Multi-Channel Signal Separation Based on Decorrelation

RLE Technical Report No. 573

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Multi-Channel Signal Separation Based on Decorrelation

Ehud Weinstein[1] Meir Feder[2] Alan V. Oppenheim[3]

Abstract

In a variety of contexts, observations are made of the outputs of an unknown multiple-input multiple-output linear system, from which it is of interest to recover the input signals. For example, in problems of enhancing speech in the presence of background noise, or separating competing speakers, multiple microphone measurements will typically have components from both sources, with the linear system representing the effect of the acoustic environment. In this paper we consider specifically the two-channel case in which we observe the outputs of a 2×2 linear time invariant system with inputs being sample functions from mutually uncorrelated stochastic processes. Our approach consists of reconstructing the input signals by making an essential use of the assumption that they are statistically uncorrelated. As a special case, the proposed approach suggests a potentially interesting modification of Widrow's least squares method for noise cancellation, when the reference signal contains a component of the desired signal.

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I. Introduction

In a variety of contexts, observations are made of the outputs of an unknown multiple-input multiple-output linear system, from which it is of interest to recover its input signals. For example, in problems of enhancing speech in the presence of background noise, or separating competing speakers, multiple microphone measurements will typically have components from both sources, with the linear system representing the effect of the acoustic environment.

In this report, we consider specifically the two-channel case in which we observe the outputs of a 2×2 linear time invariant (LTI) system with inputs being sample functions from mutually uncorrelated random processes. Our approach consists of reconstructing the input signals by making essential use of the assumption that they are statistically uncorrelated. In its most general form, this is a highly under-constrained problem. By applying appropriate constraints on the form of the reconstruction system, meaningful and useful solutions are obtained, one class of which has as a special case the well-known Widrow's least-squares method for noise cancellation. A more elaborate presentation of the results reported here can be found in [1]

II. Signal Separation Based on Decorrelation

Consider the problem in which we observe the outputs $y_1[n]$ and $y_2[n]$ of a 2×2 linear system $\mathcal{H}$ illustrated in Figure 1, whose input signals $s_1[n]$ and $s_2[n]$ are assumed to be sample functions from mutually uncorrelated stochastic processes having stationary covariance functions. To simplify the exposition, we further assume that the signals have zero mean, in which case

$$\mathrm{E}\left\{s_1[n]s_2^*[n-k]\right\} = 0 \qquad \forall k \tag{1}$$

where $\mathrm{E}\{\cdot\}$ stands for the expectation operation and $*$ denotes the complex conjugate. We note that the zero mean assumption is not necessary. The derivation and results apply equally to the more general case of non-zero possibly time-varying means since they are phrased in terms of covariances.

The system components H_{ij} $i, j = 1, 2$ are assumed to be stable single-input single-output linear time invariant (LTI) filters, and the overall frequency response is denoted by:

$$\mathcal{H}(\omega) = \begin{bmatrix} H_{11}(\omega) & H_{12}(\omega) \\ H_{21}(\omega) & H_{22}(\omega) \end{bmatrix} \tag{2}$$

where $H_{ij}(\omega)$ are the frequency responses of H_{ij}. We note that H_{ij}, $i \neq j$ represent cross-coupling from input $s_i[n]$ to sensor output $y_j[n]$, and H_{ij}, $i = j$ represent the frequency shaping applied to each individual input.

Our objective is to recover the input signals by using a 2×2 reconstruction filter $\mathcal{G}$ illustrated in Figure 2, whose outputs are denoted by $v_1[n]$ and $v_2[n]$, and whose frequency response is:

$$\mathcal{G}(\omega) = \begin{bmatrix} G_{11}(\omega) & -G_{12}(\omega) \\ -G_{21}(\omega) & G_{22}(\omega) \end{bmatrix}. \tag{3}$$

We want to adjust the components of $\mathcal{G}$ so that

$$\mathcal{H}(\omega)\mathcal{G}(\omega) = \begin{bmatrix} F_1(\omega) & 0 \\ 0 & F_2(\omega) \end{bmatrix}, \tag{4}$$

or

$$\mathcal{H}(\omega)\mathcal{G}(\omega) = \begin{bmatrix} 0 & F_1(\omega) \\ F_2(\omega) & 0 \end{bmatrix}. \tag{5}$$

If we require that $F_1(\omega) = F_2(\omega) = 1\ \forall\omega$, then in case of (4) $v_1[n] = s_1[n]$ and $v_2[n] = s_2[n]$, in case of (5) $v_1[n] = s_2[n]$ and $v_2[n] = s_1[n]$, and the input signal are exactly recovered. The equalization of F_1 and F_2 to a unity transformation requires partial knowledge of $\mathcal{H}$ (e.g. knowledge of H_{11} and H_{22}) and/or some prior knowledge concerning $s_1[n]$ and $s_2[n]$. However, since our main goal is the separation of the input signals, it may be sufficient to adjust the components of $\mathcal{G}$ so that the signals $s_1[n]$ and $s_2[n]$ are recovered up to arbitrary shaping filters F_1 and F_2. To this end we may assume, without further loss of generality, that $H_{11}(\omega) = H_{22}(\omega) = 1\ \forall\omega$, in which case the off-diagonal elements of $\mathcal{H}(\omega)\mathcal{G}(\omega)$ are zero, as required by (4), if

$$-G_{12}(\omega) + H_{12}(\omega)G_{22}(\omega) = 0 \tag{6}$$

$$H_{21}(\omega)G_{11}(\omega) - G_{21}(\omega) = 0, \tag{7}$$

and the diagonal elements of $\mathcal{H}(\omega)\mathcal{G}(\omega)$ are zero if

$$G_{11}(\omega) - H_{12}(\omega)G_{21}(\omega) = 0 \tag{8}$$

$$-H_{21}(\omega)G_{12}(\omega) + G_{22}(\omega) = 0. \tag{9}$$

Clearly, there are infinitely many combinations of G_{ij} $i,j = 1,2$ that satisfy (6) (7), or (8) (9). Therefore, we may pre-specify any pair of transfer functions, and solve for the other pair. Specifically, we choose $G_{11}(\omega) = G_{22}(\omega) = 1\ \forall\omega$, in which case the solution to (6) (7) is:

$$G_{12}(\omega) = H_{12}(\omega) \quad , \quad G_{21}(\omega) = H_{21}(\omega) \tag{10}$$

and the solution to (6) (7) is:

$$G_{12}(\omega) = \frac{1}{H_{21}(\omega)} \quad , \quad G_{21}(\omega) = \frac{1}{H_{12}(\omega)}, \tag{11}$$

under the assumption that H_{12} and H_{21} are invertible.

If the coupling systems H_{12} and H_{21} were known, then by setting the decoupling systems G_{12} and G_{21} according to (10) or (11) we obtain the desired signal separation. However, since H_{12} and H_{21} are unknown, we want to find a method or criterion that yields the desired solution. Invoking the assumption that the input signals are uncorrelated, our approach is to adjust G_{12} and G_{21} so the recovered signals are also uncorrelated, i.e.,

$$\mathrm{E}\left\{v_1[n]v_2^*[n-k]\right\} = 0 \qquad \forall k \tag{12}$$

or, equivalently, that their cross-spectrum $P_{v_1v_2}(\omega)$ is zero for all ω. Using the well-known relationship for the power spectra between inputs and outputs of an LTI system,

$$P_{v_1v_2}(\omega) = \begin{bmatrix} 1 & -G_{12}(\omega) \end{bmatrix} \begin{bmatrix} P_{y_1y_1}(\omega) & P_{y_1y_2}(\omega) \\ P_{y_2y_2}(\omega) & P_{y_2y_2}(\omega) \end{bmatrix} \begin{bmatrix} -G_{21}(\omega) & 1 \end{bmatrix}^+ \tag{13}$$

where $P_{y_iy_j}(\omega)$, $i,j = 1,2$, are the auto- and cross-spectra of the observed signals $y_1[n]$ and $y_2[n]$, and $+$ denotes the conjugate-transpose operation. Carrying out the matrix multiplication and setting the result to zero, the decorrelation condition becomes:

$$P_{y_1y_2}(\omega) - G_{12}(\omega)P_{y_2y_2}(\omega) - G_{21}^*(\omega)P_{y_1y_1}(\omega) + G_{12}(\omega)G_{21}^*(\omega)P_{y_2y_1}(\omega) = 0. \tag{14}$$

This equation does not specify a unique solution for both $G_{12}(\omega)$ and $G_{21}(\omega)$, even if $P_{y_iy_j}(\omega)$ $i,j = 1,2$ are precisely known. Any combination of G_{12} and G_{21} that satisfies (14) yields outputs $v_1[n]$ and $v_2[n]$ which are uncorrelated. We could arbitrarily choose G_{21}, in which case G_{12} is specified by:

$$G_{12}(\omega) = \frac{P_{y_1y_2}(\omega) - G_{21}^*(\omega)P_{y_1y_1}(\omega)}{P_{y_2y_2}(\omega) - G_{21}^*(\omega)P_{y_2y_1}(\omega)}, \tag{15}$$

or we could arbitrarily choose G_{12}, in which case G_{21} is specified by:

$$G_{21}(\omega) = \frac{P_{y_2y_1}(\omega) - G_{12}^*(\omega)P_{y_2y_2}(\omega)}{P_{y_1y_1}(\omega) - G_{12}^*(\omega)P_{y_1y_2}(\omega)}. \tag{16}$$

As a special case, if we choose $G_{21} = 0$ then (15) reduces to:

$$G_{12}(\omega) = \frac{P_{y_1y_2}(\omega)}{P_{y_2y_2}(\omega)} \tag{17}$$

which is recognized as Widrow's least squares solution [2] for the simplified scenario in which there is no coupling of $s_1[n]$ (the desired signal) into $y_2[n]$, i.e. $H_{21} = 0$. It conforms with the observation (e.g., see [3]) that the least squares filter causes the estimate of $s_1[n]$ to be uncorrelated from $y_2[n] = s_2[n]$ (the reference sensor signal). The least squares method has been successful in a wide variety of contexts. However, it is well known that if the assumption of zero coupling is not satisfied, its performance may seriously deteriorate. Equation (15) suggests a potentially interesting modification of the least squares method which allows the incorporation of non-zero G_{21} for the compensation of possibly non-zero coupling.

There are practical situations in which one of the coupling systems H_{12} or H_{21} may be known a-priori, or can be measured independently. For example, in speech enhancement, either the desired or the interfering signal may be in a fixed location and therefore the acoustic transfer functions that couple it to the microphones can be measured a-priori. In such cases either (15) or (16) can be used to find the other coupling system. To show this, we use the relation

$$\begin{bmatrix} P_{y_1y_1}(\omega) & P_{y_1y_2}(\omega) \\ P_{y_2y_1}(\omega) & P_{y_2y_2}(\omega) \end{bmatrix} = \begin{bmatrix} 1 & H_{12}(\omega) \\ H_{21}(\omega) & 1 \end{bmatrix} \begin{bmatrix} P_{s_1}(\omega) & 0 \\ 0 & P_{s_2}(\omega) \end{bmatrix} \begin{bmatrix} 1 & H_{12}(\omega) \\ H_{21}(\omega) & 1 \end{bmatrix}^+ \tag{18}$$

where $P_{s_1}(\omega)$ and $P_{s_2}(\omega)$ are the power spectra of $s_1[n]$ and $s_2[n]$, respectively. Substituting (18) into (14) and following straightforward algebraic manipulations, we obtain:

$$\begin{aligned} &P_{s_1}(\omega)\,[1 - G_{12}(\omega)H_{21}(\omega)]\,[H_{21}(\omega) - G_{21}(\omega)]^* + \\ &\quad P_{s_2}(\omega)\,[1 - G_{21}(\omega)H_{12}(\omega)]^*\,[H_{12}(\omega) - G_{12}(\omega)] = 0. \end{aligned} \tag{19}$$

If $G_{21} = H_{21}$ or $G_{12} = H_{21}^{-1}$ (the inverse of H_{21}), then the only solution to the equation is $G_{12} = H_{12}$ or $G_{21} = H_{12}^{-1}$, respectively, provided that $\mathcal{H}$ is invertible (i.e. $[1 - H_{21}(\omega)H_{12}(\omega)] \neq 0$ $\forall\omega$), and that $P_{s_2}(\omega)$ is strictly positive. Similarly, if $G_{12} = H_{12}$ or $G_{21} = H_{12}^{-1}$, then the only solution is $G_{21} = H_{21}$ or $G_{12} = H_{21}^{-1}$, provided that $\mathcal{H}$ is invertible and that $P_{s_1}(\omega)$ is strictly positive. Thus, if one of the coupling system is known than the decorrelation criterion yield the correct compensation for the other coupling system.

If both coupling systems H_{12} and H_{21} are unknown so that both decoupling systems G_{12} and G_{21} need to be adjusted, then the decorrelation condition is insufficient to uniquely solve the problem, and we need to use some additional information or constraints. One possibility is to assume that $s_1[n]$ and $s_2[n]$ are not only statistically uncorrelated, but statistically independent. By imposing statistical independence between the reconstructed signals, we obtain additional constraints involving higher order cumulants/spectra that can be used to specify a unique solution for both G_{12} and G_{21}. This approach is currently being developed.

Another approach consists of restricting the decoupling systems to be causal finite impulse response (FIR) filters, i.e. of the form:

$$G_{12}(\omega) = \sum_{k=0}^{q_1} a_k e^{-j\omega k} \tag{20}$$

$$G_{21}(\omega) = \sum_{k=0}^{q_2} b_k e^{-j\omega k} \tag{21}$$

where q_1 and q_2 are some pre-specified filter orders. We note that in this case the reconstructed signals $v_1[n]$ and $v_2[n]$ are given by:

$$v_1[n] = y_1[n] - \sum_{k=0}^{q_1} a_k y_2[n-k] \tag{22}$$

$$v_2[n] = y_2[n] - \sum_{k=0}^{q_2} b_k y_1[n-k]. \tag{23}$$

In Appendix A we show that if H_{12} and H_{21} are also causal FIR filters of orders less or equal to q_1 and q_2, respectively, then the only solution to the decorrelation equation is given by (10), provided that at least one of the coupling systems is of order greater than zero, and that $P_{s_1}(\omega)$ and $P_{s_2}(\omega)$ are rational spectra. There are many situations of practical interest in which H_{12} and H_{21} are closely approximated by FIR filters, and upper bounds on the filter orders are provided. We note that the finite length restriction is essential in order to obtain a unique solution. As the number of FIR coefficients increases, the solution becomes more and more ill-conditioned, and in the limit we may lose identifiability.

III. Algorithm Development

Frequency domain algorithms in the case where one of the decoupling systems G_{12} or G_{21} is pre-specified (e.g. when one of the coupling systems is known) are suggested by (15) or (16), where we note that in practice $P_{y_i y_j}(\omega)$, $i, j = 1, 2$ are replaced by their sample estimates (periodograms) based on the observed signals $y_1[n]$ and $y_2[n]$.

If both decoupling systems need to be adjusted, then the form of (15) and (16) suggests the following iterative algorithm:

$$G_{12}^{(l)}(\omega) = \frac{P_{y_1 y_2}(\omega) - [G_{21}^{(l-1)}(\omega)]^* P_{y_1 y_1}(\omega)}{P_{y_2 y_2}(\omega) - [G_{21}^{(l-1)}(\omega)]^* P_{y_2 y_1}(\omega)}, \tag{24}$$

$$G_{21}^{(l)}(\omega) = \frac{P_{y_2y_1}(\omega) - [G_{12}^{(l-1)}(\omega)]^* P_{y_2y_2}(\omega)}{P_{y_1y_1}(\omega) - G_{12}^{(l-1)}(\omega)]^* P_{y_1y_2}(\omega)}. \tag{25}$$

where $G_{12}^{(l)}(\omega)$ and $G_{21}^{(l)}(\omega)$ are the filters after l iteration cycles. Of course, in this case we must incorporate the FIR constraint by limiting the number of coefficients of the inverse Fourier transforms of the decoupling filters.

To implement these algorithms in the time domain, we note that:

$$\begin{bmatrix} P_{y_1v_1}(\omega) & P_{y_1v_2}(\omega) \\ P_{y_2v_1}(\omega) & P_{y_2v_2}(\omega) \end{bmatrix} = \begin{bmatrix} P_{y_1y_1}(\omega) & P_{y_1y_2}(\omega) \\ P_{y_2y_1}(\omega) & P_{y_2y_2}(\omega) \end{bmatrix} \begin{bmatrix} 1 & -G_{12}(\omega) \\ -G_{21}(\omega) & 1 \end{bmatrix}^+ \tag{26}$$

where $P_{y_iv_j}(\omega)$ is the cross-spectrum between $y_i[n]$ and $v_j[n]$. Using (26), Equations (15) and (16) can be represented in the form:

$$P_{y_2v_2}(\omega)G_{12}(\omega) = P_{y_1v_2}(\omega) \tag{27}$$

$$P_{y_1v_1}(\omega)G_{21}(\omega) = P_{y_2v_1}(\omega). \tag{28}$$

Inverse Fourier transforming, we obtain:

$$\sum_{l=0}^{q_1} a_l c_{y_2v_2}(k-l) = c_{y_1v_2}(k) \tag{29}$$

$$\sum_{l=0}^{q_2} b_l c_{y_1v_1}(k-l) = c_{y_2v_1}(k) \tag{30}$$

where a_k and b_k are the unit sample response coefficients of G_{12} and G_{21}, respectively, and $c_{y_iv_j}(k)$ is the covariance between $y_i[n]$ and $v_j[n]$ defined by:

$$c_{y_iv_j}(k) = \mathrm{E}\left\{y_i[n]v_j^*[n-k]\right\}. \tag{31}$$

Expressing (29) and (30) for $k = 0, 1, \ldots$, we obtain:

$$C_{\underline{y}_2\underline{v}_2}\underline{a} = \underline{c}_{y_1\underline{v}_2} \tag{32}$$

$$C_{\underline{y}_1\underline{v}_1}\underline{b} = \underline{c}_{y_2\underline{v}_1} \tag{33}$$

where $\underline{a} = [a_0\, a_1 \cdots a_{q_1}]^T$, $\underline{b} = [b_0\, b_1 \cdots b_{q_2}]^T$, and

$$C_{\underline{y}_2\underline{v}_2} = \mathrm{E}\left\{\underline{v}_2^*[n]\underline{y}_2^T[n]\right\} \tag{34}$$

$$\underline{c}_{y_1\underline{v}_2} = \mathrm{E}\left\{\underline{v}_2^*[n]y_1[n]\right\} \tag{35}$$

$$C_{\underline{y}_1\underline{v}_1} = \mathrm{E}\left\{\underline{v}_1^*[n]\underline{y}_1^T[n]\right\} \tag{36}$$

$$\underline{c}_{y_2\underline{v}_1} = \mathrm{E}\left\{\underline{v}_1^*[n]y_2[n]\right\} \tag{37}$$

where

$$\underline{y}_1[n] = [y_1[n]\, y_1[n-1] \cdots y_1[n-q_2]]^T \tag{38}$$

$$\underline{y}_2[n] = [y_2[n]\, y_2[n-1] \cdots y_2[n-q_1]]^T \tag{39}$$

$$\underline{v}_1^*[n] = [v_1^*[n]\, v_1^*[n-1] \cdots v_1^*[n-q_2]]^T \tag{40}$$

$$\underline{v}_2^*[n] = [v_2^*[n]\, v_2^*[n-1] \cdots v_2^*[n-q_1]]^T . \tag{41}$$

Equations (32) and (33) are the time domain equivalents of Equations (15) and (16), respectively. Thus, if the coefficients $\underline{b}$ of G_{21} are pre-specified, then the coefficients $\underline{a}$ of G_{12} are specified by:

$$\underline{a} = C_{\underline{y}_2\underline{v}_2}^{-1} \underline{c}_{y_1\underline{v}_2} \tag{42}$$

and if the coefficients $\underline{a}$ of G_{12} are pre-specified, then the coefficients $\underline{b}$ of G_{21} are specified by:

$$\underline{b} = C_{\underline{y}_1\underline{v}_1}^{-1} \underline{c}_{y_2\underline{v}_1} . \tag{43}$$

Since the covariance functions are unknown, they are approximated by the sample averages:

$$C_{\underline{y}_2\underline{v}_2} \approx \sum_{n=1}^{N} \beta_1^{N-n} \underline{v}_2^*[n] \underline{y}_2^T[n] \tag{44}$$

$$\underline{c}_{y_1\underline{v}_2} \approx \sum_{n=1}^{N} \beta_1^{N-n} \underline{v}_2^*[n] y_1[n] \tag{45}$$

$$C_{\underline{y}_1\underline{v}_1} \approx \sum_{n=1}^{N} \beta_2^{N-n} \underline{v}_1^*[n] \underline{y}_1^T[n] \tag{46}$$

$$\underline{c}_{y_2\underline{v}_1} \approx \sum_{n=1}^{N} \beta_2^{N-n} \underline{v}_1^*[n] y_2[n] \tag{47}$$

where β_1 and β_2 are real numbers between 0 and 1. To achieve maximal statistical stability, we choose $\beta_1 = \beta_2 = 1$. However, if the signals and/or the unknown system exhibit non-stationary behavior in time, it may be preferable to work with $\beta_1, \beta_2 < 1$. In this way we introduce exponential weighting that gives more weight to current data samples, resulting in an adaptive algorithm that is potentially capable of tracking the time-varying characteristics of the underlying system.

Replacing the covariances in (42) by their sample estimates, and following the derivation in Appendix B, we obtain the following time sequential algorithm for adjusting $\underline{a}$ (for a given $\underline{b}$):

$$\underline{a}(n) = \underline{a}(n-1) + Q(n) \underline{v}_2^*[n] v_1[n; \underline{a}(n-1)] \tag{48}$$

$$Q(n) = \frac{1}{\beta_1} \left[Q(n-1) - \frac{Q(n-1) \underline{v}_2^*[n] \underline{y}_2^T[n] Q(n-1)}{\beta_1 + \underline{y}_2^T[n] Q(n-1) \underline{v}_2^*[n]} \right] \tag{49}$$

where $v_1[n;\underline{a}(n-1)]$ is the signal $v_1[n]$ in (22) constructed using the current value $\underline{a}(n-1)$ of $\underline{a}$. Similarly, replacing the covariances in (43) by their sample estimates, we obtain a time sequential algorithm for adjusting $\underline{b}$ (for a given $\underline{a}$):

$$\underline{b}(n) = \underline{b}(n-1) + R(n)\underline{v}_1^*[n]\, v_2[n;\underline{b}(n-1)] \tag{50}$$

$$R(n) = \frac{1}{\beta_2}\left[R(n-1) - \frac{R(n-1)\underline{v}_1^*[n]\underline{y}_1^T[n]R(n-1)}{\beta_2 + \underline{y}_1^T[n]R(n-1)\underline{v}_1^*[n]}\right] \tag{51}$$

where $v_2[n;\underline{b}(n-1)]$ is the signal $v_2[n]$ in (23) constructed using the current value $\underline{b}(n-1)$ of $\underline{b}$. Thus, if one of the decoupling systems is pre-specified, we have a fully sequential algorithm to determine the other one.

If we set $\underline{b} = \underline{0}$ ($G_{21} = 0$), then (48) (49) reduce to:

$$\underline{a}(n) = \underline{a}(n-1) + Q(n)\underline{y}_2^*[n]v_1[n;\underline{a}(n-1)] \tag{52}$$

$$Q(n) = \frac{1}{\beta_1}\left[Q(n-1) - \frac{Q(n-1)\underline{y}_2^*[n]\underline{y}_2^T[n]Q(n-1)}{\beta_1 + \underline{y}_2^T[n]Q(n-1)\underline{y}_2^*[n]}\right] \tag{53}$$

which is recognized as the Recursive Least Squares (RLS) Algorithm for solving Widrow's least squares criterion [4] [5]. The algorithm in (48) (49) can therefore be viewed as a modification of the RLS method which allows the incorporation of non-zero $\underline{b}$ (non-zero G_{21}) for the compensation of non-zero coupling of $s_1[n]$ (the desired signal) into $y_2[n]$ (the reference sensor signal).

If both decoupling systems need to be adjusted, then the form of (42) and (43) suggests the following iterative algorithm:

$$\underline{a}^{(l)} = C^{-1}_{\underline{y}_2\underline{v}_2}\underline{c}_{y_1\underline{v}_2}\Big|_{\underline{b}=\underline{b}^{(l-1)}} \tag{54}$$

$$\underline{b}^{(l)} = C^{-1}_{\underline{y}_1\underline{v}_1}\underline{c}_{y_2\underline{v}_1}\Big|_{\underline{a}=\underline{a}^{(l-1)}} \tag{55}$$

where $\underline{a}^{(l)}$ and $\underline{b}^{(l)}$ denote the value of $\underline{a}$ and $\underline{b}$ after l iteration cycles. The above notation indicates that the right hand side of (54) is computed at $\underline{b} = \underline{b}^{(l-1)}$ and the right hand side of (55) is computed at $\underline{a} = \underline{a}^{(l-1)}$. In the actual implementation of the algorithm, the covariances are substituted by their sample estimates in (44)-(47). By the same considerations leading from (42) to (48) (49), and from (43) to (50) (51), each iteration of the algorithm can be performed sequentially in time as follows:

$$\underline{a}^{(l)}(n) = \underline{a}^{(l)}(n-1) + Q(n)\underline{v}_2^*[n;\underline{b}^{(l-1)}]v_1[n;\underline{a}^{(l)}(n-1)] \tag{56}$$

$$Q(n) = \frac{1}{\beta_1}\left[Q(n-1) - \frac{Q(n-1)\underline{v}_2^*[n;\underline{b}^{(l-1)}]\underline{y}_2^T[n]Q(n-1)}{\beta_1 + \underline{y}_2^T[n]Q(n-1)\underline{v}_2^*[n;\underline{b}^{(l-1)}]}\right] \tag{57}$$

$$\underline{b}^{(l)}(n) = \underline{b}^{(l)}(n-1) + R(n)\underline{v}_1^*[n;\underline{a}^{(l-1)}]v_2[n;\underline{b}^{(l)}(n-1)] \tag{58}$$

$$R(n) = \frac{1}{\beta_2}\left[R(n-1) - \frac{R(n-1)\underline{v}_1^*[n;\underline{a}^{(l-1)}]\underline{y}_1^T[n]R(n-1)}{\beta_2 + \underline{y}_1^T[n]R(n-1)\underline{v}_1^*[n;\underline{a}^{(l-1)}]}\right] \tag{59}$$

where at the end of the iteration we obtain $\underline{a}^{(l)}(N) = \underline{a}^{(l)}$ and $\underline{b}^{(l)}(N) = \underline{b}^{(l)}$.

To obtain a fully sequential algorithm, we simply suggest to use the current value of $\underline{b}$ in the recursion (48) (49) for $\underline{a}$, and the current value of $\underline{a}$ in the recursion (50) (51) for $\underline{b}$. The resulting algorithm is:

$$\underline{a}(n) = \underline{a}(n-1) + Q(n)\underline{v}_2^*[n;\underline{b}(n-1)]v_1[n;\underline{a}(n-1)] \tag{60}$$

$$Q(n) = \frac{1}{\beta_1}\left[Q(n-1) - \frac{Q(n-1)\underline{v}_2^*[n;\underline{b}(n-1)]\underline{y}_2^T[n]Q(n-1)}{\beta_1 + \underline{y}_2^T[n]Q(n-1)\underline{v}_2^*[n;\underline{b}(n-1)]}\right] \tag{61}$$

$$\underline{b}(n) = \underline{b}(n-1) + R(n)\underline{v}_1^*[n;\underline{a}(n-1)]v_2[n;\underline{b}(n-1)] \tag{62}$$

$$R(n) = \frac{1}{\beta_2}\left[R(n-1) - \frac{R(n-1)\underline{v}_1^*[n;\underline{a}(n-1)]\underline{y}_1^T[n]R(n-1)}{\beta_2 + \underline{y}_1^T[n]R(n-1)\underline{v}_1^*[n;\underline{a}(n-1)]}\right]. \tag{63}$$

This algorithm can be viewed as an approximation to the algorithm in (56)-(59), obtained by replacing the iteration index by the time index – which is a common procedure in stochastic approximation. Both (56)-(59) and (60)-(63) can be viewed as extensions of the RLS method to the case where each of the observed signals has components from both sources, and both decoupling systems need to be adjusted.

As an alternative to (54) (55), consider the following iterative algorithm for simultaneously solving (32) (33):

$$\underline{a}^{(l)} = \underline{a}^{(l-1)} + \gamma_1^{(l)}\left[\underline{c}_{y_1\underline{v}_2} - C_{\underline{y}_2\underline{v}_2}\underline{a}^{(l-1)}\right]\Big|_{\underline{b}=\underline{b}^{(l-1)}} \tag{64}$$

$$\underline{b}^{(l)} = \underline{b}^{(l-1)} + \gamma_2^{(l)}\left[\underline{c}_{y_2\underline{v}_1} - C_{\underline{y}_1\underline{v}_1}\underline{b}^{(l-1)}\right]\Big|_{\underline{a}=\underline{a}^{(l-1)}} \tag{65}$$

where $\gamma_1^{(l)}$ and $\gamma_2^{(l)}$ are gain factors (step-size) that may depend on the iteration index l. We note that

$$\underline{c}_{y_1\underline{v}_2} - C_{\underline{y}_2\underline{v}_2}\underline{a} = \mathrm{E}\left\{\underline{v}_2^*[n]\left[y_1[n] - \underline{y}_2^T[n]\underline{a}\right]\right\} = \mathrm{E}\left\{\underline{v}_2^*[n]v_1[n]\right\} \tag{66}$$

$$\underline{c}_{y_2\underline{v}_1} - C_{\underline{y}_1\underline{v}_1}\underline{b} = \mathrm{E}\left\{\underline{v}_1^*[n]\left[y_2[n] - \underline{y}_1^T[n]\underline{b}\right]\right\} = \mathrm{E}\left\{\underline{v}_1^*[n]v_2[n]\right\} \tag{67}$$

where in the actual algorithm these expectations are approximated by sample averages. An iterative-sequential algorithm in which each iteration is performed sequentially in time can also be obtained here.

Using the Robbins-Monro first order stochastic approximation method [6] [7], in which expectations are approximated by current realizations and the iteration index is replaced by the time index, the algorithm in (64) (65) may be converted into the following sequential algorithm:

$$\underline{a}(n) = \underline{a}(n-1) + \gamma_1(n)\underline{v}_2^*[n;\underline{b}(n-1)]v_1[n;\underline{a}(n-1)] \tag{68}$$

$$\underline{b}(n) = \underline{b}(n-1) + \gamma_2(n)\underline{v}_1^*[n;\underline{a}(n-1)]v_2[n;\underline{b}(n-1)] \tag{69}$$

This algorithm is similar in form to (60) (62), except that instead of the matrices $Q(n)$ and $R(n)$, we have the gains $\gamma_1(n)$ and $\gamma_2(n)$. Using well-known results from the theory of stochastic approximation [6] [7] [8], it can be shown that if these gain factors are chosen to be positive sequences such that

$$\lim_{n\to\infty}\gamma_i(n) = 0, \qquad \sum_{n=1}^{\infty}\gamma_i(n) = \infty, \qquad \sum_{n=1}^{\infty}\gamma_i^2(n) < M < \infty,$$

(e.g., $\gamma_i(n) = \gamma_i/n$) then under certain regularity conditions this algorithm converges almost surely (a.s.) and in the mean square (m.s.) to the solution of (32) (33), and the limit distribution can also be evaluated. Such an analysis is beyond the scope of this paper.

If the signals and/or the linear system exhibit changes in time, and an adaptive algorithm is required, choosing constant gains $\gamma_i(n) = \gamma_i$, $i = 1, 2$, is suggested. This corresponds to an exponential weighting that reduces the effect of past data samples relative to new data in order to track the varying parameters.

If we substitute $\underline{b}(n) = \underline{0}$ in (68), we obtain:

$$\underline{a}(n) = \underline{a}(n-1) + \gamma_1(n)\underline{y}_2^*[n]v_1\left[n;\underline{a}(n-1)\right] \tag{70}$$

which is recognized as the least mean squares (LMS) algorithm, suggested by Widrow et al [2], for solving the least squares problem under the indicated zero-coupling assumption. Thus, as observed in the case of the RLS method, the algorithm presented by (68) (69) can be viewed as an extension of the LMS method to the more general scenario in which each of the observed signals have components from both sources, and one or both coupling systems need to be identified.

IV. Experimental Results

In this section we demonstrate the performance of the proposed method on a few simulated examples. In all our experiments H_{11} and H_{22} were unity transformations, and the coupling systems H_{12} and H_{21} were non-zero FIR filters of order 9 (corresponding to 10 coefficients). In all cases the reconstructed signals $v_1[n]$ and $v_2[n]$ were filtered by the shaping filter $[1 - G_{12}G_{21}]^{-1}$ so that if $G_{12} = H_{12}$ and $G_{21} = H_{21}$ then we have applied the exact inverse filter to recover $s_1[n]$ and $s_2[n]$.

We begin with the simpler case in which $G_{21} = H_{21}$ so that we only need to adjust the coefficients $\underline{a}$ of G_{12}. For that purpose we have used (42), where the covariances are substituted by their sample estimates in (44) and (45) with $\beta_1 = 1$. We worked with a filter order of $q_1 = 99$ (100 coefficients) so that, in fact, we did not make any assumption concerning the actual order of the system H_{12} that we want to equalize. For the purpose of comparison, we have also implemented Widrow's least squares method, which corresponds to solving (42) under the incorrect choice $\underline{b} = 0$.

In the first set of experiments, $s_1[n]$ was a sampled speech signal, while $s_2[n]$ was a computer generated white noise at various spectral levels. In the three cases that the algorithm was tested, the signal-to-interference (S/I) levels at the first sensor (the primary microphone) prior to processing were -20 dB, -10 dB, and 0 dB. Using the least squares method, the measured S/I levels were 5 dB, 7 dB, and 15 dB, respectively. Using our method, the measured S/I levels were 17 dB, 20 dB and 28 dB, respectively. We have therefore achieved an improvement of 12 to 13 dB in the S/I over the least squares method. By actually listening to the recovered speech signal, the intelligibility using our method was improved compared with the least squares method by more than what would have been interpreted from the improvement in the S/I level. As shown in Figure 3, the frequency response magnitude of the decoupling filter using our method is also improved noticeably compared to the least squares method.

In the second set of experiments, both $s_1[n]$ and $s_2[n]$ were speech signals. Once again, we set $G_{21} = H_{21}$ and adjusted only the coefficients of G_{12}. This is an interesting case since separating speech signals is considered a more difficult task than enhancing speech embedded in additive background noise. In the two cases that the algorithm was tested, the S/I levels at the primary microphone were -10 dB and 0 dB, prior to processing. Using the least squares method, the resulting S/I levels were 5 dB and 11 dB, while using our method the resulting S/I levels were 15 dB and 24 dB, respectively.

Next, we have considered the case where both $s_1[n]$ and $s_2[n]$ are speech signals, and both H_{12} and H_{21} are unknown FIR filters of order 10. We have only assumed prior knowledge of the filters orders. The S/I levels at the first (primary) and the second (reference) microphones were -1.8 dB and -2 dB, indicating strong coupling effects. Since this is a more difficult and practically more interesting scenario, it will be fully presented. In Figure 4 we have plotted the original input signals $s_1[n]$ and $s_2[n]$, corresponding to the speech sentences: "He has the bluest eyes", and "Line up at the screen door", respectively. In Figure 5 we have plotted the measured signals $y_1[n]$ and $y_2[n]$, which contain the inherent coupling effects. To adjust the coefficients a_k and b_k of the decoupling systems G_{12} and G_{21}, we implemented the iterative algorithm in (54) (55) where the covariances are substituted by their sample estimates in (44)-(47) with $\beta_1 = \beta_2 = 1$. The recovered signals are plotted in Figure 6. The measured S/I levels at the first (primary) and second (reference) sensors were 7.5 dB and 8.3 dB respectively. For the purpose of comparison, we have plotted in Figure 7 the recovered signal at the primary sensor using the least squares method. The associated S/I level was 1.8 dB. We note that unlike our approach that treats the signals as being equally important, the least squares approach regards $s_2[n]$ as being an unwanted interfering signal, and therefore it makes no attempt to estimate it. By actually listening to the recovered signal using the least squares method, one can hear the reverberant quality due to the fact that the desired signal is canceled with some delay together with the interfering signal. This reverberant effect does not exist when using our method.

Appendix A: Proof of Uniqueness under the FIR Constraint

Assuming that $P_{s_1}(\omega)$ and $P_{s_2}(\omega)$ are rational spectra, that is a ratio of polynomials, then multiplying (19) by their common denominator, we obtain:

$$\begin{aligned} C(\omega)\,[1-G_{12}(\omega)H_{21}(\omega)]\,[H_{21}(\omega)-G_{21}(\omega)]^* + \\ D(\omega)\,[1-G_{21}(\omega)H_{12}(\omega)]^*\,[H_{12}(\omega)-G_{12}(\omega)] = 0 \end{aligned}$$

where $C(\omega)=\sum_{i=-p_1}^{p_1} c_i e^{-j\omega i}$, $D(\omega)=\sum_{i=-p_2}^{p_2} d_i e^{-j\omega i}$ where $c_{-i}=c_i$, $d_{-i}=d_i$ and c_{p_1}, $d_{p_2}\neq 0$ where p_1 and p_2 are finite non-negative integers.

Let H_{12} and H_{21} be causal FIR filters of orders $\tilde{q}_1 \leq q_1$ and $\tilde{q}_2 \leq q_2$, respectively (the order is defined as the index of the largest non-zero coefficient of the unit sample response). We assume that $\underline{\tilde{q}_1+\tilde{q}_2>0}$ (i.e., at least one of the coupling systems is of order greater than zero). Then $[1-G_{12}(\omega)H_{21}(\omega)]$ is of order $(q_1+\tilde{q}_2)$, $[1-G_{21}(\omega)H_{12}(\omega)]$ is of order $(q_2+\tilde{q}_1)$, $[H_{21}(\omega)-G_{21}(\omega)]$ is of order q_2 or less (if $\tilde{q}_2=q_2$ and there is a cancellation of the largest non-zero coefficient), and $[H_{21}(\omega)-G_{21}(\omega)]$ is of order q_1 or less. Hence, the first term on the right hand side of the equation is a polynomial in powers of $e^{j\omega}$, where the index of its most positive non-zero coefficient equals $(p_1+q_1+\tilde{q}_2)$, and the index of its most negative non-zero coefficient is less than or equal to (p_1+q_2). Similarly, the second term is a polynomial in powers of $e^{j\omega}$, where the index of its most positive non-zero coefficient is less than or equal to (p_2+q_1), and the index of its most negative non-zero coefficient is equal to $(p_2+q_2+\tilde{q}_1)$. To satisfy the equation all the coefficients of the sum of the two terms must be equal to zero. Therefore, necessary conditions are given by:

$$\begin{aligned} p_1+q_1+\tilde{q}_1 &\leq p_2+q_1 \\ p_2+q_2+\tilde{q}_1 &\leq p_1+q_2. \end{aligned}$$

Adding the two inequalities we obtain $\underline{\tilde{q}_1+\tilde{q}_2\leq 0}$ which is a contradiction. Therefore, the decorrelation equation is satisfied if and only if each term is equal to zero, implying that the only possible solution is given by (10).

Appendix B: Derivation of the Sequential Algorithm in (48) (49)

Let

$$
\begin{aligned}
Q(n) &= \left[\sum_{k=1}^{n} \beta_1^{n-k} \underline{v}_2^*[n]\, \underline{y}_2^T[k]\right]^{-1} = \left[\beta_1 Q^{-1}(n-1) + \underline{v}_2^*[n]\underline{y}_2^T[n]\right]^{-1} \\
&= \frac{1}{\beta_1}\left[Q(n-1) - \frac{Q(n-1)\underline{v}_2^*[n]\underline{y}_2^T[n]Q(n-1)}{\beta_1 + \underline{y}_2^T[n]Q(n-1)\underline{v}_2^*[n]}\right]
\end{aligned}
$$

$$
\underline{q}(n) = \sum_{k=1}^{n} \beta_1^{n-k} \underline{v}_2^*[n] y_1[k] = \beta_1 \underline{q}(n-1) + \underline{v}_2^*[n] y_1[n]
$$

Then

$$
\begin{aligned}
\underline{a}(n) &= Q(n)\underline{q}(n) = Q(n)\left[\beta_1 \underline{q}(n-1) + \underline{v}_2^*[n]y_1[n]\right] \\
&= Q(n)\left[\beta_1 Q^{-1}(n-1)\underline{a}(n-1) + \underline{v}_2^*[n]y_1[n]\right] \\
&= Q(n)\left\{\beta_1 \cdot \frac{1}{\beta_1}\left[Q^{-1}(n) - \underline{v}_2^*[n]\underline{y}_2^T[n]\right]\underline{a}(n-1) + \underline{v}_2^*[n]y_1[n]\right\} \\
&= \underline{a}(n-1) + Q(n)\underline{v}_2^*[n]\left[y_1[n] - \underline{y}_2^T[n]\underline{a}(n-1)\right] \\
&= \underline{a}(n-1) + Q(n)\underline{v}_2^*[n]v_1\left[n; \underline{a}(n-1)\right].
\end{aligned}
$$

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List of Figures

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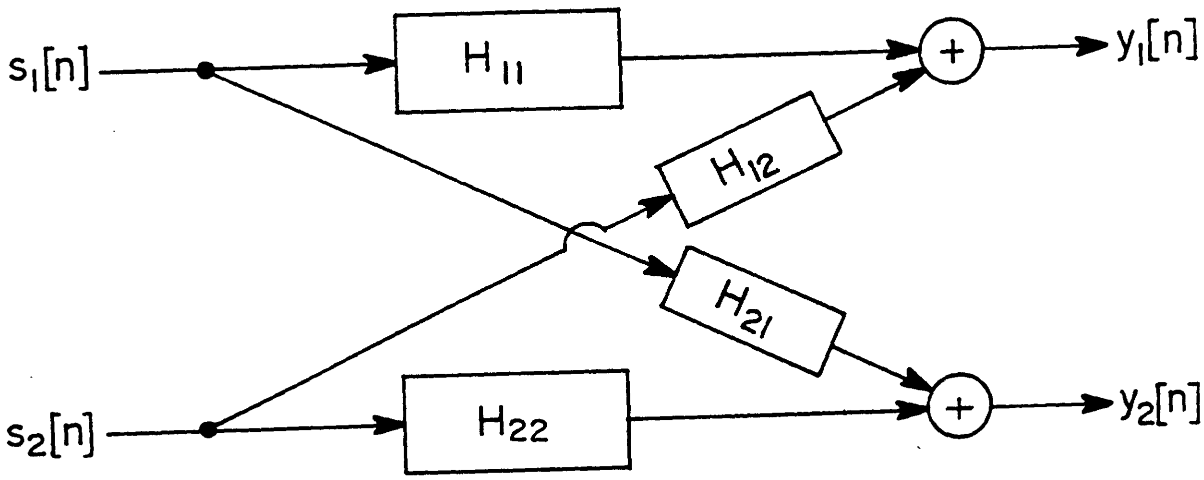


Figure 1: The signal model.

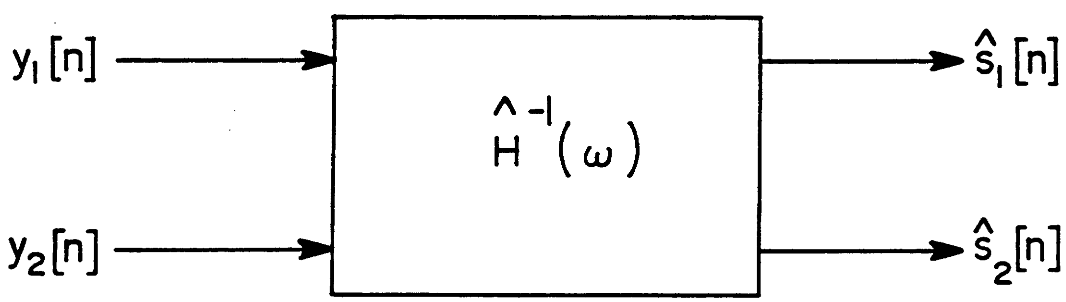


Figure 2: The reconstruction system.

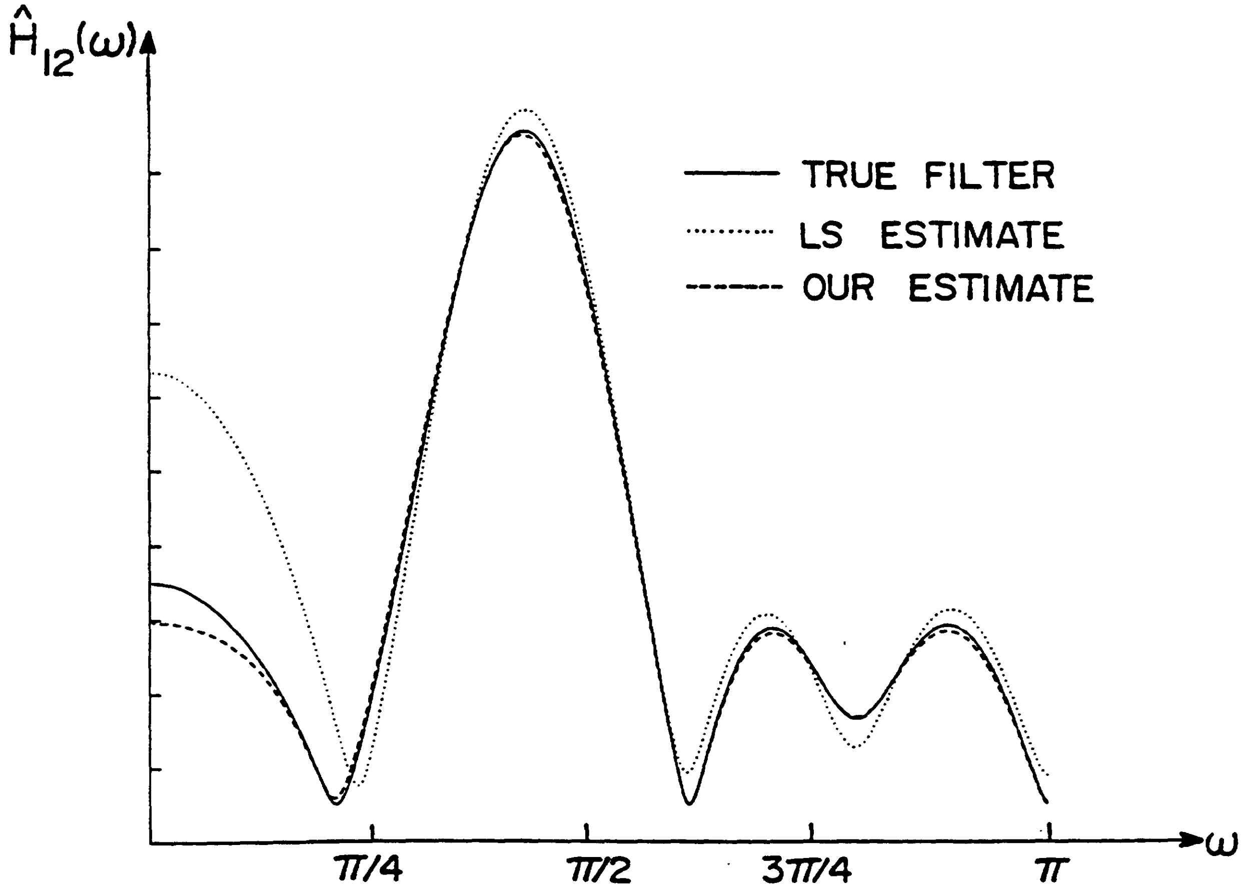
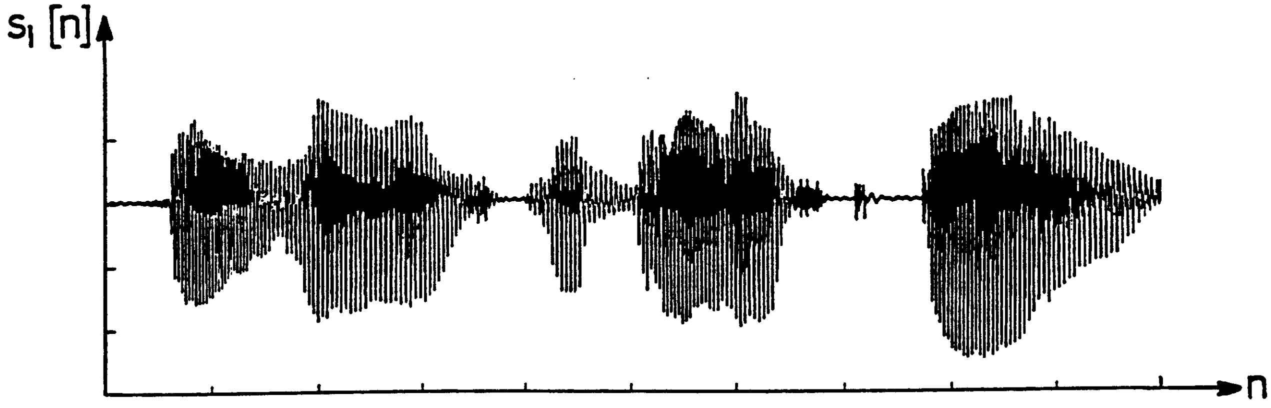
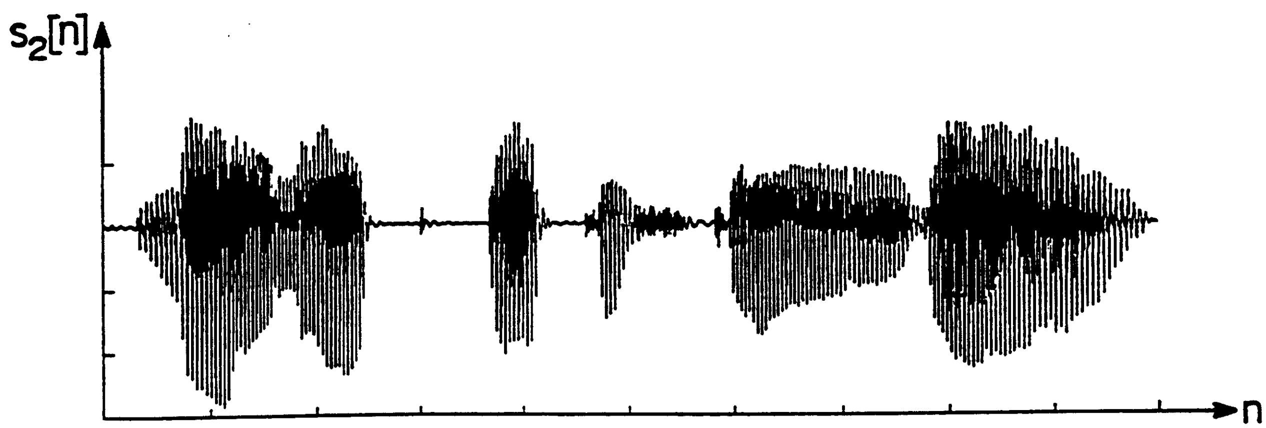


Figure 3: Frequency response magnitude of: (a) The coupling filter, (b) The decoupling filter using the proposed method, (c) The decoupling filter using the LS method.



(a)



(b)

Figure 4: The speech signals: (a) "He has the bluest eyes", (b) "Line up at the screen door".

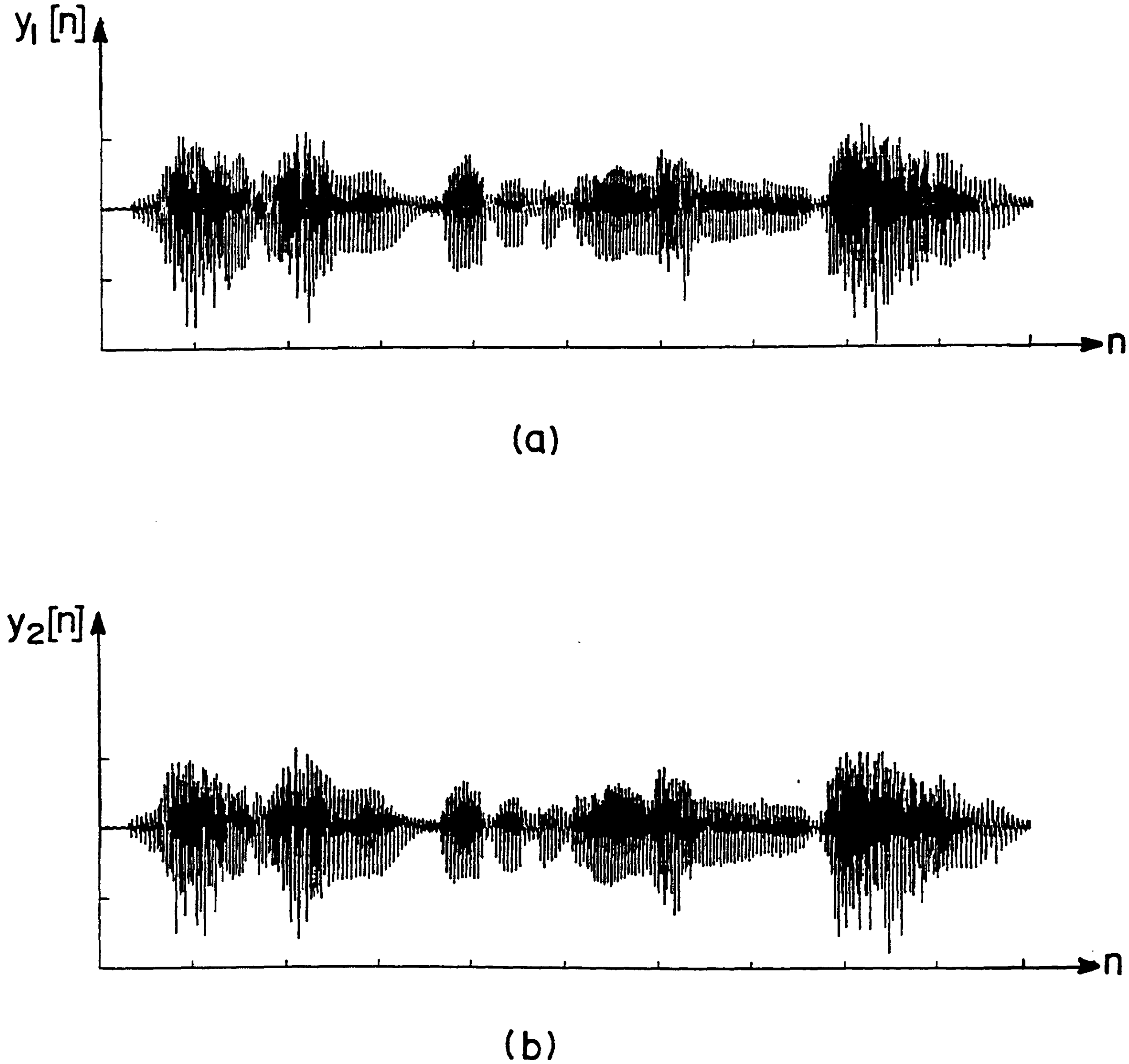


Figure 5: The measured signals.

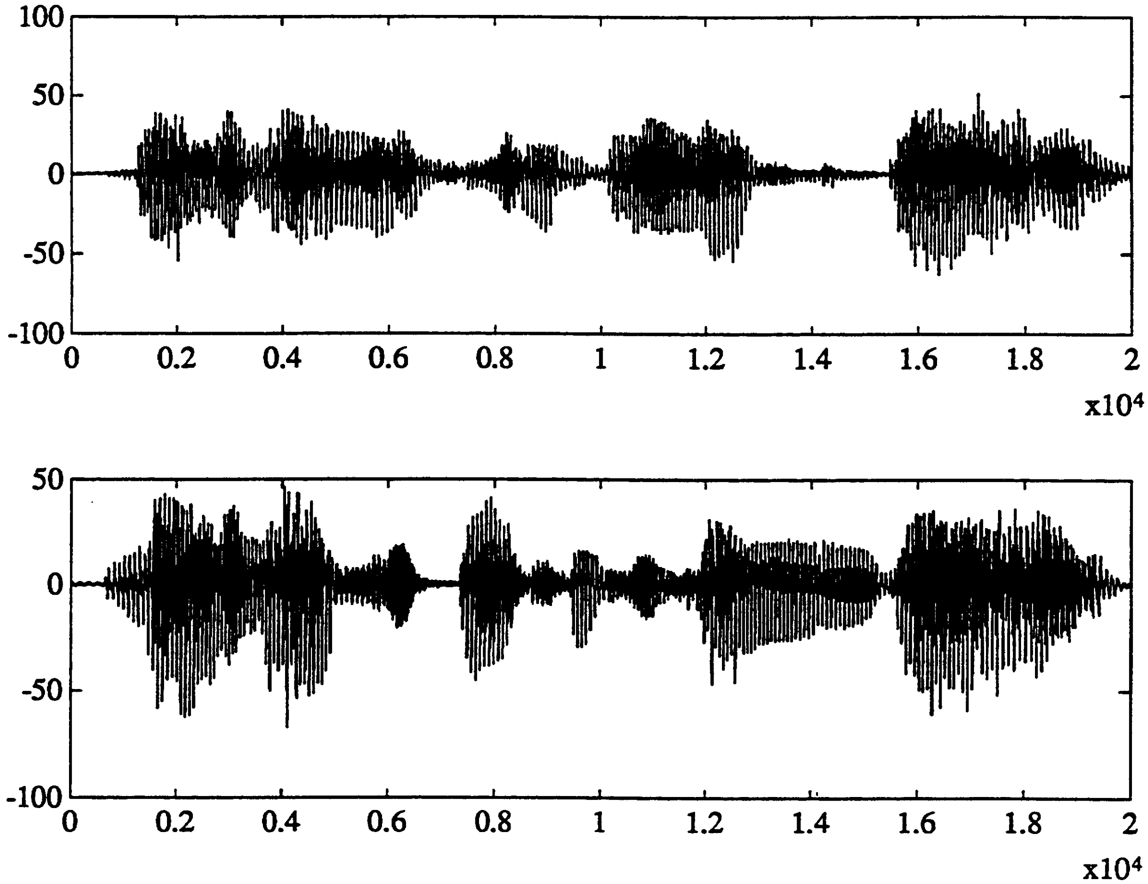


Figure 6: The recovered signals using the proposed method.

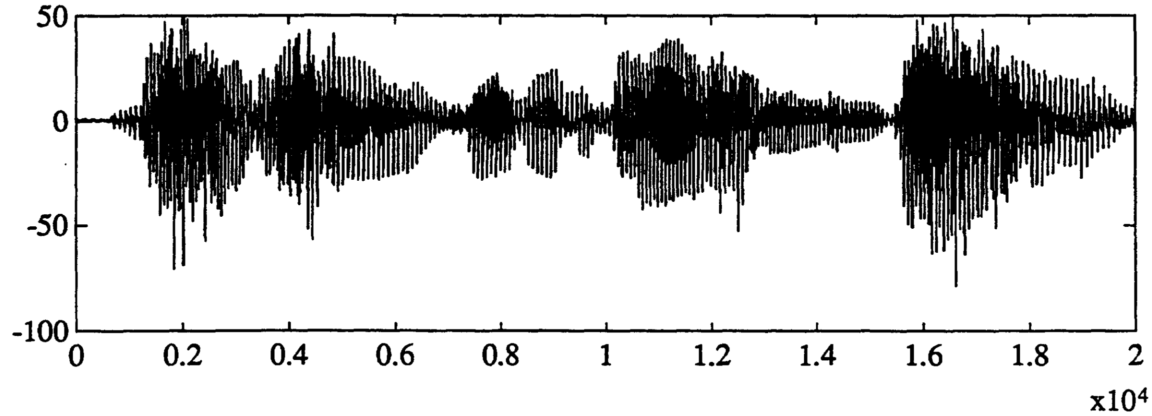


Figure 7: The recovered signal using the LS method.