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AMPLIFIERS WITH PRESCRIBED FREQUENCY CHARACTERISTICS AND ARBITRARY BANDWIDTH

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Abstract

The amplifier chain, a cascade connection of amplifier tubes connected by two-terminal or two-terminal-pair interstages, is the basic component of the amplifiers designed. Shunt capacitance in the interstages imposes a limit on the amplification per stage over a prescribed band of frequencies. The limit of amplification per stage is inversely proportional to the bandwidth. The method of design of amplifier chains presented leads to simple interstages which are economically close to the maximum in performance for the shunt capacitance present. The interstages used are simple-tuned circuits or double-tuned circuits. The technique of design of the amplifier chains is related to the stagger-tuning technique invented by Wallman. The characteristics of individual stages are nonuniform but the stages in a chain complement each other to provide an acceptably uniform characteristic. In the design procedure one chooses the amplification function for the chain being designed, according to a flexible technique presented in Technical Report No. 145, such that prescribed frequency characteristics are approximated. The amplification functions so chosen are suitable to be identified with realizable networks. This step is the key of the whole design process.

Single amplifier chains become ineffective as amplifiers when the bandwidth becomes so broad that the amplification per stage approaches one. When amplifiers are to amplify such broad bands of frequencies that this situation arises, more than one amplifier chain is used. The chains amplify different sub-bands and are connected in parallel at the input and output. The amplification functions of the individual chains are so chosen that the chains, when connected in parallel, give a desired characteristic.

A summary of the pertinent characteristics of the distributed amplifier (invented by Percival), which is capable of amplifying over bands greater than those for which single amplifier chains are useful, permits a comparison with the parallel-chain amplifier. The parallel-chain amplifier is more economical in tubes and the design is more flexible. The frequency selectivity of the parallel-chain amplifier is affected by nonuniform changes in the tube characteristics; a properly designed distributed amplifier has a frequency selectivity not altered by changing transconductances of the tubes.

AMPLIFIERS WITH PRESCRIBED FREQUENCY CHARACTERISTICS AND ARBITRARY BANDWIDTH

1.00 Introduction

The design of vacuum tube amplifiers is a broad subject because of the diverse uses of amplifiers and the widely different specifications on performance in different applications. The majority of design specifications include a prescription for the frequency characteristics of the amplifier to be designed. Ordinarily the frequency characteristics, the magnitude of amplification and the phase shift over the band of frequencies amplified, are the primary specifications for the amplifier. Further specifications may be viewed as added requirements or constraints. The technique of design of amplifiers can logically be based around a procedure for realizing amplifiers with prescribed frequency characteristics. This report presents such a procedure that is simple and flexible. The basic building block in the design procedure is the amplifier chain – a conventional cascade connection of amplifying tubes which are connected by simple two-terminal or two-terminal-pair interstages. For very broad band amplifiers two or more amplifier chains are connected in parallel; each amplifier chain amplifies a smaller, suitably chosen sub-band of the total band of frequencies being amplified.

The basic limitation which restricts the level of amplification obtainable in a given amplifier over a given band of frequencies is the shunt capacitance associated with the tubes and interstages. Parasitic capacitance is inevitable. The designer is accordingly obliged to furnish designs, with a minimum of parasitic capacitance, which give as large amplification as is feasible with the amount of capacitance present. Simple interstages are desirable in that the parasitic capacitance introduced is smaller than that introduced in complicated circuits. Moreover, simple circuits are easy to build and adjust. Circuits designed by the procedure presented here are simple and give very good amplification for the amount of shunt capacitance present.

1.10 Summary of Previous Contributions

Previous contributions in the field of amplifier design which have particular significance for the method presented here are divided into two classes. One of these classes includes contributions dealing with the nature of and a quantitative evaluation of the limit imposed by shunt capacitance. The second class of contribution deals with design techniques which have been developed earlier for various kinds of amplifiers.

1.11 The Limitation Imposed by Shunt Capacitance

The most familiar example illustrating the limitation of shunt capacitance on amplifier performance is the RC two-terminal interstage shown in Fig. 1a. The band of frequencies over which a stage of amplifier using this interstage has less than half-power variation in amplification is $1/RC$ rad/sec and the maximum impedance which the circuit presents is R ohms. If one wishes to obtain a larger bandwidth with this circuit, he must lower R and hence the amplification. To design a better stage one should try to find a circuit of the nature of Fig. 1b in which the driving-point impedance can be maintained over a corresponding bandwidth at a larger value than is possible with the RC circuit. A large number of circuits were proposed to do specifically this thing long before an ȧnalysis was completed to show the fundamental nature of the limitation imposed by shunt capacitance on the impedance level.

Several writers have suggested circuits, but Wheeler (1) made a comprehensive study of the problem and suggested a number of filter circuits which, used as interstages, would give a better amplification over a prescribed band of frequencies than could be achieved with the simple RC circuit. Wheeler stated, on an empirical basis, that the maximum uniform impedance which could be maintained over ω_0 rad/sec in a network with a shunt capacitance C is $2/C\omega_0$ ohms, which is twice the impedance level of an RC network with an ω_0 rad/sec bandwidth. This limit is obtained with filter circuits requiring an infinite number of elements.

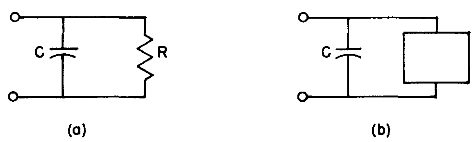

Fig. 1 Simple two-terminal circuits with shunt capacitance.

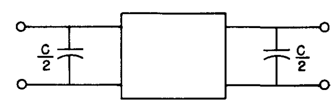

Fig. 2 A two-terminal-pair network with shunt capacitance at the terminal pairs.

Subsequent analytical work by Bode(2) verified the conclusion stated by Wheeler. Bode's work was essentially to determine the maximum constant magnitude of a driving-point impedance over ω_0 rad/sec if this impedance must approach $1/\lambda C$ (λ is the complex frequency variable) as λ approaches infinity. The prescribed behavior of the function at infinity in combination with the restriction to positive real character * limits the maximum uniform magnitude over ω_0 rad/sec to $2/\omega_0 C$ ohms and Wheeler's empirical result was substantiated on analytical grounds.

The problem of evaluating the limits imposed by parasitic capacitance in the case of two-terminal-pair networks was treated by both Wheeler and Bode. In Fig. 2 is shown a two-terminal-pair network with equal shunt capacitances whose limiting behavior can be compared with the limit obtainable with networks of the form Fig. 1b. The analytical problem solved by Bode was to determine the maximum uniform level of transfer impedance over ω_0 rad/sec for such a circuit. This he determined to be $\pi^2/2\omega_0 C$ ohms. One can say, on this basis, that splitting the shunt capacitance into two equal parts increases the potential effectiveness of a stage by 2.47 times.

The foregoing discussion applies specifically to single-stage amplifiers. Most practical cases involve amplifiers of several cascaded stages. The results stated above

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*A positive real function of λ is a rational function of λ whose real part is never negative for positive values of the real part of λ. Positive real character is the necessary and sufficient condition that a function be the driving-point impedance of a physically realizable passive network.

apply only to multi-stage amplifiers in which the amplification of every stage is uniform over the pass band. Hansen (3) determined the limit of amplification in amplifiers with a uniform over-all characteristic but with individual stages having nonuniform characteristics (stagger-tuned amplifiers). He found that for two-terminal-pair interstages the stagger-tuned case offers a slight advantage but that for two-terminal interstages the limits are the same as for the case of identical stages each with uniform amplification over the pass band.

1.12 Design Techniques

Most previous contributions in the techniques of design of amplifiers consisting of a single chain were centered around improvement in the characteristics of the RC amplifier. The use of inductance in shunt with the parasitic capacitance at the terminal pairs of the interstage or in series between the ungrounded terminals of the input and output (called respectively shunt and series compensation) is the most familiar artifice for improving the amplification-bandwidth product of the RC amplifier. This method results in a very simple circuit. The choice of parameter values is usually made either empirically or analytically to suit the particular application. The filter interstages suggested by Wheeler, which were mentioned in the last section, are theoretically more effective but also much more complicated. In practical circuits these interstages are seldom used. Experience establishes the principle that simplicity of a circuit is frequently as necessary as it is practically convenient. A complicated circuit ordinarily possesses more shunt capacitance because of its complication; consequently, its greater effectiveness in approaching the limit imposed by shunt capacitance is largely cancelled.

In many amplifiers it is necessary to employ more than one stage to obtain the desired level of amplification. The cascading of identical stages to achieve a higher level of amplification is a method commonly used. It possesses two obvious flaws which become more serious as the number of stages increases. In the first place, the pass band shrinks in width. This is easily appreciated through observing that the half-power frequencies of a stage are the quarter-power frequencies of two such identical stages connected in cascade. The half-power frequencies of the two-stage amplifier are closer together than these quarter-power frequencies. The second flaw is that irregularities in the level of amplification in the pass band are accented as the number of stages increases. For example, a ten-stage amplifier composed of identical stages which gives an amplification uniform over the band to within 30 percent must possess individual stages which are uniform to within 2.7 percent. The requirement of such extreme uniformity necessitates an undesirable complexity in the interstages or excessive shrinking of the bandwidth. A novel and extremely practical alternate solution to the problem of the design of a multi-stage amplifier is that of stagger tuning suggested by Wallman (4). A simple form of stagger-tuned amplifier is shown in Fig. 3. Each stage is tuned to a slightly different frequency and has a different damping. It turns out that an appropriately chosen set of nonuniform characteristics leads to an acceptably uniform over-all characteristic. The interstages are remarkably simple and fortunately such an amplifier provides an amplification level which is acceptably close to the limiting level set by the shunt capacitance. Wallman (4) and Baum (5) have given methods whereby one can select the tuned frequency and damping of the individual stages to give certain simple desirable over-all characteristics.

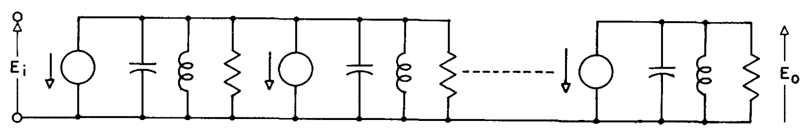

Fig. 3 Equivalent circuit of stagger-tuned amplifier.

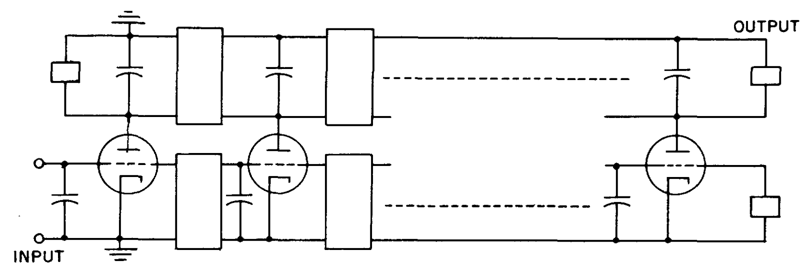

Fig. 4 Circuit of Percival's amplifier.

By the methods just discussed one can design an amplifier consisting of a chain of cascaded stages which will provide any amplification if the bandwidth is not so broad that the amplification per stage is forced down to one by the shunt capacitance. As the bandwidth required approaches this critical limit, the method of cascading to increase the amplification becomes less effective and finally fails. The theoretical problem of designing an amplifier which exceeds the bandwidth possible for conventional amplifiers has been solved before in only one way. Percival's amplifier (6, 7, 8), in which the grids and plates of vacuum tubes are distributed along two artificial lines, gives (theoretically, at least) any amplification over any bandwidth. The Percival amplifier is so constructed that the transconductances of its tubes are essentially summed without incurring an added loss due to increase in parasitic capacitance. This fact frees it from the kind of amplification-bandwidth limitation applying to single chains of cascaded amplifiers. Its operation is explained through consideration of Fig. 4. A signal applied at the input is propagated down the grid line, actuating the tubes in succession. Each of them injects a current into the plate line, and the current wave is reinforced at each tube; its size at the output is determined by the number of tubes along the line. The shunt capacitances of the transmission lines are identified with the parasitic capacitances of the tubes. To get amplification over a wide band, one designs the lines with a suitably high cut-off frequency. Increasing the cut-off frequency always reduces the impedance level of the transmission line and thereby reduces the amplification for a given number of tubes. Hence in the Percival amplifier (called the distributed amplifier by Ginzton (7), parasitic capacitance still limits the amplification-bandwidth product though it does not put a theoretical limit on the breadth of the band of frequencies to be amplified. By choosing a sufficient number of tubes for each stage (Fig. 4) and connecting a suitable number of such stages in cascade, one can theoretically obtain any level of amplification over any bandwidth. The disadvantages of the distributed amplifier are the inflexibility of the design (all sections of the lines must be identical, and the lines must be properly terminated) and the expense in the number of tubes required for a given performance. If there is much parasitic coupling between the two lines, the resulting feedback will cause instability. The amplifier has the advantage that the frequency characteristics are not influenced except in level by changes in the transconductances of the tubes.

The method, described later in this report, of paralleling simple amplifier chains, each effective over a fraction of the bandwidth, has been suggested before (9, 10) but it has never been made workable by a suitable design procedure.

1.20 New Technique of Amplifier Design

The technique of amplifier design presented here is applicable both for narrow- and broad-band cases which require respectively one chain and a number of paralleled chains. In either case the single amplifier chain is the basic building block.

A typical amplifier chain is shown in Fig. 5. For such an amplifier chain the amplification function is

$$\frac{E_o}{E_i} = (-g_m)^n Z_{i1} Z_{i2} \ldots Z_{in} = (-g_m)^n Z_o \quad . \qquad (1)$$

In Eq. 1 Z_{i1}, Z_{i2} . . . Z_{in} are the interstage impedances. They are driving-point impedances if the interstages are two-terminal networks, transfer impedances if the interstages are two-terminal-pair networks. In either case shunt capacitance always appears at the pairs of terminals.

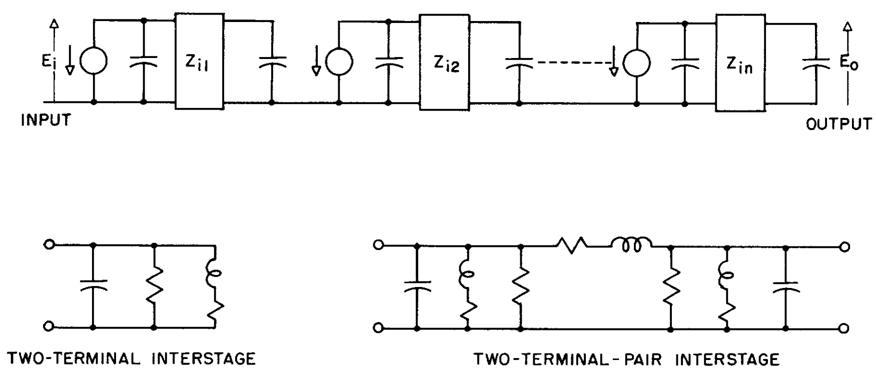

Fig. 5 Amplifier chain and typical interstages.

Amplification function of the chain is $\frac{E_o}{E_i}(\omega)$.

In a broad-band amplifier requiring more than one chain, one prescribes the individual amplification functions to be of such nature that the amplification function of the complete amplifier (the sum of the amplification functions of the paralleled chains) has the desired properties. Figure 6 shows typical amplification characteristics of a two-chain amplifier.

The salient quantities upon which attention is centered throughout the design procedure are the amplification functions of the individual chains. The magnitude and argument of the amplification function for real frequencies are the frequency characteristics of the amplifier chain. The interstage impedances are factors of the amplification function. For the interstage types used (Fig. 5) these impedances are simple functions. Element values of the interstages are easily related to the amplification function. Design constraints in the amplifier are readily translated into constraints on the amplification function. For instance, in an n-stage amplifier with two-terminal interstages each with shunt capacitance C, the amplification function is constrained to approach $(-g_m)^n/(\lambda C)^n$ as λ approaches infinity.

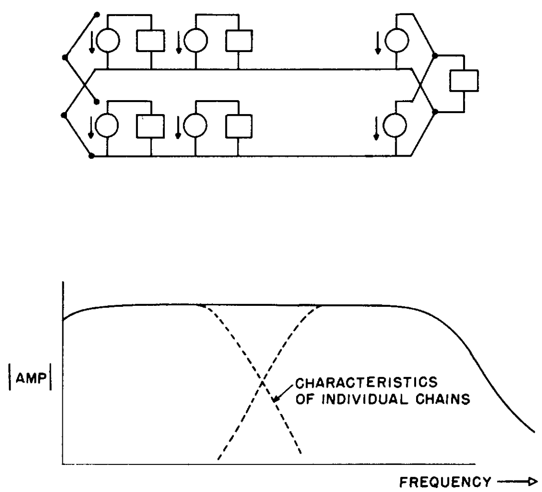

Fig. 6 Typical amplifier characteristics of two-chain amplifier.

In either the single- or multiple-chain cases the key to a successful design is the choice of the amplification functions of the individual chains. The functions chosen must be suitable to identify as the amplification functions of the amplifier networks to be realized, they must provide the frequency characteristics desired and the level of amplification in the individual chains must be economically close to the limit set by the shunt capacitance. This key problem in the design procedure, the choice of a function approximating prescribed magnitude and phase characteristics, is a familiar one which appears in almost all network design problems and is called the approximation problem. In the previously developed design methods for stagger-tuned amplifiers (4, 5) this problem was solved implicitly despite the fact that the approach was from a different point of view. A moment's reflection upon the approximation problem as viewed here in connection with Eq. 1 indicates that what one is doing here is really closely related to stagger-tuning. Through solving the approximation problem one selects Z_o both to ensure that Z_o, the desired frequency characteristics of the amplifier, will be obtained and to ensure that Z_o is factorable into realizable interstage impedances. The process automatically allocates to the various stages characteristics which are complementary in a sense similar to that in which the individual stage characteristics of Wallman's amplifiers are complementary. Here we find a desirable amplification characteristic which is factorable into realizable stage characteristics while the view previously taken is to find realizable stage characteristics which combine to give a desired over-all amplification characteristic. The approach taken here, the choice of the amplification functions of the individual chains, is adapted to a solution of the approximation problem which is presented in a companion report (11). This report gives a method whereby one can obtain rational functions whose magnitude and argument approach desired characteristics over the range of interest of the frequency variable. The procedure of solution is to start with a trial set of positions of poles and zeros of Z_o (Eq. 1). The trial set of pole and zero locations are constrained to lead to the type of network of interest. The frequency characteristics corresponding to this trial set of locations only roughly approximate the prescribed frequency characteristic. The trial set is chosen from experience with related problems or purely on the basis of a set of characteristic curves given in the report. Observing the deviations of the characteristics of the trial set from the characteristics desired, one shifts the location of poles and zeros from the first trial position to a second trial position by a systematic procedure. The shift is such that the constraints imposed earlier to lead to realizable networks are not violated, and the frequency characteristics are improved. The whole process is a systematic fitting procedure of unusual flexibility which one continues until the resulting deviations are tolerable. For most practical problems the procedure is simple and brief. For precise approximations a more complicated but more powerful algebraic fitting procedure is presented which gives results obtainable by no other known method.

As is apparent later when the problem of paralleling chains of amplifiers is discussed in detail, the practical constraints which must be imposed on the form of amplification functions of the chains makes the flexible solution to the approximation problem the most important key in the success of the design procedure presented.

2.00 The Design of Amplifier Chains

The design of the basic building block of the amplifier, the amplifier chain, Fig. 5, proceeds most effectively when the basic limitation to its performance, the shunt capacitance, is precisely understood. The following development specifically defines the amplification-bandwidth limitation imposed by parasitic capacitance on amplifier chains. Following the development, a design procedure is presented for amplifier chains in which shunt capacitances appear.

2.10 The Limitation Imposed by Parasitic Capacitance for Two-Terminal Interstages

The presence of parasitic capacitance is always most directly apparent in the behavior of the circuit at high frequencies. If the network is a two-terminal network, as shown in Fig. 7, the driving-point impedance must approach $1/C\lambda$ and become zero as infinite frequency is reached, regardless of what other passive elements are connected in the box. Moreover, the driving-point impedance Z must be a positive real rational function*. The pertinent question with regard to the amplification-bandwidth limitations is: Does the behavior of Z at infinity along with its positive real character impose a limit on the level of magnitude of Z over a prescribed frequency range? The answer is yes.

The relationship which makes the limitation most evident is an application of Cauchy's theorem. Cauchy's theorem proves that the integral of a function around a closed contour is zero if the contour encloses no singularities of the function. The positive real character of Z insures that Z has no poles in the right half-plane. If one considers a contour bounding a large semicircle in the right half of the λ-plane (Fig. 8), Z is analytic inside the semicircle. As the radius of the circle is permitted to approach infinity, the value of Z on the circular part of the contour approaches $1/\lambda C$. Any analytic function of Z integrated over the contour shown in Fig. 8 will give zero, the component of that integral on the semicircle being dependent upon C. By choosing the proper functions of Z to be integrated in this matter, illuminating results on the implications of parasitic capacitance can be deduced. Bode has given a number of results arrived at on this basis. One of them is that the largest uniform level of magnitude of Z (Fig. 7) over ω_o rad/sec is $2/C\omega_o$ ohms.

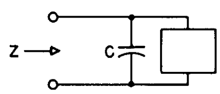

Fig. 7 Two-terminal impedance with parasitic capacitance.

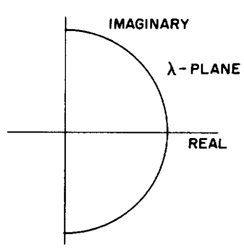

Fig. 8 Contour in λ plane.

Attention will be turned back to Eq. 1 and Fig. 5 for the case in which every interstage is a two-terminal network. Stated in the most convenient terms, the problem is: What is the maximum constant magnitude of Z_o which can be maintained fixed from zero to ω_o rad/sec in view of the fact that the factors of Z_o must be positive real functions approaching $1/\lambda C$ as λ approaches infinity?

The answer is most conveniently obtained through consideration of a relationship developed by Bode (12) which applies to Z_{i1}, Z_{i2} . . . or Z_{in} in Eq. 1. For a typical one of the impedances, one has Eq. 2.

$$\int_0^1 \frac{\alpha_{ip}}{\sqrt{1-\left(\frac{\omega}{\omega_o}\right)^2}}\, d\frac{\omega}{\omega_o} + \int_1^\infty \frac{\beta_{ip}+\frac{\pi}{2}}{\sqrt{\left(\frac{\omega}{\omega_o}\right)^2-1}}\, d\frac{\omega}{\omega_o} = \frac{\pi}{2}\ln\frac{2}{C\omega_o}\,;\ \ln Z_{ip} = \alpha_{ip} + j\beta_{ip} \quad . \quad (2)$$

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*A positive real function of λ satisfies these qualifying conditions: Re$Z \geqslant 0$ if Re $\gg 0$, Im $Z = 0$ if Im $\lambda = 0$.

Equation 2 was developed by Bode through the integration of

$$\ln \frac{\frac{1}{2}\left(\sqrt{\lambda^2 + \omega_o^2} + \lambda\right) CZ}{\sqrt{1 - \left(\frac{\lambda}{j\omega_o}\right)^2}}$$

around the contour in the complex frequency plane shown in Fig. 9. Since $|\beta_{ip}|$ can never exceed $\pi/2$ radians for $\lambda = j\omega$ (Z_{ip} being a positive real function), the second integral of Eq. 2 must not be negative and can only be zero in the limiting case in which $\beta_{ip} = -\pi/2$ for $\omega > \omega_o$.

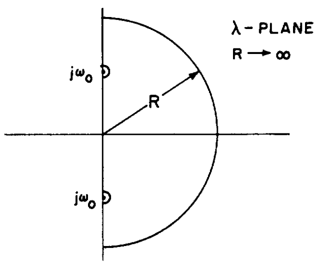


Fig. 9 Contour around which $\ln \frac{1}{2} \frac{\left(\sqrt{\lambda^2 + \omega_o^2} + \lambda\right) CZ}{\sqrt{1 - \left(\frac{\lambda}{j\omega_o}\right)^2}}$ is integrated to obtain Eq. 2.

If one writes a set of relationships of the nature of Eq. 2 and sums them for all factors of Z_o, Eq. 3 results.

$$\sum_{p=1}^{n} \int_{o}^{1} \frac{\alpha_{ip}}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}} \, d\frac{\omega}{\omega_o} + \sum_{p=1}^{n} \int_{1}^{\infty} \frac{\beta_{ip} + \frac{\pi}{2}}{\sqrt{\left(\frac{\omega}{\omega_o}\right)^2 - 1}} \, d\frac{\omega}{\omega_o} = \frac{\pi n}{2} \ln \frac{2}{C\omega_o} \quad . \quad (3)$$

The first integral of Eq. 3 is recognized to be

$$\int_{o}^{1} \frac{\ln |Z_o|}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}} \, d\frac{\omega}{\omega_o} \quad .$$

Clearly, this integral will be a maximum if the second integral of Eq. 3 is zero, as the second integral cannot be negative. This fact means that beyond ω_o rad/sec every interstage impedance should be a pure capacitive reactance if the magnitude of amplification up to ω_o rad/sec is to be maximized. The maximum constant level of $|Z_o|$ is given by Eq. 4.

$$\int_{o}^{1} \frac{\ln |Z_o| \max}{\sqrt{1 - \left(\frac{\omega}{\omega_o}\right)^2}} \, d\frac{\omega}{\omega_o} = \frac{\pi}{2} \ln |Z_o| = \frac{\pi}{2} \ln \left(\frac{2}{C\omega_o}\right)^n \quad . \quad (4)$$

Equation 4 reveals that $|Z_o|_{max}$ is $(2/C\omega_o)$ n ohms.

Equations 2, 3, and 4 all apply to cases in which the amplifier involved is of the low-pass type. That precisely the same limit applies to band-pass cases can be proved by use of a different integrand evaluated around a contour similar to that shown in Fig. 9. The integrand involves branch points at the edges of the band instead of at $\pm j\omega_o$ and the algebraic manipulations are more complicated.

The result expressed in Eq. 4 indicates that the limit of uniform amplification of n stages is just the n^{th} power of the limit of uniform amplification of a one-stage amplifier. In the proof just outlined the individual interstage impedances are not assumed to have uniform levels of impedance to ω_o rad/sec. The attainment of the limit requires only that the stages acting together provide uniform amplification over the pass band and have the maximum phase shift consistent with physical realizability beyond the pass band. At this point, one first appreciates that the method of getting a wide-band amplifier of cascaded stages through making every stage equally good and equally flat is unnecessary. This erroneous idea seems to have been widely held before Wallman's stagger-tuned amplifiers.

According to the results indicated in the foregoing discussion, the level of amplification over ω_o rad/sec which cannot be exceeded by an n-stage amplifier with two-terminal interstages is $(2g_m/C\omega_o)^n$.

2.11 Simple Amplifier Chains With Two-Terminal Interstages

The analysis just given establishes the upper limit of amplification set by parasitic capacitance, but gives little indication as to how close to that limit one can reach with practical networks. A design method for a simple low-pass amplifier gives an indication of the possibilities practically attainable. The two-terminal interstages of the design presented include both single-tuned circuits and RC circuits. For this design method the amplification is not exactly uniform; it exhibits equal maxima of deviation in the pass band. For half-power variations, the level of amplification is 50 percent of the theorectical limit. Accordingly, for 10 percent variations the amplification level over a band ω_o rad/sec wide is 0.242 $(g_m 2/C\omega_o)^n$. For an n + 1 stage amplifier, the amplification level is 0.242 $(g_m 2/C\omega_o)^{n+1}$. This result is rather interesting, in that it means that an n + 1 stage amplifier gives an amplification of an n-stage amplifier times the theoretical limit for one stage. The effectiveness per stage for a given tolerance on uniformity in the pass band increases with the number of stages.

The simple case presented also is used to illustrate the problems of design of an amplifier chain. The design of an amplifier chain can be divided into two parts: the approximation problem and the network realization. The approximation problem for this simple case is discussed first, and the network realization is taken up last.

2.12 The Approximation Problem for a Simple Amplifier Chain

The approximation problem, the problem of choosing a rational function which can be identified with Z_o, is logically attacked in two steps. The first step is the selection of a class of rational functions which satisfy the conditions of physical realizability of Z_o and are adapted to identification with the product of simple driving-point impedances. The conditions of physical realizability of Z_o are that it be the ratio of Hurwitz polynomials, that it approach $1/(C\lambda)^n$ as λ approaches infinity, and that

$$|\text{Arg } Z_o| \leqslant \frac{n\pi}{2} \quad .$$

The second step is the selection from the chosen class of functions of a type which exhibits a large approximately constant magnitude in the range of real frequency, zero to ω_o rad/sec, with an argument approaching $-n\pi/2$ beyond ω_o.

To proceed with the first step of the approximation procedure, one should select an appropriate class of functions. The function chosen as Z_o must clearly have n more poles than zeros, to exhibit the appropriate behavior at infinity. A simple class of functions which fulfills this requirement is the class of the form

$$\begin{aligned} Z_o(\lambda) &= \frac{1}{C^n(\lambda^n + a_{n-1}\lambda^{n-1} + \dots a_o)} \\ &= \frac{1}{C^n(\lambda - \lambda_{p1})(\lambda - \lambda_{p2}) \dots (\lambda - \lambda_{pn})} \end{aligned} \tag{5}$$

in which the denominator is a Hurwitz polynomial. The critical frequencies (poles) fall in conjugate pairs in the left half of the λ-plane, as shown in Fig. 10. For any point $\lambda = j\omega_a$ on the imaginary axis, Z_o is observed to be simply $1/C^n$ divided by the product of a set of vectors, each originating at one of the poles in the λ-plane, and all terminating at $j\omega_a$. This simple graphical picture verifies that $|\text{Arg } Z_o| \leqslant n\pi/2$ for any real frequency (λ is purely imaginary for real frequencies).

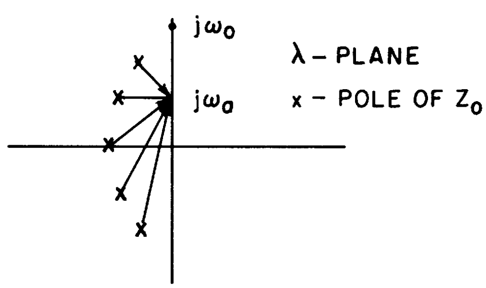


Fig. 10 Critical frequencies of Z_o as in Eq. 5.

In completing the solution of the approximation problem here, one needs to find a method of distributing the n poles so that the magnitude of the corresponding rational function will be large and uniform from $\lambda = 0$ to $\lambda = j\omega_o$, and so that Arg Z_o is nearly $-n\pi/2$ beyond $j\omega_o$. At this point, consideration of the potential analogy(11) is helpful. One recalls that the potential problem which corresponds to this approximation problem is the determination of the positions of n charged filaments (n = 5 for Fig. 10) to yield the highest potential along the line identified with the imaginary axis from zero to $j\omega_o$ in Fig. 10. From the potential analogy, one clearly sees that the poles should be brought as close to the imaginary axis as is consistent with having tolerable variations in magnitude of the function in the pass band. The distribution of poles should be such as to make the magnitude as uniform as possible.

One's acquaintance with the use of Tschebyscheff polynomials in the design of low-pass filters, along with the recollection that the poles of the transfer function were placed on a semi-ellipse there (much as they apparently need to be placed here), suggests the use of Tschebyscheff polynomials for this problem.

*The following summary presents the useful properties of Tschebyscheff polynomials for their use in the example at hand. By definition, the Tschebyscheff polynomial of order n is

$$V_n(\omega) = \cos(n \cos^{-1} \omega) \quad . \tag{6}$$

A change of variable makes clear that $V_n(\omega)$ is a polynomial

$$\cos\phi = \omega = \mathrm{Re}\, \epsilon^{j\phi} = \mathrm{Re}\left[\omega + j\sqrt{1-\omega^2}\right] \quad . \tag{7}$$

$$V_n(\omega) = \mathrm{Re}\, \epsilon^{jn\phi} = \mathrm{Re}\left[\omega + j\sqrt{1-\omega^2}\right]^n \quad , \tag{8}$$

which is a simple polynomial in ω. A recursion formula is found by use of the trigonometric identity

$$2 \cos n\phi \cos\phi = \cos(n+1)\phi + \cos(n-1)\phi \quad , \tag{9}$$

which may be written

$$\cos(n+1)\phi = 2\cos n\phi \cos\phi - \cos(n-1)\phi \quad . \tag{10}$$

The latter equation expressed in terms of ω gives

$$V_{n+1}(\omega) = 2\omega V_n(\omega) - V_{n-1}(\omega) \quad . \tag{11}$$

A few of the family of Tschebyscheff polynomials are: $V_1(\omega) = \omega$, $V_2(\omega) = 2\omega^2 - 1$, $V_3(\omega) = 4\omega^3 - 3\omega$, etc. Consideration of Eq. 6 reveals that the Tschebyscheff polynomials all oscillate in value between +1 and −1 in the range $-1 \leq \omega \leq 1$, and that beyond this range they become very large. The number of times the polynomial has the magnitude one in the range $-1 \leq \omega \leq 1$ is one greater than the order of the polynomial. The highest power of ω is always $2^{n-1}\omega^n$. In Fig. 11 sketches of the first four Tschebyscheff polynomials are shown.

In order to apply Tschebyscheff polynomials to the problem of amplifier design obtaining functions of the form of Eq. 5, one defines $|Z_o|^2_{\lambda=j\omega}$ in terms of a particular function involving Tschebyscheff polynomials. For a 1 rad/sec design, one sets

$$|Z_o|^2_{\lambda=j\omega} = \frac{M^2}{1 + \epsilon^2 V_n^2(\omega)} \quad . \tag{12}$$

---

* The presentation here follows that given by Professor Guillemin in M.I.T. Subject 6.562. An alternate enlightening derivation of the proper pole distribution to yield a Tschebyscheff approximation to constant magnitude in the pass band is given in a report by Fano (13). The method used in interesting, in that it applies results from the potential analogy after a conformal transformation has been used to make the potential problem simple to solve.

The general nature of $|Z_o|$ is indicated in Fig. 12. For Fig. 12,

$$\Delta = 1 - \sqrt{\frac{1}{1+\epsilon^2}} \quad \text{or} \quad \epsilon = \frac{[\Delta(2-\Delta)]^{1/2}}{1-\Delta} \quad . \tag{13}$$

In Eq. 19, ϵ is chosen to yield the tolerance desired (indicated as ΔM in Fig. 12). M is a constant chosen to make $Z_o \to 1/(C\lambda)^n$ as $\lambda \to \infty$. $|Z_o(\lambda)|^2_{\lambda=j\omega}$ is a function of ω^2, as is apparent from Eq. 12.

The problem at this stage is to determine Z_o from a chosen $|Z_o|^2_{\lambda=j\omega}$ as expressed in Eq. 12. That the magnitude of a rational function is not an analytic function is well known. However, $Z(\lambda)\,Z(-\lambda)$ is equal to the magnitude squared of $Z(\lambda)$ for $\lambda = j\omega$. Naturally the analytic function $Z(\lambda)\,Z(-\lambda)$ cannot be identified with $|Z(\lambda)|^2$, for $\lambda \neq j\omega$, but $Z(\lambda)\,Z(-\lambda)$ is precisely $|Z|^2$ if one restricts attention to the imaginary axis. Hence it is clear that if Eq. 12 be written as a function of λ, it must be the product of $Z_o(\lambda)$ and $Z_o(-\lambda)$. $Z_o(\lambda)$ is identified with the product of factors corresponding to poles and zeros in the left half-plane. To identify the poles of $Z(\lambda)$, one must find the roots of

$$1 + \epsilon^2 V_n^2(\omega) = 0 \quad , \tag{14}$$

which fall in the left half of the λ-plane. The roots of Eq. 14 may be most easily found through considering the corresponding function of ϕ.

$$0 = 1 + \epsilon^2 \cos^2 n\phi \quad . \tag{15}$$

The roots occur where

$$\cos n\phi = \pm j/\epsilon \quad . \tag{16}$$

Expressing ϕ in terms of its real and imaginary components, one has

$$\phi = \phi_r + j\phi_i \tag{17}$$

and

$$\begin{aligned} \cos n\phi &= \cos(n\phi_r + j\,n\,\phi_i) \\ &= \cos n\phi_r \cos j\,n\phi_i - \sin n\,\phi_r \sin j\,n\phi_i \\ &= \cosh n\,\phi_i \cos n\,\phi_r - j \sinh n\phi_i \sin n\phi_r \quad . \end{aligned} \tag{18}$$

By considering Eq. 16 and Eq. 18, one sees that the roots of Eq. 15 occur at

$$\phi_r = \frac{\pm(2k-1)\,\pi}{2n} \tag{19}$$

where k is any integer, and

$$n\,\phi_i = \sinh^{-1}\frac{1}{\epsilon} \quad . \tag{20}$$

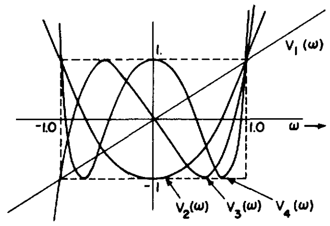


Fig. 11

Sketches of Tschebyscheff polynomials, $\cos(n \cos^{-1} \omega)$, through fourth order.

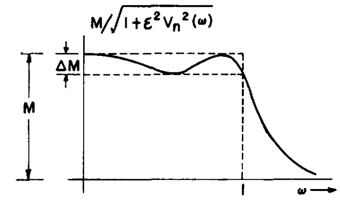


Fig. 12

Sketch of $\dfrac{M}{1 + \epsilon^2 V_n^2(\omega)}$.

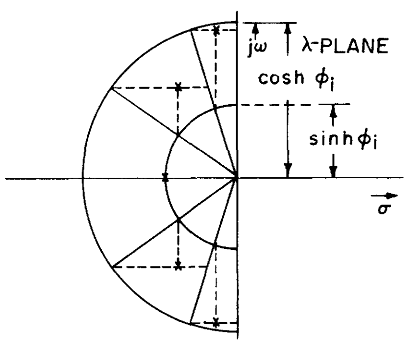


Fig. 13

Poles of $Z_o(\lambda)$ for n = 5 (see Eq. 21). Poles (crosses) lie on an ellipse.

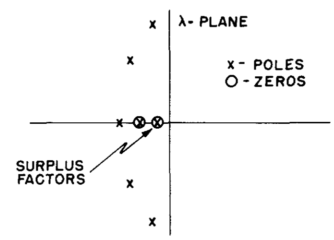


Fig. 14

Poles and zeros of $Z_o(\lambda)$ including surplus factors.

Finally, in the λ-plane the roots of $Z_o(\lambda)\, Z_o(-\lambda)$ are at

$$\omega = \frac{\lambda}{j} = \frac{\sigma + j\omega}{j} = \cosh \phi_i \cos \frac{\pm (2k-1)\pi}{2n} - j \sinh \phi_i \sin \frac{\pm (2k-1)\pi}{2n} \qquad (21)$$

where k is any integer. Figure 13 shows the location of poles of $Z_o(\lambda)$ for n = 5.

The process of finding Z_o of the form

$$Z_o(\lambda) = \frac{1}{C^n(\lambda^n + a_{n-1}\lambda^{n-1} + \ldots a_o)} \qquad (5)$$

in which

$$|Z_o|^2_{\lambda=j\omega} = \frac{M^2}{1 + \epsilon^2 V_n^2(\omega)} \qquad (12)$$

is completed when M is chosen. One recalls that the coefficient of the highest power of $V_n(\omega)$ is 2^{n-1}. For Eq. 5 and Eq. 12 to correspond at high frequencies, one must have

$$\frac{1}{C^n} = \frac{M}{\epsilon\, 2^{n-1}} \qquad (22)$$

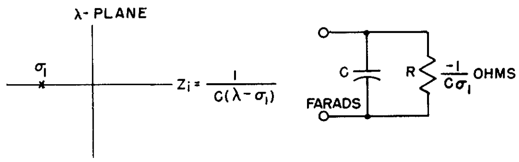


Fig. 15 Z_i with single poles.

This gives

$$M = \frac{\epsilon 2^{n-1}}{C^n} = \left(\frac{2}{C}\right)^n \times \frac{\epsilon}{2} \quad . \tag{23}$$

A summary of the approximation problem is helpful at this point. The first step is the choice of $|Z_o|^2_{\lambda=j\omega}$ with a characteristic, as shown in Fig. 12 for a one rad/sec case. In the selection of $|Z_o|^2_{\lambda=j\omega}$ one is guided by the number of stages desired in the choice of n; the tolerance permitted in the characteristic in the choice of ϵ ; the fact that $Z_o \to 1/(\lambda C)^n$ as $\lambda \to \infty$ in the choice of M. From the selected $|Z_o|^2_{\lambda=j\omega}$ one proceeds to $Z_o(\lambda)$ by identifying the location of its poles. The position of the poles is the most useful information in connection with the network realization.

Before proceeding to the network realization, it is useful to assess how close one has come to the theoretical limit imposed on $|Z_o|$ by the shunt capacitance. The theoretical maximum $|Z_o|$ for a one rad/sec case of n stages is $(2/C)^n$. The $|Z_o|$ attained for the function chosen is indicated in Fig. 12 as M. Its value, specified in Eq. 23, indicates that the fraction of the theoretical limit attained is $\epsilon/2$. If half-power variations in the pass band are permitted, $\epsilon/2$ is 0.50. If 10 percent variations in $|Z_o|$ are permitted, $\epsilon/2$ is 0.24. That the fractions indicated apply to the whole amplifier chain must be borne in mind. The individual stages of a six-stage amplifier for which $\epsilon/2$ is 0.50 really come individually to the $\sqrt[6]{0.50}$ or 0.89 of the limit for a single stage. This means that if a six-stage amplifier consisting of identical stages were to do as well as the amplifier chain mentioned, each stage would have to attain 89/100 of the limit for a single stage.

2.13 Network Realization

With Z_o chosen and expressed in terms of the position of its poles, the remaining problem is that of splitting Z_o into factors and identifying each as the driving-point impedance of an interstage.

$$Z_o = \frac{1}{C^n(\lambda - \lambda_{p1})(\lambda - \lambda_{p2}) \ldots (\lambda - \lambda_{pn})} \tag{5}$$

$$Z_o = Z_{i1} \times Z_{i2} \times \ldots Z_{in} \quad . \tag{24}$$

Each interstage impedance must approach $1/\lambda C$ as $\lambda \to \infty$; hence, the Z_i's must each have one more pole than zeros. In considering Z_o, one appreciates that the adding of surplus factors in the numerator and denominator is entirely appropriate, since the value of Z_o is unchanged by this process. With addition of factors, Eq. 5 is of the form

$$Z_o = \frac{1}{C^n(\lambda-\lambda_{p1})(\lambda-\lambda_{p2})\cdots(\lambda-\lambda_{pn})}\,\frac{(\lambda-\lambda_{s1})(\lambda-\lambda_{s2})\cdots(\lambda-\lambda_{sq})}{(\lambda-\lambda_{s1})(\lambda-\lambda_{s2})\cdots(\lambda-\lambda_{sq})} \quad . \quad (5')$$

That all of the surplus poles and zeros should be on the negative real axis for the network realization presented is seen presently as the method of choosing the superfluous factors is described. Accordingly the map of poles and zeros in the complex plane is indicated in Fig. 14.

At this point one should consider the simplest functions which could be associated with the Z_i's, bearing in mind that the function must have one more pole than zeros; in particular, must approach $1/\lambda C$ as $\lambda \to \infty$. The first case is a function with only one pole on the negative real axis of the λ-plane, as illustrated in Fig. 15. This case leads to the RC network shown, the element values of which are related to the pole position, as shown in the figure. One observes that there is no restriction on the position of the pole in Fig. 15 other than that it be on the negative real axis. In general, the position merely determines the size of R, the nearer the pole to the origin the smaller the value of R. The next case is that in which Z_i has a conjugate pair of poles. Then it must have a single zero on the negative real axis. For such a function to be positive real, $|\sigma_{Z1}| \leq 2\,|\sigma_{p1}|$, which means that the zero associated with a pair of poles in such an interstage impedance can lie anywhere in the region indicated in Fig. 16. The validity of this condition of positive real character is verified by considering the reciprocal of Z_i for Fig. 16. The consideration of that quantity also identifies the network realization.

$$\frac{1}{Z_i} = \frac{C(\lambda-\sigma_{p1}-j\omega_{p1})(\lambda-\sigma_{p1}+j\omega_{p1})}{\lambda-\sigma_{z1}}$$

$$= \frac{C(\lambda^2-2\sigma_{p1}\lambda+\sigma_{p1}^2+\omega_{p1}^2)}{\lambda-\sigma_{z1}}$$

$$= C\lambda + C(\sigma_{z1}-2\sigma_{p1}) + \frac{(\sigma_{z1}^2-2\sigma_{z1}\sigma_{p1}+\sigma_{p1}^2+\omega_{p1}^2)\,C}{\lambda-\sigma_{z1}} \quad . \quad (25)$$

For the real part of $1/Z_i$ is to be positive as $\lambda = j\omega \to j\infty$, $\sigma_{z1} - 2\sigma_{p1}$ must be greater than zero, and the condition $|\sigma_{z1}| < 2\,|\sigma_{p1}|$ is proved necessary. The condition is sufficient, since it leads to the possibility of identification of the network of Fig. 17 with Z_i. An examination of Fig. 17 reveals that the position of σ_{z1} in the range of possible values regulates the sizes of R and R_L. If $\sigma_{z1} = 2\sigma_{p1}$, $R = \infty$ and is accordingly absent. If $\sigma_{z1} = 0$, $R_L = 0$ and is absent. Any value of σ_{z1} between zero and $2\sigma_{p1}$ corresponds to a network in which R and R_L, both with finite values, are required.

The preceding discussion has shown two kinds of simple networks suitable to be identified with factors of Z_o, as shown in Eq. 24. The choice of superfluous factors as indicated in Eq. 5' and the subsequent network realization will now be outlined.

The first step is the grouping of conjugate pairs of poles. For each one of the pairs of poles, one chooses a superfluous factor for the numerator of such form that its zero $(+\sigma_{zi})$ fulfills the condition of physical realizability with the σ_{pi} of the poles, namely $|\sigma_{zi}| \leqslant 2\sigma_{pi}$. Each superfluous factor multiplied into the numerator is duplicated by a similar factor multiplied into the denominator. Finally, networks are associated with every pole on the negative real axis and with every conjugate pair of poles. Figure 18 illustrates the steps just outlined.

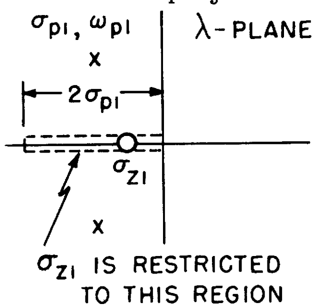


Fig. 16

Z_i with conjugate poles.

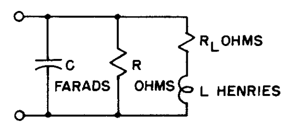


Fig. 17

Network identified with Z_i of Fig. 16.

$$Z_i = \frac{\lambda - \sigma_{z1}}{C(\lambda - \lambda_{p1})(\lambda - \lambda^*_{p1})}$$

$$\lambda_{p1} = \sigma_{p1} + j\omega_{p1} \quad |\sigma_{z1}| \leqslant 2|\sigma_{p1}| \quad .$$

$$R = \frac{1}{C(\sigma_{z1} - 2\sigma_{p1})}$$

$$L = \frac{1}{C\left[(\sigma_{z1} - 2\sigma_{p1})^2 + (\omega_{p1})^2\right]}$$

$$R_L = -\sigma_{z1} L \quad .$$

2.14 Illustrative Example

The design of a five-stage low-pass amplifier using 6AC7's and having a cut-off frequency of 16.5 megacycles is illustrated. In the design, which uses two-terminal interstages, a Tschebyscheff characteristic will be applied, giving 10 percent variations of amplification in the pass band. As indicated earlier, such a design gives an amplification which is 24 percent of the theoretical limit imposed by parasitic capacitance, or $0.24\,(g_m 2/C\omega_o)^5$, which is (using 9000 μ mhos as g_m and 25 μμf as C) 3920. Figure 19 shows a sketch of the amplification characteristic of the design. In the design, it is convenient to work first with a 1 rad/sec case. The C's here are $25 \times 10^{-12} \times 16.5 \times 10^6 \times 2\pi$, or 2.59×10^{-3} farads. The design will be transformed later to the 16.5-megacycle basis by diminishing the size of all inductances and capacitances. The poles of Z_o for the 1 rad/sec case are found through Eq. 21 and are located at -0.300, $-0.243 \pm j\,0.614$, and $-0.093 \pm j\,0.994$, as shown in Fig. 20. For the present example, Z_o is (for 1 rad/sec case)

$$\frac{1}{2.59 \times 10^{-3}\,(\lambda + 0.300)\,(\lambda^2 + 0.486\lambda + 0.243^2 + 0.614^2)\,(\lambda^2 + 0.186\lambda + (0.093)^2 + 0.994^2)} \tag{26}$$

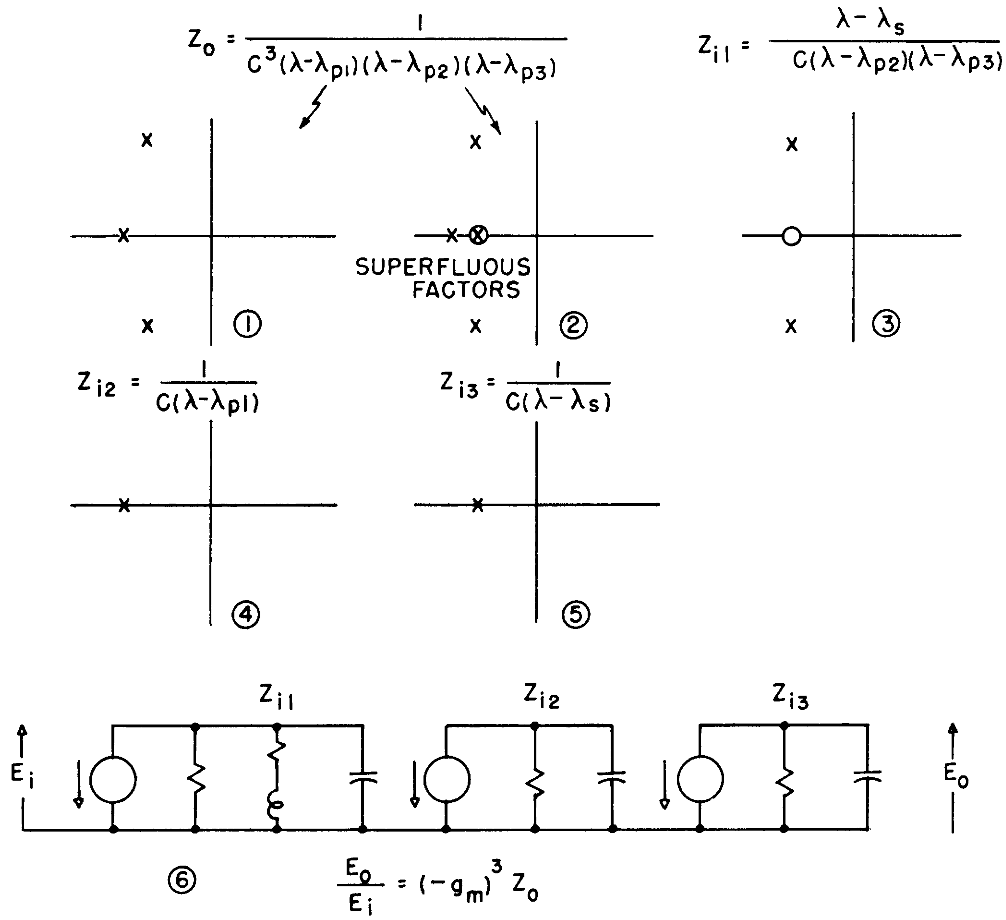


Fig. 18 Illustration of network realization of Z_o of form of Eq. 5. $Z_o = Z_{i1} Z_{i2} Z_{i3}$.

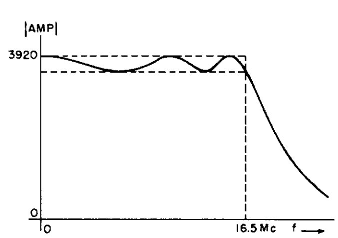


Fig. 19

Characteristic of five-stage, 16.5-Mc amplifier.

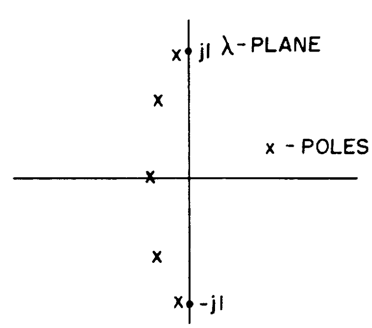


Fig. 20

Position of poles for illustrative example.

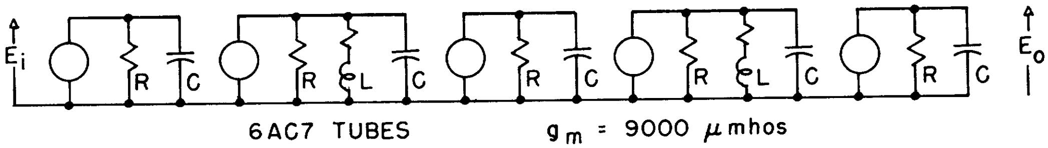


Fig. 21 Network for five-stage amplifier.

Element	1 rad/sec model	16.5×10^6 cps model
C(farads)	2.59×10^{-3}	25.0×10^{-12}
R_1 (ohms)	1287.0	1287.0
R_2	1589.0	1589.0
R_3	249.0	249.0
R_4	4150.0	4150.0
R_5	36.5	36.5
L_1 (henries)	1025.0	9.89×10^{-6}
L_2	391.0	3.75×10^{-6}

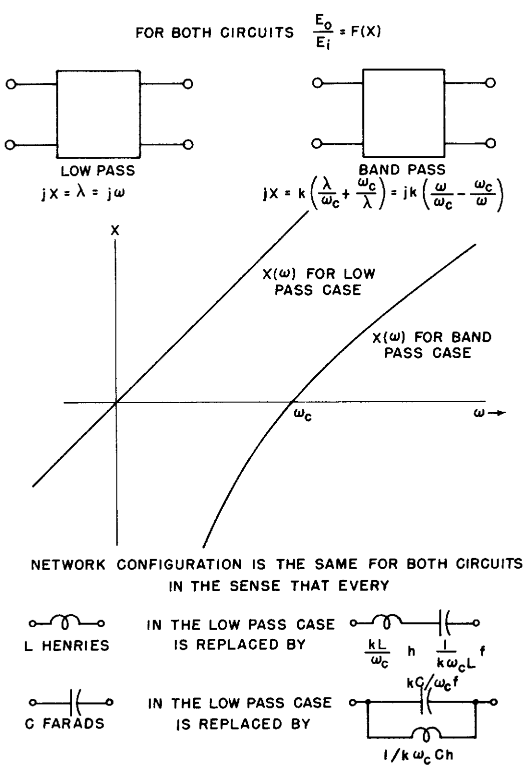


Fig. 22 Summary of low pass-band pass transformation.

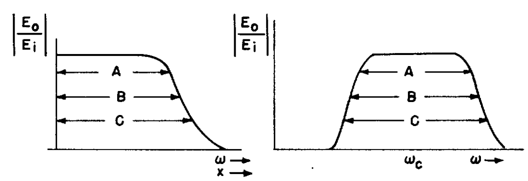


Fig. 23 Illustration of principle of conservation of bandwidth. $E_o/E_i = F(X)$. Left, low-pass case, $jX = j\omega$ Right, band-pass case, $jX = j\omega - j\omega_c^2/\omega$.

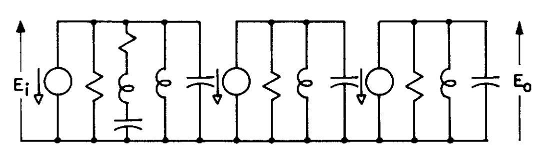


Fig. 24 Band-pass network corresponding to Fig. 18.

which is split into factors (using appropriate superfluous factors) giving

$$\frac{1}{2.59 \times 10^{-3}\,(\lambda + 0.300)} \times \frac{\lambda + 0.243}{2.59 \times 10^{-3}\,(\lambda^2 + 0.486\lambda + 0.243^2 + 0.614^2)} \times$$

$$\frac{1}{2.59 \times 10^{-3}\,(\lambda + 0.243)} \times \frac{\lambda + 0.093}{2.59 \times 10^{-3}\,(\lambda^2 + 0.186\lambda + 0.093^2 + 0.994^2} \times$$

$$\frac{1}{2.59 \times 10^{-3}\,(\lambda + 0.093)} \quad . \qquad (27)$$

According to the results of Fig. 16 and Fig. 17, this factoring leads to the equivalent circuit of Fig. 21 with the table of element values shown.

2.15 Extension of Design Method Presented to More General Specifications

The preceding development and examples have illustrated a compact design procedure for low-pass amplifiers consisting of one chain with two-terminal interstages. Such a design involving Tschebyscheff polynomials is very convenient and involves minor effort on the part of the designer. It is illuminating, in that one recognizes easily how effectively the resulting amplifier approaches the limit set by parasitic capacitance and the tolerance on uniformity in the pass band can be set in the beginning. The solution to the approximation problem is direct and explicit. The network so designed always has a low-pass characteristic and a set form of phase characteristic accompanying the choice of magnitude characteristic made. In the design process one does not directly govern the phase characteristic, but does this implicitly when he specifies the magnitude characteristic. The question to be raised at this point is: How can one design amplifiers with specifications on the characteristics which are more general than those of the low-pass amplifiers just discussed (an amplifier with band-pass characteristics or specified phase characteristics, for example)? In answering the question, two different procedures will be discussed. The first procedure applies long-established methods of transformation to yield networks with band-pass characteristics from a corresponding network with low-pass characteristics. This method turns out to be rather special and limited. The second procedure involves the use of the characteristics of a low-pass design as presented to be a very rough guide from which one proceeds to designs having much more complicated specifications to meet.

2.16 The Conventional Low-Pass to Band-Pass Transformation

In order to adapt a given low-pass design to a band-pass design, a method widely used is the replacing of every capacitance by a parallel-tuned circuit, and every inductance by a series-tuned circuit. Every tuned circuit is resonant at ω_c which is the geometric center of the pass band (14). The low-pass to band-pass transformation is really a frequency-variable transformation applied to the system function. Every λ in the low-pass function is replaced by $k\,(\lambda/\omega_c + \omega_c/\lambda)$ to obtain the band-pass function. One recalls that a system function is always equivalent to the ratio of determinants in which typical elements or components of elements are of the nature $L\lambda$ or $1/C\lambda$. The variable transformation applied to the elements of the determinants replace $L\lambda$ by $L k \lambda/\omega_c + L k \omega_c/\lambda$ and $1/C\lambda$ by

$$\frac{1}{\frac{C k \lambda}{\omega_c} + \frac{C k \omega_c}{\lambda}} ,$$

which are, respectively, the impedance of a series-tuned circuit and the impedance of a parallel-tuned circuit. The transformation represents a shift of the characteristics of the low-pass circuit to higher frequencies. Figure 22 shows the relationship between the behavior of the circuits and their elements. If k is equal to ω_c, the result is that every capacitance is replaced by the <u>same</u> capacitance in parallel with an inductance. Moreover, the behavior of such low-pass and band-pass circuits has an interesting feature called the conservation of bandwidth, which is illustrated in Fig. 23. The reason for the behavior illustrated in Fig. 23 is quite simple. $|E_o/E_i|$ is an even function of X. For the band-pass case if ω_a leads to X_a,

$$X_a = \omega_a - \frac{\omega_c^2}{\omega_a} ; \text{ then } \omega = \frac{\omega_c^2}{\omega_a} \text{ leads to } - X_a \quad . \tag{28}$$

$$\omega - \frac{\omega_c^2}{\omega} = X \quad \text{or} \quad \frac{\omega_c^2}{\omega_a} - \omega_a = - X_a \quad . \tag{29}$$

But the difference between the frequencies leading to X_a and $-X_a$ is $\omega_a - \omega_c^2/\omega_a$, which is X_a.

This case is an example of the fact that parasitic capacitance imposes the same restriction on bandwidth regardless of the position of the band in the spectrum.

To illustrate a network using the low-pass to band-pass transformation, Fig. 24 is shown. It represents the band-pass network corresponding to Fig. 18.

2.17 An Alternate View of the Low-Pass to Band-Pass Transformation

The discussion just completed illustrates the obtaining of band-pass networks from low-pass networks. The generality of the method is limited to this one problem. A second procedure will be introduced at this point which permits more general variations. It is introduced through consideration of the same low-pass to band-pass transformation considered from a different point of view. The discussion concerning the low-pass to band-pass transformation has been conducted, up to this point, in terms of the changes made in the network to get a band-pass from a low-pass network. The consideration of a shift in poles and zeros of the amplification function brought about by the same transformation is very illuminating. That poles and zeros of amplification occur for similar values of X for both the low-pass and the band-pass designs is clear from Fig. 22. For example, if for X_z there is a zero of the amplification function, then in the low-pass design there is a zero at

$$\lambda_z = jX_z = \sigma_z + j\omega_z \quad . \tag{30}$$

In the corresponding band-pass design (for which the frequency variable will be denoted by primed quantities) a zero will occur at the place where

$$\sigma_z + j\omega_z = \lambda' + \frac{\omega_c^2}{\lambda'} \quad . \tag{31}$$

Equation 31 is equivalent to :

$$\lambda'^2 - (\sigma_z + j\omega_z)\,\lambda' + \omega_c^2 = 0 \quad . \tag{32}$$

Zeros for the band-pass case are at

$$\lambda' = \frac{\sigma_z + j\omega_z}{2} \pm \sqrt{\frac{(\sigma_z + j\omega_z)^2}{4} - \omega_c^2} \quad . \tag{33}$$

The last expression indicates that if ω_c is large in comparison to σ_z and ω_z, in the band-pass case critical frequencies will be distributed around $\pm\, j\omega_c$ with half the displacement of the corresponding critical frequencies from $\omega = 0$ in the low-pass case. If σ_z and ω_z are not small compared to ω_c, the critical frequencies are displaced from

$$\pm\sqrt{\frac{(\sigma_z + j\omega_z)^2}{4} - \omega_c^2} \quad .$$

In addition to the internal critical frequencies covered by Eq. 33, it is clear that for any poles or zeros at zero in the low-pass case there will be poles or zeros at $\pm\, j\omega_c$ in the band-pass case. For any poles or zeros at infinity in the low-pass case there will be corresponding poles at both zero and infinity in the band-pass case. Figure 25

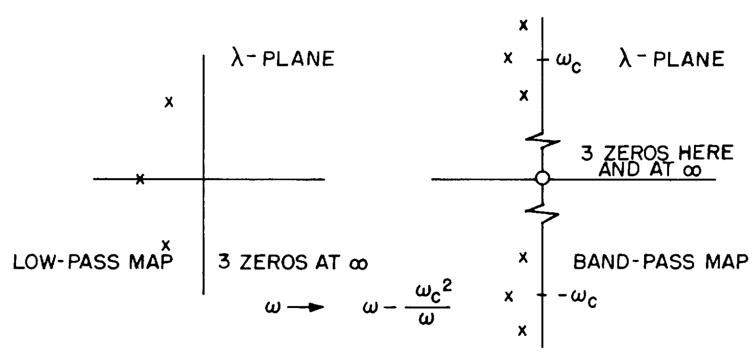


Fig. 25 Critical frequency maps for corresponding low and band-pass amplification functions.

illustrates the shift occurring in critical frequencies for a low-pass to band-pass transformation. Figure 25 and Eq. 33 indicate that if poles were placed on a semi-ellipse centered at zero in the low-pass case, corresponding poles are placed on a semi-ellipse centered at ω_c in the band-pass case if ω_c is sufficiently large. If, on the other hand, ω_c is smaller, then an elliptical distribution is somewhat warped in the shifting process, and poles are clustered toward the origin. If one views Fig. 25 in terms of the potential analogy, considering the critical frequencies to be represented by charged filaments, the warping is seen to be necessary to obtain a uniform potential near ω_c (when ω_c is small), in that the oppositely charged filaments at the origin can be counteracted by a clustering of the positively charged filaments toward the origin. For this particular case, Eq. 33 indicates exactly the kind of clustering which is required.

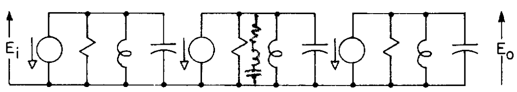


Fig. 26

Band-pass network for configuration of poles and zeros of Fig. 25.

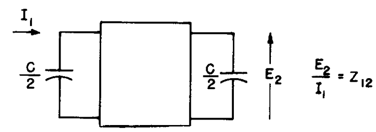


Fig. 27

Two-terminal-pair network with parasitic capacitance.

An interesting result of the consideration of the pole and zero shifting procedure for the low-pass to band-pass transformation is the fact that a band-pass network corresponding to Fig. 25 is that of Fig. 26. This is arrived at by simply considering Fig. 17. The circuit of Fig. 26 is exactly the equivalent of that of Fig. 24 and is somewhat simpler. The reason for the simplicity in this case is that the transformation has placed the right number of zeros at the origin, and the network realization can be carried out without the adding of superfluous factors as was necessary in Fig. 18. The low-pass to band-pass transformation done in terms of the network (as in Fig. 24) frequently leads to a more complicated network.

From the foregoing it is clear that the design of low-pass or band-pass amplifiers with Tschebyscheff behavior of the magnitude of amplification in the pass band is conveniently carried out through the use of Tschebyscheff polynomials to give an explicit solution. The preceding discussion has indicated that parasitic capacitance imposes the same limitation on amplification regardless of value of the center frequency of the band.

The design method just discussed is satisfactory as a final answer in a small fraction of amplifier design problems. Two examples follow, illustrating cases wherein the method discussed does not give a final answer.

If the coils used in an amplifier are appreciably lossy, the equivalent circuit of Fig. 26 cannot accurately represent the circuit. If the circuit required by the design needs lossless coils, and practical coils with loss are used in the physical circuit, then it is impossible to realize zeros of amplification at zero as indicated in Fig. 25. One would like to find an alternate solution to the approximation problem giving about the same amplification characteristics but not requiring zeros on the imaginary axis. The solution to the approximation problem, using Tschebyscheff polynomials, is fairly close to what is required but is not exactly what is needed. A practical solution to the problem is to place the zeros of amplification (Fig. 25) in the left half-plane and then to shift the poles slightly to compensate for any bad effect resulting in the characteristic arising from the zero shift. In this case the Tschebyscheff approximation has served as a first estimate, and the solution is subsequently fitted to the practical conditions. The technique of shifting pole and zero positions to achieve desired changes in the amplification function character is treated in the companion report (11). Through the use of this technique, a very wide variety of problems can be solved in which design conditions are accommodated which would be impossible in other approximation methods.

A second illustrative example in which the explicit approximation procedure using Tschebyscheff polynomials is unsatisfactory is a case in which a band-pass amplifier with approximately linear phase shift in the pass band is desired. The phase characteristics accompanying Tschebyscheff behavior of the magnitude are notably nonlinear. However, one can start with the pole and zero positions from a Tschebyscheff approximation, and make appropriate shifts of the poles and zeros to accomplish the desired changes in the phase characteristic. The procedure for this adjustment process is given in the companion report (11).

This section has presented to this point a rather complete account of the design of amplifier chains with two-terminal interstages. The limitation imposed by parasitic capacitance is very definite. It has been pointed out that the most convenient quantities through which to define network behavior and to solve the approximation problem are the pole and zero positions of the amplification function of the chain. Network realizations have been presented in which network configuration and element values are specified in terms of the pole and zero positions of the amplification function. The use of Tschebyscheff polynomials in obtaining low- or band-pass designs has been discussed. An introduction has been made of the method of altering a given first trial using such an approximation as a first estimate, to be followed by better approximations to the desired characteristics. The method described is applicable in the design of amplifiers composed of a single chain, or of amplifiers comprising several chains which are paralleled at the load. As will be seen later, the essence of a successful design procedure for a multi-chain amplifier is the ability to control the characteristics of the individual chains, thereby shaping them to fit each other in an effective manner.

2.20 Two-Terminal-Pair Interstages

The results developed and stated to this point in the present section all apply to amplifiers with two-terminal interstages. The remainder of the section is devoted to two-terminal-pair interstages. First, the limitation imposed by parasitic capacitance on the amplification bandwidth product is discussed. Unfortunately, the evaluation and interpretation of the limitation is much more difficult for the two-terminal-pair case than it is for the two-terminal case, and the results cannot be expressed in definite, conclusive form for the most general situations. However, the information available does provide a useful guide in the design of amplifiers with two-terminal-pair interstages. The significant point is that the limitation on amplification imposed by parasitic capacitance is lessened if the parasitic capacitance can be split into two parts by a coupling network. It is difficult to put a limit on the advantage which is gained by splitting. A set of simple, practical, double-tuned circuits are considered. Their configuration and element values are associated with the position of poles and zeros of transfer impedances in a manner consistent with the treatment of the two-terminal case already discussed. The presentation, as a whole, is made to facilitate the application of the approximation procedure developed in R.L.E. Report No. 145.

2.21 Limitation Imposed by Parasitic Capacitance for Two-Terminal-Pair Interstages

The amplification of a chain of n amplifiers is

$$\frac{E_o}{E_i} = (-g_m)^n Z_{i1} Z_{i2} \cdots Z_{in} \quad . \tag{1}$$

When the amplifiers are connected by two-terminal-pair interstages, the Z_i's of Eq. 1 are transfer impedances. Accordingly, the logical question to ask in connection with Eq. 1 is: What limitation is imposed on the uniform level of magnitude of $Z_{i1} Z_{i2} \cdots Z_{in}$ by the fact that parasitic capacitance shunts both the input and output terminals of each interstage? An exact answer is not available for a product of impedances. Bode has answered the question for a single interstage. His solution will be discussed at this point.

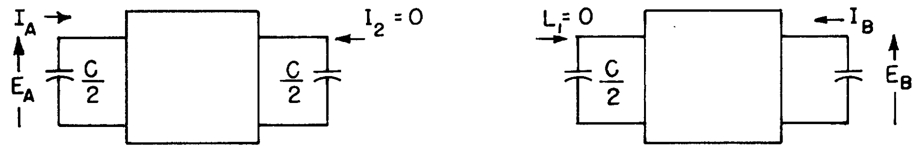
2.22 Bode's Result (15)

The maximum constant level of transfer impedance over ω_o rad/sec of a passive two-terminal-pair network is $\pi^2/2C\omega_o$ ohms. C is the sum of equal shunt capacitances at the terminals of the network (Fig. 27). Further, the network which attains the limit must be symmetrical. Though no comprehensive study will be made here of Bode's attack on the problem, it is helpful to indicate wherein the limitation arises, to better define its nature.

Gewertz has shown the necessary and sufficient conditions of physical realizability for two-terminal-pair networks. Bode applies these results in the form which states that for any real frequency ($\lambda = j\omega$) the product of real components of open-circuit impedances from the two-terminal pairs is greater than, or at least equal to, the square of the real part of the transfer impedance. These quantities are indicated in Fig. 28. The condition mentioned indicates that if there is a limit on the driving-point impedances, then there is implicitly a limit on the transfer impedance. The parasitic capacitances do impose a limit on the impedances Z_A and Z_B. This limit is in the form of the resistance integral theorem (16), which is:

$$\int_0^\infty R_A \, d\omega = \int_0^\infty R_B \, d\omega = \frac{\pi}{C} \quad . \tag{34}$$

It is improper to assign a level of Z_A or Z_B so high that Eq. 34 cannot be satisfied for the capacitance present. But the level of magnitude of Z_{12} is implicitly limited by the



In realizable networks

$$\mathrm{Re}\,\frac{E_A}{I_A}\;\mathrm{Re}\,\frac{E_B}{I_B} \geqslant \left(\mathrm{Re}\,\frac{E_2}{I_A}\right)^2$$

Fig. 28 Gewertz's condition of physical realizability as applied by Bode.

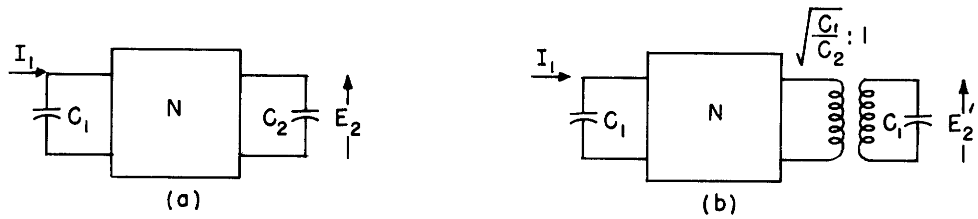


Fig. 29 Circuit providing maximum uniform $|Z_{12}|$ over ω_o rad/sec

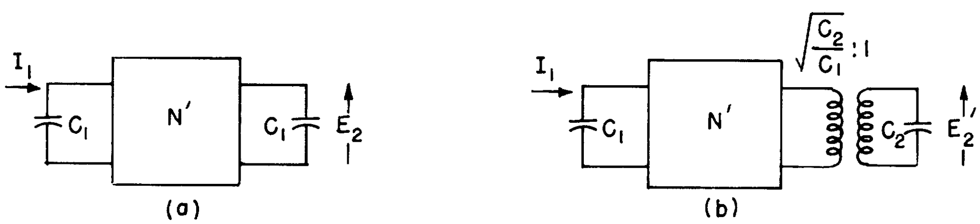


Fig. 30 Circuit fullfilling relation 36 with equality sign.

(a). $\left|\frac{E_2}{I_1}\right| = \frac{\pi^2}{4 C_1 \omega_o}$ (b). $\left|\frac{E_2'}{I_1}\right| = \frac{\pi^2}{4\sqrt{C_1 C_2}\,\omega_o}$

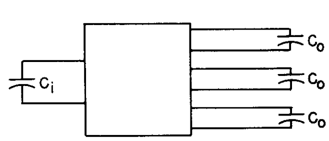


Fig. 31 Multi-terminal-pair network with parasitic capacitance.

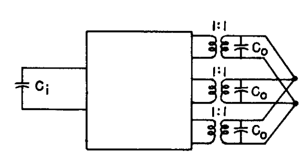


Fig. 32 Two terminal-pair network constructed from the network of Fig. 31.

condition on Z_A and Z_B, since $R_A R_B \geqslant R_{12}^2$. In particular, the constant level of $|Z_{12}|$ over any frequency range is limited. Through a clever but complicated argument, Bode used these ideas to deduce that

$$|Z_{12}| \leqslant \frac{\pi^2}{2\omega_o C} \qquad . \tag{35}$$

over a frequency band ω_o rad/sec wide.

2.23 Added Information Deduced from Bode's Result

The result indicated above is very definite and conclusive. However, it applies to a very specific problem. When one generalizes the conditions, the problem becomes much more difficult. For example, as was pointed out at the outset, the quantity in which one is interested for an amplifier chain is the product of Z_i's. The result given by Bode indicates that a chain of n stages, each of which has uniformly flat characteristics, could provide an amplification of $(-g_m)^n(\pi^2/2C\omega_o)^n$. The question as to whether it is necessary that all interstages have a flat magnitude of transfer impedance to attain the limit is not answered. Later, an argument will be given to indicate that double-tuned circuits with the same total parasitic capacitance per stage as a single-tuned circuit yield an over-all amplification of more than twice as much. However, the discussion in that connection is not at all an evaluation of the limit, but merely points out that the staggering used earlier for single-tuned circuits is effective in double-tuned circuits as well. At this point a few special cases will be discussed in which the result of Bode gives a useful indication. The application of these special cases arises at a later point in connection with the paralleling of amplifier chains and the analysis of Percival's amplifier. However, this is the most convenient point for their introduction.

The first case is that of a two-terminal-pair network with different capacitances at the two-terminal-pairs. Suppose the network in Fig. 29 (a) gives the maximum uniform magnitude of transfer impedance over ω_o rad/sec which is possible in view of its parasitic capacitance. The problem is to evaluate the maximum. Bode's result indicates that the $|Z_{12}'|$ or $|E_2'/I_1|$ for (b) of Fig. 29 is $\leqslant \pi^2/4C_1\omega_o$. Accordingly, for (a),

$$\left|\frac{E_2}{I_1}\right| \leqslant \frac{\pi^2}{4\sqrt{C_1 C_2}\,\omega_o} \qquad . \tag{36}$$

Circuit (a) can provide no larger magnitude, for if it did, an ideal transformer as in (b) would give a circuit with equal terminal capacitances which exceed the limit proved to exist by Bode. However, Eq. 36 can be fulfilled with the equality sign. Consider Fig. 30, in which circuit (a) provides the maximum Z_{12} for equal capacitance. Circuit (b) fulfills Eq. 36 with the equality sign.

The next case to be considered is illustrated in Fig. 31. For any given input, the outputs of the terminal pairs at the output are all identical. The voltage which could be uniformly maintained at these n output terminal pairs (n = 3 for Fig. 31) over a range of frequencies for a unit input current can be no greater than could be maintained at a single pair of output terminals with n C_o farads in shunt. That the limit indicated must apply is seen by considering a two-terminal-pair network (Fig. 32) constructed from the network of Fig. 31. The output capacitance of the network of Fig. 32 is n C_o farads (n = 3 for the example), and it is apparent that the voltage appearing at its terminals is the same as appears at the individual output-terminal pairs of Fig. 31. Accordingly the previous results indicate that the maximum uniform level of transfer impedance from the input to any output of Fig. 31 is $\pi^2/4\sqrt{n\,C_o\,C_i}\,\omega_o$ ohms over ω_o rad/sec.

A simple application of the result stated for Fig. 31 lies in the consideration of transfer impedance from an internal terminal pair of a recurrent structure (Fig. 33) in which capacitance shunts every terminal pair. The recurrent structure is properly terminated. The result is useful in determining the amount of voltage which is provided the output terminals by the current source shown inside the structure. The result given above indicates that the transfer impedance cannot be maintained at a uniform level over a frequency band ω_o rad/sec wide greater than $\pi^2/4C\sqrt{2}\,\omega_o$ ohms. * This quantity is one which cannot be exceeded. The argument presented cannot insure that the limit can be attained.

2.24 Simple Two-Terminal-Pair Networks Useful in Amplifier Chains

At this point one appreciates that two-terminal-pair interstages possess greater potentialities than do two-terminal networks for the problem of designing amplifier chains. However, the results just given do not supply, in themselves, a simple practical answer to the design of two-terminal-pair interstages. One is led, however, to consider symmetrical networks when the capacitances are equal at the two pairs of terminals. The use of single-tuned circuits and RC networks was successful in the case of two-terminal interstages. One accordingly is led to use double-tuned circuits for the problem of two-terminal-pair interstages. In the following, a presentation of the characteristics of practical two-terminal-pair interstages is given. One finds that the success with double-tuned circuits is just as the analysis of Bode indicates that it should be. There is a significant improvement over what is possible with two-terminal interstages.

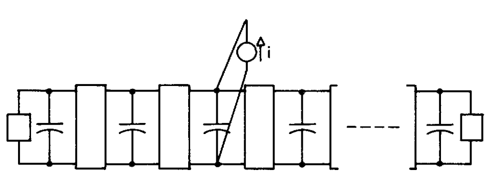


Fig. 33 Properly terminated recurrent structure supplied by internal current source.

A basic structure which one can identify with a number of double-tuned circuits is shown in Fig. 34. On the basis of the analysis of it, results can be given directly for specific examples. The steps of evaluation of E_2/E_1 are shown in Fig. 34.

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* This kind of problem arises in connection with the amplifier of Percival, which is discussed later.

Figure 35 shows a single-tuned circuit and the impedance in factored form showing the relation between element values and the pole and zero positions of the impedance. The relationships indicated there with the result of Fig. 34 lead directly to the amplification functions of the first three circuits of Fig. 36. The amplification function of the fourth circuit of Fig. 36 is arrived at in a manner similar to that used in Fig. 34. It is similarly straightforward and consequently will not be presented in detail here.

The results indicated in Fig. 36 are in a form which is convenient for the use of a design technique for two-terminal-pair interstages similar to that applied for two-terminal interstages. The design method has two steps. The first is to specify the position of poles and zeros of the amplification function. The second is the identification of networks from Fig. 36. The two steps are interrelated. In specifying the position of poles and zeros, one must be aware of the relationships illustrated in Fig. 36. For instance, (2) of Fig. 36 requires that the pairs of poles associated with the network have the same displacement from the imaginary axis. In (3) of Fig. 36, the ratio of displacement of the real-axis pole to the displacement from the imaginary axis of the poles in the plane must be no more than two, or if a lossy inductor is used, it must be less than two. Such restrictions, if they are accounted for, demand a very flexible approximation procedure. An elliptical array of the nature resulting from a Tschebyscheff approximation would frequently not be appropriate. However, small shifts from an elliptical form can be made to cause conformity with the requirements for realization, and at the same time to yield behavior of the phase and magnitude characteristics which are within tolerable limits.

2.25 An Evaluation of the Effectiveness of Circuits of Fig. 36

With the results of Fig. 36 in mind, along with the discussion which has just been given of the use of those results, the question which arises is: How effective are chains of amplifiers using these interstages in approaching the limit set by parasitic capacitance? A useful and informative way of answering the question is to make a comparison between the level of magnitude of amplification of an amplifier chain utilizing double-tuned circuits, and the level of amplification of an amplifier chain using single-tuned circuits and having the same total of shunt capacitance per stage. Such a comparison is useful, since the discussion of amplifier chains using single-tuned circuits was given earlier in this chapter. There an evaluation was made of approximately how closely they approach the limit set by shunt capacitance. The following comparison indicates that two-terminal-pair networks provide a level of amplification more than twice as large per stage as do the two-terminal networks.

In the comparison to be made between single- and double-tuned circuits, attention will be directed to (2) of Fig. 36 and to a band-pass design with which it will be associated. Precisely the same kind of argument could be presented for (1) and (4), with a difference only in the details of evaluation. All three provide the same advantage over single-tuned circuits. In the discussion, the mean frequency of the band will be assumed to be several

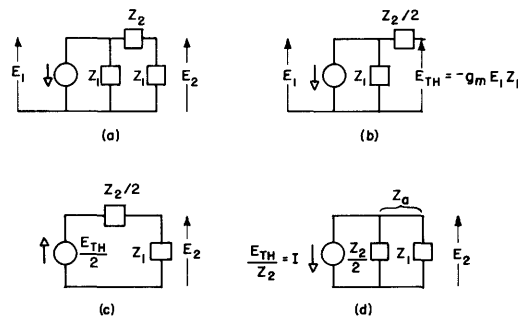


Fig. 34 Evaluation of E_2/E_1 for simple two-terminal pair network.

$$\frac{E_2}{E_1} = -g_m Z_1 \frac{Z_a}{Z_2}$$

(a) Two-terminal-pair interstage. (b) Application of Thevenin's theorem.

(c) For reasons of symmetry E_2 of (a) is E_2 of (c). (d) Circuit equivalent to (c).

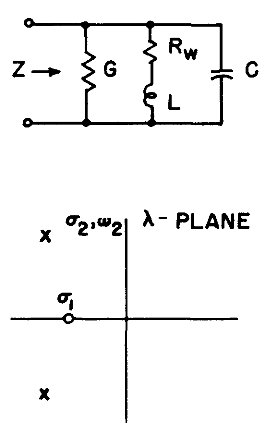


Fig. 35 Driving-point impedance of single-tuned circuit.

$$Z = \frac{\lambda + \frac{R_w}{L}}{C\left[\lambda^2 + \left(\frac{R_w}{L} + \frac{G}{C}\right)\lambda + \frac{1 + R_w G}{LC}\right]} = \frac{\lambda - \sigma_1}{C(\lambda^2 - 2\sigma_2\lambda + \sigma_2^2 + \omega_2^2)}$$

$$-\sigma_1 = \frac{R_w}{L} \qquad -\sigma_2 = \frac{1}{2}\left[\frac{R_w}{L} + \frac{G}{C}\right] \qquad \sigma_2^2 + \omega_2^2 = \frac{1 + R_w G}{LC}$$

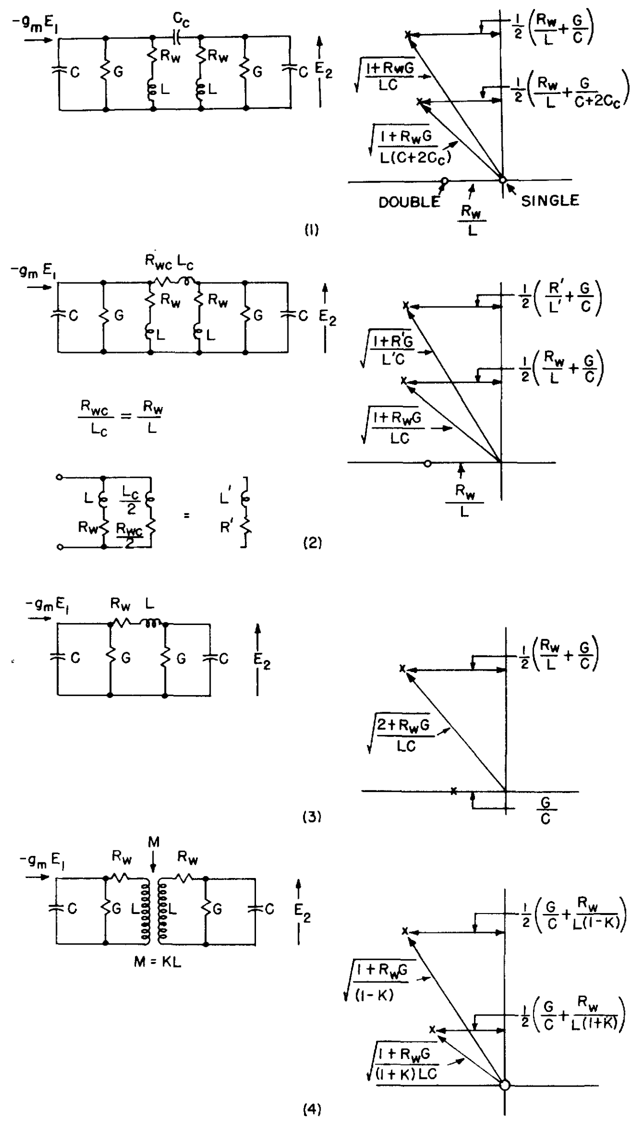


Fig. 36 Two-terminal-pair networks with amplification functions and maps of critical frequencies.

$$(1)\quad \frac{E_2}{E_1} = \frac{-g_m\, C_c\, \lambda\left(\lambda + \frac{R_w}{L}\right)^2}{C(C+2C_c)\left[\lambda^2 + \left(\frac{R_w}{L} + \frac{G}{C}\right)\lambda + \frac{1+R_wG}{LC}\right]\left[\lambda^2 + \left(\frac{R_w}{L} + \frac{G}{C+2C_c}\right)\lambda + \frac{1+R_wG}{L(C+2C_c)}\right]}$$

$$(2)\quad \frac{E_2}{E_1} = \frac{-g_m\quad 1\quad \left(\lambda + \frac{R_w}{L}\right)}{C\quad L_cC\left[\lambda^2 + \lambda\left(\frac{R_w}{L} + \frac{G}{C}\right) + \frac{1+R_wG}{LC}\right]\left[\lambda^2 + \lambda\left(\frac{R'}{L'} + \frac{G}{C}\right) + \frac{1+RG}{L'C}\right]}$$

$$(3)\quad \frac{E_2}{E_1} = \frac{-g_m}{C\quad LC\quad \left(\lambda + \frac{G}{C}\right)\left[\lambda^2 + \lambda\left(\frac{R_w}{L} + \frac{G}{C}\right) + \frac{2+R_wG}{LC}\right]}$$

$$(4)\quad \frac{E_2}{E_1} = \frac{-g_m\qquad \lambda}{C\quad CL\left(\frac{1}{K} - K\right)\left[\lambda^2 + \lambda\left(\frac{R_w}{L(1+K)} + \frac{G}{C}\right) + \frac{1+R_wG}{LC(1+K)}\right]\left[\lambda^2 + \lambda\left(\frac{R_w}{L(1-K)} + \frac{G}{C}\right) + \frac{1+R_wG}{LC(1-K)}\right]}$$

times the bandwidth. This condition is not a restriction on generality, in view of the principle of conservation of bandwidth which indicates that shunt capacitance imposes the same limit on amplification for a specified bandwidth regardless of the location of the center of the band. However, use of this assumption simplifies the comparison to be made. A comparison of (3) of Fig. 36 with a two-terminal interstage can be made along the same kind of approach as for a low-pass case, with a demonstration of about the same advantage as exhibited by (1), (2), and (4).

An evaluation of the level of magnitude of amplification for a chain of amplifiers involves two elements of information about the amplification function: the constant multiplier, the number and the manner of distribution of these critical frequencies. The significance of this statement and its implication for the present problem is clarified by a moment's reflection on the form of Eq. 5 when a Tschebyscheff characteristic is obtained for a low-pass circuit. From Eq. 23 one sees that the peak value of the amplification function of Eq. 5 is $g_m^n \epsilon 2^{n-1}/C^n$. The constant multiplier is g_m^n/C^n and it represents a constraint imposed by the tube transconductance and parasitic capacitance. The factor $\epsilon\ 2^{n-1}$ is determined by a combination of the number of critical frequencies and their distribution. For instance, if one holds the number of critical frequencies (poles here) fixed and changes their distribution by bringing them on an ellipse of smaller minor axis he finds that ϵ is increased and the level of the amplification function is raised. Alternately, if one distributes a larger number of poles on the same ellipse he finds $\epsilon\ 2^{n-1}$ or its equivalent (Eq. 20) $2^{n-1}/\sinh n\,\phi_i$ goes to $2^n/\sinh (n+1)\,\phi_i$.

The amplification functions for the networks of Fig. 36 are shown in factored form, bringing into evidence the constant multipliers and the positions of poles and zeros. To estimate the level of amplification in an amplifier made up of a number of these stages in cascade, one needs to consider the products of multiplying factors and the influence of numbers and positions of poles. The influence of number and positions of poles on the amplification function can be estimated by recalling the distribution of poles and zeros for a band-pass case with Tschebyscheff behavior. If an n^{th}-order case is considered, the critical frequencies are as shown in Fig. 37. The number of poles influences the magnitude of the function over the pass band. For a function of the form indicated in Fig. 37.

$$F = \frac{(\lambda - \lambda_{z1})(\lambda - \lambda_{z2}) \ldots (\lambda - \lambda_{zn})}{(\lambda - \lambda_{p1})(\lambda - \bar{\lambda}_{p1}) \ldots (\lambda - \bar{\lambda}_{pn})}, \quad \lambda_{z1} = \lambda_{z2} \ldots = \lambda_{zn} = 0 \qquad (37)$$

one recognizes that the magnitude near $\lambda = j\omega_o$ is $\epsilon\, 2^{n-1}/\omega_a^n$, as is seen from Eq. 12 and Eq. 23 and the discussion made in connection with those equations. Evidently the addition of two poles and a zero (and a redistribution of the original ones to lie on a different ellipse yielding the same tolerance in the pass band as the original) results in a function with its magnitude being that of the original multiplied by $2/\omega_a$. The functions associated with amplifier chains with interstages of the forms considered in Fig. 36 will not have poles distributed exactly on ellipses. However, the poles will be reasonably close to this situation, and accordingly it is very appropriate to estimate that the level of magnitude varies with the number of poles and zeros, as with the Tschebyscheff case.

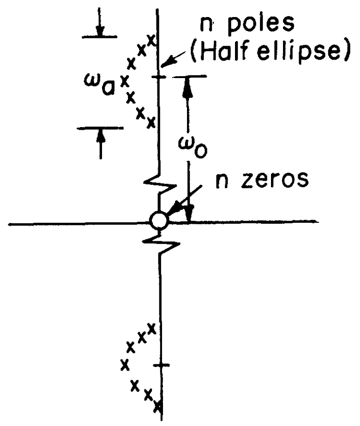


Fig. 37 Distribution of poles and zeros for Tschebyscheff behavior in pass band.

To compare the effectiveness of two-terminal-pair networks with two-terminal networks, one will consider the factor by which the amplification function is multiplied for the increase of one in the number of stages for both kinds of network. If one compares an n and n + 1-stage amplifier with two-terminal interstages and with the same tolerance on variation in magnitude over the pass band, the magnitude of amplification of an n + 1-stage amplifier will be found to be the amplification of an n-stage amplifier multiplied by $g_m/C_s \; 2/\omega_a$ where g_m is the tube transconductance and C_s is the total shunt capacitance of the stage (see Eq. 23 and accompanying discussion). The comparison of an n and an n + 1-stage amplifier using two-terminal-pair interstages of the form of (2) in Fig. 36 is somewhat complicated and involves a few approximations. The amplification function for the n + 1-stage amplifier has two more pairs of poles and one more zero than the n-stage amplifier. The new constant factor which is added is $g_m/C L_c C$. The addition of the two pairs of poles and the zero involves a multiplication of the level by $(2/\omega_a)^2 \; 1/\omega_o$. The result just given is easily verified in noting that the first factor would be appropriate for the addition of two pairs of poles and two zeros. The second factor indicates the influence of cancelling one zero. The remaining part of the evaluation consists of expressing the factor $g_m/L_c C^2 (2/\omega_a)^2 \; 1/\omega_o$ in terms which lend themselves to comparison with $g_m/C_s \; 2/\omega_a$.

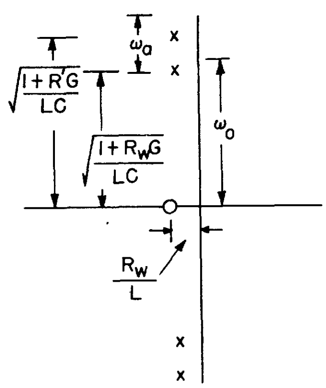


Fig. 38 Poles and zeros associated with one stage of amplifier with interstages of form of (2) in Fig. 36.

$$\omega_o \gg \omega_a \qquad \frac{R_W}{L} \ll \omega_a \qquad \sqrt{\frac{1 + R_W G}{LC}} \cong \sqrt{\frac{1}{LC}}$$

$$\sqrt{\frac{1 + R'G}{L'C}} \cong \sqrt{\frac{1}{L'C}} = \sqrt{\frac{L + L_c}{LL_c C}} = \sqrt{\frac{1}{LC}} \sqrt{1 + \frac{1}{L_c}} \cong \sqrt{\frac{1}{LC}} \left(1 + \frac{L}{2L_c}\right)$$

Figure 38 shows the position of the poles and zero added to get an n + 1-stage case along with some simplifying approximations. Clearly one will want to obtain as large a multiplier $g_m/C L_c C$ as is possible. Reference to Fig. 36 and to Fig. 38 indicates that L_c

should be as small as possible. This means that the circuit should be as closely coupled as possible if the multiplier is to be maximized. This also means that the pair of poles near $j\omega_o$ which are associated with one interstage must be as far apart as possible. The problem of obtaining a large product of constant multiplying factors is one of doing the appropriate kind of pairing of poles near $j\omega_o$. Consider Fig. 39 which illustrates the problem. Obviously, if one paired a and h, the smallest possible L_c would result, and the stage corresponding would have the largest possible factor $g_m/C\,L_c C$. However, such a pairing would accordingly greatly diminish the multiplying factors of other stages. It is easy to see that the multiplying factors of each stage should be about the same if the maximum product of them is to obtain. Accordingly the appropriate pairing for Fig. 39 is a – e, b – f, c – g, and d – h. Since the poles are clustered at the ends and are less dense in the middle near $j\omega_o$, the distance between poles in a pair is always greater than $\omega_a/2$. Referring to Fig. 38, one has

$$\sqrt{\frac{1}{LC}}\ (1 + \frac{L}{L_c}) - \sqrt{\frac{1}{LC}} > \frac{\omega_a}{2}\,, \tag{38}$$

or

$$\sqrt{\frac{1}{LC}}\ \frac{L}{L_c} > \frac{\omega_a}{2} \quad . \tag{39}$$

Since $\omega_a << \omega_o$, one can write

$$\frac{1}{\sqrt{LC}} \cong \omega_o \tag{40}$$

$$\omega_o\,\frac{LC}{L_c C} > \frac{\omega_a}{2} \tag{41}$$

$$\frac{1}{\omega_o L_c C} > \frac{\omega_a}{2} \tag{42}$$

$$\frac{1}{L_c C} > \frac{\omega_o\,\omega_a}{2} \quad . \tag{43}$$

From Eq. 43 one sees that the factor by which the amplification level of an n-stage amplification level of an n-stage amplifier is multiplied to obtain the amplification level of an n + 1-stage amplifier is greater than

$$\frac{g_m}{C}\ \frac{\omega_o\,\omega_a}{2}\ \frac{4}{\omega_o\,\omega_a^2} = \frac{2g_m}{C\,\omega_a} \quad . \tag{44}$$

But $C_s = 2C$ if the two-terminal and the two-terminal-pair interstages are to have the same shunt capacitance. Finally one has

$$\frac{2g_m}{C\,\omega_a} = \frac{4g_m}{C_s\,\omega_a} = 2\,\frac{g_m}{C_s\,\omega_a} \tag{45}$$

which indicates that the use of double-tuned circuits is more than twice as effective in providing an amplification over a prescribed bandwidth as the use of corresponding single-tuned circuits. The argument presented here has been roughly substantiated by a numerical example given in Sect. 3.20. The level of amplification of the amplifier chains approximate the expected values.

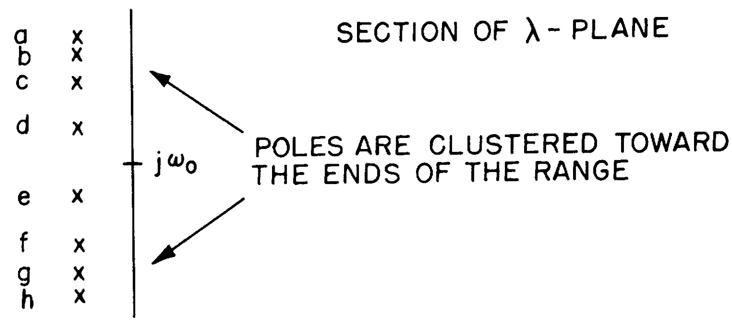


Fig. 39 An array of poles near $\lambda = j\omega_0$ to be paired for identification with (2) of Fig. 36.

The material presented in this section has indicated the limitations imposed by parasitic capacitance on the amplification of amplifier chains. The amplification functions of simple practical networks have been given in terms of their pole and zero positions in the complex plane. The problem of design of amplifier chains involves two interrelated steps: the choice of an amplification function in terms of poles and zeros to yield desired frequency characteristics, and the identification of network configuration and element values with the amplification function chosen. The design of amplifier chains employing Tschebyscheff polynomials to solve the approximation problem has been discussed in detail. The procedure employed in the design for cases in which the approximation technique of R.L.E. Report 145 is used follows essentially the same lines and one employs essentially the same technique in designing individual chains for a multichain amplifier.

3.00 Paralleled Chains of Amplifiers

It is possible to attain any desired level of amplification with a chain of amplifiers by cascading a sufficient number of stages, provided the band of frequencies to be amplified is narrow enough. (See Eq. 23 and the accompanying discussion, and Eq. 45 and the discussion accompanying it.) When one is faced with the problem of designing an amplifier with a bandwidth so broad that the maximum level of amplification per stage is less than one, clearly he must devise a circuit differing from the conventional amplifier chain. The method of design discussed in the following applies paralleled chains of amplifiers supplied from the same signal and paralleled at the load (Fig. 40). The different chains amplify different ranges of frequencies, each range being made sufficiently small that cascading of stages for the individual chain is effective in increasing the amplification. The individual chains are designed to operate effectively together.

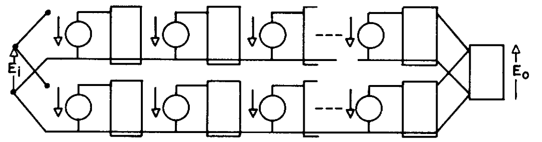


Fig. 40 A two-chain amplifier.

This requires the control of their individual frequency characteristics by methods discussed in Sect. 2.00 and Report 145. The number of chains and the number of tubes per chain are chosen to minimize the number of tubes required. The fact that one can control the frequency characteristics of the individual chains means that an amplifier using paralleled chains has frequency characteristics which can be flexibly controlled.

3.10 The Method of Paralleling Amplifier Chains

The amplifier shown in Fig. 40 has an amplification which is the sum of the amplifications of the individual amplifier chains. The amplification of the individual chains is simply

$$\left(\frac{E_o}{E_i}\right)_{\ell \text{ or } h} = (-g_m)^n Z_{i1} Z_{i2} \ldots Z_{in} \quad (46)$$

where the Z_i's are the transfer impedances of the interstages. (They reduce to driving-point impedances when the interstages are two-terminal networks.) This expression differs from those discussed earlier only in that Z_{in}, the impedance corresponding to the last interstage, is the transfer impedance from one pair of terminals to another in a three-terminal-pair network. The only influence of the extra pair of terminals in connection with the amplification of the chain is a decrease in level of amplification. This is due simply to the fact that some current from the final tube in one chain is diverted from the load to the final plate circuit of the other chain. The same sort of situation applies where there are more than two chains. As will be appreciated later, this diminished level is a loss which is easily compensated by an increase in the number of stages.

The problem of specifying frequency characteristics for the individual chains of a multi-chain amplifier is very similar to that of specifying the frequency characteristic of a single-chain amplifier, as discussed in Sect. 2.00. The reason is that each chain amplifies a different frequency range, and except for the transition ranges of frequency (where appreciable amplification comes from two chains simultaneously), the total amplification of the amplifier at any particular frequency is due essentially to only one of the amplifier chains. Accordingly the only problem unique to the design of amplifier chains for a multi-chain amplifier is that of insuring that the frequencies in the transition ranges are amplified properly. This problem is solved by properly controlling the frequency characteristics of the individual chains. One kind of frequency characteristics for individual chains which would give a uniform characteristic over the transition range is shown in Fig. 41. Clearly the characteristics indicated there are restricted far beyond the point of necessity. Tolerable variations in the characteristic of the amplifier as a whole permit the phase shift and magnitude of the lower frequency chain, for instance, to depart rather violently from the straight lines near the upper edge of the transition range, because of its small size and correspondingly slight influence on the sum there. In fact, the straight line variation in the magnitude is not at all necessary; the kind of variation indicated in Fig. 42 is equally appropriate. The phase characteristics indicated in Fig. 41 are similarly restricted far beyond practical needs. The requirements on phase shift for the two chains are simply that near the mid-point of the transition range the phase shifts should be about equivalent [differ by about n (360) degrees], and that in the remainder of the region the rate of phase shift should be about the same for the two chains. Naturally, when the magnitude of amplification becomes small, the corresponding phase shift is less important and needs less to be controlled accurately. One needs simply to start with a first estimate of the amplification functions of the two chains which have roughly the type of characteristics indicated in Fig. 41, and then to proceed with the problem of adjusting them to complement each other properly. Precisely this procedure is used in the illustrative example given later.

In connection with the network configuration, one appreciates that a problem unique to paralleled chains is the design of the multi-terminal-pair load impedance shown in Fig. 40. This network must "serve two or more masters". It must be designed to be suitable as the load impedance of a number of different amplifier chains. One possibility is that of designing a network which has equal transfer impedances from all terminal-pairs to the load (Fig. 43). If one designs the network so that every Z_{in} is uniform over the frequency range of the complete amplifier, the limit imposed by parasitic capacitance is the same as that discussed for Fig. 31 in Sect. 2.23. For such a case,

$$\left|Z_{in}\right|_{max} \quad \frac{\pi^2}{4C\sqrt{m}\,\omega_o} \text{ ohms} \qquad (47)$$

where the bandwidth of the amplifier is ω_o rad/sec.

An enlightening comparison which evaluates the expense of paralleling amplifier chains is that between the two amplifier chains described as follows. The first is terminated in an m + 1 terminal-pair network which exhibits a uniform magnitude of transfer impedance over ω_o rad/sec, the bandwidth of the m chain amplifier. The second amplifier chain is identical, except that it is terminated in a two-terminal-pair network which exhibits a uniform magnitude of transfer impedance over the bandwidth of the individual chain, ω_o/m rad/sec. A comparison between the levels of amplification of the two chains is now made. This is essentially a comparison between the transfer impedances of the last stages which can be made through Eq. 47. In the first place, the $1/\sqrt{m}$ in Eq. 47 is used in the multi-terminal-pair case, because there are m terminal-pairs feeding the load. In the second place, ω_o, applying to the bandwidth of the whole amplifier, is m times the bandwidth of a given chain. Hence the ω_o in Eq. 47 for the multi-terminal-pair case is m times as large as the corresponding quantity for the two-terminal-pair case. Accordingly, one sees that the individual chains of an

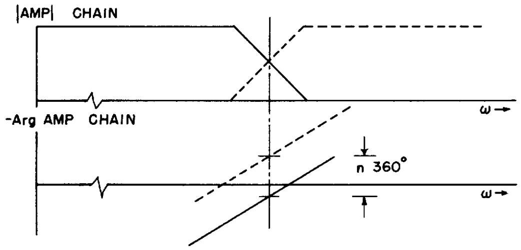


Fig. 41 Idealized characteristics of chains of amplifiers giving uniform amplification in the transition region.

– lower frequency chain
-- higher frequency chain

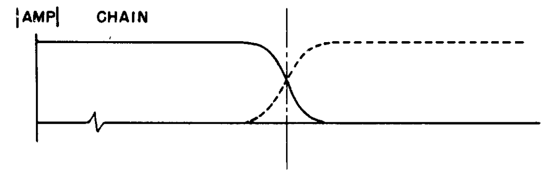


Fig. 42 Alternate magnitude characteristics serving the same purpose as those in Fig. 41

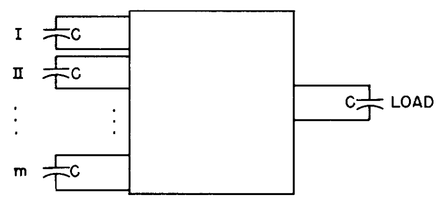
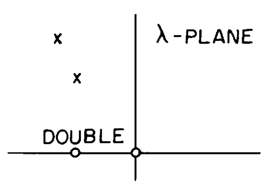
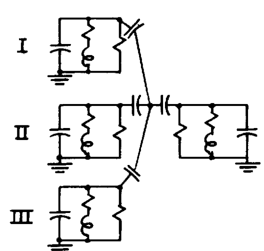


Fig. 43 Multi-terminal-pair network which might be applied in an m-chain amplifier.

$$Z_{in}^{I} = Z_{in}^{II} = \ldots = Z_{in}^{m}$$



MAP OF POLE AND ZERO POSITIONS
FOR EITHER CASE



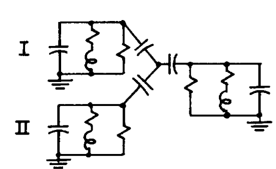


Fig. 44 Simple symmetrical multi-terminal-pair networks corresponding to Fig. 36 (1).

Left, three-chain case

$$Z_{in}^{I} = Z_{in}^{II} = Z_{in}^{III}$$

Right, two-chain case.

$$Z_{in}^{I} = Z_{in}^{II}$$

All arms of the networks are identical.

m-chain amplifier are $1/m^{3/2}$ as effective as they would be if the final stage of the chain fed into a two-terminal-pair network whose transfer impedance was held to the limit over only ω_o/m rad/sec.

The limit presented in Eq. 47 applies only where $|Z_{in}|$ is uniform over the whole frequency range. However, in connection with a practical design such a restriction is unnecessary. As has been observed before in similar situations, the simple tuned circuits are more practical than the complicated circuits which provide a uniform magnitude of transfer impedance over the band. Though the simple circuits are more practical, one finds the problem of assessing the limit of amplification much more difficult when they are used. In this case the limit of amplification set by parasitic capacitance for the simple circuits will be estimated by comparison with Eq. 47. The illustrative example presented later demonstrates that symmetrical multi-terminal-pair networks corresponding to those in Fig. 36 are simple and effective. Typical multi-terminal-pair networks corresponding to Fig. 36(1) with a map of pole and zero positions for two cases are shown in Fig. 44. In comparing Z_{in} for such a multi-terminal-pair network with that of a corresponding two-terminal-pair network, one finds two distinctions analogous to those pointed out in connection with Eq. 47. In the first place, the fact that m terminal-pairs supply the load causes the transfer impedance from any one of the terminal-pairs to the load to be $2/m + 1$ of what it would be for the two-terminal-pair case. This reduction in transfer impedance essentially comes about through the fact that current from one of the driving pairs is diverted from the load into the other $m - 1$ driving circuits. There is a parallelism between this factor and the $1/\sqrt{m}$ which arises in Eq. 47 for essentially the same reason. One observes, for instance, that for $m = 2$, $1/\sqrt{m} = 0.707$ and $2/m + 1 = 0.667$, while for $m = 3$, $1/\sqrt{m} = 0.577$ and $2/m + 1 = 0.500$. The second point of comparison of the case illustrated in Fig. 44 with that of Fig. 43 and Eq. 47 is observed in considering the poles of Z_{in} as sketched in Fig. 44. Recall that the poles of transfer impedance for single amplifier chains were distributed near the imaginary axis in the range of ω where the amplification was to be large. In Fig. 44, the poles serve all of the amplifier chains and consequently will not be placed in the most favorable position for any one of the amplifier chains. Accordingly there is a reduction in amplification for a multi-terminal-pair load below that possible with a corresponding two-terminal-pair load. Observe that this situation arising in connection with Fig. 44 is analogous to that arising in connection with Fig. 43 wherein the magnitude of transfer impedance was held fixed over the whole band of the amplifier rather than the band of the individual chains. The reduction in amplification resulting because one uses networks of the form shown in Fig. 44 rather than two-terminal pairs is difficult to assess exactly. This is particularly true, since the remainder of the amplifier chains are designed to complement the initial choice of the multi-terminal-pair network to be used at the load, and the adjustment procedure – in treating the amplifier design from this point on – focuses attention on getting the best compromise possible rather than evaluating the limitation caused by the multi-terminal-pair network at the load.

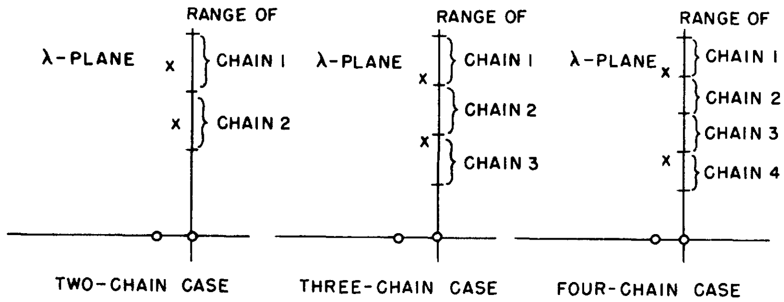


Fig. 45 Rough estimates of suitable pole positions for networks of Fig. 44 for different numbers of chains.

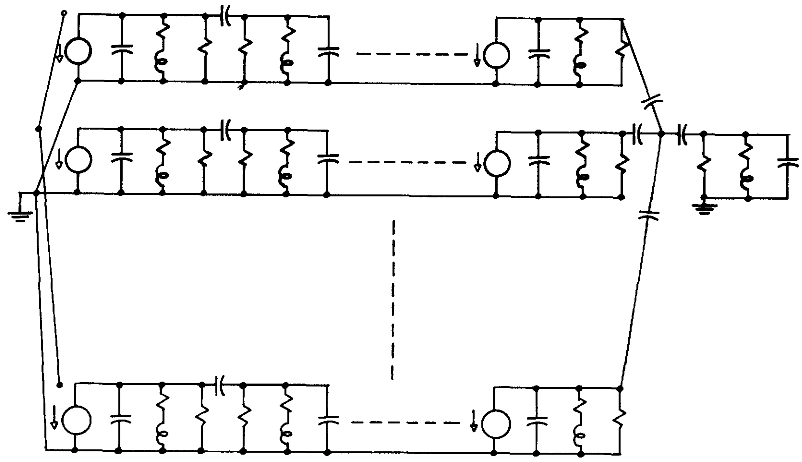


Fig. 46 Multi-chain amplifier of the type to which Eq. 48 applies.

From a practical point of view, there are two needs in this connection. These needs are a practical guide to the designing of the network in Fig. 44 (essentially a means of deciding where the poles ought to be placed for a given number of chains) and a very approximate relationship which indicates how much amplification can be expected in an m-chain amplifier using n stages per chain where the transconductance of the tubes, the parasitic capacitance per stage, and the bandwidth of the amplifier are all specified. At this point these topics will be considered.

In connection with choosing pole positions for transfer impedance for multi-terminal-pair symmetrical networks analogous to those shown in Fig. 36, there are two considerations to bear in mind. First, the multi-terminal-pair network should not favor a particular chain and its band of frequencies. Second, from the constant multipliers shown in Fig. 36, one observes that as close coupling as possible should be used to increase the constant multiplier. The close coupling is associated with as great separation as possible of the adjacent poles (see Fig. 44, for instance). Rough estimates at suitable pole positions for a number of cases based on the above remarks are shown in Fig. 45. In a particular problem, one adapts the starting point to the particular problem, using estimates of the nature of those of Fig. 45 at the outset. A great amount of freedom is permissible in that the design of the remainder of the chain complements the previous choice of the multi-terminal-pair network.

An empirical relationship giving the approximate level of amplification of paralleled chains of amplifiers is useful as a guide in the design. Such a relationship will be given at this point. The relationship applies to amplifier chains using two-terminal-pair symmetrical interstages, as shown in Fig. 36, with analogous multi-terminal-pair interstages at the output. Every terminal pair has associated with C farads (Fig. 46). The vacuum tubes have transconductances of g_m mhos. The result applies to a design in which the variation of amplification over ω_o rad/sec is less than 30 percent. For an m-chain amplifier with n stages per chain, the level of amplification is

$$\left| \text{amp} \right| = \frac{0.2}{m^{3/2}} \left(\frac{\pi^2 g_m \, m}{4C \, \omega_o} \right)^n \quad . \tag{48}$$

The factors of Eq. 48 can be identified with results previously discussed in this thesis. The quantity in parentheses is the theoretical limit of uniform amplification of an n-stage amplifier chain providing uniform amplification of all frequencies in a band ω_o/m rad/sec wide in which two-terminal-pair networks are used. The factor $1/m^{3/2}$ is used to account for the fact that the final circuit is an m + 1 terminal-pair network and the factor 0.2 is associated with the fact that simple tuned circuits are used rather than networks which have the maximum uniform level of transfer impedance over ω_o/m rad/sec (see Eq. 23 and the accompanying discussion). Equation 48 can be expected to give results with an "order of magnitude" accuracy. The application of it to the numerical examples considered indicates a reasonable accuracy. From Eq. 48, it is a simple matter to determine how many chains and how many stages are required to provide a specified amplification over a specified band (there are no restrictions on either). Further, one can determine how to do a specific job with the least number of tubes.

An enlightening problem is the determination of the number of chains to be used for a specified total number of tubes to give the maximum level of amplification, using Eq. 48 as a basis. For this problem, define the total number of tubes

$$t = m \times n \tag{49}$$

and the normalized bandwidth

$$b_n = \frac{\omega_o}{\frac{\pi^2 g_m}{4C}} \quad . \tag{50}$$

In terms of these, the level of amplification

$$\left| \text{Amp} \right| = \frac{2}{m^{3/2}} \left(\frac{m}{b_n} \right)^{t/m} \quad . \tag{51}$$

Clearly, if one chooses m such that ln |Amp| is a maximum, |Amp| will be a maximum also.

$$\ln |\mathrm{Amp}| = \ln 0.2 - \frac{3}{2} \ln m + \frac{t}{m} \ln m - \frac{t}{m} \ln b_n \quad . \qquad (52)$$

Differentiating Eq. 52 with respect to m, one has

$$\frac{d \ln |\mathrm{Amp}|}{dm} = - \frac{3}{2m} + \frac{t}{m^2} - \frac{t}{m^2} \ln m + \frac{t}{m^2} \ln b_n \quad . \qquad (53)$$

Clearly, Eq. 53 will be zero when

$$\frac{3}{2} = \frac{t}{m} (1 + \ln b_n - \ln m) = n(1 + \ln b_n - \ln m) \quad . \qquad (54)$$

Using Eq. 54, one can find the appropriate values of m and n giving the maximum amplification over the normalized band b_n for a fixed total number of tubes. Figure 47 shows a plot of optimum values of m and n for several values of the normalized bandwidth. Calculated values of amplification are shown for several points in the plot. The amplifications shown correspond to the numbers of chains and stages per chain indicated by the coordinates of the point, and to a bandwidth determined by interpolation between the curves.

In connection with all of the foregoing development, it must be borne in mind that the input voltage of the amplifier has been defined as the voltage which appears across all of the input terminal-pairs. Frequently the source supplying the amplifier provides a voltage at the grids determined by the amount of capacitance which it feeds into. If such is the case, one obtains a smaller voltage at the grids of the first tubes in the chains than would be obtained if the amplifier consists of only one chain. In any case this loss associated with paralleling can be evaluated in precisely the same manner as was done for the multi-terminal-pair circuit at the output. Further, one can take it into account in choosing the number of chains and tubes per chain to provide a given amplification and bandwidth most economically through a straight-forward extension of the technique already presented.

The procedure to be followed in the design of broad band amplifiers is clear at this point. First one determines the number of chains and the number of stages to be used, through consideration of Eq. 48. Next a suitable multi-terminal-pair network for the output is chosen and the position of its poles and zeros of transfer impedance are selected. Once the multi-terminal-pair network is chosen, a preliminary choice of pole and zero positions for the amplification functions for the amplifier chains is made. The frequency characteristics corresponding to the initial choice are determined, and one applies the adjustment procedure to improve the characteristics. Here the procedure differs from that used for a single-chain amplifier only in the fact that the phase and magnitude of any chain must be adjusted to complement the characteristics of the

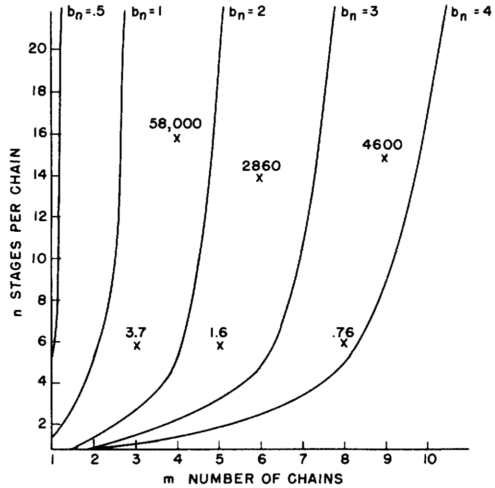


Fig. 47 Plot showing the optimum number of chains and stages per chain for a series of different normalized bandwidths.

adjacent chain in the transition region.

3.20 An Illustrative Example

To illustrate the method of designing amplifier chains which may be paralleled satisfactorily, the design of a two-chain amplifier using seven tubes will be presented at this point. (One observes that this problem statement starts beyond the usual specification which is the nature of amplification desired, the level and bandwidth. Accordingly this example does not illustrate the step involving the choice of the number of chains and stages.) The amplifier obtained, using 6AK5's, * gives an amplification of 32 over 100 megacycles with substantially linear phase shift. In the design presented, practical details – coupling condensers, by-pass condensers, and power supplies – are omitted, to focus attention on the points of importance in the present discussion. Accordingly the equivalent plate circuit is the only circuit representation used.

The low-pass chain is intended to pass the very low frequencies unattenuated except by failure due to coupling and by-pass condensers (which are omitted from consideration). The first step in the design of the low-pass chain is the selection of the type of interstages to be used. One refers to Fig. 36. The multi-terminal-pair network at the output will be one of the band-pass types (1), (2), or (4). Since very low frequencies are to be passed, the amplification function of the chain should not have zeros near the origin. This fact suggests the use of two interstages of the form of (2), Fig. 36, and two of the form of (3), Fig. 36. Such a choice permits the poles of transfer impedance of (3) on the real axis to cancel the zeros of transfer impedance of (2) on the real axis. This choice means that the multi-terminal-pair network will be a three-terminal-pair version of (2) of Fig. 36.

The preliminary choice of the amplification function of the low-pass chain (normalized to 1 rad/sec cut-off) is:

$$\frac{K}{(\lambda-\bar{\lambda}_1)(\lambda-\lambda_1)(\lambda-\lambda_2)\ldots(\lambda-\bar{\lambda}_6)} = \mathrm{Amp}\,\Big|_{\mathrm{low}} \tag{55}$$

* The 6AK5 has a transductance of 5000 μ mhos and a shunt capacitance of 11 μμf.

in which

$$\begin{aligned} \lambda_1 &= -0.2 + j.2 & \lambda_4 &= -0.1 + j.8 \\ \lambda_2 &= -0.2 + j.7 & \lambda_5 &= -0.1 + j1.55 \\ \lambda_3 &= -0.15 + j.45 & \lambda_6 &= -0.1 + j.9 \end{aligned} \quad . \qquad (56)$$

K is a constant to be associated with the constant factors shown in Fig. 36. It is determined later when the networks are finally chosen. In Eq. 56, λ_1 and λ_2 are associated with a double-tuned circuit which is inductance-coupled, and λ_4 and λ_5 are associated with the multi-terminal-pair circuit at the output. λ_3 and λ_6 are associated with the networks of Type (3) of Fig. 36. A preliminary calculation of the magnitude and phase shift for the function indicated reveals that its magnitude is reasonably uniform in the pass band, and that the phase shift is about -720 degrees at the cut-off point, $\omega = 1$.

Now one turns to the preliminary choice of the amplification function for the band-pass chain. One component of the function, that due to the multi-terminal-pair circuit, has already been chosen. The band-pass chain should, for $\omega = 1$, have a phase shift equivalent to -720 degrees, and in the range $\omega = 1$ to $\omega = 2$ should exhibit an added phase shift of -720 degrees. A very rough calculation indicates that one capacitance-coupled interstage and one inductance-coupled interstage, in addition to the multi-terminal-pair circuit already chosen, should yield approximately the result desired.

The preliminary choice of the amplification function for the band-pass chain (pass band is from $\omega = 1$ to approximately $\omega = 2$) is

$$\text{Amp}\big|_{\text{band pass}} = \frac{K'(\lambda - \sigma_{z1})(\lambda - \sigma_{z2})(\lambda - \sigma_{z3})(\lambda - \sigma_{z4})\lambda}{(\lambda - \lambda_4)(\lambda - \bar{\lambda}_4)(\lambda - \lambda_5)\ldots(\lambda - \bar{\lambda}_{10})} \qquad (57)$$

where

$$\begin{aligned} &\sigma_{z1} = \sigma_{z2} = \sigma_{z3} = \sigma_{z4} = -0.15 \\ &\lambda_4 = -0.1 + j.85 & \lambda_8 &= -0.05 + j1.2 \\ &\lambda_5 = -0.1 + j1.55 & \lambda_9 &= -0.085 + j1.95 \\ &\lambda_7 = -0.12 + j1.85 & \lambda_{10} &= -0.085 + j1.05 \end{aligned} \quad . \qquad (58)$$

(Observe that λ_4 is slightly different in Eq. 58 from the value given in Eq. 56. This is so because the values are in the process of adjustment at this point. The value chosen here leads to a more satisfactory characteristic for the band-pass chain. In the final choice, both values are the same.) In Eq. 58, λ_7, λ_8, σ_{z1}, σ_{z2}, all apply to the capacitance-coupled interstage. The inductance-coupled interstage corresponds to σ_{z3}, λ_9, and λ_{10}.

From this point, successive adjustments are made on the critical frequency positions of the amplification functions, according to the method of Report 145. In the adjustment procedure, one maintains the necessary conditions on the various positions; for instance, the real parts of λ_9 and λ_{10}, λ_4 and λ_5, λ_1 and λ_2 are kept the same for the individual pairs. This is necessary, since the pairs of poles identified with inductance-coupled circuits of the type shown in Fig. 36 (2) have the same real parts. Details of the intermediate steps of adjustment are omitted for reasons of brevity. The final critical frequency positions are shown in Fig. 48 for the low-pass chain, along with the corresponding network configurations. The pole positions apply to the 1.95-rad/sec analog of the 100-megacycle amplifier. The networks are those corresponding to the 100-megacycle amplifier. The frequency characteristics for the low-pass chain are shown in Fig. 49.

$$(\text{Amp})_{\text{low}} = \frac{K}{(\lambda - \lambda_1)(\lambda - \bar{\lambda}_1)\ldots(\lambda - \bar{\lambda}_6)} \quad . \tag{59}$$

For the 1.95-rad/sec case,

$$\begin{aligned} \lambda_1 &= -0.2 + j.133 & \lambda_4 &= -0.1 + j.85 \\ \lambda_2 &= -0.2 + j.70 & \lambda_5 &= -0.1 + j1.55 \\ \lambda_3 &= -0.179 + j.466 & \lambda_6 &= -0.050 + j.98 \end{aligned} \quad . \tag{60}$$

Parameter values in Fig. 48 are determined through the requirement that every C be 11 μμf plus the relations on Fig. 36.

Two points in connection with the low-pass chain just discussed require further clarification. The level of amplification indicated in Fig. 49 is the product of two factors from an expression similar to Eq. 55. The influence of pole positions is determined by the calculation of

$$\left| \frac{1}{(\lambda - \lambda_1)(\lambda - \bar{\lambda}_1)\ldots(\lambda - \bar{\lambda}_6)} \right| \quad .$$

The constant K is determined through the data on Fig. 36 by knowledge of the parameter values. The second point is the use of the delta connection in (d) of Fig. 48 in contrast to the Y in Fig. 46. If a Y is used, the neutral connection has an inevitable parasitic capacitance not accounted for in Fig. 46. On the other hand, any parasitic capacitance at the terminals of the delta can be associated with the capacitances shown. The delta shown in Fig. 48 is simply that network equivalent to a Y corresponding to Fig. 46.

Figure 50 shows the positions of critical frequencies for the band-pass chain and the corresponding network configurations.

$$(\text{Amp})_{\text{band pass}} = \frac{K'(\lambda - \sigma_{z1})(\lambda - \sigma_{z2})(\lambda - \sigma_{z3})(\lambda - \sigma_{z4})\lambda}{(\lambda - \lambda_4)(\lambda - \bar{\lambda}_4)(\lambda - \lambda_5)\ldots(\lambda - \bar{\lambda}_{10})} \quad . \tag{57}$$

In Eq. 57, the final choice of critical frequencies is (for the 1.95-rad/sec case)

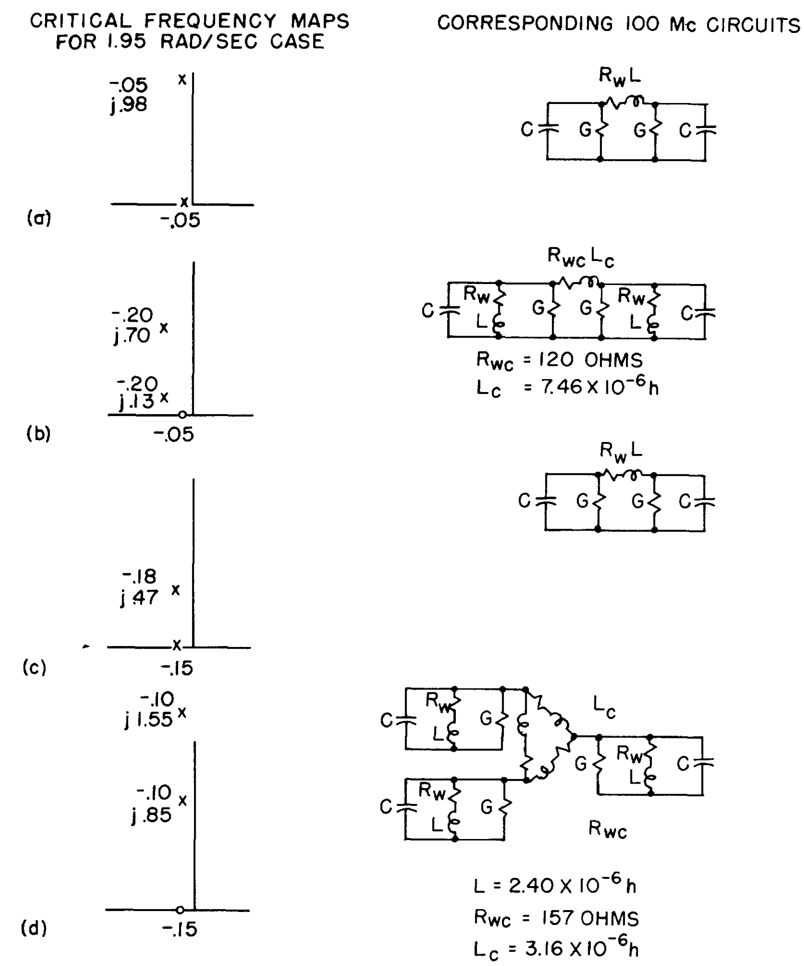


Fig. 48 Critical frequency maps for 1.95 rad/sec low-pass amplifier and corresponding 100 Mc circuits (low-pass section).

(a) $C = 5.5 \times 10^{-12}$ f

$G = 8.85 \times 10^{-5}$ mho

$R_w = 58.8$ ohms

$L = 3.54 \times 10^{-6}$ h

(b) $C = 5.5 \times 10^{-12}$ f

$G = 0.62 \times 10^{-3}$ mho

$R_w = 705.0$ ohms

$L = 4.38 \times 10^{-5}$ h

(c) $C = 5.5 \times 10^{-12}$ f

$G = 2.66 \times 10^{-4}$ mho

$R_w = 1080.0$ ohms

$L = 1.61 \times 10^{-5}$ h

(d) $C = 5.5 \times 10^{-12}$ f

$G = 8.85 \times 10^{-5}$ mho

$R_w = 120$ ohms

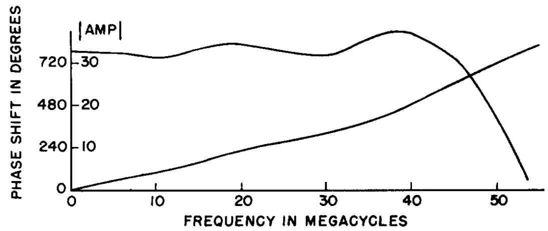


Fig. 49 Amplification characteristics of low-pass section of 100 Mc amplifier using the interstages of Fig. 48 connecting 4 6AK5 tubes.

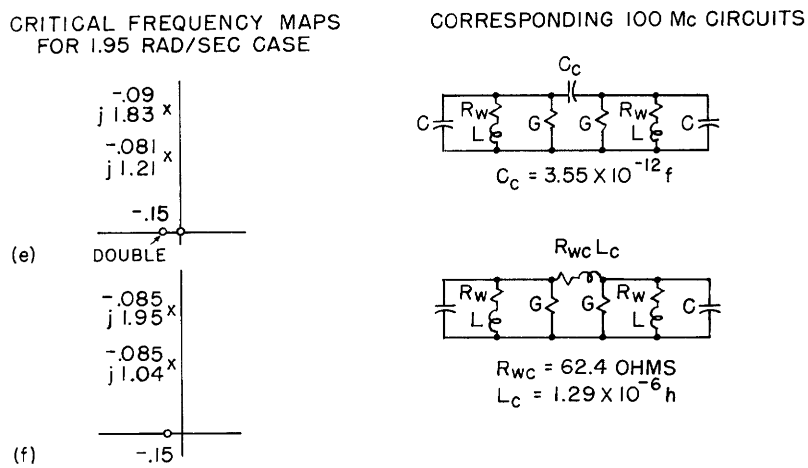


Fig. 50 Critical frequency maps for 1.95 rad/sec low-pass amplifier and corresponding 100 Mc circuits (band-pass section). The band-pass section feeds into circuit (d) of Fig. 48. The network configuration of the two chains is given in Fig. 52.

(e) $C = 5.5 \times 10^{-12}$ f

$G = 5.67 \times 10^{-5}$ mho

$R_w = 26.1$ ohms

$L = 52.5 \times 10^{-5}$ h

(f) $C = 5.5 \times 10^{-12}$ f

$G = 3.54 \times 10^{-5}$ mho

$R_w = 79.5$ ohms

$L = 1.62 \times 10^{-6}$ h

Critical frequency maps for 1.95 rad/sec case

Corresponding 100 Mc circuits

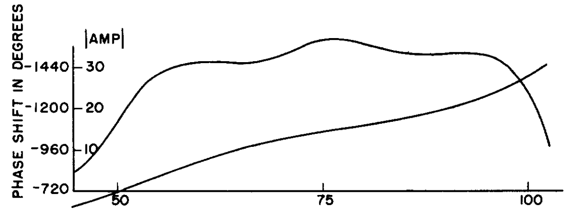


Fig. 51 Amplification characteristics of band-pass chain of 100 Mc amplifier using the interstages of Fig. 50 connecting 3 6AK5 tubes.

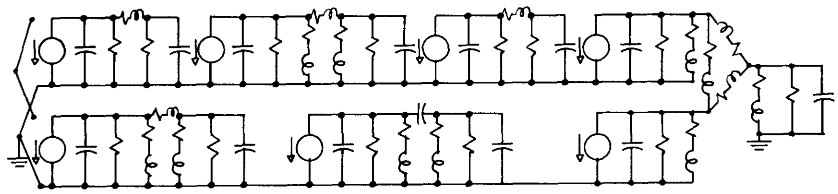


Fig. 52 Network configuration of two chain amplifier.

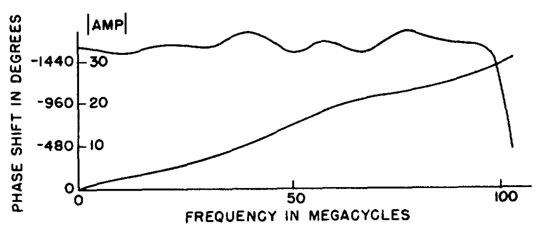


Fig. 53 Frequency characteristics of amplifier of Fig. 52.

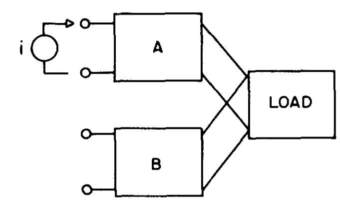


Fig. 54 Multi-terminal-pair network.

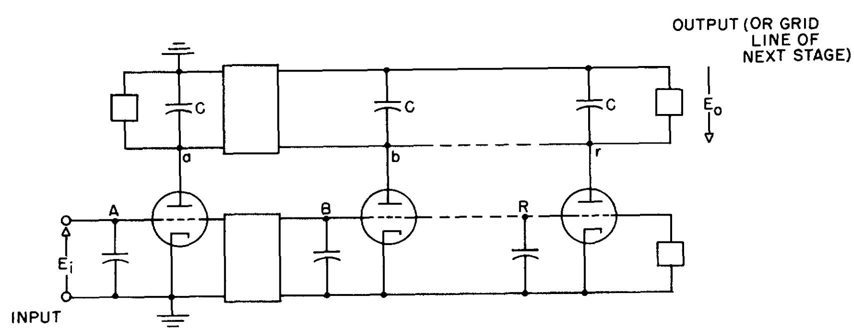


Fig. 55 Circuit of Percival's amplifier (one stage).

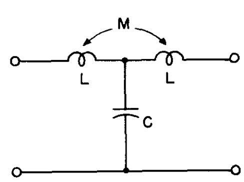


Fig. 56 Most frequently used line section in distributed amplifier. C is the tube capacitance, grid-cathode or plate-cathode.

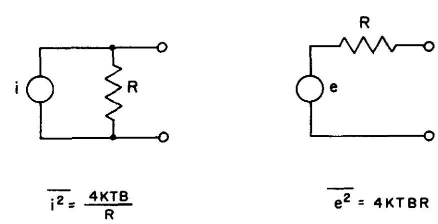


Fig. 57 Equivalent representations of thermal agitation noise associated with a resistor.
K – Boltzmann's constant – 1.38×10^{-23} watt sec per °K.
T – Absolute temperature of resistor in °K.
B – Bandwidth of the circuit affected by the noise in cycles/sec.

$$\sigma_{z1} = \sigma_{z2} = \sigma_{z3} = \sigma_{z4} = -0.15$$

$$\lambda_4 = -0.1 + j.85 \qquad \lambda_8 = -0.081 + j1.21$$

$$\lambda_5 = -0.1 + j1.55 \qquad \lambda_9 = -0.085 + j1.95$$

$$\lambda_7 = -0.09 + j1.83 \qquad \lambda_{10} = -0.085 + j1.04 \quad . \tag{61}$$

The band-pass amplification characteristics shown in Fig. 51 are those of the amplifier chain the components of which are shown in Fig. 50.

The network configuration of the two-chain amplifier, the element values of which have been given in Figs. 48 and 50, is shown in Fig. 52. The sum of amplifications of the two chains, the characteristics of which are given in Figs. 49 and 51, corresponds to the characteristics given in Fig. 53.

Through use of the method illustrated in the example just given, one can design an amplifier providing any desired amplification over any desired bandwidth, in spite of parasitic capacitance. The method permits the accommodation of numerous practical conditions. For instance, if the amplifier stages operate at high enough frequency to make transit-time loading at the inputs appreciable, this can be taken into account. Transit-time loading essentially puts a resistance in parallel with the G's of the above Figs. 48 and 50. If the loading is severe, one finds from Fig. 36 that a limit is placed on the closeness to the imaginary axis of poles of transfer impedance of the interstages. This fact simply means that in the approximation process one must restrict attention to functions with poles sufficiently removed from the imaginary axis. This will mean that to obtain a prescribed characteristic, one may need to use a more complicated function (more poles and zeros) than would otherwise be required, but the adjustment procedure is the same.

3.30 The Cost of Paralleling Amplifier Chains

The freedom obtained from the limitation of parasitic capacitance in the design method described, has incurred two costs not present in single-chain amplifiers. The first cost is the loss in amplification of the chain due to the fact that the output network is a multi-terminal-pair rather than a two-terminal-pair network. The second cost is in the fact that the selectivity of the amplifier is a function of the transconductance of the tubes, in the sense that a different deterioration of tubes in one chain from those in the other chains results in a change in level of amplification of one range of frequency from the level of other ranges.

3.31 Decrease in Level of Amplification Due to Paralleling

The decrease in level of amplification due to paralleling has been indicated to arise because of the presence of multiple driving circuits at the load. This results in the diversion of part of the plate current of any one of the driving terminal pairs from the load to the other driving circuits. In connection with Fig. 54, the presence of B decreases the level of transfer impedance from A's input to the load where A, B, and the load are identical. This fact, plus the fact that the load must serve all chains, is responsible for the factor $1/m^{3/2}$ in Eq. 48.

In connection with the decrease in level of amplification mentioned, two statements need consideration. In the first place, this cost of paralleling is finite and can always be overcome by the use of more stages when the band of each chain is below that for which the amplification is down to one per stage. In the second place, the paralleling of chains as described minimizes the cost through the fact that there is only one point of paralleling. It is pointed out in the next section that the distributed amplifier of Percival parallels at every tube and suffers a loss in effectiveness thereby.

3.32 Influence of Changing Transconductances of the Tubes

If, in the course of use of an amplifier of the form of Fig. 52, the products of transconductances of the four tubes in the upper chain change by a different percentage than the products of transconductances of the three tubes in the bottom chain, a change in the frequency characteristic results. The nature of the change is essentially a different change in levels of amplification in the high and low ranges.

The nature of the deterioration of a vacuum tube with life is ordinarily a slow drift. Abrupt changes are usually accompanied by complete failure of the tube. One manufacturer of tubes consulted indicated that life tests bear out this fact in general. (There are apparently no published data on large samples of tubes.) The RMA Standards on vacuum tube testing classify a tube as acceptable which maintains 65 percent of its rated transconductance over 500 hours of operation. The experience of the manufacturer indicated that of the few tubes out of a hundred in a test to fail, the majority of failures were complete (burn-out of a filament, shorting of elements, etc.), and that the tubes which pass maintain their transconductances essentially fixed over the 500-hour period. The variation of their transconductance is far less than the limit permitted. It is only in tubes used a very long time (several thousand hours) that the transconductance is subject to erratic behavior in general.

There are two additional facts which diminish somewhat the importance of the inherent dependence of characteristics on the tube transconductance. The first is the fact that it is the geometric mean of transconductances of the tubes in a chain which is significant. For several tubes, the geometric mean of transconductances ordinarily varies less than do the individual transconductances. In addition, the variation of performance between tubes is minimized by the fact that all tubes are supplied from the same source, and any changes affecting one affect all. A second fact is that the transconductances of tubes in a chain of amplifiers can be regulated by a single adjustment—the screen voltage or cathode bias, for instance. This means that any periodic (or even automatic) adjustment which proves to be necessary for a given application is simple to make. For most applications, the variability of the characteristics due to the non-uniform varying of transconductance would be corrected periodically, the frequency of adjustments depending upon the need for precision in the amplifier. The adjustment involves one measurement of amplification for each chain and the adjustment of the chains to a uniform level of amplification, the adjustment of any chain requiring only one operation. If a particular application needs the characteristics to be accurately controlled, and if a periodic manual adjustment is not convenient, an automatic device could be built which aligns the levels of amplification each time the power is turned on or at prescribed intervals of operation.

4.00 The Distributed Amplifier

The distributed amplifier, which was invented by Percival (6) and has recently been developed by Ginzton (7) and others, gives theoretically any amplification over any bandwidth. A description of its properties and a comparison with the parallel-chain amplifier, described in Sect. 3.00, is useful to indicate the advantages and disadvantages of both types.

The circuit diagram of Fig. 55 represents one stage of the distributed amplifier. The grids and plates of the vacuum tubes are distributed along two artificial lines. Each of the lines consists of identical recurrent sections; the vacuum tubes are placed between sections. The tube capacitances are identified with the shunt capacitance of the line. The most commonly used line employs sections of the type shown in Fig. 56. Changing the mutual inductance between the series arms of the T changes the kind of characteristic obtained. In the 6AK5, a tube frequently used for such amplifiers, the grid-cathode capacitance is larger than the plate-cathode capacitance and the impedance level of the plate line is accordingly higher than that of the grid line. In all cases the lines are designed so that the velocities of propagation of the two are the same. The lines should be properly terminated. The amplification-bandwidth limitation applying to the distributed amplifier is of a different form than that applying to a conventional single-chain amplifier and there is no theoretical limit of bandwidth beyond which the amplification is less than one provided one distributes a sufficient number of tubes along each line. The method of connection of the distributed amplifier results in summing the transconductances of the tubes in a stage essentially without increasing the effect of their shunt capacitances.

The operation of a stage of the distributed amplifier is explained as follows. As a signal is impressed on the input terminals of the grid line, a voltage wave propagates down the grid line affecting the tubes in sequence. As the voltage wave in the grid line encounters a tube, the response of the tube is to inject a corresponding current into the plate line. The current injected into the plate line splits equally, half of it going in each direction. Only that directed toward the load results in any output voltage, the other half being absorbed in the termination at the left end of the plate line. As the current wave in the plate line progresses toward the output, it is reinforced at every tube. The amplification of a stage is determined by the number of tubes, their transconductances, and the characteristic impedance of the line. A number of stages may be cascaded. To cascade two stages one simply couples the plate line of the first stage into the grid line of the second stage.

Shunt capacitance in the recurrent structure imposes a limit on the amplification due to a single tube in a stage. The limitation arises through the fact that the component of the voltage wave in the plate line due to a tube is directly proportional to the characteristic impedance of the line. The characteristic impedance for specified shunt capacitance is inversely proportional to the cut-off frequency of the line. However, by using a sufficient number of tubes in a stage one can (theoretically) build up the amplification of the stage to any desired level over any prescribed frequency range. The most economical arrangement of connection (in terms of the number of tubes) has been shown (7) to be the use of a sufficient number of tubes in a stage to build up its amplification over the prescribed range of frequencies to ϵ, the base of the natural logarithms. One cascades a sufficient number of stages to obtain the desired level of amplification.

An interesting and desirable feature of the distributed amplifier is that the frequency characteristic is not influenced except in level by the transconductance of the tubes, provided that the lines are properly terminated. This is true since the transconductance of a tube comes into the amplification of its stage in an additive sense, and a decrease of transconductance affects all frequency components uniformly.

The nature of the distributed amplifier imposes a severe inflexibility in its design. One is restricted in the choice of the line characteristics since the individual sections must be structurally simple to be practical. To insure adequate stability one must minimize the coupling between the plate and grid lines. For this reason only pentodes, with their small grid-plate capacitance, are used in distributed amplifiers. Attenuation in the lines reduces the amplification. From the following one sees that the output voltage due to the p^{th} tube of r tubes in a stage has been attenuated by $r-1$ sections of line. The grid voltage of the p^{th} tube is attenuated through $p-1$ sections before arriving at the grid, and the output voltage wave started by the tube is attenuated through $r-p$ sections. At very high frequencies where grid loading becomes effective, this attenuation is extremely severe. In fact, grid loading is neglected in arriving at the conclusion that one can amplify over any bandwidth with a distributed amplifier. The conclusion is not tenable if one considers the grid loading effects of the tubes used currently.

4.10 Quantitative Evaluation of the Shunt-Capacitance Limitation for Distributed Amplifiers

To make comparisons between distributed amplifiers and parallel-chain amplifiers one must evaluate on the same basis the limits imposed by shunt capacitance in each amplifier. To apply the same basis of evaluation to the distributed amplifier which was applied to the parallel-chain amplifier, one should consider the voltage at the $p+1^{th}$ terminal pair of the circuit of Fig. 55 due to a current from the p^{th} tube in a stage of r tubes. This voltage is limited by the maximum transfer impedance from the p^{th} to the $p+1^{th}$ terminal pair. The maximum magnitude of transfer impedance uniform over ω_o rad/sec is (see Fig. 33 and the accompanying discussion).

$$\left|Z_{p\ p+1}\right|_{max} = \frac{\pi^2}{\sqrt{2}\, 4C\omega_o} \text{ ohms} \quad . \tag{62}$$

For simple circuits the transfer impedance cannot attain this limit of magnitude but must be smaller. The transfer impedance will be called

$$\left|Z_{p\ p+1}\right| = \frac{B\pi^2}{\sqrt{2}\, 4C\omega_o} \text{ ohms} \quad . \tag{63}$$

B must be less than one. More will be said about its size at a later point. (Observe that the $\sqrt{2}$ in the denominator is present since the current from the plate can go in either direction in the plate line.) The magnitude of voltage amplification at the output due to the p^{th} tube is

$$\left|Amp\right|_p = A^{r-1} g_m \left|Z_{p\ p+1}\right| \quad . \tag{64}$$

A is the attenuation per section of the lines. From Eq. 64, one easily sees that the amplification of a distributed amplifier of s stages with r tubes per stage is

$$\left|Amp\right|_{distributed} = \left[rA^{r-1} \frac{g_m B\pi^2}{\sqrt{2}\, 4C\omega_o}\right]^s = \frac{B^s A^{s(r-1)}}{(\sqrt{2})^s} \left(\frac{r\, g_m \pi^2}{4\, \omega_o C}\right)^s \quad . \tag{65}$$

The comparison between a distributed amplifier of s stages with r tubes per stage and an m-chain amplifier with n stages per chain is made by comparing Eq. 65 and Eq. 48

$$\left|Amp\right|_{parallel\text{-}chain} = \frac{0.2}{m^{3/2}} \left(\frac{\pi^2 g_m m}{4C\omega_o}\right)^n \quad . \tag{48}$$

In Eq. 65, one observes the cost of paralleling to be indicated by the factor $1/\sqrt{2}^s$. In Eq. 48 it is indicated by the factor $1/m^{3/2}$. In the distributed amplifier the cost of paralleling increases as the required amplification of the amplifier goes up, since one cascades stages (each with an amplification of about 2.7) to increase the amplification. In the parallel-chain amplifier, however, the cost of paralleling does not increase beyond a certain point with increasing level of amplification, as is indicated in Fig. 47. Figure 47 shows that m (the number of chains) does not increase significantly for fixed bandwidth as the amplification is increased. For the distributed amplifier one also observes from Eq. 65 that the loss of amplification due to attenuation in the lines also increases with the number of vacuum tubes used. The parallel-chain amplifier does not suffer from this defect. The factor B of Eq. 65 results from the fact that the simple two-terminal-pair networks used as sections of the line will not exhibit the theoretical limit of transfer impedance for networks with the same amount of shunt capacitance. Further, if all of the stages are identical, one must use better and better individual coupling networks as the number of stages is increased, if the magnitude of amplification of the amplifier is restricted to a prescribed tolerance. The staggering of characteristics of different stages of the distributed amplifier is probably possible, but the author knows of no case in which that has been done.

In comparing the parallel-chain and distributed amplifiers on the basis of the foregoing, one may conclude that the parallel-chain amplifier is more economical in the number of tubes required, particularly if the amplification required is high. The flexibility available in the design of amplifier chains is an advantage over the distributed amplifier. The independence of the frequency selectivity characteristics of the distributed amplifier from the changing transconductances of the tubes is an advantage of the distributed amplifier.

5.00 Noise Considerations in Amplifier Design

Shunt capacitance is the primary obstacle limiting the performance of a vacuum tube as an amplifier over a specified band of frequencies. An independent and equally basic limitation restricts the smallness of a signal which can be effectively amplified. This limitation is the production of noise by an amplifier. The random variations of current and voltage which constitute noise are combined with the signal and the combination is ordinarily irresolvable. The addition of noise to the signal renders it impossible to distinguish the signal if the amount of noise added is of sufficient size. Accordingly, the smallness of signal which an amplifier can successfully amplify (referred to as the sensitivity of the amplifier) is determined by the amount of noise which the amplifier generates and combines with the signal. Different amplifiers and associated circuits have different properties as regards the generation of noise. The evaluation of the noise characteristics of the amplifiers discussed in the foregoing sections is essential if one is to design such amplifiers with the greatest possible sensitivity.

5.10 Summary of Pertinent Material on Noise

Thermal agitation noise in resistors is the most familiar kind of noise. It is well understood and its characteristics form the standard to which other noise sources are compared. In a resistor the free electrons are in random motion. At a given instant of time the average velocity of the electrons is not ordinarily zero though the average velocity of electrons in a resistor over an interval of time may be zero. The random character of the velocities of free electrons gives rise to a varying voltage across the terminals of an open resistor or to current if the terminals are short-circuited. Any circuit connected to the resistor accordingly is affected by the noise source. The distribution of harmonic content of the noise voltage (or current) is uniform. Hence the mean-square current or voltage is dependent directly upon the bandwidth of the device receiving the noise. The summary of relations regarding thermal agitation noise is presented in Fig. 57.

Vacuum tubes conduct current through the motion of electrons, the velocities of which have random distribution. Consequently associated with the plate current of the vacuum tube there are random components which have properties similar to those described in connection with the thermal agitation noise of resistors. Actually the mechanism of noise production in tubes depends upon several effects. The various components of noise associated with the plate current have been studied and designated by appropriate terms. For the present purpose, since all kinds of noise associated with the plate current cause the same effect, it is appropriate to call all such noise plate noise and to treat it as if it came from a single noise source. The harmonic content of plate noise is uniformly distributed over the spectrum. Plate noise can, for purposes of analysis, be represented by an additional current source in parallel with the current source of the incremental equivalent circuit of a vacuum tube as shown in Fig. 58. The amount of plate noise is frequently expressed in terms of a fictional resistance, R_{eq}, of such size that when connected to the grid its noise voltage would provide the same amount of plate current as results from the plate noise. It is significant to note that plate noise has no effect on the grid circuit so long as the plate current does not influence the grid voltage.

A second type of noise is important in amplifiers operating at very high frequencies. This is noise associated with transit-time loading of the grid and the effects of cathode lead inductance and the grid-to-cathode interelectrode capacitance. The noise in this case acts essentially as a current source in parallel with the grid terminals. The harmonic content of the grid-loading noise is uniform over the spectrum, the mean-square noise current per unit of bandwidth varying with the square of the frequency. Figure 59 shows the current source representing grid noise. A point to observe in connection with grid noise is that it affects both the grid and plate circuits. This fact is of importance in connection with the paralleling of tubes in the distributed and parallel-chain amplifiers.

The types of noise indicated in the preceding are the significant ones in determining the sensitivity of an amplifier. The first stages of an amplifier are the most important in determining the amplifier sensitivity if the amplification per stage is considerably larger than one. This fact is readily appreciated through noting that noise introduced at the output of the first stage must compete with an amplified signal while noise introduced at the output of the second stage must compete with a still larger signal and so on.

5.20 Noise Properties of Stagger-Tuned Amplifiers

Ordinarily the individual stages of a stagger-tuned amplifier possess a decidedly nonuniform amplification characteristic. The use of properly chosen nonuniform characteristics which properly complement each other has been indicated earlier to

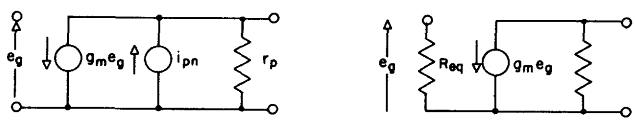


Fig. 58 Representation of plate noise.

$$\overline{i_{pn}^2} = g_m^2 \ 4KTB \ R_{eq} .$$

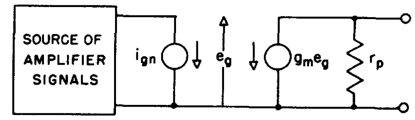


Fig. 59 Circuit showing equivalent circuit of tube and grid-noise current source.

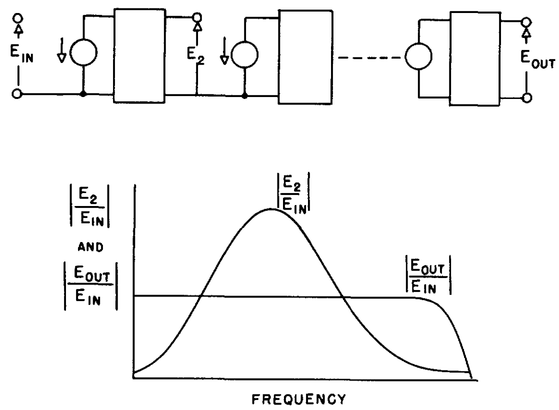


Fig. 60 Typical characteristics of first stage and complete stagger-tuned amplifier.

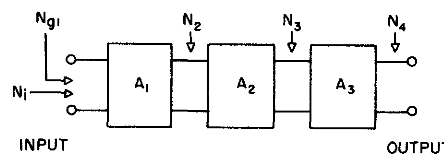


Fig. 61 Amplifier with noise sources.

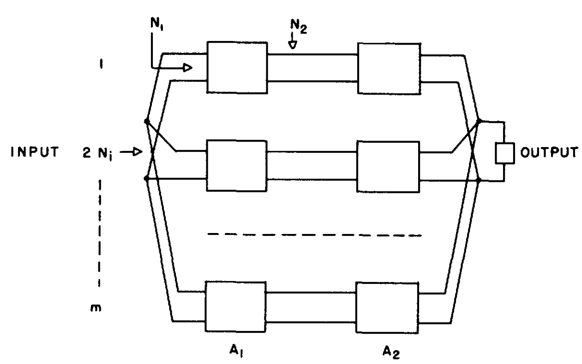


Fig. 62 m-chain amplifier with noise sources.

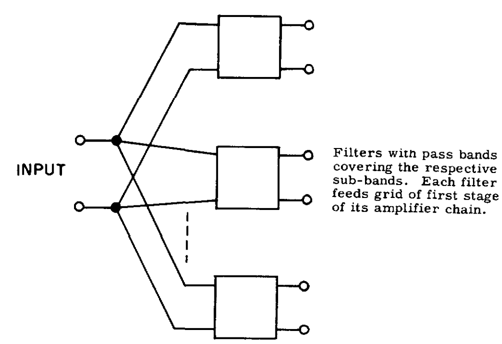


Fig. 63 Input circuit to reduce effect of grid noise in parallel-chain amplifier.

yield a particularly simple and effective circuit as far as the amplification-bandwidth product is concerned. However, the use of extremely nonuniform characteristics in the early stages of an amplifier, where the noise introduced is important, lead to very poor noise characteristics. This fact is easily appreciated through consideration of Fig. 60 which shows the characteristic of a typical first stage. The level of the signal at the output of the first stage is dependent upon the frequency of the signal. For middle frequencies the level of the signal may be very high but for the lower and upper parts of the pass band the signal may be even lower than it was at the input of the amplifier. Hence high and low frequency components of the signal compete on an unfavorable basis with new noise introduced into the amplifier. This fact may mean that for certain frequency ranges the signal must compete with the aggregate of noise from several stages and the sensitivity of the amplifier for signals in these ranges of frequency will be much lower than in the ranges of frequency over which the amplification of the early stages is large. The effective method to avoid such a situation is to use stages at the input of the amplifier which provide uniform amplification over the bandwidth of the amplifier.

5.30 Noise Properties of Broad-Band Amplifiers

The extreme bandwidth of the parallel-chain and distributed amplifiers restrict the designers use of techniques which may be helpful in narrow-band amplifiers in improving noise characteristics. For instance coupling circuits designed for very broad bands encounter limitations imposed by the presence of shunt capacitance which limitations are never encountered in narrow-band circuits. Consequently one must usually view the noise specifications as secondary in such amplifiers and turn attention to them only after the specifications on bandwidth have been met.

In describing the noise characteristics of any amplifier a significant measure is the noise factor which is defined as the ratio of the total mean square noise voltage at the output to the mean square component of that noise which is injected by the signal source. Figure 61 represents an amplifier of three cascaded sections. The amplification of each section is represented by A with the appropriate subscript. The mean-square noise voltage of the source is represented by N_i. N_{gi} is the grid noise associated with the first stage. N_2 is the combination of plate noise in the first stage, grid noise in the second stage and the noise of any resistance in the associated circuit. N_3 is the same kind of quantity for the next stage and N_4 is different only in that it includes no grid noise. The noise factor is obtained as indicated in Eq. 66.

$$\frac{\text{Total Noise Output}}{\text{Noise Output from Source}} = \frac{(N_i + N_{ig})\, A_1^2\, A_2^2\, A_3^2 + N_2\, A_2^2\, A_3^2 + N_3\, A_3^2 + N_4}{N_i\, A_1^2\, A_2^2\, A_3^2}$$

$$= 1 + \frac{N_{ig}}{N_i} + \frac{N_2}{N_i\, A_1^2} + \frac{N_3}{N_i\, A_1^2\, A_2^2} + \frac{N_4}{N_i\, A_1^2\, A_2^2\, A_3^2} \quad . \qquad (66)$$

From Eq. 66 one sees readily that if the amplification per stage is large the noise introduced in the later stages has insignificant influence on the noise factor. If the amplification per stage is low, noise introduced several stages from the input will deteriorate the noise factor seriously.

Ginzton (7) and his colleagues have analyzed the noise properties of the distributed amplifier. The results of their analysis will be explained without rigorous proof. Noise arising in a single stage of a distributed amplifier (Fig. 55) can be readily pointed out. The termination of the grid line necessitates an added resistance which emits noise. The noise from this termination propagates down the grid line in the backward sense and actuates the tubes in backward sequence. If ϕ is defined as the phase shift per section of the line (a function of frequency) it is observed that the output terminals of the plate line receive a set of termination-noise voltage components from the various tubes which are multiples of 2ϕ radians out of phase. As a consequence the noise from this termination is somewhat less effective than is noise inserted at the input terminals. A second kind of noise introduced is plate noise which is injected in parallel with each tube in the plate line. At each tube the noise current injected divides and half propagates in each direction. The noise currents emitted from each tube are independent and consequently the mean-square component of noise voltage at the output due to plate noise is merely r times the noise which appears from a single tube. Noise (and also signals) at the amplifier input are magnified in proportion to r, the mean-square values being proportional to r^2. These facts imply that the effect of plate noise is minimized in the distributed amplifier. In other words signal and input noise from the r tubes in a stage are always added in phase, the mean-square value of the sum behaving as r^2, while the plate noise components are added at random, the total plate noise at the output behaving like r. The third kind of noise introduced in the distributed amplifier is grid noise. The behavior of the distributed amplifier in connection with grid noise is much less fortunate. Every tube in a stage amplifies the grid noise from itself and from every other tube. The noise factor of the distributed amplifier as evaluated by Ginzton (7) is given in Eq. 67 (using different symbols).

$$\text{Noise Factor}_{\text{dist. amp.}} = 1 + \left(\frac{\sin r\phi}{r \sin\phi}\right)^2 + \frac{1}{r}\,\frac{R_{eq}}{Z_{ol}} + r\,\frac{Z_{ol}}{R_A}\,\frac{a}{4} \quad . \qquad (67)$$

In Eq. 67 r is the number of tubes in a stage, ϕ is the phase shift per section of the grid or plate lines, Z_{ol} is the characteristic impedance of the lines, R_{eq} is a fictitious resistance which by introducing its noise voltage in the grid provides the same noise current in the plate as is introduced by the plate noise of the tube, R_A is the equivalent shunt resistance of grid loading of a tube, and a is a constant empirically determined to be about five. The second term in Eq. 67 is associated with noise from the grid line termination. The third term is associated with the plate noise introduced.

The deterioration of noise factor due to plate noise diminishes with the number of tubes in the stage as would be expected from the statements previously made. The fourth term is associated with the noise introduced by the grids. The severity of its effect on the noise factor increases with the number of tubes per stage. However, in general, Z_{01} is much smaller than R_A and this source of deterioration is not as serious as it might otherwise appear. In connection with all of the preceding it must be remembered that a signal source of the same impedance as the characteristic impedance of the line has been assumed.

In the design of the parallel-chain amplifier a few points must be considered to obtain good noise characteristics. In the first place the amplification of the first stages of the individual chains must be reasonably uniform over the sub-bands of frequency amplified by each chain. Figure 62 depicts an m-chain amplifier with noise sources. The N's shown represent the mean-square noise voltage for a bandwidth equal to that of each amplifier chain. For the individual chains the sub-band amplified is 1/m of the total band. The boxes in the individual chains represent several stages. The first boxes represent the earlier stages the noise from which significantly influence the noise factor of the amplifier. The second boxes represent the later stages the noise from which do not significantly influence the noise factor of the network. A_1 is the amplification of the former; A_2 the amplification of the latter. N_1 represents the grid noise from each of the input tubes. N_2 represents the equivalent remaining noise for the tubes in the early stages of a chain. The influence of N_1 and N_2 are different in that N_1 affects all of the chains and N_2 affects only the chain in which it is generated. The noise factor of the parallel-chain amplifier is readily evaluated to be

$$\text{Noise factor}_{\text{parallel-chain amp}} = \frac{m N_i A_1^2 A_2^2 + m^2 N_1 A_1^2 A_2^2 + m N_2 A_2^2}{m N_i A_1^2 A_2^2} . \quad (68)$$

Increasing the number of paralleled chains decreases the number of stages at the input of the chains which significantly deteriorate the noise factor. This is true since the amplification per stage is inversely proportional to the width of the sub-band. Moreover, low-noise amplifier circuits (18) can be used to advantage in this type of amplifier. However, a disadvantage of this connection, which has a close counterpart in the distributed amplifier, is that the grid noise of the input stages of each chain is amplified by the other chains. This disadvantage can be reduced at the expense of circuit complication through use of an input circuit of the type indicated in Fig. 63. The grid noise of the upper chain, for instance, is filtered such that the essential frequency components arriving at the inputs of the other filters are sharply attenuated by those filters.

A comparison between the noise characteristics of the distributed and parallel-chain amplifiers is quite difficult without employing special examples. The type of source supplying the amplifier has an effect in both cases and the effects are different. The distributed amplifier by its nature reduces the seriousness of plate noise but it must use pentodes which generate more plate noise than do triodes. The parallel amplifier can use low-noise input stages but the grid noise of every input stage is fed to every other chain unless a rather complicated filter is employed. In a particular case one can employ the known data of tubes used to determine which kind of amplifier should be used to amplify the signals to a level above which noise causes no problem. The remaining stages of the amplifier could be of either type which best suits the needs of the problem.

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