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Abstract

This paper develops a simple two-country, two-good model, in which the real exchange rate, stock and bond prices are jointly determined. The model predicts that stock market prices are correlated internationally even though their dividend processes are independent, providing a theoretical argument in favor of financial contagion. The foreign exchange market serves as a propagation channel from one stock market to the other. The model identifies interconnections among stock, bond and foreign exchange markets and characterizes their joint dynamics as a three-factor model. Contemporaneous responses of each market to changes in the factors are shown to have unambiguous signs. These implications enjoy strong empirical support. Estimation of various versions of the model reveals that most of the signs predicted by the model indeed obtain in the data, and the point estimates are in line with the implications of our theory. Moreover, the factors we extract from daily data on stock indexes and exchange rates explain a sizable fraction of the variation in a number of macroeconomic variables, and the estimated signs on the factors are consistent with our model's implications. We also derive agents' portfolio holdings and identify economic environments under which they exhibit a home bias, and demonstrate that an international CAPM obtained in our model has two additional factors.

JEL Classifications: G12, G15, F31, F36

Keywords: Asset pricing, exchange rate, contagion, international finance, open economy macroeconomics.

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1. Introduction

Financial press has long asserted that stock prices and exchange rates are closely intertwined. In recent times, the depreciation of the dollar against the euro and other major currencies has been argued to put pressure on the investor sentiment and the US stock markets. Similarly, the previous decade associated with high productivity gains and a stock market boom in the US was accompanied by an exchange rate appreciation. Surprisingly, these connections have rarely been highlighted in workhorse models of exchange rate determination. This paper develops a framework in which the same forces that drive exchange rates, also influence countries' stock markets, and argues that a great deal can be learned about foreign exchange markets by examining stock markets, and vice versa. Identifying interrelations between these markets would also shed light on some widely-debated spillovers, such as, for example, international financial contagion.

The innovation of our modeling approach is to draw upon three separate strands of literature: international asset pricing, open economy macroeconomics and international trade, while differing from each one of them in some important dimensions. On the one hand, while encompassing a rich financial markets structure, the overwhelming majority of international asset pricing models assumes that there is a single commodity in the world, implying that the real exchange rate has to be equal to unity. Nontrivial implications on exchange rates in such a framework have been obtained by either introducing barriers to trade into a real model, or by exogenously specifying a monetary policy and focusing on the nominal exchange rate. On the other hand, the international economics literature typically concentrates either on how different patterns of international trade in goods affect the real exchange rate, or on how the nominal exchange rate is linked to bond markets, typically overlooking the implications on equity markets. Ours is a two-country, two-good model where the countries trade in goods as well as in stocks and bonds. To our knowledge, it is the first asset pricing model in which the terms of trade, exchange rate, and asset prices are jointly determined in equilibrium, thus marrying dynamic asset pricing with Ricardian trade theory.¹

The paper consists of two parts: theoretical and empirical. The first part presents our model of a dynamic world economy under uncertainty. The two countries comprising the economy specialize in producing their own good. The stock market in each country is a claim to the country's output.

¹Zapatero (1995) employs a similar two-country, two-good economy and discusses the new insights the model provides for empirical studies. Although his model offers valuable perspective on the exchange rate, it contains some counterfactual implications regarding financial markets, which we discuss in Section 2. See also Serrat (2001).

Bonds, whose interest rates are uncovered endogenously within the model, provide further opportunities for international borrowing and lending. A representative agent in each country consumes both goods, albeit with a preference bias toward the home good, and invests in the stock and bond markets. Uncertainty in the economy is due to output (supply) shocks in each country and the consumers' demand shocks. While the former are very common in models of international macroeconomics, and especially international business cycles, the latter have received considerably less attention. For the bulk of our analysis, we adopt a very general specification of the demand shocks, imposing more structure later to gain additional insight. We distinguish between the special cases where (i) there are no demand shocks, (ii) demand shocks are due to pure consumer sentiment, (iii) demand shocks are due to preferences encompassing a “catching up with the Joneses” feature or consumer confidence and (iv) preferences are state-independent and demand shocks are driven by differences of opinion. It is ultimately an empirical question – addressed in the second part of the paper – as to which of these specifications is the most plausible.

Our model is extremely tractable, allowing us to characterize, in closed-form, the patterns of responses of stock and bond markets in each country, as well as those of the foreign exchange market, to supply and demand shocks. Since, by design, our model nests a number of fundamental implications from various strands of the international economics literature, directions of some of these responses are familiar. For example, all else equal, a positive shock to a country's output leads to a deterioration of the terms of trade it enjoys and an exchange rate depreciation (consistent with the comparative advantages theory of the international trade literature). At the same time, the national stock market sees a positive return (in line with the asset pricing literature). On the other hand, a positive demand shock in a country leads to an exchange rate appreciation (as in the open economy macroeconomics literature). We unify all these implications in one model, and focus on the interconnections among the stock, bond and foreign exchange markets and on the spillovers.

One important spillover obtaining in our economy provides a natural basis for a theory of international financial contagion. The empirical literature on contagion has concluded that some factors, such as trade and financial linkages, are important in explaining the propagation of shocks. Nevertheless, these channels account for a small proportion of the observed co-movement. The argument is that the correlation in equity and bond prices is an order of magnitude larger than that implied by the correlation in real variables.² In our model, the real variables – the countries' output pro-

²See, e.g., Eichengreen, Rose, and Wyplosz (1996), Fischer (1998), Goldstein, Kaminsky, and Reinhart (2000), Kaminsky and Reinhart (2000), Claessens, Dornbusch, and Park (2001), Kaminsky, Reinhart, and Végh (2003).

cesses – are unrelated and yet stock returns on the national markets become positively correlated. Contagion is a natural response to a supply shock in one of the countries. As we discussed earlier, a positive output shock leads to a positive return on the domestic stock market; however, it has to be accompanied by an exchange rate depreciation. The latter implies a strengthening of the foreign currency, leading to a rise in the value of the foreign country’s output, thereby boosting its stock market. The foreign exchange market thus acts as a channel through which shocks are propagated across countries’ stock markets. Another spillover we uncover is what we labeled “divergence”: a response of world asset markets to a demand shock in one of the countries. As mentioned earlier, a positive shift in domestic demand causes an exchange rate appreciation. A strengthening of the domestic currency relative to the foreign leads to divergence in world financial markets: it provides a boost to the domestic stock and bond markets, while asset prices abroad fall. This pattern is very close to macroeconomic dynamics observed in the US in the 90’s when large capital inflows were pushing the interest rates down, increasing stock prices, and strengthening the dollar. These implications, together with the remaining patterns of responses to innovations in supply and demand, provide a testable theory of the interconnections among stock, bond and foreign exchange markets in different countries.

In our model, the risk premium terms in uncovered interest parity is time-varying, driven in part by demand shocks, and possibly quite volatile. We also derive portfolio strategies, shown to consist of holding a mean-variance portfolio and an additional portfolio hedging against future demand shocks. Within our model, we can identify economic environments under which countries’ portfolios exhibit a home bias, which is demonstrated to be due to the nature of the demand shocks. In particular, a home bias is induced when the demand shocks are positively correlated with the supply shocks in each country. Agents, who get enthusiastic and demand more consumption (biased toward the home good) when their country is experiencing an output increase, optimally hold a larger fraction of the domestic stock – the claim to the domestic output. This is consistent with our consumer confidence or differences in opinion interpretations of the demand shocks. Finally, due to the presence of the hedging portfolios in the countries’ investment strategies, we obtain a multi-factor CAPM in our model. In addition to the standard market factor, our model identifies two further factors: demand shocks of each country – hence pointing to a potential misspecification of the traditional international CAPM.

The second part of the paper tests the implications of our theory on daily data for the US

vis-à-vis the UK. Our model implies that the stock market prices, bond prices and exchange rates are described by a three latent-factor model with time-varying coefficients, where the latent factors correspond to our demand and two supply shocks. The model, as it is, has too many degrees of freedom to fit the data and hence is unlikely to be rejected. Therefore, we test the model using two approaches. First, we take the model literally and estimate the structural equations derived in the theoretical section. We extract the latent factors from this exercise, and test how they perform out-of-sample in forecasting macroeconomic variables. Our model provides economic meaning to each of the factors – and we compare them with macroeconomic variables they should be associated with. In the second approach, we estimate a simplified version of the model where only a minimal structure is used for identification: we set the coefficients to be constant through time and impose some sign restrictions. We then test whether the pattern of interconnections between the asset and foreign exchange markets implied by the model is indeed found in the data, and whether the signs of responses of the markets to demand and supply shocks are consistent with the theory.

Through the first test we establish that (i) demand shocks are twice as important as supply shocks in describing the behavior of asset prices and the exchange rate, (ii) the data reject the hypotheses that our demand shocks are generated by either pure sentiment or “catching up with the Joneses”-type behavior. Rather, our results support the view that the demand shocks are likely to represent differences in opinion or consumer confidence, which is positively correlated with the current performance of the domestic economy. Furthermore, the model implies that the demand and supply shocks we extract from the (daily) financial markets data should have explanatory power in forecasting several macroeconomic variables. We find that the extracted shocks indeed explain a sizable fraction of the variation in industrial production, trade balance, capital flows, consumer confidence, and business trend survey indexes. Additionally, the signs of (impulse) responses of the macro-variables to innovations in our supply and demand shocks, for the most part, are consistent with the theory. Furthermore, the signs and magnitudes of the responses (forecasts) are quite stable: they do not vary much across sub-samples. This is in contrast to the existing literature attempting to forecast macroeconomic variables using raw financial data, where parameter and forecast instability is usually the outcome (see Stock and Watson (2003) for a thorough synthesis).

Second, we estimate the pattern of responses of stock, bond and foreign exchange markets to the underlying shocks within a latent-factor model with constant coefficients where only *some* sign restrictions are imposed. To solve the identification problem, we rely on the natural heteroskedas-

ticity present in the data.³ Our estimation procedure can uniquely identify twelve coefficients in a system of simultaneous equations we consider. We find that all twelve point estimates have the signs predicted by the model, and eleven are statistically significant. Moreover, the theory entails implications as to how the coefficients are related to each other. For most of the point estimates predicted to be equal by the model, we cannot reject the hypothesis that they are the same, however we do find some rejections, especially in bond markets responses.

In terms of the modeling approach, our work is closest to the international asset pricing literature. As we mentioned earlier, however, most of the models in this literature are cast in a single-good framework, in which forces of arbitrage equate the real exchange rate to unity. Nontrivial implications on exchange rates have been obtained either by exogenously specifying a monetary policy and focusing on the nominal exchange rate instead of real (see, for example, Bakshi and Chen (1997), Basak and Gallmeyer (1999) for monetary general equilibrium models, and Solnik (1974), Adler and Dumas (1983) for the international CAPM), or by introducing barriers to trade into a real model (Dumas (1992), Sercu and Uppal (2000)), and thus impeding goods markets arbitrage. To our knowledge, the only other multi-good asset pricing models in which the exchange rate is determined through the terms of trade are Zapatero (1995) and Serrat (2001).⁴ The focus of Zapatero is, like in this paper, on asset prices and exchange rates, however, he does not allow for demand shocks, which are an essential element in our analysis. The focus of Serrat is quite different: he is primarily concerned with the home bias in portfolios, and does not compute stock prices and the real exchange rate. Within the vast international economics literature, there are numerous papers related to our work; as we mentioned earlier, implications regarding the real side of our economy are consistent with findings of others. Dynamic equilibrium models of Cole and Obstfeld (1991), Backus, Kehoe, and Kydland (1992), and Kraay and Ventura (2000) are perhaps the closest. These papers, however, do not explicitly consider stock market prices and their dynamics. In terms of the overall objective of constructing a structural model tying together exchange rates and asset prices and taking it to the data, we know of two papers that are closest to our work. In a mean-variance setting, absent international trade in goods, Hau and Rey (2002) study the relationship between

³Our approach follows the identification through heteroskedasticity literature, which is based on Philip Wright's (1928) book. It has been recently extended by Sentana and Fiorentini (2001) for the conditional heteroskedasticity case (see also Rigobon (2002)), and Rigobon (2003) for the case in which heteroskedasticity can be described by different regimes.

⁴Another analysis that inspired our work is Cass and Pavlova (2003), who employ a similar model to argue that benchmark models of financial equilibrium theory in economics and finance, commonly believed to entail similar implications, can be in fundamental disagreement.

stock market returns and the exchange rate. Brandt, Cochrane, and Santa-Clara (2001) also examine this relationship so as to argue that exchange rates fluctuate less than the implied marginal rates of substitution obtained from stock market returns.

The rest of the paper is organized as follows. Section 2 describes the economy and derives the testable implications of our model. Section 3 presents the empirical analysis. Section 4 discusses caveats and avenues for future research. Section 5 concludes and the Appendix provides all proofs.

2. The Model

2.1. The Economic Setting

We consider a continuous-time pure-exchange world economy along the lines of Lucas (1982). The economy has a finite horizon, $[0, T]$, with uncertainty represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, P)$, on which is defined a standard three-dimensional Brownian motion $\vec{w}(t) = (w(t), w^*(t), w^\theta(t))^\top$, $t \in [0, T]$. All stochastic processes are assumed adapted to $\{\mathcal{F}_t; t \in [0, T]\}$, the augmented filtration generated by \vec{w} . All stated (in)equalities involving random variables hold P -almost surely. In what follows, given our focus, we assume all processes introduced to be well-defined, without explicitly stating regularity conditions ensuring this.

There are two countries in the world economy: Home and Foreign. Each country produces its own perishable good via a strictly positive output process modeled as a Lucas' tree:

$$dY(t) = \mu_Y(t)Y(t)dt + \sigma_Y(t)Y(t)dw(t) \quad (\text{Home}), \quad (1)$$

$$dY^*(t) = \mu_Y^*(t)Y^*(t)dt + \sigma_Y^*(t)Y^*(t)dw^*(t) \quad (\text{Foreign}), \quad (2)$$

where $\mu_Y, \mu_Y^*, \sigma_Y > 0$ and $\sigma_Y^* > 0$ are arbitrary adapted processes. Note that the country-specific Brownian motions w and w^* are independent.⁵ The prices of the Home and Foreign goods are denoted by p and p^* , respectively. We fix the world numeraire basket to contain α , $\alpha \in (0, 1)$, units of the Home good and $(1-\alpha)$ units of the Foreign good, and normalize the price of this basket to be equal to unity.

Investment opportunities are represented by four securities. Located in the Home country are a

⁵It is straightforward to extend the model to the case where shocks to the countries output are multi-dimensional and correlated. Our equilibrium characterization of the stock prices and the exchange rate would be the same. Since part of our objective in this paper is to demonstrate that Home and Foreign stock returns become correlated even when the countries' output processes are unrelated, we forgo inclusion of additional (possibly common) components in the shocks structure.

bond B , in zero net supply, and a risky stock S , in unit supply. Analogously, Foreign issues a bond B^* and a stock S^* . The bonds B and B^* are money market accounts instantaneously riskless in the local good, and the stocks S and S^* are claims to the local output. The terms of trade, q , are defined as the price of the Home good relative to that of the Foreign good: $q \equiv p/p^*$. The terms of trade are positively related to the real and nominal exchange rates; however, we delay making the exact identification till Section 3, where we map nominal quantities available in the data into the variables employed in the model.

A representative consumer-investor of each country is endowed at time 0 with a total supply of the stock market of his country. Thus, the initial wealth of the Home resident is $W_H(0) = S(0)$ and that of the Foreign resident is $W_F(0) = S^*(0)$. Each consumer i chooses nonnegative consumption of each good $(C_i(t), C_i^*(t))$, $i \in \{H, F\}$, and a portfolio of the available securities $(x_i^S(t), x_i^{S^*}(t), x_i^B(t), x_i^{B^*}(t))$, where x^j denotes a fraction of wealth W_i invested in security j . The dynamic budget constraint of each consumer takes the standard form

$$\begin{aligned} \frac{dW_i(t)}{W_i(t)} &= x_i^B(t) \frac{dB(t)}{B(t)} + x_i^{B^*}(t) \frac{dB^*(t)}{B^*(t)} + x_i^S(t) \frac{dS(t) + p(t)Y(t)dt}{S(t)} + x_i^{S^*}(t) \frac{dS^*(t) + p^*(t)Y^*(t)dt}{S^*(t)} \\ &\quad - \frac{1}{W_i(t)} (p(t)C_i(t) + p^*(t)C_i^*(t)) dt, \quad i \in \{H, F\}, \end{aligned} \quad (3)$$

with $W_i(T) \geq 0$. Both representative consumers derive utility from the Home and Foreign goods

$$E \left[\int_0^T \theta_H(t) [a_H \log(C_H(t)) + (1 - a_H) \log(C_H^*(t))] dt \right] \quad (\text{Home country}), \quad (4)$$

$$E \left[\int_0^T \theta_F(t) [a_F \log(C_F(t)) + (1 - a_F) \log(C_F^*(t))] dt \right] \quad (\text{Foreign country}), \quad (5)$$

where a_H and a_F are the weights on Home goods in the utility function of each country. The objective of making a_H and a_F country-specific is to capture the possible home bias in the countries' consumption baskets. This home bias may in part be due to the presence of non-tradable goods, and by imposing an assumption that $a_H > a_F$, we would model it in a reduced form. (We elaborate on this in Section 4.) Heterogeneity in consumer tastes is required for most of our implications; otherwise demand shocks will have no effect on the real exchange rate or the terms of trade. The "demand shocks," θ_H and θ_F , are arbitrary positive adapted stochastic processes driven by \vec{w} , with $\theta_H(0) = 1$ and $\theta_F(0) = 1$. The only requirement we impose on these processes is that they be martingales. That is, $E_t[\theta_i(s)] = \theta_i(t)$, $s > t$, $i \in \{H, F\}$. This specification is very general. In the special cases we consider later in this section, we put more structure on these processes depending on the interpretation we adopt. Note that the presence of the stochastic components θ_H and θ_F

in (4)–(5) does not necessarily imply that the countries’ preferences are state-dependent. As we discuss in one of the interpretations below, the countries may disagree on probability measures they use to compute expectations. Then, state-independent utilities under their own probability measures give rise to an equivalent representation (4)–(5) of the countries’ expected utilities under the true measure.

2.2. Characterization of World Equilibrium

Financial markets in the economy are potentially dynamically complete since there are three independent sources of uncertainty and four securities available for investment. Since endowments are specified in terms of share portfolios, however, this is not sufficient to guarantee that markets are indeed complete in equilibrium, as demonstrated by Cass and Pavlova (2003) for a special case of our economy where θ_H and θ_F are deterministic. Nevertheless, one can still obtain a competitive equilibrium allocation by solving the world social planner’s problem because Pareto optimality is preserved even under market incompleteness.⁶ The planner chooses countries’ consumption so as to maximize the weighted sum of countries’ utilities, with weights λ_H and λ_F (see the Appendix), subject to the resource constraints:

$$\begin{aligned} \max_{\{C_H, C_H^*, C_F, C_F^*\}} \quad & E \left[\int_0^T \left\{ \lambda_H \theta_H(t) [a_H \log(C_H(t)) + (1 - a_H) \log(C_H^*(t))] \right. \right. \\ & \left. \left. + \lambda_F \theta_F(t) [a_F \log(C_F(t)) + (1 - a_F) \log(C_F^*(t))] \right\} dt \right] \\ & \text{with multipliers} \\ \text{s. t.} \quad & C_H(t) + C_F(t) = Y(t), \quad \eta(t), \quad (6) \\ & C_H^*(t) + C_F^*(t) = Y^*(t), \quad \eta^*(t). \quad (7) \end{aligned}$$

Solving the planner’s optimization problem, we obtain the sharing rules

$$\begin{aligned} C_H(t) &= \frac{\lambda_H \theta_H(t) a_H}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} Y(t), & C_F(t) &= \frac{\lambda_F \theta_F(t) a_F}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} Y(t), & (8) \\ C_H^*(t) &= \frac{\lambda_H \theta_H(t) (1 - a_H)}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} Y^*(t), & C_F^*(t) &= \frac{\lambda_F \theta_F(t) (1 - a_F)}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} Y^*(t). & (9) \end{aligned}$$

The competitive equilibrium prices are identified with the Lagrange multipliers associated with the resource constraints. The multiplier on (6), $\eta(t, \omega)$, is the price of one unit of the Home good

⁶For a special case of our economy where θ_H and θ_F are deterministic, Cass and Pavlova prove that any equilibrium in the economy must be Pareto optimal, and thus the allocation is a solution to the planner’s problem. When θ_H and θ_F are stochastic, we verify that the allocation is Pareto optimal.

to be delivered at time t in state ω . Similarly, $\eta^*(t, \omega)$, the multiplier on (7), is the price of one unit of the Foreign good to be delivered at time t in state ω . We find it useful to represent these quantities as products of two components: the state price and the spot good price. The former is the Arrow-Debreu state price, denoted by π , of one unit of the numeraire to be delivered at (t, ω) and the latter is either p (for the Home good) or p^* (for the Foreign good). The equilibrium terms of trade are then simply the ratio of $\eta(t, \omega)$ and $\eta^*(t, \omega)$, which is, of course, the same as the ratio of either country's marginal utilities of the Home and Foreign goods:

$$q(t) = \frac{\eta(t)}{\eta^*(t)} = \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)} \frac{Y^*(t)}{Y(t)}. \quad (10)$$

The terms of trade increase in the Foreign and decrease in the Home output. When the Home output increases, all else equal, the terms of trade deteriorate as the Home good becomes relatively less scarce. Analogously, the terms of trade improve when Foreign's output increases. This is the standard terms of trade effect that takes place in Ricardian models of trade (see Ricardo (1817) and Dornbusch, Fischer, and Samuelson (1977) for seminal contributions). So far, the asset pricing literature has ignored it by assuming a single good.

The terms of trade also depend on the relative weight of the two countries in the planner's problem and the relative demand shock θ_H/θ_F . If we make an additional assumption that each country has a preference bias for the local good, then a positive relative demand shock improves Home's terms of trade: $sign(\partial q/\partial(\theta_H/\theta_F)) = sign(a_H - a_F) > 0$. The presence of this effect relates our model to the open economy macroeconomics literature. In the "dependent economy" model (see Salter (1959), Swan (1960), and Dornbusch (1980), Chapter 6), a demand shift biased toward the domestic good raises the price of the Home good relative to that of the Foreign, thus appreciating the exchange rate. In our model, if a_H is larger than a_F then the relative demand shock does indeed represent a demand shift biased toward the domestic good.

Finally, we determine stock market prices in the economy. Using the no-arbitrage valuation principle, we obtain

$$S(t) = \int_t^T \frac{\pi(s)}{\pi(t)} p(s) Y(s) ds \quad \text{and} \quad S^*(t) = \int_t^T \frac{\pi(s)}{\pi(t)} p^*(s) Y^*(s) ds. \quad (11)$$

Explicit evaluation of these integrals, the details of which are relegated to the Appendix, yields

$$S(t) = \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t)(T - t), \quad (12)$$

$$S^*(t) = \frac{1}{\alpha q(t) + 1 - \alpha} Y^*(t)(T - t). \quad (13)$$

Consistent with insights from the asset pricing literature, each country's stock price is positively related to the national output the stock is a claim to. Recall that the innovations to the processes driving the Home and Foreign output are independent. This, however, does not imply that international stock markets are contemporaneously uncorrelated. The correlation is induced through the terms of trade present in (12)–(13). One can combine (12) and (13) to establish a simple relationship tying together the stock prices in the two countries and the prevailing terms of trade:

$$S^*(t) = \frac{1}{q(t)} \frac{Y^*(t)}{Y(t)} S(t). \quad (14)$$

The dynamics of the terms of trade and the international financial markets are jointly determined within our model. Proposition 1 characterizes these dynamics as a function of three sources of uncertainty: two country-specific shocks and the relative demand shock.

Proposition 1. *The dynamics of the Home and Foreign stock and bond markets, and the terms of trade are given by*

$$\frac{dS(t)}{S(t)} = I_1(t)dt + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} \sigma_Y^*(t)dw^*(t), \quad (15)$$

$$\frac{dS^*(t)}{S^*(t)} = I_2(t)dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} \sigma_Y^*(t)dw^*(t), \quad (16)$$

$$\frac{dB(t)}{B(t)} = I_3(t)dt + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} A(t)d\theta(t) - \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} \sigma_Y(t)dw(t) + \frac{1-\alpha}{\alpha q(t) + 1 - \alpha} \sigma_Y^*(t)dw^*(t), \quad (17)$$

$$\frac{dB^*(t)}{B^*(t)} = I_4(t)dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} A(t)d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \sigma_Y(t)dw(t) - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \sigma_Y^*(t)dw^*(t), \quad (18)$$

$$\frac{dq(t)}{q(t)} = I_5(t)dt + A(t)d\theta(t) - \sigma_Y(t)dw(t) + \sigma_Y^*(t)dw^*(t), \quad (19)$$

where $\theta(t) \equiv \theta_H(t)/\theta_F(t)$, $A(t) \equiv \lambda_H \lambda_F (a_H - a_F) / [(\lambda_H \theta(t) a_H + \lambda_F a_F)(\lambda_H \theta(t)(1 - a_H) + \lambda_F(1 - a_F))]$, and $I_j(t)$, $j = 1, \dots, 5$ are reported in the Appendix. Furthermore, if $a_H > a_F$, the diffusion coefficients of the dynamics of the Home and Foreign stock markets and the terms of trade have the following signs:

Variable/ Effects of	$d\theta(t)$	$dw(t)$	$dw^*(t)$
$\frac{dS(t)}{S(t)}$	+	+	+
$\frac{dS^*(t)}{S^*(t)}$	-	+	+
$\frac{dB(t)}{B(t)}$	+	-	+
$\frac{dB^*(t)}{B^*(t)}$	-	+	-
$\frac{dq(t)}{q(t)}$	+	-	+

(20)

Proposition 1 identifies some important interconnections between the financial and real markets in the world economy. Under the home bias assumption, it entails unambiguous directions of

contemporaneous responses of all markets to innovations in supply and demand, summarized in (20). (20) nests some fundamental implications from various strands of international economics, which we highlighted in our earlier discussion. Our goal is to unify them within a simple model, and focus on the interactions.

One such interaction sheds light on the determinants of financial contagion – a puzzling tendency of stock markets across the world to exhibit excessive co-movement (for a recent detailed account of this phenomenon, see Kaminsky, Reinhart, and Végh (2003)). Contagion in our model occurs in response to an output shock. *Ceteris paribus*, a positive output shock say in Home causes a positive return on the domestic stock market. At the same time, however, it initiates a Ricardian response of the terms of trade: the terms of trade move against the country experiencing a productivity increase. A flip side of the deterioration of the terms of trade in Home is an improvement of those in Foreign. Hence, the value of Foreign’s output has to rise, thereby providing a boost to Foreign’s stock market. Note that nothing in this argument relies on the correlation between the countries’ output processes. In fact, in a special case of our model where there are no demand shocks (discussed below), we obtain a perfect co-movement of the stock markets despite independence of the countries’ output innovations. Bond markets certainly also react to changes in productivity: a positive output shock in Home lowers bond prices in Home and lifts those in Foreign.

While supply shocks move the countries’ stock prices in the same direction, demand shocks act in the opposite way. We call this phenomenon “divergence.” Thus, a demand shock say in Home causes a relative demand shift biased toward the Home good, thereby boosting Home’s terms of trade. Improved terms of trade increase the value of Home’s output, and hence lift Home’s stock market, while decreasing that of Foreign’s output and lowering Foreign’s stock market. Similarly, since our bonds are real bonds, a strengthening of Home’s terms of trade increases the value of the Home bond and decreases that of the Foreign. A world economy without supply shocks is thus an example of perfect divergence: asset markets of different countries always move in opposite directions.

Finally, Proposition 1 provides analytical characterization of the sensitivities of each market’s responses to the supply and demand shocks, and also establishes cross-equation restrictions on how these sensitivities measure up to each other. The system of simultaneous equations describing the joint dynamics of the five markets (15)–(19) establishes a basis for our empirical analysis, carried

out in Section 3.

Our model also entails implications on the dynamics of the real variables in our economy such as consumption, current account and capital account, which can be obtained via an application of Itô's lemma to (8)–(9). Since our main focus is on financial markets, for brevity, we report just the signs of responses of the variables to the demand and supply shocks:

Variable/ Effects of	$d\theta(t)$	$dw(t)$	$dw^*(t)$
$\frac{dC_H(t)}{C_H(t)}$	+	+	0
$\frac{dC_H^*(t)}{C_H^*(t)}$	+	0	+
$\frac{dC_F(t)}{C_F(t)}$	–	+	0
$\frac{dC_F^*(t)}{C_F^*(t)}$	–	0	+
$dCA_H(t)$	–	+	–
$dKA(t)_H$	+	–	+

where CA_H and KA_H are Home's current account and capital account, respectively. Thus, for example, a demand shock deteriorates Home's current account as the agents demand more consumption while its output does not change. At the same time, there is an inflow of financial capital as Home residents sell their assets to foreigners to finance increased consumption. A supply shock in Home acts in the opposite direction. Since Home's marginal propensity to consume is unaffected, the bulk of the increased supply of the Home good is shipped for consumption abroad, and therefore the current account improves, and capital flows from Home to Foreign.

In this analysis we have measured stock and bond prices in the same numeraire – or, loosely speaking, in a single international currency. Similar conclusions are obtained if we were to define prices in terms of an equivalent domestic currency. In this model a domestic currency should reflect the price of the basket of consumption (of both local and foreign-produced goods) by nationals. It is important to keep in mind that the domestic currency is the price of the consumption basket and not the price of the production basket (local good), and therefore the terms of trade effect on the stock markets will be present regardless of whether prices are quoted in the equivalent domestic currency or the international currency. We come back to these issues in Section 3.1 where we make the mapping from the terms of trade to the real exchange rate.

2.3. Special Cases and Interpretations of θ_H and θ_F

In Proposition 1 we did not commit to specifying the dynamics of the demand shocks θ_H and θ_F . For the empirical analysis to follow, this is advantageous because the relative demand shock $\theta = \theta_H/\theta_F$, in general, depends on the underlying Brownian motions w and w^* inducing a correlation in the error structure that we do not have to impose ex-ante. In this section, we put more structure on the martingales θ_H and θ_F as we consider various economic interpretations of these processes:

$$d\theta_H(t) = \vec{\kappa}_H(t)^\top \theta_H(t) d\vec{w}(t), \quad d\theta_F(t) = \vec{\kappa}_F(t)^\top \theta_F(t) d\vec{w}(t). \quad (21)$$

A. Deterministic Preference Parameters

It is instructive to consider a special case where θ_H and θ_F are deterministic processes, i.e., $\vec{\kappa}_H$ and $\vec{\kappa}_F$ are zeros. By substituting equation (10) into equation (14), we find that

$$S^*(t) = \frac{\lambda_H \theta_H(0) a_H + \lambda_F \theta_F(0) a_F}{\lambda_H \theta_H(0) (1 - a_H) + \lambda_F \theta_F(0) (1 - a_F)} S(t).$$

As the multiplier term is non-stochastic, the two stock markets must be perfectly correlated. This implication achieves our goal of generating international financial contagion without assuming correlation between individual countries' output processes, but it is certainly rather extreme and is easily rejected empirically. (Zapatero (1995) and Cass and Pavlova (2003) obtain this implication in their models as well, dubbing the resulting equilibrium a “peculiar” financial equilibrium.) In what follows, we force θ_H and θ_F to be stochastic processes, which would provoke the divergence effect and hence guarantee imperfect correlation between Home and Foreign stock markets, but would still produce financial contagion.

B. Preference Shocks or Pure Sentiment

The next special case we consider is where θ_H and θ_F are driven by the Brownian motion w^θ , independent of w and w^* . In this case, $\vec{\kappa}_H = (0, 0, \kappa_H)^\top$ and $\vec{\kappa}_F = (0, 0, \kappa_F)^\top$, where κ_H and κ_F are arbitrary adapted processes. Then θ has dynamics

$$d\theta(t) = (\kappa_F(t)^2 - \kappa_H(t)\kappa_F(t)) \theta(t) dt + (\kappa_H(t) - \kappa_F(t)) \theta(t) dw^\theta(t).$$

The process θ can then be interpreted as a relative preference shock or a shock to consumer sentiment.

C. “Catching up with the Joneses” / Consumer Confidence

Consider the case where the processes θ_H and θ_F depend on the country-specific Brownian motions: $\vec{\kappa}_H = (\kappa_H, 0, 0)^\top$ and $\vec{\kappa}_F = (0, \kappa_F, 0)^\top$, where $\kappa_H, \kappa_F < 0$ may depend on w and w^* .⁷ Home and Foreign consumers’ preferences then display “catching up with the Joneses” behavior (Abel (1990)). The “benchmark” levels of Home and Foreign consumers, θ_H and θ_F , are negatively correlated with aggregate consumption of their country, Y and Y^* , respectively. Positive news to local aggregate consumption reduces satisfaction of the local consumer, as his consumption bundle becomes less attractive in an improved domestic economic environment. The implication of this interpretation for the relative demand shock, θ , is that

$$d\theta(t) = \kappa_F(t)^2 \theta(t) dt + \kappa_H(t) \theta(t) dw(t) - \kappa_F(t) \theta(t) dw^*(t), \quad \kappa_H(t), \kappa_F(t) < 0.$$

That is, an innovation to θ is negatively correlated with the Home output shock and positively correlated with the Foreign shock.

An alternative kind of dependence of agents’ preferences on the country-specific output shock is a form of consumer confidence. The idea is that agents get more enthusiastic when their local economy is doing well and hence demand more consumption. Formally, preferences exhibiting consumer confidence are the same as those under catching up with the Joneses, except that $\kappa_H(t), \kappa_F(t) > 0$.

D. Radon-Nikodym Derivatives Reflecting Heterogeneous Beliefs

The previous two special cases we considered assume that consumer preferences are state-dependent. State-dependence of preferences, however, is not necessarily implied by our specification (4)–(5). Under the current interpretation, the countries have state-independent preferences, but they differ in their assessment of uncertainty underlying the world economy. For example, Home residents may believe that the uncertainty is driven by the (vector) Brownian motion $\vec{w}_H \equiv (w_H, w_H^*, w_H^\theta)^\top$ and Foreign residents believe that it is driven by $\vec{w}_F \equiv (w_F, w_F^*, w_F^\theta)^\top$. All we require is that the “true” probability measure P and the country-specific measures H (Home) and F (Foreign) are equivalent; that is, they all agree on the zero probability events. Under their respective measures,

⁷The labels κ_H and κ_F are used just to save on notation, they need not be the same as in interpretation **B**. Note that under this interpretation $\vec{\kappa}_H$ and $\vec{\kappa}_F$ do not depend on the Brownian motion w^θ . This specification makes w^θ a sunspot in the sense of Cass and Shell (1983): it affects neither preferences nor aggregate endowments. There is then a potential problem with employing the planner’s problem in solving for equilibrium allocations because it only identifies nonsunspot equilibria. Since markets are dynamically complete in our economy, however, the sunspot immunity argument of Cass and Shell goes through.

the agents' expected utilities are given by

$$E^i \left[\int_0^T [a_i \log(C_i(t)) + (1 - a_i) \log(C_i^*(t))] dt \right], \quad i \in \{H, F\},$$

where E^i denotes the expectation taken under the information set of agent i . Thanks to Girsanov's theorem, the above expectations can be equivalently restated under the true probability measure in the form of (4)–(5). The multiples θ_H and θ_F , appearing in the expressions as a result of the change of measure, are the so-called density processes associated with the Radon-Nikodym derivatives of H with respect to P , $(\frac{dH}{dP})$, and F with respect to P , $(\frac{dF}{dP})$, respectively.

The densities θ_H and θ_F may reflect various economic scenarios. One is the case of incomplete information: the consumers do not observe the parameters of the output processes, and need to estimate them. While the diffusion coefficients $\sigma_Y(t)$ and $\sigma_Y^*(t)$ may be estimated by computing quadratic variations of $Y(t)$ and $Y^*(t)$, estimation of mean growth rates of output is rather nontrivial. First, the consumers may be assumed to be Bayesian, endowed with some priors of the growth rates. Then they will be updating their priors each instant as new information arrives, through solving a filtering problem.⁸ Second, the consumers may use some updating method other than Bayesian. For example, they may be systematically overly optimistic or pessimistic about the mean growth rates of output. Finally, they may explicitly account for model uncertainty in their decision-making. All these special cases result in consumers employing a probability measure different from the true one. These differences of opinion can be succinctly represented by some Radon-Nikodym derivatives $\theta_H(T)$ and $\theta_F(T)$. Under any of these interpretations, we can no longer assume that θ in Proposition 1 is uncorrelated with either w or w^* , which we have to take into account in our econometric tests.

2.4. Interest Rate Parity and Trading Strategies

Uncovered interest rate parity in its classical form is a relationship between local interest rates at Home and Foreign and the expected exchange rate (terms of trade) appreciation. In this section we show that deviations from the parity, due to a pertinent risk premium, are driven primarily

⁸We provide details of the ensuing filtering problems in the Appendix, in the context of the proof of Proposition 2. We consider the case where the agents believe there are only two independent innovation processes, w_i and w_i^* , and the case where, additionally, there is the third one, w_i^θ . The first two innovations drive the output processes Y and Y^* , respectively. Following Detemple and Murthy (1994), we impose additional regularity conditions on $\sigma_Y(t)$ and $\sigma_Y^*(t)$ to make the filtering problem tractable: both processes are bounded and are of the form $\sigma_Y(Y(t), t)$ and $\sigma_Y^*(Y^*(t), t)$, respectively. When the agents perceive that the parameters they are estimating also depend on the third innovation process, which can represent either intrinsic or extrinsic uncertainty (a sunspot), they make use of a public signal carrying information about w_i^θ .

by demand shocks. We also fully characterize equilibrium interest rates prevailing at Home and Foreign, under different interpretations of the demand shocks. These results are reported in the following proposition.

Proposition 2. *The uncovered interest rate parity relationship has the form*

$$r^F(t) - \mu_q(t) = r^H(t) + \sigma_q(t)^\top (m^H(t) + \sigma_q(t)), \quad (22)$$

where μ_q and σ_q are the mean growth and volatility parameters in the dynamic representation of q , $dq(t)/q(t) = \mu_q(t)dt + \sigma_q(t)^\top d\bar{w}(t)$, m^H is the Home market price of risk reported in the Appendix, and r^H and r^F are the Home and Foreign real interest rates, i.e., instantaneously riskless returns on local money market accounts specified in terms of the local goods, given by:

(i) under interpretations **A** and **B**,

$$r^H(t) = \mu_Y(t) - \sigma_Y(t)^2, \quad r^F(t) = \mu_Y^*(t) - \sigma_Y^*(t)^2,$$

(ii) under interpretation **C**,

$$\begin{aligned} r^H(t) &= \mu_Y(t) - \sigma_Y(t)^2 + \underbrace{\frac{\lambda_H \theta_H(t) a_H}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}}_{C_H(t)/Y_H(t)} \sigma_Y(t) \kappa_H(t), \\ r^F(t) &= \mu_Y^*(t) - \sigma_Y^*(t)^2 + \underbrace{\frac{\lambda_F \theta_F(t) (1 - a_F)}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)}}_{C_F^*(t)/Y_F^*(t)} \sigma_Y^*(t) \kappa_F(t), \end{aligned}$$

(iii) under interpretation **D** (see the Appendix for the detailed description of the economic setting),

$$\begin{aligned} r^H(t) &= \mu_Y(t) - \sigma_Y(t)^2 + \underbrace{\frac{\lambda_H \theta_H(t) a_H (\mu_H(t) - \mu_Y(t)) + \lambda_F \theta_F(t) a_F (\mu_F(t) - \mu_Y(t))}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}}_{\frac{C_H(t)}{Y(t)} (\mu_H(t) - \mu_Y(t)) + \frac{C_F(t)}{Y(t)} (\mu_F(t) - \mu_Y(t))}, \\ r^F(t) &= \mu_Y^*(t) - \sigma_Y^*(t)^2 + \underbrace{\frac{\lambda_H \theta_H(t) (1 - a_H) (\mu_H^*(t) - \mu_Y^*(t)) + \lambda_F \theta_F(t) (1 - a_F) (\mu_F(t) - \mu_Y^*(t))}{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)}}_{\frac{C_H^*(t)}{Y^*(t)} (\mu_H^*(t) - \mu_Y^*(t)) + \frac{C_F^*(t)}{Y^*(t)} (\mu_F(t) - \mu_Y^*(t))}, \end{aligned}$$

where $\mu_H(t)$ and $\mu_F(t)$ denote perceived or estimated mean growth rates of the Home output by Home and Foreign consumers, respectively, and $\mu_H^*(t)$ and $\mu_F^*(t)$ denote perceived or estimated mean growth rates of the Foreign output by Home and Foreign consumers, respectively.

In its classical form, uncovered interest rate parity is a relationship which assumes that arbitrage will enforce equality of returns on the following two investment strategies. The first one is investing a unit of wealth in the Home money market account resulting in receiving the riskless return at

rate r^H , in units of the Home good, over the next instant. The second is investing the same amount in the Foreign money market, at rate r^F , over the next instant and then converting the proceeds paid out in the Foreign good into the Home good using the prevailing terms of trade. In our model, the two strategies are not equivalent because in terms of the Home good, the first strategy is riskless, while the second one is risky. The left-hand side of (22) represents the mean return on the second strategy, given by the Foreign money market rate less the mean terms of trade appreciation. The right-hand side contains the return on the riskless first strategy, plus an additional term capturing the risk premium the risky strategy commands. This risk premium is, in general, time-varying. The only case in which it can be shown to be constant in the context of our model is within interpretation **A** (no demand shocks) under an additional assumption that the mean growths and volatilities of the output processes Y and Y^* are constant. Recall that the role of interpretation **A** is to establish a theoretical benchmark; it is easily rejected empirically. Under the remaining interpretations, the risk premium term depends on the demand shocks. Hence, uncovered interest rate parity in its classical form – (22) with no risk premium term – does not obtain in our model.

In the empirical section, which comes next, we estimate the variance-covariance matrix of the shocks, and find that the variance of demand shocks is two times larger than that of supply shocks. One implication of this finding is that the risk premium would be very volatile, too. Deviations from the classical uncovered interest rate parity relationship have been found to be very large and volatile, and the need for a very volatile risk premium has been emphasized repeatedly in the empirical literature since Fama (1984). Our model may then shed some light on these issues.

Equilibrium riskless rates of return on money market accounts in each country have to be such that agents are willing to save. In the benchmark case of no demand shocks (interpretation **A**), local interest rates must then be positively related to the growth of aggregate consumption of the local good and negatively related to the aggregate consumption volatility, reflecting the precautionary savings motive. When the demand shocks are interpreted as pure sentiment shocks (**B**), driven by an independent source of uncertainty, the local money market rates are the same as in the benchmark. This is because the sentiment shocks are martingales; otherwise if they had nonzero mean growth terms, these terms would have appeared in the expression for the interest rate as additional inducements for the agents to save, acting analogously to impatience parameters (which we do not model). Since the demand shocks do not co-vary with aggregate consumption uncertainty,

they also do not give rise to an additional precautionary savings component. Such a component appears under interpretation **C**, where aggregate consumption of the local good imposes a negative (catching up with the Joneses) or positive (consumer confidence) externality on the agents. In the former case, this gives rise to an additional precautionary savings effect, driving down the interest rates, while in the latter case the interest rates rise. The strength of this additional effect induced by consumption externalities of Home (Foreign) agents is determined by the relative importance of Home (Foreign) consumers in the world economy. This relative importance is captured by the shares of the agents in the aggregate consumption of the local good, which can also be linked to wealth distribution in the economy. An additional component over the benchmark also appears in the interest rate expressions under the differences of opinion interpretation **D**, as highlighted in Proposition 2. Perhaps a better way to present say the Home rate under this interpretation is as a weighted average of the Home rates prevailing if only Home consumers were present in the economy $(\mu_H(t) - \sigma_Y(t)^2)$ and if only Foreign consumers were present $(\mu_F(t) - \sigma_Y(t)^2)$, with weights again given by the agents' consumption shares.

We now turn to computing the agents' trading strategies. To facilitate comparison with the literature, we consider an additional security: a "world" bond B^W , locally riskless in the numeraire. This bond does not, of course, introduce any new investment opportunities in the economy; it is just a portfolio of α shares of the Home bond and $(1-\alpha)$ shares of the Foreign bond. Let r denote the interest rate on this bond.

The number of non-redundant securities differs across interpretations **A–D**, which complicates our exposition. Thus, as we discussed earlier, Home and Foreign stocks are perfectly correlated under interpretation **A**, and hence investments in individual stock markets cannot be uniquely determined. Under the remaining interpretations, we can uniquely identify agents' holdings of national stock markets. One of the bond markets, however, may or may not be redundant depending on the interpretation. In what follows, we replace the Foreign bond by the world bond in the investment opportunity set of the agents. (Investment in the Foreign bond can be recovered from the portfolio allocations to the Home and world bonds, where applicable.)

Before we proceed to reporting our results, we need to define the notion of a *home bias*. We measure a bias in portfolios relative to the (common to the agents) mean-variance portfolio. Thus, a Home (Foreign) resident's portfolio is said to exhibit a home bias if the portfolio weight he assigns to the Home (Foreign) stock is higher than that in the mean-variance portfolio.

Proposition 3. (i) Countries' portfolios are given by

$$x_i(t) = \underbrace{(\sigma(t)^\top)^{-1} \mathcal{I} m(t)}_{\text{mean-variance portfolio}} + \underbrace{(\sigma(t)^\top)^{-1} \mathcal{I} \bar{\kappa}_i(t)}_{\text{hedging portfolio}}, \quad i \in \{H, F\}, \quad (23)$$

where the compositions of the vector of the fractions of wealth invested in risky assets, x_i , of the volatility matrix of the investment opportunity set σ , of the market price of risk m and of the auxiliary matrix \mathcal{I} , different across interpretations **A–D**, are provided in the Appendix. The remaining fraction of wealth, $1 - x_i^\top \bar{\mathbf{1}}$, is invested in the world bond B^W , where $\bar{\mathbf{1}} = (1, \dots, 1)^\top$.

(ii) Under interpretation **B**, countries' portfolios do not exhibit a home bias. Assume further that $a_H > a_F$. Then, under interpretation **C**, in the case of $\kappa_H(t), \kappa_F(t) > 0$ (consumer confidence) portfolios always exhibit a home bias, while in the case of $\kappa_H(t), \kappa_F(t) < 0$ (catching up with the Joneses) the portfolio of Home exhibits a home bias if and only if $(1 - \alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t) > 0$ and that of Foreign if and only if $\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1 - \alpha)\kappa_H(t) > 0$. Under interpretation **D**, the sign of the bias is ambiguous.

(iii) The international CAPM has the form

$$\begin{aligned} \frac{E_t(dS^j(t))}{S^j(t)} - r(t) &= \text{Cov}_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{dW(t)}{W(t)} \right) - \text{Cov}_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_H}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} d\theta_H(t) \right) \\ &\quad - \text{Cov}_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_F}{\lambda_H \theta_H(t) + \lambda_F \theta_F(t)} d\theta_F(t) \right), \quad S^j \in \{S, S^*\}, \end{aligned} \quad (24)$$

where $W \equiv W_H + W_F$ is the aggregate wealth and r is the interest rate on the world bond, reported in the Appendix.

In our economy, the fact that the countries have logarithmic preferences does not imply that their investment behavior is myopic. Their trading strategies involve holding the standard mean-variance portfolio along with a hedging one. Although the countries do not hedge against changes in the investment opportunity set (which is standard for logarithmic preferences), they do hedge against their respective demand shocks.⁹ The way they do so depends on the correlation between their demand shocks and output shocks, and hence differs across our interpretations of demand shocks. Thus, if the demand shocks are independent of output shocks – pure sentiment interpretation **B** – portfolio weights the countries assign to the stocks coincide with those in the mean-variance portfolio. If a country's demand shocks are positively related to its output shocks (consumer

⁹Our references to consumers' hedging behavior made in the context of the differences of opinion interpretation of the demand shocks may appear confusing. How can consumers hedge deviations of their beliefs from the true probability measure, which they do not know? In fact, they do not. They hold a portfolio which is mean-variance efficient under *their own* probability measure, given by the product of the inverted volatility matrix $(\sigma^\top)^{-1}$ and the country-specific market prices of risk $m_i = m + \bar{\kappa}_i$, constructed from their own investment opportunity sets. The representation in equation (23) is thus formally equivalent to a mean-variance portfolio under a country's individual beliefs, except it further decomposes a country-specific market price of risk into two components: the "true" market price of risk m and the measure of deviation of agent's beliefs from the true probability measure $\bar{\kappa}_i$, both unobservable from the point of view of an agent.

confidence interpretation **C**)— that is, if a country demands more consumption (biased toward the domestic good) when it is experiencing a positive output shock – consumers increase their portfolio holdings of the domestic stock, a claim to domestic output. On the other hand, if the demand shocks are negatively correlated with the country’s output innovations – catching up with the Joneses interpretation **C** – a home bias in portfolios may or may not arise. Proposition 3 provides a necessary and sufficient condition for the bias to occur. Finally, our differences of opinion interpretation **D** is too general to entail unambiguous implications on the sign of the bias. However, if specialized further, it is likely to yield sharper predictions. For example, albeit in a different setting, Uppal and Wang (2003) argue that disparities in agents’ ambiguity about home and foreign stock returns can lead to a home bias in portfolios.

The presence of the hedging portfolios optimally held by the agents clearly rules out the traditional one-factor CAPM. In addition to the standard market (aggregate wealth) factor, our model identifies two further factors, the Home and Foreign demand shocks, that affect the risk premia on the stocks. The expression in (24) also differs from the international CAPM (see Solnik (1974), Adler and Dumas (1983)). As is standard for logarithmic preferences, the exchange rate does not appear explicitly in (24); however, its determinants – the demand shocks – do. (One would expect the exchange rate to appear as an additional factor if we relaxed the assumption of logarithmic preferences.) The presence of the demand shocks points to a possible misspecification of the CAPM widely tested empirically. Our alternative formulation might then provide an improvement over the standard specification used in the international finance literature.

3. Empirical analysis

In this section we examine the empirical implications of the model. We use monthly macro data and daily financial data for the US vis-à-vis the UK that has been collected from DataStream. The macro variables we employ are described later in the section. The financial variables needed for our estimation are stock and bond returns and the dollar-pound exchange rate. As proxies for our riskless bonds, we use data on the three-month zero-coupon government bonds for each country. The US and UK stock market indexes are taken to represent the countries’ stock prices. The dollar-pound exchange rate is used for identifying the real exchange rates and the terms of trade via a procedure described below. The data are from 1988 until the end of 2002. Figure 1 depicts the evolution of the exchange rate along with stock and bond markets values for the US vis-à-vis

the UK. Even a casual observation of the figure suggests that the bond prices do not bear too much relationship to the exchange rate. This observation is strongly supported by a regression analysis, not reported here, whose results may be interpreted as a failure of uncovered interest rate parity, well-documented in the literature and also found in our sample. On the other hand, the amount of co-movement between the stock indexes and the exchange rate is quite striking. We computed the pertinent correlations and found them to be highly statistically significant. Our theory that stock prices and exchange rates are driven by the same set of factors, then, does not appear to be empirically unfounded.

3.1. Real and Nominal Quantities

Before we proceed with our formal empirical analysis, we need to establish a mapping between the quantities employed in our model and those in the data available to us – the data are in nominal terms, while our model is real. The main issue is to impute the terms of trade from the available data on nominal exchange rates. To do so, we first compute the real exchange rate implied by our model. The real exchange rate, e , is defined as $e(t) = P_H(t)/P_F(t)$, where P_H and P_F are Home and Foreign price indexes, respectively. In practice, a variety of methods have been used for computing a country's price level, or cost of living (see Schultze (2003)). We have chosen to adopt geometric average price indexes, which address the criticism pertaining to the substitution between goods bias, and are consistent with the countries' homothetic preferences:

$$P_H(t) = \left(\frac{p(t)}{a_H}\right)^{a_H} \left(\frac{p^*(t)}{1-a_H}\right)^{1-a_H}, \quad P_F(t) = \left(\frac{p(t)}{a_F}\right)^{a_F} \left(\frac{p^*(t)}{1-a_F}\right)^{1-a_F}.$$

The real exchange rate, expressed as a function of the terms of trade, is then

$$e(t) = q(t)^{a_H-a_F} \frac{(1-a_F)^{1-a_F} a_F^{a_F}}{(1-a_H)^{1-a_H} a_H^{a_H}}.$$

Finally, to obtain the nominal exchange rate, we need to adjust the real exchange rate for inflation in Home versus Foreign. Unfortunately, daily data on inflation, required for our estimation, are not available. However, our data span a relatively short period of time when international inflation rates were very low. Consequently, as argued by Mussa (1979), our real rates are closely related to the nominal ones. We thus assume that inflation is negligibly small and simply make a level adjustment to the real rate to back out the nominal exchange rate, ε : $\varepsilon(t) = \bar{\varepsilon}e(t)$, where $\bar{\varepsilon}$ is the average nominal exchange rate in the sample.

For our empirical investigation of the dynamics reported in Proposition 1, we use the variable

$$q(t) = \left(\frac{\varepsilon(t)}{\bar{\varepsilon}} \frac{(1 - a_H)^{1-a_H} a_H^{a_H}}{(1 - a_F)^{1-a_F} a_F^{a_F}} \right)^{1/(a_H - a_F)} \quad (25)$$

as a proxy for the terms of trade. Taking logs and applying Itô's lemma, we obtain an equation to replace (19) in Proposition 1:

$$\frac{d\varepsilon(t)}{\varepsilon(t)} = I_6(t)dt + (a_H - a_F)A(t)d\theta(t) - (a_H - a_F)\sigma_Y(t)dw(t) + (a_H - a_F)\sigma_Y^*(t)dw^*(t),$$

where $I_6(t) = (a_H - a_F)I_5(t) + \frac{1}{2}(a_H - a_F)(a_H - a_F - 1)|\sigma_q(t)|^2$. Under our assumption of no inflation over the time period we are considering, the remaining equations in Proposition 1, (15)–(18), are unchanged when prices are expressed in nominal terms. Finally, since the nominal exchange rate, ε , is monotonically increasing with the terms of trade q , the signs of the coefficients identified in Proposition 1 are maintained for nominal prices. Moreover, the signs of the effects of the shocks on $\frac{d\varepsilon(t)}{\varepsilon(t)}$ are the same as those for $\frac{dq(t)}{q(t)}$ reported in (20).

For a practical implementation of this conversion of units, we need to calibrate parameters a_H , a_F and α used in the expressions. It is reasonable to assume that about three quarters of a country's consumption comes from the locally produced good. Such home bias is primarily due to the presence of non-tradable goods, which comprise a large fraction of national consumption. While we do not explicitly account for non-tradable goods, their presence is modeled in reduced form, through parameters a_H and a_F (see Section 4 for further elaboration). It is thus reasonable to set a_H equal to 0.75 and a_F equal to 0.25. We also need a sensible value for α , the weight of the US-produced goods in the world numeraire basket, or a price index. By setting $\alpha = 0.75$ we are implicitly assuming that the US economy is about three times the UK economy. Of course, we run robustness checks where we vary these parameters, which we discuss later in this section. The calibrated values of a_H and a_F are used for computing the series for the terms of trade via (25). We then use the numeraire basket (identified by α) and the terms of trade to convert stock and bond prices for each country into a common numeraire.

3.2. Empirical Strategy

Before estimating the system of equations in Proposition 1, it might be worth pointing out why the interconnections among financial and foreign exchange markets we are after cannot be analyzed by OLS. It is well-known from econometric theory that a naïve OLS estimation of equation (14) – the equation linking together stock prices and the exchange rate – would yield incorrect implications

because of the omitted variable bias due to the latent factors. It is worth demonstrating how severe this bias actually is. For example, in the case of the US vis-à-vis the UK the estimation of (log-linearized) equation (14) on monthly data yields

$$\frac{dS^*(t)}{S^*(t)} = 0.1007 \frac{d\varepsilon(t)}{\varepsilon(t)} - 0.0089 \frac{dY^*(t)}{Y^*(t)} + 0.7431 \frac{dS(t)}{S(t)} + 0.0218 \frac{dY(t)}{Y(t)},$$

where we have used the industrial production series for the US and the UK from DataStream as proxies for Y and Y^* . Several nonsensical results emerge. First, the signs on the exchange rate and both the US and the UK outputs agree with neither our theory nor the standard reasoning of policymakers. Second, the coefficients are significant only for the exchange rate and the US stock market, with the latter encompassing a very high t -statistic. In other words, the only conclusion that we derive from this regression is that contagion is extremely important. In fact, the adjusted R^2 of this regression is 54 percent, but if the US stock market is excluded, the R^2 drops to 2 percent.

The OLS results should not be interpreted as a rejection of the model. Do we believe that a positive productivity shock has no impact on stock prices? What these OLS regressions demonstrate is that they are simply not fit to capture the effects we are interested in. We thus employ an alternative estimation strategy: we consider the system of structural equations in Proposition 1 and simultaneously estimate the responses of each market to the supply and demand shocks.

Our model's implications on the dynamic behavior of asset prices and exchange rates are summarized in Proposition 1, in the system of equations (15)–(19) and the accompanying table with the predicted signs of the coefficients (20). For convenience, we reproduce this system below, in a matrix form:

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} b(t) & 1 - b(t) & b(t) \\ -1 + b(t) & 1 - b(t) & b(t) \\ b(t) & -b(t) & b(t) \\ -1 + b(t) & 1 - b(t) & -1 + b(t) \\ (a_H - a_F) & -(a_H - a_F) & (a_H - a_F) \end{bmatrix}}_{\mathcal{B}_1} \begin{bmatrix} f_\theta(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix}, \quad (\text{M1})$$

where \vec{I} is a (vector) intercept term,

$$\begin{bmatrix} f_{\theta}(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix} \equiv \begin{bmatrix} A(t) d\theta(t) \\ \sigma_Y(t) dw(t) \\ \sigma_Y^*(t) dw^*(t) \end{bmatrix},$$

$$b(t) \equiv \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha}, \quad \text{with} \quad q(t) = \left(\frac{\varepsilon(t) (1 - a_H)^{1 - a_H} a_H^{a_H}}{\bar{\varepsilon} (1 - a_F)^{1 - a_F} a_F^{a_F}} \right)^{1/(a_H - a_F)}. \quad (26)$$

Unfortunately, while we can construct all the left-hand side variables in (M1), we do not have data on the variables on the right-hand side of the system. Consequently, in our estimation we treat the innovations f_{θ} , f_w , and f_{w^*} as latent factors that we extract from stock and bond prices and exchange rates. The first factor, f_{θ} , captures the relative demand shock, while the remaining two, f_w and f_{w^*} , represent Home and Foreign supply shocks, respectively. (There is a slight abuse of terminology in this section; earlier we referred to $d\theta$, dw and dw^* as the demand and supply shocks.) Note that the factor loadings in matrix \mathcal{B}_1 are fully described by the quantity $b(t)$, which depends only on the weight of the US-produced goods in the world numeraire basket α , the nominal exchange rate ε , and the preference for domestic good parameters a_H and a_F .

Perhaps the most direct approach to testing our model would be to take Proposition 1 literally and estimate the structural latent factor model (M1). Note, however, that since the factors are in general heteroskedastic and may be correlated in our model, a priori there is no reason to put any restrictions on their variance-covariance matrix. The only set of restrictions our theory entails is those on the elements of matrix \mathcal{B}_1 . This estimation strategy, in practice, imposes too few constraints on the model: effectively, we would be fitting five series with three unrestricted factors. As a result, the fit of the model within sample would be very good and the likelihood of rejecting the model very low.¹⁰ To make this estimation more convincing, we attempt to assess the economic meaning of the latent factors we extract. We thus compare the factors with pertinent macroeconomic innovations available to us at monthly frequency. As a companion exercise, we also evaluate the performance of the factors in terms of explanatory power and explore the empirical validity of the sign restrictions implied by our theory. An alternative approach, undertaken next, is to estimate the coefficients of our system (M1) through the sign restrictions imposed by the model in combination with heteroskedasticity in the data.

¹⁰We have indeed estimated the model following this strategy. The fit within sample has been unrealistically good. Results are available from the authors upon request, and from <http://web.mit.edu/~rigobon/www/>.

f_θ	f_w	f_{w^*}
6.8397	1.9743	-2.6463
41%	3.3570	0.4023
-44%	10%	5.2112

Table 1: Variance-covariance matrix and correlations of factors. The upper-triangular matrix is the variance-covariance matrix of latent factors $f_\theta(t)$, $f_w(t)$, and $f_{w^*}(t)$; the correlations of the factors are reported in the lower left of the matrix.

3.2.1. Latent Factor Model

We take the model literally and decompose the stock prices and the exchange rate into the factor model via the specification in (M1) assuming their drifts are constant. Note that in this section we do not use bond prices to estimate the latent factors. In fact, the bond prices are some of the macroeconomic variables we intend to explain. This is a more stringent test of the model. If we use the bond prices in the estimation of (M1), the results on the remaining macro variables are roughly the same.

The five estimates reported in the upper-right of Table 1 summarize the covariance matrix of the three latent factors. First, note that the demand shocks are very important both in magnitude and significance, rejecting our interpretation **A**, where we assume that only the supply shocks drive the economy. In fact, the standard deviation of the demand shocks is twice as large as that of the supply shocks in the US. These results are robust to estimating the model using weekly instead of daily returns. Obviously, the point estimates change, but the relative importance of the shocks remains the same. Our conclusions are in line with the macroeconomics literature. For example, Clarida and Gali (1994) also find that demand shocks are large relative to supply shocks using a very different identification strategy and low frequency data.

Second, note that the estimates of the correlations constructed from the covariance matrix indicate that the relative demand shock is positively correlated with the US's supply shock, and negatively correlated with the UK's. The estimates are statistically significant, strongly rejecting the pure sentiment interpretation **B**. Furthermore, the catching up with the Joneses interpretation **C**, under which the the relative demand shock should negatively co-vary with the US supply shock and positively with the UK's, is rejected, too. The story the data is telling is the reverse: instead of becoming relatively unhappy when their country's aggregate consumption increases, agents appear to get enthusiastic when their economy is doing well. This speaks in favor of our consumer

confidence interpretation **C**. Alternatively, these results may be viewed as evidence of differences in opinion (interpretation **D**).¹¹ Overall, these conclusions are robust to using weekly instead of daily returns, and to computing the latent factors using all five variables, or by including the bond prices in the estimation of the factors.

Our next step is to use the latent factors and evaluate their predictive power on macroeconomic variables. The model we have developed implies that the latent factors identified from asset prices have economic interpretation, and therefore, we should expect that they have predictive power on macro-variables excluded from the estimation. Furthermore, the model also provides sign restrictions that can be used to evaluate these predictions.

Given the span of our data, we concentrate on macro-variables available at monthly frequency. From DataStream we obtain industrial production in the US and UK, the trade balance, consumer confidence measures (from the surveys), capital flows between the countries, and business trend surveys. The last three sets of variables require further description. The consumer confidence variables include the standard one that most market participants concentrate on, plus two of its components: the consumer confidence on the current situation and consumer expectations. The business trend surveys ask business managers about the prospects of exports, employment, and production. Lastly, the capital flows are the net flows into the US and the UK collected from the US Treasury data. These data has important limitations because it does not identify the actual capital flows by nationality of the investors, but by the nationality of the institution initiating the trade. However, at monthly frequency these are the best data available.

As was mentioned before, our model has unique sign implications on how the factors affect production, employment, expectations, and external variables such as exports. Hence, we evaluate the performance of the factors along two dimensions: the explanatory power and their signs.

The procedure is the following: we convert the factors from daily to monthly frequency by accumulating the daily changes and run the regression

$$dM(t) = b_m + \sum_{i=\{1,L\}} b_{m,i}dM(t-i) + \sum_{i=\{0,L\}} b_{\theta,i}f_{\theta}(t-i) + \sum_{i=\{0,L\}} b_{w,i}f_w(t-i) + \sum_{i=\{0,L\}} b_{w^*,i}f_{w^*}(t-i) + u(t),$$

¹¹These results change slightly when we use weekly returns: the correlations are 28, -8 and 48 percent respectively. However, even though the magnitude of the correlations depends on the horizon used to compute the returns, their signs are the same.

	With $b_{m,i} \neq 0$	With $b_{m,i} = 0$	$f_{\theta,t}$	$f_{w,t}$	$f_{w^*,t}$
Industrial Production (US)	39.1%	23.5%	–	+	+
Industrial Production (UK)	28.6%	14.7%	+	+	+
Trade Balance (US)	51.6%	23.6%	0	–	+
Consumer Confidence Total (US)	39.8%	36.7%	+	–	+
Consumer Confidence Current (US)	35.9%	24.6%	+	–	+
Consumer Confidence Expectations (US)	42.4%	40.0%	+	–	+
Capital Flows (US)	90.0%	37.1%	+	+	–
Capital Flows (UK)	91.1%	44.2%	+	–	+
Business Trend - Exports	75.4%	49.9%	–	+	0
Business Trend - Employment	80.7%	22.3%	+	+	+
Business Trend - Production	68.9%	14.2%	+	0	+
Bond Prices (US)	40.1%	28.5%	+	–	+
Bond Prices (UK)	28.8%	24.3%	0	+	–

Table 2: Regressions of macro-variables on the factors. The first two columns report the R squares of the regressions. The last three columns summarize the average signs of the impulse response: a “+” (“–”) means that more than two thirds of the response is positive (negative), otherwise we assign 0.

where $M(t)$ is a macro-variable and $u(t)$ is the error term. We allow for six lags to capture slow responses of the macro-variables. The results are presented in Table 2.

The first column reports the explained variance of a macro-variable from the full specification, while the second column indicates the proportion of the variance explained by the latent factors alone, i.e., when the lags of the macro-variable are excluded from the regression. The last three columns indicate the overall sign of the impulse response. A positive sign indicates that at least two thirds of the impulse response is positive, while a negative sign indicates that two thirds of the response is negative. Otherwise, we assign zero. The impulse responses to a permanent shock in each of the factors are shown in Figure 2. These responses were computed assuming that the shock is idiosyncratic. Hence, we did not take into account the contemporaneous correlation across factors when these responses were computed. As can be seen from Figure 2, the responses are relatively smooth with few sign changes.¹²

Before discussing our results it is important to mention that there are several papers that have attempted to estimate the macroeconomic content embedded in financial markets fluctuations, finding mixed evidence (see, for example, Campbell and Clarida (1987) and Stock and Watson (2003)). In general, the coefficients are unstable, change their signs, and significance depending on the period and country under analysis. Given our theory, this result should have been expected. If

¹²Because we estimated an MA(6) (when lags of the endogenous variables are not included), we only show the first six periods of the response.

the relative importance of the factors changes through the sample, the reduced form coefficients in the regression of macro-variables on asset prices have to change sign and significance as well. The most basic example is the case of a demand versus a supply shock where both increase output but have different effects on bond prices. Therefore, in periods where the supply shocks dominate the coefficient has the opposite sign compared to the periods where demand shocks are relatively more important. Our paper differs from this literature in that we impose the structure of our model to disentangle the underlying innovations.

From the first two columns it is clear that the latent factors we are collecting from the daily changes in stock prices and the exchange rate have significant explanatory power on the macro variables. For example, as evident from the second column, our factors explain almost 25 percent of the fluctuations of the US industrial production, and almost 15 percent of the fluctuations of the UK production. Our factors explain 24 percent of the variance of the trade balance, 36 percent of the consumer confidence index, almost 25 percent of the consumer confidence on the current situation, and more than 40 percent of the consumer expectations. In regards to the capital flows, the latent factors explain roughly 40 percent of their variation for both countries, almost 50 percent of the variation of the perceptions about exports, and close to 20 percent of the other business trend surveys.¹³

It is important to mention that these degrees of explanatory power are higher than those found using the same data and the asset returns alone. The main reason is that our model imposes economic restrictions that help the out-of-sample performance. However, it is important to mention that the improvements are of the order of one to three points in the R squares.¹⁴ However, our estimates do not suffer from the instability usually obtained in the literature and the signs of our estimates are, for the most part, in line with the theory.

In regards to the implications on the signs, our theory predicts that the factor associated with θ should be positively correlated with consumer confidence measures, and with capital inflows. Indeed, Consumer Confidence Total, current and expectations, as well as Capital Flows (US) have positive impulse responses to a shock in θ . Additionally, the theory predicts that θ should be negatively correlated with the exports (Business Trend - Exports is negative, but there is no effect

¹³We perform tests evaluating the economic significance of introducing the factors in the regression after having controlled for lagged dependent variables. Except for capital flows and business survey, the factors contribute significantly to the explanatory power of the regression at normal significance levels (5 percent).

¹⁴These results are robust to estimating the factors using (i) all four asset prices and the exchange rate, or (ii) only the two stock markets and the exchange rate. Furthermore, the signs of the impulse responses are also similar.

	(a) Raw Financial Variables		(b) Latent Factors	
	Same Sign	Sqrt Difference	Same Sign	Sqrt Difference
Industrial Production	77.8%	0.3033	83.3%	0.0328
Consumer Confidence	59.3%	0.9279	77.8%	0.1185
Business Trend	63.0%	1.6506	100.0%	0.1557
Capital Flows	63.9%	2.8024	38.9%	0.4430
Bond Prices	63.9%	1.7133	91.7%	0.2361
Simple Average	65.7%	1.6461	80.1%	0.2273

Table 3: Stability of forecasts based on raw financial variables vs. latent factors f_θ , f_w , and f_w^* .

on the Trade Balance). The theory has no predictions on the effect of θ on output, and the results on output are mixed. Furthermore, productivity shocks in the US should increase output in the US (Industrial Production and Business Trend - Employment are both positive, although Business Trend - Production is small in magnitude). Under the consumer confidence interpretation of the demand shocks, US productivity shocks should be associated with increases in consumer confidence. Here we find the opposite mainly because the impulse response is not taking into account the contemporaneous correlation between θ and the US productivity shock.

To address the issue of parameter instability we estimated the model for the sub-sample 1995 until 2002 and used it to forecast the macro-variables. We compare the impulse responses of the sub-sample with the ones from the full sample estimation. The impulse responses are compared in two dimensions. First, we evaluate the consistency on the impulse responses regarding the signs of their impulse responses. Second, we compute how different the 6 month forecasts are. We perform these comparisons for the regression using only raw financial variables – the stock markets and the exchange rate – and the ones using our factors. The results are reported in Table 3. Each macro-variable in the table combines all corresponding individual measures in its category from Table 2, e.g., “Industrial Production” combines industrial production of the US and the UK. Panel (a) shows the results for the financial variables regression, and panel (b) shows the results when the factors are used.

As evident from Table 3, panel (b), the impulse responses on the factors have (except for Capital Flows) roughly the same signs across the two samples. For example, for all the output measures, 83 percent of the impulse responses have the same signs – as opposed to 78 percent when the financial variables are used, panel (a). In regards to consumer confidence, business surveys and bond prices, the factors clearly outperform the financial variables. The only exception is Capital Flows.

More importantly, the six month forecast using raw financial variables is much noisier than using the factors. We computed the ratio between the squared difference in the six month forecast across sub-samples and the standard deviation of the macro-variable. As can be seen, the difference in the forecast using the factors is orders of magnitude better than using financial variables. For example, the differences in the six months forecasts in the two sub-samples of output variables is close to 3 percent when the factors are used and it is 30 percent when financial variables are used. For Consumer Confidence, Business Trend and Bond Prices, the forecast differences using the factors are about one tenth of the errors when using the financial variables. This is an indication of the parameter and forecast instability that arises when the financial variables are used in the regression, while the overall impact of the factors remains close across sub-samples.

The model we have evaluated is quite restrictive. It is a real model, with only three sources of shocks, and with all goods tradable. We return to several of these points below in the section devoted to the caveats. However, overall it could be claimed that the out of sample performance of the latent factors is good. They explain a sizeable proportion of the variance of several survey measures and real variables.

3.3. Sign Restrictions

The previous estimation of the model relied too heavily on the structural specification of the equations derived in the theoretical part. Of course, any theoretical model has its limitations. The purpose of this section is to estimate a less restrictive version of Model (M1) where we relax the cross-equation restrictions on the coefficients of matrix \mathcal{B}_1 , and also allow the variance of the latent factors to change through time. We assume that the coefficients in \mathcal{B}_1 are constant, and now impose only some sign restrictions arising from Proposition 1. This estimation would then allow us to test whether the model's implications regarding the signs of responses of observed prices to innovations in the latent factors, summarized in (20), are indeed present in the data. We would also be able to test whether the cross-equation restrictions (on the coefficients in \mathcal{B}_1) entailed by the model and not imposed in this estimation are rejected in our sample.

We estimate the following latent-factor model

$$\begin{bmatrix} \frac{dS(t)}{S(t)} \\ \frac{dS^*(t)}{S^*(t)} \\ \frac{dB(t)}{B(t)} \\ \frac{dB^*(t)}{B^*(t)} \\ \frac{d\varepsilon(t)}{\varepsilon(t)} \end{bmatrix} = \vec{I} + \underbrace{\begin{bmatrix} \alpha_{11} & 1 & \alpha_{13} \\ \alpha_{21} & 1 & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \\ \alpha_{41} & \alpha_{42} & \alpha_{43} \\ 1 & \alpha_{52} & \alpha_{53} \end{bmatrix}}_{\mathcal{B}_2} \begin{bmatrix} f_\theta(t) \\ f_w(t) \\ f_{w^*}(t) \end{bmatrix}, \quad (\text{M2})$$

where α_{ij} are constant. The signs of the responses of stock and bond prices and the exchange rate to innovations in the latent factors are summarized in (20). They require that α_{11} , α_{13} , α_{22} , α_{31} , α_{33} , α_{42} , α_{53} , are positive and that α_{21} , α_{32} , α_{41} , α_{43} , α_{52} , are negative. In order to estimate the model, we have to normalize some of the coefficients in matrix \mathcal{B}_2 . The coefficients set equal to unity reflect the following normalization: the productivity shock in the US has a unitary effect on the US and the foreign stock market (responses of the two markets should be identical, according to our theory) and the demand shock has a unitary effect on the exchange rate. We continue to assume that the supply shocks in the US are orthogonal to those in the foreign country, but allow the demand shocks to be correlated with the supply shocks. We also assume that heteroskedasticity can be described by two regimes: a high and a low volatility regime.

In model (M2), there is a problem of identification in the absence of heteroskedasticity. The time series for the left-hand side variables allow us to estimate their variance covariance matrix, yielding a total of 15 moments. On the other hand, once we allow for correlation between demand and supply shocks, the right-hand side has 17 moments. Hence, standard identification is not possible. Under the assumptions of constant coefficients and heteroskedasticity, however, the presence of only two heteroskedastic regimes is sufficient to solve the problem of identification – even if the covariance matrices across the two regimes are completely unrestricted. In other words, if there are two heteroskedastic regimes, we can compute two distinct variance-covariance matrices, which provide 30 moments. The right hand side has 12 unknowns in the matrix \mathcal{B}_2 . In addition, there are two factor covariance matrices, each containing five unknowns. In total, there are 22 variables to be estimated.¹⁵ Obviously, this system of equations satisfies the order condition, meaning that the

¹⁵This identification strategy is related to the recent “identification through heteroskedasticity” literature. See Wright (1928) for the original contribution, and Sentana (1992), Sentana and Fiorentini (2001) and Rigobon (2003) for recent developments. For applications see Caporale, Cipollini, and Demetriades (2002), Caporale, Cipollini, and Spagnolo (2002), Rigobon and Sack (2003).

number of unknowns is smaller than the number of equations. However, a priori it is unclear whether the 30 moments computed from the two regimes necessarily provide us with enough independent equations. Therefore, in the actual estimation we use more than two regimes; in fact, we use as many as possible.

The estimation procedure is as follows.

- (a) We compute two-day returns for each of the five variables we employ.
- (b) We run a VAR to clean for serial correlation and recover the residuals. We use five lags in all of our specifications, but the results are unaffected by increasing the number of lags.
- (c) We determine the heteroskedastic regimes using the residuals from the VAR. To do so, for each residual we first compute the rolling window variance. We also calculate the variance in the entire sample (similar to the average of the rolling window variance) and the standard deviation of the rolling window variance. We define a cut-off and identify the regime that occurs when the rolling window variance is above the average plus the cut-off times the standard deviation as highly volatile. The results we present assume the cut-off of one and a half standard deviations, but we have examined regimes defined by 0.5, 1.0 and 2 standard deviations cut-offs, and the overall message has been the same.
- (d) After computing (for each residual) the high and low volatility regimes, we compute the covariance matrix for each of the 32 possible combinations of high and low variances. Obviously not all of the regimes have enough observations to estimate a covariance matrix; in such cases, the observations are dropped.
- (e) Finally, we estimate the parameters in matrix \mathcal{B}_2 and the covariance matrices by GMM imposing the sign restrictions specified above.

We have made several assumptions in this procedure. First, we have chosen to use two-day returns. When the model is estimated on weekly returns, very similar results are found. Employing daily returns is problematic due to nonsynchronous trading in the US and the UK and some of our results do change. We discuss this issue in detail below. Second, we assume that heteroskedasticity of the latent factors can be characterized by two different regimes. In the literature, however, heteroskedasticity is typically described by an ARCH or GARCH-type model. Indeed, the identifi-

cation strategy discussed above can be carried out exactly in a GARCH setup.¹⁶ We have chosen to use the regimes instead because the estimates are consistent even if the underlying process is ARCH or GARCH, but if there were regime shifts in the data, a GARCH model would have a hard time picking them up, while our methodology would not. Since there is no reason to assume a particular structure for the evolution of the covariance matrices, we have decided to take the safe strategy and estimate the model using regimes.

In Table 4, we present the results from estimating the model for the US vis-à-vis the UK case. The first three columns present the estimates of the elements of \mathcal{B}_2 for the model where the covariance matrix of the latent factors is forced to be diagonal. The second set shows the results for the model where the covariance matrix allows for correlation between the demand shock and the supply shocks. Remember, however, that in all of the estimations we always force the supply shocks of the US and the UK to have zero correlation.

In our estimation of the first (no-correlation) model, we found 13 variance regimes with enough observations to compute covariance matrices. Of the possible 12 coefficients, six are statistically significant at more than 5 percent confidence, and the other three are statistically significant at (about) one-sided 10 percent confidence. Note that not a single one of the sign restrictions is binding, as evidenced by the coefficients being away from zero. (If a sign restriction is binding, the corresponding coefficient should be exactly zero.)

It is important to discuss the interpretation of these coefficients. Remember that our model predicts that α_{11} , α_{13} , α_{23} , α_{31} , α_{32} , α_{33} are all equal in absolute value. Note that the estimates of the coefficients in the US and the UK stock markets dynamics equations, α_{11} , α_{13} , and α_{23} , are 1.2152, 0.8237, and 1.2642, respectively, – they are close to each other and in fact are not statistically different. The only major deviation occurs in the US bond price dynamics equation. The estimates of the pertinent coefficients, α_{31} , α_{32} , and α_{33} are, in absolute value, 0.4169, 0.3497, and 0.0287. These estimates are not only statistically different from zero, but also statistically different from each other and from one. This observation suggests that even though the model performs well for the stock markets, it does a poorer job explaining the US bond returns. The model also implies that α_{12} , α_{21} , α_{22} , α_{41} , α_{42} , and α_{43} are all the same. In our estimation, α_{12} and α_{22} have been normalized to one; the absolute values of the other four are 1.844, 1.222, 0.924, and 0.094. The first three estimates are statistically different from zero but not from one. In this

¹⁶See Sentana and Fiorentini (2001) and Rigobon (2002).

US-UK						
	No Correlation			Correlations $\neq 0$		
	Point Estimate	Std. Error	t Statistic	Point Estimate	Std. Error	t Statistic
α_{11}	1.2152	0.7513	1.62	0.7123	0.0036	19.90
α_{13}	0.8237	0.5531	1.49	0.6030	0.0141	4.27
α_{21}	-1.8443	0.4542	-4.06	-0.6670	0.0053	-12.54
α_{23}	1.2642	0.9541	1.33	0.5453	0.0117	4.66
α_{31}	0.4169	0.0240	17.35	0.2846	0.0059	4.82
α_{32}	-0.3497	0.1590	-2.20	-0.1416	0.0030	-4.71
α_{33}	0.0287	0.0172	1.68	0.5916	0.0048	12.28
α_{41}	-1.2224	0.0849	-14.41	-0.4541	0.0158	-2.86
α_{42}	0.9239	0.4194	2.20	0.3949	0.0059	6.69
α_{43}	-0.0936	0.0557	-1.68	-0.5128	0.0970	-0.52
α_{52}	-1.0724	0.5246	-2.04	-0.3509	0.0052	-6.69
α_{53}	0.1633	0.1182	1.38	0.6574	0.0073	8.99

Table 4: Sign restrictions: Estimation of Model (M2) for the US vis-à-vis the UK.

case, the UK bond prices are well explained by the model and the only rejection occurs in the last coefficient. Finally, given our normalization, the coefficients in the exchange rate dynamics equation, α_{52} and α_{53} , should be close to one in absolute value. Again, one is, but the other one is not. In this estimation, the overall impact of supply shocks coming from the UK is very small.

The second set of columns presents estimates of the same model, except that now we do not restrict the covariance matrix of the factors to be diagonal. Here, all but one coefficient are statistically significant. Furthermore, the results are much closer to those predicted by the model. For instance, the absolute values of α_{21} , α_{41} , α_{42} , α_{43} are 0.67, 0.45, 0.40, and 0.51. These estimates are close to each other and different from zero, consistent with our theoretical implications. Moreover, for the second set of coefficients, predicted to be equal in absolute value, α_{11} , α_{13} , α_{23} , α_{31} , α_{32} and α_{33} , the pertinent estimates are 0.71, 0.60, 0.54, 0.28, 0.14, and 0.59. Again, the estimates are much closer to each other than in the covariance-restricted model. Even more importantly, given the normalization we have imposed and the calibrated values of the coefficients, these coefficients should be close to 0.346 (as implied by combining the estimated values of $b(t)$ from the previous section with the current normalization: $0.257/(1-0.257)$), which is indeed approximately the estimated values.

We run several robustness checks. In the interest of space, we only summarize the results.¹⁷ We have estimated the model allowing for different cut-offs to determine the volatility regimes. The results have been found to be qualitatively the same. However, it is important to mention that

¹⁷The results can be replicated by using the programs available from <http://web.mit.edu/~rigobon/www/>.

if the cut-off is too big (more than 2.5 standard deviations), then very few regimes are found. In principle, this makes the identification harder. On the other hand, if the threshold is too small (0.5), the covariance matrices are very well estimated but their differences across regimes are small. As a result, this implies that the estimates are also noisy. Nevertheless, the message in the end is similar. We have also performed several other robustness checks: we have included more than five lags in the original VAR – this has made no difference. We have also varied parameters a_H and $1-a_F$ from 0.75 to 0.9, and α from 0.65 to 0.9. This only changes the average value of the coefficients without changing dramatically the relative importance of the variances. In other words, in a variance decomposition exercise the results are (roughly) unaltered. However, remember that both a_H and $1-a_F$ have to be bounded away from $1/2$ to be consistent with the home bias in consumption.

Finally, we have varied frequencies with which we compute the returns. In the estimation presented above we used two-day returns, however we have also examined weekly and daily returns. When we have estimated the model using weekly returns we have found that the number of regimes is drastically reduced and the estimates become less precise. However, the point estimates are very close to those using two-day returns. It is important to mention that when we have used daily returns we have found different results. For example, several of the coefficients have been estimated to be zero, indicating that the sign restrictions have been binding. The reason for this is nonsynchronous trading. It is to be expected that some of these coefficients are zero because several of the US innovations occur at times when the UK and German stock markets are closed. Indeed, most GDP and productivity related announcements in the US occur in the afternoon, when European markets are closed. Our model is not designed to account for this nonsynchronous trading: all prices respond instantly to supply and demand innovations, and we do not have intraday data to compute returns over the time intervals when all pairs of markets of interest are open.

In summary, estimation of the unrestricted version of the model lends strong support to our theory. Not only are the signs of the coefficients predominantly as predicted in Section 2, but also the magnitudes of a number of the coefficients are consistent with a reasonable calibration of the model. Nevertheless, some rejections of our implications are found – mostly regarding the bond prices dynamics.

4. Caveats and Future Research

Our framework is certainly very stylized and several assumptions require further exploration. First, our model is real; both stocks and bonds are claims to a stream of goods. While perhaps this is a reasonable way to model stocks, in real life, bonds are IOU's specified in nominal terms. This is presumably the reason why our model's implications regarding stock prices and exchange rate dynamics find solid empirical support, while those for bond prices do not fare as well in the data. A natural next step is to introduce money formally into the model, which would allow for a distinction between real and nominal assets, and derive similar implications for nominal asset prices and exchange rates. We believe that the main mechanism will survive this extension – due to trade in goods and in asset markets, real exchange rates will continue to be determined by the same factors that drive real stock market returns.

Second, we have assumed that all goods are traded, and the literature on PPP and real exchange rate determination has demonstrated the importance of the non-tradable goods sector. In our model we have oversimplified this aspect by assuming different weights on Home and Foreign goods (a_H and $1-a_H$) in the utility function. This is a reduced form for a two-sector economy with a tradable and a non-tradable sectors, where the non-tradables are produced with tradable Home goods.¹⁸ However, extending the model to deal explicitly with transfers across the tradable and non-tradable sectors might find broader applications in international finance.

Third, we have employed a log-linear utility in our model. This specification has an apparent advantage of allowing us to compute stock prices in closed form. As a result, we obtain a parsimonious structural model with only three latent factors, making the problem of its estimation manageable. We are aware of very few models in the asset pricing literature capable of producing closed-form expressions for stock prices for an economy with multiple risky stocks. An apparent drawback of the logarithmic specification is, of course, the constant price-dividend ratio it induces. It would be desirable to examine different preferences specifications. This extension is likely to entail the cost of imposing further assumptions on the parameters of the countries' output processes.

¹⁸For example, consider an economy where each country additionally consumes a local non-tradable good. That is, each country's representative agent has a log-linear utility defined over the (tradable) domestic good, the non-tradable good, and the foreign good, with weights a_{1i} , a_{2i} , and $1-a_{1i}-a_{2i}$, $i \in \{H, F\}$. Furthermore, assume that the non-tradable good is produced using the local tradable good in each country with a constant returns to scale technology, which transforms one unit of the domestic good into a one unit of the non-tradable good. Due to nature of the technology employed, the utility function of each country effectively assigns a weight of $a_{1i} + a_{2i}$ to domestic goods. A reasonable calibration of the share of non-tradables in a country's consumption, would then give rise to a preference bias toward the domestic good, supporting the specification of the utility function adopted in our model.

We have attempted to relate the supply and demand shocks, treated as latent factors in our estimation, to actual output innovations in the data by moving to lower frequencies at which output data are available. A different approach would be to use survey data and macroeconomic announcements as proxies for productivity and demand innovations, and study the direct effects of these innovations on asset prices and exchange rates along the lines of Andersen, Bollerslev, Diebold, and Vega (2003), Rigobon and Sack (2002) and Rigobon and Sack (2003). Also left for future research are tests of our model's implications on capital flows and the CAPM.

5. Conclusions

The empirical literature on exchange rate determination has devoted a tremendous amount of effort to explaining its short run fluctuations by interest rate differentials. It is fair to say that this strategy has not been very successful. In this paper we attempt to offer an alternative view on the exchange rate: its movements should be influenced by the same set of factors that govern stock market returns. This new perspective not only renders new insights on exchange rates, but also identifies important interconnections between foreign exchange and financial markets and sheds light on the apparent excessive correlation of stock markets worldwide.

This paper tries to unify the international trade, open economy macroeconomics and asset pricing literatures to uncover the impact of demand and supply shocks on the joint dynamics of exchange rates, stock and bond prices. Our model has implications on the current account, the capital account, and conditions under which portfolio holdings exhibit a home bias. Our empirical strategy has used two approaches. The first one evaluates the economic meaning of the model by testing how close the latent factors – derived from financial variables – are to their macroeconomic counterparts. The second approach tests the sign and cross-equation restrictions implied by the model within sample. Our results show that in both dimensions the model performs well. Our latent factors have predictive power in explaining macroeconomic variables, and their forecasts are more stable than those obtained from financial variables by themselves. Furthermore, we find strong support for the sign, and mild support for the cross-equation implications.

Although our model is largely supported by the data, and its implications are consistent with different strands of the literature, we have made several simplifying, and sometimes unrealistic, assumptions that future research must relax. We feel that deepening our stylized framework may lead to fruitful further insights.

Appendix

The state prices densities associated with the Home and Foreign goods are proportional to the marginal utilities of the countries with respect to the corresponding good. They are given by

$$\xi(t) = \frac{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}{Y(t)} \quad (\text{Home good}), \quad (\text{A.1})$$

$$\xi^*(t) = \frac{\lambda_H \theta_H(t) (1 - a_H) + \lambda_F \theta_F(t) (1 - a_F)}{Y^*(t)} \quad (\text{Foreign good}). \quad (\text{A.2})$$

These state price densities are, of course, equal to the respective Arrow-Debreu state prices η and η^* per unit probability P . Finally, the state-price density associated with the numeraire basket, henceforth the state price density, is given by $\hat{\xi}(t) \equiv \alpha \xi(t) + (1 - \alpha) \xi^*(t)$.

Derivation of Stock Prices. Through the identification of π with the Lagrange multipliers η and η^* , $\pi(t) = \alpha \eta(t) + (1 - \alpha) \eta^*(t)$, we can derive the price of the Home stock to be

$$S(t) = \int_0^T \frac{1}{\alpha \eta(t) + (1 - \alpha) \eta^*(t)} \eta(s) Y(s) ds = E_t \left[\int_t^T \frac{1}{\hat{\xi}(t)} \xi(s) Y(s) ds \right] = \frac{q(t)}{\alpha q(t) + 1 - \alpha} Y(t) (T - t),$$

where we first employed the definition of a conditional expectation appearing on transitioning from the state prices to the state price densities associated with the respective good and then used the fact that θ_H and θ_F are martingales. The Foreign stock price is determined through an analogous procedure.

Weights in the Planner's Problem. To conform with the competitive equilibrium allocation, the weights λ_H and λ_F in the planner's problem are chosen to reflect the countries' initial endowments. In particular, they are identified with the reciprocals of Lagrange multipliers associated with each country's Arrow-Debreu (static) budget constraint. Since in equilibrium these multipliers, and hence the weights, cannot be individually determined, we adopt a normalization $a_H \theta_H(0) \lambda_H + a_F \theta_F(0) \lambda_F = 1$. To pin down λ_H , note that $W_H(0) = \frac{1}{\xi(0)} E \left[\int_0^T [\xi(t) C_H(t) + \xi^*(t) C_H^*(t)] dt \right] = \frac{1}{\xi(0)} \lambda_H \theta_H(0) T$, where we used (8) and (A.1)–(A.2). On the other hand, $W_H(0) = S(0)$. This together with (12) yields $\lambda_H = 1/\theta_H(0)$. Consequently, $\lambda_F = (1 - a_H)/(a_F \theta_F(0))$.

Proof of Proposition 1. Equation (10) can be equivalently restated as

$$\log q(t) = \log \left(\frac{\lambda_H \theta(t) a_H + \lambda_F a_F}{\lambda_H \theta(t) (1 - a_H) + \lambda_F (1 - a_F)} \right) + \log Y^*(t) - \log Y(t).$$

Applying Itô's lemma to both sides and simplifying, we obtain

$$\begin{aligned} \frac{dq(t)}{q(t)} &= \text{Itô terms } dt + \frac{\lambda_H a_H}{\lambda_H \theta(t) a_H + \lambda_F a_F} d\theta(t) - \frac{\lambda_H (1 - a_H)}{\lambda_H \theta(t) (1 - a_H) + \lambda_F (1 - a_F)} d\theta(t) + \frac{dY^*(t)}{Y^*(t)} - \frac{dY(t)}{Y(t)} \\ &= \text{Itô terms } dt + A(t) d\theta(t) - \frac{dY(t)}{Y(t)} + \frac{dY^*(t)}{Y^*(t)}. \end{aligned}$$

Equations (12)–(13) are equivalent to

$$\log S(t) = \log q(t) - \log(\alpha q(t) + 1 - \alpha) + \log Y(t) + \log(T - t), \quad (\text{A.3})$$

$$\log S^*(t) = -\log(\alpha q(t) + 1 - \alpha) + \log Y^*(t) + \log(T - t), \quad (\text{A.4})$$

respectively. Applying Itô's lemma to both sides of (A.3) and (A.4), we have

$$\begin{aligned} \frac{dS(t)}{S(t)} &= \text{Itô terms } dt + \left(1 - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha}\right) \frac{dq(t)}{q(t)} + \frac{dY(t)}{Y(t)} \\ &= \text{Itô terms } dt + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} A(t) d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dY(t)}{Y(t)} + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} \frac{dY^*(t)}{Y^*(t)} \\ \frac{dS^*(t)}{S^*(t)} &= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dq(t)}{q(t)} + \frac{dY^*(t)}{Y^*(t)} \\ &= \text{Itô terms } dt - \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} A(t) d\theta(t) + \frac{\alpha q(t)}{\alpha q(t) + 1 - \alpha} \frac{dY(t)}{Y(t)} + \frac{1 - \alpha}{\alpha q(t) + 1 - \alpha} \frac{dY^*(t)}{Y^*(t)} \end{aligned}$$

Substituting the dynamics of Y and Y^* from (1)–(2), we arrive at the expressions in Proposition 1. The Itô terms (mean growth rates) appearing in the above equations enter nowhere in our estimation procedure. Computation of these terms is straightforward but tedious, so in the interest of space we report just the end result: $I_5(t) = -\mu_Y(t) + \mu_Y^*(t) + \sigma_Y(t)^2 + A(t)\sigma_Y^*(t) \frac{d[\theta(t), w^*(t)]}{dt} - A(t)\sigma_Y(t) \frac{d[\theta(t), w(t)]}{dt}$, $I_1(t) = \mu_Y(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha} \left(I_5(t) + \sigma_Y(t)A(t) \frac{d[\theta(t), w(t)]}{dt} - \sigma_Y(t)^2 \right) - \frac{q(t)}{\alpha q(t)+1-\alpha} \frac{Y(t)}{S(t)}$ and $I_2(t) = \mu_Y^*(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} \left(I_5(t) + \sigma_Y^*(t)A(t) \frac{d[\theta(t), w^*(t)]}{dt} + \sigma_Y^*(t)^2 \right) - \frac{1}{\alpha q(t)+1-\alpha} \frac{Y^*(t)}{S^*(t)}$, where $[\theta(t), w(t)]$ and $[\theta(t), w^*(t)]$ are quadratic covariations of $\theta(t)$ with $w(t)$ and $w^*(t)$, respectively. Further simplification of the quadratic covariation terms requires committing to an interpretation of θ (see Section 2.3).

Home and Foreign bonds are riskless in terms of the local good. That is,

$$dB^i(t) = r^i(t)B^i(t)dt, \quad i \in \{H, F\},$$

where B^i is in units of good i . r^i is the local money market rate, the exact form of which need not concern us in this proposition (the rates r^H and r^F differ across our interpretations of the demand shocks, and are reported in Proposition 2). Converted into the common numeraire,

$$B(t) = p(t)B^H(t) = \frac{q(t)}{\alpha q(t) + 1 - \alpha} B^H(t), \quad B^*(t) = p^*(t)B^F(t) = \frac{1}{\alpha q(t) + 1 - \alpha} B^F(t).$$

Taking logs and then applying Itô's lemma leads to the required expressions. Again, we report the mean growth terms without providing the details of the computations: $I_3(t) = r^H(t) + \frac{1-\alpha}{\alpha q(t)+1-\alpha} I_5(t)$ and $I_4(t) = r^F(t) - \frac{\alpha q(t)}{\alpha q(t)+1-\alpha} I_5(t)$.

The sign restrictions follow from observing that $\text{sign}(A(t)) = \text{sign}(a_H - a_F) > 0$, $0 \leq \alpha \leq 1$ and $q(t)$, $\sigma_Y(t)$, $\sigma_Y^*(t) > 0$. *Q.E.D.*

Proof of Proposition 2. The state price density associated with the Home good, ξ , has a representation

$$d\xi(t) = -r^H(t)\xi(t)dt - m^H(t)\xi(t)d\vec{w}(t). \quad (\text{A.5})$$

The Home money market rate, r^H , and the market price of risk, m^H , are obtained via an application of Itô's lemma to (A.1) taking into account the dynamics of θ_H and θ_F . In particular, under interpretation **A**,

$$d\xi(t) = -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t),$$

whose drift term yields the required money market rate. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t), 0, 0)^\top$.

Under interpretation **B**,

$$d\xi(t) = -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \kappa_H(t) \theta_H(t) dw^\theta(t) + \lambda_F a_F \kappa_F(t) \theta_F(t) dw^\theta(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} \xi(t).$$

Since the quadratic covariation between the demand shocks θ_H , θ_F and Y is zero, the interest rate has the same form as under interpretation **A**. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t), 0, -\frac{\lambda_H a_H \kappa_H(t) \theta_H(t) + \lambda_F a_F \kappa_F(t) \theta_F(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F})^\top$.

Under interpretation **C**,

$$\begin{aligned} d\xi(t) &= -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \kappa_H(t) \theta_H(t) dw(t) + \lambda_F a_F \kappa_F(t) \theta_F(t) dw^*(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} \xi(t) \\ &\quad - \frac{\lambda_H a_H \kappa_H(t) \theta_H(t) \sigma_Y(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} \xi(t)dt. \end{aligned}$$

The last term reflects the nonzero quadratic covariation between θ_H and Y , lending an additional term to the Home interest rate. Note that the Home state price density does not depend on the Brownian motion w^θ , or in other words, the risk associated with w^θ is not priced (the sunspot equilibrium, in which w^θ matters does not obtain in our model). The Home market price of risk is given by $m^H(t) = (\sigma_Y(t) - \frac{\lambda_H a_H \kappa_H(t) \theta_H(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}, -\frac{\lambda_F a_F \kappa_F(t) \theta_F(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}, 0)^\top$.

We now turn to interpretation **D**. Our analysis until now, as well as our empirical tests, have not required a complete specification of information sets of the agents, all the necessary information has been captured by the density processes θ_H and θ_F . There derivatives may come from various economic settings; here, for brevity, we consider only the case of incomplete information where the agents observe all economic processes defined in Section 2, but do not have complete information about their dynamics. The description of the setting is intendedly dense, and we refer the reader to Basak (2000) for more detail. We consider two subcases: **D1** and **D2**. Under interpretation **D1**, the agents believe that there are only two independent innovations, w_i and w_i^* , $i \in \{H, F\}$. The innovation process w_i of each agent i is such that given his perceived mean growth of the Home output process, μ_i , the observed Home output process has the dynamics $dY(t) = \mu_i(t)Y(t)dt + \sigma_Y(t)Y(t)dw_i(t)$. Similarly, the innovation w_i^* is such that given μ_i^* the Foreign output has dynamics $dY^*(t) = \mu_i^*(t)Y^*(t)dt + \sigma_Y(t)Y^*(t)dw_i^*(t)$. The differences in opinion pertaining to the underlying innovation process are induced by differences in the agents' priors, which they may or may not update as new information arrives. The case of Bayesian updating is an example of a filtering problem, the details of which need not concern us here since the optimization problem is well-known to be independent of the inferencing problem. The outcome of the inferencing problem is the country-specific drift processes μ_i and μ_i^* , which we treat as exogenous. Note that the

agents differ in their assessment of the underlying innovations and the drift terms, but agree on the volatilities, which they may deduce from the quadratic variations of the observed processes. The sub-interpretation **D2** is similar to **D1**, except that now the agents believe that there are three independent innovations, w_i , w_i^* and w_i^θ , $i \in \{H, F\}$, that the parameters they estimate depend on. The dynamics of the Home and Foreign output processes are perceived to be as above. Additionally, the agents observe a public signal, s , perceived to be driven by the third innovation process, following $ds(t) = \mu_{si}(t)s(t)dt + \sigma_s(t)s(t)dw_i^\theta(t)$, $i \in \{H, F\}$, the drift component of which is not observable.

The innovation processes of agent i bear the following relationship to the ‘‘true’’ underlying Brownian motions: $dw_i(t) = dw(t) + \frac{\mu_Y(t) - \mu_i(t)}{\sigma_Y(t)}dt$, $dw_i^*(t) = dw^*(t) + \frac{\mu_Y^*(t) - \mu_i^*(t)}{\sigma_Y^*(t)}dt$ (interpretations **D1-D2**), and $dw_i^\theta(t) = dw^\theta(t) + \frac{\mu_{si}(t) - \mu_{si}(t)}{\sigma_s(t)}dt$ (interpretation **D2**). By Girsanov’s theorem, the innovation process $\vec{w}_i \equiv (w_i, w_i^*, w_i^\theta)^\top$ (the last component is absent under interpretation **D1**) is a Brownian motion under the probability measure i , $i \in \{H, F\}$. We can then identify $\vec{\kappa}_H(t)$ in the representation of (the density of) the Radon-Nikodym derivatives of H with respect to P , $\theta_H(T)$, to be $-(\frac{\mu_Y(t) - \mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_H^*(t)}{\sigma_Y^*(t)}, 0)^\top$ (interpretation **D1**), $-(\frac{\mu_Y(t) - \mu_H(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_H^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t) - \mu_{sH}(t)}{\sigma_s(t)})^\top$ (interpretation **D2**). Analogously, $\vec{\kappa}_F = -(\frac{\mu_Y(t) - \mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_F^*(t)}{\sigma_Y^*(t)}, 0)^\top$ (interpretation **D1**), $\vec{\kappa}_F = -(\frac{\mu_Y(t) - \mu_F(t)}{\sigma_Y(t)}, \frac{\mu_Y^*(t) - \mu_F^*(t)}{\sigma_Y^*(t)}, \frac{\mu_s(t) - \mu_{sF}(t)}{\sigma_s(t)})^\top$ (interpretation **D2**). Hence, under the true measure P , $d\theta_i(t) = \vec{\kappa}_i(t)\theta_i(t)d\vec{w}(t)$, $i \in \{H, F\}$.

Applying Itô’s lemma to (A.1) and using the dynamics of θ_H and θ_F , we obtain

$$\begin{aligned} d\xi(t) &= -(\mu_Y(t) - \sigma_Y(t)^2)\xi(t)dt - \sigma_Y(t)\xi(t)dw(t) + \frac{\lambda_H a_H \vec{\kappa}_H(t)^\top \theta_H(t) d\vec{w}(t) + \lambda_F a_F \vec{\kappa}_F(t)^\top \theta_F(t) d\vec{w}(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} \xi(t) \\ &\quad - \frac{\lambda_H a_H (1, 0, 0) \vec{\kappa}_H(t) \theta_H(t) \sigma_Y(t) + \lambda_F a_F (1, 0, 0) \vec{\kappa}_F(t) \theta_F(t) \sigma_Y(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F} \xi(t)dt. \end{aligned}$$

The last term is again due to the quadratic covariation of the process Y with θ_H and θ_F . Substituting the expressions for $\vec{\kappa}_H$ and $\vec{\kappa}_F$ from above, we arrive at the statement in the Proposition. The Home market price of risk is given by $m^H(t) = (\sigma_Y(t), 0, 0)^\top - \frac{\lambda_H a_H \vec{\kappa}_H(t) \theta_H(t) + \lambda_F a_F \vec{\kappa}_F(t) \theta_F(t)}{\lambda_H \theta_H(t) a_H + \lambda_F \theta_F(t) a_F}$.

The Foreign interest rate is determined through an analogous procedure, via an application of Itô’s lemma to (A.2) and then an identification of r^F with the ensuing drift term. The diffusion term yields the Foreign market price of risk: $m^F(t) = (0, \sigma_Y^*(t), 0)^\top$ (interpretation **A**), $m^F(t) = (0, \sigma_Y^*(t), -\frac{\lambda_H (1-a_H) \kappa_H(t) \theta_H(t) + \lambda_F (1-a_F) \kappa_F(t) \theta_F(t)}{\lambda_H \theta_H(t) (1-a_H) + \lambda_F \theta_F(t) (1-a_F)})^\top$ (interpretation **B**), $m^F(t) = (-\frac{\lambda_H (1-a_H) \kappa_H(t) \theta_H(t)}{\lambda_H \theta_H(t) (1-a_H) + \lambda_F \theta_F(t) (1-a_F)}, \sigma_Y^*(t) - \frac{\lambda_F (1-a_F) \kappa_F(t) \theta_F(t)}{\lambda_H \theta_H(t) (1-a_H) + \lambda_F \theta_F(t) (1-a_F)}, 0)^\top$ (interpretation **C**), $m^F(t) = (0, \sigma_Y^*(t), 0)^\top - \frac{\lambda_H (1-a_H) \vec{\kappa}_H(t) \theta_H(t) + \lambda_F (1-a_F) \vec{\kappa}_F(t) \theta_F(t)}{\lambda_H \theta_H(t) (1-a_H) + \lambda_F \theta_F(t) (1-a_F)}$ (interpretation **D**).

Finally, we obtain the uncovered interest rate parity relation through an application of Itô’s lemma to both sides of a no-arbitrage restriction $\xi^*(t) = \xi(t)/q(t)$ and matching the corresponding dt terms. This procedure yields $-r^F(t) = -r^H(t) - \mu_q(t) + \sigma_q(t)^\top (m^H(t) + \sigma_q(t))$. *Q.E.D.*

Proof of Proposition 3. We first focus on the composition of x_i and the form of σ . We also report the form of the auxiliary matrix \mathcal{I} , whose role is to modify the dimension of the vectors m and $\vec{\kappa}_i$ to make the matrices involved in the multiplications conformable.

- (i) Under interpretation **A**, one can only determine the investment in the composite (world) stock market, and not in individual stock markets. The position in the composite security is of form (23). We do not provide details for this interpretation, and refer the reader to Zapatero (1995) or Cass and Pavlova (2003).
- (ii) Under interpretation **B**, four independent investment opportunities are required to dynamically complete financial markets. We take these to be represented by the Home stock S , the Foreign stock S^* , the Home bond B and the world bond B^W . Hence, x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)$. From the dynamics of S , S^* and B , (15)–(17), and the definitions of θ_H and θ_F we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} \alpha q(t) \sigma_Y(t) & (1 - \alpha) \sigma_Y^*(t) & 0 \\ \alpha q(t) \sigma_Y(t) & (1 - \alpha) \sigma_Y^*(t) & 0 \\ (\alpha - 1) \sigma_Y(t) & (1 - \alpha) \sigma_Y^*(t) & 0 \end{pmatrix} + \frac{A(t) \theta(t)}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} 1 - \alpha \\ -\alpha q(t) \\ 1 - \alpha \end{pmatrix} \mathcal{I} (\vec{\kappa}_H(t) - \vec{\kappa}_F(t))^\top, \quad (\text{A.6})$$

with \mathcal{I} is simply a 3×3 identity matrix under this interpretation.

- (iii) Under interpretation **C**, there are only two independent sources of uncertainty, and hence three securities are sufficient to dynamically complete financial markets. We take them to be S , S^* and B^W . Hence, x_i has two components: $x_i = (x_i^S, x_i^{S^*})$. From (15)–(16) and the definitions of θ_H and θ_F we identify the matrix of the investment opportunity set:

$$\sigma(t) = \frac{1}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} \alpha q(t) \sigma_Y(t) & (1 - \alpha) \sigma_Y^*(t) \\ \alpha q(t) \sigma_Y(t) & (1 - \alpha) \sigma_Y^*(t) \end{pmatrix} + \frac{A(t) \theta(t)}{\alpha q(t) + 1 - \alpha} \begin{pmatrix} 1 - \alpha \\ -\alpha q(t) \end{pmatrix} [\mathcal{I} (\vec{\kappa}_H(t) - \vec{\kappa}_F(t))]^\top, \quad \text{with } \mathcal{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}. \quad (\text{A.7})$$

- (iv) To address interpretation **D**, we again consider subcases **D1** and **D2**. Under interpretation **D1**, three securities dynamically complete markets: for instance, S , S^* and B^W , and hence x_i has two components: $x_i = (x_i^S, x_i^{S^*})$. Under interpretation **D2**, there is an additional nonredundant security, B , and hence x_i has three components: $x_i = (x_i^S, x_i^{S^*}, x_i^B)$. The matrix σ is then obtained using (15)–(16) under interpretation **D1** to yield an expression in (A.7) and (15)–(17) under interpretation **D2** to yield an expression in (A.6). Of course, $\vec{\kappa}_H$ and $\vec{\kappa}_F$ used in the expressions for σ are interpretation-**D**-specific (reported in the proof of Proposition 2).

From now on we focus solely on interpretations **B–D**. Before we proceed to trading strategies, we derive the interest rate r on the world bond B^W , and the market price of risk m . Both are identified from the representation of the state price density $\hat{\xi}$, $d\hat{\xi}(t) = -r(t)\hat{\xi}(t)dt - m(t)\hat{\xi}(t)d\vec{w}(t)$. Using the definition of $\hat{\xi}$ and (A.1)–(A.2), we obtain

$$r(t) = \frac{\alpha r^H(t)q(t) + (1 - \alpha)r^F(t)}{\alpha q(t) + 1 - \alpha} \quad \text{and} \quad m(t) = \frac{\alpha m^H(t)q(t) + (1 - \alpha)m^F(t)}{\alpha q(t) + 1 - \alpha}.$$

(i) *Trading strategies.* It can be verified that matrix σ is invertible under interpretations **B–D**. We can then appeal to the martingale representation methodology (Cox and Huang (1989), Karatzas, Lehoczky, and Shreve (1987)) to transform the dynamic optimization problem of Home (Foreign) country to a static problem of maximizing the objective in (4) (in (5)) subject to the static budget constraint $E[\int_0^T (\xi(t)C_i(t) + \xi^*(t)C_i^*(t))dt] = W_i(0)$, $i = H$ ($i = F$). The first-order conditions for this optimization are

$$\frac{\theta_i(t)a_i}{C_i(t)} = \frac{1}{\lambda_i}\xi(t), \quad \frac{\theta_i(t)(1 - a_i)}{C_i^*(t)} = \frac{1}{\lambda_i}\xi^*(t), \quad i \in \{H, F\}. \quad (\text{A.8})$$

Note that the multipliers on the static countries' budget constraints are the reciprocals of the planner's weights λ_H and λ_F .

The optimal trading strategy of agent i is identified from the stochastic integral representation $M_i(t) = M_i(0) + \int_0^t \psi_i(s)^\top d\vec{w}(s)$ of the martingale $M_i(t) \equiv E_t \left[\int_0^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds \right]$ by modifying the standard argument to account for multiple goods (see, e.g., Karatzas and Shreve (1998), Theorem 7.3):

$$\sigma(t)^\top x_i(t) = \mathcal{I}m(t) + \mathcal{I} \frac{\psi_i(t)}{\hat{\xi}(t)W_i(t)}, \quad (\text{A.9})$$

where $W_i(t)$ is time- t optimal wealth of agent i , $W_i(t) = \frac{1}{\hat{\xi}(t)} E_t[\int_t^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds]$, and m , σ are interpretation-specific. In our model, $M_i(t) = E_t[\int_0^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds] = E_t[\int_0^T (\lambda_i\theta_i(s)a_i + \lambda_i\theta_i(s)(1 - a_i))ds] = \int_0^t \lambda_i\theta_i(s)ds + \lambda_i\theta_i(t)(T - t)$, and hence $dM_i(t) = \lambda_i\vec{\kappa}_i(t)\theta_i(t)(T - t)d\vec{w}(t)$. The diffusion coefficient of $M_i(t)$, $\vec{\kappa}_i(t)\theta_i(t)(T - t)$, is identified with $\psi_i(t)$. Note also that

$$\hat{\xi}(t)W_i(t) = E_t\left[\int_t^T (\xi(s)C_i(s) + \xi^*(s)C_i^*(s))ds\right] = \lambda_i\theta_i(t)(T - t). \quad (\text{A.10})$$

Plugging this together with $\psi_i(t)$ into (A.9) we arrive at the expression in Proposition 3.

(ii) *Home bias.* We now examine conditions under which consumers' portfolios exhibit a home bias. To do so, we need to sign the first and second components of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_i(t)$, $i \in \{H, F\}$. Under interpretation **B**, these two components are equal to zero since $\vec{\kappa}_i(t) = (0, 0, \kappa_i)$, and hence fractions of wealth both Home and Foreign consumers invest in stocks of their respective countries are exactly as in the mean-variance portfolio. Under interpretation **C**, from (A.7),

$$\begin{aligned} (\sigma(t)^\top)^{-1} &= \frac{1}{\det(\sigma(t))} \frac{1}{\alpha q(t) + 1 - \alpha} \\ &\times \begin{pmatrix} (1 - \alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t) & -(1 - \alpha)\sigma_Y^*(t) + A(t)\theta(t)(1 - \alpha)\kappa_F(t) \\ -\alpha q(t)\sigma_Y(t) + A(t)\theta(t)\alpha q(t)\kappa_H(t) & \alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1 - \alpha)\kappa_H(t) \end{pmatrix}, \end{aligned}$$

where $\det(\sigma(t))$ denotes the determinant of $\sigma(t)$. Note that

$$\begin{aligned} \det(\sigma(t)) &= [\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t)] [(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t)] \\ &\quad - \alpha q(t)[\sigma_Y(t) - A(t)\theta(t)\kappa_F(t)](1-\alpha)[\sigma_Y^*(t) - A(t)\theta(t)\kappa_H(t)] \\ &\Rightarrow \begin{cases} > 0 & \text{if } \kappa_H(t) > 0, \kappa_F(t) > 0, \\ < 0 & \text{if } \kappa_H(t) < 0, \kappa_F(t) < 0, \end{cases} \end{aligned}$$

It then follows that if $\kappa_H(t) > 0, \kappa_F(t) > 0$ (consumer confidence) then the first component of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_H(t)$ and the second component of $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_F(t)$ are unambiguously positive. On the other hand, if $\kappa_H(t) < 0, \kappa_F(t) < 0$ (catching up with the Joneses), the sign of the first component of the vector $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_H(t)$ coincides with the sign of $[(1-\alpha)\sigma_Y^*(t) + A(t)\theta(t)\alpha q(t)\kappa_F(t)]$. Similarly, the sign of the second component of $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_F(t)$ coincides with the sign of $[\alpha q(t)\sigma_Y(t) + A(t)\theta(t)(1-\alpha)\kappa_H(t)]$. Finally, under interpretation **D**, the sign of either component of $(\sigma(t)^\top)^{-1}\mathcal{I}\vec{\kappa}_i(t)$ is ambiguous.

(iii) *International CAPM*. In an arbitrage-free market, the risk premium on stock j is related to the market price of risk in the following way (e.g., Karatzas and Shreve (1998), Theorem 4.2):

$$\frac{E_t(dS^j(t)/dt)}{S^j(t)} - r(t) = \sum_{k=1}^3 \sigma_k^j(t)m_k(t),$$

where the diffusion coefficients σ_1^j, σ_2^j and σ_3^j are the loadings of stock S^j on Brownian motions w, w^* and w^θ , respectively, and m_k are components of the market price of risk vector m . On the other hand,

$$Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)} \right) = \frac{Cov_t(dS^j(t), d\hat{\xi}(t))}{S^j(t)\hat{\xi}(t)} = - \sum_{k=1}^3 \sigma_k^j(t)m_k(t)dt.$$

Hence,

$$\frac{E_t(dS^j(t))}{S^j(t)} - r(t)dt = -Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)} \right) \quad (\text{A.11})$$

From (A.10), $W_i(t) = \lambda_i\theta_i(t)(T-t)/\hat{\xi}(t)$, $i \in \{H, F\}$. Hence, the aggregate wealth is

$$W(t) = W_H(t) + W_F(t) = \frac{T-t}{\hat{\xi}(t)} [\lambda_H\theta_H(t) + \lambda_F\theta_F(t)].$$

Taking logs, applying Itô's lemma and rearranging, we have

$$\frac{d\hat{\xi}(t)}{\hat{\xi}(t)} = -\frac{dW(t)}{W(t)} + \frac{\lambda_H d\theta_H(t) + \lambda_F d\theta_F(t)}{\lambda_H\theta_H(t) + \lambda_F\theta_F(t)} + dt \text{ terms}. \quad (\text{A.12})$$

Then,

$$\begin{aligned} Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{d\hat{\xi}(t)}{\hat{\xi}(t)} \right) &= -Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{dW(t)}{W(t)} \right) + Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_H d\theta_H(t)}{\lambda_H\theta_H(t) + \lambda_F\theta_F(t)} \right) \\ &\quad + Cov_t \left(\frac{dS^j(t)}{S^j(t)}, \frac{\lambda_F d\theta_F(t)}{\lambda_H\theta_H(t) + \lambda_F\theta_F(t)} \right), \quad S^j \in \{S, S^*\}. \end{aligned}$$

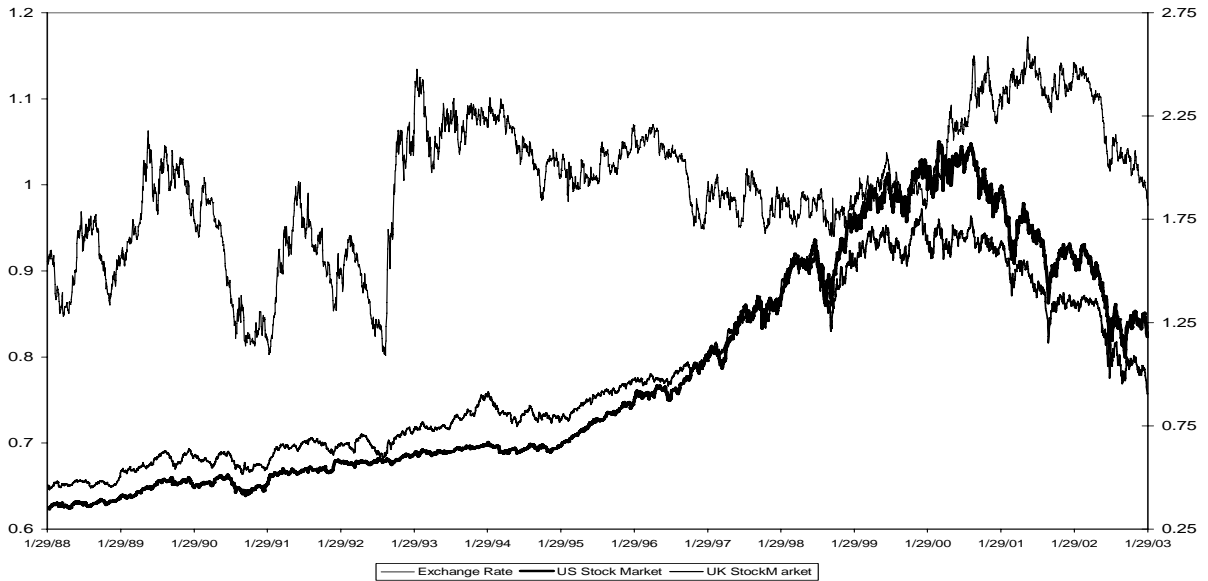
Combining this with (A.11), we obtain the required expression. *Q.E.D.*

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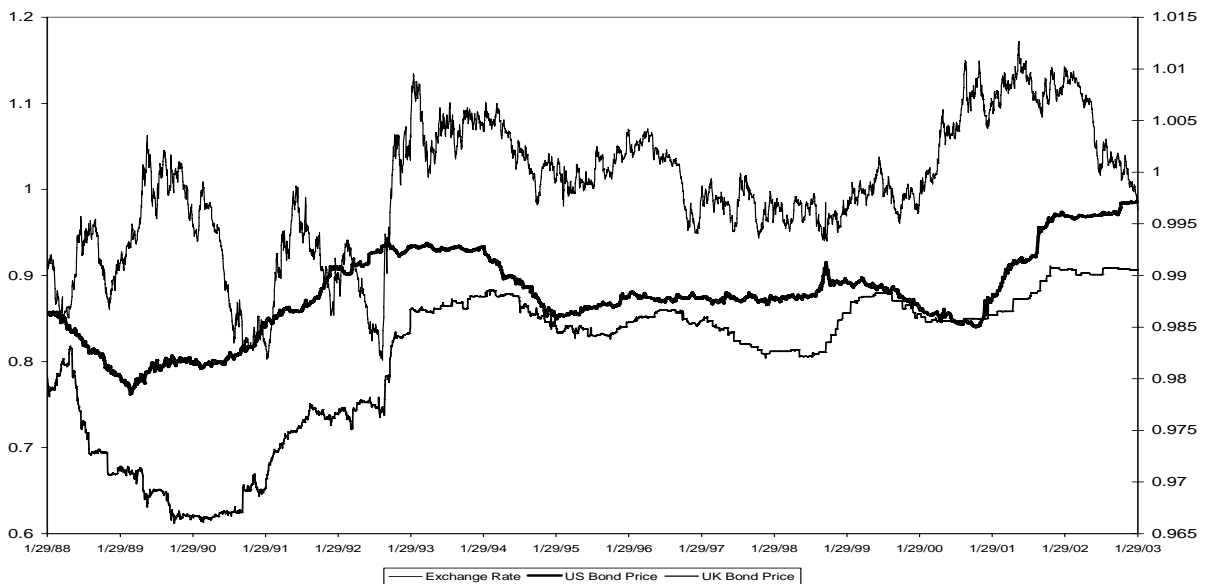
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(a) The US and the UK stock market indexes and the dollar-pound exchange rate



(b) The US and the UK three-month zero-coupon government bond prices and the dollar-pound exchange rate

Figure 1: Asset prices and exchange rates. The exchange rate is measured in the left axis. The right axis measures the US and the UK stock market indexes (panel (a)) and the bond prices (panel (b)). For demonstrative purposes, all prices are normalized so that the average exchange rate is equal to one.

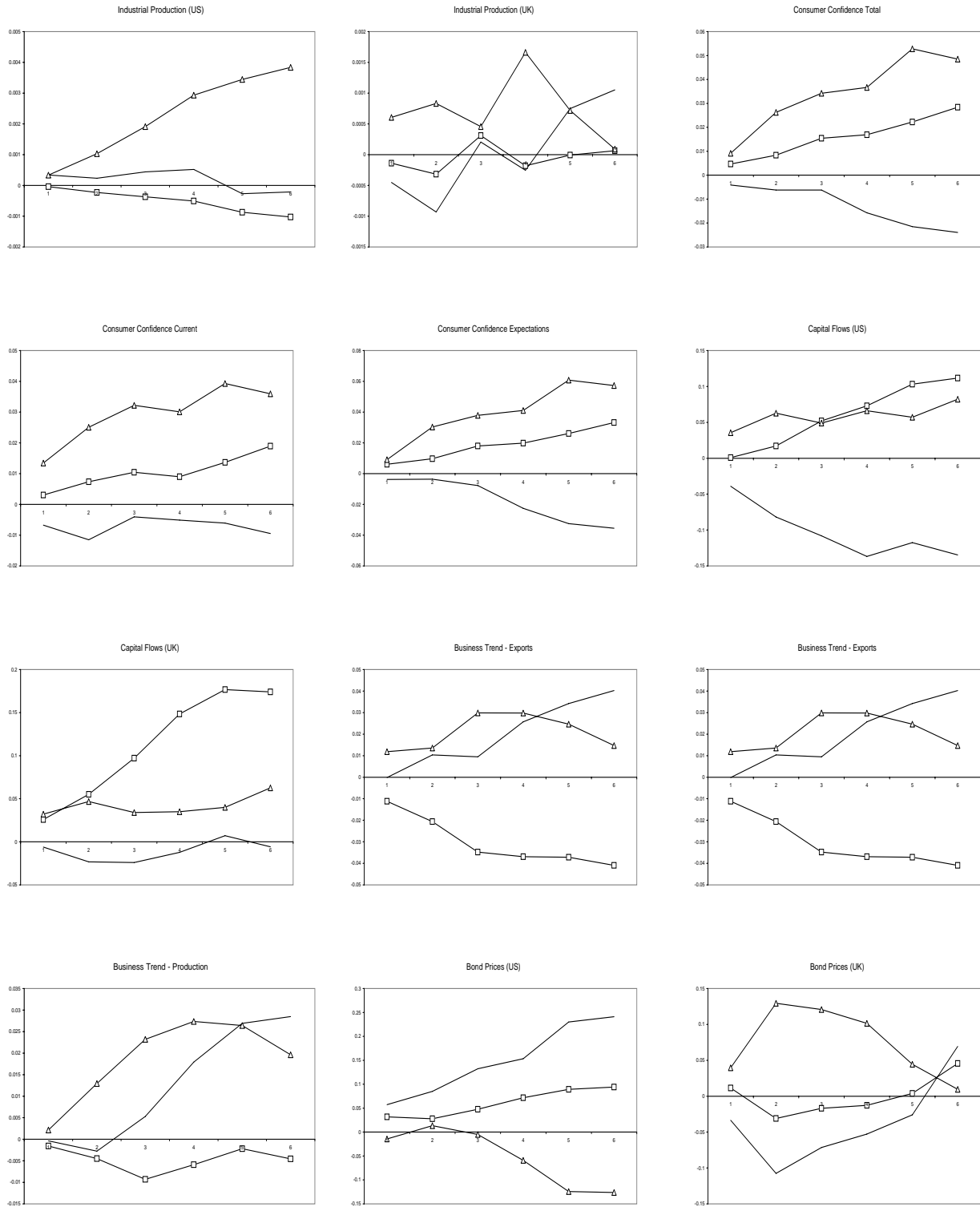


Figure 2: Impulse responses of macroeconomic variables to changes in the factors. Impulse responses to a permanent change in f_θ and f_w are marked with squares and triangles, respectively. The remaining solid plot is for impulse responses to a permanent change in f_{w^*} .