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A METHOD OF MEASURING FREQUENCY DETECTOR RESPONSE

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Abstract

A method of measuring distortion in a frequency detector, utilizing the frequency derivative of the detector's response curve, is presented. Ways in which this method reduces error are discussed. A laboratory application of the method is suggested.

A METHOD OF MEASURING FREQUENCY DETECTOR RESPONSE

I. Reduction of Error by Measuring Derivative Curve

In any measurement problem one of the chief aims is to reduce the error involved in the measurement. One type of error introduced in making point-by-point measurements may be illustrated with the help of Fig. 1. Say that C_2 of Fig. 1(a) is the response curve describing the operation of some physical frequency detector, and that an approximation of this curve is to be obtained experimentally by measuring the dc output voltage point by point as a function of the input frequency. If we assume that the error in voltage measurement is negligible in relation to that in frequency measurement, and that the maximum error in setting frequency is $\pm\delta$, the measured value of the ordinate for f_1 lies between that at points 1 and 3 along C_2. In plotting the experimental data, the ordinate measured for f_1 will be assigned to f_1 so that the experimentally determined curve is bounded by curves C_1 and C_3 in the vicinity of point 2. Similarly, if C_2 of Fig. 1(b) describes the actual operation of the detector, the experimentally determined curve is bounded in the vicinity of point 2 by curves C_1 and C_3. Now if this argument is applied to the determination of the entire experimental curve, it is evident that of the curves C_2 of Figs. 1(a) and 1(b) depicting actual operation, the one having the smaller slope will have the better experimental approximation. Thus measuring a curve with little relative variation in ordinates is desirable. It is partly for this benefit that measurement of the derivative of the response curve is under study rather than measurement of the response curve: the desired character of the derivative curve in the usual region of interest is a constant.

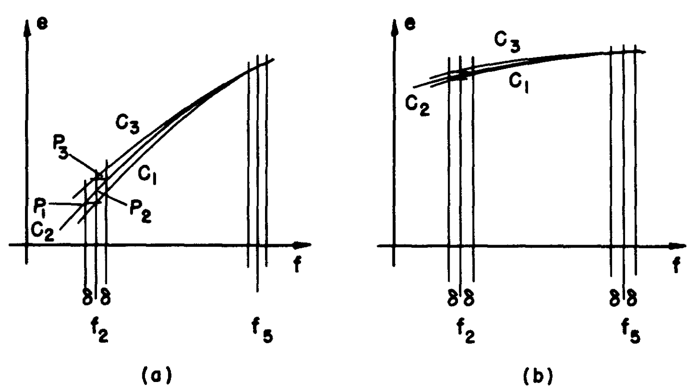

Fig. 1
Dependence of approximation on type of curve.

II. Reduction of Error by Separation of Derivative Curve into a Constant and a Remainder

When one is attempting to measure small amounts of distortion, it is often helpful to isolate the causes of the distortion from the causes of the desideratum. In the particular case of the output of a frequency detector for a sinusoidally modulated FM input signal, the output will be sinusoidal, will contain no distortion, if the response curve in the region of operation is linear. Since it is the departure of this portion of the curve from linearity that introduces the distortion, the degree of accuracy in some types of graphical measurements may be improved by focusing attention primarily on this departure rather than on the actual curve itself.

This notion could be used as presented in Fig. 2 where an experimentally determined response curve e(f), shown in Fig. 2(a), is used with an assumed sinusoidally varying frequency to obtain an output waveform, this output waveform $e_o(t)$ being used in determining the Fourier component amplitudes by graphical integration. If the distortion is small, so that alternate lobes of $e_o(t)$ are nearly identical, the result of the integrations except for the fundamental is obtained as the difference of two almost equal numbers. Now as the error introduced by planimeter measurement or square counting is essentially a percentage one, differencing two graphically determined quantities increases the final relative error, especially if the difference is relatively small.

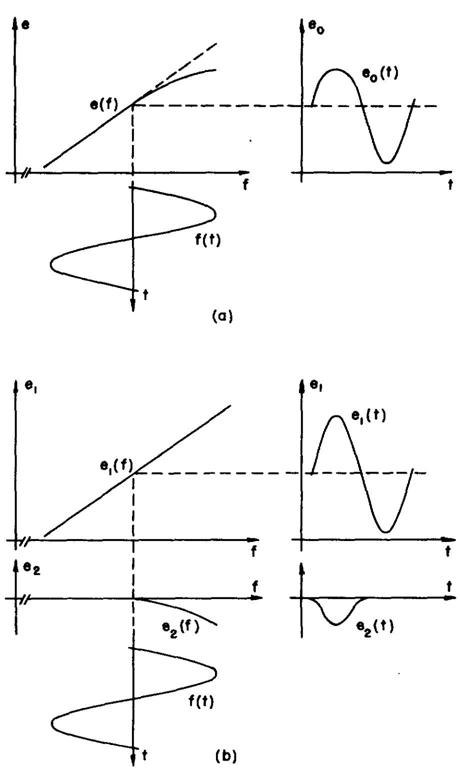

Fig. 2
Resolving a curve into two components.

The error introduced by differencing may be reduced considerably by isolating the small variations in the response curve from the major trend, as in Fig. 2(b), and calculating the output as the sum of two parts: the first, $e_1(t)$, is the major portion of the fundamental component of the output due to the trend of the response curve; the second, $e_2(t)$, is that portion of the output due to the minor variations of the transfer curve. The function $e_2(t)$ may then be analyzed graphically to obtain its harmonic amplitudes; these plus the fundamental component $e_1(t)$ are the Fourier components of the output wave $e_o(t)$. Now since the waveform $e_2(t)$ is only a meager contributor to the fundamental component and is the entire source of the distortion components, its graphical integration for these components introduces almost a minimum of differencing error. Note that the advantage of breaking the

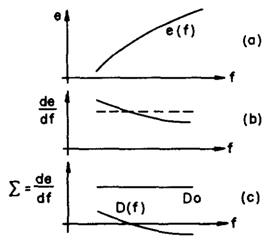

Fig. 3
Resolution of derivative curve into components.

transfer function into these two parts in order to calculate the distortion increases as the distortion decreases.

This same notion may be used in calculating the output distortion components from an experimental curve which is the derivative of the frequency detector response curve. Here the derivative curve is decomposed into a constant function, D_o, and a difference curve, D(f), as shown in Fig. 3.

It may be shown (1) that the output voltage of a discriminator for an FM input signal having sinusoidal frequency variations of the form

$$f = f_o + \Delta f \sin 2\pi f_m t \tag{1}$$

may be written

$$e = A_o + A_1 \sin 2\pi f_m t + A_2 \cos 4\pi f_m t + A_3 \sin 6\pi f_m t + \ldots \tag{2}$$

The time derivative of this output voltage is

$$\frac{de}{dt} = 2\pi f_m A_1 \cos 2\pi f_m t - 4\pi f_m A_2 \sin 4\pi f_m t + 6\pi f_m A_3 \cos 6\pi f_m t + \ldots \tag{3}$$

However, this derivative may be expressed in a different form:

$$\frac{de}{dt} = \frac{de}{df} \cdot \frac{df}{dt} \tag{4}$$

where (de)/(df) may be the experimental derivative curve of the detector response as a function of frequency, and we have

$$\frac{df}{dt} = \frac{d}{dt}(f_o + \Delta f \sin 2\pi f_m t) = 2\pi f_m \Delta f \cos 2\pi f_m t \tag{5}$$

Thus, from Eqs. 3, 4, and 5, we obtain

$$\frac{de}{dt} = \frac{de}{df} \cdot 2\pi f_m \Delta f \cos 2\pi f_m t = 2\pi f_m (A_1 \cos 2\pi f_m t - 2A_2 \sin 4\pi f_m t + \ldots) \tag{6}$$

In Eq. 6 (de)/(df) may be replaced by its two component curves:

$$\left[D_o + D(f)\right] \Delta f \cos 2\pi f_m t = A_1 \cos 2\pi f_m t - 2A_2 \sin 4\pi f_m t + \ldots \tag{7}$$

and with rearrangement, Eq. 7 becomes

$$D(f) \cdot \cos 2\pi f_m t = \left(\frac{A_1}{\Delta f} - D_o\right) \cos 2\pi f_m t - \frac{2A_2}{\Delta f} \sin 4\pi f_m t + \ldots \tag{8}$$

Introducing the variable relationship of Eq. 1 in Eq. 8, we have

$$D(f_o + \Delta f \sin 2\pi f_m t) \cdot \cos 2\pi f_m t = \left(\frac{A_1}{\Delta f} - D_o\right)\cos 2\pi f_m t - \frac{2A_2}{\Delta f} \sin 4\pi f_m t + \ldots \tag{9}$$

The coefficients of the output harmonics of Eq. 2 may now be written in the integral form from Eq. 9 by virtue of the orthogonal properties of the sine functions:

$$A_1 = \Delta f \left\{D_o + 2f_m \int_{-1/2f_m}^{1/2f_m} \left[D(f_o + \Delta f \sin 2\pi f_m t) \cdot \cos 2\pi f_m t\right] \cos 2\pi f_m t \, dt\right\}$$

$$A_2 = -\frac{\Delta f}{2} \left\{2f_m \int_{-1/2f_m}^{1/2f_m} \left[D(f_o + \Delta f \sin 2\pi f_m t) \cdot \cos 2\pi f_m t\right] \sin 4\pi f_m t \, dt\right\}$$

$$A_3 = \frac{\Delta f}{3} \left\{2f_m \int_{-1/2f_m}^{1/2f_m} \left[D(f_o + \Delta f \sin 2\pi f_m t) \cdot \cos 2\pi f_m t\right] \cos 6\pi f_m t \, dt\right\} \tag{10}$$

With an experimental D(f) curve these integrals may be calculated graphically (2).

III. Distortion Components Calculable from Series Expansion of Derivative Curve

If the experimental derivative curve is sufficiently regular, its power series representation about the center frequency may be used to obtain the output harmonic components. If the response curve, e(f), of the discriminator is expressed as

$$e(f) = a_o + a_1 (f - f_o) + a_2 (f - f_o)^2 + a_3 (f - f_o)^3 + \ldots \tag{11}$$

and the variation of input frequency is sinusoidal as in Eq. 1, the output voltage of the detector may be written

$$e(t) = a_o + a_1 (\Delta f \sin 2\pi f_m t) + a_2 (\Delta f \sin 2\pi f_m t)^2 + a_3 (\Delta f \sin 2\pi f_m t)^3 + \ldots \tag{12}$$

On expanding and collecting terms, we obtain

$$e(t) = A_o + A_1 \sin 2\pi f_m t + A_2 \cos 4\pi f_m t + A_3 \sin 6\pi f_m t + \ldots \tag{13}$$

where

$$\left.\begin{aligned} A_0 &= a_0 + \frac{a_2}{2}(\Delta f)^2 + \frac{3a_4}{8}(\Delta f)^4 + \ldots \\ A_1 &= a_1\,\Delta f + \frac{3a_3}{4}(\Delta f)^3 + \ldots \\ A_2 &= -\left[\frac{a_2}{2}(\Delta f)^2 + \frac{a_4}{2}(\Delta f)^4 + \ldots\right] \\ A_3 &= -\left[\frac{a_3}{4}(\Delta f)^3 + \ldots\right] \end{aligned}\right\} \tag{14}$$

From Eq. 11 we have

$$\left.\begin{aligned} a_0 &= e(f)\,\big|f_0 \\ a_1 &= \frac{1}{1!}\,\frac{d\,e(f)}{d\,f}\,\big|f_0 \\ a_2 &= \frac{1}{2!}\,\frac{d^2\,e(f)}{d\,f^2}\,\big|f_0 \\ a_3 &= \frac{1}{3!}\,\frac{d^3\,e(f)}{d\,f^3}\,\big|f_0 \end{aligned}\right\} \tag{15}$$

Thus the harmonic amplitudes in Eq. 13 may be calculated, by using Eqs. 14 and 15, from the known deviation and the center frequency ordinates of the response curve and the increasingly ordered derivative curves. Usually the constant term A_0 of the output is unimportant: the succeeding three or four coefficients may be determined graphically from the first and higher ordered derivative curves, the higher ordered ones being obtained graphically from the first. Note that again only the first derivative curve need be determined experimentally.

IV. Experimental Application

The idea of obtaining the distortion components from the derivative curve of a frequency detector has been investigated experimentally, and has yielded results very similar to those obtained by direct measurements with a wave analyzer (3). Knowledge of the accuracy of the derivative method was limited partly because of the lack of a sufficiently accurate standard against which the results of this method could be compared. It is felt, however, that the proposed method of measurement, by reducing from first to second order the effects of distortion arising in the oscillator and frequency modulator, has less intrinsic error than has the wave analyzer method.

References

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2. Rodgers, op. cit., pp. 36-44
3. Ibid., pp. 38, 42