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A MODEL FOR THE EFFICIENT USE
OF ENERGY RESOURCES

by

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ABSTRACT

This paper illustrates the application of mathematical programming to formulate and solve a simple energy planning problem. The energy system is represented as a network where energy flows from sources to end uses. The model formulated is dynamic and takes explicitly into account the depletion over time of fossil fuels. The problem is to determine, given an existing technology, the minimum cost flow of energy in each period, and also the optimal depletion path of energy resources. It is a subproblem of the more complex long term problem of choosing an optimal set of technologies.

1. Introduction

1.1 Definition of the Problem:

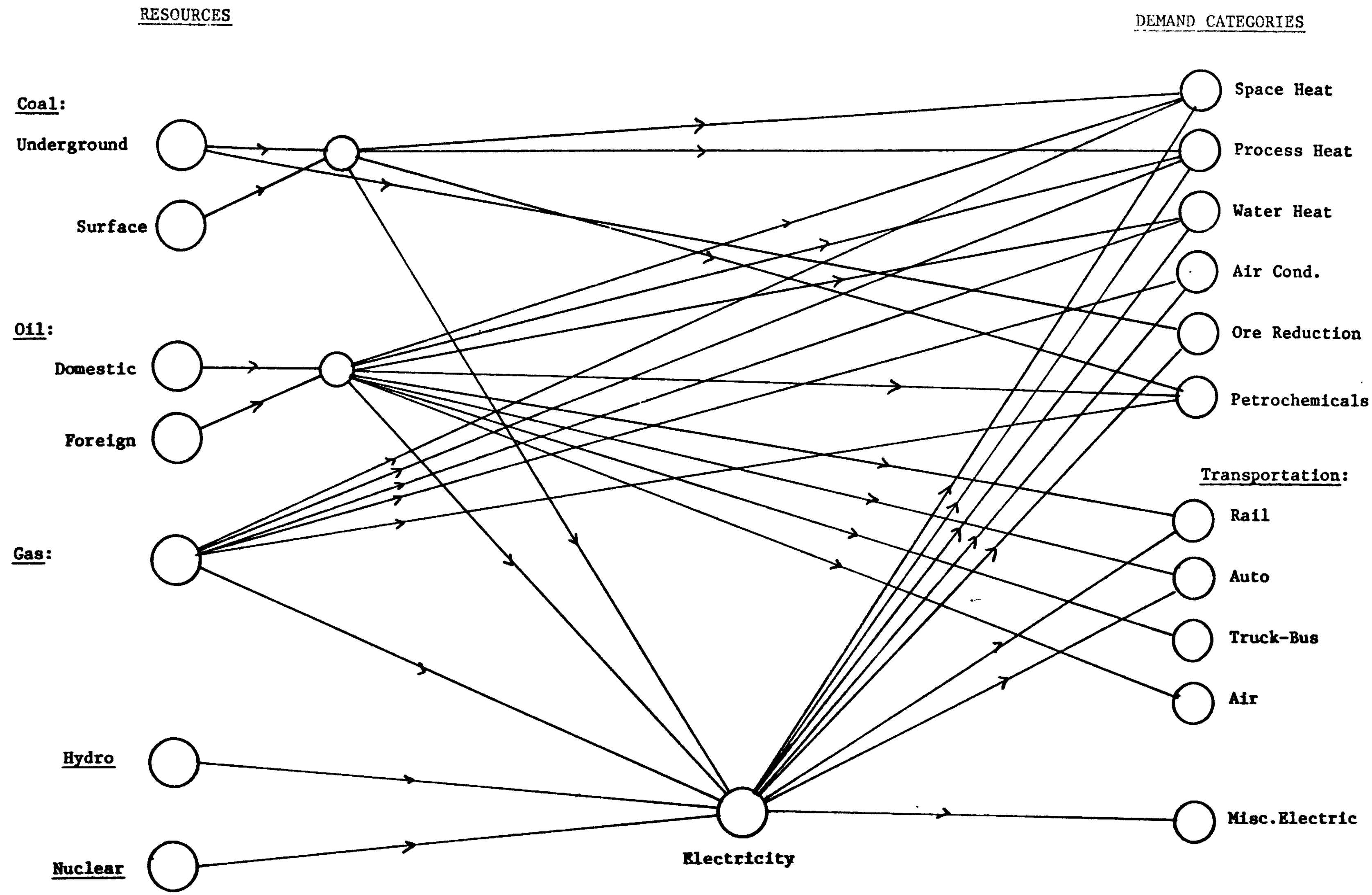
The energy system is represented as a simple network (see figure 1) where energy flows from sources to end uses; to each path is associated a cost (for extracting, processing and distributing the resource) and an efficiency of conversion. Hence the basic structure of the model is similar to that of the Brookhaven Energy System Optimization Model (BESOM) [2,4]. But, unlike BESOM, the problem considered here is dynamic, i.e., it takes into account intertemporal trade-offs. The basic structure is also similar to Nordhaus' model [7], but here we make an explicit distinction between extraction cost and processing cost. Moreover, Nordhaus assumes constant extraction costs over time, while we consider a more general structure for extraction cost functions. For depletable non-renewable resources (for instance fossil fuels), the marginal cost of extracting a unit of a resource is a non decreasing function of the cumulative quantity already extracted from that source. An example of such a function is the following, for a given resource i:

$$F_i(s_i) = \frac{K_i R_i}{R_i - s_i}$$

where R_i is the estimated quantity of recoverable resources and s_i is the cumulative quantity extracted[1]. For non-depletable resources, this cost would be constant or null.

Figure 1

Energy System Network



The corresponding data are given in the Appendix.

Most existing technologies rely on fossil fuels (coal, oil, gas) i.e., on non-renewable depletable resources. Hence, according to our assumption on extraction costs, they are characterized by non decreasing operating costs and decreasing returns to scale. Many modern technologies, such as geothermal or solar energy, use non-depletable or renewable resources. Their operating cost is thus constant, but it is assumed that some high fixed cost, corresponding to overhead cost for research and development, investment, installation, has to be paid prior to use. These technologies are hence characterized by increasing returns to scale. The problem is to determine the optimal selection and timing of new technologies, as well as the minimum total cost flow of energy to meet a demand given over time for each end use. In fact, we are considering two problems, distinct but closely related. One of them is to determine the optimal timing and sequencing of new technologies. The other one is to determine, given an existing technology, the optimal flow of energy in each period, and hence the optimal depletion path for depletable resources. Both problems are formulated in this section but only the second problem will be addressed in this paper. This problem corresponds to the present situation, or the near future, when a set of technologies exists and cannot be changed immediately. Hence it has to be solved in a first phase. It can also be considered as a subproblem of the more complex long-term problem of choosing an optimal set of technologies.

1.2 Scope of the paper:

Our goal here is to illustrate the application of mathematical programming to formulate and solve the energy planning problem described in the previous subsection. We demonstrate the possible use of the models and their possible contribution. Even with simple models, we obtain interesting results.

In section 2, the problems previously described are formulated as mathematical programs (PDR) and (PNT). Two problems closely related to (PDR) are also formulated. The approach chosen here to solve the nonlinear program (PDR) is generalized linear programming. The method is discussed in the context of problem (PDR) in section 3. We applied it to a model derived from BESOM.

Implementation issues are presented along with preliminary results in section 4. This condensation has of course weakened the model's realism significantly but the purpose of this paper is to provide a demonstration of a method, not to provide specific policy recommendations. As computations were progressing, we added to the realism and complexity of (PDR). We discuss possible extensions in section 5.

2. Some Mathematical Programming Formulations:

Unfortunately, the precise mathematical statement of the problem requires the use of a great deal of notation. For our purposes we also need to give several problem statements. They all appear in this section.

2.1 Notations and Formulation of the Basic Problem:

Subscript i corresponds to different energy resources, j to demand categories, t to time periods. All the energy quantities are expressed in

BTU. The other notations are as follows:

$\rho(t)$	Discount factor for time period t.
$d_j(t)$	Demand for j to be met in period t.
$x_{ij}(t)$	Flow from i to j in t.
c_{ij}	Processing cost for a unit of energy along the path (i,j).
w_{ij}	Efficiency factor for the path (i,j).
$s_i(t)$	Cumulative quantity extracted from source i up to period t. This variable describes the state of source i at time t.
$F_i(s_i(t))$	Marginal cost of extraction from i when the state of the source is $s_i(t)$.
T	Planning horizon assumed given for the moment.

The problem of optimally using a given technology can be presented as the following non linear program[2]:

(PDR)
$$\text{Min} \sum_{t=0}^{T} \rho(t) \sum_{i,j} [c_{ij} + F_i(s_i(t))]\, x_{ij}(t)$$

$$\text{s.t.} \quad \sum_i w_{ij} x_{ij}(t) = d_j(t) \qquad \text{all } j, \text{ all } t$$

$$s_i(t+1) = s_i(t) + \sum_j x_{ij}(t) \qquad \text{all } i, \text{ all } t$$

$$x_{ij}(t) \geq 0 \quad \text{all } i, \text{ all } j, \text{ all } t$$

$$s_i(0) \quad \text{given all } i$$

Additional notations:

E	Set of existing technologies.
M	Set of new technologies.
D	Set of depletable resources (used by technologies of E).
N	Set of resources used by technologies of M.
$y_{ij}(t)$	Flow of energy from i to j in period t using the new technology (i,j).
$\delta_{ij}(t)$	$=\begin{cases}1 & \text{if technology } (i,j) \in M \text{ is implemented in } t\\ 0 & \text{otherwise.}\end{cases}$
f_{ij}	Fixed cost of introduction of technology $(i,j) \in M$.
e_{ij}	Operating cost of technology $(i,j) \in M$.

The problem of determining an optimal set of technologies can be formulated as the following nonlinear mixed integer program:

$$\text{(PNT)} \qquad \text{Min} \sum_{t=0}^{T} \rho(t)\Big\{ \sum_{(i,j)\in E} [c_{ij} + F_i(s_i(t))]\, x_{ij}(t)$$

$$+ \sum_{(i,j)\in M} [f_{ij}\delta_{ij}(t) + e_{ij}y_{ij}(t)]\Big\}$$

$$\text{s.t.} \quad \sum_{i\in D} w_{ij}\, x_{ij}(t) + \sum_{i\in N} w_{ij}y_{ij}(t) = d_j(t) \qquad \text{all } j, \text{ all } t$$

$$s_i(t+1) = s_i(t) + \sum_j x_{ij}(t) \qquad \text{all } i, \text{ all } t$$

$$y_{ij}(t) \le Y \sum_{\tau=0}^{t} \delta_{ij}(\tau) \qquad \text{all } (i,j)\in M, \text{ all } t$$

$$\sum_{t=0}^{T} \delta_{ij}(t) \le 1 \qquad \text{all } (i,j)\in M$$

$$x_{ij}(t) \ge 0,\ y_{ij}(t) \ge 0 \quad \delta_{ij}(t) = 0,1 \qquad \text{all } i, \text{ all } j, \text{ all } t$$

This last problem will be addressed in a subsequent paper. Only the solution to problem (PDR) is discussed here.

2.2. Finding a Feasible Solution:

This solution will alternatively be called intuitive solution, recursive solution or semi dynamic solution. It is an approximation to the true optimum and is useful as a starting solution for the algorithm presented in section 2.

Define x^0 in the following way:

for each j $\quad x^0_{i*j}(t) = d_j(t)$

$$\text{for } i^* \text{ such that } c_{i*j} + F_{i*}(s_{i*}(t)) = \min_i \; c_{ij} + F_i(s_i(t))$$

and $x^0_{ij}(t) = 0 \quad \text{for } i \neq i^*$

This strategy can be interpreted as supplying each end use from the cheapest resource technology combination. It has been proved [8] that such a solution is optimal for a single end use (only one j) and for non decreasing demand over time (if the rate of increase is not too high), but this is not generally true for a multi commodity problem (a counter example is given in [8]).

This solution is feasible. It corresponds to a myopic view of the world, where the allocation in period t is done by looking only at the present, or at most the near future, which is frequently done in practice. This solution can be useful as a first step in a more sophisticated approach and, in some cases, will not be so far from the true optimum, for instance when the function F_i are very flat.

X^0 is very easy to express because of the very simple structure of (PDR), which was presented that way for clarity and ease of exposition. But in later sections, we add to the realism of (PDR) and give it a gradually more complex structure. Fortunately, the concept of this intuitive solution can be generalized very easily.

Consider problem (PDR) with some additional constraints, without specifying at the moment what these constraints are. At time t, the state of the system is characterized by $s_i(t)$ for all i, and the one period problem to determine the minimum cost flow in period t, given the state of the system, is:

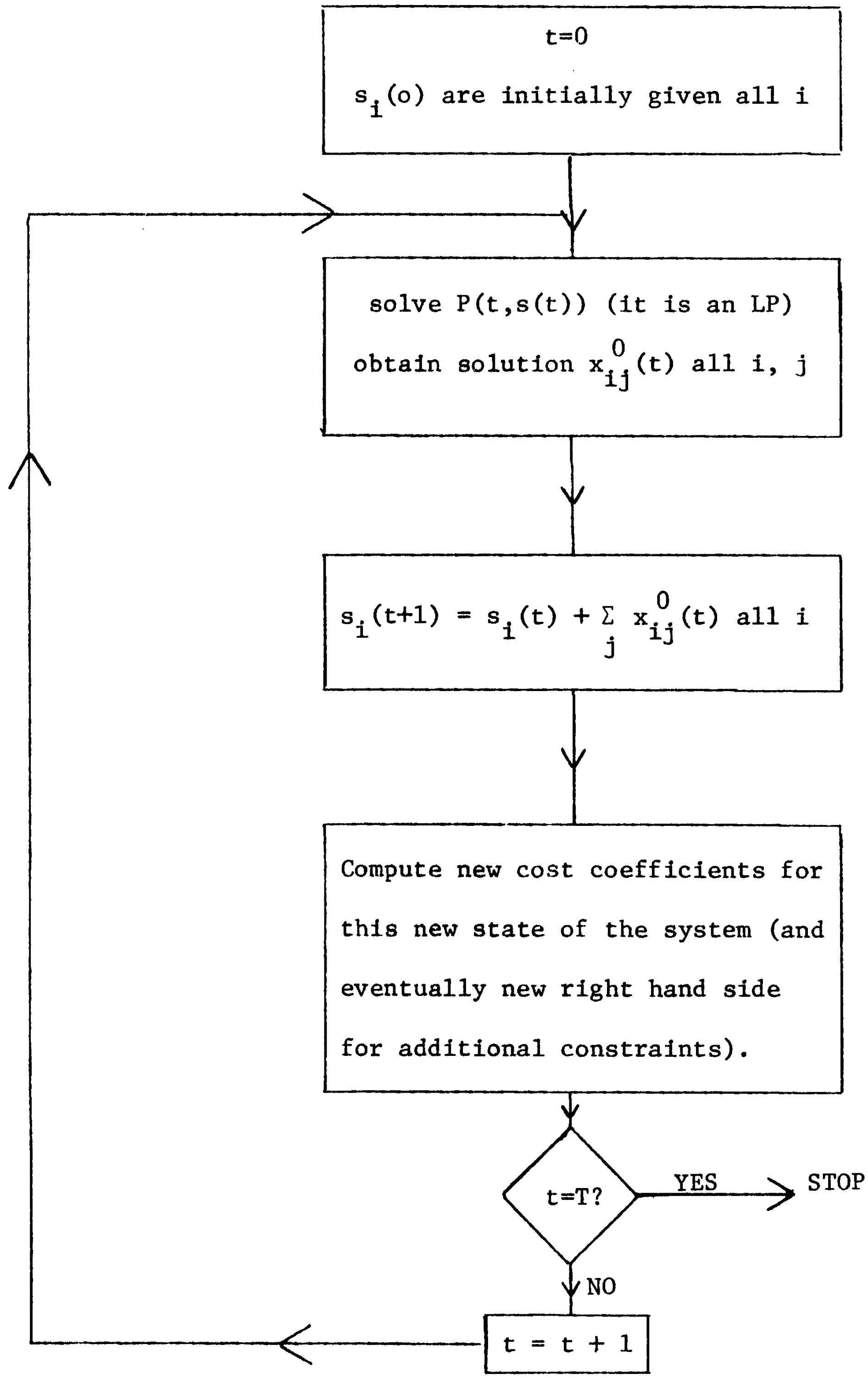
$$P(t,s(t)) \qquad \text{Min} \sum_{i,j} (c_{ij} + F_i(s_i(t)))x_{ij}(t)$$

$$\text{s.t.} \qquad \sum_i w_{ij}x_{ij}(t) = d_j(t) \quad \text{all } j$$

additional constraints for period t

$$x_{ij}(t) \geq 0 \quad \text{all } i,j$$

The solution x^0 is obtained by solving sequentially the problems P(t, s(t)) from t=0 to t=T as indicated clearly in the following diagram:


t=0
$s_i(o)$ are initially given all i
solve P(t,s(t)) (it is an LP)
obtain solution $x^0_{ij}(t)$ all i, j
$s_i(t+1) = s_i(t) + \sum_j x^0_{ij}(t)$ all i
Compute new cost coefficients for this new state of the system (and eventually new right hand side for additional constraints).
t=T?
YES
STOP
NO
t = t + 1

The advantage of this procedure is that it only requires solving a sequence of small linear programs instead of solving a large non linear program (like (PDR)). If we could know apriori how good an approximation to (PDR) this is, it would be very useful. In section 4, it will be interesting to compare the solution obtained by solving (PDR) with the intuitive solution.

2.3 A Total Extraction Cost Formulation:

In (PDR), we assumed that the marginal cost of extraction of a resource is a function of $s_i(t)$, the cumulative quantity extracted up to period t, but can be considered constant during period t, i.e. between $s_i(t)$ and $s_i(t+1)$. This may be valid in some cases, for instance if the function F_i is relatively flat in that region, or if $s_i(t+1)-s_i(t)$ is small enough, but rigorously the cost of extracting $s_i(t+1)-s_i(t)$ units of resource i should be:

$$\int_{s_i(t)}^{s_i(t+1)} F_i(\zeta_i)\, d\zeta_i$$

Then (PDR) is only an approximation to the real problem, which is:

$$(P')\text{Min} \sum_{t=0}^{T} \rho(t) \left[\sum_{i,j} c_{ij} x_{ij}(t) + \sum_i \int_{s_i(t)}^{s_i(t+1)} F_i(\zeta_i(t))d\,\zeta_i(t)\right]$$

$$\text{s.t.} \quad \sum_i w_{ij}x_{ij}(t) = d_j(t) \quad \text{all } j, \text{ all } t$$

$$\sum_j x_{ij}(t) = s_i(t+1) - s_i(t) \quad \text{all } i, \text{ all } t$$

$$x_{ij}(t) \geq 0 \quad \text{all } i,j,t$$

$$s_i(0) \text{ given all } i$$

Let $G_i(s_i) = \int_0^{s_i} F_i(\xi)\, d\xi$ be the integral of F_i.

Then the objective function of (P') is:

$$\text{Min} \sum_{t=0}^{T} \rho(t)[\sum_{i,j} c_{ij}x_{ij}(t) + \sum_i (G_i(s_i(t+1)) - G_i(s_i(t)))].$$

The objective function of (P') can alternatively be written

$$\sum_{t=0}^{T} \rho(t) \sum_{i,j} c_{ij}x_{ij}(t) + \sum_{t=0}^{T} \rho'(t)\sum_i G_i(s_i(t+1)) - \rho(0)G_i(s_i(0))$$

where

$$\rho'(t) = \rho(t) - \rho(t+1) \text{ for } t = 0\ldots T-1$$

$$\rho'(T) = \rho(T) .$$

In this form, it is easy to see that if the functions G_i are convex, which is implied by our assumption on the functions $F_i (F_i' \geq 0)$, then the objective function of (P') is convex because $\rho'(t) > 0$ for all t. For the functions F_i defined earlier,

$$G_i(s_i) = \text{cte} - K_iR_i \text{ Log}(R_i - s_i)$$

and therefore the total cost of extracting s_i units would be

$$G_i(s_i) - G_i(0) = \text{Log}\left(\frac{R_i}{R_i - s_i}\right)^{K_iR_i}$$

In a sense problem (P') is more accurate than problem (PDR); (P') is thus preferable, especially if the function is very steeply increasing in period t or has an unsmooth behavior in that range. But the functions F_i are more readily available in some cases, for instance if they are the results of an economic study or estimated econometrically.

If they have a simple analytic form, they can be integrated to obtain explicitly the functions G_i but sometimes they are available only implicitly, as, for example, the cost functions estimated for coal extraction by Zimmerman [11], and it may be necessary to work with problem (PDR). In other words, for purposes of model integration, it might be easier to work with problem (PDR). We come back to this question later.

The notion of intuitive or recursive solution can be extended to problem (P') but the problem to be solved in each period is no longer linear. Hence one of the most important advantages of this intuitive solution is lost. We will also have the occasion in section 4 to compare the solutions of problems (PDR) and (P').

3. Generalized Linear Programming [9], [5]:

To apply the algorithm, problem (PDR) is reformulated as follows (assuming $s_i(0) = 0$):

$$(P_1) \text{ Min } \sum_{t=0}^{T} \rho(t)\left[\sum_{i,j} c_{ij}x_{ij}(t) + \sum_i F_i\left(\sum_{\tau=0}^{t-1} m_i(\tau)\right)m_i(t)\right]$$

s.t.

$$(1) \quad \sum_i w_{ij}x_{ij}(t) = d_j(t) \qquad \text{all } j, \text{ all } t$$

$$(2) \quad \sum_j x_{ij}(t) - m_i(t) = 0 \qquad \text{all } i, \text{ all } t$$

$$x_{ij}(t) \geq 0 \qquad \text{all } i, j, t$$

After experimentation with different formulations, this last one appeared to be the most appropriate for application of generalized linear programming.

At iteration K of the algorithm, K points m^k have been generated and the corresponding master problem is:

$$(MP_1) \qquad \text{Min} \sum_{t=0}^{T} \rho(t) \sum_{i,j} c_{ij} x_{ij}(t) + \sum_{k=0}^{K-1} \lambda_k \left[\sum_{t=0}^{T} \rho(t) \sum_i F_i \left(\sum_{\tau=0}^{t-1} m_i^k(\tau) \right) m_i^k(t) \right]$$

s.t.

$$(1) \quad \sum_i w_{ij} x_{ij}(t) = d_j(t) \qquad \text{all } j, \text{ all } t$$

$$(2) \quad \sum_j x_{ij}(t) - m_i(t) = 0 \qquad \text{all } i, \text{ all } t$$

$$(3) \quad m_i(t) - \sum_{k=0}^{K-1} \lambda_k m_i^k(t) = 0 \quad \text{all } i, \text{ all } t$$

$$(4) \quad \sum_{k=0}^{K-1} \lambda_k = 1$$

$$x_{ij}(t) \geq 0 \qquad \text{all } i,j,t$$

The procedure is started with only one point k = 0 based on the intuitive solution:

$$m_i^0(t) = \sum_j x_{ij}^0(t) \qquad \text{all } i, t$$

where $x_{ij}^0(t)$ was defined in section 1.2.

Letting $\omega_i^k(t)$ be the dual variables to constraints (3), the corresponding subproblem is:

$$(\text{SUB } 1) \qquad \text{Min} \sum_{t=0}^{T} \sum_i \rho(t) \, F_i \left(\sum_{\tau=0}^{t-1} m_i(\tau) \right) m_i(t) - \omega_i^k(t) \, m_i(t)$$

$$m_i(t) \geq 0$$

$$\text{all } i, t$$

This problem is separable in i, thus we just have to solve one sub-problem for each resource i:

$$\text{(SUB i)} \quad \min_{\substack{m_i(t) \geq 0 \\ \text{all } t}} \sum_{t=0}^{T} \rho(t)\, F_i\Big(\sum_{\tau=0}^{t-1} m_i(\tau)\Big)\, m_i(t) - \omega_i^k(t)\, m_i(t)$$

With our assumption on the functions F_i (they are non-decreasing), this problem involves minimization of a pseudo convex function on R^T [6]. Hence any local minimum is a global minimum, and could be found by solving the optimality conditions:

$$\rho(T) \quad F_i\Big(\sum_0^{T-1} m_i(t)\Big) - \omega_i^k(T) \geq 0 \quad \text{and} = 0 \text{ if } m_i(T) > 0$$

$$\rho(t) \quad F_i\Big(\sum_0^{t-1} m_i(\tau)\Big) + \rho(t+1)\, m_i(t+1)\, F_i'\Big(\sum_0^{t} m_i(\tau)\Big) + \ldots\ldots$$

$$+ \quad \rho(T)\, m_i(T) \quad F_i'\Big(\sum_0^{T-1} m_i(\tau)\Big) - \omega_i^k(t) \geq 0 \text{ and} = 0 \text{ if } m_i(t) > 0$$

$$\text{for } t = 0 \ldots. T-1$$

In theory, this system could be solved by backward recursion, starting with period T, especially when the positivity constraints are dropped. But, due to the erratic behavior of the dual variables in the first iteration of the algorithm, it was found in practice that the procedure reacted very poorly to the vector sent back to the master problem by solving this system. Hence it was preferred to solve (SUBi) directly by dynamic programming.

For a given resource i, let $V_{it}(s_i)$ be the optimal value of the objective function of (SUBi) from period t to T, when the state of the system is s_i i.e., when T-t periods remain. The recursive relation satisfied by this function is:

$$V_{it}(s_i) = \min_{m_i(t) \geq 0} \{(\rho(t)F_i(s_i) - \omega_i(t))\, m_i(t) + V_{it+1}(s_i + m_i(t))\}$$

with $V_{iT+1} \equiv 0$

The decision variables $m_i(t)$ appearing in this minimization problem vary continuously, and not discretely. Similarly the state variables are continuous variables. But we can derive some upper bounds on the values of the state variables in each period, hence obtaining upper bounds for the variables $m_i(t)$. Thus the problem can be discretized; an appropriate grid is defined with different increments over the limits of the range in each period. The discrete approximation to the subproblem is solved by dynamic programming. The value of the objective function of the subproblem for the vector thus obtained is compared to μ, the dual variable of the convexity row (4); if it is smaller than μ, the vector is sent back to the master problem and a new column is added to (MP_1). Otherwise the grid is refined, and so on, until a suitable vector is found, or until the approximation is sufficient and the current solution is declared optimal.

Hence, at each iteration, we solve only an approximation to the subproblem. An advantage of this approach is that we do not have to know F_i explicitly, neither its derivative. We just need a procedure which, given a value for s_i, returns a value for $F_i(s_i)$; this can be another model, such as an econometric model, or Zimmerman's model for coal [11]. Whereas for solving the optimality conditions we need to know both F_i and its derivative. Besides by using dynamic programming, we do not need to make any assumptions of the functions F_i. Of course, then the subproblem may not involve minimization of a pseudo-convex function, as is the case when the functions F_i are increasing.

4. Implementation Issues and Computational Results:

For our purpose, we experimented with a simplified, condensed and aggregate version of BESOM. Most of the data were derived from BESOM or related material [1], [2], [10]. We start with a gross and global model, where we do not distinguish explicitly between different real technologies but we just consider x_{ij} to be the flow of energy from resource i to end use j, using a fictitious technology with associated cost and efficiency. (For instance, in our model, there is one variable representing the quantity of coal converted into electricity, whereas there are, in fact, several ways to produce electricity from coal). It is not conceptually harder to consider all real technologies, but the model has to be much more disaggregated, detailed and complex, thus making computations longer and the data collection task more tedious. The emphasis here is on comparing different types of fuels and other resources, examining substitution possibilities, rather than on comparing different individual technologies.

The model will be gradually refined and the improvements will be described as they are introduced. On the supply side, we start by considering only fossil fuels, to examine what would happen if the energy system relied entirely on depletable resources. The index i runs from 1 to 5, and we distinguish between two types of coal, underground and surface, two types of oil, domestic and foreign, and natural gas. For the marginal extraction cost for each resource, we use for simplicity and lack of something better the functions described above:

$$F_i(s_i) = \frac{K_i R_i}{R_i - s_i}$$

The data for those are given in Table 1. The case of foreign oil is somewhat delicate as the reserves are depleted by other agents than the United States. We have experimented with alternative treatments. These are described for each run. On the demand side, there are 11 demand categories: space heat, process heat and miscellaneous, water heat, air conditioning, ore reduction, petrochemicals, four transportation categories - rail, automobile, truck and bus, and air transportation - and miscellaneous electric. Time 0 is 1975. We experimented with a 5-period model, 1975-80, 1980-85, 1985-90, 1990-2000, 2000-2020, and a 6-period model where we added period 2020-2050.

TABLE 1

Resources i	Capital cost K_i in $\$/10^6$ BTU	Total Recoverable Resources at time 0 in 10^{15} BTU (a)
1 Underground coal	1.2	31,900
2 Surface coal	.9	9,200
3 Domestic oil	2.0	844
4 Foreign oil	2.4	5,800
5 Natural gas	1.75	780

(a) These numbers are quite arguable and controversial and we do not claim to have exact figures. See [1], [3] and [10] for different data.

The data common to all the runs are presented in the Appendix. They are the demand for all categories for all time periods, the cost and efficiency data for all the possible combinations, and the discount factors based on interest rate of 5%. Electricity is both a demand category and an intermediate source of energy. Hence the corresponding row has to be rewritten in the following way:

$$\sum_i x_{iel}(t) - \sum_j x_{elj}(t) = d_{el}(t) \quad \text{for all } t$$

The units for the data are given in the Appendix. The decision variables $x_{ij}(t)$ are in 10^{15} BTU and the cost function in 10^9\$.

For each run we discuss both computational issues and the numerical results obtained.

Run 1:

The model used for Runs 1 and 2 is the simplest, and has the formulation (PDR), except that an upper bound is imposed on the total quantity used of a resource, to make sure that no more than what is left in the last period is used. This constraint is

$$s_i(T + 1) \leq R_i \quad \text{for all } i \ .$$

The planning horizon is 5 periods long (T=5), up to 2020. For imported oil, the same type of function for marginal extraction costs is used, as for other resources. But the demand for oil from the rest of the world is assumed given, and contributes to the depletion of foreign oil. Thus, in each period the amount of remaining foreign oil is not only a function of the cumulative quantity domestically used starting at time 0. We assume the following are the demands for oil in each period of the rest of the world (in 10^{15} BTU): 425, 480, 530, 1170, 2695, leaving as the amount of recoverable resources for foreign oil at the beginning of each period: 5800, $5375 - s_4$ (1), $4895 - s_4$ (2), $4365 - s_4$ (3), $3195 - s_4$ (4), where $s_4(t)$ are decision variables in the model.

The initial solution (intuitive solution) and the best known solution are shown in Tables 2 and 3 respectively. The first thing to notice is that with this planning horizon these data (demands, cost,

TABLE 2

QUANTITIES EXTRACTED (10^{15} BTU)
FROM EACH SOURCE IN EACH PERIOD

RESOURCE i \ PERIOD t	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.16	13.23	15.715	35.876	79.675	
Surface coal 2	139.23	167.94	190.39	415.065	1029.305	
Domestic oil 3	102.87	104.385	0.0	0.0	636.745	
Foreign oil 4	0.0	0.0	138.25	321.945	902.60	
Natural gas 5	73.16	116.785	119.21	285.625	35.720	

INITIAL SOLUTION

NUMBER OF GLP INTERATIONS: 1

OBJECTIVE FUNCTION VALUE=COST: 11875.80

(10^9\$)

TABLE 3

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

PERIOD t / RESOURCE i	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.155	13.217	15.711	35.866	79.670	
Surface coal 2	139.258	107.9794	190.433	415.185	1029.340	
Domestic oil 3	102.846	122.670	18.757	294.385	0.0	
Foreign oil 4	0.0	0.0	119.463	27.560	1539.319	
Natural gas 5	73.153	98.472	119.198	285.507	35.714	

BEST SOLUTION

NUMBER OF GLP ITERATIONS: 52

OBJECTIVE FUNCTION VALUE=COST: 11613.9492

LAGRANGEAN (APPROXIMATION): 11604.7969

etc.), the depletable resources are not exhausted by the end of the 5 periods. Secondly, the best known solution gives an improvement in the objective function (as compared to the initial solution) of 261.85 units or 2.25%. These two solutions are very close, and for all practical purposes the intuitive solution might be a satisfactory approximation, at least in this case. This is investigated further in the following runs.

We now analyze in more detail the differences between the two solutions: in the first period, the quantities obtained are very close, probably due to the linearity in the first period. Then, in all periods, the last solution gives quantities extracted of coal that are very close to those dictated by the initial solution. This is due to the fact that the functions F_1 and F_2 are very flat, due to large quantities of resources. The main differences occur for oil, domestic and imported, and mostly in the distribution between domestic and imported oil in each period, except in period 2, where less gas is used than in the initial solution, and more domestic oil. The reasons for this behavior are clear: for these data and planning horizon, the resources are not exhausted and there is not much room for substitution. The most critical resource is oil and this is where sensitivity appears. Besides, the very simplistic structure of the model creates the so-called flip-flop behavior for oil; i.e., either the demand is satisfied entirely with domestic oil, or entirely with foreign oil, using the cheapest in the current period. This is not realistic, as the quantity of imported oil should be limited. This is done in later runs (run 3 and subsequent).

Run 2:

The model used for this run is the same as the one used for run 1, with the following exceptions: the planning horizon is now 5 periods long (up to 2050) and there is no constraint on the total usage of foreign oil, thus allowing the possibility of shortage in the last period. As the rest of the world demand for oil in period 5 is 2695 x 10^{15} BTU, the remaining recoverable resources at the beginning of period 6 are 500 - s_4 (5).

The results are displayed in Tables 4 and 5. The run was conducted to ε optimality where ε is .05%. The absolute difference between the initial solution and the optimal solution is 610.18, giving a relative difference of 3.9%, which is still small.

In this run, domestic oil as well as natural gas are exhausted in the last period. There is also a shortage of foreign oil in the last period if the demand for oil from the rest of the world is not lowered. By comparing Tables 4 and 5, we notice that the best solution recommends more reliance on coal starting in period 4,as well as saving domestic oil for use in the last two periods. Its' use is lowered in the first two periods and is null in periods 3 and 4. (It is also interesting to compare Tables 5 and 3.) Coal is used to produce electricity starting in period 4, and electricity is used as an alternative source of energy to oil for transportation (rail, automobile) in the last period. Hence we see a limited introduction of the electric car.

In conclusion, for this planning horizon we see the depletion effect in action, thus making the recursive solution significantly different from the optimal solution. Again here, the optimal solution

TABLE 4

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

RESOURCE i \ PERIOD t	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.16	13.23	15.715	35.876	79.676	163.301
Surface coal 2	139.23	167.94	190.39	415.065	1029.305	6590.570
Domestic oil 3	102.87	106.385	0.0	0.0	636.745	0.0
Foreign oil 4	0.0	0.0	138.25	321.945	902.60	698.02
Natural gas	73.16	116.785	119.21	285.625	35.72	149.290

INITIAL SOLUTION

NUMBER OF GLP ITERATIONS: 1
OBJECTIVE FUNCTION VALUE = COST: 16322.7734

TABLE 5

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

RESOURCE i \ PERIOD t	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.155	13.217	15.711	105.02	664.224	1129.647
Surface coal 2	139.258	175.089	242.391	539.104	1192.661	4377.262
Domestic oil 3	44.955	45.617	0.0	0.0	278.262	475.167
Foreign oil 4	47.733	58.757	118.484	280.011	394.442	512.490
Natural gas 5	83.312	109.430	85.300	128.141	130.330	243.396

BEST SOLUTION

NUMBER OF GLP ITERATIONS: 66
OBJECTIVE FUNCTION VALUE = COST: 15712.586
LAGRANGEAN (APPROXIMATION): 15708.656

advocates a heavy reliance on foreign oil, which is unrealistic and in complete disagreement with the government policy of independence. As mentioned before, this is corrected in subsequent runs by limiting the quantity of oil imported. The second drawback of this solution is that it recommends a sudden reliance on coal, thus making the extraction of coal grow at a fast rate. This is unrealistic as the productive capacity of coal has to be built progressively. This will be modified in run 5 and subsequent.

Run 3:

Runs 3 and 4 differ from runs 1 and 2 mainly by the way they treat foreign oil. Here the depletion effect of foreign oil by United States consumption is neglected; i.e., it is assumed that the United States demand for non-United States oil is small compared to the world demand. This is equivalent to saying that the cost of foreign oil is given in each period and independent of the decision variables of the model, or that the supply or foreign oil is perfectly elastic at a given price in each period. To determine these costs, we just use the function of the previous runs, using some forecasted world demand for oil (see [3]) to obtain the cost in each period, as given in Table 6.

We also impose a limit on the quantity of foreign oil imported in each period. The upper bound is equal to 40% the forecasted United States demand for oil in each period if things remain the same; i.e., the present usage of oil continues in the future, with no substitution or

TABLE 6

PERIOD	1	2	3	4	5	6
Price of foreign oil $\$/10^6$ BTU	2.24	2.415	2.654	2.976	4.066	25.984

TABLE 7

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

RESOURCE i \ PERIOD t	1 1975-80	2 1980=85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.16	13.23	15.715	35.876	79.676	
Surface coal 2	139.23	167.94	190.390	415.085	1509.382	
Domestic oil 3	61.72	62.63	82.95	193.167	443.53	
Foreign oil 4	41.141	41.754	55.30	128.778	600.25	
Natural gas 5	73.160	116.785	119.21	285.625	35.72	

INITIAL SOLUTION

NUMBER OF GLP ITERATIONS: 1

OBJECTIVE FUNCTION VALUE = COST: 12162.53

TABLE 8

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

RESOURCE i \ PERIOD t	1 1975-80	2 1080-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.155	15.782	61.732	35.866	79.670	
Surface coal 2	139.258	183.808	144.412	415.186	1777.215	
Domestic oil 3	61.698	80.916	63.184	193.107	606.561	
Foreign oil 4	41.148	41.754	55.30	128.778	266.142	
Natural gas	73.153	79.484	138.934	285.507	130.330	

BEST SOLUTION

NUMBER OF GLP ITERATIONS: 105

OBJECTIVE FUNCTION VALUE = COST: 11812.102

LAGRANGEAN: 11770.246

introduction of new technology. Maybe we would prefer to express this in the following way: the quantity of oil imported in each period cannot be more than a certain percentage of the total oil consumption in the United States in that period. But this formulation would create difficulties when domestic oil starts to be in short supply as, if there is no domestic oil remaining to be extracted, then no oil can be imported. This is not desirable, as imported oil would be very important in a transition phase.

In this run, the planning horizon is 5 periods long. The results are summarized in Tables 7 and 8. The optimal solution and the initial solution are still very close in total cost (relative difference of 3%). The quantities recommended by each solution in each period are not very different. The optimal solution recommends using more coal in periods 2 and 5. In this solution, imported oil is always used at its upper bound, except in the last period, where it is replaced by coal and gas ; less gas is used in period 2. No resource is exhausted before the end of the planning horizon. Again here, the optimal solution recommends more reliance on coal than is probably feasible.

Run 4:

It is the same as run 3 with a 6-period planning horizon. Results are summarized in Tables 9 and 10. Now the initial solution and the best solution are significantly different (12%). The optimal solution advocates even more reliance on coal than in Runs 3 and 2, calling for the same comments as before. Domestic oil and natural gas are exhausted by the end of the 6th period. Imported oil is always used at

TABLE 9

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

PERIOD t / RESOURCE i	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.16	13.23	15.715	35.876	79.675	163.30
Surface coal 2	139.23	157.94	190.39	415.065	1509.38	6582.56
Domestic oil 3	61.722	62.63	82.95	193.167	443.53	0.0
Foreign oil 4	41.141	41.754	55.30	128.78	600.25	689.74
Natural gas 5	73.16	116.785	119.21	285.625	35.72	149.29

RUN 4 INITIAL SOLUTION

NUMBER OF GLP ITERATIONS: 1

OBJECTIVE FUNCTION VALUE = COST: 17533.137

TABLE 10

QUANTITIES EXTRACTED
FROM EACH SOURCE IN EACH PERIOD

PERIOD t / RESOURCE i	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Underground coal 1	11.50	23.78	22.36	82.35	627.28	2317.24
Surface coal 2	138.92	157.41	260.02	550.57	1229.60	2662.67
Domestic oil 3	45.29	62.62	63.18	151.23	56.97	466.71
Foreign oil 4	47.148	41.75	55.30	128.78	615.74	689.74
Natural gas	89.56	116.77	60.24	139.71	130.33	243.40

RUN 4 BEST SOLUTION

NUMBER OF GLP ITERATIONS: 43

OBJECTIVE FUNCTION VALUE = COST: 15716.13

LAGRANGEAN: 15637.59

its maximum availability. If the upper bound is made tighter in the last period, the problem becomes infeasible. The quantities given in Table 10 are probably unrealistic in the sense that foreign oil would be in shortage before the end of the planning horizon. This proves again that the beginning of the 21st century is a critical time, where it will be necessary that new technologies be implemented.

The advantage of the model used in Runs 3 and 4 is that it allows for testing the policy which aims at some independence from the rest of the world by reducing the size of oil imports. The percentage of oil imports can be made smaller and smaller, until feasibility is lost. This is also a critical issue for the introduction of new technologies.

5. Discussion of the model and possible extensions:

5.1 The first extension that comes to mind is to add to the realism of the model, thus making its structure more complex. This could be done, for instance, by including additional constraints, such as pollution control, capacity limits, political and institutional constraints. The next possibility would be to disaggregate the model, taking into account more demand categories and/or more technologies, thus describing the energy system in more details. For example, the methodology described in the previous sections could be applied directly to BESOM which is quite detailed and large already. It would be straightforward to apply the algorithms and programs described above to these models.

5.2 We always assumed demands to be given and inelastic in each period. It would be very important to model demand elasticities at least within periods. This is, for instance, critical if we want to model conservation policies. Pure conservation, i.e. reduction in demand, or conservation through increased efficiency can be taken into account through model (PDR), by making the stream of demand over time more slowly increasing or constant. But, to assess the consequences of taxes or tax credits, we need to make demand elasticities explicit (at least self elasticity in this case).

Also it is very important to allow substitution in the transportation sector for instance. The transportation sector is the most reliant on oil. When oil runs out and if independence is preferred, gasoline for cars will become increasingly more scarce and more expensive;

consumers may demand less transportation by living closer to their working place, thus the importance of self elasticity, or may use increasingly more public transportation, where electricity can be used more easily, thus the importance of cross-elasticity. Commercial transportation may rely more heavily on rail and less on trucks. If demand functions could be estimated for each period, some revenue function or welfare function such as consumers' surplus could be used in the optimization problem. This function could be linearized by using generalized linear programming as is done above for the cost function.

5.3 A question which was ignored all through this paper is the problem of terminal conditions and planning horizon. The only time terminal conditions are vaguely specified is by putting upper bounds on the total quantity consumed of a resource. The problem of specifying a planning horizon is very critical and very hard in long-term planning. The optimal strategy is heavily dependent on the planning horizon specified, as a set of equations admits a feasible solution for some value of the planning horizon and does not when its value is slightly increased, thus creating serious discontinuities. But this problem would not be so serious for (PNT) as, when non-depletable resources are introduced, they would allow for feasibility to infinity. This would ensure the existence and implementation of a backstop technology as used by Nordhaus [7] and others. For problem (PDR) the time at which feasibility is lost, i.e. when exhaustion of resources occurs, for a given set of resources could be characterized and then used as planning horizon.

5.4 The next extension to problem (PDR) and also its motivation is problem (PNT) which will be discussed, together with methods to solve it and its solution, in a forthcoming paper.

Footnotes:

1. Note that for large R_i, this function is nearly constant (very flat), i.e., a resource existing in large quantity can be considered nearly non-depletable.
2. The objective function of (PDR) is not convex but it is pseudo-convex (see Mangasarian [6]).
3. This new variable will have a negative reduced cost and will enter the basis in (MP_1).

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Appendix:

Technology Matrix With Cost and Efficiency Data

processing cost ($/10⁶ BTU) (Efficiency Factor)	COAL (Under ground / Sur- face)	OIL (Domes- tic / For- eign)	GAS	HYDROPOWER	NUCLEAR	ELECTRICITY
Space Heat	3.13 (.97)	2.92 (.91)	2.79 (.91)			3.48 (.9
Process Heat	.34 (.97)	.41 (.91)	.29 (.91)			.4 (.9
Water Heat		2.11 (.91)	2.37 (.91)			2.47 (.9
Air Cond.			24.51 (.91)			24.56 (.9
Ore Reduction	.37 (.97)					.5 (.9
Petrochemicals	.13 (.97)	.65 (.91)	.22 (.91)			
Rail		5.8 (.91)				26.72 (.9
Automobile		6.18 (.91)				29.41 (.9
Truck-Bus		17.83 (.91)				
Air Transp.		4.42 (.91)				
Electricity	10.27 (.32)	8.38 (.31)	7.79 (.31)	7.30 (.37)	10.33 (.32)	

Notes: Cost includes: capital cost, operating cost, maintenance cost, fuel delivery cost, enduse cost; but not extraction cost.

Efficiency factors do not take account of end use efficiencies.

Demand Data

DEMAND (10^{15} BTU) \ PERIOD	1 1975-80	2 1980-85	3 1985-90	4 1990-2000	5 2000-20	6 2020-50
Space Heat	56.3	60.16	62.15	129.08	276.26	509.04
Process Heat & Misc.	57.83	71.84	83.06	186.41	509.5	1436.67
Water Heat	14.93	16.65	17.96	38.16	86.1	160.26
Air Cond.	1.93	3.13	4.73	13.26	32.5	61.23
Ore reduction	10.82	12.82	15.24	34.79	77.28	158.4
Petrochemicals	20.95	30.94	39.51	87.24	212.7	474.09
Rail	3.08	3.27	3.65	8.08	19.7	43.89
Automobile	45.61	49.74	52.39	117.22	256.74	521.28
Truck - Bus	18.92	24.43	28.3	66.70	163.16	295.95
Air Transportation	11.00	17.54	23.48	62.81	172.56	330.15
Misc. Electric	22.02	29.46	35.34	83.99	239.32	586.47
Discount Factor ρ (Interest rate 5%)	.885	.684	.543	.377	.181	.054

Note: Demands are corrected for end-use efficiency.