Planning and Scheduling Decisions in Supply Chains with Multiple Supply Modes: An Integrated Approach

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***Abstract* — We address the problem of jointly determining shipment planning and scheduling decisions with the presence of multiple shipment modes. We consider long lead time, less expensive sea shipment mode, and short lead time but expensive air shipment modes. Existing research on multiple shipment modes largely address the short term scheduling decisions only. Motivated by an industrial problem where planning decisions are independent of the scheduling decisions, we investigate the benefits of integrating the two sets of decisions. We develop sequence of mathematical models to address the planning and scheduling decisions. Preliminary computational results indicate improved performance of the integrated approach over some of the existing policies used in real-life situations.**

***Index Terms* —sea shipments, air shipments, planning, scheduling**

I. INTRODUCTION

FIRMS often employ multiple-mode delivery systems for demand fulfillment in globally disbursed supply chains. Consider the case of an industrial goods distributor who sources products globally and distributes these products via regional hubs and country warehouses to end customers. Typically, the regional hub places consolidated orders to the suppliers based on available demand forecasts. These orders are shipped to the hub through sea mode which is less expensive but has long lead time. By the time the orders arrive at the hub, new demand information is available and this necessitates placing additional orders by a much more expensive but short lead time air/emergency- air modes. Our discussions with several Singapore-based distributors of industrial goods indicate that traditionally, the decisions for sea and air shipments are made independently. The decision pertaining to sea shipments seeks to fulfill demand forecasts while capturing the economies of scale of the available options. The air/emergency-air shipment decision, in contrast, considers the tradeoff between inventory holding costs, shortage costs and transportation costs. In this research we propose a framework which integrates these two decisions. We believe that such an integrated approach will ensure feedback between the two decisions and enable managers to provide more responsive customer service.

The problem is motivated by the Singapore based operations of a third party logistics service provider which serves as a hub for the inbound and outbound logistics system of a large automobile parts distributor (see Figure 1). Shipments of multiple products are received at the Singapore hub from suppliers located in USA, Europe and Asia (inbound logistics). For example, there are m1 suppliers located in USA as shown is Figure 1. These products are then shipped to customers with distribution warehouses in various countries (outbound logistics). For the inbound logistics, transportation alternatives include the low cost – long lead time sea shipments, high cost-short lead time air shipments and the very high cost – short lead time emergency-air shipments. Air shipments arise primarily due to the economies of scale associated with the sea shipments while emergency-air shipments are required to meet the desired service levels in the presence of high demand variability occurring in over a shorter time horizon. In Figure 1, the shipping modes are shown from supply countries to the hub at Singapore. The outbound logistics system services distribution warehouses of various customers in countries like Japan, Korea, Taiwan, Singapore, Malaysia, Indonesia, China, Australia, New Zealand and Hong Kong. In this research we focus primarily on the inbound logistics system.

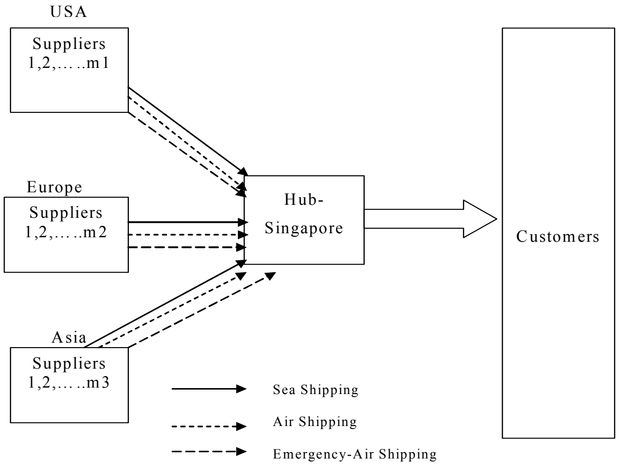

Fig. 1 Inbound and Outbound Logistics with Dual Shipping Modes

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Manuscript received November 19, 2004. This work was supported by Singapore-MIT Alliance.

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From a cost perspective, the hub's preferred strategy is to use the sea mode for shipments. The sea shipments are characterized by low costs, economies of scale, long lead times and are based on demand forecast. However the hub must plan orders based only on demand forecast. Given demand uncertainty, companies risk shortages and/or excessive inventories when there is a mismatch between the actual demand and the quantities shipped by sea. The key issue then is to complement these sea shipments by the more responsive air shipments and emergency-air shipments to balance the inventory/shortage costs and desired service levels.

In practice, the sea shipments have different lead times corresponding to the various suppliers. The sea shipment lead times are generally one week. In order to satisfy the demand of a particular period, the suppliers have to ship the products one week before the period of actual demand. Since multiple suppliers are involved in shipping multiple products, it is desirable to stagger the shipment arrivals at the hub.

The costs that must be considered are the transportation costs of products through sea, air and emergency-air modes, inventory holding costs at the hub and the backorder costs in case of shortages. The decisions to be taken at the hub are:

1. Determine quantity to be ordered through sea and air modes in each time period of the planning horizon
2. Determine quantity to be ordered through emergency-air mode in response to imminent shortages
3. Determine schedule of shipments through all supply modes

The first set of decisions is tactical in nature. Given the ratio of air to sea variable costs of shipments, economies of scale, and the capacity constraints of sea shipments (due to lease of a limited number of containers), the minimum cost split between the amount of sea and air shipments needs to be determined in order to meet the demand forecast. This is the shipment planning problem at the hub. The second and third sets of decisions are more operational in nature. Due to uncertainty of demand, inventory will fluctuate on a real time basis. Based on the inventory position at the hub, the operational problem is to determine the schedule of sea and air shipments, and to determine the timing and size of the emergency-air shipments in order to minimize the inventory and shortages costs. The decisions of the operational problem are based on the long term demand as compared to the demand forecast used in the planning problem. As seen in Figure 2, the planning problem decisions are inputs to the detailed operational decisions.

We address the shipment planning and scheduling decisions through a sequence of mathematical models. First, we develop a mixed integer program (MIP) to solve the shipment planning problem. Next, we develop a stochastic dynamic program to implement the shipment planning decisions and to obtain the schedule of sea, air and emergency-air shipments. We test the models with reference to several benchmark models which are based on typical real life heuristics used by managers. The preliminary results indicate that the proposed models lead to significant cost savings over the benchmark models. Several studies in literature address the problem of dual supply modes with some restricted assumptions about the lead times, fixed and variable transportation costs, and assumptions on the numbers and sizing of air shipments. Studies which are general in terms of lead times and transportation costs have found limited implementation in practice due to their computational complexity. In this research we consider the most general problem in terms of lead times and number of air shipments, and develop a solution procedure that can be easily implemented to determine shipment decisions and schedules.

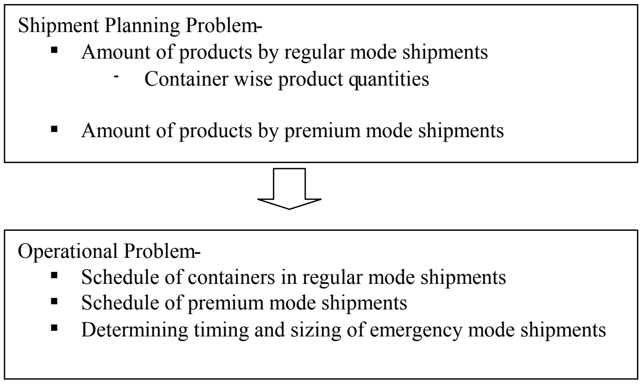

Fig. 2. Schematic of the Overall Problem

The remainder of this paper is organized as follows. In the next section we review some of the existing research on optimization policies with dual supply modes. In section 3 we describe our problem environment and develop the mathematical models that address the shipment planning and scheduling decisions. In section 4 we provide the solution procedure to solve the shipment planning and scheduling problem. We discuss the experiment design and some preliminary results of the solution procedure in section 5. Finally, in section 6 we discuss the on-going progress and future directions of this work.

II. LITERATURE REVIEW

Related research for multiple (mainly dual) supply modes includes early papers of Barankin [1] and Neuts [2]. They studied periodic review system with sea and air shipments with one period lead time and instantaneous replenishment respectively. Fukuda [3] and Veinott [4] allow longer lead times for air supplies but always differing with sea supply modes by one period. Rosenshine and Obee [5] investigate an inventory system where a sea order of constant size is shipped every period and an air order of fixed size may be placed once every period. However, the air lead time is assumed to be negligible in their analysis. Whittemore and Saunders [6] assume that the sea and air lead times are multiple of the review period and derive optimal policy for this system, but it was too complex to implement in practice.

Chiang and Gutierrez [7] analyze a periodic review inventory system with two supply modes, where at each review period a decision is made about which of the two supply modes to use. Their work is the first one that considers lead times shorter than the review period. However, they assume the variable costs of air orders to be same as those of the sea orders. In a sequel paper, Chiang and Gutierrez [8] allow multiple air orders within a review period with large variable cost of air orders. They analyze the problem within the stochastic dynamic program framework, as a result their policy is complex and difficult to implement, especially if the two lead times differ by more than one time unit. Tagaras and Vlachos [9] analyze a period review system with lead times shorter than the review period. They assume that only one air order can be placed within a review period and it can be placed near the end of the review period only. Chiang [10] analyze the period review inventory system with fixed costs as well as high variable costs of air shipments. They develop a policy which has a critical inventory level such that if the inventory position at a review period falls below this level, an air order is placed.

Continuous review inventory models with air shipments have been studied by Moinzadeh and Nahmias [11]. They propose a heuristic control policy, which is an extension of the standard (Q, R) policy. Moinzadeh and Schmidt [12] consider the (S-1, S) inventory system and propose a policy with no fixed cost of air orders. Their policy holds well when the demand variability is low. Moinzadeh and Agarwal [13] extend the results of (S-1, S) inventory policy to a multiechelon system while utilizing the information on outstanding orders.

It is seen in the literature review that most of the problems in the literature have been analyzed with some restrictive assumptions about the cost structure, and lead times. Moreover, problems relaxing the above assumptions are found to be complex for real life implementation. In the next section, we describe the development of mathematical models that address the shipment planning and scheduling decisions.

III. SHIPMENT PLANNING AND SCHEDULING MODELS

In this section we describe the problem environment, and present the mathematical models for addressing the shipment planning and scheduling decisions over a finite planning horizon with demand forecast.

The lead time for the sea shipments from different suppliers is one week. The shipments are carried in containers of standard sizes. The capacity of a container is usually measured by its volume. The unit variable transportation cost ($/volume) of container shipping is dependent on the shipment quantity. The cost is measured on the basis of Less than Container Load (LCL) and Full Container Load (FCL). Usually, there is some amount of fixed cost also for each container used by the supplier. We assume that there are no fixed costs of placing air orders. The unit variable transportation cost of placing an air order (usually measured in $/weight) is more than that of a sea shipment ($/volume). There is an inventory carrying cost of each product. Shortages are backordered and filled whenever shipments are received. The sea shipments follow a weekly review of inventory position and orders are placed every week. For illustration, the sea shipments review epochs (t_1, t_2, t_3) and air/emergency-air shipments review epochs (t_{11}, t_{12}, …t_{17}, t_{21}, t_{22},…t_{27}) are indicated in Figure 3. For illustration, the sea shipment order with a lead time of one week placed at period t_1 arrives at period t_2. The air orders (lead time of one day) placed at t_{11} will arrive at t_{12} and so on.

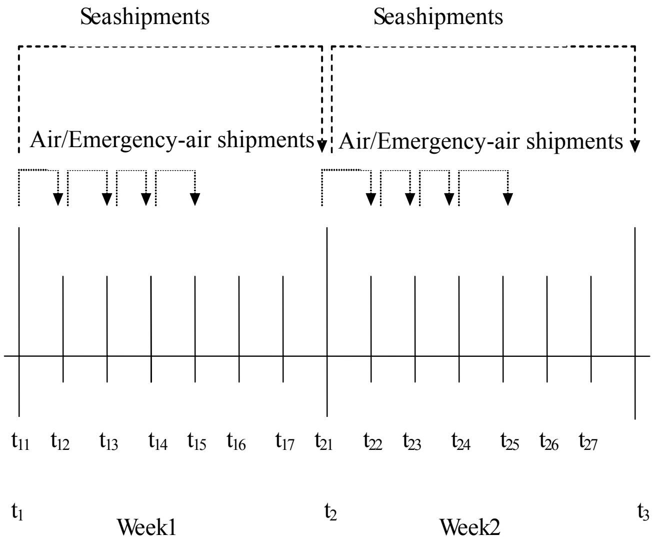

Fig.3. Sea, Air and Emergency-Air Shipment Modes Review Periods

Sea shipments orders are placed at pre-determined sea review time epochs. The shipment scheduling problem is to determine the schedule of sea and air shipments between two review periods. In case of high demand, emergency-air orders can also be placed in between the two review periods to replenish the inventory in a more responsive manner. This will avoid possible stockouts that might occur if replenishment is postponed until the next review period.

We address the shipment planning and scheduling decisions through a sequence of mathematical models. We develop shipment planning model as a MIP. The time unit of the planning model is one week. Over a finite planning horizon comprising eight to ten weeks, the decisions of the planning model are to determine the amount of sea shipments and air shipments to meet the demand forecast. The output of the planning model impose constraints on the operational model in terms of quantities of sea and air shipments. The formulation of shipment planning model is provided below.

3.1 Shipment Planning Problem Formulation

We begin with some notation used in the formulation.

Indices and Sets

P : set of products, $\{ i \mid i = 1, 2, \dots n \}$
S : set of supplier countries, $\{ j \mid j = 1, 2, \dots m \}$
T : set of weekly time periods, $\{ t \mid t = 1, 2, \dots TP \}$
K : set of containers, $\{ k \mid k = 1,2,\dots\dots N\}$

Parameters

d_{it} : demand of product i in time period t (kg), $i \in P,\ t \in T$.
cr_j : fixed cost of using a container from supplier j *(\$/container),* $j \in S$
csf_{ij} : transportation cost (FCL) of sea shipment of product i from supplier j *(\$/m^3),* $i \in P, j \in S$
csl_{ij} : transportation cost (LCL) of sea shipment of product i from supplier j *(\$/m^3),* $i \in P, j \in S$
ca_j : transportation cost of air mode from supplier j (\$/kg), $j \in S$
VA_i : value of product i *(\$),* $i \in P$
ch_i : unit inventory holding cost of product i, constant * VA_i *(\$/kg/week)*
V : volume capacity of a container *(m^3)*
N : total number of containers available
M : a large number
ρ_i : density of product i *(kg/m^3),* $i \in P$
B_j : lead time of sea shipment from supplier j, $j \in S$

Decision Variables

X_{ijt} : amount of product i shipped from supplier j with air mode in time period t, $i \in P, j \in S, t \in T$
XRF_{ijt} : amount of product i shipped from supplier j in *FCL* with sea mode in time period t, $i \in P, j \in S, t \in T$
$XRFC_{ijkt}$: amount of product i shipped from supplier j in *FCL* with sea mode in container k in time period t, $i \in P, j \in S, k \in K, t \in T$
XRL_{ijt} : amount of product i shipped from supplier j in *LCL* with sea mode in time period t, $i \in P, j \in S, t \in T$
$XRLC_{ijkt}$: amount of product i shipped from supplier j in *LCL* with sea mode in container k in time period t, $i \in P, j \in S, k \in K, t \in T$
I_{it} : ending inventory of product i in period t, $i \in P, t \in T$
N_{jt} : number of containers used by supplier j in time period t *(m^3),* $j \in S, t \in T$
y_{kjt} : *1,* if container k is used by supplier j in time period t, $j \in S, k \in K, t \in T$
0, otherwise
b_{jt} : *1,* if FCL is shipped by a supplier j in time period t, $j \in S, t \in T$
0, otherwise

The shipment planning problem can be formulated as follows:

$$\text{Minimize } Z = \sum_{i=1}^{n}\sum_{t=1}^{T} ch_i I_{it} + \sum_{j=1}^{m}\sum_{t=1}^{T} cr_j N_{jt} + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} csf_{ij}\left(\frac{XRF_{ijt}}{\rho_i}\right) + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} csl_{ij}\left(\frac{XRL_{ijt}}{\rho_i}\right) + \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{t=1}^{T} ca_j X_{ijt}$$

Subject to:

$$I_{it} = I_{it-1} + \sum_{j=1}^{m} XRF_{ijt-Bj} + \sum_{j=1}^{m} XRL_{ijt-Bj} + \sum_{j=1}^{m} X_{ijt-Bj} - d_{it} \qquad \forall i \in P, j \in S, t \in T \qquad (1)$$

$$\sum_{j=1}^{m} N_{jt} \le N \qquad \forall j \in S, t \in T \qquad (2)$$

$$\sum_{i=1}^{n}\left(\frac{XRFC_{ikjt} + XRLC_{ikjt}}{\rho_i}\right) \le y_{kjt} V \qquad \forall i \in P, k \in K, j \in S, t \in T \qquad (3)$$

$$\sum_{k=1}^{K} XRFC_{ikjt} = XRF_{ijt} \qquad \forall i \in P, k \in K, j \in S,, t \in T \qquad (4)$$

$$\sum_{k=1}^{K} XRLC_{ikjt} = XRL_{ijt} \qquad \forall i \in P, k \in K, j \in S,, t \in T \qquad (5)$$

$$\sum_{k=1}^{K} y_{kj} = N_{jt} \qquad \forall k \in K, j \in S, t \in T \qquad (6)$$

$$\sum_{j=1}^{m} y_{kjt} \le 1 \qquad \forall k \in K, j \in S, t \in T \qquad (7)$$

$$\sum_{i=1}^{n} XRF_{ijt} \le b_{jt} M \qquad \forall i \in P, j \in S, t \in T \qquad (8)$$

$$\sum_{i=1}^{n} XRL_{ijt} \le (1 - b_{jt}) V \qquad \forall i \in P, j \in S, t \in T \qquad (9)$$

$$XRF_{ijt}, XRFC_{ikjt}, XRL_{ijt}, XRLC_{ikjt}, X_{ijt}, I_{it}, N_{jt} \ge 0$$

$$y_{kjt}, b_{jt} \text{ binary}$$

Constraint 1 is the inventory balance constraint with sea shipment lead time lag and ensures that the demand of a product in each period is satisfied. Constraint 2 is the upper bound on total number of containers than can be used across all suppliers. Constraint 3 ensures that a containers' volume capacity is not exceeded. Constraints 4 and 5 equate FCL/LCL shipments over all the containers used by a supplier. Constraint 6 indicates the total number of containers used by a supplier. Constraint 7 ensures that a container is allocated to at most one supplier only. Constraints 8 and 9 ensure that either FCL or LCL, as the case may be, is shipped from a supplier.

From the planning model, we determine the composition of each container to be shipped from each supplier. We also determine the air shipment mode product quantity from each supplier. The next step is to determine the schedule of sea, air and emergency-air shipments.

3.2 Operational Model

We begin by addressing the scheduling decisions of air shipment modes. The air shipments quantity is determined in the shipment planning model. The lead time for air shipments is one day. The main decision problem is to determine the schedule of the shipments during the week to strike a right balance between the inventory carrying costs and the possible shortage costs. If the shipments are done early in the week, high inventory costs are incurred whereas if shipments are delayed to the end of the week, there may be stockouts.

We develop the operational model within the framework of stochastic dynamic program. The stage here is defined as the daily time period and state is defined as the net inventory (on hand inventory-backorders). The dynamic program to determine the optimal premium mode quantity is formulated as follows:

3.2.1 Stochastic Dynamic Program Formulation of the Operation Model

Notation

j : number of daily time periods remaining in a week (stage)

λ : mean demand during a daily time period

x_j : net inventory (on hand - backlog) at the beginning of stage j (state)

t_j : random variable of demand at stage j

$\phi(t)$: probability distribution function of demand during a period

$l(.)$: loss function at which net inventory is charged at the end of period

$L(x,r)$: expected inventory and backorders cost at the end of period

$$L(x,r) = \int_0^\infty l(x+r-t)\,d\phi(t)$$

$V_j(x)$: expected cost with j periods remaining until the end of the week, when the starting net inventory is x, and an optimal ordering policy is used at each stage.

r_j : sea shipment quantity at stage j

n : total number of periods

The optimal cost recursion with j stages to go is:

$$V_j(x_j) = \min_{r\geq 0}\left(L(x_j, r_j) + V_{j+1}\left(\tilde{x}_{j+1} \right) \right)$$

where, the transition function is

$$\tilde{x}_{j-1} = x_j + r_j - \tilde{t}_j$$

$V_{n+1}(x_{n+1})$:0

This gives the optimal order quantity of sea shipment on daily basis while minimizing the inventory cost and the backorder costs and facing uncertain demand. If the planning model has suggested air shipments higher than those determined by the operational model, the excess quantity of air shipments is scheduled at the end of the weekly period so that it can be used for future periods. There is also an option of emergency-air shipments within a week after the realization of actual demand. The lead time for an emergency-air shipment is one day. The emergency order can be made once in a week and its quantity is determined on the basis of current inventory position, number of days remaining in a week, and schedule of air shipments. In the next section we provide the solution procedure for solving the shipment planning and scheduling problem.

IV. SOLUTION PROCEDURE

The shipment planning problem is solved using the branch and bound algorithm. The MIP model is written in GAMS and is solved with commercial solver CPLEX. Weekly forecast of demand of each product is an input to the shipment planning model. The output of the planning model is the sea and air mode shipment quantity of each product in each time period of the planning horizon. This output is provided as input to the operational model.

The operational model is solved for each product for one week period of the planning horizon. The time unit of operational model is one day. At the beginning period of operational model (starting day), the inventory position is updated after arrival of sea shipments ordered earlier to meet the demand of the current week. Considering the expected daily demand, the inventory costs, and the shortage costs, daily schedule and quantity of the air shipments are determined.

The stochastic dynamic programs are computationally very intensive. As the state variable (net inventory in this problem) can have many possible values, the demand being random variable, the determination of minimum expected cost will require evaluation of many possible states of resulting inventory. To make the problem computationally tractable, we assume that actual demand in the previous stage was same as the mean demand. This allows us to evaluate only one net inventory state at a particular stage. We determine the minimum cost decision for this state considering all possibilities of demand in the coming day. This algorithm is solved in polynomial time. As indicated in Figure 4 with a 2 stage example, we start with inventory x2 (2 stages to go) and get the minimum cost order quantity r2* by evaluating all branches of the decision tree considering all possibilities of demand (d2) with 2 stages to go. At 1 stage to go, we assume that demand in the previous period was expected demand λ. This means that the current net inventory $y = x1$ *is* $x + r^* - \lambda$ at one stage to go. We can determine the best decision to minimize expected cost at one stage to go with net inventory y. This way we determine the air shipments for the entire week.

On the realization of actual daily demand, there is an option of placing an emergency air order if the inventory position is below the expected lead time demand of emergency air shipment lead time. The emergency air order can be placed only once in a week and the quantity of emergency-air shipment is determined based on remaining daily time periods in a week, quantity of air orders due to arrive and the current inventory position. The variable cost per unit of an emergency-air shipment is considerably higher than the air shipment. In the next section, we describe the experiment design of the solution procedure and discuss some preliminary results.

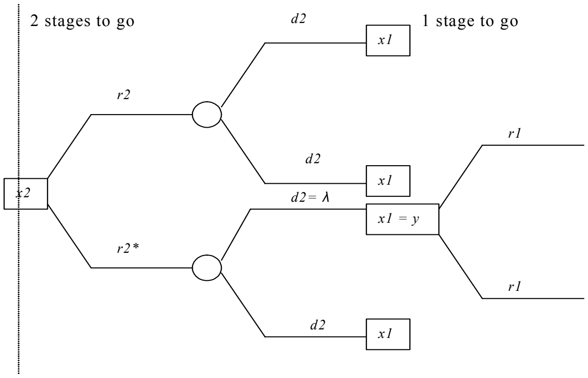

Fig. 4. Reduced Decision Tree to Determine Minimum Expected Cost in Operational Model

V. SOME PRELIMINARY RESULTS

We test the effectiveness of the proposed shipment planning and scheduling model by comparing it with some of the benchmark models based on the existing practices by the managers. The actual demand in derived through simulation. The benchmark models and the proposed model are subjected to the actual demand to compare their cost performance.

The benchmark models we consider are:

Benchmark model 1: Schedule all air shipments at the first daily period (Beginning of the week).

Benchmark model 2: Schedule all air shipments near the end of the week (Last daily period - air shipment lead time).

Benchmark model 3: Schedule all shipments in the middle of the week.

We assume daily demand to follow Poisson distribution with mean λ. The weekly demand will then follow Poisson distribution with mean 7λ. In a particular instance of single product supplied by a single supplier, we consider:

λ : 10
cr_j : \$10 /container
csf_{ij}: \$5/unit volume
ca_j : \$30/kg
ch_i : 0.01 \$/unit/day
V : 10 m^3
N : 5
ρ_i : 1 kg/m^3
bo_i : \$40/unit, backorder cost/unit
e_j : \$50/unit, emergency-air shipment cost/unit
BA : 1 day, air shipment lead time
BE : 1 day, emergency-air shipment lead time

The demand forecast for 10 weeks is generated following Poisson distribution with mean 70 and is input to the planning model. Actual daily demand is generated that follows Poisson distribution with mean $\lambda = 10$. The operation model is solved for a week with starting net inventory as the state of the system. The operational model determines the air shipment quantity and schedule for the week.

The results of the simulation are shown in Table 1. The table indicates for each model, the weekly inventory costs and backorder costs, the air and emergency-air mode shipment costs. The average weekly cost in a 10 week planning horizon is minimum in the proposed model. We consider two scenarios in the results. In scenario 1, the ratio of inventory to shortage cost is 1:4000. Average percentage savings in scenario 1 from the proposed models over the minimum cost benchmark model are 12 percent. In scenario 2, the inventory to shortage cost ratio is 1:40 and the savings from the proposed models over the minimum cost benchmark model are 17 percent.

VI. ON-GOING WORK

In this research we address the problem of shipment planning and scheduling decisions in the presence of multiple supply options- sea, air and emergency-air shipments. The option of multiple lead times gives managers the flexibility to capture the trade-offs between high inventory costs and backorder costs. The key decisions in the problem are to determine the amount of products to be shipped through each mode and the schedule of the shipments. The main objective of this study is to determine the benefits of integrating the planning and scheduling decisions of multiple transportation modes. We develop mathematical models to address these decisions and describe the solution procedure of the shipment planning and scheduling problem. We test the performance of the proposed models with some commonly used benchmark models in practice.

On going work in this research is to provide interaction between the shipment planning model and the operational model. The objective is to determine the impact of the feedback of operational model on the overall performance.

Table 1: **Shipment Planning and Scheduling Costs of Benchmark Models and Proposed Models**

Emergency-Air Shipment Cost ($/unit)	50	**Backorder Cost ($/unit)**	-40
Air Shipment Cost ($/unit)	30	**Inventory Cost ($/unit/day)**	0.01

			Benchmark Model 1				**Benchmark Model 2**			
			(1)	**(2)**	**(3)**	**(1)+(2)+(3)**	**(1)**	**(2)**	**(3)**	**(1)+(2)+(3)**
Weekly Period	**Demand Forecast**	**Actual Demand**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**
1	52	63	1.55	0.00	650.00	651.55	1.55	0.00	650.00	651.55
2	65	93	1400.74	450.00	750.00	2600.74	361.31	450.00	950.00	1761.31
3	68	71	120.79	540.00	800.00	1460.79	1.58	540.00	500.00	1041.58
4	60	63	1.35	300.00	100.00	401.35	121.80	300.00	0.00	421.80
5	74	78	1240.87	720.00	50.00	2010.87	41.80	720.00	500.00	1261.80
6	60	66	1.06	300.00	550.00	851.06	1.73	300.00	250.00	551.73
7	68	72	441.04	540.00	150.00	1131.04	41.90	540.00	50.00	631.90
8	60	80	80.95	300.00	1100.00	1480.95	1.41	300.00	1250.00	1551.41
9	74	61	1.71	720.00	0.00	721.71	3.15	720.00	0.00	723.15
10	60	64	2.18	300.00	0.00	302.18	2.78	300.00	0.00	302.78
					Average Total Cost	**1161.22**			**Average Total Cost**	**889.90**

			Benchmark Model 3				**Proposed Model**			
			(1)	**(2)**	**(3)**	**(1)+(2)+(3)**	**(1)**	**(2)**	**(3)**	**(1)+(2)+(3)**
Weekly Period	**Demand Forecast**	**Actual Demand**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**
1	52	63	1.55	0.00	650.00	651.55	2.01	990.00	0.00	992.01
2	65	93	560.98	450.00	750.00	1760.98	641.53	330.00	150.00	1121.53
3	68	71	1.30	540.00	800.00	1341.30	1.22	1200.00	0.00	1201.22
4	60	63	41.62	300.00	0.00	341.62	2.24	630.00	0.00	632.24
5	74	78	81.22	720.00	150.00	951.22	1.78	390.00	0.00	391.78
6	60	66	1.36	300.00	550.00	851.36	1.66	840.00	0.00	841.66
7	68	72	41.42	540.00	0.00	581.42	1.89	480.00	0.00	481.89
8	60	80	1.11	300.00	1250.00	1551.11	41.20	660.00	50.00	751.20
9	74	61	2.43	720.00	0.00	722.43	2.04	870.00	0.00	872.04
10	60	64	2.48	300.00	0.00	302.48	2.55	330.00	0.00	332.55
					Average Total Cost	**905.55**			**Average Total Cost**	**761.81**

Table 1: Shipment Planning and Scheduling Costs of Benchmark Models and Proposed Models

Emergency-Air Shipment Cost ($/unit)	50	**Backorder Cost ($/unit)**	-40
Air Shipment Cost ($/unit)	30	**Inventory Cost ($/unit/day)**	1

			Benchmark Model 1				**Benchmark Model 2**			
			(1)	**(2)**	**(3)**	**(1)+(2)+(3)**	**(1)**	**(2)**	**(3)**	**(1)+(2)+(3)**
Weekly Period	**Demand Forecast**	**Actual Demand**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**
1	52	63	155.00	0.00	650.00	805.00	155.00	0.00	650.00	805.00
2	65	93	1474.00	450.00	750.00	2674.00	491.00	450.00	950.00	1891.00
3	68	71	199.00	540.00	800.00	1539.00	158.00	540.00	500.00	1198.00
4	60	63	135.00	300.00	100.00	535.00	300.00	300.00	0.00	600.00
5	74	78	1327.00	720.00	50.00	2097.00	220.00	720.00	500.00	1440.00
6	60	66	106.00	300.00	550.00	956.00	173.00	300.00	250.00	723.00
7	68	72	544.00	540.00	150.00	1234.00	230.00	540.00	50.00	820.00
8	60	80	175.00	300.00	1100.00	1575.00	141.00	300.00	1250.00	1691.00
9	74	61	171.00	720.00	0.00	891.00	315.00	720.00	0.00	1035.00
10	60	64	218.00	300.00	0.00	518.00	278.00	300.00	0.00	578.00
			Average Total Cost			**1282.40**	**Average Total Cost**			**1078.10**

			Benchmark Model 3				**Proposed Model**			
			(1)	**(2)**	**(3)**	**(1)+(2)+(3)**	**(1)**	**(2)**	**(3)**	**(1)+(2)+(3)**
Weekly Period	**Demand Forecast**	**Actual Demand**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**	**Inventory and Backorder Cost**	**Air Shipment Cost**	**Emergency-Air Shipment Cost**	**Total Cost**
1	52	63	155	0	650	805	174	750	0	924
2	65	93	658	450	750	1858	958	390	450	1798
3	68	71	130	540	800	1470	98	1020	0	1118
4	60	63	202	300	0	502	182	630	0	812
5	74	78	202	720	150	1072	382	390	0	772
6	60	66	136	300	550	986	124	840	0	964
7	68	72	182	540	0	722	147	480	0	627
8	60	80	111	300	1250	1661	136	660	350	1146
9	74	61	243	720	0	963	186	690	0	876
10	60	64	248	300	0	548	213	330	0	543
			Average Total Cost			**1058.70**	**Average Total Cost**			**958.00**

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