Decomposition Methods and the Computation of Spatial Equilibria: An Application to Coal Supply and Demand Markets

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1. INTRODUCTION

Regional coal models, such as those developed at ICF [7] [8], CRA [2] [9], DRI [3], and MIT [15], forecast regional prices, production and consumption of coal, given assumptions about coal market behavior, government policies, and other key factors that affect the market. The developers of these models had a common objective -- to build structural models that reflect price, production, and consumption under alternative policies and other key market determinants. The common objective led to important similarities among the early versions of these models and their more recent variants.

First, the models estimate market clearing prices for particular market conditions subject to specified policy or other constraints. To date, the models represent competitive markets. Second, these models represent, in various levels of detail, all the components of a spatial market -- producers, consumers, transporters -- and their interaction in the market. Third, the models use the same general approach for forecasting. All the models use mathematical programming techniques to obtain equilibrium prices and quantities that satisfy mathematical relationships defining a competitive market.

As the coal models evolved, more detailed treatment of various sectors (supply, demand, transport) was attempted. This resulted in larger models or, alternatively, in the need to link models of several sectors. The expanded treatment of markets produced a need to decompose large models, or equivalently, to integrate submodels of coal supply and demand markets.

The purposes of this paper are to discuss how spatial equilibrium models can provide a useful conceptual base for comparing and contrasting the coal models listed above, and to show how mathematical programming decomposition methods can be applied to compute the equilibria. In the process, the methods provide an integrative structure for combining the various supply, transportation and demand submodels. The paper begins with a qualitative review of coal market representations. The following two sections contain an exposition of the underlying spatial equilibrium models, and decomposition methods for solving them. Section 5 discusses the practical aspects of generating and optimizing coal models using decomposition methods. The final section consists of a few concluding remarks and areas of future research.

2. COAL MARKET REPRESENTATION

Market structure is an important determinant of regional coal prices and production. Under different market structures (competitive, monopolistic, and monopsonistic), regional coal prices and production will differ. For example, in a competitive market, competition on both the producers and consumers side results in a set of equilibrium prices such that mine-mouth price equals the opportunity cost of production and delivered price equals marginal utility. In a monopolistic market, the producer is able to hold price above the opportunity cost of production, to maximize producers' revenues. In a monosponistic market, the consumer is able to hold price below the opportunity cost of production.

Since the market structure is important in price formation, each coal model utilizes an assumption about the type of market structure in predicting prices and production. The major regional coal models, such as the ICF, DRI, and CRA models reported in the Coal in Transition study [3], and subsequent versions of coal models reported separately ([7], [8], [15], [19]) assume that the coal market is a spatial competitive market. Since there are policies that constrain the use of certain coal types (SO_2 emission limits) and that affect the cost of using various coals (taxes, required scrubbing, etc.), the models represent the assumption that producers and consumers behave competitively subject to these constraints.

When the market is competitive, the market clearing price in a demand region (delivered price or demand price) and the market clearing price in a supply region (mine-mouth price or supply price) satisfy certain conditions. These conditions include:

Consumer Equilibrium

The market clearing delivered coal price in a region equals the maximum price the consumers in the region are willing to pay for the marginal unit consumed. If all delivered coal prices are greater than what consumers are willing to pay, there is no consumption in that region. Since there are differences in the combustion and clearing costs for various coals, the maximum that a consumer is willing to pay for delivered coal depends on the coal type. The difference will reflect the difference in end-use cost and BTU content.

Producer Equilibrium

The market clearing mine-mouth coal price equals the minimum price at which producers in the region are willing to supply the marginal unit (average cost of production plus normal return for the coal produced from marginal mine). If the mine-mouth price is less than this minimum, there is no production in the region.

Locational Price Equilibrium

If all coal types were the same, the market clearing delivered price in a region is less than or equal to the sum of the transportation costs plus market clearing mine-mouth supply price of coals for each region. The market clearing demand price exactly equals the transportation costs plus mine-mouth price of coal from the region supplying the demand. When there is more than one coal type, the demand price for a prototype coal minus the incremental costs required to use another coal type is greater or equal to the transportation costs plus the market clearing supply price of the alternative coal.

Market Equilibrium

There is a single market clearing demand price and market clearing supply price for each region at which all markets clear. There is no excess demand or excess supply in a region and there is efficient pricing. That is, the amount demanded at the regional market demand price is less than or equal to the amount demanded from the region. If the amount supplied to a region is greater than that demanded, the regional market demand price is zero. If the amount produced in a region is greater than the amount demanded from the region, the regional market supply price is zero.

Balance of Fuel Payments

Consumers' expenditures for coal equals producers receipts; there are no excess economic profits in the coal sector. All the coal market models represent producers within a region by a regional coal supply function that describes the quantity of coal that will be forthcoming at various price levels. These price quantity pairs represent the minimum price at which producers in the region are willing to supply the incremental quantity of coal. Several versions of the models represent consumers coal demand by a point estimate rather than a demand schedule. These point estimates describe the quantity of coal expected to be consumed in the demand region. This estimate is based on an assumption about delivered coal prices. The prices estimated by the market model should be checked against these demand levels for consistency.

The ICF model begins with a point estimate of total electric utility generation requirements and chooses among coal and other fuels within the model, thus representing the elasticity of coal demand with respect to fuel substitution. Later work carried out at CRA concerns the use of a coal supply model and an electric utility capacity expansion planning model to represent this type of elasticity of demand. In all of these models, transportation is represented as a single cost between origin destination pairs. Ideally, these transportation costs would represent the cost of the marginal unit shipped between the origin and destination. A more comprehensive treatment of transport supply would prove useful.

With one exception, the models discussed in this paper use primarily concepts from static economic analysis. One aspect of the dynamics of the market, in particular the effect of mine openings on future supply, is included in most models. However, in these cases new production is modeled using long run economic analysis, assuming that new production can adjust to demand within the time period. These models assume that decisions on mine openings are determined by the current, not future market. The work carried out at MIT is an exception; it assumes that the producer makes decisions based on perfect knowledge of future markets.

3. SPATIAL EQUILIBRIUM MODELS

In this section, we discuss briefly how the coal market representations presented in the previous section can be integrated into a spatial equilibrium model. We also discuss briefly how this model can be constructed and optimized using mathematical programming.

Suppose there are n demand regions for coal, indexed by j. Let d_j denote the variable demand in region j, measured in tons, which depends on the variable price per ton of coal delivered to that region. For the purposes of characterizing and computing spatial equilibria, we need to assume that we know the function $p_j^d(d_j)$ which gives the marginal price that consumers in region j are willing to pay for coal when they are receiving d_j. Suppose further that there are m supply regions, indexed by i, and let s_i denote the variable supply, measured in tons, from region i. Again, we need to assume that we know the function $p_i^s(s_i)$ which is the marginal price suppliers in region j are willing to sell coal when the supply level is s_i. Finally, let c_{ij} denote the cost per ton of transporting coal from supply region i to demand region j.

The mathematical programming statement of the spatial equilibrium model, assuming competitive markets, is

$$
\begin{aligned}
v^* = \max \quad & \sum_{j=1}^{n} \int_0^{d_j} p_j^d(\xi_j)\,d\xi_j - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} - \sum_{i=1}^{m} \int_0^{s_i} p_i^s(\mu_i)\,d\mu_i \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} - s_i = 0 \qquad i = 1,\ldots,m \\
& \sum_{i=1}^{m} x_{ij} - d_j = 0 \qquad j = 1,\ldots,n \qquad (1) \\
& s_i \geq 0,\ d_j \geq 0,\ x_{ij} \geq 0
\end{aligned}
$$

Problem (1) is a well known model studied in detail by Takayama and Judge [17] who also describe similar models representing a variety of market conditions (competitive, monopolistic and monopsonistic). The model can be derived in two different ways. The more direct way is to use first principles of economic theory to derive the consumers' surplus terms, $\int_0^{d_j} p_j^d(\xi_j)d\xi_j$, and the producers' surplus terms, $\int_0^{s_i} p_i^s(\mu_j)d\mu_j$ (for example, see Varian [17]). Alternatively, the Kuhn Tucker conditions characterizing optimality for problem (1) can be interpreted as competitive equilibrium conditions implying (1) correctly captures the markets we wish to study. Letting Π_i denote the dual variable (shadow price) on supply row i, and σ_j denote the dual variable on demand row j, the Kuhn-Tucker conditions require that s_i, d_j and x_{ij} be feasible in (1) and satisfy

$$p_j^d(d_j) - \sigma_j \leq 0 \text{ and } (p_j^d(d_j) - \sigma_j)d_j = 0$$

$$-p_i^s(s_i) + \Pi_i \leq 0 \text{ and } (-p_i^s(s_i) + \Pi_i)s_i = 0 \tag{2}$$

$$c_{ij} + \Pi_i - \sigma_j \geq 0 \text{ and } (c_{ij} + \Pi_i - \sigma_j)x_{ij} = 0$$

We can readily deduce from these conditions that, whenever there is a positive flow x_{ij} from supply region i to demand region j.

$$p_i^s(s_i) + c_{ij} = p_j^d(d_j) \tag{3}$$

Namely, a competitive equilibrium exists because the marginal delivered cost of coal equals the marginal value of the coal to the consumer. These are necessary conditions for optimality; they are also sufficient when

$$\frac{dp_j^d(\cdot)}{dd_j} \geq 0 \text{ and } \frac{dp_i^s(\cdot)}{ds_i} \leq 0,$$

implying the spatial equilibrium problem (1) is a concave maximization problem. Henceforth, we will assume this to be the case.

The spatial equilibrium model appears to be quite simple and possibly inadequate to describe the complexity of U.S. coal supply, transportation and demand markets. However, we should not lose sight of the fact that the marginal price functions $p_j^d(\cdot)$ and $p_i^s(\cdot)$ embody a great deal of information about their respective markets. For example, consumer j might be an electric utility whose operations are described by a large scale linear programming model. In this case, the function $p_j^d(d_j)$ gives the shadow price on the model's coal constraint as a function of the quality d_j supplied to the utility. In this way, p_j^d measures interfuel competition in meeting exogenous electricity demands, and the effect of additional constraints related to coal use, such as adherence to emission control standards.

Extensions and complications of the basic model can be made and similarly analyzed. The transportation sub-model can be extended to include capacity constraints on flows and nonlinear costs. Moreover, all of the sub-models, supply, transportation and demand, can be expanded to a multi-period planning horizon. In so doing, the models can then include capacity expansion and other capital investment decisions that have the effect of changing the basic structure of the competitive markets.

4. COMPUTING SPATIAL EQUILIBRIA BY DECOMPOSITION METHODS

There is a natural application of mathematical programming decomposition methods to the solution of the spatial equilibrium model (1). The methods are conceptually attractive because, under suitable assumptions about the model, they are guaranteed to converge to an equilibrium solution. As a practical matter, the methods can be effective in integrating diverse submodels describing the various markets. Practical considerations are discussed in the following section.

We will consider two distinct decomposition approaches, resource directive and price directive. There are several variations possible within each of these, depending on how we choose to split up the problem. We will present only one version for each decomposition approach, however, and refer the reader to Mehring and Shapiro [11] for more details.

Resource Directive Decomposition

Suppose we have previously computed or otherwise selected the trial supply and demand quantities

$$s_i^k \quad \text{for } k = 1,\ldots,K;\ i = 1,\ldots,m$$
$$d_j^k \quad \text{for } k = 1,\ldots,K;\ j = 1,\ldots,n.$$

Define the quantities

$$F^s(s_i^k) = \int_0^{s_i^k} p_i^s(\mu_i)d\mu_i$$

and

$$F^d(d_j^k) = \int_0^{d_j^k} p_j^d(\xi_j)d\xi_j.$$

The resource directive approach proceeds by solving the following linear programming approximation to problem (1), called the Resource Directive Master Problem

$$\underline{z}^K = \max \sum_{j=1}^{n} \nu_j^d - \sum_{i=1}^{m} \nu_i^s - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \tag{a}$$

$$\text{s.t. } \nu_j^d \leq F_j^d(d_j^k) + p_j^d(d_j^k)(d_j - d_j^k) \tag{b}$$

for $k=1,\ldots,K;\ j=1,\ldots,n$

$$\nu_i^s \geq F_i^s(s_i^k) + p_i^s(s_i^k)(s_i - s_i^k) \tag{c}$$

for $k=1,\ldots,K;\ i=1,\ldots,m$ (RDMP)

$$\sum_{j=1}^{n} x_{ij} - s_i = 0 \qquad i=1,\ldots,m \tag{d}$$

$$\sum_{i=1}^{m} x_{ij} - d_j = 0 \qquad j=1,\ldots,n \tag{e}$$

$$s_i \geq 0,\ x_{ij} \geq 0,\ d_j \geq 0 \tag{f}$$

Let s_i^{K+1} for i = 1,...,m and d_j^{K+1} for j = 1,...,n, denote the optimal supply and demand quantities produced by the (RDMP), and let x_{ij}^{K+1} for i = 1,...,m, and j = 1,...,n, denote the optimal transportation flows. The method proceeds by testing the approximations inherent in the constraints (b) and (c); let $\nu_j^{d,K}$ and $\nu_i^{s,K}$ denote the optimal approximating values on these constraints. Specifically, we solve the Resource Directive Supply Subproblems: Compute

$$\delta_i^{s,K} = F_i^s(s_i^{K+1}) = \int_0^{s_i^{K+1}} p_i^s(\mu_i)d\mu_i$$

and the Resource Directive Demand Subproblems: Compute

$$\delta_j^{d,K} = F_j^d(d_j^{K+1}) = \int_0^{d_j^{K+1}} p_j^d(\xi_j)d\xi_j$$

The following theorem, which is stated without proof, summarizes the information we have in hand after having solved the Resource Directive Master Problems and the Subproblems.

Theorem 1:

$$\underline{z}^K \leq v^* \leq \bar{z}^K$$

where

$$\bar{z}^K = \sum_{j=1}^{n} \delta_j^{d,K} - \sum_{i=1}^{m} \delta_i^{s,K} - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij}^{K+1}$$

In words, the theorem states that, at each iteration K, the optimal objective function value in the spatial equilibrium problem (1) is bounded below by the Master Problem objective function value, and bounded above using quantities from the Subproblems. The resource directive method has converged if $\underline{z}^K = \bar{z}^K$.

On the other hand, if $\underline{z}^K < \bar{z}^K$, then there must be some $\nu_j^{d,K} > F_j^d(d_j^{K+1})$ or $\nu_i^{s,K} > F_i^s(s_i^{K+1})$, and at least one new constraint can be added to (RDMP) which cuts off the optimal solution and improves the approximation. Of course, as a practical matter, the resource directive method can be terminated whenever the error $\min_{L=1,\ldots,K} \bar{z}^K - \underline{z}^K$ is sufficiently small.

Theorem 2: The resource directive decomposition method will converge to an optimal solution to the spatial equilibrium problem (1) if the supply and demand quantities s_i^K and d_j^K produced by (RDMP) for all K lie in a bounded set.

Price Directive Decomposition

The price directive decomposition method uses the same quantities as the resource directive method, but in a different way. The Master Problem is the linear programming problem

$$\bar{w}^K = \max \sum_{j=1}^{n} \sum_{k=1}^{K} F_j^d(d_j^k)\lambda_{jk} - \sum_{i=1}^{m} \sum_{k=1}^{K} F_i^s(s_i^k)\omega_{ik} - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \qquad \text{(a)}$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_{ij} - \sum_{k=1}^{K} s_i^k\omega_{ik} = 0 \qquad \text{(b)}$$

$$\sum_{i=1}^{m} x_{ij} - \sum_{k=1}^{K} d_j^k\lambda_{jk} = 0 \qquad \text{(c)}$$

(PDMP)

$$\sum_{k=1}^{K} \omega_{ik} = 1 \qquad \text{(d)}$$

$$\sum_{k=1}^{K} \lambda_{jk} = 1 \qquad \text{(e)}$$

$$x_{ij} \geq 0,\ \omega_{ik} \geq 0,\ \lambda_{jk} \geq 0 \qquad \text{(f)}$$

Let λ_{jk}^K, ω_{ik}^K, x_{ij}^K denote an optimal solution to this problem, and let Π_i^K and σ_j^K denote optimal linear programming shadow prices on the supply and demand balance equations. Let θ_i^K and γ_j^K denote the optimal shadow prices on the convexity constraints.

The price directive method proceeds by solving the Price Directive Supply Subproblems: Find s_i^{K+1} optimal in

$$\beta_i^{s,K} = \min_{s_i \geq 0} \{\int_0^{s_i} p_i^s (\mu_i) d\mu_i - \Pi_i^K s_i\}$$

and the Price Directive Demand Subproblems: Find d_j^{K+1} optimal in

$$\beta_j^{d,K} = \max_{d_j \geq 0} \{\int_0^{d_j} p_j^d (\xi_j) d\xi_j - \sigma_j^K d_j\}$$

For the Supply Subproblems, we have two cases

(a) If $p_i^s(0) > \Pi_i^K$, then $s_i^{K+1} = 0$.

(b) If $p_i^s(0) \leq \Pi_i^K$, then s_i^{K+1} is implicitly defined by

$$p_i^s(s_i^{K+1}) = \Pi_i^K .$$

Similarly, for the Demand Subproblems, we have the two cases

(a) If $p_j^d(0) < \sigma_j^K$, then $d_j^{K+1} = 0$.

(b) If $p_j^d(0) \geq \sigma_j^K$, then d_j^{K+1} is implicitly defined by

$$p_j^d(d_j^{K+1}) = \sigma_j^K .$$

Theorem 3:

$$\underline{w}^K \leq z^* \leq \bar{w}^K$$

where

$$\underline{w}^K = \sum_{j=1}^{n} \beta_j^{d,K} - \sum_{i=1}^{m} \beta_i^{s,K} - \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}^K$$

As in the resource directive method, this theorem states that, at each iteration K of the price directive method, the optimal objective function value in the spatial equilibrium problem (1) is bounded on both sides. The price directive method has converged if $\underline{w}^K = \bar{w}^K$. Otherwise, when $\bar{w}^K - \underline{w}^K > 0$, then there must be some $\beta_i^{s,K} < \theta_i^K$ or $\beta_j^{d,K} > \gamma_j^K$ implying the approximation inherent in the Master Problem can be improved by adding to it a new column corresponding to s_i^{K+1} or d_j^{K+1}.

Theorem 4: The price directive decomposition method will converge to an optimal solution to the spatial equilibrium problem (1) if the supply and demand shadow prices Π_i^K and σ_j^K produced by (PDMP) for all K lie in a bounded set.

5. DECOMPOSITION IN PRACTICE

We believe mathematical programming decomposition methods provide a natural and appropriate methodology capable of overcoming the practical difficulties encountered in optimizing large scale models, and in generating these models by the integration of diverse and incompatible submodels. Using a decomposition method, separate submodels can be treated largely as "black boxes" that are linked by price and quantity information which is systematically analyzed. We have seen how the methods are mechanizations of the price and quantity tatonnement process whereby coal supply and demand markets achieve a competitive equilibrium. Thus, the methods formalize intuitive procedures for updating prices and quantities, thereby providing a rigorous basis for computing equilibria (see [14] for a discussion of this point and additional references).

For our purposes, the distinction between the decomposition of large scale models and the integration of diverse submodels can be viewed as artificial. Any decomposition method naturally integrates the submodels that exist or are created for the purposes of greater efficiency and flexibility. For this reason, we can discuss coal market models exclusively from the integration point of view.

Decomposition methods have been extensively studied by academic researchers over the past twenty years, but rarely used in practice. Recently, the situation has been changing because of the flexibility of today's commercial linear programming codes, and the increasing occurrence of large scale planning models. The belief that decomposition methods provide viable model generation and optimization approaches is supported by the success of recent coal model integration efforts using them (Shapiro and White [15], Wagner [18]). More generally, the methods have been receiving growing attention as an effective means for integrating energy models (Ho and Loute [4], Ho et al [5], Hogan and Weyant [6], Modiano and Shapiro [10], Shapiro [13]).

Decomposition methods offer several practical advantages over direct optimization approaches:

<u>Advantages</u>

(1) Permit integration of large and/or incompatible mathematical programming problems by the use of price and quantity information from the problems.

Facilitate integration of statistical coal supply models with linear programming transportation and utility models;

(2) Achieve greater overall computational efficiency than brute force optimization for many large problems. In part, this is because a decomposition algorithm can be terminated prior to optimality with a good feasible solution and a bound on how far the objective function value of this solution is from optimality.

(3) Permit meaningful initializations of an integrated model for a specific policy study based on judgment about likely or representative coal supply and demand prices and quantities. Re-start of the optimization based on previous runs can also be done effectively.

(4) Allow the individual models in the integration to be separately constructed, validated and extended.

(5) Facilitate the calculation of indirect price information; e.g., supplier rents due to coal depletion effects.

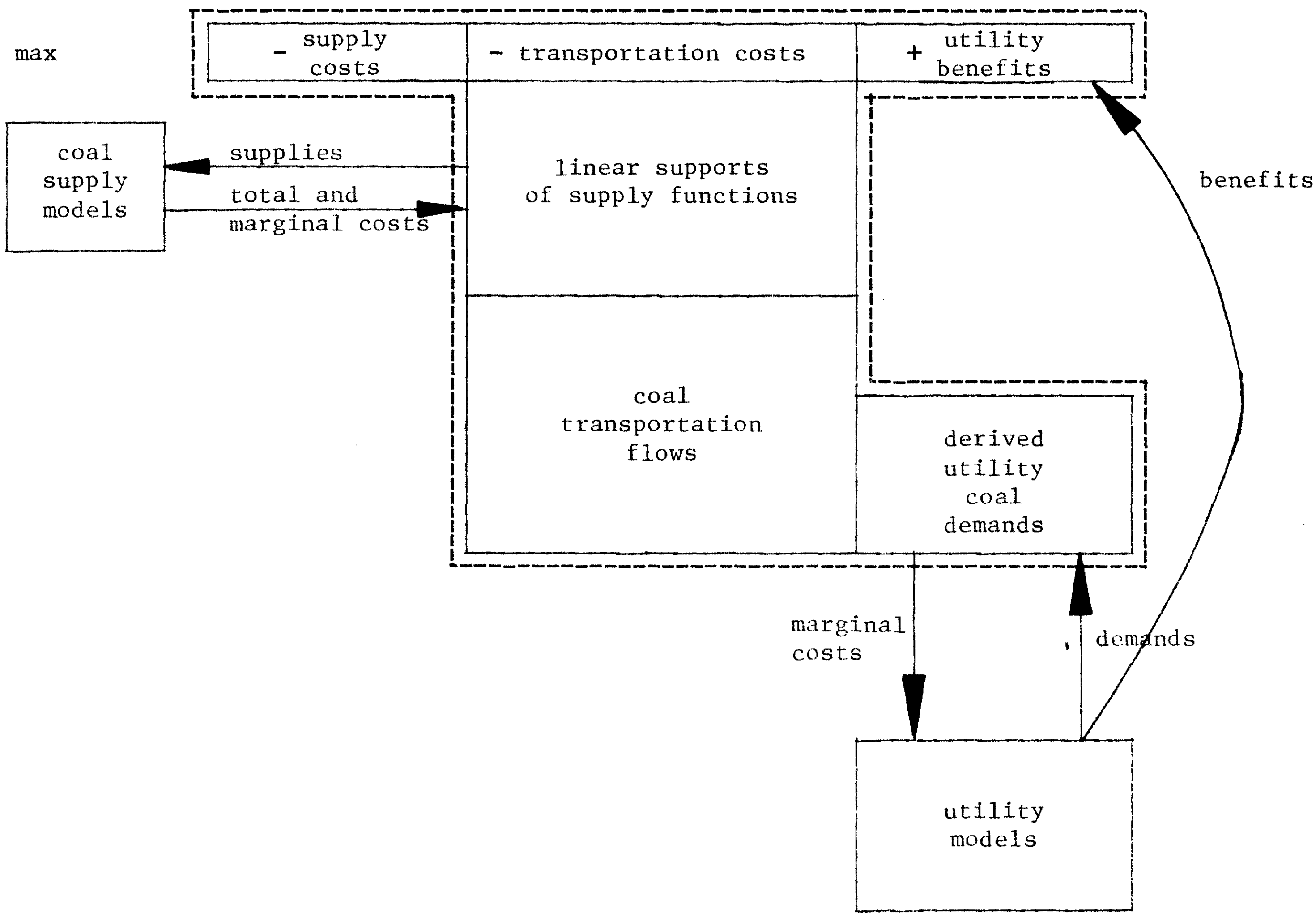
Disadvantages

(1) Requires more computation time to compute optimal solutions to reasonably sized linear programming problems.

(2) Requires greater sophistication by the user both in understanding the approach and running the computer program.

Figure 1 illustrates a successful decomposition implementation develpped by Shapiro and White [15]. Coal supply models for 6 supply regions, two types of coal, and a 10-year planning horizon were drawn from the statistical coal models developed by Zimmerman [19]. Each of 9 regional utilities was represented as a linear programming model describing how coal competes with other fuels to meet exogenous electricity demands over the 10-year planning horizon. The variables in these models reflected investment decisions in generating capacity as well as plant operating decisions. Exogenous demands for metallurgical coal were also included in the model. The supply regions were linked to the demand regions in each year by a transportation model. The objective of the integrated model was to maximize the net benefits over the 10-year period of the coal supply to the utilities. It can be shown that maximizing this objective is equivalent to computing a competitive equilibrium between the utilities and the suppliers who produce and deliver the coal.

The boxes enclosed by dotted lines in Figure 1 represent a Master Problem that communicates with the supply and utility models. As the result of computer experimentation, Shapiro and White [15] found that the most effective decomposition scheme was resource directive on the supply side and price directive on the demand side. Thus, total and marginal price information from the supply models at various levels of supply was used to generate linear supports (to the nonlinear supply functions) used in the Master Problem to approximate supply costs. These supply costs include depletion effects which are intertemporal. By contrast, the utility models responded to price information from the Master to generate derived coal demand vectors. Six iterations between the Master Problem and the supply and utility models sufficed to produce a 10-year coal supply, transport and utility demand solution that was, to all intents and purposes, optimal. The reader is referred to Shapiro and White [15] for more details about ICAM and its use in studying coal policy questions.



Integrated Coal Analysis Model (ICAM)

Optimization Schema

Figure 1

Conclusions

We have reported on the similarities among a wide variety of models which forecast coal prices and quantities on the assumption that the underlying markets are competitive. In addition, we have discussed how mathematical programming decomposition theory is appropriate for the calculation of equilibria characterizing these competitive markets. We can see several useful model extensions that would add realism to the models. The integration of any or all of these extensions would be greatly facilitated by the decomposition methods.

All of the models cited lack explicit modeling of investments in coal mining and transportation capacity expansion that must accompany expanded coal use. Decision variables relating to these investments could be added to the models without disturbing their underlying decomposable structures. This would probably have the effect of smoothing out supply patterns, particularly if there were fixed costs and returns to scale associated with the investments. Of course, non-convex cost elements would require mixed integer programming modeling and optimization techniques. Some more thought and experimentation would be required to model fixed costs and returns to scale in generating plant investments; that is, non-convexities within the utility models. Decomposition methods have also been proposed and implemented for utility capacity expansion models such as these subproblems. The question remains, however, about the effect of the subproblem non-convexities on the integrated coal supply and demand model.

Coal contracts between utilities and coal suppliers are another feature of U.S. coal markets that has been largely ignored by the coal market models. Decisions about multi-period contracts appear to be very similar to decisions about investments in generating capacity. Indeed, the two types of decisions are linked. Modeling and optimizing coal contracts is an important area of future research.

A third type of model extension would be the integration of linear programming process models of industries other than electric utilities whose use of coal might be increasingly significant. This would permit the models to evaluate the competition between industries for coal supply as well as the competition between coal and other fuels within an industry, and regional competitions due to differential freight rates.

Finally, the modeling and analysis reported on in this paper would be greatly enhanced by a modeling language for automatic generation and modification of the coal market models. The idea of the language would be to permit families of coal supply and demand models to be described in natural terms that are then translated into integrated optimization models. Decomposable structures would be automatically identified and exploited. This approach has worked extremely well in the development and use of large scale logistics planning problems whose underlying mathematical structure is similar to that of ICAM (Brown, Northup and Shapiro [1]).

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