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MARKOV ANALYSIS OF AN ALTERNATIVE TO PURE RANDOM ASSIGNMENT

by

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Abstract

There are many public policy settings in which random assignment of clients to experimental and control groups is not feasible. I will discuss one such case, and the alternative method of assignment which was actually used. The vulnerability of this method to intentional selection bias will be explored using Markov methods of analysis.

1. Introduction and Overview

The random assignment of clients to experimental and control groups is an important and controversial issue in program evaluation. On the one hand, it is generally accepted that "[A randomized] experimental design is the research approach most likely to avoid ambiguous findings and the resultant...arguments over interpretation of study results."[1] On the other hand, ethical and legal considerations--especially concern about the propriety of randomly denying a presumably beneficial treatment to the control group, or subjecting the experimental group to a potentially harmful one--are frequently an obstacle to the use of randomized designs in many situations where they might provide useful information.

One example which illustrates the weight being given to such concerns is the recent Supreme Court ruling on spot checks of automobiles:

> The Supreme Court held that a motorist who hasn't done anything to arouse suspicion can't be stopped at random [sic] by police for inspection of his driver's license and auto registration...
> While ruling out such stops that leave the choice of the vehicle up to the "unbridled discretion" of police, Justice Byron White, writing for the majority, said states could use other, more systematic methods to check compliance with traffic-safety regulations. For instance, he said, "questioning of all oncoming traffic at roadblock-type stops is one possible alternative."[2]

Interestingly enough, once we eliminate confusion over the use of the word "random", a concurring opinion seems to suggest that randomization <u>would</u> be considered a "more systematic method":

> ...Justices Harry Blackmun and Lewis Powell, while joining the majority opinion, add that they assumed that the court would also allow nonrandom [sic--nonarbitrary] stops other than roadblocks. For instance, they suggested, stopping every 10th car to pass a given point would

> "equate with" but be "less intrusive than" a 100% roadblock stop.[3]

A number of possible ways to resolve such dilemmas over randomization have been suggested. For example, many writers have provided arguments in favor of using randomization wherever possible. Campbell has pointed out that randomization could be justified as a fair way of allocating resources in situations of scarcity: "Such decision procedures as the drawing of lots have had a justly esteemed position since time immemorial."[4] Campbell and Boruch mention that randomization need not involve denying services to clients who would otherwise receive them--"...even if deprivation of treatment were a problem, one could arrange the use of evaluation budgets in a way that expands rather than decreases the number of people having access to the program."[5] Gilbert, Light and Mosteller argue that the ethical problems of randomization may if anything be overshadowed by the ethical problems of using weaker designs than are available:

> We change our social system...frequently and rather arbitrarily; that is in ways ordinarily intended to be beneficial, but with little or no evidence that the innovation will work. These changes are routinely made in such an unsystematic way that no one can assess the consequences of these haphazard adjustments... The result is that we spend our money, often put people at risk, and learn little. This haphazard approach is not "experimenting" with people; instead, it is <u>fooling around with people</u>.[6]

Another approach has involved the development of more flexible alternatives to random assignment, the best known of which are the quasi-experimental designs proposed by Campbell and Stanley.[7] Although these designs are a useful contribution in situations where true experimental designs are not feasible, they can often yield highly equivocal results. In particular, the most commonly used quasi-experiment, based on non-equivalent control groups, is extremely weak when the experimental and control groups differ on attributes closely related to treatment effectiveness.

Within the area of alternatives to randomization, another promising path is the development of "semi-random" experimental designs--assignment procedures which incorporate some element of randomness to help ensure the comparability of experimental and control groups, while permitting systematic flexibility in other respects. Precedents for designs of this type include the old and revered method of stratified sampling, and "balanced" designs such as those discussed by Blackwell and Hodges[8] and Efron[9], which are designed to yield experimental and control groups of very nearly the same size. For a discussion of other possible semi-random designs, see the proposal "New Tools for Comprehensive Evaluations in Criminal Justice," submitted to LEAA by Larson.[10]

This paper is concerned with one semi-random design in particular, that of "random time quota selection," developed and used by the Vera Institute of Justice* in their evaluation of the New York Court Employment Project.[11] To sketch the background of this evaluation, "The Court Employment Project (CEP) diverts defendants from the criminal justice system, provides them with employment services and leads to a dismissal of charges for defendants who successfully complete the program."[12]

*The Vera Institute of Justice, founded in 1961, is a non-profit organization which develops and evaluates projects and conducts research, for the purpose of furthering equal protection under the law for the indigent.

Quasi-experimental evaluations of CEP (using non-equivalent control groups) had been performed, and had yielded somewhat inconclusive results; it was felt that a true experimental design was the only way to resolve the remaining uncertainty about the program's effectiveness.

> We do not know whether the similarity of rearrest rates indicates that the Project has no more impact than normal criminal justice treatment or whether it is a product of the weakness of the research design. What is clear, however, is that the only way to find out is to initiate, after seven years of Project operations, random assignment experiments.[13]

However, the Vera Institute's proposed use of random assignment in their evaluation met with strenuous objections, on the grounds that the denial of program services to defendants in the control group might violate their rights to equal protection (Fourteenth Amendment) and due process (Fifth Amendment). The opinion of LEAA's Office of General Counsel lent additional weight to these objections:

> It is generally recognized that a prosecutor is given wide latitude in exercising his discretion in determing whether or not to prosecute...However, the prosecutor's discretion must be based upon a justifiable standard...Where a justifiable standard is not used, the exercise of discretion is arbitrary and subject to challenge as a denial of equal protection and due process. In addition, where a classification is used, it must rest upon real differences which are relevant to the purpose for which the classification is made...
>
> _This office is of the opinion that the use of a random assignment procedure to determine whether or not to prosecute is not a justifiable standard._ The selection of control group participants is not based upon real differences. [emphasis added][14]

The Vera Institute therefore needed some method of selection which would provide nearly the same degree of comparability between experimental and control groups as randomization, while not violating the principle of selection "based upon a justifiable standard." The method which they devised to satisfy these requirements--random time quota selection--was implemented as follows.

The total duration of the evaluation was divided into short time periods (roughly one to three working days) of random length. Then, proportional to the length of each time period, a quota was established for the maximum number of defendants who could be accepted into the program (i.e., the experimental group) during that period, based on the actual (time-averaged) capacity limitations of the program. Those defendants who were deemed eligible to receive program services, but were not accepted into the program under this criterion, thus formed the control group. Since the time periods were relatively short, the experimental and control groups were in effect chosen concurrently, and it was expected that the two groups would be largely free of accidental bias due to fluctuations in the characteristics of the defendant population over time.

Another dimension along which experimental designs such as the above may be assessed, in addition to accidental bias, is that of "selection bias," a term introduced by Blackwell and Hodges:

> Suppose an experimenter E wishes to compare the effectiveness of two treatments, A and B, on a somewhat vaguely defined population. As individuals arrive, E decides whether they are in the population, and if he decides that they are, he administers A or B and notes

> the result...Plainly, if E is aware, before deciding whether an individual is in the population, which treatment is to be administered next, he may, not necessarily deliberately, introduce a bias into the the experiment. This bias we call selection bias.[15]

In general, selection bias could arise in a number of ways. Perhaps the most obvious, mentioned by Blackwell and Hodges, is through slight modifications in the eligibility criteria depending on what the next assignment is likely to be. An alternative mechanism is to delay the assignment of particularly favored clients until times when they are likely to be assigned to the experimental group.

In our case, those with the most access to information about future assignments--the Vera Institute research staff--had very little opportunity to influence the composition of the experimental group, since the selection of eligible defendants from which the two groups were drawn was performed independently by the CEP screening staff, and after that point the assignment of defendants was completely determined by the experimental design. However, it is also possible that selection bias might have occurred during the phase of screening by CEP, since the CEP staff had access to the outcomes of past assignments, and might have gleaned from those some information about future assignments.

The purpose of this paper is to analyze the vulnerability of random time quota selection to selection bias in situations similar to the above. For example, the method of random time quota selection could be used in evaluating a wide range of programs, provided that clients arrive in a sequential ("trickle") manner, and that the program is unable to accommodate all clients determined to be eligible. Thus, the method is likely to meet with approval in a variety of criminal justice settings, among others, due to the legal obstacles which might be encountered by randomized experiments. The mechanisms by which selection bias might occur would depend on the particular program being evaluated, but bias can occur in some form in almost any program, unless screening and assignment procedures are either completely "blind" or very rigidly specified; for example, selection bias might be due to the actions of people other than program staff, such as social workers or other client advocates attempting to secure program services for their favored clients.

It would certainly be desirable to have some measure of vulnerability to selection bias--for example, how well one could do in attempting to intentionally influence program assignments. It would also be very interesting to explore the dependence of such measures on the exact form of the distribution for time period lengths. This paper will explore issues such as the above, in a context not necessarily limited to that encountered by the Vera Institute.

In Section 2 I will present the basic assumptions of the model used, and in Section 3 perform some preliminary analysis. Section 4 uses the techniques of Markov analysis to investigate the behavior of a somewhat simplified system. Section 5 presents a discussion of the basic results of Section 4, with several extensions. Finally, in Section 6, I discuss the implications of my analysis for actual practice.

2. Assumptions Used in the Analysis

In order to perform an analysis of the random time quota selection method, several assumptions must be made. First, there are a few relatively straightforward or notational points: I have assumed that applicants to the program arrive (in general) according to a Poisson process with rate λ, that time period lengths are independently and identically distributed according to some probability density function $f_T(\cdot)$, and that the quota for any time period is given by $q(\cdot)$, an integer-valued function of the length of the period.

More significant are the assumptions with respect to selection bias. I have assumed that most applicants make no attempt to influence their selection probabilities--these can be characterized as "naive" applicants. There is also one "opportunistic" or "gaming" applicant, who will attempt to time application to the program so as to minimize the probability of rejection, based on incomplete observation of the system. (Only one application may be made, a rejection being final.)

Obviously, the gaming applicant must have some knowledge of the selection method, and some way of making inferences about the state of the system, in order to calculate the optimal time at which to apply. With respect to the first issue, I have assumed that the gaming applicant has complete knowledge of the operating characteristics of the selection method (i.e., knows the exact forms of the functions $f_T(\cdot)$ and $q(\cdot)$ being used). This will in general be an overestimate of the true state of knowledge, and so will tend to result in overestimates of vulnerability to selection bias. (For example, in the Vera Institute case, the CEP staff knew the basic principle of random time quota selection, but not the exact forms of the functions $f_T(\cdot)$ and $q(\cdot)$.) This assumption was adopted for reasons of simplicity--other, possibly more realistic assumptions, such as allowing the gaming applicant a Bayesian prior distribution over the functions $f_T(\cdot)$ and $q(\cdot)$, would have complicated the analysis a great deal.

The final and most questionable assumption involves the gamer's basis for making inferences about the state of the system. I have permitted the gaming applicant to observe the outcome of exactly one randomly chosen call from the Poisson process, after which the gamer must calculate the optimal time to apply to the program, given the observed outcome (rejection or acceptance). The assumption that the call is randomly chosen from the stream of all calls is not entirely realistic. A more plausible scenario might be to assume that the gaming applicant enters the system at a random time, and then observes the next call to occur. However, the exact form of the assumption has little influence on the final results, so I have chosen the assumption for which the analysis is the simplest.

The limitation that only one call may be observed, on the other hand, is in fact quite restrictive compared to many plausible scenarios in which clients or their advocates may have access to a virtually complete history of calls. However, due to the previous assumption about complete knowledge of $f_T(\cdot)$ and $q(\cdot)$, some restriction on the extent of possible observation is necessary; otherwise, for many reasonable choices of $f_T(\cdot)$ and $q(\cdot)$, the probability of rejection can be made arbitrarily small by continuing to observe until some specified sequence of events has occurred. For example, if all time periods have merely non-zero quotas, then waiting until an arbitrarily long time has elapsed with no calls will bring the probability of rejection arbitrarily close to zero; if all periods have quotas of at least two, the probability of rejection immediately after a rejection/acceptance sequence will be exactly zero.

A more realistic way of imposing the needed restriction on the extent of permitted observation might be to limit the amount of time during which the favored applicant may observe, by the end of which time an application must be placed. In this case, the gamer's strategy would be given by the solution of a continuous-time, probabilistic dynamic program--to determine, after each observed call, not only the optimal time at which to call given the observed outcome, but also whether it is worthwhile to postpone calling in the hope of gaining another observation through an arrival of the Poisson process.

Under the assumption of only one observed call, the gamer would be interested in determing $P_{R|R}(t)$ and $P_{R|A}(t)$--the probabilities that a call placed t time units after the observed call would be rejected, given that the observed outcome was a rejection or an acceptance, respectively. Possible measures of vulnerability to selection bias would then be given by

$$\frac{\inf_{t \geq 0} P_{R|R}(t)}{P_R} \text{ and } \frac{\inf_{t \geq 0} P_{R|A}(t)}{P_R} ,$$

where P_R is the overall probability of rejection for a randomly chosen call from the Poisson process. In addition, it would be desirable for the functions $P_{R|R}(t)$ and $P_{R|A}(t)$ to be as close as possible to the constant value P_R, so that less optimal attempts at influencing selection probabilities would also have little effect.

3. Analysis of the General Case

Although one can derive general renewal formulas for the probabilities $P_{R|R}(t)$ and $P_{R|A}(t)$, it is not possible to obtain anything even remotely resembling closed-form results for general $f_T(\cdot)$ and $q(\cdot)$. Therefore, rather than presenting the long and cumbersome derivations for the probabilities of interest, which involve infinite sums of convolutions, I will instead present a simpler derivation, selected because it clearly illustrates some of the difficulties involved--P_R, the probability that a randomly chosen call will be rejected. (In practice, P_R will usually be a specified decision variable of the system, with $f_T(\cdot)$ and $q(\cdot)$ being chosen to yield the desired value of P_R; however, the analysis below will still hold.)

$$P_R = P\left\{\text{a randomly chosen call will be rejected}\right\} =$$

$$\frac{E[\text{number of rejections in a time period}]}{E[\text{total number of calls in a time period}]} =$$

$$\frac{\int_0^\infty f_T(t) \sum_{m=q(t)+1}^{\infty}(m-q(t)) \frac{(\lambda t)^m e^{-\lambda t}}{m!}\, dt}{\int_0^\infty f_T(t) \sum_{m=0}^{\infty} m \frac{(\lambda t)^m e^{-\lambda t}}{m!}\, dt} =$$

$$\frac{\int_0^\infty f_T(t) \sum_{m=q(t)+1}^{\infty}(m-q(t)) \frac{(\lambda t)^m e^{-\lambda t}}{m!}\, dt}{\lambda E[T]}$$

Unfortunately, however, it is not possible to simplify this formula and express the desired probability in closed form. The fundamental difficulty is that, at this level of generality in $f_T(\cdot)$ and $q(\cdot)$, the system has too much "memory." The number of acceptances so far in the current time period, when taken together with the amount of time already elapsed, provides a great deal of information about the time remaining in the period. To put it another way, noting that the beginnings of new time periods constitute the renewals of the system, the difficulty is that the distribution function for the time until the next renewal, starting with the system in some arbitrary state, is too complicated to be analytically tractable.

Therefore, the aim of systematically exploring the relationship between vulnerability to selection bias and the form of the function $f_T(\cdot)$ can not be met in the way we had originally intended. There are several alternative approaches which are still of interest, however:

1) Making the necessary simplifying assumptions to "Markovize" the system, so that the information available at any given time can be adequately represented by one of a discrete of states.

2)* Attempting to derive useful approximations for particular choices of $f_T(\cdot)$ and $q(\cdot)$.

3)* Investigating the behavior of the system numerically, again for particular choices of $f_T(\cdot)$ and $q(\cdot)$. (Note that since the expressions for $P_{R|R}(t)$ and $P_{R|A}(t)$ involve infinite sums of convolutions, this approach is not necessarily straightforward. The work of Kielson and Nunn, on numerical convolution via Laguerre transforms, might be useful here.[16])

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*) Under items 2) and 3), it would be particularly desirable to concentrate on choices of $f_T(\cdot)$ and $q(\cdot)$ which would be appropriate in actual application of the method. The most reasonable choices would be to let $q(\cdot)$ be roughly linear, and to experiment with various discrete distributions for the time period lengths, with the aim of finding a distribution for which $P_{R|R}(t)$ and $P_{R|A}(t)$ were relatively close to the constant value P_R.

In this paper I have chosen to Markovize. However, I believe that the other approaches mentioned are also quite promising, and am interested in pursuing one or more of these in the future.

The assumptions necessary to Markovize the system are as follows. First, the distribution for the lengths of time periods must be without memory. Note that the geometric distribution is <u>not</u> memoryless in this context, since time is not discrete--the arrival process takes place continuously over time. Therefore the basic choice for $f_T(\cdot)$ is exponential. By viewing each time period as being made up of a fixed number of exponential stages, the Markov analysis presented here can also be extended to permit Erlang-distributed and other, more general time periods; this extension will be discussed further in a later section of this paper.

The second, and more limiting required assumption is that the quota q(t) must be a <u>constant</u>, independent of t. To illustrate the need for this assumption, consider the following example. Suppose that q(t) were not constant, but instead were roughly proportional to t--say, q(t) = [2t] (rounded down), so that a time period five units in length would have a quota of ten acceptances. Then, if during the first two units of a time period we had already observed ten acceptances, the period would have to last at least another three time units, since its quota must have been at least ten. Thus, a system with proportional or other varying quotas is not without memory--the number of acceptances by a given time in the current period can contain a great deal of information on the time remaining in that period.

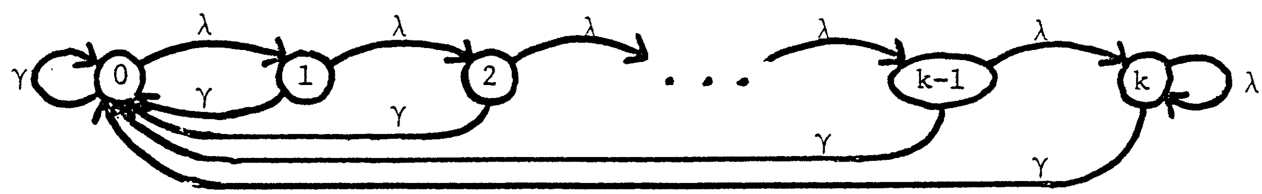
At first glance, it may appear that these Markovizing assumptions--

particularly with respect to constant quotas--are so restrictive as to render the resulting system trivial and uninteresting. This is in fact not the case; the analysis of the simplified system still yields fairly interesting results, and may give some feeling for the behavior of random time quota selection under more general assumptions.

The limitations that do exist are more in the realm of practical application. The use of constant quotas would greatly diminish the persuasive advantages of this method over randomization, since the notion of limited capacity would no longer be emphasized quite so explicitly; also, the use of continuous-length time periods might be difficult to implement in some situations. These objections confirm the potential value of the alternative approaches discussed earlier in this section.

4. Analysis of the Markovized System

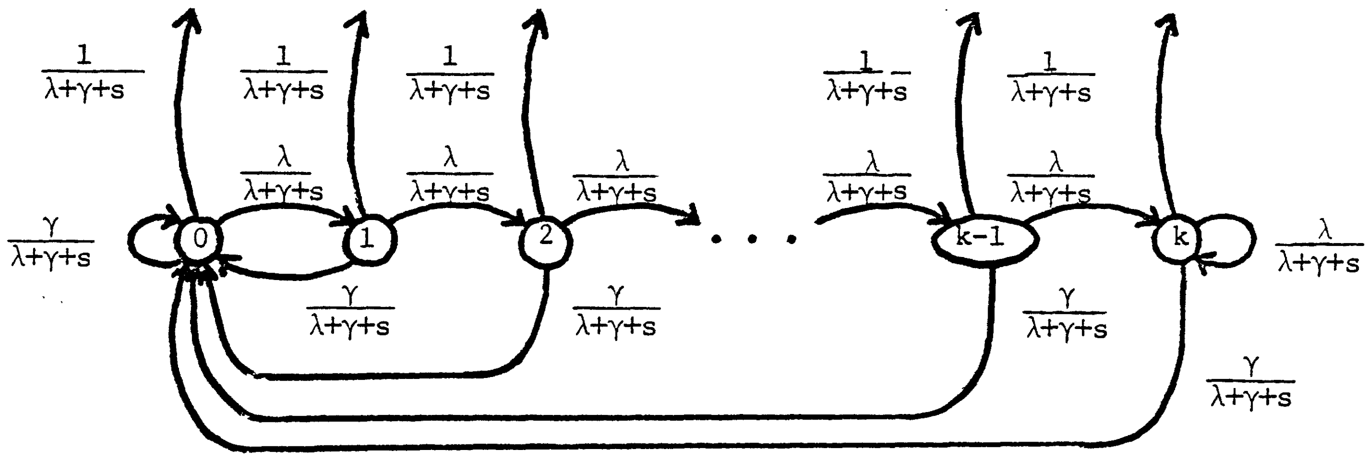
To proceed with the analysis of the Markovized system, let the time periods be exponentially distributed with mean length $1/\gamma$, let the quota be fixed at k acceptances per time period, and define the state of the system to be the total number of acceptances which have occurred so far in the current period. We can then draw the following simple state transition diagram:



There are two types of events represented here. The start of a new time period occurs with rate γ, and returns the system to state 0, independent of its previous state. The arrival of a new applicant to the program occurs with rate λ, and increments the state of the system, unless there have already been k acceptances in the current period, in which case the new applicant is rejected and the state of the system remains unchanged.

This state transition diagram can then be converted to a flow graph, using either semi-Markov or continuous-time Markov formulations, by the appropriate relabeling of the arcs and the inclusion of "tap-outs."[17]

Here I have chosen the semi-Markov formulation. Although that choice was not necessary, since all events occur exponentially, it permits a clear representation of the self-transitions at states 0 and k.



Defining our notation, let:

--p_{ij} be the probability that the next event will result in a transition (or self-transition) to state j, given that the system is currently in state i;

--$w_i(\cdot)$ be the probability density function of the waiting time until the next transition, given that the system is currently in state i; and

--$h_{ij}(\cdot)$ be the probability density function of the holding time for a transition from i to j (i.e., the waiting time in state i, <u>given</u> that the next transition is to state j).

Then we have, by the properties of the exponential distribution, that:

1)

$$P = \begin{bmatrix} \frac{\gamma}{\lambda+\gamma} & \frac{\lambda}{\lambda+\gamma} & 0 & \cdot & \cdot & \cdot & 0 \\ \frac{\gamma}{\lambda+\gamma} & 0 & \frac{\lambda}{\lambda+\gamma} & \cdot & \cdot & \cdot & 0 \\ \vdots & \vdots & \vdots & & & & \\ \frac{\gamma}{\lambda+\gamma} & 0 & 0 & \cdot & \cdot & \cdot & \frac{\lambda}{\lambda+\gamma} \\ \frac{\gamma}{\lambda+\gamma} & 0 & 0 & \cdot & \cdot & \cdot & \frac{\lambda}{\lambda+\gamma} \end{bmatrix}$$

(i.e., $p_{i0} = \frac{\gamma}{\lambda+\gamma}\ \forall_i$, $p_{i,i+1} = \frac{\lambda}{\lambda+\gamma}\ \forall\ i<k$, $p_{kk} = \frac{\lambda}{\lambda+\gamma}$, and $p_{ij} = 0$ otherwise). All events occur exponentially, so of the two types, the probability that the next to occur will be an arrival is $\frac{\lambda}{\lambda+\gamma}$; correspondingly, the probability that the next event will be the start of a new time period is $\frac{\gamma}{\lambda+\gamma}$.

2) $w_i(t) = (\lambda+\gamma)e^{-(\lambda+\gamma)t}\ \forall i$. The waiting time in state i until the next transition is the time until <u>either</u> an arrival <u>or</u> the start of a new time period, whichever occurs first; the distribution for the minimum of two exponential random variables is itself exponential, with mean rate equal to the sum of the original rates.

3) $h_{ij}(t) = (\lambda+\gamma)e^{-(\lambda+\gamma)t}\ \forall i,j$. Since transitions occur exponentially, the distribution for the holding time until the next transition from state i, <u>given</u> that it will be to state j, is the same as for the <u>unconditional</u> waiting time in state i.

Thus, each arc in the above flow graph has been labeled with the expression $p_{ij}h_{ij}{}^e(s)$, for appropriate values of i and j, where $h_{ij}{}^e(s)$ denotes the Laplace transform of $h_{ij}(t)$. Similarly, the tap-out from state i has been labeled with ${}^{cc}w_i{}^e(s)$, where ${}^{cc}w_i(t)$ is the probability that the waiting time in state i is greater than t (i.e., the complementary cumulative of $w_i(t)$).

The next step is to find the functions $\phi_{ij}(\cdot)$, where $\phi_{ij}(t)$ is the probability that the system is in state j t time units from now, given that it is currently in state i. To do this, we can either apply the techniques of flow graph analysis directly, to find $\phi_{ij}{}^e(s)$ for each pair of i and j, or we can use the matrix formula

$$\Phi^e(s) = [I-P \,\square\, H^e(s)]^{-1}\ {}^{cc}W^e(s),$$

where ${}^{cc}W^e(s)$ is a diagonal matrix with on-diagonal elements equal to ${}^{cc}w_i{}^e(s)$, $H^e(s) = (h_{ij}{}^e(s))$, and $\square$ represents the term-by-term matrix product (i.e., if $A=(a_{ij})$ and $B=(b_{ij})$, then $A \square B = (a_{ij}b_{ij})$).

The resulting matrix $\Phi^e(s)$ has the following form:

i \ j	$0 \le j < k$	$j=k$
$i=0$	$\frac{\gamma+s}{s(\lambda+\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^j$	$\frac{1}{s} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^k$
$1 \le i \le k$	$(i > j)$: $\frac{\gamma}{s(\lambda+\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^j$ $(i \le j)$: $\frac{\gamma}{s(\lambda+\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^j + \frac{1}{\lambda+\gamma+s} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^{j-i}$	$\frac{\gamma}{s(\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^k + \frac{1}{\gamma+s} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^{k-i}$

I will not invert these transforms here. The results are somewhat messy (the most complicated expression--for $\phi_{ik}(t)$, $i \geq 1$--involves the cumulative distribution function for the sum of an $\text{Erlang}_k(\lambda+\gamma)$ random variable and an exponential(λ) random variable), and would not be highly meaningful, since we are not interested in the functions $\phi_{ij}(t)$ for their own sake in any case. Instead, using the expressions for $\phi_{ij}^{e}(s)$, I will derive expressions for $P_{R|R}(t)$ and $P_{R|A}(t)$, the functions that our gaming applicant was originally interested in.

First we will need to find the steady-state probabilities, $\pi(j)$:

$$\pi(j) = s\phi_{ij}^{e}(s)\Big|_{s=0} \quad \forall i$$

$$= \begin{cases} \frac{\gamma}{\lambda+\gamma}\left(\frac{\lambda}{\lambda+\gamma}\right)^{j}, & 0 \leq j \leq k-1 \\ \left(\frac{\lambda}{\lambda+\gamma}\right)^{k}, & j=k \end{cases}$$

(Because the arrival process is Poisson, the value $\pi(j)$ is not only the probability of being in state j at a <u>random time</u>, but also the probability that a <u>randomly chosen</u> call from the process arrives while the system is in state j.)

$P_{R|R}^{e}(s)$ is now relatively simple to compute, since the observed call will be rejected only if it occurs while the system is in state k, and the state of the system therefore will not change as a result of the call:

$$P_{R|R}^{e}(s) = \frac{\pi(k)\phi_{kk}^{e}(s)}{\pi(k)} = \frac{\gamma}{s(\gamma+s)}\left(\frac{\lambda}{\lambda+\gamma+s}\right)^{k} + \frac{1}{\gamma+s}$$

Inverting we get:

$$P_{R|R}(t) = e^{-\gamma t} + \int_0^t \frac{\lambda^k v^{k-1}}{(k-1)!} e^{-\lambda v}(e^{-\gamma v} - e^{-\gamma t})dv$$

$P_{R|A}{}^e(s)$ is somewhat more complicated, since we must take into account the facts that the observed call might arrive while the system is in any of states 0,1,...,k-1, and that its acceptance would increment the state of the system:

$$P_{R|A}{}^e(s) = \frac{\sum_{i=0}^{k-1} \pi(i)\phi_{i+1,k}{}^e(s)}{\sum_{i=0}^{k-1} \pi(i)}$$

$$= \frac{\sum_{i=0}^{k-1} \left[\frac{\gamma}{\lambda+\gamma} \left(\frac{\lambda}{\lambda+\gamma}\right)^i\right] \left[\frac{\gamma}{s(\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^k\right]}{\sum_{i=0}^{k-1} \frac{\gamma}{\lambda+\gamma} \left(\frac{\lambda}{\lambda+\gamma}\right)^i}$$

$$+ \frac{\sum_{i=0}^{k-1}\left[\frac{\gamma}{\lambda+\gamma} \left(\frac{\lambda}{\lambda+\gamma}\right)^i\right] \left[\frac{1}{\gamma+s} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^{k-i-1}\right]}{\sum_{i=0}^{k-1} \frac{\gamma}{\lambda+\gamma} \left(\frac{\lambda}{\lambda+\gamma}\right)^i}$$

$$= \frac{\gamma}{s(\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^k + \frac{1}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k}\left[-\frac{\gamma}{s(\gamma+s)} \left(\frac{\lambda}{\lambda+\gamma+s}\right)^{k-1}\right.$$

$$\left. + \frac{1}{\gamma+s} \frac{\gamma}{\lambda+\gamma} \left(\frac{\lambda}{\lambda+\gamma}\right)^{k-1} + \frac{1}{s} \frac{\gamma}{\gamma+s} \left(\frac{\lambda}{\lambda+\gamma}\right)^{k-1}\right]$$

Inverting gives

$$(k \geq 2): \quad \int_0^t \frac{\lambda^k v^{k-1}}{(k-1)!} e^{-\lambda v}(e^{-\gamma v}-e^{-\gamma t})\,dv + \frac{(\frac{\lambda}{\lambda+\gamma})^{k-1}}{1-(\frac{\lambda}{\lambda+\gamma})^k}\left[1 - \frac{\lambda}{\lambda+\gamma}e^{-\gamma t}\right]$$

$$- \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k} \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}(e^{-\gamma v}-e^{-\gamma t})\,dv;$$

$$(k = 1): \quad \frac{\lambda}{\lambda+\gamma} + \frac{\gamma}{\lambda+\gamma} e^{-(\lambda+\gamma)t}$$

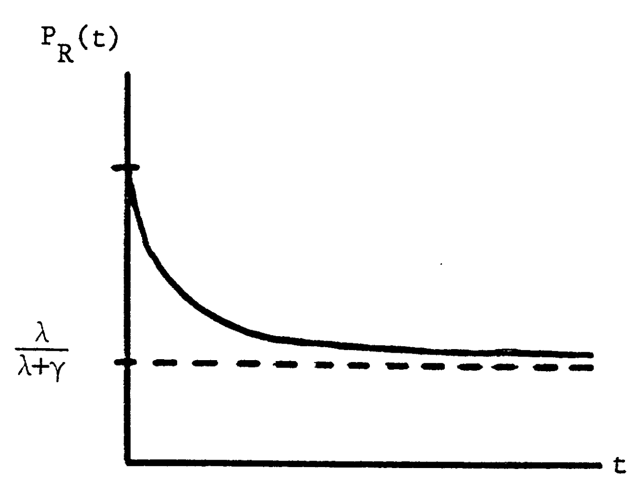
5. Interpretation of Results

Since the expressions for $P_{R|R}(t)$ and $P_{R|A}(t)$ are quite unwieldy, I will also present a variety of simpler results which help to give some qualitative understanding of the system's behavior. First, it is instructive to look at the expressions for the special case k=1 (a quota of one acceptance per time period), since they are then simple exponential functions, and the characteristics of the system are therefore particularly clear in this case.

When k=1, $P_{R|R}(t)$ and $P_{R|A}(t)$ are identical, as indeed we might have expected. With only one acceptance per time period, an observed call--whether accepted or rejected--can communicate only that the system will be in the rejection state immediately after the observation. In other words, the only information available about the state of the system through observation of calls is the knowledge that a call actually occurred at some given time. In keeping with this reasoning, evaluating either $P_{R|R}(t)$ or $P_{R|A}(t)$ at k=1 gives

$$P_R(t) \equiv P_{R|R}(t)\Big|_{k=1} = P_{R|A}(t)\Big|_{k=1} = \frac{\lambda}{\lambda+\gamma} + \frac{\gamma}{\lambda+\gamma}\, e^{-(\lambda+\gamma)t}$$

Graphing this function, we get:



Although it is not as easy to qualitatively describe the behavior of $P_{R|R}(t)$ and $P_{R|A}(t)$ when k=2, some observations can be made about the shapes of these functions. For example, $P_{R|R}(t)$ is monotonically decreasing in t, as can be shown by differentiating:

$$\frac{d}{dt} P_{R|R}(t) = -\gamma e^{-\gamma t}\left[1-\int_0^t \frac{\lambda^k v^{k-1}}{(k-1)!} e^{-\lambda v}\, dv\right] =$$

$$-\gamma e^{-\gamma t} \int_t^\infty \frac{\lambda^k v^{k-1}}{(k-1)!} e^{-\lambda v}\, dv < 0 \;\forall\; t$$

We are <u>certain</u> to be rejected if a new time period has <u>not</u> yet started since the observed rejection, but have a <u>smaller</u> probability of rejection if the system <u>is</u> in a new period, and the probability that the system is still in the same time period as the observed rejection obviously decreases over time.

$P_{R|A}(t)$, on the other hand, is at least initially increasing for $k \geq 2$:

$$\frac{d}{dt} P_{R|A}(t)\Big|_{t=0} = \gamma \frac{\left(\frac{\lambda}{\lambda+\gamma}\right)^k}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k} > 0$$

When $k \geq 2$ we can also show that $P_{R|A}(t)$ is always strictly less than $P_{R|R}(t)$:

$$P_{R|A}(t) - P_{R|R}(t) = \frac{\left(\frac{\lambda}{\lambda+\gamma}\right)^{k-1}}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k}\left[1 - \frac{\lambda}{\lambda+\gamma} e^{-\gamma t}\right]$$

$$- \frac{1}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k} \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}(e^{-\gamma v}-e^{-\gamma t})\,dv - e^{-\gamma t}$$

$$= \frac{(\frac{\lambda}{\lambda+\gamma})^{k-1}}{1-(\frac{\lambda}{\lambda+\gamma})^k} - e^{-\gamma t}\left[\frac{(\frac{\lambda}{\lambda+\gamma})^k}{1-(\frac{\lambda}{\lambda+\gamma})^k} + 1\right]$$

$$- \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k} \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}(e^{-\gamma v}-e^{-\gamma t})dv$$

$$= \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k}\left[(\frac{\lambda}{\lambda+\gamma})^{k-1} - \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-(\lambda+\gamma)v}dv\right]$$

$$- \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k} e^{-\gamma t}\left[1 - \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}dv\right]$$

$$= \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k}\left[\int_t^\infty \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-(\lambda+\gamma)v} dv - \int_t^\infty \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}e^{-\gamma t}dv\right]$$

Since $e^{-\gamma t} > e^{-\gamma v}$ when $t < v$, this gives us the inequality

$$P_{R|A}(t) - P_{R|R}(t) = \frac{1}{1-(\frac{\lambda}{\lambda+\gamma})^k} \int_t^\infty \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v}(e^{-\gamma v}-e^{-\gamma t})dv < 0$$

The intuitive justification for this result is as follows. Under the assumption of independently distributed time period lengths, the outcome of an observed call contains no information about the probability of rejection once a new time period has started. However, if the system is still in the period during which the observation took place, its outcome does make a difference: calls are sure to be rejected after an observed rejection, but have some probability of acceptance after an observed acceptance (if $k \geq 2$).

Further intuition can be gained by looking at $P_{R|R}(t)$ and $P_{R|A}(t)$ at $t=0$ (actually $t=0^+$, assuming our gaming applicant calls immediately after the observed call), and in the limit as $t \to \infty$ (assuming an arbitrarily long delay after the observed call). We would expect that $P_{R|R}(0)$, the probability of rejection immediately after an observed rejection, would equal one, since it is not possible for a new period to have begun in between the two calls, and in fact evaluating $P_{R|R}(t)|_{t=0}$ gives the desired result. $P_{R|A}(0)$ is given by

$$\frac{\frac{\gamma}{\lambda+\gamma}\left(\frac{\lambda}{\lambda+\gamma}\right)^{k-1}}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^{k}} = \frac{\pi(k-1)}{1-\pi(k)},$$

the probability that our observed call was the last acceptance in its time period, given that it was accepted at all.

After an arbitrarily long time, we would expect the observed outcome to have become uninformative, since the system will have returned to steady state. Thus we have that a call placed t time units after an observed call, in the limit as $t \to \infty$, has the same rejection probability as if it were randomly timed:

$$\lim_{t \to \infty} P_{R|R}(t) = \lim_{t \to \infty} P_{R|A}(t) = \left(\frac{\lambda}{\lambda+\gamma}\right)^k = \pi(k)$$

Finally, we can attempt to calculate our measures of vulnerability to selection bias,

$$\frac{\inf_{t \geq 0} P_{R|R}(t)}{P_R} \quad \text{and} \quad \frac{\inf_{t \geq 0} P_{R|A}(t)}{P_R}$$

Since $P_{R|R}(t)$ is monotonically decreasing,

$$\inf_{t \geq 0} P_{R|R}(t) = \lim_{t \to \infty} P_{R|R}(t) = \left(\frac{\lambda}{\lambda+\gamma}\right)^k,$$

which is also equal to the probability that a randomly chosen naive applicant is rejected, P_R. I was not able to determine

$$\inf_{t \geq 0} P_{R|A}(t) \text{ for } k \geq 2,$$

as it is not possible to analytically find the zeroes of

$$\frac{d}{dt} P_{R|A}(t) = \gamma e^{-\gamma t} \int_0^t \frac{\lambda^k v^{k-1}}{(k-1)!} e^{-\lambda v} dv$$

$$+ \frac{\left(\frac{\lambda}{\lambda+\gamma}\right)^k}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k} \gamma e^{-\gamma t} - \frac{1}{1-\left(\frac{\lambda}{\lambda+\gamma}\right)^k} \gamma e^{-\gamma t} \int_0^t \frac{\lambda^{k-1} v^{k-2}}{(k-2)!} e^{-\lambda v} dv$$

However, the needed computations could easily be performed numerically for specific values of λ, γ, and k.

It is important to note the somewhat peculiar fact that, when k=1 and/or when the observed call is a rejection, our gaming applicant can do no better than P_R, the overall probability of rejection for naive applicants. This result is not as paradoxical as it at first appears, since in both of these situations the information provided by the observed call consists only of the knowledge that the system has entered the rejection state. However, the fact that situations exist in which the gaming applicant is completely incapable of reducing the probability of rejection from the program points up the unrealistically severe restrictions we have imposed on possible gaming behavior. It is important to investigate the effect on selection bias of various means of relaxing those restrictions.

One possible relaxation is to permit the gamer to apply to the program before the arrival of the observation call. This is desirable from the gamer's perspective, since if no calls have yet arrived a long while after the gamer has entered the system, the probability of rejection for an application placed at that time will be very low. If s is the time elapsed with no arrivals since the gaming applicant has entered the system, then the probability of rejection for an application placed at time s is given by the probability that the gamer entered the system in state k, times the probability that a new time period has not yet begun:

$$\left[\left(\frac{\lambda}{\lambda+\gamma}\right)^k\right]\left[e^{-\gamma t}\right]$$

Under this model, the gamer weighs the chance of a decreased probability of rejection by delaying application, against the risk of an increased rejection probability if the observation call arrives before application to the program has been made. The optimal strategy is to apply t^* time units after arrival to the system if an observation call has not yet occurred, and an arbitrarily long time after the observation call if it occurs before t^*, where t^* is chosen to minimize

$$P\left\{\text{rejection}\right\} = \left(\frac{\lambda}{\lambda+\gamma}\right)^k e^{-\gamma t} \quad P\left\{\begin{matrix}\text{observation call does} \\ \text{not occur by } t\end{matrix}\right\}$$

$$+ \int_0^t \lambda e^{-\lambda s} \left[\pi(k) e^{-\gamma s}\right] \left[\inf_{u \geq 0} P_{R|R}(u)\right] ds$$

$$+ \int_0^t \lambda e^{-\lambda s} \left[1 - \pi(k) e^{-\gamma s}\right] \left[\inf_{u \geq 0} P_{R|A}(u)\right] ds$$

As an example, for k=1, this becomes

$$P\left\{\text{rejection}\right\} = \left(\frac{\lambda}{\lambda+\gamma}\right) \left[e^{-(\lambda+\gamma) t} + (1 - e^{-\gamma t})\right].$$

Differentiating with respect to t gives

$$\left(\frac{\lambda}{\lambda+\gamma}\right) \left[\lambda e^{-\lambda t} - (\lambda+\gamma) e^{-(\lambda+\gamma) t}\right] = 0 ,$$

implying that $t^* = -\frac{1}{\gamma} \ln \left(\frac{\lambda}{\lambda+\gamma}\right)$.

Another possible relaxation of the permitted gaming behavior, which might be a realistic model for many situations, is to assume that the gaming applicant is actually the favored client of some (gaming) client advocate, who is responsible for some amount λ_0 of the total arrival rate λ. Under this model, the gaming applicant arrives at the client advocate, and observes the outcome of the next applicant to arrive at that advocate.

Since the only possible arrivals from the time the gaming applicant enters the system to the time of the observation are from a Poisson process with rate $\lambda - \lambda_0$, the probability that the observation call will arrive with the system in state i is given by

$$\pi'(j) = \int_0^{\infty} (\lambda_0 e^{-\lambda_0 t}) \sum_{i=0}^{k} \pi(i) \; [\phi_{ij}(t)]_{\lambda-\lambda_0} \, dt$$

Then, assuming that the client advocate does not place any further calls until the optimally timed call on behalf of the favored applicant, the effective arrival rate between the observation call and the call on behalf of the gaming applicant is again only $\lambda-\lambda_0$. Thus, the probability of rejection t time units after the observed call, given that the observed call is rejected, will be $[\phi_{kk}(t)]_{\lambda-\lambda_0}$; if the observed call is accepted, the probability of rejection t time units later will be

$$\frac{\sum_{i=0}^{k-1} \pi'(i) \; [\phi_{i+1,k}(t)]_{\lambda-\lambda_0}}{\sum_{i=0}^{k-1} \pi'(i)}$$

The probability of the gaming applicant being rejected t time units after the observed call, both in the case when the observed call is rejected and when k=1, is given by

$$[\phi_{kk}(t)]_{\lambda-\lambda_0} = e^{-\gamma t} + \int_0^t \frac{(\lambda-\lambda_0)^k}{(k-1)!} v^{k-1} \; e^{-(\lambda-\lambda_0)v}(e^{-\gamma v}-e^{-\gamma t})dv.$$

This is minimized in the limit as $t \to \infty$, yielding an optimum rejection probability of

$$(\frac{\lambda-\lambda_0}{\lambda-\lambda_0+\gamma})^k,$$

compared to $(\frac{\lambda}{\lambda+\gamma})^k$ for naive applicants. Thus, the ratio

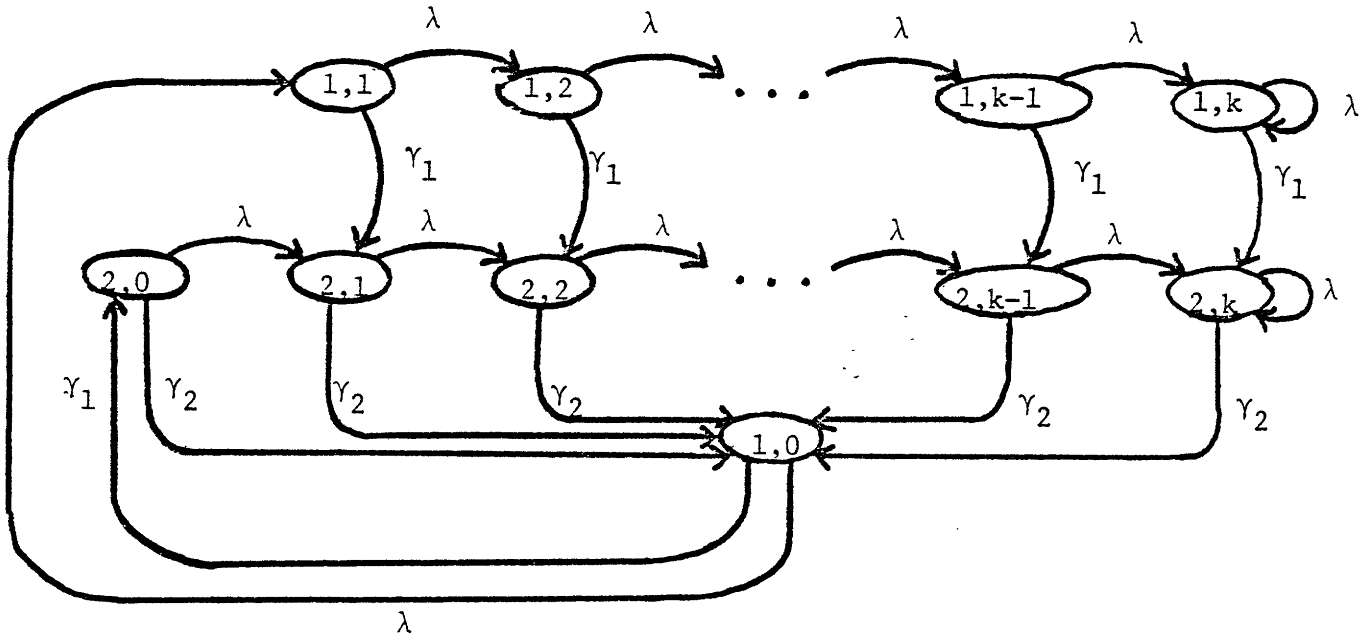
$$\frac{\inf_{t \geq 0} P \left\{ \begin{array}{l} \text{rejection at time t, given} \\ \text{a client advocate with} \\ \text{arrival rate } \lambda_0 \end{array} \right\}}{P_R} \doteq \left[\left(\frac{\lambda-\lambda_0}{\lambda-\lambda_0+\gamma}\right) \left(\frac{\lambda+\gamma}{\lambda}\right) \right]^k$$

will not be too much less than one when:

- λ_0 is small relative to λ;
- the average period length, $1/\gamma$, is small relative to λ; and
- k is small.

One additional extension--which is not related to our assumptions about gaming behavior, but can be performed fairly simply--is the generalization of the Markov analysis to allow Erlang and other, more general distributions for the lengths of time periods. This can be accomplished by viewing each period as being made up of a number of exponential stages, possibly with different parameters. (In fact, the number of stages does not even need to be fixed. Rather, at the end of stage i, one could continue to stage i+1 with probability p_i, or terminate the current time period with probability $1-p_i$, permitting the representation of an extremely wide range of continuous distributions.)

As an example, here is the state transition diagram for a system with two stages, having mean lengths $1/\gamma_1$ and $1/\gamma_2$:



The methods of analysis for this system would be parallel to those used in Section 4 of this paper, although the algebra would obviously be more complicated. I will present a sample computation here for illustrative purposes--P_R, the probability of rejection for a naive applicant.

First, examining the result for the simpler system,

$$P_R = \left(\frac{\lambda}{\lambda+\gamma}\right)^k,$$

we see that it is equal to the probability of having k arrivals before the start of a new time period. (The probability that one arrival will occur before a new period begins is given by $\frac{\lambda}{\lambda+\gamma}$, and given that one such event has occurred, the probability of the next one occurring remains unchanged.) By analogy, then, in our system we will have

$$P_R = P\left\{\text{k arrivals before the completion of the 1st stage}\right\}$$

$$+ \sum_{i=0}^{k-1} \Big[P\Big\{ \text{ i arrivals before the completion of the 1st stage} \Big\}$$

$$\times P\Big\{\text{completion of the 1st stage before the next arrival}\Big\}$$

$$\times P\Big\{\text{k-i arrivals before the completion of the 2nd stage}\Big\}\Big]$$

$$= \left(\frac{\lambda}{\lambda+\gamma_1}\right)^k + \sum_{i=0}^{k-1} \left(\frac{\lambda}{\lambda+\gamma_1}\right)^i \frac{\gamma_1}{\lambda+\gamma_1} \left(\frac{\lambda}{\lambda+\gamma_2}\right)^{k-i}$$

$$= \begin{cases} \dfrac{\gamma_1}{\gamma_1-\gamma_2} \left(\dfrac{\lambda}{\lambda+\gamma_2}\right)^k - \dfrac{\gamma_2}{\gamma_1-\gamma_2} \left(\dfrac{\lambda}{\lambda+\gamma_1}\right)^k, & \gamma_1 \neq \gamma_2 \\ \left(\dfrac{\lambda}{\lambda+\gamma}\right)^k \left[1 + k \dfrac{\lambda}{\lambda+\gamma}\right], & \gamma \equiv \gamma_1 = \gamma_2 \end{cases}$$

6. Summary and Conclusions

I set out in this paper to explore the vulnerability to selection bias of random time quota selection, as developed by the Vera Institute. In the course of pursuing that analysis, an increasing number of assumptions and restrictions became necessary, and it was not possible to investigate the method of random time quota selection in as much generality as was hoped. In this section, I will review the assumptions that were made, and present my views on which aspects of the analysis remain applicable in broader contexts. I will also mention once again several directions for further research which were discussed earlier in the paper.

The first set of assumptions was made prior to the investigation of the general case. Primary among these were:

1) that there was only one gaming applicant;
2) that the gamer had complete knowledge of the system characteristics $f_T(\cdot)$ and $q(\cdot)$; and
3) that only one call could be observed.

It is difficult to assess the impact of the first assumption on the magnitude of possible selection bias, although one can hope that it is not too great. If a number of applicants attempted to time their applications so as to influence their selection probabilities, this would be perceived by any one applicant as a non-Poisson process for arrivals (with arrivals not necessarily mutually independent). The effect might be comparable in size to that of a different distribution for time period lengths.

Assumptions 2) and 3) are obviously central to the analysis. For example, it was demonstrated that, given complete knowledge of system characteristics, failure to limit the extent of permitted observation can lead to extreme selection bias, with rejection probabilities often arbitrarily close to zero. In general, the assumptions chosen tend to counteract each other in their effects: the severe restriction on permitted observation was intended to offset the generosity in overall knowledge of the system.

Because of these two assumptions, the analysis that I have presented is clearly not an accurate model for most practical settings. However, the discussion above leads to a recommendation on how to reduce selection bias in general. In situations where clients or their advocates have access to a great deal of information about previous calls, it is especially important to provide only minimal information on the details of the selection method; similarly, in situations where there is a great deal of knowledge about the system in general, such as where the researchers deal directly with program applicants, an effort should be made to limit the possibilities for observing the outcomes of calls.

The next set of assumptions was made in order to Markovize the system. Certainly, few implementations of random time quota selection are likely to use the precise configuration we considered--constant quotas, and exponentially (or Erlang) distributed time period lengths. In general, I would conjecture that implementations similar to the memoryless system analyzed here would tend to minimize selection bias, precisely because of the Markov property: there is relatively little information to be gained through observation. However, this hypothesis should definitely be investigated further before being taken as a recommendation. It is especially important to understand the effects of proportional rather than constant quotas, since proportional quotas are virtually essential in winning approval for the method in many contexts.

Further insight into the effect of observation opportunities on selection bias is provided by one of the extensions considered in this paper. The result (for the Markovized system), for an advocate who is responsible for an amount λ_0 of the total arrival rate λ, is that the probability of rejection for a gaming applicant can be reduced by a factor of

$$\left[\left(\frac{\lambda-\lambda_0}{\lambda-\lambda_0+\gamma}\right)\left(\frac{\lambda+\gamma}{\lambda}\right)\right]^k$$

from the probability with more restricted information.

Here again, general recommendations can be made. Selection bias can be reduced by giving each advocate access to only a small proportion of all calls--for example, by having advocates work out of separate offices rather than as a group, where possible. Also, shorter time periods will tend to reduce selection bias, as might be expected, since the system will more closely approximate true randomization. Finally, while small quotas appear to minimize worst-case selection bias, they may also increase the overall variability in rejection probabilities among different times of application to the program: for k=1 the functions $P_{R|R}(t)$ and $P_{R|A}(t)$ are exponential, whereas for larger k they are "flatter" (similar to the cumulative distribution functions for Erlang random variables).

To review what I consider to be the most important recommendations to be followed in minimizing selection bias, the most obvious is that time periods should be chosen to permit the use of quite small quotas. The smaller the quotas are, the more closely the system mimics the behavior of true randomization. The other important guideline is to limit the extent of possible observation a great deal. If it is not possible to withhold information or outcomes (acceptance or rejection) from the client advocates, efforts should be made to limit observation opportunities in other ways--for example, by delaying the reporting of outcomes, by decentralizing client advocate operations so that each advocate observes only a small fraction of calls, etc.

In sum, the results of this paper, taken together with the experiences of the Vera Institute in successfully implementing random time quota selection, indicate that with care the method can be used as an alternative to random selection without incurring undue selection bias. However, further research would be extremely useful in clarifying the properties of the method. Perhaps the most promising avenue for research is the numerical solution of the general renewal equations discussed in Section 3, to permit investigation of the influence of proportional quotas. It would also be extremely worthwhile to develop and explore the variant of random time quota selection described earlier, in which calls would be accepted during a certain proportion of each time period, rather than according to fixed quotas.

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