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DEVELOPING AN OPTIMAL REPAIR-REPLACEMENT
STRATEGY FOR PALLETS

by

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I. INTRODUCTION

The problem of determining when to repair and when to replace failing equipment is a concern of management of productive resources. Inefficient management due to the use of non-optimal repair-replacement policies can have significant financial implications. The purpose of this paper is to describe the problem, analysis and results of a study which is concerned with determining the optimal repair-replacement strategy for an organization managing a large number of wooden pallets.

1. The Problem

The organization mentioned above is a wholly owned, non-profit oriented subsidiary of various corporations which use the pallet in transporting a perishable consumer product to common distribution centers. Among various other functions the organization is responsible for managing the pool of pallets which includes maintaining a sufficiently large inventory of pallets to guarantee smooth operation, purchasing new pallets when necessary, selling damaged pallets as scrap, accounting, etc. The cost associated with the pool of pallets are absorbed by the corporations according to their share of the market. These costs are of course influenced by the repair-replacement policy applied to damaged pallets. At the time of the analysis a pallet when damaged was not repaired but - if saleable - disposed of for a price of $1.50 to scrap dealers and was replaced by a new pallet costing $8.50. The policy was adopted on grounds "...that repaired pallets don't provide the same efficiency and are therefore less economical...". With the steadily rising prices for new pallets - the price had almost doubled since 1970 - and a saturation on the market for used pallets, a review of the existing policy was called for. The primary problem was therefore to determine an optimal repair-replacement policy by specifying under what conditions a damaged pallet should be repaired at an average cost of about $2.50 or be replaced by a new one. A secondary issue was to investigate the advisability of replacing the entire pool of wooden pallets by pallets of a more durable material such as plastic. Determining the optimum size of the pool of pallets, however, was not an issue to be investigated although it represents an interesting problem in light of the highly seasonable demand pattern.

2. The Criterion

In developing an optimal repair-replacement policy and evaluating the possibility of using other than wooden pallets a criterion had to be adopted. Any criterion must be defined for a specific time horizon. A horizon of one period has been chosen as the policy to be selected will be applied on an ongoing basis and the decision criterion assumes steady state. The impact of the steady state assumption is examined by investigating the transitional behavior. Due to the non-profit orientation of the organization and the stochastic nature of the problem, the criterion of minimizing the expected relevant cost of maintaining one pallet per period was selected. In choosing from different policies, the criterion can be formally expressed by

$$E(C) = \min_{k} \{ 8.5\ X(k) + 2.5\ Y(k) - 1.5\ Z(k) \} \tag{1}$$

with
- $X(k)$: probability of a pallet being new,
- $Y(k)$: probability of a pallet being repaired,
- $Z(k)$: probability of a damaged pallet being sold as scrap

when policy k (k=0,1,2,....) is used. Besides cost, convenience was also considered as important and should be used in cases of only marginal cost differentials.

3. Information and Data

Before engaging in the analysis it is useful to present the data and information which was available. Information regarding the age distribution of non-damaged and damaged pallets (pallets are identified as to their date of purchase in terms of quarter and year) was obtained through two surveys which are given in Exhibit I.

Exhibit I

Pallet Survey Results

Date of Manufacture Year	Quarter	Number of Good Pallets	Number of Damaged Pallets
73	4	133	66
	3	68	9
	2	57	4
	1	43	8
72	4	56	42
	3	72	16
	2	40	23
	1	34	20
71	4	43	27
	3	30	21
	2	15	28
	1	3	14
70	4	25	20
	3	18	11
	2	19	10
	1	18	6
69	4	8	15
	3	8	5
	2	5	9
	1	3	6
68	4	15	9
	3	5	4
	2	2	5
		720	378

The number of pallets purchased and sold as scrap during the eleven quarters prior to the analysis was available and is given in Exhibit II.

Exhibit II

Pallet Purchases and Scrappages

	Purchased	Sold as Scrap
June - August 71	19,840	7,550
September - November 71	13,220	9,580
December 71 - February 72	9,620	8,370
March - May 72	29,230	11,510
June - August 72	24,245	10,260
September - November 72	6,560	9,080
December 72 - February 73	11,190	7,900
March - May 73	14,930	10,040
June - August 73	29,350	13,220
September - November 73	11,505	8,780
December 73 - February 74	14,640	8,750

Inspecting Exhibit II suggests that the size of the pool has been steadily increasing. Management of the organization however claimed that the size of the pool had been constant at about 150,000 pallets. The discrepancies were not fully explainable but attributed to a number of reasons such as:

(a) miscounting
(b) pallets being lost in the system
(c) pallets being lent outside the pallet pool
(d) only a fraction of damaged pallets can be sold for scrap
(e) a combination of the above.

II. ANALYSIS: REPAIR-REPLACEMENT STRATEGY

Various classes of repair-replacement policies can be considered and include:

(a) Repair a pallet only if its age is less than $k(k=0,1,2,\ldots)$ quarters;
(b) Repair a pallet only $r(r=0,1,2,\ldots)$ times during its service life;
(c) Repair a pallet provided its age is less than k quarters and the number of previous repairs is less than r.

For a number of primarily administrative reasons only the first class of policies was to be considered. The index k defines the critical age of a pallet. Thus, policy k can be interpreted as a decision rule to repair a pallet if its age when damaged is less than k quarters and to replace it if its age is equal to or greater than k quarters. Of course, the policy with $k = 0$ is the no-repair policy. In order to evaluate this class of policies according to criterion (1), X(k), Y(k) and Z(k) must be determined. This is possible by modeling the stochastic behavior of a pallet in the pool.

1. The Stochastic Process

For the policies to be considered, the age of a pallet is the key variable. Since pallets are identified by the date of purchase (quarter and year), we define an index $j(j=0,1,...J)$ to represent the age of a pallet at the beginning of a period. In light of the information available from the surveys it appears only very few (if any) of the pallets currently in the pool were purchased prior to 1968. Thus, we set $J=23$. The impact of restricting the life of a pallet to 24 quarters will be examined in the section Sensitivity Analysis. Furthermore, it is necessary to know whether repaired pallets are stronger or weaker than non-repaired pallets of comparable age. Since no statistical information is available (the current policy is a no-repair policy), it is assumed that the repaired and non-repaired pallets have the same characteristics. This assumption is supported by discussions with operating people indicating that repaired pallets if anything tend to be somewhat stronger than non-repaired pallets. (Any other assumption would require classifying a pallet also by its repair status.)

We can therefore define as $\pi_j^k(t)$ the probability that a randomly selected pallet is j quarters of age (or is in state j) at the beginning of period t $(t=1,2,...)$ when policy k is used. The stochastic process which determines the probability $\pi_j^k(t)$ can be described by the transition probabilities.

Let $p_{ij}^k(t)$ be the probability that a pallet of age i at the beginning of period t will be of age j at the beginning of period t+1 if policy k is used. Since there is no reason to assume that the process by which pallets are damaged changes from period to period, we let the transition probability be independent of t. Thus, $p_{ij}^k(t) = p_{ij}^k$. For the class of repair-replacement policies to be considered, these transition probabilities are defined below:

$$(2) \qquad p_{ij}^k = \begin{cases} P_i(1-\beta) & j = 0 \\ P_i\beta + (1-P_i) & j = i+1 \\ 0 & \text{otherwise} \end{cases}$$

for i = 0,1,...k-1

$$(3) \qquad p_{ij}^k = \begin{cases} P_i & j = 0 \\ 1 - P_i & j = i+1 \\ 0 & \text{otherwise} \end{cases}$$

for i = k,...22

and

$$(4) \qquad p_{ij}^k = \begin{cases} 1 & j = 0 \\ 0 & \text{otherwise} \end{cases}$$

for i = 23

with P_i representing the probability that a pallet of age i at the beginning of a period will be damaged within the same period, and β ($0 \le \beta \le 1$) being the fraction of damaged pallets that can be repaired.

The definition of the transition probabilities implies that a pallet will be damaged at most once every quarter. Although the possibility of multiple damages within the same quarter exists for all but the no-repair policy, the aspect has not been integrated into the model. As shown in the section Sensitivity Analysis, the aspect of multiple damages has minimal impact and would only unduly complicate the formulation.

X(k), Y(k) and Z(k) representing the probabilities that a randomly selected pallet is new (i.e., has been replaced at the beginning of a period), is repaired during a period or sold as scrap can be derived using the stochastic process $\pi_j^k(t)$. Assuming the system has reached steady state, X(k), Y(k) and Z(k) can then be expressed by:

(5) $$X(k) = \sum_{j=0}^{k-1} \pi_j^k P_j (1-\beta) + \sum_{j=k}^{J} \pi_j^k P_j$$

(6) $$Y(k) = \sum_{j=0}^{k-1} \pi_j^k P_j \beta$$

(7) $$Z(k) = \sum_{j=k}^{J} \pi_j^k P_j \beta$$

2. Results

To evaluate the criterion function (1) using the relationships (5), (6) and (7) requires that the damage probability P_i - being an input into the transition probabilities p_{ij}^k - is specified. P_i must be estimated from the given information.

Let D_i represent the damage ratio which can be defined by the ratio of damaged pallets of age i per quarter to all pallets of age i. D_i can then be expressed by

(8) $$D_i = \frac{m_i M}{n_i N}$$

with m_i: Fraction of all damaged pallets which are i quarters old,

n_i: Fraction of pallets of age i in the pool,

M: total number of damaged pallets per quarter,

N: total number of pallets in the pool.

An estimate of m_i and n_i can be obtained from the survey data given in Exhibit I (e.g., $m_0 = 66/378$ and $n_0 = 133/720$). While N is stated to be 150,000 M must be estimated from Exhibit II.

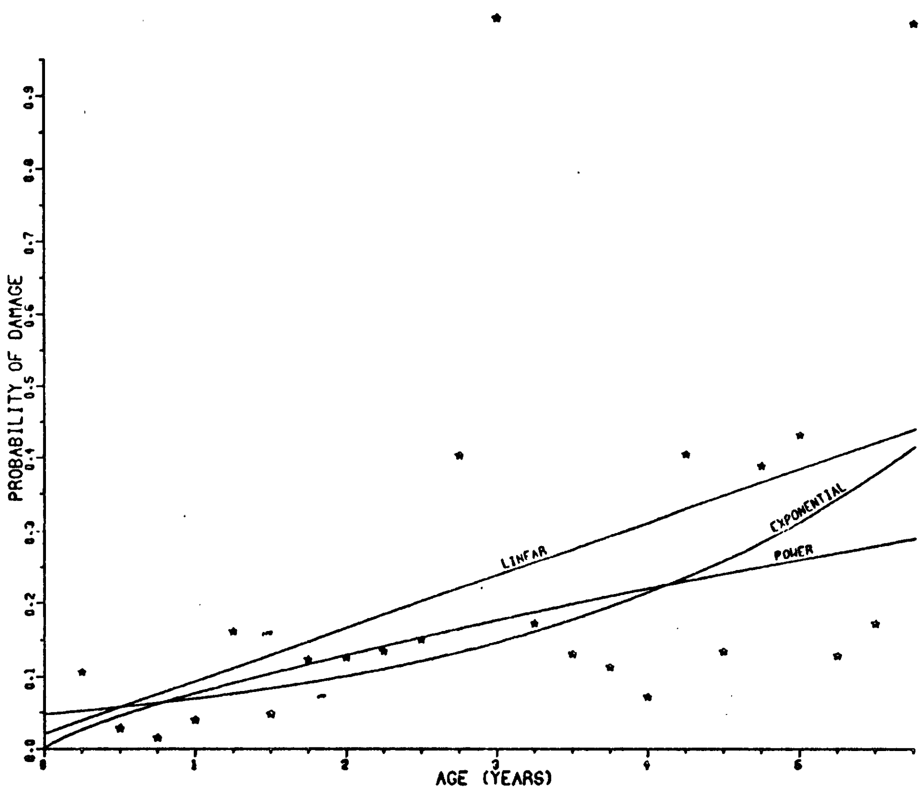
If we acccept that the size of the pool is not growing, M can be determined by averaging the number of purchases per quarter as given in Exhibit II. Thus, M = 16,757 or approximately 17,000 per quarter. The difference between purchases and scrap sales are due to the various reasons given by management. The other possibility, of course, is to assume that the pool of pallets has in fact been growing during the past years. M should now be equated with the average number of scrap sales per quarter. Thus, M = 9,549 or approximately 10,000. The difference of 7,000 pallets can be considered as the maximum growth per quarter as some of the damaged pallets cannot be repaired and therefore cannot be sold as scrap. The true value of M will fall somewhere between 10,000 and 17,000. The subsequent analysis is therefore carried out for both the no growth and the max growth situations.

No growth (M = 17,000)

The damage rates D_i according to (8) for the no growth assumption are given by asteriks in Exhibit III. Among various forms an exponential function of the type $y = a e^{bx}$ with a = .0477623 and b = .0938656 provides the best fit. This function has been used to estimate the damage probabilities P_i which are input into the transition probabilities p_{ij}^k.

Exhibit III

Probabilities of Damage: No Growth



The steady state probability of a randomly selected pallet being in state j can now be determined according to

(9) $$\pi_j^k = \sum_{i=0}^{J} \pi_i^k \, p_{ij}^k$$

and

(10) $$\sum_{j=0}^{J} \pi_j^k = 1 .$$

Exhibit IV summarizes the expected cost per pallet and quarter for different values of k and indicates that the minimum cost policy is to repair pallets if their age when being damaged is less than 12 quarters. Relative to the existing no-repair policy annual savings in the order of ($.749660 - $.634673) • 4 • 150,000 = $68,992.20 can be expected.

Exhibit IV

Cost per Pallet per Quarter for Different k: No Growth

Critical Age k	Expected Cost per Pallet	Critical Age k	Expected Cost per Pallet
0	$.74966	12	$.634673 ←
1	.731418	13	.634921
2	.714818	14	.636639
3	.699835	15	.639829
4	.686449	16	.6445
5	.674639	17	.650674
6	.664384	18	.658394
7	.655668	19	.667746
8	.648476	20	.678899
9	.642791	21	.69219
10	.638602	22	.708321
11	.635898	23	.728831

Max growth (M=10,000)

The damage rates D_i according to (8) for the max growth assumption are given by asteriks in Exhibit V. Again an exponential function of the type $y = a\ e^{bx}$ with a = .03694 and b = .07961 provides the best fit and is used to estimate the damage probabilities P_i.

The derivations of the steady state probabilities π_j^k must be modified to consider growth. Under the growth assumption the number of pallets in the pool during period t, N(t), can be expressed by

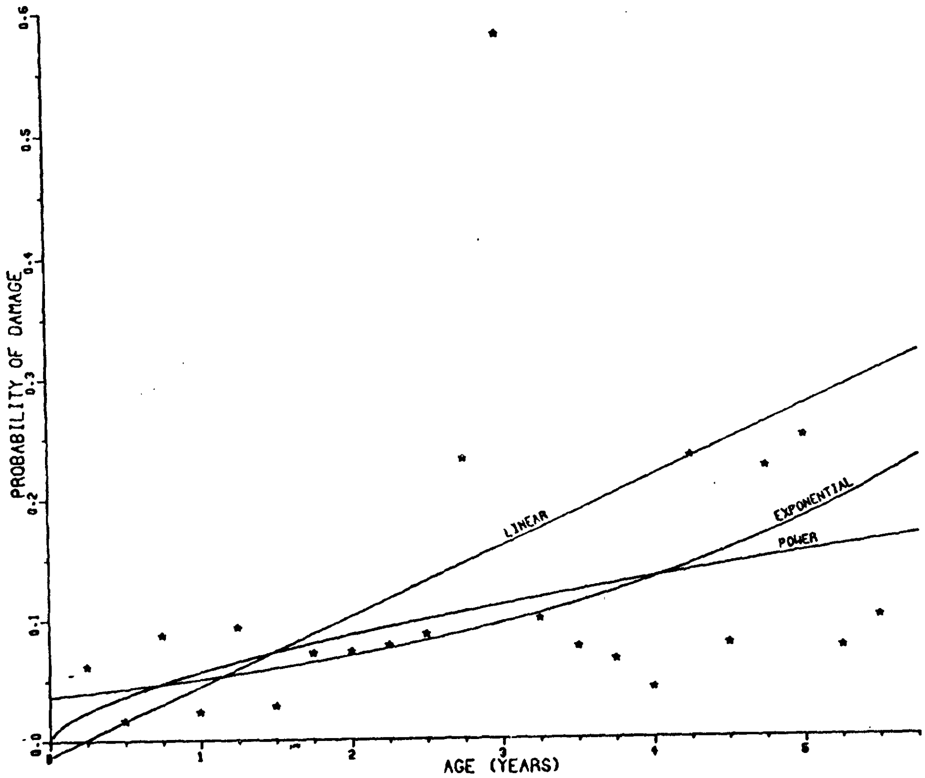
(11) $$N(t) = (1+g)\ N(t-1)$$

with g being the growth rate. (11) can be rearranged as

(12) $$\frac{N(t-1)}{N(t)} = \frac{1}{1+g}$$

Exhibit V

Probabilities of Damage: Max Growth



Let $\pi_j^k N(t)$ be the number of pallets of age j at the beginning of period t when policy k is used. Since growth materializes in state j=0 (i.e. new pallets), $\pi_j^k N(t)$ can be expressed by (13) for j=0

(13) $$\pi_0^k N(t) = \sum_{i=0}^{23} \pi_i^k N(t-1) p_{i0}^k + g N(t-1)$$

and by (14) for $j \geq 1$

(14) $$\pi_j^k N(t) = \sum_{i=0}^{23} \pi_i^k N(t-1) p_{ij}^k.$$

Dividing (13) and (14) by N(t) and substituting by (12) leads to (15)

(15) $$\pi_0^k = \frac{1}{1+g} \sum_{i=0}^{23} \pi_i^k \, p_{i0}^k + \frac{g}{1+g}$$

and

(16) $$\pi_j^k = \frac{1}{1+g} \sum_{i=0}^{23} \pi_{ij}^k \, p_{ij}^k$$

which in conjunction with (10) provides the required steady state probabilities.

Exhibit VI summarizes the expected cost per pallet and quarter for different values of k with a constant maximum growth rate

$$g = \frac{17{,}000 - 10{,}000}{150{,}000} \simeq .04 \text{ or } 4\%.$$

Exhibit VI

Cost Per Pallet and Quarter for different k: Max Growth

Critical Age k	Expected Cost Per Pallet	Critical Age k	Expected Cost Per Pallet
0	$.536107	12	$.425726
1	.520751	13	.422683
2	.506711	14	.420467
3	.493914	15	.41908
4	.482293	16	.418529 ←
5	.471789	17	.418839
6	.462348	18	.420056
7	.453923	19	.422254
8	.446471	20	.425554
9	.439955	21	.430149
10	.434343	22	.436342
11	.429607	23	.444625

The minimum cost policy is to repair a pallet if its age is less than 16 quarters. Relative to the no-repair policy annual savings are in the order of

$$(\$.536107 - \$.418529) \cdot 4 \cdot 150{,}000 \cdot (1+g)^t = \$70{,}546.80 \cdot (1+g)^t$$

3. Recommendation

Based on the above analysis a pallet should be repaired if its age is less than 12 quarters for the no growth assumption and 16 quarters for the max growth assumption. Since it is not certain whether the no growth or the max growth condition actually exists, implementing the wrong decision will result in opportunity losses. These opportunity losses can easily be determined and are given in Exhibit VII.

Exhibit VII

Opportunity Losses/Pallet/Quarter

Critical Age k	No Growth	Max Growth
12	0	\$.425726 -.418529 \$.007197
16	\$.644500 -.634673 \$.009827	0

We can conclude that the policy with critical age k = 12 is superior to the policy with k = 16 if

$$P(\text{max growth}) \cdot .007197 < [1-P(\text{max growth})] \cdot .009827.$$

Thus, the policy with k = 12 is preferable if the probability of maximum growth is approximately less than .6 while the policy with k = 16 is better for values greater than or equal to .6. Based on management's belief that the size of the pool is not growing, the policy of repairing pallets with age less than 12 quarters should be implemented. The maximum opportunity loss per year for the entire pool of pallets of using the wrong policy when the maximum growth condition exists is $\$.007197 \cdot 4 \cdot 150{,}000\ (1+g)^t \simeq \$4{,}300.00(1+g)^t$. The maximum opportunity loss per year of using the wrong policy when the no growth condition exists is $\$.009827 \cdot 4 \cdot 150{,}000 \simeq \$5{,}900.00$. In both instances, the opportunity losses are limited, indicating that financial consequences of making the wrong decision are not overly severe.

Since the actual condition may also fall somewhere between no growth and maximum growth, policies for values of k between 13 and 15 quarters could be investigated. Such refinement, however, does not appear to be warranted in view of the rather limited opportunity losses.

III. ANALYSIS: SUBSTITUTION OF MORE DURABLE PALLETS

The remaining problem is to analyze the possibility of replacing all wooden pallets by pallets of more durable and less breakable material such as plastic. This alternative is advisable if the cost of maintaining a plastic pallet is less than the same cost for a wooden pallet under the existing no-repair policy.

From the above analysis we know that the expected annual cost of maintaining a wooden pallet under the no-repair policy is

$$\$.749660 \cdot 4 = \$3.00 \qquad : \text{no growth}$$

$$\$.536107 \cdot 4 = \$2.14 \qquad : \text{max growth.}$$

In the extreme case a plastic pallet might be undamageable and would thus last forever. Therefore, only the purchasing price is relevant. The alternative of replacing the wooden pallet is advisable if the cost of purchasing a plastic pallet, R, is less than the present value of all future costs of maintaining a wooden pallet under the no-repair policy. Thus,

$$(17) \qquad R < \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^{t} \cdot \begin{cases} \$3.00 & : \text{ no growth} \\ \$2.14 & : \text{ max growth} \end{cases}$$

with ρ being the cost of capital. Since

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^{t} = \frac{1}{\rho}$$

the cost of purchasing a plastic pallet with $\rho = .15$ must be less than

$$\$3.00 \cdot \frac{1}{.15} = \$19.98 \qquad : \text{ no growth}$$

or

$$\$2.14 \cdot \frac{1}{.15} = \$14.25 \qquad : \text{ max growth}$$

respectively, to make the use of plastic pallets economically advisable. Since the cost of a plastic pallet is currently around $25.00 and such pallets certainly do not last forever, the alternative of replacing the pool of wooden pallets by plastic pallets is not recommended.

IV. SENSITIVITY ANALYSIS

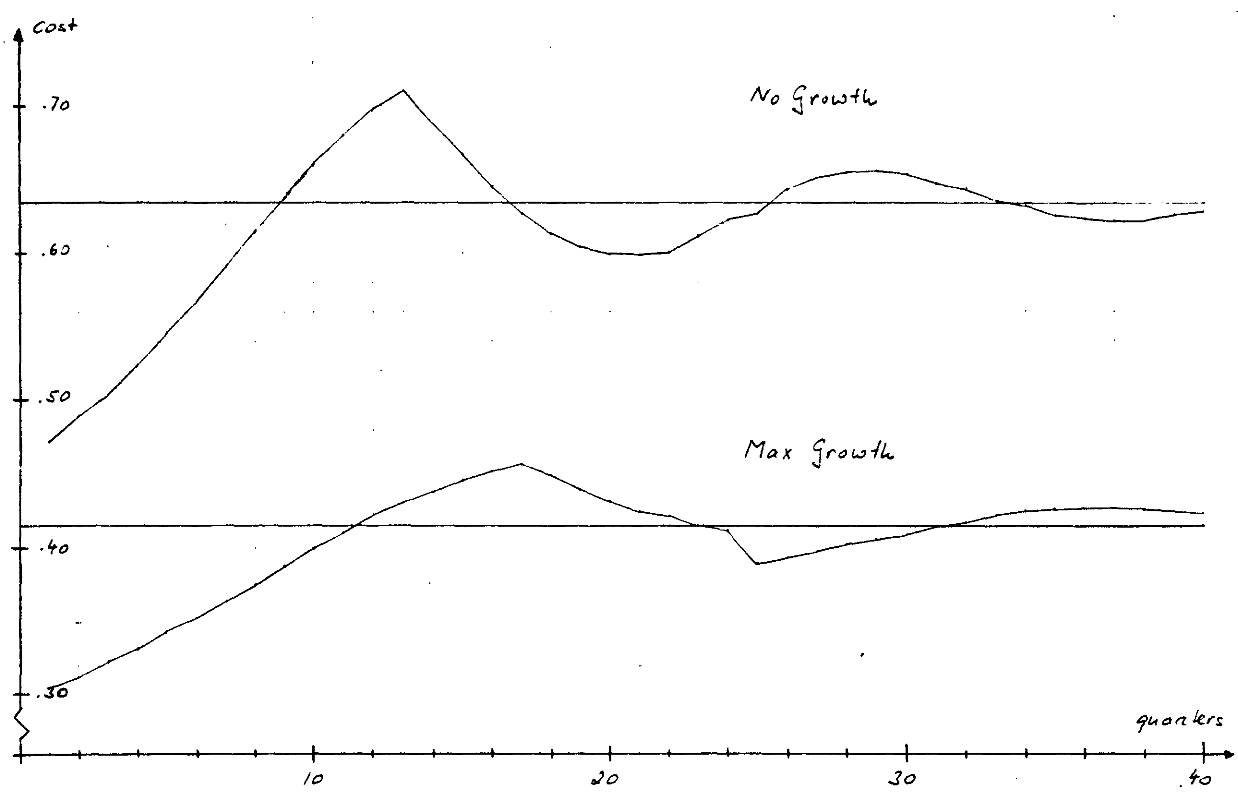
The sensitivity of the results obtained will be investigated with respect to various assumptions and certain pieces of information but is restricted to the problem of developing an optimal repair-replacement policy.

1. Transitional Behavior

The above analysis has been carried out under the steady state assumption. Exhibit VIII summarizes the transitional behavior of implementing the optimal repair-replacement policy for both the no growth and max growth condition assuming the system under the existing no-repair policy is in steady state. The transitional behavior is expressed by the expected cost per pallet and quarter. As can be expected from the structure of the problem, the time required to reach steady state is large. The fact of considerably lower than steady state cost during the first two years (no growth) and three years (max growth) is an added incentive for implementing the optimal policy.

Exhibit VIII

Sensitivity Analysis: Transitional Behavior



2. Number of State Variables

Based on the survey information the above analysis was carried out for J=23 which implies that no pallet will be older than 6 years. Extrapolating the exponential functions in Exhibits III and V allows us to evaluate the system when J>23 and to determine the effect of restricting J to 23. The results for a maximum age of 10 years (i.e., 40 quarters) are given in Exhibit IX.

Exhibit IX

Sensitivity Analysis: Number of State Variables

Number of States	No Growth		Max Growth	
	Critical Age k	Savings per Pallet/Quarter	Critical Age k	Savings per Pallet/Quarter
24	12	$.114987	16	$.117578
40*	12	$.115322	18	$.122530

*It should be noted that no pallet was older than 33 quarters under the no growth assumption.

As can be observed, the critical age and the expected savings relative to the no-repair policy do not change for the no growth assumption and vary only marginally for the max growth assumption. Restricting the age of a pallet to a maximum of 6 years appears to be justified.

3. Multiple Damages per Quarter

The models developed above assume that a pallet can only be damaged once per quarter. Under the no-repair policy the events damage and no damage represent a binomial trial. If we allow for multiple damages per quarter the process can be represented by a Poisson distribution. (Here we assume that repaired pallets are as strong as non-repaired pallets of the same age.) The probability of s=0 damages per quarter for a pallet of age i can be expressed by (18) and equated to $1-P_i$. Thus

$$P(s=0|\lambda_i) = \frac{\lambda_i^s}{s!} e^{-\lambda_i} = e^{-\lambda_i} = 1-P_i \tag{18}$$

with

$$\lambda_i = -\ln (1-P_i).$$

Naturally, the multiple damage possibility is only relevant for pallets whose age is less than k quarters as a damaged pallet aged k or more quarters will be replaced. Thus, the maximum value λ can take on is

$$\lambda_{11} = -\ln (1 - .134123) = .144 \quad : \text{no growth}$$

and

$$\lambda_{15} = -\ln (1 - .121930) = .130 \quad : \text{max growth.}$$

The probability of 2 or more damages per quarter is in both situations approximately .01 and small enough to justify the assumption of at most one damage per quarter.

4. Probability of Damage

The probabilities of damage P_i are estimated using the exponential functions in Exhibit III and V which are determined by the damage rates D_i as defined in (8). The damage rates D_i are the result of the pallet surveys in conjunction with the information given in Exhibit II and are of course subject to sampling error. The analysis have therefore also been carried out for various other values for the parameters a and b as described in Exhibit X.

The results are given in Exhibit XI. Substantial changes in parameter a produce substantial variation in the expected savings but do not affect the optimal policies significantly. Substantial changes in parameter b, however, result in slightly larger changes of the optimal policies but leave the expected savings almost unchanged.

Exhibit X

Sensitivity Analysis: Alternative Sets of Parameters

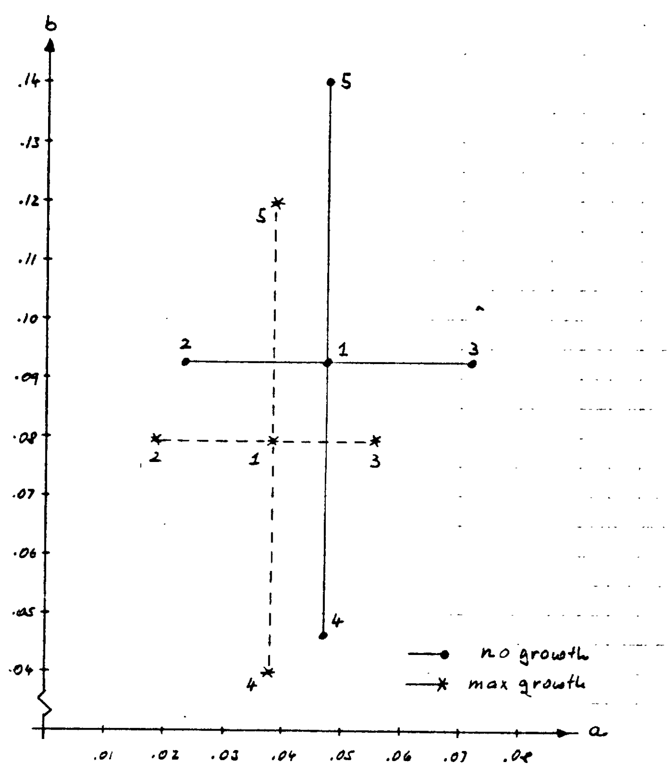


Exhibit XI

Sensitivity Analysis: Damage Probabilities

Parameter Set	No Growth		Max Growth	
	Critical Age k	Savings per Pallet/Quarter	Critical Age k	Savings per Pallet/Quarter
1(Original)	12	$.114987	16	$.117578
2	14	.055687	17	.058941
3	11	.175649	15	.171505
4	15	.104441	17	.102670
5	9	.114545	13	.117423

5. Repair Cost

Although the repair costs were stated to be $2.50 per pallet, investigating the sensitivity of this cost appears to be necessary. As can be observed from Exhibit XII, the results are more sensitive necessitating accurate information on that cost component.

Exhibit XII

Sensitivity Analysis: Repair Cost

Repair Cost	No Growth		Max Growth	
	Critical Age k	Savings per Pallet/Quarter	Critical Age k	Savings per Pallet/Quarter
2.50	12	$.114987	16	$.117578
3.00	11	.087557	14	.092702
3.50	9	.064884	12	.070786
4.00	7	.045868	11	.051811
4.50	6	.030691	9	.035821
5.00	5	.018538	7	.022690

6. Scrap Value

The indicated saturation on the market for used pallets could result in the drop of the scrap price from the existing level of $1.50. Exhibit XIII summarizes the optimum policy and the expected savings per pallet and quarter for both the no growth and maximum growth assumption. It is interesting to note that the optimal critical age increases by only three or two quarters respectively as the scrap price drops to zero. As expected, the savings per pallet per quarter are significantly larger and make the implementation of the optimal repair policy more imperative.

Exhibit XIII

Sensitivity Analysis: Scrap Value

	No Growth		Max Growth	
Scrap Price	Critical Age k	Savings per Pallet/Quarter	Critical Age k	Savings per Pallet/Quarter
$ 1.50	12	$.114987	16	$.117578
1.00	13	.136421	17	.137206
.50	14	.158908	18	.157310
.00	15	.182327	18	.177940

V. CONCLUSION

The purpose of the paper was to develop an optimal repair-replacement strategy for a pool of wooden pallets. As shown, a Markovian analysis proves useful for that task and indicates that substantial savings can be realized by implementing the suggested strategy. The possibility of replacing the wooden pallets by plastic pallets was evaluated by the model but was found to be economically not advisable.