Markov Process Modeling of A System Under WIPLOAD Control

Chao Qi, Appa Iyer Sivakumar, and Viswanath Kumar Ganesan

***Abstract*—This paper analyzes a proposed release control methodology, WIPLOAD Control (WIPLCtrl), using a transfer line case modeled by Markov process modeling methodology. The performance of WIPLCtrl is compared with that of CONWIP under 13 system configurations in terms of throughput, average inventory level, as well as average cycle time. As a supplement to the analytical model, a simulation model of the transfer line is used to observe the performance of the release control methodologies on the standard deviation of cycle time. From the analysis, we identify the system configurations in which the advantages of WIPLCtrl could be observed.**

***Index Terms*—Release control; WIPLOAD control; Markov process modeling**

I. Introduction

SINCE 1970s, an ever growing attention has been devoted by worldwide researchers and practitioners to the investigation of job release control. Job release process determines the time and amount of release of new jobs into a manufacturing system. As the link between production planning and control, job release control holds the practical significance of manufacturing industries. Its significant impacts on the system performances, especially for the complex manufacturing systems such as semiconductor wafer fabrications, have been demonstrated in the related literature [1]–[3].

To investigate the effects of the release control methodology used, in this study, we employ Markov process modeling methodology, which is well studied by Gershwin and Berman [4], Gershwin and Schick [5], and Gershwin [6]. We know that most of the analytical models of manufacturing systems are Markovian in nature [7]. A Markov chain is a natural modeling paradigm for discrete event dynamical systems because of the notions of discrete states and state transitions. Markov process modeling methodology is based on state model method to generate evaluative models for manufacturing systems. The idea behind is straightforward. Papadopoulos and Heavey [8] summarized it into three steps. First, all feasible states of the Markov chain describing the model are identified. In the second step, the transition matrix is generated from analyzing the states of the model. Then the stationary equations together with the boundary conditions can be used to solve for the stationary distribution.

In this paper, a transfer line is modeled using Markov process modeling methodology to analyze the performances of two release control methodologies, i.e. CONWIP and WIPLOAD Control (WIPLCtrl). CONWIP is proposed by Spearman et al. [9] as a new pull based production control methodology. The principle is to maintain a constant WIP level. New jobs cannot begin on a line until the WIP level has fallen below a specified level. The WIP here can be measured either in number of jobs or in time units.

WIPLCtrl, our proposed release control methodology, is described in the next section. A transfer line case is modeled subsequently in section three. In section four, the experiments carried out on the transfer line as well as the corresponding results are presented. The analysis based on the experiments is included in section five. It is followed by the conclusion drawn in section six.

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II. WIPLOAD Control

In this section, we define a new measure for the overall shop floor workload, which is named as system WIPLOAD. A closed-loop release control methodology based on WIPLOAD (WIPLCtrl) is described as well.

A. Definition of WIPLOAD

Indices

k = workstation $k = 1, \ldots, K$
i = product type $i = 1, \ldots, I$
S_i = number of operations for product type i
s = operation step $s = 1, \ldots, S_i$
$i(s)$= the s^{th} step in a route for the product type i
t = time

Parameters

$p_{i(s),k}$= processing time on workstation k to process one job undergoing the operation $i(s)$
$r_{i(s),k}$ = remaining processing time for the job undergoing the operation $i(s)$

Variables

$W_{i(s),k}(t)$= number of jobs undergoing operation $i(s)$ before machine k at time t
$L(t)$ = system WIPLOAD level at time t

Calculation

System WIPLOAD is defined as the sum of the remaining processing times of all the jobs on the shop floor. The involvement of the remaining processing times takes into account more system information when the shop load is measured. The WIP located at the front-end of the production line is considered to cause higher load for the shop floor in comparison with the WIP at the back-end. In this sense, WIPLOAD improves the conventional shop load measure.

$$L(t) = \sum_{k=1}^{K} \sum_{i=1}^{I} \sum_{s=1}^{S_i} W_{i(s),k}(t) \cdot r_{i(s),k} \qquad (1)$$

where

$$r_{i(s),k} = \sum_{s'=s}^{S_i} \sum_{k=1}^{K} p_{i(s'),k} \qquad (2)$$

B. WIPLOAD Control (WIPLCtrl)

The simplest way to control the release process based on WIPLOAD is to keep WIPLOAD at a specified level. This methodology is referred to as WIPLOAD Control (WIPLCtrl), which is depicted by the framework shown in Fig. 1. System

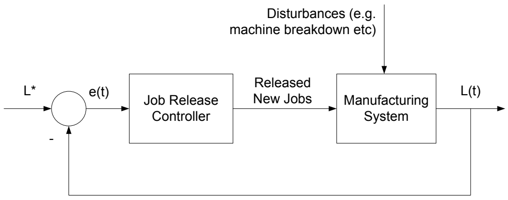

Fig. 1. Framework of WIPLOAD Control (WIPLCtrl)

WIPLOAD is the controllable variable in this framework. The value of the current WIPLOAD ($L(t)$) is continuously checked and feedback. Then $L(t)$ is compared with the reference WIPLOAD level (L^*), and their difference ($e(t)$) is computed. The release of a new job is triggered according to $e(t)$. As a result, the WIPLOAD fluctuation caused by the disturbances can be compensated so that WIPLOAD is maintained at the specified level, L^*. A simple job release controller is designed and described by the flow chart in Fig. 2.

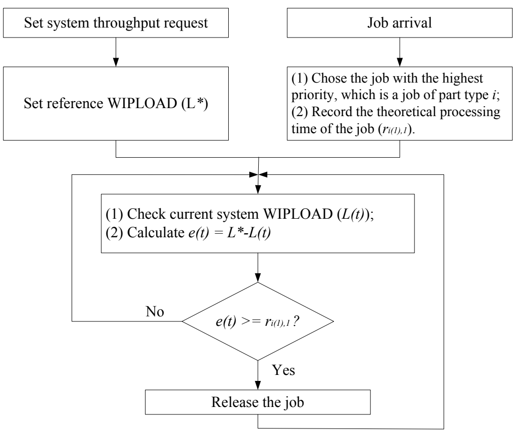

Fig. 2. Release Decision Making Process of WIPLCtrl

Assume that certain priority values have been set when the jobs arrive. The job with the highest priority, which is assumed to be a job of part type i, is considered to be released. The theoretical total processing time of the job ($r_{i(1),1}$) is recorded. In addition, we need to define a reference WIPLOAD level (L^*) that reflects a trade-off between the system throughput rate and the average cycle time level. A conceptual relationship among WIPLOAD, throughput and average cycle time is depicted in Fig. 3 to indicate that by adjusting the reference WIPLOAD level, an expected throughput level can be achieved for a specific system.

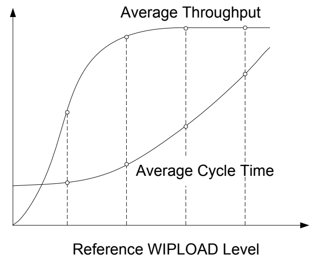

Fig. 3. Conceptual Relationship Among WIPLOAD, TH and CT

Under the assumption that the reference WIPLOAD, L^*, is given, the next step of WIPLCtrl is to determine when to trigger the release of the job. By checking the current WIPLOAD when an operation is completed, $L(t)$, which changes when a new job is released or when an operation is completed on a workstation, the difference ($e(t)$) between L^* and $L(t)$ is computed. The job with the highest priority is released when $e(t)$ is not less than its theoretical processing time ($e(t) \geq r_{i(1),1}$). In other words the reference WIPLOAD level is an upper load bound that cannot be exceeded when the release decisions are made.

III. A Transfer Line Case

A three-machine transfer line, shown in Fig. 4, is modeled using Markov process modeling methodology. It is actually a two-machine system since the first machine only represents the release process. In other words M_1 never breaks down. The second machine of the model corresponds to the first machine of the real system. This approach is described by Dallery and Gershwin [10]. With two buffers involved, this is the smallest model that can distinguish the effects of different release control methodologies.

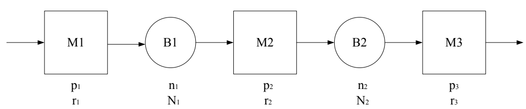

Fig. 4. A Transfer Line Case

A. Notations

M_i= machine i $\quad i = 1, \ldots, k$
B_i = buffer i $\quad i = 1, \ldots, k-1$
k = machine number of the transfer line
p_i = the probability of M_i failing during a time unit
r_i = the probability of M_i being repaired during a time unit
A_i = machine availability, $A_i = r_i/(r_i + p_i)$
$MTTF_i$= mean time to failure of M_i, $MTTF_i = 1/p_i$
$MTTR_i$= mean time to repair of M_i, $MTTR_i = 1/r_i$

α_i = machine state $\quad \alpha_i \in \{0,1\}$
N_i = buffer size of B_i
n_i = buffer level of B_i $\quad 0 \le n_i \le N_i$
$\bar{n}_i$ = average buffer level of B_i
$\bar{n}$ = average system buffer level
E_i = throughput rate of M_i
E = system throughput rate
WIP= system Work-In-Process inventory level
μ_{CT} = mean of cycle time
σ^2_{CT} = variance of cycle time
σ_{CT} = standard deviation of cycle time
$X(t)$= system state at time t
S' = system state space
S = system state space after reduction, $S \subset S'$
s = system state, $s \in S$
Φ_i = indicator of the operational rule on M_i
$w_{s,i}$ = reward of M_i at state s

B. Assumptions

It is assumed that:

- The system is synchronous.
- The system is a flow line with unreliable machines. The machine failures are operation-dependent.
- Machine status changes at the beginning of a time unit. Buffer status changes at the end of a time unit.
- The system is saturated.
- Workpieces are not destroyed or rejected at any stage in the line.
- The transportation times between stations are zero.
- A single part is modeled.
- All parts queue according to the queueing discipline of FIFO.

C. Release Control and Reward Function

The system state at time t is determined by the machine status and buffer levels:

$$X(t) = (n_1(t), ..., n_{k-1}(t); \alpha_1(t), ..., \alpha_k(t)),$$
$$X(t) \in S' = A \times B \tag{3}$$

where

$$\alpha_i \in \{0,1\}$$
$$(\alpha_1, ..., \alpha_k) \in A = \{0,1\}^k$$
$$0 \le n_i(t) \le N_i$$
$$\begin{aligned}(n_1, ..., n_{k-1}) \in B \quad &= \quad \{0,1,...,N_1\} \times \{0,1,...,N_2\} \times ... \\ &\quad\quad \times \{0,1,...,N_{k-1}\}\end{aligned}$$

State space S' has

$$n(S') = 2^k \prod_{i=1}^{2} N_i + 1 \tag{4}$$

states. After eliminating the states that will never be visited under a certain operational control policy from S', the reduced state space is referred to as S.

Tan [11] introduce Φ_i as an indicator variable that describes the effect of an operational rule on machine M_i. If the operational rule does not restrict the flow out of machine M_i, Φ_i is 1; otherwise it is 0. For the purpose of controlling the release process, we only need to consider whether or not to restrict the flow out of the release machine. In other words the release control methodologies are distinguished by their policies of controlling Φ_1. If $\Phi_1 = 1$, a new part is released, while $\Phi_1 = 0$ indicates not to release any part under the situation. Therefore, the value of Φ_i for WIPLCtrl and CONWIP could be inferred.

- WIPLCtrl (reference WIPLOAD level = L)

$$\Phi_1(t) = \begin{cases} 1 & \text{if}(k-1)n_1(t) + (k-2)n_2(t) + \ldots \\ & +n_{k-1}(t) \le L - k - 1 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$\Phi_i(t) = \begin{cases} 1 & \text{if } n_{i-1}(t) > 0 \text{ and } n_i(t) < N_i \\ 0 & \text{otherwise} \quad i = 2,3,...,k \end{cases} \tag{6}$$

- CONWIP (reference WIP level = W) [11]

$$\Phi_1(t) = \begin{cases} 1 & \text{if } n_1(t) + n_2(t) + ... + n_{k-1}(t) < W \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

$$\Phi_i(t) = \begin{cases} 1 & \text{if } n_{i-1}(t) > 0 \text{ and } n_i(t) < N_i \\ 0 & \text{otherwise} \quad i = 2,3,...,k \end{cases} \tag{8}$$

The reward of machine M_i can be determined based on Φ_i. If at state s, machine M_i is operational and control policy allows flow into downstream buffer B_i, the reward of this machine at this state $w_{s,i}$ is 1, otherwise it is 0.

$$w_{s,i} = \begin{cases} 1 & \text{if } \alpha_i = 1 \text{ and } \Phi_i = 1 \text{ at state } s \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

The rewards are stored in $n(S) \times k$ matrix $W = \{w_{s,i}\}$ $s \in S$, $i = 1,2,...,k$.

D. Performance Measures

The considered system performance measures in this paper include throughput, average buffer level, average cycle time as well as standard deviation of cycle time.

- Throughput (E)
  Throughput, which is also called production rate, is the number of parts produced per unit time in the long run. Under the assumption that the operation times on each machine is one unit time, the throughput rate of machine M_i equals to the probability that M_i produces a part in a time step [6], which is referred to as E_i.

$$\begin{aligned} E_i &= prob(\alpha_i(t+1) = 1, n_{i-1}(t) > 0, n_i(t) < N_i) \\ &= prob(I_{ui}(t+1) = 1) \\ &= (1-p_i)prob(n_{i-1}(t) > 0, \alpha_i(t) = 1, n_i(t) < \\ &\quad N_i) + r_i \; prob(n_{i-1}(t) > 0, \alpha_i(t) = 1, \\ &\quad n_i(t) < N_i) \end{aligned} \tag{10}$$

According to the conservation of flow, it can be inferred that [6]

$$E = E_1 = E_2 = ... = E_k \tag{11}$$

where E is the throughput of the system.

- Average Buffer Level ($\bar{n}$)
  The average buffer level of B_i is computed as [6]

$$\bar{n}_i = \sum_s n_i \; prob(s). \tag{12}$$

The average buffer level of the transfer line, $\bar{n}$, is calculated as the sum of the average buffer level of all the buffers (Equation 13). It can also be understood as the WIP level of the system.

$$\bar{n} = \sum_{i=1}^{k-1} \bar{n}_i \tag{13}$$

- Average Cycle Time (μ_{CT})
  The average cycle time of a transfer line is equal to the sum of the cycle times at all the individual stations. Given the throughput rate of M_i (E_i), and the average buffer level of B_{i-1} ($\bar{n}_{i-1}$), the average cycle time at M_i, μ_{CT_i}, can be obtained according to Little's Law [12].

$$\mu_{CT_i} = \bar{n}_{i-1}/E_i \tag{14}$$

In this study, the processing time and the queueing time at M_1 is not taken into account since M_1 represents the release process. In other words the cycle time of a job here is the time from the job joins the queue before M_2 until it leaves the last machine M_k, which can be computed as

$$\mu_{CT} = \sum_{i=2}^{k} \mu_{CT_i} = \sum_{i=2}^{k} \bar{n}_{i-1}/E_i. \tag{15}$$

- Standard Deviation of Cycle Time (σ_{CT})
  Another important performance measure is the standard deviation of cycle time. Reducing the standard deviation of cycle time can imply smaller WIP and finished goods inventory for a given cycle time level [9]. Meanwhile, reduced cycle time variance also improves the predictability and service level of the system. Unfortunately, there is not any existing literature that addresses the issue regarding how to calculate the standard deviation of cycle time using analytic model. In this study, the relative effect of WIPLCtrl in comparison with CONWIP on standard deviation of cycle time is observed using simulation. The simulation model of the three-machine transfer line is built using $AutoSched^{TM} AP$. The results are the average values of 20 independent simulation runs. For each run, the simulation length is 7200 unit times, in which the first 1440 unit times are treated as the warm-up period. The statistical analysis is performed using the paired-t test at a 5% level of significance.

IV. Experiment

The only source of the system randomness is machine unreliability. We design different system configurations by adjusting the parameters of machine unreliability. By observing the concerned system performance measures, the characteristics of WIPLCtrl and the impact of machine failure can be further understood.

The performances of CONWIP and WIPLCtrl are observed in 13 cases. The parameters of machines for each case are given in Table I including p_i, r_i, the distributions of MTTF and MTTR, and the availability of each machine. As mentioned earlier, M_1 is the machine representing the release process. Therefore, M_1 never breaks down, i.e. $p_1 = 0$, $r_1 = 1$, $A_1 = 100\%$.

TABLE I

Tested Cases of the Transfer Line

	M_2					M_3				
Case	p_2	MTTF$_2$	r_2	MTTR$_2$	A_2(%)	p_3	MTTF$_3$	r_3	MTTR$_3$	A_3(%)
1	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91
2	0.008	Exp(125.00)	0.080	Exp(12.05)	90.91	0.008	Exp(125.00)	0.080	Exp(12.50)	90.91
3	0.080	Exp(12.50)	0.80	Exp(1.25)	90.91	0.080	Exp(12.50)	0.800	Exp(1.25)	90.91
4	0.050	Exp(20.00)	0.200	Exp(5.00)	80.00	0.050	Exp(20.00)	0.200	Exp(5.00)	80.00
5	0.085	Exp(11.76)	0.200	Exp(5.00)	70.18	0.085	Exp(11.76)	0.200	Exp(5.00)	70.18
6	0.020	Exp(50.00)	0.080	Exp(12.50)	80.00	0.020	Exp(50.00)	0.080	Exp(12.50)	80.00
7	0.020	Exp(50.00)	0.047	Exp(21.25)	70.18	0.020	Exp(50.00)	0.047	Exp(21.25)	70.18
8	0.050	Exp(20.00)	0.200	Exp(5.00)	80.00	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91
9	0.085	Exp(11.76)	0.200	Exp(5.00)	70.18	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91
10	0.130	Exp(7.69)	0.200	Exp(5.00)	60.61	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91
11	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91	0.050	Exp(20.00)	0.200	Exp(5.00)	80.00
12	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91	0.085	Exp(11.76)	0.200	Exp(5.00)	70.18
13	0.020	Exp(50.00)	0.200	Exp(5.00)	90.91	0.130	Exp(7.69)	0.200	Exp(5.00)	60.61

M_i: Machine i
A_i: Availability of M_i
Exp(m): Exponential distribution with mean of m

In Case 1–7, the transfer line is balanced, while Case 8–13 represent the system with a distinct bottleneck. A transfer line is said to be balanced when the capacity or availability of each machine (M_2 and M_3 in our case) is same. In Case 8–10, M_2 is the bottleneck, while in Case 11–13, M_3 is the bottleneck.

The MTTF and MTTR of M_2 and M_3 are assumed to be *exponentially distributed*. Exponential distribution is widely used in the related literature as the distribution to describe MTTF and MTTR because of its analytic tractability [6]. The memoryless property of exponential distribution greatly simplifies the mathematics. Additionally, the frequent machine failures in real-life manufacturing systems are random events of random durations. Therefore, it is appropriate to set the distribution of MTTF and MTTR of unreliable machines as exponential.

Machine *availability*, A_i, is another key concept. It is a measure suitable for systems under failure and in repair, which is given by

$$A_i = \frac{MTTF_i}{MTTF_i + MTTR_i} = \frac{r_i}{r_i + p_i}. \tag{16}$$

It represents the fraction of time that M_i is operational. It is also defined as the *isolated production rate* of M_i [6]. It is what the production rate of M_i would be if it were never impeded by other machines or buffers. Availability is the measure of the capacity of a machine. The capacity of a transfer line is determined by the machine with the least availability.

A. Balanced Line

We start from the observation and analysis of the achieved system throughput and the average WIP level under WIPLCtrl and CONWIP. Average cycle time is derived based on Little's Law [12] (Equation 14 and 15). The results of the standard deviation of cycle time are got from the simulation model. Different target WIP levels of CONWIP and the reference WIPLOAD levels of WIPLCtrl are tested to get different throughput levels. The reason to observe the performance measures at different throughput levels is that the effect of a release methodology is dependent on the congestion level of the production line.

The cases when the transfer line is balanced (Case 1–7) are considered first of all. The average throughput rate and the average cycle time got using the Markov process model are listed in Table II. The last two columns of Table II show the percentage improvements of WIPLCtrl over CONWIP in terms of the mean and the standard deviation of cycle time under a certain throughput level.

In Case 1, 2 and 3, the availabilities of M_2 and M_3 are kept at around 90%. The values of MTTF and MTTR are adjusted so that different frequencies and lengths of machine failures are achieved. Among these three cases, Case 2 has the longest MTTF and the longest MTTR, while Case 3 has the shortest MTTF as well as the shortest MTTR. The experiment results show that the longer, less frequent failures bring more variabilities into the system than the shorter, more frequent failures. The increase of system variability degrades the performance measures including throughput and average cycle time. It is observed that for a certain reference WIP level (W) or WIPLOAD level (L), the less the machine failure frequency (or the longer repair time), the lower the system throughput. This observation is illustrated by Fig. 5, which compares the achieved throughput rates under different W or L levels in Case 1–3. Similar phenomenon can also be observed on the average cycle time.

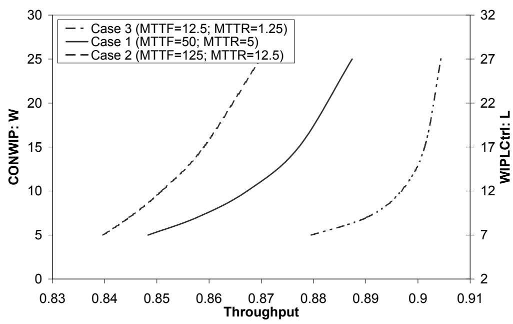

Fig. 5. Case 1–3: Impact of the Frequency and Length of Machine Failures

We know that there are two approaches to decrease the machine availability, either increasing the failure frequency or lengthening the breakdown time. The difference between the impacts of these two approaches can be observed from the results of Case 4–7. The MTTF of M_2 and M_3 are decreased in Case 4 and 5, while the MTTR of them are increased in Case 6 and 7. The effects of these two approaches can be observed from Fig. 6 and 7 that depict the achieved average cycle time at different throughput levels. We see that reducing to the same machine availability level, increasing MTTR deteriorates the system characteristic curve much more significantly than decreasing MTTF.

With regard to the performance of WIPLCtrl, its improvement over CONWIP in terms of both average WIP level and average cycle time is kept at a relatively stable level for a certain level of throughput rate. This improvement becomes more and more significant with the increase of throughput. WIPLCtrl achieves lower cycle time variance when the system is with a relatively higher variability level.

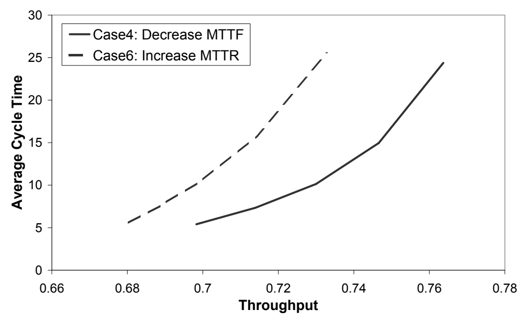

Fig. 6. Decreasing MTTF vs Increasing MTTR: Case 4 & 6 ($A_2 = A_3 = 80\%$) under WIPLCtrl

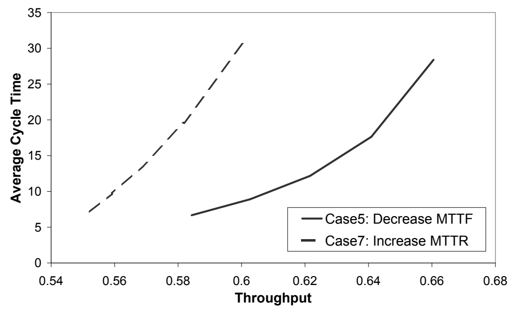

Fig. 7. Decreasing MTTF vs Increasing MTTR: Case 5 & 7 ($A_2 = A_3 = 70\%$) under WIPLCtrl

B. Bottleneck at Front-End of the Line

After observing the balanced transfer line, cases of unbalanced line are considered, in which the system has a distinct bottleneck machine. The issue regarding bottleneck location is taken into account. By adjusting machine parameters (p_i, r_i), we set M_2 as bottleneck in Case 8–10 and M_3 as bottleneck in Case 11–13. These two types of scenarios respectively represent the situations when the bottleneck is at the front-end and the back-end of the transfer line. The results are included in Table II.

TABLE II

STEADY STATE PERFORMANCE MEASURES OF THE BALANCED TRANSFER LINE CASE

	CONWIP					WIPLCtrl					P(%)	
	W	E	$\bar{n}$	μ_{CT}	$\sigma_{CT}*$	L	E	$\bar{n}$	μ_{CT}	$\sigma_{CT}*$	μ_{CT}	σ_{CT}
Case 1	5	0.8483	4.1510	4.8937	2.9723	7	0.8489	3.6110	4.2534	2.8670	15	4
	7	0.8577	6.1420	7.1607	3.3555	9	0.8580	5.0760	5.9159	3.2269	21	4
	10	0.8673	9.1320	10.5290	3.8269	12	0.8675	7.1970	8.2958	3.8351	27	0
	15	0.8772	14.1200	16.1000	4.4770	17	0.8773	11.0000	12.5380	4.5976	28	-3
	25	0.8874	24.1100	27.1720	5.4678	27	0.8874	18.4400	20.7840	6.1273	31	-11
Case 2	5	0.8397	4.1592	4.9532	5.1516	7	0.8400	3.6436	4.3376	4.8854	14	5
	7	0.8447	6.1534	7.2847	5.9310	9	0.8449	5.1237	6.0643	5.5428	20	7
	10	0.8509	9.1453	10.748	6.8634	12	0.8511	7.2202	8.4834	6.5476	27	5
	15	0.8590	14.1350	16.4550	8.0606	17	0.8591	11.0649	12.8800	7.6155	28	6
	25	0.8698	24.1240	27.7350	9.7776	27	0.8699	18.5144	21.2830	9.5230	30	3
Case 3	5	0.8796	4.1203	4.6843	1.4059	7	0.8798	3.4725	3.9469	1.3858	19	1
	7	0.8902	6.1098	6.8634	1.5715	9	0.8903	4.9350	5.5431	1.6346	24	-4
	10	0.8968	9.1032	10.1508	1.7905	12	0.8968	7.1298	7.9503	2.0766	28	-14
	15	0.9013	14.0988	15.6427	2.1201	17	0.9013	10.8819	12.0740	2.7337	30	-22
	25	0.9044	24.0956	26.6426	2.6773	27	0.9044	18.2818	20.2140	4.3039	32	-38
Case 4	5	0.6963	4.3036	6.1807	5.0514	7	0.6983	3.7709	5.4001	4.7406	14	7
	7	0.7125	6.2874	8.8244	5.6776	9	0.7139	5.2368	7.3355	5.2416	20	8
	10	0.7291	9.2709	12.7155	6.4410	12	0.7300	7.4046	10.1433	6.0012	25	7
	15	0.7461	14.2540	19.1045	7.4961	17	0.7466	11.1635	14.9525	6.9878	28	7
	25	0.7635	24.2370	31.7441	9.1792	27	0.7637	18.6135	24.3728	8.8759	30	3
Case 5	5	0.5807	4.4192	7.6101	6.6945	7	0.5843	3.8975	6.6704	6.2295	14	7
	7	0.6002	6.3998	10.6628	7.5053	9	0.6027	5.3661	8.9034	6.8332	20	10
	10	0.6200	9.3800	15.1290	8.5366	12	0.6216	7.5689	12.1765	7.7265	24	10
	15	0.6400	14.3600	22.4375	9.9590	17	0.6409	11.2991	17.6301	8.9265	27	12
	25	0.6601	24.3400	36.8729	12.0671	27	0.6605	18.7528	28.3918	11.1457	30	8
Case 6	5	0.6794	4.3203	6.3590	7.9479	7	0.6804	3.8080	5.5967	7.4499	14	7
	7	0.6882	6.3113	9.1701	9.1346	9	0.6891	5.2891	7.6754	8.3984	19	9
	10	0.6993	9.2996	13.2975	10.5393	12	0.7000	7.4503	10.6427	9.7335	25	8
	15	0.7136	14.2857	20.0178	12.3712	17	0.7141	11.2329	15.7294	11.1236	27	11
	25	0.7326	24.2672	33.1180	15.1501	27	0.7329	18.6861	25.4925	13.6530	30	11
Case 7	5	0.5510	4.4487	8.0739	14.7350	7	0.5522	3.9445	7.1432	13.7546	13	7
	7	0.5582	6.4412	11.5392	16.9127	9	0.5592	5.4331	9.7158	15.4128	19	10
	10	0.5677	9.4315	16.6121	19.5091	12	0.5686	7.6476	13.4490	17.7154	24	10
	15	0.5811	14.4183	24.8099	22.7564	17	0.5818	11.3928	19.5796	19.9914	27	14
	25	0.6011	24.3986	40.5798	27.6402	27	0.6016	18.8558	31.3374	23.9855	29	15
Case 8	5	0.7595	4.2404	5.5831	4.1815	7	0.7604	3.3715	4.4339	3.7100	26	13
	8	0.7740	7.2258	9.3357	5.0392	10	0.7746	5.0617	6.5346	4.4248	43	14
	12	0.7849	11.2150	14.2880	5.9537	14	0.7852	7.4571	9.4971	5.0766	50	17
	18	0.7929	17.2070	21.7010	7.1325	20	0.7931	10.9040	13.7482	5.9311	58	20
	28	0.7980	27.2020	34.0880	8.7402	30	0.7981	16.3430	20.4774	7.0792	66	23
Case 9	5	0.6740	4.3260	6.4184	5.3295	7	0.6751	3.2608	4.8301	4.5742	33	17
	8	0.6863	7.3136	10.6570	6.5216	10	0.6869	4.8269	7.0271	5.5002	52	19
	12	0.6954	11.3060	16.2790	7.8346	14	0.6948	7.0115	10.0914	6.2720	61	25
	18	0.6994	17.3010	24.7360	9.4545	20	0.6995	10.1740	14.5447	7.2523	70	30
	28	0.7015	27.2980	38.9140	11.7048	30	0.7015	15.2780	21.7795	8.6588	79	35
Case 10	5	0.5878	4.4122	7.5063	6.4335	7	0.5890	3.1896	5.4153	5.4456	38	18
	8	0.5972	7.4028	12.3960	7.9067	10	0.5978	4.7283	7.9095	6.6014	57	20
	12	0.6026	11.3970	18.9140	9.4677	14	0.6029	6.8158	11.3050	7.5250	67	26
	18	0.6053	17.3950	28.7370	11.4855	20	0.6054	9.8771	16.3150	8.7501	76	31
	28	0.6061	27.3940	45.1970	14.2228	30	0.6061	14.605	24.591	10.4515	84	36
Case 11	5	0.7595	4.2405	5.5833	4.1237	7	0.7604	4.0479	5.3234	4.1620	5	-1
	8	0.7740	7.2260	9.3359	4.9791	10	0.7745	6.6122	8.5374	5.0569	9	-2
	12	0.7848	11.2152	14.2905	5.8794	14	0.7851	10.1855	12.9735	6.0405	10	-3
	18	0.7928	17.2072	21.7043	7.0029	20	0.7929	15.7154	19.8202	7.3936	10	-5
	28	0.7978	27.2022	34.0965	8.6300	30	0.7978	25.2611	31.6634	9.2936	8	-7
Case 12	5	0.6740	4.3260	6.4184	5.1293	7	0.6752	4.3493	6.4415	5.3184	0	-4
	8	0.6863	7.3137	10.6567	6.2449	10	0.6869	7.1082	10.3482	6.4715	3	-4
	12	0.6944	11.3056	16.2811	7.5010	14	0.6947	10.9004	15.6908	7.8094	4	-4
	18	0.6993	17.3007	24.7400	9.1398	20	0.6994	16.7238	23.9116	9.5579	3	-4
	28	0.7013	27.2987	38.9259	11.3884	30	0.7014	26.6133	37.9431	11.9142	3	-4
Case 13	5	0.5878	4.4122	7.5063	6.1333	7	0.5890	4.6102	7.8272	6.4148	-4	-4
	8	0.5972	7.4028	12.3958	7.5126	10	0.5978	7.4752	12.5045	7.7885	-1	-4
	12	0.6026	11.3974	18.9137	9.0864	14	0.6028	11.3727	18.8665	9.4056	0	-3
	18	0.6052	17.3948	28.7422	11.0365	20	0.6052	17.3040	28.5922	11.4116	1	-3
	28	0.6060	27.3940	45.2046	13.7370	30	0.6060	27.2741	45.0068	14.0508	1	-2

W: Target WIP level of CONWIP

L: Reference WIPLOAD level of WIPLCtrl

P: Percentage improvement of WIPLCtrl over CONWIP on

*: Indicates the results are got from the simulation model

In Case 8–10, the availability of M_3 is kept at around 90%. By adjusting p_2, the availability of M_2 is set at 80%, about 70% and 60% respectively. The improvements of WIPLCtrl over CONWIP increase with the increase of the degree of system unbalance. Meanwhile, the higher the system throughput level, the more significant the improvement.

C. Bottleneck at Back-End of the Line

In Case 11–13, the availability of M_2 is set at about 90%. The availability of M_3 is 80%, about 70% and 60% respectively. By comparing the results of Case 8–10 and that of Case 11–13, it can be observed that when the bottleneck is at the back-end of the transfer line, the improvements of WIPLCtrl over CONWIP in terms of WIP level and average cycle time are not as significant as that when the bottleneck is at the front-end. This phenomenon has been intuitively inferred in Chapter 4. Under CONWIP, the last machine is the trigger machine of release of new jobs so that the variability of the last machine can be well compensated. In our transfer line case, when the availability of M_3 is lower than 60% (30% lower than M_2), the improvement of WIPLCtrl over CONWIP in terms of WIP and average cycle time cannot be observed.

Another point should not be ignored is variability propagation. The variability at the front-end of the transfer line will be propagated to the back-end. Therefore, the variability at the front-end has more significant impact on system performance than the same variability at the back-end. This can only be slightly observed by comparing Case 8–10 with Case 11–13 due to the small model size.

V. ANALYSIS

An analysis concentrating on some main issues is presented in this section. What is acquired from this study can be summarized from several perspectives. First of all, the performance of WIPLCtrl is summarized including the relative effect in comparison with CONWIP. Second, some underlying characteristics of WIPLCtrl can be understood through utilizing the Markov process model. The reason why WIPLCtrl is superior to CONWIP for some certain system configurations can be partially explained. Meanwhile, in this study, the only considered source of system variability is machine failure. Under this assumption, we have a good chance to understand the impact of machine failures not only on the isolated machines but also on the overall performance measures of a manufacturing system.

A. Relative Effect of WIPLCtrl in Comparison with CONWIP

One of the major objective of this study is to identify the system configuration, in which the advantage of WIPLCtrl can be fully embodied. This objective can be partially reached by comparing the effect of WIPLCtrl with other well studied release control methodologies. In this study, CONWIP is chosen as the referenced release methodology. This is firstly because CONWIP has been well recognized by many researchers and industrial practitioners. Secondly, the study of CONWIP started from the implementation on transfer line cases. Satisfactory performance measures of a transfer line can be achieved under CONWIP in terms of average WIP level, average cycle time and variance of cycle time.

For the cases tested in this chapter, the percentage improvements of WIPLCtrl over CONWIP are summarized as follows.

1) Average Cycle Time: Fig. 8 depicts the percentage improvements of WIPLCtrl over CONWIP in terms of average cycle time. The five tested throughput (E) levels are plotted ($E_1 < E_2 < ... < E_5$).

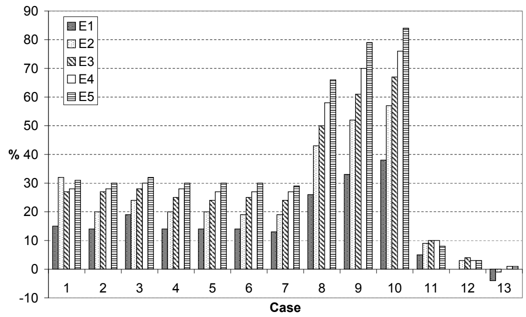

Fig. 8. Percentage Improvement of WIPLCtrl over CONWIP on Average Cycle Time

The improvement of WIPLCtrl over CONWIP on average cycle time can be observed in most tested cases. For the balanced system cases (Case 1–7), the percentage improvement of WIPLCtrl at each throughput level is relatively stable. The most distinct improvement is observed in the cases when the bottleneck is M_2 (Case 8–10). In this setting, with the increase of the difference between the availability of M_2 and M_3, the improvement becomes more remarkable. Meanwhile, higher system throughput, more percentage improvement is observed. When M_3 is the bottleneck, the predominance of WIPLCtrl becomes much less significant. In Case 13, the improvement cannot be observed at all.

2) Standard Deviation of Cycle Time: Getting from the simulation model, the data for the percentage improvements of WIPLCtrl over CONWIP in terms of standard deviation of cycle time are depicted by Fig. 9.

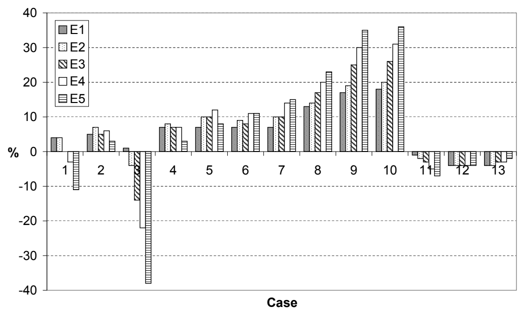

Fig. 9. Percentage Improvement of WIPLCtrl over CONWIP on Standard Deviation of Cycle Time

The relative effect of WIPLCtrl in comparison with CONWIP on the standard deviation of cycle time depends more on the system configuration. For the balanced system cases, the predominance of WIPLCtrl on the standard deviation of cycle time is indicated when the variability caused by machine outages is high enough. Meanwhile, this improvement also depends on the location of bottleneck station. Better relative performance of WIPLCtrl is achieved when M_2 is the bottleneck machine.

In summary, some inferences can be drawn from the above observations. Firstly, the choice of the release methodology has important impact on system performance measures including WIP level, mean and standard deviation of cycle time. Secondly, WIPLCtrl is a preferable release control methodology for a balanced system, under which satisfactory inventory and cycle time performance can be achieved. For a system with distinct bottleneck, WIPLCtrl predominates over CONWIP when the bottleneck locates at the front-end of the line. Additionally, a meritorious property of WIPLCtrl should not be ignored, that is WIPLCtrl is a reliable release control methodology for a manufacturing system with higher variabilities. The predominance of WIPLCtrl over CONWIP is robust to the system variability.

B. Characteristics of WIPLCtrl

To explain the reason why WIPLCtrl can achieve better system performances for certain system configurations, we need to enter on its underlying characteristics. Fig. 10 shows a sample of buffer levels for each case under CONWIP and WIPLCtrl. For Case 1–7, the scenarios when $W = 15$ and $L = 17$ are chosen as the sample, while for Case 8–13, the scenarios when $W = 18$ and $L = 20$ are chosen. The buffer levels of both B_1 and B_2 are plotted as the histogram, which are correspondingly referred to as n_1 and n_2. In Fig. 10, the number of x axis represents the case index. For each case, there are four rectangles. The first two rectangles depict the buffer levels of $B1$ and B_2 under CONWIP, while the second two depict the buffer levels under WIPLCtrl. The throughput under CONWIP is same to that under WIPLCtrl in each case.

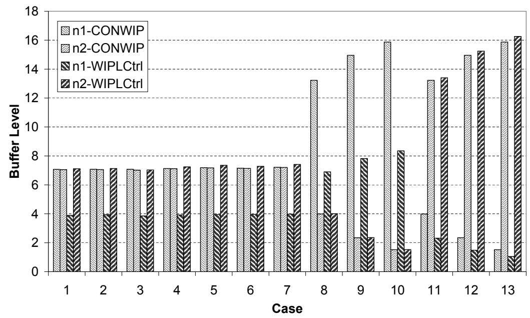

Fig. 10. Sample of Buffer Level under CONWIP and WIPLCtrl

When the balanced transfer line is controlled under CONWIP, the average buffer level of B_1 is almost equal to that of B_2. With the same throughput rate, WIPLCtrl restricts the buffer level of B_1. It is because WIPLCtrl make use of a reference WIPLOAD level to limit the release process. WIPLOAD is defined as the sum of the remaining processing times of all the jobs in the system. In our transfer line case, the remaining processing time of the jobs in B_1 is 2 unit time, while that of the jobs in B_2 is 1 unit time. Therefore, for a certain reference WIPLOAD level, L, the highest possible buffer level of B_1 and B_2 is $L/2$ and L respectively. The situation under CONWIP is different. For example, when the target WIP level of CONWIP is W, the highest possible buffer level of both B_1 and B_2 is W.

The machine parameters including p_i and r_i determine the probability of B_1 and B_2 to have a certain buffer level. To have an intuitive understanding, illustrations are utilized to observe the probabilities achieved from the Markov process model.

Fig. 11 shows the scenarios when $W = 15$ and $L = 17$ in Case 1. The x axis is the possible buffer levels (n) of B_1 and B_2. The probabilities of n_1 and n_2 under CONWIP or WIPLCtrl are plotted. Under CONWIP, the probability curves for n_1 and n_2 are exactly same. As a result, the average n_1 is almost equal to the average n_2. The probability curve of n_2 under WIPLCtrl is close to the curves under CONWIP. However under WIPLCtrl, the probability of $n_2 \leq 8$ is higher than that under CONWIP. The probability is zero for n_2 with a buffer level higher than 9 due to the restriction of the reference WIPLOAD level, L. Therefore, for the balanced transfer line cases, WIPLCtrl can achieve a certain throughput rate with lower n_1 than CONWIP. This results in a shorter average cycle time since the average cycle time is directly proportional to system WIP level.

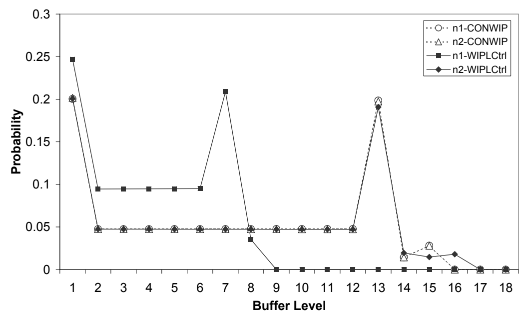

Fig. 11. Probability of n_1 and n_2 in Case 1 ($W = 15$ and $L = 17$)

The situation when the system is unbalanced can also be understood with the help of illustrations. Case 8 is used as the example when M_2 is bottleneck. Fig. 12 can be referred to understand the average buffer levels of B_1 and B_2 in Case 8 when $W = 18$ and $L = 20$ (Fig. 10). It can be observed that the major distinction between the situations under CONWIP and WIPLCtrl still lies on the buffer level of B_1. Under CONWIP, n_1 is much higher than n_2 since the increased p_2 amplifies the probability of B_1 to have a higher buffer level. Although it also amplifies n_1 under WIPLCtrl, the amplification is restricted by the given L. As a result, WIPLCtrl is able to lead to a remarkable improvement in terms of average cycle time.

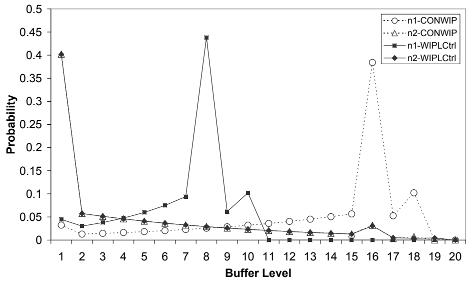

Fig. 12. Probability of n_1 and n_2 in Case 8 ($W = 18$ and $L = 20$)

WIPLCtrl does not significantly restrict the buffer level of B_2 since the remaining processing time of the jobs in it is relatively low. So the buffer level of $B2$ under WIPLCtrl is very close to that under CONWIP. When M_3 is the constraint machine in the transfer line, the buffer level of $B2$ has much more significant impact on average cycle time than that of B_1. That is why when M_3 is bottleneck, the improvement of WIPLCtrl over CONWIP on average cycle time is not that remarkable. Fig. 13 and 10 can help to understand the situation.

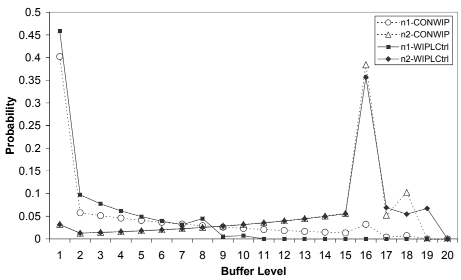

Fig. 13. Probability of n_1 and n_2 in Case 11 ($W = 18$ and $L = 20$)

C. Variability Caused by Machine Failure

Another important issue related to this study is the variability caused by machine failures. Machine failure is a significant source of variability of a manufacturing system. Not only the performance of an isolated machine is impacted by them, they also seriously influence the overall system performance measures. The longer, less frequent machine failures bring much more variabilities into the system than the shorter, more frequent ones. The variability caused by MTTR is more significant than that caused by MTTF.

VI. Conclusion

In this paper, Markov process modelling methodology is employed to study a three-machine transfer line case. The effect of WIPLCtrl and CONWIP is tested. Use of such a simple model can help to make clear the underlying characteristic of our proposed release control methodology, WIPLCtrl, so that we may understand when and why the advantages of WIPLCtrl can be observed. As a supplementation to the analytic model, a simulation model is built as well to observe the performance measure, the standard deviation of cycle time, which cannot be analytically achieved using the existing analytic models. In this study, only machine failure is considered as the stochastic factor in the system. Based on the experiments results of both the analytic model and the simulation model, the variability caused by machine failure is also observed. The result is helpful for us to get more insight into the characteristic of a manufacturing system itself.

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