i	11.468398601538773 + 26.678253568886348 i	3,12,1	17.961684710831097 + 9.390569256053851 i	308.341878849001546 +184.383959582721303 i
3,12,3	15.404057089793813 + 12.840807776783908 i	216.209795302860556 +200.121357754270491 i	3,12,5	12.472769949311884 + 15.756762008055754 i	134.858130558452416 +181.297275637237135 i
3,12,7	9.083360983590133 + 18.270880620834731 i	68.959931997293197 +136.111267542465441 i	3,12,9	5.027627249176168 + 20.423346042045431 i	23.859062207346071 + 69.313455132640200 i
3,13,1	19.583908575131371 + 9.550912817557837 i	367.523929983385074 +207.572171900797997 i	3,13,3	17.088625794576362 + 13.088050351059087 i	267.61314509968304 +231.00067079695592 i
3,13,5	14.254386548225456 + 16.099576729390915 i	177.396873893993586 +218.091256512420500 i	3,13,7	11.021610918982338 + 18.722083975890349 i	101.264748554168676 +177.422606596698586 i
3,13,9	7.259686655821556 + 21.003872116204274 i	43.884390904702897 +113.897236378449733 i	3,13,11	2.577601654531433 + 22.947400994418174 i	11.511086227578639 + 30.817084121447518 i
3,14,1	21.199907417432126 + 9.699918358953617 i	431.779599373553026 +231.256437187639307 i	3,14,3	18.758473294428583 + 13.317106353613157 i	324.265331020685551 +262.654308590228368 i
3,14,5	16.005863704607872 + 16.415744854250843 i	225.462794217398596 +255.977101184420690 i	3,14,7	12.898616655450320 + 19.135383932943519 i	139.559322678758917 +220.196553444665511 i
3,14,9	9.347745548742751 + 21.529461821955625 i	70.786830230277900 +160.351853525535489 i	3,14,11	5.142054420867715 + 23.613004550497934 i	24.192013310165358 + 80.019843182723804 i
3,15,1	22.810792404351979 + 9.839084138944621 i	501.092557860357829 +255.401877180205531 i	3,15,3	20.416442475184450 + 13.530494304365210 i	386.12631383903857 +295.02467442750594 i
3,15,5	17.733742704855722 + 16.709220534417767 i	278.971184891743803 +294.867587109786200 i	3,15,7	14.730245206042623 + 19.517006408008928 i	183.665491386707629 +264.307399387017142 i
3,15,9	11.342079354940676 + 22.010797095374063 i	104.139421181918749 +208.525482200281800 i	3,15,11	7.435639085698829 + 24.212627782008258 i	44.749366915443396 +131.194617268321821 i
3,15,13	2.618219855508352 + 26.115380603947017 i	11.549285448243696 + 34.919010628336125 i	3,16,1	24.417418730478634 + 9.969628325082066 i	575.449334661936064 +279.978226673877941 i
3,16,3	22.064662358521890 + 13.730235920122780 i	453.163748305812248 +328.062271512626694 i	3,16,5	19.442702137559742 + 16.983112917867626 i	337.855860029950975 +334.688661202058483 i
3,16,7	16.526924550178808 + 19.871677961832869 i	233.451753303012083 +309.649114257133931 i	3,16,9	13.269811129289074 + 22.455393638171302 i	143.662594018240014 +258.276408390952383 i
3,16,11	9.579457103697559 + 24.760764803367952 i	72.395756984693250 +184.310162213346416 i	3,16,13	5.242890243545027 + 26.791695297988891 i	24.488132480898500 + 90.648946677048968 i
3,17,1	26.020456555482046 + 10.092555154118541 i	654.83866763916603 +304.95896764197363 i	3,17,3	23.704764672124345 + 13.917980586280988 i	525.351050948787702 +361.724085546065339 i
3,17,5	21.136200880965717 + 17.239917175953753 i	402.063839860998030 +375.376651090602479 i	3,17,7	18.295901150989383 + 20.203093255054100 i	288.817236319590393 +356.131412994896152 i
3,17,9	15.147584030133920 + 22.868846132366631 i	189.158046072339687 +309.481522495548802 i	3,17,11	11.624569810720001 + 25.266754330716783 i	106.679563940931246 +239.232407320797620 i
3,17,13	7.591431619691627 + 27.406896952270639 i	45.519458230527917 +148.363749852497818 i	3,17,15	2.654411712763085 + 29.277688109330050 i	11.583718972926855 + 38.993073663667957 i
3,18,1	27.620439289341434 + 10.208702523359998 i	739.251031997866676 +330.320668897466987 i	3,18,3	25.338024567449448 + 14.095092644605414 i	602.666055386485368 +395.972374355933425 i
3,18,5	22.816864931646659 + 17.481669898384780 i	471.551879905734609 +416.876203820963553 i	3,18,7	20.042422753669083 + 20.514206752306773 i	349.682281468743497 +403.676586174443514 i
3,18,9	16.986458244971171 + 23.255476541928022 i	240.477666023973512 +362.034870879229231 i	3,18,11	13.598764573393613 + 25.737280409929784 i	147.300883340946513 +295.827782675505375 i
3,18,13	9.785625650778719 + 27.973656724302934 i	73.831931955682762 +208.075335231998906 i	3,18,15	5.332972743907520 + 29.962845758195332 i	24.754350109685152 +101.221696171128826 i
3,19,1	29.217797322613066 + 10.318776971482171 i	828.678290794671739 +356.042473355490017 i	3,19,3	26.965455232111108 + 14.262714222274171 i	685.090048818097330 +430.773743691304958 i
3,19,5	24.486732052770879 + 17.710057280120996 i	546.284108222449618 +459.138735603560178 i	3,19,7	21.770413089473632 + 20.807424811308522 i	415.982529259275736 +452.217060310101942 i
3,19,9	18.794198785357729 + 23.618715093389120 i	297.506693917767166 +415.844877655911660 i	3,19,11	15.519241124981983 + 26.177396144989078 i	194.044512105684817 +353.976831829753905 i
3,19,13	11.877088718574621 + 28.500244651815869 i	108.953738525063443 +269.666624359186471 i	3,19,15	7.731164866692926 + 30.591023401969704 i	46.212824498195971 +165.440137424987739 i
3,19,17	2.687016083134644 + 32.435985118016546 i	11.614991625644629 + 43.045125270698527 i	3,20,1	30.812882124918261 + 10.423379878078283 i	923.113430774801031 +382.105693601652263 i
3,20,3	28.587873362328885 + 14.421811473437222 i	772.607064459018943 +466.098428937745780 i	3,20,5	26.147412862602476 + 17.926492609450872 i	626.230361840551041 +502.121245460435205 i
3,20,7	23.482881992020630 + 21.084737461286267 i	487.665011999325316 +501.693528396952559 i	3,20,9	20.576482325632288 + 23.961341859770123 i	360.153713518744553 +470.831788244723384 i
3,20,11	17.397525402891638 + 26.591059252239738 i	246.748143006252178 +413.574846680024166 i	3,20,13	13.894087183871816 + 28.992657950441327 i	150.56797553906816 +333.01239301127595 i
3,20,15	9.971284354048393 + 31.173455334064805 i	75.128157477394822 +231.700770279088886 i	3,20,17	5.414343741745784 + 33.128575961006632 i	24.995916001103499 +111.751416894580188 i
3,21,1	32.405983804513610 + 10.523027413379433 i	1022.550359350094709 +408.493488448323319 i	3,21,3	30.205945600773159 + 14.573209273917561 i	865.203350897515179 +501.919728119481420 i
3,21,5	27.800200490110015 + 18.132173108484047 i	711.364982267621713 +545.785392457229136 i	3,21,7	25.182187336134390 + 21.347811797862370 i	564.685465985954693 +552.053506260792346 i
3,21,9	22.337590490019906 + 24.285648341513340 i	428.344300909597109 +526.925575395872556 i	3,21,11	19.241773474846639 + 26.981449629047169 i	305.284962377526199 +474.530115049488529 i
3,21,13	15.854231881410881 + 29.455486210347367 i	198.445057440522191 +397.997428895522603 i	3,21,15	12.105354194810435 + 31.717544395141164 i	111.011687870930444 +299.902720511971097 i
3,21,17	7.857810215292134 + 33.767746016209088 i	46.843007439758502 +182.446438837383795 i	3,21,19	2.716660134932555 + 35.591339311112101 i	11.643592707247077 + 47.079187829695492 i
3,22,1	33.997344127963729 + 10.618165953439814 i	1126.983746254580183 +435.190601069561126 i	3,22,3	31.820222184422196 + 14.717617694216930 i	962.866966829523221 +538.213548117823620 i
3,22,5	29.446147336989892 + 18.328122487162542 i	801.665922556156715 +590.096767220124548 i	3,22,7	26.870209128105849 + 21.598059964960605 i	647.006421554613667 +603.250204765430567 i
3,22,9	24.080834251032879 + 24.593549954114767 i	502.016788185042969 +584.064272656957542 i	3,22,11	21.057992141728349 + 27.351172890541029 i	369.553214655279874 +536.761925436689921 i
3,22,13	17.769400475330023 + 29.892364340606282 i	252.410409806815721 +464.519006004786661 i	3,22,15	14.161996374783396 + 32.228714249059877 i	153.532095569090785 +369.929084551618530 i
3,22,17	10.140116249854584 + 34.363570207705969 i	76.308851191020466 +255.220787000095072 i	3,22,19	5.488518720741094 + 36.290272079272128 i	25.216861561838179 +122.247093255020374 i
3,23,1	35.587166324615822 + 10.709184148629229 i	1236.40889841994203 +462.18314465610428 i	3,23,3	33.431161785122626 + 14.855652501505796 i	1065.587466584020149 +574.958037020693610 i
3,23,5	31.086120112777721 + 18.515223369754082 i	897.114071825132100 +635.024309059341590 i	3,23,7	28.548469063916617 + 21.836689759219716 i	734.595808680422806 +655.241637777139204 i
3,23,9	25.808826877763050 + 24.886666846858603 i	581.11932515740466 +642.19265292722037 i	3,23,11	22.850743454588219 + 27.702398027496077 i	439.469524430101330 +600.198800945827884 i
3,23,13	19.648050205643660 + 30.306242457937994 i	312.326665680507745 +532.486047206596481 i	3,23,15	16.159132867238647 + 32.711156597364081 i	202.447411821815052 +441.669209473438286 i
3,23,17	12.313591862792206 + 34.922735764388662 i	112.890521639258083 +329.989029924421345 i	3,23,19	7.973588002192327 + 36.938885218509832 i	47.420325806557564 +199.397931287015041 i
3,23,21	2.743823191317114 + 38.744468903325760 i	11.669915817636687 + 51.098157868269240 i	3,24,1	37.175622573361856 + 10.796422484083978 i	1350.821659936643073 +489.458425231126910 i
3,24,3	35.039150153621073 + 14.987851248727942 i	1173.355652648493106 +612.133282969531024 i	3,24,5	32.720840117000805 + 18.694242389767364 i	997.692735250751070 +680.539833914458427 i
3,24,7	30.218214926801700 + 22.064743026217172 i	827.425915448459932 +707.989908006746987 i	3,24,9	27.523666570653116 + 25.166383478435044 i	665.607775654612624 +701.261175284377373 i
3,24,11	24.623578118605163 + 28.036954298096873 i	514.964408637494330 +664.777033026722279 i	3,24,13	21.496446559439661 + 30.699560069957869 i	378.082914444201890 +601.817639059178418 i
3,24,15	18.108912515964473 + 33.168221729096125 i	257.572361738524535 +515.022816487562636 i	3,24,17	14.407129592764589 + 35.450182200256414 i	156.244319554255554 +406.642437488469852 i
3,24,19	10.294896714617739 + 37.546300112789829 i	77.392629613807458 +278.658640252640907 i	3,24,21	5.556652230649548 + 39.448880752199507 i	25.420332590701797 +132.715061771075653 i
3,25,1	38.762859791871300 + 10.880180934574021 i	1470.218331222680490 +517.004793874872576 i	3,25,3	36.644514320245456 + 15.114686051219016 i	1286.163378471721444 +649.721065029324336 i
3,25,5	34.350913261193135 + 18.865849878816281 i	1103.387227556674134 +726.617647849475588 i	3,25,7	31.880481578376997 + 22.283125280301672 i	925.47259471572727 +761.46062870594096 i
3,25,9	29.227062706838069 + 25.433893484678954 i	755.444171170193044 +761.225138611433749 i	3,25,11	26.379315628517336 + 28.356401580097479 i	595.979142398471822 +730.439486181289340 i
3,25,13	23.319374726093198 + 31.074364814474020 i	449.587776260132159 +672.441570884331099 i	3,25,15	20.020028686808331 + 33.602651726370063 i	318.760268213871427 +589.899985818782397 i
3,25,17	16.438891779640027 + 35.949757879636124 i	206.117507871148035 +485.075062356161254 i	3,25,19	12.505012749554950 + 38.118604896064170 i	114.618616366027481 +359.958518482919601 i
3,25,21	8.080200171933132 + 40.105702004215871 i	47.952801603423701 +216.305368014947078 i	3,25,23	2.768878907553563 + 41.895875220803667 i	11.694278928969924 + 55.104200050632102 i
3,26,1	40.349004164535600 + 10.960725153380022 i	1594.595603084332197 +544.811522476288133 i	3,26,3	38.247533554927330 + 15.236573841904363 i	1404.003389561054746 +687.704645247283679 i
3,26,5	35.976852804250008 + 19.030635503864559 i	1214.184550896685550 +773.234227448331580 i	3,26,7	33.536135937053587 + 22.492628888205616 i	1028.714649473319923 +815.622450405640279 i
3,26,9	30.920425517782913 + 25.690234091709183 i	850.595548635128466 +822.043995048135457 i	3,26,11	28.120232646152356 + 28.662082752259380 i	682.463501016651890 +797.134625961224174 i
3,26,13	25.120578798156810 + 31.432396724412449 i	526.764719126527211 +744.293059933931545 i	3,26,15	21.898952739866235 + 34.016731621734266 i	385.892064008128727 +666.220134947740235 i
3,26,17	18.421240070667480 + 36.424562199530797 i	262.31540141251998 +565.18936216733960 i	3,26,19	14.633038801219277 + 38.660326265389770 i	158.743846994721054 +443.196798163938865 i
3,26,21	10.437768677222495 + 40.723257162170050 i	78.393989075031428 +302.030752814876223 i	3,26,23	5.619643441497444 + 42.605071559379027 i	25.608822769405631 +143.159956785748960 i
3,27,1	41.934164720948843 + 11.038291520506547 i	1723.950502454565822 +572.868698492133149 i	3,27,3	39.848447927487705 + 15.353884682904884 i	1526.869194139413594 +726.068593677358763 i
3,27,5	37.599096816126981 + 19.189120823568345 i	1330.073136513124609 +820.367953148223819 i	3,27,7	35.185910740178322 + 22.693951449595733 i	1137.133349518295063 +870.446669839538913 i
3,27,9	32.604931286532356 + 25.936312930098108 i	951.03306007809044 +883.68078783782460 i	3,27,11	29.848193853625162 + 28.955163517091435 i	774.374090694719939 +864.815725533225077 i
3,27,13	26.903046689324530 + 31.775149914115946 i	609.548769766469604 +817.313680294344863 i	3,27,15	23.750649837292318 + 34.412393009121494 i	458.869175830672589 +743.910839357966324 i
3,27,17	20.363055831108198 + 36.877149984034951 i	324.683054122469628 +646.896929659609668 i	3,27,19	16.697326200482202 + 39.175014529882127 i	209.506139311785773 +528.272278497214074 i
3,27,21	12.682114074815072 + 41.307126507178573 i	116.218138111519352 +389.834587117050091 i	3,27,23	8.178979556846299 + 43.269099497254466 i	48.446784047739534 +233.176585740494346 i
3,27,25	2.792123720548651 + 45.045919036014432 i	11.716940973115751 + 59.098981723531921 i	3,28,1	43.518436191094580 + 11.113091294114819 i	1858.280347383684330 +601.167135200789858 i
3,28,3	41.447465066310826 + 15.466948561743267 i	1654.754956896125350 +764.798640050993072 i	3,28,5	39.218021764232141 + 19.341769471140779 i	1451.042635292684508 +867.998884916380575 i
3,28,7	36.830430285692771 + 22.887710537628529 i	1250.712046350780838 +925.906903890818285 i	3,28,9	34.281570705553015 + 26.172929218463285 i	1056.731279191238627 +946.101686852613852 i
3,28,11	31.564745349102875 + 29.236663215477105 i	871.673087309522657 +933.440213874749247 i	3,28,13	28.669202440441577 + 32.103918875329320 i	697.884131497386988 +891.450473459724435 i
3,28,15	25.579021932633587 + 34.791288022033442 i	537.608688239848675 +822.906737772329961 i	3,28,17	22.270976494676784 + 37.309664289128456 i	393.094090989937570 +730.117893194332623 i
3,28,19	18.710406158745631 + 39.665547450139009 i	266.702472751403093 +615.090326160569020 i	3,28,21	14.842505266994889 + 41.861481477563643 i	161.061407734181671 +479.623823652620956 i
3,28,23	10.570420971569630 + 43.895608487950888 i	79.324427171917009 +325.349116257786307 i	3,28,25	5.678205925558525 + 45.759332047225101 i	25.784337956715291 +153.585270824793980 i

Table D.4: Branch points and associated eigenvalues for $m = 3$.

Table D.5: Branch points and associated eigenvalues for $m = 4$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
4, 4 ,1	2.357663561227191 + 7.971913317957412 i	18.433063143522340 + 10.285516884986537 i	4, 5 ,1	4.496463440238013 + 8.560827522001995 i	29.782817773819936 + 27.394857362811521 i
4, 6 ,1	6.420513778117898 + 9.029346203504799 i	47.523240440325630 + 46.358165420600599 i	4, 6 ,3	2.408979269086267 + 11.429969157729531 i	18.41005595725393 + 15.23726619596111 i
4, 7 ,1	8.242463435110581 + 9.421838626464764 i	71.110818231296889 + 66.811509306571054 i	4, 7 ,3	4.682497498641788 + 12.044060825011680 i	30.079801393269918 + 39.931643564838801 i
4, 8 ,1	10.004492707374917 + 9.760832110144460 i	100.284826219045101 + 88.509228103445452 i	4, 8 ,3	6.717414759511231 + 12.558972869530987 i	48.610757674106125 + 66.536059141683666 i
4, 8 ,5	2.466495386168887 + 14.736164316478286 i	18.438667761859122 + 19.761071740637572 i	4, 9 ,1	11.727318500562561 + 10.059742988313779 i	134.892010610878401 + 111.277474457592490 i
4, 9 ,3	8.631412424642210 + 13.005237322111146 i	73.354060986312831 + 94.793360699087813 i	4, 9 ,5	4.843195268549759 + 15.371059298936938 i	30.453925598007523 + 51.515577720548492 i
4,10,1	13.422680696033217 + 10.327337196565606 i	174.831972134855647 +134.986588357994037 i	4,10,3	10.471198450272242 + 13.400221012922820 i	103.975826950115490 + 124.485757346125453 i
4,10,5	6.967206794609502 + 15.919667273324128 i	49.688194224052921 + 85.223251696585351 i	4,10,7	2.518701896497916 + 17.980895664195415 i	18.475914485130602 + 24.102262497979392 i
4,11,1	15.097865049058187 + 10.569708942714181 i	220.034243193300256 +159.535886791723499 i	4,11,3	12.260747345969603 + 13.755093607457676 i	140.271292996174026 + 155.444502594267050 i
4,11,5	8.958290207661781 + 16.405418378501562 i	75.431880777922345 +120.703278314510243 i	4,11,7	4.980502494244763 + 18.632243161808557 i	30.809123519649418 + 62.687257784331649 i
4,12,1	16.757690324283640 + 10.791289864517786 i	270.446924365338475 +184.844745511169918 i	4,12,3	14.014127925488364 + 14.077575417298078 i	182.102300954995115 + 187.536839645586042 i
4,12,5	10.865393560351134 + 16.842443223783174 i	107.303155843837317 +157.762734207252521 i	4,12,7	7.179911610398112 + 19.206508223692719 i	50.658907438237392 + 103.255119581573680 i
4,12,9	2.564976803924403 + 21.193737532932804 i	18.513244385336570 + 28.341936745751287 i	4,13,1	18.405498468020550 + 10.995415041373692 i	326.030400572316239 + 210.847033731705977 i
4,13,3	15.740325619272591 + 14.373272078801238 i	229.369692313411690 +220.655847989666000 i	4,13,5	12.714301283982611 + 17.240254755922283 i	145.06057498629343 + 196.24086364559869 i
4,13,7	9.237867380643955 + 19.722704954727881 i	77.273533220698610 +145.686857735662699 i	4,13,9	5.099520855384977 + 21.858249604336446 i	31.132532730740547 + 73.631396431845090 i
4,14,1	20.043694339311866 + 11.184663381075270 i	386.753584081812050 +237.487456561636719 i	4,14,3	17.445429302440285 + 14.646404714487886 i	281.999119085750408 + 254.713556553154632 i
4,14,5	14.520486680716576 + 17.605670347272451 i	188.537492734313957 +236.005881945172348 i	4,14,7	11.204630866647030 + 20.192710339764343 i	110.236041253793772 + 189.812045515253402 i
4,14,9	7.364492622716822 + 22.452764398382886 i	51.525630725709583 +120.922286357853906 i	4,14,11	2.606093419228999 + 24.387694139327277 i	18.548506041061778 + 32.516699293151738 i

Table D.5: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
4,15,1	21.674060521718527 + 11.361073603366693 i	452.591532137410582 +264.719047466208849 i	4,15,3	19.133751074599552 + 14.900242955573873 i	339.932979475458126 + 289.636261818944433 i
4,15,5	16.294025823618220 + 17.943787739969224 i	237.612160210672187 +276.948634047036535 i	4,15,7	13.107117923110490 + 20.624766111298051 i	149.276796911642379 + 235.478969133447293 i
4,15,9	9.481602510709159 + 22.993260701155680 i	78.909082616539621 +170.149002157370404 i	4,15,11	5.204257586819905 + 25.062937909629362 i	31.425235862027247 + 84.432521205959546 i
4,16,1	23.297950965304373 + 11.526287268272833 i	523.523865037102155 +292.501388419937484 i	4,16,3	20.808451328609390 + 15.137377497589863 i	403.125490516523541 + 325.361266633746936 i
4,16,5	18.041863747673183 + 18.258540743448346 i	292.191957284465332 +318.977413502268178 i	4,16,7	14.961727576529659 + 21.024930878789721 i	194.206433017362571 + 282.558848918072158 i
4,16,9	11.501955288172500 + 23.489951470227648 i	112.838699382485700 +221.161733445442536 i	4,16,11	7.527291892079282 + 25.673881864624370 i	52.303213915811895 + 138.359549675866020 i
4,16,13	2.642910603388425 + 27.569476359069665 i	18.581240484845722 + 36.646097807841592 i	4,17,1	24.916415321977517 + 11.681646857131694 i	599.533675991450309 + 320.799309524377122 i
4,17,3	22.471911085962702 + 15.359899762351844 i	471.539507283869568 +361.834546553184452 i	4,17,5	19.768993525959978 + 18.553040449463477 i	352.204243557985080 + 362.014112378572804 i
4,17,7	16.779286250370880 + 21.397833560029575 i	244.884647329581469 +330.942509393958858 i	4,17,9	13.453223540867372 + 23.950077351889405 i	153.022292211054975 + 273.818113429879133 i
4,17,11	9.697458991820422 + 26.234284281543655 i	80.373981867699626 +194.279975254147871 i	4,17,13	5.297640159752083 + 28.253626405357302 i	31.690920907094146 + 95.135921225870746 i
4,18,1	26.530281664660553 + 11.828265018544457 i	680.606756537041974 +349.581915487275239 i	4,18,3	24.125965369932015 + 15.569524800937403 i	545.144378648140332 + 399.009037346034063 i
4,18,5	21.479124282562488 + 18.829796563600681 i	417.590671758055180 +405.991387492412514 i	4,18,7	18.567349213542759 + 21.747112607816799 i	301.203454454053599 + 380.536629371034337 i
4,18,9	15.352449472118915 + 24.379062452467331 i	199.251820647193625 +327.993442537660087 i	4,18,11	11.766440322378278 + 26.753079814606757 i	115.171488406558424 + 252.063060066160745 i
4,18,13	7.672799429595728 + 28.878193297933407 i	53.006048355343033 +155.639839168268708 i	4,18,15	2.676156630574907 + 30.742926071182328 i	18.611501454715892 + 40.741771147827230 i
4,19,1	28.140213189477560 + 11.967074678936687 i	766.73103184321508 +378.82184147837472 i	4,19,3	25.772055618250523 + 15.767677976696648 i	623.914450137575727 + 436.843349757741066 i
4,19,5	23.175084614493972 + 19.090867174182190 i	488.30346340738862 +450.85055261400606 i	4,19,7	20.331409584375201 + 22.075691873780482 i	363.07731874843597 + 431.26063643092590 i
4,19,9	17.210998065817339 + 24.781119754165282 i	251.371393559842005 +383.579835590600567 i	4,19,11	13.762439099497827 + 27.236704033444607 i	156.385013495376683 + 311.575743688087528 i
4,19,13	9.891070160697037 + 29.455239440753335 i	81.697853110038224 +218.183805602679001 i	4,19,15	5.381818320539333 + 31.434585027625847 i	31.933371233423685 + 105.768474235364039 i
4,20,1	29.746748089910955 + 12.098866083938333 i	857.896139924269164 +408.494674007084313 i	4,20,3	27.411332661290672 + 15.955557691495246 i	707.827987787532152 + 475.300785635905186 i
4,20,5	24.859078684755211 + 19.337963588971128 i	564.302866785537162 +496.539985838749487 i	4,20,7	22.075589855872760 + 22.385963313207558 i	430.436940357968467 + 483.044283734461203 i
4,20,9	19.036874852946283 + 25.159608929667591 i	309.259875760732427 +440.483688739748743 i	4,20,11	15.702961696950497 + 27.690048365302783 i	203.790854008942148 + 372.697972494189742 i
4,20,13	12.004546448653111 + 29.992688251667190 i	117.282289500399841 +282.654856338351522 i	4,20,15	7.804274703719508 + 32.070648710789911 i	53.646121567202833 + 172.806728741763607 i
4,20,17	2.706415500967776 + 33.910411174190124 i	18.639495290984161 + 44.811205777647515 i	4,21,1	31.350328224385297 + 12.224314689700684 i	954.093112102575219 + 438.578493113372076 i
4,21,3	29.044728318122797 + 16.134181781166038 i	796.866385422579015 +514.348571510280635 i	4,21,5	26.532855455694538 + 19.572525796645497 i	645.555362667731856 + 543.013906304491684 i
4,21,7	23.803063383595571 + 22.679912975532648 i	503.225138897802594 +535.825777460523113 i	4,21,9	20.835950764934136 + 25.517264428055270 i	372.820523380309226 + 498.623232915047367 i
4,21,11	17.599771939643240 + 28.116961780122018 i	257.219810468819105 +435.323951129066472 i	4,21,13	14.041817252280044 + 30.496316286907295 i	159.433111150586001 + 348.929029985226180 i
4,21,15	10.066539855511806 + 32.661842473064119 i	82.904115045541730 +241.923174900551260 i	4,21,17	5.458401129625192 + 34.608487954456642 i	32.155899308998180 + 116.347336329969039 i
4,22,1	32.951320129748943 + 12.344002508167611 i	1055.314126324490644 +469.053505440895208 i	4,22,3	30.673006405610337 + 16.304422487918337 i	891.013568372669511 + 553.957252311670459 i
4,22,5	28.197824291351438 + 19.795778074167089 i	732.032362120635753 +590.231419793765326 i	4,22,7	25.516324315735591 + 22.959210717950434 i	581.394015689029402 + 589.550322169446758 i
4,22,9	22.612667122557578 + 25.856348760861252 i	441.974389870205584 +557.926490329163926 i	4,22,11	19.461216400901399 + 28.520549557372632 i	316.539351785767906 + 499.360660448518161 i
4,22,13	16.020733994034678 + 30.970564571328556 i	207.913272769738001 +416.891195746962751 i	4,22,15	12.221014067235274 + 33.215271189972931 i	119.208213791869156 + 313.021511566389563 i
4,22,17	7.924147665850472 + 35.254396256055180 i	54.233089114257425 +189.888151956963043 i	4,22,19	2.734150920431699 + 37.073474578813048 i	18.665459574364686 + 48.859505144126977 i
4,23,1	34.550030689861181 + 12.458434679409596 i	1161.552313932177867 +499.901746539099406 i	4,23,3	32.296799857172182 + 16.467033262242737 i	990.255536753310025 + 594.100205186157041 i
4,23,5	29.855136155300112 + 20.008770794585420 i	823.709238938119142 +638.155763311227133 i	4,23,7	27.217367471787707 + 23.225275789741524 i	664.902932994458752 + 644.168979318223364 i
4,23,9	24.370468054663792 + 26.178759685200887 i	516.655918181519723 +618.329630155241944 i	4,23,11	21.293457853927421 + 28.903365569005931 i	381.642895057669307 + 564.725994523979921 i
4,23,13	17.953363307119485 + 31.418964536443607 i	262.542072161266162 +486.438139173372406 i	4,23,15	14.296574931435901 + 33.736163715170555 i	162.218962239030390 + 385.986590732486263 i
4,23,17	10.226941819412222 + 35.857780978301520 i	84.011178192309941 +265.538535381040845 i	4,23,19	5.528618244880667 + 37.777101728189550 i	32.361278705703910 + 126.884077363940904 i
4,24,1	36.146718997966325 + 12.568052504689268 i	1272.801606342025025 +531.106836686868292 i	4,24,3	33.916638238276420 + 16.622669605473309 i	1094.580010063874170 + 634.753244851083878 i
4,24,5	31.505742015454945 + 20.212412411359335 i	920.564596105661963 +686.753699263921476 i	4,24,7	28.907812197480077 + 23.479325846543400 i	753.717035799487576 + 699.637761156676561 i
4,24,9	26.112079163766595 + 26.486107523604101 i	596.809877335927354 +679.775655188620021 i	4,24,11	23.101188184344682 + 29.267542838059434 i	452.442930717914862 + 631.347063368082331 i
4,24,13	19.848315614910021 + 31.844392617983555 i	323.177003432319168 +557.478001743607365 i	4,24,15	16.311336617732717 + 34.228577133669916 i	211.687807777157076 + 460.708310981554973 i
4,24,17	12.419417951283561 + 36.425071987584232 i	120.978179514389268 +343.217887684492723 i	4,24,19	8.034274020801250 + 38.431543517074957 i	54.774698406689730 + 206.902991222954029 i
4,24,21	2.759732673996595 + 40.233166478447288 i	18.689621406771234 + 52.890309745650498 i	4,25,1	37.741605460841676 + 12.673243815217170 i	1389.056611975624264 + 562.653778645958369 i
4,25,3	35.532968479279653 + 16.771905490133516 i	1203.976146740372315 +675.894299753224004 i	4,25,5	33.150435726703911 + 20.407494303219266 i	1022.579699903828441 + 735.995023814804767 i
4,25,7	30.588990017196913 + 23.722414277658444 i	847.806145609685132 +755.916903149694917 i	4,25,9	27.839694360994994 + 26.779772373987726 i	682.389165046385074 + 742.213349587557218 i
4,25,11	24.888067600598614 + 29.614885773055178 i	528.866393341218554 +699.158763510082053 i	4,25,13	21.711983271160069 + 32.249235100976605 i	389.702776656204435 + 629.928937338825790 i
4,25,15	18.277600425465611 + 34.695765506505737 i	267.424004966565406 +537.086335762522367 i	4,25,17	14.530674853112393 + 36.961036920553589 i	164.783258705225961 + 422.819556433017794 i
4,25,19	10.374634734100971 + 39.045554319259423 i	85.033627635244997 +289.057374334794304 i	4,25,21	5.593427877891056 + 40.941643175965837 i	32.551803544239064 + 137.386844618530176 i
4,26,1	39.334878872597180 + 12.774351307883643 i	1510.312516442192418 +594.528788656885922 i	4,26,3	37.146170726481508 + 16.915246447919738 i	1318.43432009141657 + 717.50314371197794 i
4,26,5	34.789886087656932 + 20.594710335161867 i	1129.738036622645041 +785.852163580872229 i	4,26,7	32.262008133526400 + 23.955459100725548 i	947.14391635295942 + 812.97027331152060 i
4,26,9	29.555105271879992 + 27.060947345641871 i	773.353189708117498 +805.596429763495962 i	4,26,11	26.657008952462398 + 29.946937370042242 i	610.85142291665250 + 768.10260241449771 i
4,26,13	23.549258208510089 + 32.635500850154529 i	462.024231441670167 +703.717729325065875 i	4,26,15	20.204184561512090 + 35.140400549888028 i	329.276244053751668 + 615.030352512549939 i
4,26,17	16.579030840361391 + 37.469402893885096 i	215.167835432174371 +504.238473701319663 i	4,26,19	12.602518242576961 + 39.624998681102539 i	122.615004215211925 + 373.281491312897629 i
4,26,21	8.136101086878162 + 41.603559343314330 i	55.277201937545499 +223.864553112748752 i	4,26,23	2.783458350594883 + 43.390227345141113 i	18.712184456813020 + 56.906312998382532 i
4,27,1	40.926701972633971 + 12.871679310531789 i	1636.565000814252016 +626.719154075539223 i	4,27,3	38.756570619941662 + 17.053140111761135 i	1437.945937281826446 + 759.561171563101766 i
4,27,5	36.424661187600584 + 20.774672445561311 i	1242.024960607371440 +836.299841561357084 i	4,27,7	33.927796367692622 + 24.179265650938991 i	1051.707179303603425 + 870.764887756653366 i
4,27,9	31.259793339247501 + 27.330671828346919 i	869.666652335102299 +869.882853324332473 i	4,27,11	28.410369216030656 + 30.265029410446974 i	698.345027618309928 + 838.125736622091154 i
4,27,13	25.363975444059264 + 33.004901555569766 i	540.061536902646139 +778.778560697116518 i	4,27,15	22.097664797715737 + 35.564715024729502 i	397.121476746499582 + 694.459194799477814 i
4,27,17	18.576972254992345 + 37.953180849394904 i	271.932319836466206 +587.377352156702273 i	4,27,19	14.747196333021424 + 40.174251365477076 i	167.158088754579495 + 459.476860089746481 i
4,27,21	10.511466702178430 + 42.226923335139126 i	85.983151119548808 +312.499138804887878 i	4,27,23	5.653589921246708 + 44.102979300306387 i	32.729370811833036 + 147.861579597626076 i
4,28,1	42.517215857266550 + 12.965499320482888 i	1767.810174141572361 +659.213112570130761 i	4,28,3	40.364448916092925 + 17.185984788477818 i	1562.503291622638017 + 802.051210073279208 i
4,28,5	38.055247160614563 + 20.947923201487132 i	1359.427411444924246 +887.314797949958802 i	4,28,7	35.587142476310127 + 24.394544624404904 i	1161.475427114015702 + 929.270509590213010 i
4,28,9	32.954996912488710 + 27.589857493781093 i	971.298612015475328 +935.034251657791287 i	4,28,11	30.150082711381103 + 30.570320739395427 i	791.301353271654875 + 909.180182963102538 i
4,28,13	27.159201121186594 + 33.358910648268214 i	623.746814883048614 +855.051969606368743 i	4,28,15	23.963094964087404 + 35.970601271246899 i	470.857667901738239 + 775.299744951131856 i
4,28,17	20.533483435430853 + 38.414859591708030 i	334.91757597823670 +672.14768211940304 i	4,28,19	16.827148686034928 + 40.696753678162949 i	218.395485378540059 + 547.543388449730401 i
4,28,21	12.772489592084810 + 42.817119998531140 i	124.136943112653427 +403.238992314559937 i	4,28,23	8.230778500698875 + 44.771501235924013 i	55.745691586964128 + 240.782536019430012 i
4,28,25	2.805569975138653 + 46.545194708833193 i	18.733326729532362 + 60.909568241703361 i	4,29,1	44.106543514231078 + 13.056054574983216 i	1904.044517284792619 + 691.999748923780999 i
4,29,3	41.970049110269251 + 17.314136490739010 i	1692.099440941657804 +844.957357406479673 i	4,29,5	39.682062815356574 + 21.114946011029527 i	1481.933684251733894 + 938.875554922504307 i
4,29,7	37.240719138993903 + 24.601926589868484 i	1276.430402058288337 +988.459313895344962 i	4,29,9	34.641761102396529 + 27.839308895533197 i	1078.221756989191135 +1001.015459990873751 i
4,29,11	31.877756332640299 + 30.863826969511962 i	889.680373954110223 +981.222168198961526 i	4,29,13	28.937427155979531 + 33.698807639649580 i	713.021690672730415 + 932.483975849497483 i
4,29,15	25.804453199514363 + 36.359681739227142 i	550.398861920571562 +857.485888428789849 i	4,29,17	22.455288663671098 + 38.856532094411513 i	403.993600864949315 + 758.469274334609281 i
4,29,19	18.855009752882054 + 41.195302529910663 i	276.119757582202624 +637.386926635864029 i	4,29,21	14.948579234608779 + 43.378186039105721 i	169.369138978809843 + 495.993482665025510 i
4,29,23	10.638912018877138 + 45.403167951390188 i	86.869228961497328 +335.878043402754031 i	4,29,25	5.709716451231720 + 47.261745370575262 i	32.895556276427641 + 158.312742581090248 i

Table D.5: Branch points and associated eigenvalues for $m = 4$.

Table D.6: Branch points and associated eigenvalues for $m = 5$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
5, 5 ,1	2.450444507315414 + 9.182664291501323 i	27.583139404518864 + 11.436479619199886 i	5, 6 ,1	4.662342487692823 + 9.817718575239088 i	39.529181751692839 + 30.489219805169245 i
5, 7 ,1	6.640425103767173 + 10.329717712066044 i	58.081717173997028 + 51.616148745530090 i	5, 7 ,3	2.472793551578551 + 12.718788809321248 i	27.506351542415310 + 16.586037233356752 i
5, 8 ,1	8.505337177223884 + 10.762773967232393 i	82.630878839583943 + 74.420274660631520 i	5, 8 ,3	4.805334439402123 + 13.361841671299297 i	39.600681728098301 + 43.468549428835857 i
5, 9 ,1	10.302867088113443 + 11.139627333191275 i	112.881311212210221 + 98.631465449732332 i	5, 9 ,3	6.886695967030842 + 13.906293359704238 i	58.750622370364638 + 72.403546221106907 i
5, 9 ,5	2.515148045077692 + 16.073499167961614 i	27.509332709315668 + 21.223985473887332 i	5,10,1	12.055769283117263 + 11.473969728735984 i	148.658008956032717 + 124.057718769592242 i
5,10,3	8.839191523574437 + 14.381556241838277 i	84.255483491837538 + 103.126261604752415 i	5,10,5	4.941020753197809 + 16.728686006692232 i	39.854905663150262 + 55.325348063583206 i
5,11,1	13.777074673734649 + 11.774828616189099 i	189.84548527588535 + 150.55534230266954 i	5,11,3	10.711798230703005 + 14.804618932109779 i	115.751875578527290 + 135.407755884389132 i
5,11,5	7.105286489109333 + 17.298998464188756 i	59.590834106473302 + 91.479021995539668 i	5,11,7	2.557876818342992 + 19.351951936344676 i	27.531305843658277 + 25.641194916495483 i
5,12,1	15.474947809111818 + 12.048529925073074 i	236.362134793226772 + 178.012418331632063 i	5,12,3	12.529856253484619 + 15.186525255754464 i	153.015602423927817 + 169.069298457011570 i
5,12,5	9.130696616250335 + 17.806780213767055 i	85.967411259209058 + 129.507284214283118 i	5,12,7	5.061710622724171 + 20.018436619587821 i	40.134130257440432 + 66.686127666716004 i
5,13,1	17.154835967374208 + 12.299706516239150 i	288.147477015374591 + 206.339075698259990 i	5,13,3	14.308343986119949 + 15.534973602224294 i	195.894614348007821 + 203.969619096948776 i
5,13,5	11.067721698917552 + 18.265686447170779 i	118.578481829049210 + 169.212786360463099 i	5,13,7	7.296493559157330 + 20.609438329936367 i	60.405280485407964 + 109.785837526940327 i
5,13,9	2.597653403836617 + 22.589776574316947 i	27.55848484821930 + 29.93545487613259 i	5,14,1	18.820547308967889 + 12.531867365211845 i	345.155076400757196 + 235.461406149154698 i
5,14,3	16.056891578354652 + 15.855593856465836 i	244.279277353265883 + 239.994541377601024 i	5,14,5	12.943099000611920 + 18.684997141527994 i	157.165914833924575 + 210.429359059763470 i
5,14,7	9.385244957979859 + 21.143042996881551 i	87.561353099489153 + 154.829571951700387 i	5,14,9	5.168842870792060 + 23.266065629634056 i	40.404871919147780 + 77.770081991644687 i
5,15,1	20.474842411781601 + 12.747741806674815 i	407.348293879194955 + 265.317459521127944 i	5,15,3	17.782064889205344 + 16.152651965815569 i	298.08703921482311 + 277.04985257316099 i
5,15,5	14.772958647326154 + 19.071412698854292 i	201.550795936451635 + 253.019998359303514 i	5,15,7	11.379303407480961 + 21.630673104182272 i	121.165272783498935 + 201.646202194258592 i
5,15,9	7.465295443687612 + 23.873931364699384 i	61.160651642321866 + 127.659615327810002 i	5,15,11	2.634042553060898 + 25.803052541487606 i	27.58653068810773 + 34.15135633843751 i
5,16,1	22.119781471623494 + 12.949499123284737 i	474.697582891588979 + 295.854494181216126 i	5,16,3	19.488540252213266 + 16.429470716568133 i	357.253688800629845 + 315.056421563984259 i
5,16,5	16.567856264684451 + 19.429974983253761 i	251.601929952701454 + 296.870775933047923 i	5,16,7	13.306276040979576 + 22.080319348605816 i	160.933147367179117 + 250.081356152204677 i
5,16,9	9.610249382290997 + 24.428579449187797 i	89.016831107736934 + 179.549330268739709 i	5,16,11	5.264651080960332 + 26.487744773590155 i	40.658919795088316 + 88.679112113007449 i
5,17,1	23.756939158477525 + 13.138894688144909 i	547.178687294733777 + 327.027022720341904 i	5,17,3	21.179763105522749 + 16.688696178500876 i	421.728001043197025 + 353.946782719914552 i
5,17,5	18.335100880942392 + 19.764595947307580 i	307.219178052972268 + 341.885680602560683 i	5,17,7	15.183050231369320 + 22.497894100506688 i	206.664870741235887 + 300.003136354695926 i
5,17,9	11.655604629742399 + 24.939817514509912 i	123.512419607540920 + 233.290891235576765 i	5,17,11	7.616006442815815 + 27.109613888670840 i	61.85476902447805 + 145.25842475503876 i
5,17,13	2.667272736152654 + 29.000250067844402 i	27.61389079466679 + 38.31284517617872 i	5,18,1	25.387543373684846 + 13.317370657053903 i	624.771394921832893 + 358.795382182375420 i
5,18,3	22.858341983977972 + 16.932474277884140 i	491.46827124794623 + 393.66268342942260 i	5,18,5	20.079969818534153 + 20.078384370852451 i	368.323762897777158 + 387.982729762159806 i
5,18,7	17.020817018718375 + 22.887939375713799 i	258.211717991556498 + 351.299242480355588 i	5,18,9	13.629551980582589 + 25.414646860547787 i	164.342999215552311 + 288.741011376526842 i
5,18,11	9.811539274271645 + 27.681774801092295 i	90.344638620371825 + 203.883753722047032 i	5,18,13	5.351084004376639 + 29.692616199757495 i	40.895066614453367 + 99.468418428747754 i
5,19,1	27.012568076846822 + 13.486127379698209 i	707.458648713664047 + 391.124662228753266 i	5,19,3	24.526296481157800 + 17.162572172931373 i	566.439973041932490 + 434.153279854028312 i
5,19,5	21.806399105215569 + 20.373859005045162 i	434.85222814978772 + 435.09108430022758 i	5,19,7	18.827414028545153 + 23.254042360521129 i	315.458900701105563 + 403.873373374380719 i
5,19,9	15.549506293686623 + 25.858341844847704 i	211.291127921943399 + 345.773268806222973 i	5,19,11	11.903537768133720 + 28.212808606362380 i	125.648286024570922 + 264.427151070412833 i
5,19,13	7.751956452445497 + 30.325849594133810 i	62.492737213705198 + 162.668735015152834 i	5,19,15	2.697709219824531 + 32.186300257890409 i	27.64000766782596 + 42.43421028648595 i
5,20,1	28.632797255009500 + 13.646175300460939 i	795.225896385563260 + 423.983884726407666 i	5,20,3	26.185219833198005 + 17.380464241295997 i	646.614119209324826 + 475.373779992374921 i
5,20,5	23.517401683531396 + 20.653093522051037 i	506.75247367090697 + 483.14889128921737 i	5,20,7	20.608558861294867 + 23.599097899575103 i	378.315263413654861 + 457.642168637598104 i
5,20,9	17.427115647724520 + 26.275018768019631 i	264.193420541801949 + 404.277789389914005 i	5,20,11	13.920604336966752 + 28.708932510511172 i	167.444433667637639 + 326.755766492856253 i
5,20,13	9.993493461263018 + 30.912837270777189 i	91.560613718923364 + 227.952734827358455 i	5,20,15	5.429703181124467 + 32.884456381066812 i	41.114249204065267 + 110.171011003467711 i

Table D.6: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
5,21,1	30.248870135857707 + 13.798373481630319 i	888.060604119624031 + 457.345364393291220 i	5,21,3	27.836389190530998 + 17.587394560976545 i	731.966080690566741 + 517.284403029297778 i
5,21,5	25.215333678637723 + 20.917818433808254 i	583.981043816452939 + 532.101647677280312 i	5,21,7	22.368556106561527 + 23.925483219916757 i	446.706780340184594 + 512.532792422552575 i
5,21,9	19.270608349288981 + 26.667973936003762 i	322.922609032287198 + 464.158927977450844 i	5,21,11	15.880536181411161 + 29.174894995678507 i	215.501119113892912 + 390.748776460353611 i
5,21,13	12.128264931957355 + 31.460741537172307 i	127.602534727399700 + 295.211051473422856 i	5,21,15	7.875687924878723 + 33.528259415879582 i	63.080994886104897 + 179.942630211240896 i
5,21,17	2.725711785659007 + 35.364285120642819 i	27.66471851275822 + 46.52463966218048 i	5,22,1	31.861313851781400 + 13.943458725084747 i	985.95188600458266 + 491.18420267687202 i
5,22,3	29.480842568571308 + 17.784423304082356 i	822.474714629691675 + 559.849567366056931 i	5,22,5	26.902070865227909 + 21.169494753132703 i	666.501209725827493 + 581.900939700976778 i
5,22,7	24.110729223780442 + 24.235178958919651 i	520.57223874630313 + 568.48105221208266 i	5,22,9	21.086035061354639 + 27.039900201124112 i	387.376827970323291 + 525.332914385252934 i
5,22,11	17.795321060668023 + 29.614448196994019 i	269.642465495980218 + 456.299199561017986 i	5,22,13	14.185174476094945 + 31.975143238892155 i	170.283074089941977 + 364.320683288665634 i
5,22,15	10.159423567282140 + 34.127979367061165 i	92.679774553277440 + 251.829033425807552 i	5,22,17	5.501741741925163 + 36.067885205134203 i	41.318005702735867 + 120.808067839984929 i
5,23,1	33.470567493986842 + 14.082067943364560 i	1088.890217004287933 + 525.477881545661148 i	5,23,3	31.119433867391432 + 17.972461835681269 i	918.121707010523210 + 603.037247343913691 i
5,23,5	28.579129551183737 + 21.409368478784483 i	754.281574962330524 + 632.503456986474021 i	5,23,7	25.837697738146211 + 24.529855693000520 i	599.860257653977442 + 625.429929887070784 i
5,23,9	22.877984861498387 + 27.393033121845090 i	457.472784143872843 + 587.725850024276042 i	5,23,11	19.673484091754947 + 30.030630413135452 i	329.730412968557005 + 523.312656741468231 i
5,23,13	16.182304900798464 + 32.460351068563597 i	219.357896580555121 + 435.166392353636240 i	5,23,15	12.333692101505729 + 34.690455006605113 i	129.401003974128429 + 325.738014342077634 i
5,23,17	7.989162099564954 + 36.720449440126195 i	63.62566159582632 + 197.11369902958933 i	5,23,19	2.751598118424244 + 38.536236791418460 i	27.68802479230981 + 50.59038285095635 i
5,24,1	35.077000127901208 + 14.214755591192732 i	1196.867207394215711 + 560.205933467855971 i	5,24,3	32.752873046639699 + 18.152299705941790 i	1018.891068038838625 + 646.818456667557939 i
5,24,5	30.247751717787974 + 21.638511702727584 i	847.295037639972861 + 683.870210584424967 i	5,24,7	27.551562364391639 + 24.810937427066822 i	684.527161260421735 + 683.328424572717608 i
5,24,9	24.650025169041058 + 27.729253564295401 i	533.141182310680733 + 651.272079361009219 i	5,24,11	21.521333863155565 + 30.425948646331356 i	395.653716043809197 + 591.705624218742969 i
5,24,13	18.131902920042371 + 32.919803807424643 i	274.640332664116329 + 507.644356308953490 i	5,24,15	14.427624027798448 + 35.220740953101995 i	172.89718917048759 + 401.55588776599041 i
5,24,17	10.311876266770899 + 37.331234419923732 i	93.715089790075695 + 275.559861944794704 i	5,24,19	5.568177523573321 + 39.244620292485898 i	41.507944553456348 + 131.393916253627111 i
5,25,1	36.680924490114634 + 14.342007421620504 i	1309.875423218156584 + 595.349670353384681 i	5,25,3	34.381756005197516 + 18.324625709516706 i	1124.768738616515748 + 691.166828534840192 i
5,25,5	31.908966423250721 + 21.857854161328117 i	945.518002236099960 + 735.965904737983237 i	5,25,7	29.254032747038629 + 25.079649204299454 i	774.535420833393573 + 742.130631302437905 i
5,25,9	26.404985721876759 + 28.050161981941208 i	614.323540801070067 + 715.912880580217802 i	5,25,11	23.343683497466209 + 30.802502998238801 i	467.320874747113351 + 661.403794907165548 i
5,25,13	20.042724077587586 + 33.356311534876582 i	335.982883820666473 + 581.661860229207946 i	5,25,15	16.459520724260390 + 35.722773527421523 i	222.913310534737661 + 479.172912716381916 i
5,25,17	12.522830763749665 + 37.906438221675629 i	131.065215407318931 + 356.070353289360639 i	5,25,19	8.093914778353785 + 39.904851632203496 i	64.132119490673560 + 214.204744803282011 i
5,25,21	2.775638858302242 + 41.703552579683070 i	27.709995666169380 + 54.635881953183727 i	5,26,1	38.282607543138305 + 14.464251466238057 i	1427.908241731068301 + 630.891958738779408 i
5,26,3	36.006587167283008 + 18.490044528573407 i	1235.742279150873628 + 736.058270656278296 i	5,26,5	33.563635003487271 + 22.068207819086702 i	1048.929770520560623 + 788.758426232107240 i
5,26,7	30.946518052849278 + 25.337053478000414 i	869.852486968376411 + 801.794999362860040 i	5,26,9	28.145149291401289 + 28.357133556768300 i	700.969903844742589 + 781.595411563589209 i
5,26,11	25.144295424750073 + 31.162074846862545 i	544.655366888691333 + 732.340679338576138 i	5,26,13	21.921280472352947 + 33.772212297914301 i	403.266012177235041 + 657.136079986038681 i
5,26,15	18.441830413636794 + 36.199720128521015 i	279.253270602081955 + 558.488566805157234 i	5,26,17	14.651331117281513 + 38.450673594348181 i	175.318117721378513 + 438.540322496828651 i
5,26,19	10.452845157517734 + 40.525351102879796 i	94.677453328660221 + 299.177434168038928 i	5,26,21	5.629793628268529 + 42.416170278586229 i	41.685562899736688 + 141.938661730301618 i
5,27,1	39.882278708098767 + 14.581866890406220 i	1550.959733806474333 + 666.817031673941187 i	5,27,3	37.627796803500971 + 18.649090037786969 i	1351.800620411375803 + 781.470679031431473 i
5,27,5	35.212484946337568 + 22.270286280185150 i	1157.512063676639400 + 842.218424894166446 i	5,27,7	32.630192735405025 + 25.584078349338249 i	970.449897249277797 + 862.283728928947653 i
5,27,9	29.872383996145700 + 28.651359998433151 i	793.037154656850703 + 848.271856277348320 i	5,27,11	26.926169668480000 + 31.506191292712685 i	627.59230732573269 + 804.45644549220526 i
5,27,13	23.772564565872482 + 34.169479573442011 i	476.390893933887241 + 733.992767988366950 i	5,27,15	20.383490753414627 + 36.654188820787077 i	341.761239826362669 + 639.412058173719970 i
5,27,17	16.715849348383017 + 38.967558161721676 i	226.209450314057250 + 522.865394830020023 i	5,27,19	12.698046049360430 + 41.111780370248681 i	132.61291392769712 + 386.25081864346311 i
5,27,21	8.191167274362360 + 43.083171605389836 i	64.604977416237446 + 231.231885869101944 i	5,27,23	2.798061957604203 + 44.867226423532557 i	27.730725281290297 + 58.664408425746217 i
5,28,1	41.480136357240632 + 14.695191201923823 i	1679.024567383822159 + 703.110330054394353 i	5,28,3	39.245754487026936 + 18.802236051674164 i	1472.933861843399882 + 827.383698372158733 i
5,28,5	36.856135681256482 + 22.464720299016697 i	1271.248642280708509 + 896.318965640589568 i	5,28,7	34.306045247603350 + 25.821539801786766 i	1076.302582777491125 + 923.562275294060214 i
5,28,9	31.588237629941148 + 28.933881798644077 i	890.487743790699938 + 915.898728034816941 i	5,28,11	28.691738255105783 + 31.836173437516045 i	716.076033718518147 + 877.696964163203461 i
5,28,13	25.600496904889987 + 34.549798981926038 i	555.274503525866066 + 812.165062136546908 i	5,28,15	22.291176942124544 + 37.088364852673443 i	410.310293287981096 + 721.861578477508942 i
5,28,17	18.728993217718720 + 39.460022750518178 i	283.534804771845756 + 608.948408355679135 i	5,28,19	14.858960734171463 + 41.668376348860065 i	177.571475619362502 + 475.328445595359653 i
5,28,21	10.583917144155553 + 43.712275089482468 i	95.575956784669174 + 322.704612214190433 i	5,28,23	5.687224399316383 + 45.583617788604066 i	41.852194258208755 + 152.449676869191535 i
5,29,1	43.076352985634230 + 14.804526169661694 i	1812.097927517222843 + 739.758367810078767 i	5,29,3	40.860779669854487 + 18.949905087937950 i	1599.133106665138939 + 873.778519957873868 i
5,29,5	38.495118491402465 + 22.652070306437068 i	1390.12500046450941 + 951.03523732614087 i	5,29,7	35.974914763460575 + 26.050159428684214 i	1187.388321234785053 + 985.598937723290192 i
5,29,9	33.294006484076505 + 29.205613506059315 i	993.28871223917270 + 984.43629603888246 i	5,29,11	30.443000644047395 + 32.153173298465013 i	810.058316629940691 + 952.013022812417830 i
5,29,13	27.408217186288727 + 34.914624761192748 i	639.846150591271225 + 891.592463783521339 i	5,29,15	24.170026980303682 + 37.504104699492459 i	484.795029260712795 + 805.763278959267041 i
5,29,17	20.699846501010683 + 39.930496164050233 i	347.130941635807574 + 696.700659468145659 i	5,29,19	16.954196735565681 + 42.198481308916811 i	229.280576170269882 + 566.311153819448236 i
5,29,21	12.861226983495683 + 44.308687060453863 i	134.058752490137522 + 416.309891986640821 i	5,29,23	8.281904957869079 + 46.256644009461922 i	65.048149133916837 + 248.206893328466464 i
5,29,25	2.819059133112371 + 48.027985441617702 i	27.750313406910482 + 62.678443144223586 i	5,30,1	44.671079368734908 + 14.910142721245366 i	1950.175449657061108 + 776.748616596296870 i
5,30,3	42.473150083784390 + 19.092475574545333 i	1730.390325817944813 + 920.637709823253317 i	5,30,5	40.129892082069340 + 22.832836623782367 i	1514.128117127118230 +1006.344307165943292 i
5,30,7	37.637519292934705 + 26.270578725933653 i	1303.687299850826548 +1048.364515559149140 i	5,30,9	34.990786571769426 + 29.467363800329441 i	1101.410928554613974 +1053.848109465510333 i
5,30,11	32.181620666574972 + 32.458202525811259 i	909.497003544655627 +1027.359675248176472 i	5,30,13	29.198281146571301 + 35.265222375489124 i	730.044964889533844 + 972.219977225040338 i
5,30,15	26.024091185333898 + 37.903003538709527 i	565.126538293415251 + 891.050268284484332 i	5,30,17	22.635216864784805 + 40.381025686197894 i	416.864312594851299 + 786.041576479604146 i
5,30,19	18.996489999950967 + 42.704818540980867 i	287.52842412690018 + 659.10515004037268 i	5,30,21	15.052650379807341 + 44.876321232200333 i	179.678316668712966 + 511.959274140895673 i
5,30,23	10.706374591042930 + 46.893426519891904 i	96.418199903972834 + 346.158145390383936 i	5,30,25	5.740989563226994 + 48.747761033927794 i	42.009004945971462 + 162.932493167364782 i

Table D.6: Branch points and associated eigenvalues for $m = 5$.

Table D.7: Branch points and associated eigenvalues for $m = 6$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
6, 6 ,1	2.535162563188484 + 10.371846133322299 i	38.725741591981532 + 12.519713968726423 i	6, 7 ,1	4.814351784202907 + 11.048040497610387 i	51.238119459512944 + 33.401921863215009 i
6, 8 ,1	6.842857643849872 + 11.598928156948693 i	70.562429490330999 + 56.566059600125165 i	6, 8 ,3	2.53254384284674 + 13.98324538916126 i	38.59960535020176 + 17.86493400321450 i
6, 9 ,1	8.748348740515070 + 12.068449725370911 i	96.026374365887108 + 81.583844633545141 i	6, 9 ,3	4.920200245650711 + 14.652875131722062 i	51.103341901899860 + 46.824045288693426 i
6,10,1	10.579768499604155 + 12.479522441244230 i	127.302070122522721 + 108.162099203373160 i	6,10,3	7.045253850769504 + 15.224376365290144 i	70.849254828932573 + 77.972396161371961 i
6,10,5	2.561476747754567 + 17.386340016952712 i	38.578754638637072 + 22.619435827314398 i	6,11,1	12.361654824580731 + 12.846058122931113 i	164.194087287467937 + 136.091790739400693 i
6,11,3	9.034233296839718 + 15.726233428464255 i	97.087199969174293 + 111.037504623692570 i	6,11,5	5.033785039934057 + 18.060555960016046 i	51.246466533293543 + 58.961654523911825 i
6,12,1	14.108173000302715 + 13.177291347420091 i	206.572759352136075 + 165.216107130872302 i	6,12,3	10.938158267652057 + 16.175116151539964 i	129.425711945137408 + 145.779617828390769 i
6,12,5	7.236215220685779 + 18.651103924599219 i	71.470102735778710 + 97.452880144535641 i	6,12,7	2.595632463958042 + 20.699277848062358 i	38.586316912285099 + 27.116265802738834 i
6,13,1	15.828263891222756 + 13.479731711740065 i	254.345993297150301 + 195.412653719394797 i	6,13,3	12.783593225548833 + 16.581957523095308 i	167.622129774258042 + 182.010414563816539 i
6,13,5	9.294322116480751 + 19.179404665232475 i	98.461220421892833 + 137.917745290155949 i	6,13,7	5.139508658420381 + 21.380199655875330 i	51.454365371162922 + 70.521311280293403 i
6,14,1	17.527933435802673 + 13.758172248491940 i	307.445207118587462 + 226.582952601734434 i	6,14,3	14.586331658662379 + 16.954431737531053 i	211.511212164439485 + 219.580492009705921 i
6,14,5	11.259981446063783 + 19.658706422485935 i	131.790282488025241 + 180.154354430809718 i	6,14,7	7.408052122344035 + 21.987028451595254 i	72.138215509486002 + 116.052296123929125 i
6,14,9	2.629429016569553 + 23.963153733618533 i	38.603790136645081 + 31.468904766342110 i	6,15,1	19.211406917342210 + 14.016261159617242 i	365.817505766255294 + 258.645865902034302 i
6,15,3	16.356582412567363 + 17.298183796296623 i	260.973375663282013 + 258.368769672799544 i	6,15,5	13.160802887041227 + 20.098086913731532 i	171.183160694821424 + 223.991179413727679 i
6,15,7	9.526281314250351 + 22.537047994150562 i	99.823558453226894 + 163.605772640457531 i	6,15,9	5.235771451233555 + 24.650828120798430 i	51.675078319150217 + 81.754899771826700 i
6,16,1	20.881765304806123 + 14.256849004622211 i	429.420974439352278 + 291.533261783638579 i	6,16,3	18.101340699146562 + 17.617512920650022 i	315.918269908618129 + 298.275113434383343 i
6,16,5	15.013515798756883 + 20.504142750656278 i	216.449195889465869 + 269.286117001592800 i	6,16,7	11.546551095704878 + 23.041286495606421 i	134.053815012550302 + 213.009591372637175 i
6,16,9	7.562436363789036 + 25.271465175633860 i	72.788154355853436 + 134.149459736801248 i	6,16,11	2.661407239337117 + 27.196892089845683 i	38.624856727110711 + 35.729586250430692 i
6,17,1	22.541320496508764 + 14.482211000327952 i	498.22167539566385 + 325.18703630362586 i	6,17,3	19.825611872848818 + 17.915783069810843 i	376.275394050194507 + 339.215254661315271 i
6,17,5	16.829112870757566 + 20.881864337039485 i	267.448156379218744 + 315.920609898404791 i	6,17,7	13.497113441370727 + 23.507515166089455 i	174.531320426937384 + 264.106611212061864 i
6,17,9	9.734160012538613 + 25.839592960634313 i	101.108872550405039 + 188.607782786995074 i	6,17,11	5.323313530626927 + 27.890820157842910 i	51.892035157756844 + 92.781091149129807 i
6,18,1	24.191848583200048 + 14.694195494779958 i	572.191641272141737 + 359.556994948658314 i	6,18,3	21.533099871349339 + 18.195684165691855 i	441.988336133171401 + 381.117221747298743 i
6,18,5	18.615235058927798 + 21.235141660568210 i	324.072711776826736 + 363.794493271531508 i	6,18,7	15.395308459704998 + 23.941517130364119 i	221.045618498906151 + 316.761799711854508 i
6,18,9	11.803606624016000 + 26.364663351736890 i	136.159825732921519 + 244.982613180484122 i	6,18,11	7.701975650187774 + 28.523239691301516 i	73.402493124642433 + 151.925204682477045 i
6,18,13	2.691252829534178 + 30.410627085729942 i	38.646967485438552 + 39.926540143472536 i	6,19,1	25.834741154777721 + 14.894326614930094 i	651.30748352444527 + 394.59930155681928 i
6,19,3	23.226620407536380 + 18.459405951887657 i	513.011037775164823 + 423.918771615164928 i	6,19,5	20.377419289756229 + 21.567078744987473 i	386.238213107131457 + 412.822059511258374 i
6,19,7	17.252664153796914 + 24.347756515989115 i	273.439877699064141 + 370.859761546838968 i	6,19,9	13.799458464134274 + 26.853466549052321 i	177.624703812236817 + 303.129553761424575 i
6,19,11	9.921989224420035 + 29.106694986549137 i	102.305954961735324 + 213.167741981632872 i	6,19,13	5.403247005657791 + 31.110195736615324 i	52.099594214067480 + 103.664816035625051 i
6,20,1	27.471106953232759 + 15.083877303808110 i	735.549397467897734 + 430.275314624016801 i	6,20,3	24.908361856550258 + 18.708757966693817 i	589.305265247004627 + 467.565496383242987 i
6,20,5	22.119809263070213 + 21.880199788403154 i	453.876312814132234 + 462.929117916296491 i	6,20,7	19.077278764470453 + 24.729773677930094 i	331.592766711136562 + 426.301270086720194 i
6,20,9	15.739480893269485 + 27.311143931140379 i	225.276949741692448 + 362.920370123933480 i	6,20,11	12.036235362744732 + 29.649453604381062 i	138.108216790037375 + 276.383131399446199 i
6,20,13	7.829009385535238 + 31.752762050552175 i	73.977960451810475 + 169.479471547272993 i	6,20,15	2.719013736239652 + 33.610340121271406 i	38.669002801145588 + 44.076679757495270 i
6,21,1	29.101842212005693 + 15.263922571448431 i	824.900432418133619 + 466.550695857208382 i	6,21,3	26.580056852922230 + 18.945254839176791 i	670.838836333882341 + 512.009397154591511 i
6,21,5	23.845587065465295 + 22.176589948218087 i	526.930769773429347 + 514.050786048278951 i	6,21,7	20.875077083087387 + 25.090436191673447 i	395.407726687509523 + 483.000244226410928 i
6,21,9	17.635586936279704 + 27.741725224249453 i	278.945926781610638 + 424.243164167066482 i	6,21,11	14.073700729356714 + 30.157538518648394 i	180.477673107911215 + 341.438508006152858 i
6,21,13	10.093076012244596 + 32.349366689540219 i	103.418252809795376 + 237.421587458078363 i	6,21,15	5.476623483326130 + 34.314955793559022 i	52.296092331748525 + 114.445198337328520 i
6,22,1	30.727680568834252 + 15.435379141392369 i	919.345944494216269 + 503.394714880040056 i	6,22,3	28.243099031813998 + 19.170178472501497 i	757.58434054259294 + 557.20778875332689 i
6,22,5	25.557248634376208 + 22.457994708247266 i	605.354575500413716 + 566.129809018476521 i	6,22,7	22.650531898510742 + 25.432106686363102 i	464.806187650382128 + 540.881332492824072 i
6,22,9	19.496212816147313 + 28.148449421437434 i	338.498439766632202 + 487.000246827696969 i	6,22,11	16.052459642059915 + 30.635570468145328 i	229.174311378362120 + 408.212150507558988 i
6,22,13	12.248488554152615 + 32.907352074427976 i	139.912317015443108 + 307.384990520678343 i	6,22,15	7.945459615895838 + 34.966343356245645 i	74.515949235643802 + 186.872940151001330 i
6,22,17	2.744851040847955 + 36.799813543772167 i	38.690457218769168 + 48.190904008214176 i	6,23,1	32.349229263011885 + 15.599035510357176 i	1018.873178587999064 + 540.779698354556558 i
6,23,3	29.898624736483928 + 19.384624359593445 i	849.518192554509483 + 603.122444117596615 i	6,23,5	27.256786751643357 + 22.725891943113201 i	689.107918709592241 + 619.115261457565907 i
6,23,7	24.407105497743295 + 25.756759563076212 i	539.723058363067253 + 599.878019261059421 i	6,23,9	21.327576717088515 + 28.533971288103427 i	403.827677135823990 + 551.105863818276248 i
6,23,11	17.984711873672730 + 31.087214553279157 i	284.015762450143370 + 476.595819849349084 i	6,23,13	14.324457549344046 + 33.432123598848719 i	183.115698569423785 + 379.247403702664258 i
6,23,15	10.250046507633201 + 35.574361171726686 i	104.453170031421706 + 261.452424691132705 i	6,23,17	5.544346023391250 + 37.508941441537075 i	52.481433165986076 + 125.147481953199133 i
6,24,1	33.966995889635172 + 15.755575106016357 i	1123.470943740534722 + 578.680587461622508 i	6,24,3	31.547571607599814 + 19.589536725740679 i	946.619917389272700 + 649.718915711147361 i
6,24,5	28.945816688372616 + 22.981545390812418 i	778.156703193266594 + 672.961530293462374 i	6,24,7	26.147533554530650 + 26.066064739220810 i	620.103606678697929 + 659.931139200697544 i
6,24,9	23.134405386879031 + 28.900501309436375 i	474.846149360345919 + 616.484224760322263 i	6,24,11	19.879150877538510 + 31.515448470218811 i	344.858599863333779 + 546.493740386106197 i
6,24,13	16.339288921739097 + 33.927870931445050 i	232.776922980642041 + 452.893336604498359 i	6,24,15	12.443549506201400 + 36.145607947560521 i	141.587772851956714 + 338.095157513676554 i
6,24,17	8.052877098181318 + 38.168066373072421 i	75.019336870888068 + 204.144987637353353 i	6,24,19	2.768951434123557 + 39.981566443024093 i	38.711111014701615 + 52.276630947493750 i
6,25,1	35.581408508422548 + 15.905594381478595 i	1233.129357400666322 + 617.074578056217888 i	6,25,3	33.190721484376446 + 19.785735629260738 i	1048.871600728825797 + 696.965990035288996 i
6,25,5	30.625664898773003 + 23.226045114324140 i	872.471443072578040 + 727.627506587521793 i	6,25,7	27.874015067615940 + 26.361449292912052 i	705.901231165386093 + 720.987703061817570 i
6,25,9	24.920381154452752 + 29.249904282957790 i	551.480952576334062 + 683.067885687912622 i	6,25,11	21.742222388684787 + 31.922736492215442 i	411.587057220974430 + 617.820956297468797 i
6,25,13	18.305401675125704 + 34.397949171165074 i	288.703493699930902 + 528.218267552864745 i	6,25,15	14.555348790678776 + 36.684980984390656 i	185.564319547516561 + 416.688683713675402 i

Table D.7: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
6,25,17	10.394986028037096 + 38.786091159708356 i	105.418702081441467 + 285.314495645045383 i	6,25,19	5.607168835892554 + 40.694737453905766 i	52.656142971886560 + 135.788816850004338 i
6,26,1	37.192830984516320 + 16.049617132795788 i	1347.839641363269948 + 655.940824845929683 i	6,26,3	34.828732398366633 + 19.973938161609706 i	1156.257459527267429 + 744.835243832489482 i
6,26,5	32.297433127777602 + 23.460338642306095 i	972.026422677114624 + 783.075935315232755 i	6,26,7	29.588343658758394 + 26.644143825478661 i	797.075826284114555 + 782.999960287136673 i
6,26,9	26.688430888539344 + 29.583770926259913 i	633.670466656028111 + 750.796440671633718 i	6,26,11	23.578853409298475 + 32.311148443446662 i	484.105768036926008 + 690.501744058845588 i
6,26,13	20.231689469429476 + 34.845108173903633 i	350.743046849877601 + 605.128568486319864 i	6,26,15	16.603928225194419 + 37.196307313986281 i	236.121429703157276 + 497.123471543945698 i
6,26,17	12.623936431463605 + 39.369003265890676 i	143.149363356366877 + 368.583842068141337 i	6,26,19	8.152514431586264 + 41.360704256948587 i	75.491319462835691 + 221.322478335402735 i
6,26,21	2.791495643621713 + 43.157346171394323 i	38.730884121125982 + 56.339130295587537 i	6,27,1	38.801574848881103 + 16.188105956767309 i	1467.593957078705671 + 695.260195812245570 i
6,27,3	36.462162821198646 + 20.154775243993665 i	1268.76350189206278 + 793.30067911084757 i	6,27,5	33.962045703903975 + 23.685255295466156 i	1076.79904647807848 + 839.27288616812348 i
6,27,7	31.292000853199820 + 26.915217976965753 i	893.592555631718938 + 845.924642818307802 i	6,27,9	28.440920333163209 + 29.903471190995816 i	721.361980018273925 + 819.615464045012004 i
6,27,11	25.392902336887502 + 32.682444297441556 i	562.334861954271787 + 764.468238771586016 i	6,27,13	22.124777978951471 + 35.271641970582792 i	418.771910727966031 + 683.540403737524116 i
6,27,15	18.601877968402999 + 37.682675661844996 i	293.057759133532670 + 579.298711706937297 i	6,27,17	14.769240165378950 + 39.921274683920764 i	187.846365876463295 + 453.849754980342141 i
6,27,19	10.529565283065262 + 41.987580680235489 i	106.322342990905042 + 309.044984458473891 i	6,27,21	5.665717938985815 + 43.874152782647691 i	52.820972875535041 + 146.381335659716541 i
6,28,1	40.407908578692549 + 16.321471514372700 i	1592.385271369768816 + 735.015066534631273 i	6,28,3	38.091490307252919 + 20.328805087666620 i	1386.377254331770018 + 842.338420061058741 i
6,28,5	35.620285055571259 + 23.901525441648541 i	1186.769328378856471 + 896.187318317151380 i	6,28,7	32.986223913322881 + 27.175608069006472 i	995.420913396852939 + 909.722348797961558 i
6,28,9	30.179789124201228 + 30.210194786165207 i	814.509936064509702 + 889.475650969161734 i	6,28,11	27.187451037166127 + 33.038136185260683 i	646.206524165442602 + 839.659286122252752 i
6,28,13	23.989754808464358 + 35.679491554329807 i	492.687699974196050 + 763.378477169641997 i	6,28,15	20.558250353598140 + 38.146636095039668 i	356.213057859168487 + 663.123065037839751 i
6,28,17	16.849520576880803 + 40.446423379661283 i	239.239669410854901 + 541.008402506889070 i	6,28,19	12.791665094768208 + 42.580842859807127 i	144.610173138245500 + 398.899410881046322 i
6,28,21	8.245390914722668 + 44.546213951874407 i	75.934975363968036 + 238.424473561540935 i	6,28,23	2.812647858235964 + 46.328405239149603 i	38.749766381675443 + 60.382277180993967 i
6,29,1	42.012064938387340 + 16.450080089881158 i	1722.20724592410988 + 775.18914654985201 i	6,29,3	39.717126012282343 + 20.496524091979964 i	1509.087540599593922 + 891.926459148170238 i
6,29,5	37.272818806120618 + 24.109795916883900 i	1301.919485068318863 + 953.790719025395674 i	6,29,7	34.672056169361831 + 27.426138922593527 i	1102.533994152401647 + 974.357035390765986 i
6,29,9	31.906647180112746 + 30.504982526221337 i	913.074609790361137 + 960.332114133169057 i	6,29,11	28.965002026443443 + 33.379534970492514 i	735.662506006351350 + 916.019492743871751 i
6,29,13	25.830621349975555 + 36.070318455145113 i	572.404285127092066 + 844.574875144606608 i	6,29,15	22.479815621055440 + 38.590330392887793 i	425.456615955202324 + 748.514035984901398 i
6,29,17	18.877503779528698 + 40.947307088637245 i	297.120124828814824 + 629.962643581643192 i	6,29,19	14.968430008210623 + 43.144601701934647 i	189.981519326083287 + 490.791123713921252 i
6,29,21	10.655138023807988 + 45.180980721148998 i	107.170775579764069 + 332.670380225041356 i	6,29,23	5.720514031868344 + 47.048494226435871 i	52.976725137283232 + 156.933907551391314 i
6,30,1	43.614246844455437 + 16.574259811623229 i	1857.054145584781736 + 815.767331684112946 i	6,30,3	41.339426131952031 + 20.658375750455743 i	1636.884300582418746 + 942.044442693484484 i
6,30,5	38.920220735248421 + 24.310642506765362 i	1422.233608343989999 +1012.056800960394980 i	6,30,7	36.350385024372365 + 27.667541292363278 i	1214.907916311688950 +1039.795597604934528 i
6,30,9	33.622845127024149 + 30.788750953081806 i	1017.021090049448844 +1032.143803667819157 i	6,30,11	30.727615988099227 + 33.707785947873369 i	830.652277256383172 + 993.498441152802684 i
6,30,13	27.650586782294056 + 36.445559038989977 i	657.848244573206102 + 927.068063648811176 i	6,30,15	24.371791995020114 + 39.015582071395961 i	500.679748650414069 + 835.397015450161234 i
6,30,17	20.862369784020469 + 41.426299716100409 i	361.320466127492807 + 720.623506954422737 i	6,30,19	17.078594570079581 + 43.682108859319186 i	242.158699290639788 + 584.621747114769278 i
6,30,21	12.948369645359143 + 45.783495229352631 i	145.981521698710480 + 429.076466292880127 i	6,30,23	8.332343820026825 + 47.726019696543489 i	76.353113714363715 + 255.464934364966041 i
6,30,25	2.832553333717560 + 49.495666171357939 i	38.767782512314028 + 64.409003657853759 i	6,31,1	45.214632093495794 + 16.694305811224925 i	1996.920761654832404 + 856.735577632460718 i
6,31,3	42.958701008772792 + 20.814757990969326 i	1769.758440644921620 + 992.673488479919115 i	6,31,5	40.562987199502892 + 24.504580145243779 i	1547.697398230377757 +1070.961246650531848 i
6,31,7	38.021971098913333 + 27.900465952596708 i	1332.521360947629546 +1106.007515580575728 i	6,31,9	35.329526788376711 + 31.062311934347886 i	1126.318483971916521 +1104.873024865074967 i
6,31,11	32.477010316477021 + 34.023896678359030 i	931.13162489821707 +1072.05003838184507 i	6,31,13	29.452267120275454 + 36.806465571946639 i	748.956284831044172 +1010.802038502822370 i
6,31,15	26.238298454741123 + 39.423961123705872 i	581.790668389860457 + 923.704317591959693 i	6,31,17	22.811006545397614 + 41.885406607396767 i	431.703672921790599 + 812.909986391298389 i
6,31,19	19.134994485878657 + 44.196020141001661 i	300.925622859656698 + 680.297740086270778 i	6,31,21	15.154785944298508 + 46.357558163190056 i	191.986555555833405 + 527.556263776022092 i
6,31,23	10.772813869328715 + 48.367867974574679 i	107.969842906636842 + 356.210147943565516 i	6,31,25	5.771992574225371 + 50.218730940847038 i	53.124177465191586 + 167.453194575099388 i

Table D.7: Branch points and associated eigenvalues for $m = 6$.

Table D.8: Branch points and associated eigenvalues for $m = 7$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
7, 7 ,1	2.61342222249895 + 11.54432160397778 i	51.86213180189183 + 13.54863356598100 i	7, 8 ,1	4.955171675873506 + 12.257791889607645 i	64.916029639592466 + 36.168826658909524 i
7, 9 ,1	7.031051235836070 + 12.843978325333754 i	84.978667892622141 + 61.268570332292214 i	7, 9 ,3	2.588831226419597 + 15.228034137020870 i	51.690068949154302 + 19.086236952703587 i
7,10,1	8.975028245092711 + 13.346717689237883 i	111.318457163208308 + 88.389730672011851 i	7,10,3	5.028323941470938 + 15.922287797961539 i	64.589779236720830 + 50.029685252746859 i
7,11,1	10.838861872555936 + 13.789087094061928 i	143.576665344806145 + 117.217215435410594 i	7,11,3	7.194715554183459 + 16.518800967923493 i	84.911570940754729 + 83.294173085964687 i
7,11,5	2.605722461463749 + 18.678843525392036 i	51.646864066264996 + 23.958058336736073 i	7,12,1	12.648680693960268 + 14.185184437439782 i	181.538507263889215 + 147.525842404746300 i
7,12,3	9.218419659702034 + 17.045272843293130 i	111.857831652096678 + 118.599506905914595 i	7,12,5	5.122108968849164 + 19.370999266435515 i	64.62896873067876 + 62.45148901050058 i
7,13,1	14.419663523021104 + 14.544416552765862 i	225.06087636874503 + 179.14638942167846 i	7,13,3	11.152321088053425 + 17.518101447293905 i	145.010202358259278 + 155.695433023207499 i
7,13,5	7.360886749201269 + 19.980533636575345 i	85.327543230152415 + 103.188267046435143 i	7,13,7	2.632057534645972 + 22.026479522050813 i	51.640771229578554 + 28.536596004259497 i
7,14,1	16.161447045754585 + 14.873444900621214 i	274.041730020835985 + 211.946573263676953 i	7,14,3	13.024097631918728 + 17.948122811655832 i	184.108356809970019 + 194.384377596045738 i
7,14,5	9.450255280897613 + 20.528068715788581 i	112.916617494169358 + 145.994861711629511 i	7,14,7	5.214224018086674 + 22.721202854088077 i	64.769523815075843 + 74.215861018381005 i
7,15,1	17.880539957041947 + 15.177194825851407 i	328.404788668261574 + 245.819807116218328 i	7,15,3	14.850283092788912 + 18.342987405915721 i	228.974441434075345 + 234.509122792605439 i
7,15,5	11.443399478020574 + 21.026499089470875 i	146.944077960540255 + 190.664776666601995 i	7,15,7	7.515103687575493 + 23.343058332838137 i	85.857755533494029 + 122.091304700396236 i
7,15,9	2.660322934950908 + 25.316990033468368 i	51.648955765279787 + 32.950125247952016 i	7,16,1	19.581544624288185 + 15.459429726048066 i	388.091007740702992 + 280.678151074968525 i
7,16,3	16.641612206517383 + 18.708350137092925 i	279.479361509539387 + 275.941973319035696 i	7,16,5	13.368736201307497 + 21.484724977552133 i	187.120381168044844 + 237.021196526590842 i
7,16,7	9.661638386435916 + 23.908625315985780 i	114.060963243150908 + 172.066079653147455 i	7,16,9	5.300480528174738 + 26.015673266142933 i	64.942581733360626 + 85.605645532322882 i
7,17,1	21.267833042869597 + 15.723100766569820 i	453.05343833669872 + 316.44766310484846 i	7,17,3	18.405471001086063 + 19.048536006984751 i	335.525309677666257 + 318.577135163267542 i
7,17,5	15.243547263319897 + 21.909250455026338 i	233.243564581464511 + 284.917033007689099 i	7,17,7	11.707146764511696 + 24.428585143110304 i	148.903602773529855 + 223.966793126332846 i
7,17,9	7.656222426134441 + 26.648554712163907 i	86.407500114377854 + 140.423292566028920 i	7,17,11	2.688174281459295 + 28.571923847236476 i	51.663282374573313 + 37.258166501107645 i
7,18,1	22.941946865678862 + 15.970571418183432 i	523.25394207045792 + 353.06520258581196 i	7,18,3	20.147165020200060 + 19.366941958724251 i	397.035766484197325 + 362.325438424294759 i
7,18,5	17.079227718781056 + 22.305018799462431 i	285.164705779084500 + 334.229228097436533 i	7,18,7	13.680486724119543 + 24.910514844054230 i	190.074655521707768 + 277.633468724526210 i
7,18,9	9.853750319762879 + 27.229560413132173 i	115.184803530374069 + 197.367437727418974 i	7,18,11	5.380334243841899 + 29.274867497406742 i	65.123924191385498 + 96.755585088768839 i
7,19,1	24.605846943739333 + 16.203767818597974 i	598.660991917663296 + 390.476155445058112 i	7,19,3	21.870634702741981 + 19.666293638323925 i	463.949351879312587 + 407.110619292615468 i
7,19,5	18.883724075967120 + 22.675898682805570 i	342.769559984616535 + 384.853593785128623 i	7,19,7	15.599421444553153 + 25.360082257952509 i	237.353648402205522 + 332.927940975457375 i
7,19,9	11.946464637341846 + 27.767826067204936 i	150.781020459482420 + 256.291608742418703 i	7,19,11	7.785384202598395 + 29.917485337441462 i	86.945452208054604 + 158.387054197299250 i
7,19,13	2.714822813500669 + 31.802976011832175 i	51.680283048535181 + 41.493084314229023 i	7,20,1	26.261075744515690 + 16.424282986862181 i	679.248144285950161 + 428.632767790879143 i
7,20,3	23.578885884418593 + 19.948816892741309 i	536.215832458761497 + 452.866638116156992 i	7,20,5	20.662812430603253 + 23.024988040342752 i	405.967832617354702 + 436.700824893292577 i
7,20,7	17.475792841583527 + 25.781681270577774 i	290.575913586952538 + 389.731755009362189 i	7,20,9	13.963515912798703 + 28.269953613829291 i	192.868246390780826 + 317.050160959139475 i
7,20,11	10.029075639632856 + 30.511809649750909 i	116.256937914221425 + 222.169010781585513 i	7,20,13	5.454170170417612 + 32.510079233128742 i	65.303839378514823 + 107.740047794355021 i
7,21,1	27.908866833716022 + 16.633451156172065 i	764.992944442671160 + 467.492895138920744 i	7,21,3	25.274262321421293 + 20.216356490257187 i	613.793414403069050 + 499.535699273456203 i
7,21,5	22.420828074397782 + 23.354814142837913 i	474.68647170135176 + 489.69350113969949 i	7,21,7	19.317941583413287 + 26.178809367432951 i	349.613799605392217 + 447.942811316928271 i
7,21,9	15.923001080090984 + 28.740965214944698 i	241.211058961815695 + 379.512986487437217 i	7,21,11	12.164868706170653 + 31.065826878262616 i	152.550436916692632 + 287.977937402749149 i
7,21,13	7.904068630592862 + 33.161287627887404 i	87.460679194697576 + 176.095705364038906 i	7,21,15	2.740036001404554 + 35.017128317374969 i	51.698317174980289 + 45.674355633955621 i
7,22,1	29.550220877790267 + 16.832402098060101 i	855.876122015093188 + 507.019043791591571 i	7,22,3	26.958625397709042 + 20.470460770852281 i	696.646842796664032 + 547.066757688257212 i
7,22,5	24.161109419009531 + 23.667470750649571 i	548.865251146707465 + 543.763826145875441 i	7,22,7	21.131985113347255 + 26.554308603450433 i	414.365530097444946 + 507.472369264171391 i
7,22,9	17.837082676722886 + 29.184812461588951 i	295.631780081769534 + 443.566143866577818 i	7,22,11	14.222120778852812 + 31.585385486730367 i	195.484223931958780 + 355.680783034514832 i
7,22,13	10.189990759400786 + 33.767204857870873 i	117.269511349086272 + 246.622585668970601 i	7,22,15	5.522592090705039 + 35.728143949466450 i	65.478266087888969 + 118.604156078499855 i
7,23,1	31.185959745606663 + 17.022101661544053 i	951.880986648540556 + 547.177622768721335 i	7,23,3	28.633476687770735 + 20.712443524679120 i	784.746027897882527 + 595.414370689113298 i
7,23,5	25.886281881091517 + 23.964715237585779 i	628.45374107039447 + 598.85190200460613 i	7,23,7	22.922552665935061 + 26.910527409138425 i	484.74814762981026 + 568.24262674163322 i
7,23,9	19.714392204148744 + 29.604682196547081 i	355.991465589224674 + 509.109904937757108 i	7,23,11	16.219171274044950 + 32.074991027250661 i	244.810421865127495 + 425.154833077748208 i
7,23,13	12.365444592964874 + 34.334925410281073 i	154.210316727872794 + 319.217385400892510 i	7,23,15	8.013653792228588 + 36.386958481747882 i	87.949936084366158 + 193.618429845739655 i
7,23,17	2.763797430172840 + 38.218839373467354 i	51.716559717634695 + 49.814569975045330 i	7,24,1	32.816765863800747 + 17.203382578965044 i	1052.992965412554213 + 587.938350797669386 i
7,24,3	30.300043946823966 + 20.943430196750018 i	878.065027732517024 + 644.537800272479558 i	7,24,5	27.598446840114924 + 24.248039237224482 i	713.409159288901378 + 654.904393854157433 i
7,24,7	24.693235759739864 + 27.249433524335409 i	560.692815769369531 + 630.184811054181182 i	7,24,9	21.561304956997436 + 30.003194178049856 i	422.178509788469114 + 576.056550695772899 i
7,24,11	18.168421617098272 + 32.538229756235474 i	300.340331898397494 + 496.290539034211633 i	7,24,13	14.459940144668858 + 34.869695967094373 i	197.929754108243344 + 393.758366947506715 i
7,24,15	10.338520811681240 + 37.003054916470532 i	118.222947541682345 + 270.821607063917611 i	7,24,17	5.586210318688178 + 38.933476612516053 i	65.645591923556708 + 129.377186156133263 i
7,25,1	34.443211391794790 + 17.376968250856688 i	1159.199242868407282 + 629.273779681745623 i	7,25,3	31.959342917117766 + 21.164393028787114 i	976.581278704296324 + 694.400300514137598 i
7,25,5	29.299311628597433 + 24.518721205076751 i	803.694805021180514 + 711.873481567878798 i	7,25,7	26.446878313894388 + 27.572695285692213 i	642.141565411594797 + 693.237675110510054 i
7,25,9	23.382676721160777 + 30.382535544603055 i	494.10135744338703 + 644.32842720576764 i	7,25,11	20.078724778885459 + 32.978024168470057 i	361.925302866381742 + 568.990703884203413 i
7,25,13	16.491998961136495 + 35.375598258741299 i	248.169345886097972 + 470.129674087920762 i	7,25,15	12.550741576599986 + 37.582813537515783 i	155.766983131179728 + 350.128593204539754 i
7,25,17	8.115327121878590 + 39.599059436494493 i	88.413103673222764 + 211.000392069967774 i	7,25,19	2.786175626462372 + 41.411104283110866 i	51.734583321925363 + 53.922312480077629 i
7,26,1	36.065780213357684 + 17.543491370289331 i	1270.488476960359094 + 671.158906489877040 i	7,26,3	33.612222689123179 + 21.376178220647443 i	1080.27500317312320 + 744.96854431482029 i
7,26,5	30.990281467000372 + 24.777866290707919 i	899.278890068530586 + 769.716025720286439 i	7,26,7	28.185771051371280 + 27.881741633795681 i	729.044967959159749 + 757.346305178965508 i
7,26,9	25.182297531496982 + 30.744555860400965 i	571.683608986157651 + 713.856335595891778 i	7,26,11	21.956656148090254 + 33.396799367975007 i	429.445171367543992 + 643.168932375499708 i
7,26,13	18.474205427379101 + 35.855911238645639 i	304.730893462709332 + 548.226015385809319 i	7,26,15	14.679942388763997 + 38.130988454795201 i	200.218752585000345 + 431.427976296799955 i
7,26,17	10.476342932511761 + 40.224165924879749 i	119.120657410325890 + 294.827415729130507 i	7,26,19	5.645578908194539 + 42.129071497860323 i	65.805333452515896 + 140.079115256702579 i
7,27,1	37.684884760623014 + 17.703508690456061 i	1386.850571883582916 + 713.570854552312198 i	7,27,3	35.259399605990623 + 21.579527223899785 i	1189.128746053057512 + 796.212156680423277 i
7,27,5	32.672526037133927 + 25.026437096252906 i	1000.133648987566630 + 828.392895749830359 i	7,27,7	29.911786088308659 + 28.177807361852164 i	821.360426411483218 + 822.461166883773899 i
7,27,9	26.963185338477921 + 31.090836351357115 i	654.860604002380342 + 784.578222364875160 i	7,27,11	23.807250236096635 + 33.796597217005370 i	502.800812863437613 + 718.748088611733579 i
7,27,13	20.415585163178271 + 36.313329742487120 i	367.457280204221206 + 627.952806680939261 i	7,27,15	16.744785126058179 + 38.651307053891522 i	251.310612723918126 + 514.610312769280767 i
7,27,17	12.722841298035933 + 40.814590727292241 i	157.229012437245473 + 380.789837766330038 i	7,27,19	8.210088356906549 + 42.800709648222835 i	88.851346037481392 + 228.272555663542590 i
7,27,21	2.807269484399042 + 44.596016352705931 i	51.752166934704164 + 58.003687763076670 i	7,28,1	39.300879056409428 + 17.857512866718793 i	1508.276494427220541 + 756.488608472034912 i
7,28,3	36.901483015618886 + 21.775093647451346 i	1303.127007263957694 + 848.103330632300754 i	7,28,5	34.347028683114551 + 25.265277756837733 i	1106.234649817018180 + 887.868422593891864 i
7,28,7	31.626472696777086 + 28.461967861006862 i	919.05089168820530 + 888.53733416204523 i	7,28,9	28.727783749387669 + 31.422741591153663 i	743.576960880281035 + 856.438116364096686 i
7,28,11	25.634455977703666 + 34.179157745144394 i	581.909104085773606 + 795.658943976544947 i	7,28,13	22.322872012508213 + 36.750107404164744 i	436.220801286300002 + 709.225176188641740 i
7,28,15	18.757997348712390 + 39.146786147041730 i	308.836067770274781 + 599.574425932938993 i	7,28,17	14.884539367209376 + 41.374676996744078 i	202.366234830279154 + 468.785781863437649 i
7,28,19	10.604839752953191 + 43.433849580013081 i	119.966983045394244 + 318.682262337160068 i	7,28,21	5.701182102931421 + 45.317038561644800 i	65.957541695075747 + 150.724125073601556 i
7,29,1	40.914068946165820 + 18.050594204965909 i	1634.758123947659215 + 799.892792051981701 i	7,29,3	38.538995112113867 + 21.963456832187337 i	1422.255946592920964 + 900.616508040996412 i
7,29,5	36.014623426146862 + 25.495133040419262 i	1217.560251992074200 + 948.109948089738850 i	7,29,7	33.331127046914304 + 28.735166223889784 i	1022.083879570140994 + 955.533859883163132 i
7,29,9	30.478099578155575 + 31.741458883040011 i	837.784758498290216 + 929.385262548056403 i	7,29,11	27.441432436355743 + 34.545978979912206 i	666.699384521086927 + 873.839038682474325 i
7,29,13	24.201244006289585 + 37.168155250941737 i	510.915599638665640 + 791.966767289084032 i	7,29,15	20.728741794466170 + 39.619922642670586 i	372.630536184151026 + 686.228375525737306 i
7,29,17	16.980209380866903 + 41.907851376640075 i	254.256659098919584 + 558.711836316035715 i	7,29,19	12.883447806150087 + 44.033796631466558 i	158.605177737067578 + 411.255276854847352 i
7,29,21	8.298774979969929 + 45.994123140564447 i	89.266310846298367 + 245.456987816059723 i	7,29,23	2.827184686787479 + 47.775086174638048 i	51.769201245350978 + 62.063185562824323 i

Table D.8: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
7,30,1	42.524720212253506 + 18.149187728239983 i	1766.288128678755356 + 843.765480690586060 i	7,30,3	40.172386420034854 + 22.145132862672263 i	1546.503144072762780 + 953.728112072178419 i
7,30,5	37.676023251622247 + 25.716663674548052 i	1334.091174263971425 +1009.087450513057320 i	7,30,7	35.026844029192674 + 28.998234681177845 i	1130.430705132892626 +1023.413257214374880 i
7,30,9	32.215801163791404 + 32.048028793398672 i	937.44216264524960 +1003.37341144986942 i	7,30,11	29.230748630737718 + 34.898361996013058 i	757.110893445950182 + 953.231735392150767 i
7,30,13	26.054779273977811 + 37.569112461207659 i	591.452522065328708 + 876.108611718596762 i	7,30,15	22.663879849989716 + 40.072818271055233 i	442.559604619185677 + 774.488943333064753 i
7,30,17	19.022692926708235 + 42.416904498044993 i	312.686466827295135 + 650.470795861215606 i	7,30,19	15.075702113615350 + 44.604545777184732 i	204.386401832987076 + 505.898608831917784 i
7,30,21	10.725155403758452 + 46.634472831616620 i	120.766361736270028 + 342.416364420304831 i	7,30,23	5.753437253287276 + 48.498911307861334 i	66.102517541653938 + 161.322610238315747 i
7,31,1	44.133065072383189 + 18.287601196423665 i	1902.85986289829248 + 888.09004173484482 i	7,31,3	41.802048018302877 + 22.320583582743165 i	1675.857403495038852 +1007.416321103316932 i
7,31,5	39.331842029794203 + 25.930458774656465 i	1455.810146170304279 +1070.773230693338746 i	7,31,7	36.714556441747789 + 29.251911764742911 i	1244.065877730624379 +1092.141068430004225 i
7,31,9	33.942290290436233 + 32.343369184030955 i	1042.512368720490485 +1078.360232023737581 i	7,31,11	31.004523031587617 + 35.237445522042869 i	853.090865771195354 +1033.785434037767345 i
7,31,13	27.886752898408997 + 37.954398709225892 i	677.755487309188425 + 961.588143273755577 i	7,31,15	24.568706466522109 + 40.507266160192600 i	518.511379520213723 + 864.280737422088919 i
7,31,17	21.021263327919467 + 42.904159744158100 i	377.48443510184325 + 743.97336179098136 i	7,31,19	17.200463200775896 + 45.149265244759569 i	257.027898709583098 + 602.514547096748856 i
7,31,21	13.033966151094678 + 47.242977860383199 i	159.903697547961286 + 441.563796584163640 i	7,31,23	8.382090001935827 + 49.180919586376703 i	89.659769064415016 + 262.569870050128600 i
7,31,25	2.846023544986199 + 50.949433048841563 i	51.785639202163480 + 66.104201211329340 i	7,32,1	45.739307430938176 + 18.421498926052365 i	2044.467280778453869 + 932.850997714565437 i
7,32,3	43.428321291495983 + 22.490224039223811 i	1810.308589840527702 +1061.660876286323173 i	7,32,5	40.982611716069577 + 26.137046017654793 i	1582.701623881239584 +1133.141646822651410 i
7,32,7	38.395065084009154 + 29.496856200205244 i	1362.966616608442337 +1161.685503443800826 i	7,32,9	35.658755855158098 + 32.628294378267753 i	1152.962774506628421 +1154.306822708079380 i
7,32,11	32.764523623200397 + 35.564232980241819 i	954.593086669881245 +1115.452918556553641 i	7,32,13	29.699838052086012 + 38.325253838347876 i	769.758842733774031 +1048.348356931557646 i
7,32,15	26.447406170966335 + 40.924812926138507 i	600.391262561239273 + 955.535264296551645 i	7,32,17	22.982886037175152 + 43.371583758669011 i	448.510137271276164 + 839.138051212609639 i
7,32,19	19.270664818750390 + 45.670548630980370 i	316.309332041880793 + 701.009716739890791 i	7,32,21	15.255055612240680 + 47.823334438210246 i	206.292056184813163 + 542.814624525298768 i
7,32,23	10.838243104527786 + 49.827778053371645 i	121.522990232159671 + 366.051999964260119 i	7,32,25	5.802702656728584 + 51.675833167056069 i	66.240669993668547 + 171.882394067892250 i

Table D.8: Branch points and associated eigenvalues for $m = 7$.

Table D.9: Branch points and associated eigenvalues for $m = 8$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
8, 8 ,1	2.686365197382351 + 12.703398412561532 i	66.993245738841736 + 14.532621609217433 i	8, 9 ,1	5.086727408730243 + 13.451056164498548 i	80.567627929013113 + 38.815056754753208 i
8,10,1	7.207364983300353 + 14.069648948945089 i	101.340216207095068 + 65.766223667676130 i	8,10,3	2.64212843177086 + 16.45653784926519 i	66.77798933517430 + 20.25891804137704 i
8,11,1	9.187980064644675 + 14.602962804877828 i	128.522722130864963 + 94.899342485307898 i	8,11,3	5.130653917000092 + 17.173781933241500 i	80.061708877421893 + 53.108605392055289 i
8,12,1	11.082883634074511 + 15.074225800465621 i	161.726950735166668 + 125.878337150593211 i	8,12,3	7.336340256045397 + 17.793604564945824 i	100.941636557587927 + 88.406772386951815 i
8,12,5	2.648096705083088 + 19.954127678583806 i	66.713662865848207 + 25.247894914148976 i	8,13,1	12.919643831534202 + 15.497700068098700 i	200.719644841506948 + 158.462592106215482 i
8,13,3	9.393216261062360 + 18.343029763309506 i	128.574393626162930 + 125.865574183449439 i	8,13,5	5.206502649117914 + 20.663273789146196 i	80.00290298741798 + 65.81534221173645 i
8,14,1	14.714357224494771 + 15.882941176077905 i	245.34487414969641 + 192.47104846955398 i	8,14,3	11.355888778922630 + 18.838221462332072 i	162.515730770903076 + 165.224544302604045 i
8,14,5	7.480028708021058 + 21.290717779934305 i	101.164718122245958 + 108.718250104201118 i	8,14,7	2.667244183596690 + 23.336346803622273 i	66.694575277611534 + 29.909275662412426 i
8,15,1	16.477291703973865 + 16.236740429558093 i	295.491099296849825 + 227.762008113667946 i	8,15,3	13.253059414464374 + 19.289931080719271 i	202.488332884936653 + 206.277151821628337 i
8,15,5	9.599382262318898 + 21.856380288546614 i	129.336686663341482 + 153.784545938926954 i	8,15,7	5.286142778641089 + 24.044294788123473 i	80.079544952884476 + 77.787765382732573 i
8,16,1	18.215413227218583 + 16.564135811809841 i	351.074687410506840 + 264.221175795603017 i	8,16,3	15.101941653239692 + 19.705787123836085 i	248.302224734606199 + 248.858635330321079 i
8,16,5	11.618977144633741 + 22.372844740729061 i	164.044820121289888 + 200.803120573818916 i	8,16,7	7.618088261980930 + 24.680467176999290 i	101.564253122112333 + 127.931665964691760 i
8,16,9	2.690368485439107 + 26.653760277394877 i	66.693853533572408 + 34.385349908425837 i	8,17,1	19.933668200885446 + 16.868988657998269 i	412.030712172400740 + 301.754153983821539 i
8,17,3	16.913752991476141 + 20.091451475495479 i	299.819172817715241 + 292.834928886882892 i	8,17,5	13.567983288126854 + 22.848854158911326 i	204.984662191863634 + 249.591951161007557 i
8,17,7	9.791876117121825 + 25.260816789187256 i	130.274664865816618 + 180.250181758754309 i	8,17,9	5.363132841463818 + 27.363095362977649 i	80.207069714621142 + 89.338116706807256 i
8,18,1	21.635695063644146 + 17.154335708890265 i	478.307402394404789 + 340.281490417534201 i	8,18,3	18.696238527278339 + 20.451269948076767 i	356.934204411035353 + 338.094797757278059 i
8,18,5	15.464196167000770 + 23.290829721053267 i	251.943326739023377 + 299.999072445352397 i	8,18,7	11.861743113880957 + 25.795716253830566 i	165.716753352152921 + 234.568518737919362 i
8,18,9	7.746927049672943 + 28.007741196627205 i	102.018436787741862 + 146.506296550254064 i	8,18,11	2.714346933189171 + 29.930341433868719 i	66.701671203196710 + 38.742589884619377 i
8,19,1	23.324246584057509 + 17.422615625729417 i	549.86258870411882 + 379.73527167091356 i	8,19,3	20.454980024631048 + 20.788666730632130 i	419.565014834634155 + 384.544368066240622 i
8,19,5	17.319384331426523 + 23.703671654358644 i	304.763498152322313 + 351.897041153255316 i	8,19,7	13.857119884592136 + 26.292569113203268 i	207.566514669468518 + 290.723828478216717 i
8,19,9	9.969384731774328 + 28.601082985507016 i	131.244674792519760 + 205.862832004575836 i	8,19,11	5.435804651148969 + 30.642080367505013 i	80.354166488787683 + 100.616518561690640 i
8,20,1	25.001454020659068 + 17.675820976193148 i	626.661321608940398 + 420.056698068527567 i	8,20,3	22.194135962744497 + 21.106397319556748 i	487.645445761154065 + 432.103262089361522 i
8,20,5	19.141777783693019 + 24.091228357208841 i	363.324256322783697 + 405.177760815095439 i	8,20,7	15.796167533440268 + 26.756939563151786 i	255.593749881860901 + 348.574826825131709 i
8,20,9	12.084616870538531 + 29.151972502605695 i	167.376572971540554 + 267.261807276774277 i	8,20,11	7.866404355493071 + 31.294566970848368 i	102.483095069137079 + 164.666114090398224 i
8,20,13	2.737970473753154 + 33.179246050710972 i	66.713704538414959 + 43.017338137290700 i	8,21,1	26.668999522679158 + 17.915603774999301 i	708.674214228877418 + 461.194306775235191 i
8,21,3	23.916887850895428 + 21.406717969808206 i	561.121245086660679 + 480.701805339684086 i	8,21,5	20.937374593634701 + 24.456591036244436 i	427.529856741951107 + 459.748439297983680 i
8,21,7	17.691022361228491 + 27.193157795399546 i	309.626190313959967 + 408.000037974481586 i	8,21,9	14.122221865424626 + 29.666842231923695 i	210.074899781413876 + 330.556230404835333 i
8,21,11	10.133040388758468 + 31.899369660845949 i	132.197081925731169 + 230.917832710804163 i	8,21,13	5.503902289709097 + 33.893751660173770 i	80.507345536647591 + 111.706448119642474 i
8,22,1	28.328232650497455 + 18.143350901774159 i	795.876253342801760 + 503.102637673740333 i	8,22,3	25.625723364975968 + 21.691503311916922 i	639.947188908136695 + 530.278961711405486 i
8,22,5	22.710688024345007 + 24.802288470166001 i	497.302705484586056 + 515.528531869416611 i	8,22,7	19.550252020507862 + 27.604681959529067 i	369.530058346260489 + 468.894540085430378 i
8,22,9	16.100611818088343 + 30.150606593128131 i	259.095588971650272 + 395.614079697221428 i	8,22,11	12.289738305884569 + 32.464220503032507 i	168.974654391406801 + 299.249955607051220 i
8,22,13	7.977242823046248 + 34.553374454798679 i	102.940204475336131 + 182.537020525412743 i	8,22,15	2.760756086675593 + 36.408403549718123 i	66.727826273750253 + 47.231560027713527 i
8,23,1	29.980251475663888 + 18.360239317848013 i	888.245924220885513 + 545.741210566115569 i	8,23,3	27.322623443849547 + 21.962330418486530 i	724.085058586848163 + 580.780774453523350 i
8,23,5	24.465201879944420 + 25.130420972189750 i	572.578727918087452 + 572.447427395690170 i	8,23,7	21.380154215614983 + 27.994330284150482 i	435.198522268279874 + 531.166989168294663 i
8,23,9	18.032185469156396 + 30.607146182815040 i	314.254106401518015 + 462.319361936657401 i	8,23,11	14.366213854190370 + 32.994809837734415 i	212.464158566852120 + 369.529113006083321 i
8,23,13	10.284400081442511 + 35.168319279422128 i	133.113595495013868 + 255.582452918385457 i	8,23,15	5.567631719755972 + 37.125745257653413 i	80.660313971997894 + 122.658867741387724 i
8,24,1	31.625960458739254 + 18.567277344579079 i	985.764552205176528 + 589.073726775436285 i	8,24,3	29.009190153451350 + 22.220540392872657 i	813.502177072879590 + 632.159166618134236 i
8,24,5	26.203662182569463 + 25.442755530339117 i	653.304188354554412 + 630.442678162256698 i	8,24,7	23.185504564251577 + 28.364437553129065 i	506.54437430059932 + 594.73719082145305 i
8,24,9	19.925758707830795 + 31.039600141511180 i	375.405915587294089 + 530.570319824071362 i	8,24,11	16.381060874081676 + 33.495538557553715 i	262.410019897785730 + 441.631525159447904 i
8,24,13	12.479342218213647 + 35.745454755877020 i	170.49576546476038 + 330.74204939355542 i	8,24,15	8.080338008934424 + 37.791829832080836 i	103.382215922139565 + 200.196525324079516 i
8,24,17	2.782527543822300 + 39.622920337564800 i	66.742911670772614 + 51.399500491491438 i	8,25,1	33.266112656692613 + 18.765336085544909 i	1088.415797954148729 + 633.067436028856832 i
8,25,3	30.686736576866075 + 22.467284453297054 i	908.170323716011126 + 684.371002441919245 i	8,25,5	27.928271637207942 + 25.740795190637197 i	739.43343238921534 + 689.45862723670416 i
8,25,7	24.970015560288420 + 28.716964683503509 i	583.495149832309721 + 659.534173825295284 i	8,25,9	21.787854954037094 + 31.450555938358075 i	442.434750399108623 + 600.277290501528682 i
8,25,11	18.346873350406444 + 33.969921012045596 i	318.617354980411562 + 515.446296906051202 i	8,25,13	14.591871574593108 + 36.289882894376355 i	214.724620068576542 + 407.894463831491578 i
8,25,15	10.424955907358138 + 38.415795491462589 i	133.988175795404374 + 279.960328279165140 i	8,25,17	5.627341193987862 + 40.343032322075302 i	80.810043475216773 + 133.507012258642987 i
8,26,1	34.901341088472392 + 18.955173728830037 i	1196.185264103916779 + 677.692627409061970 i	8,26,3	32.356351550997587 + 22.703559051922532 i	1008.064912734897348 + 737.377341828885392 i
8,26,5	29.640823628132551 + 26.025830800997575 i	830.927239428924736 + 749.445329354898831 i	8,26,7	26.736632533130265 + 29.053577826773385 i	665.989743453859091 + 725.494666929758864 i
8,26,9	23.623451561376864 + 31.842178946233545 i	515.245104918698189 + 671.360783873549963 i	8,26,11	20.272656057792197 + 34.420829121828646 i	380.932413993500347 + 590.874793870004169 i
8,26,13	16.640718478680991 + 36.805586848179054 i	265.534803286197473 + 486.923384929649899 i	8,26,15	12.655415288729644 + 39.003822340729435 i	171.937612633149229 + 361.868324074606278 i
8,26,17	8.176552667626810 + 41.014966258193581 i	103.806219395528444 + 217.695506200255267 i	8,26,19	2.803246147292944 + 42.826252168702808 i	66.758337775070217 + 55.530897136573614 i
8,27,1	36.532182433945913 + 19.137454608531872 i	1309.060184314776734 + 722.922215011150001 i	8,27,3	34.018947270050248 + 22.930233069440018 i	1113.164359967135624 + 791.142840255823558 i
8,27,5	31.342797138942551 + 26.298980347566744 i	927.751592147229076 + 810.357691344124305 i	8,27,7	28.487732226619986 + 29.375706913084422 i	753.975986548486276 + 792.561887914896488 i
8,27,9	25.436440014944242 + 32.216304318416896 i	593.757183534137198 + 743.749877386532489 i	8,27,11	22.165107491277666 + 34.850651912159584 i	449.230746410146310 + 667.829178985280350 i
8,27,13	18.638626511605079 + 37.295775838415246 i	322.722527337844724 + 567.722696906717488 i	8,27,15	14.801584833426721 + 39.560533254984598 i	216.859474116555191 + 445.809990777005567 i
8,27,17	10.556020381021387 + 41.647000216199565 i	134.819973281011755 + 304.119855377528268 i	8,27,19	5.683404272026205 + 43.549007793540774 i	80.955103752420570 + 144.273656553137528 i
8,28,1	38.159095206070127 + 19.312764343933782 i	1427.02917429814693 + 768.73139697675254 i	8,28,3	35.675294933065572 + 23.148069172773429 i	1223.449586998382074 + 845.635259934732744 i
8,28,5	33.035425595789398 + 26.561219360951053 i	1029.876738814337159 + 872.154779006601530 i	8,28,7	30.225260458825343 + 29.684589899846578 i	847.408866530201635 + 860.684572595948225 i
8,28,9	27.229924073766334 + 32.574504101077430 i	677.903372425234693 + 817.380903295326902 i	8,28,11	24.029364099351820 + 35.261405326642240 i	523.409565826395692 + 746.230910531298719 i
8,28,13	20.594747788004355 + 37.763094145399720 i	386.126023834667194 + 650.196827029562201 i	8,28,15	16.882300196479719 + 40.089564177783139 i	268.47994584609705 + 531.67587506512268 i
8,28,17	12.819649081461920 + 42.244751181532287 i	173.303018124472572 + 392.715137646932533 i	8,28,19	8.266658971156621 + 44.226246625614074 i	104.211478930994488 + 235.069005242207396 i
8,28,21	2.822935988299510 + 46.020830260257952 i	66.773750085227334 + 59.632685609000305 i	8,29,1	39.782473870072607 + 19.481621999182991 i	1550.082030193462970 + 815.097372143793564 i
8,29,3	37.326051869876565 + 23.357740800742352 i	1338.903627438803596 + 900.825067313526688 i	8,29,5	34.719747824434506 + 26.813404765713113 i	1137.276466317774521 + 934.799251921226983 i
8,29,7	31.950830196270580 + 29.981306821699057 i	946.249188025698686 + 929.816188735287255 i	8,29,9	29.006420736326660 + 32.918137457654964 i	767.625696099999800 + 892.196379274794253 i
8,29,11	25.869460594713512 + 35.654810767989986 i	603.382580632673012 + 826.009408654285380 i	8,29,13	22.515919539369204 + 38.209758140007033 i	455.613644870808798 + 734.259849669343566 i
8,29,15	18.910422189109287 + 40.593865794610501 i	326.587467500995899 + 619.364368281895850 i	8,29,17	14.997359720111533 + 42.812443639901019 i	218.876497879415552 + 483.380720463839907 i
8,29,19	10.678717040199691 + 44.865536730862580 i	135.610389201421356 + 328.108359708238936 i	8,29,21	5.736176469039365 + 46.746078675794898 i	81.094886457927032 + 154.975025040548218 i
8,30,1	41.402659944497380 + 19.644489949280089 i	1678.209563617243248 + 861.999102482203511 i	8,30,3	38.971782478779794 + 23.559845825533248 i	1459.511310427099716 + 956.685098445700874 i
8,30,5	36.396646444258863 + 27.056293830709436 i	1249.927527922115814 + 998.256897802019921 i	8,30,7	33.665793070814665 + 30.266806404065331 i	1050.46254673299154 + 999.91429349642056 i
8,30,9	30.768000180741343 + 33.248389022451398 i	862.873935977115707 + 968.144134610124979 i	8,30,11	27.688629156937726 + 36.032352967308242 i	689.07635928282184 + 907.10092451475032 i
8,30,13	24.407406053221127 + 38.637651086499510 i	531.076141906602402 + 819.834340123267452 i	8,30,15	20.895220100755768 + 41.075885972711788 i	391.013206498081956 + 708.782886331859800 i
8,30,17	17.108070986147368 + 43.353420056562946 i	271.259187355435813 + 576.013511164775764 i	8,30,19	12.973467286043658 + 45.472025301969332 i	174.596739236520392 + 423.342493569015971 i
8,30,21	8.351329692088658 + 47.428142336546131 i	104.598298871479230 + 252.342083680145521 i	8,30,23	2.841648905284283 + 49.208419423441107 i	66.788944988819424 + 63.709972017953646 i
8,31,1	43.019950823480762 + 19.801781958924320 i	1811.403465466597481 + 909.417112311948472 i	8,31,3	40.612974595022919 + 23.754917652542716 i	1585.259003394431375 +1013.190278337388463 i
8,31,5	38.066877252953269 + 27.290559401566494 i	1367.809186759524664 +1062.496245264404479 i	8,31,7	35.371292587672194 + 30.541927150894971 i	1160.018529664428115 +1070.940003343125227 i
8,31,9	32.516385897288558 + 33.566298697834682 i	963.604207948222324 +1045.176591950136526 i	8,31,11	29.489501792314734 + 36.395323656434364 i	780.427695193189606 + 989.447595810966618 i
8,31,13	26.273358563064679 + 39.048391351128018 i	612.421409611844524 + 906.850274987358944 i	8,31,15	22.843642974695776 + 41.537689515573994 i	461.619119609184224 + 799.847460576121989 i
8,31,17	19.164737231149253 + 43.870409114548828 i	330.232665169381505 + 670.516170511415908 i	8,31,19	15.180872773795135 + 46.049600051222811 i	220.784794848024035 + 520.679912629919158 i
8,31,21	10.794000281133096 + 48.073992339892555 i	136.361757069785938 + 351.959819877200005 i	8,31,23	5.785980696490840 + 49.936003953521137 i	81.229218244779389 + 165.623043676777996 i
8,32,1	44.634606856835759 + 19.953869853572765 i	1949.656192521487128 + 957.333317363219180 i	8,32,3	42.250052434973675 + 23.943434322179296 i	1716.134400909397755 +1070.317383682396439 i
8,32,5	39.731092030881904 + 27.516802272399975 i	1490.902847280612832 +1127.488239025764642 i	8,32,7	37.068304410594422 + 30.807414257672267 i	1274.89008262041943 +1142.85755287085294 i
8,32,9	34.253028012379716 + 33.872785151272858 i	1069.777866607571013 +1123.250172747657189 i	8,32,11	31.274251719969428 + 36.744855186978988 i	877.38164731129018 +1072.99666053705914 i
8,32,13	28.117114886203343 + 39.443382976700420 i	699.570771108205008 + 995.244060366103213 i	8,32,15	24.761059365660472 + 41.981041255673084 i	538.290171275775720 + 892.481899066807841 i
8,32,17	21.176722406755463 + 44.365687464226525 i	395.621815244473567 + 766.799023509864810 i	8,32,19	17.319918804488779 + 46.601351989274363 i	273.886770197402370 + 620.023469454551673 i
8,32,21	13.118067624220265 + 48.688377628312016 i	175.824094291123487 + 453.793632409561951 i	8,32,23	8.431145464274154 + 50.622470106141222 i	104.967471333430609 + 269.533217995818347 i
8,32,25	2.859447438375808 + 52.390334283828537 i	66.803806859900632 + 67.766619572337959 i	8,33,1	46.246857082919888 + 20.101089070421231 i	2092.960872307475256 +1005.730878267048070 i
8,33,3	43.883386939300443 + 24.125826036005463 i	1852.126349787844902 +1128.044840813457313 i	8,33,5	41.389856460257100 + 27.735561328705973 i	1619.191754525181523 +1193.205965328571438 i
8,33,7	38.757667336369764 + 31.063933323796121 i	1395.053003395059250 +1215.633924645981551 i	8,33,9	35.979158060975237 + 34.168664585737627 i	1181.360647283880780 +1202.324801448970447 i
8,33,11	33.044694876233599 + 37.081946848813864 i	979.890051595417503 +1157.699801154949000 i	8,33,13	29.941402582500011 + 39.823854094234328 i	792.456271989014908 +1084.957698452882141 i
8,33,15	26.651716568651256 + 42.407466076555366 i	620.929012904869296 + 986.616891106146795 i	8,33,17	23.151066292547701 + 44.841185866279858 i	467.282864474530584 + 864.780076644278181 i
8,33,19	19.403635493751644 + 47.129817067162840 i	333.678132444690618 + 721.279883370558650 i	8,33,21	15.353529348875574 + 49.274897349701945 i	222.593457093716466 + 557.760546996544804 i
8,33,23	10.902680920785984 + 51.274279492277508 i	137.076724927665708 + 375.699369993733796 i	8,33,25	5.833104325568234 + 53.120101630221519 i	81.358157882973686 + 176.226709056989279 i

Table D.9: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$

Table D.9: Branch points and associated eigenvalues for $m = 8$.

D.1.4 $m = [8..50]$, $n = [m..15 + m]$

Accuracy was determined by the last correction of the Newton-Raphson method.

Table D.10: Branch points and associated eigenvalues for $m = 9$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
9, 9 ,1	2.754837260868474 + 13.851441882694608 i	84.119800179347351 + 15.478560252068103 i	9,10,1	5.210456108591401 + 14.630754035290515 i	98.196512721728453 + 41.359057139269964 i
9,11,1	7.373579112214387 + 15.279366927621195 i	119.654537151703877 + 70.090277491511756 i	9,11,3	2.692813452216789 + 17.671286153248811 i	83.863592221635841 + 21.389777159761660 i
9,12,1	9.389190345506693 + 15.841057463284994 i	147.651080590635530 +101.157826801845260 i	9,12,3	5.227939150566409 + 18.410130978007146 i	97.520589959202127 + 56.078389283728782 i
9,13,1	11.313941104682295 + 16.339199871965150 i	181.769654631620227 +134.205431310708605 i	9,13,3	7.471125229994036 + 19.051816341368784 i	118.942833053701179 + 93.339048563992165 i
9,13,5	2.688782760124380 + 21.214600446608813 i	83.779187331577077 + 26.495206916215878 i	9,14,1	13.176717485589499 + 16.788205639020443 i	221.759276963529771 +168.977641584379853 i
9,14,3	9.559790268618221 + 19.622777889799718 i	147.242660310175921 +132.876348174122342 i	9,14,5	5.287389515541451 + 21.939901617171888 i	97.368793897309928 + 69.069218344212857 i
9,15,1	14.994456153865672 + 17.197762482699861 i	267.451716717069132 +205.282038662482336 i	9,15,3	11.550144886044624 + 20.138976194955255 i	181.950800232922006 +174.419873714305481 i
9,15,5	7.594241543686864 + 22.584316130873159 i	118.983106770200294 +114.068714275871287 i	9,15,7	2.701281260524614 + 24.631089355965184 i	83.747683717417019 + 31.239949526111424 i
9,16,1	16.778003329413085 + 17.574775716935406 i	318.726324914900772 +242.967901700277963 i	9,16,3	13.471840665281809 + 20.611088864125819 i	222.773532236592672 +217.754399081250995 i
9,16,5	9.742434742017080 + 23.167141115334193 i	147.724220148075659 +161.322795888552264 i	9,16,7	5.355514134990377 + 25.351737458927147 i	97.384500414137761 + 81.251390270202549 i
9,17,1	18.534744623462007 + 17.924381594180911 i	375.492412132602283 +281.913557980063217 i	9,17,3	15.342720624711641 + 21.046724753905067 i	269.509182520370871 +262.707855594354498 i
9,17,5	11.787543467969179 + 23.700683394833629 i	183.096891240541026 +210.615754921269456 i	9,17,7	7.717382672985513 + 26.001592868531940 i	119.258209124262464 +133.596435255100232 i
9,17,9	2.719605174867769 + 27.975465290228627 i	83.738403193766985 + 35.779635228164864 i	9,18,1	20.269942871047590 + 18.250526613001256 i	437.679403447894913 +322.018436877198837 i
9,18,3	17.174449160035657 + 21.451551156290350 i	322.010700707271383 +309.140016962897846 i	9,18,5	13.759445012148502 + 24.193545673214071 i	224.782191857884413 +261.760444102422525 i
9,18,7	9.917471771644243 + 26.596045204652878 i	148.465861232364887 +188.189902511613383 i	9,18,9	5.423877629875380 + 28.695116097787345 i	97.468387724795335 + 92.965164991937783 i
9,19,1	21.987482186657463 + 18.556321829702764 i	505.23089181470976 +363.1978239816498 9 i	9,19,3	18.975103874711298 + 21.829931341050525 i	380.166201692931338 +356.934395259121629 i
9,19,5	15.676415785615609 + 24.652074528061501 i	272.556644467181741 +314.600123068108928 i	9,19,7	12.010896090517948 + 27.145189694502765 i	184.495383936896303 +244.855483344969315 i
9,19,9	7.834792881718449 + 29.351087813263440 i	119.620941881890616 +152.419004661368461 i	9,19,11	2.739937855783677 + 31.273948955323327 i	83.739930027289006 + 40.187383332698218 i
9,20,1	23.690310532444354 + 18.844271201064409 i	578.100827937723807 +405.379251701642602 i	9,20,3	20.750522553837754 + 22.185312635962728 i	443.88779902824774 +405.99258077329443 i
9,20,5	17.550573432579117 + 25.081130888078707 i	326.25480035922340 +369.00306818923832 i	9,20,7	14.027630253167510 + 27.656273586326343 i	227.010117057057727 +303.427440525689065 i
9,20,9	10.081383072202973 + 29.956275273332075 i	149.288794760749596 +214.122173863183605 i	9,20,11	5.489813951463718 + 31.994267153277839 i	97.582496777286451 +104.375398382799020 i
9,21,1	25.380717175767305 + 19.116425031822093 i	656.250963518649314 +448.499929111131280 i	9,21,3	22.505066043610945 + 22.520476053349569 i	513.104722555356489 +456.230233494458048 i
9,21,5	19.390412735981403 + 25.484544105670210 i	385.74927330551554 +424.85737679283392 i	9,21,7	15.986211660957119 + 28.134767266859608 i	275.770393845243177 +363.761393556885707 i
9,21,9	12.218446216151028 + 30.519273062289063 i	185.947298594631206 +277.929136347105612 i	9,21,11	7.945194650478268 + 32.656314579657554 i	120.015117653468465 +170.780719736147546 i
9,21,13	2.760693610170558 + 34.541063717854030 i	83.747136517737246 + 44.503362482649727 i	9,22,1	27.060514166967828 + 19.374486686601553 i	739.64907043377059 +492.50485839851808 i
9,22,3	24.242079165660513 + 22.837703967177355 i	587.758839434639413 +507.574181559044007 i	9,22,5	21.202139083781262 + 25.865398452687291 i	450.939040540685482 +482.066867273869661 i
9,22,7	17.899054227448719 + 28.584942887396490 i	330.596552877001386 +425.732885841930852 i	9,22,9	14.276010445454745 + 31.046359930915763 i	229.246137817034054 +343.691479879758731 i
9,22,11	10.234101780285819 + 33.271235999590786 i	150.126191030258610 +239.439319035864287 i	9,22,13	5.552495315039050 + 35.262835012556764 i	97.709798017536571 +115.574347449086602 i
9,23,1	28.731159260883903 + 19.619888953588635 i	828.267661711721871 +537.345420435680467 i	9,23,3	25.964183634730748 + 23.138896765311959 i	667.801611217680147 +559.960273562288421 i
9,23,5	22.990432040419410 + 26.226223195353494 i	521.742109207065369 +540.547962459398377 i	9,23,7	19.774939308991193 + 29.010222414321337 i	391.34886171383829 +489.2342162623314 i
9,23,9	16.272788637976650 + 31.542352252156217 i	278.932801266700608 +411.275243433651269 i	9,23,11	12.411113053930583 + 33.846531495428444 i	187.380919897353436 +310.231069894779353 i
9,23,13	8.048636047265848 + 35.930654329700545 i	120.416085015561407 +188.819852006161909 i	9,23,15	2.781163604753508 + 37.785634885437076 i	83.757437591389575 + 48.751947128917678 i
9,24,1	30.393841654887513 + 19.853850032008797 i	922.083048985527967 +582.978290223741851 i	9,24,3	27.673471956574417 + 23.425656397793155 i	753.191953103374431 +613.331755540904510 i
9,24,5	24.758911383487526 + 26.569123729512356 i	598.090676698486618 +600.227322900947911 i	9,24,7	21.620334186416763 + 29.413401856723670 i	457.91557660812720 +554.17150550876295 i
9,24,9	18.221404122735699 + 32.011065738960561 i	334.816064306606449 +480.562381436657404 i	9,24,11	14.506291274364679 + 34.387747091933775 i	231.417898926524373 +383.022179967624538 i
9,24,13	10.376454750773650 + 36.554359954893179 i	150.950025120520621 +264.323651251739705 i	9,24,15	5.611770683437723 + 38.509219699275071 i	97.841905213748348 +126.618640052519382 i

Table D.10: Branch points and associated eigenvalues for $m = 9$.

Table D.11: Branch points and associated eigenvalues for $m = 10$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
10,10,1	2.81948507565682 + 14.990209607367 49 i	103.24235891816643 + 16.391683445935 32 i	10,11,1	5.327463550267296 + 15.799055606477189 i	117.805507012199811 + 43.814865986473905 i
10,12,1	7.531075603313457 + 16.475682283040410 i	139.927485038865967 + 74.264535271117822 i	10,12,3	2.741193306016878 + 18.874227379496350 i	102.947078388647057 + 22.484119708419950 i
10,13,1	9.580214373016309 + 17.063890194677363 i	168.712887961199812 +107.199579253734711 i	10,13,3	5.320782822391950 + 19.633473961894836 i	116.967664663069556 + 58.952785996838799 i
10,14,1	11.533697900328375 + 17.587196994302932 i	203.717939659863049 +142.244203866390933 i	10,14,3	7.599875812107518 + 20.295775310695504 i	138.917996633356466 + 98.113598141173497 i
10,14,5	2.727939006364454 + 22.462166069252422 i	102.843489636186547 + 27.704986439435562 i	10,15,1	13.421630220845062 + 18.060152891012372 i	244.674575060736544 +179.128638504466778 i
10,15,3	9.719088684478374 + 20.887048665512200 i	167.867441937917420 +139.663691070484504 i	10,15,5	5.365124724158615 + 23.202881185941006 i	116.727156382177967 + 72.225909977604147 i
10,16,1	15.261723909386927 + 18.492565606220623 i	291.402713863630709 +217.649509390609467 i	10,16,3	11.736136249012668 + 21.423076598293125 i	203.322469394829881 +183.322946004624782 i
10,16,5	7.704026910883830 + 23.863438988702118 i	138.784069374004673 +119.260388604143898 i	10,16,7	2.734251578558625 + 25.912491803406724 i	102.800079823254592 + 32.533201288186227 i
10,17,1	17.065359515225751 + 18.891442694493417 i	343.772932823713177 +257.647405022269936 i	10,17,3	13.681558043204477 + 21.914472300822641 i	244.973463813240102 +228.867676166737169 i
10,17,5	9.880024574418687 + 24.462576796638697 i	168.08171565918377 +168.63847866119616 i	10,17,7	5.422555421956821 + 26.645361714273491 i	116.684532402292675 + 84.618431127567035 i
10,18,1	18.840310767808905 + 19.262007873330962 i	401.687744634006094 +298.993554709126954 i	10,18,3	15.573784428395973 + 22.368827472146783 i	292.60742307466967 +276.11871429879147 i
10,18,5	11.949793453785180 + 25.012353402907930 i	204.104153779745729 +220.139902351725709 i	10,18,7	7.813311632017969 + 27.308330345049526 i	138.940186591375323 +139.104418808006898 i
10,18,9	2.748074475170671 + 29.283749398270174 i	102.782555418045646 + 37.137151956879869 i	10,19,1	20.592133585880898 + 19.608283099200825 i	465.071130466535976 +341.581454754924380 i
10,19,3	17.424896423769244 + 22.791814066765816 i	346.068761939624665 +324.929857824863063 i	10,19,5	13.943878983751492 + 25.521245400167999 i	246.518375832294851 +273.572471380014292 i
10,19,7	10.038835292572902 + 27.916276558571766 i	168.635757627105278 +195.911238597358988 i	10,19,9	5.482850437506527 + 30.013400523564101 i	116.726479538457752 + 96.497416787006657 i
10,20,1	22.324940049120531 + 19.933445106332488 i	533.862212438753318 +385.321337859691994 i	10,20,3	19.243281602994298 + 23.187810520213223 i	405.238905108719564 +375.179528764551947 i
10,20,5	15.881010502086063 + 25.995531945176928 i	295.090607358207819 +328.774766097769998 i	10,20,7	12.155083151153690 + 28.479044274661945 i	205.241525571142347 +254.860985228754458 i
10,20,9	7.920035275357797 + 30.680296244103651 i	139.215129567860004 +158.178427206883384 i	10,20,11	2.764964939120930 + 32.604251004815154 i	102.777994953727813 + 41.596331528962359 i
10,21,1	24.041859614495589 + 20.240055487934750 i	608.011196234297358 +430.136366847319493 i	10,21,3	21.035016542737953 + 23.560284304191644 i	470.024563690073819 +426.764927264444054 i
10,21,5	17.773633842291225 + 26.440038083540220 i	349.647496839292273 +385.610878526631723 i	10,21,7	14.192548638619845 + 29.003738929035535 i	248.408471046363758 +315.785029538484252 i
10,21,9	10.190026921597072 + 31.296883304335434 i	169.317614572951072 +222.168866884013880 i	10,21,11	5.542446988635168 + 33.332939131133159 i	116.808753148897310 +108.041866552804493 i
10,22,1	25.745328795059272 + 20.530215499965532 i	687.476646221921214 +475.959923861947232 i	10,22,3	22.804651007750746 + 23.912038688812903 i	540.35050304368303 +479.59788209019933 i
10,22,5	19.630492494606703 + 26.858575415480360 i	410.055377250443996 +443.965191922499400 i	10,22,7	16.170126621535111 + 29.495745497792797 i	297.887691011838797 +378.535922109281216 i
10,22,9	12.348288766027631 + 31.871521586909282 i	206.494132085680576 +288.323439062136458 i	10,22,11	8.021900039000988 + 34.004259676048825 i	139.541377764738542 +176.746263379366809 i
10,22,13	2.782996355333308 + 35.889802683908464 i	102.780509882606438 + 45.954592496757414 i	10,23,1	27.437280975080139 + 20.805673840704372 i	772.223585931629032 +522.733623490884270 i
10,23,3	24.555682459962558 + 24.245378880086591 i	616.154832337176686 +533.601842666555740 i	10,23,5	21.457986531742186 + 27.254221343830952 i	476.208099875252515 +503.738222181538504 i
10,23,7	18.100494049776572 + 29.959283009908670 i	353.492317409764269 +442.986266163470702 i	10,23,9	14.425262412002652 + 32.410349426511907 i	250.383505384687567 +356.492257892768976 i
10,23,11	10.332456618559755 + 34.628967810638891 i	170.044267418736979 +247.754580988510639 i	10,23,13	5.600001674338536 + 36.618701800960544 i	116.910980875040437 +119.352502745410504 i
10,24,1	29.119275209439547 + 21.067903864858078 i	862.222132586511179 +570.405820584776393 i	10,24,3	26.290858068422263 + 24.562227832010244 i	697.385801929270883 +588.709650320531068 i
10,24,5	23.260951718163600 + 27.629504972714475 i	548.019400112424137 +564.843441254885533 i	10,24,7	19.992634435730345 + 30.397740065851696 i	415.076816027459529 +509.025638304058873 i
10,24,9	16.439948760329330 + 32.918093436408370 i	300.724959092664449 +426.539408182933414 i	10,24,11	12.529233829553149 + 35.214350639302289 i	207.769496570544334 +320.948116630499442 i
10,24,13	8.118346533869008 + 37.294509967601762 i	139.888033574194679 +194.958155762191581 i	10,24,15	2.801254610803106 + 39.150076651452572 i	102.787081736754928 + 50.238640101275806 i
10,25,1	30.792586220789886 + 21.318160256794183 i	957.446489940251581 +618.930465971155286 i	10,25,3	28.012374921384058 + 24.864209340325186 i	783.999548636604345 +644.861852685650206 i
10,25,5	25.043135615336308 + 27.986535720792133 i	625.417854931570673 +627.204857686500191 i	10,25,7	21.853175328670204 + 30.813891959939298 i	482.524659114402766 +576.557640011896865 i
10,25,9	18.405185489005355 + 33.398509726490289 i	357.320727601283750 +498.344796957638152 i	10,25,11	14.642631107149461 + 35.765821647530537 i	252.346073798749615 +396.192573314276217 i
10,25,13	10.466293929748018 + 37.926726502123017 i	170.778534422013848 +272.865281356140883 i	10,25,15	5.655040213948596 + 39.879816094891737 i	117.022799352231857 +130.491442563000675 i

Table D.11: Branch points and associated eigenvalues for $m = 10$.

Table D.12: Branch points and associated eigenvalues for $m = 11$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
11,11,1	2.880815839522571 + 16.121047916478865 i	124.361375189904308 + 17.276087240552581 i	11,12,1	5.438621674254764 + 16.957622728998849 i	139.396876031271546 + 46.193475733426084 i
11,13,1	7.680952288554059 + 17.660549824117094 i	162.163758167910004 + 78.307644453577311 i	11,13,3	2.787520935446135 + 20.066898626442768 i	124.028624681267715 + 23.546183577185847 i
11,14,1	9.762296687009727 + 18.273679444101781 i	191.715660372903216 +113.051554645501312 i	11,14,3	5.409678859408797 + 20.845500221903656 i	138.403995420870416 + 61.742797480805052 i

Table D.12: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
11,15,1	11.743494408565887 + 18.820670950414787 i	227.582396830572037 +150.030484854377420 i	11,15,3	7.723253144001322 + 21.527330403259707 i	160.869528331321305 +102.748525625780672 i
11,15,5	2.765702550679174 + 23.698360356115600 i	123.906628235540055 + 28.881292524927652 i	11,16,1	13.655783667717575 + 19.316205355083614 i	269.479380192605959 +188.960789372555837 i
11,16,3	9.871892164775630 + 22.137845386532565 i	190.452791202714678 +146.253154639096977 i	11,16,5	5.440009121885860 + 24.453826920965565 i	138.078476780587494 + 75.295840561411964 i
11,17,1	15.517599013054131 + 19.770201185388594 i	317.215074915491186 +229.628490378400045 i	11,17,3	11.914729713646317 + 22.692670926492042 i	226.636671009826358 +191.967085638272323 i
11,17,5	7.809808622057471 + 25.129792983344998 i	160.568839901336474 +124.310189761862290 i	11,17,7	2.766231055468513 + 27.182019422701668 i	123.851764053638902 + 33.792815419616630 i
11,18,1	17.340818171827522 + 20.189759237600740 i	370.651577406959461 +271.865771977655129 i	11,18,3	13.883140373884853 + 23.202363964309122 i	269.096110895201377 +239.658382939437274 i
11,18,5	10.012669223080280 + 25.744488984575188 i	190.411398035704849 +175.755179909517267 i	11,18,7	5.487456803844618 + 27.926673542453525 i	137.979818804094720 + 87.898564908370815 i
11,19,1	19.133576208284374 + 20.580182362004820 i	429.684830196506368 +315.537005793810295 i	11,19,3	15.796105844100689 + 23.674501514024758 i	317.607108362990743 +289.140967130844842 i
11,19,5	12.106316265578933 + 26.309749639589086 i	227.070013244907187 +229.406003016133667 i	11,19,7	7.906156745916614 + 28.602240179933943 i	160.610764207268602 +144.471203069295143 i
11,19,9	2.775817560102565 + 30.579983011385160 i	123.826281209471233 + 38.461389641815018 i	11,20,1	20.901702169768214 + 20.945560428316128 i	494.233551739587767 +360.529961060885114 i
11,20,3	17.666097866581705 + 24.114759947888505 i	372.005793210289141 +340.262822545194865 i	11,20,5	14.121929258927844 + 26.833937517400528 i	270.197950402562356 +285.065511824788132 i
11,20,7	10.156321422872574 + 29.223131510380892 i	190.78551993057616 +203.43576333914686 i	11,20,9	5.540173848927354 + 31.319338900136184 i	137.981350604031974 + 99.943781332821302 i
11,21,1	22.649520857998802 + 21.289129083091137 i	564.232540298716913 +406.749997061759188 i	11,21,3	19.501794637957246 + 24.527531286846987 i	432.167097148286985 +392.897425013612974 i
11,21,5	16.078666194078302 + 27.323270601565767 i	319.551401991364173 +342.567697488867623 i	11,21,7	12.294717536057311 + 29.798962189337725 i	227.957085653157066 +264.612684640579630 i
11,21,9	8.002845716734798 + 31.996789106039891 i	160.801194210817158 +163.798849436553525 i	11,21,11	2.789448778726401 + 33.922517625418912 i	123.815822321420072 + 42.972637936399551 i
11,22,1	24.380331350564784 + 21.613501513086572 i	639.628377515834813 +454.116065355717865 i	11,22,3	21.309496670638339 + 24.916300926912953 i	497.992488414634181 +446.937712523967718 i
11,22,5	17.989282974677966 + 27.782541146325528 i	374.949319766536291 +401.772556052832158 i	11,22,7	14.352335026305054 + 30.336708180481445 i	271.764353403073756 +327.830452141708122 i
11,22,9	10.295565169831466 + 32.624368177278505 i	191.331659435590126 +230.022590973708674 i	11,22,11	5.593783331558379 + 34.659374279152537 i	138.032843573354484 +111.624100263254476 i
11,23,1	26.096708008792586 + 21.920824534190871 i	720.376541130967667 +502.557862197032534 i	11,23,3	23.093930997170130 + 25.283891906203039 i	569.402399436150404 +502.291797456461154 i
11,23,5	19.862757842261590 + 28.215544291598544 i	436.25192174274082 +462.5608404353947 7 i	11,23,7	16.348409654129579 + 30.841676290530962 i	321.949393580516698 +392.938578051978823 i
11,23,9	12.47444096186878 + 33.21022044413431 i	229.01805378798630 +298.4698363438883 5 i	11,23,11	8.096652650954512 + 35.339698968625669 i	161.061840093970318 +182.575819866914060 i
11,23,13	2.804886808775207 + 37.226635342513909 i	123.813774350285527 + 47.373965596149631 i	11,24,1	27.800698015670008 + 22.212887366495202 i	806.439390904577976 +552.013741216755648 i
11,24,3	24.858738427692270 + 25.632629703897926 i	646.331291393496485 +558.879854333373487 i	11,24,5	21.705674555406627 + 28.625350805344500 i	503.348069672386032 +524.829842413796996 i
11,24,7	18.295868607697173 + 31.318036733374100 i	378.318299582747045 +459.806765800321216 i	11,24,9	14.570313474359873 + 33.760355232733509 i	273.488545244525483 +368.989187279838575 i
11,24,11	10.428282421474879 + 35.973886461951395 i	191.951440027978094 +255.881574423794547 i	11,24,13	5.646472729989745 + 37.962525426557214 i	138.110747895909412 +123.048415741548695 i
11,25,1	29.493955617199642 + 22.491199631277610 i	897.784719796883110 +602.429145704261487 i	11,25,3	26.606784072066350 + 25.964457322385211 i	728.724296480074713 +616.631811644258960 i
11,25,5	23.523016255153873 + 29.014489235019674 i	576.147341326370793 +588.490156142541991 i	11,25,7	20.203887416874753 + 31.769147266697981 i	440.71987344644566 +528.32190850291818 i
11,25,9	16.602460230764848 + 34.279416550707587 i	324.474259696267211 +441.442767414930813 i	11,25,11	12.644316882247539 + 36.569027197892900 i	230.140777208406860 +331.423944848373708 i
11,25,13	8.186466180761750 + 38.646124938686377 i	161.355877509241196 +200.963905591576832 i	11,25,15	2.821029524417447 + 40.502809770261265 i	123.816706282301155 + 51.694333829887412 i
11,26,1	31.177836061882324 + 22.757048649013246 i	994.38468455357986 +653.7554063235105 6 i	11,26,3	28.340363455551138 + 26.281017985725750 i	816.534852510520636 +675.485604974491935 i
11,26,5	25.318651107000719 + 29.385072256680228 i	654.574789964518004 +653.463206253210046 i	11,26,7	22.079245716804582 + 32.197763868621323 i	509.032918855030402 +598.385149613808721 i
11,26,9	18.583924151116413 + 34.771105507064526 i	381.771026819026019 +515.708445272721519 i	11,26,11	14.775482458438024 + 37.130412272275251 i	275.249429183015764 +409.068064082881961 i
11,26,13	10.554045682653859 + 39.286618085391460 i	192.599005308255386 +281.223751472110280 i	11,26,15	5.697472767979202 + 41.238612343620701 i	138.202821731130769 +134.284163510582857 i

Table D.12: Branch points and associated eigenvalues for $m = 11$.

Table D.13: Branch points and associated eigenvalues for $m = 12$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
12,12,1	2.93923581536778 + 17.24501390600442 i	147.47722005336200 + 18.13505239200206 i	12,13,1	5.544632105740728 + 18.107759709959460 i	162.97247182574631 + 48.50369472121527 i
12,14,1	7.824098120010937 + 18.835504866723756 i	186.367198807726083 + 82.234552790701770 i	12,14,3	2.832007398777270 + 21.250536974781532 i	147.108386499700941 + 24.579420447114369 i
12,15,1	9.936451310243065 + 19.472170117524460 i	216.665550398034895 +118.735365957385852 i	12,15,3	5.495037562576078 + 22.047570496335684 i	161.830496581651516 + 64.457397546556479 i
12,16,1	11.944429186178885 + 20.041555312320618 i	253.371707104606429 +157.593010949096396 i	12,16,3	7.841807714301229 + 22.747971929051783 i	184.799479313150158 +107.258613732968598 i
12,16,5	2.802192542850233 + 24.924442705943893 i	146.968662837596725 + 30.027480502085719 i	12,17,1	13.880332913730085 + 20.558466134503007 i	296.185054249002121 +198.510360424558826 i
12,17,3	10.018853070703202 + 23.376784229192499 i	215.002158312580235 +152.665618783290711 i	12,17,5	5.512299898458958 + 25.694064509566019 i	161.423205743885774 + 78.287641587687403 i
12,18,1	15.763273052868891 + 21.032924745502783 i	344.902949361299534 +241.263140554744581 i	12,18,3	12.086652524305665 + 23.949494212562289 i	251.898447282136743 +200.379540743341380 i
12,18,5	7.911948323049684 + 26.384780766099574 i	184.338531336141756 +129.232144941489906 i	12,18,7	2.797288725904442 + 28.440892357423159 i	146.902747176929353 + 35.021961137071280 i
12,19,1	17.605592737061158 + 21.472118126364872 i	399.379268749718733 +285.675381514861215 i	12,19,3	14.077369979117545 + 24.476609675545347 i	295.148253390879290 +250.160387621540877 i
12,19,5	10.140810744815598 + 27.014359637091395 i	214.715248329652184 +182.692476512563957 i	12,19,7	5.550385365867012 + 29.196928960857647 i	161.270553109888453 + 91.099910105468553 i
12,20,1	19.415765357656372 + 21.881421688757424 i	459.503587055380251 +331.604806884922994 i	12,20,3	16.010507361445761 + 24.965695304313495 i	344.516867176862149 +301.815335948831489 i
12,20,5	12.257616297481558 + 27.594432084205035 i	251.997479721806428 +238.439343299902987 i	12,20,7	7.996163787157708 + 29.884625249853691 i	184.270511201353742 +149.709883108392802 i
12,20,9	2.802874117711047 + 31.865322183559364 i	146.869565132804894 + 39.755304019413572 i	12,21,1	21.199875861214316 + 22.264987230817212 i	525.189521622652933 +378.933638214034204 i
12,21,3	17.898904723660529 + 25.422431947697920 i	399.832357069540080 +355.186703197846271 i	12,21,5	14.294148639713791 + 28.133257034438078 i	295.825082749115268 +296.270718847252624 i
12,21,7	10.270239293616394 + 30.517964218771937 i	214.916252430882679 +210.781621921056484 i	12,21,9	5.595958586790958 + 32.614105918021394 i	161.233044979651964 +103.311818404818865 i
12,22,1	22.962448537923546 + 22.626103913221534 i	596.367678586939974 +427.562517475767777 i	12,22,3	19.751514098681973 + 25.851224116275450 i	460.963338418601722 +410.143147310660083 i
12,22,5	16.269973154384882 + 28.636996764812316 i	345.944455412863419 +356.016089817410318 i	12,22,7	12.430159486704998 + 31.106350096400263 i	252.643835232953990 +274.133867005086188 i
12,22,9	8.083394946800114 + 33.301769736457572 i	184.379375419680656 +169.292410383403706 i	12,22,11	2.813411137424572 + 35.229832245715379 i	146.853382667138703 + 44.319043416271533 i
12,23,1	24.706941717595509 + 22.967431666750898 i	672.981124557136241 +477.406322613269310 i	12,23,3	21.574847001659020 + 26.255573461064738 i	527.806179774012321 +466.573457848282601 i
12,23,5	18.198139950154552 + 29.110413530104580 i	402.167038813747752 +417.531438018034237 i	12,23,7	14.507390900796056 + 31.656639997664065 i	297.080311316164227 +339.592230606225087 i
12,23,9	10.398218743110144 + 33.939966819746608 i	215.331487132753409 +237.700088498921332 i	12,23,11	5.643897006164220 + 35.974664515442740 i	161.254723741717953 +115.129107694330116 i
12,24,1	26.436059355171373 + 23.291158515963360 i	754.982341914687936 +528.391186632115250 i	12,24,3	23.373797551355164 + 26.638320856987146 i	600.277098960607645 +524.382132591288951 i
12,24,5	20.087849786308198 + 29.557287261781042 i	464.347085690886558 +480.693962677295531 i	12,24,7	16.521495475021929 + 32.174068856159082 i	347.958916036091864 +407.003218906408222 i
12,24,9	12.597165541232345 + 34.536642429101164 i	253.520046502181117 +308.389719135721123 i	12,24,11	8.169572823944213 + 36.663741599851718 i	184.576540062951153 +188.280610685099248 i
12,24,13	2.826375493723195 + 38.552571622456441 i	146.846893295708924 + 48.764017579786895 i	12,25,1	28.151955773207938 + 23.599110297588624 i	842.331103780960120 +580.452312396575394 i
12,25,3	25.152141752240428 + 27.001810035845413 i	678.307008423038269 +583.486163008350672 i	12,25,5	21.945860446317027 + 29.980682428475962 i	532.368591461642609 +545.397802681220242 i
12,25,7	18.485639298651556 + 32.662761866132691 i	405.078857294571037 +476.233675258608230 i	12,25,9	14.711461136959604 + 35.097686750142039 i	298.562757426047483 +381.208363715732503 i
12,25,11	10.521739561145969 + 37.307123144478986 i	215.847917946882632 +263.835727518777446 i	12,25,13	5.691957920507298 + 39.295318211637728 i	161.309002727280529 +126.668572774037969 i
12,26,1	29.856374976911777 + 23.892829390922035 i	934.992944524409268 +633.532331318474689 i	12,26,3	26.912856033287600 + 27.348002035295725 i	761.838006410575758 +643.812648212407339 i
12,26,5	23.777294677356004 + 30.383126802986336 i	606.13707944454302 +611.55074045362255 i	12,26,7	20.409180941589476 + 33.126048208084413 i	468.283398863237096 +547.167851394793615 i
12,26,9	16.760649408428996 + 35.627667414975562 i	350.182799668131963 +456.016183442072418 i	12,26,11	12.756556870540516 + 37.911717062550728 i	254.495230839110604 +341.678205349129257 i
12,26,13	8.253080529197090 + 39.986521448596420 i	184.819524951659503 +206.847468597689868 i	12,26,15	2.840491714376997 + 41.844773414312087 i	146.846271425189570 + 53.121373393137382 i
12,27,1	31.550748267682394 + 24.173632595535494 i	1032.938030140685669 +687.580044594355286 i	12,27,3	28.658328916521857 + 27.678557555864849 i	850.820903873188513 +705.296988024374514 i
12,27,5	25.586134907005725 + 30.766735766893973 i	685.574166781098256 +679.071678042911230 i	12,27,7	22.299044873644018 + 33.566665838071515 i	537.446780756007911 +619.704474992799533 i
12,27,9	18.757970355235908 + 36.130234572894885 i	408.169723272884312 +532.689020034977261 i	12,27,11	14.905069214581610 + 38.482700888703320 i	300.128771263058184 +421.672554544304091 i
12,27,13	10.639827723479353 + 40.635071247045879 i	216.411421955051878 +289.413283905796220 i	12,27,15	5.739100954538860 + 42.586546326703314 i	161.381846228851458 +138.002805878776599 i

Table D.13: Branch points and associated eigenvalues for $m = 12$.

Table D.14: Branch points and associated eigenvalues for $m = 13$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
13,13,1	2.995076207889928 + 18.362954749069853 i	172.590202024164341 + 18.971257532415816 i	13,14,1	5.646069119107460 + 19.250511341004085 i	188.533832744602392 + 50.752717165245592 i
13,15,1	7.961244630830197 + 20.001777819143570 i	212.540999386986556 + 86.057472414891450 i	13,15,3	2.874830812539813 + 22.426154927852185 i	172.186500571549317 + 25.586688023601539 i
13,16,1	10.103517181783401 + 20.660762176054664 i	243.567674399005426 +124.268674712823355 i	13,16,3	5.577204031255691 + 23.240799233481841 i	187.247960310708265 + 67.104024668141179 i
13,17,1	12.137415756963254 + 21.251403750072953 i	281.093097580095105 +164.955269700307355 i	13,17,3	7.956003524899432 + 23.958921098693907 i	210.709616914180089 +111.656127001711766 i
13,17,5	2.837513020551716 + 26.141460623803702 i	172.029651784913256 + 31.146363324831579 i	13,18,1	14.096243109700696 + 21.788627985604943 i	324.801068436338994 +207.806999417205077 i
13,18,3	10.160523039730926 + 24.605190484566318 i	241.518508476034526 +158.918418227851333 i	13,18,5	5.582218777683639 + 26.924697877674287 i	186.761756835503974 + 81.208560053680742 i
13,19,1	15.999746163545025 + 22.282554938373927 i	374.478148336280412 +252.589567780550595 i	13,19,3	12.252521781950977 + 25.194970499750955 i	279.112126339523513 +208.582944960441012 i
13,19,5	8.010757392222224 + 27.629571222359132 i	210.094145719471925 +134.038044940407843 i	13,19,7	2.827487141313253 + 29.690139090493030 i	171.953046041734495 + 36.223325097056289 i
13,20,1	17.860705981247268 + 22.740452115054417 i	429.970225983033515 +299.119072875914696 i	13,20,3	14.264913015707689 + 25.738725997237893 i	323.135708202712067 +260.401835142949494 i
13,20,5	10.264830183079820 + 28.273424569240625 i	240.995033391168334 +189.466838660176620 i	13,20,7	5.611488581337776 + 30.457188131879867 i	186.556932779264855 + 94.229359781007815 i
13,21,1	19.687914315350792 + 23.167762201820402 i	491.160689248691313 +347.246775423388954 i	13,21,3	16.217691726637621 + 26.244011533012866 i	373.34410240894903 +314.17567400983523 i
13,21,5	12.404129188842244 + 28.867702554790831 i	278.889223138168063 +247.261211659047262 i	13,21,7	8.083548553479037 + 31.156586241810150 i	209.919974233421300 +154.831591284145929 i
13,21,9	2.829281766259769 + 33.140752531519517 i	171.912400885188021 + 41.021425664997686 i	13,22,1	21.487696999752824 + 23.568694091282655 i	557.958199519634263 +396.849507422134820 i
13,22,3	18.124046685837264 + 26.716512671888395 i	429.557517746306530 +369.741244369221079 i	13,22,5	14.461015684016795 + 29.420569455657954 i	323.403456422383556 +307.214335225227160 i
13,22,7	10.380860047627165 + 31.801919507150995 i	241.028988862793682 +217.964255002731335 i	13,22,9	5.650304701163352 + 33.898704497420475 i	186.481630541219403 +106.608010320447690 i
13,23,1	23.264766231428712 + 23.946586079247332 i	630.289289565931426 +447.823330963564786 i	13,23,3	19.993189060696281 + 27.160645652908265 i	491.638412649389238 +426.962501777864020 i
13,23,5	16.455443646726067 + 29.938136651061779 i	374.274554112691192 +369.151271240642586 i	13,23,7	12.561725130157681 + 32.402397991086296 i	279.303408924495557 +283.444362599231226 i
13,23,9	8.161835704329638 + 34.596266493255889 i	209.949936141124738 +174.669532177903847 i	13,23,11	2.836874005868030 + 36.527127738911140 i	171.890656619768578 + 45.637915350182311 i
13,24,1	25.022730475250565 + 24.304140888852270 i	708.093609662610561 +500.079171457229279 i	13,24,3	21.831830012603362 + 27.579927109852957 i	559.478185401887004 +485.724189424975009 i
13,24,5	18.400743374621889 + 30.425138916037767 i	431.306615938300411 +432.924063198702811 i	13,24,7	14.658069002949313 + 32.964769186657918 i	324.358674970116510 +351.094670594937952 i
13,24,9	10.498184584601507 + 35.244737075319506 i	241.317662858443441 +245.215748439605250 i	13,24,11	5.692856575953154 + 37.279751334575259 i	186.474382124762144 +118.562951005364283 i
13,25,1	26.764415985434113 + 24.643584191416078 i	791.320729858844970 +553.539706512549742 i	13,25,3	23.645021778494602 + 27.977214235941645 i	632.988946385549639 +545.927281599437038 i
13,25,5	20.306327330209214 + 30.885342165152213 i	494.348065473050951 +498.406427218489341 i	13,25,7	16.689766612229697 + 33.494201670014064 i	375.919363353163931 +420.758712384297837 i
13,25,9	12.716696466631854 + 35.851876758345099 i	280.001070601446997 +318.101486715066756 i	13,25,11	8.240770199110264 + 37.977344850391717 i	210.085559644194603 +193.870354665963617 i
13,25,13	2.847474339405069 + 39.868488697005489 i	171.879840133133087 + 50.126956066454305 i	13,26,1	28.492078309812801 + 24.966775228929443 i	879.927907154258946 +608.137085144934190 i
13,26,3	25.436668350723640 + 28.354867399285119 i	712.098149401003866 +607.485668779999628 i	13,26,5	22.179118768712449 + 31.321804960979122 i	563.278143846605872 +565.489414480115215 i
13,26,7	18.670212826188525 + 33.994778979041619 i	433.777936799514691 +492.300510010998437 i	13,26,9	14.848970336276613 + 36.423465197718855 i	325.607577659946344 +393.172248499812497 i
13,26,11	10.612973228380602 + 38.629654902206326 i	241.733959591798282 +271.630416064101837 i	13,26,13	5.736504315761035 + 40.617960590399207 i	186.505685010631851 +130.218628612754031 i
13,27,1	30.207546363561491 + 25.275286240440590 i	973.878477686170186 +663.811214611208470 i	13,27,3	27.209851603915276 + 28.714864043410962 i	796.744935412850282 +670.323695179575452 i

Table D.14: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
13,27,5	24.024373149827156 + 31.737054206242707 i	637.998413838068245 +634.078055858891730 i	13,27,7	20.608941245191758 + 34.469803757098795 i	497.772235691426943 +565.601713955471496 i
13,27,9	16.914807117049723 + 36.963999117578588 i	377.852559206617684 +470.286235556549116 i	13,27,11	12.866129531669287 + 39.243419241888780 i	280.833367881313620 +351.727949056336683 i
13,27,13	8.318268984742089 + 41.316589391741402 i	210.278941279050287 +212.617893680674229 i	13,27,15	2.859646522541507 + 43.176789611627854 i	171.875746873161660 + 54.521814925059246 i
13,28,1	31.912323486988498 + 25.570460885190574 i	1073.140668697720912 +720.508446517729681 i	13,28,3	28.967048213203871 + 29.058880751177956 i	886.877554368662913 +734.374292875428068 i
13,28,5	25.846181523114549 + 32.133207592562414 i	718.427136299981498 +704.088786493681368 i	13,28,7	22.513014701579866 + 34.921997132754178 i	567.772032337114638 +640.558675827442698 i
13,28,9	18.927636522222706 + 37.477081279874838 i	436.519401199721585 +549.317276459174650 i	13,28,11	15.031593268447958 + 39.823709584884682 i	326.984929934259071 +434.026799059080076 i
13,28,13	10.723748248357033 + 41.972989041206375 i	242.215839332869109 +297.446304936833883 i	13,28,15	5.779956863107969 + 43.924439991652903 i	186.559782888142735 +141.652639905907620 i

Table D.14: Branch points and associated eigenvalues for $m = 13$.

Table D.15: Branch points and associated eigenvalues for $m = 14$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
14,14,1	3.048611114880463 + 19.475561203675621 i	199.700581043232518 + 19.786925240028260 i	14,15,1	5.743409628958412 + 20.386729066214542 i	216.082253951873980 + 52.946514062327225 i
14,16,1	8.093002188368450 + 21.160371681035034 i	240.687848540211689 + 89.786541459239032 i	14,16,3	2.916143019289691 + 23.594593180867747 i	199.263087557549085 + 26.570385490280625 i
14,17,1	10.264197565194452 + 21.840597850981109 i	272.426344633423753 +129.666145637638778 i	14,17,3	5.656471701823944 + 24.426112262360281 i	214.657077476167899 + 69.688930100755059 i
14,18,1	12.323223336551553 + 22.451485480378402 i	310.752665548944321 +172.136766451391452 i	14,18,3	8.066235889189919 + 25.161192843479100 i	238.601476030535025 +115.951380410818032 i
14,18,5	2.871755287196822 + 27.350296143255648 i	199.089650746079712 + 32.240328075217327 i	14,19,1	14.304330380180305 + 23.008075722949236 i	355.335422255626440 +216.875331426717338 i
14,19,3	10.297373394972283 + 25.824166208964574 i	270.004411973022854 +165.026140638742504 i	14,19,5	5.649958380414025 + 28.146657493253400 i	214.094507827282939 + 84.064753590746221 i
14,20,1	16.227867417752197 + 23.520581827394111 i	405.950659543331540 +263.637767950454815 i	14,20,3	12.412866376022718 + 26.430284847294875 i	308.281456525977148 +216.596353877976213 i
14,20,5	8.106506098686731 + 28.865150189472232 i	237.836585529534432 +138.737918182017779 i	14,20,7	2.856882920949500 + 30.930635837811909 i	199.002680930703207 + 37.399209294309806 i
14,21,1	18.107029442999192 + 23.996347196460757 i	462.436487841142934 +312.232440548156887 i	14,21,3	14.446342194487563 + 26.989976098669899 i	353.063511439638262 +270.406430146541027 i
14,21,5	10.385058641792700 + 29.522726660777657 i	269.252333224976041 +196.092285558968712 i	14,21,7	5.670897213261807 + 31.708355332354149 i	213.839152541380940 + 97.292828041709313 i
14,22,1	19.950909085566902 + 24.440877460970167 i	524.670271593986172 +362.504312381270609 i	14,22,3	16.418264946015714 + 27.510786411574891 i	404.095223547057060 +326.250503132749316 i
14,22,5	12.546234174782656 + 30.130660300603125 i	307.747621086410788 +255.889739680811999 i	14,22,7	8.168501605252811 + 32.419062719000586 i	237.559671055225607 +159.845889393132381 i
14,22,9	2.855075811483945 + 34.407122250434291 i	198.954788340844544 + 42.261941678338211 i	14,23,1	21.766059878667988 + 24.858434639886923 i	592.555847444401365 +414.324966769545824 i
14,23,3	18.342154842890221 + 27.998406409156853 i	461.189125300311048 +383.959941385273396 i	14,23,5	14.622947868869156 + 30.697028581768492 i	352.936342993217977 +317.918723463550577 i
14,23,7	10.488422944380440 + 33.075975214472884 i	269.124690565451772 +224.996936346419773 i	14,23,9	5.703302724353947 + 35.173998799629913 i	213.727189361106412 +109.837967073460931 i
14,24,1	23.557371714693776 + 25.252402558685542 i	666.015782470112640 +467.586008162716496 i	14,24,3	20.227469203459080 + 28.457263531328913 i	524.201663147753266 +443.394102896957122 i
14,24,5	16.635525543701650 + 31.227896237014807 i	404.545941973193237 +381.999949097207889 i	14,24,7	12.689693575618053 + 33.688122878319938 i	307.937311030854858 +292.561230676672380 i
14,24,9	8.238305094329705 + 35.881166201550975 i	237.513148842364558 +179.939244558867216 i	14,24,11	2.859859033879725 + 37.815214011072378 i	198.927632054513396 + 46.931315772219087 i
14,25,1	25.328595123713598 + 25.625523468182536 i	744.986400401754622 +522.194517031718760 i	14,25,3	22.081108785705240 + 28.890888412994553 i	593.019382849415024 +504.433776827816132 i
14,25,5	18.597565205454735 + 31.727972837026726 i	462.373331399297342 +447.981592538945904 i	14,25,7	14.804681125915145 + 34.262151659157375 i	353.601574444202981 +362.358693402760764 i
14,25,9	10.595638993619064 + 36.539591789928195 i	269.290744057190182 +252.582048688397578 i	14,25,11	5.740725401607159 + 38.575453141962655 i	213.691829734224370 +121.930917310867926 i
14,26,1	27.082672176210785 + 25.980057733134895 i	829.414434775043560 +578.069811376148550 i	14,26,3	23.908275988925556 + 29.302153356500366 i	667.550384970421987 +566.976496217915042 i
14,26,5	20.518681387623072 + 32.201011401110271 i	526.261230243496470 +515.733952129075305 i	14,26,7	16.853561676243249 + 34.803168640001431 i	405.833560892966204 +434.229918647814713 i
14,26,9	12.833243015056881 + 37.156863881812242 i	308.462050157111833 +327.621106432507531 i	14,26,11	8.310344787839771 + 39.281341572995125 i	237.589010974126040 +199.353537497472843 i
14,26,13	2.868196005482662 + 41.175154068533999 i	198.912595737213849 + 51.464717477007682 i	14,27,1	28.821954366427082 + 26.317895328196816 i	919.254691129176877 +635.141072424029403 i
14,27,3	25.712996382365546 + 29.693434057758285 i	747.718746896600805 +630.933124500290887 i	14,27,5	22.405955190948578 + 32.650065008135805 i	596.084227889743602 +585.145051356855561 i
14,27,7	18.849949773303127 + 35.315218698071845 i	464.419117335432759 +508.036144793633468 i	14,27,9	14.983078096573790 + 37.738659172813705 i	354.624367050290004 +404.900346079689086 i
14,27,11	10.702115189256189 + 39.942332352492407 i	269.609852110527186 +279.277329339968958 i	14,27,13	5.780156423279640 + 41.931223846233820 i	213.700760556409023 +133.703549497782319 i
14,28,1	30.548351172366232 + 26.640635931087225 i	1014.468362961397474 +693.345561186750274 i	14,28,3	27.498452045326118 + 30.066723032469827 i	833.460722431430099 +696.225318006381031 i
14,28,5	24.264768631129080 + 33.077659680328161 i	671.740012567585040 +656.117224731114561 i	14,28,7	20.803546853547576 + 35.801579781829695 i	529.190766554074912 +583.656459326375170 i
14,28,9	17.065193717722540 + 38.289408310579176 i	407.485397225831434 +484.276016081496437 i	14,28,11	12.973194000713523 + 40.565003965870908 i	309.155717303450899 +361.588088231174595 i
14,28,13	8.382105162178320 + 42.637109020126665 i	237.734132523086487 +218.283136503832480 i	14,28,15	2.878500583848931 + 44.499582629312279 i	198.905109574769540 + 55.897473473207917 i
14,29,1	32.263434358898699 + 26.949647864365364 i	1115.021786872452594 +752.627250654097338 i	14,29,3	29.267203470200734 + 30.423711430066898 i	924.722060918221132 +762.783602223695084 i
14,29,5	26.099316332592824 + 33.485915139254971 i	753.143562003523016 +728.564491367207893 i	14,29,7	22.721548297724436 + 36.264957293761732 i	600.013901459292128 +660.984875204414266 i
14,29,9	19.093202618159523 + 38.812669846690838 i	466.822469855025474 +565.619946867543717 i	14,29,11	15.155237285429886 + 41.154329147896028 i	355.818735508399357 +446.148960286506679 i
14,29,13	10.805906772179624 + 43.301163794492467 i	270.012361138516155 +305.333749383793815 i	14,29,15	5.820071652066548 + 45.253018369160749 i	213.736568927415675 +145.238323321063774 i

Table D.15: Branch points and associated eigenvalues for $m = 14$.

Table D.16: Branch points and associated eigenvalues for $m = 15$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
15,15,1	3.100070327196836 + 20.583404859421176 i	228.808578659851094 + 20.583925223282463 i	15,16,1	5.837054691724287 + 21.517117054212459 i	245.618838681163538 + 55.090109529272148 i
15,17,1	8.219886192915762 + 22.312116091680085 i	270.810037228819056 + 93.430292020288277 i	15,17,3	2.956074637885490 + 24.756558509552992 i	228.338254348108904 + 27.532551473384039 i
15,18,1	10.419088753688115 + 23.012622609411697 i	303.245238073441556 +134.940122180000827 i	15,18,3	5.733092489629859 + 25.604288259373931 i	244.058454443672588 + 72.217430188026512 i
15,19,1	12.502506719789519 + 23.642852161402669 i	342.355614580070153 +179.153921182499232 i	15,19,3	8.172844795259783 + 26.355641042252849 i	268.476399432270739 +120.153152091889311 i
15,19,5	2.904999878453966 + 28.551699983282063 i	228.148712127849222 + 33.311422009208471 i	15,20,1	14.505292057763960 + 24.217958179485237 i	387.794949337370156 +225.736088987153209 i
15,20,3	10.429810539007500 + 27.034639001667617 i	300.462114050424361 +171.001206500984125 i	15,20,5	5.715687219528892 + 29.360735978800729 i	243.421803181244087 + 86.861509006582011 i
15,21,1	16.448364871140679 + 24.748243174955871 i	439.329023921460134 +274.432998884935898 i	15,21,3	12.568143611779343 + 27.656435335700543 i	339.409710046771409 +224.435998639188028 i
15,21,5	8.199430743067150 + 30.092357901114219 i	267.566664847329832 +143.340382372188088 i	15,21,7	2.885527338399176 + 32.163136137477508 i	228.051673886595580 + 38.551604723238654 i
15,22,1	18.345312940481726 + 25.241122591317335 i	496.788360267975975 +325.045461075167111 i	15,22,3	14.622154093910858 + 28.231424644940716 i	384.936058651496239 +280.194370469787998 i
15,22,5	10.501785920860993 + 30.763155189047243 i	299.488565236112208 +202.580872510654928 i	15,22,7	5.728727731809384 + 32.951209042778643 i	243.117400631533798 +100.295436001800439 i
15,23,1	20.205514336907054 + 25.702161400794935 i	560.044447651475934 +377.412290042600034 i	15,23,3	16.612753905002688 + 28.767147117770229 i	436.775825369589825 +338.064132717979874 i
15,23,5	12.684263739921079 + 31.384243151702627 i	338.574799516150165 +264.340527461404974 i	15,23,7	8.251192115145312 + 33.672864081508557 i	267.190088032290987 +164.761065134434944 i
15,23,9	2.880289196338732 + 35.665167358283611 i	227.996731547044902 + 43.478758169690039 i	15,24,1	22.035738620332850 + 26.135671443429516 i	628.996417278979493 +431.399947055482812 i
15,24,3	18.553779341137666 + 29.269298831670586 i	494.734033929179930 +397.871350045522547 i	15,24,5	14.780311912339247 + 31.963619351716090 i	384.426661737628478 +328.403132517233018 i
15,24,7	10.593140286525260 + 34.340974163652731 i	299.204248399468781 +231.891179859276377 i	15,24,9	5.755034736759656 + 36.440739526457939 i	242.969811388937131 +113.006583678654820 i
15,25,1	23.841044313909194 + 26.545079065574988 i	703.562968665196195 +486.895688633750069 i	15,25,3	20.454922319950089 + 29.742318064183440 i	558.661251324570458 +459.470878665751172 i
15,25,5	16.810613058667162 + 32.507305651755487 i	436.762400841847409 +394.585121005625524 i	15,25,7	12.814312632139098 + 34.964401775975013 i	338.546924469831083 +301.499277996532214 i
15,25,9	8.312926616086058 + 37.157239826370628 i	267.06928680363052 +185.10943429489802 i	15,25,11	2.882387194483694 + 39.094799427620295 i	227.964302088571259 + 48.201054261198479 i
15,26,1	25.625316826060203 + 26.933163314714406 i	783.677181051996740 +543.802846768592076 i	15,26,3	22.323264240581182 + 30.189748685092042 i	628.43927852785623 +522.73963786606237 i
15,26,5	18.789021702241516 + 33.019988475526311 i	495.371886600184155 +462.730867546025138 i	15,26,7	14.947504400400605 + 35.549698443804047 i	384.810957772022789 +373.402467292812162 i
15,26,9	10.690740426464568 + 37.825325006930882 i	299.251271493337640 +259.809896527255887 i	15,26,11	5.787561984246370 + 39.862486531629926 i	242.907093012581413 +125.237651759227447 i
15,27,1	27.391608249985023 + 27.302216771647728 i	869.283020965044670 +602.037465659037025 i	15,27,3	24.164150560864879 + 30.614477131595109 i	703.972295710757521 +587.571794224815108 i
15,27,5	20.725345810445205 + 33.505408995535852 i	560.092248019622616 +532.707313439276845 i	15,27,7	17.013182039278959 + 36.101914072846647 i	437.704083294366058 +447.438435970750675 i
15,27,9	12.946993180883416 + 38.452422263514080 i	338.903865774848498 +336.962545097799534 i	15,27,11	8.378387965277447 + 40.576461581127397 i	267.087025134157102 +204.737621922967662 i
15,27,13	2.888553432688683 + 42.473243747103993 i	227.945146577272254 + 52.779011859688630 i	15,28,1	29.142361291849113 + 27.654157895112803 i	960.332901959976994 +661.525835813145704 i
15,28,3	25.981722689211395 + 31.018893356255532 i	785.181079272846318 +653.875251251409281 i	15,28,5	22.626817486160544 + 33.966615239654672 i	630.79351709548962 +604.39951002630607 i
15,28,7	19.025171541376295 + 36.625057560600155 i	497.005652182466179 +523.465675185683835 i	15,28,9	15.113997391886739 + 39.044112033752953 i	385.614408398038734 +416.409726693395442 i
15,28,11	10.789285334088278 + 41.245901323723558 i	299.475897538056358 +286.786755918268966 i	15,28,13	5.822956146312108 + 43.235788056294517 i	242.894214288069691 +137.127725939266412 i
15,29,1	30.879562079145739 + 27.990611758237705 i	1056.785926214409983 +722.202641758375421 i	15,29,3	27.779258207064316 + 31.405003843887098 i	871.999060294858850 +721.569038148375398 i
15,29,5	24.498939769599737 + 34.406132397731248 i	707.369586657719537 +677.707146334241884 i	15,29,7	20.993335677915493 + 37.122383829042235 i	562.542967546066961 +601.360745911170852 i
15,29,9	17.212043313396158 + 39.604763184645719 i	439.083052612581753 +498.005744900726995 i	15,29,11	13.077894814440826 + 41.877234644780032 i	339.462811652488426 +371.271757182720705 i
15,29,13	8.444657284497017 + 43.948768861628558 i	267.185133527939342 +223.850237255258207 i	15,29,15	2.897061349051949 + 45.813794427480218 i	227.934342042181100 + 57.249961097883755 i
15,30,1	32.604847535272505 + 28.312969487475041 i	1158.606584272363534 +784.009541429967385 i	15,30,3	29.559397523126897 + 31.774513213872495 i	964.369543131489877 +790.581329194489399 i
15,30,5	26.346006317959656 + 34.826082026862245 i	789.732218695843699 +752.541872242770751 i	15,30,7	22.924997124577100 + 37.596583569805254 i	634.177124578814983 +681.015357794555030 i
15,30,9	19.254920607422662 + 40.137892898105029 i	499.081170898926757 +581.620447899845885 i	15,30,11	15.276167070991828 + 42.475341378088402 i	386.631003985641996 +458.055044603882607 i
15,30,13	10.886394928061067 + 44.620295118059495 i	299.801124163455370 +313.085301634231826 i	15,30,15	5.859475307607606 + 46.572924766700446 i	242.912162126102572 +148.763997286871728 i

Table D.16: Branch points and associated eigenvalues for $m = 15$.

Table D.17: Branch points and associated eigenvalues for $m = 16$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
16,16,1	3.149648673316651 + 21.686964764441676 i	259.914385613627871 + 21.363849119935484 i	16,17,1	5.927345278764850 + 22.642265137931808 i	277.144536311756895 + 57.187781244274277 i
16,18,1	8.342336430867860 + 23.457706021987825 i	302.909537565066671 + 96.995989750156966 i	16,18,3	2.994738944723596 + 25.912651594317300 i	259.412096030288978 + 28.474936499322364 i
16,19,1	10.568701423009793 + 24.177628883799311 i	336.027521915461818 +140.101116903125330 i	16,19,3	5.807284517267507 + 26.775989236544770 i	275.452626611809080 + 74.694093318361794 i
16,20,1	12.675828660877125 + 24.826385983069315 i	375.906430290538879 +186.020719797593642 i	16,20,3	8.276125111289824 + 27.542991810087678 i	300.335569674931776 +124.268989531653062 i
16,20,5	2.937318183749837 + 29.746317174675681 i	259.206884885598356 + 34.361417365364687 i	16,21,1	14.699729464231632 + 25.419240058541693 i	422.185545537713551 +234.406933090452924 i
16,21,3	10.558187758653387 + 28.237397953292074 i	332.893589805733768 +176.854299733073304 i	16,21,5	5.779553661739028 + 30.567614876695043 i	274.743956951570169 + 89.603407262127604 i
16,22,1	16.661868332607796 + 25.966579538187425 i	474.620617095667512 +284.996778790555709 i	16,22,3	12.718752019905612 + 28.874271449798389 i	372.499764019511190 +232.115845953488588 i

Table D.17: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
16,22,5	8.289739292645447 + 31.311917163453021 i	299.285119657120958 +147.852910396260086 i	16,22,7	2.913466888730951 + 33.388293456599023 i	259.100047648045518 + 39.682247768531290 i
16,23,1	18.576207058345890 + 26.475888644153102 i	533.034751518895405 +337.583675808229827 i	16,23,3	14.792782560639520 + 29.463978371685997 i	418.757214264052379 +289.783041216200502 i
16,23,5	10.615267333986406 + 31.995475459463936 i	331.705005280873820 +208.943059639543719 i	16,23,7	5.785084328058061 + 34.186424981937783 i	274.391856796124216 +103.241654486519494 i
16,24,1	20.452395427059113 + 26.952788647998592 i	597.293698252963168 +392.000425085675715 i	16,24,3	16.801620067111877 + 30.014054326937590 i	471.390827264319910 +349.637492184867767 i
16,24,5	12.818511257287724 + 32.629258553724789 i	371.372667112545798 +272.627117450602498 i	16,24,7	8.331771015368110 + 34.918693299514082 i	298.811679852826785 +169.584360016771143 i
16,24,9	2.904952557279039 + 36.915531290820248 i	259.038237346937365 + 44.673548797618842 i	16,25,1	22.297408623773578 + 27.401638421070100 i	667.291992130014478 +448.108480558852193 i
16,25,3	18.759403172423152 + 30.530201265655585 i	530.198272132428315 +411.500061375235077 i	16,25,5	14.933431965447259 + 33.221190192296568 i	417.877029307289263 +338.684279142151240 i
16,25,7	10.695201426680910 + 35.597648693928917 i	331.268486549409147 +238.657052921743485 i	16,25,9	5.805575324876602 + 37.699583598453188 i	274.209590279571103 +116.118162686717696 i
16,26,1	24.116465705305927 + 27.825904288864535 i	742.944554965336238 +505.790850058950127 i	16,26,3	20.676048046841117 + 31.016868056065928 i	595.024358255554603 +475.221191271940086 i
16,26,5	16.981055290648246 + 33.777252737297452 i	470.927316974284054 +406.926766087282658 i	16,26,7	12.935803454481869 + 36.231997067060362 i	371.133520677225818 +310.271458373808855 i
16,26,9	8.385811894062813 + 38.425162469042192 i	298.61861868626096 +190.18703975174282 i	16,26,11	2.904478595251654 + 40.366507678793759 i	259.000663649745036 + 49.448729568463577 i
16,27,1	25.913580521416087 + 28.228399784814812 i	824.181296822214790 +564.947204246326692 i	16,27,3	22.558808699790639 + 31.477611198042670 i	665.74624090486077 +540.67400846649980 i
16,27,5	18.975482174864343 + 34.302111328855077 i	530.306488399569730 +477.195213666539644 i	16,27,7	15.086786405952548 + 36.828201838754033 i	417.988608585723284 +384.241894356978094 i
16,27,9	10.783631850449005 + 39.102632392513883 i	331.199764236367344 +266.908894595463323 i	16,27,11	5.833420338674197 + 41.141483068973834 i	274.120208841341025 +128.487262513045494 i
16,28,1	27.691909983835522 + 28.611447605382935 i	910.943481168468338 +625.490390980065058 i	16,28,3	24.413167266922812 + 31.915330469839752 i	742.264262167070342 +607.749381520171141 i
16,28,5	20.926706227569618 + 34.799496268429991 i	595.846188648584416 +549.353262833089161 i	16,28,7	17.168897276148900 + 37.391259587561365 i	471.53328117823952 +460.40317613368234 i
16,28,9	13.058116518936938 + 39.739269283282525 i	371.327351534061393 +346.138106326307536 i	16,28,11	8.444983373597461 + 41.863348543089927 i	298.579744414918594 +210.029213863459148 i
16,28,13	2.908559544956998 + 43.763356738687939 i	258.977483349588624 + 54.071358589246536 i	16,29,1	29.453983744757831 + 28.976992782587203 i	1003.181184457096379 +687.343879579196596 i
16,29,3	26.243375832232978 + 32.332429194665927 i	824.495966960988994 +676.352315344581598 i	16,29,5	22.842104377290596 + 35.272450963754572 i	667.41197957482007 +623.28304441634486 i
16,29,7	19.196166014564348 + 37.925145641031193 i	531.540505474921474 +538.611082814770043 i	16,29,9	15.241920365581436 + 40.340563290289985 i	418.578906426913761 +427.715435427092586 i
16,29,11	10.874593033314657 + 42.541019964644327 i	331.332403474933585 +294.167809854965242 i	16,29,13	5.864942832168929 + 44.532256421729365 i	274.086045115888396 +140.495062751326742 i
16,30,1	31.201861043913382 + 29.326684044299277 i	1100.851465045483110 +750.439841009276051 i	16,30,3	28.052803238251723 + 32.730926905267971 i	912.372023107944756 +746.399267444515544 i
16,30,5	24.727295691495748 + 35.723499860231151 i	744.894031804844985 +698.881649116923995 i	16,30,7	21.178610820084650 + 38.433093409925156 i	597.832455973582228 +618.739683668213843 i
16,30,9	17.355567252685187 + 40.910825340824260 i	472.647148864502924 +511.493250766710162 i	16,30,11	13.180363639346227 + 43.180785248385838 i	371.755177310458919 +380.790598062330048 i
16,30,13	8.505988595961931 + 45.252180049458119 i	298.631999415257894 +229.325462787458974 i	16,30,15	2.915336748113825 + 47.119997114470934 i	258.963431096560669 + 58.580717542215496 i
16,31,1	32.937241001408964 + 29.661933732680573 i	1203.917008821132868 +814.717675479264017 i	16,31,3	29.844166283718110 + 33.112540768686621 i	1005.833341382597382 +817.816111367356029 i
16,31,5	26.586668751310196 + 36.154766219944683 i	828.200953941216426 +776.058400823855095 i	16,31,7	23.123676891362852 + 38.917779811103514 i	670.266006045593485 +700.678418480307982 i
16,31,9	19.413018165938791 + 41.453533799682944 i	533.297588383147399 +597.339434728054016 i	16,31,11	15.394533598109856 + 43.787436763845839 i	419.422527955847841 +469.759247399348112 i
16,31,13	10.965297210179159 + 45.931004327555421 i	331.582287133054535 +320.709588646783970 i	16,31,15	5.898196514718546 + 47.884733037378489 i	274.086535902157379 +152.233363796549952 i

Table D.17: Branch points and associated eigenvalues for $m = 16$.

Table D.18: Branch points and associated eigenvalues for $m = 17$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
17,17,1	3.197512982295725 + 22.786646902092176 i	293.018167585218805 + 22.128065940071181 i	17,18,1	6.014574080227733 + 23.762672908768973 i	310.660171196880071 + 59.243209094253928 i
17,19,1	8.460731647192855 + 24.597730105352479 i	336.988062490769721 +100.489885798006100 i	17,19,3	3.032234896764332 + 27.063387858458558 i	292.484697550920885 + 29.399057642588897 i
17,20,1	10.713476809257328 + 25.336288144544145 i	370.775948686713662 +145.158175419390403 i	17,20,3	5.879238093833929 + 27.941783398155103 i	308.840069373780238 + 77.122881151825084 i
17,21,1	12.843676929889094 + 26.002835006871514 i	411.409013710472607 +192.749197604645190 i	17,21,3	8.376334498080343 + 28.723868477038732 i	334.180034596825863 +128.305441136289971 i
17,21,5	2.968773784583193 + 30.934706618357712 i	292.26421455156884 + 35.39186102565751 i	17,22,1	14.888165369574903 + 26.612740116108998 i	458.512342269010674 +242.903062836853280 i
17,22,3	10.682814411815851 + 29.433120650066321 i	367.300587768626315 +182.594693600301468 i	17,22,5	5.841689101170195 + 31.767885108323188 i	308.061255723826264 + 92.294450138983720 i
17,23,1	16.868926906288287 + 27.176474922686534 i	511.831863605022875 +295.347627927345115 i	17,23,3	12.865041366933637 + 30.084522948766725 i	407.554164631734125 +239.648022208163951 i
17,23,5	8.377615878011065 + 32.524454915550962 i	332.992617062012812 +152.282034768339429 i	17,23,7	2.940743812206563 + 34.606678726225660 i	292.147824974253922 + 40.792664039659329 i
17,24,1	18.800280556226447 + 27.701589627583029 i	571.183427663701650 +349.869069773213425 i	17,24,3	14.958609225991699 + 30.688416645679879 i	454.530398067783381 +299.187540663379366 i
17,24,5	10.725729152566029 + 33.220351501150120 i	365.902805762832941 +215.187995710512865 i	17,24,7	5.840060595016042 + 35.414594005362787 i	307.662691368611036 +106.135415195841802 i
17,25,1	20.692135531428708 + 28.193759199790183 i	636.427168254230423 +406.294299257073362 i	17,25,3	16.985270268929035 + 31.252334291751122 i	507.944583262378785 +360.988762003312161 i
17,25,5	12.949237101017774 + 33.866407363405912 i	406.142944280717472 +280.761359476977589 i	17,25,7	8.410373587230705 + 36.157165376444041 i	332.424870448720526 +174.322147179489491 i
17,25,9	2.929094336875818 + 38.158780317137811 i	292.079314423046469 + 45.847792991295258 i	17,26,1	22.551663315763967 + 28.657387158893521 i	707.453124010392798 +464.479867625570137 i
17,26,3	18.959453083117374 + 31.781984130447558 i	567.587177095789343 +424.867440840475183 i	17,26,5	15.082596185438309 + 34.470477848218124 i	453.289801144336309 +348.776796026827355 i
17,26,7	10.794776051288457 + 36.846639764315604 i	365.318167160039081 +245.303421151429319 i	17,26,9	5.854992430014150 + 38.951109664677148 i	307.446620638486195 +119.176510866923834 i
17,27,1	24.384236225381265 + 29.095977638303022 i	784.172521708880709 +524.304624409546136 i	17,27,3	20.891288768099209 + 32.281825456260044 i	633.297343907216828 +490.669687296469306 i
17,27,5	17.147163101355055 + 35.038508836215719 i	507.043736053147256 +419.042378078503361 i	17,27,7	13.054364348751630 + 37.491576259556737 i	405.698269527514697 +318.889185086736973 i
17,27,9	8.457062154573725 + 39.685529147562455 i	332.16140522401696 +195.17820469046828 i	17,27,11	2.926152383292890 + 41.630891215770816 i	292.036716441875058 + 50.675762785628699 i
17,28,1	26.193990755428448 + 29.512376681862452 i	866.512171632186210 +585.664658216906332 i	17,28,3	22.788196700465274 + 32.755426916606474 i	704.947684820448671 +558.264906431254190 i
17,28,5	19.157276045491496 + 35.575145874949570 i	567.180918614102438 +491.395060183946498 i	17,28,7	15.222749361820023 + 38.098355813664192 i	453.136162608014956 +394.890991470379447 i
17,28,9	10.874442728684672 + 40.372127370471837 i	365.136717120959304 +273.887551394993807 i	17,28,11	5.878350366885606 + 42.413002685780882 i	307.331221006358987 +131.683404568048189 i
17,29,1	27.984183938756249 + 29.908935303511665 i	954.410693245248581 +648.469690966125995 i	17,29,3	24.655789931407760 + 33.205700977161953 i	782.434779930562058 +627.540731783145930 i
17,29,5	21.123107201652854 + 36.084109300475589 i	633.527608438404400 +565.695236046239643 i	17,29,7	17.320949632764879 + 38.671925129893637 i	507.323305252206410 +473.140814924449558 i
17,29,9	13.166766530796865 + 41.018037762035696 i	405.733294423899679 +355.158697803128860 i	17,29,11	8.510207731990958 + 43.142573483312198 i	332.067316961112510 +215.234195274503634 i
17,29,13	2.928227054061864 + 45.046026718203748 i	292.009599962990535 + 55.343115115797033 i	17,30,1	29.757428513009984 + 30.287623469828389 i	1047.815876696002988 +712.640433937753301 i
17,30,3	26.498426549993290 + 33.635063591721583 i	865.673006989071268 +698.399326018103579 i	17,30,5	23.052172720052415 + 36.568438321603701 i	705.94497825886242 +641.82216418121948 i
17,30,7	19.363192221261691 + 39.216228140655126 i	568.026384751823912 +553.491756436760852 i	17,30,9	15.367021028526235 + 41.628665528863237 i	453.518990243946007 +438.830816548987173 i
17,30,11	10.958138320184757 + 43.828272431181169 i	365.179676832266864 +301.428611823297615 i	17,30,13	5.906153372472296 + 45.821166834339813 i	307.276262175370221 +143.809051702061595 i
17,31,1	31.515853620220174 + 30.650112067669969 i	1146.682780599865737 +778.106601659642706 i	17,31,3	28.319562816813974 + 34.045546564679185 i	954.590325642782545 +770.754625811203709 i
17,31,5	24.950203158118551 + 37.030656753658576 i	784.319544474013355 +719.670380853094684 i	17,31,7	21.359645360131111 + 39.734478125467959 i	635.062532008703556 +635.815426801304284 i
17,31,9	17.495957066099631 + 42.208267110148839 i	508.179200464861083 +524.754353166373107 i	17,31,11	13.280720761971079 + 44.476254220202371 i	406.033328258830295 +390.154989960638147 i
17,31,13	8.566157766791834 + 46.547887903147327 i	332.074799382350022 +234.714421499871008 i	17,31,15	2.933334952313332 + 48.418703084466372 i	291.992366919170877 + 59.891035172402930 i
17,32,1	33.261217881839556 + 30.997833287265781 i	1250.972324712049385 +844.805388650050077 i	17,32,3	30.121988718996977 + 34.438878828473960 i	1049.125300775867117 +844.530250251292841 i
17,32,5	26.821678286287657 + 37.472889480928899 i	868.556820221101248 +799.146924115592356 i	17,32,7	23.317872415602931 + 40.229339090138588 i	708.284469373949264 +719.999027665661970 i
17,32,9	19.567701795250361 + 42.760284357543405 i	569.473659784286383 +612.795241002368016 i	17,32,11	15.510474746523359 + 45.091228643103960 i	454.194071225340110 +481.274230091977302 i
17,32,13	11.042691652685857 + 47.233846108781805 i	365.356022691675946 +328.214336145174286 i	17,32,15	5.936262601760209 + 49.188957583309943 i	307.259675612245303 +155.649748731234297 i

Table D.18: Branch points and associated eigenvalues for $m = 17$.

Table D.19: Branch points and associated eigenvalues for $m = 18$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
18,18,1	3.243807365992813 + 23.882798724858244 i	328.120069629988109 + 22.877763965401318 i	18,19,1	6.098994494417352 + 24.878767694397144 i	346.166464857783978 + 61.259587558234273 i
18,20,1	8.575400704295761 + 25.732691794189890 i	373.047111726099445 +103.917407353218479 i	18,20,3	3.068649515209791 + 28.209213346078460 i	327.556135115707320 + 30.306240453820845 i
18,21,1	10.853799165589304 + 26.489174884649607 i	407.492929529723654 +150.119151708172751 i	18,21,3	5.949120404443218 + 29.102162571796953 i	344.221207048667566 + 79.507257124675931 i
18,22,1	13.006477515798903 + 27.172839638868489 i	448.866784166951049 +199.349805162380989 i	18,22,3	8.473699627343366 + 29.898810652235500 i	370.010727878311400 +132.268234319113219 i
18,22,5	2.999423562516335 + 32.117356240751157 i	327.320743378221380 + 36.404113153457914 i	18,23,1	15.071057574485323 + 27.799159815095198 i	496.779840397455587 +251.237676872622984 i
18,23,3	10.803963171874798 + 30.622393803442417 i	403.684664872785902 +188.230501544447236 i	18,23,5	5.902210525410298 + 32.962062849127229 i	343.373961407506329 + 94.938158963737521 i
18,24,1	17.070022703121005 + 28.378687347342375 i	550.968402637660120 +305.501630123343830 i	18,24,3	13.007320578917671 + 31.287821945754498 i	444.575178497343359 +247.043140056852508 i
18,24,5	8.463224418122337 + 33.730518974587177 i	368.689763397062279 +156.633507185945945 i	18,24,7	2.967396566115626 + 35.818794126892442 i	327.195028221183463 + 41.884202912976917 i
18,25,1	19.018034036403801 + 28.919035981537139 i	611.241210556083274 +361.920737256092593 i	18,25,3	15.119971862054435 + 31.905414866049014 i	492.25865436640754 +308.42108542753772 i
18,25,5	10.833372997777255 + 34.438363709227218 i	402.083011122648088 +221.323739837607803 i	18,25,7	5.893740935712430 + 36.636236199034947 i	342.930064992690347 +108.980198467570375 i
18,26,1	20.925249147128028 + 29.425932132375600 i	677.452896821977902 +420.316132596431032 i	18,26,3	17.164065329743572 + 32.482703438054102 i	546.440969970903552 +372.133860042596893 i
18,26,5	13.076673588095669 + 35.096302362641040 i	442.887187601493679 +288.753696104325172 i	18,26,7	8.487121605643557 + 37.388821902800878 i	368.030054554761648 +178.980071984237242 i
18,26,9	2.952740923330652 + 39.395415814538758 i	327.119972627558866 + 47.002806421225912 i	18,27,1	22.799027408808573 + 29.903821909710945 i	749.489096293479520 +480.539560728438289 i
18,27,3	19.154308306204165 + 33.025402609460023 i	606.905502084502928 +437.992198285501900 i	18,27,5	15.228062063471427 + 35.712126719498734 i	490.66710613104777 +358.69358232494221 i
18,27,7	10.892016891887796 + 38.088512025348955 i	401.353995218962780 +251.838142134396890 i	18,27,9	5.903348094282521 + 40.195830473236938 i	342.680996214369713 +122.185016428999475 i
18,28,1	24.644887827300238 + 30.356245381166499 i	827.257417056340955 +542.465795708804194 i	18,28,3	21.101038373013260 + 33.537981995157644 i	673.485874423449332 +505.837953450150735 i
18,28,5	17.309214704435181 + 36.291748894583542 i	545.114408974802473 +430.947382541350180 i	18,28,7	13.170173913805151 + 38.743727588815837 i	442.242248785790252 +327.362578358536950 i
18,28,9	8.526769487180058 + 40.938867397318077 i	367.69789730083414 +200.08840129315857 i	18,28,11	2.947426709670266 + 42.888442071810736 i	327.072462189322721 + 51.883424066565347 i
18,29,1	26.467084298840152 + 30.786079395837735 i	910.681633802471879 +605.987416650725891 i	18,29,3	23.011833707152771 + 34.024022000951511 i	746.05021890912928 +575.53687500313390 i
18,29,5	19.334698604313726 + 36.839796380609457 i	605.998591657619045 +505.348425553859499 i	18,29,7	15.355593589846022 + 39.360772132236576 i	490.255122698174489 +405.362192279526710 i
18,29,9	10.963290699979973 + 41.634354017444913 i	401.062599769873032 +280.753449873286002 i	18,29,11	5.922398215953169 + 43.677544486409467 i	342.540177678934015 +134.829347408891010 i
18,30,1	28.268969718217182 + 31.195701731815412 i	999.697776711861025 +671.011083900900758 i	18,30,3	24.892433045772425 + 34.486447236232443 i	824.491424273584585 +646.973419727606824 i
18,30,5	21.314858078367259 + 37.359980387504905 i	673.140620181430563 +581.753907701231014 i	18,30,7	17.469557724387339 + 39.944545592452684 i	545.076127784271648 +485.666149455648451 i
18,30,9	13.273082677479609 + 42.289289179445035 i	442.122435274007387 +364.034045516919434 i	18,30,11	8.574131557283780 + 44.414645687091323 i	367.549893062840340 +220.357831546104194 i
18,30,13	2.947568334164072 + 46.321731527518878 i	327.041492778630072 + 56.595500351249534 i	18,31,1	30.053236400259628 + 31.587105872929069 i	1094.251396787014755 +737.454808399752324 i
18,31,3	26.747296238701789 + 34.927685661487303 i	908.720760836088402 +720.046960089730078 i	18,31,5	23.257343389608753 + 37.855336331875975 i	746.397353689739361 +660.040260333808646 i
18,31,7	19.526484199426037 + 40.498962476870901 i	606.465769529603676 +568.124905428384523 i	18,31,9	15.489457532392612 + 42.908997960158025 i	490.435717015207274 +449.767773680149787 i
18,31,11	11.040012921149644 + 45.108179951064770 i	401.018019676599067 +308.576435714325214 i	18,31,13	5.946622331274103 + 47.103001303570338 i	342.464882043387320 +147.072830718985557 i
18,32,1	31.822080465388812 + 31.961983569407185 i	1194.295593911841934 +805.245900402627171 i	18,32,3	28.579963466927595 + 35.349780692979913 i	998.663531279764584 +794.668958855759001 i
18,32,5	25.167992446815362 + 38.328387530201020 i	825.651717719235648 +740.099506289074156 i	18,32,7	21.536686334643679 + 41.027217193876467 i	674.236214895992134 +652.607644269757998 i
18,32,9	17.633386940298745 + 43.497685426775902 i	545.680620016027888 +537.803168826697629 i	18,32,11	13.379076375914812 + 45.764175734576476 i	442.297762213567182 +399.374234503825164 i
18,32,13	8.625219277446133 + 47.836381378407289 i	367.513612166926123 +240.022157101018621 i	18,32,15	2.951064206296750 + 49.710373343131039 i	327.021142326558447 + 61.182079446122799 i

Table D.19: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
18,33,1	33.577317575170639 + 32.321785672881859 i	1299.789557636362133 +874.319396240210381 i	18,33,3	30.393294987102394 + 35.754472359782056 i	1094.25599898476298 +870.76082329320195 i
18,33,5	27.051372800383866 + 38.781260441919500 i	910.806183947284580 +821.836435225214018 i	18,33,7	23.507841671232704 + 41.531961635948292 i	748.236101503272948 +738.999359626655746 i
18,33,9	19.719159448782062 + 44.058759005979596 i	607.611187194078411 +628.004232153235989 i	18,33,11	15.624116798544009 + 46.387264668386742 i	490.946365917672324 +492.611344723926834 i
18,33,13	11.118650444006688 + 48.529318045331237 i	401.122511626251196 +335.606496249446934 i	18,33,15	5.973699533058825 + 50.486061584147357 i	342.431575746362228 +159.016153700331188 i

Table D.19: Branch points and associated eigenvalues for $m = 18$.

Table D.20: Branch points and associated eigenvalues for $m = 19$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
19,19,1	3.288657290277336 + 24.975720191907815 i	365.220219646296414 + 23.613982956289540 i	19,20,1	6.180827580540467 + 25.990918209475026 i	383.66405332523266 + 63.23971212509894 i
19,21,1	8.686631255296238 + 26.863025434946195 i	411.088007677182134 +107.283304208121223 i	19,21,3	3.104059786317983 + 29.350517006922949 i	364.626477365218420 + 31.197651620374479 i
19,22,1	10.990005492412493 + 27.636784865653013 i	446.180591514553555 +154.990919935631894 i	19,22,3	6.017079233582596 + 30.257555704964442 i	381.596420206315372 + 81.850271073252543 i
19,23,1	13.164604989012879 + 28.336952798666658 i	488.282759829117026 +205.831689810470976 i	19,23,3	8.568421128302441 + 31.068288994576722 i	407.828485765388791 +136.162414516372110 i
19,23,5	3.029318620353209 + 33.294694897575937 i	364.376510537843217 + 37.399377666720042 i	19,24,1	15.248809619190109 + 28.979105191424065 i	536.992015196369607 +259.422328558503978 i
19,24,3	10.921875808497227 + 31.805729246910467 i	442.047214794139563 +193.768873110095370 i	19,24,5	5.961222608586306 + 34.150602012810104 i	380.682313802340616 + 97.537652590966104 i
19,25,1	17.265581697892454 + 29.573872206774983 i	592.035218111062818 +315.472865073820969 i	19,25,3	13.145864086091880 + 32.484720045943277 i	483.564834190729471 +254.310553651790713 i
19,25,5	8.546711570384792 + 34.930591215275832 i	406.377111311887347 +160.912424710782858 i	19,25,7	2.993460244079865 + 37.025084058552494 i	364.241679082052030 + 42.958065086484339 i
19,26,1	19.229910809904446 + 30.128928995523029 i	653.214133427302613 +373.755394354818748 i	19,26,3	15.277171095988935 + 33.115562647986224 i	531.94470784845385 +317.49532723673633 i
19,26,5	10.938379417866670 + 35.650022741001962 i	440.246571083655056 +227.357436971822267 i	19,26,7	5.946201747533484 + 37.851812110566435 i	380.194128731730814 +111.779103404868820 i
19,27,1	21.152192869892506 + 30.650051448566209 i	720.377998679949997 +434.085379041706176 i	19,27,3	17.338326993303507 + 33.705787498803474 i	586.883457599737085 +383.086821956719405 i
19,27,5	13.201029011112373 + 36.319482853118110 i	481.606810515199925 +296.613388651758214 i	19,27,7	8.562125126374099 + 38.614142665134231 i	405.627599567256539 +183.563164502430766 i
19,27,9	2.975916799689133 + 40.625884149737438 i	364.160222510824838 + 48.139765560971199 i	19,28,1	23.039967519718434 + 31.141726471055833 i	793.408130564649582 +496.309845645205883 i
19,28,3	19.344307625142157 + 34.261116643833510 i	648.157503195962931 +450.890833843037967 i	19,28,5	15.370060782665531 + 36.946704118195584 i	530.01087570249854 +368.44608145072698 i
19,28,7	10.987061979440117 + 39.323765850671521 i	439.376623364015586 +258.268220584927747 i	19,28,9	5.950699113676743 + 41.434202844587091 i	379.912808730840140 +125.146711376153903 i
19,29,1	24.898894492999769 + 31.607528432753494 i	872.208589259796099 +560.299569054097333 i	19,29,3	21.305649363616894 + 34.786029953900972 i	715.595024982995596 +520.745029772275529 i
19,29,5	17.467460249152595 + 37.537567336846216 i	585.141830157732102 +442.655467652016739 i	19,29,7	13.283393653206675 + 39.988972481994125 i	480.766452866826057 +335.700663721378760 i
19,29,9	8.595017924673336 + 42.185647440510891 i	405.22833493697556 +204.92252963626950 i	19,29,11	2.968318731186582 + 44.139600663839886 i	364.107904080361550 + 53.072854374121164 i
19,30,1	26.733340320373490 + 32.050363503268940 i	956.700173448293299 +625.943685653805233 i	19,30,3	23.230083204103259 + 35.284119275967306 i	789.059764790093027 +592.511564976405566 i
19,30,5	19.508015742032686 + 38.096683348303557 i	646.762602633108031 +519.071302356860997 i	19,30,7	15.485500396710123 + 40.615993241689260 i	529.346872389550072 +415.666589407144500 i
19,30,9	11.050283006592055 + 42.889797478516144 i	438.977856595725314 +287.513383877682145 i	19,30,11	5.965606612437224 + 44.935555550673932 i	379.747129613489790 +137.928030161216583 i
19,31,1	28.546749880421238 + 32.472634871898592 i	1046.816374887077700 +693.145853392548020 i	19,31,3	25.123468801971114 + 35.758320906435003 i	868.440985780931669 +666.071773619372948 i
19,31,5	21.502237808202203 + 38.627755013565526 i	714.688951472862527 +597.547631539281610 i	19,31,7	17.614919618529040 + 41.209684111889963 i	584.793561594353037 +497.992384894260283 i
19,31,9	13.377192085363985 + 43.553524357600516 i	480.495470550124594 +372.772867311963466 i	19,31,11	8.636819802263165 + 45.680021593336505 i	405.027622597016659 +225.404859174524546 i
19,31,13	2.966595344281428 + 47.590900973472550 i	364.073160034991474 + 57.829613869188954 i	19,32,1	30.341891864585303 + 32.876358304301796 i	1142.500549341891428 +761.821435870329083 i
19,32,3	26.990363894160964 + 36.211074281881466 i	953.646906517868842 +741.322286255050017 i	19,32,5	23.457906145491155 + 39.133814346398289 i	788.773492890660691 +677.958101810016046 i
19,32,7	19.686254224588911 + 41.773931965963548 i	646.860936295109582 +582.525891740464544 i	19,32,9	15.609374095085478 + 44.182077376799157 i	529.330076232162696 +460.536980704407483 i
19,32,11	11.120301153811923 + 46.381209854480780 i	438.847726523526831 +315.617828440038465 i	19,32,13	5.986382085999859 + 48.378193707920779 i	379.651926663115319 +150.289232575276543 i
19,33,1	32.121026698745382 + 33.263245283492637 i	1243.703876218910864 +831.895386958842437 i	19,33,3	28.834389385725963 + 36.644433865161844 i	1044.600220486203398 +818.172135220386735 i
19,33,5	25.380962223133544 + 39.617384305656309 i	868.895620833581006 +760.192261031016983 i	19,33,7	21.709958065451904 + 42.311913494763289 i	715.356274386349583 +669.133897659569584 i
19,33,9	17.768015814350566 + 44.779613056385273 i	585.152725554598760 +550.652360326221356 i	19,33,11	13.475531695608097 + 47.045028890896326 i	480.548958369099410 +408.456707657886739 i
19,33,13	8.683223776690326 + 49.118100845819320 i	404.948522715123488 +245.253225817112650 i	19,33,15	2.968532710100945 + 50.995424401564485 i	364.049752211944906 + 62.454905852995452 i
19,34,1	33.886024829778684 + 33.634764285613841 i	1350.383848442354292 +903.300628392293447 i	19,34,3	30.658473131581687 + 37.060150205964256 i	1141.23493100174278 +896.54055543065954 i
19,34,5	27.276058246498348 + 40.080592905576601 i	954.954816041208460 +844.152685501081578 i	19,34,7	23.693819182539677 + 42.826269215066731 i	790.124191021001366 +757.699216978508389 i
19,34,9	19.867562758811253 + 45.349506297856337 i	647.711848171093834 +642.981092095884378 i	19,34,11	15.735575729336567 + 47.676036176818727 i	529.680111216003752 +503.780817792151538 i
19,34,13	11.193240479333587 + 49.817868467913613 i	438.881938698290810 +342.892352690092764 i	19,34,15	6.010531932367992 + 51.776463820630113 i	379.602237784089311 +162.335299014123564 i

Table D.20: Branch points and associated eigenvalues for $m = 19$.

Table D.21: Branch points and associated eigenvalues for $m = 20$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
20,20,1	3.332172757456104 + 26.065672282752498 i	404.318731124138822 + 24.337639283940558 i	20,21,1	6.260267510930434 + 27.099445082639406 i	423.153500869671632 + 65.186046749037686 i
20,22,1	8.794676575719921 + 27.989108671205127 i	451.111923871350029 +110.591763190706644 i	20,22,3	3.138534192884632 + 30.487640331889448 i	403.695786364858691 + 32.074324754246838 i
20,23,1	11.122393236033817 + 28.779549210415155 i	486.840823055910334 +159.779539872421054 i	20,23,3	6.083245954346093 + 31.408339457812037 i	420.966051715086280 + 84.154626122253944 i
20,24,1	13.318390735943865 + 29.495655518013912 i	529.659621597178557 +212.202915567516158 i	20,24,3	8.660677566152060 + 32.232716811932313 i	447.634060786705845 +139.992455155319959 i
20,24,5	3.058505051674317 + 34.467101838166826 i	403.431552344671786 + 38.378726566602992 i	20,25,1	15.421779328412230 + 30.153103789566838 i	579.152399687994603 +267.467203437572550 i
20,25,3	11.036767850336039 + 32.983576507036979 i	482.389491122679146 +199.216149023133994 i	20,25,5	6.018819432685269 + 35.333904187886240 i	419.986532913183737 +100.095709747866124 i
20,26,1	17.455982426415172 + 30.762600392658864 i	635.036742090019175 +325.273746134154351 i	20,26,3	13.280916956493986 + 33.675701826140646 i	524.524956156057101 +261.458560757181999 i
20,26,5	8.628209152375169 + 36.125098067470951 i	446.055165928837539 +165.123330752383367 i	20,26,7	3.018966945848839 + 38.225943965999932 i	403.287798435879665 + 44.015324804795327 i
20,27,1	19.436305633198057 + 31.331879981934346 i	697.107564741563237 +385.387779568294775 i	20,27,3	15.430475860136470 + 34.319378138744057 i	573.591009169358358 +326.420603873255573 i
20,27,5	11.040910827332839 + 36.855780592084841 i	480.394351988646690 +233.295458457874076 i	20,27,7	5.997512422813267 + 39.061731796469189 i	419.455024402236575 +114.534904524563387 i
20,28,1	21.373374089413442 + 31.866766185189899 i	765.208808566170205 +447.619192039310974 i	20,28,3	17.508343580292863 + 34.922136593805860 i	629.275167895756340 +393.860102619440113 i
20,28,5	13.322490957713892 + 37.536426297548900 i	522.303100872810887 +304.348698337248891 i	20,28,7	8.635483984744971 + 39.833555005743229 i	445.217847502948871 +188.075930489451935 i
20,28,9	2.998644692855094 + 41.850584712316397 i	403.200074988491053 + 49.259727685629485 i	20,29,1	23.274900765519710 + 32.371785123045136 i	839.217551695190991 +511.810374099193666 i
20,29,3	19.529755143336139 + 35.489706706467899 i	691.34701013006554 +463.57799140107261 i	20,29,5	15.508800813236721 + 38.174712447136230 i	571.322868420535769 +378.044503830020119 i
20,29,7	11.080036531221706 + 40.552847044388578 i	479.386656447165933 +264.599933946065505 i	20,29,9	5.997097608914697 + 42.666635791038324 i	419.142147152460325 +128.064322352031269 i
20,30,1	25.146680688277765 + 32.850544033591319 i	919.034372253811171 +577.828172106599823 i	20,30,3	21.505438676796683 + 36.026578568059811 i	759.629363736770642 +535.407816551201904 i
20,30,5	17.622125610615058 + 38.776490748247646 i	627.128269769572285 +454.178849556359808 i	20,30,7	13.394170161794836 + 41.227775621396432 i	521.271800821155466 +343.911532706513015 i
20,30,9	8.661884370413580 + 43.426290473175932 i	444.75294686961763 +209.68499890925114 i	20,30,11	2.988844635209127 + 45.384763016161422 i	403.143046354073704 + 54.245083340994135 i
20,31,1	26.993188671925889 + 33.305977103668432 i	1004.577148400238343 +645.558341455529444 i	20,31,3	23.443272521333110 + 36.536355205274241 i	833.981654381485214 +609.208196384115581 i
20,31,5	19.677467872577562 + 39.346356666445203 i	689.475767700410074 +532.577966216799155 i	20,31,7	15.612634489273965 + 41.864502686241352 i	570.412687972717094 +425.814130771418093 i
20,31,9	11.135517712894014 + 44.138892461156914 i	478.882907404428465 +294.173469678014328 i	20,31,11	6.008015170107905 + 46.187438168739533 i	418.952128855991759 +140.982106958030897 i
20,32,1	28.817958033725141 + 33.740511746334121 i	1095.776880888508458 +714.901592357068353 i	20,32,3	25.349232885656388 + 37.021984214258701 i	914.289581154561802 +684.857392942010506 i
20,32,5	21.685498952790915 + 39.888005433368697 i	758.175993591034057 +613.092793813746084 i	20,32,7	17.757215422289324 + 42.467842819096269 i	626.477276788182508 +510.131366982964494 i
20,32,9	13.479210999159250 + 44.811192179272105 i	520.853054617023076 +381.383014841357351 i	20,32,11	8.698332419923625 + 46.939112110550560 i	444.500653278650077 +230.379557969620066 i
20,32,13	2.985319583694551 + 48.853923283103768 i	403.104601409445081 + 59.046451800786258 i	20,33,1	30.623830911152861 + 34.156184924378778 i	1192.574771241872668 +785.770689989827019 i
20,33,3	27.227971844342761 + 37.485916052631751 i	1000.458362918482067 +762.249339849896273 i	20,33,5	23.654123683249896 + 40.404466030442485 i	833.077387036077084 +695.594234200570668 i
20,33,7	19.842695523604103 + 43.041656892681836 i	689.213980341372235 +596.708499203582960 i	20,33,9	15.726902639898530 + 45.448367100755917 i	570.202994187868057 +471.148054387363857 i
20,33,11	11.199080710405379 + 47.647783009209327 i	478.669082636012831 +322.558708704249682 i	20,33,13	6.025462971884858 + 49.647136220491824 i	418.837421788238487 +153.460825258010232 i
20,34,1	32.413129583734921 + 34.554726860224086 i	1294.920114159506511 +858.088276015381211 i	20,34,3	29.083188094497579 + 37.930215741826210 i	1092.408128648718503 +841.290679472964143 i
20,34,5	25.589383607764539 + 40.898261205181072 i	914.055866008656267 +779.969396428430628 i	20,34,7	21.879664963140907 + 43.589104939010227 i	758.425258039124401 +685.409947850659591 i
20,34,9	17.899989165028853 + 46.054527770174460 i	626.596747714790581 +563.313339678151578 i	20,34,11	13.570179922640150 + 48.319245288311741 i	520.787376235123588 +417.409984985025801 i
20,34,13	8.740218411268899 + 50.393444545480179 i	444.379619726588601 +250.411760490035647 i	20,34,15	2.985748537113114 + 52.274234028049932 i	403.078193111231940 + 63.710474044080726 i
20,35,1	34.187777211502706 + 34.937622742562922 i	1402.768737393686024 +931.785197761522682 i	20,35,3	30.917874635882349 + 38.356643852514502 i	1190.070660396313315 +921.898511063258752 i
20,35,5	27.496012711638187 + 41.371520526141076 i	1001.00796771798798 +866.11867633195709 i	20,35,7	23.876018887928851 + 44.112816782076486 i	833.951761222343521 +776.116375267844774 i
20,35,9	20.013068934758508 + 46.633018298299035 i	689.777205992781205 +657.739058111530653 i	20,35,11	15.844958323509646 + 48.957985914225858 i	570.395973188070116 +514.791902032760618 i
20,35,13	11.266523855795453 + 51.099902972695872 i	478.634489641311973 +350.077608245204658 i	20,35,15	6.046783125655977 + 53.060544375300040 i	418.77166854525132 +165.60965957735624 i

Table D.21: Branch points and associated eigenvalues for $m = 20$.

Table D.22: Branch points and associated eigenvalues for $m = 21$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
21,21,1	3.374450825609993 + 27.152883655887006 i	445.415705349111988 + 25.049545806753212 i	21,22,1	6.337485899810973 + 28.204629087682559 i	464.635311001953767 + 67.100777212380166 i
21,23,1	8.899761009370389 + 29.111272145893381 i	493.119907744584566 +113.846498710947174 i	21,23,3	3.172133959899390 + 31.620885003789503 i	444.764118440614084 + 32.937181002102804 i
21,24,1	11.251226450708193 + 29.917845436941864 i	529.475310333098264 +164.490387805885035 i	21,24,3	6.147737952610205 + 32.554846618682994 i	462.33041176824617 + 86.42273219350257 i
21,25,1	13.468129576090883 + 30.649369167754866 i	572.999764379662224 +218.470637187219069 i	21,25,3	8.750628673955513 + 33.392459280640942 i	489.428133092858332 +143.762345688587658 i
21,25,5	3.087024586935144 + 35.634914313414185 i	444.485902480410516 + 39.343119575425945 i	21,26,1	15.590285703818724 + 31.321617954605454 i	623.264151600634591 +275.381338895562237 i
21,26,3	11.148832382742128 + 34.156332801633958 i	524.712626469509019 +204.577985404556017 i	21,26,5	6.075085914003570 + 36.512326633002210 i	461.286821015957173 +102.614819420828098 i
21,27,1	17.641563025952820 + 31.945372535566445 i	679.976937962830903 +334.915287783651422 i	21,27,3	13.412699089352044 + 34.861195563804110 i	567.457192633342856 +268.494564704885590 i
21,27,5	8.707836145853280 + 37.314418968467507 i	487.724390189997450 +169.270296760728741 i	21,27,7	3.043946102142793 + 39.421727509891141 i	444.333406267305350 + 45.056947965691052 i
21,28,1	19.637571804700649 + 32.528425364939686 i	742.926307921509192 +396.830971042087128 i	21,28,3	15.580127857025499 + 35.517319422359357 i	617.199772470029416 +335.206140326689194 i
21,28,5	11.141113941317462 + 38.056039519944612 i	522.527146522830662 +239.143515823353340 i	21,28,7	6.047736198077319 + 40.266362182046784 i	460.712885029978906 +117.250097970875501 i

Table D.22: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
21,29,1	21.589158075031687 + 33.076646257970417 i	811.950997679896318 +460.932793358927370 i	21,29,3	17.674374632565371 + 36.132237254150581 i	673.618921822680932 +404.464817825196008 i
21,29,5	13.441229064677257 + 38.747557704584928 i	564.977235885303685 +311.967033008398175 i	21,29,7	8.707289059781091 + 41.047441442714259 i	486.801116943196519 +192.522425681837632 i
21,29,9	3.020945716896309 + 43.069876505197605 i	444.239541106579736 + 50.363647304590025 i	21,30,1	23.504201789532829 + 33.594599148717343 i	886.923921061584338 +527.058585762687585 i
21,30,3	19.710925037857869 + 36.711686392278139 i	736.477484442188711 +476.066742102611158 i	21,30,5	15.644470901460663 + 39.396599024993812 i	614.604690816102448 +387.498007418261409 i
21,30,7	11.171054540121592 + 41.776154749255276 i	521.384655775348961 +270.838935052061913 i	21,30,9	6.042591524347637 + 43.893497186206424 i	460.369097247113416 +130.940312480009226 i
21,31,1	25.388628293768463 + 34.085922874049047 i	967.742235604127245 +595.071331754197672 i	21,31,3	21.700692494422420 + 37.260167133233381 i	805.593020922938194 +549.841400781081234 i
21,31,5	17.773415547872126 + 40.008988113789442 i	671.075800985314572 +465.528487498729135 i	21,31,7	13.502636967867168 + 42.460553147670794 i	563.759143545733878 +352.002474275843667 i
21,31,9	8.727439397176898 + 44.661175478123305 i	486.271950519626444 +214.379794321408497 i	21,31,11	3.009019678172835 + 46.624286738262192 i	444.177893991906672 + 55.401044062755965 i
21,32,1	27.247016698889507 + 34.553578485480820 i	1054.320950766773421 +664.853463069651411 i	21,32,3	23.651697660586354 + 37.781293465597024 i	880.820710068634298 +625.643928983579940 i
21,32,5	19.843273208924927 + 40.589306234147131 i	734.140658158238125 +545.881226331478160 i	21,32,7	15.737146011608669 + 43.106733600916876 i	613.45374923272766 +435.81378024653276 i
21,32,9	11.219084750057462 + 45.382030223055040 i	520.778148355198141 +300.739237920668756 i	21,32,11	6.049660670409248 + 47.433555835878558 i	460.155227819593449 +143.993984566059794 i
21,33,1	29.082985508239531 + 35.000016570250970 i	1146.588620637123768 +736.302792897478867 i	21,33,3	25.570029286910579 + 38.278023953850408 i	962.042744547468828 +703.349563902815021 i
21,33,5	21.864871038013181 + 41.141241654702306 i	803.60484271777113 +628.40409997604843 i	21,33,7	17.896609467523636 + 43.719471613115367 i	670.128815479471655 +522.093772413082661 i
21,33,9	13.579246025305922 + 46.062696764588324 i	563.195802218236963 +389.871590697219744 i	21,33,11	8.758724861910542 + 48.192288684296017 i	485.969129485997769 +235.285811012345107 i
21,33,13	3.003752069970225 + 50.111150487900595 i	444.135817681515334 + 60.246920097340222 i	21,34,1	30.899447612713619 + 35.427294329498316 i	1244.48433086916998 +809.32953334480419 i
21,34,3	27.460430521158504 + 38.752819683266189 i	1049.161392338862242 +782.849584294449755 i	21,34,5	23.846235032912173 + 41.667820675564009 i	879.312680048216635 +712.965303305824250 i
21,34,7	19.995984570285366 + 44.302603550560029 i	733.526834849947932 +610.685154102371371 i	21,34,9	15.842164197856111 + 46.708284352831079 i	613.055338434180499 +481.609696894592787 i
21,34,11	11.276423342632766 + 48.908279991283138 i	520.482363021281230 +329.404449094018503 i	21,34,13	6.063893424215276 + 50.910184675323499 i	460.021395811167906 +156.589945694718466 i
21,35,1	32.698784889264040 + 35.837159846632844 i	1347.955524983506166 +883.854054786488632 i	21,35,3	29.326675153457028 + 39.207755809492078 i	1142.094259834481363 +864.048280338413747 i
21,35,5	25.793503595101040 + 42.171565980728104 i	961.136664481816752 +799.449540671271166 i	21,35,7	22.045993902688117 + 44.859273760291345 i	803.445514969694727 +701.450006276174122 i
21,35,9	18.029440534895532 + 47.322859905189333 i	670.013836555643252 +575.796436496250408 i	21,35,11	13.663107086540363 + 49.587215316012021 i	563.013455213356906 +426.240945827195105 i
21,35,13	8.796247127126815 + 51.662773982658159 i	485.806993844163799 +255.501524191455530 i	21,35,15	3.002719578005766 + 53.547146080400744 i	444.106462863365039 + 64.949659701171115 i
21,36,1	34.482971301083715 + 36.231114215915809 i	1456.956395086338034 +959.805167357497112 i	21,36,3	31.171819067540881 + 39.644602487914227 i	1240.770944245335158 +946.860649857174622 i
21,36,5	27.711489835590477 + 42.654608707285185 i	1048.970434306240122 +887.755058420215278 i	21,36,7	24.054636570866105 + 45.392102006636634 i	879.721598768703757 +794.266865388781298 i
21,36,9	20.155822389421559 + 47.909738330372761 i	733.808719186192434 +672.290114947644156 i	21,36,11	15.952363144634623 + 50.233514452958801 i	613.094585315544009 +525.653002748599079 i
21,36,13	11.338558315447337 + 52.375789874146754 i	520.380349008651478 +357.167457945666342 i	21,36,15	6.082475195425329 + 54.338649427963233 i	459.93987891342510 +168.84149508091463 i

Table D.22: Branch points and associated eigenvalues for $m = 21$.

Table D.23: Branch points and associated eigenvalues for $m = 22$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
22,22,1	3.415577625869812 + 28.237555921872474 i	488.511233188460892 + 25.750427780328604 i	22,23,1	6.412635278665273 + 29.306717660089486 i	508.109935382229196 + 68.985853849646446 i
22,24,1	9.002084356659068 + 30.229807184939826 i	537.112899100093387 +117.050825269657437 i	22,24,3	3.204914076592091 + 32.750519039837137 i	487.831524887154444 + 33.787045698390727 i
22,25,1	11.376740782597487 + 31.052006203376362 i	574.085566809239481 +169.128261390054604 i	22,25,3	6.210660611137221 + 33.697372863868509 i	505.689782088303332 + 88.656749278271604 i
22,26,1	13.614085133413758 + 31.798465158421386 i	618.305338688066286 +224.641239582755787 i	22,26,3	8.838418002085470 + 34.547840854069875 i	533.211319899186947 +147.475662833288510 i
22,26,5	3.114915138695604 + 36.798433752781207 i	487.539592212602827 + 40.293420147327708 i	22,27,1	15.754614538323469 + 32.485055390093670 i	669.330107732049214 +283.172800140400795 i
22,27,3	11.258243168618877 + 35.324351078399758 i	569.017648345297516 +209.859454335646603 i	22,27,5	6.130098993690380 + 37.686188774164201 i	504.583364490375686 +105.097221980933625 i
22,28,1	17.822626989003240 + 33.122630331014221 i	726.859368039610217 +344.407319934854627 i	22,28,3	13.541408670295821 + 36.041581869188228 i	612.363038842294259 +275.425205566420573 i
22,28,5	8.785700367469154 + 38.498893235607518 i	531.385209501061240 +173.356988961859713 i	22,28,7	3.068424759526676 + 40.612752446520979 i	487.378521633338153 + 46.083807006669566 i
22,29,1	19.834026982600783 + 33.719038694909493 i	790.674682479337548 +408.096640807937604 i	22,29,3	15.726345248379273 + 36.709793702657080 i	662.773006511033827 +343.860211764961605 i
22,29,5	11.239121806926637 + 39.251159303658085 i	566.645682075514401 +244.906753796634632 i	22,29,7	6.096930878152623 + 41.466033102468600 i	503.967835365467863 +119.926939557387996 i
22,30,1	21.799873802028472 + 34.280195093216413 i	860.609668515386147 +474.039768713150011 i	22,30,3	17.836654760516097 + 37.336522112444193 i	719.917279125012101 +414.910940165710656 i
22,30,5	13.557397319712226 + 39.953257268349908 i	609.630294908242831 +319.475067152272516 i	22,30,7	8.777623346510588 + 42.256145928995601 i	530.377704899229911 +196.906317017378598 i
22,30,9	3.042839507776337 + 44.284083596214444 i	487.278631876363477 + 51.452389777874160 i	22,31,1	23.728208555005100 + 34.810700019749632 i	936.533145152697216 +542.070046574572530 i
22,31,3	19.888065507796359 + 37.927512573407277 i	783.552067889250679 +488.368814154221354 i	22,31,5	15.777242572601565 + 40.612764089159185 i	659.857815151428326 +396.814845408344695 i
22,31,7	11.260220121218925 + 42.994047946255151 i	565.371142999559993 +276.990336786819626 i	22,31,9	6.087225064399372 + 45.115119288273604 i	503.593741350856931 +133.776916075703809 i
22,32,1	25.625082335518403 + 35.314222777900092 i	1018.338906881763137 +612.046656327906931 i	22,32,3	21.891670248625221 + 38.487275585436372 i	853.489746247493940 +564.059320854103703 i
22,32,5	17.921516353904490 + 41.235479163820507 i	716.986323164713895 +476.714259913840920 i	22,32,7	13.608916094547796 + 43.687679405589272 i	608.229270256841346 +359.980083271411559 i
22,32,9	8.791747937665852 + 45.890644871519306 i	529.785552205373506 +219.010532670201229 i	22,32,11	3.028858231640836 + 47.858496009254985 i	487.212452486298901 + 56.541585444788495 i
22,33,1	27.495174885340536 + 35.793750257566245 i	1105.939143008173914 +683.848759505774410 i	22,33,3	23.855627323545875 + 39.019435915400386 i	929.581311291669408 +641.834162714942750 i
22,33,5	20.005630517613262 + 41.825970627313644 i	780.759629380757701 +558.992630606886223 i	22,33,7	15.859172273248868 + 44.343075700202512 i	658.47114896949370 +445.67365029721515 i
22,33,9	11.301066815393817 + 46.619564367272851 i	564.663953117572419 +307.215710063303845 i	22,33,11	6.090577316552479 + 48.674238256955931 i	503.356478626293608 +146.965853830346163 i
22,34,1	29.342186899434623 + 36.251755286345016 i	1199.260002516501572 +757.371319512878927 i	22,34,3	25.786134325095972 + 39.526962809964992 i	1011.705503466353321 +721.565596476237033 i
22,34,5	22.040563379357906 + 42.387920402633910 i	850.97833490188930 +643.49480947942902 i	22,34,7	18.033252167289024 + 44.964975384403807 i	715.749604740642098 +533.889266054778886 i
22,34,9	13.677395200924336 + 47.308403426099169 i	607.524291015920653 +398.245045838331123 i	22,34,11	8.818048518980596 + 49.439888366638606 i	529.433191494159246 +240.127154815254670 i
22,34,13	3.021903332444110 + 51.362902946214618 i	487.166810472639952 + 61.431845674760318 i	22,35,1	31.169099540126382 + 36.690314450929954 i	1298.23849119998090 +832.52203759275744 i
22,35,3	27.688022461620790 + 40.012327639364827 i	1099.761685770190752 +803.142285480793817 i	22,35,5	24.034458428414325 + 42.924352440127294 i	927.482709792527771 +730.086320203636092 i
22,35,7	20.146283038680433 + 45.557191688859568 i	779.801287586853618 +624.467107414832185 i	22,35,9	15.955270113139234 + 47.962206370913492 i	657.887922086445428 +491.929814402526006 i
22,35,11	11.352395461589037 + 50.163046244027640 i	564.287831913330137 +336.159944817030009 i	22,35,13	6.101700115025295 + 52.167663080084722 i	503.203878877636555 +159.678728129116905 i
22,36,1	32.978352199255461 + 37.111192914312809 i	1402.820232898767472 +909.219050895764440 i	22,36,3	29.565138123557254 + 40.477615321906121 i	1193.664981448317121 +886.466203134786042 i
22,36,5	25.993547945392258 + 43.437789507128173 i	1010.141874116573717 +818.649494683873399 i	22,36,7	22.209116248067620 + 46.122854173491191 i	850.419216547988185 +717.266942579093552 i
22,36,9	18.156492846482735 + 48.584998644246660 i	715.405067958574364 +588.111038015916165 i	22,36,11	13.754392778509184 + 50.849293415201799 i	607.227614671142533 +434.955860603696578 i
22,36,13	8.851350943300831 + 52.926418469048912 i	529.230736320749656 +260.525955379129130 i	22,36,15	3.019453503512189 + 54.814474592136115 i	487.134560342795737 + 66.173264573392572 i
22,37,1	34.771967877660494 + 37.515906958160976 i	1512.957811846423056 +987.389166956671488 i	22,37,3	31.420597988658969 + 40.924605214333589 i	1293.342837152086076 +971.450277883442482 i
22,37,5	27.922721708768847 + 43.930364334215604 i	1098.846609323134999 +909.080459031576083 i	22,37,7	24.229851935032311 + 46.664573130708000 i	927.436278597416390 +812.165207054703160 i
22,37,9	20.295956138796146 + 49.180067412270766 i	779.807750297940629 +686.645156496609502 i	22,37,11	16.057881379911954 + 51.502985563277143 i	657.776549470596478 +536.371783723930321 i
22,37,13	11.409397641306366 + 53.645864797513198 i	564.119698645374001 +364.166650786241235 i	22,37,15	6.117629041308147 + 55.611095315376559 i	503.10688284298794 +172.03287556268708 i

Table D.23: Branch points and associated eigenvalues for $m = 22$.

Table D.24: Branch points and associated eigenvalues for $m = 23$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
23,23,1	3.455629994457168 + 29.319867866408334 i	533.605396552455659 + 26.440935731671601 i	23,24,1	6.485851914833604 + 30.405930114353211 i	553.57778110533172 + 70.84302612627731 i
23,25,1	9.101825445623559 + 31.344971950574891 i	583.09174520674105 + 120.20771615392540 i	23,25,3	3.236924141779512 + 33.876781773028100 i	532.898052570194409 + 34.624661955493750 i
23,26,1	11.499147539361553 + 32.182326315930126 i	620.672957390575903 + 173.697464532692806 i	23,26,3	6.272108947240272 + 34.836182241885268 i	551.044419465411693 + 90.858622777527160 i
23,27,1	13.756494236856041 + 32.943272721047826 i	665.578284704066732 + 230.720450719240262 i	23,27,3	8.924175108363459 + 35.699151275363157 i	578.984183403505313 +151.135628785678051 i
23,27,5	3.142211264049052 + 37.957930825379435 i	532.592650600395586 + 41.230408640850122 i	23,28,1	15.915023030316414 + 33.643777635338836 i	717.352828511877533 +290.848822711005539 i
23,28,3	11.365157234065487 + 36.487946542129912 i	615.305492451391160 + 215.065126073675117 i	23,28,5	6.183928638027909 + 38.855777532392558 i	549.876335440709568 +107.544943053537637 i
23,29,1	17.999447906580496 + 34.294765643618241 i	775.687249013350424 + 353.758661504604277 i	23,29,3	13.667225041087095 + 37.217200698726920 i	659.243856372455866 +282.256467428655753 i
23,29,5	8.861899871222329 + 39.678825704699676 i	577.038015769732738 + 177.386723362836392 i	23,29,7	3.092427830177499 + 41.799305489023098 i	532.423162660792400 + 47.096693246748835 i
23,30,1	20.025957994870442 + 34.904140316786062 i	840.356590643203390 + 419.195260724076263 i	23,30,3	15.869325726726759 + 37.897164769471843 i	710.312540703695277 +352.390276855100922 i
23,30,5	11.335055509912330 + 40.441463202864725 i	612.750627953619983 + 250.589827021509819 i	23,30,7	6.145149456380799 + 42.661042302192286 i	549.219992419448999 +122.567476341258711 i
23,31,1	22.005818781956005 + 35.477859804288656 i	911.189432815672717 + 486.952307254124605 i	23,31,3	17.995396854512837 + 38.535377787534529 i	768.172571406773613 +425.207459310496574 i
23,31,5	13.671135998354744 + 41.153866642413085 i	656.263270420129516 + 326.878840919517245 i	23,31,7	8.846562870392075 + 43.459979035607056 i	575.947888565177891 +201.230933492937680 i
23,31,9	3.064344348234320 + 45.493499662006933 i	532.317358159324499 + 52.526742685879981 i	23,32,1	23.947227161005937 + 36.020560030884205 i	988.050564931483791 +556.858722875194303 i
23,32,3	20.061402077963695 + 39.137593669001397 i	832.573622858885528 + 500.494780915453191 i	23,32,5	15.907272240846227 + 41.823567371862737 i	707.08359462870817 +406.00248815810437 i
23,32,7	11.347628659078026 + 44.206850839856138 i	611.346603652201907 + 283.058782463223906 i	23,32,9	6.131039075943911 + 46.331803348513802 i	548.816158270767914 +136.576167665785107 i
23,33,1	25.856355760394084 + 36.535939749950373 i	1070.830472449100853 + 628.769945363561760 i	23,33,3	22.078607980999067 + 39.708333124243254 i	903.322956884698556 +578.073783247488677 i
23,33,5	18.066598093810189 + 42.456341235976019 i	764.861581660392062 + 487.745109833324079 i	23,33,7	13.713119390604930 + 44.909492536124553 i	654.682914281676972 +367.850350624883731 i
23,33,9	8.854869883454255 + 47.115009217755322 i	575.293947510148996 + 223.580508839813660 i	23,33,11	3.048373831930324 + 49.087685763201044 i	532.246727666547827 + 57.667482581085750 i
23,34,1	27.737981569500981 + 37.027010770075357 i	1159.438570112786238 + 702.561916063076865 i	23,34,3	24.055306297909652 + 40.251231541871022 i	980.267450272268320 +657.792783718418832 i
23,34,5	20.164721449751845 + 43.056744228674845 i	829.334845517180838 + 571.922635094838597 i	23,34,7	15.978839223704924 + 45.573881074420861 i	705.46590143310857 +455.40111235767949 i
23,34,9	11.381540149782523 + 47.851815683969257 i	610.540674121804045 + 313.607462383651125 i	23,34,11	6.130796962878585 + 49.909785553167659 i	548.555932642453172 +149.899716137281388 i
23,35,1	29.595884709248558 + 37.496267323313887 i	1253.798640864927393 + 778.126792754452254 i	23,35,3	25.997800039463563 + 40.769268601517162 i	1063.282442326403043 +739.521100432282196 i
23,35,5	22.212767477535298 + 43.628452499672598 i	900.29907587960497 + 658.37692977366589 i	23,35,7	18.167281601765218 + 46.204720008930408 i	763.340968004344063 +545.526631856315475 i
23,35,9	13.773748916844985 + 48.548643646626132 i	653.839064094554828 + 406.509261230436721 i	23,35,11	8.876351110605514 + 50.682218080803992 i	574.892975000060574 +244.906821578113409 i
23,35,13	3.039783416165814 + 52.609473166408009 i	532.197582043952252 + 62.601985840097377 i	23,36,1	31.433112318604913 + 37.945804618967593 i	1353.845644506370490 +855.369805242986558 i
23,36,3	27.911005685534182 + 41.264925657652299 i	1152.264434334876341 + 823.144818445445367 i	23,36,5	24.218993745233849 + 44.174487962022994 i	977.590543196482827 +746.970880300255317 i
23,36,7	20.293739474120390 + 46.805800695625770 i	828.038995536529114 + 638.064586583798018 i	23,36,9	16.066323083935643 + 49.210475526706958 i	704.701507902020353 +502.115616541067482 i
23,36,11	11.427058664568730 + 51.412396420726303 i	610.085742591755775 + 342.829671653213609 i	23,36,13	6.138908082401524 + 53.419867435168676 i	548.38490221645975 +162.72912815019265 i
23,37,1	33.252159376556321 + 38.377404200623964 i	1459.523414851774305 + 934.206892983215312 i	23,37,3	29.798839917658984 + 41.740297063785725 i	1247.126103582351561 +908.563627678523630 i
23,37,5	26.189723646268266 + 44.697373610537390 i	1061.075039954316708 + 837.584476704195254 i	23,37,7	22.369189586959628 + 47.380238736849783 i	899.348374483640441 +732.872457602622717 i
23,37,9	18.281259537663665 + 49.841297270368223 i	762.771449557161645 + 600.265706538099607 i	23,37,11	13.844110793679143 + 52.105802510700194 i	653.430254343495108 +443.560464459453328 i
23,37,13	8.905568200215663 + 54.184678969125066 i	574.650938041576524 + 265.488206173975755 i	23,37,15	3.035957740891358 + 56.076507248446489 i	532.162485247492782 + 67.382025016147765 i
23,38,1	35.055096285222731 + 38.792596894826310 i	1570.782954439118612 +1014.562893181329741 i	23,38,3	31.664478269997286 + 42.197171039138745 i	1347.792778497724839 +995.688417303464462 i
23,38,5	28.129921343058101 + 45.199243801955120 i	1150.640530557551301 + 930.111752548559821 i	23,38,7	24.401830385106482 + 47.930635499487131 i	977.098185636519474 +829.824603345352671 i
23,38,9	20.433593012476084 + 50.444369645902164 i	827.775573911389415 + 700.814121455481995 i	23,38,11	16.161597578536046 + 52.766730739215241 i	704.442437164304351 +546.955256628137477 i
23,38,13	11.479092011445607 + 54.910434570989814 i	609.852716620252977 + 371.079542078312272 i	23,38,15	6.152264443407581 + 56.878171988398208 i	548.27269658343187 +175.18570317858192 i

Table D.24: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$

Table D.24: Branch points and associated eigenvalues for $m = 23$.

Table D.25: Branch points and associated eigenvalues for $m = 24$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
24,24,1	3.494676805535470 + 30.399978867175516 i	580.698269600673484 + 27.121655979946844 i	24,25,1	6.557258118628709 + 31.502461863884641 i	601.039216711622430 + 72.673870901185367 i
24,26,1	9.199145063978820 + 32.456996419232233 i	631.057213259136347 + 123.319851418679818 i	24,26,3	3.268209068278944 + 34.999887929093269 i	579.963744441054700 + 35.450701855050028 i
24,27,1	11.618637042096900 + 33.309068401448513 i	669.238718374686755 + 178.201876796070195 i	24,27,3	6.332168975015834 + 35.971511663669546 i	598.394558752804187 + 93.030112624134674 i
24,28,1	13.895570555978493 + 34.084085210471144 i	714.820360427748597 + 236.713433920692495 i	24,28,3	9.008017382450159 + 36.846650501777589 i	626.747237467935520 + 154.745159218672200 i
24,28,5	3.168944558754209 + 39.113649621410723 i	579.645104685210981 + 42.154793247054876 i	24,29,1	16.071743607105375 + 34.798106939398309 i	767.334634849909889 + 298.415929012608728 i
24,29,3	11.469717026732386 + 37.647402002604444 i	663.577013880181312 + 220.199136865662865 i	24,29,5	6.236638684089989 + 40.021351728573883 i	597.165893127261029 + 109.959821642504025 i
24,30,1	18.172273409688422 + 35.462127896856828 i	826.463497858009305 + 362.977262366421428 i	24,30,3	13.790311100604914 + 38.388357104361411 i	708.100889512212234 + 288.993766906408268 i
24,30,5	8.936524133298688 + 40.854491394008342 i	624.683170924406340 + 181.362511459415146 i	24,30,7	3.115978311032026 + 42.981646357212817 i	579.467346563478941 + 48.096327197295736 i
24,31,1	20.213624846961256 + 36.084105225927587 i	891.975572542796499 + 430.136270921274729 i	24,31,3	16.009249091289714 + 39.079759123109262 i	759.820047026274779 + 360.803087795154624 i
24,31,5	11.429025617775791 + 41.627242889060732 i	660.842601625497650 + 256.196963841215904 i	24,31,7	6.192440648185693 + 43.851659603677859 i	596.469465994393772 + 125.173573028447905 i
24,32,1	22.207263097612319 + 36.670039468826111 i	963.694476197140943 + 499.681397493323743 i	24,32,3	18.150794784259531 + 39.729151356826236 i	818.386929981932781 + 435.362514320102150 i
24,32,5	13.782573304094541 + 42.349694134228940 i	704.877077488493228 + 334.183842494815792 i	24,32,7	8.914177471059588 + 44.659222276178816 i	623.511926944184324 + 205.499308731894303 i
24,32,9	3.085477282475720 + 46.698391802016445 i	579.355730588251504 + 53.587425390736868 i	24,33,1	24.161535875206614 + 37.224600958422762 i	1041.481029982577638 + 571.437205695258626 i
24,33,3	20.231140382085389 + 40.342296438657733 i	883.544766415447725 + 512.454216201514669 i	24,33,5	16.034702998980425 + 43.029333547599499 i	756.28327647208130 + 415.06772465007265 i
24,33,7	11.433367790389561 + 45.414857353320592 i	659.311490348185089 + 289.048504768952057 i	24,33,9	6.174071383971413 + 47.543823483312700 i	596.036423281158932 + 139.339926417192999 i
24,34,1	26.082733444305834 + 37.751516979836786 i	1125.222461161062938 + 645.255443080130817 i	24,34,3	22.261721178735435 + 40.923725301937019 i	955.095777912238418 + 591.895841474069130 i
24,34,5	18.208816506673038 + 43.671914962546637 i	814.703184829242332 + 498.629165921218259 i	24,34,7	13.815349670763831 + 46.126299146560619 i	703.120758231950845 + 375.618738952447188 i
24,34,9	8.916860606318005 + 48.334551193489339 i	622.797321740287089 + 228.092734980391242 i	24,34,11	3.067579230721883 + 50.312125226250878 i	579.280725567520335 + 58.779445536151798 i
24,35,1	27.975726910469294 + 38.253823431310828 i	1214.825452839756053 + 721.008877751129148 i	24,35,3	24.250958322256274 + 41.477083821952334 i	1032.882778976992086 + 673.532367545525062 i
24,35,5	20.320712525382522 + 44.281983140848013 i	879.868300682214226 + 584.680745070325884 i	24,35,7	16.096262717108068 + 46.799469032720793 i	754.43894979964637 + 465.00288936915234 i
24,35,9	11.460575212408628 + 49.079076223260202 i	658.408643835839825 + 319.918679951512672 i	24,35,11	6.170349321467843 + 51.140471820871525 i	595.753640156258371 + 152.797405829953618 i
24,36,1	29.844373258569441 + 38.734035196866024 i	1310.211458713599086 + 798.586903281369928 i	24,36,3	26.205257063692361 + 42.005361892419316 i	1116.777756037394283 + 757.230213378868257 i
24,36,5	22.381659061548454 + 44.863208989993474 i	951.56946664155384 + 673.06137786843124 i	24,36,7	18.298824880118747 + 47.439037358792206 i	812.904135115113718 + 557.013882625677411 i
24,36,9	13.868390718010248 + 49.783719268598979 i	702.140632371216952 + 414.669616725514970 i	24,36,11	8.933677030079229 + 51.919558231608946 i	622.348610855353172 + 249.627775012996665 i
24,36,13	3.057401892802253 + 53.851129058291114 i	579.228135138423340 + 63.758036313772230 i	24,37,1	31.691783476448773 + 39.194265422133675 i	1411.313424516676832 + 877.892314985458256 i
24,37,3	28.129616564688181 + 42.511050586285428 i	1206.674389553556239 + 842.872920774222280 i	24,37,5	24.400024583025850 + 45.418612679777830 i	1029.639006347649911 + 763.631345801405928 i
24,37,7	20.438490730146484 + 48.048774768837980 i	878.241497756103513 + 651.486922843939055 i	24,37,9	16.175418065059695 + 50.453403633867310 i	753.496812096557051 + 512.173700333375905 i
24,37,11	11.500470198847355 + 52.656618063166341 i	657.876337431563684 + 349.417735124102364 i	24,37,13	6.175540851504395 + 54.667068984980673 i	595.56449763202636 + 165.74294316325063 i
24,38,1	33.520506340961440 + 39.636311402777132 i	1518.073422015300366 + 958.838887895318862 i	24,38,3	30.028021652548784 + 42.996253401869446 i	1302.482945991995166 + 930.357927547007421 i
24,38,5	26.382221019477246 + 45.950717572702750 i	1113.939428962023612 + 856.268326062060396 i	24,38,7	22.526359224561912 + 48.631783675612411 i	950.234856670037857 + 748.277228558906927 i
24,38,9	18.403845546789039 + 51.092077592811940 i	812.113926194160285 + 612.268278604653688 i	24,38,11	13.932329695024920 + 53.357037766620998 i	701.621754946490910 + 452.060019791006994 i
24,38,13	8.958934784392261 + 55.437831377068179 i	622.067688812540723 + 270.391174999374130 i	24,38,15	3.052239460337641 + 57.333508359591569 i	579.190237930080343 + 68.576619297310913 i
24,39,1	35.332658134972611 + 40.061717932477201 i	1630.440896771479856 +1041.349519093929075 i	24,39,3	31.903704916533055 + 43.462767119589998 i	1404.126666129505111 +1019.594111950838624 i
24,39,5	28.333284790137117 + 46.461659689740003 i	1204.355919555873925 + 950.864285933534802 i	24,39,7	24.570724562500285 + 49.190657029127230 i	1028.709533792103457 + 847.257104005731208 i
24,39,9	20.568846704776192 + 51.702976757936653 i	877.713383945361329 + 714.806107923806053 i	24,39,11	16.263590299268301 + 54.025053035826417 i	753.092790947374624 + 557.409856957048532 i
24,39,13	11.547688315790412 + 56.169780543625436 i	657.579576497975722 + 377.910138125806895 i	24,39,15	6.186400126038833 + 58.140145972256448 i	595.43733807131605 + 178.30173084979370 i

Table D.25: Branch points and associated eigenvalues for $m = 24$.

Table D.26: Branch points and associated eigenvalues for $m = 25$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
25,25,1	3.532780069120949 + 31.478031684211729 i	629.789919745766269 + 27.793119311483625 i	25,26,1	6.626964147782888 + 32.596487866061828 i	650.494577186719425 + 74.479815734508762 i
25,27,1	9.294188386347340 + 33.566086445959670 i	681.01000074429487 + 126.38965746194705 i	25,27,3	3.298809673942294 + 36.120030991357922 i	629.028639978090155 + 36.265775740441399 i
25,28,1	11.735381408235515 + 34.432467541331818 i	719.783974055453882 + 182.645010644963975 i	25,28,3	6.390918846595823 + 37.103574609688721 i	647.740415416713518 + 95.172817478898807 i
25,29,1	14.031507627961366 + 35.221165258238322 i	766.033165123032290 + 242.624864020305210 i	25,29,3	9.090051576346154 + 37.990572771252324 i	676.500953291854216 + 158.306903179764987 i
25,29,5	3.195143994753844 + 40.265811133424087 i	628.696979665582830 + 43.067219125130471 i	25,30,1	16.224987116992967 + 35.948331885329623 i	819.277638849757750 + 305.880024463607981 i
25,30,3	11.572052230021093 + 38.802972292532473 i	713.832996612831607 + 225.265245339600710 i	25,30,5	6.288287558843803 + 41.183145753116037 i	646.452185230570763 + 112.343533660857574 i
25,31,1	18.341328468566239 + 36.625030124891765 i	879.190770114391285 + 372.070320479200348 i	25,31,3	13.910815327290049 + 39.555325985413326 i	758.935279087882464 + 295.642026767921209 i
25,31,5	9.009655059034731 + 42.026139391445206 i	674.321009987209663 + 185.287098503800451 i	25,31,7	3.139097476330631 + 44.160011175308547 i	628.511089671770264 + 49.083367237193322 i
25,32,1	20.397264084895980 + 37.259269507742871 i	945.534852278788549 + 440.928218807313556 i	25,32,3	16.146279422467224 + 40.257871039195265 i	811.297058594919690 + 369.104781862350762 i
25,32,5	11.521133407264987 + 42.808762557198641 i	710.922174143966231 + 261.732019728212492 i	25,32,7	6.238849352057397 + 45.038130407709744 i	645.716359197724046 + 127.746934210864737 i
25,33,1	22.404452797139587 + 37.857091920278314 i	1018.128612142405473 + 512.236988997187154 i	25,33,3	18.303025682422255 + 40.918155712245834 i	870.562309475031952 + 445.383503756452058 i
25,33,5	13.891826765364923 + 43.541019037991681 i	755.472561965068621 + 341.395077124929685 i	25,33,7	8.980531477281584 + 45.854131740097657 i	673.070062344058101 + 209.714216859895743 i
25,33,9	3.106254220881195 + 47.899003760665686 i	628.393759514512112 + 54.635097129769001 i	25,34,1	24.371388533899605 + 38.423201171293734 i	1096.828960512202457 + 585.816895889250532 i
25,34,3	20.397468522302056 + 41.541951607982178 i	936.467899156863723 + 524.255823550486184 i	25,34,5	16.159666145179070 + 44.230356779153674 i	807.458013230086067 + 424.016747546198303 i
25,34,7	11.517518249752397 + 46.618334907009398 i	709.266225673003419 + 294.963375476433100 i	25,34,9	6.216357086900035 + 48.751429948321039 i	645.254608182117749 + 142.069896836081199 i
25,35,1	26.304475578915870 + 38.961352243498688 i	1181.519914410831916 + 661.516047636481858 i	25,35,3	22.441207183928075 + 42.133799898080923 i	1008.811076594367137 + 605.535545060040249 i
25,35,5	18.348314630795567 + 44.882509019177732 i	866.51261871024474 + 509.37384396902013 i	25,35,7	13.915701697644677 + 47.338378237378137 i	753.543438623076554 + 383.290246337016129 i
25,35,9	8.977771413569299 + 49.549528941703144 i	672.295850431729832 + 232.549973732070299 i	25,35,11	3.086486445009429 + 51.532060923999062 i	628.314452330644258 + 59.878126821105852 i
25,36,1	28.208676246872379 + 39.474604373386278 i	1272.105465813359615 + 739.204083300919137 i	25,36,3	24.442788524572119 + 42.697356826579629 i	1087.430648951210514 + 689.064348363302656 i
25,36,5	20.473756832112844 + 45.502010126164564 i	932.361837223849761 + 597.275633362964754 i	25,36,7	16.211549602218703 + 48.020130177078855 i	805.391172802584947 + 474.485133889405176 i
25,36,9	11.538237268638337 + 50.301612743044721 i	708.268176026530796 + 326.153202427706560 i	25,36,11	6.209262148069575 + 52.366548160218514 i	644.949650159459566 + 155.660609309362258 i
25,37,1	30.087922006568022 + 39.965492415964839 i	1368.504773903388013 + 818.767671207729677 i	25,37,3	26.408717077321732 + 43.235622307464233 i	1172.195295475641842 + 774.705790673676347 i
25,37,5	22.547399840572186 + 46.092526258558543 i	1004.79172546158395 + 687.55811587858341 i	25,37,7	18.427999315674853 + 48.668229519103335 i	864.440251206842845 + 568.358352727811052 i
25,37,9	13.961398000344138 + 51.013906042022164 i	752.429476880341099 + 422.731049542667961 i	25,37,11	8.990067650732076 + 53.152165780065424 i	671.800224947286210 + 254.292740898531548 i
25,37,13	3.074767876035621 + 55.088116713815090 i	628.258472857163156 + 64.900638095572347 i	25,38,1	31.945385716518597 + 40.436146832746516 i	1470.648800505152394 + 900.107206910701620 i
25,38,3	28.344072274229777 + 43.751096895715470 i	1262.995914511616093 + 862.340903670709281 i	25,38,5	24.577720054437233 + 46.657076119288050 i	1083.630710418453191 + 780.078998829995840 i
25,38,7	20.580663210059971 + 49.286427270413121 i	930.410226693292429 + 664.742658766071713 i	25,38,9	16.282643053913283 + 51.691275598087230 i	804.274507885299840 + 522.110121496220358 i
25,38,11	11.572683371043807 + 53.895974735891592 i	707.659848109328891 + 355.927912457388572 i	25,38,13	6.211620547022292 + 55.909517001129274 i	644.74269712057514 + 168.72182992765397 i
25,39,1	33.783668286441021 + 40.888380103554077 i	1578.477881782569284 + 983.134332274362237 i	25,39,3	30.252905089949763 + 44.245892975399563 i	1359.740394988138860 + 951.864902722824354 i
25,39,5	26.571215533385384 + 47.198183574029123 i	1168.73805995780071 + 874.71367420922456 i	25,39,7	22.680759476043214 + 49.877813366564439 i	1003.080401103773170 + 763.491031675088607 i
25,39,9	18.524348170984823 + 52.337633698575857 i	863.43338492003545 + 624.12594927263160 i	25,39,11	14.019113309976003 + 54.603269787776007 i	751.802478920472026 + 460.459369641546232 i
25,39,13	9.011484331645031 + 56.686129323863071 i	671.48107684853619 + 275.23753456749216 i	25,39,15	3.068305568596970 + 58.585721417646980 i	628.217819262695457 + 69.757673881874140 i
25,40,1	35.604930463616377 + 41.323750467288463 i	1691.939929707670899 +1067.770032615403125 i	25,40,3	32.138503489938984 + 44.721815618827584 i	1462.349918988622903 +1043.184681942976567 i
25,40,5	28.532992966875515 + 47.717986349680949 i	1259.996215627741776 + 971.352067992926436 i	25,40,7	24.736675675923600 + 50.444972814863327 i	1082.272382602189964 + 864.473743441670763 i
25,40,9	20.701822691272209 + 52.956191950379626 i	929.622300282059200 + 728.629470815381069 i	25,40,11	16.363932680212308 + 55.278230341443852 i	803.728125882009522 + 567.741508915750387 i
25,40,13	11.615230439834630 + 57.424161429069684 i	707.300446863374418 + 384.662134559641913 i	25,40,15	6.220053820335877 + 59.397262914388818 i	644.60082645237753 + 181.38257831821610 i

Table D.26: Branch points and associated eigenvalues for $m = 25$.

Table D.27: Branch points and associated eigenvalues for $m = 26$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
26,26,1	3.569995842633902 + 32.554154758172672 i	680.880408494925632 + 28.455808191793086 i	26,27,1	6.695069791732281 + 33.688165458735696 i	701.944168151360032 + 76.262158267252389 i
26,28,1	9.387086998785282 + 34.672427111185137 i	732.950744131205738 + 129.419339934461050 i	26,28,3	3.328763180743584 + 37.237385999389325 i	680.092775566971568 + 37.070439991481216 i
26,29,1	11.849536878990751 + 35.552735088912584 i	772.309750648221438 + 187.030059056935301 i	26,29,3	6.448429814610633 + 38.232564214115499 i	699.082187718507726 + 97.288194985910962 i
26,30,1	14.164481395875095 + 36.354749020275669 i	819.218158990821166 + 248.458990696040786 i	26,30,3	9.170375096769732 + 39.131129986367419 i	728.245764255561767 + 161.823276505850771 i
26,30,5	3.220836210532805 + 41.414616175184356 i	679.748299056406609 + 43.968276093466081 i	26,31,1	16.374945512775774 + 37.094712029993801 i	873.183769597698415 + 313.246477469227386 i
26,31,3	11.672281297813106 + 39.954887945071427 i	766.074161620320865 + 230.266879748761056 i	26,31,5	6.338928893968697 + 42.341372644616001 i	697.735348968722292 + 114.697611755801205 i
26,32,1	18.506818173053983 + 37.783753969316493 i	933.871492059559728 + 381.044379321746248 i	26,32,3	14.028873493441933 + 40.718356045775451 i	811.748074262368959 + 302.205737634923196 i
26,32,5	9.081367843483042 + 43.193996116991109 i	725.951843764803357 + 189.162995765817215 i	26,32,7	3.161805047087397 + 45.334615341154425 i	679.554407469540138 + 50.058416959669131 i
26,33,1	20.577091636069390 + 38.429935659301130 i	1001.037376660587597 + 451.578874710113951 i	26,33,3	16.280566928744484 + 41.431766788710199 i	864.744985494438424 + 377.300958150446206 i
26,33,5	11.611471914274230 + 43.986262377676418 i	762.989874872738937 + 267.198522349379346 i	26,33,7	6.284417049460404 + 46.220678651497060 i	696.960768928766015 + 130.289123214692353 i
26,34,1	22.597612767437806 + 39.039339365283865 i	1074.495327434478213 + 524.628126922107413 i	26,34,3	18.452251888140285 + 42.102674025459059 i	924.700507940992793 + 455.277177997775766 i
26,34,5	13.999004431955161 + 44.728095273790757 i	808.050507608660610 + 348.517125362973218 i	26,34,7	9.045684290871852 + 47.044941164397002 i	724.622521745361610 + 213.878202934984557 i
26,34,9	3.126690035240780 + 49.095558665997750 i	679.431454974326812 + 55.670363908186886 i	26,35,1	24.577017426594576 + 39.616701517886838 i	1154.098399559628206 + 600.008158189127016 i
26,35,3	20.560559081212848 + 42.736858561519618 i	991.345229820637542 + 535.907544618600468 i	26,35,5	16.282282492536638 + 45.426904537439469 i	860.608872582144272 + 432.855224978272588 i

Table D.27: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
26,35,7	11.600154601051793 + 47.817527614315040 i	761.211204784169468 + 300.806947631374328 i	26,35,9	6.257928817000033 + 49.954851923594340 i	696.470781398684494 + 144.767646411652834 i
26,36,1	26.521820550833791 + 40.165804035696659 i	1239.727445161671540 + 677.563485280597661 i	26,36,3	22.617247252023617 + 43.338871823387599 i	1064.471491623905195 + 619.002064489146278 i
26,36,5	18.485224199868846 + 46.088404116887972 i	920.291259745161597 + 519.985932586077411 i	26,36,7	14.014263031277414 + 48.545984525156463 i	805.951549999434633 + 390.869460484995955 i
26,36,9	9.037649947086139 + 50.760178926460547 i	723.789699876752366 + 236.954766566872223 i	26,36,11	3.105106805374956 + 52.747719252737937 i	679.347914129366927 + 60.964127794313605 i
26,37,1	28.437072957813260 + 40.689728808958080 i	1331.283803387742637 + 757.160659941583731 i	26,37,3	24.630985509537219 + 43.912380319651660 i	1143.914145308556044 + 704.399160884610637 i
26,37,5	20.623995486691040 + 46.717118761444802 i	986.817161543808083 + 609.715240285915002 i	26,37,7	16.324798666278685 + 49.236129851051530 i	858.323390623138152 + 483.853494440321981 i
26,37,9	11.614586904117283 + 51.519669644909790 i	760.119566978958233 + 332.314563157423891 i	26,37,11	6.247561409384384 + 53.588245266854535 i	696.144010205852283 + 158.490881402751313 i
26,38,1	30.326778383463395 + 41.191030044375502 i	1428.684371770291818 + 838.683661998365324 i	26,38,3	26.608374907848077 + 44.460393811600468 i	1229.538606300937090 + 791.959564748874868 i
26,38,5	22.710139013766081 + 47.316710338944389 i	1059.96790696575363 + 701.87626551423432 i	26,38,7	18.554913444645006 + 49.892572359173293 i	917.950384559221334 + 579.566776848963855 i
26,38,9	14.052842619821719 + 52.239456646865158 i	804.706050917117068 + 430.698104212281294 i	26,38,11	9.045561598150178 + 54.380276876064919 i	723.247938184273835 + 258.904233283307633 i
26,38,13	3.091890039722647 + 56.320662797586174 i	679.288598562196853 + 66.030383372245467 i	26,39,1	32.194169713179122 + 41.671854955746930 i	1531.858156765802960 + 922.030519979586529 i
26,39,3	28.554572898347022 + 44.985422124527176 i	1321.233027557642117 + 881.561829057062369 i	26,39,5	24.752236329197380 + 47.890196345080589 i	1139.568074102101718 + 796.324170783987142 i
26,39,7	20.720373944512129 + 50.519044415081581 i	984.546518176587483 + 677.839639662694935 i	26,39,9	16.388079777538241 + 52.924352527667061 i	857.035228750911870 + 531.930455358437030 i
26,39,11	11.643747909399538 + 55.130708711273847 i	759.436495912647388 + 362.363688599403247 i	26,39,13	6.247167997150211 + 57.147441177196512 i	695.91953258110163 + 171.66731966022331 i
26,40,1	34.041898436439055 + 42.134030691550905 i	1640.743783980797843 +1007.110772023701202 i	26,40,3	30.473694736801892 + 45.489586295820253 i	1418.902952049519854 + 973.098974827593793 i
26,40,5	26.756869369945075 + 48.440101278949918 i	1225.47372950663407 + 892.93208923600253 i	26,40,7	22.832514789757049 + 51.118624139177527 i	1057.886628148083673 + 778.522846460061714 i
26,40,9	18.642857816864041 + 53.578235150611548 i	916.73065955790719 + 635.84534414377754 i	26,40,11	14.104521169059252 + 55.844747363415316 i	803.972771246359002 + 468.762983547578244 i
26,40,13	9.063248410797392 + 57.929806594205743 i	722.89118841256379 + 280.02975600018630 i	26,40,15	3.084162707777157 + 59.833371305694619 i	679.245230528380830 + 70.925768864727644 i
26,41,1	35.872168442646455 + 42.579128462340925 i	1755.28765396028416 +1093.84351837692407 i	26,41,3	32.369082196266909 + 45.974699447713391 i	1522.467530650816570 +1066.475937381962694 i
26,41,5	28.729213235844835 + 48.968564617333598 i	1317.564605273002599 + 991.587929328312498 i	26,41,7	24.899814659235165 + 51.693889038679458 i	1137.788651889000903 + 881.484658061723508 i
26,41,9	20.832619031092985 + 54.204293184504351 i	983.503374777524982 + 742.291905142423843 i	26,41,11	16.462692941705601 + 56.526518183213440 i	856.348931030241260 + 577.955681154577633 i
26,41,13	11.681759519056577 + 58.673815755289979 i	759.015491032370733 + 391.338949399597539 i	26,41,15	6.253242324749878 + 60.649749787862589 i	695.76318170527554 + 184.42974603797302 i

Table D.27: Branch points and associated eigenvalues for $m = 26$.

Table D.28: Branch points and associated eigenvalues for $m = 27$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
27,27,1	3.606374993246778 + 33.628464118142894 i	733.969792160125508 + 29.110162806680492 i	27,28,1	6.761665699006679 + 34.777636714027452 i	755.388269396835994 + 78.022082456475175 i
27,29,1	9.477960599199687 + 35.776185498087024 i	786.880026204908177 + 132.410911312552173 i	27,29,3	3.358103638742507 + 38.352111893329834 i	733.156184829646236 + 37.865203574924948 i
27,30,1	11.961245779100816 + 36.670061838562496 i	826.816988047930181 + 191.359935415362258 i	27,30,3	6.504767048868396 + 39.358655848972269 i	752.420058591420570 + 99.377578847309707 i
27,31,1	14.294652351184308 + 37.485049705266768 i	874.376679789656464 + 254.219691543919652 i	27,31,3	9.249077103102465 + 40.268514550187220 i	781.982070077870389 + 165.296490002621169 i
27,31,5	3.246045761998034 + 42.560247845057312 i	732.799084833385905 + 44.858505146885641 i	27,32,1	16.521794122202266 + 38.237481760880151 i	929.054794963346808 + 320.520186441793783 i
27,32,3	11.770512760530192 + 41.103358276538017 i	820.301173810959767 + 235.207177826398066 i	27,32,5	6.388612054202315 + 43.496226689526829 i	751.015512086070771 + 117.023462119674775 i
27,33,1	18.668930090369802 + 38.938553835906198 i	990.507887918988899 + 389.905409551991568 i	27,33,3	14.144610126805398 + 41.877673112341753 i	866.540242650010669 + 308.689010045244800 i
27,33,5	9.151731710696200 + 44.358268078482993 i	779.575961210915580 + 192.992507904881364 i	27,33,7	3.184119340556240 + 46.505655963830399 i	732.597314635097518 + 51.022031431993412 i
27,34,1	20.753305223545976 + 39.596377019913177 i	1058.485847996817711 + 462.095328805570546 i	27,34,3	16.412249524329770 + 42.601688178514976 i	920.165128348639882 + 385.396742334592261 i
27,34,5	11.700126836498614 + 45.159961413188533 i	817.046195619805076 + 272.599709818769611 i	27,34,7	6.329182153122830 + 47.399509323548415 i	750.202786335425799 + 132.801578172333649 i
27,35,1	22.786949181301615 + 40.217073064338699 i	1132.797820642234910 + 536.863064787206895 i	27,35,3	18.598622610309430 + 43.282963496119415 i	980.803184113626912 + 465.049717165050879 i
27,35,5	14.104205904887699 + 45.911154463691453 i	862.611642299636515 + 355.554192525986878 i	27,35,7	9.109690894004762 + 48.231864546578500 i	778.169518048423470 + 217.993608912452316 i
27,35,9	3.146798645170765 + 50.288261370418617 i	732.468826668840734 + 56.693784402425017 i	27,36,1	24.778635757617931 + 40.805410235259671 i	1213.293057254460336 + 614.020450351734098 i
27,36,3	20.720570846991905 + 43.927289284441969 i	1048.178796382820110 + 547.416650695935459 i	27,36,5	16.402663497904829 + 46.619220831579497 i	915.736845881683280 + 441.588361526503832 i
27,36,7	11.681345872689993 + 49.012659001012331 i	815.146797752922680 + 306.582491569764386 i	27,36,9	6.298816970614923 + 51.154299895928602 i	749.685008105495285 + 147.434620740013685 i
27,37,1	26.734987403342085 + 41.365196688223072 i	1299.849288020833910 + 693.408456359977436 i	27,37,3	22.790008319903755 + 44.539227241078159 i	1122.079458203940476 + 632.303796706551225 i
27,37,5	18.619666848286975 + 47.289856379202043 i	976.040385849737845 + 530.471666003453151 i	27,37,7	14.111114767290632 + 49.749351270967395 i	860.345648621420537 + 398.360604980890741 i
27,37,9	9.096540534143118 + 51.966718376102854 i	777.279027652139121 + 241.309458099921756 i	27,37,11	3.123451001991159 + 53.959308689346315 i	732.381117113252003 + 62.038004170275471 i
27,38,1	28.661140913512146 + 41.899536340737448 i	1392.365235556231937 + 774.890586725092021 i	27,38,3	24.815723154505022 + 45.122454046014461 i	1202.336115895185003 + 719.546360232832399 i
27,38,5	20.771558898698373 + 47.927576954447851 i	1043.235857858035160 + 622.006858556014436 i	27,38,7	16.436101455880163 + 50.447711076358132 i	913.236370130900923 + 493.113172195065772 i
27,38,9	11.689680475951500 + 52.733471487834962 i	813.963096658569839 + 338.406022723418744 i	27,38,11	6.285271433616703 + 54.805775661192804 i	749.336766326240536 + 161.289659463400454 i
27,39,1	30.561170221319291 + 42.411002185432885 i	1490.755566887706664 + 858.348167525662461 i	27,39,3	26.804410342503605 + 45.679989152306860 i	1288.810962276533019 + 809.002279685182430 i
27,39,5	22.870014577651322 + 48.536040561102141 i	1117.09991871218131 + 716.02420539443665 i	27,39,7	18.679667913053198 + 51.112318574490260 i	973.435533565846299 + 590.645357313397199 i
27,39,9	14.142791426757825 + 53.460603281779527 i	858.970782034038848 + 438.574975462948544 i	27,39,11	9.100194992661137 + 55.604109124221125 i	776.691866554769945 + 263.464577074282886 i
27,39,13	3.108776637597922 + 57.548976611979178 i	732.318515799881993 + 67.147820625672026 i	27,40,1	32.438366516429234 + 42.901757674170902 i	1594.947360167798479 + 943.676891199768306 i
27,40,3	28.761303248100848 + 46.214351464807159 i	1381.389439830151105 + 900.547659183660812 i	27,40,5	24.923717973176931 + 49.118263731413251 i	1197.453343116014139 + 812.376352307295974 i
27,40,7	20.857731530770504 + 51.746888413875318 i	1040.651620256199521 + 690.785091729357305 i	27,40,9	16.491804295438257 + 54.152874398705833 i	911.779571447755302 + 541.639849192596444 i
27,40,11	11.713710285228467 + 56.361043281497089 i	813.206492115121591 + 368.728287278352923 i	27,40,13	6.282202829410363 + 58.381053699232069 i	749.09503559891186 + 174.58083110584528 i
27,41,1	34.295430416061485 + 43.373644154150256 i	1704.877554205698289 +1030.784219963648866 i	27,41,3	30.690579661284666 + 46.727670464568227 i	1479.974775588721059 + 994.073352072804141 i
27,41,5	26.939332785587748 + 49.676771722614390 i	1284.149034429937501 + 910.934198749727557 i	27,41,7	22.981740727468537 + 52.354487516102886 i	1114.655051368040859 + 793.380944861502940 i
27,41,9	18.759458659575667 + 54.814129729076363 i	972.00653486503938 + 647.43258126231603 i	27,41,11	14.188608894458373 + 57.081699830646926 i	858.132960295797034 + 476.974997105083673 i
27,41,13	9.114256689833603 + 59.169079217409397 i	776.29810756873258 + 284.77012971899563 i	27,41,15	3.099817257867898 + 61.076666215417760 i	732.272473333485209 + 72.081442688885446 i
27,42,1	36.134607714551869 + 43.828245376099936 i	1820.491059158248390 +1119.587394189739143 i	27,42,3	32.595633693820261 + 47.221767105265251 i	1584.484115348652722 +1089.482358933017167 i
27,42,5	28.922100779940433 + 50.213705806539373 i	1377.064047761597976 +1011.583658359901392 i	27,42,7	25.060263183151307 + 52.937686302018278 i	1195.260134689433926 + 898.299186635444698 i
27,42,9	20.961327071795480 + 55.447535996582083 i	1039.357596708979372 + 755.800517606127528 i	27,42,11	16.559934831522121 + 57.770152144597752 i	910.955670922451532 + 588.057434890647528 i
27,42,13	11.747314167394252 + 59.918963984876910 i	812.724866901004020 + 397.943751705665534 i	27,42,15	6.285981562850606 + 61.897816805530084 i	748.92442434510372 + 187.44462724987321 i

Table D.28: Branch points and associated eigenvalues for $m = 27$.

Table D.29: Branch points and associated eigenvalues for $m = 28$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
28,28,1	3.641963839750421 + 34.701064976800794 i	789.058122461413177 + 29.756586157822419 i	28,29,1	6.826834496748631 + 35.865030405706946 i	810.827137887275171 + 79.760672270833894 i
28,30,1	9.566918434764135 + 36.877513014056326 i	842.798382296035129 + 135.366214160155806 i	28,30,3	3.386862288239747 + 39.464353490635347 i	788.218898910041503 + 38.650533599406067 i
28,31,1	12.070638177333175 + 37.784620675449027 i	883.306549820576038 + 195.637307167942197 i	28,31,3	6.559990333299847 + 40.482009303547350 i	807.754197262252205 + 101.442193309733725 i
28,32,1	14.422167353262914 + 38.612260526342780 i	931.509956966260916 + 259.910516859847462 i	28,32,3	9.326239445435020 + 41.402901758323281 i	837.710240402589989 + 168.728573361338420 i
28,32,5	3.270795340154363 + 43.702873617640378 i	787.849357563727494 + 45.738404012630305 i	28,33,1	16.665693580424083 + 39.376853524632082 i	986.892340144614877 + 327.705636354219337 i
28,33,3	11.866846342772124 + 42.248573987338496 i	876.514648019113338 + 240.089020621626389 i	28,33,5	6.437382593555849 + 44.647885630609522 i	806.292793729623440 + 119.322378830584341 i
28,34,1	18.827836276162834 + 40.089660376684826 i	1049.102003028806621 + 398.658877917799487 i	28,34,3	14.258139763477773 + 43.033482935786665 i	923.312679032878918 + 315.095618663391008 i
28,34,5	9.220810551922090 + 45.519144213427744 i	835.193631506449719 + 196.777756332019749 i	28,34,7	3.206057402345423 + 47.673313946251362 i	787.639825083840947 + 51.974722559047052 i
28,35,1	20.926086429544000 + 40.758841485360868 i	1117.882752023202329 + 472.484072919299706 i	28,35,3	16.541454183948357 + 43.767855540870542 i	977.558690013681939 + 393.396841679252304 i
28,35,5	11.787177313262365 + 46.330060098127241 i	873.091594264294713 + 277.938563360058367 i	28,35,7	6.373180311511914 + 48.574810613674259 i	805.442497239101840 + 135.285625803538466 i
28,36,1	22.972651593089843 + 41.390557258672864 i	1193.039034919855794 + 548.949359656155821 i	28,36,3	18.742275358451018 + 44.459258518346225 i	1038.871872269934329 + 474.706797320022247 i
28,36,5	14.207523227189418 + 47.090408547317736 i	919.156643481834294 + 362.510150939069604 i	28,36,7	9.172602291776238 + 49.415098379302698 i	833.711251207562555 + 222.062595922428613 i
28,36,9	3.166593096429835 + 51.477300461840990 i	787.505883954196065 + 57.705875042841924 i	28,37,1	24.976439758151795 + 41.989607070894664 i	1274.416348559770540 + 627.862432170969441 i
28,37,3	20.877650300027433 + 45.113491694229047 i	1106.970484244373665 + 558.789820460406986 i	28,37,5	16.520912239556150 + 47.807528956418672 i	972.842855627579638 + 450.220950319022847 i
28,37,7	11.761156111599766 + 50.203934331866272 i	871.073351672193780 + 312.293025835452227 i	28,37,9	6.339049912188957 + 52.349967707220308 i	804.897350366281103 + 150.072156555641044 i
28,38,1	26.944177952374243 + 42.559824670382646 i	1361.889341971215572 + 709.060758579989738 i	28,38,3	22.959644532130568 + 45.735127052391441 i	1181.637229673366392 + 645.448454770360968 i
28,38,5	18.751755156511962 + 48.487100215892852 i	1033.761186088067916 + 540.836786291828048 i	28,38,7	14.206332181029067 + 50.948692701071053 i	916.726255764620191 + 405.767579009201484 i
28,38,9	9.154484496842150 + 53.169347384985031 i	832.763983136650836 + 245.616217051070862 i	28,38,11	3.141529128033879 + 55.167021699886547 i	787.414067366335530 + 63.100270784469721 i
28,39,1	28.881086585424121 + 43.104335426007509 i	1455.354155694089968 + 792.404832415787268 i	28,39,3	24.997162162379045 + 46.327851359983747 i	1262.699196443982828 + 734.514723810325222 i
28,39,5	20.916567868112665 + 49.133629937448397 i	1101.619400214744246 + 634.157205881443474 i	28,39,7	16.545542993819780 + 51.655097067013912 i	970.130829555012610 + 502.268969675695530 i
28,39,9	11.763570510021374 + 53.943225151432145 i	869.799029807989086 + 344.430597892018170 i	28,39,11	6.322415046066004 + 56.019335615212775 i	804.527962984894430 + 164.058275575637907 i
28,40,1	30.791307850572380 + 43.625730597242260 i	1554.723255824580747 + 877.773358947431916 i	28,40,3	26.996989697289298 + 46.894693620953269 i	1350.015394018745610 + 825.843805596310858 i
28,40,5	23.027154463239043 + 49.750772658122578 i	1176.189535666915390 + 730.009654244134708 i	28,40,7	18.802356252059255 + 52.327700292007918 i	1030.896632928539930 + 601.599821929160612 i
28,40,9	14.231306736139379 + 54.677559892663467 i	915.224073891600483 + 446.365545236354080 i	28,40,11	9.154001665782978 + 56.823863543343116 i	832.132121236615831 + 267.975927603123921 i
28,40,13	3.125435523670608 + 58.773251889302827 i	787.348228240505478 + 68.253459070302299 i	28,41,1	32.678189628757906 + 44.126189402888386 i	1659.921817918282613 + 965.059723823722948 i
28,41,3	28.964434438126219 + 47.438181646834870 i	1443.468587650377458 + 919.309383809870155 i	28,41,5	25.092299114718465 + 50.341544175587018 i	1257.288607227604643 + 828.244287323868093 i
28,41,7	20.992836954705098 + 52.970200167341282 i	1098.726701048245786 + 703.585689208966528 i	28,41,9	16.593887530365706 + 55.377062350257440 i	968.508098757240873 + 551.243067398660855 i
28,41,11	11.782613998874876 + 57.587184758785995 i	868.970038390036166 + 375.024697927111106 i	28,41,13	6.316743558759652 + 59.610551043974290 i	804.26923728513702 + 177.46368190074654 i
28,42,1	34.544480304495700 + 44.607566958744158 i	1770.885116565792032 +1054.169339658406898 i	28,42,3	30.903735070294687 + 47.960453172247909 i	1542.959717020052267 +1014.800169499871458 i
28,42,5	27.118745298216929 + 50.908470624264432 i	1344.766391454802715 + 928.729794962423057 i	28,42,7	23.128544823178846 + 53.585652990560789 i	1173.387087130781083 + 808.072967915284266 i
28,42,9	18.874229223423757 + 56.045545792766717 i	1029.26175032292190 + 658.89332457543821 i	28,42,11	14.271428545175272 + 58.314339119908155 i	914.283358688163162 + 485.099246278391490 i
28,42,13	9.164537086261848 + 60.404147284389801 i	831.70191602133821 + 289.46078361907740 i	28,42,15	3.115275341872256 + 62.315799318725283 i	787.299549536786685 + 73.225196261024138 i
28,43,1	36.392466417100678 + 45.071459161053539 i	1887.556591539268311 +1145.017610788571346 i	28,43,3	32.818336665810357 + 48.463336784110055 i	1648.403947733027053 +1112.217251370716440 i
28,43,5	29.111799802975614 + 51.453695116631323 i	1438.497297464114354 +1031.350117673307295 i	28,43,7	25.218134542697186 + 54.176622483336836 i	1254.688508700061220 + 914.925956589656153 i
28,43,9	21.088032070820312 + 56.686155924283455 i	1097.185897711745838 + 769.161888446029593 i	28,43,11	16.655718020153909 + 59.009349958349908 i	967.548786982574939 + 598.051465652370780 i
28,43,13	11.811930682749528 + 61.159810358406432 i	868.428726896172634 + 404.479486518943247 i	28,43,15	6.318286638106481 + 63.141659089939267 i	804.08457518979753 + 190.42851852169531 i

Table D.29: Branch points and associated eigenvalues for $m = 28$.

Table D.30: Branch points and associated eigenvalues for $m = 29$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
29,29,1	3.676804696357310 + 35.772053073107244 i	846.145447042372552 + 30.395448388487672 i	29,30,1	6.890651740714856 + 36.950463664603852 i	868.261010324279027 + 81.478923318041211 i
29,31,1	9.654060524291175 + 37.976547344031857 i	900.706305603490250 + 138.286940879912436 i	29,31,3	3.415067870747245 + 40.574243163070790 i	845.280946723161492 + 39.426860054473480 i
29,32,1	12.177833301655509 + 38.896568805731640 i	941.779231743498826 + 199.864624407080754 i	29,32,3	6.614154663898653 + 41.602770633684273 i	865.08476065896241 + 103.4831655283429 1 i
29,33,1	14.547161183404389 + 39.736557186569193 i	990.619123739475867 + 265.534727667309255 i	29,33,3	9.401937470231800 + 42.534451828921839 i	895.430617906869770 + 172.121395578614482 i
29,33,5	3.295105960724113 + 44.842647129796440 i	844.899136524235814 + 46.608431913524285 i	29,34,1	16.806791483888144 + 40.513020547236309 i	1046.697903535047999 + 334.806946766570093 i
29,34,3	11.961373924673135 + 43.390709369676969 i	934.715154192208047 + 244.915061395805026 i	29,34,5	6.485282651024421 + 45.796512553588755 i	863.567305227292650 + 121.595556153942951 i
29,35,1	18.983694999121091 + 41.237283425656152 i	1109.655723664863217 + 407.309805781094212 i	29,35,3	14.369568027384510 + 44.185973568646105 i	982.066212908330954 + 321.429040045330737 i
29,35,5	9.288663479057417 + 46.676797890158248 i	892.805105896066493 + 200.520699259839262 i	29,35,7	3.227635123463868 + 48.837755773814962 i	844.681952011115868 + 52.916963703249820 i
29,36,1	21.095602468414569 + 41.917554641534650 i	1179.230381825441327 + 482.751070003209975 i	29,36,3	16.668298112052671 + 44.930470272770371 i	1036.926785703204587 + 401.305592034715175 i
29,36,5	11.872696602303300 + 47.496742357866822 i	931.126497949291888 + 283.217835349338486 i	29,36,7	6.416444675965868 + 49.746755760634272 i	862.679982527924039 + 137.742493293819848 i
29,37,1	23.154894742693010 + 42.560032485649465 i	1255.221686119496326 + 560.893952983678446 i	29,37,3	18.883337179630285 + 45.631773371588331 i	1098.907995102374343 + 484.253647031084199 i
29,37,5	14.309041657810155 + 48.266052018185412 i	977.686142944786752 + 369.388576216178990 i	29,37,7	9.234465899764905 + 50.594823571571617 i	891.247909261524001 + 226.087163482633940 i
29,37,9	3.186085631856150 + 52.662850000109607 i	844.542635838817773 + 58.707114411459330 i	29,38,1	25.170610507528224 + 43.169546765117417 i	1337.471425641723272 + 641.542058079124786 i
29,38,3	21.031932900151833 + 46.295692473326525 i	1167.722041983597819 + 570.033206427688924 i	29,38,5	16.637124267664280 + 48.992033842431674 i	1031.927762022591878 + 458.757417788686894 i
29,38,7	11.839644868321249 + 51.391542611474634 i	928.991192555118914 + 317.941343856620051 i	29,38,9	6.378654155531325 + 53.542034324216978 i	862.107867280828714 + 152.681493014745115 i
29,39,1	27.149578614326835 + 43.749956224985389 i	1425.851207041744601 + 724.529391716083865 i	29,39,3	23.126298564579532 + 46.926809861582136 i	1243.146896243415995 + 658.443144479548891 i
29,39,5	18.881593561654782 + 49.680350781751329 i	1093.454769158692670 + 551.086596822737874 i	29,39,7	14.299985291793417 + 52.144206089301335 i	975.093860676057034 + 413.093991640600336 i
29,39,9	9.211520425863279 + 54.368250730186496 i	890.244708010127283 + 249.877054405833519 i	29,39,11	3.159350720370665 + 56.371036395540237 i	844.446770876426285 + 64.151405733737533 i
29,40,1	29.097100871985312 + 44.304407153933880 i	1520.254621554324785 + 809.713472643642149 i	29,40,3	25.175451410431808 + 47.528822313665131 i	1325.005832375522232 + 749.312338379503558 i
29,40,5	21.059134542650014 + 50.335502829555210 i	1161.969163031600601 + 646.172487345581317 i	29,40,7	16.653202407555344 + 52.858493392763968 i	1029.007442655857858 + 511.325332800255580 i
29,40,9	11.836306052001339 + 55.149121706501994 i	927.627616975086880 + 350.391086704457393 i	29,40,11	6.359013691363005 + 57.229106825375503 i	861.717643066360097 + 166.797967168075445 i
29,41,1	31.017385916886756 + 44.835508600132151 i	1620.591962475532455 + 896.970416608023356 i	29,41,3	27.186267181878602 + 48.104768255932647 i	1413.154713923140662 + 842.493236421134725 i
29,41,5	23.181677529222814 + 50.961141426555372 i	1237.238412892798806 + 743.839742408528764 i	29,41,7	18.923065558329533 + 53.538931312748602 i	1090.334559176587163 + 612.435473947642549 i
29,41,9	14.318446743040491 + 55.890524100324747 i	973.466307968091087 + 454.073414786629428 i	29,41,11	9.207013353839701 + 58.039726268586577 i	889.568808740221812 + 272.440287651646770 i
29,41,13	3.141874172725245 + 59.993668353771639 i	844.377739630622500 + 69.347772524724306 i	29,42,1	32.913836807684604 + 45.345455116005446 i	1726.786527222616996 + 986.191330283296679 i
29,42,3	29.164125260205829 + 48.657184249050729 i	1507.473660618192980 + 937.857128949355456 i	29,42,5	25.258104464696945 + 51.560281850702395 i	1319.075815176465994 + 843.936053880359054 i
29,42,7	21.125784312866582 + 54.189201584626581 i	1158.772855712498085 + 716.247612413194702 i	29,42,9	16.694395737252854 + 56.597120670203772 i	1027.221342011162051 + 560.744530701390659 i
29,42,11	11.850499834741196 + 58.809324213654207 i	926.727327243700188 + 381.255699122431281 i	29,42,13	6.350807668492658 + 60.836115547234563 i	861.44216815946663 + 180.31709849397151 i
29,43,1	34.789248418944013 + 45.836115190782728 i	1838.771947657755391 + 1077.279601650227733 i	29,43,3	31.113323685633961 + 49.188216107994336 i	1607.861352056006353 + 1035.290608905974977 i
29,43,5	27.295236726703891 + 52.135451226979150 i	1407.328054435204422 + 946.327925049847636 i	29,43,7	23.273027338393149 + 54.812350418765540 i	1234.084063135039742 + 822.605991974609879 i
29,43,9	18.987242895105975 + 57.272694322963389 i	1088.497003587917106 + 670.232830325203736 i	29,43,11	14.353028924474618 + 59.542861533135977 i	972.424264136166016 + 493.139297283347446 i
29,43,13	9.214115903308882 + 61.635196533265557 i	889.10269301895096 + 294.10369894758134 i	29,43,15	3.130542832735880 + 63.550950230834061 i	844.326461191996259 + 74.357496556051473 i
29,44,1	36.645946945288522 + 46.309096503911810 i	1956.490212216251166 + 1170.148821613784321 i	29,44,3	33.037357194209257 + 49.699699872319492 i	1714.230997389383447 + 1134.692874899106755 i
29,44,5	29.298444582440094 + 52.688794553910256 i	1501.866923424178367 + 1050.897344073620161 i	29,44,7	25.373534438522874 + 55.410935200497590 i	1316.075346436684640 + 931.372958586882191 i
29,44,9	21.212813745224182 + 57.920370607409410 i	1156.989156255389389 + 782.382125125879838 i	29,44,11	16.750098452756930 + 60.244313326544123 i	1026.128698895377056 + 607.942139653121330 i
29,44,13	11.875643232137696 + 62.396544503738362 i	926.127218000779749 + 410.948896664331244 i	29,44,15	6.350171885499060 + 64.381458135974526 i	861.243655176334300 + 193.382628986528772 i

Table D.30: Branch points and associated eigenvalues for $m = 29$.

Table D.31: Branch points and associated eigenvalues for $m = 30$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
30,30,1	3.710936336100141 + 36.841515809500912 i	905.231809912961353 + 31.027090477644109 i	30,31,1	6.953186726033205 + 38.034043380176300 i	927.690105349785767 + 83.177752775378238 i
30,32,1	9.739478703528656 + 39.073414104259548 i	960.60425176731292 + 141.17465058308238 i	30,32,3	3.442746897311348 + 41.681902267523682 i	904.342355173076612 + 40.194579876199029 i
30,33,1	12.282940751878080 + 40.006049645177129 i	1002.235769145292011 + 204.044144282596534 i	30,33,3	6.667310764281315 + 42.721073739857182 i	924.411894637593718 + 105.501536178068150 i
30,34,1	14.669757879108946 + 40.858099984638272 i	1051.705227490360585 + 271.095328201389179 i	30,34,3	9.476240715711857 + 43.663311635122596 i	955.143521007034337 + 175.476682487030160 i
30,34,5	3.318997129954809 + 45.979709713793646 i	903.948439807970317 + 47.469013672875008 i	30,35,1	16.945223813150147 + 41.646159139786874 i	1108.472870373705518 + 341.827912853321664 i
30,35,3	12.054180372839166 + 44.529924192035914 i	994.903221903582789 + 249.687750436653374 i	30,35,5	6.532351295276524 + 46.942257507354867 i	922.839150780402520 + 123.844099149736564 i
30,36,1	19.136652227411325 + 42.381614488773870 i	1172.170794109623557 + 415.862819115353545 i	30,36,3	14.478992564749360 + 45.335317395791932 i	1042.801615054276908 + 327.692485074742592 i
30,36,5	9.355345306679240 + 47.831388626810316 i	952.410619314268502 + 204.223148998886870 i	30,36,7	3.248867344261541 + 49.999135058140986 i	903.723707934283425 + 53.849193683531020 i
30,37,1	21.262007717542211 + 43.072722423968941 i	1242.530858440871725 + 492.901813493111888 i	30,37,3	16.792889756566140 + 46.089717004871162 i	1098.270451795656072 + 409.126998209363592 i
30,37,5	11.956752669644024 + 48.660177429643419 i	991.151305901441333 + 288.440073515392555 i	30,37,7	6.459006135868524 + 50.915504647689900 i	921.915318519358152 + 140.173318582733373 i
30,38,1	23.333840115909108 + 43.725718394425797 i	1319.348287014612424 + 572.703239691544127 i	30,38,3	19.021925732586606 + 46.800704519941860 i	1160.912874917120234 + 493.695095973297896 i
30,38,5	14.408840346859359 + 49.438263845655655 i	1038.200731040485607 + 376.192778578912851 i	30,38,7	9.295325884670122 + 51.771207108388523 i	950.779669269183842 + 230.069166148014830 i
30,38,9	3.205287755620909 + 53.845071023119978 i	903.579090985861285 + 59.697947064632991 i	30,39,1	25.361315511237098 + 44.345462010578871 i	1402.461205776764245 + 655.066656271992770 i
30,39,3	21.183544206186070 + 47.474099490737913 i	1230.435095062802702 + 581.152492050958131 i	30,39,5	16.751388347223916 + 50.172924075893683 i	1092.992368750720971 + 467.201862318230098 i
30,39,7	11.916867623297357 + 52.575658313252561 i	988.900627046369664 + 323.530037073469089 i	30,39,9	6.417654525254094 + 54.730665374508739 i	921.31661513428787 + 155.26378150968557 i
30,40,1	27.351361992273308 + 44.935836460397468 i	1491.738215940534928 + 739.822647094385616 i	30,40,3	23.290102777379534 + 48.114494511884402 i	1306.610401305272944 + 671.294430227677708 i
30,40,5	19.009279153893750 + 50.869806091059012 i	1155.122170865431599 + 561.226008439357770 i	30,40,7	14.392139358942210 + 53.336073557015297 i	1035.448923227325395 + 420.343191567676342 i
30,40,9	9.267684423363523 + 55.563599448944458 i	949.721336729654809 + 254.093839221130395 i	30,40,11	3.176924797511966 + 57.571517975512023 i	903.479233512831229 + 65.191853989690244 i
30,41,1	29.309360685497243 + 45.500008459898446 i	1587.070390652402011 + 826.825790024899447 i	30,41,3	25.350729126906348 + 48.725596299727989 i	1389.258297778797669 + 763.946674908717114 i
30,41,5	21.199363256152274 + 51.533402841045309 i	1224.286430368704941 + 658.058449293648550 i	30,41,7	16.759153482193351 + 54.058089849623691 i	1089.866842458253132 + 520.286387360429444 i
30,41,9	11.907932978447091 + 56.351338039773708 i	987.449095471970395 + 356.290090324730272 i	30,41,11	6.395087543795734 + 58.435257871293402 i	920.905847884308173 + 169.509886282334037 i
30,42,1	31.239584962672776 + 46.040604403868194 i	1688.365877209028667 + 915.949641063857030 i	30,42,3	27.372386091945934 + 49.310452585089401 i	1478.231537873746447 + 858.958973985215266 i
30,42,5	23.333694432875209 + 52.167363015949455 i	1300.248096713574569 + 757.521073636815913 i	30,42,7	19.041877093254826 + 54.746209051380944 i	1151.750135595915253 + 623.157235416806543 i
30,42,9	14.404265890701492 + 57.099678875324038 i	1033.697845135715625 + 461.701932641526128 i	30,42,11	9.259259871518454 + 59.251870036324490 i	949.002031073454646 + 276.859522329399056 i
30,42,13	3.158099700524248 + 61.210393088351893 i	903.407053755475545 + 70.431202803360037 i	30,43,1	33.145491637078010 + 46.559833778359383 i	1795.546118196552698 + 1007.083054367048135 i
30,43,3	29.360523384829484 + 49.871608533414097 i	1573.407626091088787 + 956.200250357231994 i	30,43,5	25.421250212109165 + 52.774701575721025 i	1382.816787802209092 + 859.459133999466189 i
30,43,7	21.256661449074517 + 55.404097589844760 i	1220.791112675174645 + 728.776598082804412 i	30,43,9	16.793390918822126 + 57.813238521167520 i	1087.919803400740648 + 570.148350306086968 i
30,43,11	11.917406089292435 + 60.027638991831864 i	986.478542455070169 + 387.423879077558865 i	30,43,13	6.384411684471743 + 62.057916777288717 i	920.61385806632279 + 183.14222484507411 i
30,44,1	35.029920871878453 + 47.059578081682844 i	1908.543123233619326 + 1100.127417019097265 i	30,44,3	31.319496949466668 + 50.411217881469817 i	1674.683007985429640 + 1055.555001954321597 i
30,44,5	27.468928104678071 + 53.357946743655368 i	1471.836129502410813 + 963.736969238484448 i	30,44,7	23.415281928665067 + 56.034792090435346 i	1296.747226009838187 + 836.986586205689264 i
30,44,9	19.098568378838888 + 58.495770700235838 i	1149.712953633202687 + 681.455987485158857 i	30,44,11	14.433455854453147 + 60.767449296028580 i	1032.555960267943192 + 501.098472724992632 i
30,44,13	9.263017953411646 + 62.862399738525482 i	948.50051530748010 + 298.70072423202697 i	30,44,15	3.145625361470779 + 64.782286295694803 i	903.353210501018452 + 75.478779786349094 i
30,45,1	36.895237491455596 + 47.541456442725519 i	2027.297447582637233 + 1194.994527968894545 i	30,45,3	33.252849964718862 + 50.931123956178673 i	1781.968958934427746 + 1156.920558070972447 i
30,45,5	29.482160396078772 + 53.919245449091193 i	1567.175326578570093 + 1070.234635062041434 i	30,45,7	25.526561667169549 + 56.640843942569603 i	1379.422124278177989 + 947.647611280771798 i
30,45,9	21.335746759506790 + 59.150381614681429 i	1218.768201704926241 + 795.466909139712470 i	30,45,11	16.843128663360318 + 61.475229509842059 i	1086.695805907367912 + 617.733525617533815 i
30,45,13	11.938484018782892 + 63.629342846507605 i	985.820481832728319 + 417.354541885690082 i	30,45,15	6.381650919949120 + 65.617383096771334 i	920.401685216496162 + 196.308088470918051 i

Table D.31: Branch points and associated eigenvalues for $m = 30$.

Table D.32: Branch points and associated eigenvalues for $m = 31$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
31,31,1	3.744394387851801 + 37.909533220616296 i	966.317251831899739 + 31.651827412189551 i	31,32,1	7.014503182809345 + 39.115867394021727 i	989.114625447914477 + 84.858007917927637 i
31,33,1	9.823257523681432 + 40.168228250487239 i	1022.492642817519368 + 144.030783578318449 i	31,33,3	3.469923881098491 + 42.787442373264888 i	965.403149344372537 + 40.954060453247784 i
31,34,1	12.386061544026832 + 41.113194427670017 i	1064.676843246303861 + 208.177951969770817 i	31,34,3	6.719505532294556 + 43.837041720939276 i	985.735735055998816 + 107.498268607798352 i
31,35,1	14.790071885391301 + 41.977035608941932 i	1114.769238741311710 + 276.595093810517710 i	31,35,3	9.549213514817437 + 44.789616191397627 i	1016.849246223836303 + 178.796031883070526 i
31,35,5	3.342486990130781 + 47.114191719857970 i	964.997284420579263 + 48.320543269415715 i	31,36,1	17.081116162818891 + 42.776430663982566 i	1172.218524546810841 + 348.772040644723859 i
31,36,3	12.145344261822274 + 45.666365316200562 i	1057.079344295441842 + 254.409356476433601 i	31,36,5	6.578624826105314 + 48.085258902168192 i	984.108428081198554 + 126.069032863319265 i
31,37,1	19.286842915901328 + 43.522828867644542 i	1236.648831415384166 + 424.322191454120684 i	31,37,3	14.586503856642132 + 46.481672877165266 i	1105.519603264419857 + 333.888926965497717 i
31,37,5	9.420906973574505 + 48.983063575117917 i	1014.010391829059131 + 207.886786950627254 i	31,37,7	3.269767946949381 + 51.157593875574555 i	964.765104733368958 + 54.771820252751724 i
31,38,1	21.425445045334666 + 44.224533387522754 i	1307.786148688414642 + 502.941378300258009 i	31,38,3	16.915329691670333 + 47.245765463668349 i	1161.590653528225175 + 416.864768828927822 i
31,38,5	12.039408705784023 + 49.820521434649180 i	1053.166391928592702 + 293.607641924308155 i	31,38,7	6.500893526370159 + 52.081205186753593 i	983.148577293756944 + 142.579159315548367 i
31,39,1	23.509637300231066 + 44.887816150359527 i	1385.421168275285027 + 584.383127512111628 i	31,39,3	19.158150224490402 + 47.966232587030341 i	1224.887743417883712 + 503.035616897652631 i
31,39,5	14.506992927088694 + 50.607209134690933 i	1100.700960413636722 + 382.925830021316870 i	31,39,7	9.355223464747642 + 52.944403491007655 i	1012.306698159268421 + 234.010328004531232 i
31,39,9	3.224210291451110 + 55.024112858810980 i	964.615257719304964 + 60.678786870263231 i	31,40,1	25.548710074209481 + 45.517565981059072 i	1469.388395529700347 + 668.442996689996335 i
31,40,3	21.332600853725282 + 48.648903882812142 i	1295.111157806140682 + 592.152941036340394 i	31,40,5	16.863787109556377 + 51.350373643936891 i	1156.037428083687473 + 475.558087767532015 i
31,40,7	11.992876162829040 + 53.756442880250248 i	1050.801943966440831 + 329.061515079461515 i	31,40,9	6.456074300901534 + 55.916014485333285 i	982.52364754532528 + 157.82009424231589 i
31,41,1	27.549688258498755 + 46.117689994274230 i	1559.553461475751192 + 754.948184475307471 i	31,41,3	23.451180223300749 + 49.298382265565216 i	1372.029555684461229 + 684.008391885436822 i
31,41,5	19.134902375726774 + 52.055648844674664 i	1218.764360717145337 + 571.259579520778175 i	31,41,7	14.482855319616469 + 54.524463635912646 i	1097.791876299695787 + 427.518293009701949 i
31,41,9	9.323010319112178 + 56.755552214138184 i	1011.193996980343059 + 258.268312439804106 i	31,41,11	3.194259894888117 + 58.768619989396967 i	964.511461010574521 + 66.222030565101647 i
31,42,1	29.518030337282855 + 46.691374875485025 i	1655.804950955609456 + 843.750360194543418 i	31,42,3	25.523123921528139 + 49.918384324351962 i	1455.458712007144641 + 778.424653228982834 i
31,42,5	21.337351265580391 + 52.727521178750898 i	1288.572404117700898 + 669.820426100536338 i	31,42,7	16.863465148765286 + 55.254062084419843 i	1152.709624599086510 + 529.155970808679513 i
31,42,9	11.978494273315809 + 57.550038271089370 i	1049.263690266495587 + 362.130032143624476 i	31,42,11	6.430655607016557 + 59.637945492689326 i	982.092617206198952 + 172.195107700312178 i
31,43,1	31.458072809518434 + 47.241263955940042 i	1758.048890839066189 + 934.720548494408149 i	31,43,3	27.555479854862693 + 50.511966985511719 i	1545.248304230916801 + 875.248800626237539 i
31,43,5	23.483308396547731 + 53.369636908614503 i	1365.220034571309952 + 771.059778712233765 i	31,43,7	19.158866812512137 + 55.949716221191828 i	1215.144136641471960 + 633.769684969024865 i
31,43,9	14.488815197656722 + 58.305193999270493 i	1095.919027111452124 + 469.254219061260358 i	31,43,11	9.310769267773145 + 60.460455484626713 i	1010.431885919403385 + 281.235372126429183 i
31,43,13	3.174118883443540 + 62.423581735412419 i	964.436174409416822 + 71.504162699234399 i	31,44,1	33.373324902549399 + 47.769581284817612 i	1866.204891124102005 + 1027.745376224178244 i

Table D.32: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
31,44,3	29.553766416366472 + 51.081683886343626 i	1641.273250602175267 + 974.347414307144732 i	31,44,5	25.581844813254150 + 53.985010865637804 i	1448.513229634982281 + 874.820474323228950 i
31,44,7	21.385550517542793 + 56.615077865588297 i	1284.782439192283164 + 741.177983292359500 i	31,44,9	16.890931195061849 + 59.025591446767677 i	1150.603958088988065 + 579.458357785680619 i
31,44,11	11.983368775389984 + 61.242294043288211 i	1048.223859512335821 + 393.531653629909670 i	31,44,13	6.417571243212960 + 63.276112742050515 i	981.78433611692617 + 185.94013007806319 i
31,45,1	35.266670935920281 + 48.278221032453445 i	1980.203358740364365 +1122.724252180119720 i	31,45,3	31.522396084258784 + 51.629696539608922 i	1743.427787549535196 +1075.602919275117983 i
31,45,5	27.639932486698768 + 54.576172473684643 i	1538.292588441421003 + 980.964708605827127 i	31,45,7	23.555396233808260 + 57.253174528608646 i	1361.377748101662291 + 851.220862722515562 i
31,45,9	19.208270101176289 + 59.714956255526580 i	1212.910223610867433 + 692.567353153096633 i	31,45,11	14.512752421871577 + 61.988271918432240 i	1094.678717417941925 + 508.979874549570070 i
31,45,13	9.311266670323088 + 64.085917932649295 i	1009.89545711997175 + 303.25358754432739 i	31,45,15	3.160528326027007 + 66.009963719388054 i	964.379799775895208 + 76.589454198428285 i
31,46,1	37.140513396860257 + 48.768813468701964 i	2099.983433122259612 +1219.567203799130311 i	31,46,3	33.464959327585682 + 52.157855407740513 i	1851.621278369249239 +1178.910795375035832 i
31,46,5	29.663064340293946 + 55.145270636696381 i	1634.424754962217321 +1089.370623929892190 i	31,46,7	25.677308732905431 + 57.866551923526927 i	1444.730230539038985 + 963.756817790674063 i
31,46,9	21.456901159808218 + 60.376376038897455 i	1282.523818008957960 + 808.421536989386254 i	31,46,11	16.934858056115505 + 62.702272720691163 i	1149.250488057169605 + 627.429422738469839 i
31,46,13	12.000483433180303 + 64.858369850962347 i	1047.508654762253855 + 423.698815702788863 i	31,46,15	6.412736681611734 + 66.849591918272949 i	981.558686084164265 + 199.205954672011899 i

Table D.32: Branch points and associated eigenvalues for $m = 31$.

Table D.33: Branch points and associated eigenvalues for $m = 32$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
32,32,1	3.777211678195191 + 38.976178802931763 i	1029.401810638441475 + 32.269950925129059 i	32,33,1	7.074659875907894 + 40.196025521695475 i	1052.534758594813184 + 86.520473480381469 i
32,34,1	9.905475027524938 + 41.261095283503757 i	1086.371870600432203 + 146.856673879591597 i	32,34,3	3.496621539871962 + 43.890966319846527 i	1028.463352670867380 + 41.705642665155750 i
32,35,1	12.487289014069827 + 42.218123582482740 i	1129.103086661975567 + 212.267978771605527 i	32,35,3	6.770782428589723 + 44.950788041297756 i	1049.056408717067143 + 109.474256775850222 i
32,36,1	14.908209052812046 + 43.093498674252693 i	1179.812058952693178 + 282.036595045729882 i	32,36,3	9.620915520320825 + 45.913489935027926 i	1080.548070258118287 + 182.080926644667557 i
32,36,5	3.365592447714155 + 48.246213662342392 i	1028.045686367354620 + 49.163386930098795 i	32,37,1	17.214584809597365 + 43.903983216393414 i	1237.936058839634143 + 355.642577453940646 i
32,37,3	12.234938503225500 + 46.800168091603382 i	1121.243981537631498 + 259.081985267849234 i	32,37,5	6.624137039064572 + 49.225644721830491 i	1047.375228864669225 + 128.271310325143418 i
32,38,1	19.434392125807648 + 44.661087480300708 i	1303.091338234693922 + 432.691880973913896 i	32,38,3	14.692185928469666 + 47.625186050855476 i	1170.220847378573808 + 340.021125552379999 i
32,38,5	9.485395912772280 + 50.131958807064194 i	1077.604629927079714 + 211.513176662578758 i	32,38,7	3.290349938146998 + 52.313263932759291 i	1027.806153689907660 + 55.685223134671759 i
32,39,1	21.586046968195280 + 45.373160653389448 i	1374.998080671628713 + 512.874464840770770 i	32,39,3	17.035711389708265 + 48.398772078069641 i	1226.888291745284050 + 424.522346579587179 i
32,39,5	12.120723579133179 + 50.977918741515765 i	1117.172106638776086 + 298.722739257648414 i	32,39,7	6.542133812435248 + 53.243994524297847 i	1046.379827000722116 + 144.961000665318124 i
32,40,1	23.682425167876474 + 46.046510498782681 i	1453.442496715108746 + 595.939088231995356 i	32,40,3	19.292112231183410 + 49.128524060416503 i	1290.833750288524925 + 512.279362047345444 i
32,40,5	14.603568033954575 + 51.773040565682521 i	1165.187349311804837 + 389.590587977359860 i	32,40,7	9.414197175661711 + 54.114555992114084 i	1075.829153502302006 + 237.912255339880716 i
32,40,9	3.242863435423458 + 56.200114272690037 i	1027.651144032564844 + 61.650019933075022 i	32,41,1	25.732938500913626 + 46.686054503291039 i	1538.255511797221288 + 681.677349723817201 i
32,41,3	21.479211412485146 + 49.820281849768421 i	1361.751643908935876 + 603.039440138349505 i	32,41,5	16.974397625761906 + 52.524543448760205 i	1221.063645408953107 + 483.829632690838423 i
32,41,7	12.067718911741395 + 54.934046033446720 i	1114.695415706966287 + 334.538023235886953 i	32,41,9	6.493935345932504 + 57.098224455200935 i	1045.72901561078038 + 160.35143174342292 i
32,42,1	27.744706364430570 + 47.295723225426208 i	1629.299820433518562 + 769.913098451200995 i	32,42,3	23.609645533127615 + 50.478658686952308 i	1439.406050150734245 + 696.590674168008036 i
32,42,5	19.258547638145146 + 53.238048015880310 i	1284.382247777158227 + 581.191550902459312 i	32,42,7	14.572190176242664 + 55.709532630701851 i	1162.123127932032276 + 434.622198374915513 i
32,42,9	9.377529863336115 + 57.944256538470491 i	1074.662810099682929 + 262.402099013093959 i	32,42,11	3.211364097562372 + 59.962485445814579 i	1027.543458959627060 + 67.242323299269131 i
32,43,1	29.723262751604594 + 47.878722893032240 i	1726.461547620657029 + 860.495126110234082 i	32,43,3	25.692755691494721 + 51.107380971360591 i	1523.609054247268432 + 792.752698154121163 i
32,43,5	21.473189401192510 + 53.918034700393903 i	1354.828211257815155 + 681.463380940880370 i	32,43,7	16.966201916788954 + 56.446573011621091 i	1217.536350335430825 + 537.937660074090900 i
32,43,9	12.048030274461887 + 58.745374994460462 i	1113.071614809105768 + 367.913174549953965 i	32,43,11	6.465735804278983 + 60.837315711505504 i	1045.277989289101015 + 174.854636097717815 i
32,44,1	31.673005771352038 + 48.437713392044927 i	1829.644624237381549 + 953.291953112404826 i	32,44,3	27.735672950214077 + 51.709514723636147 i	1614.207290505408992 + 891.369942236908287 i
32,44,5	23.630615884658866 + 54.568147639117619 i	1432.155583759378942 + 784.461562211200544 i	32,44,7	19.274105835594799 + 57.149622304004737 i	1280.517291895987910 + 644.277090895559127 i
32,44,9	14.572142549310245 + 59.507227344840992 i	1160.13017779152619 + 476.73318751834324 i	32,44,11	9.361567966160514 + 61.665632296519526 i	1073.858466820628564 + 285.569464408305407 i
32,44,13	3.189938177368542 + 63.633379555180518 i	1027.465105372705693 + 72.567038616670544 i	32,45,1	33.597495800007934 + 48.974932990484234 i	1938.766848950563826 +1048.188003067981299 i
32,45,3	29.743982823080415 + 52.287621929942944 i	1711.073118674589523 + 992.306667716961329 i	32,45,5	25.739989689549333 + 55.191401712770492 i	1516.166739163353895 + 890.026538994006728 i
32,45,7	21.512528482623743 + 57.822318374338778 i	1350.747746335491456 + 753.456743886340405 i	32,45,9	16.987071132643575 + 60.234342691350271 i	1215.274256143323328 + 588.678131335636749 i
32,45,11	12.048421805838922 + 62.453443090969515 i	1111.963446040093459 + 399.581282089195213 i	32,45,13	6.450301154338034 + 64.490850953920969 i	1044.95363065141055 + 188.71181524111077 i
32,46,1	35.499660244660816 + 49.492288217918606 i	2053.757044692635191 +1145.080728055326517 i	32,46,3	31.722153028123245 + 52.843871744406485 i	1814.098589922901965 +1095.443247824893888 i
32,46,5	27.808355661897718 + 55.790327642706863 i	1606.699280541278995 + 998.018384209590408 i	32,46,7	23.693452402175044 + 58.467680060646437 i	1427.976733552212636 + 865.314520484860964 i
32,46,9	19.316408572123830 + 60.930419629499234 i	1278.089403460847734 + 703.571183650657076 i	32,46,11	14.590959198795680 + 63.205487390997440 i	1158.792793381962838 + 516.786404274240226 i
32,46,13	9.358884211011095 + 65.305901484184062 i	1073.28759019376253 + 307.76390733786491 i	32,46,15	3.175256900582551 + 67.234128572864279 i	1027.406231407789164 + 77.689902549309394 i
32,47,1	37.381938342232552 + 49.991420199195922 i	2174.552951654422486 +1243.878403495772318 i	32,47,3	33.673820234814954 + 53.380121624904490 i	1923.191176245491306 +1200.673331971215930 i
32,47,5	29.841266055414309 + 56.367076350034843 i	1703.617317179591055 +1108.313345256376351 i	32,47,7	25.825862391737910 + 59.088247700778027 i	1512.000972693883796 + 979.707015158640843 i
32,47,9	21.576342761482501 + 61.598527895261441 i	1348.256747052606897 + 821.250956199731831 i	32,47,11	17.025333157672719 + 63.925605349158104 i	1213.793107334074421 + 637.033385326153166 i
32,47,13	12.061670189900010 + 66.083779115387941 i	1111.191868055888335 + 429.983960313861701 i	32,47,15	6.443441478149228 + 68.078232343608079 i	1044.714678327809906 + 202.077219516590702 i

Table D.33: Branch points and associated eigenvalues for $m = 32$.

Table D.34: Branch points and associated eigenvalues for $m = 33$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
33,33,1	3.809418527197693 + 40.041520228891905 i	1094.485521541517983 + 32.881731870515928 i	33,34,1	7.133711124519466 + 41.274600431946006 i	1117.950679696338739 + 88.165878042636891 i
33,35,1	9.986203422843134 + 42.352112286374499 i	1152.242299764993049 + 149.653560056143647 i	33,35,3	3.522860972972818 + 44.992569133255891 i	1093.52298708478361 + 42.44964352724612 i
33,36,1	12.586709603422399 + 43.320947919444549 i	1195.515088200931359 + 216.316017822239473 i	33,36,3	6.821181816100427 + 46.062417541625351 i	1114.374034200293636 + 111.430332160195306 i
33,37,1	15.024267506406323 + 44.207613044475941 i	1246.834527322845588 + 287.422218558446161 i	33,37,3	9.691402163990929 + 47.035047836064543 i	1146.240251819101331 + 185.332746156924486 i
33,37,5	3.388329286560481 + 49.375887217351369 i	1093.093660731965201 + 49.997885832405714 i	33,38,1	17.345737643682025 + 45.028953078848460 i	1305.626583881534771 + 362.442538273912078 i
33,38,3	12.323030896467065 + 47.931457563192424 i	1187.397563873127410 + 263.707595767422333 i	33,38,5	6.668919458618248 + 50.363533579112946 i	1112.639639402734019 + 130.451819543532167 i
33,39,1	19.579416002641832 + 45.796538429888152 i	1371.499714020068041 + 440.975562666381450 i	33,39,3	14.796116971922851 + 48.765991835423087 i	1236.905973711446904 + 346.091648456720691 i
33,39,5	9.548856377544544 + 51.278200435377755 i	1143.193527660621612 + 215.103775244355631 i	33,39,7	3.310625522703695 + 53.466267585753613 i	1092.846865523777524 + 56.589756687030196 i
33,40,1	21.743936662613368 + 46.518763587411378 i	1444.168357317436630 + 522.705437235904469 i	33,40,3	17.154121898741316 + 49.548881371767465 i	1294.164208840068113 + 432.102934563845849 i
33,40,5	12.200752235747652 + 52.132503153931182 i	1183.168779417203950 + 303.787414800537476 i	33,40,7	6.582752252403992 + 54.404000096219661 i	1111.609132140250495 + 147.319762195426108 i
33,41,1	23.852332912198456 + 47.201971544928867 i	1523.414291233425047 + 607.376202153431336 i	33,41,3	19.423906418118765 + 50.287732769250908 i	1358.751970750611463 + 521.430194897206775 i
33,41,5	14.698629764506681 + 52.935899648813901 i	1231.660384531519185 + 396.189716028185671 i	33,41,7	9.472283106472576 + 55.281797754108702 i	1141.347184212379034 + 241.776447764768960 i
33,41,9	3.261256803878205 + 57.373204475347016 i	1092.686757598798749 + 62.612007168662124 i	33,42,1	25.914135148213891 + 47.851107931111031 i	1609.064900202149374 + 694.775537152679931 i
33,42,3	21.623477140840929 + 50.988396213775218 i	1430.357875693636743 + 613.816536463248326 i	33,42,5	17.083291913233914 + 53.695582627101516 i	1288.071683257135874 + 492.019795904911462 i
33,42,7	12.141441228677916 + 56.108606916776438 i	1180.581299493177539 + 339.961658137975576 i	33,42,9	6.531258223415487 + 58.277428283103283 i	1110.9327680452988 7 + 162.85872949289879 i
33,43,1	27.936555103584372 + 48.470126295387360 i	1700.979974459596860 + 784.723976062206930 i	33,43,3	23.765605695861836 + 51.655495276181931 i	1508.741466438635598 + 709.046529671252301 i
33,43,5	19.380293865466445 + 54.417160232646225 i	1351.976685863867488 + 591.025876440749357 i	33,43,7	14.660197340660238 + 56.891425811976148 i	1228.443063258572465 + 441.657618164655560 i
33,43,9	9.431272899235836 + 59.129849832652198 i	1140.127891475422302 + 266.496718578448963 i	33,43,11	3.228245070517290 + 61.153247789539151 i	1092.575232798000798 + 68.253095317097277 i
33,44,1	29.925200533528386 + 49.062252008355536 i	1799.043206387322243 + 877.067462532813806 i	33,44,3	25.859736420844563 + 52.292766107045267 i	1593.711176357314116 + 806.936788408317852 i
33,44,5	21.606962642072922 + 55.105107356974401 i	1423.054910305517978 + 692.991941479090428 i	33,44,7	17.067424258673039 + 57.635774053911859 i	1284.347549253116313 + 546.634795999058269 i
33,44,9	12.116578893968175 + 59.937490368959480 i	1178.873071798474939 + 373.641633709035546 i	33,44,11	6.500345060223579 + 62.033504821458997 i	1110.462000923160076 + 177.489412363171169 i
33,45,1	31.884529723008875 + 49.630161154196990 i	1903.156454254513164 + 971.672038683836263 i	33,45,3	27.913081722974496 + 52.903283726768208 i	1685.110628055809684 + 907.329123233038445 i
33,45,5	23.775707203611521 + 55.763066293964343 i	1501.056019183995886 + 797.731743441863841 i	33,45,7	19.387660863405621 + 58.346084837549739 i	1347.870289629697481 + 654.683440209402761 i
33,45,9	14.654292958542772 + 60.705926001194435 i	1226.33160447841033 + 484.14156363156775 i	33,45,11	9.411680891420446 + 62.867540208594299 i	1139.281863365285972 + 289.863323575379411 i
33,45,13	3.205563735746161 + 64.839922362016495 i	1092.493850393400862 + 73.620192902994901 i	33,46,1	33.818153001486039 + 50.176105898887009 i	2013.235725738793008 +1068.419947899057206 i
33,46,3	29.931292759667954 + 53.489618356071901 i	1782.809649410816064 +1010.085499297903311 i	33,46,5	25.895779846577074 + 56.394052140857205 i	1585.778817959718481 + 905.083355958688003 i
33,46,7	21.637667563660120 + 59.025982691279751 i	1418.687893471030975 + 765.617528259748724 i	33,46,9	17.081862039438764 + 61.439644360588041 i	1281.931124304404193 + 597.811018923724646 i
33,46,11	12.112597158635570 + 63.661229662205962 i	1177.697462212870505 + 405.574881249912153 i	33,46,13	6.482615457875052 + 65.702269372255984 i	1110.12176921643800 + 191.4582192968273 1 i
33,47,1	35.729039852931251 + 50.702004840090801 i	2129.208277665531568 +1167.206706153586538 i	33,47,3	31.918891263024381 + 54.053946665633482 i	1886.698129219334533 +1115.084258349265156 i
33,47,5	27.974296787743778 + 57.000597008274781 i	1677.057943127223552 +1014.904748869138075 i	33,47,7	23.829527557770675 + 59.678478194868156 i	1496.545223754266999 + 879.272883885715601 i
33,47,9	19.423040708157480 + 62.142317968258098 i	1345.251052291127280 + 714.471461951630204 i	33,47,11	14.668114441094984 + 64.419243241502912 i	1224.898434133933051 + 524.520780880145821 i
33,47,13	9.405891548402877 + 66.522491052323318 i	1138.67698380779370 + 312.23320205595883 i	33,47,15	3.189816045009247 + 68.454917681942717 i	1092.432507841692768 + 78.780484306459925 i
33,48,1	37.619665400191828 + 51.209509691760637 i	2251.010466859049302 +1267.938855476877052 i	33,48,3	33.879559070956446 + 54.598132979109607 i	1996.681668105644576 +1222.217237592242327 i
33,48,5	30.016868370462230 + 57.584853875655817 i	1774.754994378680749 +1127.070292273474706 i	33,48,7	25.972304136556343 + 60.306106593856754 i	1581.235583859341432 + 995.504217829963864 i
33,48,9	21.694133496001161 + 62.816999351687627 i	1415.967691709332030 + 833.959797089012568 i	33,48,11	17.114597844193543 + 65.145379045269891 i	1280.324008765483541 + 646.548744616981367 i
33,48,13	12.122071451691635 + 67.305714342370806 i	1176.870248038215550 + 436.212079814484923 i	33,48,15	6.473777024122310 + 69.303442805056306 i	1109.869682203162483 + 204.922814813418995 i

Table D.34: Branch points and associated eigenvalues for $m = 33$.

Table D.35: Branch points and associated eigenvalues for $m = 34$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
34,34,1	3.841043005442322 + 41.105619964491389 i	1161.568417372793419 + 33.487422292666849 i	34,35,1	7.191707254210596 + 42.351668406836097 i	1185.362551846146062 + 89.794899593669882 i
34,36,1	10.065509669276524 + 43.441368821074398 i	1220.104270376541535 + 152.422594686733163 i	34,36,3	3.548661816607783 + 46.092338822786083 i	1160.582073149053713 + 43.186358502972340 i
34,37,1	12.684403544555032 + 44.421769653583546 i	1263.913397065764912 + 220.323737770856809 i	34,37,3	6.870741257780081 + 47.172027318245824 i	1181.688722597411243 + 113.367269801537518 i

Table D.35: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
34,38,1	15.138338405308199 + 45.319492976557285 i	1315.837426743662263 + 292.754185311306344 i	34,38,3	9.760725059644878 + 48.154396362907271 i	1213.926033240443303 + 188.552776306443860 i
34,38,5	3.410712268261570 + 50.503316094621120 i	1160.141221747735699 + 50.824358475032632 i	34,39,1	17.474674984256080 + 46.151465973195123 i	1375.291135983191225 + 369.174728780893645 i
34,39,3	12.409684612784769 + 49.060349522290622 i	1255.540494309165069 + 268.288014294124821 i	34,39,5	6.713001544247314 + 51.499035638442500 i	1179.901740947777853 + 132.611389641807335 i
34,40,1	19.722022633739030 + 46.929318362368562 i	1441.875264840628006 + 449.176656375578034 i	34,40,3	14.898369892208580 + 49.904215163139952 i	1305.575568965798539 + 352.102889606800431 i
34,40,5	9.611329729597738 + 52.421905593279106 i	1210.777267673957795 + 218.659943390287964 i	34,40,7	3.330606169865636 + 54.616718734511139 i	1159.887250427925437 + 57.485752245709364 i
34,41,1	21.899228853916863 + 47.661489253366788 i	1515.298568247932508 + 532.438356603819216 i	34,41,3	17.270642439461160 + 50.696227175138063 i	1363.419194004841529 + 439.609519365505037 i
34,41,5	12.279546052924861 + 53.284398949241719 i	1251.156720192498369 + 308.803582482202103 i	34,41,7	6.622772543764389 + 55.561340554092645 i	1178.836553820643076 + 149.656303903163888 i
34,42,1	24.019480959068048 + 48.354356295989632 i	1595.338436800760292 + 618.699196844168455 i	34,42,3	19.553621176290974 + 51.444001170782307 i	1428.643412242275872 + 530.491717933144514 i
34,42,5	14.792238083657628 + 54.095917821233897 i	1300.120524047849131 + 402.725702091179073 i	34,42,7	9.529515109739666 + 56.446252753749775 i	1208.860931185702839 + 245.604308010992980 i
34,42,9	3.279399476949267 + 58.543504010234066 i	1159.722105782287144 + 63.565086576675782 i	34,43,1	26.092425352468378 + 49.012892769988810 i	1681.818751237036167 + 707.742976541137750 i
34,43,3	21.765492652220420 + 52.153397775950587 i	1500.931092291146342 + 624.488470121548175 i	34,43,5	17.190537384528785 + 54.863629704608343 i	1357.062164894654870 + 500.131658952251314 i
34,43,7	12.214085669010791 + 57.280255103041995 i	1248.459838527829106 + 345.334381245502357 i	34,43,9	6.568062300039307 + 59.453750075853982 i	1178.13495131500144 + 165.34286376911464 i
34,44,1	28.125364048766201 + 49.641074789609966 i	1774.596428394216218 + 799.386947015631563 i	34,44,3	23.919160748619646 + 52.829050892629489 i	1580.037286990813300 + 721.380856545802885 i
34,44,5	19.500214978655247 + 55.593130987236385 i	1421.548478188292165 + 600.766249870240131 i	34,44,7	14.746926940646771 + 58.070278464375960 i	1296.752046260283350 + 448.627088520258383 i
34,44,9	9.484267517704140 + 60.312460338599919 i	1207.589350917523234 + 270.553594898341203 i	34,44,11	3.244910086665352 + 62.341031765659437 i	1159.606787807816545 + 69.254687207579096 i
34,45,1	30.123976911650338 + 50.242146493248100 i	1873.552754128078732 + 893.474232232276790 i	34,45,3	26.024170888142194 + 53.474706366101266 i	1665.76681422002343 + 820.98249946133353 i
34,45,5	21.738750627214543 + 56.288891455337662 i	1493.253497064709109 + 704.410431235168176 i	34,45,7	17.167188952343320 + 58.821806232496442 i	1353.143721710539467 + 555.250504887135662 i
34,45,9	12.184175815604677 + 61.126517081012004 i	1246.668253889780999 + 379.317392631279631 i	34,45,11	6.534499375115969 + 63.226640263648378 i	1177.644687480097673 + 180.100319199188704 i
34,46,1	32.092781044728859 + 50.818799829591548 i	1978.587536499793941 + 989.868420873961782 i	34,46,3	28.087815104249909 + 54.093448127772440 i	1757.960315089699861 + 923.132614681210498 i
34,46,5	23.918667035088735 + 56.954551824707870 i	1571.922540283323769 + 810.875292427268732 i	34,46,7	19.499594550751276 + 59.539250547151319 i	1417.203780008229160 + 664.992464277709473 i
34,46,9	14.735308799255268 + 61.901427266943180 i	1294.52359900898728 + 491.48190191466097 i	34,46,11	9.461131583876453 + 64.066309903807991 i	1206.702161371842067 + 294.118380071782894 i
34,46,13	3.221001426737075 + 66.043337355278453 i	1159.522413173332325 + 74.663965920390240 i	34,47,1	34.035435598324675 + 51.373300562644438 i	2089.615011688376853 +1088.449598135199494 i
34,47,3	30.115808797122124 + 54.687854526231590 i	1856.485111171860808 +1027.690893098525521 i	34,47,5	26.049304424954911 + 57.593127566242622 i	1657.350878815103442 + 919.996557672920062 i
34,47,7	21.761035632124678 + 60.226223176216315 i	1488.603692292421329 + 777.664687156838909 i	34,47,9	17.175352228529722 + 62.641638445719799 i	1350.574967606002929 + 606.860158773465173 i
34,47,11	12.175925026064874 + 64.865788001960311 i	1245.426061152296597 + 411.514437822294440 i	34,47,13	6.514527476858326 + 66.910497240178699 i	1177.28877855476821 + 194.18022444780058 i
34,48,1	35.954951176140888 + 51.907579086431312 i	2206.560887556065609 +1189.111363619419990 i	34,48,3	32.112726550474342 + 55.260109632557068 i	1961.228950872101677 +1134.533664460178443 i
34,48,5	28.137848954586470 + 58.207152266668594 i	1749.370210949945204 +1031.630112689570979 i	34,48,7	23.963694217628273 + 60.885726831316163 i	1567.084202260930851 + 893.100936795717757 i
34,48,9	19.528220121926321 + 63.350797978812587 i	1414.395700551389609 + 725.271921956879396 i	34,48,11	14.744254267427060 + 65.629677470399969 i	1292.995874503500772 + 532.185556691411989 i
34,48,13	9.452308555912474 + 67.735818434812799 i	1206.06370483454839 + 316.66289867823787 i	34,48,15	3.204210514338237 + 69.672459419654814 i	1159.458631555810825 + 79.861537608196883 i
34,49,1	37.853837968655803 + 52.423297456113815 i	2329.360152778643169 +1291.758543781966637 i	34,49,3	34.082294391879969 + 55.812084515727975 i	2072.095582581458075 +1243.550971264721738 i
34,49,5	30.189967878074395 + 58.798781003398510 i	1847.839650926604008 +1145.648467304034739 i	34,49,7	26.116710630997432 + 61.520291932512599 i	1652.435228623861349 +1011.154056020158919 i
34,49,9	21.810331722265911 + 64.031941815037726 i	1485.657318622181265 + 846.552400892601213 i	34,49,11	17.202693546020711 + 66.361735677818714 i	1348.843521435219373 + 655.978628133244456 i
34,49,13	12.181712942260537 + 68.524310200909213 i	1244.543916264755580 + 442.385151960193525 i	34,49,15	6.503754477657149 + 70.525353218575376 i	1177.023717622060758 + 207.743617292837428 i

Table D.35: Branch points and associated eigenvalues for $m = 34$.

Table D.36: Branch points and associated eigenvalues for $m = 35$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
35,35,1	3.872111158323472 + 42.168535805737200 i	1230.650528809010211 + 34.087257236665437 i	35,36,1	7.248694991848762 + 43.427300001801306 i	1254.77052743099193 + 91.40817039999628 i
35,37,1	10.143455991774532 + 44.528947707006978 i	1289.958100213606713 + 155.164852632624076 i	35,37,3	3.574042380596181 + 47.190357077005110 i	1229.64063017502076 + 43.91606353364563 i
35,38,1	12.780445461767828 + 45.520683294884982 i	1334.298526545414688 + 224.292694756284476 i	35,38,3	6.919495778689906 + 48.279707491143810 i	1251.000578165268735 + 115.285793609228392 i
35,39,1	15.250506609382281 + 46.429244114054491 i	1386.821489037097308 + 298.034566514660185 i	35,39,3	9.828932358227412 + 49.271634325716640 i	1283.605641913455656 + 191.742218258464447 i
35,39,5	3.432755221345637 + 51.628596802440136 i	1229.188382862254684 + 51.643102765578448 i	35,40,1	17.601490296197081 + 47.271638151532756 i	1446.930684030061684 + 375.841765465610592 i
35,40,3	12.494958622093256 + 50.186951424579682 i	1325.673151003259363 + 272.824946965859851 i	35,40,5	6.756410873237363 + 52.632253425631241 i	1249.161610131570797 + 134.750796264863681 i
35,41,1	19.862312802977819 + 48.059553646682531 i	1514.219212019906536 + 457.298351334968970 i	35,41,3	14.999012791233246 + 51.039971970097540 i	1376.230183701724400 + 358.057085507919282 i
35,41,5	9.672854694666251 + 53.563183294425293 i	1280.356022123947696 + 222.182954211825063 i	35,41,7	3.350302672714701 + 55.764723610811508 i	1228.927318100970751 + 58.373520195627982 i
35,42,1	22.052030599546907 + 48.801473676787445 i	1588.390200230700430 + 542.077010195973230 i	35,42,3	17.385348932868649 + 51.840933684313995 i	1434.653987885794777 + 447.044891314578877 i
35,42,5	12.357153152990749 + 54.433721790188059 i	1321.136221018764445 + 313.773033252416838 i	35,42,7	6.662216953425567 + 56.716126581412347 i	1248.062149995098707 + 151.971431561142850 i
35,43,1	24.183981771159907 + 49.503810002765825 i	1669.216696774721186 + 629.912481051892769 i	35,43,3	19.681339185078642 + 52.597461474946030 i	1500.509020340599363 + 539.467297061785871 i
35,43,5	14.884449185011775 + 55.253217410369096 i	1370.568199367867010 + 409.200874456748124 i	35,43,7	9.585924989111680 + 57.608036652444504 i	1278.370527882179204 + 249.397150594481417 i
35,43,9	3.297300038157157 + 59.711125538612528 i	1228.757195650430049 + 64.509575255197447 i	35,44,1	26.267926248734440 + 50.171563091125478 i	1756.519114486484114 + 720.584720097292688 i
35,44,3	21.905346505604218 + 53.315426502946096 i	1573.472456896834728 + 635.059202922297686 i	35,44,5	17.296197246399082 + 56.028813609569532 i	1428.035677537504625 + 508.168105909930262 i
35,44,7	12.285692219589704 + 58.449111480762312 i	1318.331263029288039 + 350.658030938229331 i	35,44,9	6.604365839824510 + 60.627305850701333 i	1247.33560976466174 + 167.80465683478181 i
35,45,1	28.311254381235756 + 50.808731219028104 i	1850.151526431636285 + 813.907727642383406 i	35,45,3	24.070404388667303 + 53.999473000012685 i	1653.294903601276246 + 733.598231607113917 i
35,45,5	19.618380325400246 + 56.766095698689270 i	1493.098381501265976 + 610.416128491260110 i	35,45,7	14.832426093729573 + 59.246216810860588 i	1367.050421350588749 + 455.532986746386086 i
35,45,9	9.536540196419271 + 61.492207955180781 i	1277.047293005166239 + 274.574064231950217 i	35,45,11	3.261366052739340 + 63.525954186295273 i	1228.638129113658124 + 70.247418959966581 i
35,46,1	30.319716573150064 + 51.418576939907169 i	1949.992836967342100 + 909.721835186715452 i	35,46,3	26.186157295004602 + 54.653356451881763 i	1739.77759781660598 + 834.89504117752790 i
35,46,5	21.868628110750549 + 57.469528767460098 i	1565.424909768003090 + 715.722897252409325 i	35,46,7	17.265549387496087 + 60.004801128720075 i	1423.925341047986649 + 563.787717570260384 i
35,46,9	12.250854672231862 + 62.312579196093559 i	1316.457344349541700 + 384.942312767413625 i	35,46,11	6.568213892336193 + 64.416841403811034 i	1246.826082964767920 + 182.688186102639236 i
35,47,1	32.297887459751053 + 52.003807753640189 i	2055.940825435684928 +1007.888201823919871 i	35,47,3	28.259975252107594 + 55.280169617251005 i	1832.758228202983673 + 938.786276602845760 i
35,47,5	24.059574911543493 + 58.142752201963987 i	1644.756277213422663 + 823.896861649976358 i	35,47,7	19.609965839538955 + 60.729256344059223 i	1488.518377989591272 + 675.207661507837884 i
35,47,9	14.815230016189007 + 63.093859529191057 i	1364.706438792099107 + 498.756600641171929 i	35,47,11	9.509942303029600 + 65.262063804283855 i	1276.119443070080706 + 298.335978399903581 i
35,47,13	3.236256849446894 + 67.243743858897645 i	1228.550797357344663 + 75.698677892540630 i	35,48,1	34.249473938544746 + 52.566702741177373 i	2167.907975214495764 +1108.284775692500716 i
35,48,3	30.297636572369697 + 55.882498872600365 i	1932.101634609690564 +1045.129375573357493 i	35,48,5	26.200647191979993 + 58.788781994500489 i	1730.884253011961846 + 934.771417014760459 i
35,48,7	21.882696567144841 + 61.423182007631489 i	1560.495910459533889 + 789.602300049104201 i	35,48,9	17.267587255473387 + 63.840457730453828 i	1421.206170859575423 + 615.828497548468135 i
35,48,11	12.238433949510274 + 66.067243884010651 i	1315.149389306341845 + 417.401819495203654 i	35,48,13	6.546049865643814 + 68.115655829829819 i	1246.45468460400298 + 196.87866088564016 i
35,49,1	36.177526825140589 + 53.109203838601616 i	2285.818461650911104 +1210.803258937816054 i	35,49,3	32.303767587047496 + 56.462535580932652 i	2037.693446181010813 +1153.798674572640266 i
35,49,5	28.299099689994677 + 59.410153290013888 i	1823.637624580328293 +1048.200383232195463 i	35,49,7	24.096020665757585 + 62.089573330346063 i	1639.594599213179436 + 906.803352704059989 i
35,49,9	19.631997382729800 + 64.555996863820610 i	1485.523852020186951 + 735.976070049432110 i	35,49,11	14.819412820979073 + 66.836919381994406 i	1363.085338814591296 + 539.783131511233364 i
35,49,13	9.498154084586158 + 68.946007323367212 i	1275.447817802362351 + 321.054340345361936 i	35,49,15	3.218444868097973 + 70.886874413691245 i	1228.484605044768841 + 80.933381015765846 i
35,50,1	38.084590601307902 + 53.632983210564028 i	2409.605919874955816 +1315.346779512763078 i	35,50,3	34.282137583702948 + 57.022157444277653 i	2149.435577471911529 +1264.682438211897988 i
35,50,5	30.360655448603122 + 60.009023301936665 i	1922.873043955353751 +1164.054426272738510 i	35,50,7	26.259154098495877 + 62.730956158524556 i	1725.601008303345907 +1026.661809687468121 i
35,50,9	21.924992506679313 + 65.243496893321250 i	1557.326260741795977 + 859.032844740516111 i	35,50,11	17.289659432614116 + 67.574808186392104 i	1419.351959435510480 + 665.325976922734981 i
35,50,13	12.240619048921660 + 69.739693094715918 i	1314.212989700976323 + 448.505038664766346 i	35,50,15	6.533384474554056 + 71.744085693565381 i	1246.176804114295692 + 210.540453113085448 i

Table D.36: Branch points and associated eigenvalues for $m = 35$.

Table D.37: Branch points and associated eigenvalues for $m = 36$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
36,36,1	3.902647202546461 + 43.230321346586543 i	1301.731884568082478 + 34.681456338827374 i	36,37,1	7.304717811958946 + 44.501560621209215 i	1326.17474910544684 + 93.00628128263119 i
36,38,1	10.220100331538202 + 45.614925699782887 i	1361.804086793692022 + 157.881338305754525 i	36,38,3	3.599019769202020 + 48.286699873956209 i	1300.698676327444446 + 44.639016826845157 i
36,39,1	12.874904899586136 + 46.617776424123925 i	1406.670957272737724 + 228.224342927512765 i	36,39,3	6.967478097506334 + 49.385541877393649 i	1322.309698907047959 + 117.186581037524974 i
36,40,1	15.360851266376798 + 47.536964353686372 i	1459.787399576487132 + 303.265297628234066 i	36,40,3	9.896069061702098 + 50.386853615955751 i	1355.279291562186245 + 194.902196194025834 i
36,40,5	3.454471120798930 + 52.751819321172242 i	1300.235156796010870 + 52.454397865632345 i	36,41,1	17.726270822211529 + 48.389577347368920 i	1520.546135568068166 + 382.446093320720308 i
36,41,3	12.578908070714471 + 51.311363195090394 i	1397.795889385464534 + 277.319990664208945 i	36,41,5	6.799173303281159 + 53.763282541047495 i	1320.419319324817479 + 136.870766359172819 i
36,42,1	20.000380657338034 + 49.187361404755777 i	1588.532699764220297 + 465.343627726430327 i	36,42,3	15.098109395651914 + 52.173370064569383 i	1448.870335422296421 + 363.956329589010465 i
36,42,5	9.733467589892536 + 54.702135190421501 i	1351.929953507776190 + 225.674001048723056 i	36,42,7	3.369725201679067 + 56.910381474711336 i	1299.967077777728491 + 59.253351806465787 i
36,43,1	22.202441981737657 + 49.938842948440630 i	1663.444646411346866 + 551.624936997973464 i	36,43,3	17.498312468272353 + 52.983116390223600 i	1507.869286723444020 + 454.411662358523472 i
36,43,5	12.433618682611733 + 55.580579528145137 i	1393.107557496286518 + 318.697446029683476 i	36,43,7	6.701106434445274 + 57.868461615346348 i	1319.285975678810701 + 154.265901454108018 i
36,44,1	24.345940560111831 + 50.650467331631759 i	1745.050723784886031 + 641.020174504238980 i	36,44,3	19.807137911996922 + 53.748236631129863 i	1574.349684030094977 + 548.360083138996401 i
36,44,5	14.975315812303299 + 56.407912482731042 i	1443.003817642498689 + 415.617415977351413 i	36,44,7	9.641542667270219 + 58.767257548282537 i	1349.876100855744880 + 253.156209500548499 i
36,44,9	3.314966610464968 + 60.876174535823232 i	1299.792033986031811 + 65.445771191550307 i	36,45,1	26.440747497026248 + 51.327261767270116 i	1833.167911201880088 + 733.305488818052709 i
36,45,3	22.043121730424140 + 54.474612568419076 i	1647.983063226564582 + 645.532443682775579 i	36,45,5	17.400330855579401 + 57.191254566311031 i	1500.992775233791008 + 516.131840919112847 i
36,45,7	12.356298508918028 + 59.615289038684523 i	1390.195791175032355 + 355.934333213894377 i	36,45,9	6.640186088734694 + 61.798204247578994 i	1318.53478573815892 + 170.24488154933948 i
36,46,1	28.494339626662047 + 51.973246315439148 i	1927.647466413368193 + 828.291659523682256 i	36,46,3	24.219424518062119 + 55.166898759581393 i	1728.515625107364940 + 745.702939542449144 i
36,46,5	19.734855063935356 + 57.936180649375096 i	1566.627109812179697 + 619.978754134907604 i	36,46,7	14.916739152440490 + 60.419358830468667 i	1439.338514813697429 + 462.377545089910143 i
36,46,9	9.588115925181464 + 62.669204971208046 i	1348.501817410005742 + 278.559382784048012 i	36,46,11	3.277619533216764 + 64.708124612941191 i	1299.669261682670140 + 71.231591689885306 i
36,47,1	30.512536405823571 + 52.591701611910551 i	2028.365936314442024 + 925.816251817874672 i	36,47,3	26.345787826020665 + 55.828860278403035 i	1815.74506019402770 + 848.67929102933010 i
36,47,5	21.996665368608934 + 58.647151508498830 i	1639.570033686173247 + 726.933134588427379 i	36,47,7	17.362555840078564 + 61.184881735029961 i	1496.692855588874181 + 572.249186336946991 i
36,47,9	12.316647205567618 + 63.495792914935620 i	1388.240517660914293 + 390.518144329065990 i	36,47,11	6.601502959826701 + 65.604220224371701 i	1318.006220068314178 + 185.253793806939825 i
36,48,1	32.499968779430425 + 53.185350412867301 i	2135.219092165541952 +1025.738018114570195 i	36,48,3	28.429658123057255 + 56.463598631578897 i	1909.506132654340718 + 954.295595297066029 i
36,48,5	24.198505641352824 + 59.327805433164677 i	1719.558296393950059 + 836.800814155233866 i	36,48,7	19.718830257616084 + 61.916230209068580 i	1561.81466594579797 + 685.33231749216066 i
36,48,9	14.894094313888132 + 64.283343044171303 i	1436.880387763185581 + 505.967915079091426 i	36,48,11	9.558134121544507 + 66.454916777435812 i	1347.533787276821158 + 302.517384271718583 i
36,48,13	3.251335349238749 + 68.441253981605158 i	1299.579006525171280 + 76.724630555372983 i	36,49,1	34.460390372565350 + 53.756484852121964 i	2248.117682500243518 +1127.932789789693061 i
36,49,3	30.476875368201405 + 57.073708129445919 i	2009.661224274334472 +1062.407057112336815 i	36,49,5	26.349886981642122 + 59.981159076059171 i	1806.380196844263992 + 949.412879082072777 i
36,49,7	22.002710574611353 + 62.616992098121408 i	1634.365274889490365 + 801.434198562305369 i	36,49,9	17.358610132043612 + 65.036226596506225 i	1493.825100015995531 + 624.718806546439851 i
36,49,11	12.300150941594046 + 67.265715333635839 i	1386.867586809824616 + 423.238784811139112 i	36,49,13	6.577194654322401 + 69.317859107983210 i	1317.61951250233142 + 199.55431103822815 i
36,50,1	36.396891350480992 + 54.307058169239774 i	2366.984365948525465 +1232.290389684657839 i	36,50,3	32.492116590141983 + 57.661387325278291 i	2116.093865272222047 +1172.886037732633440 i

Table D.37: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
36,50,5	28.458131410551232 + 60.609749217950522 i	1899.861637935797035 +1064.621101084379461 i	36,50,7	24.226571289056395 + 63.290155458849419 i	1714.077295341793615 + 920.384521491368787 i
36,50,9	19.734420251281374 + 65.758043152015475 i	1558.635985623699753 + 746.587204294411663 i	36,50,11	14.893622415935830 + 68.041090325113274 i	1435.167041485786285 + 547.315765244008617 i
36,50,13	9.543446033608578 + 70.153173978608791 i	1346.829384964829387 + 325.408793181068177 i	36,50,15	3.232523479434889 + 72.098276179789721 i	1299.510430805960368 + 81.996315083605779 i
36,51,1	38.312049748682377 + 54.838752421227774 i	2491.751438122286345 +1338.712263629182871 i	36,51,3	34.479193452174684 + 58.228520449184721 i	2228.704154071146604 +1285.619040067087326 i
36,51,5	30.529016691078294 + 61.215735244695878 i	1999.856831920906188 +1182.294318123030052 i	36,51,7	26.399702672043173 + 63.938241801577426 i	1800.733965689864817 +1042.032438713137026 i
36,51,9	22.038167875710112 + 66.451797250685530 i	1630.975119645560881 + 871.404963910030801 i	36,51,11	17.375532580012781 + 68.784721340771299 i	1491.849622755942164 + 674.593560956153851 i
36,51,13	12.298812916193684 + 70.951981848865032 i	1385.877580903687885 + 454.573495396867770 i	36,51,15	6.562677160004798 + 72.959755168643483 i	1317.328960799893594 + 213.314101900922424 i

Table D.37: Branch points and associated eigenvalues for $m = 36$.

Table D.38: Branch points and associated eigenvalues for $m = 37$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
37,37,1	3.932673698914612 + 44.291026388716197 i	1374.812511582645584 + 35.270225229125522 i	37,38,1	7.359816241596399 + 45.574511022210423 i	1399.575350654475642 + 94.589785388531155 i
37,39,1	10.295496743484161 + 46.699374085358784 i	1435.642509166335913 + 160.572992078099020 i	37,39,3	3.623609988250024 + 49.381438018117294 i	1373.756228718442571 + 45.355460437840293 i
37,40,1	12.967846789130247 + 47.713130372002759 i	1481.031140108581667 + 232.120043721375168 i	37,40,3	7.014718831687849 + 50.489608583794762 i	1395.616177091208556 + 119.070267221591010 i
37,41,1	15.469446330853863 + 48.642744604034021 i	1534.735801379217719 + 308.448190708983248 i	37,41,3	9.962177301434362 + 51.500139857215011 i	1428.947183381216064 + 198.033764154585384 i
37,41,5	3.475872159152672 + 53.873067698342567 i	1373.281555595681994 + 53.258505826996902 i	37,42,1	17.849098142256722 + 49.505383608642717 i	1596.138342194848747 + 388.990001439418450 i
37,42,3	12.661584616705886 + 52.433677936668772 i	1471.909044051752289 + 281.774642735814496 i	37,42,5	6.841313117545806 + 54.892212289891980 i	1393.674936961889898 + 138.971982413105877 i
37,43,1	20.136314297541780 + 50.312850413899127 i	1664.816801922877403 + 473.315275693562569 i	37,43,3	15.195719437268501 + 53.304509891349603 i	1523.496511326339714 + 369.802584897846145 i
37,43,5	9.793202526699494 + 55.838856240375378 i	1425.499215408901136 + 229.134204400302394 i	37,43,7	3.388883352808498 + 58.053785232133372 i	1373.006538257717239 + 60.125520865183965 i
37,44,1	22.350556722066411 + 51.073714191483255 i	1740.463214498193565 + 561.085450308004738 i	37,44,3	17.609599719641057 + 54.122882896492925 i	1583.065746046637287 + 461.712281878045417 i
37,44,5	12.508985062140802 + 56.725072913180050 i	1467.070990050406408 + 323.578397418371480 i	37,44,7	6.739460730882296 + 59.018442486985229 i	1392.508083148270543 + 156.540424593059015 i
37,45,1	24.505455919117701 + 51.794453392141854 i	1822.842069385836339 + 652.026134191345704 i	37,45,3	19.931090057780743 + 54.896441197104210 i	1650.166240403789288 + 557.173031024122224 i
37,45,5	15.064887546565323 + 57.560109594359069 i	1517.427763566305430 + 421.977376663529185 i	37,45,7	9.696396336673802 + 59.924016643298998 i	1423.377770238526637 + 256.882645023444354 i
37,45,9	3.332406889155311 + 62.038749910793044 i	1372.826627299625898 + 66.373954859253814 i	37,46,1	26.610991928182610 + 52.480121556784539 i	1911.766945457159181 + 745.909702604993640 i
37,46,3	22.178896294541850 + 55.631077270390016 i	1724.463941278419725 + 655.911670631208835 i	37,46,5	17.502994036909666 + 58.351064885192478 i	1575.933981455774529 + 524.025403749110296 i
37,46,7	12.425939995818624 + 60.778893561985072 i	1464.053629960508943 + 361.164911211686444 i	37,46,9	6.675539351228771 + 62.966547163092400 i	1391.73251969217040 + 172.66426548378672 i
37,47,1	28.674726310354863 + 53.134760168321051 i	2007.086312513262783 + 842.543743560773919 i	37,47,3	24.366303729743361 + 56.331455993322436 i	1805.700684255930810 + 757.698998765256874 i
37,47,5	19.849700506534735 + 59.103503813321886 i	1642.135337732162270 + 629.457171780068506 i	37,47,7	14.999907924562098 + 61.589814984143281 i	1513.616636111539265 + 469.162863008985255 i
37,47,9	9.639018319105881 + 63.843556718075156 i	1421.953019197015692 + 282.510733352855937 i	37,47,11	3.293676772427689 + 65.887645965487707 i	1372.700190325977474 + 72.207489182682977 i
37,48,1	30.702546159448008 + 53.761667630350232 i	2108.674383098102226 + 941.763081127564647 i	37,48,3	26.503149148992513 + 57.001351976886717 i	1893.670645471439684 + 862.339823500328293 i
37,48,5	22.122928562770795 + 59.821883202008998 i	1715.689705271557159 + 738.044708065990903 i	37,48,7	17.458255718925294 + 62.362163210396687 i	1571.446690455964244 + 580.637500008684697 i
37,48,9	12.381583410401772 + 64.676267246970454 i	1462.017940083228041 + 396.046535502455640 i	37,48,11	6.634380186124983 + 66.788881942399982 i	1391.185130221830832 + 187.797878255648243 i
37,49,1	32.699137568054844 + 54.363581677172959 i	2216.424940232038807 +1043.424083075505905 i	37,49,3	28.596953983135233 + 57.643875400370042 i	1988.205691541040324 + 969.665716384026041 i
37,49,5	24.335529689992221 + 60.509840463120547 i	1796.329605492932160 + 849.591248512614470 i	37,49,7	19.826240187477499 + 63.100291977077731 i	1637.09319604007146 + 695.36952295239962 i
37,49,9	14.971937327272359 + 65.469990632580192 i	1511.045697263161173 + 513.117969309264026 i	37,49,11	9.605727010675889 + 67.644976766694796 i	1420.945269565313083 + 306.663791008298688 i
37,49,13	3.266242032146176 + 69.635973207950073 i	1372.607044185421728 + 77.742108636789737 i	37,50,1	34.668299919240816 + 54.942807246765710 i	2330.247014868843962 +1147.400483529346047 i
37,50,3	30.653618632421153 + 58.261628419212030 i	2089.165768982584723 +1079.529668810876501 i	37,50,5	26.497098089475664 + 61.170393040521084 i	1883.839897478181229 + 963.925589438315910 i
37,50,7	22.121134474320368 + 63.807777907363288 i	1710.212474738900710 + 813.163987347754869 i	37,50,9	17.448461519229642 + 66.229061741242674 i	1568.432103415803567 + 633.533696163698551 i
37,50,11	12.361101597057150 + 68.461313273293911 i	1460.580787825974312 + 429.026992006105502 i	37,50,13	6.607973289580369 + 70.517214332162638 i	1390.783286599565372 + 202.207913378683202 i
37,51,1	36.613161907806891 + 55.501308657826186 i	2450.061764110991589 +1253.580243473906194 i	37,51,3	32.677869822850354 + 58.856816681395529 i	2196.432328678520207 +1191.802084195777297 i
37,51,5	28.615021827638163 + 61.806079424120711 i	1978.043625045896306 +1080.897471459421695 i	37,51,7	24.355406879801539 + 64.487602230742738 i	1790.533125592325632 + 933.848573285140219 i
37,51,9	19.835533891784134 + 66.957057438154507 i	1633.732557102340706 + 757.108431591946555 i	37,51,11	14.966913670385948 + 69.242304354987326 i	1509.241187593846689 + 554.785589197702052 i
37,51,13	9.588201414830753 + 71.357427834740179 i	1420.208466374795535 + 329.727452412755326 i	37,51,15	3.246450543954862 + 73.306771690304629 i	1372.536111328478682 + 83.050623770406020 i
37,52,1	38.536334421360891 + 56.040777655777994 i	2575.800157541338649 +1361.863142350983935 i	37,52,3	34.673560751284704 + 59.431330846669233 i	2309.903669974851255 +1306.367719338506959 i
37,52,5	30.695132367589292 + 62.419061206947916 i	2078.792582297921399 +1200.373919837781159 i	37,52,7	26.538420709391474 + 65.142282346287189 i	1877.835089351678334 +1057.270609796300505 i
37,52,9	22.149907044174412 + 67.656967369393982 i	1706.604467659657075 + 883.672371709359822 i	37,52,11	17.460348122785476 + 69.991592419682135 i	1566.336798112911083 + 683.783992918085119 i
37,52,13	12.356316531270872 + 72.161288325120154 i	1459.537798202045678 + 460.592179613517885 i	37,52,15	6.591642218082538 + 74.172469982623170 i	1390.480206369777534 + 216.065300384209337 i

Table D.38: Branch points and associated eigenvalues for $m = 37$.

Table D.39: Branch points and associated eigenvalues for $m = 38$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
38,38,1	3.962211704798428 + 45.350697301684200 i	1449.892435154165469 + 35.853756772196718 i	38,39,1	7.414028129632658 + 46.646207757452480 i	1474.972457759341978 + 96.159201528062056 i
38,40,1	10.369695747760778 + 47.782359202017865 i	1511.473629505469717 + 163.240695953722451 i	38,40,3	3.647828040375054 + 50.474637614545784 i	1448.813303491754141 + 46.065621672710769 i
38,41,1	13.059331861108850 + 48.806820815839195 i	1557.379498703661511 + 235.981074073170618 i	38,41,3	7.061246679864625 + 51.591980530206875 i	1470.920099716126515 + 120.937448647974733 i
38,42,1	15.576361024318581 + 49.746669452254764 i	1611.667298743201172 + 313.584945338987382 i	38,42,3	10.027296585844525 + 52.611572979934706 i	1504.609507053852440 + 201.137912117362760 i
38,42,5	3.496969810195946 + 54.992420576133895 i	1448.327590682634536 + 54.055673047360735 i	38,43,1	17.970048670195560 + 50.619150029947839 i	1673.708104351532256 + 395.475636820502700 i
38,43,3	12.743036728450852 + 53.553982555632366 i	1548.012930458062101 + 286.190309604677225 i	38,43,5	6.882853154453900 + 56.019126241019521 i	1468.928527834702209 + 141.055086230537512 i
38,44,1	20.270196303064008 + 51.436121900274941 i	1743.072527998621581 + 481.215912169137937 i	38,44,3	15.291898992101487 + 54.433485206828699 i	1600.109170771361732 + 375.597695372419480 i
38,44,5	9.852091592300058 + 56.973435304618313 i	1501.063953170787954 + 232.564618095467097 i	38,44,7	3.407786191415853 + 59.195021984209077 i	1448.045707931750030 + 60.990285132282317 i
38,45,1	22.496462728362957 + 52.206196412179708 i	1819.447134041658728 + 570.461657719935488 i	38,45,3	17.719273317097873 + 55.260333641955100 i	1660.243983977844209 + 468.949050730516717 i
38,45,5	12.583292208827363 + 57.867296223757897 i	1543.026765085420493 + 328.417370360567361 i	38,45,7	6.777298472208082 + 60.166159991033446 i	1467.728522124353958 + 158.795670475754861 i
38,46,1	24.662620386539714 + 52.935884641279877 i	1902.592192645413888 + 662.933977627866284 i	38,46,3	20.053263953923874 + 56.042182106778029 i	1727.959478870153589 + 565.908916500412261 i
38,46,5	15.153211063383912 + 58.709908456424969 i	1593.840401090599016 + 428.282684901094854 i	38,46,7	9.750512595198394 + 61.078408837324076 i	1498.875650183403877 + 260.577549871656799 i
38,46,9	3.349628171847759 + 63.198944558847458 i	1447.860981841664398 + 67.294390646261022 i	38,47,1	26.778756119946632 + 53.630266057964477 i	1992.317914077513933 + 758.401506919884241 i
38,47,3	22.312743522664285 + 56.784933841996192 i	1802.916062489673777 + 666.200151302249765 i	38,47,5	17.604239368548814 + 59.508349663402150 i	1652.859791436387468 + 531.851183659862613 i
38,47,7	12.494650139211704 + 61.940024251973519 i	1539.904975982160295 + 366.351293715641589 i	38,47,9	6.710441059659745 + 64.132430316473091 i	1466.92885030321486 + 175.06349460225294 i
38,48,1	28.852514542326382 + 54.293403226011193 i	2088.470006531339095 + 856.668670130994997 i	38,48,3	24.511119742578071 + 57.493264035476031 i	1884.851243850050651 + 769.590184377663491 i
38,48,5	19.962974427734363 + 60.268175591146836 i	1719.623703487060993 + 638.854246137333234 i	38,48,7	15.081971871405011 + 62.757688860902590 i	1589.885079073377710 + 475.890918130456953 i
38,48,9	9.689269721068154 + 65.015362152510178 i	1497.400989104312202 + 286.429231280058673 i	38,48,11	3.309543714988134 + 67.064615067302270 i	1447.730919701100447 + 73.175379277200662 i
38,49,1	30.889849036936937 + 54.928612018883371 i	2190.920370444267519 + 957.567574454535816 i	38,49,3	26.658322863086230 + 58.170956785860412 i	1973.555716009254638 + 875.880936201318718 i
38,49,5	22.247480068405427 + 60.993839447767371 i	1793.784715891821406 + 749.060971650829970 i	38,49,7	17.552693787919058 + 63.536753552939466 i	1648.187249222726450 + 588.955097407170456 i
38,49,9	12.445691665058941 + 65.854104611790092 i	1537.789770169341182 + 401.529040697400490 i	38,49,11	6.666858491535291 + 67.970925562922616 i	1466.362843649737670 + 190.321134166681645 i
38,50,1	32.895499737688397 + 55.538644885983089 i	2299.560819692962923 +1060.952224236922348 i	38,50,3	28.761947866209223 + 58.821130873063503 i	2068.858474017066328 + 984.901474156172981 i
38,50,5	24.470713522659736 + 61.688977973395382 i	1875.071157918559265 + 862.272021056697440 i	38,50,7	19.932245108464940 + 64.281554035776239 i	1714.35449238660931 + 705.32218977140315 i
38,50,9	15.048792775951357 + 66.653908300694482 i	1587.202606848119103 + 520.208766809143754 i	38,50,11	9.652739918054326 + 68.832345356423843 i	1496.353962427660463 + 310.776325282188168 i
38,50,13	3.280981778416114 + 70.828000928754633 i	1447.634913771270931 + 78.751381186626091 i	38,51,1	34.873310862381096 + 56.125819334720305 i	2414.298684267213048 +1166.694275133376095 i
38,51,3	30.827954443819742 + 59.446396213584222 i	2170.617051106847157 +1096.502595133533305 i	38,51,5	26.642350627783856 + 62.356609526207201 i	1963.264478233284990 + 978.313919281824724 i
38,51,7	22.238021958970833 + 64.995656166239215 i	1788.038164111822880 + 824.795062731857570 i	38,51,9	17.537179901890436 + 67.419072818831211 i	1645.027512938254858 + 642.275628851490296 i
38,51,11	12.421310193608525 + 69.654142101064323 i	1536.289120869219232 + 434.768006944357296 i	38,51,13	6.638396672325069 + 71.713822585907224 i	1465.946030472117855 + 204.840165850242897 i
38,52,1	36.826448854370092 + 56.692110551484376 i	2535.053634361549030 +1274.679843060329176 i	38,52,3	32.861118065499817 + 60.048965460006478 i	2278.710837716281276 +1210.552761476522164 i
38,52,5	28.769844312808367 + 62.999274374498988 i	2058.184886149306294 +1097.034392357113347 i	38,52,7	24.482584908689315 + 65.682034655755359 i	1868.962882415361946 + 947.199399778589168 i
38,52,9	19.935381063933420 + 68.153153044213823 i	1710.814000539787685 + 767.542683044554792 i	38,52,11	15.039315627157784 + 70.440668826326942 i	1585.307973402003654 + 562.194616231793248 i
38,52,13	9.632436411908481 + 72.558872042669122 i	1495.585119961012879 + 334.011447877047601 i	38,52,15	3.260230088244617 + 74.512461885868802 i	1447.561649084176679 + 84.096575710307846 i
38,53,1	38.757556785064729 + 57.239219778145866 i	2661.755326513821274 +1384.807056206624793 i	38,53,3	34.865332658584414 + 60.630735609145695 i	2393.036350559031689 +1326.934998913333629 i
38,53,5	30.859078766710510 + 63.619136351561778 i	2159.681778514739108 +1218.298667650952439 i	38,53,7	26.675369077779738 + 66.343203004795285 i	1956.905317535114818 +1072.380720491960346 i
38,53,9	22.260256622018879 + 68.859124230826183 i	1784.214849803421430 + 895.838477294611835 i	38,53,11	17.544139392179257 + 71.195531819095891 i	1642.813759723215071 + 692.899740592394323 i
38,53,13	12.413150802200752 + 73.367717974771551 i	1535.193745876128787 + 466.562658348719992 i	38,53,15	6.620288899162609 + 75.382332388573246 i	1465.630559073156974 + 218.794745665848581 i

Table D.39: Branch points and associated eigenvalues for $m = 38$.

Table D.40: Branch points and associated eigenvalues for $m = 39$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
39,39,1	3.991280909122874 + 46.409377340599065 i	1526.971679090215730 + 36.432232169193696 i	39,40,1	7.467388885387931 + 47.716703565440831 i	1552.366188679818379 + 97.715017138328363 i
39,41,1	10.442744641627513 + 48.863942900563032 i	1589.297694528007241 + 165.885278605151001 i	39,41,3	3.671688009967502 + 51.566360488929412 i	1525.869915898511863 + 46.769714337253376 i
39,42,1	13.149417012697818 + 49.898918305644671 i	1635.716431781392430 + 239.808633707266381 i	39,42,3	7.107088584459275 + 52.692725913199666 i	1548.221548927218919 + 122.788686422342238 i
39,43,1	15.681660244470597 + 50.848817751996400 i	1690.582460488122706 + 318.677158329016777 i	39,43,3	10.091464021368241 + 53.721227730580040 i	1582.266441665766934 + 204.215571404999650 i
39,43,5	3.517774886222111 + 56.109951660414879 i	1525.373272897124252 + 54.846131569294862 i	39,44,1	18.089194096063082 + 51.730963398374215 i	1753.256175594959586 + 401.905016626942768 i
39,44,3	12.823309951303406 + 54.672358316093217 i	1626.107846440158255 + 290.568314441847349 i	39,44,5	6.923814924095699 + 57.144102723930509 i	1546.180153359184033 + 143.120682299284510 i
39,45,1	20.402104200193133 + 52.557270238972364 i	1823.300828508025916 + 489.047995817707658 i	39,45,3	15.386700783464860 + 55.560383677162960 i	1678.708747483213074 + 381.343395879820605 i

Table D.40: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
39,45,5	9.910165012523683 + 58.105955672795503 i	1578.624304506727867 + 235.966234801661386 i	39,45,7	3.426442291608077 + 60.334173517344468 i	1525.084594806708765 + 61.847887644602167 i
39,46,1	22.640242582843971 + 53.336391250749855 i	1900.397562928405478 + 579.756478868435806 i	39,46,3	17.827392179314771 + 56.395562540951175 i	1739.404584198774955 + 476.124133758957726 i
39,46,5	12.656577737155960 + 59.007337826893774 i	1620.975116028047296 + 333.215761862569423 i	39,46,7	6.814637258512835 + 61.311699394188572 i	1544.947339940637221 + 161.032270452118752 i
39,47,1	24.817520949501613 + 54.074869681850650 i	1984.302467809054178 + 673.747103510209172 i	39,47,3	20.173723918727621 + 57.185559350737478 i	1807.730144928194704 + 574.570351347902829 i
39,47,5	15.240330363952586 + 59.857402527386824 i	1672.242074971777811 + 434.535157470341971 i	39,47,7	9.803916568476321 + 62.230523258024384 i	1576.369849269066208 + 264.241954633426758 i
39,47,9	3.366637385941892 + 64.356845856376509 i	1524.895103614448999 + 68.207328135845785 i	39,48,1	26.944130912245573 + 54.777810551967889 i	2074.822415502921558 + 770.784796456043068 i
39,48,3	22.444732471455350 + 57.936288170316693 i	1883.340344365157307 + 676.400960261382465 i	39,48,5	17.704116438339433 + 60.663207408506494 i	1731.770674280344792 + 539.611431785758441 i
39,48,7	12.562460551258633 + 63.098774279293238 i	1617.750016152413991 + 371.494922768901233 i	39,48,9	6.744905837309791 + 65.295943756191505 i	1544.12381456824096 + 177.4432165649880 9 i
39,49,1	29.027798541144350 + 55.449297180373478 i	2171.800377979376208 + 870.670845868558217 i	39,49,3	24.653945791756204 + 58.652434487704191 i	1965.968402266751582 + 781.380048628603163 i
39,49,5	20.074731341602085 + 61.430299464090744 i	1799.092811639256297 + 648.172676465865266 i	39,49,7	15.162968286729869 + 63.923077754748256 i	1668.144122980468865 + 482.563576063658729 i
39,49,9	9.738891294614579 + 66.184714378357114 i	1574.845813803347255 + 290.315929790779194 i	39,49,11	3.325226024692762 + 68.239123134367915 i	1524.761454315101219 + 74.135515109749434 i
39,50,1	31.074542224178867 + 56.092662627535582 i	2275.105965001475397 + 973.234665454056199 i	39,50,3	26.811385901365139 + 59.337791840772056 i	2055.401558845982436 + 889.306673139543022 i
39,50,5	22.370378768397842 + 62.163128605212215 i	1873.855815202602344 + 759.985085766659608 i	39,50,7	17.645912366574493 + 64.708754200542003 i	1726.914915417529301 + 597.204279418225269 i
39,50,9	12.508998849670627 + 67.029401377777774 i	1615.556159244189985 + 406.967127952710825 i	39,50,11	6.698950154930764 + 69.150444375647211 i	1543.539389422281829 + 192.824218237724381 i
39,51,1	33.089155081832530 + 56.710673808767922 i	2384.629039699721034 +1078.327916591992107 i	39,51,3	28.924719986010814 + 59.995487541152919 i	2151.465962673542890 +1000.007417731635542 i
39,51,5	24.604119913004705 + 62.865331093983919 i	1955.783856876447089 + 874.846765762221594 i	39,51,7	20.036891815580251 + 65.460121949633347 i	1793.59905301650497 + 715.1930653566921 6 i
39,51,9	15.124692604127633 + 67.835195796700347 i	1665.351345035902568 + 527.242199956506397 i	39,51,11	9.699190838637952 + 70.017118279202577 i	1573.759935429929328 + 314.856052283556437 i
39,51,13	3.295559255218721 + 72.017430918405552 i	1524.662618637512878 + 79.752702775012423 i	39,52,1	35.075525286421303 + 57.305660578902412 i	2500.275247107204450 +1185.820194567355884 i
39,52,3	30.999965931478620 + 60.628139186581080 i	2254.016754919161485 +1113.330903020654205 i	39,52,5	26.785710845985648 + 63.539926318867664 i	2044.655003352974973 + 992.581987940152544 i
39,52,7	22.353423828311161 + 66.180736523654218 i	1867.842964523448927 + 836.330629427390591 i	39,52,9	17.624801747144286 + 68.606363014607524 i	1723.611645058475460 + 650.946930752625917 i
39,52,11	12.480799783820006 + 70.844300210194547 i	1613.992709109650605 + 440.463310259130651 i	39,52,13	6.668475192364700 + 72.907779260591823 i	1543.107766940865758 + 207.451728953507597 i
39,53,1	37.036856285181535 + 57.879608792327105 i	2621.962784594981713 +1295.595786400424231 i	39,53,3	33.041947040337078 + 61.237966349037059 i	2362.931283809268734 +1229.143666473536996 i
39,53,5	28.922668227552698 + 64.189456391925376 i	2140.286653201548233 +1113.036479733599435 i	39,53,7	24.608159771850158 + 66.873566408405154 i	1949.367318758912234 + 960.440673336231271 i
39,53,9	20.034002297113684 + 69.346436611802716 i	1789.880729767532785 + 777.892727761831452 i	39,53,11	15.110855863888425 + 71.636284926112538 i	1663.367586854798674 + 569.544749892189088 i
39,53,13	9.676166434575547 + 73.757603959054322 i	1572.959401605990251 + 338.261848984447681 i	39,53,15	3.273865978047907 + 75.715442136942855 i	1524.587046520487320 + 85.134425360878083 i
39,54,1	38.975822696004336 + 58.434229006279196 i	2749.620008165608397 +1407.551183601370440 i	39,54,3	35.054597203643695 + 61.826872275113288 i	2478.104299296459430 +1347.327017263977496 i
39,54,5	31.020928040808762 + 64.816087418164372 i	2242.525826218683051 +1236.073684946750518 i	39,54,7	26.810605411703349 + 67.541121407152616 i	2037.945541712304703 +1087.366920755856881 i
39,54,9	22.369260802022868 + 70.058377925738810 i	1863.806785573325442 + 907.906501677572692 i	39,54,11	17.626938041958656 + 72.396643598789851 i	1721.280770025393849 + 701.943138013827820 i
39,54,13	12.469335629500391 + 74.571370336548554 i	1612.845524331621846 + 472.486415058739681 i	39,54,15	6.648626045424765 + 76.589439017726505 i	1542.780036710303648 + 221.503098181552929 i

Table D.40: Branch points and associated eigenvalues for $m = 39$.

Table D.41: Branch points and associated eigenvalues for $m = 40$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
40,40,1	4.019899752253498 + 47.467106927239719 i	1606.050265827129351 + 37.005821939192806 i	40,41,1	7.519931690756222 + 48.786047715881793 i	1631.756654863672338 + 99.257690922654106 i
40,42,1	10.514687777010760 + 49.944182951383524 i	1669.114936761406625 + 168.507519859479117 i	40,42,3	3.695203139140318 + 52.656664560871235 i	1604.926080365543612 + 47.467939851997883 i
40,43,1	13.238155634428363 + 50.989488729483206 i	1716.042315178140143 + 243.603851631279213 i	40,43,3	7.152269877089173 + 53.791908618098063 i	1627.520602392369028 + 124.624509187449476 i
40,44,1	15.785404930271802 + 51.949263143531191 i	1771.481822853000267 + 323.726332362217136 i	40,44,3	10.154714510150356 + 54.829174124140536 i	1661.918156526936173 + 207.267619516982421 i
40,44,5	3.538297589588058 + 57.225730139004718 i	1604.418612538641128 + 55.630100242793503 i	40,45,1	18.206601781034589 + 52.840904765022472 i	1834.783266417943196 + 408.280039105638593 i
40,45,3	12.902447146359075 + 55.788881332596070 i	1706.194073580768872 + 294.909904016195071 i	40,45,5	6.964218712912320 + 58.267215272064163 i	1625.429871817367939 + 145.169340806538457 i
40,46,1	20.532110880498578 + 53.676383573691197 i	1905.502599776193847 + 496.813840346441680 i	40,46,3	15.480174453615454 + 56.685287409928456 i	1759.295651543715621 + 387.041321184178571 i
40,46,5	9.967451298251737 + 59.236495534947267 i	1658.180400052939603 + 239.339990957682886 i	40,46,7	3.444859772162892 + 61.471316741620932 i	1604.123206528612627 + 62.698557884066872 i
40,47,1	22.781973978997875 + 54.464393646211299 i	1983.315593191900689 + 588.972661236530371 i	40,47,3	17.934011811713422 + 57.528657552513266 i	1820.548098618216954 + 483.239570967582324 i
40,47,5	12.728877139108302 + 60.145280677879313 i	1700.916264273060960 + 337.974889914689811 i	40,47,7	6.851493737573364 + 62.455140890060740 i	1624.164581698265692 + 163.250820744321146 i
40,48,1	24.970239495081181 + 55.211509969667766 i	2067.974191159473321 + 684.468710118585818 i	40,48,3	20.292530577023214 + 58.326666581312658 i	1889.478943563907251 + 583.159796810609123 i
40,48,5	15.326286983110236 + 61.002679541320731 i	1752.633112173826476 + 440.736508520669702 i	40,48,7	9.856632020487730 + 63.380443735293376 i	1655.860470871226880 + 267.876832682871168 i
40,48,9	3.383441113740205 + 65.512536104638485 i	1603.928998383712496 + 69.113003258373567 i	40,49,1	27.107201869322989 + 55.922862749725383 i	2159.281957723627784 + 783.063236225913556 i
40,49,3	22.574928266690527 + 59.085239435623578 i	1965.737654642070538 + 686.516994942277620 i	40,49,5	17.802672074805532 + 61.815730594866750 i	1812.667074875861545 + 547.308272229902286 i
40,49,7	12.629401135815199 + 64.255231279109069 i	1697.588928353544361 + 376.597160510317337 i	40,49,9	6.778947555751897 + 66.457172314638640 i	1623.31744789902700 + 179.80404369946262 i
40,50,1	29.200667104216631 + 56.602555750987690 i	2257.079153113390021 + 884.554417522778749 i	40,50,3	24.794850980004323 + 59.809071890831319 i	2049.053198423027879 + 793.071939195015602 i
40,50,5	20.185022751766216 + 62.589972577730009 i	1880.543235552008355 + 657.415009848978116 i	40,50,7	15.242932458568516 + 65.086073181142311 i	1748.394033556680824 + 489.182599213729532 i
40,50,9	9.787903108391975 + 67.351701114981523 i	1654.287576140836563 + 294.171824798528519 i	40,50,11	3.340729101991603 + 69.411256215322908 i	1603.791798528262234 + 75.088136235234884 i
40,51,1	31.256717366150454 + 57.253938951893097 i	2361.233117086932907 + 988.768996806657242 i	40,51,3	26.962410893249555 + 60.501966876562044 i	2139.209391492643590 + 902.620845513113636 i
40,51,5	22.491680319149744 + 63.329852403510522 i	1955.903714200885361 + 770.820032810713656 i	40,51,7	17.737951511546139 + 65.878260568627184 i	1807.630053896188656 + 605.387219827753597 i
40,51,9	12.571530453613441 + 68.202248345727440 i	1695.317251847674697 + 412.362185601443969 i	40,51,11	6.730666856621951 + 70.327526401995883 i	1622.714795506758492 + 195.307752036155932 i
40,52,1	33.280197755419927 + 57.879793497116431 i	2471.631779769712921 +1095.556312231267839 i	40,52,3	29.085346107475793 + 61.167060169987671 i	2236.029560184020738 +1014.987834427405119 i
40,52,5	24.735808221966487 + 64.039006038714874 i	2038.468559042778452 + 887.318912055402052 i	40,52,7	20.140224617608975 + 66.636095018683577 i	1874.82735167049054 + 724.98474554395943 i
40,52,9	15.199667107692848 + 69.013947110280469 i	1745.492129995059486 + 534.220058585311335 i	40,52,11	9.745096879539783 + 71.199385872488833 i	1653.163255359795585 + 318.903980379482618 i
40,52,13	3.309978928564831 + 73.204351765336924 i	1603.690162058712758 + 80.746314574765918 i	40,53,1	35.275039558656815 + 58.482461378514593 i	2588.179116673641147 +1204.783916171414830 i
40,53,3	31.169731652988833 + 61.806976974047785 i	2339.366474105841007 +1130.019367902330259 i	40,53,5	26.927241420176841 + 64.720454011366087 i	2128.012482323409131 +1006.733683030368525 i
40,53,7	22.467388201283732 + 67.363122125861167 i	1949.627467145754281 + 847.773715548124756 i	40,53,9	17.711361648301310 + 69.791029560935712 i	1804.184801821125802 + 659.549802179669769 i
40,53,11	12.539592279010822 + 72.031880456898392 i	1693.691670659796046 + 446.114303794678108 i	40,53,13	6.698218760404115 + 74.099174490167385 i	1622.268518091068927 + 210.043228535982621 i
40,54,1	37.244482516103830 + 59.063938929501838 i	2710.791865927780236 +1316.334282346718737 i	40,54,3	33.220437794923264 + 62.423943699336903 i	2449.095456887674572 +1247.580074183242459 i
40,54,5	29.073559221609628 + 65.376740339009544 i	2224.350094861761136 +1128.908090061410121 i	40,54,7	24.732183014802125 + 68.062304427275905 i	2031.747150795096559 + 973.575864161790264 i
40,54,9	20.131436048761106 + 70.537008634315356 i	1870.933139656682215 + 788.161185293060612 i	40,54,11	15.181560593467273 + 72.829248153402318 i	1743.420208041344495 + 576.837792654418195 i
40,54,13	9.719406168519193 + 74.953715587695243 i	1652.331365223929424 + 342.479669206839958 i	40,54,15	3.287361926083083 + 76.915802661110249 i	1603.612306054696091 + 86.164414042191808 i
40,55,1	39.191232183659409 + 59.625945851534681 i	2839.397095063009601 +1430.102279638271284 i	40,55,3	35.241437655168191 + 63.019869759708378 i	2565.109507052209210 +1367.549559917036504 i
40,55,5	31.180748511409700 + 66.010033428214626 i	2327.326058949337494 +1253.703807267854017 i	40,55,7	26.944184346777991 + 68.736148219892868 i	2120.956609813171326 +1102.233132306383141 i
40,55,9	22.476961530522715 + 71.254832202534530 i	1945.380770582010200 + 919.879492144698588 i	40,55,11	17.708774163242349 + 73.595025974635547 i	1801.738080352333782 + 710.916395533270361 i
40,55,13	12.524889971860564 + 75.772339486116238 i	1692.493230269567221 + 478.364855811403118 i	40,55,15	6.676662114579044 + 77.793881299077611 i	1621.928656629628449 + 224.190984378086455 i

Table D.41: Branch points and associated eigenvalues for $m = 40$.

Table D.42: Branch points and associated eigenvalues for $m = 41$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
41,41,1	4.04808553278906 + 48.52392389961442 i	1687.12821653989204 + 37.57468679595519 i	41,42,1	7.571687689323850 + 49.854286316162728 i	1713.143961492729659 + 100.787655208679567 i
41,43,1	10.585566808229171 + 51.023133405662186 i	1750.925575679505300 + 171.108154706342731 i	41,43,3	3.718385895846933 + 53.745604176595073 i	1685.981810557084827 + 48.160488250554032 i
41,44,1	13.325597902271637 + 52.078593726410773 i	1798.357503671829363 + 247.367791938209365 i	41,44,3	7.196814408921521 + 54.889588586245445 i	1708.817333640640527 + 126.445415736265630 i
41,45,1	15.887652388508920 + 53.048074515361300 i	1854.365892093797129 + 328.733883716790444 i	41,45,3	10.217080927391027 + 55.935477847444822 i	1743.564811912919822 + 210.294884456975126 i
41,45,5	3.558547559257673 + 58.339821055709514 i	1685.463619402786890 + 56.407785768558028 i	41,46,1	18.322335111119699 + 53.949049952315548 i	1918.290047674963262 + 414.602493343764309 i
41,46,3	12.980488704824481 + 56.903623009219977 i	1788.271878442410298 + 299.216254831152241 i	41,46,5	7.004083678057992 + 59.388533019294115 i	1706.677738577728860 + 147.201600345675303 i
41,47,1	20.660284975965872 + 54.793544366958834 i	1989.678688237441747 + 504.515626397940594 i	41,47,3	15.572366807853564 + 57.808273428036649 i	1841.870271183037858 + 392.693013980718433 i
41,47,5	10.023977377429361 + 60.365128402899522 i	1739.732363871236203 + 242.686771202639903 i	41,47,7	3.463046329149811 + 62.606524084015859 i	1685.161550404089439 + 63.542512828913857 i
41,48,1	22.921730113633942 + 55.590292426872381 i	2068.202256225772089 + 598.112794279334139 i	41,48,3	18.039184574667612 + 58.659701187801062 i	1903.675049777981940 + 490.297287534416341 i
41,48,5	12.800223946748863 + 61.281202767332914 i	1782.850420042043424 + 342.695999705406450 i	41,48,7	6.887883674707526 + 63.596560007325095 i	1705.380290408583960 + 165.451885164055454 i
41,49,1	25.120853215604033 + 56.345900441864167 i	2153.608587173136584 + 695.101811759140446 i	41,49,3	20.409741147989799 + 59.465591652099995 i	1973.206542313692580 + 591.679575663209903 i
41,49,5	15.411120177098798 + 62.145821980606286 i	1835.013823141399598 + 446.888357631279064 i	41,49,7	9.908681453749322 + 64.528249226946386 i	1737.347613503261073 + 271.483104595408122 i
41,49,9	3.400045615477694 + 66.666092928933153 i	1684.962671689798071 + 70.011639329575516 i	41,50,1	27.268049695263091 + 57.065523455773047 i	2245.697965404258412 + 795.240280403068709 i
41,50,3	22.703392407038471 + 60.231880681505672 i	2050.108815047010467 + 696.550989838091823 i	41,50,5	17.899950555779917 + 62.966006161511366 i	1895.549415629530813 + 554.943712029933408 i
41,50,7	12.695500213875780 + 65.409477795522818 i	1779.421882036529723 + 381.659295328697510 i	41,50,9	6.812579387142817 + 67.616196017219721 i	1704.50978421068157 + 182.14655567959619 i
41,51,1	29.371204031045185 + 57.753285382363607 i	2344.307963045044744 + 898.323293276188906 i	41,51,3	24.933900594312885 + 60.963274324086150 i	2134.106616256425696 + 804.669015563976586 i
41,51,5	20.293897377398970 + 63.747286264349334 i	1963.975519625530296 + 666.583653120775239 i	41,51,7	15.321897815895685 + 66.246761340585536 i	1830.635063874690559 + 495.749654716830037 i
41,51,9	9.836324213018061 + 68.516405118806261 i	1735.726355363729226 + 297.997859239455693 i	41,51,11	3.356058100166388 + 70.581095588278529 i	1684.821956558137572 + 76.033469639992447 i
41,52,1	31.436460995809391 + 58.412552861701030 i	2449.303669800766784 +1004.174944083933724 i	41,52,3	27.111466491668001 + 61.663584854570075 i	2224.980367161806953 + 915.827050346352848 i
41,52,5	22.611437391000987 + 64.494106487609997 i	2039.929087995373948 + 781.568631094165539 i	41,52,7	17.828849181227667 + 67.045362532913742 i	1890.333012096532912 + 613.505975079723612 i
41,52,9	12.633310673296240 + 69.372731184144456 i	1777.073186144794363 + 417.715528284788434 i	41,52,11	6.762019717678895 + 71.502254798376413 i	1703.88908881715952 + 197.77232461076508 i
41,53,1	33.468716707588655 + 59.046121042855980 i	2560.571099915464856 +1112.642266823552291 i	41,53,3	29.243897882194123 + 62.335956451938010 i	2322.550595302628153 +1029.846770706810503 i
41,53,5	24.865834650181235 + 65.210102674089342 i	2123.126077896971765 + 899.691700818537015 i	41,53,7	20.242285516891712 + 67.809566780215206 i	1958.03983943664775 + 734.69968621861244 i
41,53,9	15.273745049916336 + 70.190250922353485 i	1827.625170181206158 + 541.144037707160010 i	41,53,11	9.790474319356370 + 72.379233490681258 i	1734.563986366750669 + 322.921065326060841 i
41,53,13	3.324245074473875 + 74.388847261182590 i	1684.717547228238118 + 81.732445341284887 i	41,54,1	35.471944764414658 + 59.656343854948133 i	2678.012574277377098 +1223.590787819751540 i
41,54,3	31.337325936397143 + 62.983021851728410 i	2426.667718551816051 +1146.572497013811017 i	41,54,5	27.067001715442007 + 65.898296594378621 i	2213.337873792249411 +1020.772678575887653 i
41,54,7	22.579960708638147 + 68.542910136670088 i	2033.392234858120901 + 859.127186133500004 i	41,54,9	17.796892455914264 + 70.973164201662698 i	1886.747271738155405 + 668.086327073082884 i
41,54,11	12.597708525951413 + 73.216970582551681 i	1775.386118844446173 + 451.722316431659522 i	41,54,13	6.727636837593918 + 75.288093544311053 i	1703.428305293684161 + 212.615258318381052 i
41,55,1	37.449420520163294 + 60.245227931254355 i	2801.543384881583734 +1336.901182546018163 i	41,55,3	33.396667049039486 + 63.607014226367944 i	2537.205052972662088 +1265.866963436104470 i
41,55,5	29.222579503405751 + 66.561234229757133 i	2310.376321017727150 +1144.653340604346113 i	41,55,7	24.854703535917110 + 69.248349453233615 i	2116.103060409309819 + 986.608255764268279 i
41,55,9	20.227718848611364 + 71.724963936120773 i	1953.971607307523527 + 798.350536851424977 i	41,55,11	15.251454755856562 + 74.019648752439636 i	1825.466009628881920 + 584.075453379587543 i
41,55,13	9.762169621275106 + 76.147293978807355 i	1733.701062837908694 + 346.665870142895983 i	41,55,15	3.300721499497303 + 78.113628901184555 i	1684.637430069418997 + 87.186770879426518 i

Table D.42: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
41,56,1	39.403879887165367 + 60.814501955387101 i	2931.089322430801531 +1452.466710812032488 i	41,56,3	35.425932871549087 + 64.209849078780792 i	2654.053860479739342 +1387.608087660768206 i
41,56,5	31.338604946241258 + 67.201086316632171 i	2414.083743286999834 +1271.193604792139240 i	41,56,7	27.076157732349028 + 69.928387701678886 i	2205.939329175281728 +1116.983066068991548 i
41,56,9	22.583398662993332 + 72.448584961038236 i	2028.937278066173121 + 931.760335276593764 i	41,56,11	17.789676389484249 + 74.790771763039672 i	1884.185931558524089 + 719.821608923691542 i
41,56,13	12.579831906514199 + 76.970714442773797 i	1774.136956850481283 + 484.199314894879421 i	41,56,15	6.704405201948408 + 78.995745839724307 i	1703.076435728184606 + 226.858999143696337 i

Table D.42: Branch points and associated eigenvalues for $m = 41$.

Table D.43: Branch points and associated eigenvalues for $m = 42$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
42,42,1	4.075854502960971 + 49.579863734164185 i	1770.20555124087241 + 38.13897843343847 i	42,43,1	7.622686155449583 + 50.921462584157211 i	1796.528207973442022 + 102.305318061087164 i
42,44,1	10.655420913703869 + 52.100844916848544 i	1834.72981872306605 + 173.68787688893676 i	42,44,3	3.741248035115589 + 54.833230406315373 i	1769.037119430657640 + 48.847539075944901 i
42,45,1	13.411791039331357 + 53.166291053992445 i	1882.662332625280214 + 251.101459005056967 i	42,45,3	7.240744667836552 + 55.985822143269118 i	1792.111812368580559 + 128.251877358479334 i
42,46,1	15.988456586688088 + 54.145316415097213 i	1939.235146818386738 + 333.701149185470740 i	42,46,3	10.278594280840499 + 57.040200619642278 i	1827.206559734966959 + 213.298148619195842 i
42,46,5	3.578533912909727 + 59.452285645696143 i	1768.508302815031357 + 57.179383636710703 i	42,47,1	18.436453814721521 + 55.055470005648949 i	2003.777153662252431 + 420.874068010478936 i
42,47,3	13.057472740959270 + 58.016650432053126 i	1872.341513681737752 + 303.488478636756156 i	42,47,5	7.043427932654397 + 60.508121055504866 i	1789.923806296091470 + 149.217970352359970 i
42,48,1	20.786691196148059 + 55.908829890100485 i	2075.829894303103174 + 512.155412206332926 i	42,48,3	15.663322034408822 + 58.929414093356662 i	1926.432974399841669 + 398.299932112664635 i
42,48,5	10.079768714348669 + 61.491923488199319 i	1823.280313906733682 + 246.007412362831303 i	42,48,7	3.481009265644580 + 63.739863841989440 i	1768.199633420359987 + 64.379957901602793 i
42,49,1	23.059580039582642 + 56.714170836319312 i	2155.058527473705453 + 607.179322079229678 i	42,49,3	18.142959925308045 + 59.788770963742650 i	1988.785933027848159 + 497.299102806242047 i
42,49,5	12.870649879202867 + 62.415177522178119 i	1866.777783164786115 + 347.380269216241857 i	42,49,7	6.923822016219177 + 64.736027976833853 i	1788.594507124614893 + 167.635997563455277 i
42,50,1	25.269434973609062 + 57.478130076740108 i	2241.206814060112720 + 705.649253496668962 i	42,50,3	20.525409704868064 + 60.602417100366303 i	2058.913574033984332 + 600.131883052039257 i
42,50,5	15.494867093401091 + 63.286907499935261 i	1919.384502957786026 + 452.992237069659950 i	42,50,7	9.960086200267515 + 65.674014201673216 i	1820.831371129196214 + 275.061642131401186 i
42,50,9	3.416456850460497 + 67.817589638494994 i	1767.996128858396332 + 70.903447988755190 i	42,51,1	27.426750608517469 + 58.205887159946549 i	2334.071786296233768 + 807.319189205591783 i
42,51,3	22.830183038413939 + 61.376299324734795 i	2136.454604693195961 + 706.505529252055680 i	42,51,5	17.995993797248662 + 64.114115958787792 i	1980.418098043961209 + 562.519650133174423 i
42,51,7	12.760784637467589 + 66.561591681415607 i	1863.249038770343986 + 386.682547416451200 i	42,51,9	6.845813851978324 + 68.773090451308747 i	1787.70085600456468 + 184.47130194754200 i
42,52,1	29.539488505065275 + 58.901585865599245 i	2433.488351044843512 + 911.981161847035310 i	42,52,3	25.071156392221930 + 62.115133941128121 i	2221.129588777234403 + 816.174263752539900 i
42,52,5	20.401401357917440 + 64.902326511637440 i	2049.390181330147243 + 675.680883608387603 i	42,52,7	15.399896061845133 + 67.405223535748064 i	1914.867455186272764 + 502.266321602289565 i
42,52,9	9.884172711198099 + 69.678904563567514 i	1819.162227328465178 + 301.794926991260809 i	42,52,11	3.371217940313155 + 71.748718119479946 i	1767.851932483850533 + 76.971730658894813 i
42,53,1	31.613854921361323 + 59.568609250728585 i	2539.319367234526908 +1019.456637134467087 i	42,53,3	27.258617669009681 + 62.822742523444084 i	2312.715579497562885 + 928.928687235102984 i
42,53,5	22.729699886167570 + 65.655980908259409 i	2125.932578325330269 + 792.233547399386680 i	42,53,7	17.918641385324360 + 68.210144863855461 i	1975.024121187031369 + 621.562493084914081 i
42,53,9	12.694362501330154 + 70.540930821982158 i	1860.824094305713970 + 423.028402391007660 i	42,53,11	6.793019336051128 + 72.674708220801620 i	1787.06229526206630 + 200.21849485741430 i
42,54,1	33.654796072787917 + 60.209766254483299 i	2651.448949769842784 +1129.590363345827427 i	42,54,3	29.400443152115855 + 63.502277590948012 i	2411.030328290853049 +1044.588051003426244 i
42,54,5	24.994252466952062 + 66.378715029841345 i	2209.757186751772224 + 911.968198807906219 i	42,54,7	20.343114372775123 + 68.980625460305575 i	2043.23694624905556 + 744.34021380826994 i
42,54,9	15.346953766948095 + 71.364191010893208 i	1911.750664925367118 + 548.015744496055731 i	42,54,11	9.835338662550379 + 73.556741877778919 i	1817.962190094989182 + 326.908214084862891 i
42,54,13	3.338361789435690 + 75.570996753335251 i	1767.744777257991018 + 82.711312301604323 i	42,55,1	35.666327100652865 + 60.827422553246464 i	2769.777779305914919 +1242.245857051773328 i
42,55,3	31.502819190039080 + 64.156379342271734 i	2515.921920480022891 +1162.994550348340908 i	42,55,5	27.205048023983728 + 67.073551986679959 i	2300.632089131760495 +1034.702451328484585 i
42,55,7	22.691184668163224 + 69.720192205720906 i	2119.137804123144178 + 870.393755360874252 i	42,55,9	17.881425397324922 + 72.152853611270800 i	1971.299330617488295 + 676.558481558558924 i
42,55,11	12.655168377117590 + 74.399653595588305 i	1859.076162454356563 + 457.288609366771254 i	42,55,13	6.756738462847593 + 76.474617184735692 i	1786.587149227567124 + 215.168382187521928 i
42,56,1	37.651758322500221 + 61.423594909991387 i	2894.219714365573054 +1357.302010025857726 i	42,56,3	33.570707509257382 + 64.787287638523523 i	2627.261681041393331 +1284.009040026171760 i
42,56,5	29.369788085741622 + 67.743039778758060 i	2398.366386900526322 +1160.276127685099937 i	42,56,7	24.975767771641294 + 70.431796513985319 i	2202.43569747658966 7+ 999.540958924739584 i
42,56,9	20.322885430336054 + 72.910392105071551 i	2038.996493146304374 + 808.463135470854695 i	42,56,11	15.320562102170957 + 75.207572104481486 i	1909.505157268498124 + 591.259354073388636 i
42,56,13	9.804470164514710 + 77.338421590979877 i	1817.068544655703136 + 350.821365209065505 i	42,56,15	3.313948126959022 + 79.309001868525115 i	1767.662420909077809 + 88.201713659732150 i
42,57,1	39.613855450641694 + 62.000020836751126 i	3024.699280070725308 +1474.650486102030300 i	42,57,3	35.608157618978986 + 65.396923997439842 i	2744.939149622196965 +1407.507761896742750 i
42,57,5	31.494558811097523 + 68.389351499071140 i	2502.80008353468384 +1288.54740258554489 i	42,57,7	27.206574825157489 + 71.117938203639198 i	2292.894469241039587 +1131.620237931113707 i
42,57,9	22.688610106092963 + 73.639728698233299 i	2114.476760275626020 + 943.551768731097241 i	42,57,11	17.869671992608147 + 75.983968783084165 i	1968.624554605170715 + 728.660767636588162 i
42,57,13	12.634178684781514 + 78.166579538219111 i	1857.776793852386845 + 489.991059911343427 i	42,57,15	6.731863061032280 + 80.195114770338989 i	1786.223390454875243 + 229.507708018280084 i

Table D.43: Branch points and associated eigenvalues for $m = 42$.

Table D.44: Branch points and associated eigenvalues for $m = 43$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
43,43,1	4.103221954085551 + 50.634959744171567 i	1855.282288868750129 + 38.698840231468047 i	43,44,1	7.672954645833265 + 51.987617091746806 i	1881.909488378731794 + 103.811065179627292 i
43,45,1	10.724286994906311 + 53.177365027582887 i	1920.527862219095823 + 176.247342130180556 i	43,45,3	3.763800654228688 + 55.919591310735058 i	1854.092019287778839 + 49.529262187447983 i
43,46,1	13.496779550911159 + 54.252634916321647 i	1968.957119466828090 + 254.805802163323932 i	43,46,3	7.284081883990256 + 57.080662293271999 i	1877.404104717833661 + 130.044339953061410 i
43,47,1	16.087868416397583 + 55.241049416216654 i	2026.090040090780803 + 338.629392292131399 i	43,47,3	10.339283854585357 + 58.143400515258982 i	1912.843544147151761 + 216.278152287803834 i
43,47,5	3.598265285113136 + 60.563181636961858 i	1853.552671661657087 + 57.945078973528467 i	43,48,1	18.549014248461646 + 56.160231596616100 i	2091.245184894172326 + 427.096359210734420 i
43,48,3	13.133435266145403 + 59.128026720924318 i	1958.403219059089452 + 307.727627393369801 i	43,48,5	7.082268622985573 + 61.626040746276651 i	1875.168125099138479 + 151.218933302462915 i
43,49,1	20.911390631919765 + 57.022312660777892 i	2163.956975848826460 + 519.735143171104028 i	43,49,3	15.753081902977817 + 60.048777486384353 i	2012.984110429090151 + 403.863455070810630 i
43,49,5	10.134849417668084 + 62.616946041929161 i	1908.824362405393231 + 249.302707049546967 i	43,49,7	3.498755518843406 + 64.871400502198313 i	1853.237462263838324 + 65.211087825599563 i
43,50,1	23.195588983758885 + 57.836107003244152 i	2243.885330659046758 + 616.174554715853446 i	43,50,3	18.245384636031005 + 60.915939809652656 i	2075.881218496152087 + 504.246738400091658 i
43,50,5	12.940184975815679 + 63.547274166200246 i	1952.698543791674865 + 352.028814271489864 i	43,50,7	6.959322947134233 + 65.873612062605389 i	1873.80727106238782 7+ 169.803664051015147 i
43,51,1	25.416053631274774 + 58.608282393984197 i	2330.769968761549080 + 716.113724391324695 i	43,51,3	20.639587409846502 + 61.737220579514279 i	2146.600639410949952 + 608.518796259906708 i
43,51,5	15.577562924701764 + 64.426009307607245 i	2005.745432400215805 + 459.049598343755576 i	43,51,7	10.010866504271950 + 66.817808984361776 i	1906.311833451670964 + 278.613271838740388 i
43,51,9	3.432680496494289 + 68.967095551716511 i	1853.029375010821923 + 71.788630048514733 i	43,52,1	27.583376680263836 + 59.344042566221020 i	2424.404697023941026 + 819.303044065291260 i
43,52,3	22.955355202307448 + 62.518577612434889 i	2224.775763159321286 + 716.383058783001616 i	43,52,5	18.090841524654046 + 65.260137150061396 i	2067.273504155263254 + 570.037885499390654 i
43,52,7	12.825279893265037 + 67.711646459042626 i	1949.070552746512931 + 391.668073793038502 i	43,52,9	6.878662862778038 + 69.927927099760765 i	1872.890694445947247 + 186.778803907742912 i
43,53,1	29.705595438910329 + 60.047550894181043 i	2524.621779133899508 + 925.531509652622844 i	43,53,3	25.206676861161210 + 63.264737450773794 i	2310.123001741539156 + 827.590509567851200 i
43,53,5	20.507578438797609 + 66.055174384265030 i	2136.787713058635745 + 684.708858831078260 i	43,53,7	15.476957294945246 + 68.561536547695312 i	2001.091437684209495 + 508.734097272146983 i
43,53,9	9.931465821794706 + 70.839273384112033 i	1904.595264695689593 + 305.563876424581224 i	43,53,11	3.386213325229165 + 72.914196588188872 i	1852.881730250541523 + 77.903123807602810 i
43,54,1	31.788976576728501 + 60.722206617978713 i	2631.281861882145222 +1034.617979299219087 i	43,54,3	27.403925985434626 + 63.979530922297535 i	2402.416066863728702 + 941.928973432207727 i
43,54,5	22.846515136709929 + 66.815560562833596 i	2213.914795855311695 + 802.817308318877508 i	43,54,7	18.007362321038705 + 69.372687618521937 i	2061.703697120122797 + 629.558621191344400 i
43,54,9	12.754707807987559 + 71.706923803776505 i	1946.570102858231849 + 428.301990985470923 i	43,54,11	6.823675819792173 + 73.844961155278583 i	1872.234439790663146 + 202.646793666503925 i
43,55,1	33.838515525011708 + 61.370832262333458 i	2744.267176826962441 +1146.404933405301335 i	43,55,3	29.555046225105102 + 64.666118827030346 i	2501.469955838430906 +1059.215294679931276 i
43,55,5	25.121112218384965 + 67.544931758319194 i	2298.362621513772865 + 924.151311673903251 i	43,55,7	20.442749050514283 + 70.149354381173595 i	2130.419082261451877 + 753.908534779972818 i
43,55,9	15.419319264207411 + 72.535846617941459 i	1997.868804978444359 + 554.836704620901969 i	43,55,11	9.879704689376233 + 74.731987505134725 i	1903.357925809179051 + 330.866288288407532 i
43,55,13	3.352333000209192 + 76.750875465021608 i	1852.771855178698843 + 83.683121962732429 i	43,56,1	35.858268232737593 + 61.995805069877115 i	2863.476778302600451 +1260.753894552377005 i
43,56,3	31.666278183866392 + 65.327148760010431 i	2607.130440019604521 +1179.289559534897762 i	43,56,5	27.341433781736054 + 68.246312512361698 i	2389.895995685068556 +1048.526295509063630 i
43,56,7	22.801101244413861 + 70.895054890993109 i	2206.864686705429449 + 881.575997598334652 i	43,56,9	17.964990185913010 + 73.330179773996022 i	2057.841242328800945 + 684.968141706620258 i
43,56,11	12.711990755140580 + 75.580008117687115 i	1944.761905984667919 + 462.814380908044598 i	43,56,13	6.785532278121020 + 77.658821988781966 i	1871.745069902157638 + 217.703136281736874 i
43,57,1	37.851579358648912 + 62.599151771386417 i	2988.823103600164814 +1377.541984881848975 i	43,57,3	33.742628154781201 + 65.964867201183154 i	2719.266869254677658 +1302.010757550900962 i
43,57,5	29.515241009434451 + 68.922252895514035 i	2488.321296833573797 +1175.780143184300186 i	43,57,7	25.095419865427559 + 71.612735361348470 i	2290.745681946963032 +1012.376924338467006 i
43,57,9	20.416968851886171 + 74.093377883752026 i	2126.008141937687014 + 818.501215217455979 i	43,57,11	15.388905271794378 + 76.393099083042841 i	1995.537809974798392 + 598.391036026427173 i
43,57,13	9.846320573058120 + 78.527176619981248 i	1902.433859143783820 + 354.947022997589499 i	43,57,15	3.327045105394979 + 80.501998455386470 i	1852.687280877201601 + 89.209449612747406 i
43,58,1	39.821243882086478 + 63.182618561232616 i	3120.229423134080662 +1496.659284913014062 i	43,58,3	35.788182860726288 + 66.581201612421822 i	2837.767074809480164 +1427.253467482854376 i
43,58,5	31.648668499262477 + 69.574928382633175 i	2593.476225984488337 +1305.769298895359043 i	43,58,7	27.335482466082635 + 72.304892620948578 i	2381.822764028132042 +1146.147983004175330 i
43,58,9	22.792631947579906 + 74.828350911528744 i	2201.999649754549864 + 955.256391931050075 i	43,58,11	17.948786971185797 + 77.174700221169431 i	2055.054171106198737 + 737.435762304145214 i
43,58,13	12.687946783248098 + 79.360014751605163 i	1943.412827822499594 + 495.741296412367547 i	43,58,15	6.759043122666227 + 81.392066059590803 i	1871.369536815790980 + 232.137649207279026 i

Table D.44: Branch points and associated eigenvalues for $m = 43$.

Table D.45: Branch points and associated eigenvalues for $m = 44$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
44,44,1	4.130202293303543 + 51.689243257402410 i	1942.358447368810630 + 39.254407891322195 i	44,45,1	7.722519135732456 + 53.052787982792324 i	1969.287891846932325 + 105.305261608668573 i
44,46,1	10.792199855327533 + 54.252738426478082 i	2008.319892211007073 + 178.787171038585115 i	44,46,3	3.786054242560683 + 57.004732180485156 i	1941.14652182006898 3+ 50.205818488682745 i
44,47,1	13.580605436184763 + 55.337676257577193 i	2057.242165027598730 + 258.481719905878549 i	44,47,3	7.326846125146418 + 58.174158983158037 i	1964.694273527283713 + 131.823225934914092 i
44,48,1	16.185935930688355 + 56.335330446321912 i	2114.931001332040978 + 343.519808891325169 i	44,48,3	10.399177338972015 + 59.245132254448883 i	2000.475902097646440 + 219.235596795605460 i
44,48,5	3.617749862006872 + 61.672563521982525 i	1940.596734418167443 + 58.705047307028153 i	44,49,1	18.660069655052922 + 57.263397383964119 i	2180.694710611894152 + 433.270877559555288 i
44,49,3	13.208410346286925 + 60.237811345419459 i	2046.457222355074009 + 311.934697752624743 i	44,49,5	7.120621998542500 + 62.742350020993655 i	1962.410742752310171 + 153.204946699775491 i
44,50,1	21.034441029780883 + 58.134060834746727 i	2254.060651366379261 + 527.256660480736400 i	44,50,3	15.841685944387812 + 61.166427747378194 i	2101.524011074729060 + 409.384889861431191 i
44,50,5	10.189242338437877 + 63.740257660979076 i	1996.364616295611086 + 252.573406913416989 i	44,50,7	3.516291684846448 + 66.001195028436655 i	1940.275043337447869 + 66.036087401564245 i
44,51,1	23.329818634647825 + 58.956174362219748 i	2334.683541608445466 + 625.100678507714179 i	44,51,3	18.346502992391130 + 62.041276432619071 i	2164.961352879115111 + 511.141825517927714 i
44,51,5	13.008857717045537 + 64.677558045032526 i	2040.612883044351065 + 356.642693106620470 i	44,51,7	6.994399943840195 + 67.009375860927975 i	1961.018619713009457 + 171.955364999592888 i

Table D.45: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
44,52,1	25.560774348438464 + 59.736435902792856 i	2422.299091468627466 + 726.497769421827570 i	44,52,3	20.752322726955381 + 62.870075247750442 i	2236.268309239604150 + 616.842283523112087 i
44,52,5	15.659241048744867 + 65.563196509233592 i	2094.096878903421384 + 465.061818130396375 i	44,52,7	10.061041597616239 + 67.959700068200007 i	1993.789086177203217 + 282.138778318163077 i
44,52,9	3.448721967763900 + 70.114676290686390 i	1940.062415073819693 + 72.667376266030061 i	44,53,1	27.737996140768633 + 60.480073066608632 i	2516.697908318223654 + 831.194761291834652 i
44,53,3	23.078961061041284 + 63.658793033022782 i	2315.072993285932171 + 726.185895696645048 i	44,53,5	18.184531428603595 + 66.404142573845547 i	2156.115997845321544 + 577.500124433055362 i
44,53,7	12.889010197032800 + 68.859711645971956 i	2036.886571243235949 + 396.616972859050406 i	44,53,9	6.911137764111895 + 71.080773643043997 i	1960.079329436730404 + 189.069556918997392 i
44,54,1	29.869595787288553 + 61.191268562193223 i	2617.709634047641885 + 938.977636269974823 i	44,54,3	25.340517453876362 + 64.412166549198346 i	2401.087696987817708 + 838.920430581465439 i
44,54,5	20.612470140585035 + 67.205906403980748 i	2226.168583817056970 + 693.669625277938167 i	44,54,7	15.553110119662076 + 69.715772975989040 i	2089.307231202506046 + 515.154403376268419 i
44,54,9	9.978219938472437 + 71.997581587905785 i	1992.025537111520862 + 309.305513628276856 i	44,54,11	3.401048752294056 + 74.077599981586488 i	1939.911353673891881 + 78.827843539398131 i
44,55,1	31.961899339399917 + 61.873437588837092 i	2725.192721346554663 +1049.662664721013471 i	44,55,3	27.547449832628750 + 65.134035833175858 i	2494.082816239933891 + 954.830957470845192 i
44,55,5	22.961928084719993 + 67.972925592838692 i	2303.876322269758475 + 813.322310517915184 i	44,55,7	18.095044497301988 + 70.533066494888345 i	2150.372041599657678 + 637.496113412267732 i
44,55,9	12.814367415746133 + 72.870782611473686 i	2034.311333014907405 + 433.537418289002346 i	44,55,11	6.853998817661952 + 75.013084217804376 i	1959.405546436815484 + 205.057725876428464 i
44,56,1	34.019950599317426 + 62.529416061383927 i	2839.027533901872630 +1163.090076447030242 i	44,56,3	29.707768125463403 + 65.827569908029687 i	2593.870615535096476 +1073.731931343940687 i
44,56,5	25.246461916958570 + 68.708836548789591 i	2388.943083203193055 + 936.243795746245269 i	44,56,7	20.541225557175526 + 71.315832329499244 i	2219.586639108355030 + 763.406744242028481 i
44,56,9	15.490866304597928 + 73.705292782240775 i	2085.979773015579212 + 561.608367998947642 i	44,56,11	9.923586501782482 + 75.905042878205961 i	1990.751250513365221 + 334.796107393436557 i
44,56,13	3.366162472999785 + 77.928554786469434 i	1939.798783940645762 + 84.648070848079101 i	44,57,1	36.047845618519446 + 63.161592615987878 i	2959.111513187323908 +1279.119415304343192 i
44,57,3	31.827766305407152 + 66.495423701088015 i	2700.294570266882147 +1195.461344887747373 i	44,57,5	27.476209765788649 + 69.416665331298901 i	2481.130419729450750 +1062.247336151274339 i
44,57,7	22.909749594574393 + 72.067580040879164 i	2296.573371249093725 + 892.676357430370786 i	44,57,9	18.047615121108183 + 74.505220327472387 i	2146.373259511958167 + 693.317090583859340 i
44,57,11	12.768193712024081 + 76.758108698235773 i	2032.443449858905524 + 468.300770839218558 i	44,57,13	6.814026551830213 + 78.840780643881899 i	1958.902086680337561 + 220.220030891357823 i
44,58,1	38.048962800222874 + 63.772003796960917 i	3085.355687104789467 +1397.626047419008273 i	44,58,3	33.912494497702092 + 67.139850244307652 i	2813.222070618462567 +1319.876336233601933 i
44,58,5	29.658991547285083 + 70.098964130422516 i	2580.242007655115231 +1191.168889476749882 i	44,58,7	25.213701822094183 + 72.791250866727736 i	2381.033605758950216 +1025.118954083347717 i
44,58,9	20.510000605701457 + 75.274001524159729 i	2215.006883720489895 + 828.466899561299783 i	44,58,11	15.456505863222056 + 77.576306376634363 i	2083.564120481267310 + 605.471965403948161 i
44,58,13	9.887733060911422 + 79.713633298039980 i	1989.797053099183358 + 359.043670337646233 i	44,58,15	3.340015606381147 + 81.692691719640450 i	1939.712012234415397 + 90.210176122943480 i
44,59,1	40.026125879857844 + 64.362404341978703 i	3217.682081880961505 +1518.498482247265883 i	44,59,3	35.966076020699276 + 67.762782876297848 i	2932.539252929599115 +1446.849833365924496 i
44,59,5	31.800989540887677 + 70.757910826726572 i	2686.113262812741141 +1322.863181711787774 i	44,59,7	27.462925241731671 + 73.489338801310353 i	2472.724914396090298 +1160.569468562732709 i
44,59,9	22.895498575340334 + 76.014534464381939 i	2291.506360524811953 + 966.876675779132711 i	44,59,11	18.027046131444973 + 78.363044962334058 i	2143.474993837980492 + 746.148391569952537 i
44,59,13	12.741151950983014 + 80.551096014568714 i	2031.045142222457116 + 501.451172125609162 i	44,59,15	6.785952512884450 + 82.586673800854541 i	1958.514890381200303 + 234.749335420255989 i

Table D.45: Branch points and associated eigenvalues for $m = 44$.

Table D.46: Branch points and associated eigenvalues for $m = 45$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
45,45,1	4.156809112664718 + 52.742743775564350 i	2031.434043765617844 + 39.805810009602723 i	45,46,1	7.771404141679417 + 54.117011168739573 i	2058.663502942833929 + 106.788253280779642 i
45,47,1	10.859192361859684 + 55.327007178519494 i	2098.106085210022684 + 181.307951732061269 i	45,47,3	3.808018726690976 + 58.088695751776854 i	2030.200638151260819 + 50.877360586188011 i
45,48,1	13.663308379245283 + 56.421463025415825 i	2147.517754752133442 + 262.130063685527261 i	45,48,3	7.369056382960906 + 59.266359340700141 i	2053.982378562533540 + 133.588935959739132 i
45,49,1	16.282704558529378 + 57.428213081691339 i	2205.758438041502814 + 348.373532224405267 i	45,49,3	10.458300948262670 + 60.345447464403705 i	2090.103763830302341 + 222.171147381956388 i
45,49,5	3.636995412865754 + 62.780482803054994 i	2029.640499175399555 + 59.459455260754787 i	45,50,1	18.769670396481342 + 58.365026337535845 i	2272.126271055433790 + 439.399054569817281 i
45,50,3	13.282430244446106 + 61.346060409496587 i	2136.503740204484984 + 316.110635112042019 i	45,50,5	7.158503475707962 + 63.857103633092066 i	2051.651704813675678 + 155.176444877694678 i
45,51,1	21.155897040117690 + 59.244138557513409 i	2346.141602818786396 + 534.721708899541113 i	45,51,3	15.929171613529393 + 62.282425383612122 i	2192.052991922205820 + 414.865476316342495 i
45,51,5	10.242969159236356 + 64.861916564720659 i	2085.901177537548392 + 255.820225594728441 i	45,51,7	3.533624041348089 + 67.129305122348484 i	2029.312382776748791 + 66.855132212059686 i
45,52,1	23.462327402723030 + 60.074442031494058 i	2427.453991716408382 + 633.959765260063932 i	45,52,3	18.446356972701960 + 63.164845646628656 i	2256.026761068967971 + 517.985911566379741 i
45,52,5	13.076695134440023 + 65.806090919751384 i	2130.520973611052796 + 361.222910510520762 i	45,52,7	7.029065822165705 + 68.143379571241269 i	2050.228588946292803 + 174.091556869950750 i
45,53,1	25.703658852784638 + 60.862664504322211 i	2515.795169718124725 + 736.803800251857751 i	45,53,3	20.863661615429237 + 64.001050118227013 i	2327.917126497833578 + 625.104212011256209 i
45,53,5	15.739933155632423 + 66.698534418262909 i	2184.439097441016656 + 471.030203651390082 i	45,53,7	10.110629768622207 + 69.099750397367600 i	2083.263211260863955 + 285.638907189278143 i
45,53,9	3.464586431308591 + 71.260394048498497 i	2029.095253788903619 + 73.539868044585305 i	45,54,1	27.890673657368723 + 61.614057166880833 i	2610.952569760699134 + 842.997104411303440 i
45,54,3	23.201050102514625 + 64.797018686475113 i	2407.346963720517579 + 735.916238312573455 i	45,54,5	18.277099306682690 + 67.546201071002884 i	2246.945926041945313 + 584.907987233209155 i
45,54,7	12.951998579867961 + 70.005853050338601 i	2126.697235052906763 + 401.530288533649014 i	45,54,9	6.943249369332463 + 72.231694233515938 i	2049.266789683535990 + 191.344032106895241 i
45,55,1	30.031556831102996 + 62.332821812010948 i	2712.753232643452975 + 952.322668396835525 i	45,55,3	25.472730802577974 + 65.557498309431509 i	2494.024475474180690 + 850.166566967756012 i
45,55,5	20.716115912919694 + 68.354594893083572 i	2317.533240771000692 + 702.565126369410223 i	45,55,7	15.628381747377617 + 70.868001546798794 i	2179.515045860846385 + 521.528591150831971 i
45,55,9	10.024450683465066 + 73.153895537873368 i	2081.453111376391462 + 313.020605344618957 i	45,55,11	3.415728525427559 + 75.238993763000892 i	2028.940806444662940 + 79.746074934817344 i
45,56,1	32.132692819293226 + 63.022389383518615 i	2821.053434422515693 +1064.594193975300641 i	45,56,3	27.689244655691869 + 66.286338188898654 i	2587.716766768752223 + 967.637531496076917 i
45,56,5	23.075981446637780 + 69.128151743163656 i	2395.817712188420046 + 823.750830043744941 i	45,56,7	18.181718848307046 + 71.691353152832065 i	2241.029442970803757 + 645.376636994878140 i
45,56,9	12.873361167619141 + 74.032575956678343 i	2124.047900976922847 + 438.735753754306756 i	45,56,11	6.883997547351213 + 76.179144427307762 i	2048.575638361189249 + 207.451772054102662 i
45,57,1	34.199172984243873 + 63.685608999313580 i	2935.731685898129399 +1179.649677099101837 i	45,57,3	29.858666822143242 + 66.986715514952124 i	2688.233389942555732 +1088.141214712161172 i
45,57,5	25.370347214509604 + 69.870508501927958 i	2481.499240258166992 + 948.248268725083335 i	45,57,7	20.638578165897794 + 72.480133890167849 i	2310.739991064652713 + 772.836833791304116 i
45,57,9	15.561618489377809 + 74.872600641329598 i	2176.083744103822937 + 568.332114033847006 i	45,57,11	9.966997565676202 + 77.075976815701168 i	2080.142219063293396 + 338.698451555819361 i
45,57,13	3.379853822055783 + 79.104102540286874 i	2028.825566414749119 + 85.606346169670886 i	45,58,1	36.235132803295841 + 64.324880524008805 i	3056.683828716424159 +1297.346697690175688 i
45,58,3	31.987343793096372 + 67.661292485446097 i	2795.415541894740727 +1211.513530840638396 i	45,58,5	27.609424274656625 + 70.584692828314687 i	2574.336149186337479 +1075.868541207189537 i
45,58,7	23.017167001898830 + 73.237845140433365 i	2388.264324727870644 + 903.697158771183808 i	45,58,9	18.129327180099317 + 75.678048874862569 i	2236.895624233120088 + 701.607024673583282 i
45,58,11	12.823794483634760 + 77.934026100543363 i	2122.120890639399931 + 473.748864399756371 i	45,58,13	6.842229200566463 + 80.020562216027770 i	2048.058218301218858 + 222.719552194007548 i
45,59,1	38.243983851555278 + 64.942250168226266 i	3183.819492855972840 +1417.558879050283167 i	45,59,3	34.080368820407266 + 68.312328620287772 i	2909.128668141165690 +1337.609779963247320 i
45,59,5	29.801090390442116 + 71.273259078042088 i	2674.129431848414697 +1206.445692982841138 i	45,59,7	25.330653649115305 + 73.967423379557900 i	2473.300034598213188 +1037.769712045711003 i
45,59,9	20.602010719806874 + 76.452339109897167 i	2305.993034673550937 + 838.362209000603116 i	45,59,11	15.523384499237272 + 78.757266782545557 i	2173.584235572956459 + 612.503538344331105 i
45,59,13	9.928719314595948 + 80.897862166769343 i	2079.158171719005168 + 363.112095091195670 i	45,59,15	3.352862682199363 + 82.881151144793250 i	2028.736617196993166 + 91.204081380958584 i
45,60,1	40.228578130248728 + 65.539481080418341 i	3317.059470540855273 +1540.173171437816336 i	45,60,3	36.141901224037156 + 68.941763070421658 i	3029.257223142706152 +1466.301251260995969 i
45,60,5	31.951574794421823 + 71.938387559860800 i	2780.712235642794440 +1339.832743794898761 i	45,60,7	27.588945632432555 + 74.671359915240927 i	2565.601590129134820 +1174.887705808292367 i
45,60,9	22.997242786619704 + 77.198357918468545 i	2382.997289180198095 + 978.414971506283791 i	45,60,11	18.104473161804208 + 79.549077891887308 i	2233.887227215441271 + 754.800368320546795 i
45,60,13	12.793809253160452 + 81.739895489447761 i	2120.673817567085280 + 507.121780816134162 i	45,60,15	6.812598069583621 + 83.779008474126854 i	2047.659466293806872 + 237.343255552515387 i

Table D.46: Branch points and associated eigenvalues for $m = 45$.

Table D.47: Branch points and associated eigenvalues for $m = 46$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
46,46,1	4.183055251467214 + 53.795489117795753 i	2122.509094228930280 + 40.353168597601403 i	46,47,1	7.819632832292854 + 55.180320504587527 i	2150.036401985166776 + 108.260368413729523 i
46,48,1	10.925295590650430 + 56.400210932303146 i	2189.886608876815444 + 183.810242212609154 i	46,48,3	3.829703511326163 + 59.171522401080630 i	2121.254378875546081 + 51.544033386473060 i
46,49,1	13.744925921929591 + 57.504040407880851 i	2239.784159796797212 + 265.751641352966260 i	46,49,3	7.410730651243137 + 60.357307889456536 i	2145.268476725168966 + 135.341850487974568 i
46,50,1	16.378217298979344 + 58.519747812233341 i	2298.572737358757422 + 353.191637495671387 i	46,50,3	10.516679527406723 + 61.444394915332978 i	2181.727253341927735 + 225.085435784319344 i
46,50,5	3.656009318885411 + 63.886988214370660 i	2120.683973663563393 + 60.208461183873027 i	46,51,1	18.877864165318680 + 59.465174029702482 i	2365.540379525910339 + 445.482248433665802 i
46,51,3	13.355525550369915 + 62.452826908412348 i	2228.542978856512946 + 320.256337292120463 i	46,51,5	7.195927695771314 + 64.970353395659529 i	2142.891054775170232 + 157.133840635821144 i
46,52,1	21.275810442373501 + 60.352606280761378 i	2440.200478232417936 + 542.131943815357772 i	46,52,3	16.015574437419549 + 63.396827546748746 i	2284.571353443860971 + 420.306391908902810 i
46,52,5	10.296050475380033 + 65.981977845492324 i	2177.434143443465473 + 259.043841403876229 i	46,52,7	3.550758568443748 + 68.255785459983670 i	2120.349486464962411 + 67.668389262699961 i
46,53,1	23.593170656840915 + 61.190975153011927 i	2522.197471091362786 + 642.753780635001476 i	46,53,3	18.544986411365013 + 64.286708669710620 i	2349.077847638347976 + 524.780466160953551 i
46,53,5	13.143722910874626 + 66.932931232693761 i	2222.422980292275497 + 365.770421589390878 i	46,53,7	7.063332781371479 + 69.275680241973035 i	2141.437213106678428 + 176.212673870219078 i
46,54,1	25.844765685308079 + 61.987037853992398 i	2611.259142112334303 + 747.034104974625734 i	46,54,3	20.973647705679898 + 65.130210375198274 i	2421.547608237546228 + 633.306355065465368 i
46,54,5	15.819669363912652 + 67.832084837143512 i	2276.772331333103921 + 476.955997559166804 i	46,54,7	10.159648425048932 + 70.238019623623543 i	2174.734287132236659 + 289.114367790291453 i
46,54,9	3.480278822222272 + 72.404307832337309 i	2120.127895721232562 + 74.406278072953668 i	46,55,1	28.041470587214757 + 62.746068869923789 i	2707.169774093952583 + 854.712695333824627 i
46,55,3	23.321669326671181 + 65.933323618706964 i	2501.598311238496990 + 745.576174519300821 i	46,55,5	18.368579192854188 + 68.686377781030686 i	2339.763619818411371 + 592.263014237305242 i
46,55,7	13.014266967092754 + 71.150133038859437 i	2218.502678876449409 + 406.409014021218070 i	46,55,9	6.975007994335614 + 73.380749744910091 i	2140.453102761470291 + 193.602678016109820 i
46,56,1	30.191542435975347 + 63.472288837552071 i	2809.753826814659533 + 965.569572487941173 i	46,56,3	25.603366914113767 + 66.700805533163347 i	2588.934100048595610 + 861.331331336018820 i
46,56,5	20.818553275164849 + 69.501308285473584 i	2410.882110662102150 + 711.397209693928971 i	46,56,7	15.702798088793369 + 72.018287392611995 i	2271.715082658572555 + 527.857946279139583 i
46,56,9	10.070172956949639 + 74.308278209718239 i	2172.878051602408505 + 316.709881645771759 i	46,56,11	3.430256766199023 + 76.398440116342158 i	2119.970092133206435 + 80.657994331266380 i
46,57,1	32.301423121790819 + 64.169144239132734 i	2918.865416625493253 +1079.415888217977226 i	46,57,3	27.829363155505948 + 67.436514441508180 i	2683.318812991519280 + 980.351442451935327 i
46,57,5	23.188715863370540 + 70.281310687460945 i	2489.739494921050209 + 834.105030787865871 i	46,57,7	18.267414837444939 + 72.847615505566594 i	2333.676177039830236 + 653.201778402070317 i
46,57,9	12.931707989890915 + 75.192369046568089 i	2215.779918216568603 + 443.898015783865787 i	46,57,11	6.913680821546270 + 77.343205454452075 i	2139.744737891420755 + 209.829390120976001 i
46,58,1	34.376250788280487 + 64.839497216365672 i	3034.381215960573172 +1196.087420872513803 i	46,58,3	30.007797437029698 + 68.143635646292097 i	2784.559310309724424 +1102.446235188358514 i
46,58,5	25.492811560371067 + 71.030022469049612 i	2576.031730645714106 + 960.167219400192380 i	46,58,7	20.734839529706338 + 73.642329749308203 i	2403.879496113445384 + 782.200698621947822 i
46,58,9	15.631598332416342 + 76.037837706446709 i	2268.180886136215122 + 575.009256393873670 i	46,58,11	10.009950749892337 + 78.244854704090841 i	2171.530884272456206 + 342.574064256651411 i
46,58,13	3.393410517722257 + 80.277583223784703 i	2119.852205393905024 + 86.558126442889815 i	46,59,1	36.420199688790603 + 65.485758704359654 i	3156.195479266561506 +1315.439800782380871 i
46,59,3	32.145067949372724 + 68.824838556290842 i	2892.494527358964206 +1227.449559949705190 i	46,59,5	27.741123293180930 + 71.750472965759457 i	2669.513936104091044 +1089.392732553193127 i
46,59,7	23.123388997994905 + 74.405923625799815 i	2481.937993780144097 + 914.640613165510672 i	46,59,9	18.210152102065823 + 76.848735268923846 i	2329.408568593050404 + 709.839559733704846 i
46,59,11	12.878809539916169 + 79.107827562833776 i	2213.794321224959731 + 479.159695921228888 i	46,59,13	6.870147809252997 + 81.198232394989830 i	2139.213482902678606 + 225.202163841971185 i
46,60,1	38.436714020396806 + 66.109984439340778 i	3284.216449707937954 +1437.344921213596990 i	46,60,3	34.246310392545922 + 69.482389117840000 i	3006.987979540921060 +1355.214891754689688 i
46,60,5	29.941585818982851 + 72.445218742509283 i	2769.984440409692979 +1221.613716489585613 i	46,60,7	25.446313486168233 + 75.141329052846601 i	2567.545509516356105 +1050.331733417502619 i
46,60,9	20.693027850698297 + 77.628462849425446 i	2398.966897917866390 + 848.189068018624653 i	46,60,11	15.589560886964028 + 79.936049474776482 i	2265.598296398051389 + 619.487085618293918 i
46,60,13	9.969290524009979 + 82.079930326510592 i	2170.517258667443002 + 367.153048711187864 i	46,60,15	3.365589271572779 + 84.067442877861637 i	2119.761097935884209 + 92.191344980211369 i
46,61,1	40.428673579221126 + 66.713945854028381 i	3418.363695374248891 +1561.688184726789132 i	46,61,3	36.315719517127135 + 70.118232232574513 i	3127.922452097465793 +1485.611892599704788 i
46,61,5	32.100474621935746 + 73.116442557391039 i	2877.274138809630131 +1356.681496337998169 i	46,61,7	27.713584148000542 + 75.851034792413273 i	2660.453431853191432 +1189.105560586241381 i
46,61,9	23.097895888423331 + 78.379895836051929 i	2476.472815899500802 + 989.873518746666832 i	46,61,11	18.181090701552680 + 80.732870170531839 i	2326.291067737000958 + 763.393325381017121 i
46,61,13	12.845933111410377 + 82.926481823504773 i	2212.298931556775642 + 512.754165820377466 i	46,61,15	6.838986358078901 + 84.969137185714814 i	2138.803279277963611 + 239.919876225890675 i

Table D.47: (continued)

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$

Table D.47: Branch points and associated eigenvalues for $m = 46$.

Table D.48: Branch points and associated eigenvalues for $m = 47$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
47,47,1	4.208952852636513 + 54.847505550087966 i	2215.583614133620358 + 40.896599552394242 i	47,48,1	7.867227128561436 + 56.242747947563579 i	2243.406665344276007 + 109.721918777662779 i
47,49,1	10.990538959212060 + 57.472387106879836 i	2283.661622641180657 + 186.294572520360475 i	47,49,3	3.851117516496135 + 60.253250321262790 i	2214.307754092641062 + 52.205974638461029 i
47,50,1	13.825493620492068 + 58.585451046990798 i	2334.041638028497346 + 269.347220275323082 i	47,50,3	7.451885997085442 + 61.447046743217477 i	2238.552622243942096 + 137.082331205871412 i
47,51,1	16.472514897364590 + 59.609982280374915 i	2393.374267484129177 + 357.975146023382536 i	47,51,3	10.574336649130446 + 62.542020733960172 i	2275.346488799964845 + 227.979062593350733 i
47,51,5	3.674798599478546 + 64.992125923452292 i	2213.727165274398885 + 60.952215724600329 i	47,52,1	18.984696176614808 + 60.563892898160141 i	2460.937524261350063 + 451.521749266848474 i
47,52,3	13.427725298346461 + 63.558160961166699 i	2322.575134869095109 + 324.372657878198822 i	47,52,5	7.232908577876245 + 66.082148395169980 i	2236.128834192438717 + 159.077526729675071 i
47,53,1	21.394230349694027 + 61.459521047736331 i	2536.237894055210745 + 549.488937632554780 i	47,53,3	16.100928150017044 + 64.509688283790979 i	2379.079382008583707 + 425.708756131273503 i
47,53,5	10.348505869050282 + 67.100493695851185 i	2270.963606971944500 + 262.244899761654551 i	47,53,7	3.567700967738928 + 69.380687906869142 i	2213.386360046977643 + 68.476017566638120 i
47,54,1	23.722400939259458 + 62.305835199146266 i	2618.914731418487918 + 651.484591744068666 i	47,54,3	18.642429147689743 + 65.406923392796031 i	2444.114998196247143 + 531.526886583590681 i
47,54,5	13.209965472084576 + 68.058134348629423 i	2316.319060501718468 + 370.286135193882103 i	47,54,7	7.097212444467742 + 70.406331994094842 i	2234.644525102117314 + 178.319129469050552 i
47,55,1	25.984150423756006 + 63.109621688396992 i	2708.691901704838074 + 757.190856952315585 i	47,55,3	21.082322459750269 + 66.257617660114931 i	2517.160247312362571 + 641.450398777965233 i
47,55,5	15.898478326638823 + 68.963906312431902 i	2371.096812987525936 + 482.840382385821897 i	47,55,7	10.208114151785937 + 71.374564339666506 i	2268.202388904353938 + 292.565835640206672 i
47,55,9	3.495803857694570 + 73.546473684962052 i	2213.160345268036963 + 75.266770909252244 i	47,56,1	28.190445207516010 + 63.876178021729935 i	2805.350561145439315 + 866.344024483983162 i
47,56,3	23.440863415641115 + 67.067773124199135 i	2597.827642863886467 + 755.167689514850963 i	47,56,5	18.459003475745209 + 69.824734410910480 i	2434.569395402522332 + 599.566671325502853 i
47,56,7	13.075836250547116 + 72.292610780607646 i	2312.303031687526982 + 411.254095247214025 i	47,56,9	7.006423488635754 + 74.527997999709385 i	2233.638295173847837 + 195.845922119576471 i
47,57,1	30.349613287931696 + 64.609743448322234 i	2908.712607965146162 + 978.721166217799123 i	47,57,3	25.732473348176082 + 67.842157069196688 i	2685.817297980380772 + 872.417017668700282 i
47,57,5	20.919817945034467 + 70.646111408926885 i	2506.215601107922339 + 720.167633599096348 i	47,57,7	15.776383838630602 + 73.166692306666079 i	2365.907534022898290 + 534.143693326192192 i
47,57,9	10.115400982455906 + 75.460789426473866 i	2266.300419360105479 + 320.374038379009335 i	47,57,11	3.444637424157513 + 77.555998169267610 i	2212.999214193912849 + 81.563769898903061 i
47,58,1	32.468153087712849 + 65.313779790816440 i	3018.630015227348849 +1094.130902019519681 i	47,58,3	27.967855473638952 + 68.584636895848149 i	2780.889807805764576 + 992.975302251857784 i
47,58,5	23.300170037683063 + 71.432470323460180 i	2585.642176077602471 + 844.386972194129839 i	47,58,7	18.352160552616542 + 74.001917984760581 i	2428.312507830368759 + 660.973048770368109 i
47,58,9	12.989425949805248 + 76.350223826307953 i	2309.507491740077967 + 449.025175127254442 i	47,58,11	6.943057072029512 + 78.505327848850925 i	2232.912866560371640 + 212.191016840673683 i
47,59,1	34.551248783144004 + 65.991162042662367 i	3134.977631080566425 +1212.406808403492732 i	47,59,3	30.155212435374612 + 69.298405966055867 i	2882.849359968902263 +1116.649931298708225 i
47,59,5	25.613896346189168 + 72.187449360035018 i	2672.541163798917751 + 972.003016504570951 i	47,59,7	20.830040785930656 + 74.802486969994220 i	2499.005496930819845 + 791.500144040568557 i
47,59,9	15.700827328484179 + 77.201068113167111 i	2362.271360235081374 + 581.641047378873054 i	47,59,11	10.052458362173773 + 79.411738730080756 i	2264.917297012172185 + 346.423654705462695 i
47,59,13	3.406835893988704 + 81.449058230637291 i	2212.878703594535637 + 87.503582049652451 i	47,60,1	36.603112778891408 + 66.644312058094371 i	3257.648135016345350 +1333.402580028849570 i
47,60,3	32.300993335642872 + 69.986140841901715 i	2991.532644743995661 +1243.272705624782702 i	47,60,5	27.871350643629015 + 72.914079603600385 i	2766.664498936494056 +1102.822596016318585 i
47,60,7	23.228449475069247 + 75.571885170307112 i	2577.594805939851085 + 925.508827364142121 i	47,60,9	18.290114465660977 + 78.017345871022428 i	2423.912315291716823 + 718.016236151243106 i
47,60,11	12.933254631226600 + 80.279577036675775 i	2307.463831036583542 + 484.534252155774027 i	47,60,13	6.897789649873711 + 82.373853718691137 i	2232.367898043500190 + 227.668308416826562 i
47,61,1	38.627221365388777 + 67.275294964283916 i	3386.548394156039194 +1456.988392533382466 i	47,61,3	34.410375669661086 + 70.650113836965531 i	3106.801261550092477 +1372.695287805466933 i
47,61,5	30.080523858312731 + 73.614919869336248 i	2867.807865480285272 +1236.675970375701581 i	47,61,7	25.560717724107068 + 76.313040139417311 i	2663.770548423238779 +1062.807433364403323 i
47,61,9	20.783079368815979 + 78.802441343482741 i	2493.928764260544085 + 857.949311443564511 i	47,61,11	15.655053873276993 + 81.112720248836524 i	2359.606438759779394 + 626.423876894360660 i
47,61,13	10.009457411038928 + 83.259901664531711 i	2263.874356140231157 + 371.167248586447556 i	47,61,15	3.378198205094179 + 85.251629947364236 i	2212.785456576124943 + 93.172138464328725 i
47,62,1	40.626481680989144 + 67.885890357284993 i	3521.596762022022631 +1583.048111934537474 i	47,62,3	36.487589069156328 + 71.292275544465852 i	3228.536338702012472 +1504.785723940711023 i
47,62,5	32.247737049966574 + 74.292155384568474 i	2975.799922356338811 +1373.412781416339158 i	47,62,7	27.836879452489029 + 77.028438227768646 i	2757.281052802922659 +1203.225763168027015 i
47,62,9	23.197487789941366 + 79.559219055743398 i	2571.933305385597123 +1001.254452920561221 i	47,62,11	18.256920404225365 + 81.914489485760569 i	2420.686704400640792 + 771.928820730151187 i
47,62,13	12.897537341189730 + 84.110920381630464 i	2305.920559203613266 + 518.349323286148319 i	47,62,15	6.865123685635814 + 86.157123887953745 i	2231.946343649583923 + 242.479643202898075 i

Table D.48: Branch points and associated eigenvalues for $m = 47$.

Table D.49: Branch points and associated eigenvalues for $m = 48$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
48,48,1	4.234513413823534 + 55.898817902282474 i	2310.657618114255001 + 41.436213085067266 i	48,49,1	7.914207794792153 + 57.304323700527675 i	2338.774365713258703 + 111.173200846979867 i
48,50,1	11.054950346330829 + 58.543571060591674 i	2379.431278266527443 + 188.761446691663480 i	48,50,3	3.872269211429058 + 61.333915681284708 i	2309.360773439910645 + 52.863315427339067 i
48,51,1	13.905045187929208 + 59.665735231736463 i	2430.290434934609493 + 272.917530171023600 i	48,51,3	7.492538625636681 + 62.535615782305840 i	2333.834866849756963 + 138.810722319424116 i
48,52,1	16.565636005454110 + 60.698961496931510 i	2490.163378973094041 + 362.725029013196263 i	48,52,3	10.631294702393877 + 63.638368597087101 i	2370.961582924696813 + 230.852599397497443 i
48,52,5	3.693369936339198 + 66.095939714242405 i	2308.770081081628632 + 61.690862353116657 i	48,53,1	19.090209342501463 + 61.661232483432599 i	2558.318170146349530 + 457.518783876041653 i
48,53,3	13.499057074648151 + 64.662110021244061 i	2418.600395744268553 + 328.460409263887755 i	48,53,5	7.269459367430496 + 67.192535185774744 i	2331.365082804379654 + 161.007877229402766 i
48,54,1	21.511203395282834 + 62.564936751206623 i	2634.254437306438831 + 556.794185583562353 i	48,54,3	16.185264815207240 + 65.621058764608719 i	2475.577350805720471 + 431.073634481064175 i
48,54,5	10.400353977075369 + 68.217513615162275 i	2366.489656998539431 + 265.424015425295238 i	48,54,7	3.584456679925121 + 70.504061713925537 i	2308.423008942414526 + 69.278168678423114 i
48,55,1	23.850068161591409 + 63.419080250008527 i	2717.606488570028432 + 660.153974050294437 i	48,55,3	18.738721161745811 + 66.525544623497439 i	2541.138580628900399 + 538.226502753497130 i
48,55,5	13.275446070394658 + 69.181752774012935 i	2412.209364726918011 + 374.770917045751332 i	48,55,7	7.130715895224503 + 71.535386224807866 i	2329.850556486514961 + 180.411317777995691 i
48,56,1	26.121865886411374 + 64.230478120922513 i	2808.094299088288608 + 767.276122851617629 i	48,56,3	21.189725317886118 + 67.383330331058332 i	2614.755513958875781 + 649.537947985160372 i
48,56,5	15.976387328435621 + 70.094054366725359 i	2467.412764581310967 + 488.684484602529722 i	48,56,7	10.256042763799190 + 72.509438291777968 i	2363.667588567114308 + 295.993954688661404 i
48,56,9	3.511166049996740 + 74.686944886883168 i	2308.192606666606935 + 76.121503515077777 i	48,57,1	28.337652925682136 + 65.004450624348536 i	2905.495921406810339 + 877.893460009984551 i
48,57,3	23.558674889267593 + 68.200429020460959 i	2696.035537810465030 + 764.692672857672505 i	48,57,5	18.548403006963472 + 70.961329479550329 i	2531.363555104149327 + 606.820354943396183 i
48,57,7	13.136726354941842 + 73.433342469163776 i	2408.098417069358675 + 416.066433998193361 i	48,57,9	7.037505264010088 + 75.673493976748389 i	2328.822392408147152 + 198.074172199482575 i
48,58,1	30.505827108664210 + 65.745255398777900 i	3009.630711091922876 + 991.780128901155535 i	48,58,3	25.860095380309769 + 68.981618102307507 i	2784.674763277756483 + 883.425809463583050 i
48,58,5	21.019943956445502 + 71.789065741767217 i	2603.534101796745745 + 728.878073206708450 i	48,58,7	15.849162553396150 + 74.313274974829127 i	2462.092584315559179 + 540.386999792027041 i
48,58,9	10.160148348691404 + 76.611486072693083 i	2361.720273815393739 + 324.013739404726891 i	48,58,11	3.458874286445253 + 78.711724197290238 i	2308.028175969569929 + 82.463562168309821 i
48,59,1	32.632942512647652 + 66.456369416647050 i	3120.348513851846747 +1108.742235032255849 i	48,59,3	28.104769361588904 + 69.730774012197770 i	2880.430565173116975 +1005.511597043690699 i
48,59,5	23.410380860151722 + 72.581695041524767 i	2683.526239048250318 + 854.598616293577948 i	48,59,7	18.435982793776137 + 75.154321782177149 i	2524.938688282037674 + 668.691888899250785 i
48,59,9	13.046532308692305 + 77.506199200445849 i	2405.230724332395766 + 454.118157991159478 i	48,59,11	6.972134371990985 + 79.665569246936130 i	2328.080045142516610 + 214.537069182443958 i
48,60,1	34.724228626279626 + 67.140680357856326 i	3237.522367212410700 +1228.611168401118448 i	48,60,3	30.300961800214882 + 70.451098119560029 i	2983.104477445905559 +1130.755100109486875 i
48,60,5	25.733641038763242 + 73.342856423391012 i	2771.028122397544848 + 983.757916795259462 i	48,60,7	20.924211652155854 + 75.960669243552047 i	2596.118321795297106 + 800.736891411541819 i
48,60,9	15.769326016150927 + 78.362352850328463 i	2458.355321127098250 + 588.228681918573557 i	48,60,11	10.094532182435268 + 80.576688093329210 i	2360.301506306019292 + 350.247900043315497 i
48,60,13	3.420133155564821 + 82.618586053989816 i	2307.905063658292875 + 88.442875755237196 i	48,61,1	36.783935404550661 + 67.800620850529228 i	3361.043387590143084 +1351.238701512618036 i
48,61,3	32.455171950961869 + 71.145274084004129 i	3092.530961285386638 +1258.986083728134872 i	48,61,5	28.000148124379475 + 74.075582790587276 i	2865.788524637206592 +1116.160690525998689 i
48,61,7	23.332380789121082 + 76.735795945275243 i	2675.235170772998117 + 936.303810250437095 i	48,61,9	18.369237760394657 + 79.183943787734506 i	2520.407078152877148 + 726.138523845518989 i
48,61,11	12.987144831158146 + 81.449335405181529 i	2403.129506191975452 + 489.873475328028405 i	48,61,13	6.925161698892311 + 83.547485778858999 i	2327.521480725011770 + 230.118408764700519 i
48,62,1	38.815570722698972 + 68.438265283743718 i	3490.817076513148095 +1476.493304422651363 i	48,62,3	34.572618475344044 + 71.815580529328713 i	3208.569713858590603 +1390.054410302548604 i
48,62,5	30.217948422796265 + 74.782435247273568 i	2967.600502766374120 +1251.635322858120389 i	48,62,7	25.673901114396958 + 77.482625261998052 i	2761.975647464656959 +1075.199114952019045 i
48,62,9	20.872191437314431 + 79.974339829385599 i	2590.878912885565114 + 867.644690273154538 i	48,62,11	15.719881495997960 + 82.287341745797605 i	2455.608793390018946 + 633.315124650660209 i
48,62,13	10.049230256108556 + 84.437837064226414 i	2359.229504926492154 + 375.155380194656686 i	48,62,15	3.390692210361625 + 86.433772463417753 i	2307.809695196572648 + 94.146625830271617 i
48,63,1	40.822068625815042 + 69.055401301135760 i	3626.760582218460968 +1604.257317432713307 i	48,63,3	36.657565357056164 + 72.463973683571552 i	3331.100218495357694 +1523.826521010173110 i
48,63,5	32.393407917315024 + 75.465601508691520 i	3076.290494789024706 +1390.029783351170863 i	48,63,7	27.958868478996159 + 78.203641260637141 i	2856.085040452882822 +1217.250917196748333 i
48,63,9	23.296047087757465 + 80.736394944449287 i	2669.379107736940568 +1012.559811996598341 i	48,63,11	18.331982996163735 + 83.094000281973393 i	2517.074319093207123 + 780.408342283927027 i
48,63,13	12.948635186435981 + 85.293273459707109 i	2401.538772951424107 + 523.908205148125262 i	48,63,15	6.891016115054970 + 87.343029580946293 i	2327.088673326547450 + 245.022982686791380 i

Table D.49: Branch points and associated eigenvalues for $m = 48$.

Table D.50: Branch points and associated eigenvalues for $m = 49$.

m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$	m,n,r	$c_{\circ;r}^{m,n} = c_{\circ;r+1}^{m,n+2}$	$\lambda_{\circ;r}^{m,n} = \lambda_{\circ;r+1}^{m,n+2}$
49,49,1	4.259747833811788 + 56.949449674063509 i	2407.731120114903206 + 41.972114110774299 i	49,50,1	7.960594521260379 + 58.365076341854824 i	2436.139572355408745 + 112.614496849562229 i
49,51,1	11.118556201121281 + 59.613796243965851 i	2477.195720365108173 + 191.211344542689147 i	49,51,3	3.893166645459350 + 62.413552771293887 i	2406.413446121845936 + 53.516180625059789 i
49,52,1	13.983612623524238 + 60.744931072846960 i	2528.530784453623710 + 276.463265692073549 i	49,52,3	7.532703939198716 + 63.623052813750441 i	2431.115259936107577 + 140.527351734858883 i
49,53,1	16.657617327368008 + 61.786728036594660 i	2588.940405918119268 + 367.442210995410107 i	49,53,3	10.687574973132465 + 64.733479907444348 i	2468.572643339603928 + 233.706590739765147 i
49,53,5	3.711729695501132 + 67.198471153837986 i	2405.812727859856295 + 62.424537839313111 i	49,54,1	19.194444431368563 + 62.757239643973115 i	2657.682760273320582 + 463.474520101436667 i
49,54,3	13.569547115661631 + 65.764719068065332 i	2516.618940510565153 + 332.520365428173534 i	49,54,5	7.305592680442519 + 68.301557966260788 i	2428.599838643366184 + 162.925248761350929 i
49,55,1	21.626773902420805 + 63.668904367129635 i	2734.250667540246468 + 564.049111022642705 i	49,55,3	16.268614939198024 + 66.730987488644686 i	2574.065520692005066 + 436.402042099347625 i
49,55,5	10.451612553017487 + 69.333084597766430 i	2464.012378565130348 + 268.581774522911644 i	49,55,7	3.601030900968897 + 71.625953696266905 i	2405.459438357814633 + 70.074987182508806 i
49,56,1	23.976219783713258 + 64.530765244690429 i	2818.273424989939031 + 668.763617655339090 i	49,56,3	18.833896698599492 + 67.642624307594403 i	2640.148946237382461 + 544.880581763527857 i
49,56,5	13.340186861442868 + 70.303836356692159 i	2510.094036953462819 + 379.225592595765420 i	49,56,7	7.163853712198874 + 72.662891793468376 i	2427.055337536277875 + 182.489614816714891 i
49,57,1	26.257962318285990 + 65.349665909634908 i	2909.467145215814980 + 777.291869963411159 i	49,57,3	21.295893832663118 + 68.507403698471677 i	2714.333857246439948 + 657.570531737189413 i
49,57,5	16.053422374485720 + 71.222581709930154 i	2565.720398688110890 + 494.489378330427201 i	49,57,7	10.303449354797214 + 73.642692573944998 i	2461.129955166556101 + 299.399339375460727 i
49,57,9	3.526369718505860 + 75.825772141243078 i	2405.224684001866535 + 76.970625745023298 i	49,58,1	28.483146471461851 + 66.130949119542308 i	3007.606799305131062 + 889.363256172464844 i
49,58,3	23.675144247520812 + 69.331349897448519 i	2796.222549261765835 + 774.152924901896654 i	49,58,5	18.636807200449439 + 72.096218540461763 i	2630.146388169168858 + 614.025396693927405 i

Table D.50: (continued)

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
49,58,7	13.196956298856444 + 74.572381525433599 i	2505.888953526594378 + 420.846890796695163 i	49,58,9	7.068262320938600 + 76.817290001099224 i	2426.005418988455858 + 200.287817613184757 i
49,59,1	30.660238852486941 + 66.878890686931172 i	3112.509218517054705 +1004.749010985290170 i	49,59,3	25.986276151272609 + 70.119250415785814 i	2885.507158813261867 + 894.359787165318380 i
49,59,5	21.118963767676060 + 72.930229646702585 i	2702.837985587525509 + 737.530125912289009 i	49,59,7	15.921156722893608 + 75.458091187315361 i	2560.270410301665379 + 546.588979823292334 i
49,59,9	10.204428048120356 + 77.760422290387936 i	2459.137671857460646 + 327.629618648396331 i	49,59,11	3.472970986751705 + 79.865671810776632 i	2405.056980695615039 + 83.357524514818621 i
49,60,1	32.795848347765414 + 67.596982550422283 i	3224.022136676354876 +1123.252742618526327 i	49,60,3	28.240150335956493 + 70.874990681376943 i	2981.941862603076970 +1017.962695666345667 i
49,60,5	23.519383524822686 + 73.729045969341470 i	2783.392146365902590 + 864.741834136841248 i	49,60,7	18.518907153465548 + 76.304885070315819 i	2623.554960896684406 + 676.359673820335502 i
49,60,9	13.103043570965669 + 78.660351235467161 i	2502.949714785314882 + 459.177848891363226 i	49,60,11	7.000920456708450 + 80.823984562448317 i	2425.246293688522655 + 216.867945572900879 i
49,61,1	34.895249064711578 + 68.288124917359227 i	3342.016793952325770 +1244.703669441968032 i	49,61,3	30.445093192388637 + 71.601780020555168 i	3085.325559313215763 +1144.764406736170713 i
49,61,5	25.852083302095636 + 74.496307501453046 i	2871.49316400734597 + 995.43407244262107 i	49,61,7	21.017380514530050 + 77.116937119048814 i	2695.218285428867148 + 809.912583604064594 i
49,61,9	15.837114035800054 + 79.521749969495218 i	2556.432917492445085 + 594.773301239737066 i	49,61,11	10.136183493555611 + 81.739759201416234 i	2457.683559418924688 + 354.047447365862638 i
49,61,13	3.433305384516568 + 83.786222472870435 i	2404.931288153868081 + 89.376163183355928 i	49,62,1	36.962727929966618 + 68.954761050235646 i	3466.382755220475701 +1368.951654942012738 i
49,62,3	32.607653396059547 + 72.302309136382462 i	3195.490496601717496 +1274.592663161941118 i	49,62,5	28.127555637413508 + 75.235049029609328 i	2966.886670588106881 +1129.409456483113900 i
49,62,7	23.435213854959390 + 77.897718858201856 i	2774.859480928475932 + 947.027479184805657 i	49,62,9	18.447544452490842 + 80.348589087351613 i	2618.893062612004996 + 734.207826765861682 i
49,62,11	13.040494576156201 + 82.617160683026768 i	2500.791429669582612 + 495.178265938204902 i	49,62,13	6.952270653469248 + 84.719185409822671 i	2424.674247412158365 + 232.552869223953110 i
49,63,1	39.001823913923232 + 69.598974476230310 i	3597.024166560417598 +1495.863475296205024 i	49,63,3	34.733090168549893 + 72.978862907848836 i	3312.294482732555934 +1407.295539112895085 i
49,63,5	30.353901447869291 + 75.947833983455865 i	3069.363113766967217 +1266.494509362662257 i	49,63,7	25.785896869925999 + 78.650149659910824 i	2862.161282295978253 +1087.508976406638112 i
49,63,9	20.960389084758752 + 81.144220404600205 i	2689.817611995863444 + 877.276877018210371 i	49,63,11	15.784061031262009 + 83.459973657704211 i	2553.605486205883153 + 640.161987768375411 i
49,63,13	10.088618922856529 + 85.613794597206663 i	2456.582744467994416 + 379.118099082353297 i	49,63,15	3.403073916836012 + 87.613927801696931 i	2404.833815829902960 + 95.114963991478916 i
49,64,1	41.015497549102463 + 70.222560775626903 i	3733.856979934304945 +1625.319955606766598 i	49,64,3	36.825701335480318 + 73.633403143214309 i	3435.615367659599087 +1542.737881518927225 i
49,64,5	32.537531011064900 + 76.636852583673956 i	3178.746725613613307 +1406.535539102954999 i	49,64,7	28.079586535477706 + 79.376711429706475 i	2956.865958025359305 +1231.183507882344429 i
49,64,9	23.393601144516481 + 81.911487627961321 i	2768.810559257251043 +1023.791542696195279 i	49,64,11	18.406298330699890 + 84.271463971467412 i	2615.454086954912782 + 788.833312290386857 i
49,64,13	12.999239351738357 + 86.473600480554907 i	2499.153642789955029 + 529.431721864852079 i	49,64,15	6.916669477380432 + 88.526912498083973 i	2424.230281839424606 + 247.550302518608959 i

Table D.50: Branch points and associated eigenvalues for $m = 49$.

Table D.51: Branch points and associated eigenvalues for $m = 50$.

m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$	m,n,r	$c^{m,n}_{\circ;r} = c^{m,n+2}_{\circ;r+1}$	$\lambda^{m,n}_{\circ;r} = \lambda^{m,n+2}_{\circ;r+1}$
50,50,1	4.284666454747461 + 57.999423131179185 i	2506.804133434685355 + 42.504402604727730 i	50,51,1	8.006405999465372 + 59.425032943314051 i	2535.502351330471356 + 114.046075724370112 i
50,52,1	11.181381642398893 + 60.683094338457195 i	2576.955086869174465 + 193.644723297298555 i	50,52,3	3.913817476279100 + 63.492194134701009 i	2505.465780937154705 + 54.164689302079850 i
50,53,1	14.061226330982475 + 61.823074661373511 i	2628.762909734839468 + 279.985088780879721 i	50,53,3	7.572396591240508 + 64.709393717091586 i	2530.393848706421068 + 142.232532137666453 i
50,54,1	16.748493752737598 + 62.873322215325913 i	2689.705667029728829 + 372.127572962106228 i	50,54,3	10.743197718088073 + 65.827393953760506 i	2568.179772893072823 + 236.541555906483154 i
50,54,5	3.729883947590005 + 68.299759744615699 i	2504.855112102056410 + 63.153372690080019 i	50,55,1	19.297440213242300 + 63.851958751377069 i	2759.031717370869956 + 469.390070780193241 i
50,55,3	13.639220397670066 + 66.866030781255105 i	2616.630940257766724 + 336.553264474278933 i	50,55,5	7.341320544194978 + 69.409258741522223 i	2527.833138137017158 + 164.829981644678185 i
50,56,1	21.740984039861615 + 64.771472166743521 i	2836.227118642391815 + 571.255070257489251 i	50,56,3	16.351007573417537 + 67.839520473069967 i	2674.544140969244381 + 441.694947096784517 i
50,56,5	10.502298524137391 + 70.447251304687981 i	2563.531853109981512 + 271.718736416218007 i	50,56,7	3.617428597044404 + 72.746408396670716 i	2504.495653298025900 + 70.866611141057319 i
50,57,1	24.100900977403082 + 65.640942209348879 i	2920.916191876386620 + 677.315133037752616 i	50,57,3	18.927988382122724 + 68.758211730653684 i	2741.146430782273001 + 551.490332027881095 i
50,57,5	13.404208974599694 + 71.424432468152148 i	2609.973215056247227 + 383.650949640689589 i	50,57,7	7.196636000066805 + 73.788895191603103 i	2526.258897321468339 + 184.554379672992667 i
50,58,1	26.392487561540015 + 66.467240700524115 i	3012.811213983759444 + 787.239972883438668 i	50,58,3	21.400863791933887 + 69.629890239767377 i	2815.895706408661226 + 665.549608299060651 i
50,58,5	16.129608272241914 + 72.349538432061692 i	2666.019918856782169 + 500.256088739023085 i	50,58,7	10.350348342031348 + 74.774375805389568 i	2560.589554971204961 + 302.782576519202962 i
50,58,9	3.541419000851338 + 76.963003743170475 i	2504.256581213538993 + 77.814280796067720 i	50,59,1	28.626976072924428 + 67.255732646407765 i	3111.684096197975123 + 900.755561001000274 i
50,59,3	23.790310101123136 + 70.460591344659051 i	2898.389206006029781 + 783.550162681710731 i	50,59,5	18.724244123752865 + 73.229454384983129 i	2730.918171566831461 + 621.183067543005222 i
50,59,7	13.256544250899328 + 75.709778783142767 i	2605.674754774446683 + 425.596287537954481 i	50,59,9	7.098703273042078 + 77.959435918048825 i	2525.187398524647506 + 202.487230455607317 i
50,60,1	30.812900886849594 + 68.010711825608482 i	3217.349163305259481 +1017.630242713745702 i	50,60,3	26.111056804116810 + 71.255112630511618 i	2988.315118276242174 + 905.220934961038438 i
50,60,5	21.216908360784764 + 74.069658584249865 i	2804.127609524037325 + 746.125316422363426 i	50,60,7	15.992387836082600 + 76.601194032329829 i	2660.441181584121750 + 552.750697617780475 i
50,60,9	10.248252512597604 + 78.907649658579544 i	2558.552668218291274 + 331.222281985103564 i	50,60,11	3.486931013660595 + 81.017892126550677 i	2504.085631504266985 + 84.245803603779507 i
50,61,1	32.956924883987270 + 68.735684965863740 i	3329.652052280436692 +1137.665145551226942 i	50,61,3	28.374041820947912 + 72.017348475302967 i	3085.424443435032572 +1030.330857383082048 i
50,61,5	23.627211635582775 + 74.874581195271887 i	2885.240340962465325 + 874.818411686268973 i	50,61,7	18.600958091011233 + 77.453663204235596 i	2724.161558336951202 + 683.977716989046485 i
50,61,9	13.158975529373782 + 79.812733345425031 i	2602.664558110295275 + 464.205093272489648 i	50,61,11	7.029422742737498 + 81.980626161288242 i	2524.411631558085446 + 219.184027047091263 i
50,62,1	35.064366122115878 + 69.433564648836935 i	3448.462218825081891 +1260.687330735462638 i	50,62,3	30.587652097574342 + 72.750516112773397 i	3189.513462811763020 +1158.680393039420551 i
50,62,5	25.969259109702044 + 75.647863263358630 i	2973.93682259155821 +1007.03353779939164 i	50,62,7	21.109574509157707 + 78.271348213224513 i	2796.305689775768769 + 819.028789989662755 i
50,62,9	15.904210183215072 + 80.679314776937858 i	2656.504292290140256 + 601.275996235200068 i	50,62,11	10.177423109917111 + 82.901005848836149 i	2557.063501941214327 + 357.822915584095597 i
50,62,13	3.446355546492617 + 84.952020723549893 i	2503.957379578883319 + 90.303593253535865 i	50,63,1	37.139547941911964 + 70.106804637212292 i	3573.667687478512562 +1386.544765507346256 i
50,63,3	32.758485024153480 + 73.457313236942852 i	3300.412225664898870 +1290.095275550757833 i	50,63,5	28.253611305696307 + 76.392541519969612 i	3069.959566377390729 +1142.571223426999040 i
50,63,7	23.536978233820257 + 79.057713770670205 i	2876.468113110935974 + 957.681665826092967 i	50,63,9	18.525056045732420 + 81.511338998329734 i	2719.370466170571945 + 742.225487024234326 i
50,63,11	13.093317702225152 + 83.783108200104081 i	2600.449681462832359 + 500.449485340766330 i	50,63,13	6.979122946573871 + 85.889006862110790 i	2523.826214053935473 + 234.972076755733468 i
50,64,1	39.186039937109172 + 70.757497477345112 i	3705.171258726373708 +1515.102543543897355 i	50,64,3	34.891839797523687 + 74.140030928819314 i	3417.976664340646948 +1424.421802475036202 i
50,64,5	30.488423011740267 + 77.111181754648726 i	3173.096427828445030 +1281.256141109087821 i	50,64,7	25.896736758009446 + 79.815675414781353 i	2964.327909261168770 +1099.739117777868842 i
50,64,9	21.047696272308816 + 82.312142231685883 i	2790.745119410793905 + 886.847470613018459 i	50,64,11	15.847609037396365 + 84.630672916195749 i	2653.596638550474836 + 646.965574838065550 i
50,64,13	10.127632881080574 + 86.787829699962955 i	2555.934112915842888 + 383.056032688696064 i	50,64,15	3.415345860434988 + 88.792150772816754 i	2503.857820462818836 + 96.077303204858850 i
50,65,1	41.206828723628441 + 71.387446579723573 i	3842.887697008214218 +1646.239984969783109 i	50,65,3	36.992047593268865 + 74.800636524297701 i	3542.083006708559424 +1561.523236884768039 i
50,65,5	32.680148191950622 + 77.805976709934725 i	3283.169447675388255 +1422.932947793684889 i	50,65,7	28.199067402411387 + 80.547713006333026 i	3059.624345886107676 +1245.025909522229995 i
50,65,9	23.490176161653391 + 83.084558202352952 i	2870.227983208623300 +1034.951506195630373 i	50,65,11	18.479885438355844 + 85.446939128202033 i	2715.826176720832336 + 797.205091373938785 i
50,65,13	13.049362032238641 + 87.651958174163084 i	2598.765236363406075 + 534.920744940274972 i	50,65,15	6.942089383796844 + 89.708828276912740 i	2523.371182342393695 + 250.061993281049865 i

Table D.51: Branch points and associated eigenvalues for $m = 50$.

D.2 Asymptotic Expansions of the Prolate Angular Spheroidal Wave Functions, Their Derivatives, and their Characteristic Eigenvalues

All *prolate*-type expansions (except those marked with an asterisk) are of the type described in Section 4.3.1, while all *oblate*-type expansions are of the type described in Section 4.3.2. An asterisk next to a value indicates that the asymptotic expansion for the radial spheroidal wave function Eq. (3.11) in [3] was used and normalized using Eq. (5.15). Accuracy is difficult to determine due to the lack of comparable tables. Some entries ($S^{(a)}_{mn}(c, \eta)$ and $\lambda^{(a)}_{mn}$ with $\arg(c) = 0, \pi/4, \pi/2$ and $|c| \leq 500$) for were compared to values produced by code

found at [84]. These verified entries include $S_{mn}^{(a)}(c,\eta)$ and $\lambda_{mn}^{(a)}$ for $m = 0, n = [0..1]$ for $\eta = [0, \pi/4, \pi/2]$. Seven digits are listed for reference.

$S_{mn}^{(a)}(c,\eta)$ and $S_{mn}^{(a)\prime}(c,\eta)$ are listed for the prolate case only. Values for the oblate SWFs can be computed using the methods outlined in Chapter 4.

"Type" in the tables below refers to either the Pr $\Rightarrow$ *prolate*-type expansion of Section 4.3.1, or Ob $\Rightarrow$ *oblate*-type expansion of Section 4.3.2.

Some values are unable to be represented in double precision (for example $S_{0,0}^{(a)}(5000\angle 0, .99)$ and $S_{0,0}^{(a)}(5000\angle \pi/2, .99)$). Those values that too small are set to ± 0 while those values that are too large are set to $\pm\infty$. In addition, if the ratio of the real to the imaginary (or the reciprocal) part of $S_{mn}^{(a)}(c,\eta)$ or $S_{mn}^{(a)\prime}(c,\eta)$ is smaller than $\approx 1 \times 10^{-10}$, then the smaller value should probably be 0. An example of this can be seen in the values for $S_{0,0}^{(a)}(250\angle 90, 0.25)$ where the $\Im\{S_{0,0}^{(a)}(250\angle 90, 0.25)\} \to 0$.

D.2.1 Asymptotic expansions of the PASWF, $S_{mn}^{(a)}(c,\eta)$ and the corresponding characteristic eigenvalues, $\lambda_{mn}^{(a)}$, for $m = [0..1]$, $n = [m..6]$.

Table D.52: $S_{0n}^{(a)}(c,\,\eta)$ for $n = [0..6]$.

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
0,0	$25\angle 0^\circ$	Pr	2.42421e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.98789e−01 − 0.00000e+00i	8.85585e−01 − 0.00000e+00i	4.63264e−01 − 0.00000e+00i	3.91021e−02 − 0.00000e+00i	2.85622e−04 − 0.00000e+00i	1.73861e−09 − 0.00000e+00i
0,0	$25\angle 15^\circ$	Pr	2.33906e+01 + 6.47263e+00i	1.00000e+00 − 0.00000e+00i	9.98831e−01 − 3.23261e−04i	8.88905e−01 − 2.88502e−02i	4.65943e−01 − 9.71392e−02i	2.83544e−02 − 3.34193e−02i	−2.21683e−04 − 3.09923e−04i	2.73064e−09 + 2.35142e−09i
0,0	$25\angle 30^\circ$	Pr	2.08940e+01 + 1.25041e+01i	1.00000e+00 − 4.33681e−19i	9.98956e−01 − 6.24568e−04i	8.98804e−01 − 5.64089e−02i	4.75145e−01 − 1.99244e−01i	−6.38800e−03 − 6.09009e−02i	−4.08388e−04 + 7.87578e−04i	−7.03803e−09 + 2.96538e−08i
0,0	$25\angle 45^\circ$	Pr	1.69224e+01 + 1.76834e+01i	1.00000e+00 + 4.33681e−19i	9.99154e−01 − 8.83443e−04i	9.15074e−01 − 8.13245e−02i	4.94731e−01 − 3.11224e−01i	−7.46693e−02 − 7.27486e−02i	3.25564e−03 + 9.90839e−04i	−8.00933e−07 − 4.33226e−07i
0,0	$25\angle 60^\circ$	Ob	−2.70208e+02 + 5.16261e+02i	1.00000e+00 + 1.38778e−16i	1.01343e+00 − 2.59301e−02i	1.26329e+00 − 3.69384e+00i	−8.93631e+01 − 1.28199e+00i	1.66522e+04 + 4.90860e+02i	−3.19320e+06 − 1.43870e+05i	4.98236e+08 + 9.36905e+07i
0,0	$25\angle 75^\circ$	Ob	−4.93980e+02 + 2.99556e+02i	1.00000e+00 + 8.32667e−17i	1.02476e+00 − 1.51019e−02i	4.08786e+00 − 3.03203e+00i	−8.00296e+00 − 1.66581e+02i	−5.77612e+04 + 5.53875e+03i	2.96100e+06 + 2.05703e+07i	5.95426e+09 − 7.51026e+08i
0,0	$25\angle 90^\circ$	Ob	−5.76010e+02 + 7.34781e−14i	1.00000e+00 + 7.42368e−17i	1.02894e+00 + 7.26758e−17i	5.57240e+00 − 4.26060e−16i	2.06321e+02 − 6.36999e−14i	8.88126e+04 − 6.14256e−11i	3.93549e+07 − 4.22854e−08i	1.39420e+10 − 2.01037e−05i
0,0	$50\angle 0^\circ$	Pr	4.92462e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.97541e−01 − 0.00000e+00i	7.81265e−01 − 0.00000e+00i	2.09413e−01 − 0.00000e+00i	1.37190e−03 − 0.00000e+00i	6.01237e−08 − 0.00000e+00i	8.01364e−19 − 0.00000e+00i
0,0	$50\angle 15^\circ$	Pr	4.75426e+01 + 1.29420e+01i	1.00000e+00 − 0.00000e+00i	9.97625e−01 − 6.45578e−04i	7.86307e−01 − 5.10814e−02i	2.02648e−01 − 8.83091e−02i	−2.79907e−04 − 1.70074e−03i	−3.47424e−08 + 1.01238e−07i	4.32009e−19 + 3.43224e−18i
0,0	$50\angle 30^\circ$	Pr	4.25480e+01 + 2.50020e+01i	1.00000e+00 + 1.08420e−19i	9.97874e−01 − 1.24747e−03i	8.01604e−01 − 1.00989e−01i	1.81606e−01 − 1.84727e−01i	−3.29290e−03 + 6.95211e−04i	−3.33093e−07 − 4.75554e−07i	−2.20072e−16 − 1.22503e−16i
0,0	$50\angle 45^\circ$	Pr	3.46027e+01 + 3.53581e+01i	1.00000e+00 + 2.16840e−19i	9.98270e−01 − 1.76489e−03i	8.27597e−01 − 1.48234e−01i	1.44419e−01 − 3.00481e−01i	2.41413e−04 + 9.75442e−03i	7.08220e−06 + 4.79411e−06i	1.18050e−13 + 1.98617e−13i
0,0	$50\angle 60^\circ$	Ob	−1.16440e+03 + 2.11506e+03i	1.00000e+00 + 3.05311e−16i	1.05690e+00 − 1.07808e−01i	−2.76527e+01 − 2.06277e+01i	2.00653e+04 + 6.44857e+02i	8.38393e+08 + 5.42465e+07i	3.60049e+13 + 3.51444e+12i	9.62721e+17 + 3.84667e+17i
0,0	$50\angle 75^\circ$	Ob	−2.06948e+03 + 1.22412e+03i	1.00000e−00 − 3.88578e−16i	1.10464e+00 − 6.33384e−02i	1.55156e+01 − 5.46790e+01i	−6.96578e+04 + 6.58104e+03i	1.00256e+10 − 1.90948e+09i	−1.47123e+15 + 4.26422e+14i	1.41159e+20 − 3.55228e+19i
0,0	$50\angle 90^\circ$	Ob	−2.40101e+03 + 3.00038e−13i	1.00000e−00 − 7.26897e−18i	1.12248e+00 − 2.37692e−17i	6.74008e+01 − 2.11272e−14i	1.07113e+05 − 8.27776e−11i	2.39193e+10 − 3.67949e−05i	5.49615e+15 − 1.26618e+01i	7.86088e+20 − 2.38860e+06i
0,0	$100\angle 0^\circ$	Pr	9.92481e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.95050e−01 − 0.00000e+00i	6.08061e−01 − 0.00000e+00i	4.28000e−02 − 0.00000e+00i	1.69033e−06 − 0.00000e+00i	2.67211e−15 − 0.00000e+00i	1.77027e−37 − 0.00000e+00i
0,0	$100\angle 15^\circ$	Pr	9.58408e+01 + 2.58824e+01i	1.00000e+00 − 5.42101e−20i	9.95218e−01 − 1.28797e−03i	6.13338e−01 − 8.00222e−02i	3.24702e−02 − 3.49297e−02i	−2.52772e−06 + 8.54525e−07i	−6.68085e−15 − 5.20547e−15i	−3.20708e−36 + 7.91822e−37i
0,0	$100\angle 30^\circ$	Pr	8.58509e+01 + 5.00010e+01i	1.00000e+00 − 0.00000e+00i	9.95713e−01 − 2.48940e−03i	6.29979e−01 − 1.61285e−01i	−1.10961e−03 − 6.54843e−02i	9.30729e−06 − 4.10835e−06i	−8.55258e−14 + 2.34159e−13i	9.03604e−33 + 1.50843e−32i
0,0	$100\angle 45^\circ$	Pr	6.99594e+01 + 7.07120e+01i	1.00000e−00 − 0.00000e+00i	9.96502e−01 − 3.52334e−03i	6.60442e−01 − 2.44415e−01i	−6.77563e−02 − 8.47187e−02i	−8.54277e−05 + 4.17737e−06i	2.00048e−11 + 5.02806e−11i	−7.37981e−27 + 1.29201e−26i
0,0	$100\angle 60^\circ$	Ob	−4.82780e+03 + 8.56025e+03i	1.00000e−00 − 0.00000e+00i	1.21925e+00 − 4.62407e−01i	7.44132e+02 + 2.51339e+03i	1.00807e+09 + 6.64838e+07i	2.10844e+18 + 2.79697e+17i	4.51651e+27 + 9.06204e+26i	3.10860e+36 + 2.97957e+36i
0,0	$100\angle 75^\circ$	Ob	−8.46807e+03 + 4.94824e+03i	1.00000e+00 − 4.44089e−16i	1.44306e+00 − 2.83670e−01i	−6.05587e+03 − 3.74027e+03i	1.20603e+10 − 2.28890e+09i	2.92212e+20 − 1.15031e+20i	6.99178e+30 − 4.40732e+30i	7.49185e+40 − 4.00575e+40i
0,0	$100\angle 90^\circ$	Ob	−9.80100e+03 + 1.21240e−12i	1.00000e+00 − 2.25777e−17i	1.53142e+00 − 1.05597e−16i	1.00074e+04 − 6.35400e−12i	2.87727e+10 − 4.46969e−05i	1.72531e+21 − 5.32138e+06i	1.06432e+32 − 4.91200e+17i	2.47842e+42 − 1.50805e+28i
0,0	$250\angle 0^\circ$	Pr	2.49249e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.87615e−01 − 0.00000e+00i	2.86685e−01 − 0.00000e+00i	3.65451e−04 − 0.00000e+00i	3.16398e−15 − 0.00000e+00i	1.95537e−93 − 0.00000e+00i	1.95537e−93 − 0.00000e+00i
0,0	$250\angle 15^\circ$	Pr	2.40731e+02 + 6.47050e+01i	1.00000e+00 − 0.00000e+00i	9.88030e−01 − 3.19661e−03i	2.83593e−01 − 9.53468e−02i	−2.22813e−04 − 4.23984e−04i	−7.20744e−15 − 6.79504e−15i	−4.19022e−36 − 3.54797e−37i	1.66212e−90 + 2.42914e−90i
0,0	$250\angle 30^\circ$	Pr	2.15756e+02 + 1.25000e+02i	1.00000e+00 − 0.00000e+00i	9.89251e−01 − 6.18307e−03i	2.74680e−01 − 1.98834e−01i	−7.16217e−04 + 7.79513e−04i	−1.42618e−13 + 2.42365e−13i	−1.79754e−33 + 1.96880e−32i	5.18478e−81 − 3.20875e−81i
0,0	$250\angle 45^\circ$	Pr	1.76026e+02 + 1.76777e+02i	1.00000e+00 − 0.00000e+00i	9.91198e−01 − 8.76151e−03i	2.61711e−01 − 3.20548e−01i	2.93035e−03 + 2.32049e−03i	6.99796e−12 + 5.72070e−11i	−1.35544e−26 + 2.18597e−27i	2.02131e−66 − 3.48570e−66i
0,0	$250\angle 60^\circ$	Ob	−3.08180e+04 + 5.38766e+04i	1.00000e+00 + 5.55112e−16i	1.37865e+00 − 4.03928e+00i	1.14632e+09 + 7.60879e+07i	1.26706e+23 + 2.11926e+22i	3.25138e+46 + 1.11919e+46i	8.32069e+69 + 4.51699e+69i	−1.03039e+92 + 2.89073e+92i
0,0	$250\angle 75^\circ$	Ob	−5.36446e+04 + 3.11206e+04i	1.00000e+00 + 9.99201e−16i	4.45487e+00 − 3.31151e+00i	1.37164e+10 − 2.60010e+09i	−5.90105e+25 + 2.98516e+25i	5.39912e+51 − 7.34058e+51i	−2.13279e+77 + 1.27384e+78i	5.67280e+102 − 1.58014e+103i
0,0	$250\angle 90^\circ$	Ob	−6.20010e+04 + 7.62343e−12i	1.00000e+00 + 7.87257e−17i	6.07228e+00 − 4.38820e−16i	3.27235e+10 − 4.75176e−05i	5.56276e+26 − 2.08510e+12i	6.44759e+53 − 4.88428e+39i	7.68728e+80 − 8.76534e+66i	7.71910e+106 − 1.16376e+93i
0,0	$500\angle 0^\circ$	Pr	4.99250e+02 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	9.75346e−01 − 0.00000e+00i	8.18790e−02 − 0.00000e+00i	1.30364e−07 − 0.00000e+00i	8.99708e−30 − 0.00000e+00i	4.09510e−74 − 0.00000e+00i	1.07905e−186 − 0.00000e+00i
0,0	$500\angle 15^\circ$	Pr	4.82213e+02 + 1.29410e+02i	1.00000e−00 − 6.77626e−21i	9.76157e−01 − 6.31645e−03i	7.10652e−02 − 5.38756e−02i	−1.27011e−07 + 1.84425e−07i	5.18613e−30 + 8.80322e−29i	1.29147e−71 + 2.20469e−72i	−8.88925e−181 + 2.27814e−180i
0,0	$500\angle 30^\circ$	Pr	4.32262e+02 + 2.50000e+02i	1.00000e+00 − 0.00000e+00i	9.78542e−01 − 1.22327e−02i	3.57788e−02 − 1.08820e−01i	−9.23915e−08 − 1.08994e−06i	−3.45081e−26 − 6.21356e−26i	−2.84784e−64 − 5.25193e−65i	4.70912e−162 − 9.38484e−162i
0,0	$500\angle 45^\circ$	Pr	3.52803e+02 + 3.53554e+02i	1.00000e+00 − 0.00000e+00i	9.82361e−01 − 1.73681e−02i	−3.41291e−02 − 1.67150e−01i	3.12545e−06 + 1.32751e−05i	−2.89748e−21 + 7.19277e−22i	1.32610e−52 − 4.38588e−53i	−2.26474e−132 − 3.99166e−132i
0,0	$500\angle 60^\circ$	Ob	−1.24135e+05 + 2.16006e+05i	1.00000e−00 − 2.02616e−15i	−3.01289e+01 − 2.24988e+01i	2.87880e+18 + 3.84261e+17i	3.90293e+46 + 1.34450e+46i	2.79700e+93 + 2.18667e+93i	1.70927e+140 + 2.63497e+140i	−2.90523e+185 − 2.37623e+185i
0,0	$500\angle 75^\circ$	Ob	−2.15541e+05 + 1.24741e+05i	1.00000e−00 + 5.16254e−15i	1.69280e+01 − 5.96013e+01i	3.99153e+20 − 1.56935e+20i	6.48297e+51 − 8.81190e+51i	−7.42104e+103 − 2.38047e+104i	−5.52640e+156 − 1.90602e+156i	−8.66642e+206 − 7.14896e+206i
0,0	$500\angle 90^\circ$	Ob	−2.49001e+05 + 3.05549e−11i	1.00000e−00 + 4.17943e−19i	7.34745e+01 − 2.24623e−14i	2.35647e+21 − 7.21364e+06i	7.74075e+53 − 5.92449e+39i	1.24840e+108 − 1.91101e+94i	2.07098e+162 − 4.75533e+148i	2.37503e+214 − 7.19862e+200i
0,0	$750\angle 0^\circ$	Pr	7.49250e+02 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	9.63230e−01 − 0.00000e+00i	2.33852e−02 − 0.00000e+00i	4.65042e−11 − 0.00000e+00i	2.55851e−44 − 0.00000e+00i	7.13375e−111 − 0.00000e+00i	5.96162e−280 − 0.00000e+00i
0,0	$750\angle 15^\circ$	Pr	7.23694e+02 + 1.94114e+02i	1.00000e+00 − 0.00000e+00i	9.64416e−01 − 9.36088e−03i	1.49602e−02 − 2.19715e−02i	1.03950e−10 + 1.24548e−11i	5.04060e−43 − 6.01920e−43i	−3.95167e−107 − 1.02445e−107i	−1.98185e−270 + 4.57732e−271i
0,0	$750\angle 30^\circ$	Pr	6.48769e+02 + 3.75000e+02i	1.00000e+00 − 0.00000e+00i	9.67911e−01 − 1.81509e−02i	−1.17650e−02 − 3.68655e−02i	8.93922e−10 + 6.91710e−10i	1.79587e−38 + 4.48880e−40i	1.14637e−96 − 4.08442e−96i	−1.57393e−243 − 1.80313e−242i
0,0	$750\angle 45^\circ$	Pr	5.29580e+02 + 5.30330e+02i	1.00000e+00 − 0.00000e+00i	9.73525e−01 − 2.58212e−02i	−6.22763e−02 − 3.26817e−02i	−2.11303e−08 + 4.50512e−08i	−5.51951e−32 − 1.44465e−31i	−1.26114e−78 + 6.54976e−79i	−5.23103e−198 − 6.43552e−200i
0,0	$750\angle 60^\circ$	Ob	−2.79952e+05 + 4.86389e+05i	1.00000e+00 − 7.32747e−15i	−2.68894e+02 + 1.87299e+02i	7.19689e+27 + 1.45200e+27i	1.16540e+70 + 6.33203e+69i	1.99419e+140 + 3.07488e+140i	8.07533e+209 + 1.03853e+211i	3.93065e+278 − 2.36809e+278i
0,0	$750\angle 75^\circ$	Ob	−4.85691e+05 + 2.80862e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$750\angle 90^\circ$	Ob	−5.61001e+05 + 6.87945e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1000\angle 0^\circ$	Pr	9.99250e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.51264e−01 − 0.00000e+00i	6.67895e−03 − 0.00000e+00i	1.65893e−14 − 0.00000e+00i	7.27571e−59 − 0.00000e+00i	1.24275e−147 − 0.00000e+00i	±0
0,0	$1000\angle 15^\circ$	Pr	9.65176e+02 + 2.58819e+02i	1.00000e+00 − 0.00000e+00i	9.52806e−01 − 1.23312e−02i	2.13962e−03 − 7.62854e−03i	−1.74537e−14 − 4.57302e−14i	−6.94141e−57 + 8.20553e−58i	1.19997e−142 + 4.22091e−143i	±0
0,0	$1000\angle 30^\circ$	Pr	8.65275e+02 + 5.00000e+02i	1.00000e−00 − 0.00000e+00i	9.57358e−01 − 2.39396e−02i	−1.05220e−02 − 7.75763e−03i	−1.15129e−12 + 1.96585e−13i	−2.40006e−51 + 3.85440e−51i	5.80565e−128 + 2.21762e−128i	−1.97626e−323 − 2.47033e−323i
0,0	$1000\angle 45^\circ$	Pr	7.06357e+02 + 7.07107e+02i	1.00000e+00 − 0.00000e+00i	9.64694e−01 − 3.41222e−02i	−2.66737e−02 + 1.13664e−02i	−1.62489e−10 + 8.09993e−11i	7.08135e−42 − 3.74611e−42i	1.16090e−104 − 8.61873e−105i	−3.06900e−264 + 5.11434e−264i
0,0	$1000\angle 60^\circ$	Ob	−4.98269e+05 + 8.65025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1000\angle 75^\circ$	Ob	−8.64095e+05 + 4.99482e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1000\angle 90^\circ$	Ob	−9.98001e+05 + 1.22342e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1500\angle 0^\circ$	Pr	1.49925e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.27777e−01 − 0.00000e+00i	5.44808e−04 − 0.00000e+00i	2.11104e−21 − 0.00000e+00i	5.88379e−88 − 0.00000e+00i	3.77163e−221 − 0.00000e+00i	±0
0,0	$1500\angle 15^\circ$	Pr	1.44814e+03 + 3.88229e+02i	1.00000e+00 − 0.00000e+00i	9.29975e−01 − 1.80549e−02i	−2.57965e−04 − 6.54924e−04i	1.03964e−20 + 2.52762e−21i	−9.72758e−86 − 5.45428e−85i	1.07952e−213 + 6.00117e−214i	±0
0,0	$1500\angle 30^\circ$	Pr	1.29829e+03 + 7.50000e+02i	1.00000e−00 − 6.77626e−21i	9.36487e−01 − 3.51356e−02i	−1.21606e−03 + 8.64177e−04i	3.12976e−19 + 1.20717e−18i	2.89712e−76 + 1.44997e−77i	−1.13896e−191 − 6.94148e−192i	±0
0,0	$1500\angle 45^\circ$	Pr	1.05991e+03 + 1.06066e+03i	1.00000e+00 + 6.77626e−21i	9.47050e−01 − 5.02733e−02i	2.79967e−03 + 4.05528e−03i	−1.54534e−15 − 1.85849e−15i	−1.60215e−62 + 1.43339e−62i	8.61008e−157 − 1.22432e−156i	±0
0,0	$1500\angle 60^\circ$	Ob	−1.12240e+06 + 1.94706e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1500\angle 75^\circ$	Ob	−1.94566e+06 + 1.12422e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$1500\angle 90^\circ$	Ob	−2.24700e+06 + 2.75362e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,0	$2500\angle 0^\circ$	Pr	2.49925e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	8.82527e−01 − 0.00000e+00i	3.62506e−06 − 0.00000e+00i	3.41852e−35 − 0.00000e+00i	3.84792e−146 − 0.00000e+00i	±0	±0
0,0	$2500\angle 15^\circ$	Pr	2.41406e+03 + 6.47048e+02i	1.00000e+00 − 0.00000e+00i	8.85830e−01 − 2.86694e−02i	−5.52719e−06 + 5.64477e−07i	−6.42953e−35 − 5.07154e−34i	1.00920e−141 + 3.33140e−141i	±0	±0
0,0	$2500\angle 30^\circ$	Pr	2.16431e+03 + 1.25000e+03i	1.00000e+00 − 0.00000e+00i	8.95679e−01 − 5.60544e−02i	1.94259e−05 + 3.39629e−07i	−5.83376e−31 − 1.29659e−30i	−6.75259e−127 + 9.72438e−127i	−3.75979e−319 − 4.85923e−319i	±0
0,0	$2500\angle 45^\circ$	Pr	1.76702e+03 + 1.76777e+03i	1.00000e−00 − 3.38813e−21i	9.11864e−01 − 8.08107e−02i	−1.20317e−04 − 7.60602e−05i	3.92055e−25 + 1.72597e−25i	−5.37179e−104 + 1.45186e−103i	−4.11304e−262 − 1.60352e−260i	±0

Table D.52: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
0,0	$2500\angle 60^\circ$	Ob	$-3.12067e{+}06+5.41016e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$2500\angle 75^\circ$	Ob	$-5.40783e{+}06+3.12371e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$2500\angle 90^\circ$	Ob	$-6.24500e{+}06+7.65098e{-}10i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 0^\circ$	Pr	$4.99925e{+}03-0.00000e{+}00i$	$1.00000e{+}00-0.00000e{+}00i$	$7.78825e{-}01-0.00000e{+}00i$	$1.30916e{-}11-0.00000e{+}00i$	$1.14076e{-}69-0.00000e{+}00i$	$1.33093e{-}291-0.00000e{+}00i^{*}$	± 0	± 0
0,0	$5000\angle 15^\circ$	Pr	$4.82888e{+}03+1.29410e{+}03i$	$1.00000e{+}00-0.00000e{+}00i$	$7.83844e{-}01-5.07906e{-}02i$	$3.01176e{-}11-6.21647e{-}12i$	$-2.47035e{-}67+6.36595e{-}68i$	$-9.06049e{-}282+6.04410e{-}282i^{*}$	± 0	± 0
0,0	$5000\angle 30^\circ$	Pr	$4.32938e{+}03+2.50000e{+}03i$	$1.00000e{+}00-0.00000e{+}00i$	$7.99069e{-}01-1.00410e{-}01i$	$3.75832e{-}10+1.31457e{-}11i$	$-1.30883e{-}60+1.47671e{-}60i$	$-4.40138e{-}253-1.18050e{-}252i^{*}$	± 0	± 0
0,0	$5000\angle 45^\circ$	Pr	$3.53478e{+}03+3.53553e{+}03i$	$1.00000e{+}00+1.69407e{-}21i$	$8.24934e{-}01-1.47371e{-}01i$	$8.65836e{-}09+1.82338e{-}08i$	$1.20961e{-}49+1.32107e{-}49i$	$-1.63536e{-}206-1.40212e{-}206i^{*}$	± 0	± 0
0,0	$5000\angle 60^\circ$	Ob	$-1.24913e{+}07+2.16456e{+}07i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 75^\circ$	Ob	$-2.16410e{+}07+1.24974e{+}07i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 90^\circ$	Ob	$-2.49900e{+}07+3.06100e{-}09i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$25\angle 0^\circ$	Pr	$7.32096e{+}01-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.98814e{-}03-0.00000e{+}00i$	$8.87835e{-}02-0.00000e{+}00i$	$1.17706e{-}01-0.00000e{+}00i$	$2.09716e{-}02-0.00000e{+}00i$	$2.58465e{-}04-0.00000e{+}00i$	$3.04243e{-}09-0.00000e{+}00i^{*}$
0,1	$25\angle 15^\circ$	Pr	$7.06557e{+}01+1.94227e{+}01i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.98856e{-}03-3.23349e{-}06i$	$8.91162e{-}02-2.89306e{-}03i$	$1.18384e{-}01-2.46869e{-}02i$	$1.52026e{-}02-1.79266e{-}02i$	$-2.00771e{-}04-2.80289e{-}04i$	$4.78608e{-}09+4.09846e{-}09i^{*}$
0,1	$25\angle 30^\circ$	Pr	$6.31681e{+}01+3.75213e{+}01i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.98981e{-}03-6.24734e{-}06i$	$9.01083e{-}02-5.65655e{-}03i$	$1.20716e{-}01-5.06339e{-}02i$	$-3.43977e{-}03-3.26567e{-}02i$	$-3.68503e{-}04+7.12851e{-}04i$	$-1.19994e{-}08+5.18474e{-}08i^{*}$
0,1	$25\angle 45^\circ$	Ob	$3.43483e{+}01+5.89637e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.99432e{-}03-9.82425e{-}05i$	$6.66196e{-}02-9.13042e{-}02i$	$-1.21804e{+}00+6.12146e{-}01i$	$-1.63576e{+}01-9.27150e{+}01i$	$6.64708e{+}03+7.80595e{+}02i$	$-2.26162e{+}05+3.41007e{+}05i$
0,1	$25\angle 60^\circ$	Ob	$-2.70208e{+}02+5.16261e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.00452e{-}02-8.62792e{-}05i$	$1.25326e{-}01-1.08911e{-}01i$	$-3.13897e{+}00-1.96557e{+}00i$	$5.79552e{+}02+3.75223e{+}02i$	$-1.10065e{+}05-7.37149e{+}04i$	$1.56426e{+}07+1.40263e{+}07i$
0,1	$25\angle 75^\circ$	Ob	$-4.93980e{+}02+2.99556e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.00828e{-}02-5.01750e{-}05i$	$1.95435e{-}01-7.89694e{-}02i$	$1.54950e{+}00-6.76894e{+}00i$	$-2.37809e{+}03-4.26456e{+}02i$	$-1.12239e{+}05+8.57999e{+}05i$	$2.47165e{+}08+3.67410e{+}07i$
0,1	$25\angle 90^\circ$	Ob	$-5.76010e{+}02+7.34781e{-}14i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.00966e{-}02+5.62872e{-}19i$	$2.28579e{-}01-7.97660e{-}18i$	$8.60433e{+}00-2.25536e{-}15i$	$3.70385e{+}03-2.38898e{-}12i$	$1.64126e{+}06-1.68694e{-}09i$	$5.81439e{+}08-8.11292e{-}07i$
0,1	$50\angle 0^\circ$	Pr	$1.48231e{+}02-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.97566e{-}03-0.00000e{+}00i$	$7.83238e{-}02-0.00000e{+}00i$	$5.32025e{-}02-0.00000e{+}00i$	$7.35477e{-}04-0.00000e{+}00i$	$5.43403e{-}08-0.00000e{+}00i$	$1.39654e{-}18-0.00000e{+}00i$
0,1	$50\angle 15^\circ$	Pr	$1.43120e{+}02+3.88281e{+}01i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.97650e{-}03-6.45630e{-}06i$	$7.88293e{-}02-5.12132e{-}03i$	$5.14830e{-}02-2.24365e{-}02i$	$-1.50150e{-}04-9.11742e{-}04i$	$-3.13720e{-}08+9.15056e{-}08i$	$7.60204e{-}19+5.97919e{-}18i$
0,1	$50\angle 30^\circ$	Pr	$1.28137e{+}02+7.50100e{+}01i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.97899e{-}03-1.24757e{-}05i$	$8.03627e{-}02-1.01250e{-}02i$	$4.61351e{-}02-4.69322e{-}02i$	$-1.76515e{-}03+3.73029e{-}04i$	$-3.01244e{-}07-4.29567e{-}07i$	$-3.83593e{-}16-2.12479e{-}16i^{*}$
0,1	$50\angle 45^\circ$	Ob	$6.97071e{+}01+2.42929e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.98380e{-}03+4.04586e{-}04i$	$-2.90436e{-}01-1.24371e{-}01i$	$-1.03553e{+}01-5.49141e{+}01i$	$-1.24535e{+}05+2.95724e{+}05i$	$1.57836e{+}09-1.04988e{+}09i$	$-7.57455e{+}12-2.78232e{+}12i$
0,1	$50\angle 60^\circ$	Ob	$-1.16440e{+}03+2.11506e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.01918e{-}02-3.56627e{-}04i$	$-2.70508e{-}01-6.47884e{-}01i$	$3.44869e{+}02+2.19151e{+}02i$	$1.41269e{+}07+9.63517e{+}06i$	$5.94409e{+}11+4.34542e{+}11i$	$1.28824e{+}16+1.67121e{+}16i$
0,1	$50\angle 75^\circ$	Ob	$-2.06948e{+}03+1.22412e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.03476e{-}02-2.08277e{-}04i$	$5.99709e{-}01-9.92360e{-}01i$	$-1.40586e{+}03-2.45654e{+}02i$	$2.07523e{+}08+1.64242e{+}07i$	$-3.12408e{+}13+4.66286e{+}11i$	$2.96841e{+}18+6.13174e{+}16i$
0,1	$50\angle 90^\circ$	Ob	$-2.40101e{+}03+3.00038e{-}13i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.04053e{-}02-2.74980e{-}19i$	$1.37566e{+}00-3.64897e{-}16i$	$2.18645e{+}03-1.58415e{-}12i$	$4.88252e{+}08-7.27505e{-}07i$	$1.12190e{+}14-2.53043e{-}01i$	$1.60460e{+}19-4.79827e{+}04i$
0,1	$100\angle 0^\circ$	Pr	$2.98240e{+}02-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.95075e{-}03-0.00000e{+}00i$	$6.09593e{-}02-0.00000e{+}00i$	$1.08731e{-}02-0.00000e{+}00i$	$9.06015e{-}07-0.00000e{+}00i$	$2.41373e{-}15-0.00000e{+}00i$	$3.07775e{-}37-0.00000e{+}00i^{*}$
0,1	$100\angle 15^\circ$	Pr	$2.88019e{+}02+7.76482e{+}01i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.95243e{-}03-1.28801e{-}05i$	$6.14883e{-}02-8.02248e{-}03i$	$8.24873e{-}03-8.87376e{-}03i$	$-1.35482e{-}06+4.58086e{-}07i$	$-6.03539e{-}15-4.70120e{-}15i$	$-5.57463e{-}36+1.37940e{-}36i^{*}$
0,1	$100\angle 30^\circ$	Pr	$2.58049e{+}02+1.50055e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.95738e{-}03-2.48949e{-}05i$	$6.31565e{-}02-1.61693e{-}02i$	$-2.82240e{-}04-1.66358e{-}02i$	$4.98834e{-}06-2.20247e{-}06i$	$-7.71934e{-}14+2.11522e{-}13i$	$1.57312e{-}32+2.62023e{-}32i^{*}$
0,1	$100\angle 45^\circ$	Ob	$1.40420e{+}02+9.85858e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.89606e{-}03-1.63896e{-}03i$	$5.38803e{+}00+2.34805e{-}02i$	$-7.33208e{+}04+1.76893e{+}05i$	$-7.58677e{+}12+4.89605e{+}10i$	$-1.22045e{+}20-2.84132e{+}20i$	$6.34614e{+}27-6.22358e{+}26i$
0,1	$100\angle 60^\circ$	Ob	$-4.82780e{+}03+8.56025e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.07615e{-}02-1.49557e{-}03i$	$-6.30687e{+}00+2.56784e{+}01i$	$8.44222e{+}06+5.70826e{+}06i$	$1.69419e{+}16+1.31642e{+}16i$	$3.47290e{+}25+3.08738e{+}25i$	$1.19162e{+}34+4.17697e{+}34i$
0,1	$100\angle 75^\circ$	Ob	$-8.46807e{+}03+4.94824e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.14510e{-}02-8.96472e{-}04i$	$-4.91545e{+}01-5.24393e{+}01i$	$1.23592e{+}08+9.52060e{+}06i$	$3.15177e{+}18-3.49973e{+}17i$	$7.97806e{+}28-2.45047e{+}28i$	$8.35948e{+}38-1.92707e{+}38i$
0,1	$100\angle 90^\circ$	Ob	$-9.80100e{+}03+1.21240e{-}12i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.17158e{-}02-6.48506e{-}19i$	$1.01090e{+}02-5.93246e{-}14i$	$2.90648e{+}08-4.37533e{-}07i$	$1.74283e{+}19-5.29161e{+}04i$	$1.07513e{+}30-4.91018e{+}15i$	$2.50358e{+}40-1.51133e{+}26i$
0,1	$250\angle 0^\circ$	Pr	$7.48246e{+}02-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.87639e{-}03-0.00000e{+}00i$	$2.87407e{-}02-0.00000e{+}00i$	$9.28381e{-}05-0.00000e{+}00i$	$1.69569e{-}15-0.00000e{+}00i$	$2.12305e{-}37-0.00000e{+}00i$	$3.39558e{-}93-0.00000e{+}00i^{*}$
0,1	$250\angle 15^\circ$	Pr	$7.22691e{+}02+1.94115e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.88055e{-}03-3.19670e{-}05i$	$2.84306e{-}02-9.55868e{-}03i$	$-5.66032e{-}05-1.07708e{-}04i$	$-3.86280e{-}15-3.64163e{-}15i$	$-3.78384e{-}36-3.20181e{-}37i$	$2.88710e{-}90+4.21761e{-}90i^{*}$
0,1	$250\angle 30^\circ$	Pr	$6.47766e{+}02+3.75002e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.89275e{-}03-6.18324e{-}05i$	$2.75370e{-}02-1.99335e{-}02i$	$-1.81944e{-}04+1.98026e{-}04i$	$-7.64288e{-}14+1.29894e{-}13i$	$-1.62132e{-}33+1.77783e{-}32i$	$9.00049e{-}81-5.57498e{-}81i^{*}$
0,1	$250\angle 45^\circ$	Ob	$3.52553e{+}02+6.21464e{+}04i$	$-0.00000e{+}00-0.00000e{+}00i$	$6.77210e{-}03-9.84868e{-}03i$	$-3.29064e{+}04+8.02353e{+}04i$	$2.09927e{+}16+1.36365e{+}16i$	$3.04160e{+}35+1.15883e{+}35i$	$4.30307e{+}54+6.59157e{+}53i$	$-1.39272e{+}72+1.01024e{+}73i$
0,1	$250\angle 60^\circ$	Ob	$-3.08180e{+}04+5.38766e{+}04i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.30163e{-}02-1.17038e{-}02i$	$3.82690e{+}06+2.57282e{+}06i$	$3.97254e{+}20+3.28762e{+}20i$	$9.03514e{+}43+1.04341e{+}44i$	$1.97933e{+}67+3.24409e{+}67i$	$-9.39943e{+}89+7.96174e{+}89i$
0,1	$250\angle 75^\circ$	Ob	$-5.36446e{+}04+3.11206e{+}04i$	$-0.00000e{+}00-0.00000e{+}00i$	$2.05068e{-}02-8.47907e{-}03i$	$5.59000e{+}07+4.22873e{+}06i$	$-2.59966e{+}23+5.41850e{+}22i$	$2.85945e{+}49-2.28309e{+}49i$	$-2.15607e{+}75+4.71696e{+}75i$	$3.84833e{+}100-5.53533e{+}100i$
0,1	$250\angle 90^\circ$	Ob	$-6.20010e{+}04+7.62343e{-}12i$	$-0.00000e{+}00-0.00000e{+}00i$	$2.40538e{-}02-6.94243e{-}19i$	$1.31421e{+}08-1.84578e{-}07i$	$2.23406e{+}24-8.26759e{+}09i$	$2.58942e{+}51-1.94925e{+}37i$	$3.08729e{+}78-3.50555e{+}64i$	$3.10007e{+}104-4.65900e{+}90i$
0,1	$500\angle 0^\circ$	Pr	$1.49825e{+}03-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.75370e{-}03-0.00000e{+}00i$	$8.20849e{-}03-0.00000e{+}00i$	$3.31172e{-}08-0.00000e{+}00i$	$4.82170e{-}30-0.00000e{+}00i$	$3.69757e{-}74-0.00000e{+}00i^{*}$	$1.87309e{-}186-0.00000e{+}00i^{*}$
0,1	$500\angle 15^\circ$	Pr	$1.44714e{+}03+3.88229e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.76181e{-}03-6.31661e{-}05i$	$7.12439e{-}03-5.40110e{-}03i$	$-3.22653e{-}08+4.68505e{-}08i$	$2.77978e{-}30+4.71779e{-}29i$	$1.16610e{-}71+1.99035e{-}72i^{*}$	$-1.54265e{-}180+3.95467e{-}180i^{*}$
0,1	$500\angle 30^\circ$	Pr	$1.29729e{+}03+7.50001e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.78566e{-}03-1.22330e{-}04i$	$3.58687e{-}03-1.09094e{-}02i$	$-2.34718e{-}08-2.76883e{-}07i$	$-1.84940e{-}26-3.32990e{-}26i$	$-2.57137e{-}64-4.74071e{-}65i^{*}$	$8.17093e{-}162-1.62916e{-}161i^{*}$
0,1	$500\angle 45^\circ$	Ob	$7.06106e{+}02+2.49293e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$-3.14183e{-}02-1.29975e{-}02i$	$-2.05966e{+}12+2.27974e{+}10i$	$1.82374e{+}35+6.92549e{+}34i$	$3.95146e{+}73-2.41049e{+}72i$	$7.31141e{+}111-3.85063e{+}111i$	$-4.48945e{+}148+2.53845e{+}148i$
0,1	$500\angle 60^\circ$	Ob	$-1.24135e{+}05+2.16006e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$-2.96417e{-}02-6.92509e{-}02i$	$4.60638e{+}15+3.55513e{+}15i$	$5.41870e{+}43+6.24792e{+}43i$	$2.65587e{+}90+6.59852e{+}90i$	$3.19838e{+}136+6.28438e{+}137i$	$-2.65332e{+}182-7.03583e{+}182i$
0,1	$500\angle 75^\circ$	Ob	$-2.15541e{+}05+1.24741e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$6.37325e{-}02-1.06567e{-}01i$	$8.54041e{+}17-9.63022e{+}16i$	$1.71257e{+}49-1.36850e{+}49i$	$-1.99209e{+}100-4.99261e{+}101i$	$-9.70491e{+}153-6.56054e{+}153i$	$-1.30574e{+}204-1.83390e{+}204i$
0,1	$500\angle 90^\circ$	Ob	$-2.49001e{+}05+3.05549e{-}11i$	$-0.00000e{+}00-0.00000e{+}00i$	$1.47230e{-}01-3.79826e{-}17i$	$4.72239e{+}18-1.42305e{+}04i$	$1.55126e{+}51-1.17986e{+}37i$	$2.50181e{+}105-3.81774e{+}91i$	$4.15027e{+}159-9.50991e{+}145i$	$4.75959e{+}211-1.44034e{+}198i$
0,1	$750\angle 0^\circ$	Pr	$2.24825e{+}03-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.63254e{-}03-0.00000e{+}00i$	$2.34439e{-}03-0.00000e{+}00i$	$1.18137e{-}11-0.00000e{+}00i$	$1.37113e{-}44-0.00000e{+}00i$	$6.44102e{-}111-0.00000e{+}00i^{*}$	$1.03473e{-}279-0.00000e{+}00i^{*}$
0,1	$750\angle 15^\circ$	Pr	$2.17158e{+}03+5.82343e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.64440e{-}03-9.36111e{-}05i$	$1.49978e{-}03-2.20267e{-}03i$	$2.64070e{-}11+3.16392e{-}12i$	$2.70129e{-}43-3.22577e{-}43i$	$-3.56795e{-}107-9.24908e{-}108i^{*}$	$-3.43973e{-}270+7.94683e{-}271i^{*}$
0,1	$750\angle 30^\circ$	Pr	$1.94681e{+}03+1.12500e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.67935e{-}03-1.81514e{-}04i$	$-1.17945e{-}03-3.69581e{-}03i$	$2.27087e{-}10+1.75718e{-}10i$	$9.62424e{-}39+2.40443e{-}40i$	$1.03491e{-}96-3.68780e{-}96i^{*}$	$-2.73566e{-}243-3.12947e{-}242i^{*}$
0,1	$750\angle 45^\circ$	Ob	$1.05966e{+}03+5.61439e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$-2.57905e{-}02+1.30270e{-}01i$	$-2.60511e{+}19-5.98681e{+}19i$	$2.00576e{+}54+3.04410e{+}53i$	$5.68382e{+}111-2.99582e{+}111i$	$9.10461e{+}168-1.88457e{+}169i$	$-3.19822e{+}224-1.37219e{+}224i$
0,1	$750\angle 60^\circ$	Ob	$-2.79952e{+}05+4.86389e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$-4.35886e{-}01+3.67648e{-}02i$	$7.34641e{+}24+6.48695e{+}24i$	$9.23609e{+}66+1.51045e{+}67i$	$2.49794e{+}136+4.88584e{+}137i$	$-6.00641e{+}207+1.25408e{+}208i$	$6.12460e{+}275-1.10048e{+}274i$
0,1	$750\angle 75^\circ$	Ob	$-4.85691e{+}05+2.80862e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$750\angle 90^\circ$	Ob	$-5.61001e{+}05+6.87945e{-}11i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 0^\circ$	Pr	$2.99825e{+}03-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.51288e{-}03-0.00000e{+}00i$	$6.69573e{-}04-0.00000e{+}00i$	$4.21424e{-}15-0.00000e{+}00i$	$3.89911e{-}59-0.00000e{+}00i$	$1.12205e{-}147-0.00000e{+}00i^{*}$	± 0
0,1	$1000\angle 15^\circ$	Pr	$2.89603e{+}03+7.76457e{+}02i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.52830e{-}03-1.23315e{-}04i$	$2.14499e{-}04-7.64771e{-}04i$	$-4.43384e{-}15-1.16170e{-}14i$	$-3.71995e{-}57+4.39758e{-}58i$	$1.08343e{-}142+3.81082e{-}143i^{*}$	± 0
0,1	$1000\angle 30^\circ$	Pr	$2.59633e{+}03+1.50000e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.57382e{-}03-2.39402e{-}04i$	$-1.05484e{-}03-7.77712e{-}04i$	$-2.92466e{-}13+4.99400e{-}14i$	$-1.28619e{-}51+2.06561e{-}51i$	$5.24181e{-}128+2.00209e{-}128i^{*}$	$-3.45846e{-}323-4.44659e{-}323i^{*}$
0,1	$1000\angle 45^\circ$	Ob	$1.41321e{+}03+9.98586e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$5.83283e{-}01-1.03852e{-}03i$	$1.60642e{+}27-1.68601e{+}27i$	$2.36951e{+}73-1.45841e{+}72i$	$7.21031e{+}149-9.25648e{+}149i$	$-1.10173e{+}226-5.86227e{+}226i$	$-6.41997e{+}299-2.56320e{+}300i$
0,1	$1000\angle 60^\circ$	Ob	$-4.98269e{+}05+8.65025e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 75^\circ$	Ob	$-8.64095e{+}05+4.99482e{+}05i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 90^\circ$	Ob	$-9.98001e{+}05+1.22342e{-}10i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 0^\circ$	Pr	$4.49825e{+}03-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.27800e{-}03-0.00000e{+}00i$	$5.46177e{-}05-0.00000e{+}00i$	$5.36277e{-}22-0.00000e{+}00i$	$3.15316e{-}88-0.00000e{+}00i$	$3.40527e{-}221-0.00000e{+}00i^{*}$	± 0
0,1	$1500\angle 15^\circ$	Pr	$4.34492e{+}03+1.16469e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.29999e{-}03-1.80553e{-}04i$	$-2.58613e{-}05-6.56570e{-}05i$	$2.64105e{-}21+6.42099e{-}22i$	$-5.21316e{-}86-2.92298e{-}85i$	$9.74666e{-}214+5.41815e{-}214i^{*}$	± 0
0,1	$1500\angle 30^\circ$	Pr	$3.89536e{+}03+2.25000e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$9.36511e{-}03-3.51365e{-}04i$	$-1.21911e{-}04+8.66349e{-}05i$	$7.95070e{-}20+3.06661e{-}19i$	$1.55258e{-}76+7.76956e{-}78i$	$-1.02833e{-}191-6.26700e{-}192i^{*}$	± 0
0,1	$1500\angle 45^\circ$	Ob	$2.12032e{+}03+2.24788e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 60^\circ$	Ob	$-1.12240e{+}06+1.94706e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 75^\circ$	Ob	$-1.94566e{+}06+1.12422e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 90^\circ$	Ob	$-2.24700e{+}06+2.75362e{-}10i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$2500\angle 0^\circ$	Pr	$7.49825e{+}03-0.00000e{+}00i$	$-0.00000e{+}00-0.00000e{+}00i$	$8.82549e{-}03-0.00000e{+}00i$	$3.63417e{-}07-0.00000e{+}00i$	$8.68420e{-}36-0.00000e{+}00i$	$2.06211e{-}146-0.00000e{+}00i^{*}$	± 0	± 0
0,1	$2500\angle 15^\circ$	Pr	$7.24269e{+}03+1.94114e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$8.85853e{-}03-2.86702e{-}04i$	$-5.54108e{-}07+5.65895e{-}08i$	$-1.63332e{-}35-1.28834e{-}34i$	$5.40835e{-}142+1.78530e{-}141i^{*}$	± 0	± 0
0,1	$2500\angle 30^\circ$	Pr	$6.49344e{+}03+3.75000e{+}03i$	$-0.00000e{+}00-0.00000e{+}00i$	$8.95702e{-}03-5.60558e{-}04i$	$1.94747e{-}06+3.40480e{-}08i$	$-1.48197e{-}31-3.29377e{-}31i$	$-3.61870e{-}127+5.21133e{-}127i^{*}$	$-3.39458e{-}319-4.38715e{-}319i^{*}$	± 0
0,1	$2500\angle 45^\circ$	Ob	$3.53453e{+}03+6.24646e{+}06i$	$-0.00000e{+}00-0.00000e{+}00i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.52: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
0,1	2500∠60°	Ob	−3.12067e+06 + 5.41016e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	2500∠75°	Ob	−5.40783e+06 + 3.12371e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	2500∠90°	Ob	−6.24500e+06 + 7.65098e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	5000∠0°	Pr	1.49982e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	7.78845e−03 − 0.00000e+00i	1.31245e−12 − 0.00000e+00i	2.89791e−70 − 0.00000e+00i	7.13244e−292 − 0.00000e+00i*	±0	±0
0,1	5000∠15°	Pr	1.44871e+04 + 3.88229e+03i	−0.00000e+00 − 0.00000e+00i	7.83864e−03 − 5.07919e−04i	3.01932e−12 − 6.23209e−13i	−6.27551e−68 + 1.61717e−68i	−4.85552e−282 + 3.23904e−282i*	±0	±0
0,1	5000∠30°	Pr	1.29886e+04 + 7.50000e+03i	−0.00000e+00 − 0.00000e+00i	7.99089e−03 − 1.00412e−03i	3.76777e−11 + 1.31787e−12i	−3.32487e−61 + 3.75134e−61i	−2.35871e−253 − 6.32630e−253i*	±0	±0
0,1	5000∠45°	Ob	7.07007e+03 + 2.49929e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	5000∠60°	Ob	−1.24913e+07 + 2.16456e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	5000∠75°	Ob	−2.16410e+07 + 1.24974e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,1	5000∠90°	Ob	−2.49900e+07 + 3.06100e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	25∠0°	Pr	1.21126e+02 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.96975e−01 − 0.00000e+00i	−2.27201e−01 − 0.00000e+00i	4.92801e−01 − 0.00000e+00i	2.52995e−01 − 0.00000e+00i	5.50678e−03 − 0.00000e+00i*	1.28090e−07 − 0.00000e+00i*
0,2	25∠15°	Pr	1.16872e+02 + 3.23877e+01i	−5.00000e−01 − 0.00000e+00i	−4.97081e−01 + 8.08027e−04i	−2.33773e−01 + 6.55277e−02i	5.10688e−01 + 9.67982e−02i	2.38696e−01 − 1.55316e−01i	−2.47721e−03 − 6.93883e−03i*	1.48278e−07 + 2.20503e−07i*
0,2	25∠30°	Pr	1.04401e+02 + 6.25665e+01i	−5.00000e−01 − 8.67362e−19i	−4.97391e−01 + 1.56130e−03i	−2.53707e−01 + 1.29424e−01i	5.63809e−01 + 2.14797e−01i	1.83659e−01 − 3.58486e−01i	−1.48371e−02 + 8.87192e−03i*	−1.56335e−06 + 1.62168e−06i*
0,2	25∠45°	Ob	3.43483e+01 + 5.89637e+02i	−5.00000e−01 + 2.77556e−17i	−4.99069e−01 + 1.47328e−02i	2.71426e−01 + 1.25697e+00i	4.73532e+00 − 1.58681e+01i	9.56506e+02 + 6.27775e+02i	−6.22879e+04 + 5.23006e+04i	−1.13243e+06 − 4.84199e+06i
0,2	25∠60°	Pr	1.17465e+01 + 2.16575e+01i	−5.00000e−01 + 2.16840e−19i	−4.99706e−01 + 5.41132e−04i	−4.68639e−01 + 5.10760e−02i	−2.66151e−01 + 2.18646e−01i	1.01290e−01 + 2.47792e−02i	−4.88087e−03 + 8.51231e−03i	−3.73098e−05 − 9.61043e−06i*
0,2	25∠75°	Ob	−4.01534e+02 + 2.73631e+02i	−5.00000e−01 + 6.93889e−18i	−5.10057e−01 + 6.88684e−03i	−1.66357e+00 + 1.22791e+00i	2.65321e+00 + 4.89671e+01i	9.21557e+03 − 1.00220e+03i	−2.29415e+05 − 1.33644e+06i	1.56363e+07 + 6.23097e+06i
0,2	25∠90°	Ob	−4.80164e+02 + 6.73446e−14i	−5.00000e−01 − 4.73800e−17i	−5.12053e−01 − 4.68250e−17i	−2.26940e+00 + 1.25364e−16i	−6.06068e+01 + 1.75563e−14i	−1.41586e+04 + 9.54498e−12i	−2.55658e+06 + 2.71099e−09i	3.67940e+07 − 4.78231e−08i
0,2	50∠0°	Pr	2.46191e+02 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.93859e−01 − 0.00000e+00i	−4.01883e−03 − 0.00000e+00i	5.60816e−01 − 0.00000e+00i	1.87312e−02 − 0.00000e+00i	2.38943e−06 − 0.00000e+00i	1.19454e−16 − 0.00000e+00i*
0,2	50∠15°	Pr	2.37675e+02 + 6.47207e+01i	−5.00000e−01 − 0.00000e+00i	−4.94070e−01 + 1.61118e−03i	−1.08587e−02 + 1.03434e−01i	5.94194e−01 − 5.75067e−02i	2.63752e−03 − 2.34297e−02i	−2.40054e−06 + 3.51632e−06i	−7.16882e−17 + 5.10863e−16i*
0,2	50∠30°	Pr	2.12706e+02 + 1.25030e+02i	−5.00000e−01 − 4.33681e−19i	−4.94689e−01 + 3.11395e−03i	−3.26924e−02 + 2.08783e−01i	7.05971e−01 − 1.21805e−01i	−4.36189e−02 − 1.54927e−02i	−1.72722e−06 − 2.30919e−05i	−1.91452e−14 − 3.23737e−14i*
0,2	50∠45°	Ob	6.97071e+01 + 2.42929e+03i	−5.00000e−01 + 3.05311e−16i	−4.97030e−01 + 6.06536e−02i	7.20199e+00 − 2.99311e+00i	1.14880e+03 + 7.60007e+02i	−3.09002e+06 − 7.28077e+06i	−8.54350e+09 + 4.59383e+10i	1.79282e+14 − 8.61445e+13i
0,2	50∠60°	Pr	2.42482e+01 + 4.33046e+01i	−5.00000e−01 − 0.00000e+00i	−4.99393e−01 + 1.08133e−03i	−4.32388e−01 + 9.53597e−02i	−4.50308e−02 + 2.27166e−01i	−1.73129e−02 − 9.04440e−03i	7.13189e−05 + 1.23243e−04i	−7.39423e−10 − 3.95625e−10i*
0,2	50∠75°	Ob	−1.88036e+03 + 1.17233e+03i	−5.00000e−01 + 4.85723e−17i	−5.47460e−01 + 3.02354e−02i	−6.29774e+00 + 2.22874e+01i	2.06948e+04 − 2.01154e+03i	−1.63572e+09 + 3.21222e+08i	1.00279e+14 − 3.03948e+13i	1.65272e+16 + 9.58889e+16i
0,2	50∠90°	Ob	−2.20508e+03 + 2.87787e−13i	−5.00000e−01 + 1.86102e−17i	−5.56149e−01 + 2.81624e−17i	−2.74614e+01 + 9.43731e−15i	−3.18180e+04 + 2.55625e−11i	−3.90360e+09 + 6.12709e−06i	−3.75309e+14 + 8.76948e−01i	3.04881e+16 + 3.04857e+01i
0,2	100∠0°	Pr	4.96221e+02 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.87649e−01 − 0.00000e+00i	3.02475e−01 − 0.00000e+00i	2.52739e−01 − 0.00000e+00i	4.73540e−05 − 0.00000e+00i	2.15106e−13 − 0.00000e+00i	5.30544e−35 − 0.00000e+00i*
0,2	100∠15°	Pr	4.79185e+02 + 1.29417e+02i	−5.00000e−01 − 0.00000e+00i	−4.88068e−01 + 3.20776e−03i	3.04914e−01 + 1.22487e−01i	2.42946e−01 − 1.44348e−01i	−7.46915e−05 + 4.31349e−06i	−4.09434e−13 − 5.45532e−13i	−9.89972e−34 − 2.07937e−35i*
0,2	100∠30°	Pr	4.29238e+02 + 2.50015e+02i	−5.00000e−01 − 2.16840e−19i	−4.89296e−01 + 6.20248e−03i	3.09612e−01 + 2.58081e−01i	2.05716e−01 − 3.33661e−01i	2.83928e−04 + 3.43670e−05i	−1.54945e−11 + 1.28071e−11i	7.22469e−32 + 5.27299e−30i*
0,2	100∠45°	Ob	1.40420e+02 + 9.85858e+03i	−5.00000e−01 + 1.04951e−15i	−4.76285e−01 + 2.45232e−01i	−1.88623e+02 + 1.89687e+02i	−3.69894e+06 − 8.75776e+06i	2.62695e+14 − 2.69953e+14i	1.42998e+22 + 5.58786e+21i	−1.99181e+29 + 2.46073e+29i
0,2	100∠60°	Pr	4.92491e+01 + 8.66042e+01i	−5.00000e−01 − 5.42101e−20i	−4.98766e−01 + 2.15983e−03i	−3.54394e−01 + 1.64298e−01i	9.67731e−02 + 3.99543e−02i	−3.91218e−04 − 5.63081e−04i	1.50202e−08 − 2.59918e−08i	−2.11730e−19 − 3.33255e−19i*
0,2	100∠75°	Ob	−8.08574e+03 + 4.84470e+03i	−5.00000e−01 + 3.88578e−16i	−7.10847e−01 + 1.38033e−01i	2.47349e+03 + 1.52590e+03i	−3.60053e+09 + 6.88094e+08i	−4.81719e+19 + 1.91143e+19i	−4.87166e+29 + 3.11057e+29i	−8.32206e+37 + 7.62952e+37i
0,2	100∠90°	Ob	−9.40504e+03 + 1.18790e−12i	−5.00000e−01 + 3.05494e−17i	−7.54150e−01 + 8.06554e−17i	−4.08585e+03 + 2.75213e−12i	−8.59049e+09 + 1.36784e−05i	−2.84621e+20 + 8.89002e+05i	−7.43729e+30 + 3.46208e+16i	−3.08167e+39 + 1.90635e+25i
0,2	250∠0°	Pr	1.24624e+03 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.69190e−01 − 0.00000e+00i	5.74813e−01 − 0.00000e+00i	5.69566e−03 − 0.00000e+00i	2.24939e−13 − 0.00000e+00i	4.76720e−35 − 0.00000e+00i	1.46922e−90 − 0.00000e+00i*
0,2	250∠15°	Pr	1.20365e+03 + 3.23527e+02i	−5.00000e−01 − 0.00000e+00i	−4.70208e−01 + 7.91476e−03i	6.06338e−01 + 1.41290e−03i	−1.57972e−03 − 7.30524e−03i	−3.68502e−13 − 6.00393e−13i	−8.01836e−34 − 2.90477e−34i	7.32983e−88 + 2.08674e−87i*
0,2	250∠30°	Pr	1.07877e+03 + 6.25006e+02i	−5.00000e−01 − 0.00000e+00i	−4.73203e−01 + 1.53245e−02i	7.08075e−01 + 1.33429e−02i	−1.59025e−02 + 4.68658e−03i	−1.74680e−11 + 9.77954e−12i	−2.32140e−30 + 3.27145e−30i	4.58060e−78 − 1.35934e−79i*
0,2	250∠45°	Ob	3.52553e+02 + 6.21464e+04i	−5.00000e−01 − 1.58207e−15i	2.92206e−01 + 1.37960e+00i	−4.19986e+06 − 9.96025e+06i	−3.05031e+18 + 6.57049e+17i	−3.69746e+37 + 1.66998e+37i	−4.36449e+56 + 3.22412e+56i	−7.70536e+74 − 1.01098e+75i
0,2	250∠60°	Pr	1.24250e+02 + 2.16507e+02i	−5.00000e−01 − 0.00000e+00i	−4.96874e−01 + 5.37918e−03i	−1.25175e−01 + 2.37220e−01i	−8.02910e−03 + 5.39705e−03i	2.20631e−08 − 1.98288e−08i	1.41442e−19 − 2.43488e−19i	3.39408e−47 − 2.38946e−47i*
0,2	250∠75°	Ob	−5.26827e+04 + 3.08618e+04i	−5.00000e−01 − 7.49401e−16i	−2.18385e+00 + 1.62234e+00i	−5.60685e+09 + 1.06402e+09i	1.76665e+25 − 8.94790e+24i	−8.95068e+50 + 1.21962e+51i	1.49036e+76 − 9.02660e+76i	−9.40714e+99 + 3.27242e+100i
0,2	250∠90°	Ob	−6.10050e+04 + 7.56219e−12i	−5.00000e−01 − 5.42850e−17i	−2.97661e+00 + 1.26035e−16i	−1.33765e+10 + 1.90254e−05i	−1.66568e+26 + 6.19399e+11i	−1.07027e+53 + 8.07599e+38i	−5.44361e+79 + 6.19106e+65i	−1.54884e+104 + 2.33285e+90i
0,2	500∠0°	Pr	2.49624e+03 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.38976e−01 − 0.00000e+00i	3.69899e−01 − 0.00000e+00i	4.13498e−06 − 0.00000e+00i	1.28560e−27 − 0.00000e+00i	1.66483e−71 − 0.00000e+00i*	1.62304e−183 − 0.00000e+00i*
0,2	500∠15°	Pr	2.41106e+03 + 6.47049e+02i	−5.00000e−01 − 1.35525e−20i	−4.40923e−01 + 1.54867e−02i	3.78949e−01 − 1.41736e−01i	−5.42915e−06 + 4.58624e−06i	−2.55603e−27 + 1.23410e−26i	4.83847e−69 + 2.22797e−69i*	−2.17915e−177 + 2.96340e−177i*
0,2	500∠30°	Pr	2.16131e+03 + 1.25000e+03i	−5.00000e−01 − 5.42101e−20i	−4.46664e−01 + 3.00524e−02i	4.10967e−01 − 3.28449e−01i	1.50532e−05 − 3.13503e−05i	1.94122e−25 − 1.01608e−23i	−8.95249e−62 − 7.65163e−62i*	1.31978e−158 − 8.67840e−159i*
0,2	500∠45°	Ob	7.06106e+02 + 2.49293e+05i	−5.00000e−01 − 2.78944e−15i	7.84955e+00 − 3.25724e+00i	3.59039e+14 − 3.68119e+14i	−4.43909e+37 + 2.00315e+37i	−6.53937e+75 + 7.41018e+75i	−6.08125e+113 + 1.97126e+114i	3.42645e+150 − 1.24110e+151i
0,2	500∠60°	Pr	2.49250e+02 + 4.33013e+02i	−5.00000e−01 + 1.35525e−20i	−4.93692e−01 + 1.06907e−02i	8.09027e−02 + 1.18329e−01i	−6.89953e−05 + 1.69195e−04i	−1.68484e−16 + 1.57286e−15i	5.81712e−38 + 1.02114e−37i*	−3.32987e−94 + 9.16504e−94i*
0,2	500∠75°	Ob	−2.13614e+05 + 1.24224e+05i	−5.00000e−01 − 3.80251e−15i	−8.29578e+00 + 2.92094e+01i	−1.63223e+20 + 6.41933e+19i	−1.94248e+51 + 2.64165e+51i	1.23651e+103 + 3.95911e+103i	3.93257e+155 + 1.35139e+155i	2.01274e+204 + 1.56494e+204i
0,2	500∠90°	Ob	−2.47005e+05 + 3.04325e−11i	−5.00000e−01 + 1.07890e−17i	−3.60084e+01 + 1.18004e−14i	−9.63638e+20 + 2.97111e+06i	−2.32005e+53 + 1.78080e+39i	−2.07649e+107 + 3.18322e+93i	−1.47292e+161 + 3.38535e+147i	−5.36709e+211 + 1.62829e+198i
0,2	750∠0°	Pr	3.74625e+03 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−4.09441e−01 − 0.00000e+00i	1.64402e−01 − 0.00000e+00i	2.22532e−09 − 0.00000e+00i	5.49279e−42 − 0.00000e+00i*	4.35387e−108 − 0.00000e+00i*	1.34547e−276 − 0.00000e+00i*
0,2	750∠15°	Pr	3.61847e+03 + 9.70572e+02i	−5.00000e−01 + 1.35525e−20i	−4.12229e−01 + 2.27246e−02i	1.44195e−01 − 1.19635e−01i	4.64677e−09 + 1.87761e−09i	1.38071e−40 − 9.67072e−41i	−2.16738e−104 − 1.22916e−104i*	−4.58785e−267 − 1.60518e−268i*
0,2	750∠30°	Pr	3.24384e+03 + 1.87500e+03i	−5.00000e−01 + 2.71051e−20i	−4.20477e−01 + 4.41963e−02i	6.81122e−02 − 2.66283e−01i	2.02401e−08 + 5.02472e−08i	3.28894e−36 + 2.01748e−36i	1.85423e−93 − 1.80788e−93i*	1.72835e−239 − 3.70164e−239i*
0,2	750∠45°	Ob	1.05966e+03 + 5.61439e+05i	−5.00000e−01 + 4.99600e−16i	−2.77187e+01 − 4.13146e+01i	2.27698e+22 + 8.93711e+21i	−6.11574e+56 + 4.51292e+56i	−7.09920e+113 + 2.30004e+114i	2.58757e+171 + 7.40203e+171i	1.21031e+227 − 4.84884e+226i
0,2	750∠60°	Pr	3.74250e+02 + 6.49519e+02i	−5.00000e−01 + 6.77626e−21i	−4.90472e−01 + 1.59345e−02i	7.61065e−02 − 8.72654e−03i	7.01142e−07 + 3.37913e−06i	−4.93753e−23 − 6.83943e−23i	−4.90486e−56 − 4.31693e−58i*	−5.92365e−141 − 2.21417e−140i*
0,2	750∠75°	Ob	−4.82798e+05 + 2.80085e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	750∠90°	Ob	−5.58005e+05 + 6.86108e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	1000∠0°	Pr	4.99625e+03 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−3.80572e−01 − 0.00000e+00i	6.37358e−02 − 0.00000e+00i	1.06147e−12 − 0.00000e+00i	2.08438e−56 − 0.00000e+00i	1.01172e−144 − 0.00000e+00i*	±0
0,2	1000∠15°	Pr	4.82588e+03 + 1.29410e+03i	−5.00000e−01 − 0.00000e+00i	−3.84117e−01 + 2.96371e−02i	3.95287e−02 − 6.46195e−02i	−3.14594e−13 − 3.11701e−12i	−1.98167e−54 − 2.88906e−55i	8.54521e−140 + 5.85053e−140i*	±0
0,2	1000∠30°	Pr	4.32637e+03 + 2.50000e+03i	−5.00000e−01 − 0.00000e+00i	−3.94640e−01 + 5.77686e−02i	−4.72577e−02 − 1.16458e−01i	−7.00547e−11 − 2.62691e−11i	−1.14870e−48 + 6.11290e−49i	3.18857e−125 + 3.92930e−125i*	−1.08299e−320 − 9.33043e−320i*
0,2	1000∠45°	Ob	1.41321e+03 + 9.98586e+05i	−5.00000e−01 + 2.38698e−15i	−2.05563e+02 + 2.06588e+02i	2.89414e+28 + 1.16321e+30i	−7.85001e+75 + 8.89238e+75i	7.27040e+151 + 5.81726e+152i	2.46160e+229 + 1.68017e+229i	1.13289e+303 + 6.77966e+302i
0,2	1000∠60°	Pr	4.99250e+02 + 8.66026e+02i	−5.00000e−01 − 0.00000e+00i	−4.87216e−01 + 2.11108e−02i	1.48571e−02 − 3.81485e−02i	4.65937e−08 + 4.55814e−08i	4.39622e−30 + 9.53117e−31i	1.04448e−74 − 1.76080e−74i*	4.12384e−187 + 3.46803e−187i*
0,2	1000∠75°	Ob	−8.60235e+05 + 4.98447e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	1000∠90°	Ob	−9.94005e+05 + 1.22097e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	1500∠0°	Pr	7.49625e+03 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−3.24784e−01 − 0.00000e+00i	7.93674e−03 − 0.00000e+00i	2.03191e−19 − 0.00000e+00i	2.53049e−85 − 0.00000e+00i	4.60762e−218 − 0.00000e+00i*	±0
0,2	1500∠15°	Pr	7.24069e+03 + 1.94114e+03i	−5.00000e−01 + 6.77626e−21i	−3.29607e−01 + 4.25188e−02i	−1.07011e−03 − 1.02110e−02i	9.03058e−19 + 4.95418e−19i	2.04039e−83 − 2.37416e−82i	1.08392e−210 + 1.04975e−210i*	±0
0,2	1500∠30°	Pr	6.49144e+03 + 3.75000e+03i	−5.00000e−01 − 1.35525e−20i	−3.44019e−01 + 8.32460e−02i	−2.17733e−02 + 1.67755e−03i	−3.23609e−17 + 1.15684e−16i	1.04755e−73 + 6.78003e−74i	−7.80488e−189 − 1.43058e−188i*	±0
0,2	1500∠45°	Ob	2.12032e+03 + 2.24788e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	1500∠60°	Pr	7.49250e+02 + 1.29904e+03i	−5.00000e−01 − 3.38813e−21i	−4.80599e−01 + 3.12606e−02i	−1.13891e−02 + 2.64658e−03i	2.13326e−11 − 9.25070e−12i	4.02727e−45 − 1.21417e−44i	−3.56599e−111 − 6.33280e−113i*	2.58260e−280 − 1.48327e−280i*
0,2	1500∠75°	Ob	−1.93987e+06 + 1.12267e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	1500∠90°	Ob	−2.24101e+06 + 2.74994e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	2500∠0°	Pr	1.24962e+04 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−2.20687e−01 − 0.00000e+00i	8.92427e−05 − 0.00000e+00i	5.49646e−33 − 0.00000e+00i	2.75998e−143 − 0.00000e+00i*	±0	±0
0,2	2500∠15°	Pr	1.20703e+04 + 3.23524e+03i	−5.00000e−01 − 0.00000e+00i	−2.27204e−01 + 6.47337e−02i	−1.35009e−04 − 2.25304e−05i	1.11924e−32 − 8.14392e−32i	8.01212e−140 + 2.49553e−138i*	±0	±0
0,2	2500∠30°	Pr	1.08216e+04 + 6.25000e+03i	−5.00000e−01 − 0.00000e+00i	−2.46970e−01 + 1.27860e−01i	4.08574e−04 + 2.51264e−04i	2.34022e−29 − 2.27505e−28i	−7.68476e−124 + 3.61551e−124i*	−1.68032e−316 − 1.24013e−315i*	±0
0,2	2500∠45°	Ob	3.53453e+03 + 6.24646e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.52: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
0,2	2500∠60°	Pr	1.24925e+03 + 2.16506e+03i	−5.00000e−01 − 3.38813e−21i	−4.66974e−01 + 5.07511e−02i	1.35979e−04 − 9.44031e−04i	−2.76371e−18 − 1.05688e−18i	−5.26361e−74 + 8.90576e−74i*	5.68771e−185 − 9.20960e−185i*	±0
0,2	2500∠75°	Ob	−5.39817e+06 + 3.12112e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	2500∠90°	Ob	−6.23501e+06 + 7.64486e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	5000∠0°	Pr	2.49962e+04 − 0.00000e+00i	−5.00000e−01 − 0.00000e+00i	−3.89520e−05 − 0.00000e+00i	6.51231e−10 − 0.00000e+00i	3.67457e−67 − 0.00000e+00i	1.91019e−288 − 0.00000e+00i*	±0	±0
0,2	5000∠15°	Pr	2.41444e+04 + 6.47048e+03i	−5.00000e−01 + 1.69407e−21i	−6.82117e−03 + 1.02310e−01i	1.52745e−09 + 9.31227e−11i	−8.21744e−65 − 8.24430e−67i	−1.48069e−278 + 5.01166e−279i*	±0	±0
0,2	5000∠30°	Pr	2.16469e+04 + 1.25000e+04i	−5.00000e−01 − 0.00000e+00i	−2.84665e−02 + 2.06509e−01i	1.58349e−08 + 1.00085e−08i	−6.03259e−58 + 2.00680e−58i	3.00533e−250 − 1.78320e−249i*	±0	±0
0,2	5000∠45°	Ob	7.07007e+03 + 2.49929e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	5000∠60°	Pr	2.49925e+03 + 4.33013e+03i	−5.00000e−01 − 0.00000e+00i	−4.30961e−01 + 9.47941e−02i	1.73885e−06 + 5.11544e−07i	−1.27311e−35 − 1.14051e−35i	9.27751e−147 + 1.68548e−146i*	±0	±0
0,2	5000∠75°	Ob	−2.16217e+07 + 1.24922e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	5000∠90°	Ob	−2.49700e+07 + 3.05978e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	25∠0°	Pr	1.67953e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.49585e−02 − 0.00000e+00i	−1.12038e−01 − 0.00000e+00i	3.47462e−03 − 0.00000e+00i	1.11615e−01 − 0.00000e+00i	4.64438e−03 − 0.00000e+00i	2.19511e−07 − 0.00000e+00i*
0,3	25∠15°	Pr	1.62005e+02 + 4.53798e+01i	−0.00000e+00 − 0.00000e+00i	−1.49600e−02 + 1.13250e−05i	−1.13029e−01 + 9.48754e−03i	7.34850e−03 + 5.01960e−02i	1.10603e−01 − 6.27147e−02i	−1.98752e−03 − 5.90322e−03i	2.53596e−07 + 3.77745e−07i*
0,3	25∠30°	Ob	3.36495e+02 + 4.97956e+02i	−0.00000e+00 − 0.00000e+00i	−1.49162e−02 + 1.24075e−04i	−5.50772e−02 + 8.46040e−02i	2.07506e−01 − 5.15854e−01i	9.98146e+00 − 3.34596e+00i	1.75421e+02 + 1.06435e+02i	−2.18449e+02 + 3.61525e+03i
0,3	25∠45°	Pr	5.12567e+01 + 5.30623e+01i	−0.00000e+00 − 0.00000e+00i	−1.49877e−02 + 1.32550e−05i	−1.37608e−01 + 1.22324e−02i	−1.88523e−01 + 1.18630e−01i	6.00872e−02 + 5.84726e−02i	−4.41745e−03 − 1.33625e−03i	2.09584e−06 + 1.11875e−06i*
0,3	25∠60°	Ob	−1.87732e+02 + 4.66179e+02i	−0.00000e+00 − 0.00000e+00i	−1.50472e−02 + 1.16767e−04i	−1.71253e−01 + 1.37049e−01i	2.91566e+00 + 1.98593e+00i	−2.95655e+02 − 2.02530e+02i	2.35010e+04 + 1.57063e+04i	−3.24521e+04 + 2.17484e+05i
0,3	25∠75°	Ob	−4.01534e+02 + 2.73631e+02i	−0.00000e+00 − 0.00000e+00i	−1.51010e−02 + 6.86852e−05i	−2.61049e−01 + 9.96910e−02i	−1.62479e+00 + 6.48008e+00i	1.23971e+03 + 2.40312e+02i	2.39075e+04 − 1.83165e+05i	1.78437e+06 + 1.44001e+06i
0,3	25∠90°	Ob	−4.80164e+02 + 6.73446e−14i	−0.00000e+00 − 0.00000e+00i	−1.51208e−02 − 1.36812e−18i	−3.03337e−01 − 1.26046e−18i	−8.30720e+00 + 1.84882e−15i	−1.94074e+03 + 1.17804e−12i	−3.50436e+05 + 3.48070e−10i	5.04343e+06 − 6.21657e−09i
0,3	50∠0°	Pr	3.43111e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.49149e−02 − 0.00000e+00i	−7.91325e−02 − 0.00000e+00i	8.75181e−02 − 0.00000e+00i	9.20131e−03 − 0.00000e+00i	2.08477e−06 − 0.00000e+00i	2.05582e−16 − 0.00000e+00i*
0,3	50∠15°	Pr	3.31191e+02 + 9.06247e+01i	−0.00000e+00 − 0.00000e+00i	−1.49178e−02 + 2.25735e−05i	−8.03254e−02 + 1.55209e−02i	9.77819e−02 + 8.57807e−03i	1.58533e−03 − 1.15190e−02i	−2.12568e−06 + 3.05254e−06i	−1.25015e−16 + 8.79080e−16i*
0,3	50∠30°	Ob	1.29900e+03 + 2.07846e+03i	−0.00000e+00 − 0.00000e+00i	−1.46725e−02 + 5.12889e−04i	1.67261e−01 − 2.18920e−02i	5.91755e+00 − 2.07639e+00i	4.53417e+02 + 2.66713e+03i	−1.20067e+06 − 1.22522e+03i	2.41729e+08 − 3.50559e+08i
0,3	50∠45°	Pr	1.04303e+02 + 1.06080e+02i	−0.00000e+00 − 0.00000e+00i	−1.49744e−02 + 2.64755e−05i	−1.24452e−01 + 2.22924e−02i	−5.50269e−02 + 1.14508e−01i	−1.96253e−04 − 7.84300e−03i	−9.60302e−06 − 6.48957e−06i	−3.09555e−13 − 5.17426e−13i*
0,3	50∠60°	Ob	−9.95259e+02 + 2.01502e+03i	−0.00000e+00 − 0.00000e+00i	−1.52455e−02 + 5.08782e−04i	3.26220e−01 + 8.28055e−01i	−3.15518e+02 − 2.08167e+02i	−7.13579e+06 − 4.98672e+06i	−1.27179e+11 − 9.27729e+10i	−8.00652e+13 + 3.34811e+13i
0,3	50∠75°	Ob	−1.88036e+03 + 1.17233e+03i	−0.00000e+00 − 0.00000e+00i	−1.54733e−02 + 2.98638e−04i	−7.77010e−01 + 1.25563e+00i	1.30349e+03 + 2.39864e+02i	−1.05875e+08 − 9.01736e+06i	6.67850e+12 − 1.03019e+11i	−6.69766e+14 + 6.16615e+15i
0,3	50∠90°	Ob	−2.20508e+03 + 2.87787e−13i	−0.00000e+00 − 0.00000e+00i	−1.55579e−02 + 6.81900e−19i	−1.75455e+00 + 4.91893e−16i	−2.03324e+03 + 1.50458e−12i	−2.49448e+08 + 3.75717e−07i	−2.39830e+13 + 5.45181e−02i	1.94825e+15 + 2.07163e+00i
0,3	100∠0°	Pr	6.93182e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.48279e−02 − 0.00000e+00i	−3.09463e−02 − 0.00000e+00i	5.29804e−02 − 0.00000e+00i	2.43482e−05 − 0.00000e+00i	1.90996e−13 − 0.00000e+00i	9.16656e−35 − 0.00000e+00i*
0,3	100∠15°	Pr	6.69333e+02 + 1.81191e+02i	−0.00000e+00 − 0.00000e+00i	−1.48337e−02 + 4.49613e−05i	−3.12330e−02 + 2.03441e−02i	5.32028e−02 − 2.75082e−02i	−3.84892e−05 + 1.79076e−06i	−3.61624e−13 − 4.86284e−13i	−1.71049e−33 − 3.77781e−35i*
0,3	100∠30°	Ob	5.09900e+03 + 8.48705e+03i	−0.00000e+00 − 0.00000e+00i	−1.36710e−02 + 2.01384e−03i	−2.50872e−01 + 9.85260e−01i	2.81918e+02 + 1.59245e+03i	5.92761e+07 − 3.56621e+08i	−4.02047e+13 + 7.27609e+13i	1.17792e+19 + 1.66292e+18i
0,3	100∠45°	Pr	2.10375e+02 + 2.12139e+02i	−0.00000e+00 − 0.00000e+00i	−1.49479e−02 + 5.28520e−05i	−9.93155e−02 + 3.67550e−02i	2.58202e−02 + 3.22822e−02i	6.86791e−05 − 3.36744e−06i	−2.71267e−11 − 6.81068e−11i	1.91874e−26 − 3.36998e−26i*
0,3	100∠60°	Ob	−4.48542e+03 + 8.36023e+03i	−0.00000e+00 − 0.00000e+00i	−1.60571e−02 + 2.18362e−03i	8.16937e+00 − 3.19441e+01i	−7.65899e+06 − 5.26951e+06i	−8.51066e+15 − 6.68600e+15i	−7.43720e+24 − 6.60334e+24i	−1.31091e+32 − 1.58786e+32i
0,3	100∠75°	Ob	−8.08574e+03 + 4.84470e+03i	−0.00000e+00 − 0.00000e+00i	−1.70743e−02 + 1.31168e−03i	6.10925e+01 + 6.58272e+01i	−1.12902e+08 − 9.17640e+06i	−1.59218e+18 + 1.72318e+17i	−1.70721e+28 + 5.24962e+27i	−3.10720e+36 + 1.58649e+36i
0,3	100∠90°	Ob	−9.40504e+03 + 1.18790e−12i	−0.00000e+00 − 0.00000e+00i	−1.74649e−02 + 1.40955e−18i	−1.26400e+02 + 7.72766e−14i	−2.65756e+08 + 4.06623e−07i	−8.80504e+18 + 2.69545e+04i	−2.30080e+29 + 1.05672e+15i	−9.53348e+37 + 5.83818e+23i
0,3	250∠0°	Pr	1.74322e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.45689e−02 − 0.00000e+00i	2.87401e−02 − 0.00000e+00i	1.35108e−03 − 0.00000e+00i	1.18621e−13 − 0.00000e+00i	4.27582e−35 − 0.00000e+00i	2.54469e−90 − 0.00000e+00i*
0,3	250∠15°	Pr	1.68359e+03 + 4.52940e+02i	−0.00000e+00 − 0.00000e+00i	−1.45832e−02 + 1.11119e−04i	3.22122e−02 + 9.74877e−03i	−3.42885e−04 − 1.74463e−03i	−1.93098e−13 − 3.17621e−13i	−7.18905e−34 − 2.61829e−34i	1.26783e−87 + 3.61508e−87i*
0,3	250∠30°	Ob	3.14990e+04 + 5.36936e+04i	−0.00000e+00 − 0.00000e+00i	−5.44562e−03 + 9.26025e−03i	1.31100e+02 + 7.21406e+02i	8.48168e+10 + 2.91290e+10i	−2.57897e+24 + 1.05687e+24i	3.97297e+37 − 7.97503e+37i	−7.90214e+50 − 2.76706e+50i
0,3	250∠45°	Pr	5.28577e+02 + 5.30333e+02i	−0.00000e+00 − 0.00000e+00i	−1.48683e−02 + 1.31426e−04i	−3.93552e−02 + 4.82032e−02i	−1.11663e−03 − 8.84220e−04i	−5.62795e−12 − 4.59878e−11i	1.83582e−26 − 2.96346e−27i	−5.25903e−66 + 9.08035e−66i*
0,3	250∠60°	Ob	−2.99560e+04 + 5.33766e+04i	−0.00000e+00 − 0.00000e+00i	−1.93434e−02 + 1.72630e−02i	−4.71460e+06 − 3.19496e+06i	−3.58512e+20 − 2.98598e+20i	−4.52263e+43 − 5.24458e+43i	−4.24124e+66 − 6.94709e+66i	5.30495e+87 − 5.68813e+87i
0,3	250∠75°	Ob	−5.26827e+04 + 3.08618e+04i	−0.00000e+00 − 0.00000e+00i	−3.04048e−02 + 1.25085e−02i	−6.90806e+07 − 5.35897e+06i	2.35455e+23 − 4.86768e+22i	−1.43655e+49 + 1.14449e+49i	4.61625e+74 − 1.01030e+75i	−2.14271e+98 + 3.53533e+98i
0,3	250∠90°	Ob	−6.10050e+04 + 7.56219e−12i	−0.00000e+00 − 0.00000e+00i	−3.56409e−02 − 5.51744e−19i	−1.62474e+08 + 2.20980e−07i	−2.02318e+24 + 7.39752e+09i	−1.29997e+51 + 9.72843e+36i	−6.61194e+77 + 7.47869e+63i	−1.88125e+102 + 2.82184e+88i
0,3	500∠0°	Pr	3.49324e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.41441e−02 − 0.00000e+00i	2.88330e−02 − 0.00000e+00i	1.01625e−06 − 0.00000e+00i	6.83484e−28 − 0.00000e+00i	1.49815e−71 − 0.00000e+00i*	2.81368e−183 − 0.00000e+00i*
0,3	500∠15°	Pr	3.37398e+03 + 9.05870e+02i	−0.00000e+00 − 0.00000e+00i	−1.41716e−02 + 2.18040e−04i	3.08299e−02 − 8.78095e−03i	−1.34589e−06 + 1.11671e−06i	−1.37293e−27 + 6.56004e−27i	4.35282e−69 + 2.00885e−69i*	−3.77916e−177 + 5.13655e−177i*
0,3	500∠30°	Ob	1.25499e+05 + 2.15640e+05i	−0.00000e+00 − 0.00000e+00i	1.80116e−02 − 2.61808e−03i	1.55693e+07 − 9.70410e+07i	−1.54535e+24 + 6.35706e+23i	4.95412e+50 + 1.87277e+51i	2.29140e+78 − 2.92402e+77i	−2.15647e+104 + 8.55010e+103i
0,3	500∠45°	Pr	1.05891e+03 + 1.06066e+03i	−0.00000e+00 − 0.00000e+00i	−1.47358e−02 + 2.60528e−04i	5.13225e−03 + 2.51355e−02i	−1.19099e−06 − 5.05849e−06i	2.32918e−21 − 5.78263e−22i	−1.79596e−52 + 5.94130e−53i*	5.89906e−132 + 1.03908e−131i*
0,3	500∠60°	Ob	−1.22407e+05 + 2.15006e+05i	−0.00000e+00 − 0.00000e+00i	4.35270e−02 + 1.02258e−01i	−5.66307e+15 − 4.38720e+15i	−4.88111e+43 − 5.64571e+43i	−1.32690e+90 − 3.30635e+90i	−6.87075e+135 − 1.34631e+137i	2.09046e+180 + 4.76162e+180i
0,3	500∠75°	Ob	−2.13614e+05 + 1.24224e+05i	−0.00000e+00 − 0.00000e+00i	−9.42298e−02 + 1.57192e−01i	−1.05194e+18 + 1.17609e+17i	−1.54690e+49 + 1.23408e+49i	9.84854e+99 + 2.50122e+101i	2.07916e+153 + 1.40530e+153i	9.26939e+201 + 1.22811e+202i
0,3	500∠90°	Ob	−2.47005e+05 + 3.04325e−11i	−0.00000e+00 − 0.00000e+00i	−2.17336e−01 + 5.77646e−17i	−5.81679e+18 + 1.75739e+04i	−1.40044e+51 + 1.06626e+37i	−1.25343e+105 + 1.91371e+91i	−8.89093e+158 + 2.03798e+145i	−3.23972e+209 + 9.80874e+195i
0,3	750∠0°	Pr	5.24324e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.37275e−02 − 0.00000e+00i	1.41254e−02 − 0.00000e+00i	5.53115e−10 − 0.00000e+00i	2.92804e−42 − 0.00000e+00i	3.92196e−108 − 0.00000e+00i*	2.33305e−276 − 0.00000e+00i*
0,3	750∠15°	Pr	5.06435e+03 + 1.35880e+03i	−0.00000e+00 − 0.00000e+00i	−1.37673e−02 + 3.20868e−04i	1.29485e−02 − 9.77982e−03i	1.15319e−09 + 4.73711e−10i	7.36860e−41 − 5.14596e−41i	−1.95202e−104 − 1.10850e−104i*	−7.95606e−267 − 2.79696e−268i*
0,3	750∠30°	Ob	2.81999e+05 + 4.85840e+05i	−0.00000e+00 − 0.00000e+00i	−2.82261e−02 − 3.12446e−02i	−8.41178e+12 + 1.54327e+13i	1.84597e+37 − 3.72121e+37i	1.78170e+78 − 2.28084e+77i	5.23868e+118 + 6.03209e+118i	−3.95861e+157 + 7.60561e+157i
0,3	750∠45°	Pr	1.58924e+03 + 1.59099e+03i	−0.00000e+00 − 0.00000e+00i	−1.46032e−02 + 3.87328e−04i	9.36492e−03 + 4.91457e−03i	8.05168e−09 − 1.71669e−08i	4.43712e−32 + 1.16129e−31i	1.70794e−78 − 8.87125e−79i*	1.36179e−197 + 1.65107e−199i*
0,3	750∠60°	Ob	−2.77358e+05 + 4.84889e+05i	−0.00000e+00 − 0.00000e+00i	6.42423e−01 − 5.33253e−02i	−9.02524e+24 − 7.98889e+24i	−8.31322e+66 − 1.36268e+67i	−1.23397e+136 − 2.44583e+137i	1.28663e+207 − 2.68699e+207i	−4.35617e+273 + 2.27661e+272i
0,3	750∠75°	Ob	−4.82798e+05 + 2.80085e+05i	−0.00000e+00 − 0.00000e+00i	1.46390e−01 + 1.35634e+00i	−2.06776e+28 + 6.37628e+27i	9.05688e+74 − 1.98129e+75i	3.77675e+153 + 2.55435e+153i	−5.21443e+231 + 5.48222e+231i	6.79283e+305 − 2.04037e+305i
0,3	750∠90°	Ob	−5.58005e+05 + 6.86108e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1000∠0°	Pr	6.99324e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.33192e−02 − 0.00000e+00i	5.71666e−03 − 0.00000e+00i	2.65300e−13 − 0.00000e+00i	1.11259e−56 − 0.00000e+00i	9.11888e−145 − 0.00000e+00i*	±0
0,3	1000∠15°	Pr	6.75472e+03 + 1.81174e+03i	−0.00000e+00 − 0.00000e+00i	−1.33700e−02 + 4.19700e−04i	3.74722e−03 − 5.70958e−03i	−7.53424e−14 − 7.79838e−13i	−1.05776e−54 − 1.55323e−55i	7.70013e−140 + 5.27698e−140i*	±0
0,3	1000∠30°	Ob	5.00999e+05 + 8.64293e+05i	−0.00000e+00 − 0.00000e+00i	−2.64060e−02 + 1.07054e−01i	2.63189e+18 − 2.36251e+18i	2.97881e+50 + 1.12303e+51i	7.88555e+102 − 1.87399e+105i	−8.22407e+158 + 2.99828e+159i	8.24352e+210 + 3.46906e+211i
0,3	1000∠45°	Pr	2.11957e+03 + 2.12132e+03i	−0.00000e+00 − 0.00000e+00i	−1.44708e−02 + 5.11846e−04i	4.01111e−03 − 1.70925e−03i	6.19164e−11 − 3.08650e−11i	−5.69236e−42 + 3.01141e−42i	−1.57217e−104 + 1.16729e−104i*	7.98735e−264 − 1.33146e−263i*
0,3	1000∠60°	Ob	−4.94809e+05 + 8.63025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1000∠75°	Ob	−8.60235e+05 + 4.98447e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1000∠90°	Ob	−9.94005e+05 + 1.22097e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1500∠0°	Pr	1.04932e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.25264e−02 − 0.00000e+00i	7.40776e−04 − 0.00000e+00i	5.10641e−20 − 0.00000e+00i	1.35251e−85 − 0.00000e+00i	4.15532e−218 − 0.00000e+00i*	±0
0,3	1500∠15°	Pr	1.01355e+04 + 2.71760e+03i	−0.00000e+00 − 0.00000e+00i	−1.25966e−02 + 6.05761e−04i	−8.12887e−05 − 9.57680e−04i	2.26682e−19 + 1.25190e−19i	1.09935e−83 − 1.26900e−82i	9.77284e−211 + 9.47021e−211i*	±0
0,3	1500∠30°	Ob	1.12650e+06 + 1.94596e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1500∠45°	Pr	3.18023e+03 + 3.18198e+03i	−0.00000e+00 − 0.00000e+00i	−1.42061e−02 + 7.54118e−04i	−4.21005e−04 − 6.09821e−04i	5.88853e−16 + 7.08177e−16i	1.28788e−62 − 1.15225e−62i	−1.16601e−156 + 1.65811e−156i*	±0
0,3	1500∠60°	Ob	−1.11721e+06 + 1.94406e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1500∠75°	Ob	−1.93987e+06 + 1.12267e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	1500∠90°	Ob	−2.24101e+06 + 2.74994e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	2500∠0°	Pr	1.74932e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.10329e−02 − 0.00000e+00i	8.58145e−06 − 0.00000e+00i	1.38732e−33 − 0.00000e+00i	1.47673e−143 − 0.00000e+00i*	±0	±0
0,3	2500∠15°	Pr	1.68969e+04 + 4.52933e+03i	−0.00000e+00 − 0.00000e+00i	−1.11311e−02 + 9.34069e−04i	−1.29779e−05 − 2.31557e−06i	2.86010e−33 − 2.05553e−32i	4.23205e−140 + 1.33532e−138i*	±0	±0
0,3	2500∠30°	Ob	3.12750e+06 + 5.40833e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	2500∠45°	Pr	5.30155e+03 + 5.30330e+03i	−0.00000e+00 − 0.00000e+00i	−1.36783e−02 + 1.21219e−03i	1.80929e−05 + 1.14377e−05i	−1.49393e−25 − 6.57679e−26i	4.31806e−104 − 1.16708e−103i*	5.57336e−262 + 2.17160e−260i*	±0

Table D.52: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
0,3	$2500\angle 60^\circ$	Ob	$-3.11201_{e+06}+5.40516_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$2500\angle 75^\circ$	Ob	$-5.39817_{e+06}+3.12112_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$2500\angle 90^\circ$	Ob	$-6.23501_{e+06}+7.64486_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$5000\angle 0^\circ$	Pr	$3.49932_{e+04}-0.00000_{e+00}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-7.78923_{e-03}-0.00000_{e+00}i$	$6.39677_{e-11}-0.00000_{e+00}i$	$9.30474_{e-68}-0.00000_{e+00}i$	$1.02286_{e-288}-0.00000_{e+00}i^{*}$	± 0	± 0
0,3	$5000\angle 15^\circ$	Pr	$3.38007_{e+04}+9.05867_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-7.90724_{e-03}+1.53107_{e-03}i$	$1.50094_{e-10}+9.96200_{e-12}i$	$-2.08103_{e-65}-2.26118_{e-67}i$	$-7.92947_{e-279}+2.68206_{e-279}i^{*}$	± 0	± 0
0,3	$5000\angle 30^\circ$	Ob	$1.25050_{e+07}+2.16420_{e+07}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$5000\angle 45^\circ$	Pr	$1.06049_{e+04}+1.06066_{e+04}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-1.23743_{e-02}+2.21062_{e-03}i$	$-1.30202_{e-09}-2.74195_{e-09}i$	$-4.60924_{e-50}-5.03395_{e-50}i$	$1.31459_{e-206}+1.12709_{e-206}i^{*}$	± 0	± 0
0,3	$5000\angle 60^\circ$	Ob	$-1.24740_{e+07}+2.16356_{e+07}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$5000\angle 75^\circ$	Ob	$-2.16217_{e+07}+1.24922_{e+07}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,3	$5000\angle 90^\circ$	Ob	$-2.49700_{e+07}+3.05978_{e-09}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$25\angle 0^\circ$	Pr	$2.13649_{e+02}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.71001_{e-01}-0.00000_{e+00}i$	$4.04840_{e-02}-0.00000_{e+00}i$	$-3.59501_{e-01}-0.00000_{e+00}i$	$5.03353_{e-01}-0.00000_{e+00}i$	$4.46423_{e-02}-0.00000_{e+00}i$	$4.40392_{e-06}-0.00000_{e+00}i^{*}$
0,4	$25\angle 15^\circ$	Pr	$2.06014_{e+02}+5.84142_{e+01}i$	$3.75000_{e-01}-8.67362_{e-19}i$	$3.71143_{e-01}-1.09147_{e-03}i$	$4.59575_{e-02}-7.54101_{e-02}i$	$-4.03797_{e-01}+7.56639_{e-02}i$	$5.92103_{e-01}-6.59095_{e-02}i$	$-9.15683_{e-04}-6.03768_{e-02}i$	$2.77663_{e-06}+8.72630_{e-06}i^{*}$
0,4	$25\angle 30^\circ$	Ob	$3.36495_{e+02}+4.97956_{e+02}i$	$3.75000_{e-01}+3.33067_{e-16}i$	$3.68670_{e-01}-9.28462_{e-03}i$	$-3.67215_{e-01}-4.42539_{e-01}i$	$2.21273_{e+00}+2.58926_{e+00}i$	$-1.05481_{e+01}+6.36830_{e+01}i$	$-1.08052_{e+03}+6.44552_{e+02}i$	$-1.89573_{e+04}-1.15690_{e+04}i$
0,4	$25\angle 45^\circ$	Pr	$8.45599_{e+01}+8.84789_{e+01}i$	$3.75000_{e-01}+8.67362_{e-19}i$	$3.73414_{e-01}-1.65653_{e-03}i$	$2.15658_{e-01}-1.42209_{e-01}i$	$-4.86213_{e-01}-2.85620_{e-01}i$	$-3.25609_{e-02}+5.23021_{e-01}i$	$-2.19891_{e-02}-4.51573_{e-02}i$	$1.37698_{e-05}+4.84873_{e-05}i^{*}$
0,4	$25\angle 60^\circ$	Ob	$-1.87732_{e+02}+4.66179_{e+02}i$	$3.75000_{e-01}+5.55112_{e-17}i$	$3.78492_{e-01}-8.76844_{e-03}i$	$3.82380_{e-01}-1.11756_{e+00}i$	$-1.97683_{e+01}-5.73437_{e-02}i$	$2.00778_{e+03}+1.11566_{e+01}i$	$-1.58362_{e+05}+9.33870_{e+02}i$	$-5.32447_{e+05}-1.11097_{e+06}i$
0,4	$25\angle 75^\circ$	Pr	$5.71888_{e+00}+2.41556_{e+01}i$	$3.75000_{e-01}-2.16840_{e-19}i$	$3.74893_{e-01}-4.52798_{e-04}i$	$3.61738_{e-01}-4.40150_{e-02}i$	$2.25224_{e-01}-2.17106_{e-01}i$	$-1.74579_{e-01}+1.65938_{e-02}i$	$-1.80743_{e-02}-5.36922_{e-02}i^{*}$	$-1.14247_{e-03}-5.48500_{e-03}i^{*}$
0,4	$25\angle 90^\circ$	Ob	$-3.88753_{e+02}+6.11785_{e-14}i$	$3.75000_{e-01}+1.65893_{e-17}i$	$3.82313_{e-01}+1.57582_{e-17}i$	$1.37483_{e+00}-1.44562_{e-16}i$	$2.59579_{e+01}-8.92851_{e-15}i$	$3.15498_{e+03}-2.31396_{e-12}i$	$2.04323_{e+05}-2.31853_{e-10}i$	$1.92979_{e+05}-2.04458_{e-10}i$
0,4	$50\angle 0^\circ$	Pr	$4.38973_{e+02}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.66800_{e-01}-0.00000_{e+00}i$	$-1.88167_{e-01}-0.00000_{e+00}i$	$1.15213_{e-01}-0.00000_{e+00}i$	$1.04888_{e-01}-0.00000_{e+00}i$	$4.36899_{e-05}-0.00000_{e+00}i$	$8.59043_{e-15}-0.00000_{e+00}i^{*}$
0,4	$50\angle 15^\circ$	Pr	$4.23651_{e+02}+1.16545_{e+02}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.67083_{e-01}-2.16955_{e-03}i$	$-1.89148_{e-01}-9.35679_{e-02}i$	$1.17779_{e-01}+2.54732_{e-01}i$	$5.77806_{e-02}-1.20568_{e-01}i$	$-6.07921_{e-05}+4.91823_{e-05}i$	$-1.49202_{e-14}+3.40293_{e-14}i^{*}$
0,4	$50\angle 30^\circ$	Ob	$1.29900_{e+03}+2.07846_{e+03}i$	$3.75000_{e-01}+1.11022_{e-16}i$	$3.50239_{e-01}-3.81294_{e-02}i$	$-7.79059_{e-01}+1.91059_{e+00}i$	$-1.30161_{e+01}+7.65275_{e+01}i$	$-3.15985_{e+04}-1.10902_{e+04}i$	$7.21583_{e+06}-1.29929_{e+07}i$	$2.34561_{e+09}+4.72027_{e+09}i$
0,4	$50\angle 45^\circ$	Pr	$1.72987_{e+02}+1.76819_{e+02}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.71756_{e-01}-3.30519_{e-03}i$	$5.50059_{e-02}-2.37546_{e-01}i$	$-7.02056_{e-01}+1.43414_{e-01}i$	$7.25887_{e-02}-7.09647_{e-02}i$	$-4.40662_{e-05}-2.53077_{e-04}i$	$6.57122_{e-12}-2.50777_{e-11}i^{*}$
0,4	$50\angle 60^\circ$	Ob	$-9.95259_{e+02}+2.01502_{e+03}i$	$3.75000_{e-01}+1.11022_{e-16}i$	$3.93176_{e-01}-3.84085_{e-02}i$	$-8.46962_{e+00}-6.28850_{e+00}i$	$4.47791_{e+03}+1.20848_{e+02}i$	$1.03017_{e+08}+5.53896_{e+06}i$	$1.86042_{e+12}+1.37931_{e+11}i$	$5.96299_{e+14}-8.37921_{e+14}i$
0,4	$50\angle 75^\circ$	Pr	$1.21901_{e+01}+4.83000_{e+01}i$	$3.75000_{e-01}-1.08420_{e-19}i$	$3.74770_{e-01}-9.05094_{e-04}i$	$3.42486_{e-01}-8.45772_{e-02}i$	$9.43988_{e-03}-2.54544_{e-01}i$	$7.23953_{e-02}-1.37613_{e-02}i$	$-5.05861_{e-03}+3.80650_{e-03}i$	$-1.61771_{e-05}+1.21595_{e-05}i$
0,4	$50\angle 90^\circ$	Ob	$-2.01335_{e+03}+2.75523_{e-13}i$	$3.75000_{e-01}-2.36542_{e-17}i$	$4.13389_{e-01}-3.14170_{e-17}i$	$1.67056_{e+01}-6.18244_{e-15}i$	$1.40059_{e+04}-1.16353_{e-11}i$	$9.26987_{e+08}-1.48219_{e-06}i$	$3.54399_{e+13}-8.40369_{e-02}i$	$-7.42187_{e+15}+2.30186_{e+01}i$
0,4	$100\angle 0^\circ$	Pr	$8.89116_{e+02}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.58454_{e-01}-0.00000_{e+00}i$	$-3.79227_{e-01}-0.00000_{e+00}i$	$4.71492_{e-01}-0.00000_{e+00}i$	$6.05599_{e-04}-0.00000_{e+00}i$	$8.31862_{e-12}-0.00000_{e+00}i$	$7.80930_{e-33}-0.00000_{e+00}i^{*}$
0,4	$100\angle 15^\circ$	Pr	$8.58454_{e+02}+2.32973_{e+02}i$	$3.75000_{e-01}-1.08420_{e-19}i$	$3.59012_{e-01}-4.30470_{e-03}i$	$-4.03536_{e-01}-3.57031_{e-02}i$	$5.49052_{e-01}-4.81118_{e-02}i$	$-9.34733_{e-04}-2.24713_{e-04}i$	$-9.50952_{e-12}-2.46594_{e-11}i$	$-1.39816_{e-31}-4.15281_{e-32}i^{*}$
0,4	$100\angle 30^\circ$	Ob	$5.09900_{e+03}+8.48705_{e+03}i$	$3.75000_{e-01}-2.35922_{e-16}i$	$2.72698_{e-01}-1.45647_{e-01}i$	$-1.82616_{e+01}-1.74987_{e+01}i$	$-3.79327_{e+04}-1.34026_{e+04}i$	$6.99537_{e+09}+5.65182_{e+09}i$	$-1.08292_{e+15}-1.76178_{e+15}i$	$-1.80293_{e+20}+2.34671_{e+20}i$
0,4	$100\angle 45^\circ$	Pr	$3.49784_{e+02}+3.53574_{e+02}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.68441_{e-01}-6.58814_{e-03}i$	$-2.30882_{e-01}-3.14809_{e-01}i$	$-8.59208_{e-02}+4.87020_{e-01}i$	$1.31964_{e-03}+1.23975_{e-03}i$	$1.32398_{e-09}-3.00023_{e-09}i$	$3.23428_{e-24}-8.71035_{e-25}i^{*}$
0,4	$100\angle 60^\circ$	Ob	$-4.48542_{e+03}+8.36023_{e+03}i$	$3.75000_{e-01}-1.11022_{e-16}i$	$4.50881_{e-01}-1.68430_{e-01}i$	$2.28706_{e+02}+7.69620_{e+02}i$	$2.25922_{e+08}+1.43446_{e+07}i$	$2.61428_{e+17}+3.32935_{e+16}i$	$2.37965_{e+26}+4.49881_{e+25}i$	$4.72451_{e+33}+1.67878_{e+33}i$
0,4	$100\angle 75^\circ$	Pr	$2.51314_{e+01}+9.65944_{e+01}i$	$3.75000_{e-01}-5.42101_{e-20}i$	$3.74525_{e-01}-1.80891_{e-03}i$	$2.92614_{e-01}-1.53898_{e-01}i$	$-1.68419_{e-01}-1.25394_{e-02}i$	$1.21108_{e-02}-4.76514_{e-03}i$	$2.21284_{e-05}-7.60205_{e-05}i$	$8.02288_{e-11}-2.82304_{e-10}i^{*}$
0,4	$100\angle 90^\circ$	Ob	$-9.01317_{e+03}+1.16340_{e-12}i$	$3.75000_{e-01}-2.98571_{e-17}i$	$5.57084_{e-01}-6.95969_{e-17}i$	$2.49687_{e+03}-1.72874_{e-12}i$	$3.82542_{e+09}-6.16453_{e-06}i$	$6.94362_{e+19}-2.18277_{e+05}i$	$7.51065_{e+29}-3.51304_{e+15}i$	$-1.20309_{e+37}+7.24365_{e+22}i$
0,4	$250\angle 0^\circ$	Pr	$2.23920_{e+03}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.33809_{e-01}-0.00000_{e+00}i$	$-7.34908_{e-02}-0.00000_{e+00}i$	$3.83304_{e-02}-0.00000_{e+00}i$	$7.72288_{e-12}-0.00000_{e+00}i$	$4.75819_{e-33}-0.00000_{e+00}i$	$5.47942_{e-88}-0.00000_{e+00}i^{*}$
0,4	$250\angle 15^\circ$	Pr	$2.16253_{e+03}+5.82357_{e+02}i$	$3.75000_{e-01}-5.42101_{e-20}i$	$3.35140_{e-01}-1.05212_{e-02}i$	$-9.93038_{e-02}+2.20565_{e-01}i$	$5.08299_{e-03}-5.04630_{e-02}i$	$-6.62226_{e-12}-2.33033_{e-11}i$	$-6.95880_{e-32}-4.91300_{e-32}i$	$6.06553_{e-86}+8.22901_{e-85}i^{*}$
0,4	$250\angle 30^\circ$	Ob	$3.14990_{e+04}+5.36936_{e+04}i$	$3.75000_{e-01}+6.66134_{e-16}i$	$-3.93182_{e-01}+4.91902_{e-01}i$	$-4.31116_{e+04}-1.52674_{e+04}i$	$-4.20597_{e+12}+3.68789_{e+12}i$	$2.27419_{e+25}-1.72355_{e+26}i$	$3.08504_{e+39}+4.62269_{e+39}i$	$3.94737_{e+52}-3.41941_{e+52}i$
0,4	$250\angle 45^\circ$	Pr	$8.80126_{e+02}+8.83892_{e+02}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.58497_{e-01}-1.63122_{e-02}i$	$-6.76240_{e-01}-4.36212_{e-02}i$	$-4.01360_{e-03}-4.39726_{e-02}i$	$1.91547_{e-09}-2.41451_{e-09}i$	$1.69080_{e-24}+1.23024_{e-24}i$	$-2.19640_{e-63}+5.81013_{e-64}i^{*}$
0,4	$250\angle 60^\circ$	Ob	$-2.99560_{e+04}+5.33766_{e+04}i$	$3.75000_{e-01}+4.44089_{e-16}i$	$5.06933_{e-01}-1.48382_{e+00}i$	$3.51485_{e+08}+2.31918_{e+07}i$	$2.84670_{e+22}+4.73351_{e+21}i$	$4.05293_{e+45}+1.38589_{e+45}i$	$4.43479_{e+68}+2.38253_{e+68}i$	$-1.05387_{e+89}+4.69387_{e+89}i$
0,4	$250\angle 75^\circ$	Pr	$6.39546_{e+01}+2.41482_{e+02}i$	$3.75000_{e-01}+2.71051_{e-20}i$	$3.73775_{e-01}-4.51334_{e-03}i$	$9.59614_{e-02}-2.54672_{e-01}i$	$9.09930_{e-03}-4.83778_{e-02}i$	$4.24871_{e-05}-5.77571_{e-05}i$	$1.54509_{e-10}-1.16759_{e-11}i$	$9.62803_{e-25}-7.18795_{e-26}i^{*}$
0,4	$250\angle 90^\circ$	Ob	$-6.00131_{e+04}+7.50096_{e-12}i$	$3.75000_{e-01}+2.61558_{e-17}i$	$2.18853_{e+00}-1.77455_{e-16}i$	$8.19513_{e+09}-1.19749_{e-05}i$	$7.46509_{e+25}-2.80513_{e+11}i$	$2.65035_{e+52}-2.01034_{e+38}i$	$5.70086_{e+78}-6.50659_{e+64}i$	$2.02296_{e+101}-3.08060_{e+87}i$
0,4	$500\angle 0^\circ$	Pr	$4.48922_{e+03}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$2.93995_{e-01}-0.00000_{e+00}i$	$4.42678_{e-01}-0.00000_{e+00}i$	$6.12139_{e-05}-0.00000_{e+00}i$	$9.02921_{e-26}-0.00000_{e+00}i$	$3.35789_{e-69}-0.00000_{e+00}i^{*}$	$1.21617_{e-180}-0.00000_{e+00}i^{*}$
0,4	$500\angle 15^\circ$	Pr	$4.33589_{e+03}+1.16469_{e+03}i$	$3.75000_{e-01}-2.71051_{e-20}i$	$2.96453_{e-01}-2.02604_{e-02}i$	$5.16394_{e-01}+9.44851_{e-02}i$	$-9.65486_{e-05}+4.26940_{e-05}i$	$-4.02597_{e-25}+7.89063_{e-25}i$	$8.24784_{e-67}+6.89102_{e-67}i^{*}$	$-2.15439_{e-174}+1.71992_{e-174}i^{*}$
0,4	$500\angle 30^\circ$	Ob	$1.25499_{e+05}+2.15640_{e+05}i$	$3.75000_{e-01}+8.88178_{e-16}i$	$-8.49400_{e-01}+2.08394_{e+00}i$	$9.53584_{e+09}+7.72622_{e+09}i$	$2.73808_{e+25}-2.06862_{e+26}i$	$-2.33573_{e+53}-6.29496_{e+52}i$	$-1.10986_{e+80}+2.66254_{e+80}i$	$4.16824_{e+105}-2.86670_{e+106}i$
0,4	$500\angle 45^\circ$	Pr	$1.76401_{e+03}+1.76777_{e+03}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.41930_{e-01}-3.21036_{e-02}i$	$-3.67493_{e-01}+4.72783_{e-01}i$	$1.74964_{e-04}-2.75190_{e-04}i$	$2.73897_{e-19}+1.66254_{e-19}i$	$-3.80413_{e-50}-1.92160_{e-50}i^{*}$	$-1.38111_{e-129}+4.99112_{e-129}i^{*}$
0,4	$500\angle 60^\circ$	Ob	$-1.22407_{e+05}+2.15006_{e+05}i$	$3.75000_{e-01}-4.57967_{e-16}i$	$-1.10745_{e+01}-8.26941_{e+00}i$	$8.82995_{e+17}+1.17688_{e+17}i$	$8.77589_{e+45}+3.01853_{e+45}i$	$3.49293_{e+92}+2.72507_{e+92}i$	$9.15315_{e+138}+1.40437_{e+139}i$	$-5.22333_{e+182}-3.81229_{e+182}i$
0,4	$500\angle 75^\circ$	Pr	$1.28659_{e+02}+4.82963_{e+02}i$	$3.75000_{e-01}+1.35525_{e-20}i$	$3.72487_{e-01}-8.99684_{e-03}i$	$-1.47839_{e-01}-1.29850_{e-01}i$	$-5.87659_{e-03}-2.29195_{e-03}i$	$-3.66713_{e-09}-1.17645_{e-08}i$	$4.69177_{e-20}-7.10664_{e-21}i$	$6.97024_{e-49}-1.01098_{e-49}i^{*}$
0,4	$500\angle 90^\circ$	Ob	$-2.45013_{e+05}+3.03100_{e-11}i$	$3.75000_{e-01}-1.58493_{e-17}i$	$2.64695_{e+01}-9.22206_{e-15}i$	$5.90853_{e+20}-1.83398_{e+06}i$	$1.04192_{e+53}-8.01911_{e+38}i$	$5.16684_{e+106}-7.93150_{e+92}i$	$1.56043_{e+160}-3.58983_{e+146}i$	$1.39236_{e+209}-4.23112_{e+195}i$
0,4	$750\angle 0^\circ$	Ob	$6.73923_{e+03}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$2.55722_{e-01}-0.00000_{e+00}i$	$4.06491_{e-01}-0.00000_{e+00}i$	$5.08996_{e-08}-0.00000_{e+00}i$	$5.82964_{e-40}-0.00000_{e+00}i$	$1.32175_{e-105}-0.00000_{e+00}i^{*}$	$1.51458_{e-273}-0.00000_{e+00}i^{*}$
0,4	$750\angle 15^\circ$	Pr	$6.50923_{e+03}+1.74703_{e+03}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$2.59111_{e-01}-2.92465_{e-02}i$	$4.56076_{e-01}-1.38561_{e-01}i$	$9.06921_{e-08}+7.05090_{e-08}i$	$1.68428_{e-38}-6.06032_{e-39}i$	$-5.38153_{e-102}-5.31818_{e-102}i$	$-4.94107_{e-264}-1.51535_{e-264}i^{*}$
0,4	$750\angle 30^\circ$	Ob	$2.81999_{e+05}+4.85840_{e+05}i$	$3.75000_{e-01}+7.77156_{e-16}i$	$7.72135_{e+00}-1.66011_{e+00}i$	$-1.71945_{e+15}-2.80886_{e+15}i$	$4.31652_{e+39}+6.47682_{e+39}i$	$-1.29553_{e+80}+3.10639_{e+80}i$	$-1.46931_{e+121}+2.86657_{e+120}i$	$-8.64868_{e+159}-1.35392_{e+160}i$
0,4	$750\angle 45^\circ$	Pr	$2.64790_{e+03}+2.65165_{e+03}i$	$3.75000_{e-01}+2.71051_{e-20}i$	$3.25376_{e-01}-4.73810_{e-02}i$	$9.45987_{e-02}+3.67154_{e-01}i$	$1.69006_{e-06}-5.95487_{e-07}i$	$-1.02262_{e-29}+2.27306_{e-29}i$	$6.20264_{e-76}+1.97019_{e-76}i$	$6.18238_{e-195}+6.34176_{e-195}i^{*}$
0,4	$750\angle 60^\circ$	Ob	$-2.77358_{e+05}+8.84289_{e+05}i$	$3.75000_{e-01}-3.52496_{e-15}i$	$-9.88354_{e+01}+6.88460_{e+01}i$	$2.20770_{e+27}+4.45117_{e+26}i$	$2.62117_{e+69}+1.42312_{e+69}i$	$2.49242_{e+139}+3.83749_{e+139}i$	$4.39507_{e+208}+5.54918_{e+209}i$	$6.82741_{e+275}-4.45190_{e+275}i$
0,4	$750\angle 75^\circ$	Pr	$1.93364_{e+02}+7.24445_{e+02}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.71148_{e-01}-1.34500_{e-02}i$	$-1.25542_{e-01}+6.69194_{e-02}i$	$-4.27837_{e-04}+6.85740_{e-04}i$	$-2.00208_{e-12}-6.90673_{e-13}i$	$1.41653_{e-29}-3.24727_{e-30}i$	$5.02139_{e-73}-1.09483_{e-73}i^{*}$
0,4	$750\angle 90^\circ$	Ob	$-5.55013_{e+05}+6.84271_{e-11}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$1000\angle 0^\circ$	Pr	$8.98924_{e+03}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$2.18948_{e-01}-0.00000_{e+00}i$	$2.38247_{e-01}-0.00000_{e+00}i$	$3.28426_{e-11}-0.00000_{e+00}i$	$2.96047_{e-54}-0.00000_{e+00}i$	$4.10219_{e-142}-0.00000_{e+00}i^{*}$	± 0
0,4	$1000\angle 15^\circ$	Pr	$8.68257_{e+03}+2.32937_{e+03}i$	$3.75000_{e-01}-1.35525_{e-20}i$	$2.23088_{e-01}-3.75070_{e-02}i$	$2.25530_{e-01}-1.78864_{e-01}i$	$1.66457_{e-11}-9.56438_{e-11}i$	$-2.61026_{e-52}-1.13269_{e-52}i$	$2.72897_{e-137}+3.19235_{e-137}i^{*}$	± 0
0,4	$1000\angle 30^\circ$	Ob	$5.00999_{e+05}+8.64293_{e+05}i$	$3.75000_{e-01}+9.43690_{e-16}i$	$-1.98855_{e+01}+1.90703_{e+01}i$	$1.83171_{e+20}+8.64545_{e+20}i$	$-2.80305_{e+53}-7.56051_{e+52}i$	$4.04747_{e+107}+2.35487_{e+107}i$	$-5.46552_{e+161}-5.52092_{e+161}i$	$-8.53911_{e+213}-2.54287_{e+213}i$
0,4	$1000\angle 45^\circ$	Pr	$3.53178_{e+03}+3.53554_{e+03}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.08842_{e-01}-6.21513_{e-02}i$	$1.92600_{e-01}+8.59143_{e-02}i$	$8.26632_{e-09}+2.82216_{e-09}i$	$-1.64533_{e-39}-5.09945_{e-40}i$	$-8.73519_{e-102}-1.29916_{e-102}i$	$1.30642_{e-260}-3.26086_{e-261}i^{*}$
0,4	$1000\angle 60^\circ$	Ob	$-4.94809_{e+05}+8.63025_{e+05}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$1000\angle 75^\circ$	Pr	$2.58069_{e+02}+9.65926_{e+02}i$	$3.75000_{e-01}-6.77626_{e-21}i$	$3.69760_{e-01}-1.78724_{e-02}i$	$1.32702_{e-02}+1.01998_{e-01}i$	$7.62197_{e-05}+7.01219_{e-05}i$	$-2.99536_{e-16}+2.06798_{e-16}i$	$4.25146_{e-39}-1.31698_{e-39}i$	$3.59852_{e-97}-1.05657_{e-97}i^{*}$
0,4	$1000\angle 90^\circ$	Ob	$-9.90013_{e+05}+1.21852_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$1500\angle 0^\circ$	Pr	$1.34892_{e+04}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$1.49748_{e-01}-0.00000_{e+00}i$	$4.96803_{e-02}-0.00000_{e+00}i$	$9.56529_{e-18}-0.00000_{e+00}i$	$5.41090_{e-83}-0.00000_{e+00}i$	$2.80717_{e-215}-0.00000_{e+00}i^{*}$	± 0
0,4	$1500\angle 15^\circ$	Pr	$1.30292_{e+04}+3.49406_{e+03}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$1.54882_{e-01}-5.19584_{e-02}i$	$1.35427_{e-02}-6.33925_{e-02}i$	$3.48162_{e-17}+3.38179_{e-17}i$	$1.74485_{e-80}-4.78864_{e-80}i$	$4.71694_{e-208}+7.89189_{e-208}i^{*}$	± 0
0,4	$1500\angle 30^\circ$	Ob	$1.12650_{e+06}+1.94596_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$1500\angle 45^\circ$	Pr	$5.29955_{e+03}+5.30330_{e+03}i$	$3.75000_{e-01}-1.35525_{e-20}i$	$2.75856_{e-01}-9.01970_{e-02}i$	$1.11043_{e-02}-5.32615_{e-02}i$	$-1.67107_{e-14}+1.73974_{e-13}i$	$6.92644_{e-60}+3.93084_{e-61}i$	$-1.35151_{e-153}+2.34653_{e-154}i^{*}$	± 0
0,4	$1500\angle 60^\circ$	Ob	$-1.11721_{e+06}+1.94406_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$1500\angle 75^\circ$	Pr	$3.87479_{e+02}+1.44889_{e+03}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$3.66840_{e-01}-2.66228_{e-02}i$	$2.99739_{e-02}-4.46378_{e-02}i$	$-7.47568_{e-07}-1.52741_{e-06}i$	$8.46420_{e-24}+6.62956_{e-24}i$	$3.75805_{e-58}-1.81777_{e-58}i$	$1.81743_{e-145}-8.28439_{e-146}i^{*}$
0,4	$1500\angle 90^\circ$	Ob	$-2.23501_{e+06}+2.74627_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$2500\angle 0^\circ$	Pr	$2.24892_{e+04}-0.00000_{e+00}i$	$3.75000_{e-01}-0.00000_{e+00}i$	$2.76976_{e-02}-0.00000_{e+00}i$	$1.00769_{e-03}-0.00000_{e+00}i$	$4.36104_{e-31}-0.00000_{e+00}i$	$9.86473_{e-141}-0.00000_{e+00}i^{*}$	± 0	± 0
0,4	$2500\angle 15^\circ$	Pr	$2.17226_{e+04}+5.82343_{e+03}i$	$3.75000_{e-01}+6.77626_{e-21}i$	$3.30779_{e-02}-7.32927_{e-02}i$	$-1.39340_{e-03}-6.84107_{e-04}i$	$2.55759_{e-30}-6.00409_{e-30}i$	$-2.04209_{e-136}+8.68869_{e-136}i^{*}$	± 0	± 0
0,4	$2500\angle 30^\circ$	Ob	$3.12750_{e+06}+5.40833_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$2500\angle 45^\circ$	Pr	$8.83508_{e+03}+8.83884_{e+03}i$	$3.75000_{e-01}-6.77626_{e-21}i$	$2.10382_{e-01}-1.40497_{e-01}i$	$5.43924_{e-04}+2.58777_{e-03}i$	$-1.86156_{e-23}-4.82403_{e-23}i$	$7.57064_{e-101}-3.47510_{e-101}i^{*}$	$-1.68849_{e-257}+1.77615_{e-257}i^{*}$	± 0

Table D.52: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
0,4	$2500\angle 60^\circ$	Ob	−3.11201e+06 + 5.40516e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$2500\angle 75^\circ$	Pr	6.46298e+02 + 2.41481e+03i	3.75000e−01 − 0.00000e+00i	3.60432e−01 − 4.37326e−02i	1.31523e−02 + 6.54843e−03i	1.30483e−10 − 4.39500e−10i	−9.36354e−39 − 5.64612e−40i*	2.68519e−96 − 2.50557e−96i*	4.28822e−242 − 3.70251e−242i*
0,4	$2500\angle 90^\circ$	Ob	−6.22501e+06 + 7.63873e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$5000\angle 0^\circ$	Pr	4.49892e+04 − 0.00000e+00i	3.75000e−01 − 0.00000e+00i	−1.94638e−01 − 0.00000e+00i	1.55381e−08 − 0.00000e+00i	5.87967e−65 − 0.00000e+00i	1.36846e−285 − 0.00000e+00i*	± 0	± 0
0,4	$5000\angle 15^\circ$	Pr	4.34559e+04 + 1.16469e+04i	3.75000e−01 − 0.00000e+00i	−1.95672e−01 − 9.09373e−02i	3.45311e−08 + 1.20772e−08i	−1.26625e−62 − 3.55866e−63i	−1.11767e−275 + 7.16276e−277i*	± 0	± 0
0,4	$5000\angle 30^\circ$	Ob	1.25050e+07 + 2.16420e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$5000\angle 45^\circ$	Pr	1.76739e+04 + 1.76777e+04i	3.75000e−01 + 3.38813e−21i	5.15625e−02 − 2.34948e−01i	2.58482e−07 − 7.09735e−07i	1.95698e−48 − 4.32505e−47i	1.76761e−204 + 2.31236e−203i*	± 0	± 0
0,4	$5000\angle 60^\circ$	Ob	−1.24740e+07 + 2.16356e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,4	$5000\angle 75^\circ$	Pr	1.29335e+03 + 4.82963e+03i	3.75000e−01 − 0.00000e+00i	3.41317e−01 − 8.40642e−02i	3.45631e−04 + 4.57617e−04i	−4.58488e−19 − 2.98560e−19i	2.09398e−76 + 2.53486e−77i*	1.84428e−192 − 2.65977e−191i*	± 0
0,4	$5000\angle 90^\circ$	Ob	−2.49500e+07 + 3.05856e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$25\angle 0^\circ$	Pr	2.58164e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.86701e−02 − 0.00000e+00i	1.17229e−01 − 0.00000e+00i	−9.06018e−02 − 0.00000e+00i	1.44672e−01 − 0.00000e+00i	3.39934e−02 − 0.00000e+00i	7.25765e−06 − 0.00000e+00i*
0,5	$25\angle 15^\circ$	Pr	2.48858e+02 + 7.15098e+01i	−0.00000e+00 − 0.00000e+00i	1.86729e−02 − 2.22899e−05i	1.18765e−01 − 1.71568e−02i	−1.05968e−01 − 3.38025e−02i	1.92747e−01 + 3.43626e−02i	1.89688e−03 − 4.65015e−02i	4.37567e−06 + 1.44825e−05i*
0,5	$25\angle 30^\circ$	Pr	1.44563e+02 + 8.76615e+01i	−0.00000e+00 − 0.00000e+00i	1.87055e−02 − 2.73515e−05i	1.45024e−01 − 2.33496e−02i	−2.31513e−02 − 1.33818e−01i	−1.28146e−01 + 1.93592e−01i	1.61179e−02 − 8.88237e−03i	3.34861e−06 − 3.45141e−06i*
0,5	$25\angle 45^\circ$	Ob	1.00961e+02 + 5.18815e+02i	−0.00000e+00 − 0.00000e+00i	1.87187e−02 − 1.61971e−04i	1.19067e−01 − 1.41882e−01i	−1.46528e+00 + 6.44242e−01i	−8.43804e+00 − 5.98904e+01i	1.77301e+03 + 2.04277e+02i	5.78375e+03 + 2.25131e+03i
0,5	$25\angle 60^\circ$	Pr	3.57328e+01 + 6.49866e+01i	−0.00000e+00 − 0.00000e+00i	1.87395e−02 − 2.02974e−05i	1.76183e−01 − 1.92060e−02i	2.53528e−01 − 2.08341e−01i	−2.03651e−01 − 4.96728e−02i	1.64821e−02 − 2.88772e−02i	2.42776e−04 + 6.35786e−05i*
0,5	$25\angle 75^\circ$	Ob	−3.13497e+02 + 2.47571e+02i	−0.00000e+00 − 0.00000e+00i	1.88487e−02 − 7.76112e−05i	2.90902e−01 − 1.04105e−01i	1.38510e+00 − 5.09254e+00i	−5.11630e+02 − 9.88191e+01i	−1.87657e+03 + 2.76693e+04i	−6.77248e+02 + 3.40563e+04i
0,5	$25\angle 90^\circ$	Ob	−3.88753e+02 + 6.11785e−14i	−0.00000e+00 − 0.00000e+00i	1.88723e−02 + 1.58084e−18i	3.35811e−01 + 7.36349e−19i	6.59306e+00 − 1.48024e−15i	8.01425e+02 − 4.91993e−13i	5.19019e+04 − 5.26912e−11i	4.90203e+04 − 4.60766e−11i
0,5	$50\angle 0^\circ$	Pr	5.33757e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.85843e−02 − 0.00000e+00i	6.11024e−02 − 0.00000e+00i	−5.93572e−02 − 0.00000e+00i	4.58070e−02 − 0.00000e+00i	3.66271e−05 − 0.00000e+00i	1.45521e−14 − 0.00000e+00i*
0,5	$50\angle 15^\circ$	Pr	5.15038e+02 + 1.42488e+02i	−0.00000e+00 − 0.00000e+00i	1.85901e−02 − 4.42902e−05i	6.22024e−02 − 2.50224e−02i	−6.93852e−02 + 5.50954e−02i	2.91508e−02 − 5.15881e−02i	−5.20180e−05 + 4.02750e−05i	−2.56597e−14 + 5.77613e−14i
0,5	$50\angle 30^\circ$	Pr	2.96242e+02 + 1.75072e+02i	−0.00000e+00 − 0.00000e+00i	1.86581e−02 − 5.45318e−05i	1.05067e−01 − 3.88938e−02i	−1.64657e−01 − 2.19339e−02i	2.67116e−02 + 1.09133e−02i	1.44485e−06 + 2.53465e−05i	4.09482e−14 + 6.98412e−14i
0,5	$50\angle 45^\circ$	Ob	2.07079e+02 + 2.28781e+03i	−0.00000e+00 − 0.00000e+00i	1.86778e−02 − 7.13444e−04i	−4.51325e−01 − 2.08120e−01i	−1.04060e+01 − 6.33899e+01i	−8.18626e+04 + 1.86184e+05i	4.21455e+08 − 2.81310e+08i	−4.20477e+10 + 4.38685e+10i
0,5	$50\angle 60^\circ$	Pr	7.32410e+01 + 1.29921e+02i	−0.00000e+00 − 0.00000e+00i	1.87277e−02 − 4.05530e−05i	1.62554e−01 − 3.58518e−02i	4.28832e−02 − 2.16415e−01i	3.48042e−02 + 1.81675e−02i	−2.41963e−04 − 4.17227e−04i	4.82727e−09 + 2.56364e−09i*
0,5	$50\angle 75^\circ$	Ob	−1.69543e+03 + 1.12049e+03i	−0.00000e+00 − 0.00000e+00i	1.92830e−02 − 3.56135e−04i	8.37240e−01 − 1.32098e+00i	−9.98275e+02 − 1.90959e+02i	4.38633e+07 + 3.67874e+06i	−1.10364e+12 + 4.29275e+10i	−1.75071e+14 − 9.73937e+12i
0,5	$50\angle 90^\circ$	Ob	−2.01335e+03 + 2.75523e−13i	−0.00000e+00 − 0.00000e+00i	1.93862e−02 − 5.03517e−19i	1.86159e+00 − 4.84268e−16i	1.56114e+03 − 1.12503e−12i	1.03325e+08 − 1.53834e−07i	3.95025e+12 − 8.93209e−03i	−8.27265e+14 + 2.47465e+00i
0,5	$100\angle 0^\circ$	Pr	1.08401e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.84138e−02 − 0.00000e+00i	−6.59776e−03 − 0.00000e+00i	6.54268e−02 − 0.00000e+00i	2.96938e−04 − 0.00000e+00i	7.25280e−12 − 0.00000e+00i	1.33954e−32 − 0.00000e+00i*
0,5	$100\angle 15^\circ$	Pr	1.04654e+03 + 2.84764e+02i	−0.00000e+00 − 0.00000e+00i	1.84252e−02 − 8.80209e−05i	−8.74523e−03 − 2.40725e−02i	8.48173e−02 + 1.58973e−02i	−4.57285e−04 − 1.22428e−04i	−8.09758e−12 − 2.16080e−11i	−2.39670e−31 − 7.20243e−32i
0,5	$100\angle 30^\circ$	Pr	5.99410e+02 + 3.50034e+02i	−0.00000e+00 − 0.00000e+00i	1.85637e−02 − 1.08658e−04i	4.05472e−02 − 5.26589e−02i	−6.56929e−02 + 8.44875e−02i	−1.83121e−04 − 2.61533e−05i	1.73586e−11 − 1.41019e−11i	−1.32955e−31 − 1.13936e−29i
0,5	$100\angle 45^\circ$	Ob	4.19238e+02 + 9.57571e+03i	−0.00000e+00 − 0.00000e+00i	1.84771e−02 − 2.97674e−03i	8.37072e+00 + 1.48176e−01i	−8.56575e+04 + 2.00214e+05i	−4.77543e+12 − 4.72103e+09i	−3.27263e+19 − 7.59998e+19i	3.73406e+25 − 2.49948e+25i
0,5	$100\angle 60^\circ$	Pr	1.48245e+02 + 2.59816e+02i	−0.00000e+00 − 0.00000e+00i	1.87042e−02 − 8.09966e−05i	1.33232e−01 − 6.17676e−02i	−9.21917e−02 − 3.80589e−02i	7.86447e−04 + 1.13155e−03i	−5.08255e−08 + 8.80430e−08i	1.38261e−18 + 2.16819e−18i*
0,5	$100\angle 75^\circ$	Ob	−7.70749e+03 + 4.74114e+03i	−0.00000e+00 − 0.00000e+00i	2.12168e−02 − 1.59864e−03i	−6.31933e+01 − 6.87379e+01i	8.55359e+07 + 7.21064e+06i	6.61311e+17 − 7.20915e+16i	2.93021e+27 − 9.34377e+26i	−2.35957e+34 − 6.58307e+33i
0,5	$100\angle 90^\circ$	Ob	−9.01317e+03 + 1.16340e−12i	−0.00000e+00 − 0.00000e+00i	2.16970e−02 − 1.27817e−18i	1.31508e+02 − 7.74307e−14i	2.01482e+08 − 3.03813e−07i	3.65715e+18 − 1.11177e+04i	3.95580e+28 − 1.80932e+14i	−6.33658e+35 + 3.74954e+21i
0,5	$250\angle 0^\circ$	Pr	2.73416e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.79083e−02 − 0.00000e+00i	−3.61071e−02 − 0.00000e+00i	8.34568e−03 − 0.00000e+00i	4.00404e−12 − 0.00000e+00i	4.23827e−33 − 0.00000e+00i	9.46421e−88 − 0.00000e+00i*
0,5	$250\angle 15^\circ$	Pr	2.64046e+03 + 7.11777e+02i	−0.00000e+00 − 0.00000e+00i	1.79360e−02 − 2.16344e−04i	−4.21864e−02 + 1.22653e−02i	1.64344e−03 − 1.10221e−02i	−3.32975e−12 − 1.21276e−11i	−6.18583e−32 − 4.40039e−32i	1.02979e−85 + 1.42168e−84i
0,5	$250\angle 30^\circ$	Pr	1.50877e+03 + 8.75013e+02i	−0.00000e+00 − 0.00000e+00i	1.82818e−02 − 2.68857e−04i	−5.41374e−02 − 2.67150e−02i	4.81505e−03 − 1.23273e−03i	1.15931e−11 − 6.36689e−12i	2.61712e−30 − 3.66249e−30i	−9.91982e−78 + 2.85055e−79i
0,5	$250\angle 45^\circ$	Ob	1.05565e+03 + 6.14393e+04i	−0.00000e+00 − 0.00000e+00i	1.26416e−02 − 1.81449e−02i	−5.13830e+04 + 1.23454e+05i	2.36512e+16 + 1.55122e+16i	1.90428e+35 + 7.31817e+34i	1.15221e+54 + 1.76041e+53i	1.84076e+69 + 8.40143e+70i
0,5	$250\angle 60^\circ$	Pr	3.73248e+02 + 6.49522e+02i	−0.00000e+00 − 0.00000e+00i	1.86332e−02 − 2.01725e−04i	4.70583e−02 − 8.91811e−02i	7.64871e−03 − 5.14151e−03i	−4.43374e−08 + 3.98526e−08i	−4.78769e−19 + 8.24527e−19i	−2.20835e−46 + 1.55687e−46i
0,5	$250\angle 75^\circ$	Ob	−5.17248e+04 + 3.06029e+04i	−0.00000e+00 − 0.00000e+00i	3.75670e−02 − 1.53764e−02i	7.10909e+07 + 5.63987e+06i	−1.77334e+23 + 3.64582e+22i	5.97647e+48 − 4.76467e+48i	−8.05915e+73 + 1.78199e+74i	9.94841e+94 − 1.09317e+96i
0,5	$250\angle 90^\circ$	Ob	−6.00131e+04 + 7.50096e−12i	−0.00000e+00 − 0.00000e+00i	4.40081e−02 + 7.20650e−19i	1.67266e+08 − 2.27347e−07i	1.52365e+24 − 5.56993e+09i	5.40947e+50 − 4.04799e+36i	1.16357e+77 − 1.31615e+63i	4.12894e+99 − 6.24550e+85i
0,5	$500\angle 0^\circ$	Pr	5.48420e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.70853e−02 − 0.00000e+00i	1.54422e−02 − 0.00000e+00i	1.45026e−05 − 0.00000e+00i	4.76113e−26 − 0.00000e+00i	3.01150e−69 − 0.00000e+00i*	2.10556e−180 − 0.00000e+00i*
0,5	$500\angle 15^\circ$	Pr	5.29680e+03 + 1.42352e+03i	−0.00000e+00 − 0.00000e+00i	1.71374e−02 − 4.20665e−04i	2.08200e−02 + 1.82703e−02i	−2.31387e−05 + 9.69470e−06i	−2.14191e−25 + 4.15409e−25i	7.38740e−67 + 6.19432e−67i	−3.73188e−174 + 2.97544e−174i*
0,5	$500\angle 30^\circ$	Pr	3.02433e+03 + 1.75001e+03i	−0.00000e+00 − 0.00000e+00i	1.78161e−02 − 5.28586e−04i	−4.69940e−02 + 2.74540e−02i	−4.81029e−06 + 9.59790e−06i	−1.56253e−25 + 6.75927e−24i	1.00605e−61 + 8.62740e−62i	−2.86124e−158 + 1.87948e−158i*
0,5	$500\angle 45^\circ$	Ob	2.11632e+03 + 2.47879e+05i	−0.00000e+00 − 0.00000e+00i	−5.78360e−02 − 2.41161e−02i	−3.16788e+12 + 2.68934e+10i	2.05445e+35 + 7.85305e+34i	2.47337e+73 − 1.47335e+72i	1.95781e+111 − 1.03160e+111i	−3.77707e+146 + 2.51922e+146i
0,5	$500\angle 60^\circ$	Pr	7.48249e+02 + 1.29904e+03i	−0.00000e+00 − 0.00000e+00i	1.85139e−02 − 4.00911e−04i	−3.04148e−02 − 4.44849e−02i	6.57258e−05 − 1.61180e−04i	3.38496e−16 − 3.16091e−15i	−1.96986e−37 − 3.45721e−37i	2.16522e−93 − 5.96558e−93i*
0,5	$500\angle 75^\circ$	Ob	−2.11690e+05 + 1.23706e+05i	−0.00000e+00 − 0.00000e+00i	1.16098e−01 − 1.93218e−01i	1.07933e+18 − 1.19740e+17i	1.16295e+49 − 9.26758e+48i	−4.11861e+99 − 1.04150e+101i	−3.69170e+152 − 2.48505e+152i	−4.73815e+199 − 4.93897e+199i
0,5	$500\angle 90^\circ$	Ob	−2.45013e+05 + 3.03100e−11i	−0.00000e+00 − 0.00000e+00i	2.67348e−01 − 6.59874e−17i	5.96837e+18 − 1.79189e+04i	1.05247e+51 − 7.99335e+36i	5.21916e+104 − 7.95878e+90i	1.57624e+158 − 3.61017e+144i	1.40646e+207 − 4.25968e+193i
0,5	$750\angle 0^\circ$	Pr	8.23422e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.62860e−02 − 0.00000e+00i	2.65666e−02 − 0.00000e+00i	1.23598e−08 − 0.00000e+00i	3.09084e−40 − 0.00000e+00i	1.18803e−105 − 0.00000e+00i*	2.62417e−273 − 0.00000e+00i*
0,5	$750\angle 15^\circ$	Pr	7.95311e+03 + 2.13527e+03i	−0.00000e+00 − 0.00000e+00i	1.63596e−02 − 6.13362e−04i	3.27162e−02 − 4.07299e−03i	2.18496e−08 + 1.74229e−08i	8.94234e−39 − 3.18928e−39i	−4.83204e−102 − 4.78643e−102i	−8.56000e−264 − 2.62914e−264i*
0,5	$750\angle 30^\circ$	Pr	4.53988e+03 + 2.62500e+03i	−0.00000e+00 − 0.00000e+00i	1.73559e−02 − 7.79372e−04i	−1.00172e−02 + 2.87261e−02i	−6.13417e−09 − 1.57289e−08i	−2.18953e−36 − 1.35112e−36i	−2.09087e−93 + 2.03412e−93i	−3.74940e−239 + 8.02344e−239i*
0,5	$750\angle 45^\circ$	Ob	3.17698e+03 + 5.59318e+05i	−0.00000e+00 − 0.00000e+00i	−4.79386e−02 + 2.39778e−01i	−3.98752e+19 − 9.20682e+19i	2.25925e+54 + 3.46242e+53i	3.55751e+111 − 1.87078e+111i	2.43774e+168 − 5.04761e+168i	−2.93541e+222 − 1.09637e+222i
0,5	$750\angle 60^\circ$	Pr	1.12325e+03 + 1.94856e+03i	−0.00000e+00 − 0.00000e+00i	1.83932e−02 − 5.97560e−04i	−2.86117e−02 + 3.28070e−03i	−6.67942e−07 − 3.21904e−06i	9.92296e−23 + 1.37446e−22i	1.66066e−55 + 1.45180e−57i	3.85818e−140 + 1.44087e−139i*
0,5	$750\angle 75^\circ$	Ob	−4.79908e+05 + 2.79309e+05i	−0.00000e+00 − 0.00000e+00i	−1.78676e−01 − 1.66621e+00i	2.11954e+28 − 6.52286e+27i	−6.80673e+74 + 1.48766e+75i	−1.57309e+153 − 1.06368e+153i	9.25626e+230 − 9.75603e+230i	−3.36484e+303 + 1.26161e+303i
0,5	$750\angle 90^\circ$	Ob	−5.55013e+05 + 6.84271e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$1000\angle 0^\circ$	Pr	1.09842e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.55099e−02 − 0.00000e+00i	1.81426e−02 − 0.00000e+00i	8.06949e−12 − 0.00000e+00i	1.57387e−54 − 0.00000e+00i	3.69115e−142 − 0.00000e+00i*	± 0
0,5	$1000\angle 15^\circ$	Pr	1.06094e+04 + 2.84702e+03i	−0.00000e+00 − 0.00000e+00i	1.56021e−02 − 7.94807e−04i	1.88447e−02 − 1.21969e−02i	4.30618e−12 − 2.34923e−11i	−1.38682e−52 − 6.05241e−53i	2.45329e−137 + 2.87500e−137i	± 0
0,5	$1000\angle 30^\circ$	Pr	6.05542e+03 + 3.50000e+03i	−0.00000e+00 − 0.00000e+00i	1.69010e−02 − 1.02139e−03i	4.59689e−03 + 1.36169e−02i	2.18681e−11 + 8.40595e−12i	7.67662e−49 − 4.06560e−49i	−3.58951e−125 − 4.43112e−125i	2.34187e−320 + 2.02300e−319i*
0,5	$1000\angle 45^\circ$	Ob	4.23764e+03 + 9.95757e+05i	−0.00000e+00 − 0.00000e+00i	1.07350e+00 − 4.01935e−04i	2.47087e+27 − 2.58662e+27i	2.66877e+73 − 1.61343e+72i	4.51374e+149 − 5.78619e+149i	−2.95234e+225 − 1.57006e+226i	−6.66941e+297 − 2.31198e+298i
0,5	$1000\angle 60^\circ$	Pr	1.49825e+03 + 2.59808e+03i	−0.00000e+00 − 0.00000e+00i	1.82710e−02 − 7.91675e−04i	−5.58539e−03 + 1.43416e−02i	−4.43866e−08 − 4.34219e−08i	−8.83483e−30 − 1.91528e−30i	−3.53603e−74 + 5.96170e−74i	−2.68403e−186 − 2.25644e−186i*
0,5	$1000\angle 75^\circ$	Ob	−8.56379e+05 + 4.97412e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$1000\angle 90^\circ$	Ob	−9.90013e+05 + 1.21852e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$1500\angle 0^\circ$	Pr	1.64842e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.40250e−02 − 0.00000e+00i	4.23629e−03 − 0.00000e+00i	2.37723e−18 − 0.00000e+00i	2.88433e−83 − 0.00000e+00i	2.52874e−215 − 0.00000e+00i*	± 0
0,5	$1500\angle 15^\circ$	Pr	1.59220e+04 + 4.27052e+03i	−0.00000e+00 − 0.00000e+00i	1.41466e−02 − 1.12540e−03i	1.43924e−03 − 5.39302e−03i	8.61063e−18 + 8.46171e−18i	9.33813e−81 − 2.55181e−80i	4.24523e−208 + 7.11219e−208i	± 0
0,5	$1500\angle 30^\circ$	Pr	9.08651e+03 + 5.25000e+03i	−0.00000e+00 − 0.00000e+00i	1.60073e−02 − 1.47983e−03i	2.57535e−03 − 1.01383e−04i	1.03785e−17 − 3.63391e−17i	−6.99527e−74 − 4.54066e−74i	8.79076e−189 + 1.61343e−188i	± 0
0,5	$1500\angle 45^\circ$	Ob	6.35896e+03 + 2.24364e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$1500\angle 60^\circ$	Pr	2.24825e+03 + 3.89711e+03i	−0.00000e+00 − 0.00000e+00i	1.80229e−02 − 1.17230e−03i	4.28165e−03 − 9.94961e−04i	−2.03220e−11 + 8.81250e−12i	−8.09307e−45 + 2.44004e−44i	1.20733e−110 + 2.14051e−112i	−1.68052e−279 + 9.65419e−280i*
0,5	$1500\angle 75^\circ$	Ob	−1.93408e+06 + 1.12112e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$1500\angle 90^\circ$	Ob	−2.23501e+06 + 2.74627e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,5	$2500\angle 0^\circ$	Pr	2.74842e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.13108e−02 − 0.00000e+00i	9.23997e−05 − 0.00000e+00i	1.09354e−31 − 0.00000e+00i	5.26968e−141 − 0.00000e+00i*	± 0	± 0
0,5	$2500\angle 15^\circ$	Pr	2.65472e+04 + 7.11753e+03i	−0.00000e+00 − 0.00000e+00i	1.14629e−02 − 1.66708e−03i	−1.26650e−04 − 6.62551e−05i	3.64170e−31 − 1.50403e−30i	−1.09484e−136 + 4.64107e−136i*	± 0	± 0
0,5	$2500\angle 30^\circ$	Pr	1.51487e+04 + 8.75000e+03i	−0.00000e+00 − 0.00000e+00i	1.42840e−02 − 2.29902e−03i	−4.87535e−05 − 3.14441e−05i	−7.62231e−30 + 7.18175e−29i	5.14269e−124 − 2.41452e−124i*	1.89060e−316 + 1.39881e−315i*	± 0
0,5	$2500\angle 45^\circ$	Ob	1.06016e+04 + 6.23939e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.52: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
0,5	2500∠60°	Pr	3.74825e+03 + 6.49519e+03i	–0.00000e+00 – 0.00000e+00i	1.75120e–02 – 1.90321e–03i	–5.11202e–05 + 3.54901e–04i	2.63278e–18 + 1.00681e–18i	1.05778e–73 – 1.78973e–73i*	–1.92561e–184 + 3.11809e–184i*	±0
0,5	2500∠75°	Ob	–5.38852e+06 + 3.11853e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	2500∠90°	Ob	–6.22501e+06 + 7.63873e–10i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	5000∠0°	Pr	5.49842e+04 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	5.84357e–03 – 0.00000e+00i	1.49343e–09 – 0.00000e+00i	1.48403e–65 – 0.00000e+00i	7.32190e–286 – 0.00000e+00i*	±0	±0
0,5	5000∠15°	Pr	5.31102e+04 + 1.42350e+04i	–0.00000e+00 – 0.00000e+00i	5.95126e–03 – 2.44057e–03i	3.31096e–09 + 1.20074e–09i	–3.19523e–63 – 9.03785e–64i	–5.98055e–276 + 3.80778e–277i*	±0	±0
0,5	5000∠30°	Pr	3.03041e+04 + 1.75000e+04i	–0.00000e+00 – 0.00000e+00i	1.03450e–02 – 3.83664e–03i	–1.93700e–09 – 1.25255e–09i	1.91131e–58 – 6.32404e–59i	–2.01655e–250 + 1.19362e–249i*	±0	±0
0,5	5000∠45°	Ob	2.12082e+04 + 2.49788e+07i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	5000∠60°	Pr	7.49825e+03 + 1.29904e+04i	–0.00000e+00 – 0.00000e+00i	1.61615e–02 – 3.55487e–03i	–6.53706e–07 – 1.92311e–07i	1.21280e–35 + 1.08648e–35i	–1.86444e–146 – 3.38717e–146i*	±0	±0
0,5	5000∠75°	Ob	–2.16024e+07 + 1.24871e+07i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	5000∠90°	Ob	–2.49500e+07 + 3.05856e–09i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	25∠0°	Pr	3.01441e+02 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–3.07802e–01 – 0.00000e+00i	5.22634e–02 – 0.00000e+00i	1.31161e–01 – 0.00000e+00i	1.49025e–01 – 0.00000e+00i	1.89347e–01 – 0.00000e+00i	9.10373e–05 – 0.00000e+00i*
0,6	25∠15°	Pr	2.90487e+02 + 8.46911e+01i	–3.12500e–01 – 3.46945e–18i	–3.07971e–01 + 1.31689e–03i	4.93004e–02 + 7.68026e–02i	1.65735e–01 – 1.82550e–01i	1.95284e–01 + 4.11419e–01i	1.04782e–01 – 2.44214e–01i	–2.23649e–06 + 1.91372e–04i*
0,6	25∠30°	Pr	1.83626e+02 + 1.12833e+02i	–3.12500e–01 – 0.00000e+00i	–3.09634e–01 + 1.75756e–03i	–5.27621e–02 + 1.26334e–01i	4.56005e–01 – 1.14993e–01i	–7.74238e–01 + 1.74772e–01i	1.21046e–01 + 1.31910e–02i	6.32380e–05 – 1.52361e–05i*
0,6	25∠45°	Ob	1.00961e+02 + 5.18815e+02i	–3.12500e–01 – 1.38778e–17i	–3.10889e–01 + 8.09302e–03i	1.45020e–01 + 6.26470e–01i	1.66282e+00 – 5.89782e+00i	1.97831e+02 + 1.20842e+02i	–4.92159e+03 + 4.75104e+03i	–2.07464e+04 + 1.16394e+04i
0,6	25∠60°	Ob	–1.09596e+02 + 4.15850e+02i	–3.12500e–01 – 9.71445e–17i	–3.14191e–01 + 6.50969e–03i	–2.51030e–01 + 7.43144e–01i	9.51524e+00 – 2.41637e–01i	–5.09498e+02 + 3.07185e+01i	1.49505e+04 – 2.70713e+03i	1.10449e+04 – 1.19287e+04i
0,6	25∠75°	Ob	–3.13497e+02 + 2.47571e+02i	–3.12500e–01 – 4.16334e–17i	–3.17403e–01 + 3.88864e–03i	–8.37866e–01 + 6.16356e–01i	1.22426e+00 + 1.75139e+01i	1.71569e+03 – 2.49710e+02i	–2.43995e+04 – 8.89891e+04i	–3.51609e+04 – 1.07743e+05i
0,6	25∠90°	Ob	–3.02167e+02 + 5.49418e–14i	–3.12500e–01 – 1.68079e–17i	–3.17233e–01 – 1.61996e–17i	–9.16730e–01 + 8.70341e–17i	–1.19122e+01 + 4.07227e–15i	–7.11006e+02 + 5.27905e–13i	–1.30397e+04 + 1.55742e–11i	–2.80644e+03 + 5.36126e–12i
0,6	50∠0°	Pr	6.27445e+02 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–3.02747e–01 – 0.00000e+00i	2.53025e–01 – 0.00000e+00i	–3.32933e–01 – 0.00000e+00i	2.98646e–01 – 0.00000e+00i	4.85020e–04 – 0.00000e+00i	3.94622e–13 – 0.00000e+00i*
0,6	50∠15°	Pr	6.05334e+02 + 1.68460e+02i	–3.12500e–01 – 4.33681e–19i	–3.03086e–01 + 2.60553e–03i	2.62899e–01 + 6.69684e–02i	–4.25679e–01 – 7.65747e–02i	3.08301e–01 – 2.51073e–01i	–8.20579e–04 + 3.08658e–04i	–1.10376e–12 + 1.31064e–12i*
0,6	50∠30°	Pr	3.78729e+02 + 2.25146e+02i	–3.12500e–01 – 0.00000e+00i	–3.06594e–01 + 3.49535e–03i	1.57834e–01 + 1.64418e–01i	–9.11031e–02 – 5.02238e–01i	1.22208e–01 + 1.90746e–01i	–1.73868e–04 + 3.14057e–04i	–2.89250e–14 + 2.26803e–12i*
0,6	50∠45°	Ob	2.07079e+02 + 2.28781e+03i	–3.12500e–01 + 4.06467e–16i	–3.08590e–01 + 3.56198e–02i	3.67029e+00 – 1.53646e+00i	4.29988e+02 + 2.80077e+02i	–6.57322e+05 – 1.48582e+06i	–6.10816e+08 + 4.00147e+09i	–3.22801e+10 – 4.84343e+11i
0,6	50∠60°	Ob	–8.30279e+02 + 1.91488e+03i	–3.12500e–01 + 2.77556e–17i	–3.25082e–01 + 3.03336e–02i	5.75422e+00 + 4.22932e+00i	–2.19909e+03 – 3.33917e+01i	–2.74553e+07 – 7.19755e+05i	–2.00381e+11 – 1.04005e+09i	1.07616e+13 + 6.51877e+12i
0,6	50∠75°	Ob	–1.69543e+03 + 1.12049e+03i	–3.12500e–01 – 4.16334e–17i	–3.39202e–01 + 1.80066e–02i	–3.16580e+00 + 1.13097e+01i	7.59414e+03 – 7.86121e+02i	–3.23486e+08 + 6.84355e+07i	7.84607e+12 – 2.69652e+12i	1.28043e+15 – 3.08708e+14i
0,6	50∠90°	Ob	–1.82596e+03 + 2.63234e–13i	–3.12500e–01 + 1.78227e–17i	–3.41468e–01 + 2.37143e–17i	–1.12340e+01 + 4.10027e–15i	–6.75818e+03 + 5.59062e–12i	–2.36319e+08 + 3.77887e–07i	–3.36671e+12 + 8.02168e–03i	–1.37048e+14 + 4.04455e–01i
0,6	100∠0°	Pr	1.27787e+03 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–2.92747e–01 – 0.00000e+00i	2.94597e–01 – 0.00000e+00i	1.45587e–01 – 0.00000e+00i	4.65378e–03 – 0.00000e+00i	2.05597e–10 – 0.00000e+00i	7.51580e–31 – 0.00000e+00i*
0,6	100∠15°	Pr	1.23359e+03 + 3.36568e+02i	–3.12500e–01 – 0.00000e+00i	–2.93409e–01 + 5.15041e–03i	3.29709e–01 – 5.32610e–02i	1.81235e–01 + 3.52377e–01i	–6.32052e–03 – 3.98069e–03i	–5.26527e–11 – 6.53598e–10i	–1.18962e–29 – 7.48989e–30i*
0,6	100∠30°	Pr	7.68559e+02 + 4.50069e+02i	–3.12500e–01 – 0.00000e+00i	–3.00543e–01 + 6.94069e–03i	3.96464e–01 + 7.97879e–02i	–7.01179e–01 + 1.20207e–01i	–2.36528e–03 – 2.01105e–03i	6.46389e–10 – 9.12937e–11i	3.23126e–28 – 5.62632e–28i*
0,6	100∠45°	Ob	4.19238e+02 + 9.57571e+03i	–3.12500e–01 + 6.93889e–16i	–2.94089e–01 + 1.48202e–01i	–9.61869e+01 + 9.70036e+01i	–1.39375e+06 – 3.26859e+06i	5.39329e+13 – 5.62404e+13i	1.26512e+21 + 4.71659e+20i	–1.26640e+26 + 7.22173e+26i
0,6	100∠60°	Ob	–4.14712e+03 + 8.16017e+03i	–3.12500e–01 + 1.11022e–16i	–3.70539e–01 + 1.36262e–01i	–1.56511e+02 – 5.22571e+02i	–1.11998e+08 – 6.50197e+06i	–7.12375e+16 – 8.15831e+15i	–2.70957e+25 – 4.26059e+24i	–1.40886e+30 + 2.57725e+31i
0,6	100∠75°	Ob	–7.70749e+03 + 4.74114e+03i	–3.12500e–01 – 1.38778e–17i	–4.37712e–01 + 8.39239e–02i	1.26071e+03 + 7.75780e+02i	–1.33594e+09 + 2.59237e+08i	–9.77680e+18 + 3.95440e+18i	–4.06657e+28 + 2.69024e+28i	3.88359e+35 – 1.43354e+33i
0,6	100∠90°	Ob	–8.62546e+03 + 1.13889e–12i	–3.12500e–01 + 2.18917e–17i	–4.57247e–01 + 5.24984e–17i	–1.69154e+03 + 1.15569e–12i	–1.88150e+09 + 3.01600e–06i	–1.85406e+19 + 5.81544e+04i	–8.09075e+28 + 3.78214e+14i	–2.88721e+34 + 1.62385e+20i
0,6	250∠0°	Pr	3.22810e+03 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–2.63416e–01 – 0.00000e+00i	–2.37061e–01 – 0.00000e+00i	1.42780e–01 – 0.00000e+00i	1.70438e–10 – 0.00000e+00i	3.11597e–31 – 0.00000e+00i	1.35195e–85 – 0.00000e+00i*
0,6	250∠15°	Pr	3.11737e+03 + 8.41201e+02i	–3.12500e–01 – 0.00000e+00i	–2.64965e–01 + 1.24645e–02i	–2.80291e–01 – 1.77191e–01i	8.93820e–02 – 1.71747e–01i	3.68687e–12 – 5.36343e–10i	–3.53218e–30 – 4.32723e–30i	–3.90256e–83 + 1.99946e–82i*
0,6	250∠30°	Pr	1.93776e+03 + 1.12503e+03i	–3.12500e–01 – 0.00000e+00i	–2.82553e–01 + 1.70033e–02i	1.65639e–01 – 4.09525e–01i	9.06721e–02 + 3.14799e–02i	5.76070e–10 + 2.06949e–11i	3.05568e–28 – 1.36338e–28i	–1.25248e–75 – 6.81914e–76i*
0,6	250∠45°	Ob	1.05565e+03 + 6.14393e+04i	–3.12500e–01 – 1.07553e–15i	1.79265e–01 + 8.44061e–01i	–2.14930e+06 – 5.08936e+06i	–1.14203e+18 + 2.47599e+17i	–7.67132e+36 + 3.49130e+36i	–3.85567e+55 + 2.88502e+55i	–2.52871e+72 – 2.37895e+72i
0,6	250∠60°	Ob	–2.90980e+04 + 5.28766e+04i	–3.12500e–01 – 3.19189e–16i	–4.14152e–01 + 1.21119e+00i	–2.39331e+08 – 1.55959e+07i	–1.41886e+22 – 2.32920e+21i	–1.11848e+45 – 3.76453e+44i	–5.20918e+67 – 2.72113e+67i	–4.79957e+86 – 1.05848e+87i
0,6	250∠75°	Ob	–5.17248e+04 + 3.06029e+04i	–3.12500e–01 – 2.35922e–16i	–1.33809e+00 + 9.93436e–01i	–2.86247e+09 + 5.44472e+08i	6.59539e+24 – 3.34939e+24i	–1.84140e+50 + 2.52230e+50i	1.25413e+75 – 7.89192e+75i	7.86951e+96 + 4.41539e+97i
0,6	250∠90°	Ob	–5.90252e+04 + 7.43972e–12i	–3.12500e–01 – 2.79215e–17i	–1.78777e+00 + 1.09781e–16i	–5.57391e+09 + 8.03631e–06i	–3.70909e+25 + 1.38663e+11i	–7.25176e+51 + 5.48707e+37i	–6.53708e+77 + 7.44974e+63i	4.03904e+98 – 5.99049e+84i
0,6	500∠0°	Pr	6.47818e+03 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–2.16677e–01 – 0.00000e+00i	–1.12306e–01 – 0.00000e+00i	5.59639e–04 – 0.00000e+00i	4.15427e–24 – 0.00000e+00i	4.47942e–67 – 0.00000e+00i*	6.05218e–178 – 0.00000e+00i*
0,6	500∠15°	Pr	6.25670e+03 + 1.68234e+03i	–3.12500e–01 + 2.71051e–20i	–2.19430e–01 + 2.36032e–02i	–1.75242e–01 + 3.04484e–01i	–9.66682e–04 + 1.05080e–04i	–2.76433e–23 + 3.00292e–23i	8.19763e–65 + 1.17742e–64i*	–1.25875e–171 + 5.46609e–172i*
0,6	500∠30°	Pr	3.88634e+03 + 2.25001e+03i	–3.12500e–01 – 0.00000e+00i	–2.53113e–01 + 3.28702e–02i	–6.49757e–01 – 2.02234e–01i	–3.69442e–04 + 2.29078e–04i	–3.13777e–22 + 5.07439e–22i	6.51345e–60 + 1.87217e–59i*	–9.85423e–156 + 5.49975e–157i*
0,6	500∠45°	Ob	2.11632e+03 + 2.47879e+05i	–3.12500e–01 – 2.13718e–15i	4.80880e+00 – 1.99547e+00i	1.83498e+14 – 1.88242e+14i	–1.66306e+37 + 7.51793e+36i	–1.35825e+75 + 1.54354e+75i	–5.35970e+112 + 1.75637e+113i	7.22273e+147 – 3.69753e+148i
0,6	500∠60°	Ob	–1.20683e+05 + 2.14006e+05i	–3.12500e–01 – 8.32667e–16i	9.04571e+00 + 6.75383e+00i	–6.01661e+17 – 7.99457e+16i	–4.38184e+45 – 1.50219e+45i	–9.68109e+91 – 7.51602e+91i	–1.08852e+138 – 1.64954e+138i	1.86420e+180 + 8.22804e+179i
0,6	500∠75°	Ob	–2.11690e+05 + 1.23706e+05i	–3.12500e–01 – 6.73073e–16i	–5.08143e+00 + 1.78930e+01i	–8.33952e+19 + 3.28182e+19i	–7.26493e+50 + 9.89074e+50i	2.57448e+102 + 8.20759e+102i	3.47674e+154 + 1.18330e+154i	4.83962e+201 + 2.91234e+201i
0,6	500∠90°	Ob	–2.43025e+05 + 3.01875e–11i	–3.12500e–01 + 8.63197e–18i	–2.16185e+01 + 7.21556e–15i	–4.02368e+20 + 1.24307e+06i	–5.19341e+52 + 3.98961e+38i	–1.42459e+106 + 2.18484e+92i	–1.82386e+159 + 4.19340e+145i	–2.56588e+206 + 7.81665e+192i
0,6	750∠0°	Pr	9.72820e+03 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–1.72535e–01 – 0.00000e+00i	3.24644e–01 – 0.00000e+00i	7.39661e–07 – 0.00000e+00i	4.07748e–38 – 0.00000e+00i	2.66093e–103 – 0.00000e+00i*	1.13375e–270 – 0.00000e+00i*
0,6	750∠15°	Pr	9.39598e+03 + 2.52350e+03i	–3.12500e–01 – 0.00000e+00i	–1.76173e–01 + 3.34824e–02i	4.35365e–01 + 2.29464e–01i	9.73310e–07 + 1.36514e–06i	1.24947e–36 – 9.60577e–38i	–7.65517e–100 – 1.31760e–99i*	–3.27668e–261 – 2.05830e–261i*
0,6	750∠30°	Pr	5.83491e+03 + 3.37501e+03i	–3.12500e–01 – 0.00000e+00i	–2.24348e–01 + 4.76353e–02i	–5.34247e–01 + 3.29863e–01i	1.75639e–07 – 1.02615e–06i	–1.60810e–34 – 3.01763e–34i	–6.36074e–91 + 1.59503e–91i*	–3.14506e–236 + 2.19263e–236i*
0,6	750∠45°	Ob	3.17698e+03 + 5.59318e+05i	–3.12500e–01 + 4.23273e–15i	–1.69813e+01 – 2.53095e+01i	1.16423e+22 + 4.56717e+21i	–2.29165e+56 + 1.69262e+56i	–1.47307e+113 + 4.78863e+113i	2.31904e+170 + 6.59093e+170i	3.54894e+224 – 1.63099e+224i
0,6	750∠60°	Ob	–2.74768e+05 + 4.83389e+05i	–3.12500e–01 + 1.25594e–15i	8.07267e+01 – 5.62352e+01i	–1.50465e+27 – 3.02951e+26i	–1.30955e+69 – 7.09852e+68i	–6.92141e+138 – 1.06200e+139i	–5.44558e+207 – 6.56161e+208i	–2.16311e+273 + 1.83586e+273i
0,6	750∠75°	Ob	–4.79908e+05 + 2.79309e+05i	–3.12500e–01 + 1.14841e–10i	7.53307e+01 + 1.94007e+02i	–2.33042e+30 + 1.46786e+30i	3.34868e+76 – 2.00401e+77i	2.23036e+155 + 7.66475e+154i	–7.94246e+232 + 1.46927e+233i	3.62648e+305 – 2.60136e+305i
0,6	750∠90°	Ob	–5.52025e+05 + 6.82434e–11i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1000∠0°	Pr	1.29782e+04 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–1.30899e–01 – 0.00000e+00i	4.06063e–01 – 0.00000e+00i	6.54064e–10 – 0.00000e+00i	2.77921e–52 – 0.00000e+00i	1.10447e–139 – 0.00000e+00i*	±0
0,6	1000∠15°	Pr	1.25353e+04 + 3.36466e+03i	–3.12500e–01 + 1.35525e–20i	–1.35128e–01 + 4.21648e–02i	5.24092e–01 – 8.05328e–02i	8.52208e–10 – 1.74196e–09i	–2.08466e–50 – 1.67315e–50i	4.84988e–135 + 1.02190e–134i*	±0
0,6	1000∠30°	Pr	7.78347e+03 + 4.50001e+03i	–3.12500e–01 + 2.71051e–20i	–1.96256e–01 + 6.13319e–02i	–1.24289e–01 + 3.97297e–01i	1.20074e–09 + 1.52465e–09i	1.54118e–46 + 6.17596e–48i	–2.65027e–123 – 1.69036e–122i*	–4.67980e–317 + 1.07901e–316i*
0,6	1000∠45°	Ob	4.23764e+03 + 9.95757e+05i	–3.12500e–01 + 3.99680e–15i	–1.25929e+02 + 1.26561e+02i	1.48786e+28 + 5.94738e+29i	–2.94168e+75 + 3.33452e+75i	1.52219e+151 + 1.21097e+152i	2.19680e+228 + 1.49454e+228i	3.50736e+300 + 1.92713e+300i
0,6	1000∠60°	Ob	–4.91353e+05 + 8.61025e+05i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1000∠75°	Ob	–8.56379e+05 + 4.97412e+05i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1000∠90°	Ob	–9.86025e+05 + 1.21607e–10i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1500∠0°	Pr	1.94782e+04 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	–5.47919e–02 – 0.00000e+00i	1.70557e–01 – 0.00000e+00i	2.93425e–16 – 0.00000e+00i	7.66959e–81 – 0.00000e+00i	1.13717e–212 – 0.00000e+00i*	±0
0,6	1500∠15°	Pr	1.88138e+04 + 5.04698e+03i	–3.12500e–01 – 1.35525e–20i	–5.94173e–02 + 5.61811e–02i	1.28233e–01 – 1.89122e–01i	7.47242e–16 + 1.29108e–15i	4.16755e–78 – 5.90480e–78i	1.01277e–205 + 3.58495e–205i*	±0
0,6	1500∠30°	Pr	1.16806e+04 + 6.75000e+03i	–3.12500e–01 – 0.00000e+00i	–1.42090e–01 + 8.56487e–02i	1.00578e–01 + 6.11989e–02i	3.42771e–15 – 3.26051e–15i	–1.00624e–71 – 1.98523e–71i	–2.14210e–187 + 8.27481e–186i*	±0
0,6	1500∠45°	Ob	6.35896e+03 + 2.24364e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1500∠60°	Ob	–1.11202e+06 + 1.94106e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1500∠75°	Ob	–1.93408e+06 + 1.12112e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	1500∠90°	Ob	–2.22903e+06 + 2.74260e–10i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	2500∠0°	Pr	3.24782e+04 – 0.00000e+00i	–3.12500e–01 – 0.00000e+00i	7.10742e–02 – 0.00000e+00i	6.87109e–03 – 0.00000e+00i	2.27608e–29 – 0.00000e+00i	2.34256e–138 – 0.00000e+00i*	±0	±0
0,6	2500∠15°	Pr	3.13708e+04 + 8.41162e+03i	–3.12500e–01 – 0.00000e+00i	6.81968e–02 + 7.23973e–02i	–7.61475e–03 – 7.50883e–03i	2.12376e–28 – 2.66757e–28i	–1.00648e–133 + 1.86595e–133i*	±0	±0
0,6	2500∠30°	Pr	1.94748e+04 + 1.12500e+04i	–3.12500e–01 – 0.00000e+00i	–4.17713e–02 + 1.22852e–01i	–1.95565e–03 – 4.15261e–03i	–8.98335e–27 + 1.22069e–26i	2.52171e–121 + 2.17497e–122i*	–4.03222e–313 + 9.81068e–313i*	±0
0,6	2500∠45°	Ob	1.06016e+04 + 6.23939e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.52: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
0,6	$2500\angle 60^\circ$	Ob	−3.10336e+06 + 5.40016e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$2500\angle 75^\circ$	Ob	−5.38852e+06 + 3.11853e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$2500\angle 90^\circ$	Ob	−6.21503e+06 + 7.63261e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$5000\angle 0^\circ$	Pr	6.49782e+04 − 0.00000e+00i	−3.12500e−01 − 0.00000e+00i	2.59540e−01 − 0.00000e+00i	2.36445e−07 − 0.00000e+00i	6.23078e−63 − 0.00000e+00i	6.52470e−283 − 0.00000e+00i*	± 0	± 0
0,6	$5000\angle 15^\circ$	Pr	6.27634e+04 + 1.68232e+04i	−3.12500e−01 − 3.38813e−21i	2.69344e−01 + 6.21847e−02i	4.53357e−07 + 3.26797e−07i	−1.19640e−60 − 7.16576e−61i	−5.23540e−273 − 1.05479e−273i*	± 0	± 0
0,6	$5000\angle 30^\circ$	Pr	3.89604e+04 + 2.25000e+04i	−3.12500e−01 − 3.38813e−21i	1.63441e−01 + 1.59922e−01i	−1.65633e−07 − 3.36738e−07i	8.30636e−56 + 1.74080e−56i	−6.88780e−247 + 8.31729e−247i*	± 0	± 0
0,6	$5000\angle 45^\circ$	Ob	2.12082e+04 + 2.49788e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$5000\angle 60^\circ$	Ob	−1.24567e+07 + 2.16256e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$5000\angle 75^\circ$	Ob	−2.16024e+07 + 1.24871e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,6	$5000\angle 90^\circ$	Ob	−2.49300e+07 + 3.05733e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.52: $S^{(a)}_{0n}(c,\eta)$ for $n = [0..6]$.

Table D.53: $S^{(a)}_{1n}(c,\eta)$ for $n = [1..6]$.

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
1,1	$25\angle 0^\circ$	Pr	2.52634e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.98788e−01 − 0.00000e+00i	8.85490e−01 − 0.00000e+00i	4.62940e−01 − 0.00000e+00i	3.89730e−02 − 0.00000e+00i	3.05244e−04 − 3.09994e−36i*	3.08826e−09 − 3.81590e−41i*
1,1	$25\angle 15^\circ$	Pr	2.44110e+01 + 6.46678e+00i	1.00000e+00 − 4.33681e−19i	9.98830e−01 − 3.22969e−04i	8.88814e−01 − 2.88210e−02i	4.65649e−01 − 9.69846e−02i	2.82950e−02 − 3.32878e−02i	−2.36008e−04 − 3.32102e−04i*	4.67252e−09 + 4.49191e−09i*
1,1	$25\angle 30^\circ$	Pr	2.19119e+01 + 1.24930e+01i	1.00000e+00 − 8.67362e−19i	9.98955e−01 − 6.24011e−04i	8.98727e−01 − 5.63533e−02i	4.74939e−01 − 1.98953e−01i	−6.26500e−03 − 6.07430e−02i	−4.42149e−04 + 8.40700e−04i*	−1.83679e−08 + 5.38431e−08i*
1,1	$25\angle 45^\circ$	Pr	1.79365e+01 + 1.76680e+01i	1.00000e+00 + 1.73472e−18i	9.99153e−01 − 8.82672e−04i	9.15016e−01 − 8.12477e−02i	4.94660e−01 − 3.10830e−01i	−7.43330e−02 − 7.27687e−02i	3.48447e−03 + 1.09081e−03i*	−1.47669e−06 − 1.01023e−06i*
1,1	$25\angle 60^\circ$	Pr	1.27558e+01 + 2.16391e+01i	1.00000e+00 + 3.46945e−18i	9.99412e−01 − 1.08135e−03i	9.37243e−01 − 1.02061e−01i	5.32402e−01 − 4.36837e−01i	−2.02147e−01 − 5.00632e−02i	1.06703e−02 − 1.82130e−02i*	1.48724e−04 + 4.43354e−05i*
1,1	$25\angle 75^\circ$	Ob	−4.46714e+02 + 2.86606e+02i	1.00000e+00 + 2.77556e−16i	1.02244e+00 − 1.44379e−02i	3.69672e+00 − 2.73319e+00i	−6.34369e+00 − 1.28466e+02i	−3.31031e+04 + 3.24745e+03i	1.13039e+06 + 7.70068e+06i	4.17456e+08 − 5.40169e+07i
1,1	$25\angle 90^\circ$	Ob	−5.27043e+02 + 7.04144e−14i	1.00000e+00 + 1.02726e−16i	1.02652e+00 + 1.01899e−16i	5.03933e+00 − 2.39599e−16i	1.59089e+02 − 4.46354e−14i	5.08908e+04 − 3.37745e−11i	1.47320e+07 − 1.54191e−08i	9.77365e+08 − 1.38222e−06i
1,1	$50\angle 0^\circ$	Pr	5.02565e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.97540e−01 − 0.00000e+00i	7.81224e−01 − 0.00000e+00i	2.09342e−01 − 0.00000e+00i	1.36971e−03 − 0.00000e+00i	6.46214e−08 + 5.50096e−40i*	1.55813e−18 − 0.00000e+00i*
1,1	$50\angle 15^\circ$	Pr	4.85525e+01 + 1.29392e+01i	1.00000e+00 + 2.16840e−19i	9.97625e−01 − 6.45441e−04i	7.86268e−01 − 5.10679e−02i	2.02590e−01 − 8.82620e−02i	−2.78753e−04 − 1.69825e−03i	−3.75015e−08 + 1.08781e−07i*	7.13874e−19 + 6.71032e−18i*
1,1	$50\angle 30^\circ$	Pr	4.35568e+01 + 2.49967e+01i	1.00000e+00 − 4.33681e−19i	9.97874e−01 − 1.24721e−03i	8.01571e−01 − 1.00964e−01i	1.81585e−01 − 1.84642e−01i	−3.28897e−03 + 6.91574e−04i	−3.56919e−07 − 5.12490e−07i*	−4.24401e−16 − 2.56473e−16i*
1,1	$50\angle 45^\circ$	Pr	3.56098e+01 + 3.53507e+01i	1.00000e+00 − 8.67362e−19i	9.98269e−01 − 1.76452e−03i	8.27573e−01 − 1.48198e−01i	1.44458e−01 − 3.00377e−01i	2.30002e−04 + 9.74404e−03i	7.60554e−06 + 5.18993e−06i*	2.16172e−13 + 4.07110e−13i*
1,1	$50\angle 60^\circ$	Pr	2.52530e+01 + 4.32957e+01i	1.00000e+00 + 8.67362e−19i	9.98786e−01 − 2.16221e−03i	8.64764e−01 − 1.90676e−01i	9.01803e−02 − 4.54233e−01i	3.45750e−02 + 1.81231e−02i	−1.52548e−04 − 2.66363e−04i*	2.90804e−09 + 1.76138e−09i*
1,1	$50\angle 75^\circ$	Ob	−1.97390e+03 + 1.19823e+03i	1.00000e−00 − 0.00000e+00i	1.09977e+00 − 6.19027e−02i	1.40084e+01 − 4.94158e+01i	−5.38436e+04 + 5.11871e+03i	5.76761e+09 − 1.10428e+09i	−5.53458e+14 + 1.61157e+14i	9.95254e+18 − 2.51972e+18i
1,1	$50\angle 90^\circ$	Ob	−2.30202e+03 + 2.93914e−13i	1.00000e−00 − 2.25101e−17i	1.11738e+00 − 4.04192e−17i	6.09079e+01 − 2.00237e−14i	8.27931e+04 − 6.52562e−11i	1.37611e+10 − 2.13814e−05i	2.06795e+15 − 4.79619e+00i	5.54323e+19 − 1.69300e+05i
1,1	$100\angle 0^\circ$	Pr	1.00253e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.95050e−01 − 0.00000e+00i	6.08046e−01 − 0.00000e+00i	4.27929e−02 − 0.00000e+00i	1.68901e−06 − 0.00000e+00i	2.87965e−15 − 5.56977e−47i*	3.56475e−37 + 4.70247e−69i*
1,1	$100\angle 15^\circ$	Pr	9.68456e+01 + 2.58811e+01i	1.00000e−00 − 1.08420e−19i	9.95218e−01 − 1.28790e−03i	6.13323e−01 − 8.00161e−02i	3.24665e−02 − 3.49227e−02i	−2.52598e−06 + 8.53356e−07i	−7.19663e−15 − 5.61512e−15i*	−6.47924e−36 + 1.54208e−36i*
1,1	$100\angle 30^\circ$	Pr	8.68553e+01 + 4.99984e+01i	1.00000e−00 − 4.33681e−19i	9.95713e−01 − 2.48927e−03i	6.29968e−01 − 1.61273e−01i	−1.10394e−03 − 6.54750e−02i	9.30263e−06 − 4.10188e−06i	−9.25274e−14 + 2.52318e−13i*	1.77926e−32 + 3.08106e−32i*
1,1	$100\angle 45^\circ$	Pr	7.09629e+01 + 7.07084e+01i	1.00000e−00 − 0.00000e+00i	9.96502e−01 − 3.52316e−03i	6.60435e−01 − 2.44399e−01i	−6.77384e−02 − 8.47169e−02i	−8.53834e−05 + 5.42673e−11i*	2.14771e−11 + 5.42673e−11i*	−1.55942e−26 + 2.59472e−26i*
1,1	$100\angle 60^\circ$	Pr	5.02515e+01 + 8.65998e+01i	1.00000e+00 − 0.00000e+00i	9.97531e−01 − 4.31943e−03i	7.08787e−01 − 3.28577e−01i	−1.93519e−01 − 7.99300e−02i	7.81371e−04 + 1.12626e−03i	−3.25399e−08 + 5.60238e−08i*	8.31001e−19 + 1.38785e−18i*
1,1	$100\angle 75^\circ$	Ob	−8.27589e+03 + 4.89647e+03i	1.00000e−00 − 2.22045e−16i	1.43236e+00 − 2.79859e−01i	−5.47569e+03 − 3.38099e+03i	9.33208e+09 − 1.77378e+09i	1.68400e+20 − 6.63805e+19i	2.63607e+30 − 1.66382e+30i	5.29586e+39 − 2.83621e+39i
1,1	$100\angle 90^\circ$	Ob	−9.60201e+03 + 1.20015e−12i	1.00000e+00 − 4.19768e−17i	1.51984e+00 − 1.33884e−16i	9.04779e+03 − 5.92050e−12i	2.22643e+10 − 3.50198e−05i	9.94403e+20 − 3.08643e+06i	4.01391e+31 − 1.86032e+17i	1.75242e+41 − 1.06973e+27i
1,1	$250\angle 0^\circ$	Pr	2.50251e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.87614e−01 − 0.00000e+00i	2.86683e−01 − 0.00000e+00i	3.65426e−04 − 0.00000e+00i	3.16299e−15 − 0.00000e+00i	2.34864e−37 − 0.00000e+00i	3.28224e−93 + 8.28941e−125i*
1,1	$250\angle 15^\circ$	Pr	2.41733e+02 + 6.47044e+01i	1.00000e+00 − 0.00000e+00i	9.88030e−01 − 3.19659e−03i	2.83590e−01 − 9.53451e−02i	−2.22791e−04 − 4.23961e−04i	−7.20473e−15 − 6.79358e−15i	−4.52268e−36 − 3.84136e−37i*	3.39654e−90 + 4.99750e−90i*
1,1	$250\angle 30^\circ$	Pr	2.16757e+02 + 1.24999e+02i	1.00000e+00 + 1.08420e−19i	9.89251e−01 − 6.18302e−03i	2.74678e−01 − 1.98831e−01i	−7.16202e−04 + 7.79444e−04i	−1.42618e−13 + 2.42278e−13i	−1.80613e−33 + 1.96696e−32i	8.40001e−81 − 5.48064e−81i
1,1	$250\angle 45^\circ$	Pr	1.77028e+02 + 1.76776e+02i	1.00000e+00 + 1.08420e−19i	9.91198e−01 − 8.76144e−03i	2.61711e−01 − 3.20544e−01i	2.93010e−03 + 2.32052e−03i	6.98380e−12 + 5.71961e−11i	−1.46357e−26 + 2.34966e−27i	4.22364e−66 − 7.14354e−66i*
1,1	$250\angle 60^\circ$	Pr	1.25251e+02 + 2.16505e+02i	1.00000e+00 − 0.00000e+00i	9.93748e−01 − 1.07583e−02i	2.50352e−01 − 4.74435e−01i	1.60583e−02 − 1.07928e−02i	−4.41300e−08 + 3.96397e−08i	−2.83174e−19 + 4.86476e−19i	−1.16092e−46 + 7.89182e−47i
1,1	$250\angle 75^\circ$	Ob	−5.31627e+04 + 3.09912e+04i	1.00000e−00 + 3.33067e−16i	4.41116e+00 − 3.27798e+00i	1.24047e+10 − 2.35206e+09i	−4.56890e+25 + 2.31187e+25i	3.11441e+51 − 4.23588e+51i	−8.04354e+76 + 4.81082e+77i	4.01450e+101 − 1.11915e+102i
1,1	$250\angle 90^\circ$	Ob	−6.15020e+04 + 7.59281e−12i	1.00000e−00 + 9.35674e−17i	6.01256e+00 − 3.45023e−16i	2.95941e+10 − 4.25345e−05i	4.30715e+26 − 1.60808e+12i	3.72001e+53 − 2.81254e+39i	2.90300e+80 − 3.30583e+66i	5.46643e+105 − 8.23329e+91i
1,1	$500\angle 0^\circ$	Pr	5.00251e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.75346e−01 − 0.00000e+00i	8.18786e−02 − 0.00000e+00i	1.30366e−07 − 0.00000e+00i	8.99568e−30 − 0.00000e+00i	4.58735e−74 − 1.71106e−105i*	2.31076e−186 − 0.00000e+00i*
1,1	$500\angle 15^\circ$	Pr	4.83214e+02 + 1.29409e+02i	1.00000e+00 − 0.00000e+00i	9.76157e−01 − 6.31644e−03i	7.10650e−02 − 5.38752e−02i	−1.27009e−07 + 1.84418e−07i	5.18180e−30 + 8.80192e−29i	1.44421e−71 + 2.50195e−72i*	−1.92028e−180 + 4.86575e−180i*
1,1	$500\angle 30^\circ$	Pr	4.33263e+02 + 2.50000e+02i	1.00000e+00 − 0.00000e+00i	9.78542e−01 − 1.22327e−02i	3.57789e−02 − 1.08820e−01i	−9.23709e−08 − 1.08991e−06i	−3.44986e−26 + 6.07969e−65i*	−3.29090e−64 − 6.07969e−65i*	1.04739e−161 − 2.07171e−161i*
1,1	$500\angle 45^\circ$	Pr	3.53804e+02 + 3.53553e+02i	1.00000e−00 − 5.42101e−20i	9.82361e−01 − 1.73681e−02i	−3.41284e−02 − 1.67150e−01i	3.12507e−06 + 1.32749e−05i	−2.89725e−21 + 7.18880e−22i	1.42869e−52 − 4.28279e−53i*	−4.36735e−132 − 8.30553e−132i*
1,1	$500\angle 60^\circ$	Pr	2.50250e+02 + 4.33012e+02i	1.00000e+00 − 0.00000e+00i	9.87383e−01 − 2.13813e−02i	−1.61804e−01 − 2.36658e−01i	1.37998e−04 − 3.38381e−04i	3.37365e−16 − 3.14542e−15i	−1.29894e−37 − 2.28243e−37i*	1.44629e−93 − 3.91871e−93i*
1,1	$500\angle 75^\circ$	Ob	−2.14577e+05 + 1.24482e+05i	1.00000e+00 + 2.44249e−15i	1.67592e+01 − 5.90076e+01i	3.61012e+20 − 1.41949e+20i	5.02035e+51 − 6.82460e+51i	−4.28435e+103 − 1.37388e+104i	−2.08799e+156 − 7.19875e+155i	−6.14116e+205 − 5.06451e+205i
1,1	$500\angle 90^\circ$	Ob	−2.48002e+05 + 3.04937e−11i	1.00000e−00 − 1.06122e−17i	7.27428e+01 − 2.30410e−14i	2.13131e+21 − 6.54790e+06i	5.99475e+53 − 4.59479e+39i	7.20523e+107 − 1.10375e+94i	7.82420e+161 − 1.79744e+148i	1.68277e+213 − 5.10228e+199i
1,1	$750\angle 0^\circ$	Pr	7.50250e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.63230e−01 − 0.00000e+00i	2.33851e−02 − 0.00000e+00i	4.65032e−11 − 0.00000e+00i	2.55824e−44 − 0.00000e+00i	9.25553e−111 − 6.71346e−143i*	1.48131e−279 + 5.72134e−311i*
1,1	$750\angle 15^\circ$	Pr	7.24695e+02 + 1.94114e+02i	1.00000e+00 + 1.35525e−20i	9.64416e−01 − 9.36087e−03i	1.49602e−02 − 2.19714e−02i	1.03948e−10 + 1.24551e−11i	5.04025e−43 − 6.01846e−43i	−4.97125e−107 − 1.24735e−107i	−4.75817e−270 + 1.13357e−270i*
1,1	$750\angle 30^\circ$	Pr	6.49769e+02 + 3.75000e+02i	1.00000e+00 + 2.71051e−20i	9.67911e−01 − 1.81509e−02i	−1.17649e−02 − 3.68654e−02i	8.93897e−10 + 6.91707e−10i	1.79571e−38 + 4.49770e−40i	1.47217e−96 − 5.23084e−96i*	−3.75207e−243 − 4.42650e−242i*
1,1	$750\angle 45^\circ$	Pr	5.30580e+02 + 5.30330e+02i	1.00000e+00 − 0.00000e+00i	9.73525e−01 − 2.58212e−02i	−6.22761e−02 − 3.26818e−02i	−2.11307e−08 + 4.50502e−08i	−5.51806e−32 − 1.44458e−31i	−1.53658e−78 + 8.22930e−79i*	−1.23020e−197 − 2.64286e−200i*
1,1	$750\angle 60^\circ$	Pr	3.75250e+02 + 6.49519e+02i	1.00000e−00 − 5.42101e−20i	9.80943e−01 − 3.18691e−02i	−1.52213e−01 + 1.74526e−02i	−1.40214e−06 − 6.75820e−06i	9.87333e−23 + 1.36790e−22i	1.21401e−55 + 1.10027e−57i	2.77779e−140 + 1.05240e−139i*
1,1	$750\angle 75^\circ$	Ob	−4.84244e+05 + 2.80474e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,1	$750\angle 90^\circ$	Ob	−5.59502e+05 + 6.87027e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,1	$1000\angle 0^\circ$	Pr	1.00025e+03 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	9.51264e−01 − 0.00000e+00i	6.67893e−03 − 0.00000e+00i	1.65890e−14 − 0.00000e+00i	7.27515e−59 − 0.00000e+00i	1.72152e−147 + 1.86555e−179i*	± 0
1,1	$1000\angle 15^\circ$	Pr	9.66176e+02 + 2.58819e+02i	1.00000e+00 − 0.00000e+00i	9.52806e−01 − 1.23312e−02i	2.13962e−03 − 7.62852e−03i	−1.74532e−14 − 4.57295e−14i	−6.94091e−57 + 8.20352e−58i	1.65445e−142 + 5.81050e−143i*	± 0
1,1	$1000\angle 30^\circ$	Pr	8.66276e+02 + 5.00000e+02i	1.00000e+00 − 0.00000e+00i	9.57358e−01 − 2.39395e−02i	−1.05219e−02 − 7.75762e−03i	−1.15127e−12 + 1.96573e−13i	−2.40005e−51 + 3.85404e−51i	7.97365e−128 + 3.07205e−128i*	−4.94066e−323 − 6.42285e−323i*
1,1	$1000\angle 45^\circ$	Pr	7.07357e+02 + 7.07107e+02i	1.00000e−00 − 2.71051e−20i	9.64694e−01 − 3.41222e−02i	−2.66737e−02 + 1.13663e−02i	−1.62488e−10 + 8.09965e−11i	7.08117e−42 − 3.74552e−42i	1.59501e−104 − 1.18329e−104i*	−8.11717e−264 + 1.34583e−263i*
1,1	$1000\angle 60^\circ$	Pr	5.00250e+02 + 8.66025e+02i	1.00000e−00 − 5.42101e−20i	9.74432e−01 − 4.22216e−02i	−2.97142e−02 + 7.62968e−02i	−9.31853e−08 − 9.11634e−08i	−8.79196e−30 − 1.90675e−30i	−2.77129e−74 + 4.67085e−74i*	−2.09559e−186 − 1.77096e−186i*
1,1	$1000\angle 75^\circ$	Ob	−8.62164e+05 + 4.98965e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,1	$1000\angle 90^\circ$	Ob	−9.96002e+05 + 1.22220e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.53: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
1,1	$1500\angle 0^\circ$	Pr	1.50025e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.27777e−01 − 0.00000e+00i	5.44807e−04 − 0.00000e+00i	2.11102e−21 − 0.00000e+00i	5.88349e−88 − 0.00000e+00i	5.97565e−221 − 0.00000e+00i*	±0
1,1	$1500\angle 15^\circ$	Pr	1.44914e+03 + 3.88229e+02i	1.00000e+00 − 0.00000e+00i	9.29975e−01 − 1.80549e−02i	−2.57964e−04 − 6.54923e−04i	1.03963e−20 + 2.52762e−21i	−9.72638e−86 − 5.45403e−85i	1.71670e−213 + 9.54459e−214i*	±0
1,1	$1500\angle 30^\circ$	Pr	1.29929e+03 + 7.50000e+02i	1.00000e+00 − 1.35525e−20i	9.36487e−01 − 3.51356e−02i	−1.21606e−03 + 8.64175e−04i	3.12967e−19 + 1.20716e−18i	2.89699e−76 + 1.45065e−77i	−1.74735e−191 − 1.08135e−191i*	±0
1,1	$1500\angle 45^\circ$	Pr	1.06091e+03 + 1.06066e+03i	1.00000e+00 − 2.71051e−20i	9.47050e−01 − 5.02733e−02i	2.79966e−03 + 4.05528e−03i	−1.54531e−15 − 1.85848e−15i	−1.60214e−62 + 1.43328e−62i	1.31946e−156 − 1.87347e−156i*	±0
1,1	$1500\angle 60^\circ$	Pr	7.50250e+02 + 1.29904e+03i	1.00000e+00 − 0.00000e+00i	9.61198e−01 − 6.25212e−02i	2.27782e−02 − 5.29311e−03i	−4.26651e−11 + 1.85009e−11i	−8.05542e−45 + 2.42824e−44i	1.05302e−110 + 1.86037e−112i*	−1.46567e−279 + 8.38968e−280i*
1,1	$1500\angle 75^\circ$	Ob	−1.94276e+06 + 1.12345e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$1500\angle 90^\circ$	Ob	−2.24400e+06 + 2.75178e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$2500\angle 0^\circ$	Pr	2.50025e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	8.82527e−01 − 0.00000e+00i	3.62506e−06 − 0.00000e+00i	3.41850e−35 − 0.00000e+00i	4.12808e−146 + 2.12833e−177i*	±0	±0
1,1	$2500\angle 15^\circ$	Pr	2.41506e+03 + 6.47048e+02i	1.00000e+00 − 0.00000e+00i	8.85830e−01 − 2.86694e−02i	−5.52719e−06 + 5.64475e−07i	−6.42940e−35 − 5.07150e−34i	1.11601e−141 + 3.68430e−141i*	±0	±0
1,1	$2500\angle 30^\circ$	Pr	2.16531e+03 + 1.25000e+03i	1.00000e+00 − 0.00000e+00i	8.95679e−01 − 5.60544e−02i	1.94259e−05 + 3.39638e−07i	−5.83368e−31 − 1.29658e−30i	−7.18564e−127 + 1.03554e−126i*	−6.74592e−319 − 8.71314e−319i*	±0
1,1	$2500\angle 45^\circ$	Pr	1.76802e+03 + 1.76777e+03i	1.00000e+00 − 0.00000e+00i	9.11864e−01 − 8.08107e−02i	−1.20317e−04 − 7.60602e−05i	3.92052e−25 + 1.72598e−25i	−5.70819e−104 + 1.54111e−103i*	−7.24286e−262 − 2.86782e−260i*	±0
1,1	$2500\angle 60^\circ$	Pr	1.25025e+03 + 2.16506e+03i	1.00000e+00 + 1.35525e−20i	9.33947e−01 − 1.01502e−01i	−2.71960e−04 + 1.88806e−03i	5.52739e−18 + 2.11379e−18i	1.05649e−73 − 1.78290e−73i	−1.92323e−184 + 3.10586e−184i*	±0
1,1	$2500\angle 75^\circ$	Ob	−5.40300e+06 + 3.12241e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$2500\angle 90^\circ$	Ob	−6.24000e+06 + 7.64792e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$5000\angle 0^\circ$	Pr	5.00025e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	7.78825e−01 − 0.00000e+00i	1.30916e−11 − 0.00000e+00i	1.14075e−69 − 0.00000e+00i	1.75672e−291 − 1.48220e−323i*	±0	±0
1,1	$5000\angle 15^\circ$	Pr	4.82988e+03 + 1.29410e+03i	1.00000e+00 − 0.00000e+00i	7.83844e−01 − 5.07906e−02i	3.01175e−11 − 6.21647e−12i	−2.47034e−67 + 6.36591e−68i	−1.19551e−281 + 7.97400e−282i*	±0	±0
1,1	$5000\angle 30^\circ$	Pr	4.33038e+03 + 2.50000e+03i	1.00000e+00 − 0.00000e+00i	7.99069e−01 − 1.00410e−01i	3.75832e−10 + 1.31458e−11i	−1.30883e−60 + 1.47670e−60i	−5.74852e−253 − 1.52633e−252i*	±0	±0
1,1	$5000\angle 45^\circ$	Pr	3.53578e+03 + 3.53553e+03i	1.00000e+00 − 0.00000e+00i	8.24934e−01 − 1.47371e−01i	8.65835e−09 + 1.82338e−08i	1.20961e−49 + 1.32107e−49i	−2.05753e−206 − 1.77232e−206i*	±0	±0
1,1	$5000\angle 60^\circ$	Pr	2.50025e+03 + 4.33013e+03i	1.00000e+00 + 6.77626e−21i	8.61923e−01 − 1.89588e−01i	−3.47769e−06 − 1.02309e−06i	2.54621e−35 + 2.28102e−35i	−2.31993e−146 − 4.08736e−146i*	±0	±0
1,1	$5000\angle 75^\circ$	Ob	−2.16313e+07 + 1.24948e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$5000\angle 90^\circ$	Ob	−2.49800e+07 + 3.06039e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$25\angle 0^\circ$	Pr	7.42764e+01 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.99644e−02 − 0.00000e+00i	2.66321e−01 − 0.00000e+00i	3.52859e−01 − 0.00000e+00i	6.26976e−02 − 0.00000e+00i	7.74206e−04 − 6.91810e−36i*	1.53291e−08 − 0.00000e+00i*
1,2	$25\angle 15^\circ$	Pr	7.17195e+01 + 1.94035e+01i	−0.00000e+00 − 0.00000e+00i	2.99657e−02 − 9.69090e−06i	2.67320e−01 − 8.66966e−03i	3.54920e−01 − 7.39348e−02i	4.55110e−02 − 5.35577e−02i	−5.99000e−04 − 8.41977e−04i*	2.32650e−08 + 2.22792e−08i*
1,2	$25\angle 30^\circ$	Pr	6.42230e+01 + 3.74853e+01i	−0.00000e+00 − 0.00000e+00i	2.99694e−02 − 1.87240e−05i	2.70301e−01 − 1.69517e−02i	3.61989e−01 − 1.51668e−01i	−1.01076e−02 − 9.77118e−02i	−1.15774e−03 + 2.21125e−03i*	−9.27957e−08 + 2.78792e−07i*
1,2	$25\angle 45^\circ$	Pr	5.22985e+01 + 5.30134e+01i	−0.00000e+00 − 0.00000e+00i	2.99753e−02 − 2.64856e−05i	2.75199e−01 − 2.44404e−02i	3.77000e−01 − 2.36950e−01i	−1.19620e−01 − 1.16997e−01i	9.15922e−03 + 2.83980e−03i*	−7.73106e−06 − 5.08549e−06i*
1,2	$25\angle 60^\circ$	Ob	−2.27933e+02 + 4.91244e+02i	−0.00000e+00 − 0.00000e+00i	3.01150e−02 − 2.46193e−04i	3.59117e−01 − 2.99729e−01i	−7.43606e+00 − 4.87315e+00i	1.02150e+03 + 6.89341e+02i	−1.26826e+05 − 8.82758e+04i	3.36493e+06 + 3.12291e+06i
1,2	$25\angle 75^\circ$	Ob	−4.46714e+02 + 2.86606e+02i	−0.00000e+00 − 0.00000e+00i	3.02252e−02 − 1.43951e−04i	5.53580e−01 − 2.17611e−01i	3.91573e+00 − 1.62865e+01i	−4.25568e+03 − 8.07416e+02i	−1.40973e+05 + 1.00373e+06i	5.41463e+07 + 8.54966e+06i
1,2	$25\angle 90^\circ$	Ob	−5.27043e+02 + 7.04144e−14i	−0.00000e+00 − 0.00000e+00i	3.02657e−02 + 2.13730e−18i	6.45338e−01 − 1.15500e−17i	2.07907e+01 − 5.08756e−15i	6.65083e+03 − 4.17529e−12i	1.92530e+06 − 1.94602e−09i	1.27730e+08 − 1.76057e−07i
1,2	$50\angle 0^\circ$	Pr	1.49262e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.99270e−02 − 0.00000e+00i	2.34959e−01 − 0.00000e+00i	1.59552e−01 − 0.00000e+00i	2.20283e−03 − 0.00000e+00i	1.69548e−07 − 2.62848e−39i*	7.94953e−18 − 1.39986e−49i*
1,2	$50\angle 15^\circ$	Pr	1.44151e+02 + 3.88195e+01i	−0.00000e+00 − 0.00000e+00i	2.99295e−02 − 1.93646e−05i	2.36476e−01 − 1.53598e−02i	1.54404e−01 − 6.72726e−02i	−4.48533e−04 − 2.73114e−03i	−9.82402e−08 + 2.85444e−07i*	3.74827e−18 + 3.42160e−17i*
1,2	$50\angle 30^\circ$	Pr	1.29164e+02 + 7.49936e+01i	−0.00000e+00 − 0.00000e+00i	2.99370e−02 − 3.74190e−05i	2.41078e−01 − 3.03670e−02i	1.38390e−01 − 1.40731e−01i	−5.28909e−03 + 1.11306e−03i	−9.37570e−07 − 1.34336e−06i*	−2.17112e−15 − 1.29337e−15i*
1,2	$50\angle 45^\circ$	Pr	1.05324e+02 + 1.06057e+02i	−0.00000e+00 − 0.00000e+00i	2.99488e−02 − 5.29395e−05i	2.48898e−01 − 4.45737e−02i	1.10085e−01 − 2.28936e−01i	3.73634e−04 + 1.56694e−02i	1.99633e−05 + 1.35807e−05i*	1.11939e−12 + 2.06089e−12i*
1,2	$50\angle 60^\circ$	Ob	−1.07881e+03 + 2.06505e+03i	−0.00000e+00 − 0.00000e+00i	3.05331e−02 − 1.04371e−03i	−7.28142e−01 − 1.79562e+00i	8.09251e+02 + 5.25251e+02i	2.46802e+07 + 1.71566e+07i	6.79215e+11 + 5.05325e+11i	2.74733e+15 + 3.62370e+15i
1,2	$50\angle 75^\circ$	Ob	−1.97390e+03 + 1.19823e+03i	−0.00000e+00 − 0.00000e+00i	3.09946e−02 − 6.11044e−04i	1.67342e+00 − 2.73599e+00i	−3.32331e+03 − 5.97927e+02i	3.65338e+08 + 3.06218e+07i	−3.59860e+13 + 3.79724e+11i	6.40754e+17 + 1.59149e+16i
1,2	$50\angle 90^\circ$	Ob	−2.30202e+03 + 2.93914e−13i	−0.00000e+00 − 0.00000e+00i	3.11659e−02 − 1.11306e−18i	3.80790e+00 − 1.04099e−15i	5.17684e+03 − 3.79319e−12i	8.60443e+08 − 1.28921e−06i	1.29304e+14 − 2.92722e−01i	3.46604e+18 − 1.03937e+04i
1,2	$100\angle 0^\circ$	Pr	2.99256e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.82873e−01 − 0.00000e+00i	3.26137e−02 − 0.00000e+00i	2.71589e−06 − 0.00000e+00i	7.54774e−15 + 1.59915e−46i*	1.80701e−36 − 0.00000e+00i*	
1,2	$100\angle 15^\circ$	Pr	2.89033e+02 + 7.76441e+01i	−0.00000e+00 − 0.00000e+00i	2.98573e−02 − 3.86384e−05i	1.84460e−01 − 2.40656e−02i	2.47434e−02 − 2.66159e−02i	−4.06164e−06 + 1.37235e−06i	−1.88661e−14 − 1.47121e−14i*	−3.28230e−35 + 7.87445e−36i*
1,2	$100\angle 30^\circ$	Pr	2.59063e+02 + 1.49997e+02i	−0.00000e+00 − 0.00000e+00i	2.98721e−02 − 7.46809e−05i	1.89466e−01 − 4.85044e−02i	−8.42321e−04 − 4.99002e−02i	1.49575e−05 − 6.59688e−06i	−2.42146e−13 + 6.61378e−13i*	9.06655e−32 + 1.55733e−31i*
1,2	$100\angle 45^\circ$	Pr	2.11386e+02 + 2.12128e+02i	−0.00000e+00 − 0.00000e+00i	2.98958e−02 − 1.05699e−04i	1.98629e−01 − 7.35050e−02i	−5.16267e−02 − 6.45632e−02i	−1.37287e−04 + 6.65332e−06i	5.63806e−11 + 1.42155e−10i*	−7.82335e−26 + 1.31680e−25i*
1,2	$100\angle 60^\circ$	Ob	−4.65560e+03 + 8.46025e+03i	−0.00000e+00 − 0.00000e+00i	3.21993e−02 − 4.42688e−03i	−1.76048e+01 + 7.01799e+01i	1.97107e+07 + 1.34616e+07i	2.94599e+16 + 2.30950e+16i	3.95074e+25 + 3.54097e+25i	2.51618e+33 + 8.95134e+33i
1,2	$100\angle 75^\circ$	Ob	−8.27589e+03 + 4.89647e+03i	−0.00000e+00 − 0.00000e+00i	3.42507e−02 − 2.65633e−03i	−1.34274e+02 − 1.43986e+02i	2.89697e+08 + 2.30209e+07i	5.50525e+18 − 5.98844e+17i	9.12221e+28 − 2.78087e+28i	1.79175e+38 − 4.09245e+37i
1,2	$100\angle 90^\circ$	Ob	−9.60201e+03 + 1.20015e−12i	−0.00000e+00 − 0.00000e+00i	3.50386e−02 − 2.37599e−18i	2.77002e+02 − 1.65888e−13i	6.81631e+08 − 1.03432e−06i	3.04441e+19 − 9.28032e+04i	1.22888e+30 − 5.62726e+15i	5.36512e+39 − 3.24526e+25i
1,2	$250\angle 0^\circ$	Pr	7.49252e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.96292e−02 − 0.00000e+00i	8.62211e−02 − 0.00000e+00i	2.78496e−04 − 0.00000e+00i	5.08549e−15 − 0.00000e+00i	6.36257e−37 − 0.00000e+00i	1.70742e−92 − 0.00000e+00i*
1,2	$250\angle 15^\circ$	Pr	7.23697e+02 + 1.94114e+02i	−0.00000e+00 − 0.00000e+00i	2.96416e−02 − 9.59002e−05i	8.52910e−02 − 2.86755e−02i	−1.69793e−04 − 3.23105e−04i	−1.15840e−14 − 1.09225e−14i	−1.22572e−35 − 1.03908e−36i*	1.77572e−89 + 2.60813e−89i*
1,2	$250\angle 30^\circ$	Pr	6.48771e+02 + 3.74993e+02i	−0.00000e+00 − 0.00000e+00i	2.96783e−02 − 1.85496e−04i	8.26106e−02 − 5.97995e−02i	−5.45821e−04 + 5.94027e−04i	−2.29286e−13 + 3.89541e−13i	−4.88741e−33 + 5.32848e−32i	4.37815e−80 − 2.83954e−80i*
1,2	$250\angle 45^\circ$	Pr	5.29582e+02 + 5.30328e+02i	−0.00000e+00 − 0.00000e+00i	2.97367e−02 − 2.62850e−04i	7.87105e−02 − 9.64051e−02i	2.23308e−03 + 1.76846e−03i	1.12330e−11 + 9.19580e−11i	−3.96556e−26 + 6.38439e−27i*	2.19535e−65 − 3.73216e−65i*
1,2	$250\angle 60^\circ$	Ob	−3.03860e+04 + 5.36266e+04i	−0.00000e+00 − 0.00000e+00i	3.88678e−02 − 3.48179e−02i	1.04057e+07 + 7.02498e+06i	9.24631e+20 + 7.68058e+20i	1.56641e+44 + 1.81513e+44i	2.24462e+67 + 3.69210e+67i	−2.00673e+89 + 1.69456e+89i
1,2	$250\angle 75^\circ$	Ob	−5.31627e+04 + 3.09912e+04i	−0.00000e+00 − 0.00000e+00i	6.11644e−02 − 2.52265e−02i	1.52243e+08 + 1.16705e+07i	−6.06345e+23 + 1.25783e+23i	4.97227e+49 − 3.96291e+49i	−2.45666e+75 + 5.36287e+75i	8.21729e+99 − 1.17996e+100i
1,2	$250\angle 90^\circ$	Ob	−6.15020e+04 + 7.59281e−12i	−0.00000e+00 − 0.00000e+00i	7.17207e−02 − 4.98740e−19i	3.57999e+08 − 4.94947e−07i	5.21035e+24 − 1.91677e+10i	4.50009e+51 − 3.37769e+37i	3.51175e+78 − 3.97983e+64i	6.61272e+103 − 9.92360e+89i
1,2	$500\angle 0^\circ$	Pr	1.49925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.92611e−02 − 0.00000e+00i	2.46253e−02 − 0.00000e+00i	9.93482e−08 − 0.00000e+00i	1.44628e−29 − 0.00000e+00i	1.28092e−73 − 1.00232e−105i*	1.24172e−185 + 1.03743e−217i*
1,2	$500\angle 15^\circ$	Pr	1.44814e+03 + 3.88228e+02i	−0.00000e+00 − 0.00000e+00i	2.92854e−02 − 1.89498e−04i	2.13731e−02 − 1.62032e−02i	−9.67940e−08 + 1.40546e−07i	8.33235e−30 + 1.41513e−28i	3.91317e−71 + 6.76541e−72i*	−9.99507e−180 + 2.53764e−179i*
1,2	$500\angle 30^\circ$	Pr	1.29829e+03 + 7.49999e+02i	−0.00000e+00 − 0.00000e+00i	2.93570e−02 − 3.66990e−04i	1.07606e−02 − 3.27279e−02i	−7.03997e−08 + 8.30626e−07i	−5.54667e−26 − 9.98881e−26i	−8.91787e−64 − 1.64507e−64i*	5.45114e−161 − 1.08066e−160i*
1,2	$500\angle 45^\circ$	Pr	1.05991e+03 + 1.06066e+03i	−0.00000e+00 − 0.00000e+00i	2.94716e−02 − 5.21055e−04i	−1.02643e−02 − 5.02710e−02i	2.38169e−06 + 1.01168e−05i	−4.65798e−21 + 1.15589e−21i	3.87732e−52 − 1.19871e−52i*	−2.32732e−131 − 4.32562e−131i*
1,2	$500\angle 60^\circ$	Ob	−1.23270e+05 + 2.15506e+05i	−0.00000e+00 − 0.00000e+00i	−8.79850e−02 − 2.06129e−01i	1.25113e+16 + 9.67524e+15i	1.25984e+44 + 1.45530e+44i	4.59716e+90 + 1.14494e+91i	3.57515e+136 + 7.13582e+137i	−5.63871e+181 − 1.49865e+182i
1,2	$500\angle 75^\circ$	Ob	−2.14577e+05 + 1.24482e+05i	−0.00000e+00 − 0.00000e+00i	1.89825e−01 − 3.17034e−01i	2.32195e+18 − 2.60654e+17i	3.98810e+49 − 3.18379e+49i	−3.41835e+100 − 8.66158e+101i	−1.10181e+154 − 7.45481e+153i	−2.77936e+203 − 3.90683e+203i
1,2	$500\angle 90^\circ$	Ob	−2.48002e+05 + 3.04937e−11i	−0.00000e+00 − 0.00000e+00i	4.38170e−01 − 1.14780e−16i	1.28393e+19 − 3.87411e+04i	3.61131e+51 − 2.74815e+37i	4.34052e+105 − 6.62532e+91i	4.71339e+159 − 1.08021e+146i	1.01372e+211 − 3.06811e+197i
1,2	$750\angle 0^\circ$	Pr	2.24925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.88976e−02 − 0.00000e+00i	7.03316e−03 − 0.00000e+00i	3.54403e−11 − 0.00000e+00i	4.11297e−44 − 0.00000e+00i	2.43534e−110 − 6.65367e−142i*	7.49770e−279 − 1.36416e−310i*
1,2	$750\angle 15^\circ$	Pr	2.17258e+03 + 5.82343e+02i	−0.00000e+00 − 0.00000e+00i	2.89332e−02 − 2.80833e−04i	4.49933e−03 − 6.60800e−03i	7.92193e−11 + 9.49200e−12i	8.10332e−43 − 9.67613e−43i	−1.36147e−106 − 3.63916e−107i*	−2.52543e−269 + 5.61414e−270i*
1,2	$750\angle 30^\circ$	Pr	1.94781e+03 + 1.12500e+03i	−0.00000e+00 − 0.00000e+00i	2.90381e−02 − 5.44541e−04i	−3.53833e−03 − 1.10874e−02i	6.81244e−10 + 5.27150e−10i	2.88701e−38 + 7.22763e−40i	3.98476e−96 − 1.41760e−95i*	−1.97303e−242 − 2.30701e−241i*
1,2	$750\angle 45^\circ$	Pr	1.59024e+03 + 1.59099e+03i	−0.00000e+00 − 0.00000e+00i	2.92065e−02 − 7.74655e−04i	−1.87298e−02 − 9.82916e−03i	−1.61036e−08 + 3.43329e−08i	−8.87189e−32 − 2.32248e−31i	−4.12487e−78 + 2.21187e−78i*	−6.35283e−197 − 6.48012e−200i*
1,2	$750\angle 60^\circ$	Ob	−2.78654e+05 + 4.85639e+05i	−0.00000e+00 − 0.00000e+00i	−1.29621e+00 + 1.08459e−01i	1.99460e+25 + 1.76352e+25i	2.14639e+67 + 3.51492e+67i	4.28347e+136 + 8.47092e+137i	−6.82431e+207 + 1.42293e+208i	1.30363e+275 − 2.27680e+273i
1,2	$750\angle 75^\circ$	Ob	−4.84244e+05 + 2.80474e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$750\angle 90^\circ$	Ob	−5.59502e+05 + 6.87027e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 0^\circ$	Pr	2.99925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.85386e−02 − 0.00000e+00i	2.00872e−03 − 0.00000e+00i	1.26425e−14 − 0.00000e+00i	1.16964e−58 − 0.00000e+00i	4.64974e−147 − 5.13260e−179i*	±0
1,2	$1000\angle 15^\circ$	Pr	2.89703e+03 + 7.76457e+02i	−0.00000e+00 − 0.00000e+00i	2.85849e−02 − 3.69946e−04i	6.43498e−04 − 2.29431e−03i	−1.33012e−14 − 3.48506e−14i	−1.11591e−56 + 1.31895e−57i	4.48951e−142 + 1.57919e−142i*	±0
1,2	$1000\angle 30^\circ$	Pr	2.59733e+03 + 1.50000e+03i	−0.00000e+00 − 0.00000e+00i	2.87215e−02 − 7.18204e−04i	−3.16451e−03 − 2.33314e−03i	−8.77388e−13 + 1.49811e−13i	−3.85855e−51 + 6.19626e−51i	2.16109e−127 + 8.31427e−128i*	−2.56914e−322 + 3.45846e−322i*
1,2	$1000\angle 45^\circ$	Pr	2.12057e+03 + 2.12132e+03i	−0.00000e+00 − 0.00000e+00i	2.89416e−02 − 1.02369e−03i	−8.02221e−03 + 3.41847e−03i	−1.23832e−10 + 6.17279e−11i	1.13844e−41 − 6.02187e−42i	4.32063e−104 − 3.20827e−104i*	−4.22307e−263 + 7.01612e−263i*
1,2	$1000\angle 60^\circ$	Ob	−4.96538e+05 + 8.64025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 75^\circ$	Ob	−8.62164e+05 + 4.98965e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 90^\circ$	Ob	−9.96002e+05 + 1.22220e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.53: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
1,2	$1500\angle 0^\circ$	Pr	4.49925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.78340e−02 − 0.00000e+00i	1.63853e−04 − 0.00000e+00i	1.60881e−21 − 0.00000e+00i	9.45899e−88 − 0.00000e+00i	1.58482e−220 − 0.00000e+00i*	±0
1,2	$1500\angle 15^\circ$	Pr	4.34592e+03 + 1.16469e+03i	−0.00000e+00 − 0.00000e+00i	2.79000e−02 − 5.41660e−04i	−7.75838e−05 − 1.96971e−04i	7.92306e−21 + 1.92630e−21i	−1.56375e−85 − 8.76851e−85i	4.65362e−213 + 2.58574e−213i*	±0
1,2	$1500\angle 30^\circ$	Pr	3.89636e+03 + 2.25000e+03i	−0.00000e+00 − 0.00000e+00i	2.80953e−02 − 1.05409e−03i	−3.65734e−04 + 2.59904e−04i	2.38514e−19 + 9.19976e−19i	4.65752e−76 + 2.33197e−77i	−4.73352e−191 − 2.92310e−191i*	±0
1,2	$1500\angle 45^\circ$	Pr	3.18123e+03 + 3.18198e+03i	−0.00000e+00 − 0.00000e+00i	2.84122e−02 − 1.50824e−03i	8.42008e−04 + 1.21964e−03i	−1.17768e−15 − 1.41635e−15i	−2.57576e−62 + 2.30432e−62i	3.57330e−156 − 5.07834e−156i*	±0
1,2	$1500\angle 60^\circ$	Ob	−1.11981e+06 + 1.94556e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1500\angle 75^\circ$	Ob	−1.94276e+06 + 1.12345e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1500\angle 90^\circ$	Ob	−2.24400e+06 + 2.75178e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 0^\circ$	Pr	7.49925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.64765e−02 − 0.00000e+00i	1.09025e−06 − 0.00000e+00i	2.60524e−35 − 0.00000e+00i	6.59355e−146 − 1.50635e−177i*	±0	±0
1,2	$2500\angle 15^\circ$	Pr	7.24369e+03 + 1.94114e+03i	−0.00000e+00 − 0.00000e+00i	2.65756e−02 − 8.60105e−04i	−1.66232e−06 + 1.69768e−07i	−4.89987e−35 − 3.86500e−34i	1.79488e−141 + 5.92395e−141i*	±0	±0
1,2	$2500\angle 30^\circ$	Pr	6.49444e+03 + 3.75000e+03i	−0.00000e+00 − 0.00000e+00i	2.68711e−02 − 1.68167e−03i	5.84242e−06 + 1.02147e−07i	−4.44586e−31 − 9.88126e−31i	−1.15533e−126 + 1.66385e−126i*	−1.82702e−318 − 2.36018e−318i*	±0
1,2	$2500\angle 45^\circ$	Pr	5.30255e+03 + 5.30330e+03i	−0.00000e+00 − 0.00000e+00i	2.73566e−02 − 2.42438e−03i	−3.61858e−05 − 2.28754e−05i	2.98783e−25 + 1.31537e−25i	−9.17501e−104 + 2.47778e−103i*	−1.99528e−261 − 7.77015e−260i*	±0
1,2	$2500\angle 60^\circ$	Ob	−3.11634e+06 + 5.40766e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 75^\circ$	Ob	−5.40300e+06 + 3.12241e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 90^\circ$	Ob	−6.24000e+06 + 7.64792e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 0^\circ$	Pr	1.49993e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.33653e−02 − 0.00000e+00i	3.93736e−12 − 0.00000e+00i	8.69369e−70 − 0.00000e+00i	2.95095e−291 − 0.00000e+00i*	±0	±0
1,2	$5000\angle 15^\circ$	Pr	1.44881e+04 + 3.88229e+03i	−0.00000e+00 − 0.00000e+00i	2.35159e−02 − 1.52376e−03i	9.05796e−12 − 1.86963e−12i	−1.88265e−67 + 4.85146e−68i	−1.92216e−281 + 1.28247e−281i*	±0	±0
1,2	$5000\angle 30^\circ$	Pr	1.29896e+04 + 7.50000e+03i	−0.00000e+00 − 0.00000e+00i	2.39727e−02 − 3.01237e−03i	1.13033e−10 + 3.95364e−12i	−9.97461e−61 + 1.12540e−60i	−9.18585e−253 − 2.44444e−252i*	±0	±0
1,2	$5000\angle 45^\circ$	Pr	1.06059e+04 + 1.06066e+04i	−0.00000e+00 − 0.00000e+00i	2.47486e−02 − 4.42125e−03i	2.60403e−09 + 5.48390e−09i	9.21843e−50 + 1.00679e−49i	−3.30912e−206 − 2.84888e−206i*	±0	±0
1,2	$5000\angle 60^\circ$	Ob	−1.24827e+07 + 2.16406e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 75^\circ$	Ob	−2.16313e+07 + 1.24948e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 90^\circ$	Ob	−2.49800e+07 + 3.06039e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$25\angle 0^\circ$	Pr	1.22245e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.49092e+00 − 0.00000e+00i	−6.80885e−01 − 0.00000e+00i	1.47941e+00 − 0.00000e+00i	7.57038e−01 − 0.00000e+00i	1.65761e−02 + 2.79784e−34i*	6.54483e−07 − 7.05259e−39i*
1,3	$25\angle 15^\circ$	Pr	1.17985e+02 + 3.23516e+01i	−1.50000e+00 − 6.93889e−18i	−1.49123e+00 + 2.42137e−03i	−7.00649e−01 + 1.96322e−01i	1.53278e+00 + 2.89456e−01i	7.14482e−01 − 4.64397e−01i	−7.44355e−03 − 2.08808e−02i*	7.05249e−07 + 1.18602e−06i*
1,3	$25\angle 30^\circ$	Pr	1.05495e+02 + 6.24998e+01i	−1.50000e+00 − 6.93889e−18i	−1.49217e+00 + 4.67889e−03i	−7.60586e−01 + 3.87790e−01i	1.69133e+00 + 6.42757e−01i	5.50747e−01 − 1.07216e+00i	−4.46596e−02 + 2.65878e−02i*	−9.27179e−06 + 8.00595e−06i*
1,3	$25\angle 45^\circ$	Ob	6.86826e+01 + 5.54258e+02i	−1.50000e+00 − 2.22045e−16i	−1.49473e+00 + 4.15235e−02i	7.47072e−01 + 3.37713e+00i	1.08517e+01 − 3.68027e+01i	1.65504e+03 + 1.07317e+03i	−6.97896e+04 + 5.94462e+04i	−2.47514e+05 − 1.02063e+06i
1,3	$25\angle 60^\circ$	Ob	−2.27933e+02 + 4.91244e+02i	−1.50000e+00 + 9.71445e−17i	−1.51705e+00 + 3.69849e−02i	−1.71135e+00 + 4.98661e+00i	1.03453e+02 + 1.22618e+00i	−1.43346e+04 − 3.63694e+02i	1.79657e+06 + 7.15363e+04i	−5.25557e+07 − 9.55511e+06i
1,3	$25\angle 75^\circ$	Ob	−3.56423e+02 + 2.60632e+02i	−1.50000e+00 − 2.77556e−16i	−1.52684e+00 + 1.96646e−02i	−4.49203e+00 + 3.30743e+00i	6.67785e+00 + 1.11870e+02i	1.53504e+04 − 1.85966e+03i	−2.79605e+05 − 1.37699e+06i	9.90898e+06 + 2.28359e+06i
1,3	$25\angle 90^\circ$	Ob	−4.33359e+02 + 6.42691e−14i	−1.50000e+00 − 1.80091e−16i	−1.53270e+00 − 1.79161e−16i	−6.13251e+00 + 1.81276e−16i	−1.38364e+02 + 3.67617e−14i	−2.35631e+04 + 1.53584e−11i	−2.63341e+06 + 2.74512e−09i	2.32882e+07 − 3.05850e−08i
1,3	$50\angle 0^\circ$	Pr	2.47245e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.48157e+00 − 0.00000e+00i	−1.17996e−02 − 0.00000e+00i	1.68230e+00 − 0.00000e+00i	5.61128e−02 − 0.00000e+00i	7.20200e−06 + 1.22264e−37i*	6.63294e−16 + 6.45421e−48i*
1,3	$50\angle 15^\circ$	Pr	2.38727e+02 + 6.47056e+01i	−1.50000e+00 − 0.00000e+00i	−1.48220e+00 + 4.83240e−03i	−3.23371e−02 + 3.10197e−01i	1.78235e+00 − 1.72584e−01i	7.92549e−03 − 7.01877e−02i	−7.25072e−06 + 1.05911e−05i*	−4.42444e−16 + 2.83910e−15i*
1,3	$50\angle 30^\circ$	Pr	2.13752e+02 + 1.25002e+02i	−1.50000e+00 − 3.46945e−18i	−1.48406e+00 + 9.33970e−03i	−9.78894e−02 + 6.26151e−01i	2.11747e+00 − 3.65439e−01i	−1.30656e−01 − 4.65064e−02i	−5.27990e−06 − 7.21826e−05i*	−1.06366e−13 − 1.91279e−13i*
1,3	$50\angle 45^\circ$	Ob	1.39407e+02 + 2.35856e+03i	−1.50000e+00 + 1.55431e−15i	−1.48616e+00 + 1.76463e−01i	1.95297e+01 − 8.12872e+00i	2.66853e+03 + 1.75950e+03i	−5.37165e+06 − 1.25674e+07i	−9.48817e+09 + 5.19635e+10i	3.79239e+13 − 1.83997e+13i
1,3	$50\angle 60^\circ$	Ob	−1.07881e+03 + 2.06505e+03i	−1.50000e+00 − 1.02696e−15i	−1.57903e+00 + 1.57668e−01i	3.75047e+01 + 2.79444e+01i	−2.32728e+04 − 7.22015e+02i	−7.23994e+08 − 4.55043e+07i	−2.03395e+13 − 1.93689e+12i	−1.02026e+17 − 4.04429e+16i
1,3	$50\angle 75^\circ$	Ob	−1.78686e+03 + 1.14643e+03i	−1.50000e+00 − 5.55112e−17i	−1.63527e+00 + 8.85667e−02i	−1.69828e+01 + 6.03074e+01i	4.77129e+04 − 4.74620e+03i	−2.78159e+09 + 5.60949e+08i	1.08746e+14 − 3.44128e+13i	8.27873e+16 + 2.19916e+16i
1,3	$50\angle 90^\circ$	Ob	−2.10817e+03 + 2.81658e−13i	−1.50000e+00 + 7.70664e−17i	−1.66099e+00 + 1.07213e−16i	−7.42815e+01 + 2.65942e−14i	−7.33486e+04 + 6.00058e−11i	−6.63964e+09 + 1.05236e−05i	−4.07739e+14 + 9.59682e−01i	4.34920e+17 − 1.28762e+03i
1,3	$100\angle 0^\circ$	Pr	4.97247e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.46295e+00 − 0.00000e+00i	9.07498e−01 − 0.00000e+00i	7.58132e−01 − 0.00000e+00i	1.41956e−04 − 0.00000e+00i	6.73029e−13 − 0.00000e+00i*	3.13224e−34 + 2.14177e−66i*
1,3	$100\angle 15^\circ$	Pr	4.80210e+02 + 1.29410e+02i	−1.50000e+00 − 8.67362e−19i	−1.46420e+00 + 9.62275e−03i	9.14807e−01 + 3.67420e−01i	7.28759e−01 − 4.32982e−01i	−2.23914e−04 + 1.28891e−05i	−1.28046e−12 − 1.70748e−12i*	−5.84934e−33 − 1.56444e−34i*
1,3	$100\angle 30^\circ$	Pr	4.30260e+02 + 2.50001e+02i	−1.50000e+00 − 0.00000e+00i	−1.46789e+00 + 1.86064e−02i	9.28877e−01 + 7.74167e−01i	6.17110e−01 − 1.00084e+00i	8.51185e−04 + 1.03344e−04i	−4.85203e−11 + 4.00440e−11i*	9.10443e−32 + 3.12464e−29i*
1,3	$100\angle 45^\circ$	Ob	2.80836e+02 + 9.71715e+03i	−1.50000e+00 − 2.45637e−15i	−1.42024e+00 + 7.23508e−01i	−5.11509e+02 + 5.14741e+02i	−8.60449e+06 − 2.03305e+07i	4.53892e+14 − 4.67580e+14i	1.61998e+22 + 6.30095e+21i	−4.21889e+28 + 5.23162e+28i
1,3	$100\angle 60^\circ$	Ob	−4.65560e+03 + 8.46025e+03i	−1.50000e+00 − 1.16573e−15i	−1.81619e+00 + 6.83642e−01i	−1.01005e+03 − 3.40856e+03i	−1.17028e+09 − 7.65623e+07i	−1.82350e+18 − 2.40272e+17i	−2.55637e+27 − 5.09930e+26i	−3.30233e+35 − 3.15702e+35i
1,3	$100\angle 75^\circ$	Ob	−7.89559e+03 + 4.79292e+03i	−1.50000e+00 + 4.44089e−16i	−2.11676e+00 + 4.08453e−01i	6.70390e+03 + 4.13181e+03i	−8.33450e+09 + 1.60171e+09i	−8.26654e+19 + 3.30249e+19i	−5.39824e+29 + 3.48875e+29i	4.63610e+36 + 1.10753e+37i
1,3	$100\angle 90^\circ$	Ob	−9.20808e+03 + 1.17566e−12i	−1.50000e+00 + 1.06487e−16i	−2.24536e+00 + 2.61750e−16i	−1.10704e+04 + 7.56735e−12i	−1.98863e+10 + 3.18659e−05i	−4.88714e+20 + 1.53159e+06i	−8.26376e+30 + 3.85607e+16i	1.35131e+37 − 3.39523e+20i
1,3	$250\angle 0^\circ$	Pr	1.24725e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.40757e+00 − 0.00000e+00i	1.72444e+00 − 0.00000e+00i	1.70860e−02 − 0.00000e+00i	6.74610e−13 − 0.00000e+00i	1.42869e−34 − 0.00000e+00i	7.39656e−90 − 0.00000e+00i*
1,3	$250\angle 15^\circ$	Pr	1.20466e+03 + 3.23524e+02i	−1.50000e+00 − 0.00000e+00i	−1.41062e+00 + 2.37441e−02i	1.81901e+00 + 4.23333e−03i	−4.73861e−03 − 2.19145e−02i	−1.10504e−12 − 1.80073e−12i	−2.51094e−33 − 9.10039e−34i*	4.35813e−87 + 1.24867e−86i*
1,3	$250\angle 30^\circ$	Pr	1.07978e+03 + 6.25001e+02i	−1.50000e+00 + 8.67362e−19i	−1.41961e+00 + 4.59731e−02i	2.12421e+00 + 4.00202e−02i	−4.77054e−02 + 1.40578e−02i	−5.23945e−11 + 2.93229e−11i	−6.96308e−30 + 9.80201e−30i	2.25015e−77 − 8.88875e−79i*
1,3	$250\angle 45^\circ$	Ob	7.05104e+02 + 6.17929e+04i	−1.50000e+00 − 1.88738e−15i	8.68452e−01 + 4.09502e+00i	−1.13988e+07 − 2.70232e+07i	−7.08584e+18 + 1.52845e+18i	−6.39979e+37 + 2.89419e+37i	−4.94361e+56 + 3.65664e+56i	−1.63904e+74 − 2.14730e+74i
1,3	$250\angle 60^\circ$	Ob	−3.03860e+04 + 5.36266e+04i	−1.50000e+00 + 6.66134e−16i	−2.04784e+00 + 5.99689e+00i	−1.55509e+09 − 1.03075e+08i	−1.47172e+23 − 2.45849e+22i	−2.81448e+46 − 9.67728e+45i	−4.71498e+69 − 2.55692e+69i	1.09314e+91 − 3.07167e+91i
1,3	$250\angle 75^\circ$	Ob	−5.22028e+04 + 3.07324e+04i	−1.50000e+00 + 8.74301e−16i	−6.48700e+00 + 4.81758e+00i	−1.52056e+10 + 2.88806e+09i	4.09862e+25 − 2.07792e+25i	−1.54337e+51 + 2.10692e+51i	1.65606e+76 − 1.01613e+77i	−9.72356e+98 + 5.48036e+99i
1,3	$250\angle 90^\circ$	Ob	−6.05080e+04 + 7.53158e−12i	−1.50000e+00 − 1.33390e−16i	−8.84171e+00 + 5.47697e−16i	−3.62768e+10 + 5.23104e−05i	−3.86493e+26 + 1.44484e+12i	−1.84749e+53 + 1.39774e+39i	−6.12409e+79 + 6.97731e+65i	−2.46555e+103 + 3.72483e+89i
1,3	$500\angle 0^\circ$	Pr	2.49725e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.31693e+00 − 0.00000e+00i	1.10969e+00 − 0.00000e+00i	1.70860e−02 − 0.00000e+00i	3.85620e−27 − 0.00000e+00i	5.50539e−71 − 1.08360e−102i*	1.02813e−182 − 1.08115e−214i*
1,3	$500\angle 15^\circ$	Pr	2.41206e+03 + 6.47048e+02i	−1.50000e+00 − 0.00000e+00i	−1.32277e+00 + 4.64600e−02i	1.13684e+00 − 4.25207e−01i	−1.62871e−05 + 1.37582e−05i	−7.66841e−27 + 3.70172e−26i	1.62294e−68 + 7.49778e−69i*	−1.40522e−176 + 1.90243e−176i*
1,3	$500\angle 30^\circ$	Pr	2.16231e+03 + 1.25000e+03i	−1.50000e+00 − 0.00000e+00i	−1.33999e+00 + 9.01570e−02i	1.23290e+00 − 9.85342e−01i	4.51596e−05 − 9.40476e−05i	5.84622e−25 − 3.04783e−23i	−3.10636e−61 − 2.65433e−61i*	8.78462e−158 − 5.75667e−158i*
1,3	$500\angle 45^\circ$	Ob	1.41221e+03 + 2.48586e+05i	−1.50000e+00 + 8.90954e−15i	2.33145e+01 − 9.67440e+00i	9.74163e+14 − 9.98928e+14i	−1.03134e+38 + 4.65572e+37i	−1.13208e+76 + 1.28345e+76i	−6.88657e+113 + 2.23474e+114i	7.27492e+149 − 2.63871e+150i
1,3	$500\angle 60^\circ$	Ob	−1.23270e+05 + 2.15506e+05i	−1.50000e+00 − 3.60822e−15i	4.47437e+01 + 3.34118e+01i	−3.90567e+18 − 5.21146e+17i	−4.53416e+46 − 1.56143e+46i	−2.42189e+93 − 1.89275e+93i	−9.69028e+139 − 1.49312e+140i	3.08849e+184 + 2.52483e+184i
1,3	$500\angle 75^\circ$	Ob	−2.12651e+05 + 1.23965e+05i	−1.50000e+00 − 3.21965e−15i	−2.46383e+01 + 8.67538e+01i	−4.42781e+20 + 1.74179e+20i	−4.50952e+51 + 6.13513e+51i	2.14123e+103 + 6.84533e+103i	4.44393e+155 + 1.52207e+155i	3.93154e+203 + 2.83812e+203i
1,3	$500\angle 90^\circ$	Ob	−2.46008e+05 + 3.03712e−11i	−1.50000e+00 + 4.80701e−17i	−1.06947e+02 + 3.61678e−14i	−2.61416e+21 + 8.08745e+06i	−5.38732e+53 + 4.14080e+39i	−3.59055e+107 + 5.50804e+93i	−1.66364e+161 + 3.82548e+147i	−1.01423e+211 + 3.07894e+197i
1,3	$750\angle 0^\circ$	Pr	3.74725e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.22832e+00 − 0.00000e+00i	4.93206e−01 − 0.00000e+00i	6.67581e−09 − 0.00000e+00i	1.64767e−41 − 0.00000e+00i	1.68533e−107 − 2.06167e−139i*	9.98790e−276 + 8.14083e−308i*
1,3	$750\angle 15^\circ$	Pr	3.61947e+03 + 9.70572e+02i	−1.50000e+00 + 1.08420e−19i	−1.23669e+00 + 6.81738e−02i	4.32585e−01 − 3.58904e−01i	1.39400e−08 + 5.63279e−09i	4.14180e−40 − 2.90082e−40i	−8.14640e−104 − 4.74396e−104i*	−3.32742e−266 − 1.56485e−267i*
1,3	$750\angle 30^\circ$	Pr	3.24484e+03 + 1.87500e+03i	−1.50000e+00 + 2.16840e−19i	−1.26143e+00 + 1.32589e−01i	2.04337e−01 − 7.98848e−01i	6.07176e−08 + 1.50739e−07i	9.86563e−36 + 6.05242e−36i	7.13210e−93 − 6.95348e−93i*	1.27838e−238 − 2.72982e−238i*
1,3	$750\angle 45^\circ$	Ob	2.11932e+03 + 5.60379e+05i	−1.50000e+00 − 5.32907e−15i	−8.23288e+01 − 1.22709e+02i	6.17864e+22 + 2.42481e+22i	−1.42094e+57 + 1.04874e+57i	−1.22880e+114 + 3.98335e+114i	2.93514e+171 + 8.39082e+171i	2.57306e+226 − 1.03154e+226i
1,3	$750\angle 60^\circ$	Ob	−2.78654e+05 + 4.85639e+05i	−1.50000e+00 + 2.33147e−15i	3.99325e+02 − 2.78153e+02i	−9.76429e+27 − 1.96967e+27i	−1.35396e+70 − 7.35539e+69i	−1.72698e+140 − 2.66222e+140i	−4.58561e+209 − 5.88641e+210i	−4.17792e+277 + 2.51802e+277i
1,3	$750\angle 75^\circ$	Ob	−4.81352e+05 + 2.79697e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$750\angle 90^\circ$	Ob	−5.56508e+05 + 6.85190e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 0^\circ$	Pr	4.99725e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.14172e+00 − 0.00000e+00i	1.91207e−01 − 0.00000e+00i	3.18435e−12 − 0.00000e+00i	6.25265e−56 − 0.00000e+00i	4.18876e−144 + 7.06448e−176i*	±0
1,3	$1000\angle 15^\circ$	Pr	4.82688e+03 + 1.29410e+03i	−1.50000e+00 − 0.00000e+00i	−1.15235e+00 + 8.89111e−02i	1.18586e−01 − 1.93858e−01i	−9.43730e−13 − 9.35089e−12i	−5.94455e−54 − 8.66771e−55i	3.53808e−139 + 2.42156e−139i*	±0
1,3	$1000\angle 30^\circ$	Pr	4.32738e+03 + 2.50000e+03i	−1.50000e+00 − 0.00000e+00i	−1.18392e+00 + 1.73306e−01i	−1.41773e−01 − 3.49374e−01i	−2.10160e−10 − 7.88078e−11i	−3.44595e−48 + 1.83362e−48i	1.31360e−124 + 1.62445e−124i*	−8.39121e−320 − 7.39157e−319i*
1,3	$1000\angle 45^\circ$	Ob	2.82643e+03 + 9.97172e+05i	−1.50000e+00 − 3.71925e−15i	−6.10548e+02 + 6.13597e+02i	7.86349e+28 + 3.15637e+30i	−1.82390e+76 + 2.06639e+76i	1.26031e+152 + 1.00745e+153i	2.79106e+229 + 1.90442e+229i	2.40908e+302 + 1.44111e+302i
1,3	$1000\angle 60^\circ$	Ob	−4.96538e+05 + 8.64025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 75^\circ$	Ob	−8.58306e+05 + 4.97929e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 90^\circ$	Ob	−9.92008e+05 + 1.21975e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.53: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
1,3	1500∠0°	Pr	7.49725e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−9.74353e−01 − 0.00000e+00i	2.38102e−02 − 0.00000e+00i	6.09568e−19 − 0.00000e+00i	7.59109e−85 − 0.00000e+00i	2.14088e−217 − 0.00000e+00i*	±0
1,3	1500∠15°	Pr	7.24169e+03 + 1.94114e+03i	−1.50000e+00 − 5.42101e−20i	−9.88820e−01 + 1.27556e−01i	−3.21032e−03 − 3.06330e−02i	2.70914e−18 + 1.48625e−18i	6.12178e−83 − 7.12213e−82i	5.17927e−210 + 5.01266e−210i*	±0
1,3	1500∠30°	Pr	6.49244e+03 + 3.75000e+03i	−1.50000e+00 − 0.00000e+00i	−1.03206e+00 + 2.49738e−01i	−6.53198e−02 + 5.03260e−03i	−9.70837e−17 + 3.47049e−16i	3.14246e−73 + 2.03400e−73i	−3.57373e−188 − 6.62703e−188i*	±0
1,3	1500∠45°	Ob	4.24064e+03 + 2.24576e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	1500∠60°	Ob	−1.11981e+06 + 1.94556e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	1500∠75°	Ob	−1.93697e+06 + 1.12189e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	1500∠90°	Ob	−2.23801e+06 + 2.74811e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	2500∠0°	Pr	1.24972e+04 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−6.62061e−01 − 0.00000e+00i	2.67728e−04 − 0.00000e+00i	1.64893e−32 − 0.00000e+00i	8.86264e−143 − 7.99671e−175i*	±0	±0
1,3	2500∠15°	Pr	1.20713e+04 + 3.23524e+03i	−1.50000e+00 − 0.00000e+00i	−6.81613e−01 + 1.94201e−01i	−4.05026e−04 − 6.75911e−05i	3.35774e−32 − 2.44316e−31i	2.66677e−139 + 8.28161e−138i*	±0	±0
1,3	2500∠30°	Pr	1.08226e+04 + 6.25000e+03i	−1.50000e+00 − 1.08420e−19i	−7.40909e−01 + 3.83580e−01i	1.22572e−03 + 7.53791e−04i	7.02084e−29 − 6.82510e−28i	−2.45377e−123 + 1.15448e−123i*	−9.07518e−316 − 6.67566e−315i*	±0
1,3	2500∠45°	Ob	7.06907e+03 + 6.24293e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	2500∠60°	Ob	−3.11634e+06 + 5.40766e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	2500∠75°	Ob	−5.39335e+06 + 3.11982e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	2500∠90°	Ob	−6.23001e+06 + 7.64180e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	5000∠0°	Pr	2.49972e+04 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.16833e−04 − 0.00000e+00i	1.95369e−09 − 0.00000e+00i	1.10237e−66 − 0.00000e+00i	7.70939e−288 − 0.00000e+00i*	±0	±0
1,3	5000∠15°	Pr	2.41454e+04 + 6.47048e+03i	−1.50000e+00 − 0.00000e+00i	−2.04635e−02 + 3.06930e−01i	4.58235e−09 + 2.79368e−10i	−2.46522e−64 − 2.47349e−66i	−5.86278e−278 + 1.98579e−278i*	±0	±0
1,3	5000∠30°	Pr	2.16479e+04 + 1.25000e+04i	−1.50000e+00 + 5.42101e−20i	−8.53993e−02 + 6.19527e−01i	4.75046e−08 + 3.00255e−08i	−1.80977e−57 + 6.02035e−58i	1.13595e−249 − 6.88448e−249i*	±0	±0
1,3	5000∠45°	Ob	1.41401e+04 + 2.49859e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	5000∠60°	Ob	−1.24827e+07 + 2.16406e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	5000∠75°	Ob	−2.16120e+07 + 1.24896e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	5000∠90°	Ob	−2.49600e+07 + 3.05917e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,4	25∠0°	Pr	1.69132e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.47925e−02 − 0.00000e+00i	−5.60004e−01 − 0.00000e+00i	1.83578e−02 − 0.00000e+00i	1.09943e+00 − 1.65594e−32i*	8.47535e−02 − 2.52293e−33i*	6.79684e−06 − 2.65498e−37i*
1,4	25∠15°	Pr	1.63172e+02 + 4.53217e+01i	−0.00000e+00 − 0.00000e+00i	−7.47999e−02 + 5.65526e−05i	−5.64975e−01 + 4.73676e−02i	3.75144e−02 + 2.50272e−01i	5.51721e−01 − 3.12604e−01i	−9.84036e−03 − 2.95378e−02i*	1.97424e−06 + 3.41250e−06i*
1,4	25∠30°	Pr	1.45698e+02 + 8.75570e+01i	−0.00000e+00 − 0.00000e+00i	−7.48217e−02 + 1.09275e−04i	−5.79965e−01 + 9.32724e−02i	9.28057e−02 + 5.34055e−01i	5.11948e−01 − 7.71811e−01i	−6.47617e−02 + 3.51954e−02i*	−2.72028e−05 + 2.27149e−05i*
1,4	25∠45°	Ob	6.86826e+01 + 5.54258e+02i	−0.00000e+00 − 0.00000e+00i	−7.49160e−02 + 6.92377e−04i	−4.88553e−01 + 6.24876e−01i	7.35814e+00 − 3.44086e+00i	6.17193e+01 + 4.12714e+02i	−1.92008e+04 − 2.73618e+03i	1.27128e+05 − 1.82216e+05i
1,4	25∠60°	Pr	3.67591e+01 + 6.49296e+01i	−0.00000e+00 − 0.00000e+00i	−7.49578e−02 + 8.11183e−05i	−7.04706e−01 + 7.67532e−02i	−1.01434e+00 + 8.32492e−01i	8.12919e−01 + 2.00823e−01i	−6.67675e−02 + 1.14351e−01i	−1.33805e−03 − 3.97071e−04i*
1,4	25∠75°	Ob	−3.56423e+02 + 2.60632e+02i	−0.00000e+00 − 0.00000e+00i	−7.54497e−02 + 3.26967e−04i	−1.23301e+00 + 4.56285e−01i	−6.76141e+00 + 2.58014e+01i	3.60981e+03 + 7.14259e+02i	3.85587e+04 − 3.32151e+05i	2.07261e+06 + 1.24917e+06i
1,4	25∠90°	Ob	−4.33359e+02 + 6.42691e−14i	−0.00000e+00 − 0.00000e+00i	−7.55466e−02 − 6.29800e−18i	−1.42800e+00 + 8.95576e−19i	−3.32394e+01 + 7.55040e−15i	−5.66097e+03 + 3.47136e−12i	−6.32669e+05 + 6.35095e−10i	5.59492e+06 − 7.13206e−09i
1,4	50∠0°	Pr	3.44189e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.45743e−02 − 0.00000e+00i	−3.95596e−01 − 0.00000e+00i	5.16104e−01 − 6.15873e−33i*	1.54167e−01 + 3.77699e−33i*	6.24153e−05 + 9.06273e−37i*	1.14057e−14 + 1.91649e−46i*
1,4	50∠15°	Pr	3.32265e+02 + 9.06021e+01i	−0.00000e+00 − 0.00000e+00i	−7.45891e−02 + 1.12839e−04i	−4.01565e−01 + 7.75781e−02i	4.88892e−01 + 4.27924e−02i	7.93495e−03 − 5.75110e−02i	−1.07019e−05 + 1.53246e−05i*	−1.29088e−15 + 8.19513e−15i*
1,4	50∠30°	Pr	2.97306e+02 + 1.75030e+02i	−0.00000e+00 − 0.00000e+00i	−7.46325e−02 + 2.18074e−04i	−4.20218e−01 + 1.55526e−01i	6.58458e−01 + 8.76043e−02i	−1.06676e−01 − 4.36686e−02i	−5.55443e−06 − 1.01966e−04i*	−2.95012e−13 − 5.34764e−13i*
1,4	50∠45°	Ob	1.39407e+02 + 2.35856e+03i	−0.00000e+00 − 0.00000e+00i	−7.47949e−02 + 2.94404e−03i	1.98445e+00 + 8.83107e−01i	5.65821e+01 + 3.23889e+02i	5.61681e+05 − 1.28880e+06i	−4.55901e+09 + 2.95619e+09i	4.05305e+12 + 1.54044e+12i
1,4	50∠60°	Pr	7.42551e+01 + 1.29893e+02i	−0.00000e+00 − 0.00000e+00i	−7.49108e−02 + 1.62178e−04i	−6.50205e−01 + 1.43374e−01i	−1.71764e−01 + 8.65475e−01i	−1.39013e−01 − 7.28129e−02i	1.00276e−03 + 1.74368e−03i*	−3.71793e−08 − 2.18790e−08i*
1,4	50∠75°	Ob	−1.78686e+03 + 1.14643e+03i	−0.00000e+00 − 0.00000e+00i	−7.72491e−02 + 1.45885e−03i	−3.60999e+00 + 5.76340e+00i	5.11342e+03 + 9.62688e+02i	−3.06784e+08 − 2.65230e+07i	1.23748e+13 − 2.62933e+11i	7.94938e+15 + 4.81798e+15i
1,4	50∠90°	Ob	−2.10817e+03 + 2.81658e−13i	−0.00000e+00 − 0.00000e+00i	−7.76670e−02 + 2.86453e−18i	−8.08757e+00 + 2.20155e−15i	−7.98764e+03 + 5.84823e−12i	−7.23055e+08 + 1.08389e−06i	−4.44026e+13 + 1.00694e−01i	4.73627e+16 − 1.36152e+02i
1,4	100∠0°	Pr	6.94219e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.41393e−02 − 0.00000e+00i	−1.54711e−01 − 0.00000e+00i	2.64875e−01 − 0.00000e+00i	1.21649e−04 − 0.00000e+00i	9.96480e−13 − 2.62729e−44i*	9.06123e−34 − 0.00000e+00i*
1,4	100∠15°	Pr	6.70369e+02 + 1.81181e+02i	−0.00000e+00 − 0.00000e+00i	−7.41686e−02 + 2.24794e−04i	−1.56147e−01 + 1.01710e−01i	2.65986e−01 − 1.37524e−01i	−1.92307e−04 + 8.91084e−06i	−1.88606e−12 − 2.53774e−12i*	−1.69230e−32 − 4.55181e−34i*
1,4	100∠30°	Pr	6.00441e+02 + 3.50015e+02i	−0.00000e+00 − 0.00000e+00i	−7.42548e−02 + 4.34610e−04i	−1.62176e−01 + 2.10616e−01i	2.62748e−01 − 3.37901e−01i	7.31960e−04 + 1.04814e−04i	−7.24970e−11 + 5.88313e−11i*	2.47123e−31 + 9.04218e−29i*
1,4	100∠45°	Ob	2.80836e+02 + 9.71715e+03i	−0.00000e+00 − 0.00000e+00i	−7.40644e−02 + 1.20993e−02i	−3.67978e+01 − 4.15706e−01i	4.35425e+05 − 1.03139e+06i	3.30464e+13 − 1.33915e+10i	3.43268e+20 + 8.11893e+20i	−3.39372e+27 + 3.14157e+26i
1,4	100∠60°	Pr	1.49253e+02 + 2.59803e+02i	−0.00000e+00 − 0.00000e+00i	−7.48167e−02 + 3.23970e−04i	−5.32926e−01 + 2.47056e−01i	3.68715e−01 + 1.52277e−01i	−3.14150e−03 − 4.52666e−03i	2.20001e−07 − 3.79950e−07i*	−1.09832e−17 − 1.80642e−17i*
1,4	100∠75°	Ob	−7.89559e+03 + 4.79292e+03i	−0.00000e+00 − 0.00000e+00i	−8.51189e−02 + 6.47633e−03i	2.77964e+02 + 3.00979e+02i	−4.40025e+08 − 3.65741e+07i	−4.60459e+18 + 4.95908e+17i	−3.19363e+28 + 9.91270e+27i	7.70375e+34 + 6.19877e+35i
1,4	100∠90°	Ob	−9.20808e+03 + 1.17566e−12i	−0.00000e+00 − 0.00000e+00i	−8.70561e−02 + 5.92081e−18i	−5.76832e+02 + 3.45095e−13i	−1.03619e+09 + 1.57201e−06i	−2.54649e+19 + 7.76323e+04i	−4.30590e+29 + 1.97250e+15i	7.04113e+35 + 4.23745e+19i
1,4	250∠0°	Pr	1.74424e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.28445e−02 − 0.00000e+00i	1.43702e−01 − 0.00000e+00i	6.75503e−03 − 0.00000e+00i	5.92924e−13 − 0.00000e+00i	2.13570e−34 − 0.00000e+00i	2.13923e−89 + 2.02293e−121i*
1,4	250∠15°	Pr	1.68461e+03 + 4.52936e+02i	−0.00000e+00 − 0.00000e+00i	−7.29162e−02 + 5.55591e−04i	1.61062e−01 + 4.87421e−02i	−1.71422e−03 − 8.72268e−03i	−9.65079e−13 − 1.58771e−12i	−3.75293e−33 − 1.36723e−33i*	1.26101e−86 + 3.61237e−86i*
1,4	250∠30°	Pr	1.50978e+03 + 8.75006e+02i	−0.00000e+00 − 0.00000e+00i	−7.31270e−02 + 1.07542e−03i	2.16549e−01 + 1.06857e−01i	−1.92593e−02 + 4.93015e−03i	−4.63638e−11 + 2.54537e−11i	−1.04668e−29 + 1.46314e−29i	6.49804e−77 − 2.10617e−78i*
1,4	250∠45°	Ob	7.05104e+02 + 6.17929e+04i	−0.00000e+00 − 0.00000e+00i	−5.06792e−02 + 7.32209e−02i	2.25330e+05 − 5.45175e+05i	−1.22063e+17 − 7.97369e+16i	−1.31896e+36 − 5.06113e+35i	−1.22209e+55 − 1.90014e+54i	7.53584e+71 − 5.38085e+72i
1,4	250∠60°	Pr	3.74251e+02 + 6.49517e+02i	−0.00000e+00 − 0.00000e+00i	−7.45330e−02 + 8.06893e−04i	−1.88236e−01 + 3.56721e−01i	−3.05950e−02 + 2.05636e−02i	1.77366e−07 − 1.59338e−07i	1.91706e−18 − 3.29472e−18i	1.50926e−45 − 1.02479e−45i*
1,4	250∠75°	Ob	−5.22028e+04 + 3.07324e+04i	−0.00000e+00 − 0.00000e+00i	−1.51144e−01 + 6.20225e−02i	−3.13458e+08 − 2.46067e+07i	9.14321e+23 − 1.88370e+23i	−4.15036e+49 + 3.30530e+49i	8.67312e+74 − 1.90319e+75i	−4.83292e+97 + 1.02223e+98i
1,4	250∠90°	Ob	−6.05080e+04 + 7.53158e−12i	−0.00000e+00 − 0.00000e+00i	−1.77116e−01 − 2.76591e−18i	−7.37383e+08 + 1.00280e−06i	−7.85606e+24 + 2.87241e+10i	−3.75530e+51 + 2.81031e+37i	−1.24482e+78 + 1.40803e+64i	−5.01162e+101 + 7.53018e+87i
1,4	500∠0°	Pr	3.49424e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.07203e−02 − 0.00000e+00i	1.44165e−01 − 0.00000e+00i	5.08110e−06 − 0.00000e+00i	3.41689e−27 − 0.00000e+00i	8.62472e−71 − 2.34346e−102i*	3.10606e−182 − 0.00000e+00i*
1,4	500∠15°	Pr	3.37498e+03 + 9.05868e+02i	−0.00000e+00 − 0.00000e+00i	−7.08582e−02 + 1.09020e−03i	1.54149e−01 − 4.39047e−02i	−6.72931e−06 + 5.58335e−06i	−6.86492e−27 + 3.27950e−26i	2.43271e−68 + 1.12775e−68i*	−4.06642e−176 + 5.50012e−176i*
1,4	500∠30°	Pr	3.02533e+03 + 1.75000e+03i	−0.00000e+00 − 0.00000e+00i	−7.12645e−02 + 2.11434e−03i	1.87975e−01 − 1.09815e−01i	1.92412e−05 − 3.83903e−05i	6.27003e−25 − 2.70334e−23i	−4.65703e−61 − 3.99080e−61i*	2.54124e−157 − 1.66623e−157i*
1,4	500∠45°	Ob	1.41221e+03 + 2.48586e+05i	−0.00000e+00 − 0.00000e+00i	2.33482e−01 + 9.69735e−02i	1.39918e+13 − 1.35874e+11i	−1.06033e+36 − 4.04204e+35i	−1.71317e+74 + 1.02471e+73i	−2.07611e+112 + 1.09045e+112i	2.39085e+148 − 1.34848e+148i
1,4	500∠60°	Pr	7.49251e+02 + 1.29904e+03i	−0.00000e+00 − 0.00000e+00i	−7.40556e−02 + 1.60364e−03i	1.21658e−01 + 1.77940e−01i	−2.62917e−04 + 6.44704e−04i	−1.35558e−15 + 1.26425e−14i	8.51783e−37 + 1.49377e−36i*	−1.81160e−92 + 4.93838e−92i*
1,4	500∠75°	Ob	−2.12651e+05 + 1.23965e+05i	−0.00000e+00 − 0.00000e+00i	−4.67763e−01 + 7.79395e−01i	−4.76574e+18 + 5.30644e+17i	−6.00014e+49 + 4.78349e+49i	2.83749e+100 + 7.22278e+101i	3.92343e+153 + 2.64904e+153i	3.07896e+201 + 3.79474e+201i
1,4	500∠90°	Ob	−2.46008e+05 + 3.03712e−11i	−0.00000e+00 − 0.00000e+00i	−1.07801e+00 + 2.76494e−16i	−2.63528e+19 + 7.93735e+04i	−5.43085e+51 + 4.12986e+37i	−3.61956e+105 + 5.52296e+91i	−1.67708e+159 + 3.84268e+145i	−1.02242e+209 + 3.09546e+195i
1,4	750∠0°	Pr	5.24425e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−6.86377e−02 − 0.00000e+00i	7.06268e−02 − 0.00000e+00i	2.76552e−09 − 0.00000e+00i	1.46387e−41 − 0.00000e+00i	2.53551e−107 − 5.11285e−139i*	2.89451e−275 − 7.78186e−307i*
1,4	750∠15°	Pr	5.06536e+03 + 1.35880e+03i	−0.00000e+00 − 0.00000e+00i	−6.88364e−02 + 1.60434e−03i	6.47422e−02 − 4.88990e−02i	5.76581e−09 + 2.36854e−09i	3.68400e−40 − 2.57263e−40i	−1.21625e−103 − 7.17935e−104i*	−9.59939e−266 − 5.02795e−267i*
1,4	750∠30°	Pr	4.54088e+03 + 2.62500e+03i	−0.00000e+00 − 0.00000e+00i	−6.94235e−02 + 3.11749e−03i	4.00686e−02 − 1.14904e−01i	2.45356e−08 + 6.29148e−08i	8.75708e−36 + 5.40444e−36i	1.07238e−92 − 1.04386e−92i*	3.69612e−238 − 7.89926e−238i*
1,4	750∠45°	Ob	2.11932e+03 + 5.60379e+05i	−0.00000e+00 − 0.00000e+00i	1.92596e−01 − 9.68031e−01i	1.76523e+20 + 4.06671e+20i	−1.16608e+55 − 1.77989e+54i	−2.46412e+112 + 1.29630e+112i	−2.58680e+169 + 5.34430e+169i	1.70105e+224 + 7.31267e+223i
1,4	750∠60°	Pr	1.12425e+03 + 1.94856e+03i	−0.00000e+00 − 0.00000e+00i	−7.35726e−02 + 2.39024e−03i	1.14447e−01 − 1.31224e−02i	2.67149e−06 + 1.28761e−05i	−3.96849e−22 − 5.49791e−22i	−8.22241e−55 − 7.08244e−57i*	−3.63398e−139 − 1.37011e−138i*
1,4	750∠75°	Ob	−4.81352e+05 + 2.79697e+05i	−0.00000e+00 − 0.00000e+00i	7.23276e−01 + 6.72304e+00i	−9.36303e+28 + 2.88418e+28i	3.51230e+75 − 7.67903e+75i	1.09045e+154 + 7.37601e+153i	−9.83914e+231 + 1.03512e+232i	2.23141e+305 − 7.21359e+304i
1,4	750∠90°	Ob	−5.56508e+05 + 6.85190e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,4	1000∠0°	Pr	6.99425e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−6.65958e−02 − 0.00000e+00i	2.85832e−02 − 0.00000e+00i	1.32648e−12 − 0.00000e+00i	5.56255e−56 − 0.00000e+00i	6.31470e−144 − 0.00000e+00i*	±0
1,4	1000∠15°	Pr	6.75573e+03 + 1.81173e+03i	−0.00000e+00 − 0.00000e+00i	−6.68502e−02 + 2.09850e−03i	1.87361e−02 − 2.85478e−02i	−3.76691e−13 − 3.89913e−12i	−5.28838e−54 − 7.76662e−55i	5.31060e−139 + 3.63879e−139i*	±0
1,4	1000∠30°	Pr	6.05643e+03 + 3.50000e+03i	−0.00000e+00 − 0.00000e+00i	−6.76040e−02 + 4.08557e−03i	−1.83875e−02 − 5.44673e−02i	−8.74708e−11 − 3.36240e−11i	−3.07051e−48 + 1.62601e−48i	1.97185e−124 + 2.44198e−124i*	−2.42947e−319 − 2.13851e−318i*
1,4	1000∠45°	Ob	2.82643e+03 + 9.97172e+05i	−0.00000e+00 − 0.00000e+00i	−4.33414e+00 + 4.65877e−03i	−1.09131e+28 + 1.14382e+28i	−1.37749e+74 + 8.39007e+72i	−3.12636e+150 + 4.00869e+150i	3.11559e+226 + 1.66289e+227i	3.40780e+299 + 1.36366e+300i
1,4	1000∠60°	Pr	1.49925e+03 + 2.59808e+03i	−0.00000e+00 − 0.00000e+00i	−7.30842e−02 + 3.16670e−03i	2.23416e−02 − 5.73664e−02i	1.77542e−07 + 1.73689e−07i	3.53374e−29 + 7.66321e−30i	1.87546e−73 − 3.16440e−73i*	2.73118e−185 + 2.30323e−185i*
1,4	1000∠75°	Ob	−8.58306e+05 + 4.97929e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,4	1000∠90°	Ob	−9.92008e+05 + 1.21975e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.53: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)}_{mn}$	$S^{(a)}_{mn}(c,\eta{=}0)$	$S^{(a)}_{mn}(c,\eta{=}0.01)$	$S^{(a)}_{mn}(c,\eta{=}0.1)$	$S^{(a)}_{mn}(c,\eta{=}0.25)$	$S^{(a)}_{mn}(c,\eta{=}0.5)$	$S^{(a)}_{mn}(c,\eta{=}0.75)$	$S^{(a)}_{mn}(c,\eta{=}0.99)$
1,4	$1500\angle 0^\circ$	Pr	$1.04942_{e+04}-0.00000_{e+00}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-6.26319_{e-02}-0.00000_{e+00}i$	$3.70388_{e-03}-0.00000_{e+00}i$	$2.55318_{e-19}-0.00000_{e+00}i$	$6.76222_{e-85}-0.00000_{e+00}i$	$3.24135_{e-217}-3.76206_{e-249}i^*$	± 0
1,4	$1500\angle 15^\circ$	Pr	$1.01365_{e+04}+2.71760_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-6.29830_{e-02}+3.02881_{e-03}i$	$-4.06442_{e-04}-4.78839_{e-03}i$	$1.13340_{e-18}+6.25948_{e-19}i$	$5.49731_{e-83}-6.34467_{e-82}i$	$7.78903_{e-210}+7.54136_{e-210}i^*$	± 0
1,4	$1500\angle 30^\circ$	Pr	$9.08751_{e+03}+5.25000_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-6.40293_{e-02}+5.91932_{e-03}i$	$-1.03014_{e-02}+4.05527_{e-04}i$	$-4.15143_{e-17}+1.45355_{e-16}i$	$2.79794_{e-73}+1.81626_{e-73}i$	$-5.36559_{e-188}-9.95176_{e-188}i^*$	± 0
1,4	$1500\angle 45^\circ$	Ob	$4.24064_{e+03}+2.24576_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1500\angle 60^\circ$	Pr	$2.24925_{e+03}+3.89711_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-7.20917_{e-02}+4.68921_{e-03}i$	$-1.71266_{e-02}+3.97982_{e-03}i$	$8.12877_{e-11}-3.52491_{e-11}i$	$3.23758_{e-44}-9.75976_{e-44}i$	$-7.13111_{e-110}-1.22581_{e-111}i^*$	$1.90714_{e-278}-1.09383_{e-278}i^*$
1,4	$1500\angle 75^\circ$	Ob	$-1.93697_{e+06}+1.12189_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1500\angle 90^\circ$	Ob	$-2.23801_{e+06}+2.74811_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 0^\circ$	Pr	$1.74942_{e+04}-0.00000_{e+00}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-5.51643_{e-02}-0.00000_{e+00}i$	$4.29072_{e-05}-0.00000_{e+00}i$	$6.93657_{e-33}-0.00000_{e+00}i$	$7.87479_{e-143}-8.49114_{e-175}i^*$	± 0	± 0
1,4	$2500\angle 15^\circ$	Pr	$1.68980_{e+04}+4.52933_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-5.56553_{e-02}+4.67035_{e-03}i$	$-6.48895_{e-05}-1.15779_{e-05}i$	$1.43006_{e-32}-1.02776_{e-31}i$	$2.48942_{e-139}+7.08082_{e-138}i^*$	± 0	± 0
1,4	$2500\angle 30^\circ$	Pr	$1.51497_{e+04}+8.75000_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-5.71361_{e-02}+9.19610_{e-03}i$	$1.95014_{e-04}+1.25776_{e-04}i$	$3.04900_{e-29}-2.87268_{e-28}i$	$-2.18941_{e-123}+1.02687_{e-123}i^*$	$-1.35989_{e-315}-1.00419_{e-314}i^*$	± 0
1,4	$2500\angle 45^\circ$	Ob	$7.06907_{e+03}+6.24293_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 60^\circ$	Pr	$3.74925_{e+03}+6.49519_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-7.00478_{e-02}+7.61285_{e-03}i$	$2.04482_{e-04}-1.41960_{e-03}i$	$-1.05311_{e-17}-4.02729_{e-18}i$	$-4.24384_{e-73}+7.15863_{e-73}i$	$1.30095_{e-183}-2.10194_{e-183}i^*$	± 0
1,4	$2500\angle 75^\circ$	Ob	$-5.39335_{e+06}+3.11982_{e+06}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 90^\circ$	Ob	$-6.23001_{e+06}+7.64180_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 0^\circ$	Pr	$3.49942_{e+04}-0.00000_{e+00}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-3.89461_{e-02}-0.00000_{e+00}i$	$3.19838_{e-10}-0.00000_{e+00}i$	$4.65235_{e-67}-0.00000_{e+00}i$	$6.89540_{e-288}-5.89371_{e-320}i^*$	± 0	± 0
1,4	$5000\angle 15^\circ$	Pr	$3.38017_{e+04}+9.05867_{e+03}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-3.95362_{e-02}+7.65534_{e-03}i$	$7.50472_{e-10}+4.98101_{e-11}i$	$-1.04051_{e-64}-1.13067_{e-66}i$	$-5.23430_{e-278}+1.77174_{e-278}i^*$	± 0	± 0
1,4	$5000\angle 30^\circ$	Pr	$3.03051_{e+04}+1.75000_{e+04}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-4.13798_{e-02}+1.53466_{e-02}i$	$7.74799_{e-09}+5.01020_{e-09}i$	$-7.64522_{e-58}+2.52960_{e-58}i$	$1.02542_{e-249}-6.13980_{e-249}i^*$	± 0	± 0
1,4	$5000\angle 45^\circ$	Ob	$1.41401_{e+04}+2.49859_{e+07}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 60^\circ$	Pr	$7.49925_{e+03}+1.29904_{e+04}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$-6.46458_{e-02}+1.42195_{e-02}i$	$2.61482_{e-06}+7.69245_{e-07}i$	$-4.85117_{e-35}-4.34591_{e-35}i$	$9.26884_{e-146}+1.64605_{e-145}i^*$	± 0	± 0
1,4	$5000\angle 75^\circ$	Ob	$-2.16120_{e+07}+1.24896_{e+07}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 90^\circ$	Ob	$-2.49600_{e+07}+3.05917_{e-09}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,5	$25\angle 0^\circ$	Pr	$2.14898_{e+02}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.85498_{e+00}-0.00000_{e+00}i$	$2.00828_{e-01}-0.00000_{e+00}i$	$-1.79577_{e+00}-0.00000_{e+00}i$	$2.83940_{e+00}-5.18626_{e-32}i^*$	$4.41184_{e-01}-1.67346_{e-32}i^*$	$7.31555_{e-05}-3.12719_{e-36}i^*$
1,5	$25\angle 15^\circ$	Pr	$2.07243_{e+02}+5.83274_{e+01}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.85570_{e+00}-5.44921_{e-03}i$	$2.28376_{e-01}-3.76321_{e-01}i$	$-2.01657_{e+00}+3.78944_{e-01}i$	$2.95586_{e+00}-3.31641_{e-01}i$	$-3.68393_{e-03}-3.01949_{e-01}i^*$	$7.78328_{e-05}+7.72048_{e-05}i^*$
1,5	$25\angle 30^\circ$	Pr	$1.84802_{e+02}+1.12682_{e+02}i$	$1.87500_{e+00}-2.77556_{e-17}i$	$1.85778_{e+00}-1.05312_{e-02}i$	$3.15617_{e-01}-7.56737_{e-01}i$	$-2.73186_{e+00}+6.90355_{e-01}i$	$4.63236_{e+00}-1.04739_{e+00}i$	$-7.19092_{e-01}-8.24944_{e-02}i$	$-6.72270_{e-04}+7.66956_{e-05}i^*$
1,5	$25\angle 45^\circ$	Pr	$8.56282_{e+01}+8.83908_{e+01}i$	$1.87500_{e+00}-1.38778_{e-17}i$	$1.86707_{e+00}-8.27439_{e-03}i$	$1.07786_{e+00}-7.10250_{e-01}i$	$-2.42946_{e+00}-1.42569_{e+00}i$	$-1.66975_{e-01}+2.60877_{e+00}i$	$-1.08603_{e-01}-2.27307_{e-01}i^*$	$9.41638_{e-05}+4.77494_{e-04}i^*$
1,5	$25\angle 60^\circ$	Ob	$-1.47588_{e+02}+4.41071_{e+02}i$	$1.87500_{e+00}+2.22045_{e-16}i$	$1.88880_{e+00}-4.14539_{e-02}i$	$1.70772_{e+00}-5.00317_{e+00}i$	$-7.55147_{e+01}+5.38199_{e-01}i$	$5.61608_{e+03}-9.50852_{e+01}i$	$-2.77427_{e+05}+1.69870_{e+04}i$	$-7.12834_{e+05}-8.59499_{e+05}i$
1,5	$25\angle 75^\circ$	Pr	$6.72295_{e+00}+2.41357_{e+01}i$	$1.87500_{e+00}-3.46945_{e-18}i$	$1.87446_{e+00}-2.26213_{e-03}i$	$1.80867_{e+00}-2.19890_{e-01}i$	$1.12668_{e+00}-1.08465_{e+00}i$	$-8.72603_{e-01}+8.02356_{e-02}i$	$-8.75373_{e-02}-2.68576_{e-01}i$	$-1.62055_{e-02}-3.19566_{e-02}i^*$
1,5	$25\angle 90^\circ$	Ob	$-3.44291_{e+02}+5.80751_{e-14}i$	$1.87500_{e+00}+2.28886_{e-16}i$	$1.90746_{e+00}+2.27373_{e-16}i$	$6.16407_{e+00}-1.65988_{e-16}i$	$9.71350_{e+01}-2.61302_{e-14}i$	$8.38672_{e+03}-5.55716_{e-12}i$	$3.06237_{e+05}-3.30690_{e-10}i$	$7.29000_{e+05}-8.87953_{e-10}i$
1,5	$50\angle 0^\circ$	Pr	$4.40077_{e+02}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.83399_{e+00}-0.00000_{e+00}i$	$-9.41240_{e-01}-0.00000_{e+00}i$	$5.77158_{e-01}-0.00000_{e+00}i$	$9.45647_{e-01}-1.13499_{e-32}i^*$	$7.07283_{e-04}-0.00000_{e+00}i^*$	$2.59038_{e-13}+2.20150_{e-45}i^*$
1,5	$50\angle 15^\circ$	Pr	$4.24750_{e+02}+1.16514_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.83540_{e+00}-1.08448_{e-02}i$	$-9.46099_{e-01}-4.67605_{e-01}i$	$5.89971_{e-01}+1.27296_{e+00}i$	$2.88667_{e-01}-6.02047_{e-01}i$	$-3.05861_{e-04}+2.46810_{e-04}i^*$	$-1.45058_{e-13}+3.17703_{e-13}i^*$
1,5	$50\angle 30^\circ$	Pr	$3.79813_{e+02}+2.25088_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.83956_{e+00}-2.09667_{e-02}i$	$-9.47222_{e-01}-9.86073_{e-01}i$	$5.47557_{e-01}+3.01174_{e+00}i$	$-7.31805_{e-01}-1.14339_{e+00}i$	$1.05379_{e-03}-1.89273_{e-03}i^*$	$9.86559_{e-13}-2.58346_{e-11}i^*$
1,5	$50\angle 45^\circ$	Pr	$1.74022_{e+02}+1.76780_{e+02}i$	$1.87500_{e+00}-6.93889_{e-18}i$	$1.85878_{e+00}-1.65223_{e-02}i$	$2.74892_{e-01}-1.18739_{e+00}i$	$-3.50932_{e+00}+7.16791_{e-01}i$	$3.62894_{e-01}-3.54070_{e-01}i$	$-2.26738_{e-04}-1.31962_{e-03}i^*$	$7.28331_{e-11}-2.43563_{e-10}i^*$
1,5	$50\angle 60^\circ$	Ob	$-9.11733_{e+02}+1.96498_{e+03}i$	$1.87500_{e+00}+1.11022_{e-15}i$	$1.95818_{e+00}-1.87018_{e-01}i$	$-3.82549_{e+01}-2.82975_{e+01}i$	$1.72297_{e+04}+3.91545_{e+02}i$	$2.93067_{e+08}+1.29283_{e+07}i$	$3.39887_{e+12}+1.73584_{e+11}i$	$-2.19290_{e+14}-8.54797_{e+14}i$
1,5	$50\angle 75^\circ$	Pr	$1.31924_{e+01}+4.82902_{e+01}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.87385_{e+00}-4.52455_{e-03}i$	$1.71243_{e+00}-4.22797_{e-01}i$	$4.76049_{e-02}-1.27261_{e+00}i$	$3.61951_{e-01}-6.82330_{e-02}i$	$-2.73973_{e-02}+2.04064_{e-02}i^*$	$-1.59352_{e-04}+1.13221_{e-04}i^*$
1,5	$50\angle 90^\circ$	Ob	$-1.91859_{e+03}+2.69383_{e-13}i$	$1.87500_{e+00}-1.17471_{e-16}i$	$2.05786_{e+00}-1.54999_{e-16}i$	$7.51266_{e+01}-2.77998_{e-14}i$	$5.34618_{e+04}-4.44451_{e-11}i$	$2.58488_{e+09}-4.13828_{e-06}i$	$6.12659_{e+13}-1.45660_{e-01}i$	$3.17172_{e+15}-8.70997_{e+00}i$
1,5	$100\angle 0^\circ$	Pr	$8.90164_{e+02}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.79227_{e+00}-0.00000_{e+00}i$	$-1.89617_{e+00}-0.00000_{e+00}i$	$2.67479_{e+00}-2.71565_{e-32}i^*$	$8.45801_{e-03}-1.21751_{e-34}i^*$	$2.03507_{e-10}-5.00426_{e-42}i^*$	$3.63426_{e-31}+4.13140_{e-63}i^*$
1,5	$100\angle 15^\circ$	Pr	$8.59501_{e+02}+2.32960_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.79506_{e+00}-2.15222_{e-02}i$	$-2.01769_{e+00}-1.78450_{e-01}i$	$2.74511_{e+00}-2.40670_{e-01}i$	$-4.67026_{e-03}-1.12355_{e-03}i$	$-4.96141_{e-11}-1.28731_{e-10}i^*$	$-1.38802_{e-30}-4.18282_{e-31}i^*$
1,5	$100\angle 30^\circ$	Pr	$7.69599_{e+02}+4.50043_{e+02}i$	$1.87500_{e+00}-3.46945_{e-18}i$	$1.80325_{e+00}-4.16418_{e-02}i$	$-2.37874_{e+00}-4.78609_{e-01}i$	$4.20658_{e+00}-7.21331_{e-01}i$	$1.41786_{e-02}+1.20632_{e-02}i$	$-4.04965_{e-09}+5.70539_{e-10}i^*$	$-3.91097_{e-27}+6.69962_{e-27}i^*$
1,5	$100\angle 45^\circ$	Pr	$3.50801_{e+02}+3.53555_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.84220_{e+00}-3.29389_{e-02}i$	$-1.15441_{e+00}-1.57391_{e+00}i$	$-4.29716_{e-01}+2.43478_{e+00}i$	$6.59135_{e-03}+6.19882_{e-03}i$	$6.92369_{e-09}-1.56456_{e-08}i^*$	$3.21896_{e-23}-8.16613_{e-24}i^*$
1,5	$100\angle 60^\circ$	Ob	$-4.31525_{e+03}+8.26021_{e+03}i$	$1.87500_{e+00}+1.88738_{e-15}i$	$2.23881_{e+00}-8.29831_{e-01}i$	$1.03577_{e+03}+3.47539_{e+03}i$	$8.72259_{e+08}+5.36554_{e+07}i$	$7.49546_{e+17}+9.20507_{e+16}i$	$4.42907_{e+26}+7.89199_{e+25}i$	$1.27645_{e+33}-8.00028_{e+32}i$
1,5	$100\angle 75^\circ$	Pr	$2.61327_{e+01}+9.65895_{e+01}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.87262_{e+00}-9.04409_{e-03}i$	$1.46308_{e+00}-7.69450_{e-01}i$	$-8.42052_{e-01}-6.28290_{e-02}i$	$6.05605_{e-02}-2.37756_{e-02}i$	$1.20463_{e-04}-4.10130_{e-04}i^*$	$9.12722_{e-10}-2.88309_{e-09}i^*$
1,5	$100\angle 90^\circ$	Ob	$-8.81828_{e+03}+1.15115_{e-12}i$	$1.87500_{e+00}-1.42687_{e-16}i$	$2.76442_{e+00}-3.34873_{e-16}i$	$1.12627_{e+04}-7.76017_{e-12}i$	$1.47159_{e+10}-2.36689_{e-05}i$	$1.97250_{e+20}-6.19580_{e+05}i$	$1.36366_{e+30}-6.37725_{e+15}i$	$-4.16515_{e+36}+2.55908_{e+22}i$
1,5	$250\angle 0^\circ$	Pr	$2.24022_{e+03}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.66904_{e+00}-0.00000_{e+00}i$	$-3.67421_{e-01}-0.00000_{e+00}i$	$1.91643_{e-01}-0.00000_{e+00}i$	$3.86029_{e-11}-0.00000_{e+00}i$	$2.37664_{e-32}-0.00000_{e+00}i$	$4.75475_{e-87}-1.86790_{e-118}i^*$
1,5	$250\angle 15^\circ$	Pr	$2.16355_{e+03}+5.82352_{e+02}i$	$1.87500_{e+00}-8.67362_{e-19}i$	$1.67570_{e+00}-5.26057_{e-02}i$	$-4.96483_{e-01}+1.10281_{e+00}i$	$2.54157_{e-02}-2.52302_{e-01}i$	$-3.30929_{e-11}-1.16485_{e-10}i$	$-3.63351_{e-31}-2.56553_{e-31}i^*$	$5.99686_{e-85}+8.23680_{e-84}i^*$
1,5	$250\angle 30^\circ$	Pr	$1.93878_{e+03}+1.12502_{e+03}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.69532_{e+00}-1.02019_{e-01}i$	$-9.93789_{e-01}+2.45712_{e+00}i$	$-5.44002_{e-01}-1.88880_{e-01}i$	$-3.45549_{e-09}-1.24641_{e-10}i$	$-1.83220_{e-27}+8.16351_{e-28}i$	$1.22893_{e-74}+6.67429_{e-75}i^*$
1,5	$250\angle 45^\circ$	Pr	$8.81133_{e+02}+8.83884_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.79248_{e+00}-8.15606_{e-02}i$	$-3.38117_{e+00}-2.18097_{e-01}i$	$-2.00585_{e-02}-2.19853_{e-01}i$	$9.57785_{e-09}-1.20679_{e-08}i$	$9.13205_{e-24}+6.64619_{e-24}i^*$	$-2.27279_{e-62}+5.89009_{e-63}i^*$
1,5	$250\angle 60^\circ$	Ob	$-2.95260_{e+04}+5.31266_{e+04}i$	$1.87500_{e+00}-3.33067_{e-16}i$	$2.50978_{e+00}-7.34287_{e+00}i$	$1.58887_{e+09}+1.04355_{e+08}i$	$1.10125_{e+23}+1.82262_{e+22}i$	$1.16726_{e+46}+3.96906_{e+45}i$	$8.34231_{e+68}+4.43897_{e+68}i$	$-5.96111_{e+87}+1.39781_{e+89}i$
1,5	$250\angle 75^\circ$	Pr	$6.49551_{e+01}+2.41480_{e+02}i$	$1.87500_{e+00}+4.33681_{e-19}i$	$1.86888_{e+00}-2.25665_{e-02}i$	$4.79818_{e-01}-1.27335_{e+00}i$	$4.55111_{e-02}-2.41882_{e-01}i$	$2.12505_{e-04}-2.88699_{e-04}i$	$8.34517_{e-10}-6.22527_{e-11}i^*$	$9.97646_{e-24}-6.31270_{e-25}i^*$
1,5	$250\angle 90^\circ$	Ob	$-5.95181_{e+04}+7.47034_{e-12}i$	$1.87500_{e+00}+1.48197_{e-16}i$	$1.08343_{e+01}-7.77414_{e-16}i$	$3.70262_{e+10}-5.37618_{e-05}i$	$2.88371_{e+26}-1.08097_{e+12}i$	$7.60390_{e+52}-5.76092_{e+38}i$	$1.06120_{e+79}-1.21030_{e+65}i$	$9.23884_{e+99}-1.50811_{e+86}i$
1,5	$500\angle 0^\circ$	Pr	$4.49023_{e+03}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.46998_{e+00}-0.00000_{e+00}i$	$2.12340_{e+00}-0.00000_{e+00}i$	$9.44165_{e-04}-7.34916_{e-36}i^*$	$3.07783_{e-24}-0.00000_{e+00}i^*$	$1.94521_{e-67}-2.33903_{e-99}i^*$	$1.35233_{e-178}-0.00000_{e+00}i^*$
1,5	$500\angle 15^\circ$	Pr	$4.33690_{e+03}+1.16469_{e+03}i$	$1.87500_{e+00}+4.33681_{e-19}i$	$1.48226_{e+00}-1.01302_{e-01}i$	$2.58197_{e+00}+4.72418_{e-01}i$	$-4.82731_{e-04}+2.13461_{e-04}i$	$-2.01284_{e-24}+3.94465_{e-24}i$	$4.60692_{e-66}+3.86052_{e-66}i^*$	$-2.31616_{e-173}+1.84217_{e-173}i^*$
1,5	$500\angle 30^\circ$	Pr	$3.88735_{e+03}+2.25001_{e+03}i$	$1.87500_{e+00}-8.67362_{e-19}i$	$1.51868_{e+00}-1.97221_{e-01}i$	$3.89853_{e+00}+1.21339_{e+00}i$	$2.21661_{e-03}-1.37440_{e-03}i$	$1.88264_{e-21}-3.04409_{e-21}i$	$-4.53044_{e-59}-1.29977_{e-58}i^*$	$1.31376_{e-154}-7.31808_{e-156}i^*$
1,5	$500\angle 45^\circ$	Pr	$1.76502_{e+03}+1.76777_{e+03}i$	$1.87500_{e+00}-8.67362_{e-19}i$	$1.70965_{e+00}-1.60518_{e-01}i$	$-1.83746_{e+00}+2.36390_{e+00}i$	$8.74829_{e-04}-1.37590_{e-03}i$	$1.36924_{e-18}+8.31330_{e-19}i$	$-2.01004_{e-49}-1.06298_{e-49}i^*$	$-1.52026_{e-128}+5.07529_{e-128}i^*$
1,5	$500\angle 60^\circ$	Ob	$-1.21544_{e+05}+2.14506_{e+05}i$	$1.87500_{e+00}+4.55191_{e-15}i$	$-5.48209_{e+01}-4.09336_{e+01}i$	$3.99256_{e+18}+5.31534_{e+17}i$	$3.39715_{e+46}+1.16706_{e+46}i$	$1.00752_{e+93}+7.84669_{e+92}i$	$1.72917_{e+139}+2.64172_{e+139}i$	$-1.76276_{e+182}-1.11626_{e+182}i$
1,5	$500\angle 75^\circ$	Pr	$1.29660_{e+02}+4.82962_{e+02}i$	$1.87500_{e+00}+2.16840_{e-19}i$	$1.86243_{e+00}-4.49841_{e-02}i$	$-7.39189_{e-01}-6.49253_{e-01}i$	$-2.93823_{e-02}-1.14606_{e-02}i$	$-1.83261_{e-08}-5.88229_{e-08}i$	$2.62140_{e-19}-3.95729_{e-20}i^*$	$7.48057_{e-48}-1.04188_{e-48}i^*$
1,5	$500\angle 90^\circ$	Ob	$-2.44018_{e+05}+3.02488_{e-11}i$	$1.87500_{e+00}-6.59873_{e-17}i$	$1.31024_{e+02}-4.47230_{e-14}i$	$2.67089_{e+21}-8.27155_{e+06}i$	$4.03015_{e+53}-3.09899_{e+39}i$	$1.48701_{e+107}-2.28165_{e+93}i$	$2.92714_{e+160}-6.73207_{e+146}i$	$3.65716_{e+208}-1.11213_{e+195}i$
1,5	$750\angle 0^\circ$	Pr	$6.74024_{e+03}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.27861_{e+00}-0.00000_{e+00}i$	$2.03245_{e+00}-0.00000_{e+00}i$	$2.54493_{e-07}-0.00000_{e+00}i$	$2.91452_{e-39}-0.00000_{e+00}i$	$8.43662_{e-105}-0.00000_{e+00}i^*$	$1.85651_{e-272}+2.32724_{e-304}i^*$
1,5	$750\angle 15^\circ$	Pr	$6.51024_{e+03}+1.74703_{e+03}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.29556_{e+00}-1.46232_{e-01}i$	$2.28038_{e+00}-6.92806_{e-01}i$	$4.53450_{e-07}+3.52540_{e-07}i$	$8.42064_{e-38}-3.02964_{e-38}i$	$-3.32205_{e-101}-3.37093_{e-101}i^*$	$-5.91068_{e-263}-1.90005_{e-263}i^*$
1,5	$750\angle 30^\circ$	Pr	$5.83591_{e+03}+3.37501_{e+03}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.34608_{e+00}-2.85812_{e-01}i$	$3.20547_{e+00}-1.97917_{e+00}i$	$-1.05387_{e-06}+6.15678_{e-06}i$	$9.64681_{e-34}+1.81046_{e-33}i$	$4.89651_{e-90}-1.23108_{e-90}i^*$	$4.65009_{e-235}-3.24315_{e-235}i^*$
1,5	$750\angle 45^\circ$	Pr	$2.64890_{e+03}+2.65165_{e+03}i$	$1.87500_{e+00}+4.33681_{e-19}i$	$1.62688_{e+00}-2.36905_{e-01}i$	$4.72989_{e-01}+1.83576_{e+00}i$	$8.45019_{e-06}-2.97726_{e-06}i$	$-5.11356_{e-29}+1.13641_{e-28}i$	$3.77569_{e-75}+1.15033_{e-75}i^*$	$7.27086_{e-194}+7.30679_{e-194}i^*$
1,5	$750\angle 60^\circ$	Ob	$-2.76062_{e+05}+4.84139_{e+05}i$	$1.87500_{e+00}+2.55351_{e-15}i$	$-4.89248_{e+02}+3.40804_{e+02}i$	$9.98322_{e+27}+2.01179_{e+27}i$	$1.01488_{e+70}+5.50686_{e+69}i$	$7.19427_{e+139}+1.10632_{e+140}i$	$8.42342_{e+208}+1.04625_{e+210}i$	$2.21538_{e+275}-1.57426_{e+275}i$
1,5	$750\angle 75^\circ$	Pr	$1.94364_{e+02}+7.24444_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.85574_{e+00}-6.72501_{e-02}i$	$-6.27711_{e-01}+3.34594_{e-01}i$	$-2.13925_{e-03}+3.42864_{e-03}i$	$-1.00098_{e-11}-3.45427_{e-12}i$	$8.76548_{e-29}-2.00576_{e-29}i^*$	$5.97109_{e-72}-1.27800_{e-72}i^*$
1,5	$750\angle 90^\circ$	Ob	$-5.53518_{e+05}+6.83353_{e-11}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,5	$1000\angle 0^\circ$	Pr	$8.99024_{e+03}-0.00000_{e+00}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.09474_{e+00}-0.00000_{e+00}i$	$1.19124_{e+00}-0.00000_{e+00}i$	$1.64211_{e-10}-0.00000_{e+00}i$	$1.48012_{e-53}-0.00000_{e+00}i$	$2.83691_{e-141}+3.35144_{e-173}i^*$	± 0
1,5	$1000\angle 15^\circ$	Pr	$8.68357_{e+03}+2.32937_{e+03}i$	$1.87500_{e+00}+2.16840_{e-19}i$	$1.11544_{e+00}-1.87535_{e-01}i$	$1.12765_{e+00}-8.94318_{e-01}i$	$8.32290_{e-11}-4.78212_{e-10}i$	$-1.30502_{e-51}-5.66328_{e-52}i$	$1.88067_{e-136}+2.20107_{e-136}i^*$	± 0
1,5	$1000\angle 30^\circ$	Pr	$7.78447_{e+03}+4.50000_{e+03}i$	$1.87500_{e+00}+4.33681_{e-19}i$	$1.17754_{e+00}-3.67991_{e-01}i$	$7.45735_{e-01}-2.38378_{e+00}i$	$-7.20430_{e-09}+9.14782_{e-09}i$	$-9.24647_{e-46}-3.70886_{e-47}i$	$2.17755_{e-122}+1.39609_{e-121}i^*$	$7.44280_{e-316}-1.71086_{e-315}i^*$
1,5	$1000\angle 45^\circ$	Pr	$3.53278_{e+03}+3.53553_{e+03}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.54421_{e+00}-3.10756_{e-01}i$	$9.62997_{e-01}+4.29572_{e-01}i$	$4.13310_{e-08}+1.41111_{e-08}i$	$-8.22608_{e-39}-2.55003_{e-39}i$	$-6.00462_{e-101}-8.90116_{e-102}i^*$	$1.72420_{e-259}-4.29105_{e-260}i^*$
1,5	$1000\angle 60^\circ$	Ob	$-4.93080_{e+05}+8.62025_{e+05}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,5	$1000\angle 75^\circ$	Pr	$2.59069_{e+02}+9.65926_{e+02}i$	$1.87500_{e+00}-0.00000_{e+00}i$	$1.84880_{e+00}-8.93620_{e-02}i$	$6.63498_{e-02}+5.09990_{e-01}i$	$3.81091_{e-04}+3.50614_{e-04}i$	$-1.49773_{e-15}+1.03386_{e-15}i$	$2.81929_{e-38}-8.72742_{e-39}i^*$	$4.58675_{e-96}-1.33299_{e-96}i^*$
1,5	$1000\angle 90^\circ$	Ob	$-9.88018_{e+05}+1.21730_{e-10}i$	$-0.00000_{e+00}-0.00000_{e+00}i$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.53: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
1,5	$1500\angle 0^\circ$	Pr	1.34902e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	7.48739e−01 − 0.00000e+00i	2.48401e−01 − 0.00000e+00i	4.78259e−17 − 0.00000e+00i	2.70531e−82 − 0.00000e+00i	2.18804e−214 − 2.88950e−246i*	±0
1,5	$1500\angle 15^\circ$	Pr	1.30302e+04 + 3.49406e+03i	1.87500e+00 − 0.00000e+00i	7.74409e−01 − 2.59792e−01i	6.77133e−02 − 3.16962e−01i	1.74079e−16 + 1.69088e−16i	8.72413e−80 − 2.39419e−79i	3.66272e−207 + 6.04077e−207i*	±0
1,5	$1500\angle 30^\circ$	Pr	1.16816e+04 + 6.75000e+03i	1.87500e+00 − 0.00000e+00i	8.52542e−01 − 5.13892e−01i	−6.03467e−01 − 3.67193e−01i	−2.05662e−14 + 1.95627e−14i	6.03689e−71 + 1.19110e−70i	2.25485e−186 − 7.63860e−185i*	±0
1,5	$1500\angle 45^\circ$	Pr	5.30055e+03 + 5.30330e+03i	1.87500e+00 − 2.16840e−19i	1.37928e+00 − 4.50985e−01i	5.55217e−02 − 2.66307e−01i	−8.35595e−14 + 8.69862e−13i	3.46308e−59 + 1.96661e−60i	−1.03545e−152 + 1.79902e−153i*	±0
1,5	$1500\angle 60^\circ$	Ob	−1.11462e+06 + 1.94256e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	$1500\angle 75^\circ$	Pr	3.88479e+02 + 1.44889e+03i	1.87500e+00 − 0.00000e+00i	1.83420e+00 − 1.33114e−01i	1.49870e−01 − 2.23189e−01i	−3.73775e−06 − 7.63706e−06i	4.23188e−23 + 3.31495e−23i	2.87844e−57 − 1.39177e−57i*	2.67633e−144 − 1.21394e−144i*
1,5	$1500\angle 90^\circ$	Ob	−2.23202e+06 + 2.74443e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	$2500\angle 0^\circ$	Pr	2.24902e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.38488e−01 − 0.00000e+00i	5.03846e−03 − 0.00000e+00i	2.18051e−30 − 0.00000e+00i	5.25836e−140 − 4.50847e−172i*	±0	±0
1,5	$2500\angle 15^\circ$	Pr	2.17236e+04 + 5.82343e+03i	1.87500e+00 − 0.00000e+00i	1.65389e−01 − 3.66464e−01i	−6.96700e−03 − 3.42053e−03i	1.27879e−29 − 3.00202e−29i	−1.12895e−135 + 4.80741e−135i*	±0	±0
1,5	$2500\angle 30^\circ$	Pr	1.94758e+04 + 1.12500e+04i	1.87500e+00 + 2.16840e−19i	2.50627e−01 − 7.37113e−01i	1.17339e−02 + 2.49156e−02i	5.39000e−26 − 7.32409e−26i	−1.60955e−120 − 1.39562e−121i*	4.33818e−312 − 1.05680e−311i*	±0
1,5	$2500\angle 45^\circ$	Pr	8.83608e+03 + 8.83883e+03i	1.87500e+00 − 1.08420e−19i	1.05191e+00 − 7.02486e−01i	2.71961e−03 + 1.29388e−02i	−9.30765e−23 − 2.41201e−22i	4.01912e−100 − 1.84412e−100i*	−1.51024e−256 + 1.59009e−256i*	±0
1,5	$2500\angle 60^\circ$	Ob	−3.10769e+06 + 5.40266e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	$2500\angle 75^\circ$	Pr	6.47298e+02 + 2.41481e+03i	1.87500e+00 + 5.42101e−20i	1.80216e+00 − 2.18663e−01i	6.57615e−02 + 3.27422e−02i	6.52428e−10 − 2.19749e−09i	−4.96943e−38 − 2.99800e−39i*	2.40102e−95 − 2.24003e−95i*	7.37408e−241 − 6.35258e−241i*
1,5	$2500\angle 90^\circ$	Ob	−6.22002e+06 + 7.63567e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	$5000\angle 0^\circ$	Pr	4.49902e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	−9.73191e−01 − 0.00000e+00i	7.76904e−08 − 0.00000e+00i	2.93982e−64 − 0.00000e+00i	9.30447e−285 − 7.43285e−317i*	±0	±0
1,5	$5000\angle 15^\circ$	Pr	4.34569e+04 + 1.16469e+04i	1.87500e+00 − 0.00000e+00i	−9.78360e−01 − 4.54687e−01i	1.72656e−07 + 6.03860e−08i	−6.33124e−62 − 1.77933e−62i	−7.38025e−275 + 4.75505e−276i*	±0	±0
1,5	$5000\angle 30^\circ$	Pr	3.89614e+04 + 2.25000e+04i	1.87500e+00 − 0.00000e+00i	−9.80647e−01 − 9.59532e−01i	9.93794e−07 + 2.02043e−06i	−4.98380e−55 − 1.04449e−55i	5.31368e−246 − 6.42798e−246i*	±0	±0
1,5	$5000\angle 45^\circ$	Pr	1.76749e+04 + 1.76777e+04i	1.87500e+00 + 5.42101e−20i	2.57812e−01 − 1.17474e+00i	1.29241e−06 − 3.54867e−06i	9.78540e−48 − 2.16252e−46i	1.09396e−203 + 1.45779e−202i*	±0	±0
1,5	$5000\angle 60^\circ$	Ob	−1.24654e+07 + 2.16306e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	$5000\angle 75^\circ$	Pr	1.29435e+03 + 4.82963e+03i	1.87500e+00 − 0.00000e+00i	1.70658e+00 − 4.20321e−01i	1.72815e−03 + 2.28808e−03i	−2.29244e−18 − 1.49280e−18i	1.31898e−75 + 1.59712e−76i*	1.95883e−191 − 2.82248e−190i*	±0
1,5	$5000\angle 90^\circ$	Ob	−2.49400e+07 + 3.05794e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$25\angle 0^\circ$	Pr	2.59497e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.30690e−01 − 0.00000e+00i	8.20046e−01 − 0.00000e+00i	−6.35275e−01 − 0.00000e+00i	1.01394e+00 − 0.00000e+00i	3.34299e−01 + 1.16352e−32i*	1.16595e−04 + 2.83787e−36i*
1,6	$25\angle 15^\circ$	Pr	2.50157e+02 + 7.13852e+01i	−0.00000e+00 − 0.00000e+00i	1.30710e−01 − 1.55757e−04i	8.30872e−01 − 1.19847e−01i	−7.42102e−01 − 2.35101e−01i	1.34782e+00 + 2.36943e−01i	2.83551e−02 − 4.02196e−01i*	3.52767e−05 + 2.20474e−04i*
1,6	$25\angle 30^\circ$	Ob	3.60481e+02 + 4.54627e+02i	−0.00000e+00 − 0.00000e+00i	1.30467e−01 − 9.90957e−04i	4.84839e−01 − 6.62156e−01i	−1.55584e+00 + 3.50176e+00i	−5.17036e+01 + 1.55697e+01i	−5.77871e+02 − 3.74087e+02i	2.00652e+02 − 2.26870e+03i
1,6	$25\angle 45^\circ$	Pr	1.17904e+02 + 1.23828e+02i	−0.00000e+00 − 0.00000e+00i	1.30999e−01 − 2.70535e−04i	1.05898e+00 − 2.37990e−01i	−3.05033e−01 − 1.56080e+00i	−6.08800e−01 + 2.87561e+00i	−1.94120e−01 − 4.68782e−01i*	3.70966e−04 + 1.96131e−03i*
1,6	$25\angle 60^\circ$	Ob	−1.47588e+02 + 4.41071e+02i	−0.00000e+00 − 0.00000e+00i	1.31578e−01 − 9.66308e−04i	1.43167e+00 − 1.09501e+00i	−1.99831e+01 − 1.41764e+01i	1.49709e+03 + 1.04017e+03i	−7.62852e+04 − 4.81463e+04i	−2.47096e+04 − 3.61562e+05i
1,6	$25\angle 75^\circ$	Ob	−2.70703e+02 + 2.34458e+02i	−0.00000e+00 − 0.00000e+00i	1.31849e−01 − 5.14287e−04i	1.92515e+00 − 6.64087e−01i	7.96683e+00 − 2.81505e+01i	−2.02521e+03 − 3.77627e+02i	4.01912e+01 + 6.30811e+04i	6.71424e+04 + 9.95959e+04i
1,6	$25\angle 90^\circ$	Ob	−3.44291e+02 + 5.80751e−14i	−0.00000e+00 − 0.00000e+00i	1.32011e−01 + 1.24598e−17i	2.21494e+00 + 3.41075e−17i	3.66592e+01 − 7.74929e−15i	3.16584e+03 − 1.91480e−12i	1.15599e+05 − 1.18150e−10i	2.75185e+05 − 3.19285e−10i
1,6	$50\angle 0^\circ$	Pr	5.34889e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.30090e−01 − 0.00000e+00i	4.27557e−01 − 0.00000e+00i	−4.15159e−01 − 0.00000e+00i	4.18383e−01 + 5.07093e−36i*	5.91323e−04 + 5.07093e−36i*	4.38413e−13 + 8.87477e−45i*
1,6	$50\angle 15^\circ$	Pr	5.16163e+02 + 1.42448e+02i	−0.00000e+00 − 0.00000e+00i	1.30130e−01 − 3.09943e−04i	4.35272e−01 − 1.75081e−01i	−4.85276e−01 + 3.85560e−01i	2.03861e−01 − 3.60653e−01i	−3.66402e−04 + 2.82879e−04i*	−3.52112e−13 + 7.53409e−13i*
1,6	$50\angle 30^\circ$	Ob	1.34799e+03 + 1.99184e+03i	−0.00000e+00 − 0.00000e+00i	1.28285e−01 − 4.29870e−03i	−1.34037e+00 + 1.53757e−01i	−4.06793e+01 + 1.35390e+01i	−2.10551e+03 − 1.36150e+04i	4.00045e+06 + 6.04878e+04i	−1.53961e+08 + 2.16961e+08i
1,6	$50\angle 45^\circ$	Pr	2.41694e+02 + 2.47530e+02i	−0.00000e+00 − 0.00000e+00i	1.30728e−01 − 5.40044e−04i	7.93958e−01 − 4.08510e−01i	−1.74912e+00 − 2.64770e−01i	4.55164e−01 − 3.90804e−01i	−3.82748e−04 − 2.63397e−03i*	2.88550e−10 − 9.52359e−10i*
1,6	$50\angle 60^\circ$	Ob	−9.11733e+02 + 1.96498e+03i	−0.00000e+00 − 0.00000e+00i	1.33218e−01 − 4.33771e−03i	−2.55366e+00 − 6.68368e+00i	2.15248e+03 + 1.44409e+03i	3.61050e+07 + 2.53577e+07i	4.16818e+11 + 2.97087e+11i	4.13827e+13 − 1.26108e+14i
1,6	$50\angle 75^\circ$	Ob	−1.60407e+03 + 1.09454e+03i	−0.00000e+00 − 0.00000e+00i	1.34781e−01 − 2.43302e−03i	5.44016e+00 − 8.47872e+00i	−5.45713e+03 − 1.06082e+03i	1.75492e+08 + 1.41930e+07i	−2.74374e+12 + 1.63040e+11i	3.25808e+13 + 1.28795e+14i
1,6	$50\angle 90^\circ$	Ob	−1.91859e+03 + 2.69383e−13i	−0.00000e+00 − 0.00000e+00i	1.35494e−01 − 2.57643e−18i	1.20028e+01 − 3.02412e−15i	8.54413e+03 − 6.09250e−12i	4.13109e+08 − 6.12507e−07i	9.79136e+12 − 2.21209e−02i	5.06897e+14 − 1.33205e+00i
1,6	$100\angle 0^\circ$	Pr	1.08507e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.28896e−01 − 0.00000e+00i	−4.62187e−02 − 0.00000e+00i	4.58012e−01 − 0.00000e+00i	4.16099e−03 − 0.00000e+00i*	1.78678e−10 + 2.03753e−42i*	6.29863e−31 − 0.00000e+00i*
1,6	$100\angle 15^\circ$	Pr	1.04760e+03 + 2.84748e+02i	−0.00000e+00 − 0.00000e+00i	1.28976e−01 − 6.16110e−04i	−6.12449e−02 − 1.68483e−01i	5.93709e−01 + 1.11216e−01i	−3.19864e−03 − 8.56916e−04i	−5.70202e−11 − 1.52429e−10i*	−3.22632e−30 − 9.83299e−31i*
1,6	$100\angle 30^\circ$	Ob	5.19800e+03 + 8.31384e+03i	−0.00000e+00 − 0.00000e+00i	1.19449e−01 − 1.72449e−02i	2.06092e+00 − 7.81743e+00i	−1.83306e+03 − 1.08558e+04i	−3.13964e+08 + 1.80670e+09i	1.35148e+14 − 2.40633e+14i	−7.32627e+18 − 1.08381e+18i
1,6	$100\angle 45^\circ$	Pr	4.89203e+02 + 4.94996e+02i	−0.00000e+00 − 0.00000e+00i	1.30186e−01 − 1.07695e−03i	3.12219e−01 − 5.83678e−01i	−4.10192e−01 + 1.24890e+00i	7.78762e−03 + 7.77387e−03i	1.48012e−08 − 3.24273e−08i*	1.30447e−22 − 3.30904e−23i*
1,6	$100\angle 60^\circ$	Ob	−4.31525e+03 + 8.26021e+03i	−0.00000e+00 − 0.00000e+00i	1.40133e−01 − 1.88469e−02i	−6.63410e+01 + 2.54451e+02i	5.20794e+07 + 3.60999e+07i	4.30264e+16 + 3.38705e+16i	2.45028e+25 + 2.15352e+25i	1.09221e+32 − 1.65713e+30i
1,6	$100\angle 75^\circ$	Ob	−7.51941e+03 + 4.68934e+03i	−0.00000e+00 − 0.00000e+00i	1.48082e−01 − 1.10480e−02i	−4.02267e+02 − 4.39599e+02i	4.65565e+08 + 3.98008e+07i	2.65893e+18 − 2.93601e+17i	7.51948e+27 − 2.46653e+27i	−1.26633e+34 + 2.36226e+33i
1,6	$100\angle 90^\circ$	Ob	−8.81828e+03 + 1.15115e−12i	−0.00000e+00 − 0.00000e+00i	1.51416e−01 − 6.79061e−18i	8.39555e+02 − 4.82196e−13i	1.09697e+09 − 1.63857e−06i	1.47036e+19 − 4.44994e+04i	1.01651e+29 − 4.63724e+14i	−3.10483e+35 + 1.87202e+21i
1,6	$250\angle 0^\circ$	Pr	2.73518e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.25358e−01 − 0.00000e+00i	−2.52747e−01 − 0.00000e+00i	5.84170e−02 − 0.00000e+00i	2.80199e−11 − 0.00000e+00i	2.96372e−32 − 0.00000e+00i	1.12413e−86 − 1.02234e−118i*
1,6	$250\angle 15^\circ$	Pr	2.64148e+03 + 7.11771e+02i	−0.00000e+00 − 0.00000e+00i	1.25552e−01 − 1.51440e−03i	−2.95301e−01 + 8.58577e−02i	1.15040e−02 − 7.71508e−02i	−2.32951e−11 − 8.48705e−11i	−4.52283e−31 − 3.21742e−31i*	1.43845e−84 + 1.99633e−83i*
1,6	$250\angle 30^\circ$	Ob	3.17480e+04 + 5.32606e+04i	−0.00000e+00 − 0.00000e+00i	4.76995e−02 − 8.02026e−02i	−1.07051e+03 − 5.72395e+03i	−5.75268e+11 − 1.99590e+11i	1.30655e+25 − 5.31000e+24i	−1.32322e+38 + 2.63799e+38i	4.90427e+50 + 1.73175e+50i
1,6	$250\angle 45^\circ$	Pr	1.23168e+03 + 1.23745e+03i	−0.00000e+00 − 0.00000e+00i	1.28562e−01 − 2.66983e−03i	−5.60201e−01 − 3.33620e−01i	−5.16799e−03 − 1.24995e−01i	1.20137e−08 − 1.47859e−08i	1.84450e−23 + 1.35597e−23i*	−8.89650e−62 + 2.30485e−62i*
1,6	$250\angle 60^\circ$	Ob	−2.95260e+04 + 5.31266e+04i	−0.00000e+00 − 0.00000e+00i	1.68469e−01 − 1.49784e−01i	3.73814e+07 + 2.54290e+07i	2.43263e+21 + 2.03152e+21i	2.28515e+44 + 2.65185e+44i	1.40250e+67 + 2.28738e+67i	−2.15832e+87 + 3.33498e+87i
1,6	$250\angle 75^\circ$	Ob	−5.12469e+04 + 3.04735e+04i	−0.00000e+00 − 0.00000e+00i	2.61448e−01 − 1.06738e−01i	4.51451e+08 + 3.61935e+07i	−9.63034e+23 + 1.97576e+23i	2.40961e+49 − 1.92313e+49i	−2.09609e+74 + 4.67136e+74i	−2.12741e+95 − 2.37639e+95i
1,6	$250\angle 90^\circ$	Ob	−5.95181e+04 + 7.47034e−12i	−0.00000e+00 − 0.00000e+00i	3.06176e−01 + 7.88388e−18i	1.06239e+09 − 1.43405e−06i	8.27419e+24 − 3.01708e+10i	2.18177e+51 − 1.63068e+37i	3.04488e+77 − 3.44158e+63i	2.65088e+98 − 4.30011e+84i
1,6	$500\angle 0^\circ$	Pr	5.48522e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.19597e−01 − 0.00000e+00i	1.08097e−01 − 0.00000e+00i	1.01516e−04 − 0.00000e+00i	3.33228e−25 − 0.00000e+00i	2.33052e−68 − 2.74189e−100i*	3.13090e−179 − 0.00000e+00i*
1,6	$500\angle 15^\circ$	Pr	5.29781e+03 + 1.42351e+03i	−0.00000e+00 − 0.00000e+00i	1.19962e−01 − 2.94465e−03i	1.45741e−01 + 1.27891e−01i	−1.61967e−04 + 6.78599e−05i	−1.49923e−24 + 2.90737e−24i	5.78262e−66 + 4.86033e−66i*	−5.62435e−173 + 4.47257e−173i*
1,6	$500\angle 30^\circ$	Ob	1.25998e+05 + 2.14774e+05i	−0.00000e+00 − 0.00000e+00i	−1.56240e−01 + 2.24495e−02i	−1.24619e+08 + 7.68573e+08i	1.04901e+25 − 4.29609e+24i	−2.49109e+51 − 9.47235e+51i	−7.58536e+78 + 9.57427e+77i	1.33929e+104 − 5.28986e+103i
1,6	$500\angle 45^\circ$	Pr	2.46912e+03 + 2.47488e+03i	−0.00000e+00 − 0.00000e+00i	1.25856e−01 − 5.26534e−03i	−4.59907e−01 + 4.05603e−01i	5.25720e−04 − 7.85111e−04i	1.69674e−18 + 1.04335e−18i	−4.17261e−49 − 2.20484e−49i*	−6.02408e−128 + 2.03105e−127i*
1,6	$500\angle 60^\circ$	Ob	−1.21544e+05 + 2.14506e+05i	−0.00000e+00 − 0.00000e+00i	−3.76826e−01 − 8.87752e−01i	4.48579e+16 + 3.48139e+16i	3.30946e+44 + 3.83287e+44i	6.70233e+90 + 1.67091e+91i	2.31059e+136 + 4.44510e+137i	−1.35477e+180 − 2.61087e+180i
1,6	$500\angle 75^\circ$	Ob	−2.10729e+05 + 1.23447e+05i	−0.00000e+00 − 0.00000e+00i	8.06840e−01 − 1.34121e+00i	6.84445e+18 − 7.56528e+17i	6.31125e+49 − 5.02743e+49i	−1.67394e+100 − 4.20504e+101i	−9.72599e+152 − 6.52699e+152i	−2.00372e+199 − 1.66589e+199i
1,6	$500\angle 90^\circ$	Ob	−2.44018e+05 + 3.02488e−11i	−0.00000e+00 − 0.00000e+00i	1.85649e+00 − 4.45573e−16i	3.78481e+19 − 1.13375e+05i	5.71095e+51 − 4.33354e+37i	2.10718e+105 − 3.21187e+91i	4.14792e+158 − 9.49766e+144i	5.18240e+206 − 1.57069e+193i
1,6	$750\angle 0^\circ$	Pr	8.23523e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.14002e−01 − 0.00000e+00i	1.85966e−01 − 0.00000e+00i	8.65167e−08 − 0.00000e+00i	2.16337e−39 − 0.00000e+00i	1.03298e−104 + 1.96794e−136i*	4.38467e−272 + 5.57419e−304i*
1,6	$750\angle 15^\circ$	Pr	7.95412e+03 + 2.13526e+03i	−0.00000e+00 − 0.00000e+00i	1.14517e−01 − 4.29353e−03i	2.29013e−01 − 2.85111e−02i	1.52944e−07 + 1.21959e−07i	6.25908e−38 − 2.23211e−38i	−4.21525e−101 − 4.22745e−101i*	−1.44108e−262 − 4.52479e−263i*
1,6	$750\angle 30^\circ$	Ob	2.82748e+05 + 4.84541e+05i	−0.00000e+00 − 0.00000e+00i	2.44370e−01 + 2.71127e−01i	6.67538e+13 − 1.22148e+14i	−1.25453e+38 + 2.52234e+38i	−9.00693e+78 + 1.14419e+78i	−1.73162e+119 − 1.99755e+119i	2.46074e+157 − 4.71776e+157i
1,6	$750\angle 45^\circ$	Pr	3.70656e+03 + 3.71231e+03i	−0.00000e+00 − 0.00000e+00i	1.23151e−01 − 7.78745e−03i	5.57385e−02 + 4.00609e−01i	4.96034e−06 − 1.66143e−06i	−6.42349e−29 + 1.41333e−28i	7.93211e−75 + 2.47527e−75i*	2.93125e−193 + 2.97633e−193i*
1,6	$750\angle 60^\circ$	Ob	−2.76062e+05 + 4.84139e+05i	−0.00000e+00 − 0.00000e+00i	−5.57193e+00 + 4.58780e−01i	7.14661e+25 + 6.33332e+25i	5.63465e+67 + 9.24505e+67i	6.22097e+136 + 1.23584e+138i	−4.24498e+207 + 8.87949e+207i	2.53768e+273 − 2.32911e+272i
1,6	$750\angle 75^\circ$	Ob	−4.78464e+05 + 2.78921e+05i	−0.00000e+00 − 0.00000e+00i	−1.23588e+00 − 1.15625e+01i	1.34346e+29 − 4.13058e+28i	−3.69355e+75 + 8.06974e+75i	−6.35422e+153 − 4.29492e+153i	2.43816e+231 − 2.57461e+231i	−1.39645e+303 + 6.12678e+302i
1,6	$750\angle 90^\circ$	Ob	−5.53518e+05 + 6.83353e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1000\angle 0^\circ$	Pr	1.09852e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.08569e−01 − 0.00000e+00i	1.26998e−01 − 0.00000e+00i	5.64855e−11 − 0.00000e+00i	1.10163e−53 − 0.00000e+00i	3.56605e−141 + 4.62062e−173i*	±0
1,6	$1000\angle 15^\circ$	Pr	1.06104e+04 + 2.84701e+03i	−0.00000e+00 − 0.00000e+00i	1.09215e−01 − 5.56365e−03i	1.31912e−01 − 8.53781e−02i	3.01434e−11 − 1.64444e−10i	−9.70695e−52 − 4.23657e−52i	2.37229e−136 + 2.77689e−136i*	±0
1,6	$1000\angle 30^\circ$	Ob	5.01998e+05 + 8.62561e+05i	−0.00000e+00 − 0.00000e+00i	2.29669e−01 + 9.27662e−01i	−2.08560e+19 + 1.86902e+19i	−2.01392e+51 − 7.61660e+51i	−4.67005e+103 + 9.47096e+103i	2.72768e+159 − 9.91792e+159i	−5.10111e+210 − 2.15290e+211i
1,6	$1000\angle 45^\circ$	Pr	4.94400e+03 + 4.94975e+03i	−0.00000e+00 − 0.00000e+00i	1.20450e−01 − 1.02371e−02i	2.01748e−01 + 1.10506e−01i	2.41261e−08 + 8.55004e−09i	−1.02485e−38 − 3.20855e−39i	−1.26386e−100 − 1.88258e−101i*	6.98325e−259 − 1.74155e−259i*
1,6	$1000\angle 60^\circ$	Ob	−4.93080e+05 + 8.62025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1000\angle 75^\circ$	Ob	−8.54452e+05 + 4.96894e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1000\angle 90^\circ$	Ob	−9.88018e+05 + 1.21730e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.53: (continued)

$m,\ n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)}$	$S_{mn}^{(a)}(c,\eta{=}0)$	$S_{mn}^{(a)}(c,\eta{=}0.01)$	$S_{mn}^{(a)}(c,\eta{=}0.1)$	$S_{mn}^{(a)}(c,\eta{=}0.25)$	$S_{mn}^{(a)}(c,\eta{=}0.5)$	$S_{mn}^{(a)}(c,\eta{=}0.75)$	$S_{mn}^{(a)}(c,\eta{=}0.99)$
1,6	$1500\angle 0^\circ$	Pr	1.64852e+04 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	9.81750e–02 – 0.00000e+00i	2.96540e–02 – 0.00000e+00i	1.66404e–17 – 0.00000e+00i	2.01893e–82 – 0.00000e+00i	2.73555e–214 – 3.55291e–246i*	±0
1,6	$1500\angle 15^\circ$	Pr	1.59230e+04 + 4.27052e+03i	–0.00000e+00 – 0.00000e+00i	9.90259e–02 – 7.87778e–03i	1.00747e–02 – 3.77511e–02i	6.02736e–17 + 5.92315e–17i	6.53660e–80 – 1.78617e–79i	4.64104e–207 + 7.66353e–207i*	±0
1,6	$1500\angle 30^\circ$	Ob	1.12800e+06 + 1.94336e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1500\angle 45^\circ$	Pr	7.41887e+03 + 7.42462e+03i	–0.00000e+00 – 0.00000e+00i	1.15057e–01 – 1.49225e–02i	1.54558e–02 – 5.87194e–02i	–5.30736e–14 + 5.11344e–13i	4.32183e–59 + 2.53529e–60i	–2.18049e–152 + 3.78310e–153i*	±0
1,6	$1500\angle 60^\circ$	Ob	–1.11462e+06 + 1.94256e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1500\angle 75^\circ$	Ob	–1.93119e+06 + 1.12034e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$1500\angle 90^\circ$	Ob	–2.23202e+06 + 2.74443e–10i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$2500\angle 0^\circ$	Pr	2.74852e+04 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	7.91759e–02 – 0.00000e+00i	6.46797e–04 – 0.00000e+00i	7.65470e–31 – 0.00000e+00i	3.92667e–140 + 4.78808e–172i*	±0	±0
1,6	$2500\angle 15^\circ$	Pr	2.65482e+04 + 7.11753e+03i	–0.00000e+00 – 0.00000e+00i	8.02400e–02 – 1.16695e–02i	–8.86549e–04 – 4.63785e–04i	4.52731e–30 – 1.05281e–29i	–8.05254e–136 + 3.44969e–135i*	±0	±0
1,6	$2500\angle 30^\circ$	Ob	3.13000e+06 + 5.40400e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$2500\angle 45^\circ$	Pr	1.23686e+04 + 1.23744e+04i	–0.00000e+00 – 0.00000e+00i	1.04339e–01 – 2.34632e–02i	5.30098e–04 + 2.95959e–03i	–5.42714e–23 – 1.42575e–22i	5.02256e–100 – 2.29813e–100i*	–3.18138e–256 + 3.34908e–256i*	±0
1,6	$2500\angle 60^\circ$	Ob	–3.10769e+06 + 5.40266e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$2500\angle 75^\circ$	Ob	–5.38370e+06 + 3.11724e+06i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$2500\angle 90^\circ$	Ob	–6.22002e+06 + 7.63567e–10i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$5000\angle 0^\circ$	Pr	5.49852e+04 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	4.09050e–02 – 0.00000e+00i	1.04540e–08 – 0.00000e+00i	1.03882e–64 – 0.00000e+00i	6.83470e–285 + 7.50049e–317i*	±0	±0
1,6	$5000\angle 15^\circ$	Pr	5.31112e+04 + 1.42350e+04i	–0.00000e+00 – 0.00000e+00i	4.16588e–02 – 1.70840e–02i	2.31767e–08 + 8.40517e–09i	–2.23665e–62 – 6.32649e–63i	–5.53026e–275 + 3.54172e–276i*	±0	±0
1,6	$5000\angle 30^\circ$	Ob	1.25100e+07 + 2.16333e+07i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$5000\angle 45^\circ$	Pr	2.47430e+04 + 2.47487e+04i	–0.00000e+00 – 0.00000e+00i	7.82029e–02 – 4.03072e–02i	3.09953e–07 – 8.14029e–07i	6.07765e–48 – 1.27879e–46i	1.35646e–203 + 1.82237e–202i*	±0	±0
1,6	$5000\angle 60^\circ$	Ob	–1.24654e+07 + 2.16306e+07i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$5000\angle 75^\circ$	Ob	–2.15927e+07 + 1.24845e+07i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	$5000\angle 90^\circ$	Ob	–2.49400e+07 + 3.05794e–09i	–0.00000e+00 – 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.53: $S_{1n}^{(a)}(c,\,\eta)$ for $n\,=\,[1..6]$.

D.2.2 Asymptotic expansions of the derivative of the PASWF, $S_{mn}^{(a)\prime}(c,\eta)$ and the corresponding characteristic eigenvalues, $\lambda_{mn}^{(a)}$, for $m=[0..1]$, $n=[m..6]$.

Table D.54: $S_{0n}^{(a)\prime}(c,\,\eta)$ for $n\,=\,[0\,..6]$.

$m,\ n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,0	$25\angle 0^\circ$	Pr	2.42421e+01 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	–2.42139e–01 – 0.00000e+00i	–2.15737e+00 – 0.00000e+00i	–2.89720e+00 – 0.00000e+00i	–5.45079e–01 – 0.00000e+00i	–7.74910e–03 – 0.00000e+00i	–2.51398e–07 – 0.00000e+00i
0,0	$25\angle 15^\circ$	Pr	2.33906e+01 + 6.47263e+00i	–0.00000e+00 – 0.00000e+00i	–2.33664e–01 – 6.45782e–02i	–2.10815e+00 – 5.10442e–01i	–2.97386e+00 – 1.92570e–01i	–5.06218e–01 + 3.43463e–01i	3.52500e–03 + 9.73686e–03i	–2.69429e–07 – 4.53501e–07i
0,0	$25\angle 30^\circ$	Pr	2.08940e+01 + 1.25041e+01i	–0.00000e+00 – 0.00000e+00i	–2.08810e–01 – 1.24786e–01i	–1.95802e+00 – 1.01110e+00i	–3.20404e+00 – 4.60246e–01i	–3.62999e–01 + 7.77361e–01i	2.07006e–02 – 1.25864e–02i	3.53489e–06 – 2.98901e–06i*
0,0	$25\angle 45^\circ$	Pr	1.69224e+01 + 1.76834e+01i	–0.00000e+00 – 0.00000e+00i	–1.69245e–01 – 1.76543e–01i	–1.70057e+00 – 1.48803e+00i	–3.58002e+00 – 9.00718e–01i	–1.74343e–02 + 1.46911e+00i	–4.14433e–02 – 8.39631e–02i	2.27577e–05 + 1.44173e–04i*
0,0	$25\angle 60^\circ$	Ob	–2.70208e+02 + 5.16261e+02i	–0.00000e+00 – 0.00000e+00i	2.66982e+00 – 5.20942e+00i	–2.23683e+01 – 9.43630e+01i	–1.87817e+03 + 1.09088e+03i	3.55436e+05 – 1.97938e+05i	–6.90905e+07 + 3.68945e+07i	1.17059e+10 – 4.24834e+09i
0,0	$25\angle 75^\circ$	Ob	–4.93980e+02 + 2.99556e+02i	–0.00000e+00 – 0.00000e+00i	4.96581e+00 – 3.04525e+00i	7.30710e+01 – 9.77547e+01i	–1.26525e+03 – 3.83544e+03i	–1.31995e+06 + 5.03898e+05i	2.02929e+08 + 4.65680e+08i	1.35901e+11 – 5.62900e+10i
0,0	$25\angle 90^\circ$	Ob	–5.76010e+02 + 7.34781e–14i	–0.00000e+00 – 0.00000e+00i	5.81593e+00 – 3.17243e–16i	1.31954e+02 – 1.91518e–14i	4.99010e+03 – 1.85672e–12i	2.16027e+06 – 1.63012e–09i	9.61115e+08 – 1.09294e–06i	3.41471e+11 – 5.13731e–04i
0,0	$50\angle 0^\circ$	Pr	4.92462e+01 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	–4.91275e–01 – 0.00000e+00i	–3.86656e+00 – 0.00000e+00i	–2.66164e+00 – 0.00000e+00i	–3.89299e–02 – 0.00000e+00i	–3.33611e–06 – 0.00000e+00i	–2.57925e–16 – 0.00000e+00i
0,0	$50\angle 15^\circ$	Pr	4.75426e+01 + 1.29420e+01i	–0.00000e+00 – 0.00000e+00i	–4.74404e–01 – 1.28812e–01i	–3.82333e+00 – 7.78703e–01i	–2.78161e+00 + 4.06390e–01i	–5.04079e–03 + 4.86802e–02i	3.34644e–06 – 4.91204e–06i	1.78997e–16 – 1.10324e–15i
0,0	$50\angle 30^\circ$	Pr	4.25480e+01 + 2.50020e+01i	–0.00000e+00 – 0.00000e+00i	–4.24908e–01 – 2.48970e–01i	–3.68136e+00 – 1.58244e+00i	–3.18663e+00 + 8.56040e–01i	9.07429e–02 + 3.04943e–02i	2.47002e–06 + 3.22196e–05i	3.89726e–14 + 7.24439e–14i*
0,0	$50\angle 45^\circ$	Pr	3.46027e+01 + 3.53581e+01i	–0.00000e+00 – 0.00000e+00i	–3.46069e–01 – 3.52376e–01i	–3.40464e+00 – 2.42550e+00i	–4.03276e+00 + 1.36455e+00i	1.94321e–01 – 1.99260e–01i	–8.31674e–05 – 4.70383e–04i	2.35276e–11 – 7.29952e–11i*
0,0	$50\angle 60^\circ$	Ob	–1.16440e+03 + 2.11506e+03i	–0.00000e+00 – 0.00000e+00i	1.11134e+01 – 2.19722e+01i	–1.68756e+03 – 1.83526e+02i	8.68811e+05 – 4.74296e+05i	3.70976e+10 – 1.86492e+10i	1.62625e+15 – 7.50022e+14i	5.08183e+19 – 7.60684e+18i
0,0	$50\angle 75^\circ$	Ob	–2.06948e+03 + 1.22412e+03i	–0.00000e+00 – 0.00000e+00i	2.11598e+01 – 1.30981e+01i	2.71662e+01 – 2.79183e+03i	–3.22287e+06 + 1.21409e+06i	4.52759e+11 – 2.20693e+11i	–6.46907e+16 + 9.93899e+16i	6.28644e+21 – 3.52450e+21i
0,0	$50\angle 90^\circ$	Ob	–2.40101e+03 + 3.00038e–13i	–0.00000e+00 – 0.00000e+00i	2.49841e+01 – 3.42669e–15i	3.30782e+03 – 1.24345e–12i	5.26927e+06 – 4.40010e–09i	1.17991e+12 – 1.88828e–03i	2.71648e+17 – 6.42641e+02i	3.89074e+22 – 1.20630e+08i
0,0	$100\angle 0^\circ$	Pr	9.92481e+01 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	–9.87617e–01 – 0.00000e+00i	–6.06510e+00 – 0.00000e+00i	–1.09655e+00 – 0.00000e+00i	–9.67642e–05 – 0.00000e+00i	–2.99775e–13 – 0.00000e+00i	–1.19201e–34 + 7.29323e–51i*
0,0	$100\angle 15^\circ$	Pr	9.58408e+01 + 2.58824e+01i	–0.00000e+00 – 0.00000e+00i	–9.54206e–01 – 2.56365e–01i	–6.11585e+00 – 8.24686e–01i	–1.03676e+00 + 6.47193e–01i	1.52498e–04 – 9.46434e–06i	5.70918e–13 + 7.59945e–13i	2.22680e–33 + 6.87101e–35i*
0,0	$100\angle 30^\circ$	Pr	8.58509e+01 + 5.00010e+01i	–0.00000e+00 – 0.00000e+00i	–8.56116e–01 – 4.95754e–01i	–6.24598e+00 – 1.77426e+00i	–8.20826e–01 + 1.46554e+00i	–5.79412e–04 – 6.52773e–05i	2.15718e–11 – 1.78635e–11i	6.13752e–32 – 1.19131e–29i*
0,0	$100\angle 45^\circ$	Pr	6.99594e+01 + 7.07120e+01i	–0.00000e+00 – 0.00000e+00i	–6.99672e–01 – 7.02217e–01i	–6.38049e+00 – 2.97520e+00i	–3.23225e–01 + 2.76694e+00i	3.61634e–03 + 3.31916e–03i	2.45166e–09 – 5.57502e–09i	9.86968e–24 – 2.38147e–24i*
0,0	$100\angle 60^\circ$	Ob	–4.82780e+03 + 8.56025e+03i	–0.00000e+00 – 0.00000e+00i	3.91531e+01 – 9.93442e+01i	1.89439e+05 + 1.78164e+05i	8.98162e+10 – 4.47007e+10i	1.95172e+20 – 8.13889e+19i	4.33865e+29 – 1.47869e+29i	4.16627e+38 + 1.01106e+38i
0,0	$100\angle 75^\circ$	Ob	–8.46807e+03 + 4.94824e+03i	–0.00000e+00 – 0.00000e+00i	9.25352e+01 – 6.42558e+01i	–6.76232e+05 – 2.01123e+05i	1.09600e+12 – 5.31405e+11i	2.50528e+22 – 1.85974e+22i	5.57276e+32 – 6.04153e+32i	6.16205e+42 – 5.78813e+42i
0,0	$100\angle 90^\circ$	Ob	–9.80100e+03 + 1.21240e–12i	–0.00000e+00 – 0.00000e+00i	1.14831e+02 – 1.89790e–14i	9.91600e+05 – 6.90877e–10i	2.85416e+12 – 4.60998e–03i	1.71377e+23 – 5.39143e+08i	1.05822e+34 – 4.94903e+19i	2.46594e+44 – 1.51563e+30i
0,0	$250\angle 0^\circ$	Pr	2.49249e+02 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	–2.46174e+00 – 0.00000e+00i	–7.18152e+00 – 0.00000e+00i	–2.35170e–02 – 0.00000e+00i	–4.55135e–13 – 0.00000e+00i	–6.63641e–35 – 0.00000e+00i	–3.37563e–90 + 2.06631e–106i*
0,0	$250\angle 15^\circ$	Pr	2.40731e+02 + 6.47050e+01i	–0.00000e+00 – 0.00000e+00i	–2.38068e+00 – 6.31641e–01i	–7.48130e+00 + 4.62594e–01i	6.76467e–03 + 3.00736e–02i	7.47491e–13 + 1.21329e–12i	1.11628e–33 + 4.04154e–34i	–1.66686e–87 – 4.80318e–87i*
0,0	$250\angle 30^\circ$	Pr	2.15756e+02 + 1.25000e+02i	–0.00000e+00 – 0.00000e+00i	–2.14220e+00 – 1.22329e+00i	–8.45410e+00 + 8.60713e–01i	6.50539e–02 – 2.03050e–02i	3.52488e–11 – 1.98846e–11i	3.22966e–30 – 4.55491e–30i	–1.05473e–77 + 2.36468e–79i*
0,0	$250\angle 45^\circ$	Pr	1.76026e+02 + 1.76777e+02i	–0.00000e+00 – 0.00000e+00i	–1.76034e+00 – 1.73688e+00i	–1.03250e+01 + 1.02106e+00i	–2.72523e–02 – 2.39205e–01i	5.12788e–09 – 6.52496e–09i	3.13884e–24 + 2.28140e–24i	–6.77575e–63 + 1.71859e–63i*
0,0	$250\angle 60^\circ$	Ob	–3.08180e+04 + 5.38766e+04i	–0.00000e+00 – 0.00000e+00i	–2.29420e+02 – 1.06199e+03i	2.56653e+11 – 1.26887e+11i	2.99802e+25 – 1.12669e+25i	8.41675e+48 – 1.64860e+48i	2.36135e+72 – 6.47157e+70i	1.38773e+94 + 7.53207e+94i
0,0	$250\angle 75^\circ$	Ob	–5.36446e+04 + 3.11206e+04i	–0.00000e+00 – 0.00000e+00i	8.36232e+02 – 1.09307e+03i	3.13153e+12 – 1.51303e+12i	–1.22711e+28 + 1.10030e+28i	8.25212e+53 – 2.11706e+54i	3.10430e+79 + 3.20680e+80i	3.44595e+104 – 4.17486e+105i
0,0	$250\angle 90^\circ$	Ob	–6.20010e+04 + 7.62343e–12i	–0.00000e+00 – 0.00000e+00i	1.49140e+03 – 2.05742e–13i	8.15107e+12 – 1.23371e–02i	1.38623e+29 – 5.28120e+14i	1.60759e+56 – 1.22768e+42i	1.91742e+83 – 2.19809e+69i	1.92589e+109 – 2.91536e+95i
0,0	$500\angle 0^\circ$	Pr	4.99250e+02 – 0.00000e+00i	–0.00000e+00 – 0.00000e+00i	–4.86965e+00 – 0.00000e+00i	–4.10838e+00 – 0.00000e+00i	–1.68040e–05 – 0.00000e+00i	–2.59284e–27 – 0.00000e+00i	–2.31679e–71 + 1.41690e–87i*	–3.75609e–183 + 2.29953e–199i*

Table D.54: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,0	$500\angle 15^\circ$	Pr	4.82213e+02 + 1.29410e+02i	−0.00000e+00 − 0.00000e+00i	−4.71556e+00 − 1.23284e+00i	−4.14481e+00 + 1.68673e+00i	2.19754e−05 − 1.87173e−05i	5.13374e−27 − 2.48913e−26i	−6.73345e−69 − 3.09977e−69i*	5.05703e−177 − 6.85032e−177i*
0,0	$500\angle 30^\circ$	Pr	4.32262e+02 + 2.50000e+02i	−0.00000e+00 − 0.00000e+00i	−4.26066e+00 − 2.39360e+00i	−4.28857e+00 + 3.82857e+00i	−6.00440e−05 + 1.27606e−04i	−3.58339e−25 + 2.04844e−23i	1.24597e−61 + 1.06452e−61i*	−3.06443e−158 + 1.99938e−158i*
0,0	$500\angle 45^\circ$	Pr	3.52803e+02 + 3.53554e+02i	−0.00000e+00 − 0.00000e+00i	−3.52738e+00 − 3.41207e+00i	−4.72929e+00 + 7.13949e+00i	9.27154e−04 − 1.49452e−03i	7.36854e−19 + 4.44977e−19i	−7.05861e−50 − 3.56325e−50i*	−4.34858e−129 + 1.54118e−128i*
0,0	$500\angle 60^\circ$	Ob	−1.24135e+05 + 2.16006e+05i	−0.00000e+00 − 0.00000e+00i	−1.86385e+04 − 2.19382e+03i	1.34000e+21 − 5.53660e+20i	2.02302e+49 − 3.94625e+48i	1.75594e+96 + 2.46146e+95i	1.39790e+143 + 7.12149e+142i	−1.85060e+188 − 3.01436e+187i
0,0	$500\angle 75^\circ$	Ob	−2.15541e+05 + 1.24741e+05i	−0.00000e+00 − 0.00000e+00i	4.43749e+02 − 3.09202e+04i	1.72104e+23 − 1.27305e+23i	1.98550e+54 − 5.08772e+54i	−6.65969e+106 − 1.05205e+107i	−2.91254e+159 − 2.04279e+158i	−5.10635e+209 − 2.32757e+209i
0,0	$500\angle 90^\circ$	Ob	−2.49001e+05 + 3.05549e−11i	−0.00000e+00 − 0.00000e+00i	3.66611e+04 − 1.34593e−11i	1.17609e+24 − 3.67240e+09i	3.86418e+56 − 2.98120e+42i	6.23368e+110 − 9.58054e+96i	1.03431e+165 − 2.38129e+151i	1.18632e+217 − 3.60296e+203i
0,0	$750\angle 0^\circ$	Pr	7.49250e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.21736e+00 − 0.00000e+00i	−1.76095e+00 − 0.00000e+00i	−8.99625e−09 − 0.00000e+00i	−1.10662e−41 − 0.00000e+00i	−6.05813e−108 + 3.70649e−124i*	−3.12117e−276 + 1.91092e−292i*
0,0	$750\angle 15^\circ$	Pr	7.23694e+02 + 1.94114e+02i	−0.00000e+00 − 0.00000e+00i	−6.99794e+00 − 1.80442e+00i	−1.51676e+00 + 1.30621e+00i	−1.87991e−08 − 7.53721e−09i	−2.78039e−40 + 1.94973e−40i	3.01585e−104 + 1.71009e−104i*	1.06440e−266 + 3.85535e−268i*
0,0	$750\angle 30^\circ$	Pr	6.48769e+02 + 3.75000e+02i	−0.00000e+00 − 0.00000e+00i	−6.34789e+00 − 3.51209e+00i	−6.22305e−01 + 2.84716e+00i	−8.27628e−08 − 2.02419e−07i	−6.62856e−36 − 4.05629e−36i	−2.57964e−93 + 2.51577e−93i*	−4.03235e−239 + 8.58288e−239i*
0,0	$750\angle 45^\circ$	Pr	5.29580e+02 + 5.30330e+02i	−0.00000e+00 − 0.00000e+00i	−5.29280e+00 − 5.02641e+00i	1.57268e+00 + 5.05880e+00i	9.05808e−06 − 3.26653e−06i	−2.73601e−29 + 6.10626e−29i	1.15072e−75 + 3.65296e−76i*	1.90830e−194 + 1.97069e−194i*
0,0	$750\angle 60^\circ$	Ob	−2.79952e+05 + 4.86389e+05i	−0.00000e+00 − 0.00000e+00i	−1.04147e+05 + 2.22304e+05i	5.21247e+30 − 1.75705e+30i	9.93466e+72 − 2.62539e+71i	2.44701e+143 + 1.24732e+143i	4.41855e+213 + 6.43672e+213i	1.66302e+281 − 3.01093e+281i
0,0	$750\angle 75^\circ$	Ob	−4.85691e+05 + 2.80862e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$750\angle 90^\circ$	Ob	−5.61001e+05 + 6.87945e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1000\angle 0^\circ$	Pr	9.99250e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−9.50598e+00 − 0.00000e+00i	−6.70754e−01 − 0.00000e+00i	−4.28003e−12 − 0.00000e+00i	−4.19708e−56 − 0.00000e+00i	−1.40766e−144 + 8.61405e−161i*	± 0
0,0	$1000\angle 15^\circ$	Pr	9.65176e+02 + 2.58819e+02i	−0.00000e+00 − 0.00000e+00i	−9.22863e+00 − 2.34714e+00i	−4.05986e−01 + 6.84339e−01i	1.29349e−12 + 1.25624e−11i	3.99030e−54 + 5.80047e−55i	−1.18897e−139 − 8.13952e−140i*	± 0
0,0	$1000\angle 30^\circ$	Pr	8.65275e+02 + 5.00000e+02i	−0.00000e+00 − 0.00000e+00i	−8.40390e+00 − 4.57988e+00i	5.25186e−01 + 1.20338e+00i	2.82586e−10 + 1.04712e−10i	2.31153e−48 − 1.23248e−48i	−4.43680e−125 − 5.46649e−125i*	2.47725e−320 + 2.16850e−319i*
0,0	$1000\angle 45^\circ$	Pr	7.06357e+02 + 7.07107e+02i	−0.00000e+00 − 0.00000e+00i	−7.05582e+00 − 6.58073e+00i	2.70137e+00 + 1.08870e+00i	4.44223e−08 + 1.48940e−08i	−4.41684e−39 − 1.36344e−39i	−1.62044e−101 − 2.40791e−102i*	4.05235e−260 − 1.00066e−260i*
0,0	$1000\angle 60^\circ$	Ob	−4.98269e+05 + 8.65025e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1000\angle 75^\circ$	Ob	−8.64095e+05 + 4.99482e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1000\angle 90^\circ$	Ob	−9.98001e+05 + 1.22342e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1500\angle 0^\circ$	Pr	1.49925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.39104e+01 − 0.00000e+00i	−8.20917e−02 − 0.00000e+00i	−8.17184e−19 − 0.00000e+00i	−5.09264e−85 − 0.00000e+00i	−6.41043e−218 + 3.92358e−234i*	± 0
0,0	$1500\angle 15^\circ$	Pr	1.44814e+03 + 3.88229e+02i	−0.00000e+00 − 0.00000e+00i	−1.35381e+01 − 3.34914e+00i	1.19909e−02 + 1.05385e−01i	−3.63389e−18 − 1.98722e−18i	−4.09291e−83 + 4.77797e−82i	−1.50806e−210 − 1.46042e−210i*	± 0
0,0	$1500\angle 30^\circ$	Pr	1.29829e+03 + 7.50000e+02i	−0.00000e+00 − 0.00000e+00i	−1.24224e+01 − 6.56782e+00i	2.23814e−01 − 2.10962e−02i	1.28854e−16 − 4.65263e−16i	−2.10864e−73 − 1.36317e−73i	1.08597e−188 + 1.99022e−188i*	± 0
0,0	$1500\angle 45^\circ$	Pr	1.05991e+03 + 1.06066e+03i	−0.00000e+00 − 0.00000e+00i	−1.05716e+01 − 9.51260e+00i	1.34061e−01 − 7.30433e−01i	−8.60675e−14 + 9.31806e−13i	1.85810e−59 + 1.04040e−60i	−2.50695e−153 + 4.35484e−154i*	± 0
0,0	$1500\angle 60^\circ$	Ob	−1.12240e+06 + 1.94706e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1500\angle 75^\circ$	Ob	−1.94566e+06 + 1.12422e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$1500\angle 90^\circ$	Ob	−2.24700e+06 + 2.75362e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$2500\angle 0^\circ$	Pr	2.49925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.20577e+01 − 0.00000e+00i	−9.10556e−04 − 0.00000e+00i	−2.20597e−32 − 0.00000e+00i	−5.55212e−143 + 3.39722e−159i*	± 0	± 0
0,0	$2500\angle 15^\circ$	Pr	2.41406e+03 + 6.47048e+02i	−0.00000e+00 − 0.00000e+00i	−2.15711e+01 − 5.03990e+00i	1.37773e−03 + 2.22483e−04i	−4.46531e−32 + 3.26852e−31i	−1.62005e−139 − 5.01999e−138i*	± 0	± 0
0,0	$2500\angle 30^\circ$	Pr	2.16431e+03 + 1.25000e+03i	−0.00000e+00 − 0.00000e+00i	−2.00870e+01 − 9.98330e+00i	−4.18289e−03 − 2.51435e−03i	−9.24706e−29 + 9.12840e−28i	1.54554e−123 − 7.27747e−124i*	2.33825e−316 + 1.72524e−315i*	± 0
0,0	$2500\angle 45^\circ$	Pr	1.76702e+03 + 1.76777e+03i	−0.00000e+00 − 0.00000e+00i	−1.75422e+01 − 1.46924e+01i	7.85389e−03 + 3.48840e−02i	−1.00090e−22 − 2.57693e−22i	2.02979e−100 − 9.32837e−101i*	−3.13180e−257 + 3.29472e−257i*	± 0
0,0	$2500\angle 60^\circ$	Ob	−3.12067e+06 + 5.41016e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$2500\angle 75^\circ$	Ob	−5.40783e+06 + 3.12371e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$2500\angle 90^\circ$	Ob	−6.24500e+06 + 7.65098e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 0^\circ$	Pr	4.99925e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.89374e+01 − 0.00000e+00i	−6.57781e−09 − 0.00000e+00i	−1.47248e−66 − 0.00000e+00i	−3.84140e−288 + 2.35129e−304i*	± 0	± 0
0,0	$5000\angle 15^\circ$	Pr	4.82888e+03 + 1.29410e+03i	−0.00000e+00 − 0.00000e+00i	−3.85101e+01 − 7.69146e+00i	−1.54252e−08 − 9.00153e−10i	3.29275e−64 + 3.17173e−66i	2.97756e−278 − 1.00808e−278i*	± 0	± 0
0,0	$5000\angle 30^\circ$	Pr	4.32938e+03 + 2.50000e+03i	−0.00000e+00 − 0.00000e+00i	−3.71068e+01 − 1.56304e+01i	−1.60229e−07 − 1.00151e−07i	2.41627e−57 − 8.05871e−58i	−6.03776e−250 + 3.58596e−249i*	± 0	± 0
0,0	$5000\angle 45^\circ$	Pr	3.53478e+03 + 3.53553e+03i	−0.00000e+00 − 0.00000e+00i	−3.43717e+01 − 2.39578e+01i	3.40316e−06 − 9.55435e−06i	1.01989e−47 − 2.30993e−46i	4.75314e−204 + 6.19955e−203i*	± 0	± 0
0,0	$5000\angle 60^\circ$	Ob	−1.24913e+07 + 2.16456e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 75^\circ$	Ob	−2.16410e+07 + 1.24974e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,0	$5000\angle 90^\circ$	Ob	−2.49900e+07 + 3.06100e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$25\angle 0^\circ$	Pr	7.32096e+01 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.96443e−01 − 0.00000e+00i	6.76072e−01 − 0.00000e+00i	−2.49679e−01 − 0.00000e+00i	−2.43832e−01 − 0.00000e+00i	−6.48883e−03 − 0.00000e+00i	−4.17265e−07 + 2.54474e−23i*
0,1	$25\angle 15^\circ$	Pr	7.06557e+01 + 1.94227e+01i	1.00000e−00 + 5.55112e−17i	9.96570e−01 − 9.69323e−04i	6.84348e−01 − 8.02644e−02i	−2.66368e−01 − 1.50960e−01i	−2.36303e−01 + 1.42785e−01i	2.78808e−03 + 8.24096e−03i	−4.37257e−07 − 7.62131e−07i
0,1	$25\angle 30^\circ$	Pr	6.31681e+01 + 3.75213e+01i	1.00000e−00 − 0.00000e+00i	9.96943e−01 − 1.87296e−03i	7.09362e−01 − 1.58244e−01i	−3.15260e−01 − 3.26193e−01i	−2.02498e−01 + 3.41435e−01i	1.79680e−02 − 9.96396e−03i	6.07582e−06 − 4.86996e−06i
0,1	$25\angle 45^\circ$	Ob	3.43483e+01 + 5.89637e+02i	1.00000e+00 + 4.44089e−16i	9.98238e−01 − 2.94666e−02i	−5.36305e−01 − 2.52232e+00i	−9.72395e+00 + 3.18547e+01i	−1.91777e+03 − 1.28732e+03i	1.27479e+05 − 1.04188e+05i	2.14593e+06 + 9.85451e+06i
0,1	$25\angle 60^\circ$	Ob	−2.70208e+02 + 5.16261e+02i	1.00000e+00 − 3.88578e−16i	1.01353e+00 − 2.59309e−02i	1.27532e+00 − 3.70388e+00i	−9.00031e+01 − 1.70134e+00i	1.68488e+04 + 6.23488e+02i	−3.24114e+06 − 1.77224e+05i	5.06108e+08 + 1.00994e+08i
0,1	$25\angle 75^\circ$	Ob	−4.93980e+02 + 2.99556e+02i	1.00000e+00 − 1.38778e−16i	1.02486e+00 − 1.51024e−02i	4.10640e+00 − 3.03932e+00i	−7.66603e+00 − 1.67982e+02i	−5.85801e+04 + 5.38345e+03i	2.90777e+06 + 2.09496e+07i	6.08090e+09 − 7.31046e+08i
0,1	$25\angle 90^\circ$	Ob	−5.76010e+02 + 7.34781e−14i	1.00000e+00 + 5.69686e−17i	1.02904e+00 + 5.49135e−17i	5.59402e+00 − 5.22993e−16i	2.08110e+02 − 6.77271e−14i	9.00923e+04 − 6.37816e−11i	4.00825e+07 − 4.37112e−08i	1.42407e+10 − 2.07606e−05i
0,1	$50\angle 0^\circ$	Pr	1.48231e+02 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	9.92703e−01 − 0.00000e+00i	3.99572e−01 − 0.00000e+00i	−4.56375e−01 − 0.00000e+00i	−1.91706e−02 − 0.00000e+00i	−2.90543e−06 − 0.00000e+00i	−4.39369e−16 − 0.00000e+00i
0,1	$50\angle 15^\circ$	Pr	1.43120e+02 + 3.88281e+01i	1.00000e+00 − 0.00000e+00i	9.92956e−01 − 1.93389e−03i	4.08985e−01 − 1.29544e−01i	−4.93960e−01 + 1.05474e−02i	−3.04709e−03 + 2.39904e−02i	2.95984e−06 − 4.25537e−06i	3.15123e−16 − 1.87935e−15i
0,1	$50\angle 30^\circ$	Pr	1.28137e+02 + 7.50100e+01i	1.00000e−00 + 5.55112e−17i	9.93700e−01 − 3.73753e−03i	4.38626e−01 − 2.60415e−01i	−6.18956e−01 + 2.35839e−02i	4.45690e−02 + 1.72011e−02i	1.63912e−06 + 2.82473e−05i	6.53378e−14 + 1.24469e−13i
0,1	$50\angle 45^\circ$	Ob	6.97071e+01 + 2.42929e+03i	1.00000e+00 − 1.60115e−15i	9.94159e−01 − 1.21311e−01i	−1.44306e+01 + 5.97440e+00i	−2.29955e+03 − 1.53116e+03i	6.13681e+06 + 1.46607e+07i	1.77762e+10 − 9.23237e+10i	−3.62356e+14 + 1.70848e+14i
0,1	$50\angle 60^\circ$	Ob	−1.16440e+03 + 2.11506e+03i	1.00000e−00 − 6.66134e−16i	1.05701e+00 − 1.07811e−01i	−2.76771e+01 − 2.06878e+01i	2.01349e+04 + 6.90062e+02i	8.43143e+08 + 5.75481e+07i	3.62616e+13 + 3.70542e+12i	9.69148e+17 + 3.93144e+17i
0,1	$50\angle 75^\circ$	Ob	−2.06948e+03 + 1.22412e+03i	1.00000e−00 − 3.33067e−16i	1.10474e+00 − 6.33404e−02i	1.55717e+01 − 5.47706e+01i	−6.99438e+04 + 6.52959e+03i	1.00959e+10 − 1.90359e+09i	−1.48483e+15 + 4.26565e+14i	1.42658e+20 − 3.54856e+19i
0,1	$50\angle 90^\circ$	Ob	−2.40101e+03 + 3.00038e−13i	1.00000e−00 − 2.15044e−17i	1.12258e+00 − 3.97509e−17i	6.75284e+01 − 2.21204e−14i	1.07559e+05 − 8.46247e−11i	2.40848e+10 − 3.73819e−05i	5.54502e+15 − 1.28502e+01i	7.94196e+20 − 2.42403e+06i
0,1	$100\angle 0^\circ$	Pr	2.98240e+02 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.85248e−01 − 0.00000e+00i	4.63389e−03 − 0.00000e+00i	−2.33651e−01 − 0.00000e+00i	−4.97722e−05 − 0.00000e+00i	−2.65918e−13 − 0.00000e+00i	−2.05027e−34 + 1.25441e−50i*
0,1	$100\angle 15^\circ$	Pr	2.88019e+02 + 7.76482e+01i	1.00000e−00 + 2.77556e−17i	9.85751e−01 − 3.85183e−03i	4.86156e−03 − 1.63307e−01i	−2.29302e−01 + 1.27755e−01i	7.86076e−05 − 4.01824e−06i	5.03618e−13 + 6.76895e−13i	3.83116e−33 + 1.27434e−34i
0,1	$100\angle 30^\circ$	Pr	2.58049e+02 + 1.50005e+02i	1.00000e+00 − 5.55112e−17i	9.87227e−01 − 7.44735e−03i	8.58150e−03 − 3.40384e−01i	−2.09686e−01 + 3.03584e−01i	−2.99035e−04 − 4.00485e−05i	1.93246e−11 − 1.57133e−11i	1.99798e−31 − 2.05180e−29i
0,1	$100\angle 45^\circ$	Ob	1.40420e+02 + 9.85858e+03i	1.00000e+00 − 9.43690e−16i	9.52668e−01 − 4.90479e−01i	3.77738e+02 − 3.79368e+02i	7.38288e+06 + 1.75510e+07i	−5.27934e+14 + 5.39907e+14i	−2.86515e+22 − 1.12985e+22i	4.01538e+29 − 4.92440e+29i
0,1	$100\angle 60^\circ$	Ob	−4.82780e+03 + 8.56025e+03i	1.00000e+00 + 1.49880e−15i	1.21935e+00 − 4.62422e−01i	7.43543e+02 + 2.51574e+03i	1.00976e+09 + 6.76424e+07i	2.11411e+18 + 2.84141e+17i	4.53144e+27 + 9.19587e+26i	3.11448e+36 + 3.00051e+36i
0,1	$100\angle 75^\circ$	Ob	−8.46807e+03 + 4.94824e+03i	1.00000e+00 + 6.66134e−16i	1.44318e+00 − 2.83679e−01i	−6.06037e+03 − 3.74510e+03i	1.20852e+10 − 2.28692e+09i	2.93272e+20 − 1.15146e+20i	7.02626e+30 − 4.41783e+30i	7.53376e+40 − 4.01532e+40i
0,1	$100\angle 90^\circ$	Ob	−9.80100e+03 + 1.21240e−12i	1.00000e+00 − 3.63542e−17i	1.53154e+00 − 1.26702e−16i	1.00167e+04 − 6.49731e−12i	2.88314e+10 − 4.51816e−05i	1.73117e+21 − 5.36294e+06i	1.06897e+32 − 4.94788e+17i	2.49097e+42 − 1.51904e+28i
0,1	$250\angle 0^\circ$	Pr	7.48246e+02 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	9.63071e−01 − 0.00000e+00i	−4.31102e−01 − 0.00000e+00i	−5.59065e−03 − 0.00000e+00i	−2.40007e−13 − 0.00000e+00i	−5.94999e−35 − 0.00000e+00i	−5.83758e−90 + 3.57331e−106i*
0,1	$250\angle 15^\circ$	Pr	7.22691e+02 + 1.94115e+02i	1.00000e+00 − 0.00000e+00i	9.64297e−01 − 9.51344e−03i	−4.64272e−01 + 4.96929e−02i	1.48466e−03 + 7.19483e−03i	3.91697e−13 + 6.41828e−13i	1.00041e−33 + 3.64256e−34i	−2.87543e−87 − 8.30986e−87i
0,1	$250\angle 30^\circ$	Pr	6.47766e+02 + 3.75002e+02i	1.00000e−00 + 5.55112e−17i	9.67902e−01 − 1.84168e−02i	−5.70778e−01 − 1.14052e−01i	1.57744e−02 − 4.34023e−03i	1.87141e−11 − 1.03575e−11i	2.91267e−30 − 4.07750e−30i	−1.82492e−77 + 3.77574e−77i
0,1	$250\angle 45^\circ$	Ob	3.52553e+02 + 6.21464e+04i	1.00000e−00 − 1.57513e−15i	−5.84344e−01 − 2.75930e+00i	8.39669e+06 + 1.99278e+07i	6.10483e+18 − 1.31135e+18i	7.40508e+37 − 3.33605e+37i	8.74745e+56 − 6.44537e+56i	1.54037e+75 + 2.02699e+75i
0,1	$250\angle 60^\circ$	Ob	−3.08180e+04 + 5.38766e+04i	1.00000e+00 + 2.38698e−15i	1.37878e+00 − 4.03939e+00i	1.14667e+09 + 7.63232e+07i	1.26785e+23 + 2.12587e+22i	3.25440e+46 + 1.12268e+46i	8.32917e+69 + 4.53094e+69i	−1.03509e+92 + 2.89469e+92i
0,1	$250\angle 75^\circ$	Ob	−5.36446e+04 + 3.11206e+04i	1.00000e+00 − 2.77392e−15i	4.45508e+00 − 3.31160e+00i	1.37215e+10 − 2.59971e+09i	−5.90627e+25 + 2.98624e+25i	5.40869e+51 − 7.34820e+51i	−2.14207e+77 + 1.27587e+78i	5.69203e+102 − 1.58290e+103i
0,1	$250\angle 90^\circ$	Ob	−6.20010e+04 + 7.62343e−12i	1.00000e+00 + 6.48600e−17i	6.07251e+00 − 5.23022e−16i	3.27355e+10 − 4.79882e−05i	5.56724e+26 − 2.09447e+12i	6.45625e+53 − 4.89974e+39i	7.70056e+80 − 8.79107e+66i	7.73457e+106 − 1.16715e+93i
0,1	$500\angle 0^\circ$	Pr	1.49825e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	9.26721e−01 − 0.00000e+00i	−3.29372e−01 − 0.00000e+00i	−4.13200e−06 − 0.00000e+00i	−1.37841e−27 − 0.00000e+00i	−2.08443e−71 + 1.27478e−87i*	−6.50667e−183 + 3.98347e−199i*

Table D.54: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,1	$500\angle 15^\circ$	Pr	1.44714e+03 + 3.88229e+02i	1.00000e+00 + 5.55112e−17i	9.29073e−01 − 1.86457e−02i	−3.43920e−01 + 1.14814e−01i	5.44922e−06 − 4.56131e−06i	2.75756e−27 − 1.32307e−26i	−6.05634e−69 − 2.79467e−69i*	8.76603e−177 − 1.18637e−176i*
0,1	$500\angle 30^\circ$	Pr	1.29729e+03 + 7.50001e+02i	1.00000e+00 + 5.55112e−17i	9.36008e−01 − 3.61702e−02i	−3.93886e−01 + 2.74176e−01i	−1.53501e−05 + 3.12726e−05i	−2.34556e−25 + 1.09010e−23i	1.11986e−61 + 9.60157e−62i*	−5.31268e−158 + 3.45982e−158i*
0,1	$500\angle 45^\circ$	Ob	7.06106e+02 + 2.49293e+05i	1.00000e−00 + 1.03806e−14i	−1.56994e+01 + 6.51434e+00i	−7.18266e+14 + 7.36241e+14i	8.88183e+37 − 4.00491e+37i	1.30919e+76 − 1.48212e+76i	1.21939e+114 − 3.94418e+114i	−6.87526e+150 + 2.48346e+151i
0,1	$500\angle 60^\circ$	Ob	−1.24135e+05 + 2.16006e+05i	1.00000e+00 − 4.77396e−15i	−3.01292e+01 − 2.24995e+01i	2.87922e+18 + 3.84585e+17i	3.90401e+46 + 1.34575e+46i	2.79788e+93 + 2.18887e+93i	1.70941e+140 + 2.63766e+140i	−2.90655e+185 − 2.37973e+185i
0,1	$500\angle 75^\circ$	Ob	−2.15541e+05 + 1.24741e+05i	1.00000e−00 + 8.43769e−15i	1.69286e+01 − 5.96023e+01i	3.99231e+20 − 1.56944e+20i	6.48640e+51 − 8.81464e+51i	−7.42170e+103 − 2.38214e+104i	−5.53056e+156 − 1.90884e+156i	−8.67293e+206 − 7.15809e+206i
0,1	$500\angle 90^\circ$	Ob	−2.49001e+05 + 3.05549e−11i	1.00000e+00 − 1.31457e−17i	7.34760e+01 − 2.34593e−14i	2.35690e+21 − 7.24689e+06i	7.74386e+53 − 5.93735e+39i	1.24924e+108 − 1.91398e+94i	2.07276e+162 − 4.76222e+148i	2.37740e+214 − 7.20902e+200i
0,1	$750\angle 0^\circ$	Pr	2.24825e+03 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	8.91127e−01 − 0.00000e+00i	−1.52976e−01 − 0.00000e+00i	−2.23656e−09 − 0.00000e+00i	−5.89882e−42 − 0.00000e+00i	−5.45686e−108 + 3.33861e−124i*	−5.40985e−276 + 3.31215e−292i*
0,1	$750\angle 15^\circ$	Pr	2.17158e+03 + 5.82343e+02i	1.00000e−00 − 2.77556e−17i	8.94507e−01 − 2.74062e−02i	−1.36984e−01 + 1.08811e−01i	−4.66653e−09 − 1.90164e−09i	−1.48380e−40 + 1.03744e−40i	2.71582e−104 + 1.54211e−104i	1.84495e−266 + 6.73638e−268i*
0,1	$750\angle 30^\circ$	Pr	1.94681e+03 + 1.12500e+03i	1.00000e+00 + 5.55112e−17i	9.04503e−01 − 5.32740e−02i	−7.42408e−02 + 2.48288e−01i	−2.00866e−08 − 5.06955e−08i	−3.53011e−36 − 2.17320e−36i	−2.32696e−93 + 2.26410e−93i*	−6.99862e−239 + 1.48749e−238i*
0,1	$750\angle 45^\circ$	Ob	1.05966e+03 + 5.61439e+05i	1.00000e+00 − 2.78666e−14i	5.54371e+01 + 8.26304e+01i	−4.55419e+22 − 1.78797e+22i	1.22355e+57 − 9.02523e+56i	1.42174e+114 − 4.60108e+114i	−5.17122e+171 − 1.48121e+172i	−2.42222e+227 + 9.69084e+226i
0,1	$750\angle 60^\circ$	Ob	−2.79952e+05 + 4.86389e+05i	1.00000e+00 + 1.02696e−14i	−2.68899e+02 + 1.87299e+02i	7.19756e+27 + 1.45259e+27i	1.16558e+70 + 6.33505e+69i	1.99427e+140 + 3.07651e+140i	8.04954e+209 + 1.03907e+211i	3.93370e+278 − 2.36815e+278i
0,1	$750\angle 75^\circ$	Ob	−4.85691e+05 + 2.80862e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$750\angle 90^\circ$	Ob	−5.61001e+05 + 6.87945e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 0^\circ$	Pr	2.99825e+03 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	8.56273e−01 − 0.00000e+00i	−6.05145e−02 − 0.00000e+00i	−1.06986e−12 − 0.00000e+00i	−2.24024e−56 − 0.00000e+00i	−1.26868e−144 + 7.76358e−161i*	± 0
0,1	$1000\angle 15^\circ$	Pr	2.89603e+03 + 7.76457e+02i	1.00000e+00 − 0.00000e+00i	8.60589e−01 − 3.58042e−02i	−3.85448e−02 + 6.09196e−02i	3.10276e−13 + 3.14329e−12i	2.12984e−54 + 3.11857e−55i	−1.07132e−139 − 7.34116e−140i*	± 0
0,1	$1000\angle 30^\circ$	Pr	2.59633e+03 + 1.50000e+03i	1.00000e+00 − 0.00000e+00i	8.73389e−01 − 6.97413e−02i	4.20491e−02 + 1.12824e−01i	7.05783e−11 + 2.68066e−11i	1.23579e−48 − 6.55734e−49i	−3.99543e−125 − 4.93141e−125i*	4.28009e−320 + 3.76028e−319i*
0,1	$1000\angle 45^\circ$	Ob	1.41321e+03 + 9.98586e+05i	1.00000e−00 − 1.59317e−14i	4.11132e+02 − 4.13176e+02i	−5.77366e+28 − 2.32656e+30i	1.57048e+76 − 1.77851e+76i	−1.45167e+152 − 1.16376e+153i	−4.92366e+229 − 3.36286e+229i	−2.26610e+303 − 1.35721e+303i
0,1	$1000\angle 60^\circ$	Ob	−4.98269e+05 + 8.65025e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 75^\circ$	Ob	−8.64095e+05 + 4.99482e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1000\angle 90^\circ$	Ob	−9.98001e+05 + 1.22342e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 0^\circ$	Pr	4.49825e+03 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	7.88739e−01 − 0.00000e+00i	−7.68087e−03 − 0.00000e+00i	−2.05377e−19 − 0.00000e+00i	−2.72189e−85 − 0.00000e+00i	−5.78087e−218 + 3.53825e−234i*	± 0
0,1	$1500\angle 15^\circ$	Pr	4.34492e+03 + 1.16469e+03i	1.00000e−00 − 0.00000e+00i	7.94661e−01 − 5.15484e−02i	9.42188e−04 + 9.90511e−03i	−9.12221e−19 − 5.02169e−19i	−2.20538e−83 + 2.55379e−82i	−1.35962e−210 − 1.31746e−210i*	± 0
0,1	$1500\angle 30^\circ$	Pr	3.89536e+03 + 2.25000e+03i	1.00000e−00 − 0.00000e+00i	8.12330e−01 − 1.00818e−01i	2.12124e−02 − 1.24421e−03i	3.30615e−17 − 1.16926e−16i	−1.12645e−73 − 7.30340e−74i	9.78433e−189 + 1.79561e−188i*	± 0
0,1	$1500\angle 45^\circ$	Ob	2.12032e+03 + 2.24788e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 60^\circ$	Ob	−1.12240e+06 + 1.94706e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 75^\circ$	Ob	−1.94566e+06 + 1.12422e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$1500\angle 90^\circ$	Ob	−2.24700e+06 + 2.75362e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$2500\angle 0^\circ$	Pr	7.49825e+03 − 0.00000e+00i	1.00000e−00 − 0.00000e+00i	6.62011e−01 − 0.00000e+00i	−8.76320e−05 − 0.00000e+00i	−5.56803e−33 − 0.00000e+00i	−2.97063e−143 + 1.81766e−159i*	± 0	± 0
0,1	$2500\angle 15^\circ$	Pr	7.24269e+03 + 1.94114e+03i	1.00000e−00 + 2.77556e−17i	6.70180e−01 − 7.90718e−02i	1.32550e−04 + 2.28729e−05i	−1.14108e−32 + 8.24992e−32i	−8.55750e−140 − 2.68610e−138i*	± 0	± 0
0,1	$2500\angle 30^\circ$	Pr	6.49344e+03 + 3.75000e+03i	1.00000e+00 − 0.00000e+00i	6.94872e−01 − 1.55894e−01i	−3.99767e−04 − 2.51725e−04i	−2.41027e−29 + 2.30531e−28i	8.27421e−124 − 3.88801e−124i*	2.10441e−316 + 1.55675e−315i*	± 0
0,1	$2500\angle 45^\circ$	Ob	3.53453e+03 + 6.24646e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$2500\angle 60^\circ$	Ob	−3.12067e+06 + 5.41016e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$2500\angle 75^\circ$	Ob	−5.40783e+06 + 3.12371e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$2500\angle 90^\circ$	Ob	−6.24500e+06 + 7.65098e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$5000\angle 0^\circ$	Pr	1.49982e+04 − 0.00000e+00i	1.00000e+00 − 0.00000e+00i	3.89500e−01 − 0.00000e+00i	−6.46243e−10 − 0.00000e+00i	−3.72863e−67 − 0.00000e+00i	−2.05696e−288 + 1.25905e−304i*	± 0	± 0
0,1	$5000\angle 15^\circ$	Pr	1.44871e+04 + 3.88229e+03i	1.00000e+00 − 0.00000e+00i	3.98792e−01 − 1.27711e−01i	−1.51605e−09 − 9.65049e−11i	8.33877e−65 + 8.72516e−67i	1.59455e−278 − 5.39486e−279i*	± 0	± 0
0,1	$5000\angle 30^\circ$	Pr	1.29886e+04 + 7.50000e+03i	1.00000e+00 + 5.55112e−17i	4.28052e−01 − 2.56725e−01i	−1.56844e−08 − 1.00271e−08i	6.12440e−58 − 2.03169e−58i	−3.24104e−250 + 1.92026e−249i*	± 0	± 0
0,1	$5000\angle 45^\circ$	Ob	7.07007e+03 + 2.49929e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$5000\angle 60^\circ$	Ob	−1.24913e+07 + 2.16456e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$5000\angle 75^\circ$	Ob	−2.16410e+07 + 1.24974e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,1	$5000\angle 90^\circ$	Ob	−2.49900e+07 + 3.06100e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
0,2	$25\angle 0^\circ$	Pr	1.21126e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	6.04364e−01 − 0.00000e+00i	4.88734e+00 − 0.00000e+00i	2.90633e+00 − 0.00000e+00i	−2.26531e+00 − 0.00000e+00i	−1.26592e−01 + 7.46495e−18i*	−1.65917e−05 + 1.01141e−21i*
0,2	$25\angle 15^\circ$	Pr	1.16872e+02 + 3.23877e+01i	−0.00000e+00 − 0.00000e+00i	5.83279e−01 + 1.61272e−01i	4.82725e+00 + 1.02523e+00i	3.22737e+00 − 1.03973e+00i	−2.62076e+00 + 3.99851e−01i	3.43765e−03 + 1.71085e−01i*	−8.00346e−06 − 3.42512e−05i*
0,2	$25\angle 30^\circ$	Pr	1.04401e+02 + 6.25665e+01i	−0.00000e+00 − 0.00000e+00i	5.21422e−01 + 3.11686e−01i	4.62927e+00 + 2.08165e+00i	4.27889e+00 − 2.02762e+00i	−3.96054e+00 + 1.17802e+00i	4.11224e−01 + 4.08335e−02i*	3.15761e−04 − 3.17550e−05i*
0,2	$25\angle 45^\circ$	Ob	3.43483e+01 + 5.89637e+02i	−0.00000e+00 − 0.00000e+00i	2.00620e−01 + 2.94492e+00i	2.81201e+01 + 1.81439e+01i	−2.00964e+02 − 3.51518e+02i	2.73670e+04 − 6.24011e+03i	−1.40415e+05 + 1.99584e+06i	−1.05059e+08 − 6.31217e+07i
0,2	$25\angle 60^\circ$	Ob	1.17465e+01 + 2.16575e+01i	−0.00000e+00 − 0.00000e+00i	5.88178e−02 + 1.08165e−01i	6.64284e−01 + 9.59786e−01i	2.02847e+00 + 8.26351e−01i	−3.71436e−01 − 1.43330e+00i	2.72394e−01 + 9.51523e−03i	1.18097e−03 + 6.29046e−03i*
0,2	$25\angle 75^\circ$	Ob	−4.01534e+02 + 2.73631e+02i	−0.00000e+00 − 0.00000e+00i	−2.01496e+00 + 1.38660e+00i	−2.58885e+01 + 3.72125e+01i	3.74229e+02 + 1.01718e+03i	1.84193e+05 − 8.06937e+04i	−1.30723e+07 − 2.33944e+07i	1.87871e+09 − 1.50115e+08i
0,2	$25\angle 90^\circ$	Ob	−4.80164e+02 + 6.73446e−14i	−0.00000e+00 − 0.00000e+00i	−2.42022e+00 + 1.12813e−16i	−4.86233e+01 + 6.46390e−15i	−1.33155e+03 + 4.78920e−13i	−3.04990e+05 + 2.27401e−10i	−4.97725e+07 + 5.67606e−08i	4.33878e+09 − 6.12003e−06i
0,2	$50\angle 0^\circ$	Pr	2.46191e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.22562e+00 − 0.00000e+00i	7.79137e+00 − 0.00000e+00i	−1.62799e+00 − 0.00000e+00i	−4.41759e−01 − 0.00000e+00i	−1.22802e−04 − 0.00000e+00i	−3.67082e−14 + 2.24371e−30i*
0,2	$50\angle 15^\circ$	Pr	2.37675e+02 + 6.47207e+01i	−0.00000e+00 − 0.00000e+00i	1.18378e+00 + 3.20870e−01i	7.87109e+00 + 1.08562e+00i	−1.73524e+00 − 2.12043e+00i	−2.35805e−01 + 5.09853e−01i	1.70406e−04 − 1.38664e−04i	6.77890e−14 − 1.44386e−13i*
0,2	$50\angle 30^\circ$	Pr	2.12706e+02 + 1.25030e+02i	−0.00000e+00 − 0.00000e+00i	1.06096e+00 + 6.20431e−01i	8.06397e+00 + 2.37109e+00i	−1.95401e+00 − 4.99051e+00i	6.36166e−01 + 9.39463e−01i	−5.79764e−04 + 1.06065e−03i	−7.14598e−13 + 1.18097e−11i*
0,2	$50\angle 45^\circ$	Ob	6.97071e+01 + 2.42929e+03i	−0.00000e+00 − 0.00000e+00i	8.39431e−01 + 1.21130e+01i	1.41465e+02 − 3.57442e+02i	6.65659e+04 − 1.43628e+04i	−3.64617e+08 − 1.43278e+08i	1.32712e+12 + 1.89989e+12i	3.20232e+15 − 9.34112e+15i
0,2	$50\angle 60^\circ$	Pr	2.42482e+01 + 4.33046e+01i	−0.00000e+00 − 0.00000e+00i	1.21568e−01 + 2.16009e−01i	1.46864e+00 + 1.64951e+00i	2.82169e+00 − 9.17545e−01i	1.52863e−02 + 5.58985e−01i	4.11610e−03 − 6.84764e−03i	−1.12380e−08 + 2.83774e−07i*
0,2	$50\angle 75^\circ$	Ob	−1.88036e+03 + 1.17233e+03i	−0.00000e+00 − 0.00000e+00i	−9.58203e+00 + 6.23399e+00i	3.42610e+00 + 1.09189e+03i	9.11817e+05 − 3.59226e+05i	−6.92901e+10 + 3.56188e+10i	3.91490e+15 − 2.60901e+15i	3.72830e+19 − 4.61650e+18i
0,2	$50\angle 90^\circ$	Ob	−2.20508e+03 + 2.87787e−13i	−0.00000e+00 − 0.00000e+00i	−1.14358e+01 + 1.97221e−15i	−1.29110e+03 + 5.27974e−13i	−1.49612e+06 + 1.29949e−09i	−1.81933e+11 + 2.97529e−04i	−1.67666e+16 + 4.03304e+01i	1.97880e+20 − 6.01299e+05i
0,2	$100\angle 0^\circ$	Pr	4.96221e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.45924e+00 − 0.00000e+00i	9.17439e+00 − 0.00000e+00i	−4.21002e+00 − 0.00000e+00i	−2.48808e−03 − 0.00000e+00i	−2.32582e−11 − 0.00000e+00i	−3.49590e−32 + 2.13886e−48i*
0,2	$100\angle 15^\circ$	Pr	4.79185e+02 + 1.29417e+02i	−0.00000e+00 − 0.00000e+00i	2.37704e+00 + 6.36036e−01i	9.67516e+00 − 3.15167e−01i	−4.83367e+00 + 6.10554e−01i	3.84231e−03 + 8.99644e−04i	2.66736e−11 + 6.88979e−11i	6.24893e−31 + 1.93186e−31i*
0,2	$100\angle 30^\circ$	Pr	4.29238e+02 + 2.50015e+02i	−0.00000e+00 − 0.00000e+00i	2.13542e+00 + 1.23095e+00i	1.11808e+01 − 2.16021e−01i	−7.17146e+00 + 1.71082e+00i	−1.17319e−02 − 9.74904e−03i	2.16635e−09 − 3.11543e−10i	1.81111e−27 − 3.00465e−27i*
0,2	$100\angle 45^\circ$	Ob	1.40420e+02 + 9.85858e+03i	−0.00000e+00 − 0.00000e+00i	8.77389e+00 + 4.86670e+01i	2.47880e+02 + 2.65781e+04i	−8.77874e+08 − 3.50678e+08i	−6.89199e+14 − 3.74839e+16i	1.39809e+24 − 6.19242e+23i	3.41626e+30 + 3.13605e+31i
0,2	$100\angle 60^\circ$	Pr	4.92491e+01 + 8.66042e+01i	−0.00000e+00 − 0.00000e+00i	2.47520e−01 + 4.30910e−01i	3.18410e+00 + 2.27148e+00i	−3.36638e−01 − 2.67181e+00i	−1.70524e−02 + 3.55409e−02i	−3.38597e−06 − 3.26240e−08i	−1.34349e−16 + 2.36327e−16i*
0,2	$100\angle 75^\circ$	Ob	−8.08574e+03 + 4.84470e+03i	−0.00000e+00 − 0.00000e+00i	−4.39024e+01 + 3.11094e+01i	2.71122e+05 + 7.88525e+04i	−3.19332e+11 + 1.57641e+11i	−3.99624e+21 + 3.02920e+21i	−3.64513e+31 + 4.10397e+31i	1.27890e+40 − 6.06402e+38i
0,2	$100\angle 90^\circ$	Ob	−9.40504e+03 + 1.18790e−12i	−0.00000e+00 − 0.00000e+00i	−5.47544e+01 + 1.12828e−14i	−3.96521e+05 + 2.92111e−10i	−8.33660e+11 + 1.38003e−03i	−2.75049e+22 + 8.76540e+07i	−7.04818e+32 + 3.32654e+18i	3.16018e+41 − 1.89713e+27i
0,2	$250\angle 0^\circ$	Pr	1.24624e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	6.09321e+00 − 0.00000e+00i	3.63988e−02 − 0.00000e+00i	−3.17948e−01 − 0.00000e+00i	−3.13107e−11 − 0.00000e+00i	−1.32638e−32 − 0.00000e+00i	−2.51529e−87 + 1.53966e−103i*
0,2	$250\angle 15^\circ$	Pr	1.20365e+03 + 3.23527e+02i	−0.00000e+00 − 0.00000e+00i	5.89868e+00 + 1.54856e+00i	3.77586e−01 − 4.90716e+00i	−3.78393e−02 + 4.18309e−01i	2.70480e−11 + 9.43902e−11i	1.94037e−31 + 1.36846e−31i	−2.63344e−85 − 3.78062e−84i*
0,2	$250\angle 30^\circ$	Pr	1.07877e+03 + 6.25006e+02i	−0.00000e+00 − 0.00000e+00i	5.32475e+00 + 3.00519e+00i	1.80733e+00 − 1.09151e+01i	9.02716e−01 + 2.94145e−01i	2.79973e−09 + 8.87282e−11i	1.02143e−27 − 4.56933e−28i	−6.88464e−75 − 3.81805e−75i*
0,2	$250\angle 45^\circ$	Ob	3.52553e+02 + 6.21464e+04i	−0.00000e+00 − 0.00000e+00i	3.07230e+02 + 2.08704e+02i	−2.49937e+09 − 1.00923e+09i	−4.20630e+20 + 6.54852e+20i	−3.55944e+39 + 9.47726e+39i	−1.99092e+58 + 1.33965e+59i	−3.14542e+77 − 4.19954e+76i
0,2	$250\angle 60^\circ$	Pr	1.24250e+02 + 2.16507e+02i	−0.00000e+00 − 0.00000e+00i	6.29042e−01 + 1.06914e+00i	6.72493e+00 − 2.38444e−01i	5.59246e−01 + 2.75727e−01i	−4.06011e−06 − 1.33655e−06i	−7.96534e−17 − 5.04248e−19i	−6.51326e−44 − 3.12846e−44i*
0,2	$250\angle 75^\circ$	Ob	−5.26827e+04 + 3.08618e+04i	−0.00000e+00 − 0.00000e+00i	−4.05269e+02 + 5.32454e+02i	−1.26863e+12 + 6.16618e+11i	3.63517e+27 − 3.27758e+27i	−1.34230e+53 + 3.48353e+53i	−2.31967e+78 − 2.22948e+79i	1.27812e+102 + 4.51130e+102i
0,2	$250\angle 90^\circ$	Ob	−6.10050e+04 + 7.56219e−12i	−0.00000e+00 − 0.00000e+00i	−7.24774e+02 + 7.87847e−14i	−3.30482e+12 + 4.90521e−03i	−4.11520e+28 + 1.55577e+14i	−2.63987e+55 + 2.00837e+41i	−1.33272e+82 + 1.52404e+68i	−1.91898e+106 + 2.91706e+92i
0,2	$500\angle 0^\circ$	Pr	2.49624e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.19315e+01 − 0.00000e+00i	−1.03019e+01 − 0.00000e+00i	−4.98297e−04 − 0.00000e+00i	−3.64534e−25 − 0.00000e+00i	−9.35154e−69 + 5.71908e−85i*	−5.62639e−180 + 3.44455e−196i*

Table D.54: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,2	$500\angle 15^\circ$	Pr	2.41106e+03 + 6.47049e+02i	−0.00000e+00 − 0.00000e+00i	1.15779e+01 + 2.96176e+00i	−1.18774e+01 − 1.45007e+00i	7.83797e−04 − 3.50931e−04i	1.62173e−24 − 3.18694e−24i	−2.29742e−66 − 1.91842e−66i*	9.98506e−174 − 7.93943e−174i*
0,2	$500\angle 30^\circ$	Pr	2.16131e+03 + 1.25000e+03i	−0.00000e+00 − 0.00000e+00i	1.05277e+01 + 5.77425e+00i	−1.74862e+01 − 3.75271e+00i	−3.57966e−03 + 2.26362e−03i	−1.51421e−21 + 2.46014e−21i	2.17965e−59 + 6.25464e−59i*	−5.47736e−155 + 2.84867e−156i*
0,2	$500\angle 45^\circ$	Ob	7.06106e+02 + 2.49293e+05i	−0.00000e+00 − 0.00000e+00i	1.60906e+03 − 3.92081e+03i	−3.53709e+15 − 2.56755e+17i	−8.57680e+39 + 2.27607e+40i	3.12244e+77 + 4.92697e+78i	4.82290e+116 + 9.10825e+116i	−3.17825e+153 − 5.59315e+153i
0,2	$500\angle 60^\circ$	Pr	2.49250e+02 + 4.33013e+02i	−0.00000e+00 − 0.00000e+00i	1.27688e+00 + 2.11121e+00i	3.12299e+00 − 6.48501e+00i	2.33565e−02 − 3.17391e−03i	4.17450e−13 − 1.84133e−13i	3.37170e−35 − 5.73859e−35i*	3.36009e−90 − 5.70409e−91i*
0,2	$500\angle 75^\circ$	Ob	−2.13614e+05 + 1.24224e+05i	−0.00000e+00 − 0.00000e+00i	−2.00641e+02 + 1.50949e+04i	−7.00443e+22 + 5.19374e+22i	−5.90584e+53 + 1.51944e+54i	1.10541e+106 + 1.73887e+106i	2.05388e+158 + 1.36772e+157i	9.54905e+206 + 3.14261e+206i
0,2	$500\angle 90^\circ$	Ob	−2.47005e+05 + 3.04325e−11i	−0.00000e+00 − 0.00000e+00i	−1.78946e+04 + 6.96771e−12i	−4.78992e+23 + 1.50634e+09i	−1.15321e+56 + 8.92270e+41i	−1.03131e+110 + 1.58734e+96i	−7.28857e+163 + 1.67972e+150i	−2.08170e+214 + 6.33240e+200i
0,2	$750\angle 0^\circ$	Pr	3.74625e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.75033e+01 − 0.00000e+00i	−8.84020e+00 − 0.00000e+00i	−4.11909e−07 − 0.00000e+00i	−2.35034e−39 − 0.00000e+00i	−3.67983e−105 + 2.25138e−121i*	−7.02483e−273 + 4.30091e−289i*
0,2	$750\angle 15^\circ$	Pr	3.61847e+03 + 9.70572e+02i	−0.00000e+00 − 0.00000e+00i	1.70241e+01 + 4.24478e+00i	−9.77305e+00 + 3.26277e+00i	−7.35328e−07 − 5.68166e−07i	−6.78813e−38 + 2.44794e−38i	1.49849e−101 + 1.48030e−101i*	2.29121e−263 + 7.05931e−264i*
0,2	$750\angle 30^\circ$	Pr	3.24384e+03 + 1.87500e+03i	−0.00000e+00 − 0.00000e+00i	1.55902e+01 + 8.31465e+00i	−1.32262e+01 + 9.07248e+00i	1.64575e−06 − 9.95842e−06i	−7.79715e−34 − 1.45833e−33i	−2.12464e−90 + 5.33571e−91i*	−1.75470e−235 + 1.21678e−235i*
0,2	$750\angle 45^\circ$	Ob	1.05966e+03 + 5.61439e+05i	−0.00000e+00 − 0.00000e+00i	−3.65833e+04 − 7.17122e+03i	1.67944e+25 − 7.34400e+24i	−8.45126e+58 + 5.63309e+59i	8.43762e+116 + 1.59474e+117i	5.29631e+174 + 2.54902e+174i	3.84108e+229 − 8.98771e+229i
0,2	$750\angle 60^\circ$	Pr	3.74250e+02 + 6.49519e+02i	−0.00000e+00 − 0.00000e+00i	1.93918e+00 + 3.12623e+00i	−3.43227e+00 − 4.63994e+00i	4.98949e−04 − 4.44095e−04i	−1.49819e−20 + 3.32902e−20i	2.04792e−53 + 3.63066e−53i*	−8.55056e−137 + 8.46533e−137i*
0,2	$750\angle 75^\circ$	Ob	−4.82798e+05 + 2.80085e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$750\angle 90^\circ$	Ob	−5.58005e+05 + 6.86108e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$1000\angle 0^\circ$	Pr	4.99625e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.28160e+01 − 0.00000e+00i	−5.05260e+00 − 0.00000e+00i	−2.65019e−10 − 0.00000e+00i	−1.19275e−53 − 0.00000e+00i	−1.14189e−141 + 6.98766e−158i*	±0
0,2	$1000\angle 15^\circ$	Pr	4.82588e+03 + 1.29410e+03i	−0.00000e+00 − 0.00000e+00i	2.22425e+01 + 5.40268e+00i	−4.69926e+00 + 3.86451e+00i	−1.32574e−10 + 7.71846e−10i	1.05183e−51 + 4.55763e−52i	−7.59750e−137 − 8.88501e−137i*	±0
0,2	$1000\angle 30^\circ$	Pr	4.32637e+03 + 2.50000e+03i	−0.00000e+00 − 0.00000e+00i	2.05124e+01 + 1.06336e+01i	−2.79824e+00 + 1.00835e+01i	1.16759e−08 + 1.46965e−08i	7.44943e−46 + 2.90874e−47i	−8.86717e−123 − 5.64569e−122i*	−2.62055e−316 + 6.01121e−316i*
0,2	$1000\angle 45^\circ$	Ob	1.41321e+03 + 9.98586e+05i	−0.00000e+00 − 0.00000e+00i	9.28933e+02 + 2.91230e+05i	8.42949e+32 + 8.00988e+32i	7.43355e+77 + 1.18315e+79i	4.62704e+155 + 3.59545e+155i	2.92727e+232 − 5.53512e+231i	1.27990e+306 − 3.22019e+305i
0,2	$1000\angle 60^\circ$	Pr	4.99250e+02 + 8.66026e+02i	−0.00000e+00 − 0.00000e+00i	2.61538e+00 + 4.11422e+00i	−4.06587e+00 + 6.21005e−01i	4.18636e−06 − 1.62941e−05i	−7.90374e−28 − 2.47278e−27i	−2.31998e−71 − 2.94910e−73i*	6.72274e−184 − 3.71358e−183i*
0,2	$1000\angle 75^\circ$	Ob	−8.60235e+05 + 4.98447e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$1000\angle 90^\circ$	Ob	−9.94005e+05 + 1.22097e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$1500\angle 0^\circ$	Pr	7.49625e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	3.26917e+01 − 0.00000e+00i	−1.03090e+00 − 0.00000e+00i	−7.69677e−17 − 0.00000e+00i	−2.17853e−82 − 0.00000e+00i	−7.81272e−215 + 4.78186e−231i*	±0
0,2	$1500\angle 15^\circ$	Pr	7.24069e+03 + 1.94114e+03i	−0.00000e+00 − 0.00000e+00i	3.20160e+01 + 7.36254e+00i	−2.66773e−01 + 1.31606e+00i	−2.80490e−16 − 2.71657e−16i	−7.01797e−80 + 1.92815e−79i	−1.31298e−207 − 2.19627e−207i*	±0
0,2	$1500\angle 30^\circ$	Pr	6.49144e+03 + 3.75000e+03i	−0.00000e+00 − 0.00000e+00i	2.99371e+01 + 1.46357e+01i	2.51760e+00 + 1.46474e+00i	3.29838e−14 − 3.15512e−14i	−4.86723e−71 − 9.58643e−71i	−7.10434e−187 + 2.76346e−185i*	±0
0,2	$1500\angle 45^\circ$	Ob	2.12032e+03 + 2.24788e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$1500\angle 60^\circ$	Pr	7.49250e+02 + 1.29904e+03i	−0.00000e+00 − 0.00000e+00i	4.00718e+00 + 6.00925e+00i	1.20316e+00 + 1.28765e+00i	−7.22957e−09 − 5.36562e−09i	−1.08482e−41 + 2.23113e−42i	2.93504e−108 + 5.30638e−108i*	−2.70435e−276 − 1.57788e−276i*
0,2	$1500\angle 75^\circ$	Ob	−1.93987e+06 + 1.12267e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$1500\angle 90^\circ$	Ob	−2.24101e+06 + 2.74994e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$2500\angle 0^\circ$	Pr	1.24962e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	4.96333e+01 − 0.00000e+00i	−2.05860e−02 − 0.00000e+00i	−3.50130e−30 − 0.00000e+00i	−3.96959e−140 + 2.42889e−156i*	±0	±0
0,2	$2500\angle 15^\circ$	Pr	1.20703e+04 + 3.23524e+03i	−0.00000e+00 − 0.00000e+00i	4.90484e+01 + 9.98770e+00i	2.85216e−02 + 1.37989e−02i	−2.04899e−29 + 4.82165e−29i	8.20969e−136 − 3.49643e−135i*	±0	±0
0,2	$2500\angle 30^\circ$	Pr	1.08216e+04 + 6.25000e+03i	−0.00000e+00 − 0.00000e+00i	4.71198e+01 + 2.02874e+01i	−4.88995e−02 − 1.00930e−01i	−8.63135e−26 + 1.17696e−25i	1.21760e−120 + 1.04510e−121i*	−1.34609e−312 + 3.27614e−312i*	±0
0,2	$2500\angle 45^\circ$	Ob	3.53453e+03 + 6.24646e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$2500\angle 60^\circ$	Pr	1.24925e+03 + 2.16506e+03i	−0.00000e+00 − 0.00000e+00i	6.93281e+00 + 9.47674e+00i	−2.22491e−01 + 8.89383e−02i	3.00617e−16 + 1.88586e−15i	1.49283e−70 + 1.56683e−72i*	−3.06639e−181 − 9.20693e−183i*	±0
0,2	$2500\angle 75^\circ$	Ob	−5.39817e+06 + 3.12112e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$2500\angle 90^\circ$	Ob	−6.23501e+06 + 7.64486e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$5000\angle 0^\circ$	Pr	2.49962e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	7.78806e+01 − 0.00000e+00i	−3.13985e−07 − 0.00000e+00i	−4.71271e−64 − 0.00000e+00i	−5.50449e−285 + 3.36924e−301i*	±0	±0
0,2	$5000\angle 15^\circ$	Pr	2.41444e+04 + 6.47048e+03i	−0.00000e+00 − 0.00000e+00i	7.86775e+01 + 1.05313e+01i	−6.98186e−07 − 2.42048e−07i	1.01500e−61 + 2.84786e−62i	4.49561e−275 − 2.88594e−276i*	±0	±0
0,2	$5000\angle 30^\circ$	Pr	2.16469e+04 + 1.25000e+04i	−0.00000e+00 − 0.00000e+00i	8.06128e+01 + 2.30331e+01i	−4.05324e−06 − 8.13223e−06i	7.98893e−55 + 1.66739e−55i	−3.32362e−246 + 4.01510e−246i*	±0	±0
0,2	$5000\angle 45^\circ$	Ob	7.07007e+03 + 2.49929e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$5000\angle 60^\circ$	Pr	2.49925e+03 + 4.33013e+03i	−0.00000e+00 − 0.00000e+00i	1.48762e+01 + 1.62928e+01i	−2.14149e−04 − 8.85227e−04i	−4.53592e−33 + 2.15935e−32i	2.87506e−143 − 4.75134e−143i*	±0	±0
0,2	$5000\angle 75^\circ$	Ob	−2.16217e+07 + 1.24922e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,2	$5000\angle 90^\circ$	Ob	−2.49700e+07 + 3.05978e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$25\angle 0^\circ$	Pr	1.67953e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.48757e+00 − 0.00000e+00i	−4.28237e−01 − 0.00000e+00i	1.48110e+00 − 0.00000e+00i	−6.34954e−01 − 0.00000e+00i	−9.61377e−02 − 0.00000e+00i	−2.67252e−05 + 1.62830e−21i*
0,3	$25\angle 15^\circ$	Pr	1.62005e+02 + 4.53798e+01i	−1.50000e+00 − 8.32667e−17i	−1.48802e+00 + 3.39353e−03i	−4.49175e−01 + 2.50080e−01i	1.59882e+00 + 8.19723e−04i	−8.48527e−01 − 1.30688e−01i	−4.74575e−03 + 1.31430e−01i	−1.15903e−05 − 5.57783e−05i*
0,3	$25\angle 30^\circ$	Ob	3.36495e+02 + 4.97956e+02i	−1.50000e+00 + 1.72085e−15i	−1.47483e+00 + 3.71396e−02i	1.46329e+00 + 1.77776e+00i	−8.80500e+00 − 1.04604e+01i	4.56040e+01 − 2.55760e+02i	4.39699e+03 − 2.52974e+03i	7.56686e+04 + 4.80957e+04i
0,3	$25\angle 45^\circ$	Pr	5.12567e+01 + 5.30623e+01i	−1.50000e+00 − 1.11022e−16i	−1.49631e+00 + 3.97440e−03i	−1.12732e+00 + 3.46764e−01i	5.85498e−01 + 8.33475e−01i	1.52389e−01 − 1.04653e+00i	4.74578e−02 + 1.11102e−01i	−4.69168e−05 − 3.69854e−04i*
0,3	$25\angle 60^\circ$	Ob	−1.87732e+02 + 4.66179e+02i	−1.50000e+00 − 0.00000e+00i	−1.51412e+00 + 3.50747e−02i	−1.54424e+00 + 4.48027e+00i	7.92843e+01 + 4.19762e−01i	−7.92982e+03 + 2.67647e+01i	5.80109e+05 − 4.44322e+04i	1.41383e+07 + 1.66814e+07i
0,3	$25\angle 75^\circ$	Ob	−4.01534e+02 + 2.73631e+02i	−1.50000e+00 − 5.55112e−16i	−1.53032e+00 + 2.06611e−02i	−5.01231e+00 + 3.69111e+00i	7.78407e+00 + 1.47432e+02i	2.72188e+04 − 3.09199e+03i	−7.60178e+05 − 3.57066e+06i	2.50550e+08 + 5.60441e+07i
0,3	$25\angle 90^\circ$	Ob	−4.80164e+02 + 6.73446e−14i	−1.50000e+00 − 1.37397e−16i	−1.53631e+00 − 1.35631e−16i	−6.83301e+00 + 3.97313e−16i	−1.82525e+02 + 5.33881e−14i	−4.18056e+04 + 2.83633e−11i	−6.82242e+06 + 7.32220e−09i	5.94726e+08 − 7.98951e−07i
0,3	$50\angle 0^\circ$	Pr	3.43111e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.47450e+00 − 0.00000e+00i	3.67268e−01 − 0.00000e+00i	6.32154e−01 − 0.00000e+00i	−1.92178e−01 − 0.00000e+00i	−1.02699e−04 − 0.00000e+00i	−6.16556e−14 + 3.76823e−30i*
0,3	$50\angle 15^\circ$	Pr	3.31191e+02 + 9.06247e+01i	−1.50000e+00 + 1.11022e−16i	−1.47538e+00 + 6.75553e−03i	3.58879e−01 + 3.43889e−01i	6.79261e−01 − 6.03952e−01i	−1.19574e−04 + 1.21474e−01i	1.45574e−04 − 1.13199e−04i	1.16279e−13 − 2.41811e−13i*
0,3	$50\angle 30^\circ$	Ob	1.29900e+03 + 2.07846e+03i	−1.50000e+00 + 2.83107e−15i	−1.40110e+00 + 1.52523e−01i	3.13163e+00 − 7.64411e+00i	5.32647e+01 − 3.06510e+02i	1.26533e+05 + 4.52590e+04i	−2.93817e+07 + 5.19640e+07i	−9.25897e+09 − 1.90551e+10i
0,3	$50\angle 45^\circ$	Pr	1.04303e+02 + 1.06080e+02i	−1.50000e+00 − 1.11022e−16i	−1.49233e+00 + 7.93374e−03i	−7.38824e−01 + 5.88812e−01i	1.30944e+00 − 4.69501e−02i	−1.56664e−01 + 1.42136e−01i	9.38020e−05 + 6.24314e−04i	−6.31132e−11 + 1.86963e−10i*
0,3	$50\angle 60^\circ$	Ob	−9.95259e+02 + 2.01502e+03i	−1.50000e+00 + 9.43690e−16i	−1.57286e+00 + 1.53639e−01i	3.39006e+01 + 2.52170e+01i	−1.79337e+04 − 4.99901e+02i	−4.09660e+08 − 2.04382e+07i	−7.16144e+12 − 3.34149e+11i	7.21938e+15 + 1.66036e+16i
0,3	$50\angle 75^\circ$	Ob	−1.88036e+03 + 1.17233e+03i	−1.50000e+00 − 0.00000e+00i	−1.64253e+00 + 9.07088e−02i	−1.89539e+01 + 6.69560e+01i	6.21802e+04 − 6.01392e+03i	−4.87116e+09 + 9.66907e+08i	2.85689e+14 − 9.11870e+13i	2.36574e+18 + 3.71304e+17i
0,3	$50\angle 90^\circ$	Ob	−2.20508e+03 + 2.87787e−13i	−1.50000e+00 + 5.86536e−17i	−1.66860e+00 + 8.76339e−17i	−8.25175e+01 + 2.85037e−14i	−9.56053e+04 + 7.69780e−11i	−1.16259e+10 + 1.82755e−05i	−1.07142e+15 + 2.50926e+00i	1.26449e+19 − 3.76225e+04i
0,3	$100\angle 0^\circ$	Pr	6.93182e+02 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.44849e+00 − 0.00000e+00i	1.21362e+00 − 0.00000e+00i	−5.65908e−01 − 0.00000e+00i	−1.21877e−03 − 0.00000e+00i	−2.02557e−11 − 0.00000e+00i	−5.97349e−32 + 3.65462e−48i*
0,3	$100\angle 15^\circ$	Pr	6.69333e+02 + 1.81191e+02i	−1.50000e+00 − 5.55112e−17i	−1.45024e+00 + 1.34213e−02i	1.26601e+00 + 2.56802e−01i	−7.38425e−01 − 1.25957e−01i	1.87819e−03 + 4.91795e−04i	2.26725e−11 + 6.03161e−11i	1.06694e−30 + 3.34740e−31i*
0,3	$100\angle 30^\circ$	Ob	5.09900e+03 + 8.48705e+03i	−1.50000e+00 − 5.10703e−15i	−1.09093e+00 + 5.82608e−01i	7.30227e+01 + 7.00846e+01i	1.51785e+05 + 5.39306e+04i	−2.79608e+10 − 2.27265e+10i	4.31415e+15 + 7.07830e+15i	7.27049e+20 − 9.37814e+20i
0,3	$100\angle 45^\circ$	Pr	2.10375e+02 + 2.12139e+02i	−1.50000e+00 − 1.11022e−16i	−1.48437e+00 + 1.58191e−02i	−3.86788e−02 + 8.16813e−01i	2.29819e−01 − 9.21011e−01i	−2.74906e−03 − 2.67587e−03i	−3.37334e−09 + 7.41658e−09i	−2.55776e−23 + 5.99982e−24i*
0,3	$100\angle 60^\circ$	Ob	−4.48542e+03 + 8.36023e+03i	−1.50000e+00 + 5.55112e−17i	−1.80368e+00 + 6.73741e−01i	−9.14189e+02 − 3.08079e+03i	−9.04211e+08 − 5.77599e+07i	−1.04286e+18 − 1.30908e+17i	−9.35732e+26 − 1.65206e+26i	−1.03317e+34 + 2.50194e+34i
0,3	$100\angle 75^\circ$	Ob	−8.08574e+03 + 4.84470e+03i	−1.50000e+00 + 6.66134e−16i	−2.13271e+00 + 4.14112e−01i	7.42489e+03 + 4.58254e+03i	−1.08095e+10 + 2.06353e+09i	−1.43979e+20 + 5.72864e+19i	−1.42379e+30 + 9.21889e+29i	3.85884e+38 + 8.73254e+37i
0,3	$100\angle 90^\circ$	Ob	−9.40504e+03 + 1.18790e−12i	−1.50000e+00 + 9.30730e−17i	−2.26262e+00 + 2.44129e−16i	−1.22668e+04 + 8.27366e−12i	−2.57901e+10 + 4.10882e−05i	−8.50894e+20 + 2.65873e+06i	−2.18043e+31 + 1.01553e+17i	9.77635e+39 − 5.80816e+25i
0,3	$250\angle 0^\circ$	Pr	1.74322e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.37151e+00 − 0.00000e+00i	1.01318e+00 − 0.00000e+00i	−6.90469e−02 − 0.00000e+00i	−1.62298e−11 − 0.00000e+00i	−1.18095e−32 − 0.00000e+00i	−4.33814e−87 + 2.65545e−103i*
0,3	$250\angle 15^\circ$	Pr	1.68359e+03 + 4.52940e+02i	−1.50000e+00 − 5.55112e−17i	−1.37570e+00 + 3.29159e−02i	1.11252e+00 − 4.39614e−01i	−1.27880e−02 + 9.12012e−02i	1.35968e−11 + 4.91148e−11i	1.72400e−31 + 1.22538e−31i	−4.43749e−85 − 6.52264e−84i*
0,3	$250\angle 30^\circ$	Ob	3.14990e+04 + 5.36936e+04i	−1.50000e+00 − 8.93730e−15i	1.57267e+00 + 1.96770e+00i	1.72458e+05 + 6.11353e+04i	1.68408e+13 − 1.47457e+13i	−9.18296e+25 + 6.89769e+26i	−1.23230e+40 − 1.85249e+40i	−1.58288e+53 + 1.36638e+53i
0,3	$250\angle 45^\circ$	Pr	5.28577e+02 + 5.30333e+02i	−1.50000e+00 − 1.11022e−16i	−1.46050e+00 + 3.91971e−02i	1.15711e+00 + 3.30917e−01i	5.77246e−03 + 8.74972e−02i	−4.13499e−09 + 5.13934e−09i	−4.21481e−24 − 3.09539e−24i	1.76054e−62 − 4.41995e−63i*
0,3	$250\angle 60^\circ$	Ob	−2.99560e+04 + 5.33766e+04i	−1.50000e+00 − 1.47105e−15i	−2.02792e+00 + 5.93546e+00i	−1.40628e+09 − 9.29980e+07i	−1.13892e+23 − 1.89545e+22i	−1.61966e+46 − 5.52596e+45i	−1.76482e+69 − 9.37922e+68i	−2.82799e+89 − 1.28765e+90i
0,3	$250\angle 75^\circ$	Ob	−5.26827e+04 + 3.08618e+04i	−1.50000e+00 − 1.77636e−15i	−6.55184e+00 + 4.86715e+00i	−1.68255e+10 + 3.19167e+09i	5.30156e+25 − 2.68469e+25i	−2.68039e+51 + 3.65502e+51i	4.37040e+76 − 2.68612e+77i	6.32868e+98 + 5.69244e+100i
0,3	$250\angle 90^\circ$	Ob	−6.10050e+04 + 7.56219e−12i	−1.50000e+00 − 1.63190e−16i	−8.93018e+00 + 3.76103e−16i	−4.01411e+10 + 5.70829e−05i	−4.99842e+26 + 1.85859e+12i	−3.20645e+53 + 2.41947e+39i	−1.61875e+80 + 1.84107e+66i	−2.33084e+104 + 3.52864e+90i
0,3	$500\angle 0^\circ$	Pr	3.49324e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.24656e+00 − 0.00000e+00i	−3.29883e−01 − 0.00000e+00i	−1.17989e−04 − 0.00000e+00i	−1.92202e−25 − 0.00000e+00i	−8.38488e−69 + 5.12785e−85i*	−9.73363e−180 + 5.95906e−196i*

Table D.54: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,3	$500\angle 15^\circ$	Pr	3.37398e+03 + 9.05870e+02i	−1.50000e+00 − 0.00000e+00i	−1.25439e+00 + 6.37535e−02i	−4.63974e−01 − 4.03027e−01i	1.87798e−04 − 7.96405e−05i	8.62794e−25 − 1.67763e−24i	−2.05733e−66 − 1.72425e−66i*	1.72869e−173 − 1.37229e−173i*
0,3	$500\angle 30^\circ$	Ob	1.25499e+05 + 2.15640e+05i	−1.50000e+00 + 1.73767e−14i	3.39778e+00 − 8.33578e+00i	−3.81419e+10 − 3.09137e+10i	−1.09833e+26 + 8.27577e+26i	9.34456e+53 + 2.52423e+53i	4.44927e+80 − 1.06514e+81i	−1.67804e+106 + 1.14710e+107i
0,3	$500\angle 45^\circ$	Pr	1.05891e+03 + 1.06066e+03i	−1.50000e+00 − 0.00000e+00i	−1.42074e+00 + 7.72364e−02i	7.62757e−01 − 8.20994e−01i	−3.58211e−04 + 5.48593e−04i	−5.86963e−19 − 3.59023e−19i	9.52395e−50 + 4.83717e−50i*	1.13532e−128 − 4.00534e−128i*
0,3	$500\angle 60^\circ$	Ob	−1.22407e+05 + 2.15006e+05i	−1.50000e+00 − 6.43929e−15i	4.42984e+01 + 3.30786e+01i	−3.53238e+18 − 4.71065e+17i	−3.51068e+46 − 1.20779e+46i	−1.39673e+93 − 1.08892e+93i	−3.65987e+139 − 5.58849e+139i	1.89383e+183 + 1.00212e+183i
0,3	$500\angle 75^\circ$	Ob	−2.13614e+05 + 1.24224e+05i	−1.50000e+00 − 6.23113e−15i	−2.48883e+01 + 8.76298e+01i	−4.89742e+20 + 1.92588e+20i	−5.82850e+51 + 7.92578e+51i	3.70921e+103 + 1.18690e+104i	1.17529e+156 + 4.02384e+155i	5.07043e+204 + 3.33144e+204i
0,3	$500\angle 90^\circ$	Ob	−2.47005e+05 + 3.04325e−11i	−1.50000e+00 + 3.17905e−17i	−1.08027e+02 + 3.53604e−14i	−2.89133e+21 + 8.91348e+06i	−6.96108e+53 + 5.34284e+39i	−6.22528e+107 + 9.54303e+93i	−4.39959e+161 + 1.01120e+148i	−1.25657e+212 + 3.81463e+198i
0,3	$750\angle 0^\circ$	Pr	5.24324e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.12570e+00 − 0.00000e+00i	−5.67088e−01 − 0.00000e+00i	−9.99986e−08 − 0.00000e+00i	−1.24607e−39 − 0.00000e+00i	−3.30685e−105 + 2.02317e−121i*	−1.21644e−272 + 7.44755e−289i*
0,3	$750\angle 15^\circ$	Pr	5.06435e+03 + 1.35880e+03i	−1.50000e+00 + 2.77556e−17i	−1.13666e+00 + 9.25838e−02i	−6.96954e−01 + 9.74206e−02i	−1.77113e−07 − 1.40401e−07i	−3.60386e−38 + 1.28813e−38i	1.34532e−101 + 1.33218e−101i*	3.96744e−263 + 1.22467e−263i*
0,3	$750\angle 30^\circ$	Ob	2.81999e+05 + 4.85840e+05i	−1.50000e+00 − 1.04361e−14i	−3.08857e+01 + 6.64014e+00i	6.87704e+15 + 1.12368e+16i	−1.72624e+40 − 2.59147e+40i	5.18806e+80 − 1.24263e+81i	5.87947e+121 − 1.14404e+121i	3.45750e+160 + 5.41947e+160i
0,3	$750\angle 45^\circ$	Pr	1.58924e+03 + 1.59099e+03i	−1.50000e+00 − 1.11022e−16i	−1.38100e+00 + 1.14133e−01i	−1.42374e−01 − 7.11334e−01i	−3.41833e−06 + 1.17381e−06i	2.20954e−29 − 4.88181e−29i	−1.55502e−75 − 4.96438e−76i*	−4.95901e−194 − 5.12927e−194i*
0,3	$750\angle 60^\circ$	Ob	−2.77358e+05 + 4.84889e+05i	−1.50000e+00 − 1.05471e−14i	3.95348e+02 − 2.75385e+02i	−8.83145e+27 − 1.78104e+27i	−1.04853e+70 − 5.69338e+69i	−9.96929e+139 − 1.53418e+140i	−1.78580e+209 − 2.21390e+210i	−2.27706e+276 + 1.77344e+276i
0,3	$750\angle 75^\circ$	Ob	−4.82798e+05 + 2.80085e+05i	−1.50000e+00 + 5.51381e−10i	3.68899e+02 + 9.50115e+02i	−1.36814e+31 + 8.61440e+30i	2.68863e+77 − 1.60533e+78i	3.21928e+156 + 1.10883e+156i	−2.68649e+234 + 4.95552e+234i	±∞
0,3	$750\angle 90^\circ$	Ob	−5.58005e+05 + 6.86108e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1000\angle 0^\circ$	Pr	6.99324e+03 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−1.00885e+00 − 0.00000e+00i	−3.81560e−01 − 0.00000e+00i	−6.51070e−11 − 0.00000e+00i	−6.34078e−54 − 0.00000e+00i	−1.02737e−141 + 6.28685e−158i*	±0
0,3	$1000\angle 15^\circ$	Pr	6.75472e+03 + 1.81174e+03i	−1.50000e+00 + 1.11022e−16i	−1.02244e+00 + 1.19475e−01i	−3.92562e−01 + 2.61166e−01i	−3.43274e−11 + 1.89563e−10i	5.58807e−52 + 2.43531e−52i	−6.82910e−137 − 8.00117e−137i*	±0
0,3	$1000\angle 30^\circ$	Ob	5.00999e+05 + 8.64293e+05i	−1.50000e+00 − 3.69704e−14i	7.95418e+01 + 7.62821e+01i	−7.32443e+20 − 3.45840e+21i	1.12128e+54 + 3.02645e+53i	−1.61899e+108 − 9.42575e+107i	2.18585e+162 + 2.20965e+162i	3.41605e+214 + 1.01888e+214i
0,3	$1000\angle 45^\circ$	Pr	2.11957e+03 + 2.12132e+03i	−1.50000e+00 − 0.00000e+00i	−1.34131e+00 + 1.49900e−01i	−3.65911e−01 − 1.80894e−01i	−1.66714e−08 − 5.80284e−09i	3.53738e−39 + 1.10292e−39i	2.19140e−101 + 3.28377e−102i*	−1.05429e−259 + 2.59678e−260i*
0,3	$1000\angle 60^\circ$	Ob	−4.94809e+05 + 8.63025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1000\angle 75^\circ$	Ob	−8.60235e+05 + 4.98447e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1000\angle 90^\circ$	Ob	−9.94005e+05 + 1.22097e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1500\angle 0^\circ$	Pr	1.04932e+04 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−7.86755e−01 − 0.00000e+00i	−8.76381e−02 − 0.00000e+00i	−1.91274e−17 − 0.00000e+00i	−1.16126e−82 − 0.00000e+00i	−7.03741e−215 + 4.30732e−231i*	±0
0,3	$1500\angle 15^\circ$	Pr	1.01355e+04 + 2.71760e+03i	−1.50000e+00 + 5.55112e−17i	−8.04257e−01 + 1.67702e−01i	−2.87669e−02 + 1.11701e−01i	−6.93656e−17 − 6.79720e−17i	−3.75589e−80 + 1.02746e−79i	−1.18159e−207 − 1.97919e−207i*	±0
0,3	$1500\angle 30^\circ$	Ob	1.12650e+06 + 1.94596e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1500\angle 45^\circ$	Pr	3.18023e+03 + 3.18198e+03i	−1.50000e+00 − 1.11022e−16i	−1.26210e+00 + 2.18108e−01i	−2.43909e−02 + 1.03711e−01i	3.52281e−14 − 3.52140e−13i	−1.49067e−59 − 8.62818e−61i	3.39275e−153 − 5.86508e−154i*	±0
0,3	$1500\angle 60^\circ$	Ob	−1.11721e+06 + 1.94406e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1500\angle 75^\circ$	Ob	−1.93987e+06 + 1.12267e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$1500\angle 90^\circ$	Ob	−2.24101e+06 + 2.74994e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$2500\angle 0^\circ$	Pr	1.74932e+04 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	−3.86491e−01 − 0.00000e+00i	−1.88582e−03 − 0.00000e+00i	−8.77936e−31 − 0.00000e+00i	−2.12051e−140 + 1.29748e−156i*	±0	±0
0,3	$2500\angle 15^\circ$	Pr	1.68969e+04 + 4.52933e+03i	−1.50000e+00 − 0.00000e+00i	−4.07028e−01 + 2.43697e−01i	2.59014e−03 + 1.33766e−03i	−5.18158e−30 + 1.20780e−29i	4.40155e−136 − 1.86759e−135i*	±0	±0
0,3	$2500\angle 30^\circ$	Ob	3.12750e+06 + 5.40833e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$2500\angle 45^\circ$	Pr	5.30155e+03 + 5.30330e+03i	−1.50000e+00 + 3.33067e−16i	−1.10476e+00 + 3.41617e−01i	−9.99205e−04 − 5.13080e−03i	3.75224e−23 + 9.79221e−23i	−1.63065e−100 + 7.47176e−101i*	4.24134e−257 − 4.45762e−257i*	±0
0,3	$2500\angle 60^\circ$	Ob	−3.11201e+06 + 5.40516e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$2500\angle 75^\circ$	Ob	−5.39817e+06 + 3.12112e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$2500\angle 90^\circ$	Ob	−6.23501e+06 + 7.64486e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$5000\angle 0^\circ$	Pr	3.49932e+04 − 0.00000e+00i	−1.50000e+00 − 0.00000e+00i	3.89189e−01 − 0.00000e+00i	−3.01719e−08 − 0.00000e+00i	−1.18948e−64 − 0.00000e+00i	−2.94515e−285 + 1.80269e−301i*	±0	±0
0,3	$5000\angle 15^\circ$	Pr	3.38007e+04 + 9.05867e+03i	−1.50000e+00 − 0.00000e+00i	3.81082e−01 + 3.35356e−01i	−6.69317e−08 − 2.40747e−08i	2.56121e−62 + 7.23274e−63i	2.40555e−275 − 1.53424e−276i*	±0	±0
0,3	$5000\angle 30^\circ$	Ob	1.25050e+07 + 2.16420e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$5000\angle 45^\circ$	Pr	1.06049e+04 + 1.06066e+04i	−1.50000e+00 − 0.00000e+00i	−7.21906e−01 + 5.80449e−01i	−5.24842e−07 + 1.40920e−06i	−4.07667e−48 + 8.78118e−47i	−3.79058e−204 − 4.98090e−203i*	±0	±0
0,3	$5000\angle 60^\circ$	Ob	−1.24740e+07 + 2.16356e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$5000\angle 75^\circ$	Ob	−2.16217e+07 + 1.24922e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,3	$5000\angle 90^\circ$	Ob	−2.49700e+07 + 3.05978e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$25\angle 0^\circ$	Pr	2.13649e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.98335e−01 − 0.00000e+00i	−5.46812e+00 − 0.00000e+00i	2.21490e+00 − 0.00000e+00i	−6.35966e−01 − 0.00000e+00i	−8.15057e−01 − 0.00000e+00i	−5.01156e−04 + 3.05162e−20i*
0,4	$25\angle 15^\circ$	Pr	2.06014e+02 + 5.84142e+01i	−0.00000e+00 − 0.00000e+00i	−7.70127e−01 − 2.17534e−01i	−5.50438e+00 − 8.99966e−01i	2.02036e+00 + 2.71686e+00i	−1.09592e+00 − 2.98704e+00i	−4.42892e−01 + 1.10541e+00i	1.14402e−04 − 1.07441e−03i*
0,4	$25\angle 30^\circ$	Ob	3.36495e+02 + 4.97956e+02i	−0.00000e+00 − 0.00000e+00i	−1.27030e+00 − 1.84652e+00i	−1.52136e+01 + 2.40575e−01i	8.21650e+01 − 1.71525e+01i	1.25455e+03 + 9.81784e+02i	1.07305e+03 + 3.10870e+04i	−4.77922e+05 + 2.71753e+05i
0,4	$25\angle 45^\circ$	Pr	8.45599e+01 + 8.84789e+01i	−0.00000e+00 − 0.00000e+00i	−3.17173e−01 − 3.30816e−01i	−3.18243e+00 − 2.38693e+00i	−4.73444e+00 + 2.07231e+00i	5.63931e+00 − 2.20340e+00i	−5.86319e−01 + 1.10777e+00i	5.09460e−03 − 5.86094e−03i*
0,4	$25\angle 60^\circ$	Ob	−1.87732e+02 + 4.66179e+02i	−0.00000e+00 − 0.00000e+00i	6.92610e−01 − 1.75920e+00i	−7.94701e+00 − 2.64408e+01i	−3.69101e+02 + 2.46998e+02i	3.67076e+04 − 2.50279e+04i	−2.55196e+06 + 2.02799e+06i	−1.17985e+08 − 3.29946e+07i
0,4	$25\angle 75^\circ$	Pr	5.71888e+00 + 2.41556e+01i	−0.00000e+00 − 0.00000e+00i	−2.15500e−02 − 9.05360e−02i	−3.14657e−01 − 8.52917e−01i	−1.68547e+00 − 1.08530e+00i	7.98219e−01 + 2.38105e+00i	−1.35898e+00 + 8.25179e−01i*	−6.31862e−01 + 4.53356e−01i*
0,4	$25\angle 90^\circ$	Ob	−3.88753e+02 + 6.11785e−14i	−0.00000e+00 − 0.00000e+00i	1.46736e+00 − 1.67501e−16i	2.61224e+01 − 5.11559e−15i	5.09491e+02 − 2.15596e−13i	5.84222e+04 − 4.77917e−11i	2.73498e+06 − 3.45051e−09i	8.89480e+07 − 1.22289e−07i
0,4	$50\angle 0^\circ$	Pr	4.38973e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.63399e+00 − 0.00000e+00i	−6.87792e+00 − 0.00000e+00i	7.30623e+00 − 0.00000e+00i	−1.87755e+00 − 0.00000e+00i	−2.05622e−03 − 0.00000e+00i	−2.51217e−12 + 1.53523e−28i*
0,4	$50\angle 15^\circ$	Pr	4.23651e+02 + 1.16545e+02i	−0.00000e+00 − 0.00000e+00i	−1.57823e+00 − 4.30782e−01i	−7.27334e+00 + 5.96412e−02i	8.63637e+00 + 1.50601e−02i	−1.93114e+00 + 1.59472e+00i	3.47154e−03 − 1.32211e−03i	7.29705e−12 − 8.18520e−12i*
0,4	$50\angle 30^\circ$	Ob	1.29900e+03 + 2.07846e+03i	−0.00000e+00 − 0.00000e+00i	−5.03156e+00 − 7.45772e+00i	6.56609e+01 + 7.99705e+01i	2.99927e+03 + 2.41543e+03i	−1.24909e+06 + 1.09854e+06i	−3.86388e+08 − 6.29854e+08i	2.61863e+11 + 1.40557e+10i
0,4	$50\angle 45^\circ$	Pr	1.72987e+02 + 1.76819e+02i	−0.00000e+00 − 0.00000e+00i	−6.48845e−01 − 6.59007e−01i	−6.16821e+00 − 3.02688e+00i	1.32932e+00 + 7.24605e+00i	−2.56002e+00 − 4.13185e−01i	−8.62449e−03 + 1.05787e−02i	−7.59576e−09 + 3.50643e−09i*
0,4	$50\angle 60^\circ$	Ob	−9.95259e+02 + 2.01502e+03i	−0.00000e+00 − 0.00000e+00i	3.53708e+00 − 7.80681e+00i	−4.98845e+02 − 4.19704e+01i	1.83615e+05 − 1.07179e+05i	4.25076e+09 − 2.35752e+09i	7.41645e+13 − 4.14457e+13i	−1.76544e+17 − 1.21930e+17i
0,4	$50\angle 75^\circ$	Pr	1.21901e+01 + 4.83000e+01i	−0.00000e+00 − 0.00000e+00i	−4.61241e−02 − 1.80913e−01i	−8.30054e−01 − 1.55895e+00i	−3.20410e+00 + 6.82098e−01i	−8.89277e−01 − 1.92276e+00i	2.76643e−01 + 2.25780e−01i	5.00065e−03 + 4.35111e−03i*
0,4	$50\angle 90^\circ$	Ob	−2.01335e+03 + 2.75523e−13i	−0.00000e+00 − 0.00000e+00i	7.80642e+00 − 1.59609e−15i	7.50162e+02 − 3.29021e−13i	6.27385e+05 − 5.64207e−10i	4.05851e+10 − 6.77445e−05i	1.40000e+15 − 3.43024e+00i	1.20529e+18 − 3.51282e+03i
0,4	$100\angle 0^\circ$	Pr	8.89116e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.28411e+00 − 0.00000e+00i	−2.43687e+00 − 0.00000e+00i	−1.17194e+00 − 0.00000e+00i	−2.87433e−02 − 0.00000e+00i	−8.64741e−10 − 0.00000e+00i	−5.03198e−30 + 3.07853e−46i*
0,4	$100\angle 15^\circ$	Pr	8.58454e+02 + 2.32973e+02i	−0.00000e+00 − 0.00000e+00i	−3.17595e+00 − 8.48292e−01i	−3.32986e+00 + 3.66432e+00i	−1.83299e+00 − 5.04956e+00i	3.91568e−02 + 2.43433e−02i	2.27003e−10 + 2.74783e−09i	7.92001e−29 + 5.11960e−29i*
0,4	$100\angle 30^\circ$	Ob	5.09900e+03 + 8.48705e+03i	−0.00000e+00 − 0.00000e+00i	−2.17007e+01 − 2.64404e+01i	−2.41201e+03 + 7.22304e+02i	−3.02696e+06 + 2.62579e+06i	8.34570e+11 − 3.27013e+11i	−2.06103e+17 + 6.70409e+15i	1.13994e+22 + 2.72295e+22i
0,4	$100\angle 45^\circ$	Pr	3.49784e+02 + 3.53574e+02i	−0.00000e+00 − 0.00000e+00i	−1.31194e+00 − 1.30937e+00i	−1.02333e+01 − 6.31269e−01i	9.94378e+00 − 2.93918e+00i	3.63217e−03 − 9.81633e−02i	−3.39825e−07 + 1.18603e−07i	−1.89966e−21 − 1.21128e−21i*
0,4	$100\angle 60^\circ$	Ob	−4.48542e+03 + 8.36023e+03i	−0.00000e+00 − 0.00000e+00i	1.34423e+01 − 3.60100e+01i	5.76221e+04 + 5.29432e+04i	1.96158e+10 − 1.00987e+10i	2.34275e+19 − 1.03038e+19i	2.16179e+28 − 8.24774e+27i	−9.69425e+34 − 6.51963e+35i
0,4	$100\angle 75^\circ$	Pr	2.51314e+01 + 9.65944e+01i	−0.00000e+00 − 0.00000e+00i	−9.58754e−02 − 3.61333e−01i	−2.23305e+00 − 2.45206e+00i	7.79226e−01 + 4.28177e+00i	−4.40804e−01 − 6.06531e−01i	−8.94938e−03 − 2.83904e−04i	−2.03541e−07 − 1.12152e−08i*
0,4	$100\angle 90^\circ$	Ob	−9.01317e+03 + 1.16340e−12i	−0.00000e+00 − 0.00000e+00i	3.91129e+01 − 8.87918e−15i	2.37168e+05 − 1.79505e−10i	3.62904e+11 − 6.08248e−04i	6.51976e+21 − 2.09209e+07i	6.75398e+31 − 3.20528e+17i	−1.12439e+39 + 7.13100e+24i
0,4	$250\angle 0^\circ$	Pr	2.23920e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−8.08069e+00 − 0.00000e+00i	1.61742e+01 − 0.00000e+00i	−1.76270e+00 − 0.00000e+00i	−1.03801e−09 − 0.00000e+00i	−1.30443e−30 − 0.00000e+00i	−9.30168e−85 + 5.69371e−101i*
0,4	$250\angle 15^\circ$	Pr	2.16253e+03 + 5.82357e+02i	−0.00000e+00 − 0.00000e+00i	−7.83531e+00 − 2.02561e+00i	1.80835e+01 + 4.23374e+00i	−1.09449e+00 + 2.12710e+00i	−1.42669e−11 + 3.26547e−09i	1.47997e−29 + 1.81011e−29i	2.74403e−82 − 1.37529e−81i*
0,4	$250\angle 30^\circ$	Ob	3.14990e+04 + 5.36936e+04i	−0.00000e+00 − 0.00000e+00i	−1.67192e+02 − 1.79144e−01i	−8.65524e+06 + 7.43947e+06i	2.76077e+14 + 1.36866e+15i	−3.44885e+28 − 2.63531e+28i	1.38471e+42 − 9.27385e+40i	−2.48890e+54 − 1.28034e+55i
0,4	$250\angle 45^\circ$	Pr	8.80126e+02 + 8.83892e+02i	−0.00000e+00 − 0.00000e+00i	−3.30055e+00 − 3.21052e+00i	−4.37738e+00 + 1.43258e+01i	−1.86693e+00 + 1.81099e+00i	−4.32157e−07 + 3.84993e−08i	−8.34461e−23 − 5.79079e−22i	3.35259e−60 + 2.02907e−60i*
0,4	$250\angle 60^\circ$	Ob	−2.99560e+04 + 5.33766e+04i	−0.00000e+00 − 0.00000e+00i	−8.54971e+01 − 3.87414e+02i	7.79655e+10 − 3.89842e+10i	6.67127e+24 − 2.54758e+24i	1.03718e+48 − 2.11220e+47i	1.23508e+71 − 5.09196e+69i	5.53350e+91 + 5.98898e+91i
0,4	$250\angle 75^\circ$	Pr	6.39546e+01 + 2.41482e+02i	−0.00000e+00 − 0.00000e+00i	−2.49958e−01 − 8.99760e−01i	−6.79763e+00 − 6.92105e−01i	−3.16658e+00 + 2.31262e−01i	−9.61895e−03 − 3.79410e−03i	−1.43479e−08 − 4.14644e−08i	−5.32078e−22 − 1.60120e−21i*
0,4	$250\angle 90^\circ$	Ob	−6.00131e+04 + 7.50096e−12i	−0.00000e+00 − 0.00000e+00i	5.28224e+02 − 7.82509e−14i	2.00801e+12 − 3.05961e−03i	1.82825e+28 − 6.98426e+13i	6.46578e+54 − 4.94499e+40i	1.36905e+81 − 1.57128e+67i	−3.71481e+103 + 5.53730e+89i
0,4	$500\angle 0^\circ$	Pr	4.48922e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.55780e+01 − 0.00000e+00i	6.72499e+00 − 0.00000e+00i	−6.82773e−03 − 0.00000e+00i	−2.51779e−23 − 0.00000e+00i	−1.87253e−66 + 1.14515e−82i*	−4.19848e−177 + 2.57036e−193i*

Table D.54: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)\prime}_{mn}$	$S^{(a)\prime}_{mn}(c,\eta{=}0)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.01)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.1)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.25)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.5)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.75)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.99)$
0,4	$500\angle 15^\circ$	Pr	4.33589e+03 + 1.16469e+03i	−0.00000e+00 − 0.00000e+00i	−1.51670e+01 − 3.74419e+00i	8.37734e+00 − 1.17623e+01i	1.17843e−02 − 1.32714e−03i	1.67281e−22 − 1.82181e−22i	−3.42887e−64 − 4.92001e−64i*	8.74191e−171 − 3.77523e−171i*
0,4	$500\angle 30^\circ$	Ob	1.25499e+05 + 2.15640e+05i	−0.00000e+00 − 0.00000e+00i	7.06245e+02 + 8.88883e+02i	5.72084e+12 − 2.20462e+12i	−8.27509e+28 − 6.34063e+28i	−8.54954e+55 + 8.54447e+55i	8.76085e+82 + 1.14470e+83i	−1.13732e+109 − 8.95723e+108i
0,4	$500\angle 45^\circ$	Pr	1.76401e+03 + 1.76777e+03i	−0.00000e+00 − 0.00000e+00i	−6.61124e+00 − 6.21406e+00i	2.27005e+01 + 7.05776e+00i	−3.96225e−02 + 6.77967e−03i	−2.05684e−17 − 8.89977e−17i	7.34770e−48 + 2.28536e−47i*	1.57528e−125 − 8.74573e−126i*
0,4	$500\angle 60^\circ$	Ob	−1.22407e+05 + 2.15006e+05i	−0.00000e+00 − 0.00000e+00i	−6.82861e+03 − 7.89676e+02i	4.09180e+20 − 1.70136e+20i	4.52894e+48 − 8.95795e+47i	2.18210e+95 + 2.97650e+94i	7.42727e+141 + 3.72030e+141i	−2.66224e+185 + 1.06354e+184i
0,4	$500\angle 75^\circ$	Pr	1.28659e+02 + 4.82963e+02i	−0.00000e+00 − 0.00000e+00i	−5.22717e−01 − 1.78749e+00i	−4.39125e+00 + 8.85505e+00i	−9.06201e−02 + 8.08940e−01i	−3.00820e−06 + 1.89578e−06i	−1.07201e−17 − 2.46592e−17i*	−9.56201e−46 − 2.27357e−45i*
0,4	$500\angle 90^\circ$	Ob	−2.45013e+05 + 3.03100e−11i	−0.00000e+00 − 0.00000e+00i	1.31010e+04 − 5.37557e−12i	2.92495e+23 − 9.25982e+08i	5.15665e+55 − 4.00072e+41i	2.55231e+109 − 3.93382e+95i	7.64952e+162 − 1.76458e+149i	3.34460e+211 − 1.02136e+198i
0,4	$750\angle 0^\circ$	Pr	6.73923e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.24753e+01 − 0.00000e+00i	−9.31463e+00 − 0.00000e+00i	−8.97786e−06 − 0.00000e+00i	−2.46722e−37 − 0.00000e+00i	−1.11177e−102 + 6.80191e−119i*	−7.88602e−270 + 4.82816e−286i*
0,4	$750\angle 15^\circ$	Pr	6.50923e+03 + 1.74703e+03i	−0.00000e+00 − 0.00000e+00i	−2.19716e+01 − 5.17264e+00i	−1.31949e+01 − 7.99384e+00i	−1.18558e−05 − 1.65318e−05i	−7.55905e−36 + 5.87556e−37i	3.20008e−99 + 5.50375e−99i*	2.27764e−260 + 1.43495e−260i*
0,4	$750\angle 30^\circ$	Ob	2.81999e+05 + 4.85840e+05i	−0.00000e+00 − 0.00000e+00i	1.80511e+03 − 5.63111e+03i	−2.46764e+18 + 6.60515e+16i	5.82206e+42 − 3.80039e+41i	1.53270e+83 + 2.00429e+83i	−3.63961e+123 + 1.06168e+124i	−1.20329e+163 + 5.47082e+161i
0,4	$750\angle 45^\circ$	Pr	2.64790e+03 + 2.65165e+03i	−0.00000e+00 − 0.00000e+00i	−9.91409e+00 − 9.01472e+00i	1.69053e+01 − 1.69573e+01i	−2.98595e−04 − 1.55058e−04i	1.00388e−26 − 3.71234e−27i	−2.51266e−73 − 4.90434e−73i*	8.55189e−193 − 4.63445e−191i*
0,4	$750\angle 60^\circ$	Ob	−2.77358e+05 + 4.84889e+05i	−0.00000e+00 − 0.00000e+00i	−3.80819e+04 + 8.15742e+04i	1.59439e+30 − 5.40085e+29i	2.22848e+72 − 6.27811e+70i	3.04963e+142 + 1.54505e+142i	2.36418e+212 + 3.41088e+212i	2.01779e+278 − 5.00115e+278i
0,4	$750\angle 75^\circ$	Pr	1.93364e+02 + 7.24445e+02i	−0.00000e+00 − 0.00000e+00i	−8.15146e−01 − 2.66289e+00i	7.31208e+00 + 7.84017e+00i	1.49626e−01 + 4.57944e−02i	−6.54801e−11 + 9.14454e−10i	−5.76831e−27 − 1.09251e−26i*	−1.22664e−69 − 2.40687e−69i*
0,4	$750\angle 90^\circ$	Ob	−5.55013e+05 + 6.84271e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$1000\angle 0^\circ$	Pr	8.98924e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.87959e+01 − 0.00000e+00i	−1.24281e+01 − 0.00000e+00i	−7.91753e−09 − 0.00000e+00i	−1.68029e−51 − 0.00000e+00i	−4.61339e−139 + 2.82310e−155i*	±0
0,4	$1000\angle 15^\circ$	Pr	8.68257e+03 + 2.32937e+03i	−0.00000e+00 − 0.00000e+00i	−2.82660e+01 − 6.32720e+00i	−1.62717e+01 + 2.34606e+00i	−1.06251e−08 + 2.11031e−08i	1.26090e−49 + 1.01069e−49i	−2.02673e−134 − 4.26771e−134i*	±0
0,4	$1000\angle 30^\circ$	Ob	5.00999e+05 + 8.64293e+05i	−0.00000e+00 − 0.00000e+00i	−2.64390e+04 + 7.70266e+03i	8.40137e+23 + 2.72856e+23i	−2.05404e+56 + 2.05009e+56i	4.06042e+110 − 2.32935e+110i	−7.51089e+164 + 1.97598e+164i	−6.46746e+216 + 6.12493e+216i
0,4	$1000\angle 45^\circ$	Pr	3.53178e+03 + 3.53554e+03i	−0.00000e+00 − 0.00000e+00i	−1.32054e+01 − 1.16165e+01i	−3.49476e+00 − 1.81454e+01i	−9.23515e−07 − 2.00084e−06i	4.55105e−37 + 8.77292e−37i	5.91638e−99 + 8.03860e−99i*	−8.04592e−257 − 4.87867e−257i*
0,4	$1000\angle 60^\circ$	Ob	−4.94809e+05 + 8.63025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$1000\angle 75^\circ$	Pr	2.58069e+02 + 9.65926e+02i	−0.00000e+00 − 0.00000e+00i	−1.12693e+00 − 3.52567e+00i	9.55771e+00 − 3.93375e+00i	1.24101e−02 − 2.36813e−02i	1.59940e−13 + 1.36244e−13i	−2.68502e−36 − 4.27152e−36i*	−1.35980e−93 − 2.25042e−93i*
0,4	$1000\angle 90^\circ$	Ob	−9.90013e+05 + 1.21852e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$1500\angle 0^\circ$	Pr	1.34892e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.97972e+01 − 0.00000e+00i	−5.24938e+00 − 0.00000e+00i	−3.54213e−15 − 0.00000e+00i	−4.63318e−80 − 0.00000e+00i	−4.74851e−212 + 2.90637e−228i*	±0
0,4	$1500\angle 15^\circ$	Pr	1.30292e+04 + 3.49406e+03i	−0.00000e+00 − 0.00000e+00i	−3.93898e+01 − 7.87668e+00i	−3.93231e+00 + 5.84168e+00i	−9.04141e−15 − 1.55696e−14i	−2.51595e−77 + 3.56794e−77i	−4.23154e−205 − 1.49688e−204i*	±0
0,4	$1500\angle 30^\circ$	Ob	1.12650e+06 + 1.94596e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$1500\angle 45^\circ$	Pr	5.29955e+03 + 5.30330e+03i	−0.00000e+00 − 0.00000e+00i	−1.97389e+01 − 1.62396e+01i	−6.65848e+00 + 3.38878e+00i	5.20845e−11 − 4.15905e−11i	−3.96544e−57 − 4.48029e−57i	1.90057e−150 + 1.34444e−150i*	±0
0,4	$1500\angle 60^\circ$	Ob	−1.11721e+06 + 1.94406e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$1500\angle 75^\circ$	Pr	3.87479e+02 + 1.44889e+03i	−0.00000e+00 − 0.00000e+00i	−1.80725e+00 − 5.21220e+00i	−7.66737e+00 − 2.62644e+00i	−4.96618e−04 + 4.32470e−04i	3.65267e−21 − 8.56318e−21i	−4.63621e−55 − 5.37602e−55i*	−1.33246e−141 − 1.62461e−141i*
0,4	$1500\angle 90^\circ$	Ob	−2.23501e+06 + 2.74627e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$2500\angle 0^\circ$	Pr	2.24892e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−5.58353e+01 − 0.00000e+00i	−2.09915e−01 − 0.00000e+00i	−2.74142e−28 − 0.00000e+00i	−1.41424e−137 + 8.65330e−154i*	±0	±0
0,4	$2500\angle 15^\circ$	Pr	2.17226e+04 + 5.82343e+03i	−0.00000e+00 − 0.00000e+00i	−5.62190e+01 − 8.34166e+00i	2.33496e−01 + 2.28412e−01i	−2.55483e−27 + 3.21485e−27i	6.07308e−133 − 1.12662e−132i*	±0	±0
0,4	$2500\angle 30^\circ$	Ob	3.12750e+06 + 5.40833e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$2500\angle 45^\circ$	Pr	8.83508e+03 + 8.83884e+03i	−0.00000e+00 − 0.00000e+00i	−3.25161e+01 − 2.32763e+01i	3.75006e−01 − 5.03614e−01i	−1.36805e−20 + 3.01067e−20i	−1.12348e−97 − 4.19776e−98i*	6.93589e−254 − 1.66412e−255i*	±0
0,4	$2500\angle 60^\circ$	Ob	−3.11201e+06 + 5.40516e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$2500\angle 75^\circ$	Pr	6.46298e+02 + 2.41481e+03i	−0.00000e+00 − 0.00000e+00i	−3.38569e+00 − 8.42154e+00i	7.34984e−01 − 3.61739e+00i	−2.95803e−07 − 8.01808e−09i	2.70622e−36 + 1.32652e−35i*	−8.82748e−93 − 5.51713e−93i*	−8.20990e−238 − 5.59632e−238i*
0,4	$2500\angle 90^\circ$	Ob	−6.22501e+06 + 7.63873e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$5000\angle 0^\circ$	Pr	4.49892e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−6.81516e+01 − 0.00000e+00i	−7.16272e−06 − 0.00000e+00i	−7.49183e−62 − 0.00000e+00i	−3.93710e−282 + 2.40985e−298i*	±0	±0
0,4	$5000\angle 15^\circ$	Pr	4.34559e+04 + 1.16469e+04i	−0.00000e+00 − 0.00000e+00i	−7.20586e+01 + 1.24229e+00i	−1.37536e−05 − 9.86415e−06i	1.43884e−59 + 8.60913e−60i	3.15914e−272 + 6.36040e−273i*	±0	±0
0,4	$5000\angle 30^\circ$	Ob	1.25050e+07 + 2.16420e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$5000\angle 45^\circ$	Pr	1.76739e+04 + 1.76777e+04i	−0.00000e+00 − 0.00000e+00i	−6.16899e+01 − 2.94563e+01i	−3.38891e−04 + 1.45888e−04i	−4.12524e−44 + 3.73293e−44i	4.36018e−200 − 5.06909e−200i*	±0	±0
0,4	$5000\angle 60^\circ$	Ob	−1.24740e+07 + 2.16356e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,4	$5000\angle 75^\circ$	Pr	1.29335e+03 + 4.82963e+03i	−0.00000e+00 − 0.00000e+00i	−8.47482e+00 − 1.53979e+01i	1.77198e−01 − 2.27252e−01i	−2.19200e−16 + 6.71437e−16i	−8.56672e−74 − 6.02811e−73i*	−1.48361e−187 + 2.88969e−188i*	±0
0,4	$5000\angle 90^\circ$	Ob	−2.49500e+07 + 3.05856e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$25\angle 0^\circ$	Pr	2.58164e+02 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.85104e+00 − 0.00000e+00i	−5.01680e−02 − 0.00000e+00i	−1.32269e+00 − 0.00000e+00i	9.15480e−01 − 0.00000e+00i	−5.29904e−01 − 0.00000e+00i	−7.67004e−04 + 4.66718e−20i*
0,5	$25\angle 15^\circ$	Pr	2.48858e+02 + 7.15098e+01i	1.87500e+00 − 1.55431e−15i	1.85190e+00 − 6.67559e−03i	−2.73678e−02 − 4.20551e−01i	−1.54282e+00 + 7.42264e−01i	1.02450e+00 − 1.48190e+00i	−3.92842e−01 + 6.70690e−01i	2.55606e−04 − 1.65534e−03i*
0,5	$25\angle 30^\circ$	Pr	1.44563e+02 + 8.76615e+01i	1.87500e+00 + 3.33067e−16i	1.86164e+00 − 8.19690e−03i	6.42734e−01 − 6.24106e−01i	−2.46565e+00 − 7.76786e−02i	2.07895e+00 + 2.29321e−01i	−4.01515e−01 − 8.11860e−02i	−6.51282e−04 + 3.69904e−05i*
0,5	$25\angle 45^\circ$	Ob	1.00961e+02 + 5.18815e+02i	1.87500e+00 + 2.22045e−16i	1.86552e+00 − 4.85594e−02i	−8.59147e−01 − 3.76888e+00i	−1.00962e+01 + 3.54157e+01i	−1.18455e+03 − 7.04305e+02i	2.54666e+04 − 2.93373e+04i	3.78784e+05 − 3.88913e+05i
0,5	$25\angle 60^\circ$	Ob	3.57328e+01 + 6.49866e+01i	1.87500e+00 + 2.22045e−16i	1.87183e+00 − 6.08704e−03i	1.52099e+00 − 5.53939e−01i	−8.85377e−01 − 1.64810e+00i	2.77892e−01 + 2.76664e+00i	−8.90074e−01 − 8.88611e−02i	−6.69677e−03 − 4.06347e−02i*
0,5	$25\angle 75^\circ$	Ob	−3.13497e+02 + 2.47571e+02i	1.87500e+00 + 1.11022e−16i	1.90461e+00 − 2.33324e−02i	5.04758e+00 − 3.70370e+00i	−7.38774e+00 − 1.04785e+02i	−9.70688e+03 + 1.63817e+03i	1.73918e+05 + 3.64920e+05i	7.99696e+06 + 5.72274e+06i
0,5	$25\angle 90^\circ$	Ob	−3.88753e+02 + 6.11785e−14i	1.87500e+00 + 1.58966e−16i	1.91175e+00 + 1.56309e−16i	6.89716e+00 − 4.43664e−16i	1.29435e+02 − 3.92886e−14i	1.48404e+04 − 1.03661e−11i	6.94738e+05 − 7.93451e−10i	2.25945e+07 − 2.83630e−08i
0,5	$50\angle 0^\circ$	Pr	5.33757e+02 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.82538e+00 − 0.00000e+00i	−1.22943e+00 − 0.00000e+00i	1.29916e+00 − 0.00000e+00i	−6.64895e−01 − 0.00000e+00i	−1.64061e−03 − 0.00000e+00i	−4.14580e−12 + 2.53331e−28i*
0,5	$50\angle 15^\circ$	Pr	5.15038e+02 + 1.42488e+02i	1.87500e+00 − 2.22045e−16i	1.82710e+00 − 1.32397e−02i	−1.26151e+00 − 4.37125e−01i	1.58675e+00 + 8.87668e−01i	−8.28614e−01 + 4.66901e−01i	2.82956e−03 − 9.54616e−04i	1.22929e−11 − 1.34207e−11i
0,5	$50\angle 30^\circ$	Pr	2.96242e+02 + 1.75072e+02i	1.87500e+00 + 5.55112e−17i	1.84746e+00 − 1.63239e−02i	−4.07918e−01 − 8.88813e−01i	−1.08953e+00 + 1.75954e+00i	−2.96564e−01 − 5.80743e−01i	6.60743e−04 − 1.09794e−03i	1.94329e−12 − 2.49075e−11i*
0,5	$50\angle 45^\circ$	Ob	2.07079e+02 + 2.28781e+03i	1.87500e+00 − 6.66134e−16i	1.85172e+00 − 2.13725e−01i	−2.20543e+01 + 9.20167e+00i	−2.58031e+03 − 1.68504e+03i	3.97258e+06 + 8.85222e+06i	2.69883e+09 − 2.34211e+10i	7.34868e+12 + 5.87405e+12i
0,5	$50\angle 60^\circ$	Pr	7.32410e+01 + 1.29921e+02i	1.87500e+00 − 2.22045e−16i	1.86831e+00 − 1.21565e−02i	1.08160e+00 − 9.80484e−01i	−2.51101e+00 − 1.98367e−02i	4.93392e−02 − 1.08161e+00i	−1.44103e−02 + 2.23666e−02i	1.01215e−07 − 1.83215e−06i*
0,5	$50\angle 75^\circ$	Ob	−1.69543e+03 + 1.12049e+03i	1.87500e+00 − 0.00000e+00i	2.03540e+00 − 1.08043e−01i	1.90465e+01 − 6.79325e+01i	−4.55008e+04 + 4.72755e+03i	1.89393e+09 − 4.15493e+08i	−4.11977e+13 + 1.61649e+13i	2.07593e+16 + 1.97645e+16i
0,5	$50\angle 90^\circ$	Ob	−2.01335e+03 + 2.75523e−13i	1.87500e+00 − 4.02027e−17i	2.06714e+00 − 7.10298e−17i	8.36366e+01 − 2.74615e−14i	6.99304e+04 − 5.51890e−11i	4.52374e+09 − 7.05295e−06i	1.56048e+14 − 3.65164e−11i	1.34346e+17 − 3.76758e+02i
0,5	$100\angle 0^\circ$	Pr	1.08401e+03 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.77451e+00 − 0.00000e+00i	−1.83670e+00 − 0.00000e+00i	7.62590e−01 − 0.00000e+00i	−1.32872e−02 − 0.00000e+00i	−7.38490e−10 − 0.00000e+00i	−8.53307e−30 + 5.22037e−46i*
0,5	$100\angle 15^\circ$	Pr	1.04654e+03 + 2.84764e+02i	1.87500e+00 − 5.55112e−17i	1.77789e+00 − 2.62135e−02i	−2.00333e+00 + 6.94638e−02i	8.53647e−01 − 1.19408e+00i	1.77216e−02 + 1.22073e−02i	1.57450e−10 + 2.35522e−09i	1.33850e−28 + 8.77623e−29i*
0,5	$100\angle 30^\circ$	Pr	5.99410e+02 + 3.50034e+02i	1.87500e+00 + 2.22045e−16i	1.81919e+00 − 3.24524e−02i	−1.77226e+00 − 7.27739e−01i	1.73680e+00 + 2.80771e−01i	6.99889e−03 + 6.41942e−03i	−2.37762e−09 + 2.96565e−10i	−3.92803e−27 + 6.40155e−27i*
0,5	$100\angle 45^\circ$	Ob	4.19238e+02 + 9.57571e+03i	1.87500e+00 + 2.31672e−15i	1.76472e+00 − 8.89241e−01i	5.77727e+02 − 5.81996e+02i	8.35610e+06 + 1.96251e+07i	−3.21992e+14 + 3.37456e+14i	−7.52263e+21 − 2.66232e+21i	−5.22585e+27 − 3.86674e+27i
0,5	$100\angle 60^\circ$	Pr	1.48245e+02 + 2.59816e+02i	1.87500e+00 − 0.00000e+00i	1.86123e+00 − 2.42598e−02i	1.41985e−01 − 1.47475e+00i	−6.00995e−02 + 2.38806e+00i	3.60778e−02 − 6.88223e−02i	1.13642e−05 + 2.82935e−07i	8.82426e−16 − 1.52515e−15i*
0,5	$100\angle 75^\circ$	Ob	−7.70749e+03 + 4.74114e+03i	1.87500e+00 + 4.44089e−16i	2.62648e+00 − 5.03558e−01i	−7.56757e+03 − 4.65844e+03i	8.01001e+09 − 1.55587e+09i	5.79756e+19 − 2.36583e+19i	2.29284e+29 − 1.56987e+29i	−2.12396e+36 + 2.21797e+36i
0,5	$100\angle 90^\circ$	Ob	−9.01317e+03 + 1.16340e−12i	1.87500e+00 − 7.61120e−17i	2.78563e+00 − 2.39293e−16i	1.24915e+04 − 8.16063e−12i	1.91139e+10 − 3.00563e−05i	3.43391e+20 − 1.06632e+06i	3.55726e+30 − 1.65135e+16i	−5.92207e+37 + 3.69450e+23i
0,5	$250\angle 0^\circ$	Pr	2.73416e+03 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.62490e+00 − 0.00000e+00i	5.26956e−01 − 0.00000e+00i	−3.36803e−01 − 0.00000e+00i	−5.28362e−10 − 0.00000e+00i	−1.15317e−30 − 0.00000e+00i	−1.59977e−84 + 9.79243e−101i*
0,5	$250\angle 15^\circ$	Pr	2.64046e+03 + 7.11777e+02i	1.87500e+00 − 5.55112e−17i	1.63288e+00 − 6.36990e−02i	5.91963e−01 + 1.08631e+00i	−2.59385e−01 + 3.93925e−01i	−3.11478e−11 + 1.66556e−09i	1.30036e−29 + 1.60882e−29i	4.77808e−82 − 2.36491e−81i*
0,5	$250\angle 30^\circ$	Pr	1.50877e+03 + 8.75013e+02i	1.87500e+00 − 2.77556e−16i	1.73505e+00 − 7.97463e−02i	−1.83187e+00 + 1.20521e+00i	−2.45501e−01 − 1.05087e−01i	−1.82197e−09 − 8.91641e−11i	−1.14262e−27 + 5.01485e−28i	1.48299e−74 + 8.28445e−75i*
0,5	$250\angle 45^\circ$	Ob	1.05565e+03 + 6.14393e+04i	1.87500e+00 − 2.22045e−16i	−1.07547e+00 − 5.06454e+00i	1.28920e+07 + 3.05449e+07i	6.85375e+18 − 1.48452e+18i	4.59643e+37 − 2.09725e+37i	2.28854e+56 − 1.73502e+56i	1.65423e+73 + 4.88266e+72i
0,5	$250\angle 60^\circ$	Pr	3.73248e+02 + 6.49522e+02i	1.87500e+00 − 2.22045e−16i	1.83983e+00 − 6.02672e−02i	−2.05524e+00 − 8.06661e−01i	−5.01161e−01 − 2.83901e−01i	8.05734e−06 + 2.77760e−06i	2.68739e−16 + 3.32200e−18i	4.22533e−43 + 2.04486e−43i*
0,5	$250\angle 75^\circ$	Ob	−5.17248e+04 + 3.06029e+04i	1.87500e+00 + 3.33067e−15i	8.02889e+00 − 5.96077e+00i	1.71785e+10 − 3.26652e+09i	−3.95606e+25 + 2.00939e+25i	1.09877e+51 − 1.50857e+51i	−7.13089e+75 + 4.64948e+76i	−2.19254e+98 + 9.50022e+97i
0,5	$250\angle 90^\circ$	Ob	−6.00131e+04 + 7.50096e−12i	1.87500e+00 + 2.04892e−16i	1.09431e+01 − 4.54739e−16i	4.09843e+10 − 5.82665e−05i	3.73153e+26 − 1.38744e+12i	1.31969e+53 − 9.95827e+38i	2.79428e+79 − 3.17852e+65i	−7.58207e+101 + 1.12245e+88i
0,5	$500\angle 0^\circ$	Pr	5.48420e+03 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.38514e+00 − 0.00000e+00i	1.44801e+00 − 0.00000e+00i	−1.54885e−03 − 0.00000e+00i	−1.31632e−23 − 0.00000e+00i	−1.67323e−66 + 1.02326e−82i*	−7.25370e−177 + 4.44080e−193i*

Table D.54: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,5	$500\angle 15^\circ$	Pr	5.29680e+03 + 1.42352e+03i	1.87500e+00 − 2.77556e−17i	1.39954e+00 − 1.21466e−01i	1.82232e+00 − 6.78010e−01i	2.69899e−03 − 2.10562e−04i	8.81681e−23 − 9.47528e−23i	−3.05283e−64 − 4.40611e−64i*	1.51126e−170 − 6.50528e−171i*
0,5	$500\angle 30^\circ$	Pr	3.02433e+03 + 1.75001e+03i	1.87500e+00 − 1.66533e−16i	1.59696e+00 − 1.54965e−01i	1.17929e+00 + 1.22701e+00i	1.09714e−03 − 6.37420e−04i	1.01368e−21 − 1.61684e−21i	−2.42085e−59 − 7.02530e−59i*	1.18507e−154 − 5.98158e−156i*
0,5	$500\angle 45^\circ$	Ob	2.11632e+03 + 2.47879e+05i	1.87500e+00 − 9.24261e−15i	−2.88534e+01 + 1.19726e+01i	−1.10122e+15 + 1.12945e+15i	9.97974e+37 − 4.51022e+37i	8.14122e+75 − 9.26077e+75i	3.17361e+113 − 1.05162e+114i	−1.62678e+147 + 1.98483e+149i
0,5	$500\angle 60^\circ$	Pr	7.48249e+02 + 1.29904e+03i	1.87500e+00 − 0.00000e+00i	1.80360e+00 − 1.19265e−01i	−1.47974e+00 + 1.99090e+00i	−2.19785e−02 + 2.35784e−03i	−8.38139e−13 + 3.62770e−13i	−1.14539e−34 + 1.93610e−34i*	−2.18517e−89 + 3.67680e−90i*
0,5	$500\angle 75^\circ$	Ob	−2.11690e+05 + 1.23706e+05i	1.87500e+00 + 1.55431e−15i	3.04897e+01 − 1.07360e+02i	5.00426e+20 − 1.96915e+20i	4.35819e+51 − 5.93383e+51i	−1.54429e+103 − 4.91407e+103i	−2.06848e+155 − 6.98080e+154i	−1.82729e+202 − 4.15915e+201i
0,5	$500\angle 90^\circ$	Ob	−2.45013e+05 + 3.03100e−11i	1.87500e+00 − 4.64106e−18i	1.32350e+02 − 4.08449e−14i	2.95457e+21 − 9.05329e+06i	5.20888e+53 − 3.98829e+39i	2.57816e+107 − 3.94746e+93i	7.72699e+160 − 1.77460e+147i	3.37848e+209 − 1.02827e+196i
0,5	$750\angle 0^\circ$	Pr	8.23422e+03 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	1.15702e+00 − 0.00000e+00i	4.37427e−02 − 0.00000e+00i	−2.12428e−06 − 0.00000e+00i	−1.30083e−37 − 0.00000e+00i	−9.96883e−103 + 6.09899e−119i*	−1.36445e−269 + 8.35376e−286i*
0,5	$750\angle 15^\circ$	Pr	7.95311e+03 + 2.13527e+03i	1.87500e+00 + 1.11022e−16i	1.17641e+00 − 1.73571e−01i	−1.64666e−01 − 1.03760e+00i	−2.73426e−06 − 3.98093e−06i	−3.98915e−36 + 2.92775e−37i	2.86102e−99 + 4.94123e−99i*	3.93914e−260 + 2.48603e−260i*
0,5	$750\angle 30^\circ$	Pr	4.53988e+03 + 2.62500e+03i	1.87500e+00 + 5.55112e−17i	1.46156e+00 − 2.25784e−01i	1.47823e+00 − 4.90085e−01i	−5.74982e−07 + 3.03097e−06i	5.12226e−34 + 9.70695e−34i	2.38966e−90 − 5.94269e−91i*	3.80210e−235 − 2.63083e−235i*
0,5	$750\angle 45^\circ$	Ob	3.17698e+03 + 5.59318e+05i	1.87500e+00 − 2.05391e−14i	1.01887e+02 + 1.51859e+02i	−6.98569e+22 − 2.74095e+22i	1.37514e+57 − 1.01555e+57i	8.82655e+113 − 2.87255e+114i	−1.39669e+171 − 3.94372e+171i	−1.83360e+225 + 1.10640e+225i
0,5	$750\angle 60^\circ$	Pr	1.12325e+03 + 1.94856e+03i	1.87500e+00 + 3.33067e−16i	1.76669e+00 − 1.76996e−01i	1.00278e+00 + 1.77732e+00i	−4.78070e−04 + 4.09761e−04i	3.03360e−20 − 6.65845e−20i	−6.90097e−53 − 1.22918e−52i*	5.56617e−136 − 5.50003e−136i*
0,5	$750\angle 75^\circ$	Ob	−4.79908e+05 + 2.79309e+05i	1.87500e+00 − 6.89051e−10i	−4.51986e+02 − 1.16406e+03i	1.39836e+31 − 8.80748e+30i	−2.00876e+77 + 1.20231e+78i	−1.33664e+156 − 4.58814e+155i	4.72142e+233 − 8.76939e+233i	−1.39541e+306 + 1.30947e+306i
0,5	$750\angle 90^\circ$	Ob	−5.55013e+05 + 6.84271e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$1000\angle 0^\circ$	Pr	1.09842e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	9.40168e−01 − 0.00000e+00i	−6.23219e−01 − 0.00000e+00i	−1.90976e−09 − 0.00000e+00i	−8.89605e−52 − 0.00000e+00i	−4.14364e−139 + 2.53563e−155i*	±0
0,5	$1000\angle 15^\circ$	Pr	1.06094e+04 + 2.84702e+03i	1.87500e+00 − 2.77556e−17i	9.63204e−01 − 2.20271e−01i	−1.01003e+00 − 2.06695e−01i	−2.55563e−09 + 5.06521e−09i	6.66144e−50 + 5.37459e−50i	−1.81549e−134 − 3.83635e−134i*	±0
0,5	$1000\angle 30^\circ$	Pr	6.05542e+03 + 3.50000e+03i	1.87500e+00 + 3.33067e−16i	1.32884e+00 − 2.92324e−01i	4.62871e−01 − 9.75291e−01i	−3.52474e−09 − 4.59771e−09i	−4.95490e−46 − 2.13583e−47i	9.86569e−123 + 6.35421e−122i*	5.68687e−316 − 1.30170e−315i*
0,5	$1000\angle 45^\circ$	Ob	4.23764e+03 + 9.95757e+05i	1.87500e+00 + 1.21569e−14i	7.55586e+02 − 7.59365e+02i	−8.90963e+28 − 3.56861e+30i	1.76518e+76 − 2.00072e+76i	−9.14821e+151 − 7.26386e+152i	−1.31745e+229 − 8.93354e+228i	−2.04548e+301 − 9.19010e+300i
0,5	$1000\angle 60^\circ$	Pr	1.49825e+03 + 2.59808e+03i	1.87500e+00 − 2.22045e−16i	1.72912e+00 − 2.33459e−01i	1.47240e+00 − 8.93240e−02i	−4.17136e−06 + 1.53429e−05i	1.56804e−27 + 4.96495e−27i	7.84765e−71 + 1.11519e−72i*	−4.39022e−183 + 2.41513e−182i*
0,5	$1000\angle 75^\circ$	Ob	−8.56379e+05 + 4.97412e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$1000\angle 90^\circ$	Ob	−9.90013e+05 + 1.21852e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$1500\angle 0^\circ$	Pr	1.64842e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	5.38829e−01 − 0.00000e+00i	−3.88230e−01 − 0.00000e+00i	−8.70064e−16 − 0.00000e+00i	−2.46303e−80 − 0.00000e+00i	−4.27242e−212 + 2.61497e−228i*	±0
0,5	$1500\angle 15^\circ$	Pr	1.59220e+04 + 4.27052e+03i	1.87500e+00 + 5.55112e−17i	5.65541e−01 − 2.98452e−01i	−3.51652e−01 + 4.09641e−01i	−2.18879e−15 − 3.84961e−15i	−1.34187e−77 + 1.89446e−77i	−3.79558e−205 − 1.34730e−204i*	±0
0,5	$1500\angle 30^\circ$	Pr	9.08651e+03 + 5.25000e+03i	1.87500e+00 + 5.55112e−17i	1.07141e+00 − 4.13050e−01i	−2.61435e−01 − 1.87217e−01i	−1.03886e−14 + 9.71636e−15i	3.22827e−71 + 6.40086e−71i	8.36193e−187 − 3.11242e−185i*	±0
0,5	$1500\angle 45^\circ$	Ob	6.35896e+03 + 2.24364e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$1500\angle 60^\circ$	Pr	2.24825e+03 + 3.89711e+03i	1.87500e+00 − 0.00000e+00i	1.65211e+00 − 3.42588e−01i	−4.09285e−01 − 4.94082e−01i	6.80314e−09 + 5.14782e−09i	2.17821e−41 − 4.42762e−42i	−9.91329e−108 − 1.79650e−107i*	1.75876e−275 + 1.02730e−275i*
0,5	$1500\angle 75^\circ$	Ob	−1.93408e+06 + 1.12112e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$1500\angle 90^\circ$	Ob	−2.23501e+06 + 2.74627e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$2500\angle 0^\circ$	Pr	2.74842e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	−1.43926e−01 − 0.00000e+00i	−1.81349e−02 − 0.00000e+00i	−6.82782e−29 − 0.00000e+00i	−7.54253e−138 + 4.61504e−154i*	±0	±0
0,5	$2500\angle 15^\circ$	Pr	2.65472e+04 + 7.11753e+03i	1.87500e+00 − 2.77556e−17i	−1.21841e−01 − 4.00442e−01i	1.94031e−02 + 2.08653e−02i	−6.40983e−28 + 7.97923e−28i	3.24703e−133 − 6.00553e−133i*	±0	±0
0,5	$2500\angle 30^\circ$	Pr	1.51487e+04 + 8.75000e+03i	1.87500e+00 − 0.00000e+00i	5.88468e−01 − 6.08321e−01i	5.11124e−03 + 1.20153e−02i	2.73463e−26 − 3.67776e−26i	−8.13272e−121 − 7.11213e−122i*	1.51986e−312 − 3.69183e−312i*	±0
0,5	$2500\angle 45^\circ$	Ob	1.06016e+04 + 6.23939e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$2500\angle 60^\circ$	Pr	3.74825e+03 + 6.49519e+03i	1.87500e+00 + 1.11022e−16i	1.49130e+00 − 5.45718e−01i	8.31301e−02 − 2.98688e−02i	−2.75502e−16 − 1.79236e−15i	−2.99759e−70 − 3.56015e−72i*	1.03779e−180 + 3.17815e−182i*	±0
0,5	$2500\angle 75^\circ$	Ob	−5.38852e+06 + 3.11853e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$2500\angle 90^\circ$	Ob	−6.22501e+06 + 7.63873e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$5000\angle 0^\circ$	Pr	5.49842e+04 − 0.00000e+00i	1.87500e+00 − 0.00000e+00i	−1.26522e+00 − 0.00000e+00i	−6.72102e−07 − 0.00000e+00i	−1.88473e−62 − 0.00000e+00i	−2.10483e−282 + 1.28834e−298i*	±0	±0
0,5	$5000\angle 15^\circ$	Pr	5.31102e+04 + 1.42350e+04i	1.87500e+00 − 2.77556e−17i	−1.29721e+00 − 4.13920e−01i	−1.27678e−06 − 9.52531e−07i	3.61542e−60 + 2.17581e−60i	1.68885e−272 + 3.41120e−273i*	±0	±0
0,5	$5000\angle 30^\circ$	Pr	3.03041e+04 + 1.75000e+04i	1.87500e+00 + 1.66533e−16i	−4.36908e−01 − 8.67011e−01i	4.68286e−07 + 9.93881e−07i	−2.52101e−55 − 5.34718e−56i	2.22548e−246 − 2.68412e−246i*	±0	±0
0,5	$5000\angle 45^\circ$	Ob	2.12082e+04 + 2.49788e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$5000\angle 60^\circ$	Pr	7.49825e+03 + 1.29904e+04i	1.87500e+00 − 0.00000e+00i	1.05835e+00 − 9.66501e−01i	7.39376e−05 + 3.30862e−04i	4.37112e−33 − 2.05256e−32i	−5.78205e−143 + 9.54059e−143i*	±0	±0
0,5	$5000\angle 75^\circ$	Ob	−2.16024e+07 + 1.24871e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,5	$5000\angle 90^\circ$	Ob	−2.49500e+07 + 3.05856e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$25\angle 0^\circ$	Pr	3.01441e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	9.37308e−01 − 0.00000e+00i	5.38473e+00 − 0.00000e+00i	−5.00320e+00 − 0.00000e+00i	5.45264e+00 − 0.00000e+00i	−2.38658e+00 − 0.00000e+00i	−8.86764e−03 + 5.39151e−19i*
0,6	$25\angle 15^\circ$	Pr	2.90487e+02 + 8.46911e+01i	−0.00000e+00 − 0.00000e+00i	9.03788e−01 + 2.62097e−01i	5.54081e+00 + 5.87364e−01i	−5.56488e+00 − 2.28829e+00i	8.13292e+00 − 1.09081e+00i	−3.26625e+00 + 2.04177e+00i	9.42638e−03 − 1.73988e−02i*
0,6	$25\angle 30^\circ$	Pr	1.83626e+02 + 1.12833e+02i	−0.00000e+00 − 0.00000e+00i	5.72762e−01 + 3.50425e−01i	4.64447e+00 + 1.61449e+00i	−9.52315e−01 − 5.05248e+00i	2.32457e+00 + 7.14606e+00i	−1.58027e+00 − 1.96646e+00i	−7.23361e−03 − 4.46297e−03i*
0,6	$25\angle 45^\circ$	Ob	1.00961e+02 + 5.18815e+02i	−0.00000e+00 − 0.00000e+00i	3.29003e−01 + 1.61590e+00i	1.43023e+01 + 7.93207e+00i	−8.04328e+01 − 1.16192e+02i	4.96898e+03 − 1.79164e+03i	2.48376e+04 + 1.46815e+05i	2.28994e+05 + 2.06818e+06i
0,6	$25\angle 60^\circ$	Ob	−1.09596e+02 + 4.15850e+02i	−0.00000e+00 − 0.00000e+00i	−3.34095e−01 + 1.30434e+00i	6.31483e+00 + 1.61419e+01i	1.52361e+02 − 1.24434e+02i	−7.38941e+03 + 6.97688e+03i	1.21883e+05 − 2.32977e+05i	−1.62969e+06 − 2.46693e+06i
0,6	$25\angle 75^\circ$	Ob	−3.13497e+02 + 2.47571e+02i	−0.00000e+00 − 0.00000e+00i	−9.81640e−01 + 7.81804e−01i	−1.09047e+01 + 1.74598e+01i	1.37959e+02 + 3.21194e+02i	2.87045e+04 − 1.57752e+04i	−9.45993e+05 − 9.56121e+05i	−3.13916e+07 − 9.22436e+06i
0,6	$25\angle 90^\circ$	Ob	−3.02167e+02 + 5.49418e−14i	−0.00000e+00 − 0.00000e+00i	−9.49063e−01 + 1.22390e−16i	−1.49885e+01 + 3.08094e−15i	−2.03919e+02 + 8.86460e−14i	−1.07680e+04 + 9.15639e−12i	−6.03122e+04 + 1.02657e−10i	2.65635e+06 − 3.36611e−09i
0,6	$50\angle 0^\circ$	Pr	6.27445e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.94026e+00 − 0.00000e+00i	4.76295e+00 − 0.00000e+00i	−1.88360e+00 − 0.00000e+00i	−3.14483e+00 − 0.00000e+00i	−2.05827e−02 − 0.00000e+00i	−1.09418e−10 + 6.68531e−27i*
0,6	$50\angle 15^\circ$	Pr	6.05334e+02 + 1.68460e+02i	−0.00000e+00 − 0.00000e+00i	1.87407e+00 + 5.15789e−01i	5.39412e+00 − 1.37103e+00i	−3.42236e+00 + 5.70794e+00i	−5.38074e+00 − 2.71572e−01i	3.79031e−02 − 2.36033e−04i	4.13159e−10 − 2.46808e−10i*
0,6	$50\angle 30^\circ$	Pr	3.78729e+02 + 2.25146e+02i	−0.00000e+00 − 0.00000e+00i	1.17868e+00 + 6.94568e−01i	7.00942e+00 + 2.48842e−01i	−1.09811e+01 − 1.00323e+00i	1.13913e+00 − 4.58597e+00i	1.58383e−02 − 7.45244e−03i	4.07355e−10 − 5.52499e−10i*
0,6	$50\angle 45^\circ$	Ob	2.07079e+02 + 2.28781e+03i	−0.00000e+00 − 0.00000e+00i	9.16716e−01 + 7.09748e+00i	6.39807e+01 − 1.79413e+02i	2.38393e+04 − 6.14430e+03i	−7.36178e+07 − 2.42501e+07i	1.23622e+11 + 1.42083e+11i	−7.42358e+13 + 1.17078e+13i
0,6	$50\angle 60^\circ$	Ob	−8.30279e+02 + 1.91488e+03i	−0.00000e+00 − 0.00000e+00i	−2.43744e+00 + 6.14919e+00i	3.25965e+02 + 1.77595e+01i	−8.46635e+04 + 5.38658e+04i	−1.03722e+09 + 6.62851e+08i	−6.63135e+12 + 5.06227e+12i	9.46814e+13 − 2.04406e+15i
0,6	$50\angle 75^\circ$	Ob	−1.69543e+03 + 1.12049e+03i	−0.00000e+00 − 0.00000e+00i	−5.38246e+00 + 3.70182e+00i	1.00642e+01 + 5.30033e+02i	3.17078e+05 − 1.32448e+05i	−1.27249e+10 + 7.08633e+09i	2.61379e+14 − 2.05412e+14i	−1.92091e+17 − 9.72616e+16i
0,6	$50\angle 90^\circ$	Ob	−1.82596e+03 + 2.63234e−13i	−0.00000e+00 − 0.00000e+00i	−5.88170e+00 + 1.20886e−15i	−4.80147e+02 + 2.09956e−13i	−2.87264e+05 + 2.58464e−10i	−9.65043e+09 + 1.61634e−05i	−1.13629e+14 + 2.81585e−01i	5.13684e+15 − 1.23000e+01i
0,6	$100\angle 0^\circ$	Pr	1.27787e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	3.90822e+00 − 0.00000e+00i	−4.14985e+00 − 0.00000e+00i	9.47868e+00 − 0.00000e+00i	−1.94945e−01 − 0.00000e+00i	−2.04893e−08 − 0.00000e+00i	−4.73222e−28 + 2.89502e−44i*
0,6	$100\angle 15^\circ$	Pr	1.23359e+03 + 3.36568e+02i	−0.00000e+00 − 0.00000e+00i	3.78148e+00 + 1.00853e+00i	−3.62975e+00 − 5.40409e+00i	1.35662e+01 − 2.11353e+00i	1.91555e−01 + 2.56712e−01i	−1.42168e−08 + 6.41749e−08i	5.84342e−27 + 6.70205e−27i*
0,6	$100\angle 30^\circ$	Pr	7.68559e+02 + 4.50069e+02i	−0.00000e+00 − 0.00000e+00i	2.38124e+00 + 1.36988e+00i	5.22008e+00 − 5.97934e+00i	3.37763e+00 + 1.18005e+01i	3.55506e−02 + 1.49068e−01i	−6.25756e−08 − 2.85997e−08i	−3.75548e−25 + 1.96187e−25i*
0,6	$100\angle 45^\circ$	Ob	4.19238e+02 + 9.57571e+03i	−0.00000e+00 − 0.00000e+00i	6.04195e+00 + 2.92815e+01i	3.42701e+02 + 1.33765e+04i	−3.25620e+08 − 1.22911e+08i	−3.45224e+14 − 7.60294e+15i	1.16255e+23 − 5.86414e+22i	1.04541e+29 − 1.79780e+28i
0,6	$100\angle 60^\circ$	Ob	−4.14712e+03 + 8.16017e+03i	−0.00000e+00 − 0.00000e+00i	−1.01895e+01 + 2.89914e+01i	−3.89168e+04 − 3.48102e+04i	−9.45084e+09 + 5.07399e+09i	−6.14396e+18 + 2.90885e+18i	−2.28901e+27 + 1.03363e+27i	4.19841e+33 + 3.55105e+32i
0,6	$100\angle 75^\circ$	Ob	−7.70749e+03 + 4.74114e+03i	−0.00000e+00 − 0.00000e+00i	−2.59922e+01 + 1.88194e+01i	1.35549e+05 + 3.83956e+04i	−1.15473e+11 + 5.83108e+10i	−7.82267e+20 + 6.11160e+20i	−2.81995e+30 + 3.38451e+30i	2.28258e+37 − 4.30026e+37i
0,6	$100\angle 90^\circ$	Ob	−8.62546e+03 + 1.13889e−12i	−0.00000e+00 − 0.00000e+00i	−3.10019e+01 + 6.82243e−15i	−1.57147e+05 + 1.17738e−10i	−1.74342e+11 + 2.91005e−04i	−1.68910e+21 + 5.41183e+06i	−6.86654e+30 + 3.25989e+16i	−1.76607e+37 + 1.07136e+23i
0,6	$250\angle 0^\circ$	Pr	3.22810e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	9.54891e+00 − 0.00000e+00i	−1.21231e+01 − 0.00000e+00i	−4.86038e+00 − 0.00000e+00i	−2.20661e−08 − 0.00000e+00i	−8.41354e−29 − 0.00000e+00i	−2.27545e−82 + 1.39283e−98i*
0,6	$250\angle 15^\circ$	Pr	3.11737e+03 + 8.41201e+02i	−0.00000e+00 − 0.00000e+00i	9.27440e+00 + 2.35925e+00i	−1.63454e+01 + 6.48397e+00i	−6.10304e+00 + 3.88568e+00i	−2.05741e−08 + 6.66745e−08i	6.01967e−28 + 1.38595e−27i	1.54171e−79 − 3.06851e−79i*
0,6	$250\angle 30^\circ$	Pr	1.93776e+03 + 1.12503e+03i	−0.00000e+00 − 0.00000e+00i	5.92318e+00 + 3.28672e+00i	−1.98341e+01 − 1.00250e+01i	−2.37223e+00 − 4.20792e+00i	−6.48103e−08 − 4.40169e−08i	−9.14963e−26 − 1.11211e−26i	1.23352e−72 + 2.09627e−72i*
0,6	$250\angle 45^\circ$	Ob	1.05565e+03 + 6.14393e+04i	−0.00000e+00 − 0.00000e+00i	1.88030e+02 + 1.26261e+02i	−1.27335e+09 − 5.04780e+08i	−1.54756e+20 + 2.44933e+20i	−7.13267e+38 + 1.96169e+39i	−1.51593e+57 + 1.17679e+58i	−6.22954e+74 + 3.45992e+74i
0,6	$250\angle 60^\circ$	Ob	−2.90980e+04 + 5.28766e+04i	−0.00000e+00 − 0.00000e+00i	7.08115e+01 + 3.14015e+02i	−5.25767e+10 + 2.66204e+10i	−3.29089e+24 + 1.28133e+24i	−2.82473e+47 + 6.06006e+46i	−1.41694e+70 + 8.91257e+68i	−2.49565e+89 + 1.62969e+89i
0,6	$250\angle 75^\circ$	Ob	−5.17248e+04 + 3.06029e+04i	−0.00000e+00 − 0.00000e+00i	−2.45437e+02 + 3.24174e+02i	−6.41763e+11 + 3.14006e+11i	1.34234e+27 − 1.21856e+27i	−2.70313e+52 + 7.13024e+52i	−2.20436e+77 − 1.90924e+78i	7.61602e+99 − 6.10868e+99i
0,6	$250\angle 90^\circ$	Ob	−5.90252e+04 + 7.43972e−12i	−0.00000e+00 − 0.00000e+00i	−4.27666e+02 + 5.52452e−14i	−1.35435e+12 + 2.03801e−03i	−9.00368e+27 + 3.42279e+13i	−1.74945e+54 + 1.33483e+40i	−1.53884e+80 + 1.76370e+66i	1.32290e+101 − 2.01676e+87i
0,6	$500\angle 0^\circ$	Pr	6.47818e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.81096e+01 − 0.00000e+00i	2.10354e+01 − 0.00000e+00i	−5.70036e−02 − 0.00000e+00i	−1.13859e−21 − 0.00000e+00i	−2.47968e−64 + 1.51642e−80i*	−2.08063e−174 + 1.27378e−190i*

Table D.54: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
0,6	$500\angle 15^\circ$	Pr	6.25670e+03 + 1.68234e+03i	−0.00000e+00 − 0.00000e+00i	1.76933e+01 + 4.20145e+00i	2.77576e+01 + 1.06543e+01i	9.79115e−02 + 2.27015e−02i	9.55043e−21 − 5.86802e−21i	−2.65170e−62 − 7.49351e−62i*	4.67330e−168 − 6.70535e−169i*
0,6	$500\angle 30^\circ$	Pr	3.88634e+03 + 2.25001e+03i	−0.00000e+00 − 0.00000e+00i	1.16101e+01 + 6.12580e+00i	−1.32963e+01 + 2.52203e+01i	4.97578e−02 + 2.39039e−03i	1.48634e−19 − 7.65752e−20i	2.16988e−57 − 1.08647e−56i*	3.03515e−152 + 1.56498e−152i*
0,6	$500\angle 45^\circ$	Ob	2.11632e+03 + 2.47879e+05i	−0.00000e+00 − 0.00000e+00i	9.75947e+02 − 2.39791e+03i	−2.21585e+15 − 1.30879e+17i	−3.17296e+39 + 8.51577e+39i	7.00512e+76 + 1.02079e+78i	4.34263e+115 + 8.01419e+115i	−1.16008e+151 − 1.16922e+151i
0,6	$500\angle 60^\circ$	Ob	−1.20683e+05 + 2.14006e+05i	−0.00000e+00 − 0.00000e+00i	5.55933e+03 + 6.31202e+02i	−2.77530e+20 + 1.16198e+20i	−2.25070e+48 + 4.52650e+47i	−6.01286e+94 − 7.88935e+93i	−8.73149e+140 − 4.25985e+140i	5.93455e+182 − 3.74935e+182i
0,6	$500\angle 75^\circ$	Ob	−2.11690e+05 + 1.23706e+05i	−0.00000e+00 − 0.00000e+00i	−1.12460e+02 + 9.21087e+03i	−3.56159e+22 + 2.64796e+22i	−2.19180e+53 + 5.66680e+53i	2.29006e+105 + 3.58138e+105i	1.79829e+157 + 1.09932e+156i	1.54529e+204 − 6.28132e+202i
0,6	$500\angle 90^\circ$	Ob	−2.43025e+05 + 3.01875e−11i	−0.00000e+00 − 0.00000e+00i	−1.06565e+04 + 4.21931e−12i	−1.98370e+23 + 6.25161e+08i	−2.55918e+55 + 1.98187e+41i	−6.99888e+108 + 1.07775e+95i	−8.85596e+161 + 2.04174e+148i	1.90880e+207 − 4.22913e+193i
0,6	$750\angle 0^\circ$	Pr	9.72820e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.56682e+01 − 0.00000e+00i	1.55218e+01 − 0.00000e+00i	−1.23703e−04 − 0.00000e+00i	−1.70643e−35 − 0.00000e+00i	−2.22739e−100 + 1.36273e−116i*	−5.88684e−267 + 3.60418e−283i*
0,6	$750\angle 15^\circ$	Pr	9.39598e+03 + 2.52350e+03i	−0.00000e+00 − 0.00000e+00i	2.52275e+01 + 5.56527e+00i	2.19507e+01 − 1.82845e+01i	−8.74470e−05 − 2.68568e−04i	−5.15249e−34 − 1.01302e−34i	3.28584e−97 + 1.23325e−96i*	1.36220e−257 + 1.47822e−257i*
0,6	$750\angle 30^\circ$	Pr	5.83491e+03 + 3.37501e+03i	−0.00000e+00 − 0.00000e+00i	1.70273e+01 + 8.53749e+00i	2.40764e+01 + 1.85740e+01i	−1.26084e−04 + 1.37404e−04i	−6.61308e−33 + 1.45031e−31i	5.30380e−88 + 1.54481e−88i*	1.99283e−232 − 1.59323e−233i*
0,6	$750\angle 45^\circ$	Ob	3.17698e+03 + 5.59318e+05i	−0.00000e+00 − 0.00000e+00i	−2.23775e+04 − 4.34207e+03i	8.56228e+24 − 3.76559e+24i	−3.10953e+58 + 2.10801e+59i	1.76327e+116 + 3.30480e+116i	4.71333e+173 + 2.23154e+173i	6.33579e+226 − 2.59310e+227i
0,6	$750\angle 60^\circ$	Ob	−2.74768e+05 + 4.83389e+05i	−0.00000e+00 − 0.00000e+00i	3.09412e+04 − 6.65176e+04i	−1.08345e+30 + 3.68978e+29i	−1.11013e+72 + 3.36256e+70i	−8.43657e+141 − 4.23850e+141i	−2.80914e+211 − 3.99367e+211i	−1.89602e+275 + 1.62880e+276i
0,6	$750\angle 75^\circ$	Ob	−4.79908e+05 + 2.79309e+05i	−0.00000e+00 − 0.00000e+00i	9.18563e+04 + 1.24955e+05i	−1.39177e+33 + 1.50848e+33i	−1.48118e+79 − 1.50663e+80i	1.75115e+158 + 1.17706e+157i	−2.82413e+235 + 1.20425e+236i	±∞
0,6	$750\angle 90^\circ$	Ob	−5.52025e+05 + 6.82434e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1000\angle 0^\circ$	Pr	1.29782e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	3.22780e+01 − 0.00000e+00i	−4.33554e+00 − 0.00000e+00i	−1.51858e−07 − 0.00000e+00i	−1.56437e−49 − 0.00000e+00i	−1.23762e−136 + 7.57341e−153i*	±0
0,6	$1000\angle 15^\circ$	Pr	1.25353e+04 + 3.36466e+03i	−0.00000e+00 − 0.00000e+00i	3.19156e+01 + 6.48741e+00i	−1.00057e+01 − 1.96511e+01i	−3.07314e−07 + 3.31998e−07i	8.82341e−48 + 1.22045e−47i	−2.24809e−132 − 1.24796e−131i*	±0
0,6	$1000\angle 30^\circ$	Pr	7.78347e+03 + 4.50001e+03i	−0.00000e+00 − 0.00000e+00i	2.21755e+01 + 1.05414e+01i	2.69630e+01 − 9.40728e+00i	−5.08587e−08 − 4.70168e−07i	−7.37707e−44 − 4.75216e−44i	−7.00567e−120 + 1.79448e−119i*	6.60397e−313 − 4.85461e−313i*
0,6	$1000\angle 45^\circ$	Ob	4.23764e+03 + 9.95757e+05i	−0.00000e+00 − 0.00000e+00i	8.23843e+02 + 1.78159e+05i	4.31021e+32 + 4.08279e+32i	2.86419e+77 + 4.42816e+78i	9.63411e+154 + 7.44607e+154i	2.59887e+231 − 5.04280e+230i	3.48346e+303 − 1.33216e+303i
0,6	$1000\angle 60^\circ$	Ob	−4.91353e+05 + 8.61025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1000\angle 75^\circ$	Ob	−8.56379e+05 + 4.97412e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1000\angle 90^\circ$	Ob	−9.86025e+05 + 1.21607e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1500\angle 0^\circ$	Pr	1.94782e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	4.28538e+01 − 0.00000e+00i	−1.29198e+01 − 0.00000e+00i	−1.06114e−13 − 0.00000e+00i	−6.53141e−78 − 0.00000e+00i	−1.91899e−209 + 1.17453e−225i*	±0
0,6	$1500\angle 15^\circ$	Pr	1.88138e+04 + 5.04698e+03i	−0.00000e+00 − 0.00000e+00i	4.29028e+01 + 7.14538e+00i	−1.76292e+01 + 7.93428e+00i	−1.30737e−13 − 5.24941e−13i	−4.74976e−75 + 3.92007e−75i	−7.22241e−204 − 6.28776e−202i*	±0
0,6	$1500\angle 30^\circ$	Pr	1.16806e+04 + 6.75000e+03i	−0.00000e+00 − 0.00000e+00i	3.16691e+01 + 1.34019e+01i	−3.92032e+00 − 1.31485e+01i	−1.72343e−12 + 3.74110e−13i	−1.14796e−69 + 1.90529e−68i	7.35065e−183 − 1.19297e−182i*	±0
0,6	$1500\angle 45^\circ$	Ob	6.35896e+03 + 2.24364e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1500\angle 60^\circ$	Ob	−1.11202e+06 + 1.94106e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1500\angle 75^\circ$	Ob	−1.93408e+06 + 1.12112e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$1500\angle 90^\circ$	Ob	−2.22903e+06 + 2.74260e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$2500\angle 0^\circ$	Pr	3.24782e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	5.47474e+01 − 0.00000e+00i	−1.26095e+00 − 0.00000e+00i	−1.41143e−26 − 0.00000e+00i	−3.34748e−135 + 2.04821e−151i*	±0	±0
0,6	$2500\angle 15^\circ$	Pr	3.13708e+04 + 8.41162e+03i	−0.00000e+00 − 0.00000e+00i	5.63098e+01 + 4.60093e+00i	8.30027e−01 + 1.82983e+00i	−1.71624e−25 + 1.24054e−25i	2.08584e−130 − 2.19864e−130i*	±0	±0
0,6	$2500\angle 30^\circ$	Pr	1.94748e+04 + 1.12500e+04i	−0.00000e+00 − 0.00000e+00i	4.74872e+01 + 1.50391e+01i	−1.85179e−01 + 9.75714e−01i	8.81113e−24 − 3.71801e−24i	−2.97060e−118 − 2.08966e−118i*	2.37669e−309 − 1.82787e−309i*	±0
0,6	$2500\angle 45^\circ$	Ob	1.06016e+04 + 6.23939e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$2500\angle 60^\circ$	Ob	−3.10336e+06 + 5.40016e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$2500\angle 75^\circ$	Ob	−5.38852e+06 + 3.11853e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$2500\angle 90^\circ$	Ob	−6.21503e+06 + 7.63261e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$5000\angle 0^\circ$	Pr	6.49782e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	4.54517e+01 − 0.00000e+00i	−1.03761e−04 − 0.00000e+00i	−7.88701e−60 − 0.00000e+00i	−1.87414e−279 + 1.14714e−295i*	±0	±0
0,6	$5000\angle 15^\circ$	Pr	6.27634e+04 + 1.68232e+04i	−0.00000e+00 − 0.00000e+00i	5.15911e+01 − 1.46508e+01i	−1.48446e−04 − 1.97252e−04i	1.22233e−57 + 1.27535e−57i	1.37350e−269 + 6.83766e−270i*	±0	±0
0,6	$5000\angle 30^\circ$	Pr	3.89604e+04 + 2.25000e+04i	−0.00000e+00 − 0.00000e+00i	6.94286e+01 + 1.33075e+00i	−1.95953e−05 + 1.74349e−04i	−8.02357e−53 − 7.27911e−53i	2.91575e−243 − 1.07705e−243i*	±0	±0
0,6	$5000\angle 45^\circ$	Ob	2.12082e+04 + 2.49788e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$5000\angle 60^\circ$	Ob	−1.24567e+07 + 2.16256e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$5000\angle 75^\circ$	Ob	−2.16024e+07 + 1.24871e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
0,6	$5000\angle 90^\circ$	Ob	−2.49300e+07 + 3.05733e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞

Table D.54: $S_{0n}^{(a)\prime}(c,\,\eta)$ for $n = [0\,..\,6]$.

Table D.55: $S_{1n}^{(a)\prime}(c,\,\eta)$ for $n = [1\,..\,6]$.

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,1	$25\angle 0^\circ$	Pr	2.52634e+01 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.42352e−01 − 0.00000e+00i	−2.15906e+00 − 0.00000e+00i	−2.89789e+00 − 0.00000e+00i	−5.43923e−01 − 0.00000e+00i	−7.68225e−03 − 4.25126e−19i*	−4.59844e−07 − 1.86544e−23i*
1,1	$25\angle 15^\circ$	Pr	2.44110e+01 + 6.46678e+00i	−0.00000e+00 − 0.00000e+00i	−2.33868e−01 − 6.45197e−02i	−2.10975e+00 − 5.09864e−01i	−2.97429e+00 − 1.91713e−01i	−5.05223e−01 + 3.42572e−01i	3.27836e−03 + 9.76564e−03i*	−4.75778e−07 − 8.33512e−07i*
1,1	$25\angle 30^\circ$	Pr	2.19119e+01 + 1.24930e+01i	−0.00000e+00 − 0.00000e+00i	−2.08988e−01 − 1.24675e−01i	−1.95936e+00 − 1.01000e+00i	−3.20372e+00 − 4.58704e−01i	−3.62722e−01 + 7.75363e−01i	2.13439e−02 − 1.17008e−02i*	6.60748e−06 − 5.05733e−06i*
1,1	$25\angle 45^\circ$	Pr	1.79365e+01 + 1.76680e+01i	−0.00000e+00 − 0.00000e+00i	−1.69385e−01 − 1.76389e−01i	−1.70151e+00 − 1.48651e+00i	−3.57851e+00 − 8.98806e−01i	−1.92313e−02 + 1.46577e+00i	−3.68715e−02 − 8.81319e−02i*	1.16253e−05 + 2.61546e−04i*
1,1	$25\angle 60^\circ$	Pr	1.27558e+01 + 2.16391e+01i	−0.00000e+00 − 0.00000e+00i	−1.17728e−01 − 2.16147e−01i	−1.32902e+00 − 1.91776e+00i	−4.05386e+00 − 1.65083e+00i	7.35801e−01 + 2.86208e+00i	−5.63632e−01 − 6.18748e−02i*	1.05523e−03 − 2.45933e−02i*
1,1	$25\angle 75^\circ$	Ob	−4.46714e+02 + 2.86606e+02i	−0.00000e+00 − 0.00000e+00i	4.49711e+00 − 2.90915e+00i	6.19581e+01 − 8.55200e+01i	−9.73450e+02 − 2.81797e+03i	−7.11532e+05 + 2.86235e+05i	7.39088e+07 + 1.56533e+08i	−1.14607e+10 − 1.26466e+09i
1,1	$25\angle 90^\circ$	Ob	−5.27043e+02 + 7.04144e−14i	−0.00000e+00 − 0.00000e+00i	5.32734e+00 − 1.69374e−16i	1.13804e+02 − 1.36880e−14i	3.67573e+03 − 1.27518e−12i	1.16951e+06 − 8.54132e−10i	3.26003e+08 − 3.63773e−07i	−2.51814e+10 + 3.41153e−05i
1,1	$50\angle 0^\circ$	Pr	5.02565e+01 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−4.91377e−01 − 0.00000e+00i	−3.86718e+00 − 0.00000e+00i	−2.66133e+00 − 0.00000e+00i	−3.88786e−02 − 0.00000e+00i	−3.45696e−06 − 2.02156e−22i*	−4.96947e−16 − 2.56339e−32i*
1,1	$50\angle 15^\circ$	Pr	4.85525e+01 + 1.29392e+01i	−0.00000e+00 − 0.00000e+00i	−4.74503e−01 − 1.28784e−01i	−3.82391e+00 − 7.78446e−01i	−2.78123e+00 + 4.06402e−01i	−5.04802e−03 + 4.86156e−02i	3.52944e−06 − 5.05898e−06i*	3.83752e−16 − 2.12271e−15i*
1,1	$50\angle 30^\circ$	Pr	4.35568e+01 + 2.49967e+01i	−0.00000e+00 − 0.00000e+00i	−4.24996e−01 − 2.48917e−01i	−3.68181e+00 − 1.58195e+00i	−3.18603e+00 + 8.55953e−01i	9.06138e−02 + 3.05087e−02i	1.85806e−06 + 3.36366e−05i*	7.05592e−14 + 1.43257e−13i*
1,1	$50\angle 45^\circ$	Pr	3.56098e+01 + 3.53507e+01i	−0.00000e+00 − 0.00000e+00i	−3.46139e−01 − 3.52302e−01i	−3.40491e+00 − 2.42481e+00i	−4.03182e+00 + 1.36414e+00i	1.94280e−01 − 1.98872e−01i	−7.25229e−05 − 4.96242e−04i*	5.36242e−11 − 1.40545e−10i*
1,1	$50\angle 60^\circ$	Pr	2.52530e+01 + 4.32957e+01i	−0.00000e+00 − 0.00000e+00i	−2.43184e−01 − 4.31928e−01i	−2.93731e+00 − 3.29819e+00i	−5.64202e+00 + 1.83397e+00i	−2.92591e−02 − 1.11700e+00i	−9.23983e−03 + 1.41840e−02i*	1.22940e−07 − 1.12411e−06i*
1,1	$50\angle 75^\circ$	Ob	−1.97390e+03 + 1.19823e+03i	−0.00000e+00 − 0.00000e+00i	2.01603e+01 − 1.27822e+01i	9.46848e+00 − 2.47269e+03i	−2.43299e+06 + 9.34576e+05i	2.52675e+11 − 1.25767e+11i	−2.30592e+16 + 1.44851e+16i	−5.71191e+19 − 1.22604e+20i
1,1	$50\angle 90^\circ$	Ob	−2.30202e+03 + 2.93914e−13i	−0.00000e+00 − 0.00000e+00i	2.39259e+01 − 3.70748e−15i	2.92705e+03 − 1.14904e−12i	3.98400e+06 − 3.39368e−09i	6.60403e+11 − 1.06825e−03i	9.74751e+16 − 2.32406e+02i	−4.20719e+19 − 4.12359e+04i
1,1	$100\angle 0^\circ$	Pr	1.00253e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−9.87667e−01 − 0.00000e+00i	−6.06526e+00 − 0.00000e+00i	−1.09643e+00 − 0.00000e+00i	−9.66948e−05 − 0.00000e+00i	−3.17286e−13 − 1.90055e−29i*	−2.38158e−34 − 1.34856e−50i*
1,1	$100\angle 15^\circ$	Pr	9.68456e+01 + 2.58811e+01i	−0.00000e+00 − 0.00000e+00i	−9.54254e−01 − 2.56352e−01i	−6.11599e+00 − 8.24584e−01i	−1.03666e+00 + 6.47103e−01i	1.52393e−04 − 9.43153e−06i	6.00356e−13 + 8.08177e−13i*	4.45286e−33 + 1.81008e−34i*
1,1	$100\angle 30^\circ$	Pr	8.68553e+01 + 4.99984e+01i	−0.00000e+00 − 0.00000e+00i	−8.56159e−01 − 4.95728e−01i	−6.24606e+00 − 1.77406e+00i	−8.20784e−01 + 1.46534e+00i	−5.79017e−04 − 6.54311e−05i	2.30933e−11 − 1.87222e−11i*	5.78304e−31 − 2.39099e−29i*
1,1	$100\angle 45^\circ$	Pr	7.09629e+01 + 7.07084e+01i	−0.00000e+00 − 0.00000e+00i	−6.99707e−01 − 7.02181e−01i	−6.38047e+00 − 2.97493e+00i	−3.23365e−01 + 2.76660e+00i	3.61273e−03 + 3.31912e−03i	2.69772e−09 − 5.89878e−09i*	2.00544e−23 − 4.26590e−24i*

Table D.55: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,1	$100\angle 60^\circ$	Pr	5.02515e+01 + 8.65998e+01i	−0.00000e+00 − 0.00000e+00i	−4.95064e−01 − 8.61775e−01i	−6.36805e+00 − 4.54266e+00i	6.72680e−01 + 5.34310e+00i	3.41349e−02 − 7.10310e−02i	7.24144e−06 + 1.98803e−07i*	5.78926e−16 − 9.43110e−16i*
1,1	$100\angle 75^\circ$	Ob	−8.27589e+03 + 4.89647e+03i	−0.00000e+00 − 0.00000e+00i	9.01638e+01 − 6.32331e+01i	−6.05871e+05 − 1.78329e+05i	8.38020e+11 − 4.09560e+11i	1.42105e+22 − 2.24183e+32i	2.04020e+32 − 2.06375e+22i	1.69334e+41 − 2.67072e+41i
1,1	$100\angle 90^\circ$	Ob	−9.60201e+03 + 1.20015e−12i	−0.00000e+00 − 0.00000e+00i	1.12162e+02 − 2.08377e−14i	8.87338e+05 − 6.36044e−10i	2.18472e+12 − 3.57272e−03i	9.74470e+22 − 3.08546e+08i	3.89910e+33 − 1.83168e+19i	8.62958e+42 − 5.37506e+28i
1,1	$250\angle 0^\circ$	Pr	2.50251e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.46176e+00 − 0.00000e+00i	−7.18151e+00 − 0.00000e+00i	−2.35156e−02 − 0.00000e+00i	−4.54998e−13 − 0.00000e+00i	−6.62970e−35 − 0.00000e+00i	−5.64514e−90 − 3.35554e−106i*
1,1	$250\angle 15^\circ$	Pr	2.41733e+02 + 6.47044e+01i	−0.00000e+00 − 0.00000e+00i	−2.38070e+00 − 6.31636e−01i	−7.48128e+00 + 4.62604e−01i	6.76390e−03 + 3.00720e−02i	7.47182e−13 + 1.21299e−12i	1.19567e−33 + 4.35766e−34i*	−3.36998e−87 − 9.83821e−87i*
1,1	$250\angle 30^\circ$	Pr	2.16757e+02 + 1.24999e+02i	−0.00000e+00 − 0.00000e+00i	−2.14222e+00 − 1.22328e+00i	−8.45406e+00 + 8.60726e−01i	6.50510e−02 − 2.03022e−02i	3.52425e−11 − 1.98742e−11i	3.22910e−30 − 4.54928e−30i	−1.72784e−77 + 7.70952e−79i*
1,1	$250\angle 45^\circ$	Pr	1.77028e+02 + 1.76776e+02i	−0.00000e+00 − 0.00000e+00i	−1.76036e+00 − 1.73686e+00i	−1.03249e+01 + 1.02106e+00i	−2.72419e−02 − 2.39195e−01i	5.12813e−09 − 6.52245e−09i	3.35763e−24 + 2.47020e−24i*	−1.39527e−62 + 3.37584e−63i*
1,1	$250\angle 60^\circ$	Pr	1.25251e+02 + 2.16505e+02i	−0.00000e+00 − 0.00000e+00i	−1.25809e+00 − 2.13826e+00i	−1.34497e+01 + 4.76860e−01i	−1.11842e+00 − 5.51489e−01i	8.11822e−06 + 2.67479e−06i	1.59223e−16 + 1.14761e−18i	2.17667e−43 + 1.09861e−43i*
1,1	$250\angle 75^\circ$	Ob	−5.31627e+04 + 3.09912e+04i	−0.00000e+00 − 0.00000e+00i	8.23332e+02 − 1.07892e+03i	2.81946e+12 − 1.36611e+12i	−9.45174e+27 + 8.49586e+27i	4.71755e+53 − 1.21593e+54i	1.19348e+79 + 1.20002e+80i	4.15255e+102 − 2.39428e+104i
1,1	$250\angle 90^\circ$	Ob	−6.15020e+04 + 7.59281e−12i	−0.00000e+00 − 0.00000e+00i	1.47038e+03 − 1.81451e−13i	7.34162e+12 − 1.10049e−02i	1.06874e+29 − 4.05607e+14i	9.22556e+55 − 7.03199e+41i	7.17453e+82 − 8.21451e+68i	1.08916e+108 − 1.64881e+94i
1,1	$500\angle 0^\circ$	Pr	5.00251e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−4.86966e+00 − 0.00000e+00i	−4.10836e+00 − 0.00000e+00i	−1.68035e−05 − 0.00000e+00i	−2.59244e−27 − 0.00000e+00i	−2.58604e−71 − 1.57675e−87i*	−8.02785e−183 − 4.84437e−199i*
1,1	$500\angle 15^\circ$	Pr	4.83214e+02 + 1.29409e+02i	−0.00000e+00 − 0.00000e+00i	−4.71557e+00 − 1.23284e+00i	−4.14480e+00 + 1.68673e+00i	2.19749e−05 − 1.87165e−05i	5.13395e−27 − 2.48874e−26i	−7.49535e−69 − 3.48127e−69i*	1.08603e−176 − 1.45785e−176i*
1,1	$500\angle 30^\circ$	Pr	4.33263e+02 + 2.50000e+02i	−0.00000e+00 − 0.00000e+00i	−4.26067e+00 − 2.39359e+00i	−4.28856e+00 + 3.82856e+00i	−6.00443e−05 + 1.27601e−04i	−3.59837e−25 + 2.04816e−23i	1.43288e−61 + 1.22943e−61i*	−6.78082e−158 + 4.38587e−158i*
1,1	$500\angle 45^\circ$	Pr	3.53804e+02 + 3.53553e+02i	−0.00000e+00 − 0.00000e+00i	−3.52739e+00 − 3.41206e+00i	−4.72928e+00 + 7.13946e+00i	9.27164e−04 − 1.49446e−03i	7.36726e−19 + 4.45007e−19i	−7.39854e−50 − 4.02433e−50i*	−9.92248e−129 + 3.11532e−128i*
1,1	$500\angle 60^\circ$	Pr	2.50250e+02 + 4.33012e+02i	−0.00000e+00 − 0.00000e+00i	−2.55377e+00 − 4.22241e+00i	−6.24598e+00 + 1.29700e+01i	−4.67123e−02 + 6.34646e−03i	−8.34884e−13 + 3.68127e−13i	−7.56608e−35 + 1.27744e−34i*	−1.43944e−89 + 2.34347e−90i*
1,1	$500\angle 75^\circ$	Ob	−2.14577e+05 + 1.24482e+05i	−0.00000e+00 − 0.00000e+00i	4.22439e+02 − 3.05532e+04i	1.55292e+23 − 1.15002e+23i	1.53210e+54 − 3.93297e+54i	−3.83855e+106 − 6.05341e+106i	−1.09562e+159 − 7.54089e+157i	−3.30966e+208 − 1.39420e+208i
1,1	$500\angle 90^\circ$	Ob	−2.48002e+05 + 3.04937e−11i	−0.00000e+00 − 0.00000e+00i	3.62230e+04 − 1.37025e−11i	1.06156e+24 − 3.32663e+09i	2.98618e+56 − 2.30716e+42i	3.58820e+110 − 5.51873e+96i	3.88974e+164 − 8.95977e+150i	7.55978e+215 − 2.29733e+202i
1,1	$750\angle 0^\circ$	Pr	7.50250e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.21736e+00 − 0.00000e+00i	−1.76095e+00 − 0.00000e+00i	−8.99607e−09 − 0.00000e+00i	−1.10650e−41 − 0.00000e+00i	−7.84135e−108 − 4.78778e−124i*	−7.74506e−276 − 4.69674e−292i*
1,1	$750\angle 15^\circ$	Pr	7.24695e+02 + 1.94114e+02i	−0.00000e+00 − 0.00000e+00i	−6.99795e+00 − 1.80441e+00i	−1.51676e+00 + 1.30620e+00i	−1.87987e−08 − 7.53716e−09i	−2.78017e−40 + 1.94947e−40i	3.79307e−104 + 2.11481e−104i*	2.55689e−266 + 7.58179e−268i*
1,1	$750\angle 30^\circ$	Pr	6.49769e+02 + 3.75000e+02i	−0.00000e+00 − 0.00000e+00i	−6.34789e+00 − 3.51208e+00i	−6.22306e−01 + 2.84716e+00i	−8.27592e−08 − 2.02416e−07i	−6.62777e−36 − 4.05626e−36i	−3.30370e−93 + 3.20965e−93i*	−9.95174e−239 + 2.10098e−238i*
1,1	$750\angle 45^\circ$	Pr	5.30580e+02 + 5.30330e+02i	−0.00000e+00 − 0.00000e+00i	−5.29280e+00 − 5.02640e+00i	1.57267e+00 + 5.05879e+00i	9.05798e−06 − 3.26635e−06i	−2.73625e−29 + 6.10562e−29i	1.41392e−75 + 4.31786e−76i*	4.52570e−194 + 4.58817e−194i*
1,1	$750\angle 60^\circ$	Pr	3.75250e+02 + 6.49519e+02i	−0.00000e+00 − 0.00000e+00i	−3.87837e+00 − 6.25245e+00i	6.86451e+00 + 9.27987e+00i	−9.97902e−04 + 8.88162e−04i	2.99681e−20 − 6.65742e−20i	−5.04203e−53 − 8.98741e−53i*	4.07564e−136 − 3.99891e−136i*
1,1	$750\angle 75^\circ$	Ob	−4.84244e+05 + 2.80474e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$750\angle 90^\circ$	Ob	−5.59502e+05 + 6.87027e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$1000\angle 0^\circ$	Pr	1.00025e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−9.50598e+00 − 0.00000e+00i	−6.70753e−01 − 0.00000e+00i	−4.27996e−12 − 0.00000e+00i	−4.19676e−56 − 0.00000e+00i	−1.94649e−144 − 1.18933e−160i*	±0
1,1	$1000\angle 15^\circ$	Pr	9.66176e+02 + 2.58819e+02i	−0.00000e+00 − 0.00000e+00i	−9.22863e+00 − 2.34714e+00i	−4.05986e−01 + 6.84337e−01i	1.29342e−12 + 1.25622e−11i	3.98999e−54 + 5.80082e−55i	−1.63621e−139 − 1.12007e−139i*	±0
1,1	$1000\angle 30^\circ$	Pr	8.66276e+02 + 5.00000e+02i	−0.00000e+00 − 0.00000e+00i	−8.40391e+00 − 4.57988e+00i	5.25184e−01 + 1.20337e+00i	2.82581e−10 + 1.04713e−10i	2.31142e−48 − 1.23231e−48i	−6.06266e−125 − 7.52745e−125i*	6.25685e−320 + 5.71530e−319i*
1,1	$1000\angle 45^\circ$	Pr	7.07357e+02 + 7.07107e+02i	−0.00000e+00 − 0.00000e+00i	−7.05582e+00 − 6.58072e+00i	2.70137e+00 + 1.08870e+00i	4.44216e−08 + 1.48943e−08i	−4.41652e−39 − 1.36360e−39i	−2.22247e−101 − 3.33919e−102i*	1.06781e−259 − 2.60344e−260i*
1,1	$1000\angle 60^\circ$	Pr	5.00250e+02 + 8.66025e+02i	−0.00000e+00 − 0.00000e+00i	−5.23076e+00 − 8.22844e+00i	8.13173e+00 − 1.24199e+00i	−8.37311e−06 + 3.25879e−05i	1.58036e−27 + 4.94548e−27i	6.14894e−71 + 8.82362e−73i*	−3.48324e−183 + 1.88883e−182i*
1,1	$1000\angle 75^\circ$	Ob	−8.62164e+05 + 4.98965e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$1000\angle 90^\circ$	Ob	−9.96002e+05 + 1.22220e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$1500\angle 0^\circ$	Pr	1.50025e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.39104e+01 − 0.00000e+00i	−8.20916e−02 − 0.00000e+00i	−8.17175e−19 − 0.00000e+00i	−5.09239e−85 − 0.00000e+00i	−1.01444e−217 − 6.20275e−234i*	±0
1,1	$1500\angle 15^\circ$	Pr	1.44914e+03 + 3.88229e+02i	−0.00000e+00 − 0.00000e+00i	−1.35381e+01 − 3.34914e+00i	1.19908e−02 + 1.05385e−01i	−3.63384e−18 − 1.98721e−18i	−4.09335e−83 + 4.77773e−82i	−2.39466e−210 − 2.32071e−210i*	±0
1,1	$1500\angle 30^\circ$	Pr	1.29929e+03 + 7.50000e+02i	−0.00000e+00 − 0.00000e+00i	−1.24224e+01 − 6.56782e+00i	2.23814e−01 − 2.10960e−02i	1.28855e−16 − 4.65258e−16i	−2.10851e−73 − 1.36316e−73i	1.64859e−188 + 3.07530e−188i*	±0
1,1	$1500\angle 45^\circ$	Pr	1.06091e+03 + 1.06066e+03i	−0.00000e+00 − 0.00000e+00i	−1.05716e+01 − 9.51260e+00i	1.34061e−01 − 7.30432e−01i	−8.60739e−14 + 9.31798e−13i	1.85803e−59 + 1.04104e−60i	−3.83581e−153 + 6.60268e−154i*	±0
1,1	$1500\angle 60^\circ$	Pr	7.50250e+02 + 1.29904e+03i	−0.00000e+00 − 0.00000e+00i	−8.01435e+00 − 1.20185e+01i	−2.40631e+00 − 2.57530e+00i	1.44590e−08 + 1.07313e−08i	2.16960e−41 − 4.46118e−42i	−8.64720e−108 − 1.56682e−107i*	1.53115e−275 + 8.97530e−276i*
1,1	$1500\angle 75^\circ$	Ob	−1.94276e+06 + 1.12345e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$1500\angle 90^\circ$	Ob	−2.24400e+06 + 2.75178e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$2500\angle 0^\circ$	Pr	2.50025e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.20577e+01 − 0.00000e+00i	−9.10555e−04 − 0.00000e+00i	−2.20595e−32 − 0.00000e+00i	−5.94682e−143 − 3.63703e−159i*	±0	±0
1,1	$2500\angle 15^\circ$	Pr	2.41506e+03 + 6.47048e+02i	−0.00000e+00 − 0.00000e+00i	−2.15711e+01 − 5.03990e+00i	1.37773e−03 + 2.22483e−04i	−4.46534e−32 + 3.26850e−31i	−1.76450e−139 − 5.54324e−138i*	±0	±0
1,1	$2500\angle 30^\circ$	Pr	2.16531e+03 + 1.25000e+03i	−0.00000e+00 − 0.00000e+00i	−2.00870e+01 − 9.98330e+00i	−4.18288e−03 − 2.51435e−03i	−9.24730e−29 + 9.12834e−28i	1.64353e−123 − 7.72950e−124i*	4.18957e−316 + 3.09238e−315i*	±0
1,1	$2500\angle 45^\circ$	Pr	1.76802e+03 + 1.76777e+03i	−0.00000e+00 − 0.00000e+00i	−1.75422e+01 − 1.46924e+01i	7.85387e−03 + 3.48840e−02i	−1.00089e−22 − 2.57692e−22i	2.15388e−100 − 9.85990e−101i*	−5.60348e−257 + 5.88439e−257i*	±0
1,1	$2500\angle 60^\circ$	Pr	1.25025e+03 + 2.16506e+03i	−0.00000e+00 − 0.00000e+00i	−1.38656e+01 − 1.89535e+01i	4.44982e−01 − 1.77876e−01i	−6.01212e−16 − 3.77171e−15i	−2.98755e−70 − 3.49313e−72i	1.03445e−180 + 3.29270e−182i*	±0
1,1	$2500\angle 75^\circ$	Ob	−5.40300e+06 + 3.12241e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$2500\angle 90^\circ$	Ob	−6.24000e+06 + 7.64792e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$5000\angle 0^\circ$	Pr	5.00025e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.89374e+01 − 0.00000e+00i	−6.57780e−09 − 0.00000e+00i	−1.47248e−66 − 0.00000e+00i	−5.06629e−288 − 3.10031e−304i*	±0	±0
1,1	$5000\angle 15^\circ$	Pr	4.82988e+03 + 1.29410e+03i	−0.00000e+00 − 0.00000e+00i	−3.85101e+01 − 7.69146e+00i	−1.54252e−08 − 9.00154e−10i	3.29274e−64 + 3.17199e−66i	3.92598e−278 − 1.32801e−278i*	±0	±0
1,1	$5000\angle 30^\circ$	Pr	4.33038e+03 + 2.50000e+03i	−0.00000e+00 − 0.00000e+00i	−3.71068e+01 − 1.56304e+01i	−1.60229e−07 − 1.00151e−07i	2.41627e−57 − 8.05865e−58i	−7.67553e−250 + 4.64129e−249i*	±0	±0
1,1	$5000\angle 45^\circ$	Pr	3.53578e+03 + 3.53553e+03i	−0.00000e+00 − 0.00000e+00i	−3.43717e+01 − 2.39578e+01i	3.40316e−06 − 9.55434e−06i	1.01994e−47 − 2.30992e−46i	5.76436e−204 + 7.81270e−203i*	±0	±0
1,1	$5000\angle 60^\circ$	Pr	2.50025e+03 + 4.33013e+03i	−0.00000e+00 − 0.00000e+00i	−2.97525e+01 − 3.25857e+01i	4.28297e−04 + 1.77045e−03i	9.07194e−33 − 4.31869e−32i	−6.87635e−143 + 1.16880e−142i*	±0	±0
1,1	$5000\angle 75^\circ$	Ob	−2.16313e+07 + 1.24948e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,1	$5000\angle 90^\circ$	Ob	−2.49800e+07 + 3.06039e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$25\angle 0^\circ$	Pr	7.42764e+01 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.98932e+00 − 0.00000e+00i	2.02739e+00 − 0.00000e+00i	−7.50659e−01 − 0.00000e+00i	−7.30052e−01 − 0.00000e+00i	−1.78434e−02 − 9.77105e−19i*	−2.17841e−06 − 8.62281e−23i*
1,2	$25\angle 15^\circ$	Pr	7.17195e+01 + 1.94035e+01i	3.00000e+00 − 1.11022e−16i	2.98970e+00 − 2.90510e−03i	2.05226e+00 − 2.40510e−01i	−8.00400e−01 − 4.51674e−01i	−7.07483e−01 + 4.27352e−01i	7.03792e−03 + 2.29955e−02i*	−2.21992e−06 − 3.97437e−06i*
1,2	$25\angle 30^\circ$	Pr	6.42230e+01 + 3.74853e+01i	3.00000e+00 + 4.44089e−16i	2.99082e+00 − 5.61347e−03i	2.12744e+00 − 4.74199e−01i	−9.46149e−01 − 9.76398e−01i	−6.06625e−01 + 1.02156e+00i	5.36949e−02 − 2.61276e−02i*	3.30535e−05 − 2.43662e−05i*
1,2	$25\angle 45^\circ$	Pr	5.22985e+01 + 5.30134e+01i	3.00000e+00 − 0.00000e+00i	2.99260e+00 − 7.94146e−03i	2.25420e+00 − 6.92805e−01i	−1.16995e+00 − 1.66421e+00i	−3.06652e−01 + 2.08752e+00i	−7.80686e−02 − 2.25911e−01i*	2.08273e−05 + 1.30336e−03i*
1,2	$25\angle 60^\circ$	Ob	−2.27933e+02 + 4.91244e+02i	3.00000e+00 − 2.66454e−15i	3.03440e+00 − 7.39721e−02i	3.45549e+00 − 9.99815e+00i	−2.07953e+02 − 3.23508e+00i	2.86806e+04 + 7.55759e+02i	−3.48611e+06 − 7.17205e+04i	−5.89065e+07 − 1.32995e+08i
1,2	$25\angle 75^\circ$	Ob	−4.46714e+02 + 2.86606e+02i	3.00000e+00 + 1.11022e−16i	3.06761e+00 − 4.33151e−02i	1.11395e+01 − 8.21773e+00i	−1.83673e+01 − 3.87807e+02i	−9.94058e+04 + 9.69116e+03i	3.49893e+06 + 2.22696e+07i	−1.38581e+09 − 5.78070e+08i
1,2	$25\angle 90^\circ$	Ob	−5.27043e+02 + 7.04144e−14i	3.00000e+00 + 2.15362e−16i	3.07986e+00 + 2.10439e−16i	1.51751e+01 − 1.18727e−15i	4.80385e+02 − 1.49411e−13i	1.52841e+05 − 1.06141e−10i	4.26048e+07 − 4.60122e−08i	−3.29091e+09 + 4.34039e−06i
1,2	$50\angle 0^\circ$	Pr	1.49262e+02 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.97810e+00 − 0.00000e+00i	1.19840e+00 − 0.00000e+00i	−1.36912e+00 − 0.00000e+00i	−5.74359e−02 − 0.00000e+00i	−8.71912e−06 − 5.08778e−22i*	−2.47815e−15 − 1.27278e−31i*
1,2	$50\angle 15^\circ$	Pr	1.44151e+02 + 3.88195e+01i	3.00000e+00 − 1.66533e−16i	2.97886e+00 − 5.80037e−03i	1.22666e+00 − 3.88509e−01i	−1.48177e+00 + 3.18019e−02i	−9.14931e−03 + 7.18747e−02i	9.05353e−06 − 1.26855e−05i*	1.95001e−15 − 1.05863e−14i*
1,2	$50\angle 30^\circ$	Pr	1.29164e+02 + 7.49936e+01i	3.00000e+00 − 0.00000e+00i	2.98110e+00 − 1.12101e−02i	1.31564e+00 − 7.81011e−01i	−1.85646e+00 + 7.09520e−02i	1.33511e−01 + 5.16155e−02i	2.99222e−06 + 8.54791e−05i*	3.48472e−13 + 7.18405e−13i*
1,2	$50\angle 45^\circ$	Pr	1.05324e+02 + 1.06057e+02i	3.00000e+00 − 2.22045e−16i	2.98465e+00 − 1.58641e−02i	1.47750e+00 − 1.17730e+00i	−2.61804e+00 + 9.39009e−02i	3.13227e−01 − 2.83677e−01i	−1.50500e−04 − 1.27385e−03i*	2.74232e−10 − 7.02660e−10i*
1,2	$50\angle 60^\circ$	Ob	−1.07881e+03 + 2.06505e+03i	3.00000e+00 + 8.88178e−16i	3.15836e+00 − 3.15346e−01i	−7.50669e+01 − 5.60410e+01i	4.36557e+04 + 1.52073e+03i	1.44813e+09 + 9.13324e+07i	4.01012e+13 + 3.45181e+12i	7.01155e+16 − 9.57123e+16i
1,2	$50\angle 75^\circ$	Ob	−1.97390e+03 + 1.19823e+03i	3.00000e+00 − 2.77556e−16i	3.29963e+00 − 1.85714e−01i	4.21692e+01 − 1.48477e+02i	−1.61997e+05 + 1.52649e+04i	1.73069e+10 − 3.31132e+09i	−1.63002e+15 + 4.83010e+14i	−1.37188e+18 − 8.33204e+18i
1,2	$50\angle 90^\circ$	Ob	−2.30202e+03 + 2.93914e−13i	3.00000e+00 − 9.26673e−17i	3.35246e+00 − 1.49356e−16i	1.83046e+02 − 6.16892e−14i	2.49109e+05 − 1.98382e−10i	4.12933e+10 − 6.45048e−05i	6.09487e+15 − 1.41937e+01i	−2.63065e+18 − 2.72427e+03i
1,2	$100\angle 0^\circ$	Pr	2.99256e+02 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.95574e+00 − 0.00000e+00i	1.38061e−02 − 0.00000e+00i	−7.00880e−01 − 0.00000e+00i	−1.49209e−04 − 0.00000e+00i	−8.16205e−13 − 4.88668e−29i*	−1.19423e−33 − 6.75626e−50i*
1,2	$100\angle 15^\circ$	Pr	2.89033e+02 + 7.76441e+01i	3.00000e+00 − 5.55112e−17i	2.95725e+00 − 1.15549e−02i	1.44985e−02 − 4.89872e−01i	−6.87835e−01 + 3.83219e−01i	2.35659e−04 − 1.20054e−05i	1.53545e−12 + 2.08788e−12i*	2.23324e−32 + 9.33750e−34i*
1,2	$100\angle 30^\circ$	Pr	2.59063e+02 + 1.49997e+02i	3.00000e+00 − 1.11022e−16i	2.96168e+00 − 2.23409e−02i	2.56857e−02 − 1.02106e+00i	−6.29007e−01 + 9.10627e−01i	−8.96485e−04 − 1.20374e−04i	6.00107e−11 − 4.77444e−11i*	3.15911e−30 − 1.19989e−28i*
1,2	$100\angle 45^\circ$	Pr	2.11386e+02 + 2.12128e+02i	3.00000e+00 + 2.22045e−16i	2.96874e+00 − 3.16366e−02i	7.73424e−02 − 1.63350e+00i	−4.59691e−01 + 1.84177e+00i	5.49244e−03 + 5.35154e−03i	7.17498e−09 − 1.51734e−08i*	1.00796e−22 − 2.11460e−23i*

Table D.55: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,2	$100\angle 60^\circ$	Ob	−4.65560e+03 + 8.46025e+03i	3.00000e+00 − 1.55431e−15i	3.63269e+00 − 1.36733e+00i	2.01859e+03 + 6.82288e+03i	2.34322e+09 + 1.55000e+08i	3.64707e+18 + 4.80744e+17i	5.07899e+27 + 9.89858e+26i	5.37775e+35 + 1.95062e+35i
1,2	$100\angle 75^\circ$	Ob	−8.27589e+03 + 4.89647e+03i	3.00000e+00 + 1.11022e−15i	4.29743e+00 − 8.39601e−01i	−1.64381e+04 − 1.01549e+04i	2.80359e+10 − 5.31789e+09i	5.05231e+20 − 1.99136e+20i	7.83068e+30 − 4.96765e+30i	7.15351e+39 − 6.51335e+39i
1,2	$100\angle 90^\circ$	Ob	−9.60201e+03 + 1.20015e−12i	3.00000e+00 − 1.46945e−16i	4.55988e+00 − 4.33615e−16i	2.71662e+04 − 1.79653e−11i	6.68862e+10 − 1.05669e−04i	2.98338e+21 − 9.28072e+06i	1.19373e+32 − 5.54154e+17i	2.64198e+41 − 1.63094e+27i
1,2	$250\angle 0^\circ$	Pr	7.49252e+02 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.88921e+00 − 0.00000e+00i	−1.29331e+00 − 0.00000e+00i	−1.67710e−02 − 0.00000e+00i	−7.19804e−13 − 0.00000e+00i	−1.78318e−34 − 0.00000e+00i	−2.92434e−89 − 1.73804e−105i*
1,2	$250\angle 15^\circ$	Pr	7.23697e+02 + 1.94114e+02i	3.00000e+00 + 1.66533e−16i	2.89289e+00 − 2.85401e−02i	−1.39282e+00 − 1.49071e−01i	4.45346e−03 + 2.15833e−02i	1.17460e−12 + 1.92501e−12i	3.21576e−33 + 1.17839e−33i*	−1.75114e−86 − 5.11712e−86i*
1,2	$250\angle 30^\circ$	Pr	6.48771e+02 + 3.74999e+02i	3.00000e+00 − 0.00000e+00i	2.90371e+00 − 5.52498e−02i	−1.71232e+00 − 3.42142e−01i	4.73210e−02 − 1.30188e−02i	5.61320e−11 − 3.10559e−11i	8.73647e−30 − 1.22173e−29i	−8.95930e−77 + 3.56122e−78i*
1,2	$250\angle 45^\circ$	Pr	5.29582e+02 + 5.30328e+02i	3.00000e+00 − 0.00000e+00i	2.92101e+00 − 7.83936e−02i	−2.31420e+00 − 6.61821e−01i	−1.15377e−02 − 1.74987e−01i	8.27033e−09 − 1.02747e−08i	9.02092e−24 + 6.70273e−24i*	−7.26030e−62 + 1.75089e−62i*
1,2	$250\angle 60^\circ$	Ob	−3.03860e+04 + 5.36266e+04i	3.00000e+00 + 3.99680e−15i	4.09607e+00 − 1.19941e+01i	3.11102e+09 + 2.06726e+08i	2.94469e+23 + 4.92739e+22i	5.62896e+46 + 1.93551e+46i	9.41072e+69 + 5.08231e+69i	−1.20799e+91 + 5.31718e+91i
1,2	$250\angle 75^\circ$	Ob	−5.31627e+04 + 3.09912e+04i	3.00000e+00 − 3.60822e−15i	1.32341e+01 − 9.83418e+00i	3.72264e+10 − 7.05521e+09i	−1.37149e+26 + 6.93729e+25i	9.34336e+51 − 1.27077e+52i	−2.39211e+77 + 1.43866e+78i	8.03760e+101 − 2.78221e+102i
1,2	$250\angle 90^\circ$	Ob	−6.15020e+04 + 7.59281e−12i	3.00000e+00 + 2.59691e−16i	1.80384e+01 − 1.16140e−15i	8.88114e+10 − 1.28266e−04i	1.29285e+27 − 4.83586e+12i	1.11601e+54 − 8.44550e+39i	8.67901e+80 − 9.88957e+66i	1.31755e+106 − 1.98735e+92i
1,2	$500\angle 0^\circ$	Pr	1.49925e+03 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.78016e+00 − 0.00000e+00i	−9.88113e−01 − 0.00000e+00i	−1.23956e−05 − 0.00000e+00i	−4.13461e−27 − 0.00000e+00i	−7.19509e−71 − 4.38687e−87i*	−4.30498e−182 − 2.59774e−198i*
1,2	$500\angle 15^\circ$	Pr	1.44814e+03 + 3.88228e+02i	3.00000e+00 − 1.66533e−16i	2.78722e+00 − 5.59369e−02i	−1.03176e+00 + 3.44441e−01i	1.63473e−05 − 1.36834e−05i	8.27299e−27 − 3.96860e−26i	−2.02321e−68 − 9.41161e−69i*	5.65020e−176 − 7.58674e−176i*
1,2	$500\angle 30^\circ$	Pr	1.29829e+03 + 7.49999e+02i	3.00000e+00 + 1.11022e−16i	2.80802e+00 − 1.08511e−01i	−1.18165e+00 + 8.22525e−01i	−4.60505e−05 + 9.38144e−05i	−7.06049e−25 + 3.26985e−23i	3.86557e−61 + 3.32710e−61i*	−3.52946e−157 + 2.28220e−157i*
1,2	$500\angle 45^\circ$	Pr	1.05991e+03 + 1.06066e+03i	3.00000e+00 − 2.22045e−16i	2.84148e+00 − 1.54473e−01i	−1.52551e+00 + 1.64198e+00i	7.16429e−04 − 1.09715e−03i	1.17372e−18 + 7.18094e−19i	−2.01465e−49 − 1.08012e−49i*	−5.05541e−128 + 1.63248e−127i*
1,2	$500\angle 60^\circ$	Ob	−1.23270e+05 + 2.15556e+05i	3.00000e+00 − 8.99281e−15i	−8.94882e+01 − 6.68256e+01i	7.81235e+18 + 1.04308e+18i	9.07001e+46 + 3.12482e+46i	4.84378e+93 + 3.78552e+93i	1.93774e+140 + 2.98013e+140i	−5.90207e+184 − 4.31903e+184i
1,2	$500\angle 75^\circ$	Ob	−2.14577e+05 + 1.24482e+05i	3.00000e+00 + 2.02061e−14i	5.02796e+01 − 1.77026e+02i	1.08322e+21 − 4.25869e+20i	1.50664e+52 − 2.04781e+52i	−1.28530e+104 − 4.12166e+104i	−6.25455e+156 − 2.15325e+156i	−1.70685e+206 − 1.32889e+206i
1,2	$500\angle 90^\circ$	Ob	−2.48002e+05 + 3.04937e−11i	3.00000e+00 − 5.17210e−17i	2.18233e+02 − 7.05706e−14i	6.39498e+21 − 1.96892e+07i	1.79891e+54 − 1.38000e+40i	2.16157e+108 − 3.31269e+94i	2.34323e+162 − 5.38462e+148i	4.55410e+213 − 1.38144e+200i
1,2	$750\angle 0^\circ$	Pr	2.24925e+03 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.67338e+00 − 0.00000e+00i	−4.58926e−01 − 0.00000e+00i	−6.70953e−09 − 0.00000e+00i	−1.76946e−41 − 0.00000e+00i	−2.05832e−107 − 1.25676e−123i*	−3.91480e−275 − 2.37398e−291i*
1,2	$750\angle 15^\circ$	Pr	2.17258e+03 + 5.82343e+02i	3.00000e+00 + 1.11022e−16i	2.68352e+00 − 8.22185e−02i	−4.10950e−01 + 3.26433e−01i	−1.39993e−08 − 5.70486e−09i	−4.45103e−40 + 3.11190e−40i	1.03115e−103 + 5.96700e−104i*	1.34979e−265 + 6.09589e−267i*
1,2	$750\angle 30^\circ$	Pr	1.94781e+03 + 1.12500e+03i	3.00000e+00 − 3.33067e−16i	2.71351e+00 − 1.59822e−01i	−2.22722e−01 + 7.44861e−01i	−6.02571e−08 − 1.52084e−07i	−1.05891e−35 − 6.51957e−36i	−8.94168e−93 + 8.67190e−93i*	−5.18016e−238 + 1.09380e−237i*
1,2	$750\angle 45^\circ$	Pr	1.59024e+03 + 1.59099e+03i	3.00000e+00 − 2.22045e−16i	2.76201e+00 − 2.28265e−01i	2.84746e−01 + 1.42267e+00i	6.83658e−06 − 2.34748e−06i	−4.41946e−29 + 9.76258e−29i	3.78894e−75 + 1.16194e−75i*	2.33521e−193 + 2.36672e−193i*
1,2	$750\angle 60^\circ$	Ob	−2.78654e+05 + 4.85639e+05i	3.00000e+00 + 1.31006e−14i	−7.98662e+02 + 5.56306e+02i	1.95302e+28 + 3.94078e+27i	2.70821e+70 + 1.47155e+70i	3.45396e+140 + 5.32445e+140i	9.22959e+209 + 1.17606e+211i	7.72029e+277 − 5.02493e+277i
1,2	$750\angle 75^\circ$	Ob	−4.84244e+05 + 2.80474e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$750\angle 90^\circ$	Ob	−5.59502e+05 + 6.87027e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 0^\circ$	Pr	2.99925e+03 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.56882e+00 − 0.00000e+00i	−1.81543e−01 − 0.00000e+00i	−3.20954e−12 − 0.00000e+00i	−6.72022e−56 − 0.00000e+00i	−5.24798e−144 − 3.20657e−160i*	±0
1,2	$1000\angle 15^\circ$	Pr	2.89703e+03 + 7.76457e+02i	3.00000e+00 − 0.00000e+00i	2.58177e+00 − 1.07412e−01i	−1.15634e−01 + 1.82758e−01i	9.30776e−13 + 9.42974e−12i	6.38903e−54 + 9.35629e−55i	−4.43022e−139 − 3.03891e−139i*	±0
1,2	$1000\angle 30^\circ$	Pr	2.59733e+03 + 1.50000e+03i	3.00000e+00 − 1.11022e−16i	2.62017e+00 − 2.09224e−01i	1.26147e−01 + 3.38470e−01i	2.11731e−10 + 8.04203e−11i	3.70719e−48 − 1.96693e−48i	−1.63946e−124 − 2.03732e−124i*	3.26617e−319 + 2.97555e−318i*
1,2	$1000\angle 45^\circ$	Pr	2.12057e+03 + 2.12132e+03i	3.00000e+00 + 2.22045e−16i	2.68262e+00 − 2.99801e−01i	7.31820e−01 + 3.61789e−01i	3.33423e−08 + 1.16059e−08i	−7.07425e−39 − 2.20610e−39i	−6.01396e−101 − 9.08665e−102i*	5.55949e−259 − 1.35647e−259i*
1,2	$1000\angle 60^\circ$	Ob	−4.96538e+05 + 8.64025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 75^\circ$	Ob	−8.62164e+05 + 4.98965e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1000\angle 90^\circ$	Ob	−9.96002e+05 + 1.22220e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1500\angle 0^\circ$	Pr	4.49925e+03 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	2.36622e+00 − 0.00000e+00i	−2.30426e−02 − 0.00000e+00i	−6.16124e−19 − 0.00000e+00i	−8.16526e−85 − 0.00000e+00i	−2.68724e−217 − 1.64310e−233i*	±0
1,2	$1500\angle 15^\circ$	Pr	4.34592e+03 + 1.16469e+03i	3.00000e+00 − 1.11022e−16i	2.38398e+00 − 1.54645e−01i	2.82655e−03 + 2.97153e−02i	−2.73663e−18 − 1.50650e−18i	−6.61678e−83 + 7.66098e−82i	−6.48275e−210 − 6.28313e−210i*	±0
1,2	$1500\angle 30^\circ$	Pr	3.89636e+03 + 2.25000e+03i	3.00000e+00 + 1.11022e−16i	2.43699e+00 − 3.02454e−01i	6.36371e−02 − 3.73258e−03i	9.91854e−17 − 3.50774e−16i	−3.37913e−73 − 2.19101e−73i	4.46171e−188 + 8.31584e−188i*	±0
1,2	$1500\angle 45^\circ$	Pr	3.18123e+03 + 3.18198e+03i	3.00000e+00 − 0.00000e+00i	2.52420e+00 − 4.36216e−01i	4.87820e−02 − 2.07422e−01i	−7.04609e−14 + 7.04273e−13i	2.98122e−59 + 1.72666e−60i	−1.03864e−152 + 1.78352e−153i*	±0
1,2	$1500\angle 60^\circ$	Ob	−1.11981e+06 + 1.94556e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1500\angle 75^\circ$	Ob	−1.94276e+06 + 1.12345e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$1500\angle 90^\circ$	Ob	−2.24400e+06 + 2.75178e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 0^\circ$	Pr	7.49925e+03 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	1.98603e+00 − 0.00000e+00i	−2.62896e−04 − 0.00000e+00i	−1.67040e−32 − 0.00000e+00i	−9.48327e−143 − 5.79989e−159i*	±0	±0
1,2	$2500\angle 15^\circ$	Pr	7.24369e+03 + 1.94114e+03i	3.00000e+00 − 5.55112e−17i	2.01054e+00 − 2.37215e−01i	3.97650e−04 + 6.86188e−05i	−3.42327e−32 + 2.47496e−31i	−2.80209e−139 − 8.89938e−138i*	±0	±0
1,2	$2500\angle 30^\circ$	Pr	6.49444e+03 + 3.75000e+03i	3.00000e+00 + 1.11022e−16i	2.08461e+00 − 4.67682e−01i	−1.19930e−03 − 7.55174e−04i	−7.23099e−29 + 6.91590e−28i	2.63904e−123 − 1.23752e−123i*	1.13043e−315 + 8.37133e−315i*	±0
1,2	$2500\angle 45^\circ$	Pr	5.30255e+03 + 5.30330e+03i	3.00000e+00 + 2.22045e−16i	2.20952e+00 − 6.83234e−01i	1.99840e−03 + 1.02616e−02i	−7.50435e−23 − 1.95844e−22i	3.46061e−100 − 1.57979e−100i*	−1.51761e−256 + 1.59343e−256i*	±0
1,2	$2500\angle 60^\circ$	Ob	−3.11634e+06 + 5.40766e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 75^\circ$	Ob	−5.40300e+06 + 3.12241e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$2500\angle 90^\circ$	Ob	−6.24000e+06 + 7.64792e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 0^\circ$	Pr	1.49993e+04 − 0.00000e+00i	3.00000e+00 − 0.00000e+00i	1.16850e+00 − 0.00000e+00i	−1.93873e−09 − 0.00000e+00i	−1.11859e−66 − 0.00000e+00i	−8.50358e−288 − 5.20375e−304i*	±0	±0
1,2	$5000\angle 15^\circ$	Pr	1.44881e+04 + 3.88229e+03i	3.00000e+00 + 5.55112e−17i	1.19638e+00 − 3.83133e−01i	−4.54814e−09 − 2.89515e−10i	2.50162e−64 + 2.61775e−66i	6.30810e−278 − 2.13335e−278i*	±0	±0
1,2	$5000\angle 30^\circ$	Pr	1.29896e+04 + 7.50000e+03i	3.00000e+00 − 1.11022e−16i	1.28416e+00 − 7.70175e−01i	−4.70533e−08 − 3.00812e−08i	1.83732e−57 − 6.09502e−58i	−1.23648e−249 + 7.42447e−249i*	±0	±0
1,2	$5000\angle 45^\circ$	Pr	1.06059e+04 + 1.06066e+04i	3.00000e+00 − 2.22045e−16i	1.44381e+00 − 1.16090e+00i	1.04968e−06 − 2.81839e−06i	8.15372e−48 − 1.75623e−46i	9.22562e−204 + 1.25554e−202i*	±0	±0
1,2	$5000\angle 60^\circ$	Ob	−1.24827e+07 + 2.16406e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 75^\circ$	Ob	−2.16313e+07 + 1.24948e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,2	$5000\angle 90^\circ$	Ob	−2.49800e+07 + 3.06039e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$25\angle 0^\circ$	Pr	1.22245e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.81487e+00 − 0.00000e+00i	1.46732e+01 − 0.00000e+00i	8.70708e+00 − 0.00000e+00i	−6.79251e+00 − 0.00000e+00i	−3.45257e−01 − 1.86680e−17i*	−8.85277e−05 − 3.40777e−21i*
1,3	$25\angle 15^\circ$	Pr	1.17985e+02 + 3.23516e+01i	−0.00000e+00 − 0.00000e+00i	1.75151e+00 + 4.83276e−01i	1.44919e+01 + 3.07073e+00i	9.66831e+00 − 3.11909e+00i	−7.85243e+00 + 1.20476e+00i	−6.61922e−03 + 4.69938e−01i*	−4.16992e−05 − 1.80851e−04i*
1,3	$25\angle 30^\circ$	Pr	1.05495e+02 + 6.24998e+01i	−0.00000e+00 − 0.00000e+00i	1.56567e+00 + 9.34058e−01i	1.38953e+01 + 6.23580e+00i	1.28176e+01 − 6.08049e+00i	−1.18522e+01 + 1.85208e−01i*	1.14181e+00 + 1.85208e−01i*	1.62725e−03 − 1.13736e−04i*
1,3	$25\angle 45^\circ$	Ob	6.86826e+01 + 5.54258e+02i	−0.00000e+00 − 0.00000e+00i	1.09092e+00 + 8.29543e+00i	7.60391e+01 + 4.58602e+01i	−4.80624e+02 − 7.73596e+02i	4.49126e+04 − 1.24689e+04i	1.77951e+04 + 2.11491e+06i	−9.86170e+06 + 3.81430e+07i
1,3	$25\angle 60^\circ$	Ob	−2.27933e+02 + 4.91244e+02i	−0.00000e+00 − 0.00000e+00i	−3.38676e+00 + 7.42530e+00i	3.25409e+01 + 1.22787e+02i	2.05943e+03 − 1.27033e+03i	−2.86004e+05 + 1.72187e+05i	3.46374e+07 − 2.11268e+07i	1.41052e+09 + 9.35413e+08i
1,3	$25\angle 75^\circ$	Ob	−3.56423e+02 + 2.60632e+02i	−0.00000e+00 − 0.00000e+00i	−5.37653e+00 + 3.95640e+00i	−6.44179e+01 + 9.70166e+01i	8.62897e+02 + 2.19278e+03i	2.83336e+05 − 1.36217e+05i	−1.37319e+07 − 2.00462e+07i	5.91962e+07 − 1.67623e+08i
1,3	$25\angle 90^\circ$	Ob	−4.33359e+02 + 6.42691e−14i	−0.00000e+00 − 0.00000e+00i	−6.56300e+00 + 1.90141e−16i	−1.24184e+02 + 1.40338e−14i	−2.88218e+03 + 9.79330e−13i	−4.73384e+05 + 3.45022e−10i	−4.40437e+07 + 5.01140e−08i	1.20897e+08 − 2.56268e−07i
1,3	$50\angle 0^\circ$	Pr	2.47245e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	3.67767e+00 − 0.00000e+00i	2.33767e+01 − 0.00000e+00i	−4.88920e+00 − 0.00000e+00i	−1.32384e+00 − 0.00000e+00i	−3.55094e−04 − 2.06721e−20i*	−2.01948e−13 − 1.03246e−29i*
1,3	$50\angle 15^\circ$	Pr	2.38727e+02 + 6.47056e+01i	−0.00000e+00 − 0.00000e+00i	3.55211e+00 + 9.62384e−01i	2.36155e+01 + 3.25506e+00i	−5.21068e+00 − 6.35768e+00i	−7.06843e−01 + 1.52775e+00i	4.99308e−04 − 3.95124e−04i*	3.84274e−13 − 7.90050e−13i*
1,3	$50\angle 30^\circ$	Pr	2.13752e+02 + 1.25002e+02i	−0.00000e+00 − 0.00000e+00i	3.18356e+00 + 1.86086e+00i	2.41931e+01 + 7.10985e+00i	−5.86591e+00 − 1.49633e+01i	1.90523e+00 + 2.81577e+00i	−1.83091e−03 + 3.16194e−03i*	−5.92402e−12 + 6.75620e−11i*
1,3	$50\angle 45^\circ$	Ob	1.39407e+02 + 2.35856e+03i	−0.00000e+00 − 0.00000e+00i	3.45934e+00 + 3.52013e+01i	3.62887e+02 − 9.61355e+02i	1.51566e+05 − 3.54648e+04i	−6.23548e+08 − 2.29159e+08i	1.52913e+12 + 2.02398e+12i	−1.23470e+15 − 1.05756e+15i
1,3	$50\angle 60^\circ$	Ob	−1.07881e+03 + 2.06505e+03i	−0.00000e+00 − 0.00000e+00i	−1.54078e+01 + 3.20907e+01i	2.24989e+03 + 2.19122e+02i	−9.82092e+05 + 5.52073e+05i	−3.10340e+10 + 1.62242e+10i	−8.70925e+14 + 4.30233e+14i	−2.50379e+17 + 2.85249e+18i
1,3	$50\angle 75^\circ$	Ob	−1.78686e+03 + 1.14643e+03i	−0.00000e+00 − 0.00000e+00i	−2.72883e+01 + 1.82343e+01i	3.01928e+01 + 2.89131e+03i	2.04839e+06 − 8.28360e+05i	−1.13780e+11 + 6.04960e+10i	3.95610e+15 − 2.81294e+15i	7.53315e+18 − 3.03897e+18i
1,3	$50\angle 90^\circ$	Ob	−2.10817e+03 + 2.81658e−13i	−0.00000e+00 − 0.00000e+00i	−3.27634e+01 + 6.21022e−15i	−3.41484e+03 + 1.45077e−12i	−3.36827e+06 + 2.98049e−09i	−3.00243e+11 + 4.96253e−04i	−1.72045e+16 + 4.17525e+01i	4.11889e+19 − 1.28392e+05i
1,3	$100\angle 0^\circ$	Pr	4.97247e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	7.37812e+00 − 0.00000e+00i	2.75229e+01 − 0.00000e+00i	−1.26297e+01 − 0.00000e+00i	−7.45922e−03 − 0.00000e+00i	−7.13882e−11 − 4.27188e−27i*	−2.04735e−31 − 1.15722e−47i*
1,3	$100\angle 15^\circ$	Pr	4.80210e+02 + 1.29410e+02i	−0.00000e+00 − 0.00000e+00i	7.13148e+00 + 1.90800e+00i	2.90251e+01 + 9.45962e−01i	−1.45002e+01 + 1.83213e+00i	1.15189e−02 + 2.69884e−03i	8.07321e−11 + 2.12121e−10i*	3.65475e−30 + 1.15878e−30i*
1,3	$100\angle 30^\circ$	Pr	4.30260e+02 + 2.50001e+02i	−0.00000e+00 − 0.00000e+00i	6.40660e+00 + 3.69264e+00i	3.35414e+01 − 6.48884e−01i	−2.15120e+01 + 5.13274e+00i	−3.51650e−02 − 2.92393e−02i	6.68166e−09 − 8.86922e−10i*	1.08914e−26 − 1.75103e−26i*
1,3	$100\angle 45^\circ$	Ob	2.80836e+02 + 9.71715e+03i	−0.00000e+00 − 0.00000e+00i	2.76601e+01 + 1.43266e+02i	1.21641e+03 + 7.15790e+04i	−2.03001e+09 − 7.91071e+08i	−1.87854e+15 − 6.42227e+16i	1.54474e+24 − 7.18013e+23i	2.85752e+30 + 4.02726e+30i

Table D.55: (continued)

m,n	$\lvert c\rvert\angle\arg(c)$	type	$\lambda^{(a)\prime}_{mn}$	$S^{(a)\prime}_{mn}(c,\eta{=}0)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.01)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.1)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.25)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.5)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.75)$	$S^{(a)\prime}_{mn}(c,\eta{=}0.99)$
1,3	$100\angle 60^\circ$	Ob	−4.65560e+03 + 8.46025e+03i	−0.00000e+00 − 0.00000e+00i	−5.62452e+01 + 1.46519e+02i	−2.55970e+05 − 2.38116e+05i	−1.02986e+11 + 5.20306e+10i	−1.66280e+20 + 7.08523e+19i	−2.39575e+29 + 8.51202e+28i	−2.76233e+37 + 5.19525e+36i
1,3	$100\angle 75^\circ$	Ob	−7.89559e+03 + 4.79292e+03i	−0.00000e+00 − 0.00000e+00i	−1.28225e+02 + 9.18245e+01i	7.27872e+05 + 2.09095e+05i	−7.29935e+11 + 3.64044e+11i	−6.73948e+21 + 5.17484e+21i	−3.89939e+31 + 4.50450e+31i	4.48776e+39 − 1.75592e+39i
1,3	$100\angle 90^\circ$	Ob	−9.20808e+03 + 1.17566e−12i	−0.00000e+00 − 0.00000e+00i	−1.60346e+02 + 3.48179e−14i	−1.06300e+06 + 7.94447e−10i	−1.90832e+12 + 3.17973e−03i	−4.65639e+22 + 1.48921e+08i	−7.63513e+32 + 3.61343e+18i	1.32724e+41 − 8.10102e+26i
1,3	$250\angle 0^\circ$	Pr	1.24725e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.82798e+01 − 0.00000e+00i	1.08753e−01 − 0.00000e+00i	−9.53798e−01 − 0.00000e+00i	−9.39046e−11 − 0.00000e+00i	−3.97510e−32 − 0.00000e+00i	−1.26150e−86 − 7.49660e−103i*
1,3	$250\angle 15^\circ$	Pr	1.20466e+03 + 3.23524e+02i	−0.00000e+00 − 0.00000e+00i	1.76962e+01 + 4.64565e+00i	1.13229e+00 − 1.47213e+01i	−1.13522e−01 + 1.25486e+00i	8.11003e−11 + 2.83095e−10i	6.02489e−31 + 4.26804e−31i*	−1.50370e−84 − 2.25281e−83i*
1,3	$250\angle 30^\circ$	Pr	1.07978e+03 + 6.25001e+02i	−0.00000e+00 − 0.00000e+00i	1.59744e+01 + 9.01549e+00i	5.42148e+00 − 3.27449e+01i	2.70800e+00 + 8.82439e−01i	8.39696e−09 + 2.67305e−10i	3.06228e−27 − 1.36806e−27i	−3.38656e−74 − 1.84268e−74i*
1,3	$250\angle 45^\circ$	Ob	7.05104e+02 + 6.17929e+04i	−0.00000e+00 − 0.00000e+00i	9.12062e+02 + 6.16037e+02i	−6.77028e+09 − 2.71007e+09i	−9.69175e+20 + 1.51996e+21i	−6.06898e+39 + 1.63717e+40i	−2.13375e+58 + 1.50988e+59i	−5.86149e+76 + 1.91369e+75i
1,3	$250\angle 60^\circ$	Ob	−3.03860e+04 + 5.36266e+04i	−0.00000e+00 − 0.00000e+00i	3.43043e+02 + 1.57122e+03i	−3.46582e+11 + 1.72270e+11i	−3.46618e+25 + 1.31199e+25i	−7.24685e+48 + 1.44230e+48i	−1.32696e+72 + 4.30947e+70i	−2.02771e+93 − 6.45776e+93i
1,3	$250\angle 75^\circ$	Ob	−5.22028e+04 + 3.07324e+04i	−0.00000e+00 − 0.00000e+00i	−1.19687e+03 + 1.57660e+03i	−3.42485e+12 + 1.66993e+12i	8.38826e+27 − 7.58654e+27i	−2.29121e+53 + 5.98778e+53i	−2.70175e+78 − 2.48479e+79i	4.72230e+101 + 2.61828e+101i
1,3	$250\angle 90^\circ$	Ob	−6.05080e+04 + 7.53158e−12i	−0.00000e+00 − 0.00000e+00i	−2.14347e+03 + 2.75757e−13i	−8.92576e+12 + 1.34261e−02i	−9.50714e+28 + 3.61325e+14i	−4.53211e+55 + 3.45710e+41i	−1.48511e+82 + 1.70139e+68i	−7.79714e+104 + 1.23270e+91i
1,3	$500\angle 0^\circ$	Pr	2.49725e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	3.57945e+01 − 0.00000e+00i	−3.09058e+01 − 0.00000e+00i	−1.49485e−03 − 0.00000e+00i	−1.09344e−24 − 0.00000e+00i	−3.08129e−68 − 1.87864e−84i*	−3.55709e−179 − 2.14638e−195i*
1,3	$500\angle 15^\circ$	Pr	2.41206e+03 + 6.47048e+02i	−0.00000e+00 − 0.00000e+00i	3.47338e+01 + 8.88525e+00i	−3.56321e+01 − 4.35013e+00i	2.35133e−03 − 1.05275e−03i	4.86485e−24 − 9.55924e−24i	−7.66965e−66 − 6.43308e−66i*	6.42135e−173 − 5.07835e−173i*
1,3	$500\angle 30^\circ$	Pr	2.16231e+03 + 1.25000e+03i	−0.00000e+00 − 0.00000e+00i	3.15832e+01 + 1.73227e+01i	−5.24584e+01 − 1.12580e+01i	−1.07388e−02 + 6.79053e−03i	−4.54257e−21 + 7.37912e−21i	7.50192e−59 + 2.16458e−58i*	−3.63619e−154 + 1.79880e−155i*
1,3	$500\angle 45^\circ$	Ob	1.41221e+03 + 2.48586e+05i	−0.00000e+00 − 0.00000e+00i	4.75553e+03 − 1.16356e+04i	−1.06280e+16 − 6.95676e+17i	−1.98102e+40 + 5.28369e+40i	5.57824e+77 + 8.51451e+78i	5.48590e+116 + 1.02719e+117i	−7.12642e+152 − 1.05621e+153i
1,3	$500\angle 60^\circ$	Ob	−1.23270e+05 + 2.15506e+05i	−0.00000e+00 − 0.00000e+00i	2.76345e+04 + 3.22444e+03i	−1.81399e+21 + 7.51759e+20i	−2.34524e+49 + 4.60339e+48i	−1.51705e+96 − 2.10326e+95i	−7.90112e+142 − 4.00015e+142i	1.81181e+187 + 1.93014e+186i
1,3	$500\angle 75^\circ$	Ob	−2.12651e+05 + 1.23965e+05i	−0.00000e+00 − 0.00000e+00i	−5.70754e+02 + 4.47458e+04i	−1.89559e+23 + 1.40736e+23i	−1.36591e+54 + 3.52204e+54i	1.90999e+106 + 2.99693e+106i	2.31008e+158 + 1.48618e+157i	1.60164e+206 + 3.34702e+205i
1,3	$500\angle 90^\circ$	Ob	−2.46008e+05 + 3.03712e−11i	−0.00000e+00 − 0.00000e+00i	−5.30408e+04 + 2.12147e−11i	−1.29676e+24 + 4.09184e+09i	−2.67207e+56 + 2.07030e+42i	−1.77848e+110 + 2.73925e+96i	−8.19403e+163 + 1.88929e+150i	−3.28284e+213 + 9.99890e+199i
1,3	$750\angle 0^\circ$	Pr	3.74725e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	5.25100e+01 − 0.00000e+00i	−2.65206e+01 − 0.00000e+00i	−1.23570e−06 − 0.00000e+00i	−7.05030e−39 − 0.00000e+00i	−1.42101e−104 − 8.67626e−121i*	−5.20786e−272 − 3.15805e−288i*
1,3	$750\angle 15^\circ$	Pr	3.61947e+03 + 9.70572e+02i	−0.00000e+00 − 0.00000e+00i	5.10724e+01 + 1.27343e+01i	−2.93191e+01 + 9.78830e+00i	−2.20593e−06 − 1.70448e−06i	−2.03626e−37 + 7.34256e−38i	5.58851e−101 + 5.65555e−101i*	1.65398e−262 + 5.32062e−263i*
1,3	$750\angle 30^\circ$	Pr	3.24484e+03 + 1.87500e+03i	−0.00000e+00 − 0.00000e+00i	4.67707e+01 + 2.49439e+01i	−3.96786e+01 + 2.72174e+01i	4.93745e−06 − 2.98747e−05i	−2.33872e−33 − 4.37471e−33i	−8.15767e−90 + 2.03802e−90i*	−1.29481e−234 + 8.94447e−235i*
1,3	$750\angle 45^\circ$	Ob	2.11932e+03 + 5.60379e+05i	−0.00000e+00 − 0.00000e+00i	−1.08575e+05 − 2.11762e+04i	4.55081e+25 − 1.99543e+25i	−1.94731e+59 + 1.30779e+60i	1.46328e+117 + 2.75619e+117i	5.99811e+174 + 2.86934e+174i	6.86920e+228 − 1.85927e+229i
1,3	$750\angle 60^\circ$	Ob	−2.78654e+05 + 4.85639e+05i	−0.00000e+00 − 0.00000e+00i	1.54265e+05 − 3.29859e+05i	−7.06197e+30 + 2.38606e+30i	−1.15272e+73 + 3.13634e+71i	−2.11659e+143 − 1.07622e+143i	−2.50394e+213 − 3.63455e+213i	−1.55734e+280 + 3.07442e+280i
1,3	$750\angle 75^\circ$	Ob	−4.81352e+05 + 2.79697e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$750\angle 90^\circ$	Ob	−5.56508e+05 + 6.85190e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 0^\circ$	Pr	4.99725e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	6.84479e+01 − 0.00000e+00i	−1.51578e+01 − 0.00000e+00i	−7.95046e−10 − 0.00000e+00i	−3.57799e−53 − 0.00000e+00i	−4.71922e−141 − 2.88348e−157i*	±0
1,3	$1000\angle 15^\circ$	Pr	4.82688e+03 + 1.29410e+03i	−0.00000e+00 − 0.00000e+00i	6.67275e+01 + 1.62080e+01i	−1.40978e+01 + 1.15935e+01i	−3.97724e−10 + 2.31550e−09i	3.15522e−51 + 1.36725e−51i	−3.13878e−136 − 3.67301e−136i*	±0
1,3	$1000\angle 30^\circ$	Pr	4.32738e+03 + 2.50000e+03i	−0.00000e+00 − 0.00000e+00i	6.15372e+01 + 3.19009e+01i	−8.39472e+00 + 3.02505e+01i	3.50268e−08 + 4.40892e−08i	2.23468e−45 + 8.73411e−47i	−3.59421e−122 − 2.32815e−121i*	−2.08905e−315 + 4.75254e−315i*
1,3	$1000\angle 45^\circ$	Ob	2.82643e+03 + 9.97172e+05i	−0.00000e+00 − 0.00000e+00i	3.37315e+03 + 8.64381e+05i	2.28735e+33 + 2.17023e+33i	1.74869e+78 + 2.74699e+79i	8.01237e+155 + 6.21239e+155i	3.31223e+232 − 6.32386e+231i	2.60023e+305 − 7.57598e+304i
1,3	$1000\angle 60^\circ$	Ob	−4.96538e+05 + 8.64025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 75^\circ$	Ob	−8.58306e+05 + 4.97929e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1000\angle 90^\circ$	Ob	−9.92008e+05 + 1.21975e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1500\angle 0^\circ$	Pr	7.49725e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	9.80752e+01 − 0.00000e+00i	−3.09269e+00 − 0.00000e+00i	−2.30901e−16 − 0.00000e+00i	−6.53527e−82 − 0.00000e+00i	−3.62577e−214 − 2.21695e−230i*	±0
1,3	$1500\angle 15^\circ$	Pr	7.24169e+03 + 1.94114e+03i	−0.00000e+00 − 0.00000e+00i	9.60481e+01 + 2.20876e+01i	−8.00319e−01 + 3.94819e+00i	−8.41460e−16 − 8.14965e−16i	−2.10536e−79 + 5.78415e−79i	−6.26476e−207 − 1.04789e−206i*	±0
1,3	$1500\angle 30^\circ$	Pr	6.49244e+03 + 3.75000e+03i	−0.00000e+00 − 0.00000e+00i	8.98113e+01 + 4.39072e+01i	7.55280e+00 + 4.39421e+00i	9.89511e−14 − 9.46523e−14i	−1.46003e−70 − 2.87584e−70i	−3.97682e−186 + 1.27525e−184i*	±0
1,3	$1500\angle 45^\circ$	Ob	4.24064e+03 + 2.24576e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1500\angle 60^\circ$	Ob	−1.11981e+06 + 1.94556e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1500\angle 75^\circ$	Ob	−1.93697e+06 + 1.12189e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$1500\angle 90^\circ$	Ob	−2.23801e+06 + 2.74811e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$2500\angle 0^\circ$	Pr	1.24972e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	1.48900e+02 − 0.00000e+00i	−6.17581e−02 − 0.00000e+00i	−1.05038e−29 − 0.00000e+00i	−1.27263e−139 − 7.78327e−156i*	±0	±0
1,3	$2500\angle 15^\circ$	Pr	1.20713e+04 + 3.23524e+03i	−0.00000e+00 − 0.00000e+00i	1.47145e+02 + 2.99631e+01i	8.55648e−02 + 4.13966e−02i	−6.14694e−29 + 1.44648e−28i	2.72399e−135 − 1.15843e−134i*	±0	±0
1,3	$2500\angle 30^\circ$	Pr	1.08226e+04 + 6.25000e+03i	−0.00000e+00 − 0.00000e+00i	1.41359e+02 + 6.08623e+01i	−1.46698e−01 − 3.02790e−01i	−2.58940e−25 + 3.53084e−25i	3.88217e−120 + 3.36339e−121i*	−7.24053e−312 + 1.76263e−311i*	±0
1,3	$2500\angle 45^\circ$	Ob	7.06907e+03 + 6.24293e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$2500\angle 60^\circ$	Ob	−3.11634e+06 + 5.40766e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$2500\angle 75^\circ$	Ob	−5.39335e+06 + 3.11982e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$2500\angle 90^\circ$	Ob	−6.23001e+06 + 7.64180e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$5000\angle 0^\circ$	Pr	2.49972e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	2.33642e+02 − 0.00000e+00i	−9.41954e−07 − 0.00000e+00i	−1.41381e−63 − 0.00000e+00i	−2.21979e−284 − 1.35840e−300i*	±0	±0
1,3	$5000\angle 15^\circ$	Pr	2.41454e+04 + 6.47048e+03i	−0.00000e+00 − 0.00000e+00i	2.36033e+02 + 3.15939e+01i	−2.09456e−06 − 7.26142e−07i	3.04499e−61 + 8.54358e−62i	1.77879e−274 − 1.14205e−275i*	±0	±0
1,3	$5000\angle 30^\circ$	Pr	2.16479e+04 + 1.25000e+04i	−0.00000e+00 − 0.00000e+00i	2.41838e+02 + 6.90992e+01i	−1.21597e−05 − 2.43967e−05i	2.39667e−54 + 5.00220e−55i	−1.27683e−245 + 1.55205e−245i*	±0	±0
1,3	$5000\angle 45^\circ$	Ob	1.41401e+04 + 2.49859e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$5000\angle 60^\circ$	Ob	−1.24827e+07 + 2.16406e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$5000\angle 75^\circ$	Ob	−2.16120e+07 + 1.24896e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,3	$5000\angle 90^\circ$	Ob	−2.49600e+07 + 3.05917e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,4	$25\angle 0^\circ$	Pr	1.69132e+02 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−7.43779e+00 − 0.00000e+00i	−2.13627e+00 − 0.00000e+00i	7.40554e+00 − 0.00000e+00i	−2.94858e+00 − 3.92543e−17i*	−1.56900e+00 − 8.35147e−17i*	−8.72587e−04 − 3.25341e−20i*
1,4	$25\angle 15^\circ$	Pr	1.63172e+02 + 4.53217e+01i	−7.50000e+00 − 6.66134e−16i	−7.44001e+00 + 1.69459e−02i	−2.24140e+00 + 1.24836e+00i	7.99189e+00 − 1.83881e−04i	−4.23972e+00 − 6.43113e−01i	−4.78990e−02 + 5.92161e−01i*	−1.00658e−04 − 4.92975e−04i*
1,4	$25\angle 30^\circ$	Pr	1.45698e+02 + 8.75570e+01i	−7.50000e+00 − 8.88178e−16i	−7.44653e+00 + 3.27485e−02i	−2.56763e+00 + 2.49276e+00i	9.85432e+00 + 3.04535e−01i	−8.29028e+00 − 9.05642e−01i	1.46464e+00 + 4.15666e−01i*	4.48110e−03 − 9.81663e−05i*
1,4	$25\angle 45^\circ$	Ob	6.86826e+01 + 5.54258e+02i	−7.50000e+00 + 1.77636e−15i	−7.47442e+00 + 2.07623e−01i	3.68858e+00 + 1.69369e+01i	5.53554e+01 − 1.84417e+02i	8.26844e+03 + 5.37333e+03i	−3.32787e+05 + 2.99094e+05i	−7.42838e+06 + 3.77997e+06i
1,4	$25\angle 60^\circ$	Pr	3.67591e+01 + 6.49296e+01i	−7.50000e+00 − 4.44089e−16i	−7.48732e+00 + 2.43268e−02i	−6.08342e+00 + 2.21365e+00i	3.53445e+00 + 6.58539e+00i	−1.08583e+00 − 1.10443e+01i	3.53661e+00 + 3.82929e−01i	−2.64465e−02 + 2.18440e−01i*
1,4	$25\angle 75^\circ$	Ob	−3.56423e+02 + 2.60632e+02i	−7.50000e+00 − 2.66454e−15i	−7.63498e+00 + 9.83258e−02i	−2.25547e+01 + 1.65664e+01i	3.30718e+01 + 5.59757e+02i	7.41625e+04 − 9.86453e+03i	−1.62411e+06 − 5.54976e+06i	2.57834e+07 − 3.35402e+07i
1,4	$25\angle 90^\circ$	Ob	−4.33359e+02 + 6.42691e−14i	−7.50000e+00 − 6.33253e−16i	−7.66423e+00 − 6.22843e−16i	−3.07703e+01 + 1.99753e−15i	−6.92477e+02 + 2.08543e−13i	−1.13729e+05 + 7.85022e−11i	−1.05814e+07 + 1.16314e−08i	2.90452e+07 − 6.04468e−08i
1,4	$50\angle 0^\circ$	Pr	3.44189e+02 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−7.37245e+00 − 0.00000e+00i	1.83781e+00 − 0.00000e+00i	6.93307e+00 + 5.80436e−16i*	−2.79608e+00 − 1.52003e−16i*	−2.94186e−03 − 1.70827e−19i*	−3.38889e−12 − 1.72414e−28i*
1,4	$50\angle 15^\circ$	Pr	3.32265e+02 + 9.06021e+01i	−7.50000e+00 − 0.00000e+00i	−7.37687e+00 + 3.37692e−02i	1.79572e+00 + 1.71872e+00i	3.39356e+00 − 3.01861e+00i	−5.97310e−01 + 1.08612e+00i	7.09858e−04 − 5.35001e−04i*	1.10383e−12 − 2.21895e−12i*
1,4	$50\angle 30^\circ$	Pr	2.97306e+02 + 1.75030e+02i	−7.50000e+00 − 0.00000e+00i	−7.38982e+00 + 6.52798e−02i	1.63260e+00 + 3.55389e+00i	4.35494e+00 − 7.03496e+00i	1.18406e+00 + 2.32053e+00i	−2.68559e−03 + 4.19348e−03i*	−1.93769e−11 + 1.84545e−10i*
1,4	$50\angle 45^\circ$	Ob	1.39407e+02 + 2.35856e+03i	−7.50000e+00 − 2.38698e−15i	−7.43154e+00 + 8.82342e−01i	9.78110e+01 − 4.05649e+01i	1.33487e+04 + 8.84266e+03i	−2.68428e+07 − 6.28423e+07i	−4.36069e+10 + 2.57265e+11i	−7.95663e+12 − 1.67038e+14i
1,4	$50\angle 60^\circ$	Pr	7.42551e+01 + 1.29893e+02i	−7.50000e+00 − 1.77636e−15i	−7.47322e+00 + 4.86156e−02i	−4.32631e+00 + 3.92097e+00i	1.00405e+01 + 8.07755e−02i	−2.02288e−01 + 4.32229e+00i	6.25144e−02 − 8.95382e−02i*	−1.16314e−06 + 1.41211e−05i*
1,4	$50\angle 75^\circ$	Ob	−1.78686e+03 + 1.14643e+03i	−7.50000e+00 − 1.55431e−15i	−8.17710e+00 + 4.42847e−01i	−8.51672e+01 + 3.01916e+02i	2.38599e+05 − 2.37118e+04i	−1.36955e+10 + 2.82534e+09i	4.97854e+14 − 1.72120e+14i	8.77245e+17 − 8.64310e+16i
1,4	$50\angle 90^\circ$	Ob	−2.10817e+03 + 2.81658e−13i	−7.50000e+00 + 2.41895e−16i	−8.30573e+00 + 3.77262e−16i	−3.71952e+02 + 1.26012e−13i	−3.66804e+05 + 2.93055e−10i	−3.26964e+10 + 5.12322e−05i	−1.87357e+15 + 4.38584e+00i	4.48546e+18 − 1.35965e+04i
1,4	$100\angle 0^\circ$	Pr	6.94219e+02 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−7.24245e+00 − 0.00000e+00i	6.06837e+00 − 0.00000e+00i	−2.82968e+00 − 0.00000e+00i	−6.08972e−03 − 0.00000e+00i	−1.03610e−10 − 6.19672e−27i*	−5.85679e−31 − 3.30732e−47i*
1,4	$100\angle 15^\circ$	Pr	6.70369e+02 + 1.81181e+02i	−7.50000e+00 + 4.44089e−16i	−7.25116e+00 + 6.71028e−02i	6.33025e+00 + 1.28377e+00i	−3.69204e+00 − 6.29394e−01i	9.38434e−03 + 2.45871e−03i	1.14252e−10 + 3.09495e−10i*	1.04501e−29 + 3.35126e−30i*
1,4	$100\angle 30^\circ$	Pr	6.00441e+02 + 3.50015e+02i	−7.50000e+00 − 0.00000e+00i	−7.27677e+00 + 1.29802e−01i	7.08905e+00 + 2.91052e+00i	−6.94630e+00 − 1.12250e+00i	−2.79702e−02 − 2.56698e−02i	9.77787e−09 − 1.10880e−09i*	3.15338e−26 − 5.00085e−26i*
1,4	$100\angle 45^\circ$	Ob	2.80836e+02 + 9.71715e+03i	−7.50000e+00 + 7.99361e−15i	−7.10195e+00 + 3.61766e+00i	−2.56056e+03 + 2.57362e+03i	−4.29605e+07 − 1.01791e+08i	2.26959e+15 − 2.33776e+15i	8.07026e+22 + 3.08144e+22i	−4.55073e+28 + 2.46248e+29i

Table D.55: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,4	$100\angle 60^\circ$	Pr	1.49253e+02 + 2.59803e+02i	−7.50000e+00 − 8.88178e−16i	−7.44491e+00 + 9.70342e−02i	−5.68044e−01 + 5.89861e+00i	2.41336e−01 − 9.55110e+00i	−1.44425e−01 + 2.75084e−01i	−4.86369e−05 − 2.05775e−06i*	−7.56236e−15 + 1.22471e−14i*
1,4	$100\angle 75^\circ$	Ob	−7.89559e+03 + 4.79292e+03i	−7.50000e+00 + 1.77636e−15i	−1.05846e+01 + 2.04233e+00i	3.35369e+04 + 2.06784e+04i	−4.16739e+10 + 8.00799e+09i	−4.10195e+20 + 1.64804e+20i	−2.58505e+30 + 1.71027e+30i	2.49458e+38 − 2.50713e+37i
1,4	$100\angle 90^\circ$	Ob	−9.20808e+03 + 1.17566e−12i	−7.50000e+00 + 3.71426e−16i	−1.12277e+01 + 1.06779e−15i	−5.53887e+04 + 3.66703e−11i	−9.94348e+10 + 1.57200e−04i	−2.42625e+21 + 7.55269e+06i	−3.97835e+31 + 1.84887e+17i	6.91570e+39 − 4.16211e+25i
1,4	$250\angle 0^\circ$	Pr	1.74424e+03 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−6.85755e+00 − 0.00000e+00i	5.06583e+00 − 0.00000e+00i	−3.45218e−01 − 0.00000e+00i	−8.11249e−11 − 0.00000e+00i	−5.89874e−32 − 0.00000e+00i	−3.63308e−86 − 2.15872e−102i*
1,4	$250\angle 15^\circ$	Pr	1.68461e+03 + 4.52936e+02i	−7.50000e+00 + 1.11022e−16i	−6.87850e+00 + 1.64578e−01i	5.56251e+00 − 2.19806e+00i	−6.39400e−02 + 4.55984e−01i	6.79466e−11 + 2.45508e−10i	8.92292e−31 + 6.37030e−31i*	−4.26237e−84 − 6.49136e−83i*
1,4	$250\angle 30^\circ$	Pr	1.50978e+03 + 8.75006e+02i	−7.50000e+00 − 8.88178e−16i	−6.94020e+00 + 3.18983e−01i	7.32737e+00 − 4.82080e+00i	9.81947e−01 + 4.20344e−01i	7.28593e−09 + 3.57601e−10i	4.56747e−27 − 2.00186e−27i	−9.69254e−74 − 5.39075e−74i*
1,4	$250\angle 45^\circ$	Ob	7.05104e+02 + 6.17929e+04i	−7.50000e+00 − 8.77076e−15i	4.34175e+00 + 2.04758e+01i	−5.69755e−07 − 1.35160e+08i	−3.54455e+19 + 7.63141e+18i	−3.19991e+38 + 1.44706e+38i	−2.46135e+57 + 1.82992e+57i	−8.56245e+74 − 8.11324e+74i
1,4	$250\angle 60^\circ$	Pr	3.74251e+02 + 6.49517e+02i	−7.50000e+00 − 0.00000e+00i	−7.35931e+00 + 2.41067e−01i	8.22084e+00 + 3.22663e+00i	2.00451e+00 + 1.13566e+00i	−3.22213e−05 − 1.11171e−05i	−1.07439e−15 − 1.42271e−17i	−2.81718e−42 − 1.43664e−42i*
1,4	$250\angle 75^\circ$	Ob	−5.22028e+04 + 3.07324e+04i	−7.50000e+00 − 3.55271e−15i	−3.24365e+01 + 2.40885e+01i	−7.60471e+10 + 1.44387e+10i	2.04932e+26 − 1.03896e+26i	−7.68893e+51 + 1.05125e+52i	7.97586e+76 − 5.01455e+77i	7.85740e+99 + 7.65409e+99i
1,4	$250\angle 90^\circ$	Ob	−6.05080e+04 + 7.53158e−12i	−7.50000e+00 − 8.15423e−16i	−4.42103e+01 + 1.86329e−15i	−1.81430e+11 + 2.58023e−04i	−1.93247e+27 + 7.18596e+12i	−9.21220e+53 + 6.95152e+39i	−3.01871e+80 + 3.43358e+66i	−1.58489e+103 + 2.49264e+89i
1,4	$500\angle 0^\circ$	Pr	3.49424e+03 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−6.23278e+00 − 0.00000e+00i	−1.64943e+00 − 0.00000e+00i	−5.89930e−04 − 0.00000e+00i	−9.60864e−25 − 0.00000e+00i	−4.80961e−68 − 2.93233e−84i*	−1.07239e−178 − 6.47070e−195i*
1,4	$500\angle 15^\circ$	Pr	3.37498e+03 + 9.05868e+02i	−7.50000e+00 + 1.11022e−16i	−6.27196e+00 + 3.18767e−01i	−2.31988e+00 − 2.01512e+00i	9.38969e−04 − 3.98185e−04i	4.31366e−24 − 8.38676e−24i	−1.14413e−65 − 9.64097e−66i*	1.85482e−172 − 1.46391e−172i*
1,4	$500\angle 30^\circ$	Pr	3.02533e+03 + 1.75000e+03i	−7.50000e+00 + 2.22045e−16i	−6.38785e+00 + 6.19859e−01i	−4.71715e+00 − 4.90798e+00i	−4.38847e−03 + 2.54956e−03i	−4.05468e−21 + 6.46621e−21i	1.11195e−58 + 3.24259e−58i*	−1.05023e−153 + 5.11120e−155i*
1,4	$500\angle 45^\circ$	Ob	1.41221e+03 + 2.48586e+05i	−7.50000e+00 − 1.15463e−14i	1.16575e+02 − 4.83710e+01i	4.87195e+15 − 4.99465e+15i	−5.15811e+38 + 2.32731e+38i	−5.66041e+76 + 6.41723e+76i	−3.42552e+114 + 1.11643e+115i	2.47187e+150 − 1.25361e+151i
1,4	$500\angle 60^\circ$	Pr	7.49251e+02 + 1.29904e+03i	−7.50000e+00 − 0.00000e+00i	−7.21439e+00 + 4.77061e−01i	5.91897e+00 − 7.96356e+00i	8.79128e−02 − 9.42883e−03i	3.35248e−12 − 1.45052e−12i	4.96430e−34 − 8.33849e−34i*	1.81080e−88 − 2.95136e−89i*
1,4	$500\angle 75^\circ$	Ob	−2.12651e+05 + 1.23965e+05i	−7.50000e+00 − 4.97380e−14i	−1.23196e+02 + 4.33777e+02i	−2.21420e+21 + 8.70929e+20i	−2.25477e+52 + 3.06757e+52i	1.07043e+104 + 3.41783e+104i	2.20843e+156 + 7.51857e+155i	1.47027e+204 + 7.46669e+203i
1,4	$500\angle 90^\circ$	Ob	−2.46008e+05 + 3.03712e−11i	−7.50000e+00 + 9.01276e−17i	−5.34747e+02 + 1.70131e−13i	−1.30724e+22 + 4.01803e+07i	−2.69366e+54 + 2.06501e+40i	−1.79285e+108 + 2.74673e+94i	−8.26024e+161 + 1.89780e+148i	−3.30937e+211 + 1.00526e+198i
1,4	$750\angle 0^\circ$	Pr	5.24425e+03 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−5.62851e+00 − 0.00000e+00i	−2.83544e+00 − 0.00000e+00i	−4.99983e−07 − 0.00000e+00i	−6.22969e−39 − 0.00000e+00i	−2.13271e−104 − 1.30216e−120i*	−1.50717e−271 − 9.13936e−288i*
1,4	$750\angle 15^\circ$	Pr	5.06536e+03 + 1.35880e+03i	−7.50000e+00 − 1.11022e−16i	−5.68331e+00 + 4.62919e−01i	−3.48477e+00 + 4.87106e−01i	−8.85545e−07 − 7.01998e−07i	−1.80177e−37 + 6.43957e−38i	8.29766e−101 + 8.50788e−101i*	4.75773e−262 + 1.56046e−262i*
1,4	$750\angle 30^\circ$	Pr	4.54088e+03 + 2.62500e+03i	−7.50000e+00 − 4.44089e−16i	−5.84624e+00 + 9.03134e−01i	−5.91292e+00 + 1.96034e+00i	2.30000e−06 − 1.21236e−05i	−2.04853e−33 − 3.88254e−33i	−1.22372e−89 + 3.03105e−90i*	−3.74273e−234 + 2.58340e−234i*
1,4	$750\angle 45^\circ$	Ob	2.11932e+03 + 5.60379e+05i	−7.50000e+00 + 6.90559e−14i	−4.11642e+02 − 6.13556e+02i	3.08946e+23 + 1.21273e+23i	−7.10624e+57 + 5.24348e+57i	−6.14399e+114 + 1.99168e+115i	1.46978e+172 + 4.19083e+172i	1.20360e+227 − 5.51421e+226i
1,4	$750\angle 60^\circ$	Pr	1.12425e+03 + 1.94856e+03i	−7.50000e+00 − 8.88178e−16i	−7.06674e+00 + 7.07982e−01i	−4.01109e+00 − 7.10927e+00i	1.91229e−03 − 1.63899e−03i	−1.21361e−19 + 2.66314e−19i	3.40108e−52 + 6.08545e−52i*	−5.30410e−135 + 5.20435e−135i*
1,4	$750\angle 75^\circ$	Ob	−4.81352e+05 + 2.79697e+05i	−7.50000e+00 + 2.75685e−09i	1.82613e+03 + 4.70320e+03i	−6.18620e+31 + 3.89556e+31i	1.03978e+78 − 6.21407e+78i	9.28052e+156 + 3.19231e+156i	−5.04523e+234 + 9.33169e+234i	1.10308e+308 − 8.46234e+307i
1,4	$750\angle 90^\circ$	Ob	−5.56508e+05 + 6.85190e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1000\angle 0^\circ$	Pr	6.99425e+03 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−5.04423e+00 − 0.00000e+00i	−1.90780e+00 − 0.00000e+00i	−3.25530e−10 − 0.00000e+00i	−3.17015e−53 − 0.00000e+00i	−7.10162e−141 − 4.33912e−157i*	± 0
1,4	$1000\angle 15^\circ$	Pr	6.75573e+03 + 1.81173e+03i	−7.50000e+00 − 2.22045e−16i	−5.11221e+00 + 5.97373e−01i	−1.96281e+00 + 1.30583e+00i	−1.71638e−10 + 9.47801e−10i	2.79381e−51 + 1.21762e−51i	−4.69931e−136 − 5.51019e−136i*	± 0
1,4	$1000\angle 30^\circ$	Pr	6.05643e+03 + 3.50000e+03i	−7.50000e+00 − 2.22045e−16i	−5.31535e+00 + 1.16930e+00i	−1.85148e+00 + 3.90116e+00i	1.40986e−08 + 1.83907e−08i	1.98183e−45 + 8.55027e−47i	−5.33536e−122 − 3.49328e−121i*	−6.04101e−315 + 1.37289e−314i*
1,4	$1000\angle 45^\circ$	Ob	2.82643e+03 + 9.97172e+05i	−7.50000e+00 + 2.10498e−13i	−3.05278e+03 + 3.06799e+03i	3.92290e+29 + 1.57828e+31i	−9.12135e+76 + 1.03320e+77i	6.30155e+152 + 5.03723e+153i	1.39526e+230 + 9.50787e+229i	1.18793e+303 + 6.54074e+302i
1,4	$1000\angle 60^\circ$	Pr	1.49925e+03 + 2.59808e+03i	−7.50000e+00 − 4.44089e−16i	−6.91647e+00 + 9.33834e−01i	−5.88958e+00 + 3.57285e−01i	1.66861e−05 − 6.13707e−05i	−6.27058e−27 − 1.98594e−26i	−4.16086e−70 − 6.41729e−72i*	4.52976e−182 − 2.45837e−181i*
1,4	$1000\angle 75^\circ$	Ob	−8.58306e+05 + 4.97929e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1000\angle 90^\circ$	Ob	−9.92008e+05 + 1.21975e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1500\angle 0^\circ$	Pr	1.04942e+04 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−3.93377e+00 − 0.00000e+00i	−4.38190e−01 − 0.00000e+00i	−9.56358e−17 − 0.00000e+00i	−5.80600e−82 − 0.00000e+00i	−5.48297e−214 − 3.35251e−230i*	± 0
1,4	$1500\angle 15^\circ$	Pr	1.01365e+04 + 2.71760e+03i	−7.50000e+00 + 1.11022e−16i	−4.02128e+00 + 8.38511e−01i	−1.43834e−01 + 5.58503e−01i	−3.46824e−16 − 3.39858e−16i	−1.87792e−79 + 5.13704e−79i	−9.40447e−207 − 1.57485e−206i*	± 0
1,4	$1500\angle 30^\circ$	Pr	9.08751e+03 + 5.25000e+03i	−7.50000e+00 − 8.88178e−16i	−4.28566e+00 + 1.65220e+00i	1.04574e+00 + 7.48869e−01i	4.15543e−14 − 3.88649e−14i	−1.29119e−70 − 2.56026e−70i	−6.09611e−186 + 1.91294e−184i*	± 0
1,4	$1500\angle 45^\circ$	Ob	4.24064e+03 + 2.24576e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1500\angle 60^\circ$	Pr	2.24925e+03 + 3.89711e+03i	−7.50000e+00 + 4.44089e−16i	−6.60844e+00 + 1.37035e+00i	1.63714e+00 + 1.97633e+00i	−2.72122e−08 − 2.05914e−08i	−8.71268e−41 + 1.77061e−41i	5.84658e−107 + 1.06075e−106i*	−1.99295e−274 − 1.16753e−274i*
1,4	$1500\angle 75^\circ$	Ob	−1.93697e+06 + 1.12189e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$1500\angle 90^\circ$	Ob	−2.23801e+06 + 2.74811e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 0^\circ$	Pr	1.74942e+04 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	−1.93245e+00 − 0.00000e+00i	−9.42911e−03 − 0.00000e+00i	−4.38965e−30 − 0.00000e+00i	−1.12895e−139 − 6.90453e−156i*	± 0	± 0
1,4	$2500\angle 15^\circ$	Pr	1.68980e+04 + 4.52933e+03i	−7.50000e+00 + 1.11022e−16i	−2.03514e+00 + 1.21848e+00i	1.29507e−02 + 6.68830e−03i	−2.59079e−29 + 6.03898e−29i	2.30059e−135 − 9.89603e−135i*	± 0	± 0
1,4	$2500\angle 30^\circ$	Pr	1.51497e+04 + 8.75000e+03i	−7.50000e+00 − 0.00000e+00i	−2.35387e+00 + 2.43328e+00i	−2.04449e−02 − 4.80611e−02i	−1.09385e−25 + 1.47109e−25i	3.45652e−120 + 1.03502e−121i*	−1.09073e−311 + 2.64868e−311i*	± 0
1,4	$2500\angle 45^\circ$	Ob	7.06907e+03 + 6.24293e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 60^\circ$	Pr	3.74925e+03 + 6.49519e+03i	−7.50000e+00 + 4.44089e−16i	−5.96518e+00 + 2.18287e+00i	−3.32520e−01 + 1.19475e−01i	1.10196e−15 + 7.16941e−15i	1.19891e−69 + 1.53487e−71i	−6.99727e−180 − 2.25540e−181i*	± 0
1,4	$2500\angle 75^\circ$	Ob	−5.39335e+06 + 3.11982e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$2500\angle 90^\circ$	Ob	−6.23001e+06 + 7.64180e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 0^\circ$	Pr	3.49942e+04 − 0.00000e+00i	−7.50000e+00 − 0.00000e+00i	1.94594e+00 − 0.00000e+00i	−1.50859e−07 − 0.00000e+00i	−5.94740e−64 − 0.00000e+00i	−1.98382e−284 − 1.21399e−300i*	± 0	± 0
1,4	$5000\angle 15^\circ$	Pr	3.38017e+04 + 9.05867e+03i	−7.50000e+00 + 1.11022e−16i	1.90541e+00 + 1.67678e+00i	−3.34658e−07 − 1.20373e−07i	1.28060e−61 + 3.61637e−62i	1.58681e−274 − 1.01223e−275i*	± 0	± 0
1,4	$5000\angle 30^\circ$	Pr	3.03051e+04 + 1.75000e+04i	−7.50000e+00 + 2.22045e−16i	1.74763e+00 + 3.46804e+00i	−1.87314e−06 − 3.97552e−06i	1.00840e−54 + 2.13888e−55i	−1.14156e−245 + 1.38097e−245i*	± 0	± 0
1,4	$5000\angle 45^\circ$	Ob	1.41401e+04 + 2.49859e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 60^\circ$	Pr	7.49925e+03 + 1.29904e+04i	−7.50000e+00 + 4.44089e−16i	−4.23341e+00 + 3.86601e+00i	−2.95750e−04 − 1.32345e−03i	−1.74847e−32 + 8.21023e−32i	2.78202e−142 − 4.68467e−142i*	± 0	± 0
1,4	$5000\angle 75^\circ$	Ob	−2.16120e+07 + 1.24896e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,4	$5000\angle 90^\circ$	Ob	−2.49600e+07 + 3.05917e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,5	$25\angle 0^\circ$	Pr	2.14898e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.99632e+00 − 0.00000e+00i	−2.73592e+01 − 0.00000e+00i	1.11317e+01 − 0.00000e+00i	2.62323e+00 + 5.07303e−16i*	−7.10345e+00 − 3.70577e−16i*	−8.88713e−03 − 3.19396e−19i*
1,5	$25\angle 15^\circ$	Pr	2.07243e+02 + 5.83274e+01i	−0.00000e+00 − 0.00000e+00i	−3.85489e+00 − 1.08605e+00i	−2.75371e+01 − 4.48723e+00i	1.01586e+01 + 1.35551e+01i	−5.54049e+00 − 1.48636e+01i	−2.22814e+00 + 4.60374e+00i*	1.05262e−03 − 9.50534e−03i*
1,5	$25\angle 30^\circ$	Pr	1.84802e+02 + 1.12682e+02i	−0.00000e+00 − 0.00000e+00i	−3.43985e+00 − 2.09972e+00i	−2.78735e+01 − 9.66521e+00i	5.76879e+00 + 3.02511e+01i	−1.39604e+01 − 4.26739e+01i	9.36656e+00 + 1.17164e+01i	6.70573e−02 + 4.53931e−02i*
1,5	$25\angle 45^\circ$	Pr	8.56282e+01 + 8.83908e+01i	−0.00000e+00 − 0.00000e+00i	−1.58714e+00 − 1.65243e+00i	−1.59167e+01 − 1.19196e+01i	−2.36382e+01 + 1.03509e+01i	2.81168e+01 − 1.09715e+01i	−3.23264e+00 + 5.05739e+00i*	4.86978e−02 − 4.44352e−02i*
1,5	$25\angle 60^\circ$	Ob	−1.47588e+02 + 4.41071e+02i	−0.00000e+00 − 0.00000e+00i	2.73209e+00 − 8.31145e+00i	−3.86884e+01 − 1.13628e+02i	−1.31538e+03 + 9.59986e+02i	9.30749e+04 − 7.25444e+04i	−3.54466e+06 + 3.83359e+06i	−2.26235e+07 + 2.85441e+07i
1,5	$25\angle 75^\circ$	Pr	6.72295e+00 + 2.41357e+01i	−0.00000e+00 − 0.00000e+00i	−1.07825e−01 − 4.52308e−01i	−1.57313e+00 − 4.26092e+00i	−8.41803e+00 − 5.42391e+00i	3.95346e+00 + 1.18964e+01i	−6.80471e+00 + 4.04868e+00i	−8.50968e+00 + 1.64504e+00i*
1,5	$25\angle 90^\circ$	Ob	−3.44291e+02 + 5.80751e−14i	−0.00000e+00 − 0.00000e+00i	6.51178e+00 − 3.06625e−16i	1.09239e+02 − 1.37932e−14i	1.78846e+03 − 6.33398e−13i	1.41850e+05 − 1.07333e−10i	2.96841e+06 − 3.78012e−09i	3.10163e+07 − 4.39275e−08i
1,5	$50\angle 0^\circ$	Pr	4.40077e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−8.17185e+00 − 0.00000e+00i	−3.43890e+01 − 0.00000e+00i	3.65306e+01 − 0.00000e+00i	−1.43421e+01 − 7.57793e−16i*	−3.17653e−02 − 1.83941e−18i*	−7.50472e−11 − 3.79833e−27i*
1,5	$50\angle 15^\circ$	Pr	4.24750e+02 + 1.16514e+02i	−0.00000e+00 − 0.00000e+00i	−7.89298e+00 − 2.15333e+00i	−3.63651e+01 + 3.01313e−01i	4.31765e+01 + 6.38630e−02i	−9.64620e+00 + 7.96792e+00i	1.68036e−02 − 6.08472e−03i*	6.87931e−11 − 7.51257e−11i*
1,5	$50\angle 30^\circ$	Pr	3.79813e+02 + 2.25088e+02i	−0.00000e+00 − 0.00000e+00i	−7.07365e+00 − 4.16633e+00i	−4.20521e+01 − 1.48751e+00i	6.58651e+01 + 5.99658e+00i	−6.82811e+00 + 2.74827e+01i	−9.35107e−02 + 4.06121e−02i*	−4.72940e−09 + 6.07532e−09i*
1,5	$50\angle 45^\circ$	Pr	1.74022e+02 + 1.76780e+02i	−0.00000e+00 − 0.00000e+00i	−3.24487e+00 − 3.29429e+00i	−3.08405e+01 − 1.51286e+01i	6.64801e+00 + 3.62111e+01i	−1.27837e+01 − 2.07223e+00i	−4.56123e−02 + 5.23131e−02i*	−7.44465e−08 + 3.08442e−08i*
1,5	$50\angle 60^\circ$	Ob	−9.11733e+02 + 1.96498e+03i	−0.00000e+00 − 0.00000e+00i	1.61530e+01 − 3.79623e+01i	−2.21123e+03 − 1.55612e+02i	6.81585e+05 − 4.16088e+05i	1.16185e+10 − 6.84582e+09i	1.25246e+14 − 7.94860e+13i	−8.37148e+16 − 1.30410e+16i
1,5	$50\angle 75^\circ$	Pr	1.31924e+01 + 4.82902e+01i	−0.00000e+00 − 0.00000e+00i	−2.30663e−01 − 9.04380e−01i	−4.14982e+00 − 7.79306e+00i	−1.60170e+01 + 3.40602e+00i	−4.43034e+00 − 9.61427e+00i	1.43124e+00 + 1.26610e+00i*	4.93943e−02 + 5.24619e−02i*
1,5	$50\angle 90^\circ$	Ob	−1.91859e+03 + 2.69383e−13i	−0.00000e+00 − 0.00000e+00i	3.71569e+01 − 7.70706e−15i	3.29321e+03 − 1.45017e−12i	2.33441e+06 − 2.10510e−09i	1.09438e+11 − 1.83179e−04i	2.25244e+15 − 5.54854e+00i	1.10365e+18 − 3.32271e+03i
1,5	$100\angle 0^\circ$	Pr	8.90164e+02 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.64214e+01 − 0.00000e+00i	−1.21806e+01 − 0.00000e+00i	6.18442e+00 + 1.09824e−15i*	−3.79559e−01 − 2.22447e−17i*	−2.07283e−08 − 1.23904e−24i*	−2.32243e−28 − 1.31021e−44i*
1,5	$100\angle 15^\circ$	Pr	8.59501e+02 + 2.32960e+02i	−0.00000e+00 − 0.00000e+00i	−1.58806e+01 − 4.24121e+00i	−1.66451e+01 + 1.83213e+01i	−9.16984e+00 − 2.52432e+01i	1.95649e−01 + 1.21666e−01i	1.07586e−09 + 1.40749e−08i*	7.77879e−28 + 5.08954e−28i*
1,5	$100\angle 30^\circ$	Pr	7.69599e+02 + 4.50043e+02i	−0.00000e+00 − 0.00000e+00i	−1.42882e+01 − 8.21882e+00i	−3.13151e+01 + 3.58750e+01i	−2.02696e+01 − 7.07893e+01i	−2.12951e−01 − 8.93916e−01i	3.83514e−07 + 1.80644e−07i*	4.47770e−24 − 2.27981e−24i*
1,5	$100\angle 45^\circ$	Pr	3.50801e+02 + 3.53555e+02i	−0.00000e+00 − 0.00000e+00i	−6.56004e+00 − 6.54648e+00i	−5.11644e+01 − 3.15510e+00i	4.97118e+01 − 1.46935e+01i	1.83718e−02 − 4.90536e−01i	−1.75984e−06 + 5.84761e−07i*	−1.84523e−20 − 1.22885e−20i*

Table D.55: (continued)

m,n	$\|c\|\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,5	100∠60°	Ob	−4.31525e+03 + 8.26021e+03i	−0.00000e+00 − 0.00000e+00i	6.41690e+01 − 1.76988e+02i	2.59424e+05 + 2.35365e+05i	7.47062e+10 − 3.91996e+10i	6.59846e+19 − 2.99604e+19i	3.89531e+28 − 1.59499e+28i	−2.02352e+35 − 2.90351e+35i
1,5	100∠75°	Pr	2.61327e+01 + 9.65895e+01i	−0.00000e+00 − 0.00000e+00i	−4.79399e−01 − 1.80658e+00i	−1.11647e+01 − 1.22597e+01i	3.89293e+00 + 2.14075e+01i	−2.20128e+00 − 3.03349e+00i	−4.80653e−02 − 2.47449e−03i*	−2.08841e−06 − 1.92150e−07i*
1,5	100∠90°	Ob	−8.81828e+03 + 1.15115e−12i	−0.00000e+00 − 0.00000e+00i	1.90774e+02 − 4.28805e−14i	1.05812e+06 − 7.98094e−10i	1.37992e+12 − 2.30965e−03i	1.82479e+22 − 5.85282e+07i	1.19250e+32 − 5.66082e+17i	6.71551e+38 − 3.93744e+24i
1,5	250∠0°	Pr	2.24022e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−4.04038e+01 − 0.00000e+00i	8.08716e+01 − 0.00000e+00i	−8.81319e+00 − 0.00000e+00i	−5.18858e−09 − 0.00000e+00i	−6.51554e−30 − 0.00000e+00i	−8.04066e−84 − 4.77702e−100i*
1,5	250∠15°	Pr	2.16355e+03 + 5.82352e+02i	−0.00000e+00 − 0.00000e+00i	−3.91769e+01 − 1.01279e+01i	9.04177e+01 + 2.11677e+01i	−5.47224e+00 + 1.06351e+01i	−7.24387e−11 + 1.63228e−08i	7.65270e−29 + 9.39972e−29i*	2.76121e−81 − 1.37088e−80i*
1,5	250∠30°	Pr	1.93878e+03 + 1.12502e+03i	−0.00000e+00 − 0.00000e+00i	−3.55394e+01 − 1.97202e+01i	1.19004e+02 + 6.01481e+01i	1.42325e+01 + 2.52464e+01i	3.88726e−07 + 2.64086e−07i	5.48461e−25 + 6.69427e−26i	−1.20391e−71 − 2.04960e−71i*
1,5	250∠45°	Pr	8.81133e+02 + 8.83884e+02i	−0.00000e+00 − 0.00000e+00i	−1.65029e+01 − 1.60525e+01i	−2.18862e+01 + 7.16279e+01i	−9.33447e+00 + 9.05424e+00i	−2.16035e−06 + 1.92013e−07i	−4.31779e−22 − 3.11458e−21i*	3.43854e−59 + 2.11707e−59i*
1,5	250∠60°	Ob	−2.95260e+04 + 5.31266e+04i	−0.00000e+00 − 0.00000e+00i	−4.26158e+02 − 1.91046e+03i	3.50766e+11 − 1.76440e+11i	2.56792e+25 − 9.89346e+24i	2.96867e+48 − 6.18467e+47i	2.29872e+71 − 1.15257e+70i	2.33537e+91 + 4.54904e+90i
1,5	250∠75°	Pr	6.49551e+01 + 2.41480e+02i	−0.00000e+00 − 0.00000e+00i	−1.24980e+00 − 4.49876e+00i	−3.39878e+01 − 3.46069e+00i	−1.58326e+01 + 1.15534e+00i	−4.80849e−02 − 1.89830e−02i	−7.55908e−08 − 2.24133e−07i*	−5.25062e−21 − 1.66375e−20i*
1,5	250∠90°	Ob	−5.95181e+04 + 7.47034e−12i	−0.00000e+00 − 0.00000e+00i	2.60339e+03 − 3.62301e−13i	9.03459e+12 − 1.36850e−02i	7.03133e+28 − 2.67988e+14i	1.84476e+55 − 1.40928e+41i	2.52345e+81 − 2.89426e+67i	−1.87256e+103 + 2.81809e+89i
1,5	500∠0°	Pr	4.49023e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−7.78902e+01 − 0.00000e+00i	3.36245e+01 − 0.00000e+00i	−1.00948e−01 − 6.01596e−18i*	−8.50951e−22 − 5.17788e−38i*	−1.08080e−64 − 6.58927e−81i*	−4.65930e−175 − 2.81129e−191i*
1,5	500∠15°	Pr	4.33690e+03 + 1.16469e+03i	−0.00000e+00 − 0.00000e+00i	−7.58351e+01 − 1.87209e+01i	4.18862e+01 − 5.88115e+01i	5.89199e−02 − 6.63521e−03i	8.36316e−22 − 9.10740e−22i	−1.90417e−63 − 2.74647e−63i*	9.37606e−170 − 4.02329e−170i*
1,5	500∠30°	Pr	3.88735e+03 + 2.25001e+03i	−0.00000e+00 − 0.00000e+00i	−6.96606e+01 − 3.67547e+01i	7.97768e+01 − 1.51321e+02i	−2.98539e−01 − 1.43452e−02i	−8.91720e−19 + 4.59326e−19i	−1.51172e−56 + 7.51895e−56i*	−4.03713e−151 − 2.08712e−151i*
1,5	500∠45°	Pr	1.76502e+03 + 1.76777e+03i	−0.00000e+00 − 0.00000e+00i	−3.30562e+01 − 3.10703e+01i	1.13502e+02 + 3.52885e+01i	−1.98109e−01 + 3.38938e−02i	−1.02784e−16 − 4.44951e−16i	3.65073e−47 + 1.22426e−46i*	1.62892e−124 − 8.57095e−125i*
1,5	500∠60°	Ob	−1.21544e+05 + 2.14506e+05i	−0.00000e+00 − 0.00000e+00i	−3.37476e+04 − 3.86745e+03i	1.84596e+21 − 7.70091e+20i	1.74917e+49 − 3.48624e+48i	6.27732e+95 + 8.42160e+94i	1.39633e+142 + 6.91875e+141i	−7.58005e+184 + 1.73148e+184i
1,5	500∠75°	Pr	1.29660e+02 + 4.82962e+02i	−0.00000e+00 − 0.00000e+00i	−2.61359e+00 − 8.93742e+00i	−2.19564e+01 + 4.42750e+01i	−4.53220e−01 + 4.04465e+00i	−1.50418e−05 + 9.47626e−06i	−5.92942e−17 − 1.37875e−16i*	−1.00627e−44 − 2.44431e−44i*
1,5	500∠90°	Ob	−2.44018e+05 + 3.02488e−11i	−0.00000e+00 − 0.00000e+00i	6.47180e+04 − 2.61058e−11i	1.31948e+24 − 4.16811e+09i	1.99028e+56 − 1.54277e+42i	7.32556e+109 − 1.12858e+96i	1.42815e+163 − 3.29353e+149i	5.24156e+210 − 1.60937e+197i
1,5	750∠0°	Pr	6.74024e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.12377e+02 − 0.00000e+00i	−4.65733e+01 − 0.00000e+00i	−4.48884e−05 − 0.00000e+00i	−1.23349e−36 − 0.00000e+00i	−7.07924e−102 − 4.32231e−118i*	−9.65348e−269 − 5.85373e−285i*
1,5	750∠15°	Pr	6.51024e+03 + 1.74703e+03i	−0.00000e+00 − 0.00000e+00i	−1.09858e+02 − 2.58632e+01i	−6.59747e+01 − 3.99690e+01i	−5.92776e−05 − 8.26577e−05i	−3.77916e−35 + 2.93653e−36i	1.94931e−98 + 3.46212e−98i*	2.70859e−259 + 1.75910e−259i*
1,5	750∠30°	Pr	5.83591e+03 + 3.37501e+03i	−0.00000e+00 − 0.00000e+00i	−1.02164e+02 − 5.12249e+01i	−1.44458e+02 − 1.11444e+02i	7.56495e−04 − 8.24405e−04i	3.97181e−32 − 8.70105e−31i	−4.07433e−87 − 1.18938e−87i*	−2.94353e−231 + 2.33927e−232i*
1,5	750∠45°	Pr	2.64890e+03 + 2.65165e+03i	−0.00000e+00 − 0.00000e+00i	−4.95705e+01 − 4.50735e+01i	8.45265e+01 − 8.47860e+01i	−1.49294e−03 − 7.75301e−04i	5.01917e−26 − 1.85568e−26i	−1.55134e−72 − 2.95388e−72i*	4.91554e−192 − 5.38941e−190i*
1,5	750∠60°	Ob	−2.76062e+05 + 4.84139e+05i	−0.00000e+00 − 0.00000e+00i	−1.88017e+05 + 4.03467e+05i	7.19935e+30 − 2.44497e+30i	8.61630e+72 − 2.51070e+71i	8.78793e+142 + 4.43616e+142i	4.46438e+212 + 6.40175e+212i	4.72594e+277 − 1.61442e+278i
1,5	750∠75°	Pr	1.94364e+02 + 7.24444e+02i	−0.00000e+00 − 0.00000e+00i	−4.07573e+00 − 1.33144e+01i	3.65602e+01 + 3.92009e+01i	7.48122e−01 + 2.28986e−01i	−3.27845e−10 + 4.57211e−09i	−3.54877e−26 − 6.76530e−26i*	−1.44222e−68 − 2.86604e−68i*
1,5	750∠90°	Ob	−5.53518e+05 + 6.83353e−11i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	1000∠0°	Pr	8.99024e+03 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.43980e+02 − 0.00000e+00i	−6.21405e+01 − 0.00000e+00i	−3.95871e−08 − 0.00000e+00i	−8.40079e−51 − 0.00000e+00i	−3.18469e−138 − 1.94585e−154i*	±0
1,5	1000∠15°	Pr	8.68357e+03 + 2.32937e+03i	−0.00000e+00 − 0.00000e+00i	−1.41330e+02 − 3.16360e+01i	−8.13582e+01 + 1.17303e+01i	−5.13423e−08 + 1.05514e−07i	6.30394e−49 + 5.05319e−49i	−1.39260e−133 − 2.93780e−133i*	±0
1,5	1000∠30°	Pr	7.78447e+03 + 4.50000e+03i	−0.00000e+00 − 0.00000e+00i	−1.33053e+02 − 6.32484e+01i	−1.61777e+02 + 5.64437e+01i	3.05129e−07 + 2.82097e−06i	4.42584e−43 + 2.85127e−43i	5.80155e−119 − 1.47861e−118i*	−1.04794e−311 + 7.67751e−312i*
1,5	1000∠45°	Pr	3.53278e+03 + 3.53553e+03i	−0.00000e+00 − 0.00000e+00i	−6.60269e+01 − 5.80825e+01i	−1.74737e+01 − 9.07267e+01i	−4.61742e−06 − 1.00041e−05i	2.27517e−36 + 4.38634e−36i	4.05718e−98 + 5.52166e−98i*	−1.06005e−255 − 6.44786e−256i*
1,5	1000∠60°	Ob	−4.93080e+05 + 8.62025e+05i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	1000∠75°	Pr	2.59069e+02 + 9.65926e+02i	−0.00000e+00 − 0.00000e+00i	−5.63464e+00 − 1.76283e+01i	4.77886e+01 − 1.96686e+01i	6.20522e−02 − 1.18405e−01i	7.99633e−13 + 6.81264e−13i	−1.77420e−35 − 2.83453e−35i*	−1.72062e−92 − 2.87176e−92i*
1,5	1000∠90°	Ob	−9.88018e+05 + 1.21730e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	1500∠0°	Pr	1.34902e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−1.98986e+02 − 0.00000e+00i	−2.62469e+01 − 0.00000e+00i	−1.77105e−14 − 0.00000e+00i	−2.31647e−79 − 0.00000e+00i	−3.69679e−211 − 2.26036e−227i*	±0
1,5	1500∠15°	Pr	1.30302e+04 + 3.49406e+03i	−0.00000e+00 − 0.00000e+00i	−1.96949e+02 − 3.93834e+01i	−1.96615e+01 + 2.92084e+01i	−4.52064e−14 − 7.78473e−14i	−1.25794e−76 + 1.78386e−76i	−3.31682e−204 − 1.14685e−203i*	±0
1,5	1500∠30°	Pr	1.16816e+04 + 6.75000e+03i	−0.00000e+00 − 0.00000e+00i	−1.90014e+02 − 8.04115e+01i	2.35219e+01 + 7.88909e+01i	1.03405e−11 − 2.24459e−12i	6.89033e−69 − 1.14312e−67i	−6.82567e−182 + 1.09734e−181i*	±0
1,5	1500∠45°	Pr	5.30055e+03 + 5.30330e+03i	−0.00000e+00 − 0.00000e+00i	−9.86947e+01 − 8.11980e+01i	−3.32924e+01 + 1.69438e+01i	2.60422e−10 − 2.07949e−10i	−1.98257e−56 − 2.24014e−56i	1.45417e−149 + 1.03025e−149i*	±0
1,5	1500∠60°	Ob	−1.11462e+06 + 1.94256e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	1500∠75°	Pr	3.88479e+02 + 1.44889e+03i	−0.00000e+00 − 0.00000e+00i	−9.03625e+00 − 2.60610e+01i	−3.83368e+01 − 1.31323e+01i	−2.48311e−03 + 2.16232e−03i	1.82652e−20 − 4.28144e−20i	−3.54438e−54 − 4.12075e−54i*	−1.95414e−140 − 2.39484e−140i*
1,5	1500∠90°	Ob	−2.23202e+06 + 2.74443e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	2500∠0°	Pr	2.24902e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−2.79177e+02 − 0.00000e+00i	−1.04957e+00 − 0.00000e+00i	−1.37070e−27 − 0.00000e+00i	−7.52633e−137 − 4.60299e−153i*	±0	±0
1,5	2500∠15°	Pr	2.17236e+04 + 5.82343e+03i	−0.00000e+00 − 0.00000e+00i	−2.81095e+02 − 4.17083e+01i	1.16748e+00 + 1.14206e+00i	−1.27741e−26 + 1.60741e−26i	3.35632e−132 − 6.22269e−132i*	±0	±0
1,5	2500∠30°	Pr	1.94758e+04 + 1.12500e+04i	−0.00000e+00 − 0.00000e+00i	−2.84923e+02 − 9.02345e+01i	1.11107e+00 − 5.85428e+00i	−5.28666e−23 + 2.23078e−23i	1.89180e−117 + 1.33439e−117i*	−2.55799e−308 + 1.96760e−308i*	±0
1,5	2500∠45°	Pr	8.83608e+03 + 8.83883e+03i	−0.00000e+00 − 0.00000e+00i	−1.62580e+02 − 1.16381e+02i	1.87503e+00 − 2.51807e+00i	−6.84029e−20 + 1.50533e−19i	−5.95432e−97 − 2.23356e−97i*	6.20354e−253 − 1.48513e−254i*	±0
1,5	2500∠60°	Ob	−3.10769e+06 + 5.40266e+06i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	2500∠75°	Pr	6.47298e+02 + 2.41481e+03i	−0.00000e+00 − 0.00000e+00i	−1.69285e+01 − 4.21077e+01i	3.67493e+00 − 1.80869e+01i	−1.47901e−06 − 4.00997e−08i	1.42457e−35 + 7.03951e−35i*	−7.88737e−92 − 4.93805e−92i*	−1.40883e−236 − 9.63443e−237i*
1,5	2500∠90°	Ob	−6.22002e+06 + 7.63567e−10i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	5000∠0°	Pr	4.49902e+04 − 0.00000e+00i	−0.00000e+00 − 0.00000e+00i	−3.40758e+02 − 0.00000e+00i	−3.58136e−05 − 0.00000e+00i	−3.74590e−61 − 0.00000e+00i	−2.67475e−281 − 1.63681e−297i*	±0	±0
1,5	5000∠15°	Pr	4.34569e+04 + 1.16469e+04i	−0.00000e+00 − 0.00000e+00i	−3.60293e+02 + 6.21147e+00i	−6.87681e−05 − 4.93207e−05i	7.19416e−59 + 4.30456e−59i	2.08453e−271 + 4.19400e−272i*	±0	±0
1,5	5000∠30°	Pr	3.89614e+04 + 2.25000e+04i	−0.00000e+00 − 0.00000e+00i	−4.16572e+02 − 7.98447e+00i	1.17572e−04 − 1.04609e−03i	4.81412e−52 + 4.36746e−52i	−2.24982e−242 + 8.32281e−243i*	±0	±0
1,5	5000∠45°	Pr	1.76749e+04 + 1.76777e+04i	−0.00000e+00 − 0.00000e+00i	−3.08450e+02 − 1.47281e+02i	−1.69445e−03 + 7.29441e−04i	−2.06262e−43 + 1.86646e−43i	2.75322e−199 − 3.18819e−199i*	±0	±0
1,5	5000∠60°	Ob	−1.24654e+07 + 2.16306e+07i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,5	5000∠75°	Pr	1.29435e+03 + 4.82963e+03i	−0.00000e+00 − 0.00000e+00i	−4.23741e+01 − 7.69893e+01i	8.85991e−01 − 1.13626e+00i	−1.09601e−15 + 3.35718e−15i	−5.36441e−73 − 3.79671e−72i*	−1.57435e−186 + 3.05982e−187i*	±0
1,5	5000∠90°	Ob	−2.49400e+07 + 3.05794e−09i	−0.00000e+00 − 0.00000e+00i	±∞	±∞	±∞	±∞	±∞	±∞
1,6	25∠0°	Pr	2.59497e+02 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	1.29570e+01 − 0.00000e+00i	−3.64761e−01 − 0.00000e+00i	−9.23488e+00 − 0.00000e+00i	6.36104e+00 − 0.00000e+00i	−4.54517e+00 − 2.30803e−16i*	−1.33610e−02 − 4.60135e−19i*
1,6	25∠15°	Pr	2.50157e+02 + 7.13852e+01i	1.31250e+01 − 1.33227e−15i	1.29631e+01 − 4.66475e−02i	−2.03153e−01 − 2.93685e+00i	−1.07708e+01 + 5.19261e+00i	7.11022e+00 − 1.03392e+01i	−3.44623e+00 + 4.87276e+00i*	4.89062e−03 − 2.49167e−02i*
1,6	25∠30°	Ob	3.60481e+02 + 4.54627e+02i	1.31250e+01 + 3.85025e−13i	1.28900e+01 − 2.96580e−01i	−1.18375e+01 − 1.36843e+01i	6.00211e+01 + 7.01606e+01i	−2.05131e+02 + 1.28354e+03i	−1.36726e+04 + 8.91320e+03i	−5.68165e+04 + 8.24509e+04i
1,6	25∠45°	Pr	1.17904e+02 + 1.23828e+02i	1.31250e+01 − 0.00000e+00i	1.30496e+01 − 8.10927e−02i	5.50344e+00 − 6.50216e+00i	−2.32056e+01 − 2.56604e+00i	3.34792e+01 + 5.67447e−01i	−7.57756e+00 + 8.81411e+00i*	2.03304e−01 − 1.61573e−01i*
1,6	25∠60°	Ob	−1.47588e+02 + 4.41071e+02i	1.31250e+01 − 6.21725e−15i	1.32229e+01 − 2.90185e−01i	1.20699e+01 − 3.50911e+01i	−5.28672e+02 + 3.33275e+00i	3.82718e+04 − 1.45163e+03i	−1.66065e+06 + 3.37292e+05i	−1.13721e+07 + 3.22635e+06i
1,6	25∠75°	Ob	−2.70703e+02 + 2.34458e+02i	1.31250e+01 − 4.44089e−15i	1.33047e+01 − 1.54568e−01i	3.16238e+01 − 2.31554e+01i	−4.60023e+01 − 5.47749e+02i	−3.51717e+04 + 7.51719e+03i	4.92987e+05 + 5.85002e+05i	5.66723e+06 + 1.11142e+06i
1,6	25∠90°	Ob	−3.44291e+02 + 5.80751e−14i	1.31250e+01 + 1.25147e−15i	1.33536e+01 + 1.23493e−15i	4.32858e+01 − 2.30946e−15i	6.75242e+02 − 2.00023e−13i	5.35460e+04 − 3.74223e−11i	1.12052e+06 − 1.36218e−09i	1.17081e+07 − 1.59053e−08i
1,6	50∠0°	Pr	5.34889e+02 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	1.27775e+01 − 0.00000e+00i	−8.60879e+00 − 0.00000e+00i	9.09954e+00 − 0.00000e+00i	−5.03511e+00 − 2.54287e−16i*	−2.52138e−02 − 1.45556e−18i*	−1.23738e−10 − 6.22809e−27i*
1,6	50∠15°	Pr	5.16163e+02 + 1.42448e+02i	1.31250e+01 − 0.00000e+00i	1.27896e+01 − 9.26510e−02i	−8.83286e+00 − 3.05793e+00i	1.11111e+01 + 6.20660e+00i	−5.79437e+00 + 3.26741e+00i	1.91290e−02 − 6.07527e−03i*	1.62778e−10 − 1.71766e−10i*
1,6	50∠30°	Ob	1.34799e+03 + 1.99184e+03i	1.31250e+01 − 4.44089e−15i	1.22308e+01 − 1.27793e+00i	−2.47583e+01 + 6.03779e+01i	−3.54395e+02 + 2.07501e+03i	−6.38075e+05 − 2.21899e+05i	9.11883e+07 − 1.71909e+08i	1.33611e+10 + 1.07932e+09i
1,6	50∠45°	Pr	2.41694e+02 + 2.47530e+02i	1.31250e+01 − 1.77636e−15i	1.29683e+01 − 1.61728e−01i	−2.16260e+00 − 9.79734e+00i	−1.24558e+01 + 2.33294e+01i	−1.39003e+01 − 4.53039e+00i	−9.44310e−02 + 9.60463e−02i*	−2.89813e−07 + 1.12622e−07i*
1,6	50∠60°	Ob	−9.11733e+02 + 1.96498e+03i	1.31250e+01 + 7.54952e−15i	1.37086e+01 − 1.30916e+00i	−2.67928e+02 − 1.98533e+02i	1.20614e+05 + 2.75881e+03i	2.02690e+09 + 7.24943e+07i	2.23102e+13 + 6.03542e+10i	−9.55687e+15 − 8.42849e+15i
1,6	50∠75°	Ob	−1.60407e+03 + 1.09454e+03i	1.31250e+01 + 6.21725e−15i	1.41872e+01 − 7.37684e−01i	1.19195e+02 − 4.27922e+02i	−2.42827e+05 + 2.64088e+04i	7.31875e+09 − 1.71070e+09i	−9.42367e+13 + 4.23028e+13i	3.16596e+16 + 1.36008e+16i
1,6	50∠90°	Ob	−1.91859e+03 + 2.69383e−13i	1.31250e+01 − 1.91386e−16i	1.44064e+01 − 3.92575e−16i	5.26478e+02 − 1.69461e−13i	3.73080e+05 − 2.92304e−10i	1.74902e+10 − 2.72065e−05i	3.59980e+14 − 8.44176e−01i	1.76383e+17 − 5.10165e+02i
1,6	100∠0°	Pr	1.08507e+03 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	1.24215e+01 − 0.00000e+00i	−1.28568e+01 − 0.00000e+00i	5.33656e+00 − 0.00000e+00i	−1.75412e−01 − 1.02414e−17i*	−1.78160e−08 − 1.06434e−24i*	−3.97875e−28 − 2.24240e−44i*
1,6	100∠15°	Pr	1.04760e+03 + 2.84748e+02i	1.31250e+01 − 8.88178e−16i	1.24452e+01 − 1.83483e−01i	−1.40230e+01 + 4.86641e−01i	5.97344e+00 − 8.35767e+00i	1.23965e−01 + 8.54119e−02i	9.77435e−10 + 1.62907e−08i*	1.78234e−27 + 1.18272e−27i*
1,6	100∠30°	Ob	5.19800e+03 + 8.31384e+03i	1.31250e+01 + 1.95399e−14i	9.50214e+00 − 4.98574e+00i	−5.78063e+02 − 5.54258e+02i	−1.02843e+06 − 3.63735e+05i	1.41402e+11 + 1.13909e+11i	−1.44690e+16 − 2.30483e+16i	−8.83318e+19 + 6.35308e+20i
1,6	100∠45°	Pr	4.89203e+02 + 4.94996e+02i	1.31250e+01 − 2.66454e−15i	1.28059e+01 − 3.21914e−01i	−1.43973e+01 − 9.24924e+00i	2.69924e+01 + 2.74870e+00i	5.99824e−02 − 5.77778e−01i	−3.65148e−06 + 1.10779e−06i*	−7.38314e−20 − 5.00507e−20i*

Table D.55: (continued)

$m,\,n$	$\lvert c\rvert\angle\arg(c)$	type	$\lambda_{mn}^{(a)\prime}$	$S_{mn}^{(a)\prime}(c,\eta{=}0)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.01)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.1)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.25)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.5)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.75)$	$S_{mn}^{(a)\prime}(c,\eta{=}0.99)$
1,6	$100\angle 60^\circ$	Ob	−4.31525e+03 + 8.26021e+03i	1.31250e+01 + 8.88178e−16i	1.56730e+01 − 5.80900e+00i	7.24577e+03 + 2.43437e+04i	6.10589e+09 + 3.75784e+08i	5.21787e+18 + 6.21086e+17i	3.01504e+27 + 4.74365e+26i	−1.64035e+33 − 2.56095e+34i
1,6	$100\angle 75^\circ$	Ob	−7.51941e+03 + 4.68934e+03i	1.31250e+01 − 5.21805e−15i	1.82495e+01 − 3.47610e+00i	−4.78121e+04 − 2.93825e+04i	4.31029e+10 − 8.46510e+09i	2.29327e+20 − 9.50932e+19i	5.67909e+29 − 4.03146e+29i	1.79913e+36 + 1.67669e+36i
1,6	$100\angle 90^\circ$	Ob	−8.81828e+03 + 1.15115e−12i	1.31250e+01 − 3.50469e−16i	1.93524e+01 − 1.38829e−15i	7.88758e+04 − 5.04479e−11i	1.02864e+11 − 1.60373e−04i	1.36026e+21 − 4.20690e+06i	8.88925e+30 − 4.11782e+16i	5.00595e+37 − 2.87769e+23i
1,6	$250\angle 0^\circ$	Pr	2.73518e+03 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	1.13743e+01 − 0.00000e+00i	3.68892e+00 − 0.00000e+00i	−2.35754e+00 − 0.00000e+00i	−3.69747e−09 − 0.00000e+00i	−8.06398e−30 − 0.00000e+00i	−1.89284e−83 − 1.12440e−99i*
1,6	$250\angle 15^\circ$	Pr	2.64148e+03 + 7.11771e+02i	1.31250e+01 − 0.00000e+00i	1.14301e+01 − 4.45889e−01i	4.14396e+00 + 7.60401e+00i	−1.81562e+00 + 2.75737e+00i	−2.18778e−10 + 1.16556e−08i	9.41428e−29 + 1.16974e−28i*	6.72756e−81 − 3.30743e−80i*
1,6	$250\angle 30^\circ$	Ob	3.17480e+04 + 5.32606e+04i	1.31250e+01 − 8.88178e−16i	−1.36455e+01 − 1.70142e+01i	−1.36489e+06 − 4.83550e+05i	−1.14047e+14 + 9.99746e+13i	4.57391e+26 − 3.48192e+27i	4.09524e+40 + 6.08698e+40i	7.39057e+52 − 9.33234e+52i
1,6	$250\angle 45^\circ$	Pr	1.23168e+03 + 1.23745e+03i	1.31250e+01 − 0.00000e+00i	1.23186e+01 − 7.93563e−01i	−1.98294e+01 + 1.42741e+01i	−5.61517e+00 + 4.30317e+00i	−2.64636e−06 + 1.71407e−07i	−8.07033e−22 − 6.28984e−21i*	1.33947e−58 + 8.30474e−59i*
1,6	$250\angle 60^\circ$	Ob	−2.95260e+04 + 5.31266e+04i	1.31250e+01 − 1.99840e−15i	1.75701e+01 − 5.14015e+01i	1.11244e+10 + 7.32076e+08i	7.70873e+23 + 1.27587e+23i	8.15553e+46 + 2.76048e+46i	5.79151e+69 + 3.02743e+69i	5.06033e+89 + 4.47553e+89i
1,6	$250\angle 75^\circ$	Ob	−5.12469e+04 + 3.04735e+04i	1.31250e+01 − 4.44089e−15i	5.56463e+01 − 4.13001e+01i	1.08653e+11 − 2.06921e+10i	−2.13828e+26 + 1.08814e+26i	4.39601e+51 − 6.06139e+51i	−1.78076e+76 + 1.20680e+77i	−9.18326e+97 + 1.08286e+98i
1,6	$250\angle 90^\circ$	Ob	−5.95181e+04 + 7.47034e−12i	1.31250e+01 + 1.55455e−15i	7.58431e+01 − 2.45344e−15i	2.59228e+11 − 3.66180e−04i	2.01749e+27 − 7.48322e+12i	5.29313e+53 − 3.98956e+39i	7.24050e+79 − 8.23047e+65i	−5.37290e+101 + 8.03101e+87i
1,6	$500\angle 0^\circ$	Pr	5.48522e+03 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	9.69598e+00 − 0.00000e+00i	1.01361e+01 − 0.00000e+00i	−1.08417e−02 − 0.00000e+00i	−9.21288e−23 − 0.00000e+00i	−1.29012e−65 − 7.86530e−82i*	−1.07646e−175 − 6.49484e−192i*
1,6	$500\angle 15^\circ$	Pr	5.29781e+03 + 1.42351e+03i	1.31250e+01 + 4.44089e−16i	9.79679e+00 − 8.50258e−01i	1.27562e+01 − 4.74608e+00i	1.88924e−02 − 1.47380e−03i	6.17112e−22 − 6.63150e−22i	−2.37613e−63 − 3.44532e−63i*	2.27268e−169 − 9.73492e−170i*
1,6	$500\angle 30^\circ$	Ob	1.25998e+05 + 2.14774e+05i	1.31250e+01 − 4.18138e−14i	−2.94380e+01 + 7.22021e+01i	3.01887e+11 + 2.44629e+11i	7.42671e+26 − 5.60836e+27i	−4.71945e+54 − 1.27047e+54i	−1.46009e+81 + 3.52118e+81i	3.77902e+105 − 6.85329e+106i
1,6	$500\angle 45^\circ$	Pr	2.46912e+03 + 2.47488e+03i	1.31250e+01 − 0.00000e+00i	1.15071e+01 − 1.55015e+00i	1.77557e+01 + 1.86106e+01i	−1.13128e−01 + 1.34268e−02i	−1.20696e−16 − 5.51604e−16i	7.50079e−47 + 2.53629e−46i*	6.49985e−124 − 3.43018e−124i*
1,6	$500\angle 60^\circ$	Ob	−1.21544e+05 + 2.14506e+05i	1.31250e+01 − 0.00000e+00i	−3.83750e+02 − 2.86544e+02i	2.79506e+19 + 3.72288e+18i	2.37801e+47 + 8.16947e+46i	7.04821e+93 + 5.48150e+93i	1.20968e+140 + 1.83386e+140i	−1.04618e+183 − 3.27141e+182i
1,6	$500\angle 75^\circ$	Ob	−2.10729e+05 + 1.23447e+05i	1.31250e+01 + 8.17124e−14i	2.11284e+02 − 7.44008e+02i	3.16680e+21 − 1.24662e+21i	2.35866e+52 − 3.21390e+52i	−6.23817e+103 − 1.97826e+104i	−5.42446e+155 − 1.81443e+155i	−5.73412e+201 + 1.11599e+201i
1,6	$500\angle 90^\circ$	Ob	−2.44018e+05 + 3.02488e−11i	1.31250e+01 + 5.53261e−17i	9.17185e+02 − 2.76921e−13i	1.86978e+22 − 5.71687e+07i	2.82034e+54 − 2.15759e+40i	1.03807e+108 − 1.58874e+94i	2.02376e+161 − 4.64659e+147i	7.42759e+208 − 2.27304e+195i
1,6	$750\angle 0^\circ$	Pr	8.23523e+03 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	8.09914e+00 − 0.00000e+00i	3.06176e−01 − 0.00000e+00i	−1.48697e−05 − 0.00000e+00i	−9.10492e−37 − 0.00000e+00i	−8.64680e−102 − 5.27936e−118i*	−2.27679e−268 − 1.38059e−284i*
1,6	$750\angle 15^\circ$	Pr	7.95412e+03 + 2.13526e+03i	1.31250e+01 + 2.22045e−16i	8.23486e+00 − 1.21499e+00i	−1.15269e+00 − 7.26318e+00i	−1.91394e−05 − 2.78661e−05i	−2.79214e−35 + 2.04851e−36i	2.47580e−98 + 4.34406e−98i*	6.60814e−259 + 4.23152e−259i*
1,6	$750\angle 30^\circ$	Ob	2.82748e+05 + 4.84541e+05i	1.31250e+01 + 3.14859e−13i	2.67562e+02 − 5.75238e+01i	−5.44332e+16 − 8.89290e+16i	1.17020e+41 + 1.75601e+41i	−2.61641e+81 + 6.27696e+81i	−1.94185e+122 + 3.81350e+121i	−2.26639e+160 − 3.12801e+160i
1,6	$750\angle 45^\circ$	Pr	3.70656e+03 + 3.71231e+03i	1.31250e+01 − 0.00000e+00i	1.06968e+01 − 2.27039e+00i	2.17136e+01 − 1.13499e+01i	−8.43679e−04 − 4.73885e−04i	6.24673e−26 − 2.25534e−26i	−3.20779e−72 − 6.23551e−72i*	3.33109e−191 − 2.18185e−189i*
1,6	$750\angle 60^\circ$	Ob	−2.76062e+05 + 4.84139e+05i	1.31250e+01 − 3.75255e−14i	−3.42479e+03 + 2.38563e+03i	6.98869e+28 + 1.40864e+28i	7.10417e+70 + 3.85481e+70i	5.03559e+140 + 7.73595e+140i	6.04335e+209 + 7.29318e+210i	1.14358e+276 − 1.08634e+276i
1,6	$750\angle 75^\circ$	Ob	−4.78464e+05 + 2.78921e+05i	1.31250e+01 − 4.82344e−09i	−3.13239e+03 − 8.06704e+03i	8.85059e+31 − 5.57560e+31i	−1.08659e+78 + 6.51347e+78i	−5.39029e+156 − 1.84637e+156i	1.23708e+234 − 2.30742e+234i	−4.58337e+305 + 5.56569e+305i
1,6	$750\angle 90^\circ$	Ob	−5.53518e+05 + 6.83353e−11i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1000\angle 0^\circ$	Pr	1.09852e+04 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	6.58117e+00 − 0.00000e+00i	−4.36254e+00 − 0.00000e+00i	−1.33682e−08 − 0.00000e+00i	−6.22676e−51 − 0.00000e+00i	−3.99598e−138 − 2.44154e−154i*	± 0
1,6	$1000\angle 15^\circ$	Pr	1.06104e+04 + 2.84701e+03i	1.31250e+01 − 0.00000e+00i	6.74243e+00 − 1.54190e+00i	−7.07023e+00 − 1.44686e+00i	−1.78893e−08 + 3.54559e−08i	4.66259e−49 + 3.76203e−49i	−1.75167e−133 − 3.70061e−133i*	± 0
1,6	$1000\angle 30^\circ$	Ob	5.01998e+05 + 8.62561e+05i	1.31250e+01 + 9.50351e−14i	−6.89067e+02 − 6.60822e+02i	5.79820e+21 + 2.73710e+22i	−7.59937e+54 − 2.04998e+54i	8.17884e+108 + 4.75698e+108i	−7.23312e+162 − 7.29286e+162i	−2.09363e+214 − 5.25411e+213i
1,6	$1000\angle 45^\circ$	Pr	4.94400e+03 + 4.94975e+03i	1.31250e+01 + 8.88178e−16i	9.88830e+00 − 2.95494e+00i	3.19385e−01 − 1.91513e+01i	−2.53549e−06 − 5.86038e−06i	2.79781e−36 + 5.47015e−36i	8.50676e−98 + 1.16255e−97i*	−4.29013e−255 − 2.61095e−255i*
1,6	$1000\angle 60^\circ$	Ob	−4.93080e+05 + 8.62025e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1000\angle 75^\circ$	Ob	−8.54452e+05 + 4.96894e+05i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1000\angle 90^\circ$	Ob	−9.88018e+05 + 1.21730e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1500\angle 0^\circ$	Pr	1.64852e+04 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	3.77180e+00 − 0.00000e+00i	−2.71761e+00 − 0.00000e+00i	−6.09038e−15 − 0.00000e+00i	−1.72404e−79 − 0.00000e+00i	−4.61630e−211 − 2.82258e−227i*	± 0
1,6	$1500\angle 15^\circ$	Pr	1.59230e+04 + 4.27052e+03i	1.31250e+01 − 0.00000e+00i	3.95879e+00 − 2.08916e+00i	−2.46156e+00 + 2.86749e+00i	−1.53213e−14 − 2.69471e−14i	−9.39280e−77 + 1.32605e−76i	−4.18926e−204 − 1.45313e−203i*	± 0
1,6	$1500\angle 30^\circ$	Ob	1.12800e+06 + 1.94336e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1500\angle 45^\circ$	Pr	7.41887e+03 + 7.42462e+03i	1.31250e+01 − 0.00000e+00i	8.27909e+00 − 4.21945e+00i	−7.51397e+00 + 2.72976e+00i	1.53946e−10 − 1.19012e−10i	−2.45921e−56 − 2.79999e−56i	3.05719e−149 + 2.17096e−149i*	± 0
1,6	$1500\angle 60^\circ$	Ob	−1.11462e+06 + 1.94256e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1500\angle 75^\circ$	Ob	−1.93119e+06 + 1.12034e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$1500\angle 90^\circ$	Ob	−2.23202e+06 + 2.74443e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$2500\angle 0^\circ$	Pr	2.74852e+04 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	−1.00748e+00 − 0.00000e+00i	−1.26944e−01 − 0.00000e+00i	−4.77945e−28 − 0.00000e+00i	−5.61115e−137 − 3.43168e−153i*	± 0	± 0
1,6	$2500\angle 15^\circ$	Pr	2.65482e+04 + 7.11753e+03i	1.31250e+01 − 0.00000e+00i	−8.52890e−01 − 2.80309e+00i	1.35821e−01 + 1.46057e−01i	−4.48686e−27 + 5.58542e−27i	2.39985e−132 − 4.45906e−132i*	± 0	± 0
1,6	$2500\angle 30^\circ$	Ob	3.13000e+06 + 5.40400e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$2500\angle 45^\circ$	Pr	1.23686e+04 + 1.23744e+04i	1.31250e+01 − 1.77636e−15i	5.10655e+00 − 6.34783e+00i	4.50847e−01 − 5.28533e−01i	−4.09989e−20 + 8.80474e−20i	−7.42276e−97 − 2.80307e−97i*	1.30606e−252 − 3.05082e−254i*	± 0
1,6	$2500\angle 60^\circ$	Ob	−3.10769e+06 + 5.40266e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$2500\angle 75^\circ$	Ob	−5.38370e+06 + 3.11724e+06i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$2500\angle 90^\circ$	Ob	−6.22002e+06 + 7.63567e−10i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$5000\angle 0^\circ$	Pr	5.49852e+04 − 0.00000e+00i	1.31250e+01 − 0.00000e+00i	−8.85653e+00 − 0.00000e+00i	−4.70471e−06 − 0.00000e+00i	−1.31931e−61 − 0.00000e+00i	−1.96319e−281 − 1.20136e−297i*	± 0	± 0
1,6	$5000\angle 15^\circ$	Pr	5.31112e+04 + 1.42350e+04i	1.31250e+01 − 4.44089e−16i	−9.08050e+00 − 2.89744e+00i	−8.93746e−06 − 6.66771e−06i	2.53078e−59 + 1.52306e−59i	1.56057e−271 + 3.14947e−272i*	± 0	± 0
1,6	$5000\angle 30^\circ$	Ob	1.25100e+07 + 2.16333e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$5000\angle 45^\circ$	Pr	2.47430e+04 + 2.47487e+04i	1.31250e+01 − 8.88178e−16i	−2.38368e+00 − 9.56416e+00i	−3.90250e−04 + 1.54028e−04i	−1.22212e−43 + 1.09575e−43i	3.44434e−199 − 3.97904e−199i*	± 0	± 0
1,6	$5000\angle 60^\circ$	Ob	−1.24654e+07 + 2.16306e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$5000\angle 75^\circ$	Ob	−2.15927e+07 + 1.24845e+07i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$
1,6	$5000\angle 90^\circ$	Ob	−2.49400e+07 + 3.05794e−09i	−0.00000e+00 − 0.00000e+00i	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$	$\pm\infty$

Table D.55: $S_{1n}^{(a)\prime}(c,\,\eta)$ for $n = [1\,..\,6]$.

Publications

Several parts of this thesis are either published, submitted for publication, or being prepared for publication. The following table summarizes the status of these publications.

Chapter 2	B. E. Barrowes, C. O. Ao, F. L. Teixeira, J. A. Kong, and L. Tsang, "Monte Carlo simulation of electromagnetic wave propagation in dense random media with dielectric spheroids," *IEICE Trans. on Electronics*, vol. E83-C, no. 12, pp. 1797–1802, December 2000.
Chapter 3	B. E. Barrowes, C. O. Ao, F. L. Teixeira, and J. A. Kong, "Sparse matrix/canonical grid method applied to 3-D dense medium simulations," *IEEE Trans. on Antennas and Propagation*, vol. 51, no. 1, pp. 48–58, Jan. 2003.
Chapter 4	B. E. Barrowes, K. O'Neill, T. M. Grzegorczyk, and J. A. Kong, "Asymptotic expansions of the prolate angular spheroidal wave function for complex size parameter," *Studies in Applied Mathematics*, Oct. 2003, submitted for publication.
Chapter 5	B. E. Barrowes, K. O'Neill, T. M. Grzegorczyk, X. Chen, and J. A. Kong, "Broadband electromagnetic induction solution for a conducting and permeable spheroid," *IEEE Trans. on Geoscience and Remote Sensing*, Jan. 2004, submitted for publication.
Chapter 6	B. E. Barrowes, K. O'Neill, T. M. Grzegorczyk, and J. A. Kong, "Simultaneous analytical solution for the broadband magnetoquasistatic electromagnetic induction (emi) response from multiple conducting and permeable spheroids," *IEEE Trans. on Geoscience and Remote Sensing*, Feb. 2004, submitted for publication.
Appendix A	B. E. Barrowes, F. L. Teixeira, and J. A. Kong, "Fast algorithm for matrix-vector multiply of asymmetric multilevel block-toeplitz matrices in 3-D scattering," *Microwave Opt. Technol. Lett.*, vol. 31, no. 1, pp. 28–32, Oct. 2001.
Appendix B	B. E. Barrowes, C. O. Ao, J. A. Kong, and K. O'Neill, "Electromagnetic induction (EMI) response from conducting and permeable spheroidal shells," in *Progress In Electromagnetics Research Symposium*, J. A. Kong, Ed., 2002, p. 227.

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Biographical note

Benjamin Earl Barrowes was born in Baton Rouge, Louisiana on July 11, 1973. He is the son of Steven C. Barrowes, Ph.D., and Hope R. Barrowes of Salt Lake City, Utah and is the youngest of six children who are in order: Wilford, Thomas, Edward, Stephanie, Winston, and Benjamin.

Benjamin studied Electrical Engineering at Brigham Young University (BYU) from September 1991 to June 1999, receiving both a B. S. and M. S. degrees in June 1999. From September 1992 until September 1994, he served a volunteer proselyting mission for the Church of Jesus Christ of Latter-Day Saints in Tokyo, Japan. While at BYU, he interned at Seino Joho Service in Ōgaki, Gifu prefecture, Japan from June 1996 to December 1996, as well as at Hewlett Packard (now Agilent) in Santa Rosa, CA from June 1998 to August 1998. From July 1999, he has been a research assistant at the Center for Electromagnetic Theory and Application within the Research Laboratory of Electronics in the Department of Electrical Engineering and Computer Science at the Massachusetts Institute of Technology, Cambridge, Massachusetts under Professor Jin Au Kong. There, his research has focused on efficient scattering calculations for dielectric spheroidal inclusions and induction models for conducting and permeable spheroids.

Benjamin Barrowes was named Sterling Scholar (or top student) in mathematics for the State of Utah during his senior year at Cottonwood High School, 1991. Thereafter he was awarded a full tuition four year scholarship to attend Brigham Young University. From 1997-9, he was the recipient of the Rocky Mountain Space Grant Consortium (RMSGC) grant during which time he worked on radar remote sensing of the sea state under Professor David G. Long. He was also the recipient of a graduate fellowship from the National Science Foundation September 1999 through September 2002. He has been a student member of the IEEE since 1998.

Benjamin married Masako I on February 20, 1996 in South Jordan, Utah. They currently live in Cambridge, Massachusetts with their two children, Leia and Earl.