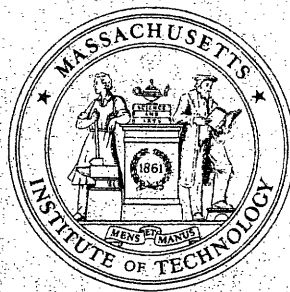


# OPERATIONS RESEARCH CENTER

working paper



**MASSACHUSETTS INSTITUTE  
OF TECHNOLOGY**

A SURVEY OF NETWORK DESIGN PROBLEMS

by

Richard T. Wong

OR 080-78

August 1978

Supported in part by the U.S. Department of Transportation under contract  
DOT-TSC-1058, Transportation Advanced Research Program (TARP).

## ABSTRACT

Network design problems arise in many different application areas such as air freight, highway traffic, and communication systems.

The intention of this survey is to present a coherent unified view of a number of papers in the network design literature. We discuss suggested solution procedures, computational experience, relations between various network models, and potential application areas. Promising topics of research for improving, solving, and extending the models reviewed in this survey are also indicated.

Titles of Figures

<u>Number</u>	<u>Title</u>	<u>Page</u>
1	Plant Location as an Arc Design Problem	9

## 1. Introduction

The selection of an optimal configuration or design of a network occurs in many different application contexts including transportation (airline, railroad, traffic, and mass transit), communication (telephone and computer networks), electric power systems, and oil and gas pipelines. For example, consider a traffic network whose nodes represent both origin and destination areas for the vehicular traffic of a city and also intersections in the road network. The arcs correspond to streets in the city, and the arc flows denote the amount of traffic traversing the streets. A typical network design problem would be to select a subset of the possible road improvements subject to a budget constraint. The design objective would be to minimize the total travel cost for all travelers in the city network.

In this survey, we will introduce a basic network design model which frequently occurs in the network literature. Although most real-world network design problems are more complicated than our general model, we believe that our basic framework embodies many of the most essential features of network design problems. Thus, any sophisticated design model will have to deal with the issues represented in our general framework.

We will discuss a number of network design papers in terms of this basic model. Although this general framework is applicable to many different problem domains, we will concentrate mainly on transportation network problems since most of the work concerning network design has focused on these applications. Our goal is to present a coherent unified view of these papers and their contribution to the network design literature. We will review suggested solution procedures, computational experience, relations between various network models, and potential application areas. We also indicate promising areas of research for improving, solving, and extending the models reviewed in this survey.

Previous survey work in the area of network design problems includes reports by MacKinnon [39], Schwartz [59], Stairs [62], and Steenbrink [64].

## 2. Problem Formulation

This section gives a general framework for the network design problems that will be discussed in this survey.

Our basic network design model has the following description: we have a set  $N$  of nodes, a set  $A$  of arcs, and between each pair  $(k, \ell) \in N \times N$  of nodes there is a required flow  $R_{k\ell}$  that must be routed through the network. Let  $f_{ij}^{k\ell}$  be the amount of required flow between nodes  $k$  and  $\ell$  on arc  $(i, j)$ .

For each node  $i \in N$  we can write the flow conservation equations:

$$\sum_{j \in N} f_{ji}^{k\ell} - \sum_{j \in N} f_{ij}^{k\ell} = \begin{cases} -R_{k\ell} & k=i \\ R_{k\ell} & \ell=i \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

$(k, \ell) \in N \times N.$

For each arc in the network we assume there is an initial given capacity  $u_{ij}$  and a set of possible capacity improvement levels  $L_{ij}$ . Thus we can write the following arc capacity constraints:

$$\sum_{(k, \ell) \in N \times N} f_{ij}^{k\ell} = \hat{f}_{ij} \leq u_{ij} + l_{ij} \quad (i, j) \in A \quad (2)$$

$$l_{ij} \in L_{ij}$$

For example, suppose all capacities are initially zero and an arc  $(i, j)$ 's capacity can either remain zero or be increased to a value  $K_{ij}$ . Then  $L_{ij} = \left\{ 0, K_{ij} \right\}$  and (2) can be represented by:

$$\sum_{(k,\ell) \in N \times N} f_{ij}^{k\ell} = \hat{f}_{ij} \leq 0 + K_{ij} y_{ij} \quad (i,j) \in A$$

$$y_{ij} = 0 \text{ or } 1$$

where  $y_{ij}$  is a 0-1 variable indicating the capacity level of arc  $(i,j)$ .

Our general framework includes two types of costs. The first kind, denoted by  $RC_{ij}(\hat{f}_{ij})$ , is the routing cost for arc  $(i,j)$  associated with satisfying the required flow constraints (1). In various applications the routing costs may correspond to travel time, risk of accidents or any other "costs" which vary with the amount of traffic on the arc.

The second type of cost, the construction cost for the capacity improvement of arc  $(i,j)$ , is denoted by  $CC_{ij}(\ell_{ij})$  and includes capital construction costs, maintenance fees, and any other costs that depend solely on the arc capacity level.

In general, our objective will be to minimize the total routing and construction costs. With the above information, we can state our general network design problem as:

$$\text{Minimize} \quad \sum_{(i,j) \in A} RC_{ij}(\hat{f}_{ij}) + CC_{ij}(\ell_{ij})$$

subject to: (1), (2) and any special problem constraints

$$f_{ij}^{k\ell} \geq 0 \quad (i,j) \in A$$

$$(k,\ell) \in N \times N.$$

We will further classify our network design problems according to their routing cost functions  $RC_{ij}(\hat{f}_{ij})$ . If, for a particular network design, all routing cost functions are linear and every arc capacity is either zero or infinite (we usually represent an "infinite" arc capacity value as some sufficiently large number such as the total amount of required

flow in the problem), then we will refer to the model as a network design problem without congestion costs. This terminology is chosen to reflect the fact that if an arc is present in the network (i.e., has nonzero capacity) then any amount of flow can be routed through it and the marginal cost for routing an additional unit of flow is always constant, independent of flow conditions in the network.

If a network design model has convex routing cost functions and/or some finite nonzero arc capacities, then we will refer to it as a network design problem with congestion costs. These congestion costs are reflected in the convex routing cost functions (i.e., increasing marginal costs) and/or the prohibition of additional flow through an arc after a certain limit has been reached.

In the following sections we describe and analyze a number of different problems in terms of this basic network design model.

### 3. Network Design Problems Without Congestion Costs

Network design problems without congestion costs often model underutilized systems such as a communications network where the amount of information transmitted is always below the capacity of a standard trunk. In this case, the arc capacity is effectively infinite. Another use for this type of system is to gain insight into more complicated networks with congestion costs by studying this simpler network model. Also network design problems without congestion costs can be used as sub-problems in a procedure for solving more complicated network design models.

The first network problem that we consider was formulated by Billheimer and Gray [7]. Initially all arcs have zero capacity. The arc



routing costs are linear functions of the total arc flow. The construction cost required to build an arc with "infinite" capacity is a fixed charge. The objective is to minimize the sum of routing and construction costs. (We will refer to this network design model as the "fixed charge design problem.")

Since all arc capacities can only take on discrete values (either zero or "infinity"), we can formulate the fixed charge design problem as the following mixed integer program:

$$\begin{aligned} \text{Minimize} \quad & \sum_{(i,j) \in A} d_{ij} \hat{f}_{ij} + c_{ij} y_{ij} \\ \text{subject to: (1)} \quad & \\ & f_{ij}^{kl} \leq R_{kl} \cdot y_{ij} \quad (i,j) \in A \\ & f_{ij}^{kl} \geq 0 \quad (k,l) \in N \times N \\ & y_{ij} = 0 \text{ or } 1. \end{aligned}$$

$y_{ij}$  indicates whether or not arc  $(i,j)$  is present in the network. Let  $d_{ij}$  be the cost of routing a unit of flow through arc  $(i,j)$  and  $c_{ij}$  be the cost of adding arc  $(i,j)$  to the network (i.e., setting its capacity to "infinity"). Note that the second constraint is a specialization of (2) to the fixed charge design model.

Note that the above network design formulation gives rise to large mixed integer programs. For example, a network design with 50 nodes and 200 possible directed arcs will be formulated with 12,500 rows, 10,000 continuous variables and 200 binary variables.

Magnanti and Wong [42] applied Benders' decomposition to the above formulation of the fixed charge design problem. They specify a technique for accelerating the convergence of Benders procedure. Their computational experience includes satisfactorily solving networks with 10 nodes and 45 arcs in about 60 seconds of IBM 370/168 computer time.

Since this problem is very complex, Billheimer and Gray propose a heuristic solution procedure. Each iteration of this procedure consists of either deleting or adding an arc to the network so that the total cost (routing and construction) is reduced. The iterations are continued until a local optimum is reached where no further addition or deletion of a single arc reduces the cost of the network configuration.

The heuristic procedure has been tested on a problem with 68 nodes and 476 arcs. The method reached a local optimum after about 3 minutes of computation time on an IBM 360/67 computer. It is difficult to judge the quality of the heuristic's solution since no satisfactory method is known for optimally solving problems of that size.

It is interesting to see the wide range of network models that are related to the fixed charge design problem. Many combinatorial network problems are special cases of it. If all arc construction costs are set to zero, then the fixed charge design model becomes a series of shortest path problems. If all arc routing costs are set to zero, the fixed charge design model becomes a Steiner tree problem on a graph (Steiner's problem) [16, 27]. The Steiner problem occurs because the required flows will necessitate that there be a path between every pair of nodes in some subset of the nodes in the network.

Since the fixed charge design problem contains the Steiner problem as a special case, we can be confident that it is very difficult to solve. Karp [33] has shown that the Steiner tree problem on a graph is NP-complete. This implies that the Steiner problem is as difficult to solve as such combinatorial problems as the traveling salesman problem [6], the maximum clique problem [28], and the 0-1 integer programming problem (see [33, 34] for a full discussion of the various NP-complete problems). In view of the lack of success in solving any of these problems on a large scale, it appears unlikely that there is an efficient algorithm for the Steiner problem or for the fixed charge design problem. In fact, the fixed charge design problem itself is NP-complete. (This result follows from the fact that the Steiner problem is a special case of fixed charge network design).

If the arc construction costs are all equal and totally dominate the routing costs (i.e., the optimal network design must be a tree), then the fixed charge design problem becomes the optimum communication spanning tree problem defined by Hu [31].

Another special case of Billheimer and Gray's problem is the fixed charge plant location problem [17, 12]. The plant location problem is normally associated with the placement of facilities on the nodes of a graph. The objective is to minimize the sum of the fixed charges for locating the various plants and the routing costs for servicing customers from the constructed plants. However, it is possible to convert the plant location problem to a network synthesis problem. This can be done in the following way: add a special node to the plant location network. This node will be the source of all the flow required by the customer nodes. Also, add a set of special arcs leading from the special node to each potential plant

site (see Figure 1). A special arc connecting the special node to a plant site has a construction cost equal to the fixed charge associated with opening the site. These special arcs will have no routing costs. Arcs connecting plant locations with customers have no construction costs. However, they will have a routing cost equal to the transportation cost from the plant location to the customer. So now the corresponding synthesis problem is to design the minimum total cost (construction plus routing cost) network so that all the flow requirements between the special node and the customers are satisfied. Thus, the fixed charge plant location problem is a special case of the fixed charge design problem.

Viewing the fixed charge plant location problem as a special case of the fixed charge design model gives us additional insight into the network synthesis problem. For instance, Billheimer and Gray describe some methods for partially characterizing the optimal network configuration. These techniques can be shown to be generalizations of procedures given by Efroymson and Ray [17] for characterizing the optimal set of sites in the plant location problem.

By using a similar transformation we can show that many other different facility location problems are special cases of various network design problems. For example, if we have a capacitated plant location problem, the node capacity constraint can be represented by a capacity constraint on one of the "special" arcs added to the network. Since there has been so much work done in the area of facility location problems (see [19, 12]), it may be possible to generalize some of these other techniques in order to apply them to network design problems. The rules given by Billheimer and Gray and Efroymson and Ray are one example of such a generalization.

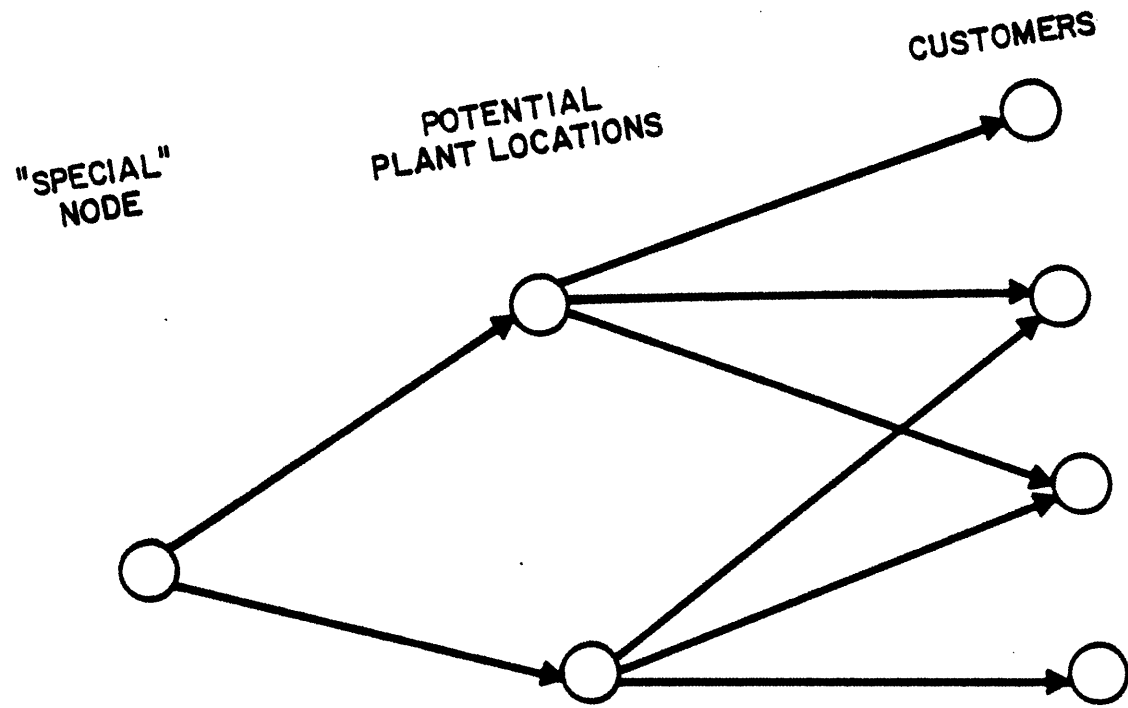


FIGURE 1 PLANT LOCATION AS AN ARC DESIGN PROBLEM

Scott [60, 61] has introduced another network synthesis problem, called the "optimal network" problem, that is closely related to the fixed charge design problem. The arc routing costs in this problem are all linear functions of the total flow. Arc capacities, which are all initially zero, can be raised to infinity. The objective is to minimize total routing cost subject to the usual capacity and flow routing constraints and the added constraint that the total construction costs cannot exceed a given budget.

The optimal network problem can be formulated as the following mixed integer program:

$$\text{Minimize} \quad \sum_{(i,j) \in A} d_{ij} \hat{f}_{ij}$$

$$\text{subject to:} \quad (1)$$

$$f_{ij}^{kl} \leq R_{kl} y_{ij} \quad (i,j) \in A$$

$$\sum_{(i,j) \in A} c_{ij} y_{ij} \leq \text{BUDGET} \quad (k,l) \in N \times N$$

$$f_{ij}^{kl} \geq 0$$

$$y_{ij} = 0 \text{ or } 1.$$

All variables and constants have the same interpretation as in the formulation of the fixed charge design problem.

Many researchers have considered this problem since its solution could be useful to the design of various transportation (highway, rail or air) systems. As noted by Dionne and Florian [15], since these systems usually

have many more operating constraints, "the justification for studying this problem is that its solution may be used as a measuring standard for the efficiency of proposed designs."

Boyce et al. [8] utilized a branch and bound algorithm to solve the optimal network problem. They were able to solve problems with 10 nodes and 45 arcs in 3 to 400 seconds of IBM 360/75 computer time depending on the value of the construction budget. Hoang [30] presented another branch and bound procedure that has been modified and improved by Dionne and Florian [15]. Their procedure produced computation times that were comparable to the Boyce et al. results. Dionne [14] has shown that the computation time of the Dionne and Florian procedure increases exponentially with decreasing construction budget. It is believed that the algorithm of Boyce et al. should behave in a similar manner. Geoffrion [23] has presented another branch and bound procedure that is based on Lagrangian relaxation techniques.

In order to address large-scale optimal network problems, several researchers have suggested using heuristic procedures. Scott [61] and Dionne and Florian [15] proposed heuristic algorithms that are closely related to the Billheimer and Gray procedure for the fixed charge design problem. Dionne and Florian have solved test problems containing up to 29 nodes and 54 arcs. The computational results were very promising with the average error relative to the optimal solution less than one percent. Computation times ranged from .1 to 12 seconds on the CDC Cyber 74 computer. However, Wong [69] has presented analyses that indicate the maximum error for such heuristics could be very large. Further computational tests should be performed in order to resolve this issue.

It is unlikely that there exists an efficient optimal algorithm for the optimal network problem since Johnson, Lenstra, and Rinooy Kan [32] have shown that the optimal network problem is NP-complete.

Another group of network design problems without congestion costs concerns network improvement where we start with an initial feasible network and then attach additional arcs. As in the case of the optimal network problem, there are the usual capacity and flow routing constraints and also a construction budget for the added arcs.

Ridley [55] suggested a network-based branch and bound approach for these problems. Stairs [62] indicated that Ridley's method has been used to solve problems containing up to 12 nodes. Goldman and Nemhauser [25] consider a special case of the network improvement problem where the objective is to improve the shortest path between a single pair of nodes. They show how to transform the problem into a shortest route problem on an expanded network. Wollmer [68] and Ridley [54] give efficient procedures for solving special cases of the shortest path improvement problem. However, these techniques are just special cases of Goldman and Nemhauser's procedure.

Stairs [62] presented a network improvement problem that is related to Billheimer and Gray's network synthesis problem. She described an interactive computer solution procedure which has been successfully applied to a test problem containing 35 nodes and 10 possible arcs that could be improved.

Note that our network improvement models are all special cases of the network synthesis models presented earlier. So it should be possible to adapt the previously described network synthesis techniques to network improvement applications.



#### 4. Network Design Problems with Congestion Costs

A more complex type of network design problem incorporates congestion costs for the routing of the network flows. These congestion costs can be represented by i) convex flow routing costs that could reflect such effects as highway traffic congestion or communication network queuing delays; ii) finite arc capacities that could represent physical, environmental or political limits on the total traffic that can pass through an arc.

Some of the models described here have been used to help design traffic network, rail network and communication network systems. All of the models that we will discuss are network improvement problems. Unless specified otherwise, we assume that the initial arc capacities constitute a feasible network design solution.

The first type of network improvement problem that we consider is similar to the uncongested design problems of the previous section. In addition to the usual capacity and flow routing constraints that must be satisfied, we must select the arc capacities from a discrete set of values. Thus, the problem is essentially a combinatorial one as was the case for the uncongested design problems. Roberts and Funk [56], Carter and Stowers [10], and Hershdorfer [29] described work in this area. Hershdorfer utilized a branch and bound procedure with networks containing up to 12 nodes.

Agarwal [1] considered a different kind of network improvement problem where the possible capacity of an arc  $(i,j)$  ranges continuously between zero and some upper bound  $K_{ij}$ . Construction costs are linear functions of the arc capacity increase. Routing costs are convex piecewise linear functions of the flow. The objective is to minimize the total routing cost subject to all the usual constraints and a construction budget constraint.

Agarwal conducted computational tests on a network with 24 nodes and 38 arcs that was formulated as a linear program with 667 rows and 1938 variables. The results were quite discouraging since the simplex method, Dantzig-Wolfe decomposition and the Boxstep method [43] all failed to solve the problem in a reasonable amount of time. Agarwal concluded that none of the methods was effective because of the arc capacity upper bounds present in the problem.

The difficulty caused by the capacity constraints should not be surprising. Note that the problem of computing the routing cost for a particular proposed network solution requires the solution of a difficult capacitated multicommodity flow problem [2, 66]. Since the problem of evaluating a proposed solution is so difficult, it should be expected that the problem of finding the optimal network improvement solution is also very difficult. Next we review several models that are similar to Agarwal's problem and discuss some approaches for dealing with the difficulty of the embedded routing problem.

Steenbrink [63, 64] used a model similar to Agarwal's for the design of a Dutch roadway network. The capacity of an arc  $(i,j)$  is restricted to be between zero and  $K_{ij}$ . Routing costs are convex but the construction costs are nonlinear. The objective is to minimize the total routing and construction costs subject to all the usual flow routing constraints.

Steenbrink formulated his model as an optimization problem with linear constraints and a nonlinear objective function. He suggests decomposing the problem into a master problem and a series of subproblems. Each subproblem concerns finding the optimal capacity for an arc given the total

flow through it. The master problem is to route the required flows through the network with a modified flow cost structure. (This master problem is again a capacitated multicommodity flow problem.) Steenbrink's heuristic procedure for solving the master problem, as was noted by Nguyen [49], is closely related to the well-known incremental loading traffic assignment procedure [44]. So Steenbrink's technique for dealing with the embedded routing problem is to solve it heuristically.

Steenbrink applied this method to a Dutch roadway design problem containing 2000 nodes and 6000 arcs. The heuristic procedure required about 50 minutes of IBM 360/65 computer time. Due to the size of the problem, there is no way to evaluate the quality of Steenbrink's solution.

Dantzig et al. [13] consider a network improvement problem identical to Agarwal's except for a crucial assumption that there is no upper limit on an arc capacity. (Note that congestion costs are still present due to the convex routing costs.) They dualize with respect to the budget constraint and then use Steenbrink's decomposition. The master problem is a convex cost multicommodity flow problem which can be solved very efficiently using the Frank-Wolfe algorithm. The procedure required 10.68 seconds of IBM 370/168 computer time on a test problem with 24 nodes and 76 arcs and produced a solution 2.5 percent away from optimality. In contrast, the simplex method, implemented on the MPS/360 package, required 40.8 minutes to obtain an optimal solution. The authors also report experience on a problem with 394 nodes and 1042 arcs which required 5.63 minutes of IBM 370/168 computer time.

Note that the use of a convex routing cost function to "represent" a finite flow capacity constraint greatly improved the computational performance for this type of congested network improvement problem. So, slightly altering the modeling of congestion avoids a difficult embedded routing problem.

McCallum [45] described a capacitated network planning problem concerning the location of circuits in a communication (telephone) network. This capacitated network is similar to Agarwal's except that between every pair of nodes only a few paths are allowable as flow routes. Thus, the difficult embedded routing problem is avoided. After formulating the model as a linear program, McCallum used a specialized implementation of the generalized upper bounding technique to solve problems containing up to 563 arcs and 1857 required flows between pairs of nodes. The computation time required for a problem of this size was 173 seconds on an IBM 370/165 computer.

##### 5. Network Design Problems with User Equilibrium Routing

In the network design problems with congestion costs discussed in the previous sections, all flows were routed according to a "system optimal" policy which minimized the total routing cost of all flows. In this section we consider problems where the flows are routed according to Wardrop's "Principle of Equal Travel Times" [67]. That is, the traffic is assigned so that the path or paths actually used between each origin and destination will have the smallest travel costs. (Under certain circumstances [5, 41] the user equilibrium routing problem can be transformed into a system optimal routing problem.) The User Equilibrium Routing (UER) policy has been demonstrated to be a useful method of modeling behavior in transportation systems [18].

We begin by describing a major difference between network design problems (with congestion costs) that have UER and those with system optimal routing. For a network with system optimal routing, the addition of an arc to the network never increases the total flow routing costs. Since we can always choose to use the previously determined flow routing pattern, the total routing cost can never increase and will usually decrease. Somewhat surprisingly, for a network with UER, the addition of an arc can lead to an increase in the total flow routing costs. This phenomenon, known as Braess' paradox [9, 47], indicates that great care should be used in evaluating proposed improvements to a network with UER. Knödel [36, 47], described an actual situation in an urban street system where such a phenomenon occurred.

Leblanc [37] considered a network design with UER where all arcs can have either zero or infinite capacity. The objective is to minimize total routing costs subject to all the usual constraints and a construction budget. The branch and bound solution procedure proposed for this model has solved a network problem containing 24 nodes, 76 arcs, and 5 arcs that could be added to the network. Computation time was about 136 seconds on the CDC 6400 computer.

Morlok and LeBlanc [46] address the same network design problem but with a heuristic procedure. The technique is based on marginal analysis of the traffic flows. The heuristic procedure essentially solved the same 24 node problem in 17.8 seconds of Cyber 70 computer time.

Ochoa and Silva [51] and Chan [11] also discuss similar types of network improvement problems.

Barbier [4, 62] considered a problem similar to LeBlanc's except that the objective is to minimize the total routing and construction costs without a budget constraint. His heuristic procedure for obtaining proposed solutions has been used to study additions to the Paris rail network. Computational experience includes analyzing a network with 36 nodes, over 30 arcs and over 50 candidate arcs. Steenbrink [64] reported that Haubrich used a revised version of Barbier's method to study the Dutch rail network. Haubrich's procedure solved a design network problem with about 1250 nodes and about 8000 arcs in less than 40 minutes of IBM 360/75 computer time.

## 6. Conclusion

In this paper we have reviewed a large number of network design problems and their proposed solution techniques. Table 1 summarizes this information.

There still remains a great deal of work to be done on network design problems. Most of the network design problems without congestion costs that we have considered are known to be difficult (NP-complete) combinatorial optimization problems. All of the known exact solution techniques are limited to small and medium sized networks. In order for these models to be useful in applications such as transportation planning, large-scale problems will have to be solved. Branch and bound methods appear inadequate for this task.

Recent work by several authors [24, 42] has shown that Benders decomposition could be a useful tool.

TABLE 1

AUTHORS	ARC CAPACITY VARIABLES/ CONGESTED OR UNCONGESTED PROBLEM	SOLUTION ALGORITHM	# of nodes in test net- work, # of arcs, Comp. time, Machine) COMPUTATIONAL EXPERIENCE
1. Billheimer and Gray [7]	Discrete/ uncongested	Heuristic	(68 nodes, 476 arcs, 180 seconds, IBM 360/67)
2. Magnanti and Wong [42]	Discrete/ uncongested	Benders Decomposition	(10 nodes, 45 arcs, 60 seconds, IBM 370/168)
3. Boyce et al [8]	Discrete/ uncongested	Branch and Bound	(10 nodes, 45 arcs, 200 seconds, IBM 360/75)
4. Hoang [30]	Discrete/ uncongested	Branch and Bound	(8 nodes, 20 arcs, ?, ?)
5. Dionne and Florian [15]	Discrete/ uncongested	1) Branch and Bound 2) Heuristic	(29 nodes, 54 arcs, 12 seconds, Cyber 74)
6. Scott [61]	Discrete/ uncongested	Heuristic	(10 nodes, 45 arcs, 60 seconds, IBM 360/65)
7. Ridley [55]	Discrete/ uncongested	Branch and Bound	(12 nodes, ?,?,?)
8. Stairs [62]	Discrete/ uncongested	Interactive Computer System	(35 nodes, ?,?,?)
9. Agarwal [1]	Continuous/ congested	1) Simplex Method 2) Dantzig- Wolfe De- Composition 3) Boxstep	(24 nodes, 38 arcs, 840 seconds, CDC 6400) for simplex method
10. Steenbrink [63, 64]	Continuous/ Congested	Special de- composition with a heuristic	(2000 nodes, 6000 arcs, 2880 seconds, IBM 360/65)

TABLE 1 (continued)

AUTHORS	ARC CAPACITY VARIABLES/ CONGESTED OR UNCONGESTED PROBLEM	SOLUTION ALGORITHM	# of nodes in test net- work, # of arcs, Comp. time, Machine) COMPUTATIONAL EXPERIENCE
11. Dantzig et al [13]	Continuous/ congested	Special de- composition with Frank- Wolfe decom- position	(394 nodes, 1042 arcs, 340 seconds, IBM 370/168)
12. McCallum [45]	Continuous/ congested	Generalized Upper Bounding	(?, 563 arcs, 173 seconds, IBM 370/165)
13. LeBlanc [37]	Discrete/ congested	Branch and Bound	(24 nodes, 76 arcs, 135 seconds, CDC 6400)
14. Morlok and LeBlanc [46]	Discrete/ congested	Heuristic	(24 nodes, 76 arcs, 18 seconds, Cyber 70)
15. Barbier [4,62]	Discrete/ congested	Heuristic	(36 nodes, 8 arcs, ?,?)
16. Haubrich [64]	Discrete/ congested	Heuristic	(1250 nodes, 8000 arcs, 2400 seconds, IBM 360/65)



Another promising approach is to use heuristic algorithms as approximate solution techniques. Further work is required in evaluating the accuracy and reliability of these procedures. For example, see [12, 22, 57, 69] for work in analyzing heuristics for various network optimization problems.

Also, recent advances in large scale system methodology, such as list processing techniques and network flow algorithms, may have some impact on the size of problems that can be solved practically. The reader may consult a recent report by Magnanti [40] for a survey of these new advances.

The network design problem with congestion costs and discrete arc capacities, is even more difficult than the uncongested case and appears to be a formidable problem. For a network with congestion costs and continuous arc capacities, there have been some successful efforts. Although the embedded multicommodity routing problem poses difficulties for some versions of this problem, Dantzig et al. and McCallum have successfully avoided this obstacle. Utilizing special problem structures in formulating their mathematical programs, they were able to apply linear and convex programming techniques to solve problems whose size is of practical interest. It would be interesting to see if these techniques could be used to solve other versions of network design models with congestion costs.

There are also other kinds of basic network design models that could be explored in future research. For example, Yaged [70] and Zadeh [72] considered network design problems with concave objective functions. Soukoup [65], Newell [48], Bansal and Jacobsen [3], and Rothfarb and Goldstein [58] have also explored various other network design models.

Another promising area for future research is to extend these network design problems to dynamic situations. The basic models considered here are all static in that the network is optimized for a single time period with all changes to the network made instantaneously.

There are several types of time-varying elements that could be incorporated into network design problems. One kind of model of this nature involves networks where the required flows between nodes can be time-varying. For example, in an urban transportation system or a communications network, the traffic demands could vary greatly according to the time of day or season of the year. Gomory and Hu [26] and Oettli and Prager [52] have investigated this kind of network problem.

Another type of time varying design problem concerns network improvements that must be sequenced over a number of time periods. In most real situations the network can only change gradually over a given time span. Ochoa-Rosso [50], Funk and Tillman [21] and Yaged [71] have considered this type of problem.

The third type of time-dependence is related to the previous two and concerns the changes in traffic demands when the network is modified. For example, the evolution of a transportation network will influence the development of the surrounding geographic region. Therefore, future traffic demands by region will be dependent on changes to the transportation network in previous time periods. See Frey and Nemhauser [20] and Los [38] for examples of this type of problem. Also MacKinnon [39] discusses these last two types of time-dependent problems in his survey.

## 7. Acknowledgments

I am indebted to my doctoral thesis advisor, Professor Thomas L. Magnanti of MIT, for his invaluable encouragement and assistance in the

preparation of this report. This research was supported in part by the Department of Transportation (Contract DOT - TSC - 1058).

#### REFERENCES

- [1] S. K. Agarwal, "Optimizing Techniques for the Interactive Design of Transportation Networks Under Multiple Objectives," Ph.D. Thesis, Department of Civil Engineering, Northwestern University, 1973.
- [2] A. Assad, "Multicommodity Network Flows - A Survey," To appear in Networks.
- [3] P. Bansal and S. Jacobsen, "An Algorithm for Optimizing Network Flow Capacity Under Economies of Scale," J. of Opt. Th. and Applic., Vol. 15, No. 5, pp. 565-586, 1975.
- [4] M. Barbier, "Le Future Réseau de Transports en Région de Paris," Cahiers de l'Institute d'Aménagement et d'Urbanisme de la Région Parisienne, 4-5, No. 4, 1966.
- [5] M. Beckmann, C. B. McGuire, and C. Winsten, Studies in the Economics of Transportation, Yale University Press, New Haven, CT., 1956.
- [6] M. Bellmore and G. L. Nemhauser, "The Traveling Salesman Problem - A Survey," Opns. Res., Vol. 16, pp. 538-558, 1968.
- [7] J. Billheimer and P. Gray, "Network Design with Fixed and Variable Cost Elements," Trans. Sci., Vol. 7, pp. 49-74, 1973.
- [8] D. E. Boyce, A. Farhi, and R. Weischedel, "Optimal Network Problem: A Branch-and-Bound Algorithm," Environment and Planning, Vol. 5, pp. 519-533, 1973.
- [9] D. Braess, "Über ein Paradoxen der Verkehrsplanung," Unternehmensforschung, Vol. 12, pp. 258-268, 1968.
- [10] E. C. Carter and J. R. Stowers, "Model for Funds Allocation for Urban Highways Systems Capacity Improvements," Highway Research Record, No. 20, pp. 84-102, 1963.
- [11] Y. P. Chan, "Optimal Travel Time Reduction in a Transport Network: An Application of Network Aggregation and Branch-and-Bound Techniques. Report No. R69-39, Department of Civil Engineering, M.I.T., Cambridge, MA, July 1969.
- [12] G. Cornuejols, M. L. Fisher, and G. L. Nemhauser, "An Analysis of Heuristics and Relaxations for the Uncapacitated Location Problem," Man. Sci., Vol. 23, pp. 789-810, 1977.

- [13] G. B. Dantzig, S. F. Maier, Z. F. Lansdowne, R. P. Harvey, and D. W. Robinson, "Application of Decomposition to Transportation Network Analysis," Tech Report DOT-TSC-OST-76-26, Control Analysis Corporation, October 1976.
- [14] R. Dionne, "Une Analyse Théorique et numérique du Problème du Choix Optimal D'un Réseau de Transport sans congestion," Publication #198, Département d'informatique, Université de Montreal, October 1974.
- [15] R. Dionne and M. Florian, "Exact and Approximate Algorithms for Optimal Network Design," Publication #41, Centre de Recherche Sur les Transports, Université de Montreal, 1977.
- [16] S. E. Dreyfus and R. A. Wagner, "The Steiner Problem in Graphs," Networks, Vol. 1, pp. 195-207, 1972.
- [17] M. A. Efroymson and T. L. Ray, "A Branch-Bound Algorithm for Plant Location," Opns. Res., Vol. 14, pp. 361-368, 1966.
- [18] M. Florian and S. Nguyen, "An Application and Validation of Equilibrium Trip Assignment Methods," Trans. Sci., Vol. 10, pp. 372-390, 1976.
- [19] R. L. Francis and J. M. Goldstein, "Location Theory: A Selective Bibliography," Opns. Res., Vol. 22, pp. 400-410, 1974.
- [20] S. C. Frey, and G. L. Nemhauser, "Temporal Expansion of a Transportation Network-1," Trans Sci., Vol. 6, 306-323, 1972.
- [21] M. L. Funk and F. A. Tillman, "A Dynamic Programming Approach to Optimal Construction Staging," American Society of Civil Engineers, Journal of the Highway Division, 94, pp. 255-265, 1968.
- [22] M. R. Garey and D. S. Johnson, "Approximation Algorithms for Combinatorial Problems: An Annotated Bibliography," in Algorithms and Complexity (J. F. Taub, ed.), Academic Press, New York, 1976.
- [23] A. M. Geoffrion, "How Can Specialized Discrete and Convex Optimization Methods be Married," UCLA Western Management Science Institute, Working Paper, No. 238, 1975.
- [24] A. M. Geoffrion and G. Graves, "Multicommodity Distribution System Design by Benders Decomposition," Man. Sci., Vol. 5, pp. 822-844, 1974.
- [25] A. J. Goldman and G. L. Nemhauser, "A Transport Improvement Problem Transformable to a Best-Path Problem," Trans. Sci., Vol. 1, pp. 295-307, 1967.

- [26] R. Gomory and T. Hu, "Synthesis of a Communication Network," Journal of SIAM, Vol. 12, pp. 348-369, 1964.
- [27] S. L. Hakimi, "Steiner's Problem in a Graph and Its Implications," Networks, Vol. 1, pp. 113-133, 1971.
- [28] F. Harary, Graph Theory, Addison-Wesley, Reading, MA 1969.
- [29] A. M. Hershdorfer, "Optimal Routing of Urban Traffic," Ph.D. Thesis, Department of City and Regional Planning, M.I.T., Cambridge, MA, May, 1965.
- [30] H. H. Hoang, "A Computational Approach to the Selection of an Optimal Network," Man. Sci., Vol. 19, pp. 488-498, 1973.
- [31] T. C. Hu, "Optimum Communication Spanning Trees," SIAM Journal of Computing, Vol. 3, No. 3, pp. 188-195, Sept. 1974.
- [32] D. S. Johnson, J. K. Lenstra, A. H. G. Rinnooy Kan, "The Complexity of the Network Design Problem," (to appear in Networks).
- [33] R. M. Karp, "Reducibility Among Combinatorial Problems," Complexity of Computer Computations, (R. E. Miller and J. W. Thatcher, eds.), Plenum Press, New York, pp. 85-104, 1972.
- [34] R. M. Karp, "On the Computational Complexity of Combinatorial Problems," Networks, Vol. 5, pp. 45-68, 1975.
- [35] J. Kennington, "Multicommodity Flows: A State-of-the-Art Survey of Linear Models and Solution Techniques," Opns Res., Vol. 26, pp. 209-236, 1978.
- [36] W. Knödel, Graphentheoretische Methoden und ihre Anwendungen, Springer-Verlag, Berlin, pp. 56-59, 1969.
- [37] L. J. LeBlanc, "An Algorithm for the Discrete Network Design Problem," Trans. Sci., Vol. 9, No. 3, pp. 283-287, Aug. 1975.
- [38] M. Los, "Simultaneous Optimization of Land Use and Transportation: A Synthesis of the Quadratic Assignment Problem and the Optimal Network Problem," Publication #31, Centre de Recherche Sur Les Transports, Université de Montreal, 1975.
- [39] R. D. MacKinnon, "Optimization Models of Transportation Network Improvement: Review and Future Prospects," Working Paper WP-76, University of Toronto, Feb. 1976.

- [40] T. L. Magnanti, "Optimization for Sparse Systems," in Sparse Matrix Computations, (J. R. Bunch and D. J. Rose, eds.), Academic Press, New York, 1976.
- [41] T. L. Magnanti and B. L. Golden, "Transportation Planning: Network Models and Their Implementation," Mass. Inst. of Tech., Cambridge, MA, 1977.
- [42] T. L. Magnanti and R. T. Wong, "Accelerating Benders Decomposition for Network Design," Discussion Paper, Center for Operations Research and Econometrics, Université Catholique de Louvain, Belgium, 1978.
- [43] R. E. Marsten, W. W. Hogan, and J. W. Blankenship, "The BOXSTEP Method for Large-Scale Optimization," Opns. Res., Vol. 23, pp. 389-405, 1975.
- [44] B. V. Martin and M. L. Manheim, "A Research Program for Comparison of Traffic Assignment Techniques," Highway Research Record, 88, 1965.
- [45] C. J. McCallum, "A Generalized Upper Bounding Approach to a Communications Network Planning Problem," Networks, Vol. 7, pp. 1-23, 1976.
- [46] E. K. Morlok and L. J. Leblanc, "A Marginal Analysis Technique for Determining Improvements to an Urban Road Network," Paper presented at ORSA/TIMS National Meeting, Las Vegas, NV, Fall, 1975.
- [47] J. D. Murchland, "Braess' Paradox of Traffic Flow," Trans. Res., Vol. 4, pp. 391-394, 1970.
- [48] G. F. Newell, "Optimal Network Geometry," Sixth International Symp. on Trans. and Traffic Theory, Elsevier, New York, pp. 561-580, 1974.
- [49] S. Nguyen, "Book Review of Optimization of Transport Networks," Trans. Sci., Vol. 9, pp. 283-284, 1975.
- [50] F. Ochoa-Rosso, "Applications of Discrete Optimization Techniques to Capital Investment and Network Synthesis Problems," Research Report, R68-42, M.I.T., Department of Civil Engineering, June 1968.
- [51] F. Ochoa-Rosso and A. Silva, "Optimum Project Addition in Urban Transportation Networks Via Descriptive Traffic Assignment Models," Research Report, R68-44, M.I.T., Department of Civil Engineering, June, 1968.
- [52] W. Oettli and W. Prager, "Optimal and Suboptimal Capacity Allocation in Communication Networks," Jour. of Opt. Th. and Applic., Vol. 8, pp. 396-411, 1971.
- [53] C. Revelle, D. Marks, and J. Liebman, "An Analysis of Private and Public Sector Location Models," Man. Sci., Vol. 16, pp. 692-707, 1970.

- [54] T. M. Ridley, "Investment in a Network to Reduce the Length of the Shortest Route," Proceedings of the Third International Symposium on the Theory of Traffic Flow," Elsevier, New York, pp. 235-236, June, 1965.
- [55] T. M. Ridley, "An Investment Policy to Reduce the Travel Time in a Transportation Network," Transportation Research, Vol. 2, pp. 409-424, 1968.
- [56] P. O. Roberts and M. L. Funk, "Toward Optimum Methods of Link Addition in Transportation Networks," M.I.T., Department of Civil Engineering, Sept. 1964.
- [57] D. J. Rosenkrantz, R. E. Stearns, and P. M. Lewis, "Approximate Algorithms for the Traveling Salesperson Problem," 15th Annual IEEE Symposium on Switching and Automata Theory, pp. 33-52, 1974.
- [58] B. Rothfarb and M. Goldstein, "The One-Terminal Telpak Problem," Opns. Res., Vol. 19, pp. 156-169, 1971.
- [59] J. G. Schwartz, "Network Flow and Synthesis Models for Transportation Planning: A Survey," Research Report, R68-49, M.I.T., Department of Civil Engineering, June, 1968.
- [60] A. J. Scott, "A Programming Model of an Integrated Transportation Network," Pap. Reg. Sci. Ass., 19, pp. 215-222, 1967.
- [61] A. J. Scott, "The Optimal Network Problem: Some Computational Procedures," Trans. Res., Vol. 3, pp. 201-210, 1969.
- [62] S. Stairs, "Selecting a Traffic Network," J. Transport. Econ. and Policy, Vol. 2, pp. 218-231, 1968.
- [63] P. A. Steenbrink, "Transport Network Optimization in the Dutch Integral Transportation Study," Transportation Research, Vol. 8, pp. 11-27, 1974.
- [64] P. A. Steenbrink, Optimization of Transport Networks, John Wiley & Sons, London, 1974.
- [65] J. Soukup, "On Minimum Cost Networks with Nonlinear Costs," SIAM J. of Applied Mathematics, Vol. 29, pp. 571-581, 1975.
- [66] J. Tomlin, "Minimum Cost Multicommodity Network Flow", Opns. Res. Vol. 14, No. 1, pp. 45-51, 1966.
- [67] J. G. Wardrop, "Some Theoretical Aspects of Road Traffic Research," Proc. Inst. Civ. Eng., 1, Part II, pp. 325-362, 1952.



- [68] R. Wollmer, "Removing Arcs from a Network," Opns. Res., Vol. 12, pp. 934-940, 1964.
- [69] R. Wong, "Accelerating Benders Decomposition for Network Design," Ph.D. Thesis, Department of Elec. Eng. and Comp. Sci., Massachusetts Institute of Technology, 1978.
- [70] B. Yaged, Jr. "Minimum Cost Routing for Static Network Models," Networks, Vol. 1, pp. 139-172, 1971.
- [71] B. Yaged, Jr., "Minimum Cost Routing for Dynamic Network Models," Networks, Vol. 3, pp. 193-224, 1973.
- [72] N. Zadeh, "On Building Minimum Cost Communication Networks," Networks, Vol. 3, pp. 315-331, 1973.

Dr. Richard T. Wong is a Member of the Technical Staff at Bell Laboratories in Holmdel, New Jersey. His research interests are in the area of network optimization and mathematical programming.

Dr. Wong received his Ph.D. in Operations Research from the Massachusetts Institute of Technology. He is affiliated with ORSA, TMS, and the ACM.